National School Curriculum

INSTRUCTIONAL GUIDE FOR MATHEMATICS

CLASS: IX-X

School Curriculum Division
Department of School Education
Ministry of Education and Skills Development
Royal Government of Bhutan

“Your parents, relatives, and friends would be very proud of what you have achieved. At your age, to have completed your studies is your personal accomplishment. Your knowledge and capabilities are a great asset for the nation. I congratulate you for your achievements.

Finally, your capabilities and predisposition towards hard work will invariably shape the future of Bhutan. You must work with integrity, you must keep learning, keep working hard, and you must have the audacity to dream big.”

- His Majesty Jigme Khesar Namgyel Wangchuck

National School Curriculum

INSTRUCTIONAL GUIDE FOR MATHEMATICS

CLASSES IX-X

School Curriculum Division
Department of School Education
Ministry of Education and Skills Development
Royal Government of Bhutan

Published by

Department of School Education
Ministry of Education and Skills Development
Royal Government of Bhutan
Thimphu, Bhutan.

Provisional Edition 2021

First Edition 2022

Second Edition 2023

www.education.gov.bt

ISBN 978-99936-0-634-5

Acknowledgements

The School Curriculum Division (SCD) at the Department of School Education (DSE), Ministry of Education and Skill Development (MoESD) would like to acknowledge the contributions of the following professionals and teachers who participated in the process of the development of the Instructional Guide for the implementation of the mathematics Curriculum.

The SCD also sincerely acknowledges the retrieval and use of contents and resources, either in part or whole, from relevant websites and other forms of sources. However, SCD reassures that these resources are used exclusively for educational learning purposes.

Research and writing 2021

1. Mr. Geewanath Sharma, Curriculum Developer, REC
2. Mr. Tashi Dendup, Curriculum Developer, REC
3. Mr. Kinley Wangdi, Drukgyel CS, Paro
4. Mr. Pema Tshering, Bajothang HSS, Wangdue Phodrang
5. Mr. Passang Dorji, Gesarling CS, Dagana
6. Mr. Yeashi Rinzin, Chapcha MSS, Chhukha

Review and writing 2022

1. Mr. Tashi Dendup, Curriculum Developer, DCPD
2. Mr. Geewanath Sharma, Project Officer, BYDF
3. Mr. Kinley Wangdi, Drukgyel HSS, Paro
4. Mr. Norbu Wangdi, Kuzhugchen MSS, Thimphu
5. Mr. Ugyen Kelzang, Gelephu MSS, Sarpang

Review and writing 2023

1. Mr. Tashi Dendup, Curriculum Developer, SCD
2. Mr. Kinley Wangdi, Drukgyel HSS, Paro
3. Mr. Karma Jigme, Dechencholing HSS, Thimphu
4. Mr. Yeashi Rinzin, Chapcha MSS, Chhukha

Advisers

1. Mr. Karma Galay, Director General, DSE

Foreword

COVID-19 has caused unforgiving disruptions in public education all over the world, and brought about threats of fragmentation in the society due to disparities in accessibility and connectivity in many systems. In Bhutan too, continuity of education and learning has been severely affected as a result of sporadic nationwide school closures, restrictions and health protocols. The disruptions exposed the limitation of the existing ideologies and practises in education. This has deprived children living in poverty worldwide, who rely on the physical settings of their schools for educational materials and guidance, of the learning and other essential educational services. Cognizant of the global trend to embrace competency based learning as education for the 21st century, the current priority of the Government is to transform the knowledge and textbook based learning to competency based learning through open source and experiential learning.

In the new normal education, human interaction and well-being is a priority. Technology, particularly digital technology that enables communication, collaboration and learning across distance, is a formidable tool though not a panacea but a source of innovation and expanded potential. As we embrace this exceptional opportunity to transform education, it is imperative to reimagine the organisation of our educational institutions and learning environments. In the post COVID 19 era, we must prioritise the development of the whole person, not just the acquisition of academic knowledge. Inspiration for the change can be drawn from the 1996 Delors report, learning the treasure within. Its four pillars of learning as “learning to know”, “learning to do”, “learning to be”, and “learning to live together” are the current global ethos of teaching and learning. Therefore, curricula must be increasingly perceived as an integrated, themes based and problems based orientation that allows learners to develop a strong base of knowledge about one’s self and about the world, and find purpose in life and be better able to participate in social and political milieu.

The National School Curriculum is, not just a mere response to the pandemic, but also culmination of the curriculum reform work for the last four years by the erstwhile Royal Education Council. It is an attempt to transform education from the teaching of “what” to learning of “how” and “why” towards empowering learners with the transversal competencies and the 21s t century skills, and preparing them to be lifelong learners. In tandem with this initiative, we are optimistic that the paradigm shift in mathematics education orients our education process in empowering the young generation with the mathematics mind-set and disposition, and skills towards nurturing nationally rooted and globally competent citizens.

With this guide, we are optimistic that our learners and teachers are ushered through a life enriching experiential mathematics education.

Tashi Delek

Karma Galay
Director General

Table of Contents

Introduction

The 21st Century Education framework emphasises on the theme-based learning approach that broadens opportunities for experiential learning contextualised to the learner's physical, social, political, economic, spiritual and cultural setting. This requires learning through active engagement of learners. The role of teachers therefore, is transformed from knowledge transmitter to facilitation, guide, evaluator, researcher and motivator.

The conventional education system is predominantly knowledge based and examination centred. This system comprises the development of psychomotor and affective domains of learning thereby affecting the holistic development of students.

Despite the devastating effect caused by COVID-19 pandemic, it presented scopes for creation, innovation, generally perceived as more efficient and effective in work and social activities. The pandemic situation explicated that the old ways of working, teaching and learning, and lifestyle have limitations. Consequently, new normal ways of how we work and live, teach and learn must be critically analysed and embraced.

Therefore, the education system needs to be transformed to meet contemporary requirements. Students should learn to critically filter information that is flooded on the internet. Classroom instruction should facilitate learners to construct knowledge, develop essential skills and values which are crucial for learners to realise their potential towards becoming locally rooted and globally competent citizens who would contribute towards making a just and harmonious society. Accordingly, classroom instruction from teacher centred to learner centred calls for the following adjustment, or even the overhaul of a few practises.

i. Reduction of learning content to facilitate deep learning as opposed to the width of the teaching through the active engagement of students.

ii. Integration of ICT as tools and ends of learner's education. The use of multimedia and ICT software is commonly utilised in teaching and learning as innovation to introduce variation in stimuli and sustain learner's interest and zeal in learning.

iii. Adoption of theme-based learning content, which facilitates to broaden the horizon of learning beyond the four walls, and stimulates the transfer of learnt concepts to the learner's immediate environment. This arrangement makes students aware of the realities of the social, political, economic and cultural practices and ethos of the society. Being aware of the immediate environment of the scopes and challenges, students are sensitised to the opportunities and issues, which may need attention for a better future for the society.

iv. Consideration to ground the curriculum design and instruction approaches the epistemological theories is imperative to facilitate deep learning as opposed to factual learning. However, the selection and use of them is subject to the nature of the respective subject. For instance, constructivism is more apt for science, while connectivism is relevant for languages and ICT curricula.

v. Active engagement of students is imperative for competency-based education and learning. Inevitably, summative assessment has limitations in gauging the progressive development of the learner. This is achieved objectively by the use of the continuous formative assessment (CFA). However, if summative assessment evidence is used to provide feedback to help students in learning, it can serve as one of the techniques of CFA.

Purpose of the Instructional Guide

This instructional guide provides a suggestive direction to the teachers to facilitate them to transform the classroom instruction to the contemporary requirements stated above. The content of the instructions in the guide are aligned with the mathematics curriculum framework with references to the existing textbooks.

The instructional guides are developed to achieve the following objectives:

i. Facilitate learning anywhere, any time with the learner being responsible for the learning.

ii. Facilitate deep learning with awareness and sensitivity of the realities of the world around.

iii. Strengthen competency based learning and experiential learning to foster sensitivity of realities of life and environment.

iv. Strengthen blended learning and flip classroom with multimedia, digital pedagogies and ICT devices and websites as the tools and learning content.

v. Guide parents in facilitating learning of their children.

vi. Inspire teachers to assume the roles of facilitation, guide, motivator and evaluator.

vii. Helps in the prioritisation of learning content with emphasis to create time and space for active engagement of learners. Facilitate the use of CFA for learning through objective observation and guidance.

The effective and efficient use of this guide is subject to the nature of the topic(s) and the target class.

ORGANISATION OF THE INSTRUCTIONAL GUIDE

This is the main part of the instructional guide. It contains suggested approaches to teaching to guide students to achieve the desired competency (ies) through the identified topic(s).

a. Broad theme /Strand/Chapter/Topic

Under this heading, the topic/topics under one strand or under different strands that can be addressed together is/are listed. The topics are taken from the framework and not from the textbooks.

b. Introduction, Utility and Scope

Knowing history, utility and scope of topic(s) or concept (s) may provide an inspiration to learn certain things. It arouses curiosity, interest, and motivates the learners if they know why and how something was invented or discovered. Due to these reasons, the teachers are expected to explore and express the history, utility, and scope of the topic(s). It is advisable to do while introducing the topic or concept because if students know the history and utility of what they learn, learning becomes much more engaging and captivating.

c. Competency (ies)

Under this heading, the main competency (ies) associated with the topic(s) is/are listed.

d. Objective(s)

Lesson objective(s) which are directly linked with the competency is/are given.

e. Essential Skills/Process

When students discuss and do mathematics, they use various skills and at the same time learn new skills. All learning processes are intended to facilitate students to acquire and develop certain skills. Each lesson in this guide contains few suggestive skills for each topic. However, since the list of essential skills for each topic is not exhaustive, teachers may explore helping students to acquire or develop some other essential skills through the topics covered.

f. Learning Experiences

In each lesson, learning experiences linked to the objectives which are further linked to the competency (ies) are suggested. These suggested learning experiences are to provide directions for the teachers to leverage the lesson to enable students to achieve the desired competency (ies). It can be used for both face to face and virtual mode to make the teaching learning more authentic. Both indoor and outdoor learning experiences (experiential learning) with possible cross pollination and thematic approaches are suggested for each lesson.

Teachers are expected to use the approaches of cross pollination (multidisciplinary), thematic (intra and inter topic(s)/concept(s)), place-based education pedagogies, project based learning, and flip classroom to offer greater flexibility and opportunities to enable students to generate new knowledge and associated skills. Students should also be provided opportunities to connect, communicate, and represent mathematical ideas and be given avenues for thinking divergently and reflecting on their learning.

g. Assessment

Achievement of learning shall be recorded based on bands of achievement for all students in all classes from IV to XII. The evidence from assessment is to identify individual learning needs, design, and deliver appropriate interventions to support students falling in the beginning and approaching category and to further provide appropriate support to the ones falling under the higher bands.

Teachers shall assess the objectives in a cumulative manner and ensure that competency (ies) are achieved at the end of the lesson. In each class, marks obtained for each strand from the formative assessment can be converted using appropriate conversions for summative purposes.

$$CA\ Marks = \frac{Sum\ of\ scores\ obtained\ of\ all\ competencies}{Total\ scores\ of\ all\ competencies} \times CA\ for\ the\ term.$$

- Performance Task

Each lesson consists of one or two suggestive performance tasks. The performance tasks are intended to assess the students' achievement of the desired competency(ies) through that particular lesson. Through performance tasks students can demonstrate knowledge and skills that they have acquired or developed, and how they can apply the knowledge and skills in real situations. Teachers should try and design performance tasks where students will be able to use the immediate objects around them to show their ability to transfer/apply the learning and assess higher order thinking.

The performance tasks should be assessed through Performance-based Assessment (PBA) wherein not only knowledge and skills are assessed but the ability to apply the knowledge and skills are also assessed.

- Reflective Questions

  Suggestive reflective questions that can be included in a performance task are mentioned below each performance task. Also, in some cases, reflective questions are mentioned after the completion of the performance tasks. These reflective questions are suggested, mainly, to help students to reflect on their learning and make personal connections to the topics being learnt.

h. Resources

All resources required for the suggested learning experiences and tasks are listed under each lesson both for face to face and virtual mode. Some lengthy worksheets are provided as annexure at the end.

i. Annexure

For the effective implementation of the NSC, certain worksheets are suggested for each lesson and annexed at the end of each lesson. However, teachers are expected to use it or further develop similar authentic worksheets for the teaching learning purposes.

Instructional Guide
Class IX
Mathematics

Topic: IX-A1 Exponent Laws: Integral Exponents [500 minutes]

Introduction

In mathematics, there are different laws of exponents. All the rules of exponents are used to solve many mathematical problems which involve repeated multiplication processes. The laws of exponents simplify the multiplication and division operations and help to solve the problems easily.


Muhammad Musa

The history of exponents or powers is pretty old. In the 9th century, a Persian mathematician Muhammad Musa introduced the square of a number. Later in the 15th century, they introduced a cube of a number. The symbols to represent these indices are different, but the method of calculation was the same. The term 'exponent' was first used in 1544 and the term 'indices' was first used in 1696. In the 17th century, the exponential notation matured and mathematicians all over the world started using them in problems.
History of Exponents and History of Exponents (extension)

Utility and Scope

Scientific field uses a scale, like the pH scale or the Richter scale, where you will find exponents. Both the pH scale and the Richter scale are logarithmic relationships with each whole number representing a ten-fold increase from the number before it. Sometimes scientists must use exceptionally large or small numbers. Scientific notation relies on exponents to write these numbers in a simpler way.

One of the most common real-world applications of exponents involves taking measurements and calculating multi-dimensional quantities. Area is the measure of space in two dimensions (length x width), so you always measure it in square units like square feet or square metres. Similarly, volume is the measure of space in three dimensions (length x width x height), so you always measure it in cubic units like cubic feet or cubic metres.

A. Competency

- Deduce and apply the exponent laws for positive, negative and fractional powers to solve related problems.

B. Objectives

- Demonstrate understanding of the laws of exponents in solving related problems
- Apply the following exponent rules/laws:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad (a^m)^n = a^{mn} \qquad a^0 = 1$$

$$a^{-n} = \frac{1}{a^n} \qquad a^{\frac{1}{2}} = \sqrt{a} \qquad a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

C. Essential Skills/Processes

- Applying
- Analysing
- Reasoning

D. Learning Experiences

- Recap and explain briefly about the three components of a power.
  - The weblink Concept of exponents explains the basic concepts of the exponents: Base and Exponent. It also has opportunities for students to play around with the exponent concepts and the exponent laws.
- Use the video link: Laws of exponents (video) to introduce and demonstrate the laws of exponents by either using examples (inductive approach) or using proofs (deductive approach).
  Refer Inductive and Deductive Reasoning Sample under the annexure:
  - Product Law: $a^m \times a^n = a^{m+n}$
    Use the pre-knowledge of multiplication operations on numbers to demonstrate the product law with adequate examples.
  - Quotient Law: $a^m \div a^n = a^{m-n}$

Use the pre-knowledge of division operations on numbers to demonstrate the quotient law with adequate examples.

- Power of product law: $(ab)^n = a^n b^n$

  Use the pre-knowledge of exponents and multiplication operations on numbers to demonstrate the power of product law with adequate examples.

- Power of quotient law: $(\frac{a}{b})^n = \frac{a^n}{b^n}$

  Use the pre-knowledge of exponents and division operations on numbers to demonstrate the power of quotient law with adequate examples.

- Zero and Negative Exponents: $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$

  Use the pre-knowledge of exponents and division operations on numbers to demonstrate the zero exponents with adequate examples.

  Refer 'Inductive and Deductive Reasoning Sample' following the annexure for demonstrating the Negative Exponents using both Inductive and Deductive approach.
  Refer the video lesson: Zero and Negative Exponents

- Fractional Exponents: $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

  Revise the concept and meaning of square roots and cube roots and relate to the concept of fractional exponents to deduce the general fractional exponents using the power law concepts.
  Refer to the weblink: Fractional Exponents to understand and get more ideas to teach the concept of fractional exponents.

E. Assessment

Performance Task 1

Design and carry out learning activities to:

- express powers using the Product and Quotient laws.
- compare and order exponential quantities.
- solve equations involving whole numbers, negative, zero and fractional exponents.

  Refer Understanding mathematics IX textbook for developing relevant activities.

Performance Task 2

Explore real life applications of exponents in measurements of very large or small quantities around us (E.g., light year, mass of an electron etc.). (*Note on Assessment tools and templates*)

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

Number Match Game:
To play this game you first need to create ten pairs of matching cards. One number in each pair is one of the numbers below and the other is a calculation that results

Number Cards

$\pi - 2$	$6\sqrt{29}$	$-3\frac{2}{5}$	4.1×10^{4}	3.24×10^{-3}
10π	$8(\sqrt{2} + \sqrt{3})$	-8.9	$5^{\frac{2}{3}}$	42.7×10^{-1}

in that number (see the instructions below for making the other cards).
Instructions for creating the matching cards:

1. at least two cards must include negative exponents
2. at least two cards must include the exponent 0
3. at least two cards must use a fractional exponent
4. at least two cards must include three operations (three of +, –, ×, and ÷)
5. at least two cards must involve a square root
6. at least two cards must include both rational and irrational numbers,
7. at least two cards must require the use of an exponent law

Note that some cards will meet several of the conditions above
How to play the game:

- Play with a partner. Shuffle all 20 cards. Place them face down on a table and make four rows of five cards.
- Take turns. On your turn, turn two cards over. If they match, keep the cards and take another turn. If they do not match, turn them face down again. A player may take no more than two turns in a row.
- Once all the cards have been removed, the player with the most cards wins.

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics
- Introduction to Exponent Laws - History of Exponents and History of Exponents (extension)
- Laws of Exponents - Concept of exponents
- Law of Exponents (mathsAntics) - Laws of exponents (video)
- Zero and Negative Exponents - Zero and Negative Exponents
- Fractional Exponents - Fractional Exponents
- Exponential Expressions - Exponential Expressions

G. Annexure

i. Sample Assessment ToolTemplate to record assessment

CATEGORY	Beginning	Approaching	Meeting	Advancing	Exceeding
Mathematical Concepts	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written.	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows complete understanding of the mathematical concepts and is able to synthesise the idea in new problem situations.
Mathematical Errors	More than 75% of the steps and solutions have mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	90-100% of the steps and solutions have no mathematical errors.	Exhibits no mathematical error and is able to identify and correct errors of peers.
Mathematical Terminology and Notation	There is little use, or a lot of inappropriate use, of terminology and notation.	Correct terminology and notation are used, but it is sometimes not easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are always used, and are able to derive meaning and justify.

Neatness and organisation	The work appears sloppy and unorganised. It is hard to know what information goes together.	The work is presented in an organised fashion but may be hard to read at times.	The work is presented in a neat and organised fashion that is usually easy to read.	The work is presented in a neat, clear, organised fashion that is easy to read.	The work is presented in a neat, clear, organised fashion with key notes for self-reflection (underlining key points/ highlighting key points)
Completion	75% of the problems are not completed.	50% of the problems are not completed.	25% of the problems are not completed.	All problems are completed.	All problems are completed and explored related extended activities.
Team Work	Student did not work effectively with others.	Student cooperated with others, but needed prompting to stay on-task.	Student was an engaged partner but had trouble listening to others and/or working cooperatively.	Student was an engaged partner, listening to suggestions of others and observed working cooperatively throughout lesson.	Student consistently demonstrated exceptional collaboration and cooperation, actively contributing to the teams' success.

Strand(s): IX-A1	**Topic(s): Exponent Laws**				
Competency: Deduce and apply the exponent laws for positive, negative and fractional powers to solve related problems.					
Name of the student	Level of achievement				
	Beginning	Approaching	Meeting	Advancing	Exceeding

ii. Inductive and Deductive Reasoning Sample:

- Deductive Reasoning is used to *prove* that something is always true using information, knowledge and established facts

  For example: To show the value of Negative Deductively;

  Let 'a' be any number with a negative exponent of 'n'. We can use the interpretation of the *quotient law* and *zero exponent* to determine the value of a power with negative exponent deductively as follows:

  $a^{-n} = a^{0-n} = a^{0} \div a^{n} = 1 \div a^{n} = \frac{1}{a^{n}}$ Therefore: $a^{-n} = \frac{1}{a^{n}}$

- Inductive Reasoning is used to *verify* (but not *prove*) that something is true using examples and evaluating their general pattern.

  For example: To show the value of Negative Exponents inductively;
  let's observe the given set of examples and its pattern:

Power	3^{3}	3^{2}	3^{1}	3^{0}	3^{-1}	3^{-2}
Value	27	9	3	1	$\frac{1}{3}$	?

Observe that each value in the right-hand column is one third (divided by the base number) of the value above it. So, if you extend the pattern to 3^{-1} and 3^{-2},

3^{-1} is $\frac{1}{3}$ and 3^{-2} is $\frac{1}{9}$ or $\frac{1}{3^{2}}$

Therefore, it looks like $3^{-n} = \frac{1}{3^{n}}$ and, for a general base *'a'*: $a^{-n} = \frac{1}{a^{n}}$

For more information and examples of Inductive and Deductive reasonings refer Reasoning in Understanding mathematics textbook for Class X or other online resources.

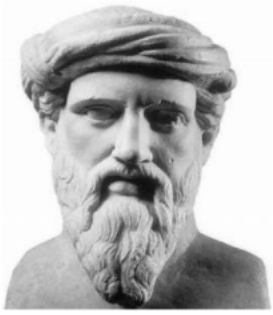
Topic: IX-A2 Rational, Irrational and Real Numbers [450 minutes]

Introduction

A rational number is a number that can be expressed as a quotient of two integers. All real numbers which are positive or negative are rational numbers. A number which is not rational is referred to as irrational. An irrational number is a real number that cannot be expressed as a ratio of integers, for example, 2 is an irrational number. Again, the decimal expansion of an irrational number is neither terminating nor recurring. Real numbers are simply the combination of rational and irrational numbers, in the number system.

Hippassus of Metapontum

Hippassus of Metapontum, a Greek philosopher of the Pythagorean school of thought, is widely regarded as the first person to recognize the existence of irrational numbers. Supposedly, he tried to use his teacher's famous theorem $a^2 + b^2 = c^2$ to find the length of the diagonal of a unit square. This revealed that a square's sides are incommensurable with its diagonal, and this length cannot be expressed as the ratio of two integers. Since the other Pythagoreans believed that only positive rational numbers could exist, what happened next has been the subject of speculation for centuries. In short, Hippassus may have died because of his discovery.



Pythagoras

So, what happened to Hippassus? No one will probably ever know for sure, but below are some better-known stories.

- Some believe that the Pythagoreans were so horrified by the idea of incommensurability that they threw Hippassus overboard on a sea voyage and vowed to keep the existence of irrational numbers a secret.
- Hippassus discovered irrational numbers, the Pythagoreans ostracised him, and the gods were so disgusted by his discovery that they scuttled his boat on the high seas.
- Hippassus discovered irrational numbers, and then died on an ocean voyage as the result of a natural accident (the sea is a treacherous place). Nonetheless, his

colleagues were still so displeased with his discovery that they wished they had been the ones to throw him overboard.

Another possibility is that none of the stories above are true, and they are tales invented and embellished through the ages to illustrate a pivotal moment in history.

History of Irrational Numbers and https://www.youtube.com/watch?v=sbGjr_awePE

Utility and Scope

Money: Irrational numbers are used for calculating the compound interest on loans, where the sum of infinite series is used.

Construction: In constructions where there is a need to build structures that are cylindrical in shape, irrational numbers can be used to calculate the structure using pi. Also, the circumference of any circular object is calculated with the help of irrational numbers.

Design and Engineering: The concept of '*e*' or Euler's number is quite popular but these components are not used directly but have indirect applications in fields like engineering and design. It is also used for the processing of signals, calculations, speedometres, and uses this concept. Apart from these, irrational numbers have many other indirect uses in our real life.

Imaginary numbers are used in real-life applications, such as electricity, as well as quadratic equations. In quadratic planes, imaginary numbers show up in equations that don't touch the x axis. Imaginary numbers become particularly useful in advanced calculus.

5 Uses of Irrational Numbers in our Daily Life

A. Competency

- Evaluate the meaning of real numbers through their representations and operations to apply in our everyday understanding of numbers.

B. Objectives

- Justify if a given number is a rational or irrational number.
- Place irrational numbers on a number line relative to known rational numbers.

- Demonstrate that the set of real numbers include both rational and irrational numbers.
- Apply knowledge of order of operations on rational numbers.

C. Essential Skills/Processes

- Applying
- Evaluating
- Reasoning

D. Learning Experiences

- Introduce the meaning of what rational and irrational numbers are and discuss the differences and similarities between them and relate them to real numbers.
  - The link: https://youtu.be/cLP7INqs3JM?s contains a video that explains the concept of rational and irrational numbers, shows different forms of representing the rational number and also explains the differences and similarities of rational and irrational numbers.
  - Explain how to classify rational and irrational numbers with the help of some examples. Suggested link (https://youtu.be/classification)

- Discuss the number system and let the students discover that all rational and irrational numbers belong to real numbers. Refer the given link: (https://youtu.be/VEY08fMY)

- Demonstrate an understanding of placing the real numbers on the number line with the help of examples.
  Refer 'Representing Real Numbers' in Understanding mathematics IX textbook.
  Suggested link: (https://youtu.be/6) and (https://www.mathssopenref.com).

- Assess the prior knowledge on application of order of operations with integers. Design an activity about operating with rational and irrational numbers for the students to practise and to test their understanding on the application of order of operations with irrational and rational numbers. Refer 'Order of Operations' in Understanding mathematics Textbook IX to select and assign the related questions. Suggested link: maths Antics - Order Of Operations - YouTube.

E. Assessment

Performance Task 1

Work out an activity to classify a variety of numbers into rational and irrational groups.
Refer 'Representing Real Numbers' in Understanding mathematics Textbook IX to select the related questions. Or use link Properties of Rational Numbers vs Irrational.

Performance Task 2

Carry out group activity on locating irrational and rational numbers on a number line. Refer 'Representing Real Numbers' in Understanding mathematics Textbook IX to select the related questions.

Performance Task 3

practise and test their understanding on the application of order of operations. Refer practising and applying activities on 'Order of Operations' in Understanding of mathematics Textbook IX assign related questions.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions
  Classify the numbers as rational or irrational numbers and justify.

Number	Type and Explanation
π	Irrational; result in non-repeating, non-terminating decimal.
$\frac{22}{7}$	Rational; result in repeating decimal.
$\sqrt{3}$	Irrational; result in non-repeating, non-terminating decimal.

0.3	Rational; terminating decimal and can be represented in $\frac{a}{b}$ form.
1.010010001….	Irrational; non-repeating, non-terminating decimal.
$\frac{6}{3}$	Rational; result in whole number.

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics textbook for Class IX
- National School Curriculum Framework for mathematics
- Introduction to rational and irrational numbers - Introduction to rational and irrational numbers
- Introduction to Real Numbers - History of Irrational Numbers and Making sense of irrational numbers
- Utility and Scope - 5 Uses of Irrational Numbers in our Daily Life
- Rational and Irrational Numbers Introduction to Rational and Irrational
- Classification of Rational and Irrational Numbers- Classification of Rational and Irrational.
- Displaying Real Numbers - https://youtu.be/VEY08fMYmEU?si
- Estimating values of Irrational Numbers - https://youtu.be/6htWeCE2wro
- Number line - https://www.mathssopenref.com
- Properties of Rational and Irrational Numbers - Properties of Rational Numbers vs Irrational
- Order of operations - maths Antics - Order Of Operations

G. Annexure

Refer IX-A1 for template to record achievements

Topic: IX-A3 Set Theory [250 minutes]

Introduction


Richard Dedekind


Georg Cantor

The field of "set theory" is a branch of mathematics that is concerned with describing collections of objects. Set theory is fundamental to probability theory, which is the cornerstone of the field of Statistics. Thus, we need to understand some basic set theory as a prerequisite to understanding probability and statistics.

The modern study of set theory was initiated by the German mathematicians Richard Dedekind and Georg Cantor in the 1870s. In particular, Georg Cantor is commonly considered the founder of set theory. The non-formalized systems investigated during this early stage go under the name of naive set theory.
Set theory - Wikipedia.

Utility and Scope

Applications of Set theory are used throughout mathematics. It is used as a foundation for many subfields of mathematics. In the areas pertaining to statistics, it is particularly used in probability. Much of the concepts in probability are derived from the consequences of set theory.

Because of its very general or abstract nature, set theory has many applications in other branches of mathematics. In the branch called analysis, of which differential and integral calculus are important parts, an understanding of limit points and what is meant by the continuity of a function are based on set theory.

A. Competency

- Demonstrate understanding of sets through their representations and operations to apply in sorting of data in real life.

B. Objectives

- Use set language and notation to describe sets of numbers, shapes and objects, etc.

- Represent sets using Venn diagrams and carry out different operations on sets (union, intersection and complementary).
- Apply set theory to solve problems in real life context.

C. Essential Skills/Processes

- Applying
- Classifying
- Representing

D. Learning Experiences

- Define sets and types of sets (Use the idea of Flipped Classroom approach to understanding Sets).
  - Day prior to the class: Provide the students with the introductory notes and resourceful links on 'Sets' and ask them to study. Refer to the web link: Introduction.
  - During face to face teaching: Ask some questions to check students' understanding on sets. Discuss the definition of sets: "A set is a collection of well-defined objects or things." E.g. Set of even numbers, set of odd numbers, set of number cards, set of fruits, set of vegetables, etc. Each item of a set is called an element.
  - Notations: Explain that the set is usually denoted by a capital letter. E.g. A set of even numbers can be denoted as: E= {2, 4, 6, 8, ...}

    Teach the following concepts with some examples:

    Cardinality of sets: The number of elements in a set.

    Denotation: n(A) or |A|.

    e.g. If A={3,6,9,12,15}, then n(A)=5.

    If A={3,6,9,12,15,.....}, then n(A)= infinity or ∞.

  - Types of sets and denotations: You should explain the following types of sets with relevant examples:
    - Finite set

        - Infinite set
        - Universal set
    - Universal sets are generally denoted by U or S or (Xi). Use to avoid confusion with the symbol of union (U).
        - Subset
        - Proper subset
        - Empty set or null set: If set A is an empty set, then it can be denoted as A={ } or simply, A= ∅
        - Equal and Equivalent sets
        - Overlapping set
        - Disjoint set

    Refer to this web link: Types of sets to know more about the types of sets and their denotations.

- Design an appropriate set of questions for students to practise.
- Represent sets using Venn diagrams and carry out different operations on sets (union, intersection and complementary).

  Suggestions
    - Introduce the concept of Venn Diagram - an illustration that uses circles to show relationships among things or a finite set of things.
    - Explain the following concepts and notations:
        - Union Set (OR) - A set formed by combining two or more sets (This OR That). Denoted by A∪B.
        - Intersection Set (AND/BOTH/ALL) -A set formed by the elements that are common in the given sets (This AND That). Denoted by A∩ B.
        - Complementary set - Complementary set of set A is a set formed by those elements which are in the universal set, but not in the set A. Denoted by A' or A^c.
    - Refer to the additional notes provided in the annexure to understand more on the operations on set.

- Apply set theory to solve problems in context
  - Demonstrate operations on sets with the help of some examples. Refer the web link: Set operations and Set operations, Video.
  - Develop some questions related to the operations on set theory involving intersection, union and complementary sets.

E. Assessment

Performance Task 1

Solve questions to check the understanding of sets and the types of sets from the flipped classroom approach. Refer to the sample practise questions provided in the annexure IX-A3 (practise questions on sets) questions I and II.

Performance Task 2

Carry out group activity on locating irrational and rational numbers on a number line. Refer 'Representing Real Numbers' in Understanding mathematics Textbook IX to select the related questions.

Performance Task 3

Solve questions to assess the understanding of operations on sets. Refer to the sample practice questions provided in the annexure IX-A3 (practise questions on sets) questions III.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

  S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18}
  A= {2, 4, 6, 8, 10, 12, 14,16, 18}
  B= {3, 6, 9, 12, 15, 18}
  C= {1, 3, 5, 7, 9, 11, 13, 15, 17}
  Let students solve the following questions based on the above sets.

1. A∪B	2. A∪C	3. B∪C	4. A∩ B
5. A∩C	6. B∩C	7. A∪B∪C	8. A∩B∪C
9. A'∪B	10. (A∩B)'	11. (B∪C)'	12. B∩C'

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics textbook for Class IX
- National School Curriculum Framework for mathematics
- Introduction to Set Theory -Set theory - Wikipedia
- Introduction to Sets - Introduction.
- Types of Sets - Types of sets.
- Set Operations - Set operations and Set operations, Video.

G. Annexure

i. Refer IX-A1 for template to record achievements

ii. Operations on Sets (Intersection, Union and Complementary sets):

Universal set: Any set which is a superset of all the sets under consideration is said to be universal set and it is denoted by $\cup$ or Xi (ξ)

Operations on sets

Intersection ($\cap$)/Union ($\cup$) set: The **intersection** of two sets A and B is the set which consists of all those elements which belong to both A and B.
Symbolically, we write $A \cap B = \{x: x \in A \text{ and } x \in B\}$

The **union** of any two given sets A and B consists of all those elements which are either in A or in B.
In symbols, we write $A \cup B = \{x: x \in A \text{ or } x \in B\}$

Example: Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $B = \{2, 4, 6, 8, 10\}$
$A \cap B = \{2, 4, 6, 8, 10\}$
$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Complement set: The complement of A is a set of those elements which are in the universal set but which are not in A.
It is denoted by A'.
Example: Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $B = \{2, 4, 6, 8, 10, 12\}$
Complement of B = $B' = \{1, 3, 5, 7, 9\}$

Disjoint set: Two sets which have no elements in common.
That is $A \cap B = \{\ \}$.

Difference set: The difference of two sets, A – B, is the set of elements belonging to set A and not to set B:
$A - B = \{x: x \in A \text{ and } x \notin B\}$

iii. Practise Questions:

Practice

I. *Read each question below. Select the best answer from the four possible choices.*

1) G = {liquids, solid, gases}
   What type of set is G?
   a) Empty set
   b) Finite set
   c) Infinite set
   d) Undefined set

2) H = {... -2, -1, 0, 1, 2, ...}
   What type of set is H?
   a) Empty set
   b) Finite set
   c) Infinite set
   d) Universal set

3) Which of the following set(s) is/are finite set?
   a) {vowels}
   b) {days of the week}
   c) {even numbers less than 10}
   d) All of the above

4) Which of the following is an empty set?
   a) {tiger, lion, leopard}
   b) {eighth day in a week}
   c) {prime numbers}
   d) {dzongkhag *tshechus*}

5) If A = {2, 4, 6, 8, 10} and B = {a, e, i, o, u}.
   Which of the following is correct?
   a) A is subset of B
   b) A is equal to B
   c) A is equivalent to B
   d) A = B'

II. If A = {set of natural numbers}
B = {2, 4, 6, 8,10}
C = {1, 3, 5, 7, 9}
D = {Multiples of 2 less than 12}
Identify the following based on the above sets;
a) universal set (ξ)
b) equal sets
c) equivalent set
d) finite set
e) infinite set
f) subset of A

III. If
ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {2, 3, 5, 7}, B = {1, 3, 5, 7, 9}
List the following sets:
a) $(A \cup B)$ b) $(A \cap B)$
c) $A \cap B'$ d) $(A \cap B)'$

Topic: IX-A4 Commercial Mathematics [600 minutes]

Introduction

Commercial maths, also known as business mathematics or applied mathematics in a business context, involves the application of mathematical techniques to solve practical problems in the business and financial world. It plays a crucial role in various aspects of commerce, finance, and economics, providing tools and methods to analyse, interpret, and make informed decisions.

Financial literacy is the ability to understand and effectively use various financial skills, including personal financial management, budgeting and investing. Financial literacy is the foundation of your relationship with money, and it is a lifelong journey of learning. The earlier you start, the better off you will be because education is the key to success when it comes to money.

Becoming financially literate involves learning and practising a variety of skills related to budgeting, managing and paying off debts, and understanding credit and investment products. Basic steps to improve your personal finances include creating a budget, keeping track of expenses, being diligent about timely payments, being prudent about saving money, periodically checking your credit report, and investing for your future.
Financial Literacy Definition (investopedia.com)

Utility and Scope

- The term financial literacy refers to a variety of important financial skills and concepts.
- People who are financially literate are generally less vulnerable to debts or bankruptcy.
- Understanding and applying these mathematical concepts are crucial for individuals involved in business, finance, economics, and related fields. Commercial maths provides the analytical tools necessary for making sound financial decisions and managing resources effectively in the business world.

A. Competency

- Use ideas of income to calculate taxation and evaluate interests as a way of saving money.

- Solve problems related to day to day buying and selling situations to evaluate purchasing decisions.

B. Objectives

- Explain income and explore various ways to earn money.
- Estimate and calculate deductions from income.
- Estimate and calculate taxes on income using a tax slab.
- Calculate simple interest, rates, time, principal and amount.
- Solve problems involving purchases using the idea of percentage.

C. Essential Skills/Processes

- Reasoning
- Applying
- Analysing

D. Learning Experiences

- Explain income and expenditure.
  - Explain the concept of income and expenditure by citing examples. (Refer Income and Expenditure, Understanding mathematics textbook for class IX).

- Explore and discuss various ways to earn money.
  - Let students explore various ways to earn money. One way to earn money is by saving it in banks to earn interest.
  - Facilitate experiential learning opportunities for the students through community as a classroom approach. Conduct a simple data collection on ways to earn money, income/expenses from shops/offices in the community. Use the collected data to represent, analyse and interpret their findings (Use MS Excel).

- Estimate and calculate deductions from income.
  - Explain the meaning of deductions and demonstrate how to calculate deductions from various income sources (refer Income Deductions, Understanding mathematics textbook for class IX).

- Estimate and calculate taxes on income using a tax slab.

- Explain the meaning of tax deducted at source (TDS) and demonstrate how to calculate personal income tax (PIT). Suggestion: Provide the latest tax slab. (Refer Reporting Income and Taxes, Understanding mathematics textbook for Class IX).

- Calculate simple interest, rates, time, principal and amount
  - The web link: Introduction to Simple Interest contains videos which can help to introduce simple interests.
  - Contextualise the problem to find the simple interest after introducing the formula. For example, it could be the loan borrowed by the teacher or the loan borrowed by the student's parent. Use the link Simple Interest Calculation on simple interest calculations.
  - Show to find simple interest for the interest charged for less than one year (Refer Simple Interest from Understanding mathematics textbook for Class VIII).
  - Let students explore in their groups to find the rate/time/principal given amount of simple interest and any two using inverse operation. For example, $I = Prt,\ t = \frac{I}{Pr}$

- Recall the concept of consumer problems (markup, mark down, cost price, selling price and commission). Discuss terms related to Purchasing Decisions (Refer Purchasing Decisions in Understanding mathematics textbook for Class X or other relevant resources). Refer to the link Key stage 4 - Purchasing Decision on purchasing decision.

E. Assessment

Performance Task 1

Solve a set of questions based on the competency to calculate deductions on income from various sources (Refer questions from Reporting Income and Taxes, and Income Deductions from Understanding mathematics textbook for class IX or from other resources).

Performance Task 2

Solve questions related to Purchasing Decisions (markup, mark down, cost price, selling price and commission). Refer to purchasing Decision in Understanding mathematics textbook for X and other relevant resources.

F. Resources

- Understanding mathematics Textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics
- Introduction to Household Finance -Financial Literacy Definition (investopedia.com)
- Simple Interest - Introduction to Simple Interest
- Simple Interest calculation - Simple Interest Calculation
- Purchasing Decision - Key stage 4 - Purchasing decision

G. Annexure

i. Refer IX-A1 for template to record achievements

Topic: IX-B1 Polynomial Expressions [600 minutes]

Introduction

A polynomial is an expression containing constants and variables connected only through basic operations of algebra. A polynomial is a finite expression constructed from variables and constants, using the operations of addition, subtraction, multiplication, and taking non-negative integer powers.

Determining the roots of polynomials, or "solving algebraic equations", is among the oldest problems in mathematics. However, the elegant and practical notation we use today only developed beginning in the 15th century. Before that, equations were written out in words. For example, an algebra problem from the Chinese Arithmetic in Nine Sections, circa 200 BCE, begins "Three sheaves of good crop, two sheaves of mediocre crop, and one sheaf of bad crop are sold for 29 dou." We would write; $3x + 2y + z = 29$.
Polynomials | CK-12 Foundation (ck12.org) / History - Polynomials (weebly.com)

Utility and Scope

In the real world, algebra and calculus concepts are essential to career paths in the areas of construction, architecture, aerospace and financial planning.

The most obvious of these are mathematicians, but they can also be used in fields ranging from construction to meteorology. real_world_~_polynomials.pdf (weebly.com)

A. Competency

- Demonstrate understanding of polynomials through their classifications and representations to operate polynomials pictorially and symbolically.

B. Objectives

- Interpret and classify different types of polynomials.
- Add and subtract polynomials pictorially and symbolically.
- Multiply pictorially and symbolically: a polynomial by a monomial, multiply a binomial by a binomial and a monomial by a scalar, a polynomial by a scalar.
- Divide pictorially and symbolically: monomial by a monomial; polynomial by a

scalar; polynomial by a monomial.

C. Essential Skills/Processes

- Applying
- Analysing
- Comprehending
- Understanding

D. Learning Experiences

- Classify different types of polynomials and identify their applications.
  - o Define polynomial, degree of a polynomial and terms. Explain the types of polynomials.

  Suggestion:
  - o The web link https://byjus.com/mathsss/polynomial/ and concept contains concepts on definition of polynomials, degree of polynomial, terms and types of polynomials (monomial, binomial and trinomial).

- Add and subtract polynomials pictorially and symbolically.
  - o Demonstrate addition and subtraction of polynomials pictorially
  - o Introduce the concept of 'Like terms' and 'Unlike terms' to add and subtract polynomials symbolically. Use the link Polynomial Tiles to add and subtract polynomials pictorially.

  Suggestion:
  - o The web link Key Stage 4 : Operations on Linear Polynomials - YouTube contains a video tutorial on addition and subtraction of polynomials pictorially and symbolically.
  - o Design polynomial tiles and demonstrate concrete ideas of addition and subtraction of polynomials.

- Multiply pictorially and symbolically using area model: a polynomial by a monomial, multiply a binomial by a binomial, a monomial by a scalar and a polynomial by a scalar.

- Multiplying polynomials symbolically.

  Suggestion:

- Refer Multiplying a Polynomial by a Monomial, Multiplying a Binomial by a Binomial, and Multiplying Polynomials Symbolically from Understanding mathematics textbook for Class IX or the link Multiplying Polynomials | CK-12 Foundation (ck12.org).

- Divide pictorially, and symbolically using area model: monomial by a monomial; polynomial by a scalar; polynomial by a monomial; polynomial by a binomial.
  Suggestion:
  - Refer Dividing a Polynomial by a Monomial, Dividing a Polynomial by a Binomial from Understanding Mathematics textbook for Class IX or the links Dividing Polynomials | CK-12 Foundation (ck12.org) / Division of Polynomials | CK-12 Foundation (ck12.org).

E. Assessment

Performance Task 1

Classify and apply polynomials in terms of the number of terms and the highest power of the terms.
Suggestion:
- Design online quiz using multiple choice questions to evaluate learnt concepts on degree and types of polynomials.

Performance Task 2

Solve a set of questions based on competency to add and subtract polynomials using various methods (pictorially and symbolically). Refer questions from Adding and Subtracting Polynomials, Understanding mathematics textbook for Class IX or from other relevant resources.

Performance Task 3

Solve a set of questions related to competency to multiply and divide (pictorially and symbolically). Refer questions from Multiplying Polynomials, Multiplying Polynomials Symbolically and Dividing Polynomials from Understanding mathematics textbook for Class IX or from the links provided in the resources.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions
  i. For each question, use at least five positive and five negative tiles. Each time, sketch your model and write an algebraic equation to describe your calculation.
     a. Model and complete an addition.
     b. Model and complete a subtraction.
     c. Model and complete a multiplication by 3x + 2.
     d. Model and complete a division by y.
     e. Model and complete a factoring.

  ii. For each question, the answer must be $2x^2 + x - 1$. Each time, sketch your model and write an algebraic equation to describe your calculation.
     a. Add two polynomials that are modelled with 12 tiles altogether.
     b. Subtract two polynomials that are modelled with more than 10 tiles altogether.
     c. Multiply two polynomials.

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics
- Introduction to Polynomials - Polynomials | CK-12 Foundation (ck12.org)/ History - Polynomials (weebly.com)
- Utility and Scope -real_world_~_polynomials.pdf (weebly.com)
- Polynomials - https://byjus.com/mathsss/polynomial/
- Polynomials in Standard form - Write and Classify Polynomials in Standard Form
- Operations on Linear Polynomials - mathematics Key Stage 4 : Operations on Linear Polynomials (1/2) - YouTube
- Operations on linear polynomials using polynomial tiles. Polynomial Tiles
- Multiplying Polynomials - Multiplying Polynomials | CK-12 Foundation (ck12.org)
- Dividing Polynomials (tabular method) - Dividing Polynomials | CK-12 Foundation (ck12.org)
- Dividing Polynomials (long method) - Division of Polynomials | CK-12 Foundation (ck12.org

G. Annexure

i. Refer IX-A1 for template to record achievements

ii. Chart:

Polynomial	Degree	Number of terms	Type of Polynomial
Eg: $3x^2 - 1$	2	2	Binomial

Topic: IX-B2 Linear and Non-Linear relations and Graphs [650 minutes]

Introduction

A straight line graph shows a linear relationship, where one variable changes by consistent amounts as you increase the other variable. A curve graph shows a nonlinear relationship, where one variable changes by inconsistent amounts as you increase the other variable.

Utility and Scope

The following are some of the examples in which applications of linear equations are used in real life:

- It can be used to solve age related problems.
- It is used to calculate speed, distance and time of a moving object.
- Geometry related problems can be solved.
- It is used to calculate money and percentage related problems.
- Work, time and wages problems can be solved.
- Problems based on force and pressure can be solved. Application of Linear Equations

A. Competencies

- Classify linear from non-linear relations and demonstrate the properties of linear relations in its two forms through a table of values, graphical representations, and equations.
- Evaluate the nature of data (discrete/continuous) from a scatter plot and use interpolation/extrapolation to draw conclusions and predict trends.

B. Objectives

- Describe verbally, symbolically, patterns given in tables, charts, pictures, and by problem situations.
- Explain why the data in a table represents a linear, quadratic, or exponential relationship.
- Derive the formula of slope from the graph as rise/run (rise the vertical change, and run the horizontal change) and relate the y-intercept to the value of the y-coordinate where the graph crosses the y-axis.

- Determine the slope and y-intercept by examining a table, graph, or equations.
- Determine the two forms of linear equations: (y = mx+b) given the slope (m) and y-intercept(b), and standard form (ax + by = c).
- Distinguish between independent and dependent variables in a scatter plot to identify if the data represented are continuous or discrete.
- Demonstrate how interpolation and extrapolation using a line of best fit is meaningful for a given set of real life data.

C. Essential Skills/Processes

- Applying
- Analysing
- Modelling
- Representing

D. Learning Experiences

- Ask students to predict the missing values from the pattern provided to them, encourage students to predict the next figures without drawing and ask them to justify their prediction.
  - Let the students describe the patterns symbolically. Refer Linear and Non-Linear Relation Graphs from Understanding mathematics textbook for Class IX.
  - Get the students to identify the type of relationships presented in tables, graphs and equations. This link: Identifying relations contains a video explaining if a function is Linear, Quadratic or Exponential from a table of values (by finding first and second differences). Further refer Linear and Non-Linear Relation Graphs from Understanding mathematics Textbook for Class IX.

- Graphing and determining the two forms of linear relations:
  - Explain the meaning of slope and y-intercept (y = m*x* +b) where *b* -starting point/origin of the line and *m*- rise/run or $\frac{change\ in\ value\ of\ y}{change\ in\ value\ of\ \ x}$.
  - Demonstrate how to graph linear relations given in slope and y-intercept form and standard form with the help of examples. Refer the video link: Graphing a line in slope intercept and Graphing Lines in Standard Form
  - Determining the equations in Slope and y-intercept form and Standard form from a given linear graph with help of examples.

Refer Slope and y-intercept Form and Standard Form from Understanding mathematics textbook for Class IX or Determining the equations from graphs and other relevant resources.

- Let the students identify dependent and independent variables in a scatter plot. Refer link: Independent vs. Dependent Variables . Show how to create the scatter plot using a table of values manually and visually (using GeoGebra or relevant software).
- Explain the meaning of discrete and continuous data, evaluate their differences using examples (Note: if the independent variable/input variable is continuous, the data is a continuous data and is represented by a solid line graph. Corollary, if the independent variable/input variable is discrete, the data is a discrete data and is represented by a dashed line graph). Refer to the video link: Continuous and discrete data/ maths Antics - Data And Graphs and Continuous and discrete data 2.
- Explain the significance of interpolating and extrapolating using a line of best fit to predict values in a scatter plot. Refer Scatter Plots of Discrete and Continuous Data from Understanding mathematics textbook for Class IX or use link: Interpolation and Extrapolation

E. Assessment

Performance Task 1

Work on a set of questions related to the competency of representing patterns and evaluating the type of relation using a table of values. Refer Patterns and Relations in Tables from Understanding mathematics textbook for Class IX.

Performance Task 2

Solve a set of questions based on competency of evaluating and comparing the graphical representation of linear and non-linear relations. Refer Graphs of Linear and Non-Linear Relations from Understanding mathematics textbook for Class IX or from other resources.

Performance Task 3

Solve problems related to graphing the slope and y-intercept and standard form of linear relations. Refer the chapter on Equation of a Line from Understanding mathematics textbook for Class IX or from other resources. Or use the link: Deriving the Equation y = mx+b .

Performance Task 4

Solve questions based on competency of identifying discrete and continuous data using table of values and scatter plot to solve real life problems. Refer Scatter Plots of Discrete and Continuous Data from Understanding mathematics textbook for Class IX or from other resources.
Suggested activity: Refer questions given in the link Modelling with Functions .

Performance Task 5

Carry out an activity to test the competency of predicting values using the line of best fit by interpolating and extrapolating in real life context.
Determine the equation of the line of best fit for a given set of data. (Refer The Line of Best Fit in Understanding mathematics textbook for Class IX)
Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions
  The International Telecommunications Union (ITU) provides estimates of the number of Internet users in Bhutan each year since 1999. Before 1999, the ITU indicates there were none.

Year	1999	2000	2001	2002	2003
Number of Internet users	750	2250	5000	10000	15000

a. Use the ITU data to create a graph and an equation that describes the relation between the year and the number of Internet users in Bhutan.
b. Use the equation to predict the number of Bhutanese Internet users in 2010.

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics

- Utility and Scope - Application of Linear Equations
- Using Tables to identify equations - Identifying relations
- Graphing equations: Graphing a line in slope intercept and Graphing Lines in Standard Form
- Determining equations from graphs - Determining the equations from graphs
- Independent versus dependent variables: Independent vs. Dependent Variables
- Continuous and discrete data: Continuous and discrete data/ maths Antics - Data And Graphs and Continuous and discrete data 2
- Interpolation and extrapolation: Interpolation and Extrapolation
- Deriving the equations: Deriving the Equation y = mx+b
- Modelling with functions - Modeling with Functions

G. Annexure

Refer IX-A1 for template to record achievements

Topic: IX-B3 Linear Equations and Inequalities [550 minutes]

Introduction

In mathematics, a statement of an order relationship that is greater than, greater than or equal, less than or less than or equal to- between two numbers or algebraic expressions is called an inequality. We see in real life that inequalities are more than equalities. It plays a vital role in almost all branches of mathematics as well as other areas of science like economics, finance, statistics, physics, etc.


Godfrey Harold Hardy

Inequalities have existed in different branches of mathematics but it was not a systemic discipline. In 1934, the first book "Inequalities" was written by Godfrey Harold Hardy, J. E. Littlewood, and J. Polya. The second book on this topic was written by E. Bechanbach and R. Bellman in 1961. The third manuscript name "Analytic Inequalities" was written by D. S. Mitrinovic in 1970. These books transform the field of inequalities into a systematic field and provide motivations, ideas, techniques, and applications for the new research. What are Inequalities?

Utility and Scope

A system of linear inequalities is often used to determine the best solution to a problem. This solution could be as simple as determining how many of a product should be produced to maximise a profit or as complicated as determining the correct combination of drugs to give a patient. Regardless of the problem, there is a theorem in mathematics that is used, with a system of linear inequalities, to determine the best solution to the problem.

A. Competency

- Solve linear equations and linear inequalities algebraically and graphically to investigate polynomial behaviour.

B. Objectives

- Solve linear equation problems algebraically and graphically.
- Represent inequality using symbols of inequality and solve linear inequalities algebraically.

- Describe inequalities using graphs.
- Explore graphing for given information in a variety of formats using MS Excel/ GeoGebra/ Graphmatica and others.

C. Essential Skills/Processes

- Analysing
- Representing
- Graphing
- Computing

D. Learning Experiences

- Ask questions to students to check their prior knowledge on the difference between the equation and expression and then explain the concept of equation - "An equation is a mathematical statement in which the value on the left side of the equal sign is the same as the value on the right side of the equation sign."
  - Explain the meaning of the solution to a linear equation - refer Solving Linear Equations Algebraically from Understanding mathematics textbook for Class IX.
  - Watch the video: Balancing Equations which explains the balancing of equations in a simple way.
  - Explain the strategies to solve linear equations using Inverse Operation, refer to the link: Solving Equations by Using Inverse Operations . Remind students that solving a linear equation is to isolate the given variable using the Inverse Operation.
  - Show that a linear equation has only one unique solution. Explain that "a solution of a linear equation is the value which makes the equation true or that satisfies the equation." Refer Solving Linear Equations Algebraically from Understanding mathematics textbook for Class IX to provide some hands-on practice questions in class.

- Solving linear equations graphically: refer solving linear equations graphically from Understanding mathematics textbook for Class IX.
  - Revise graphing linear equations using slope and y-intercept for slope and y-intercept form and x-intercept and y-intercept for standard form.

- Show that using a graph, we can find the value of y for any value of x and find the value of x for any value of y.
- Provide some hands-on practice questions from the Understanding mathematics textbook for class IX.
- Give some practice and apply questions from the Understanding of mathematics textbook for class IX or from other resources.

- Represent inequality using symbols of inequality and solve linear inequalities.
  - Explain the meaning of inequality, and the inequality signs: $\leq$, $<$, $>$ and $\geq$
  - Compare and contrast with the linear equation. Refer Solving Linear Inequalities from Understanding mathematics textbook for class IX or from other resources.
  - Explain the strategies to solve linear inequalities using Inverse Operation. Convince students that an inequality sign flips when multiplying or dividing by a negative number: E.g. $1 < 4$, but $-1 > -4$.
  - Show that a linear inequality has many solutions i.e., there are many values that satisfy the given inequality. Give some hands-on practice questions in the class.
  - Share the video link: Solving linear inequalities which explains solving linear inequalities. Discuss examples of using an inequality to solve a problem and solving an inequality with variables on both sides from Understanding mathematics textbook for class IX.
  - This video link Key Stage 4 - Linear Inequalities explains how to solve linear inequation problems algebraically and graphically.
  - Provide some practise questions from Solving Linear Inequalities, Understanding mathematics textbook for Class IX.

- Graphs of Linear Inequalities: Refer Graphs of Linear Inequalities from Understanding mathematics Textbook for class X.
  - Introduce graphing of linear inequalities. Using an equality, show that there are many ordered pairs which satisfy the given inequality. Thus, the shaded region shows all the ordered pairs which satisfy the given inequality.
  - Demonstrate graphing an inequality by specifying the steps required. Refer Graphing an Inequality in Slope and y-intercept form and Standard Form

from Understanding mathematics textbook for Class X or use link: Graphing an Inequality

- Use GeoGebra or any other graphing software to verify an inequality graph.
- Demonstrate writing an inequality statement from a given inequality graph and vice versa using the link, Writing Inequality from the graph. Specify the steps required. Refer Expressing an Inequality Algebraically From its Graph from Understanding mathematics textbook for Class X.

E. Assessment

Performance Task 1

Work out questions related to solving linear equations: Direct equations as well as some word problems.
Answer questions related to solving linear equations graphically. Refer Understanding mathematics textbook for class IX.

Performance Task 2

Work on questions related to solving linear inequalities: Direct inequalities as well as situational word problems.

Performance Task 3

Carry out activities relating to interpreting linear inequalities to graphs and vice versa.
Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions
  Karma has two part-time jobs. Last month he made at least Nu 25000. Write an equality to represent his total income from both jobs last month. Graph the inequality.

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics
- Introduction to Linear Inequalities - What are Inequalities?

- Balancing/Solving linear equations - Balancing Equations
- Solving linear inequalities - Key stage 4 - Linear inequalities
- Solving Equations using inverse operations - Solving Equations by Using Inverse Operations
- Solving linear inequalities - Solving linear inequalities
- Graphs of linear equalities - Graphing an Inequality
- Solving linear equations - Writing Inequality from the graph

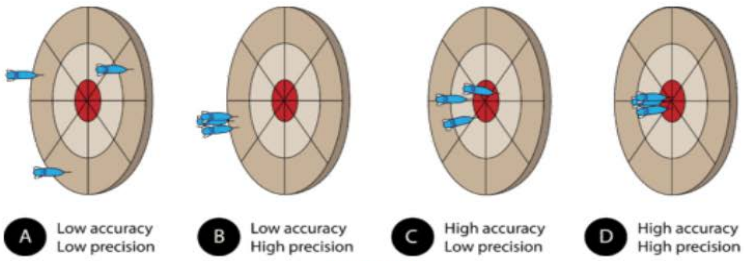
G. Annexure

Refer IX-A1 for template to record achievements

Topic: IX-C1 Precision and Accuracy [300 minutes]

Introduction

The ability of an instrument to measure the accurate value is known as accuracy. In other words, it is the closeness of the measured value to a standard or true value. The closeness of two or more measurements to each other is known as the precision of a substance. If you weigh a given substance five times and get 3.2 kg each time, then your measurement is very precise but not necessarily accurate. Precision is independent of accuracy. Accuracy is the degree of closeness between a measurement and the measurement's true value. Precision is the degree to which repeated measurements under the same conditions are unchanged.



[Figure 2]

The distribution of darts on a dartboard shows the difference between accuracy and precision.

Darts thrown at a dartboard are helpful in illustrating accuracy and precision:
What's the difference between accuracy and precision? - Matt Anticole - YouTube

Utility and Scope

We may not use much accuracy and precision in our everyday lives, but engineers and scientists often require accuracy on microscopic levels with a high certainty of being right every time. Factories and laboratories increase precision through better equipment and more detailed procedures. These improvements can be expensive, so managers must decide what the acceptable uncertainty for each project is. However, investments in precision can take us beyond what was previously possible in the field of medicine, space travel and engineering.

A. Competency

- Demonstrate understanding of properties and measures of precision and accuracy to evaluate and enhance the credibility of a measurement.

B. Objectives

- Demonstrate understanding that precision depends on how finely an instrument is calibrated (or graduated) (e.g., measuring length using cm ruler or mm ruler).
- Demonstrate understanding that accuracy depends upon how correctly the measurement is taken.
- Express measurements using significant figures.

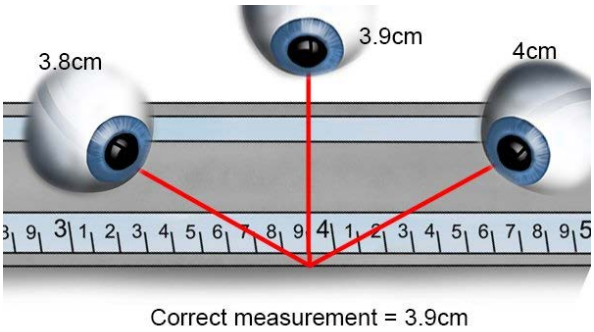
C. Essential Skills/Processes

- Estimating
- Analysing
- Reasoning

D. Learning Experiences

- Demonstrate understanding that precision depends on how finely an instrument is calibrated (or graduated) (e.g. measuring length using cm ruler or mm ruler).
  Suggestion:
  - The web link Precision of the Instrument contains a video on understanding the concept precision with examples.
  - Demonstrate the concept of precision using various instruments. Example, to measure the diameter of a pencil, one can use centimetre ruler, vernier callipers and screw gauge. Demonstrate that the measurement taken by the screw gauge (least count = 0.001 cm) is more precise (it can take measurement up to more decimal places) than measurements taken by vernier calliper (least count = 0.01 cm) and centimetre ruler (least count = 0.1 cm).
- Demonstrate understanding that accuracy depends upon how correctly the measurement is taken.
  Suggestion:
  - Demonstrate that the accuracy depends on how correctly the measurement is taken. Example, the diagram below shows different ways of taking measurements. (Include other factors that affect accuracy).

- Show that accuracy of measurement is proportional to error in measurement.



Correct measurement = 3.9cm

- Express measurements using significant figures.
  - Explain the concept of significant figures. Explain the rules for counting significant figures. Express measurements using significant figures.
  - Suggestion: the link https://www.omnicalculator.com/mathss/sig-fig can be used to understand the meaning of significant figures and rules for counting significant figures. The link also contains an online significant figure calculator.

E. Assessment

Performance Task 1

Solve a set of questions based on real life context of understanding that precision depends on how finely an instrument is calibrated and understanding that accuracy depends upon how correctly the measurement is taken. (Refer Precision and Accuracy from Understanding mathematics textbook for Class X or from other resources).

Performance Task 2

Solve a set of questions based on competency of counting significant figures and expressing measurements using significant figures (Refer Understanding mathematics for class X mathematics textbook from other resources).

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

i. Yoesel measured the volume of a liquid three times and got these results: 66.71 mL, 66.70 mL, 66.69 mL. The actual volume of the liquid is 69.70 mL. Are Yoesel's measurements precise? Are they accurate? Explain.

ii. Meto measured the side of a triangle to be 73 cm using a metre stick that measures to the nearest centimetre.
   a. If she had used a metre stick that measured to the nearest 10 cm, what might she have found the length to be?
   b. If she had used a metre stick that measured to the nearest millimetre, what might she have found the length to be?

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics
- Introduction to Linear Precision and Accuracy - What's the difference between accuracy and precision? - Matt Anticole - YouTube
- Precision on an instrument - https://www.youtube.com/watch?v=4VBbgNsdvcg
- Significant figures - https://www.omnicalculator.com/mathss/sig-fig

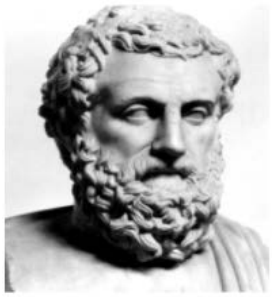
G. Annexure

Refer IX-A1 for template to record achievements

Topic: IX-C2 Properties of 2-D Efficiency [300 minutes]

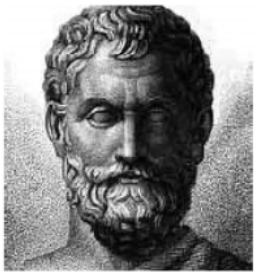
Introduction

The term efficiency refers to the peak level of performance that uses the least amount of inputs to achieve the highest amount of output. Efficiency requires reducing the number of unnecessary resources used to produce a given output, including personal time and energy.



Aristotle

Efficient shapes problems emerged when someone asked the question: Of all plane figures with the same perimeter, which has the greatest area? Aristotle (384 b.c.e.–322 b.c.e.) may have known this problem, having noted that a geometer knows why round wounds heal the slowest. Zenodorus (c. 180 b.c.e.) wrote a treatise on isometric figures and included the following propositions, not all of which were proved completely:



Zenodorus

- Of all regular polygons with equal perimeter, the one with the most sides has the greatest area.
- A circle has a greater area than any regular polygon of the same perimeter.

Utility and Scope

Efficiency can be understood as achieving maximum benefits or profits with minimum wastage. For example, for a garden, it is beneficial to have a garden that requires less fence and more area; less fence to save cost for fencing and more area to have more space for plantations.

A. Competency

- Verify 2-D efficiency inductively to apply properties of 2-D efficiency in real life situations.

B. Objectives

- Understand the meaning of efficiency of any 2-D shape.
- Examine maximising area while restricting perimeter and examine minimising perimeter while restricting area.

- Explore efficiency design of 2-D shapes.

C. Essential Skills/Processes

- Communicating
- Evaluating
- Applying

D. Learning Experiences

- Explain the meaning of Efficiency and relate to 2-D shape efficiency (Refer 2-D Efficiency from Understanding mathematics textbook for Class X).
- Revise the concept and meaning of perimeter and area of a 2-D shape:
  - Perimeter is the sum of the lengths of all sides, and the area is the amount of space within a 2-D shape.
  - Provide some questions to find the perimeter and area of some simple shapes such as: rectangles, squares, triangles and circles. (Note that the perimeter of a circle is called a circumference of a circle).
  - Relate perimeter and area to the real-life application such as length of a fence of a garden as perimeter and the space available to plant vegetables as the area of the garden.
  - Explain that a square is also a rectangle, since it has four sides and each corner has 90°. It is a special rectangle with all sides equal (regular rectangle).
- Exploring the change in area while restricting the perimeter:
  Suggested activity:
  - Take students outside in groups. Assign each group with an equal length of rope of 2 m. Explain that the rope of 2 m is the perimeter of any 2-D shape which is going to be created and will remain constant throughout the experiment.
  - Ask each group to make different shaped rectangles (narrow rectangles, wider rectangles and square shape), let students calculate and note the area for each shape. At the end of the experiment, students should realise that as the shape becomes more square-like, the area increases and the square has the greatest area among the rectangles.

- Repeat the same experiment with triangle shape. Students will realise that as the triangle becomes more equilateral-like, the area increases and the equilateral triangle has the greatest area of the triangles.
- Now ask students to compare the area of the equilateral triangle and the area of the square from the two experiments. As they find that the area of the square will be greater than the area of the equilateral triangle, let students come up with conjectures on the change in area while restricting the perimeter.
- Share the conjecture, "keeping the perimeter constant, the area increases as the number of sides of 2-D shapes increases or the 2-D shapes will have greater area as it becomes more circle-like". Highlight upon the inductive reasoning being used to draw conclusion on the conjecture (Refer Inductive and Deductive Reasoning Sample given below the annexure A1).

- Exploring the change is perimeter while restricting the area:
  Suggested Activity:
  - Ask students to draw different rectangles and different triangles all with the same area.
  - Let students compare the perimeter lengths and the areas of each shape. Students will find that the areas are the same, but the perimetres are different for each shape.
  - Conclude that when the area remains constant, the perimeter decreases as the number of sides of a 2-D shape increases or the 2-D shapes will have lesser perimeter as it becomes more circle-like. Highlight upon the inductive reasoning being used to draw conclusion on the conjecture.
- Use GeoGebra software to explore more on the 2-D efficiency by drawing different shapes, restricting either a perimeter or an area.
  - Refer to the web link: Regular Polygons to learn how to construct regular polygons in GeoGebra.
  - Refer to the web link: Area of 2-D shapes to learn how to find the area and perimeter of a shape in GeoGebra.
- Present these final notes to the students:
  - For any 2-D shapes, the circle is the most efficient shape.
  - Among the rectangles, square is the most efficient as it is more circle-like

- Among the triangles, the equilateral triangle is the most efficient as it is more circle-like.
- Efficiency of 2-D shapes increase as the number of sides increases or as they become more circle-like.

- Assign some practise questions: Refer questions on 2-D Efficiency from Understanding mathematics textbook for class X.

E. Assessment

Performance Task 1

Work out questions related to 2-D efficiency on examining situations of maximising area while restricting perimeter and examine minimising perimeter while restricting area. (Refer 2-D Efficiency from Understanding mathematics textbook for class X).

Performance Task 2

Work out questions related to application of 2-D efficiency properties in real life contexts (e.g., fencing of garden, table trims etc.). (Refer 2-D Efficiency from Understanding mathematics textbook for class X).

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

  These triangles have the same area but different perimeters.

  a. Predict which triangle has the shorter perimeter. Explain your prediction.
  b. Check your prediction. Show your work.

F. Resources

- Understanding mathematics Textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics
- Using GeoGebra - Regular Polygons and Area of 2-D shapes

G. Annexure

Refer IX-A1 for template to record achievements

Topic: IX-D1 Relate Congruence and Similarity of Triangles [500 minutes]

Introduction

All congruent figures are similar, but not all similar figures are congruent. Congruence means two objects (whether two dimensional or three dimensional) are identical in size and shape. Everything about them and their angles, lengths of sides, and overall dimensions are identical. Similar figures have the same shape and proportions but are not necessarily the same size.

Utility and Scope

In the setup for a camera lens, we have a "Bow Tie" shaped pair of Similar Triangles and when light passes through a camera lens the original image ends up upside down or "inverted". This is why cameras have a mirror inside them to put the image right up so we can view it while taking the photo.

Similar Triangles are also very useful for indirectly determining the sizes of items which are difficult to measure by hand. Typical examples include building heights, tree heights, river width and tower heights.
https://youtu.be/ZZTqi4qnj1o?si=FtmfC_6S7UOVVA4h

A. Competency

- Analyse measures of sides and angles to evaluate the properties of congruence and similarity of triangles to solve real-life based problems.

B. Objectives

- Demonstrate understanding of the meaning of congruence and similarity.
- Examine the conditions necessary for congruence and similarity.
- Compare and contrast congruence and similarity as they relate to triangles.
- Solve problems based on real life context related to congruence and similarity by applying related properties.

C. Essential Skills/Processes

- Applying
- Analysing

- Evaluating

D. Learning Experiences

- Deductively explain the meaning of congruence and similarity with respect to triangles:
  - o Refer Congruent Triangles and Similar Triangles from Understanding mathematics Textbook for Class IX.
  - o Explain and clarify that all congruent triangles are similar but all similar triangles are not congruent.
  - o Create a few congruent triangles and similar triangles out of chart paper and use them to conjecture possible properties for congruence and similarity.
  - o Highlight upon the deductive reasoning being used to prove the Congruence and similarity (Refer Inductive and Deductive Reasoning Sample given below the annexure A1).

- Explain and verify four conditions to verify if triangles are congruent: SSS, ASA, AAS, and SAS. Explain why fulfilling the conditions SAA and SSA cannot be used to conclude the triangles are congruent. Note that before teaching the conditions, make sure that students understand the meaning of corresponding sides and angles of triangles, and also the orientation of triangles. This web link: https://www.youtube.com/watch?v=vGuiy7NnJlM explains each condition of congruence in detail with pictorial animations.
  - o Discuss some example questions on similar triangles from Congruent Triangles, Understanding mathematics textbook for Class IX.

- Explain the meaning of similar triangles:
  - o Explain the meaning of similar triangles and its link to the dilation.
  - o Refer similar triangles Understanding mathematics textbook for class IX or use link: Applications of Similar Triangles | CK-12 Foundation (ck12.org)
  - o Show the model of similar triangles made out of chart paper to conjecture possible conditions for similarity.
  - o Explain the three conditions for similarity: AAA, SSS, SAS. This web link https://www.youtube.com/watch?v=reGN77SiESA explains about similar triangles' properties and conditions for similarity.

- Discuss some practice example questions on similar triangles from Understanding mathematics textbook for class IX.

- Carry out an outdoor learning experience activity to solve a missing length problem using similarity. Refer to the activity in the exposition part of Using Similarity Properties to Solve Problems in Understanding mathematics textbook for Class X.

- Explain the real-life applications of similar triangles and congruent triangles properties.
  - Refer and discuss the example questions from Solving Problems With Similarity in Understanding mathematics textbook for class IX and Using Similarity Properties to Solve Problems in Understanding mathematics textbook for Class X.

E. Assessment

Performance Task 1

Carry out activities related to congruent triangles and its application to real life. Refer to the practise and applying questions of Understanding mathematics textbook for class IX.

Performance Task 2

Carry out activities related to similar triangle properties and their applications to real life. Refer Understanding mathematics textbook for class IX and X for sample questions.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

Mosaics are used in many cultures to decorate floors and walls. A mosaic is

usually made up of many small tiles.

a. Choose two congruent triangles in the mosaic. Describe four different ways you could establish that they are congruent.
b. Choose two triangles in the mosaic that are similar but not congruent. Describe three different ways you could establish that they are similar.

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics
- Utility and Scope - https://youtu.be/ZZTqi4qnj1o?si=FtmfC_6S7UOVVA4h
- Triangle Congruence Theorems - https://www.youtube.com/congruence-throrems
- Application of similar triangles - Applications of Similar Triangles
- Properties and problems of similarity - https://www.youtube.com/watch?v=reGN77SiESA

G. Annexure

Refer IX-A1 for template to record achievement

Topic: IX-C3 Trigonometry [500 minutes]

Introduction


Hipparchus of Nicaea

Trigonometry is a branch of mathematics concerned with angle measurement and problems involving related angles. The Greek terms 'trigonon' and 'metron' are used to form the word trigonometry. The terms trigonon and metron refer to triangle and measure, respectively. As a result, trigonometry refers to the study of relationships between a triangle's sides and angles.

Hipparchus is considered the founder of trigonometry. He was a Greek astronomer, geographer and mathematician born about 180 B.C. in northern Asia Minor. Hipparchus produced a table of chords, an early example of a trigonometric table. He did this by using the supplementary angle theorem, half angle formulas, and linear interpolation. Hipparchus was not only the founder of trigonometry but also the man who transformed Greek astronomy from a purely theoretical into a practical predictive science. He also introduced the division of a circle 360 degrees into Greece. The subject was developed by him as a tool to help in his astronomical work.

History of trigonometry1 and History of Trigonometry

Utility and Scope

The early applications of Trigonometry were to astronomy, surveying, navigation and engineering. In modern times, it finds wide application in warfare, all sorts of vibratory phenomena, sound, light, electricity, etc. Applications of Trigonometry

A. Competencies

- Use appropriate properties of triangles to determine the basic trig-ratios in right angled triangles and use the ratios to deduce trigonometric identities.
- Demonstrate the application of trigonometry to deduce relations and calculate areas of polygons and heights and distances in real life context.

B. Objectives

- Develop primary trigonometric ratios applying properties of similarity and side-angle relationships.

- Use calculators to determine the trig ratios sin θ, cos θ, and tan θ.
- Use the sine and cosine ratios to articulate the relationships between the sides and angles of a triangle.
- Prove the trigonometric identities and apply in appropriate situations.
- Explore angles of elevation (measured from the horizon up) and angles of depression (measured from the horizon down) in real world settings.
- Find areas of polygons using trigonometric ratios.

C. Essential Skills/Processes

- Applying
- Analysing
- Classifying
- Representing

D. Learning Experiences

- Construct a right triangle to discuss how to name the sides with respect to any of the acute angles of the right triangle.
  - The video link: Bing Videos contains how to name the sides with respect to the given angle. Provide some practise questions from Understanding mathematics Textbook for class X.

- Demonstrate how to determine the basic T-ratios from a set of given diagrams. (Use similarity properties to establish that the ratio is constant for a given angle).
  - The video link: Trigonometry For Beginners (definition and trig ratios defined in terms of an acute angle, easy way to remember the trig ratios with examples). Demonstrate and solve some more relevant questions in the class. Refer a few more relevant questions from Understanding mathematics Textbook for class X.
  - The video link : Key Stage 4 - Trigonometry demonstrates the basic concepts of trigonometry.

- Demonstrate how to use calculators to determine the trig ratios for the given angle.

- Assign a few questions to the students to investigate and explore the applications of calculators in determining the values of trig ratios; sin θ, cos θ, and tan θ (with the given angle).

For practice refer relevant questions from The Sine, Cosine, and Tangent Ratios in Understanding mathematics Textbook for class X.

- Recall prior knowledge on their knowledge of using Pythagorean Theorem by providing the worksheet and intervene accordingly. Use the link provided to download the worksheet. Pythagorean Theorem Worksheet and Finding Sides and Angles worksheet

- Deduce the Pythagorean identity and other two identities (making sine or cosine as subject). Highlight upon the deductive reasoning being used to draw and prove the identities (Refer Inductive and Deductive Reasoning Sample given below the annexure A1).

$$Sin^2\theta + Cos^2\theta = 1$$

$$Cos\,(90° - \theta) = Sin\,\theta \quad OR \quad Sin\,(90° - \theta) = Cos\,\theta$$

$$\tan\theta = \frac{Sin\,\theta}{Cos\,\theta}$$

Derive the reciprocal identities by relating it to the primary trigonometric ratios.

- Use the video link: Trig Identities for the explanation of trig identities.
- Use the weblink: Trig Identities - Steps, Examples & Worksheet for the delivery and practise.

  Further refer Trigonometric Identities from Understanding mathematics textbook for class X.

- Design learning activities involving both direct and situational problems of Applying Trigonometric Ratios in Calculating Side Lengths and Angles (Refer Calculating Side Lengths and Angles from Understanding mathematics textbook for Class X or from other resources).

- Disseminate the meaning of angles of elevation and angles of depression by using the link: Angles Of Elevation & Depression. For additional information

refer to the Angles of Elevation and Angles of Depression from Understanding mathematics textbook for Class X or from other resources.

- Demonstrate how to deductively derive the formula to calculate the area of parallelogram: A= ab Sin x (and relate this formula in finding the area of any triangle). Connect to find the areas of regular polygons applying trigonometry. Refer Areas of Polygons from Understanding mathematics textbook for class X or from other resources.

E. Assessment

Performance Task 1

Attend a short test using the link: https://www.liveworksheets and ask students to self-assess their work and honestly share their score and provide necessary feedback. Use other similar links if necessary.
Conduct an assessment on the practical application in the usage of scientific calculators to find the value of trig ratios with the given angle.

- Calculating the value of trig ratios (use the *Printable worksheet*)- Trigonometric Ratios Worksheets. Ask students to share their work to provide feedback.
- Work on the worksheets using the given links:Trigonometry - Finding Sides and Angles worksheet (live sheet) and share findings.

Performance Task 2

Carry out activities involving both direct and situational problems of Applying Trigonometric Ratios (Refer Calculating Side Lengths and Angles from Understanding mathematics textbook for Class X).

Performance Task 3

Work on pictorial, direct and situational problems to evaluate the understanding of the concept of angles of elevation and angles of depression. (Refer Angles of Elevation and Angles of Depression from Understanding mathematics textbook for Class X and from other resources).

Performance Task 4

Solve diagrammatic problems with simple and composite shapes to evaluate the area with the trigonometric-ratio based formulae (Refer problems from Understanding mathematics textbook for Class X or other resources).

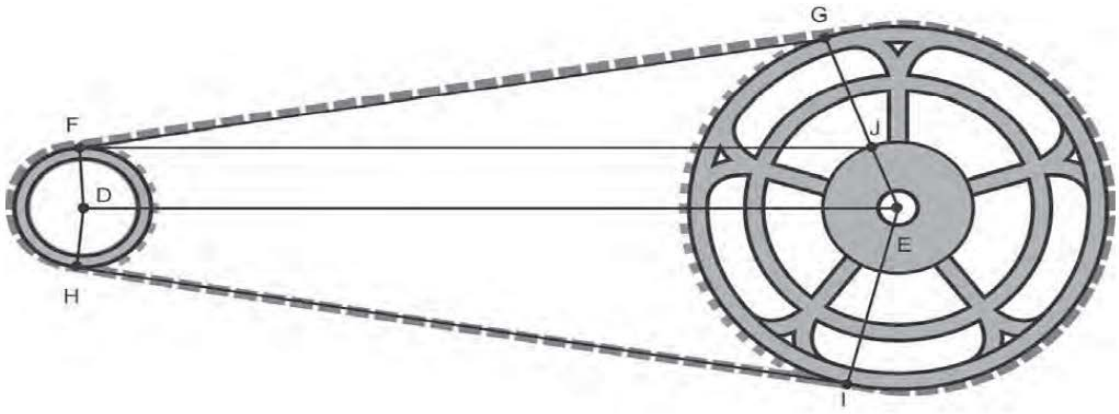
Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

  Look at the photo of a chain on a bicycle.

  a. Your task is to figure out, to the nearest millimetre, how long the chain needs to be.
  b. The diameter of the larger sprocket wheel is 176 mm and the diameter of the smaller sprocket wheel is 90 mm. The centres of the two sprockets are 420 mm apart.
  c. Assume that the straight lengths of the chain meet the sprocket wheels at 90° angles.

  This diagram below is a model of the chain connecting the two sprocket wheels.



  Use trigonometry to determine the length of the chain.

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics
- Introduction to Trigonometry - History of Trigonometry

- Utility and Scope - Applications of Trigonometry
- Sides of Right Triangles - Bing Videos
- Trigonometric Ratios - Trigonometry For Beginners
- Pythagorean Theorem activity - Pythagorean Theorem Worksheet
- Trigonometric Identities - Trig Identities
- Trig identities activities - Trig Identities - Steps, Examples & Worksheet /
- Angle of Elevation/Depression - Angles Of Elevation & Depression
- Naming the sides: https://www.liveworksheets
- Trigonometric ratios activities: Trigonometric Ratios Worksheets and Trigonometry - Finding Sides and Angles worksheet

- Concept of Trigonometry - Key Stage 4 - Trigonometry

G. Annexure

Refer IX-A1 for template to record achievements

Topic: IX-D2 Transformations [600 minutes]

Introduction

A transformation is an operation that moves, flips, or changes a figure to create a new figure. A rigid transformation is a transformation that preserves size and shape. The rigid transformations are: translations (discussed here), reflections, and rotations. The new figure created by a transformation is called the image. The original figure is called the preimage. Another word for a rigid transformation is isometry. Rigid transformations are also called congruence transformations.


Felix Klein

A dilation is a type of transformation that enlarges or reduces a figure (called the preimage) to create a new figure (called the image). The scale factor, r, determines how much bigger or smaller the dilation image will be compared to the preimage.

The first systematic effort to use transformations as the foundation of geometry was made by Felix Klein in the 19th century, under the name Erlangen programme. For nearly a century this approach remained confined to mathematics research circles. In the 20th century efforts were made to exploit it for mathematical education. Andrei Kolmogorov included this approach (together with set theory) as part of a proposal for geometry teaching reform in Russia. These efforts culminated in the 1960s with the general reform of mathematics teaching known as the New maths movement.


Andrei Kolmogorov

Transformation geometry - Wikipedia

Utility and Scope

Transformations are very present in our surroundings and they have been part of our daily lives ever since. From simple objects such as mirrors and toys to skyscrapers and cars, the four basic transformations are very evident. They function to make our lives easier and more convenient. Their uses are truly relevant as they add efficiency in our work and lifestyle. Not only that, they are also used in many fields as a form of art and design, so that objects possess more beauty and personality. Likewise, regularity in objects can be easily checked by applying the

ideas of transformations. Many of the things we enjoy nowadays are products of transformation concepts such as the developments in science and technology, architecture, communications, and transportation. Transformations in Real Life

A. Competency

- Demonstrate the interpretation of transformations using mapping notations to graphically represent dynamics of transformations on a coordinate plane.

B. Objectives

- Apply translation, reflection, rotation, and dilation to shapes on the coordinate plane using mapping notation.
- Describe the nature of a transformation based on a given mapping notation.
- Evaluate the transformations when image and the preimage are given.
- Use graphing software to explore the characteristics of transformations.

C. Essential Skills/Processes

- Applying
- Reasoning
- Evaluating

D. Learning Experiences

- Recap the lessons students have learned in lower classes on transformations.
  - Remind students that “transformation” means change. Transformation of a shape is a change in size, orientation, appearance or any other change in the shape. Translation, Reflection, Rotation and Dilation are the transformations of a shape. Use link: Transformation and its types to understand the concept of types of transformations.
    Refer Translation from Understanding mathematics textbook for class IX or use web link: Translation and Translation; Video.

- Explain the mapping rule and mapping notations.

- Translations: Show how to use mapping notations or translation rules to translate a shape and how to write mapping notations or translation rules for a

given transformation. Discuss and deduce the properties of translations with regard to change in position, size and orientation.

- User GeoGebra with a visible grid to show the translations using a translation rule and mapping notation.
- Discuss and explain relevant examples from Translations in Understanding mathematics textbook for class IX.
- Assign some practise questions from Understanding mathematics textbook for class IX. Use the web link for resources: Rules for Translations.
- Remind students to remember the mapping notation for translations.

- Reflections: Discuss and deduce the properties of reflections with regard to change in position, size and orientation. Refer Understanding mathematics textbook for class IX or use web link: Rules for Reflections and Reflection, Video
  - Using GeoGebra, show students a reflection of a shape and their corresponding changes in coordinates. Comparing the coordinates of the original and the reflected shape, derive the mapping notation of reflection.
  - Explain the mapping notation of reflection on each axis.
  - Discuss and explain relevant examples in Understanding mathematics textbook for class IX.
  - Provide some hands-on practice questions related to reflections.

- Rotations: Discuss and deduce the properties of rotations with regard to change in position, size and orientation. Refer Understanding mathematics textbook for class IX or use web link: Rules for Rotations and Rotation, Vido

- Using GeoGebra, show students the rotation of a shape (triangle) and their corresponding changes in coordinates. Comparing the coordinates of the original and the reflected shape, derive the mapping notation of rotation.
  - Rotate the shape CW (90° and 180°), derive and explain the mapping notation.
  - Rotate the shape CCW (90° and 180°), derive and explain the mapping notation.
  - Discuss and explain relevant examples in Understanding mathematics textbook for class IX.
  - Provide some hands-on practice questions related to rotations of different angles with different directions.

- Dilations: Discuss and deduce the properties of dilations with regard to change in position, size and orientation. Refer Dilations in Understanding mathematics textbook for class IX or use web link: Rules for Dilations
  - Using GeoGebra, show students a dilation of a shape (triangle) and their corresponding changes in coordinates. Comparing the coordinates of the original and the dilated shape, derive the mapping notation of dilation.
  - Explain clearly on the scale factor of dilation: Enlargement and Reduction.
  - Discuss and explain relevant examples in Understanding mathematics textbook for class IX.

- Combining Transformations: Refer Combining Transformations in Understanding mathematics textbook for Class IX or use web link: Notation for Composite Transformations
  - Start with easier composite transformations such as reflection and dilation. Derive mapping notation for each transformation and then derive composite mapping notation of transformation from original shape to the final shape.
  - Use GeoGebra to show the transformations.
  - Explain how to combine two mapping notations into a single mapping notation.
  - Discuss relevant examples given in the Understanding mathematics Textbook for class IX and provide some practise questions.

- Note that the knowledge of mapping notations of transformations from this lesson will be linked to the transformations of parabola in class X.

E. Assessment

Performance Task 1

Solve questions related to representing different types of transformations from mapping notations (and vice versa) from the Understanding mathematics textbook for class IX or from the links shared or from other resources.

Performance Task 2

Work out questions relevant to combining transformations. Refer relevant questions of describing composite mapping notations from a series of

transformations (and vice versa) from the Understanding mathematics textbook for class IX or from the links shared or other resources.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

  Mosaics are used in many cultures to decorate floors and walls. A mosaic is usually made up of many small tiles.

  In this task, you will perform transformations on a triangle to construct a mosaic. Read through all the instructions and sketch out a plan before you begin your mosaic. Your writing about your constructions should be done on a separate piece of paper.

  a. Start your mosaic by drawing any triangle on grid paper. Label the vertices A, B, and C. Decide which point on the grid you will use as the origin of your coordinate grid and label it O. Draw very faint lines (which you will erase when you are done) to show the x- and y-axes. Use words to list the coordinates of the vertices of a triangle.
  b. Choose a translation rule and use it to translate your original triangle. Label the vertices of your image triangle. Write about the translation using words, a translation rule, and mapping notation.
  c. Reflect the original triangle in the x- or y-axis. Label the vertices of your image triangle. Use words and mapping notation to describe the reflection.
  d. Rotate the original triangle 90º or 180º using your origin as the centre. Label the vertices of your image triangle. Use words and mapping notation to describe the rotation.

e. Dilatate the original triangle using a scale factor of your choice and your origin as the centre. Label the vertices of your image triangle. Use words and mapping notation to describe the dilatation.
f. Choose two of your image triangles from parts a), b), c), and d). Use mapping notation and words to explain how you could map one onto the other.
g. Add other triangles, polygons, and colour to your design to make it look attractive.

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics
- Introduction to Transformations - Transformation geometry - Wikipedia
- Utility and Scope - Transformations in Real Life
- Introduction to types of Transformations - Transformation and its types
- Translations - Translation, Translation; Video, and Rules for Translations.
- Reflections - Rules for Reflections and Reflection, Video
- Rotations - Rules for Rotations and Rotation, Vido
- Dilation - Rules for Dilations and Dilations, Video
- Composite Transformation - Notation for Composite Transformations

G. Annexure

Refer IX-A1 for template to record achievements

Topic: IX-E1 Collecting, Displaying and Analysing Data [600 minutes]

Introduction

The goal of collecting and analysing data is to understand the world around us. How data is collected is very important. The goal of collecting data is to get “unbiased” data that represents the population. Analysing biassed data may result in incorrect conclusions and lead to a misguided view of the world around us. It is also important to have a goal in mind when you collect data. Whether we are trying to find a population percentage from categorical data or a population average from quantitative data, or whether we are trying to show that two variables are related or showing cause and effect, data needs to be collected differently depending on what goal we have in mind.

Utility and Scope

Security: Data analytics applications or, more specifically, predictive analysis has also helped in dropping crime rates in certain areas. In a few major cities like Los Angeles and Chicago, historical and geographical data has been used to isolate specific areas where crime rates could surge. On that basis, while arrests could not be made on a whim, police patrols could be increased. Thus, using applications of data analytics, crime rates dropped in these areas.

Transportation: Data analytics can be used to revolutionise transportation. It can be used especially in areas where you need to transport a large number of people to a specific area and require seamless transportation. This data analytical technique was applied in the London Olympics a few years ago.

Reasonable Expenditure: When one is building Smart cities, it becomes difficult to plan it out in the right way. Remodelling of the landmark or making any change would incur large amounts of expenditure, which might eventually turn out to be a waste. Data analytics can be used in such cases. With data analytics, it will become easier to direct the tax money in a cost-efficient way to build the right infrastructure and reduce expenditure.
Internet searching: When you use Google, you are using one of their many data analytics applications employed by the company. Most search engines like Google,

Bing, Yahoo, etc. use data analytics. These search engines use different algorithms to deliver the best result for a search query, and they do so within a few milliseconds. Google is said to process about 20 petabytes of data every day.
Application of data analytics

A. Competencies

- Apply appropriate methods of data representation for a set of real life based ungrouped data collected to evaluate the data distributions and draw conclusions.
- Demonstrate an understanding of the properties of the various methods of displaying data to evaluate and correct misleading features.

B. Objectives

- Compute the 5 number summary for a given set of ungrouped data.
- Identify and explain the basic steps of Data Management.
- Determine, discuss and justify, why a particular display is suited to a specific type of data, or to a given context or purpose.
- Use relevant software (such as MS Excel, Graphmatica, GeoGebra, etc.) to display data.
- Compare various methods of displaying data.
- Draw inferences and conclusions from a number of data displays.
- Identify and explain the features that might mislead the graphs.

C. Essential Skills/Processes

- Collecting data
- Displaying data appropriately
- Applying
- Analysing
- Representation

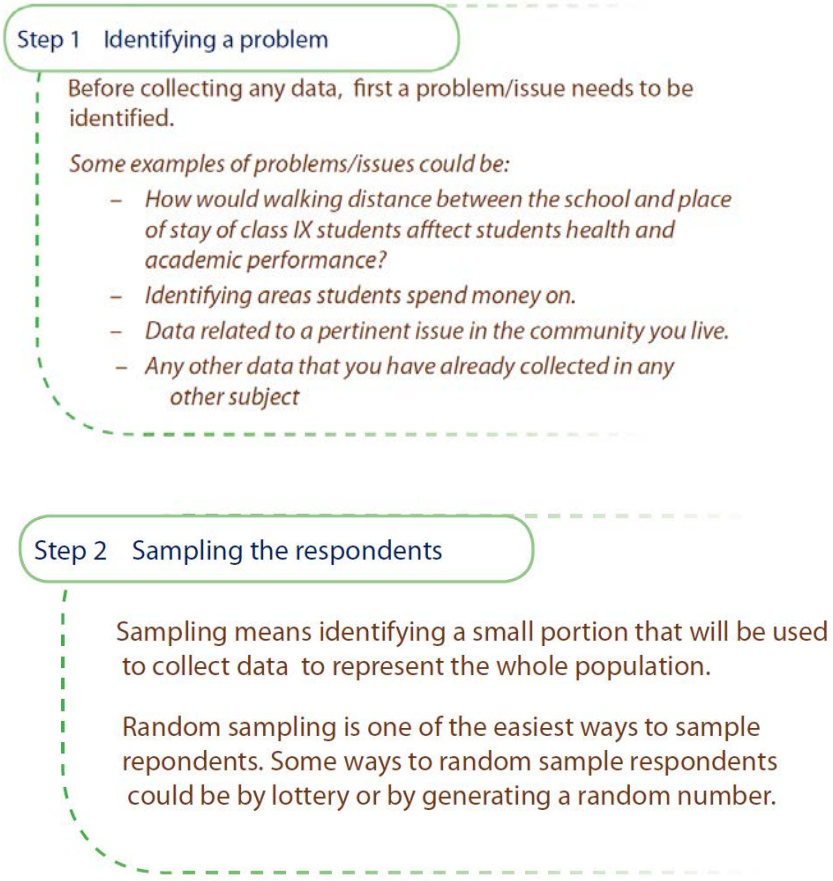
D. Learning Experiences

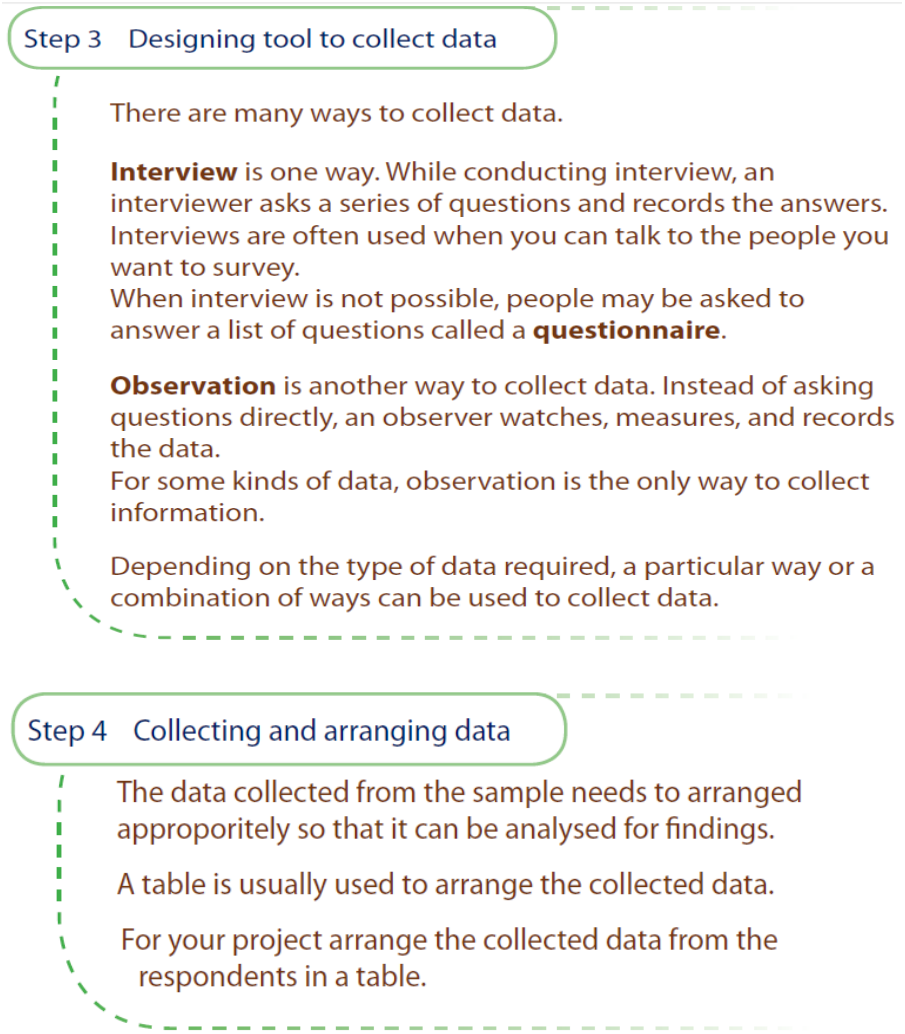
- Types of Data Display (Refer examples from constructing Data Displays in Understanding mathematics textbook for Class IX and Histograms and Box and Whisker Plots in Understanding mathematics textbook for Class X) or refer the given links:

- Stem and leaf plot: Use the link: Stem and leaf plot to understand the process of stem and leaf construction.
- Box and whisker plot: Use the video Box plot to demonstrate the construction of box plot.
- Circle graph: watch the video Circle graph to construct a circle graph.
- Bar graph: Use the link Bar graph to understand the process of stem and leaf construction.
- Histogram: Use the video Histogram to understand the construction of the histogram.

- Data Management comprises a series of steps. It is the process of data collection, data organisation, data display, data analysis, drawing conclusions and making recommendations. All these processes need to be followed strictly to get evidence-based results.

- Activity: Design a project-based learning activity for collecting and displaying data.
    - Steps for data collection:





- Demonstrate, discuss and justify, why a particular display method is suited to a specific type of data, or to a given context or purpose.
  - Ask learners to choose a data display to represent their data collection pictorially (from the project work). Discuss the importance of choosing the right type of graph.
  - The web link Choosing right graph type contains concepts related to questions to ask when deciding which type of chart to use (Limit to stem and leaf plot, box and whisker plot, circle graph, bar graph and line graph).
  - Demonstrate the construction of different data displays: stem and leaf plot, box and whisker plot, circle graph, bar graph and line graph.
  - Explore the use of relevant software (such as MS Excel, Graphmatica, GeoGebra, etc.) to display data.

- Allow students to compare and analyse different methods of data display. Refer Using Graphs to Compare and organise Data from Understanding mathematics textbook for class IX.

- Draw inferences and conclusions from a number of data displays.
  - Instruct learners to analyse and draw conclusions from the data collection and representation (**from the project work**).
  - Demonstrate how to draw conclusions from a data display (Refer Drawing Conclusions from Graph in Understanding mathematics textbook for class IX).
- Identify and explain the features of misleading graphs. Describe the features of misleading graph: the web link: Misleading graph explains the meaning and features of misleading graph or refer Misleading Graphs in Understanding mathematics textbook for Class IX.

E. Assessment

Performance Task 1

To give learners the basics of research, assign Project work to learners. Refer to the project calendar and rubrics for assessment.

Suggested Topics

Sl. No.	Topics	Objectives
1	Survey and Graphical Representation	Conduct a survey on a topic of your interest (e.g., favourite sports, hobbies, movie genres) among classmates. Collect data and create various types of graphs to represent the findings (bar graphs, pie charts, etc.).
2	Weather Data Analysis	Gather weather data over a period of time and analyse trends. Create graphs to represent temperature changes, rainfall, or other relevant weather variables.
3	Sports Statistics	Choose a sport and collect statistics on player performance, team standings, or game outcomes. Analyse the data to identify patterns and trends.
4	Population Growth	Investigate the population growth of a city or country over the years. Use the data to create line graphs and discuss the factors influencing population change.

5	Health and Nutrition	Collect data on the eating habits of classmates or analyse nutritional information of different food items. Create graphs to compare and draw conclusions about healthy eating.
6	Consumer Price Index (CPI)	Study the concept of CPI and analyse how it has changed over the years. Discuss the impact of inflation on the prices of goods and services.
7	Environmental Impact	Investigate environmental data such as pollution levels, deforestation rates, or wildlife populations. Analyse the data and discuss the implications for the environment.
8	School Performance Analysis	Analyse the academic performance of students in different subjects. Create visual representations of the data and explore correlations between study habits and grades.
9	Travel and Distance	Collect data on the distances students travel to school. Use the data to create histograms, calculate averages, and discuss transportation trends.
10	Social Media Usage	Survey classmates about their social media usage habits. Analyse the data to understand trends in social media preferences and usage patterns.

Sample Project Work Calendar

Sl.	Date	Activity	Recommendation	Remarks
1	1st week of March	Draw project plan	Plans must contain schedules with intended activity spread across the year. Can refer to this calendar as a sample.	
2	2nd week of March.	Identifying a working title	Problem/ issue/ situation/opportunity/ project/new venture that is identified as a working title must be endorsed by the teacher. Only after approval of title, students can proceed working on it	
3	3rd and 4th week of march	Literature Review	Refer different books and online literature , which have link with project title	
4	Month of April	Data collection.	Data can be collected on identified topics using various tools (interview, questionnaires, observation, and	

			document) or gather available data from various sources with acknowledgement.	
5	1st Week of May	Data organisation	Organise the collected data in appropriate form	
6	2nd week of May	Data display (1st Draft).	Represent the organised data using appropriate data display(s): · bar graph · circle graph · histogram · stem and leaf plot · box plot.	
4	3rd week of May	Data display (Final Draft).	Represent data display using relevant software (ICT tools). Subject teacher facilitates the availability of computers and software for data display.	
5	4th week of May	Data analysis, drawing conclusions and making recommendations	Analyse the data display, draw conclusion and make recommendations as per the analysis and conclusion supported by data	
6	1st week of June	Compilation and submission of Project Work.	Compile the work and submit it to the respective subject teacher with proper binding.	
7	2nd week of June	Project Evaluation and Awarding of Marks (maximum 35 marks).		

Rubrics for Project Work Assessment

Criteria	Exceeding (4.1-5)	Advancing (3.1-4)	Meeting (2.1-3)	Approaching (1.1-2)	Beginning (0.1-1)
Plan	Plan is detail with 5 components (schedules, activities, materials requirement, data collection source, representations) and endorsed by teacher	Plan is incomplete by two components and not endorsed by teacher	Plan is incomplete by three components and not endorsed by teacher	Plan is incomplete by 4 components and not endorsed by teacher	Plan is sketchy without required components and not endorsed by teacher

problem	problem is new, meaningful and will have positive impact on community	problem is new, meaningful but will not have any impact on community	problem is not new but meaningful	problem is new but not meaningful	problem is not stated
literature	Literature study is researched and aligns with problem showing deep understanding of concept	Literature study is researched and aligns with problem but lacks understanding of concept	Literature study is well researched but do not align well with the problem	Literature study is not well researched and do not align well with problem	Literature study is not stated but visible from other part of project
data collection	Data collection is systematic with collection tools, processes and authentic source and mentioned in plan	Data collection is systematic with collection tools, processes and authentic source but not mentioned in plan	Data collection has collection tools but without logical process and authentic source	Data collection is not systematic but source mentioned	Data collection is not described and source not mentioned
data representation and analysis	Data representation is appropriate, neat with topics and legends and analysis is meaningful	Data representation is appropriate with incomplete components but analysis is meaningful	Data representation is appropriate but analysis is not meaningful	Data representation inappropriate and analysis is not meaningful	Data representation inappropriate but no analysis
Findings	Finding aligns as solution to the problem with strong argument supported by data	Finding aligns as solution to the problem but argument is weak and not supported by data	Finding is described well but not addressed to solve the problem	Finding is not well described but align as solution to the problem	Finding is brief and do not align with problem
References	More than five references are cited in APA format and referenced throughout the project	Four to five references are cited and referenced throughout the project	Two to three references are cited and referenced throughout the project	At least one reference is cited and referenced throughout the project	No references

Performance Task 2

Assess competencies of collecting, representing and analysing data at different phases of the project work or refer relevant problems from Understanding mathematics textbook for Class IX.

Performance Task 3

Explore and list down the features of misleading graphs and cite real life examples of misleading graphs (Refer discussion questions from Understanding mathematics textbook for Class IX or other resources).

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

F. Resources

- Understanding mathematics textbook for Class IX and X
- Teachers Guide to Understanding mathematics, Class IX and X
- National School Curriculum Framework for mathematics
- Utility and Scope - Application of data analytics
- Stem and leaf plot - Stem and leaf plot.
- Box and whisker plot - Box plot.
- Circle Graph - Circle graph.
- Bar graph - Bar graph.
- Histogram - Histogram
- Choosing the right graph - Choosing the right graph type.
- Features of misleading graphs - Misleading graph.

G. Annexure

Refer IX-A1 for template to record achievements

Topic: IX-E2 Probability [400 minutes]

Introduction

Probability is the measure of uncertainty of any event. Probability is the chance that something will happen, how likely it is that some event will occur. Sometimes we can measure a probability with a number like “10%”, or we can use words to describe such as impossible, unlikely, possible, even chance, likely and certain. For example, “it is unlikely to rain tomorrow”.


Pierre de Fermat

Probability, chance and randomness have been around since the ancient days. They could be found in fortune telling, games of chance, philosophy, law, insurance, and errors of prediction in astronomy and medicine (Hald, 1990). In about 1200 B.C., an ancient game was played with four astragali (heel bones of hooved animals). They would grind down the bone into a rough, cube like shape and then make small depressions of various numbers. This is where the idea of the pips, the dots on the dice, we use today came from.

There is evidence that in the late 15th century and the early 16th century, mathematicians started to experiment with the idea of probability. A gambler’s dispute in 1654 led to the creation of a mathematical theory of probability by two famous mathematicians, Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665). The links provided will help you further explore the History of probability: https://www.youtube.com/History


Blaise Pascal

Utility and Scope

Meteorologists collect the database related to weather and its changes worldwide by using different instruments and tools. They collect the weather information worldwide to estimate the temperature changes around the world and the weather conditions for a particular hour, day, week, month and year.

The marketing persons or salespersons promote the products to increase sales. They use the probability technique to check how much the particular product is

going well in the market or not. The probability technique helps to forecast the business in future.

Insurance companies provide insurance policies or premiums based on the future forecast to the persons, vehicles etc. Insurance companies generally use theoretical probability or theory of probability to frame any particular policy and complete the policy at the premium rate.

Cricket, volleyball, soccer, football, tennis, badminton, poker, gambling and all the board games use the concept of probability, which gives the idea about how likely a particular person or team is going to win or lose.
https://www.youtube.com/watch or https://www.youtube.com/application

A. Competencies

- Determine Experimental probability and Theoretical probability of an event to evaluate their differences based on randomness in real life context.
- Use outcome charts (space diagrams) and tree diagrams to determine the theoretical probabilities of individual and/or two independent events.

B. Objectives

- Relate the experimental probability of an event with the theoretical probability for that event.
- Determine the number of possible outcomes for independent events using outcome charts and tree diagrams.
- Define independent events with examples and reasoning.
- Calculate probabilities of two independent events; *P(A) x P(B) = P(A and B)*

C. Essential Skills/Processes

- Computing
- Applying
- Reasoning
- Analysing

D. Learning Experiences

- Check students' prior knowledge by asking some questions related to probability that they have learned in lower classes.

Example

- Suppose Pema rolls a die once. List all possible outcomes. What is the probability of getting an odd number?
- Explain that the possible outcomes are also called sample space.

- Experimental Probability and Theoretical Probability:
  - Take a die or a coin and demonstrate on how to determine experimental probability practically and compare it with theoretical probability.
  - Let students conduct experimental probability practically in groups. Refer to determining probability in Understanding mathematics textbook for class IX.
  - Use this link https://mathigon.org/polypad#random to access probability devices and conduct experimental probability.

- Watch these videos explaining probability: https://www.youtube.com/watch?v=KzfWUEJjG18 & Key Stage 4 - Probability

- Determine probabilities using space diagram (outcome chart) or tree diagram:
  - Students must know how to determine probability by using both a space diagram and a tree diagram. Discuss the example given on determining probability in Understanding mathematics textbook for class IX.

- Calculating probabilities of two independent events:
  - Introduce independent events with reasoning and some examples.
  - Demonstrate calculating the probability of two independent events by multiplying their individual probabilities; *P(A) x P(B) = P(A and B).*
  - Refer to Calculating Probability of Two Independent events in Understanding mathematics textbook for class IX and discuss the examples given.
  - Provide a few questions on calculating probability of two independent events.

- Note that the knowledge of independent events from this lesson will be linked to the Dependent events in class X.

E. Assessment

Performance Task 1

Solve relevant questions related to evaluating theoretical and experimental probability for an event. Refer relevant questions from the Understanding mathematics textbook for class IX.

Performance Task 2

Work on a variety of appropriate questions to test the competency to determine the theoretical probability of two events using both the tree diagram and outcome chart. Refer the questions in the Understanding mathematics textbook for Class IX.

Performance Task 3

Answer questions relevant to identifying independent events and determining probabilities of two independent events using both reasoning and calculation. Refer relevant questions from the Understanding mathematics textbook for class IX.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

  Write the theoretical probability of each.

  In basketball practise, Khamsum has the following free-throw accuracy:

  a. Standing 5 m from the basket, he is able to score the ball in the basket 50% of the time.
  b. From 10 m, he is able to score the ball half as often as from 5 m.
  c. From 15 m, he is able to score the ball with the same frequency he does from 10 m.

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics
- Introduction to Probability - https://www.youtube.com/History
- Utility and Scope - https://www.youtube.com/application or https://www.youtube.com/reallife
- Probability Devices - https://mathigon.org/polypad#random
- Basic Probability - https://www.youtube.com/watch?v=KzfWUEJjG18

G. Annexure

Refer IX-A1 for template to record achievements

Topic: X-A1 Radicals [300 minutes]

Introduction

Radicals are a common concept in algebra. We think of radicals as reversing the operation of an exponent. Hence, to negate the "square" of a number, we "square root" the number and to cancel the "cube" of a number, we "cube root" the number to find the base. Square roots are the most common type of radical used in algebra.

In the thirteenth century, the word "radix" was for square root and was abbreviated as "R" or R with a slash through the right leg of R, which looks like the Rx symbol at pharmacies.

The symbol that looks like a check or the radical sign with the "roof", originated in Germany in the 1500's. At first it looks like a musical note. if you had a long expression under a radical sign, the expression was put in parentheses, and later, placed with a line over it. https://www.purplemathss.com/Intro/Radical

Utility and Scope

Radicals have real world applications in fields like architecture, carpentry and masonry. Radical expressions are utilised in financial industries to calculate formulas for depreciation, home inflation and interest. Electrical engineers also use radical expressions for measurements and calculations.

Radical is also applied in the formula to calculate the speed of a tsunami with the relation $s = \sqrt{gd}$ (where $g = 9.8ms^{-2}$ and d is the depth of the ocean). https://www.youtube.com/application

A. Competency

- Demonstrate the understanding of properties of radicals in relation to rational numbers, and perform arithmetic operations on radicals.

B. Objectives

- Relate radicals with irrational numbers
- Convert an entire radical to mixed radical and vice versa

- Perform basic arithmetic operations on radical expressions

C. Essential Skills/Processes

- Estimating
- Analysing
- Computing

D. Learning Experiences

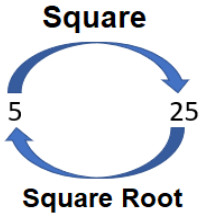
- Recap the concepts of rational and irrational numbers. Give some examples of irrational number such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt[3]{3}$ etc. Let students note that most of the radicals are irrational numbers.

- Introduce the concept of radicals: A radical form is also called a root form. Some of the radical forms are square root, cube root, fourth root and so on.

- Recall the concept of fractional exponents and relate them to the radicals:
  $5^{\frac{1}{2}} = \sqrt{5}$, $5^{\frac{3}{2}} = \sqrt[3]{5}$ and etc.
  Refer to Simplifying Radicals in Understanding Mathematics Textbook for Class X.

- Demonstrate interpretation of the entire radical to mixed radical and vice versa. Refer to Simplifying Radicals in Understanding Mathematics Textbook for Class X.
  - Provide relevant questions to practise the concept of converting the entire radical to mixed radical and vice versa. Ensure that students get enough practice since it is the basic part of operations with radicals.

- Before teaching operations on radicals, teach the following concepts:
  - Radical is the mathematical expression of an exponent.
    Square and square root cancel out each other because they are inverse operations,



using the power law: $\sqrt{25} = 25^{\frac{1}{2}} = (5^2)^{\frac{1}{2}} = 5^{2\times\frac{1}{2}} = 5$

Similarly, cube and cube root cancel out each other, i.e. using the power law:

$\sqrt[3]{27} = 27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3^{3\times\frac{1}{3}} = 3$

- Watch the video https://www.youtube.com/simplification . It contains a video lesson explaining the meaning of radical and simplifying square roots.
- Discuss the examples of Simplifying Radicals in Understanding mathematics textbook for Class X or use web link: Radical Expressions | CK-12 Foundation (ck12.org) or refer the notes below:

• One way to simplify a radical is to look for **perfect powers** as factors under the root sign and then take their roots outside the root sign.

For example: $\sqrt{18} = \sqrt{9\times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

$\sqrt{150} = \sqrt{25\times 6} = \sqrt{25} \times \sqrt{6} = 5\sqrt{6}$

$\sqrt[3]{54} = \sqrt[3]{27\times 2} = \sqrt[3]{27} \times \sqrt[3]{2} = 3\sqrt[3]{2}$

• The principles that apply to numerical **radical expressions** also apply to algebraic radical expressions.

For example: $\sqrt{b^8} = \sqrt{b^4 \times b^4} = b^4$ $\quad \sqrt[4]{b^8} = \sqrt[4]{b^2 \times b^2 \times b^2 \times b^2} = b^2$

$\sqrt[3]{b^8} = \sqrt[3]{b^2 \times b^2 \times b^2 \times b^2} = \sqrt[3]{b^2 \times b^2 \times b^2} \times \sqrt[3]{b^2} = b^2 \times \sqrt[3]{b^2}$

Assign relevant questions from Understanding mathematics textbook for class X.

- Demonstrate and explain the four operations involving radicals with relevant examples.
  - Explain 'like terms' in radicals by comparing them to the like terms in polynomials for addition and subtraction of radicals.
  - Demonstrate multiplying and dividing of both numerical and algebraic radicals.
    Refer to Operations with Radicals in Understanding mathematics Textbook for Class X.

E. Assessment

Performance Task 1

Interpret radicals from entire to mixed form (and vice versa).
Solve both numerical and algebraic problems to test the competencies.

Refer relevant questions from Understanding mathematics Textbook for Class X.

Performance Task 2

Carry out the four operations on radicals.
Solve both numerical and algebraic problems to test the competencies.
Refer relevant questions from Understanding mathematics Textbook for Class X.
Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure.

(*Note on Assessment tools and templates*)

- Reflective Questions

  The area of a rectangular room is 126 m^2 and the width is 22m. What is the length of the room

F. Resources

- Understanding mathematics textbook for Class
- Teachers Guide to Understanding mathematics, Class X
- National School Curriculum Framework for mathematics
- Introduction to Radicals - https://www.purplemaths.com/Intro/Radical
- Utility and Scope - https://www.youtube.com/application
- Simplifying Radicals - https://www.youtube.com/simplfying.
- Simplifying operation with Radicals - Radical Expressions | CK-12 Foundation
- Simplifying Radicals - https://www.youtube.com/watch?v=2mejAHKMBiM
- Simplifying Radicals - https://www.youtube.com/watch?v=Ef2gOQbDv7M
- Operations with Radicals - https://www.youtube.com/watch?v=4Gq3LPORQ-U

G. Annexure

i. Sample Assessment ToolTemplate to record assessment

CATEGORY	Beginning	Approaching	Meeting	Advancing	Exceeding
Mathematical Concepts	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written.	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows complete understanding of the mathematical concepts and is able to synthesise the idea in new problem situations.

Mathematical Errors	More than 75% of the steps and solutions have mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	90-100% of the steps and solutions have no mathematical errors.	Exhibits no mathematical error and is able to identify and correct errors of peers.
Mathematical Terminology and Notation	There is little use, or a lot of inappropriate use, of terminology and notation.	Correct terminology and notation are used, but it is sometimes not easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are always used, and are able to derive meaning and justify.
Neatness and organisation	The work appears sloppy and unorganised. It is hard to know what information goes together.	The work is presented in an organised fashion but may be hard to read at times.	The work is presented in a neat and organised fashion that is usually easy to read.	The work is presented in a neat, clear, organised fashion that is easy to read.	The work is presented in a neat, clear, organised fashion with key notes for self-reflection (underlining key points/ highlighting key points)
Completion	75% of the problems are not completed.	50% of the problems are not completed.	25% of the problems are not completed.	All problems are completed.	All problems are completed and explored related extended activities.
Team Work	Student did not work effectively with others.	Student cooperated with others, but needed prompting to stay on-task.	Student was an engaged partner but had trouble listening to others and/or working cooperatively.	Student was an engaged partner, listening to suggestions of others and observed working cooperatively throughout lesson.	Student consistently demonstrated exceptional collaboration and cooperation, actively contributing to the teams' success.

Strand(s): X-A1		Topic(s): Radical			
Competency: Demonstrate the understanding of radicals and performing arithmetic operations on radicals.					
Name of the student	Level of achievement				
	Beginning	Approaching	Meeting	Advancing	Exceeding

ii. Inductive and Deductive Reasoning Sample:

- Deductive Reasoning is used to *prove* that something is always true using information, knowledge and established facts.

  For example: To show the value of Negative Deductively;

  Let 'a' be any number with a negative exponent of 'n'. We can use the interpretation of the *quotient law* and *zero exponent* to determine the value of a power with negative exponent deductively as follows:

  $a^{-n} = a^{0-n} = a^0 \div a^n = 1 \div a^n = \frac{1}{a^n}$ Therefore: $a^{-n} = \frac{1}{a^n}$

- Inductive Reasoning is used to *verify* (but not *prove*) that something is true using examples and evaluating their general pattern.

  For example: To show the value of Negative Exponents inductively; let's observe the given set of examples and its pattern.

Power	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}
Value	27	9	3	1	$\frac{1}{3}$	?

  Observe that each value in the right-hand column is one third (divided by the base number) of the value above it. So, if you extend the pattern to 3^{-1} and 3^{-2}, 3^{-1} is $\frac{1}{3}$ and 3^{-2} is $\frac{1}{9}$ or $\frac{1}{3^2}$

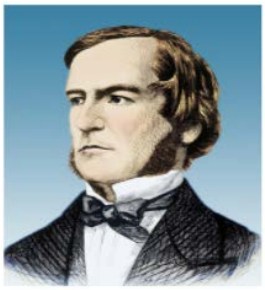
  Therefore, it looks like $3^{-n} = \frac{1}{3^n}$ and, for a general base *'a'*: $a^{-n} = \frac{1}{a^n}$

For more information and examples of Inductive and Deductive reasonings refer Reasoning in Understanding mathematics textbook for Class X or other online resources.

Topic: X-A2 Matrices [400 minutes]

Introduction

In mathematics, a matrix (plural matrices) is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns. Matrices are commonly written in box brackets. The horizontal and vertical lines of entries in a matrix are called rows and columns, respectively. The matrix has a long Introduction of application in solving linear equations. They were known as arrays until the 1800's.


Cuthbert Edmund Cullis


James Joseph Sylvester

The term "matrix" (Latin for "womb", derived from *mater*—mother) was coined by James Joseph Sylvester in 1850, who understood a matrix as an object giving rise to a number of determinants today called minors, that is to say, determinants of smaller matrices that are derived from the original one by removing columns and rows. An English mathematician named Cuthbert Edmund Cullis was the first to use modern bracket notation for matrices in 1913. Introduction to matrix

Utility and Scope

- Graphic software such as Adobe Photoshop on your personal computer uses matrices to process linear transformations to render images.
- In physics related applications, matrices are used in the study of electrical circuits, quantum mechanics and optics.
- Engineers use matrices to model physical systems and perform accurate calculations needed for complex mechanics to work. Electronics networks, aeroplanes and spacecraft, and in chemical engineering all require perfectly calibrated computations which are obtained from matrix transformations.
- Many IT companies also use matrices as data structures to track user information, perform search queries, and manage databases. In the world of information security, many systems are designed to work with matrices.
- Matrices are used in the compression of electronic information, for example in the storage of biometric data in the new Identity Card in Mauritius.

- In geology, matrices are used for making seismic surveys. They are used for plotting graphs, statistics and also to do scientific studies and research in almost different fields.

A. Competencies

- Evaluate properties of matrices to classify into different types and use operations on matrices to apply to real life situations.
- Interpret digraphs to matrices and use the relation to solve real-life based network problems involving one-stopover and/or two-stopover trips.

B. Objectives

- Describe matrices and identify the parts of the matrix (e.g., row, column, dimension, location and element).
- Classify types of matrices (square matrix, column matrix, row matrix, identity matrix).
- Justify inductively if two matrices can be added, subtracted, or multiplied by checking the orders of the matrices.
- Apply operations on matrices (addition, subtraction and multiplication) in problem situations.
- Represent a network as a matrix and interpret a matrix in terms of a corresponding network situation.

C. Essential Skills/Processes

- conceptualising
- Reasoning
- Computing
- Applying
- Analysing

D. Learning Experiences

- Use examples in and around the class to introduce matrices, such as the class time table, calendar etc. Describe matrices and identify the parts of matrix, e.g., rows, column, dimension [limit the dimension of matrix up to 3 by 3], element and its location.

- Design a group activity to conceptualise the concept of matrix and parts of matrices (refer to Introducing Matrices in Understanding mathematics textbook for class X mathematics textbook).
- Define matrix and name the parts through the activity. Use the video link Parts of matrix to understand the concept better (Includes definition and parts of matrix, and also explains how to perform addition, subtraction and multiplication of matrices).

- Classify types of matrices (limit to square matrix, column matrix, row matrix and identity matrix)
  - Design a task by listing examples of matrices (limit to square matrix, column matrix, row matrix, identity matrix) and let children generalise different features from various examples. Deduce the types of matrices based on the features. Use the link Types of matrices to understand the concept better.

- Verify the addition and subtraction rule inductively to check if two matrices can be added or subtracted.
  - Explain the limitations involved while adding and subtracting matrices (condition for adding and subtracting matrices).
  - Design an activity to let learners verify inductively if matrices can be added or subtracted. (Refer Multiplying Matrices in Understanding mathematics textbook for Class X).
  - Highlight upon the inductive reasoning being used to draw conclusion on the addition and subtraction rules (Refer Inductive and Deductive Reasoning Sample given below the annexure A1).

- Verify the multiplication rule inductively to check if two matrices can be multiplied:
  - Explain the limitations involved while multiplying matrices (condition for multiplying matrices).
  - Design an activity to let learners verify inductively if matrices can be multiplied or not. (Refer Multiplying Matrices in Understanding mathematics textbook for Class X).
  - Highlight upon the inductive reasoning being used to draw conclusion on the multiplication rule (Refer Inductive and Deductive Reasoning Sample given below the annexure A1).

- Apply operations on matrices (addition, subtraction and multiplication) in problem situations.
  - The web link Adding matrices contains video lessons on adding matrices. Relate the concept to subtracting matrices.
  - The web link Multiplying matrices contains video lessons multiplying matrices.
  - Design a task to evaluate learners' understanding on operation with matrices or use web link: Adding and Subtracting Matrices and Multiplying Matrices.

- Represent a network as a matrix and interpret a matrix in terms of a corresponding network situation.
  - Demonstrate understanding on the relationship between network and matrix. Represent the network as a matrix and vice versa (include real life situations like the network of a bus visiting various stations).
  - Discuss the determination of one-stopover and two-stopover trips by both calculations and tracing, and relate the findings to real life situations. Use school-based examples like the path of a student moving in the school from one point to another in a day. The link Matrices and networks contains a video that explains conversion of networks into matrices and vice versa. (Refer to Understanding mathematics textbook for class X).

E. Assessment

Performance Task 1

Carry out activities to identify and describe different types of matrices. Refer to Introducing Matrices in Understanding mathematics textbook for class X or other resources.

Performance Task 2

Solve tasks on adding, subtracting and multiplying of matrices including both direct and situational problems. Refer to Understanding mathematics textbook for class X or other resources.

Performance Task 3

Work on questions to interpret matrices to networks and vice versa based on real life context such as transportation terminals, games, trips etc. Solve problems on determining one-stopover and two-stopover trips by both calculations and tracing, and relate the findings to real life context.

Discuss real-world based questions like 'why would more one way traffic routes restrict access from one point to another in a town'. Refer to Networks in Understanding mathematics textbook for class X or other resources.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

i. Lugar theatre charges the following prices of a ticket: Nu 200 for balcony, Nu 150 for the first class and Nu 120 for the second class. Whereas in Trowa theatre, the charges are: Nu 180 for the balcony, Nu 150 for the first class and Nu 120 for the second class. How could you organise this data to easily compare the prices? Write the dimension of the matrix you have created.

ii. Travelling in Bhutan:

a. Select four communities (cities, towns, or villages) in Bhutan. Sketch the outline of Bhutan below and then place the communities in their approximate locations.

b. Decide on four communities you would like to connect by roads. Create a network of four roads to connect the communities. The communities do not have to be connected by roads in reality.

c. Build a network to display the roads between those pairs of adjacent communities you chose in part ii.a.

d. Create an adjacency matrix to describe your network in part c.

e. Which places CANNOT be connected by two-stopover trips? Explain.

f. Create two other problems you could solve with your matrix or network. Provide a solution for each problem.

F. Resources

- Understanding mathematics textbook for Class X
- Teachers Guide to Understanding mathematics, Class X

- National School Curriculum Framework for mathematics
- Introduction to Matrices - Introduction to matrix
- Parts of a matrix – Parts of matrix
- Types of matrices - Types of matrices
- Adding matrices – Adding matrices
- Multiplying matrices - Multiplying matrices
- Activity - Adding and Subtracting Matrices and Multiplying Matrices.
- Conversion of Networks into Matrices and vice versa - Matrices and networks
- Complete package lesson on solving systems of linear equations from e-learning Mathematics key stage 4 - Network

G. Annexure

Refer X-A1 for template to record achievements

Topic: X-A3 Commercial Mathematics [400 minutes]

Introduction

Commercial maths deals with all those concepts that people use in their everyday life. The word commercial itself means the economical world or something relating to buying or selling. It involves the concept of profit and loss, discounts, marked price, simple and compound interest, taxes, ratio and proportion, percentages, and everything that revolves around money.

Introduction to Commercial maths

Utility and Scope

Commercial mathematics is used in accounting, inventory management, marketing, sales forecasting, and financial analysis. mathematics typically used in commerce includes elementary arithmetic, elementary algebra, statistics and probability.

A. Competencies

- Solve situational problems on simple and compound interests, dividends and stocks to evaluate better money-making options.

B. Objectives

- Demonstrate understanding of the long-term differences between simple and compound interest.
- Investigate both investments and financing situations.
- Solve problems related to dividends and stocks using concepts of dividend, stock, dividend rate, face value, market value and yield percentage.

C. Essential Skills/Processes

- conceptualising
- Applying
- Reasoning
- Describing

D. Learning Experiences

- Recall the concept of Simple Interest with a few examples. Refer to the video link: Simple Interest. Refer to Simple Interest in Understanding mathematics textbook for class VIII or other relevant resources.

- Introduce and explain the terminologies of Compound Interest. Refer the video link: Compound Interest Formula
  - Discuss and explain the solved examples of compound interest in Understanding mathematics textbook for class X.
  - Assign relevant questions as a practice in the class (Refer to compound Interest in Understanding mathematics textbook for class X or other relevant resources).
  - Discuss about payment options and investments to relate commercial maths in real life situations (Refer to Using Commercial maths in Understanding mathematics textbook for class X or other relevant resources).
  - This link Key Stage 4 - Compound Interest explains the concepts of compound interest.
  - Suggested activity:
    Invite a resource person to class from any financial institution to give insights on banking.

- Discuss to compare between the simple interest and compound interest in terms of making money. Refer to the link: Simple vs. Compound Interest.

- Introduce and explain the terminologies of Dividends and Stocks and relate the terminologies in solving problems. Refer to the video link: Dividend Stocks Explained for Beginners (Refer to Dividends and Stocks in Understanding mathematics textbook for class X or other relevant resources).
  - Introduce the concept of dividends and stocks using real life examples.
  - Demonstrate calculations on dividends and stocks.
  - Let students practise the concepts by solving the questions as a classwork (Refer to Dividends and Stocks in Understanding mathematics textbook for X and other relevant resources).

E. Assessment

Performance Task 1

Answer questions based on real life context related to finding Compound Interest, amount, principal, rate and time. (Refer to Compound Interest in Understanding mathematics for class X and other relevant resources).

Performance Task 2

Carry out relevant questions on Dividends and Stocks. (Refer to Dividends and Stocks in Understanding mathematics for class X and other relevant resources).

Performance Task 3

Work on questions based on real life situations involving both payment options (on loans) and investments (money making opportunities) related to commercial maths concepts learned. (Refer to Using Commercial maths in Understanding mathematics textbook for class X and other relevant resources).

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions
  i. Suppose that when you were born, your relatives had been able to invest Nu 10,000 in an interest-earning account or in stocks. The money would have grown in value until you finished school at age 18.
     a. Would it be possible to invest Nu 10,000 in an interest bearing account earning 6% p.a. compounded annually so that it would grow to a value of Nu 40,000 in 18 years? Explain.
     b. Would more frequent compounding allow the investment to grow to Nu 40,000? How do you know?
     c. What interest rate would cause the investment to grow to Nu 40,000? Show your work. Find at least one more answer.
     d. Why is there more than one answer to part c)?

  ii. Suppose Nu 10,000 was invested in a stock that yielded an annual dividend of 8.33% for 18 years. The stock had a face value of Nu 100 and was

purchased at par. The stock was sold at Nu 150 per share after 18 years. The dividend earnings were not reinvested each year.

a. Would the value of the investment reach Nu 40,000 in 18 years? Explain.
b. If the answer to part a) is no, determine a selling price for the stock that would result in a total investment value of Nu 40,000. Show your work.

iii. Would you choose the investment option described in question i. a. or the option in question ii. a.? Why?
Why might someone else choose the option you did not choose?

F. Resources

- Understanding mathematics textbook for Class X
- Teachers Guide to Understanding mathematics, Class X
- National School Curriculum Framework for mathematics
- Introduction to Commercial mathematics – Introduction to Commercial maths
- Simple interest: Simple Interest
- Compound Interest: Compound Interest Formula
- Concept of Compound Interest - Key Stage 4 - Compound Interest
- Simple Interest versus of Compound Interest: Simple vs. Compound Interest
- Dividend and Stock: Dividend Stocks Explained for Beginners

G. Annexure

Refer X-A1 for template to record achievements

Topic: X-B1 Linear Functions and Relations [200 minutes]

Introduction

A linear relationship (or linear association) is a statistical term used to describe a straight-line relationship between two variables. Linear relationships can be expressed either in a graphical format where the variable and the constant are connected via a straight line or in a mathematical format where the independent variable is multiplied by the slope coefficient, added by a constant, which determines the dependent variable; y=mx+b, (the slope and y-intercept form). Basic Linear Functions and What are functions?

Utility and Scope

The following are some of the examples in which applications of linear equations are used in real life:

- Geometry related problems can be solved
- It can be used to solve age related problems
- It is used to calculate speed, distance and time of a moving object
- Work, time and wages problems can be solved
- Problems based on force and pressure can be solved
- It is used to calculate money and percentage related problems
  Application of Linear Equations

A. Competencies

- Identify properties of relations and functions to classify through algebraic expressions, word statements, tables of values and graphs.
- Devise applications of linear functions and its graphical representations to interpret and solve real life problems.

B. Objectives

- Demonstrate an understanding of a relation and a function through examples.
- Convert equations of line from one form to another (standard form to slope and y-intercept form and vice versa).
- Devise applications of linear functions in real life situations.

- Create graphs for given information in a variety of formats using MS Excel/ GeoGebra/ Graphmatica and others.

C. Essential Skills/Processes

- Applying
- Graphing
- Reasoning
- Communicating

D. Learning Experiences

- Recall the concept of dependent and independent variables from class IX. Explain how to identify dependent variables and independent variables from an equation or a function. Refer the video link: Dependent and Independent Variables
- Introduce three forms of relations in function notation: Linear, Quadratic and Exponential functions. Students should be able to recognize a form of a function by looking at the function expressions. Refer to the video link : Linear, Quadratic, and Exponential Models. Or refer to the exposition under Linear Functions in Understanding mathematics textbook for class X.
- Inductively discuss the properties of functions and relations (using table of values, set of ordered pairs, function rule using words, an algebraic equation and a graph as presented below).
    - Explain the concept of relation: “Relation describes how two things (variables) are connected (related)”. Explain with some examples of relations such as $y > 2x$, $y = 2x$, $y < x+1$, etc.
    - Show the value/s of y for each value of x. For instance, consider the relation $y > 2x$. For $x = 3$, $y > 2(3) \rightarrow y > 6$, which means, $y = 6.01, 6.1, 7, 8, 9, 10, 10.1, \ldots$ are all solutions to the relation. Students should also know that x=3 is a value of input variable and y values are the output variables.
    - From the above examples, connect to the introduction of the concept of a function: “A function is a special type of relation where there is only one output variable for one input variable.” Introduce the function notation and

let students note that, y = 2x+1, for instance, is an equation and f(x) = 2x+1 is the equation in function form.

- Explain how to identify a function relation from mapping notations, table of values, set of ordered pairs and graphs (vertical line test).
- Highlight upon the deductive reasoning being used to identify a function from a relation. Refer to the video link: Function vs. Not a Function . Or refer to the Inductive and Deductive Reasoning Sample given below the annexure A1.

- Recall and revise the concept of slope and y-intercept form and standard form. Discuss how to transform standard form to slope and y-intercept form with a few examples.
  - Introduce the concept of 'one variable as a function of the other': x as a function of y and y as a function of x. Discuss relevant examples and provide at least 3 equations for students to practise.
  - Note that if a function is in 'y as a function of x' form then, x is the independent variable (known), and y is the dependent variable (unknown). Discuss relevant examples and explain which variable is independent (known) and which variable is dependent (unknown).

- Contextualise the application of Linear Functions to solve real life problems. Recall and demonstrate the use of the line of best fit for a graphical representation to make predictions using interpolation and extrapolation with relevant example questions. (Refer to Application of Linear Functions from Understanding mathematics textbook for class X).
  - Facilitate students to practise graphing equations using GeoGebra or any other software.

E. Assessment

Performance Task 1

Carry out activities to express differences in properties of relations and functions. Classify functions from relations through algebraic expressions, word statements, tables of values and graphs. Refer to Linear Functions in Understanding mathematics textbook for class X.

Performance Task 2

Solve questions to interpret a given situational problem into a linear equation and use applications of linear functions and graphs to solve problems. Refer to Application of Linear Functions in Understanding mathematics textbook for class X. Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

Women's 100 m Sprint Winning Times

Year	Name	Country	Time (s)
1928	Elizabeth Robinson	USA	12.2
1932	Stanislawa Walasiewicz	POL	11.9
1936	Helen Stephens	USA	11.5
1948	Fanny Blankers-Koen	NED	11.9
1952	Marjorie Jackson	AUS	11.5
1956	Betty Cuthbert	AUS	11.5
1960	Wilma Rudolph	USA	11.0
1964	Wyomia Tyus	USA	11.4
1968	Wyomia Tyus	USA	11.0
1972	Renate Stecher	GDR	11.07
1976	Annegret Richter	FRG	11.08
1980	Lyudmila Kondratyeva	URS	11.06
1984	Evelyn Ashford	USA	10.97
1988	Florence Griffith-Joyner	USA	10.54
1992	Gail Devers	USA	10.82
1996	Gail Devers	USA	10.94
2000	Marion Jones	USA	10.75
2004	Yuliya Nesterenko	BLR	10.93

Men's 100 m Sprint Winning Times

Name	Country	Time (s)
Percy Williams	CAN	10.8
Eddie Tolan	USA	10.3
Jesse Owens	USA	10.3
Harrison Dillard	USA	10.3
Lindy Remigino	USA	10.4
Bobby Morrow	USA	10.5
Armin Hary	FRG	10.2
Robert Hayes	USA	10.0
Jim Hines	USA	9.9
Valeriy Borzov	URS	10.14
Hassely Crawford	URS	10.08
Allan Wells	GBR	10.25
Carl Lewis	USA	9.99
Carl Lewis	USA	9.92
Linford Christie	GBR	9.96
Donovan Bailey	CAN	9.84
Maurice Greene	USA	9.87
Justin Gatlin	USA	9.85

The winner of the Olympic 100 m sprint is often called the fastest human on earth. For many years, the winning times for this race have been declining for both men and women. Some people claim that the female sprinters' times have been improving more quickly than the male times. They say that eventually the fastest human on earth will be a woman.

a. Use the data to create a scatter plot for the 100 m winning times for women.
   i. Draw a line of best fit.
   ii. Determine the equation of the line of best fit.
b. Repeat part 'a.' for men on the same grid.
c. i. Use the equations from parts a. and b. to predict the Olympic year in which the fastest human on earth will be a woman.

ii. What do you predict the men's and women's winning times will be that year?
iii. Do you think this is likely to happen? Why or why not?

F. Resources

- Understanding mathematics textbook for Class X
- Teachers Guide to Understanding mathematics, Class X
- National School Curriculum Framework for mathematics
- Introduction to Linear Functions and Relations - Basic Linear Functions and What are functions?
- Utility and Scope - Application of Linear Equations
- Independent and dependent variables - Dependent and Independent Variables
- Three forms of relations: Linear, Quadratic, and Exponential Models
- Function and not a function: Function vs. Not a Function

G. Annexure

Refer X-A1 for template to record achievements

Topic: X-B2 Solving Systems of Linear Equations [450 minutes]

Introduction

In mathematics, a system of linear equations (or linear system) is a collection of one or more linear equations involving the same set of variables. It is also a system of three equations in the three variables x, y and z. A solution to a linear system is an assignment of values to the variables such that all the equations are simultaneously satisfied.


Girolamo Cardano

The study of systems of linear equations first introduced by the Babylonians around 1800 BC. For solving linear equations systems, Girolamo Cardano, an Italian physician, mathematician and astrologer, constructed a simple rule for two linear equations with two unknowns around 1550 AD. He also wrote a book named *Ars magna* (*The Great Art; or, The Rules of Algebra*) which is one of the cornerstones in the history of algebra. v07i01-12.pdf (tojsat.net)

Utility and Scope

Some of these applications of linear equations are:

- Geometry problems by using two variables.
- Money problems by using two variables
- A mixture of problems by using two variables.
- Distance-Rate-Time problems by using two variables.
- Application of linear equation in business and economics https://applications-of-linear-equations

A. Competency

- Demonstrate graphical and algebraic solutions to a system of linear equations and apply its interpretation to real life contexts.

B. Objectives

- Demonstrate graphical solutions for a system of linear equations.
- Analyse a variety of situations and model them into algebraic equations.

- Solve a system of linear equations by comparison method/ substitution method/ elimination method and relate to the real-life contexts.

C. Essential Skills/Processes

- Applying
- Graphing
- Reasoning
- Communicating

D. Learning Experiences

- Solving system of linear equations graphically:
  - Revise the methods to graph linear equations of slope and y-intercept form and standard form. One may use GeoGebra for graphing to save time but make sure students know how to graph.
  - Spell out that "Two or more linear equations form a system of linear equations."
  - Explain that the solution of a system of linear equations is the intersection point of the graphs of the equations. The point of intersection is the point where the values of the equations become equal. Then relate to solving the system of linear equations algebraically.
  - Demonstrate how to solve the system of linear equations graphically using examples and verify that the point of intersection correlates to the solution of the two equations.
  - Provide some hands-on practice questions from Solving a System of Linear Equations from Understanding mathematics textbook for class IX. https://www.youtube.com/graphing
  - Share the video link: Graphically . This video explains the processes to find the solutions of the system of equations graphically.

- Solve systems of linear equations by comparison/ substitution/ elimination strategy. (note: avoid specifying a strategy while developing test items).
  - Demonstrate solving systems of linear equations by comparison strategy, substitution strategy and elimination strategy.
  - Design a task to let learners explore other methods of solving a system of linear equations (E.g.; matrix).

- Refer the links: Comparison Method, Substitution Method , and Elimination Method

- Discuss the appropriateness of the different methods for a given system of linear equations (relate to the two forms of the linear equations).
- Analyse a variety of situational problems and model them into algebraic equations.
  - Demonstrate process of modelling a situation into algebraic equations.
  - Evaluate the process of modelling situations into algebraic equations using relevant questions from Solving Systems of Linear Equations from Understanding mathematics textbook for Class X.
- Evaluate the meaning of the solutions derived from the system of linear equations in terms of the real-life context.

E. Assessment

Performance Task 1

Carry out activities to test the competency of applying the graphical, matrix and the three strategies to a given system of linear equations. Refer to Understanding mathematics textbook for Class IX and X. Use the template given in the annexure to let learners choose a suitable strategy to approach the solution of the system of linear equations. (The template also provides room for learners to explore other methods of solving the problem)

Performance Task 2

Carry out activities to test the competency of interpreting real life situational problems into systems of linear equations and make sense of the solutions in the real-life context. Refer to Understanding mathematics textbook for Class X.
Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions
  On Friday, an ice cream shop sold 15 small chocolate cones and 25 large chocolate cones. It also sold 20 small vanilla cones and 50 large vanilla cones. The shop's chocolate sales for the day totalled Nu 1700 and its vanilla sales

totalled Nu 3150. How much did the shop charge for a small cone versus an X-large cone?

F. Resources

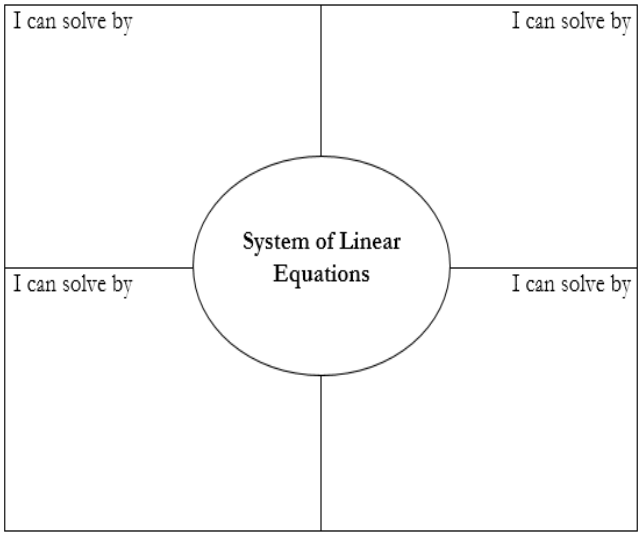
- Understanding mathematics textbook for Class X
- Teachers Guide to Understanding mathematics, Class X
- National School Curriculum Framework for mathematics
- Introduction to Systems of Linear Equations - v07i01-12.pdf (tojsat.net)
- Utility and Scope - https://applications-of-linear-equations
- Graphical Solutions to Systems of Equations - https://www.youtube.com/graphing or https://www.youtube.com/watch?
- Solving a System of Linear Equations using Comparison Method: Comparison Method
- Solving a System of Linear Equations using Substitution Method :Substitution Method
- Solving a System of Linear Equations using Elimination Method : Elimination Method
- Complete package lesson on solving systems of linear equations from e-learning mathematics Key Stage 4 - Solving Systems of Linear Equations

G. Annexure

Refer X-A1 for template to record achievements

- Template:

Template



Topic: X-B3 Graphs of Quadratic Functions [450 minutes]

Introduction

The single defining feature of quadratic functions is that they are of the second order, or of degree two polynomials. This means that in all quadratic functions, the highest exponent of x in a non-zero term is equal to two.

It is often claimed that the Babylonians (about 400 BC) were the first to solve quadratic equations. This is an oversimplification, for the Babylonians had no notion of 'equation'. What they did develop was an algorithmic approach to solving problems which, in our terminology, would give rise to a quadratic equation. The method is essentially one of completing the square. However, all Babylonian problems had answers which were positive (more accurately unsigned) quantities since the usual answer was a length.


Euclid of Alexandria

In about 300 BC Euclid developed a geometrical approach which, although later mathematicians used it to solve quadratic equations, amounted to finding a length which in our notation was the root of a quadratic equation. Euclid had no notion of equation, coefficients etc. but worked with purely geometrical quantities. Introduction to Quadratic Functions Quadratic Functions.

Utility and Scope

- Quadratic equations are often used to calculate business profit. Even when dealing with small products, you will need to solve a quadratic equation to determine how many of them will make a profit.
- Whenever construction is taking place, constructors use quadratic equations to determine the area. People also calculate the areas of other things such as a piece of land and boxes.
- Finding the speed of a given object may lead to the formation of a quadratic equation. For example, kayakers use these equations when determining the amount of speed they have to apply while going up or down a river.
- One of the biggest applications of quadratic equations in Agriculture is in the arrangement of boundaries. For example, calculating the areas of pens that will produce high yields involves finding the areas.
- Quadratic equations are often used by the military or law enforcement to determine the speed of moving objects such as cars and planes. The military can

also use them to determine the distance between them and an approaching enemy. In addition, the military uses quadratic equations to predict where tanks or artillery will land.

- Engineers apply quadratic equations more than any other career. Quadratic equations are important when designing curved equipment such as auto-bodies. Brake systems are designed by automotive engineers by solving equations that arise.

A. Competency

- Use graphs of quadratic functions to interpret real life situations and correlate the graphical solutions to real-life contexts.

B. Objectives

- Classify and interpret the three forms of quadratic functions to evaluate if they are equivalent functions.
- Sketch the graph of quadratic functions in standard, factored, and vertex form and graphically solve related real-life problems.
- Transform quadratic functions in standard and vertex form into mapping notations and vice versa to represent the function graphically.
- Create an equation from the graph of quadratic function.

C. Essential Skills/Processes

- Computing
- Applying
- Graphing
- Recognizing
- Creating
- Evaluating

D. Learning Experiences

- Classify the three forms of quadratic functions and study their differences in algebraic form. Demonstrate how to check if a given set of functions (in different forms) are equivalent by evaluating their graphs, algebraic expressions and comparing their input/output values. Refer to Forms of Quadratic Functions in

Understanding mathematics textbook for Class X. Refer the link: Forms of Quadratic function.

- Demonstrate graphing of quadratic functions in factored form manually and verify using appropriate software such as GeoGebra, graphmatica, geometer's sketchpad, etc. Interpret real life situational problems into quadratic equations and use graphical solutions to relate to real life context. Refer to Graphs of Quadratic Functions in Factored Form in Understanding mathematics textbook for class X or the video link Factored Form.

- Demonstrate graphing of quadratic functions in vertex form manually and verify using appropriate graphing software such as GeoGebra, graphmatica, geometer's sketchpad, etc. the web link: Vertex form contains a video on graphing quadratic functions in vertex form.
  - Use $y=x^2$ as the basis to connect algebraic and geometric transformations to draw the graph.
  - Demonstrate geometric transformation of dilation (a), horizontal translation (h) and vertical translation (v) manually and verify using appropriate graphing software such as GeoGebra, graphmatica, gsp, etc.
  - Use mapping notations to frame quadratic equations in vertex form (and vice versa).
  - Discuss the appropriateness of the name 'vertex form'. Highlight upon the inductive reasoning being used to draw conclusion on deriving the coordinates of the vertex directly from the vertex form of a quadratic equation (Refer Inductive and Deductive Reasoning Sample given below the annexure A1).

- Refer relevant example questions from Relating Graphs of Quadratic Functions in Understanding mathematics textbook for class X.

- Refer the link Creating Equation to explain how to create an equation from the given graph of quadratic function.

E. Assessment

Performance Task 1

Carry out activities to classify and interpret the three forms of quadratic functions to check if a given set of functions (in different forms) are equivalent by evaluating their graphs, algebraic expressions and comparing their input/output values. Refer questions from Forms of Quadratic Functions in Understanding mathematics textbook for Class X.

Performance Task 2

Carry out activities to sketch the graph of quadratic functions in factored form, interpret situational problems to factored form of quadratic functions and find graphical solutions which relate to real life context.

Refer questions from Graphs of Quadratic Functions in Factored Form in Understanding mathematics textbook for class X.

Performance Task 3

Solve questions to transform quadratic functions in vertex form into mapping notations (and vice versa) to represent the function graphically.

Refer questions from Relating Graphs of Quadratic Functions in Understanding mathematics textbook for class X.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

  A firecracker is fired from the ground. The height of the firecracker at a given time is modelled by the following function, where h is the height in metres and t is the time in seconds:

  $h(t) = -5t^2 + 50t$

  i. Predict how the graph of this function will compare to the graph of $y = x^2$

  ii. a. Represent this function both in factored form and in vertex form.

  b. What does each form quickly tell you?

  iii. State the coordinates of: a) the x-intercepts b) the vertex

  iv. What do the zeros of the function represent in this situation?

v. Create and solve two problems related to this situation.
   a. One problem should be easy to solve using the graph.
   b. The other problem should be solved using a quadratic equation.
   c. One problem should have only one solution and the other should have two solutions.

vi. Suppose the height of a different firecracker were given by the following function: **$h(t) = -6t^2 + 48t - 54$**
   a. How would the graph of the path look different from the graph of the first firecracker?
   b. How is the situation different from the first situation?

F. Resources

- Understanding mathematics textbook for Class X
- Teachers Guide to Understanding mathematics textbook for Class X
- National School Curriculum Framework for mathematics
- Introduction to Quadratic Functions - Quadratic Functions.
- Quadratic Function - Forms, Examples, Conversion from one form to another - Forms of Quadratic function.
- Graphing Quadratic Functions in factored form - Factored Form.
- Graphing Quadratic Functions in vertex form - Vertex form.
- Creating equation from graphs Quadratic Functions - Creating Equation.

G. Annexure

Refer X-A1 for template to record achievements

Topic: X-B4 Solving Non-Linear Equations [500 minutes]

Introduction

Non-linear equations, as it says in its name, are any functions that are not linear, for example, quadratic, circle and exponential functions. A nonlinear function is any function that can be expressed using a degree of 2 or more single variable polynomials.

Although as successful the resulting scientific paradigm of linear equations and its applications proved to be, most of the patterns in life are non-linear. After all, it took only a few small shifts in the genes of bacteria in Chinese fowl sometime around 1917 to cause the influenza epidemic that ravaged America and Europe at the end of World War I. Small causes can have enormous effects. But non-linear equations were much more time-consuming to solve.
With the desktop computer revolution of the late 1970s, it was suddenly possible for scientists to model their topics with non-linear equations, with which small causes can have large effects. The scientists using them quickly made two discoveries. First, non-linear equations created a more accurate picture of how things in the world behaved. Second, non-linear modelling produced strikingly similar results across disciplines, which would drive the creation of complexity theory. And the result was a non-linear world view, 'a new way of seeing reality'.
https://www.cuemaths.com/nonlinear-functions/

Utility and Scope

Solving Non-linear Equations are used in interpreting non-linear patterns in everyday life and relating its solutions to real life contexts, such as calculating areas of a certain land, determining a product's profit or formulating the speed of an object like throwing a ball etc. Nonlinear application.

A. Competency

- Demonstrate factorization of non-linear equations to determine their roots and/or solutions, and to apply their interpretation in solving real life problems.

B. Objectives

- Develop factoring strategies for polynomials in one variable that are products of binomials (degree one).

- Use the roots (x-intercepts) to determine the solutions of quadratic equations and apply its interpretation to real life contexts.

C. Essential Skills/Processes

- Strategize
- Applying
- Analysing
- Interpreting

D. Learning Experiences

- Conduct a pre-assessment test on dividing a trinomial of degree 2 by a binomial. Connect to the factoring of polynomials: Explain that the factors are represented by the dimensions of the rectangle and the product is the area of the rectangle.

- Demonstrate the factoring of the polynomials of each type:
  - Factoring by using a common factor: e.g., $6x^2 - 3x$. Students should note that this method can be used only for binomials of degree 2 having common factors. Refer to Factoring Quadratic Expressions in Understanding mathematics Textbook for class X.
  - This link contains a video lesson on solving quadratic equations by factoring - Key Stage 4 - Solving quadratic equations by factoring.
  - The following link contains a video lesson on factoring by using a common factor. Common Factorization Method
  - Factoring the difference of squares using the formula, $a^2 - b^2 = (a + b)(a - b)$: e.g., $x^2 - 4$. (Recall on how to factor using tiles before introducing the formula). Refer to Factoring Quadratic Expressions in Understanding mathematics Textbook for class X.
  - The following link provides an explanation of how to factor differences of squares. https://www.youtube.com/watch
  - Factoring of trinomials of degree 2: First show the factoring of this type using tiles. Then introduce the various strategies of factoring:
    - Algebraic model. (Refer to Factoring Quadratic Expressions from Understanding mathematics textbook for class X)

- Sum product rule. Refer to the link: https://www.youtube.com/watch?v=TlV2XSJure8 to know more about factoring using the sum product rule.
- The following link contains a video lesson on factoring and solving of polynomials using a different strategy: Solving and Factoring

- Solving Quadratic Equations by Factoring:
  - Demonstrate examples of solving quadratic equations by factoring using the zero-product rule. After solving, graph the quadratic equation (you may use GeoGebra or Graphmatica) and show that the solutions are the x-intercepts of the parabola. Students should note that the x-intercepts are also called as the roots/zeros of the functions.
  - Interpret situational problems into quadratic functions to evaluate their solutions (roots) and relate to real-life contexts.
    Refer to Solving Quadratic Equations by Factoring from Understanding mathematics textbook for Class X.

Refer relevant example questions from Relating Graphs of Quadratic Functions in Understanding mathematics textbook for class X.

E. Assessment

Performance Task 1

Solve questions on developing factoring strategies for polynomials in one variable that are products of binomials (degree one). Refer questions from Factoring Quadratic Expressions in Understanding mathematics Textbook for class X or other relevant resources.

Performance Task 2

Work on questions to determine the solutions/roots of quadratic equations and interpret situational problems into quadratic functions to relate their solutions (roots) to real-life contexts. Refer questions from Understanding mathematics textbook for class X or other resources.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

  A ladder is leaning against a wall. The top of the ladder is 9.4 m up the wall. The ladder is 6.6 m longer than the distance from the bottom of the wall to the bottom of the ladder. How far is it from the bottom of the wall to the bottom of the ladder?

F. Resources

- Understanding mathematics textbook for Class X
- Teachers Guide to Understanding mathematics textbook for Class X
- National School Curriculum Framework for mathematics
- Introduction Nonlinear Equations - https://www.cuemaths.com/nonlinear-functions/
- Utility and Scope - Nonlinear applications
- Solving quadratic equation by factoring - Key Stage - Solving quadratic equation by factoring.
- Factoring by using a common factor - Common Factorization Method
- How to factor differences of squares - Differences of squares
- Factoring using the sum product rule - https://www.youtube.com/watch?v=TlV2XSJure8
- Factoring and solving of quadratic equations using a different strategy - Solving and Factoring/Formula
- Graphical calculation of roots - https://www.youtube.com/watch?v=Na3po6pA958

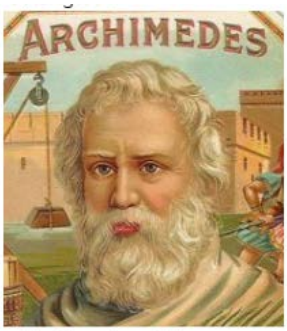
G. Annexure

Refer X-A1 for template to record achievements

Topic: X-C1 Surface Area of 3-D Shapes [600 minutes]

Introduction

The area of the outside surface of a three-dimensional shape or a solid is called Surface Area of that surface. For example, a rectangular prism has six rectangular bases and lateral faces. Thus, the total surface area is equal to the sum of the areas of all six rectangles.



Archimedes of Syracuse

The first recorded use of areas was in ancient Babylon, where they used it to measure the amount of land that was owned by different populations for taxation purposes. Later in 287 BC, the great mathematician Archimedes of Syracuse from Greece discovered the area and the perimeter of the circle and the relationship between spheres. Although Archimedes wasn't the first to realise the fact. He was, as far as we know, the first to prove it formally. He also gave the earliest proofs for the volume of the sphere and surface area. Surface Area. Click the link 3-D shape nets to demonstrate the concept surface area using nets of the shape.

Utility and Scope

Application of surface area of 3-D shapes can be used in estimating and calculating the resources needed in real life situations:

- How much paint will it take to cover the object.
- How much wallpaper it takes to paper a room .
- How quickly will the object lose or gain heat. (Especially if you know the volume as well) . By reducing the surface area relative to the volume (say by curling up in a ball) you stay warmer in cold weather.
- How much material you need to make an item of clothing.
- How many materials you need to clad the outside of a building or tile a roof.

A. Competency

- Calculate surface area of prisms, cylinders, pyramids, cones and spheres and relate to the composite 3-D shapes around us.

B. Objectives

- Calculate the surface area of a variety of 3-D shapes.
- Solve problems related to finding:
  - surface area when dimensions are given
  - unknown dimension when surface area is given
- Calculate surface area of real life based composite shapes using appropriate formulas.

C. Essential Skills/Processes

- Analysing
- Applying
- Reasoning

D. Learning Experiences

- Explain how the total surface area of 3-D shapes are determined based on lateral faces and bases (recall the idea of nets).

- Introduce the general formula to find the total surface area of prisms. Refer Surface Area of Prisms in Understanding mathematics textbook for Class IX.
  - Discuss questions related to calculation of surface area of prisms (when dimensions are given) and missing dimensions (when surface area is given). Refer Understanding mathematics textbook for Class IX or other resources or refer the link: Surface Area of Prisms.

- Connect the concept of area of circle and rectangle in derivation of the surface area of Cylinder. Refer Surface Area of Cylinders in Understanding mathematics textbook for Class IX and the link: Surface area of cylinder.
  - Discuss questions on calculation of surface area of a cylinder (when dimensions are given) and missing dimensions (when surface area is given). Refer Understanding mathematics textbook for Class IX.
  - Watch the video link: Surface area of Sphere and discuss the surface area of the cylinder and relate it to derive the total surface area of Sphere. Refer to Surface Area of Spheres from Understanding mathematics textbook for Class IX.

- Discuss questions on calculation of surface area of a sphere (when dimensions are given) and missing dimensions (when surface area is given). Refer to Understanding mathematics textbook for Class IX.

- Introduce the general formula to find the surface area of pyramids. Refer to Surface Area of Pyramids from Understanding mathematics for Class IX.
  - Discuss questions on calculation of surface area of a pyramid from given dimensions, and a missing dimension when surface area is given. Refer to Understanding mathematics textbook for Class IX or use the link: Surface area of pyramid.

- Demonstrate how to find the surface area of cones by evaluating its curved lateral surface and its circular base. Refer to Surface Area of Cones from Understanding mathematics textbook for Class IX or watch the video link: Surface area of cone.
  - Discuss questions on calculation of surface area of cones (when dimensions are given) and missing dimensions (when surface area is given). Refer Understanding mathematics textbook for Class IX.

- Let students work in groups to explore and to calculate the total surface area of different composite shapes relating to the 3-D shapes around us and ask them to present their strategies applied in finding the surface area. Refer Understanding mathematics textbook for Class IX or use the link Surface area of composite shapes.

E. Assessment

Performance Task 1

Solve questions on calculating surface area of prisms, cylinders, pyramids, cones and spheres. Refer to Understanding mathematics textbook for Class IX or other resources.

Performance Task 2

Work on questions to solve problems related to finding: surface area when dimensions are given, unknown dimension when surface area is given. Refer to Understanding mathematics textbook for Class IX or other resources.

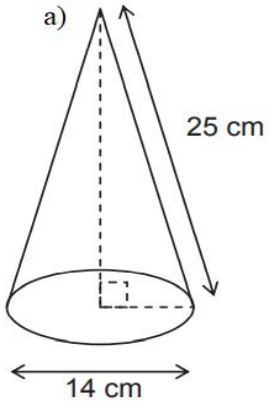
Performance Task 3

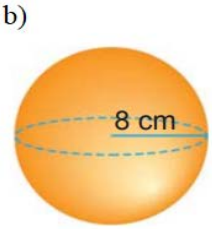
Solve problems of calculating the total surface area of different composite shapes relating to the 3-D shapes around us and elaborate on the strategies applied in finding the surface area. Refer questions from Understanding mathematics textbook for Class IX.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions
  i. A cup of paint covers about 22 m^2. Karma needs to paint all faces of a cubic box. Each edge of the box is 7 m long. How much paint will he need to buy? If a m^2 costs Nu 50, find the cost of painting the box?

  ii. a. Determine the total surface area of each figure.





  b. For the two shapes above, change the length of only one dimension so that the new total surface area will be between 1300 cm^2 and 1700 cm^2. Explain the reasoning you used to determine which length to change and by how much. Show that the new total surface area is in the 1300 cm^2 to 1700 cm^2 range.

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics
- Introduction to Surface Area of 3-D shapes - Surface Area.
- Nets of 3-D shapes - 3-D shape nets.
- Calculation of Surface Area of Prisms - Surface Area of Prisms.

- Derivation of the Surface Area of Cylinder - Surface area of cylinder.
- Surface Area of Sphere - Surface area of Sphere.
- Calculation of Surface Area of a Pyramid - Surface area of pyramid.
- Finding the Surface Area of Cones - Surface area of cone.
- Surface area of composite shapes - Surface area of composite shapes.

G. Annexure

Refer X-A1 for template to record achievements

Topic: X-C2 Volume of 3-D Shapes [600 minutes]

Introduction

Volume of a 3-D shape is defined as the total space enclosed/occupied by any 3-dimensional object or solid shape. It also can be defined as the number of unit cubes that can be fit into a shape. The SI unit of volume is cubic metres.

Early civilizations used standard measuring pottery to measure volume. For instance, in the city of Herclea Pontica, special amphorae for measuring grains and liquids were found. The amphorae were used as units of volume in all Greek territories. They came in various forms and sizes, from 2 to 26 litres.
The ancient Greeks had a wide variety of containers for measuring volume, and they differed from each other depending on their intended use. The Hydria, for example, was used exclusively to measure water, while a phial was a metallic or ceramic container for measuring wine. Large measures were also available, for example the wine was commonly measured by Old Dutch measures - the largest one being with a 232 litre barrel. Introduction to volume of 3-D Shapes.

Utility and Scope

- When you fuel up your vehicle, the volume of gasoline your gas tank holds determines your purchase. Whether you fill up with gallons or litres of gasoline or other fuels, the amount is a volume calculation.
- Volume is used in most house-cleaning chores. When washing clothes, you add a specific amount of laundry liquid to the washing machine based on the load or volume of the tub and the clothes being washed. When using concentrated cleaning detergents, you add a specified amount of the cleaner to a clearly defined amount of water. Many times, you apply a fixed volume of liquid to whatever you are cleaning.
- When a pool or hot tub is filled with water, maintenance begins. Detailed volumes of cleaners and chemicals are needed at critical intervals. The volume of chemicals, such as chlorine, are adjusted to the pool's capacity for water.
  Applications of Volume

A. Competency

- Calculate volume of prisms, cylinders, pyramids, cones and spheres and relate to the composite 3-D shapes around us.

B. Objectives

- Estimate and calculate the volume of prisms, cylinders, cones and spheres (memorization of formulas is not intended at this level).
- Calculate volume of real life based composite shapes using appropriate formulas.

C. Essential Skills/Processes

- Applying
- Analysing
- Reasoning

D. Learning Experiences

- Students must be encouraged to understand the formulas of calculating volume instead of mere memorising.
  - Before teaching the volume of any shape, teach the concept of why volume is expressed in cubic units: The volume of 10 cm^3 means 10 cubes of 1cm × 1cm × 1cm.
  - Similarly, the volume of 10 m^3 means 10 cubes of 1m × 1m × 1m. Use linking-cubes to demonstrate the concept of volume.
    Refer to Volume of Prisms and Cylinders in Understanding mathematics Textbook for class IX or other resources.

- Volume of prisms and cylinders:
  - Watch the video Volume of prisms & cylinders to discuss the volume of prisms and cylinders. This video is about the volume of prisms, cylinders, cones and spheres.
  - Explain using the linking cubes why the volume of a prism = Area of Base × Height. Refer to Volume of Prisms and Cylinders in Understanding mathematics Textbook for class IX or other resources.
  - Discuss deriving the formula of the volume of cylinders on the basis Volume = Area of Base × Height. Refer to example questions Volume of Prisms and

Cylinders in Understanding mathematics Textbook for class IX or other resources.

- Volume of Pyramids and Cones:
  - Demonstrate how the volume of a pyramid is $\frac{1}{3}$ of the that of a prism and the volume of cone is $\frac{1}{3}$ of that of a cylinder with same base and the height, i.e V_{prism} =Area of Base × height

    $\Longrightarrow V_{pyramid} = \frac{1}{3}$ Area of Base × height

    Similarly: $V_{cylinder}$ =Area of circular Base × height

    $\Longrightarrow V_{cone} = \frac{1}{3}$ Area of circular Base × height

  Watch the video links: Volume of pyramids & cone or volume of pyramids
  - Discuss relevant questions to calculate the volumes of prisms, pyramids, cones and cylinders from Understanding mathematics textbook for Class IX or other resources.

- Volume of spheres:
  - To derive the formula for the volume of spheres, first show that volume of a sphere = 2 × volume of a cone (Note: Both sphere and cone have the same radius, but the height of the cone is equal to the diameter of the sphere). Refer to the link https://www.youtube.com/watch? to see the derivation of the volume of the sphere using the volume of a cone.
  - Highlight upon the deductive reasoning being used to derive the relation of volumes of different 3-D shapes (Refer Inductive and Deductive Reasoning Sample given below the annexure A1).
  - Discuss relevant questions of calculating volumes of spheres and hemispheres from Understanding mathematics textbook for Class IX.

- Composite shapes: Demonstrate finding the volumes of real-life based composite 3-D shapes around us. Refer to Volumes of Spheres and Composite Shapes in Understanding mathematics textbook for Class IX.

E. Assessment

Performance Task 1

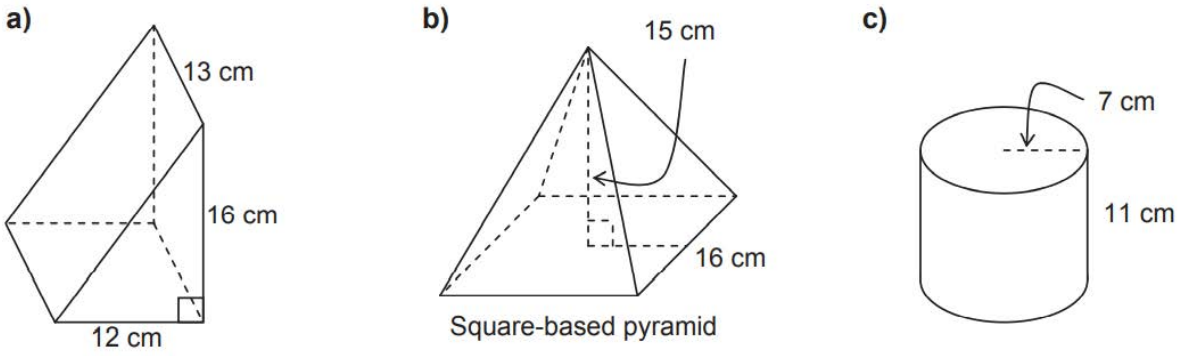
Solve questions on calculating volumes of prisms, cylinders, pyramids, cones and spheres. Refer to Understanding mathematics textbook for Class IX or other resources.

Performance Task 2

Solve problems of calculating the volume of different composite shapes relating to the 3-D shapes around us and elaborate on the strategies applied in finding the volumes. Refer questions from Understanding mathematics textbook for Class IX.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions



i. Determine the volume of each figure.

ii. For each of the three shapes from question 1, change the length of only one dimension so that the new volume will be between 400 cm^3 and 600 cm^3. Explain the reasoning you used to determine which length to change and by how much. Show that the new volume is in the 400 cm^3 to 600 cm^3 range. (For example, for part 1b), you could keep the 16 cm × 16 cm square base and change the height, or you could keep the height at 15 cm and change the side length of the square base).

F. Resources

- Understanding mathematics textbook for Class IX
- Teachers Guide to Understanding mathematics, Class IX
- National School Curriculum Framework for mathematics
- Introduction to Volume – Introduction to volume
- Utility and Scope - Applications of volume
- Volume of Prisms and Cylinders - https://www.youtube.com/watch?v=qJwecTgce6c
- Volume of Pyramids - Volume of pyramids & cone & Volume of pyramids
- Derivation of formula for the volume of Spheres - Volume of sphere

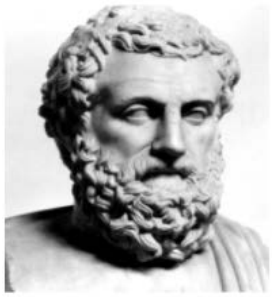
G. Annexure

Refer X-A1 for template to record achievements

Topic: X-C3 Properties of 3-D Efficiency [200 minutes]

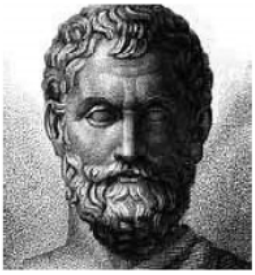
Introduction

The term efficiency refers to the peak level of performance that uses the least amount of input to achieve the highest amount of output. Efficiency requires reducing the number of unnecessary resources used to produce a given output, including personal time and energy.



Aristotle

Efficient shapes problems emerged when someone asked the question: Of all plane figures with the same perimeter, which has the greatest area? Aristotle (384 B.C.E.–322 B.C.E.) may have known this problem, having noted that a geometer knows why round wounds heal the slowest. Zenodorus (180 B.C.E.) wrote a treatise on isometric figures and included the following propositions, not all of which were proved completely:



Zenodorus

- Of all regular polygons with equal perimeter, the one with the most sides has the greatest area.
- A circle has a greater area than any regular polygon of the same perimeter.
- A sphere has a greater volume than solid figures with the same surface area.

Shapes, Efficient | Encyclopedia.com

Utility and Scope

Efficiency can be understood as achieving maximum benefits or profits with minimum wastage. For example, if you are manufacturing cans for canning food items and you want to keep your costs down, you would want to make the can using as little metal as possible to hold a given volume of food items.

A. Competency

- Verify 3-D efficiency inductively and apply properties of 3-D efficiency in real life situations.

B. Objectives

- Examine maximising volume while restricting surface area and minimising surface area while restricting volume.
- Explore efficiency design of 3-D shapes.

C. Essential Skills/Processes

- Communicating
- Evaluating
- Applying

D. Learning Experiences

- Examine maximising volume while restricting surface area and minimising surface area while restricting volume.
  - Demonstrate efficiency of 3-D shapes in terms of volume;
    Note: With a constant surface area, a 3-D shape with a greater volume is more efficient.
    - Suggested Activity: Design an activity where learners have different sizes of two or more cylinders with the same surface area but different diameters and heights. Ask them to calculate their volumes and compare the answers to evaluate the efficiency of the cylinders.
      Allow the students to form a conjecture based on their findings. Highlight upon the inductive reasoning being used to draw conclusion on their conjecture (Refer Inductive and Deductive Reasoning Sample given below the annexure A1).
      (Note: Cylinders that have equal height and diameters are the most efficient cylinders).
  - Demonstrate efficiency of 3-D shapes in terms of surface area;
    Note: With a constant volume, a 3-D shape with lesser surface area is more efficient.
    - Suggested Activity: Design a task where learners have two or more rectangular prisms with the same volume but different dimensions of bases. Ask them to calculate their surface areas and compare the answers to evaluate the efficiency of the prisms.

Allow the students to form a conjecture based on their findings and verify inductively.
(Note: Cube is the most efficient rectangular prism)

- Demonstrate relevant example questions with regard to both maximising volume while restricting surface area and minimising surface area while restricting volume in 3-D Efficiency from Understanding mathematics textbook for Class X or other resources.

- Explore efficiency design of 3-D using relevant software (such as GeoGebra, graphmatica, etc. to visualise the efficiency of 3-D shapes).

E. Assessment

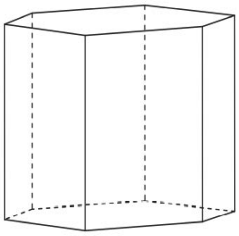
Performance Task 1

Solve questions involving both maximising volume while restricting surface area and minimising surface area while restricting volume in 3-D Efficiency. Refer questions from 3-D Efficiency in Understanding mathematics textbook for Class X or other resources.

Performance Task 2

Explore the understanding of 3-D efficiency by evaluating properties of efficiency to conjecture the most efficient shapes for prisms, cylinders, pyramids and cones.
(Note: Sphere is the most efficient 3D shape).
Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions
  i. Your task is to find the dimensions of a regular hexagon-based prism with the least surface area that will hold 1.0 L.

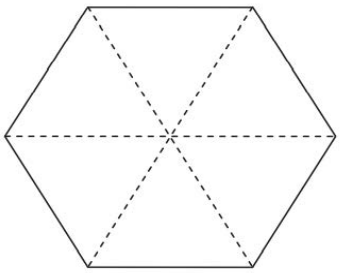


  ii. Show all your calculations for this question.
  a. Determine the area of a regular hexagon with sides 5.0 cm.

b. Determine the height of a prism that will hold 1.0 L. Use the base area from part a).

c. Determine the surface area of the regular hexagon-based prism

iii. Enter your results from question (i) into the chart below, and repeat for the various side lengths. (You do not have to show your work for each prism)



5.0 cm

Base side length (cm)	5.0	6.0	7.0	8.0
Area of base (cm^2)				
Height (cm)				
Surface area (cm^2)				

iv. Which regular hexagon-based prism in question 2 has the least surface area? Explain why you might have predicted this result.

F. Resources

- Understanding mathematics textbook for Class X
- Teachers Guide to Understanding mathematics textbook for Class X
- National School Curriculum Framework for mathematics
- Introduction to 3-D efficiency - Shapes, Efficient | Encyclopedia.com

G. Annexure

Refer X-A1 for template to record achievements

Topic: X-C4 Trigonometry [850 minutes]

Introduction

Trigonometry is a branch of mathematics concerned with angle measurement and problems involving related angles. The Greek terms 'trigonon' and 'metron' are used to form the word trigonometry. The terms trigonon and metron refer to triangle and measure, respectively. As a result, trigonometry refers to the study of relationships between a triangle's sides and angles.

Hipparchus of Nicaea is considered the founder of trigonometry. He was a Greek astronomer, geographer and mathematician born about 180 B.C. in northern Asia Minor. Hipparchus produced a table of chords, an early example of a trigonometric table. He did this by using the supplementary angle theorem, half angle formulas, and linear interpolation. Hipparchus was not only the founder of trigonometry but also the man who transformed Greek astronomy from a purely theoretical into a practical predictive science. He also introduced the division of a circle into 360 degrees into Greece. The subject was developed by him as a tool to help in his astronomical work.

Hipparchus of Nicaea

History of trigonometry1 and History of Trigonometry

Utility and Scope

The early applications of Trigonometry were to astronomy, surveying, navigation and engineering. In modern times, it finds wide application in warfare, all sorts of vibratory phenomena, sound, light, electricity, etc. Applications of Trigonometry

A. Competency

- Use degrees and radians as measures of angles to demonstrate understanding of trigonometric functions (types, sign conventions, magnitude, periods) and to apply in different contexts.

B. Objectives

- Demonstrate the understanding of conventions of signs of angles
- Deduce the relationship between degrees and radians
- Use fundamental trigonometric relations to prove simple trigonometric identities

- Demonstrate the understanding of signs of trigonometric functions (types, sign conventions, magnitude, periods)
- Apply t-ratios of standard angles and allied angles to solve problems
- Examine graphs of trigonometric functions to determine periods using relevant graphing software (GeoGebra)

C. Essential Skills/Processes

- Estimating
- Applying
- Reasoning
- Analysing

D. Learning Experiences

- Discuss the convention of signs of angles with respect to its initial position and terminal position. Refer to the video link: Angle and its Measurement - Trigonometry Basics and Angles in Trigonometry. Show how to name the quadrants on a coordinate plane. Refer to Convention of signs of angles in BHSEC mathematics Book I and other resources.

- Highlight upon the deductive reasoning being used to draw conclusions and derive trigonometric ratios and relations given in the following learning experiences (Refer Inductive and Deductive Reasoning Sample given below the annexure A1).

- Demonstrate the understanding of Radians and its relationship with degrees by screening the video to the class using the link: Radians and Degrees. (Derives the relation; π radians = 180 degrees and define radians) or refer to Relation Between Radians and Degrees in BHSEC mathematics Book I or other resources.

- Discuss relevant questions to convert the radians to degrees and vice versa. Refer BHSEC mathematics Book I or other resources. Note: Exclude minutes and seconds while converting the radian to degree form.

- Recapitulate and discuss the relationship between the trigonometric ratios (reciprocal relations, quotient relations and square relation/Pythagorean identities). Derive and illustrate the other two Pythagorean identities (secant and tangent, cosecant and cotangent square relations/ Pythagorean identities).

Explain the procedures to prove trigonometric identities, refer the link: Verifying Trigonometric Identities Solve questions related to the trigonometric identities, refer Relationship between the trigonometric ratios in BHSEC mathematics Book I or other resources. Note: Limit to dealing with solving simple questions.

- Using flipped classrooms share the links: https://youtu.be/IQ5TuVdtKNU (convention of signs) and https://www.youtube.com/watch?v=qkYfvMbJDP8 (explains signs of trigonometric functions based on x and y coordinates) via google classroom and explain why they need to watch the videos before they attend the next lesson on sign conventions. During Face-to-Face teaching, recall and check their understanding on convention of signs, explain the concept if required. After the lesson you may discuss Oral Examples for class discussion to strengthen their understanding. Suggestion: Refer to Signs of trigonometric ratios in BHSEC mathematics Book I or discuss the video link: Other Trigonometric Functions From One Given Ratio and other five Trigonometric Functions given one. (finding the t-ratios from one of the given t-ratios) to students. Note: Limit to dealing with solving simple questions.

- Illustrate how to determine the t-ratios of standard angles (0°, 30°, 45°, 60° and 90°) using 30-60-90 and 45-45-90 special triangles. Refer the link: Simple trick to remember Trigonometric Ratio and Exact Trig Values - Trigonometry or T-ratios of standard angles in BHSEC mathematics Book I.
  - Discuss example questions in BHSEC mathematics Book I.

- Screen the video lesson: https://youtu.be/ZUlTCDc01KE (Explains about allied angles of t-functions) or refer to t-ratios of allied angles BHSEC mathematic Book I to derive the t-ratio of allied angles using geometry.
  - Solve questions on how to apply the t-ratio of allied angles by solving problems from BHSEC mathematics Book I. Note: Limit to dealing with solving simple questions.

- Graphing trigonometric functions.
  - Demonstrate on graphing sine function using the table of trigonometric functions. Refer the video link: Graphing Sine and Cosine Trig Function

- Provide the worksheet for cosine and tangent function, ask students to complete the table of trigonometric functions. Note: Students can use a scientific calculator to find the values of the t-function.
- Instruct students to plot the graph for cosine and tangent function.

- Explain the Periods of trigonometric functions using the graphs of t-functions.
  - Define what is Period and explain how to determine the period of a given trig-function from the graph. Refer to Periods of trigonometric functions in BHSEC mathematics Book I or the given link: The Period of a periodic function (Defines period and pictorial illustration on how to determine the period from a t-function graph).
  - Suggestion: You may use GeoGebra to graph the t-function and to explain about the periodicity of the t-functions. Refer to the link: https://youtu.be/QmxMPPkZpME (Explains about determining the periods of trig-functions and further shows how the graph changes when some parameters are introduced in the function).

- Show the simulation through this link: Trig Tour (Simulation of the trig-function graph) to enhance the understanding of the concept further.

E. Assessment

Performance Task 1

Work out questions related to conventions of signs of angles. Refer to Examples for Class discussion in BHSEC mathematics Book I or other resources.

Performance Task 2

Carry out activities to convert degrees to radians and vice versa. Refer to Relation between radians and degrees in BHSEC mathematics Book I or other resources.

Performance Task 3

Work out questions on fundamental trigonometric relations to prove simple trigonometric identities. Refer to Relationship between the T-ratios and proving simple trigonometric identities in BHSEC mathematics Book I or other resources.

Performance Task 4

Solve questions related to signs of trigonometric functions (types, sign conventions, magnitude, periods). Refer to BHSEC mathematics Book I or other resources.

Performance Task 5

Carry out questions related to t-ratios of standard angles and allied angles to solve problems. Refer to Trigonometric ratios of allied angles in BHSEC mathematics Book I or other relevant resources.

Performance Task 6

Workout to check your competency to examine graphs of trigonometric functions to determine periods using relevant graphing software (GeoGebra). Refer to Graphs of trigonometric functions in BHSEC mathematics Book I or other resources. Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions
  i. If $\sin\theta = \frac{5}{13}$ and is in quadrant II, find $sin2\theta$, $cos2\theta$ and $tan2\theta$.
  ii. Prove that: $\tan\theta + \cot\theta = \sec\theta \csc\theta$
  iii. If $\sin A = \frac{3}{5}$, prove that $\tan A + \frac{1}{Cos\ A} = 2$ or -2

F. Resources

- BHSEC mathematics Book I
- National School Curriculum Framework for mathematics
- Introduction to Trigonometry – History of trigonometry1 and History of Trigonometry
- Utility and Scope - Applications of Trigonometry
- Convention of signs of angles: Angle and its Measurement - Trigonometry Basics and Angles in Trigonometry
- Relation between π Radians = 180 Degrees - Radians and Degrees
- Procedures to prove trigonometric identities – Verifying Trigonometric Identities
- Convention of Signs - https://youtu.be/IQ5TuVdtKNU
- Trigonometric Functions based on x and y coordinates - https://www.youtube.com/watch?v=qkYfvMbJDP8

- Finding the T-ratios of from one of the given T-ratios - Other Trigonometric Functions From One Given Ratio and other five Trigonometric Functions given one
- T-ratios of standard angles (0°, 30°, 45°, 60° and 90°) – Simple trick to remember Trigonometric Ratio and Exact Trig Values - Trigonometry
- Allied angles of T-functions - https://youtu.be/ZUlTCDc01KE
- Graphing the trigonometric functions: Graphing Sine and Cosine Trig Function
- Period of T-functions - The Period of a periodic function
- Determining the periods of T-functions and further shows how the graph changes when some parametres are introduced in the function - https://youtu.be/QmxMPPkZpME
- Simulation of the T-function graph - Trig Tour

G. Annexure

Refer X-A1 for template to record achievements

Topic: X-D1 Symmetry [200 minutes]

Introduction

The word "Symmetry" comes from the Greek word which implies "to measure together". The two objects are claimed to be symmetrical, if they have the identical size and shape with one object having a different orientation from the first.

There are those who claim that the concept of symmetry has been known in all cultures since the earliest times, even though it is an inherent part of human consciousness. However, it is also believed that the concept of symmetry originated in Italy in the 15th century at the outset of the Renaissance. On the other hand, we do not really know who first discovered this concept, or what the train of ideas and circumstances led to its discovery. Symmetry.

Utility and Scope

Symmetry is used in Physics to understand the properties of the molecules. It is used in understanding the mirror image of the molecules by any combinations of translations and rotations.

The idea of symmetry is applicable in tessellation of art and craft forms such as wallpaper, ceramic tilework such as in Islamic geometric decoration, carpet-making, and many kinds of textile and embroidery patterns. Symmetry is also used in designing logos.

A. Competency

- Demonstrate properties of mirror and turn symmetry on 2-D/3-D shapes to relate to symmetry in nature around us.

B. Objectives

- Demonstrate and compare mirror symmetry for 2-D and 3-D shapes.
- Use lines of symmetry to inductively verify names of regular polygons.
- Demonstrate and compare mirror symmetry for 2-D and 3-D shapes.
- Compare properties of mirror and turn symmetry for both 2-D and 3-D shapes using technology.

C. Essential Skills/Processes

- Comparing
- Reasoning
- Applying
- Evaluating

D. Learning Experiences

- Demonstrate and compare mirror symmetry for 2-D and 3-D shapes (reflectional symmetry):
  - Use the web link Reflection Symmetry to help explain the concept of 2-D reflectional symmetry.
  - Use the web link Plane of symmetry to help explain the concept of 3-D reflectional symmetry.
  - Design quiz questions to evaluate learner's understanding of the concept of 2-D and 3-D mirror/ reflectional symmetry. Refer 2-D and 3-D Reflectional Symmetry from Understanding mathematics textbook for Class X.
  - Explore 2-D and 3-D Reflectional symmetry using technology.

- Use lines of symmetry to inductively verify names of regular polygons:
  - Design a worksheet which will enable learners to name regular polygons based on the number of lines of symmetry.

Sl. No.	No. of edges/ vertices	No. of lines of symmetry	Name of regular polygon
1	3	3	Triangle
2	4	4	Square
3	5	5	Pentagon
4	6	6	Hexagon

Highlight upon the inductive reasoning being used to draw and verify conclusions (Refer Inductive and Deductive Reasoning Sample given below the annexure A1).

- Demonstrate and compare Rotational symmetry for 2-D and 3-D shapes:
  - Describe the concept of 2-D rotational symmetry (point symmetry). Animate the concept using relevant software or use the web link 2-D Rotational Symmetry 1 and 2-D Rotational Symmetry 2 to help explain the 2-D rotational symmetry with animation.
  - Explain the concept of 3D rotational symmetry (axis of symmetry). Animate the concept using relevant software or use the web links 3-D rotational symmetry 1 and 3-D rotational symmetry 2 to help explain the 3-D rotational symmetry with animation.
  - Demonstrate how to determine and describe the order of turn symmetry for 2-D and 3-D rotational symmetry. Design a task where learners can use different types of 3-D shapes to calculate and describe their order of turn symmetry.
  - Explore 2-D and 3-D Rotational symmetry using technology.
- Compare properties of mirror and rotational symmetry.
- Analyse the difference between reflectional and rotational symmetry. The web link Reflectional Vs. Rotational Symmetry contains an explanation on comparing rotational and reflectional symmetry.

E. Assessment

Performance Task 1

Answer the questions on identifying mirror symmetry and turn symmetry in both 2-D and 3-D shapes from Understanding mathematics textbook for Class X.

Performance Task 2

Work on questions based on deductive reasoning for evaluating the properties of turn and mirror symmetry in 2-D/3-D shapes. Refer relevant questions from Understanding mathematics textbook for Class X.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions
  a) What do you notice about the number of lines of symmetry in a regular polygon? Explain why this property applies to all regular polygons.

  b) How does this property relate to the number of planes of symmetry of a prism with a regular polygon base?

  c) How does this property relate to the number of planes of symmetry of a pyramid with a regular polygon base?

  (Note: The number of planes of symmetry in a prism is one more than the number of lines of symmetry in the base. Similarly, the number of planes of symmetry in a pyramid is equal to the number of lines of symmetry in the base. However, this inductive rule is NOT applicable to cubes and tetrahedrons).

F. Resources

- Understanding mathematics textbook for Class X
- Teachers Guide to Understanding mathematics textbook for Class X
- National School Curriculum Framework for mathematics
- Introduction to Symmetry – Symmetry.
- Concept of 2-D Reflectional Symmetry - Reflection Symmetry.
- Concept of 3-D Reflectional Symmetry - Plane of symmetry.
- 2-D Rotational Symmetry with animation - 2-D Rotational Symmetry 1 and 2-D Rotational Symmetry 2.
- 3-D Rotational Symmetry with animation - 3-D rotational symmetry 1 and 3-D rotational symmetry 2.
- Comparing Rotational and Reflectional symmetry - Reflectional Vs. Rotational. Symmetry.

G. Annexure

Refer X-A1 for template to record achievements

Topic: X-D2 Constructions [450 minutes]

Introduction

The word construction in geometry has a very specific meaning: the drawing of geometric items such as lines and circles using only compasses and straightedge.


Euclid of Alexandria

In antiquity, geometric constructions of figures and lengths were restricted to the use of only a straightedge and compasses. The Greeks formulated much of what we think of as geometry over 2000 years ago. In particular, the mathematician Euclid documented it in his book “Elements”. Because of the prominent place Greek geometric constructions held in Euclid’s Elements, these constructions are known as Euclidean constructions. The History of Geometric Construction - PPT

Utility and Scope

The best use of geometrical constructions in daily life is the construction of buildings, dams, rivers, roads, monuments, etc. For ages, geometry has been exceptionally used to make monuments that hold the heritage of our country. Some of these famous monuments are even counted as miraculous developments done by people with primitive instruments.

Another excellent use of geometrical construction is how artists use the concepts to design the best paintings and express their ideologies. The use of colours, brushes, and different strokes give birth to excellent artwork. Artists also design apparel, accessories, and other items we use.

The applications of geometrical construction in daily life can also be found in interior design. Setting new items in an open space is done perfectly using the concepts of coordinate geometry. Geometry in Daily Life

A. Competency

- Use bisectors, medians and altitudes to explore the different geometric centres of triangles through constructions.

B. Objectives

- Construct perpendiculars and angle bisectors.
- Locate in-centres and circum-centres to construct incircles and circumcircles.
- Construct medians and altitudes of triangles.
- Locate centroids (centres of gravity) and ortho centres using medians and altitudes constructions.

C. Essential Skills/Processes

- Communicating
- Constructing
- Creating
- Applying

D. Learning Experiences

- Use the interactive geometry tools to demonstrate constructions. Interactive tool 1 and Interactive tool 2.

- Revise the constructions of some standard angles that students have learned in class VII: 30°, 45°, 60°, 90° and 120°.

- Demonstrate the construction of perpendicular and angle bisectors. Let students practise the construction with some examples. Refer to the links: Perpendicular Bisector of a line. and Angle Bisector Construction

- Demonstrate the construction of an incircle of a triangle using the angle bisectors with an example. Refer to the video link: Circumcircle and incircle construction.
  - Specify that the intersection point of the angle bisectors of a triangle is the incenter of the triangle. Incircle is constructed from the incenter.
  - Any two angle bisectors would suffice to locate the incenter of a triangle but it is advised to construct all three angle bisectors to get the incenter accurately. Verify your constructions using relevant drawing software such as GeoGebra or GSP (Geometer's Sketchpad).
  - Provide example questions for students to facilitate hands-on practise.

- Demonstrate the construction of a circumcircle using the perpendicular bisectors with an example. Refer to the video link: Circumcircle and incircle construction.
    - Specify that the intersection point of the perpendicular bisectors of the sides of the triangle is the circumcenter of the triangle.
    - Any two perpendicular bisectors of the triangle are sufficient to locate the circumcenter. However, it is advised to construct all three perpendicular bisectors of the triangle to get the accurate circum-centre.
    - Provide example questions for students to facilitate hands-on practice.

- Demonstrate the construction of medians of a triangle and locating the centre of gravity or centroid of the triangle. Refer to the video link: Construction - Centroid
    - Specify that the intersection point of the medians of a triangle is the centre of gravity or the centroid of the triangle. Clarify that perpendicular bisector constructions are needed for both the circumcenter and centroid, but for the centroid the perpendiculars are constructed just to locate the mid points of the sides of the triangle to get the medians.
    - Provide example questions for students to facilitate hands-on practice.

- Demonstrate the construction of altitudes of a triangle and locating the orthocenter of the triangle. Refer to the video link: Construct the Orthocenter of a Triangle
    - Before teaching the construction of an altitude of a triangle, let students practise the construction of a perpendicular line from a point to a line segment. Refer to Understanding mathematics Textbook for Class X.
    - Show that the intersection point of the altitudes of a triangle is the orthocenter of the triangle. To locate the orthocenter of a triangle, the construction of any two altitudes is sufficient. However, to get the accurate orthocenter, it is necessary to construct all the altitudes.
    - Provide example questions for students to facilitate hands-on practice.
    - Show the location of the orthocenters of different types of triangles: acute triangle, right triangle and obtuse triangle.
    - Demonstrate relevant examples to determine the area of a triangle using the constructed altitude.

E. Assessment

Performance Task 1

Carry out questions to construct incircle, circumcircle, centroid and orthocentre. Refer questions from Understanding mathematics textbook for Class X.

Performance Task 2

Carry out questions to determine the area of a triangle using a constructed altitude. Refer questions from Understanding mathematics textbook for Class X.

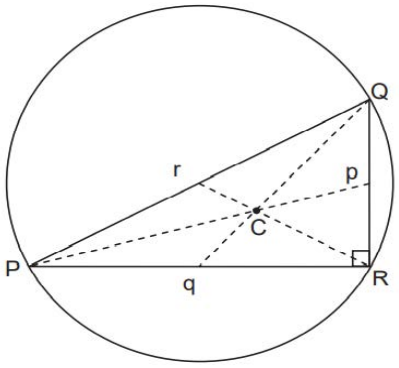
Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions
  In this task, you will explore some properties of triangles.
  i. Construct any right triangle, ΔPQR, and perform the following constructions:

  a. Construct the circumcentre and circumcircle.

  b. Find the midpoint of each side length and label it with the small letter of the vertex that is opposite to it. For example, if the vertex is P, label the opposite midpoint p.

  c. Construct all three medians, locate the centre of gravity, and label it C.
  For example:



  ii.

  a. For your right triangle from part A, measure the hypotenuse and measure the distance from the triangle's centre of gravity to its circumcentre.

  b. What is the ratio of these distances?

c. Repeat part a) for other right triangles until you can make a conjecture about the ratio of these distances in any right triangle.

iii. Draw any triangle that is not a right triangle.

a. Make the constructions as described above in part A.

b. Measure to find the ratio of the lengths of the two parts of any median (separated by the centre of gravity). For example, in the triangle above, you would find the ratio of RC to rC, PC to pC, and QC to qC.

c. Repeat parts i) and ii) using different triangles until you can make a conjecture about the ratio of these distances in any triangle. Use examples of scalene, isosceles, and equilateral triangles.

F. Resources

- Understanding mathematics textbook for Class X
- Teachers Guide to Understanding mathematics, Class X
- National School Curriculum Framework for mathematics
- Introduction to geometrical constructions -The History of Geometric Construction - PPT
- Utility and Scope – Geometry in Daily Life
- Interactive tools - Interactive tool 1 and Interactive tool 2.
- Bisectors: Perpendicular Bisector of a line. and Angle Bisector Construction
- Circumcircle and incircle constructions - Circumcircle and incircle construction
- Medians - Construction - Centroid
- Altitudes - Construct the Orthocenter of a Triangle

G. Annexure

Refer X-A1 for template to record achievements

Topic: X-E1 Collecting, Displaying and Analysing Data [500 minutes]

Introduction

The word data refers to observations and measurements which have been collected in some way, often through research. Data management concerns the dealing with data in the scientific context. Often, more importance is given to results, analysis and derived conclusion than to the data themselves. Data that is recorded as numbers (and therefore measures quantities) is quantitative data, while data that is recorded as text (and therefore records qualities) is qualitative data.

Utility and Scope

Online streaming services use databases to keep track of which TV shows and movies are available and your viewing preferences, so it can provide better watching recommendations every time you log in to the service.

Gaming done across social networks is extremely data-intensive. Gathering individual player information from around the globe and serving it to other players on-demand requires high availability database software.

If you save photos or documents to your smartphone or tablet, or even just to any online backup solution, your data is being transferred to the cloud, a large central storage environment, where just a small portion of space is dedicated to you. Dropbox, Google Drive, Microsoft OneDrive, and iCloud are just some examples of personal cloud storage services available to you. All of them use complex data models and powerful data warehouses to ensure your data is stored securely and is available to be called upon at a moment's notice, regardless of where you are.

Fan participation in national sports doesn't just utilise the power of the database; it depends on it. From fantasy football leagues to March Madness brackets, the sports industry depends on massive cloud databases and data collection to keep track of everything that's going on. Such databases store and analyse player statistics, game performances, injury reports, and more always calculating the odds of a win on a weekly basis.

Every social media platform stores an abundance of user information in databases used to recommend friends, businesses, products, and topics to you (the end user). This cross-referencing of data is immensely complex and uses highly reliable and capable database software.

Predicting the weather across the globe is incredibly complex. Weather organisations use prediction models that depend on various factors, all gathered, stored, and analysed within databases. The databases allow the weather data to be always accessible and easily deliverable to your local TV station or smartphone app. what -is-the-role-of-data-science?

A. Competency

- Apply appropriate methods of data representation for a set of real-life based data (both grouped and ungrouped) collected to evaluate the data distributions, draw conclusions and predict trends.

B. Objectives

- Identify and explain the basic steps of Data Management.
- Compare various methods of displaying data which are grouped in intervals and evaluate their effectiveness: stem and leaf plots, box and whisker plots and histograms.
- Derive the 5 number summary from a grouped data to create a box plot and relate to its histogram.
- Calculate the measures of central tendencies of a grouped data.
- Predict, observe and explain how the different measures of central tendency are affected by extreme data values (or outliers) and discuss their appropriate use in different contexts.
- Demonstrate an understanding of the properties of different data distributions with relevant situational examples.

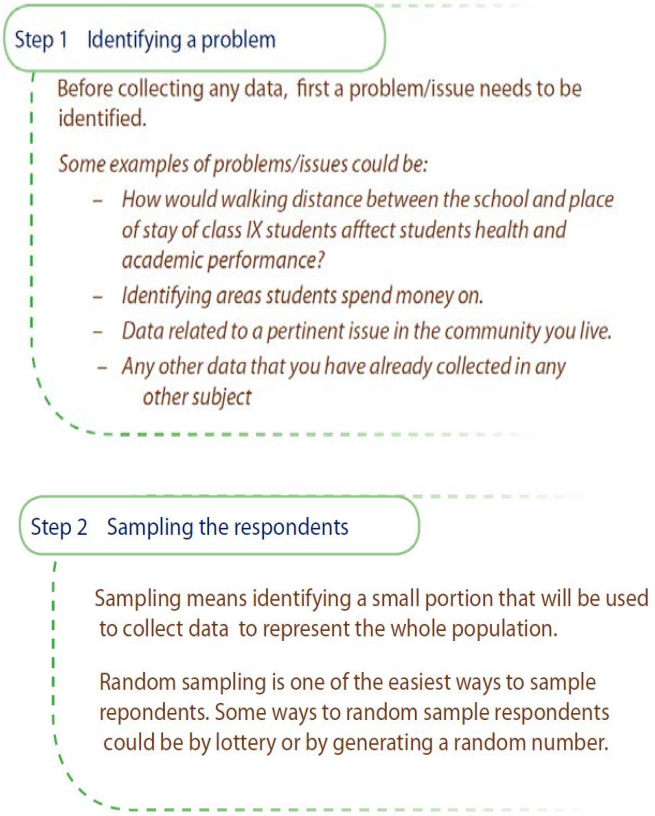
C. Essential Skills/Processes

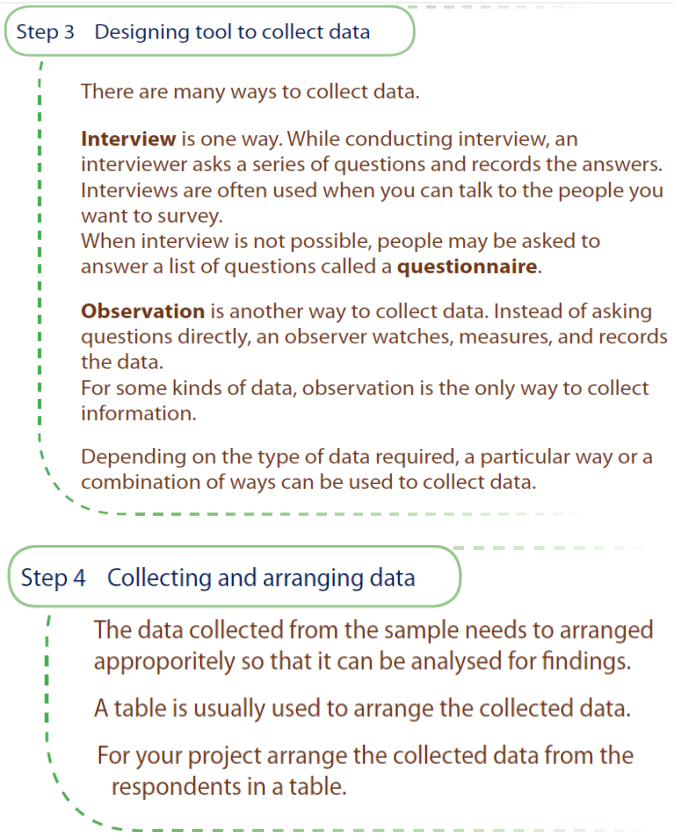
- Collecting data
- Communicating
- Applying
- Analysing

- Representing
- Reasoning
- Creating

D. Learning Experiences

- Discuss that Data Management comprises a series of steps. It is the process of data collection, data organisation, data display, data analysis, drawing conclusions and making recommendations. All these processes need to be followed strictly to get evidence-based results.





- Compare various methods of displaying data which are grouped in intervals and evaluate their effectiveness: stem and leaf plots, box and whisker plots and histograms.
  - Use the web links: How To Choose The Right Graph and Understanding Box and Whisker Plot to help recall and explain concepts on which graph (Limit to stem and leaf plot, box and whisker plot and histogram) to choose for a given set of data.
  - Use the web link Stem and leaf plot to help recall and explain the process of creating stem and leaf plots from an ungrouped set of data.
  - Demonstrate how to create a histogram from a given set of ungrouped data. Use the web link Histogram to help explain the process of creating histogram. Allow students to practise using relevant questions from Histograms and Box-Plots, Understanding mathematics textbook for Class X.
  - Use the web link Box plot to help recall and explain the process of creating box plots from an ungrouped set of data.
  - Demonstrate how to derive the 5 number summary from a grouped data to create a box plot and relate to its histogram. For the formula to calculate the

median and the quartiles refer to the topic on 'Median in a grouped frequency distribution' in BHSEC mathematics Book-1. Refer to Histograms and Box-Plots in Understanding mathematics textbook for Class X for more examples and practise questions or use the link Box plot for grouped data Part 1 and Box plot for grouped data Part 2.

- Predict, observe and explain how the different measures of central tendency are affected by extreme data values (or outliers) for both grouped and ungrouped data. Use the link to understand how to calculate the central tendencies of grouped data Central Tendencies of Grouped data.
  - Demonstrate methods of calculating central tendency (mean, median and mode).
  - Design a task where learners calculate the central tendency of the data set collected earlier.
  - Discuss the appropriate use of the measures of central tendency in different contexts (When is it best to use a particular central tendency).
    - When is the mean the best measure of central tendency?
      The mean is usually the best measure of central tendency to use when your data distribution is continuous and symmetrical, such as when your data is normally distributed. However, it all depends on what you are trying to show from your data.
    - When is the mode the best measure of central tendency?
      The mode is the least used of the measures of central tendency and can only be used when dealing with nominal data. For this reason, the mode will be the best measure of central tendency (as it is the only one appropriate to use) when dealing with nominal data. The mean and/or median are usually preferred when dealing with all other types of data, but this does not mean it is never used with these data types.
    - When is the median the best measure of central tendency?
      The median is usually preferred to other measures of central tendency when your data set is skewed (i.e., forms a skewed distribution) or you are dealing with ordinal data. However, the mode can also be appropriate in these situations, but is not as commonly used as the median.
  - Explain the concept of outliers (anomalies/extreme values).

- Demonstrate an understanding of the properties of the data distribution.
  - Demonstrate an understanding of the properties of a normal distribution and identify situations that give rise to it.
  - Demonstrate an understanding of the properties of skewed distribution, U-shaped distribution, uniform distribution and identify real life situations that give rise to it.
  - Design a task where learners will explore real life examples of skewed, U-shaped and uniform distribution.
    Refer Data Distribution from Understanding mathematics textbook for class X for examples and practise questions.

E. Assessment

Performance Task 1

To give learners the basics of research, assign Project work to learners. Refer to the project calendar and rubrics for assessment.

Suggested Topics

Sl. No.	Topics	Objectives
1	Survey and Graphical Representation	Conduct a survey on a topic of your interest (e.g., favourite sports, hobbies, movie genres) among classmates. Collect data and create various types of graphs to represent the findings (bar graphs, pie charts, etc.).
2	Weather Data Analysis	Gather weather data over a period of time and analyse trends. Create graphs to represent temperature changes, rainfall, or other relevant weather variables.
3	Sports Statistics	Choose a sport and collect statistics on player performance, team standings, or game outcomes. Analyse the data to identify patterns and trends.
4	Population Growth	Investigate the population growth of a city or country over the years. Use the data to

		create line graphs and discuss the factors influencing population change.
5	Health and Nutrition	Collect data on the eating habits of classmates or analyse nutritional information of different food items. Create graphs to compare and draw conclusions about healthy eating.
6	Consumer Price Index (CPI)	Study the concept of CPI and analyse how it has changed over the years. Discuss the impact of inflation on the prices of goods and services.
7	Environmental Impact	Investigate environmental data such as pollution levels, deforestation rates, or wildlife populations. Analyse the data and discuss the implications for the environment.
8	School Performance Analysis	Analyse the academic performance of students in different subjects. Create visual representations of the data and explore correlations between study habits and grades.
9	Travel and Distance	Collect data on the distances students travel to school. Use the data to create histograms, calculate averages, and discuss transportation trends.
10	Social Media Usage	Survey classmates about their social media usage habits. Analyse the data to understand trends in social media preferences and usage patterns.

Sample Project Work Calendar

Sl.	Date	Activity	Recommendation	Remarks
1	1st week of March	Draw project plan	Plans must contain schedules with intended activity spread across the year. Can refer to this calendar as a sample.	
2	2nd week of March.	Identifying a working title	Problem/ issue/ situation/opportunity/ project/new venture that is identified as a working title must be endorsed by the teacher. Only after approval of Title, students can proceed working on it	
3	3rd and 4th week of march	Literature Review	Refer different books and online literature , which have link with project title	
4	Month of April	Data collection.	Data can be collected on identified topics using various tools (interview, questionnaires, observation, and document) or gather available data from various sources with acknowledgement.	
5	1st Week of May	Data organisation	organise the collected in appropriate form	
6	2nd week of May	Data display (1st Draft).	Represent the organised data using appropriate data display(s): · stem and leaf plot · histogram · box plot	
4	3rd week of May	Data display (Final Draft).	Represent data display using relevant software (ICT tools). Subject teacher facilitates the	

			availability of computers and software for data display.	
5	4th week of May	Data analysis, drawing conclusions and making recommendations	Analyse the data display, draw conclusion and make recommendations as per the analysis and conclusion supported by data	
6	1st week of June	Compilation and submission of Project Work.	Compile the work and submit it to the respective subject teacher with proper binding.	
7	2nd week of June	Project Evaluation and Awarding of Marks (maximum 35 marks).		

Rubrics for Project Work Assessment

Criteria	Exceeding (4.1-5)	Advancing (3.1-4)	Meeting (2.1-3)	Approaching (1.1-2)	Beginning (0.1-1)
Plan	Plan is detail with 5 components (schedules, activities, materials requirement, data collection source, representations) and endorsed by teacher	Plan is incomplete by two components and not endorsed by teacher	Plan is incomplete by three components and not endorsed by teacher	Plan is incomplete by 4 components and not endorsed by teacher	Plan is sketchy without required components and not endorsed by teacher
problem	problem is new, meaningful and will have positive impact on community	problem is new, meaningful but will not have	problem is not new but meaningful	problem is new but not meaningful	problem is not stated

		any impact on community			
literature	Literature study is researched and aligns with problem showing deep understanding of concept	Literature study is researched and aligns with problem but lacks understanding of concept	Literature study is well researched but do not align well with the problem	Literature study is not well researched and do not align well with problem	Literature study is not stated but visible from other part of project
data collection	Data collection is systematic with collection tools, processes and authentic source and mentioned in plan	Data collection is systematic with collection tools, processes and authentic source but not mentioned in plan	Data collection has collection tools but without logical process and authentic source	Data collection is not systematic but source mentioned	Data collection is not described and source not mentioned
data representation and analysis	Data representation is appropriate, neat with topics and legends and analysis is meaningful	Data representation is appropriate with incomplete components but analysis is meaningful	Data representation is appropriate but analysis is not meaningful	Data representation inappropriate and analysis is not meaningful	Data representation inappropriate but no analysis
Findings	Finding aligns as solution to the problem with strong argument supported by data	Finding aligns as solution to the problem but argument is weak and not supported by data	Finding is described well but not addressed to solve the problem	Finding is not well described but align as solution to the problem	Finding is brief and do not align with problem

References	More than five references are cited in APA format and referenced throughout the project	Four to five references are cited and referenced throughout the project	Two to three references are cited and referenced throughout the project	At least one reference is cited and referenced throughout the project	No references

Performance Task 2

Workout questions related to comparing various methods of displaying data which are grouped in intervals and evaluate their appropriateness: stem and leaf plots, box and whisker plots and histograms. Refer to Understanding mathematics Textbook for class X or other resources.

Performance Task 3

Carry out activities to derive the 5 number summary from a grouped data to create a box plot and relate to its histogram. Refer to Histograms and Box and Whisker Plots in Understanding mathematics Textbook for class X or other resources.

Performance Task 4

Solve questions related to predict, observe and explain how the different measures of central tendency are affected by extreme data values (or outliers) and discuss their appropriate use in different contexts. Refer to example questions of Data Distribution and other relevant questions in mathematics Textbook for class X or other resources.

Performance Task 5

Work out questions to demonstrate an understanding of the properties of different data distributions with relevant situational examples. Refer to Data Distribution in mathematics Textbook for class X or other resources.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

  People around the world have become more health-conscious over the last 25 years. They have become better educated about the benefits of healthy eating

and an active lifestyle. Fruits and nuts are a very important part of a healthy diet. The chart below shows the amount of carbohydrates and dietary fibre in a variety of fruits and nuts as well as in some common North American snack foods. Nutritionists and doctors have found that a healthy diet tends to be low in carbohydrates and high in dietary fibre.

How do fruits and nuts compare to snack foods with respect to health benefits?

Nutritional Information for Selected Fruits and Nuts (per serving)

Fruits and Nuts	Carbo-hydrates (g)	Dietary fibre (g)
Apples	22	5
Almonds	20	3
Bananas	29	4
Blueberries	27	3
Cashews	28	1
Cherries	22	3
Grapefruit	16	6
Grapes	24	1
Hazelnuts	17	4
Kiwi Fruit	24	4
Lemons	5	1
Limes	7	2
Mangos	17	1
Nectarines	16	2
Oranges	21	7
Peaches	10	2
Peanuts	21	5
Plums	19	2
Strawberries	12	4
Tangerines	15	3
Walnuts	14	5

Nutritional Information for Selected Snack Foods (per serving)

Snack Foods	Carbo-hydrates (g)	Dietary fibre (g)
Chocolate bars	31	2
Chocolate cookies	10	1
Crackers	2	0
Cup cakes	36	1
Noodles	42	2
Tortilla chips	11	2
Granola bars	16	2
Molasses cookies	11	0
Pie	42	2
Popcorn	10	4
Chocolate Bars (nuts)	35	1.7
Doughnuts	23	0.8
Muffins	10	0.2
Sugar cookies	10	0.2
Fig Bars	11	0.7
Pretzels	4	0.1
Potato chips	10	0.8
Brownies	36	1.7
Fudge	14	0.3
Pizza	21	0.3
Yogurt	12	0

i. Work with the carbohydrate data for both groups of foods.
   a. Create a double stem and leaf plot comparing the fruit and nut data with the snack food data. State at least two observations about the data.
   b. Create two box plots using the same scale, one for the fruit and nut data, and one for the snack food data. State at least two observations about the data.

c. Create two histograms using the same scale you used for the box plots, one for the fruit and nut data, and one for the snack food data. State at least two observations about the data.

d. What can you conclude about the health benefits of eating fruits and nuts compared to eating snack foods with respect to carbohydrate content?

ii. Repeat part i) using the dietary fibre data. What advice would you give a friend about his or her consumption of fruits and nuts compared to snack foods? Support your advice with evidence from the data.

F. Resources

- Understanding mathematics textbook for Class X
- Teachers Guide to Understanding mathematics textbook for Class X
- National School Curriculum Framework for mathematics
- Utility and Scope - what-is-the-role-of-data science?
- Choosing an appropriate graph type - How To Choose The Right Graph and Understanding Box and Whisker Plot.
- Process of creating stem and leaf plots from an ungrouped set of data - Stem and leaf plot
- Process of creating histogram - Histogram.
- Process of creating box plot from an ungrouped set of data - Box plot.
- Process of creating box plot from grouped set of data - Box plot for grouped data Part 1 and Box plot for grouped data Part 2.
- Central Tendencies of grouped data - Central Tendencies of Grouped data.

G. Annexure

Refer X-A1 for template to record achievements

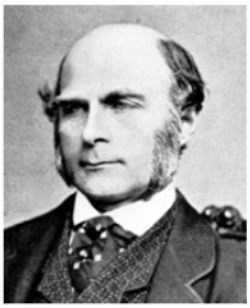
Topic: X-E2 Data Involving Two Variables [300 minutes]

Introduction

Variables are the characteristics or attributes that you are observing, measuring and recording data for, such as height, weight, climate, electrical conductivity, customer service satisfaction and class attendance. A set of data involving two different variables is often collected and then graphed to determine whether a relationship exists between the independent and dependent variables.


Karl Pearson

The complete name of the correlation coefficient is Pearson Product Moment Correlation (PPMC) which deceives many students into a belief that Karl Pearson developed this statistical measure himself. Although Pearson did develop a rigorous treatment of the mathematics of the PPMC, it was the imagination of Sir Francis Galton that originally conceived modern notions of correlation and regression.


Sir Francis Galton

The thoughts that prompted the development of the PPMC began with a then vexing problem of heredity; understanding how strongly the characteristics of one generation of living things manifested in the following generation. Galton initially approached this problem by examining characteristics of the sweet pea plant. He chose the sweet pea because that species could self-fertilise; i.e. daughter plants express genetic variations from mother plants without contribution from a second parent. This characteristic eliminated, or at least postponed, having to deal with the problem of statistically assessing genetic contributions from multiple sources.

Galton's first insights about regression sprang from a two-dimensional diagram plotting the sizes of daughter peas against the sizes of mother peas. As described below, Galton used this representation of his data to illustrate basic foundations of what statisticians still call regression. The generalisation of these efforts into the Product-Moment Correlation and the more complex multiple regression came much later. Current books of behavioural science statistics typically reverse this order: the PPMC is presented first and linear regression is covered later. Journal of Statistics Education, V9N3: Stanton (amstat.org)

Utility and Scope

The Correlations between two variables can be used to evaluate their relationship, draw conclusions and predict trends. For example:

- Ecommerce:
  Time spent on E-commerce website vs Money spent by a customer
  Number of unique user vs Sales in a day
- Real Estate:
  Salary/income of person vs Area of their Home
- Education Ministry:
  Number of years of study vs Income
  Applications of Correlation. How to use correlations?

A. Competency

- Use strengths of correlations between two variables from scatter plots of real-life data to draw conclusions and predict trends.

B. Objectives

- Demonstrate understanding that a correlation coefficient is a description of the strength of the correlation represented by a linear pattern.
- Identify the difference between a strong and weak correlation and between a negative and positive correlation based on the scatter plot and the value of the correlation coefficient.
- Explore the line and curve fitting for a scatter plot to draw appropriate conclusions.

C. Essential Skills/Processes

- Conceptualising
- Communicating
- Reasoning
- Applying
- Analysing

D. Learning Experiences

- Recall and explain the concept of correlation that students have learned in class IX.
  - Recall and discuss the meaning of Dependent and Independent variables and relate to continuous and discrete data.
  - Explain the concept of scatter plot. Demonstrate with an example of scatter plot and a line of best fit.

    Note: The line of best fit is appropriate if the data is continuous hence, a solid line is used. However, a dashed line is used for discrete data.
    - Suggested activity: Collect the two-variable data in the class such as age of students and their heights. Use the collected real life data to plot the graph.
  - Refer to Correlation and the lines of best fit in Understanding mathematics textbook for Class X or other resources.
  - The web link https://www.youtube.com/scatterplots contains how to make scatter plots.

- Explain the types of correlation and their respective correlation coefficients:
  - Show the basic two types of correlation; Positive and Negative Correlation based on real life situations.
  - Explain the perfect, strong and weak correlation and their associate correlation coefficient for both Positive and Negative correlations.
  - Explain the concept of no correlation and the correlation coefficient 0.
  - Let students identify each type of correlation and relate the value of correlation coefficient from a scatter plot.

    The web link https://www.youtube.com/typesofcorrelation contains the types of correlation. Demonstrate the use of a line of best fit in a scatter plot to predict the value of one quantity given the other quantity.

    Note: The accuracy of the predictions is proportional to the strength of the correlation between the variables.
  - Discuss relevant examples of Correlation and Line of Best Fit in Understanding mathematics Textbook for Class X. the web link https://www.youtube.com/watch? explains how to draw a line of best fit.

- Curves of Best Fit:
  - Relate to the line of best fit and explain the concept of a curve of best fit for non-linear scatter plot. Use this link https://www.youtube.com/cruve of best fit to explain how to prepare a curve of best fit.
  - Explain the different types of non-linear curves: quadratic curve, exponential curve, periodic curve and cubic curve.
    Note: Since exponential curve and cubic curve look alike, teachers should specifically differentiate between the two curves: Cubic curve can pass through the origin and below x-axis, whereas exponential curve will never touch the x-axis. Watch the video link Cubic, Exponential, and hyperbola Graphs - YouTube which compares cubic and exponential graphs (ignore the hyperbolic graph).
  - Let the students identify different non-linear curves for a given set of graphs.
    Refer relevant questions from Understanding of mathematics textbook for Class X.

E. Assessment

Performance Task 1

Work out questions to identify the correlation coefficient based on the strength of the correlation represented by a linear pattern and relate to real-life based examples. Refer to Correlation and Lines of Best Fit in Understanding of mathematics textbook for Class X or other resources.

Performance Task 2

Carry out questions to identify different non-linear curves for a given set of distribution and relate to real-life based examples. Refer to Non-linear Data and Curves of Best Fit in Understanding of mathematics textbook for Class X or other resources.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions

  A movie theatre monitors weekly attendance during the first 10 weeks of a movie's showing.

Movie Attendance

Week	Attendance
1	2246
2	2115
3	1935
4	1675
5	1440
6	1200
7	995
8	722
9	664
10	590

a. Create a scatter plot for the data. Is a line of best fit appropriate? Explain.
b. Estimate the correlation coefficient. Justify your decision.
c. The movie will close when weekly attendance drops below 350. Predict when the movie will close. Explain your prediction.

F. Resources

- Understanding mathematics textbook for Class X
- Teachers Guide to Understanding mathematics, Class X
- National School Curriculum Framework for mathematics
- Introduction to Data - Journal of Statistics Education, V9N3: Stanton (amstat.org)
- Utility and Scope - Application of correlation
- Scatter Plot - How to prepare a scatter plot
- Types of Correlation - https://www.youtube.com/types
- Line of Best Fit - https://www.youtube.com/line of best fit
- Curve of Best Fit - https://www.youtube.com/curve of best fit
- Comparison between Cubic, Exponential and hyperbolic Graphs (ignore the hyperbolic graph) - Cubic, Exponential, and hyperbola Graphs - YouTub

G. Annexure

Refer X-A1 for a template to record achievements.

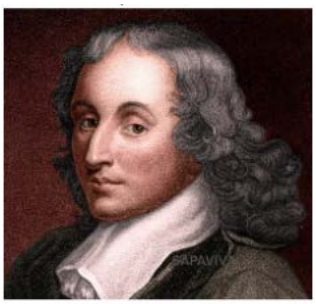
Topic: X-E3 Probability [300 minutes]

Introduction

Probability is the measure of uncertainty of any event, Probability is the chance that something will happen, how likely it is that some event will occur. Sometimes we can measure a probability with a number like “10%”, or we can use words to describe such as impossible, unlikely, possible, even chance, likely and certain. For example, “it is unlikely to rain tomorrow”.

Pierre de Fermat

Probability, chance and randomness have been around since the ancient days. They could be found in fortune telling, games of chance, philosophy, law, insurance, and errors of prediction in astronomy and medicine (Hald, 1990). In about 1200 B.C., an ancient game was played with four astragali (heel bones of hooved animals). They would grind down the bone into a rough, cube like shape and then make small depressions of various numbers. This is where the idea of the pips, the dots on the dice, we use today came from.



Blaise Pascal

There is evidence that in the late 15th century and the early 16th century, mathematicians started to experiment with the idea of probability. A gambler’s dispute in 1654 led to the creation of a mathematical theory of probability by two famous mathematicians, Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665). The links provided will help you further explore the History of probability: A Brief History of Probability Theory

Utility and Scope

Meteorologists collect the database related to weather and its changes worldwide by using different instruments and tools. They collect the weather information worldwide to estimate the temperature changes around the world and the weather conditions for a particular hour, day, week, month and year.

The marketing persons or salespersons promote the products to increase sales. They use the probability technique to check how much the particular product is going well in the market or not. The probability technique helps to forecast the business in future.

Insurance companies provide insurance policies or premiums based on the future forecast to the persons, vehicles etc. Insurance companies generally use theoretical probability or theory of probability to frame any particular policy and complete the policy at the premium rate.

Cricket, volleyball, soccer, football, tennis, badminton, poker, gambling and all the board games use the concept of probability, which gives the idea about how likely a particular person or team is going to win or lose. https://www.youtube.com/watch or https://www.youtube.com/application

A. Competency

- Differentiate and determine the theoretical probabilities for dependent and independent events and apply to both common and conditional real-life events.

B. Objectives

- Distinguish between two events that are dependent or independent using reasoning and calculations.
- Calculate the probability of two independent events, A and B, as P(A) × P(B).
- Calculate probability of dependent and independent events.

C. Essential Skills/Processes

- Computing
- Applying
- Reasoning
- Analysing

D. Learning Experiences

- Recapitulate the concept of probability and the terminologies related to probability: events, outcomes (favourable and possible) and sample space.
- This link Key stage 4 - Probability contains the basic concepts of probability.

- Explain the meaning of dependent and independent events. Refer to the video link: Probability of Independent and Dependent Events. Further refer to Dependent and Independent Events in Understanding mathematics Textbook for class X.
  - Explain each type using an appropriate example.

- Show how to check if the events are dependent and independent using reasoning.
  - Additional ideas for the reasoning part: Relate to the sets and Venn diagrams that students have learned in class IX: Represent the favourable outcomes of each event in separate sets. If there are common elements in the two sets ($A \cap B = \emptyset$ or $n(A \cup B) \neq 0$), then the events are dependent because the outcome of the first event will affect the outcome of the second event. Refer to the video link: Probability - Independent and Dependent Events

    Discuss relevant examples from Understanding mathematics textbook for class X or from other resources.

- Show how to check if the events are dependent and independent by calculation; i.e. Two events are independent if $P(A \text{ and } B) = P(A) \times P(B)$. Determining Whether Events Are Independent or Dependent and Independence of Two Events or Independent and Dependent Events
  - Recall and demonstrate the use of outcome charts and tree diagrams to evaluate the probabilities of events to be used in the calculation.

  Discuss relevant examples from Understanding mathematics textbook for class X or from other resources.

  Highlight upon the deductive reasoning being used to draw conclusion on whether the two events are independent or dependent (Refer Inductive and Deductive Reasoning Sample given below the annexure A1).

- Conditional Probability.
  - Explain the concept of conditional probability and the notations, $P(A|B)$ and $P(B|A)$ with examples. Refer the video link: Intro to Conditional Probability
  - Explain the following relations of conditional probability with examples:
    - $P(A \text{ and } B) = P(B) \times P(A|B)$
    - $P(A \text{ and } B) = P(A) \times P(B|A)$

    Refer to the video link: Conditional Probability

- Discuss relevant examples from Understanding mathematics textbook for class X or from other resources. Watch the video link; https://www.youtube.com/watch?v=_XAqXGJ7fu0 which contains a video lesson on conditional probability with some examples.

E. Assessment

Performance Task 1

Carry out activities of differentiating independent events and dependent events using both reasoning and calculations. Refer to Dependent and Independent Events and Calculating Probabilities in Understanding mathematics Textbook for class X or other resources.

Performance Task 2

Carry out activities of evaluating conditional probabilities and applying its calculations to real-life based situations. Refer to Calculating Probabilities in Understanding mathematics Textbook for class X or other resources.

Design appropriate assessment tools for each performance task and record feedback and achievement based on the template given in the annexure A1.

- Reflective Questions
  You have a bag of blue and red marbles. You select two marbles one after another without replacing the first one. The probability of selecting a blue marble is 0.6. The probability of selecting two blue marbles is 0.3. What is the probability of selecting a blue marble second if the first marble was blue? Explain your thinking.

F. Resources

- Understanding mathematics textbook for Class X
- Teachers Guide to Understanding mathematics, Class X
- National School Curriculum Framework for mathematics
- Introduction to Probability - A Brief History of Probability Theory
- Utility and Scope - https://www.youtube.com/watch or https://www.youtube.com/application
- Basic concept of probability - Key stage 4 - Probability

- Dependent and independent events: Probability of Independent and Dependent Events
- Dependent and independent events using reasoning: Probability - Independent and Dependent Events
- Dependent and independent events using reasoning: Determining Whether Events Are Independent or Dependent and Independence of Two Events or Independent and Dependent Events
- Introduction to conditional probability: Intro to Conditional Probability
- Examples on conditional probability: Conditional Probability
- Conditional probability with some examples - https://www.youtube.com/watch?v=_XAqXGJ7fu0
- Complete package lesson on Probability from e-learning - mathematics Key Stage 4 : Probability

G. Annexure

Refer X-A1 for template to record achievements

Appendix

Assessment Structures for each Strand

KS-IV (Class IX & X)

Key Stage	Assessment									
	Term I					Term II				
	CA (15)			Mid Term Examination	Total	CA (15)			Annual Examination	Total
	CW	HW	PW			CW	HW	PW		
IV	6	6	3	35	**50**	6	6	3	35	**50**

For both Term I and Term II, assess each competency through appropriate performance tasks and assessment tools.

Performance Tasks: quiz, question and answer, presentation, making models, small projects, etc.

Assessment Tools: checklist, rating scale or rubrics.

Assessment Areas:

Content: Formulating situations mathematically, applying concepts, facts, and procedures, and interpreting mathematical results.

Skills and attitude: Collaboration, communication, creativity, time management, learning attitude, feedback reception,etc.

NOTE: Project work assessment is mandatory for class IX and X

Sample Rubrics for Classwork

CATEGORY	Beginning	Approaching	Meeting	Advancing	Exceeding
Mathematical Concepts	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written.	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows complete understanding of the mathematical concepts and is able to synthesise the idea in new problem situations.
Mathematical Errors	More than 75% of the steps and solutions have mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	90-100% of the steps and solutions have no mathematical errors.	Exhibits no mathematical error and is able to identify and correct errors of peers.

Mathematical Terminology and Notation	There is little use, or a lot of inappropriate use, of terminology and notation.	Correct terminology and notation are used, but it is sometimes not easy to understand what was done.	Correct terminology and notation are usually used, making it fairly easy to understand what was done.	Correct terminology and notation are always used, making it easy to understand what was done.	Correct terminology and notation are always used, and are able to derive meaning and justify.
Neatness and organisation	The work appears sloppy and unorganised. It is hard to know what information goes together.	The work is presented in an organised fashion but may be hard to read at times.	The work is presented in a neat and organised fashion that is usually easy to read.	The work is presented in a neat, clear, organised fashion that is easy to read.	The work is presented in a neat, clear, organised fashion with key notes for self-reflection (underlining key points/ highlighting key points)
Completion	75% of the problems are not completed.	50% of the problems are not completed.	25% of the problems are not completed.	All problems are completed.	All problems are completed and explored related extended activities.
Team Work	Student did not work effectively with others.	Student cooperated with others, but needed prompting to stay on-task.	Student was an engaged partner but had trouble listening to others and/or working cooperatively.	Student was an engaged partner, listening to suggestions of others and observed working cooperatively throughout lesson.	Student consistently demonstrated exceptional collaboration and cooperation, actively contributing to the teams' success.

Sample Rubrics for Homework

CATEGORY	Beginning	Approaching	Meeting	Advancing	Exceeding
Accuracy	Numerous errors; solution is unclear or incorrect.	Multiple errors that impact the overall solution.	Some errors that do not significantly impact the overall solution.	Minor errors that do not affect the correctness of the solution	All calculations are correct, and the solution is clearly presented with appropriate steps.
Understanding of the concepts	Shows a limited	Demonstrates a basic	Shows a good grasp of the	Demonstrates a deep	Demonstrates a profound

	understanding of the concepts; struggles with fundamental aspects.	understanding of the concepts but may struggle with some advanced elements.	concepts but may struggle with some advanced elements.	understanding of the mathematical concepts involved.	understanding of concepts; applies them in creative and advanced ways.
Presentations and neatness	Homework is disorganised and messy	Homework is somewhat organised but lacks neatness, making it challenging to follow.	Homework is generally organised and presented neatly but may have some disorganisation.	Homework is well-organised, generally neat, and easy to follow.	Homework is exceptionally well-organised, neatly presented, and aesthetically pleasing.
Clarity of communication	Ideas are poorly communicated, making it challenging to understand the solution.	Ideas are presented but lack clarity, making it challenging to follow the thought process.	Ideas are presented clearly but may lack some precision in mathematical language.	Ideas are mostly clear, with some minor issues in communication.	Ideas are communicated exceptionally clearly, using precise mathematical language and excellent organisation.
Timeliness	Homework is consistently submitted late.	Homework is often submitted late.	Homework is usually submitted on time with occasional delays.	Homework is consistently submitted on time with occasional delays.	Consistently submits homework well before the deadline, demonstrating excellent time management skills.
Problem solving	Shows little to no attempt at problem-solving.	Attempts to use problem-solving strategies but struggles with the process.	Applies problem-solving strategies with some guidance.	Applies problem-solving strategies effectively, considering various approaches.	Applies advanced problem-solving strategies effectively, considering multiple approaches.
Overall effort and engagement	Little effort and engagement evident in the homework.	Inconsistent effort with limited engagement in the assignment.	Demonstrates effort with occasional lapses in engagement or participation.	Shows consistent effort but may lack engagement in some areas.	Consistently demonstrates a high level of effort and engagement in completing the homework.

Sample Rubrics for Project Work

Criteria	Exceeding (4.1-5)	Advancing (3.1-4)	Meeting (2.1-3)	Approaching (1.1-2)	Beginning (0.1-1)
Plan	Plan is detail with 5 components (schedules, activities, materials requirement, data collection source, representations) and endorsed by teacher	Plan is incomplete by two components and not endorsed by teacher	Plan is incomplete by three components and not endorsed by teacher	Plan is incomplete by 4 components and not endorsed by teacher	Plan is sketchy without required components and not endorsed by teacher
Problem	problem is new, meaningful and will have positive impact on community	problem is new, meaningful but will not have any impact on community	problem is not new but meaningful	problem is new but not meaningful	problem is not stated
Literature	Literature study is researched and aligns with problem showing deep understanding of concept	Literature study is researched and aligns with problem but lacks understanding of concept	Literature study is well researched but do not align well with the problem	Literature study is not well researched and do not align well with problem	Literature study is not stated but visible from other part of project
Data collection	Data collection is systematic with collection tools, processes and authentic source and mentioned in plan	Data collection is systematic with collection tools, processes and authentic source but not mentioned in plan	Data collection has collection tools but without logical process and authentic source	Data collection is not systematic but source mentioned	Data collection is not described and source not mentioned
Data representation and analysis	Data representation is appropriate, neat with topics and legends and analysis is meaningful	Data representation is appropriate with incomplete components but analysis is meaningful	Data representation is appropriate but analysis is not meaningful	Data representation inappropriate and analysis is not meaningful	Data representation inappropriate but no analysis

Findings	Finding aligns as solution to the problem with strong argument supported by data	Finding aligns as solution to the problem but argument is weak and not supported by data	Finding is described well but not addressed to solve the problem	Finding is not well described but align as solution to the problem	Finding is brief and do not align with problem
References	More than five references are cited in APA format and referenced throughout the project	Four to five references are cited and referenced throughout the project	Two to three references are cited and referenced throughout the project	At least one reference is cited and referenced throughout the project	No references

Weighting and Time Allocations for each Strand

Key Stage IV (Class IX & X)

Strand	Time Allocation (Mins.)		Weighting(%)	
	IX	X	IX	X
Strand A: Numbers and Operations	1800	1100	25	19
Strand B: Patterns and Algebra	1800	1600	25	25
Strand C: Measurement	1100	2250	18	27
Strand D: Geometry	1100	650	15	12
Strand E: Data and Probability	1000	1100	17	17