National School Curriculum

INSTRUCTIONAL GUIDE FOR BUSINESS MATHEMATICS

CLASS: XI-XII

School Curriculum Division
Department of School Education
Ministry of Education and Skills Development
Royal Government of Bhutan

“Your parents, relatives, and friends would be very proud of what you have achieved. At your age, to have completed your studies is your personal accomplishment. Your knowledge and capabilities are a great asset for the nation. I congratulate you for your achievements.

Finally, your capabilities and predisposition towards hard work will invariably shape the future of Bhutan. You must work with integrity, you must keep learning, keep working hard, and you must have the audacity to dream big.”

- His Majesty Jigme Khesar Namgyel Wangchuck

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Foreword

COVID-19 has caused unforgiving disruptions in public education all over the world, and brought about threats of fragmentation in the society due to disparities in accessibility and connectivity in many systems. In Bhutan too, continuity of education and learning has been severely affected as a result of sporadic nationwide school closures, restrictions and health protocols. The disruptions exposed the limitation of the existing ideologies and practices in education. This has deprived children living in poverty worldwide, who rely on the physical settings of their schools for educational materials and guidance, of the learning and other essential educational services. Cognizant of the global trend to embrace competency based learning as education for the 21st century, the current priority of the Government is to transform the knowledge and textbook based learning to competency based learning through open source and experiential learning.

In the new normal education, human interaction and well-being is a priority. Technology, particularly digital technology that enables communication, collaboration and learning across distance, is a formidable tool though not a panacea but a source of innovation and expanded potential. As we embrace this exceptional opportunity to transform education, it is imperative to reimagine the organisation of our educational institutions and learning environments. In the post COVID 19 era, we must prioritise the development of the whole person, not just the acquisition of academic knowledge. Inspiration for the change can be drawn from the 1996 Delors report, learning the treasure within. Its four pillars of learning as “learning to know”, “learning to do”, “learning to be”, and “learning to live together” are the current global ethos of teaching and learning. Therefore, curricula must be increasingly perceived as an integrated, themes based and problems based orientation that allows learners to develop a strong base of knowledge about one’s self and about the world, and find purpose in life and be better able to participate in social and political milieu.

The National School Curriculum is, not just a mere response to the pandemic, but also culmination of the curriculum reform work for the last four years by the erstwhile Royal Education Council. It is an attempt to transform education from the teaching of “what” to learning of “how” and “why” towards empowering learners with the transversal competencies and the 21s t century skills, and preparing them to be lifelong learners. In tandem with this initiative, we are optimistic that the paradigm shift in Mathematics education orients our education process in empowering the young generation with the Mathematics mind-set and disposition, and skills towards nurturing nationally rooted and globally competent citizens.

With this guide, we are optimistic that our learners and teachers are ushered through a life enriching experiential Mathematics education.

Tashi Delek

(Karma Galay**)**
DIRECTOR GENERAL

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Introduction

The 21st Century Education framework emphasises on the theme-based learning approach that broadens opportunities for experiential learning contextualised to the learner's physical, social, political, economic, spiritual and cultural setting. This requires learning through active engagement of learners. The role of teachers therefore, is transformed from knowledge transmitter to facilitation, guide, evaluator, researcher and motivator.

The conventional education system is predominantly knowledge based and examination centred. This system comprises the development of psychomotor and affective domains of learning thereby affecting the holistic development of students.

Despite the devastating effect caused by COVID-19 pandemic, it presented scopes for creation, innovation, generally perceived as more efficient and effective in work and social activities. The pandemic situation explicated that the old ways of working, teaching and learning, and lifestyle have limitations. Consequently, new normal ways of how we work and live, teach and learn must be critically analysed and embraced.

Therefore, the education system needs to be transformed to meet contemporary requirements. Students should learn to critically filter information that is flooded on the internet. Classroom instruction should facilitate learners to construct knowledge, develop essential skills and values which are crucial for learners to realise their potential towards becoming locally rooted and globally competent citizens who would contribute towards making a just and harmonious society. Accordingly, classroom instruction from teacher centred to learner centred calls for the following adjustment, or even the overhaul of a few practices.

i. Reduction of learning content to facilitate deep learning as opposed to the width of the teaching through the active engagement of students.

ii. Integration of ICT as tools and ends of learner's education. The use of multimedia and ICT software is commonly utilised in teaching and learning as innovation to introduce variation in stimuli and sustain learner's interest and zeal in learning.

iii. Adoption of theme-based learning content, which facilitates to broaden the horizon of learning beyond the four walls, and stimulates the transfer of learnt concepts to the learner's immediate environment. This arrangement makes students aware of the realities of the social, political, economic and cultural practices and ethos of the society. Being aware of the immediate environment of the scopes and challenges, students are sensitised to the opportunities and issues, which may need attention for a better future for the society.

iv. Consideration to ground the curriculum design and instruction approaches the epistemological theories is imperative to facilitate deep learning as opposed to factual learning. However, the selection and use of them is subject to the nature of the respective subject. For instance, constructivism is more apt for science, while connectivism is relevant for languages and ICT curricula.

v. Active engagement of students is imperative for competency-based education and learning. Inevitably, summative assessment has limitations in gauging the progressive development of the learner. This is achieved objectively by the use of the continuous formative assessment (CFA). However, if summative assessment evidence is used to provide feedback to help students in learning, it can serve as one of the techniques of CFA.

Purpose of the Instructional Guide

This instructional guide provides a suggestive direction to the teachers to facilitate them to transform the classroom instruction to the contemporary requirements stated above. The content of the instructions in the guide are aligned with the mathematics curriculum framework with references to the existing textbooks.

The instructional guides are developed to achieve the following objectives:

i. Facilitate learning anywhere, any time with the learner being responsible for the learning.

ii. Facilitate deep learning with awareness and sensitivity of the realities of the world around.

iii. Strengthen competency based learning and experiential learning to foster sensitivity of realities of life and environment.

iv. Strengthen blended learning and flip classroom with multimedia, digital pedagogies and ICT devices and websites as the tools and learning content.

v. Guide parents in facilitating learning of their children.

vi. Inspire teachers to assume the roles of facilitation, guide, motivator and evaluator.

vii. Helps in the prioritisation of learning content with emphasis to create time and space for active engagement of learners. Facilitate the use of CFA for learning through objective observation and guidance.

The effective and efficient use of this guide is subject to the nature of the topic(s) and the target class.

ORGANISATION OF THE INSTRUCTIONAL GUIDE

This is the main part of the instructional guide. It contains suggested approaches to teaching to guide students to achieve the desired competency (ies) through the identified topic(s).

a. Broad theme /Strand/Chapter/Topic

Under this heading, the topic/topics under one strand or under different strands that can be addressed together is/are listed. The topics are taken from the framework and not from the textbooks.

b. Introduction, Utility and Scope

Knowing history, utility and scope of topic(s) or concept (s) may provide an inspiration to learn certain things. It arouses curiosity, interest, and motivates the learners if they know why and how something was invented or discovered. Due to these reasons, the teachers are expected to

explore and express the history, utility, and scope of the topic(s). It is advisable to do so while introducing the topic or concept because if students know the history and utility of what they learn, learning becomes much more engaging and captivating.

c. Competency (ies)

Under this heading, the main competency (ies) associated with the topic(s) is/are listed.

d. Objective(s)

Lesson objective(s) which are directly linked with the competency is/are given.

e. Essential Skills/Process

When students discuss and do mathematics, they use various skills and at the same time learn new skills. All learning processes are intended to facilitate students to acquire and develop certain skills. Each lesson in this guide contains few suggestive skills for each topic. However, since the list of essential skills for each topic is not exhaustive, teachers may explore helping students to acquire or develop some other essential skills through the topics covered.

f. Learning Experiences

In each lesson, learning experiences linked to the objectives which are further linked to the competency (ies) are suggested. These suggested learning experiences are to provide directions for the teachers to leverage the lesson to enable students to achieve the desired competency (ies). It can be used for both face to face and virtual mode to make the teaching learning more authentic. Both indoor and outdoor learning experiences (experiential learning) with possible cross pollination and thematic approaches are suggested for each lesson.

Teachers are expected to use the approaches of cross pollination (multidisciplinary), thematic (intra and inter topic(s)/concept(s)), place-based education pedagogies, project based learning, and flip classroom to offer greater flexibility and opportunities to enable students to generate new knowledge and associated skills. Students should also be provided opportunities to connect, communicate, and represent mathematical ideas and be given avenues for thinking divergently and reflecting on their learning.

g. Assessment

Achievement of learning shall be recorded based on bands of achievement for all students in all classes from IV to XII. The evidence from assessment is to identify individual learning needs, design, and deliver appropriate interventions to support students falling in the beginning and approaching category and to further provide appropriate support to the ones falling under the higher bands.

Teachers shall assess the objectives in a cumulative manner and ensure that competency (ies) are achieved at the end of the lesson. In each class, marks obtained for each strand from the formative assessment can be converted using appropriate conversions for summative purposes.

$$CA\ Marks = \frac{Sum\ of\ scores\ obtained\ of\ all\ competencies}{Total\ scores\ of\ all\ competencies} \times CA\ for\ the\ term.$$

- Performance Task

Each lesson consists of one or two suggestive performance tasks. The performance tasks are intended to assess the students' achievement of the desired competency (ies) through that particular lesson. Through performance tasks students can demonstrate knowledge and skills that they have acquired or developed, and how they can apply the knowledge and skills in real situations. Teachers should try and design performance tasks where students will be able to use the immediate objects around them to show their ability to transfer/apply the learning and assess higher order thinking.

The performance tasks should be assessed through Performance-based Assessment (PBA) wherein not only knowledge and skills are assessed but the ability to apply the knowledge and skills are also assessed.

- Reflective Questions

  Suggestive reflective questions that can be included in a performance task are mentioned below each performance task. Also, in some cases, reflective questions are mentioned after the completion of the performance tasks. These reflective questions are suggested, mainly, to help students to reflect on their learning and make personal connections to the topics being learnt.

h. Resources

All resources required for the suggested learning experiences and tasks are listed under each lesson both for face to face and virtual mode. Some lengthy worksheets are provided as annexure at the end.

i. Annexure

For the effective implementation of the NSC, certain worksheets are suggested for each lesson and annexed at the end of each lesson. However, teachers are expected to use it or further develop similar authentic worksheets for the teaching learning purposes.

Instructional Guide
Class XI
(BMT)

Topic: XIB-A1 Determinants *[350 minutes]*

Introduction

The concept of a determinant emerged as a means of determining, hence the term 'determinant,' whether a system of equations has a unique solution. Interestingly, the attribution of the determinant goes to the Japanese mathematician Seki Takakazu, who introduced it around the beginning of the 1700s. Seki Takakazu is colloquially known as the 'Japanese Newton.' In fact, he independently worked on infinitesimal calculus, similar to Newton and Leibniz, who were his contemporaries.

It's important to note that earlier forms of the determinant were utilised as early as the third century BC by Chinese scholars. The term 'determinant' was coined by Gauss at the start of the 1800s in the context of the discriminant of homogeneous polynomials. It was only fifty years later, in 1858, that matrices themselves were introduced by Cayley in his work "A memoir on the theory of matrices." Eight years before that, Sylvester had introduced the term 'matrix,' but he considered a matrix as an array of numbers from which determinants of its submatrices (nowadays called minors) could be derived. In contrast, Cayley was the first to describe how matrices could be treated as algebraic objects that can be added and multiplied together.

For further information click here: History of Determinant

Utility and Scope

Learning about determinants has several utility and scope applications in real-world fields such as technology and business. Here are five important applications:

1. **Linear Algebra in Computer Graphics and Gaming**:
Determinants play a crucial role in linear algebra, a fundamental concept in computer graphics. They are utilised in transformations, such as scaling, rotation, and translation, which are essential in creating realistic images in computer graphics and gaming.

2. **Data Analysis and Machine Learning**:

Determinants are employed in various algorithms and models within the realm of data analysis and machine learning. They are utilised in techniques like Principal Component Analysis (PCA) to reduce the dimensionality of datasets and extract essential features for predictive modelling.

3. **Cryptographic Systems**:

Determinants are applied in the field of cryptography, particularly in the development of cryptographic algorithms and systems. Matrix operations involving determinants are used in encryption and decryption processes, contributing to the security of data in technologies such as secure communication protocols.

4. **Economics and Business Decision-Making**:

Determinants find applications in economic modelling and business decision-making. In economics, they are used in input-output models and Leontief models, providing insights into the interdependencies of economic activities. In business, determinants can be employed in optimization problems, resource allocation, and risk assessment.

5. **Network Analysis and Optimization**:

Determinants play a role in network analysis and optimization problems. In technology and business, understanding determinants is valuable in analysing complex systems, such as transportation networks or communication networks, to optimise efficiency and resource allocation.

Explore the web link: utility and scope of determinant for more information.

A. Competency

- Demonstrate the understanding of determinants and apply them to solve real-world problems involving a system of equations with two or three variables.

B. Objectives

- Evaluate the determinant of orders two and three.
- Find the minors and cofactors of every element of a determinant.
- Examine the conditions for consistency for a system of equations.
- Solve the system of equations in two and three unknowns related to real-life scenarios.

C. Essential Skills/Processes

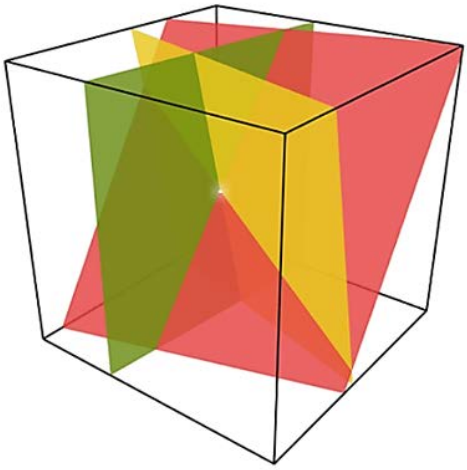
- Reading Comprehension

- Information Recall
- Knowledge Application
- Problem Solving

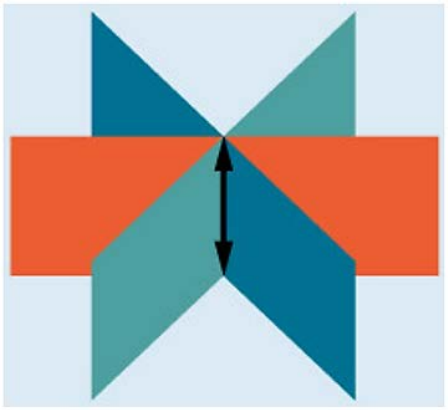
D. Learning Experiences

- Project the video from the web link Introduction of Determinant and allow students to write their understanding and share it in class.
- Demonstrate and explain how to compute determinants of order 2 and order 3 during interactive teaching.
  - Refer to the web links Determinant Order 2 and Determinant Order 3 on expanding determinants of order 2 and order 3 respectively.
  - Assign similar questions to test their understanding of expanding determinants of order 2 and 3.
- Display a video given in the web link: Minor Cofactor Introduction to find minor and co-factor of determinants of order 2 and 3.
  - Project a series of questions on the screen and allow a few students to take turns to complete on the board.
- Allow students to solve a system of equations with two variables using Crammer's rule. Assign a question to each group and facilitate their presentations, aligning with the discussed rubrics.
  - After each presentation, allocate time for question and answer sessions. suggestion - allow students to explore the following web links:
    - ➢ https://www.youtube.com/watch?v=LprQ_Id-8hE (Crammer's rule for a system of equations with two variables).
    - ➢ https://www.youtube.com/watch?v=vXqlIOX2itM (Crammer's rule for a system of equations with two variables).
    - ➢ https://www.youtube.com/watch?v=Ot87qLTODdQ&t=5s (Crammer's rule for a system of equations with three variables).
- Reflective question: Provide two systems of linear equations on the board.
  - Instruct students to solve the linear equations using all the methods they have learned (substitution, elimination, comparison and Cramer's rule) to solve the systems.
  - Allow them to write a reflection on which method they would use to solve each kind of system with justification.

- Explain the meaning of the inconsistent system in terms of the solutions of the system and its geometrical meaning using relevant software like *GeoGebra* or any other 3-D graphing software (refer the web link to know how to graph the planes on GeoGebra: https://www.youtube.com/watch?v=--gpbRvnIFE).
  The consistent and inconsistent systems can be graphically shown as below:



Consistent system with unique solution



Consistent system with infinite solutions

Inconsistent system with no solution

- Display a flow chart explaining the steps to check for consistency and solve for an infinite number of solutions.
  - Allow students to write their own interpretation of the flowchart and let volunteers to read their interpretations to make sure all the students are on the same page.

- Share the web link: https://www.youtube.com/watch?v=Ix8Nne-a-KQ with the students to further amplify their understanding on the conditions of consistency of a system of linear equations using the determinants method.
- Refer BHSEC Mathematics book II to assign relevant questions to the students.

E. Assessment

Performance Task 1

Assign the questions from the worksheet Worksheet on Determinant and evaluate based on the rubrics.

Performance Task 2

Place Based Approach

Design a place-based task where students will need to write a real life situation into a system of linear equations, and solve using the determinant method.

Sample:

- Put the students into groups of three members each.
- Discuss with the shopkeeper in advance to ensure that the students are not informed about the price of each item.
- The groups will visit any nearby shop or school canteen, and purchase three different items as follows:
  - 1st Member: purchase any number of three different types of items worth Nu 10.
  - 2nd Member: purchase a different number of the same types of items purchased by 1st member of worth Nu 15.
  - 3rd Member: purchase a different number of the same types of items purchased by the 1st member worth Nu 20.
- The group will come together and formulate the situation into a system of equations. Then they will solve using a determinant method to determine the price of each item.

OR

- Divide the students into four to five groups.
- Pack three different items each, such as pencils, erasers and sharpeners, in three plastic bags. The number of each item can be different in each plastic bag.

- Provide three plastic bags to each group, and remind them not to open the bags.
- Instruct the students to go to the science lab, weigh their bags, and determine the weight of each item by solving a system of equations using the determinant method.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure.

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum, Mathematics
- Introduction of determinant- History of Determinant
- Utility and scope- utility and scope of determinant
- Definition of determinants- https://www.youtube.com/watch?v=YFGTpSkfT40
- Determinant of order 2- Determinant_Order_2
- Determinant of order 3- Determinant_Order_2
- Minors and Cofactors- Minor_Cofactor Introduction
- Crammer's rule for 2x2 - https://www.youtube.com/watch?v=LprQ_Id-8hE
- Crammer's rule for 2x2 - https://www.youtube.com/watch?v=vXqlIOX2itM
- Crammer's rule for 3x3 - https://www.youtube.com/watch?v=Ot87qLTODdQ&t=5s
- Consistency of a system - https://www.youtube.com/watch?v=--gpbRvnIFE
- Determinant worksheet - Worksheet on Determinant

G. Annexure

i) Template to record assessment

Strand(s): XIB-A1		**Topic(s): Determinant**			
Competency: Demonstrate an understanding of determinants, and apply in solving real-life problems involving a system of equations in two or three variables.					
Name of the student	**Level of achievement**				
	Beginning	Approaching	Meeting	Advancing	Exceeding

ii) Assessment Rubrics Sample

*(**Note:** This is just a sample of one of the assessment tools. Teacher must design other appropriate assessment tools as per the competency and performance tasks)*

	Beginning	Approaching	Meeting	Advancing	Exceeding
Completion	Several of the problems are not completed.	About three of the problems are completed.	About two of the problems are completed.	One of the problems is not completed.	All problems are completed.
Neatness and Organization	The work appears sloppy and unorganised. It is hard to know what information goes together.	The work is presented in an organised fashion but may be hard to read at times.	The work is presented in a neat and organised fashion that is usually easy to read.	The work is presented in a neat and organised fashion that is mostly easy to read.	The work is presented in a neat, clear, organised fashion that is easy to read.
Working with others	Students did not work effectively with others.	Students cooperated with others, but needed prompting to stay on-task.	Student was engaged with a partner/group but had trouble listening to others and/or working cooperatively.	Student was engaged with a partner/group, listening to some suggestions of others and working quite cooperatively throughout the lesson.	Student was engaged with a partner/group, listening to suggestions of others and working cooperatively throughout the lesson.
Mathematizing the problem	The student did not attempt to mathematize the problem	Student mathematics the problem mostly incorrectly	Students wrote part of the system of linear equations correctly.	Students wrote most parts of the system of linear equations correctly.	Student wrote the whole system of linear equations accurately
Accurateness of application	Student did not attempt to solve	Student solved only 1	Student solved only 2 determinants	Student solved only 3	Student solved all the 4

of Cramer's rule	the system of linear equations	determinant out 4 correctly	out of 4 correctly	determinants out of 4 correctly	determinants correctly
Interpretation of the solutions	Student did not attempt to interpret the solutions	Student's interpretation of the solutions was mostly incorrect	Student's interpretation of the solutions was partially correct	Student's interpretation of the solutions was mostly correct	All the interpretation of the solution was correct

Topic: XIB-A2 Logarithm *[400 minutes]*

Introduction

The concept of logarithms was introduced by the Scottish mathematician John Napier in 1614 in his work "Mirifici Logarithmorum Canonis Descriptio." Napier's logarithms, based on the natural logarithm with the base e, aimed to simplify complex arithmetic calculations. Recognizing the practical utility of logarithms, English mathematician Henry Briggs collaborated with Napier to develop logarithmic tables. In 1624, Briggs published "Arithmetica Logarithmica," which included base-10 logarithms, making calculations more intuitive. Logarithmic tables became essential tools for scientists, engineers, and mathematicians, facilitating complex computations by reducing them to simpler addition and subtraction operations. The widespread use of logarithms persisted through the centuries, finding applications in fields such as navigation, astronomy, and engineering, until the advent of electronic calculators in the mid-20th century. Despite the decline in everyday use, logarithms continue to be fundamental in advanced mathematics and scientific disciplines.

For more information, https://www.cuemath.com/algebra/logarithms/

Utility and Scope

The common application of the logarithmic function is to find the compound interest, exponential growth, and decay, to find the pH level of substance, to know the magnitude of an earthquake, etc. Logarithms are used to know the magnitude of earthquakes. Logarithmic functions are used to measure the pH level of chemicals where pH = log10 [H+]. The exponential growth can be measured using the logarithmic functions. For example, the money growth rate at a given fixed interest rate can be determined. For example, you have Rs.40,000 in your bank account at an interest of 4%. With a logarithm, you may know when your money will reach Rs.42,000. Logarithms are used in specific calculations where multiplications are converted into additions. Logarithms are used to determine radioactive decay.

sources: https://application of log function.

(Teachers can explore further using the link:
https://numberdyslexia.com/applications-of-logarithms-in-real-life/).

A. Competency

- Exhibit an ability to show the relationship between logarithmic and exponential expressions, and apply the properties and laws of logarithm in solving problems.

B. Objectives

- Establish connections between exponent laws and logarithmic forms.
- Develop a comprehensive understanding of the meaning and concept of logarithms.
- Apply theorems and laws of logarithms proficiently to solve logarithmic problems.

C. Essential Skills/Processes

- Information Recall
- Interpreting Information
- Analysing
- Problem Solving

D. Learning Experiences

- Recapitulation: Let students revise the laws of exponents learned in lower classes. Refer to BHSEC Mathematics Book I, Understanding Mathematics textbook for class IX or the web link https://www.exponent-rules/ .

- Understand the definition of the logarithm of a number using the link https://www.defination of log (pause the video whenever it requires elaboration). Allow students to refer to BHSEC Mathematics Book I or share the link https://www.defination of log.

- Discuss the laws and theorems of the logarithm (excluding change of base) with an illustration. Refer to BHSEC Mathematics Book I or explore the suggested links:
  - https://youtu.be/bowrJ31ojOg
  - https://youtu.be/7sh2_AsKCyA
- Demonstrate and solve logarithmic equations using the laws of logarithms. Additionally, explore the following links for further insights into the laws of logarithms:
  - laws of logarithm
  - https://logarithm-questions/

- Demonstrate the solution of problems involving exponential such as $1.3^n = 15,\ 13.01^x = 29,$ etc using logarithms.

- Encourage students to explore the real-world applications of logarithms and present their findings. Refer to the link https://youtu.be/KNEdVMCQIYY for additional insights.

E. Assessment

Performance Task 1

Practise some of the questions to show the relationship between logarithmic and exponential expressions by referring to BHSEC Mathematics Book I.

Practice questions referring to BHSEC Mathematics Book I (To check the understanding of laws of logarithm and its properties) or practise some of the questions from the link https://www.Logarithm-Problems.php.

Performance Task 2

In the group (group activity), students discuss the application of logarithm in practical situations. Browse the link https://youtu.be/UQm9JaY7MLQ .

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIB-A1.

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction of logarithms- https://www.cuemath.com/algebra/logarithms/
- Utility and scope- https://application of log function
  https://numberdyslexia.com/applications-of-logarithms-in-real-life/
- Exponent test- https://www.exponent-rules/
- Definition of logarithms- https://www.defination of log
  https://youtu.be/TOoXH_IwUx0
- Laws and properties of logarithms- https://youtu.be/bowrJ31ojOg
  https://youtu.be/7sh2_AsKCyA
- Questions on logarithms- https://youtu.be/N0e7SdWaSUw
  https://logarithm-questions/
- Application of logarithms https://youtu.be/KNEdVMCQIYY
- Practice questions https://www.Logarithm-Problems.php.

https://youtu.be/UQm9JaY7MLQ

G. Annexure

Refer XIB-A1 for template to record achievement

Topic: XIB-A3 Binomial Theorem *[400 minutes]*

Introduction

The binomial theorem was known since at least the 4th century BC when Greek mathematician Euclid mentioned the special case of the binomial theorem for exponent 2. There is evidence that the binomial theorem for cubes was known by the 6th century AD in India. However, one solution known as Pascal's triangle was determined in China as early as the 13th-century by the mathematician Yang Hui. His solution was independently discovered in Europe 300 years later by Blaise Pascal whose name has been permanently associated with it since.

The binomial theorem is an algebraic method of expanding a binomial expression. Essentially, it demonstrates what happens when you multiply a binomial by itself (as many times as you want). For example, consider the expression $(4x+y)^7$. It would take quite a long time to multiply the binomial (4x+y) out seven times. The binomial theorem provides a shortcut or a formula that yields the expanded form of this expression.

Source: https://www.cuemath.com/algebra/binomial-theorem/

Utility and Scope

Higher mathematics: The binomial theorem is used in the calculation and higher mathematics to solve highly complex and nearly impossible calculations. Many of the equations, laws, and theories suggested by Sir Albert Einstein used a lot of binomial theorems.

Internet protocol (IP): The random generation and distribution of IP addresses in electronic devices, such as the Internet of Things (IoT), such complex issues are easily done using the power of binomial theorem.

Architecture: The binomial theorem is of great use in Architectural Space. It allows Engineers to calculate and estimate the various magnitudes which make their work a lot easier and it also has a positive impact on costs which would be rather high had it not been for the method of binomial theorem. This results in a faster construction workflow and the contractors have a bigger margin of profit due to the minimization of costs.

Probability: One major field in which the theorem plays a crucial role is in probability learning. The concept is called *binomial probability*, and it helps us in finding the probability of *x* successes in *n* repeated trials which has two possible outcomes: like the probability of tails 7 times when you toss a coin 15 times.

For extra information, explore the web link: Applying Binomial Theorem

A. Competency

- Display an ability to expand expressions with positive indices using the concept of the binomial theorem.

B. Objectives

- Understand the concepts of binomial theorem.
- Expand any binomials with positive exponents using the binomial theorem.

C. Essential Skills/Processes

- Defining key Concepts
- Information Recall
- Problem Solving
- Analysing

D. Learning Experiences

- Recapitulation: Students expand $(a + b)^2$, $(a + b)^3$ and $(a + b)^4$ using the concept studied in classes IX and X.
- Students work in pairs or groups to understand the meaning of factorial notation, the symbol ${}^nC_r$ and results associated with the ${}^nC_r$ symbol. Refer to the BHSEC Mathematics Book I or the suggested web links:
  - https://www.youtube.com/watch?v=8Ti0mCEpli4
  - https://www.youtube.com/watch?v=x83Yx6jffik
- Investigate the use of Pascal's triangle in expansion of binomial expression with positive indices by referring to BHSEC Mathematics Book I or the web link: https://www.youtube.com/watch?v=LiFeg_xKi3I.
- Demonstrate to the students how to apply Pascal's triangle in the expansion of the following expressions:
  - $(x+y)^2 = x^2 + 2xy + y^2$

- o $(x-y)^2 = x^2 - 2xy + y^2$
- o $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
- o $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$
- o Allow students to expand the following two expressions $(a+b)^4$ and $(a+b)^5$ using Pascal's Triangle.

- Deduce the Binomial Theorem by studying the patterns of expansion using Pascal's Triangle.
  - o Share the link: properties of binomial theorem which contains notes on the properties of binomial expansion with positive indices.
  - o Allow students to write the notes and solve questions to expand binomial expressions with positive indices using Binomial Theorem. You can refer to BHSEC Book I, Exercise 2(a).

- Reflective question: How can you use the binomial theorem to solve the powers of the type: 105^5 or 27.5^6?

E. Assessment

Performance Task 1

Design a quiz referring to the sample in the link https://www.bionomial-quiz.com to assess the students' knowledge of binomial theorems in expansions of expressions with integral powers.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIB-A1.

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction of Binomial Theorem-https://www.cuemath.com/algebra/binomial-theorem/
- Utility and scope-Applying Binomial Theorem
- Meaning of factorial notation- https://www.youtube.com/watch?v=8Ti0mCEpli4
- Binomial Theorem- https://www.youtube.com/watch?v=x83Yx6jffik
- Properties of binomial theorem: properties of binomial theorem
- Expansion of binomial-https://www.youtube.com/watch?v=LiFeg_xKi3I

- Quiz on Binomial Theorem- Binomial Theorem quiz

G. Annexure

Refer XIB-A1 for the template to record achievement

Topic: XIB-A4 Banking *[250 minutes]*

Introduction

The term bank is derived from the Old Italian word banca or from the French word banque both mean a Bench or money exchange table. Similarly, in the olden days, European money lenders or money changers used to display coins of different countries in big heaps (quantity) on benches or tables for the purpose of lending or exchanging. However, some experts recount the history of modern banking, asserting that The Bank of Venice, established in 1157, holds the distinction of being the first public banking institution.

In the same way, Bhutan's commercial bank, Bank of Bhutan (BOB), was established on 28 May 1968 as a joint venture with the Chartered Bank of India, Australia, and China, which owned 25% of the bank. In 1970, this share was transferred to the State Bank of India. The bank was restructured in the year 1971. Government departments were required to deposit all of their accounts with this bank to ensure that it had sufficient funds until 1982, when the Royal Monetary Authority was established. Since then, the Bank of Bhutan has been the retail banking agent for the Royal Monetary Authority. The Royal Monetary Authority was established in 1983; before then, Bank of Bhutan was the country's central bank.

Source: Introduction to banking

Utility and Scope

Banks are closely linked with our everyday lives and activities. Drawing salaries, paying bills, buying homes, building up savings and taking out loans all involve transactions with banks. Businesses also rely on the banking system for settlement of their transactions and meeting other financial needs. [Visit a nearby bank if possible].

A. Competency

- Exhibit an understanding of different types of bank accounts in Bhutan and calculate its interests.

B. Objectives

- Understand different types of deposit accounts - saving, fixed, current, recurring and loan.

- Compute interest on saving deposit account and fixed deposit.

C. Essential Skills/Processes

- Conceptualising
- Applying
- Recalling Information
- Analysing

D. Learning Experiences

- Introduce different types of deposit accounts, including Current/Savings Accounts, Recurring Deposits, and Fixed Deposits. It's important to note that account types may vary among banks. To explore the offerings, visit the websites of banks in Bhutan such as Bank of Bhutan, Bhutan National Bank, Bhutan Development Bank, Tashi Bank, and Druk Punjab National Bank. Check their websites for details on various deposit accounts, and explain to students. Some of the most common types include
    - **Current Deposit Account**: A current deposit account is a deposit account for business owners, large institutions, entrepreneurs, and traders, traders who require frequent transactions. For instance, Central School Budgets are deposited in their respective current accounts so that any amount can be withdrawn to procure items at any time. Current accounts do not accrue interest and necessitate maintaining a minimum balance for operation.
    - **Savings Deposit Account**: A savings Deposit account is a regular deposit account, where an account holder has the freedom to deposit or withdraw at any time. Savings accounts earn a minimum interest rate depending on the banks. Saving interest rates are usually 5%. It does not have fixed tenure. Unlike current accounts, saving accounts have withdrawal limits.
    - **Recurring deposit accounts**: In a recurring deposit account, individuals are required to deposit (invest) a fixed sum of money regularly, usually on monthly, quarterly, half-yearly, or yearly basis. The interest rates offered are slightly higher compared to savings accounts. In the event of premature withdrawal, a penalty of a certain percentage, usually 0.5%, will be charged. The maturity period for a RD can range from six months to ten years. Depending on the banks, there are different types of RDs. Visit the websites of different Bhutanese Banks and show the different types of RDs and their

interest rates. From this, students should get ideas to save money in the future. (Calculation of maturity amount of RD will be taught in class XII under the topic "Future value of an annuity")

- **Fixed Deposit Accounts:** Fixed deposit accounts are usually chosen to earn a higher rate of interest compared to saving accounts and recurring deposits. However, in this account, one must deposit a specific sum of money for a fixed time period (tenure). Visit the websites of different Bhutanese Banks to compare the different interest rates offered on fixed deposits.
- Explore more on the different types of accounts and disseminate the idea of saving in the future.

- **Calculating the interest on a savings account:** In Bhutanese banks, the interest rate for a savings deposit is usually 5% p.a, calculated using simple interest at the end of each month. Prior to the teaching of interest calculation, refer to the following links to learn about savings account in Bhutanese banks:
  - BoB: https://www.bob.bt/interest-service-charges/
  - BNB: https://bnb.bt/interest-rates/
  - BDBL: https://bdb.bt/retail-banking/
  - T-Bank: https://www.tbankltd.com/savings-account
- Normally, the interest for a savings account is calculated on a daily basis based on the balance at the end of the day. Here is an example to demonstrate how interest is calculated:
  - For example, Mr. Dorji has Nu 100,000 in his account on Day 1. He withdraws Nu 50,000 after 7 days. And then deposits Nu 30,000 on the 14th day and thereafter, there are no transactions. His transaction records can be displayed as shown below:

Date	Particular	Withdrawal (Debit)	Deposit (Credit)	Balance
01-01-2021	Opening Balance			100,000
07-01-2021	Transferred to Ms. Chimi	50,000		50,000
14-01-2021	Salary		30,000	80,000

17-01-2021	Grocery Shopping	5000		75,000
31-01-2021	TADA		1000	76,000

- Assuming the rate of interest as 5%, the interest he has earned for the month of January can be calculated as follows (Interest is calculated using the simple interest formula: I=Prt):
  - From 01-01-2021 till 06-01-2021, the outstanding balance was Nu 100,000. Thus, the interest will be calculated on Nu 100,000 for the period of 6 days: $100,000 \times 0.05 \times \frac{6}{365} = Nu\,82.19$
  - From 07-01-2021 till 13-01-2021, the outstanding balance was Nu 50,000. Thus the interest will be calculated on Nu 50,000 for the period of 7 days:

    $$50,000 \times 0.05 \times \frac{7}{365} = Nu\,47.95$$
  - From 14-01-2021 till 16-01-2021, the outstanding balance was Nu 80,000. Thus the interest will be calculated on Nu 80,000 for the period of 3 days:

    $$80,000 \times 0.05 \times \frac{3}{365} = Nu\,32.87$$
  - From 17-01-2021 till 30-01-2021, the outstanding balance was Nu 75,000. Thus, the interest will be calculated on Nu 75,000 for the period of 14 days: $75,000 \times 0.05 \times \frac{14}{365} = Nu\,143.84$
  - At the end of 31-01-2021, the outstanding balance was Nu 76,000. Thus, the interest will be calculated on Nu 76,000 for one day:

    $76,000 \times 0.05 \times \frac{1}{365} = Nu\,10.41$ Thus, the total interest earned for the month of January will be $82.19 + 47.95 + 32.87 + 143.84 = Nu\,306.85$

- In the bank statement, interest will be reflected as shown below:

Date	Particular	Withdrawal (Debit)	Deposit (Credit)	Balance
01-02-2021	Opening Balance		306.85	76,306.85

- Provide about three examples similar to the above example for practice.
- Demonstrate the calculation of interest of fixed deposits. Most banks calculate interest and maturity amount using simple interest.
- Visit the websites of respective banks to view the fixed deposit interest rates for familiarisation. Usually, banks display their interest rates in the following format. Let's illustrate with an example:

Term	0-6 months	6 months to less than 1 year	1 to less than 2 years	2 to less than 3 years	3 to less than 4 years	4 to less than 5 years	5 to less than 7 years	More than 7 years
Rates	2.5	3	6	6.5	7.25	7.5	7.75	8

- In case of premature closure of the deposit (closing before the agreed term), the banks deduct interest of 1% of the prevailing interest rate.
    - For example, Ms. Wangmo invested Nu 300,000 in a fixed deposit for a period of 5 years.
        - ➢ Case I: What amount will she receive if she withdraws after 5 years?

          In this case, she completes the tenure, thus, she will receive:

          $$A = 300,000 + 300,000 \times 0.0725 \times 5 = Nu\,408,750.$$

    - Verify the answer using any of the banks' calculators. BNB has its own fixed deposit calculator on its bank's website. Refer https://www.bnb.bt/calc/ .
        - ➢ Case 2: What amount will she receive if she closes her fixed deposit account after 4 years and 6 months?

          In this case, she is closing her account after 4 years. The prevailing interest rate at that term is 7.5%, but because of early closure, the bank will deduct 1% as a penalty. Thus, the interest rate she will get is $7.5\% - 1\% = 6.5\%$.

          Therefore, the total amount she will receive will be

          $$A = 300,000 + 300,000 \times 0.065 \times 4 = Nu\,378,000.$$

- Design about three similar questions for students to practise.

E. Assessment

Performance Task

Frame at least three questions to assess the understanding of different types of deposits and calculation of interest for saving and fixed deposits.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIB-A1.

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction of Banking Introduction to banking
- Saving Account in Bhutanese Bank:
  - BoB: https://www.bob.bt/interest-service-charges/
  - BNB: https://www.bnb.bt/savings-account/
  - BDBL: https://bdb.bt/retail-banking/
  - T-Bank: https://www.tbankltd.com/savings-account
- EMI calculator BOB -https://www.bob.bt/service-and-support/emi-calculator/

G. Annexure

Refer XIB-A1 for template to record achievement

Topic: XIB-B1 Sequence and Series *[400 minutes]*

Introduction

An arithmetic sequence and series is a simple pattern of numbers that has a constant difference between consecutive numbers. Some sources say that the Arithmetic sequence was first found in the Ahmes Papyrus which is dated 1550 BC. There is no specific and detailed history on the sequence and series.

In one of the schools of Germany, the students were asked by a teacher to find the sum of all integers from 1 to 100. One eight-year-old boy came up with the answer almost at once. It was Carl Fredrick Gauss (1777-1855), who later became a famous Mathematician. His strategies to find the sum of the integers leads to the development of the formula to add the sum of the Arithmetic series.

Geometric sequences have been found on Babylonia tables dating back to 2100 BC. Later, the detailed works of the geometric sequence were found in a book of Euclid of Alexandria in 300 BC. Euclid, the father of Geometry, was the main contributor to the theory of geometric sequence and series.

Source: https://prezi.com/hzq60rtpgfmt/sequences-and-series/ ;
https://www.math.toronto.edu/mathnet/questionCorner/arithgeom.html

Utility and Scope

The utility and scope of learning sequences and series extend across various disciplines, offering valuable insights and applications in mathematics and beyond. Some key aspects include:

1. **Mathematical Foundation:** Sequences and series provide a fundamental foundation in mathematics, serving as building blocks for more advanced topics. Understanding these concepts is essential for progressing in algebra, calculus, and other branches of mathematics.

2. **Financial Applications**: Arithmetic and geometric progressions find practical use in finance, particularly in calculating interest, loan repayments, and investment returns. Understanding these sequences is crucial for financial planning and decision-making.

3. **Computer Science**: Algorithms, data structures, and computational problems often involve sequences and series. Understanding these concepts is beneficial in designing efficient algorithms, analysing time complexity, and optimising code.

4. **Economic Modelling**: Economic models frequently incorporate sequences and series to represent trends, growth rates, and economic indicators. Analysing economic data and making predictions often rely on understanding and manipulating these mathematical concepts.

5. **Statistical Analysis**: In statistics, time series analysis utilises sequences to understand patterns and trends over time. This is crucial in fields such as economics, epidemiology, and environmental science.

Source:
http://www.tutor-homework.com/Math_Help/college_algebra/m6l4notes1.pdf

A. Competencies

- Analyse and categorise real-life sequences as either arithmetic or geometric progressions, and address problems by utilising the concepts of n^{th} term and the sum of n terms in a series.
- Utilise the concept of summation notation to evaluate the sum of a series involving natural numbers.

B. Objectives

- Relate the Arithmetic and Geometric Progressions with the patterns.
- Understand the concept of Arithmetic Progression (A.P) and Geometric Progression (G.P).
- Find n^{th} term (T_n) of A.P and G.P.
- Find the sum of the *n* terms of series (S_n) of A.P and G.P.

C. Essential Skills/Processes

- Reading Comprehension
- Information Recall
- Knowledge Application
- Problem Solving
- Analysing

D. Learning Experiences

- Discuss the general understanding of sequences and series by referring to the web link: https://www.youtube.com/watch?v=6VeUsMqonsE.

- Allow students to explore the link: ntroduction to A.P. which defines an arithmetic progression and discuss their understanding through presentation.
  - Examine the n^{th} term of an Arithmetic Progression by referring to BHSEC Mathematics Book I, or alternatively, refer to the video formula for nth term, which elucidates the derivation of the formula for the n^{th} term of an Arithmetic Progression.
  - Allow students to explore a few examples related to the topic. Refer to the web links for questions:
    - ➢ https://byjus.com/maths/arithmetic-progression-questions/
    - ➢ morechallenging questions on A.P.
  - Selected questions from Exercise 1(a), BHSEC Mathematics Book I and students will be directed to present their findings and understanding based on those questions.

- During interactive teaching, illustrate the process of finding the sum of n^{th} terms in an Arithmetic Progression (A.P), utilising the resources provided in the web link: https://www.youtube.com/watch?v=savKcKBC1AU.
  - To assess comprehension, assign students the task of solving specific questions from exercise 1(b), BHSEC Mathematics Book I.

- Commence the discussion on Geometrical Progression and its n^{th} term, providing illustrative examples.
  - Direct students to BHSEC Mathematics Book I for reference or utilise the web links for further insights: G.P Introduction, which features a video explaining the definition of Geometric progression with examples, and https://www.youtube.com/watch?v=3xbormMmuK4, which features a video solving examples of Geometric progression.
  - Foster active learning by encouraging students to apply their understanding through the practice of additional questions, as provided in BHSEC Mathematics Book I.

- Design a group work with questions to find the sum of Geometric Progressions (refer to BHSEC Mathematics Book I) and direct students to explore the resources provided below to learn how to solve the questions provided:

- https://www.youtube.com/watch?v=6G0-aqZsAMU contains video explaining the derivation of the formula of sum of n^{th} term in a GP.
- https://www.youtube.com/watch?v=J2XAcw-ZzdM contains video on solved examples of finding sum of n^{th} terms in a GP.
- https://www.youtube.com/watch?v=eRRjDCHlDwg contains a video explaining the formula of sum of infinite terms in a GP with examples.
- Allow the group to present their findings through powerpoint (ppt) and discuss it thoroughly.

- Reflective questions: Write the differences between Arithmetic and Geometric Progression. Give a relevant real life example each.

- Discuss on the topic summation notation referring to BHSEC Mathematics Book I. Provide a few questions related to the topics and allow students to solve.

- Demonstrate and explain how to find the sum of first *n* natural numbers referring to the web links : sum-natural-numbers and https://www.youtube.com/watch?v=tR9MeNGyGMA.

- Assign a pair task on finding the sum of the squares and cubes of the first n natural numbers.
  - Allow children to explore the links https://www.youtube.com/watch?v=aI0M4XRiz4I and sum of cubes-formula.
  - Select random pairs to present their learning and address any recurring doubts.

E. Assessment

Performance Task 1

Design competency questions and ask them to solve and find the solution.

a) Sonam's parents want to have Nu 500,000 saved up to pay for college by the time Sonam graduates from high school (16 years from now). If the investment plan they choose to invest in claims to yield 7% growth per year, how much should they invest today?

b) If a piece of machinery depreciates at a rate of 6% per year, what was its initial value if it is 10 years old and worth Nu 100,000?

c) The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of the 2nd hour, 4th hour, and nth hour?

What will Nu 500 amount to in 10 years after its deposit in a bank which pays an annual interest rate of 10% compounded annually?

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - https://prezi.com/hzq60rtpgfmt/sequences-and-series/ ,
- Introduction- history of binomial theorem
- Differences between Sequence and Series: https://www.youtube.com/watch?v=6VeUsMqonsE
- Arithmetic progression introduction - https://www.youtube.com/watch?v=gua96ju_FBk
- n^{th} term of A.P.: https://www.youtube.com/watch?v=IshPbnP3vk8
- Problems on A.P. - https://byjus.com/maths/arithmetic-progression-questions/
- Problems on A.P. - https://www.hitbullseye.com/AP Questions
- Sum of n th term of A.P. - https://www.youtube.com/watch?v=savKcKBC1AU
- Concepts on GP - https://www.youtube.com/watch?v=3xbormMmuK4
- Finding the term of G.P. - https://www.youtube.com/watch?v=TKtO3C9xpsQ
- Sum of first n term of G.P. - https://www.youtube.com/watch?v=6G0-aqZsAMU
- First n term of G.P. - https://www.youtube.com/watch?v=J2XAcw-ZzdM
- Sum of infinite G.P. - https://www.youtube.com/watch?v=eRRjDCHlDwg
- n term of natural numbers - https://testbook.com/maths/sum-of-n-natural-numbers
- n term of natural numbers - https://www.youtube.com/watch?v=tR9MeNGyGMA.
- Squares of natural numbers - https://www.youtube.com/watch?v=aI0M4XRiz4I
- cube of natural numbers - sum of cubes of natural numbers
- Technological gadgets for learning (smart phone, laptop, desktop...)

• Annexure

Refer X-A1 for a template to record achievement.

Topic: XIB-B2 Remainder and Factor Theorem *[200 minutes]*

Introduction

The remainder factor theorem is actually two theorems that relate the roots of a polynomial with its linear factors. The theorem is often used to help factorise polynomials without the use of long division. Especially when combined with the rational root theorem, this gives us a powerful tool to factor polynomials.

Etienne Bezout (1730-1783) was a French mathematician who is best known for his remainder theorem on the number of solutions of polynomial equations. The Factor theorem has its origin in the work of the 3rd-century-ad Chinese mathematician Sun Zi; the theorem was first given in 1247 by Qin Jiushao (1202-1261) who is also a Chinese mathematician.
Source: https://prezi.com/p/rrner7wc1atz/etienne-bezout/

Utility and Scope

The remainder theorem provides a more efficient avenue for testing whether certain numbers are roots of polynomials. This theorem can increase efficiency when applying other polynomial tests, like the rational roots test.
Source: https://khanacademy.fandom.com/wiki/Remaindertheoremofpolynomials

A. Competency

- Exhibit an ability to apply concepts of remainder and factor theorems in factoring quadratic and cubic polynomials.

B. Objectives

- Understand the concept of the remainder theorem, and evaluate the remainder when a polynomial is divided by a monomial.
- Understand the factor theorem and factorise a polynomial using the factor theorem.

C. Essential Skills/Processes

- Conceptualising
- Communication
- Analysing
- Reasoning
- Applying
- Problem Solving

D. Learning Experiences

- Demonstrate and explain the concepts of rational integral functions and the value of a function, as outlined in BHSEC Mathematics Book I.

- Empower students to deepen their understanding by exploring the Remainder and Factor Theorem through the following links:
  - https://www.remainder-factor theorem-video1(contains video explaining the Remainder and Factor Theorem).
  - https://www.remainder-factortheorem-video2 (contains videos explaining solved problems using Remainder and Factor Theorem).
  - Let students draw the differences between two Theorems.
  - Provide additional practice questions, referring to BHSEC Mathematics Book I, Exercise 4(a), to help students reinforce their understanding.

- Demonstrate factorisation of cubic polynomials using Remainder and Factor Theorems, watch the tutorial available in the web link: https://www.youtube.com/watch?v=7sFrHa6uNUk&t=1s.
  - For homework, encourage students to engage in factoring additional cubic polynomials, either from BHSEC Mathematics Book I, Exercise 4(a), or by exploring online resources.
  - Further allow students to explore the real life application of Remainder and Factor Theorem. Visit the suggested link: https://sciencing.com/do-math-activities-real-life-8489684.html for the insightful examples.

E. Assessment

Performance Task

Design a worksheet containing questions to evaluate students' comprehension on Remainder and Factor Theorems, assessing their ability to determine remainders and factors of polynomials, as well as their proficiency in factoring quadratic and cubic polynomials.

Sample worksheet: https://Worksheet-factor_and_remainder_theorem.pdf

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIB-A1.

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction
  https://prezi.com/p/rrner7wc1atz/etienne-bezout/
- Utility and scope
  https://khanacademy.Remainder_theorem_of_polynomials
- Remainder and factor theorem - https://www.remainder-factor theorem-video1
- Remainder and factor theorem - https://www.remainder-factortheorem-video2
- Factorisation of cubic polynomials using Remainder and Factor theorems
  https://www.youtube.com/watch?v=7sFrHa6uNUk&t=1s.
- Application for remainder and factor theorem
  https://sciencing.com/do-math-activities-real-life-8489684.html
- Worksheet for remainder and factor theorem
  https://Worksheet-factor_and_remainder_theorem.pdf
- Technological gadgets for learning (mobile, desktop, laptop...)

G. Annexure

Refer XIB-A1 for template to record achievements

Topic: XIB-B3 Quadratic Equations and Functions

[200 minutes]

Introduction

Our knowledge of ancient civilizations is based only on what survives today. The earliest known problems that led to quadratic equations are on Babylonian tablets dating from 1700 BCE. In these problems, the Babylonians were interested in finding the dimensions x and y of a rectangle with a given area c and a given perimeter 2b.

The historian Victor Katz suggests that maybe there were some people who believed that if you knew the area of a rectangle, then you knew its perimeter. In solving these problems, these Babylonians may have been trying to show that many rectangles with different dimensions have the same area.

At the end of the 16th Century the mathematical notation and symbolism was introduced by amateur-mathematician François Viète, in France. In 1637, when René Descartes published La Géométrie, modern Mathematics was born, and the quadratic formula has adopted the form we know today.

Source: A brief history on Quadratic equations for maths educators
https://www.youtube.com/watch?v=GeTTLbJ3B10

Utility and Scope

Quadratic equations are widely used in science, business, engineering and in situations where two things are multiplied together and they both depend on the same variable. For example, when working with area, if both dimensions are written in terms of the same variable, you use a quadratic equation. Additionally, in business contexts, where the quantity of a product sold is linked to its price, quadratic equations are utilised to represent revenue as a product of the price and the quantity sold.

Quadratic equations are also used when gravity is involved, such as the path of a ball or the shape of cables in a suspension bridge. A very common and easy-to-understand application is the height of a ball thrown at the ground off a building. Because gravity will make the ball speed up as it falls, a quadratic equation

can be used to estimate its height any time before it hits the ground which explains the concept of objects in free fall.

Source: https://www.youtube.com/watch?v=p1CqdKhkjRA
Suggested web link: https://www.youtube.com/watch?v=Bx4PUeoYldg
https://www.youtube.com/watch?v=Vi6q7pQnmkk

A. Competency

- Demonstrate proficiency in solving both quadratic equations and quadratic inequalities.

B. Objectives

- Determine the solutions of a quadratic equation using both the factoring method and the quadratic formula.
- Determine the solutions of quadratic inequalities using both the factoring method and the quadratic formula.

C. Essential Skills/Processes

- Conceptualising
- Communication
- Analysing
- Evaluating
- Applying
- Problem Solving
- Exploring

D. Learning Experiences

- Pre-assessment: Allow students to solve a few quadratic equations using both factorization and graphical method (refer the web link quadratic equations sample worksheets for practice questions).

- Demonstrate how to solve a quadratic equation using formula.
  - Refer to the video from the web link: Use of formula on how to use the formula to solve for roots of quadratic equations.
  - Assign problems with a) equal real roots, b) unequal real roots and c) complex roots for further practice (refer the worksheet: quadratic equations sample worksheets).

- Demonstrate the different types of roots by graphing relevant examples on GeoGebra or any graphing tool.
  - Allow students to examine the characteristics of each graph obtained and take notes.

- Reflective question: Under what circumstances do you obtain: a) identical real roots, b) distinct real roots, and c) complex roots when applying the quadratic formula? [Encourage students to infer the nature of roots based on the discriminant value.]

- Play the video in the web link: Discriminant and nature of roots which explains the nature or character of roots i.e. real roots, complex roots and equal roots depending on different values of discriminants.
  - Allow students to make notes and apply their knowledge to check the nature of roots of the quadratic equations in the worksheet: Nature of roots.

- Elaborate on the concept of quadratic inequalities using examples: *A quadratic inequality is an equation of second degree that uses an inequality sign instead of an equal sign*.
  - Allow students to watch the following links on solutions of Quadratic Inequalities with example:
    - ➢ https://www.youtube.com/watch?v=gC9zS9u7ZJ0 contains the first way to solve for the roots of a quadratic inequality.
    - ➢ https://www.youtube.com/watch?v=t54ccHYVhoo contains the second way to solve for the roots of a quadratic inequality.

- Demonstrate the use of the Method of Interval to solve for the roots of a quadratic inequality (refer to BHSEC Mathematics Book I). To enhance learning experiences, use the worksheet: Worksheets on quadratic inequalities.
  - Assign additional practice questions from BHSEC Mathematics Book I.

- Reflective question:
  - Describe the difference between the solutions or roots of a quadratic equality and a quadratic inequality.
  - How can you solve a quadratic inequality graphically?
  - Refer to the web link: https://www.youtube.com/watch?v=yBtJAVA cNkw.

E. Assessment

Performance Task

Competency based assessment

Provide the worksheet in the web link: Worksheets on quadratic inequalities to assess the competency of the students in applying their knowledge of quadratic inequalities in finding a range of solutions of a problem in real world context.

Assess, provide feedback and record student achievement based on the template given in Annexure XIB-A1.

F. Resources

- BHSEC Mathematics Book-I
- National School Curriculum Framework for Mathematics
- Introduction - A brief history on Quadratic equations for maths educators
  https://www.youtube.com/watch?v=GeTTLbJ3B10
- Utility and scope - https://www.youtube.com/watch?v=p1CqdKhkjRA
  https://www.youtube.com/watch?v=Bx4PUeoYldg
  https://www.youtube.com/watch?v=Vi6q7pQnmkk
- Worksheet on quadratic - quadratic equations sample worksheets
- Quadratic formula methods - Use of formula
- Quadratic worksheet - quadratic equations sample worksheets
- Nature of roots - Discriminant and nature of roots
- Worksheet on nature of roots - Nature of roots
- Quadratic inequalities - https://www.youtube.com/watch?v=gC9zS9u7ZJ0
  https://www.youtube.com/watch?v=t54ccHYVhoo
- Worksheet - Worksheets on quadratic inequalities
- Quadratic inequality graph -https://www.youtube.com/watch?v=yBtJAVXcNkw
- Worksheet on quadratic inequalities - Worksheets on quadratic inequalities
- Technological gadgets for learning (mobile, desktop, laptop...)

G. Annexure

Refer XIB-A1 for the template to record achievements

Topic: XIB-B4 Partial Fraction *[500 minutes]*

Introduction

Partial fractions are the fractions used for the decomposition of a rational expression. When an algebraic expression is split into a sum of two or more rational expressions, then each part is called a partial fraction. Hence, it is the reverse of the addition of rational expressions. Similar to fractions, a partial fraction will have a numerator and denominator, where the denominator represents the decomposed part of a rational function. In mathematics, we can see many complex rational expressions. If we try to solve the problems in a complex form, it will take a lot of time to find the solution. To avoid this complexity, we have to continue the problem by reducing the complex form of the rational expression into the simpler form. Partial fraction decomposition is one of the methods, which is used to decompose rational expressions into simpler partial fractions. This process is more useful in the integration process. In this article, you will learn the definition of the partial fraction, partial fraction decomposition, partial fractions of an improper fraction with solved examples in detail.

Thus, Partial Fraction is a way of "breaking apart" fractions with polynomials in them. In partial fractions when you split up a single fraction into a number of fractions whose denominators are the factors of the denominator of that fraction. These fractions are called Partial fractions.

The concept of partial fraction was discovered in 1702 by Johann Bernoulli (1667-1748), Swiss mathematician and Gottfried Leibniz (1646-1716), German mathematician independently.

Source: https://Introduction-partial and http://www.history-partial

Utility and Scope

Partial Fractions are used to decompose a complex rational expression into two or more simpler fractions. A partial fraction is a reverse of the process of the addition of rational expressions. In the normal process, we perform arithmetic operations across algebraic fractions to obtain a single rational expression.

For example if you are working in electronics, use of partial fractions will help you convert from LaPlace / Fourier (frequency) response to time-domain response. Solving an n by n linear system purely algebraically, with the variable s in tow, is a lot easier than solving an n by n system of differential equations, which is what you would have to do to keep everything in the time domain. Using partial fraction decomposition will perform inverse LaPlace transforms in an easy way.

Source:
https://numberdyslexia.com/8-examples-of-partial-fractions-application-in-real-life/

A. Competency

- Demonstrate an ability to resolve rational fractions of different types into partial fractions.

B. Objectives

- Comprehend the meaning and concept of rational functions in the form $\frac{f(x)}{g(x)}$.
- Resolve partial fractions for proper rational fractions.
- Resolve partial fractions for rational fractions with denominators containing repeating linear factors.
- Resolve partial fractions for rational fractions with denominators containing quadratic factors that cannot be factored into linear terms.
- Resolve partial fractions for improper rational fractions.

C. Essential Skills/Processes

- Conceptualising
- Connection
- Communication
- Critical Thinking
- Applying
- Problem Solving

D. Learning Experiences

- Define rational fractions, both proper and improper, by drawing connections with numerical fractions.
- Introduce the concept of Partial Fractions, referring to BHSEC Mathematics Book I or the notes provided in the link: https:/partial-fraction-expansion-1.

- Demonstrate how to resolve a given fraction into partial fractions in various cases:
  - Case 1. Degree of numerator < degree of denominator

    Type 1. Non Repeated linear factors

    ➢ Refer to the link: https://non repeated linear factor for solved examples.

    Type 2. Repeated linear factor

    ➢ Refer to the link: https://repeated linear function.com for solved examples.

    Type 3. Quadratic factors not resolvable into linear factor

    ➢ Refer to the link: https://www.uEXgK8fvS6M for solved examples.

  - Case 2. Degree of numerator ≥ degree of denominator

    Type 1. Non Repeated linear factors

    ➢ Refer to the link: https://www.Mke71wzJIGY for solved examples.

    Type 2. Repeated linear factor

    ➢ Allow students to devise the method to resolve improper rational fraction with repeated linear factors, employing concepts from long division and handling repeated linear factors.

- For each type, provide opportunities for students to apply their understanding by solving similar questions, referring to BHSEC Mathematics Book I.

E. Assessment

Performance Task

Assign questions to solve in a group on decomposing a given fraction depending on the nature of factors of the denominator.
Students can use PowerPoint presentations, chart papers or just the whiteboard to present their solutions with justification on why they used the particular method to resolve the fraction.

Refer BHSEC Mathematics Book I or link
https://tutorial.math.lamar.edu/problems/alg/partialfractions.aspx for assignment questions.
[A teacher can design appropriate assessment tools to examine the student's learning based on the template in annexure XIB-A1].

F. Resources

- BHSEC Mathematics Book-I
- National School Curriculum Framework for Mathematics
- Introduction -
  https://Introduction-partial and http://www.history-partial
- Utility and scope -
  https://numberdyslexia.com/8-examples-of-partial-fractions-application-in-real-life/
- Meaning of partial fraction -
  https://www.khanacademy.org/math/algebra-home/alg-rational-expr-eq-func/alg-partial-fraction/v/partial-fraction-expansion-1
- Non-repeated linear factors -https://www.non repeated linear factor.com
- Repeated linear factors - https://www.repeated linear function.com
- Quadratic factors - https://www.youtube.com/watch?v=uEXgK8fvS6M
- Improper fractions with non-repeated linear factors
  https://www.youtube.com/watch?v=Mke71wzJIGY
- Partial fractions questions -
  https://tutorial.math.lamar.edu/problems/alg/partialfractions.aspx
- Technological gadgets (mobile phones, laptops, computers, etc.)

G. Annexure

Refer XIB-A1 for template to record achievement

Topic: XIB-B5 Limits

[550 minutes]

Introduction

Limits in maths are defined as the values that a function approaches the output for the given input values. Limits play a vital role in calculus and mathematical analysis and are used to define integrals, derivatives, and continuity.

Archimedes of Syracuse was the first to develop the idea of limits to measure curved figures and the volume of a sphere in the third-century B.C. By carving these figures into small pieces that can be approximated, then increasing the number of pieces, the limit of the sum of pieces can give the desired quantity.

Source: https://byjus.com/maths/limits/

Utility and Scope

Limits are essential to calculus (and mathematical analysis in general) and are used to define continuity, derivatives, and integrals. The concept of a limit of a sequence is further generalised to the concept of a limit of a topological net, and is closely related to limit and direct limit in category theory.

A. Competency

- Demonstrate an understanding of fundamental theorems of limits, and evaluate limits of algebraic functions.

B. Objectives

- Understand the meaning of limits and the fundamental theorem of limits.
- Compute left-hand limit and right-hand limit to check if limit at a point exists.
- Evaluate the limit of a function using direct substitution method.
- Evaluate the limit of a function using the factorisation method.
- Evaluate the limit of a function using rationalisation.

C. Essential Skills/Processes

- Conceptualising
- Critical thinking
- Evaluating
- Analysing
- Applying

- Problem Solving

D. Learning Experiences

- Illustrate the idea of a limit through examples like the relationship between the area of a polygon inscribed in a circle and the area of the circle itself. As the number of sides of the polygon increases, its area grows, yet it never surpasses the area of the circle. Thus, we express the limit of the polygon's area as the number of sides approaches infinity to be the area of the circle. This can be symbolically represented as:

  $\lim_{sides \to \infty} (area\ of\ polygon) = area\ of\ the\ circle.$

  - Provide some other relevant examples of limits.

- Explain the meaning of x tends to a, symbolically, $x \to a$.
  - Refer to the link: https://www.youtube.com/watch?v=rs-jpEw3Iic or BHSEC Mathematics Book I to learn the concepts of limits and its terminologies.

- Explain evaluating the limits of a function using a table of values: x approaching from both sides, and introducing the right-hand limit and left-hand limit. Refer to the link: https://www.youtube.com/watch?v=jeWY537C8cw or refer to BHSEC Mathematics Book I.

- Demonstrate finding of right-hand limit and left-hand limit. Refer to the link: https://www.youtube.com/watch?v=IZw2tdiYyfk and provide similar practice questions or refer to BHSEC Mathematics Book I.

- Explain the existence of a limit – the limit of a function exists only if the right hand limit is equal to the left hand limit. Refer example of checking for limit existence or BHSEC Mathematics Book I. Or refer to the following links to learn more:
  - Using graphing software (Geogebra), show the graphs of each function and explain the limits.
  - Assign some questions for practice. Refer BHSEC Mathematics Book I, exercise 14(a), or find relevant questions from other sources.

- Refer BHSEC Mathematics Book I or https://tutorial.math.lamar.edu/classes/calcI/defnoflimit.aspx to understand the meaning of limits.

- Explore the theorems on limits using BHSEC Mathematics Book I or watch the videos from the web links:
  - https://www.youtube.com/watch?v=py8JWwQSOck
  - https://www.youtube.com/watch?v=q7kxe6T0E14

- Demonstrate on evaluating algebraic limits by the following methods:
  - Direct substitution method: limit by direct substitution method.
  - Factorization method: https://www.youtube.com/watch?v=fOrOeZA-vdY.
  - Rationalisation method: https://www.youtube.com/watch?v=iVLyE_UEfxs.
  - Expansion: https://www.youtube.com/watch?v=epaHyrGisWM.

- Provide practice questions for each type of method of evaluating algebraic limits . Refer to BHSEC Mathematics Book I, exercises 14(b), 14(c), 14(d), and 14(e).

- Demonstrate finding of limit at infinity. Refer to the link: limit at infinity or BHSEC Mathematics Book I.
  - Assign some questions from other sources or refer to BHSEC Mathematics Book I, exercise 14(f).

E. Assessment

Performance Task 1

Use the worksheet in the web link: Live worksheet on limits to assess the students' comprehension of concepts in limits.

Performance Task 2

Assign some questions on evaluating limits of all the types and limits at infinity. Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIB-A1.

F. Resources

- BHSEC Mathematics Book-I
- National School Curriculum Framework for Mathematics
- Introduction of Limits- https://byjus.com/maths/limits/
- Utility and scope- https://www.youtube.com/watch?v=5L5o8j_iRYc
- Concept of x tends to a - https://www.youtube.com/watch?v=rs-jpEw3Iic
- Evaluating of limits- https://www.youtube.com/watch?v=IZw2tdiYyfk
  https://www.youtube.com/watch?v=IZw2tdiYyfk
- Existence of limits- example of checking for limit existence

- Meaning of limits- https://tutorial.math.lamar.edu/classes/calcI/defnoflimit.aspx
- Theorem of limits- https://www.youtube.com/watch?v=py8JWwQSOck https://www.youtube.com/watch?v=q7kxe6T0E14
- Direct substitution method: limit by direct substitution method
- Factorization method: https://www.youtube.com/watch?v=fOrOeZA-vdY
- Rationalisation method: https://www.youtube.com/watch?v=iVLyE_UEfxs
- Expansion: https://www.youtube.com/watch?v=epaHyrGisWM
- limit at infinity - https://www.youtube.com/watch?v=NmLljBAg82o
- Graphing software.

G. Annexure

Refer XIB-A1 for template to record achievement

Topic: XIB-B6 Continuity and Discontinuity of Functions

[150 minutes]

A. Competency

- Exhibit an ability to recognize the continuity of a function both theoretically and geometrically.

B. Objectives

- Understand the concept of continuity and removable discontinuity of a function theoretically as well as graphically.
- Examine the continuity of the function by evaluating the right-hand limit and left-hand limit.

C. Essential Skills/Processes

- Analysing
- Conceptualising
- Applying
- Information recall
- Knowledge Application

D. Learning Experiences

- Illustrate the definition of continuous function. Simultaneously, deliver the concept of discontinuity of function.
- Show some examples of functions and graphs. Refer to the links Continuity and discontinuity 1 and Continuity and discontinuity 2 to learn the basic concept of continuity of a function. You can refer to BHSEC Mathematics Book I.
- Discuss the conditions and processes involved in testing for continuity. Refer to the link https://www.youtube.com/watch?v=Myxxxit7Zlo or BHSEC Mathematics Book I. Provide some functions to the students for practising.
- Introduce removable discontinuity. Refer to the link Removable discontinuity or BHSEC Mathematics Book I. If possible, let students explore on their own.
- Explain the continuity of a function at an interval. Refer to the link Continuity over an interval or BHSEC Mathematics Book I.
- Assign relevant questions from BHSEC Mathematics Book I, exercise 15 or from other sources.

E. Assessment

Performance Task

Assign practice questions to examine the continuity of a function, and to evaluate the understanding of continuous function at a point. Refer to the following links for the questions:

- Sample Problems 1
- https://byjus.com/maths/continuity-and-discontinuity/

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIB-A1.

F. Resources

- BHSEC Mathematics Book-I
- National School Curriculum Framework for Mathematics
- Basic concepts of continuity and discontinuity of functions-
  Continuity and discontinuity 1
  Continuity and discontinuity 2
- Testing of continuity- https://www.youtube.com/watch?v=Myxxxit7Zlo
- Removable of discontinuity- Removable discontinuity
- Continuity functions- Continuity over an interval
- Practice questions - Sample Problems 1
  https://byjus.com/maths/continuity-and-discontinuity/
- Graphing software.

G. Annexure

Refer XIB-A1 for a template to record achievement

Topic: XIB-B7 Differentiation *[600 minutes]*

Introduction

Differentiation is the process of finding the derivative *f'(x)* of a function *f*. It is a part of maths under the branch of Calculus. The derivative is the gradient of the tangent of the graph of f at the point x. The discovery of calculus is often attributed to two men, Isaac Newton and Gottfried Leibniz, who independently developed its foundations. Although they both were instrumental in its creation, they thought of the fundamental concepts in very different ways.
While Newton considered variables changing with time, Leibniz thought of the variables x and y as ranging over sequences of infinitely close values. He introduced dx and dy as differences between successive values of these sequences. Leibniz knew that dy/dx gives the tangent but he did not use it as a defining property. On the other hand, Newton used quantities x and y, which were finite velocities, to compute the tangent. Neither Leibniz nor Newton thought in terms of functions, but both always thought in terms of graphs. For Newton the calculus was geometrical while Leibniz took it towards analysis.

Explore more on the history of differentiation and calculus, and you may consider using the provided web link: https://www.youtube.com/watch?v=IMj5dgGWxSM.
Suggested web links:

https://www.youtube.com/watch?v=BrH1fz-jZOo
https://www.youtube.com/watch?v=dfr23VIQPCo
https://www.youtube.com/watch?v=b7vIKO-uung&t=2s

Utility and Scope

Differentiation in mathematics, particularly in calculus, involves studying rates at which quantities change. It has a wide range of real-world applications and is crucial in various fields such as science, technology, business, and more. Here are some examples illustrating the application and scope of differentiation:

1. **Physics and Engineering:**

   a) **Motion Analysis**: Differentiation is used to analyse the motion of objects. In physics and engineering, it helps calculate velocities, accelerations, and study the behaviour of particles.

   b) **Electric Circuits**: In electrical engineering, differentiation is applied to analyse voltage and current changes over time in circuits.

2. **Computer Science and Technology**:
   a) **Algorithm Analysis**: Differentiation helps analyse the efficiency and performance of algorithms, especially in terms of their time complexity.

   b) **Signal Processing**: In image and audio processing, differentiation is used to analyse and enhance signals.

3. **Economics and Finance**:

   a) **Marginal Analysis**: Differentiation is used in economics to analyse marginal cost, marginal revenue, and marginal utility, providing insights into optimal decision-making.

   b) **Options Pricing**: In finance, differentiation is used in option pricing models, such as the Black-Scholes model, to determine the rate of change of option prices concerning various factors.

4. **Biology and Medicine**:
   a) **Population Dynamics**: In biology, differentiation is used to model and analyse population growth and dynamics.

   b) **Medical Imaging**: Differentiation plays a role in the analysis of medical images, such as identifying edges and contours in diagnostic imaging.

5. **Environmental Science**:
   a) **Rate of Change in Environmental Variables**: Differentiation is used to study the rates of change in environmental variables, such as temperature, pollution levels, and ecosystem dynamics.

6. **Operations Research and Management**:
   a) **Optimization**: Differentiation is a key tool in optimization problems, such as finding the minimum or maximum values of functions. This is applied in operations research and management decision-making.

   b) **Supply Chain Management**: Differentiation helps optimise inventory levels, production rates, and distribution schedules.

7. **Statistics and Data Analysis**:

   a) **Regression Analysis**: Differentiation is used in regression analysis to find the best-fit line that represents the relationship between variables.

   b) **Time Series Analysis**: Differentiation is applied to study trends and patterns in time-series data.

8. **Telecommunications**:

a) **Signal Transmission**: In telecommunications, differentiation is used to analyse and optimise signal transmission, such as the modulation and demodulation of signals.

9. **Business and Marketing**:

a) **Marginal Cost and Revenue**: In business, differentiation helps analyse marginal cost and marginal revenue, aiding in pricing decisions.

b) **Market Research**:** Differentiation is applied in market research to analyse how variables such as pricing, advertising, and product features impact sales.

10. **Automotive and Transportation**:

a) **Vehicle Dynamics**: Differentiation is used to analyse the motion and performance of vehicles, including aspects like acceleration, braking, and turning.

Explore more from the web link: utility and scope of differentiation.

A. Competencies

- Demonstrate a comprehensive understanding of derivatives both theoretically and geometrically.
- Exhibit proficiency in differentiating functions using the first principle and other methods, and apply these skills to solve real-life problems.

B. Objectives

- Explore the meaning of derivatives and its geometrical interpretation.
- Differentiate functions using the first principle.
- Differentiate the algebraic functions of any form and simple logarithmic functions $logx$ and $log(ax + b)$.

C. Essential Skills/Processes

- Defining key concepts
- Making Connections
- Conceptualisation
- Knowledge Application
- Problem Solving

D. Learning Experiences

- Recap the previous knowledge on slope of a straight line by letting students solve the following question:

- Give one linear equation and its graph on the board and instruct students to find the slope of the graph using various methods. Students should figure out the main two methods to find the slope: $\frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$.
- After completing the above task, ask the following questions:
  i. What is the meaning of the slope of the graph?
  ii. What is its significance in real life?

- Relate the knowledge of slope with the derivatives of a function. Inform students that in real life, the functions we get are not always linear. Write one non-linear function ($y = x^2$) with its graph and ask students to find the slope at a particular point.
- Relate the slope of a line learned in lower classes with the slope of a curve at a particular point. Differentiation is similar to finding the slope, but differentiation will help us to find the slope at a particular point – a very small change in y and a small change in x.
  - Refer to links: https://www.youtube.com/watch?v=lowavG2SXsQ and https://www.youtube.com/watch?v=N2PpRnFqnqY to know the basic concepts of derivatives or refer to BHSEC Mathematics Book I.
  - Use the link: https://www.geogebra.org/m/kHAkeTCw to show the derivative graphically.
- Explain that the derivative of a function can also be called as slope, rate of change, and gradient.
- Discuss the process to find the slope of a function with a very small change ($\Delta x\ or\ h$) and derive the First Principle. Refer to the links: https://youtu.be/d1_PQJcQog0 and https://youtu.be/cdisv5VksuY or BHSEC Mathematics Book I to learn about differentiation from the first principle or definition. Explain that the first principle is the same as $\frac{change\ in\ y}{change\ in\ x}$, but in the first principle, the change is almost 0, i.e. $\lim_{\Delta x \to 0}$ or $\lim_{h \to 0}$.
- Allow students to practise finding the derivative of some simple algebraic function from the first principle to get a sense of the meaning of differentiation. Provide functions such as $y = 2x^2$; $y = x^3$; $y = 2x^3$; $y = \sqrt{x}$ etc.

- Relate the derivatives from the first principal with the shortcuts. Introduce the general formulas to find the derivatives of functions.
  - Watch the video https://youtu.be/OLyeTUZDH-o till 4:30 minutes to know some of the formulas. You may refer to the link: https://youtu.be/AdLAkD-r9Rs or BHSEC Mathematics Book I. (Rather than simply presenting the formula, provide explanations and derivations for each formula).
  - Demonstrate application of the formulae to differentiate the related functions. Assign some questions from BHSEC Mathematics Book I, exercise 16(b), or relevant questions from other sources.
- Introduce the basic chain rule for differentiating simple functions such as: $y=(ax+b)^n$, $y=\sqrt{(ax+b)}$,and $y=\log(ax+b)$.
- Explain that the chain rule can be used in differentiating composite functions.
  - Refer to the link: https://www.youtube.com/watch?v=H-ybCx8gt-8 for more information.
- Demonstrate the differentiation of some simple composite functions and assign questions for practice. Refer BHSEC Mathematics Book I, exercise 16(b).
- Illustrate the product rule and quotient rule for derivatives, providing a derivation for each. Refer the following links:
  - Product Rule- https://www.youtube.com/watch?v=L5ErlC0COxI
    https://www.cuemath.com/calculus/uv-differentiation-formula/
  - Quotient Rule: https://www.youtube.com/watch?v=jxxzbMxihjQ
    https://www.youtube.com/watch?v=ho87DN9wO70
    https://www.cuemath.com/calculus/quotient-rule/
- Demonstrate applying product rule and quotient rule to differentiate some algebraic functions. Refer https://youtu.be/gWa9KpteZ_g or BHSEC Mathematics Book I to get some examples.
  - Assign some questions for practice. Refer to BHSEC Mathematics Book I, exercise 16(c) or find some relevant questions from other sources.

E. Assessment

Performance Task 1

Design functions to differentiate using both the first principle and direct methods.
Assess, provide feedback, and record achievement based on the template given in annexure XIB-A1.

Performance Task 2

Design functions to differentiate using the chain rule, product rule, and quotient rule.
Assess, provide feedback, and record achievement based on the template given in annexure XIB-A1.

F. Resources

- BHSEC Mathematics Book-I
- National School Curriculum Framework for Mathematics
- Introduction of differentiation-
  https://www.youtube.com/watch?v=IMj5dgGWxSM
  https://www.youtube.com/watch?v=BrH1fz-jZOo
  https://www.youtube.com/watch?v=dfr23VIQPCo
  https://www.youtube.com/watch?v=b7vIKO-uung&t=2s
- Utility and scope- utility and scope of differentiation.
- Basics concept of derivative- https://www.youtube.com/watch?v=lowavG2SXsQ
  https://www.geogebra.org/m/kHAkeTCw
- First principles- https://youtu.be/d1_PQJcQog
  https://youtu.be/cdisv5VksuY
- Product rule and quotient rule:
- Product Rule- https://www.youtube.com/watch?v=L5ErlC0COxI
  https://www.cuemath.com/calculus/uv-differentiation-formula/
- Quotient Rule- https://www.youtube.com/watch?v=jxxzbMxihjQ
  https://www.youtube.com/watch?v=ho87DN9wO70
  https://www.cuemath.com/calculus/quotient-rule/
  https://youtu.be/gWa9KpteZ_g
- Graphing Software

G. Annexure

Refer XIB-A1 for template to record achievement

Topic: XIB-B8 Integration *[600 minutes]*

Introduction

In Mathematics, integration is a method of adding or summing up the parts to find the whole. It is a reverse process of differentiation, where we reduce the functions into parts. This method is used to find the summation under a vast scale.
Today, it is generally believed that calculus was discovered independently in the late 17th century by two great mathematicians: Isaac Newton and Gottfried Leibniz. The result also shows that Leibniz started first with integration and Newton with differentiation. It is Leibniz, however, who gave the new discipline its name. Newton called his calculus "the science of fluxions".

Source: history of integration

Utility and Scope

Learning about integration in mathematics holds significant utility and scope in various applications within the realms of business and technology. Here are five important aspects where understanding integration proves beneficial:

1. **Data Analysis and Decision-Making in Business:**

Integration is used in statistics and data analysis for calculating cumulative distribution functions and probability density functions. In business, this can help in making informed decisions based on the analysis of trends, patterns, and probabilities.

2. **Optimization and Resource Allocation**:

Integration is fundamental to optimization problems, where businesses aim to maximise profits or minimise costs. In areas like supply chain management, integration techniques are applied to optimise the allocation of resources, such as time, inventory, and workforce.

3. **Financial Modeling and Risk Management**:

Integration is utilised in financial modelling to calculate metrics such as net present value (NPV) and to analyse cash flows. In risk management, it plays a role in calculating expected values and understanding the distribution of risks and returns.

4. **Physics and Engineering Applications**:

Integration is extensively used in physics and engineering applications, including areas like fluid dynamics, electrical circuits, and structural analysis. In technology, understanding integration is crucial for designing and optimising systems and processes.

5. **Algorithm Design and Machine Learning**:

Integration concepts are applied in algorithm design and machine learning, particularly in the development of algorithms for optimization problems and pattern recognition. Integrative techniques are often used to improve the efficiency and accuracy of machine learning models.

For more information, explore the link: applications of integration

A. Competencies

- Demonstrate an understanding of integration as the reverse process of differentiation.
- Display an ability to integrate functions using different methods and apply the concept in solving real-life problems.

B. Objectives

- Understand the meaning of integration, recognizing it as the reverse process of differentiation.
- Evaluate integrals of algebraic functions using basic rules.
- Integrate algebraic functions using the substitution method.

C. Essential Skills/Processes

- Conceptualising
- Connection
- Critical Thinking
- Analysing
- Applying
- Problem Solving
- Evaluating

D. Learning Experiences

- Recapitulation: students can revise the concept of “differentiation”. Ask some questions on differentiation.

- Differentiate a simple function and inquire from students how to recover the original function from its derivative. Subsequently, elucidate how integration serves as the reverse process of differentiation. Refer to the link: Integration as a reverse process.

- Show that if the derivative is the rate of flow of water in a tank at any point of time, then the integral of the derivative will be the volume of water at any point of time in the tank. Refer to the link: meaning of integration.

- Introduce the notation of integration and explain that the symbol is an elongated letter of S, which implies for ‘sum’. For more detailed exploration of the integration symbol, you can refer to the link: symbol of integration.

- Demonstrate the process of integration through simple examples and understand why we add the constant 'C' at the end of every integral. For a detailed explanation, you can refer to the link: constant in integration or BHSEC Mathematics Book I.
  - Explore and allow students to practise the integration of simple monomials to become acquainted with the integration sign and process.

- Encourage students to explore the meaning of integration and standard forms of integrals. Refer to BHSEC Mathematics Book I for comprehensive explanations, or visit the link: basic integral formulas to gain insights into the origin of each integration formula and how they are related to differentiation.

- Delve into the principles of integration, which are elucidated as "general theorems" in BHSEC Mathematics Book I. In other references, these principles may be referred to as 'integration rules'. Refer to the link: integration formulas to familiarise with specific integration formulas.
  - Provide some questions to practise. Use the link: : practise questions on integration or refer to BHSEC Mathematics Book I, exercise 17(a).

- Discuss the integration of $\frac{1}{x}$ and $\frac{1}{ax+b}$, and practise some related examples.

- Demonstrate the process of integration using the substitution method, with reference to BHSEC Mathematics Book I.
  - Utilise the instructional videos available through the provided links to enhance understanding of integration through substitution.
    - ➢ https://www.youtube.com/watch?v=sdYdnpYn-1o
    - ➢ https://www.youtube.com/watch?v=D0QJvYk0OkM
  - Allow students to practise questions on integration by substitution. You can refer to BHSEC Book I, exercise 17(b), or explore other sources to find relevant problems.

E. Assessment

Performance Task

Assign questions on general integrations and integration by substitution of algebraic functions. Refer to BHSEC Mathematics Book I or refer to link questions on integration for the questions.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIB-A1.

F. Resources

- BHSEC Mathematics Book-I
- National School Curriculum Framework for Mathematics
- Introduction of integration- history of integration
- Utility and scope- applications of integration
- Concept of integration- Integration as a reverse process
- Meaning of integration- meaning of integration
- Notation of integration- symbol of integration
- Constant in the integral- constant in integration
- Meaning of integration and standard form of integral- basic integral formulas
- Rule of integration- integration formulas
- Practice questions 1- practice questions on integrations
- Practice questions 2- questions on integration
- Integration by substitutions- https://www.youtube.com/watch?v=sdYdnpYn-1o https://www.youtube.com/watch?v=D0QJvYk0OkM

G. Annexure

Refer XIB-A1 for template to record achievement

Topic: XIB-CD1 Points and their Coordinates in 2-Dimension

[400 minutes]

Introduction

The coordinate system we commonly use is called the Cartesian system, after the French mathematician René Descartes (1596-1650), who developed it in the 17th century. Legend has it that Descartes, who liked to stay in bed until late, was watching a fly on the ceiling from his bed. He wondered how to best describe the fly's location and decided that one of the corners of the ceiling could be used as a reference point.

Refer https://wild.maths.org/ren%C3%A9-descartes-and-fly-ceiling

Utility and Scope

In the technical fields: The concept of 3-D geometry is also applied in the fields of robotics, computer, and video games. The way and the design of the characters that move through their virtual worlds requires geometric computations to create paths around the obstacles concentrating around the virtual world.

Astronomy & Physics: Geometry is used in the field of astronomy, helping to map the positions of stars and planets on the celestial sphere and describing the relationship between movements of celestial bodies.

Geographic Information Systems: Geometry concepts are used in satellites in GPS systems, it calculates the position of the satellite and location of GPS gauged by the latitudes and longitudes.

Refer https://studiousguy.com/examples-of-geometry-in-everyday-life/

A. Competency

- Demonstrate a comprehensive understanding of the coordinate system in 2-Dimension, and apply distance, section, and midpoint formulas to describe a point and a line within a two-dimensional coordinate system.

B. Objectives

- Understand the meaning of the coordinate plane and the coordinates (x-coordinate and y-coordinate).

- Find the distance between two points in the 2-D plane and solve related problems.
- Find the coordinates of the points using distance, section and midpoint formula, and solve related problems.

C. Essential Skills/Processes

- Conceptualising
- Applying
- Analysing
- Creative Thinking
- Information recall

D. Learning Experiences

- Allow students to watch a video Cartesian coordinate system to recapitulate the Cartesian coordinate. After watching the video, ask students to define Cartesian coordinates. Ensure that students understand the meaning of x -coordinate and y-coordinate.

- Screen the video https://www.youtube.com/watch?v=ZVv1KiGkx50 on how to find the distance between the points and distance of a point from the origin using the distance formula. Then, ask students to take notes and assign questions from the link: https://www.youtube.com/watch?v=nyZuite17Pc or from Exercise 18 (b), BHSEC Mathematics Book I.

- To introduce the section formula, first demonstrate the derivation of the internal section formula. Refer to the web link: Section formula or BHSEC Mathematics Book I.
- Discuss the application on the internal section division formula to solve the problem.
    - Refer https://www.youtube.com/watch?v=Em3Au5 dVXc or BHSEC Mathematics Book I to learn how to apply the section formula.
    - Assign practice questions from the links: problems on distance formula and worksheet on section formula or refer to the BHSEC Mathematics Book I, exercise 18 (b).

- Before moving to the external section formula, derive the midpoint formula from the internal division since the dividing point will lie exactly at the middle

of the two points. In this case, the ratio *m* and *n* will be 1:1. Let students practise a few questions related to the midpoint formula.

- Allow students to explore the external section formula and make notes. Refer to the link: https://www.youtube.com/watch?v=a27bzhIKpWM or BHSEC Mathematics Book I.
    - Discuss how to apply the external section division formula to solve the problem.
    - Assign practice questions from the link: https://www.vedantu.com/maths/external-division-formula or BHSEC Mathematics Book I, exercise 18(b).

E. Assessment

Performance Task

Assign questions on points that divide internally and externally, as well as the midpoint between two points, from exercise 18, BHSEC Mathematics Book I.

Conduct online tests using Mentimeter. Questions could be used from the link Sample quiz questions.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIB-A1.

F. Resources

- BHSEC Mathematics Book-I
- National School Curriculum Framework for Mathematics
- Introduction of Points and their coordinates in 2-Dimension-https://wild.maths.org/ren%C3%A9-descartes-and-fly-ceiling
- Utility and scope- https://studiousguy.com/examples-of-geometry-in-everyday-life/
- Cartesian coordinates- Cartesian coordinate system
- Distance formula- https://www.youtube.com/watch?v=ZVv1KiGkx50 https://www.youtube.com/watch?v=nyZuite17Pc
- Section formula - Section formula https://www.youtube.com/watch?v=Em3Au5pdVXc
- Practice questions - problems on distance formula worksheet on section formula

- External section formula- https://www.youtube.com/watch?v=a27bzhIKpWM
- External division- https://www.vedantu.com/maths/external-division-formula
- Online test- Sample quiz questions
- Laptops, graphing software

G. Annexure

Refer XIB-A1 for template to record achievement

Topic: XIB-CD2 The Straight Line *[250 minutes]*

Introduction

A straight line is an endless one-dimensional figure that has no width. A straight line does not have any curve in it. It can be horizontal, vertical, or slanted. When geometry was first formulated by Euclid (325 - 265 BC) in the Elements, he defined a general line (straight or curved) to be "breadthless length" with a straight line being a line "which lies evenly with the points on itself".

Source: https://www.math.tamu.edu/ehmwk/Tan_chpt1.pdf

Utility and Scope

The concept of straight lines from mathematics has numerous practical applications in real life. For instance, in architecture and engineering, straight lines are used to design and construct buildings, bridges, and other structures. In physics, straight lines are used to represent the path of objects in motion.

The concept of straight lines can be applied in mathematical analysis for business and economics. Additionally, a straight line graph is used in medicine and pharmacy to figure out the accurate strength of drugs. Straight line graphs are used in the research process and the preparation of the government budget.

Source: https://www.utility and scope

A. Competency

- Demonstrate an ability to write the equations of straight lines in different forms, and apply the concepts in solving problems related to commerce and economics.

B. Objectives

- Express the equations of the straight lines in different forms: slope-intercept, point-slope, two-points and general.
- Apply the concept of the equation of a straight line to find the relations between two quantities in commerce and economics.

C. Essential Skills/Processes

- Distinguishing differences
- Defining key concepts

- Knowledge application
- Analysing

D. Learning Experiences

- Pose the questions on a straight line that students have learned in lower classes: equation of a line in standard form (general form) and slope and y-intercept form.
    - Encourage students to review the concepts of slope and y-intercept by watching the video available at https://www.youtube.com/watch?v=lz8zVJxRFX8.

- Discuss the meaning of slope and y-intercept and the equation in slope and y-intercept form. Provide students with some straight-line graphs and ask them to find the equation in slope and y-intercept form.

- Ask students to revise the finding of slope from two given points using the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

- Demonstrate derivation of equations of line in other forms: point-slope form using the web link: https://www.google.point slope form and two-points form using the web link: https://www.google.two point form. Alternatively, you can refer to BHSEC Mathematics Book I.
    - Students can discuss the problems on the above topics and assign a task to find equations of straight lines in different forms. Refer Mathematics Book I, exercise 19(a).

- Demonstrate a few questions that require the application of the equations of straight lines in commerce and economics. Refer to the suggested links:
    - https://www.youtube.com/watch?v=LznoJOZMGhA
    - https://www.youtube.com/watch?v=XcFq_I-if7Y (you can ask students to graphically find the solution of the problem solved in video).
    - Explore application of straight lines applied to commerce and economics.

E. Assessment

Performance Task 1

Provide some points of a straight line and ask students to find the equation of the straight line in different forms: general, point-slope, two-points, and slope and y-intercept form.

Performance Task 2

Find real-life data or example questions from the internet and have students work collaboratively in groups to solve them during class. Teachers should closely monitor students for active participation, engagement in discussions, and the demonstration of active involvement.

Assess, provide feedback and record achievement based on the template given in the annexure XIB- A1.

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum for Mathematics
- Introduction of straight line - https://www.math.tamu.edu/ehmwk/Tan_chpt1.pdf
- Utility and scope- http://bit.do/usestraightlinereallife
- Forms of equations:
  - Slope-intercept form - https://www.youtube.com/watch?v=lz8zVJxRFX8
  - point-slope form- https://www.google.point slope form
  - Two-points form - https://www.google.two point form.
- Applications of straight line -https://www.youtube.com/watch?v=LznoJOZMGhA - https://www.youtube.com/watch?v=XcFq_J-if7Y

G. Annexure

Refer XIB-A1 for template to record achievement

Topic: XIB-CD3 The Circle and Its Theorem *[450 minutes]*

Introduction

There is no way to be certain, but anthropologists generally agree that the circle was created long before recorded history. It is quite likely that it was drawn by a stick in the sand. With the sun being a constant in early man's existence and the source of all life, it is quite likely that the first circle represented the sun.

Through the years, man's understanding of the circle has evolved substantially with Euclidean geometry being its crowning point of technological understanding.
Without knowing the properties of a circle, the world would not be anything like it is today. Without circles, there would be no wheel. And of course, we would never have the pleasure of driving a car, riding a Ferris wheel, or watching the moon landing on our television set.

Source: https://circlesonly.wordpress.com/circles-are-the-mother-of-all-inventions/

Utility and Scope

Learning circle theorems in mathematics provides students with essential tools for understanding and analysing the properties of circles. The utility and scope of this knowledge extend across various mathematical and practical applications. Here are key aspects of the utility and scope of learning circle theorems:

1. **Architectural and Engineering Design**: Circle theorems find practical applications in architectural and engineering design. Understanding the properties of circles is essential for designing circular structures, arcs, and curves in various construction projects.

2. **Computer Graphics**: In computer graphics and programming, circle theorems are essential for rendering and manipulating circular shapes. Understanding these theorems is crucial for creating accurate and visually appealing graphics.

3. **Problem-Solving in Competitive Exam**s: Knowledge of circle theorems is often tested in competitive exams, making it a valuable skill for students preparing for mathematics assessments and entrance examinations.

4. **Advanced Mathematics**: Circle theorems serve as a stepping stone to more advanced mathematical concepts, including calculus and differential geometry.

Understanding circular functions and their properties becomes important in these advanced studies.

5. **Real-WorldApplications:** Circle theorems find applications in real-world scenarios, such as navigation, where understanding angles and bearings is crucial. They are also relevant in surveying, where circular measurements may be encountered.

A. Competencies

- Demonstrate proficiency in identifying and representing equations of circles in various forms, adeptly generating circles under specified conditions.
- Display the ability to analyse geometric configurations involving circles and apply the theorems to deduce unknown angles, arc lengths, and other geometric properties.
- Demonstrate the ability to engage in logical reasoning and systematically construct rigorous mathematical proofs, allowing them to present a sequence of logical arguments to establish the truth of a given theorem.

B. Objectives

- Define a circle and its parts.
- Represent the equation of a circle in standard form, and solve related problems.
- Illustrate theorems of circles using relevant diagrams, and apply the theorems in solving logical problems related to circles.

C. Essential Skills/Processes

- Conceptualising
- Representing
- Critical Thinking
- Deductive Reasoning
- Computing
- Knowledge Application

D. Learning Experiences

- Introduce the definition of circle and components of circle by displaying the videos in the following links and instruct students to make notes.
  - https://www.youtube.com/watch?v=q-3w7o4zbWQ (contains video defining a circle and locus).
  - https://www.youtube.com/watch?v=Y5Z69g7A0eI (contains video explaining the components or basics of circle.)

- Explain the Equation of the circle in centre-radius form . Refer to the link: https://www.youtube.com/watch?v=cyKFBmTjyio explaining the derivation of the equation of circle.

  - Allow students to use the GeoGebra app https://www.geogebra.org/calculator to visualise and confirm the particular cases (BHSEC Mathematics Book I, page 21-1 to 21-2) by typing in relevant examples.

- Discuss the diameter form of the equation of circle with students and demonstrate finding the equation of circle when two ends of the diameters are given. Refer to BHSEC Mathematics Book I.

- Demonstrate finding the centre and radius of a circle when the general equation of the circle is given.
  - First method: Refer to the link: general to centre-radius form which explains how to determine the centre and radius of a circle by reducing the general equation to centre-radius form.
  - Second method: Refer to BHSEC Mathematics Book I, for the prescribed formula to find centre and radius of a circle from the general form of equation of circle i.e. $x^2 + y^2 + 2gx + 2fy + c = 0$. Then, radius = $\sqrt{(g^2 + f^2 - c)}$ and the centre= $(-g, -f)$.
  - Assess students' learning by providing further practice questions from BHSEC Mathematics Book I.

- Discuss examples on determining the equation of a circle when any 3 conditions are given. Refer to BHSEC Mathematics Book I for the insightful examples.

- Theorems on Circles: Prepare a Powerpoint presentation on Theorems of Circle and explain each theorem diagrammatically on the board or using a graphing tool (GeoGebra) to help students comprehend the theorems visually as well as prove them if possible.
    - Discuss all the eighteen theorems with proofs. Use illustrative diagrams in Geogebra for the complicated theorems. Some of the resources of the theorems are given below:

1. **Theorem on chords of a circle**
   Theorem 1 - https://www.geogebra.org/m/NMMEsDAQ
   Theorem 2 - https://www.geogebra.org/m/YaQ3mEjY
   Theorem 3 - https://www.geogebra.org/m/bUJVWeYn
   Theorem 4 & 5 - https://www.geogebra.org/m/pSn7Jd76

2. **Theorem on Arcs and Angles**
   Theorem 6 - https://www.geogebra.org/m/NkK6vSVr
   Theorem 7 - https://www.geogebra.org/m/JjDd2TKw
   Theorem 8 - https://www.geogebra.org/m/wCGeV5ku
   Theorem 9 - https://www.geogebra.org/m/vpxurwwk
   Theorem 10 - https://www.geogebra.org/m/xZeN5f4D

3. **Theorems on congruent arcs and chords**
   Theorem 11, 12, 13, 14 - explain with relevant diagrammatic representations.

4. **Theorems on Tangent lines and circles**
   Theorem 15 - https://www.geogebra.org/m/u6Ev7bHg

   https://www.youtube.com/watch?v=IcgycGSq9Us

   Theorem 16 - https://www.youtube.com/watch?v=3jQ_vWb3OFE

5. **Theorems on Angles in alternate Segment**

6. **Theorems on segments of a chord-** explain with relevant diagrammatic representations.

- Discuss how the theorems can be used in solving problems relevant to real life applications after studying each theorem (refer to exercise questions in BHSEC Mathematics Book I).

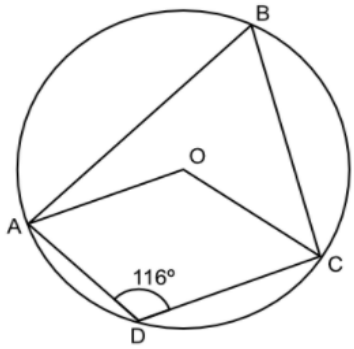
E. Assessment

Performance Task 1

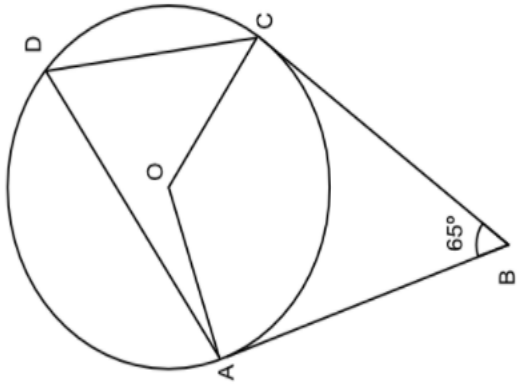
Design a circle diagram where students will apply circle theorems to solve the problem.

Sample:

1. ABCD is a cyclic quadrilateral. O is the centre of the circle. The angle ∠ADC is 116º: what is the value of the angle ∠AOC?



2. AB and BC are tangents to the circle with centre O. D is a further point on the



circumference. The angle ABC is 65º. What is the value of the angle ∠ADC?

3. **Performance Task 2**

Group work

Divide the students into six groups.

Select higher-order-thinking problems for the application of each Circle Theorem by referring to BHSEC Mathematics Book I and divide the questions of each theorem among each group for the students to explore and solve.

Students can also explore the application of circle theorems in their daily experiences and provide two examples in each group.

Design an appropriate rubric for each performance task and record feedback and achievement based on the templates given in the annexure XIB-A1

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - History of Circle
- Definition of circle and locus - definition of circle and locus
- Components of basic circle - https://www.youtube.com/watch?v=Y5Z69g7A0eI
- Derivation of equation of circle - circle equation derivation
- Equation of circle in centre radius form- centre-radius form
- Determining centre and radius of a circle- general to centre-radius
- Theorem 1 - https://www.geogebra.org/m/NMMEsDAQ
- Theorem 2 - https://www.geogebra.org/m/YaQ3mEjY
- Theorem 3 - https://www.geogebra.org/m/bUJVWeYn
- Theorem 4 & 5 - https://www.geogebra.org/m/pSn7Jd76
- Theorem 6 - https://www.geogebra.org/m/NkK6vSVr
- Theorem 7 - https://www.geogebra.org/m/JjDd2TKw
- Theorem 8 - https://www.geogebra.org/m/wCGeV5ku
- Theorem 9 - https://www.geogebra.org/m/vpxurwwk
- Theorem 10 - https://www.geogebra.org/m/xZeN5f4D
- Theorem 15 - https://www.geogebra.org/m/u6Ev7bHg
- Theorem 16 - https://www.youtube.com/watch?v=3jQ_vWb3OFE
- Technological gadgets (Smart phone, projector and laptop).

G. Annexure

Refer 11 B-A1 for template to record achievement

Topic: XIB-E1 Measures of Central Tendency *[450 minutes]*

Introduction

Central Tendency is defined as the statistical measure that identifies a single value as representative of an entire distribution. It aims to provide an accurate description of the entire data. There are three main measures of central tendency: the mode, the median, and the mean. Each of these measures describes a different indication of the typical or central value in the distribution.However, the term is first found in the mid-1690s in the writings of Edmund Halley (1656-1742), and it has been used to summarise observations of a variable since the time of Galileo (1564-1642).

Source: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3127352/

Utility and Scope

Measures of Central Tendency provide a summary measure that attempts to describe a whole set of data with a single value that represents the middle or centre of its distribution. For instance, the measures of central tendency allow researchers to determine the typical numerical point in a set of data. The data points of any sample are distributed on a range from lowest value to the highest value. Measures of central tendency tell researchers where the centre value lies in the distribution of data.

Source: Applications of Mean, Median and Mode

A. Competency

- Demonstrate proficiency in determining the central tendencies for diverse data distributions through the application of various strategies.

B. Objectives

- Understand the meaning of central tendencies and their significance in data analysis.
- Know the different types of data distributions: simple distribution, grouped frequency distribution, and ungrouped frequency distribution.
- Determine central tendencies (Mean, Median, Mode) of each type of data distributions.

C. Essential Skills/Processes

- Defining key concepts
- Distinguishing differences
- Drawing inferences
  - Knowledge Application
- Problem Solving

D. Learning Experiences

- Ask questions on the definition of mean, median and mode that the learners have learned in lower classes to assess their prior knowledge.
- Elaborate why mean, median and mode are called as the measures of central tendencies.
  - Refer to the link: https://www.youtube.com/watch?v=6DYtC7lrVuY or BHSEC Mathematics Book I.
  - For in depth understanding of the central tendencies, allow learners to refer the following links:
    - ➢ https://www.youtube.com/watch?v=B1HEzNTGeZ4
    - ➢ https://www.youtube.com/watch?v=o8OHiGilABE
- Revise the type of distribution: simple distribution (Individual Observation), ungrouped distribution (Discrete series), and group distribution (Continuous Series).
- Allows students to recall the direct method of finding mean of simple distributions. Then introduce the concept of using an assumed mean to find the mean of a data, which is called the shortcut method.
  - Refer to the video https://www.youtube.com/watch?v=sFQmPKMwnpo explaining the concept of direct and shortcut method of finding the means of different data types.
- Discuss on the step-deviation method to find means of grouped data. Refer to the video https://www.youtube.com/watch?v=9yNPMVa0NQU or BHSEC Mathematics Book I.
  - Provide practice questions on finding the mean of different types of data distribution. Refer BHSEC Mathematics Book I exercise 23(a).

- Discuss the approach to calculating the median for simple data series, considering situations with both even and odd data counts. Refer to BHSEC Mathematics Book I, example 15.
- Discuss the steps involved in determining the median for discrete series, offering clarity on the concept of cumulative frequency. For a more comprehensive understanding, refer to BHSEC Mathematics Book I, example 20 or access the instructional video link: median of discrete series.
- Explain the formula to find the median of grouped data. Refer to BHSEC Mathematics Book I or refer to the link: https://www.youtube.com/watch?v=1b1Tnp79Emk.
- Allow students to explore the definition of mode and practice finding modes of simple distribution.
- Discuss on finding the mode of discrete series: by inspection method only. Refer to BHSEC Mathematics Book I and allow students to practise some relevant questions.
- Explain the formula to find the mode of grouped data. Refer to BHSEC Mathematics Book I, page number 23-38 or alternatively refer to the link: https://www.youtube.com/watch?v=R0Zgml8VHT8.
    - Provide some relevant questions for practice.

E. Assessment

Performance Task 1

Collect the marks of the students in the mathematics midterm exam and let students find mean, median and mode, by organising the data collected into different types: simple, discrete, and continuous. Based on the central tendencies obtained, ask students to describe and draw conclusions on the midterm performance.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIB-A1.

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics

- Introduction - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3127352/
- Utility and scope - Applications of Mean, Median and Mode
- Meaning of central tendencies:
    - https://www.youtube.com/watch?v=B1HEzNTGeZ4
    - https://www.youtube.com/watch?v=6DYtC7lrVuY
    - https://www.youtube.com/watch?v=o8OHiGilABE
- Shortcut method for mean:
    - https://www.youtube.com/watch?v=sFQmPKMwnpo
- Step-deviation method of mean:
    - https://www.youtube.com/watch?v=9yNPMVa0NQU
- Simple distribution mean, median and mode:
    - https://www.youtube.com/watch?v=zjHfAhcU6kE
- Discrete data mean, median and mode -
    - https://www.youtube.com/watch?v=fBG1T7OwGQc&t=15
- Computing median of discrete series:
    - https://www.youtube.com/watch?v=AuwPFjwloJc
- Median of grouped data:
    - https://www.youtube.com/watch?v=1b1Tnp79Emk
- Mode of grouped data:
    - https://www.youtube.com/watch?v=R0Zgml8VHT8
- Notes on central tendencies:
  https://byjus.com/mean-median-mode-formula/
- Technological gadgets (Smart phone, projector and laptop)

G. Annexure

Refer XIB-A1 for a template to record achievement

Topic: XIB-E2 Dispersion *[500 minutes]*

Introduction

Dispersion was originally proposed by R.A. Fisher (17 February 1890 – 29 July 1962) for the processing of the results of agricultural trials, aimed at establishing the conditions under which a given agricultural crop yields a maximal harvest. He was a British polymath who was active as a mathematician, statistician, geneticist and academic.

Source: https://protonstalk.com/statistics/measures-of-dispersion

Utility and Scope

Measures of Dispersion are used to estimate "normal" values of a dataset, measures of dispersion are important for describing the spread of the data, or its variation around a central value. It is usually used in conjunction with a measure of central tendencies, such as the mean or median, to provide an overall description of a set of data.

Source: https://protonstalk.com/statistics/measures-of-dispersion/

A. Competency

- Demonstrate the ability to measure the dispersion of various types of data using an appropriate method.

B. Objectives

- Understand the meaning of dispersion and its significance in data analysis.
- Know different methods to measure dispersion: quartile deviation, interquartile deviation, standard deviation, mean deviations and coefficient of quartile deviation.
- Compute quartile deviation, interquartile deviation and coefficient of quartile deviation.
- Compute standard deviation using various approaches.
- Compute mean deviation about mean or median.

C. Essential Skills/Processes

- Defining key concepts

- Distinguishing differences
- Drawing inferences
- Knowledge Application
- Problem Solving

D. Learning Experiences

- Recap the previous knowledge on central tendencies and their purposes in describing data. Refer to the link https://www.concept of central tendency to recap the concept of central tendencies.

- Explain the limitations of central tendencies in describing data, and the need for other ways to describe a data set.

- Introduce the term dispersion and its applications in analysing data. Refer to the link: https://www.concept of dispersion to know the meaning and concepts of dispersions.
  - Watch the video from the link: https://www.youtube.com/watch?v=hEErJHx--qY to understand the limitations of central tendencies and importance of dispersion.

- Discuss on finding range, interquartile range and semi-interquartile range of different data sets. Refer to BHSEC Mathematics Book I and BHSEC Mathematics Book II, or the following link: https://youtu.be/06lxbMpYxm4 .
  - Allow students to practise finding range, interquartile range and semi-interquartile range of different data sets. Refer to BHSEC Mathematics Book I, exercise 24 (a) and (b).

- Discuss on quartile deviation (Q.D.) and coefficient of quartile deviation (C.Q.D.), and their significance. Ensure that students understand the appropriate situations for applying these measures.
  - Refer to BHSEC Mathematics Book I and BHSEC Mathematics Book II, or the following links:
    - ➢ https://www.youtube.com/watch?v=hEErJHx--qY
    - ➢ https://www.quartile deviation
    - ➢ https://www.youtube.com/watch?v=qdJlHO6tLkw
  - Assign a few relevant questions on the quartile deviation and coefficient of quartile deviation for practice. Refer to BHSEC Mathematics Book II, exercise 15 (a).

- Discuss on mean deviation (M.D.) and Coefficient of mean deviation (C.M.D.). Refer https://www.youtube.com/watch?v=Kc1aiVApB8M or BHSEC Mathematics Book II. Ensure that students understand the appropriate situations for applying these measures.
  - Assign some relevant questions on the mean deviation and coefficient of mean deviation to practise. Refer BHSEC Mathematics Book II, exercise 15 (b).

- Explain the meaning of standard deviation and variance. Refer to BHSEC Mathematics Book I and BHSEC Mathematics Book II, or the web link: https://www.standard deviation and variance.

- Demonstrate calculating standard deviation for different data sets, simple distribution and frequency distribution using various methods, variance and coefficient of variation. Refer to BHSEC Mathematics Book I and BHSEC Mathematics Book II.
  - Assign a few questions on the standard deviations and variance for practice. Refer BHSEC Mathematics Book I, exercise 24 (b) and BHSEC Mathematics Book II, and exercise 15(c).

- Discuss on combined mean and standard deviation of combined mean.
  - Refer to BHSEC Mathematics Book II or the following links:
    - ➢ https://youtu.be/Djl6XbBsHLA
    - ➢ https://youtu.be/tM9cQW97ZkY
  - Assign a few questions on the combined mean for practice. Refer to BHSEC Mathematics Book II, exercise 15(d).

E. Assessment

Performance Task 1

Collect the marks of the students in the mathematics midterm exam and let students find range, interquartile range, semi-interquartile range, quartile deviation and coefficient of quartile deviation by organizing the data collected into different types: simple, discrete and continuous. Based on the dispersion value obtained, ask students to describe and draw conclusions on the midterm performance.

Assess, provide feedback and record achievement based on the template given in the annexure 11B-A1.

Performance Task 2

For the data collected above, let students find standard deviation and variance by organizing the data into different types: simple, discrete and continuous. Based on the dispersion value obtained, ask students to describe and draw conclusions on the data collected.

Assess, provide feedback and record achievement based on the template given in the annexure 11B-A1.

Performance task 3

Collect the marks of the students in the test and midterm examination. Now there are two different sets of data in hand. Ask students which method of measuring dispersion would be best to compare the performance of students in test and midterm exam. Students should know that they must use any one of the deviations and coefficient of deviations.

Ask students to find the quartile deviation, coefficient of quartile deviation, mean deviation, and coefficient of mean deviation, variance and coefficient of variance.

Assess, provide feedback and record achievement based on the template given in the annexure XIB-A1.

F. Resources

- BHSEC Mathematics Book I
- National School Curriculum Framework for Mathematics
- Introduction - https://protonstalk.com/statistics/measures-of-dispersion
- Utility and scope - https://protonstalk.com/statistics/measures-of-dispersion/
- Concepts on central tendency - https://www.concept of dispersion
- Limitation and importance of central tendency https://www.youtube.com/watch?v=hEErJHx--qY
- Finding range - https://youtu.be/06lxbMpYxm4
- Finding quartile deviation for simple distribution- https://www.youtube.com/watch?v=hEErJHx--qY

https://www.quartile deviation

https://www.youtube.com/watch?v=qdJlHO6tLkw

- Mean deviation (M.D.) and Coefficient of mean deviation (C.M.D.) - https://www.youtube.com/watch?v=Kc1aiVApB8M
- Meaning of standard deviation and variance - https://www.standard deviation and variance
- Finding combined mean - https://youtu.be/DjI6XbBsHLA https://youtu.be/tM9cQW97ZkY

G. Annexure

Refer XIB-A1 for template to record achievement

Topic: XIB-E3 Probability *[600 minutes]*

Introduction

Probability is the chance that something will happen. How likely it is that some event will occur. Sometimes we can measure a probability with a number like "10% chance", or we can use words such as impossible, unlikely, and possible, even chance, likely and certain. Example: "It is unlikely to rain tomorrow".

A gambler's dispute in 1654 led to the creation of a mathematical theory of probability by two famous French mathematicians, Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665). Probability, chance and randomness have been around since the ancient days. They could be found in fortune telling, games of chance, philosophy, law, insurance, and errors of prediction in astronomy and medicine (Hald, 1990). There is evidence that in the late 15th century and the early 16th century, mathematicians started to experiment with the idea of probability. The link helps to learn additional history of probability.

Source: http://history of probability

Utility and Scope

Probability is an essential tool in applied mathematics and mathematical modelling. Moreover, probability is used in daily life to make decisions when you don't know for sure what the outcome will be. Most of the time, you won't perform actual probability problems, but you'll use subjective probability to make judgement calls and determine the best course of action. For example, to check the weather forecast, playing cards, lottery tickets, etc.

Source: https://www.probability-real-life-examples/

A. Competency

- Demonstrate the ability to calculate probabilities for various events by applying the laws of probability and set theory.

B. Objectives

- Know the meaning of basic terms of probability: random experiments, outcomes, sample space, sample point and events.

- Know different types of events: simple event, compound event, sure event, impossible event, exhaustive events, mutually exclusive events, not mutually exclusive events, and complementary events.
- Solve simple probability problems by drawing sample space diagrams, outcome charts or sets.
- Solve probability problems involving 'at least', 'at most', 'exactly', and 'not all'.
- Differentiate between dependent and independent events, and calculate probabilities using the addition (OR) rule and multiplication (AND) rule.

C. Essential Skills/Processes

- Reading Comprehension
- Interpretation Information
- Information Recall
- Knowledge Applying
- Analysing
- Critical Thinking

D. Learning Experiences

- Revise terminologies related to probabilities learned in lower classes.
  - Refer https://www.youtube.com/watch?v=KzfWUEJjG18&t=297s to learn the basic concepts of probability.

- Allow students to watch the video given in the link: https://www.youtube.com/watch?v=SnzUoLWeFkA to understand the concept of random experiments, outcomes, sample space, and types of events.

- Recapitulate the set theory concepts learned by students in class IX and draw connections to the representation of sample points, events, and outcomes through a set.
  - Refer to BHSEC Mathematics Book II or explore the following web links:
    - ➢ https://www.youtube.com/watch?v=1WONKtD2-Yw (set theory)
    - ➢ https://www.youtube.com/watch?v=719OsAWmq0A (representing sample space, events and outcomes in sets)
    - ➢ https://www.youtube.com/watch?v=fQqQCVkY_Ig (Intersection and union)
  - Let students solve questions from exercise 18 (a), BHSEC Mathematics Book II, to check understanding on random experiments, outcomes, sample space, and types of events.

- Explain the probability of an event and failure of the event. Refer to BHSEC Mathematics Book II or the web link: https://www.youtube.com/watch?v=0T-CaQCiSf4.

- Discuss the concept of 'odds' against the happening of an event. Refer to BHSEC Mathematics Book II or the link: https://www.youtube.com/watch?v=Vu4x2DKn12g&t=1s.
  - Assign a few probability questions for practice. Refer to BHSEC Mathematics Book II, exercise 18(b), or the link : https://www.indiabix.com/aptitude/probability/.

- Explain the meaning of 'at least', 'exactly', 'at most' and 'not all' terms and solve related problems.
  - Assign relevant questions from BHSEC Mathematics Book II, exercise 18(c) and 18(d), or the link: https://www.indiabix.com/aptitude/probability/ .

- Explain the 'addition rule' of probability (**mutually exclusive** events and **not mutually exclusive**). Discuss the addition theorem on probability and related set notations with the Venn diagrams or by drawing sample space diagrams and outcome charts.
  - Refer combining probability part in BHSEC Mathematics Book II or check the following web links:
    - ➢ https://www.addition rule-video1
    - ➢ https://www.addition rule-video2
    - ➢ https://www.addition rule-video3
  - Assign a few questions for practice. Refer to BHSEC Mathematics Book II, exercise 18(e) and 18(f), or the web link: https://www.quiz/addition-rule-probability.

- Recall the definition of dependent and independent events learned in lower classes to introduce the 'multiplication rule'.
  - Discuss the multiplication theorem on probability and related set notations with Venn diagrams or by drawing sample space diagrams and outcome charts. Refer to BHSEC Mathematics Book II or the following links:
    - ➢ https://www.1. multiplication rule
    - ➢ https://www.2. multiplication rule
    - ➢ https://www.3. multiplication rule

- Discuss a few examples involving the multiplication rule and assign a set of practice questions. Refer to BHSEC Mathematics Book II, exercise 18(g), or the web link: https://www.practice question.pdf.

E. Assessment

Performance Task 1

In a single throw of three fair dice, find the probability of getting

i. a total of 6
ii. a triplet or a total of 6
iii. a total of at most 6
iv. a total of at least 6
v. a total of 6 or at least 6.
vi. the same number on all the dice.
vii. not getting the same number on all the dice.

Assess, provide feedback and record achievement based on the template given in the annexure XIB-A1

Performance Task 2

Place Based Approach
Take students to a basketball court of your school, and let each student shoot a basketball 7 times. Let the individual student record their own shots. Using their records:

i. Ask students to find the probability of making at least 2 shots if they shoot four times continuously.
ii. Instruct students to get into a group of three members and ask them to find the probability that one of them will make a shot, if they shoot once each.

Assess, provide feedback and record achievement based on the template given in the annexure XIB-A1

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction of probability- http://history of probability
- Utility and scope- https://www.probability-real-life-examples/
- Concepts of probability- https://www.youtube.com/watch?v=KzfWUEJjG18&t=297s
- Concepts of probability - https://www.youtube.com/watch?v=0T-CaQCiSf4

- Set theory - https://www.youtube.com/watch?v=1WONKtD2-Yw
- Set theory - https://www.youtube.com/watch?v=719OsAWmq0A
- Set theory - https://www.youtube.com/watch?v=fQqQCVkY_lg
- Probability events-https://www.youtube.com/watch?v=0T-CaQCiSf4
- Meaning of 'odds' against the happening of an event-https://www.youtube.com/watch?v=Vu4x2DKn12g&t=1s
- Practice questions-https://www.indiabix.com/aptitude/probability/
- Addition rule- https://www.addition rule-video1
- Addition rule - https://www.addition rule-video2
- Addition rule - https://www.addition rule-video3
- Practice question on addition rule - https://www.quiz/addition-rule-probability
- Multiplication rule-https://www.1. multiplication rule
- Multiplication rule-https://www.2. multiplication rule
- Multiplication rule-https://www.3. multiplication rule
- Practice question on multiplication rule- https://www.practice question.pdf

G. Annexure

Refer XIB-A1 for template to record achievement

Instructional Guide
Class XII
(BMT)

Topic: XIIB-A1 Matrices *[750 minutes]*

Introduction

A matrix (plural matrices) is a rectangular array or table of numbers, symbols, or expressions, arranged in rows and columns, which is used to represent a mathematical object or a property of such an object. Only gradually did the idea of the matrix as an algebraic entity emerge. The term matrix was introduced by the 19th-century English mathematician, James Sylvester (1814–1897), but it was his friend the mathematician Arthur Cayley (1821-1895) who developed the algebraic aspect of matrices in two papers in the 1850s.

(Teacher can explore further for interesting history).

Source: https://www.youtube.com/watch?v=s4VFKg8OKTM

Utility and Scope

Matrices have widespread applications in various real-world fields due to their ability to efficiently represent and manipulate complex data. Here are some examples of how matrices are applied in different domains:

1. **Computer Graphics:**

Matrices are extensively used in computer graphics to represent transformations, such as translation, rotation, and scaling, enabling the creation and manipulation of 2D and 3D graphics.

2. **Data Analysis and Statistics**:

In statistics, matrices are used to represent multivariate data sets, and techniques like covariance matrices are employed to analyse relationships and variability between variables.

3. **Engineering**:

Structural engineers use matrices to analyse and design complex structures by representing them as systems of linear equations. Finite element analysis involves extensive matrix calculations to model and analyse structural behaviour.

4. **Physics**:

Matrices play a vital role in quantum mechanics, where they represent operators corresponding to physical observables. The manipulation of matrices is essential for describing the behaviour of quantum systems.

5. **Computer Science and Information Technology**:

Matrices are used in cryptography for encryption and decryption algorithms. Linear algebra, including matrix operations, is fundamental in various algorithms and data structures.

6. **Economics and Finance**:

Input-output matrices are used to represent economic relationships between different sectors in an economy. Markowitz's portfolio theory utilises matrices to optimise investment portfolios.

7. **Electrical Engineering**:

Matrices are employed in circuit analysis to model electrical networks and solve systems of linear equations. They are also used in control systems for analysing and designing dynamic systems.

8. **Geography and GIS (Geographic Information Systems)**:

Matrices are used to represent spatial data and transformations in GIS applications. They help analyse and visualise geographic information.

9. **Machine Learning and Data Mining:**

Matrices play a crucial role in machine learning algorithms, especially in areas like linear regression, neural networks, and dimensionality reduction. Matrices are used to represent datasets and model parameters.

10. **Medical Imaging:**

Matrices are utilised in medical imaging for tasks like image reconstruction, where they represent the transformation of data from the spatial domain to the image domain.

11. Environmental Science:

Matrices are used in environmental modelling to represent the interactions between different components of ecosystems, such as the transfer of nutrients in a food web.

For more information, refer to the link:
https://www.vedantu.com/maths/application-of-matrices

A. Competencies

- Show understanding of matrices and apply matrix multiplication to solve practical problems.
- Demonstrate the ability to solve real-life problems with simultaneous equations in two or three unknowns using the matrix method.

B. Objectives

- Identify the types of matrices based on the number of elements, their arrangement and order.
- Solve problems involving addition, subtraction and multiplication of matrices.
- Apply matrix multiplications in solving real-life problems.
- Find transpose, adjoint and inverse matrices.
- Solve real-life problems involving systems of equations with two and three variables using matrices.
- Examine the consistency of a given system of equations.

C. Essential Skills/Processes

- Reading Comprehension
- Knowledge Application
- Reasoning
- Conceptualising
- Problem Solving

D. Learning Experiences

- Conduct pre-assessment on the Matrix. Let students complete the questions given in the worksheets for self assessment.
  - Worksheet on matrix addition and subtraction
  - Worksheet on Matrix multiplications

- Define Matrix from the relevant sources. Then, briefly describe the types of matrices including rectangular, row, column, square, diagonal, scalar, null matrix, unit matrix or identity matrix. Refer BHSEC Mathematics Book II on the topic "Types of Matrices" or find relevant resources online.

- Let students explore operations on matrices: addition, subtraction & multiplication of matrices to complete the problem solving.
  - Assign relevant questions on operation of matrix from the BHSEC Mathematics Book II.

- Display the video from the link: https://www.youtube.com/watch?v=P5GJJ02OG08 on properties of addition and multiplication of matrices. Instruct them to make notes from the video or refer to BHSEC Mathematics Book II.

  - Suggested questions - Assign question 2 & 7 from Exercise 3(b), BHSEC Mathematics Book II, to check their understanding on properties of addition.

  - Assign question 3, 4 & 5 from Exercise 3(b), BHSEC Mathematics Book II, and evaluate their understanding on properties of multiplication.

- Design a group activity on Application of Matrix Multiplication.
  - Encourage students to work collaboratively within their assigned groups to explore, solve, and interpret the problems. Refer BHSEC Mathematic Book II to assign questions on Application of Matrix Multiplication.
  - Students will then present their work in the classroom for assessment and feedback.

- Display a video given in the web link: Transpose-of-matrix that contains methods to find transpose matrices of order 2 and 3.

  - Allow students to define the transpose of a matrix after watching the video and assign practice questions to enhance their understanding.

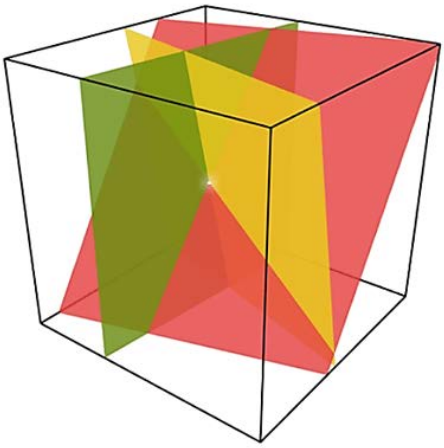
  - Suggested questions - Assign question 2, 3, 4 and 7 from exercise 3(e), BHSEC Mathematics Book II.

- Demonstrate how to find the adjoint matrix of order 2 and 3.

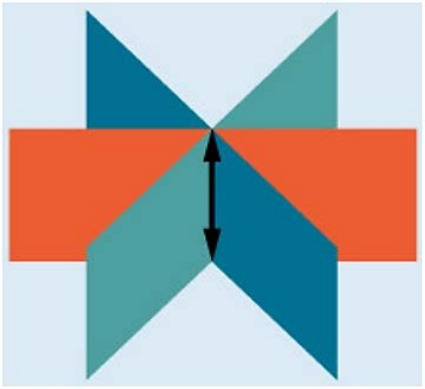
  - Refer to the web link: https://www.youtube.com/watch?v=hiuqyvR-f_4).

  - Suggested question: Question 1 from exercise 3(g), BHSEC Mathematics Book II.

- Project the video from the link: https://www.youtube.com/watch?v=AMLUikdDQGk on the inverse of a matrix.
    - Discuss on finding the inverse of matrices of order 2 and 3. Refer the web links:
    - inverse of order 2 matrix on finding the inverse of 2x2 matrix.
    - inverse of order 3 on finding the inverse of 3x3 matrix.
    - Assign questions from the link: Practice Questions for Inverse Matrices or refer BHSEC Mathematics Book II.
- Demonstrate the processes to solve a system of linear equations using Martin's Rule. Clarify all the steps involved, and explain the meaning of the solutions.
    - Refer to the link: https://www.youtube.com/watch?v=NNmiOoWt86M.
    - Use *geogebra*, *maple, mathematica* or other relevant software and display the solutions of simultaneous equations graphically. Check the video https://www.youtube.com/watch?v=AkMogY8p0Qo to learn how to graph equations of 3-D planes in Geogebra.
- Assign simultaneous equations having 2 and 3 variables using Martin's rule (refer BHSEC Mathematics book I and II or relevant online resources).
    - Reflective question: Display a system of 2 linear equations on the board and instruct students to use all the methods they have learned in lower classes (substitution, elimination, and comparison), determinant's method (class XI) and the matrix method to find the solutions of the equations.
    - Allow students to write a reflection on use of each method and justify which and why they would use it to solve each type of system.
- Explain the meaning of the inconsistent system in terms of the solutions of the system and its geometrical meaning using relevant software like *geogebra* or any other 3-D graphing software.
- The consistent and inconsistent systems can be graphically shown as below:



Consistent system with unique solution



Consistent system with infinite solutions

Inconsistent system with no solution

- Display a flow chart explaining the steps to check for consistency and solve for an infinite number of solutions.
  - Allow students to write their own interpretation of the flowchart and let volunteers read their interpretations to make sure all the students are on the same page.
  - Refer to BHSEC Mathematics book II to assign questions to students.
- Demonstrate solving of real world application word problems involving systems of equations.

- Refer BHSEC mathematics book II question 26 of the exercise 3(i) as a sample question. Teachers can search for relevant questions from other sources, or refer to Practise Word Problem for sample questions.

E. Assessment

Performance Task 1

Design about three real life practical questions where students will apply their knowledge of multiplying matrices to solve the questions. Refer the sample questions given below:

Sample:

Mr. Bokto sells different items in a town. His sales are recorded in the table given below:

	Khabzey (pkt.)	Dzaw (pkt.)	Seep (pkt.)
Monday	5	7	10
Tuesday	3	6	9
Wednesday	6	8	11
Thursday	2	2	8
Friday	7	1	5
Saturday	5	0	7

If one packet of khabzey costs Nu 50, one packet of Dzaw costs Nu 80, and one packet of Seep costs Nu 250, find the total amount he has earned in that week.

Performance Task 2

Place Based Approach

Design a place-based task where students will need to write a real life situation into a system of linear equations, and solve using the matrix method.

Sample

- Put the students into groups of three members each.
- Talk with a shopkeeper beforehand to not mention the price of each item to the students.
- The groups will visit any nearby shop or school canteen, and purchase three different items as follows:

- 1st Member: purchase any number of three different types of items worth Nu 10.
- 2nd Member: purchase a different number of the same types of items purchased by 1st member worth Nu 15.
- 3rd Member: purchase a different number of the same types of items purchased by the 1st member worth Nu 20.

- The group will come together and formulate the situation into a system of equations. Then they will solve using the matrix method to determine the price of each item.

OR

- Divide the students into four to five groups.
- Pack three different items each, such as pencils, erasers, and sharpeners, in three plastic bags. The number of each item can be different in each plastic.
- Provide three plastic bags to each group, and remind them not to open the bags.
- Instruct the students to go to the science lab, weigh their bags, and determine the weight of each item by solving a system of equations using the matrix method.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure.

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction of Matrices - https://www.youtube.com/watch?v=s4VFKg8OKTM
- Utility and Scope of Matrix- https://www.vedantu.com/maths/application-of-matrices
- Worksheet of Matrices - Worksheet on matrix addition and subtraction
- Worksheet on matrices - Worksheet on Matrix multiplications
- Matrix Multiplication Properties - https://www.youtube.com/watch?v=P5GJJ02OG08
- Transpose of Matrices - https://www.youtube.com/watch?v=g_Rz94DXvNo
- Adjoint of a matrix - https://www.youtube.com/watch?v=hiuqyvR-f_4
- Inverse of 2x2 matrix: https://www.youtube.com/watch?v=HYWeEx21WWw

- Inverse of 3x3 matrix: https://www.youtube.com/watch?v=xfhzwNkMNg4
- Practice questions on inverse matrix: Practice Questions for Inverse Matrices
- System of linear equations - https://www.youtube.com/watch?v=NNmiOoWt86M.
- Graphing in 3D Plane - https://www.youtube.com/watch?v=AkMogY8p0Qo
- Word Problem: Practice Word Problem
- Technological gadgets for learning (smart phone, desktop, laptop...)

G. Annexure

i. Template to record assessment

Strand(s): XIIB-A1	**Topic(s): Matrix**
Competency: Demonstrate an understanding of matrices and apply matrix multiplication in real-life problems	

Name of the student	**Level of achievement**				
	Beginning	**Approaching**	**Meeting**	**Advancing**	**Exceeding**

ii. Assessment Tool Sample: Rubrics.

*(**Note:** This is just a sample of one of the assessment tools. Teacher must design other appropriate assessment tools as per the competency and performance tasks)*

	Beginning	**Approaching**	**Meeting**	**Advancing**	**Exceeding**
Completion	Several of the problems are not completed.	About three of the problems are completed.	About two of the problems are completed.	One of the problems is not completed.	All problems are completed.
Neatness and Organization	The work appears sloppy and unorganised. It is hard to know what	The work is presented in an organised fashion but may be hard to read at times.	The work is presented in a neat and organised fashion that is	The work is presented in a neat and organised fashion that is mostly easy to read.	The work is presented in a neat, clear, organised fashion that is easy to read.

	information goes together.		usually easy to read.		
Working with others	Students did not work effectively with others.	Students cooperated with others, but needed prompting to stay on-task.	Student was engaged with a partner/group but had trouble listening to others and/or working cooperatively.	Student was engaged with a partner/group, listening to some suggestions of others and working quite cooperatively throughout the lesson.	Student was engaged with a partner/group, listening to suggestions of others and working cooperatively throughout the lesson.
Mathematizing the problem	The student did not attempt to mathematize the problem	Student mathematics the problem mostly incorrectly	Students wrote part of the system of linear equations correctly.	Students wrote most parts of the system of linear equations correctly.	Student wrote the whole system of linear equations accurately
Accurateness of application of Cramer's rule	Student did not attempt to solve the system of linear equations	Student solved only 1 determinant out 4 correctly	Student solved only 2 determinants out of 4 correctly	Student solved only 3 determinants out of 4 correctly	Student solved all the 4 determinants correctly
Interpretation of the solutions	Student did not attempt to interpret the solutions	Student's interpretation of the solutions was mostly incorrect	Student's interpretation of the solutions was partially correct	Student's interpretation of the solutions was mostly correct	All the interpretation of the solution was correct

Topic: XIIB-A2 Annuities *[750 minutes]*

Introduction

Every day we witness a number of transactions taking place like sale and purchase of goods such as refrigerators, washing machines, television sets, cars, etc., hire and purchase lands and buildings. Some of these things are so expensive that it may not be possible to pay the total cost all at once and therefore, these transactions may involve payments in instalments over a period of time. These instalments are so determined that the seller is more than fully compensated for his waiting time and are calculated on the basis of compound interest.

Similarly, the insurance companies fix the insurance premiums in equal instalments at equal intervals of time. The bank loans are also paid back in equal instalments at equal intervals of time and they are calculated at compound interest.

An annuity is a sequence of equal payments made at equal intervals of time, with compound interest on these payments. These intervals may be a year, a half year, a month, etc. Additionally, annuities can trace their origins back to Roman times. Contracts during the Emperor's time were known as annua, or "annual stipends" in Latin. Back then, Roman citizens would make a one-time payment to the annua, in exchange for lifetime payments made once a year. [Teacher can explore the related history of annuities].

Source: https://www.investopedia.com/terms/a/annuity.asp

Utility and Scope

Annuities find practical applications in various fields due to their ability to provide a steady stream of income over a specified period or for the lifetime of an individual. Here are real-life applications of annuities in different fields:

1. Retirement Planning:

Immediate Annuities: Individuals can convert a lump sum of retirement savings into an immediate annuity, ensuring a guaranteed income stream throughout their retirement.

2. **Insurance Industry**:

Structured Settlements: In legal cases where individuals receive compensation for injuries or damages, annuities can be used to provide a series of payments rather than a lump sum, ensuring long-term financial stability.

3. **Education Funding**:

Education Savings Annuities: Parents or guardians can use annuities to save for a child's education, providing a predetermined income stream to cover tuition or other expenses.

4. **Real Estate**:

Land Lease Annuities: In real estate, annuities can be used for land lease agreements, providing a steady income to landowners over a specified period.

5. **Pensions and Employee Benefits**:

Defined Benefit Pension Plans: Many pension plans utilise annuities to provide retired employees with a consistent income for the rest of their lives.

6. **Lottery Winnings**:

Annuity Payments: Some lottery winners choose to receive their winnings as annuity payments over time instead of a lump sum, providing a more stable and predictable income.

7. **Business Succession Planning**:

Buyout Agreements: In business, annuities can be part of buyout agreements, providing a predictable income stream to the selling party over time.

8. **Endowment Funds**:

Endowment Annuities: Institutions such as universities may use annuities as part of their endowment funds, ensuring a consistent income to support ongoing operations and initiatives.

A. Competency

- Showcase comprehension of annuities by recognizing their future and present values, and adeptly solve associated problems.
- Demonstrate practical knowledge of annuities in various financial sectors like banking, insurance, and pensions, showcasing their real-world applications.

B. Objectives

- Define key terms associated with annuities to establish a foundational understanding of the concepts.
- Differentiate between the present and future value of annuities, and proficiently apply relevant formulas to solve associated problems.
- Apply the understanding of future value of annuities to solve practical problems in real-life situations, including recurring deposits, insurance schemes, and instalment plans.
- Understand and apply the present value of annuities to solve real-life problems such as computing loan Equated Monthly Instalments (EMI) and determining loan amounts.
- Comprehend and solve problems related to annuity perpetuity, and apply this knowledge in practical scenarios.

C. Essential Skills/Processes

- Reading Comprehension
- Interpreting Information
- Information Recall
- Knowledge Application
- Problem Solving

D. Learning Experiences

- Introduce the terminologies related to annuities, and explain their utilities in real-life applications.
  - Let students explore the types of annuities and ask to give relevant examples: Annuity certain, annuity contingent and annuity perpetual.
  - Refer https://www.youtube.com/watch?v=lW36jYkv0lM to know the basics of annuity and https://www.youtube.com/watch?v=jQk13MA1wm4 to know the different types of annuities.
- Elaborate the classes of annuities with relevant examples: Annuity Due, Immediate/Ordinary Annuity, and Deferred Annuity.
- Clarify the differences between the future value and present value of annuity. Refer BHSEC Mathematics Book II or refer the notes provided in Present value vs Future value of annuity.

- Derive formula for future value of annuity due and annuity certain. Solve relevant examples and discuss their applications in the banks and insurances. Refer to BHSEC Mathematics Book II or the video link: Future Value Formula Derivation to learn about the derivations. Find some other sources on derivation of the formulas.
- Relate the 'future value of an annuity due' with the 'Recurring Deposits' in the Banks of Bhutan.
- Open one of the websites of any banks of Bhutan, and familiarise students with the terms related to banking. Show the types of recurring deposits, their rate of interests as per the terms, etc.
- Demonstrate the use of formulas in practical questions (refer to BHSEC Mathematics Book II or use the web links:
    - https://www.youtube.com/watch?v=izMPyNixoJM contains a video explaining Future Value for immediate annuity with two examples.
    - https://www.youtube.com/watch?v=joBu9TnFngQ&t=26s contains video explaining Future value of annuity due with an example.
    - https://www.youtube.com/watch?v=q-cfrQEiXGI contains an example on how to find the instalment needed to be deposited to have a certain future value.
- Provide enough time for the students to get acquainted with word problems and in applying the formula.
- Relate the 'future value of an annuity due' with the 'Recurring Deposits' in Banks of Bhutan: $A = \frac{a}{i}(1+i)\left[(1+i)^n - 1\right]$.
    - The future value or Amount (A) is called the Maturity Amount in recurring deposits.
    - The amount of annuity, "a", is the instalment.
    - The "i" is the interest (note: Banks in Bhutan usually compute interests monthly).
    - Example: Ms. Pema wants to open a recurring account in one of the banks of Bhutan. She wishes to deposit Nu 1000 every month for two years. If the bank pays an interest rate of 7.25% p.a., find the maturity amount she would receive after two years.

Using Compound Interest:

$$a = Nu\ 1000, i = \frac{0.0725}{12} = 0.00604,\ n = 24.$$

$$\text{Maturity Amount (A)} = \frac{1000}{0.00604}(1+0.00604)\left[(1+0.00604)^{24} - 1\right]$$

$$= Nu\ 25,898.77$$

- Open one of the banks' websites and cross check the maturity amount or use one of the links below:
  - https://www.paisabazaar.com/rd-recurring-deposit-calculator/
  - https://www.tbankltd.com/calculator
  - https://www.bnb.bt/calc/
- Provide some questions and allow students to practise the application of the formulas in groups. Refer to exercise 19(a), BHSEC Mathematics Book II.
- Introduce 'Present Value or Present Worth of an annuity', and explain the formulas of finding present values of immediate, due and perpetual annuity. Refer to BHSEC Mathematics Book II.
- Demonstrate the derivation of the formula. Refer the BHSEC Mathematics Book II or the following links:
  - https://www.youtube.com/watch?v=deTCMwdtbv0&t=256s
  - https://www.youtube.com/watch?v=wS2WPlHu9jY, explaining the relation between future value and present value, and derives the formula for present value).
  - https://www.youtube.com/watch?v=88-B0vXTTIU , explaining the meaning of perpetual annuity and its formula to find the present value of a perpetual annuity with an example.
- Demonstrate the use of formulas in practical questions (refer BHSEC Mathematics Book II, and assign relevant questions from exercise 19(b))
  - May use the web link: https://www.youtube.com/watch?v=RU-osjAs6hE which contains a video explaining two examples of finding present value for immediate annuity using different methods.

- May use the web line: https://www.youtube.com/watch?v=Tr6SmLk4Qc4 which contains a video explaining the present value of an annuity due with two examples.

- Relate the concept of "Present Value of Immediate annuity" with the loan systems in banks:

$$P = \frac{a}{i}\left[1-(1+i)^{-n}\right]$$

- Based on the loan amount, the banks will calculate the monthly payments using the present value formula.
  - 'P' is the loan amount (the present value)
  - 'a' is the monthly payment. Banks call it EMI - Equated Monthly Instalment).
  - 'i' is the interest (Note: Banks in Bhutan compute interests monthly).
  - Allow students to see the application of present value concepts in calculating the EMI of loans. EMI formula can be derived from the Present Value formula:
    - $P = \frac{a}{i}\left[1-(1+i)^{-n}\right]$
    - $EMI\,(a) = \frac{Pi}{\left[1-(1+i)^{-n}\right]}$
  - Find the EMI of a loan using relevant examples.
- Allow students to work in groups to utilise the formulas in solving different types of questions under present value: (Refer BHSEC Mathematics Book II, and assign relevant questions from exercise 19(b). You may design some application questions in Bhutanese context and assign them to the students.

E. Assessment

Performance Task 1

Placed Based Approach and Role Play

Take students outside with a book, pen, and calculator. Five students will act as bankers and any one of the students will act as a customer.
The customer wishes to open a recurring deposit account and tells the amount of instalment and the number of years. The bankers shall then calculate the maturity amount for the customer. The activity will continue until all the students get a chance to act as bankers.

Performance Task 2

Follow the same procedure of performance task 1. This time, the customer wishes to loan some amount of money from the bank. The customer will tell the amount and the number of years he will take to clear the loan. The bankers shall then calculate EMI.

- https://www.paisabazaar.com/rd-recurring-deposit-calculator/
- https://www.tbankltd.com/calculator
- https://www.bnb.bt/calc/

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIIB-A1.

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum Framework for Mathematics
- Introduction to Annuity - https://www.investopedia.com/terms/a/annuity.asp
- Annuity Concepts - https://www.youtube.com/watch?v=lW36jYkv0lM
- Types of Annuity - https://www.youtube.com/watch?v=jQk13MA1wm4
- Difference between Present and Future Value - Present value vs Future value of annuity
- Future Value of Annuity - Future Value Formula Derivation
- Future Value Examples - https://www.youtube.com/watch?v=izMPyNixoJM https://www.youtube.com/watch?v=joBu9TnFngQ&t=26s https://www.youtube.com/watch?v=q-cfrQEiXGI
- Banking online Calculators: https://www.paisabazaar.com/rd-recurring-deposit-calculator/ https://www.tbankltd.com/calculator https://www.bnb.bt/calc/
- Present Value - https://www.youtube.com/watch?v=deTCMwdtbv0&t=256s

https://www.youtube.com/watch?v=wS2WPlHu9jY

- Perpetual Annuity - https://www.youtube.com/watch?v=88-B0vXTTIU
- Present Value Examples - https://www.youtube.com/watch?v=RU-osjAs6hE https://www.youtube.com/watch?v=Tr6SmLk4Qc4

G. Annexure

Refer XIIB-A1 for template to record achievements

Topic: XIIB-A3 Discount *[500 minutes]*

Introduction

While purchasing an item, we come across various terms like cost price, marked price, discount, selling price. In order to increase the sale of goods, shopkeepers offer discounts to customers. The rebate or the offer given to the customers on the marked price of products is termed as discount.

In addition, the term discount is the reduction in the price of goods or services offered by shopkeepers at the marked price. This percentage of the rebate is usually offered to increase the sales or clear the old stock of goods. The List price or Marked price is the price of an article as declared by the seller or the manufacturer, without any reduction in price. Selling price is the actual price at which an article is sold after any reduction or discounts in the list price. "Off", "Reduction" are some common terms used to describe discounts. It should be noted that discount is always calculated on the marked price (List price) of the article.

Source: https://www.cuemath.com/commercial-math/discounts/

Refer https://www.youtube.com/watch?v=PnxLKhrz8xU to know the history of the bill of exchange or teacher could search from other sources.

Utility and Scope

These concepts can be used in day to day buying and selling, business and bill of exchange in trade.

A. Competencies

- Exhibit comprehension of various types of discounts and apply this knowledge in the context of trade and business transactions.
- Demonstrate understanding of the bill of exchange and apply its principles in the realms of trade, business, commerce, and economics.

B. Objectives

- Define and understand the terms related to trade discounts.
- Solve practical problems related to trade discounts in buying and selling.
- Define and understand the terms related to the Bill of Exchange.

- Solve problems related to the Bill of Exchange, and understand its applications in the trades and business.

C. Essential Skills/Processes

- Reading Comprehension
- Making Connections
- Defining key concepts
- Knowledge Application
- Interpreting information
- Analysing
- Problem Solving

D. Learning Experiences

- Conduct pre-assessment tasks on finding cost price, selling price, discount, and marked price. Refer to the link: worksheet on discounts or Understanding Mathematics Textbook for Class X.
- Reintroduce the terms related to trade discounts, and solve related problems. Refer to BHSEC Mathematics Book I exercise 26(a), or refer to link: reference to discounts and bankers gain topics.
- Explain the concept of true present worth and true discounts. These concepts are to be used in the Bill of Exchange. Assign some questions for the students to practise.
- Introduce the concept of Bill of Exchange. You may refer to BHSEC Mathematics Book I or Accountancy Textbook of class XII. Refer to this link: concept of bills of exchange to understand the concept of bill of exchange.
- Display this video https://www.youtube.com/watch?v=PnxLKhrz8xU to understand how the bill of exchange concept is used internationally.
- Allow students to explore the meaning of trade discount, present value, true discount, banker's discount, banker's gain, discounted value of bill, face value of bill and days of grace.
- Demonstrate processes in solving Bill of Exchange problems. Allow students to practise a few questions. Refer to the exercise 26(c) of BHSEC Mathematics Book I, and assign relevant questions. Or get the reference from the link: reference to discounts and bankers gain topics

E. Assessment

Performance Task 1

Design a place-based task where students will have to compute cost price, selling price, marked price, and discounts.

Sample

- Take students to a shop or school canteen. Or conduct a role-play in a shop and canteen is not available.
- Talk with a shopkeeper to give a discount on a certain item and let students calculate the discount percent.
- Choose any item, ask the shopkeeper for the selling price and his profit percent. Then let students find the cost price of the item.

Performance Task 2

Design a question which asks to determine banker's discount, true discount, banker's gain and the discounted value of the bill, and assign it as a class activity. Students will solve and present their answers to the class.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIIB-A1.

F. Resources

- National School Curriculum, Mathematics for PP – XII
- BHSEC Mathematics Book II
- Understanding Mathematics for class X
- Introduction to discounts:
  https://www.cuemath.com/commercial-math/discounts/
  https://www.youtube.com/watch?v=PnxLKhrz8xU
- Worksheet on Discount - worksheet on discounts
- Bill of Exchange concepts- https://www.youtube.com/watch?v=QbMDlJHS1Yw
- True discount vs Bankers discount- true discount vs bankers discount
- Bill of Exchange examples - https://www.youtube.com/watch?v=PnxLKhrz8xU
- Reference of Discount- reference to discounts and bankers gain topics

G. Annexure

Refer XIIB-A1 for template to record achievement

Topic: XIIB-A4 Permutations and Combinations *[800 minutes]*

Introduction

Permutation and combination form the principles of counting and they are applied in various situations. A permutation is a count of the different arrangements which can be made from the given set of things. In permutation the details matter, as the order or sequence is important. Writing the names of three countries {USA, Brazil, Australia} or {Australia, USA, Brazil) or { Brazil, Australia, USA} is different and this sequence in which the names of the countries are written is important. In combinations, the name of three countries is just a single group, and the sequence or order does not matter.

Sources: https://www.cuemath.com/data/permutations-and-combinations/

- https://www./science/permutation

https://www.predicting-blaise-pascal-pierre-de-fermat - to explore further about the history of permutation and combination.

Utility and Scope

Application Areas of Combinatorics: Permutations are frequently used in communication networks and parallel and distributed systems. Many communication networks require secure transfer of information, which drives development in cryptography and network security because of the increased use of internet information transfers. Encryption process involves manipulations of sequences of codes such as digits, characters, and words. Hence, they are closely related to combinatorics, possibly with intelligent encryption processes. Permutations of fast Fourier transforms are employed in speech encryption

Computer architecture: Design of computer chips involves consideration of possible permutations of input to output pins. Field-programmable interconnection chips provide user programmable interconnection for a desired permutation. Arrangement of logic gates is a basic element for computer architecture design

Computational molecular biology: This field involves many types of combinatorial and sequencing problems such as atoms, molecules, DNAs, genes, and proteins. One-dimensional sequencing problems are essentially permutation problems under certain constraints.

Pattern analysis: Patterns can have many forms; for example, visual images, acoustic signals, and other physical quantities such as electrical, pressure, temperature, etc., that appear in engineering problems. Some of these types of patterns can be associated with combinatorics. Computer music can be a specialised application domain of combinatorics of acoustic signals.

Source: https://www.use-of-permutation-and-combination (explore the web link further for more utilities of permutation and combination).

A. Competency

- Apply skills in using counting principles, P(n, r), and C(n, r) formulas to determine the number of arrangements and selections in various real-life situations.
- Exhibit the ability to analyse situations and determine the appropriate combinatorial approach.

B. Objectives

- Comprehend the fundamental principle of counting.
- Differentiate between permutations and combinations through illustrative examples.
- Solve permutation problems across various cases.
- Solve combination problems across various cases.
- Solve problems involving both permutations and combinations.

C. Essential Skills/Processes

- Reading comprehension
- Conceptualising
- Knowledge Application
- Analysing
- Reasoning
- Problem Solving

D. Learning Experiences

- Recapitulate on the concept of factorial; definition of $n!$ and solve one or two examples: $5!$, $9!$, etc. Or design a short pre-assessment quiz on the concept of factorial, to check the previous knowledge of students.

- Define and differentiate between permutations as arrangement and combinations as selection.
  - Allow students to manually count the number of combinations and permutations of 3 digits out of 4 given digits to understand the difference between permutation and combination.

- Discuss an example (refer to BHSEC Mathematics Book II from the topic: Fundamental Principle of Counting); suggested question - Ex. 1) and arrive at the 'Fundamental principle of counting' together with the students.

  - Refer to the link : https://youtu.be/3lmEqp8VhAU explaining the fundamental principle of counting. Instruct students to make notes.

  - Suggested learning activity - solve example 6 from BHSEC Mathematics Book-II, and allow students to list the outcomes using a tree diagram. Subsequently, collaboratively determine the total number of outcomes by applying the fundamental principle of counting. This exercise demonstrates two distinct methods for calculating the number of outcomes when tossing 3 coins.

- Explain the meaning of $P(n, r)$ and different methods to calculate permutation of n things taken r at a time (refer to BHSEC Mathematics Book II from the topic: Permutation).

  - Refer to the link: https://www.youtube.com/watch?v=viKDzyeCHr0 on different methods to calculate permutation of n things taken r at a time.

- Explain each type of permutation: restricted permutation, permutations of alike things, permutation of repeated things and circular permutations with examples (refer to BHSEC Mathematics Book II).

  - Suggestion: Refer the following the web links for further understanding:
    - ➢ https://youtu.be/XPPYYM6WCuE (contains a video explaining an example of formation of numbers when repetition is not allowed and when repetition is allowed.)
    - ➢ https://www.youtube.com/watch?v=tBQhcP9Zr2E

    (contains a video explaining an example of the arrangement of people on chairs.)
    - ➢ https://www.youtube.com/watch?v=L81vWvqGvJ8

(contains a video explaining an example of forming anagram (permutation of alike things).

➢ https://youtu.be/gxeP3PeA09I (contains a video explaining circular permutations with examples).

- Explain the meaning of $C(n, r)$ and the corollaries (refer to BHSEC Mathematics Book II) :
  - o $C(n, n) = 1$,
  - o $C(n, r) = C(n, n - r)$,
  - o $If\ C(n, x) = C(n, y) then\ either\ x = y\ or\ x = n - y,\ i.e.\ x + y = n$
  - o $C(n, r) + C(n, r - 1) = C(n + 1, r)$

  Discuss problems using the formula and corollaries of $C(n, r)$ with the students.

- Display and discuss the video in the web link to explain different methods of calculating combinations: https://combinations.com.

- Design a PowerPoint presentation on different types of combinations: distribution of different things into groups; open selection of items from different things and from alike things with examples.
  - o Discuss examples of each types of combinations from BHSEC Mathematics Book II (suggested questions - Ex. 44, 45, 47, 48, 50, etc).
  - o For supplementary, refer to the link: https://youtu.be/B6xDGgoSMJY on the selection of a group of boys and girls using combination.

- Allow students in pairs to examine the examples under 'permutations and combinations occurring simultaneously' (refer to BHSEC Mathematics Book II, examples 52 and 53).

E. Assessment

Performance Task 1

Design a place-based assessment allowing students to employ their knowledge of permutations and combinations in their immediate environment.

Place Based Approach
Sample
In groups, allow students to work on the following tasks:

1) At the school parking area, count the number of cars parked and total number of parking slots (including the empty ones).

a. Calculate in how many other ways the cars could have been parked?
b. In how many ways would all the big cars be always parked together?
c. How many ways can the cars be parked if one of these parking slots must be occupied by the school bus? Students must consider the possibility of 'alike things' if two or more cars are of the same model and colour.

Performance Task 2

Divide students into a group of 5 members and let them visit any of the school buildings where there are different flower pots available.

a. Let the groups find permutations by providing different cases such as: when two flower pots are always together, one flower pot is always in 3rd place, etc.
b. Let the groups find combinations by providing different cases such as: two flower pots are always selected, two flower pots are not selected, etc.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIIB-A1.

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction (permutation and combination) –https://www./science/permutation
- Introduction of permutation and combination- https://www.predicting-blaise-pascal-pierre-de-fermat
- Utility and scope of permutation and combination - https://www.use-of-permutation-and-combination
- Fundamental principle of counting - https://youtu.be/3lmEqp8VhAU
- Calculating Permutation - https://www.youtube.com/watch?v=viKDzyeCHr0
- Permutation of repetition - https://youtu.be/XPPYYM6WCuE
- Permutation - https://www.youtube.com/watch?v=tBQhcP9Zr2E
- Permutation - https://www.youtube.com/watch?v=L81vWvqGvJ8
- Circular permutation - https://youtu.be/gxeP3PeA09I
- Combination - https://combinations.com
- Combination - https://youtu.be/B6xDGgoSMJY
- BHSEC Mathematics Book II
- Self-Instructional Material
- Technological gadgets for learning (smart phone, desktop, laptop)

G. Annexure

Refer XIIB-A1 for template to record achievement

Topic: XIIB-B1 Differentiation *[700 minutes]*

Introduction

Differentiation is the process of finding the derivative *f'(x)* of a function *f*. It is a part of maths under the branch of Calculus. The derivative is the gradient of the tangent of the graph of *f* at the point *x*. The discovery of calculus is often attributed to two men, Isaac Newton and Gottfried Leibniz, who independently developed its foundations. Although they both were instrumental in its creation, they thought of the fundamental concepts in very different ways.

While Newton considered variables changing with time, Leibniz thought of the variables x and y as ranging over sequences of infinitely close values. He introduced dx and dy as differences between successive values of these sequences. Leibniz knew that dy/dx gives the tangent but he did not use it as a defining property. On the other hand, Newton used quantities x and y, which were finite velocities, to compute the tangent.
Neither Leibniz nor Newton thought in terms of functions, but both always thought in terms of graphs. For Newton the calculus was geometrical while Leibniz took it towards analysis.

Explore more on the history of differentiation and calculus. May use the suggested web link: https://www.youtube.com/watch?v=IMj5dgGWxSM.

Suggested web links: https://www.youtube.com/watch?v=BrH1fz-jZOo
https://www.youtube.com/watch?v=dfr23VIQPCo
https://www.youtube.com/watch?v=b7vIKO-uung&t=2s

Utility and Scope

Differentiation in mathematics, particularly in calculus, involves studying rates at which quantities change. It has a wide range of real-world applications and is crucial in various fields such as science, technology, business, and more. Here are some examples illustrating the application and scope of differentiation:

1. Physics and Engineering:

a) **Motion Analysis**: Differentiation is used to analyse the motion of objects. In physics and engineering, it helps calculate velocities, accelerations, and study the behaviour of particles.

b) **Electric Circuits**: In electrical engineering, differentiation is applied to analyse voltage and current changes over time in circuits.

2. **Computer Science and Technology**:

a) **Algorithm Analysis**: Differentiation helps analyse the efficiency and performance of algorithms, especially in terms of their time complexity.

b) **Signal Processing**: In image and audio processing, differentiation is used to analyse and enhance signals.

3. **Economics and Finance**:

a) **Marginal Analysis**: Differentiation is used in economics to analyse marginal cost, marginal revenue, and marginal utility, providing insights into optimal decision-making.

b) **Options Pricing**: In finance, differentiation is used in option pricing models, such as the Black-Scholes model, to determine the rate of change of option prices concerning various factors.

4. **Biology and Medicine**:

a) **Population Dynamics**: In biology, differentiation is used to model and analyse population growth and dynamics.

b) **Medical Imaging**: Differentiation plays a role in the analysis of medical images, such as identifying edges and contours in diagnostic imaging.

5. **Environmental Science**:

a) **Rate of Change in Environmental Variables**: Differentiation is used to study the rates of change in environmental variables, such as temperature, pollution levels, and ecosystem dynamics.

6. **Operations Research and Management**:

a) **Optimization**: Differentiation is a key tool in optimization problems, such as finding the minimum or maximum values of functions. This is applied in operations research and management decision-making.

b) **Supply Chain Management**: Differentiation helps optimise inventory levels, production rates, and distribution schedules.

7. **Statistics and Data Analysis**:

a) **Regression Analysis**: Differentiation is used in regression analysis to find the best-fit line that represents the relationship between variables.

b) **Time Series Analysis**: Differentiation is applied to study trends and patterns in time-series data.

8. **Telecommunications**:

a) **Signal Transmission**: In telecommunications, differentiation is used to analyse and optimise signal transmission, such as the modulation and demodulation of signals.

9. **Business and Marketing**:

a) **Marginal Cost and Revenue**: In business, differentiation helps analyse marginal cost and marginal revenue, aiding in pricing decisions.

b) **Market Research**:** Differentiation is applied in market research to analyse how variables such as pricing, advertising, and product features impact sales.

10. **Automotive and Transportation**:**

a) **Vehicle Dynamics**: Differentiation is used to analyse the motion and performance of vehicles, including aspects like acceleration, braking, and turning.

Explore more from the web link: utility and scope of differentiation.

A. Competencies

- Demonstrate an understanding of the meaning of differentiation and its real-life applications.

- Showcase skills in differentiating algebraic functions of various forms using appropriate rules.

B. Objectives

- Comprehend the concept of differentiation.
- Calculate the first and second derivatives of algebraic functions, including composite, implicit, and parametric functions.
- Apply differentiation to algebraic functions with respect to other functions.
- Evaluate the derivatives of a function at a specified point and its applications.

C. Essential Skills/Processes

- Defining key Concepts
- Making Connection
- Critical thinking
- Knowledge Application
- Problem Solving

D. Learning Experiences

- Derivative of algebraic functions:
  - Discuss the concepts of the following topics by selecting one or two examples: the meaning and geometrical interpretation of derivatives, differentiation from the first principle, a derivative of simple functions, a derivative of sums, differences, products and quotients of functions, and application of derivatives.
  - Suggestions: Discuss the following web links: https://youtu.be/N2PpRnFqnqY and https://www.general_concepts that contains general concepts on the topic.

- Algebraic Differentiation:
  - Demonstrate and discuss at least four or five relevant examples of algebraic differentiation. Explore examples from other sources or refer to BHSEC Mathematics Book II. Include the differentiation of the topics: i. Derivative of the algebraic sum of two functions, ii. differentiation of product of two functions, iii. derivative of the quotient of two functions, iv. function of a function or composite function and differentiating by using chain rule method.

- Refer the following links:
  - ➢ https://www.youtube.com/watch?v=x8j-4WYLM6Y (contains derivatives using difference quotient)
  - ➢ https://www.youtube.com/watch?v=4s7G7nkMYHM (contains derivatives using chain rule)
  - ➢ https://www.youtube.com/watch?v=n66nYARUyzs (contains derivatives of constant)
  - ➢ https://www.youtube.com/watch?v=7TXDubwGOSk (contains derivative using quotient rule)
  - ➢ https://www.youtube.com/watch?v=QeOTfHLG13c (contains derivative using product rule)
- Design some questions or refer to exercise 5(a) and 5(b), BHSEC Mathematics Book II, and assign some questions to practise.

- Discuss and explain the meaning of implicit functions.
  - Explore differentiation of implicit functions using the following links:
    - ➢ https://youtu.be/M0SMSWM2oZA
    - ➢ https://www.youtube.com/watch?v=Zo630jMysKk
  - Assign the questions from exercise 5(h), BHSEC Mathematics Book II.

- Define and explain parametric function. Let students explore from the online or other sources to understand why these functions are named parametric.
  - You may refer to the link: https://www.youtube.com/watch?v=kf2dZWqLnqE (contains derivatives of the parametric functions).
  - Assign questions from online sources or BHSEC Mathematics Book II, exercise 5(i).

- Discuss the definition of differentiation of a function with respect to another function. Compare with the parametric function and explain the similarities and differences.
  - Refer to the link: https://www.youtube.com/watch?v=M8iMROLjf-I on differentiating a function with respect to another function.
  - Provide examples and let students investigate the methods/ steps involved in differentiating functions with respect to other functions.
  - Assign questions from Exercise 5(j), BHSEC Mathematics Book II.

- Let students go to the IT lab and explore higher derivatives (successive differentiation). Share the video links: https://youtu.be/m-s_SfJaoc4 and https://www.youtube.com/watch?v=tBtF3Lr-VLk on higher derivatives.
    - Assign questions from BHSEC Mathematics Book II to check their understanding on higher derivatives.

E. Assessment

Performance Task 1

Design a homework task with questions involving simple derivatives and direct students to differentiate the functions using both methods of first principle and basic rules.
Students will also write a reflection on the use of the two methods.

Performance Task 2

Design a quiz on Google Form and bring students to the school IT laboratory to sit the quiz. Include questions on use of derivative rules to differentiate algebraic functions.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIIB-A1.

F. Resources

- National School Curriculum, Mathematics for PP – XII
- BHSEC Mathematics Book II
- Introduction to differentiation

https://www.youtube.com/watch?v=IMj5dgGWxSM
https://www.youtube.com/watch?v=BrH1fz-jZOo
https://www.youtube.com/watch?v=dfr23VIQPCo
https://www.youtube.com/watch?v=b7vIKO-uung&t=2s

- Utility and Scope - utility and scope of differentiation.
- Differentiation concepts -
    https://youtu.be/N2PpRnFqnqY
    https://www.general concepts
- Differentiation of quotient of two functions - https://www.youtube.com/watch?v=x8j-4WYLM6Y

- Differentiation using chain rule - https://www.youtube.com/watch?v=4s7G7nkMYHM
- Derivatives of constant- https://www.youtube.com/watch?v=n66nYARUyzs
- Quotient rule- https://www.youtube.com/watch?v=7TXDubwGOSk
- Product rule - https://www.youtube.com/watch?v=QeOTfHLG13c
- Differentiation of implicit functions
  https://youtu.be/M0SMSWM2oZA
  https://www.youtube.com/watch?v=Zo630jMysKk
- Differentiate parametric functions
  -https://www.youtube.com/watch?v=kf2dZWqLnqE
- Differentiation of a function with respect to other function - https://www.youtube.com/watch?v=M8iMROLjf-I
- Successive differentiation -
  https://youtu.be/m-s_SfJaoc4
  https://www.youtube.com/watch?v=tBtF3Lr-VLk

G. Annexure

Refer XIIB-A1 for template to record achievement

Topic: XIIB-B2 Maxima and Minima *[300 minutes]*

Introduction

Maxima and minima are known as the extrema of a function. Maxima and minima are the maximum or the minimum value of a function within the given set of ranges. The function, under the entire range, the maximum value of the function is known as the absolute maxima and the minimum value is known as the absolute minima. Pierre de Fermat (1626-1665) was one of the first mathematicians to propose a general technique, adequately, for finding the maxima and minima of functions. (Teacher can further explore a detailed history of Maxima and Minima).

Source: https://www.cuemath.com/calculus/maxima-and-minima/

Utility and Scope

There are numerous practical applications in which it is desired to find the maximum or minimum value of a particular quantity. Such applications exist in economics, business, and engineering. Finding maxima or minima also has important applications in linear algebra and game theory. For example, linear programming consists of maximising.

For further information on application of maxima minima in optimization and economics, please click link: Maxima - minima - optimization.pdf

A. Competency

- Demonstrate proficiency in employing the knowledge of turning points in determining the maximum and minimum point of a function and apply in business such as cost and revenue.

B. Objectives

- Understand the different turning points of a function.
- Apply higher-order derivatives to check for maximum, minimum, and inflexion points.
- Solve simple problems related to maxima and minima.

C. Essential Skills/Processes

- Reading Comprehension
- Analysing
- Making Connections

- Problem Solving

D. Learning Experiences

- Introduce the three types of turning points of a function: maxima, minima and inflexion.
  - Refer to the web link: https://www.youtube.com/watch?v=8j8hQNb1_eU to learn about the concept of turning points.
  - Display the turning points using graphing software like Geogebra, Maple, Mathematica, etc.
- Demonstrate how to find three types of turning points of a function using successive order of differentiation.
  - Discuss some simple applications of maxima and minima. Refer to the exercise 6(a) of BHSEC Mathematics Book II or explore the web links: https://www.youtube.com/watch?v=FdtmqkbItT8 and https://www.youtube.com/watch?v=nMd8KAQjYL0&t=432s to know about the turning points.
- Explain the conditions for maxima, minima and inflexion clearly.
  - Refer https://www.youtube.com/watch?v=8MSMaCjWn7k to know about the point of inflexion.
- Discuss some word problems of applying maxima and minima.
  - Assign relevant questions from the exercise 6(b), BHSEC Mathematics Book II. Suggested questions: 1 – 7.
- Discuss the real-life applications of maxima and minima in commerce and economics. Refer https://youtu.be/p3ssxTiGNUg to know how the maxima and minima are applied in commerce and economics.

E. Assessment

Performance Task

Search for algebraic functions having different turning points, and provide them to the students in the class. Let students examine for turning points and present their work to the class.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIIB-A1.

F. Resources

- Introduction to Maxima and Minima – https://www.cuemath.com/calculus/maxima-and-minima/
- Utility and scope - Maxima - minima - optimization.pdf
- Turning points concepts - https://www.youtube.com/watch?v=8j8hQNb1_eU
- Applications of turning points- https://www.youtube.com/watch?v=FdtmqkbItT8
- https://www.youtube.com/watch?v=nMd8KAQjYL0&t=432s
- Inflexion point - https://www.youtube.com/watch?v=8MSMaCjWn7k
- Applications of maxima and minima - https://youtu.be/p3ssxTiGNUg

G. Annexure

Refer XIIB-A1 for template to record achievement.

Topic: XIIB-B3 Integration *[700 minutes]*

Introduction

The history of integration unfolds over centuries, with numerous mathematicians paving the way for the development of calculus. Archimedes, in ancient Greece, initiated the exploration of volumes and areas using the method of exhaustion, predating the formalisation of derivatives. Luca Valerio in the 1600s furthered these ideas, employing inscribed rectangles to approximate areas under curves. Descartes and Fermat in the 17th century delved into tangents and extremal values, resembling early limit concepts. Cavalieri introduced the notion of indivisibles in 1639, contributing to the understanding of geometric figures in terms of infinitesimals. John Wallis in 1656 emphasised the limit in integration, extending the power formula. Mathematicians like James Gregory expanded integration to trigonometric functions. Isaac Barrow, Newton's teacher, made the crucial discovery that integration and derivation were inverse processes. Newton and Leibniz, independently and using different symbolism, established the calculus as a general systematic method, solidifying the connection between derivatives and integrals. While Newton and Leibniz are credited with much of calculus, the contributions of these predecessors played a vital role in laying the foundation for their groundbreaking work.

Source: https://www.youtube.com/watch?v=qrEt-0SQATg

Utility and Scope

In real life, integrations are used in various fields such as engineering, where engineers use integrals to find the shape of a building. In Physics, used in the centre of gravity etc. In the field of graphical representation, where three-dimensional models are demonstrated.

A. Competencies

- Demonstrate the understanding of integration and determine integrals of algebraic functions using various methods.

B. Objectives

- Understand the concept of integration.
- Evaluate integrals of algebraic functions.

- Apply the substitution method to evaluate integrals.
- Employ the partial fraction method to evaluate integrals of rational fractions.

C. Essential Skills/Processes

- Critical Thinking
- Analysing
- Knowledge Connections
- Knowledge Application
- Problem Solving

D. Learning Experiences

- Recapitulate the concepts learnt previously under Integral Calculus: integration as the reverse process of differentiation, and integral as the antiderivative.
  - Refer to the link: https://www.youtube.com/watch?v=W1uKClY3sN4 to know the differences between them.
- Assign questions on integration of algebraic and logarithmic functions in groups, and let the groups solve them.
- Use the web link below if required to understand about the basic integral rules for algebraic functions:
  - https://www.youtube.com/watch?v=e1nxhJQyLYI (till 11:35 minutes) is a video on basic integral rules for algebraic functions.
- Revise finding of integrals by method of substitution.
  - Demonstrate the method of substitution to evaluate the integral of algebraic functions with appropriate examples. Refer to BHSEC Mathematics Book I or the links:.
  - https://www.youtube.com/watch?v=1zUsgXiy4HY
  - https://www.youtube.com/watch?v=8B31SAk1nD8 (till 22:35 minutes) is a video on substitution method for simple algebraic functions.
  - Allow students to discuss similar examples and practise some questions.
- Introduce Integration using partial fraction.
  - Allow students to write notes from the web link: https://www.slideshare.net/slister07/5-3-partial-fractions that contains a PowerPoint presentation on partial fraction decompositions.
  - Explain the use of partial fractions in evaluating integral with the help of examples for different types of partial fractions - non-repeated linear factors,

repeated linear factors, and quadratic factors not resolvable into linear factors. Explore for examples of each type from external sources or refer to BHSEC Mathematics Book II.

- Use the web link: https://www.youtube.com/watch?v=GIGJdvdrdhs&t=60s to learn about integration using partial fractions of non-repeated linear factors.
- Provide a group task to evaluate integrals using partial fractions from each type for discussion. Assign relevant questions from BHSEC Book II exercise 7(g) or from any external sources.

E. Assessment

Performance Task

Frame questions on integration of each type of partial fractions, and assign them as homework. Or use the questions given in this worksheet: https://tutorial.math.lamar.edu/problems/calcii/partialfractions.aspx

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIIB-A1.

F. Resources

- National School Curriculum, Mathematics for PP – XII
- BHSEC Mathematics Book II
- Introduction https://www.youtube.com/watch?v=qrEt-0SQATg
- Differences between differentiation and integration - https://www.youtube.com/watch?v=W1uKClY3sN4
- Integration rules - https://www.youtube.com/watch?v=e1nxhJQyLYI
- Substitution method - https://www.youtube.com/watch?v=8B31SAk1nD8 https://www.youtube.com/watch?v=1zUsgXiy4HY
- Integration using partial fraction: https://www.slideshare.net/slister07/5-3-partial-fractions https://www.youtube.com/watch?v=GIGJdvdrdhs&t=60s
- Worksheet on integration - https://problems/partialfractions

G. Annexure

Refer XIIB-A1 for the template to record achievement

Topic: XIIB-B4 Application of Calculus in Commerce and Economics *[500 minutes]*

Introduction

Calculus has a wide range of applications in real life mostly in engineering and computer science. However, in this topic, we will learn how calculus is applied in commerce and economics.

Source: Application-of-calculus-in-everyday-life

Utility and Scope

In economics, calculus is used to study and record complex information – commonly on graphs and curves. Calculus allows for the determination of a maximal profit by providing an easy way to calculate marginal cost and marginal revenue. It can also be used to study supply and demand curves.

Source: Application-of-calculus-in-everyday-life

A. Competency

- Exhibit comprehension of total cost, total revenue, demand functions, and adeptly solve associated problems.
- Show proficiency in applying calculus to optimise costs, revenues, and profits in the realms of commerce and economics.

B. Objectives

- Define and write the functions of Total Cost, Variable Cost, Average Cost, Marginal Cost, Total Revenue, Marginal Revenue and Average Revenue.
- Find the break-even points, average cost and average revenue.
- Find the marginal cost and marginal revenue by applying the idea of differentiation.
- Determine the functions that maximise profits and revenues, and minimise costs, using the concept of maxima and minima.
- Determine total cost and total revenue function using the concept of integration.

C. Essential Skills/Processes

- Reading Comprehension

- Information Recall
- Interpreting Information
- Knowledge Application
- Problem Solving

D. Learning Experiences

- Let students explore the applications of integral calculus (differentiation and integration) in commerce and economics through online sources. Have them share their findings in the class, supplementing their findings with any aspects they may have missed.
- Define functions (cost function, demand function, revenue function, profit function and break-even point), and explain the equations along with an example of each function.
  - Refer to the link: Applications-of-Calculus-in-economics.pdf to discuss relevant examples.
  - Also assign relevant questions from exercise 20(a), BHSEC Mathematics Book II. (*Do not assign questions having complex functions).*
- Upon solving each example, plot the functions using graphing software, and present the graphs to the students. This approach will help students visualise the graphical representation of cost functions, demand functions, and revenue functions.
- Graph cost function and revenue function together, and show the parts: lost, break-even point and profit.
- Assign students to explore on average and marginal functions on their own and present in the class. Explain the parts left out by the students or any additional information. Use the reference given in the link: Applications-of-Calculus-in-economics.pdf or BHSEC Mathematics Book II.
  - Discuss relevant examples and assign relevant questions to practise. Refer to exercise 20(b) of BHSEC Mathematics Book II.
- Instruct students to explore on average revenue and marginal revenue on their own. Students may use their Accountancy textbook or online source. Ask students to bring one application question, and present it in the class.
  - Discuss and demonstrate additional information or relevant examples that students may not have covered. Refer to the link:

Applications-of-Calculus-in-economics.pdf or BHSEC Mathematics Book II to assign the questions.

- Discuss the minimization of average cost or total cost and the Maximization of total revenue and total profit with respect to the level of outputs illustrated under the title: *Minimization of Average cost or total cost and Maximization of total revenue and total profit.*
  - If possible, allow students to explore the concept by referring to the link: Applications-of-Calculus-in-economics.pdf.
  - Discuss and assign questions from exercises 20(b) and 20(c), BHSEC Mathematics Book II.
- Graph a revenue function and show the maximum revenue point from the graph. And also graph the profit function and show the point where the profit is maximum. This will create a better sense of the functions related to business.
- Discuss and assign questions from exercises 20(b) and 20(c), BHSEC Mathematics Book II.
- Provide some idea to the students on using integration to find marginal cost and marginal revenue. Let students explore the link: Applications-of-Calculus-in-economics.pdf or other sources to find marginal cost and marginal revenue.
- Assign relevant questions from BHSEC Mathematics Book II or from other sources.

E. Assessment

Performance Task

To assess the competencies of this topic, assign the questions given in the worksheet: Applications-of-Calculus-in-economics.pdf.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIIB-A1.

F. Resources

- National School Curriculum, Mathematics for PP – XII
- BHSEC Mathematics Book II
- Introduction - Application-of-calculus-in-everyday-life

- Utility and Scope - Applications-of-Calculus-in-economics.pdf
- Cost function examples - Applications-of-Calculus-in-economics.pdf
- Marginal function examples - Applications-of-Calculus-in-economics.pdf
- Optimization examples - Applications-of-Calculus-in-economics.pdf
- Applying Integration - Applications-of-Calculus-in-economics.pdf
- Worksheet - Applications-of-Calculus-in-economics.pdf

G. Annexure

Refer XIIB-A1 for a template to record achievements.

Topic: XIIB-CD1 The Conic Section *[550 minutes]*

Introduction

The early history of conic sections is joined to the problem of "doubling the cube." According to Eratosthenes of Cyrene (c. 276–190 BC), the people of Delos consulted the oracle of Apollo for aid in ending a plague (c. 430 BC) and were instructed to build Apollo a new altar of twice the old altar's volume and with the same cubic shape. Perplexed, the Delians consulted Plato, who stated that "the oracle meant, not that the god wanted an altar of double the size, but that he wished, in setting them the task, to shame the Greeks for their neglect of mathematics and their contempt for geometry." Hippocrates of Chios (c. 470–410 BC) first discovered that the "Delian problem" can be reduced to finding two mean proportional between 'a' and '2a' (the volumes of the respective altars). From there, the concept of parabola was born.

Source: https://www.britannica.com/science/conic-section

Utility and Scope

Parabola: The features of parabola have a wide range of applications. For instance, the bulb in the headlights of the flash lights is located at the focus and light from that point is reflected outward parallel to the axis of symmetry. The satellite dishes are parabolic shaped, and the receivers are placed at the focus to get strong signals.

Ellipse: All planets in the solar system revolve around the Sun in elliptic orbits with the Sun at one of the foci. Some comets have elliptical orbits with the Sun at one of the foci as well. Our satellite also moves around the Earth in an elliptical orbit, thus, astronomers and scientists can keep track of their positions and motion paths at any time. Elliptic arches are often built for its beauty and stability.

Hyperbola: Some Comets travel in hyperbolic paths with the Sun at one focus, such comets pass by the Sun only one time unlike those in elliptical orbits, which reappear at intervals. Some cooling towers are built in a hyperbolic shape due to their strength and efficiency. We also see hyperbolas in architecture, such as Mumbai Airport terminal, Tokyo Tower, etc.

Source: https://www.cuemath.com/learn/mathematics/conics-in-real-life/

Suggested web link: https://www.youtube.com/watch?v=8nPMlW5NZSo to investigate utility and scope.

A. Competencies

- Demonstrate an understanding of the meaning of conic sections and the ability to identify their constituent parts.
- Exhibit the capacity to apply the concepts of conic sections in solving problems related to real-life situations.

B. Objectives

- Identify various types of conic sections through analysis of graphs and equations.
- Determine the components of conic sections centred at the origin, including focus, directrix, vertex, axes, centre, latus rectum, and eccentricity.
- Formulate the equation of conic sections centred at the origin in standard form.
- Apply the principles of conic sections to solve problems related to real-life scenarios.

C. Essential Skills/Processes

- Conceptualising
- Representing
- Analysing
- Connecting
- Applying

D. Learning Experiences

- Introduce conics as sections of a plane and a right circular cone. Explain the concept by demonstration: use clay/mud or any other improvised material to make a right circular cone (a double right circular cone may not be possible). Then cut the cone in different angles using a string or scissor to get each conic section. You may get the idea from the video link https://www.youtube.com/watch?v=HO2zAU3Eppo.
    - Refer the link: https://www.geogebra.org/m/pCg8NFVT which contains a GeoGebra worksheet on conic sections whereby one can manipulate the graph and conics by changing the angle.

- Allow students to deduce the definition of parabola by visually demonstrating a parabola as the locus of points equidistant from a fixed point (focus) and fixed line (directrix). Then, explain the eccentricity of parabola.
  - Use the web link: https://www.youtube.com/watch?v=_71biRTkDSg&t=277s and GeoGebra worksheet: illustrative diagram of parabola.

- Demonstrate the derivation of the equation of horizontal parabola (right handed parabola) and how to find focus, directrix, latus rectum, and axes from the equation of the right handed parabola and vice versa using the link: https://www.youtube.com/watch?v=naNWM_6VCgo.

- From the right handed parabola, relate to the left handed parabola and vertical parabola (upward and downward). Refer to the link: https://www.youtube.com/watch?v=gLE6mBuCPYk to understand the standard equation of left handed, upward and downward parabola and their parts.
  - Assign questions related to parabola whose vertex at origin, from the exercise 12(a), BHSEC Mathematics Book II.
  - You may get the questions from the suggested web links:
    - ➢ https://www.youtube.com/watch?v=dozDc9p1o6E
    - ➢ https://www.youtube.com/watch?v=_fPanTRrVJ4.

- Assign questions from relevant sources to discuss the concept of parabola in solving real-life problems. Refer the following suggested links:
  - https://www.youtube.com/watch?v=tfIgcdAq9I4
  - https://www.youtube.com/watch?v=oXKkgIRnfEU
  - https://www.youtube.com/watch?v=dEyA8gx1zIl

- Introduce Ellipse- its definitions and its parts: foci, directrices, vertices, axes (major and minor axis), latus rectum and eccentricity.
  - Use the GeoGebra worksheets https://www.geogebra.org/m/epGhevZJ to allow students to visualise to define the ellipse. In continuation, explain the eccentricity of the ellipse.
  - Refer to the link: https://www.youtube.com/watch?v=cRY50CTdVvE explaining the parts of the ellipse.

- Demonstrate the derivation of the equation of horizontal ellipse.
  - Refer BHSEC Mathematics Book II or the link: equation of ellipse to know how the equation of ellipse is derived: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ (a > b)$.

- Display some graphs of horizontal ellipses using Geogebra so that students can have clear visualisations and understanding. Or refer to the link: https://www.geogebra.org/m/epGhevZJ to show an interactive ellipse diagram.
    - Explain the parts of the horizontal ellipse: vertex, focus, directrix, eccentricity, latus rectum and axes of ellipse.

- Using the knowledge of horizontal ellipse, let students derive the equation of vertical ellipse: $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, $(a > b)$. Further, allow them to explain the parts of the vertical ellipse: vertex, focus, directrix, eccentricity, latus rectum and axes of ellipse.
    - Display some graphs of vertical ellipses using Geogebra so that students can have clear visualisations and understanding. Or use to show an interactive ellipse diagram.
    - Demonstrate finding the parts of the ellipse: vertex, focus, directrix, eccentricity, latus rectum and axes of ellipse from an equation, and vice versa. Refer to BHSEC Mathematics Book II or refer to the suggested web links:
        - ➢ https://www.youtube.com/watch?v=dKK7wFN8oE0,
        - ➢ https://www.youtube.com/watch?v=li_rtf6b1so,
        - ➢ https://www.youtube.com/watch?v=B0YZC3mwKzM
        - ➢ https://www.youtube.com/watch?v=3qckea8OuN8
    - Assign questions related to the ellipse whose centre at (0, 0), from the exercise 12(b), BHSEC Mathematics Book II.

- Discuss the concept of the ellipse in solving real-life problems. Refer to the following suggested links:
    - https://www.youtube.com/watch?v=y8hCdHqS768
    - https://www.youtube.com/watch?v=leuhGEVUt_k
    - https://www.youtube.com/watch?v=ng_mQo4aqZY
    - https://www.youtube.com/watch?v=h9f_nUD911w

- Explain the concept of hyperbola by graphing one function in Geogebra. Refer https://www.geogebra.org/m/zSvbuhbT to allow students to visualise how hyperbola change when the components change and vice versa.

- Demonstrate deriving equation of horizontal hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ using the web link: https://www.youtube.com/watch?v=a2niebD-3CA.
- Explain the parts of the horizontal hyperbola: vertex, focus, directrix, eccentricity, latus rectum, and axes of a horizontal hyperbola. Refer to BHSEC Mathematics Book II or the link: https://www.youtube.com/watch?v=vTaslh6DK_Y&t=267s.
- Demonstrate finding the parts of the horizontal hyperbola: vertex, focus, directrix, eccentricity, latus rectum, and axes of a horizontal hyperbola from an equation, and vice versa.
    - Assign questions on hyperbola whose centres are at (0, 0) from the exercise 12(c), BHSEC Mathematics Book II.
- Using the knowledge of horizontal hyperbola, let students derive the equation of vertical hyperbola: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.
    - Display some graphs of vertical hyperbola using Geogebra so that students can have clear visualisations and understanding.
- Discuss the parts of the vertical hyperbola, and demonstrate finding the parts: vertex, focus, directrix, eccentricity, latus rectum, and axes of a vertical hyperbola from an equation, and vice versa.
    - Assign questions on hyperbola, whose centres are at (0, 0), from the exercise 12 (c) of BHSEC Mathematics Book II.
- Discuss the applications of hyperbola in real-life from the relevant sources or refer to the suggested links: https://www.youtube.com/watch?v=8fZiUhb-WP0.
- Explore: Allow students to explore the applications of conics in real life from different resources (suggested web link: https://www.cuemath.com/learn/mathematics/conics-in-real-life/).
- Take students to the IT Laboratory and let them explore with the Geogebra worksheets on conic sections. Let them explore and understand the nature of conics. Refer the following links for the worksheets:
    - Parabola- https://www.geogebra.org/m/Dhh8qyNt
    - Ellipse- https://www.geogebra.org/m/vZ6T6S23

- Hyperbola- https://www.geogebra.org/m/FVfq4Sz3

E. Assessment

Performance Task 1

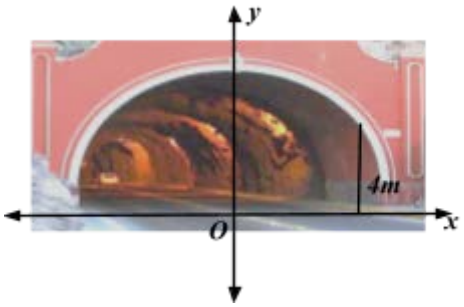
Design some questions which encompass all the conic sections and their parts, and assign them as homework.

Performance Task 2

Design one application question where students have to apply their knowledge on conic sections in real life. Refer the sample given below:

Sample

The tunnel of Tsirang to Wangdi highway is in an elliptical opening. The total width of the tunnel is 14 m, and the highest point of the tunnel is approximately 5 m. The height at the edge of the road is sufficient for a truck 4 m high to clear. How wide is the road?



Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIIB-A1.

F. Resources

- National School Curriculum, Mathematics for PP – XII
- BHSEC Mathematics Book II
- Introduction of Conics - https://www.britannica.com/science/conic-section
- Utility and Scope - https://www.conics-in-real-life/
  https://www.youtube.com/watch?v=8nPMlW5NZSo
- Conic Sections Concepts- https://www.youtube.com/watch?v=HO2zAU3Eppo.

- Conic section Geogebra worksheet - https://www.geogebra.org/m/pCg8NFVT
- Parabola Equation - https://www.youtube.com/watch?v=NDh57N7U2BU

Parabola Geogebra worksheet- illustrative diagram of parabola.

- Horizontal Parabola https://www.youtube.com=naNWM_6VCgo
- Horizontal and vertical parabola https://www.youtube.=gLE6mBuCPYk
- Problem solving on parabola https://www.youtube.com/watch?v=dozDc9p1o6E https://www.youtube.com/watch?v=_fPanTRrVJ4.
- concept of parabola in solving real life problems https://www.youtube.com/tfIgcdAq9I4 https://www.youtube.com/watch?v=oXKkgIRnfEU https://www.youtube.com/watch?v=dEyA8gx1zil
- Equation of Ellipse - equation of ellipse
- Ellipse Geogebra worksheet-https://www.geogebra.org/m/epGhevZ
- Parts of ellipse- https://www.youtube.com/watch?v=cRY50CTdVvE
- Problem solving on ellipse - https://www.youtube.com/watch?v=dKK7wFN8oE0, https://www.youtube.com/watch?v=li_rtf6b1so, https://www.youtube.com/watch?v=B0YZC3mwKzM and https://www.youtube.com/watch?v=3qckea8OuN8
- concept of parabola in solving real life problems https://www.youtube.com/watch?v=y8hCdHqS768 https://www.youtube.com/watch?v=leuhGEVUt_k https://www.youtube.com/watch?v=ng_mQo4aqZY https://www.youtube.com/watch?v=h9f_nUD911w
- Hyperbola Geogebra worksheet - https://www.geogebra.org/m/zSvbuhbT
- Equation of hyperbola - https://www.youtube.com/watch?v=a2niebD-3CA
- Parts of hyperbola - https://www.youtube.com/watch?v=vTaslh6DK_Y&t=267s.
- Hyperbola Applications - https://www.youtube.com/watch?v=8fZiUhb-WP0
- Conic Section Worksheets:
  - Parabola- https://www.geogebra.org/m/Dhh8qyNt
  - Ellipse- https://www.geogebra.org/m/vZ6T6S23
- Hyperbola- https://www.geogebra.org/m/FVfq4Sz3

G. Annexure

Refer XIIB-A1 for template to record achievement levels

Topic: XIIB-CD2 Points and Their Coordinates in 3-Dimensions

[500 minutes]

Introduction

In class XI, our knowledge on geometry was in a plane, that is, in two dimensions. It is easier to analyse a point or a shape on a plane, but we live in a world of three dimensions. If we are to apply our geometric knowledge to physical problems, we must be able to extend our concepts to 3-D space. A three-dimensional coordinate system has an x-axis, a y-axis, and a z-axis. The point (x, y, z) in three dimensions specifies the signed distance from the origin along the x, y, and z-axes, respectively.

Refer: https://wild.maths.org/ren%C3%A9-descartes-and-fly-ceiling to learn the history of geometry.

Utility and Scope

The concept of 3-D geometry is also applied in the fields of robotics, computer, and video games. Geometry provides handy concepts both for computer and video game programmers. The way & the design of the characters that move through their virtual worlds requires geometric computations to create paths around the obstacles concentrating around the virtual world. Video game engines typically put to use ray casting, which is a technique that simulates a 3-D world using a 2-D map. Using this form of geometry helps speed up processing because calculations are only done for the vertical lines on the screen.

Refer - https://www.toppr.com/bytes/geometry-in-daily-life/
https://studiousguy.com/examples-of-geometry-in-everyday-life/

A. Competency

- Demonstrate comprehension of the distance, section, and midpoint formula by applying them in relevant scenarios.
- Display an understanding of the relationship between direction cosines and direction ratios to illustrate conditions of parallelism and perpendicularity between two lines in real-world situations.

B. Objectives

- Comprehend the 3-D coordinate system and articulate the positioning of a point in 3-D space.

- Solve problems related to distance formula, section formulae, and midpoint formula.
- Calculate direction ratios and direction cosines of a line in 3-D space.
- Determine the angle between two lines using direction cosines and direction ratios.
- Assess the parallelism and perpendicularity of two lines in 3-D space.

C. Essential Skills/Processes

- Information Recall
- Critical Thinking
- Analysing
- Knowledge Application
- Problem Solving

D. Learning Experiences

- Direct the students to explore the 3-D coordinate system for three-space and the application of 3-D coordinate systems in real life in the IT lab. Also ask students to compare the 3-D coordinate system with respect to the 2-D coordinate system as learned in class XI.
    - Share the web link: https://www.youtube.com/watch?v=yPysmMXI_Is which contains a video explaining a point in 3-D coordinate system for students' reference.
    - Use the corners of your class as reference axes: x-axis, y-axis, and z-axis. Demonstrate how to describe the location of that object using the coordinates.
    - Use Geogebra or any other 3-D graphing program to show how a 3-D coordinate system looks. Plot one point in 3-D space and explain the distances of the point from each axis.
- Discuss distance formula, division or section formula and midpoint formula with reference to 2D-coordinate system, and their use in solving problems. Refer to BHSEC Mathematics Book II or refer to the following video links:
    - distance formula (3:15 minutes onward) which contains a video explaining the deriving of distance formulas in 3-D space.

- https://www.youtube.com/watch?v=cBbWbz0kQaQ which contains a video explaining the section formula and solving problems using section formula.
- https://www.youtube.com/watch?v=_ZRT4bmFYDQ which contains a video on more questions solved using the section formula.
- Assign questions from the exercise 13(a) and 13 (b) of BHSEC Mathematics Book II.

- Introduce the concept of direction cosines and direction ratios Refer to the web link: Direction-cosines-and-ratios-of-lines which contains the notes of direction cosines and direction ratios. Or refer to BHSEC Mathematics Book II.
  - May also use the explanatory video in the web links:
    - ➢ https://www.youtube.com/watch?v=HNTYvIVoT-U
    - ➢ https://www.youtube.com/watch?v=RWJsy9ufxtc
  - Keeping the three corners of your classroom as reference axes, tie one string or thread in one of the corners and let it as the origin. Then explain the direction angles formed by the thread.

- Exhibit finding direction ratios of a line joining two points, angle between two lines and conditions of perpendicularity and parallelism.
  - Solve problems related to direction ratios, directions cosines, perpendicularity and parallelism. Refer BHSEC Mathematics Book II or the web link: https://www.youtube.com/watch?v=oi3bpxklbP4 to learn about the conditions.
  - Assign relevant questions from BHSEC Mathematics Book II, exercise 13(c) or any relevant questions from other sources. *(Exclude question involving proving of geometric figures)*

E. Assessment

Performance Task 1

- Provide two points in 3-D coordinates. From the points, let the students find the following:
  - Distance between the points
  - Midpoint of the line
  - Direction ratios of the lines

- Direction angles of the line
- Direction cosines of the lines.

Performance Task 2

- Frame some questions to assess students' understanding of perpendicularity, parallelism, and finding the angle between two lines.

Performance Task 3

- Druk Air is flying at an altitude of 9 km and it is currently at the longitude of 88^{o} and the latitude of 27^{o}. The air traffic controller building at Paro International airport, which is 12 m tall, is located at the longitude of 89.4^{o} and the latitude of 27.4^{o}. Find the distance between the air traffic controller and the Druk Air.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIIB-A1.

F. Resources

- National School Curriculum, Mathematics for PP – XII
- BHSEC Mathematics Book II
- Introduction to coordinate geometry
  https://wild.maths.org/ren%C3%A9-descartes-and-fly-ceiling
- Utility and Scope - https://www.toppr.com/bytes/geometry-in-daily-life/
  https://studiousguy.com/examples-of-geometry-in-everyday-life/
- 3-D space concepts - https://www.youtube.com/watch?v=yPysmMXI_Is
- Distance formula - https://www.youtube.com/watch?v=5sJdfciNM20
- Applying section formula- https://www.youtube.com/watch?v=cBbWbz0kQaQ
  https://www.youtube.com/watch?v=_ZRT4bmFYDQ
- Direction cosines and direction ratios- Direction-cosines-and-ratios-of-lines
  https://www.youtube.com/watch?v=HNTYvIVoT-U
  https://www.youtube.com/watch?v=RWJsy9ufxtc
- Conditions for parallelism and perpendicularity -
  https://www.youtube.com/watch?v=oi3bpxklbP4

G. Annexure

Refer XIIB-A1 for template to record achievement level

Topic: XIIB-E1 Correlation *[600 minutes]*

Introduction

The history of correlation in statistics and mathematics can be traced back to the late 19th century. Sir Francis Galton, an English polymath, is often credited as one of the pioneers in the development of correlation. In the 1880s, Galton studied the relationship between the heights of parents and their offspring. He introduced the concept of correlation and developed the method of "regression toward the mean" to describe the tendency of extreme traits in parents to be less extreme in their children.

Karl Pearson, a contemporary of Galton, further advanced the field of correlation. Pearson developed the correlation coefficient, which quantifies the strength and direction of a linear relationship between two variables. His work laid the foundation for modern correlation analysis.

In the early 20th century, the concept of correlation continued to evolve, with scholars like Udny Yule contributing to its refinement. The field expanded to include various types of correlation coefficients, such as the product-moment correlation coefficient and the rank correlation coefficient.

Correlation became an essential tool in statistics, providing a quantitative measure of the association between variables. Over the years, advancements in technology and computing further facilitated the application of correlation analysis to large datasets and complex problems.

Today, correlation is a fundamental concept in statistics, data analysis, and machine learning, playing a crucial role in understanding relationships between variables and making informed decisions in various fields. The historical contributions of Galton, Pearson, and others have shaped correlation into a powerful and widely used tool in both theoretical and applied aspects of mathematics and statistics.

For more information visit: History of Correlation.

Utility and Scope

Correlation and Regression Analysis are forms of statistical analysis and have been traditionally reserved for statisticians and mathematicians.

1. **Improve Operations**: Improves business performance by impacting operational efficiency, such as discovering innovative material substitutions to reduce manufacturing costs.
2. **Sales Forecasting**: Maximise profits by making adjustments to resources and marketing strategies based on forecasted market trends.
3. **Analysing Results**: Accurately test decision making results to determine how your hypothesis impacts your business.
4. **Improve Employee Efficiency**: Connect employee behaviours to specific software or technology implementations, and drive efficiency improvements.
5. **Develop New Strategies**: Bring to light previously undiscovered relationships between data, such as customer demand increases based on a specific sales event.
6. **Correct Mistakes**: Analyse the findings of your decisions and reveal the exact reasons behind your results.

Source: https://www.researchoptimus.com/article/what-is-correlation.php

Further explore the link:
https://www.researchoptimus.com/article/what-is-correlation.php

A. Competency

- Demonstrate understanding of the correlation coefficients, analyse and describe a data set by finding an appropriate correlation coefficient.

B. Objectives

- Understand the significance of Correlation Coefficients in different fields.
- Calculate and interpret Karl Pearson's coefficient of correlation for ungrouped data collected from real life experiments.
- Calculate and interpret Spearman's Rank correlation coefficient for ungrouped data.

C. Essential Skills/Processes

- Reading Comprehension
- Information Recall

- Information Interpreting
- Knowledge Application
- Problem Solving

D. Learning Experiences

- Check students' previous knowledge on correlation by asking questions on the types of correlation and correlation coefficients.

- Draw different scatter plots and ask students to recall the type of correlation and correlation coefficient for each scatter plot. If some students have forgotten the concept, provide a review of what they learned in classes X and XI.
  - Refer to the web link: https://www.youtube.com/watch?v=8nxXOEfZeHs.

- Explain the two methods to measure the degree of correlation between two variables. Clarify to students that, in contrast to the estimations they encountered in class X, these methods are designed to precisely determine the correlation coefficient.
  - Suggestion: Explain the utility of the concepts of correlation coefficients in real-life to arouse students' interests and motivation.

- Demonstrate the concept of Covariance and Karl Pearson's first formula with an example. Explain the interpretations of correlation coefficients. Refer BHSEC Book II or the link: https://www.youtube.com/watch?v=mG__Wpp9dns.

  - Collect data from real-life situations from other sources, and discuss how to compute correlation coefficient using the first formula. Or discuss the relevant examples from BHSEC Mathematics Book II.
  - Explore : Derivation of correlation coefficient formula using the web links:
    - ➢ https://www.youtube.com/watch?v=ecvI02c6SuM
    - ➢ https://www.youtube.com/watch?v=N7bZnC_a01M

- Compute Karl Pearson's correlation coefficient using MS Excel. In MS Excel, there is a function called 'Pearson' to get the correlation directly.
  - Refer to https://youtu.be/Ev86DMtLXOk to learn how to find Pearson's correlation coefficient using Ms Excel.

- Assign questions for practice from the exercise 16(a), BHSEC Mathematics Book II.

- Demonstrate Karl Pearson's second formula (Direct Method) to find correlation coefficients using the link: https://www.youtube.com/watch?v=Gxw4-1AnLy4.
  - Instruct students to compute correlation coefficients using both the first formula and second formula of Karl Pearson. Assign questions from BHSEC Mathematics Book II.
  - Allow students to identify the advantages and disadvantages of the first formula and second formula of Karl Pearson.

- Recap the lesson on finding the mean of a data using assumed mean, and then explain Karl Pearson's third formula. Students could use the third formula if the means of the data are not exact.
  - Assign some data of real life or from online sources for practice. Or assign some relevant questions from the exercise 16(a), BHSEC Mathematics Book II.
  - Instruct students to complete performance task 1.

- Introduce the Spearman's Rank correlation coefficient (Derivation is not necessary at this level). Though Pearson and Spearman's methods are both used to find the correlation coefficient between two variables, in some situations, Spearman's method will give a more accurate correlation than Pearson.
  - Relate between the Pearson and Spearman's correlation coefficient. Refer https://youtu.be/9dr8rJ9fE7o to know the differences between them.
  - Refer to this video https://youtu.be/mmTs_zkox6w to check how the Spearman's rank formula is applied to find the rank correlation coefficient between two variables.

- Discuss determining the ranks for repeated ranks, and adding the correction factor.
  - Refer to this video link https://www.youtube.com/watch?v=rW9vYelxNyk to learn how to compute Spearman's rank correlation coefficient for repeated ranks. Note that in the video, they use the letter t in the correction factor, whereas, in BHSEC Mathematics Book II, they have used the letter m.

- Allow students in groups to examine a few questions on Spearman's rank correlation coefficient for tied rank. Refer to the exercise 16(b), BHSEC Mathematics Book II or search for real-life data from other sources.
- Instruct students to complete performance task 2.

E. Assessment

Performance Task 1

Direct students to gather data comprising two variables, with approximately 30 observations. Offer examples such as marks in two subjects for 30 students in the same stream, or the height vs. weight of 30 students in the school. You may refer to the examples or exercise questions from BHSEC Mathematics Book II. Check their work and discourage any form of plagiarism.

Performance Task 2

Group work:

In the school IT lab, demonstrate an example on an Excel sheet to punch in data and generate a scatter plot.

Allow students to do the same for their data set.

Instruct students to either print or copy the scatter plot and attach into their project work with an analysis of the plot to describe the direction and degree of the correlation between the data.

Performance Task 3

Group work:

Using an Excel sheet, demonstrate how to find the values of x^2, y^2, xy, Σx, Σy, Σxy, d_x, d_y, etc. (all the necessary values for applying Karl Pearson's correlation coefficient formula).

Instruct the students to use an appropriate formula of Karl Pearson's for their data to find the correlation coefficient and interpret the result. (Encourage students to manually find the values such as $\bar{x}$, $\bar{y}$, σ_x, σ_y, cov(x, y), and r, using relevant formula and the Sums (Σ) compiled from the spreadsheet).

Instruct the students to attach a printed copy of the excel sheet and the solution and interpretation of the solution, into their project work.

Performance Task 4

Group work:

Using an Excel sheet, demonstrate how to assign ranks (tied or not) for their data and find the values of D, D^2, ΣD^2 (all the necessary values for applying the formula to calculate the Rank correlation coefficient.).

Instruct the students to use Spearman's rank correlation coefficient to find and interpret the correlation between the variables of their data set. Encourage students to calculate the values of r manually using the information extracted from the spreadsheet.

Additionally, request students to write reflections on the correlation coefficients obtained through both methods (Karl Pearson's coefficient and Spearman's rank coefficient), and instruct them to include all completed activities in their project work.

Students may read the article in the link:
http://geoinfo.amu.edu.pl/qg/archives/2011/QG302_087-093.pdf to compare between Pearson's and Spearman's correlation coefficient

Note:
Teachers must design an assessment tool to assess the project work.
Sample:

Expectations	Met (✓) Not met(X	Comments
1. Collected relevant data with two variables		
• Correct scatter plot generated		
• Interpreted the direction and degree of correlation correctly from the scatter plot		
• All the values accurately calculated on the excel sheet with right use of formulas (PT 3).		
• Appropriate formula used to calculate the Pearson's coefficient of correlation		
• Analysis and interpretation of the correlation coefficient done correctly		
• All the values accurately calculated on the excel sheet with right use of formulas (PT 4).		
• Accurate rank correlation coefficient found.		
• Analysis and interpretation of the Rank correlation coefficient done correctly		
• Reflection on the use of two formulas drew the correct conclusion.		

★ If 9-10 expectations are met - Exceeding
★ If 7-8 expectations are met - Advancing
★ If 5-6 expectations are met - Meeting
★ If 3-4 expectations are met - Approaching
★ If 1-2 expectations are met - Beginning

F. Resources

- National School Curriculum, Mathematics for PP – XII
- BHSEC Mathematics Book II
- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction to Correlation - History of Correlation
- Utility and Scope
  https://www.researchoptimus.com/article/what-is-correlation.php
  https://www.researchoptimus.com/article/what-is-correlation.php
- Correlation Concepts- https://www.youtube.com/watch?v=8nxXOEfZeHs
- Covariance - https://www.youtube.com/watch?v=mG__Wpp9dns

- Correlation concepts - https://www.youtube.com/watch?v=8nxXOEfZeHs
- Derivation of correlation coefficient formula: https://www.youtube.com/watch?v=ecvI02c6SuM https://www.youtube.com/watch?v=N7bZnC_a01M
- Finding correlation using MS Excel- https://youtu.be/Ev86DMtLXOk
- Difference between Pearson and Spearman's correlation coefficient: https://youtu.be/9dr8rJ9fE7o
- Application of Spearman's rank correlation coefficient : https://youtu.be/mmTs_zkox6w https://www.youtube.com/watch?v=rW9vYelxNyk

G. Annexure

Refer XIIB-A1 for a template to record achievement levels.

Topic: XIIB-E2 Regression *[600 minutes]*

Introduction

In 1885, Sir Francis Galton first defined the term “regression” and completed the theory of bivariate correlation (Rodgers, 1988). Similarly, Sir Francis Galton is commonly regarded as the founder of the statistical techniques of correlation and linear regression (Millar, 1996). A decade later, Karl Pearson developed the index that we still use to measure correlation, Pearson's r.

"Regression" comes from "regress" which in turn comes from Latin "regressus" - to go back (to something). In that sense, regression is the technique that allows "to go back" from messy, hard to interpret data, to a clearer and more meaningful model. Regression takes a group of random variables, thought to be predicting Y, and tries to find a mathematical relationship between them. This relationship is typically in the form of a straight line (linear regression) that best approximates all the individual data points.

Source: https://byjus.com/maths/correlation-and-regression/

Utility and Scope

Regression analysis is used to estimate the relationship between a dependent variable and one or more independent variables. This technique is widely applied to predict the outputs, forecasting the data, analysing the time series, and finding the causal effect dependencies between the variables.

Researchers use regression to indicate the strength of the impact of multiple independent variables on a dependent variable on different scales. Regression has numerous applications. For example, consider a data set consisting of weather information recorded over the past few decades. Using that data, we could forecast weather for the next couple of years.

Regression is also widely used in organisations and businesses to assess risk and growth based on previously recorded data. You can find the implementation of regression analysis directly as a deployable code chunk.

Source: https://builtin.com/data-science/linear-regression-tensorflow

A. Competency

- Demonstrate an ability to analyse relationships between variables, make informed predictions and decisions based on statistical patterns in data.

B. Objectives

- Interpret the meaning of regression coefficients and understand the significance of the regression line.
- Determine regression coefficients and equations.
- Utilise regression lines to interpret data distribution means and standard deviations.
- Apply this knowledge to generate regression lines of real-life data, facilitating accurate estimations.

C. Essential Skills/Processes

- Interpreting Information
- Reading Comprehension
- Defining key concepts
- Knowledge Application
- Problem Solving

D. Learning Experiences

- Recap the previous knowledge on the scatter plots and lines of best fit. Explain how lines of best fit can be used to predict unknown values using interpolating and extrapolating.

- Clarify the drawbacks of the line of best fit drawn by hands, and then introduce the regression line and its equation.

- Define the regression line and relate it to the line of best fit: regression coefficients are the slopes of the line of best fit.
  - Refer to the link: https://youtu.be/P8hT5nDai6A to learn the line of regression in *least squares sense*.

- Explain the formulas of two regression coefficients (derived if required), and their applications. Explain the scenarios where each type of regression coefficients will be used.

- Refer BHSEC Mathematics Book II or the link: https://www.cuemath.com/data/regression-coefficients/ for the derivation of regression coefficients.

- Discuss the forms of lines of regression and demonstrate how to predict unknown data values from the regression equations.
  - Refer to the video link https://www.youtube.com/watch?v=sKfAmFK6u8A or BHSEC Mathematics Book II.

- Explain all the formulae for regression analysis. You can find the list of tables in BHSEC Mathematics Book II.

- Discuss the relation between regression coefficients and Pearson correlation coefficient.

- Discuss and solve examples to determine each type of regression equations: y on x, and x on y. Demonstrate how to predict x values when y value is known, and vice versa.
  - Assign some questions from the exercise 17 of BHSEC Book II, or data collected from other sources. Students must be able to find the regression coefficients and equations from a table of values, or from a given scatter plot.

- Carry out the Performance task.

E. Assessment

Performance Task 1

Group work:

Demonstrate generating line of best fit using Excel sheets.

Instruct students to generate scatter plots and the line of best fit i) when first variable (x) is taken as independent and the second variable (y) is taken as dependent variable, ii) when y is taken as independent and x is taken as a dependent variable.

Find the slope and the y –intercepts of each of the regression lines $y\ on\ x$ and $x\ on\ y$ and find the equations of the lines: $y = mx + c\ and\ x = my + c$. Ask the students to attach their findings to their project work.

Performance Task 2

Group work:

Instruct students to use Excel sheets to manipulate their data collected and find the equations of regression lines of $y\ on\ x$ and $x\ on\ y$ by using the relation: $y = \bar{y} + b_{yx}(x - \bar{x})$ and $x = \bar{x} + b_{xy}(y - \bar{y})$ (Students should be able to complete this task without much assistance since they have already been working on Excel sheet for similar tasks before)

Instruct students to compare the results with that in performance task 1.

Students will make predictions using their regression lines for an assumed value of x and an assumed value of y.

Students should also describe the interdependence of the two variables of their data by analysing the results obtained in correlation and regression.

Allow students to compile all the work from each task into the project work and submit for assessment of each competency.

Note:

Teachers may set assessment tools to assess the competencies.

Sample:

Expectations	Met (✓) Not met(X)	Comments
1. Regression line of y on x accurately generated in excel sheet		
2. Regression line of x on y accurately generated in excel sheet		
3. Values of slope and y-intercept correctly worked out		
4. Found correct equation of regression line of y on x		
5. Found correct equation of regression line of x on y		
6. All the values accurately calculated on the excel sheet with right use of formulas (PT 2).		
7. Appropriate formula used to calculate the regression coefficients (bxy and byx) and found equations of the regression line correctly.		
8. Analysis and interpretation of the correlation coefficient done correctly		
9. Used a relevant regression line to predict the values of y for assumed value of x and vice versa.		
10. Correct deduction of dependency of the two variables made.		

If 9-10 expectations are met - Exceeding
If 7-8 expectations are met - Advancing
If 5-6 expectations are met - Meeting
If 3-4 expectations are met - Approaching
If 1-2 expectations are met – Beginning

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIIB-A1.

F. Resources

- National School Curriculum, Mathematics for PP – XII
- BHSEC Mathematics Book II
- Introduction to Regression - https://byjus.com/maths/correlation-and-regression/
- Utility and Scope - https://builtin.com/data-science/linear-regression-tensorflow
- Regression in Least Square Sense - https://youtu.be/P8hT5nDai6A
- Derivation of regression coefficients - https://www.cuemath.com/data/regression-coefficients/
- Application of regression coefficients https://www.youtube.com/watch?v=sKfAmFK6u8A

G. Annexure

Refer XIIB-A1 for template to record achievement levels

Topic: XIIB-E3 Probability *[750 minutes]*

Introduction

Probability is the chance that something will happen. How likely it is that some event will occur. Sometimes we can measure a probability with a number like "10% chance", or we can use words such as impossible, unlikely, and possible, even chance, likely and certain. Example: "It is unlikely to rain tomorrow".

A gambler's dispute in 1654 led to the creation of a mathematical theory of probability by two famous French mathematicians, Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665). Probability, chance and randomness have been around since the ancient days. They could be found in fortune telling, games of chance, philosophy, law, insurance, and errors of prediction in astronomy and medicine (Hald, 1990). There is evidence that in the late 15th century and the early 16th century, mathematicians started to experiment with the idea of probability.

Source: http://history of probability

Utility and Scope

Probability is an essential tool in applied mathematics and mathematical modelling. Moreover, probability is used in daily life to make decisions when you don't know for sure what the outcome will be. Most of the time, you won't perform actual probability problems, but you'll use subjective probability to make judgement calls and determine the best course of action. For example, to check the weather forecast, playing cards, lottery tickets, etc.

Source: https://www.probability-real-life-examples/

A. Competency

- Demonstrate the skills to solve real-life probability problems by applying the concepts of independent and dependent events, conditional probabilities and permutations and combinations.

B. Objectives

- Use the concept of permutation and combination in calculating probability.
- Determine probability by applying laws (addition (OR) rule, multiplication (AND) rule) for the problems containing selection/arrangement of two or more things.
- Solve probability problems by using both the addition and multiplication theorems of probability.

- Determine the conditional probability of two dependent events.

C. Essential Skills/Processes

- Identifying
- Applying
- Analysing
- Problem Solving
- Interpreting Information

D. Learning Experiences

- Recapitulate the addition and multiplication theorems on probability that students have learned in class XI. Refer the followings links:
  - https://www.youtube.com/watch?v=mppk8je2BQA (video on addition rule).
  - https://www.youtube.com/watch?v=eUYNeK60ql8 (video on multiplication rule of independent events).
  - Provide a few questions from BHSEC Mathematics Book II, exercise 18(f) and 18(g) to recall their previous knowledge on the addition and multiplication theorems of probability.

- Revisit the concepts of 'at least', 'at most', 'exactly' and 'not all'. Solve some related problems to refresh their understanding.

- Demonstrate solving probability problems by using both the addition and multiplication theorems of probability. Refer to BHSEC Mathematics Book II or the link: https://byjus.com/question-answer.
  - Assign relevant questions from BHSEC Mathematics Book II, exercise 18(h).

- Revise concepts of permutation and combination learned in the previous lessons.
  - Use the link: https://www.youtube.com/watch?v=tnF9f3zCCKI (suggested timing - till up to 10 minutes).

- Calculating probability using Permutation and combination:
  - Play the video in the web link: https://www.youtube.com/watch?v=RNH_O2QvkWA which contains a lesson on the concepts of calculating probability using permutation and combination.
  - Apply a blended instructional approach and pause the video wherever required to supplement on the explanation.

- Discuss the questions on finding probability using permutation from the given links:
  - ➢ https://www.youtube.ordering
  - ➢ https://www.letter/ arrangement
    https://www.digit/ arrangement
- Discuss the questions on finding probability using combination from the given links:
  - ➢ https://www.lottery
  - ➢ https://www.deck-card
  - ➢ https://www.committee-members
- Assign further practice questions from the BHSEC Mathematics Book II, exercise 18(d) as homework tasks.

- Conditional probability:
  - Recap the concepts of conditional probability learned in class X. Refer https://www.youtube.com/watch?v=evyT3_8Dnhs to learn about the basic concept of conditional probability.
  - Refer to the link: https://www.youtube.com/watch?v=JGeTcRfKgBo which contains a video lesson on conditional probability. Conduct hands-on experiments using two bowls and marbles, as in the video, so that students' will have an in-depth understanding.
  - Investigate and explore the link: https://www.youtube.com/watch?v=ES9HFNDu4Bs&t=834s which contains notes and video lessons on conditional probability.
  - Conduct an active interactive session and discuss the relevant examples of conditional probability. Refer https://www.onlinemathlearning.com/conditional-probability.html for relevant questions or BHSEC Mathematics Book II.

- Multiplication Theorem of probability for dependent events:
  - Prove three multiplication theorems for dependent events and its example from BHSEC Mathematics Book II, or refer to the link: https://testbook.com/maths/multiplication-theorem-of-probability.
  - Discuss problems related to the multiplication of two dependent events. Refer https://testbook.com/objective-questions/ for relevant questions for practice or refer to BHSEC Mathematics Book II, exercise 18(j).

E. Assessment

Performance Task 1

Assign the task on probability given in the link https://www.toppr.com/

Performance Task 2

Place Based Approach

Take students to a basketball court of your school, and let each student shoot a basketball 7 times. Let the individual student record their own shots. Using their records:

i. Instruct students to get into a group of three members and ask them to find the probability that their group will make a shot, if they shoot three times each.

ii. In the same group, ask them to find the probability that the group will make two shots if they shoot once each.

Design an appropriate assessment tool for each performance task, and record feedback and achievement as per the template attached in the annexure XIIB-A1.

F. Resources

- BHSEC Mathematics Book II
- National School Curriculum for Mathematics
- Introduction - http://history of probability
- Utility and scope - https://www.probability-real-life-examples/
- Addition rule https://www.youtube.com/watch?v=mppk8je2BQA
- Multiplication rule https://www.youtube.com/watch?v=eUYNeK60ql8
- Practice question involving both addition and multiplication rule https://byjus.com/question-answer
- Permutation and combination of probability - https://www.youtube.com/watch?v=tnF9f3zCCKI
- Permutation and combination of probability - https://www.youtube.com/watch?v=RNH_O2QvkWA
- Probability using permutation - https://www.youtube.ordering
- Probability using permutation - https://www.letter/ arrangement
- Probability using permutation - https://www.digit/ arrangement
- Conditional probability - https://www.youtube.com/watch?v=evyT3_8Dnhs
- Conditional probability - https://www.youtube.com/watch?v=JGeTcRfKgBo

- Conditional probability - https://www.youtube.com/watch?v=ES9HFNDu4Bs&t=834s
- Conditional probability – https://www.onlinemathlearning.com/conditional-probability.html
- Theorem of Multiplication dependent probability - https://testbook.com/maths/multiplication-theorem-of-probability
- Problems on multiplication probability https://testbook.com/objective-questions/
- Technological gadgets for learning (smart phone, laptop, desk top)

G. Annexure

Refer XIIB-A1 for template to record achievement levels

Appendix A

Project Work

Background

Mathematics, as a discipline, goes beyond numbers and equations; it is the language of patterns, structures and logical reasoning. Traditionally, mathematics has been taught through textbooks and lectures, emphasising problem-solving skills and theoretical understanding. However, the integration of project work in mathematics education has emerged as a pivotal method to enhance learning outcomes and foster a deeper understanding of the subject.

Mathematics project work stands as a fundamental tool in modern education, enriching students' mathematical understanding, nurturing critical skills, and promoting a holistic approach to learning mathematics. By integrating project-based learning, educators can empower students to become proficient problem solvers, critical thinkers and enthusiasts of the intricate world of mathematics. It also equips students with fundamental research skills to gather, analyse, and present data, which is a crucial skill that the education system should prepare them for.

Why project work in mathematics?

- **Real - world Relevance**: Mathematics projects bridge the gap between theoretical concepts learned in class and their practical applications in real- life scenarios. They allow students to see the tangible relevance and usefulness of mathematics principles in various fields such as science, engineering, economics and technology.
- **Problem-solving proficiency**: Project work cultivates critical thinking and problem-solving skills. Students are challenged to apply mathematical theories to solve complex problems, encouraging them to think analytically and logically.
- **Creativity and Innovation**: Projects encourage students to explore mathematics beyond routine calculations, promoting creativity and innovative thinking in finding solutions to mathematical challenges.

- **Depth of Understanding**: Project-based learning enables students to delve deeper into mathematical concepts. It allows them to explore topics beyond the curriculum, leading to a richer understanding of mathematical theories and their applications.
- **Conceptual connections**: Through project work, students often discover the interconnectedness of mathematical topics, recognizing how different concepts relate and interact, thereby solidifying their understanding of the subject.
- **Technology Integration**: Projects often involve the use of mathematical software, simulations, or data analysis tools. The exposure to technology aids students in becoming proficient in utilising digital resources for mathematical analysis.
- **Independent Inquiry:** Through project work, students develop research skills, learn to conduct independent inquiries, and manage their time effectively to complete tasks.

Expected fields of project work:

- Research analysis report on application of concepts like measures of central tendency, dispersions, correlation and regression.
- Solving problems through codings.
- Development of model and simulations.
- Demonstration of use of any online mathematical softwares.
- Concepts of mathematics used in real life and other fields of study.
- Mathematics in nature (Bio-maths).
- Mathematics behind Artificial Intelligence (AI).
- Recent inventions or discoveries in mathematics.
- A particular mathematician and their contributions.
- Extension of any concepts that they have learnt.

Components of Project Report

Components	Details
Title of the project	Title: Clearly stating the projects topic or theme
Brief Overview	Summarise the project's objectives, methods used, major findings and conclusions. Highlight the most significant aspects or outcomes of the project.
Introduction	Background and Context: introduce the project topic, explaining its importance and relevance. Objectives: Clearly state the goals and aims of the project. Scope; Define the boundaries and limitations of the study.
Literature Review (If Applicable)	Review of sources: Discuss relevant literature, theories, or existing research related to the project. Critical Analysis: Evaluate and compare various viewpoints explored in the literature.
Methodology	Research Methods: Describe the methods used for data collection, analysis or mathematical modelling. If students work on topics that don't require collection of data, they can mention how they explored the topic. Tools and Techniques: Explain the mathematical tools, software or technology used in the project.
Data Collection and Analysis	Data Description: Present any collected data, observations, measurements, exploration, case study or calculations used in the project. Mathematical Analysis: Show mathematical formulas, equations or models applied in analysing the data or patterns observed. Visual Representations: Include figures, graphs, charts, diagrams or tables to illustrate mathematical patterns or findings.
Results and Fundings	Summary of Findings: Summarise the key outcomes and discoveries. Interpretation: Explain the significance of the results in relation to the project objectives.
Discussions	Analysis and Explanation: interpret the results in detail, discussing their implications and relevance. Comparative Analysis: Compare findings with literature (If applicable) Limitations: Address any limitations or constraints encountered during the project.
Conclusion	Conclusive remarks: Recap the main points discussed in the report and state overall findings, their significance and their contribution to the field of study. Recommendations: Provide recommendations for further research extensions or improvement related to the project topic.
References:	Citations: Include a list of references, books, articles, websites or mathematical sources cited in the report. Proper Formatting: Follow a standard citation style (APA - American Psychological Association 7th edition)

Assessment :

- Develop suitable tools or rubrics that align with the topic chosen by the students.

Sample 1: Here is a sample rubric designed for individuals exploring the different fields of project work.

Category	Beginning (1)	Approaching (2)	Meeting (3)	Advancing (4)	Exceeding (5)
Understanding of Topic	Limited understanding of the topic, with major misconceptions.	Basic understanding of the topic, with some misconceptions.	Adequate understanding of the topic, demonstrating a grasp of fundamental concepts.	Thorough understanding of the topic, showcasing a comprehensive knowledge base.	Exceptional understanding of the topic, demonstrating advanced insights and a deep understanding.
Research and Information:	Limited or inaccurate use of sources with minimal relevance to the topic.	Basic use of sources with some relevance, but with limitations.	Effective use of diverse sources demonstrating a good understanding of the topic.	Skillful use of authoritative sources, showcasing a high level of research competency.	Exceptional use of a wide range of credible sources, revealing insightful and sophisticated research.
Problem Solving:	Limited attempt at problem-solving with significant errors or omissions.	Basic problem-solving with some correct steps, but with noticeable errors.	Competent problem-solving, demonstrating a clear understanding of the steps involved.	Proficient problem-solving with accuracy and efficiency in approach.	Exceptional problem-solving, showing creativity and originality in tackling challenges.
Critical Thinking:	Limited application of critical thinking skills, with minimal analysis or evaluation.	Basic application of critical thinking skills, with some analysis but lacking depth.	Sound critical thinking skills, demonstrating a good level of analysis and evaluation.	Strong application of critical thinking, showcasing a high level of analysis, evaluation, and synthesis.	Exceptional critical thinking, revealing advanced and insightful analysis, evaluation, and synthesis.
Presentation:	Disorganised or ineffective presentation with minimal use of appropriate visuals.	Basic organisation with some visuals, but lacking clarity or creativity.	Well-organised presentation with clear visuals enhancing understanding.	Professionally presented work, showcasing clarity, creativity, and effective use of visuals.	Exceptional presentation, demonstrating exceptional clarity, creativity, and innovation.
Communication:	Limited or unclear communication of ideas, with poor expression.	Basic communication with some clarity, but lacking in precision and coherence.	Clear and effective communication of ideas, demonstrating appropriate language use.	Proficient communication, with precise and coherent expression of ideas.	Exceptional communication, showcasing eloquence, precision, and sophistication.
Overall Quality:	Overall quality of the project is significantly below expectations.	Overall quality is basic, meeting minimal expectations.	Overall quality is good, meeting expectations.	Overall quality is excellent, surpassing expectations.	Overall quality is outstanding, showcasing exemplary work and surpassing all expectations.

★ *Sample 2: Here is a general sample rubric designed for project work assessment.*

Criteria	Excelling (4.1-5)	Advancing (3.1-4)	Meeting (2.1-3)	Approaching (1.1-2)	Beginning (0.1-1)
Plan	Plan is detail with 5 components (schedules, activities, materials requirement, data collection source, representations) and endorsed by teacher	Plan is incomplete by two components and not endorsed by teacher	Plan is incomplete by three components and not endorsed by teacher	Plan is incomplete by 4 components and not endorsed by teacher	Plan is sketchy without required components and not endorsed by teacher
Problem	problem is new, meaningful and will have positive impact on community	problem is new, meaningful but will not have any impact on community	problem is not new but meaningful	problem is new but not meaningful	problem is not stated
Literature	Literature study is researched and aligns with problem showing deep understanding of concept	Literature study is researched and aligns with problem but lacks understanding of concept	Literature study is well researched but do not align well with the problem	Literature study is not well researched and do not align well with problem	Literature study is not stated but visible from other part of project
Data collection	Data collection is systematic with collection tools, processes and authentic source and mentioned in plan	Data collection is systematic with collection tools, processes and authentic source but not mentioned in plan	Data collection has collection tools but without logical process and authentic source	Data collection is not systematic but source mentioned	Data collection is not described and source not mentioned
Data representation and analysis	Data representation is appropriate, neat with topics and legends and analysis is meaningful	Data representation is appropriate with incomplete components but analysis is meaningful	Data representation is appropriate but analysis is not meaningful	Data representation inappropriate and analysis is not meaningful	Data representation inappropriate but no analysis
Findings	Finding aligns as solution to the problem with strong argument supported by data	Finding aligns as solution to the problem but argument is weak and not supported by data	Finding is described well but not addressed to solve the problem	Finding is not well described but align as solution to the problem	Finding is brief and do not align with problem
References	More than five references are cited in APA format and referenced throughout the project	Four to five references are cited and referenced throughout the project	Two to three references are cited and referenced throughout the project	At least one reference is cited and referenced throughout the project	No references

Sample Project Work Calendar

Sl.#	Date	Activity	Recommendation	Remarks
1	1st week of April	Draw project plan	Plans must contain schedules with intended activity spread across the year. Can refer to this calendar as a sample.	
2	2nd week of April	Identifying a working title	Problem/ issue/ situation/opportunity/ project/new venture that is identified as a working title must be endorsed by the teacher. Only after approval of Title, students can proceed working on it	
3	3rd and 4th week May	Literature Review	Refer different books and online literature , which have link with project title	
4	Month of June - July	Data collection.	Data can be collected on identified topics using various tools (interview, questionnaires, observation, and document) or gather available data from various sources with acknowledgement.	
5	1st Week of August	Data organisation	Organise the collected data in appropriate form	
6	2nd week of August	Data display (1st Draft).	Represent the organised data using appropriate data display(s) using any graphs.	
4	3rd week of August	Data display (Final Draft).	Represent data display using relevant software (ICT tools). Subject teacher facilitates the availability of computers and software for data display.	
5	4th week of August	Data analysis, drawing conclusions and making recommendations	Analyse the data display, draw conclusions and make recommendations as per the analysis and conclusion supported by data collected.	
6	1st week of September	Compilation and submission of Project Work.	Compile the work and submit it to the respective subject teacher with proper binding.	
7	2nd week of September	Project Evaluation and Awarding of Marks (maximum 35 marks).		

Appendix B

Assessment Structures for KS- 5

Assessment Structure

Key Stage	Assessment									
	Term I					Term II				
	CA (10)			Mid Term Examination	Total	CA (10)			Annual Examination	Total
	CW	HW	PW			CW	HW	PW		
IV	3	4	3	40	**50**	3	4	3	40	**50**

For both Term I and Term II, assess each competency through appropriate performance tasks and assessment tools.

Performance Tasks: quiz, question and answer, presentation, making models, small projects, etc.

Assessment Tools: checklist, rating scale or rubrics.

Assessment Areas: Formulating situations mathematically, applying concepts, facts, and procedures, and interpreting mathematical results.

NOTE: Project work assessment is mandatory for class XI and XII

Appendix C

Weighting for each Strand

Weighting and time allocations for Class XI BMT

Strand	Time Allocation (Mins.)	Weighting(%)
Strand A: Numbers and Operations	1400	24
Strand B: Patterns and Algebra	3200	36
Strand CD: Measurement & Geometry	1100	18
Strand E: Data and Probability	1550	22

Weighting and time allocations for Class XII BMT

Strand	Time Allocation (Mins.)	Weighting(%)
Strand A: Numbers and Operations	2800	32
Strand B: Patterns and Algebra	2200	32
Strand CD: Measurement & Geometry	1050	16
Strand E: Data and Probability	1950	20

Appendix D

Sample - Class work Assessment Rubrics – Classes XI - XII

Criteria	Exceeding (5)	Advancing (4)	Meeting (3)	Approaching (2)	Beginning (1)
Understanding	➔ Demonstrates a deep and thorough understanding of the concepts covered. ➔ Consistently applies knowledge to solve problems.	➔ Shows a good understanding of the concepts. ➔ Applies knowledge effectively in most situations.	➔ Demonstrates a basic understanding of the concepts. ➔ Struggles with consistent application.	➔ Limited understanding of the concepts. ➔ Inconsistently applied knowledge.	➔ Minimal understanding of the concept. ➔ Unable to apply knowledge effectively.
Effort and participation	➔ Actively engages in class discussions and activities. ➔ Displays an exceptional commitment to learning, actively seeking opportunities to contribute beyond expectations	➔ Engages in class activities with enthusiasm. ➔ Demonstrates a strong commitment to learning, willing to take on additional responsibilities when appropriate ➔	➔ Participates at a basic level. ➔ Effort is inconsistent. ➔ Demonstrates a commitment to learning, however lacks consistency in engaging with course content.	➔ Shows limited effort and participation. ➔ Often disengaged in class. ➔ Demonstrates a basic commitment to learning, with room for improvement in engagement.	➔ Minimal effort and participation Frequently disengaged.
Independence	➔ Demonstrates a high level of independence in completing classwork. ➔ Rarely requires assistance.	➔ Generally works independently but may seek clarification when needed.	➔ Works somewhat independently but often requires assistance.	➔ Requires frequent assistance to complete classwork.	➔ Constantly relies on others to complete classwork.
Seeking support	➔ Proactively seeks support when faced with challenging concepts or problems. ➔ Collaborates effectively with peers and teachers to enhance understanding.	➔ Willingly seeks support when needed. ➔ Demonstrates effective collaboration with peers and teachers.	➔ Occasionally seeks support but may be hesitant. ➔ Limited collaboration with peers and teachers.	➔ Rarely seeks support.- ➔ Minimal collaboration with peers and teachers.	➔ Does not seek support. ➔ No collaboration with peers and teachers.
Collaboration	➔ Demonstrates excellent collaboration skills in group activities. ➔ Encourages and supports peers, fostering a positive and inclusive class atmosphere	➔ Collaborates well with peers in group activities. ➔ Generally fostering a positive class environment.	➔ Participates in group activities but with limited collaboration ➔ Inconsistent contributions.	➔ Struggles to collaborate effectively in group activities. ➔ Offers occasional contributions, with room for improvement in depth of insights	➔ Unable to collaborate effectively. ➔ Rarely contributes ideas or insights to class conversations.

Appendix E

Sample - Homework Assessment Rubrics – Classes XI - XII

Criteria	Exceeding (5)	Advancing (4)	Meeting (3)	Approaching (2)	Beginning (1)
Understanding	➔ Demonstrates a deep and thorough understanding of the homework assigned. ➔ Consistently applies knowledge to solve problems.	➔ Shows a good understandin g of the homework concepts. ➔ Applies knowledge effectively in most situations.	➔ Demonstrat es a basic understandi ng of the homework concepts. ➔ Struggles with consistent application.	➔ LLimited understanding of the homework concepts. ➔ Inconsistently applies knowledge.	➔ Minimal understandin g. Unable to apply knowledge effectively.
Completion	➔ All homework are completed accurately and thoroughly.- ➔ Consistently submits high-quality work.	➔ Most homework tasks are completed accurately and thoroughly. ➔ Few minor errors present.	➔ Some homework tasks are completed accurately, but there are notable gaps. ➔ Several errors are present.	➔ Numerous incomplete or inaccurately completed homework tasks. ➔ Completion is inconsistent.	➔ Virtually all homework tasks are incomplete or inaccurately completed.
Accuracy of response	➔ All calculations and solutions are accurate and precise. ➔ Demonstrates meticulous attention to detail.	➔ Most calculations and solutions are accurate and precise. ➔ Few minor errors present.	➔ Some calculations and solutions are accurate but lack precision. ➔ Several errors are present.	➔ Numerous errors in calculations and solutions. ➔ Accuracy and precision are major issues.	➔ Virtually all calculations and solutions are incorrect or imprecise.
Neatness and organization	➔ Homework is exceptionally well-organized and neatly presented. ➔ All text is highly legible, and there are no smudges or unintended marks. ➔ Clear headings, labels, and steps enhance the overall organization	➔ Overall organization is good, with a clear presentation. ➔ Most text is legible, and there are minimal smudges or unintended marks. ➔ Headings, labels, and steps contribute to effective organization.	➔ Organizatio n is acceptable but may lack some neatness. ➔ Legibility varies, and there may be occasional smudges or unintended marks. ➔ Clear headings and labels help maintain a	➔ Organization is somewhat lacking, and there is some difficulty in following the work. ➔ Legibility issues are noticeable, and there are frequent smudges or unintended marks. ➔ Headings and labels are consistently not clear.	➔ Poor organization makes it challenging to follow the homework. ➔ Legibility is compromised, and there are significant smudges or unintended marks throughout. ➔ Chaotic presentation hinders understandin g, and

			basic level of organization		headings and labels may be unclear or absent.
Follow up and improvement	➔ Actively seeks feedback on homework. ➔ Demonstrates a commitment to improving based on feedback. ➔ Makes corrections and improvements on subsequent submissions.	➔ Open to feedback and uses it to make improvements in subsequent homework. ➔ Shows a willingness to learn from mistakes.	➔ Occasionally seeks feedback but inconsistently incorporates it into subsequent work. ➔ Limited improvement over time.	➔ Rarely seeks feedback and seldom makes improvements. ➔ Little evidence of learning from mistakes.	➔ Does not seek feedback or make improvements. ➔ Repeated mistakes persist
Timeline	➔ Submits homework/assignments consistently on time.	➔ Generally submits homework on time but may occasionally be late.	➔ Submits homework somewhat late on a regular basis.	➔ Frequently submits homework late.	➔ Consistently submits homework/ assignments late.