National School Curriculum

INSTRUCTIONAL GUIDE FOR MATHEMATICS

CLASS: XI-XII

School Curriculum Division
Department of School Education
Ministry of Education and Skills Development
Royal Government of Bhutan

“Your parents, relatives, and friends would be very proud of what you have achieved. At your age, to have completed your studies is your personal accomplishment. Your knowledge and capabilities are a great asset for the nation. I congratulate you for your achievements.

Finally, your capabilities and predisposition towards hard work will invariably shape the future of Bhutan. You must work with integrity, you must keep learning, keep working hard, and you must have the audacity to dream big.”

- His Majesty Jigme Khesar Namgyel Wangchuck

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Published by

Department of School Education
Ministry of Education and Skills Development
Royal Government of Bhutan
Thimphu, Bhutan.

Provisional Edition 2021

First Edition 2022

Second Edition 2023

www.education.gov.bt

ISBN 978-99936-0-635-2

Acknowledgements

The School Curriculum Division (SCD) at the Department of School Education (DSE), Ministry of Education and Skill Development (MoESD) would like to acknowledge the contributions of the following professionals and teachers who participated in the process of the development of the Instructional Guide for the implementation of the Mathematics Curriculum.

The SCD also sincerely acknowledges the retrieval and use of contents and resources, either in part or whole, from relevant websites and other forms of sources. However, SCD reassures that these resources are used exclusively for educational learning purposes.

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Foreword

COVID-19 has caused unforgiving disruptions in public education all over the world, and brought about threats of fragmentation in the society due to disparities in accessibility and connectivity in many systems. In Bhutan too, continuity of education and learning has been severely affected as a result of sporadic nationwide school closures, restrictions and health protocols. The disruptions exposed the limitation of the existing ideologies and practices in education. This has deprived children living in poverty worldwide, who rely on the physical settings of their schools for educational materials and guidance, of the learning and other essential educational services. Cognizant of the global trend to embrace competency based learning as education for the 21st century, the current priority of the Government is to transform the knowledge and textbook based learning to competency based learning through open source and experiential learning.

In the new normal education, human interaction and well-being is a priority. Technology, particularly digital technology that enables communication, collaboration and learning across distance, is a formidable tool though not a panacea but a source of innovation and expanded potential. As we embrace this exceptional opportunity to transform education, it is imperative to reimagine the organisation of our educational institutions and learning environments. In the post COVID 19 era, we must prioritise the development of the whole person, not just the acquisition of academic knowledge. Inspiration for the change can be drawn from the 1996 Delors report, learning the treasure within. Its four pillars of learning as “learning to know”, “learning to do”, “learning to be”, and “learning to live together” are the current global ethos of teaching and learning. Therefore, curricula must be increasingly perceived as an integrated, themes based and problems based orientation that allows learners to develop a strong base of knowledge about one’s self and about the world, and find purpose in life and be better able to participate in social and political milieu.

The National School Curriculum is, not just a mere response to the pandemic, but also culmination of the curriculum reform work for the last four years by the erstwhile Royal Education Council. It is an attempt to transform education from the teaching of “what” to learning of “how” and “why” towards empowering learners with the transversal competencies and the 21s t century skills, and preparing them to be lifelong learners. In tandem with this initiative, we are optimistic that the paradigm shift in Mathematics education orients our education process in empowering the young generation with the Mathematics mind-set and disposition, and skills towards nurturing nationally rooted and globally competent citizens.

With this guide, we are optimistic that our learners and teachers are ushered through a life enriching experiential Mathematics education.

Tashi Delek

(Karma Galay)
DIRECTOR GENERAL

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Introduction

The 21st Century Education framework emphasises on the theme-based learning approach that broadens opportunities for experiential learning contextualised to the learner's physical, social, political, economic, spiritual and cultural setting. This requires learning through active engagement of learners. The role of teachers therefore, is transformed from knowledge transmitter to facilitation, guide, evaluator, researcher and motivator.

The conventional education system is predominantly knowledge based and examination centred. This system comprises the development of psychomotor and affective domains of learning thereby affecting the holistic development of students.

Despite the devastating effect caused by COVID-19 pandemic, it presented scopes for creation, innovation, generally perceived as more efficient and effective in work and social activities. The pandemic situation explicated that the old ways of working, teaching and learning, and lifestyle have limitations. Consequently, new normal ways of how we work and live, teach and learn must be critically analysed and embraced.

Therefore, the education system needs to be transformed to meet contemporary requirements. Students should learn to critically filter information that is flooded on the internet. Classroom instruction should facilitate learners to construct knowledge, develop essential skills and values which are crucial for learners to realise their potential towards becoming locally rooted and globally competent citizens who would contribute towards making a just and harmonious society. Accordingly, classroom instruction from teacher centred to learner centred calls for the following adjustment, or even the overhaul of a few practices.

i. Reduction of learning content to facilitate deep learning as opposed to the width of the teaching through the active engagement of students.

ii. Integration of ICT as tools and ends of learner's education. The use of multimedia and ICT software is commonly utilised in teaching and learning as innovation to introduce variation in stimuli and sustain learner's interest and zeal in learning.

iii. Adoption of theme-based learning content, which facilitates to broaden the horizon of learning beyond the four walls, and stimulates the transfer of learnt concepts to the learner's immediate environment. This arrangement makes students aware of the realities of the social, political, economic and cultural practices and ethos of the society. Being aware of the immediate environment of the scopes and challenges, students are sensitised to the opportunities and issues, which may need attention for a better future for the society.

iv. Consideration to ground the curriculum design and instruction approaches the epistemological theories is imperative to facilitate deep learning as opposed to factual learning. However, the

selection and use of them is subject to the nature of the respective subject. For instance, constructivism is more apt for science, while connectivism is relevant for languages and ICT curricula.

v. Active engagement of students is imperative for competency-based education and learning. Inevitably, summative assessment has limitations in gauging the progressive development of the learner. This is achieved objectively by the use of the continuous formative assessment (CFA). However, if summative assessment evidence is used to provide feedback to help students in learning, it can serve as one of the techniques of CFA.

Purpose of the Instructional Guide

This instructional guide provides a suggestive direction to the teachers to facilitate them to transform the classroom instruction to the contemporary requirements stated above. The content of the instructions in the guide are aligned with the mathematics curriculum framework with references to the existing textbooks.

The instructional guides are developed to achieve the following objectives:

i. Facilitate learning anywhere, any time with the learner being responsible for the learning.

ii. Facilitate deep learning with awareness and sensitivity of the realities of the world around.

iii. Strengthen competency based learning and experiential learning to foster sensitivity of realities of life and environment.

iv. Strengthen blended learning and flip classroom with multimedia, digital pedagogies and ICT devices and websites as the tools and learning content.

v. Guide parents in facilitating learning of their children.

vi. Inspire teachers to assume the roles of facilitation, guide, motivator and evaluator.

vii. Helps in the prioritisation of learning content with emphasis to create time and space for active engagement of learners. Facilitate the use of CFA for learning through objective observation and guidance.

The effective and efficient use of this guide is subject to the nature of the topic(s) and the target class.

ORGANISATION OF THE INSTRUCTIONAL GUIDE

This is the main part of the instructional guide. It contains suggested approaches to teaching to guide students to achieve the desired competency (ies) through the identified topic(s).

a. Broad theme /Strand/Chapter/Topic

Under this heading, the topic/topics under one strand or under different strands that can be addressed together is/are listed. The topics are taken from the framework and not from the textbooks.

b. Introduction, Utility and Scope

Knowing history, utility and scope of topic(s) or concept (s) may provide an inspiration to learn certain things. It arouses curiosity, interest, and motivates the learners if they know why and how something was invented or discovered. Due to these reasons, the teachers are expected to explore and express the history, utility, and scope of the topic(s). It is advisable to do so while introducing the topic or concept because if students know the history and utility of what they learn, learning becomes much more engaging and captivating.

c. Competency (ies)

Under this heading, the main competency (ies) associated with the topic(s) is/are listed.

d. Objective(s)

Lesson objective(s) which are directly linked with the competency is/are given.

e. Essential Skills/Process

When students discuss and do mathematics, they use various skills and at the same time learn new skills. All learning processes are intended to facilitate students to acquire and develop certain skills. Each lesson in this guide contains few suggestive skills for each topic. However, since the list of essential skills for each topic is not exhaustive, teachers may explore helping students to acquire or develop some other essential skills through the topics covered.

f. Learning Experiences

In each lesson, learning experiences linked to the objectives which are further linked to the competency (ies) are suggested. These suggested learning experiences are to provide directions for the teachers to leverage the lesson to enable students to achieve the desired competency (ies). It can be used for both face to face and virtual mode to make the teaching learning more authentic. Both indoor and outdoor learning experiences (experiential learning) with possible cross pollination and thematic approaches are suggested for each lesson.

Teachers are expected to use the approaches of cross pollination (multidisciplinary), thematic (intra and inter topic(s)/concept(s)), place-based education pedagogies, project based learning, and flip classroom to offer greater flexibility and opportunities to enable students to generate new knowledge and associated skills. Students should also be provided opportunities to connect, communicate, and represent mathematical ideas and be given avenues for thinking divergently and reflecting on their learning.

g. Assessment

Achievement of learning shall be recorded based on bands of achievement for all students in all classes from IV to XII. The evidence from assessment is to identify individual learning needs, design, and deliver appropriate interventions to support students falling in the beginning and approaching category and to further provide appropriate support to the ones falling under the higher bands.

Teachers shall assess the objectives in a cumulative manner and ensure that competency (ies) are achieved at the end of the lesson. In each class, marks obtained for each strand from the formative assessment can be converted using appropriate conversions for summative purposes.

$$CA\ Marks = \frac{Sum\ of\ scores\ obtained\ of\ all\ competencies}{Total\ scores\ of\ all\ competencies} \times CA\ for\ the\ term.$$

- Performance Task

Each lesson consists of one or two suggestive performance tasks. The performance tasks are intended to assess the students' achievement of the desired competency (ies) through that particular lesson. Through performance tasks students can demonstrate knowledge and skills that they have acquired or developed, and how they can apply the knowledge and skills in real situations. Teachers should try and design performance tasks where students will be able to use the immediate objects around them to show their ability to transfer/apply the learning and assess higher order thinking.

The performance tasks should be assessed through Performance-based Assessment (PBA) wherein not only knowledge and skills are assessed but the ability to apply the knowledge and skills are also assessed.

- Reflective Questions
  Suggestive reflective questions that can be included in a performance task are mentioned below each performance task. Also, in some cases, reflective questions are mentioned after the completion of the performance tasks. These reflective questions are suggested, mainly, to help students to reflect on their learning and make personal connections to the topics being learnt.

h. Resources

All resources required for the suggested learning experiences and tasks are listed under each lesson both for face to face and virtual mode. Some lengthy worksheets are provided as annexure at the end.

i. Annexure

For the effective implementation of the NSC, certain worksheets are suggested for each lesson and annexed at the end of each lesson. However, teachers are expected to use it or further develop similar authentic worksheets for the teaching learning purposes.

Instructional Guide
Class XI
Mathematics

Topic: XIP-A1 Determinants *[350 minutes]*

Introduction

The concept of a determinant emerged as a means of determining, hence the term 'determinant,' whether a system of equations has a unique solution. Interestingly, the attribution of the determinant goes to the Japanese mathematician Seki Takakazu, who introduced it around the beginning of the 1700s. Seki Takakazu is colloquially known as the 'Japanese Newton.' In fact, he independently worked on infinitesimal calculus, similar to Newton and Leibniz, who were his contemporaries.

It's important to note that earlier forms of the determinant were utilised as early as the third century BC by Chinese scholars. The term 'determinant' was coined by Gauss at the start of the 1800s in the context of the discriminant of homogeneous polynomials. It was only fifty years later, in 1858, that matrices themselves were introduced by Cayley in his work "A memoir on the theory of matrices." Eight years before that, Sylvester had introduced the term 'matrix,' but he considered a matrix as an array of numbers from which determinants of its submatrices (nowadays called minors) could be derived. In contrast, Cayley was the first to describe how matrices could be treated as algebraic objects that can be added and multiplied together.

Refer the link: history of determinant for more information.

Utility and Scope

Determinants of a matrix serve as a valuable tool, as the name suggests, by "determining" various aspects. In the realm of matrix algebra or linear algebra, the determinant plays a crucial role in assessing whether a system of equations has a unique solution. Determinants are employed in solving linear equations, capturing how linear transformations affect area or volume, and facilitating variable changes in integrals. Additionally, determinants can be viewed as a function with a square matrix as input and a numerical output.

When dealing with mathematical problem-solving involving a large set of independent yet interconnected quantities, linear algebra is often utilised to find exact or approximate solutions. In the course of any linear algebra work, it is nearly inevitable to encounter the need to find the determinant of a matrix—whether for matrix inversion, eigenvalue determination, or more complex problem-solving.

In essence, determinants can manifest in any field that addresses problems with numerous variables that interact meaningfully. This encompasses a wide range of disciplines such as science, engineering, big data, data analysis, business calculus, and more. While not everyone in these fields may require determinants, many individuals across these domains have undoubtedly found them to be a valuable tool in their work.

Source: https://www.toppr.com/guides/maths/determinants/

A. Competencies

- Demonstrate the ability to evaluate determinants and apply the concept to determine the areas of triangles and quadrilaterals.
- Solve real-life problems involving a system of linear equations in two or three variables using determinants and their properties.

B. Objectives

- Define determinant, and evaluate determinants of orders 2 and 3 by expanding it along any rows or columns.
- Determine minors and cofactors for each element of a determinant.
- Apply determinants in computing the areas of triangles and quadrilaterals.
- Expand determinants efficiently using their relevant properties.
- Solve systems of equations in two or three variables using Cramer's rule.
- Examine the consistency of a system of equations.

C. Essential Skills/Processes

- Computing
- Applying
- Analysing
- Recognising

D. Learning Experiences

- Project the video from the web link: Introduction of Determinant and allow students to write their understanding and share it in class.
- Demonstrate and explain how to compute determinants of order 2 and order 3 during interactive teaching.

- Refer to the web links: Determinant_Order_2 and Determinant_Order_3 on expanding determinants of order 2 and order 3 respectively.
- Assign similar questions to test their understanding of expanding determinants of order 2 and 3.

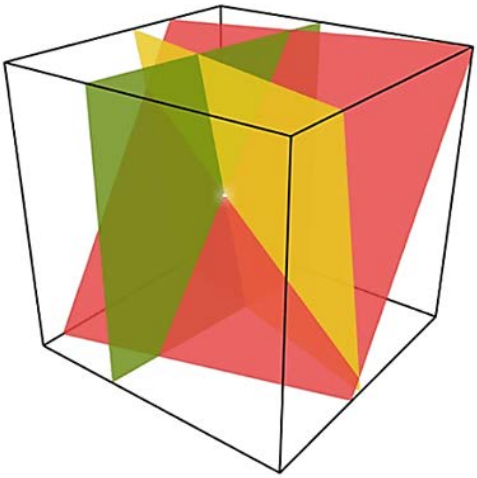
- Display a video given in the web link: Minor_Cofactor Introduction to find minor and co-factor of determinants of order 2 and 3.
  - Project a series of questions on the screen and allow a few students to take turns to complete on the board.

- Allow students to solve a system of equations with two variables using Crammer's rule. Assign a question to each group and facilitate their presentations, aligning with the discussed rubrics.
  - After each presentation, allocate time for question and answer sessions.
    suggestion - allow students to explore the following web links:
    - ➢ https://www.youtube.com/watch?v=LprQ_Id-8hE (Crammer's rule for a system of equations with two variables).
    - ➢ https://www.youtube.com/watch?v=vXqlIOX2itM (Crammer's rule for a system of equations with two variables).
    - ➢ https://www.youtube.com/watch?v=Ot87qLTODdQ&t=5s (Crammer's rule for a system of equations with three variables).

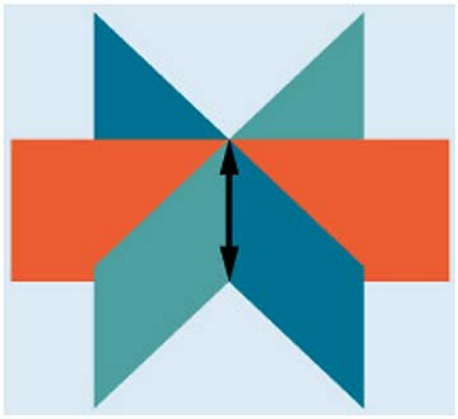
- Reflective question: Provide two systems of linear equations on the board.
  - Instruct students to solve the linear equations using all the methods they have learned (substitution, elimination, comparison and Cramer's rule) to solve the systems.
  - Allow them to write a reflection on which method they would use to solve each kind of system with justification.

- Explain the meaning of the inconsistent system in terms of the solutions of the system and its geometrical meaning using relevant software like *GeoGebra* or any other 3-D graphing software (refer the web link to know how to graph the planes on GeoGebra: https://www.youtube.com/watch?v=--gpbRvnIFE).

- The consistent and inconsistent systems can be graphically shown as below:



Consistent system with unique solution



Consistent system with infinite solutions

Inconsistent system with no solution

- Display a flow chart explaining the steps to check for consistency and solve for an infinite number of solutions.
  - Allow students to write their own interpretation of the flowchart and let volunteers to read their interpretations to make sure all the students are on the same page.

- Share the web link: https://www.youtube.com/watch?v=lx8Nne-a-KQ with the students to further amplify their understanding on the conditions of consistency of a system of linear equations using the determinants method.
- Refer BHSEC Mathematics book II to assign relevant questions to the students.

- Introduce the application of determinants to find the area of triangles and quadrilaterals. Explore the video from the link: area of the triangle by determinant or refer BHSEC Mathematics book II.

E. Assessment

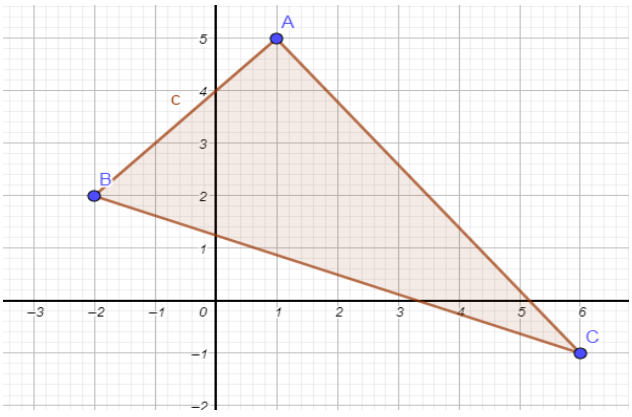
Performance Task 1

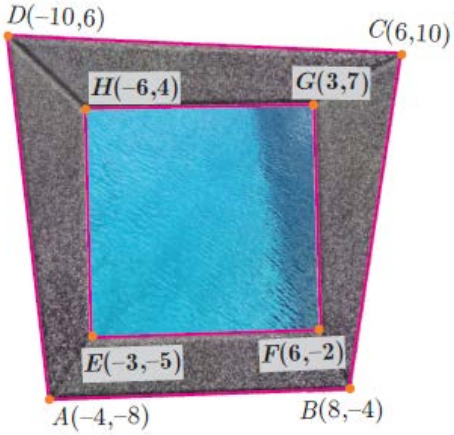
Competency based assessments;

Design a homework task to assess the students' competency in applying the concept of determinants in finding areas of triangles and quadrilaterals.

Sample questions:

1. You are planning to make a flower garden, fenced at three different strategic points as portrayed in the figure below. Determine the area of the garden where you can plant flowers.





2. In the figure, the quadrilateral swimming pool shown is surrounded by a concrete patio. Find the area of the patio.
3. A triangular shaped glass with vertices at A (-5,-4), B (1, 6) and C (7,-4) has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied?

Performance Task 2

Place Based Approach:

Design a place-based task where students will need to write a real-life situation into a system of linear equations and solve using the determinant method.

Sample:
Divide students into groups and send them off to different shops.
Instruct students to inquire about the total cost (not individual cost) of purchasing 3 sets of following items: water (500 mL), pens and books.
Example,
First set: 3 waters, 5 pens and 2 books
Second set: 5 waters, 7 pens and 1 book
Third set: 2 water, 3 pens and 5 books

Students will then represent the problem into a system of linear equations and solve the equation to determine the individual price of a water, a pen and a book using the determinant's method. After comparing their results with other groups, they may also reflect and share their findings on which shop may be the best choice for purchasing items at lower cost and for maximising savings.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1.

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum Framework for Mathematics
- Introduction – https://www.quora.com/How-did-the-concept-of-determinants-come-about
- Introduction – https://man-nigeria.org.ng/issues/ABA-SCI-2019-54.pdf
- Utility and scope - https://www.toppr.com/guides/maths/determinants/
- Concepts on determinant - https://www.youtube.com/watch?v=YFGTpSkfT40
- Calculating determinant of order 2- https://www.youtube.com/watch?v=wrIAFfZBoBM
- Calculating determinant of order 3- https://www.youtube.com/watch?v=lhgFVZOgleg
- Minor and cofactor of determinant - https://www.youtube.com/watch?v=KMKd993vG9Q

- Calculating cramer's rules - https://www.youtube.com/watch?v=LprQ_Id-8hE (Crammer's rule for a system of equations with two variables).

https://www.youtube.com/watch?v=vXqlIOX2itM (Crammer's rule for a system of equations with two variables).

https://www.youtube.com/watch?v=Ot87qLTODdQ&t=5s (Cramer's rule for a system of equations with three variables).

- Consistency check graphing method-https://www.youtube.com/watch?v=--gpbRvnIFE
- Condition of consistency - https://www.youtube.com/watch?v=lx8Nne-a-KQ
- calculating minor and cofactor of determinant - https://www.youtube.com/watch?v=KMKd993vG9Q
- Area of triangle: https://www.youtube.com/watch?v=bkJX3q7wvJc&t=11s

G. Annexure

i. Template to record assessment

Strand(s): XIP-A1			**Topic(s): Determinant**		
Competency: Demonstrate an understanding of the concepts of the determinants and their properties					
Name of the student	**Level of achievement**				
	Beginning	**Approaching**	**Meeting**	**Advancing**	**Exceeding**

ii. Assessment sample tool

*(**Note:** This is just a sample of one of the assessment tools. Teacher must design other appropriate assessment tools as per the competency and performance tasks)*

	Beginning	**Approaching**	**Meeting**	**Advancing**	**Exceeding**
Completion	Several of the problems are not completed.	About three of the problems are completed.	About two of the problems are completed.	One of the problems is not completed.	All problems are completed.
Neatness and Organization	The work appears sloppy and unorganised. It is hard to know	The work is presented in an organised fashion but may be hard	The work is presented in a neat and organised fashion that is	The work is presented in a neat and organised fashion that is	The work is presented in a neat, clear, organised fashion

	what information goes together.	to read at times.	usually easy to read.	mostly easy to read.	that is easy to read.
Working with others	Students did not work effectively with others.	Students cooperated with others, but needed prompting to stay on-task.	Student was engaged with a partner/group but had trouble listening to others and/or working cooperatively.	Student was engaged with a partner/group, listening to some suggestions of others and working quite cooperatively throughout the lesson.	Student was engaged with a partner/group, listening to suggestions of others and working cooperatively throughout the lesson.
Mathematizing the problem	The student did not attempt to mathematize the problem	Student mathematics the problem mostly incorrectly	Students wrote part of the system of linear equations correctly.	Students wrote most parts of the system of linear equations correctly.	Student wrote the whole system of linear equations accurately
Accurateness of application of Cramer's rule	Student did not attempt to solve the system of linear equations	Student solved only 1 determinant out 4 correctly	Student solved only 2 determinants out of 4 correctly	Student solved only 3 determinants out of 4 correctly	Student solved all the 4 determinants correctly
Interpretation of the solutions	Student did not attempt to interpret the solutions	Student's interpretation of the solutions was mostly incorrect	Student's interpretation of the solutions was partially correct	Student's interpretation of the solutions was mostly correct	All the interpretation of the solution was correct

Topic: XIP-A2 Logarithms *[300 minutes]*

Introduction

Have you ever imagined yourself solving problems in the form of 102.5 without a calculator? This can be really painful and difficult to manage. Until 1500 AD there was no efficient way to solve this kind of problem. The only alternative was using some calculation tables, very common by that time. But many of them were full of errors and imprecisions. They also usually take a long time to be made, but were strictly necessary to Astronomy studies.

John Napier (1550–1617), a protestant and also a great inventor, was pretty unsatisfied with calculation tables. Luckily, he met the Prosthaphaeresis method, mainly used by Tycho Brahe and came up with a tricky idea: why not associate the terms of an Arithmetic Progression with a Geometric Progression? And then this idea gave birth to Logarithms. Napier used to handle his associations on base 1/e. That's why some people call logex (or ln x) as Napierian Logarithm. Napier initially named his structure as "artificial numbers", and only later they started being called logarithms. Henry Briggs (1561–1630), a mathematician and also an Oxford professor, spread Logarithms ideas to Europe. Briggs also started to work with base 10, and it quickly became popular. He also published several logarithm tables — from 1 to 20000 and 90 to 100000.

Source: https://www.history/log

Utility and Scope

Logarithms have many different applications:

- Seismologists use logarithms to calculate the magnitude of earthquakes.
- Financial institutions make use of logarithms to calculate the length of loan repayments.
- Scientists use logarithms to determine the rate of radioactive decay.
- Biologists use logarithms to calculate population growth rates.
- Scientists use logarithms to determine pH levels.

Source: https:applications-of-logarithms

A. Competency

- Exhibit an ability to show the relationship between logarithmic and exponential expressions, and apply the properties and laws of logarithm in solving problems.

B. Objectives

- Establish connections between exponent laws and logarithmic forms.
- Develop a comprehensive understanding of the meaning and concept of logarithms.
- Apply theorems and laws of logarithms proficiently to solve logarithmic problems.

C. Essential Skills/Processes

- Conceptualising
- Computing
- Applying
- Recognising

D. Learning Experiences

- To check the basic concept of exponents, conduct a test on laws of exponent. Refer to BHSEC Mathematics Book I, Understanding Mathematics textbook for class IX or the web link: https:/quiz-powers-and-exponents.

- Introduce the definition of logarithm of a number using the video in the web link: https://youtu.be/TOoXH_IwUx0.
  - Pause the video whenever it requires elaboration and instruct students to take notes and try to answer the questions imposed in the video.

- Reflective questions: Explore and write the differences between common logarithm and natural logarithm.
  - Refer to the web link: https://difference-between-ln-and-log (contains notes explaining the differences between common log and natural log).

- Play the video in the web link https://www.fundamental properties of log to understand the fundamental properties of logarithm and conditions for argument and base. Instruct the students to make notes from the video.

- Direct students to watch the videos given in the following links on the laws and theorems of logarithm and write notes.

- https://www.product,quotient,power-law (contains video explaining the proofs of the product, quotient and power law of logarithm)
- https://www.log-change of base (contains video explaining the law of 'change of base')
- Provide practice questions for students to manifest their understanding by referring to the web links https://www.practice question-1 and https://www.practice question-2 (contains variety of questions on product, quotient and power law, and law of 'change of base') or referring to BHSEC Mathematics Book I, Exercise 3(c).

- Discuss the relationship between natural and common logarithm as $ln\, x = 2.303\, log\, x$.
- Use the video in the web link https://www.real life application to give a few examples of how the concept of logarithm can be used in real life applications.

E. Assessment

Performance Task 1

Cross-pollination Approach

Design a summative assessment at the end of the chapter to assess students' knowledge on logarithmic Laws in real world problem solving.

Sample questions:

1. **Biology**: A biologist is researching a newly-discovered species of bacteria. At time $t = 0$ hours, he puts one hundred bacteria into what he has determined to be a favourable growth medium. Six hours later, he measures 450 bacteria. Assuming exponential growth is given by $A = Pe^{kt}$ where A is the end population at end time t and P is the initial population, what is the growth constant "*k*" for the bacteria? (Round *k* to two decimal places.)

   Solution: https://www.solution/Question-1

2. **Chemistry**: Radio-isotopes of different elements have different half-lives. Magnesium-27 has a half-life of 9.45 minutes. What is the decay constant for Magnesium-27 if the decay equation can be written a $0.5 = e^{9.45k}$ where k is the decay constant? Round to five decimal places.

   Solution: https://www.solution/Question-2

3. **Medical**: Some people are frightened of certain medical tests because the tests involve the injection of radioactive materials. A hepatobiliary scan of my gallbladder involved an injection of 0.5 cc's (or about one-tenth of a teaspoon) of Technetium-99 m, which has a half-life of almost exactly 6 hours which gives the decay equation as $0.5 = e^{6k}$. While undergoing the test, I heard the technician telling somebody on the phone that "in twenty-four hours, you'll be down to background radiation levels." Figure out just how much radioactive material remained from my gallbladder scan after twenty-four hours. Show your work below.

   Solution: https://www.solution/Question-3

4. **Seismology:** Earthquake intensity is measured by the Richter scale. The formula for the Richter rating of a given quake is given by "$R = log[I \div I0]$" where $I0$ is the "threshold quake", or movement that can barely be detected, and the intensity I is given in terms of multiples of that threshold intensity.

   You have a seismograph set up at home, and see that there was an event while you were out that had an intensity of I = 989I_0. Given that a heavy truck rumbling by can cause a microquake with a Richter rating of 3 or 3.5, and "moderate" quakes have a Richter rating of 4 or more, what was likely the event that occurred while you were out?

   Solution:https://www. solution /Question 4

Design an appropriate tool to assess each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - https://www.history/log
- Utility and scope - https:applications-of-logarithms
- Exponents of law - https:/quiz-powers-and-exponents
- Introduction of logarithm - https://youtu.be/TOoXH_IwUx0
- Common log and natural log –https://difference-between-ln-and-log
- Properties of log - https://www.fundamental properties of log
- Laws of log - https://www.product,quotient,power-law
- Laws of log - https://www.log-change of base

- Practice questions of log - https://www.practice question-1
  https://www.practice question-2
- Use of log in real life - https://www.real life application
- Log in real life application - https://www.solution/Question-1
- log in real life application - https://www.solution/Question-2
- Log in real life application - https://www.solution/Question-3
- Log in real life application - https://www. solution /Question 4
- Self-instructional Material
- Technological gadgets (Smart phone, projector and laptop).

G. Annexure

Refer XIP-A1 for template to record achievement

Topic: XIP-A3 Complex Numbers *[300 minutes]*

Introduction

Complex numbers were introduced by the Italian famous gambler and mathematician Gerolamo Cardano (1501--1576) in 1545 while he found the explicit formula for all three roots of a cube equation. Many mathematicians contributed to the full development of complex numbers. The rules for addition, subtraction, multiplication, and division of complex numbers were developed by the Italian mathematician Rafael Bombelli (baptised on 20 January 1526; died 1572). The notations 1 and i for unit vectors in horizontal positive direction and vertical positive direction, respectively, were introduced by Leonhard Euler (1707--1783). He visualised complex numbers as points with rectangular coordinates, but did not give a satisfactory foundation for complex numbers theory. He also suggested dropping the unit vector 1 in presenting vectors on the plane.

It was Carl Friedrich Gauss (1777--1855) who introduced the term complex number. Cauchy, a French contemporary of Gauss, extended the concept of complex numbers to the notion of complex functions. University of Rhode Island professor Orlando Merino (born in 1954) has written an essay on the history of discovery of complex numbers.

Source: the origin of complex numbers
Suggested sources: https://www.youtube.com/watch?v=gHUHZXjpwOE
https://www.youtube.com/watch?v=0OHiSZUvn0I

Utility and Scope

- Complex numbers are used in real-life applications, such as electricity, as well as quadratic equations. In quadratic planes, imaginary numbers show up in equations that don't touch the x axis. Imaginary numbers become particularly useful in advanced calculus. Usually denoted by the symbol *i*, imaginary numbers are denoted by the symbol *j* in electronics (because *i* already denotes "current").

- Imaginary numbers are particularly applicable in electricity, specifically alternating current (AC) electronics. AC electricity changes between positive and negative in a sine wave. Combining AC currents can be very difficult because they may not match properly on the waves. Using imaginary currents and real numbers helps those working with AC electricity do the calculations and avoid electrocution.

- Imaginary numbers can also be applied to signal processing, which is useful in cellular technology and wireless technologies, as well as radar and even biology (brain waves). Essentially, if what is being measured relies on a sine or cosine wave, the imaginary number is used.

  Source: utility and scope of complex numbers

A. Competencies

- Demonstrate an understanding of the concept of imaginary and complex numbers and its representation on the argand plane.
- Display proficiency in applying the properties of complex numbers and their conjugates in carrying out operations on complex numbers.

B. Objectives

- Understand the concept of imaginary number and its integral powers.
- Define the meaning of complex numbers in cartesian form and in ordered-pair form to represent on an argand plane.
- Define the meaning and properties of conjugate of a complex number.
- Apply the properties of complex numbers and their conjugates to perform operations on complex numbers.

C. Essential Skills/Processes

- Conceptualising
- Representing
- Applying
- Analysing

D. Learning Experiences

- Project the video in the web link: https://www.youtube.com/watch?v=hqr1DtXXHpY to introduce the concept of Imaginary Numbers.

- Allow students to watch the video in the web link: https://www.youtube.com/watch?v=KTNcYYHuBTY which explains the integral powers of imaginary numbers and how to evaluate roots of negative numbers using the unit imaginary number.

- Students will make notes and demonstrate their understanding by solving questions from exercise 21(a), BHSEC Mathematics Book II.

- In a flipped classroom approach, share the web links: complex numbers and complex planes for students to explore as part of their homework.
  - In class, allow students to define complex numbers and give the differences between real numbers and complex numbers.
  - Instruct students to write at least five examples of complex numbers on the board.
  - Suggest students to explore online worksheets, similar to the one provided in the link: worksheet to assess their learning progress.

- Project the video in a link: https://byjus.com/jee/complex-numbers/ to understand the geometrical representation of complex numbers in the Argand diagram.
  - Provide examples of complex numbers and instruct them to represent in the Argand diagram.

- Allow students to watch a video in a link: https://www.?v=BZxZ_eEuJBM on conjugate complex numbers.
  - Assign practice questions from Exercise 21 (b), BHSEC Mathematics Book II.

- Using the video in the link: https://www=wQnXN0xljR0&t=669s explain the operations on complex numbers: addition, subtraction, multiplication, division and multiplicative inverse of complex numbers.
  - Assign practice questions from Exercise 21 (b), BHSEC Mathematics Book II.

- Use the video in the link: https://www.nagwa.com/en/videos/145198074381/ to solve the problems involving equations.

E. Assessment

Performance Task 1

Cross-pollination with ICT

Create a quiz on Google Form to evaluate students' understanding of imaginary numbers, their proficiency in operations and simplifications of complex numbers. Take the students to the school's IT laboratory for them to attempt the quiz and experience online testing.

Sample questions: refer the web link: worksheet

Performance Task 2

Assign questions to solve equations involving complex numbers. Allow students to do short write-ups on methods and strategy incorporated.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1.

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum Framework for Mathematics
- Introduction – the origin of complex numbers
- Introduction - https://www.youtube.com/watch?v=gHUHZXjpwOE
  https://www.youtube.com/watch?v=0OHiSZUvn0I
- Utility and scope - utility and scope of complex numbers
- Introduction to imaginary numbers-
  https://www.youtube.com/watch?v=hqr1DtXXHpY
- Integral powers of imaginary numbers-
  https://www.youtube.com/watch?v=KTNcYYHuBTY
- Concepts of real and complex numbers - complex numbers
- Concepts of real and complex numbers - complex plane
- Summary sheet of complex number - worksheet
- Complex number in argand plane - complex numbers
- Conjugate of complex numbers - Conjugate of complex numbers
- Operation of complex number - Operations of complex numbers
- Complex numbers worksheet - worksheet
- Solving equation of complex numbers
  -https://www.nagwa.com/en/videos/145198074381/
- Technological gadgets (Smart phone, projector and laptop).

G. Annexure

Refer XIP-A1 for template to record achievements

Topic: XIP-A4 Binomial Theorem *[350 minutes]*

Introduction

The binomial theorem provides a simple method for determining the coefficients of each term in the series expansion of a binomial with the general form $(A + B)^n$. A series expansion or Taylor series is a sum of terms, possibly an infinite number of terms, that equals a simpler function. The binomial theorem has been used extensively in the areas of probability and statistics. The main argument in this theorem is the use of the combination formula to calculate the desired coefficients. The question of expanding an equation with two unknown variables called a binomial was posed early in the history of mathematics. One solution (for real n), known as Pascal's triangle, was determined in China as early as the thirteenth century by the mathematician Yang Hui.

His solution was independently discovered in Europe 300 years later by Blaise Pascal (1623–1662), whose name has been permanently associated with it since. The binomial theorem, a simpler and more efficient solution to the problem, was first suggested by Isaac Newton (1642–1727). He developed the theorem as an undergraduate at Cambridge and first published it in a letter written for Gottfried Leibniz (1646–1716), a German mathematician.

Source:https://www.encyclopedia.com/science-and-technology/mathematics/mathematics/binomial

Utility and Scope

Higher mathematics: The binomial theorem is used in the calculation and higher mathematics to solve highly complex and nearly impossible calculations. Many of the equations, laws, and theories suggested by Sir Albert Einstein used a lot of binomial theorems.

Internet protocol (IP): The random generation and distribution of IP addresses in electronic devices, such as the Internet of Things (IoT), such complex issues are easily done using the power of binomial theorem.

Architecture: The binomial theorem is of great use in Architectural Space. It allows Engineers to calculate and estimate the various magnitudes which make their work a lot easier and it also has a positive impact on costs which would be rather high had it not

been for the method of binomial theorem. This results in a faster construction workflow and the contractors have a bigger margin of profit due to the minimization of costs.

Probability: One major field in which the theorem plays a crucial role is in probability learning. The concept is called *binomial probability*, and it helps us in finding the probability of *x* successes in *n* repeated trials which has two possible outcomes: like the probability of tails 7 times when you toss a coin 15 times.

Source: https://www.uplarn.com/how-to-apply-the-binomial-theorem-in-real-life/

A. Competencies

- Demonstrate an understanding of the meaning of the symbol C (n, r) and apply the formula in solving related problems.
- Express an understanding of binomial theorem and its application in expanding expressions with positive indices.

B. Objectives

- Define the meaning of the symbol of C (n, r) and apply the formula to solve related problems.
- Expand binomial expressions with positive indices using binomial theorem.
- Evaluate the general, middle term(s), Coefficient of a particular power of x and term independent of x of a binomial expansion.

C. Essential Skills/Processes

- Selecting
- Arranging
- Applying
- Analysing

D. Learning Experiences

- Let students explore expansion of $(a+b)^2, (a+b)^3, (a+b)^4$ using the concept that they have studied in class IX and X.
- In a group instruct students to explore meaning of factorial notation, meaning of the symbol $C(n,r)$ and important results relating to the symbol.
    - Refer to the BHSEC Mathematics Book I or the web links:
        - ➢ https://www.youtube.com/watch?v=8Ti0mCEpli4

- ➢ https://www.youtube.com/watch?v=x83Yx6jffik
- Students will then present their findings in the class in the form of a Powerpoint presentation.

- Investigate the use of Pascal's triangle in expansion of binomial expression with positive indices by referring to BHSEC Mathematics Book I or the web link: https://www.youtube.com/watch?v=LiFeg_xKi3I.

- Demonstrate to the students, how to apply the Pascal's triangle in the expansion of following expressions:
  - $(x + y)^2 = x^2 + 2xy + y^2$
  - $(x - y)^2 = x^2 - 2xy + y^2$
  - $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
  - $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

  - Allow students to expand the following two expressions $(x + y)^4$ *and* $(x + y)^5$ using Pascal's Triangle.

- Deduce the Binomial Theorem by studying the patterns of expansion using Pascal's Triangle.
  - Share the link: https://www.pascal triangle -concepts.com which contains notes on the properties of binomial expansion with positive indices.
  - Allow students to write the notes and solve a few examples related to the topics and ask them to share their findings with their classmates.
  - Provide further practice questions to expand binomial expressions with positive indices using Binomial Theorem (refer to BHSEC Book I Exercise 2(a)).

- Divide the topics: general term, middle term, coefficient of a particular power of x and term independent of x , as group work for students to explore and present in class with solved questions.
  - Suggested references: BHSEC Mathematics Book I or the video weblinks:
    - ➢ https://www.general-term -concepts.com (contains a video explaining how to find a general term in binomial expansion.)
    - ➢ https://www.youtube.com/watch?v=5AYyOp69Qx8 (contains video explaining how to find the term independent of x.)

- ➢ https://www.middle-term.com (contains video on how to find the middle term)
- ➢ https://www.youtube.com/watch?v=Kc_A3QTg24M (contains video explaining how to find the coefficient of a particular power of x)
- ➢ https://www.youtube.com/watch?v=XPmrvRiTruw (contains video explaining how to find the coefficient of a particular powers of x and y)

- Reflective question: How can you use the binomial theorem to solve the powers of the type: 105^5 or 27.5^6?

E. Assessment

Performance Task 1

Design a quiz referring to the sample in the link: https://www.bionomial-quiz.com.
to assess the students' knowledge of binomial theorems in expansions of expressions with integral powers.

Performance Task 2

Construct competency based questions and divide them amongst different groups of students for discussion and presentation in class. Assess the competency of students in applying the knowledge of binomial theorems and its general terms in broader applications.

Refer to the web link for resources: https://www.youtube.com/watch?v=R27JLs4suMo.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1.

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction- https://www.encyclopedia.com/science-and-technology/mathematics/mathematics/binomial
- Utility and scope – https://www.uplarn.com/how-to-apply-the-binomial-theorem-in-real-life/
- Binomial expansion - https://www.youtube.com/watch?v=8Ti0mCEpli4 https://www.youtube.com/watch?v=x83Yx6jffik
- Pascal triangle - https://www.pascal triangle -concepts.com

- Properties of binomial theorem - https://www.google.bionomial.com
- General term of binomial expansion - https://www.general-term -concepts.com
- Middle term of binomial expansion -https://www.youtube.com/watch?v=5AYyOp69Qx8
- coefficient of particular power of x - https://www.middle-term.com
- Coefficient of particular power of -https://www.youtube.com/watch?v=Kc_A3QTg24M
- Term independent of x - https://www.youtube.com/watch?v=XPmrvRiTruw
- Quiz questions - https://www.bionomial-quiz.com
- Use of binomial theorem - https://www.youtube.com/watch?v=R27JLs4suMo
- Technological gadgets (Smart phone, projector and laptop)

G. Annexure

Refer XIP-A1 for template to record achievement

Topic: XIP-B1 Sequences and Series *[400 minutes]*

Introduction

An arithmetic sequence and series represent a straightforward numerical pattern characterised by a constant difference between consecutive numbers. Historical traces of arithmetic sequences date back to the Ahmes Papyrus from 1550 BC. However, a comprehensive and detailed history of these sequences remains elusive.

In a German school, students were once tasked by a teacher to determine the sum of all integers from 1 to 100. Remarkably, an eight-year-old named Carl Fredrick Gauss swiftly provided the correct answer. Gauss, who later became a renowned mathematician (1777-1855), employed a clever observation: he noted that there were 50 pairs of numbers, each summing up to 101. Consequently, the sum of all numbers equated to 50 * 101, resulting in 5050. Gauss's innovative approach in solving this problem laid the foundation for the formula to calculate the sum of arithmetic series.

Geometric sequences, on the other hand, can be traced back to Babylonian tables from 2100 BC. Subsequently, the detailed exploration of geometric sequences emerged in Euclid of Alexandria's book around 300 BC. As the father of Geometry, Euclid played a pivotal role in advancing the theory of geometric sequences and series.

Source: https://prezi.com/hzq60rtpgfmt/sequences-and-series/
https://www.math.toronto.edu/mathnet/questionCorner/arithgeom.html

Utility and Scope

The utility and scope of learning sequences and series extend across various disciplines, offering valuable insights and applications in mathematics and beyond. Some key aspects include:

1. **Mathematical Foundation:** Sequences and series provide a fundamental foundation in mathematics, serving as building blocks for more advanced topics. Understanding these concepts is essential for progressing in algebra, calculus, and other branches of mathematics.
2. **Problem Solving**: Mastery of sequences and series enhances problem-solving skills, enabling individuals to analyse and solve complex mathematical problems.

This skill is applicable not only in academic settings but also in various real-world scenarios.

3. **Financial Applications**: Arithmetic and geometric progressions find practical use in finance, particularly in calculating interest, loan repayments, and investment returns. Understanding these sequences is crucial for financial planning and decision-making.

4. **Physics and Engineering**: Sequences and series play a significant role in physics and engineering, particularly in the analysis of oscillatory motion, waveforms, and signal processing. Engineers use these concepts in designing circuits, analysing data, and solving dynamic systems.

5. **Computer Science**: Algorithms, data structures, and computational problems often involve sequences and series. Understanding these concepts is beneficial in designing efficient algorithms, analysing time complexity, and optimising code.

6. **Economic Modelling**: Economic models frequently incorporate sequences and series to represent trends, growth rates, and economic indicators. Analysing economic data and making predictions often rely on understanding and manipulating these mathematical concepts.

7. **Statistical Analysis**: In statistics, time series analysis utilises sequences to understand patterns and trends over time. This is crucial in fields such as economics, epidemiology, and environmental science.

8. **Mathematical Research**: Sequences and series are subjects of ongoing mathematical research, with applications in number theory, combinatorics, and analysis. Exploring these concepts contributes to the advancement of mathematical knowledge and the development of new theories.

A. Competencies

- Analyse and categorise real-life sequences as either arithmetic or geometric progressions, and address problems by utilising the concepts of n^{th} term and the sum of n terms in a series.
- Utilise the concept of summation notation to evaluate the sum of a series involving natural numbers.

B. Objectives

- Define an arithmetic series and find its n^{th} term.
- Calculate the sum of n terms in an arithmetic series.
- Define a geometric progression and determine the n^{th} term of the series.
- Calculate the sum of n terms, and sum to infinity of a G.P ($|r|<1$).
- Differentiate the sequences in real life practices into A.P or G.P and describe their properties.
- Illustrate the meaning of summation notation.
- Calculate the sum of series involving natural numbers.

C. Essential Skills/Processes

- Analysing
- Applying
- Computing

D. Learning Experiences

- Discuss the general understanding of sequences and series by referring to the video https://www.youtube.com/watch?v=6VeUsMqonsE.

- Allow students to explore the link: Introduction to A.P. which defines an arithmetic progression and discuss their understanding through presentation.

- Examine the n^{th} term of an Arithmetic Progression by referring to BHSEC Mathematics Book I, or alternatively, refer to the video formula for nth term, which elucidates the derivation of the formula for the n^{th} term of an Arithmetic Progression.
  - Allow students to explore examples related to the topic. Refer the web link for questions:
    - ➢ https://byjus.com/maths/arithmetic-progression-questions/
    - ➢ morechallenging questions on A.P.
  - Assign questions from Exercise 1(a), BHSEC Book I, instruct students to present their findings and understanding on those questions.

- During interactive teaching, illustrate the process of finding the sum of *n* terms in an Arithmetic Progression (A.P), utilising the resources provided in the web link: https://www.youtube.com/watch?v=savKcKBC1AU.

- To assess comprehension, assign questions from exercise 1(b), BHSEC Mathematics Book I.

- Commence the discussion on Geometrical Progression and its nth term, providing illustrative examples.
  - Direct students to refer BHSEC Mathematics Book I, or utilise the web links for further insights: G.P. Introduction, a video explaining the definition of Geometric progression with examples, and https://www.youtube.com/watch?v=3xbormMmuK4, a video solving problems on Geometric progression.
  - Foster active learning by encouraging students to apply their understanding through the practice of additional questions, as found in BHSEC Mathematics Book I.

- Design a group work with questions to find the sum of Geometric Progressions (refer to BHSEC Mathematics Book I) and direct students to explore the resources provided below to learn how to solve the questions provided:
  - https://www.youtube.com/watch?v=6G0-aqZsAMU, a video explaining the derivation of the formula of sum of n terms in a GP.
  - https://www.youtube.com/watch?v=J2XAcw-ZzdM, a video on solved examples of finding sum of n terms in a GP.
  - https://www.youtube.com/watch?v=eRRjDCHlDwg, a video explaining the formula of sum of infinite terms in a GP with examples.
  - Allow the group to present their findings through powerpoint (ppt) and discuss it thoroughly.

- Reflective questions: Write the differences between Arithmetic and Geometric Progression. Give a relevant real life example each.

- Discuss on the topic summation notation referring to BHSEC Mathematics Book I. Design a few questions related to the topics and allow students to solve.

- Demonstrate and explain how to find the sum of first *n* natural numbers referring to the web links: sum-natural-numbers and https://www.youtube.com/watch?v=tR9MeNGyGMA.

- Assign a pair task on finding the sum of the squares and cubes of the first n natural numbers.
  - Allow children to explore the links: https://www.youtube.com/watch?v=aI0M4XRiz4I and sum of cubes-formula.
  - Select random pairs to present their learning and address any recurring doubts.

E. Assessment

Performance Task 1

Design a homework task with competency-based questions for students to solve individually whereby students are assessed on their ability to employ the concepts of A.P and G.P in solving problems in different real life situations.

Sample questions:

a. Sonam's parents want to have Nu. 500,000 saved up to pay for college by the time Sonam graduates from high school (16 years from now). If the investment plan they choose to invest in claims to yield 7% growth per year, how much should they invest today?
b. If a piece of machinery depreciates (loses value) at a rate of 6% per year, what was its initial value if it is 10 years old and worth Nu 100,000?
c. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of the 2nd hour, 4th hour and nth hour?
d. What will Nu. 500 amount to in 10 years after its deposit in a bank which pays an annual interest rate of 10% compounded annually?

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1.

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - https://prezi.com/hzq60rtpgfmt/sequences-and-series/ ,
- Introductionhttps://www.math.toronto.edu/mathnet/questionCorner/arithgeom.html
- Differences between Sequence and Series: https://www.youtube.com/watch?v=6VeUsMqonsE
- Arithmetic progression introduction - https://www.youtube.com/watch?v=gua96ju_FBk

- n^{th} term of A.P.: https://www.youtube.com/watch?v=IshPbnP3vk8
- Problems on A.P. - https://byjus.com/maths/arithmetic-progression-questions/
- Problems on A.P. - https://www.hitbullseye.com/AP Questions
- Sum of n th term of A.P. - https://www.youtube.com/watch?v=savKcKBC1AU
- Concepts on GP - https://www.youtube.com/watch?v=3xbormMmuK4
- Finding the term of G.P. - https://www.youtube.com/watch?v=TKtO3C9xpsQ
- Sum of first n term of G.P. - https://www.youtube.com/watch?v=6G0-aqZsAMU
- First n term of G.P. - https://www.youtube.com/watch?v=J2XAcw-ZzdM
- Sum of infinite G.P. - https://www.youtube.com/watch?v=eRRjDCHlDwg
- n term of natural numbers - https://testbook.com/maths/sum-of-n-natural-numbers
- n term of natural numbers - https://www.youtube.com/watch?v=tR9MeNGyGMA.
- Squares of natural numbers - https://www.youtube.com/watch?v=aI0M4XRiz4I
- cube of natural numbers - https://www.math-only-math.com/sum-of-the-cubes-of-first-n-natural-numbers.html
- Technological gadgets for learning (smart phone, laptop, desktop…)

G. Annexure

Refer XIP-A1 for template to record achievement

Topic: XIP-B2 Remainder and Factor Theorem *[250 minutes]*

Introduction

The Remainder and Factor Theorems are two mathematical principles that establish a connection between the roots of a polynomial and its linear factors. These theorems are frequently employed to facilitate the factorization of polynomials without resorting to long division, especially when combined with the Rational Root Theorem. This combination provides a powerful tool for polynomial factorization. Etienne Bezout (1730-1783), a French mathematician, is renowned for his work on the remainder theorem, which addresses the number of solutions of polynomial equations.

The Factor Theorem can trace its origins to the contributions of Sun Zi, a Chinese mathematician from the 3rd century AD. Qin Jiushao (1202-1261), also a Chinese mathematician, is credited with first presenting the Factor Theorem in 1247. These theorems, along with the Rational Root Theorem, offer valuable methods for factoring polynomials.

Source: https://prezi.com/p/rrner7wc1atz/etienne-bezout/

Utility and Scope

The Remainder Theorem offers a more efficient approach for testing whether specific numbers are roots of polynomials. When applied in conjunction with other polynomial tests, such as the rational roots test, this theorem can significantly improve efficiency.

Source: https://khanacademy.Remainder_theorem_of_polynomials

A. Competency

- Exhibit the knowledge of implementing remainder and factor theorems in factorization of quadratic and cubic polynomials.

B. Objectives

- Apply the remainder and factor theorem in determining remainders and factors of polynomials.
- Factorize quadratic and cubic polynomials using the factor theorem.

C. Essential Skills/Processes

- Analysing
- Applying
- Computing

D. Learning Experiences

- Demonstrate and explain the concepts of rational integral functions and the value of a function, as outlined in BHSEC Mathematics Book I.

- Empower students to deepen their understanding by exploring the Remainder and Factor Theorem through the following links:
  - o https://www.remainder-factor theorem-video1, a video explaining the Remainder and Factor Theorem.
  - o https://www.remainder-factortheorem-video2 , a videos solving problems using Remainder and Factor Theorem.
  - o Let students draw the differences between two Theorems.
  - o Provide additional practice questions, referring to BHSEC Mathematics Book I, Exercise 4(a), to help students reinforce their understanding.

- Demonstrate factorisation of cubic polynomials using Remainder and Factor Theorems, watch the tutorial available in the web link: https://www.youtube.com/watch?v=7sFrHa6uNUk&t=1s.
  - o For homework, encourage students to engage in factoring additional cubic polynomials, either from BHSEC Mathematics Book I, Exercise 4(a), or by exploring online resources.
  - o Further allow students to explore the real life application of Remainder and Factor Theorem. Visit the suggested link: https://sciencing.com/do-math-activities-real-life-8489684.html for the insightful examples.

E. Assessment

Performance Task 1

Design a worksheet containing questions to evaluate students' comprehension on Remainder and Factor Theorems, assessing their ability to determine remainders and

factors of polynomials, as well as their proficiency in factoring quadratic and cubic polynomials.

Sample worksheet: https://Worksheet-factor_and_remainder_theorem.pdf

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction
  https://prezi.com/p/rrner7wc1atz/etienne-bezout/
- Utility and scope
  https://khanacademy.Remainder_theorem_of_polynomials
- Remainder and factor theorem - https://www.remainder-factor theorem-video1
- Remainder and factor theorem - https://www.remainder-factortheorem-video2
- Factorisation of cubic polynomials using Remainder and Factor theorems
  https://www.youtube.com/watch?v=7sFrHa6uNUk&t=1s.
- Application for remainder and factor theorem
  https://sciencing.com/do-math-activities-real-life-8489684.html
- Worksheet for remainder and factor theorem
  https://Worksheet-factor_and_remainder_theorem.pdf
- Technological gadgets for learning (mobile, desktop, laptop...)

G. Annexure

Refer XIP-A1 for template to record achievements

Topic: XIP-B3 Quadratic Equations *[250 minutes]*

Introduction

Our knowledge of ancient civilizations is based only on what survives today. The earliest known problems that led to quadratic equations are on Babylonian tablets dating from 1700 BCE. In these problems, the Babylonians were interested in finding the dimensions x and y of a rectangle with a given area c and a given perimeter $2b$. The historian Victor Katz suggests that maybe there were some people who believed that if you knew the area of a rectangle, then you knew its perimeter. In solving these problems, these Babylonians may have been trying to show that many rectangles with different dimensions have the same area.

At the end of the 16th Century the mathematical notation and symbolism was introduced by amateur-mathematician François Viète, in France. In 1637, when René Descartes published La Géométrie, modern Mathematics was born, and the quadratic formula has adopted the form we know today.

Source: A brief history on Quadratic equations for maths educators
https://www.youtube.com/watch?v=GeTTLbJ3B10

Utility and Scope

Quadratic equations are widely used in science, business, engineering and in situations where two things are multiplied together and they both depend on the same variable. For example, when working with area, if both dimensions are written in terms of the same variable, you use a quadratic equation. Additionally, in business contexts, where the quantity of a product sold is linked to its price, quadratic equations are utilised to represent revenue as a product of the price and the quantity sold.

Quadratic equations are also used when gravity is involved, such as the path of a ball or the shape of cables in a suspension bridge. A very common and easy-to-understand application is the height of a ball thrown at the ground off a building. Because gravity will make the ball speed up as it falls, a quadratic equation can be used to estimate its height any time before it hits the ground which explains the concept of objects in free fall.

Source: https://www.youtube.com/watch?v=p1CqdKhkjRA
Suggested web link: https://www.youtube.com/watch?v=Bx4PUeoYldg
https://www.youtube.com/watch?v=Vi6q7pQnmkk

A. Competency

- Employ the formula methods to solve for the roots of quadratic equations occurring in everyday situations and classify the nature of roots.
- Display the knowledge of determining and interpreting the roots of quadratic inequalities derived from real world problems.

B. Objectives

- Factorise quadratic equations and find the roots using different methods.
- Determine the nature of roots using the value of discriminant.
- Determine the solutions of quadratic inequalities and interpret the roots.

C. Essential Skills/Processes

- Evaluating
- Computing
- Analysing
- Exploring

D. Learning Experiences

- Pre-assessment: Allow students to solve questions on quadratic equations using both factorization and graphical method (refer the web link quadratic equations sample worksheets for practice questions).

- Demonstrate how to solve a quadratic equation using formula.
  - Refer to the video from the web link: Use of formula on how to use the formula to solve roots of quadratic equations.
  - Assign problems with a) equal real roots, b) unequal real roots and c) complex roots for further practice (refer the worksheet: quadratic equations sample worksheets).

- Demonstrate the different types of roots by graphing relevant examples on GeoGebra or any graphing tool.
  - Allow students to examine the characteristics of each graph obtained and take notes.

- Reflective question:
  - Under what circumstances do you obtain: a) identical real roots, b) distinct real roots, and c) complex roots when applying the quadratic formula? [Encourage students to infer the nature of roots based on the discriminant value.]

- Play the video in the web link: Discriminant and nature of roots which explains the nature or character of roots i.e. real roots, complex roots and equal roots depending on different values of discriminants.
  - Allow students to make notes and apply their knowledge to check the nature of roots of the quadratic equations in the worksheet: Nature of roots.

- Elaborate on the concept of quadratic inequalities using examples: *A quadratic inequality is an equation of second degree that uses an inequality sign instead of an equal sign*.
  - Allow students to watch the following links on solutions of Quadratic Inequalities with example:
    - ➢ https://www.youtube.com/watch?v=gC9zS9u7ZJ0 contains the first way to solve the roots of a quadratic inequality.
    - ➢ https://www.youtube.com/watch?v=t54ccHYVhoo contains the second way to solve the roots of a quadratic inequality.

- Demonstrate the use of the Method of Interval to solve the roots of a quadratic inequality (refer to BHSEC Mathematics Book I). To enhance learning experiences, use the worksheet : Worksheets on quadratic inequalities.
  - Assign additional practice questions from BHSEC Mathematics Book I.

- Reflective question:
  - Describe the difference between the solutions or roots of a quadratic equality and a quadratic inequality.
  - How can you solve a quadratic inequality graphically?
  - Refer to the web link: https://www.youtube.com/watch?v=yBtJAVA cNkw.

E. Assessment

Performance Task 1

Competency based assessment

Design a homework task for the students with application based questions whereby students will need to demonstrate their ability to solve quadratic equations derived from any real world situations and interpret the results.

Sample questions:

For the following questions, write the relationships as equations and solve the equations using the formula method.

1. The sum of two numbers is 22, and the product of these two numbers is 120. What are the numbers?
2. The difference of two numbers is 8, and the sum of the squares of these two numbers is 320. What are the numbers?
3. The difference of the squares of two consecutive even integers is 60. What are these numbers?
4. Find three consecutive even integers such that the product of the first two is 38 more than the third integer.
5. The product of the ages of Alan and Terry is 80 more than the product of their ages 4 years prior. If Alan is 4 years older than Terry, what are their current ages?
6. The product of the ages of James and Susan in 5 years is 230 more than the product of their ages today. What are their ages if James is one year older than Susan?

Performance Task 2

Competency based assessment

Provide the worksheet in the web link: Worksheets on quadratic inequalities to assess the competency of the students in applying their knowledge of quadratic inequalities in finding a range of solutions of a problem in real world context.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - A brief history on Quadratic equations for maths educators
  https://www.youtube.com/watch?v=GeTTLbJ3B10
- Utility and scope - https://www.youtube.com/watch?v=p1CqdKhkjRA
  https://www.youtube.com/watch?v=Bx4PUeoYldg
  https://www.youtube.com/watch?v=Vi6q7pQnmkk
- Worksheet on quadratic - quadratic equations sample worksheets
- Quadratic formula methods - Use of formula
- Quadratic worksheet - quadratic equations sample worksheets
- Nature of roots - Discriminant and nature of roots
- Worksheet on nature of roots - Nature of roots
- Quadratic inequalities - https://www.youtube.com/watch?v=gC9zS9u7ZJ0
  https://www.youtube.com/watch?v=t54ccHYVhoo
- Worksheet - Worksheets on quadratic inequalities
- Quadratic inequality graph -https://www.youtube.com/watch?v=yBtJAVXcNkw
- Worksheet on quadratic inequalities - Worksheets on quadratic inequalities
- Technological gadgets for learning (mobile, desktop, laptop...)

G. Annexure

Refer XIP-A1 for template to record achievements

Topic: XIP-B4 Partial Fractions *[350 minutes]*

Introduction

Partial fractions are the fractions used for the decomposition of a rational expression. When an algebraic expression is split into a sum of two or more rational expressions, then each part is called a partial fraction. Hence, it is the reverse of the addition of rational expressions. Similar to fractions, a partial fraction will have a numerator and denominator, where the denominator represents the decomposed part of a rational function. In mathematics, we can see many complex rational expressions. If we try to solve the problems in a complex form, it will take a lot of time to find the solution. To avoid this complexity, we have to continue the problem by reducing the complex form of the rational expression into the simpler form. Partial fraction decomposition is one of the methods, which is used to decompose rational expressions into simpler partial fractions. This process is more useful in the integration process. In this article, you will learn the definition of the partial fraction, partial fraction decomposition, partial fractions of an improper fraction with solved examples in detail.

Thus, Partial Fraction is a way of "breaking apart" fractions with polynomials in them. In partial fractions when you split up a single fraction into a number of fractions whose denominators are the factors of the denominator of that fraction. These fractions are called Partial fractions.

The concept of partial fraction was discovered in 1702 by Johann Bernoulli (1667-1748), Swiss mathematician and Gottfried Leibniz (1646-1716), German mathematician independently.

Source: https://Introduction-partial and http://www.history-partial

Utility and Scope

Partial Fractions are used to decompose a complex rational expression into two or more simpler fractions. A partial fraction is a reverse of the process of the addition of rational expressions. In the normal process, we perform arithmetic operations across algebraic fractions to obtain a single rational expression.

For example if you are working in electronics, use of partial fractions will help you convert from LaPlace / Fourier (frequency) response to time-domain response. Solving an n by n linear system purely algebraically, with the variable s in tow, is a lot easier than solving an n by n system of differential equations, which is what you would have to do to keep

everything in the time domain. Using partial fraction decomposition will perform inverse LaPlace transforms in an easy way.

Source:
https://numberdyslexia.com/8-examples-of-partial-fractions-application-in-real-life/

A. Competency

- Apply effective methods to resolve different types of rational fractions into partial fractions for its utility in higher mathematical applications.

B. Objectives

- Comprehend the meaning and concept of rational functions in the form $\frac{f(x)}{g(x)}$.
- Resolve partial fractions for proper rational fractions.
- Resolve partial fractions for rational fractions with denominators containing repeating linear factors.
- Resolve partial fractions for rational fractions with denominators containing quadratic factors that cannot be factored into linear terms.
- Resolve partial fractions for improper rational fractions.

C. Essential Skills/Processes

- Applying
- Computing

D. Learning Experiences

- Define rational fractions, both proper and improper, by drawing connections with numerical fractions.

- Introduce the concept of Partial Fractions, referring to BHSEC Mathematics Book I or the notes provided in the link: https:/partial-fraction-expansion-1.

- Demonstrate how to resolve a given fraction into partial fractions in various cases:
  - Case 1. Degree of numerator < degree of denominator

    Type 1. Non Repeated linear factors
    - ➢ Refer to the link: https://non repeated linear factor for solved examples.

    Type 2. Repeated linear factor
    - ➢ Refer to the link: https://repeated linear function.com for solved examples.

Type 3. Quadratic factors not resolvable into linear factor

- ➢ Refer to the link: https://www.uEXgK8fvS6M for solved examples.

o Case 2. Degree of numerator ≥ degree of denominator

Type 1. Non Repeated linear factors

- ➢ Refer to the link: https://www.Mke71wzJIGY for solved examples.

Type 2. Repeated linear factor

- ➢ Allow students to devise the method to resolve improper rational fraction with repeated linear factors, employing concepts from long division and handling repeated linear factors.

- For each type, provide opportunities for students to apply their understanding by solving similar questions, referring to BHSEC Mathematics Book I.

E. Assessment

Performance Task 1

Assign questions to solve in a group on decomposing a given fraction depending on the nature of factors of the denominator.
Students can use PowerPoint presentations, chart papers or just the whiteboard to present their solutions with justification on why they used the particular method to resolve the fraction.

Refer BHSEC Mathematics Book I or link
https://tutorial.math.lamar.edu/problems/alg/partialfractions.aspx for assignment questions.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction -
  https://Introduction-partial and http://www.history-partial
- Utility and scope -
  https://numberdyslexia.com/8-examples-of-partial-fractions-application-in-real-life/

- Meaning of partial fraction - https://www.khanacademy.org/math/algebra-home/alg-rational-expr-eq-func/alg-partial-fraction/v/partial-fraction-expansion-1
- Non-repeated linear factors -https://www.non repeated linear factor.com
- Repeated linear factors - https://www.repeated linear function.com
- Quadratic factors - https://www.youtube.com/watch?v=uEXgK8fvS6M
- Improper fractions with non-repeated linear factors https://www.youtube.com/watch?v=Mke71wzJIGY
- Partial fractions questions - https://tutorial.math.lamar.edu/problems/alg/partialfractions.aspx
- Technological gadgets (mobile phones, laptops, computers, etc.)

G. Annexure

Refer XIP-A1 for template to record achievement

Topic: XIP-B5 Functions *[350 minutes]*

Introduction

Functions play a fundamental role in both mathematics and science, serving as a conceptual backbone that helps describe relationships and interactions within various contexts. In mathematics, a function is a rule or relationship between a set of inputs and outputs, where each input corresponds to exactly one output. This concept enables mathematicians to model real-world phenomena, analyse patterns, and solve problems. In science, functions are essential for expressing the dependencies and behaviours of physical processes. For instance, in physics, functions describe the motion of objects, changes in energy, or the trajectory of particles. In biology, functions can represent the relationship between variables in ecological systems, growth patterns, or physiological processes. Whether in mathematical equations or scientific models, functions provide a powerful and versatile tool for understanding and representing the complexities of the world around us.

The history of functions in mathematics is a fascinating journey that has evolved over several centuries. The concept of a function has its roots in ancient mathematical traditions, with early mathematicians exploring relationships between quantities. However, it wasn't until the 17th century that the notion of a mathematical function began to take a more formal shape. Mathematicians like Pierre de Fermat and John Wallis made initial contributions, laying the groundwork for future developments. The 18th-century mathematician Leonhard Euler made significant strides in advancing the theory of functions, introducing the notation f(x) and delving into the study of various types of functions. The 19th century saw further refinement, with Augustin-Louis Cauchy and Karl Weierstrass providing rigorous definitions and emphasising concepts such as continuity and limits. The 20th century witnessed the expansion of the function concept into more abstract realms, particularly in functional analysis. Today, functions are pervasive in mathematics and its applications, serving as a foundational tool for modelling relationships, solving equations, and advancing our understanding of the mathematical landscape. The historical development of functions reflects the continual quest for precision and abstraction in mathematical thought.

Explore the weblinks to learn more about the history of functions:
History of functions I and History of functions II

Utility and Scope

The study of functions in mathematics holds immense utility and has a broad scope, playing a pivotal role across various mathematical disciplines and real-world applications. Understanding functions provides a powerful tool for modelling relationships between quantities, analysing patterns, and solving a myriad of problems. Here are some key aspects of the utility and scope of learning functions in mathematics:

1. **Modelling Real-world Phenomena**: Functions serve as effective tools for representing and modelling real-world phenomena. They can describe physical processes, economic relationships, population growth, and more, allowing mathematicians and scientists to make predictions and understand the underlying mechanisms.

2. **Problem Solving**: Functions are essential for solving equations and inequalities. They enable mathematicians to express relationships between variables and formulate mathematical models that represent various problem scenarios. The ability to work with functions is crucial in solving problems across diverse fields, from physics to engineering to finance.

3. **Calculus and Analysis**: Functions are at the heart of calculus, a branch of mathematics that deals with rates of change and accumulation. The concept of limits, derivatives, and integrals, which are fundamental in calculus, relies heavily on understanding and manipulating functions.

4. **Graphical Representation**: Graphs of functions provide a visual representation of mathematical relationships. The study of functions involves interpreting and creating graphs, enabling a deeper understanding of the behaviour of variables and the overall structure of mathematical models.

5. **Optimization**: Functions are frequently used in optimization problems, where the goal is to maximise or minimise a certain quantity. This is applicable in fields such as economics, engineering, and operations research.

6. **Computer Science**: Functions are fundamental in computer science and programming. Algorithms, data structures, and software development often involve the use of functions to encapsulate and modularize code, making it more manageable and scalable.

7. **Statistics and Probability**: Functions play a crucial role in statistics and probability theory. Probability density functions, cumulative distribution functions, and other statistical models are expressed in terms of functions, providing a foundation for understanding uncertainty and variability.

8. **Engineering Applications**: Engineers use functions to model and analyse systems in various branches such as electrical engineering, mechanical engineering, and civil engineering. Understanding functions is essential for designing and optimising engineering systems.

In essence, learning functions in mathematics equips individuals with a versatile and essential set of tools that extend far beyond the confines of the mathematical realm. The applications of functions are vast and span numerous disciplines, making them a cornerstone of mathematical understanding and problem-solving.

A. Competencies

- Utilise the concept of functions and their classifications to find solutions to relevant real-life problems and make informed decisions.
- Demonstrate an understanding of the domain and range of a function by analysing the input and output values of the function.

B. Objectives

- Define and differentiate between various types and classifications of functions.
- Determine the inverse of a given function.
- Demonstrate an understanding of undefined functions.
- Perform algebraic operations on functions.
- Determine the domain and range of a given function.
- Mathematise the real life problem into different kinds of functions and interpret the functions to make decisions.

C. Essential Skills/Processes

- Conceptualising
- Computing
- Applying
- Recognising
- Creating

D. Learning Experiences

- Define a function and explain how to classify a relation as a function, with the help of mapping notations and vertical line test. Refer to BHSEC Mathematics Book I or the following weblinks:
  - https://www.youtube.com/watch?v=cqSZnON00OQ (contains video explaining the definition of a function with examples and non-examples. Play the video till 3:24 minutes).
  - https://www.youtube.com/watch?v=Mxe2lX1htNk (contains video explaining the vertical line test to check whether a relation is a function).
  - https://www.youtube.com/watch?v=V2C-wU5-7NY (contains video explaining the mapping notations which show a function).
  - Examine the students' learning through the quiz in the web link below: Suggestive questions for quiz

- Explain the concept of piece functions with graphical representations (refer the weblink: https://www.youtube.com/watch?v=Uzw9tsGq2Pw) .

- Demonstrate how to determine the 'value' of a real-valued function and piece function. Refer the web links:
  - ➢ https://www.youtube.com/watch?v=_e0EdFGpcvc (contains video explaining how to evaluate the value of a function with solved examples).
  - ➢ https://www.youtube.com/watch?v=OYOXMyFKotc (contains video explaining examples of piece functions).

- Explain the classifications of functions with examples by referring to BHSEC Book I. Alternatively, refer to the video from the link: Classification of Functions for the classification of functions, and https://www.youtube.com/watch?v=fKyBOLsqRlo for even and odd functions.

- Divide students into five groups and assign one type of special function to each group to explore and prepare a presentation: identity function, modulus function, greatest integer function, even function and odd function.
  - Direct students to refer either BHSEC Mathematics Book I or the web links:

- ➢ https://www.youtube.com/watch?v=oORSfuL_5SE&t=17s (contains video explaining an identity function).
- ➢ https://www.youtube.com/watch?v=tmPJdARPMS4 (contains video explaining a modulus function).
- ➢ https://www.youtube.com/watch?v=teaD5isBTfk&t=195s (contains video explaining the greatest integer or step function).
- ➢ https://www.youtube.com/watch?v=fKyBOLsqRlo&t=79s (contains video explaining the even and odd functions).

- Demonstrate with examples the definition of inverse functions and method to find inverse $f^{-1}(x)$ of a given function $f(x)$.
  - Refer to BHSEC Mathematics Book I or the web links: Inverse Function Notes to explore the concepts and examples of inverse functions.
  - Refer to https://tutorial.math.lamar.edu/problems/alg/inversefunctions.aspx for practice problems on inverse functions.
  - https://www.youtube.com/watch?v=2zeYEx4eTdc (contains a video explaining an example on determining the inverse of a function).

- Explain the meaning of 'undefined functions' with relevant examples (refer to BHSEC Mathematics Book I).

- Prepare a Powerpoint slide which meaningfully exhibits the different operations on functions with an example each. Refer to BHSEC Mathematics Book I or the web link: https://www.mathsisfun.com/sets/functions-operations.html (contains notes and examples for operations on functions).

- Introduce the concept of domain and range of a function. Refer BHSEC Book I or alternatively the following links:
  - https://www.youtube.com/watch?v=jw742vZRkOI (explains the differences between domain, co-domain and range).
  - https://www.youtube.com/watch?v=FdK9_Fp76cw (explains the concepts of domain and range).
  - https://www.youtube.com/watch?v=djT6-YamHaA&t=3s (examples of finding domain and range).

- Demonstrate the procedure to determine the domain and range of different types of functions.
  - Refer to BHSEC Mathematics Book I or the web links:
    - ➢ https://www.youtube.com/watch?v=GQGFMUfr10M (contains relevant methods to evaluate the domain of each type of function.)
    - ➢ https://www.youtube.com/watch?v=ryCKagPlzM8 (contains a video explaining a number of examples on finding the range of different kinds of functions.)
  - Provide practice questions to find domain and range of various types of functions (refer to BHSEC Mathematics Book I, Exercise 13(b)).

E. Assessment

Performance Task 1

Group work:
Design competency-based questions where students (in groups) will develop functions for real life situations and employ it to make decisions. Assess the students' learning through a presentation task.

Few examples are given below:

1. Question: At Dorji's pizzeria, a pizza costs Nu 300 with the first topping, and then an additional Nu 25 for each additional topping. If x represents the number of toppings on a pizza, what function represents the cost of a pizza with at least one topping?
   a. How much will you be charged if you ordered a pizza with 6 toppings?

Piecewise function

Question: B-mobile charges Nu 499 a month for a postpaid data plan. After 8.5 GB of data use, the plan charges Nu 300 for each additional GB used. Write the function showing the monthly charge.

a) How much will be the monthly charge if the data used a month is 6 GB?
b) Calculate how much data was used by Karma if she paid Nu 1249 in the month of December.
c) Compare the plan with the current plan that you are using and justify which and why you will choose to use it, for your data plan.

Step function

Question: An electrician works at a job site at a rate of Nu 500 per hour or any portion of an hour. In other words, he will charge you Nu 500 as soon as he comes up to the first hour, and then Nu 500 for the second hour, and so on.

a) Graph the amount the electrician charges in Ngultrum as a function of the number of hours he works.
b) How much does he charge for working 3.5 hours? Circle the point on the graph that shows this answer.

Performance Task 2

Worksheet

Design a worksheet (or use the worksheet in the web link) with questions on functions and their domains and ranges to assess the students individually on their learning experiences.

Refer to BHSEC Mathematics Book I or the web link:
https://www.mathworksheets4kids.com/domain-range.php which contains a sample of the worksheet on function and its domain and range.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - History of functions II
- Definition of functions - https://www.youtube.com/watch?v=cqSZnON00OQ
- Definition of functions - https://www.youtube.com/watch?v=Mxe2lX1htNk
- Mapping notation of functions - https://www.youtube.com/watch?v=V2C-wU5-7NY
- Quiz - Suggestive questions for quiz
- Piecewise function - https://www.youtube.com/watch?v=OYOXMyFKotc
- Value of functions - https://www.youtube.com/watch?v=_e0EdFGpcvc
- Classification of function -

https://www.youtube.com/watch?v=Qbd5_xcgBpU&t=139s

- Identity function: https://www.youtube.com/watch?v=oORSfuL_5SE&t=17s
- Modulus function: https://www.youtube.com/watch?v=tmPJdARPMS4
- Greatest Integer function: https://www.youtube.com/watch?v=teaD5isBTfk
- Even and odd functions - https://www.youtube.com/watch?v=fKyBOLsqRlo
- Inverse of function - https://www.youtube.com/watch?v=2zeYEx4eTdc
- Notes on inverse functions: Inverse Function Notes
- Practice problems on inverse functions: https://tutorial.math.lamar.edu/problems/alg/inversefunctions.aspx
- Video on inverse function: https://www.youtube.com/watch?v=2zeYEx4eTdc
- Operation of functions - https://www.mathsisfun.com/sets/functions-operations.html
- Domain, co-domain and range of functions https://www.youtube.com/watch?v=jw742vZRkO
- Concept of domain and range: https://www.youtube.com/watch?v=FdK9_Fp76cw
- Domain of function - https://www.youtube.com/watch?v=GQGFMUfr10M
- Types of functions- https://www.youtube.com/watch?v=ryCKagPlzM8
- Domain and range of functions -https://www.mathworksheets4kids.com/domain-range.php
- Self-instructional material
- Technological gadgets (Smart phone, computer and laptop)

G. Annexure

Refer XIP-A1 for template to record achievement

Topic: XIP-B6 Limits and Continuity of a Functions *[350 minutes]*

Introduction

The concept of limits is a foundational and transformative idea in mathematics, particularly within the realm of calculus. Introduced to tackle questions of continuity, change, and infinity, limits provide a systematic approach to understanding the behaviour of functions as they approach specific values. Originating in the 17th century through the pioneering work of mathematicians like Isaac Newton and Gottfried Wilhelm Leibniz, the concept of limits revolutionised mathematical thinking. Limits are instrumental in defining derivatives, integrals, and exploring the fundamental notions of convergence and divergence. This fundamental concept has applications across diverse mathematical domains, making it an essential tool for analysing functions and understanding the subtleties of mathematical continuity and change.

Ancient Greek mathematician Archimedes of Syracuse (287–212 BC) first developed the idea of limits to measure curved figures and the volume of a sphere in the third century BC. By carving these figures into small pieces that can be approximated, then increasing the number of pieces, the limit of the sum of pieces can give the desired quantity. Archimedes' thesis,*The Method,* was lost until 1906, when mathematicians discovered that Archimedes came close to discovering infinitesimal calculus. As Archimedes' work was unknown until the twentieth century, others developed the modern mathematical concept of limits. English physicist and mathematician Sir Isaac Newton (1642–1727) and German mathematician Gottfried Wilhelm Leibniz (1646–1716) independently developed the general principles of calculus (of which the theory of limits is an important part) in the seventeenth century.
Source: https://www. History and Inroduction.com

Utility and Scope

Learning about limits in mathematics is essential for understanding the foundations of calculus and has broad applications in various mathematical and scientific disciplines. Here are some of the utilities and scope of learning limits:

1. **Calculus Foundation**: Limits are a fundamental concept in calculus, serving as the building blocks for derivatives and integrals. Mastery of limits is crucial for a deep understanding of the principles and applications of calculus.
2. **Understanding Continuity**: Limits play a key role in defining continuity in mathematical functions. Learning about limits helps students comprehend the behaviour of functions and identify points of discontinuity.
3. **Infinite Series and Sequences**: Limits are used to define and analyse infinite series and sequences. This is particularly important in areas like number theory, where the convergence or divergence of series is a fundamental consideration.
4. **Physics and Engineering Applications**: Limits are extensively used in physics and engineering to model and analyse physical systems. Concepts like velocity, acceleration, and the trajectory of objects are often described using limits.
5. **Economics and Finance**: Understanding limits is beneficial in economics and finance, where it is employed to analyse rates of growth, optimise functions, and model economic behaviours over time.
6. **Computer Science**: Limits are relevant in computer science and programming, especially in algorithms, data analysis, and computational mathematics. They are used to analyse the efficiency and complexity of algorithms.
7. **Probability and Statistics**: Limits play a role in probability theory and statistics, particularly in defining probability distributions and understanding the behaviour of random variables.
8. **Mathematical Modelling**: Limits are crucial for mathematical modelling, allowing mathematicians and scientists to create accurate representations of real-world phenomena and make predictions based on mathematical models.

A. Competency

- Demonstrate understanding of fundamental theorems on limits to evaluate limits of algebraic and trigonometric functions and examine the continuity of a function.

B. Objectives

- Understand the fundamental theorems on limits.
- Compute the left-hand limit and right-hand limit to check if the limit at a point exists.
- Use various methods to evaluate the limit of algebraic functions.
- Evaluate the limit of trigonometric functions.
- Check for the continuity of a function both algebraically and graphically.

C. Essential Skills/Processes

- Conceptualising
- Computing
- Applying
- Recognising
- Creating

D. Learning Experiences

- Illustrate the idea of a limit through examples like the relationship between the area of a polygon inscribed in a circle and the area of the circle itself. As the number of sides of the polygon increases, its area grows, yet it never surpasses the area of the circle. Thus, we express the limit of the polygon's area as the number of sides approaches infinity to be the area of the circle. This can be symbolically represented as:

  $\lim_{sides \to \infty} Area\ of\ polygon = Area\ of\ circle$.

  - Provide some other relevant examples of limits.

- Explain the meaning of x tends to a, symbolically, $x \to a$.
  - Refer to the link: https://www.youtube.com/watch?v=rs-jpEw3Iic or BHSEC Mathematics Book I to learn the concepts of limits and its terminologies.

- Demonstrate finding of right-hand limit and left-hand limit. Refer to the link: https://www.youtube.com/watch?v=IZw2tdiYyfk and provide similar practice questions or refer to BHSEC Mathematics Book I.

- Explain the existence of a limit – the limit of a function exists only if the right hand limit is equal to the left hand limit. Refer to the link: example of checking for limit existence or BHSEC Mathematics Book I.

- Using graphing software (Geogebra), show the graphs of each function and explain the limits.
- Assign some questions for practice. Refer BHSEC Mathematics Book I, exercise 14(a), or find relevant questions from other sources.

- Refer BHSEC Mathematics Book I or the weblink: https://tutorial.math.lamar.edu/classes/calcI/defnoflimit.aspx to understand the meaning of limits.

- Explore the theorems on limits using BHSEC Mathematics Book I or watch the videos from the web links:
  - https://www.youtube.com/watch?v=py8JWwQSOck
  - https://www.youtube.com/watch?v=q7kxe6T0E14

- Demonstrate on evaluating algebraic limits by the following methods:
  - Direct substitution method: limit by direct substitution method.
  - Factorization method: https://www.youtube.com/watch?v=fOrOeZA-vdY .
  - Rationalisation method: https://www.youtube.com/watch?v=iVLyE_UEfxs .
  - Expansion: https://www.youtube.com/watch?v=epaHyrGisWM .
  - Provide practice questions for each type of method of evaluating algebraic limits . Refer to BHSEC Mathematics Book I, exercises 14(b), 14(c), 14(d), and 14(e).

- Demonstrate finding of limit at infinity. Refer the link: limit at infinity or BHSEC Mathematics Book I.
  - Assign questions from other sources or refer to BHSEC Mathematics Book I, exercise 14(f).

- Demonstrate and explain the trigonometric limits and its basic standard results.
  - Refer to BHSEC Mathematics Book I or the link: trig limits.
  - Allow students to explore questions related to the topic and let them present in the class.

- Discuss the definition of continuity of a function.
  - Suggested weblinks:
    - https://byjus.com/maths/continuity-and-discontinuity/ (contains notes),
    - https: limits-to-define-continuity (contains video lesson).

- Allow students to explore more in depth from the web link shared on how graphs are used to test the continuity of a function.

E. Assessment

Performance Task 1

Use the worksheet in the web link: https://www.liveworksheets.com/w/en/math/375129 to assess the students' comprehension of concepts in limits.

Performance Task 2

Design a homework task with questions checking the continuity of a function (refer to BHSEC Mathematics Book I).

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - https://www. History and Inroduction.com
- Concepts on limits - https://www.youtube.com/watch?v=rs-jpEw3Iic
- Right hand and left hand limits - https://www.youtube.com/watch?v=IZw2tdiYyfk
- Existence of limits - https://www.youtube.com/watch?v=aujuSVomE7k
- Theorem on limits - https://theorem on limits.com.
- Direct substitution methods - limit-by-substitution
- Factorization methods - https://www.youtube.com/watch?v=fOrOeZA-vdY
- Rationalisation method: https://www.youtube.com/watch?v=iVLyE_UEfxs
- Expansion: https://www.youtube.com/watch?v=epaHyrGisWM
- Infinity of limits - https://www.youtube.com/watch?v=NmLljBAg82o
- Trigonometry limits - https://www.trig limits
- Continuity of functions - https://byjus.com/maths/continuity-and-discontinuity/ https: limits-to-define-continuity
- Worksheet - https://www.liveworksheets.com/w/en/math/375129
- Technological gadgets (Smart phone, computer and laptop)

G. Annexure

Refer XIP-A1 for template to record achievement

Topic: XIP-B7 Differentiation *[500 minutes]*

Introduction

Differentiation is the process of finding the derivative $f'(x)$ of a function f. It is a part of maths under the branch of Calculus. The derivative is the gradient of the tangent of the graph of f at the point x. The discovery of calculus is often attributed to two men, Isaac Newton and Gottfried Leibniz, who independently developed its foundations. Although they both were instrumental in its creation, they thought of the fundamental concepts in very different ways.

While Newton considered variables changing with time, Leibniz thought of the variables x and y as ranging over sequences of infinitely close values. He introduced dx and dy as differences between successive values of these sequences. Leibniz knew that dy/dx gives the tangent but he did not use it as a defining property. On the other hand, Newton used quantities x and y, which were finite velocities, to compute the tangent. Neither Leibniz nor Newton thought in terms of functions, but both always thought in terms of graphs. For Newton the calculus was geometrical while Leibniz took it towards analysis.

Explore more on the history of differentiation and calculus. May use the suggested web link: https://www.youtube.com/watch?v=IMj5dgGWxSM

Suggested web links:

https://www.youtube.com/watch?v=BrH1fz-jZOo

https://www.youtube.com/watch?v=dfr23VIQPCo

https://www.youtube.com/watch?v=b7vIKO-uung&t=2s

Utility and Scope

Differentiation in mathematics, particularly in calculus, involves studying rates at which quantities change. It has a wide range of real-world applications and is crucial in various fields such as science, technology, business, and more. Here are some examples illustrating the application and scope of differentiation:

1. **Physics and Engineering:**

   a) **Motion Analysis**: Differentiation is used to analyse the motion of objects. In physics and engineering, it helps to calculate velocities, accelerations, and study the behaviour of particles.

   b) **Electric Circuits**: In electrical engineering, differentiation is applied to analyse voltage and current changes over time in circuits.

2. **Computer Science and Technology**:

   a) **Algorithm Analysis**: Differentiation helps analyse the efficiency and performance of algorithms, especially in terms of their time complexity.

   b) **Signal Processing**: In image and audio processing, differentiation is used to analyse and enhance signals.

3. **Economics and Finance**:

   a) **Marginal Analysis**: Differentiation is used in economics to analyse marginal cost, marginal revenue, and marginal utility, providing insights into optimal decision-making.

   b) **Options Pricing**: In finance, differentiation is used in option pricing models, such as the Black-Scholes model, to determine the rate of change of option prices concerning various factors.

4. **Biology and Medicine**:

   a) **Population Dynamics**: In biology, differentiation is used to model and analyse population growth and dynamics.

   b) **Medical Imaging**: Differentiation plays a role in the analysis of medical images, such as identifying edges and contours in diagnostic imaging.

5. **Environmental Science**:

   a) **Rate of Change in Environmental Variables**: Differentiation is used to study the rates of change in environmental variables, such as temperature, pollution levels, and ecosystem dynamics.

6. **Operations Research and Management**:

   a) **Optimization**: Differentiation is a key tool in optimization problems, such as finding the minimum or maximum values of functions. This is applied in operations research and management decision-making.

   b) **Supply Chain Management**: Differentiation helps optimise inventory levels, production rates, and distribution schedules.

7. **Statistics and Data Analysis**:

   a) **Regression Analysis**: Differentiation is used in regression analysis to find the best-fit line that represents the relationship between variables.

   b) **Time Series Analysis**: Differentiation is applied to study trends and patterns in time-series data.

8. **Telecommunications**:

   a) **Signal Transmission**: In telecommunications, differentiation is used to analyse and optimise signal transmission, such as the modulation and demodulation of signals.

9. **Business and Marketing**:

   a) **Marginal Cost and Revenue**: In business, differentiation helps analyse marginal cost and marginal revenue, aiding in pricing decisions.

   b) **Market Research**:** Differentiation is applied in market research to analyse how variables such as pricing, advertising, and product features impact sales.

10. **Automotive and Transportation**:**

    a) **Vehicle Dynamics**: Differentiation is used to analyse the motion and performance of vehicles, including aspects like acceleration, braking, and turning.

Explore more from the web link: [utility and scope of differentiation.]

A. Competencies

- Demonstrate an understanding of the definition of derivative of a function, theoretically and graphically to visualise the slope of a curve at any given point.
- Employ the knowledge of first principle and basic rules of differentiation to evaluate the equation of tangent and normal to the curve.

B. Objectives

- Explore the meaning and geometrical interpretation of derivatives.
- Differentiate functions using the first principle.
- Differentiate algebraic functions using basic rules.
- Differentiate trigonometric functions using basic rules.
- Apply the concept of derivatives to find the equations of tangent and normal to the curve.

C. Essential Skills/Processes

- Exploring
- Applying
- Computing
- Evaluating

D. Learning Experiences

- Introduce the meaning of differentiation as the instantaneous rate of change of a function which gives the slope of the tangent line at any point on the curve of the function.
  - Play the video in the following web links: Introduction to derivatives 1 and Introduction to derivatives 2 (contains video introducing the meaning of derivatives. May pause and explain further wherever necessary.)
  - Or refer to BHSEC Mathematics Book I and explain the meaning of differentiation geometrically with the help of the interactive design of the GeoGebra sheet: https://www.geogebra.org/m/kHAkeTCw.

- Prepare a Powerpoint presentation on the process of evaluation of derivatives of functions using definition or first principle. Refer to BHSEC Mathematics Book I.
  - Provide a simple function (e.g. x^2) and instruct students to apply the procedures displayed on the Powerpoint slide to evaluate the derivative.
  - Supplement on the activity by addressing the common doubts and solving further examples using the first principle to evaluate the derivatives.

- Write and display the basic rules of differentiation: constants, x^n and $(axe + b)^n$(power rules) and addition and subtraction rules of derivatives, on a chart paper (refer to BHSEC Mathematics Book I).
  - Provide one question each under each rule and pick random students to apply the rule to find the derivative.
  - Intrigue students to analyse the meaning of the derivative obtained each time (in terms of slope of the curve and rate of change).
  - May also play the video in the web link: General methods to show how to apply the power, addition and subtraction rule on differentiation of polynomial functions.

- Explain the product and quotient rule of differentiation on polynomial functions. Refer the links to learn how to apply product and quotient rule with examples.
  - Application of product rule
  - Examples of application of product rule
  - Quotient rule
  - Refer to BHSEC Mathematics Book I to provide practice questions.

- Direct students to watch the video in the link: Derivatives of trigonometric functions which explains the rule of differentiation of trigonometric functions with solved examples.
  - Students will make notes from the video or refer to BHSEC Mathematics Book I.
  - Provide questions from Exercise 16(d), BHSEC Mathematics Book I and direct students to solve and check their understanding of differentiating trigonometric functions.
  - Assess their works and address the common doubts.

- Discuss tangents and normals of a curve using derivatives.
  - Assign questions from Exercise 16(e), BHSEC Mathematics Book I.

- Instruct students to explore and solve a derivative of a function derived from a practical situation and present in the class.
  - Provide the web link: https://www.youtube.com/watch?v=eHDUI3pNEn8 for students to explore examples on the application in derivatives.

E. Assessment

Performance Task 1

Design a homework task comprising questions involving simple derivatives . Instruct students to differentiate the functions using both the methods of first principle and basic rules. Students will also write a reflection on the use of the two methods.

Performance Task 2

Create a Google Form quiz and bring students to the school's IT laboratory for quiz participation. Include questions on use of derivative rules to differentiate algebraic and trigonometric functions and give geometrical interpretation of the derivatives and questions to derive equations of tangent lines and normal lines to a curve.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - https://www.youtube.com/watch?v=IMj5dgGWxSM
  https://www.youtube.com/watch?v=BrH1fz-jZOo
  https://www.youtube.com/watch?v=dfr23VIQPCo
  https://www.youtube.com/watch?v=b7vIKO-uung&t=2s
- Utility and scope:https://www.cuemath.com/calculus/applications-of-derivatives/
- Meaning of derivatives - Introduction to derivatives 1, Introduction to derivatives 2
- Geometrical interpretation of derivatives - https://www.geogebra.org/m/kHAkeTCw.
- Derivatives of polynomial functions - General methods
- Derivatives of product rule - Application of product rule
  Examples of application of product rule
- Derivatives of Quotient rule - Quotient rule
- Derivatives of trig- functions - Derivatives of trigonometric functions
- Application derivatives - https://www.youtube.com/watch?v=eHDUI3pNEn8
- Self-Instructional Material
- Technological gadgets for learning (smart phone, laptop, desktop)

G. Annexure

Refer XIP-A1 for template to record achievement

Topic: XIP-B8 Integration *[450 minutes]*

Introduction

A fundamental concept in mathematics, integration includes the sum of a quantity over a given break or intermission. As an opposite process to differentiation, it comes in handy while calculating areas, volumes and other quantities by totalling infinitely smaller parts. The process plays a crucial role in mathematics, serving as a fundamental calculus tool, solving differential equations, as well as aiding in the calculation of complex shapes in analytical geometry.

Today, it is generally believed that calculus was discovered independently in the late 17th century by two great mathematicians: Isaac Newton and Gottfried Leibniz. The result also shows that Leibniz started first with integration and Newton with differentiation. It is Leibniz, however, who gave the new discipline its name. Newton called his calculus "the science of fluxions".

Source: history of integration.

Utility and Scope

Learning integration in mathematics can have significant real-world applications across various domains, including science, technology, arts, and more. Here are some examples illustrating how learning integration in mathematics can be applied in different fields:

1. **Science:**
    a. **Biostatistics:** Integrating mathematics with biology allows for the analysis of biological data, such as genetic sequences or population dynamics, using statistical methods.
    b. **Physics and Engineering**: Mathematical models are crucial in physics and engineering for understanding physical phenomena, designing experiments, and optimising processes.

2. **Technology**:
    a. **Computer Science**: Algorithms, data structures, and cryptography heavily rely on mathematical principles. Integrating mathematics with computer science enhances problem-solving and algorithmic thinking.
    b. **Artificial Intelligence**: Machine learning algorithms involve mathematical concepts, such as linear algebra and calculus, to analyse and make predictions based on data.
3. **Arts**:
    a. **Computer Graphics and Animation**: Mathematical concepts like geometry and linear algebra are essential in creating realistic computer-generated imagery (CGI) and special effects in the film and gaming industries.
    b. **Music and Sound Engineering**: Mathematical principles like Fourier analysis are used in the synthesis and processing of sound in music production.
4. **Economics**:
    a. **Econometrics**: Integration of mathematics and economics allows for the development of statistical models to analyse economic trends, forecast future conditions, and make informed policy decisions.
    b. **Financial Mathematics**: Mathematical models are used in finance for risk assessment, portfolio optimization, and the pricing of financial derivatives.
5. **Environmental Science**:
    a. **Climate Modelling**: Mathematics is integrated into climate models to simulate and predict climate patterns, allowing for the understanding of climate change and its potential impacts.
6. **Health Sciences**:
    a. **Medical Imaging**: Mathematics plays a crucial role in the development of algorithms for image processing and analysis in medical imaging technologies like MRI and CT scans.

b. Epidemiology: Mathematical modelling is used to analyse the spread of diseases and predict the impact of public health interventions.

Explore the web link: https://www.utility and scope of integration for more information.

A. Competencies

- Demonstrate an understanding of integration as the reverse process of differentiation.
- Display the ability to apply basic rules of integration to integrate functions derived from real-life applications.

B. Objectives

- Understand the meaning of integration, recognizing it as the reverse process of differentiation.
- Evaluate integrals of algebraic and trigonometric functions using basic rules.
- Integrate algebraic and trigonometric functions using the substitution method.

C. Essential Skills/Processes

- Evaluating
- Applying
- Creating

D. Learning Experiences

- Recapitulation: students can revise the concept of "differentiation". Ask some questions on differentiation.

- Differentiate a simple function and inquire from students how to recover the original function from its derivative. Subsequently, elucidate how integration serves as the reverse process of differentiation. Refer to the link: Integration as a reverse process.

- Show that if the derivative is the rate of flow of water in a tank at any point of time, then the integral of the derivative will be the volume of water at any point of time in the tank. Refer to the link: meaning of integration.

- Introduce the notation of integration and explain that the symbol is an elongated letter of S, which implies for 'sum'. For more detailed exploration of the integration symbol, you can refer to the link: symbol of integration.

- Demonstrate the integration process with simple examples and explain the reason for putting the constant 'C' at the end of every integral.
  - Refer to the link: https://www.puttingC or BHSEC Book I.
  - Let students practise integration of simple monomials to get acquainted with the integration sign and process.

- Students can explore the meaning of integration and standard forms of integrals.
  - Refer to BHSEC Mathematics Book I or the link: https://www.standard formula to know some of the basic integration formulae.
  - Students should understand how each integration formula came by relating with differentiation.

- Explore the basic rules of integration.
  - Refer to BHSEC Mathematics Book I or the link: https://www.basic rule of integration to find information about the rules.

- Demonstrate on evaluating integration on algebraic and t-functions.
  - Refer to the video link: https://www.algebraic and t-functions.
  - Provide questions to practise from the link: https://byjus.com/maths/integration-questions/ or refer to BHSEC Mathematics Book I, exercise 17(a).

- Demonstrate the method of integration by substitution.
  - Refer to BHSEC Mathematics Book I. Additionally, watch instructional videos on integration by substitution to enhance your understanding of this technique.
    - ➢ https://www.youtube.com/watch?v=sdYdnpYn-1o
    - ➢ https://www.youtube.com/watch?v=D0QJvYk0OkM
    - ➢ https://www.youtube.com/watch?v=3pXALn2ovIE
  - Allow students to practise some questions on integration by substitution. Refer to BHSEC Book I, exercise 17(b), or find some relevant questions from other sources.

E. Assessment

Performance Task 1

Students can assess their comprehension of fundamental integral formulas by solving questions from BHSEC Mathematics Book I, Exercise 17(a). This exercise focuses on

obtaining the integration of algebraic and trigonometric functions using the substitution method..

Performance Task 2

Design a task where students engage in discussion and write a reflection report on methods and strategy incorporated solving integration of algebraic and trigonometric functions by substitution methods. Allow students to refer to the link https://testbook.com/objective-questions/mcq-on-integration-using-substitution--5eea6a1439140f30f369f107 to carry out the performance task 2.

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - https://www.introduction - Integration
- Utility and scope - https://www.utility and scope of integration
- Definition of integration - https://www.youtube.com/watch?v=B_sj1gWR2oE
- Integration notes and symbols - htt://introduction-to-integral-calculus
- Integration notes and constant - https://www.puttingC
- Standard form of integration - https://www.standard formula
- Basic rule of integration- https://www.basic rule of integration
- Integration on trig -function - https://www.algebraic and t-functions
- Practice questions on integration on trig-function - https://byjus.com/maths/integration-questions/
- Integration by substitution - https://www.youtube.com/watch?v=sdYdnpYn-1o https://www.youtube.com/watch?v=D0QJvYk0OkM https://www.youtube.com/watch?v=3pXALn2ovIE
- Integration by substitution - https://testbook.com/objective-questions/mcq-on-integration-using-substitution--5eea6a1439140f30f369f107
- Technological gadgets (Smart phone, computer and laptop)

G. Annexure

Refer XIP-A1 for a template to record achievement.

Topic: XIP-C1 Compound and Multiple Angles *[450 minutes]*

Introduction

Trigonometry began in the work of Greek astronomers, especially Aristarchus of Samos, and was developed further by the Greek mathematical astronomers Claudius Ptolemy and Menelaus, in Alexandria, Egypt in the first few centuries of the Common Era. For these scholars, there was one major function, the length of the chord of a circle as a function of the length of the circular arc joining its endpoints. It was Indian astronomers (500-900 C.E.) who decided to study what we now call the sine and cosine as functions of an angle in a right triangle. After this knowledge was brought to the Arab and Persian world in mediaeval times and developed further, it was transmitted to Europe through contacts among merchants and as a result of warfare.

The word “sinus” arose from a misinterpretation of the Arabic word for this concept. Thomas Flinck of Flensburg, Germany, introduced the names “tangent” and “secant” into trigonometry in his book of 1583. Between 1624 and 1636, Edmund Gunter invented “cosine” and “cotangent,” with the prefix “co-” meaning “complement.” (In a right triangle, the cosine of an acute angle is the sine of its complementary angle.)

A compound angle is an algebraic sum of two or more angles. We use trigonometric identities to connote compound angles through trigonometric functions. The sum and difference of functions in trigonometry can be solved using the compound angle formula or the addition formula.

Source: https://www.youtube.history

Utility and Scope

Astronomy: Astronomers use trigonometry to calculate how far stars and planets are from Earth.

Architecture: Architects and people working in construction use trigonometry in a number of different ways. It’s used to work out the height and the basic structure of a building. An architect can use the functions to calculate loads and forces of a building. Without these calculations, buildings wouldn’t be safe.

Construction: Trigonometry is also used while a building is being designed, but it is also used in the construction of a building. The correct slope of a roof and the proper height and rise of a stairway can both be calculated with this mathematical technique.

Music: If you're interested in music theory and production, then trigonometry is very useful. For those who want to work in music production, knowing how to manipulate sine waves will help you create your own sounds. Sine waves are a single note, whereas multiple sine waves make a chord. Understanding how these work together helps sound engineers create the desired sound they require. It is also useful in live environments, as the angles that the speakers are placed can impact the quality of the sound we hear.

Game Development: It's also used in many aspects of game development. To design a character, make them move or design the environment they will move in, you'll need to learn trigonometry. While it might appear daunting to learn at first, taking the time to learn trigonometry can really have a positive impact in lots of different careers.

Source: https:/applications-of-trigonometry/

A. Competency

- Demonstrate an ability to apply the concept of compound and multiple angle formulas in solving trigonometric problems.

B. Objectives

- Comprehend addition and subtraction formulas for trigonometric ratios.
- Solve problems using the concept of addition and subtraction formulas.
- Understand the product formulas and apply them to convert sums and differences to products, as well as products to sums and differences.
- Be familiar with double-angle, triple-angle, half-angle, and one-third angle formulas.
- Apply double-angle, triple-angle, half-angle, and one-third angle formulas in solving related problems.

C. Essential Skills/Processes

- Conceptualising
- Applying
- Reasoning
- Analysing

D. Learning Experiences

- Demonstrate the derivation of the addition formula on $sin(A + B)$ and allow students to derive the addition formula of $cos(A + B)$ and $tan(A + B)$.

- Use the same concept of addition formulae to introduce subtraction formula $sin(A - B)$, $cos(A - B)$, $tan(A - B)$.
  - Accordingly, apply the formulae in problem solving.
  - Assign practice questions from Exercise 9(a), BHSEC Mathematics Book I.
  - Recommend students to refer to the following web links to supplement their knowledge:
    - ➢ https://www.youtube.com/watch?v=X2oZ_Lg5VbY (contains the addition and subtraction formula).
    - ➢ https://www.youtube.com/watch?v=zcEMKv5yIYs (contains the derivation of sum and subtraction formula)
    - ➢ https://www.youtube.com/watch?v=v4JrGnWgJLg (contains the ways to use addition or subtraction rules).

- First, allow students to explore on converting products into sums or differences using the web link: https://www.product-of-sum-or-difference.
  - Assign the task using the link: https://www.youtube.com/watch?v=wJkObGypLWk.

- A similar task will be assigned in groups converting sums or differences into product form the link: https://www.youtube.com/watch?v=J6rqasnRyGg and allow students to present their work and discuss with the entire class.
  - Suggest students to refer to BHSEC Mathematics Book I.
  - Discuss the application of this formula using the link: https://www.rJNFbP7K3r.
  - Allow students to explore questions from the exercise 9(b), BHSEC Mathematics Book I.

- Discuss the derivation of the double and half angle formula.
  - Refer the web links:
    - ➢ https://www.youtube.com/watch?v=SoC54lTgawU
    - ➢ https://www.youtube.com/watch?v=8KTkJTuMfPI.
  - For practice, the students will explore example questions from the web link: https://double-angle-half-angle.

- Demonstrate and explain the triple angle and one third angle formula.
  - Suggestion - refer to web link: https://www.youtube.com/watch?v=25xbE1gFXf8 and allow students to explore for one third angle formula by themselves and
  - Assign practice questions from the web link: https://www.youtube.com/watch?v=I4bgpHj7ekY

E. Assessment

Performance Task 1

To help students apply the formula of compound and multiple angles, assign a task from Exercise 9 (a), 9(b) and 9(c) from BHSEC Mathematics Book I.

Performance Task 2

Allow students to visit the web link https://compound-angle-practice-questions and solve the example questions. This will allow students to self-assess their solution with the answer provided in the website.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - https://www.youtube.history
- Utility and scope - https:/applications-of-trigonometry/
- Addition and subtraction of trig function - https://www.youtube.com/watch?v=X2oZ_Lg5VbY
- Derivation of sum and subtraction formula- https://www.youtube.com/watch?v=zcEMKv5ylYs
- Practice questions on addition and subtraction formula - https://www.youtube.com/watch?v=v4JrGnWgILg
- Product formula - https://www.product-of-sum-or-difference.
- Practice question on product formula- https://www.youtube.com/watch?v=wJkObGypLWk.
- Sum formula - https://www.youtube.com/watch?v=J6rqasnRyGg
- Practice question on sum formula- https://www.youtube.com/watch?v=rJNFbP7K3r.
- Double angle formula -

https://www.youtube.com/watch?v=SoC54ITgawU

- Half angle formula https://www.youtube.com/watch?v=8KTkJTuMfPI.
- Practice question on double and half angle - https://double-angle-half-angle
- Triple angle formula - https://www.youtube.com/watch?v=25xbE1gFXf8
- Practice question on third angle formula- https://www.youtube.com/watch?v=I4bgpHj7ekY
- Practice question on compound angle- https://compound-angle-practice-questions
- Technological gadgets (Smart phone, computer and laptop)

G. Annexure

Refer XIP-A1 for template to record achievement

Topic: XIP-D1 Points and their Coordinates in 2-Dimensions

[450 minutes]

Introduction

The coordinate system we commonly use is called the Cartesian system, after the French mathematician René Descartes (1596-1650), who developed it in the 17th century. Legend has it that Descartes, who liked to stay in bed until late, was watching a fly on the ceiling from his bed. He wondered how to best describe the fly's location and decided that one of the corners of the ceiling could be used as a reference point.

Source: https://wild.maths.org/ren%C3%A9-descartes-and-fly-ceiling

Utility and Scope

In the technical fields: The concept of 3-D geometry is also applied in the fields of robotics, computer, and video games. The way and the design of the characters that move through their virtual worlds requires geometric computations to create paths around the obstacles concentrating around the virtual world.

Astronomy & Physics: Geometry is used in the field of astronomy, helping to map the positions of stars and planets on the celestial sphere and describing the relationship between movements of celestial bodies.

Geographic Information Systems: Geometry concepts are used in satellites in GPS systems, it calculates the position of the satellite and location of GPS gauged by the latitudes and longitudes.

Source: https://www.toppr.com/bytes/geometry-in-daily-life/
https://studiousguy.com/examples-of-geometry-in-everyday-life/

A. Competencies

- Demonstrate a comprehensive understanding of the coordinate system in 2-Dimension, and apply distance, section, and midpoint formulas to delineate a point and a line within a two-dimensional coordinate system.
- Demonstrate an ability to apply formulas to find the coordinates of centroids and the incenter of a triangle with specified vertices.
- Interpret the slopes and angles of two lines to articulate the nature of their relationship, whether parallel or perpendicular.

B. Objectives

- Apply distance formula to find distance between two points in 2D.
- Use the division and midpoint formulas to find the coordinates of points dividing the join of two points.
- Determine the coordinates of the centroid and incenter of a triangle with given vertices.
- Determine the slopes of a straight line and draw the conditions for parallelism and perpendicularity of two lines
- Use formula to find the angle between two lines.

C. Essential Skills/Processes

- Conceptualising
- Comparing
- Reasoning
- Applying
- Evaluating

D. Learning Experiences

- Allow students to watch a video from the links: Cartesian Coordinate System and Cartesian Coordinate Plane to recapitulate the cartesian coordinate.
  - After watching the video, ask students to define cartesian coordinates. Ensure that students understand the meaning of *x*-coordinate and *y*-coordinate.

- Screen the video from the link: https://www.youtube.com/watch?v=0e-MHaMk7lw on how to find distance between the points and distance of a point from the origin using distance formula.
  - Ask students to take notes and assign questions from Exercise 18 (b), BHSEC Mathematics Book I.

- To introduce the section formula, first demonstrate the derivation of the internal section formula.
  - Refer web link: Section Formula Derivation or BHSEC Mathematics Book I.

- Discuss how to apply the internal section division formula to solve the problem from the BHSEC Mathematics Book I. Assign practice questions from the BHSEC Mathematics Book I, exercise 18 (b).

- Before moving on to the external section formula, derive the midpoint formula from the internal division, as the dividing point will be exactly in the middle of the two points.
  - In this case, the ratio *m* and *n* will be 1:1. Let students practise a few questions related to the midpoint formula.

- Allow students to explore the external section formula and make notes.
  - Refer to the link: https://www.youtube.com/watch?v=CJ0WPS65sIo or BHSEC Mathematics Book I.
  - Discuss how to apply the external section division formula to solve the problem.
  - Assign practice questions from BHSEC Mathematics Book I, exercise 18(b).

- Demonstrate and explain the derivation of finding the centroid of a triangle referring to the link: https://www.youtube.com/watch?v=GXKFAvbtfQc and BHSEC Mathematic Book I.
  - Allow students to explore a few questions from the exercise 18(b), BHSEC Mathematics Book I.

- Discuss on the slope of a straight line and the angle between two straight lines. Refer the links:
  - https://www.youtube.com/watch?v=jlkE4VCnhdE
  - https://www.youtube.com/watch?v=ILzOxUNXuSs

- Discuss angle between two lines and condition for parallel and perpendicular lines referring to BHSEC Mathematics Book I.
  - Suggestion - allow students to explore the web link: https://www.youtube.com/watch?v=BPgM3q6e-bk&t=219s.

E. Assessment

Performance Task 1

Allow students to explore questions from BHSEC Mathematics Book I, Exercise 18 (c) and (d).

Performance Task 2

Lead the students outdoors, randomly choose one student from the entire class, and instruct the selected student to walk 5 metres north from the starting point. Then, let him/her turn left at a 90-degree angle and walk an additional 12 metres before stopping. The remaining students will be tasked with calculating the distance of that student from the original point. (Encourage students to provide their answers in metres).

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - https://wild.maths.org/ren%C3%A9-descartes-and-fly-ceiling
- Utility and scope - https://www.toppr.com/bytes/geometry-in-daily-life/
- Utility and scope - https://studiousguy.com/examples-of-geometry-in-everyday-life/
- Cartesian coordinate system - Cartesian Coordinate System
- Cartesian coordinate plane: Cartesian Coordinate Plane
- Distance between two points - https://www.youtube.com/watch?v=0e-MHaMk7lw
- Concepts on section formula and derivation - https://www.youtube.com/watch?v=y4X01NzRM9k
- External section formula - https://www.youtube.com/watch?v=CJ0WPS65sIo
- Centroid and incentre of triangle - https://www.youtube.com/watch?v=GXKFAvbtfQc
- Slope of straight lines - https://www.youtube.com/watch?v=jlkE4VCnhdE
- Angle between lines - https://www.youtube.com/watch?v=ILzOxUNXuSs
- Condition of parallelism and perpendicularity - https://www.youtube.com/watch?v=BPgM3q6e-bk&t=219s
- Measuring tape and cone (for demarcation)

G. Annexure

Refer XIP-A1 for template to record achievement

Topic: XIP-D2 The Straight lines *[450 minutes]*

Introduction

A straight line is an endless one-dimensional figure that has no width. A straight line does not have any curve in it. It can be horizontal, vertical, or slanted.

When geometry was first formulated by Euclid (325 - 265 BC) in his work “Elements”, he defined a general line (straight or curved) to be "breadthless length" with a straight line being a line "which lies evenly with the points on itself"

Source: https://www.math.tamu.edu/ehmwk/Tan_chpt1.pdf

Utility and Scope

The concept of straight lines from mathematics has numerous practical applications in real life. For instance, in architecture and engineering, straight lines are used to design and construct buildings, bridges, and other structures. In physics, straight lines are used to represent the path of objects in motion.

The concept of straight lines can be applied in mathematical analysis for business and economics. Additionally, a straight line graph is used in medicine and pharmacy to figure out the accurate strength of drugs. Straight line graphs are used in the research process and the preparation of the government budget.

Source: https://www.utility and scope

A. Competencies

- Classify and represent an equation of a line in various forms for utility in higher applications of mathematics.
- Display an ability to visualise and calculate the distance, angle and bisector between two lines occurring on any 2-D plane.

B. Objectives

- Express the equation of the straight line in general form, intercept form and normal form.
- Reduce the general equation to the normal form.
- Calculate distance of a point from a line and distance between two parallel lines.
- Calculate angles between two lines using the relevant formula.

- Find the equation of the bisector between the lines.
- Discuss the family of lines and apply it in the relevant context.

C. Essential Skills/Processes

- Conceptualising
- Computing
- Creating
- Applying

D. Learning Experiences

- Using flipped classroom, share video links: https://www.google.solpe intercept form, https://www.google.point slope form and https://www.google.two point form on slope-intercept form, point-slope form and two points form of equation of straight line respectively.
  - In a class, discuss the problem solving on the above topics and assign a task.
  - For additional information refer to BHSEC Mathematics Book I.
  - Suggestion: Assign questions from Exercise 19(a), BHSEC Mathematics Book I.

- Allow students to provide a general equation of a line and identify slope and y-intercept.
  - Similarly, ask them to reduce the general equation $ax + by + c = 0$ of the straight line and identify the slope $(-\ a/b)$ and y-intercept$(-\ c/b)$.

- Instruct students to explore the derivation of intercept form and normal form of a straight line along with examples.
  - Refer the links:
    - ➢ https://www.intercept form of straight lines
    - ➢ https://www.normal form of equation.
  - In a group, assign a set of questions from Exercise 19(b), BHSEC Mathematics Book I. Allow students to present at least two questions randomly picked by the teacher.

- Recapitulate angle between two lines and allow students to explore questions that require the application of the angle between two lines formula.

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

- Demonstrate and explain how to reduce the general equation to the perpendicular form, concepts of identical lines and intersection of straight lines.
  - Allow students to explore questions from BHSEC Mathematics Book I.

- Allow students to explore the derivation of distance of a point from a line and distance between two parallel lines.
  - Refer the links:
    - ➢ https://www.point from a line
    - ➢ https://www.distance between two parallel lines
  - Assign practice questions from BHSEC Mathematics Book I.

- Present the notes on angular bisectors from the BHSEC Mathematics Book I, and give students an opportunity to share their understanding.
  - Allow students to explore questions from exercise 19(f), BHSEC Mathematics Book I.

- Explain the family of lines and help them define it.
  - Demonstrate how to find the equation of line parallel and perpendicular to the given line.
  - Assign similar practice questions from Exercise 19(h), BHSEC Mathematics Book I.

- Demonstrate how to apply the equation of intersection of two given lines. Solve questions from Exercise 19(h), BHSEC Mathematics Book I.

E. Assessment

Performance Task 1

Give some points of a straight line and ask students to find the equation of the straight line in different forms: general, point-slope, two-point and slope and y-intercept form.

Performance Task 2

Design a group task to facilitate a discussion on the real-life applications of straight lines. Encourage students to present their work, incorporating the tools provided by the teacher. Allocate dedicated time for question and answer sessions during the presentation.

Supplement: Allow students to explore the link https://www.com/watch?v=b-GAxKAyV0o.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - https://www.math.tamu.edu/ehmwk/Tan_chpt1.pdf
- Utility and scope - https://www.utility and scope
- Slope intercept form - https://www.google.solpe intercept form
- Point slope form - https://www.google.point slope form
- Two point form - https://www.google.two point form
- Derivation of normal form of equation - https://www.normal form of equation
- Derivation of intercept form - https://www.intercept form of straight lines
- Distance of point form a line - https://www.point from a line
- Distance between two parallel lines -https://www.distance between two parallel line
- Application straight lines - https://www.youtube.com/watch?v=b-GAxKAyV0o
- Technological gadgets for learning (smart phone, laptop, desktop)

G. Annexure

Refer 11-A1 for template to record achievement

Topic: XIP-D3 Circle and Theorems on Circles *[500 minutes]*

Introduction

The history of the circle and its associated theorems is a tale woven through the fabric of mathematical exploration spanning millennia. Ancient civilizations, including the Greeks, Egyptians, and Babylonians, recognized the geometric significance of the circle, employing it in architectural designs and practical measurements. The Greek mathematician Euclid, around 300 BCE, formalised the study of circles in his influential work "Elements," presenting fundamental theorems about their properties. Archimedes, in the 3rd century BCE, made notable contributions to understanding the measurement of circles and approximating the value of π. During the Islamic Golden Age, scholars further refined circle theorems, integrating geometric and algebraic approaches. The Renaissance witnessed a resurgence of interest in mathematical rigour, with figures like Descartes and Kepler contributing to the study of circles. In modern times, the circle remains a central theme in geometry, and its theorems form the basis for understanding angles, arcs, and relationships between geometric elements. The rich history of the circle and its theorems underscores its enduring significance in the development of mathematical thought.

Explore more on:
https://circlesonly.wordpress.com/circles-are-the-mother-of-all-inventions/

Utility and Scope

Learning circle theorems in mathematics provides students with essential tools for understanding and analysing the properties of circles. The utility and scope of this knowledge extend across various mathematical and practical applications. Here are key aspects of the utility and scope of learning circle theorems:

1. **Geometry Mastery**: Circle theorems contribute to a comprehensive understanding of geometry, particularly the properties and relationships associated with circles. Students learn to identify and apply theorems that govern angles, arcs, and lengths within circles.

2. **Problem-Solving Skills:** Mastery of circle theorems enhances students' problem-solving skills. By applying these theorems, students can analyse complex geometric situations, deduce unknown quantities, and solve problems involving circles.

3. **Coordinate Geometry**: Understanding circle theorems provides a foundation for working with circles in coordinate geometry. Students can apply their knowledge to derive equations of circles and solve geometric problems involving coordinates.

4. **Trigonometry Applications**: Circle theorems are closely tied to trigonometry, especially when dealing with angles and arc lengths. This knowledge is valuable for applications in physics, engineering, and other fields where trigonometric functions are prevalent.

5. **Architectural and Engineering Design**: Circle theorems find practical applications in architectural and engineering design. Understanding the properties of circles is essential for designing circular structures, arcs, and curves in various construction projects.

6. **Computer Graphics**: In computer graphics and programming, circle theorems are essential for rendering and manipulating circular shapes. Understanding these theorems is crucial for creating accurate and visually appealing graphics.

7. **Physics and Astronomy**: The study of circles is foundational in physics, where circular motion, orbits, and rotational dynamics are prevalent. Circle theorems play a role in understanding these phenomena and predicting their behaviour.

8. **Problem-Solving in Competitive Exam**s: Knowledge of circle theorems is often tested in competitive exams, making it a valuable skill for students preparing for mathematics assessments and entrance examinations.

9. **Advanced Mathematics**: Circle theorems serve as a stepping stone to more advanced mathematical concepts, including calculus and differential geometry. Understanding circular functions and their properties becomes important in these advanced studies.

10. **Real-WorldApplications:** Circle theorems find applications in real-world scenarios, such as navigation, where understanding angles and bearings is crucial. They are also relevant in surveying, where circular measurements may be encountered.

In essence, learning circle theorems equips students with a versatile set of tools applicable in various mathematical domains and real-world situations. The understanding of circles and their properties is foundational, forming an integral part of a well-rounded mathematical education.

A. Competencies

- Demonstrate proficiency in identifying and representing equations of circles in various forms, adeptly generating circles under specified conditions.
- Display the ability to analyse geometric configurations involving circles and apply the theorems to deduce unknown angles, arc lengths, and other geometric properties.
- Demonstrate the ability to engage in logical reasoning and systematically construct rigorous mathematical proofs, allowing them to present a sequence of logical arguments to establish the truth of a given theorem.

B. Objectives

- Define a circle and its parts.
- Represent the equation of a circle in standard form, and solve related problems.
- Illustrate theorems of circles using relevant diagrams, and apply the theorems in solving logical problems related to circles.

C. Essential Skills/Processes

- Conceptualising
- Representing
- Proving
- Computing
- Applying

D. Learning Experiences

- Introduce the definition of circle and components of circle by displaying the videos in the following links and instruct students to make notes.
  - o https://www.youtube.com/watch?v=q-3w7o4zbWQ (contains video defining a circle and locus)
  - o https://www.youtube.com/watch?v=Y5Z69g7A0eI (contains video explaining the components or basics of circle)
- Explain the Equation of the circle in centre-radius form . Refer to the link https://www.youtube.com/watch?v=cyKFBmTjyio explaining the derivation of the equation of circle.

- Allow students to use the GeoGebra app https://www.geogebra.org/calculator to visualise and confirm the particular cases (BHSEC Mathematics Book I, page 21-1 to 21-2) by typing in relevant examples.

- Discuss the diameter form of the equation of circle with students and demonstrate finding the equation of circle when two ends of the diameters are given. Refer to BHSEC Mathematics Book I.

- Demonstrate finding the centre and radius of a circle when the general equation of the circle is given.
  - First method: Refer to the link general to centre-radius form which explains how to determine the centre and radius of a circle by reducing the general equation to centre-radius form.
  - Second method: Refer to BHSEC Mathematics Book I, for the prescribed formula to find centre and radius of a circle from the general form of equation of circle i.e. $x^2 + y^2 + 2gx + 2fy + c = 0$. Then, radius = $\sqrt{(g^2 + f^2 - c)}$ and the centre= $(-g, -f)$.
  - Assess students' learning by providing further practice questions from BHSEC Mathematics Book I.

- Discuss examples on determining the equation of a circle when any 3 conditions are given. Refer to BHSEC Mathematics Book I for the insightful examples.

- Theorems on Circles: Prepare a Powerpoint presentation on Theorems of Circle and explain each theorem diagrammatically on the board or using a graphing tool (GeoGebra) to help students comprehend the theorems visually as well as prove them if possible.
  - Discuss all the eighteen theorems with proofs. Use illustrative diagrams in Geogebra for the complicated theorems. Some of the resources of the theorems are given below:

1. **Theorem on chords of a circle**

   Theorem 1 - https://www.geogebra.org/m/NMMEsDAQ
   Theorem 2 - https://www.geogebra.org/m/YaQ3mEjY
   Theorem 3 - https://www.geogebra.org/m/bUJVWeYn
   Theorem 4 & 5 - https://www.geogebra.org/m/pSn7Jd76

2. **Theorem on Arcs and Angles**
   Theorem 6 - https://www.geogebra.org/m/NkK6vSVr
   Theorem 7 - https://www.geogebra.org/m/JjDd2TKw
   Theorem 8 - https://www.geogebra.org/m/wCGeV5ku
   Theorem 9 - https://www.geogebra.org/m/vpxurwwk
   Theorem 10 - https://www.geogebra.org/m/xZeN5f4D

3. **Theorems on congruent arcs and chords**
   Theorem 11, 12, 13, 14 - explain with relevant diagrammatic representations.

4. **Theorems on Tangent lines and circles**

   Theorem 15 - https://www.geogebra.org/m/u6Ev7bHg

   https://www.youtube.com/watch?v=IcgycGSq9Us

   Theorem 16 - https://www.youtube.com/watch?v=3jQ_vWb3OFE

5. **Theorems on Angles in alternate Segment**

6. **Theorems on segments of a chord-** explain with relevant diagrammatic representations.

- Discuss how the theorems can be used in solving problems relevant to real life applications after studying each theorem (refer to exercise questions in BHSEC Mathematics Book I).

E. Assessment

Performance Task 1

Group work

Divide the students into six groups.

Select higher-order-thinking problems for the application of each Circle Theorem by referring to BHSEC Mathematics Book I and divide the questions of each theorem among each group for the students to explore and solve.

Students can also explore the application of circle theorems in their daily experiences and provide two examples in each group.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1.

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - History of Circle
- Definition of circle and locus - definition of circle and locus
- Components of basic circle - https://www.youtube.com/watch?v=Y5Z69g7A0eI
- Derivation of equation of circle - circle equation derivation
- Equation of circle in centre radius form- centre-radius form
- Determining centre and radius of a circle- general to centre-radius
- Theorem 1 - https://www.geogebra.org/m/NMMEsDAQ
- Theorem 2 - https://www.geogebra.org/m/YaQ3mEjY
- Theorem 3 - https://www.geogebra.org/m/bUJVWeYn
- Theorem 4 & 5 - https://www.geogebra.org/m/pSn7Jd76
- Theorem 6 - https://www.geogebra.org/m/NkK6vSVr
- Theorem 7 - https://www.geogebra.org/m/JjDd2TKw
- Theorem 8 - https://www.geogebra.org/m/wCGeV5ku
- Theorem 9 - https://www.geogebra.org/m/vpxurwwk
- Theorem 10 - https://www.geogebra.org/m/xZeN5f4D
- Theorem 15 - https://www.geogebra.org/m/u6Ev7bHg
- Theorem 16 - https://www.youtube.com/watch?v=3jQ_vWb3OFE
- Technological gadgets (Smart phone, projector and laptop).

G. Annexure

Refer 11 P-A1 for template to record achievement

Topic: XIP-E1 Measures of Central Tendency *[450 minutes]*

Introduction

Central tendency is defined as the statistical measure that identifies a single value as representative of an entire distribution. It aims to provide an accurate description of the entire data. There are three main measures of central tendency: the mode, the median and the mean. Each of these measures describes a different indication of the typical or central value in the distribution.

However, the term is first found in the mid-1690s in the writings of Edmund Halley (1656-1742), and it has been used to summarise observations of a variable since the time of Galileo (1564-1642).

Source: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3127352/

Utility and Scope

Measures of Central Tendency provide a summary measure that attempts to describe a whole set of data with a single value that represents the middle or centre of its distribution. For instance, the measures of central tendency allow researchers to determine the typical numerical point in a set of data. The data points of any sample are distributed on a range from lowest value to the highest value. Measures of central tendency tell researchers where the centre value lies in the distribution of data.

Source: Applications of Mean, Median and Mode

A. Competency

- Demonstrate proficiency in determining the central tendencies for diverse data distributions through the application of various strategies.

B. Objective

- Understand the meaning of central tendencies and their significance in data analysis.
- Know the different types of data distributions: simple distribution, grouped frequency distribution, and ungrouped frequency distribution.
- Determine central tendencies (Mean, Median, Mode) of each type of data distributions.

C. Essential Skills/Processes

- Displaying

- Analysing
- Applying

D. Learning Experiences

- Ask questions on the definition of mean, median and mode that the learners have learned in lower classes to assess their prior knowledge.

- Elaborate why mean, median and mode are called as the measures of central tendencies.
  - Refer to the link https://www.youtube.com/watch?v=6DYtC7lrVuY or BHSEC Mathematics Book I.
  - For in depth understanding of the central tendencies, allow learners to refer the following links:
    - ➢ https://www.youtube.com/watch?v=B1HEzNTGeZ4
    - ➢ https://www.youtube.com/watch?v=o8OHiGilABE

- Revise the type of distribution: simple distribution (Individual Observation), ungrouped distribution (Discrete series), and group distribution (Continuous Series).

- Allows students to recall the direct method of finding mean of simple distributions. Then introduce the concept of using an assumed mean to find the mean of a data, which is called the shortcut method.
  - Refer to the link :https://www.youtube.com/watch?v=sFQmPKMwnpo explaining the concept of direct and shortcut method of finding the means of different data types.

- Discuss on the step-deviation method to find means of grouped data. Refer to the video link: https://www.youtube.com/watch?v=9yNPMVa0NQU or BHSEC Mathematics Book I.
  - Provide practice questions on finding the mean of different types of data distribution. Refer BHSEC Mathematics Book I exercise 23(a).

- Discuss the approach to calculating the median for simple data series, considering situations with both even and odd data counts. Refer to BHSEC Mathematics Book I, example 15.

- Discuss the steps involved in determining the median for discrete series, offering clarity on the concept of cumulative frequency. For a more comprehensive

understanding, refer to BHSEC Mathematics Book I, example 20 or access the instructional video link: median of discrete series.

- Explain the formula to find the median of grouped data. Refer to BHSEC Mathematics Book I or refer to the link : https://www.youtube.com/watch?v=1b1Tnp79Emk.
- Allow students to explore the definition of mode and practice finding modes of simple distribution.
- Discuss on finding the mode of discrete series: by inspection method only. Refer to BHSEC Mathematics Book I and allow students to practise some relevant questions.
- Explain the formula to find the mode of grouped data. Refer to BHSEC Mathematics Book I, page number 23-38 or alternatively refer to the link: https://www.youtube.com/watch?v=R0Zgml8VHT8.
    - Provide some relevant questions for practice.

E. Assessment

Performance Task 1

Collect the marks of the students in the mathematics for midterm exam and let students find mean, median and mode, by organising the data collected into different types: simple, discrete and continuous. Based on the central tendencies obtained, ask students to describe and draw conclusions on the midterm performance.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1.

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3127352/
- Utility and scope - Applications of Mean, Median and Mode
- Meaning of central tendencies:
    - https://www.youtube.com/watch?v=B1HEzNTGeZ4
    - https://www.youtube.com/watch?v=6DYtC7lrVuY
    - https://www.youtube.com/watch?v=o8OHiGilABE
- Shortcut method for mean:

- https://www.youtube.com/watch?v=sFQmPKMwnpo
- Step-deviation method of mean:
  - https://www.youtube.com/watch?v=9yNPMVa0NQU
- Simple distribution mean, median and mode:
  - https://www.youtube.com/watch?v=zjHfAhcU6kE
- Discrete data mean, median and mode -
  - https://www.youtube.com/watch?v=fBG1T7OwGQc&t=15
- Computing median of discrete series:
  - https://www.youtube.com/watch?v=AuwPFjwloJc
- Median of grouped data:
  - https://www.youtube.com/watch?v=1b1Tnp79Emk
- Mode of grouped data:
  - https://www.youtube.com/watch?v=R0Zgml8VHT8
- Notes on central tendencies:
  https://byjus.com/mean-median-mode-formula/
- Technological gadgets (Smart phone, projector and laptop)

G. Annexure

Refer XIP-A1 for template to record achievement

Topic: XIP-E2 Dispersion *[450 minutes]*

Introduction

Dispersion was originally proposed by R.A. Fisher (17 February 1890 – 29 July 1962) for the processing of the results of agricultural trials, aimed at establishing the conditions under which a given agricultural crop yields a maximal harvest. He was a British polymath who was active as a mathematician, statistician, geneticist and academic.

Source: https://protonstalk.com/statistics/measures-of-dispersion

Utility and Scope

Measures of Dispersion are used to estimate "normal" values of a dataset, measures of dispersion are important for describing the spread of the data, or its variation around a central value. It is usually used in conjunction with a measure of central tendencies, such as the mean or median, to provide an overall description of a set of data.

Source: https://protonstalk.com/statistics/measures-of-dispersion/

A. Competency

- Demonstrate the ability to measure the dispersion of various types of data using an appropriate method.

B. Objectives

- Understand the meaning of dispersion and its significance in data analysis.
- Know different methods to measure dispersion: quartile deviation, interquartile deviation, standard deviation, mean deviations, coefficient of quartile deviation.
- Compute quartile deviation, interquartile deviation and coefficient of quartile deviation.
- Compute standard deviation using various approaches.
- Compute mean deviation about mean or median.
- Calculate mean and standard deviation of combined distributions (two groups only).

C. Essential Skills/Processes

- Conceptualising
- Applying
- Analysing

- Representing
- Reasoning

D. Learning Experiences

- Recap the previous knowledge on central tendencies and their purposes in describing data. Refer to the link https://www.concept of central tendency to recap the concept of central tendencies.

- Explain the limitations of central tendencies in describing data, and the need for other ways to describe a data set.

- Introduce the term dispersion and its applications in analysing data. Refer the link https://www.concept of dispersion to know the meaning and concepts of dispersions.
  - Watch the video from the link https://www.youtube.com/watch?v=hEErJHx--qY to understand the limitations of central tendencies and importance of dispersion.

- Discuss on finding range, interquartile range and semi-interquartile range of different data sets. Refer to BHSEC Mathematics Book I and BHSEC Mathematics Book II, or the following link: https://youtu.be/06lxbMpYxm4
  - Allow students to practise finding range, interquartile range and semi-interquartile range of different data sets. Refer to BHSEC Mathematics Book I, exercise 24 (a) and (b).

- Discuss on quartile deviation (Q.D.) and coefficient of quartile deviation (C.Q.D.), and their significance. Ensure that students understand the appropriate situations for applying these measures.
  - Refer to BHSEC Mathematics Book I and BHSEC Mathematics Book II, or the following links:
    - ➢ https://www.youtube.com/watch?v=hEErJHx--qY
    - ➢ https://www.quartile deviation
    - ➢ https://www.youtube.com/watch?v=qdJlHO6tLkw
  - Assign a few relevant questions on the quartile deviation and coefficient of quartile deviation for practice. Refer to BHSEC Mathematics Book II, exercise 15 (a).

- Discuss on mean deviation (M.D.) and Coefficient of mean deviation (C.M.D.). Refer https://www.youtube.com/watch?v=Kc1aiVApB8M or BHSEC Mathematics Book II.

Ensure that students understand the appropriate situations for applying these measures.

- Assign some relevant questions on the mean deviation and coefficient of mean deviation to practise. Refer BHSEC Mathematics Book II, exercise 15 (b).

- Explain the meaning of standard deviation and variance. Refer to BHSEC Mathematics Book I and BHSEC Mathematics Book II, or the web link: https://www.standard deviation and variance.

- Demonstrate calculating standard deviation for different data sets, simple distribution and frequency distribution using various methods, variance and coefficient of variation. Refer to BHSEC Mathematics Book I and BHSEC Mathematics Book II.
  - Assign a few questions on the standard deviations and variance for practice. Refer BHSEC Mathematics Book I, exercise 24 (b) and BHSEC Mathematics Book II, and exercise 15(c).

- Discuss on combined mean and standard deviation of combined mean.
  - Refer to BHSEC Mathematics Book II or the following links:
    https://youtu.be/Djl6XbBsHLA
    https://youtu.be/tM9cQW97ZkY
  - Assign a few questions on the combined mean for practice. Refer to BHSEC Mathematics Book II, exercise 15(d).

E. Assessment

Performance Task 1

Collect the marks of the students in the mathematics midterm exam and let students find range, interquartile range, semi-interquartile range, quartile deviation and coefficient of quartile deviation by organising the data collected into different types: simple, discrete and continuous. Based on the dispersion value obtained, ask students to describe and draw conclusions on the midterm performance.

Performance Task 2

For the data collected above, let students find standard deviation and variance by organising the data into different types: simple, discrete and continuous. Based on the dispersion value obtained, ask students to describe and draw conclusions on the data collected.

Performance Task 3

Collect the marks of the students in the test and midterm examination. Now there are two different sets of data in hand. Ask students which method of measuring dispersion would be best to compare the performance of students in test and midterm exam. Students should know that they must use any one of the deviations and coefficient of deviations. Ask students to find the quartile deviation, coefficient of quartile deviation, mean deviation, and coefficient of mean deviation, variance and coefficient of variance.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - https://protonstalk.com/statistics/measures-of-dispersion
- Utility and scope - https://protonstalk.com/statistics/measures-of-dispersion/
- Concepts on central tendency - https://www.concept of dispersion
- Limitation and importance of central tendency
  https://www.youtube.com/watch?v=hEErJHx--qY
- Finding range - https://youtu.be/06lxbMpYxm4
- Finding quartile deviation for simple distribution-
  https://www.youtube.com/watch?v=hEErJHx--qY
  https://www.quartile deviation
  https://www.youtube.com/watch?v=qdJlHO6tLkw
- Mean deviation (M.D.) and Coefficient of mean deviation (C.M.D.) -
  https://www.youtube.com/watch?v=Kc1aiVApB8M
- Meaning of standard deviation and variance - https://www.standard deviation and variance
- Finding combined mean -
  https://youtu.be/DjI6XbBsHLA
  https://youtu.be/tM9cQW97ZkY

G. Annexure

Refer XIP-A1 for template to record achievement

Topic: 11 P-E3 Probability *[300 minutes]*

Introduction

Probability is the chance that something will happen. How likely it is that some event will occur. Sometimes we can measure a probability with a number like "10% chance", or we can use words such as impossible, unlikely, and possible, even chance, likely and certain. Example: "It is unlikely to rain tomorrow".

A gambler's dispute in 1654 led to the creation of a mathematical theory of probability by two famous French mathematicians, Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665). Probability, chance and randomness have been around since the ancient days. They could be found in fortune telling, games of chance, philosophy, law, insurance, and errors of prediction in astronomy and medicine (Hald, 1990). There is evidence that in the late 15th century and the early 16th century, mathematicians started to experiment with the idea of probability. The link helps to learn additional history of probability.

Source: http://history of probability

Utility and Scope

Probability is an essential tool in applied mathematics and mathematical modelling. Moreover, probability is used in daily life to make decisions when you don't know for sure what the outcome will be. Most of the time, you won't perform actual probability problems, but you'll use subjective probability to make judgement calls and determine the best course of action. For example, to check the weather forecast, playing cards, lottery tickets, etc.

Source: https://www.probability-real-life-examples/

A. Competency

- Demonstrate the ability to calculate probabilities for various events by applying the laws of probability and set theory.

B. Objectives

- Understand the meaning of basic terms of probability: random experiments, outcomes, sample space, sample point and events.

- Differentiate the different types of events: simple event, compound event, sure event, impossible event, exhaustive events, mutually exclusive events, not mutually exclusive events, and complementary events.
- Solve simple probability problems by drawing sample space diagrams, outcome charts or sets.
- Solve probability problems involving 'at least', 'at most', 'exactly', and 'not all'.
- Differentiate between dependent and independent events, and calculate probabilities using the addition (OR) rule and multiplication (AND) rule. *(Exclude probability problems involving the selection/arrangement of two or more things and combining addition and multiplication rule).*

C. Essential Skills/Processes

- Computing
- Demonstrating
- Applying

D. Learning Experiences

- Revise terminologies related to probabilities learned in lower classes. Refer https://www.youtube.com/watch?v=KzfWUEJjG18&t=297s to learn the basic concepts of probability.

- Allow students to explore the link: https://www.youtube.com/watch?v=SnzUoLWeFkA to understand the concept of random experiments, outcomes, sample space, and types of events.

- Recapitulate the set theory concepts learned by students in class IX and draw connections to the representation of sample points, events, and outcomes through a set. Refer to BHSEC Mathematics Book II or explore the following web links:
  - https://www.youtube.com/watch?v=1WONKtD2-Yw (set theory)
  - https://www.youtube.com/watch?v=719OsAWmq0A (representing sample space, events and outcomes in sets)
  - https://www.youtube.com/watch?v=fQqQCVkY_Ig (Intersection and union

- Let students solve questions from exercise 18 (a), BHSEC Mathematics Book II, to check understanding on random experiments, outcomes, sample space, and types of events.

- Explain the probability of an event and failure of the event. Refer to BHSEC Mathematics Book II or the web link: https://www.youtube.com/watch?v=0T-CaQCiSf4.

- Discuss the concept of 'odds' against the happening of an event. Refer to BHSEC Mathematics Book II or the link: https://www.youtube.com/watch?v=Vu4x2DKn12g&t=1s.
  - Assign a few probability questions for practice. Refer to BHSEC Mathematics Book II, exercise 18(b), or the link: https://www.indiabix.com/aptitude/probability/ .

- Explain the meaning of 'at least', 'exactly', 'at most' and 'not all' terms and solve related problems. Assign relevant questions from BHSEC Mathematics Book II, exercise 18(c) and 18(d), or the link: https://www.indiabix.com/aptitude/probability/ .

- Explain the 'addition rule' of probability (**mutually exclusive** events and **not mutually exclusive**). Discuss the addition theorem on probability and related set notations with the Venn diagrams or by drawing sample space diagrams and outcome charts.
  - Refer combining probability part in BHSEC Mathematics Book II or check the following web links:
    - ➢ https://www.addition rule-video1
    - ➢ https://www.addition rule-video2
    - ➢ https://www.addition rule-video3
  - Assign a few questions for practice. Refer to BHSEC Mathematics Book II, exercise 18(e) and 18(f), or the web link: https://www.quiz/addition-rule-probability.

- Recall the definition of dependent and independent events learned in lower classes to introduce the 'multiplication rule'.
  - Discuss the multiplication theorem on probability and related set notations with Venn diagrams or by drawing sample space diagrams and outcome charts. Refer to BHSEC Mathematics Book II or the following links:
    - ➢ https://www.1. multiplication rule
    - ➢ https://www.2. multiplication rule
    - ➢ https://www.3. multiplication rule
  - Discuss a few examples involving the multiplication rule and assign a set of practice questions. Refer to BHSEC Mathematics Book II, exercise 18(g), or the web link: https://www.practice question.pdf.

E. Assessment

Performance Task 1

In a single throw of three fair dice, find the probability of getting

i. a total of 6
ii. a triplet or a total of 6
iii. a total of at most 6
iv. a total of at least 6
v. a total of 6 or at least 6.
vi. the same number on all the dice.
vii. not getting the same number on all the dice.

Performance Task 2

Place Based Approach

Take students to a basketball court of your school, and let each student shoot a basketball 7 times. Let the individual student record their own shots. Using their records:

i) Ask students to find the probability of making at least 2 shots if they shoot four times continuously.
ii) Instruct students to get into a group of three members and ask them to find the probability that one of them will make a shot, if they shoot once each.

Design an appropriate tool for each performance task and record feedback and achievement based on the templates given in the annexure XIP-A1.

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction of probability- http://history of probability
- Utility and scope- https://www.probability-real-life-examples/
- Concepts of probability- https://www.youtube.com/watch?v=KzfWUEJjG18&t=297s
- Concepts of probability - https://www.youtube.com/watch?v=0T-CaQCiSf4
- Set theory - https://www.youtube.com/watch?v=1WONKtD2-Yw
- Set theory - https://www.youtube.com/watch?v=719OsAWmq0A
- Set theory - https://www.youtube.com/watch?v=fQqQCVkY_lg
- Probability events-https://www.youtube.com/watch?v=0T-CaQCiSf4
- Meaning of 'odds' against the happening of an event-https://www.youtube.com/watch?v=Vu4x2DKn12g&t=1s

- Practice questions-https://www.indiabix.com/aptitude/probability/
- Addition rule- https://www.addition rule-video1
- Addition rule - https://www.addition rule-video2
- Addition rule - https://www.addition rule-video3
- Practice question on addition rule - https://www.quiz/addition-rule-probability
- Multiplication rule-https://www.1. multiplication rule
- Multiplication rule-https://www.2. multiplication rule
- Multiplication rule-https://www.3. multiplication rule
- Practice question on multiplication rule- https://www.practice question.pdf

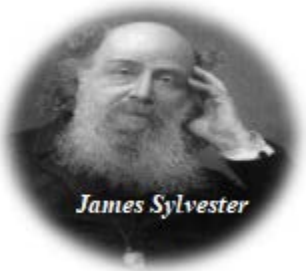
G. Annexure

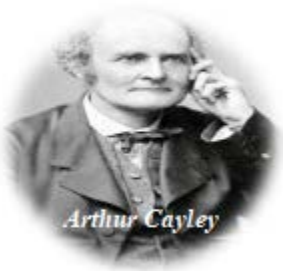
Refer 11 P-A1 for template to record achievement

Instructional Guide
Class XII
Mathematics

Topic: XIIP-A1 Matrices *[500 minutes]*

Introduction





The history of matrices in mathematics is a journey that spans centuries and continents. The concept of arrays of numbers for solving systems of linear equations can be traced back to ancient China and Japan. However, it wasn't until the 19th century that the term "matrix" was coined by James Joseph Sylvester in England, and Arthur Cayley made significant contributions to matrix multiplication. In the early 20th century, matrices became essential in the development of quantum mechanics, with David Hilbert and Wilhelm Ackermann formalising "matrix mechanics" in 1928. The 1930s saw the emergence of matrix algebra and linear algebra as distinct branches, and matrices found practical applications in computer science in the 1940s. Throughout the 20th and into the 21st century, matrices have become a fundamental and versatile tool in mathematics, playing a central role in diverse fields such as physics, engineering, computer science, and data analysis.

Explore more on https://www.youtube.com/watch?v=s4VFKg8OKTM.

Utility and Scope

Matrices have widespread applications in various real-world fields due to their ability to efficiently represent and manipulate complex data. Here are some examples of how matrices are applied in different domains:

1. Computer Graphics:

Matrices are extensively used in computer graphics to represent transformations, such as translation, rotation, and scaling, enabling the creation and manipulation of 2D and 3D graphics.

2. **Data Analysis and Statistics**:

In statistics, matrices are used to represent multivariate data sets, and techniques like covariance matrices are employed to analyse relationships and variability between variables.

3. **Engineering**:

Structural engineers use matrices to analyse and design complex structures by representing them as systems of linear equations. Finite element analysis involves extensive matrix calculations to model and analyse structural behaviour.

4. **Physics**:

Matrices play a vital role in quantum mechanics, where they represent operators corresponding to physical observables. The manipulation of matrices is essential for describing the behaviour of quantum systems.

5. **Computer Science and Information Technology**:

Matrices are used in cryptography for encryption and decryption algorithms. Linear algebra, including matrix operations, is fundamental in various algorithms and data structures.

6. **Economics and Finance**:

Input-output matrices are used to represent economic relationships between different sectors in an economy. Markowitz's portfolio theory utilises matrices to optimise investment portfolios.

7. **Electrical Engineering**:

Matrices are employed in circuit analysis to model electrical networks and solve systems of linear equations. They are also used in control systems for analysing and designing dynamic systems.

8. **Chemistry**:

Matrices are used to represent molecular structures and transformations in chemistry. Quantum chemistry involves matrix computations to study molecular properties.

9. **Geography and GIS (Geographic Information Systems)**:

Matrices are used to represent spatial data and transformations in GIS applications. They help analyse and visualise geographic information.

10. **Machine Learning and Data Mining:**

Matrices play a crucial role in machine learning algorithms, especially in areas like linear regression, neural networks, and dimensionality reduction. Matrices are used to represent datasets and model parameters.

11. **Telecommunications:**

Matrices are used in signal processing for tasks like encoding and decoding information. In communication systems, matrices represent the transformation of signals during transmission.

12. **Medical Imaging:**

Matrices are utilised in medical imaging for tasks like image reconstruction, where they represent the transformation of data from the spatial domain to the image domain.

13. **Environmental Science:**

Matrices are used in environmental modelling to represent the interactions between different components of ecosystems, such as the transfer of nutrients in a food web.

For more information, refer to the link:
https://www.vedantu.com/maths/application-of-matrices.

A. Competencies

- Demonstrate understanding of matrices and apply matrix multiplication in real-life problems.
- Exhibit an ability to solve real-life problems involving simultaneous equations in two or three unknown variables by matrix method.

B. Objectives

- Identity and describe the dimensions of matrices and perform operations on matrices.
- Apply matrix multiplications in solving real-life problems.

- Evaluate the transpose, adjoint and inverse of a matrix.
- Solve real-life problems involving systems of equations with two and three variables using matrices.
- Examine the consistency of a given system of equations.

C. Essential Skills/Processes

- Conceptualising
- Applying
- Reasoning
- Describing

D. Learning Experiences

- Conduct pre-assessment on the Matrix. Let students complete the questions given in the worksheets for self assessment.
  - o Worksheet on matrix addition and subtraction
  - o Worksheet on Matrix multiplications
- Define Matrix from the relevant sources. Then, briefly describe the types of matrices including rectangular, row, column, square, diagonal, scalar, null matrix, unit matrix or identity matrix. Refer BHSEC Mathematics Book II on the topic "Types of Matrices" or find relevant online resources.
- Let students explore operations on matrices: addition, subtraction & multiplication of matrices to complete the problem solving.
  - o Assign relevant questions on operation of matrix from the BHSEC Mathematics Book II.
- Display the video from the link: https://www.youtube.com/watch?v=P5GJJ02OG08 on properties of addition and multiplication of matrices. Instruct them to make notes from the video or refer to BHSEC Mathematics Book II.
  - o Suggested questions - Assign question 2 & 7 from Exercise 3(b), BHSEC Mathematics Book II, to check their understanding on properties of addition.
  - o Assign question 3, 4 & 5 from Exercise 3(b), BHSEC Mathematics Book II, and evaluate their understanding on properties of multiplication.
- Design a group activity on Application of Matrix Multiplication.

- Encourage students to work collaboratively within their assigned groups to explore, solve, and interpret the problems. Refer BHSEC Mathematic Book II to assign questions on Application of Matrix Multiplication.
- Students will then present their work in the classroom for assessment and feedback.

- Display a video given in the web link: Transpose-of-matrix that contains methods to find transpose matrices of order 2 and 3.
  - Allow students to define the transpose of a matrix after watching the video and assign practice questions to enhance their understanding.
  - Suggested questions - Assign question 2, 3, 4 and 7 from exercise 3(e), BHSEC Mathematics Book II.

- Demonstrate how to find the adjoint matrix of order 2 and 3.
  - Refer to the web link: https://www.youtube.com/watch?v=hiuqyvR-f_4.
  - Suggested question: Question 1 from exercise 3(g), BHSEC Mathematics Book II.

- Project the video from the link: https://www.youtube.com/watch?v=AMLUikdDQGk on the inverse of a matrix. Discuss on finding the inverse of matrices of order 2 and 3. Refer the web links:
  - inverse of order 2 matrix on finding the inverse of 2x2 matrix.
  - inverse of order 3 on finding the inverse of 3x3 matrix.
  - Assign questions from the link: Practice Questions for Inverse Matrices or refer BHSEC Mathematics Book II.

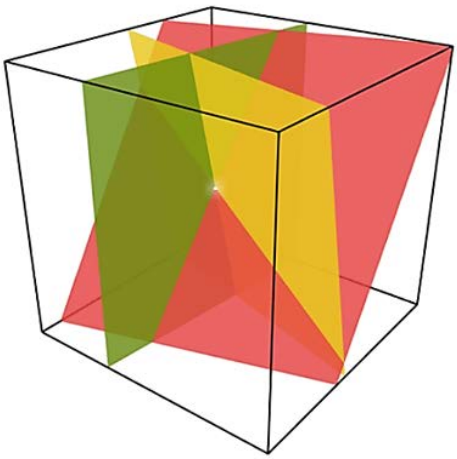
- Demonstrate the processes to solve a system of linear equations using Martin's Rule. Clarify all the steps involved, and explain the meaning of the solutions.
  - Refer to the link: https://www.youtube.com/watch?v=NNmiOoWt86M.
  - Use *geogebra*, *maple, mathematica* or other relevant software and display the solutions of simultaneous equations graphically. Check the video https://www.youtube.com/watch?v=AkMogY8p0Qo to learn how to graph equations of 3-D planes in Geogebra.

- Assign simultaneous equations having 2 and 3 variables using Martin's rule (refer BHSEC Mathematics book I and II or relevant online resources).
  - Reflective question:
  - Display a system of 2 linear equations on the board and instruct students to use all the methods they have learned in lower classes (substitution, elimination, and comparison), determinant's method (class XI) and the matrix method to find the solutions of the equations.

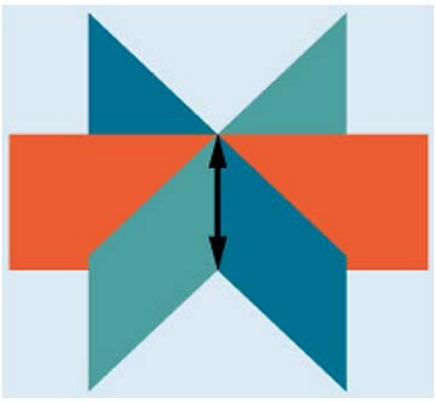
- Allow students to write a reflection on use of each method and justify which and why they would use it to solve each type of system.

- Explain the meaning of the inconsistent system in terms of the solutions of the system and its geometrical meaning using relevant software like *geogebra* or any other 3-D graphing software.

- The consistent and inconsistent systems can be graphically shown as below:



Consistent system with unique solution



Consistent system with infinite solutions

Inconsistent system with no solution

- Display a flow chart explaining the steps to check for consistency and solve for an infinite number of solutions.

- Allow students to write their own interpretation of the flowchart and let volunteers read their interpretations to make sure all the students are on the same page.
- Refer to BHSEC Mathematics book II to assign questions to students.

- Demonstrate solving of real world application word problems involving systems of equations.
  - Refer BHSEC mathematics book II, question 26 of the exercise 3(i) as a sample question. Teachers can search for relevant questions from other sources, or refer to the link: Practise Word Problem.

E. Assessment

Performance Task 1

To enhance their understanding on addition, subtraction and multiplication of matrices assign practice questions from the worksheets from the link given below:

- Worksheet on matrix addition and subtraction
- Worksheet on Matrix multiplications

Performance Task 2

Group work
Instruction: In your group, apply your knowledge of matrix multiplication to design two application questions relating to your everyday experiences.
Use matrices to represent the problem and perform multiplication operations to determine your desired solution.
Write your interpretation of the solution and reflect on any other method you could have used to solve the problem.

Performance Task 3

Place Based Approach

Design a place-based task where students will need to write a real life situation into a system of linear equations and solve using the matrix method.

Sample:
Divide students into groups and send them off to different shops.
Instruct students to inquire about the total cost (not individual cost) of purchasing 3 sets of following items: water (500 mL), pens and books. Example,
First set: 3 waters, 5 pens and 2 books

Second set: 5 waters, 7 pens and 1 book
Third set: 2 water, 3 pens and 5 books: Students will then represent the problem into a system of linear equations and solve the equation to determine the individual price of a water, a pen and a book using the matrix method. After comparing their results with other groups, they may also reflect and share their findings on which shop may be the best choice to buy items at lower cost and for more savings.

Design an appropriate assessment tool for each performance task and record feedback and achievement based on the templates given in the annexure XIIP-A1

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction of Matrices - https://www.youtube.com/watch?v=s4VFKg8OKTM
- Utility and Scope of Matrix- https://www.vedantu.com/maths/application-of-matrices
- Worksheet of Matrices - Worksheet on matrix addition and subtraction
- Worksheet on matrices - Worksheet on Matrix multiplications
- Matrix Multiplication Properties - https://www.youtube.com/watch?v=P5GJJ02OG08
- Transpose of Matrices - https://www.youtube.com/watch?v=g_Rz94DXvNo
- Adjoint of a matrix - https://www.youtube.com/watch?v=hiuqyvR-f_4
- Inverse of 2x2 matrix: https://www.youtube.com/watch?v=HYWeEx21WWw
- Inverse of 3x3 matrix: https://www.youtube.com/watch?v=xfhzwNkMNg4
- Practice questions on inverse matrix: Practice Questions for Inverse Matrices
- System of linear equations - https://www.youtube.com/watch?v=NNmiOoWt86M.
- Graphing in 3D Plane - https://www.youtube.com/watch?v=AkMogY8p0Qo
- Word Problem: Practice Word Problem
- Technological gadgets for learning (smart phone, desktop, laptop...)

G. Annexure

Sample: Assessment tools for Performance Task 3

	Beginning	Approaching	Meeting	Advancing	Exceeding
Completion	Several of the problems are not completed.	About three of the problems are completed.	About two of the problems are completed.	One of the problems is not completed.	All problems are completed.
Neatness and Organization	The work appears sloppy and unorganised. It is hard to know what information goes together.	The work is presented in an organised fashion but may be hard to read at times.	The work is presented in a neat and organised fashion that is usually easy to read.	The work is presented in a neat and organised fashion that is mostly easy to read.	The work is presented in a neat, clear, organised fashion that is easy to read.
Working with others	Students did not work effectively with others.	Students cooperated with others, but needed prompting to stay on-task.	Student was engaged with a partner/group but had trouble listening to others and/or working cooperatively.	Student was engaged with a partner/group, listening to some suggestions of others and working quite cooperatively throughout the lesson.	Student was engaged with a partner/group, listening to suggestions of others and working cooperatively throughout the lesson.
Mathematizing the problem	The student did not attempt to mathematize the problem	Student mathematics the problem mostly incorrectly	Students wrote part of the system of linear equations correctly.	Students wrote most parts of the system of linear equations correctly.	Student wrote the whole system of linear equations accurately
Accurateness of application of Cramer's rule	Student did not attempt to solve the system of linear equations	Student solved only 1 determinant out 4 correctly	Student solved only 2 determinants out of 4 correctly	Student solved only 3 determinants out of 4 correctly	Student solved all the 4 determinants correctly
Interpretation of the solutions	Student did not attempt to interpret the solutions	Student's interpretation of the solutions was mostly incorrect	Student's interpretation of the solutions was partially correct	Student's interpretation of the solutions was mostly correct	All the interpretation of the solution was correct

*(**Note:** This is just a sample of one of the assessment tools. Teacher must design other appropriate assessment tools as per the competency and performance tasks)*

Sample: Template to record assessment

Strand(s): 12B-A1	**Topic(s): Matrix**				
Competency: Demonstrate an understanding of matrices and apply matrix multiplication in real-life problems					
Name of the student	Level of achievement				
	Beginning	Approaching	Meeting	Advancing	Exceeding

Topic: XIIP-A2 Complex Numbers *[450 minutes]*

Introduction

Complex numbers were first conceived and defined by the Italian mathematician Gerolamo Cardano, who called them “fictitious”, during his attempts to find solutions to cubic equations. This ultimately led to the fundamental theorem of algebra, which shows that with complex numbers, a solution exists to every polynomial equation of degree one or higher. Complex numbers thus form an algebraically closed field, where any polynomial equation has a root.

The rules for addition, subtraction and multiplication of complex numbers were later developed by the Italian mathematician Rafael Bombelli. A more abstract formalism for the complex numbers was further developed by the Irish mathematician William Rowan Hamilton.

Source: https://rossroessler.tripod.com/

Utility and Scope

Imaginary numbers have essential concrete applications in a variety of sciences and related areas such as signal processing, control theory, electromagnetism, quantum mechanics, cartography, vibration analysis, and many others.

Applications of Complex Numbers in Physics: An important and fascinating field of modern physics is quantum mechanics, which focuses on the behaviour of subatomic particles. At the base of quantum physics are complex numbers, which are used to do things like predict a particle's position or momentum using equations – most famously, the Schrödinger Equation. Without complex numbers, physicists and chemists wouldn't be able to understand how atoms work and how they interact. For example, the complex numbers behind quantum physics helped scientists to understand that how atoms bond to form molecules is by sharing electrons. This has been an integral part of both chemistry and physics for most of the past century. Another area of physics where complex numbers become very important is the field of optics.

Applications of Complex Numbers in Engineering: Complex numbers are used whenever a quantity has both a phase and a magnitude. While this does include optics and quantum mechanics in physics, it also includes electronics. Electronic engineers working with sinusoidal oscillating voltage and currents use complex numbers. Complex numbers, and the imaginary numbers that they contain, aren't limited only to electronics. Any engineer

is likely to encounter them, Kaiser science explains, and have to work with them, when oscillation needs to be considered. This includes the construction of bridges, which can oscillate in the wind if they aren't carefully designed, and even the design of the shock absorbers in your car.

Applications of Complex Numbers in Finance and Economics: Complex numbers and complex analysis, are an important part of economic models that use different equations in analysing capital. Just as with bridges, waves and photons, finance and economics are also influenced by oscillations such as market fluctuations or economic seasonal changes, which require the use of complex numbers to understand them properly.

Sources: complex numbers used in real life
https://work.chron.com/careers-use-complex-numbers-29845.html

A. Competencies

- Employ the knowledge of modulus and argument of complex numbers to describe the cartesian and polar form of complex numbers.
- Express the understanding of complex numbers in describing various loci and calculating square roots of complex numbers and cube roots of unity.

B. Objectives

- Find the modulus and argument of a complex number.
- Represent the cartesian form of complex numbers in polar form and vice versa.
- Solve locus problems related to complex numbers.
- Calculate square root of complex numbers.
- Investigate cube roots of unity and its properties.

C. Essential Skills/Processes

- Computing
- Applying
- Analysing
- Illustrating

D. Learning Experiences

- Student will revisit the following topics in complex number previously learnt: definition, integral powers of *i*, properties of complex numbers, a geometrical representation of

complex number in the argand plane, operations on complex numbers, conjugate of a complex number and additive and multiplicative inverse of a complex number.
  - Carry out the following activity (or you can design worksheets) to assess the students' knowledge on complex numbers that they have learned in previous grade: https://www.baamboozle.com/game/784975 .

- Use the video in the web link: https://youtu.be/ZSvbgs6SJO8 to explain the definition of modulus and method to calculate the modulus of a complex number with examples.
  - Allow students to explore the properties of modulus using the link: https://www.youtube.com/watch?v=rMdsJNdkfBk.
  - Exercise the idea of modulus and its properties by providing practice questions (refer BHSEC Mathematics Book II, Ch 21).

- Create a PowerPoint slide to explain the concept and problem-solving aspects of the argument of a complex number. Include graphical representations of arguments in each quadrant. For additional information, refer to the BHSEC Mathematics Book II or visit the suggested web link: https://youtu.be/nWJhQnzPZiM**.**

- Play the video given in the link : https://youtu.be/pvPRTHWvMU4 to explain the conversion of complex numbers from cartesian (or rectangular) form to polar form and with examples.
  - Allow students to explore the conversion of complex numbers between cartesian and polar form by providing practice questions (refer BHSEC Mathematics Book II or suggested web link: Sample worksheet).

- Demonstrate examples of locus problems involving complex numbers, providing a step-by-step explanation of the procedure and describing the resulting locus.
  - Suggested question: BHSEC Mathematics Book II, Chapter 21, example 35, 38 and 39.
  - Assign group-work to students to exercise and explore locus problems and interpretations of different kinds of loci. For questions, refer to BHSEC Mathematics Book II or suggested web link: Geometry and Locus of Complex Numbers .

- Instruct students to watch the video in the web link: https://www.youtube.com/watch?v=zuYifqD9wqE in the IT laboratory and explore

supplementary resources to learn the method of determining square roots of a complex number.
  - Provide questions from BHSEC Mathematics Book II to demonstrate their learning.

- Discuss the cube roots of unity and its properties. The properties may be written and displayed on a chart paper.
  - For notes, refer to BHSEC Mathematics Book II or the suggested web link: https://testbook.com/maths/cube-root-of-unity .
  - Allow students to solve questions from BHSEC Mathematics Book II, as a classwork task.

E. Assessment

Performance Task 1

Develop competency-based questions to design a quiz on Google Form on the concepts of modulus, argument and polar form of complex numbers and bring the students to sit the quiz in the school IT laboratory.
For sample questions, refer the weblink: MCQ on polar form of complex numbers

Performance Task 2

Group work: Demonstrate only a few examples in the class on finding loci of complex numbers.

Assign group-work to students to explore and present in class, on all types of the questions involving loci of complex numbers. For questions, refer to BHSEC Mathematics Book II or suggested web link: Locus problems

Design assessment tools to assess the students' knowledge of employing correct procedures and their understanding of each type of locus and its descriptions and record feedbacks and achievement using the templates given in the annexure XIIP-A1

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction of Complex Number - https://rossroessler.tripod.com/
- Utility and scope - complex numbers used in real life
  https://work.chron.com/careers-use-complex-numbers-29845.html

- Game on complex exercise - https://www.baamboozle.com/game/784975
- Modulus of Complex - https://youtu.be/ZSvbgs6SJO8
- Modulus of complex - https://www.youtube.com/watch?v=rMdsJNdkfBk
- Argument of Complex - https://youtu.be/nWJhQnzPZiM
- Polar form of complex - https://youtu.be/pvPRTHWvMU4
- Polar form of complex - Sample worksheet
- Locus problem - Geometry and Locus of Complex Numbers
- Square roots of complex - https://www.youtube.com/watch?v=zuYifqD9wqE
- Cube roots of complex - https://testbook.com/maths/cube-root-of-unity
- Modulus and argument - MCQ on polar form of complex numbers
- Locus of Complex number - Locus problems
- GeoGebra tool
- Self-Instructional Material (SIM)
- Technological gadgets for learning (smart phone, desktop, laptop, etc.)

G. Annexure

Use a template given in XIIP-AI to record student achievements under each competency.

Topic: XIIP-A3 Permutations and Combinations *[600 minutes]*

Introduction

Permutation and combination form the principles of counting and they are applied in various situations. A permutation is a count of the different arrangements which can be made from the given set of things. In permutation the details matter, as the order or sequence is important. Writing the names of three countries {USA, Brazil, Australia} or {Australia, USA, Brazil) or { Brazil, Australia, USA} is different and this sequence in which the names of the countries are written is important. In combinations, the name of three countries is just a single group, and the sequence or order does not matter.

Sources: https://www.cuemath.com/data/permutations-and-combinations/

- https://www./science/permutation

https://www.predicting-blaise-pascal-pierre-de-fermat - to explore further about the history of permutation and combination.

Utility and Scope

Application Areas of Combinatorics: Permutations are frequently used in communication networks and parallel and distributed systems. Many communication networks require secure transfer of information, which drives development in cryptography and network security because of the increased use of internet information transfers. Encryption process involves manipulations of sequences of codes such as digits, characters, and words. Hence, they are closely related to combinatorics, possibly with intelligent encryption processes. Permutations of fast Fourier transforms are employed in speech encryption

Computer architecture: Design of computer chips involves consideration of possible permutations of input to output pins. Field-programmable interconnection chips provide user programmable interconnection for a desired permutation. Arrangement of logic gates is a basic element for computer architecture design

Computational molecular biology: This field involves many types of combinatorial and sequencing problems such as atoms, molecules, DNAs, genes, and proteins. One-dimensional sequencing problems are essentially permutation problems under certain constraints.

Pattern analysis: Patterns can have many forms; for example, visual images, acoustic signals, and other physical quantities such as electrical, pressure, temperature, etc., that appear in engineering problems. Some of these types of patterns can be associated with combinatorics. Computer music can be a specialised application domain of combinatorics of acoustic signals.

Source: https://www.use-of-permutation-and-combination (explore the web link further for more utilities of permutation and combination)

A. Competencies

- Apply skills in using counting principles, P(n, r), and C(n, r) formulas to determine the number of arrangements and selections in various real-life situations.
- Exhibit the ability to analyse situations and determine the appropriate combinatorial approach.

B. Objectives

- Comprehend the fundamental principle of counting.
- Differentiate between permutations and combinations through illustrative examples.
- Solve permutation and combination problems across various cases.
- Solve problems involving both permutations and combinations.

C. Essential Skills/Processes

- Conceptualising
- Computing
- Applying
- Selecting
- Arranging
- Analysing

D. Learning Experiences

- Recapitulate on the concept of factorial; definition of $n!$ and solve one or two examples: $5!$, $9!$, etc. Or design a short pre-assessment quiz on the concept of factorial, to check the previous knowledge of students.

- Define and differentiate between permutations as arrangement and combinations as selection.

- Allow students to manually count the number of combinations and permutations of 3 digits out of 4 given digits to understand the difference between permutation and combination.

- Discuss an example (refer to BHSEC Mathematics Book II from the topic: Fundamental Principle of Counting); suggested question - Ex. 1) and arrive at the 'Fundamental principle of counting' together with the students.
  - Refer to the link: https://youtu.be/3lmEqp8VhAU explaining the fundamental principle of counting. Instruct students to make notes.
  - Suggested learning activity - solve example 6 from BHSEC Mathematics Book-II, and allow students to list the outcomes using a tree diagram. Subsequently, collaboratively determine the total number of outcomes by applying the fundamental principle of counting. This exercise demonstrates two distinct methods for calculating the number of outcomes when tossing 3 coins.

- Explain the meaning of $P(n,r)$ and different methods to calculate permutation of n things taken r at a time (refer to BHSEC Mathematics Book II from the topic: Permutation).
  - Refer to the link: https://www.youtube.com/watch?v=viKDzyeCHr0 on different methods to calculate permutation of n things taken r at a time.

- Explain each type of permutation: restricted permutation, permutations of alike things, permutation of repeated things and circular permutations with examples (refer to BHSEC Mathematics Book II). For supplement, use following the web links where relevant:
  - https://youtu.be/XPPYYM6WCuE (contains a video explaining an example of formation of numbers when repetition is not allowed and when repetition is allowed.)
  - https://www.youtube.com/watch?v=tBQhcP9Zr2E (contains a video explaining an example of the arrangement of people on chairs.)
  - https://www.youtube.com/watch?v=L81vWvqGvJ8 (contains a video explaining an example of forming anagram (permutation of alike things).
  - https://youtu.be/gxeP3PeA09I (contains a video explaining circular permutations with examples.)

- Explain the meaning of $C(n,r)$ and the corollaries (refer to BHSEC Mathematics Book II):
  - $C(n,\ n) = 1,$

- $C(n, r) = C(n, n - r)$,
- $If\ C(n, x) = C(n, y) then\ either\ x = y\ or\ x = n - y,\ i.e.\ x + y = n$
- $C(n, r) + C(n, r - 1) = C(n + 1, r)$

Discuss problems using the formula and corollaries of $C(n, r)$ with the students.

- Display and discuss the video in the web link to explain different methods of calculating combinations: https://combinations.com
- Design a PowerPoint presentation on different types of combinations: distribution of different things into groups; open selection of items from different things and from alike things with examples.
  - Discuss examples of each types of combinations from BHSEC Mathematics Book II (suggested questions - Ex. 44, 45, 47, 48, 50, etc).
  - For supplementary, refer to the link: https://youtu.be/B6xDGgoSMJY on the selection of a group of boys and girls using combination.
- Allow students in pairs to examine the examples under 'permutations and combinations occurring simultaneously' (refer to BHSEC Mathematics Book II, suggested questions examples 52 and 53).

E. Assessment

Performance Task 1

Place-based approach:

Design a place-based assessment allowing students to employ their knowledge of permutations and combinations in their immediate environment.

Sample:

In groups, allow students to work on the following tasks.

1. At the school parking area, count the number of cars parked and total number of parking slots (including the empty ones).
   a. Calculate in how many other ways the cars could have been parked?
   b. In how many ways would all the big cars be always parked together?
   c. How many ways can the cars be parked if one of these parking slots must be occupied by the school bus?
   Students must consider the possibility of 'alike things' if two or more cars are of the same model and colour.

2. Find a collection of flower pots with different flowers growing in them.

a. Determine in how many ways 5 flower pots can be selected.
b. How many combinations of 4 pots can be made if at least one flower pot containing Rose variety has to be included?
c. In how many ways can you select one or more flower pot(s) to decorate the stage during a cultural show?

Design an appropriate assessment tool for each performance task and record feedbacks and achievements using the templates given in the annexure XIIP-A1

F. Resources

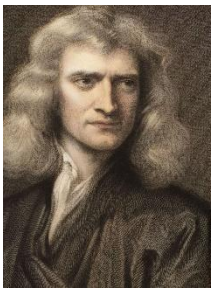
- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction (permutation and combination) –https://www./science/permutation
- Introduction of permutation and combination- https://www.predicting-blaise-pascal-pierre-de-fermat
- Utility and scope of permutation and combination - https://www.use-of-permutation-and-combination
- Fundamental principle of counting - https://youtu.be/3lmEqp8VhAU
- Calculating Permutation - https://www.youtube.com/watch?v=viKDzyeCHr0
- Permutation of repetition - https://youtu.be/XPPYYM6WCuE
- Permutation - https://www.youtube.com/watch?v=tBQhcP9Zr2E
- Permutation - https://www.youtube.com/watch?v=L81vWvqGvJ8
- Circular permutation - https://youtu.be/gxeP3PeA09I
- Combination - https://combinations.com
- Combination - https://youtu.be/B6xDGgoSMJY
- Self-Instructional Material
- Technological gadgets for learning (smart phone, desktop, laptop)

G. Annexure

Use a template given in XIIP-AI to record student achievements under each competency.

Topic: XIIP-B1 Differentiation *[850 minutes]*

Introduction



Calculus is one of the most important fields of Mathematics. Calculus is a study of rates of change and motion, which we can see by the slope of a line or a curve. There are two major branches of calculus, Differential and Integral calculus, and they are inverses of each other. Integral calculus is used to find the areas under a curve, surface area or volume, and linear distance travel. Differential calculus (which concerns the derivative) mostly goes over the problem of finding the rate of change that is instantaneous, for example, the speed , velocity or an acceleration of an object. Differentiation is especially important in natural sciences, engineering and technology. An example of differential calculus is if you wanted to find the velocity or the acceleration of an object, for example, a car. To find the velocity of a car, you would take the first derivative of a function (position at time t : dx/dt) and to find the acceleration you would take the second derivative of a function (dv/dt : change in velocity/change in time . This leads us to Newton's law of motion, which is Force = Mass x Acceleration, where in this context, acceleration is the second derivative of a function.

Who was the person behind the development of calculus? Well, it wasn't actually just one person. Sir Isaac Newton and Gottfried Wilhelm Leibniz were both credited with the development of calculus. Throughout their lives, they both argued on who came up with the idea first, both have accused each other of plagiarism. Those two weren't the only ones who contributed to the discovery of Calculus. There have been many other known mathematicians of that time that also helped with the development of calculus. For example, Rene Descartes indirectly helped create differential calculus by introducing variable magnitude.
Source: https://history of differentiation

Utility and Scope

Differentiation in mathematics, particularly in calculus, involves studying rates at which quantities change. It has a wide range of real-world applications and is crucial in various fields such as science, technology, business, and more. Here are some examples illustrating the application and scope of differentiation:

1. **Physics and Engineering:**

   a) **Motion Analysis**: Differentiation is used to analyse the motion of objects. In physics and engineering, it helps calculate velocities, accelerations, and study the behaviour of particles.

b) **Electric Circuits**: In electrical engineering, differentiation is applied to analyse voltage and current changes over time in circuits.

2. **Computer Science and Technology**:

   a) **Algorithm Analysis**: Differentiation helps analyse the efficiency and performance of algorithms, especially in terms of their time complexity.

   b) **Signal Processing**: In image and audio processing, differentiation is used to analyse and enhance signals.

3. **Economics and Finance**:

   a) **Marginal Analysis**: Differentiation is used in economics to analyse marginal cost, marginal revenue, and marginal utility, providing insights into optimal decision-making.

   b) **Options Pricing**: In finance, differentiation is used in option pricing models, such as the Black-Scholes model, to determine the rate of change of option prices concerning various factors.

4. **Biology and Medicine**:

   a) **Population Dynamics**: In biology, differentiation is used to model and analyse population growth and dynamics.

   b) **Medical Imaging**: Differentiation plays a role in the analysis of medical images, such as identifying edges and contours in diagnostic imaging.

5. **Environmental Science**:

   a) **Rate of Change in Environmental Variables**: Differentiation is used to study the rates of change in environmental variables, such as temperature, pollution levels, and ecosystem dynamics.

6. **Operations Research and Management**:

   a) **Optimization**: Differentiation is a key tool in optimization problems, such as finding the minimum or maximum values of functions. This is applied in operations research and management decision-making.

   b) **Supply Chain Management**: Differentiation helps optimise inventory levels, production rates, and distribution schedules.

7. **Statistics and Data Analysis**:

   a) **Regression Analysis**: Differentiation is used in regression analysis to find the best-fit line that represents the relationship between variables.

   b) **Time Series Analysis**: Differentiation is applied to study trends and patterns in time-series data.

8. **Telecommunications**:

   a) **Signal Transmission**: In telecommunications, differentiation is used to analyse and optimise signal transmission, such as the modulation and demodulation of signals.

9. **Business and Marketing**:

   a) **Marginal Cost and Revenue**: In business, differentiation helps analyse marginal cost and marginal revenue, aiding in pricing decisions.

   b) **Market Research**:** Differentiation is applied in market research to analyse how variables such as pricing, advertising, and product features impact sales.

Explore more from the weblink: : utility and scope of differentiation.

A. Competencies

- Demonstrate proficiency in employing appropriate methods of differentiation to calculate both first and higher-order derivatives across various functions.
- Apply the understanding of turning points to effectively address optimization challenges in practical, geometrical, and mensuration problems within real-world scenarios.

B. Objectives

- Evaluate first derivatives of algebraic, trigonometric, inverse trigonometric (including 'by transformation'), logarithmic and exponential functions in simple, composite, absolute and implicit forms.
- Understand the method of differentiation of parametric functions, differentiation of a function with respect to another function and logarithmic differentiation.
- Evaluate higher order derivatives (successive differentiation) of all the types of functions.
- Determine and interpret the turning points of a function using higher derivatives
- Apply the concept of maxima and minima in solving and optimising practical, geometrical and mensuration problems in real life.

C. Essential Skills/Processes

- Conceptualising
- Computing
- Applying
- Analysing
- Evaluating

D. Learning Experiences

- Design a quiz on basic rules of derivatives, application of rules in simple problems and geometrical interpretation of derivatives, to assess the students' previous knowledge on the topic. For questions, refer BHSEC Mathematics Book I and BHSEC Mathematics Book II, Exercise 5(a).

- Illustrate the definition of a composite function with examples and refer to the tutorials in the following web links to understand and demonstrate the procedure(s) of 'Chain Rule - to evaluate the derivative of a composite function'.
  - https://www.youtube.com/watch?v=U0m4MsOgETw
  - https://www.youtube.com/watch?v=z-tEsz0bSrA (contains tutorial videos explaining the chain rule.)
  - https://www.youtube.com/watch?v=4s7G7nkMYHM (contains an explanatory video solving examples on chain rule.)
  - For notes, refer BHSEC Mathematics Book II or the suggested web link: https://www.calculus/derivatives-of-composite-functions/

- Display and discuss the basic rules of derivatives of each type of function (algebraic, trigonometric, logarithmic, exponential and inverse trigonometric functions) on chart papers or other TLMs.
  Refer the web link for all the basic rules of differentiation:
  - https://youtu.be/zmnh448y_ZU
  - https://youtu.be/KbYW9FDm-Zk

- Allow students in groups to explore the application of the rules in determining respective derivatives with relevant questions. For questions, refer to BHSEC Mathematics Book II [Exercises 5(c - f)].

- Take students to the IT laboratory to watch the video tutorial in the web link: https://www.youtube.com/watch?v=ZHgtb8NoZbQ. Let students explore the method of differentiation of inverse trigonometric functions by transformation/substitution:
  - Allow students to make notes and practise their understanding on similar questions (refer BHSEC Mathematics Book II, topic- 'Differentiation by transformation').

- Direct students to watch the video in the web link: https://youtu.be/M0SMSWM2oZA which explains the differentiation of implicit functions with examples.
  - Design questions of higher-order-thinking on implicit differentiations for group work (refer to BHSEC Mathematics Book II) and assign them to students to assess their learning.

- Use the video in the web link to explain the concept of differentiation of parametric function: https://www.youtube.com/watch?v=hljazIjVnz8 (the link contains an explanation of derivatives of the parametric of functions).
  - Provide practice questions as classwork (refer to BHSEC Mathematics Book II or the web link: https://youtu.be/kf2dZWqLnqE which contains questions and solutions on derivatives of parametric functions).

- Relate the process of differentiation of a function with respect to another function with that of differentiation of parametric functions.
  - Allow the students to solve relevant questions (refer BHSEC Mathematics Book II, Exercise 5(j)) as homework.

- Demonstrate the method of taking logarithm before derivative in solving problems of the type $u^{v}[variable^{variable}\ or\ function^{function}]$ with appropriate examples. Refer BHSEC Mathematics Book II or the weblink: https://youtu.be/Dp9sgIvaKPk for notes and examples

- Share the web link: https://youtu.be/s7rd9YPJrNc which contains the video lesson on successive differentiation and its applications.
  - Discuss a few questions with the students to enhance their learning from the video tutorial.

- Cross Pollination: Instruct students to explore different topics in Physics and Chemistry where they apply the concept of first and higher order derivatives and have them reflect on the significance of the concept of Differentiation.

- Use the video in the web link: https://youtu.be/cXxW6mHhVXY to provide the definitions of the three turning points: Maxima, Minima and Point of inflection and how second derivatives can predict the occurrence of the turning points. For an alternate understanding of the three turning points, refer the web link: https://www.turning points with example .
  - Design a PowerPoint slide on the procedure of determining and differentiating the turning points of a function with relevant examples. Refer BHSEC Mathematics Book II for questions.
  - Allow students to discuss the scope of applications of the concept of maxima and minima in everyday life to make connections with the next topic, 'Application of Maxima and Minima'.

- Demonstrate with examples, the practical, geometrical and mensuration applications of maxima and minima; showing how to translate word problems into mathematical equations i.e. to write one variable as a function of the other.
  - Allow students to find the maximum or minimum value and give their interpretation of the results obtained.
  - Suggested link: https://byjus.com/ncert-solutions-class-12-maths/chapter-6-applications-of-derivative/ (the links containing an application of derivatives in algebraic problem, an application of derivatives on the geometrical figure, an application of derivatives on mensuration)

E. Assessment

Performance Task 1

Design a test to assess the knowledge of students on first and higher derivatives of various types of functions in different forms each time a topic is completed or as a summative assessment at the end of all the topics. For questions, refer BHSEC Mathematics Book II or the following weblinks:

- Worksheet -
  https://www.liveworksheets.com/w/en/math/483122
  https://www.liveworksheets.com/w/en/math/2284091
  https://www.liveworksheets.com/w/en/math/645393
  https://www.liveworksheets.com/w/en/math/389718

Performance Task 2

Place-based approach: Group-work

Sample tasks:

1. Take the dimensions of any rectangular figure around your school campus (window, brick, wall, etc; different figures for each group) and find the perimeter. Show how you would maximise the area of the figure.
2. Measure surface area of the school Mani-Dungkor.
   If the steel sheet costs Nu 350 per square metre, what should be the radius and height of the Mani-Dungkor so that the cost of covering the Mani-dungkor with steel sheet is minimum?
3. Your school would like to install a hand-washing station for the sanitation purpose of the students. For that purpose, assume that you need to build a tank to reserve water of 500 litres. Design a tank of different shapes (cylindrical, cubic or cuboidal) with dimensions such that the cost of materials used to build the tanks of volume 500 litres would be minimum.

   Reflection: Out of the different shaped tanks designed, which one would you recommend to the school to build?

Design an appropriate assessment tool for each performance task and record feedback and achievement based on the templates given in the annexure XIIP-A1

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction - https://history of differentiation
- Utility and scope - utility and scope of differentiation.
- Derivatives of chain rule - https://www.youtube.com/watch?v=U0m4MsOgETw
- Derivatives of chain rule - https://www.youtube.com/watch?v=z-tEsz0bSrA
- Derivatives of chain rule - https://www.youtube.com/watch?v=4s7G7nkMYHM
- Derivatives of composite-
  https://www.calculus/derivatives-of-composite-functions/
- Rule of derivatives - https://youtu.be/zmnh448y_ZU
  https://youtu.be/KbYW9FDm-Zk
- Transformation derivatives -https://www.youtube.com/watch?v=ZHgtb8NoZbQ

- Implicit derivatives - https://youtu.be/M0SMSWM2oZA
- Parametric derivatives - https://www.youtube.com/watch?v=hljazIjVnz8
- Parametric derivatives - https://youtu.be/kf2dZWqLnqE
- Logarithm derivative - https://youtu.be/Dp9sgIvaKPk
- Successive derivative - https://youtu.be/s7rd9YPJrNc
- Maxima and minima derivatives - https://youtu.be/cXxW6mHhVXY
- Turning point on derivatives - https://www.turning points with example
- Application of derivatives - https://byjus.com/ncert-solutions-class-12-maths/chapter-6-applications-of-derivative/
- Worksheet -
  https://www.liveworksheets.com/w/en/math/483122
  https://www.liveworksheets.com/w/en/math/2284091
  https://www.liveworksheets.com/w/en/math/645393
  https://www.liveworksheets.com/w/en/math/389718
- Self-Instructional Material
- Teacher's guide book for class XII
- Technological gadgets for learning (smart phone, desktop, laptop...)

G. Annexure

Use a template given in XIIP-AI to record student achievements in each competency.

Topic: XIIP-B2 Integration *[850 minutes]*

Introduction

The history of integration unfolds over centuries, with numerous mathematicians paving the way for the development of calculus. Archimedes, in ancient Greece, initiated the exploration of volumes and areas using the method of exhaustion, predating the formalisation of derivatives. Luca Valerio in the 1600s furthered these ideas, employing inscribed rectangles to approximate areas under curves. Descartes and Fermat in the 17th century delved into tangents and extremal values, resembling early limit concepts. Cavalieri introduced the notion of indivisibles in 1639, contributing to the understanding of geometric figures in terms of infinitesimals. John Wallis in 1656 emphasised the limit in integration, extending the power formula. Mathematicians like James Gregory expanded integration to trigonometric functions. Isaac Barrow, Newton's teacher, made the crucial discovery that integration and derivation were inverse processes. Newton and Leibniz, independently and using different symbolism, established the calculus as a general systematic method, solidifying the connection between derivatives and integrals. While Newton and Leibniz are credited with much of calculus, the contributions of these predecessors played a vital role in laying the foundation for their groundbreaking work.

Source: https://www.youtube.com/watch?v=qrEt-0SQATg

Utility and Scope

Engineering: An architect or engineer uses integration in determining the amount of the necessary materials to construct curved shape constructions (e.g. dome over a sports arena) and also to measure the weight of that structure.

Electrical Engineering: In electrical engineering calculus (Integration) is used to determine the exact length of power cable needed to connect two substations, which are miles away from each other. Space flight engineers frequently use calculus when planning for long missions. To launch an exploratory probe, they must consider the different orbiting velocities of the Earth and the planet the probe is targeted for, as well as other gravitational influences like the sun and the moon.

Physics: In Physics, Integration is very much needed. For example, to calculate the Centre of Mass, Centre of Gravity and Mass Moment of Inertia of a sports utility vehicle. To

calculate the velocity and trajectory of an object, predict the position of planets, and understand electromagnetism.

Chemistry: In chemistry it is used to determine the rate of a chemical reaction and to determine some necessary information of Radioactive decay reaction.

Source: https://www.youtube.com/watch?v=_xlUYWyahc4

Suggested weblink: https://www.youtube.com/watch?v=iGJxxlyqrRM (Explore further on the Utility and Scope of Integration).

A. Competencies

- Indicate understanding of applying relevant methods to evaluate the indefinite integrals of various types of functions.
- Employ the methods of limit of sum and by use of properties to evaluate definite integrals.
- Show an ability to utilise integration for finding the area under curves and volumes of irregular shapes, quantifying spatial relationships within real-world scenarios.

B. Objectives

- Evaluate integrals of different types of functions using relevant methods: by substitution, by parts and using partial fractions.
- Evaluate definite integrals using basic rules, by substitution, as limit of sums and by using properties.
- Employ the concept of definite integral in calculation of area under a curve and area between two curves.
- Apply the concept of definite integrals to find the volume of revolution about the x-axis or y-axis or about a line.
- Find the area and volume of regular and irregular shapes in our surroundings using definite integrals.

C. Essential Skills/Processes

- Conceptualising
- Evaluating
- Computing
- Analysing
- Exploring

D. Learning Experiences

- Recapitulate on the concepts learnt previously under Integral Calculus like integration as the inverse of differentiation or anti-derivative; integrals of algebraic functions, trigonometric functions and integration by substitution for simple algebraic and trigonometric functions. Refer to BHSEC Mathematics Book I or BHSEC Mathematics Book II or the videos in the web links:
  - o https://youtu.be/7bJXbDqi9FE is a video on basic integral rules for algebraic and trigonometric functions.
  - o https://youtu.be/sci1pls4Lc8 is a video on substitution method for simple algebraic and trigonometric functions.

- Introduce and explain the rules for integration of exponential functions (e^x, a^x, e^{axe}) and other trigonometric functions ($tan\ x$, $cot\ x$, $cosec\ x$ and $sec\ x$). Use the 'Table of standard forms of Integrals and other standard integrals' referring to BHSEC Mathematics Book II.
  - o Assign homework questions to students to practise evaluating integral of exponential and other t-functions (refer to BHSEC Mathematics Book II, exercise 7(c, d, and e)).

- Advance further on Integration by method of substitution.
  - o Demonstrate the use of method of substitution to evaluate the integral of algebraic, trigonometric and exponential functions with appropriate examples (refer to BHSEC Mathematics Book II, from the topic: Methods of Integration)
  - o Refer a suggested web links:
  - o https://www.youtube.com/watch?v=tUpzydgMUDg (integral of algebraic) https://www.youtube.com/watch?v=Fpe88FbIXUI contains a video explaining integration by method of substitution with examples.
  - o Allow students to practise further with similar questions (refer to BHSEC Mathematics Book II) suggested exercise; exercise 7(c, d and e).

- Introduce Integration by parts
  - o Explain the formula: $\int uvdx = \mathrm{u}\int vdx - \int\left[\frac{du}{dx}\int vdx\right]dx$ and the concept of ILATE with examples (refer to BHSEC Mathematics Book II, from the topics Integration by Parts)

- Could also refer to the web link: https://youtu.be/zNU8iK8sGD0 contains description on integration by parts and the concept of ILATE with some solved examples.
- Allow students to practise further assigning examples and exercise questions (refer to BHSEC Mathematics Book II, from the topics Integration by Parts).

- Introduce Integration using partial fraction
  - Share the web link below to the student to recap and to make notes from the IT laboratory on the concept of Partial Fractions.
    https://www.slideshare.net/slister07/5-3-partial-fractions
    contains a PowerPoint presentation on partial fraction decompositions.
  - Explain use of partial fractions in evaluating integral with the help of examples for different types of partial fractions - non-repeated linear factors, repeated linear factors and quadratic factors (not resolvable into linear factors) referring to BHSEC Mathematics Book II, from the topic Integration using Partial Fraction.
  - Use the suggested web link;
    https://www.youtube.com/watch?v=GIGJdvdrdhs&t=60s
    contains video explaining an example of integration using partial fractions (non-repeated linear factor)
  - Provide a group task to evaluate integrals using partial fractions from each type for discussion and presentation to the class on roll call. (Refer to BHSEC Mathematics Book II, exercise 7(g)).

- Introduce the concept of Definite integral
  - Use the suggested web link:
    https://www.mathsisfun.com/calculus/integration-definite.html
    to introduce the concepts of Definite Integral and its properties.
  - After discussion, allow students to investigate and find methods and strategy involved in evaluating the definite integral.
  - Show how to evaluate definite integrals by substitution with appropriate examples (refer to BHSEC Mathematics Book II, from the topic - Evaluation of definite Integral by Substitution).
  - Allow students to demonstrate their understanding of the concepts by providing homework questions and allowing them to present in the class. (Refer to BHSEC Mathematics Book II, exercise 8(a and b))

- Demonstrate definite integral as a limit of sum

- Define the definite integral as a limit of sum as: $\int_a^b f(x)dx = \lim_{h\to 0}\left[h\sum_{r=0}^{n-1} f(a+rh)\right]$ allow students to explore the proof for the definition online and make note or present to the class; suggested web link https://www.youtube.com/watch?v=CMG6ucGUQiU contains an explanation of proof of definite integral as the limit of sum.
- Solve related examples in the class (refer to BHSEC Mathematics Book II) and allow students to demonstrate their understanding through assigning few related examples.

- Introduce properties of definite integrals with an example using the web link https://www.youtube.com/watch?v=2ODALMaUmc0 or refer to BHSEC Mathematics Book II.

- Explain the geometrical interpretation of definite integral and arrive at the formula for the area under a curve:

  $\int_a^b f(x)dx =$ area bounded by the curve $y = f(x)$, the $x-axis$ and the ordinates $x = a,\ x = b.$

  - Use suggested web link to explain the proof: https://www.youtube.com/watch?v=FsC3do74UIo contains a video explaining the area under the curve as a sum of infinite rectangles under the curve.

- Demonstrate and solve examples on use of definite integral to calculate the area under the curve (refer to BHSEC Mathematics Book II, from the topic - calculation of the area under a curve)
  - Use *GeoGebra* to sketch the curve, find the limits on the x or y axes and show the required area.
  - Discuss and explore the videos on solved examples with the help of following web links:
    https://www.youtube.com/watch?v=c8W9_iaERnU (2:40 minutes onwards) explains a step-by-step method of finding an area bounded by a curve and x-axis.
    https://www.youtube.com/watch?v=su9-sz5p3vc explains the finding of the area of a region under the x-axis.
    https://www.youtube.com/watch?v=UyM3qudctYs explains how to find the area between the curve and x-axis when regions are both above and below the x-axis. https://www.youtube.com/watch?v=Z3Vgv5cna1w explains how to find the area between the curve and the y-axis.

- Allow students to explore examples on finding the area between two curves (refer to BHSEC Mathematics Book II, from the topic - Area defined by two curves) $\int_a^b [f(x) - g(x)]\,dx$ – area bounded by the curve $f(x)$ and $g(x)$ where $f(x) > g(x)$
  - Recommend children to use *GeoGebra* to sketch the given curves, find the intersection points and show the required area and calculate the area using formula on the board.
  - A suggested web link - Allow children to explore the tutorial video on finding areas defined by two curves https://youtu.be/xec6HTcn2M8.
  - Allow students to solve questions from the web link questions on the area between two curves, or refer to BHSEC Mathematics Book II.

- Explain the volume of solids of revolution of the region between a curve and axes using the web links https://www.curve and y-axis and https://www.curve and x-axis.

- Explain the volume of solids of revolution of the region between two curves using the web link https://www.youtube.com/watch?v=E0JJ_tgl59o (refer to BHSEC Mathematics Book II, from the topic - Volume of solids revolution) $\pi \int_a^b y^2 dx$ - volume of revolution of $y = f(x)$ between $x = a$ and $x = b$;

  $\pi \int_a^b (y_1^2 - y_2^2) dx$ – volume of revolution of the region between curve $y_1 and\ y_2$
  - Use *GeoGebra* (or other relevant software) to animate the solid of revolution by revolving the region formed by a given curve(s) around any of the axes and to find the limits.
    Suggested web link:
    https://www.geogebra.org/m/qgcz82ae shows formation of cones from revolving the line.
    https://www.geogebra.org/m/kM7hwJBa shows formation of a vase from revolving a curve.
    Calculate the volume using formula on the board.
  - Assign practice questions from the web link https://tutorial.math.lamar.edu/problems/calci/volumewithrings.aspx

E. Assessment

Performance Task 1

Conduct test(s) aligned to different methods of evaluating integrals (may conduct a test every time after each method is taught) to assess and record the level of achievement (for questions refer to BHSEC Mathematics Book II)

Performance Task 2

Instruction: As a homework, evaluate the following definite integral i) using basic rules and ii) using limit of sum.

$1.\int_0^4 (2x-1)\,dx \quad 2.\int_0^3 (3x^2-4)\,dx \quad 3.\int_0^2 (x^2+x)\,dx$

Let students write their reflection on the use of the two methods in solving the same question.

Performance Task 3

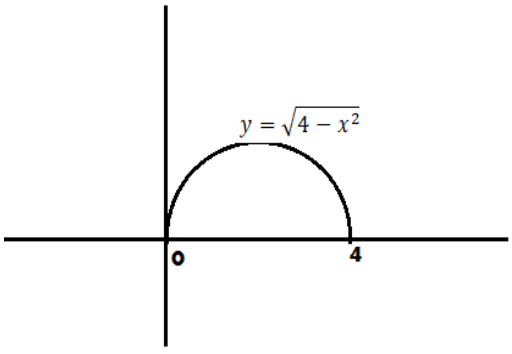
Instruction: As a homework, use the properties of definite integral to evaluate the following:

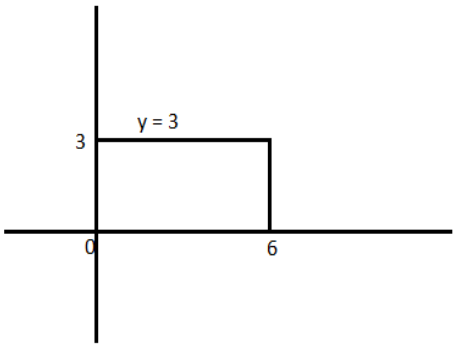
$1.\int_0^6 |x-2|\,dx \quad 2.\int_{-\pi}^{\pi} (1-x^2)\sin x\cos^2 x\,dx \quad 3.\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

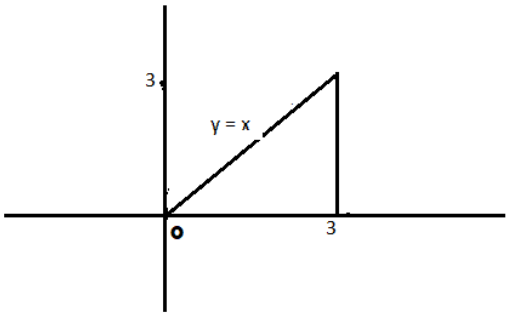
Q. Explain why you used the particular property to evaluate each integral. Could you use any other methods or properties to evaluate the same integral?

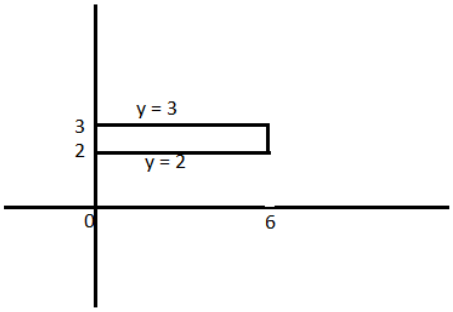
Performance Task 4:

Instruction: Calculate the area of the following shapes with the use of formula learned in lower classes and with that of definite integral.







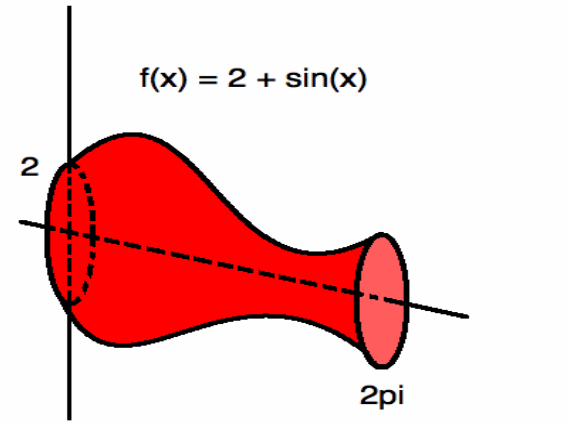


Performance Task 5

Instruction: If the shapes in performance task 4 were revolves once around x -axis, find the volume of revolution by the use of i) formula learnt in lower classes and ii) by that of definite integral. Write your reflection on the use of the two methods in solving the same question.

Performance Task 6

Sample - Bring into class, different 3D shapes like, a vase, a birthday cone, cylindrical glass, a bowl and a ball. Distribute one shape to each group and allow them to discuss the equation(s) of the curve which when revolved around an axis gives a shape similar to the one allocated to their group. For example, if the curve $y = 2 + \sin x$ is revolved around $x - axis$ in the limit, 0 to 2π, we get a shape similar to a vase.



May also use the GeoGebra worksheet https://www.geogebra.org/m/kM7hwJBa which shows formation of a vase from revolving a curve.

Allow each group to i) find the area of the region formed by each curve with the x-axis and ii) find the volume of the given shape using definite integral both within the same limits and present their results to the class.

Design an appropriate assessment tool for each performance task and record feedback and achievement based on the templates given in the annexure XIIP-A1

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction - https://www.youtube.com/watch?v=qrEt-0SQATg
- Utility and scope - https://www.youtube.com/watch?v=_xlUYWyahc4
- Utility and scope - https://www.youtube.com/watch?v=iGJxxlyqrRM
- Basic integration - https://youtu.be/7bJXbDqi9FE
- Integration by substitution - https://www.youtube.com/watch?v=tUpzydgMUDg
- Integration by substitution - https://www.youtube.com/watch?v=Fpe88FbIXUI
- Integration by parts - https://youtu.be/zNU8iK8sGD0
- Integration by partial fractions – https://www.slideshare.net/slister07/5-3-partial-fractions
- Integration by partial fraction - https://www.youtube.com/watch?v=GIGJdvdrdhs&t=60s
- Definite integral - https://www.mathsisfun.com/calculus/integration-definite.html
- Integral as limit sum - https://www.youtube.com/watch?v=CMG6ucGUQiU
- Geometrical interpretation on area under a curve: https://www.youtube.com/watch?v=FsC3do74UIo
- Properties of definite integral Area bounded by curve - https://www.youtube.com/watch?v=2ODALMaUmc0
- Area bounded by curve - https://www.youtube.com/watch?v=FsC3do74UIo
- Area bounded by curve - https://www.youtube.com/watch?v=c8W9_iaERnU
- Area bounded by curve - https://www.youtube.com/watch?v=su9-sz5p3vc
- Area bounded by curves- https://www.youtube.com/watch?v=UyM3qudctYs
- Area bounded by curves - https://www.youtube.com/watch?v=Z3Vgv5cna1w
- Area bounded by two curves - https://youtu.be/xec6HTcn2M8.
- Practice questions on area bounded by curves https://tutorial.math.lamar.edu/ProblemsNS/CalcI/AreaBetweenCurves.asp
- Revolving curves in GeoGebra - https://www.geogebra.org/m/qgcz82ae
- Revolving curves in GeoGebra - https://www.geogebra.org/m/kM7hwJBa
- Volume of solid revolution - https://www.curve and y-axis
- Volume of solid revolution - https://www.curve and x-axis
- Volume of solid revolution-https://www.youtube.com/watch?v=E0JJ_tgl59o
- GeoGebra link of revolving curve - https://www.geogebra.org/m/kM7hwJBa
- Practice question on volume of solid revolution https://tutorial.math.lamar.edu/problems/calci/volumewithrings.aspx
- Self-Instructional Material

- Teacher’s guide book for class XII
- Technological gadgets for learning (smart phone, desktop, laptop…)

G. Annexure

Refer XIIP-A1 for template to record achievement

Topic: XIIP-B3 Differential Equations *[600 minutes]*

Introduction

'Differential equations' began with Leibniz, the Bernoulli brothers and others from the 1680s, not long after Newton's `fluxional equations' in the 1670s. Applications were made largely to geometry and mechanics; isoperimetric problems were exercises in optimisation. Most 18th-century developments consolidated the Leibnizian tradition, extending its multivariate form, thus leading to partial differential equations. Generalisation of isoperimetric problems led to the calculus of variations.

New figures appeared, especially Euler, Daniel Bernoulli, Lagrange and Laplace. Development of the general theory of solutions included singular ones, functional solutions and those by infinite series. Many applications were made to mechanics, especially to astronomy and continuous media.

In the 19th century: general theory was enriched by development of the understanding of general and particular solutions, and of existence theorems. More types of equations and their solutions appeared; for example, Fourier analysis and special functions. Among new figures, Cauchy stands out. Applications were now made not only to classical mechanics but also to heat theory, optics, electricity and magnetism, especially with the impact of Maxwell. Later Poincar\'e introduced recurrence theorems, initially in connection with the three-body problem. In the 20th century: general theory was influenced by the arrival of set theory in mathematical analysis; with consequences for theorisation, including further topological aspects. New applications were made to quantum mathematics, dynamical systems and relativity theory.

Source: history of differential equations.

Utility and Scope

Differential equations find widespread application across various scientific, engineering, and mathematical fields due to their ability to model and describe dynamic processes. Here are some notable applications in different domains:

1. **Physics:**
   a) **Mechanics**: Differential equations model the motion of objects, fluid flow, and the behaviour of mechanical systems. Newton's second law of motion, describing the relationship between force and acceleration, is a differential equation.

b) **Electrodynamics**: Maxwell's equations, a set of partial differential equations, describe the behaviour of electric and magnetic fields in space.

2. **Engineering**:
   a) **Control Systems**: Engineers use differential equations to model and analyse dynamic systems, helping design controllers for processes ranging from automotive systems to industrial automation.
   b) **Electrical Circuits**: Differential equations describe the behaviour of electrical circuits, aiding in the analysis and design of electronic systems.
3. **Biology**:
   a) **Population Dynamics**: Differential equations model the growth and decline of biological populations, considering factors such as birth rate, death rate, and migration.
   b) **Neuroscience**: Differential equations are employed to understand and model neural activity and the dynamics of biological systems in the brain.
4. **Economics**:
   a) **Macroeconomics**: Differential equations model economic phenomena such as economic growth, inflation, and the dynamics of financial markets.
   b) **Microeconomics**: Differential equations are used to study individual and firm behaviour, including optimization problems and game theory.
5. **Chemistry**:
   a) **Chemical Kinetics**: Differential equations describe the rates of chemical reactions, helping chemists understand reaction mechanisms and predict reaction outcomes.
6. **Environmental Science**:
   a) **Ecology**: Differential equations are applied to model ecological systems, including predator-prey interactions, population dynamics, and the spread of diseases in ecosystems.
   b) **Climate Modelling**: Differential equations play a crucial role in modelling climate systems, describing factors like temperature, ocean currents, and atmospheric circulation.
7. **Computer Science**:
   a) **Machine Learning**: Differential equations are employed in neural network architectures, optimising parameters during the training process.
8. **Medicine**:
   a) **Physiology:** Differential equations model physiological processes in the human body, including the circulation of blood, nerve impulses, and drug kinetics.

b) Epidemiology: Differential equations are crucial for modelling and predicting the spread of infectious diseases in populations.

A. Competency

- Demonstrate an understanding of differential equations in various forms to effectively employ suitable methods for solving each type of differential equation.

B. Objectives

- Identify the order, degree and linearity of Differential Equation (DE).
- Evaluate the general and particular solutions for variable separable, homogeneous, linear and second order linear differential equations.

C. Essential Skills/Processes

- Conceptualising
- Applying
- Computing
- Analysing

D. Learning Experiences

- Introduce the definition, order and degree of a differential equation by playing the video in the web link: https://www.youtube.com/watch?v=ShTSx_B_J3k and instruct students to make notes.
  - Enhance their understanding of the order and degree of differential equations by discussing further examples like in the web link: https://www.youtube.com/watch?v=QARoeV5DG1w**.**

- Prepare a Powerpoint slide on classifying a Differential equation into linear or nonlinear by emphasising on the conditions for linearity.
  - Refer to the example in the web link: Linearity of Differentiation Equations or refer to BHSEC Mathematics Book II.
  - Provide further examples from BHSEC Mathematics Book II, Exercise 10(a) for students to classify as linear or nonlinear.

- Prepare PowerPoint presentations on solutions of a differential equation explaining the number of arbitrary constants in a solution and solutions of differential equations in general and particular form with relevant examples. Refer to BHSEC Mathematics Book II or the idea in the web links:
  - https://www.youtube.com/watch?v=AmPSBmxw0y0

- https://www.youtube.com/watch?v=BwghiuIfdPs

- Discuss the methods to solve the following standard form of differential equations: a) Variable separable, b) Homogeneous equation of first order and c) Linear equations.

- Demonstrate the working rule of solving the separable differential equations of the types 1. $\frac{dy}{dx} = f(x)$ and 2.$\frac{dy}{dx} = f(y)$ with relevant examples and allow students to apply their understanding in solving higher-order-thinking questions as homework. Refer to BHSEC Mathematics Book II for notes and questions.
  - Instruct students to explore and learn the methods of solving the separable differential equations of type 3. $\frac{dy}{dx} = f(x).\emptyset(y)$and on the initial value problems by referring the following web links:
    - ➢ https://www.youtube.com/watch?v=nNHlSB6b1HU (solving separable DE of type 3)
    - ➢ https://www.youtube.com/watch?v=XExEixAPK6s (solving separable DE of type 3)
    - ➢ https://www.youtube.com/watch?v=CzMAVLrJ3MM (solving initial value problems)
    - ➢ https://www.youtube.com/watch?v=3jpiW_oueaA (solving initial value problems)
  - Demonstrate with few examples, use of appropriate substitution to reduce an inseparable differential equation into separable form and let students solve similar problems in class (refer to BHSEC Mathematics Book II for questions).

- Discuss the method of solving homogeneous differential equations with examples referring to the notes given from the web link: https://www.youtube.com/watch?v=ZEJVyybsiT4 .
  - Provide further practice questions to enhance the student's understanding of solving homogeneous differential equations.
  - Present the slide on Equation reducible to a homogenous form from the web link: equations reducible to homogeneous form and explain the procedure step-by-step to reduce an equation to a homogeneous form.
  - Demonstrate the procedure with an example on the board and solve the differential equation.
  - Allow students to practise their understanding with similar questions and suggest them to refer to BHSEC Mathematics Book II.

- Play the video in the web link: https://www.youtube.com/watch?v=v5C0CtRAK10 to demonstrate the use of the Integrating-factor method in solving first order linear differential equations with an example and instruct students to make notes.
  - Divide the students into groups to explore and work on different problems involving linear differential equations and assess the students' knowledge through presentation tasks. Refer to BHSEC Mathematics Book II for questions.
- Demonstrate how to solve second-order linear differential equations with a few examples and allow students to practise their learning by providing further questions (refer to BHSEC Mathematics Book II) in the class to discuss in pairs.

E. Assessment

Performance Task 1

Cross pollination with ICT

Design an MCQ quiz on Google Form on questions to classify the differential equations under different orders, degrees and linearities.
Bring the students to the school IT laboratory to sit the quiz.

Performance Task 2

Design a worksheet with questions on all the different kind of differential equations and assess students on their knowledge on:
1. Identifying the correct method for solving each question
2. Use of procedures accurately and coming to correct solution

Design an appropriate assessment tool for each performance task and record feedback and achievement based on the templates given in the annexure XIIP-A1

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction - history of differential equations
- Order and degree of differential equation - https://www.youtube.com/watch?v=ShTSx_B_J3k
- Order and degree of differential equation - https://www.youtube.com/watch?v=QARoeV5DG1w
- Linearity of differential equation - https://www.youtube.com/watch?v=bVy1F5T8QE4

- Solution of differential equations I - https://www.youtube.com/watch?v=BwghiuIfdPs
- Solution of differential equations II: https://www.youtube.com/watch?v=BwghiuIfdPs
- Separable first order differential equations- https://www.youtube.com/watch?v=nNHlSB6b1HU
- Separable differential equation - https://www.youtube.com/watch?v=XExEixAPK6s
- Initial value problem- https://www.youtube.com/watch?v=CzMAVLrJ3MM
- Separable differential equation - https://www.youtube.com/watch?v=3jpiW_oueaA
- Homogeneous differential equation - equations reducible to homogeneous form
- Integrating differential equations - https://www.youtube.com/watch?v=v5C0CtRAK10
- Technological gadgets for learning (smart phone, desktop, laptop)

G. Annexure

Refer XIIP-A1 for template to record achievement

Topic: XIIP-C1 Inverse Trigonometric Functions *[500 minutes]*

Introduction

Inverse trigonometric functions were considered early in the 1700s by Daniel Bernoulli, who used "A.sin" for the inverse sine of a number, and in 1736, Euler wrote "A t" for the inverse tangent. As early as 1772, J.L. Lagrange used the symbols "arcsin" and "arctan." These writers were identifying an angle with the arc it subtends when placed at the centre of a circle. John Herschel introduced the $Sin^{-1}$and tan^{-1} notations in an article in the Philosophical Transactions of London in 1813.

In mathematics, the inverse trigonometric functions (occasionally also called arcus functions, anti-trigonometric functions or cyclometric functions are the inverse functions of the trigonometric functions (with suitably restricted domains). Specifically, they are the inverses of the sine, cosine, tangent, cotangent, secant, and cosecant functions, and are used to obtain an angle from any of the angle's trigonometric ratios.

Source: https://www.brainkart.com/article/Introduction_39137/

Utility and Scope

Inverse trigonometric functions have applications in many real life situations and in different fields of studies.
Some applications are:

1. To measure the height of objects such as buildings or mountains
2. In video games, the projectile of Mario jumping over an obstacle is thanks to incorporation of trigonometry in the IT and computer sciences
3. In construction we need trigonometry in measuring fields, lots and areas, making walls parallel and perpendicular, installing ceramic tiles; roof inclination;
4. Architects use trigonometry to calculate structural load, roof slopes, ground surfaces and many other aspects, including sun shading and light angles
5. In physics, trigonometry is used to find the components of vectors, model the mechanics of waves and oscillations, sum the strength of fields, use dot and cross products and in projectile motion.

6. In criminology, trigonometry can help to calculate a projectile's trajectory, to estimate what might have caused a collision in a car accident or how an object fell down from somewhere, or at which angle was a bullet shot.
7. Trigonometry is used to set directions such as the north, south, east and west and tells you what direction to take with the compass to get in a straight direction. It is used in navigation in order to pinpoint a location.

Source: Applications of inverse trigonometric functions

A. Competency

- Exhibit the meaning and principal values of inverse trigonometric functions and apply the properties to solve related problems.

B. Objectives

- Discuss the definition and principal values of inverse trigonometric functions.
- Apply properties of inverse trigonometric functions to solve problems.

C. Essential Skills/Processes

- Conceptualising
- Applying
- Reasoning
- Analysing

D. Learning Experiences

- Define 'Inverse Trigonometric Function' and show the graphs for each of the inverse trigonometric functions using relevant software (e.g., GeoGebra).
  - https://www.youtube.com/watch?v=YXWKpgmLgHk (contains a video which explains the definition of inverse trigonometric functions and the graphs of inverse t-functions.)
- Discuss 'Principal values of inverse trigonometric functions' with the students with examples. (refer to BHSEC Mathematics Book II)
  - Use the web link to supplement the knowledge:
    Principal values of inverse trigonometric functions [contains powerpoint presentation(ppt)]

- Introduce each of the properties of inverse trigonometric functions and apply the properties to solve problems. (refer to BHSEC Mathematics Book II, from the topic - Properties of Inverse t-functions).
  - o May also refer to the notes from the link: Inverse trigonometric functions properties which explains the use of different properties of inverse trigonometric functions in questions.
  - o Suggested link: Refer the link: problems and solutions to problem solving on inverse trigonometric functions.

E. Assessment

Performance Task 1

Perform a short quiz related to the concept of principle values and other values of an inverse trigonometric function (refer to BHSEC Mathematics Book II, from the topic - Principal Values of Inverse Trigonometric Functions) using any online platforms like Quizizz, Kahoot, etc. The sample questions are provided with the web link; https://quizizz.com/print/quiz/5f8deaa2ad2e89001c717266

Performance Task 2

Assignment: Devise 5-6 higher-order level questions (refer to BHSEC Mathematics Book II suggested questions - Exercise 4 (9 (iv), 10, 17 and 30) and instruct students to solve the questions as an assignment. Encourage students to explore the use of different properties to solve one question and assess them on every method used.

Design an appropriate assessment tool for each performance task and record feedback and achievement based on the templates given in the annexure XIIP-A1

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction - https://www.brainkart.com/article/Introduction_39137/
- Utility and scope - Applications of inverse trigonometric functions
- Inverse trigonometric function - https://www.youtube.com/watch?v=YXWKpgmLgHk
- Finding range and domain - Principal values of inverse trigonometric functions
- Properties of inverse trigonometric function - Inverse trigonometric functions properties
- Solved resources - problems and solutions
- Sample Quiz - https://quizizz.com/print/quiz/5f8deaa2ad2e89001c717266

- GeoGebra
- Self-Instructional Material
- Technological gadgets for learning (smart phone, laptop, desk top)

G. Annexure

i. Refer XIIP-A1 for template to record achievement

ii. GeoGebra Resource: https://www.geogebra.org/m/S9mgw5vv (explore for different inverse t-functions

Topic: XIIP-D1 Points and Their Coordinates in 3-Dimensions

[550 minutes]

Introduction

The three-dimensional coordinate system is an extension of the two-dimensional coordinate system invented in 1637 by French mathematician René Descartes (1596–1650). Soon after Descartes wrote about his two-dimensional coordinate system, other mathematicians took Descartes's idea and expanded it from a two-dimensional plane to three-dimensional space. This new development greatly enlarged the uses of the coordinate system.

With the enhancement of the two-dimensional coordinate system to three dimensions, it was possible to locate any object in three-dimensional space. For example, with Descartes's two-dimensional coordinate system, you could describe the location of a coin on the floor of a room by referring to how many feet the coin was located from a front wall and a side wall. However, in a two-dimensional coordinate system, it is impossible to describe the location of an object that is off the plane of the floor, such as the location of a crystal on a chandelier that is hanging from the ceiling. In a three-dimensional coordinate system, it is possible to refer to the location of a chandelier crystal with reference to the two walls of the room and also the floor.

Source: introduction of 3-D coordinate

Utility and Scope

Graphic artists, fashion, tool and die makers, and blueprint designers are examples of people who use a three-dimensional coordinate system to map three-dimensional objects, in this case, on a two-dimensional computer monitor or a drawing board. A three-dimensional coordinate system helps the artist draw the object in proper perspective. This system also ensures that any object drawn by the artist can be properly interpreted by anyone familiar with three-dimensional coordinates.

In the technical fields: The concept of 3-D geometry is also applied in the fields of robotics, computer, and video games. Geometry provides handy concepts both for computer and video game programmers. The way & the design of the characters that move through their virtual worlds requires geometric computations to create paths

around the obstacles concentrating around the virtual world. Video game engines typically put to use ray casting, which is a technique that simulates a 3-D world using a 2-D map. Using this form of geometry helps speed up processing because calculations are only done for the vertical lines on the screen.

Astronomy & Physics: Here, geometry is used in the field of astronomy, helping to map the positions of stars and planets on the celestial sphere and describing the relationship between movements of celestial bodies.

In the field of Physics, there is a deep link between pseudo-Riemannian geometry and general relativity.

Geographic Information Systems: Geometry concepts are used in satellites in GPS systems, it calculates the position of the satellite and location of GPS gauged by the latitudes and longitudes.

Source: https://www.toppr.com/bytes/geometry-in-daily-life/
Suggested weblink https://studiousguy.com/examples-of-geometry-in-everyday-life/

A. Competencies

- Demonstrate the understanding of the distance, section and midpoint formula by applying in relevant scenarios.
- Display an understanding of the relationship between direction cosine and direction ratio to draw the conditions of parallelism and perpendicularity between two lines in real world situations.

B. Objectives

- Comprehend the 3-D coordinate system and articulate the positioning of a point in 3-D space.
- Solve problems related to distance, section and midpoint formula.
- Calculate direction ratios and direction cosines of a line in 3-D space.
- Determine the angle between two lines using direction cosines and direction ratios.
- Assess the parallelism and perpendicularity of two lines in 3-D space.

C. Essential Skills/Processes

- Conceptualising
- Applying
- Analysing

- Illustrating

D. Learning Experiences

- Direct the students to explore the 3-D coordinate system and to compare the 3-D coordinate system to the 2-D coordinate system that they have learned in class XI.
  - Share the web link: https://www.youtube.com/watch?v=yPysmMXI_Is which contains a video explaining a point in a 3D-coordinate system for students' reference.
  - Assume the three corner-lines of the classroom as the three coordinate axes. Describe how you would describe a point with coordinates (2, 3, 1) using the classroom context.

- Recall the distance formula, division or section formula and midpoint formula in the 2-D coordinate system and help them to relate it in 3-D . Solve problems referring to BHSEC Mathematics Book II or display the video in the web links:
  - https://www.youtube.com/watch?v=5sJdfciNM20 (3:15 minutes onward) which contains a video explaining the deriving of distance formulas in 3D space.
  - https://www.youtube.com/watch?v=cBbWbz0kQaQ which contains a video explaining the section formula and solving problems using section formula.
  - https://www.youtube.com/watch?v=_ZRT4bmFYDQ which contains a video on more questions solved using the section formula.

- Demonstrate and explain the concept of Direction Cosines and Direction Ratios (for supplement refer the web link: Direction Cosine & Ratios which contains the notes on direction cosines and direction ratios.)
  - May also use the explanatory video in the web link: https://www.youtube.com/watch?v=HNTYvIVoT-U which contains the introduction of direction cosines and direction ratios.

- Exhibit finding direction ratios of a line joining two points, angle between two lines and conditions of perpendicularity and parallelism using direction cosines and ratios.
  - Refer to BHSEC Mathematics Book II or the link: https://www.youtube.com/watch?v=oi3bpxklbP4 to learn about the conditions
  - Assign relevant questions from BHSEC Mathematics Book II, exercise 13(c) or any relevant questions from other sources.

E. Assessment

Performance Task 1

- Provide two points in 3-D coordinates. From the points, let the students find the following:
  - Distance between the points and Midpoint of the line
  - Direction ratios of the lines
  - Direction angle of the lines
  - Direction cosines of the lines.

Performance Task 2

- Assignment: Frame questions which asks students to check for perpendicularity and parallelism between two lines and finding the angle. Ask students to write reflection on each step used to solve the problems.

Design an appropriate assessment tool for each performance task and record feedback and achievement based on the templates given in the annexure XIIP-A1

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction - introduction of 3-D coordinate
- Utility and scope - https://www.toppr.com/bytes/geometry-in-daily-life/
- Utility and scope - https://studiousguy.com/examples-of-geometry-in-everyday-life/
- Point in a 3D coordinate system - https://www.youtube.com/watch?v=yPysmMXI_Is
- Deriving 3D -distance formula - https://www.youtube.com/watch?v=5sJdfciNM20
- Section formula - https://www.youtube.com/watch?v=cBbWbz0kQaQ
- Section formula - https://www.youtube.com/watch?v=_ZRT4bmFYDQ
- Direction cosines and direction ratios - https://www.youtube.com/watch?v=RWJsy9ufxtc
- Direction cosines and direction ratios - https://www.youtube.com/watch?v=HNTYvIVoT-U
- Parallelism and perpendicularity - https://www.youtube.com/watch?v=oi3bpxklbP4
- Technological gadgets for learning (smart phone, laptop, desk top)

G. Annexure

Refer XIIP-A1 for template to record achievement level

Topic: XIIP-D2 Pairs of Straight Lines *[550 minutes]*

Introduction

In geometry, the notion of line or straight line was introduced by ancient mathematicians to represent straight objects (i.e., having no curvature) with negligible width and depth. Lines are an idealisation of such objects, which are often described in terms of two points.

Until the 17th century, lines were defined as the first species of quantity, which has only one dimension, namely length, without any width nor depth, and is nothing else than the flow or run of the point which will leave from its imaginary moving some vestige in length, exempt of any width. The straight line is that which is equally extended between its points. Euclid described a line is "breadthless length" which "lies equally with respect to the points on itself"; he introduced several postulates as basic unprovable properties from which he constructed all of geometry, which is now called Euclidean geometry to avoid confusion with other geometries which have been introduced since the end of the 19th century (such as non-Euclidean, projective and affine geometry).

A second-degree equation in two variables represents a pair of straight lines. A pair of straight lines can also be represented as a product of two linear equations in x and y, representing a straight line. The concept of a pair of straight lines is very helpful in the mathematical world as it simplifies our complex problems more easily.

Source: who invented the straight line
Introduction to pairs of straight lines

Utility and Scope

The concepts of pairs of straight lines is applicable to conic sections, which have further interesting applications in engineering, science and industry. When a double napped cone is cut vertically through the vertex by a plane, a pair of straight lines is obtained.

Sources: https://www.youtube.com/watch?v=aYF5ufwre_4

The concept of a pair of straight lines can have the same utility as a straight line in everyday practice.

Explore the applications of straight lines using the link: Applications of straight lines.

A. Competencies

- Demonstrate an understanding of concepts related to pairs of straight lines and use of graphing tools to represent each theorem of pairs of straight lines.
- Apply the conditions to show that the given general equation represents a pair of straight lines.
- Utilise the general equation of a pair of straight lines to determine angles between two lines, point of intersection, equation of the bisector between lines and individual equations in the pair.

B. Objectives

- Demonstrate using graphing tools, the concepts and theorems related to pairs of straight lines using relevant examples.
- Employ the conditions to show that the given general equation represents a pair of straight lines.
- Use the general equation of a pair of lines to determine point of intersection, angles between two lines and equation of the bisector between the lines.
- Resolve the general equation of a pair of straight lines into two individual equations of straight lines.
- Find the distance between the two parallel lines.

C. Essential Skills/Processes

- Applying
- Analysing
- Evaluating
- Illustrating

D. Learning Experiences

- Recapitulation: Let children come up with a short presentation on the general equation of a straight line, angle between two lines, distance between two parallel lines and angular bisector which they have already covered in class XI. Refer to BHSEC Mathematics Book I - chapter 19.
  - Recommend children to use the web link: https://www.youtube.com/watch?v=dQM_IX77Irc which contains a video summarising the concepts of straight lines.

- Display a GeoGebra sheet: Graphing Calculator - GeoGebra and type a general equation of a pair of straight lines to show graphically how the equation represents two straight lines. (Use GeoGebra throughout the chapter whenever graphical representation is desired).

- Introduce briefly the homogeneous equation of a pair of straight lines, its condition of perpendicularity, angle between the lines and equation of bisector of the angles. Refer to BHSEC Mathematics Book II or use the web link: https://www.youtube.com/watch?v=-jUxTuFWOhA explaining the angle between the lines and the equation of bisector of angles for the homogeneous equation of a pair of straight lines.

- Reflective task: Students can give an example each when a concept or theorem is discussed and use the GeoGebra calculator to check if their examples verify the theorems.

- Take the students in the IT laboratory and direct them to explore the web link: https://www.youtube.com/watch?v=10hrYwiS2z0 (contains video explaining the conditions to check whether a general second degree equation represents a pair of straight lines).
  - Allow students to explore further by referring to BHSEC Mathematics Book II and present their findings and solved examples through presentation tasks.

- Relate to the homogeneous equation and discuss angle between the lines and equation of the bisector of the angle for the general equation of a pair of lines.
  - Solve related examples with the students. (Refer to BHSEC Mathematics Book II, suggested examples - Examples 8 and 12).
  - Or use the web link below to explain the angle between the pair of lines and the equation of the bisector of the angles for the general equation of the pair of straight lines: https://www.youtube.com/watch?v=-jUxTuFWOhA .

- Demonstrate different methods of separating the pair of straight lines in a general equation into individual equations with examples.
  - Refer to BHSEC Mathematics Book II, suggested questions - Examples 9 or 10 or refer to suggested weblink: splitting of a pair of straight lines.

- Allow students to discuss and prepare a presentation on the topics - the pairs of parallel straight lines and finding distance between them referring to BHSEC Mathematics Book II, suggested examples - example 15 and Exercise 11(b) Q18.

E. Assessment

Performance Task 1

Devise questions aligned to the objective from the source Problems with solutions which contain different questions and their solutions from a pair of straight lines or refer to BHSEC Mathematics Book II, (suggested questions: Exercise 11(b) - Q1, 5, 10).

Performance Task 2

Assignment: Prepare questions aligned to the objective from the source Problems with solutions which contains different questions and their solutions from pair of straight lines or refer to BHSEC Mathematics Book II, (suggested questions: Exercise 11(b) - Q3, 11 (iii), 13, 15).

Performance Task 3

Test: Prepare questions aligned to the objective from the source Problems with solutions which contain different questions and their solutions from a pair of straight lines or refer BHSEC Mathematics Book II, (suggested questions: Exercise 11(b) - Q8(ii), 17, 19) . Conduct a short test to assess the students' understanding.

Design an appropriate assessment tool for each performance task and record feedback and achievement based on the templates given in the annexure XIIP-A1.

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction - who invented the straight line
  Introduction to pairs of straight lines
- Utility and scope - https://www.youtube.com/watch?v=aYF5ufwre_4
  Applications of straight lines
- Concepts on straight lines - https://www.youtube.com/watch?v=dQM_IX77Irc
- Straight lines calculator - Graphing Calculator – GeoGebra
- Angle between lines - https://www.youtube.com/watch?v=-jUxTuFWOhA
- Second degree equations - https://www.youtube.com/watch?v=10hrYwiS2z0

- General equations - https://www.youtube.com/watch?v=-jUxTuFWOhA
- Pair of straight lines - splitting of a pair of straight lines
- Problem on pair of straight lines - Problems with solutions
- Technological gadgets for learning (smart phone, laptop, desk top)

G. Annexure

Refer XIIP-A1 for template to record achievement level

Topic: XIIP-D3 The Conic Section *[650 minutes]*

Introduction

Greek origins: The early history of conic sections is joined to the problem of "doubling the cube." According to Eratosthenes of Cyrene (c. 276–190 BC), the people of Delos consulted the oracle of Apollo for aid in ending a plague (c. 430 BC) and were instructed to build Apollo a new altar of twice the old altar's volume and with the same cubic shape. Perplexed, the Delians consulted Plato, who stated that "the oracle meant, not that the god wanted an altar of double the size, but that he wished, in setting them the task, to shame the Greeks for their neglect of mathematics and their contempt for geometry."
Hippocrates of Chios (c. 470–410 BC) first discovered that the "Delian problem" can be reduced to finding two mean proportional between a and $2a$ (the volumes of the respective altars)—that is, determining x and y such that $a : x = x : y = y : 2a$. This is equivalent to solving simultaneously any two of the equations $x^2 = ay$, $y^2 = 2ax$, and $xy = 2a^2$, which correspond to two parabolas and a hyperbola, respectively.
Later, Archimedes (c. 290–211 BC) showed how to use conic sections to divide a sphere into two segments having a given ratio. Diocles (c. 200 BC) demonstrated geometrically that rays—for instance, from the Sun—that are parallel to the axis of a paraboloid of revolution (produced by rotating a parabola about its axis of symmetry) meet at the focus.

Archimedes is said to have used this property to set enemy ships on fire. The focal properties of the ellipse were cited by Anthemius of Tralles, one of the architects for Hagia Sophia Cathedral in Constantinople (completed in AD 537), as a means of ensuring that an altar could be illuminated by sunlight all day.

Post-Greek applications: Conic sections found their first practical application outside of optics in 1609 when Johannes Kepler derived his first law of planetary motion: A planet travels in an ellipse with the Sun at one focus.

Galileo Galilei published the first correct description of the path of projectiles—a parabola—in his *Dialogues of the Two New Sciences* (1638).
In 1639 the French engineer Girard Desargues initiated the study of those properties of conics that are invariant under projections (*see* projective geometry).

Eighteenth-century architects created a fad for whispering galleries—such as in the U.S. Capital and in St. Paul's Cathedral in London—in which a whisper at one focus of an ellipsoid (an ellipse rotated about one axis) can be heard at the other focus, but nowhere

else. From the ubiquitous parabolic satellite dish (*see* the figure) to the use of ultrasound in lithotripsy, new applications for conic sections continue to be found.

Source: https://www.britannica.com/science/conic-section

Utility and Scope

Conic sections are widely applied in different fields. Here are some illustrations of the application of conic sections:

1. **Astronomy:**

Celestial Orbits: Conic sections play a crucial role in describing the orbits of celestial bodies. The paths of planets around the sun and satellites around planets often follow elliptical trajectories. Comets, with their elongated orbits, are another example of conic sections in celestial mechanics.

2. **Optics**:

Telescope and Camera Lenses: Conic sections, particularly lenses with shapes derived from ellipses and hyperbolas, are used in optical systems. Elliptical and hyperbolic mirrors are employed to correct aberrations in telescopes and camera lenses, ensuring clear and focused images.

3. **Engineering**:

Satellite Dish Design: Parabolic reflectors, a specific type of conic section, are commonly used in satellite dishes. The parabolic shape allows the dish to focus incoming parallel radio waves onto a single point, enhancing signal reception.

4. **Architecture**:

Architectural Design: Conic sections find applications in architecture, especially in the design of domes and arches. For instance, the Pantheon in Rome features a dome shaped like the top half of a sphere, showcasing the use of conic sections in architectural aesthetics and stability.

5. **Physics**:

Projectile Motion: Conic sections, particularly parabolas, describe the trajectory of projectiles in physics. When an object is launched with an initial velocity under the influence of gravity, its path can be modelled using a parabola. This application is relevant in various fields, including ballistics and sports.

These examples illustrate the versatility of conic sections across astronomy, optics, engineering, architecture, and physics, showcasing their importance in diverse scientific and practical applications.

Suggested web link: https://www.youtube.com/watch?v=8nPMlW5NZSo to investigate utility and scope.

A. Competencies

- Show an understanding of conic sections and their geometric components by utilising interactive tools for deriving the equation of each conic section.
- Demonstrate the ability to apply the concepts of conic sections effectively in solving real-life problems.

B. Objectives

- Define and comprehend the three types of conic sections based on eccentricity.
- Define and determine the elements of a parabola, including the vertex, focus, directrix, axis, latus rectum, and eccentricity, along with its equations in both standard and general forms.
- Explore the components of ellipses and hyperbolas, encompassing the centre, vertices, foci, directrices, axes, latus recta, and eccentricity, along with their equations in standard and general forms.
- Examine the conditions necessary for classifying general conics.
- Establish connections between the concepts of conics and their applications in solving real-life problems.

C. Essential Skills/Processes

- Conceptualising
- Representing
- Computing
- Connecting
- Applying

D. Learning Experiences

- Introduce conics as sections of a plane and a right circular cone. Refer the links:
    - https://www.geogebra.org/m/pCg8NFVT which is a GeoGebra worksheet on conic sections whereby one can manipulate the graph and conics by changing the angle.

- https://www.youtube.com/watch?v=HO2zAU3Eppo which contains videos explaining conics on a circular cone.

- Allow students to deduce the definition of parabola by visually demonstrating a parabola as locus of points equidistant from a fixed point, focus and fixed line, directrix. Use the GeoGebra worksheet: illustrative diagram of parabola.

- Demonstrate finding focus, directrix, eccentricity, latus rectum and axes for any two (right handed and downward) of the four parabolas in standard forms with examples.
  - Use relevant software (like GeoGebra) for graphical explanations.
  - Refer to the video in the web link to understand the derivation of the equation of parabola in standard form: https://www.youtube.com/watch?v=YJG_CUAG7tc
  - Refer to the following weblinks for better understanding the concepts:
    - ➢ https://www.youtube.com/watch?v=r5_mRxOrFaM (contains video explaining the components of parabola graphically)
    - ➢ https://byjus.com/maths/standard-equations-of-parabola/ (contains notes on finding components in a standard form of parabola)

- Discuss on finding parts of parabola centred at origin: focus, equation of axis, directrix, length of latus rectum, and vertex. Allow students to practise some questions from the BHSEC Mathematics Book II or refer to the links:
  - ➢ https://www.youtube.com/watch?v=tfIgcdAq9I4
  - ➢ https://www.youtube.com/watch?v=oXKkgIRnfEU

- Interesting fact: https://www.youtube.com/watch?v=UQrKtiKfmMU - focus on a parabola and its application.

- Allow students to discuss and present how to find the equation of parabola when vertex is at origin and when vertex is not at the origin for given conditions.
  - Refer to Examples 2 and 3 along with Exercise 12(a) Questions 7 and 11, BHSEC Mathematics Book II.

- Discuss on finding the components of parabola from an equation in general form using relevant examples.
  - Refer to Examples 4, 5, 6 and 7, along with Exercise 12(a) Questions 15 and 16, BHSEC Mathematics Book II.
  - Suggest students to explore the web link: parabola-calculator (the online calculators to obtain the equation of parabola step by step).

- Introduce Ellipse and Hyperbola simultaneously - its definitions, equations of conics and components: foci, directrices, vertices, (major, minor, transverse and conjugate) axes, latus rectum and eccentricity.
  - Use the GeoGebra worksheets:
    - ➢ https://www.geogebra.org/m/epGhevZJ and
    - ➢ https://www.geogebra.org/m/zSvbuhbT to allow students to visualise how ellipse and hyperbola change when the components change and vice versa.
  - Refer suggested web link to understand the concepts further:
    - ➢ https://www.youtube.com/watch?v=cRY50CTdVvE contains a video explaining ellipse and its parts
    - ➢ https://www.youtube.com/watch?v=a2niebD-3CA contains a video explaining hyperbola and its parts
    - ➢ https://www.shelovesmath.com/precal/conics/#TableofConics contains a summary table of all the conics and their parts and equations.

- Interesting fact: https://www.youtube.com/watch?v=4KHCuXN2F3I - foci of an elliptical pool.

- Discuss with students, problems related to finding the equations of ellipse and hyperbola with given condition(s). Refer BHSEC Mathematics Book II.
  - Use GeoGebra or other relevant software for any graphical work.

- Assign students a task to find centre, vertices, foci, directrices, lateral recta, etc. of ellipse or hyperbola when the equation of ellipse or hyperbola is given (refer BHSEC Mathematics Book II).
  - Use GeoGebra or other relevant apps for any graphical work.

- Explore: Allow students to explore the applications of conics in real life from different resources. Suggested web link: https://www.cuemath.com/learn/mathematics/conics-in-real-life/

- Discuss the conditions to differentiate the conics by examining the given second degree equations.
  - Refer BHSEC Mathematics Book II, topic: General Conic or the web links: identifying conics short video and identifying conics explained in detail (contains notes on conditions to classify the conics by examining the general equations).

- Reflective question: Recollect the conditions of equations of circle (from class XI) and pair of straight lines (in previous chapter) and compare with the conditions of general conics. Write your findings in a paragraph.

E. Assessment

Performance Task 1

Take students to the school IT laboratory and guide them each to work through the following GeoGebra sheets to manipulate the focus, directrix, and vertex and notice how the conics are changing.

Provide relevant problems from each conic (refer BHSEC Mathematics Book II, chapter 12, exercise 12(a, b and c)) to solve manually and by the use of GeoGebra sheets. Instruct students to write a reflection on the use of these two methods.

- https://www.geogebra.org/m/Dhh8qyNt contains a GeoGebra worksheet on parabola
- https://www.geogebra.org/m/vZ6T6S23 contains a GeoGebra worksheet on ellipse
- https://www.geogebra.org/m/FVfq4Sz3 contains a GeoGebra worksheet on hyperbola

Performance Task 2

Competency-based assessment

Instruction: At the end of the chapter, design a test aligned to the competency of being able to connect the concepts of conics in solving real life problems.
For sample questions, refer the web links:

- https://www.youtube.com/watch?v=Bzl_6LkuO5Y (contains competency-based questions using concept of parabola with solutions)
- https://www.purplemath.com/modules/ellipse4.htm (contains competency-based questions using concept of ellipse with solutions)
- application of hyperbola question (contains competency-based question using concept of hyperbola with solutions)

Design an appropriate assessment tool for each performance task and record feedback and achievement based on the templates given in the annexure XIIP-A1

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics

- Introduction - https://www.britannica.com/science/conic-section
- Utility and scope - https://www.youtube.com/watch?v=8nPMlW5NZSo
- Geogebra worksheet on conic section - https://www.geogebra.org/m/pCg8NFVT
- Concepts on conic section - https://www.youtube.com/watch?v=HO2zAU3Eppo
- Geogebra worksheet on conic section - https://www.geogebra.org/m/HpQ3DdSc
- Parts of parabola - https://www.youtube.com/watch?v=r5_mRxOrFaM
- Components of standard Parabola - https://byjus.com/maths/standard-equations-of-parabola/
- Parabola and focus application - https://www.youtube.com/watch?v=UQrKtiKfmMU
- Equation of standard parabola - https://www.youtube.com/watch?v=YJG_CUAG7tc
- Parabola online calculator- parabola-calculator
- Geogebra worksheet on ellipse and hyperbola https://www.geogebra.org/m/epGhevZJ
- Geogebra worksheet on ellipse and hyperbola https://www.geogebra.org/m/zSvbuhbT
- Concepts on hyperbola and ellipse- https://www.youtube.com/watch?v=cRY50CTdVvE
- Concepts on hyperbola and ellipse – https://www.youtube.com/watch?v=a2niebD-3CA
- Concepts on hyperbola and ellipse - https://www.shelovesmath.com/precal/conics/#TableofConics
- Ellipse and focus application - https://www.youtube.com/watch?v=4KHCuXN2F3I
- Application of conics – https://www.cuemath.com/learn/mathematics/conics-in-real-life/
- Identifying Conics I: identifying conics short video
- Identifying Conics II: identifying conics explained in detail
- GeoGebra worksheet on Parabola - https://www.geogebra.org/m/Dhh8qyNt
- Geogebra worksheet on Ellipse - https://www.geogebra.org/m/vZ6T6S23
- Geogebra worksheet on Hyperbola - https://www.geogebra.org/m/FVfq4Sz3
- Application of hyperbola problem: application of hyperbola question
- Problem on conic section – https://www.ck12.org/c/calculus/ https://www.youtube.com/watch?v=Bzl_6LkuO5Y
- Technological gadgets for learning (smart phone, laptop, desk top)

G. Annexure

Refer XIIP-A1 for template to record achievement levels

Topic: XIIP-D4 Plane *[500 minutes]*

Introduction

Plane geometry, and much of solid geometry also, was first laid out by the Greeks some 2000 years ago. Euclid in particular made great contributions to the field with his book "Elements" which was the first deep, methodical treatise on the subject. In particular, he built a layer-by-layer sequence of logical steps, proving beyond doubt that each step followed logically from those before.

Source: https://www.mathopenref.com/planegeometry.html

Utility and Scope

Computer Aided Design (CAD) and Architecture: Before a contractor builds a structure, someone must design the building's shape and create blueprints. A computer outfitted with computer-aided design software contains the maths to render the visual images on the screen. Some CAD programs can also create a simulation that allows you to see what the finished space looks like in a simulated walk-through.

Robotics, Computer and Video Games: Though most gamers appreciate speed over real life effects, geometry provides both for computer and video game programmers. The way that characters move through their virtual worlds requires geometric computations to create paths around the obstacles populating the virtual world. Video game engines typically employ ray casting, which is a technique that simulates a 3-D world using a 2-D map. Using this form of geometry helps speed up processing because calculations are only done for the vertical lines on the screen. The same geometry helps a robot to see.

Geographic Information Systems: Geometry plays a significant role in global positioning systems which require three coordinates to calculate location. A satellite equipped with a GPS system uses a form of geometry not unlike that used to calculate a right triangle. It involves the position of the satellite in the sky, the location of the GPS position on Earth identified by longitude and latitude, and the distance from that location to the place on Earth that equates to the satellite's position in the sky.

Star Maps and Space Travel: Geometry plays a role in calculating the location of galaxies, solar systems, planets, stars and other moving bodies in space. Geometry calculations between coordinates also help to chart a trajectory for a space vehicle's journey and its entry point into a planet's atmosphere. NASA scientists use geometry to compute the

journey of a vehicle sent to Mars. They calculate the elliptical orbits and the correct angle to enter a planet's atmosphere and land on the surface.

Source: https://sciencing.com/geometry-used-real-life-8698204.html

A. Competencies

- Visualise and demonstrate an understanding of equations of planes in various forms and generate the equations of planes for given conditions.
- Apply the formula to calculate the distance and angle between two planes and illustrate using examples in real world space.

B. Objectives

- Use visual representations (interactive tools) to understand the concept of planes in each form: one-point form, normal form and intercept form.
- Calculate distance and angle between two planes with relevant examples in real world space.
- Generate the equation of the planes when different conditions are given.

C. Essential Skills/Processes

- Conceptualising
- Analysing
- Representing
- Reasoning

D. Learning Experiences

- Define a plane using the video in the web link: https://youtu.be/RTfx9UV5EMc.
- Prepare a Powerpoint slide to introduce the general equation of a plane and the formula or method to:
  1. Find angle between two planes
  2. Check parallelism and perpendicularity of two planes
  3. Determine the angle between a line and a plane
  4. Determine the distance of a point from a plane in the general form
  5. Determine the distance between two parallel plane

     With the help of an example each.
  - Refer BHSEC Mathematic Book II or the following weblinks:

- ➢ https://www.youtube.com/watch?v=aW-Nn19ZBNY (contains video explaining formula for angle between two planes and the condition for parallelism or perpendicularity)
- ➢ https://www.youtube.com/watch?v=P7WrLUe4PHI (contains video explaining formula to find the angle between a line and a plane)
- ➢ https://www.youtube.com/watch?v=u3G1uJXIe5E (contains a video explaining an example on how to find the distance of a point from a plane)
- ➢ https://www.youtube.com/watch?v=Tl6x2u88c1w (contains a video explaining an example on how to find the distance between two parallel planes)

  - o Allow students to reflect on the similarities of these methods and formulas in finding angles and distance between two lines.

- Present the equation of the plane in one-point form when the plane is passing through a given point.

- Demonstrate with an example, how to determine the equation of a plane when three points on the plane are given and an example to check whether four points are coplanar (refer to BHSEC Mathematics Book II).
  - o Must explain the questions using appropriate diagrammatic representation on the board or using a relevant interactive tool.
  - o Provide practice questions as homework.

- Display visually using an appropriate interactive tool, the meaning of a normal to the plane, length of the normal and direction cosines of the normal and demonstrate the derivation of the normal form of equation of a plane.
  - o Play the video in the web link: https://www.youtube.com/watch?v=7uFUUk8CqTo to let students learn how to reduce the general equation to the normal form and find the length of the normal to the plane.
  - o Demonstrate more relevant questions under normal form of the equation of plane and provide students with further practice questions (refer to BHSEC Mathematics Book II).

- Allow students to watch the video in the web link: https://www.youtube.com/watch?v=w5KmwxMQJ3o to understand the derivation of the equation of plane in intercept form.
  - o Solve examples to show how to reduce the general form of the equation of plane to the intercept form (refer BHSEC Mathematics Book II).

- Divide students into pairs and assign practice questions to write equations in intercept form.

- Design Powerpoint presentation on 'Equations of some particular planes' and 'Conditions for a line to lie on or be perpendicular to a given plane' and discuss with students (*refer to BHSEC Mathematics Book II).*

- Discuss one question from each similar type of problem and demonstrate using visual illustration, how to determine the equation of the plane for given conditions.

E. Assessment

Performance Task 1

Set forth a set of questions which involves finding the equations of planes under different conditions as group work so that each group member will be required to explore different resources, solve and present their solution with diagrammatic interpretations.

Performance Task 2

Project-based assessment - Instruction:

1. List down 3 examples where you may be able to apply this formula of finding angle and distance between planes in and around your space.
   E.g., you can use the formula of distance between two parallel planes to find the distance between the two opposite walls of your classroom.
2. Design 2 competency-based questions with real world application using concepts of 'angle and distance between two planes'.
3. Make a simple project (model, presentation, video tutorials, etc.) to best simulate the context in your competency-based questions and their solutions.

Design an appropriate assessment tool for each performance task and record feedback and achievement based on the templates given in the annexure XIIP-A1

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction - https://www.mathopenref.com/planegeometry.html
- Utility and Scope - https://sciencing.com/geometry-used-real-life-8698204.html
- Definition - https://youtu.be/RTfx9UV5EMc
- Angle between plane - https://www.youtube.com/watch?v=aW-Nn19ZBNY

- Angle between line and plane - https://www.youtube.com/watch?v=P7WrLUe4PHI
- Distance of point from a plane - https://www.youtube.com/watch?v=u3G1uJXIe5E
- Distance between two parallel planes https://www.youtube.com/watch?v=Tl6x2u88c1w
- Rescue general equation to normal form - https://www.youtube.com/watch?v=7uFUUk8CqTo
- Intercept form of equation of plane - https://www.youtube.com/watch?v=w5KmwxMQJ3o
- Technological gadgets for learning (smart phone, laptop, desk top)

G. Annexure

Refer XIIP-A1 for template to record achievement for the students

Topic: XIIP-E1 Correlation *[450 minutes]*

Introduction

The history of correlation in statistics and mathematics can be traced back to the late 19th century. Sir Francis Galton, an English polymath, is often credited as one of the pioneers in the development of correlation. In the 1880s, Galton studied the relationship between the heights of parents and their offspring. He introduced the concept of correlation and developed the method of "regression toward the mean" to describe the tendency of extreme traits in parents to be less extreme in their children.

Karl Pearson, a contemporary of Galton, further advanced the field of correlation. Pearson developed the correlation coefficient, which quantifies the strength and direction of a linear relationship between two variables. His work laid the foundation for modern correlation analysis.

In the early 20th century, the concept of correlation continued to evolve, with scholars like Udny Yule contributing to its refinement. The field expanded to include various types of correlation coefficients, such as the product-moment correlation coefficient and the rank correlation coefficient.

Correlation became an essential tool in statistics, providing a quantitative measure of the association between variables. Over the years, advancements in technology and computing further facilitated the application of correlation analysis to large datasets and complex problems.

Today, correlation is a fundamental concept in statistics, data analysis, and machine learning, playing a crucial role in understanding relationships between variables and making informed decisions in various fields. The historical contributions of Galton, Pearson, and others have shaped correlation into a powerful and widely used tool in both theoretical and applied aspects of mathematics and statistics.

For more information visit: History of Correlation.

Utility and Scope

Correlation and Regression Analysis are forms of statistical analysis and have been traditionally reserved for statisticians and mathematicians.

1. **Improve Operations**: Improves business performance by impacting operational efficiency, such as discovering innovative material substitutions to reduce manufacturing costs.
2. **Sales Forecasting**: Maximise profits by making adjustments to resources and marketing strategies based on forecasted market trends.
3. **Analysing Results**: Accurately test decision making results to determine how your hypothesis impacts your business.
4. **Improve Employee Efficiency**: Connect employee behaviours to specific software or technology implementations, and drive efficiency improvements.
5. **Develop New Strategies**: Bring to light previously undiscovered relationships between data, such as customer demand increases based on a specific sales event.
6. **Correct Mistakes**: Analyse the findings of your decisions and reveal the exact reasons behind your results.

Source: https://www.researchoptimus.com/article/what-is-correlation.php

A. Competency

- Determine and interpret the coefficient of correlation between two variables in data collected in real world context to study the correlation between them.

B. Objectives

- Understand the significance of Correlation Coefficients in different fields.
- Calculate and interpret Karl Pearson's coefficient of correlation for ungrouped data collected from real life experiments.
- Calculate and interpret Spearman's Rank correlation coefficient for ungrouped data.

C. Essential Skills/Processes

- Decision making
- Analysing
- Interpreting
- Evaluating

D. Learning Experiences

- Check students' previous knowledge on correlation by asking questions on the types of correlation and correlation coefficients.

- Draw different scatter plots and ask students to recall the types of correlation and correlation coefficient for each scatter plot. If some students have forgotten the concept, provide a review of what they learned in classes X and XI.
  - Refer to the web link: https://www.youtube.com/watch?v=8nxXOEfZeHs.

- Explain the two methods to measure the degree of correlation between two variables. Inform students that, unlike the estimations the students have learned in class X, these methods are designed to precisely determine the correlation coefficient.
  - Suggestion: Explain the utility of the concepts of correlation coefficients in real-life to arouse students' interests and motivation.

- Demonstrate the concept of Covariance and Karl Pearson's first formula with an example. Explain the interpretations of correlation coefficients. Refer BHSEC Book II or the link: https://www.youtube.com/watch?v=mG__Wpp9dns.
  - Collect data from real-life situations from other sources, and discuss how to compute correlation coefficient using the first formula. Or discuss the relevant examples from BHSEC Mathematics Book II.
  - Explore : Derivation of correlation coefficient formula using the web links:
    - ➢ https://www.youtube.com/watch?v=ecvI02c6SuM
    - ➢ https://www.youtube.com/watch?v=N7bZnC_a01M

- Compute Karl Pearson's correlation coefficient using MS Excel. In MS Excel, there is a function called 'Pearson' to get the correlation directly.
  - Refer to the link: https://youtu.be/Ev86DMtLXOk to learn how to find Pearson's correlation coefficient using Ms Excel.
  - Assign questions from the exercise 16(a), BHSEC Mathematics Book II.

- Demonstrate Karl Pearson's second formula (Direct Method) to find correlation coefficients using the link: https://www.youtube.com/watch?v=Gxw4-1AnLy4.
  - Instruct students to compute correlation coefficients using both the first formula and second formula of Karl Pearson. Assign questions from BHSEC Mathematics Book II.
  - Allow students to identify the advantages and disadvantages of the first formula and second formula of Karl Pearson.

- Recap the lesson on finding the mean of a data using assumed mean, and then explain Karl Pearson's third formula. Students could use the third formula if the means of the data are not exact.

- Assign some data of real life or from online sources for practice. Or assign some relevant questions from the exercise 16(a), BHSEC Mathematics Book II.
- Instruct students to complete performance task 1.

- Introduce the Spearman's Rank correlation coefficient (Derivation is not necessary at this level). Though Pearson and Spearman's methods are both used to find the correlation coefficient between two variables, in some situations, Spearman's method will give a more accurate correlation than Pearson.
  - Relate between the Pearson and Spearman's correlation coefficient. Refer to the link: https://youtu.be/9dr8rJ9fE7o to know the differences between them.
  - Refer to this video https://youtu.be/mmTs_zkox6w to check how the Spearman's rank formula is applied to find the rank correlation coefficient between two variables.

- Discuss determining the ranks for repeated ranks, and adding the correction factor.
  - Refer to this video link: https://www.youtube.com/watch?v=rW9vYelxNyk to learn how to compute Spearman's rank correlation coefficient for repeated ranks. Note that in the video, they use the letter t in the correction factor, whereas, in BHSEC Mathematics Book II, they have used the letter m.
  - Allow students in groups to examine questions on Spearman's rank correlation coefficient for tied rank. Refer to the exercise 16(b), BHSEC Mathematics Book II or search for real-life data from other sources.
  - Instruct students to complete performance task 2.

E. Assessment

Performance Task 1

- Direct students to gather data comprising two variables, with approximately 30 observations. Offer examples such as marks in two subjects for 30 students in the same stream, or the height vs. weight of 30 students in the school. You may refer to the examples or exercise questions from BHSEC Mathematics Book II. Check their work and discourage any form of plagiarism.

Performance Task 2

- Group work:
  - In the school IT lab, demonstrate an example on an Excel sheet to punch in data and generate a scatter plot.

- Allow students to do the same for their data set.
- Instruct students to either print or copy the scatter plot and attach into their project work with an analysis of the plot to describe the direction and degree of the correlation between the data.

Performance Task 3

- Group work:
  - Using an Excel sheet, demonstrate how to find the values of x^2, y^2, xy, Σx, Σy, Σxy, d_x, d_y, etc. (all the necessary values for applying Karl Pearson's correlation coefficient formula).
  - Instruct the students to use an appropriate formula of Karl Pearson's for their data to find the correlation coefficient and interpret the result. (Encourage students to manually find the values such as $\bar{x}$, $\bar{y}$, σ_x, σ_y, cov(x, y), and r, using relevant formula and the Sums (Σ) compiled from the spreadsheet).
  - Instruct the students to attach a printed copy of the excel sheet and the solution and interpretation of the solution, into their project work.

Performance Task 4

- Group work:
  - Using an Excel sheet, demonstrate how to assign ranks (tied or not) for their data and find the values of D, D^2, ΣD^2 (all the necessary values for applying the formula to calculate the Rank correlation coefficient.).
  - Instruct the students to use Spearman's rank correlation coefficient to find and interpret the correlation between the variables of their data set. Encourage students to calculate the values of r manually using the information extracted from the spreadsheet.
  - Additionally, request students to write reflections on the correlation coefficients obtained through both methods (Karl Pearson's coefficient and Spearman's rank coefficient), and instruct them to include all completed activities in their project work.
  - Students may read the article in the link: http://geoinfo.amu.edu.pl/qg/archives/2011/QG302_087-093.pdf to compare between Pearson's and Spearman's correlation coefficient

Note:
Teachers must design an assessment tool to assess the project work.

Sample:

Expectations	Met (✓) Not met(X	Comments
1. Collected relevant data with two variables		
• Correct scatter plot generated		
• Interpreted the direction and degree of correlation correctly from the scatter plot		
• All the values accurately calculated on the excel sheet with right use of formulas (PT 3).		
• Appropriate formula used to calculate the Pearson's coefficient of correlation		
• Analysis and interpretation of the correlation coefficient done correctly		
• All the values accurately calculated on the excel sheet with right use of formulas (PT 4).		
• Accurate rank correlation coefficient found.		
• Analysis and interpretation of the Rank correlation coefficient done correctly		
• Reflection on the use of two formulas drew the correct conclusion.		

★ If 9-10 expectations are met - Exceeding
★ If 7-8 expectations are met - Advancing
★ If 5-6 expectations are met - Meeting
★ If 3-4 expectations are met - Approaching
★ If 1-2 expectations are met - Beginning

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction - https://journals.physiology.org/doi/full/10.1152/advan.00068.2010
- Utility and scope - https://www.researchoptimus.com/article/what-is-correlation.php
- Direction and degree of correlation - https://shiny.rit.albany.edu/stat/corrsim/
- Karl Pearson's correlation - https://www.youtube.com/watch?v=2CEGh1emkzM
- Spearman's rank correlation - https://www.youtube.com/watch?v=HdU 6 ocg
- Spearman's rank correlation(Repeated ranks) - https://www.youtube.com/watch?v=FNVdFV4fLfs

- Spearman's rank correlation (repeated ranks) - https://www.youtube.com/watch?v=Sdp3PIfyhM4
- Technological gadgets for learning (smart phone, laptop, desk top)

G. Annexure

Refer XIIP-A1 for template to record achievement levels

Topic: XIIP-E2 Regression *[450 minutes]*

Introduction

Sir Francis Galton (1885) introduced the idea of "regression" to the research community in a study examining the relationship of fathers' and sons' heights. In his study he observed that sons do not tend toward their fathers' heights but instead "regress to" the mean of the population. He thus formulated the idea of "regression toward mediocrity", and with the development of the method of least squares procedures by Carl Friedrich Gauss (Myers, 1990), multiple regression analysis using ordinary least squares procedures (OLS) has become one of the most common statistical techniques for investigating and modelling relationships among variables.

Source: https://history and introduction.

Explore - https://www.tandfonline.com/doi/pdf/10. for more introduction on the regression.

Utility and Scope

Regression analysis is used to estimate the relationship between a dependent variable and one or more independent variables. This technique is widely applied to predict the outputs, forecasting the data, analysing the time series, and finding the causal effect dependencies between the variables. There are several types of regression techniques at hand based on the number of independent variables, the dimensionality of the regression line, and the type of dependent variable. Out of these, the two most popular regression techniques are linear regression and logistic regression.

Regression has numerous applications. For example, consider a data set consisting of weather information recorded over the past few decades. Using that data, we could forecast weather for the next couple of years. Regression is also widely used in organisations and businesses to assess risk and growth based on previously recorded data.

You can find the implementation of regression analysis directly as a deployable code chunk. In modern machine learning frameworks like TensorFlow and PyTorch, in-built libraries are available to directly proceed with the implementation of our desired application.

Source: https://builtin.com/data-science/linear-regression-tensorflow

Refer the link: https://utility and scope for more information.

A. Competency

- Demonstrate an ability to analyse relationships between variables, make informed predictions and decisions based on statistical patterns in data.

B. Objectives

- Interpret the meaning of regression coefficients and understand the significance of the regression line.
- Determine regression coefficients and equations.
- Utilise means and standard deviations to find lines of regression.
- Apply this knowledge to generate regression lines for real-life data, facilitating accurate estimations.

C. Essential Skills/Processes

- Decision making
- Analysing
- Interpreting
- Evaluating

D. Learning Experiences

- Recap the previous knowledge on the scatter plots and lines of best fit. Explain how lines of best fit can be used to predict unknown values using interpolating and extrapolating.
- Clarify the drawbacks of the line of best fit drawn by hands, and then introduce the regression line and its equation.
- Define the regression line and relate it to the line of best fit: regression coefficients are the slopes of the line of best fit.
  - Refer to https://youtu.be/P8hT5nDai6A to learn the line of regression in *least squares sense*.
- Explain the formulas of two regression coefficients (derived if required), and their applications. Explain the scenarios where each type of regression coefficients will be used.

- Refer BHSEC Mathematics Book II or https://www.cuemath.com/data/regression-coefficients/ for the derivation of regression coefficients.

- Discuss the forms of lines of regression and demonstrate how to predict unknown data values from the regression equations.
  - Refer to the video link https://www.youtube.com/watch?v=sKfAmFK6u8A or BHSEC Mathematics Book II.

- Explain all the formulae for regression analysis. You can find the list of tables in BHSEC Mathematics Book II.

- Discuss the relation between regression coefficients and Pearson correlation coefficient.

- Discuss and solve examples to determine each type of regression equations: y on x, and x on y. Demonstrate how to predict x values when y value is known, and vice versa.
  - Assign some questions from the exercise 17 of BHSEC Book II, or data collected from other sources. Students must be able to find the regression coefficients and equations from a table of values, or from a given scatter plot.

- Carry out the Performance task.

E. Assessment

Performance Task 1

- Group work:

  - Demonstrate generating line of best fit using Excel sheets.
  - Instruct students to generate scatter plots and the line of best fit i) when first variable (x) is taken as independent and the second variable (y) is taken as dependent variable, ii) when y is taken as independent and x is taken as a dependent variable.
  - Find the slope and the y –intercepts of each of the regression lines $y\,on\,x$ and $x\,on\,y$ and find the equations of the lines: $y = mx + c\,and\,x = my + c$. Ask the students to attach their findings to their project work.

Performance Task 2

- Group work:
  - Instruct students to use Excel sheets to manipulate their data collected and find the equations of regression lines of $y\ on\ x$ and $x\ on\ y$ by using the relation: $y = \overline{y} + b_{yx}(x - \overline{x})$ and $x = \overline{x} + b_{xy}(y - \overline{y})$ (Students should be able to complete this task without much assistance since they have already been working on Excel sheet for similar tasks before)
  - Instruct students to compare the results with that in performance task 1.
  - Students will make predictions using their regression lines for an assumed value of x and an assumed value of y.
  - Students should also describe the interdependence of the two variables of their data by analysing the results obtained in correlation and regression.
  - Allow students to compile all the work from each task into the project work and submit for assessment of each competency.

Note:
Teachers may set assessment tools to assess the competencies.

Sample:

Expectations	Met (✓) Not met(X)	Comments
1. Regression line of y on x accurately generated in excel sheet		
2. Regression line of x on y accurately generated in excel sheet		
3. Values of slope and y-intercept correctly worked out		
4. Found correct equation of regression line of y on x		
5. Found correct equation of regression line of x on y		
6. All the values accurately calculated on the excel sheet with right use of formulas (PT 2).		
7. Appropriate formula used to calculate the regression coefficients (bxy and byx) and found equations of the regression line correctly.		
8. Analysis and interpretation of the correlation coefficient done correctly		

9. Used a relevant regression line to predict the values of y for assumed value of x and vice versa.		
10. Correct deduction of dependency of the two variables made.		

If 9-10 expectations are met - Exceeding
If 7-8 expectations are met - Advancing
If 5-6 expectations are met - Meeting
If 3-4 expectations are met - Approaching
If 1-2 expectations are met – Beginning

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction - https://history and introduction
- Introduction - https://www.tandfonline.com/doi/pdf/10.
- Utility and scope - https://builtin.com/data-science/linear-regression-tensorflow
- Utility and scope - https://utility and scope
- Regression y on x - https://youtu.be/zPG4NjIkCjc
- Technological gadgets for learning (smart phone, laptop, desk top)

G. Annexure

Refer XIIP-A1 for template to record achievement levels

Topic: XIIP-E3 Probability *[500 minutes]*

Introduction

"A gambler's dispute in 1654 led to the creation of a mathematical theory of probability by two famous French mathematicians, Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665). Probability, chance and randomness have been around since the ancient days. They could be found in fortune telling, games of chance, philosophy, law, insurance, and errors of prediction in astronomy and medicine (Hald, 1990). There is evidence that in the late 15th century and the early 16th century, mathematicians started to experiment with the idea of probability. The link helps to learn additional history of probability.

Source: http://history of probability

Utility and Scope

Probability is an essential tool in applied mathematics and mathematical modelling. Moreover, probability is used in daily life to make decisions when you don't know for sure what the outcome will be. Most of the time, you won't perform actual probability problems, but you'll use subjective probability to make judgement calls and determine the best course of action. For example, to check the weather forecast, playing cards, lottery tickets, etc.

Source: https://www.probability-real-life-examples/

A. Competencies

- Utilise the concepts of permutations and combinations in determining probabilities of various events related to real-life situations.
- Solve probability problems involving the use of both addition and multiplication theorems.
- Utilise different methods to compute the conditional probability of two dependent events.

B. Objectives

- Use the concept of permutation and combination in calculating probability.
- Solve probability problems by using both the addition and multiplication theorems of probability.
- Determine probability by applying laws (addition (OR) rule, multiplication (AND) rule) for the problems containing selection/arrangement of two or more things.

- Determine the conditional probability of two dependent events.

C. Essential Skills/Processes

- Identifying
- Applying
- Analysing
- Problem Solving
- Interpreting Information

D. Learning Experiences

- Recapitulate the addition and multiplication theorems on probability that students have learned in class XI. Refer the followings links:
  - o https://www.youtube.com/watch?v=mppk8je2BQA (contains video on addition rule)
  - o https://www.youtube.com/watch?v=eUYNeK60ql8 (contains video on multiplication rules of independent events).
  - o Provide questions from BHSEC Mathematics Book II, exercise 18(f) and 18(g) to recall their previous knowledge on the addition and multiplication theorems of probability.

- Revisit the concepts of 'at least', 'at most', 'exactly' and 'not all'. Solve some related problems to refresh their understanding.

- Demonstrate solving probability problems by using both the addition and multiplication theorems of probability. Refer to BHSEC Mathematics Book II or the link: https://byjus.com/question-answer
  - o Assign relevant questions from BHSEC Mathematics Book II, exercise 18(h).

- Revise concepts of permutation and combination learned in the previous lessons.
  - o Use the link: https://www.youtube.com/watch?v=tnF9f3zCCKI (suggested timing - till up to 10 minutes).

- Calculating probability using Permutation and combination.
  - o Play the video in the web link: probability using permutations and combinations which contains a lesson on the concepts of calculating probability using permutation and combination.
  - o Apply a blended instructional approach and pause the video wherever required to supplement on the explanation.

- Discuss the questions on finding probability using permutation from the given links:
  - ➢ https://www.youtube.ordering
  - ➢ https://www.letter/ arrangement
  - ➢ https://www.digit/ arrangement
- Discuss the questions on finding probability using combination from the given links:
  - ➢ https://www.lottery
  - ➢ https://www.deck-card
  - ➢ https://www.committee-members
- Assign further practice questions from the BHSEC Mathematics Book II, exercise 18(d) as homework tasks.

- Conditional probability:
  - Recap the concepts of conditional probability learned in class X. Refer https://www.youtube.com/watch?v=evyT3_8Dnhs to learn about the basic concept of conditional probability.
  - Use the link: https://www.youtube.com/watch?v=JGeTcRfKgBo which contains a video lesson on conditional probability. Conduct hands-on experiments using two bowls and marbles, as in the video, so that students' will have an in-depth understanding.
  - Investigate and explore the link: https://www.youtube.com/watch?v=ES9HFNDu4Bs&t=834s (contains notes and video lesson on conditional probability).
  - Conduct an active interactive session and discuss the relevant examples of conditional probability. Refer https://www.onlinemathlearning.com/conditional-probability.html for relevant questions or BHSEC Mathematics Book II.

- Multiplication Theorem of probability for dependent events:
  - Prove three multiplication theorems for dependent events and its example from BHSEC Mathematics Book II, or refer https://testbook.com/maths/multiplication-theorem-of-probability.
  - Discuss a few problems related to the multiplication of two dependent events. Refer https://testbook.com/objective-questions/ for relevant questions for practice or refer to BHSEC Mathematics Book II, exercise 18(j).

E. Assessment

Performance Task 1

- Assign the task on probability given in the link https://www.toppr.com/

Performance Task 2

- Place Based Approach

Take students to a basketball court of your school, and let each student shoot a basketball 7 times. Let the individual student record their own shots. Using their records:

  - o Instruct students to get into a group of three members and ask them to find the probability that the group will make a shot, if they shoot three times each.
  - o In the same group, ask them to find the probability that the group will make two shots if they shoot once each.

Design an appropriate assessment tool for each performance task and record feedback and achievement based on the templates given in the annexure XIIP-A1.

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction - http://history of probability
- Utility and scope - https://www.probability-real-life-examples/
- Addition rule https://www.youtube.com/watch?v=mppk8je2BQA
- Multiplication rule https://www.youtube.com/watch?v=eUYNeK60ql8
- Practice question involving both addition and multiplication rule https://byjus.com/question-answer
- Permutation and combination of probability - https://www.youtube.com/watch?v=tnF9f3zCCKI
- Permutation and combination of probability - https://www.youtube.com/watch?v=RNH_O2QvkWA
- Probability using permutation - https://www.youtube.ordering
- Probability using permutation - https://www.letter/ arrangement
- Probability using permutation - https://www.digit/ arrangement
- Conditional probability - https://www.youtube.com/watch?v=evyT3_8Dnhs
- Conditional probability - https://www.youtube.com/watch?v=JGeTcRfKgBo
- Conditional probability - https://www.youtube.com/watch?v=ES9HFNDu4Bs&t=834s

- Conditional probability – https://www.onlinemathlearning.com/conditional-probability.html
- Theorem of Multiplication dependent probability - https://testbook.com/maths/multiplication-theorem-of-probability
- Problems on multiplication probability https://testbook.com/objective-questions/
- Technological gadgets for learning (smart phone, laptop, desk top)

G. Annexure

Refer 12B-A1 for template to record achievement levels

Appendix A

Project Work

Background

Mathematics, as a discipline, goes beyond numbers and equations; it is the language of patterns, structures and logical reasoning. Traditionally, mathematics has been taught through textbooks and lectures, emphasising problem-solving skills and theoretical understanding. However, the integration of project work in mathematics education has emerged as a pivotal method to enhance learning outcomes and foster a deeper understanding of the subject.

Mathematics project work stands as a fundamental tool in modern education, enriching students' mathematical understanding, nurturing critical skills, and promoting a holistic approach to learning mathematics. By integrating project-based learning, educators can empower students to become proficient problem solvers, critical thinkers and enthusiasts of the intricate world of mathematics. It also equips students with fundamental research skills to gather, analyse, and present data, which is a crucial skill that the education system should prepare them for.

Why project work in mathematics?

- **Real - world Relevance**: Mathematics projects bridge the gap between theoretical concepts learned in class and their practical applications in real- life scenarios. They allow students to see the tangible relevance and usefulness of mathematics principles in various fields such as science, engineering, economics and technology.
- **Problem-solving proficiency**: Project work cultivates critical thinking and problem-solving skills. Students are challenged to apply mathematical theories to solve complex problems, encouraging them to think analytically and logically.
- **Creativity and Innovation**: Projects encourage students to explore mathematics beyond routine calculations, promoting creativity and innovative thinking in finding solutions to mathematical challenges.

- **Depth of Understanding**: Project-based learning enables students to delve deeper into mathematical concepts. It allows them to explore topics beyond the curriculum, leading to a richer understanding of mathematical theories and their applications.
- **Conceptual connections**: Through project work, students often discover the interconnectedness of mathematical topics, recognizing how different concepts relate and interact, thereby solidifying their understanding of the subject.
- **Technology Integration**: Projects often involve the use of mathematical software, simulations, or data analysis tools. The exposure to technology aids students in becoming proficient in utilising digital resources for mathematical analysis.
- **Independent Inquiry:** Through project work, students develop research skills, learn to conduct independent inquiries, and manage their time effectively to complete tasks.

Expected fields of project work:

- Research analysis report on application of concepts like measures of central tendency, dispersions, correlation and regression.
- Solving problems through codings.
- Development of model and simulations.
- Demonstration of use of any online mathematical softwares.
- Concepts of mathematics used in real life and other fields of study.
- Mathematics in the nature (Bio-maths)
- Mathematics behind Artificial Intelligence (AI).
- Recent inventions or discoveries in mathematics.
- A particular mathematician and their contributions.
- Extension of any concepts that they have learnt.

Components of Project Report

Components	Details
Title of the project	Title: Clearly stating the projects topic or theme
Brief Overview	Summarise the project's objectives, methods used, major findings and conclusions. Highlight the most significant aspects or outcomes of the project.
Introduction	Background and Context: introduce the project topic, explaining its importance and relevance. Objectives: Clearly state the goals and aims of the project. Scope; Define the boundaries and limitations of the study.
Literature Review (If Applicable)	Review of sources: Discuss relevant literature, theories, or existing research related to the project. Critical Analysis: Evaluate and compare various viewpoints explored in the literature.
Methodology	Research Methods: Describe the methods used for data collection, analysis or mathematical modelling. If students work on topics that don't require collection of data, they can mention how they explored the topic. Tools and Techniques: Explain the mathematical tools, software or technology used in the project.
Data Collection and Analysis	Data Description: Present any collected data, observations, measurements, exploration, case study or calculations used in the project. Mathematical Analysis: Show mathematical formulas, equations or models applied in analysing the data or patterns observed. Visual Representations: Include figures, graphs, charts, diagrams or tables to illustrate mathematical patterns or findings.
Results and Fundings	Summary of Findings: Summarise the key outcomes and discoveries. Interpretation: Explain the significance of the results in relation to the project objectives.
Discussions	Analysis and Explanation: interpret the results in detail, discussing their implications and relevance. Comparative Analysis: Compare findings with literature (If applicable) Limitations: Address any limitations or constraints encountered during the project.
Conclusion	Conclusive remarks: Recap the main points discussed in the report and state overall findings, their significance and their contribution to the field of study. Recommendations: Provide recommendations for further research extensions or improvement related to the project topic.
References:	Citations: Include a list of references, books, articles, websites or mathematical sources cited in the report. Proper Formatting: Follow a standard citation style (APA - American Psychological Association 7th edition)

Assessment :

- Develop suitable tools or rubrics that align with the topic chosen by the students.

Sample 1: Here is a sample rubric designed for individuals exploring the different fields of project work.

Category	Beginning (1)	Approaching (2)	Meeting (3)	Advancing (4)	Exceeding (5)
Understanding of Topic	Limited understanding of the topic, with major misconceptions.	Basic understanding of the topic, with some misconceptions.	Adequate understanding of the topic, demonstrating a grasp of fundamental concepts.	Thorough understanding of the topic, showcasing a comprehensive knowledge base.	Exceptional understanding of the topic, demonstrating advanced insights and a deep understanding.
Research and Information:	Limited or inaccurate use of sources with minimal relevance to the topic.	Basic use of sources with some relevance, but with limitations.	Effective use of diverse sources demonstrating a good understanding of the topic.	Skillful use of authoritative sources, showcasing a high level of research competency.	Exceptional use of a wide range of credible sources, revealing insightful and sophisticated research.
Problem Solving:	Limited attempt at problem-solving with significant errors or omissions.	Basic problem-solving with some correct steps, but with noticeable errors.	Competent problem-solving, demonstrating a clear understanding of the steps involved.	Proficient problem-solving with accuracy and efficiency in approach.	Exceptional problem-solving, showing creativity and originality in tackling challenges.
Critical Thinking:	Limited application of critical thinking skills, with minimal analysis or evaluation.	Basic application of critical thinking skills, with some analysis but lacking depth.	Sound critical thinking skills, demonstrating a good level of analysis and evaluation.	Strong application of critical thinking, showcasing a high level of analysis, evaluation, and synthesis.	Exceptional critical thinking, revealing advanced and insightful analysis, evaluation, and synthesis.
Presentation:	Disorganised or ineffective presentation with minimal use of appropriate visuals.	Basic organisation with some visuals, but lacking clarity or creativity.	Well-organised presentation with clear visuals enhancing understanding.	Professionally presented work, showcasing clarity, creativity, and effective use of visuals.	Exceptional presentation, demonstrating exceptional clarity, creativity, and innovation.
Communication:	Limited or unclear communication of ideas, with poor expression.	Basic communication with some clarity, but lacking in precision and coherence.	Clear and effective communication of ideas, demonstrating appropriate language use.	Proficient communication, with precise and coherent expression of ideas.	Exceptional communication, showcasing eloquence, precision, and sophistication.
Overall Quality:	Overall quality of the project is significantly below expectations.	Overall quality is basic, meeting minimal expectations.	Overall quality is good, meeting expectations.	Overall quality is excellent, surpassing expectations.	Overall quality is outstanding, showcasing exemplary work and surpassing all expectations.

★ *Sample 2: Here is a general sample rubric designed for project work assessment.*

Criteria	Excelling (4.1-5)	Advancing (3.1-4)	Meeting (2.1-3)	Approaching (1.1-2)	Beginning (0.1-1)
Plan	Plan is detail with 5 components (schedules, activities, materials requirement, data collection source, representations) and endorsed by teacher	Plan is incomplete by two components and not endorsed by teacher	Plan is incomplete by three components and not endorsed by teacher	Plan is incomplete by 4 components and not endorsed by teacher	Plan is sketchy without required components and not endorsed by teacher
Problem	problem is new, meaningful and will have positive impact on community	problem is new, meaningful but will not have any impact on community	problem is not new but meaningful	problem is new but not meaningful	problem is not stated
Literature	Literature study is researched and aligns with problem showing deep understanding of concept	Literature study is researched and aligns with problem but lacks understanding of concept	Literature study is well researched but do not align well with the problem	Literature study is not well researched and do not align well with problem	Literature study is not stated but visible from other part of project
Data collection	Data collection is systematic with collection tools, processes and authentic source and mentioned in plan	Data collection is systematic with collection tools, processes and authentic source but not mentioned in plan	Data collection has collection tools but without logical process and authentic source	Data collection is not systematic but source mentioned	Data collection is not described and source not mentioned
Data representation and analysis	Data representation is appropriate, neat with topics and legends and analysis is meaningful	Data representation is appropriate with incomplete components but analysis is meaningful	Data representation is appropriate but analysis is not meaningful	Data representation inappropriate and analysis is not meaningful	Data representation inappropriate but no analysis
Findings	Finding aligns as solution to the problem with strong argument supported by data	Finding aligns as solution to the problem but argument is weak and not supported by data	Finding is described well but not addressed to solve the problem	Finding is not well described but align as solution to the problem	Finding is brief and do not align with problem
References	More than five references are cited in APA format and referenced throughout the project	Four to five references are cited and referenced throughout the project	Two to three references are cited and referenced throughout the project	At least one reference is cited and referenced throughout the project	No references

Appendix B

Assessment Structures for KS- 5

Assessment Structure

Key Stage	Assessment									
	Term I					Term II				
	CA (10)			Mid Term Examination	Total	CA (10)			Annual Examination	Total
	CW	HW	PW			CW	HW	PW		
IV	3	4	3	40	**50**	3	4	3	40	**50**

For both Term I and Term II, assess each competency through appropriate performance tasks and assessment tools.

Performance Tasks: quiz, question and answer, presentation, making models, small projects, etc.

Assessment Tools: checklist, rating scale or rubrics.

Assessment Areas: Formulating situations mathematically, applying concepts, facts, and procedures, and interpreting mathematical results.

NOTE: Project work assessment is mandatory for class XI and XII

Appendix C

Weighting for each Strand

Weighting and time allocations for class XI Mathematics (PMT)

Strand	Time Allocation (Mins.)	Weighting(%)
Strand A: Numbers and Operations	1300	20
Strand B: Patterns and Algebra	2900	40
Strand C: Measurement	450	6
Strand D: Geometry	1400	20
Strand E: Data and Probability	1200	14

Weighting and time allocations for class XII Mathematics(PMT)

Strand	Time Allocation (Mins.)	Weighting (%)
Strand A: Numbers and Operations	1550	20
Strand B: Patterns and Algebra	2300	31
Strand C: Measurement	500	7
Strand D: Geometry	2250	26
Strand E: Data and Probability	1400	16

Appendix D

Sample - Class work Assessment Rubrics – Classes XI - XII

Criteria	Exceeding (5)	Advancing (4)	Meeting (3)	Approaching (2)	Beginning (1)
Understanding	→ Demonstrates a deep and thorough understanding of the concepts covered. → Consistently applies knowledge to solve problems.	→ Shows a good understanding of the concepts. → Applies knowledge effectively in most situations.	→ Demonstrates a basic understanding of the concepts. → Struggles with consistent application.	→ Limited understanding of the concepts. → Inconsistently applied knowledge.	→ Minimal understanding of the concept. → Unable to apply knowledge effectively.
Effort and participation	→ Actively engages in class discussions and activities. → Displays an exceptional commitment to learning, actively seeking opportunities to contribute beyond expectations	→ Engages in class activities with enthusiasm. → Demonstrates a strong commitment to learning, willing to take on additional responsibilities when appropriate →	→ Participates at a basic level. → Effort is inconsistent. → Demonstrates a commitment to learning, however lacks consistency in engaging with course content.	→ Shows limited effort and participation. → Often disengaged in class. → Demonstrates a basic commitment to learning, with room for improvement in engagement.	→ Minimal effort and participation Frequently disengaged.
Independence	→ Demonstrates a high level of independence in completing classwork. → Rarely requires assistance.	→ Generally works independently but may seek clarification when needed.	→ Works somewhat independently but often requires assistance.	→ Requires frequent assistance to complete classwork.	→ Constantly relies on others to complete classwork.
Seeking support	→ Proactively seeks support when faced with challenging concepts or problems. → Collaborates effectively with peers and teachers to enhance understanding.	→ Willingly seeks support when needed. → Demonstrates effective collaboration with peers and teachers.	→ Occasionally seeks support but may be hesitant. → Limited collaboration with peers and teachers.	→ Rarely seeks support.- → Minimal collaboration with peers and teachers.	→ Does not seek support. → No collaboration with peers and teachers.
Collaboration	→ Demonstrates excellent collaboration skills in group activities. → Encourages and supports peers, fostering a positive and inclusive class atmosphere	→ Collaborates well with peers in group activities. → Generally fostering a positive class environment.	→ Participates in group activities but with limited collaboration → Inconsistent contributions.	→ Struggles to collaborate effectively in group activities. → Offers occasional contributions, with room for improvement in depth of insights	→ Unable to collaborate effectively. → Rarely contributes ideas or insights to class conversations.

Appendix E

Sample - Homework Assessment Rubrics – Classes XI - XII

Criteria	Exceeding (5)	Advancing (4)	Meeting (3)	Approaching (2)	Beginning (1)
Understanding	➔ Demonstrates a deep and thorough understanding of the homework assigned. ➔ Consistently applies knowledge to solve problems.	➔ Shows a good understanding of the homework concepts. ➔ Applies knowledge effectively in most situations.	➔ Demonstrates a basic understanding of the homework concepts. ➔ Struggles with consistent application.	➔ LLimited understanding of the homework concepts. ➔ Inconsistently applies knowledge.	➔ Minimal understanding. Unable to apply knowledge effectively.
Completion	➔ All homework are completed accurately and thoroughly.- ➔ Consistently submits high-quality work.	➔ Most homework tasks are completed accurately and thoroughly. ➔ Few minor errors present.	➔ Some homework tasks are completed accurately, but there are notable gaps. ➔ Several errors are present.	➔ Numerous incomplete or inaccurately completed homework tasks. ➔ Completion is inconsistent.	➔ Virtually all homework tasks are incomplete or inaccurately completed.
Accuracy of response	➔ All calculations and solutions are accurate and precise. ➔ Demonstrates meticulous attention to detail.	➔ Most calculations and solutions are accurate and precise. ➔ Few minor errors present.	➔ Some calculations and solutions are accurate but lack precision. ➔ Several errors are present.	➔ Numerous errors in calculations and solutions. ➔ Accuracy and precision are major issues.	➔ Virtually all calculations and solutions are incorrect or imprecise.
Neatness and organization	➔ Homework is exceptionally well-organized and neatly presented. ➔ All text is highly legible, and there are no smudges or unintended marks. ➔ Clear headings, labels, and steps enhance the overall organization	➔ Overall organization is good, with a clear presentation. ➔ Most text is legible, and there are minimal smudges or unintended marks. ➔ Headings, labels, and steps contribute to effective organization.	➔ Organization is acceptable but may lack some neatness. ➔ Legibility varies, and there may be occasional smudges or unintended marks. ➔ Clear headings and labels help maintain a	➔ Organization is somewhat lacking, and there is some difficulty in following the work. ➔ Legibility issues are noticeable, and there are frequent smudges or unintended marks. ➔ Headings and labels are consistently not clear.	➔ Poor organization makes it challenging to follow the homework. ➔ Legibility is compromised, and there are significant smudges or unintended marks throughout. ➔ Chaotic presentation hinders understanding, and

			basic level of organization		headings and labels may be unclear or absent.
Follow up and improvement	➔ Actively seeks feedback on homework. ➔ Demonstrates a commitment to improving based on feedback. ➔ Makes corrections and improvements on subsequent submissions.	➔ Open to feedback and uses it to make improvements in subsequent homework. ➔ Shows a willingness to learn from mistakes.	➔ Occasionally seeks feedback but inconsistently incorporates it into subsequent work. ➔ Limited improvement over time.	➔ Rarely seeks feedback and seldom makes improvements. ➔ Little evidence of learning from mistakes.	➔ Does not seek feedback or make improvements. ➔ Repeated mistakes persist
Timeline	➔ Submits homework/assignments consistently on time.	➔ Generally submits homework on time but may occasionally be late.	➔ Submits homework somewhat late on a regular basis.	➔ Frequently submits homework late.	➔ Consistently submits homework/ assignments late.