CONTENTS

PP – Strand A – Numbers

KSO - Numeration: *by the end of **Class 3** students should:*

- *have number sense with respect to whole number meanings & multiple representations, including place value to 4 place, in order to deal with numbers in the real world and perform meaningful operations*
- *efficiently select & apply appropriate estimation strategies, to answer real life questions and check for reasonableness of answer in calculation*
- *have developed a wide variety of relationships with other numbers and across strands so as to support strong reasoning in new situations, both mathematical and real life*
- *understand simple fractions & decimals to tenths, and the relationship between them, so as to provide a strong foundation for higher level fractional ideas and computation*

Toward this, students in class **PP** will be expected to investigate the following **SO** (Specific Outcomes):

PP-A1 Counting to determine number 1-10

Counting principles – concrete experiences:

- one to one correspondence (one count for each item, no item counted twice)
- counting order doesn't change the amount
- last number said is the count
- stable sequence (rote counting)
- sets of actual objects, and pictures

PP-A2 Sets: sorting, based on number

- distinguish between sets that have a given number of items and those that do not
- restrict to counting situations to 10
- sets of actual objects and pictures
- oral, not symbolic at this point

PP-A3 Representing numbers physically

- represent numbers to 10 with various dot configurations
- important to recognize simple amounts without counting

- configurations develop number relationships and help with counting on from an easily recognizable set
- identify verbally number names

PP-A4 Counting in a variety of ways

- through games, rhymes, songs, meaningful real-life experiences
- to 10: forward, backward, counting on from a number

PP-A5 Comparing Sets: more, less, same

- more, less, same, by: counting, matching**,** lining up
- compare sets where size of objects vary but count is the same
- compare sets where smaller objects give a bigger count
- estimate before counting and justify estimation

PP-A6 Numerals: matching to quantities, printing

- match printed numerals to sets of objects
- build sets to match printed numerals
- trace numerals in sand, modeling clay
- copy numerals from a sample
- develop a motor plan for printing numerals

PP-A7 Ordinal Numbers: to 10

- ordinal numbers meaning
- ordinal number changes depending on where the count is started
- connect to real life situations, stories, actions (situational, not symbolic)

PP-A8 Halves: meaning (in context)

- build on students' prior knowledge of sharing
- essential idea: 2 fair (equal) shares
- part of a whole (partitioning)
- part of a set (sharing one by one)
- size of the share depends on what is being

PP – Strand B – Operations

KSO - Operations: *by the end of **Class 3** students should:*

- *have visual images for understanding the four basic operations of whole numbers (through concrete experiences)*
- *have developed relationships among the 4 operations*
- *efficiently use a variety of simple algorithms for the four basic operations of whole numbers by connecting concrete models to symbolic and use appropriate mathematical language to communicate and justify procedures*
- *have developed strategies for efficient estimation and mental math calculations and know when each is appropriate*

Toward this, students in **PP** will be expected to investigate the following **SO** (Specific Outcomes):

PP-B1 Combining Sets: small groups - (counting totals)

- how many altogether (oral, not symbolic)
- adding objects to an existing group
- combining 2 existing groups
- total is reached by: a) counting all items b) counting on from one of the sets

PP-B2 Subtraction Meaning: separating small groups - (counting results)

- taking away: how many left?
- physically removing objects (in context), then counting
- counting backwards from total
- counting forward from the missing part to the total
- predicting results

PP-B3 Subtraction Meaning: comparing small groups - (counting results)

- comparison concept: how many more/less?
- matching one-to-one to see how many more or less in a set
- comparing 2 existing sets
- adjusting sets to make more/less/same
- link to number (orally)

PP – Strand C – Pattern

KSO - Pattern: *by the end of **Class 3** students should:*

- *recognize and create patterns in mathematical and real world contexts for purposes of solving problems and developing relationships*
- *apply pattern to problems based on number, geometry and measurement*
- *use patterns to assist in mental math strategies*

Toward this, students in **PP** will be expected to investigate the following **SO** (Specific Outcomes):

PP-C1 Copy, Extend, Create patterns based on size, shape, colour, actions, sounds, attributes of shapes:

- exploring patterns that repeat
- exploring patterns that grow
- extending patterns in different ways

PP-C2 Copy, Extend, Create patterns based on measurement attributes of length, mass, capacity, time:

- comparing informally (no units)

PP-C3 Represent patterns concretely: (in different ways)

- reading patterns different ways (e.g. ABC can be read as 1 2 3)
- representing patterns in different ways (e.g. a snap, clap, snap, clap pattern could be represented by a blue, red, blue, red pattern or by a 1, 2, 1, 2 pattern)
- connecting patterns to real world

PP – Strand D – Measurement

KSO - Measurement: *by the end of **Class 3** students should:*

- *understand concepts of length, capacity, time, mass and area, and use this understanding to perform measurements in non-standard and standard units*
- *justify the necessity for common (standard) units as an effective communication tool*
- *have measurement sense, including a sense of relative size of a unit, so as to effectively estimate and measure and check for reasonableness of answer*
- *choose appropriate units to solve real life problems in measurement*

Toward this, students in **PP** will be expected to investigate the following **SO** (Specific Outcomes):

PP-D1 Length: compare, order, sort (directly)

- exploring vocabulary: longer, taller, wider, narrow, shorter
- comparing directly (no units)

PP-D2 Length: compare, order, sort: (directly, indirectly)

- comparing indirectly (compare each of the 2 objects to a 3rd)

PP-D3 Capacity: compare, order, sort (directly)

- exploring vocabulary: holds more/less than
- comparing capacity of containers by pouring one into the other

PP-D4 Mass: compare, order, sort: (direct)

- exploring vocabulary: heavier/ lighter than
- comparing directly by hand balance or balance scale (focus on comparison – not measuring with units)

PP – Strand E – Geometry

KSO - Geometry: *by the end of **Class 3** students should:*

- *through exploration, understand and name geometric shapes according to their attributes, and develop relationships among them so as to make sense of the world around them*
- *draw, model and classify 2-D and 3-D shapes by attributes to foster spatial sense for later ideas in geometry and for real life situations*
- *predict and verify results of transforming, combining and subdividing shapes to understand other shapes and how shapes change in the world around them*
- *develop other aspects of spatial sense including: visual memory, perceptual constancy, visual discrimination and position in space, in order to understand everyday events and objects as well as higher geometrical ideas*

Toward this, students in **PP** will be expected to investigate the following **SO** (Specific Outcomes):

PP-E1 Spatial Sense: position in space

- exploring position in space, relative to the child

- include the relative position of :
  - -one object to another
  - -the object to the observer
- exploring vocabulary: beside, above, below, between, in front of, through, behind

PP-E2 Spatial Sense: eye-motor coordination

- connecting perception to action (experiential)
  - -where the child moves himself to accommodate his world
  - -where the child moves objects to effect change in his/her world

PP-E3 2-D & 3-D Shapes: sort & build

- hands-on experiences only
- focusing on attributes through sorting and building
- include situations where the child: a) builds b) sorts in different ways by attributes
- discussing the way shapes interact with other shapes

PP-E4 2-D & 3-D Shapes: comparing

- exploring vocabulary: square, triangle, circle, rectangle….. cylinder, cone, cube
- comparing shapes through hands-on experiences and discussion of discoveries
- using (not memorizing) shape names
- orally comparing attributes of 2-D and 3-D shapes (similarities, differences)
- exploring non-examples

PP-E5 2-D & 3-D Shapes: in real life

- recognizing examples of 2-D and 3-D shapes in the environment
- recognizing shapes inside other shapes in the environment

PP-E6 2-D Shapes: subdividing & changing

- exploring through hands-on experiences and dialogue
- exploring shape vocabulary
- focusing on how shapes can be transformed into other shapes

PP-E7 Transformation of Shapes: repositioning shapes to understand constancy

- developing perceptual constancy (a shape can be moved by sliding, flipping or turning, and still be exactly the same shape)
- exploring through hands-on experiences and accompanying dialogue (concrete objects, cut-out shapes only)

PP – Strand F – Data Management

KSO – Data Management: *by the end of **Class 3** students should:*

- *collect, record, organize and describe data to answer questions of personal interest*
- *design and implement simple surveys for real world issues and make predictions*
- *construct concrete and pictorial graphs as a way to communicate ideas about the data and check predictions*
- *interpret data from graphs and tables in a factual way as well as through interpolation and extrapolation (draw conclusions about things not represented in the data)*

Toward this, students in **PP** will be expected to investigate the following **SO** (Specific Outcomes):

PP-F1 Collect & Organize Data: pictorially, chart form

- investigating questions of interest to students (weather, favourite snack, more boys or girls, etc.)
- collecting data (eg by dropping counters in a jar)
- organizing data pictorially (pictures, charts)
- predicting results and discussing findings

PP-F2 People Graphs: create, interpret

- investigating questions of interest to students
- using students themselves to form people graphs
- focusing on one to one correspondence
- understanding importance of common start line
- predicting first then checking findings

PP-F3 Concrete & Picture Graphs: create, interpret

- organizing data gathered, or drawing conclusions
- using real objects to create graphs
- using pictures to represent real objects

- interpreting graphs through discussions

Class 1 – Strand A – Numbers

KSO - Numeration: *by the end of **Class 3** students should:*

- *have number sense with respect to whole number meanings & multiple representations, including place value to 4 place, in order to deal with numbers in the real world and perform meaningful operations*
- *efficiently select & apply appropriate estimation strategies, to answer real life questions and check for reasonableness of answer in calculation*
- *have developed a wide variety of relationships with other numbers and across strands so as to support strong reasoning in new situations, both mathematical and real life*
- *understand simple fractions & decimals to tenths, and the relationship between them, so as to provide a strong foundation for higher level fractional ideas and computation*

Toward this, students in **Class 1** will be expected to master the following **SO** (Specific Outcomes):

1-A1 Sets: sorting & creating, based on number

- identify sets of a given number
- create sets of a given number, orally and for numerals

1-A2 Counting: beyond 10, by 2,5,10 (2-digit numbers only)

- counting by 2's, 5's, and 10's
- counting on from a given number
- counting back from a given number

1-A3 Compare Sets: for size

- more, less, same
- creating equivalent sets
  -include sets where items are; (a) added (b) taken away

1-A4 Numbers to 100: representing with symbols

- matching quantities with numerals
- associating with concrete materials (base ten)

1-A5 Ordinal Numbers: to 20

- using to identify position

- relating ordinal words to the symbols
- recognizing the relative aspect of ordinals

1-A6 Estimating: amounts to 100

- estimating quantity in a collection
- developing estimation strategies: chunking, referents
- include small objects and equivalent set of larger objects

1-A7 Number Meaning: 10-20

- understanding of "teen" numbers as 10 and more

1-A8 Place Value: modeling whole numbers to 2 places

- grouping 10s and units
- understanding that digit placement denotes value
- developing understanding from groupable to pre-grouped models

1-A9 Comparing & ordering 2-digit whole numbers

- establishing benchmarks (16 < 37 since 37 is close to 40)
- understanding any 2-digit number is greater than any 1-digit number
- include examples with
  -more tens
  -equal number of tens

1-A10 Fractional Parts: simple denominators

- understanding that parts of a fraction must be equal in size
- understanding developed through concrete models and related discussion (symbol recording delayed)

1-A11 Money

- exploring relationship of coins, through trading activities
- identifying value of coins
- counting coins in increments of 1, 5 and 10

Class 1 – Strand B – Operations

KSO - Operations: *by the end of **Class 3** students should:*

- *have visual images for understanding the four basic operations of whole numbers (through concrete experiences)*
- *have developed relationships among the 4 operations*
- *efficiently use a variety of simple algorithms for the four basic operations of whole numbers by connecting concrete models to symbolic and use appropriate mathematical language to communicate and justify procedures*
- *have developed strategies for efficient estimation and mental math calculations and know when each is appropriate*

Toward this, students in **Class 1** will be expected to master the following **SO** (Specific Outcomes):

1-B1 Addition Meaning: joining two groups

- +0, +1,
- including situations where:
  - -the result is unknown
  - -the change is unknown
  - -the initial is unknown
- understanding the meaning of the addition operation
- associate with related subtraction

1-B2 Addition Properties

- commutative
- associative
- +0**, +1**

1-B3 Addition Facts

- developing strategies for sums to 10
- memorizing sums to 10
- applying strategies and previous knowledge to access sums to 18 (see 1-B11)

1-B4 Recording Addition: models & symbols

- connecting models, diagrams and symbols
- relating each part of a number sentence to the context (sums to 10)

1-B5 Subtraction Meanings

- ♦ separation
- ♦ comparison

1-B6 Subtraction Meaning: missing addend

- ♦ understanding 'how many more' idea

1-B7 Addition & Subtraction: relationship

- ♦ subtraction and addition "undo" each other
- ♦ same context may be seen as addition or subtraction

1-B8 Addition & Subtraction Facts: sums to 10

- ♦ result unknown, change unknown, initial unknown
- ♦ recalling from memory or through strategies

1-B9 Addition & Subtraction: solving problems

- ♦ moving easily from one operation to another
- ♦ taking away or counting up
- ♦ connecting concrete models to contexts

1-B10 1 and 2-Digit Subtraction: models & symbols (no regrouping)

- ♦ connecting models, diagrams and symbols
- ♦ relating each part of a number sentence to the context

1-B11 Mental Strategies: sums & differences to 18

- ♦ developing strategies: doubles, benchmarks, relating to a known fact, counting on
- ♦ choosing appropriate strategy for a given fact

Class 1 – Strand C – Pattern

KSO - Pattern: *by the end of **Class 3** students should:*

- *recognize and create patterns in mathematical and real world contexts for purposes of solving problems and developing relationships*
- *apply pattern to problems based on number, geometry and measurement*
- *use patterns to assist in mental math strategies*
- *represent mathematical patterns informally via open sentences as a foundation for later algebraic ideas*

Toward this, students in **Class 1** will be expected to master the following **SO** (Specific Outcomes):

1-C1 Sequence Events: in real life

- before/after
- first, then, last

1-C2 Physical Configurations: creating & recognizing multiple ways

- recognize without counting, various configurations, or spatial patterns for small number of items
- application to addition and place value

1-C3 Create patterns: based on 2-D & 3-D shapes

- focusing on attributes of shapes
- investigating pattern rules: shape name, number of sides, type of side, orientation

1-C4 Copy, Extend, Create Patterns

- investigate number patterns based on numerals or actual objects which must be counted to determine the pattern

1-C5 Use patterns: to solve addition & subtraction

- fact family patterns: e.g., 2 + 4 = 6, 4 + 2 = 6, 6 – 2 = 4, 6 – 4 = 2
- other fact patterns: e.g., **2** + 6 = 8, **3** + 6 = 9, **4** + 6 = 10
- investigating hundred chart patterns

1-C6 Place Value patterns: copy & extend

- connecting to models: eg, what happens to the numeral when 10 is consistently added/taken away (concretely)

Class 1 – Strand D – Measurement

KSO - Measurement: *by the end of **Class 3** students should:*

- *understand concepts of length, capacity, time, mass and area, and use this understanding to perform measurements in non-standard and standard units*
- *justify the necessity for common (standard) units as an effective communication tool*
- *have measurement sense, including a sense of relative size of a unit, so as to effectively estimate and measure and check for reasonableness of answer*
- *choose appropriate units to solve real life problems in measurement*

Toward this, students in **Class 1** will be expected to master the following **SO** (Specific Outcomes):

1-D1 Compare, Order: lengths (direct, indirect)

- longer, taller, wider
- direct & indirect
- include misconceptions (not straight, wide)

1-D2 Principles: of measurement

- understanding, through hands-on experiences:
  - -bigger unit = smaller count
  - -understanding importance of a common starting point
  - -units must be the same size
  - -reiteration

1-D3 Non-Standard Units: estimate & measure length

- estimating and measuring in non-standard units to answer questions of interest
- choosing appropriate unit for contexts

1-D4 Capacity: compare, order

- predict, then test the amount a container is capable of holding
- direct & indirect

1-D5 Measuring Process: measure length, capacity, surface space

- estimating and measuring objects (non-standard units)
- choosing appropriate units
- understand measurement principles (D2)

1-D6 Mass: compare (no units)

- developing meaning of mass (how heavy)
- using measurement vocabulary
- investigating common misconceptions: same mass/different size, same size/different mass, large/light, small/heavy
- measuring directly & indirectly
- **1-D7 Area: compare (directly)**
- understanding meaning as surface space
- include direct & indirect measurement

1-D8 Durations of Time: compare (no units)

- compare directly (not clock reading)

1-D9 Hours: read hours on an analogue clock

- reading hours only (one-handed clock)
- using approximate language e.g., "a little past six o'clock" or "almost ten o'clock"

Class 1 – Strand E – Geometry

KSO - Geometry: *by the end of **Class 3** students should:*

- *through exploration, understand and name geometric shapes according to their attributes, and develop relationships among them so as to make sense of the world around them*
- *draw, model and classify 2-D and 3-D shapes by attributes to foster spatial sense for later ideas in geometry and for real life situations*
- *predict and verify results of transforming, combining and subdividing shapes to understand other shapes and how shapes change in the world around them*
- *develop other aspects of spatial sense including: visual memory, perceptual constancy, visual discrimination and position in space, in order to understand everyday events and objects as well as higher geometrical ideas*

Toward this, students in **Class 1** will be expected to master the following **SO** (Specific Outcomes):

1-E1 Spatial Sense: visual memory

- recalling an object or drawing no longer in view

1-E2 Spatial Sense: figure-ground perception

- identifying figures against a complex background or assemble parts to make a whole

1-E3 2-D & 3-D Shapes: sort, build, pattern (visually)

- manipulating 2-D and 3-D shapes to develop familiarity with characteristics
- naming informally

1-E4 2-D & 3-D Shapes: use attributes to name, describe, represent

- naming according to attributes: rhombus, trapezoid, hexagon, cylinder, sphere, cone and cube
- concept of prism, pyramid
- exploring according to attributes: rectangular prism, triangular prism, triangular pyramid and square pyramid

1-E5 2-D Figures in 3-D Shapes: recognize

- understanding 3-D shapes by 2-D faces
- recognizing similar faces in different solids

1-E6 Identify Shapes: in the environment (2-D, 3-D)

- variety of sizes and proportion possible for circles, rectangles, triangles, squares, hexagons

1-E7 2-D Shapes: build, subdivide & change

- exploring results when shapes are subdivided and combined

1-E8 Slides: 2-D Shapes

- slides (translations) move a shape, down, right, left or diagonally without changing its orientation

1-E9 Angles: recognize informally (small, big, square)

- informal language: small, big, square, corners/angles

Class 1 – Strand F – Data Management

KSO – Data Management: *by the end of **Class 3** students should:*

- *collect, record, organize and describe data to answer questions of personal interest*
- *design and implement simple surveys for real world issues and make predictions*
- *construct concrete and pictorial graphs as a way to communicate ideas about the data and check predictions*
- *interpret data from graphs and tables in a factual way as well as through interpolation and extrapolation (draw conclusions about things not represented in the data)*

Toward this, students in **Class 1** will be expected to master the following **SO** (Specific Outcomes):

1-F1 Collect & Organize Data: tallies

- concept of tallies (groups of 5)
- using tallies / charts to organize data in investigations of interest

1-F2 Pose Questions: for conducting, interpreting data (oral)

- asking and answering questions in conducting surveys & interpreting the results

1-F3 Concrete & Picture Graphs: create, interpret

- creating concrete and picture graphs
- common base line
- one-to-one matching of objects

1-F4 Pictographs & Symbolic Graphs: create, interpret

- pictographs
- symbolic bar graphs (equal size squares)
- horizontal & vertical

Class 1 – Strand G – Probability

KSO - Probability: *by the end of **Class 3** students should:*

- *understand and apply the probability of everyday events to effectively predict outcomes*
- *conduct informal investigations of chance to determine fairness of a game or situation, in order to make effective decisions*
- *express probability outcomes in a variety of ways including as simple fractions, so as to facilitate communication of ideas*

Toward this, students in **Class 1** will be expected to master the following **SO** (Specific Outcomes):

1-G1 Always, Sometimes, Never: probability of everyday events

- predicting whether events will always, sometimes, or never occur

Class 2 – Strand A – Numbers

KSO - Numeration: *by the end of **Class 3** students should:*

- *have number sense with respect to whole number meanings & multiple representations, including place value to 4 place, in order to deal with numbers in the real world and perform meaningful operations*
- *efficiently select & apply appropriate estimation strategies, to answer real life questions and check for reasonableness of answer in calculation*
- *have developed a wide variety of relationships with other numbers and across strands so as to support strong reasoning in new situations, both mathematical and real life*
- *understand simple fractions & decimals to tenths, and the relationship between them, so as to provide a strong foundation for higher level fractional ideas and computation*

Toward this, students in **Class 2** will be expected to master the following **SO** (Specific Outcomes):

2-A1 Counting: (beyond 10) by 2, 5, 10, 25, 100

- backwards & forwards
- by 2's, 3's, 4's, 5's, 10's, 25's, and 100's

- from various starting points

2-A2 Numbers to 100: describing as combinations of smaller numbers

- describe as combinations of smaller numbers

2-A3 Ordinal Numbers

- use of ordinal language to 31st (calendar)

2-A4 Estimating: to nearest 10

- in context only ("About how much money/time?")
-

2-A5 Base Ten: groupings to 3 digits

- connect concrete models to corresponding symbols

2-A6 Place Value

- build & interpret models for numbers to 3 places
- proportional materials
- include numbers with zeros, e.g. 108, 340
- record interpretation of models

2-A7 Comparing

- compare & order 3-digit whole numbers to 3 places
- symbols < and >

2-A8 Simple Fractions: modeling numerators/denominators

- number of fair shares: halves, fourths, thirds, fifths, and tenths
- natural language
- part of a whole, part of a set

2-A9 Money

- explore relationship between coins through trading activities
- identify coins by value to 1 ngultrum
- count coins in increments to 1 ngultrum

Class 2 – Strand B – Operations

KSO - Operations: *by the end of **Class 3** students should:*

- *have visual images for understanding the four basic operations of whole numbers (through concrete experiences)*
- *have developed relationships among the 4 operations*
- *efficiently use a variety of simple algorithms for the four basic operations of whole numbers by connecting concrete models to symbolic and use appropriate mathematical language to communicate and justify procedures*
- *have developed strategies for efficient estimation and mental math calculations and know when each is appropriate*

Toward this, students in **Class 2** will be expected to master the following **SO** (Specific Outcomes):

2-B1 Addition Properties

- commutative, associative, zero, one
- use properties as fact strategies

2-B2 Fact Strategies: sums to 20 - learning & applying

- doubles
- associative principle
- bridging to ten
- compensation
- select appropriate strategy

2-B3 3 Addends: adding 3 single-digit numbers

- associative principle

2-B4 2-Digit Addition

- with & without regrouping
- estimation as the first step in computation to determine reasonableness of answer

2-B5 Subtraction Properties

- change both numbers
- reverse of addition – relationship between addition and subtraction
- change one number and adjust at the end

2-B6 Addition & Subtraction Facts

- memorize sums to ten by mid year
- memorize sums to 20 by year end
- efficient use of strategies

2-B7 2-Digit Subtraction: with & without regrouping

- base ten models to show separation / comparison
- relate models to symbols
- estimation as the first step to determine reasonableness of answer

2-B8 Add & Subtract (mental): numbers rounded to 10

- add & subtract numbers rounded to 10

Class 2 – Strand C – Patterns

KSO - Pattern: *by the end of **Class 3** students should:*

- *recognize and create patterns in mathematical and real world contexts for purposes of solving problems and developing relationships*
- *apply pattern to problems based on number, geometry and measurement*
- *use patterns to assist in mental math strategies*
- *represent mathematical patterns informally via open sentences as a foundation for later algebraic ideas*

Toward this, students in **Class 2** will be expected to master the following **SO** (Specific Outcomes):

2-C1 Even & Odd numbers

- build rectangles (width = 2) & connect to symbols

2-C2 Compare patterns: (repeating & growing)

- similarities & differences

2-C3 Continue patterns: in different ways

- a pattern can be extended in many different ways unless a pattern rule is given

2-C4 Addition Table patterns: finding & using

- identify & explain patterns in addition table

2-C5 Open Sentences: simple patterns in addition & subtraction

- of the form: $a + b = \_\_$, $a + \_\_ = c$, $\_\_ + b = c$, $c - a = \_\_$, $c - \_\_ = b$, $\_\_ - a = b$

2-C6 Place Value patterns: 10s and 100s (addition / subtraction) - concretely & symbolically

- explain with models what happens when a number is changed to another where only one digit is different (214, 224, 234… or 302, 402, 502… etc)
- explain what happens to the number when the model is changed
- explore patterns that increase and decrease

Class 2 – Strand D – Measurement

KSO - Measurement: *by the end of **Class 3** students should:*

- *understand concepts of length, capacity, time, mass and area, and use this understanding to perform measurements in non-standard and standard units*
- *justify the necessity for common (standard) units as an effective communication tool*
- *have measurement sense, including a sense of relative size of a unit, so as to effectively estimate and measure and check for reasonableness of answer*
- *choose appropriate units to solve real life problems in measurement*

Toward this, students in **Class 2** will be expected to master the following **SO** (Specific Outcomes):

2-D1 Appropriate Units: choosing

- small and large measures

2-D2 Standard Units: justify

- investigate situations to justify use of standard units

2-D3 cm, m

- develop a sense of the length of a metre and a centimeter
- estimate & measure in cm, m

2-D4 cm, m: develop relationship

- ♦ recognize that a metre is 100 cm long

2-D5 Capacity: estimate & measure (non-standard)

- ♦ order containers by capacity

2-D6 Litre: sense of size

- ♦ use a variety of containers
- ♦ explore common items found in litres
- ♦ explore how much it takes to make a litre

2-D7 Mass: estimate & measure (non-standard & standard units)

- ♦ explore non-standard units on a balance scale

2-D8 Kilogram: sense of size

- ♦ develop a sense of what a kilogram feels like when lifted

2-D9 Area: compare (directly & indirectly)

- ♦ directly (no units)
- ♦ indirectly (no units)
- ♦ order a number of shapes or flat objects by area

2-D10 Area: estimate & measure (non-standard units)

- ♦ use a variety of non-standard units to fill outlines of shapes

2-D11 Perimeter: estimate & measure (non-standard units)

- ♦ estimate & measure in non-standard units

2-D12 Durations of Time: estimate & measure

- ♦ investigate non-standard units

2-D13 Hours & Half Hours

- ♦ read time in hours and half hours on an analogue clock and a digital clock
- ♦ explore a one-handed clock before a two-handed clock

2-D14 Calendar: explore properties

- ♦ explore number concepts
- ♦ investigate days of the week, months of the year, seasons (in context)

Class 2 – Strand E – Geometry

KSO - Geometry: *by the end of **Class 3** students should:*

- *through exploration, understand and name geometric shapes according to their attributes, and develop relationships among them so as to make sense of the world around them*
- *draw, model and classify 2-D and 3-D shapes by attributes to foster spatial sense for later ideas in geometry and for real life situations*
- *predict and verify results of transforming, combining and subdividing shapes to understand other shapes and how shapes change in the world around them*
- *develop other aspects of spatial sense including: visual memory, perceptual constancy, visual discrimination and position in space, in order to understand everyday events and objects as well as higher geometrical ideas*

Toward this, students in **Class 2** will be expected to master the following **SO** (Specific Outcomes):

2-E1 Spatial Sense: perceptual constancy

- continue to develop the ability to recognize figures or objects in space regardless of size, position, or orientation (shapes viewed from a different distance or different viewpoint)
- recognize that a shape or size is stable even if it appears to be different to the observer

2-E2 Spatial Sense: visual discrimination

- identify the similarities and differences between or among objects
- separate figures from complex background

2-E3 2-D & 3-D Shapes: sort, build, pattern

- explore attributes of shapes through sorting, patterning, building

2-E4 3-D Shapes: recognize, name, describe prisms, pyramids

- distinguish between prisms & pyramids: triangular, square, and rectangular
- recognize shapes from description

2-E5 3-D Shapes

- ♦ recognize shapes from drawings and alternate perspectives

2-E6 2-D Shapes: build, subdivide & change

- ♦ relate polygons/triangles, parallelograms/squares, triangles/right triangles – through exploration

2-E7 Nets

- ♦ cut & assemble 2-D plans (nets), into 3-D shapes
- ♦ explore a cube, triangular prism, square prism, rectangular prism

2-E8 Slides & Reflections: 2-D Shapes

- ♦ sort, build, pattern using slides & reflections

2-E9 Parallel Lines & Parallelograms: name & represent

- ♦ investigate parallel lines
- ♦ investigate and develop personal definition for parallel lines

2-E10 Reflective Symmetry: identify & describe in 2-D shapes & real life

- ♦ understand that half of the shape is the mirror image of the other half
- ♦ explore vertical, horizontal, or slanted line of symmetry
- ♦ investigate symmetry in real life

Class 2 – Strand F – Data Management

KSO – Data Management: *by the end of **Class 3** students should:*

- ♦ *collect, record, organize and describe data to answer questions of personal interest*
- ♦ *design and implement simple surveys for real world issues and make predictions*
- ♦ *construct concrete and pictorial graphs as a way to communicate ideas about the data and check predictions*
- ♦ *interpret data from graphs and tables in a factual way as well as through interpolation and extrapolation (draw conclusions about things not represented in the data)*

Toward this, students in **Class 2** will be expected to master the following **SO** (Specific Outcomes):

2-F1 Collect & Organize Data

- develop strategies to collect and record information
- use tallies to record information
- create questions

2-F2 Pose Questions & Predict

- ask & answer oral, and simple written questions to conduct surveys & interpret results
- make & modify predictions based on data collected or presented

2-F3 Pictographs

- create, interpret
- 1 symbol = 1 unit
- use both vertical & horizontal orientations

2-F4 Bar Graphs

- create, interpret
- 1 symbol = 1 unit
- use both vertical & horizontal orientations

Class 2 – Strand G – Probability

KSO - Probability: *by the end of **Class 3** students should:*

- *understand and apply the probability of everyday events to effectively predict outcomes*
- *conduct informal investigations of chance to determine fairness of a game or situation, in order to make effective decisions*
- *express probability outcomes in a variety of ways including as simple fractions, so as to facilitate communication of ideas*

Toward this, students in **Class 2** will be expected to master the following **SO** (Specific Outcomes):

2-G1 More / Less Likely Events

- investigate every day & fictional events

2-G2 Exceptions: predictions may not prove true

- realize that theoretical predictions may not prove true given a set of tries

Class 3 – Strand A – Numbers

KSO - Numeration: *by the end of **Class 3** students should:*

- *have number sense with respect to whole number meanings & multiple representations, including place value to 4 place, in order to deal with numbers in the real world and perform meaningful operations*
- *efficiently select & apply appropriate estimation strategies, to answer real life questions and check for reasonableness of answer in calculation*
- *have developed a wide variety of relationships with other numbers and across strands so as to support strong reasoning in new situations, both mathematical and real life*
- *understand simple fractions & decimals to tenths, and the relationship between them, so as to provide a strong foundation for higher level fractional ideas and computation*

Toward this, students in **Class 3** will be expected to master the following **SO** (Specific Outcomes):

3-A1 Numbers to 4 digits: reading in several ways

- eg. 1542 (1 thousand 5 hundred forty two, fifteen forty two, fifteen hundred forty two)
- the word "and" is used for the decimal only

3-A2 Estimating: to nearest 10 or 100

- investigate real life problem solving situations

3-A3 Base Ten: understand groupings to 1000s

- one hundred = ten tens, one thousand = ten hundreds

3-A4 Place Value: to 4 places

- interpret & model numbers in different ways
- include numbers with zero

3-A5 Comparing: ordering 4-digit whole numbers

- identify the greater of two numbers
- compare numbers by placing them on a number line
- order a set of numbers, by comparing concrete models
- relate models to symbols for purposes of comparing

3-A6 Simple Fractions: using them in real contexts

- relate simple fractions such as $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{10}$ to real world situations
- include part of a whole / part of a set

3-A7 Money

- explore value of denominations to 100 ngultrums through trading activities
- count combinations of money to 100 ngultrums

3-A8 Tenths: model and record

- investigate concept of tenths in place-value system
- develop as a result of continuing pattern of dividing by 10
- relate tenths to models

3-A9 Tenths: compare and order numbers to tenths

- compare whole number part first, decimal part second
- use symbols < and >

Class 3 – Strand B – Operations

KSO - Operations: *by the end of **Class 3** students should:*

- *have visual images for understanding the four basic operations of whole numbers (through concrete experiences)*
- *have developed relationships among the 4 operations*
- *efficiently use a variety of simple algorithms for the four basic operations of whole numbers by connecting concrete models to symbolic and use appropriate mathematical language to communicate and justify procedures*
- *have developed strategies for efficient estimation and mental math calculations and know when each is appropriate*

Toward this, students in **Class 3** will be expected to master the following **SO** (Specific Outcomes):

3-B1 3-Digit Addition: whole numbers

- with & without regrouping
- continue estimating as a first step
- develop, explain and use alternative paper-and-pencil algorithms to solve problems

3-B2 3-Digit Subtraction: with & without regrouping - models, symbols (including estimating)

- in meaningful contexts
- with & without regrouping
- continue estimating as a first step
- develop, explain and use alternative paper-and-pencil algorithms to solve problems
- solve & create problems (include comparison, missing addend, take away)

3-B3 Early Multiplication: repeated addition

- focus on idea of adding groups of equal size – not memorizing facts
- use common language to verbalize (" three groups of six; that's eighteen")

3-B4 Multiplication Meanings: arrays, groups, number lines

- understand as total number in an array
- understand as repeated addition (groups)

3-B5 Multiplication Properties: commutative, associative, zero, 1

- explore properties: commutative, associative, zero, 1

3-B6 Multiplication Facts: develop & practice strategies

- to 9×9
- turn-around, double/half, half/double, doubles, doubles +/–1, compensation, relate to known fact, benchmarks

3-B7 Multiplication Facts: select appropriate strategies

- to 9×9
- choose most efficient strategy for a fact
- select facts for specific strategies

3-B8 2-Digit x 1-Digit Multiplication: concretely, symbolically

- no regrouping
- include products less than 100
- continue estimating as a first step
- connect alternate & standard algorithms to a model

3-B9 Division Meanings: groups, shares (small numbers)

- understand division as groups, and as shares

- understand division as repeated subtraction

3-B10 Mult & Div: relationship

- understand every multiplication as a related division & vice versa

3-B11 Division Facts: relate to multiplication

- relate concrete models to symbolic representation
- relate division facts to corresponding multiplication facts
- master facts to at least 6×6 by the end of class 3

3-B12 Add & Subtract (mental): 1 & 2-digit numbers

- add and subtract rounded numbers mentally
- develop strategies: make ten, front-end, counting on, subtract 10 and compensate, balancing, using the nearest multiple of ten then compensating, partner numbers

Class 3 – Strand C – Pattern

KSO - Pattern: *by the end of **Class 3** students should:*

- *recognize and create patterns in mathematical and real world contexts for purposes of solving problems and developing relationships*
- *apply pattern to problems based on number, geometry and measurement*
- *use patterns to assist in mental math strategies*
- *represent mathematical patterns informally via open sentences as a foundation for later algebraic ideas*

Toward this, students in **Class 3** will be expected to master the following **SO** (Specific Outcomes):

3-C1 Repeated Addition pattern: record using multiplication notation

- recognize repeated addition problems as multiplication
- recognize the meaning of each factor

3-C2 Multiplication Table patterns: find & use

- find and explain patterns in the multiplication table

3-C3 Open Sentences

- explore simple patterns in multiplication: missing products, missing factors

3-C4 Place Value pattern: base ten system to thousands (concretely, symbolically)

- develop from concrete to symbolic
- understand hundreds are recorded to the left of tens
- understand that 10 of any unit = 1 of the unit to the left

Class 3 – Strand D – Measurement

KSO - Measurement: *by the end of **Class 3** students should:*

- *understand concepts of length, capacity, time, mass and area, and use this understanding to perform measurements in non-standard and standard units*
- *justify the necessity for common (standard) units as an effective communication tool*
- *have measurement sense, including a sense of relative size of a unit, so as to effectively estimate and measure and check for reasonableness of answer*
- *choose appropriate units to solve real life problems in measurement*

Toward this, students in **Class 3** will be expected to master the following **SO** (Specific Outcomes):

3-D1 dm: sense of size

- explore relationship 1 dm = 10 cm or 1/10 metre (concretely & symbolically)
- estimate and measure metres, centimeters, decimeters
- include perimeter contexts

3-D2 km: sense of size

- explore relationship 1 kilometre = 1000 metres

3-D3 Capacity: estimate & measure

- explore non-standard units
- estimate & order containers by capacity
- verify estimations by measuring in non-standard units

3-D4 Litres, mL: estimate & measure

- estimate and measure capacities, using the litre
- introduce the millilitre
- realize that millilitres are extremely small units

3-D5 kg, grams: estimate & measure mass

- estimate and measure masses in kg
- introduce the gram (g)
- understand that grams are used to measure very light objects

3-D6 Capacity & Mass

- choose appropriate units (g or kg) for a given situation

3-D7 Area: estimate & measure (square cm - concrete)

- estimate & measure the amount of surface space of common objects in non-standard units and square centimetres (concrete)
- use the centimetre grid as a measuring tool

3-D8 Arrays & Dimensions (rectangles): develop relationship (concrete materials)

- explore squares & rectangles
- explore arrays as a pictorial way to show multiplication
- understand that the number of rows= one dimension, the number in each row = other dimension
- connect concrete models to corresponding symbols

3-D9 Perimeter

- investigate distance around regular objects, using standard units
- use different units to measure the same object

3-D10 Minutes: reading clocks

- relate digital & analogue clocks
- read time to the nearest 5 minutes

3-D11 Calendar: solve problems

- explore problems relating to days of the week, months of the year and the four seasons

Grade 3 – Strand E – Geometry

KSO - Geometry: *by the end of **Class 3** students should:*

- *through exploration, understand and name geometric shapes according to their attributes, and develop relationships among them so as to make sense of the world around them*
- *draw, model and classify 2-D and 3-D shapes by attributes to foster spatial sense for later ideas in geometry and for real life situations*
- *predict and verify results of transforming, combining and subdividing shapes to understand other shapes and how shapes change in the world around them*
- *develop other aspects of spatial sense including: visual memory, perceptual constancy, visual discrimination and position in space, in order to understand everyday events and objects as well as higher geometrical ideas*

Toward this, students in **Class 3** will be expected to master the following **SO** (Specific Outcomes):

3-E1 Spatial Sense: perceptual constancy

- use hands-on materials to explore and continue to develop spatial sense
- recognize a shape as stable even if it appears to be different to the observer

3-E2 Polygons: name, describe for concave, convex, regular

- concave, convex, regular
- kite: quadrilateral with two pairs of congruent adjacent sides
- regular shapes: all sides & all angles are congruent

3-E3 Squares & Rectangles: relationship

- understand a square as a special rectangle; conclude by examining attributes

3-E4 Parallelograms: name, describe, represent

- name, describe, represent
- investigate and develop personal definition for a parallelogram

3-E5 Prisms & Pyramids: name, describe, generalize

- ♦ discover that the shape of the base determines the name of prism or pyramid
- ♦ include trapezoidal, pentagonal, hexagonal, and octagonal
- ♦ generalize patterns in attributes of prisms & pyramids (e.g. the number of vertices for all prisms is two times the number associated with its name –
  a triangular prism has 6 vertices)

3-E6 Polygons, Prisms, Pyramids

- ♦ identify in real world situations

3-E7 Combining Shapes: predict results for triangles and quadrilaterals

- ♦ predict results for combining triangles & quadrilaterals by visualizing
- ♦ build a variety of polygons & 3-D shapes to check predictions

3-E8 Nets: cut & assemble for prisms and pyramids (pentagonal, hexagonal)

- ♦ cut and fold nets for pentagonal and hexagonal prisms and pyramids
- ♦ examine the 2-D shapes that make up each net

3-E9 Skeletons: prisms & pyramids

- ♦ build to focus on number of edges, faces, vertices

3-E10 Turns: 2-D shapes

- ♦ half, quarter turns
- ♦ quarter turn: connection to right angle

3-E11 Angles: right, less/more than right

- ♦ compare angles by sight
- ♦ describe angles as less or more than a right angle

3-E12 Reflective Symmetry: find various lines of reflection in polygons

- ♦ find various lines of reflection in polygons
- ♦ develop personal definition for a line of symmetry

3-E13 Congruence: angles

- ♦ understand that congruent polygons are a perfect match

- understand that congruent angles in shapes occur when the corners match

Grade 3 – Strand F – Data Management

KSO – Data Management: *by the end of **Class 3** students should:*

- *collect, record, organize and describe data to answer questions of personal interest*
- *design and implement simple surveys for real world issues and make predictions*
- *construct concrete and pictorial graphs as a way to communicate ideas about the data and check predictions*
- *interpret data from graphs and tables in a factual way as well as through interpolation and extrapolation (draw conclusions about things not represented in the data)*

Toward this, students in **Class 3** will be expected to master the following **SO** (Specific Outcomes):

3-F1 Collect, Organize & Describe Data: choose strategies

- focus on selection of appropriate strategies for collecting and displaying data

3-F2 Collection Plans: implement

- explore considerations when collecting data: Where is a good source? Where should I conduct the survey? Does it matter when the survey is conducted? How should the questions be phrased?

3-F3 Pictographs

- investigate construction where each symbol represents more than one item
- interpret pictographs before creating them

3-F4 Bar Graphs

- interpret graphs for which each section represents a value greater than one
- introduce simple scales for larger numbers
- construct both horizontally and vertically

Grade 3 – Strand G – Probability

KSO - Probability: *by the end of **Class 3** students should:*

- *understand and apply the probability of everyday events to effectively predict outcomes*
- *conduct informal investigations of chance to determine fairness of a game or situation, in order to make effective decisions*
- *express probability outcomes in a variety of ways including as simple fractions, so as to facilitate communication of ideas*

Toward this, students in **Class 3** will be expected to master the following **SO** (Specific Outcomes):

3-G1 Extremes

- explore real-life or mathematical events with very low or very high probabilities

3-G2 Exceptions

- understand that events in theory may not prove true in a given set of tries

3-G3 Experiments: predict & record results

- conduct experiments and record outcomes
- use common language (e.g. "2 out of 5") to describe results

Class 4 – Strand A – Numbers

KSO - Numeration: *by the end of **Class 6** students should:*

- *have strong number sense with respect to whole numbers and decimals, and be able to draw on a wide variety of relationships & strategies within number to solve problems in new situations*
- *have a strong sense of the base ten system to millions and thousandths, and use place value patterns to understand new ideas and apply reasoning to computational problems and mental mathematics within mathematics itself and in real world situations*
- *efficiently select & apply appropriate estimation strategies, to answer real life questions and check for reasonableness of answer in calculation*
- *understand fractions & decimals to thousandths, and the relationship between them, and to move freely from one form of representation to another, as might be appropriate in a given situation, and to provide a strong foundation for higher level fractional ideas and computation*
- *understand meanings and appropriate application of integers, ratios and percent in real world situations*
- *apply number theory concepts in relevant situations as a way to solve problems with respect to whole numbers, fractions, decimals*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-A1 Place Value: model whole numbers to 5 places

- recognize actual value of each digit
- read numbers several ways and record numbers
- include numbers with zero

4-A2 Compare & Order Whole Numbers to 5-digits

- order two or more numbers
- justify order
- identify numbers greater or less than a given number
- identify numbers between given numbers

4-A3 Mixed Numbers: modeling

- develop visual images for fractions & mixed numbers through concrete materials
- use contexts which include part of a whole, part of a group

4-A4 Renaming Fractions

- understand concretely that two or more fractions can have different names but the same value
- find number patterns in equivalent fractions (delay use of rules until understanding is fully developed)

4-A5 Compare & Order Fractions

- compare visually, in a variety of ways
- compare fractions with same denominators
- compare fraction with same numerators
- develop and apply benchmarks

4-A6 Hundredths: model and record

- develop concept of hundredths in our place-value system
- develop as a result of continuing pattern of dividing by 10
- relate to models
- explore relationship between decimals and fractions

4-A7 Hundredths: compare & order

- compare whole number part first, decimal part second
- explore relationship between decimals and fractions

Class 4 – Strand B – Operations

KSO - Operations: *by the end of **Class 6** students should:*

- *model & solve computational problems involving whole numbers and decimals by selecting appropriate operations and procedures for computation, estimation and mental math*
- *choose appropriate method of computation in given situations (including pencil/paper, mental math, estimation, technology)*
- *model & solve problems involving the addition and subtraction of simple fractions and be able to justify answers through reasoning*
- *informally explore simple algebraic situations*
- *demonstrate flexibility in procedures chosen to solve computational problems relevant to the real world and the study of mathematics*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-B1 Addition & Subtraction of Decimals & Wholes: 10ths and 100ths

- explore mixed numbers (decimals & wholes) to 5 digits
- recognize actual value of each digit
- develop strategies for adding & subtracting mixed numbers
- relate addition & subtraction of decimals to addition & subtraction of whole numbers
- continue estimating as a first step

4-B2 Multiplication Meanings

- explore combinations, rate times a quantity, comparison

4-B3 Multiplication Properties

- explore commutative, distributive, associative, zero, 1

4-B4 Multiplication Facts

- recall to 9×9
- develop facts through concrete & pictorial representations
- develop & practice strategies for recall

4-B5 3-Digit × 1-Digit Multiplication: with / without regrouping

- develop alternate & standard algorithms (from understanding)
- connect concrete to symbolic

4-B6 Division Meanings (small numbers)

- understand division as groups or shares
- recognize division in contexts of rate, comparison, combinations

4-B7 Division Properties: zero, 1

- zero, 1
- understand that order matters (opposite to multiplication)

4-B8 Multiplication & Division Facts: relate through properties

- understand multiplication and division as two ways of looking at the same situation
- division facts mastered by 'thinking multiplication'

4-B9 2, 3-Digit ÷ 1-Digit: with / without regrouping

- develop sharing or grouping algorithms

- connect algorithms to models
- understand remainders in real life as a fraction, ignored, rounded, addressed specifically (depending on context)
- continue estimating as a first step

4-B10 Add & Subtract Mentally: to 4 digits

- develop & use mental strategies: front end, compensation, counting on/back, compatibles
- determine if a problem can be solved mentally

4-B11 Multiply Mentally: by 10 or 100

- explore 2-digit numbers
- develop visual images of whole numbers multiplied by 10 or 100 (base ten materials - that number of rods or flats)
- read numbers different ways (e.g.5300 is often read as 53 hundred, rather than 5 thousand, 3 hundred)

4-B12 Open Frame: as numbers or digits

- understand that an open frame can represent a number or a digit **(□ × 5 = 30)**

Class 4 – Strand C – Patterns

KSO - Pattern: *by the end of **Class 6** students should:*

- *describe, extend and create patterns to solve problems in real world situations and mathematical contexts (number, geometry, measurement)*
- *use patterns to generalize for mathematical situations to aid in solving problems and understanding relationships*
- *explore and generalize how a change in one quantity in a relationship affects another, in order to efficiently solve similar(but new) problems*
- *represent mathematical patterns and relationships in a variety of ways (charts, tables, graphs, numerically)*
- *use patterns to assist in mental math strategies*
- *informally (through reasoning) solve linear equations via open sentences as a foundation for later algebraic ideas*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-C1 Apply Pattern in Computation

- explore and apply patterns to solve computation problems (e.g. × 9, 11, 10)

4-C2 Open Sentences & Computation Patterns: multiplication & division

- generate rules about how a change in one variable affects the result (e.g. □ × 10 : as □ increases by 1 the product increases by 10)

4-C3 × 10, × 100, × 1000: apply pattern visually & symbolically

- identify and continue patterns with increasing powers of ten

4-C5 Whole Numbers & Decimals: relationship in computation

- understand relationship concretely & symbolically
- discover, through investigation, that the process of adding/subtracting tenths or hundredths is the same as adding/subtracting whole numbers

Class 4 – Strand D – Measurement

KSO - Measurement: *by the end of **Class 6** students should:*

- *understand relationships among common SI units and choose appropriate units to solve measurement problems in given situations*
- *move freely among common SI units to effectively communicate measurement ideas appropriate to a given measurement situation*
- *estimate effectively using a variety of strategies to solve measurement problems and understand when estimation is close enough*
- *use relationships and reasoning to develop and apply procedures for measuring in real situations and mathematical contexts*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-D1 mm, cm, dm, m, km: estimate & measure

- estimate & measure in mm, cm, dm , m and km
- develop a sense of longer units
- investigate and develop unit relationships

- explore roots of words: milli, centi, deci, kilo, deca, hecto

4-D2 Volume: estimate & measure

- explore meaning through non-standard units
- understand volume as the number of units it takes to build a solid
- estimate & measure volume in non-standard units

4-D3 Volume (rectangular prisms): estimate & measure with cm cubes

- estimate then verify the volume of rectangular prisms using centimetre cubes
- determine the volume of a rectangular prism and to build prisms with a specified volume
- connect volume to dimensions (dimensions of first layer × number of layers)

4-D4 Area: estimate & measure (square cm - symbols)

- understand that area is expressed as the number of units required to cover a given surface

4-D5 Constant Area - Different Perimeters

- explore concept concretely
- understand that objects of different shapes can have the same area
- understand that area and perimeter are independent of each other

4-D6 Dimensions & Area - Factors & Products (rectangles): relate

- relate dimensions of rectangles to area (product) concretely
- develop personal formula for area

4-D7 Angles: (meaning) amount of turn

- develop meaning concretely
- understand angle as a turn, measure of angle as amount of turn (smaller angle = smaller turn)
- investigate to discover that length of arms of an angle does not influence angle size
- differentiate and describe right, acute and obtuse angles

Class 4 – Strand E – Geometry

KSO - Geometry: *by the end of **Class 6** students should:*

- *identify, draw, compare and build physical models of 2-D and 3-D shapes to focus on their attributes and understand how they affect everyday life*
- *predict and verify results of transforming, combining and subdividing shapes to understand other shapes and explain other geometrical ideas*
- *use geometric relationships and spatial reasoning to solve problems and understand everyday events and objects, as well as higher geometrical ideas*
- *appreciate the importance of geometry in understanding mathematical ideas and the world around*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-E1 Isometric Drawings

- build shapes from isometric drawings
- include shapes that have "hidden cubes"

4-E2 Quadrilaterals: discover properties (concretely)

- investigate a variety of quadrilaterals to discover properties (sides, angles, diagonals, symmetry)

4-E3 Quadrilaterals: sort by properties & make generalizations (concretely)

- use properties to make generalizations
- include properties that relate sides and those that relate angles

4-E4 Triangles: discover properties, name, construct (concretely)

- concrete
- equilateral, isosceles, scalene
- sort by various properties (e.g. number of lines of symmetry) or (e.g. number of identical angles)
- recognize and construct
- generalize to properties of sets: common to all members of the set

4-E5 Prisms, Pyramids, Cones, Cylinders

- explore relationships concretely to identify properties (e.g. prisms: number of vertices for all prisms is the number associated with its name – triangular prism has 6 vertices)
- include relationships that deal with faces, edges, vertices
- understand why those relationships make sense
- examine the similarities and differences between any pair of 3-D shapes

4-E6 Composite Figures: find all possibilities from a given set

- find all possibilities from a given set of figures
- predict first, then verify by combining

4-E7 Nets: draw for rectangular prisms & cubes

- draw a variety of nets for rectangular prisms, square prisms (including cubes)

4-E8 Models: constructing for cylinders, cones, prisms, pyramids

- construct, from given nets, cylinders and cones
- build skeletal models for prisms and pyramids

4-E9 Slides, Flips, Turns (half, quarter): predict & confirm results for 2-D shape

- predict & confirm results for 2-D shapes under tranformations

4-E10 Angles: acute, obtus

- name, describe & construct
- explore the effects of varies arm lengths of angles

4-E11 Reflective Symmetry: generalize for properties of various quadrilaterals

- explore properties of various quadrilaterals
- make generalizations (focus on reflective symmetry property)

4-E12 Congruence: polygons

- understand that congruent polygons are a perfect match
- explore through variety of materials (pattern blocks, tangrams, pictures of shapes) & methods (including tracing)

Class 4 – F Strand – Data Management

KSO – Data Management: *by the end of **Class 6** students should:*

- *collect, record, organize and describe data in multiple ways to draw conclusions about everyday issues*
- *construct a variety of data displays and choose most appropriate*
- *predict, read, interpret and modify predictions for a variety of data displays, including interpolation and extrapolation (draw conclusions about things not specifically represented by the data)*
- *develop and apply measures of central tendency to data reflecting relevant situations, in order to draw conclusions and make decisions*
- *design and implement strategies for the collection of data, including question design, population sampling, first and second hand data and bias*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-F1 Collect, Organize & Describe Data: real world issues

- explore many ways to collect data – (e.g. open question or choose from options)
- consider how survey design affects data: question construction, sampling, external factors, sample size, bias
- choose most appropriate method
- make decisions about format of presentation (charts, tables, graphs)

4-F2 Bar Graphs & Pictographs: construct & interpret

- pictographs: choose symbol and decide how much each represents (scale)
- bar graphs: decide value of each square
- include vertical & horizontal representations
- interpret results and extrapolate from data

4-F3 Ordered Pairs: position on a grid

- introduce coordinate grid
- explore method for naming points and why order is significant

- investigate the differences on a map and specific points
- use terminology informally: “axes,” “ coordinates,” “plot,” and “origin”

4-F4 Mean

- understand as summary statistic
- understand mean in terms of balancing data
- relate calculation of mean to division

4-F5 Maxima, Minima, Range, Frequency

- determine, given numerical data
- investigate method for naming points and why order is significant
- understand how each gives a different piece of information and when each might be important (eg money saved: greatest, least, difference between greatest & least, amount cited most often)
- relate frequency to the heights of bar graphs

Class 4 – Strand G – Probability

KSO - Probability: *by the end of **Class 6** students should:*

- *explore, interpret and make predictions for everyday events by estimating and conducting experiments*
- *understand the difference between theoretical and experimental probability and when each is relevant*
- *begin to conduct simulations to understand real life probability situations*
- *understand the relationship between the numerical representations of probability and the events they represent*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-G1 Simple Outcomes: more / less likely

- predict whether an outcome is more or less likely than another by investigating with e.g.spinners, dice, coloured cubes

4-G2 Predict Probability: near 0, near 1, near ½

- probability 0: an event rarely occurs
- probability 1: an event almost always occurs

- probability ½: event has an equal chance of occurring or not occurring
- investigate through simple experiments and everyday events

4-G3 Experiments: predict & record results (concrete materials)

- investigate concretely
- predict outcomes, verify by experiments, record outcomes, compare findings
- devise ways to track spins & tosses
- compare results of a few experiments with those of many
- use common language: e.g. "2 out of 5"

4-G4 Describe Results: as fractions

- express simple experimental results as fractions (restrict the total number of possible events to 6, 8, or 10)

Class 5 – A Strand – Numbers

KSO - Numeration: *by the end of **Class 6** students should:*

- *have strong number sense with respect to whole numbers and decimals, and be able to draw on a wide variety of relationships & strategies within number to solve problems in new situations*
- *have a strong sense of the base ten system to millions and thousandths, and use place value patterns to understand new ideas and apply reasoning to computational problems and mental mathematics within mathematics itself and in real world situations*
- *efficiently select & apply appropriate estimation strategies, to answer real life questions and check for reasonableness of answer in calculation*
- *understand fractions & decimals to thousandths, and the relationship between them, and to move freely from one form of representation to another, as might be appropriate in a given situation, and to provide a strong foundation for higher level fractional ideas and computation*
- *understand meanings and appropriate application of integers, ratios and percent in real world situations*
- *apply number theory concepts in relevant situations as a way to solve problems with respect to whole numbers, fractions, decimals*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-A1 Factors: of whole numbers

- generalize that factors of a number are never greater than the number
- conclude that the number is always a multiple of any of its factors
- find factors by dividing
- understand, through investigation, that the greatest factor is always the number itself
- understand, through investigation, that the least factor is always 1
- understand, through investigation, that the second greatest factor is always ½ the number or less

5-A2 Millions: interpret as a mixed number

- interpret in different ways (eg- 1 500 000 = 1 ½ million, 1.5 million)
- justify interpretation
- develop a sense of how big a million is, through meaningful investigations

5-A3 Place Value: whole numbers to 7 digits

- read and represent
- generalize place-value pattern as groups of 3 digits

5-A4 Comparing: order 5-digit whole numbers

- in standard notation (34 256 876 > 34 255 996)
- in decimal notation (34.25 million > 34.3 million)
- both (34 256 876 < 35.2 million)
- with different units (3423 thousand > 3 325 146)

5-A5 Fraction Meaning: division

- develop relationship between fractions and division
- use division meaning to change an improper fraction to a mixed number
- link concrete materials to symbols to develop reasoning

5-A6 Rename Fractions: with and without models (conceptual)

- develop understanding through concrete materials, then link to symbolic

- understand equivalent fractions as the same region partitioned in different ways
- understand the relationship between numerator and denominator
- investigate as part of a whole; part of a set; part of a linear measurement

5-A7 Compare & Order fractions (using reasoning)

- develop referents
- with same denominator
- with same numerator
- as mixed numbers

5-A8 Thousandths: model and record

- develop referents (0.432 is a little less than half a metre)
- place decimal numbers on a number line and justify
- read quantitative value of each digit in decimal numbers (16.5 as "sixteen and 5 tenths" or "sixteen and a half")

5-A9 Thousandths: compare and order numbers to thousandths

- compare whole number parts first
- understand that decimal numbers do not need the same number of places after the decimal to be compared ($0.7 > 0.423$)
- understand that the number of places after the decimal does not dictate size
- understand that the size of thousandths is relative to context (in population numbers e.g. 3.014 million: thousandths are not as significant as 3.014 m in a measurement situation)

5-A10 Ratio & Rate: exploring informally (ideas, not symbolically)

- understand ratio as a multiplicative comparison of two numbers or quantities of the same type
- understand rate as a multiplicative comparison of two quantities described in different units
- explore common ratio and rate situations in geometric, numerical and measurement situations

Class 5 – B Strand – Operations

KSO - Operations: *by the end of **Class 6** students should:*

- *model & solve computational problems involving whole numbers and decimals by selecting appropriate operations and procedures for computation, estimation and mental math*
- *choose appropriate method of computation in given situations (including pencil/paper, mental math, estimation, technology)*
- *model & solve problems involving the addition and subtraction of simple fractions and be able to justify answers through reasoning*
- *informally explore simple algebraic situations*
- *demonstrate flexibility in procedures chosen to solve computational problems relevant to the real world and the study of mathematics*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-B1 Addition & Subtraction of Decimals & Wholes: 5 digits - to 1000ths

- perform addition and subtraction presented horizontally and vertically
- choose best method for computation: mentally, pencil paper, calculator
- continue estimating as a first step in computation

5-B2 2-Digit × 2-Digit Multiplication: with / without regrouping

- relate models to algorithms
- develop personal and standard algorithms
- continue estimating as a first step

5-B3 Decimals × Whole Numbers: simple products

- link concrete models to algorithm
- estimate as a first step (e.g. 4×2.45 - whole number first then decimal part)

5-B4 4-Digit ÷ 1-Digit: with / without regrouping

- develop focus on whole number (rather than digits)
- link concrete models to algorithms

- understand procedure is the same as for whole numbers – only the value is different
- express remainders as fractions where appropriate (depends on context)
- continue estimating as a first step

5-B5 4-Digit ÷ 2-Digit: introduce

- explore divisors which are multiples of 10 only (10, 20, 30…)
- place value focus in estimation – verbalize in common language (e.g. 869 ÷ 20: "2 tens × 4 tens = 800, 69 remains giving 3 more twenties… estimate 43")

5-B6 Addition & Subtraction: simple fractions with common denominators

- link concrete models to symbols
- use common language to build understanding (2 fourths + 1 fourth = 3 fourths)
- use decimal computation for tenths

5-B7 Multiply Mentally: to 4 digits x 1 digit (using strategies)

- understand the difference between estimation and mental math
- understand that strategies used in estimation can often be used to calculate mentally
- develop efficiency mentally multiplying by 10, 100, 1000
- apply distributive strategy ($25 \times 30 = 25 \times 3 \times 10 = 750$)
- apply double/half strategy: ($50 \times 16 = 100 \times 8$)
- apply front end strategy: ($3 \times 325 = 900 + 60 + 15 = 975$) – focus on digit value
- choose appropriate strategy (depends on numbers being calculated)

5-B8 Divide Mentally

- use prior knowledge of basic facts
- divide by 10, 100, 1000
- link to concrete models to focus on place value

5-B9 Multiply Mentally whole numbers by 0.1, 0.01, 0.001

- use prior knowledge of basic facts
- multiply by 0.1, 0.01, 0.001
- link to concrete models to focus on place value

5-B10 Open Sentence applying number sense

- apply number sense
- explore numerical situations which are always, sometimes, never true (324 + □ > 300 is always true if □ is a whole number)
- include the four basic operations
- understand that □ can also be expressed as a letter or a triangle

Class 5 – C Strand – Patterns

KSO - Pattern: *by the end of **Class 6** students should:*

- *describe, extend and create patterns to solve problems in real world situations and mathematical contexts (number, geometry, measurement)*
- *use patterns to generalize for mathematical situations to aid in solving problems and understanding relationships*
- *explore and generalize how a change in one quantity in a relationship affects another, in order to efficiently solve similar(but new) problems*
- *represent mathematical patterns and relationships in a variety of ways (charts, tables, graphs, numerically)*
- *use patterns to assist in mental math strategies*
- *informally (through reasoning) solve linear equations via open sentences as a foundation for later algebraic ideas*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-C1 Open Sentences: patterns in addition, subtraction, multiplication & division

- generate rules about how a change in one variable affects the result (e.g. □ × 10 : as □ increases by 1 the product increases by 10)
- include 4 basic operations

5-C2 Computation patterns ×, ÷: how a change in either factor affects the computation

- rearrange factors to simplify computation (28 × 250 is more difficult that 7 × 1000)

- understand that dividing one factor and multiplying the other by the same amount produces no change in the final result

5-C3 Equivalent Fractions: multiplicative relationship

- investigate the multiplicative relationship of numerator/denominator (constant for equivalent fractions)
- explore by equal subdividing (for ¾: subdivide each fourth into 3 parts = 9/12)
- explore by equally grouping the fractional pieces that make up the whole (group 4 sixths in groups of 2 = 2 thirds)
- investigate result when numerators of equivalent fractions differ by a constant amount

5-C4 Area/Perimeter: changing rectangle dimensions

- use concrete models to discover patterns: eg longer length = shorter width
- conclude, through investigation, that rectangles of the same area can have different perimeters
- connect models to symbols: if one dimension is multiplied by a factor, the other must be divided by that factor (e.g. $24 \times 5 = 12 \times 10$)

5-C5 Place Value pattern: base ten system to millions

- recognize as groups of three digits
- investigate patterns in dividing by 10, 100 and 1000
- investigate patterns in multiplying by 0.1, 0.01 and 0.001
- develop a personal, conceptual rule for placement of the decimal point

5-C6 SI Measurement: pattern in changing units

- understand that a smaller measurement unit increases the number of those units and that a larger measurement unit decreases the number of those units
- apply the above relationship to reason through conversions

Class 5 – D Strand – Measurement

KSO - Measurement: *by the end of **Class 6** students should:*

- *understand relationships among common SI units and choose appropriate units to solve measurement problems in given situations*
- *move freely among common SI units to effectively communicate measurement ideas appropriate to a given measurement situation*
- *estimate effectively using a variety of strategies to solve measurement problems and understand when estimation is close enough*
- *use relationships and reasoning to develop and apply procedures for measuring in real situations and mathematical contexts*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-D1 Volume & Capacity: solve simple problems

- understand volume as the amount of space an object occupies or how much it takes to build it
- understand capacity as how much a container is capable of holding
- understand cubic measurement as 2 dimensional layers stacked up
- develop a sense of size and referents for a cubic cm, mm, m
- discover, through investigation, that 1 cm^3 holds 1 millilitre, 1 dm^3 holds 1 litre

5-D2 Area: irregular shapes - estimate & measure

- use transparent grids to measure area of irregular shapes (include squares and half squares)
- explore scaled drawings to measure large areas (e.g. 1 cm = 1 km)

5-D3 Perimeter: polygons

- solve perimeter problems in context
- understand perimeter as the total distance around an object
- develop generalizations for perimeter of regular polygons – (e.g. for equilateral triangles, the perimeter is 3 times the side length, square is 4 times, etc)

5-D4 Perimeter & Area: rectangles & squares

- develop from concrete to symbolic

- develop various personal formulas
- understand that all squares with the same perimeter have the same area & vice versa
- understand that rectangles with the same area can have different perimeters
- understand that rectangles with the same perimeter can have different areas

5-D5 SI Units: reinforce relationships among various SI units

- apply relationships among km / metres / decimeters / centimeters / millimeters, litres / milliliters, kilograms/grams, decameters, hectometers
- use relationships to rename measures
- apply referents for various measurement standards (30 cm is like a ruler, 1 dm is about a small hand span, etc.)

5-D6 Angles: estimate and measure

- explore in non-standard units (as wedges)
- understand that a smaller wedge = greater measure
- link wedges to degrees (degree is just a very small wedge)
- create and use a modified protractor for 45, 90, 135, 180 degrees

5-D7 Angles: estimate size

- estimate angles relative to common referents: 45, 90, 180 degrees (about the same as, more than, less than)

Class 5 – Strand E – Geometry

KSO - Geometry: *by the end of **Class 6** students should:*

- *identify, draw, compare and build physical models of 2-D and 3-D shapes to focus on their attributes and understand how they affect everyday life*
- *predict and verify results of transforming, combining and subdividing shapes to understand other shapes and explain other geometrical ideas*
- *use geometric relationships and spatial reasoning to solve problems and understand everyday events and objects, as well as higher geometrical ideas*
- *appreciate the importance of geometry in understanding mathematical ideas and the world around*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-E1 Isometric Drawings: make & interpret drawings of shapes made from cubes

- make and interpret drawings of shapes made from cubes
- focus on perceptual constancy (same shape whether seen from right, left, front, back)

5-E2 Triangles

- explore equilateral, isosceles, scalene triangles
- discover properties through concrete experiences; sort by nature of angles, name, construct concretely (sticks, straws)
- develop personal referent for 90 degrees

5-E3 Cross Sections: cubes & rectangular prisms

- investigate concretely
- explore cuts parallel to faces and oblique (not parallel)
- explore cuts from a vertex or a different point along the edge of the prism

5-E4 Combine Triangles

- predict results through visualization
- develop spatial sense using:
  - two congruent equilateral triangles
  - two congruent isosceles right triangles
  - two congruent isosceles triangles
  - two congruent right triangles
  - two congruent acute/obtuse triangles
  - two different isosceles triangles with a base of the same length

5-E5 Nets: prisms and pyramids

- develop various nets for prisms and pyramids
- understand that a net as a reflection or a rotation is still the original net

5-E6 Translations & Reflections: generalize & apply

- identify properties of: a shape and its reflected image, a shape and its congruent image

- compare orientation, corresponding parallel sides of shape and reflected image
- understand, through investigation, that corresponding points of shape and reflected image are equidistant from the mirror line
- understand, through investigation, that a mirror line is the perpendicular bisector
- of all segments joining corresponding points

5-E7 Rotations: 1/4, 1/2, 3/4 turns: predict & investigate

- investigate a variety of rotations
- predict, draw & identify quarter, half and 3-quarter turns
- explore the results of a variety of turn centres (pivot point)
- associate 90 degrees to quarter turns, 180 degrees to half turns

5-E8 Tessellations

- understand that, to tessellate, a shape must cover a surface with replications without gaps or overlaps
- describe, predict, investigate a variety of shapes for tessellating properties

5-E9 Perpendicular lines / segments

- construct with straws or straight sticks and use appropriate mathematical terminology, lines which are:
  - parallel to one another
  - intersecting
  - perpendicular at an end point
  - perpendicular at end points
  - perpendicular to another line at its midpoint
  - bisecting another line but not perpendicular
  - bisecting another line and perpendicular
  - bisect each other and are perpendicular

5-E10 Diagonal properties: squares & rectangles

- investigate by folding and develop generalizations for diagonals:
- of squares
- which bisect each other
- which intersect to form four right angles (perpendicular bisectors)
- which form two pairs of equal opposite angles at the point of intersection

- which form two angles at each vertex of the rectangle that sum to 90 degrees and have the same measures as the two angles at the other vertices
- which form two pairs of congruent isosceles triangles

5-E11 Rotational Symmetry properties: squares & rectangles

- understand the centre of rotation as the centre of the square and the intersection point of its two diagonals
- understand, through concrete investigation, when a shape has rotational symmetry (can be turned so that it matches exactly the original shape)
- discover, through concrete investigations that a square has rotational symmetry of "order 4" while a rectangle has rotational symmetry of "order 2"
- combine rotational symmetry of squares and rectangles with other properties of squares and rectangles

5-E12 Similarity: name, describe & represent

- understand when shapes are similar (corresponding angles equal, pairs of corresponding sides equal multiples of each other)
- understand that dilatation images of a shape are always similar (shape has all the same properties but may be different in size)
- understand that similar figures are not always dilatations (can be on different planes and/or be the result of a dilatation in combination with other transformations)

Class 5 – Strand F – Data Management

KSO – Data Management: *by the end of **Class 6** students should:*

- *collect, record, organize and describe data in multiple ways to draw conclusions about everyday issues*
- *construct a variety of data displays and choose most appropriate*
- *predict, read, interpret and modify predictions for a variety of data displays, including interpolation and extrapolation (draw conclusions about things not specifically represented by the data)*
- *develop and apply measures of central tendency to data reflecting relevant situations, in order to draw conclusions and make decisions*
- *design and implement strategies for the collection of data, including question design, population sampling, first and second hand data and bias*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-F1 Collect, Organize & Describe Data

- choose the best way to display the data
- interpret displays/presentations of data to draw conclusions about real world issues
-

5-F2 Bar & Double Bar Graphs: construct and interpret

- interpret displays/presentations of data to draw conclusions about real world issues
- construct and interpret simultaneous displays - 2 sets of data from same population (perhaps taken at different times)

5-F3 Coordinate Graphs: construct and interpret

- use coordinate graphs for purposes of location
- construct a number of coordinate graphs using appropriate labels and scales

5-F4 Mean: effect of change in data

- understand mean as balance through concrete materials (paper squares)
- understand that the mean of a set of data increases if any piece of data increases
- understand that the mean of a set of data decreases in any piece of data decreases
- understand that the mean increases if a piece of data below the existing mean is removed
- understand that the mean decreases if a piece of data above the existing mean is removed

5-F6 Interpolate Data: from a graph

- extract information from a graph or table which was not specifically provided (e.g. information between given values)

Class 5 – Strand G – Probability

KSO - Probability: *by the end of **Class 6** students should:*

- *explore, interpret and make predictions for everyday events by estimating and conducting experiments*
- *understand the difference between theoretical and experimental probability and when each is relevant*
- *begin to conduct simulations to understand real life probability situations*
- *understand the relationship between the numerical representations of probability and the events they represent*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-G1 Describe Probability

- understand that experimental probabilities are calculated by performing experiments
- understand that theoretical probabilities describe what will happen if the experiment is completed many times
- use fractions and decimals to describe theoretical probability & experimental results

5-G2 Experiments

- conduct simple experiments with coins and dice to determine probability
- predict & record results as fractions and decimals:
- understand the results of theoretical probability as the number of favourable outcomes/number of possible outcomes
- understand the results of experimental probability as the number of times favourable outcomes occur/number of trials in the experiment
- use common language to describe probability (e.g. for 15/20: "15 out of 20 trials were red")
- record results in tables & charts

Class 6 – A Strand – Numbers

KSO - Numeration: *by the end of **Class 6** students should:*

- *have strong number sense with respect to whole numbers and decimals, and be able to draw on a wide variety of relationships & strategies within number to solve problems in new situations*
- *have a strong sense of the base ten system to millions and thousandths, and use place value patterns to understand new ideas and apply reasoning to computational problems and mental mathematics within mathematics itself and in real world situations*
- *efficiently select & apply appropriate estimation strategies, to answer real life questions and check for reasonableness of answer in calculation*
- *understand fractions & decimals to thousandths, and the relationship between them, and to move freely from one form of representation to another, as might be appropriate in a given situation, and to provide a strong foundation for higher level fractional ideas and computation*
- *understand meanings and appropriate application of integers, ratios and percent in real world situations*
- *apply number theory concepts in relevant situations as a way to solve problems with respect to whole numbers, fractions, decimals*

Toward this, students in **Class 6** will be expected to master the following **SO** (Specific Outcomes):

6-A1 Common Factors: whole numbers

- find factors by dividing by smaller numbers and looking for a remainder of 0
- use "common" in the sense of "joint", rather than "ordinary"
- understand that 1 is always a common factor of any two numbers

6-A2 Prime Numbers: distinguish from composites

- cut rectangles from grids to investigate numbers which can be arranged in squares in only one way
- understand a prime number to be that for which there is only one possible rectangle (has only two factors: 1 and itself)
- understand that 1 is not a prime (doesn't have 2 factors)
- understand that ideas of prime apply only to whole numbers

6-A3 Large Numbers: reading & writing

- as words (e.g. 345 million, 321 thousand, 400)
- as rounded decimals (345.3 million)
- investigate simple exponential notation (e.g. 3422 as $3 \times 10^3 + 4 \times 10^2 + 22$
- include numbers with internal zeros

6-A4 Place Value: checking for understanding

- understand that our place value system follows patterns
  each position represents 10 times as much as the position to its left
  each position represent 1/10 as much as the position to its right
- understand that positions are grouped in 3s for purpose of reading them, both before and after the decimal point

6-A5 Rename Mixed Numbers and Improper Fractions

- move easily between improper and mixed number formats
- focus on meaning through concrete models

6-A6 Renaming: simple fractions to decimals

- use models to make connection between fractions and division
- investigate repeating decimals through concrete models only (no symbolism at this level)

6-A7 Comparing Fractions: develop procedures

- compare using benchmarks
- compare based on common denominator
- compare based on common numerator
- understand place value comparison of decimals

6-A8 Ratio: part to part, part to whole

- use concrete models to introduce concept of ratio
- understand that ratios and fractions are both comparisons
- compare a part to a whole (e.g. in a group of 6 boys and 4 girls, the ratio 6:10 describes the ratio of boys to the whole group)
- compare part to part (e.g. in a group of 6 boys and 4 girls, the ratio 6:4 describes the ratio of boys to girls)

6-A9 Equivalent Ratios: using models and symbols

- connect models and symbols to develop multiplicative relationship (3:5, 6:10, 12:20… etc.)
- simplify ratios to make interpretation of situations easier (36:9 = 4:1)

6-A10 Percent: developing benchmarks (number sense)

- understand that percent is viewed as a special ratio where the second term is 100 (not higher than 100 at this level)
- understand percent as equivalent ratios
- use 100 grids to represent percentages visually
- recognize the relationship between the percent and decimal names of ratios (37% = 0.37 = 37 hundredths)
- find percent equivalents for common ratios like ¼, ½ and ¾ (benchmarks)
- recognize everyday situations in which percent is used

6-A11 Integers: negative and positive

- develop meaning with models and symbols
- explore negative integers in context (e.g. temperature, money, sea level heights)
- understand that each negative integer is the mirror image of a positive integer with respect to the 0 mark (+3 and –3 are closer to each other than +8 and –8)
- understand that 0 is neither positive or negative
- generalize that negative integers are all less than any positive integer

Class 6 – B Strand – Operations

KSO - Operations: *by the end of **Class 6** students should:*

- *model & solve computational problems involving whole numbers and decimals by selecting appropriate operations and procedures for computation, estimation and mental math*
- *choose appropriate method of computation in given situations (including pencil/paper, mental math, estimation, technology)*
- *model & solve problems involving the addition and subtraction of simple fractions and be able to justify answers through reasoning*
- *informally explore simple algebraic situations*
- *demonstrate flexibility in procedures chosen to solve computational problems relevant to the real world and the study of mathematics*

Toward this, students in **Class 6** will be expected to master the following **SO** (Specific Outcomes):

6-B1 Addition & Subtraction: decimals and wholes choosing most appropriate method (pencil, mental, calculator, estimation)

- recognize that different situations need different computation approaches
- choose among written, mental calculations, calculator, estimation
- continue to estimate first in all computations
- apply strategies: front-end, compensation (e.g. 14.95 + 1.99 + 10.98 = 15 + 2 + 11 – 8 = 20)

6-B2 Decimals × Whole Numbers: concretely, symbolically

- compute products of whole numbers using an algorithm
- know when to use a pencil/paper algorithm, a mental procedure or the calculator
- continue to estimate as a first step

6-B3 Decimals × Decimals: concretely, symbolically

- use patterns to understand the placement of the decimal in the product of two decimal amounts ($34 \times 2 = 68$, $3.4 \times 2 = 6.8$, $3.4 \times 0.2 = 3.4 \times 2$ tenths = 68 tenths or 0.68)
- interpret the symbolism in meaningful ways rather than simply giving a rule
- continue to estimate as a first step

6-B4 Whole Numbers & Decimals: ÷ single digit

- connect to whole number division
- link concrete models to algorithms
- continue to estimate as a first step

6-B5 ×, ÷ Estimation Strategies: whole numbers and decimals

- apply estimation strategies: rounding, front-end

6-B6 Decimals ÷ Decimals: estimating & developing algorithms through reasoning

- develop algorithm through reasoning
- relate the division of a decimal by a decimal to the corresponding division of a decimal by a whole number (e.g. 32.5 ÷ 0.5 = 325 tenths ÷ 5 tenths = 65)

6-B7 Percent: number sense- estimating for familiar fractions

- estimate for familiar fractions (benchmarks) visually (shaded portions of grids or rectangles)

6-B8 Addition & Subtraction: simple fractions - various denominators

- develop conceptual understanding by exploring models (pattern blocks, fraction circles)
- avoid algorithm (although some may see the pattern that leads to algorithm)
- solve problems in context

6-B9 Multiply & Divide Mentally: whole numbers to 4 digits ×, ÷ 1 digit

- apply strategies:
  - -by 10, 100, 1000 (20 × 600: "tens × hundreds = thousands. How many thousands? 2 × 6 = 12 thousands")
  - -distributive strategy (25 × 30 = 25 × 3 × 10 = 750)
  - -double/half strategy: (50 × 16 = 100 × 8)
  - -front end strategy: (3 × 325 = 900 + 60 + 15 = 975) – focus on digit value
- mental math must be a consistent part of instruction and not an "add-on"
- choose strategies which make sense personally and are efficient
- justify choice of strategies

6-B10 Divide Mentally: whole numbers by 0.1, 0.01, 0.001

- understand that dividing by 0.1 makes the quotient larger, not smaller (division usually associated with smaller quotient) as it is asking "how many tenths are in the number"
- recognize the pattern of changes produced by dividing by 0.1, 0.01, 0.001 is the same as that produced by multiplying by 10, 100, 1000
- describe these patterns in terms of place value changes, not just in terms of a rule involving moving the decimal point "*n*" places

6-B11 Function: input/output

- determine function given input and output (e.g. input 14 output 28: function is × 2)
- understand that the result depends on the original number chosen

Class 6 – C Strand – Patterns

KSO - Pattern: *by the end of **Class 6** students should:*

- *describe, extend and create patterns to solve problems in real world situations and mathematical contexts (number, geometry, measurement)*
- *use patterns to generalize for mathematical situations to aid in solving problems and understanding relationships*
- *explore and generalize how a change in one quantity in a relationship affects another, in order to efficiently solve similar(but new) problems*
- *represent mathematical patterns and relationships in a variety of ways (charts, tables, graphs, numerically)*
- *use patterns to assist in mental math strategies*
- *informally (through reasoning) solve linear equations via open sentences as a foundation for later algebraic ideas*

Toward this, students in **Class 6** will be expected to master the following **SO** (Specific Outcomes):

6-C1 Open Frames: substituting for letters

- use letters to represent variable quantities
- draw parallel between open sentences and "*n*"
- understand irrelevance of letter choice
- understand that the multiplication sign is not written when a letter is used

6-C2 Equivalent Ratios: change in one term affects the other term

- explore symbolically how a change in one term affects the other
- understand that, when a particular quantity is added to or subtracted from each term, the resulting ratios are not equivalent; the relationship needs to be multiplicative (e.g. 2:4 is not equivalent to 3:5)

6-C3 Equivalent Ratios: represent in tables and graphs

- represent in tables and graphs to more clearly see relationship between the ratio elements (line graph where x = first term, $y = 2^{nd}$ term)

6-C4 Area Patterns: explore

- explore concretely how changes in base/height affect area of rectangles, parallelograms, triangles
- link concrete to symbols which represent the changes e.g. parallelograms: $A = bh$ so if b and h are both doubled, area is quadrupled; if b is doubled but h is halved the area remains the same

6-C5 Volume Patterns: explore

- explore how changes in one dimension of the formula affects the volume of a rectangular prism

6-C6 Square & Triangular Numbers: represent pictorially and symbolically

- represent pictorially and symbolically to show both geometric and numerical significance
- understand that square numbers may be represented in square arrays and are the products of numbers multiplied by themselves
- understand that each triangular number is half the number in an array with dimensions that are one unit apart (adding of consecutive numbers)
- display triangular numbers visually (as though a rectangle cut on a diagonal)

6-C7 Linear Equations: using open frames

- solve simple linear open frame equations in context (23 students, 8 are absent, others are sitting in groups of 3-how many groups? $3 \times \square + 8 = 23$)
- replace open frames with letters

Class 6 – D Strand – Measurement

KSO - Measurement: *by the end of **Class 6** students should:*

- *understand relationships among common SI units and choose appropriate units to solve measurement problems in given situations*
- *move freely among common SI units to effectively communicate measurement ideas appropriate to a given measurement situation*
- *estimate effectively using a variety of strategies to solve measurement problems and understand when estimation is close enough*
- *use relationships and reasoning to develop and apply procedures for measuring in real situations and mathematical contexts*

Toward this, students in **Class 6** will be expected to master the following **SO** (Specific Outcomes):

6-D1 Volume & Capacity: relationships

- explore relationship between the cubic units of volume and capacity ($1cm^3 = 1$ mL, $1\ dm^3 = 1$ L, $1\ m^3 = 1$ kL
- understand that capacity and volume are both measures of the size of a 3-D region of space
- understand that capacity is for measuring fluids and/or the containers which hold fluids
- understand that volume is used to measure how much space is occupied by a solid

6-D2 Mass: tonnes

- tonne: equivalent to 1000 kg

6-D3 Area: calculate to solve problems

- calculate area in square cm, m, km
- choose appropriate units for actual situations

6-D4 Parallelograms: relate bases, heights, areas

- understand that the area of a parallelogram is the same as the area of a related rectangle (with same base and height)
- determine the base or height, given the area and the other dimension
- understand that a variety of parallelograms can have the same area

6-D5 Area (of a Triangle): relate to area of a parallelogram

- explore with models
- understand that any triangle is one half of a parallelogram
- understand that the area of a triangle is one-half of the related parallelogram
- understand that as long as the base and height are the same, the areas of visually-different triangles are the same

6-D6 SI Units: Relationships

- compare objects
- investigate the relationship between linear SI units and the relationship between corresponding SI area and volume units (not the same)

6-D7 Time: solve problems

- solve problems involving time
- read and record time using the 24-hour clock
- explore idea of time zones in context

6-D8 Angles: estimate, measure and draw

- introduce protractor: degrees as very small wedges
- estimate, measure & draw angles between 0 and 90 degrees
- understand the importance of positioning the 0 degree line on the protractor so that it coincides with the first arm of the angle to produce an accurate drawing

Class 6 – E Strand –Geometry

KSO - Geometry: *by the end of **Class 6** students should:*

- *identify, draw, compare and build physical models of 2-D and 3-D shapes to focus on their attributes and understand how they affect everyday life*
- *predict and verify results of transforming, combining and subdividing shapes to understand other shapes and explain other geometrical ideas*
- *use geometric relationships and spatial reasoning to solve problems and understand everyday events and objects, as well as higher geometrical ideas*
- *appreciate the importance of geometry in understanding mathematical ideas and the world around*

Toward this, students in **Class 6** will be expected to master the following **SO** (Specific Outcomes):

6-E1 Orthographic Drawings: make and interpret shapes

- make and interpret shapes built from cubes
- understand that orthographic drawings are a series of 2-D views of a 3-D shape drawn by looking at from sides, top , back

6-E2 Cross Sections: 3-D shapes (cones, cylinders, prisms, pyramids)

- understand that a cross-section is the 2-D face produced when a plane cut is made through a 3-D shape
- examine properties of cuts concretely (e.g. right circular cone: if a cut is made in any plane parallel to its base, the face produced is a circle; through its vertex, exposed face is a triangle, etc.)

6-E3 Sum of Angles: make generalizations

- use the results of a variety of concrete investigations to generalize about the sum of the angles in triangles and quadrilaterals

6-E4 Quadrilaterals: sort by attributes

- sort concretely by attributes, including angles

6-E5 Combining Transformations: predict and confirm results

- predict and confirm results
- understand that two congruent angles on the same plane are images of one another under a translation, reflection, rotation or any combination of these three transformations
- understand that if two similar shapes are on the same plane, they are dilatation images or dilatations in combination with translations, reflections, or rotations

6-E6 Rotations: 1/4, 1/2, 3/4 turns

- use a variety of centres

6-E7 Bisectors: of angle, segments

- recognize and describe
- include perpendicular bisectors

6-E8 Diagonal Properties: generalize

- generalize about diagonals for a rhombus: the diagonals are perpendicular bisectors of each other, form four congruent right triangles, bisect the angles of the rhombus, and are its two lines of reflective symmetry
- generalize about diagonals for a parallelogram: the diagonals bisect each other and form two pairs of congruent triangles
- generalize about diagonals for a kite: the diagonals are perpendicular and form two pairs of congruent right triangles; one of the diagonals is bisected, and the other diagonal is a line of reflective symmetry and bisects two opposite angles of the kite
- understand that for a trapezoid, there are no special properties of its diagonals

6-E9 Rotational Symmetry: properties

- generalize for quadrilaterals and regular polygons

- understand that, for a shape to have rotational symmetry, it must be turned about a point so that it exactly coincides with its original position at least once in less than a complete rotation
- understand that the number of times it appears in the identical position during one compete rotation is the order of rotational symmetry
- understand that if a shape has to be rotated 360 degrees before it fits its traced shape then it does not have rotational symmetry

6-E10 Planes of Symmetry: 3-D shapes

- generalize for 3-D shapes
- understand that some 3-D shapes have planes of reflective symmetry – planes that bisect 3-D shapes such that all points in one-half are mirror images of the corresponding points in the other half
- understand, through investigation, that a cube has 9 different planes of symmetry
- investigate cones, cylinders, prisms, and pyramids for planes of symmetry (models should be right)

6-E11 Dilations: of 2-D figures

- understand that a dilatation image is one where lines through all corresponding vertices of two shapes on a plane converge at a single point
- understand that the point of convergence is the centre of dilatation
- connect the centre of dilatation to the concept of vanishing point in art

Class 6 – Strand F – Data Management

KSO – Data Management: *by the end of **Class 6** students should:*

- *collect, record, organize and describe data in multiple ways to draw conclusions about everyday issues*
- *construct a variety of data displays and choose most appropriate*
- *predict, read, interpret and modify predictions for a variety of data displays, including interpolation and extrapolation (draw conclusions about things not specifically represented by the data)*
- *develop and apply measures of central tendency to data reflecting relevant situations, in order to draw conclusions and make decisions*
- *design and implement strategies for the collection of data, including question design, population sampling, first and second hand data and bias*

Toward this, students in **Class 6** will be expected to master the following **SO** (Specific Outcomes):

6-F1 Collect, Organize & Describe Data: real world issues

- draw conclusions

6-F2 Evaluate Data: choose appropriate samples

- develop the concept of sampling: e.g. when each person cannot be asked
- interpret second-hand data (examining statistics)
- interpret first-hand data (direct observation or interview)
- determine how such data is collected both first and second-hand (consider the source to determine bias)

6-F3 Line Graphs: construct and interpret

- understand that the purpose of a line graph is to focus on trends implicit in the data (e.g. for temperature)

6-F4 Bar & Double Bar Graphs: construct and interpret

- construct and interpret

6-F5 Stem & Leaf Plots: grouping data

- construct to display grouped numerical data (e.g. heights of students in a class)

6-F6 Coordinates: plotting

- plot data in all four quadrants
- understand that a negative number for the second coordinate indicates that the point is below the horizontal axis
- understand that the point at which the axes intersect has coordinates (0, 0) and is known as the origin

6-F7 Mean, Median, Mode: concepts

- understand conceptually
  -mean – the average calculated by taking a total amount and sharing it equally
  -median is another type of average, it is the middle number in a set of data
- understand that the mean and median may be the same or may be different
- understand that the mode is a type of average that appears most often

6-F8 Inference: interpret data

- interpolate (describe data between existing pieces of information)
- extrapolate (extend beyond the existing data)
- use different graphs to be able to "tell a story"

Class 6 – Strand G – Probability

KSO - Probability: *by the end of **Class 6** students should:*

- *explore, interpret and make predictions for everyday events by estimating and conducting experiments*
- *understand the difference between theoretical and experimental probability and when each is relevant*
- *begin to conduct simulations to understand real life probability situations*
- *understand the relationship between the numerical representations of probability and the events they represent*

Toward this, students in **Class 6** will be expected to master the following **SO** (Specific Outcomes):

6-G1 Theoretical Probability: determine

- understand that theoretical probability is number of favourable outcomes/number of possible outcomes
- use percentage and decimals to describe probabilities
- identify events that might be associated with a particular theoretical probability

6-G2 Simulations: design and implement

- design & implement to determine probability
- introduce simulations, experiments which directly model situations
- understand that probabilities may be stated as fractions, decimals and percent

6-G3 Reliability: evaluate

- evaluate sampling results
- understand that data from larger samples generally produces more reliable probabilities

Class 7 – A Strand – Numbers

KSO - Numeration: *by the end of **Class 8** students should:*

- *understand meanings and appropriate application (number sense) with respect to integers, rational and common irrational numbers and be able to draw on a wide variety of relationships & strategies within number to solve problems in relevant situations*
- *move flexibly from one form of representation of numbers to another, as might be appropriate in a given situation to understand or solve a particular problem*
- *interpret numbers in many ways, through reading, writing, illustrating, modeling and talking about numbers*
- *apply reasoning to order integers, rational and common irrational number to aid in estimation and developing strong number sense*

Toward this, students in **Class 7** will be expected to master the following **SO** (Specific Outcomes):

7-A1 GCF: using common factors and greatest common factors to solve problems

- understand that common factors and GCF are helpful renaming fractions in lowest terms
- review terms factor, common factor, and prime number
- use prime factorization method and the listing of factors in developing GCF
- expose students to a variety of methods to find common factors and GCF

7-A2 LCM: using common multiples and least common multiples to solve problems

- use various methods to find LCM s: prime factorization and listing of multiples
- variations may include using a calculator
- understand that LCM helps when adding and subtracting fractions

7-A3 Divisibility: develop and apply rules for 3, 4, 6, 9

- develop personal divisibility rules through exploration and models (focus on why they work)
- understand the importance of divisibility rules for mental computations

7-A4 Large Numbers: model

- develop models using power, base, and exponents to represent repeated multiplication
- understand exponents as a means of expressing factors in a compact form
- understand terms “squared” and “cubed” to describe powers of two and powers of three
- connect “squared” with a 2-D object and “cubed” with a 3-D object
- use Base 10 Blocks to relate the understanding to powers of 10
- use the models to extend to numbers less than one

7-A5 Large Numbers: rename

- investigate exponential, expanded and standard forms
- use expanded forms of numbers to demonstrate understanding of place value as well as exponents
- use Base 10 Blocks to relate understanding to powers of 10

7-A6 Large Numbers: representation

- switch from standard to scientific form and vice versa

- focus on when large numbers, typically written in scientific notation occur
- understand why scientific notation is important
- understand reasonableness of using a calculator to multiply large numbers
- explore positive exponents only (informally)

7-A7 Rename: Mixed Numbers and Fractions

- rename as decimals
- concept developed using concrete and pictorial models
- introduce the terminology "repeating" and "period" as well as bar notation to show repeated decimal
- explore patterns in various fractions especially 7ths

7-A8 Rename: Repeating Decimals to Fractions

- explore 1- and 2-digit repeating decimals
- use pattern to rename and make predictions

7-A9 Compare & Order: decimals, proper / improper fractions and mixed numbers

- order fractions on a number line
- compare fractions relative to benchmarks
- compare fractions based on common denominator
- compare fractions based on common numerator
- change to decimal values and compare using place value

7-A10 Equivalent Ratios & Rates: solve problems

- solve problems involving equivalent ratios and rates

7-A11 Percent: as a special ratio

- focus on intuitive understanding of percent as a special ratio
- use benchmarks
- relate visually to fractions
- understand that parts should always add up to 100%
- recognize the relative percentage of a figure and match with visual representation

7-A12 Integers: compare and order

- represent integers in a variety of ways (6- A11)

- ♦ compare & order integers with coloured counters, number lines and real life situations
- ♦ understand the zero principle: balance of positive and negative values

Class 7 – B Strand – Operations

KSO - Operations: *by the end of **Class 8** students should:*

- ♦ *establish the relationship between algebraic and arithmetic operations and use this relationship in solving computational problems with algebraic expressions*
- ♦ *model, explain & use rational numbers and integers to solve problems*
- ♦ *model & solve computational problems involving fractions, ratios, percent, proportion, integers, exponents by selecting appropriate operations and procedures for computation, estimation and mental math*
- ♦ *efficiently select & apply appropriate estimation strategies to problems involving rational numbers and integers, to answer real life questions, make predictions and check for reasonableness of answer in calculation*

Toward this, students in **Class 7** will be expected to master the following **SO** (Specific Outcomes):

7-B1 Add, Subtract, Mult, Div: whole numbers and decimals

- ♦ choose appropriate method (pencil, mental, calculator, estimation) for a given situation
- ♦ (addition and multiplication facts essential and should be reviewed as needed)

7-B2 Properties of Operations : decimals and integers

- ♦ review commutative, associative and distributive properties
- ♦ determine when these three properties are used
- ♦ explore the concept of "closure" using situations such as: addition within the set of even numbers; addition within the set of odd numbers; and division within the set of integers
- ♦ apply distributive, associative and commutative principles in mental computation

7-B3 Order of Operations : whole numbers and decimals

- understand why order is important (brackets, exponents, division/multiplication and addition/subtraction)

7-B4 Percent: number sense

- estimate & calculate for familiar fractions
- explore concretely & symbolically

7-B5 Percent: develop algorithms

- link to models
- estimate & calculate symbolically
- identify percent from a picture
- use a variety of strategies in calculating percent of a number (including invented strategies):
  - change percent to a decimal and multiply
  - computing 1% and then multiplying
  - changing to a fraction and dividing
  - using the percent key on a calculator

7-B6 Add & Subtract: **simple fractions of various denominators**

- develop algorithm (pictorially and symbolically)
- estimate the sum or difference of fractions when appropriate

7-B7 Multiply and Divide Mentally: fraction by a whole number

- develop & apply strategies necessary to mental calculation of percents
- use concrete models and pictorial representations (visualization of the fraction being multiplied is critical)

7-B8 Add & Subtract Integers & Decimals Mentally: develop & use strategies

- develop and use strategies
  - -front-end
  - -compatible numbers
  - -compatible factors
  - -working by parts
  - -double and halve

7-B9 Multiply & Divide Integers & Decimals Mentally

- develop and use strategies
  - -front-end

-compatible numbers
-compatible factors
-working by parts
-double and halve

7-B10 Add & Subtract Integers: to solve problems

- connect visual models to symbols
- use counters, number lines and real-life contexts
- understand that, when adding two integers, it is necessary to first model each integer, then match positive and negative values to make zeros
- understand that the concept of net worth can also be used to solve addition and subtraction situations

7-B11 Multiply & Div Integers: to solve problems

- connect visual models to symbols
- understand multiplication as repeated addition
- understand that net worth can also be used
- model on a number line
- understand patterning as one of the best methods for investigating division of integers
- compare multiplication and division situations
- understand that multiplication and division are inverse operations
- understand that using a missing factor can also be used

7-B12 Order of Operations for Integers: solve problems

- focus on models and pictorial representations
- numbers should be kept small so computation can be handled mentally
- emphasize proper use of brackets in writing expressions

7-B13 Simple Variable Expressions: relate to numerical expressions

- recognize that the four operations apply in the same way as they do for numerical expressions
- understand that quantities that change are called variables
- develop a sense of why we need variables
- use simple patterning
- evaluate simple variable expressions by substituting a variable in the expression

- understand that what was true in evaluating numerical expressions applies to variable expressions, once the variable has been given a numerical value
- use primarily decimals and integers as replacement values or coefficients

7-B14 Like & Unlike Terms: develop meaning

- develop meaning visually (not symbolically)
- distinguish between like and unlike terms
- add and subtract like terms by recognizing the parallel with numerical situations, using concrete and pictorial models

Class 7 – C Strand – Patterns

KSO - Pattern: *by the end of **Class 8** students should:*

- *represent patterns as algebraic expressions, equations, inequalities and exponents*
- *interpret patterns through algebraic description and apply generalizations to make predictions of unknown values and solve real world and mathematical problems*
- *explore and generalize how a change in one quantity in a function affects another, in order to efficiently solve similar problems*
- *solve linear equations and inequalities through algebraic methods*
- *understand the meaning of non-linear equations*

Toward this, students in **Class 7** will be expected to master the following **SO** (Specific Outcomes):

7-C1 Summarize Patterns: make predictions

- use constants, variables, algebraic expressions and equations to make predictions
- variables can represent a changing quantity ($x = 4y$) or a single value ($x + 3 = 9$)
- use tables to organize the information that a pattern provides- pattern easier to see
- patterns should include counting, exponential growth, geometric figures
- understand that a number sentence is called an equation

- understand that a number sentence with a variable is an algebraic equation
- understand that an equation contains a verb ($p = 3$); an expression has no verb
  ($p + 3$)

7-C2 Single Variable Linear Equations: represent solutions

- show solution concretely, pictorially (one step, two step)
- use envelopes and counters to show a solution to a simple equation ($e + 3 = 7$; how many are in the envelope)
- add and subtract the same values from both sides of an equation maintains balance
- neutralize the positives or negatives on one side of the balance
- the cover-up method can be used as an extension ($4m + 5 = 25$; what + 5 gives 25? (20) Now what $\times$ 4 = 20?)

7-C3 Single Variable Linear Equations: one and two step

- understand what a systematic trial means
- at first, students may use the guess and check strategy
- from the results of trials (too large, too small), students can become more systematic (narrow it down) in the guesses they make…called a systematic trial e.g. $t = 3s - 2$; if $t = 82$ a trial might be 30 ($3 \times 30 - 2$… too large); next trial might be 20 ($3 \times 20 - 2$… too small); next trial will be somewhere between 20 and 30

7-C4 Linear Equations: graph using table of values

- use the *x*-axis and *y*-axis for the horizontal and vertical axes
- understand the axis as two number lines that are perpendicular to each other, intersecting at the origin
- use a table of values for graphing
- interpolate (find a point between two known points)
- extrapolate (find a point that lies beyond the existing data) (done mainly by sighting)
- determine if an ordered pair satisfies a given equation;
  - by plotting the points to see if they are in keeping with the rest of the points in the pattern
  - by substituting them into the equation to see if they make the equation true or false
- equate an ordered pair that makes an equation true with the fact that it is a solution to the equation

7-C5 Graphs: **linear & non-linear**

- understand how changing one quantity affects the other: data only produces a line when equal increases in *x* result in equal increases in *y*
- develop sense of how the value of an expression changes with the value of the variable

Class 7 – Strand D – Measurement

KSO - Measurement: *by the end of **Class 8** students should:*

- *use concepts of rate to solve real-life & mathematical problems*
- *use direct and indirect measurement to make comparisons and interpret scales*
- *understand how a change in one measurement affects another in problems of rate*
- *understand relationships and move freely among all SI units, and choose appropriate units to solve measurement problems in given situations*
- *estimate effectively using a variety of strategies to solve measurement problems and understand when estimation is appropriate*
- *use relationships and reasoning to develop and apply procedures for measuring in a wide variety of measurement problems*

Toward this, students in **Class 7** will be expected to master the following **SO** (Specific Outcomes):

7-D1 Volume: **rectangular prisms**

- relate volume to dimensions
- estimate before calculating
- understand that each of the three dimensions of a prism affects the volume and surface area

7-D2 Area: **composite figures**

- estimate & calculate (grids)
- recognize that certain shapes are made up of other standard shapes
- understand that figures can be broken down into components
- understand that area can be found separately, then combined

7-D3 Circles: solve problems with diameter, radii, circumference

- relate diameter, radii, circumference to solve problems
- investigate π through measurement and charting of value of C/d for a number of circular objects
- develop the formulas for $C = \pi d$ and $C = \pi r$
- 3 can be used as an approximation for π, but students must understand that 3.14 is also an approximation (all measure is approximate)

7-D4 SI Units: identify, use & convert

- identify, use, and convert to measure, estimate, and solve problems
- understand the approximate nature of measurement
- introduce deca- and hecto
- examine milli, centi, deci, deca, hecto, and kilo as the study of length, mass and capacity
- use benchmarks
- apply principles of conversion within common units (relate the size of a number to the size of the unit rather than concentrating on moving the decimal)
- establish link between cubic units and capacity

7-D5 Rate: compare 2 quantities

- construct and analyse graphs to show change
- understand rate as the comparison of two quantities
- write as a ratio: e.g. m/s, km/h, beats per minute
- compare linear and non-linear graphs
- investigate actual contexts
- solve indirect problems

7-D6 Angles: estimate and measure using a protractor

- use appropriate scale on a double scale protractor
- estimate angle first to know which measure is correct

Class 7 – Strand E – Geometry

KSO - Geometry: *by the end of **Class 8** students should:*

- *build & analyse physical and pictoral models of 2-D and 3-D shapes to understand relationships & properties, and enhance spatial sense in mathematical and real world situations*
- *analyse the results of transforming shapes to understand & apply transformation properties to mathematical & real world situations and to explain geometrical ideas*
- *compare, classify and apply geometric properties to figures*
- *appreciate the importance of geometry in understanding mathematical ideas, in art, and in the world around them*

Toward this, students in **Class 7** will be expected to master the following **SO** (Specific Outcomes):

7-E1 3-D Shapes: sketch, build

- sketch & build, given information
- represent two- and three-dimensional figures in various positions by drawing and constructing (cubes)

7-E2 Angles: sum

- understand through investigating models that ,for triangles, the sum of angles =180◦
- tear off paper vertices and line them up to make 180 degrees

7-E3 Triangles: classify (combinations)

- decide and justify which combinations of triangle classifications are possible, through construction
- continue to classify scalene, isosceles, equilateral, acute-angled, obtuse-angled, right-angled, and equiangular
- determine if certain combinations of classifications can exist at the same time
- explore relationships among triangles

7-E4 Relationships: triangles

- explore the relationship: angle measure to side length
- draw conclusions about angle measures within an isosceles triangle
- establish relationships between the longest side and the largest angle

- establish relationships between the shortest side and the smallest angle
- make associations between side length and side angle

7-E5 Transformations – properties

- use formal language: translations, reflections and rotations for slides, flips and turns
- emphasis on what changes and what stays the same as a result of a transformation
- use tessellations as a good context for transformations
- formally discuss similarity and congruency
- describe and create art-related designs

7-E6 Bisectors: of **angles – constructs**

- construct (include perpendicular bisectors)
- use materials to introduce angle and line segment bisection
- review concept of congruency
- explore basic use of compass and straightedge

Class 7 – F Strand – Data Management

KSO – Data Management: *by the end of **Class 8** students should:*

- *understand issues in data collection, including bias, repeated sample variability, randomness, collect, record, organize and describe data in multiple ways to draw conclusions about everyday issues*
- *understand, choose & apply appropriate data collection methods (real or simulated data) to answer questions on meaningful issues*
- *predict, read, and draw inferences for a variety of data displays, including interpolation and extrapolation (draw conclusions about things not specifically represented by the data)*
- *construct & analyse a variety of data displays, including circle graphs, box and whisker plots and scatter plots, and choose the most appropriate display for a given situation*
- *analyse measures of central tendency in terms of the effect on mean, median and mode when changes in data occur, in order to draw conclusions and make decisions*
- *examine interpretations of data displays for validity, considering data collection issues to form opinions and make predictions*

Toward this, students in **Class 7** will be expected to master the following **SO** (Specific Outcomes):

7-F1 Data Collection Methods: select & defend

- select, defend and use appropriate data collection methods in real world applications
- interview, observation, questionnaire, simulation (advantages/disadvantages of each)
- formulate questions (consider bias and accuracy of data)
- consider sensitivities (privacy, cost, political agenda)

7-F2 Formulate Questions: for real world application

- consider:
  - accuracy of question
  - simplicity of question
  - information desired
  - how data will be displayed
- explore issue of bias
- test questions before implementing

7-F3 Bias: determine in questions and samples

- understand first-, second-hand data
- understand biased, unbiased sampling
- consider data sources

7-F4 Circle Graphs: meaning

- represent proportions (as percentage of total circle)
- model pictorially: circle divided into tenths and hundredths
- identify appropriate situations for circle graphs

7-F5 Histograms: construct, interpret

- construct and label
- construct to show the frequency distribution of grouped data
- introduce using grouped whole number data (e.g. how many in each age group)
- explore intervals of equal size (e.g. 0-9, 10-19, 20-29, etc.)
- identify appropriate situations for histograms

7-F6 Scatter Plots: interpret

- understand that scatter plots are used to show relationship between 2 quantities (e.g. age/mass, time of day/temperature)

- understand that each ordered pair is plotted as a point
- generalize relationships

7-F7 Central Tendency: examine the effect of changing data

- understand mean by linking to real life examples of average
- understand that median is the middle value when the values are arranged in order of smallest to largest
- understand that mode is the score that occurs most often
- understand that it is uncommon to find sets of data that are bimodal
- understand that the measure of central tendency best suited to a particular situation is dependent on the situation (e.g. median not affected by outliers as much as mean)

7-F8 Variability: make inferences and predictions

- use range, outliers, gaps, clusters to make inferences and predictions
- investigate the impact of extreme values
- understand that range is the difference between two extreme values
- find gaps and clusters by observing and analyzing the data
- relate range to mean, mode and median
- discuss the effect on mean, median, mode if outliers are removed

Class 7 – Strand G – Probability

KSO - Probability: *by the end of **Class 8** students should:*

- *explore, interpret and make predictions for everyday events by estimating and conducting experiments*
- *determine theoretical and experimental probability, understand the difference between the two and determine when each is relevant to a particular situation*
- *express probability as ratios, fractions, decimals, percents and choose appropriate expressions given a particular situation*
- *conduct simulations & experiments to determine the probability of single & complementary events in real life situations*
- *use real life data to establish broad probability patterns for the purpose of planning & making decisions (e.g. patterns in population growth, traffic)*

Toward this, students in **Class 7** will be expected to master the following **SO** (Specific Outcomes):

7-G1 Describe Probability: identify situations near 0, near 1, near 1/2, 1/4, ¾

- understand that more trials usually represents greater accuracy
- understand that impossible events have a probability of 0
- understand that events that are certain to occur have a probability of 1
- understand that all uncertain events have a probability between 0 and 1

7-G2 Simulations & Experiments: to solve probability problems

- understand that a simulation is an experiment which model the actual situation (e.g. probability of 3 children in a family being all girls: toss 3 coins – heads for girls, tails for boys)
- understand that simulations must reflect the situation being simulated
- select model to generate the necessary outcomes
- understand that a large number of trials should be conducted and recorded
- summarize information to draw conclusions
- understand why simulations are appropriate for specific situations (where direct experiments would be too costly, time-consuming or dangerous)

7-G3 Compare Results: theoretical, experimental

- understand theoretical probability as: *p*(event) = # of favorable outcomes/ total # of outcomes
- understand that it can only be used when dealing with equally likely events (e.g. probability of rolling 1, 2, 3, 4, 5, 6, on a die)
- understand experimental probability as: (actual trials) where *p* = # of times favoured outcome occurs/# of total trials

7-G4 Independent Events: identify all possible outcomes

- identify all possible outcomes of two independent events
- construct tree diagrams to identify possible outcomes
- use the area model where one event is represented by one dimension, the other event by the other dimension of the rectangle

Class 8 – A Strand – Numbers

KSO - Numeration: *by the end of **Class 8** students should:*

- *understand meanings and appropriate application (number sense) with respect to integers, rational and common irrational numbers and be able to draw on a wide variety of relationships & strategies within number to solve problems in relevant situations*
- *move flexibly from one form of representation of numbers to another, as might be appropriate in a given situation to understand or solve a particular problem*
- *interpret numbers in many ways, through reading, writing, illustrating, modeling and talking about numbers*
- *apply reasoning to order integers, rational and common irrational number to aid in estimation and developing strong number sense*

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-A1 Square Root: modeling and representing

- model perfect squares and square roots using blocks or grid paper
- establish link between concrete and numerical representations (100 – 81 = 19)
- on grids, view the area as the perfect square, and either dimension of the square as the square root

8-A2 Negative Exponents: develop meaning concretely and symbolically

- encountered in the context of place value charts (tenths, hundredths, thousandths as 10^{-1}, 10^{-2}, 10^{-3})
- introduce using a pattern ($100 = 10^2$, $10 = 10^1$, $1 = 10^0$, $0.1 = 10^{-1}$, $0.01 = 10^{-2}$)
- work with base-ten models, but can include base 2 and base 3 (used in exponential growth)

8-A3 Perfect Squares: patterns between 1 and 144

- recognize each of the perfect squares from 1 through 144
- expose to perfect squares up to 400
- relate patterns to perfect squares
- understand that the differences in perfect squares increase in a constant way

- work with patterns related to perfect squares of any size
- graph increases in perfect squares
- understand that the sum of the square roots of two consecutive perfect squares is equal to the difference between those two perfect squares ($\sqrt{25} + \sqrt{36} = 6 + 5 = 11$ and $36 - 25 = 11$)

8-A4 Large Numbers: scientific notation to standard form and vice versa

- establish connection with multiplying by 0.1, 0.01, and 0.001
- make real-life connections (diameter of cell, diameter of electrons, mass of a bug)
- relate small numbers to large numbers so students can see the difference between the two in scientific notation form
- translate numbers from one form to another

8-A5 Square Root: exact square root and its decimal approximation

- emphasize the difference between exact square root and the decimal approximation
- recognize the difference between the decimal value of the square root of a non-perfect square and the decimal value of a rational number
- understand that square root of a non-perfect has a decimal portion which does not repeat
- construct personal definition of an irrational number
- model square root for non-perfect squares

8-A6 Square Root: find using an appropriate number

- identify where the square root will fall (between 1–144)
- approximate to the point where students can identify which whole number is closer to the square root (square root of 22 is between 4 and 5 and is closer to 5 than 4)
- use patterns to determine whether the square root of a number is another number (16 is 4 therefore 1600 is 40 and 2200 is between 40 and 50)
- apply prime factorization ($\sqrt{576}$ is $576 = 4 \times 144 = 2 \times 2 \times 144 = 2 \times 2 \times 12 \times 12 = 24 \times 24$, then $\sqrt{576} = 24$) in a variety of ways

A7 Percent: greater than 100

- use 100% as a baseline (if ? represents 100%, then ? represents 200%)

- relate percent greater than 100 to other subjects (social studies and inflation rates, population growth, etc.)

8-A8 Integers & Rational Numbers: comparing & ordering (fractional and decimal form)

- understand that placement of the negative does not affect the value ($\frac{-2}{3}$, $\frac{2}{-3}$, $-\frac{2}{3}$ are the equivalent)
- understand that a negative is always less than a positive
- understand that positive fractions with common denominators can be compared by examining numerators ($\frac{3}{8}$ is less than $\frac{5}{8}$ because 3 is less than 5)
- understand that positive fractions with common numerators can be compared by examining denominators ($\frac{3}{5}$ is greater than $\frac{3}{6}$ because 5 is less than 6)
- use reference points (1, $\frac{1}{2}$, –1, etc.)
- change numbers to a common form

Class 8 – B Strand – Operations

KSO - Operations: *by the end of **Class 8** students should:*

- *establish the relationship between algebraic and arithmetic operations and use this relationship in solving computational problems with algebraic expressions*
- *model, explain & use rational numbers and integers to solve problems*
- *model & solve computational problems involving fractions, ratios, percent, proportion, integers, exponents by selecting appropriate operations and procedures for computation, estimation and mental math*
- *efficiently select & apply appropriate estimation strategies to problems involving rational numbers and integers, to answer real life questions, make predictions and check for reasonableness of answer in calculation*

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-B1 Percent: solving & creating real problems in context (including estimation)

- estimate and calculate a percentage of a given number (*a*% of *b* = *c*, 25% of 1500)
- find the percentage one number is of another number (what percent of 20 is 15?)
- find the whole when a specified percentage is given (28% of what number is 42?)
- use mental strategies when an exact answer is required (28% of 1200 = 20% of 1200 + 8% of 1200)

8-B2 Percent: increase and decrease

- investigate through real-life situations
- investigate mark-ups and mark-downs of retail items (a dress cost 7 Nu to make and is being sold for 15Nu. What is the percent of mark-up?)
- develop formula (% increase = increase/original amount × 100%)
- develop formula (% decrease = decrease/original amount ×100%)

8-B3 Add & Subtract: fractions- develop algorithm (pictorially and symbolically)

- apply prior understanding of equivalent fractions, lowest terms, and LCM
- use a variety of manipulatives to develop operations with fractions concretely (fraction circles, pattern blocks, tangrams, money, number line, etc.)
- record equivalent fractions when moving from the concrete to symbolic
- represent both fractions using the same subdivision of the whole

8-B4 Multiply: fractions- develop algorithm (pictorially and symbolically)

- construct concrete and pictorial models to develop meaning
- understand that "of" means multiplication and can be shown by comparing results in questions such as $\frac{1}{4}$ of 8 and $\frac{1}{4} \times 8$

- multiply a whole number by a fraction less than one ($4 \times \frac{1}{3}$ uses repeated addition)
- multiply a fraction less than one by another fraction (especially when the numerator is 1 ($\frac{1}{4}$ of $\frac{2}{3}$))

8-B5 Divide: fractions- develop algorithm (pictorially and symbolically)

- derive a personal algorithm from carefully chosen examples
  e.g.:
  - a simple fraction divided by a whole number ($\frac{1}{2} \div 3$, divide $\frac{1}{2}$ into 3 equal parts, what does each part represent?)
  - a whole number divided by a simple fraction ($4 \div \frac{1}{2}$, ask how many $\frac{1}{2}$'s there are in 4)
  - a simple fraction divided by simple fraction where the numerator of the divisor in one and both denominators are the same ($\frac{5}{6} \div \frac{1}{6}$ asks how many $\frac{1}{6}$'s are there in $\frac{5}{6}$)
  - a simple fraction divided by a simple fraction where the numerator of the divisor is one and the fractions are compatible ($\frac{1}{2} \div \frac{1}{4}$)
- use a number line to model division
- explore two common algorithms:
  -the common-denominator algorithm involves finding the common denominator and dividing the numerators
  -the invert-and-multiply algorithm involves inverting the divisor and multiplying by it
- apply prior knowledge of reciprocal

8-B6 Order of Operations: fractions

- understand that the order is the same as for whole numbers, and why that makes sense

- understand how improper order impacts results

8-B7 Proportion: solve problems

- use a variety of strategies to solve problems of proportionality
  - -find relationships between the various terms of proportion and use these relations to solve for missing values ($\frac{2.2}{5}=\frac{x}{35}$; students in this situation should consider equivalent fractions)
  - -multiply each side of the proportion by a different form of one
  - -apply the unitary method
- recognize uses for and importance of proportion
- investigate problem solving opportunities (study of scale, transformational geometry – dilatations)

8-B8 Add & Subtract: fractions – mentally

- attempt mental calculation first when denominators are the same or easily determined ($\frac{1}{2}+\frac{1}{4}$)
- when addition or subtraction can not be done mentally, determine if estimation is sufficient or an exact answer is required

8-B9 Fractions: estimate and mentally compute products and quotients

- appropriate problems to use for multiplication include:
  - -a fraction by a whole number when the numbers are compatible;
  - -any two proper fractions when the numerators and denominators are relatively simple to work with;
  - -a whole number by a mixed number (distributive property should be used)
- appropriate problems to use for division include:
  - -a simple fraction divided by a whole number
  - -a whole number divided by a fraction
  - -a simple fraction divided by a simple fraction when the denominators are the same
- estimate prior to computation; use estimation to check reasonableness of results
- round to nearest whole and sometimes to nearest half to reach rough estimates

8-B10 Properties of Operations integers: commutative, associative, and distributive

- apply properties: commutative (order, $-5 \times 4 = 4 \times (-5)$), associative (grouping, $((-2 \times 4) \times (-3) = (-3 \times 4) \times -2)$, and distributive properties $(-2\ (3 + (-2)) = -2(3) + -2(3))$
- understand usefulness of these properties
- use formal names to discuss properties
- notation of closure ($2 - 5$ is not defined within the set of whole numbers, therefore needing the introduction to integers)

8-B11 Operations: positive and negative decimal numbers

- use prior experience to construct concrete and pictorial representations
- connect visual representations to symbols
- use a variety of models to illustrate the operations (coloured counters, number lines)
- develop computational algorithms with decimals, using estimation, mental computation, pencil and paper, and calculator
- apply prior knowledge of order of operations in the context of positive and negative decimals $(-0.2 + 4.5\ (-5 + 2.24) - (-6) \div 0.2$
- continue to estimate before calculating

8-B12 Like Terms: numerical situations represented visually

- use concrete materials such as algebra tiles to show like and unlike terms (difference between x and y… use these materials to develop an understanding that $2x$ refers to how many x's)

8-B13 Add & Subtract (simple algebraic terms): to solve problems

- establish a parallel between a measurement situation and a variable situation (3m and 20cm – need to be like terms before you can add or subtract – 3m + 0.2m)
- place value can also be used as an analogy (2 tens and 5 ones to get 7 of something)
- use concrete materials such as algebra tiles to show like and unlike terms (difference between x and y… use these materials to develop an understanding that $2x$ refers to how many x's)
- add and subtract simple expressions with concrete materials (know which like terms can and cannot be combined)

8-B14 Polynomial Expressions: Add &Subtract visually

- use a variety of concrete materials for conceptual development
- for subtraction, consider different representations of subtraction, including the following:
    - comparison (which refers to comparing and finding the difference between two quantities)
    - taking away (which refers to starting with a quantity and removing a specific amount
    - adding the opposites (which refers to subtracting by first changing the question to an addition and then adding the opposite of a quantity (subtracting x instead of $-x$)
    - missing addend (What would be added to the number being subtracted to get the starting amount ex. $(3x - 2) - (2x + 1)$, what is added to $2x + 1$ to get $3x - 2$?

8-B15 Multiplication by a Scalar (polynomials): Multiplying visually and symbolically

- develop with concrete materials and diagrams using repeated addition
  (**3**$(2x + 1) = 2x + 1 + 2x + 1 + 2x + 1$; model the binomial three times and combine the like terms using Algebra Tiles)
- explore the area model to associate repeated multiplication

Class 8 – C Strand – Pattern

KSO - Pattern: *by the end of **Class 8** students should:*

- *represent patterns as algebraic expressions, equations, inequalities and exponents*
- *interpret patterns through algebraic description and apply generalizations to make predictions of unknown values and solve real world and mathematical problems*
- *explore and generalize how a change in one quantity in a function affects another, in order to efficiently solve similar problems*
- *solve linear equations and inequalities through algebraic methods*
- *understand the meaning of non-linear equations*

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-C1 Patterns and Relations: represent in a variety of formats

- move interchangeably among a variety of formats which describe relationships
- describe in words, and use expressions and equations to represent patterns given in tables, graphs, charts, pictures and/or problems situations
- use information presented in a variety of formats to derive mathematical expressions and predict unknown values
- investigate linear situations and those which create a regular pattern (broken line or curved graph)
- predict unknown values once algebraic description of a pattern is established
- interpolate and extrapolate to predict unknown values when patterns are not regular

8-C2 Linear Equations: create & solve problems

- create and solve relevant problems for which algebraic solutions are required
- justify strategies used to create and solve problems
- appreciate the use of an algebraic equation in problems involving large numbers (as opposed to a guess and check approach)

8-C3 Single Variable Equations: solve algebraically

- use prior knowledge developed through concrete experiences to transfer to symbolic representation of single variable equations
- solve one- and two-step equations symbolically by the end of class 8
- explore whole and integer coefficients
- connect to fraction and decimal ideas
- use the "balance method" to solve problems
- use algebra tiles to solve problems

8- C4 Graphs (linear & non-linear): interpret

- understand, when looking at tabular data, that when an equal spacing between the values of one variable produces an equal spacing between values of another variable, the relationship will be linear
- interpret graphs of non-smooth situations

- use information from tables, diagrams, pictures, graphs or equations to describe change
- match situations to corresponding graphs
- sketch graphs for a variety of situations, leading to linear and broken-line graphs
- understand that a variety of representations may be used to show relationships and that choices are available (when students have difficulty recognizing a relationship from a table, they should realize that the same information can be represented and understood another way, i.e. a graph)

8-C5 Graphs & Tables (linear & non-linear): how changing one quantity affects the other

- use information from tables, diagrams, pictures, graphs or equations to investigate the impact of changing related quantities
- explore patterns associated with parameter changes in a linear equation (e.g. understand how changes in the equation affect the slant of the graph or its placement on a coordinate plane)

8-C6 Slope: link visual characteristics with numerical values

- understand that, for linear relationships, the ratio of vertical change to horizontal change is constant anywhere along the line
- understand that vertical change is the rise
- understand that horizontal change is the run
- investigate practical situations: slope of a staircase, slope of a roof, and the steepness of roads (do not limit strictly to graphs)
- determine the slope of a line and link to ratio ideas (magnitude of the ratio should be linked with the steepness of the line)
- understand that ratios for a graph that rises to the right are positive
- understand that ratios for a graph that rises to the left are negative

8-C7 Intersection of Two Lines: solve problems

- compare tables of values, equations or verbal descriptions of two linear situations to identify where lines will intersect
- use tables of values to generate ordered pairs for each equation and identify coordinates for points of intersection (each ordered pair produces a graph)
- graph both equations directly without generating a table of values, using a graphing calculator

Class 8 – D Strand – Measurement

KSO - Measurement: *by the end of **Class 8** students should:*

- *use concepts of rate to solve real-life & mathematical problems*
- *use direct and indirect measurement to make comparisons and interpret scales*
- *understand how a change in one measurement affects another in problems of rate*
- *understand relationships and move freely among all SI units, and choose appropriate units to solve measurement problems in given situations*
- *estimate effectively using a variety of strategies to solve measurement problems and understand when estimation is appropriate*
- *use relationships and reasoning to develop and apply procedures for measuring in a wide variety of measurement problems*

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-D1 Pythagorean Relationship: understanding (with models)

- investigate, using paper squares, the side relationships of a variety of right angled triangles
- explore the 3 – 4 – 5 rule for establishing a right angle
- understand that the longest side is the hypotenuse
- understand, through investigation, that if a square is made on each side of a right triangle, the sum of the two smaller squares will equal the area of the longer side ($c^2 = a^2 + b^2$ where a, b and c are sides of a right triangle)
- explore triangles of a variety of orientations to discover that the hypotenuse is the side opposite the right angle regardless of orientation

8-D2 Pythagorean Relationship: application

- understand usefulness of Pythagorean relationships to solve problems in real life (whenever a triangle has a right angle and 2 known sides)
- investigate real world problems to determine the length of the hypotenuse, as well as length of the other side when the hypotenuse and one side is given

- understand that the Pythagorean relationship can be used if only one side is given as long as the right triangle is isosceles
- use the Pythagorean relationship to determine if three given side lengths are, or are not, the sides of a right triangle
- find distance between two points using Pythagorean relationship; e.g. determine the reach of a ladder

8-D3 Volume & Surface Area: estimate and calculate right prisms and cylinders

- estimate volume in a variety of situations (how many cans will fit in a small box)
- cylinders can be treated as if they were rectangular prisms for rough estimation (length × width × height)
- find surface area of actual objects (cereal boxes, cracker boxes for rectangular prisms, bathroom toilet rolls for cylinders)

8-D4 Volume & Surface Area: measure and calculate composite 3-D shapes

- explore volume and surface area of a variety of constructed composite figures

8-D5 Area (circles): estimate

- understand why it makes sense to estimate by squaring the diameter
- understand why a closer estimate is $3 \times r^2$

8-D6 Area (circles): develop formula

- apply prior knowledge of area for a parallelogram to develop personal formulae for the area of a circle
- investigate to determine the radius when the area of a circle is given
- apply prior knowledge of square root
- understand that 3.14 is an approximation and that a calculator must be used for more precision

8-D7 Area and Perimeter: patterns and relationships of quadrilaterals and circles

- understand, through investigation, that area can vary when perimeter is fixed (perimeter of rectangle is 16 cm, students can determine all possible whole-number dimensions)

- understand, through investigation, that perimeter can vary when the area of a rectangle is fixed
- determine what happens to the area of a regular polygon as the number of sides increase (perimeter is 24 cm, what is the area when the figure has 4 sides? 6 sides?)

8-D8 SI Units: solve measurement problems

- integrate measurement problems as other mathematical ideas are explored (scale, volume, and surface area)
- develop sense of relative size of units (e.g. compare 1 cm^3 to 1 m^3; 1 kL to 1 L)
- apply relationships between capacity and volume (1mL = 1 cm^3; 1 L = 1000 cm^3) to solve problems
- when choosing between capacity and volume, understand which unit is most appropriate in a given situation
- establish link between capacity and mass of pure water (1mL of water has a mass of 1 g)
- include the prefixes kilo, hector, deca, deci, centi, and milli in problems and understand what each prefix represents
- investigate and create problems involving length and perimeter (mm, cm, m, km), area (mm^2, cm^2, m^2, km^2, and hectare (1 hm^2)), volume (cm^3 and m^3), mass (mg, g, kg) and capacity (mL, L, kL)
- continue to make decisions in real world situations about when estimating is close enough

8-D9 Proportion: solve indirect measurement problems

- link proportion to ideas of ratio and rate
- read, interpret and discuss scale drawings
- connect ideas of scale drawings to concepts of enlargements and reductions in transformational geometry
- understand usefulness of proportion ideas in relevant real-world problems

Class 8 – E Strand – Geometry

KSO - Geometry: *by the end of **Class 8** students should:*

- *build & analyse physical and pictoral models of 2-D and 3-D shapes to understand relationships & properties, and enhance spatial sense in mathematical and real world situations*
- *analyse the results of transforming shapes to understand & apply transformation properties to mathematical & real world situations and to explain geometrical ideas*
- *compare, classify and apply geometric properties to figures*
- *appreciate the importance of geometry in understanding mathematical ideas, in art, and in the world around them*

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-E1 Interpret Plans: (orthographic, mat, isometric) to represent more than one 3-D shape

- apply prior interpretation knowledge of 2-D pictures to enhance mathematical experience with 3-D objects
- use interlocking cubes to explore attributes of 3-D shapes
- use cubes to construct figures from a set of plans (orthographic plans or drawings)
- compare constructions to determine how they are different/the same
- construct structures from isometric drawings

8-E2 Polygons: properties and interrelationships

- develop a table to observe and extend patterns and generalize about the sum of the measures of the interior angles of various polygons, and the measure of each interior angle of a regular polygon
- understand, through investigating, that the sum of the measures of the interior angles of a polygon is found by dividing the polygon into triangles (since the sum of the measure of the angles is 180°, the sum of the interior angles in a pentagon is $5 \times 180°$, subtract the 360° at the centre: $5 \times 180 - 2 \times 180 = (5 - 2) \times 180$

# of sides	sum of interior angles	measure of each angle
3	180(1 × 180)	60
4	360(2 × 180)	90
5	540(3 × 180)	108
and so on….		

- construct graphs to determine if a relationship is linear (graph the sum of the interior angles against the number of sides)

8-E3 Dilatations: represent, analyse and apply

- understand that when a ratio is used to represent an enlargement or a reduction, the format is "New:Original" (a ratio of 2:1 means that the new figure is an enlargement twice the original)
- understand that centre of dilatation must be identified in order to locate the position of the dilatation image
- understand that it is when the issue of dilatation centre is considered that an enlargement or reduction can be described as a dilatation
- use projectors to explore dilatations
- explore combinations of transformations that include dilatations, such as an enlargement followed by a reflection

8-E4 Transformations (3-D Shapes): draw, describe, apply

- continue to develop visualization skills by physically exploring the results of moving objects/structures in a variety of ways (use real world contexts such as predicting the result of moving furniture around a room in a variety of ways)
- record and compare predictions and results of movements sketched on isometric paper
- sketch a variety of 3-D objects as they are reflected about a horizontal or vertical line to produce an image
- explore a variety of 3-D reflections and translations

8-E5 Angle Pair Relationships: parallel and non-parallel lines

- understand that corresponding angles and alternate angles can exist when the lines are not parallel, and are only equal when a transversal intersects two parallel lines
- provide evidence, through investigation, that parallelism exists in a situation
- use a variety of techniques to construct parallel lines (folding paper, transparent mirror, tracing paper, geoboards, etc.)

- apply transformational geometry to discover why the various angle pairs are equal (use a mirror mid-way between the parallel lines to reflect angles formed at one intersection onto angles formed at the other intersection)

Class 8 – F Strand – Data

KSO – Data Management: *by the end of **Class 8** students should:*

- *understand issues in data collection, including bias, repeated sample variability, randomness, collect, record, organize and describe data in multiple ways to draw conclusions about everyday issues*
- *understand, choose & apply appropriate data collection methods (real or simulated data) to answer questions on meaningful issues*
- *predict, read, and draw inferences for a variety of data displays, including interpolation and extrapolation (draw conclusions about things not specifically represented by the data)*
- *construct & analyse a variety of data displays, including circle graphs, box and whisker plots and scatter plots, and choose the most appropriate display for a given situation*
- *analyse measures of central tendency in terms of the effect on mean, median and mode when changes in data occur, in order to draw conclusions and make decisions*
- *examine interpretations of data displays for validity, considering data collection issues to form opinions and make predictions*

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-F1 Repeated Sampling (same population): variability

- understand that survey results of two different samples of the same population will not exactly be the same
- recognize the variability among repeated samples and provide a basic and informal introduction to the notion of sampling distribution
- conduct probability experiments to demonstrate variability of repeated sampling)
- use real and simulated data in interesting investigations

8-F2 Randomness: concepts

- explore real world situations where incorrect predictions have been made on the basis of non-random samples (create fictional situations as well)
- understand that a random sample is a sample collected from a population so that every member of the population has an equal chance of being selected
- understand that members are chosen independently of each other
- understand that common methods used in selecting random sample are coins, dice, sampling boxes, a table of random numbers, or a computer generated sample

8-F3 Circle Graphs: construct, interpret

- understand usefulness of circle graphs in situations where a comparison of the part to the whole is needed (e.g. budgets)
- apply prior knowledge about percent and using a protractor in construction of circle graphs
- use calculators, where appropriate, to assist the tedious calculations associated with percentages and conversions to degree measures
- use technology to create circle graphs where appropriate (focus on when a circle graph is the most appropriate data display)

8-F4 Box and Whisker Plots: construct, interpret

- understand that a box-and-whisker plot is also referred to as a box plot
- understand that this is an easy method for visually displaying the median statistic and the range and distribution
- construct plots
    - identify the median score and the median of the upper half of the data (upper quartile)
    - identify the median of the lower half of the data (lower quartile)
    - identify the extremes, that is the lower value and the higher value
    - draw a box between the two quartiles, mark the median line, and draw the whiskers to the extremes

8-F5 Scatter Plots: construct, interpret, find line of best fit

- use data collected by students to construct scatter plots

- understand that the line of best fit is one of the most commonly used graphs in displaying scientific data because it takes scientific error into account r
- line of best fit is drawn to show relationship between two variables

8-F6 Variations: on mean, median, and mode

- consider and compare, through investigation, the impact of alterations to data sets in each of mean, median and mode
- continue to make decisions about most appropriate measure of central tendency for a particular situation

8-F7 Extrapolate & Interpolate: from a data display

- explore through line of best fit
- estimate values which lie between data (interpolation)
- make predictions beyond the set given (extrapolation)

Class 8 – G Strand – Probability

KSO - Probability: *by the end of **Class 8** students should:*

- *explore, interpret and make predictions for everyday events by estimating and conducting experiments*
- *determine theoretical and experimental probability, understand the difference between the two and determine when each is relevant to a particular situation*
- *express probability as ratios, fractions, decimals, percents and choose appropriate expressions given a particular situation*
- *conduct simulations & experiments to determine the probability of single & complementary events in real life situations*
- *use real life data to establish broad probability patterns for the purpose of planning & making decisions (e.g. patterns in population growth, traffic)*

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-G1 Theoretical Probability: single & complementary events

- apply formula used in class 7

$$P(E) = \frac{\text{\# of favorable outcomes}}{\text{Total \# of possible outcomes}}$$

- understand that this formula can only be used when dealing with equally likely outcomes or events
- find the probability of a complementary event using the formula $1-P(E)$
- understand that, if the probability of an event occurring is $\frac{1}{4}$, then the probability of it not occurring is $1 - \frac{1}{4} = \frac{3}{4}$

8-G2 Simulations & Experiments: single & complimentary events

- understand that the need to determine that something will not occur is as important as that it will occur
- understand that an event is described as (E)
- understand that the probability of an event is described as $P(E)$
- understand that the probability of an event not happening is Not $P(E)$
- understand that Not $P(E)$ is the complement of $P(E)$
- understand that the sum of the probabilities of an event and its compliment is always equal to 1 $P(E)$ + not $P(E) = 1$)
- in situations for which the probability of various events occurring is not equally likely, experimentation is often the only method of determining probability

8-G3 Compare Results: theoretical, experimental

- compare theoretical and experimental probability for a given situation and discuss results
- investigate strategies to allow greater accuracy in experimental results (e.g. sample size)

Class 9 - Strand A – Numbers

KSO - Numeration: *by the end of **Class 10** students should:*

- *demonstrate an understanding of number meanings with respect to real numbers*
- *order real numbers, represent them in multiple ways, and apply appropriate representations to solve problems*
- *demonstrate an understanding of the real number system and its subsystems by applying a variety of number theory concepts in relevant situations*

Toward this, students in **Class 9** will be expected to master the following **SO** (Specific Outcomes):

9-A1 Square Root: solve problems

- determine if the solution to a problem involves both values of the square root or just the principal square root

9-A2 Integers & Real Numbers: write solution sets for equations and inequalities

- relate the language of inequality to the symbols of inequality
- include less than $<$, less than or equal to $\leq$, greater than $>$, or greater than or equal to $\geq$
- graph when given a set notation and produce the set notation when given a graph

9-A3 Irrational Numbers: demonstrate and understand meaning

- examine the use of π and the decimal representation for it
- place irrational numbers on a number line relative to known rational numbers

9-A4 Real Numbers: interrelationships of subsets

- determine and justify if any given number is rational or irrational
- give examples and explain why numbers fit the conditions of natural, whole, integers, rational and irrational numbers

9-A5 Real Numbers: compare and order

- use irrational numbers in ordering activities involving square roots

9-A6 Matrices: represent problems

- understand that matrices are used as a means of storing data (interpretation often includes simple addition and subtraction)
- understand how the rows and columns of a matrix are identified

Class 9 - Strand B – Operations

KSO - Operations: *by the end of **Class 10** students should:*

- *Explain how algebraic and arithmetic operations are related, use them in problem-solving situations, and explain and demonstrate t he power of mathematical symbolism*
- *Derive, analyse, and apply computational procedures in situations involving all representations of real numbers*
- *Derive, analyse, and apply algebraic procedures in problem situations*

Toward this, students in **Class 9** will be expected to master the following **SO** (Specific Outcomes):

9-B1 Exponent Laws: integral exponents

- develop an understanding of the law of exponents
- understand and apply the following rules:

$$a^m \times a^n = a^{m+n},\ a^m \div a^n = a^{m-n},\ \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n},\ (ab)^n = a^n b^n,\ (a^m)^n = a^{mn},\ a^0 = 1, a^{-n} = \frac{1}{a^n}$$

9-B2 Scientific Notation: model, solve, and create problems

- explore situations involving addition, subtraction, multiplication and division with numbers in scientific notation
- apply the laws of exponents to numbers written in scientific notation

9-B3 Reasonableness of Results: square root, rational numbers, scientific notation

- continue to use estimation skills developed in class 8 with irrational and rational numbers and in class 9 with large numbers

9-B4 Matrices: scalar multiplication - model, solve, create problems

- understand that to add or subtract two matrices there must be an equal number of rows and columns

- understand that to multiply a matrix by a scalar, a real number, multiply each entry in the matrix by the scalar

9-B5 Add, Subt, mult & div: rational numbers in fractional and decimal form

- use mental computation first when attempting to solve problems
- continue to develop computational fluency by solving pencil and paper problems where mental calculations are not appropriate

9-B6 Real Numbers: model, solve, create problems

- investigate the importance of decimal fractions in real world situations

9-B7 Order of Operations: rational number computation

- continue to develop a strong knowledge of operation with rational numbers

9-B8 Polynomial Expressions: Add & Subtract symbolically, to solve problems

- continue to develop an understanding of addition and subtraction of polynomial equations

9-B9 Polynomials: Products - concretely, pictorially, and symbolically

- multiply: monomial by a monomial, a scalar by a monomial, a scalar by a polynomial, monomial by a polynomial and a binomial by a binomial

9-B10 Algebraic Expressions: commutative , associative, distributive

- investigate and apply properties together
- examine properties in the context of simplifying expressions and solving equations

9-B11 Algebraic Expressions: factoring - concretely, pictorially, and symbolically

- understand that factoring and multiplying are reverse operations

9-B12 Polynomials: Factors – dimensions of a rectangle

- explore dimensions through Algebra Tiles
- review and apply distributive property (focus on area)

9-B13 Polynomials: Quotients - with monomial divisors

- ♦ divide: monomial by a monomial; polynomial by a scalar; then polynomial by a monomial
- ♦ understand a common method to solve division of a polynomial by a monomial is to separate the monomial and solve each monomial division question (student must understand why this works)

9-B14 Polynomial Expressions: evaluate

- ♦ examine the evaluation of polynomials prior to and after being simplified, to show the advantage of simplifying

9-B15 Income and Deductions: estimate and calculate

- ♦ estimate and calculate various ways to make money
- ♦ estimate and calculate deductions from income

9-B16 Budgets: solve problems

- ♦ solve problems relating to personal budgets

9-B17 Purchases: analyse situations and make decisions involving financing

- ♦ analyse situations and make decisions involving financing of purchases with respect to personal budgets

9B-C18 Simple and Compound Interest: demonstrate an understanding

- ♦ understand the long term difference between simple and compound interest
- ♦ investigate both investments and financing situations

9B-C19 Consumer Data: demonstrate understanding of real-world relationships

- ♦ translate between various representations of consumers data (graphs, tables, and written descriptions)

Class 9 - Strand C – Pattern

KSO - Patterns: *by the end of **Class 10** students should:*

- *Model-real world problems using functions, equations, inequalities, and discrete structure*
- *Represent functional relationships in multiple ways and describe connections among those representations*
- *solve problems involving relationships, using graphing technology as well as paper and pencil techniques*
- *perform operations on and between functions*
- *analyse and explain the behaviors, transformations, and general properties of types of equations and relations*
- *interpret algebraic equations and inequalities geometrically and geometric relationships algebraically*

Toward this, students in **Class 9** will be expected to master the following **SO** (Specific Outcomes):

9-C1 Patterns and Relationships: justify unknown values

- explore linear, exponential and parabolic curves
- describe verbally and symbolically, patterns given in tables, graphs, charts, pictures, and /or by problem situations

9-C2 Patterns and Relationships: connections among different representations

- use models such as tables, graphs, and symbolic statements to assist in examining patterns and relationships.
- explain why the data in a table represents a linear, parabolic, or exponential relationship

9-C3 Single Variable Equations: solve algebraically

- investigate problems involving integers and fractions

9-C4 Inequalities: first-degree single -variables - solve and verify

- begin to solve inequalities
- inequalities should have a variable with an exponent of 1

9- C5 Graphs (linear & non-linear): interpret

- examine patterns which can be described algebraically and represented by smooth curves

- develop familiarity with the terms exponential and parabolic

9-C6 Equation of a Line: determine using slope and y-intercept from graphs

- understand slope as, slope = rise/run
- make the connection between rise and vertical change, and run and horizontal change
- determine the equation of a linear relationship by calculating the slope and the
  y- intercept from the graph
- realize that the *y*-intercept is the place where the graph crosses the *y*-axis

9-C7 Slope and *Y*-intercept: determine from a table or graph

- determine the slope and *y*-intercept by examining a table or graph
- in a linear situation, determine the slope and *y*-intercept from a table of values or graph
- determine if a table represents a linear model by plotting the points

9-C8 Equation of a Line: determine using slope and y-intercept

- determine the equation of a line ($y = mx + b$) given the slope (m) and
  y-intercept(b)
- determine the equation of a line ($y = mx + b$) if the slope (m) and *y*-intercept(b) can be determined

9-C9 Two Linear Equations: find solutions to a problem by graphing

- solve problems by graphing pairs of linear equations (including identifying
  conditions where one option is better than another for solving a complex linear equation

Class 9 - Strand D – Measurement

KSO - Measurement: *by the end of **Class 10** students should:*

- *measure quantities indirectly, using techniques of algebra, geometry and trigonometry*
- *determine measurements in a wide variety of problem situations, and determine specified degrees of precision, accuracy, and error of measurement*
- *apply measure formulas and procedures in a wide variety of*

Toward this, students in **Class 9** will be expected to master the following **SO** (Specific Outcomes):

9-D1 Volume & Surface Area: estimate and measure pyramids, cones and spheres

- estimate and find the volume and surface area of a variety of hollow shapes including but not limited to: the pyramid, cone, and sphere
- problem situations should be included where students are given the opportunity to find the dimensions when surface area or volume is given
- memorization of formulas is not intended at this level

9-D2 SI Units: solve measurement problems involving conversion

- apply prior measurement skills (integrated throughout the course of study)
- solve problems involving mass and capacity units, as well as linear, area, and volume units, throughout the full SI scale

9-D3 Similar Triangles: understand and apply proportions

- understand that in similar triangles the ratios of side lengths within one triangle are equal to the ratio of the corresponding side lengths within the second triangle

Class 9 - Strand E – Geometry

KSO - Geometry: *by the end of **Class 10** students should:*

- *make and test conjectures about, and deduce properties of and relationships between, 2- and 3-dimensional figures in multiple contexts*
- *interpret and classify geometric figures, translate between synthetic and coordinate representations, and apply geometric properties and relationships*
- *analyse and apply Euclidean transformations, including representing and applying translations as vectors*
- *represent problem situations with geometric models and apply properties of figures*
- *demonstrate an understanding of the operations of axiomatic systems, and the connections among reasoning, justification, and proof*

Toward this, students in **Class 9** will be expected to master the following **SO** (Specific Outcomes):

9-E1 Congruent Triangles: properties and minimum sufficient conditions

- ♦ understand, through investigation, that if two triangles are congruent through: SSS; SAS; ASA; or ASS, then the other corresponding parts of the triangle are also congruent
- ♦ understand that the symbol " ≅" which is read as "is congruent to."

9-E2 Congruent Triangles and Angle Properties: informal deductions

- ♦ distinguish between inductive and deductive reasoning using both mathematical and non-mathematical reasoning

9-E3 Triangles: relate congruency and similarity

- ♦ compare and contrast congruence and similarity as they relate to triangles

9-E4 Similar Triangles: apply properties

- ♦ understand properties of similar triangles; that the corresponding angles are congruent and the corresponding sides are in proportion
- ♦ understand that two triangles are similar when two pairs of corresponding sides are in proportion and the pair of included corresponding angles are congruent
- ♦ understand that two triangles are also similar when two angles of one triangle are congruent to two corresponding angles of another triangle

9-E5 Unique Triangles: minimum sufficient conditions

- ♦ examine what pieces of information are needed to guarantee a unique triangle
- ♦ understand that the following are necessary in order to produce unique triangles: three sides; two sides and contained angle; two angles and contained side; and two angles and non-contained side
- ♦ discover that AAA and SSA are not ways to give a unique triangle

9-E6 Transformations (mapping notation): represent and interpret

- enhance knowledge of translations, reflections, rotation, and dilatations to include these on the coordinate plane, using mapping notations
- understand that dilatation center is restricted to the origin and rotations are restricted to 90° and 180°
- describe the nature of a transformation based on a given mapping

9-E7 Transformations (mapping notation): analyse and represent combinations

- analyse a transformation given in mapping notation and to represent a transformation using mapping notation
- recognize the transformations when given an image and the pre-image after a combination of transformations

9-E8 Transformations: investigate and apply effects on congruence, similarity, and orientation

- understand, through hands-on investigation, properties for each transformation:
  - Reflections – line segments joining points to their images are perpendicular to the reflection line and have their midpoint on the reflection line; The reflection image of any figure is a congruent figure; The orientation of a reflection image is the opposite of the original figure

  - Translations – The line segments joining points to their images are parallel and equal in length; The translation image of any figure is a congruent figure; The orientation of a translation image is the same as that of the original figure; The translation images of lines or segments are parallel or collinear to their pre-images.. For 90° rotations students should know that: horizontal segments become vertical and vertical segments become horizontal; any segment and its image are perpendicular. For 180° rotations, students should be aware that segments are parallel or collinear to their images

  - Rotations – A rotation of $A°$ about a point X is such that a segment joining a point to X and a segment joining its image to X are equal in length and form an angle of $a°$; The rotation image of any figure is a congruent figure; The orientation of a rotation image is the same as that of the original figure

- Dilatations – The dilatation center, a point, and its image form a line; The ratio of the distance between the dilatation center and the figure, to the distance between the dilatation center and the image, is the same as the dilatation ratio; The ratio of the length of a segment in the original figure to the length of a segment in the image is the same as the dilatation ratio; The image is similar to the figure; Angle measures in the figure are the same as angle measures in the image

Class 9 - Strand F – Data Management

KSO – Data Management: *by the end of **Class 10** students should:*

- *design and conduct experiments using statistical methods and scientific inquiry*
- *use curve fitting to determine the relationship between, and make predictions from, sets of data and be aware of bias in the interpretation of results*
- *determine, interpret and apply as appropriate a wide variety of statistical measures and distributions*

Toward this, students in **Class 9** will be expected to master the following **SO** (Specific Outcomes):

9-F1 Scatter Plots: characteristics of relationships

- consider whether data represented by a scatter plot are continuous or discrete
- understand that interpolating between the data points of discrete data may lead to conclusions that are not meaningful
- understand that when constructing scatter plots, the independent variable is always placed on the *x*-axis and the dependent variable is placed on the *y*-axis
- understand that in lines of best fit, where the data is grouped closely around the line, the relationship is said to be a strong positive or negative (depending on a positive or negative trend)
- understand that if the data were dispersed but still showed a trend it would be a weak one
- understand that if a positive or negative trend is not apparent then there is no relationship

9-F2 Lines of Best Fit: sketch and determine equations

- use the eyeball method to draw the line and then use the slope and *y*-intercept to determine the equation of the line

9-F3 Curves of Best Fit: relationships that appear to be non-linear

- plot points on a graph and sketch curves of best fit – non linear
- recognize when the pattern for the data does not appear to be linear, and a curve would best represent the data
- the graph can be represented using a piece of string (avoid equations)

9-F4 Data Analysis: evaluate arguments and interpretations

- compare various methods of displaying data and evaluating their effectiveness
- examine how the choice of certain graphs can lead to errors in judgment

9-F5 Displaying Data: most appropriate methods

- determine, debate and defend, why a particular display is best suited to a specific type of data, or to a given context

9-F6 Displaying Data: draw inferences and make predictions

- draw inferences and conclusions on a number of data displays, with particular attention paid to scatterplots
- understand that if the scatterplot approximates a line, the slope is: positive, the correlation is positive, negative slope gives a negative correlation, a zero correlation shows no apparent relationship, a relationship of "+1" or "–1" shows a perfect correlation
- predict data for values not determined between two given pieces of data (interpolate) and beyond the data given (extrapolate)

Class 9 - Strand G – Probability

KSO – Probability: *by the end of **Class 10** students should:*

- *represent and solve problems involving uncertainty*
- *make predictions and carry out simulations to answer real world issues of interest*
- *predict for dependent and independent events in simulations*
- *determine theoretical probability for dependent and independent events and apply to real life issues*

Toward this, students in **Class 9** will be expected to master the following **SO** (Specific Outcomes):

9-G1 Theoretical Probability: independent and dependent variables

- ♦ determine when two events are dependent or independent
- ♦ understand that the probability of two independent events, *A* and *B*, is equal to $P(A) \times P(B)$

9-G2 Simulations & Experiments: dependent and independent events

- ♦ understand that if the probability of the second event is affected by the outcome of the first event, then the two events are dependent
- ♦ understand that if the probability of the second event is not influenced by the outcome of the first event, then the two events are independent

9-G3 Compare Results: theoretical, experimental

- ♦ relate the experimental probability of an event with results achieved using the definition of theoretical probability for that event

Class 10 - Strand A – Numbers

KSO - Numeration: *by the end of **Class 10** students should:*

- ♦ *demonstrate an understanding of number meanings with respect to real numbers*
- ♦ *order real numbers, represent them in multiple ways, and apply appropriate representations to solve problems*
- ♦ *demonstrate an understanding of the real number system and its subsystems by applying a variety of number theory concepts in relevant situation*

Toward this, students in **Class 10** will be expected to master the following **SO** (Specific Outcomes):

10-A1 Square Root: approximate

- ♦ develop an awareness that square roots are often irrational
- ♦ understand that appropriate approximations in some situations are beneficial

10-A2 Square Root: apply properties to operations

- develop and apply properties for operations involving square roots (limit to conversion between entire and mixed radicals dealing with Pythagorean Theorem)

10-A3 Equations: zero product property and its relationship

- understand the zero product rule: if " $a \bullet b = 0$, then either $a = 0$ or $b = 0$"
- apply the zero product rule to solving quadratic equations by factoring
- convert a quadratic equation to two linear equations by the factoring method

10-A4 Equations: apply properties of numbers upon expressions

- solve equations by applying associative, distributive, identity and inverse properties

10-A5 Inequalities: relate sets of numbers to solutions

- solve problems with a restricted solution set
- include choice of number systems

10-A6 Irrational Numbers: understand role

- develop awareness of errors in decimal approximations and rounding off
- understand when to approximate and when to continue with radical expressions
- convert between entire and mixed radicals, within the application of Pythagorean Theorem

Class 10 - Strand B – Operations

KSO - Operations: *by the end of **Class 10** students should:*

- *explain how algebraic and arithmetic operations are related, use them in problem-solving situations, and explain and demonstrate the power of mathematical symbolism*
- *derive, analyse, and apply computational procedures in situations involving all representations of real numbers*
- *derive, analyse, and apply algebraic procedures in problem situations*

Toward this, students in **Class 10** will be expected to master the following **SO** (Specific Outcomes):

10-B1 Matrix Multiplication: develop and apply procedures

- develop and apply the algorithm for multiplication
- focus on labels used for rows and columns
- understand the importance of the dimensions
- understand that matrices can only be multiplied if the number of rows in the first matrix is the same as the number of columns in the second

10-B2 Matrices: solve network problems

- represent and solve network problems using matrices
- represent network problems, using matrices

10-B3 Algebraic Expressions and Equations: model relationships

10-B4 Irrational Numbers: develop algorithms, perform operations

- understanding of the way approximate values for irrational numbers are displayed on the calculator

Class 10 - Strand C – Pattern

KSO - Patterns: *by the end of **Class 10** students should:*

- *Model-real world problems using functions, equations, inequalities, and discrete structure*
- *Represent functional relationships in multiple ways and describe connections among those representations*
- *solve problems involving relationships, using graphing technology as well as paper and pencil techniques*
- *perform operations on and between functions*
- *analyse and explain the behaviors, transformations, and general properties of types of equations and relations*
- *interpret algebraic equations and inequalities geometrically and geometric relationships algebraically*

Toward this, students in **Class 10** will be expected to master the following **SO** (Specific Outcomes):

10-C1 Transformations: express algebraically; mapping rules

- express transformations either algebraically or with a mapping rule when given an image of a known graph
- express mapping rules algebraically and vice versa

10-C2 Pattern: identify and apply

- identify patterns in written descriptions, diagrams, graph, and/or table of data
- generalize and apply identified patterns (e.g. simple interest, vehicle depreciation, financing of road construction)

10-C3 Inequalities: write and describe its graph

- express inequalities algebraically by examining their graphs
- describe a given graphical model using inequalities

10- C4 Graphs& Tables: construct and analyse relating two variables

- analyse graphs and tables to determine mathematical characteristics
- include slope/rate of change and intercepts
- interpret the characteristics in relation to given contexts

10-C5 Graphs& Tables: explore dynamics of change

- determine how changes in one variable affect another through the analysis of tables or graphs
- recognize the differences between linear, quadratic and other relationships (compound interest)

10-C6 Graphs: sketch

- translate among tabular, written, symbolic and graphical representations of functions
- create graphs given information in a variety of formats

10-C7 Graphs: determine if linear by plotting points

- determine both visually and numerically, whether there is a constant ratio by plotting points
- a constant ratio of rise to run determines if the graph is linear

10-C8 Graphs: create by constructing a table of values and graphing technology

- construct a graph from a table of values (essential)
- understand when to choose to graph by the *y*-intercept slope method

10-C9 Real-World Relationships: describe

- use graphs, tables of values, and written descriptions to describe

10-C10 Conjectures: (steepness and direction of line)- investigate, make and test (Now C10)

- conjectures should include that the sign of the slope determines whether a line rises or falls and the magnitude of the slope is related to the steepness of the line
- discover that whether a line rises or falls relates to the sign of the slope
- discover that the magnitude of the slope is related to the steepness of the line

10-C11 Systems of Linear Equations: solve

- realize that the graphing method will not always give exact solutions easily
- solve linear equations by substitution method, including comparison of equations

10-C12 Non-Linear Equations: evaluate and interpret using graphing technology

- calculate the roots of quadratic equations from the corresponding graph
- decide whether a non-linear scatter plot represents a quadratic or exponential relationship

10-C13 Equations: solve for linear and simple radical, exponential, and absolute value equations and linear inequalities

- use prior knowledge developed in previous grades
- encourage proficiency with algebraic manipulation of all kinds
- use strategies to check answers for reasonableness within the problem context

10-C14 Equations and Inequalities: graph and analyse graphs

- graph equations and inequalities by constructing a table of values, using knowledge of transformations and by identifying characteristics

10-C15 Linear and non-linear Functions: compare regression models

- understand that that not all data can be represented using a regression model
- understand that the nature of the data will determine if a linear or non-linear model will be used

10-C16 Problems: express in terms of equations

- analyse and interpret variety of situations and express as equations
- include written problems, pictorial patterns and tables of data
- express equations are linear, quadratic, or exponential
- create scatter plots and determine equation of best fit

10-C17 Equations: rearrange

- transform equations from one form to another
- include transformation of linear equations in standard form to slope/*y*-intercept form and vice versa

10-C18 Equations: solve using graphs

- use the *x*-intercept to determine the solution of both linear and quadratic graphs
- determine solution to an equation by graphing one side of the equation against the other and identifying the intersection point

10-C19 Quadratic Equations: solve by factoring

- apply the zero product property
- solve equations including those which involve common factors, regular equations ($ax^2 + bx + c$), perfect square trinomials and difference of squares

10-C20 Functional Relationships and Notation: explore formally and informally

- understand the relationship between a relation and a function
- start with functional relationships then apply the mathematical concept of function
- use mathematical notation and vocabulary

10-C21 Quadratic Functions: analyse and describe transformations and apply them to absolute value functions

- apply graphical transformations (reflections, stretches, and translations) resulting from changes in the parameters of the function
- apply same techniques to absolute value functions

10-C22 Scatterplots: create and analyse

- use graphing technology to create scatter plots
- examine characteristics such as the strength, linear or non-linear and decreasing or non-decreasing

10-C23 Network Problems: represent using matrices and vice versa

- represent a network as a matrix and interpret the matrix in terms of a corresponding network situation

10-C24 Problems: solve using graphing technology

- analyse data, construct data displays, create scatter plots and lines of best fit, analyse graphs, and perform matrix operations using graphing technology
- (outcome should be developed throughout the course of study)

10-C25 Data: gather, plot and demonstrate understanding of independent and dependent variables, and domain and range

- gather and represent data graphically
- make decisions regarding independent and dependent variables
- consider the domain and range when determining scales

Class 10 - Strand D – Measurement

KSO - Measurement: *by the end of **Class 10** students should:*

- *measure quantities indirectly, using techniques of algebra, geometry and trigonometry*
- *determine measurements in a wide variety of problem situations, and determine specified degrees of precision, accuracy, and error of measurement*
- *apply measurement formulas and procedures in a wide variety of contexts*

Toward this, students in **Class 10** will be expected to master the following **SO** (Specific Outcomes):

10-D1 Measurement: determine accuracy and precision

- understand that accuracy depends upon the usage of the measurement instrument
- understand that precision depends on how finely an instrument is graduated
- address precision issues when performing calculations on measurement data

- understand that measurement of side lengths should be made with appropriate precision
- express answers to problems with significant digits

10-D2 Measurement Differences: significant or accidental

- understand that repeated experiments will yield differences in results
- consider whether differences in results are accidental or the result of an unidentified variable

10-D3 Volume & Area: explore and apply properties

- examine maximizing the area while restricting perimeter
- examine maximizing volume while restricting surface area

10-D4 Volume & Surface Area: demonstrate understanding

- real understand that surface area and volume deal with 3-D shapes
- understand connection between volume and surface area
- compare capacities of prisms

10-D5 Relationships: perimeter and area; surface area; surface area and volume

- understand that ratio between the perimeter of similar figures is equivalent to the ratio between any pair of sides

10-D6 Area, Perimeter, Surface Area, Volume: determine and apply

- apply formulas in a variety of contexts
- investigate precision and accuracy issues
- develop non-routine formulas to determine areas and volume
- understand that areas of regular polygons can be determined by dividing the area into familiar shapes

10-D7 Similar Triangles: apply properties

- apply side and angle relationships when developing the primary trig ratios
- examine ratios to determine if sides are proportional

10-D8 Similar Triangles& Right Triangles solve problems

- solve problems using the proportionality relationship among sides in similar triangles
- apply Pythagorean theorem in appropriate problems

10-D9 Trigonometric Functions: relate to ratios in similar right triangles

- ♦ understand that primary trig ratios are equivalent for the equal angles in similar right triangles
- ♦ investigate the three ratios between length of pairs of sides in right angle triangles
- ♦ investigate concepts of sine, cosine, and tangent

10-D10 Trigonometric Values: use calculators

- ♦ use calculators to determine the trig ratios sin θ, cos θ, tan θ

10-D11 Trigonometric Values: (right triangles) - apply to solve problems

- ♦ find areas of polygons using right triangle trigonometry
- ♦ explore angle of elevation (measured from the horizon up) in real world setting

10-D12 Trigonometric Ratios: solve problems

- ♦ calculate side length and angles using trig ratios (use of calculators is required)

10-D13 Vectors & Bearings: solve problems

- ♦ solve bearing and vector problems using the Pythagorean theorem and/or trigonometric ratios
- ♦ use trigonometry to solve bearing and vector questions

Class 10 - Strand E – Geometry

KSO - Geometry: *by the end of **Class 10** students should:*

- ♦ *make and test conjectures about, and deduce properties of and relationships between, 2- and 3-dimensional figures in multiple contexts*
- ♦ *interpret and classify geometric figures, translate between synthetic and coordinate representations, and apply geometric properties and relationships*
- ♦ *analyse and apply Euclidean transformations, including representing and applying translations as vectors*
- ♦ *represent problem situations with geometric models and apply properties of figures*
- ♦ *demonstrate an understanding of the operations of axiomatic systems, and the connections among reasoning, justification, and proof*

Toward this, students in **Class 10** will be expected to master the following **SO** (Specific Outcomes):

10-E1 Geometric Reasoning: inductive and deductive

- use inductive and deductive reasoning in situations such as developing operational procedures, generalizing relationships and proving theorems

10-E2 2-D & 3-D Figures: explore properties and test conjectures

- understand that making conjectures is inductive in nature, while testing them is deductive
- understand connection between line and point symmetry in regular polygons
- define polygonal shapes by using symmetry

10-E3 Polygons & Polyhedra: solve problems

- understand that the sides of a regular polygon is the same as the number of lines of symmetry
- understand that rotational order of symmetry is the same as the number of sides in a regular polygon
- include locating centres of triangles to determine the most efficient shapes for 3-D packages

10-E4 Transformations: solve problems

- apply transformational technique to solve problems with coordinate geometry and 3-D dilation
- solve an equation such as $3x - 5 = 4$ by finding the intercept of $y = 3x - 5$ and $y = 4$

10-E5 Transformations: draw graphs

- connect algebraic and geometric transformations to draw graphs
- using $y = x^2$ as a basis, practice sketching the image under certain transformations, equations or mapping rules

10-E6 Bisectors: examine intersection points (altitudes, medians, angle bisectors, perpendicular bisectors)

- include study of incentres and circumcentres, and centres of gravity
- include the concepts of perpendicular and angle bisectors, medians and altitudes of triangles and their effect on rigidity

Class 10 - Strand F – Data Management & Probability

KSO – Data Management & Probability: *by the end of **Class 10** students should:*

- *design and conduct experiments using statistical methods and scientific inquiry*
- *use curve fitting to determine the relationship between, and make predictions from, sets of data and be aware of bias in the interpretation of results*
- *determine, interpret and apply as appropriate a wide variety of statistical measures and distributions*
- *make predictions and carry out simulations to answer real world issues*
- *predict for dependent and independent events in simulations*
- *determine theoretical probability for dependent and*

Toward this, students in **Class 10** will be expected to master the following **SO** (Specific Outcomes):

10-F1 Data Collection: concerns and issues

- understand which variables have an effect on data outcomes
- investigate issues of accuracy and precision of data
- understand how data is impacted when the experiments is repeated

10-F2 Experiments: design and conduct

- investigate issues relating to independent vs. dependent variables – controlling variables – data collection
- design experiments to determine which variables affect the outcome

10-F3 Curves of Best Fit: (non-linear data) - power and exponential regression

- explore curve fitting for non-linear data
- understand that non-linear models often show a better relationship than linear models

10-F4 Normal Curves: explore measurement issues

- understand that a normal curve is an extension of histograms in some cases
- understand that 68% of data lies within one standard deviation of the mean in a normal distribution
- understand that 95% of data lies within two standard deviation of the mean in a normal distribution

10-F5 Correlations: develop an intuitive understand

- understand that correlation is a description of "how well" data fits a pattern
- identify the difference between a strong and weak correlation – negative and positive
- understand that correlation coefficients can be misleading

10-F6 Data Analysis: results about distribution of data

- examine results with respect to how they were generated(situation/experiment)
- understand that box and whisker plots are useful when comparing data

10-F7 Displaying Data: construct

- apply prior knowledge to construct : stem and leaf, box and whisker plots, and histograms

Class 11B - Strand A – Numbers

KSO - Numeration: *by the end of* ***Class 12 Business*** *students should:*

- *order real numbers, represent them in multiple ways and apply appropriate representations to solve problems*
- *demonstrate an understanding of the real number system and it's subsystems by applying a variety of number theory concepts in relevant situations*

Toward this, students in **Class 11B** will be expected to master the following **SO** (Specific Outcomes):

11B-A1 Exponents: develop and demonstrate understanding

- apply properties of exponents to rewrite, simplify and evaluate expressions using exponents

11B-A2 Irrational Numbers: demonstrate understanding

- knowledge of whether an exact or an approximate solution is appropriate

11B-A3 Zero and Negative Exponents: demonstrate understanding

- apply properties of zero and negative exponents to rewrite and evaluate expressions

11B-A4 Factorial Notation: develop understanding and apply

- develop in connection with tree diagrams and the fundamental counting principle recognition that n! (n-factorial) represents the number of ways to arrange n distinct objects

11B-A5 Matrices: demonstrate an understanding of properties and apply them

- realization that only a square matrix can have an inverse
- understanding that an identity matrix is a result of the product of a square matrix and its inverse
- properties for real numbers, with exception of commutativity under multiplication, apply to matrices

11B-A6 Matrices: (identities and inverses) demonstrate an understanding of the conditions

- ability to identify if a matrix has an inverse or not
- ability to figure out whether a non-zero value can be found for the determinant

11B-A7 Matrix: develop and apply the procedure to obtain the inverse (Now B3)

- determine how to find the inverse of a 2 x 2 matrix with various determinants

11B-A8 Inverse Matrices: solve systems of equations (Now B4)

- two, three, or more equations with two, three or more variables will be included

11B-A9 Algebraic and Matrix Equations: understanding of the relationship

- apply algebraic skills to the solving of systems of equations with matrices
- isolating of a variable matrix develops from the process of isolating a variable

Class 11B - Strand B – Operations

KSO - Operations: *by the end of Class **12Business** students should:*

- *Explain how algebraic and arithmetic operations are related, use them in problem-solving situations, and explain and demonstrate t he power of mathematical symbolism*
- *Derive, analyze, and apply computational procedures in situations involving all representations of real numbers*
- *Derive, analyze, and apply algebraic procedures (including those involving algebraic expressions and matrices) in problem situations*

Toward this, students in **Class 11B** will be expected to master the following **SO** (Specific Outcomes):

11B-B1 Algebraic Expressions and Equations: perform operations

- ability to rewrite expressions and equations
- ability to solve expressions and equations

11B-B2 Arithmetic Operations: demonstrate understanding of the relationships with operations used when solving equations

- use of these properties and principals in relation with trigonometric expressions with exponents

11B-B3 Annuities: determine the amount and present value

- solve problems with respect to annuities
- use of time diagrams, formulas and technology should be incorporated

11B-B4 Quadratic Formula: apply

- apply the formula to equations, determine the roots and decide if the roots are admissible or inadmissible

11B-B5 Probabilities: calculate solve problems

- solve problems in a number of probability situations

11B-B6 Probabilities: determine using permutations and combinations

- apply to situations that can be solved without the use of formulas

11B-B7 Income and Deductions: estimate and calculate

- estimate and calculate various ways of making money
- estimate and calculate deductions form income

11B-B8 Budgets: solve problems

- solve problems relating to personal budgets

11B-B9 Purchase: analyze situations and make decisions involving finance

- with respect to personal budgets

Class 11B - Strand C - Patterns & Relations

KSO - Patterns: ***by*** *the end* ***of Class 12Business*** *students should:*

- *Model-real world problems using functions, equations, inequalities, and discrete structure*
- *Represent functional relationships in multiple ways and describe connections among those representations*
- *solve problems involving relationships, using graphing technology as well as paper and pencil techniques*
- *perform operations on and between functions*
- *analyze and explain the behaviours, transformations, and general properties of types of equations and relations*
- *interpret algebraic equations and inequalities geometrically and geometric relationships algebraically*

Toward this, students in **Class 11B** will be expected to master the following SO (Specific Outcomes):

11B-C1 Patterns: demonstrate understanding of arithmetic, power and geometric

- knowledge of levels of common differences between terms in arithmetic, power and geometric patterns
- understanding that geometric patterns do not show common differences but have common multiples
- comparison of arithmetic, power and geometric patterns to linear, power and
- exponential relations

11B-C2 Patterns: determine and describe to solve problems

- describe: numeric and other patterns, properties of exponents and exponential growth
- identify: patterns in table of differences and symmetric patterns

11B-C3 Tables and Graphs: analyze distinguish between linear, quadratic and exponential relationships

- ability to differentiate between linear, quadratic and exponential relationships
- study of tables and graphs should be addressed

11B-C4 Quadratic Equations: solve problems

- solve a variety of problems involving quadratic equations by graphing or using the quadratic formula
- ability to identify maximum/ minimum values, find roots, and interpolate/ extrapolate should be studied

11B-C5 Quadratic Relationships: describe and translate between different forms

- situations that are presented in words, words and equations, graphs and tables of values should be examined

11B-C6 Exponential Equations: solve problems

- solve real - world exponential equations

11B-C7 Exponential Relationships: describe and translate between different forms

- describing and translating between various representations of exponential and logarithmic relationships is vital
- focus on patterns with exponential functions and the nature of the inverse relationship between exponential and logarithmic functions

11B-C8 Quadratic Equations: determine and interpret x-intercepts

- determine x-intercepts as a means of solving quadratic equations
- interpretation of solutions is important in relation to the given context to determine if the solutions are admissible or not

11B-C9 Quadratic Relationships: describe and apply

- examine characteristics such as: common second - level differences, parabolic
  shapes, symmetry, maximum / minimum values

11B-C10 Exponential Relationships: describe and apply

- examine characteristics such as: common multiples, accelerating growth/ decelerating decay, asymptotes, focal point

11B-C11 Scatterplots: create, analyze and determine equations for curves of best fit

- use of appropriate technology to determine equations for curves of best fit
- scatter plots should be created from assembled data

11B-C12 Trigonometric Equations: solve

- ability to substitute trigonometric values for known angles to solve unknown measurements
- ability to use known information to determine trigonometric values for unknown angles
- ability to solve angles using inverse trigonometric relationships

11B-C13 Quadratic Equations: model real-world phenomena

- describe real world situations using quadratic functions

11B-C14 Interpolate and Extrapolate: solve problems

- interpolate and extrapolate to solve problems with respect to aspects of personal budget situations
- interpolate and extrapolate situations with functions to answer question relating to a given situation

11B-C15 Systems of Equations: solve problems

- develop and apply in contextual situations

11B-C16 Infinity: investigate and apply the concept by examining sequences and series

- identify a situation when a sequence or series would approach infinity
- define infinity, examples would include a bank balance compounding
  monthly, or sequences whose terms continue to grow or decay will approach infinity
- by examining limits students will come across sequences that have no limits

- understanding that infinite sequences and series are those that continue
  indefinitely

Class 11B - Strand D – Measurement

KSO - Measurement: *by the end of **Class** 12 **Business** students should:*

- *measure quantities indirectly, using techniques of algebra, geometry and trigonometry, including angle and arc relationships, and degree and radian measures*

Toward this, students in **Class 11B** will be expected to master the following **SO** (Specific Outcomes):

11B-D1 Trigonometry: (sine, cosine, tangent ratios) - solve problems

- use of primary trigonometric ratios with respect to the sine law, cosinelaw, and area of a triangle
- Include problems with non-acute angles

11B-D2 Trigonometry: (Sines, the Law of Cosines and the formula 'area of a triangle ABC = .5 bcsinA') - apply the laws to solve problems

- apply Law of Sines, Law of Cosines and the formula for the area of triangle in a number of problem situations
- knowledge of the formulas for:

Law or Sines $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ also expressed as $\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$

Law of Cosines $a^2 = b^2 + c^2 - 2bc\ CosA$

Area of a triangle, Area = $\frac{1}{2}$ bcSinA

Class 11B - Strand E – Geometry

KSO - Geometry: *by the end of **Class 12 Business** students should:*

- *extend spatial sense in a variety of mathematical contexts*
- *interpret and classify geometric figures, translate between synthetic(Euclidean) and coordinate representations, and apply geometric properties and relationships*

Toward this, students in **Class 11B** will be expected to master the following **SO** (Specific Outcomes):

11B-E1 Symmetry: describe and apply

- identify in tables and on graphs the symmetric nature of quadratic relationships
- connection between symmetry and maximum/minimum values should be made

Class 11B - Strand F - Data & Probability

KSO - Data & Probability: *by the end of **Class 12Business** students should:*

- *use curve fitting to determine the relationship between, and make predictions from, sets of data*
- *be aware of bias in the interpretation of results*

Toward this, students in Class **11B** will be expected to master the following **SO** (Specific Outcomes):

11B-F1 Bias: identify in data collection, interpolation and presentation

- understanding that some sampling methods may produce results that are not representative of the population
- identification that bias can enter the interpretation of results based on sampling
- realization that bias is the difference between the results obtained and the truth

  about the whole population when an error occurs due to bad sampling

11B-F2 Levels of Confidence: interpret normal curves and standard deviation

- use previous knowledge to give levels of confidence with experimental/survey data
- determination of how much deviation from the expected is reasonable

11B-F3 Statistics: calculate, analyze and interpret

- calculation of mean, median, range and standard deviation is expected
- use of mean, median, range and standard deviation in making inferences concerning data

11B-F6 Graphs and Tables: draw inferences

- draw inferences based on: data trends, data distribution and data variability
- examination of graphs, tables and displays of samples of data should be included

11B-F7 Interpolate and Extrapolate: predict and solve problems

- examine graphs and equations to interpolate and extrapolate to solve problems

11B-F8 Samples: draw inferences about a population

- knowledge of issues with respect to sampling strategies and sample size is
  essential to properly draw inferences
- realization that there is a large number of possible samples from a population

11B-F9 Sample Means and Dispersions: demonstrate understanding of what can be inferred about a population

- understanding of the relationship between a population and a sample
- knowledge of how sample mean and dispersion relate to the characteristics of the population
- understanding that experimental results will be different than expected results in most cases
- understanding of how much deviation from the expected results is considered normal

11B-F10 Sample: demonstrate understanding of how the size affects the variation in sample results

- realization that a large sample size gives a greater chance that the statistical results will approximate expected values

11B-F11 Simulations: develop and apply to solve problems

- design simulations to model and solve problem situations
- a connection between simulation results to the original problem

11B-F12 Area Diagrams and Tree Diagrams: apply to interpret and determine probabilities of independent and dependent events (Now F12)

- use of diagrams to assist with quantifying outcomes and determining probabilities

11B-F13 Probability and Odds: demonstrate an understanding (Now F13)

- understanding that odds are a specific way a express probabilities
- aility to translate from odds to probabilities and vice versa is essential.

Class 12 B - Strand A – Numbers

KSO - Numeration: *by the end* ***of Class 12 Business*** *students should:*

- *order real numbers, represent them in multiple ways and apply appropriate representations to solve problems*
- *demonstrate an understanding of the real number system and it 's subsystems by applying a variety of number theory concepts in relevant situations*

Toward this, students in **Class 12B** will be expected to master the following SO (Specific Outcomes):

12B-A1 Irrational numbers: demonstrate an understanding in applications

- distinguish between exact and approximate values of irrational numbers
- perform operations on expressions involving radicals

Class 12 B - Strand B – Operations

KSO - Operations: ***by*** *the end* ***of Class 12Business*** *students should:*

- *Explain how algebraic and arithmetic operations are related, use them in problem-solving situations, and explain and demonstrate t he power of mathematical symbolism*
- *Derive, analyze, and apply computational procedures in situations involving all representations of real numbers*
- *Derive, analyze, and apply algebraic procedures (including those involving algebraic expressions and matrices) in problem situations*

Toward this, students in **Class 12B** will be expected to master the following SO (Specific Outcomes):

12B-B1 Rational Algebraic Expressions and Operations Fractions: demonstrate an understanding of the relationship

- simplify algebraic expressions while keeping mind earlier work with fractions
- this will be used in proving trigonometric identities

12B-B2 Equation of a Plane: determine given three points on the plane (Now B5)

- create the equation of a plane
- use the equation of a plane Ax + By + Cz + D = 0 and three points on the plane to determine the equation of the plane

12B-B3 Systems of "m" Equations in "n" Variables: solve **(Now** B6)

- solve systems of equations of various dimensions

Class 12B - Strand C - Patterns & Relations

KSO - Patterns: *by the end of **Class 12Business** students should:*

- *Model-real world problems using functions, equations, inequalities, and discrete structure*
- *Represent functional relationships in multiple ways and describe connections among those representations*
- *solve problems involving relationships, using graphing technology as well as paper and pencil techniques*
- *perform operations on and between functions*
- *analyze and explain the behaviors, transformations, and general properties of types of equations and relations*
- *interpret algebraic equations and inequalities geometrically and geometric relationships algebraically*

Toward this, students in **Class 12B** will be expected to master the following SO (Specific Outcomes):

12B-C1 Periodic Data: create and analyze scatter plots

- plot data points and interpret whether the data is sinusoidal or not

12B-C2 Systems of Equations: solve problems

- develop elimination method for solving 2x2 systems, higher order systems (3 x 3) and apply matrix methods to solve equations
- develop and apply in contextual situations

12B-C3 Non-Acute Angles: demonstrate an understanding of sine and12B-C6 Trigonometric Equations: apply function notation to trigonometric equations

- recognition that a trigonometric relationship is a function and can be expressed in function notation

12B-C7 Trigonometric Equations: analyze and solve

- algebraically solve trigonometric equations
- work with angles in both degrees and radians should be included

12B-C8 Trigonometric Functions: model situation

- graph sine and cosine
- model real-world situations

12B-C9 Polar Plane: construct and examine graphs

12B-C10 Rectangular and Polar Coordinates: translate between each

12B-C11 Continuity, Limits and Functions: explore and describe the connections

- recognize the conditions for which a limit exists
- focus on continuity with respect to rational and piece-wise functions
- realization of the connection between discontinuities with asymptotes **of** graphs
- make a connection between the characteristics of the defining algebraic expressions with characteristics of the graphs
- use of limits to help identify the nature and location of discontinuities
- step functions and one-sided limits

12B-C12 Limit: demonstrate an intuitive understanding for the concept

- show an understanding of limits in everyday situations
- understanding of the connection between limits and infinite sequences and series should be developed
- solve a limit question like the above using an algebraic approach and by using

technology $\lim_{x\to\infty}\left[\frac{4n+1}{n}\right]$

12B-C13 Average Rate of Change: calculate (Intro)

12B-C14 Derivative: explore as the instantaneous rate of change (Intro)

12B-C15 Asymptotic Behaviour: demonstrate an understanding (intro)

12B-C16 Maximum, Minimum, And Points Of Inflection Of A Graph: determine in applications related to Economics and Commerce (Intro) cosine ratios and functions

- compare trigonometric values of first quadrant angles to those in the other three quadrants
- work in both degree and radians
-

12B-C4 Trigonometric Identities: explore and verify

- ♦ derive compound angle identities
- ♦ use compound angle identities, once derived, to prove other identities and solve problems

12B-C5 Pythagorean Identities: derive and apply the reciprocal

- ♦ derive or prove quotient and Pythagorean identities

Class 12B - Strand D – Measurement

KSO - Measurement: *by the end of Class 12 Business students should:*

- ♦ *measure quantities indirectly, using techniques of algebra, geometry and trigonometry, including angle and arc relationships, and degree and radian measures*

Toward this, students in **Class 12B** will be expected to master the following **SO** (Specific Outcomes):

12B-D1 Angle and Arc Length Relationships: derive, analyze, and apply

- ♦ within the context of the unit circle determination of arc length in connection with rotating points should be developed
- ♦ determination of how the arc length is affected by changes in circle radius should be made

12B-D2 Degree & Radian Measures: demonstrate an understanding of the connection and apply them

- ♦ define angle measures in radians as the ratio of arc length to radius
- ♦ development of the ability to interchangeably use degrees and radians should be made

Class 12B - Strand E – Geometry

KSO - Geometry: *by the end of Class 12 Business students should:*

- ♦ *extend spatial sense in a variety of mathematical contexts*
- ♦ *interpret and classify geometric figures, translate between synthetic(Euclidean) and coordinate representations, and apply geometric properties and relationships*

Toward this, students in **Class 12B** will be expected to master the following **SO** (Specific Outcomes):

12B-E1 3-space: demonstrate an understanding of the position of axes

- understanding that the three axes (x, y, z) intersect at right angles at a single point
- point of intersect is described as the origin, (0, 0, 0)

12B-E2 3-Space: locate and identify points and planes

- locate points and planes by concrete modeling and sketching
- realization that 3 points define a plane and practice techniques to sketch the planes should be developed

Class 12B - Strand F - Data and Probability

- **11B-F1 Index Numbers**
- **11B-F2 Methods of Aggregates**
- **12B-F3Correlation and Regression: meanings and application**
- **12B-F4 Correlations and Regression: Karl Pearson's Coefficient**

Class 11P - Strand A – Numbers

KSO - Numeration: *by the end of* ***Class 12 Pure*** *students should:*

- *order real numbers, represent them in multiple ways and apply appropriate representations to solve problems*
- *explain and apply relationships among real and complex numbers*

Toward this, students in **Class IIP** will be expected to master the following **SO** (Specific Outcomes):

11P-A1 Roots of Quadratic Equations: demonstrate an understanding

- understanding that there are three possibilities for the roots of a quadratic equation: two real, distinct roots, 2 real, equal roots and no real roots (complex roots)
- comparison of the value of the discriminant to the nature of the graph should be made

11P-A2 Irrational Numbers: understanding in applications

- knowledge of whether an exact or an approximate solution is appropriate
- exact answers should be expressed in simplified form Note: This would include changing an entire radical into mixed radical, for example

$$\sqrt{16x3} = \sqrt{16} \text{ x } \sqrt{3} = 4\sqrt{3}$$

11P-A3 Factorial Notation: permutations and combinations

- develop in connection with tree diagrams and the fundamental counting principle recognize that n! (n-factorial) represents the number of ways to arrange n distinct objects

11P-A4 Real Numbers: (exponential and logarithmic expressions and equations) demonstrate an understanding

- knowledge of real number (rational and irrational) exponents will be explored for rational exponents include:
- $a^{\frac{1}{n}} = \sqrt[n]{a}$, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ $a^{-\frac{m}{n}} = \left(\frac{1}{n}\right)^{\frac{m}{n}}$ $a^0 = 1$,

$a \neq 0$ and m and n are both positive integers

11P-A5 Domain and Ranges: describe and interpret using set notation

- include **quadratic,** exponential and logarithmic functions
- domain from 1 to 20 should be symbolized as $\{ 1 < x < 20, x \in R\}$, or by interval notation, $x \in [1, 20]$
- domain of greater than 1 but less than 20 should be symbolized as $\{1 < x < 20, x \in R\}$, or by interval notation, ie(l, 20)

11P-A6 Recursive Formula: demonstrate an understanding

- express a sequence of numbers or a diagram of an arithmetic or geometric pattern as a recursive relation
- knowledge that in a recursive formula any given term in a sequences depends on one)or more) of the previous terms

11P-A7 Arithmetic and Geometric Sequences: represent as ordered pairs and discrete graphs

- make a table of values and graph a recursive routine
- represent sequences as ordered pairs and as 2-D graphs
- realization that graphs are discrete, rather than continuous, given that the first element is restricted to the natural numbers

Class IIP - Strand B – Operations

KSO - Operations: ***by*** *the end of* ***Class 12 Pure*** *students* ***should:***

- *Explain how algebraic and arithmetic operations are related, use them in problem-solving situations, and explain and demonstrate the power of mathematical symbolism*
- *Derive, analyze, and apply computational procedures in situations involving all representations of real numbers*
- *Derive, analyze, and apply algebraic procedures (including those involving algebraic expressions and matrices) in problem situations*
- *Apply operations on complex numbers to problems situations*

Toward this, students in **Class IIP** will be expected to master the following SO (Specific Outcomes):

11P-B1 Real Number Exponents: apply in expressions and equations

- restate expressions and/or equations in equivalent forms by applying exponent laws involving real numbers

11P-B2 Arithmetic Operations: relationships with operations used when solving equations

- use of these arithmetic properties and principals in relation with trigonometric expressions with exponents

11P-B3 Quadratic Formula: Derive and apply

- understanding the connection to the nature of the roots is important

11P-B4 Exponential Growth: demonstrate an intuitive understanding of the recursive nature

- successive terms in an exponential sequence are generated by multiplying each preceding term by a constant factor

11P-B5 Trigonometry: (the Law of Sines, Law of Cosines, and the formula 'area of triangle ABC = 0.5 bcsinA')- derive and analyze

- ability to develop both laws from basic principals is important
- area of a triangle should be derived using previous knowledge of geometry and basic trigonometry

Note: Include the ambiguous case (when two sides and a non-contained angle are given) in the applications of the Law of Sines.

11P-B6 Logarithms: demonstrate and apply properties

- development of logarithms properties based on experience with understanding of exponent laws. (Includes laws of logarithms for products, quotients, powers, and roots) Note: Should include using the properties to simplify or evaluate expressions containing logarithms.
- connections should be made between exponential and logarithms relations

11P-B7 Terms in a Sequence: develop, analyze and apply algorithms to generate

- read a word problem and generate terms for an arithmetic sequence
- from a word problem come up with a formula and find the n^{th} term of the sequence
- examine sequences to develop algorithms

Class IIP - Strand C - Patterns & Relations

KSO - Patterns: ***by*** *the end of* ***Class 12 Pure*** *students should:*

- *model-real world problems using functions, equations, inequalities, and discrete structure*
- *represent functional relationships in multiple ways and describe connections among those representations*
- *solve problems involving relationships, using graphing technology as well as paper and pencil techniques*
- *perform operations on and between functions*
- *analyze and explain the behaviors, transformations, and general properties of types of equations and relations*
- *interpret algebraic equations and inequalities geometrically and geometric relationships algebraically*
- *describe and explore the concept of continuity of a function*
- *investigate limiting processes by examining infinite sequences and series*
- *make connections among trigonometric functions, polar coordinates, complex numbers and series*

Toward this, students in **Class IIP** will be expected to master the following SO (Specific Outcomes):

11P-C1 Patterns: (arithmetic, power and geometry) demonstrate understanding and relate corresponding functions

- knowledge of levels of common differences between terms in arithmetic, power and geometric patterns
- understanding that geometric patterns do not show common differences but have common multiples
- comparison of arithmetic, power and geometric patterns to linear, power and exponential relations

11P-C2 Tables and Graphs: analyze to distinguish between linear, quadratic and exponential relationships

- ability to differentiate between linear, **quadratic** and exponential relationships
- study of tables and graphs should be addressed

11P-C3 Quadratic Equations: analyze and describe the characteristics

- analyze representations of quadratic functions to describe characteristics such as symmetry, vertices and intercepts. Note: Describe characteristics from given graphs. Find characteristics from equations.
- express symmetry in terms of equations of axes of symmetry, vertices as maximum/minimum values and range and intercepts as the roots of the equation

11P-C4 Graphs: sketch from descriptions, tables and collected data

- sketch scatter plots from descriptions, tables and collected data
- based on scatter plots, ability to distinguish patterns, translating between representations and fitting curves and equations to the data should be developed

11P-C5 Quadratic Functions: model real-world phenomena

- describe real world situations using quadratic functions

11P-C6 Exponential Functions: model real-world phenomena

- describe real world situations using exponential functions

11P-C7 Quadratic Equations: solve

- use of general quadratic formula to solve quadratic equations is new at this level

Note: i) Need to emphasize the connection between a quadratic function $y = ax + bx + c$, $a \neq 0$, and a quadratic equation $ax^2 + bx + c = 0$. By letting _y = 0, we are solving for the x-intercepts of the related graph.
ii) Not all quadratic equations require the quadratic formula. Some equations can be factored. Factoring quadratic equations should be included.

- factoring quadratic trinomials of the form $x + bx + c = 0$, $ax + bx + c = 0$, $a \neq 0$ or 1, perfect square trinomials, and difference of squares

11P-C8 Quadratic Equation: relate the nature of the roots and the x-intercepts of the graphs of corresponding functions

- connection that: two x-intercepts give two distinct roots, one x-intercept gives two
- equal roots and no x-intercepts gives no real roots

11P-C9 Exponential and Logarithmic Relationships: describe and translate between different forms

- describing and translating between various representations of exponential and logarithmic relationships is vital
- focus on patterns with exponential functions and the nature of the inverse relationship between exponential and logarithmic functions

11P-C10 Exponential and Logarithmic Equations: solve

- solve a variety of exponential and logarithmic equations

11P-C11 Quadratic Equations: solve problems

- solve a variety of problems involving quadratic equations by graphing or using the quadratic formula
- ability to identify maximum/ minimum values, find roots, and interpolate/ extrapolate should be studied

11P-C12 Quadratic Functions: parameter changes affect the graphs of quadratic functions

- altering one of the parameters in quadratic functions produces related families of quadratic functions
- connect with stretches, reflections and translations of graphs

11P-C13 Quadratic Functions: translate and describe between different forms

- sketching graphs and analyzing tables and graphs should be included
- situations that are presented in words, words and equations, graphs and tables of values should be examined

11P-C14 Exponential and Logarithmic Equations: solve problems

- solve real - world exponential and logarithmic equations. Include factored form

11P-C15 Quadratic Equations: translate between different forms

- translate between general, transformational and standard forms of quadratic equation

Note: i) The Method of Completing the Square is required to put y — $\boldsymbol{ax}$ + bx + c, a ^0 into vertex or transformational form $y = SL(x$ - p) + q. Relate the form of the equation with information it may or may not provide for graphing. For example, when the equation is in vertex form we know the vertex, the axis of symmetry, the concavity, and the stretch

factor. With this information we can easily graph a parabola by using the vertex, and symmetry with one other point.

11P-C16 Exponential Function: (algebraically and graphically)

- understanding that exponential and logarithmic functions are inverses
- knowledge that the domain of a function is the range of its inverse and vice versa

11P-C17 Exponential and Logarithmic Functions: analyze and describe characteristics

- analyze exponential and logarithmic functions and describe characteristics such as shape, location of asymptotes, domain and range
- values of "a" should be $a > 1$ and $0 < a < 1$

IIP C18 Exponential Functions: how parameter changes affect the graphs

- altering the parameters of an exponential function produces families of exponential functions

11P-C19 Exponential Functions: Write in transformational form and as mapping rules to visualize and sketch graph

- connect algebraic representations of exponential functions to their graphs
- continuation of previous knowledge of transformations and mapping rules

11P-C20 Trigonometric Equations: analyze and solve, with and without technology

- solve for unknown measurements and unknown angles

11P-C21 Trigonometric Identities: prove and apply

- prove key identities and apply them as proofs

11P-C22 Quadratic Equations: explore and describe the connections and their inverses

- explain the steps of obtaining the inverse of a function
- apply previous knowledge regarding functions and their inverses to quadratic graphs
- examine the non-functional nature of the inverse

11P-C23 Problem Situations: model using discrete structures such as sequences and recursive formulas

- model situations involving arithmetic and geometric sequences, general rules, and discrete graphs
- ability to construct a sequence of numbers or write a recursive formula from a word problem should be developed. Note: requires knowledge of vertical line test (VLT), and understanding of how a function is different from a relation.

11P-C24 Recursive Formulas: demonstrate an understanding, and how they relate to a variety of sequences

- experience discovering, and applying not only arithmetic and geometric sequences as well as other sequences such as Fibonacci, Harmonic

11PC25 Continuity, Limits and Functions: explore and describe the connections

- recognize the conditions for which a limit exists
- focus on continuity with respect to rational and piece-wise functions
- realization of the connection between discontinuities with asymptotes of graphs
- make a connection between the characteristics of the defining algebraic expressions with characteristics of the graphs
- use of limits to help identify the nature and location of discontinuities
- step functions and one-sided limits

11P-C26 Slope Function: demonstrate an understanding and their connection to derivative function

- match a function with its slope function
- understanding that the slope function is derived from the original function
- sketch a slope function given the graph of the function
- include graphs of functions that have non-differentiable points such as the absolute value function
- Use preceding, or following intervals to calculate instantaneous rates of change of data given in a table, graph the new "function" and call it the slope function
- Identify features of the slope function

- Introduce the derivative function as the name given to the slope function

11P-C27 Limit, The Area Under A Curve, Rate Of Change, Slope Of A Tangent Line: demonstrate an understanding for the conceptual foundations and their applications

- calculate the area under a curve
- instantaneous rate of change can be determined by calculating the slope of smaller and smaller secant lines
- determine average and instantaneous rates of change from data using preceding, following and centered intervals
- determine average and instantaneous rates of change from equations using the difference quotient

11P-C28 Limit: demonstrate an intuitive understanding for the concept

- show an understanding of limits in everyday situations
- an understanding of the connection between limits and infinite sequences and series should be developed
- solve a limit question like the below using an algebraic approach and by using

  technology $\lim_{x\to\infty}\left(\frac{4n+1}{n}\right)$

11P-C29 Derivative of a Function: determine and apply

- find the expression for the slope of the tangent line, then determine the slope the given a-values on the curve: example $f(x) = 2x^2 + 3x - 2; a = 1$
- given the slope determine the tangent point on the curve with that slope
- given a point not on the curve, determine the equation(s) of the tangent line from the curve

Class IIP - Strand D – Measurement

KSO - Measurement: ***by*** *the end* ***of Class 12 Pure*** *students should:*

- *Demonstrate an understanding of the meaning of area under a curve*
- *Understand how to approximate using limits*

Toward this, students in **Class IIP** will be expected to master the following SO (Specific Outcomes):

11P-D1 Distance and Midpoint: develop and apply formula

- ♦ connection between Pythagorean Theorem and the distance formula should be developed. Note: Begin with distance from the origin to a point, then do distance between two points. Should include the development of the formulas based on Pythagorean theorem.
- ♦ midpoint formula should be connected with averaging

$$Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

11P-D2 Trigonometry: (Law of Sines, the Law of Cosines and the formula 'area of a triangle ABC = .5 bcsinA') - apply to solve problems

- ♦ apply Law of Sines, Law of Cosines and the formula for the area of triangle in a number of problem situations
- ♦ knowledge of the formulas for:

Law or Sines $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$ also expressed as $\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$

Law of Cosines $a^2 = b^2 + c^2 - 2bc\ CosA$

Area of a triangle, Area = $\frac{1}{2}$ bc SinA

11P-D3 Trigonometry: (sine and cosine ratios) - apply to situations involving non-acute angles

- ♦ sine and cosine values for obtuse angles when working with obtuse triangles

Class IIP - Strand E – Geometry

KSO - Geometry: ***by*** *the end of* ***Class 12 Pure*** *students should:*

- *Demonstrate an understanding of the operations of axiomatic systems and the connections among reasoning, justification and proof*

Toward this, students in **Class IIP** will be expected to master the following **SO** (Specific Outcomes):

11P-E1 Proofs: write using various axiomatic systems and assess the validity of deductive arguments

- exposure to Euclidean, transformational and analytical systems of proofs is required
- write and asses synthetic and analytical proofs

11P-E2 Converse: demonstrate understanding

- understanding that not all theorems have converses that are true
- use of "if and only **if** terminology in cases where a theorem and its converse are both true

11P-E3 Circles, Ellipses and Hyperbolas: write equations in transformational form

- key characteristics can be identified from transformational form of circles,
  ellipses and hyperbolas and through mapping rules

11P-E4 Circles: apply properties

- apply properties of circles to practical situations and proofs

11-E5 Circles: investigate, make and prove conjectures associated with chord properties

- make, prove and apply conjectures regarding the properties of chords in circles

11P-E6 Circles: investigate, make and prove conjectures associated with angle relationships

- make, prove and apply conjectures regarding angles within circles

11P-E7 Circles: investigate, make and prove conjectures associated with tangent properties

- make, prove and apply conjectures regarding angles within circles

Class IIP - Strand F - Data & Probability

KSO - Data & Probability: *by the end of Class **12 Pure** students should:*

- *Demonstrate an understanding of combinations and permutations and the difference between the two.*
- *Develop and apply formulas for finding permutations and combinations and the probabilities for each*
- *Identify and apply patterns involving exponents in binomial expansion*
- *use curve fitting to determine the relationship between, and make predictions from, sets of data*
- *be aware of bias in the interpretation of results*

Toward this, students in **Class IIP** will be expected to master the following **SO** (Specific Outcomes):

11P-F1 Combinations and Permutations: distinguish between situations

- distinguish between situations that involve combinations and those that involve permutations
- knowledge that combinations are the unordered collections of objects, while permutations involve an ordering of object is important

IIP F2 Combinations and Permutations: develop and apply formulae to evaluate

- develop and apply formulae to calculate permutations and combinations
- determine probabilities involving permutations and combinations

IIP F3 Binomial Expansion: demonstrate understanding and its connection to combinations

- apply pattern of exponents in binomial expansion
- understanding of the connection between coefficients and combinations

Note: Find terms in the binomial expansions by expanding and looking at patterns using Pascal's Triangle.

11P-F4 Bias: identify in data collection, interpolation and presentation

- ♦ understanding that some sampling methods may produce results that are not representative of the population
- ♦ identification that bias can enter the interpretation of results based on sampling
- ♦ realization that bias is the difference between the results obtained and the truth
  about the whole population when an error occurs due to bad sampling

11P-F5 Levels of Confidence: interpret normal curves and standard deviation

- ♦ use previous knowledge to give levels of confidence with experimental/survey data
- ♦ determination of how much deviation from the expected is reasonable

11P-F6 Statistics: calculate, analyze and interpret

- ♦ calculation of mean, median, range and standard deviation is expected
- ♦ use of mean, median, range and standard deviation in making inferences concerning data

11P-F7 Graphs and Tables: draw inferences

- ♦ draw inferences based on: data trends, data distribution and data variability
- ♦ examination of graphs, tables and displays of samples of data should be included

11P-F8 Interpolate and Extrapolate: predict and solve problems

- ♦ examine graphs and equations to interpolate and extrapolate to solve problems

11 P-F9 Samples: draw inferences about a population

- ♦ knowledge of issues with respect to sampling strategies and sample size is
  essential to properly draw inferences
- ♦ realization that there is a large number of possible samples from a population

11P-F10 Sample Means and Dispersions: demonstrate understanding of what can be inferred about a population

- understanding of the relationship between a population and a sample
- knowledge of how sample mean and dispersion relate to the characteristics of the population
- understanding that experimental results will be different than expected results in most cases
- understanding of how much deviation from the expected results is considered normal

11P-F11 Sample: demonstrate understanding of how the size affects the variation in sample results

- realization that a large sample size gives a greater chance that the statistical results will approximate expected values

11P-F12 Simulations: develop and apply to solve problems

- design simulations to model and solve problem situations
- a connection between simulation results to the original problem

11P-F13 Area Diagrams and Tree Diagrams: apply to interpret and determine probabilities of independent and dependent events (Now F12)

- use of diagrams to assist with quantifying outcomes and determining probabilities

11P-F14 Probability and Odds: demonstrate an understanding (Now F13)

- understanding that odds are a specific way a express probabilities
- ability to translate from odds to probabilities and vice versa is essential.

Class 12P - Strand A – Numbers

KSO - Numeration: *by the end of **Class 12 Pure** students should:*

- *order real numbers, represent them in multiple ways and apply appropriate representations to solve problems*
- *explain and apply relationships among real and complex numbers*

Toward this, students in **Class 12P** will be expected to master the following **SO** (Specific Outcomes):

12P-A1 Value of 'e': determine, describe and apply

- graph $f(x) = e^x$
- state the domain, range and the equation of its inverse
- arithmetic and Geometric Sequences: represent as ordered pairs and discrete graphs **that "e" is an irrational number**

12P-A2 Series: represent in expanded form and using sigma notation

- understand and use the sigma notation
- knowledge of how to write an expanded form using sigma notion

12P-A3 Real and Complex Numbers: explain the connection between the two

- describe what complex numbers are
- knowledge that complex numbers are in the form a + bi
- knowledge that real numbers are a subset of complex numbers when b = 0

12P-A4 Polar and Rectangular Representation: Translate between the two

- able to change complex numbers from polar to rectangular vice versa

12P-A5 Quadratic Equations Roots: (non-real) as complex numbers

- representation of $i^2 = -1$ should be made

Class 12P - Strand B – Operations

KSO - Operations: *by the end of **Class 12 Pure** students should:*

- *Explain how algebraic and arithmetic operations are related, use them in problem-solving situations, and explain and demonstrate the power of mathematical symbolism*
- *Derive, analyze, and apply computational procedures in situations involving all representations of real numbers*
- *Derive, analyze, and apply algebraic procedures (including those involving algebraic expressions and matrices) in problem situations*
- *Apply operations on complex numbers to problems*

Toward this, students in **Class 12P** will be expected to master the following **SO** (Specific Outcomes):

12P-B1 Arithmetic Operations: describe the relationship on rational algebraic expressions and equations

- add or subtract multiply and divide rational algebraic expressions and understand the application of work with numeric fractions

12P-B2 Sum of a Series: develop, analyze and apply algorithms

- understand and use $S_n = \frac{n}{2}(t_1 + t_n)$ and $S_n = \frac{t_1(r^n - 1)}{r - 1}$
- include the development of each formula; other forms of each of these formulas; real world applications for each formula

12P-B3 Complex Numbers: apply operations, both in rectangular and polar form

- add, subtract, multiply, divide, and evaluate powers of complex numbers in rectangular form
- find products, quotients and powers when complex numbers are expressed in polar form and exponential forms

12P-B4 DeMoivre's Theorem : develop and apply for powers

- use DeMoivre's theorem to determine powers of complex numbers in polar form
- examples could include: $(-3+i)^8$ or $(3cis210^\circ)^9$ Note: include the proof of DeMoivre's theorem
- roots of complex numbers

Class 12P - Strand C – Patterns & Relations

KSO - Patterns: *by the end of **Class 12 Pure** students should:*

- *model-real world problems using functions, equations, inequalities, and discrete structure*
- *represent functional relationships in multiple ways and describe connections among those representations*
- *solve problems involving relationships, using graphing technology as well as paper and pencil techniques*
- *perform operations on and between functions*
- *analyze and explain the behaviors, transformations, and general properties of types of equations and relations*
- *interpret algebraic equations and inequalities geometrically and geometric relationships algebraically*
- *describe and explore the concept of continuity of a function*
- *investigate limiting processes by examining infinite sequences and series*
- *make connections among trigonometric functions, polar coordinates, complex numbers and series*

Toward this, students in **Class 12P** will be expected to master the following **SO** (Specific Outcomes):

12P-C1 Tables and Graphs: use as tools to interpret expressions

- use of tables and graphs should be used through out the course of study to assist with the understanding of algebraic relationships and function characteristics

12P-C2 Relations, Functions and their Graphs: analyze

- ability to visualize and identify the graphs of functions, stating the domain, range, and position should be developed
- using tables, graphs, and algebraic techniques analyze a variety of relation and functions

12P-C3 Complex and Polar Planes: construct and examine graphs

- plot complex numbers on the Argand plane and construct graphs of polar equations

Note: i) include polar equations of the form $r = 3 + 5\cos\theta$ which produce a limacon, and equations of the form $r = 4\cos 2\theta$ which produce a four-leaved rose.

ii) use the properties of symmetry for the construction of the above graphs.

12P-C4 Problem Situations: model with combinations and compositions of functions

- develop and analyze a function made from the combination or composition of two other functions
- include irrational and absolute value functions
- combinations and compositions should be related to contexts to make them more meaningful

12P-C5 Polynomial Functions and Rational Functions: model real-world phenomena

- model real-world situations to make work with polynomial and rational functions meaningful

12P-C6 Polynomial and Rational Functions: determine the equations

- from the roots of a polynomial function, determine the equation
- given a word problem ,determine a polynomial function and solve the equation to solve the problem

12P-C7 Polynomial, Rational, Irrational, and Absolute Value Equations: analyze and solve

- solve for the x and y intercepts of various equations
- write equations in general form and graph the equations.
- use a variety of methods to determine the roots of various equations

12P-C8 Polynomial, Rational, Irrational, And Absolute Value Inequalities: solve

- solve a number of inequality questions using a variety of methods

12P-C9 Polynomial Expressions: factors

- factor polynomial expressions using a variety of methods

12P-C10 Functions: analyze the effect of parameter changes on the graphs and express the changes using transformations

- transformations

- recognize different transformations in functions and be able to visualize the transformation from the equation
- connect parameter changes with variations in graphs of corresponding functions

12P-C11 Functions: investigate and interpret combinations and compositions

- examine a variety of combinations and composition of functions involving linear and quadratic functions
- describe rational functions as quotients of two functions
- describe irrational and absolute value functions as compositions

12P-C12 Asymptotic Behavior: demonstrate an understanding

- understanding of what an asymptote is and indicate one on a graph
- examine horizontal, vertical and oblique asymptotes

12P-C13 Exponential Growth and Decay: extend an understanding through multiple contexts (Now C16)

- revisit exponential and logarithmic functions
- find derivatives of exponential functions
- find derivatives of logarithmic functions

Note: i) Include non-derivative applications related to growth and decay problems, the Richter scale, PH scale, Decibel scale etc.)
ii) Include applications that involve derivatives of these functions.

12P-C14 Complex Numbers: represent in a variety of ways

- represent complex numbers in rectangular and polar forms

12P-C15 Rectangular and Polar Coordinates: translate between each

- convert between Cartesian and polar coordinates and vice-versa

12P-C16 Continuity and Discontinuity of Limits of Functions: explore the connection

- identify the conditions that make a function continuous or discontinuous
- determine one-sided limits
- use limits to identiy points of discontinuity given the graph of a function

- given the graphs of continuous and discontinuous functions, determine limits at different points on the graphs
- use limits to determine if a function is continuous or discontinuous given its equation

12P-C17 Discontinuities: demonstrate an understanding of the three types (removable, infinite, and jump)

- understand and identify removable, infinite and jump discontinuities from given graph
- determine if the graph of a function will have a removable, infinite, or jump discontinuity given its equation

12P-C18 Average Rate of Change: calculate

- determine average rate of change from data and graphs;
- determine average rate of change from equations

12P-C19 Derivative and the Slope Function: understand the connection

- use the slope of the tangent to sketch the graph of the derivative function from a given graph of a function
- sketch the slope function of the graphs of continuous, piecewise linear, and discontinuous functions

12P-C20 Derivative: explore as the instantaneous rate of change

- determine the instantaneous rate of change given data in a table or a graph by using preceding, following, or centered intervals
- determine the instantaneous rate of change from equations by using the difference quotient
- establish the relationship between instantaneous rate of change and the derivative; identify the derivative as a new function

Note: i) Make the connection between the slope of the tangent line as the limit of the slopes of secant lines as the distance between the two points on the secant line approaches 0.
iii) Relate instantaneous rate of change to the derivative.
iv) Introduce the derivative notation, which will be studied in detail in the next chapter.
v) Include finding the equation of the tangent line to a given point on a curve (either graph is given or equation is given, the slope will be an estimate)

12P-C21A + B Derivatives: find using first principle definitions

- use the definition $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to prove the sum, difference, power and product rules for derivatives
- use the definition $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to prove the derivative of $y = \sin x$
- use the definition $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to derive and justify the derivatives of polynomial and rational functions

12P-C22 Functions: Analyze and interpret their graphs

- use the first derivative to determine the intervals of increase and decrease of polynomial, rational, and root or irrational functions
- use the first derivative to determine the critical values, minimum and maximum points of polynomial, rational, and root or irrational functions
- use the second derivative to determine the intervals of concavity and points of inflection of polynomial, rational, and root or irrational functions
- determine the domain and range of polynomial, rational, and root or irrational functions
- determine the point discontinuities, and/or asymptotes of rational and root or irrational functions
- sketch the graphs of polynomial functions, rational functions, and root or irrational functions and identify their characteristics

12P-C23A, B, + C Differentiation: derive and apply the sum, difference, power, product, quotient and chain rule for using algebraic, trigonometric and exponential functions and logarithmic functions

- derive, using first principles, and apply the sum, difference, and power rule
- derive, using first principles, and apply the product rule
- derive, using first principles, and apply the quotient rule
- apply the chain rule

- derive, using first principles, and apply derivatives of trigonometric functions

Note: Use $y = \sin x$ and its derivative to prove the derivatives of the other trigonometric trig ratios. We should include all 6 trig ratios (if students learned about these in 11P.)

- derive and apply derivatives of exponential functions
- derive and apply derivatives of logarithmic functions

12P-C24 Differentiation: implicit differentiation

- understand the difference between explicit and implicit differentiation and when each is used
- use derivative rules to determine y' for relations such as $x^2 + y^2 = 25$
- determine the slope and equations of tangent lines to relations need to be differentiated implicitly

12P-C25 Differentiation: higher order derivatives

- use the derivative rules to determine the first, second, third, and nth derivative of functions
- determine the second derivative of relations using implicit differentiation

12P-C26 Differentiation: solving related rate and optimization problems

- identify a related rate problem as one in which two or more quantities that vary are related
- apply implicit differentiation to solve related rate problems involving areas, volumes and perimeters
- apply implicit differentiation to solve related rate problems involving the Pythagorean theorem and quantities not related by measurement formulas
- understand that an optimization problem is a problem in which a quantity is to maximized or minimized
- apply derivative rules to solve optimization problems

Note: Use the second derivative test to verify maximums and minimums in the optimization problems.

12P-C27 Anti-derivatives: understand the process as the integration of a function

- identify the relationship between the derivative and the anti-derivative of a function
- find the anti-derivative of functions by using the related derivative

12P-C28 Anti-derivatives: area between curves

- identify the relationship between anti-derivatives and the area under a curve
- determine the area between the *x*-axis and a curve
- determine the points of intersection of two curves
- determine the area between two curves

12P-C29 Anti-derivatives: Fundamental Theorem of Calculus

- state the Fundamental Theorem of Calculus for continuous functions
- apply the Fundamental Theorem of Calculus for continuous functions

12P-C30 Anti-derivatives: volumes of revolution (application)

- recognize the relationship between the volumes of a solids and integrals
- apply integration to evaluate the volumes of solids formed by rotating a plane region about a line.

12P-C31 Integration: understand the process using algebraic functions

- identify the relationship between differentiation and integration
- use the derivative rules for algebraic functions to determine simple integration rules
- recognize the difference between a definite and indefinite integral
- apply simple integration rules to determine definite and indefinite integrals

12P-C32 Integration: use several methods such as substitution and integration by parts

- understand the relationship between the product rule for differentiation and the method of substitution for integration
- understand the relationship between the chain rule for derivatives and the method of integration by parts

- recognize when to use the method of substitution or the method of integration by parts
- apply the method of substitution to integrate functions
- apply the method of integration by parts to integrate functions
- apply both methods to definite and indefinite integrals

12P-C33 Divergent and Convergent Geometric Series

- identify divergence or convergence by graphing or showing limits
- note a connection between sequences and series and convergence and divergence
- knowledge that limits will be applied to determine sums of infinite geometric series
- explore the equation $S_n = \frac{t_1}{1-r}$ to determine sums of infinite series
- compare convergent and divergent geometric series

12P-C34 Infinity

- identify a situation when a sequence or series would approach infinity
- define infinity. examples would include a bank balance compounding monthly, or sequences whose terms continue to grow or decay will approach infinity
- by examining limits students will come across sequences that have no limits
- understanding that infinite sequences and series are those that continue indefinitely

12P-C35 Trigonometric Identities: Prove and apply trigonometric identities

- such as $\sin(x + y)$, $\cos(x + y)$

Class 12P - Strand D – Measurement

KSO - Measurement: *by the end of **Class 12 Pure** students should:*

- *Demonstrate an understanding of the meaning of area under a curve*
- *Understand how to approximate using limits*

Toward this, students in **Class 12P** will be expected to master the following **SO** (Specific Outcomes):

12P-D1 Area Under Curve: demonstrate an understanding of how to approximate using limits

- approximate the area under a curve using sequences of rectangles
- determine the exact values for areas with the use of limits

Class 12P - Strand E – Geometry

KSO - Geometry: *by the end of **Class 12 Pure** students should:*

- *Demonstrate an understanding of the operations of axiomatic systems and the connections among reasoning, justification and proof*

Toward this, students in **Class 12P** will be expected to master the following **SO** (Specific Outcomes):

12P-E1 Mathematical Arguments and Proofs: develop and evaluate

- prove and evaluate, conjectures using mathematical induction
- prove problems where the conjecture is given (there are a large variety of problems, need to decide the extent we wish to include i.e. equalities, inequalities, and divisibility to name a few)
- prove problems where the conjecture must be first be found (these are more difficult)

Class 12B - Strand F - Data and Probability

- **11P-F1 Index Numbers**
- **11P-F2 Methods of Aggregates**
- **12P-F3Correlation and Regression: meanings and application**
- **12P-F4 Correlations and Regression: Karl Pearson's Coefficient**

Class 12 General Graduating Outcome

Students who graduate from Class 12 will have a strong conceptual understanding of mathematics to pursue a secondary education in various fields that require high-level mathematics such as engineering (civil, electrical and mechanical), architecture, and computer programming. Graduates will also have a strong mathematical foundation to pursue a post-secondary education in the field of business and business administration. Students will have the skills to become more autonomous, but yet able to work with others, becoming more reflective, and developing personal and intellectual competencies that will aid in their journey through post-secondary education and in life.

Throughout their high school journey, students have had the opportunity to build on prior knowledge, learn more-varied and more-sophisticated problem solving techniques; increase their ability to visualize, describe, and analyze situations in mathematical terms to allow them to have a richer understand of the mathematical implications of the world around them. Graduates have the skill set to use a wide range of explicit and recursive defined functions, allowing them to understand their properties and give them insight into the modeled real-world phenomena.

Bhutanese graduates will be able to use many strategies and processes needed to solve a wide variety of mathematical problem. Students will be able to process, interpret information critically to make informed decisions; solve problems individually and collaboratively; identify, describe, formulate and reformulate problems; frame and test hypothesis; ask questions, observe relationships, make inferences and draw conclusions.

Communication of ideas is a key characteristic of a strong and competent mathematics student. Changes in the workforce continuously demand teamwork, collaboration and communication. College-level mathematics are emphasizing the need to convey ideas clearly, both orally and written. Graduates will have the ability to structure logical chains of thought, express themselves coherently and clearly listen to the ideas of others. The relationships students wish to express symbolically and with graphs, as well as the notation and representations for expressing them are extremely sophisticated. Students have the ability to justify claims, provide conjectures, and use symbols in reasoning to provide careful reasoned arguments in support of their claims. Oral and written claims can me made and interpreted so that they can communicate effectively

while working with others and can convey the results of their work with clarity and power.

Students, who have successfully met the challenges in mathematics throughout their high school career, will be privy to important connections in the world of work. Graphic designers routinely use geometry; carpenters apply the principals of trigonometry, as do surveyors, architects and engineers. Algebra pervades computing and business modeling, from everyday spreadsheets to sophisticated scheduling systems and financial strategies. By making connections and solving problems from a wide variety of contexts, students are able to adapt to the changing needs of the workplace

Conclusion And Recommendations

The mathematical concepts outlined in this document represent a sequentially and developmentally logical framework for Mathematics instruction in the schools of Bhutan from PP-12. This framework adheres to guidelines for Mathematics instruction in a modern world, as set down by the NCTM.

Presentations of this framework to practicing teachers in Bhutan were positively received. The vast majority of teachers welcomed the changes and expressed, in conversation and through written feedback, high hopes of a speedy implementation. Upon examining the content of the framework, teachers almost unanimously agreed that the teaching sequence PP-10 and accompanying methodology was logical and achievable. At the Class 11-12 level, there was some concern expressed about the omission of some higher Calculus concepts. This point is to be thoroughly examined by senior high school teachers and policy makers and, if deemed necessary, minor revisions will be made.

Although this framework represents a curriculum necessary to produce a numerate and mathematically competitive society, it is but one factor in successful implementation. The following recommendations help in formulating a complete vision of change.

1. With this framework, the greatest departure from the way Mathematics is currently taught in Bhutan is in the conceptual delivery of concepts. Substantive training will be required for teachers to learn these new methodologies and techniques.

It is recommended that substantial consideration be given to the re-training and on-going coaching of teachers, and that a long term commitment to a system of support be maintained.

2. This framework is based on a minimum class instruction time. To accomplish all outcomes at each class level, this instruction time must be consistent.
   It is recommended that at least one hour of uninterrupted instruction time be devoted to the subject of Mathematics per day. This recommendation is applicable to all levels.

3. A conceptual approach to teaching Mathematics requires developing a strong ability to visualize mathematical ideas. This visualization is developed, to a large degree, through the manipulation of concrete materials which represent the ideas in question. However, it is not sufficient to simply demonstrate the ideas; students must gain personal experience through the manipulation of these concrete materials.
   It is recommended that sufficient materials be supplied to all classrooms and students be given ready access to them. The principle materials to be used in the construction of each idea, and the techniques for using them should be clearly outlined in the teacher manuals.

4. As with any new curricula, it is anticipated that, after practical application, the need for minor adjustments may be identified. Where this curriculum represents a substantial departure from traditional methods, sufficient time should be allowed before adjustments/revisions are made.
   It is recommended that a test period of two years be allowed before revisions are made. Accurate records of concerns, suggestions and other feedback should be maintained to facilitate decisions around revisions after this test period.

5. This framework focuses on the gradual construction of foundational ideas over a period of years. Where the curriculum is implemented at several levels at one time, it is expected that teachers will encounter some difficulties in the resulting gaps at higher class levels. For example, teachers of Class 2 will be required to teach outcomes for which some key ideas will not be in place, the students not having investigated the preceding ideas in Classes PP and 1. As the class level increases, so will the gap of preceding ideas. However, with

each subsequent year, that gap in foundational ideas will decrease as students are exposed to more of these concepts.
It is recommended that a plan for specific strategies to support teachers be implemented and maintained during the initial implementation years, and that special consideration be given to expectations at higher levels, where they may need to pick up foundational ideas from previous levels.

6. Mathematics has traditionally been a subject to be feared by students and teachers alike. This framework proposes a methodology that fosters a deep understanding of concepts, ultimately producing students who are confident of their ability in the field of Mathematics and who appreciate the beauty of the subject. Although learning with understanding will certainly promote an improved attitude towards Mathematics, a true appreciation of the subject is gained as students see the relevancy of the ideas in their own lives.
It is recommended that, as teacher manuals and textbooks are developed in future phases that utmost consideration be given to including contexts which are specifically Bhutanese, allowing students to understand the ideas relative to their own lives.

With serious consideration given to the above recommendations and with an appropriate span of time and resources, Bhutan will experience positive gains in the mathematical competency of its citizens. Without doubt, the positive impact of this on its human resource bank will be great.