Understanding Mathematics

TextBook for Class 3

Department of School Education
Ministry of Education and Skills Development
Royal Government of Bhutan
Thimphu

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ROYAL GOVERNMENT OF BHUTAN
ཤེས་རིག་ལྷན་ཁག།
MINISTRY OF EDUCATION
THIMPHU : BHUTAN

Cultivating the Grace of Our Mind.

ཤེས་རིག

MINISTER

Foreword

I am at once awed and fascinated by the magic and potency of numbers. I am amazed at the marvel of the human mind that conceived of fantastic ways of visualizing quantities and investing them with enormous powers of representation and symbolism. As my simple mind struggles to make sense of the complexities that the play of numbers and formulae presents, I begin to realize, albeit ever so slowly, that, after all, all mathematics, as indeed all music, is a function of forming and following patterns and processes. It is a supreme achievement of the human mind as it seeks to reduce apparent anomalies and to discover underlying unity and coherence.

Abstraction and generalization are, therefore, at the heart of meaning-making in Mathematics. We agreed, propped up as by convention, that a certain figure, a sign, or a symbol, would carry the same meaning and value for us in our attempt to make intelligible a certain mass or weight or measure. We decided that for all our calculations, we would allow the signifier and the signified to yield whatever value would result from the tension between the quantities brought together by the nature of their interaction.

One can often imagine a mathematical way of ordering our surrounding and our circumstance that is actually finding a pattern that replicates the pattern of the universe - of its solid and its liquid and its gas. The ability to engage in this pattern-discovering and pattern-making and the inventiveness of the human mind to anticipate the consequence of marshalling the power of numbers give individuals and systems tremendous privilege to the same degree which the lack of this facility deprives them of.

Small systems such as ours cannot afford to miss and squander the immense power and privilege the ability to exploit and engage the resources Mathematics

Thank you, Teacher. I can read this!

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have to present. From the simplest act of adding two quantities to the most complex churning of data, the facility of calculation can equip our people with special advantage and power. How intelligent a use we make of the power of numbers and the precision of our calculations will determine, to a large extent, our standing as a nation.

I commend the good work done by our colleagues and consultants on our new Mathematics curriculum. It looks current in content and learner-friendly in presentation. It is my hope that this initiative will give the young men and women of our country the much-needed intellectual challenge and prepares them for life beyond school. The integrity of the curriculum, the power of its delivery, and the absorptive inclination of the learner are the eternal triangle of any curriculum. Welcome to Mathematics.

Imagine the world without numbers! Without the facility of calculation!

Tashi Delek.

Thakur S Powdyel

INTRODUCTION

BACKGROUND

Mathematics is a beautiful subject. It is also a useful subject for our daily lives. For example, we are using mathematics when we make an estimation of how much rice to cook for a family dinner. Mathematics is a necessary subject in order to study other subjects such as science, technology, engineering and business studies at the higher levels.

Mathematics is studied in all the modern schools around the world. The kind of mathematics that you will study in class 3 is more or less the same the children of your age and class level around the world study. It builds progressively on the mathematics that you have studied so far up to class 2, including the ways in which you studied. It will then lead you nicely to the mathematics that you will study in class 4 next year.

Mathematics is not a difficult subject. Everyone of you can learn, understand and do well in it. An important part of learning mathematics is for you to really understand and make sense of the mathematics that you study. When you understand the mathematics, you will find it more interesting and easier to learn. There are so many ways to make that happen for you, such as requiring you to work in pairs and small groups as well as to work individually on problems and activities, using concrete models and materials, relating mathematics to everyday life and other subjects, and asking you to explain your thinkings. Once you understand the ideas, concepts and procedures, it would also be useful for you to memorise certain facts, such as multiplication facts. Above all, your teacher will support you with all these.

This is the first time that you will be using a textbook to study your mathematics.

USING YOUR TEXTBOOK

The textbook has 11 chapters. Each chapter has the following components: a *Chapter Overview*, several *Lessons*, and a *Chapter Review*.

Chapter Overview

The *Chapter Overview* talks about the main mathematics concepts in the chapter, as well as why it is important for you to learn them. The key points of the chapter are highlighted in the form of a bulleted list under the subheading *Basic Principles.* Then you will find a list of learning goals that you should achieve from studying the chapter. These appear under the subheading *Chapter Goals*.

Lessons
There are several lessons in a chapter, as you can see in the *Table of Contents*. The lessons are designed for you to achieve the essential mathematical concepts and skills outlined in the *Chapter Goals*. A lesson may be covered in a day or more than a day.

A lesson generally has the following components: *Try This*, *Exposition*, *Examples*, and *Practising and Applying*. However, a few of the lessons do not have the *Examples*. In such cases, the examples are adequately covered within the lesson *Exposition*.

Try This
The *Try This* is in a grey box that appears right after the lesson title. It consists of a simple question or an activity that you can do with some or no help. It is related to the mathematics you will study in the lesson. Later, towards the end of the lesson, you will be asked to revisit the *Try This* problem and relate it to the lesson.

Exposition
The *Exposition* comes after the *Try This* section, enclosed in a rectangular box. It presents and explains the main ideas of the lesson. The important mathematical words, that appear for the first time, are in **bold** text. In some cases, you will find *Notes* contained in small boxes, on the right part of the *Exposition.* These *Notes* provide additional and useful information related to the ideas in the lesson.

Examples
The *Examples* prepare you for the *Practising and Applying* questions. An example consists of a question with its solution. Sometimes, a question will be solved in more than one way, showing that a problem can be solved in more than one way. What is special about the ***Examples*** is that the ***Solution*** column shows you what you should write when you solve a problem, and the ***Thinking*** column shows you how you might be thinking as you solve the problem.

You should work through the ***Examples*** sometimes on your own, sometimes with a friend, and sometimes with your teacher.

Practising and Applying
The questions and problems in the ***Practising and Applying*** allow you to apply the ideas you have learned during the lesson. Many of the questions ask you to explain your solutions besides simply solving them. Your teacher might ask you to work on some of the questions in the class by yourself, with a partner, or in a group. Your teacher might give some questions as homework.

It is important that you understand each question or problem before you start solving them. If a question is not clear to you, discuss it with a friend, or ask your teacher to explain the question.

Chapter Review

The ***Chapter Review*** consists of one to two pages of questions covering the entire chapter. It helps you to review the lessons in the chapter.

THE CLASSROOM ENVIRONMENT

Working in pairs or groups and communicating with each other is an important aspect of learning mathematics in the classroom. In almost every lesson you will have a chance to work with other students. You should always share your responses, even if they are different from the answers offered by other students. It is only in this way that you will really be engaged in the mathematical thinking.

Pair and Group Work

There are many reasons why you should work in pairs or groups. It allows you to have more opportunities to communicate mathematically; it makes it easier for you to discuss an answer you are not sure of; it allows you to see the different mathematical ideas of other students; and it allows you to share materials more readily with others.

When you work in a group, it is important to contribute and follow your teacher's rules for working in groups. Some sample rules are shown here on the right.

Rules for Group Work

- Make sure you understand all the work produced by the group.
- If you have a question, ask your group members first, before asking your teacher.
- Find a way to work out disagreements without arguing.
- Listen to and help others.
- Make sure everyone is included and encouraged.
- Speak just loudly enough to be heard.

YOUR NOTEBOOK

It is important for you to have a well organised, and neat notebook to work on as you learn your mathematics. It will help you in reviewing the main ideas that you have learned by looking back at them later.

Your teacher will sometimes show important points to write down in your notebook. You should also make your own decisions about which ideas to include in your notebooks.

When you solve problems, you should do your rough work in the same notebook. Do not do your rough work elsewhere and then waste valuable time copying it neatly in your notebook.

MATHEMATICS COURSE OBJECTIVES AND CONTENTS FOR CLASS 3

Numbers

The students should be able to:

- Represent 4-digit numbers using base ten blocks.
- Rename 4-digits numbers a combinations of groups of 100s, 100s, 10s, and 1s.
- Write numbers in expanded forms.
- Compare numbers.
- Round numbers to the nearest thousand, hundred, and ten.
- Write numbers using Dzongkha and Roman numerals.
- Understand the various meanings of fractions (as part of a whole, and part of a set).
- Represent fractions with shapes and models.
- Describe and represent mixed numbers.
- Understand decimal tenths as another form writing fraction tenths.
- Write numbers from fractions to decimals and vice-versa.

Operations (Addition, Subtraction, Multiplication, and Division)

The students should be able to:

- Estimate sums and differences.
- Add and subtract 2-digit numbers using various strategies, and using addition and subtraction strategies
- Solve simple addition and subtraction problems.
- Understand the various meanings of multiplication (as repeated addition, equal sets, and arrays).
- Multiply using various strategies such as skip counting, double facts, and multiplication tables.
- Understand the various meanings of division (as equal sharing, equal grouping, and repeated subtraction).
- Relate division to multiplication.
- Perform simple divisions using various strategies.
- Solve simple division and multiplication problems.

Measurement (Length, Area, Mass, Capacity, and Time)

The students should be able to:

- Measure lengths in centimetres, metres, and millimetres.
- Get a sense of how long a kilometre is and compare lengths and distances to a kilometre.
- Estimate lengths in terms of millimetres, centimetres, metres and kilometres.
- Measure and describe a length using combinations standard units.
- Choose an appropriate standard unit for measuring a length.
- Measure and describe the perimeters of shapes.
- Measure areas using non-standard units.
- Measure areas using square centimetres.
- Measure mass in kilograms and grams.
- Describe the relationship between a kilogram and a gram.

- Measure capacities of containers in litres and millilitres, and *drey* (a Bhutanese capacity unit)
- Describe time taken in days, weeks, months, seasons and years.
- Read and tell times from both analog and digital clocks.
- Describe the relationships among different units of time such as minutes, hours, days, weeks, months, seasons and years.

Geometry

The students should be able to:

- Identify and describe polygons (triangles, quadrilaterals, pentagons, hexagons).
- Identify and describe various quadrilaterals (rectangles, squares, rhombuses, kite, and trapezoids).
- Describe an angle, a right angle and identify angles less than and more than right angles
- Identify and describe symmetrical shapes.
- Identify and describe similar and congruent shapes.
- Describe the movement of shapes as turns, slides, and flips.
- Identify and describe 3-D shapes (cones, pyramids, cylinders, and prisms).
- Identify the nets for the 3-D shapes.
- Create 3-D shapes from their nets.

Data Management and Probability

The students should be able to:

- Collect, organise and describe data.
- Interpret and create pictographs with a scale.
- Interpret and create bar graphs with a scale.
- Describe the likelihood of various events using probability terms (certain, impossible, possible, likely, unlikely).
- Conduct simple probability experiments.
- Predict results in simple probability experiments tossing a coin, or drawing out coloured cubes from a bag.

Patterns

The students should be able to:

- Recognise, describe, extend, and create simple repeating patterns.
- Translate simple repeating patterns using letter codes.
- Recognise, describe, extend, and create simple growing patterns.
- Recognise, describe, extend, and create simple shrinking patterns.
- Solve simple problems using patterns.

MODE OF ASSESSMENT FOR CLASS 3

There are basically two types of assessment: formative assessment and summative assessment.

Formative assessment is observation to guide further instruction; and the observation is not normally measured, or its measurement is not recorded to grade students. It is also called assessment for learning. Various ideas and techniques of formative assessment for class 3 have been provided in an integrated manner within this textbook. Further, formative assessment should be carried out in a continuous manner.

Summative assessment is used to determine a mark. It is also called assessment of learning. The assessment discussed here pertains to the summative assessment of the students in class 3.

The summative assessment of students in class 3 is to be done through the following means:

- Interview-based Performance Task (for Continuous Summative Assessment)
- Homework (for Continuous Summative Assessment)
- Chapter Tests (for Continuous Summative Assessment)
- Half-yearly Examination
- Annual Examination

Interview-based Performance Task (For Continuous Summative Assessment)
An Interview-based Performance Task is a small task, usually hands-on, which the teacher gives to a student or to a group of students. This allows the teacher to see if the student(s) understand(s) certain concepts and can perform the associated mathematical skills. The task should be interactive, and carried out in an informal setting.

The teacher should conduct at least one Interview-based Performance Task for the students on one of the chapters during Term I and at least one during Term II. The task setting and the marking criteria could be adapted from the samples in the Understanding Mathematics, Teacher's Guide for Class 2. The marks obtained should then be converted to 10% for each of the Interview-based Performance Tasks conducted during each term for entering into the Student Progress Report Card.

Homework (For Continuous Summative Assessment)
Homework should be assigned to the students on a regular basis. However, care should be taken not to overload the students with too much of homework. Also, the homework should be checked with proper feedback provided to the students in a timely manner. The teacher should check at least two times each student's homework during each of terms. The teacher can devise his/her own marking scheme for the homework. The marks for the homework during each term should be converted to 5% for entering into the Student Progress Report Card.

Chapter Test (For Continuous Summative Assessment)
A chapter test could be conducted at the end of teaching a chapter. The total marks obtained by a student for the chapter tests during each of the terms should be converted to 15% for entering into the Student Progress Report Card.

Half-yearly Examination
The question paper for the half-yearly examination should be set out of 15 marks, with a writing time 1 hour. The paper should not be divided into any sections. There could be a total number of 15 to 20 questions in the paper. The questions should be simple and clear set from the content chapters covered up to the point before the examination. The questions should be similar to the ones included in the textbook. Some of the students may require help with explanations of the questions during the examinations, which should be provided.

Annual Examination
The question paper for the annual examination should be set out of 25 marks, with a writing time 1 hour. The paper should not be divided into any sections. There could be a total number of 15 to 20 questions in the paper. The questions should be simple and clear covering the entire syllabus. The rest should be the same as half-yearly examination paper.

Weighting and the Student Progress Report Card
The scores from the above five areas of summative assessments will then be used to generate the Student Progress Report Card. The weighting among them is as given in the table below:
The total CA marks obtained before the half-yearly break, depending on the number

Summative Assessment Activities	TERM 1	TERM 2
Interview-based Performance Task	10%	10%
Home Work	5%	5%
Chapter Test	15%	15%
Half-yearly Exams	15%	
Annual Exams		25%
	100	

of chapters covered by then, should be converted to be out of 30%, to be entered in the Student Progress Report Card. The CA marks after the half-yearly break should be converted to be out of 30% in a similar manner. This gives a total of 60% for the CA for the entire year. The 15% for the Half-yearly Examination and the 25% for the Annual Examination are straight forward as the examinations would be set for a maximum of 15 and 25 marks respectively.

CHAPTER 1 NUMBERS

Chapter Overview

Numbers talk about quantities. A number tells us of how many or how much. We use numerals to write and represent numbers. There are only ten numerals (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). We can represent any number, however large it may be, by using a combination of these numerals.

The way we use, say and write our numbers is called the 'base ten system'. This means we count quantities in groups of tens. For instance, ten single items are, in fact, counted as one ten, being written with numerals as 10; fifteen single items are, in fact, counted as and mean, one ten and five ones, being written with numerals as 15; one hundred, in fact, means ten tens, being written with numerals as 100; and one thousand, really, means ten hundreds, being written with numerals as 1000, and so on.

The individual numerals in a number are called digits. For example, the number 225 (two hundred twenty-five) has 3 digits. The first two digits from the left are the same, but they have different values. The digit 2 on the left stands for 2 hundreds, and the 2 on its right stands for 2 tens. The digit 5 stands for 5 ones. So, each digit in a number has a value according to the place, or position, it occupies. This is called place value. The place value of a digit increases as it moves leftward in the number.

The understanding of the idea of place value allows you to see and describe a number in more than one way. This, in turn, helps you to appreciate and gain a deep sense of what a particular number means. The use of concrete models to represent and talk about numbers is essential in the early stages of understanding numbers.

In this chapter, we will review some of the important concepts, skills and experiences you have had with numbers in your previous classes. These ideas include representing numbers with concrete models, naming and describing numbers in more than one way, looking at and describing patterns in numbers, comparing numbers, estimating numbers, and writing numbers with place value tables. These will be extended to and dealt with in the context of 4-digit numbers.

This chapter has 7 lessons as detailed in the Table of Contents.

Basic Principles about Numbers

- Numbers tell how many or how much.
- In counting, one, and only one, number is said for each object, and the last number spoken tells how many.
- Representing a number in different ways increases the understanding of numbers.
- Relating numbers to benchmark numbers (like 100, 200, ...) helps us interpret and compare them, and supports work in addition and subtraction.
- The number of digits in a number gives information about its size.
- We write our numbers using the base ten system.
- We write numbers using a place value system not only to be efficient (we only need 10 symbols to represent all numbers) but also to provide benchmarks to compare numbers.
- On a number line, numbers to the right are greater than those to the left.

Chapter Goals

By the end of this chapter, you will be able to:

- Model (or represent) numbers up to 4-digit numbers with base ten blocks.
- Sketch the base ten models of up to 4-digit numbers.
- Rename (and/or describe) a 4-digit number as groups of thousands, hundreds, tens and ones.
- Rename (and/or describe) a 4-digit number as groups of hundreds, tens and ones.
- Write a 4-digit number on place value tables.
- Write a 4-digit number from standard form to expanded form and vice versa.
- Round numbers to the nearest ten, hundred, or thousand.
- Compare numbers and use the “greater than” and “less than” signs correctly.
- Locate 4-digit numbers on a number line.

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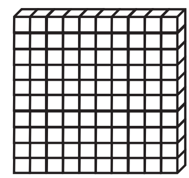
Lesson 1 Representing and Interpreting 3-digit Numbers

Try This

How many tens are there in 42?
How many hundreds are there in 142?
How many tens are there in 142?

The individual numerals in a number are called its digits. For example, 152 (one hundred fifty-two) is a 3-digit number as there are three digits in it. The digits in this number are 1, 5, and 2. An example of a 2-digit number is 99, and an example of a 1-digit number is 6.

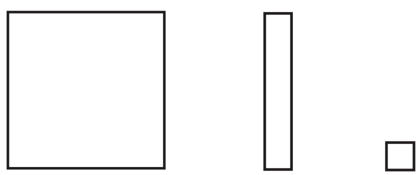
You can represent a 3-digit number using base ten blocks. You can use hundreds blocks, tens blocks and ones blocks.


a hundreds block

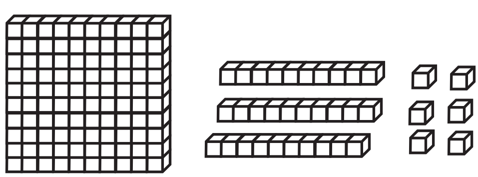

a tens block


a ones block

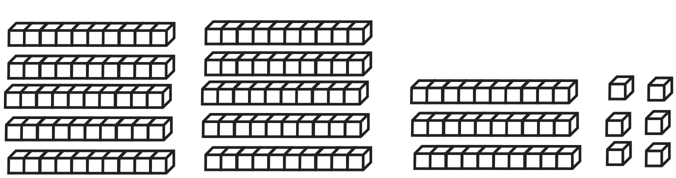
You can sketch the base ten blocks as shown below.


hundred ten one

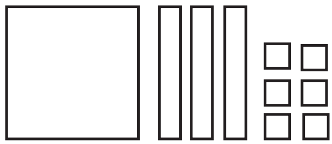
136 (say: one hundred thirty-six) is made up of 1 hundred, 3 tens, and 6 ones. As such, you can represent 136 with the base ten blocks as shown below.



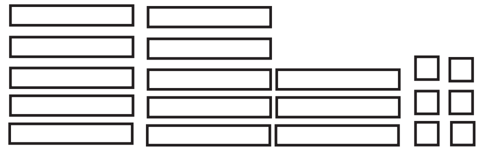
136 (one hundred thirty-six) is also 13 tens and 6 ones. As such, it can be represented by using only the tens and ones blocks as shown below.



You can sketch the base ten blocks that represent 136 as shown below:



136 is 1 hundred 3 tens 6 ones

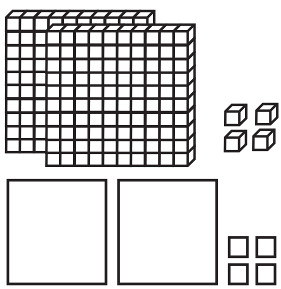


136 is 13 tens 6 ones

Examples

Example 1 Represent 204 with base ten blocks and sketch the blocks.

Solution 1

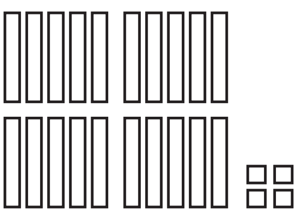


Thinking

204 (Two hundred four) is 2 hundreds, 0 tens and 4 ones. So I needed 2 hundreds blocks and 4 ones blocks to represent it.

Then, I sketched the blocks representing 204 as shown.

Solution 2

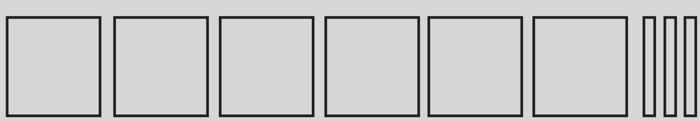


Thinking

204 (Two hundred four) is also 20 tens and 4 ones. So, I can represent it with 20 tens blocks and 4 ones blocks.

Then, I sketched the blocks representing 204 with tens and ones blocks as shown.

Example 2 What number does this set of sketches represent?



Solution

630

Thinking

There are 6 hundreds blocks, 3 tens blocks and 0 or no ones blocks.
So, the number is 6 hundred 3 tens 0 ones or 630.

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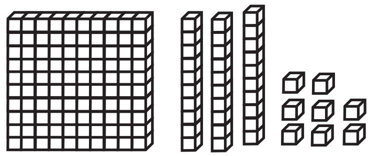
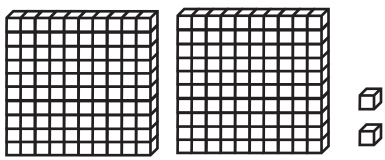
Practising and Applying

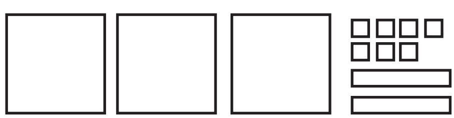
1. What number does the following set of base ten blocks represent?



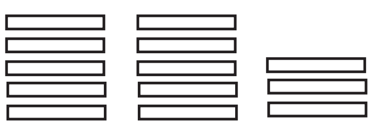
2. What number does the following set of base ten blocks represent?



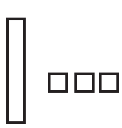
3. What number does the following diagram represent?



4. What number does the following diagram represent? Represent this number in a different way?



5. What number does the following diagram represent? Represent the number with the sketch of only ones blocks.



6. Represent each number with base ten blocks. Sketch the diagrams of the blocks.
   a) 234
   b) 468
   c) 500
   d) 71
   e) 303

7. You have 14 tens and 2 ones. What number would you write?

8. A number has more tens than hundreds or ones. What might the number be?

9. Which is greater: 412 or 189? How do you know?

10. Describe each number as groups of hundreds, tens and ones.
    a) 263
    b) 458
    c) 507
    d) 200

11. Describe each number as groups of tens and ones.
    a) 24
    b) 139
    c) 237
    d) 200

12. Write an example each for a 3-digit number, a 2-digit number and a 1-digit number.

Lesson 2   Renaming Numbers

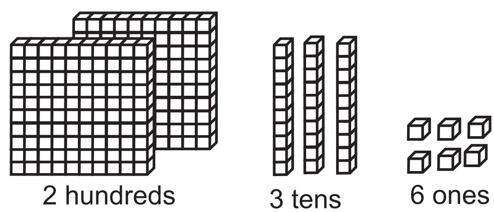
Try This

Use base ten blocks to show the number 134,
- using hundreds, tens and ones blocks;
- using only tens and ones blocks;
- using only ones blocks.

We can call a number by different names. For example, we can call 24 with any of these names: twenty-four, 2 tens 4 ones, or 24 ones.

Representing (or modelling) a number with base ten blocks in different ways helps us in renaming a number in different ways. For example, we can model 236 (two hundred thirty-six) as shown below.

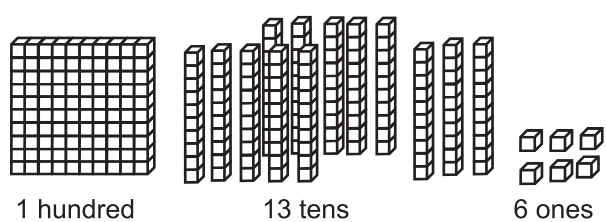
With this model on the right, we can rename 236 as 2 hundreds 3 tens 6 ones. You can write it as a sentence, as done below.



236 = 2 hundreds 3 tens 6 ones

We can regroup one of the hundreds as 10 tens and show the same number as done below.

With this model, we can rename (or describe) 236 as 1 hundred 13 tens 6 ones. Then, write the following sentence.



236 = 1 hundred 13 tens 6 ones

Use base ten blocks and represent 236 in another different way, and rename it accordingly. Also write a sentence as per your model. Sketch your model.

Rename 134, the number in the above **Try This** problem, in four different ways.

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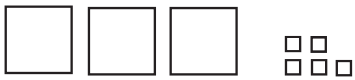
Examples

Example 1 Model 305 (three hundred five) with base ten blocks in 3 different ways. Sketch your models, and rename 305 accordingly each time.

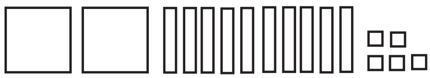
Solution

Model 1



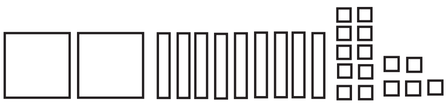
305 = 3 hundreds 5 ones

Model 2



305 = 2 hundreds 10 tens 5 ones

Model 3



305 = 2 hundreds 9 tens 15 ones

Thinking

I represented 305 with 3 hundreds blocks and 5 ones blocks. Then, I sketched the model. I renamed it as 3 hundreds 5 ones.

I replaced 1 hundreds block with 10 tens. Then, I sketched and renamed 305 as shown.

Then, I replaced 1 ten block with 10 ones, after which I sketched and renamed 305 as shown.

Example 2 Write the normal name for 643. Then, rename it using groups of hundreds, tens, and ones.

Solution

Normal name:
643 = Six hundred forty-three

Renamed:
643 = 6 hundreds 4 tens 3 ones

Thinking

The name we normally say for 643 is six hundred forty-three. Since it has 6 hundreds, 4 tens and 3 ones, it can also be said as 6 hundreds 4 tens 3 ones.

Practising and Applying

1. Write the normal names for the numbers.
   a) 136 b) 544 c) 728
   d) 205 e) 912 f) 801
   g) 600 h) 357 i) 570

2. Model each number with base ten blocks and sketch the models. Rename each number accordingly.
   a) 136 b) 544
   c) 205 d) 412

3. Rename each number each number in three different ways.
   a) 713
   b) 302
   c) 31

4. Model 113 with only tens and ones blocks. Sketch the model and rename it accordingly.

Lesson 3 Introducing 1000

Try This

What number comes right after 9?
What number comes right after 99?
What number would come right after 999?

Count the numbers in 10s by saying them aloud:

10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, ...

Count the numbers in 100s, by saying them aloud:

100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 1100, 1200, ...

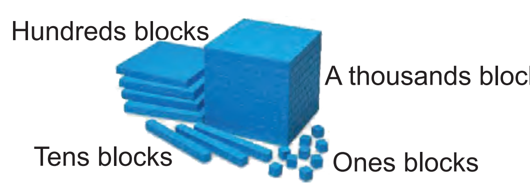
You would say the above sequence as: 1 hundred, 2 hundred, 3 hundred, ... 8 hundred, 9 hundred, 10 hundred, 11 hundred, 12 hundred, and so on.

Recall that we have a number name for 10 ones, which is 1 ten, and the name for 10 tens is 1 hundred. Similarly, we have a name for 10 hundreds, and it is called **one thousand**.

10 ones = 1 ten (10)   10 tens = 1 hundred (100)   10 hundreds = 1 thousand (1000)

So, you could count aloud the above number sequence in 100s, as: 1 hundred, 2 hundred, ... 9 hundred, 1 thousand, 1 thousand 1 hundred, 1 thousand 2 hundred, and so.

The base ten model for 1000 (1 thousand) is a large cube. In fact, it is the same as 10 hundreds blocks stacked together.



Just like you need to sketch the ones, tens and hundreds blocks, you will need to sketch the thousands block. You can sketch a thousands block, as suggested here. Draw a rectangle which has all its sides equal. Next, draw another such rectangle a little up and right of the first one. Then, joins the four corresponding corners. Practise sketching the thousands blocks.

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Lesson 4 Representing 4-digit Numbers

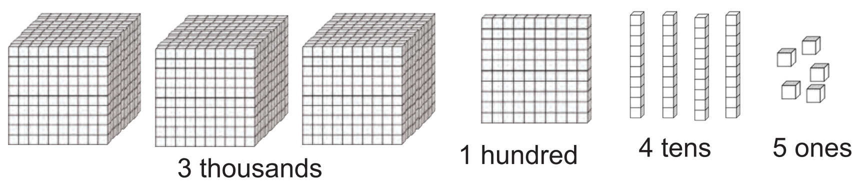
Try This

How many digits are there in each number?

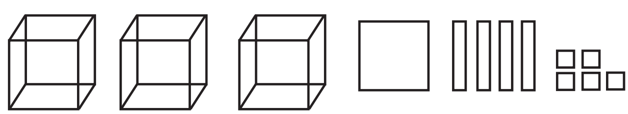
a) 356 b) 1000 c) 57 d) 2014 e) 6

The numbers from 1000 to 9999 are all 4-digit numbers. A 4-digit number has at least 1 thousand in it.

You can model or represent a 4-digit numbers with base ten blocks. For example, the model for 3145 (say: three thousand one hundred forty-five) is as shown here.

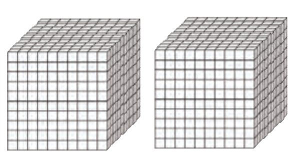


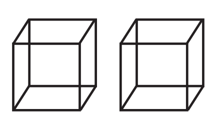
You can sketch the model as:



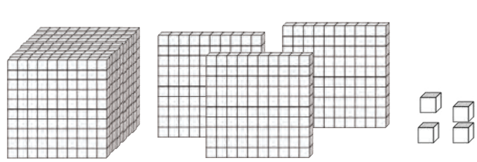
3145 = 3 thousands 1 hundred 4 tens 5 ones.

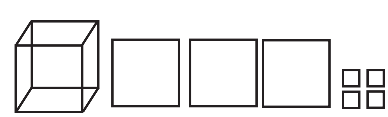
The model and its sketch for 2000 (two thousand) is:





The model and its sketch for 1304 (one thousand three hundred four) is:

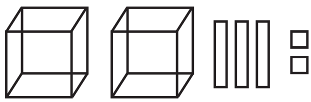




Example

Example 1 Model 2032 (two thousand thirty-two) with base ten blocks. Sketch your model. How many blocks did you use? Rename the number.

Solution



I used a total of 7 blocks.

2032 = 2 thousands 3 tens 2 ones

Thinking

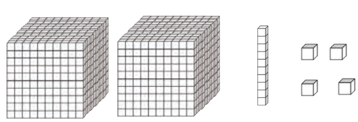
I represented the number with base ten blocks. I did not use hundreds blocks as there are no hundreds in it. I sketched the blocks as shown.

2032 is the same as 2 thousands 3 tens 2 ones.

Practising and Applying

1. Write the normal names for the numbers.
   a) 1362　　b) 5471
   c) 3005　　d) 2014
   e) 6000　　f) 7890

2. What number does the model show? How many blocks are used in the model? Rename the number.



3. Model each number with base ten blocks and sketch the models. Rename each number accordingly.
   a) 1362　　b) 2471
   c) 3005　　d) 2015
   e) 6000　　f) 4440

4. Model and sketch the model for 1100. How many blocks did you use?

5. Model 1100 without using thousands blocks. Sketch the model. How many blocks did you use?

6. How many hundreds make 1 thousand?

7. How many tens make 1 thousand?

8. Look at the picture in question 2. Model the same number without using the tens block. Sketch you model. How many blocks did you use?

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Lesson 5 Writing Numbers in Expanded Form

Try This

How would you say the number word for 279?
How would you say it in terms of groups of hundreds, tens and ones?

The **standard form** of a number is the way we normally say or write. For example, 3275 (say: three thousand two hundred seventy-five), is a number in its standard form.

The **expanded form** of a number shows its various groups such as thousands, hundreds, tens and ones, joined by addition signs. For example, the expanded form for 3275 is as shown below.

3275 = 3 thousands + 2 hundreds + 7 tens + 5 ones
3275 = 3000 + 200 + 70 + 5

Number in standard form   Number in expanded form

Writing a number on a place value table helps in writing it in its expanded form.

Thousands	Hundreds	Tens	Ones
2	0	1	5

2015 = 2 thousands + 0 hundreds + 1 tens + 5 ones
= 2000 + 0 + 10 + 5
= 2000 + 10 + 5

Practising and Applying

1. Write each number in its expanded form.

   a) 4263   b) 4510
   c) 787   d) 2003
   e) 9500   f) 1324
   g) 26   h) 1616

2. Write each number in its standard form.

   a) 5000 + 300 + 60 + 4
   b) 6000 + 400 + 8
   c) 7000 + 500 + 70
   d) 100 + 10 + 1
   e) 20 + 5

Lesson 6 Rounding Numbers

Try This

Count by 100s up to 1900: 100, 200, 300, ...
Count by 1000s up to 9000: 1000, 2000, 3000, ...

Rounding numbers to the nearest ten, hundred, or thousand helps us in comparing numbers, estimating quantities, and estimating the results of calculations such as sums and differences.

First, let us take up rounding a number to the nearest ten. For example, take 87. It is clear that 87 is closer to 90 than 80. So, to round 87 to the nearest ten is 90. Similarly, to round 83 to the nearest ten is 80, as it is closer to 80 than 90. But, how about 85? It is exactly in the middle of 80 and 90. As a rule, if a number is exactly in the middle, then we round it up to the higher ten. So, 85 would be rounded to 90.

We use the same rules, as above, in rounding a number to the nearest hundred or thousand. For example, 850 would be rounded to 900 if we are rounding it to the nearest hundred. And, 3500 would be rounded to 4000 if we are rounding it to the nearest thousand.

Practising and Applying

1. Round each number to the nearest ten.
   a) 28 b) 382
   c) 65 d) 813
   e) 725 f) 8
   g) 4652 h) 3

2. Round each number to the nearest hundred.
   a) 395 b) 321
   c) 75 d) 850
   e) 4612 f) 4682
   g) 50 h) 31

3. Round each number to the nearest thousand.
   a) 3720 b) 867
   c) 5207 d) 1800
   e) 4612 f) 4500
   g) 500 h) 150

4. Write two numbers that you can round to 70 if they are rounded to the nearest ten.

5. A newspaper reporter says, “There were about 900 people at the festival.” How many people do you think might have been there?

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Lesson 7 Comparing Numbers

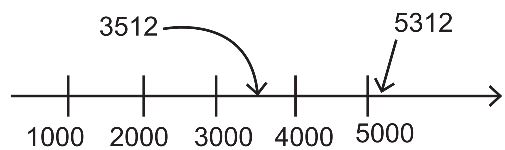
Try This

Which number is greater, 4812 or 5312? How do you know

When you compare two 4-digit numbers, the one that has more thousands is the greater one. If they have the same number of thousands, the one with more hundreds is greater. If they have the same number of thousands and hundreds, then, the one with more tens is greater, and so on.

We can use a number line to compare numbers. A number that is on the right is greater than a number that is on its left on the number line.

To compare 3512 and 5312, locate them on the number line. Since 5312 is to theright of 3512, it is the greater number.



5312 > 3512

If 5312 is greater than 3512, then, it means that 3512 is less than 5312.

3512 < 5312

Use a number line to compare the numbers in the above **Try This** problem.

Note

As an aid for remembering the use of the sign >, think of it as an open mouth of a snake. The snake is always after a larger number of rats.

Practising and Applying

1. Compare the pairs of numbers, and write “is greater than” or “is less than” between them.

   a) 13, 17 b) 305, 310

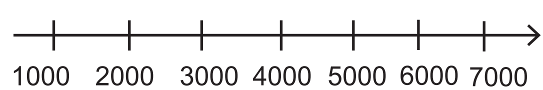
   c) 305, 250 d) 8000, 7999

2. Compare the pairs of numbers, and insert > or < between them.

   a) 65, 40 b) 382, 400

   c) 3250, 4000 d) 8130, 7550

3. Make a number line as shown here. Indicate the numbers 3850, 3900, 1500, and 5450 on it.



4. Explain why 3900 is greater than 3850 in Question 3.

5. Order the 4 numbers in Question 3 from the least to the greatest.

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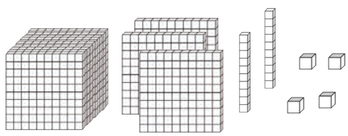
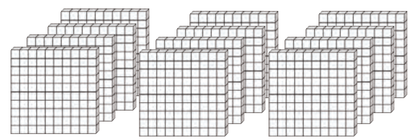
Chapter Review

1. Write the number words for the numbers.
   a) 1350 b) 1356 c) 4043
   d) 6800 e) 8135 f) 9999

2. What number does each model show?
   a)  b) 

3. Sketch the base ten models for the numbers.
   a) 1350 b) 2743 c) 427
   d) 3200 e) 42 f) 9999

4. Rename the numbers as groups of thousands, hundreds, tens and ones. Writing the numbers in place value tables will help you with this.
   a) 1350 b) 1356 c) 2743
   d) 6800 e) 8135 f) 2014

5. Write the numbers in their expanded forms.
   a) 1350 b) 1356 c) 2743
   d) 6800 e) 8135 f) 2014

6. Make a number line. Indicate the numbers 7100 and 6990 on it. Compare the two numbers and insert the appropriate sign (< or >) between them.

7. What is the least 4-digit number?

8. What is the greatest 4-digit number?

9. Identify the number being described in each case.
   a) I have 6 thousands, 5 hundreds, 3 tens and 7 ones.
   b) I have 3 thousands, no hundreds, 8 tens and no ones.
   c) I have 4 hundreds and 5 tens.

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CHAPTER 2 ADDITION AND SUBTRACTION

Chapter Overview

There are basically two situations where addition is involved. One is called active addition, in which some objects join a set and, as a result, the number of objects in the set is increased, and we determine the total number of objects in the set. The other is called static addition, in which a whole is made up of two or more parts, and we determine the number in each part.

There are basically three situations where subtraction is involved. In the first situation, some objects from a set are removed, or taken away, and as a result, the number of objects in the set is decreased, and we determine how many are left in the set. This will be called active subtraction. In the second situation, we compare two sets, and determine which set has more items and how many more. We call this static subtraction. In the third situation, we determine how many we need to add to a set to make it grow from its initial number to a bigger number. We call this kind of situation as finding a missing addend. We shall elaborate upon each of these subtraction situations in the specific lessons in this chapter.

There are many ways to add and subtract numbers such as “Counting Forward and Counting Backward”, “Using Double Facts” and “Facts for 10”. You have learnt these in classes 1 and 2. You have also learnt to look at and describe numbers as groups of hundreds, tens and ones. You will be using these ideas and skills in adding and subtracting 2-digit and 3-digit numbers in this chapter.

This chapter has 12 lessons as detailed in the Table of Contents.

Basic Principles about Addition and Subtraction

- Addition and subtraction both involve changes in quantity.
- Addition involves combining. The combining can be active, where some objects join other objects, or static, where a whole is made of two or more parts. Subtraction involves removal or taking-away or separation, comparison, or finding a missing addend.
- Addition and subtraction are intrinsically related. In any situation involving an addition, there is an equivalent subtraction situation and vice versa.

- There are many strategies to solve any addition or subtraction problem.
- It is useful to estimate sums and differences of numbers by using nearby multiples of 10.
- The strategy used to solve an addition or a subtraction problem may be affected by the numbers themselves, by personal preferences, or by the situation described in the problem.

Chapter Goals

By the end of this chapter, you will be able to:

- Add numbers up to 3-digit numbers using more than one way.
- Estimate sums of up to 3-digit numbers.
- Understand and describe meanings of subtraction.
- Subtract numbers up to 3-digit numbers in more than one way.
- Estimate differences of up to 3-digit numbers.
- Solve simple problems involving addition and subtraction.
- Calculate change in purchasing situations.

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Lesson 1 Adding 2-digit Numbers using Various Strategies

Try This

What is the **sum** of 46 + 32? Explain how you solved it.

The result of adding numbers is called **sum**. You have learned how to add numbers using various strategies in class 2. You will review these strategies to add 2-digit numbers in this lesson. For example, you can add 47 and 32 in any of the following ways:

- Adding the tens to get 70 (4 tens and 3 tens is 7 tens) and adding the ones to get 9 (7 ones and 2 ones is 9 ones), and then adding the total of ones to the total of tens to get 79. So the sum is 79. You could record the process as shown below.

$$\begin{aligned} 47 + 32 &= (4 \text{ tens} + 7 \text{ ones}) + (3 \text{ tens} + 2 \text{ ones}) \\ &= 7 \text{ tens} + 9 \text{ ones} \\ &= 70 + 9 \\ &= 79 \end{aligned}$$

- Adding, first, 30 to 47 to get 77, and, then, 2 to 77 to get 79. You could record the process as:

$$\begin{aligned} 47 + 32 &= 47 + 30 + 2 \\ &= 77 + 2 \\ &= 79 \end{aligned}$$

- You could think of 47 as 50 and add it to 32 to get 82. Then subtract 3 from 82 to get 79. You could record the process as:

$$\begin{aligned} 47 + 32 &= (50 + 32) - 3 \\ &= 82 - 3 \\ &= 79 \end{aligned}$$

How would you add 46 + 32 in a different way than you used earlier under the above **Try This** problem?

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Example

Example 1 Find the sum of 89 + 36.		
Solution 1 89 + 36 = (80 + 30) + (9 + 6) = 110 + 15 = 110 + 10 + 5 = 120 + 5 = 125	**Thinking** I separated the numbers into their **tens** and **ones**. I added the tens to get 110 and the ones to get 15. I added 15 to 110 by first adding 10 and then 5 to get 125.	
Solution 2 89 + 36 = (90 + 36) - 1 = 126 - 1 = 125	**Thinking** I changed 89 to 90. I added 36 to 90 to get 126. Then, I subtracted 1 from 126 for changing 89 to 90 at the start. The sum is then 125.	

Practising and Applying

1. Find the **sums** using any strategy. Show your work.
   a) 41 + 27
   b) 36 + 39
   c) 25 + 25
   d) 72 + 52
   e) 88 + 44

2. Will the sum of 27 + 53 be the same as the sum of 20 + 50 + 7 + 3? Why? What is the sum?

3. Jamba solved 19 + 29 in the following way. Do you think the sum is correct? Why?

   19 + 29 = (20 + 30) - 2
   = 50 - 2
   = 48

4. How would you solve the above addition (19 + 29) in a different way?

5. Add. Show your work.
   a) 62 + 23 + 14
   b) 35 + 20 + 20
   c) 56 + 45 + 34
   d) 20 + 30 + 40
   e) 50 + 50 + 50

6. What is the smallest 2-digit number?

7. What is the greatest 2-digit number?

8. There are two small villages nearby each other. One village has 84 people living in it and the other village has 92 people. How many people are there altogether?

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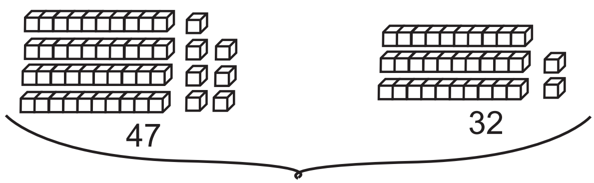
Lesson 2 Adding with Base Ten Blocks

Try This

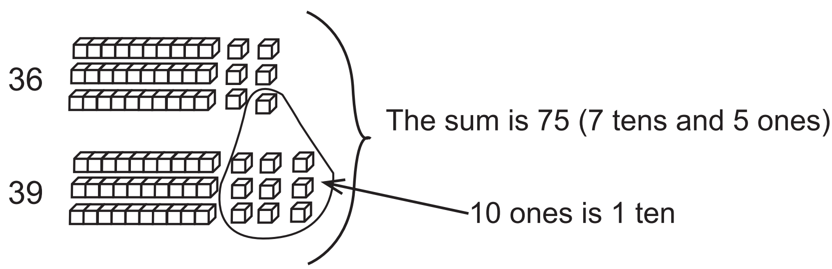
Make a set with base ten blocks to represent 46. Make another set to represent 32. Put the two sets together. What number does the big set represent?

You can use base ten blocks to help you add numbers. For example, to add 47 + 32, represent each number with base ten blocks separately. Then, put the blocks together, and determine the number represented by the blocks together. That is the **sum**.



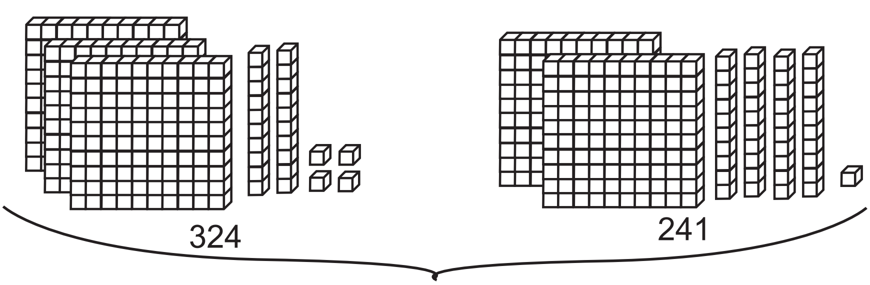
The sum is 79 (7 tens and 9 ones)

Similarly, 36 + 39 could be solved using base ten blocks, as shown below.



Notice that we have to regroup 15 ones to 1 ten and 5 one in 36 + 39.

To add 3-digit numbers you would also need hundreds blocks. For example, 324 + 241 would be added as shown below.

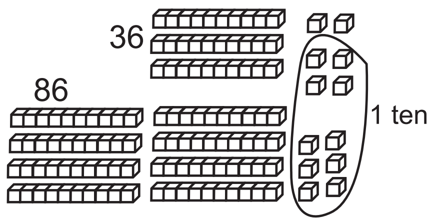


The sum is 565 (5 hundreds 6 tens and 5 ones)

Examples

Example 1 Use base ten blocks to add 36 + 86.

Solution

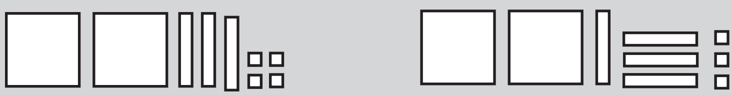


36 + 86 = 122

Thinking

I represented 36 and 86 with base ten blocks. I counted the tens and ones blocks. Altogether, there are 11 tens and 12 ones. I regrouped 12 ones to 1 ten and 2 ones. Now there are 12 tens and 2 ones. 12 tens is 120 and 2 ones is 2. So, the sum is 122.

Example 2 What numbers are represented by the two sets of sketches. Write an addition sentence for them.



Solution

The numbers are 234 and 243.

234 + 243 = 477

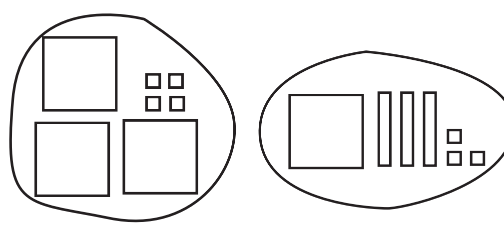
Thinking

I counted the number of hundreds blocks, tens blocks and ones blocks in each set. The numbers are 234 and 243. Altogether there are 4 hundreds blocks, 7 tens blocks, and 7 ones blocks. So, the sum is 477. Then, I wrote the addition sentence.

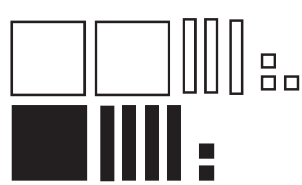
Practising and Applying

1. Solve the following additions using base ten blocks. Sketch the diagrams and write the addition sentence for each.
   a) 325 + 132
   b) 536 + 33
   c) 464 + 16
   d) 706 + 107
   e) 200 + 300
   f) 135 + 102 + 142

2. Write an addition sentence for the two sets of sketches.



3. Write an addition sentence for the set of diagrams. What does each number in your sentence tell?



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Lesson 3 Adding with Place Value Tables

Try This

Choden is trying to find the sum for 25 + 46 by using a place value table. She is having trouble completing the addition. Can you help her complete it?

Tens	Ones
2 + 4	5 6
6	11

You have learnt how to add 2-digit numbers using place value tables in class 2. Let us review how to add 2-digit numbers on a place value chart. For example, 25 + 43 and 25 + 47.

Tens	Ones
2 + 4	5 3
6	8

25 + 43 = 68

Tens	Ones
2 + 4	5 7
6 1 ←	12 (10 + 2)
7	2

25 + 47 = 72

In the second place value table above, you notice that 5 ones and 7 ones make 12 ones. Then, 12 is separated into 10 + 2, after which the 10 ones is regrouped as 1 ten and shifted under the tens column. Why do you think that is necessary?

Similarly, we can add 3-digit numbers using place value tables. For example, let us add 385 + 246 as shown below. Notice that the tables for the 3-digit numbers have one more column for the hundreds digits.

Hundreds	Tens	Ones
3 + 2	8 4	5 6
5 1 ←	12 (10 + 2) 1 ←	11 (10 + 1)
6	3	1

385 + 246 = 631

Examples

Example 1 Add 453 + 172 using a place value table.

Solution

Hundreds	Tens	Ones
4 + 1	5 7	3 2
5 1 ←	12 (10 + 2)	5
6	**2**	**5**

453 + 172 = 625

Thinking

I put the numbers in the place value table appropriately. I added the hundreds, tens and ones separately. I noticed that there are 12 tens, so I regrouped it as 10 tens and 2 tens. 10 tens is 1 hundred, so I shifted it to the hundreds column. Now there are 6 hundreds, 2 tens, and 5 ones. So, the sum is 625.

Example 2 Add 806 + 51 using a place value table.

Solution

Hundreds	Tens	Ones
8 +	0 5	6 1
8	**5**	**7**

806 + 51 = 857

Thinking

I put the numbers in the table appropriately the hundreds, tens and ones separately. There was no need of any regrouping, so the sum is 857.

Practising and Applying

1. Add each of the following using place value tables. Write the addition sentence for each after finding the sum.
   a) 32 + 43
   b) 26 + 57
   c) 528 + 371
   d) 364 + 418
   e) 298 + 8

2. Solve the following additions using place value tables.
   a) 300 + 250 + 127
   b) 502 + 210 + 46

3. Surjay added 36 + 47 using a place value table as shown below and wrote the sum as 713. Can his sum be correct? What mistake did Surjay make for his sum? What should be the correct sum?

Tens	Ones
3 + 4	6 7
7	13

36 + 47 = 713

4. Use a place value table to add 938 + 83. What do you notice?

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Lesson 4 Estimating Sum

Try This

Do not add, but tell about how many marbles there will be if you put the marbles from containers A and B together. Container A has 68 marbles and container B has 174 marbles in them.

A B

It is always good to estimate a sum before you add the numbers. This will help you to determine whether the sum you get after adding the numbers is reasonable or not. Remember how Surjay had added 36 + 47 on a place value table to get a sum of 713 during the last lesson? If Surjay had estimated, he would have realized that 713 cannot be the sum for 36 + 47, because it is too large. Both 36 and 47 are less than 50, so their sum cannot be more than 100.

How could you estimate the sum for 36 and 47? One way is to round each number to the nearest ten: 36 is about 40 and 47 is about 50. So, the sum would be about 90 (40 + 50). It is easier to add 40 and 50 in your head. The actual sum would be a little less than 90, as you took the estimates on the higher side of both the numbers.

For 3-digit numbers, we can estimate the sum by rounding the numbers to the nearest hundred. For example, 315 + 289 is about 300 + 300, which is about 600. We can also round them to the nearest ten, so that it is about 320 + 290, which is about 610. Sometimes, we can leave one number as it is and round only the other number to estimate the sum. For example, 315 + 289 is about 315 + 300, which is about 615. All of these estimates are correct. You should be able to carry out the estimations in your head using mental maths.

Now, estimate the total number of marbles in the above **Try This** problem, by estimating the sum for 68 + 174.

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Example

Example 1 Estimate the sum for 207 + 476. Show your working.		
Solution 1 207 + 476 is about 700	Working 200 + 500 = 700	**Thinking** I rounded each number to the nearest hundreds: 207 is close to 200 and 476 is close to 500. So, I estimated the sum as 700. 
Solution 2 207 + 476 is about 690	Working 210 + 480 = (200 + 400) + (10 + 80) = 600 + 90 = 690	**Thinking** I rounded each number to the nearest tens: 207 is close to 210 and 476 is close to 480. So, I estimated the sum as 690. 

Practising and Applying

1. Round each number to the nearest ten.

   a) 59 c) 55 e) 878
   b) 52 d) 117 f) 113

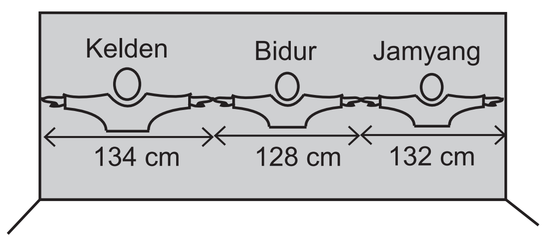
2. Round each number to the nearest hundred.

   a) 878 c) 420 e) 85
   b) 823 d) 150 f) 31

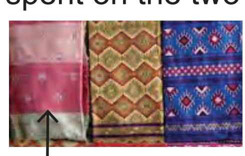
3. Estimate the sum. Show your work.

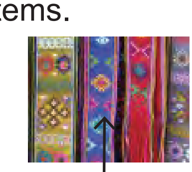
   a) 44 + 28 e) 174 + 226
   b) 19 + 19 + 13 f) 99 + 399
   c) 317 + 383 g) 500 + 200
   d) 777 + 17 h) 615 + 51

4. When Kelden, Bidur and Jamyang joined their arm spans they cover the wall of their classroom from one end to the other. Estimate the length of the wall in centimetres.



5. Dil Maya bought a kira for Nu 475 and a kera for Nu 125. Estimate the total amount of money she spent on the two items.


A kira (Nu 475)


A kera (Nu 125)

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Lesson 5 Addition Algorithm

Try This

Estimate the sum of 489 + 243. Show your working.
Calculate the sum of 489 + 243. Show your working.
How close is your estimate to the actual sum?

You have learnt how to add 3-digit numbers in the earlier lessons using various ways such as using mental maths, using base ten blocks, and using place value tables.

Let us review how you add 3-digit numbers using a place value table, to add 489 + 243. Draw a place value table, put the digits in their appropriate places, add the digits in the same columns, and regroup the numbers as necessary.

Hundreds	Tens	Ones
4 + 2	8 4	9 3
6 1	12 (10 + 2) 1	12 (10 + 2)
7	**3**	**2**

10 tens is 1 hundred   10 ones is 1 ten

We can also add numbers in a way similar to using the place value tables, but without using the tables. First, we need to place the numbers in such a way that the digits having the same place value are in the same columns. For example, we add 489 + 243 in the following way.

489	
+ 243	
600	Add the hundreds (4 hundreds and 2 hundred is 6 hundreds)
+ 120	Add the tens (8 tens and 4 tens is 12 tens)
+ 12	Add the ones (9 ones and 3 ones is 12 ones)
732	Add the hundreds, tens and ones for the final sum

Note

A step-by-step way of adding numbers is called Addition Algorithm.

Examples

Example 1 Add 612 + 144 + 108.

Solution	Thinking
612 + 144 + 108 800 + 50 + 14 864	I lined up the numbers so that the digits which belong to the same place value are in the same column. I added the hundreds first (600 + 100 + 100 = 800). Then, I added the tens (10 + 40 = 50). Then, I added ones (2 + 4 + 8 = 14). I repeated the same process for 800 + 50 + 14 to get the final sum of 864.

Example 2 Add 491 + 37

Solution	Thinking
491 + 37 400 + 120 + 8 528	I lined up the numbers so that the digits which belong to the same place value are in the same column. The second number has no hundreds, so I wrote down 400 from the first number. Then, I added the tens (90 + 30 = 120). Then I added the ones (1 + 7 = 8). The final sum is, then, 400 + 120 + 8 = 528.

Practising and Applying

1. Add:

a) 137 + 421 b) 752 + 132 c) 250 + 250

d) 506 + 236 e) 644 + 23 f) 72 + 713

g) 333 + 444 + 222 h) 15 + 273 + 500 i) 56 + 48

2. Purna added 25 + 75 in the following way. He added the ones first, and, then, he added the tens. Is the sum correct? How do you know?

$$\begin{array}{r} 25 \\ +\ 75 \\ \hline 10 \\ 90 \\ \hline 100 \end{array}$$

3. Lhamo added 425 + 31 in the following way. How do you know that the sum is wrong? Where did she make the mistake? How would you solve it?

$$\begin{array}{r} 425 \\ +\ 31 \\ \hline 700 \\ 30 \\ 5 \\ \hline 735 \end{array}$$

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Lesson 6 Meanings of Subtraction

Try This

Daza wants to buy a geometry box which costs Nu 80. Daza has only Nu 50 with her. So, she asks her mom for the additional money needed to buy the geometry box. How many Ngultrums does she require from her mom?

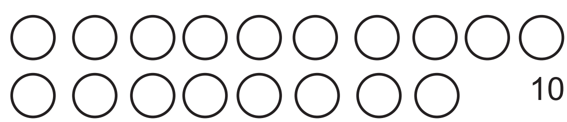
There are three situations in which we use subtraction. In the first situation, we remove or take away some items from a set and we determine how many are left in the set. For example: You have 10 marbles. You give away 3 marbles to your friend. Now, how many marbles are left with you? Here, the set is 10 marbles. When you take away 3 marbles from the set, it is left with 7 marbles. We subtract 3 from 10 to get a **difference** of 7.

10 - 3 = 7

In the second situation, we compare two sets, and determine which set has more items and by how many more. For example: You have 10 marbles, and your friend has 8 marbles. Now, who has more marbles and how many more? Here, the two sets are the set of 10 marbles and the set of 8 marbles. You compare the two sets to say that you have 2 more marbles than your friend. We subtract 8 from 10 to get a difference of 2.

Your marbles

Your friend's marbles



10 - 8 = 2

In the third situation, we determine how many items we need to add to a set to make it grow from its initial number to a bigger number. For example: You have 10 marbles in your hand. But you want to increase the number of marbles to 15. Now, how many more marbles do you need? We can represent this situation and the subtraction involved in it as shown below.

10 + ? = 15 | 15 - 10 = 5

The above situations of subtraction are called "take away", "comparison" and "finding the missing addend" situations respectively. Which one of these three situations of subtraction is the case with additional money required by Daza in the above **Try This** problem?

Examples

Example 1 Write a subtraction sentence for the situation represented by the following diagram. What situation is involved in this subtraction?

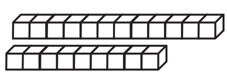
Solution	Thinking
13 - 7 = 6 It is a take away situation of subtraction.	There are 13 sticks in the set, and 7 are being removed or taken away from it. 6 sticks would be left in the set then. So, the subtraction is 13 - 7 = 6.

Example 2 Use subtraction to find the missing addend in the addition sentence 8 + __ = 12.

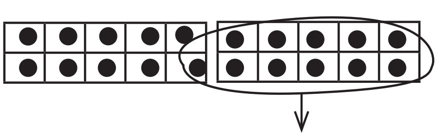
Solution	Thinking
8 + __ = 12 12 - 8 = 4	First, I write the addition sentence, 8 + __ = 12. To find the missing addend, I subtract 8 from 12 to get the difference of 4, or 12 - 8 = 4. So, 4 is the missing addend.

Practising and Applying

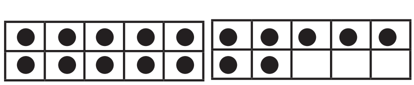
1. Write a subtraction sentence for the two trains of cubes. What kind of a subtraction situation is this?



2. Write a subtraction sentence for the ten frames. What kind of a subtraction situation is this?



3. Write a subtraction sentence for the ten frames. What kind of a subtraction situation is this?



4. The lengths of the two lines below are as indicated. Write a subtraction sentence based on them.

5 cm

2 cm

5. Draw a diagram to represent the subtraction sentence, 10 - 6 = 4.

6. Use subtraction to find the missing addend in each of the following addition sentences.

   a) 7 + __ = 10
   b) 10 + __ = 16
   c) __ + 18 = 20
   d) __ + 6 = 12

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Lesson 7 Subtracting 2-digit Numbers using Various Strategies

Try This

What is the **difference** of 36 - 22? Explain how you solved it.

The result of subtracting numbers is called **difference**. You have learned how to subtract 2-digit numbers using various strategies in Class 2. You will review these strategies in this lesson. For example, you can find the difference of 55 - 32 (that is to subtract 32 from 55) in any of the following ways:

- Subtracting the tens and the ones separately, and then adding the differences of the tens and the ones. 5 tens minus 3 tens is 2 tens, and 5 ones minus 2 ones is 3 ones. Now, 2 tens and 3 ones is 23. So, 55 - 32 is 23.

$$\begin{aligned} 55 - 32 &= (50 - 30) + (5 - 2) \\ &= 20 + 3 \\ &= 23 \end{aligned}$$

- Counting on from 32 to 55 in the following ways: 32 to 40 is 8, 40 to 50 is 10, and 50 to 55 is 5. So, altogether, it would be 8 plus 10 plus 5, which is 23. You could write the process as below.

$$\begin{aligned} 55 - 32 &= 8 + 10 + 5 \\ &= 18 + 5 \\ &= 23 \end{aligned}$$

- Keep the first number intact as 55, think of 32 as 30, and subtract 30 from 55 to get 25. Then, subtract 2 from 25 to get 23. You could write the process as below.

$$\begin{aligned} 55 - 32 &= (55 - 30) - 2 \\ &= 25 - 2 \\ &= 23 \end{aligned}$$

How did you solve 36 - 22 in the above **Try This** problem? How would you solve it in a different way?

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Examples

Example 1 Find the difference of 68 - 25.	
Solution 1 68 - 25 = (60 - 20) + (8 - 5) = 40 + 3 = 43	**Thinking** I subtracted the tens and the ones separately, to get 40 and 3. Then, I added 40 and 3 to get the overall difference of 43.
Solution 2 68 - 25 = (68 - 20) - 5 = 48 - 5 = 43	**Thinking** First, I subtracted only 20 from 68 to get 48. Then, I subtracted the remaining 5 from 48 to get 43.
Solution 3 68 - 25 = 5 + 30 + 8 = 35 + 8 = 43	**Thinking** I counted on from 25 to 30 as 5, then 30 to 60 as 30, and then from 60 to 68 as 8. So, altogether, it would be 5 plus 30 plus 8 from 25 to 68, which is 43.

Example 2 If 50 - 30 = 20, what would be the difference for 50 - 32?	
Solution 1 50 - 32 = 18	**Thinking** If 50 - 30 = 20, then I know the difference will be 2 less than 20 for 50 - 32, which is 18.

Practising and Applying

1. Find the **difference** for each of the following subtractions using any strategy. Show your work.
   a) 68 - 25
   b) 76 - 55
   c) 85 - 75
   d) 90 - 75
   e) 32 - 18

2. One school has 97 students. Another school has 72 students. How many more students does the larger school have?

3. If 50 - 20 = 30, what would be the difference for 50 - 22?

4. If 50 - 20 = 30, what would be the difference for 50 - 19?

5. Kaka solved 63 - 41 as shown below.

   63 - 41 = 9 + 10 + 3
   = 19 + 3
   = 22

   Do you think that Kaka has done the subtraction correctly? Why?

6. If 63 - 41 = 22, what would be the difference for 63 - 31? Why?

7. If 63 - 41 = 22, what would be the difference for 63 - 51? Why?

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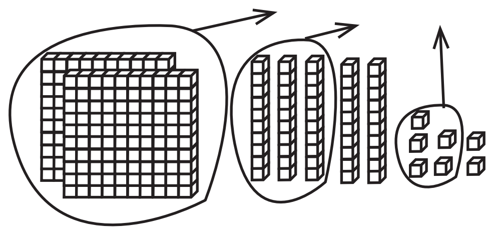
Lesson 8 Subtracting with Base Ten Blocks

Try This

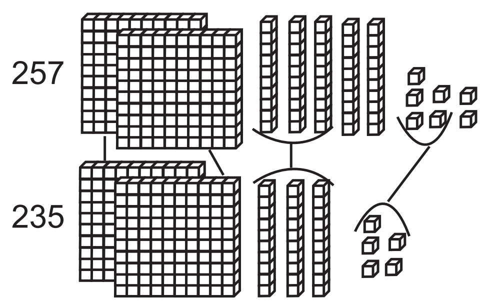
Make a set with base ten blocks to represent 46. Remove 1 tens block and 5 ones blocks from the set. What number do the remaining blocks in the set represent?

Recall that there are three meanings or situations for subtraction. We can use base ten blocks to represent each situation and find the difference. For example, let us subtract 257 - 235 relating it to each situation, one by one.

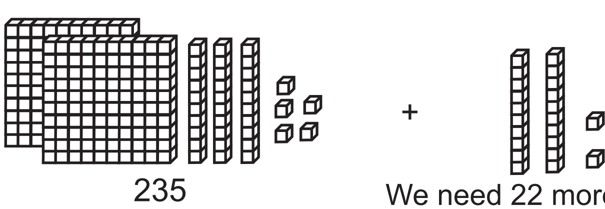
First, let us consider it as a 'take away' situation. Make a set to represent 257 with blocks. Then, we take away 235 from it. That leaves 2 tens and 2 ones (or 22) in the set. So, 257 - 235 = 22.



Second, let us consider it as a 'comparison' situation. Here, we should make two sets of blocks to represent each number. Then, compare the two sets to see how many more blocks 257 has than 235. We see that 257 has 2 tens and 2 ones more (or 22). So, again, 257 - 235 = 22.



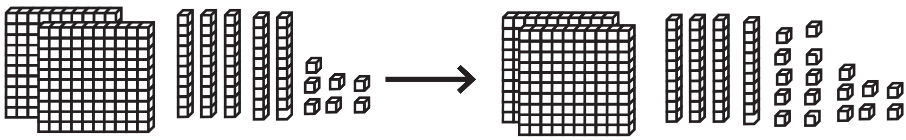
And, third, let us consider it as a 'missing addend' situation. Here, we make a set with the blocks to represent the smaller number, which is 235. Then, find out how many more blocks are required to make the set to represent 257. This is like counting on from 235 to 257.



We need 22 more to make 235 to 257.

Which subtraction situation is involved in the above **Try This** problem? Write a subtraction sentence for it.

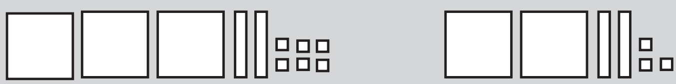
Sometimes, we have to **regroup** the larger number in order to do the subtraction. This is required when there are more ones, tens or both in the smaller number than in the larger number. For example, to solve 257 - 239, first, represent 257 with blocks. Although, we can take away 2 hundreds and 3 tens from the set of blocks, we will have a problem in taking away 9 ones because there are only 7 ones in the set. So, we need to regroup 1 tens block into 10 ones blocks. Then, we can take away 9 ones, 3 tens, and 2 hundreds from the set.



2 hundreds 5 tens 7 ones is regrouped as 2 hundreds 4 tens 17 ones

Example

Example 1 Write a subtraction sentence for the two sets below.



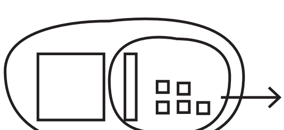
Solution	**Thinking**
326 - 223 = 103	The first set shows 326 and the second set shows 223. I compared the sets and find out that the first set has 1 hundred and 3 ones more than the second set. So, the difference of the two numbers is 103.

Practising and Applying

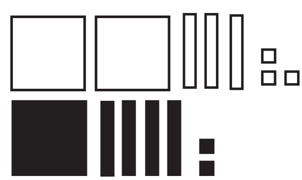
1. Solve each subtraction using base ten blocks. Sketch the diagrams and write the subtraction sentence for each.

   a) 468 - 125 d) 255 - 228
   b) 536 - 33 e) 150 - 45
   c) 325 - 132 f) 123 - 89

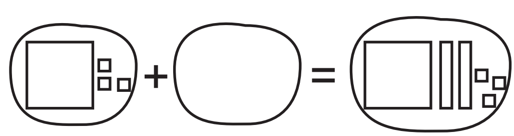
2. Write a subtraction sentence for the diagram below.



3. Write a subtraction sentence for the set of diagrams. What does each number in your sentence tell?



4. Write an open addition sentence for the three sets of diagrams. What is the missing addend?



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Lesson 9 Subtracting with Place Value Tables

Try This

Choden is trying to find the difference for 25 - 19 by using a place value table. She calculated the difference as 14, which is not correct. What did Choden do wrong in her place value table? How would you solve it correctly?

Tens	Ones
1 - 2	9 5
1	4

You have learnt how to subtract 2-digit numbers using place value tables in class 2. Let us review this here. For example, to solve 56 - 32, first, write the numbers in the place value table appropriately with the larger number placed on the top of the smaller number. Then, subtract the ones and the tens separately. The difference is the number obtained in the next row of the place value table.

Tens	Ones
5 - 3	6 2
2	4

56 - 32 = 24

However, in some cases, you can not subtract the tens and the ones straight away. This happens when the digit in the ones place of the smaller number is greater than the digit in the ones place of the larger number. In such a case, you have to first regroup the larger number. For example, to solve 56 - 39, you realise that 9 is greater than 6, so you regroup 56, which is **5 tens 6 ones** as **4 tens 16 ones**. Now, you can subtract the tens and the ones separately as shown on the right to get the difference of 17.

Tens	Ones
~~5~~ 4 - 3	~~6~~ 16 9
1	7

56 - 39 = 17

Similarly, we can subtract 3-digit numbers using the place value tables. For example, let us solve 362 - 148 as shown below. Notice that the digit 8 in ones place in 148 is greater than the digit 2 in 362. So, 362 is first regrouped from **3 hundreds 6 tens 2 ones** to **3 hundreds 5 tens 12 ones** before the separate subtractions of ones, tens and hundreds are carried out.

Hundreds	Tens	Ones
3 - 1	~~6~~ 5 4	~~2~~ 12 8
2	1	4

362 - 148 = 214

Examples

Example 1 Subtract using a place value table: 753 - 321.

Solution

Hundreds	Tens	Ones
7 - 3	5 2	3 1
4	3	2

753 - 321 = 432

Thinking

I put the numbers in the table. There was no need to regroup 753 since the digits in both the ones and the tens places in it are greater than those in 321. So, I simply subtracted the numbers separately in the ones, tens, and hundreds.

Example 2 Subtract using a place value table: 753 - 388.

Solution

Hundreds	Tens	Ones
~~7~~ 6 - 3	~~5~~ ~~4~~ 14 8	~~3~~ 13 8
3	6	5

753 - 388 = 365

Thinking

I put the numbers in the table. Here, both 5 and 3 in 753 are less than the digits in the same places of 388. So, I regrouped 753 as 6 hundreds 14 tens and 13 ones. Then, I subtracted 8 ones from 13 ones, 8 tens from 14 tens and 3 hundreds from 6 hundreds to get the difference of 365.

Practising and Applying

1. Subtract each of the following using a place value table. Write the subtraction sentence for each.

   a) 95 - 72
   b) 864 - 621
   c) 842 - 627
   d) 842 - 687
   e) 555 - 44
   f) 555 - 77

2. What are the three meanings of subtraction? From these three meanings, which one is used when subtracting using place value tables?

3. Wangyal subtracted 31 - 19 using a place value table as shown below and wrote the difference as 28, which is wrong. What mistake did Wangyal make?

Tens	Ones
3 - 1	1 9
2	8

31 - 19 = 28

4. Solve the 31 - 19 using a place value table.

5. How would you solve 31 - 19 without using a place value table? Show your workings.

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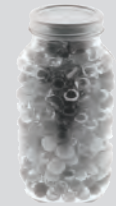
Lesson 10 Estimating Difference

Try This

Do not subtract, but tell about how many more marbles there will be in container B than in container A. Container A has 68 marbles in it. Container B has 174 marbles in it.

A



B

It is also always good to estimate a difference before you actually carry out a subtraction. This will help you to determine whether the difference you get after subtracting the numbers is reasonable or not. For example, in problem 3 of the last lesson, if Wangyal had estimated, he would have realised that 28 is too large a difference for 31 - 19, because 31 is very close to 30 and 19 is very close to 20, so the difference should be around 10 and not 28.

One way to estimate a difference is to round the numbers to the nearest ten. For example, to estimate the difference for 63 - 28, we round 63 to 60 and 28 to 30, so, the estimated difference is 30.

For 3-digit numbers, we can estimate by rounding to the nearest hundred. For example, 315 - 189 is about 300 - 200, which is about 100. You could also estimate by rounding the numbers to their nearest tens. For example, 315 - 189 is about 320 - 190, which is about 130. Sometimes, we can leave one number as it is and round only the other number to estimate the difference. For example, 315 - 189 is about 315 - 200, which is about 115. All of these estimates are correct.

You should be able to carry out the estimates in your head using mental maths.

In some situations you do not even need to calculate the exact difference as an estimate is all that you need. For example, if a school has 312 students and another school has 199 students, we could simply say that the first school has about 100 more students than the second school.

Example

Example Estimate the difference for 486 - 206. Show your working.		
Solution 1 486 - 206 is about 300	Working 500 - 200 = 300	**Thinking** I rounded each number to the nearest hundred: 486 is close to 500 and 206 is close to 200. So, the difference is about 300. 
Solution 2 486 - 206 is about 280	Working 490 - 210 = 280	**Thinking** I rounded each number to the nearest ten: 486 is close to 490 and 206 is close to 210. So, the estimated difference is 280. 

Practising and Applying

1. Round each number to the nearest ten.

   a) 78 c) 73 e) 35
   b) 116 d) 566 f) 404

2. Round each number to the nearest hundred.

   a) 828 c) 650 e) 26
   b) 875 d) 76 f) 331

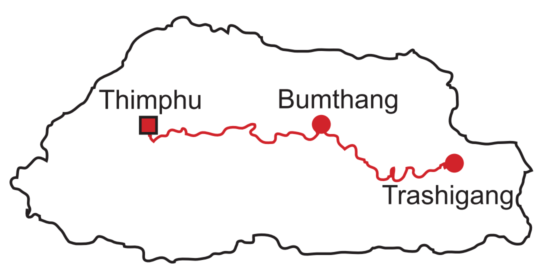
3. Estimate each difference. Show your work by writing the subtraction sentences you used.

   a) 300 - 185 d) 893 - 314
   b) 517 - 88 e) 903 - 289
   c) 138 - 43 f) 500 - 92

4. Pema's estimate for 517 - 286 and Lhamo's estimate for 768 - 575 are the same. What could that estimate be?

5. Write a subtraction sentence for which you could have the same estimate as in question 4.

6. The distance from Thimphu to Trashigang is 555 km. Khotsha is travelling from Thimphu to Trashigang and has reached Bumthang, which is 267 km from Thimphu. About how many km are left for Khotsha to travel?



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Lesson 11 Subtraction Algorithm

Try This

Estimate the difference for 496 - 321. Show your working.
Calculate the difference for 496 - 321. Show your working.
How close is your estimate to the actual difference?

You have learnt how to subtract 3-digit numbers in the earlier lessons using various ways such as using 'counting on', 'base ten blocks', and 'place value tables'. There is also another method of subtraction called subtraction algorithm. Let us solve 493 - 321 using this method. It involves the following steps.

Write the larger number above the smaller number. The digits in the same place value of the two numbers should be aligned vertically. Draw a horizontal line below the smaller number. Then, subtract the digits in hundreds, tens, and ones separately. Add up these differences to get the final difference.

$$
\begin{array}{rl}
493 & \\
-\ 321 & \\
\hline
100 & \leftarrow \text{4 hundreds minus 3 hundreds is 1 hundred.} \\
+\ \ 70 & \leftarrow \text{9 tens minus 2 tens is 7 tens.} \\
+\ \ \ \ 2 & \leftarrow \text{3 ones minus 1 one is 2 ones.} \\
\hline
172 & \leftarrow \text{100 + 70 + 2 is the difference.}
\end{array}
$$

Now, let us solve 493 - 328. Notice that we cannot directly subtract 8 ones from 3 ones, in this case, since 3 is less than 8. So, we have to first regroup the larger number and then proceed with the subtraction as shown on the right.

$$
\begin{array}{rl}
4\ 8\ \underline{13} & \text{Regroup 493 as 4 hundreds 8 tens 13 ones.} \\
\cancel{493} & \\
-\ 328 & \\
\hline
100 & \leftarrow \text{4 hundreds minus 3 hundreds is 1 hundred.} \\
+\ \ 60 & \leftarrow \text{8 tens minus 2 tens is 6 tens.} \\
+\ \ \ \ 5 & \leftarrow \text{13 ones minus 8 ones is 5 ones.} \\
\hline
165 & \leftarrow \text{100 + 60 + 5 is the difference.}
\end{array}
$$

Notice that we have subtracted the numbers from left to right. In other words, we have first subtracted the hundreds, then, the tens, and then, the ones, before adding up the separate difference for the final difference. You can also do it from the right, by subtracting the ones first, then the tens, and then the hundred, as shown below.

$$
\begin{array}{r}
493 \\
-\ 321 \\
\hline
2 \\
+\ \ 70 \\
+100 \\
\hline
172
\end{array}
\qquad\qquad
\begin{array}{r}
4\ 8\ \underline{13} \\
\cancel{493} \\
-\ 328 \\
\hline
5 \\
+\ \ 60 \\
+100 \\
\hline
165
\end{array}
$$

Examples

Example 1 Subtract 637 - 285.

Solution

5 13 7
~~637~~
- 285
300
+ 50
+ 2
352

Thinking

I see that the digit in the tens place, which is 3 in the number 632 is less than the digit 8 in the number 286. So, I regrouped 637 to 5 hundreds 13 tens, 7 ones. Then, I subtracted the hundreds, tens and ones separately. I added up the individual differences to get the final difference of 352.

Example 2 Subtract 523 - 78.

Solution

4 11 13
~~523~~
- 78
400
+ 40
+ 5
445

Thinking

I lined up the numbers so that the digits which belong to the same place values are in the same columns. I see that the digits in both the tens and ones place of 524 are less than the digits in the same places of 78. So, I regrouped the 523 to 4 hundreds, 11 tens, 13 ones. Then, I subtracted the hundreds, tens, and ones separately. I added up the individual differences to get the final difference of 345.

Practising and Applying

1. Subtract:

a) 421 - 357 b) 752 - 129 c) 752 - 291

d) 506 - 236 e) 250 - 250 f) 312 - 88

g) 333 - 222 h) 510 - 490 i) 950 - 80

2. Jigme solved 75 - 23 in the following way. He subtracted the ones first and the tens next. Is the final difference correct? How do you know?

75
- 23
2
+ 50
52

3. Tika Ram solved 458 - 32 in the following way, which is wrong. What mistake did Tika make? How would you solve it?

458
- 32
100
30
8
138

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Lesson 12   Calculating Change

Try This

Gaki Wangmo is buying a bunch of spinach, which costs Nu 15 at the vegetable market. Gaki hands over a Nu 50 note to the seller. How much money should Gaki get back?

You have learnt how to subtract numbers using various ways. In this lesson, you will use your skills of subtracting numbers to calculate **change** involving money in purchasing situations.

In our lives, we buy and sell things. We buy things such as our cloths, groceries, vegetables, notebooks and pencils. We also buy services from other people such as to take us from one place to another in cars and buses, to carry things for us, and to do certain works for us. Sometimes, we also sell things and services to others as shopkeepers and farmers. When we sell things and services, we get money in return. When we buy things and services, we pay in money.

Buying and selling things and services with money happens in all countries. The name of the money is different for different countries. The money of United States of America is called US Dollar. The money of India is called Indian Rupee. In Bhutan, our money is called Ngultrum. The short form of writing Ngultrum is Nu. Our government issues Ngultrums as paper notes as well as coins. People generally prefer to carry paper notes over the coins. Why do you think people do that? The paper notes come with the values of Nu 1, Nu 2, Nu 5, Nu 10,   Nu 20, Nu 50, Nu 100, Nu 500 and Nu 1000.

Change here means the money you get back when you hand out more money than the cost of something that you buy. For example, you go to a shop and buy a pen which costs Nu 15. But you have only a Nu 20 note. So, you give it to the shopkeeper. The shopkeeper then gives you Nu 5. Nu 5 is your change.

Example

Example 1 Kuenley has a Nu 100 note. How much change should he get if he buys a pair of goggles which cost Nu 72?

Solution 1	**Thinking**
$\begin{array}{r} 0\;9\;10 \\ \cancel{100} \\ -\;72 \\ \hline 20 \\ +\;8 \\ \hline 28 \end{array}$ Kuenley should get Nu 28 in change.	The change for Kuenley should be Nu 100 - 72. I solved the subtraction by using subtraction algorithm. I regrouped 100 to 9 tens and 10 ones. Then, I subtracted the tens and the ones separately, which gives a total difference of 28. 
Solution 2	**Thinking**
$100 - 72 = 8 + 10 + 10$ $= 28$ Kuenley should get Nu 28 in change.	I also solved 100 - 72 as shown here. I counted on 8 from 72 to 80, then 10 from 80 to 90, and then another 10 from 90 to 100. So, the total counting from 72 to 100 is 28. 

Practising and Applying

1. Calculate your change in each case. Show your workings.

   a) You have a Nu 20 note. You buy a packet of wai wai which costs Nu 12.

   

   b) You have a Nu 20 note and a Nu 10 note. You buy an ice cream which costs Nu 25.

   

   c) You have a Nu 500 note. You buy a pair of bangles that costs Nu 280.

   

   d) You have two notes of Nu 100. You buy a football for Nu 175.

   

   e) You have a Nu 100 note. You buy a pen for Nu 35.

   

2. Yangzom went with her mother to buy her new school uniforms at the beginning of the year. They bought the following dresses.

   a kira for Nu 430
   a tego for Nu 150
   a wonju for Nu 120
   a pair of socks for Nu 80

   Her mother paid Nu 800 to the shopkeeper. How much change should she get back?

3. Aum Zangmo was travelling from Paro to Thimphu in a taxi. Her taxi fare was Nu 160. After reaching Thimphu, she paid Nu 200 to the taxi driver. The taxi driver gave her change of Nu 30. Did she get the correct amount of change? How do you know?

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Chapter Review

1. Estimate each sum. Write the addition sentences you wrote to estimate the sums.

   a) 413 + 387 b) 208 + 312 c) 717 + 34 d) 500 + 244

2. Estimate each difference. Write the subtraction sentences you wrote to estimate the differences.

   a) 413 - 287 b) 808 - 312 c) 737 - 35 d) 500 - 214

3. Calculate each sum using the addition algorithm.

   a) 413 + 365 b) 642 + 312 c) 717 + 34 d) 500 + 344

4. Calculate each difference using the subtraction algorithm.

   a) 864 - 342 b) 864 - 328 c) 723 - 378 d) 500 - 366

5. Use any method you want to calculate the sum or difference. Show your workings.

   a) 364 - 299 b) 364 + 199 c) 522 - 98 d) 522 + 98

6. Fill in the missing numbers to make the sums correct.

a)
$$\begin{array}{rccc} & 5 & 2 & 6 \\ + & \square & \square & \square \\ \hline & \square & \square & \square \\ + & & \square & \square \\ + & & & \square \\ \hline & 7 & 5 & 8 \end{array}$$

b)
$$\begin{array}{rccc} & \square & \square & \square \\ + & \square & \square & \square \\ \hline & \square & \square & \square \\ + & & \square & \square \\ + & & & \square \\ \hline & 7 & 6 & 8 \end{array}$$

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7. Fill in the missing numbers to make the differences correct.

a)
$$\begin{array}{r} 7\ 6\ 5 \\ -\ \square\square\square \\ \hline \square\square\square \\ +\ \square\square \\ +\ \square \\ \hline 3\ 1\ 4 \end{array}$$

b)
$$\begin{array}{r} \square\square\square \\ -\ \square\square\square \\ \hline \square\square\square \\ +\ \square\square \\ +\ \square \\ \hline 3\ 2\ 5 \end{array}$$

8. Fill in the missing numbers so the the sum is correct.

$\square\square\square + \square\square\square = 544$

9. Fill in the missing numbers in the subtraction sentences.

a) $\square\square\square - \square\square\square = 544$

b) $\square\square\square - \square\square = 544$

10. You are at a shop buying a packet of crayons that costs Nu 65. You hand over the shopkeeper a Nu 100 note. How much money will you get back as change?

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CHAPTER 3 MEASUREMENT: ANGLE, LENGTH AND TIME

Chapter Overview

Measurement involves comparison. For example, when we measure and say that a length is 5 centimetres, we are, in fact, saying that it is 5 times longer than a length which is 1 centimetre.

Through your experiences in the earlier classes, you would be familiar with the measurements of length, time, area, mass and capacity. In this chapter, we will review your ideas and skills related to measuring length and time. Additionally, you will learn about angles, how to represent and compare the angles.

This chapter has 9 lessons as detailed in the Table of Contents.

Basic Principles about Measuring Length and Time

- Any measurement comparison can be stated in two ways. For example, if A is more than B, it means B is less than A.
- The units used for measurements should be appropriate to the contexts.
- The length of an object is the measure of how long it is.
- The measurement of time is not so much about reading clocks as about how long an event takes or its duration.

Chapter Goals

By the end of this chapter, you will be able to:

- Describe angles as turns, represent and compare angles as greater than or less than the right angles.
- Measure lengths correctly and express them in metres, centimetres, millimetres and combinations of these units.
- Gain a sense of how long a kilometre is.
- Measure and calculate perimeters of various shapes.
- Read and tell times from analog and digital clocks.
- Describe the relationships among different units of time.
- Tell the number of days for each month.
- Calculate and describe the duration of various events.

Lesson 1 Turns and Angles

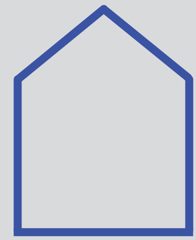
Try This

Look at the picture.

A. Mark all the corners in the picture.

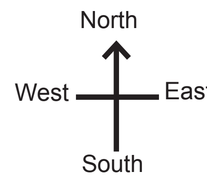
i) How many corners does the shape have?

ii) Are all corners same? How do you know?



Turn

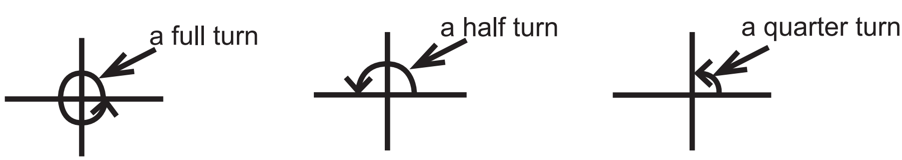
Stand facing the east. Then, turn your body leftward by moving your feet until you are back to your initial position. You have made a full turn.



Again, stand facing east, turn your body leftward and stop at the position when you are facing west. At this position, you have made a half turn.

Again, stand facing east, and turn leftward and stop at the position when you are facing north. You have made a quarter turn.

Next, stand facing east, and turn rightward and stop at the postion when you are facing south. How much of a turn did you make?



Angle

The amount of turn is called **angle**. If you make a big turn, then, you make a big angle. Similarly, if you make a small turn, then, you make a small angle.

An angle is made of two arms that meet at a point called the **vertex.**

You can think about angle as if the two arms started out together and then one arm is turned at the vertex.

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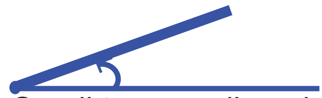
The more the arm is turned, the greater is the angle.

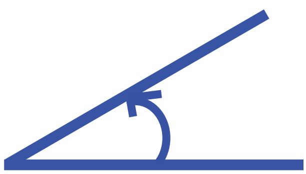
No turn no angle



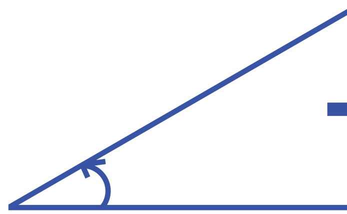
Small turn small angle

Big turn big angle

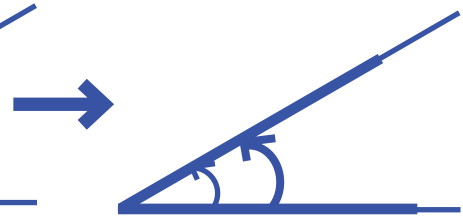
It does not matter how long the arms are or if both are of the same length. Angles with different arm lengths can be the same size.



Angle A



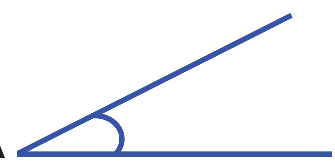
Angle B



Angle A and angle B are of the same size

Representing an angle

You can show an angle by drawing part of a circle to represent the turn between the two arms. You can name an angle by giving its vertex a letter name.



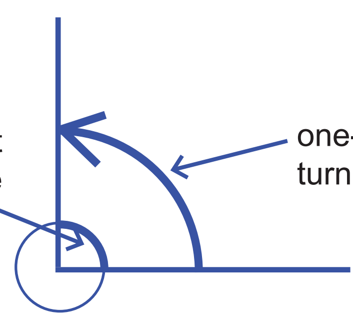
This is angle A

Right Angle

When you stand facing the east and turn leftward till you face north you make a quarter turn. You have turned one-fourth of the full turn.

The angle made by a quarter turn is called **right angle**.

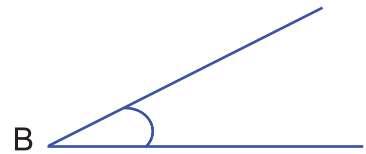
Using the two arms, when one arm is turned one-fourth away from the other arm, a right angle is formed. The letter 'L' makes a right angle at the corner.



Comparing angles

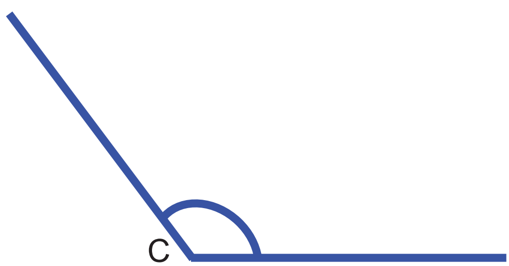
Look at angles A, B and C.

Angle A is a right angle because one arm is turned one-fourth away from the other arm.



Angle B is less than right angle because one arm is turned less than one-fourth away from the other arm.

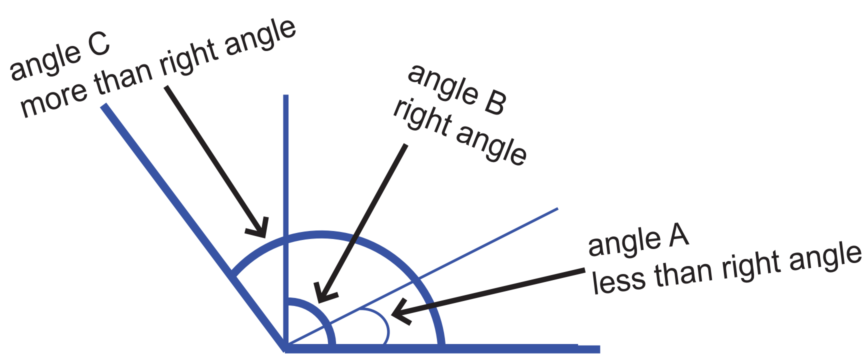
So, angle B is less than angle A. In other words angle B is less than right angle.



Angle C is more than right angle because one arm is turned more than one-fourth away from the other arm.

So, angle C is more than angle A. In other words angle C is more than right angle.

You can compare the size of angles A, B and C by overlapping them.



B. i) How many corners in the shape in "Try This" are right angles?

ii) How many corners are greater than right angle?

iii) How many corners are less than right angle?

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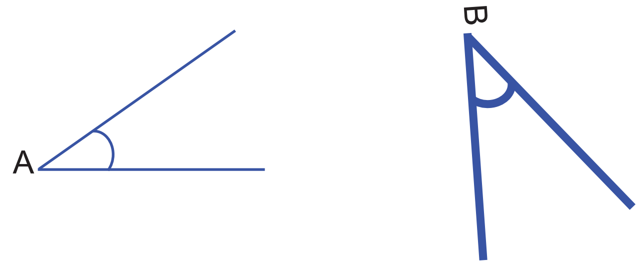
Examples

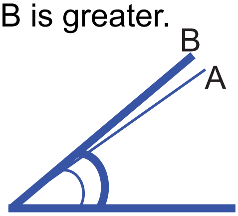
Example 1 Comparing Angles

Which angle is greater?
How do you know?



Solution

Angle B is greater.



The top arm of angle B is turned away more than the top arm of angle A

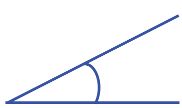
Thinking

• Because the angles opened different ways, I couldn't compare them by just looking at them.

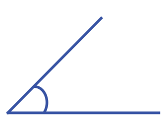
• I traced angle B and then put the tracing on top of angle A.
• I made sure the bottom arm of angle B matched the bottom arm of the angle A.

Practising and Applying

1. For each angle below,
• draw a bigger angle
• draw a smaller angle

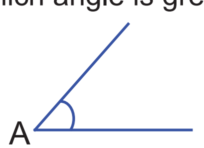
a)  b) 

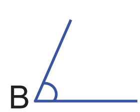
2. Determine the angles greater than or less than the right angle.

a)  b) 

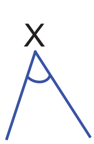
3. Which angle is greater in each pair?

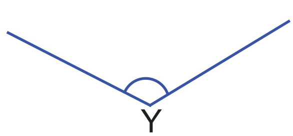
a)





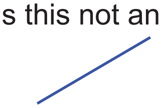
b)





4. Draw an angle to match each description.
a) an angle that is a little less than a right angle
b) an angle that is about half a right angle
c) an angle that is almost twice the size of a right angle.

5. Why is this not an angle?



6. Pema says that angle A is greater than angle B.
a) What mistake do you think he is making?
b) How would you explain to him why he is wrong?

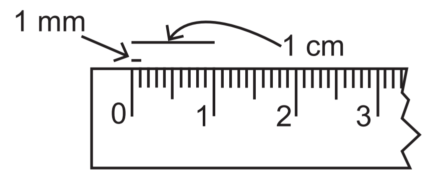
Lesson 2 Measuring Lengths in Centimetre and Millimetre

Try This

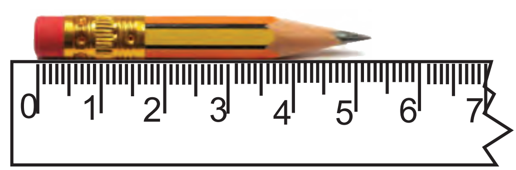
Measure the length of your pencil with a ruler. How long is it in centimetres? Describe how you measured the length.

The length of something is the measure of how long it is. We use units like the **centimetre (cm)** to express lengths.

If you observe the cm marking on a ruler, you will see that each cm is made up of 10 small equal parts. Each of these small lengths is called a **millimetre (mm)**. There are 10 mm in 1 cm (1 cm = 10 mm).



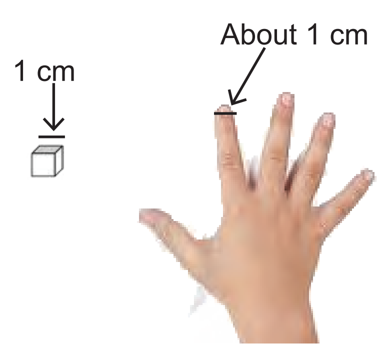
When you measure the length of an object, remember to line up the 0 cm mark on the ruler with one end of the object. The pencil in the diagram is about 5 cm. If you use the millimetre as the unit, it is about 50 mm.



It is a good and useful mathematical habit to estimate lengths, in terms of the various units, before you actually measure them. Sometimes, an estimate is all that you might require.

Your estimation of the lengths will be good if you have a good sense how long a cm and a mm are. 1 cm might be about how wide your fingertip is. 1 cm is also about the length of a ones block in your set of base ten blocks. How would you confirm these? What else might be about 1 cm long?

1 mm might be about the thickness of a coin or a spoon. How would you confirm these? What else might be about 1 mm long?



Describe the length of your pencil measured in the above **Try This** problem, using both cm and mm.

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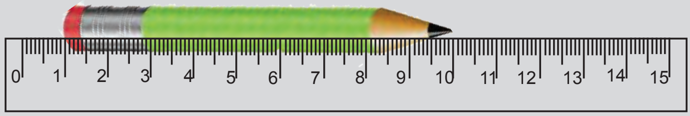
Examples

Example 1 Pema says that the length of her pencil is 10 cm. Is she correct? Why? What should be the correct length of her pencil? How many millimetres is that?



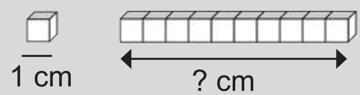
Solution

No, Pema is not correct, because she is reading the 10 cm mark without lining the end of the pencil with the 0 cm mark of the ruler. The correct length of the pencil is 9 cm. That is 90 mm.

Thinking

I know that the correct way to measure the length of an object is to line up the 0 cm mark with an end of the object. However, Pema could deduct 1 cm from 10 cm and get the length correctly as 9 cm since she has lined the pencil with the 1 cm mark. I know that 1 cm has 10 mm. So, I skip counted in 10s for 9 times to get 90 mm for 9 cm.

Example 2 If a ones block is 1 cm long, how long will a tens block be? Why? How will you confirm it?



Solution

A tens block will be 10 cm long, because a tens block is the same as 10 ones blocks joined.
I will confirm it by measuring it with a ruler.

Thinking

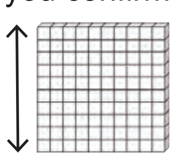
A tens block is 10 times longer than a ones block. So, if the ones block is 1 cm long, then the tens block has to be 10 cm long.

Practising and Applying

1. Use a ruler to draw straight lines of the following lengths.

a) 1 cm	b) 2 cm	c) 3 cm
d) 5 cm	e) 10 cm	f) 5 mm
g) 10 mm	h) 50 mm	i) 35 mm

2. What do you think is the length of a hundreds block? Why? How would you confirm it?



3. First, estimate and record your estimates in cm or mm. Then, measure and write the lengths.

a) Your pencil sharpener

Estimate: ___
Actual length: ___

b) A Nu 5 note

Estimate: ___
Actual length: ___

c) Your handspan

Estimate: ___
Actual length: ___

Lesson 3   Measuring Lengths in Metre

Try This

Estimate the height of the classroom door in centimetres.

To measure longer lengths or distances, we use a unit called **metre (m)**. As you might recall from class 2, a metre has 100 centimetres (1 m = 100 cm).

You should have a good sense of how long a metre is. One way to gain this sense is to measure yourself against 1 m in various ways using a metre stick or a measuring tape. For instance, 1 m could be about:

a metre stick

Measuring tapes

- the height of your chest from the floor when you stand,
- 1 giant step that you take,
- the length between your left elbow and the tip of your right hand with your hands stretched.

Check each of the above situations. Also, see about how many of your handspans make 1 metre.

You should experience making your own length measuring tool that is 1 m long. You could make it easily in two ways. One way is to measure and cut a rope or string that is 1 metre in length.

The other way is to cut out or fold a strip of paper, which is about 2 cm wide. Then, make cm marks from 0 cm to 10 cm along an edge of the paper with the help of a ruler. Take a similar paper strip, and make cm marks from 11 cm to 20 cm. Continue on a third strip from 21 cm to 30 cm. Continue this way until you get to 100 cm. Then, carefully join the paper strips with glue ensuring that the cm markings are in proper sequence. You could, then, wrap your paper strips with transparent sellotape, for longer lasting use.

Go about in the classroom and around the school to measure various lengths such as the heights and widths of doors, desks, tables, and windows in metres.

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An activity

Work in pairs or small groups to estimate and measure the following lengths. You should have, at least, a measuring tape, or a metre stick, or a length of 1 metre that you might have made, as suggested on the previous page. Make a table as shown below in your notebook. If possible, sketch a diagram for the specified lengths in the table below.

When you record the lengths, round them to the nearest metre. For example, if a length is between 3 and 4 m, but closer to 4 m, then consider it as 4 metres.

Notes

Metre, **centimetre** and **millimetre** are different units of measuring length.

The short forms for writing **metre** is **m**, **centimetre** is **cm**, and **millimetre** is **mm**.

1 m = 100 cm

1 cm = 10 mm

Length or Distance	Estimated Length (in metres)	Measured Length (to the nearest metre)
Height of the classroom door		
Height of a classroom window		
Height of the classroom ceiling		
Length around the chalkboard		
Distance around the classroom floor		
Distance around the main school building		
Distance from one end to the other of the school ground		

Practising and Applying

1. Name 3 things that have a length of less than 1 m.
2. Name 3 things that have a length of more than 1 m.
3. Name 2 things that are about 1 m long.
4. Name 1 thing that is about 2 m long or tall.
5. Calculate the number of centimetres in each.

   a) 2 m b) 3 m

   c) 4 m d) Half a metre
6. The tallest person in 2012, Sultan Kosen, was just over 251 cm tall. Do you think Sultan could walk into your class without bending through the door? How do you know?

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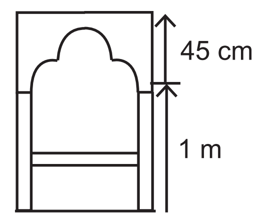
Lesson 4 Combining Units to Measure Lengths

Try This

Measure the length of the line below to the nearest centimetre.

Often the lengths that we measure are not exact to a unit. For instance, the various lengths that you have measured during the last lesson, such as the height of a classroom window might not have an exact number of metres. That is why you have been asked to round them to the nearest metre.

We can describe a length more precisely by using a combination of two units. For example, to measure the height of a window, which is between 1 and 2 m, mark the 1 m level from the base of it. Then, measure in centimetres up to the top from the 1 m mark. If it is 45 cm, then, we can describe the height of the window as: 1 m 45 cm.



Height = 1 m 45 cm

Now measure the various lengths as you did during the last lesson in your groups. This time, express these lengths more precisely as a combination of metres and centimetres. Make a table as shown on the right your notebook.

Length or Distance	Measured Length (in m and cm)
Height of the classroom door	
Height of a classroom window	
Length of the of the classroom floor	
Length around the chalkboard	
Distance around the classroom floor	

For shorter lengths, you should use a combination of centimetres and millimetres to measure and describe them. Make a table as shown on the right. Then measure the various lengths as specified, and express them as a combination of cm and mm.

Length or Distance	Measured Length (in cm and mm)
Length of your pencil or pen	
Length of a pencil sharpener	
Length of your maths textbook	
Thickness of your maths textbook	
Width of your classroom door	
Distance around your wrist	

Could you now express the length of the line in the above **Try This** problem as a combination of cm and mm? Then, draw a line that is 8 cm 8 mm long.

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Lesson 5 Comparing Lengths to a Kilometre

Try This

Name three different units for measuring length that you know from the earlier lessons. Which is the longest unit? Which is the shortest unit?

There is another unit for measuring length. It is called **kilometre (km)**. A kilometre represents a length of 1000 metres (1 km = 1000 m).

As you can sense, a kilometre is a long unit. It is used for measuring long distances such as the distance between two towns. The distance between Thimphu and Paro is about 60 km. The distance between Thimphu and Trashigang is about 500 km.

If possible, take a whole class walk from your school along a road for 1 km. This will give you a good sense of how long 1 km is. Note the time at the start of the walk, and again it at the end of the 1 km walk. Then, walk back to the school. You should observe traffic safety rules if you have to walk along a busy road with cars moving on it. At the end of the walk, discuss about it. Is 1 km a long distance? How long does it take you to walk 1 km? How many km do you think your home is from the school?

Measure and cut a rope that is 1000 metres. Go to the school ground. Tie the rope at one corner of the ground, and stretch it along the boundary of the ground by walking along with it. How many times did the rope go around the ground? How many metres did you walk? How many kilometres did you walk?

Practising and Applying

1. Describe a distance that is less than 1 km.

2. Describe a distance that is more than 1 km.

3. You know that 1 km = 1000 m. Calculate the number of metres for each.

   a) 2 km b) 3 km
   c) 5 km d) Half a km

4. Maya's school ground is about 100 m long. She walked from one end of the ground to the other end and back to the starting point. She repeated this 6 times. Would Maya have walked more than or less than 1 km? How do you know?

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Lesson 6 Choosing an Appropriate Length Unit

Try This

Could you use a kilometre to describe the length of an ant? Why? Why not?

Although we could use any unit for length, such as km, m, cm and mm, to describe any length, some units result in more comfortable numbers than other units depending on the situation.

For instance, we could describe the distance between Thimphu and Paro in terms of metres, which would be about 60,000 metres. This is a large number. If we were to describe the same distance in centimetres, it would be 6,000,000 cm, which is even larger. In terms of kilometres, it is about 60 km. The last one, in kilometres, gives a better number. So, we use kilometres to describe long distances.

Similarly, it is more appropriate to describe the length of a pencil in terms of centimetres, or a combination of centimetres and millimetres.

What unit would you choose for each of the following lengths?

The length of your classroom.
The length of your desk.
The length of an ant.
Your height.

Practising and Applying

1. Sonam was collecting information on the average body length of various animals, and wrote the following. But, he forgot to write the units. Write the missing unit for each (as m, cm, or mm).

Animal	Average body length
Elephant	6
Ant	12
Earthworm	8
python	5
cat	42
housefly	10

2. Complete each statement using one of the units (mm, cm, m, km).

   a) Your *kera* is about 2 ___ long.
   b) A coin is about 1 ___ thick.
   c) A prayer flag is about 8 ___ tall.
   d) The distance from Thimphu to Trongsa is about 200 ___.
   e) A cell phone is about 11 ___ long.

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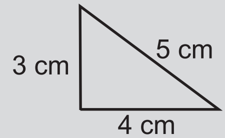
Lesson 7 Measuring and Calculating Perimeter

Try This

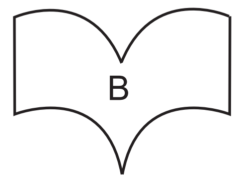
The triangle on the right has the lengths of its 3 sides as shown. What is the sum of the side lengths?



The total length around the boundary of a shape is called its **perimeter**.

If the outside of a shape is made up of straight sides, you could directly measure the length of each side and add them all up to get the perimeter. What is the perimeter of shape A?

If the outside of a shape is curved, you could place a thread all along its boundary. Once the thread has made a complete round, it could be cut and straightened to measure its length. The length of the thread would be the same as the perimeter of the shape. What is the perimeter of shape B?

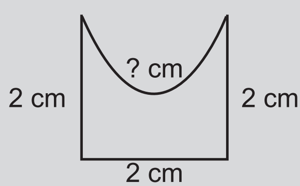


Could you tell the perimeter of your classroom floor, from the measuring activity you might have done during lesson 2?

What is the perimeter of the triangle in the above **Try This** problem?

Examples

Example 1 The perimeter of the shape is 9 cm. What is the length of the curved line? Show how you calculated it.



Solution

2 + 2 + 2 + ? = 9

6 + ? = 9

3

So, the length of the curved line = 3 cm.

Thinking

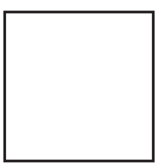
I know that the perimeter is the sum of all the side lengths, which means, 2 + 2 + 2 + ? equals 9, or 6 + ? = 9. I know from my addition facts that 6 + 3 equals 9. So, the length of the curved line must be 3 cm.

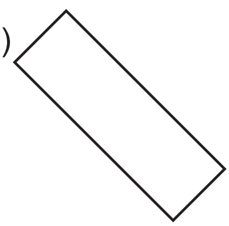
Example 2 The shape is made up of 5 sides. All sides are of the same length. Karma measured one side with a ruler and found it to be 3 cm. Karma says that he does not need to measure the rest of the sides, and says that the perimeter of the shape is 15 cm. Is he correct? How do you know?

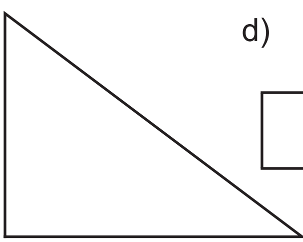
Solution	**Thinking**
Yes, Karma is correct. As the sides are the same, we can simply add 3 repeatedly for 5 times to give the perimeter of 15 cm. Perimeter = 3 + 3 + 3 + 3 + 3 = 15 cm	Since all the sides are of the same length, I need to measure only one side, as Karma did. Then , I can add up the same number repeatedly for 5 times to calculate the perimeter.

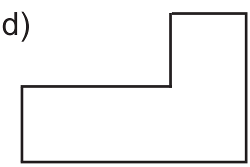
Practising and Applying

1. Use a ruler to measure the perimeter of each shape to the nearest cm.

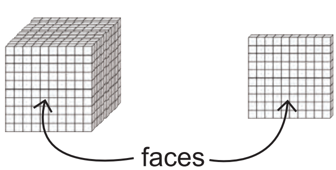
a) 

b) 

c) 

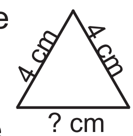
d) 

2. Take out a thousands and a hundreds block. Measure the perimeter of a face of each block, as indicated below, with a ruler. What is the perimeter of each?

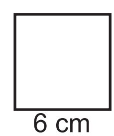


3. Draw a shape that has a perimeter of 15 cm.

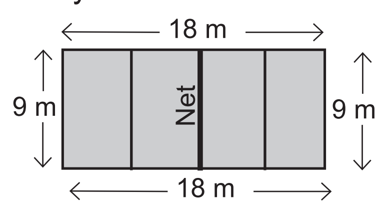
4. The perimeter of a triangle is 12 cm. Two of its sides are 4 cm each. Calculate the length of the third side.



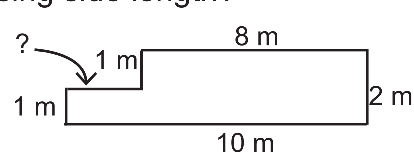
5. A square has a side length of 6 cm. Calculate its perimeter.



6. Calculate the perimeter of a volleyball court.



7. The perimeter of the following shape is 24 m. What is the missing side length?



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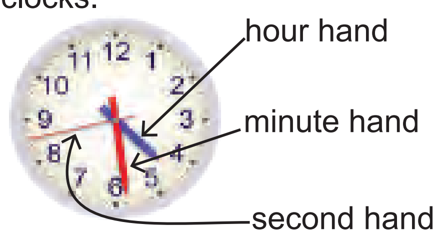
Lesson 8 Using Analog Clock and Digital Clock

Try This

A. Sagar and Sither are looking at the time on two types of clocks.
What type of clocks are they?
Do the clocks show the same time?
How do you know?

A clock is an instrument that measures and shows time.

There are two types of clocks - analog clocks and digital clocks.



An analog clock has numbers from 1 to 12 arranged in a circle. It has hour, minute and second hands that are used to tell the time

A digital clock has numbers separated by colon. The first number tells the hour. The second number tells the minute.

Notes

A **watch** is the same as a **clock**, in measuring and showing time.

A watch is smaller, and is usually worn around the wrist.

A clock is bigger, and is often hung on the wall.

A watch

Telling time using analog clock

You know that when the hour hand moves from one number to the next number, the time is one hour. For example, when the hour hand moves from 12 to 6, the time is six hours.

Similarly, when the minute hand moves from one number to the next the time is five minutes. For example, when the minute hand moves from 12 to 1 or from 1 to 2 or from 2 to 3, the time is five minutes. When the minute hand moves from 12 back to 12 the time is 60 minutes.

If the hour hand and the minute hand start out together at 12, the minute hand completes one round and reaches back to 12. During this time the hour hand moves from 12 to 1. This shows that **1 hour = 60 minutes.**

Notes

1 hour = 60 minutes

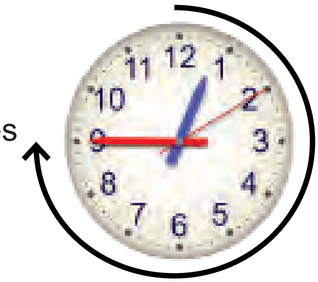
There are **two ways** to tell the time from an analog clock.

Way one

See how many minutes have passed after an hour.

Tell the time as "__ minutes past __ o'clock".

For example, the clock on the right shows that 45 minutes have passed after 12 o'clock. So, the time is **45** minutes past **12** o'clock.



Way two

See how many minutes are required for the hour hand to strike the next hour.

Tell the time as "__ minutes to __ o'clock".

For example, the analog clock above, shows that the minute hand has to travel 15 more minutes to strike the number 12, in order for the hour hand to strike 1 o'clock. So, the time is **15** minutes to **1** o'clock.

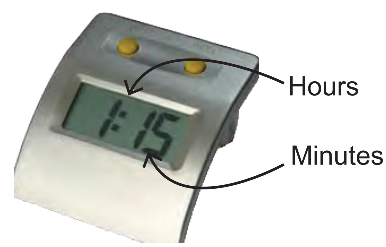
From way one and way two, **45** minutes past **12** o'clock and **15** minutes to **1** o'clock mean the same time.

Telling time using digital clock

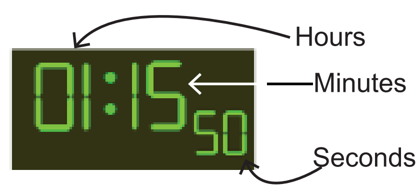
A digital clock shows time with numbers, which are separated by a colon.

The first number, or the number before the colon, tells the hour.

The second number, or the number after the colon, tells how many minutes have passed after that hour.



Some digital clocks, or digital watches, show 3 numbers. The third number tells how many **seconds** have passed. A second is a very short period of time. We shall ignore seconds in our lesson.



The time on the digital clock above is **1:15** or **one fifteen** or **15** minutes past **1** o'clock.

It also means that 45 minutes are required to be 2 o'clock, since there are 60 minutes in an hour.

So, **one fifteen** is also the same as **45** minutes to **2** o'clock.

Notes

For the time less than 10 minutes after an hour like 3:07, we do not say three-seven but rather say three-o-seven.

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B. i. How many minutes have passed after 10 o' clock in the digital clock in the **Try this** above?

ii. How would you tell the time?

iii. How many minutes would it require for the time to become 11 o'clock in the digital clock?

iv. Repeat steps i., ii., and iii. for the analog clock.

Examples

Example 1 Tell the time that the clock shows in two ways.

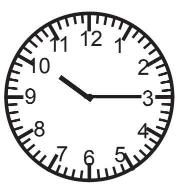
Solution	**Thinking**
35 minutes past 4 o'clock. or 25 minutes to 5 o'clock.	The hour hand had crossed 4, so the time is later than 4 o' clock. The minute hand is at 7. I skip counted the minutes by 5s after 12 as 5, 10, 15, 20, 25, 30, 35. This means 35 minutes have passed after 4 o' clock. So, the time is 35 minutes past 4'o clock. To write the time in another way, I knew that the time is after 4 o' clock. So, I skip counted the minutes by 5s from 7 to 12 as 5, 10, 15, 20, 25. This means that 25 minutes more are required for the time to be 5 o' clock. So, the time is 25 minutes to 5 o' clock.

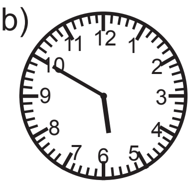
Example 2 What time does the digital clock show?
What does it mean?
Show this time on an Analog clock.

Solution	**Thinking**
The time is twelve thirty four. It means 34 minutes have passed after 12 o'clock. An analog clock showing the time 	I read the hour and the minutes separately. I know that it means 25 minutes have passed after 12 o'clock. In an analog clock, the minute hand would point to the 25 minute mark after the number 12. I know that when the minute hand strikes 12, the hour hand will strike 1. Then, it will be 1 o'clock. There are 60 minutes in an hour. So, 60-25=35 minutes remains before 1 o'clock.

Practising and Applying

1. Look at each clock and write the time in two ways

a) 

b) 

c) 

d) 

2. Write the time shown in digital clock format in words in two ways.

	Numbers	Words
a)	7:15	15 minutes past 7 o' clock 45 minutes to 8 o' clock
b)	10:50	
c)	4:45	
d)	2:07	

3. Write the time shown in words in digital format.

	Words	Numbers
a)	Eight twenty-five	
b)	Twelve forty	
c)	25 minutes past 2 o' clock	
d)	8 o' clock	

4. A baby slept from 1:30 to 3:30.

a) How many hours did the baby sleep?

b) How many minutes did the baby sleep?

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Lesson 9 Relationships Among Different Units of Time

Try This

How many months have passed since you started school this year?
How many weeks have passed since you started school this year?
(You could use a calendar to answer these two questions.)

You know that an hour and a minute are units to measure time. You could measure any duration of time with these units. But it is not always convenient to use these units, especially for measuring long durations, such as your age.

What unit of time would you use to measure your age? Why do you think it is not convenient to use hours to describe your age?

Just like we use kilometres to measure very long distances, we use days, weeks, months and years to measure long durations of time.

The following describe the relationships among different units of time.

1 hour = 60 minutes
1 day = 24 hours
1 week = 7 days

1 month = 30 days (roughly, not exactly for every month)
1 month = 4 weeks (roughly, not exactly for every month)

1 year = 12 months
1 year = 365 days

Use a calendar to discuss and confirm the above relationships among years, months, weeks and days.

You should be able to say and write the names of the 7 days of the week and the names of the 12 months in the year in their correct sequence, as below.

The names of the days are: *Sunday, Monday, Tuesday, Wednesday, Thursday, Friday,* and *Saturday*.

The names of the months are: *January, February, March, April, May, June, July, August, September, October, November,* and *December.*

Use the calendar to see how many days are in each month, and make and fill up a table as shown, with the names of the months and the number of days for each.

Months	Number of Days
January	31
February	
March	
April	
May	

Describe the pattern that you notice in the table. Which months have 31 days? Which months have 30 days? What is different about February from the rest of the months?

Except for February, the number of days for the other 11 months are the same every year. That means January will always have 31 days, March will always have 31 days, April will always have 30 days, and so on. Also, all months except February have 30 or more days.

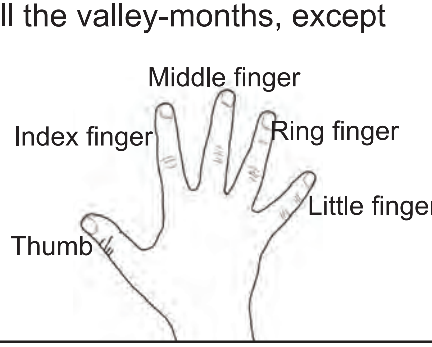
February has 28 days normally, but it will have 29 days every fourth year. Such a year, when February has 29 days, is called a **leap year**. The year 2012 was a leap year. The next leap year will be 2016. What year will be the leap year after 2016?

The rhyme on the right will help you in remembering the number of days in each month. It will be useful to learn to sing and memorise it. Sing it in unison in the class often.

30 days hath September

30 days hath September,
April, June and November,
All the rest have 31,
Excepting February alone,
And that has 28 days clear,
With 29 in each leap year.

Another way to remember the number of days in each month is to use your hands. Make fists of both your hands, so that you can see the knuckles. Let us call the dip between two knuckles as a valley. Arrange the fists next to each other, as shown below. Start with the knuckle of the little finger on the left hand, and say January for it. The valley next to it is, then, February. The knuckle of the ring finger on the left hand is March. Continue this way on to the knuckles and valleys on the right hand. All the knuckle-months have 31 days. All the valley-months, except February, have 30 days.



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Example

Example 1 The people of Laya move to Punakha during the winter, because it is very cold to live in Laya. Ap Chimi and his family moved out of their village on 23 December 2012. After spending their winter time in Punakha, the family reached back home in Laya on 21 March 2013. How many months was the family away from their home? About how many weeks was that? About how many days was that?

Solution	**Thinking**
Number of months the family was away from home = 3 months	The family was out for the whole of January and February. Adding the days of December from 23 to 31, which is 8 days to the 21 days is about 1 month. So, the family was away for about 3 months.
Number of weeks = 4 + 4 + 4 = 12 weeks	1 month is about 4 weeks. So, 3 months is the same as 4 + 4 + 4 weeks.
Number of days = 30 + 30 + 30 = 90 days	1 month has about 30 days. So 3 months will be the same as about 30 + 30 + 30 = 90 days.

Practising and Applying

1. Calculate the number of weeks in each. Use addition.
   a) 2 months b) 3 months
   c) 5 months d) 10 months

2. Calculate the number of months in each. Use addition.
   a) 2 years b) 3 years
   c) 4 years d) 1 and a half year

3. How many days are there in 2 years?

4. The year 2008 was a leap year. Write 3 years that are leap years after 2008.

5. How many days would have been in February of 2008? Why?

6. How many days would have been in February of 2009? Why?

7. In 2014 how many days would there be altogether in January and February? How do you know?

8. How many days would there be if you combine the days of July and August?

9. Sonam's family went on a holiday from 25 December to 7 January. How many days did the holiday last? How many weeks was that?

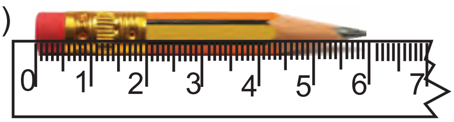
Chapter Review

1. Draw straight lines of the following lengths.

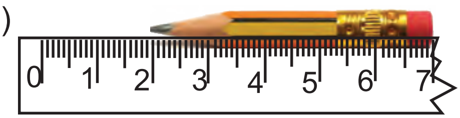
   a) 7 cm   b) 10 mm   c) 3 cm 8 mm   d) 25 mm

2. How many centimetres long is each pencil?

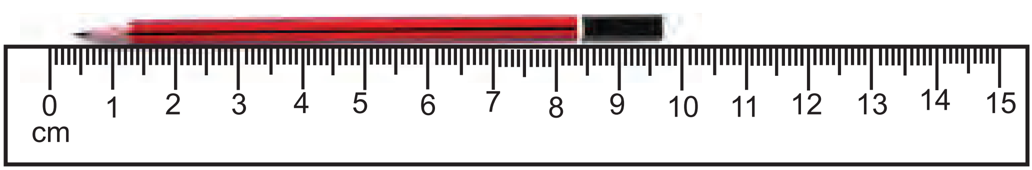
   a)



   b)



3. How long is the pencil. Use both cm and mm to express the length..



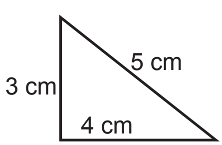
4. Write the correct number in each equation. Show how you worked out the numbers. (1 km = 1000 m; 1 m = 100 cm; 1 cm = 10 mm)

   a) 2 km = ___ m   b) 3 m = ___ cm   c) 6 cm = ___ mm

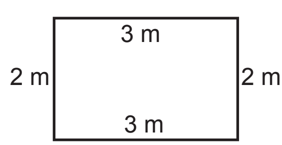
   e) 1 cm 5 mm = ___ mm   f) Half a m = ___ cm   g) Half a km = ___ m

5. Calculate the perimeter of each shape.

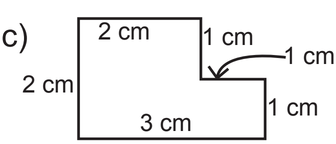
   a)



   b)



   c)



6. The schools in Bhutan close for winter vacation from the 18th of December to the 1st of February. About how many weeks is the winter vacation? About how many days is that?

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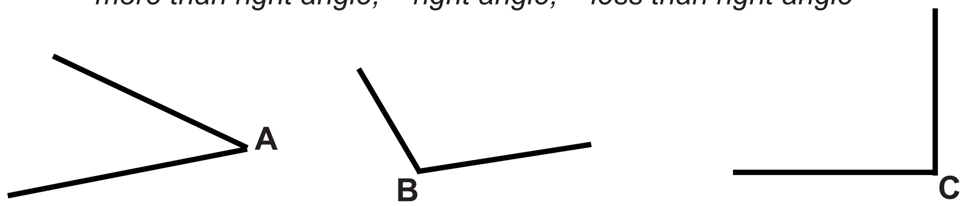
7. Label angle A, B and C by choosing the phrases given below

more than right angle, *right angle,* *less than right angle*



…………………………………… …………………………………… ……………………………………

8. Draw an angle that is
   i) almost a right angle.
   ii) almost two right angles

9. How many quarter turns make one full turn?

10. How many half turns make one full turn?

11. How many degrees are there in a quarter turn?

12. Write the time shown by each watch in two ways.

a) 

b) 

13. One Sunday, a Druk Air plane took off from Paro airport at 1:40. It landed at the Bangkok airport at 5:20.
    How many minutes did the Druk Air plane take to reach Bangkok from Paro on that day?

CHAPTER 4   MULTIPLICATION

Chapter Overview

Multiplication is a way of describing repeated addition. The repeated addition should involve adding a number over and over onto itself. For example, let us consider the following addition: 5 + 5 + 5 = 15. We can describe this addition with multiplication as: 3 x 5 = 15.

Since multiplication is related to addition, we can use our knowledge of addition, such as double facts, to multiply numbers. Multiplication can also be performed using skip counting. However, it will be helpful, and therefore, important for you to also memorise multipication facts (also known as times table). It may be necessary to carry out the memorisation practice quite regularly over an extended period of time.

This chapter has 9 lessons as detailed in the Table of Contents.

Basic Principles about Multiplication

- Multiplication is a way to describe repeated addition.
- Only situations that involve repeatedly adding the same amount, not different amounts, can be described using multiplication.
- When you multiply, the first factor tells the number of equal groups and the second factor tells the size of the equal groups.
- Multiplication can be performed using skip counting.

Chapter Goals

By the end of this chapter, you will be able to:

- Represent a multiplication sentence by diagrams of equal sets, including arrays, and vice versa.
- Use repeated addition, double facts, and skip counting to multiply.
- Generalise the products when a number is multiplied by 1 and 0.
- Relate various multiplication facts.
- Solve simple multiplication problems.
- Commit to memory some multiplication facts, at least up to 6 x 10.

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Lesson 1 Multiplication as Repeated Addition

Try This

How many momos are there? Write an addition sentence for it.

Let us consider the following addition sentence.

2 + 2 + 2 + 2 + 2 = 10.

We notice that 2 is repeatedly added 5 times, and the sum is 10.

We can describe such a repeated addition, using multiplication, as: 5 x 2 = 10. We say, or read, this multiplication sentence as: **5 times 2 equals 10**, or **5 multiplied by 2 equals 10**. 5 and 2 are called *factors* and 10 is called their *product*.

5 x 2 = 10 multiplication sign

factors product

The first factor tells how many times a number is added, and the second factor tells what number is being repeatedly added.

We can also write an addition sentence for a multiplication sentence. For example, 4 x 3 = 12 means that 3 is repeatedly added 4 times, and so, the addition sentence for this multiplication is 3 + 3 + 3 + 3 = 12.

In order to use multiplication for a repeated addition, the numbers being added must all be the same. As such we can not use a multiplication sentence for an addition like 2 + 2 + 2 + 3 = 9.

How would you write the multiplication sentence for the addition sentence you wrote for the above **Try This** problem?

Notes

The result of adding numbers is called their **sum**.

The numbers which are added are called **addends**.

The result of multiplying numbers is called their **product**.

The numbers that are multiplied are called **factors**.

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Example

Example 1 Write a multiplication sentence for the addition sentence:

3 + 3 + 3 + 3 + 3 = 15

What does each number in your multiplication sentence tell?

Solution	**Thinking**
5 x 3 = 15 5 tells the number of times 3 is added repeatedly. 3 tells the number that is added repeatedly. 15 is the product, which is the same as the sum for the addition.	 I know that the first factor to write down is the number of times a number is repeatedly added. Since 3 is added 5 times, the first factor should be 5. The second factor is the number that is added, which is 3. Then, I should write the product, which is the same as the sum in the addition, after the equals sign.

Practising and Applying

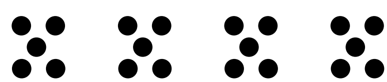
1. Write a multiplication sentence for each addition sentence.
   a) 5 + 5 + 5 = 15
   b) 5 + 5 + 5 + 5 = 20
   c) 4 + 4 + 4 + 4 = 16
   d) 10 + 10 + 10 = 30
   e) 2 + 2 + 2 + 2 + 2 + 2 + 2 = 14
   f) 3 + 3 + 3 + 3 = 12

2. Write an addition sentence for each multiplication sentence.
   a) 4 x 2 = 8
   b) 4 x 3 = 12
   c) 6 x 2 = 12
   d) 2 x 10 = 20
   e) 10 x 2 = 20
   f) 5 x 2 = 10

3. Which number is the product in the multiplication sentence, 3 x 2 = 6? Which numbers are the factors in it?

4. The multiplication sentence, 2 x 6 = 12 represents the following sets of dots. What does each number in it tell?

5. Write a multiplication sentence for the sets of dots below. What does each number in it tell?



6. Calculate each product.
   a) 2 x 3 = __
   b) 2 x 5 = __
   c) 3 x 4 = __
   d) 3 x 3 = __
   e) 3 x 6 = __
   f) 2 x 6 = __

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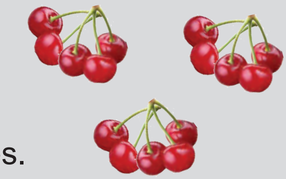
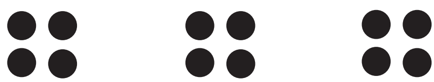
Lesson 2 Multiplication as Equal Sets

Try This

How many groups of cherries are there?
Are the groups equal? How do you know?
What is the total count of cherries?
Write a multiplication sentence for the groups of cherries.



Multiplication also describes equal sets. For example, the three sets below are all equal, and are described by the multiplication sentence under them.



3 x 4 = 12

Notes

A **set** is a group of items or things.

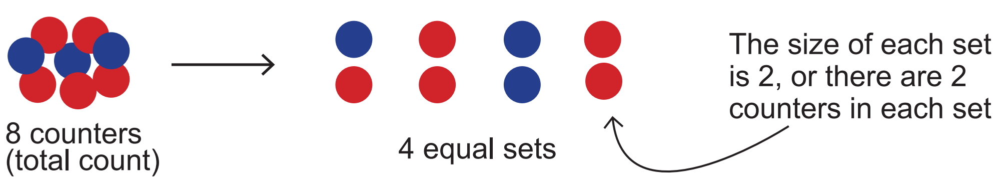
This is 1 set of 3 cherries.

These are 3 sets of 2 cherries.

The first factor, 3, in the multiplication sentence tells the number of equal sets, the second factor, 4, tells the size of the sets, and the product, 12, tells the total count in the sets.

The above multiplication sentence makes sense, because, the three set can also be described by addition as 4 + 4 + 4 = 12. And, you also know that this repeated addition is the same as 3 x 4 = 12.

As each number in a multiplication sentence tells one thing about a collection of equal sets, we can use this knowledge to determine one of the pieces of information if two of them are known. For example, if there are 4 equal sets of counters, and the total count is 8, we can find the size of the sets, by writing an open multiplication sentence as: 4 x □ = 8. You can model this situation by counters. Put out 8 counters, and make 4 equal groups out of them. The size of each set is then 2 counters.



Example

Example 1 Write a multiplication sentence for the sets of flowers. What does each number in your multiplication sentence tell?

Solution

5 x 3 = 15

5 tells the number of bunches, or sets of flowers.
3 tells the number of flowers in each set.
15 tells the total number of flowers.

Thinking

I see that there are 5 sets of flowers. Each set has 3 flowers, so, they are equal sets. So, I can represent the sets with a multiplication sentence as 5 x 3 = 15.

Practising and Applying

1. Write a multiplication sentence for the sets of cherries. What does each number in it tell?

2. Deepak has 3 equal sets of counters. He counted the total number of counters to be 18.
   a) Write an open multiplication sentence for it.
   b) Draw diagram to represent the situation.
   c) How many counters are in each set?

3. What would 4 x 2 = 8 mean? Describe it with drawings. Write an addition sentence for it.

4. Dema made some equal sets with counters. There are 2 counters in each set, and the total number of counters is 12.
   a) Write an open multiplication sentence for it.
   b) Draw diagram to represent the situation.
   c) How many sets did she make?

5. Four friends contributed equal amount of money to go to a restaurant. They collected a total of Nu 40. How much did each person contribute?

6. Determine the missing factor in each open sentence.
   a) $2 \times \square = 8$
   b) $3 \times \square = 12$
   c) $\square \times 2 = 10$
   d) $\square \times 4 = 16$

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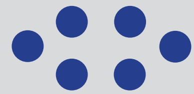
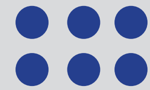
Lesson 3 Multiplication as Arrays

Try This

Are the two sets of counters equal? Why?
Which set looks like a rectangle? Why?



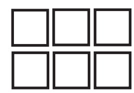
Set A



Set B

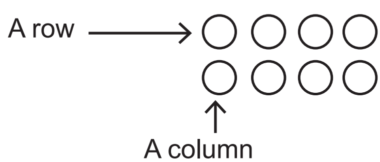
When a set of things is arranged in a rectangular format, it is called an **array**.

An array of momos

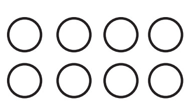


An array of squares

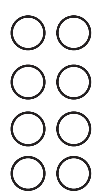
We describe an array in terms of its **rows** and **columns**. For example, the array below is a 2 x 4 (say: **2 by 4**) array. The first number tells the number of rows and the second number tells the number of columns.



An array is like a collection of equal sets. Each row is like a set. The number of items in a row is like the size of an equal set. So, we can describe an array with multiplication just like we did for the collection of equal sets in the earlier lesson.



2 x 4 = 8



4 x 2 = 8

The first factor tells the number of rows (equal sets), the second factor tells the number of columns (size of the sets), and the product tells the total count in the array.

Notes

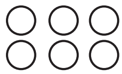
A **row** is a line of items that go left-right in an array.

A **column** is a line of items that go up-down in an array.

Example

Example 1 Take out 6 counters. Arrange them in an array. Sketch your array. Write a multiplication sentence for your array. Describe what each number in your sentence means.

Solution 1

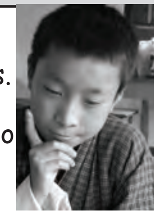


2 x 3 = 6

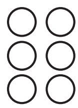
2 shows the number of rows.
3 shows the number of columns.
6 shows the total count.

Thinking

I made an array with 2 rows and 3 columns. I know that the first factor should be the number of rows and the second factor is to be the number of columns. The product is the total count.



Solution 2



3 x 2 = 6

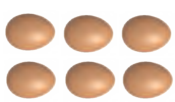
3 shows the number of rows.
2 shows the number of columns.
6 shows the total count.

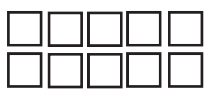
Thinking

I could also make an array with 3 rows and 2 columns. The product is the same as before, because the total count is 6 in both the cases.

Practising and Applying

1. Write a multiplication sentence for each array.

   a) 

   b) 

   c) 

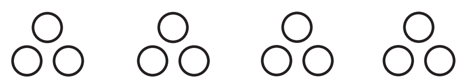
   d) 

2. Describe each array in question 1, as you could say it.
   Example: a) 2 by 3

3. Make two different arrays with 20 counters. Sketch them and write a multiplication sentence for each.

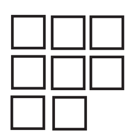
4. Sketch an array for each mutiplication.
   a) 2 x 7 = 14
   b) 7 x 2 = 14

5. Represent the following collection of equal sets with multiplication.



6. Sketch an array for the equal sets of circles in question 5 above.

7. Why is the following arrangment of squares not an array?



8. How will you rearrange the squares in question 7 into an array? Sketch your array, and write a multiplication sentence for it.

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Lesson 4 Skip Counting to Multiply

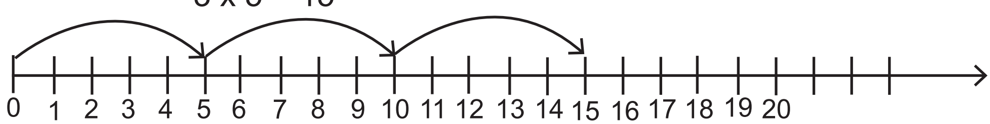
Try This

Skip count the numbers as far as you can:

2, 4, 6, 8, 10, ...	5, 10, 15, ...
3, 6, 9, 12, ...	6, 12, 18, ...
4, 8, 12, ...	10, 20, 30. ...

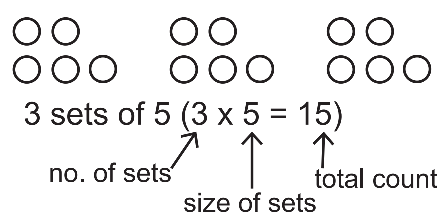
One way to multiply, or to find the product of two numbers, is to skip count the second factor a number of times equal to the first factor. It is useful to use a number line to do the skip counting in the beginning. For example, to find the product of 3 x 5, you skip count 5 three times, as 5, 10, 15. The last count, which is 15, is the product.

3 x 5 = 15

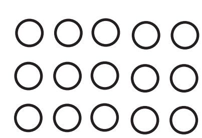


Skip counting to multiply makes sense because it is like repeated addtion. For instance, when you skip count in 5s for 3 x 5 on the number line, it is like adding 5 + 5 + 5 and getting the sum of 15. From Lesson 1 we know that 3 x 5 is the same as 5 + 5 + 5.

Skip counting to multiply also makes sense because it is like representing equal sets with multiplication. For instance, you know from Lesson 2 that 3 x 5 means **3 sets of 5,** or **3 equal sets with each set having 5 items**. Each skip count, or each jump, on the number line is like a set of 5. So, 3 such jumps is a total of 15.

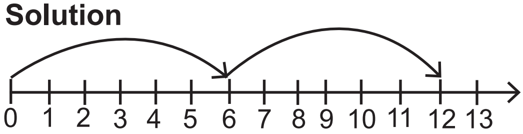


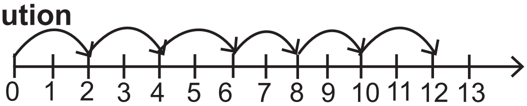
Skip counting is also directly related to arrays. You know from Lesson 3 that 3 x 5 is an array of **3 rows by 5 columns.** Because each row has 5 items, and there are 3 such rows, the total count can be found by skip counting to 15 as 5, 10, 15.



3 x 5 = 15

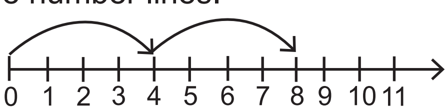
Examples

Example 1 Use skip counting on a number line to multiply 2 x 6.	
Solution  2 x 6 = 12	**Thinking** I drew a number line. I know 2 x 6 is like 2 equal sets of 6. So, I have to skip count 6 two times as 6, 12. The product is 12. 

Example 2 Multiply 6 x 2 by skip counting on a number line. What is different about this multiplication and the one in Example 1 above? What is the same about them?	
Solution  6 x 2 = 12 The difference between the two multiplications is that the orders of the factors are changed. The thing that is the same about them is that the products are the same.	**Thinking** 6 x 2 is like 6 sets of 2. So, I have to skip count in 2s for 6 times, as 2, 4, 6, 8, 10, 12. 

Practising and Applying

1. Write a multiplication sentence for the skip counting shown on each of the number lines.

a)


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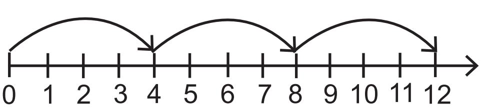

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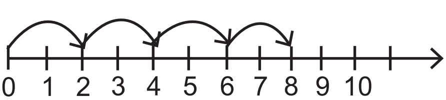

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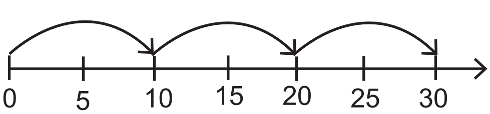
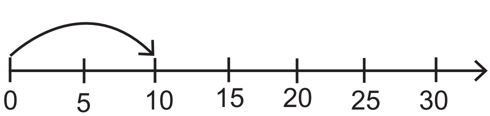


e)


2. Multiply the following by showing skip counting on number lines.

a) 2 x 5
b) 5 x 2
c) 3 x 4
d) 4 x 3
e) 4 x 4
f) 5 x 5
g) 2 x 6
h) 6 x 2
i) 6 x 3
j) 2 x 10
k) 10 x 2
l) 2 x 7

3. Represent 3 x 7 = 21 with:
   a) Skip counting on a number line
   b) An array
   c) Equal sets

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Lesson 5 Using Double Facts to Multiply

Try This

A person has 2 legs.
How many legs are there with 4 persons?
How many legs are there with 5 persons?

In class 2 you learnt that when a number is added onto itself, you get its double. For example, the double of 2 is 4, because 2 + 2 = 4; and the double of 3 is 6 because 3 + 3 = 6.

In terms of multiplication, the double of a number is the product of 2 and the number (2 x the number). For example, the double of 2 is the product of 2 x 2, because it is the same as 2 + 2; the double of 3 is the product of 2 x 3, because it is the same as 3 + 3.

Let us review the double facts for numbers, as we did in class 2, by adding the numbers onto themselves, as shown below.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Double	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32

Let us write the double facts for numbers as multiplication facts, as shown below.

2 x 1 = 2	2 x 5 = 10	2 x 9 = 18	2 x 13 = 26
2 x 2 = 4	2 x 6 = 12	2 x 10 = 20	2 x 14 = 28
2 x 3 = 6	2 x 7 = 14	2 x 11 = 22	2 x 15 = 30
2 x 4 = 8	2 x 8 = 16	2 x 12 = 24	2 x 16 = 32

It is important for you to be able to recall some of the above double facts. This will help you to multiply numbers, where you can use double facts.

If you double a factor in a mutiplication, it will double the product. For example, you know that 2 x 3 = 6. Now, if you double 2, so that it is now 4 x 3, the product 6 is also doubled to 12. Therefore, 4 x 3 = 12. This, in turn, will help you to calculate the product for 8 x 3. You know that the double of 4 is 8, which means that 8 x 3 is the double of 4 x 3. So, the product for 8 x 3 should be the double of 12, which is 24. Therefore, 8 x 3 = 24.

Examples

Example 1 If 3 x 5 = 15, calculate the product for 6 x 5.

Solution	**Thinking**
6 x 5 = 30	6 is double 3. It means 6 x 5 equals double 3 x 5. So, the product should be the double 15, which is 30.

Example 2 If 3 x 5 = 15, calculate the product of 3 x 10.

Solution	**Thinking**
3 x 10 = 30	10 is double 5. So 3 x 10 is double 3 x 5. The product should be double 15, which is 30.

Practising and Applying

1. If 2 x 3 = 6, calculate the product of 4 x 3.

2. If 4 x 3 = 12, calculate the product of 4 x 6.

3. If 4 x 3 = 12, calculate the product of 8 x 3.

4. If you double one factor in a multiplication sentence, what happens to the product?

5. Sketch an array for 2 x 4. Write the multiplication sentence for it.

6. Double the array made for question 5 in the following ways:
   a) First, double the rows. Write the multiplication sentence for this new array.
   b) Double the columns instead of the rows. Write the multiplication sentence for this new array.

7. Complete each doubled fact.
   a) **2** x 3 = 6, so **4** x 3 = ___
   b) **2** x 6 = 12, so **4** x 6 = ___
   c) **2** x 7 = 14, so **4** x 7 = ___
   d) **3** x 3 = 9, so **6** x 3 = ___
   e) **3** x 4 = 12, so **6** x 4 = ___
   f) 6 x **1** = 6, so 6 x **2** = ___
   g) 5 x **3** = 15, so 5 x **6** = ___
   h) 8 x **2** = 16, so 8 x **4** = ___
   i) 3 x **8** = 24, so 3 x **16** = ___
   j) 4 x **5** = 20, so 4 x **10** = ___

8. If 3 x 5 = 15, how can you use doubling to calculate 6 x 10?

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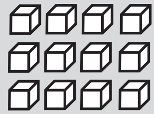
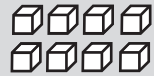
Lesson 6 Multiplying with 1 and 0

Try This

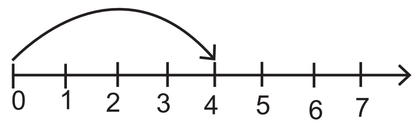
Write a multiplication sentence for each array.


What could 1 x 4 mean? It could mean all of the following situations.

1 set of 4

An array of 1 row by 4 columns



1 time skip counting in 4s

As you know, the product of the multiplication is the total count. The total count in each of the above situations is 4. So, whatever may be the situation, the product of 1 x 4 is always 4. Or, 1 x 4 = 4.

Show how you can represent 1 x 3 with all of the above situations. What is the product for 1 x 3 in each situation? What can you say about 1 x 3 in general?

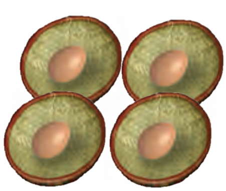
Similarly, repeat the above process for 1 x 2, and 1 x 5.

Now, let us list all the above multiplications as shown on the right. Do you see a pattern there? Describe the pattern.

1 x 2 = 2
1 x 3 = 3
1 x 4 = 4
1 x 5 = 5

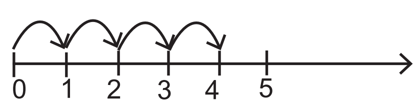
When the first factor is 1, the product is the same as the second factor.

What could 4 x 1 mean? It could mean all of the following situations.



4 sets of 1

An array of 4 rows by 1 column



4 times skip counting in 1s

Again, as you know the product in each situation is the total count. The total count in each of these situation is 4. So, whatever may be the situation, the product of 4 x 1 is 4. Or, 4 x 1 = 4.

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Could you represent 3 x 1, 2 x 1, and 5 x 1 with each of the three situations? Write a multiplication sentence for each situation. Then, list the multiplication sentences as shown on the right. Describe the pattern in it.

2 x 1 = 2
3 x 1 = 3
4 x 1 = 4
5 x 1 = 5

The above patterns in multiplying a number with 1 give us the following fact.

If you multiply a number with 1, the product is the number itself.

What could 4 x 0 mean? Let us discuss this with a situation related to equal sets. Recall that the first factor tells the number of equal sets, and the second factor tells the size of the sets. So, 4 x 0 means 4 equal sets with zero or no items in each set, as shown here.

4 sets of 0 (4 empty bangchungs)
4 x 0 = 0

Also recall that the product is the total count. In this case, the total count is 0, as there are no eggs in the bangchungs. So, 4 x 0 = 0.

What could 0 x 4 mean? It could mean that there are no sets with 4 items in each set. If there are no sets, then the total count (the product) will be 0. So, 0 x 4 = 0.

You notice that whether the 0 is the first factor or the second factor, when you multiply it with 4 the product is 0. This is also true with multiplying any number with 0. This gives the following fact.

If you multiply a number with 0, the product is always 0.

Practising and Applying

1. Calculate the product for each.

   a) 4 x 1
   b) 6 x 1
   c) 16 x 1
   d) 1 x 4
   e) 1 x 6
   f) 1 x 20
   g) 5 x 0
   h) 10 x 0
   i) 8 x 0
   j) 0 x 2
   k) 0 x 20
   l) 0 x 23

2. Sketch a diagram to represent 7 x 0. Write the multiplication sentence.

3. Why do 5 x 0 and 3 x 0 have the same product?

4. Why do 6 x 1 and 1 x 6 have the same product?

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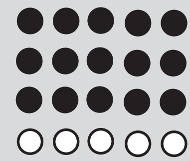
Lesson 7 Relating Multiplication Facts

Try This

Write a multiplication sentence for the part of the array formed by the black counters.

Write a multiplication sentence for the part of the array formed by the white counters.



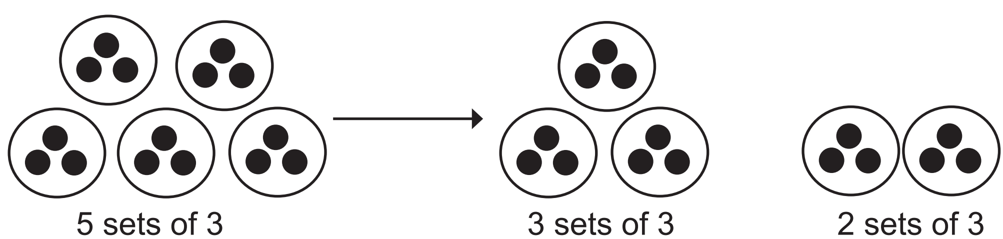
Write a multiplication sentence for the whole array.

What relationship do you notice among the three multiplication sentences?

Now you know some basic multiplication facts, including multiplying a number with 1 and 0. You have also learnt to determine the product of a multiplication using double facts.

In this lesson, you will relate simple known multiplication facts to determine some unknown facts. In particular, you will learn to break down a multiplication into small parts for which you know the products, and then add up these products to calculate the product for the original multiplication. For example, let us assume that you do not yet know the fact for 5 x 3, but you already know the facts for 3 x 3 = 9 and 2 x 3 = 6.

Recall that 5 x 3 is 5 sets of 3. Now, 5 sets of 3 is the same as 3 sets of 3 plus 2 sets of 3.



$$
\begin{aligned}
5 \times 3 &= (3 \times 3) + (2 \times 3) \\
&= 9 + 6 \\
&= 15
\end{aligned}
$$

Find the product of 4 x 5 by breaking it into 3 x 5 and 1 x 5, based on the array in the above **Try This** problem.

Examples

Example 1 If 2 x 7 = 14, calculate the product of 4 x 7. Then, calculate the product of 6 x 7.

Solution 1

4 x 7 = (2 x 7) + (2 x 7)
= 14 + 14
= 28

6 x 7 = (4 x 7) + (2 x 7)
= 28 + 14
= 42

Thinking

4 is the double of 2. So, the product of 4 x 7 should be double the product of 2 x 7.

6 is 4 + 2. So, I can break 6 x 7 into 4 x 7 and 2 x 7. I already know the products of these two simpler multiplications. I just have to add them up.

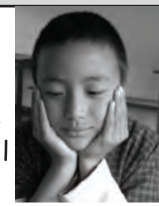
Example 2 Calculate the product of 5 x 7.

Solution 1

5 x 7 = (2 x 7) + (2 x 7) + (1 x7)
= 14 + 14 + 7
= 28 + 7
= 35

Thinking

5 x 7 is 5 sets of 7, which is the same as 2 sets of 7 plus 2 sets of 7 plus 1 set of 7. I know the fact for 2 x 7 as 14, and 1 x7 as 7. So, I added 14 + 14 + 7 to get the overall product of 35.



Practising and Applying

1. Find the product by first breaking the multiplication into small parts . Show your workings.
   a) 6 x 4
   b) 4 x 7
   c) 3 x 8
   d) 5 x 8
   e) 5 x 5
   f) 5 x 6
   g) 4 x 10
   h) 4 x 12
   i) 4 x 9
   j) 3 x 7

2. If 5 x 5 = 25, how do you know that 6 x 5 is 5 more than 25?

3. Write a multiplication sentence each for the shaded and the unshaded stars. Then, use their products to calculate the product of 5 x 8.

4. If you know that 5 x 3 = 15, how would this help you to find the products for:
   a) 10 x 3?
   b) 6 x 3?
   c) 15 x 3?

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Lesson 8 Multiplication Table

Try This

Could you complete the multiplication facts for 3?

3 x 0 =	3 x 2 =	3 x 4 =	3 x 6 =	3 x 8 =
3 x 1 =	3 x 3 =	3 x 5 =	3 x 7 =	3 x 9 =

In this lesson, let us develop a multiplication table. The multiplication table below will show the product of each single digit number from 0 to 9 with each other number from 0 to 9.

The numbers in the shaded row and the shaded column are the factors. The unshaded part of the table will be filled with the products. The first row in the product area has been filled for you, as well as the products for 2 and 5, and 7 and 7.

1st column of the products area

1st row of the products area

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1										
2						10				
3										
4										
5										
6										
7								49		
8										
9										

Make a table like the one shown here. Fill up the table with the products that you already know. Then, work out the products that are not familiar to you by using the known facts.

After the table has been filled, discuss the following.

- Why do you think that the numbers in the first row of the product area are all zeros?

- Compare the 2nd row to the 2nd column in the products area. What do you notice? Similarly, compare the 3rd row to the 3rd column and describe the relationship.

- In which row do the numbers increase by 5?

- In which row do the numbers increase by 7?

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- Look at the column that includes the numbers 12, 18, 24. What number comes next? Why?

- Which rows and columns contain only even numbers?

- Which rows start with an odd number after the 0?

Making and Memorising Multiplication Facts for Numbers

The list of multiplication sentences on the right show the multiplication facts for 4 up to 4 x 10 = 40.

For 4 x 0 = 0, say '4 times 0 equals 0'
for 4 x 1 = 4, say '4 times 1 equals 4'
for 4 x 2 = 8, say '4 times 2 equals 8'
for 4 x 3 = 12, say '4 times 3 equals 12', and so on.

Make a similar list of multiplication facts for the numbers 2, 3, 4, 5, and 6 in your notebook. If possible, it would be nice if you could also have them printed from the computer for your reference.

It will be useful if you could begin to memorise these multiplication facts over time during the next several weeks.

4 x 0 = 0
4 x 1 = 4
4 x 2 = 8
4 x 3 = 12
4 x 4 = 16
4 x 5 = 20
4 x 6 = 24
4 x 7 = 28
4 x 8 = 32
4 x 9 = 36
4 x 10 = 40

One good way to memorise these multiplication facts is for you to practise reciting them together aloud in the class regularly over the next several days.

Another good way is to write them down over and over again in your notebook and on paper.

Such practices and memorisation will be very useful for you with multiplication in the future. If you can, you should also write and practise memorising the multiplication facts for 7, 8, 9 and 10, but that is not necessary at this stage. You may already know many of the multiplication facts for 10.

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Lesson 9 Solving Multiplication Problems

Try This

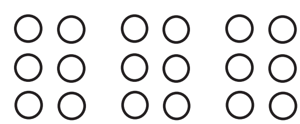
Three friends went to a restaurant. Each person ordered a plate of momos. A plate of momos contain 6 momos. How many momos are there in all?

A plate of momos

It is important for you to be able to multiply numbers, recall multiplication facts, and represent or describe what a multiplication could mean. Additionally, you should be able to identify what problems and situations involve multiplication. These problems and situations would normally appear as stories and word problems.

A general strategy to solve such a problem is to read carefully and understand it, represent it with sketches and concrete materials such as counters, recognise that it involves multiplication, and write down the multiplication sentences to solve the problem. Then, say what the solution is as per the question asked in the problem.

For example, in the **Try This** problem above it asks you to tell the total number of momos that the 3 friends ordered in a restaurant. We know that there would be 3 plates. Also, we know that there will be 6 momos on each plate. You could quickly sketch diagrams to represent this situation, as shown here. This is clearly a case of 3 equal sets of 6, which means the total number of momos will be the product in the multiplication sentence 3 x 6 = 18. So, there are 18 momos in all.



3 plates of 6 momos

You should also be able to create or tell stories based on simple multiplication sentences. A general strategy for you with this would be to think of the first factor as the number of equal sets and the second factor as the size of the equal sets. Then, describe a story or a situation based on these numbers. For example, what could the multiplication sentence, 4 x 3 = 12 mean to you? Write a short situation based on it.

Examples

Example 1 Sushma bought 3 packets of pencils. A packet contains 6 pencils. How many pencils did she buy in all?

Solution

|||||| |||||| ||||||

3 x 6 = 18

Sushma bought 18 pencils.

Thinking

I had to find the total number of pencils in the 3 packets. I drew 6 lines to represent the pencils in a packet. I repeated this for the other two packets.

This is like 3 equal sets of 6. So, the total number of pencils can be found by the multiplication, 3 x 6. I know the product is 18 from the multiplication facts for 3.

Example 2 Write a problem for the multiplication sentence 4 x 5 = 20.

Solution 1

Norbu walked 5 Km from his home to town to buy something. Then, he walked back home. After reaching home, he realised that he forgot something in the town. So, again he walked to the town and back home. How many kilometres did he walk?

Solution 2

A father gave Nu 5 each to his 4 children. How much money did he give in all?

Thinking

I know that 4 tells the number of equal sets, 5 tells the size of each set, and 20 tells the total count of all 4 sets.

I can think of many stories or problems to write for this multiplication sentence. I wrote two of them as shown on the left.

Practising and Applying

1. A chair has 4 legs. There are 10 chairs in a room. How many chair legs are there in the room?

2. Karma started putting Nu 5 in his piggy bank every week. How much money would he have saved after 6 weeks?

3. A match box contains 50 match sticks. How many match sticks will there be in 2 boxes?

4. Write a problem about 4 x 6, and solve it.

5. The sum of two numbers is 6. What could the numbers be? What is their product?

6. Write a problem for the multiplication sentence 7 x 1 = 7.

7. A kettleful of tea can fill 6 cups, which are the same size. How many such cups will 2 such kettles fill?

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Chapter Review

1. Write each addition as multiplication and calculate the products.

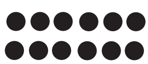
   a) 2 + 2 + 2 + 2
   b) 3 + 3 + 3 + 3 + 3
   c) 9 + 9
   d) 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1

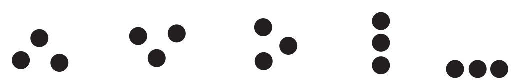
2. Write each multiplication as addition and calculate the sums.

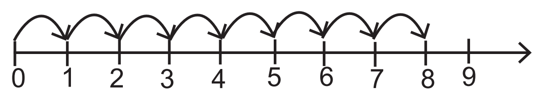
   a) 4 x 6
   b) 3 x 7
   c) 5 x 1
   d) 5 x 0

3. Write a multiplication sentence for each set of diagrams.

   a) 

   b) 

   c) 

   d) 

4. Sketch diagrams of equal sets to represent 3 x 7, and find its product.

5. Sketch an array to represent 4 x 5, and find its product.

6. Show skip counting on a number line to solve 5 x 2.

7. You can make 10 butter lamps of the same size with 1 kg of dalda. How many such butter lamps can you make with 3 kg of dalda?


A butter lamp


1 kg of dalda

8. Determine the missing factor.

a) 2 x ☐ = 8
b) 3 x ☐ = 18
c) 4 x ☐ = 0
d) 6 x ☐ = 6
e) 1 x ☐ = 7

f) ☐ x 6 = 30
g) ☐ x 3 = 21
h) ☐ x 9 = 9
i) ☐ x 5 = 20
j) ☐ x 6 = 0

9. Write a multiplication problem for 7 x 3.

10. Explain why you cannot represent 6 + 6 + 6 + 7 + 6 as 5 x 6.

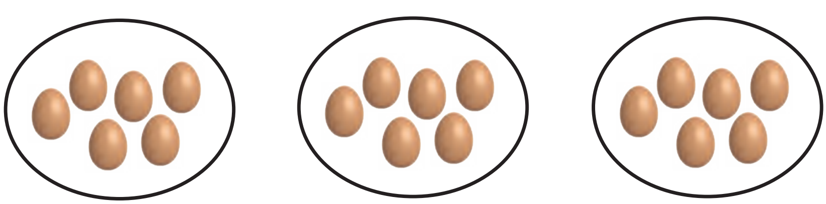
11. Write all the pairs of numbers for which the sum is 7. Write the products for each pair.

12. If 3 x 3 = 9, calculate the product for 6 x 3.

13. What is same about the products for 5 x 0, 8 x 0, and 3 x 0?

14. What is same about the products for 4 x 1 and 1 x 4?

15. Write a multiplication sentence for the sets below.



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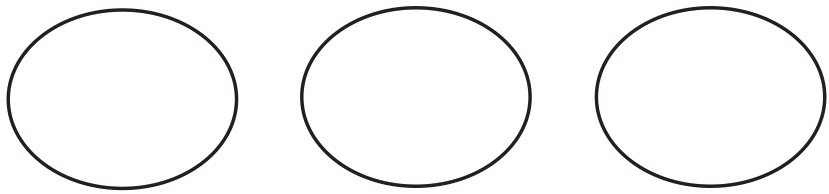
16. Write a multiplication sentence for the empty sets below.



17. Why is the set below not an array?

18. Calculate the products based on the facts provided.

a) 2 x 6 = 12, so 4 x 6 = □
b) 3 x 5 = 15, so 3 x 10 = □
c) 4 x 6 = 24, so 4 x 12 = □
d) 7 x 1 = 7, so 14 x 1 = □

e) 2 x 3 = 6 and 4 x 3 = 12, so 6 x 3 = □
f) 2 x 9 = 18 and 2 x 9 = 18, so 4 x 9 = □
g) 1 x 6 = 6 and 2 x 6 = 12, so 3 x 6 = □
h) 5 x 0 = 0 and 4 x 0 = 0, so 9 x 0 = □

CHAPTER 5 GEOMETRY

Chapter Overview

Geometry is the study of shapes, both 2-dimensional and 3-dimensional. 2-dimensional (or 2-D) shapes are flat and can be drawn on paper. Triangles, rectangles, rhombi, and circles are examples of 2-D shapes. 3-dimensional (or 3-D) shapes are solid shapes and occupy space. Prisms, pyramids, cylinders, cones, and spheres are examples of 3-D shapes.

You have had experiences with both the 2-D shapes and 3-D shapes, in terms of exploring, naming, describing, comparing, drawing and creating them during your earlier classes. You have also dealt with the ideas of parallel lines, symmetrical shapes and lines of symmetry in class 2. We will review these concepts and experiences in this chapter. Additionally, you will learn about polygons, similarity, congruence, and slides, flips, and turns of shapes.

This chapter has 9 lessons as detailed in the Table of Contents.

Basic Principles about Geometry

- The characteristics of a shape are related to the type of shape, rather than size.
- Shapes can be classified in a variety of ways.
- It is often useful to think of a shape in terms of its component parts.
- The net of a 3-D shape focuses on how the faces of the shape touch each other.
- Shapes can be moved by sliding, flipping and turning them.
- Shapes do not change by simply moving them in any way.

Chapter Goals

By the end of this chapter, you will be able to:

- Describe what a polygon is, and name various polygons.
- Identify polygons as concave or convex, and regular or irregular.
- Describe what a quadrilateral is, and name various quadrilaterals.
- Identify a symmetrical shape, and determine the lines of symmetry in it.

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- Describe and identify congruent shapes.
- Identify and describe the movements of a 2-D shape as slide, flip, or turn.
- Combine various 2-D shapes to form a bigger shape.
- Identify and describe the shape features of prisms and pyramids.
- Name prisms and pyramids based on the shapes of their bases, up to hexagonal prisms and hexagonal pyramids.
- Identify and draw nets for prisms and pyramids.
- Identify and describe the shape features of cylinders, cones and spheres.

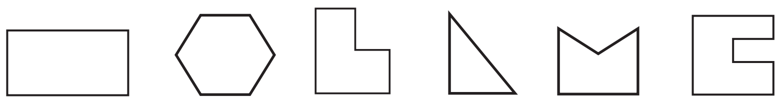
Lesson 1 Polygons

Try This

Draw a triangle.
How will you describe what a triangle is to someone?

Polygon

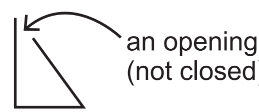
A polygon is a closed plane (or flat) shape made up of straight lines joined together. The following shapes are all polygons.



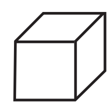
Properties of polygons

If we look carefully at the above sentence, you can see words like **closed**, **plane** (or **flat**), and **straight lines**. Each of these words is important to talk about a polygon. Let us discuss each one of them.

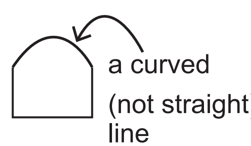
A polygon should be a **closed** shape. That means the shape on the right is not a polygon, as it has an opening.



A polygon should be a **plane** (or **flat**) shape. That means the shape on the right is not a polygon, as it is a solid shape. It is a cube.

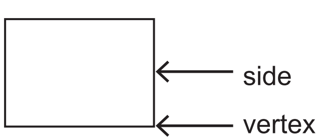


A polygon should be made up of **straight lines**. That means the shape on the right is not a polygon, as it has a curved line.

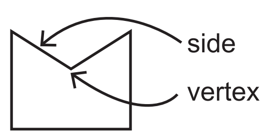


Parts of a polygon

The straight lines that make a polygon are called its **sides** (or edges). The point where two sides join is called a **vertex** (or a corner). The plural of vertex is **vertices**.



This polygon has 4 sides and 4 vertices.



This polygon has 5 sides and 5 vertices.

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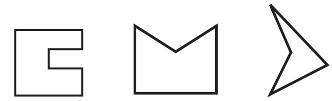
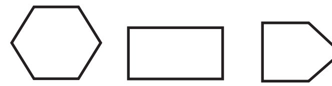
Types of polygons

We can classify polygons as concave, convex, regular, or irregular.

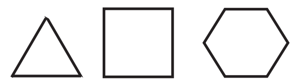
Concave polygon - A polygon which has a bent in part is called a concave polygon. You can think of the bent in part as a cave, or an open mouth.


Concave polygons

Convex polygon - A polygon which does not have any bent in part is called a convex polygon.


Convex polygons

Regular polygon - A polygon in which all sides are equal and all angles are equal is called a regular polygon.

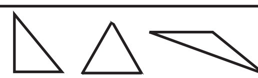
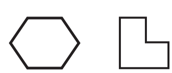

Regular polygons

Irregular polygon - A polygon which does not have all of its sides and angles equal is called an irregular polygon.


Irregular polygons

Names of polygons

Polygons are named according to their number of sides, as shown below. It will be helpful if you can learn the names of these polygons.

Name of polygon	Number of sides (or edges)	Examples
Triangle	3	
Quadrilateral	4	
Pentagon	5	
Hexagon	6	
Heptagon	7	
Octagon	8	
Nonagon	9	
Decagon	10	

Examples

Example 1 How are these two shapes the same?
How are they different?

shape A shape B

Solution

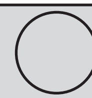
<u>Same</u>
Both the shapes are plane or flat.
Both the shapes are made of 3 straight lines.

<u>Different</u>
Shape A is a closed shape, but shape B is not closed.
So, shape A is a polygon, and shape B is not a polygon.
Shape A is a triangle, but shape B is not a triangle.

Thinking

First I wrote what is the same about these two shapes. Then, I wrote what is different about them.

Example 2 Is a circle a polygon? Why or why not?

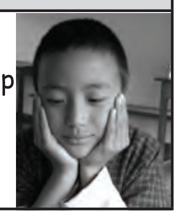


Solution

No, a circle is not a polygon, because it is not made up of straight lines.

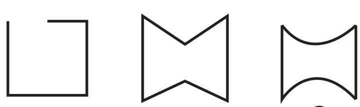
Thinking

I know that a polygon should be made up of straight lines. A circle is made by one round curved line, so it is not a polygon.



Practising and Applying

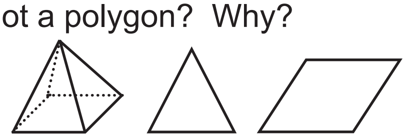
1. Which one of the following shapes is a polygon? Why is it a polygon?



A B C

2. What is the name of the polygon in question 1 above? Why is it called that?

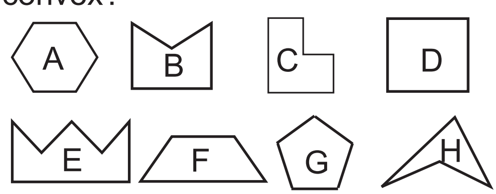
3. Which one of the following shapes is not a polygon? Why?



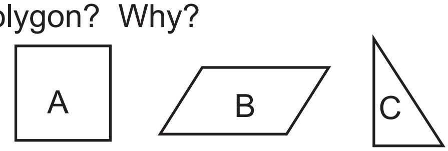
A B C

4. Draw two different pentagons.

5. Which of the following polygons are concave and which are convex?



6. Which of the following is a regular polygon? Why?



7. Try to draw a polygon with only 2 straight lines. Was that possible?

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Lesson 2 Quadrilaterals

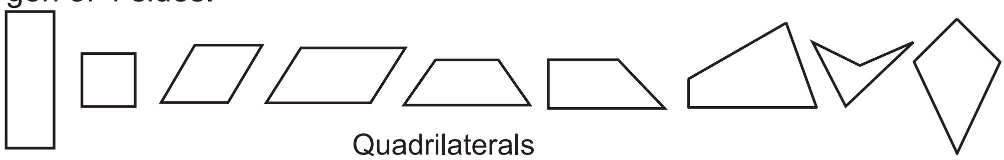
Try This

What is the same about a rectangle and a trapezoid? What is different about them?


a rectangle


a trapezoid

As you now know from Lesson 1, a quadrilateral is a closed plane shape made up of 4 straight lines joined together. In other words, a quadrilateral is a polygon of 4 sides.


Quadrilaterals

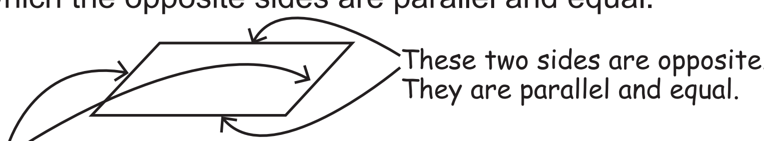
Note
Parallel lines are lines that remain the same distance apart.


Parallel lines

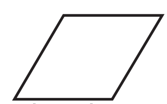

Non-parallel lines

Special quadrilaterals

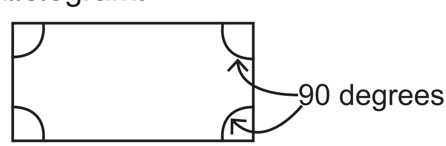
Parallelogram - A parallelogram is a quadrilateral in which the opposite sides are parallel and equal.



Rhombus - A rhombus is a quadrilateral in which the opposite sides are parallel and the lengths of all 4 sides are equal. In fact, a rhombus is a special parallelogram.

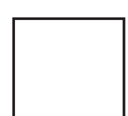

a rhombus

Rectangle - A rectangle is a quadrilateral in which the opposite sides are parallel and equal, and also has all of its angles at the 4 corners measuring 90 degrees.
Use a protractor to see that these angles are 90 degrees.



a rectangle

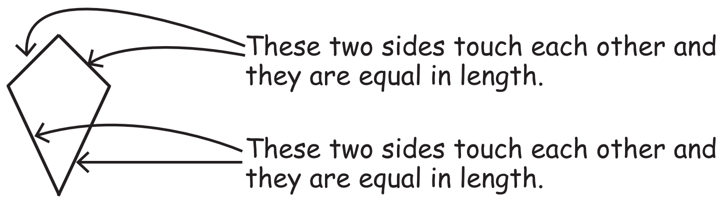
Square - A square is a special rectangle which has 4 equal sides. A square is a regular quadrilateral.


a square

Trapezoid - A trapezoid is a quarilateral in which one pair of opposite sides is parallel.

Trapezoids

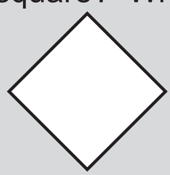
Kite - A kite is a quadrilateral in which two pairs of sides that touch are equal.



a kite

Examples

Example 1 Is the following shape a square? Why, or why not? How will you confirm it?



Solution	Thinking
Yes, it is a square, because all 4 sides are equal, and also all the angles at the corners are 90 degrees.	A square should have all sides equal in length. It should also have an angle of 90 degrees at all 4 corners. So, I checked these conditions for the shape. I measured the side lengths with my ruler. They are all equal. I measured the angles at the corners with my protractor. They are all 90 degrees. So, the shape is a square. 

Example 2 Lekzin says that the following shape is a rectangle. Her reason is that the opposite sides are equal and parallel. Do you agree with her? Why, or why not?

Solution	Thinking
I agree that the opposite sides are equal and parallel. But, I don't agree that the shape is a rectangle, because the angles are not 90 degrees.	A rectangle should have the opposite sides equal and parallel. It should also have an angle of 90 degrees at all 4 corners. The angles in this shape are clearly not 90 degrees. I checked one angle with my protractor, and it is not 90 degrees. So, the shape is not a rectangle. It is a parallelogram. 

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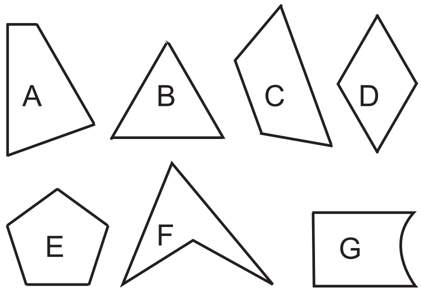
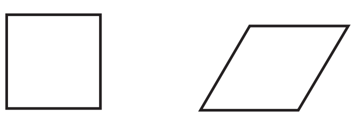
Practising and Applying

1. Which of the following shapes are quadrilaterals? Why?



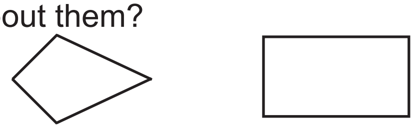
2. The two shapes below have 4 lines each. Explain why each shape is not a quadrilateral.

3. What is the same about a square and a rhombus? What is different about them?



a square   a rhombus

4. What is the same about a kite and a rectangle? What is different about them?

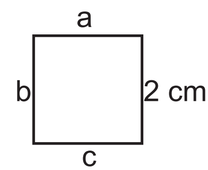


a kite   a rectangle

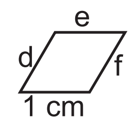
5. What is different about a parallelogram and a trapezoid?

a parallelogram   a trapezoid

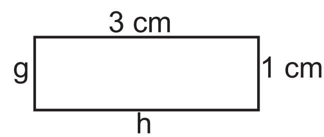
6. Without actually measuring the sides, determine the side lengths of the following shapes in centimetres (cm) which are designated with letters.



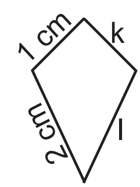
square



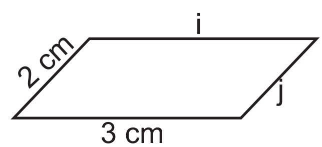
rhombus



rectangle

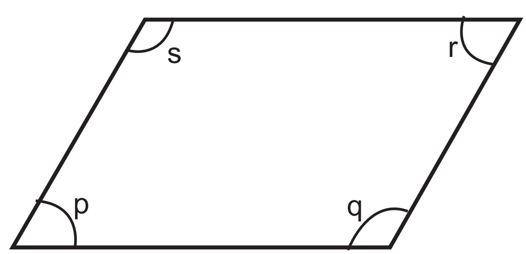


kite



parallelogram

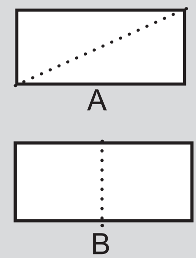
7. Measure the angles at the corners of the following shape and express them in degrees.



Lesson 3 Symmetrical Shapes

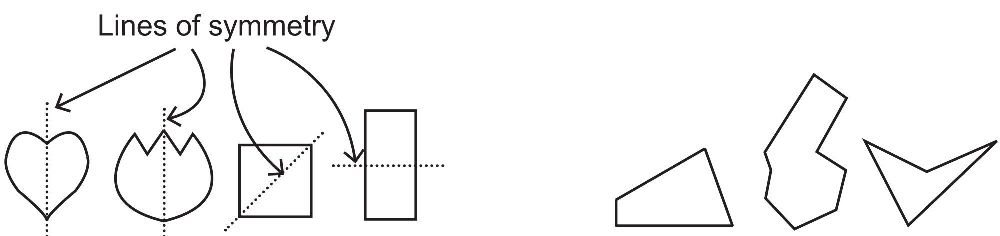
Try This

The two rectangles are the same. If you fold them along the dotted lines and then fold over one half on the other, in which case will the two halves created by the fold line fit exactly onto each other?



Try them with two paper rectangles of the same size, folding each in the same manner as shown here.

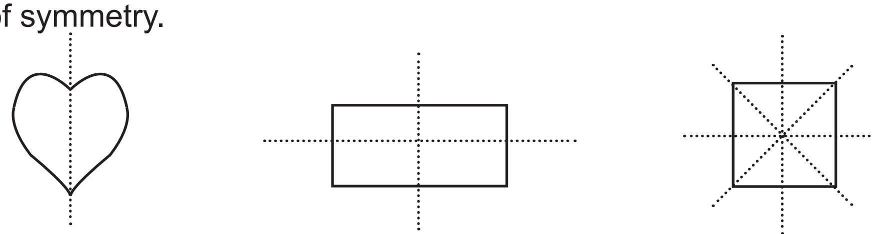
If you can fold a shape into two parts, along a line, so that one part fits exactly onto the other part, the shape is called a **symmetrical shape.** The fold line is called the **line of symmetry**.



Symmetrical shapes

Non-symmetrical shapes

Symmetrical shapes may have only one line of symmetry, two lines of symmetry, or many lines of symmetry. **Non-symmetrical shapes** do not have any line of symmetry.



This shape has only 1 line of symmetry.

This shape has 2 lines of symmetry.

This shape has 4 lines of symmetry.

Many shapes, or bodies, in nature are symmetrical. A butterfly is an example. Where would be the line of symmetry for the butterfly shown here? It is believed that we find symmetrical shapes beautiful and pleasing to look at.

Human-made things are also mostly symmetrical. For example, cars.

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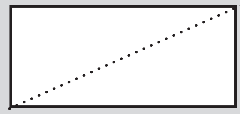
Examples

Example 1 Lhaden says that the line from a corner to its opposite corner in a rectangle as shown below is a line of symmetry, saying that the line divides the rectangle into two equal parts. Is Lhaden correct in saying the line is a line of symmetry? Why?



Solution

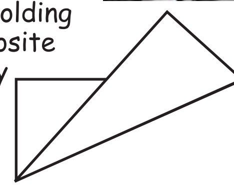
No, the line is not a line of symmetry, because if you fold the rectangle along this line, one half of the rectangle will not exactly cover the other half.

It will look like in the picture shown on the right.

Thinking

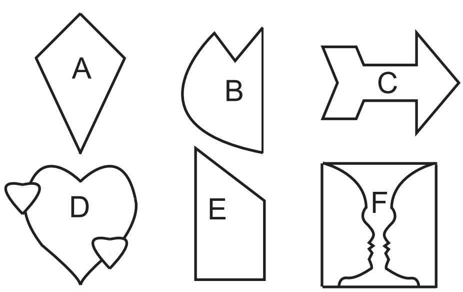
It does not look like the two halves of the rectangle will fit exactly onto each other when folded along this line from corner to corner. So I think that the line is not a line of symmetry.

I checked it with a paper rectangle, by folding it along a line from one corner to its opposite corner. The two halves do not fit exactly onto each other as shown here. So, the line is not a line of symmetry.

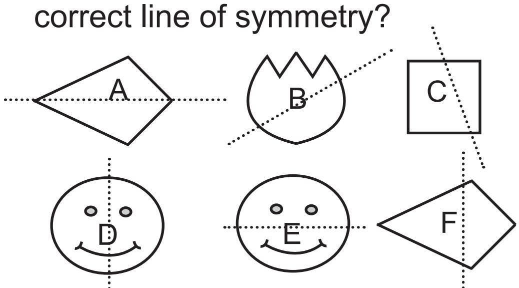


Practising and Applying

1. Which of the following shapes are symmetrical? Which are not symmetrical?



2. Which shapes are shown with a correct line of symmetry?



3. Draw a square and a rhombus and show all the lines of symmetry on each.

4. Explain why the square has more lines of symmetry than the rhombus.

5. Write your name in block letters (all capital letters) and draw the lines of symmetry through the letters that are symmetrical.

   Example: K A R M A

6. Name two things that are symmetrical in nature. You may draw each picture with a line of symmetry.

Lesson 4 Similar and Congruent Shapes

Try This

Which two teddy bears are similar?
Describe how they are similar.

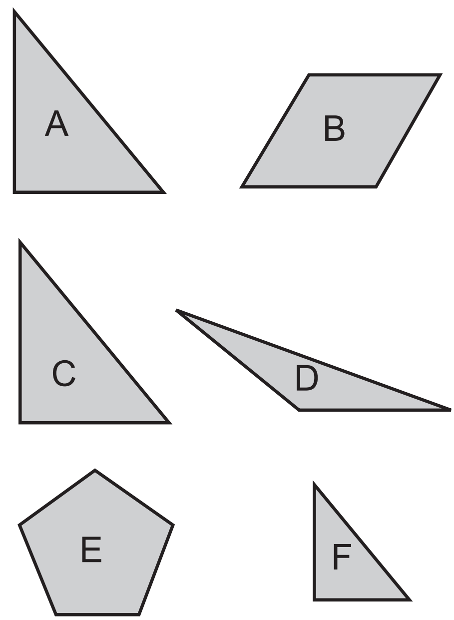
Two shapes are **similar** if they look the same. It is not necessary for the similar shapes to be the same size. Shapes A, C and F are similar.

Two shapes are **congruent** if they are exactly the same in shape and size. Shapes A and C are congruent.

You could check if two shapes are congruent or not by placing one shape onto the other. If, when you place a shape on top of another, and they fit exactly, then, the shapes are congruent.

If the shapes are solid, like the pattern blocks, then you could easily place the shapes onto each other to check if they are congruent.

But, if you have to check if a shape which is drawn is congruent to another drawn shape, you could trace one shape on a tracing paper, cut it out and place the cut-out shape onto the other shape to see if it fits exactly. For example, to check whether shapes A and C above are congruent, you could use tracing paper to trace shape A. Then,see if the traced shape fits exactly onto shape C.



Note

Congruent shapes are also called **identical** shapes.

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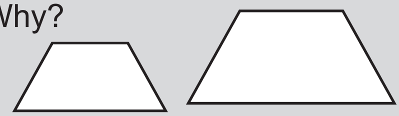
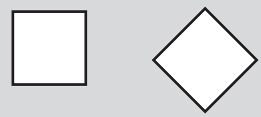
Examples

Example 1 Are these two trapezoids similar? Why?
Are they congruent? Why?



Solution	**Thinking**
Yes, the two trapezoids are similar, because they look the same. No, they are not congruent, because they are not the same size.	The two trapezoids look the same in all respects even though one is larger than the other. So, they are similar. Since one is clearly larger than the other, the two shapes can not be congruent. 

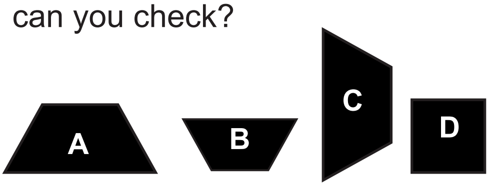
Example 2 Are the rectangles congruent?
How do you know?



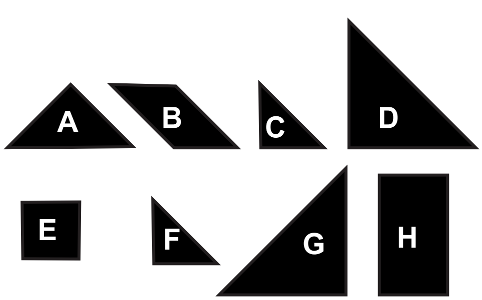
Solution	**Thinking**
Yes, the two rectangles are congruent, because they are the same size and shape.	When I look carefully, the two rectangles look the same both shape wise and size wise. I traced the first rectangle on a tracing paper. Then, I checked if the traced shape fit exactly on the second shape. It fit exactly. So the two shapes are congruent.  Also, in this case, the sides of both rectangles are all the same length. So, they must be congruent.

Practising and Applying

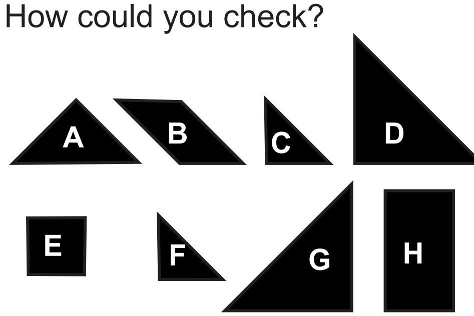
1. Which shapes are congruent? How can you check?



2. Which shapes are similar?



3. Which shapes are congruent? How could you check?

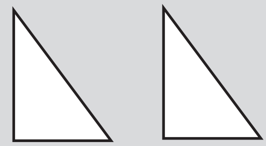


4. Find at least one pair of congruent shapes in your classroom. Describe the shapes.

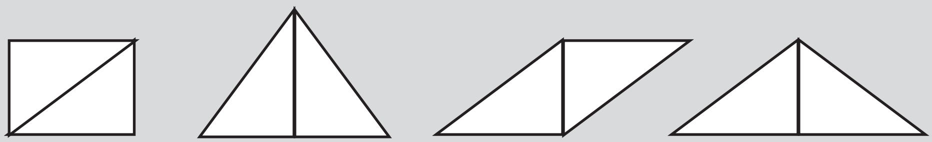
Lesson 5 Combining Polygons

Try This

Cut two congruent triangles out of paper like the ones shown here.

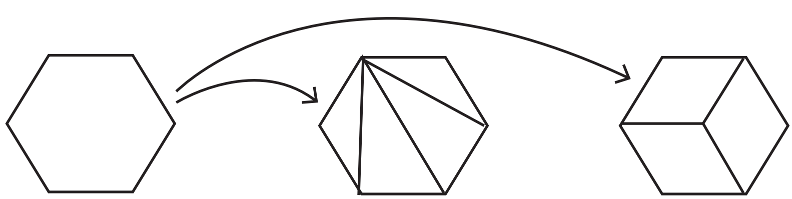


Combine the two triangles to create the following shapes. Name each of the shapes created.



Most of the polygons can be created by combining simpler polygons. For example, in the above **Try This** section, you created two bigger triangles, a rectangle and a parallelogram by combining two congruent triangles.

In fact, a polygon can be seen as a combination of simpler polygons. For example, the hexagon below can be seen as being made up of 4 triangles. It could also be seen as being made of up 3 rhombuses. Can you see it as created of any other shapes?



You should now review the types and names of polygons which you have learned in Lesson 1 of this chapter. For instance, what do we call a polygon which has 4 sides? Name some quadrilaterals. What is a parallelogram?

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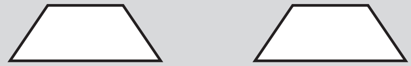
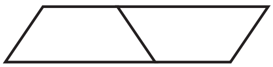
Examples

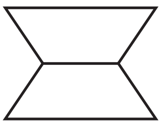
Example 1 Create two different polygons by combining two identical (or congruent) trapezoids. Draw the shapes and name each of them.

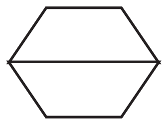


Solution



Parallelogram





Hexagon Hexagon

Thinking

I created a parallelogram and a hexagon by joining the two trapezoids in different ways. In fact, I created the hexagon in two different ways, as shown here.

Practising and Applying

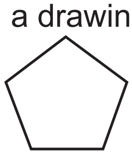
1. What is a polygon?

2. What is the common name for all 3-sided polygons?

3. What is the common name for all 4-sided polygons?

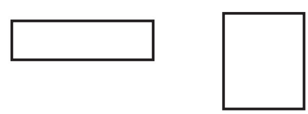
4. Name at least four quadrilaterals. Draw their diagrams.

5. How many triangles would make the pentagon shown below? Show this with a drawing.



6. Show with a drawing that the above pentagon also could be made up of a triangle and a trapezoid.

7. What kind of a polygon can you create by joining two rectangles like the ones shown below? Draw your polygon and name it.



8. What shape will you create if you join two congruent squares?

9. Is it possible to create a bigger square by joining two congruent squares?

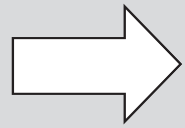
10. Is it possible to make a big parallelogram out of two small ones? If so, show this with a diagram.

11. Is it possible to make a big trapezoid out of two small ones? If so, show this with a diagram.

Lesson 6 Moving Shapes

Try This

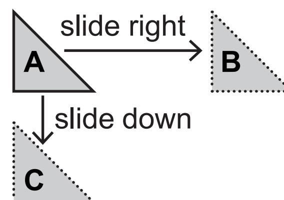
Trace the shape on the right, or draw one like it, on paper and cut it out.



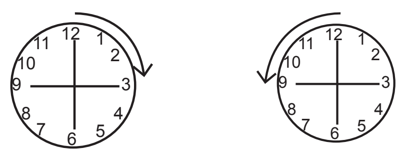
Place the cutout shape on a flat surface, such as your table. Move the shape in different ways, such as by sliding it in different directions (right, left, up, down etc.), turning it in any direction, and flipping it. Discuss or describe what has happened to the shape with each movement. Does the shape change with the movements? Is it still the same shape? How is it the same shape even after all the movements?

We can move a shape in three ways: by **sliding**, **turning**, and **flipping**.

Sliding a shape is like pushing or dragging it from one position to another. For example, you can move a triangle from its position A to B by sliding it right, or from A to C by sliding it down.

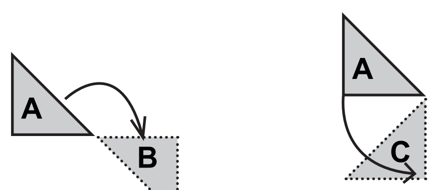


You can **turn** a shape in either a **clockwise** or **counterclockwise** direction. Clockwise direction is the direction in which the hour and minute hands of a clock turns. The opposite direction is counterclockwise.



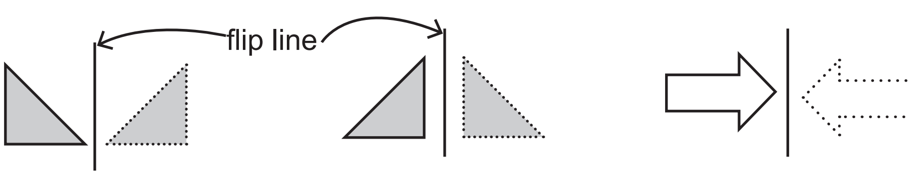
clockwise turn counterclockwise turn

For example, you can move a triangle from its position A to B by turning it clockwise, or from A to C by turning it counterclockwise.



clockwise turn counterclockwise turn

Flipping a shape is like seeing its reflection in a mirror. For example, the following pairs of shapes show flips across the flip lines. You can check them by placing a mirror upright along the flip line to see the image of the shapes.



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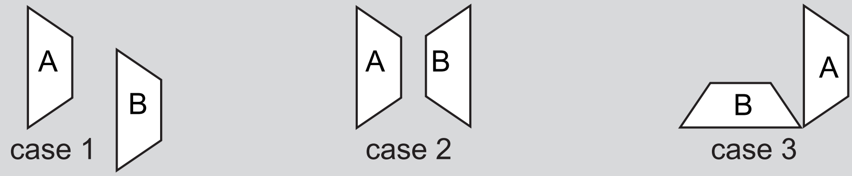
Examples

Example 1 What kind of movement (slide, turn, flip) is applied to move shape A to B in each case below? How do you know?



Solution

It is a **slide** in case 1, because orientation of the two shapes is the same.

It is a **flip** in case 2, because shape B looks like a mirror image of A.

It is a **turn** in case 3, because shape A can be turned counterclockwise to fit into shape B.

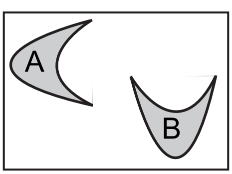
Thinking

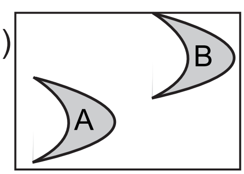
In case 1, shapes A and B appear in the same manner. I traced shape A and cut it out. I could slide it onto shape B.

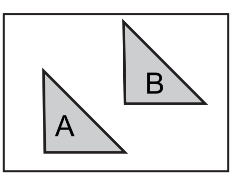
I used the same cutout for both cases 2 and 3 since the shapes are all congruent. Sliding the cutout from A to fit onto B did not work in both cases 2 and 3. It worked with flipping in case 2 and turning in case 3.

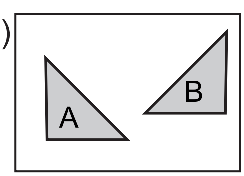
Practising and Applying

1. Can you slide shape A onto shape B in each case below?

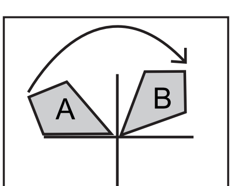
a) 

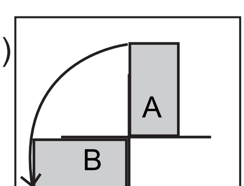
b) 

c) 

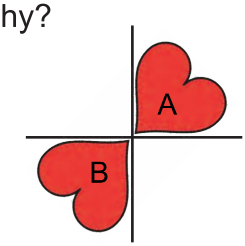
d) 

2. Shape A is turned to B. What is the direction of the turn in each case?

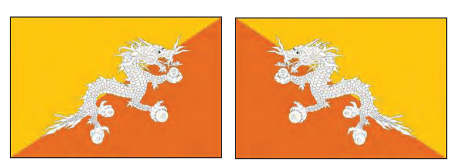
a) 

b) 

3. The heart shape has been moved from A to B. Tshering says that it could be done by turning, Tenzin says it could done by sliding, and Jigme says that it could be done by flipping. Who is right and who is wrong? Why?



4. What type of movement would change the national flag from A to B?

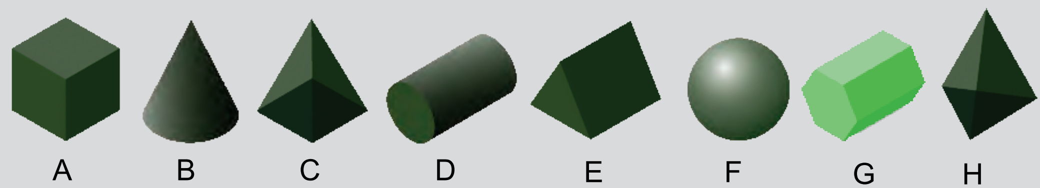


A B

Lesson 7 Prisms and Pyramids

Try This

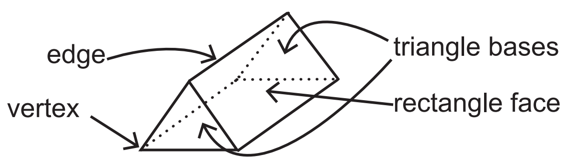
Name each of the following shapes (as prism, pyramid, cone, cylinder, sphere).



Prism

A prism is a 3-D shape with two congruent and parallel polygonal bases which are joined by rectangular faces.

Prisms are named according to the shape of their bases. For example, a prism with triangular bases is called a triangular prism, or triangle-based prism.



triangular prism

Face

The polygons on the surface of a prism are called its faces. A triangular prism has two triangular faces (its bases) and three rectangular faces. The two triangular faces are congruent and parallel.

Edge

The line along which two faces of a prism meet is called an edge. A triangular prism has 9 edges.

Vertex

The point where three or more edges meet is called a vertex. A triangular prism has 6 vertices.

Notes

1. **3-D** (short form for 3 dimensional) shapes are solid shapes or objects. Cups, boxes, books, cars, trees, stones etc. are all 3-D shapes.

2. **Congruent** (also called **identical**) shapes are shapes which are exactly the same in terms of size and looks.

3. **Polygons** are 2-D shapes made up of straight lines.

4. **2-D shapes** are flat shapes such as polygons and circles.

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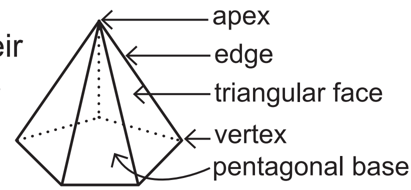
Pyramid
A pyramid is a 3-D shape with one base that is a polygon and triangular faces which join at a point called an apex.

Pyramids are named according to the shape of their bases. For example, a pyramid with a base that is a pentagon is called pentagonal pyramid.
As with prisms, pyramids have faces and edges.



pentagonal pyramid

Apex
The vertex which is opposite to its base in a pyramid is called its apex.

Examples

Example 1 What would be the name of the shape below? Why?

Solution

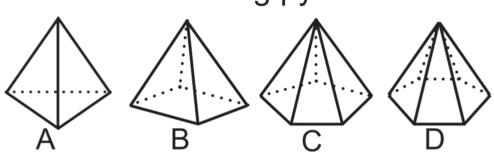
It is a trapezoidal prism, because it has two parallel and identical trapezoids as its bases.

Thinking

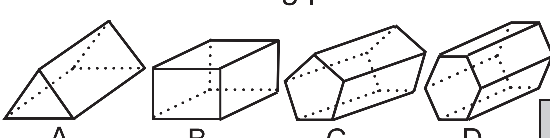
I know that the shape is not a pyramid, as it does not have an apex. It is a prism. I see that it has two trapezoids, which are identical and parallel. Therefore, I can say that it can be called as trapezoidal prism.

Practising and Applying

1. Name the following pyramids.

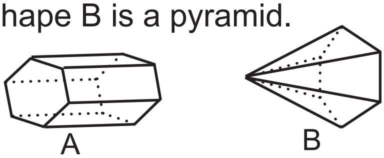


2. Name the following prisms.



3. Draw and label the pyramids and prisms shown above.

4. Describe why shape A is a prism and shape B is a pyramid.



5. What is the same about a pyramid and a prism. What is different about them?

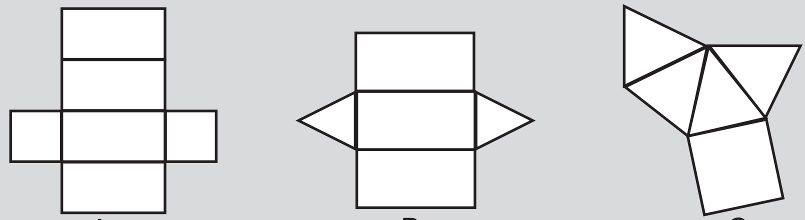
6. Determine the required numbers for the shapes below. You should look at their diagrams or physical models to help you with this task.

Number of:	vertices	edges	triangular faces	rectangular faces
Triangular pyramid				
Rectangular pyramid				
Pentagonal pyramid				
Hexagonal pyramid				
Triangular prism				
Rectangular prism				
Pentagonal prism				
Hexagonal prism				

Lesson 8 Nets of Prisms and Pyramids

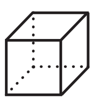
Try This

Each shape below is a net for a 3-D shape. Name the shape that each net will form if you fold it.

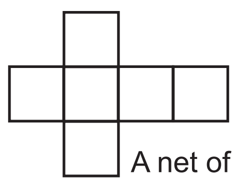


A B C

If you have a paper model of a prism, which is empty inside, and cut it along some edges and flatten it, you get the net of the prism. For example, the following shapes show a cube and its net.



cube

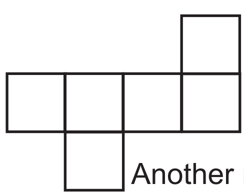


A net of the cube

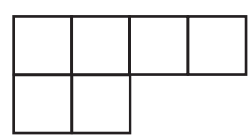
Note

A **cube** is a rectangular prism in which all 6 faces are congruent squares.

There can be more than one net for a 3-D shape. A net has to be able to form the 3-D shape when folded up. For example, of the two shapes below, one is another net for the above cube and the other is not. Trace, or draw similar shapes as the ones below, cut them along their boundaries, and fold up to see if each net forms a cube. You may wish to work in pairs or small groups.

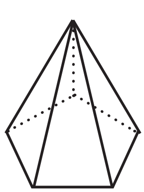


Another net of the cube

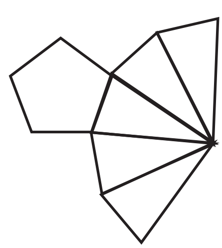


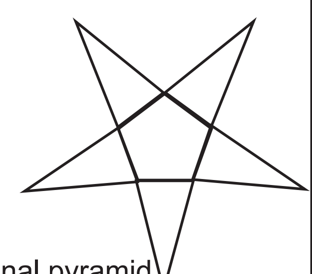
This is not a net of the cube

Similarly, the following shapes show two different nets for a pentagonal pyramid.



pentagonal pyramid





Two different nets for the pentagonal pyramid

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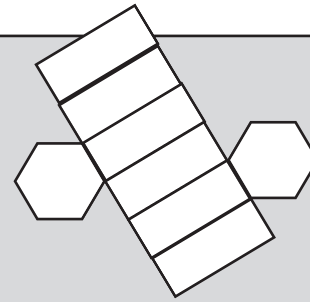
Example

Example 1 The shape on the right is a net of a 3-D shape. Name the 3-D shape. How do you know?

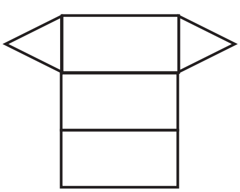


Solution	Thinking
The 3-D shape is a hexagonal prism, because it has two congruent hexagonal bases and 6 rectangular faces.	There are two congruent hexagons. So, it has to be a prism. It looks like the 6 rectangles will fold to form the side faces of the prism. The two hexagons are the bases of the prism. So, it is a hexagonal prism. To confirm, I traced the net, cut it along the boundaries, and folded it up. And, yes it forms a hexagonal prism.

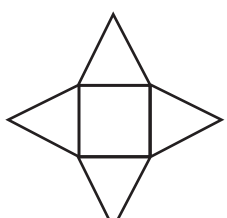
Practising and Applying

1. The shapes at the back of this page are nets of some 3-D shapes. Trace each net on plain paper, cut them out, and fold along the lines. Name the 3-D shapes formed. You could join the faces with Sellotape. (If possible, your teacher will provide you photocopies of them.)

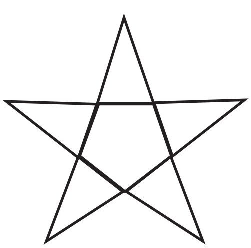
2. Name the 3-D shapes for each of the nets below.



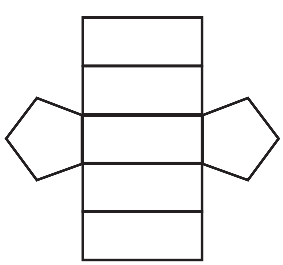
A



B



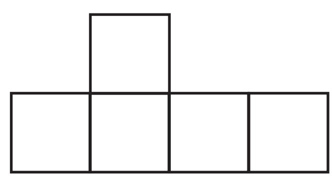
C



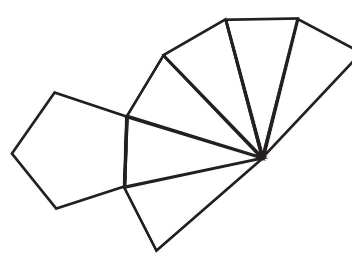
D

3. Draw a sketch of a net for each of the following 3-D shapes.
   a) A triangular prism
   b) A hexagonal prism
   c) A cube
   d) A hexagonal pyramid

4. The shape below cannot be a net for a cube. Why not?



5. The shape below can not be a net for a pentagonal pyramid. Why not?



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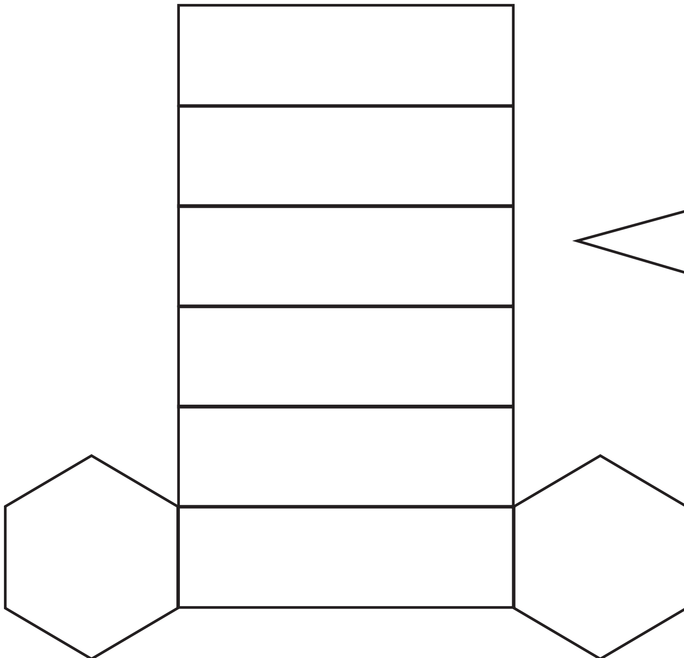

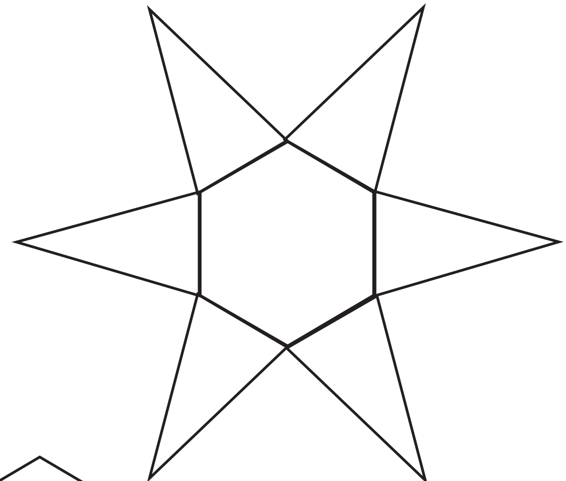

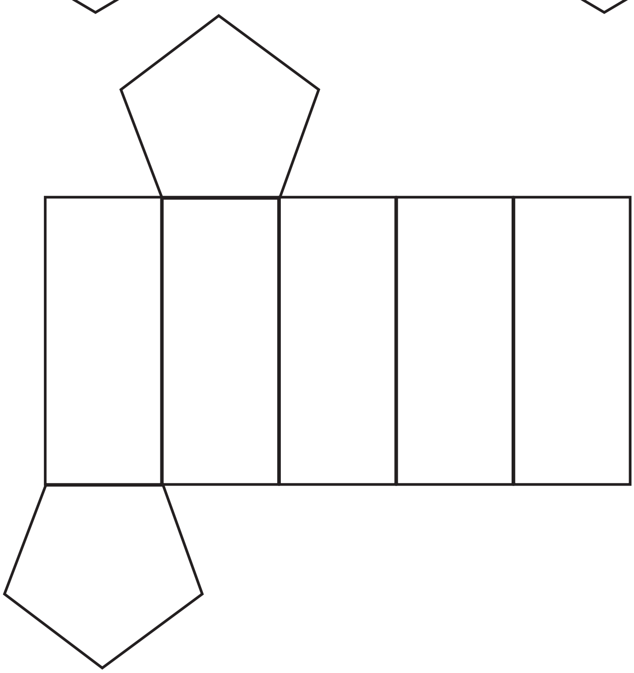

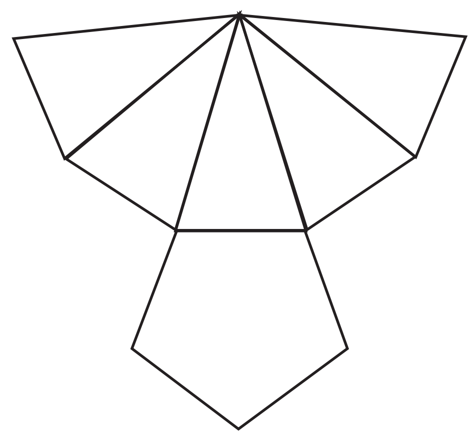

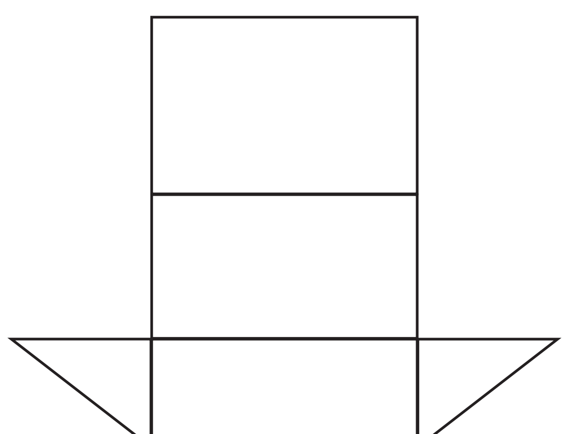

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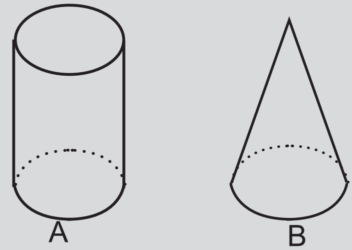
Lesson 9 Cylinders and Cones

Try This

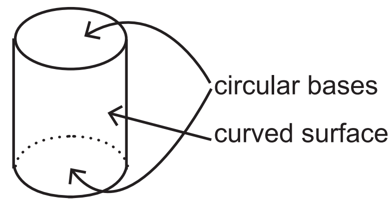
These shapes on the right are not pyramids but one of them looks like a pyramid. Which one is it? Why?

Why is the other shape not like a pyramid?



Cylinders

A cylinder is a 3-D shape with two congruent and parallel circular bases which are joined by a curved surface.



A cylinder

A cylinder is like a prism, because both of them have two congruent and parallel bases.

A cylinder is different from a prism, because the bases in a cylinder are circles while the bases in a prism are polygons. The two bases in a cylinder are joined by a curved surface, but bases in a prism are joined by rectangular faces.

Notes

A **polygon** is a closed 2-D shape made up of straight lines.

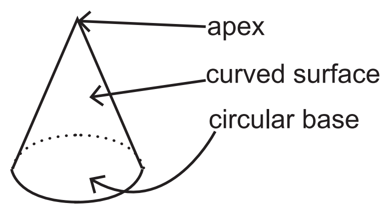
Triangles, rectangles, pentagons, and hexagons are all polygons.

Cones

A cone is a 3-D shape with one circular base and an apex joined by a curved surface.

A cone is like a pyramid, because both of them have one base and an apex.

A cone is different from a pyramid, because the base of a cone is a circle, while the base of a pyramid is a polygon. The base and the apex in a cone are joined by a curved surface, but the base and the apex in a pyramid are joined by triangular faces.



A cone

It would be good for you to practise drawing sketches of cylinders, cones, prisms and pyramids.

Example

Example 1 Describe what is similar and what is different about cylinders and prisms.

Solution

Similarities:
- Cylinders and prisms are both 3-D shapes.
- Both have two congruent and parallel bases.

Differences:
- The bases of cylinders are circles, but the bases of prisms are polygons.
- The bases of a cylinder are joined by a curved surface, but the bases of a prism are joined by rectangle faces.
- There are different types of prisms named as per the shape of their bases, but this does not apply to cylinders.

Thinking

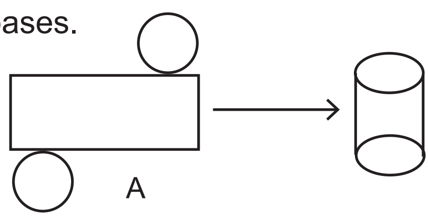
I compared the different shapes and features of cylinders and prisms to talk about their similarities and differences.

Practising and Applying

1. Describe what is similar and what is different about cones and pyramids.

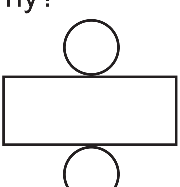
2. Describe what is similar and what is different about cones and cylinders.

3. The shape A below is a net of a cylinder, because if you roll the rectangle, it will form its curved surface and the two circles will form its bases.

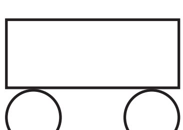


A

Which of the shapes below is a net of a cylinder, and which is not? Why?

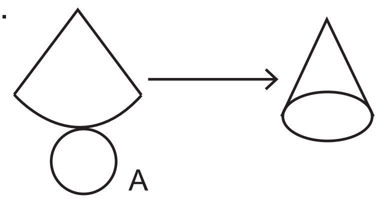


B



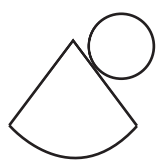
C

4. The shape A below is a net for a cone.

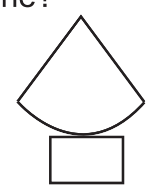


A

Why can shapes B and C below not be nets of a cone?



B



C

5. Name two things that you find in your classroom, school or home that are like a cylinder.

6. Name one thing that you find in your classroom, school or home that is like a cone.

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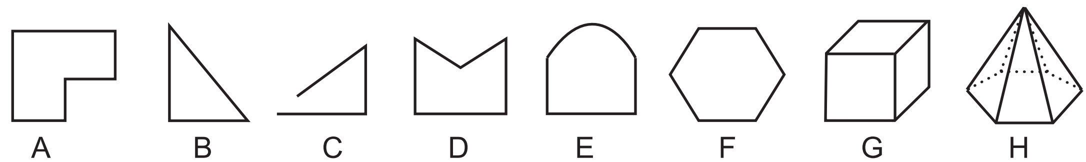
Chapter Review

1. Which of the following shapes are polygons and which are not?



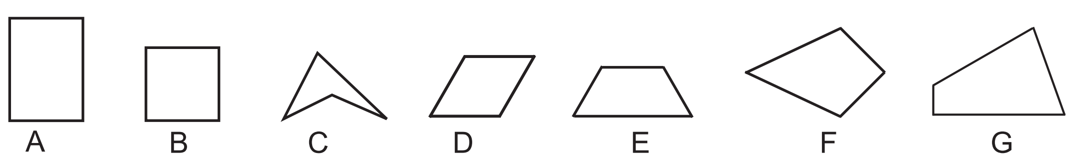
2. What is a polygon?

3. For each shape that is not a polygon in question 1, explain why it is not a polygon.

4. Which shape below is a convex polygon and which is a concave polygon? Why?

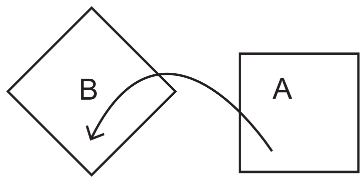
5. Kinley says that both polygons A and B in question 4 above are hexagons. Do you agree with Kinley? Why or why not?

6. What is the same about all the polygons below?

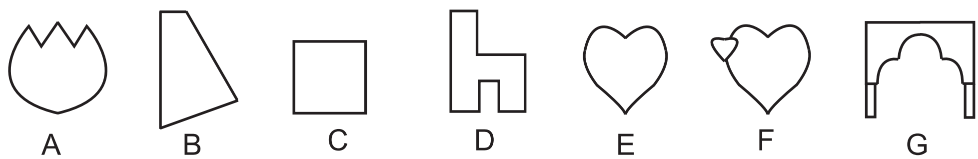


7. Name shapes A, B, D, E and F in question 6 above.

8. A square has been turned from position A to B as shown below. Is the direction of the turn clockwise or counterclockwise? How do you know?



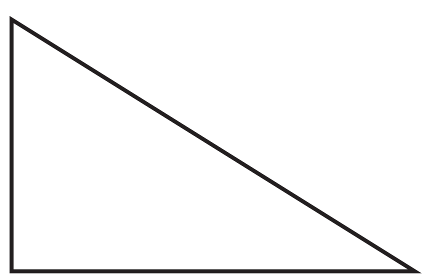
9. Which of the following shapes are symmetrical and which are not?



10. Draw the sketches of the symmetrical shapes in question 9 above, and draw a line of symmetry through each of them.

11. Which symmetrical shape in question 9 above has more than one line of symmetry? Draw the shape with all its lines of symmetry.

12. Trace, or draw on paper a triangle like the one shown below. Make another triangle which is congruent to it. Cut out the two triangles. Join the two triangles in different ways so that you make a larger triangle, and then a rectangle, and finally, a parallelogram. Draw the sketches for these newly formed shapes.



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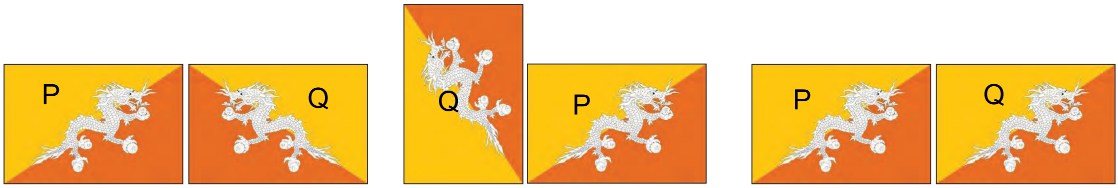
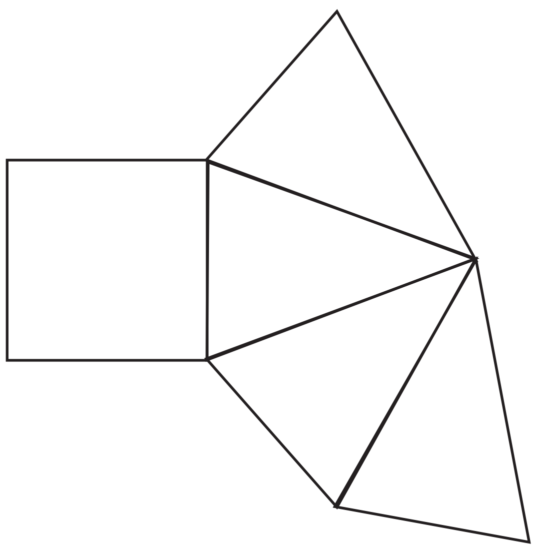
13. What type of movement (slide, flip, turn) has been applied to move the national flag from position P to Q in each case below? How do you know?



Case 1 Case 2 Case 3

14. Nima says that the shape below is a net for a square based prism. Is Nima correct? Why or why not?



CHAPTER 6 DIVISION

Chapter Overview

Division is a way of describing equal sharing, equal grouping, and repeated subtraction. We will discuss each of these, one by one, in this chapter. You will realise that these three situations are all related.

Division is related to multiplication. They are the opposite of each other. As such, you can use your knowledge of multiplication facts to determine the related division facts. Just like it is important for you to memorise basic multiplication facts, it is also important for you to memorise basic division facts. It may be necessary for you to carry out the memorisation practice over an extended period of time.

This chapter has 7 lessons as detailed in the Table of Contents.

Basic Principles about Division

- Division is a way to describe equal sharing, equal grouping, and repeated subtraction.
- Division is the opposite of multiplication. As such, they are related.
- Division can be performed using backward skip counting.

Chapter Goals

By the end of this chapter, you will be able to:

- Describe fair sharing situations using division.
- Describe forming equal groups with division sentences.
- Describe repeated subtraction with division and vice versa.
- Describe what each number in a division sentence means.
- Use a multiplication fact to determine the related division facts.
- Solve simple division problems.
- Commit to memory some basic division facts.

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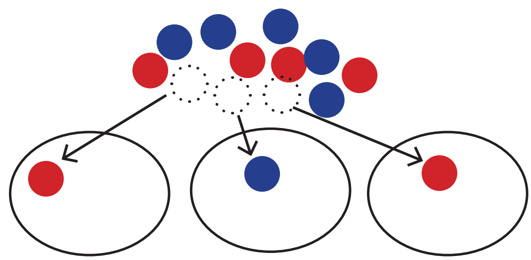
Lesson 1   Division as Equal Sharing

Try This

Tenzin and Jigme are brothers. Their mother has put 12 plums in a *bangchung* for them to eat. The two brothers decide to share the plums equally. How might they share the plums? How many plums will each brother get?

You will need counters in this lesson to represent things to share.

For example, to share 12 plums among 3 friends, take out 12 counters to represent the 12 plums. Then, actually share the counters among the 3 friends, so that each person gets the same number of counters. One way to share is that each friend takes 1 counter at a time, until there are no counters left.



You will notice that each friend gets 4 counters. We can represent this sharing with a division sentence as, $12 \div 3 = 4$. (Say, '12 divided by 3 equals 4').

$12 \div 3 = 4$

Dividend Divisor Quotient

Division sign

12 is called the dividend, 3 is called the divisor, and 4 is called the quotient.

In this case, 12 is the total number of plums, 3 is the number of friends to share the plums, and 4 is the share for each friend.

Now, can you write a division sentence to represent the sharing of the plums between Tenzin and Jigme in the above **Try This** problem? What is the dividend, the divisor and the quotient in your division sentence? What does each number tell?

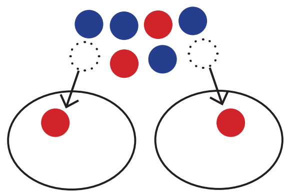
Notes

The result of dividing numbers is called the **quotient**.

Examples

Example 1 Model with counters how 8 apples could be shared by 2 friends. Sketch pictures of it. How many apples does each friend get? Write a division sentence for this sharing. Describe what each number in your sentence tells.

Solution



Each friend gets 4 apples.

The division sentence is, $8 \div 2 = 4$.

8 tells the total number of apples to share.
2 tells the number of persons to share the apples.
4 tells the share of each person.

Thinking

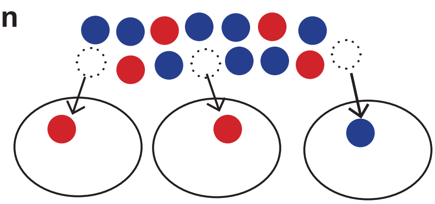
I took out 8 counters to represent the apples. Then, I separated the apples into two groups by putting one counter each into the groups, one by one, until there were no counters left.

Each group has now 4 counters. So, the share of apples for each person is 4 apples.

Then, I write the division sentence as shown on the left and describe what each number in it tells.

Example 2 Calculate the quotient for $15 \div 3 = \square$ Show your workings.

Solution



$15 \div 3 = 5$

Thinking

I modelled the division with counters. I took out 15 counters, and separated them into 3 equal groups.

Practising and Applying

1. Represent each sharing situation with counters. Sketch your counters, and write a division sentence for each. Describe what each number in each of your division sentences means.

   a) 8 chocolates shared by 4 friends
   b) 20 momos shared by 4 friends
   c) 16 plums shared by 2 friends
   d) 6 mangoes shared by 6 friends
   e) 5 apples shared by 1 person

2. Calculate each quotient. You are encouraged to use counters and drawings in solving these.

   a) $12 \div 4 = \square$
   b) $14 \div 2 = \square$
   c) $10 \div 2 = \square$
   d) $20 \div 5 = \square$
   e) $16 \div 4 = \square$

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Lesson 2 Division as Equal Grouping

Try This

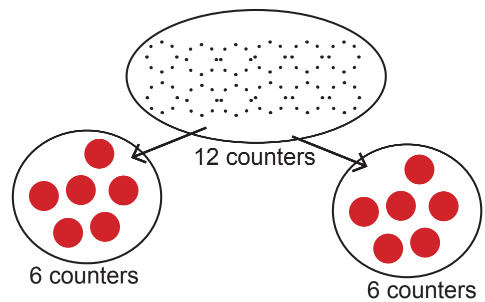
Take out 12 counters. Make 2 equal groups out of them. How many counters are in each group? Put the counters back together. Then, make 3 equal groups out of them. How many counters are in each group?

12 counters

You will need counters in this lesson to represent things to make into groups.

A **group** is a collection of things put together. A group is also called a **set**.

Division is also a process of breaking a collection of things into smaller and equal groups. For example, if you break 12 counters into 2 equal groups, each group will have 6 counters. We represent this situation with division as $12 \div 2 = 6$.



In the above division sentence, 12, the dividend,tells the total number of counters; 2, the divisor,tells the number of equal groups; and 6, the quotient, tells the size of the groups.

$12 \div 2 = 6$

Total number ↑   ↑ Number of equal groups   ↑ Size of the groups

We can also represent the same situation with division in a different way, by exchanging the places of 6 and 2, as $12 \div 6 = 2$. Here, the size of the groups is the divisor, and the number of equal groups is the quotient.

$12 \div 6 = 2$

Total number ↑   ↑ Size of the groups   ↑ Number of equal groups

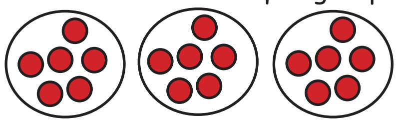
Could you now write a division sentence for each of the situations in the above **Try This** problem?

Examples

Example 1 Make 3 equal groups from 18 counters. Sketch the situation. Write a division sentence. What does the quotient tell?

Solution

18 counters into 3 equal groups



$18 \div 3 = 6$

Thinking

I took out 18 counters, and separated them into 3 equal groups. I drew 3 circles for the 3 groups. Then, I drew 6 counters in each circle.
I wrote the division sentenc. The quotient, which is 6, tells the number of counters in each group.

Example 2 Jigme made some equal groups out of 12 apples. There are 4 apples in each group. Write a division sentence for this. What does each number in the sentence tell?

Solution

$12 \div 4 = 3$

12 is the total number of apples.
4 is the size of each group.
3 is the number of equal groups.

Thinking

I know that in this situation, the quotient will be the number of groups.

I took out 12 counters. Since I know the group size, I made a group of 4 counters. Then, I made another group of 4. Then, I could make a third group of 4. So, now there are 3 such groups.

Practising and Applying

1. Tashi is making 3 equal groups out of 21 persons.
   a) Represent this situation with counters, and sketch them.
   b) Write a division sentence.
   c) What does each number in the division sentence tell?

2. Deki wants to divide 14 persons into equal groups of 7 persons.
   a) Represent this with situation with counters, and sketch them.
   b) Write a division sentence.
   c) How many groups can Deki make?

3. Calculate each quotient. You are encouraged to use counters and drawings in solving these.
   a) $20 \div 2 = \square$
   b) $20 \div 10 = \square$
   c) $8 \div 2 = \square$
   d) $8 \div 1 = \square$
   e) $16 \div 4 = \square$
   f) $9 \div 3 = \square$
   g) $4 \div 2 = \square$

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Lesson 3 Division as Repeated Subtraction

Try This

There are 14 biscuits on a plate. Children can come, one at a time, and each can take 2 biscuits from the plate. How many children can take biscuits?

Division is also a way to represent repeated subtraction. For example, if we subtract 2 from 8, we get a difference of 6; subtract 2 from 6, we get a difference of 4; subtract 2 from 4, we get a difference of 2, subtract 2 from 2, we get a difference of 0. Now, we do not have any numbers to subtract from. We can represent this repeated subtraction of 2 as shown below.

$8 - 2 = 6$
$6 - 2 = 4$
$4 - 2 = 2$
$2 - 2 = 0$

Notes

The result of subtracting numbers is called **difference.**

It can also be written as $8 - 2 - 2 - 2 - 2 = 0$.

As you can see 2 is subtracted 2 repeatedly for 4 times altogether, to get to the difference of 0. We can represent this repeated subtraction with division as $8 \div 2 = 4$.

We can also write a division sentence as a repeated subtraction sentence. For example, $15 \div 3 = 5$ can be written as, $15 - 3 - 3 - 3 - 3 - 3 = 0$.

Now, could you write a repeated subtraction sentence for the situation of taking away 2 biscuits at a time from the plate in the above **Try This** problem? Then, write the division sentence.

Examples

Example 1 What division does $20 - 4 - 4 - 4 - 4 - 4 = 0$ represent?

Solution	**Thinking**
It represents $20 \div 4 = 5$.	4 is subtracted 5 times from 20 to get to the difference of 0. So, I know the divisor is 4, and the quotient is 5 for the division sentence.

Example 2 How could you use subtraction to solve 18 ÷ 3 = □?

Solution

I could subtract 3 repeatedly starting from 18, until the difference is 0, as shown. The quotient will be the number of times 3 is subtracted, which is 6 in this case.

18 - 3 = 15
15 - 3 = 12
12 - 3 = 9
9 - 3 = 6
6 - 3 = 3
3 - 3 = 0

So, 18 ÷ 3 = 6

Thinking

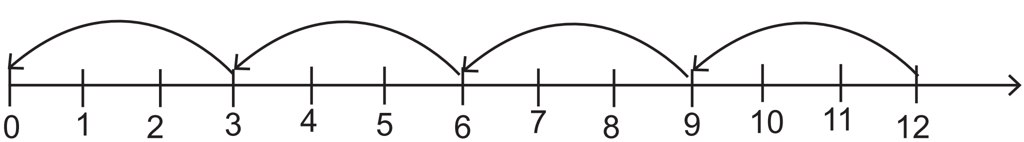
I will subtract 3 from 18, to get the difference of 15. Then, I will subtract 3 from 15 to get 12. I will continue this way until the difference is 0.

Practising and Applying

1. Represent each repeated subtraction with a division sentence.

   a) 12 - 2 - 2 - 2 - 2 - 2 - 2 = 0
   b) 12 - 6 - 6 = 0
   c) 12 - 12 = 0
   d) 24 - 8 - 8 - 8 = 0
   e) 20 - 5 - 5 - 5 - 5 = 0
   f) 15 - 5 - 5 - 5 = 0
   g) 5 - 1 - 1 - 1 - 1 - 1 = 0
   h) 16 - 8 - 8 = 0
   i) 6 - 2 - 2 - 2 = 0
   j) 6 - 6 = 0

2. Write a repeated subtraction sentence and, then, a division sentence for the skip counting shown on the number line.



3. Use repeated subtraction to calculate each quotient.

   a) 8 ÷ 2 = □
   b) 9 ÷ 3 = □
   c) 20 ÷ 5 = □
   d) 20 ÷10 = □
   e) 50 ÷ 10 = □
   f) 12 ÷ 4 = □
   g) 16 ÷ 4 = □
   h) 21 ÷ 7 = □
   i) 8 ÷ 1 = □
   j) 7 ÷ 1 = □

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Lesson 4 Relating Division to Multiplication

Try This

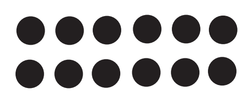
Calculate the quotient for $12 \div 6 = \square$
Calculate the product for $6 \times 2 = \square$
What relation do you see between the above division and multiplication?

Multiplication and division are related. When you multiply two numbers you get their product. If you divide the product by one of the factors, you get the other factor as the quotient.

$2 \times 6 = 12$
Factors
Product

$12 \div 2 = 6$ ← Quotient *(The other factor in the multiplication)*
Divisor *(One of the factors in the multiplication)*
Dividend *(the product in the multiplication)*

This makes sense, because, as you know, we can represent an array by a multiplication. For example, the array on the right shows the multiplication $2 \times 6 = 12$.

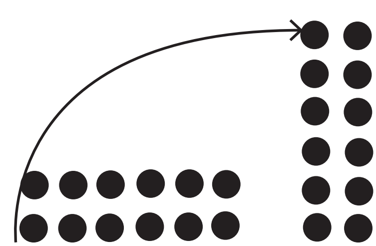


A 2 x 6 array
(2 rows by 6 columns)

The same array also shows the division $12 \div 2 = 6$. This means that if the total number of counters is arranged into 2 equal rows, each row has 6 counters.

If we turn the array, as shown on the right, the number of rows and columns are interchanged. But it is still the same array, in term of its size, or the total number of items.

The turned array, then, shows the multiplication, $6 \times 2 = 12$ and the division, $12 \div 6 = 2$.



A 2 x 6 array
(2 rows by 6 columns)

A 6 x 2 array
(6 rows by 2 columns)

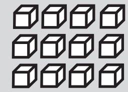
So, in fact, we can write 2 multiplication sentences and 2 division sentences, which are all related for the above array.

$2 \times 6 = 12$ $\quad$ $6 \times 2 = 12$ $\quad$ $12 \div 2 = 6$ $\quad$ $12 \div 6 = 2$

We call these a **multiplication-division fact family.**

Examples

Example 1 What multiplication-division fact family does this array show?



Solution

The array shows the fact family:

$3 \times 4 = 12$

$4 \times 3 = 12$

$12 \div 3 = 4$

$12 \div 4 = 3$

Thinking

This is a 3 by 4 array. The total count is 12, which will be the product for the multiplication. For the multiplications, I just have to change the order of thefactors. For the divisions, I just have to interchange the divisor and the quotient.

Example 2 How could you use multiplication to figure out the quotient for $21 \div 7$?

Solution

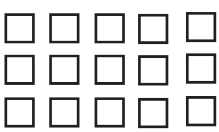
$3 \times 7 = 21$, so $21 \div 7 = 3$

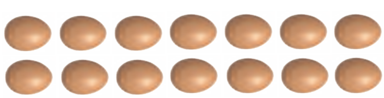
Thinking

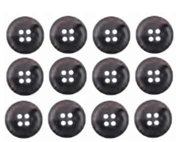
I know from my multiplication fact table that $3 \times 7 = 21$. This means, 21 divided by 7 equals 3.

Practising and Applying

1. Write the multiplication-division fact family for each array shown below.

   a) 

   b) 

   c) 

   d) 

   e) 

2. Calculate the quotient based on the given multiplication fact.

   a) $6 \times 3 = 18$, so $18 \div 6 = \square$

   b) $6 \times 3 = 18$, so $18 \div 3 = \square$

   c) $3 \times 3 = 9$, so $9 \div 3 = \square$

   d) $5 \times 6 = 30$, so $30 \div 6 = \square$

   e) $9 \times 1 = 9$, so $9 \div 1 = \square$

3. Calculate the product, based on the division fact.

   a) $12 \div 3 = 4$, so $4 \times 3 = \square$

   b) $18 \div 2 = 9$, so $9 \times 2 = \square$

   c) $7 \div 7 = 1$, so $7 \times 1 = \square$

   d) $32 \div 4 = 8$, so $8 \times 4 = \square$

   e) $28 \div 7 = 4$, so $4 \times 7 = \square$

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Lesson 5 Dividing with Multiplication Tables

Try This

How does knowing 3 x 7 = 21 help you solve $21 \div 7$ and $21 \div 3$?

Now that you know that division and multiplication are related, you could use the multiplication table, as you made in the last chapter, to help you solve division problems.

For example, to divide 36 by 9 ($36 \div 9$) look right along row number 9, locate 36, then, look up along the column to its heading, which is 4 in this case. This tells that $36 \div 9 = 4$. This is because 9 x 4 = 36.

You could also look down along column number 9, locate 36, then, look left across to the row heading, which is 4 in this case. This also tells that $36 \div 9 = 4$. This is because 4 x 9 = 36.

Columns

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	**36**
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	**36**	45	54	63	72	81

Rows

Multiplication table

Note that 36 is the dividend in both cases.

Practising and Applying

1. Use the multiplication table to solve:

   a) $30 \div 5$ b) $48 \div 8$

   c) $25 \div 5$ d) $27 \div 3$

   e) $27 \div 3$ f) $36 \div 6$

   g) $56 \div 7$ h) $81 \div 9$

   e) $63 \div 7$ i) $7 \div 7$

2. What is same about the divisions in parts c, f, and h of question 1?

3. Use the multiplication table to write four division sentences each having 18 as the dividend.

Lesson 6 Using Double Facts to Divide

Try This

How would you solve $32 \div 4$? How would you solve it in a different way?

Recall that there are three meanings of division, such as equal sharing, equal grouping, and repeated subtraction. You can use any of these meanings to find the quotient in a division sentence. Also, as you have learnt, you can use multiplication facts to solve related division facts. Additionally, you can also use **double facts** to find the quotient in a division sentence.

If you double the dividend without changing the divisor, it will double the quotient. For example, if you know that $\mathbf{8} \div 4 = \mathbf{2}$, then, $\mathbf{16} \div 4 = \mathbf{4}$.

Now can you use double facts of 16 and 4 to find the quotient for the division in the above **Try This** problem?

Examples

Example 1 If $18 \div 9 = 2$, solve $36 \div 9 = \square$.

Solution	**Thinking**
$36 \div 9 = 4$	36 is the double of 18, and since the divisors are the same, the quotient for $36 \div 9$ will be double of 2, which is 4.

Practising and Applying

1. Complete each doubled fact.
   a) $\mathbf{8} \div 4 = 2$, so $\mathbf{16} \div 4 = \square$
   b) $\mathbf{9} \div 3 = 3$, so $\mathbf{18} \div 3 = \square$
   c) $\mathbf{15} \div 3 = 5$, so $\mathbf{30} \div 3 = \square$
   d) $\mathbf{21} \div 7 = 3$, so $\mathbf{42} \div 7 = \square$
   e) $\mathbf{35} \div 5 = 7$, so $\mathbf{70} \div 5 = \square$
   f) $\mathbf{24} \div 8 = 3$, so $\mathbf{48} \div 8 = \square$
   g) $\mathbf{30} \div 5 = 6$, so $\mathbf{60} \div 5 = \square$

2. How would knowing $\mathbf{10} \div 2 = \mathbf{5}$ help you to find the quotient of $\mathbf{20} \div 2$?

3. If $12 \div 3 = 4$, what will the quotient of $24 \div 3$?

4. Use your quotient of $24 \div 3$ to calculate the quotient of $48 \div 3$. (Hint: 48 is double of 24)

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Lesson 7 Solving Division Problems

Try This

A person has 2 legs. There are 12 legs in a room. How many persons are there?

It is important for you to be able to divide numbers, recall division facts, and represent or describe what a division sentence could mean. Additionally, you should be able to identify what problems and situations involve division. These problems and situations would normally appear as stories and word problems.

A general strategy to solve such a problem is to first read carefully and understand it, represent it with sketches and concrete materials such as counters, recognise that it involves division, and write down the division sentences to solve the problem. Then, say what the solution is as per the question asked in the problem.

Example 1 A class has 30 students. The class teacher wants to form 3 dance teams involving all the students so that each team has the same number of students. How many students will be on each team?

Solution	Thinking
$30 \div 3 = 10$ Each dance team will have 10 students.	3 equal teams mean 3 equal groups. I know the total number, and the number of groups. I have to find the size of the groups. It is a division problem. If I divide 30 by 3, the quotient will be the number of students in each team.

Practising and Applying

1. Sarita puts 3 flowers together to make a bunch. How many bunches of flowers would she make with 15 flowers?

2. A room in a hotel can accommodate 2 persons. How many such rooms are required to accommodate 18 persons?

3. A car can carry 5 persons. How many such cars are required to carry 30 persons?

4. One week is 7 days. How many weeks is 28 days?

5. Dema divided a number by another, and got 4 as the quotient. But she forgot what the numbers were. What could the numbers be?

6. A bag contains equal number of blue, red and yellow counters, making a total of 21. How many of them are blue?

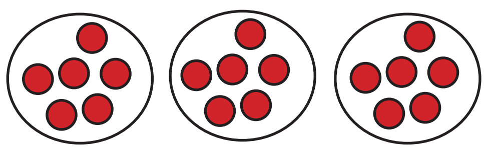
Chapter Review

1. Write each repeated subtraction sentence as a division sentence.

    a) $28 - 7 - 7 - 7 - 7 = 0$　　b) $28 - 4 - 4 - 4 - 4 - 4 - 4 - 4 = 0$

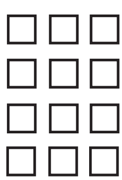
    c) $21 - 3 - 3 - 3 - 3 - 3 - 3 - 3 = 0$　　d) $5 - 5 = 0$

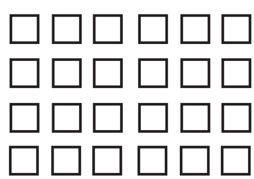
2. Kuenley represented the diagram below with the division sentence $18 \div 3 = 6$. What does each number in Kuenley's division sentence tells?

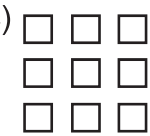


3. Namgay represented the same diagram in question 2 above with the division sentence $18 \div 6 = 3$. This is also correct. What does each number in Namgay's division tell?

4. Write the complete multiplication-division fact family for each of the arrays.

    a) 

    b) 

    c) 

5. Divide. In each case, show your working. You may use sketches, multiplication facts, or any other means.

    a) $35 \div 5$　　b) $36 \div 6$　　c) $28 \div 4$　　d) $21 \div 3$

    e) $7 \div 7$　　f) $7 \div 1$　　g) $42 \div 6$　　h) $6 \div 6$

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6. In the division sentence below, the quotient and the divisor are the same. Write 2 other division sentences in which the divisor and the quotient are the same.

$$9 \div \underbrace{\mathbf{3}}_{\text{divisor}} = \mathbf{3} \longleftarrow \text{quotient}$$

7. Write a division problem for the sentence $20 \div 2 = 10$.

8. Tshering Yangchen reads 2 books in a week from her school library. How many weeks will take for her to read 4 books?

CHAPTER 7 FRACTIONS AND DECIMALS

Chapter Overview

A fraction is a number that represents a part of a whole. For a fraction to represent a part of whole, the whole has to be divided into a certain number of equal sizes. The whole can be either a single shape or object, or a set or group.

A fraction number has two parts, one number on top of the other, separated by a short line. The number on the top is called **numerator** and the number on the bottom is called **denominator**. The denominator tells how many equal parts the whole is divided into. The numerator tells how many of those equal parts are being considered for a particular situation.

When a fraction represents a part of a set, it is not necessary that the individual items that make up a set be identical or equal in size. That is because it is the numbers, represented by the individual items, that are considered equal in size.

It is important to know that a fraction always has a complementary fraction. A fraction and its complementary fraction make up the whole.

A fraction with a denominator such as 10, 100 and 1000 is called a decimal fraction.

This chapter has 5 lessons as detailed in the Table of Contents.

Basic Principles about Fractions and Decimals

- A fraction represents a comparison of a part to a whole. The whole can be a single shape, or it can be a set.
- The denominator in a fraction tells the total number of parts the whole is divided into, and the numerator tells how many such parts are under consideration.
- The parts of a whole must be equal in size in the case of a single shape or object.
- The parts of a whole need not be equal in size in the case of a set or group, for example, a set of people.

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- A fraction always has a complementary fraction. These two fractions make up the whole.
- Fractions with denominators such as 10, 100 and 1000 are called decimal fractions.

Chapter Goals

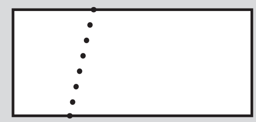
By the end of this chapter, you will be able to:

- Identify and describe the indicated parts of single shapes with fractions using both fraction numbers and fraction words.
- Represent or model fractions with various diagrams.
- Identify and describe the indicated parts of a set with fractions.
- Represent or model fractions with set diagrams.
- Use fractions to show parts of measures such as masses, capacities and lengths.
- Determine the complementary fraction of a fraction.
- Create or identify smaller shapes as fractions of a larger shape.
- Understand what a decimal fraction represents.
- Write a decimal tenth fraction in decimal form, and vice versa.
- Say and write correctly the names for decimal tenths.

Lesson 1 Fractions as Parts of Single Shapes

Try This

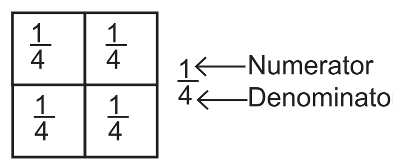
Does the dotted line divide the rectangle into two equal parts? How do you know?



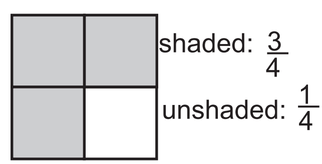
If a shape is divided into equal parts, each part is called a fraction. The name of the fraction depends upon how many parts the shape is divided into. If the shape is divided into 2 equal parts, each part is called **one half;** if it is divided into 3 equal parts, each part is called **one third**; if it is divided into 4 equal parts, each part is called **one fourth (**or **one quarter)**, and so on.

A fraction number has two parts, one number on top of the other, separated by a short horizontal line. The number at the top is called the **numerator** and the number at the bottom is called the **denominator**. The denominator tells how many equal parts the whole is divided into. The numerator tells how many of those equal parts are being considered for a particular situation.

For example, the shape on the right is divided into 4 equal parts. Each part is one fourth. One fourth is represented with number as $\frac{1}{4}$. How many one fourths make one whole shape?



If 3 of those equal parts are shaded, then, three fourths of the shape is shaded.



A fraction always has a complementary fraction. A complementary fraction describes what is not discussed. For example, if the fraction $\frac{3}{4}$ describes the shaded part of a shape, then, the fraction that describes the unshaded part is $\frac{1}{4}$. Here, $\frac{1}{4}$ is the complementary fraction of $\frac{3}{4}$.

A fraction and its complementary fraction make up the whole. In the above example, $\frac{3}{4}$ and $\frac{1}{4}$ make $\frac{4}{4}$, which is 1 whole.

Notes

Remember that for a part of a shape to be represented by a fraction, all the parts into which the shape is divided have to be **equal in size.**

Can you describe either of the two parts of the rectangle in the above **Try This** problem with a fraction? Explain.

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Example

Example 1 Look at the rectangle below.

Write a fraction to describe the black part of the rectangle.
Write a fraction to describe the white part of the rectangle.

Solution	**Thinking**
Black part: $\frac{2}{5}$ White part: $\frac{3}{5}$	The rectangle is divided into 5 equal parts. Each part is one fifth. There are two fifths which are black. Three fifths are white. 

Practising and Applying

1. Write the fraction word for each fraction number.

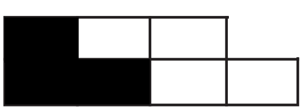
   a) $\frac{2}{3}$ b) $\frac{2}{4}$ c) $\frac{4}{5}$

   d) $\frac{1}{6}$ e) $\frac{3}{7}$ f) $\frac{5}{10}$

2. Write the fraction number for each fraction word.

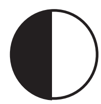
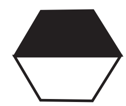
   a) One third b) One fifth
   c) Three thirds d) Two sixths
   e) Two sevenths f) Five eighths
   g) Four ninths h) Six tenths

3. Answer the questions related to the shape below.



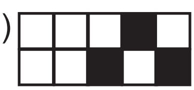
   a) Represent the black part of the shape with a fraction.
   b) What does the denominator tell?
   c) What does the numerator tell?

4. Represent the shaded part of each shape with a fraction.

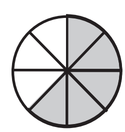
a)  b) 

c)  d) 

e)  f) 

g)  h) 

5. Consider the following shape.



   a) Write a fraction for the shaded part of the shape.
   b) What does the denominator tell?
   c) What does the numerator tell?
   d) Write the complementary fraction for your fraction.

Lesson 2   Fractions as Parts of Sets

Try This

How many fruit items are in the banchung?
How many of them are cherries?
How many of them are apples?

Fractions also describe parts of a set (or group).

For example, we say that $\frac{2}{5}$ (two fifth) of the set of shapes on the right are rectangles. The **numerator** tells the number of items in the set that is being discussed (rectangles in this case), and the **denominator** tells the total number of items in the set (total shapes in this case)

What fraction would describe the traingles in the above set?

Use a fraction to describe the cherries in the above **Try This** problem. What does the numerator in your fraction tell? What does the denominator tell?

Notes

In describing parts of sets with fractions, it is not necessary that the items in the sets be identical. The reason is that it is the numbers represented by the items that are considered equal.

Examples

Example 1 Look at the set of shapes below and answer the questions.

a) What fraction would describe the rhombus?
b) What fraction would describe the black shapes?
c) What fraction would describe the circles in the set?

Solution

a) The fraction that describes the rhombus is $\frac{1}{4}$.

b) The fractions that describes the black shapes is $\frac{2}{4}$.

c) The fraction that describes circles is $\frac{2}{4}$.

Thinking

There are 4 shapes in the set. So the denominators in the fractions will be 4. The numerator in each fraction will depend on what is being considered in each case.

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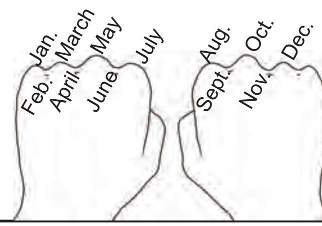
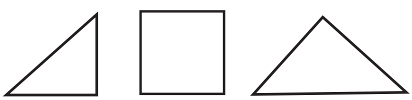
Example 2 What fraction describes the number of months in a year that have 31 days?

Solution	**Thinking**
The fraction that describes the number of months that have 31 days is $\frac{7}{12}$.	I know that there are 12 months in a year. So, the denominator in the fraction should be 12. I said aloud the names of months on knuckles and valleys on my fists from the knuckle of the left hand. I counted 7 months which fall on the knuckles. They all have 31 days.

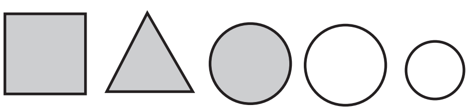


Practising and Applying

1. Write a fraction that describes the set of shapes below. What does the numerator in your fraction tell? What does the denominator tell?



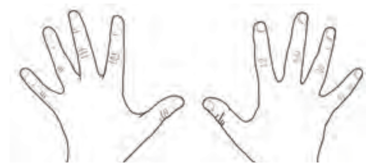
2. What does $\frac{3}{5}$ represent about the set below?



3. What else could $\frac{3}{5}$ represent about the set of shapes in question 2 above?

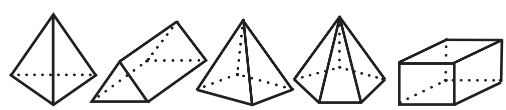
4. Write two fractions that describe the group of children below.

5. What fraction of the fingers on two hands are thumbs?



6. What fraction of the months in the year have 30 days?

7. Look at the set of 3-D shapes and answer the questions.



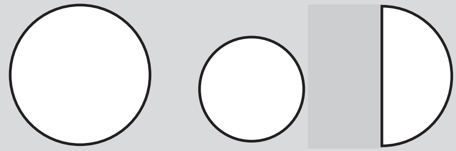
   a) Write a fraction to describe the pyramids.
   b) Write a fraction to describe the prisms.
   c) Why are the denominators in the two fractions the same?

8. Sketch a set of shapes to represent $\frac{2}{7}$.

Lesson 3 Further Work with Fractions

Try This

The semi-circle is a half of one of the circles. Which circle's half is the semi-circle? How do you know?



When we describe a part of a whole with a fraction, we must always know what the whole is. In other words, the part must always be related to its whole. For example, in the above **Try This** problem, the semi-circle is $\frac{1}{2}$ of a circle. But, it is $\frac{1}{2}$ of circle A, and not of circle B.

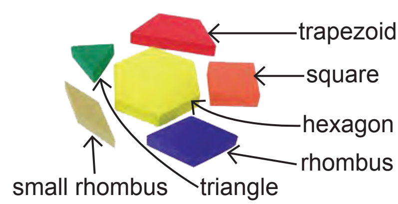
Notes

The word, **semi** means **half**. So, a **semi-circle** means a **half-circle**.

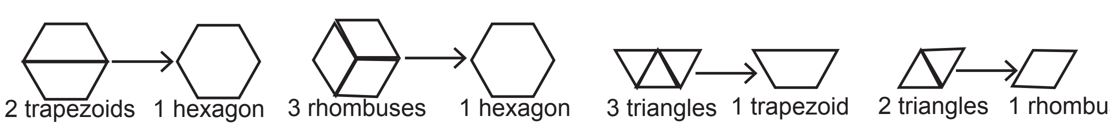
In this lesson, we will mostly use pattern blocks to help you better understand the relationship between a part and a whole. You should be using actual pattern blocks during this lesson.


Pattern blocks

A set of pattern blocks usually has 6 different blocks, as shown and labelled on the right. But, we will use only 4 of them - hexagon, trapezoid, rhombus and triangle, as the square and small rhombus are not suitable for the purpose.



When you play with these pattern blocks, you will realise that it takes 2 trapezoids to make a hexagon, 3 rhombuses to make a hexagon, 3 triangles to make a trapezoid, 2 triangles to make a rhombus, and so on.



So, if a hexagon is 1 whole, then a trapezoid is $\frac{1}{2}$, because it takes 2 trapezoids to make 1 hexagon. For the same hexagon, a rhombus is $\frac{1}{3}$, because it takes 3 rhombuses to make 1 hexagon. Similarly, if a trapezoid is 1 whole, then, a triangle is $\frac{1}{3}$. But, if the whole is a rhombus, then, a triangle is $\frac{1}{2}$.

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Examples

Example 1 If the trapezoid is 1 whole, then, describe each shape below with a fraction. Give reasons.

a) The triangle
b) The rhombus

Solution

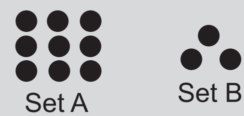
a) The triangle is $\frac{1}{3}$, because it takes 3 triangles to cover 1 trapezoid exactly.

b) The rhombus is $\frac{2}{3}$, because it is the same as 2 triangles.

Thinking

I think 3 triangles will cover 1 trapezoid. exactly. I confirmed this with pattern blocks.

Example 2 If set A is 1 whole, what part of it is set B? Describe set B with a fraction.



Solution

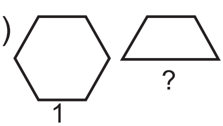
Set B would be $\frac{1}{3}$, because it would take 3 sets of 3 to make a set of 9.

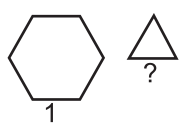
Thinking

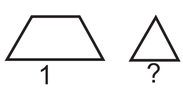
If I divide Set A into small sets that are the same as Set B, then, there would be 3 such sets. So, Set B is one third of Set A.

Practising and Applying

1. Describe the smaller shape with a fraction if the larger shape is 1 whole in each pair of shapes. Give reason for your fractions.

a) 

b) 

c) 

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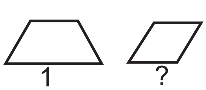
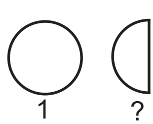
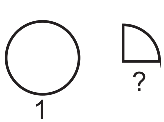
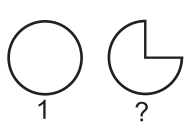
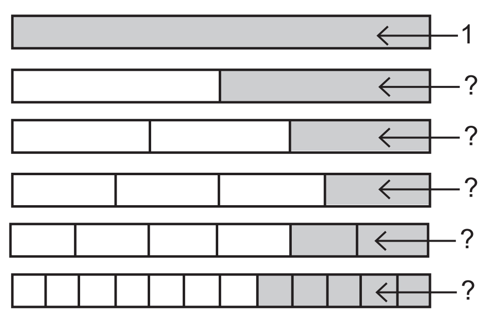
e) 

f) 

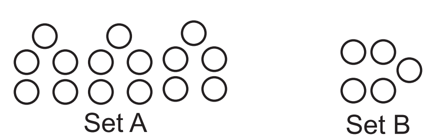
g) 

h) 

2. The completely shaded rectangle is 1 whole shape. Write a fraction for the shaded part in each of the other rectangles.



3. Describe Set B as a fraction of Set A.



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Lesson 4 Mixed Numbers

Try This

If a triangle is 1 whole shape, what number would describe 3 such triangles?

A number that describes a part of a whole shape or a set is called a **fraction**. A number that describes the whole shapes or groups is called a **whole number**.

When a set of whole shapes, which are identical, and a part of a whole are presented together, we can represent them with a **mixed number**, as shown in the examples below.

$1\frac{1}{3}$ $2\frac{1}{3}$ $1\frac{2}{3}$

Note

A **mixed number** consists of a whole number and a fraction.

Example: $2\frac{1}{3}$ Whole number Fraction

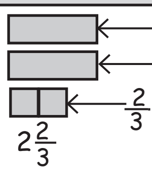
This is how you should say the mixed numbers: $1\frac{1}{3}$ as **one and one third**; $2\frac{1}{3}$ as **two and one third**; and $1\frac{2}{3}$ as **one and two thirds**.

Is the number that you used to describe the 3 triangles in the above **Try This** problem a mixed number? Why?

Examples

Example 1 Sketch a drawing to represent the mixed number $2\frac{2}{3}$.

Solution

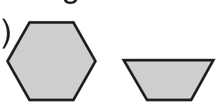


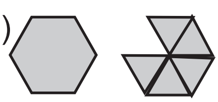
Thinking

I need to show 2 identical whole shapes and two third of a shape. I choose to sketch rectangles for my shapes.

Practising and Applying

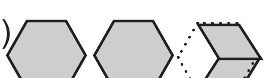
1. Write a mixed number for each set of diagrams.

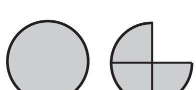
a) 

b) 

c) 

d) 

e) 

f) 

2. Sketch diagrams to represent each mixed number.
   a) $2\frac{1}{3}$ b) $3\frac{1}{4}$
   c) $1\frac{2}{5}$ d) $5\frac{1}{2}$

3. What is wrong with representing $2\frac{1}{2}$ with the following diagram?

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Lesson 5 Decimal Tenths

Try This

Write the next two numbers in the pattern: 10, 100, 1000, _____, _____ .

A fraction with a denominator of 10 or 100 or other **powers of 10** is called a **decimal fraction.** The following are examples of decimal fractions.

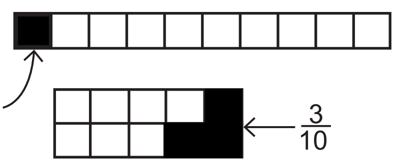
$\frac{1}{10}$	$\frac{7}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{11}{1000}$
one tenth	seven tenths	one hundredth	two hundredths	eleven thousandths

A fraction with a denominator of 10 is called a **decimal tenth fraction**. A fraction with a denominator of 100 is called a **decimal hundredth fraction**. However, in this lesson, we will discuss only the decimal tenth fractions.

Notes

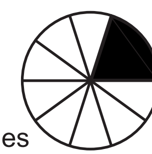
The numbers such as 10, 100, 1000, ... are called **powers of 10**.

Recall that when a whole is divided into 10 equal parts, each part is called one tenth. The following diagrams all show decimal tenth fractions.



$\frac{1}{10}$ $\frac{3}{10}$

$\frac{7}{10}$ of the buttons have 4 button holes



$\frac{2}{10}$

We can write decimal tenth fractions in **decimal forms**, as shown in the table on the right. The names for both a fraction and its decimal form are the same. Both $\frac{3}{10}$ and 0.3 should be read and said as 'three-tenths'

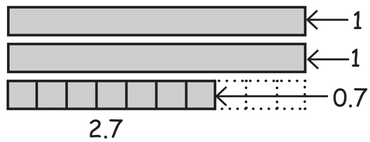
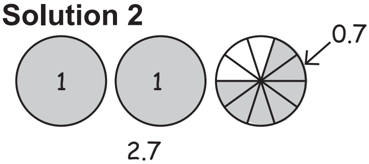
Fraction	Decimal	Name
$\frac{1}{10}$	0.1	one tenth
$\frac{7}{10}$	0.7	seven tenths
$\frac{2}{10}$	0.2	two tenths

Ten tenths, or $\frac{10}{10}$, is the same as 1, or 1 whole. This would be written in its decimal form as 1.0. Twelve tenths, or $\frac{12}{10}$, is the same as $\frac{10}{10}$ and $\frac{2}{10}$. This, in turn, is the same as 1 and $\frac{2}{10}$. This would be written as 1.2 in its decimal form, and should be said as, **one and two tenths**.

The table on the right shows a few decimal tenth fractions which have numerators greater than 10, in their decimal forms, as well as how to say them.

Fraction	Fraction Name	Decimal	Decimal Name
$\frac{11}{10}$	eleven tenths	1.1	One and one tenth
$\frac{16}{10}$	sixteen tenths	1.6	One and six tenths
$\frac{22}{10}$	twenty-two tenths	2.2	Two and two tenths

Example

Example 1 Sketch a diagram to represent the decimal 2.7.	
Solution 1 	**Thinking**  I know that 2.7 is **two and seven tenths**. I sketched 2 rectangles, and seven tenths of a third rectangle. The rectangles should be the same.
Solution 2 	**Thinking** I could also draw 2 circles for the whole number part of the decimal. I shaded the 2 circles. Then, I divided another circle into 10 equal parts, and shaded 7 parts to represent 7 tenths. All the circles have to be the same.

Practising and Applying

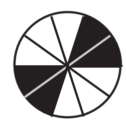
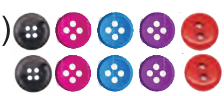
1. Write the decimal number for each.
   a) Two tenths
   b) Seven tenths
   c) One and four tenths
   d) Two and five tenths
   e) Four and one tenth

2. Write the name for each decimal number.
   a) 0.2 b) 0.3 c) 1.3
   d) 1.9 e) 5.4 f) 6.7

3. Write each decimal tenth fraction in its decimal form.
   a) $\frac{1}{10}$ = ___ b) $\frac{2}{10}$ = ___
   c) $\frac{5}{10}$ = ___ d) $\frac{9}{10}$ = ___
   e) $\frac{13}{10}$ = ___ f) $\frac{24}{10}$ = ___

4. Write each decimal in its decimal fraction form.
   a) 0.3 = ___ b) 0.4 = ___
   c) 0.8 = ___ d) 1.3 = ___
   e) 2.1 = ___ f) 3.5 = ___

5. Write a decimal tenth fraction for each model. Explain what each of your fractions means.

   a) 
   b) 
   c) 
   d) 

6. Sketch diagrams to represent the decimals.
   a) 0.2
   b) 1.3
   c) 1.9
   d) 5.4

7. What does 2 tell and what does 5 tell in the decimal number 2.5?

8. Which of these decimal numbers, 0.9 and 0.5 is closer to 1? Why?

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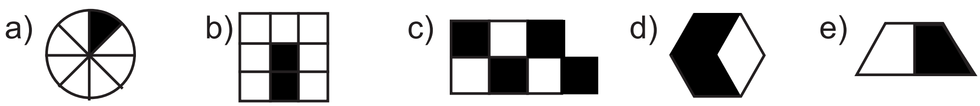
Chapter Review

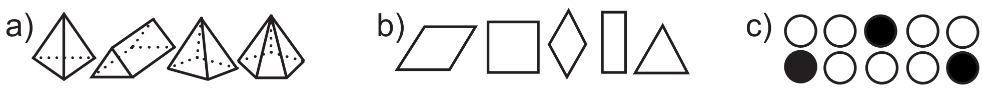
1. Write the fraction name for each.
   a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) $\frac{2}{5}$ d) $\frac{5}{6}$ e) $\frac{6}{10}$

2. Write the fraction number for each.
   a) Three fourths   b) One fifth   c) Four sevenths
   d) Five thirds   e) Twelve tenths   f) Three halves

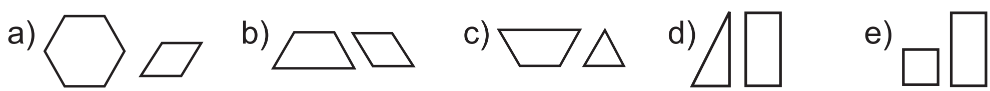
3. Describe the shaded part in each with a fraction.



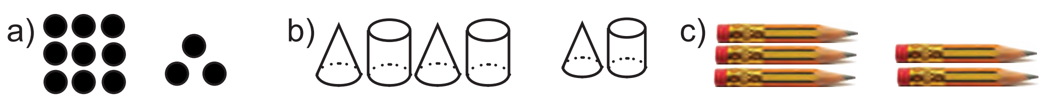
4. Describe each set with a fraction. Explain what your fractions mean.



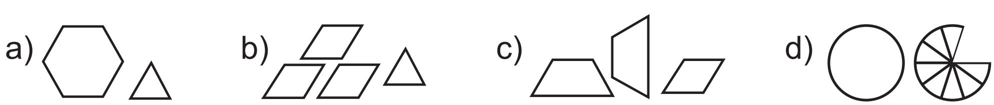
5. Describe the smaller shape as a fraction of the larger shape in each pair.



6. Describe the smaller set as a fraction of the larger set in each pair of sets.



7. Describe each set with a mixed number.



8. Write each decimal tenth fraction in its decimal form.
   a) $\frac{2}{10}$ b) $\frac{3}{10}$ c) $\frac{10}{10}$ d) $\frac{11}{10}$ e) $\frac{15}{10}$

9. Write each decimal tenth in its fraction form.
   a) 0.5   b) 0.1   c) 1.2   d) 1.7   e) 1.0

10. What is a mixed number? Give two examples.

CHAPTER 8 DATA AND PROBABILITY

Chapter Overview

Data is a collection of information. We collect data for many purposes, such as to understand particular situations, to predict future events, to confirm certain assumptions, and to help make decisions.

We can collect data through various means, such as observation, interviews, questionnaires, polls and surveys. After data has been collected, it has to be organised and presented. Then, we analyze the data to understand it, and use it for various purposes. One way of organising and presenting data is by making graphs. Graphs are very helpful, because they can display data visually and are easy to interpret quickly.

Probability is the study of the chance of something happening. It helps in predicting an event occurring in the future. Generally, we base our prediction on the pattern of what has already happened.

You have had some experiences with data and probability in your earlier classes, such as collecting simple data, organising them with tally marks, interpreting and creating concrete graphs, pictographs and bar graphs, conducting probability experiments, and using probability terms. We shall review these concepts and experiences for you in this chapter. Additionally, you will learn to interpret and create pictographs and bar graphs with scales in this chapter.

This chapter has 8 lessons as detailed in the Table of Contents.

Basic Principles about Data and Probability

- Data is collected for various purposes.
- There are various ways to collect data.
- Once the data has been collected, there are various ways to organise and present the data.
- Graphs are powerful data displays since visual displays are easy to interpret quickly.
- Pictographs and bar graphs are used to compare frequency within

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categories.

- When a pictograph is created, it is important for a single symbol to be used and the symbol must be lined up properly.
- When a bar graph is created, the bars should be separated and should start at the same base line.
- Scales for graphs are used when the values are large enough that it would be awkward to create a graph where one element of the graph represents one actual item.
- Probability is about predicting the chances of an event happening.
- We use data to predict the likelihood or chance of an event happening.
- Although we can predict the likelihood of an event, it is only a prediction. We do not really know whether something will actually happen or not.

Chapter Goals

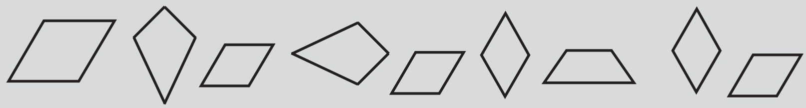
By the end of this chapter, you will be able to:

- Plan a survey, collect data and organise the data using tables and tally marks.
- Describe a set of data, either from a collection or a presentation.
- Examine and interpret a given pictograph with a scale.
- Create a pictograph with a suggested scale.
- Examine and interpret a given bar graph with a scale.
- Create a bar graph with a suggested scale.
- Use probability terms, such as likely, unlikely, possible, impossible, and certain, to predict future events.
- Conduct simple probabilty experiments and predict future events based on the experimental results collected.

Lesson 1 Collecting and Organising Data

Try This

Sort these quadrilaterals into sets. What is your sorting rule? How many sets did you make?



> **Notes**
>
> Recall that a **quadrilateral** is a polygon with 4 sides.

Data is a collection of information. For example, the set of numbers below is data on the ages of students in a class. Answer the following questions based on it:

What is the youngest age in the class?
What is the oldest age?
How many students are 7 years old?
Which age is the most common?
How many students are in the class?

5	6	7	7	6
6	7	6	5	8
5	6	6	6	6
6	7	6	6	
	6		8	6

If data is not organised it could be difficult to understand. But, if you organise data, it helps you understand it better.

The above data is not organised. It then takes a longer time to answer the questions. One way to organise the data is to make a tally chart, as shown on the right. Your teacher will explain how to make a tally chart.

Ages of students in a class

Age	How many student?
5	|||
6	𝍸 𝍸 |||
7	||||
8	||

Now, answer the above questions, once again, by referring to the tally chart. Discuss and describe the difference you felt in answering the questions earlier and now.

Name the quadrilaterals in the above **Try This** problem, and make a tally chart as a way to sort or organise them?

> **Notes**
>
> Sorting is a way of organising data.

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The example on the previous page, with the data on the ages of students in a class, demonstrated two things: how to organise data, and the benefits of organising data.

But, you should also know how to collect data in the first place. In this part of the lesson, you will learn to collect data by asking questions to your classmates. To collect data by asking questions, you have to remember to do the following things:

- You should first decide on what question to ask your friends. Your question has to be clear to you and others. Write down the question, and ask it to a few friends and your teacher to see that the question is clear and correct.

- The question should also have a choice of answers for your friends to choose from. There should be only a few choices. (As a rule, there should not be more than 4 options.)

- You should also decide on a format to record the responses, such as using a tally chart. This will be helpful for organising your data, as you collect it.

- Then, ask your questions to your friends, one by one, and record their responses, or answers, in your data recording format.

Write a question to ask your classmates to find out something about them. Use the above guideline in writing your question. You may write your question based on one of the ideas suggested below. Write the question in your notebook.

- the number of persons living in their house
- the number of sisters they have
- the number of brothers they have
- the number of hours of TV they watch at home
- their favourite fruit
- their favourite pastime
- the time they take to walk to school
- their age
- the number of times they brush their teeth a day

Then, ask your question to each of your classmate, and record their responses.

Lesson 2 Describing Data

Try This

Tell two things about the data on the right.

Weather during the last 2 weeks (recorded by Dorji)	
Sunny	卌 卌 |
Cloudy	|||
Rainy	

The purpose of collecting data is to use it in some ways. You should be able to describe or talk about a data that you have collected or that is presented to you. So, in this lesson, you will practise doing that. For example, let us, first,discuss the data given below.

Weather during the last 2 weeks (recorded by Sonam)	
Sunny	||
Cloudy	|||
Rainy	卌 ||||

The data shows that it was rainy for much of the last 2 weeks. Out of 14 days, it rained for 9 days, it was cloudy for 3 days, and sunny for only 2 days. It is likely that it will rain over the next several days too. So, it would be advisible for Sonam and people at the same place to take their umbrellas when they go out. It looks like the time that Sonam recorded the weather was during summer. Could you say anything more about the data?

Now, tell some more things about the data in the above **Try This** problem.

Now, look at the data you have collected during the previous lesson and describe it. You could do that in the following manner.

Pair up with a friend next to you. Decide who will talk first. If you are the first to go, talk to your friend of what your data is about. Then, describe what your data tells. Encourage your friend to ask questions when he or she is not clear about what you said. After you have finished talking about your data, two of you should reverse your roles.

Then, write down a few things that your data tells in your notebook.

Finally, you should verbally share with the whole class about your data.

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Lesson 3 Interpreting Pictographs with a Scale

Try This

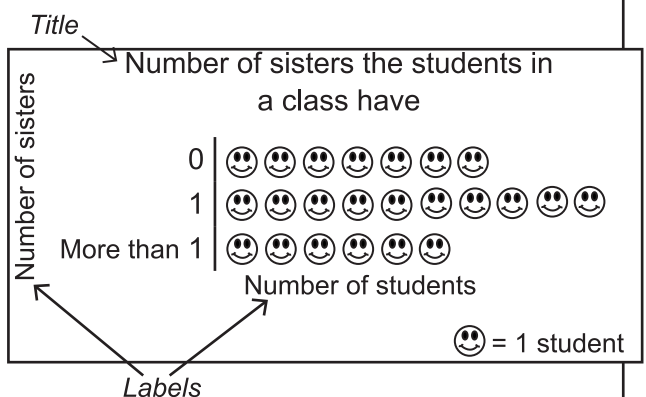
If one counter represents 2 persons (● = 2 persons), how many persons does each set of counters below represent?

a) ● ● b) ● ● ● c) ● ● ◖

A graph is a visual display of data. It helps in understanding data at a glance, especially in comparing various groups of information within the data set.

A pictograph is a graph that uses a picture or a symbol to represent the number of times a data item is repeated. In this lesson, you will learn to interpret pictographs.

What information does the pictograph on the right show? How many students have 0 sisters? What is the title of this graph? What are its labels?



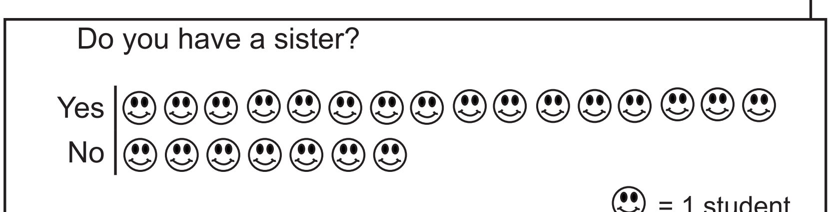
As you can see, there are 23 students in the class. If someone asked these students the question, “Do you have a sister?” and recorded the responses one by one, the data would look similar to that one shown on the right.

Do you have a sister?

Yes	卌 卌 卌 |	16
No	卌 ||	7

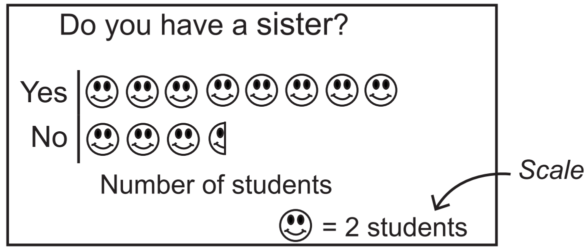
Below is a pictograph representing the data for the question “Do you have a sister?”

In this pictograph, the row of symbols for “Yes” is quite long. And, if there were more students, the row would be even longer. If the row becomes very long, it could create some difficulty in reading the graph quickly.



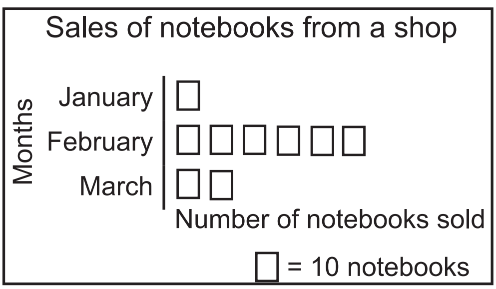
When such a case arises, we use an appropriate scale for the graph. In the earlier graphs, 1 symbol represents 1 student (☺ = 1 student). Now, let us use the scale, ☺ = 2 students, and create the pictograph as shown below.

This graph and the graph before it look quite different, but they show the same data on the question, "Do you have a sister?" Which of the two graphs looks better? In what ways?



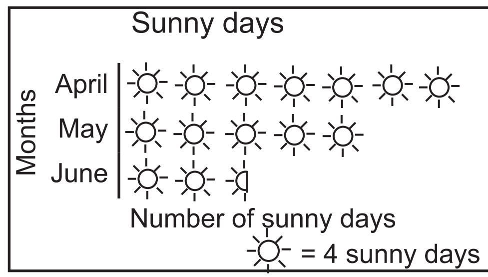
Practising and Applying

1. Look at the pictograph below, and answer the questions.



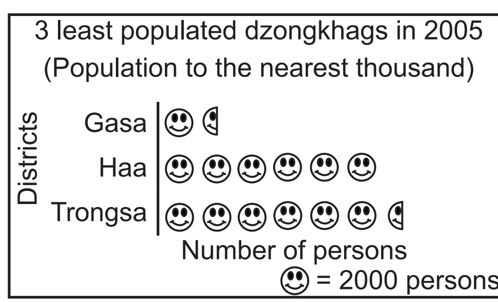
   a) In which month did the shop sell the most notebooks?
   b) How many notebooks were sold in March?
   c) How many notebooks were sold during the 3 months?
   d) What could be the reason for selling the most notebooks in February?
   e) What is the title of this graph?

2. Purna kept a record of the sunny days for 3 months, and created a pictograph, as shown above on the right.



   a) What scale is used in this graph?
   b) How many days were sunny in April?
   c) How many days were sunny in June?

3. Answer the questions on the pictograph below.



   a) How many people were there in Gasa dzongkhag?
   b) How many more people were there in Trongsa than in Haa?

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Lesson 4 Creating Pictographs with a Scale

Try This

Represent 15 persons with less than 6 counters. Draw your counters. How many persons does 1 counter represent?

In this lesson, you will be creating pictographs for the sets of data provided. The following are the basic points that you should remember while creating your pictographs.

- Your pictograph should have a title, labels, a symbol, and a scale.
  Recall that a title tells what the graph is about, labels tell the category of data and the number of times the data items are repeated, a symbol represents the data items, and a scale tells how many of the data items are represented by 1 symbol.

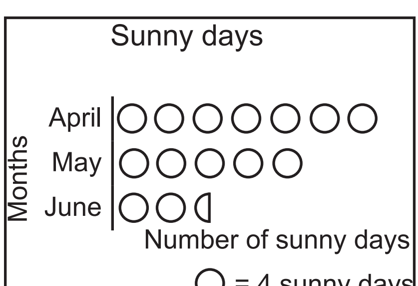
- You should choose a convenient scale depending upon the numbers in your data.

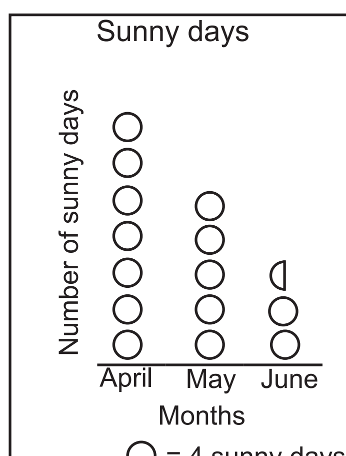
- The symbols representing the data items for each type of data should all start from the same baseline.

- The symbols should be of uniform size, and should be lined up in one-to-one correspondence.

- You do not have to draw a specific sketch for the symbols. For example, the symbol for sunny days does not have to be a picture of the sun. You could simply draw circles for it.

- You could line up the symbols either horizontally or vertically, as demonstrated here for a same set of data.



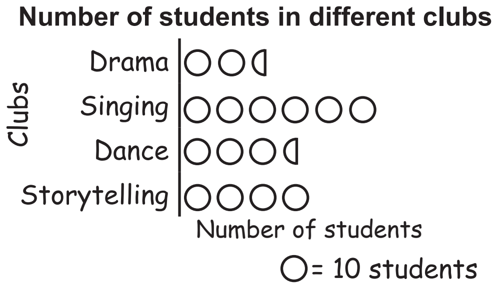


Examples

Example 1 The data below shows the number of students in different clubs in a school. Make a pictograph for the data.

Clubs	Drama	Singing	Dance	Storytelling
Number of students	25	60	35	40

Solution



Thinking

The greatest number is 60, and looking at the other numbers, I decided to choose a scale of 1 symbol to represent 10 students. I drew circles for my symbols.

I created my pictograph horizontally. I made sure to write the title, the labels, and the scale in my graph.

Practising and Applying

1. Create a pictograph for each of the following data sets.
   a) Number of girls and boys in a class

      Girls: 16
      Boys: 20

   b) Number of girls and boys in a school

      Girls: 120
      Boys: 115

   c) Number of students in a primary school

      Class PP: 20
      Class 1: 18
      Class 2: 18
      Class 3: 17
      Class 4 : 10
      Class 5: 13
      Class 6: 13

2. The data below shows the favourite colours of students in a class. Create a pictograph for it.

Colour	Number of students
Blue	8
Green	5
Red	10
Purple	6
Yellow	4

3. Create a pictograph for the data given below.

Weather during the last 2 weeks (recorded by Yangden)	
Sunny	𝍸 |||
Cloudy	||||
Rainy	||

4. Create a pictograph for the data you have collected about your classmates during lesson 1.

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Lesson 5 Interpreting Bar Graphs with a Scale

Try This

Write the next three numbers in each sequence:

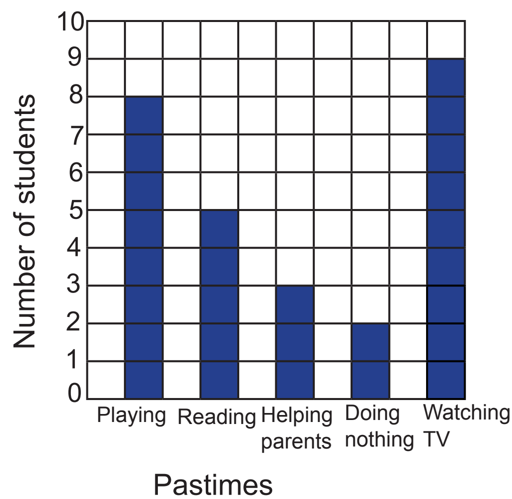
2, 4, 6, 8, ...
4, 8, 12, 16, ...
5, 10, 15, 20, ...

A bar graph uses bars to show and compare the numbers in different categories of information in a data set. In this lesson, you will learn to interpret bar graphs.

Let us interpret the bar graph on the right.

As the title of it says, the bar graph tells us about the pastimes of students of a class on Sundays. From the 5 pastimes, watching TV is the most popular among the students of the class. Eight students play, five students read,three students help their parents, and two students enjoy doing nothing on the sundays.

Pastimes of students of a class on Sundays



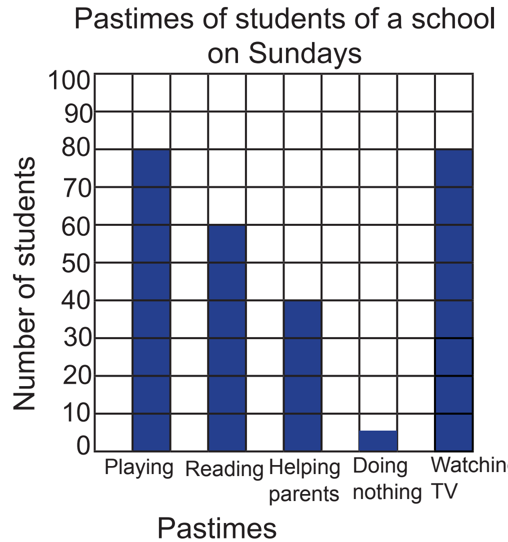
What is your own pastime on Sundays?

There are 27 students in the class. You can tell that by adding the numbers represented by each bar.

As you can see, each square in the grid represents 1 student in the above graph. Now, if we were to create a bar graph for the pastimes of all the students in the school, in the same way, the bars would become very long or tall, because the number of students would be much greater. We should, then, use a scale, so that we can fit the number of students with the available grid. For example, the bar graph on the next page uses **a scale of 10**, which means 1 square of the grid represents 10 students.

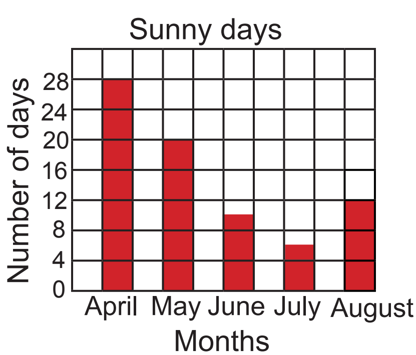
The bar graph on the right shows about the pastimes of students in a school. Answer the follwing questions.

Which two pastimes are the most popular among the studens?
How many students enjoy reading?
How many students does 1 square represent?
How many students do nothing?
How many students are there in the school?
What is the title of the bar graph?



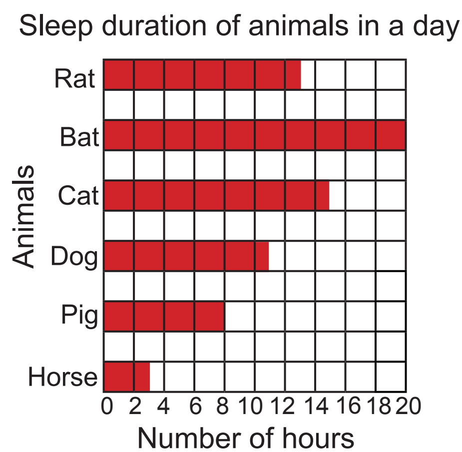
Practising and Applying

1. Answer the questions about the bar graph below.



   a) What is the title of the graph?
   b) What scale does the graph use?
   c) How many sunny days were in May?
   d) How many sunny days were in July?
   e) Do you expect more sunny days in September than in July? Why?

2. Bar graphs can also be horizontal, as shown above on the right. Answer the questions based on it.



   a) Which animal sleeps for the longest time in a day?
   b) What scale does the graph use?
   c) How many hours does a pig sleep in a day?
   d) How many hours does a horse sleep in a day?
   e) How many more hours does a cat sleep than a dog in a day?
   f) About how many hours do you sleep in a day?

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Lesson 6 Creating Bar Graphs with a Scale

Try This

What does it mean when you say a bar graph uses a scale of 10?

In this lesson, you will be creating bar graphs. You should remember the following points while creating a bar garph.

- A bar graph should have a title, labels, and a scale.
- All the bars should start from the same baseline
- The spaces between the bars should be the same.
- The bars could be either horizontal or vertical.
- You should use square grids to create your bar graphs, to help you make the bars properly.

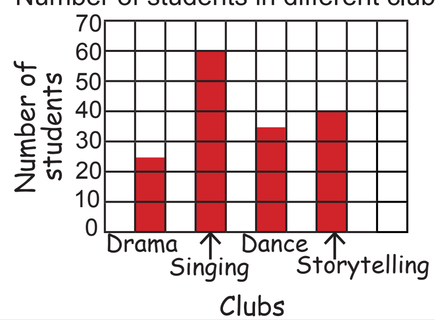
Example

Example 1 The data below shows the number of students in different clubs in a school. Make a bar graph for it.

Clubs	Drama	Singing	Dance	Storytelling
Number of students	25	60	35	40

Solution 1

Number of students in different clubs



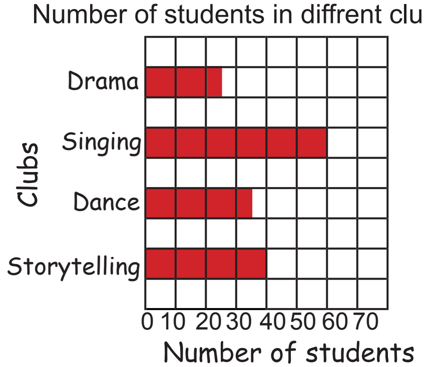
Thinking

I chose a scale of 10. I made my bar graph vertically.

I wrote the title and the lables for my graph as shown.

Solution 2

Number of students in diffrent clubs



Thinking

I could also make my graph horizontally, using a similar grid paper. The title, labels and the scale are the same, except that the positions of the two labels are interchanged.

Practising and Applying

1. Create a bar graph for each of the following data sets.

   a) Number of girls and boys in a class

      Girls: 16
      Boys: 20

   b) Number of girls and boys in a school

      Girls: 120
      Boys: 115

   c) Number of students in a primary school

      Class PP: 20
      Class 1: 18
      Class 2: 18
      Class 3: 17
      Class 4 : 10
      Class 5: 13
      Class 6: 13

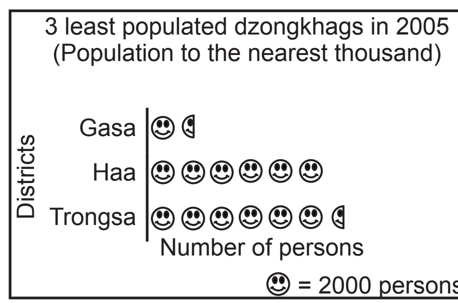
2. The data below shows the favourite colours of students in a class. Create a bar graph for the data using a scale of 2.

Colour	Number of students
Blue	8
Green	5
Red	10
Purple	6
Yellow	4

3. Create a bar graph for the data given below.

Weather during the last 2 weeks (recorded by Yangden)	
Sunny	~~||||~~ |||
Cloudy	||||
Rainy	||

4. The pictograph below shows the population of 3 dzongkhags in 2005. Change the pictograph into a bar graph.



5. The data below shows the number of people in Bhutan who lived with various disabilities in 2005. The numbers are rounded to the nearest thousand. Make a bar graph using a scale of 1000.

Disability	Number of persons
Seeing	6000
Hearing	9000
Speaking	4000
Moving	4000
Mental	1000

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Lesson 7 Using Probability Language

Try This

Answer each question with a ‘yes’, ‘no’, or ‘maybe’. Give reasons for each.

a) Can you turn yourself into a tiger?
b) If you put your finger in fire, would it burn?
c) Will you slip and fall on the ground when you walk home today?

Probability is the study of the chance of something happening. It helps in predicting an event occurring in the future. In this lesson, you will learn to describe your prediction of a future event using five words.

If we are sure that an event will take place in the future, then we say that the event is **certain**. For example, it is certain that the sun will rise tomorrow.

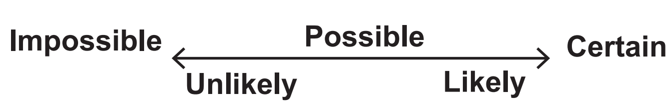
If we are sure that an event will not take place in the future, then we say that the event is **impossible**. For example, it is impossible for us to live forever.

If we think that an event may or may not take place, then we say that the event is **possible**. For example, it is possible that you will become a millionaire.

We use the word **likely** when we think that the chances of something happening is higher than it not happening. For example, it is likely that many people will eat dinner today, as usual.

We use the word **unlikely** when the chances of something happening is lower than it not happening. For example, it is unlikely that a teacher will cheat his or her students.

We can write these five **probability words** graphically as shown below. The words *unlikely* and *likely* are parts of *possible*, with *unlikely* toward the direction of *impossible* and *likely* toward the direction of *certain*.



Describe each question in the above **Try This** section with one of these five probability words.

Example

Example 1 Describe the probability of each result when you roll a die once, using a probability word. *(The plural of die is dice. For example: 1 die, 2 dice.)*

a) You will get number 6 on the top.

b) You will get a number greater than 1 on the top.

c) You will get number 7 on the top.

Solution

a) **Unlikely**, because the chance of getting 6 is less as compared to getting any of the 5 other numbers on the die.

b) **Likely**, because there are 5 numbers which are greater than 1 on the die. So, the chances of getting one of them is very high.

c) **Impossible**, because I will never get number 7 when I roll the die, as it does not exist on the die.

Thinking

I could also use the word **possible** for a) and b), but **unlikey** and **likely** are more precise.

Practising and Applying

1. If you call one side of a coin **head**, then the other side is called **tail**. Describe each result of flipping a coin once using a probability word (possible, certain).

   a) It will land with head on top.
   b) It will land with either head or tail on top.

2. Describe the probability of each result when you roll a die once using a word (likely, unlikely, impossible, or certain). Explain your choice of word.
   a) You will get a number from 1 to 6 on the top.
   b) You will get a number greater than 6 on the top.
   c) You will get a number that is less than 5 on the top.
   d) You will get a number that is less than 2 on the top.

3. Describe the probability of each event below using a word (likely, unlikely, impossible, or certain). Explain your choice of word.
   a) Tomorrow will be a sunny day.
   b) When you drop a nail in water, it will sink.
   c) When you heat ice, it will melt.
   d) When you put salt in water and stir it, the salt will not dissolve.
   e) Every person in Bhutan is happy.
   f) You will become a movie star in the future.
   g) It will snow this winter in your school.

4. Describe something for which the probabiltiy is **impossible**.

5. Describe something for which the probabiltiy is **certain.**

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Lesson 8 Conducting Probability Experiments

Try This

What probability word would describe the chance of getting both the head and the tail on the top when you toss a coin?

Conducting an event again and again and recording its result each time is called an experiment. Doing experiments helps you to improve your predictions and/or confirm them.

In this lesson, you will perform some probability experiments. You must actually use the following things to do the experiments: coins, dice, coloured snap cubes, and spinners.


coins


dice


snap cubes


spinners

Carry out the following experiments, in pairs or small groups.

Experiment 1: Get a coin. Designate one side of it as head, and the other side as tail. Make a tally chart as shown below.

a) *Make a prediction:* If you toss the coin 20 times, about how many times will it land with its head on the top? Explain.

b) *Perform the experiment:* Toss the coin 20 times, and record the result each time in the tally chart.

c) How close was your prediction to the result?

d) If you toss your coin one more time, could you be sure of which side it will land? Why?

Heads	Tails

Experiment 2: Get a die. Make a tally chart as shown here.

a) *Make a prediction:* If you roll the die 20 times, about how many times will you get a number that is less than 3 on the top? Explain.

b) *Perform the experiment:* Roll the die 20 times, and record the result each time in the tally chart.

c) How many times did you get a number that is less than 3?

d) How many times did you get a number that is more than 3?

Less than 3	More than 3

Experiment 3: Get some snap cubes and a container. Put 1 black cube, 2 red cubes and 7 blue cubes in the container and mix them up thoroughly. (The three colours, black, red, and blue are examples only; you could use any three colours with the same numbers.)

a) *Make a prediction:* About how many times do you think you will draw out each colour (black, red, and blue), if you draw out a cube from the container without looking, put it back, mix the cubes again, and repeat the process for 20 times? Write your prediction in a table as shown on the right.

Prediction table

Black cube	Red cube	Blue cube

b) *Perform the experiment:* Draw out a cube from the bag without looking. Record the result in a tally chart. Put back the cube, and mix them thoroughly. Repeat the process 20 times, and record the results in the tally chart.

Tally chart to record your experimental results

Black cube	Red cube	Blue cube

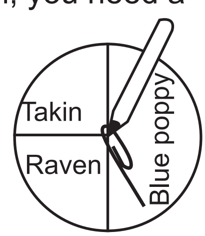
c) How many times did you draw out each colour? How do these results compare with your predictions?

d) What probability word would describe the chance of drawing out a black cube?

e) What probability word would describe the chance of drawing out a blue cube?

f) What probability word would describe the chance of drawing out a white cube?

Experiment 4: Make a spinner like the one shown here. You can make it by drawing a circle and dividing it into three parts as shown. Then, you need a pencil and a paper clip to use it.



a) *Make a prediction:* About how many times do you think you will spin blue poppy, if you spin the spinner 20 times? Explain.

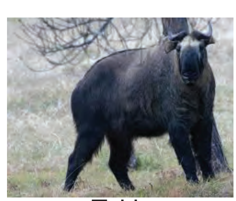
b) *Perform the experiment:* Spin the spinner 20 times, and record the results each time in a tally chart.

c) How close is the result for blue poppy to your prediction?

d) What probability word describes best the probability of spinning blue poppy? Why?

Takin	Raven	Blue poppy


Blue poppy


Takin


Raven

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Chapter Review

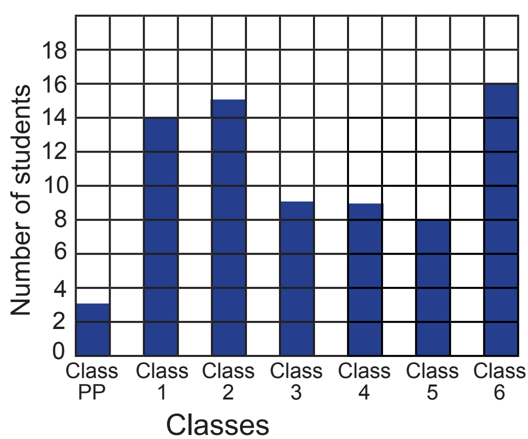
1. The students in a class were asked to write their ages on the board, one by one, which were written as shown below.

10	11	10	9	10	10	
11	11	10	9	10	10	
11	10	10	10	10	10	
11	10	10	9	9	9	10
9	11	9	11	11	10	

   a) Organise the data using a tally chart.
   b) Create a pictograph using a scale of 2.
   c) Create a bar graph on a square grid using a scale of 2.

2. The bar graph below shows the number of students in a primary school in 2013.

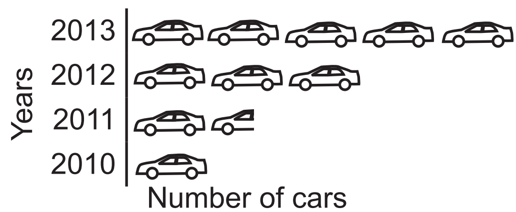
Number of students in Bemji Primary School in 2013



   a) What is the title of the graph?
   b) What scale does the graph use?
   c) How many students were there in class 6?
   d) How many students were there in the school altogether?
   e) How many students would you predict to be in class 1 in 2014? Explain your answer.

3. The pictograph shows the number of cars in a town over some years.

Number of cars in a town



Each  represents 10 cars.

a) What do you notice about the number of cars in the town over the years?
b) How many cars does one  represent?
c) How many cars were there in 2011?
d) What could be making the number of cars increase over the years?

4. Daw Tenzin wants to roll a die once. He wonders what number he will get. Describe the probability of each result with a word (*impossible, certain, likely, unlikely*). Explain your choice of words.

a) He will get number 1.
b) He will get one of these numbers: 1, 2, 3, 4, 5, 6.
c) He will get one of these numbers: 1, 2, 3, 4, 5.
d) He will get a number that is more than 6.

A die

5. Pushpalal Chhetri wants to toss a coin once. He wonders how the coin will land. Describe the probability of each result with a word (*impossible, certain, likely, unlikely*). Explain your choice of word.

a) The coin will land with either the head or the tail on its top.
b) The coin will land with both its head and tail on the top.
c) The coin will land on its edge.

A coin

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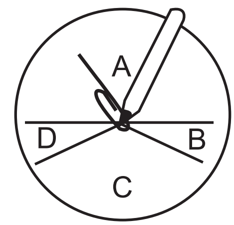
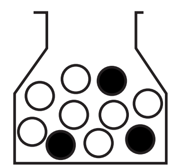
6. Eden Dema wants to spin the spinner (shown below) once. She wonders what letter she will spin. Describe the probability of each result with a word (*impossible, certain, likely, unlikely*). Explain your choice.

   a) She will spin A.
   b) She will spin B.
   c) She will spin one of these letters: A, B, C, or D.
   d) She will spin E.



7. There are ten counters in a container. Three of them are black and seven are white. If you draw a counter without looking, what is the probability of each?

   a) You will draw a black counter.
   b) You will draw a white counter.
   c) You will draw either a black or a white counter.
   d) You will draw a purple counter.



CHAPTER 9 MEASUREMENT: MASS, CAPACITY AND AREA

Chapter Overview

The **mass** of an object is the amount of matter in it. How heavy or light an object is depends on its mass. An object which is heavier has more mass than an object which is lighter.

The **capacity** of a container is the amount of something it can hold or contain. How much a container can hold depends on the space it has inside. A container which has more space inside has greater capacity than a container which has less space inside.

The **area** of a shape is the amount of surface it covers. The area of a shape depends on its size. A larger shape has a greater area than a smaller shape.

Measurement involves comparison. For example, when we measure and say that the mass of an object is 5 kg, we are, in fact, saying that it is 5 times heavier than an object which is 1 kg.

You have had some experiences with measurement concepts and skills such as measuring length, mass, capacity, time, and angle in your previous classes and in chapter 6 of this class. In this chapter, we will review these ideas and skills related to measuring mass, capacity and area, and extend them to using new units of measurement including grams, millilitres and square centimetres.

This chapter has 8 lessons as detailed in the Table of Contents.

Basic Principles about Mass, Capacity and Area

- Any measurement comparison can be stated in two ways. For example, if A is more than B, it means that B is less than A.
- Whether an object has greater mass, capacity, or area depends on different features than shape alone. For example, a long thin rectangle and a small square rectangle could have the same area.
- It is useful to have standard units to measure mass, capacity, and area to make communication clearer and easier.
- The units used for measurements should be appropriate to the contexts.

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- It is useful to have both large and small units so that a convenient unit can be selected for a particular situation.

Chapter Goals

By the end of this chapter, you will be able to:

- Measure the mass of objects and express them in kilograms (kg) and grams (g), and combinations of these two units.
- Convert kilograms to grams and vice versa.
- Measure the capacity of containers and express them in litres (L) and millilitres (mL), and combinations of these two units.
- Convert litres to millitres and vice versa.
- Measure the areas of various shapes in non-standard units.
- Measure the areas of various shapes in square centimetres.
- Estimate the areas of various shapes in square centimetres.
- Solve simple problems involving mass, capacity and area.

Lesson 1 Measuring Mass in Kilograms

Try This

About how many kilograms of mass can you lift? How do you know?

The **mass** of an object is the amount of matter in it. How heavy or light an object is depends on its mass. An object which is heavier has more mass than an object which is lighter.

The mass of an object does not depend on its size. For example, a block of stone, about the size of a pumpkin, would be heavier than a sack of wool. This is because the stone has more mass than the sack of wool. Even though the sack of wool is much bigger in size than the stone, it has lots of open spaces in between, whereas the matter in the stone is compact.

We use a **pan balance** (also called **common balance**) to measure the mass of things.

Common balances

As you have learnt in class 2, we measure mass in a unit called **kilogram** (**kg**).

There are objects, usually in blocks of iron, which come with a fixed mass, such as 1 kg, 2 kg, and 5 kg. You would find these in shops and stores. These blocks are useful in measuring the masses of things

5 kg 2 kg 1 kg

To measure a specific mass of something, say 5 kg of rice, we place a 5 kg iron mass on one pan of the balance. Then we put rice on the other pan until its mass equals the 5 kg on the first pan. It is important to note that the beam of the balance must be horizontal for the masses on the two pans to be the same.

There are also different machines to measure mass. They are called **weighing machines**. When you place a thing on the appropriate part of a machine, it shows the mass with numbers.

If possible, bring some common balances and weighing machines to the class, and experiment measuring the mass of various things. Alternatively, you could visit some shops to see how shopkeepers measure mass.

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It is important for you to become familiar with different masses, and to be able to estimate a mass. In order to do that, you should experience lifting objects to feel how heavy they are. Then you should measure their masses using pan balances and/or weighing machines.

For example, you should know and be able to describe about how heavy 1 kg is, how heavy 2 kg is, how heavy 5 kg is, how heavy 10 kg is, and so on. For instance, a *banchungful* of rice would be about 1 kg.

Many things that are sold in the market come in packets that show the mass of the contents. For instance, sugar, salt, butter, cheese and dalda come in 1 kg packets. Sacks of rice come in 5 kg, 25 kg, or 50 kg. Look for the mass on labels of various things in the market place or at home, and feel the masses.

Example

Example 1 Six balls of cheese together have a mass of 1 kg. What will be the total mass of twelve such cheese balls?

Solution	**Thinking**
○○○○○○←1 kg ○○○○○○←1 kg So, 12 balls = 2 kg.	I drew 6 circles in a row to represent 6 cheese balls. Then, I drew another row of 6 cheese balls. That is a total of 12 cheese balls. If 1 row of cheese balls represents 1 kg, then 2 rows would be 2 kg. 

Practising and Applying

1. Name an object that would have each of the following masses:
   a) A mass of about 1 kg
   b) A mass of less than 1 kg
   c) A mass of about 5 kg
   d) A mass of more than 10 kg

2. Five maths textbooks, which are all the same, together have a mass of 1 kg. What will be the mass of fifteen such textbooks altogether? Draw sketches.

3. A cheese maker makes uniform cheese blocks. One kg of cheese consists of 4 blocks. How many blocks will be needed for 3 kg of cheese? Use multiplication to solve this, and draw sketches.

4. The body mass of babies are measured and recorded at the hospitals. A baby weighed 3 kg at the time of birth. She weighed 5 kg after one month. By how many kg has the baby grown?

Lesson 2 Measuring Mass in Grams

Try This

Five identical mugs together have a mass of 1 kg.
How would you describe the mass of one mug in kg, using a fraction?

1 kg

Just like there are more than one unit for measuring length, such as kilometre, metre, centimetre, and millimetre, there are many units for measuring mass. However, we shall use only one of them beside the kilogram during this lesson. This unit is called **gram.** The short form for gram is **g**.

A gram is a very small unit of mass. You can hardly feel the weight of one gram on your palm. A gram is one thousandth of a kilogram. This means it takes 1000 grams to make 1 kilogram. (1000 g = 1 kg, or 1 kg = 1000 g)

There are standard masses made of metals such as 1 g, 2 g, 5 g, 10 g, 50 g, and 100 g. Because these are very small masses, they are usually contained in boxes, as shown here.

You should feel how heavy each of these masses is, and gain a sense of them.

You should experience measuring the masses of light objects in grams, using pan balances and/or weighing machines, and become familiar with their approximate masses. For example, each of the following things would have a mass of about 1 gram: a paper clip, a pen cap, an empty envelope, and a needle.

Many things that come in packets, such as toothpastes, soaps, cheese, butter, facial creams, glue sticks, and biscuits have their mass labeled on them in grams. So, the next time you see such things look for the labels showing their masses. If possible, bring some of them to the class to show to each other and to discuss.

How many grams is the mass of each cup in the above **Try This** problem? How do you know?

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Examples

Example 1 A packet of wheat flour weighs 2 kg. How many kg would half of the packet weigh? How many grams is that?

Solution	**Thinking**
Half of the wheat flour would weigh 1 kg. That would be 1000 g.	Half of 2 kg is 1 kg. I know that 1 kg = 1000 g.

Example 2 Add 700 g + 600 g. Show how you added. Write your sum as a combination of kg and g.

Solution	**Thinking**
$\begin{array}{r} 700 \\ +\ 600 \\ \hline 1300 \end{array}$ 1300 g = 1 kg 300 g	I used column addition to add 700 g and 600 g. The sum is 1300 g. I know that 1300 g is 1000 g and 300 g, or 1 kg and 300 g.

Practising and Applying

1. Name an object that would have each of the following masses:
   a) A mass of about 1 g
   b) A mass of more than 1 g and less than 100 g
   c) A mass of about 500 g
   d) A mass of about 1000 g

2. Estimate the mass of each of the following objects in grams. If possible, measure each mass, and compare with your estimates.
   a) An apple
   b) A tomato
   c) A chilli
   d) Your maths textbook
   e) Your eraser
   f) Your pencil
   g) Your toothbrush
   h) One of your shoes

3. Write each mass in grams.
   a) 1 kg
   b) 2 kg
   c) $\frac{1}{2}$ kg
   d) 1 kg 200 g
   e) 2 kg 300 g
   f) $1\frac{1}{2}$ kg

4. Write each mass in kg.
   a) 1000 g
   b) 5000 g
   c) 500 g
   d) 1500 g

5. Add each, and write your sum as a combination of kg and g.
   a) 700 g + 400 g
   b) 800 g + 600 g
   c) 850 g + 150 g
   d) 973 g + 73 g
   e) 884 g + 424 g
   f) 1500 g + 2100 g

6. A soap bar has a mass of 250 g. How many such soap bars would make 1 kg?

Lesson 3 Choosing an Appropriate Mass Unit

Try This

Could you use kilogram as a unit to measure the mass of a pen? Why or why not?

Although we could use either the kilogram or the gram as the unit to measure and describe any mass, sometimes it is better to use one unit over the other, depending upon the situation.

For example, it is more convenient to describe the mass of a pencil in gram, rather than in kilogram. That is because a pencil is a light object, and describing its mass in gram would give a comfortable number. The mass of a pencil might be about 20 g. The number to describe the same mass in kilogram is 0.02 kg, which is not as nice and convenient as 20 g.

Similarly, it is more convenient to measure and describe the mass of heavy objects in kilogram. For example, the body mass of an adult person would be about 70 kg. The same mass, in terms of gram, would have to be a large number, as 70,000 g.

Discuss what unit you would choose to measure and describe the mass of each of these things: a notebook, a spoon, a dog, a beetle, and yourself.

Practising and Applying

1. Sonam was collecting information on the average body mass of various animals, and wrote the following. But, he forgot to write the units. Write the missing unit (g, kg) for each.

Animal	Average body mass
Cow	465
Goat	28
Elephant	2547
Hen	2000
Pigeon	300
Horse	521
Mouse	23

2. Complete each statement using one of the mass units (g, kg).

   a) A sheep weighs about 56 ___.
   b) A rabbit weighs about 2500 ___.
   c) A rat weighs about 200 ___.
   d) A crow weighs about 341 ___.
   e) A baby weighs about 4 ___.
   f) A button weighs about 25 ___ .
   g) Six apples weigh about 1 ___.
   h) A car weighs about 1000 ___.

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Lesson 4   Measuring Capacity in Litres

Try This

Sonam drinks a mug of tea every morning during breakfast. Does she drink more than or less than one litre of tea? How do you know?

The **capacity** of a container is the amount of something it can hold or contain. How much a container can hold depends on the space it has inside it. A container which has more space inside has more capacity than a container which has less space inside.

Which has more capacity: the mug or the cup? Why?

As you have learnt in class 2, we measure capacity in a unit called **litre (L).** A mineral water bottle that you normally see has a capacity of 1 L. This means it holds 1 L of water. An Amul Taaza milk packet that you might find in shops also has a capacity of 1 L. Some fruit juices also come in bottles of 1 L. Check the labels on such containers.


1 L of water

1 L of milk


1 L of juice

You should become familiar with various containers that have capacity of 1 L.

You should also become familiar with containers of various capacities in litres such as 5 L, 10 L, and 20 L. One way to gain such familiarity is to actually measure the capacities of various containers by pouring water into or out of them using a container of 1 L. The other way is to read the labels, as many containers have their capacities written on them in litres, such as jerry cans, buckets, water filters, water bottles, water boilers, rice cookers, and water tanks.

It is a good mathemtical habbit for you to always estimate the capacities before you measure them. This will improve your estimation skills. For example, take out some containers such as a pot and a bucket, and and estimate the capacity of each in litres. Then, actually measure their capacities.


A pot


A bucket

Example

Example 1 One litre of juice can fill exactly five glasses of the same capacity, for five persons.

a) What fraction of a litre is the capacity of 1 glass?

b) If you have to fill 10 glasses of the juice, how many litres of juice would be needed?

1 L of juice

Solution

a) *Capacity of 1 glass =* $\frac{1}{5}$ *L*

b) *10 glasses would need 2 L of the juice.* | $10 \div 5 = 2$

Thinking

It takes 5 glasses to make 1 litre. So the capacity of 1 glass would be one fifth of a litre.

If 1 litre can make 5 glasses of juice, it would require 2 litres to make 10 glasses. I used division to solve this.

Practising and Applying

1. Name a container that would have each of the following capacities:
   a) 1 L
   b) More than 1 L
   c) Less than 1 L
   d) About 5 L
   e) More than 5 L
   f) More than 20 L
   g) About $\frac{1}{2}$ L

2. A pot can be filled with one and a half bottles of water. The bottle has a capacity of 1 L. What is the capacity of the pot?

Pot

Bottle

3. A teapot, which has a capacity of one and a half litres, can fill six identical mugs with tea. What is the capacity of the six mugs altogether?

4. One litre of milk can fill exaclty four identical mugs.
   a) What is the capacity of the 4 mugs altogether?
   b) What is the capacity of 1 mug?
   c) How many such mugs can 3 litres of milk fill?

5. A container can hold 1 L of milk.
   a) How many litres of water can the container hold?
   b) How many litres of oil can the container hold?

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Lesson 5   Measuring Capacity in Millilitres

Try This

Two identical containers together have a capacity of 1 litre.
How would you describe the capacity of one container in litres?

Just like we use a small unit of mass, called gram, to measure small masses, we also use a small unit called **millilitre (mL)** to measure small capacities.

One millilitre is one thousandth of a litre. This means it takes 1000 millilitres to make 1 litre (1000 mL = 1 L, or 1 L = 1000 mL).

A millilitre is a very small amount. A cap of a mineral water bottle would hold about 5 mL of water. A spoon might hold about 7 mL of water.   A drinking glass may hold about 200 mL. A mineral water bottle holds 1000 mL of water, because it is the same as 1 L.

Many small containers, sold in the market have labels showing their capacities in millilitres, such as inkpots, juice containers, drinking water bottles, body lotion containers, shampoo containers, and liquid medicine containers. You should read the labels on such containers so that you gain a better sense of various capacities in millilitres. If possible, bring some containers to the class for sharing and discussion.

Some containers, such as beakers,show millilitre markings at regular intervals on them. If possible, your teacher will bring them to the class, for you to use to measure various amounts of water in millilitres.You should be careful, in handling them if they are made of glass.

Read the capacities of containers such as these.

Beakers

How many millilitres would be the capacity of one container in the above **Try This** problem? How do you know?

Examples

Example 1 A container holds 6 L of water. How many litres of water can half of the container hold? How do you know? How many millilitres is that?

Solution	Thinking
Half of the container holds 3 L of water, because 3 is half of 6. That would be 3000 mL.	Half of 6 L is 3 L. I know that 1 L = 1000 mL. So 3 L = 3 x 1000 = 3000 mL.

Example 2 Add 600 mL + 700 mL. Show how you added. What is the sum in mL? Write the sum as a combination of L and mL.

Solution	Thinking
The sum = 1300 mL $\quad \begin{array}{r} 600 \\ +\ 700 \\ \hline 1300 \end{array}$ 1300 mL = 1000 mL + 300 mL = 1 L + 300 mL = 1 L 300 mL	I used column addition to add: 600 plus 700 is 1300. So the sum is 1300 mL, which is the same as 1 L 300 mL.

Practising and Applying

1. Name a container that would have each of the following capacities:
   a) About 5 mL
   b) About 200 mL
   c) About 500 mL
   d) About 1000 mL
   e) About 2000 mL

2. Estimate the capacity of each of the following containers in mL. If possible, measure each capacity, and compare against your estimates.
   a) A mug
   b) A ladle
   c) A spoon
   d) A cup
   e) Your mouth

3. Convert each into mL.
   a) 1 L
   b) 2 L
   c) $\frac{1}{2}$ L
   d) 1 L 200 mL
   e) 2 L 500 mL
   f) $1\frac{1}{2}$ L

4. Convert each into L.
   a) 1000 mL
   b) 5000 mL
   c) 1500 mL
   d) 500 mL

5. Add each, and write your sum as a combination of L and mL.
   a) 700 mL + 400 mL
   b) 800 mL+ 600 mL
   c) 850 mL + 150 mL
   d) 973 mL + 73 mL
   e) 884 mL + 424 mL
   f) 1500 mL + 2100 mL

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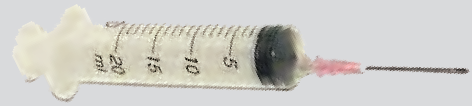
Lesson 6 Choosing an Appropriate Capacity Unit

Try This

Could you use litre as a unit to measure the capacity of this syringe? Why or why not?



Although we could use either the litre or the millilitre as a unit to measure and describe the capacity of any container, sometimes it is better to use unit one over the other, depending upon the situation.

For example, it is more convenient to describe the capacity of a small container, such as a syringe, in millilitre rather than in litre. This is because that would give us a comfortable number. The capacity of a syring would be about 20 mL. The same capacity in litre would be 0.02 L, which is not as comfortable a number as 20 mL.

Similarly, it is more convenient to measure and describe the capacity of large containers in litre than in millilitre. For example, the capacity of a big jerry can would be about 20 L. The same capacity in millilitre is 20, 000 mL, which is a very large number, and so not as comfortable a number as 20 L.

Discuss what unit you would choose to measure and describe the capacity of each of these things: a cup, a tub, an ink pen, a water tank, and a rice cooker.

Practising and Applying

1. Poonam was collecting the capacities of various containers, and wrote the following. But, she forgot to write the units. Write the missing unit for each as either L or mL.

Container	Capacity
A body lotion container	250
The fuel tank of a car	35
A mineral water bottle	1000
A bucket	20
A mineral water bottle	1
An ink bottle	60
A syringe	15

2. Complete each statement using one of the capacity units (L, mL).
   a) A water tank contains about 1000 ___ of water.
   b) A cooking oil container contains about 10 ___ of oil.
   c) A cough syrup bottle contains about 100 ___ of cough syrup.
   d) A bottle cap holds about 5 ___ of water.
   e) A mug holds about 230 ___ of tea.
   f) A jug holds about 1000 ___ of water.

Lesson 7 Measuring Area in Non-standard Units

Try This

Which shape has the greater area?
How do you know?

A B

Area is the amount of surface a shape covers. In this lesson, we will review your experiences of measuring areas in non-standard units as you have done in your earlier classes.

The following are some points that you should remember while measuring the area of a shape with non-standard units:

- You should choose a unit which is neither too big nor too small for the area.
- The units should be uniform in size and shape.
- The units should fit on the shape without leaving gaps in between.
- When it is not possible to cover an area with an exact number of the unit chosen, you could describe it using an approximate language. For example, “the area of the tabletop is **about** 12 notebooks”.

You should actually measure and describe the area of various shapes, such as your books, geometry boxes, handkerchiefs, tabletops, chalkboard, classroom door, room floors, and even your palms and feet after choosing some appropriate units for each. You could use pattern blocks as units for measuring small areas, and notebooks and chart papers for the larger areas.

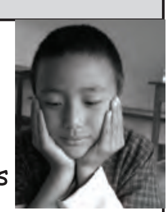
Example

Example 1 Sonam and Reshma want to measure the area of a book together. Sonam says they could use circular counters to cover the book, while Reshma suggests to go for square pattern blocks. If they ask you for a suggestion, which one would you suggest? Why?

Solution

I would suggest that they use the square pattern blocks, as they can arrange these without leaving gaps in between. It is not possible to lay the circular counters without leaving gaps.

Thinking



I know that there should not be gaps between the units to measure an area. This is not possible with circular counters.

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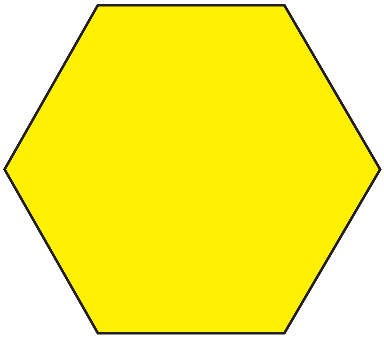
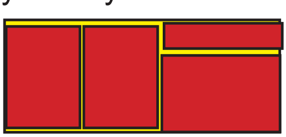
Practising and Applying

1. Use pattern blocks to measure the area of the shape below, and complete the sentences.

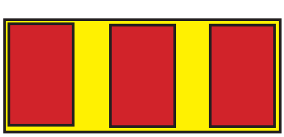


   a) The area of the shape is ___ traingles.
   b) The area of the shape is ___ trapezoids.
   c) The area of the shape is ___ rhombuses.
   d) The area of the shape is ___ hexagon.

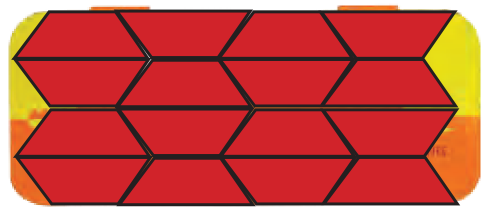
2. Norbu measured the area of a shape by placing some rectangles on it while trying to leave no gaps in between. He says that the area is about 4 rectangles. Is he correct? Why or why not?



3. Tek Bahadur placed identical rectangles on Norbu's shape, and says that the area of the shape is 3 rectangles. Is Tek correct? Why or why not?



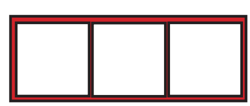
4. Yangchen covered her geometry box with trapezoids, as shown below, and said, "the area of the top of my geometry box is about 16 trapezoids".



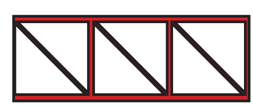
   a) Yangchen is reasonable in her statement. Why is that?
   b) Would the area of the geometry box be slightly more than 16 trapezoids? How do you know?

5. Measure the area of one of your palms using an appropriate unit, and describe it.

6. Tashi says that the area of a rectangular shape is 3 squares. Karma says it is 6 traingles. Who is correct? How do you know?



This is how Tashi measured the area



This is how Karma measured the area

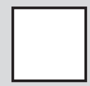
7. Suppose you know that the area of a shape is 10 squares, and that the area of another shape is 7 triangles. Would you know which shape has the greater area? Explain.

Lesson 8 Measuring Area in Square Centimetres

Try This

Use a ruler to measure and describe the sides of the larger rectangle in centimetres.

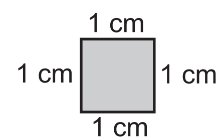
Use a ruler to measure and describe the sides of the smaller rectangle in centimetres.



How many of the smaller rectangles will cover the larger rectangle exactly? How would you confirm this?

Just like in the cases of measuring mass and capacity, we need standard units for measuring area that are understood the same way by people all over the world. In this lesson, we will use one standard unit for area. It is called a **square centimetre**. The short form for square centimetre is **sq. cm**.

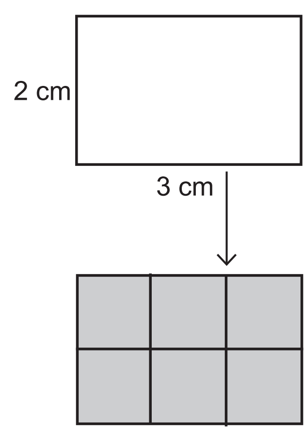
As you know, a rectangle which has all four of its sides equal in length is called a square. A square with sides of 1 cm is called a **square centimetre.** This is what we mean by an area of 1 square centimetre.



1 square centimetre

Make and cut out a square of 1 cm (1 square centimetre) on paper. If possible, use stiff paper. Make several copies of it (about 15 of them).

Draw a rectangle of sides **2 cm by 3 cm**. Place the cutout squares on it without leaving gaps. How many such squares cover the rectangle exactly? How would you describe the area of the rectangle? (The area of the rectangle is **6 square centimetres.**)



Make a rectangle of sides 5 cm by 3 cm. Repeat the above process and describe its area in terms of square centimetres.

Using the same process, describe the area of the rectangle in the above **Try This** problem.

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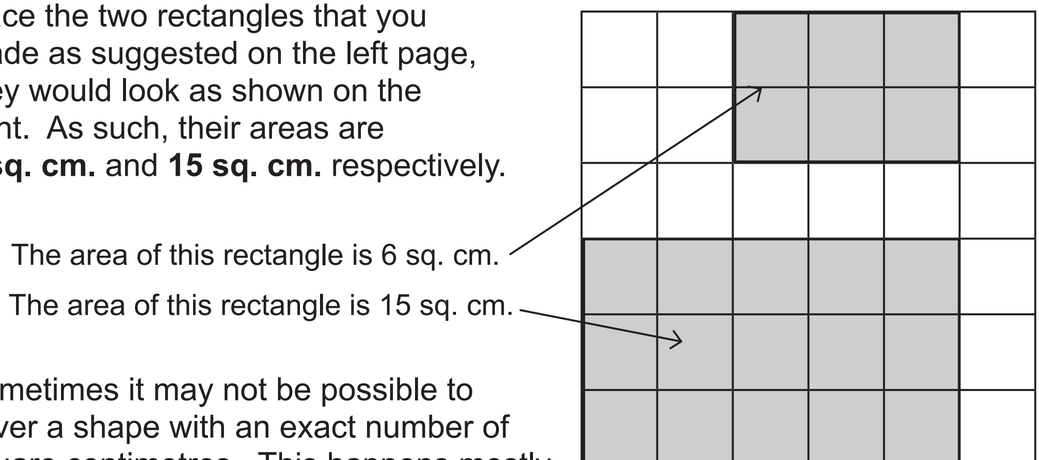
Placing the cutout 1 square centimetres on the shapes every time to measure their areas would be tedious. To overcome this, you could use **centimetre grid paper**. A centimetre grid paper shows lines of rows and columns that are 1 cm wide, as shown on the back of this page. You should have a copy of centimetre grid paper. If possible, you should have it on a transparent sheet of paper or plastic. You could make a sheet of paper with the centimetre grid transparent by smearing some oil lightly on it.

To measure the area of a shape, you can either place the shape on the grid or place the grid on the shape, and count the number of square centimetres the shape covers. For example, if you place the two rectangles that you made as suggested on the left page, they would look as shown on the right. As such, their areas are **6 sq. cm.** and **15 sq. cm.** respectively.

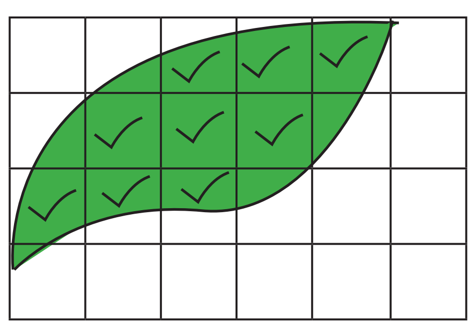


A centimetre grid

Sometimes it may not be possible to cover a shape with an exact number of square centimetres. This happens mostly with **irregular shapes,** but it could also happen with r**egular shapes.** When this happens, we take an approximate number of square centimetres for the area. Here is a way to do this is: Count all the squares that are totally within the shape, plus the squares that are more than half within the shape, and leave out the rest of the squares.

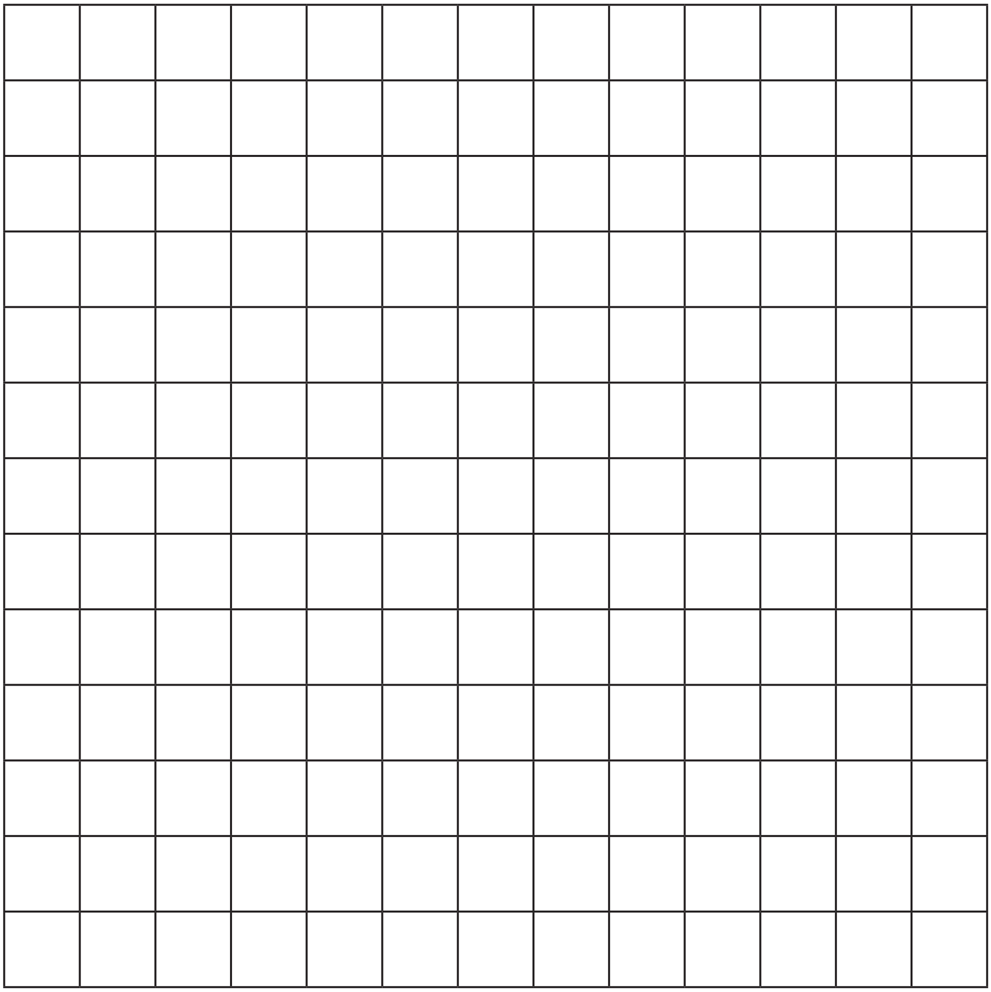
For example, the area of the shape on the right is measured in the following way:

- Place it on the centimetre grid.
- Put a tick mark on each of the squares that is either totally or more than half within the shape.
- Count all the tick marks.



The area of the shape is about 9 sq. cm.

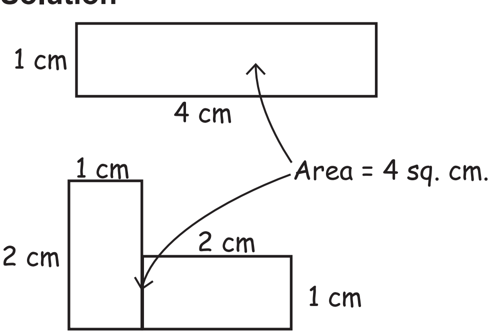
A centimetre grid



Example

Example 1 Draw two shapes that have the same area of 4 sq. cm.

Solution



Thinking

I arranged 4 cutouts of 1 square centimetre alongside one another. That is like drawing a rectangle of 1 cm by 4 cm, as shown in the first drawing.

Next, I rearranged the 4 cutouts and drew a shape like the one shown in the second drawing.

I labelled the lengths of the sides of shapes.

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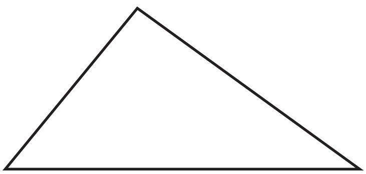
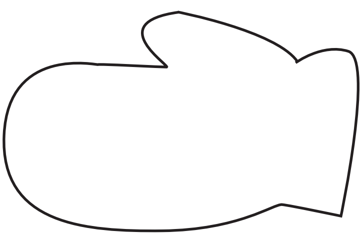
Practising and Applying

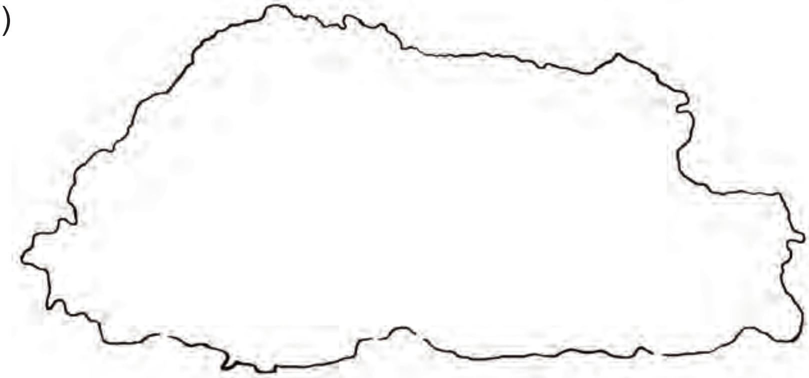
1. Use centimetre grid paper to measure the area of each shape, and describe them in sq. cm.

   a) 

   b) 

   c) 

   d) 

   e) 

2. Draw a shape that has an area of 7 sq. cm.

3. Draw two shapes each having an area of 10 sq. cm.

4. Draw an irregular shape that has an area of about 9 sq. cm.

5. About how many square centimetres will cover the roof of your geometry box? How would you confirm this?

6. About how many square centimetres will cover the base of a mineral water bottle? How will you find that out?

7. Is it clearer to say that something has an area of 10 sq. cm than simply saying that it has an area of 10 squares? Explain.

Chapter Review

1. Give an example of an object for each of the following masses:

   a) about 1 kg b) about 5 kg c) about 500 g d) about 50 g

2. About how many kilograms is your body mass? Do you think it will increase over the next few months? Why?

3. Fill in the blanks. Show your work. (1 kg = 1000 g)

   a) 2 kg = ______ g b) 5 kg = _____ g c) 1 kg 100 g = _____ g
   d) 1000 g = ____ kg e) 500 g = ____ kg f) 3500 g = ____ kg _____ g

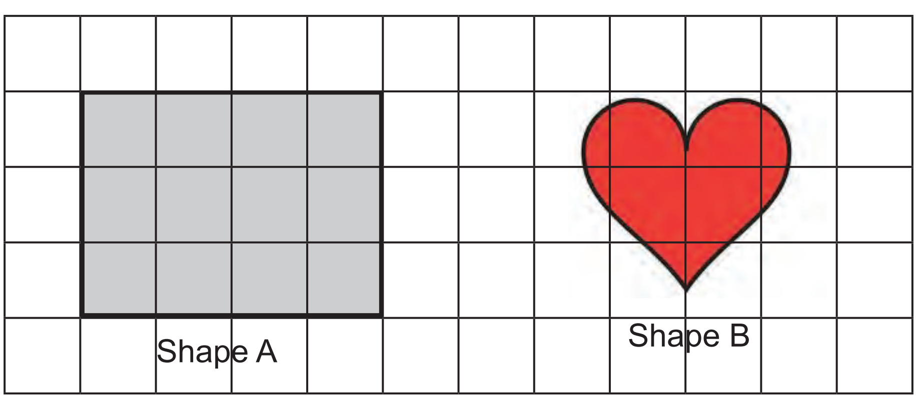
4. Give an example of a container for each of the following capacities:

   a) about 1 L b) about 5 L c) about 15 mL d) about 200 mL

5. Fill in the blanks. Show your work. (1 L = 1000 mL)

   a) 2 L = ______ mL b) 5 L = _____ mL c) 1 L 100 mL = _____ mL
   d) 3000 mL = ____ L e) 2300 mL = ____ L _____ mL

6. What is the area of shapes A and B.



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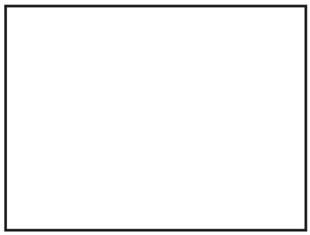
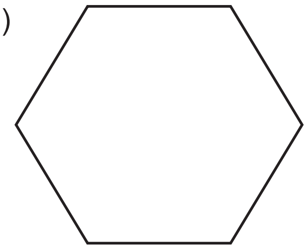
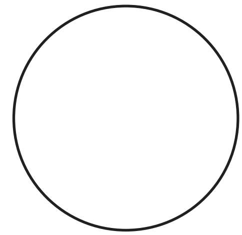
7. Use centimetre grid paper to measure and describe the area of each shape below:

a) 

b) 

c) 

8. Draw three shapes each having an area of 9 sq. cm.

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CHAPTER 10 DZONGKHA AND ROMAN NUMERALS

Chapter Overview

Numbers show how much or how many. We write numbers with numerals. There are ten numerals in English. They are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. We can write any number, however large or small, with these ten numerals. We write a large number such as *seven thousand two hundred sixty-three* with numerals as 7263, and a small number, a fraction, such as *3 tenths* as either 0.3 or $\frac{3}{10}$.

Just like some languages use different alphabets from other languages, they also use different numerals. For example, as you know, the alphabets for the dzongkha language are called *Seljed Sumchu* and *Yang Zhi*. In this chapter, you will learn to write numbers in Dzongkha numerals and Roman numerals.

It is important for you to learn the Dzongkha numerals as Dzongkha is our national language, and we use it in our daily lives. You would have already learnt them in the Dzongkha subject. In this chapter we will relate them to the English numerals.

Rome is the capital city of Italy. A long time ago, the Roman Empire was powerful. It ruled over many parts of the world. It was advanced in the knowledge and application of science, mathematics, language, arts, and other ideas. Although the conditions in the world have changed now, we still use some of the ideas and practices that originated in there, including the Roman numerals. For example, the pages for this textbook, from the beginning to the page before chapter 1 is marked in Roman numerals. Many books use them in a similar manner. You will also find Roman numerals used in other situations, such as for the numbers on the faces of clocks and watches, in sciences (later on for you), in the titles and production years of movies, and in the newspapers. So, it is important for you to be able to recognise, read and write Roman numerals.

This chapter has 2 lessons as detailed in the Table of Contents.

Basic Principles about Dzongkha and Roman Numerals

- Some languages have their own unique alphabets and numerals.

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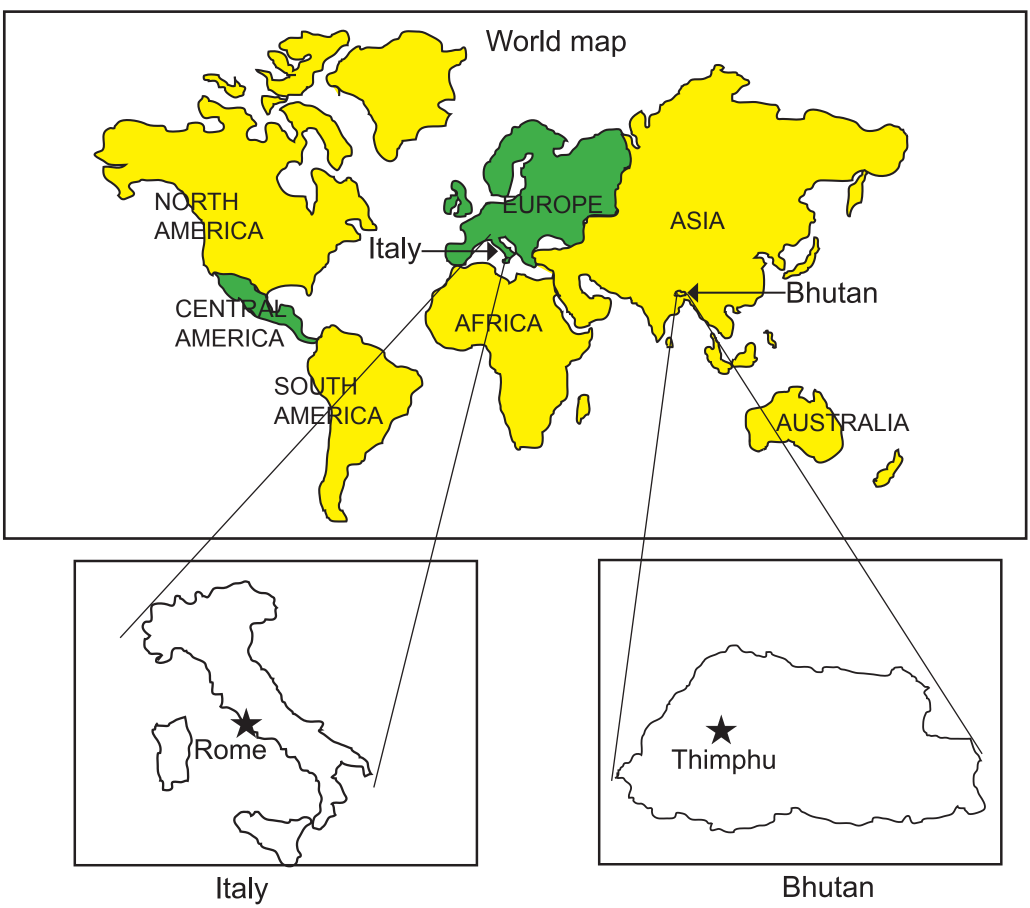
- It is important to learn the Dzongkha numerals and the Roman numerals as they are both used in the lives of the Bhutanese people, besides the English numerals.

Chapter Goals

By the end of this chapter, you will be able to:

- Understand that the Dzongkha and the English languages use the same system of writing numbers.
- Read and write numbers in Dzongkha numerals for their English counterparts.
- Read and write Dzongkha number words in English up to number ten.
- Read and write numbers in the Roman numerals up to number thirty.



Lesson 1 Dzongkha Numerals

Try This

Write the following numbers in Dzongkha numerals.

a) Three b) Twenty-five c) One hundred sixty-seven

Just as with English numerals, there are ten Dzongkha numerals. They are ༡ ༢ ༣ ༤ ༥ ༦ ༧ ༨ ༩ and ༠.

In fact, the way we write numbers in Dzongkha is exactly the same as we write them in English. This is because both follow the same place value system.

The table on the right shows numbers in Dzongkha numerals and words along with those in English from zero to nine.

Look at the numerals, and discuss which numerals are similar and which are different between the Dzongkha and the English.

English		Dzongkha	
Numeral	Word	Numeral	Word
0	zero	༠	lekor
1	one	༡	chi
2	two	༢	nyi
3	three	༣	sum
4	four	༤	zhi
5	five	༥	nga
6	six	༦	dru
7	seven	༧	duen
8	eight	༨	gay
9	nine	༩	gu

Practising and Applying

1. Write the numbers from one to fifty in Dzongkha numerals.

2. Write the number words (in English) for the following numbers.

   a) ༡༥ b) ༢༣༦

   c) ༨༠༤ d) ༤༧༡༠

   e) ༩༩༠༠ f) ༥༠༤༢

3. Write the following numbers in both English and Dzongkha numerals.

   a) Fifty-three
   b) Two hundred thirty-four
   c) Six hundred eight
   d) Seven hundred twenty
   e) One thousand one hundred ten
   f) Three thousand ninety-two
   g) Five thousand two hundred four

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Lesson 2 Roman Numerals

Try This

If 1 is represented by one counter, how many counters will represent 2?

1 ⟶ ● 2 ⟶ how many counters?

The Roman numerals consists of the letters (they are usually capital letters) I, V, X, L, C, D and M.

I is for 1, V is for 5, X is for 10, L is for 50, C is for 100, D is for 500, and M is for 1000. Other numbers are written by combining these numerals appropriately. However, we will use only the combinations of I, V, and X to write numbers up to 30 in this lesson. There is no Roman numeral for zero.

The tables below show the numbers from one up to twenty written using both English numerals and Roman numerals.

Number word	English number	Roman number
one	1	I
two	2	II
three	3	III
four	4	IV
five	5	V
six	6	VI
seven	7	VII
eight	8	VIII
nine	9	IX
ten	10	X

Number word	English number	Roman number
eleven	11	XI
twelve	12	XII
thirteen	13	XIII
fourteen	14	XIV
fifteen	15	XV
sixteen	16	XVI
seventeen	17	XVII
eighteen	18	XVIII
nineteen	19	XIX
twenty	20	XX

If you observe the above Roman numerals, there is a clear pattern in them. The numerals for 1, 2, and 3 are respectively I, II and III. For 4, an I is placed before V (for 5) as IV. For 6, an I is placed after V as VI. For 7, two Is are placed after V as VII. For 8, 3 Is are placed after V as VIII. But, for 9, since it is just before X (for 10), an I is placed in front of X as IX. The pattern continues in this manner up to XIX for 19. For 20, it is two Xs for two 10s as XX.

The pattern would continue after XX (for 20) as XXI for 21, and so on. 30 would be written as XXX.

Example

Example 1 How would you write the following numbers in Roman numerals? a) 25 b) 29	
Solution a) 25 = XXV b) 29 = XXIX	**Thinking** I know that for 25 I have to first write XX for 20, and then write V to make it XXV for 25. For 29, I have to first write XX for 20, and then IX for 9 to make it XXIX for 29. 

Practising and Applying

1. Write the numbers from one to nine in Roman numerals.

2. Write the numbers from ten to twenty in Roman numerals.

3. Write the number words for the following numbers.

   a) XXI
   b) XXII
   c) XXIV
   d) XXVI
   e) XXVII
   f) XXVIII
   g) XXIX
   h) XXX

4. Write the following numbers in Roman numerals.

   a) one
   b) five
   c) ten
   d) fifteen
   e) three
   f) seven
   g) twenty-five
   h) twenty-six
   i) thirteen
   j) twenty
   k) nineteen
   l) seventeen
   m) eleven
   n) two
   o) nine

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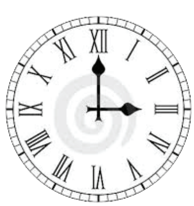
Chapter Review

1. Write the following numbers in all three three different types of numerals (English, Dzongkha, and Roman).

	Number word	English numeral	Dzongkha numeral	Roman numeral
a)	one			
b)	two			
c)	three			
d)	four			
e)	five			
f)	ten			
g)	eleven			
h)	fifteen			
i)	twenty			
j)	twenty-seven			
k)	twenty-nine			
l)	thirty			

2. What times do the following clocks show?

a) 

b) 

c) 

3. Write the number *Two thousand fourteen* in both English and Dzongkha numerals.

4. Write your birth year in both English and Dzongkha numerals.

5. Write your age in Roman numerals.

CHAPTER 11 PATTERNS

Chapter Overview

When there is a systematic and regular happening of something over and over again, there is a pattern. If you can see a pattern in something, you can predict the elements in a pattern.

There are many patterns in the natural world. For example, the day and the night always alternate. The day follows the night, and the night follows the day in a cyclic manner. Because of this pattern, you know that if it is day now, it will be night next. You can also see a pattern in the way lightning and thunder occur. It is always the lightning that you see first just before you hear the thunder in the sky.

You may also see patterns in a person's behaviour. If you have known a person for a long time, you usually know how he or she behaves or acts under certain situations.

There are many patterns in the mathematics. There are patterns in shapes and numbers. There are patterns in addition, subtraction, multiplication and division facts. For instance, you know that when you multiply a number with 0, the product is always 0.

Observing patterns is a very useful mathematical skill. You can learn this skill. After recognising a pattern, you should be able to describe it.

You have had some experience with the concept of patterns in your earlier classes. We will review and extend these experiences in this chapter. There are basically three types of patterns. They are repeating patterns, growing patterns, and shrinking patterns.

This chapter has 3 lessons as detailed in the Table of Contents.

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Basic Principles about Patterns

- There are patterns all around in nature.
- There are patterns in mathematical concepts.
- Recognising a pattern helps in predicting the elements in it.
- There are basically three types of patterns: repeating, growing, and shrinking patterns.
- Pattern discovery is a very important mathematical skill.

Chapter Goals

By the end of this chapter, you will be able to:

- Recognise and describe a repeating, growing, or shrinking pattern.
- Extend a repeating, growing, or shrinking pattern.
- Create a pattern with a given rule.
- Solve simple problems using patterns.

Lesson 1 Repeating Patterns

Try This

Extend the line of shapes by drawing the next three shapes.

A **repeating pattern** is a pattern in which a part of the pattern, called the **core,** repeats over and over again.

For example, the pattern on the right has this part () of it,repeating over and over again.

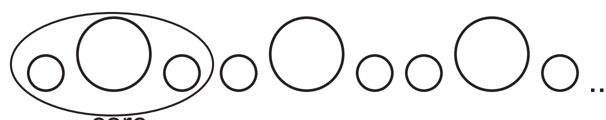
A shape pattern

Patterns are based on a single **attribute** of the items in it. Attributes are the features of the items such as shape, size, colour, and number. For example, the above pattern is based on the attribute of shape, as it is made up of rhombus, triangle, triangle; rhombus, triangle, triangle, ...

The pattern on the right is a colour pattern. The colours (black, white) repeat over and over again.

A colour pattern

The pattern on the right is a based on the attribute of size of circles. The core is made up of *small, big, small* circles.



A size pattern

We can **translate** a repeating pattern into **letter codes**, by looking at how the items in the core are structured or arranged. For example, the shape pattern above is an ABB pattern, if we assign the letter A to the rhombus, and the letter B to the triangle. The colour pattern above is an AB pattern, by assigning A to the black colour and B to the white colour. The size pattern above is an ABA pattern, by assigning A to the small circle and B to the big circle. Using letter codes to represent the core of a pattern helps us talk about it.

On what attribute is the pattern in the above **Try This** problem based? Use letter codes to describe it.

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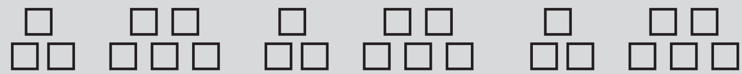
Example

Example 1 On what attribute is the repeating pattern below based? How do you know? Translate it into letter codes.



Solution	**Thinking**
The pattern is based on the attribute of number, because it consists of sets of 3 and sets of 5 in a repeated manner. It is an AB pattern.	 The items in the pattern have the same shape, colour and size. But the squares are clearly grouped into sets. I see that the sets of 3 and 5 squares repeat over and over again. So, this pattern is based on the attribute of number. I assigned A to the set of 3, and B to the set of 5, making this pattern an AB pattern.

Practising and Applying

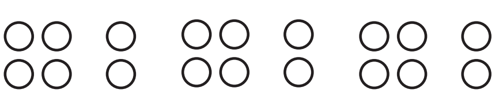
1. On what attribute is each of the following patterns based? (The attributes may be shape, size, colour, or number.)

   a) 

   b) 

   c) 

   d) 

   e) 

   f) 

2. Translate the core of each pattern in question 1 into letter codes.

3. Copy and extend each of the patterns in question 1.

4. Copy and extend the following word patterns.
   a) lock key lock key lock key ...
   b) mouse rat cat mouse rat cat mouse rat cat ...
   c) honey honey bee honey honey bee honey honey bee ...

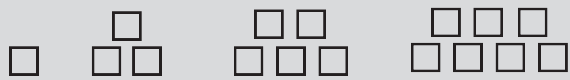
5. Copy and extend the following number patterns.
   a) 7 5 3 7 5 3 7 5 3 ...
   b) I V X I V X I V X ...
   c) ༧ ༥ ༣ ༧ ༥ ༣ ༧ ༥ ༣ ...

6. Create a repeating pattern, and explain why it is a pattern.

Lesson 2 Growing and Shrinking Patterns

Try This

Look at the sets of squares. Make the next set of squares in the pattern.



A **growing pattern** is a pattern, in which each successive item is bigger by a regular amount. The pattern on the right is a growing pattern, in which each successive number increases by 2. A growing pattern is also called an **increasing pattern**.

2, 4, 6, 8, ...
A growing pattern

The opposite of a growing pattern is a **shrinking pattern**, in which each successive item is smaller by a regular amount. The pattern on the right is a shrinking pattern. A shrinking pattern is also called a **decreasing pattern**.

100, 90, 80, 70, ...
A shrinking pattern

There is no repeating core in a growing or shrinking pattern. Also, you cannot use letter codes to describe a growing pattern or a shrinking pattern unlike a repeating pattern.

Represent the sets of squares in the above **Try This** problem with numbers. Is it a growing pattern or a shrinking pattern? Why?

Practising and Applying

1. Extend each pattern below by copying and writing the next three numbers.
   a) 3, 6, 9, 12, ...
   b) 12, 11, 10, 9, ...
   c) 10, 20, 30, 40, ...
   d) 100, 90, 80, 70, ...

2. Which of the patterns in question 1 are growing patterns? Why?

3. Which of the patterns in question 1 are shrinking patterns? Why?

4. Extend the pattern below by writing the next 4 numbers in Dzongkha numerals.

   ༢ ༤ ༦ ༨ ...

5. Extend the pattern below by writing the next 4 numbers in Roman numerals.

   XV XIII XI IX ...

6. Describe a difference between a repeating pattern and a growing pattern.

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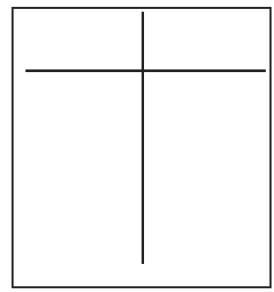
Lesson 3 Solving Simple Problems using Patterns

Try This

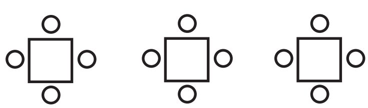
Tshencho Wangdi drinks plenty of water, because it is good for his health. He drinks 3 litres of water daily. How many litres of water does he drink in 4 days?

You can use our knowlege of patterns in solving problems. A good way to do this is to write down the pieces of information contained in a problem, and look for a pattern in them. Once you have found a pattern, you could use it to predict the solution for the problem. One way to write the information is to use a **t-chart**. A t-chart is simply a chart, or a table, with two columns that looks like the letter “t”. For example, let us solve the following problem using a pattern:



An empty t-chart

> Four chairs can be arranged around a square table. There are three such tables. What will be the total number of chairs if there were 4 tables?



Number of tables	Number of chairs
1	4
2	8
3	12
4	?

First, make a t-chart. Then, fill in the given information. Now, if you observe the numbers in the second column, they increase by 4 each time. That is a pattern. The next number, after 12, will be 16. That corresponds with 4 tables. So there will be a total of 16 chairs if there are 4 tables.

You could continue the pattern for both the number of tables and chairs, and answer the questions like: How many chairs will be there for 5 tables? How many tables will be required for 12 chairs?

Solve the above **Try This** problem using a t-chart?

Practising and Applying

1. An ant has 6 legs. Three ants are shown below.

Number of ants	Total number of ant legs
1	6
2	12
3	

a) Complete the above t-chart.

b) Describe the pattern for the total number of ant legs.

c) Extend the numbers in the t-chart up to 5 ants. What is the total number of ant legs for 5 ants?

2. A spider has 8 legs, as shown below.

a) Make a t-chart for the number of spiders and the total number of spider legs.

b) Describe the pattern for the total number of spider legs.

c) Is it a growing pattern or a repeating pattern? Why?

d) Extend the numbers in the t-chart up to 6 spiders. What is the total number of legs for 6 spiders?

e) If there are 32 spider legs, how many spiders are there?

3. Sonam's family takes eggs during breakfast. Sonam checks the number of eggs in the fridge every morning after breakfast, and makes a t-chart.

Days	Number of eggs
1	6
2	3
3	0
4	6
5	3
6	0

a) Describe the pattern for the number of eggs.

b) Extend the t-chart. How many eggs would Sonam find in the fridge on the 14th day?

c) How many eggs are used for each breakfast?

4. Four chairs can be placed around one square table. Six chairs can be placed around two tables when they are joined, as shown below in figure 2.

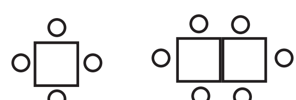


figure 1 figure 2 figure 3

a) Draw figure 3 for placing chairs around 3 tables joined in a row.

b) Make a t-chart for the number of tables and number of chairs.

c) How many chairs can be placed around 6 tables joined in a row?

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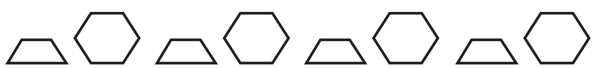
Chapter Review

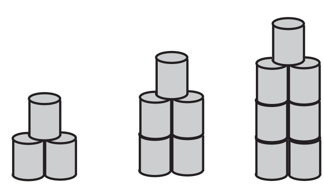
1. What type of a pattern is each?
   a) 1, 3, 5, 7, 9, ...
   b) 60, 50, 40, 30, ...
   c) 7, 1, 7, 1, 7, 1, ...

2. Write the next three numbers, letters, or words to extend each pattern.
   a) 15, 14, 13, 15, 14, 13, 15, 14, 13, ...
   b) P, Q, R, S, P, Q, R, S, P, Q, R, S, ...
   c) Gyalyong, Gakid, Pelzom, Gyalyong, Gakid, Pelzom, ...

3. Translate each repeating pattern below using letter codes.
   a) 3, 6, 9, 3, 6, 9, 3, 6, 9, ...
   b) red, red, blue, blue, red, red, blue, blue, red, red, blue, blue, ...
   c)  ...

4. Make each of the following patterns with numbers:
   a) A repeating pattern starting with 30.
   b) A growing pattern starting with 30.
   c) A shrinking pattern starting with 30.

5. Tashi has built a series of 4 towers with cylindrical blocks. The first 3 towers look as shown below.
   a) How many blocks would be in tower 4? Draw tower 4.
   b) Make a t-chart for the tower numbers and the number of blocks in the towers.
   c) How many blocks would be in tower 10?



tower 1 tower 2 tower 3 tower 4