National School Curriculum

INSTRUCTIONAL GUIDE FOR MATHEMATICS

CLASS: VII-VIII

ཤེས་རིག

School Curriculum Division
Department of School Education
Ministry of Education and Skills Development
Royal Government of Bhutan

“Your parents, relatives, and friends would be very proud of what you have achieved. At your age, to have completed your studies is your personal accomplishment. Your knowledge and capabilities are a great asset for the nation. I congratulate you for your achievements.

Finally, your capabilities and predisposition towards hard work will invariably shape the future of Bhutan. You must work with integrity, you must keep learning, keep working hard, and you must have the audacity to dream big.”

- His Majesty Jigme Khesar Namgyel Wangchuck

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Foreword

COVID-19 has caused unforgiving disruptions in public education all over the world, and brought about threats of fragmentation in the society due to disparities in accessibility and connectivity in many systems. In Bhutan too, continuity of education and learning has been severely affected as a result of sporadic nationwide school closures, restrictions and health protocols. The disruptions exposed the limitation of the existing ideologies and practices in education. This has deprived children living in poverty worldwide, who rely on the physical settings of their schools for educational materials and guidance, of the learning and other essential educational services. Cognizant of the global trend to embrace competency based learning as education for the 21st century, the current priority of the Government is to transform the knowledge and textbook based learning to competency based learning through open source and experiential learning.

In the new normal education, human interaction and well-being is a priority. Technology, particularly digital technology that enables communication, collaboration and learning across distance, is a formidable tool though not a panacea but a source of innovation and expanded potential. As we embrace this exceptional opportunity to transform education, it is imperative to reimagine the organisation of our educational institutions and learning environments. In the post COVID 19 era, we must prioritise the development of the whole person, not just the acquisition of academic knowledge. Inspiration for the change can be drawn from the 1996 Delors report, learning the treasure within. Its four pillars of learning as "learning to know", "learning to do", "learning to be", and "learning to live together" are the current global ethos of teaching and learning. Therefore, curricula must be increasingly perceived as an integrated, themes based and problems based orientation that allows learners to develop a strong base of knowledge about one's self and about the world, and find purpose in life and be better able to participate in social and political milieu.

The National School Curriculum is, not just a mere response to the pandemic, but also culmination of the curriculum reform work for the last four years by the erstwhile Royal Education Council. It is an attempt to transform education from the teaching of "what" to learning of "how" and "why" towards empowering learners with the transversal competencies and the 21s t century skills, and preparing them to be lifelong learners. In tandem with this initiative, we are optimistic that the paradigm shift in Mathematics education orients our education process in empowering the young generation with the Mathematics mind-set and disposition, and skills towards nurturing nationally rooted and globally competent citizens.

With this guide, we are optimistic that our learners and teachers are ushered through a life enriching experiential Mathematics education.

Tashi Delek

(Karma Galay**)**

DIRECTOR GENERAL

Table of Contents

Introduction

The 21st Century Education framework emphasises on the theme-based learning approach that broadens opportunities for experiential learning contextualized to the learner's physical, social, political, economic, spiritual and cultural setting. This requires learning through active engagement of learners. The role of teachers therefore, is transformed from knowledge transmitter to facilitation, guide, evaluator, researcher and motivator.

The conventional education system is predominantly knowledge based and examination centred. This system compromised the development of psychomotor and affective domains of learning thereby affecting the holistic development of students.

Despite the devastating effect caused by COVID-19 pandemic, it presented scopes for creation, innovation, generally perceived as more efficient and effective in work and social activities. The pandemic situation explicated that the old ways of working, teaching and learning, and lifestyle have limitations. Consequently, new normal ways of how we work and live, teach and learn must be critically analysed and embraced.

Therefore, the education system needs to be transformed to meet contemporary requirements. Students should learn to critically filter information that is flooded on the internet. Classroom instruction should facilitate learners to construct knowledge, develop essential skills and values which are crucial for learners to realise their potential towards becoming locally rooted and globally competent citizens who would contribute towards making a just and harmonious society. Accordingly, classroom instruction from teacher centred to learner centred calls for the following adjustment, or even the overhaul of a few practices.

i. Reduction of learning content to facilitate deep learning as opposed to the width of the teaching through the active engagement of students.

ii. Integration of ICT as tools and ends of learner's education. The use of multimedia and ICT software is commonly utilized in teaching and learning as innovation to introduce variation in stimuli and sustain learner's interest and zeal in learning.

iii. Adoption of theme-based learning content, which facilitates to broaden the horizon of learning beyond the four walls, and stimulates the transfer of learnt concepts to the learner's immediate environment. This arrangement makes students aware of the realities of the social, political, economic and cultural practices and ethos of the society. Being aware of the immediate environment of the scopes and challenges, students are sensitized of the opportunities and issues, which may need attention for a better future for the society.

iv. Consideration to ground the curriculum design and instruction approaches the epistemological theories is imperative to facilitate deep learning as opposed to factual learning. However, the selection and use of them is subject to the nature of the respective subject. For instance, constructivism is more apt for science, while connectivism is relevant for languages and ICT curricula.

v. Active engagement of students is imperative for competency-based education and learning. Inevitably, summative assessment has limitations in gauging the progressive development of the learner. This is achieved objectively by the use of the continuous formative assessment (CFA). However, if summative assessment evidence is used to provide feedback to help students in learning, it can serve as one of the techniques of CFA.

Purpose of the Instructional Guide

This instructional guide provides a suggestive direction to the teachers to facilitate them to transform the classroom instruction to the contemporary requirements stated above. The content of the instructions in the guide are aligned with the mathematics curriculum framework with references to the existing textbooks.

The instructional guides are developed to achieve the following objectives:

i. Facilitate learning anywhere, any time with the learner being responsible for the learning.
ii. Facilitate deep learning with awareness and sensitivity of the realities of the world around.
iii. Strengthen competency based learning and experiential learning to foster sensitivity of realities of life and environment.
iv. Strengthen blended learning and flip classroom with multimedia, digital pedagogies and ICT devices and websites as the tools and learning content.
v. Guide parents in facilitating learning of their children.
vi. Inspire teachers to assume the roles of facilitation, guide, motivator and evaluator.
vii. Helps in the prioritization of learning content with emphasis to create time and space for active engagement of learners. Facilitate the use of CFA for learning through objective observation and guidance.

The effective and efficient use of this guide is subject to the nature of the topic(s) and the target class. It should be understood that class VII and VIII are in the same key stage (KS -III), Therefore if there are similar topics in both classes, teachers have the privilege to introduce basics in class VII and go deeper in class VIII.

Instructional Guide
Class VII
Mathematics

Topic: VII-A1 Divisibility Test [300 minutes]

Introduction

One way to see if a number is divisible or not by the other number is by using the divisibility rule. It is a shortened way to identify if the given number is divisible by a fixed divisor without performing the division process. May use the link provided below for further learning:
Divisibility Tests: A History and User's Guide | Mathematical Association of America

Utility and Scope

Divisibility test is used in everyday life. For example, if you are at school and want to know how many students will be left when divided into three groups from 1024 students, you can easily say that 1 student will be left out without dividing. It is a quick way to find the factors of a large number. The given web address offers examples and questions demonstrating the application of a divisibility test in real-life scenarios.
Test of Divisibility

A. Competency

- Investigate and deduce the divisibility rules from 2-12 to apply in appropriate situations.

B. Objectives

- Explore divisibility rules using models (e.g. base 10 block) focusing on how the rules work.
- Investigate the divisibility test rules from 2 to 12.
- Apply divisibility rules in mental calculations and also relate to real life situations.

C. Learning Experiences

- Try dividing a number by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 (For example: 128 ÷?) and see which number (2-12) can divide the number selected without leaving a remainder.
- Represents base ten blocks to show whether 128 is divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12.

The link to the video below provides insight into the process of dividing a number using base ten blocks.
https://www.youtube.com/watch?v=6mhbMLV0-yE

- Explore and investigate the divisibility rule for 2, 5 & 10 and then by 3 and 9.
- Explore the divisibility rule for 4, 6, 7, 8, 11 and 12.
  - Solve some examples to apply divisibility rules in real life situations (May refer Understanding Mathematics for class VIII for relevant questions).
- May use the link given below:
  - Divisibility rule from 2 to 12- https://www.mathsisfun.com/divisibility-rules.html
  - Video Divisibility test rules for numbers 2–to- 12 symbolically https://www.youtube.com/watch?v=98p4lQBfc6E

D. Assessment

Performance Task 1

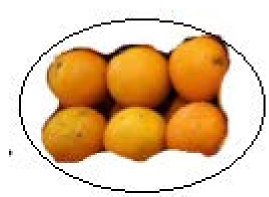
Solve a set of questions that involves divisibility rules based on the objectives.
May use the link provided to check students' basic understanding of the concept: https://www.math-only-math.com/worksheet-on-divisibility-rules.html

Performance Task 2

Develop situational questions related to divisibility rules and find the solution. Present the same to the whole class (Group work).
May ask the group to first project the question and allow others to try solving it.

- Sample Questions

  i. Aum Dema sells oranges in a market. She procures 741 pieces of oranges. She then packs oranges into smaller packets each containing 6 pieces. Will she have any leftover oranges after packing them into those smaller packets?

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  i. How does the divisibility test help grouping in real life? Share some situations where you can apply a divisibility test.

- Template to record assessment

Strand(s): Number and Operation			Topic(A-1): Divisibility Test		
Competency: Develop and apply divisibility rules for 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 in appropriate situations					
Name of the student	Level of achievement				
	Beginning	Approaching	Meeting	Advancing	Excelling

- Assessment tool (Sample)

Beginning	Approaching	Meeting	Advancing	Excelling
E (1)	D (2)	C (3)	B (4)	A(5)
Struggles to understand divisibility rules. Understands divisibility rule for only 2, 5 and 10 and finds difficulty in applying it.	Understands the divisibility rule for 2, 3, 4, 5, 6, 9, 10 and can apply the rule for solving problems with some difficulties.	Understands the divisibility rule for 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and can apply the rule for solving problems without difficulties.	Understands the divisibility rule and comes up with some rules developed on their own.	Develops divisibility rules from 2-12 on their own, and applies it in any circumstances

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
    - Divisibility test introduction- http://surl.li/bathi
    - Divisibility test scope and utility- https://flexbooks.ck12.org/cbook/ck-12-cbse-maths-class-8/section/16.1/primary/lesson/tests-of-divisibility/
    - Divisibility test using base ten blocks- https://www.youtube.com/watch?v=6mhbMLV0-yE
    - Divisibility rule from 2-12. https://www.mathsisfun.com/divisibility-rules.html
    - Divisibility rules for 2, 4, and 8 symbolically- https://www.youtube.com/watch?v=-Liq8hlGeOk
    - Divisibility rules for 3, 6 and 9 symbolically- https://www.youtube.com/watch?v=L0n_jJDNvrE
    - Divisibility test rules for numbers 2–to-12 symbolically- https://www.youtube.com/watch?v=98p4lQBfc6E
    - Divisibility test concepts- https://www.math-only-math.com/worksheet-on-divisibility-rules.html

Topic: VII-A2 Lowest Common
VII A3 Greatest Common Factor [300 minutes]

Introduction

The least common multiple (LCM) is the smallest number that is a common multiple of two or more numbers, and the greatest common factor (GCF) is the largest number that is a common factor of two or more numbers. May use the link provided below for further information;
Source: https://byjus.com/maths/hcf-and-lcm/

Utility and Scope

LCM helps to see how many times the event will repeat over time. Example if you are doing two things on different days, you can find out when you will be doing both tasks at the same time. It also helps us in adding/subtracting fractions easily by finding common denominators.
GCF allows us to:

- split things into smaller sections,
- equally distribute 2 or more sets of items into their largest group.
- easily find the lowest term of a fraction.

A. Competency

- Understand the concepts of common multiples, least common multiples, common factors, and greatest common factors and apply to solve relevant real-life problems.

B. Objectives

- Recall the procedure to list down common multiples and common factors.
- Apply various methods (Prime Factorization, listing the multiples and repeated division) in developing LCM.
- Apply various methods (Prime Factorization, listing the factors and division) in developing GCF.
- Implement the concept of LCM and GCF in real life situations.

C. Learning Experiences

- List down the multiples and factors for a set of numbers (e.g: 6 and 9).
- Identify their common multiples and common factors and from that identified common multiples and factors, find the LCM and GCF.
- Suggestions:
  - Use different methods like prime factorization, listing the multiples or factors, repeated division etc.
  - Provide a few examples.
- Solve real-world word problems using LCM and GCF.

  Example:

  To celebrate a birthday party, Tashi needs cups and plates to serve food to the guests. He went to a store to find plates available in sets of 6 and cups available in sets of 4. Assuming that there must be no leftovers, what should be the minimum number of each of these packages he should buy?
  (Answer: 2 sets of plates and 3 sets of cups).
  May use the link for questions but need to contextualize it:
  https://www.ixl.com/math/grade-6/gcf-and-lcm-word-problems
  https://sac.edu/StudentServices/EOPS/Documents/Math%20Study%20Guide%203%20-%20GCF%20and%20LCM.pdf
- Video link to supplement on LCM and GCF.
  - The video link https://youtu.be/3W8SeYgZcMo shows how to determine LCM and GCF (Clarify to your students that dot denotes multiplication in the video provided).

D. Assessment

Performance Task 1

Ask students to solve a set of questions related to the objectives (May use the *Plicker app* to assess students' understanding).
May use the link below https://www.liveworksheets.com/da1546168rj to check students' understanding.

Performance Task 2

Explore the real-life application of LCM/GCF and present it to the whole class for discussion (Group Work).

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  1. Three buses ply from Trashigang to Thimphu. One bus ply every fourth day, the other every fifth day and the third bus ply every sixth day. If all buses plied from Trashigang to Thimphu on 5th July when would all buses ply together again? Discuss how LCM enables you to plan such a journey.
  2. Discuss how LCM can be applied in real life; like probability of getting a ticket for bus looking at their schedule, glowing of all electric decorating blinking bulbs together which might be blinking at different rates, etc.

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - Introducing LCM & GCF https://byjus.com/maths/hcf-and-lcm/
  - Questions on LCM & GCF- https://sac.edu/StudentServices/EOPS/Documents/Math%20Study%20Guide%203%20-%20GCF%20and%20LCM.pdf
  - LCM contextualization - https://www.ixl.com/math/grade-6/gcf-and-lcm-word-problems
  - Determining LCM and GCF https://youtu.be/3W8SeYgZcMo
  - LCM & GCF assessment https://www.liveworksheets.com/da1546168rj

Topic: VII-A4 Powers [200 minutes]

Introduction

The power is another way of representing repeated multiplication with the same number. Usually, power is represented with a base and an exponent. The exponent tells us how many times we multiply the base by itself. The link below provides further information on power and exponent.
https://www.splashlearn.com/math-vocabulary/algebra/exponent

Utility and Scope

Power helps to simplify repeated multiplication and help in representing large/small numbers. It is also used in many real-life situations such as pH scale, Richter scale, measuring area (square unit) and volume (cubic unit), exponential growth/ exponential decay and other topics in Mathematics.

A. Competency

- Understand the number systems to represent and convert numbers in standard, expanded and exponential forms.

B. Objectives

- Represent repeated multiplication as power with base and exponent
- Recognize exponents as a means of expressing factors in a compact form. Connect "squared" with an area of a 2-D object and "cubed" with a volume of a 3-D object.
- Interconvert numbers in standard, expanded and exponential form
- Connect expanded forms of numbers to digits in the place value chart.

C. Learning Experiences

- Conduct pre-requisites on prime factors using repeated multiplication (May write down the prime factors of 4, 9, 16, 25.. etc. (use only the perfect square numbers).
- Describe a base and an exponent as a Power using repeated multiplication (to further augment may use this suggestive video link): https://www.youtube.com/watch?v=-zUmvpkhvW8 - introduction to Powers.

https://www.youtube.com/watch?v=1M3ICk8FyrU - describes and expresses numbers into an exponential form.

- Use audio records to help students read powers correctly (2^5 as “two to the fifth” or “two raised to the fifth power” or “2 raised to the power of 5”).
- Investigate the connection of “squared” and “cubed” with area of 2-D shape and volume of a 3-D shape.
- Visually show how an area of 2-D shape and volume of a 3-D shape can be inferred using the power squared and cubed (Area is measured as m^2 /Volume as m^3).
- Investigate exponential, expanded, and standard form. Express Standard form to expanded and exponential form and vice versa.
  - o *Watch a video on exponential form and standard form:* https://www.youtube.com/watch?v=sBAOJ9NdVdM&t=183s

▪ Game: Rolling Power

Materials required: Dice.

Number of players: 2 or more

How to Play:

- ✔ Take turn to roll a die
- ✔ Roll a die to get the base of your power
- ✔ Predict whether the value of your power will be greater or less than a number (say 28).
- ✔ Roll a die to get the exponent
- ✔ Calculate the value of your power
- ✔ Score 1 point for correct prediction
- ✔ The first player to score 10 points wins.

Example:

Roll a die and the number you get is a base (let’s say you got 3),

Predict the value of your power, less than 28

Roll a die again and the number you get is power (Let’s say you got 2)

Calculate the value (3^2 = 3 X 3 = 9)

You score 1 point since your prediction is correct.

D. Assessment

Performance Task 1

Ask students to solve a set of questions related to the objectives (May refer to Understanding Mathematics Textbook for class VII or other relevant resources).

- Sample Questions
  i. Gaylek has 5 boxes, each box has 5 packets, and each packet contains 5 chocolates. Show in exponents how many chocolates are there in total. (Answer $5^3 = 125$)

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  i. How learning exponents help individuals?

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - Definition and examples of exponents (power)- https://www.splashlearn.com/math-vocabulary/algebra/exponent
  - Introducing Powers- https://www.youtube.com/watch?v=-zUmvpkhvW8
  - Describing and expressing number into an exponential form- https://www.youtube.com/watch?v=1M3ICk8FyrU
  - Exponential and expanded form- https://www.youtube.com/watch?v=sBAOJ9NdVdM&t=183s

Topic: VII-A5 Decimal Operations [400 minutes]

Introduction

"A decimal mark is a symbol used to separate the integer part from the fraction part of a number written in the decimal form". For example, instead of writing $\frac{1}{4}$, you can express the fraction as the decimal 0.25, where the zero is in the one's place and the two is in the tenth place and five in the hundredth place. The decimal system, therefore, has 10 as its base and is sometimes called a base-10 system.

May use the link for more information:

History and Metric System

Utility and Scope

We use decimals in our everyday life such as money, mass, length etc.

Example:

While measuring the mass of an item, it is not necessary for the mass to be in the whole number. For example, while measuring the mass of cauliflower with a weighing machine, the mass may not be a whole number, it may be 1.2kg.

A. Competency

- Explore and understand different strategies of decimal multiplication and division to apply accordingly in problem situations.

B. Objectives

- Multiply and divide decimals pictorially and symbolically
- Apply multiplying and dividing decimals in real life situations.
- Apply order of operations to problems related to the four basic operations in decimals.

C. Learning Experiences

- Conduct pre-assessment on decimals to represent them using different models (number line and place value chart).
- Use visual methods to multiply and divide decimals (Grid paper, thousands grid, place value chart, etc.).
- Investigate different methods of multiplying and dividing decimals using various resources.
- Suggestion
    - o Rename decimals (rename decimals using same place value to divide, eg. 0.5 ÷ 0.2 = 5tenth ÷ 2 tenth = 2.5)
    - o To multiply 0.5 x 0.2 = 5 tenth x 2 tenth = 10 hundredth = 0.1
- Provide a real word problem involving multiplying and dividing decimals to enhance students' understanding of the concepts taught (May use Understanding Mathematics Textbook for class VII or the online resources).
- May use the following links;
    - o The weblink https://youtu.be/STyoP3rCmb0 shows how to multiply decimals.
    - o The weblink https://youtu.be/7JPIX3odZrY shows how to divide decimals.
    - o The weblink https://m.youtube.com/watch?v=Htt09rY9I_A shows how to multiply and divide decimals.
    - o The video link https://www.youtube.com/watch?v=dAgfnK528RA shows how to use operation rules to operate decimals.

D. Assessment

Performance Task 1

Ask students to solve a set of competency-based questions related to the objectives (May refer to Understanding Mathematics Textbook for class VII or other relevant resources).

Performance Task 2

Why might someone say that knowing how to multiply or divide the whole numbers will ease the process of multiplying or dividing decimal numbers?

- Sample Questions
    i. A plant grows 2.5 metres every year. If it grows at the same rate, how tall will it be after two and half years ?

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
    i. Decimals and fractions are interchangeable, how would operating decimals provide alternatives to operating fractions? For example 0.5 + 0.4 would be easier to add then adding half and two fifth (like $\frac{1}{2}$ + $\frac{2}{5}$).

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
    - introducing decimal operations https://arindambose.home.blog/2014/04/03/did-you-know-the-history-of-decimal-point/
    - introducing decimal operations- https://extranet.education.unimelb.edu.au/SME/TNMY/Decimals/Decimals/backinfo/metric.htm
    - Multiplying decimals- https://youtu.be/STyoP3rCmb0
    - Dividing decimals- https://youtu.be/7JPIX3odZrY
    - Multiplying and dividing https://m.youtube.com/watch?v=Htt09rY9I_A
    - Decimal's operation rules- https://www.youtube.com/watch?v=dAgfnK528RA

Topic:VII-A6 Fractions [600 minutes]

Introduction

A fraction is a part of a whole thing. It represents an equal part of a whole. The word 'fraction' has been derived from the Latin 'fractus' which means "broken". Fraction has been in existence since the Egyptian era which is known to be one of the oldest civilizations in the world. However, fractions were not regarded as numbers, in fact, they were used to compare the whole numbers with one another. For more information may use the link below:

https://nrich.maths.org/2515

https://www.basic-mathematics.com/history-of-fractions.html

Utility and Scope

Fractions help students understand the meaning of division, nature of number and their interaction. Fractions are used in many other mathematics concepts, such as decimal, percent, probability and it will also enable children to learn algebra better in the later stage.

Fractions are seen in our daily lives such as in recipes, pizza and sharing of fruits, exams/test marks, time, medical prescription and others.

A. Competencies

- Demonstrate understanding of the concept of fractions and apply different strategies to compare and order fractions, pictorially and symbolically.
- Explore various strategies to add and subtract fractions to apply while solving real-world problems related to fractions.

B. Objectives

- Arrange fractions on a number line to compare and order them.
- Compare fractions relative to benchmarks, common denominator, the common numerator and decimal equivalents.
- Convert fractions to decimals and vice versa (terminating decimals and recurring decimals). Introduce the terminology "repeating" and "period" as well as bar notation to show repeated decimal (up to 2-digit repeating decimals)
- Use estimation before carrying out any operations.

- Add and subtract fractions pictorially and symbolically (Recall concept of LCM to add and Subtract Fractions).

C. Learning Experiences

- Conduct prerequisite knowledge on comparing fractions using number lines. https://www.youtube.com/watch?v=62u9A6kKhkI
- Explore various methods to compare fractions.
  Suggestion:
  - o Show how to relate fractions and decimals. Convert fractions to decimals and the other way round. Discuss about terminating and recurring decimals during the conversion to fraction. May use the following link for reference.
    https://www.varsitytutors.com/hotmath/hotmath_help/topics/terminating-and-repeating-decimals
  - o Use a visual fraction model to compare fractions (divided objects, shapes, divided bars, area model, number line and fraction strips).
  - o The video link https://www.youtube.com/watch?v=N8dIOmk_lHs shows how to compare and order fractions by finding common denominators.
  - o Compare and order fractions by finding common numerators.
- Demonstrate how to add and subtract fractions.
- *Suggestion:*
  - o The video link https://www.youtube.com/wtch?v=WrvDWD9HvOs contains video on adding fractions visually (rectangle model)
  - o The video link https://www.youtube.com/watch?v=YjEwB3dqe1A contains video on subtracting fractions visually (rectangle model).
  - o The video link https://www.youtube.com/watch?v=TaOqIGKvPZI contains video on adding and subtracting fractions using LCM

D. Assessment

Performance Task 1

Prepare a set of questions aligned with objectives and let students solve the questions (May refer to Understanding Mathematics Textbook for class VII or other relevant resources).

Performance Task 2

In a bag there are 24 marbles of three different colours: red, black, and blue. If six marbles are red and eight marbles are black, what fractions of the marbles are blue?

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  i. Cow's milk is a good source of different nutrients including vitamin B12 and iodine.. Kinjal drinks $\frac{1}{2}$ litre of milk daily. Deb drinks $\frac{2}{3}$ litres daily. Their father has a cow which gives 3 litres of milk daily. How many litres of milk will be available for sale if they reserve $\frac{1}{5}$ of litres for preparing tea daily?
  ii. How do you know how many fractional parts make 2 (take *any number here, 1,2 3 etc. but a whole number only*)?
  iii. Where do you think the fraction best applied in your daily life?

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - introducing fraction- https://nrich.maths.org/2515
  - introducing fraction
    https://www.basic-mathematics.com/history-of-fractions.html
  - comparing and ordering fractions-
    https://www.youtube.com/watch?v=62u9A6kKhkI
    https://www.youtube.com/watch?v=N8dIOmk_IHs
  - Converting fractions to decimals- Terminating and Repeating Decimals
  - Adding fractions https://www.youtube.com/watch?v=WrvDWD9HvOs
  - Subtracting fractions https://www.youtube.com/watch?v=YjEwB3dqe1A
  - Adding and Subtracting fractions using LCM
    https://www.youtube.com/watch?v=TaOqIGKvPZI

Topic: VII-A7 Ratio, Rate and Percent [600 minutes]

Introduction

A ratio is a comparison of two numbers or measurements. A rate is a special ratio in which the two terms are in different units. And a percent is a ratio expressed as a fraction with a denominator of 100. Percent is usually represented by %. Use the link provided below for more information. Refer https://en.wikipedia.org/wiki/Ratio for further reading

Utility and Scope

Rate and ratio are fundamental units across numerous topics in mathematics and science. In mathematics, they are essential to developing concepts and skills related to fractions, constant rate of change, algebraic concepts, and skills and in many real-life situations.

Similarly, the percent is applied in most real-life situations such as consumer problems, and in interpreting data.

A. Competency

- Use the concepts of ratio, rate, and percent to solve real-life problem situations.

B. Objectives

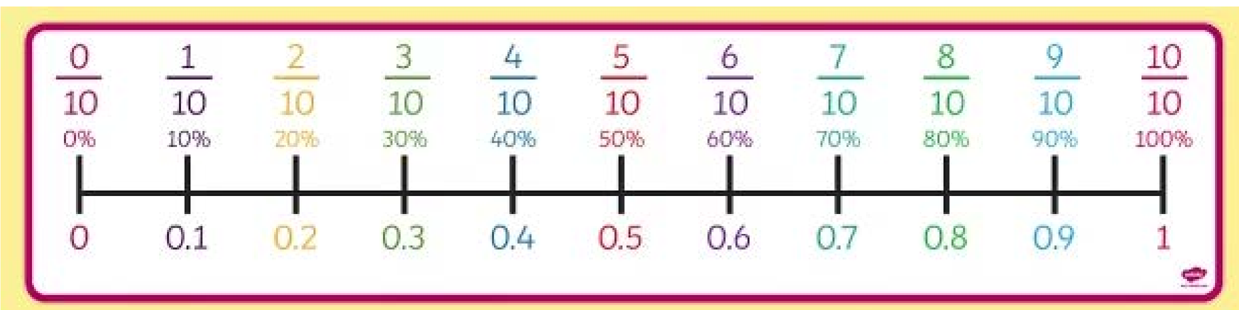
- Comprehend ratio as the comparison of numbers or quantities in the same units.
- Comprehend rate as the comparison of two quantities with different units.
- Solve problems involving ratios and rates.
- Recognize percent as a special ratio.
- Relate visual and symbolic representation of percent.
- Relate percent to fraction and decimal equivalent.
- Estimate and calculate percent for familiar fractions pictorially and symbolically.
- Use a variety of strategies to calculate percent.

C. Learning Experiences

- Revisit the concept of ratio, equivalent ratio and lowest term of a ratio.
- Solve ratio and rate problems. (May use ratio/rate tables to solve problems).

- o The video link https://www.youtube.com/watch?v=s9XFvTkHQ9s shows how to write ratios in different ways and how to solve ratio problems by using tables.
- o Let students experience the real problem related to ratio and rate; May ask students to role-play or take them to the nearby shops to calculate cost of items. E.g: 2 apples cost Nu 15, what is the cost of 10 apples?

- Present a few real-world examples of ratio and rate.
- Show percent as a special ratio.
- Relate visual and symbolic representation of percent.
- Represent and describe percent using the hundred grids. The following link provides worksheets for percent.
  https://www.mathworksheets4kids.com/percent.php
- Relate percentages, fractions, and decimal.
  - o Represent percent symbolically as fraction and decimal.
  - o Use a number line to show how fractions, decimals and percent relate.



- Provide ratio, rate, and percent problems in our day-to-day life. For example, comparing prices per item while grocery shopping, calculating the proper amounts for ingredients in recipes etc.
- Explore and play Games (May refer to Understanding Mathematics Textbook for class VII).

D. Assessment

Performance Task 1

Prepare quiz questions to test the concept learned based on the objectives (May use Kahoot/Nearpod/Plickers app or any other to create a short quiz).

Performance Task 2

Assign a set of competency-based questions aligned with objectives. May refer to Understanding Mathematics Textbook for class VII or other relevant resources).

- Sample Questions
    i. To prepare a curry in a mess for 100 students, a cook uses 125 grams of salt, 250 grams of onion and 500 grams of potatoes and other vegetables. One day 25 students from a class go for out-school learning. How many grams of salt and onion should be used for preparing curry on that day?

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
    i. We see discounts given in percentages, discuss this practice as a better option both for customer and shopkeeper.

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
    - Introducing ratio- https://en.wikipedia.org/wiki/Ratio
    - Writing ratios-https://www.youtube.com/watch?v=s9XFvTkHQ9s
    - Percent worksheets- https://www.mathworksheets4kids.com/percent.php

Topic: VII-A8 Integer [500 minutes]

Introduction

Integers are a set of whole numbers and their opposites. (example ...,-3, -2, -1, 0, 1, 2, 3 ...) Source: https://www.youtube.com/watch?v=OSfDRqxmXAE
May use the following links to share its history.
https://463431396329892656.weebly.com/history-of-integers.html

Utility and Scope

Integers help in computing the efficiency in positive or negative numbers in almost every field. Integers are used in our everyday life such as representing debit/credit of money, profit and loss, geographical level; above/below the sea level and describing temperature.

A. Competencies

- Understand the concept of integers through representation and apply it to compare and order integers pictorially or symbolically.
- Investigate different strategies to add and subtract integers and apply them in real-life situations.

B. Objectives

- Develop an understanding of the need to introduce integers.
- Represent integers in a variety of ways (number line and counters).
- Compare and order integers using different strategies.
- Recognize the balance of positive and negative values based on the zero property.
- Add and subtract integers pictorially (number line and counters) and symbolically.
- Relate integers to real life context (calculating time zone, temperature).

C. Learning Experiences

- Review the meaning of integers.
- Show on the number line where each integer would be located and use counters to represent integers.
    - Why do you think that the opposite integers have different signs?

- Work with integer models for comparing and ordering integers.
    - May use Understanding Mathematics Textbook for class VII for related questions.
- Represent zero property using counter/number line.Subtraction can be also termed taking away. When we don't have enough counters/numbers to take away, we bring pair(s) of opposite counters or equal numbers of opposite numbers. This is where we apply zero property.
- Demonstrate adding and subtracting integers using different strategies.
- *Suggestion:*
    - Use visual representations like Counters in two colours, (e.g., black and white counters) and Number lines (BLM).
    - The video link https://www.youtube.com/watch?v=Q9fPuX3lWXY contains video on adding and subtracting integers pictorially (using counters).
    - The video link https://www.youtube.com/watch?v=A186iWp5vKQ contains video on adding and subtracting integers pictorially (number line).
- Let students watch videos on adding and subtracting integers symbolically.
    - The video link https://www.youtube.com/watch?v=jVvvUiExjes contains video on adding and subtracting integers symbolically.
- Relate adding or subtracting integers to time calculation in geography.

D. Assessment

Performance Task 1

Relate this topic to time calculation learned in geography and solve questions related to time calculation, may consult geography teacher for the level of questions (May refer to Understanding Mathematics Textbook for class VII or other relevant resources).

Performance Task 2

Prepare questions related to integers and solve the problems. Present the same to the whole class (Group work).

Sample Questions:

i. Karma took 6 steps forward and 8 steps backwards. How far is Karma from the original position?

ii. A sea animal is 800 feet below sea level. If it ascends 250 feet to feed, what is the new position?

iii. Gasa records a temperature of -7° C. The temperature further drops 3° C. What will be the new temperature? or Mark integers on an edge line of a basketball court or any flat ground. Let the student move certain steps in front from a point and note the position. Let the student then move backwards and note the final position. Ask students to draw the same movement on a paper(notebook) and find the final position with an integer.

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  i. Water freezes at 0° Celsius and orange juice freezes at around -10 degree Celsius. If both milk and orange juice are kept in a refrigerator, which liquid freezes faster. The refrigerator has the capacity to reduce 2° Celsius hourly. If water gets frozen at 10 am, when will the Juice get frozen?
  ii. How does the invention of negative integers help handle real life problems? *(Suggested answer: Helped in dealing with dues, credits, negative balance, negative temperature, etc)*

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - Introducing integers- https://www.youtube.com/watch?v=OSfDRqxmXAE
  - History of integers- https://463431396329892656.weebly.com/history-of-integers.html
  - History of integers – https://bit.ly/3r3m1QZ.
  - Adding and subtracting integers (counters)- https://www.youtube.com/watch?v=Q9fPuX3lWXY
  - Adding and subtracting integers (number line)- linkhttps://www.youtube.com/watch?v=A186iWp5vKQ
  - Adding and subtracting integers (symbolically) https://www.youtube.com/watch?v=jVvvUiExjes

Topic: VII-B1 Patterns and Relations [350 minutes]

Introduction

The concept of patterns was first described by Christopher Alexander in A Pattern Language: Towns, Buildings, Construction. The book describes a "language" for designing the urban environment. The units of this language are patterns. They may describe how high windows should be, how many levels a building should have, how large green areas in a neighbourhood are supposed to be, and so on.
In Mathematics, a pattern is a repeated arrangement of numbers, shapes, colours, etc. If the set of numbers are related to each other in a specific rule, then the rule or manner is called a pattern. Sometimes, patterns are also known as a sequence.
Refer https://en.wikipedia.org/wiki/A_Pattern_Language for further reading.

Utility and Scope

Patterns are important because they offer visual clues to an underlying order. If you can unlock a pattern, then you have the ability to alter or shape it in order to achieve some effect. Patterns can also be used as a template that will enable one to quickly analyse a situation and understand how it works.
Patterns help us organize thoughts and establish order in our lives. As we begin to connect patterns in nature and life, they bring a sense of harmony to our minds. Patterns are excellent in helping us establish priorities.

Competency

- Represent patterns as linear algebraic expressions and simplify expressions to solve everyday problems.

Objectives

- Develop an understanding of constants, variables, coefficients, expressions and equations.
- Use pictures, series of numbers, table of values and explore patterns rules to make predictions.
- Create algebraic expressions from given words and vice versa.
- Evaluate expressions visually using algebraic tiles.
- Explore patterns in nature and your immediate environment.

• Learning Experiences

- Pre-assessment on the terms related to algebra (true and false sentence).
- Introduce the terms variable, coefficient, constant, expressions and equations.
- Visually comprehend the effect of variables in an expression using an interactive tool (Sample video link: hts://www.youtube.com/watch?v=tHYis-DP0oU, the link contains how a variable can represent a changing quantity).
- Use a set of pictures to make predictions for unknown values (may watch this video link to help students make predictions). https://www.youtube.com/watch?v=KSrnZMAfwTM
- Create a table of values using a pattern.
- Investigate pattern rules to match the given situations (This video link: https://www.youtube.com/watch?v=J0Q8XtSuZx8 contains how to draw a pattern rule from the description).
- Create single variable algebraic expressions from the given situation (Example: A dozen eggs cost Nu 120 and an apple cost Nu 100 per kilogram. Write an algebraic expression for the total cost of 'n' dozen of eggs and a kg of apples).
- Use an example to describe an algebraic expression.
- Visually represent algebra tiles to simplify expressions:
  - A rectangle represents a variable and a square represents a constant value. White can represent a positive value and grey can represent negative values. (can use improvised models)
  - Distinguish like and unlike terms.
- Show how to evaluate simple expressions using models (concretely and pictorially) or may share a video on simplifying expressions (sample video link on simplifying expressions: https://www.youtube.com/watch?v=ztXllVU-E-g).
- Explore the idea of patterns in other subjects especially in science and geography.

• Assessment

Performance Task 1

Ask students to solve the set of questions on modelling expressions using algebra tiles.

Performance Task 2

Provide a set of questions on simplifying simple algebraic expressions.

E.g: a) (7n + 3) + (-4n + 5)

Performance Task 3

Carry out some tasks where patterns can be related with nature/environment. Let students explore patterns from petals of flowers, leaves of a plant, doors and windows in buildings, threads used in weaving cloths etc.

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  i. A creeper may grow twice the earlier day in summer months. It is 15 cm on the first day of observation. Show its daily length in the table to find how long the creeper will be, if you go to observe on the 7th day.
  ii. Discuss on pattern rules how observing patterns in the real world help us in understanding the world better.

- Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - History of Patterns- https://en.wikipedia.org/wiki/A_Pattern_Language
  - Represent a changing quantity- https://www.youtube.com/watch?v=tHYis-DP0oU
  - Making predictions for unknown values- https://www.youtube.com/watch?v=KSrnZMAfwTM
  - Draw pattern rule from a description- https://www.youtube.com/watch?v=J0Q8XtSuZx8
  - Simplifying expressions- https://www.youtube.com/watch?v=ztXllVU-E-g

Topic: VII-B2 Solving Single Variable Linear Equations [200 minutes]

Introduction

Systems of linear equations arose in Europe with the introduction in 1637 by René Descartes of Coordinates in geometry. In fact, in this new geometry, now called Cartesian geometry, lines and planes are represented by linear equations, and computing their intersections amounts to solving systems of linear equations.
An equation that has the highest degree of 1 is known as a linear equation. This means that no variable in a linear equation has an exponent of more than 1. The graph of a linear equation always forms a straight line.

Utility and Scope

The applications of linear equations are vast and are applicable in numerous real-life situations. To handle real-life situations using algebra, we change the given situation into mathematical statements. So that it clearly illustrates the relationship between the unknown variables and the known information.
Linear equations are used in real-life situations to

- Solve age related problems.
- Calculate speed, distance, and time of a moving object.
- Solve geometry related problems.
- Calculate consumer and percentage related problems.
- Solve problems related to work, time, and wages.
- Solve problems based on force and pressure.

A. Competency

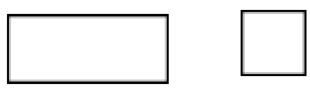
- Apply linear equations using various strategies (models and inverse operation) in relevant fields.

B. Objectives

- Realize the differences between an equation and an expression.
- Model and solve equations using algebraic tiles.
- Recognise an equation maintains balance on both sides and solve it using inverse operations.
- Explore the relevance of linear equations in real life applications.

C. Learning Experiences

- Conduct pre-assessment on exponents and algebraic expression to relate with linear equations.
- Use an example to differentiate an equation and an expression.
  Sample:
  - *An equation is a mathematical sentence with an equal sign (=), like 2 + 3 = 5*
  - *An expression is a number, a variable, or a combination of numbers and variables and operation symbols without the equal sign, like 2x+ 3, 6n-4*
- Show the pictorial representation of solving algebraic equations using models like algebraic tiles (rectangle - variable and square-constant), Rectangular Model and Number line.
  Eg:



X 1

- Classify like and unlike terms to conduct simple algebraic operations like addition and Subtraction.
- Use the concept of balance to explain the inverse operation in solving equations.
  Suggestions:
  - Inverse *operation as undoing operations (opposites).*
  - Sketch *a balance to represent an equation.*
  - May *watch a video on solving equations using the idea of balancing (suggested link:* https://youtube.com/watch?v=XN48Ba9Ncr8*).*
- Discuss how to solve linear equations symbolically. May use the suggested link: https://youtube.com/watch?v=jWpiMu5LNdg that shows how to solve simple linear equations symbolically (*Trim the video to teach only the contents needed).*
- Explore the cross pollination of linear equations with other relevant concepts. The link is an example of linear equations and its applications in real life situations. https://byjus.com/maths/applications-of-linear-equations/

D. Assessment

Performance Task 1

Assign a set of questions on representing algebraic equations using various models like tile, number line or rectangle (May refer to Understanding Mathematics Textbook for Class VII or other relevant resources).

Performance Task 2

Assign a set of questions on determining solutions for an algebraic equation using the inverse operation (May refer to Understanding Mathematics Textbook for Class VII or other relevant resources).

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  1. Dawa meets his friend who was his classmate in Sherubtse college. After their departure from college, they are meeting after 11 years. Dawa tells his friend that he married at the age of 25 years and one year after his marriage he became a father. His child is 5 years old today. How old was Dawa when he departed with his friend from college. (Answer: 20 years)
  2. Equation shows balance. Check its importance in the real world.

E. Resources

- Understanding Mathematics Textbook for class VII.
- Teacher's Guide to Understanding Mathematics Textbook for Class VII.
- National School Curriculum Mathematics Framework for PP-XII.
- Online resources
  - o Solving linear equations- https://m.youtube.com/watch?v=l3XzepN03KQ)
  - o S*olving equations using the idea of balancing-* https://youtube.com/watch?v=XN48Ba9Ncr8
  - o Solve linear equations symbolically- https://youtube.com/watch?v=jWpiMu5LNdg
  - o Linear equations and real life applications- https://byjus.com/maths/applications-of-linear-equations/

Topic: VII-B3 Graphical Representation [350 minutes]

Introduction

Graphical representation refers to the use of charts and graphs to visually display, analyze, clarify, and interpret numerical data, functions, and other qualitative structures.

The graphic presentation of data and information offers a quick and simple way of understanding the features and drawing comparisons. Data that are represented through graphs are represented by a variety of symbols, such as lines on a line chart, bars on a bar chart, or slices on a pie chart, from which users can gain greater insight than by numerical analysis alone.

Utility and Scope

The purpose of a graph is to present data that are too numerous or complicated to be described adequately in the text and in less space.

Graphs are beneficial because they summarize and display information in a manner that is easy for most people to comprehend. Graphs are used in many academic disciplines, including maths, hard sciences and social sciences. They make appearances in corporate settings, serving as useful tools to convey financial information and facilitate data analysis.

Graphic visual representation of information is a crucial component in understanding and identifying patterns and trends in the ever-increasing flow of data. Graphical representation enables the quick analysis of large amounts of data at one time and can aid in making predictions and informed decisions.

A. Competency

- Create a graph and examine or describe a change on the graph to solve related problems.

B. Objectives

- Construct the axes as two number lines that are perpendicular to each other, intersecting at the origin.
- Use a table of values for graphing.
- Interpolate and extrapolate on a graph.

- Determine if an ordered pair satisfies a given equation.
  - by plotting the points to see if they are keeping with the rest of the points in the pattern.
  - by substituting them into the equation to see if they make the equation true or false.
- Construct a graph to describe a change on a graph.
- Evaluate single variable expressions by substituting a variable in the expression.
- Create a straight line graph related to daily life.

C. Learning Experiences

- Conduct pre-assessment on graphs: discuss the components of a drawing a linear graph (example: using x-axis and y-axis for horizontal and vertical axes intersecting at the origin).
- Use the visual interactive tool to draw straight line graph using a table of values on GeoGebra (sample link to draw a straight line graph: https://www.geogebra.org/m/JDhRRHFq).
- Show how to locate a point between two known points (interpolating) and a point that lies beyond the existing data (extrapolating).
- Use a graph to show the linear relationship formed by the points (ordered pair).
- Show how to simplify equations by substituting a variable in the equation.
- Construct a graph to describe a change on a graph (example: Drawing a graph that could be used to determine rate over time).
- Let students collect data that shows linear relationships related to their daily life situation and ask them to create a straight-line graph (e.g: Number of periods to a number of minutes in a day).

D. Assessment

Performance Task 1

Use an equation to draw a table of values. Use the table of values to create a graph and to describe a change on a graph (May refer to Understanding Mathematics Textbook for Class VII or other relevant resources).

Performance Task 2

Provide a real-life problem to create a graph.

- Sample Questions:

Namgay hired a taxi to go home. For every 4 km, a taxi charges Nu 50. Create a graph to find the amount needed to pay as a hiring charge if Namgay travels 16 km.

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
    i. A Lhakhang in a village is a certain distance away from Ram Bahadur's Camp. Sangye's camp is 5 km further away from Ram Bahadur's camp. Write an equation to represent the distance between Lhakhang and Sangye's camp. Create a table of values and plot in graphs to see how distance changes as they shift their camp, keeping the conditions the same.
    ii. Extrapolating graphs and pattern rules are connected. Discuss how both of these help us predict future events. Show examples wherever possible.

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
    - Constructing a straight-line graph - https://www.geogebra.org/m/Adc44ZZq
    - drawing straight-line graph - https://www.geogebra.org/m/JDhRRHFq

Topic: VII-C1 SI Units [200 minutes]

Introduction

The International System of Units (SI) was established in 1960 by the 11th General Conference on Weights and Measures (CGPM, Conférence Générale des Poids et Mesures). It is the modern form of the metric system and the world's most widely used system of measurement. It is the only system of measurement with official status in nearly every country in the world employed in science, technology, industry, and everyday commerce.

The SI comprises a coherent system of units of measurement starting with seven base units, which are the second (symbol s, the unit of time), metre (m, length), kilogram (kg, mass), ampere (A, electric current), kelvin (K, thermodynamic temperature), mole (mol, amount of substance), and candela (cd, luminous intensity). Here in class VII, we will emphasize on units of capacity, mass and length, and their conversion, both within the same and with other standard and non-standard units.

Utility and Scope

SI unit is an international system of measurements that are used universally in technical and scientific research to avoid confusion with the units. Having a standard unit system is important because it helps the entire world to understand the measurements in one set of unit systems.

A. Competency

Understand the relations and convert SI units from one to another unit and vice versa in real life contexts.

B. Objectives

- Identify, use, and convert SI units (metre, litre and gram) to measure, estimate, and solve problems.
- Determine the special relationship between volume, capacity and mass.
- Explore and convert common imperial units into metric units.

C. Learning Experiences

- Recapitulate the prior knowledge on basic units for measurement for length, capacity, and mass.
- Discuss why it is important to have different units for the same attribute.
  Example: Which is clearer to understand?
  A. Drive 15 km and walk 40 m
  B. Drive 15,000 m and walk 40 m
  C. Drive 15 km and walk 0.04 km
  Conclude why it is important to have different units for the same attribute.
- Explore how to convert metric units to one another using different prefixes.
  Suggestions:
  - o Prepare a chart or PowerPoint presentation that shows the prefix, symbol and meaning of the metric units (kilo, hecto, deca, deci, centi, and milli).
  - o Explore the meanings of the special units, hectare and tonne, and tell students some examples of how they are used.
  - o Use the chart or PowerPoint presentation to convert metric units to one another (demonstrate a few examples).
- May use a video link for unit conversion.
  The weblink https://www.youtube.com/watch?v=pEDVddQviml contains a video on metric unit conversions.
- Determine the special relationship between volume, capacity and mass ($1cm^3$ = 1ml=1gram) and solve related problems (May refer to Understanding Mathematics Textbook for class VII).
- Explore common imperial units such as inch, feet, mile, acre and decimals that are used commonly in real life situations by converting them to metric units. Refer to the following weblink for further reading.
  https://www.cuemath.com/measurement/imperial-system/

  Eg: Tashi is purchasing 30 cubic feet (cft) of planks from a sawmill to make a wooden bathtub. Convert the volume of plank in cubic metres(m^3).

D. Assessment

Performance Task 1

Design a question to estimate and measure any available item in school and express the measurement in different equivalent units. For example, a football field is 10,000 m^2 or one hectare or 0.01 km^2.

Performance Task 2

Prepare/use an interactive worksheet for converting units from one form to another form.

https://www.liveworksheets.com/w/en/math/1849769

https://cutt.ly/zkn6xYi)

Performance Task 3

Explore the use of non-standard measurement units in your locality and relate the concept to metric units. Share the advantages and disadvantages of using non-standard units in daily activities.

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
    i. A circular garden has a 1-metre-wide footpath around it. The outer boundary of the foot path is fenced and has a 2 m wide gate for entering the garden. If the radius of the garden is 2.5 metres, find the length of the fence. (Sketch before solving)
    ii. Share your views on SI units.

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
    - Unit Conversion in the Metric System - CLEAR & SIMPLE - YouTube contains a video on metric unit conversions.
    - Conversion of Imperial units to metric units- https://www.cuemath.com/measurement/imperial-system/

- https://www.liveworksheets.com/w/en/math/1849769 worksheet for converting units from one form to another form.
- https://cutt.ly/zkn6xYi worksheet for converting units from one form to another form.

Topic: VII-C2 Perimeter of Polygons and Composite Shapes
[200 minutes]

Introduction

Polygons have been known to people since historic times. Greeks studied non-convex regular polygons in the seventh century BC on a krater by Aristophanes. Thomas Bradwardine was the first person to learn about non-convex polygons in the 14th century.

Polygons got their name in Ancient Greece since the word “polygon” was derived from the Greek language, *polus means many* and *gon means corner or angle*.

Utility and Scope

Polygons are an important part in the field of measurement that shows how to make patterns, tessellations, how to make other shapes, build on polygons to form 3D shapes and about symmetry.

We use different types of polygons in our everyday life. Different polygons form patterns and build structures. Polygons are used in different areas such as;

- The tiles that you walk on are squared in shape, which implies that they are polygons.
- The truss of a building or bridge, the walls of a building, etc., are examples of polygons.
- The rectangular part of a chair on which you are sitting is an example of a polygon.

A. Competency

- Confirm relations to calculate the perimeter of regular and irregular polygons through investigation and apply in real-life situations.

B. Objectives

- Investigate the perimeter of a regular polygon.
- Calculate the perimeter of irregular polygon and composite shapes.

C. Learning Experiences

- Recall the meaning of the perimeter of a shape.
- Investigate whether the perimeter of a regular polygon is always equal to n x s (n= number of sides, s = side length).
- *Suggestion:*
  - o *Measure the side length of different regular polygons and record it in the table shown.*
  - o *Link the number of sides to the perimeter.*

Polygon (regular)	Number of sides	Side length	Perimeter
Triangle	3	Example If s=15 cm	(P = ns) 3x 15 cm = 45 cm
Square	4		
Pentagon	5		
Hexagon	6		

- Investigate that the perimeter of an irregular polygon is always the sum of the lengths of the sides.
- *Suggestions:*
  - o *Provide pictures of different irregular polygons and let them measure the side length and find the perimeter.*

    Suggested link: https://www.liveworksheets.com/w/en/math/932242
- Teachers may share relevant video links to calculate the perimeter of regular and irregular polygons.

  Suggested link: https://www.youtube.com/watch?v=Ewir65KTBp8
- Use an interactive tool (PhET simulation: https://bit.ly/2MGQ15g) to determine the perimeter of the composite shape.
- Demonstrate how to determine the side length of a few quadrilaterals from the given perimeter.

D. Assessment

Performance Task 1

Design activity that helps to show that perimeter is the sum of individual sides added together.

Sample Questions:

i. Create composite polygons on geoboards using rubber bands and find the perimeter.
ii. Measure the perimeter of real objects (doors frame, table top, floor, window frame, books etc).

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)




- Reflective Questions

i. A straight 14 m long rod is bent to form a pillar square bar. The excess from the rod is then bent into pillar circular bars. What will be the radius of the circle? Show the side length of square pillar bars you made. (Answer: radius of circle depends on the side length of square pillar bars)

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII

- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - Calculating perimeter of composite shapes- https://www.youtube.com/watch?v=4c_-Q5gADcQ&t=33s
  - Worksheet for calculating perimeter of Irregular shapes- https://www.liveworksheets.com/w/en/math/932242
  - Calculating perimeter of polygons- https://www.youtube.com/watch?v=Ewir65KTBp8
  - Determining perimeter of the composite shape-Area Builder 1.1.31

Topic: VII-C3 Area of Regular Polygons and Composite Shapes [400 minutes]

Introduction

A polygon is a flat two-dimensional shape with straight sides that are fully closed. The sides must be straight, not curved. However, polygons can have any number of sides.

Polygons can be either regular or irregular. A regular polygon is a polygon in which all sides are the same length and at the same angle. An irregular polygon is a polygon with sides and/or angles of differing lengths and sizes. Although it must still have straight sides that are all joined up, or it would not be a polygon.

The composite shape is a shape in which a few polygons are put together to form a required shape. These shapes or figures can be made up of a combination of triangles, squares, and quadrilaterals, etc.

Utility and Scope

Any form of tiling involves polygons. The tiles need to tessellate to cover an area without leaving any gaps. This is directly connected to the angle properties of polygons. Architects include polygons with every plan of a house - rooms usually have 90° corners, but not always. Rooms on a plan are polygons.

The cost of building any structure depends on the lengths of the walls and the size of the angles that involve properties of polygons.

The greatest ratio between the area and the perimeter is found in a circle and the smallest ratio is found in a triangle.

A. Competency

- Explore relations to calculate the area of regular polygons (Triangles and quadrilaterals) through investigation and apply in related problems.

B. Objectives

- Explore the relations (formula) to calculate the area of quadrilaterals (rectangle, square, parallelogram, rhombus, trapezoid and kite).
- Solve problems involving the area of composite shapes (mm^2, cm^2, m^2, km^2, hectare) by breaking into familiar shapes (triangle and quadrilaterals).

C. Learning Experiences

- Pre assessment on the properties of triangles and quadrilaterals with the help of diagrams.
- Demonstrate how to calculate the area of quadrilaterals by breaking them into triangles using an interactive tool (may use GeoGebra).
  - o The web link https://www.geogebra.org/m/WMVY3esJ can be used for calculating the area of triangles and quadrilaterals. (Discuss quadrilaterals relevant to 7th grade).
- Use an interactive tool(s) to investigate how to determine the area of composite shapes by composing and decomposing into familiar shapes.
  - o The web link https://www.geogebra.org/m/ukfzs6pn can be used for calculating areas of composite shapes.
  - o Use PhET interactive simulation https://bit.ly/2MGQ15g to build different shapes and find the relationship between area and perimeter.
- Calculate the area of composite shapes with given side lengths.
- Relate the area of composite shapes to the area of land measurement (Land survey).

D. Assessment

Performance Task 1

Determine the area of composite shapes by providing pictures or using appropriate software (PhET simulation, GeoGebra, etc).

- o The web link https://www.geogebra.org/m/WMVY3esJ is an interactive tool that can be used for calculating the area of triangles and quadrilaterals.
- o The web link https://www.geogebra.org/m/ukfzs6pn is an interactive tool that can be used for calculating areas of composite shapes.
- o The web link https://bit.ly/2MGQ15g can be used for building shapes and determining area and perimeter.
- o The web link https://www.liveworksheets.com/w/en/math/1510513 provides a worksheet to calculate the Area and Perimeter of composite shapes.

- Sample Questions

  Karma Dema has a square field that measures 49 square metres. She wants to construct a circular water tank. What will be the greatest area covered by the tank?

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  1. Area means surface covered. Discuss how this concept is applied in tailoring, white washing, floor concreting and land transaction. How would these be affected if the area is not understood properly?

E. Resources

- Understanding Mathematics, Textbook for class VII
- Teacher's Guide to Understanding Mathematics, Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - Calculating the area of triangles and quadrilaterals- https://www.geogebra.org/m/WMVY3esJ
  - Calculating areas of composite shapes- https://www.geogebra.org/m/ukfzs6pn
  - Simulation to build different shapes and to find the relationship between area and perimeter- https://bit.ly/2MGQ15g
  - Worksheet to calculate the area and perimeter of composite shapes- https://www.liveworksheets.com/w/en/math/1510513

Topic: VII- C4 Circumference of a Circle [200 minutes]

Introduction

The circumference of a circle is the length of the boundary of the circle. The perimeter of a circle is called the circumference and is the linear distance around the edge of a circle. The circumference of a circle is proportional to its diameter, d, and its radius, r, and relates to the famous mathematical constant pi (π).

Utility and Scope

Finding the circumference of a circle can also be used for many reasons in day-to-day life. Think about everyday items that are circular - car wheels, cups and saucers, roundabouts and even the Earth, moon, sun, etc.

Because the circumference measures a distance around something, it's an invaluable tool for lots of work that involves area, shape and measurement. The concept of the circumference is also used in farming activities and plumbing works.

A. Competencies

- Conclude that the value of π is a ratio of circumference to the diameter of the circle.
- Deduce the formula to calculate the circumference of a circle and apply it to related problems.

B. Objectives

- Relate diameter, radii, circumference to solve problems.
- Record the value of $\frac{C}{d}$ for several circles through measurement to investigate the value of π.
- Develop the formulas for C = πd and C = 2πr.
- Compute lengths of different parts of a circle (semicircle and quadrants).

C. Learning Experiences

- Investigate the value of π through measurement and charting the value of $\frac{C}{d}$ for various circles.
- Develop the formula for the circumference of the circle (C = πd and C = 2πr).

Suggestions:

- o Sketch a circle on the board and label the parts.
    - o Mention the term radius and its plural, radii.
    - o Determine the relationship between radius, diameter, and circumference.
    - o Assure students that they may use either 3.14 or $\frac{22}{7}$ as an approximation of pi.
    - o Use an interactive tool (GeoGebra) through this web link: https://www.geogebra.org/m/DGyTYERs#material/y5H8Hd2d, where they can manipulate the radius of the circle to find the corresponding circumference.
- Model the procedure of measuring the diameter and circumference concretely.
    - o Measure the diameter of the different sizes of round objects (base of bottle or bucket, cup, plate etc.) and find the circumference of it.
    - o Show how to find radius when the circumference is given.
    - o Provide real-world problems to use circumference formulas. Example, the diameter of a cylindrical water tank is 9m. Find its circumference.
- Use the video link https://www.youtube.com/watch?v=DLcjed7qy4I to show why the circumference of a circle is $2\pi r$ and pi as the ratio of circumference to the diameter.
- The weblink https://www.liveworksheets.com/w/en/math/1570866 provides a worksheet to find the circumference of a circle.

D. Assessment

Performance Task 1

Ask students to solve a set of questions related to the objectives. (May refer to Understanding Mathematics Textbook for Class VII or provide competency-based questions from other sources).

- Sample Questions

    The measurement from the centre of the tyre of a cycle to its outer edge is 35cm. If the tyre makes 5 complete rotations on the ground, how far will the cycle travel?

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
    i. The diameter of a circular fountain in a city park is 8 m. A sidewalk that is 1 m was built around the fountain. Find the circumference of the fountain and the sidewalk. Use 22/7 as an approximation for π.
    ii. How would painting of football and basketball be affected if circles were not there. Discuss how circles are applied in sports.

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
    - Manipulating the radius of the circle to find the corresponding circumference- https://www.geogebra.org/m/DGyTYERs#material/y5H8Hd2d
    - Circumference of circle- https://www.youtube.com/watch?v=DLcjed7qy4I
    - Worksheet to find the circumference of a circle- https://www.liveworksheets.com/w/en/math/1570866

Topic: VII-D1 Angle Relationships [200 minutes]

Introduction

The history of the mathematical measurement of angles, possibly dates to 1500 BC in Egypt, where measurements were taken of the Sun's shadow against graduations marked on stone tables. Use the link to learn about the history of angles https://www.newworldencyclopedia.org/entry/Angle_(mathematics).
Slowly properties and relationships between angles evolved. When we talk of angle relationships, we are comparing the position, measurement, and congruence between two or more angles. Here we learn angle relations with the side length of the triangle and how they are inter-connected.

Utility and Scope

Triangles have many fundamental advantages that make them great for architects as well as interested students: they are quite common, structurally sound, and simple to apply and utilise in everyday life. A triangle's strength comes from its shape, which distributes pressures evenly along its three sides. Inside the triangle, the angles are also significant. The total angle is always 180 degrees. Triangles are unique in that they are extremely powerful. Only a triangle is stiff out of all the two-dimensional structures we can create out of straight metal struts.
More of this can be learnt from the website:
How Triangles are Used in Real Life Situations

A. Competency

- Establish relationships between angles and side lengths of triangles to investigate their relations.

B. Objectives

- Conclude through investigating models that the sum of the interior angle of a triangle is always 180°.
- Investigate to establish relationships in a triangle between the longest side & the largest angle.
- Investigate to establish relationships in a triangle between the shortest side & the smallest angle.

C. Learning Experiences

- Pre-assessment on the types of triangles based on angle and side lengths (Acute, Obtuse, Right, Isosceles, equilateral and scalene triangles).
- Use concrete materials to investigate the sum of angles in a triangle is equal to 180°.

  Suggestion:
  - o *Facilitate different ways to investigate the sum of angle in a triangle =180°.*
  - o *Example, cutting out a triangle and tearing the corners off and placing them together so that they form a straight angle.*
  - o The video link https://www.youtube.com/watch?v=_yRDwYYjgOY demonstrates the sum of interior angles in a triangle is 180^0.
- Measure side lengths and angles to show that the largest angle in the triangle is always opposite to the longest side and the smallest angle is always opposite to the shortest side (encourage students to use protractors and rulers).

  This lesson can practically be demonstrated using sticks laid on the ground.
  Provide six or more sticks; three of the same length and other sticks of different lengths.
  Arrange three of these sticks to form an equilateral triangle.
  Replace one of the sticks with the shortest of the remaining sticks and then note the angle change opposite to the added side.
  Continue replacing sticks in ascending order and note what happens to the opposite angle each time the stick gets longer.
  This should enable us to discover that as the side gets longer the angle gets larger.
- You can make site visits to telephone towers and high-tension electric pylon sites, to see the application of angle relationships. The economic advantage of where more materials would be required to build a tower can be explored; having shorter angles on top of a triangle requires a shorter base compared with a larger angle on top with a longer base.
- The weblink https://yoshiwarabooks.org/trig/Side-and-Angle-Relationships.html contains details about side and angle relationships.
- The weblink https://yoshiwarabooks.org/trig/Side-and-Angle-Relationships.html contains definitions of various angles and triangles.

D. Assessment

Performance Task 1

Ask students to solve a set of questions related to the objectives.

OR

Prepare/use an interactive worksheet related to objectives using https://www.liveworksheets.com/w/en/math/765587 (Sample worksheet is available at https://bit.ly/3p7SbHX) Practical work observing angle change can be tried as suggested above by using sticks of varied lengths.

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  i. Will you be able to draw a triangle if the sum of any two sides of a triangle is less than the third side? Prove your statement.
  ii. A triangle is formed with three sides. The angle opposite to the longest side is the largest. If you go on increasing the longest side, what will happen to the other sides? If you continue increasing all three sides in equal proportion, what will happen to the angles?

E. Resources

- Understanding Mathematics, Textbook for class VII
- Teacher's Guide to Understanding Mathematics, Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - History of angles- https://www.newworldencyclopedia.org/entry/Angle_(mathematics)
  - Scope and utility- How Triangles are Used in Real Life Situations
  - Interior angles in a triangle- https://www.youtube.com/watch?v=_yRDwYYjgOY
  - Side of a triangle and angle relationships- https://yoshiwarabooks.org/trig/Side-and-Angle-Relationships.html
  - Definition of various angles and triangles- https://yoshiwarabooks.org/trig/Side-and-Angle-Relationships.html
  - Sum of interior angle worksheet- https://www.liveworksheets.com/w/en/math/765587

- Worksheet on naming angles- https://bit.ly/3p7SbHX

Topic: VII-D2 Constructing and Bisecting Angles [200 minutes]

Introduction

Archimedes (c. 285–212/211 BC) made use of neusis (the sliding and manoeuvring of a measured length or marked straightedge) to solve one of the great problems of ancient geometry: constructing an angle that is one-third the size of a given angle. For details visit https://bit.ly/3zATKFa.

The angle bisector theorem later appears as Proposition 3 of Book VI in Euclid's Elements. According to Heath (1956, p. 197 (vol. 2), the corresponding statement for an external angle bisector was given by Robert Simson who noted that Pappus assumed this result without proof. This can be further read from the site; https://bit.ly/3pYOrfH.

Utility and Scope

Angle construction is widely used in architecture. This is to assure symmetry and balance when designing the look of the structure. It is used to ensure that window panes, windows, and doors are proportionate and even. Angle construction later became a handy tool in constructing triangles and other polygons.

A. Competency

- Demonstrate an ability to construct and bisect angles to further construct other angles.

B. Objectives

- Construct 30°, 45°, 60°, 90°, 120° and 180°.
- Bisect angles and create new angles after bisecting.

C. Learning Experiences

- Distinguish between the verbs construct, draw and sketch.
- Demonstrate how to construct 30^0, 45^0, 60^0, 90^0 and 120^0 angles. The weblink https://www.youtube.com/watch?v=5l8bltVe_IE contains video on how to construct those angles.
- Demonstrate how to construct an 180^0 angle. The weblink https://www.youtube.com/watch?v=n38QlhxFFWU contains a video on how to construct an 180^0 angle.

- You may use the video link https://www.youtube.com/watch?v=IlwvSzfUkOY which shows how to construct angle bisectors.
- Demonstrate how to construct an angle bisector using a compass or use interactive teaching tool (GeoGebra: https://www.geogebra.org/m/WqpC3XD7) to demonstrate and bisect angles.
- Explore how you can make other angles using 60^0, 90^0 and 120^o angles and bisectors. Show one way of making a 105^0 angle (example, making a 90^0 angle, adding a 60^0 angle, and bisecting the 90^0 angle), and then ask students to suggest other ways of constructing a 120^0 angle.
- Demonstrate splitting of bamboo/logs with wedges as application of bisection.
- Use a perpendicular bisector to find the midpoint of the basketball court/ plane field.

D. Assessment

Performance Task 1

This lesson is practical, and students need a mathematical instrument box. Teachers can organise competitions on who can construct families of angles of 15^o like $30\,^o$, $45\,^o$, $60\,^o$, $75\,^o$, $90\,^o$, $105\,^o$ and $120\,^o$ in short duration with accuracy and neatness.

Short video recording/competition may be held where students can record videos explaining construction steps. It may be assessed by peers or viewed by parents, shared in a close social media forum (say class telegram group) or assessed as competition.

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
    i. To bisect an angle, first, you keep a compass point on the vertex and cut arms with an arc. Next, you take a compass point on the point where the arc cuts the arms.
    - What will happen if you change the radius of the compass and then make arcs?
    - Will the angle still get bisected?

ii. Why do you think bisection of angle is important? How will it help you in your future learning and where do you think it will be applied?

iii. While bisecting angles, see how changing radius of compass affects result.

E. Resources

- Understanding Mathematics, Textbook for class VII
- Teacher's Guide to Understanding Mathematics, class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - Introduction/History- https://bit.ly/3zATKFa
  - Introduction/History https://bit.ly/3pYOrfH.
  - Constructing 30^0, 45^0, 60^0, 90^0 and 120^0 angles- https://www.youtube.com/watch?v=5l8bltVe_IE
  - Constructing 180^0 angle- https://www.youtube.com/watch?v=n38QlhxFFWU
  - Construct angle bisectors- https://www.youtube.com/watch?v=lIwvSzfUkOY
  - Demonstrating angle bisection-https://www.geogebra.org/m/WqpC3XD7

Topic: VII-D3 Properties of Transformation [500 minutes]

Introduction

The first systematic effort to use transformations as the foundation of geometry was made by Felix Klein in the 19th century, under the name Erlangen programme. For nearly a century this approach remained confined to mathematics research circles. His work can be learnt from site:
https://www.britannica.com/biography/Felix-Klein

Utility and Scope

Geometric transformations are needed to give an object the required position, orientation, or shape starting from an existing position, orientation, or shape. The basic transformations are translation, rotation, reflection, dilation, and other important types of transformations are projections and mappings.

The symmetry of our face, a body of butterfly, production of shoes, spectacle frames, flipping images on a computer, mirror images of the chemical structure of the sugar molecules, glucose (in sugarcane) and fructose (in fruit) all exhibit transformation properties. Transformation is also present in the tessellation, where the creation of floor and mosaic design happens. This gives students the ability to manipulate figures in the plane freely.

A. Competency

- Apply the properties of transformation to transform regular polygons through translation, reflection and rotation.

B. Objectives

- Use formal language: translations, reflections, and rotations for slides, flips and turns.
- Deduce the properties of transformation and apply transformations (Rotation, reflection and translation).
- Use tessellations as a context for transformations and create art-related designs using translations, reflections, and rotations.

C. Learning Experiences

- *Translation, Reflection and Rotation may be explained separately but emphasis should be there to see them in combination. Tessellation requires combined transformation which helps create design.*

  Translation
- Describe translation as a slide.
- Demonstrate how to translate a shape first on a grid visually and then without grid. Emphasise on their properties.
  - o Draw a diagram of a 2D shape and its image after a translation.
  - o Use the diagram to talk about the different ways you can describe the translation, including using an arrow and referring to the endpoints of a line segment.
  - o Identify the similarities between the original shape and the image.
  - o Show that the orientation is the same in the translation (to help students understand, draw an example of two shapes with opposite orientation).
- Use appropriate interactive tools (GeoGebra) through this web link: https://www.geogebra.org/m/GPyAmqWt#material/x3hFZQZb to translate a 2D shape.

  Reflection
- Describe reflection as a flip/mirror image.
- Demonstrate how to reflect a shape on a grid visually (considering reflection lines) and emphasise its properties.
  - o Use the diagrams to show the reflection line.
  - o Show how to locate a reflection line for a shape.
  - o To reflect on paper, draw a polygon on one side of the paper and fold the paper to form a reflection line. When it is folded, the original shape points match up with their image points.
  - o Help Students describe the reflection seen and notice that the reflected image has an opposite orientation to the original shape.
- Use appropriate interactive tools (GeoGebra) through this weblink https://www.geogebra.org/m/Q558rky7 to reflect a 2D shape.

Rotation

- Describe rotation as turns.
- Demonstrate how to rotate the shape visually and emphasise its properties.
  - o Draw any 2D shape on the board and demonstrate how you would use a compass and protractor to rotate it 57^0 in both clockwise and counterclockwise directions, around one of the vertices. The weblink https://www.youtube.com/watch?v=5va-z_ViXLU contains video on how to rotate using compass and protector given the turn centre and degree of turn.
  - o Use your rotation to show how you could measure the angle of rotation, with and without tracing paper.
  - o See if the image is congruent to the original shape, and explain. Be sure that you use the formal terminology (e.g. image and original shape).
  - o See if the image has the same orientation as the original shape, and explain.
- Use appropriate interactive tools (GeoGebra) to investigate and practice rotation visually.
  - o The weblink https://www.geogebra.org/m/GPyAmqWt#material/qfXNnhs6 shows how to rotate an image on a grid.
  - o The weblink https://www.geogebra.org/m/GPyAmqWt#material/C7GxC4uG can be used to investigate the rotation of the 2D shapes.
- Explore real life application of transformation (Example… floor tiles arrangement through tessellation).
- Let students watch videos on transforming shape (translation, reflection, rotation) by sharing appropriate video links.
  - o The video link https://www.youtube.com/watch?v=VJTxv-tRKj0&t=1s contains a presentation on three basic transformations (translation, reflection, rotation).
- Ask students to practice translation, reflection and rotation by using interactive tools like GeoGebra (provide instruction).
  - o The video link here https://www.geogebra.org/m/hMNc4eES can be used for investigating transformation (translation, reflection, Rotation).
- Let students watch videos on creating tessellations using transformations.
  - o The video link https://www.youtube.com/watch?v=0xvP_QDFr-s contains video on identifying transformations in tessellations.

- Explore Games (see Understanding Mathematics, student textbook VII) related to transformation to enrich learning.

D. Assessment

Performance Task 1

Let students draw out the properties of each transformation by performing the transformation.

Performance Task 2

Prepare questions to perform a transformation (translation, reflection and rotation).

OR

Provide a tessellated picture and let them identify the transformations (translation, reflection, and rotation) used in it.

Performance Task 3

Students can take a campus tour to identify applications of transformation and make observations. Teachers can design an observation record sheet. May use the sample given below:

Site	Observation made	Type of transformation applied
X (sample site)	Two sides of roof look similar	Reflection

Students can also compare the use of transformation related to cultural aspects like in Chorten, Bhutanese traditional houses with modern structures and equipment.

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  i. Will it be possible to construct a house without using the concept of transformations? Explain your thinking.

E. Resources

- Understanding Mathematics, Textbook for class VII
- Teacher's Guide to Understanding Mathematics, class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - Introducing transformation- https://www.britannica.com/biography/Felix-Klein
  - Translating 2-D shape- https://www.geogebra.org/m/GPyAmqWt#material/x3hFZQZb
  - Reflecting 2-D shape https://www.geogebra.org/m/Q558rky7
  - Rotating a shape using compass and protector- https://www.youtube.com/watch?v=5va-z_ViXLU
  - Rotating image on a grid- https://www.geogebra.org/m/GPyAmqWt#material/qfXNnhs6
  - Rotation of the 2D shapes https://www.geogebra.org/m/GPyAmqWt#material/C7GxC4uG
  - Three basic transformations- https://www.youtube.com/watch?v=VJTxv-tRKj0&t=1s
  - Investigating transformation https://www.geogebra.org/m/hMNc4eES
  - Tessellations https://www.youtube.com/watch?v=0xvP_QDFr-s

Topic: VII-D4 Isometric Drawings [200 minutes]

Introduction

Originally Buddhist monks created isometric exercises which were introduced to monks in China in the twelfth century. Isometric drawing is also called isometric projection and in Mathematical sense, it was formally invented by Professor William Farish (1759–1837) in 19^{th} century.

The most recent and noteworthy adoption of isometric drawing can be traced to the Bauhaus, as with much of modern design and architecture. For further reading browse site https://www.bricsys.com/en-eu/blog/what-is-isometric-drawing

Utility and Scope

Isometric drawing has been an invaluable tool for architects and engineers. Isometric drawing is a method of graphic representation of three-dimensional objects, used by engineers, technical illustrators, and, occasionally, architects. Isometric drawings are commonly used in technical drawing to show an item in 3D on a 2D page. Isometric drawings are a good way of showing measurements and how components fit together.

A. Competency

- Draw regular 3D shapes in 2D design to represent concrete structures made from cubes.

B. Objectives

- Draw 3-D shapes to interpret its representation in 2-D design.
- Justify the efficiency of 2-D design by observing isometric drawing that larger 3-D shapes can be accommodated in smaller space.

C. Learning Experiences

- A 3-D object fixed on the floor or kept on a tabletop can be represented in 2D design as isometric views.
- Following video links may be used to explain isometric drawings:
  - o https://bit.ly/3f3ahIK contains an introduction on isometric drawings.
  - o https://bit.ly/3F70pbp shows how to create isometric drawings.

- Demonstrate or use interactive tools https://bit.ly/3zw6VHG to make isometric drawings of the 3-D structures.
- Draw 3-D structure made from cubes on isometric dot paper.

D. Assessment

Performance Task 1

Create isometric drawings of the 3-D structures on the isometric dot paper or use cubes to build structures from isometric drawings concretely.

Performance Task 2

Provide some number of cubes. Let them design a structure and represent the same on isometric dot paper. Pass the isometric design to another group and let them construct the shape using the cubes. Finally, compare the shape to the original one.

- Sample Questions
  i. Using linking cubes to make a structure. Make isometric drawings of the structure in isometric dot paper.

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  i. How would have been the development in the world if isometric drawing were not invented.

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - History on isometric drawing- https://www.bricsys.com/en-eu/blog/what-is-isometric-drawing
  - Using Isometric dot paper-https://www.youtube.com/watch?v=obAQJZUr--w
  - Isometric drawings- https://www.geogebra.org/geometry?lang=en)
  - Introduction on isometric drawings-https://bit.ly/3f3ahIK
  - Creating isometric drawings- https://bit.ly/3F70pbp
  - Isometric drawings tool- https://bit.ly/3zw6VHG

Topic: VII-E1 Data Handling [600 minutes]

Introduction

The history of data handling is as old as civilization where information was always stored. It is therefore confusing to point out who exactly started data handling. Internet search however points out that modern systematics methods of data handling were begun by Herman Hollerith, a German born American Statistician, inventor, and businessman to summarize information.

Utility and Scope

Data handling offers immense scope in modern mathematics, science and in multidisciplinary areas. Research and statistical analysis implement data handling to derive conclusions and findings of the studies. Data science offers millions of jobs across the globe and has immense potential for job creation when combined with technology. This topic must be taught in a project-based model such that it proliferates for smart decisions to identify probable solutions for scientific and wide range of social issues.

A. Competency

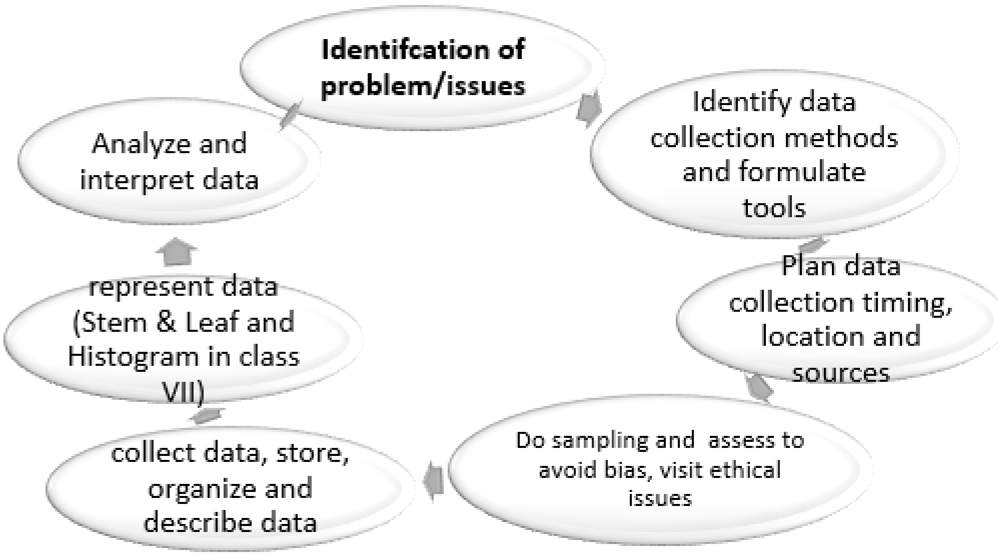
- Collect and organize data to describe and interpret through central tendencies and different graphs.

B. Objectives

- Collect data through appropriate data collection tools as an approach to solve real life problems. (Questionnaire/ interview/observation).
- Discuss sampling and bias while collecting data.
- Regroup the data in ascending order and construct stem and leaf plot
- Organize the collected data in a grouped frequency distribution table.
- Identify appropriate situations to construct histogram and to interpret it.
- Calculate central tendencies (mean, median and mode) and data range

C. Learning Experiences

- Data handling applies across all disciplines for research and analysis. The cycle of the data handling process could be presented prior to the lesson for getting the overall picture and it can be then explained with examples in each step. Processes should be revisited after completion of all tasks to rule out missing steps. The cycle is as:



- Prior to embarking into data collection, sampling methods, target population and tools for data collection must be clearly discussed. Explanation to collect data through questionnaire, observation and interview considering advantages and disadvantages of different data collection methods and sensitivities such as privacy, ethics, cost, and political agenda should be discussed.
- Explanation to identify a problem statement are suggested:
  - o https://www.youtube.com/watch?app=desktop&v=uO7c2tvrPj0 the link contains a video that explains the importance of the problem statement, list of relevant questions and to formulate a problem statement.
  - o Identify a relevant problem statement (For example: Number of hours spent doing homework in a week).
- Suggestion:
  - o https://m.youtube.com/watch?v=7onVHIkS1YY contains a video that explains how to frame a good question for doing research with examples.

- o https://m.youtube.com/watch?v=OGXdorYr4Bg contains a video that explains how to do research through observation (definition, types and its usefulness).
- o https://m.youtube.com/watch?v=7P4nURgH43A contains a video that explains how to do research through an interview (definition, types, and its usefulness).

- Discussion on Sampling and Bias
  - o Provide examples of biassed and unbiased sampling. For example: In terms of a number of hours spent doing homework.
    - ▪ asking only girls in a class is a biassed sample and
    - ▪ asking both boys and girls is an unbiased sample
  - o This video link https://www.youtube.com/watch?v=PK8aeeVx3io contains examples of biassed and unbiased sampling.
- Deliver the concept of stem and leaf plot and construct it (Can also use MS Excel for data arrangement and construction).
  May also use this video link to help students construct stem and leaf plot: https://www.youtube.com/watch?v=OaJXJduRiIE
- The weblink https://www.youtube.com/watch?v=2YtYsaKelmY contains video on how to create intervals in a grouped frequency distribution table. Find the range of the data as difference of maximum and minimum value and use it in creating bin-width and scale while representing in histogram.
- Make a frequency distribution table and use it to create Histograms. May use the link below to learn more: https://bit.ly/3q8520U
- Use data to explain the measures of central tendency. The video link: https://www.youtube.com/watch?v=B1HEzNTGeZ4 provides a brief explanation on mean, median and mode with examples (mean to find real life examples of average, median as the middle value when the values are arranged in order from smallest to largest and mode as an item that occurs most often).
- Use an interactive tool (GeoGebra) to determine measures of central tendency and range (sample link: https://www.geogebra.org/m/ZJztkKaz)
- Explore:
  What is the limitation of sample size?
  How would data handling help you make decisions?

D. Assessment

Performance Task 1

Let students conduct a study comprising the following attributes.

- o Collect the data (may use Google form/other tools)
- o Follow the cyclic process shared above
- o Describe using the measures of central tendency
- o Represent on stem and leaf plot

- Sample Questions

  Use a data set (minimum of 25 items; may use height/weight/mathematics marks from classmates) to find its mean, median, mode and range.

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  - i. Discuss the importance of data in research and development.

E. Resources

- Understanding Mathematics, Textbook for class VII
- Teacher's Guide to Understanding Mathematics, class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - o https://www.youtube.com/watch?v=PdXDLNNXPik for data handling
  - o problem statement- https://youtu.be/uO7c2tvrPj0.
  - o Framing question- https://www.youtube.com/watch?app=desktop&v=7onVHIkS1YY
  - o Research through observation https://m.youtube.com/watch?v=OGXdorYr4Bg
  - o Research through an interview- https://m.youtube.com/watch?v=7P4nURgH43A
  - o Sampling https://www.youtube.com/watch?v=PK8aeeVx3io
- Stem and leaf plotting-https://www.youtube.com/watch?v=OaJXJduRiIE)
- Measures of central tendency- https://www.youtube.com/watch?v=B1HEzNTGeZ4
- Measures of central tendency and range- https://www.geogebra.org/m/ZJztkKaz
- Frequency distribution and creating histogram- https://bit.ly/3q8520U

Topic: VII-E2 Probability [300 minutes]

Introduction

The modern mathematical theory of probability has its roots in attempts to analyse games of chance by Gerolamo Cardano in the 16th century, and by Pierre de Fermat and Blaise Pascal in the seventeenth century.
These details can be found at https://bit.ly/3q3oZpr.

Utility and Scope

Probability is widely used in all sectors in daily life like sports, weather reports, blood samples, predicting the sex of the baby in the womb, congenital disabilities, and statics. Probability is also used as the basis of Quantum Mechanics and some other physics. Probability also helps to make decisions while conducting events where a large population could gather. For more reading visit; https://www.britannica.com/science/probability.

A. Competency

- Calculate theoretical and experimental probability of events to predict and match with real & natural events.

B. Objectives

- Match events that might be associated with a particular theoretical probability.
- Deduce that theoretical probability of equally likely events as
  $P(E) = \frac{Number\ of\ favourable\ outcome}{Total\ number\ of\ possible\ outcomes}$
- Calculate theoretical probability for both single and double experiments (only for equally likely independent events).
- Conduct experiments to determine experimental probability and compare with their theoretical probabilities.

C. Learning Experiences

- Pre assessment on probability as likelihood, chances, possibility, etc.
- Prepare video lesson or use relevant video links to address the aspects of Theoretical Probability (sample: https://youtu.be/KzfWUEJjG18 contain video that explains the concept of basic probability)
- Demonstrate how to conduct an experimental probability using varieties of probability devices.
- Conclude through experimentation/investigation that:
  - more trials usually represent greater accuracy
  - impossible events have a probability of 0
  - events that are certain to occur have a probability of 1
  - all uncertain events have a probability between 0 and 1
- Identify all possible outcomes of independent events by
  - Selecting models and finding outcomes of single experiments. (Provide questions like find the probability of getting Khorlo if you toss a coin, etc)
  - Constructing tree diagrams for double experiment (e.g., rolling two dice to get the sum/product).
  - Using the area/rectangle model (where one event is represented by one dimension, the other event by the other dimension of a rectangle)
- Let students explore the difference between theoretical and experimental probability (may watch a video on comparing theoretical and experimental probability- expected probabilities to what really happens when we run experiments: https://www.youtube.com/watch?v=tXlcE_K_C-Y).
- Use available probability devices/models to compare outcomes with theoretical probability (conduct more trials to represent greater accuracy).

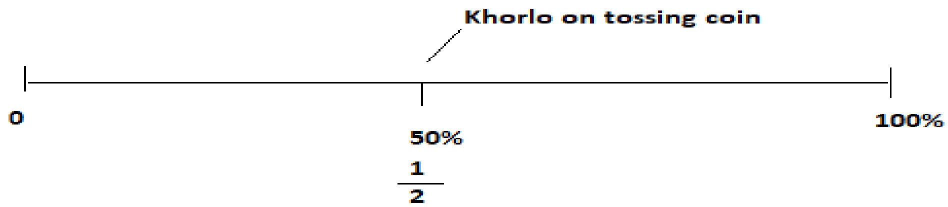
D. Assessment

Performance Task 1

Solve theoretical probability problems using tree diagram and area/rectangle model, assess, provide feedback, and record achievement level based on the template provided.

Performance Task 2

Create events and represent them on a probability line using fractions and percentages Example: The probability of getting khorlo on tossing a coin is $\frac{1}{2}$ or 50%).



- Sample Questions

  A deck of cards consists of 4 suits. In each suit there is an Ace, 3 face cards and 9 numbered cards. Calculate the probability for drawing each from the deck.

  a) face card

  b) numbered card

(Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in Annexure VII-A1)

- Reflective Questions
  i. Explore: Why can't the probability be more than 1?
  ii. How is probability both evil and good for gamblers?

E. Resources

- Understanding Mathematics Textbook for class VII
- Teacher's Guide to Understanding Mathematics Textbook for Class VII
- National School Curriculum Mathematics Framework for PP-XII
- Online resources
  - o Probability explanation -https://www.youtube.com/watch?v=uzkc-qNVoOk-
  - o Probability explanation https://www.youtube.com/watch?v=9M4VvNlinok-
  - o Probability introduction https://timesofmalta.com/articles/view/the-origins-of-probability.684474
  - o Utility & Scope- https://www.britannica.com/science/probability.
  - o Basic concept of probability-https://youtu.be/KzfWUEJjG18

Assessment Structures for KS- 3 (Class VII)

Assessment Structure

Key Stage 3	Assessment					
	Term I			Term II		
	CA	Mid Term Examination	Total	CA	Annual Examination	Total
	20	30	50	20	30	50

For both Term I and Term II, assess each competency through appropriate performance tasks and assessment tools.
Performance Tasks: quiz, question and answer, presentation, making models, etc.
Project Work: One mandatory project must be completed annually. Refer rubrics and planning sample at the end of IG).
Assessment Tools: checklist, rating scale or rubrics.
Assessment Areas: Formulating situations mathematically, applying concepts, facts, and procedures, and interpreting mathematical results.
Project Work: One mandatory progressive project in a year.

Weighting for Key stage 3 (Class VII)

Strand	**Time Allocation (Mins.)**	**Weighting(%)**
Strand A: Numbers and Operations	3000	42
Strand B: Patterns and Algebra	950	13
Strand C: Measurement	1100	15
Strand D: Geometry	1200	17

Strand E: Data and Probability	950	13

Instructional Guide Class VIII Mathematics

Topic: VIII-A1 Negative Exponents [200 minutes]

Introduction

The word exponent comes from Latin, expo, meaning out of, and ponere, meaning place. A negative exponent is defined as the multiplicative inverse of the base, raised to the power which is opposite to the given power. It is written as a reciprocal of the number and then solved like positive exponents.

For example, $(\frac{2}{3})^{-3} = (\frac{3}{2})^{3} = \frac{27}{8} = 3.375$

For more explore https://sciencing.com/history-exponents-5134780.html

Utility and Scope

Exponents are essential in Mathematics because they enable us to abbreviate something that would otherwise be really tedious to write. Exponents can be used in a variety of ways to represent length. Negative exponents are specifically used to represent small quantities. Scientists use negative exponents to measure small organisms, for example, the world's smallest bat, the bumblebee bat, weighs about 7×10^{-2} ounces.

Negative exponents will enable learners to work more effectively with decimals when they understand the connections between negative exponents and place value less than 1 (Tenth, hundredth and so on). This prepares them for further work with exponents including scientific notations.

Exponents are used in Computer games, pH and Richter Scales, in Science, Engineering, Economics, Accounting, Finance, and many other disciplines.

A. Competency

- Develop the meaning of negative exponents to compare and convert the exponential form to standard or expanded form and vice versa (base 10).

B. Objectives

- Develop concept of negative exponents through patterns using place value charts (tenths, hundredths, thousandths... as 10^{-1}, 10^{-2}, 10^{-3})
- Investigate and relate negative exponents 10^{-1}, 10^{-2}, 10^{-3} to multiplying by 0.1, 0.01, and 0.001.

- Use base as 2 OR 3 with different negative exponents and write them as their equivalent fraction;($2^{-1} = \frac{1}{2}$, $3^{-2} = \frac{1}{9}$,etc) to show the concept of exponential growth.
- Convert exponential form (including negative exponent) to standard and expanded form and vice versa.

C. Learning Experiences

- Conduct pre-assessment on exponents (let students use https://www.liveworksheets.com/worksheets/en/Math/Exponents to assess themselves; (*From the Left hand side of worksheet, select language as English, Subject at Maths and then select Exponent*)
- Develop the concept of negative exponents
- *The weblink contains a video that introduces the negative exponents* (https://m.youtube.com/watch?v=PYd99ItlLEc)

  Suggestions:

  - *Use a place value chart that includes the power of 10 to introduce the concept of negative exponent. As power of 10 decreases to negative exponents , fraction or decimal equivalents appear*

Place	*Hundreds*	*Tens*	*Ones*	*Tenth*	*Hundredth*
Power of 10	10^2	10^1	10^0	10^{-1}	10^{-2}
value	*100*	*10*	*1*	$\frac{1}{10}$ *or 0.1*	$\frac{1}{100}$ *or 0.01*

  - *relate negative exponents 10^{-1}, 10^{-2}, 10^{-3} to multiplying by 0.1, 0.01, and 0.001. May use example like 2×10^{-2} can be written as $2 \times 0.01 = 0.02$*
  - *Demonstrate using a pattern table that includes base 2 or base 3 and let students make similar pattern tables. For example*

Power of 2	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}
Value	8	4	2	?	?	?

  - Demonstrate and explain how to convert exponential form to expanded and standard form and vice versa. (May refer to Understanding Mathematics for class VIII).

- The web link www.johncmccloskey.com/math-topics/scientific-notation-and-powers-of-ten / gives information where exponents are applied. This link can also be used while teaching the next lesson (Scientific Notation).

D. Assessment

Performance Task 1

Prepare a set of questions to check the understanding of the concept based on the objectives. (May refer to Understanding Mathematics Textbooks for Class VIII or online resources).

- Sample Questions
  1000 millimetres = 1 metre. Write 1 millimetre in terms of metre in exponential form. (Answer 1 mm= 10^{-3}m)

Performance Task 2

Let students identify and list applications of exponents in our day-to-day life. (Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1).

- Reflective Questions
  i. If $2 \times 2 \times 2 = 2^3$ What will be the product of $2^2 \times 2^2 \times 2^2$?
  ii. How does negative exponent help in writing fractions? Discuss with examples.
- Template to record assessment

<table>
<tr><td colspan="3">Strand(s): Numbers and Operations</td><td colspan="3">Topic(s): Negative Exponents</td></tr>
<tr><td colspan="6">Competency:
Develop meaning of negative exponents through patterns in place value charts</td></tr>
<tr><td rowspan="2">Name of the student</td><td colspan="5">Level of achievement</td></tr>
<tr><td>Beginning</td><td>Approaching</td><td>Meeting</td><td>Advancing</td><td>Excelling</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

- Suggested Assessment tools

<table>
<tr><td colspan="2">Strand: Numbers and Operation</td><td colspan="3">Topic: Negative Exponent</td></tr>
<tr><td colspan="5">Competency:
Develop the meaning of negative exponents to compare and convert the exponential form to standard or expanded form and vice versa (base 10)</td></tr>
<tr><td>Beginning</td><td>Approaching</td><td>Meeting</td><td>Advancing</td><td>Exceeding</td></tr>
<tr><td>E (1)</td><td>D (2)</td><td>C (3)</td><td>B (4)</td><td>A (5)</td></tr>
<tr><td>Struggles to develop meaning of negative exponent with place value chart. Writes simple numbers in powers ($10 \times 10 \times 10 = 10^3$).</td><td>Develops meaning through patterns in place value charts, Need support to convert numbers from one form to another.</td><td>Relates base 10 exponents to decimals (eg. $10^{-1} = 0.1$) inter converts exponential to standard and expanded form without any error. progresses to</td><td>Completes all requirements of performance level C 'Meeting' and moves to higher exponential numbers with base 4 and 5. ($4^{-2} = \frac{1}{4^2} = \frac{1}{16}$, etc)</td><td>Completes all requirements of performance level B 'Advancing' Coaches peer and confidently shares in-depth meaning of negative exponents.</td></tr>
</table>

		exponential growth with base 2 and 3		

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Pattern Table
- Online
  - History of exponent - ps://sciencing.com/history-exponents-5134780.html
  - Worksheet to assess previous knowledge on exponent - https://www.liveworksheets.com/worksheets/en/Math/Exponents
  - *A video that introduces the negative exponents -* https://m.youtube.com/watch?v=PYd99ItlLEc
  - Application of exponents - www.johncmccloskey.com/math-topics/scientific-notation-and-powers-of-ten/...

Topic: VIII-A2 Scientific Notations [200 minutes]

Introduction

The idea of scientific notation was developed by Archimedes in the 3rd century BC, where he outlined a system for calculating the number of grains of sand in the universe, which he found to be 1 followed by 63 zeros. Scientific notation is a way of expressing numbers that are too large or too small (usually would result in a long string of digits) to be conveniently written in decimal form.

Source: https://www.mometrix.com/academy/scientific-notation/

Utility and Scope

Scientific notation allows us to write extremely large numbers and extremely small numbers into manageable sizes. It ensures accuracy and reduces the possibility of error when using very small or very large numbers. Scientists use this notation to describe astronomical distances, such as the distance between planets, or microscopic distances, such as the length of a blood cell.

Example:

- ✔ Value of planck constants is 0.000000000000000000000000000000000016 = 1.6 X 10^{-35}
- ✔ Estimated number of cells in human body 37,200,000,000,000 = 3.72 X 10^{13}.

Link http://passyworldofmathematics.com/exponents-in-the-real-world/ shows exponent in real world.

A. Competency

- Flexibly move between numbers in scientific notations and standard form to compare very large numbers/ very small numbers and understand the role of scientific notation.

B. Objectives

- Convert numbers from standard form to scientific notation and vice versa.
- Compare numbers written in scientific notation as applied in real life situations.

C. Learning Experiences

- Conduct pre-assessment on power of 10.

  Example:

  (1) $5 \times 10^{2} = 5\times100 = 500$

  (ii) $3 \times 10^{-2} = 3 \times 0.01 = 0.03$
- Express numbers in a scientific notation. The weblink (http://m.youtube.com/watch?v=bXkewQ7WEdI&app=m&persist_app=1) shows the steps to write numbers in scientific notation and recall negative exponents.
- Clarify the terms multiplier and power of 10 and their significance in scientific notation.
- Let students explore to convert standard numbers into scientific notation and vice versa. (May refer to Understanding Mathematics Textbooks for Class VIII.
- Use a worksheet from https://www.liveworksheets.com/w/en/math/2196894 which connects exponents and scientific notation.
- Compare numbers written in scientific notation. Emphasize on the process of comparison, that when exponents are same, multipliers are compared and conversely when exponents are different, only the exponents are compared. The link https://youtu.be/xYTYQRXofQU?si=B7Ej5ZIBgUorKjCn explains the process of comparing numbers written in scientific notation.

D. Assessment

Performance Task 1

Prepare a set of questions to check the understanding of the concept based on the objectives (express numbers in scientific notation and compare numbers). May use apps such as PowerPoint, Kahoot, Plickers, Nearpod or other relevant apps to develop quizzes.

Performance Task 2

Explore very large numbers or very small numbers and write those numbers in the form of scientific notation (eg. Distance from earth to the sun, the mass of Jupiter, the diameter of red blood cells, etc.).

Sample question:

i. The sun has a mass of about 1,988,000,000,000,000,000,000,000,000,000 kg. Express the number in scientific notation.

ii. A tiny space inside a computer chip has been measured to be 0.00000256 m wide and 0.00000014 m long. Express the area of the chip in scientific notation.

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*)

- Reflective Questions
  i. The speed of light is $3 \times 10^{8} metre/second$. If the sun is 1.5×10^{11} metres away from earth, how many seconds does it take for sunlight to reach the earth? Write your answer in scientific notation.
  ii. What would have been the problem if Scientific notations were not invented?

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Online
  - Introduction on Scientific notation – https://www.mometrix.com/academy/scientific-notation/ or https://www.mathsisfun.com/numbers/scientific-notation.html
  - Steps to write numbers in scientific notation - http://m.youtube.com/watch?v=bXkewQ7WEdI&app=m&persist_app=1
  - Comparing numbers written in scientific notation https://youtu.be/xYTYQRXofQU?si=B7Ej5ZIBgUorKjCn

Topic: VIII-A3 Perfect Squares [300 minutes]

Introduction

Regiomontanus is considered the inventor of the square root symbol. When a square is made with the same number of rows and columns it is called a perfect square. Any number multiplied by itself creates a perfect square

Explore and share related history. May use the link provided below:

https://programminginsider.com/3-applications-of-square-and-square-roots-in-daily-life/

This($\sqrt{\ }$) is the special symbol that means "square root", it is like a tick, and actually started hundreds of years ago as a dot with a flick upwards. It is called the *radical*.

Utility and Scope

Square/ square roots are used in our daily life. Example: Square/square root can be used when you want to know how many tiles you would need to cover a floor that has an area of $25m^2$.

Square root helps us apply Pythagorean Theorem to calculate the side length of a triangle. This is used by Architecture and engineers (Finding length of trusses to hold up bridges and buildings).

Having a list of perfect squares will be of greater importance through all levels of mathematics. When you don't have a calculator at hand, you must rely on your knowledge of the most common square roots to get you through challenging equations and word problems through estimation.

A. Competency

- Recognize square numbers and calculate their square roots to apply concepts in solving real life problems.

B. Objectives

- Identify each of the perfect squares from 1 through 400 using factors, or prime factorization or observing digits in one's place.
- Demonstrate that the differences in perfect squares follow a pattern.

- Show that the sum of the square roots of two consecutive perfect squares is equal to the difference between those two perfect squares
- Approximate which whole number is closer to the square root of non-perfect squares.
- Estimate and calculate the square root of the larger number.
- Apply prime factorization to calculate square roots.
- Calculate square root using square root algorithm or Long division method.

C. Learning Experiences

- Write square numbers from 1 to 400. (i.e 1,4,9,16.....400).
- Define a perfect square and find the difference of square root from 1 to 400 (to see the pattern).
- The web link (https://m.youtube.com/watch?v=k6KhHKYu4UY) contains a video that explains the concept of square root, shows perfect square till 900 (30 x 30), how to identify perfect square and how to estimate and calculate perfect square.

  Suggestion:
    - Highlight the unit digits of perfect squares from 1 to100.
    - Write perfect squares from 11-to-20 and ask what they notice in the unit digit.
    - Conclude that the number with 2, 3, 7 and 8 in unit digit is not a perfect square.
    - *Prepare a worksheet for students to practise on square root in their group.*
- Show $\sqrt{25}+\sqrt{36}=5+6=36-25=11$. That is, the sum of square roots of two consecutive perfect squares is equal to the difference between those two perfect squares.

  Suggestion: Give one example and let students try for others.
- Explore and show how to calculate/estimate the square root of a given number.

  Suggestion:
    - *Using patterns.*

$$\Rightarrow \sqrt{1600} = \sqrt{16\times 100} = \sqrt{16}\times\sqrt{100} = 4\times 10 = 40$$

$$\Rightarrow \sqrt{1645} = \sqrt{16.45}\times\sqrt{100} \approx 4.1\times 10 = 41).$$

- *Prime factorization method.*

  $\sqrt{576} = \sqrt{2\times2\times2\times2\times2\times2\times3\times3} = 2\times2\times2\times3 = 24$
- *Estimating which whole number is closer to the square root.*

  $\sqrt{22}$ *is in between 4 and 5, but closer to 5.*

- Explore finding square roots using the algorithm method and calculate the square root of some numbers (may go up to 5-digit numbers). The link https://www.cuemath.com/algebra/square-root-by-long-division-method/ explains steps to calculate square root by long division method. The video from the link https://www.youtube.com/watch?v=x4D5bPqONAE explains how to calculate square root using the long division method.

D. Assessment

Performance Task 1

Prepare a set of competency-based questions to use the concept of square and estimate the square root of different numbers. (May refer to Understanding Mathematics for class VIII).

Performance Task 2

Solve questions related to calculating the square root of a given number using different strategies.

- Sample Questions
  i. Karma wants to plant 1225 potatoes in the garden in such a way that each row contains as many potatoes as the number of rows. Find the number of rows and the number of potatoes in each row.
  ii. Is 5400 a perfect square? If not, find the smallest number by which 5400 must be multiplied so that the product is a perfect square. Find the square root of the new number.
  iii. Find the smallest number by which 2700 could be divided so as to get a perfect square. Also, find the square root of the new number.

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Reflective Questions
  i. If 25 × 25 = 625, 35 × 35 = 1225 and 45 × 45 = 2025, what will be the square of 55, 65, 75, 85, and 95? Explore the tricks to find the square of numbers ending in 5.
  ii. Perfect squares form patterns. Elaborate it through a short panel discussion.

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Online
  - Application of square and square root - https://programminginsider.com/3-applications-of-square-and-square-roots-in-daily-life/
  - Concept of square root https://m.youtube.com/watch?v=k6KhHKYu4UY
  - Calculate square root using long division method- https://www.youtube.com/watch?v=x4D5bPqONAE

Topic: VIII-A4 Proportion and Percent [350 minutes]

Introduction

Mediaeval writers used the word *proportio* ("proportion") to indicate ratio and *proportionalitas* ("proportionality") for the equality of ratios. Euclid collected the results appearing in the Elements from earlier sources. The Pythagoreans developed a theory of ratio and proportion as applied to numbers. A percent proportion is an equation where a percent is equal to an equivalent ratio. For more information explore and share relevant history. May use the link https://bit.ly/3qYoE6F.

Utility and Scope

Proportions are basics across numerous topics in mathematics and science. In mathematics, they are essential for developing concepts and skills related to fractions, constant rate of change, algebraic concepts and skills and in many real-life situations such as density, speed and slope.
Similarly, the percent is applied in most real-life situations such as consumer problems, and in interpreting data.

A. Competency

- Understand the concept of proportion and percent to apply and solve real life problems.

B. Objectives

- Apply the concept of terms *means and extremes* and calculate missing values in proportions.
- Explore and solve problems related to proportion using a variety of strategies.
- Use different strategies to solve percent greater than 100.
- Solve problems involving fractional percents

C. Learning Experiences

- Explore the concept of proportion and methods of solving proportions.
- Introduce the terms means and extremes. (when 2:3= 8:12, 3 & 8 are means and 2 &12 are extremes). From this show that the product of means is equal to the product of extremes. 3×8=2×12= 24. Use this concept to find missing terms in

proportion. Example 3:x= 9: 15. 9x=45⇒ x= 5 May use the link below to explore further on means and extremes and their application: https://m.youtube.com/watch?v=AsNYLh9CF_0&pp=ygUgbWVhbnMgYW5kIGV4dHJlbWVzIGluIHBvcnBvcnRpb24%3D

- May use the web link below that contains a video that defines proportion with examples by relating fractions and ratios.
  - https://m.youtube.com/watch?v=qYjiVWwefto
  - https://m.youtube.com/watch?v=USmit5zUGas
- Explore ways to solve percent problems
- Suggestion:
  - The weblink (https://m.youtube.com/watch?v=Uo8HgcyfRFI) contains a video on percent proportion (relating percent and proportion) where students can apply cross multiplication to find the unknown values.
  - Percent greater than 100 (use 100% as baseline)
  - Using familiar percent
  - The weblink (https://m.youtube.com/watch?v=vh8HJITCCBE) contains a video that introduces the concept of percent (%) greater than 100 and less than 1 along with examples.
- Recall percent to fraction and fraction to percent and solve related problems.

  Example

  $$8\%=\frac{8}{100} = \frac{2}{25}\text{ ; conversely } \frac{2}{8} = \frac{2\times12.5}{8\times12.5} = \frac{25}{100} = 25\%$$

- Explore fractional percent (May use Understanding Mathematics Textbook for class VIII)

  Example

  $$\frac{1}{2}\% = \frac{\frac{1}{2}}{100} = \frac{1}{2}\times\frac{1}{100} = \frac{1}{200} = 0.005$$

D. Assessment

Performance Task 1

Ask students to solve a set of questions related to the objectives. (May refer to Understanding Mathematics Textbook for class VIII)

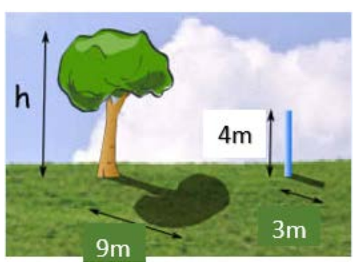
Performance Task 2

Explore real life problems where proportion and percent are used for calculation and solve with examples (Group Work). Present to the whole class.

- Sample Questions
    i. Bhagi and Dorji shared a packet of sweets in the ratio 3:2. If Bhagi got 18 sweets, how many sweets will Dorji get?
    ii. Wangmo has some oranges and some apples in her bag. The ratio of oranges to apples is 2:3. If there are 20 fruits in total, How many apples will be in her bag?
    iii. An alloy contains 26 % of copper. What quantity of alloy is required to get 260 g of copper? (hint: 26% of copper = 260g)
    iv. In a basket of apples, 12% of them are rotten and 66 are in good condition. Find the total number of apples in the basket. (hint: 88% of total =66)

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*)

- Reflective Questions
    i. Karma tried using a ladder, tape measure, ropes and various other things, but still couldn't work out how tall the tree was. But then Karma has a clever idea ... similar triangles! Karma measures a stick and its shadow (in metres), and also the shadow of the tree, and this is what he gets. What will be the height of a tree?



    ii. How proportion is important. Connect to real life applications and discuss.

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Online

- Introduction to proportion and percent - https://math.libretexts.org/Courses/Barton_Community_College/Book:_Technical_Mathematics_(Turner)/04:_Business_Math/4.01:_Ratio_and_Proportion
- Video that defines proportion with examples by relating fractions and ratios. https://m.youtube.com/watch?v=qYjiVWwefto OR https://m.youtube.com/watch?v=USmit5zUGas
- Video on percent and proportion https://m.youtube.com/watch?v=Uo8HgcyfRFI
- Video on percent greater than 100 https://m.youtube.com/watch?v=vh8HJITCCBE
- Video on proportion: https://www.mathsisfun.com/algebra/proportions.html

Topic: VIII-A5 Consumer Math [300 minutes]

Introduction

Commerce in Bhutan was traditionally carried out through a system of bartering with rice, butter, cheese, meat, wool, hand-woven cloth and other local produce. Bhutan first began to produce coins in silver towards the end of the 18th century, mainly for use in trade with the plains.

These were followed by coins struck in alloyed silver, copper or brass, which were used for minor local purchases. Today, with booming trade industries, commercial mathematics and consumer maths find a stronghold in the life of every adult.

Consumer maths deal with the concepts used in businesses like simple and compound interest, profit and loss, taxes, loans, percentages, etc.

Utility and Scope

Consumer maths are basic maths skills in real life situations like shopping, calculating taxes, estimating monthly budget, calculating interest rate for a loan, income related calculation, etc. Teaching kids about spending, saving and other aspects of "money maths" will prepare them to make better financial decisions.

A. Competency

- Explore and understand the basics of consumer maths and effectively apply them in real life situations

B. Objectives

- Investigate and identify cost price, selling price, mark-up and discount/ Markdown through the problem situation.
- Calculate selling price, Mark-up/Markdown amount.
- Apply formula to calculate percent increase and percent decrease (Mark-up % or Markdown %).
- Explore and solve simple problems related to simple interest and commission.

C. Learning Experiences

- Explore and identify the terms such as Cost price (CP), Regular Selling Price (RSP), Sale Price (SP), Mark-up amount and Markdown amount.

- May use the following
  - o Exposition from Understanding Mathematics Textbook for Class VIII.
  - o The weblink https://youtu.be/CddjCVHEIjQ?si=09_yjG-gNuUxMoPs contains a video that introduces markup and discount and shows how to calculate markup and discount with an example.
  - o https://youtu.be/8-UGRS8FBRk contains a video that shows the examples of calculating Cost price (CP), Regular Selling Price (RSP), Sale Price (SP), Mark-up /Markdown amount and Mark-up/Markdown percent.
- Markups and discounts are often expressed in percent. The general method of change by percent could be instructed. $change\,\% = \frac{actual\ change}{base\ of\ change} \times 100$. For example if original price is Nu 250 and raised by Nu 15, the markup percent is $= \frac{15}{250} \times 100 = 6\%$
- Explore the concept of Simple interest and solve related problems.
  Suggestions:
  - o The weblink https://www.youtube.com/watch?v=qezL48FNvcY introduces simple interest.
  - o The weblink https://www.youtube.com/watch?v=2hCXQQ5HeQc contains videos that can help to calculate simple interest.
  - o Demonstrate how to calculate simple interest charged for less than one year (May refer Understanding Mathematics Textbook for class).
  - o Let students explore in their groups to find the rate/time/principal when the amount of simple interest and any two variables are given. For example, $I = PRT \quad T = \frac{I}{PR}$
  - o Contextualize the problem to find the *simple interest* after introducing the formula. For example, it could be the loan borrowed by the teacher or the loan borrowed by the student's parent.
- Explore to calculate commission;
  - o The weblink (https://m.youtube.com/watch?v=W-K7q1dfzps) and https://youtu.be/Iq875LdH2BU contains a video that defines and explains about commission. May refer to the Understanding Mathematics Textbook for Class VIII.

D. Assessment

Performance Task 1

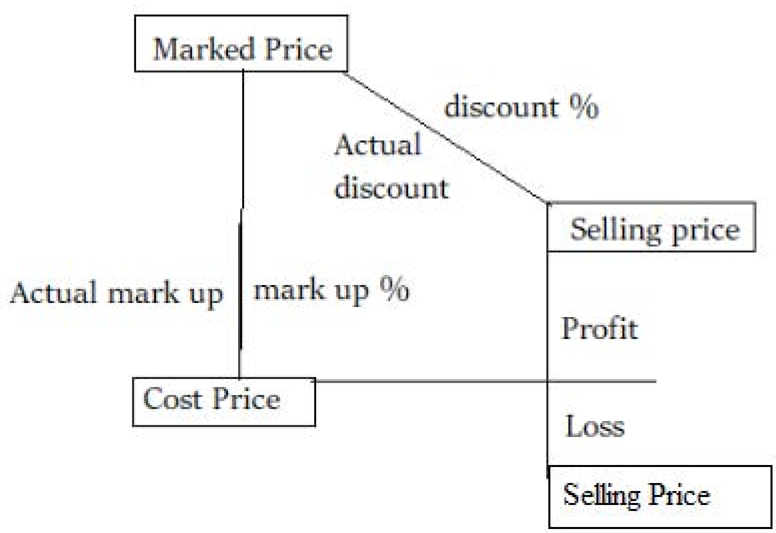
Ask students to solve a set of questions related to the objectives. (May refer to the Understanding Mathematics Textbook for class VIII).

Performance Task 2

Let students role play on consumer maths problems (Mark-up/Mark down, CP, RSP, SP, Mark-up/Markdown %, Simple Interest and Commission.

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*)

- Sample Questions
  i. A shopkeeper sells a rice cooker for Nu. 3500 with a loss of Nu. 1500. Find the price at which he had bought it from the dealer. Also, calculate the loss percent. (hint: CP = SP + Loss)
  ii. A shopkeeper buys t-shirts from a dealer at Nu. 700 per t-shirt. He sells them for Nu. 875 per t-shirt. Calculate the actual profit and profit percent.
- Reflective Questions
  i. The schema below shows how the retail market gets operated. Explain how it would help retailers manage their business.



Sample Answer: Profit margin depends on differences between marked price and discount

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Online
  - Introduces markup and discount - https://m.youtube.com/watch?v=h1YcEQKdOr4
  - Example of calculating CP, RSP, Markup, Discount - https://youtu.be/8-UGRS8FBRk
  - Introduces simple interest - https://www.youtube.com/watch?v=qezL48FNvcY
  - Video on calculating Simple Interest - https://www.youtube.com/watch?v=2hCXQQ5HeQc
  - Explaining commission - https://m.youtube.com/watch?v=W-K7q1dfzps) and https://youtu.be/Iq875LdH2BU

Topic: VIII-A6 Integers [300 minutes]

Introduction

The number system notation which we use today is said to be given by the two great mathematicians from ancient India, Aryabhatt (5th century BC) and Brahmagupta (6th century BC).The positive and negative numbers did not actually become part of a single "number line" (today's "set of integers") until the 1700's or 1800's though Indian mathematicians such as Brahmagupta were describing the use of negative numbers in the 7th century. The letter (Z) is the symbol used to represent integers. An integer can be 0, a positive number to infinity, or a negative number to negative infinity. Further read from https://bit.ly/3q4OI0C.

Utility and Scope

Integers help in computing the efficiency in positive or negative numbers in almost every field. Integers are used in our everyday life such as representing debit/credit of money, profit and loss, geographical level; above/below the sea level and describing temperature.

A. Competency

- Understand the fundamentals of operating integers and logically apply the process in multiplying and dividing Integers using various strategies.

B. Objectives

- Multiply and divide integers pictorially and symbolically.
- Estimate to check the reasonableness of results
- Apply properties for multiplying integers such as Commutative (order), Associative (grouping) and Distributive properties.

C. Learning Experiences

- The weblink (https://m.youtube.com/watch?v=K_tPbVPfHgk) contains a video that shows how to derive multiplication and division rules for integers.
- Demonstrate how to multiply pictorially and symbolically.

  Suggestions:

  - o Use counters/number lines to multiply integers with the same or different sign.

- o Use a pattern to show why negative multiplied by negative gives positive. (Known to unknown).

  Example: $(-3) \times (-2) = ?$

  $(-3) \times (2) = (-6)$

  $(-3) \times (1) = (-3)$

  $(-3) \times (0) = 0$

  $(-3) \times (-1) = 3$

  $(-3) \times (-2) = 6$

  (The difference in the product increases by 3)

- o Rename factors to multiply mentally such as compatible factors, doubling and halving and work by part. (May refer to the Understanding Mathematics Textbook for class VIII).

● Demonstrate how to divide integers pictorially and symbolically.

Suggestions;

- o Use counters/number line to divide different signs of integer
- o Use a pattern to show why positive divided by negative gives negative. (Known to unknown).

  Example $(9) \div (-3) = ?$

  $(-9) \div (-3) = (3)$

  $(-6) \div (-3) = (2)$

  $(-3) \div (-3) = (1)$

  $(0) \div (-3) = 0$

  $(3) \div (-3) = (-1)$

  $(6) \div (-3) = (-2)$

  $(9) \div (-3) = (-3)$

  (The difference in the quotient decreases by 1)

● Relate division of integers to multiplication

● Apply properties to calculate and estimate the product of integers;

- o The web link (https://m.youtube.com/watch?v=UI-ThJlkLVQ) contains a video that explains three properties of addition and multiplication (commutative, associative and distributive properties).
- o May refer to the Understanding Mathematics Textbook for Class VIII.

- The web links below contain video that shows how to *multiply and divide* integers concretely (using counters) and pictorially (using number line).
  - o https://m.youtube.com/watch?v=fW3FWuLfpFc
  - o https://m.youtube.com/watch?v=lDR-B_OhUQo

  (note that we use white counter for positive integers and black counter for negative integers for easy representation in the notebook).
- Solve questions related to order of operations in integers.
- Let students play the game.

  (May refer to Understanding Mathematics Textbook for class VIII for clearer instruction)

  Materials:

  In a group, prepare 42 integer cards from -10 to 10 (2each).

  Number of players: 2-8

  How to Play:

  Shuffle the cards and deal five cards to each player and face it up. You cannot rearrange the cards once it has been dealt.

  Each player multiply for three rounds with his/her own cards

  Round 1

  Select an integer that is not the first or last card. Multiply the integers by the integers to its right and to its left. Record the product as the score and remove the card selected for multiplication.

  Round 2

  Select an integer from the remaining cards that are not the first or last card. Multiply the integers by the integers to its right and to its left. Record the product as a score and remove the card selected for multiplication.

  Round 3

  Multiply the remaining three integers. Record the product as a score.

  Final score: Find the total of three scores. The player with the greatest score wins.

D. Assessment

Performance Task 1

Make a list of about 5 questions based on certain contexts to assess students' competency to multiply and divide integers using various strategies (pictorially and symbolically). May refer to the Understanding Mathematics textbook for class VIII sample questions.

- Sample Questions
    i. What will be the sign of the product if you multiply eight negative integers with one positive integer?
    ii. Karma has a debt of Nu 35. If Nu 5 is paid weekly, for how many weeks will he have to pay to clear the debt? Show your work using integers.

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Sample Questions
  Is (-2.5) an integer? Explore and discuss.
- Reflective Question
    i. Discuss how a negative integer plays its role in the real world. Elaborate some of their applications.

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Counters
- Technological gadgets for learning (smartphone, desktop, laptop…)
- Online
    - Introduction - https://bit.ly/3q4OI0C
    - Multiplying and dividing integers - https://m.youtube.com/watch?v=K_tPbVPfHgk
    - Properties of multiplication - https://m.youtube.com/watch?v=UI-ThJlkLVQ
    - Multiplying and dividing integers - https://m.youtube.com/watch?v=fW3FWuLfpFc and https://m.youtube.com/watch?v=lDR-B_OhUQo

Topic: VIII-A7 Fractions and Rational Numbers [400 minutes]

Introduction

The concept of fractional numbers dates to prehistoric times. Even the Ancient Egyptians wrote maths texts describing how to convert general fractions into their special notation. Pythagoras is an ancient Greek mathematician who mainly invented rational numbers. A rational number is a number that is especially expressed as a quotient or fraction $\frac{p}{q}$ of 2 integers.

Explore and share relevant history. May use the link below:
https://www.basic-mathematics.com/history-of-fractions.html

Utility and Scope

Fractional numbers are indispensable in any number operation. Without fractions, it will be inconvenient to represent numbers with parts. Similarly, rational numbers are very essential in our day-to-day life. It helps us in our daily calculations such as measuring length, mass, capacity, time, and distance. It is used in all activities of life such as shopping, banking, farming, engineering etc.

A. Competency

- Demonstrate the ability to identify rational numbers, multiply and divide fractions pictorially/symbolically and apply the essentials for order of operations to seamlessly apply in solving related real-world problems.

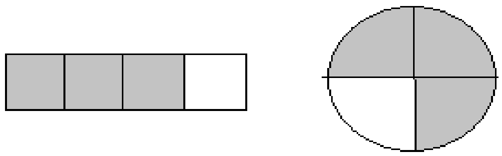
B. Objectives

- Construct concrete or pictorial models of fractions to develop meaning.
- Multiply and divide fractions pictorially and symbolically.
- Define and identify rational numbers.
- Apply order of operations to problems related to rational numbers.

C. Learning Experiences

- Construct concrete or pictorial models to develop the meaning of fraction.

  For example $\frac{3}{4}$ could be



- Explore how to multiply fractions using model and symbolically;
  Suggestion:
  - o Using models to multiply two fractions. May refer to the weblink (http://m.youtube.com/watch?v=qg2u0bvHBGU) that contains a video to multiply fractions pictorially using an area model.
  - o Multiply fractions symbolically.
- Relate mixed numbers to whole numbers concretely. For example, if a plate of momo has 6 pieces, then $2\frac{1}{2}$ plates of momo will have 15 momos.
- Explore different strategies to divide fractions:
  - o The weblink (http://m.youtube.com/watch?v=N-SXqAAF9F0 contains video to divide fractions pictorially using the number line.
  - o Show different methods to divide fractions (Rectangular model, number line and symbolically) May refer Understanding Mathematics Textbook for class VIII.
- The web link https://www.youtube.com/watch?v=Z44ID9KpEKY contains a video that shows the concept of rational numbers pictorially.
- Give expression (of mixed operations) to apply the order of operations. (May refer to the Understanding Mathematics Textbook for class VIII).

D. Assessment

Performance Task 1

Prepare about 5 to 6 sets of questions based on the objectives of the topic. Administer each set to the group of students (a group may consist of 5 to 6 students). May refer to Understanding Mathematics Textbook for class VIII.

Performance Task 2

Discuss and show pictorially why $\frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$.

(Hint: How many halves are there in one fourth?)

Sample Questions

i. Penjor eats $\frac{1}{4}$ cake every day. If he eats the same amount for 7 days, how much cake will he eat?

ii. Lemo has $3\frac{1}{3}$ pizza. She shares the pizza equally with her friends. If each friends get $\frac{2}{3}$ of the total Pizza, how many friends are there?

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Reflective Question
  - i. What would be the result of operations if BEDMAS is not implemented? Discuss with example(s).

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Fraction stripes, Number line
- Area model/grid mode
- Technological learning gadgets (smartphone, desktop, laptop…)
- Online
  - o History of the fraction - https://www.basic-mathematics.com/history-of-fractions.html
  - o Multiplying fraction - http://m.youtube.com/watch?v=qg2u0bvHBGU
  - o Dividing fraction - http://m.youtube.com/watch?v=N-SXqAAF9F0
  - o Concept of rational number - https://www.youtube.com/watch?v=Z44ID9KpEKY
  - o Decimal, fraction and percent - https://www.mathsisfun.com/decimal-fraction-percentage.html

Topic: VIII-B1 Describing Relationships and Patterns [300 minutes]

Introduction

The concept of patterns was first described by Christopher Alexander in A Pattern Language: Towns, Buildings, Construction. The book describes a "language" for designing the urban environment. The units of this language are patterns.

The pattern is a repeated arrangement of numbers, shapes, colours, etc. If the set of numbers are related to each other in a specific rule, then the rule or manner is called a pattern. Sometimes, patterns are also known as a sequence. Patterns are finite or infinite in numbers.

Utility and Scope

There are many daily-life activities in which children engage with mathematical patterns without formal instruction. Patterns help us organize thoughts and establish order in our lives. As we begin to connect patterns in nature and life, they bring a sense of harmony to our minds. Patterns lead to and build maths, vocabulary and cognitive concepts. Patterns are excellent in helping us establish priorities.

The pattern is fundamental to our understanding of the world; it is an important element in every field of study. The importance of patterns usually gets lost in a repeating pattern of two-dimensional shapes.

Some of the areas where patterns can be found:

- the brick pattern on a building or home.
- the pattern on the sidewalk or driveway.
- the tree rings and the leaf.
- the number of petals on flowers.
- colours, shape and size of structures.
- the shadows of people, trees, buildings.

A. Competency

- Describe relationships using words, expressions and equations to represent patterns given in tables, graphs or charts, pictures and problems in real life situations.

B. Objectives

- Describe in words and use expressions and equations to represent patterns given in tables, graphs or charts, pictures and problem situations.

- Use the information presented in a variety of formats to derive Linear mathematical expressions and predict unknown values .
- Identify linear relationships from derived expressions
- Sketch graphs for a variety of situations leading to linear graphs

C. Learning Experiences

- Draw some pictures showing linear relationships. Then observe and describe its patterns.
- Demonstrate how to find pattern rules or algebraic equations of linear relationships from pictures, table of values and graphs. The weblink https://m.youtube.com/watch?v=Ywomf4-5Puo contains a video that represents the linear pattern from the blocks and derives the pattern rule pictorially.
  Suggestions:
    - Use patterns to create a table of values.
    - Represent the pattern in a graph.
    - Formulate the pattern rule (*may use concept of slope and y-intercepts for developing linear algebraic expression*s)
    - Use the formulated pattern rule to find unknown values
- The weblink https://m.youtube.com/watch?v=J0Q8XtSuZx8 contains a video to write algebraic expressions for the given description of pattern rule pictorially and using tables.
- Identify relations as linear or nonlinear relations looking at graphs. Linear relation relations will have straight line graphs, whereas non-linear relations will have parabola or exponential graphs. Display different graph to identify them as linear on non-linear.
- Provide an equation and let learners create situations represented by the equation. For example 2x+3 = 9 could be said as there are two bags with the same number of fruits and 3 fruits kept outside the bag. There are 9 fruits in total.

D. Assessment

Performance Task 1

Provide a set of questions to describe patterns using expressions and equations from a table of values, graphs, pictures and word problems.

Performance Task 2

Carry out an activity based on real life problems to predict and find unknown values.

- Sample Questions

  Gaylek uses trapezoid shaped tiles to create a design. He uses a different number of tiles in this pattern.

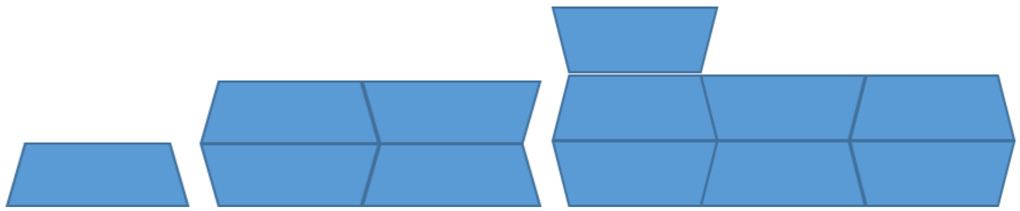


Figure 1 Figure 2 Figure 3

i. Create an algebraic expression to find the number of tiles in figure 8?

ii. Draw a graph to show the relationship between figure number and number of tiles?

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Reflective Questions
  - o Let students sketch hills/mountains/areas in their locality from where the sun rises and sets. Let the student use the same sketch to note with a dot, how the sun changes this position every month. At the end of the year let students see the pattern formed and connect it to Nyinlo (Solstice) & Equinox.
  - o Visit the school campus/flower garden/forest nearby to observe patterns and discuss in the class.
  - o Seasons and weather exhibit patterns. Discuss how mathematising such patterns of the real world help us.

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII

- Online
  - Pattern rule - https://m.youtube.com/watch?v=Ywomf4-5Puo
  - Creating algebraic expression - https://m.youtube.com/watch?v=J0Q8XtSuZx8

Topic: VIII-B2 Solving Linear Equations [400 minutes]

Introduction

Systems of linear equations arose in Europe with the introduction of Coordinate geometry by René Descartes in 1637. In fact, in this new geometry, now called Cartesian geometry, lines and planes are represented by linear equations, and computing their intersections amounts to solving systems of linear equations.

An equation that has the highest degree of 1 is known as a linear equation. This means that no variable in a linear equation has an exponent more than 1. The graph of a linear equation always forms a straight line.

An equation is like a weighing balance with equal weights on both sides. If we add or subtract the same number from both sides of an equation, it still holds true. Similarly, if we multiply or divide the same number on both sides of an equation, it is correct. We bring the variables to one side of the equation and the constant to the other side and then find the value of the unknown variable.

Utility and Scope

Linear equations are an important tool in science and many everyday applications. They allow scientists to describe relationships between two variables in the physical world, make predictions, calculate rates, and make conversions, among other things. Graphing linear equations helps make trends visible.

Almost any situation where there is an unknown quantity can be represented by a linear equation, like figuring out income over time, calculating mileage rates, or predicting profit. Many people use linear equations every day, even if they do the calculations in their head without drawing a line graph.

One of the most helpful ways to apply linear equations in everyday life is to make predictions about what will happen in the future. While real world factors certainly impact how accurate predictions are, they can be a good indication of what to expect in the future. Linear equations are a tool that makes this possible.

A. Competency

- Use the concept of linear equations and apply them in solving real life problems algebraically and graphically.

B. Objectives

- Create and solve relevant problems for which algebraic solutions are required using concrete or pictorial models and algebraic representations
- Determine the solution for two linear equations graphically and algebraically
- Define and calculate slope through various strategies.
- Investigate slope in practical situations. (e.g: slope of a staircase, slope of a roof, and the steepness of roads.

C. Learning Experiences

- Provide examples of different situations and show how algebraic expressions and equations can be created. For example if we say 2 bags full and 5 Kg more would be 105 Kg. For this we can assume the weight of each bag to be 'x' and create equation $2x + 5 = 105$
- Show algebraic equations pictorially and then find unknown value using inverse operation (refer to the Understanding Mathematics Textbook for Class VIII). The link https://youtu.be/LDIiYKYvvdA?si=ttdk9cQ1oeNs0f6n shows video on inverse operation. Further explanation is in the links below.
- Discuss how to solve linear relationships.

  May use the link (https://m.youtube.com/watch?v=l3XzepN03KQ) that shows how to solve simple linear equations symbolically.

  The web link (https://m.youtube.com/watch?v=rd03arcRbXk) contains a video that shows how to write phrases as equations and its solution, symbolically. It also shows its application.
- Explore the meaning of slope.

  The weblink https://m.youtube.com/watch?v=ADLoWIxKsyQ explains the meaning of slope pictorially.
- Investigate the slope (Estimate the slope of the staircase or a roof or the steepness of roads).

  May use the suggested link

  https://www.geogebra.org/m/WHxmVN3F#material/GGR6pcxC

- Solve problems involving two linear relationships by making a table of values and plotting graphs in the same grid. The coordinates at the point of intersection are solutions to two linear equations.

D. Assessment

Performance Task 1

Provide a real-life question to assess student competency related to solving linear equations. (Refer Understanding Mathematics Textbook for Class VIII or other appropriate sources).

Performance Task 2

Work in groups to compare the steepness (slope) of different real structures (staircase, roof, ramp, steep footpaths or road, etc.).

- Sample Questions
  i. The two angles are supplementary. Their difference is 24°. Find the measure of each angle.

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).Reflective *Questions*

  i. Physical quantities like; speed & distance, temperature & humidity are related. How do equations help us understand these relations?

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Online
  - Solve simple linear equations symbolically - https://m.youtube.com/watch?v=l3XzepN03KQ
  - Writing equation - https://m.youtube.com/watch?v=rd03arcRbXk
  - Meaning of slope - https://m.youtube.com/watch?v=ADLoWIxKsyQ
  - Investigating slope - geogebra.org/m/WHxmVN3F#material/GGR6pcxC

Topic: VIII-B3 Linear Polynomials [300 minutes]

Introduction

A linear polynomial is a type of polynomial where the highest degree of the variable is 1. In other words, the highest exponent of the variable is 1. Polynomials are algebraic expressions where the variables have non-negative integer powers. The expression consists of one or more terms such as a variable, constant, and a variable with a non-zero coefficient. Linear polynomials are the simplest form of polynomials.

Utility and Scope

Polynomials are an important part of the "language" of mathematics and algebra. They are used in nearly every field of mathematics to express numbers as a result of mathematical operations. Polynomials are also "building blocks" in other types of mathematical expressions, such as rational expressions.

Many mathematical processes that are done in everyday life can be interpreted as polynomials.

- o Summing the cost of items on a grocery bill can be interpreted as a polynomial.
- o Calculating the distance travelled by a vehicle or object can be interpreted as a polynomial.
- o Calculating the perimeter, area, and volume of geometric figures can be interpreted as polynomials.

A. Competency

- Demonstrate the ability to add and subtract polynomials using various strategies (pictorially and symbolically) and apply the concept in real life situations.

B. Objectives

- Add and subtract polynomials pictorially using algebra tiles or symbolically.
- Use Zero property for adding and subtracting polynomials.

C. Learning Experiences

- Conduct pre-assessment on identifying tiles with polynomials. Review the concept of variables, constant, coefficient, identifying polynomials (monomials, binomials, trinomials), like terms and degree of polynomials.
- Add polynomials pictorially and symbolically. Prepare a PowerPoint presentation or use relevant software to identify and add polynomials pictorially and symbolically.
- Subtract polynomials pictorially and symbolically.
- Suggestion:
  - o Demonstrate the subtracting strategies or explain through relevant video clips.
  - o Divide students into three groups and provide a question in each group.
  - o Assign a subtracting polynomials strategy (Take away, comparison and missing addend) to each group and let them work in groups. May use Understanding Mathematics Textbook for Class VIII to learn the strategies.
  - o Let groups take turns to present their task to the whole class.
- Apply zero property for adding and subtracting linear polynomials pictorially (refer to the Understanding Mathematics Textbook for Class VIII).
- Further, use the link to demonstrate how to add, subtract and use zero property in polynomials.
  - o Video link https://www.youtube.com/watch?v=pw5R4NOdC-4 shows how to add and subtract polynomials.

D. Assessment

Performance Task 1

Prepare worksheets to assess students' understanding on adding and subtracting polynomials pictorially or symbolically and using zero property. (May use related online worksheets).

- Sample Questions
  - i. The length of a rectangle is 5 cm more than the width of a rectangle. Calculate the perimeter of a rectangle.

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Reflective Questions
  i. Explore the idea of adding polynomials pictorially and discuss them in the class. (try using other than tiles).
  ii. Share your thoughts on the role of linear polynomials in real life and share its implication if it was not discovered.

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Online resources:
  - Add and subtract polynomials – https://www.youtube.com/watch?v=pw5R4NOdC-4

Topic: VIII-C1 Pythagorean Theorem [350 minutes]

Introduction

The Pythagorean Theorem was first known in ancient Babylon and Egypt (beginning about 1900 B.C.). The relationship was shown on a 4000 years old Babylonian tablet now known as Plimpton 322. However, the relationship was not widely publicised until Pythagoras stated it explicitly.

Pythagoras lived during the 6th century B.C. on the island of Samos in the Aegean Sea. He also lived in Egypt, Babylon and southern Italy. Pythagoras was a teacher and a philosopher.

In mathematics, the Pythagorean Theorem, also called Pythagoras' theorem, is a fundamental relation in Euclidean geometry among the three sides of a right triangle. It states that the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares on the other two sides. This theorem can be written as an equation relating the lengths of the sides *a*, *b* and *c*, where $a^2 + b^2 = c^2$, often called the Pythagorean equation.

Utility and Scope

The Pythagorean Theorem is a useful tool that shows how the sum of the areas of three intersecting squares can determine the side lengths of a right triangle. This theorem is an extremely useful tool that provides the basis for more complex trigonometry theories such as the converse of the Pythagorean Theorem. This theorem, however, isn't a generalization about all triangles. The Pythagoras theorem doesn't apply to obtuse or isosceles triangles that don't contain right angles. Some of the application of Pythagoras theorem in real life:

- To calculate the length of staircase required to reach a window.
- To find the steepness of the hills or mountains.
- To determine heights and measurements in the construction sites.

A. Competency

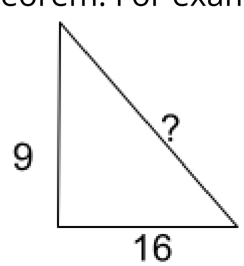
- Explore and understand the concept of Pythagorean Theorem to apply and solve real life problems.

B. Objectives

- Prove, through investigation, that if a square is made on each side of a right triangle, the sum of the area of two smaller squares will be equal to the area of the square on the longer side.
- Explore various Pythagorean triplets.
- Apply Pythagorean Theorem.
    - o to calculate the length of the hypotenuse, as well as length of the other side when the hypotenuse and one side is given
    - o to find distance between two points in real life situations (e.g. determine the reach of a ladder)

C. Learning Experiences

- Conduct pre-assessment on types of triangles by angle. (Acute, right and obtuse triangles). Identify and discuss similarities and differences between different triangles.
- Use chart paper or some hard covers to prove the Pythagorean Theorem. (*Teacher needs to prepare a right triangle and three squares attached on each side*). Demonstrate how the area of square on two smaller sides combined are equivalent to the area of square on the greater side.
    - o Use the web link (https://m.youtube.com/watch?v=tTHhBE5lYTg) to prove Pythagoras Theorem, concretely.
- Explore different Pythagorean triplets by drawing different right triangles or symbolically
    - o May use the given links to find different sets of Pythagorean triplets
    - https://m.youtube.com/watch?v=F9YtVqyps1s
    - https://m.youtube.com/watch?v=F9YtVqyps1s&pp=ygUTcHl0aGFnb3JlYW4gdHJpcGxldA%3D%3D
- Demonstrate to find the missing side length given any two sides of a right triangle using Pythagoras Theorem. For example:



- Explore beyond classroom situations (For example: investigate the reach of ladder, investigate whether triangular fields/places or triangular structures have Pythagorean triples).

D. Assessment

Performance Task 1

Prepare and provide tasks related to applying Pythagoras theorem (May refer Understanding Mathematics Textbook for Class VIII).

Performance Task 2

Confirm if volleyball courts and football ground penalty areas follow Pythagoras theorem.

- o First measure side lengths and diagonals.
- o Note their measurements.
- o Use Pythagorean Theorem to confirm the measurement.

- Sample Questions
  i. The height of two buildings is 34 m and 29 m respectively. If the distance between the two buildings at the base is 12 m, find the distance between their tops.

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Reflective Questions
  i. Prepare a model to explain Pythagorean Theorem. (May use chart/ cardboard/any other materials).
  ii. Move around the school campus and see where Pythagorean theorems can be applied. Discuss how the theory is significant.

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Online Resources:
  - To prove Pythagoras Theorem - https://m.youtube.com/watch?v=tTHhBE5lYTg)
  - Video for students to visualize the Pythagoras Theorem, concretely- https://m.youtube.com/watch?v=CAkMUdeB06o
  - Sets of Pythagorean triple - https://m.youtube.com/watch?v=F9YtVqyps1s

- Explore Pythagorean Theorem - https://www.geogebra.org/m/wdQ5VRW9
- Solutions for problems of Pythagorean theorem - https://www.mathsisfun.com/pythagoras.html
- Pythagoras Theorem in their daily life - https://bit.ly/3naaWfN

Topic: VIII-C2 Area and Perimeter Relationships [350 minutes]

Introduction

Perimeter and area are two important and fundamental mathematical topics. They help us to quantify physical space and also provide a foundation for more advanced mathematics found in algebra, trigonometry, and calculus.

Area is the amount of surface covered by a two-dimensional figure. In other words, it is the quantity that measures the number of unit squares that cover the surface of a closed figure. The standard unit of area is square units which is generally represented as square metres, square centimetres and square millimetres.

Perimeter is the distance around the outside of a two-dimensional shape. The perimeter of a shape is the total length of the shape's sides. Perimeter is measured in linear units such as kilometre, metre, centimetre, millimetre, etc.

Utility and Scope

Knowledge of area and perimeter is applied practically by people on a daily basis, such as architects, engineers, and graphic designers.

Some of the essential uses of the area and the perimeter are listed below:

- We have essential applications in agricultural fields, such as finding the land area required for agriculture and knowing the field's dimensions, etc.
- We also have its applications in engineering fields such as fencing the lands, surveying the fields, painting the room's walls, finding the space of the building etc.
- The perimeter and area have their applications in mapping and navigation.
- To find the surface occupied by the parks or grounds in the shape of square, triangle, circular, etc.

A. Competency

- Understand and examine the relationship between area and perimeter and apply the concept to solve real world problems.

B. Objectives

- Calculate the area and perimeter of 2-D shapes (triangles and quadrilaterals).
- Investigate that area can vary when perimeter is fixed
- Investigate that perimeter can vary when the area is fixed

C. Learning Experiences

- Conduct pre-assessment on understanding the area and perimeter of a 2-D shape. Relate area to total surface covered by the garden and perimeter to the fencing around the garden.
- Explore and investigate the formula to calculate the area and the perimeter of 2-D shapes.
- Demonstrate how to find the area and perimeter of classroom objects and introduce the linear and square units accordingly. (For example, the top surface of a desk/book/teacher's table, floor of the classroom...)
- Investigate the effect of change in area and perimeter using related software (eg; GeoGebra).
  - keeping the area fixed and changing the perimeter using the following link https://www.geogebra.org/m/gwSeBhQx
  - keeping the perimeter fixed and changing the area using the following link https://www.geogebra.org/m/r6XXP7bv
- Divide students into groups and let them find the area and perimeter of different fields (Eg: volleyball court, basketball court, football field, assembly ground, agriculture garden, etc). Relate the area and perimeter concept with related contents from other subjects.

D. Assessment

Performance Task 1

Design some questions to relate the area and perimeter of the rectangle.

Performance Task 2

Let a few groups of students draw at least three rectangles with the same perimeter and ask them to find the area of each. Similarly let other groups draw at least three rectangles with the same area and ask them to find the perimeter of each. Share their findings.

- Sample Questions
  i. Each student tries to make a different rectangle using a 30 m long thread. Draw and all possible types of rectangles showing their length and width. Which rectangle will have the largest area?

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Reflective Questions
  i. How learning area and perimeter help you in purchasing land?

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Real objects/ field or furniture
- Online resources:
  - keeping the area fixed and changing the perimeter using the following link https://www.geogebra.org/m/gwSeBhQx
  - keeping the perimeter fixed and changing the area using the following link https://www.geogebra.org/m/r6XXP7bv

Topic: VIII-C3 Volume and Surface area of a Rectangular Prism
[400 minutes]

Introduction

Archimedes, who was the famous mathematician in ancient Greece had invented a method for determining the volume and the surface area of an object. He was well known for his discovery of the relation between the surface area and volume.

Utility and Scope

We use the idea of volume and surface area of rectangular prisms in our everyday life. Many real objects and structures that we see around in our environment are made up of rectangular prisms. You could use surface area to find out how much cardboard was used to make a box, or how much fabric was used to make a pillow. Surface area can be used to calculate things like the area of the outside of machines.

Volume is used basically in every chemical experiment. It is used to find out how much space there is to fill something up, such as a room in a house, or a pool. Knowing the volume and surface area of a rectangular prism would help us to solve many real-life problems.

A. Competency

- Explore and examine the concept of volume and surface area of rectangular prisms and apply to solve real life problems.

B. Objectives

- Investigate that the volume of a rectangular prism is the area of its base multiplied by its height.
- Identify appropriate units while finding volume and capacity in a given situation.
- Compare the size of an object by comparing their volumes.
- Estimate before calculating dimensions.
- Use nets of a rectangular prism to deduce the formula to calculate total surface area (TSA) and apply it in related problems.
- Investigate changes in total surface area based on changes in dimensions.

- Calculate the missing side length of a rectangular prism when total surface area or Volume, and other two side lengths are given.

C. Learning Experiences

- Provide linking cubes to investigate that the volume is equal to area of base multiplied by its height.
- Use the link https://www.geogebra.org/m/SYj783n2 or similar link to further explore the volume of rectangular prisms. What happens to the volume of a rectangular prism when you triple each dimension?
- Calculate the volume of packages which are in rectangular prism shape (e.g. Milk/Juice) and compare with capacity labelled on it. (Highlight the units of volume and capacity)
- (If you have a jerry can of water which is irregular in shape, how would you find the volume of water in the jerry can?)
- Estimate the volume of the real objects (rectangular shape) and compare with their actual volume.
    - Estimating exercise: Take the students out of the class and allow them to come with different containers filled with water. Let them estimate the capacity. Then pour the water into a rectangular prism and calculate the volume. Then compare the estimated volume with actual volume.
- Deduce the formula for the total surface area of a rectangular prism using the following steps
    - Draw its net
    - Find area of each face of the net
    - Combine areas of all faces to get total surface area.

      May use the link *https://youtu.be/hwo-aHSbiqg* that shows how to find surface area of a rectangular prism by combining the area of the nets/faces.
- Investigate how the total surface area changes when there is a change in any dimensions. Investigate how a change in dimension changes total surface area.
- Calculate missing side lengths when total surface and two other side lengths of a rectangular prism are given.

D. Assessment

Performance Task 1

Assign a set of questions to calculate the volume of the rectangular prism. Refer to class VIII mathematics textbook or other relevant sources.

Prepare a worksheet to find the total surface area and missing dimension (side length) when the total surface area and other two side lengths are given.

May use appropriate worksheets from

https://www.liveworksheets.com/qc1980282oc

- Sample Questions
  i. Calculate the amount of paint required to paint the classroom walls at a given rate (say 5 litre cost Nu 450 and can paint 3 m^2).

Performance Task 2

How does the combined surface area of two halves of a rectangular prism compare to the original surface area? How does the combined volume of two halves of a rectangular prism compare to the volume of the original rectangular prism? Why does this happen?

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Reflective Questions
  i. Discuss how knowledge on volume and surface area help in packaging and transporting industries.

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Linking cubes/books/relevant classroom objects.
- Online resources
  - Volume of a rectangular prism - https://www.geogebra.org/m/SYj783n2
  - Surface area of a rectangular prism - *https://youtu.be/hwo-aHSbiqg*
  - Finding total Surface area from missing dimension https://www.liveworksheets.com/qc1980282oc

Topic: VIII-C4 Area of a Circle [400 minutes]

Introduction

The study of the circle goes back beyond the recorded history. The invention of the wheel is a fundamental discovery of the properties of a circle. Thales found the first theorems relating to circles around 650 BC.

Ancient Greeks discovered the area of a circle by relating it with the area of a disk. Eudoxus of Cnidus in the fifth century B.C. had found that the area of a disk is proportional to its radius squared. Archimedes used the tools of Euclidean geometry to show that the area inside a circle is equal to that of a right triangle whose base has the length of the circle's circumference and whose height equals the circle's radius. The circumference is $2\pi r$, and the area of a triangle is half the base times the height, yielding the area πr^2 for the disk.

Utility and Scope

There are many practical applications in everyday life where we need to use a circle's area. Architects use the symmetrical properties of a circle to design Ferris-wheels, buildings, athletic tracks, roundabouts, etc. These circular measurements are also significant for engineers in designing aeroplanes, bicycles, rockets, etc.

The circle is used in designing a simple machine such as a clock to develop a complex nuclear reactor; circular calculations play a significant role. The area of a circle is useful for measuring the region occupied by a circular field or a plot. Suppose, if you have a circular table, then the area formula will help us to know how much cloth is needed to cover it completely. The area formula will also help us to know the boundary length i.e., the circumference of the circle. The area of a circle will also help us find the volume of a cylinder.

A. Competency

- Determine the relation to calculate the area of a circle and its parts (semi-circle and quadrants) to apply in solving related problems.

B. Objectives

- Explore various ways to conclude the formula for the area of the circle.
- Compute area of a circle, semi-circles and quarter-circles (quadrants).
- Use the formula for area of the circle to determine the radius when the area is given.

C. Learning Experiences

- Pre-assessment on lessons learnt in class VII about value of pi (π)and circumference of a circle and then relate it with a lesson on 'Area of a circle'.may use the link below to explain and connect to future lesson: https://www.youtube.com/watch?v=O-cawByg2aA.
- Deduce the formula for area of a circle and calculate the area of a circle accordingly (refer Understanding Mathematics Textbook for Class VIII).
  May use the link https://youtu.be/YokKp3pwVFc?si=PEAQZ8jg_k2jifOY which explains the concept and way to find the area of a circle. Bent circle to form Triangles and quadrilaterals and to show that area can vary while perimeter remains fixed.
- Determine the area of the semicircle and the area of the quadrant (quarter circle) by relating it to the area of the circle. Show a larger circle and fold it into halves and one-fourths to find the area of parts of the circle.
- Area of semi-circle as the half of area of a circle (Area of Semi-circle = $\frac{\pi r^2}{2}$)
- Area of a quadrant as the one-fourth of the area of a circle.
  (Area of quadrant =$\frac{\pi r^2}{4}$)
- Apply the concept of area of a circle, semi-circle, quadrants and radius, in real life problems. (eg: making tracks, pipe fitting, tyre size, etc.)
- Outdoor Activity: Take students to the football ground/basketball court and let them calculate the area of the circle in the field.

D. Assessment

Performance Task 1

Assign a set of questions for finding the area of the circle. (Refer class VIII mathematics textbook or other relevant sources).

Provide the worksheet from the given link or let students do it online https://www.liveworksheets.com/search.asp?content=area+of+circle (worksheet for finding diameter, radius and area of a circle).

- Sample Questions
  i. Lids of most kitchen utensils are circular. If the circumference of the pot's lid is 154 cm, what could be the radius ?
  ii. If a cow is tethered with 3.5m long rope, what could be the maximum area that it would graze?
  iii. A wifi operating on certain bandwidth has a signal range area of 31,400 square feet outdoors. What is the farthest distance in metres it can cover from the point it is installed?

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Reflective Questions
  i. Imagine a world without a circle and discuss your insights.

E. Resources

- Understanding Mathematics, Textbook for class VIII.
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Circular object
- Online resources:
  - Concept of the area of a circle - https://youtu.be/YokKp3pwVFc?si=PEAQZ8jg_k2jifOY
  - worksheet for finding diameter, radius and area of circle - https://www.liveworksheets.com/search.asp?content=area+of+circle
  - Area and circumference of a circle - https://www.youtube.com/watch?v=O-cawByg2aA.

Topic: VIII-D1 Transformation [450 minutes]

Introduction

The first systematic effort to use transformations as the foundation of geometry was made by Felix Klein in the 19th century, under the name Erlangen programme. For nearly a century this approach remained confined to mathematics research circles. His views were geometry as the study of the properties of a space that are invariant. His work can be learnt from the site https://www.britannica.com/biography/Felix-Klein.

Utility and Scope

Geometric transformations are needed to give an object the required position, orientation, or shape starting from an existing position, orientation, or shape. The basic transformations are translation, rotation, reflection, dilation, and other important types of transformations are projections and mappings.

The symmetry of our face, the body of the butterfly, production of shoes, spectacle frames, flipping images on a computer and mirror images all exhibit transformation properties. Dilation offers invaluable support to zoom images of objects which enables users to see things with clarity. Dilation enables projectors and cameras to enlarge or diminish images for kaleidoscopic and required view. The tessellation, which is used in the creation of floor and mosaic design often uses the concepts of transformation.

A. Competency

- Demonstrate understanding of dilation of 2-D shapes and apply to transform regular polygons through combined transformations.

B. Objectives

- Explain dilation as enlargement or reduction of a shape based on scale factor.
- Investigate dilations of 2-D objects by applying different scale factors
- Analyse how an object dilates based on the position of its dilation centre using relevant software.

- Explore combinations of transformations that include dilations, such as an enlargement/reduction followed by a reflection/rotation /translation and vice versa.

C. Learning Experiences

- Conduct pre-assessment on types of transformation based on class VII lesson and introduce dilation.
- Introduce the concept of *dilation.*

  Suggestion:
  - Show an image on a computer or projector screen and use slider/percentile to zoom (zoom in and zoom out).
  - *Use interactive software (GeoGebra).*
  - *Use interactive web link-* https://www.geogebra.org/m/NujwnT5Z
- The weblink https://www.geogebra.org/m/SWUD4jav shows how the shape dilates as we change its centre.
- Dilate shapes manually (hands on experience).
- Demonstrate the combination of transformation manually and using interactive software (GeoGebra).

D. Assessment

Performance Task 1

Compile and assign questions related to dilation and combining transformations

- Sample Questions
  1. A microscope has four eyepieces, 4X, 10X, 40X and 100X. What are these 4, 10, 40 and 100 with the letter X here? An organism is 0.2mm in actuality. If you use a 10X eyepiece, how large will you see it? Do you need a 40X eyepiece here to see the organism?
  2. Design a question on reduction/enlargement of photographs when scale factor and dilation centre are given.
  3. Design a question to find how the area changes when a polygon (Triangles and quadrilaterals) is reduced or enlarged. Find the ratio between the original area to the area of an image. (May refer to the Understanding Mathematics Textbook for class VIII).

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Reflective Questions
  - Explore how would the function of the camera be undermined if there is no zoom function in it?
  - How does dilation of pupils in our eyes benefit us?
  - How dilatation affects us. Discuss the impact of its presence as well as its absence.

E. Resources

- Understanding Mathematics Textbook for class VIII
- Teachers Guide to Understanding Mathematics Textbook for class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Online
  - Introduction to Transformation - https://www.britannica.com/biography/Felix-Klein
  - Interactive weblink - https://www.geogebra.org/m/NujwnT5Z
  - Dilatation - https://www.geogebra.org/m/SWUD4jav

Topic: VIII-D2 Angles in a Polygons and Triangle Construction
[400 minutes]

Introduction

Polygons have been known since ancient times. The regular polygons were known to the ancient Greeks, with the pentagram, a non-convex regular polygon (star polygon), appearing as early as the 7th century B.C. on a krater (Vase) by Aristophanes, found at Caere (present-day Cerveteri, Italy) and now in the Capitoline Museum (in Rome, Italy). In 1952, Shephard generalised the idea of polygons to the complex plane, where each real dimension is accompanied by an imaginary one, to create complex polygons.

Thomas Bradwardine was the first known person to study non-convex polygons in the 14th century. The concept of polygons was generalized in 1952 by Geoffrey Colin. Explore further in https://www.storyofmathematics.com/polygons-definition to learn about polygons.

Utility and Scope

Polygons are all around us; from the tiles on the footpath to the truss on the roof, polygons are used. Polygons are an important part in architecture as they help to make patterns, tessellations, to make other shapes and to form 3D shapes. We use polygons at almost every moment in our daily life from fruits to honeycomb, from floor designs to rectangular or square shaped buildings. We see the traffic signals which can be rectangular, square or triangular in shape, which are all polygons.

A. Competency

- Deduce rules to calculate interior and exterior angles of regular polygons and apply it to solve related problems in real life situations.
- Construct and draw triangles appropriately exploring different ways when side lengths and angles are given.

B. Objectives

- Develop a relationship through investigation, to find the sum of the interior and exterior angles of a polygon.

- Apply the relationship to calculate the sum of the interior angles and the measure of each interior angle in a regular polygon.
- Draw and Construct triangles appropriately using given side length and angle(s).

C. Learning Experiences

- Conduct pre-assessment for the sum of the angles in a triangle.
- Explore ways to calculate the sum of the interior angles/ exterior angles of a polygon and each interior angle of a regular polygon.
    - The web link https://m.youtube.com/watch?v=BG1HpadfiKw contains a video that shows the concept of interior and exterior angles, how to find the interior and exterior angles and shows how interior and exterior angle supplements, pictorially.
    - Demonstrate to find interior and exterior angles for regular polygons by applying formula or refer https://m.youtube.com/watch?v=otzb_GkRiqQ to derive formula for finding interior angles for regular polygons.
    - Investigate visually, interior, and exterior angles through interactive software https://www.geogebra.org/m/xHD5f4PF.
- Constructing Triangles
    - Differentiate between drawing triangles and constructing triangles.
    - Practise the skills of angle construction to apply in triangle construction.
    - Show the video to construct a triangle. Use the link below to assist understanding- https://www.youtube.com/watch?v=UzNWf737nNk. Draw rough sketch before constructing actual triangle.

D. Assessment

Performance Task 1

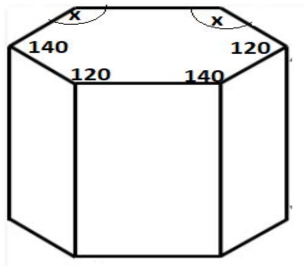
Design a presentation on any polygon and show how to find

- The sum of interior angles
- The sum of exterior angles
- measures of each interior and exterior angle
- show one real life application

Performance Task 2

Make a site visit or a campus tour to note polygons and to calculate their angles.

- Sample Questions
    i. Calculate the value of angle x shown in hexagonal prism below



(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Sample Questions

Explore what challenges would it pose if angles of polygons on trusses of a roof were inappropriate.

- Reflective Questions
    i. Express your views on the role of angles in polygons in the construction industry.

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Online
    - Introduction to polygons - https://www.storyofmathematics.com/polygons-definition
    - Concept of interior and exterior angle - https://m.youtube.com/watch?v=BG1HpadfiKw
    - Calculating interior and exterior angle - https://m.youtube.com/watch?v=otzb_GkRiqQ
    - Interior, and exterior angles through interactive software - https://www.geogebra.org/m/xHD5f4PF
    - Constructing triangles - https://www.youtube.com/watch?v=UzNWf737nNk
    - Exterior angle - https://www.youtube.com/watch?v=9-m16WDgLUQ

Topic: VIII-D3 Angles in Parallel and Intersecting Lines [300 minutes]

Introduction

Ancient mathematicians introduced the concept of lines to represent straight objects which had negligible width and depth. When two rays (part of a straight line) intersect each other in the same plane, they form an angle. Explore further in https://e-gmat.com/blogs/lines-and-angles

Utility and Scope

Parallel line examples in real life are railroad tracks, the edges of sidewalks, marking on the streets, zebra crossing on the roads, the surface of pineapple and strawberry fruit, staircase, and railings, etc.

A. Competency

- Investigate angles when parallel lines are cut by a transversal to find the relation between various angles formed and apply the concept in finding angles in figures without measuring.

B. Objectives

- Investigate how angles change when a pair of lines are cut by a transversal line.
- Determine the relationship between corresponding angles and alternate angles when a transversal intersects a pair of parallel lines.
- Apply transformational geometry to discover why the various angle pairs are equal.

C. Learning Experiences

- Explain *opposite, adjacent, alternate, corresponding and supplementary angles* when parallel lines are cut by transversal. Use the link below https://m.youtube.com/watch?v=bq97X0ZTmnk that define angles pictorially.
- Investigate the changes in the angles when two lines are cut by a transversal using interactive software (GeoGebra). Use weblink https://www.geogebra.org/m/zKaH7DEm.

- Calculate values of corresponding angles, interior angles, and alternate angles visually or pictorially when there is transversal in parallel lines. https://www.geogebra.org/m/pxk6bZWF
- Share notes to determine corresponding angles, interior angles, and alternate angles.
- *Suggestion:*
  - o Confirm through transformational geometry why the various angle pairs are equal when parallel lines are cut by a transversal line. Use a mirror mid-way between the parallel lines to reflect angles formed at one intersection onto angles formed at the other intersection.
  - o Read page 10 and 12 from the link given below https://www.mathed.page/transformations/proof/transformational-proof.pdf

D. Assessment

Performance Task 1

Ask students to solve a set of questions aligned with objectives and competency.

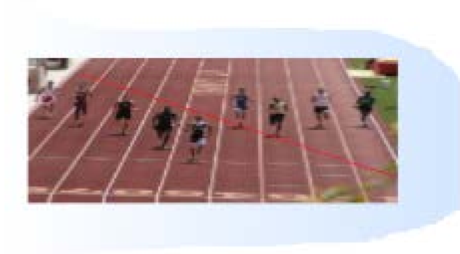
Example:

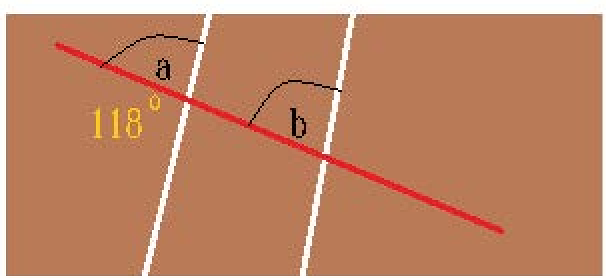
Share the web link https://m.youtube.com/watch?v=bq97X0ZTmnk and ask them to identify corresponding angles, interior angles and alternate angles.

Performance Task 2

Lay three sticks on the cemented floor. Make practical attempts at arranging sticks like parallel lines and transversal. Let students trace lines on the cemented floor and use a protractor to measure angles and confirm if the sticks were kept as parallel lines.

- Sample Questions
  i. Calculate the angles 'a' and 'b' shown below.





(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Sample Questions

  Explore to answer following questions

  - o Are the electricity transmission wires parallel lines? If not, why?
  - o Are longitudinal or latitudinal lines parallel ? Why?
  - o How can you ascertain that two lines are parallel?

- Reflective Questions

  i. Move around campus to observe parallel lines. Discuss how you assure that the lines are parallel. Draw conclusions of your observations.

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, Textbook for class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Online resources:
  - Introducing parallel lines and angles - https://e-gmat.com/blogs/lines-and-angles
  - Defines angles pictorially https://m.youtube.com/watch?v=bq97X0ZTmnk
  - using interactive software (GeoGebra) - https://www.geogebra.org/m/zKaH7DEm)
  - Angles in parallel lines - https://www.geogebra.org/m/pxk6bZWF
  - Angles (corresponding angles, interior angles and alternate angles) - https://m.youtube.com/watch?v=bq97X0ZTmnk...

Topic: VIII-D4 Representing Objects [350 minutes]

Introduction

The first systematic effort to use transformations as the foundation of geometry was made by Felix Klein in the 19th century, under the name Erlangen programme. For nearly a century this approach remained confined to mathematics research circles. His views were geometry as the study of the properties of a space that are invariant. His work can be learnt from the site https://www.britannica.com/biography/Felix-Klein.

Utility and Scope

Geometric transformations are needed to give an object the required position, orientation, or shape starting from an existing position, orientation, or shape. The basic transformations are translation, rotation, reflection, dilation, and other important types of transformations are projections and mappings.

The symmetry of our face, a body of butterfly, production of shoes, spectacle frames, flipping images on a computer, mirror images of the chemical structure of the sugar molecules, glucose (in sugarcane) and fructose (in fruit) all exhibit transformation properties.

In the tessellation, where the creation of floor and mosaic design happens, transformation is present. This gives students the ability to manipulate figures in the plane freely, which sets the foundation for verification of perpendicular segments, the derivation of the equation of a circle, congruence, and similarity.

A. Competency

- Represent face views of regular 3-D shapes in 2-D design to visualize their spatial movement.

B. Objectives

- Apply prior interpretation knowledge of 2-D pictures to enhance the mathematical experience with 3-D objects.
- Construct structures from isometric drawings in various face views.
- Create orthographic drawings of 3-D figures, apply the skill to make structure using linking cubes from a given set of orthographic plans or drawings.
- Compare isometric plans and orthographic face views

C. Learning Experiences

- Conduct pre-assessment on drawing 3-D shapes.
- *Suggestion:*
  - o Draw 3-D shapes formed by cubes on isometric dot paper.
  - o Provide isometric dot paper to draw cubes.
- Making 3-D figures(can use linking cubes) from orthographic drawing and creating orthographic drawings of 3-D shapes.
- *Suggestion:*
  - o *Provide linking cubes along with orthographic drawings.*
  - o https://youtu.be/SdLegfoMXNA *shows how to draw orthographic drawings from a linking cube.*
  - o *Provide structures and ask students to draw orthographic drawings.*
- Explore to relate structure to orthographic drawings and to isometric drawing (Issue isometric dot papers, linking cubes and specified orthographic drawings)
- *Suggestion:*
  - o Provide orthographic drawing and ask students to create structure and isometric drawing.
  - o Provide structure and ask students to create isometric and orthographic drawings.
  - o Provide isometric drawing and ask students to create structure and orthographic drawing.
  - o https://www.geogebra.org/m/uhXY6U9v shows orthographic projection visually.

D. Assessment

Performance Task 1

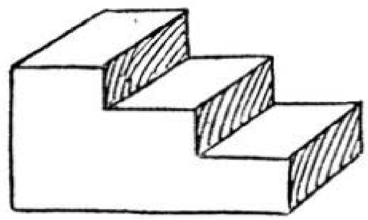
Hang a shape made from linking cubes from ceiling reaching to height easy to see (eye height from ground). Let children see the object from different viewpoints (front, right side, back and left side) and draw the face views. Raise the height of the object and let the child see the bottom view and draw. Finally, lower the height of the object, and let children draw the top view.

Performance Task 2

Build the structure for the set of orthographic drawings provided.

Use the structure built from orthographic drawing to draw isometric drawing.

- Sample Questions
  i. Draw an orthographic view to show the top and front view of the staircase given below.



(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Sample Questions

  Explore and find what challenge do you encounter if you draw isometric drawings in square grid paper instead of isometric dot paper?

- Reflective Questions
  i. From campus or community, identify a few rectangular objects. Discuss how you would represent them in 2-D. Share your opinion on what mathematics you used.

E. Resources

- Understanding Mathematics. Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Online
  - History - https://www.britannica.com/biography/Felix-Klein
  - Orthographic drawing - https://youtu.be/SdLegfoMXNA
  - Orthographic projection visually - https://www.geogebra.org/m/uhXY6U9v

Topic: VIII-E1 Data Handling [500 minutes]

Introduction

Representing data in pictorial form provides an immediate picture of the whole data. In 1644, Michael Florent Van Langren, a Flemish astronomer, is believed to have provided the first visual representation of statistical data. Prior to the 17th century, data visualization existed mainly in maps, displaying land markers, cities, roads, and resources. As the demand grew for more accurate mapping and physical measurement, better visualizations were needed.

The 18th century saw the beginning of thematic mapping. Abstract graphs of functions, measurement error, and collection of empirical data were introduced at this time. In this period William Playfair, invented many of the most popular graphs we use today (line, bar, circle, and pie charts). Many statistical chart types, including histograms, time series plots, contour plots, scatterplots, and others were invented during this period. More information is available at https://bit.ly/3r6q7YL. This topic deals with how to represent data in a circle graph and histogram.

Utility and Scope

Data representation at glance presents a comparative view of organized data. Data represented in appropriate form enables researchers and scholars to draw conclusions and analyse situations thereby providing possible clues and partial solutions to problems studied. The scope of data representation is multifaceted in studying areas like market scenario, lifestyles, production management and capturing trends.

Data handling offers immense scope in modern mathematics, science and in multidisciplinary areas. Data science offers millions of jobs across the globe and has immense potential for job creation when combined with technology.

A. Competency

- Demonstrate the understanding of the data collection process and implement it to collect, organize, represent and interpret data.

B. Objectives

- Generate samples using random sampling avoiding bias for fairness while collecting data.
- Collect data using developed tool(s) as a process to solve problems identified.
- Organise the collected data in tabular form.
- Represent the collected data in a circle graph and make comparative analysis.
- Decide when a circle graph is the most appropriate representation to display data.
- Display the data in histogram.
- Compare and contrast histogram and circle graph to show data can be represented in multiple ways.

C. Learning Experiences

- Pre-assessment of data collection process
  - o problem identification
  - o formulating questions
  - o sampling and bias
  - o data collection tools (questionnaire, observation and interview
- Introduce Random sampling
  - In a random sample, each member of the population is as likely to be selected as any other member
  - Use probability devices such as dice or coins, Computers and scientific calculators or random number table to generate random samples
  - https://www.youtube.com/watch?v=yx5KZi5QArQ shows how to carry out simple random sampling.
  - https://bit.ly/3FcqyWc for more understanding on random sampling
- Concept of circle graph (pie graph/pie chart)
  - *o* The link https://m.youtube.com/watch?v=AEhK5eujjZI&noapp=1 contains a video that explains the meaning of circle graphs and its advantages.
  - *o* Demonstrate how to draw a circle graph manually and using software (MS Excel/GeoGebra) . Use both central angles and percentages for the sectors in different pie charts.
  - *o* Get relevant data for the demonstration to draw a circle graph.

- o https://m.youtube.com/watch?v=KnOvvTXl9C8 contains a video that shows how to construct a pie chart using GeoGebra.
- o https://m.youtube.com/watch?v=DPle9gDpNxY contains a video that shows how to construct pie charts using GeoGebra.

- Review on how to draw histograms manually and using software (GeoGebra)
- *Suggestion:*
  - o *May use the data set that was used in drawing the circle graph.*
  - o https://m.youtube.com/watch?v=Qpi0mZZhPxs *contains a video that shows how to draw a histogram using GeoGebra.*
- Create a circle graph from histogram and explore to create a histogram from a circle graph.
- *Suggestion:*
  - o *Show circle graph into percent form (divide circle into 100 parts or 100%)*
  - o *https://youtu.be/eztM13u76Ec?si=Y3445WJHDpeMoHTi contains a video that shows visually how to create a histogram from a circle graph (pie chart). Watch and then manually create a histogram from a circle. Note that representation depends on type of data, therefore selecting appropriate representation for data is paramount.*
  - o *The link* https://m.youtube.com/watch?v=p_nPxTRuLxo or similar web link that shows steps/procedures to draw a circle graph or pie graph from the tabular data pictorially.
- Explore when to choose histogram and when to choose pie charts to represent data.

D. Assessment

Performance Task 1

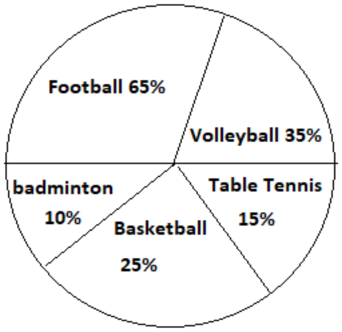
Create a circle graph and histogram from the data collected (may use data collected for other subjects too).

Performance Task 2

Find out a set of data represented in a circle graph and represent the same data in Histogram.

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Sample Questions
  i. Students were surveyed about their favourite sports to play. The results are represented in this circle graph. If 48 students were surveyed, how many chose:
     a. Football
     b. basketball
     c. volleyball



- Reflective Questions
  i. Share your opinion on issues the world would face if Data management is not taught in schools.

E. Resources

- Understanding Mathematics, Textbook for class VIII
- Teachers Guide to Understanding Mathematics, class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Online resources:
  - History - https://www.dundas.com/resources/blogs/introduction-to-business-intelligence/brief-history-data-visualization
  - how to carry out simple random sampling https://www.youtube.com/watch?v=yx5KZi5QArQ
  - understanding on random sampling - https://bit.ly/3FcqyWc
  - Meaning of circle graph - https://m.youtube.com/watch?v=AEhK5eujjZI&noapp=1
  - Video that shows how to construct a pie chart using GeoGebra - https://m.youtube.com/watch?v=KnOvvTXl9C8 contains a
  - Video that shows how to construct pie charts using GeoGebra - https://m.youtube.com/watch?v=DPle9gDpNxY
  - Video that shows how to draw a histogram using GeoGebra - https://m.youtube.com/watch?v=Qpi0mZZhPxs
  - Video that shows visually how to create a histogram from a circle graph - *https://youtu.be/eztM13u76Ec?si=Y3445WJHDpeMoHTi*
  - Random Sampling - https://www.youtube.com/watch?v=pTuj57uXWlk
  - Creating Histogram - https://m.youtube.com/watch?v=9V8Fdcp32Ao

Topic: VIII-E2 Probability [200 minutes]

Introduction

The modern mathematical theory of probability has its roots in attempts to analyse games of chance by Gerolamo Cardano in the 16^{th} century, and by Pierre de Fermat and Blaise Pascal in the seventeenth century. These details can be found in https://timesofmalta.com/articles/view/the-origins-of-probability.684474

Utility and Scope

Probability is widely used in daily life like sports, weather reports, blood samples, predicting the sex of the baby in the womb, congenital disabilities, statics, and many. Probability is also used as the basis of Quantum Mechanics and some other physics. Probability also helps to make decisions while conducting events where a large population could gather. For more reading visit; https://www.britannica.com/science/probability

A. Competency

- Calculate theoretical probability of events to find complementary events.

B. Objectives

- Deduce that theoretical probability of equally likely events as $P(E) = \frac{Number\ of\ favourable\ outcome}{Total\ number\ of\ possible\ outcomes}$
- Calculate theoretical probability for both single and double experiments (only for equally likely independent events).
- Conduct experiments to determine experimental probability and compare with their theoretical probabilities.
- Explain the meaning of complementary events.
- Deduce that the probability of a complementary event is calculated using 1−P(E) and show that P(E) + P (not E) = 1.

C. Learning Experiences

- Conduct pre-assessment on theoretical probability
- *Suggestion:*
  - Definition

  - o How to calculate the theoretical probability of a given event using a table or tree diagram. (Example; Rolling two dice to get the sum/product).
- Explore the concept of the *complement* of an event
- *Suggestion:*
  - o https://m.youtube.com/watch?v=Ae_CbicRvu4 contains a video that explains the complement of an event with examples.
  - o Investigate that the sum of events happened (P [E]) and not happened (P [not E) is a whole (1).
- Link to https://m.youtube.com/watch?v=KzfWUEJjG18 or similar web link that contains video about the basic probability.
- Explore the use of real life data to establish broad probability patterns for the purpose of planning and decision making. (e.g., patterns in population growth, traffic)

D. Assessment

Performance Task 1

Solve a set of questions that is aligned with the objectives and competency.

Performance Task 2

Let student come up with past experiences to make probability on event management; For example, it is Fete Day in school; What could be the probability:

- o that the weather be fine?
- o that people would litter an area?
- o of time where maximum visitors would come?

- Sample Questions
  i. There were 12 students participating in the selection round for an event. The complementary probability of the event is $\frac{1}{6}$ which represents the disqualified number in that event. What is the theoretical probability of the event? How many students qualified from the event?

(*Design an appropriate assessment tool for each performance task to provide feedback and record achievement based on the template provided in annexure VIII-A1*).

- Reflective Questions
  i. Express your views on why you should learn probability.

(*Note: suggested for further discussion only*) Explore introducing complement of set using Venn diagram as exposure for class IX Set theorem. If S= {1,2,3,4,5,6}, A= {1,2,3} then A' = {4,5,6} :P(A)+P(A') = 1. Here in set theory P (Not A) is equivalent to complement of Set A. (A' is read as A complement). Visit https://www.ck12.org/probability/complement-rule-for-probability/lesson/Complement-Rule-for-Probability-ADV-PST/ for more details on complement set using Venn diagram.

E. Resources

- Understanding Mathematics Textbook for class VIII
- Teachers Guide to Understanding Mathematics Textbook for class VIII.
- National School Curriculum Mathematics Framework for PP – XII
- Dice/spinner/cards
- Online
  - History of probability - https://bit.ly/3thiGR5 Or https://www.britannica.com/science/probability
  - *Video that explains the complement of an event with examples -* https://m.youtube.com/watch?v=Ae_CbicRvu4
  - Complementary event - https://m.youtube.com/watch?v=Ae_CbicRvu4

Assessment Structures for KS- 3 (Class VIII)

Assessment Structure

<table>
<tr><th rowspan="3">Key Stage</th><th colspan="6">Assessment</th></tr>
<tr><th colspan="3">Term I</th><th colspan="3">Term II</th></tr>
<tr><th>CA</th><th>Mid Term Examination</th><th>Total</th><th>CA</th><th>Annual Examination</th><th>Total</th></tr>
<tr><td>III</td><td>20</td><td>30</td><td>50</td><td>20</td><td>30</td><td>50</td></tr>
<tr><td colspan="7">For both Term I and Term II, assess each competency through appropriate performance tasks and assessment tools.
Performance Tasks: Worksheets, quiz, question and answer, presentation, making models, small projects, etc.
Project Work: One mandatory project must be completed annually. Refer rubrics and planning sample at the end of IG).
Assessment Tools: checklist, rating scale or rubrics.
Assessment Areas: Content: Formulating situations mathematically, applying concepts, facts, and procedures, and interpreting mathematical results.
Skills and attitude: Collaboration, communication, creativity, time management, learning attitude, feedback reception,etc.</td></tr>
</table>

Weighting for Key stage 3 (Class VIII)

Strand	Time Allocation (Mins.)	Weighting(%)
Strand A: Numbers and Operations	2150	30
Strand B: Patterns and Algebra	1050	14
Strand C: Measurement	1600	22
Strand D: Geometry	1600	22

Strand E: Data and Probability	800	12

A. Rubrics for Assessing Class Activity

Criteria	Exceeding (5)	Advancing(4)	Meeting (3)	Approaching (2)	Beginning (1)
Understanding of the mathematical concept	Demonstrates a thorough understanding of the mathematical concept being taught and goes beyond class level.	Demonstrates a thorough understanding of the mathematical concept being taught.	Demonstrates a good understanding of the mathematical concept being taught.	Demonstrates a partial understanding of the mathematical concept being taught.	Demonstrates little or no understanding of the mathematical concept being taught.
Problem-solving skills	Applies the mathematical concept to solve problems accurately and efficiently. Displays the ability to connect mathematical concepts to real world problems.	Applies the mathematical concept to solve problems accurately and efficiently. Struggles to connect mathematical concepts to real world problems.	Applies the mathematical concept to solve problems accurately.	Struggles in applying the mathematical concept to solve problems	Unable to apply the mathematical concept to solve problems

Communication	Communicates mathematical ideas clearly and effectively using appropriate mathematical language and symbols and brings new issues for discussions.	Communicates mathematical ideas clearly and effectively using appropriate mathematical language and symbols.	Communicates mathematical ideas effectively but may have some errors in mathematical language or symbols.	Has difficulty communicating mathematical ideas effectively using mathematical language and symbols.	Does not communicate mathematical ideas effectively using mathematical language and symbols.
Collaboration	Works effectively with classmates to complete the activity and contributes significantly to the group's success. Goes beyond to help peers during classroom learning.	Works effectively with classmates to complete the activity and contributes significantly to the group's success.	Works collaboratively and contributes to the group's success, but may have some difficulty working effectively within the group.	Works collaboratively but does not contribute significantly to the group's success.	Does not work effectively with classmates and does not contribute significantly to the group's success.
Neatness and organisation	Work is neat, organised, easy to read and have footnotes and comments	Work is neat, organised, and easy to read.	Work is mostly neat, organised, and easy to read.	Work is somewhat messy or disorganised, but still readable.	Work is messy or disorganised and difficult to read.

B. Rubrics for Assessing Homework

Criteria	Exceeding (5)	Advancing (4)	Meeting (3)	Approaching (2)	Beginning (1)
Understanding	→ Demonstrates a deep and thorough understanding of the homework assigned. → Consistently applies knowledge to solve problems.	→ Shows a good understanding of the homework concepts. → Applies knowledge effectively in most situations.	→ Demonstrates a basic understanding of the homework concepts. → Struggles with consistent application.	→ LLimited understanding of the homework concepts. → Inconsistently applies knowledge.	→ Minimal understanding. Unable to apply knowledge effectively.
Completion	→ All homework are completed accurately and thoroughly.- → Consistently submits high-quality work.	→ Most homework tasks are completed accurately and thoroughly. → Few minor errors present.	→ Some homework tasks are completed accurately, but there are notable gaps. → Several errors are present.	→ Numerous incomplete or inaccurately completed homework tasks. → Completion is inconsistent.	→ Virtually all homework tasks are incomplete or inaccurately completed.
Accuracy of response	→ All calculations and solutions are accurate and precise. → Demonstrates meticulous attention to detail.	→ Most calculations and solutions are accurate and precise. → Few minor errors present.	→ Some calculations and solutions are accurate but lack precision. → Several errors are present.	→ Numerous errors in calculations and solutions. → Accuracy and precision are major issues.	→ Virtually all calculations and solutions are incorrect or imprecise.
Neatness and organization	→ Homework is exceptionally well-organized and neatly presented. → All text is highly legible, and there are no smudges or unintended marks. → Clear headings, labels, and steps enhance the overall organization	→ Overall organization is good, with a clear presentation. → Most text is legible, and there are minimal smudges or unintended marks. → Headings, labels, and steps contribute to effective organization.	→ Organization is acceptable but may lack some neatness. → Legibility varies, and there may be occasional smudges or unintended marks. → Clear headings and labels help maintain a basic level of organization	→ Organization is somewhat lacking, and there is some difficulty in following the work. → Legibility issues are noticeable, and there are frequent smudges or unintended marks. → Headings and labels are consistently not clear.	→ Poor organization makes it challenging to follow the homework. → Legibility is compromised, and there are significant smudges or unintended marks throughout. → Chaotic presentation hinders understanding, and headings and labels may be unclear or absent.

Follow up and improvement	→ Actively seeks feedback on homework. → Demonstrates a commitment to improving based on feedback. → Makes corrections and improvements on subsequent submissions.	→ Open to feedback and uses it to make improvements in subsequent homework. → Shows a willingness to learn from mistakes.	→ Occasionally seeks feedback but inconsistently incorporates it into subsequent work. → Limited improvement over time.	→ Rarely seeks feedback and seldom makes improvements. → Little evidence of learning from mistakes.	→ Does not seek feedback or make improvements. → Repeated mistakes persist
Timeline	→ Submits homework/assignments consistently on time.	→ Generally submits homework on time but may occasionally be late.	→ Submits homework somewhat late on a regular basis.	→ Frequently submits homework late.	→ Consistently submits homework/ assignments late.

C. Rubrics for Assessing Project Work

Criteria	Exceeding (4.1-5)	Advancing (3.1-4)	Meeting (2.1-3)	Approaching (1.1-2)	Beginning (0.1-1)
Plan	Plan is detailed with 5 components (schedules, activities, materials requirement, data collection source, representations) and endorsed by teacher	Plan is detailed with 4 components and endorsed by teacher	Plan is detailed with 3 components and endorsed by teacher	Plan is detailed with 2 components and endorsed by teacher	Plan is sketchy without required components and not endorsed by teacher
Problem	Problem is new, meaningful and will have a positive impact on the community.	Problem is new , meaningful but does not have any impact on the community.	Problem is not new but meaningful.	Problem is new but not meaningful.	Problem is not meaningfully stated.
Literature	Literature study is researched and aligns with problem showing deep understanding of the concept.	Literature study is researched and aligns with problem but lacks understanding of concept.	Literature study is well researched but does not align with the problem.	Literature study is not well researched and does not align well with problem.	Literature study is not stated but visible from other part of the project.
Data collection	Data is systematic with collection tools, processes and authentic sources mentioned in the plan.	Data is systematic with collection tools, processes and authentic sources but not mentioned in plan.	Data has collection tools but without logical process and authentic source.	Data collection is not systematic but sources are mentioned.	Data collection is not described and sources are not mentioned.

Data representation and analysis	Data representation is appropriate, neat with topics and legends and analysis is meaningful.	Data representation is appropriate with incomplete components but analysis is meaningful.	Data representation is appropriate but analysis is not meaningful.	Data representation is inappropriate and analysis is not meaningful.	Data representation inappropriate but no analysis.
Findings	Finding aligns as a solution to the problem with strong arguments supported by data.	Finding aligns as a solution to the problem but argument is weak and not supported by data.	Finding is described well but not addressed to solve the problem.	Finding is not well described but aligns as a solution to the problem.	Finding is brief and does not align with the problem.
References	More than five references are cited in APA format and referenced throughout the project.	Four to five references are cited and referenced throughout the project.	Three to two references are cited and referenced throughout the project.	At least one reference is cited and referenced throughout the project.	No references
Deadlines	Meets all deadline as per schedule endorsed	3 days behind schedule	5 days behind schedule	Week behind schedule	More than week behind schedule

D. Planning Chart for project work (Sample)

Sl.	Date	Activity	Recommendation	Remarks
1	1st week of March	Draw project plan	Plans must contain schedules with intended activity spread across the year. Can refer to this calendar as a sample.	
2	2nd week of March.	Identifying a working title	Problem/ issue/ situation/opportunity/ project/new venture that is identified as a working title must be endorsed by the teacher. Only after the approval of Title, students can proceed working on it	
3	3rd and 4th week of march	Literature Review	Referred to different books and online literature, which have links with project titles.	
4	Month of April	Data collection.	Data can be collected on identified topics using various tools (interview, questionnaires, observation, and document) or gather available data from various sources with acknowledgement.	
5	1st Week of May	Data organisation	Organise the collected data in appropriate form.	

6	2^{nd} week of May	Data display (1^{st} Draft).	Represent the organised data using appropriate data display(s): • bar graph • circle graph • histogram • stem and leaf plot • box plot.	
7	3^{rd} week of May	Data display (Final Draft).	Represent data display using relevant software (ICT tools). Subject teacher facilitates the availability of computers and software for data display.	
8	4^{th} week of May	Data analysis, drawing conclusions and making recommendations	Analyse the data display, draw conclusion and make recommendations as per the analysis and conclusion supported by data	
9	1^{st} week of June	Compilation and submission of Project Work.	Compile the work and submit it to the respective subject teacher with proper binding.	
10	2^{nd} week of June	Project Evaluation and Awarding of Marks (maximum 35 marks).		