Teacher's Guide to

Understanding Mathematics

Textbook for Class IV

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Department of School Education
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ROYAL GOVERNMENT OF BHUTAN

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MINISTRY OF EDUCATION

THIMPHU :BHUTAN

Cultivating the Grace of Our Mind

MINISTER

December 15, 2008

Foreword

I am at once awed and fascinated by the magic and potency of numbers. I am amazed at the marvel of the human mind that conceived of fantastic ways of visualizing quantities and investing them with enormous powers of representation and symbolism. As my simple mind struggles to make sense of the complexities that the play of numbers and formulae presents, I begin to realize, albeit ever so slowly, that, after all, all mathematics, as indeed all music, is a function of forming and following patterns and processes. It is a supreme achievement of the human mind as it seeks to reduce apparent anomalies and to discover underlying unity and coherence.

Abstraction and generalization are, therefore, at the heart of meaning-making in Mathematics. We agreed, propped up as by convention, that a certain figure, a sign, or a symbol, would carry the same meaning and value for us in our attempt to make intelligible a certain mass or weight or measure. We decided that for all our calculations, we would allow the signifier and the signified to yield whatever value would result from the tension between the quantities brought together by the nature of their interaction.

One can often imagine a mathematical way of ordering our surrounding and our circumstance that is actually finding a pattern that replicates the pattern of the universe – of its solid and its liquid and its gas. The ability to engage in this pattern-discovering and pattern-making and the inventiveness of the human mind to anticipate the consequence of marshalling the power of numbers give individuals and systems tremendous privilege to the same degree which the lack of this facility deprives them of.

Small systems such as ours cannot afford to miss and squander the immense power and privilege the ability to exploit and engage the resources of Mathematics have to present. From the simplest act of adding two quantities to the most complex churning of data, the facility of calculation can equip our people with special advantage and power. How intelligent a use we make of the power of numbers and the precision of our calculations will determine, to a large extent, our standing as a nation.

I commend the good work done by our colleagues and consultants on our new Mathematics curriculum. It looks current in content and learner-friendly in presentation. It is my hope that this initiative will give the young men and women of our country the much-needed intellectual challenge and prepares them for life beyond school. The integrity of the curriculum, the power of its delivery, and the absorptive inclination of the learner are the eternal triangle of any curriculum. Welcome to Mathematics.

Imagine the world without numbers! Without the facility of calculation!

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INTRODUCTION

HOW MATHEMATICS HAS CHANGED

Mathematics is a subject with a long history. Although newer mathematical ideas are always being created, much of what your students will be learning is mathematics that has been known for hundreds of years, if not longer.

There are some changes in the content that you will teach. It may be that the content is new to your class, but not to your curriculum. Or, it may be new to your curriculum. For example, work on building shapes from isometric drawings in geometry is new.

What you may notice most is a change in the approach to mathematics. Worldwide, there is now a greater emphasis on the need for students to understand the mathematics they learn rather than to memorize rote procedures. There are many reasons for this.

Worldwide, there is now a greater emphasis on the need for students to understand the mathematics they learn rather than to memorize rote procedures.

- In the long run, it is very unlikely that students will remember the mathematics they learn unless it is meaningful. It is much harder to memorize "nonsense" than to remember something that relates to what they already know.
- Some approaches to mathematics have not been successful; there are many adults who are not comfortable with mathematics even though they were successful in school.

In this new program, you will find many ways to make mathematics more meaningful.

It is important always to talk about why something is true, not simply that it is true.

- We will always talk about why something is true, not simply that it is true. For example, the reason why 25 × 100 is 25 with two zeros added to the end (25 × 100 = 2500) is explained and not just stated.
- Mathematics should be taught using contexts that are meaningful to the students. They can be mathematical contexts or real-world contexts. These contexts will help students see and appreciate the value of mathematics.

For example:

- In Unit 4 (Multiplication and Division with Greater Numbers), a task with a real-world context involves arrays of plants, cards, and stamps.

3. How many items are there altogether in each array?
a) 5 rows with 28 carrots in each row
b) 6 rows with 18 cards in each row
c) 6 rows with 157 stamps in each row
d) 4 rows with 132 potato plants in each row

• A task with a broader context in Unit 1 (Numeration, Addition and Subtraction) involves comparing populations in different parts of Bhutan.

Try This

The 2005 census told how many people lived in each dzongkhag.

Dzongkhag	Population (2005)
Ha	11,648
Samtse	60,100
Trongsa	13,419

A. Which of the dzongkhags in the chart has the most people? How do you know?

• When discussing mathematical ideas, we expect students to use the processes of problem solving, communication, reasoning, making connections (connecting mathematics to the real world and connecting mathematical topics to each other) and representation (representing mathematical ideas in different ways, such as manipulatives, graphs, and tables). For example, students use number lines to represent repeated multiplication. This will help them see how multiplication relates to repeated addition.

We expect students to use the processes of problem solving, communication, reasoning, making connections, and representation.

• There is an increased emphasis on problem solving because the reason we learn mathematics is to solve problems. Once students are adults, they are not told when to factor or when to multiply; they need to know how to apply those skills to solve certain problems. Students will be given opportunities to make decisions about which concepts and skills they need in certain situations.

A significant amount of research evidence has shown that these more meaningful approaches work. Scores on international tests are higher when the emphasis is on higher level thinking and not only on the application of skills.

Scores on international tests are higher when the emphasis is on higher level thinking and not only on the application of skills.

THE DESIGN OF THE STUDENT TEXTBOOK

Each unit of the textbook has the following features:

• a *Getting Started* to review prerequisite content

• two or three chapters, which cluster the content of the unit into sections that contain related content

• regular lessons and at least one *Explore* lesson

• a *Game*

• at least one *Connections* feature

• a *Unit Revision*

Getting Started

There are two parts to the *Getting Started*. They are designed to help you know whether students are missing critical prerequisites and to remind students of knowledge and terminology they have already learned that will be useful in the unit.

The Getting Started is designed to help you know whether students are missing critical prerequisites and to remind students of knowledge and terminology they will need for the unit.

• The *Use What You Know* section is an activity that takes 20 to 30 minutes. Students are expected to work in pairs or in small groups. Its purpose is consistent with the rest of the text's approach, that is, to engage students in learning by working through a problem or task rather than being told what to do and then just carrying it through.

• The *Skills You Will Need* section is a more straightforward review of required prerequisite skills for the unit. This should usually take about 20 to 30 minutes.

Regular Lessons

• Each lesson might be completed in one or two hours (i.e., one or two class periods), although some are shorter. The time is suggested in this *Teacher's Guide*, but it is ultimately at your discretion.

• Lessons are numbered #.#.#. The first number tells the unit, the second number the chapter, and the third number the lesson within the chapter. For example, Lesson 4.2.1 is Unit 4, Chapter 2, Lesson 1.

Lessons are numbered #.#.#. The first number tells the unit, the second number the chapter, and the third number the lesson within the chapter.

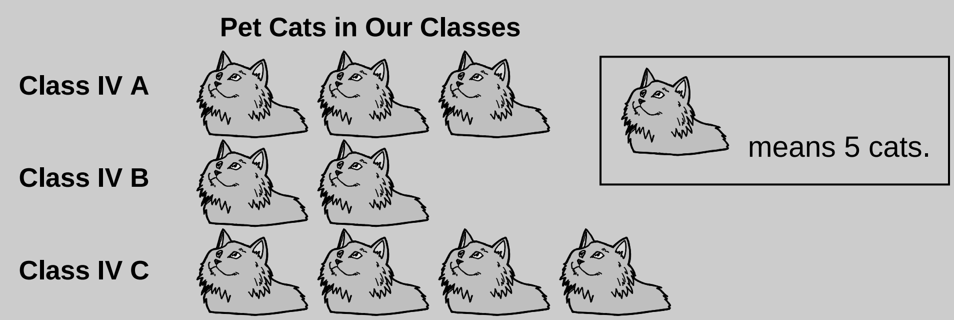
• Every regular lesson is divided into five parts:
  - A *Try This* task or problem
  - The exposition (the main points of the lesson)
  - A question that revisits the *Try This* task, called *Revisiting the Try This* in this guide
  - one or more *Examples*
  - *Practising and Applying* questions

Try This

• The *Try This* task is in a shaded box, like the one below from lesson 2.1.1 on page 37.

Try This

This pictograph shows the number of pet cats in three different Class IVs.



A. How many pet cats are there altogether in the three classes?

• The *Try This* is a brief task or problem that students complete in pairs or small groups. It serves to motivate new learning. Students can do the *Try This* without the new concepts or skills that are the focus of the lesson, but the problem is related to the new learning. It should be completed in 5 to 10 minutes. The reason to start with a *Try This* is that we believe students should do some mathematics independently before you intervene.

The Try This is a brief task or problem that students complete in pairs or small groups to motivate new learning.

• The answers to the *Try This* problems or tasks are not found in the back of the student book (but they are in this *Teacher's Guide*).

The Exposition

• The exposition presents the main concepts and skills of the lesson. Examples are often included to clarify the points being made.

• You will help the students through the exposition in different ways (as suggested in this *Teacher's Guide*). Sometimes you will present the ideas first, using the exposition as a reference. Other times students will work through the exposition independently, in pairs, or in small groups.

• Key mathematical terms are introduced and described in the exposition. When a key term first appears in a unit of the textbook, it is highlighted in **bold type** to indicate that it is found in the glossary (at the back of the student textbook).

• Students are not expected to copy the exposition into their notebooks either directly from the book or from your recitation.

The exposition presents the main concepts and skills of the lesson and is taught in different ways, as suggested in this guide. The Revisiting the Try This question links the Try This task to the new ideas presented in the exposition.

Revisiting the Try This

• The *Revisiting the Try This* question follows the exposition and appears in a shaded lozenge, like this example from lesson 2.1.1 on page 38.

B. Use skip counting to solve each. Tell how you skip counted.
i) How many pet cats are there in each class?
ii) How many pet cats are there in all three classes?

C. Write a multiplication fact for each.
i) The number of pet cats in each class
ii) The number of pet cats in all three classes

• The *Revisiting the Try This* question links the *Try This* task or problem to the new ideas presented in the exposition. This is designed to build a stronger connection between the new learning and what students already understand.

Examples

• The *Examples* are designed to provide additional instruction by modelling how to approach some of the questions students will meet in *Practising and Applying*. Each example is a bit different from the others so that students have multiple models from which to work.

• The *Examples* show not only the formal mathematical work (in the left hand *Solution* column), but also student reasoning (in the right hand *Thinking* column). This model should help students learn to think and communicate mathematically. Photographs of students are used to further reinforce this notion.

• Some of the *Examples* present two or three different solutions. The example in lesson 2.1.1 on page 39 shows three possible ways to solve the problem posed about the number of wheels on five cards, *Solution 1*, *Solution 2*, and *Solution 3*.

• The treatment of *Examples* varies and is discussed in the *Teacher's Guide*. Sometimes the students work through these independently, sometimes they work in pairs or small groups, and sometimes you are asked to lead them through some or all of the examples.

• A number of the questions in the *Practising and Applying* section are modelled in the *Examples* to make it more likely that students will be successful.

The Examples model some of the questions that students will meet in Practising and Applying.

The Examples show the formal mathematical work in the Solution column and model student reasoning in the Thinking column. This model should help students learn to think and communicate mathematically.

Practising and Applying

• Students work on the *Practising and Applying* questions independently, with a partner, or in a group, using the exposition and *Examples* as references.

• The questions usually start like the work in the *Examples* and get progressively more conceptual, with more explanations and more problem solving required later in the exercise set.

• The last question in the section always brings closure to the lesson by asking students to summarize the main learning points. This question could be done as a whole class.

Students work on the Practising and Applying questions independently, with a partner, or in a group, using the exposition and Examples as references.

Explore Lessons

• *Explore* lessons provide an opportunity for students to work in pairs or small groups to investigate some mathematics in a less directed way. Often, but not always, the content is revisited more formally in a regular lesson immediately before or after the *Explore* lesson. The *Teacher's Guide* indicates whether the *Explore* lesson is optional or essential.

• There is no exposition or teacher lecture in an *Explore* lesson, so the parts of the regular lesson are not there. Instead, a problem or task is posed and students work through it by following a sequence of questions or instructions that direct their investigation.

• The answers for these lessons are not found in the back of the textbook, but are found in this *Teacher's Guide*.

Explore lessons provide an opportunity for students to work in pairs or small groups to investigate some mathematics in a less directed way.

Connections

• The *Connections* is an optional feature that relates the content of the unit to something else.

• There are always one or more *Connections* features in a unit. The placement of a *Connections* feature in a unit is not fixed; it depends on the content knowledge required. Sometimes it will be early in the unit and sometimes later.

• The *Connections* feature always gives students something to do beyond simply reading it.

• Students usually work in pairs or small groups to complete these activities.

The Connections feature takes many forms. Sometimes it is a relevant and interesting historical note. Sometimes it relates the mathematical content of a unit to the content of a different unit. Other times it relates the mathematical content to a real world application.

Game

• There is at least one *Game* per unit.

• The *Game* provides an enjoyable way to practise skills and concepts introduced in the unit.

• Its placement in the unit is based on where it makes most sense in terms of the content required to play the *Game*.

• In most *Games* students work in pairs or small groups, as indicated in the instructions.

• The required materials and rules are listed in the student book. Usually, there is a sample shown to make sure that students understand the rules.

• Most *Games* require 15 to 20 minutes, but students can often benefit from playing them more than once.

The Game provides an enjoyable way to practise skills and concepts introduced in the unit.

Unit Revision

• The *Unit Revision* provides an opportunity for review for students and for you to gather informal assessment data. *Unit Revisions* review all lesson content except the *Getting Started* feature, which is based on previous class content. There is always a mixture of skill, concept, and problem solving questions.

• The order of the questions in the *Unit Revision* generally follows the order of the lessons in the unit. Sometimes, if a question reflects more than one lesson, it is placed where questions from the later lesson would appear.

• Students can work in pairs or on their own, as you prefer.

• The *Unit Revision*, if done in one sitting, requires one or more hours. If you wish, you might break it up and assign some questions earlier in the unit and some questions later in the unit.

The Unit Revision provides an opportunity for review for students and allows you to gather informal assessment data.

Glossary

• At the end of the student textbook, there is a glossary of key mathematical terminology introduced in the units. When new terms are introduced in the units, they are in **bold** type. All of these terms are found in the glossary.

• The glossary also contains important mathematical terms from previous classes that students might need to refer to.

• In addition, there is a set of instructional terms commonly used in the *Practising and Applying* questions (for example, explain, predict,…) along with descriptions of what those terms require the student to do.

The glossary contains key mathematical terminology introduced in the units, important terms from previous classes, and a set of instructional terms.

Answers

• Answers to most numbered questions are provided in the back of the student textbook. In most cases, only the final answers are shown, not full solutions and explanations. For example, if students are asked to solve a problem and then "Show your work" or "Explain your thinking", only the final answer to the problem will be included, not the work or the reasoning.

• There is often more than one possible answer. This is indicated by the phrase *Sample Response*.

• Full solutions to the questions and explanations that show reasoning are provided in this *Teacher's Guide*, as are the answers to the lettered questions (such as A or B) in the *Try This* and the *Explore* lessons. When an answer or any part of an answer is enclosed in square brackets, this indicates that it has been omitted from the answers at the back of the student textbook.

The answers to most of the numbered questions are found in the back of the student text. This Teacher's Guide contains a full set of answers.

THE DESIGN OF THE TEACHER'S GUIDE

The *Teacher's Guide* is designed to complement and support the use of the student textbook.

• The sequencing of material in the guide is identical to the sequencing in the student textbook.

• The elements in the *Teacher's Guide* for each unit include:

- a *Unit Planning Chart*
- *Math Background* for the unit
- a *Rationale for Teaching Approach*
- support for each lesson
- a *Unit Test*
- a *Performance Task*
- an *Assessment Interview* (in Units 5 and 7)

The Teacher's Guide is designed to complement and support the use of the student textbook.

The support for each lesson includes:

- *Curriculum outcomes* covered in that lesson
- *Outcome relevance* (*Lesson relevance* in the case of optional *Explore* lessons)
- *Pacing* in terms of minutes or hours
- *Materials* required to teach the lesson
- *Prerequisites* that the lesson assumes students possess
- *Main Points to be Raised* explicitly in the lesson
- suggestions for working through the parts of the lesson
- *Suggested assessment* for the lesson
- *Common errors* to be alert for
- *Answers,* often with more complete solutions than are found in the student text
- suggestions for *Supporting Students* who are struggling and/or for enrichment

Unit Planning Chart

This chart provides an overview of the unit and indicates, for each lesson, which curriculum outcomes are being covered, the pacing, the materials required, and suggestions for which questions to use for formative assessment.

The Unit Planning Chart provides an overview of the unit.

Math Background and Rationale for Teaching Approach

This explains the organization of the unit or provides some background for you that you might not have, particularly in the case of less familiar content. In addition, there is an indication of why the material is approached the way it is.

This section provides information about the math behind the unit, and an explanation of why the math is approached the way it is.

Regular Lesson Support

Regular lesson support includes grouping and instructional strategies, alerts for common errors, suggestions for assessment, and teaching tips.

• Suggestions for grouping and instructional strategies are offered under the headings *Try This, Revisiting the Try This, The Exposition — Presenting the Main Ideas, Using the Examples,* and *Practising and Applying — Teaching Tips.*

• *Common errors* are sometimes included. If you are alert for these and apply some of the suggested remediation, it is less likely that students will leave the lessons with mathematical misunderstandings.

• A number of *Suggested assessment questions* are listed for each lesson. This is to emphasize the need to collect data about different aspects of the students' performance — sometimes the ability to apply skills, sometimes the ability to solve problems or communicate mathematical understanding, and sometimes the ability to show conceptual understanding.

• It is not necessary to assign every *Practising and Applying* question to each student, but they are all useful. You should go through the questions and decide where your emphasis will be. It is important to include a balance of skills, concepts, and problem solving. You might use the *Suggested assessment questions* as a guide for choosing questions to assign.

• You may decide to use the last *Practising and Applying* question to focus a class discussion that revisits the main ideas of the lesson and to bring closure to the lesson.

Explore Lesson Support

• As with regular lessons, for *Explore* lessons there is an indication of the curriculum outcomes being covered, the relevance of the lesson, and whether the lesson is optional or essential.

• Because the style of the lesson is different, the support provided is different than for the regular lessons. There are suggestions for grouping the students for the exploration, a list of *Observe and assess* questions to guide your informal formative assessment, and *Share and reflect* ideas on how to consolidate and bring closure to the exploration.

Because the style of an Explore lesson is different than a regular lesson, the support provided in the Teacher's Guide is also different.

Unit Test

A pencil-and-paper unit test is provided for each unit. It is similar to the unit revision, but not as long. If it seems that the test might be too long (for example, if students would require more than one class period to complete it) some questions may be omitted. It is important to balance the items selected for the test to include questions involving skills, concepts, and problem solving, and to include at least one question requiring mathematical communication.

If the test seems too long, some questions may be omitted but it is important to ensure a balance of questions involving skills, concepts, problem solving, and communication.

Performance Task

• The *Performance Task* is designed as a summative assessment task. Performance on the task can be combined with performance on a *Unit Test* to give a mark for a student on a particular unit.

• The task requires students to use both problem solving and communication skills to complete it. Students have to make mathematical decisions to complete the task.

• It is not appropriate to mark a performance task using percentages or numerical grades. For that reason, a rubric is provided to guide assessment. There are four levels of performance that can be used to describe each student’s work on the task. A level is assigned for each different aspect of the task and, if desired, an overall profile can be assigned. For example, if a student’s performance is level 2 on most of the aspects of the task, but level 3 on one aspect, an overall profile of level 2 might be assigned.

• A sample solution is provided for each task.

The Performance Task is designed as a summative assessment task that can be combined with a student’s performance on the Unit Test to give an overall mark.

Unit Assessment Interviews

• Selected units (5 and 7) provide a structure for an interview that can be used with one or several students to determine their understanding of the outcomes.

• Interviews are a good way to collect information about students because they allow you to interact with the students and to follow up if necessary. Interviews are particularly appropriate for students whose performance is inconsistent or who do not perform well on written tests, but who you feel really do understand the content.

• You may use the data you collect in combination with class work or even a unit test mark.

Interviews are a good way to collect information about students, since interviews allow you to interact with the students and to follow up if necessary.

ALTERNATIVE UNIT SEQUENCES

Although the book is designed to be followed in the order that the units appear, it is possible to follow a different sequence. You may want to do all the number units first (Units 1, 2, 4, and 6) together in any order as long as Unit 2 precedes Units 4 and 6. Then you can do the non-number units (Units 3, 5, and 7) in any order (note that Unit 3 must follow Unit 2).

ASSESSING MATHEMATICAL PERFORMANCE

Forms of Assessment

It is important to consider both formative (continuous) and summative assessment.

Formative

• Formative assessment is observation to guide further instruction. For example, if you observe that a student does not understand an idea, you may choose to re-teach that idea to that student using a different approach.

Formative assessment is observation to guide further instruction.

• Formative assessment opportunities are provided through
- prerequisite or diagnostic assessment in the *Getting Started*
- suggestions for assessment questions in each regular lesson
- questions that might be asked while students work on the *Try This* or during an *Explore* lesson
- the *Unit Revision*
- the unit *Assessment Interview* (for the units with interviews)

• Formative assessment can be supplemented by
- everyday observation of students' mathematical performance
- formal or informal interviews to reveal students' understanding
- journals in which students comment on their mathematical learning
- short quizzes
- projects
- a portfolio of work so students can see their progress over time, for example, in problem solving or mathematical communication (see *Portfolios* below)

Summative

• Summative assessment is used to see what students have learned and is often used to determine a mark or grade.

Summative assessment is used to see what students have learned and is often used to determine a mark.

• Summative assessment opportunities are provided through
- the *Unit Test*
- the *Performance Task*
- the *Assessment Interview*

• Summative assessment can be supplemented with
- short quizzes
- projects
- a portfolio that is assessed with respect to progress in, for example, problem solving or communication

Portfolios

One of the advantages of using portfolios for assessment is that students can observe their own growth over time, leading to greater confidence. In each unit, the section on math background identifies items that pertain to the various mathematical processes. The portfolio could be made up of student work items related to one of the mathematical processes: problem solving, communication, reasoning, or representation.

One of the advantages of using portfolios for assessment is that students can observe their own growth over time, leading to greater confidence.

Assessment Criteria

• It is right and fair to inform students about what will be assessed and how it will be assessed. For example, students should know whether the intent of a particular assessment task is to focus on application or on problem solving.

It is right and fair to inform students about what will be assessed and how it will be assessed.

• A student's mark and all assessments should reflect the curriculum outcomes for Class IV. The proportions of the mark assigned for each unit should reflect both the time spent on the unit and the importance of the unit. The modes of assessment used for a particular unit should be appropriate for the content. For example, if the unit focuses mostly on skills, the main assessment might be a paper-and-pencil test or quizzes. If the unit focuses on concepts and application, more of the student's mark should come from activities like performance tasks.

The modes of assessment used for a particular unit should be appropriate for the content.

• The focus of this curriculum is not on procedures for their own sake but on a conceptual understanding of mathematics so that it can be applied to solve problems. Procedures are important too, but only in the context of solving problems. All assessment should balance procedural, conceptual, and problem solving items, although the proportions will vary in different situations.

All assessment should balance procedural, conceptual, and problem solving items, although the proportions may vary in different situations.

• Students should be informed whether a test is being marked numerically, using letter grades, or with a rubric. If a rubric is being used, then it should be shared with students before they begin the task to which it is being applied.

Determining a Mark

• In determining a student's mark, you can use the tools described above along with other information, such as work on a project or poster. It is important to remember that the mark should, as closely as possible, reflect student competence with the mathematical outcomes of the course. The mark should not reflect behaviour, neatness, participation, and other non-mathematical aspects of the student's learning. These are important to assess as well, but not as part of the mathematics mark.

It is important to remember that the mark should reflect student competence with the mathematical outcomes of the course and not behaviour, neatness, participation, and so on.

• In looking at a student's mathematics performance, the most recent data might be weighted more heavily. For example, suppose a student does poorly on some of the quizzes given early in Unit 1, but you later observe that he or she has a better understanding of the material in the unit. You might choose to give the early quizzes less weight in determining the student's mark for the unit.

• At present, you are required to produce a numerical mark for a student, but that should not preclude your use of rubrics to assess some mathematical performance. One of the values of rubrics is their reliability. For example, if a student performs at level 2 on a particular task one day, he or she is very likely to perform at the same level on that task another day. On the other hand, a student who receives a test mark of 45 one day might have received a mark of 60 on a different day if one question on the test had changed or if he or she had read an item more carefully.

• You can combine numerical and rubric data using your own judgment. For example, if a student's marks on tests average 50%, but the rubric performance is higher, for example, level 3, it is fair and appropriate to use a higher average for that student's class mark.

THE CLASSROOM ENVIRONMENT

This new curriculum requires a change in the classroom environment to include more pair and group work and an increased emphasis on communication. It is only in this way that students will really become engaged in mathematical thinking instead of being spectators.

This new curriculum requires a change in the classroom environment to include more pair and group work and an increased emphasis on communication.

• In every lesson, students should be engaged in some pair or small group work (for the *Try This*, selected *Practising and Applying* questions, or during an *Explore* lesson).

• Students should be encouraged to communicate with each other to share responses that are different from those offered by other students or from the responses you might expect. Communication involves not only talking and writing, but also listening and reading.

Pair and Group Work

• There are many reasons why students should be working in pairs or groups, including

- to ensure that students have more opportunities to communicate mathematically (instead of competing with the whole class for a turn to talk)
- to make it easier for students to take the risk of giving an answer they are not sure of (rather than being embarrassed in front of so many other people if they are incorrect)
- to see the different mathematical viewpoints of other students
- to share materials more easily

• Sometimes students can work with the students who sit near them, but other times you might want to form the groups so that students who are struggling are working together. Then you can help them while the other students move forward. Students who need enrichment can also work together so that you can provide an extra challenge for them all at once.

• For students who are not used to working in pairs or groups, you should set down rules of behaviour that require them to attend to the task and to participate fully. You need to avoid a situation where four students are working together, but only one of them is really doing the work. You might post Rules for Group Work, as shown here to the right.

• Once students are used to working in groups, you might sometimes be able to base assessment on group performance rather than on individual performance.

Rules for Group Work

- Make sure you understand all of the work produced by the group.
- If you have a question, ask your group members first, before asking your teacher.
- Find a way to work out disagreements without arguing.
- Listen to and help others.
- Make sure everyone is included and encouraged.
- Speak just loudly enough to be heard.

Communication

Students should be communicating regularly about their mathematical thinking. It is through communication that they clarify their own thinking as well as show you and their classmates what they do or do not understand. When they give an answer to a question, you can always be asking questions like, *How did you get that? How do you know? Why did you do that next?*

It is through communication that students clarify their own thinking as well as show you and their classmates what they do or do not understand.

• Communication is practised in small group settings, but is also appropriate when the whole class is working together.

• Students will be reluctant to communicate unless the environment is risk-free. In other words, if students believe that they will be reprimanded or made to feel bad if they say the wrong thing, they will not want to speak in class. Instead, show your students that good thinking grows out of clarifying muddled thinking. It is reasonable for students to have some errors in their thinking and it is your job to help shape that thinking. If a student answers incorrectly, it is your job to ask follow-up questions that will help the student clarify his or her own thinking. For example, suppose a problem requires a student to use the digits 6, 5, 1, 9, and 2 to create two different 5-digit numbers that are about 20,000 apart. The student hesitates or answers inappropriately. Follow up by asking questions like the following:

Students will be reluctant to communicate unless the environment is risk-free.

- *What is the greatest 5-digit number you can make with those digits? Why?*
- *What is the least 5-digit number you can make with those digits? Why?*
- *How far apart are those numbers?*
- *What could you do now?*

• Many of the questions in the textbook require students to explain their thinking. The sample *Thinking* in the *Examples* is designed to provide a model for mathematical communication.

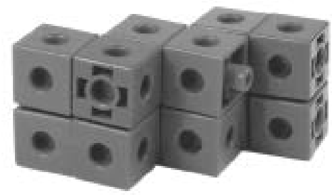
• One of the ways students communicate mathematically is by describing how they know an answer is right. Even when a question does not ask students to check their work, you should encourage them to think about whether their answer makes sense. When they check their work, they should usually check using a different way than the way they used to find their answer so that they do not make the same reasoning error twice. This will enhance their mathematical flexibility.

The sample Thinking in the Examples is designed to provide a model for mathematical communication.

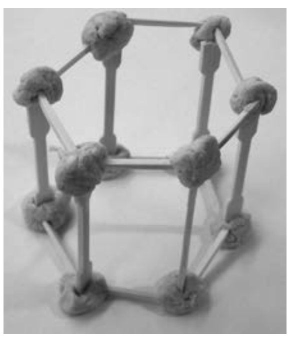
MATHEMATICAL TOOLS

Manipulatives

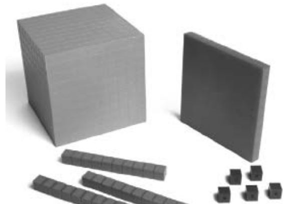
There is great value in using manipulatives in mathematics instruction. Sometimes, they are essential. For example, some of the lessons in Units 3 and 5 cannot be completed without using linking cubes. In Unit 5, students will need sticks and clay to create skeleton models of 3-D shapes. Other times, even though some students can be successful without manipulatives, all students will benefit from using them. For example, if students use base ten blocks on place value charts in Unit 1 to model and calculate with whole numbers, they will learn not only how to perform the calculations, but they will see why the calculations are done the way they are.



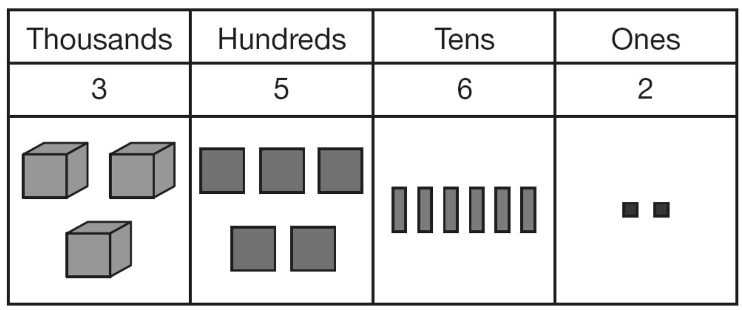
Linking cubes



A skeleton model



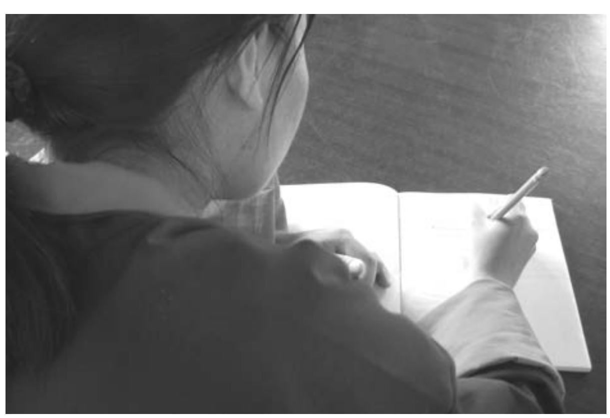
Base ten blocks



Modelling 3562 with base ten blocks on a place value mat

THE STUDENT NOTEBOOK

It is valuable for students to have a well-organized, neat notebook to look back at to review the main mathematical ideas they have learned. However, it is also important for students to feel comfortable doing rough work in that notebook or doing rough work elsewhere without having to waste time copying it neatly into their notebooks. In addition to the things you tell students to include in their notebooks, they should be allowed to make some of their own decisions about what to include in their notebooks.



Students should be allowed to make some of their own decisions about what to include in their notebooks.

CLASS IV CURRICULUM

STRAND A: NUMBER

KSO Number *By the end of* ***Class 6*** *students should*
♦ *have strong number sense with respect to whole numbers and decimals, and be able to draw on a wide variety of relationships and strategies within number to solve problems in new situations*
♦ *have a strong sense of the base ten system to millions and thousandths, and use place value patterns to understand new ideas and apply reasoning to computational problems and mental mathematics within mathematics itself and in real world situations*
♦ *efficiently select and apply appropriate estimation strategies, to answer real life questions and check for reasonableness of answers in calculation*
♦ *understand fractions and decimals to thousandths, and the relationship between them, and move freely from one form of representation to another, as might be appropriate in a given situation, to provide a strong foundation for higher level fractional ideas and computation*
♦ *understand meanings and appropriate application of integers, ratios, and percent in real world situations*
♦ *apply number theory concepts in relevant situations as a way to solve problems*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-A1 Place Value: model whole numbers to five places
• recognize value of each digit
• read numbers several ways and record numbers
• write numbers in expanded form
• estimate the value of numbers

4-A2 Compare and Order: whole numbers to five digits
• order two or more numbers and justify order
• identify numbers greater or less than a given number and numbers between given numbers

4-A3 Fractions and Mixed Numbers: model
• develop visual images for fractions and mixed numbers through concrete materials
• use contexts which include part of a whole and part of a group

4-A4 Renaming Fractions: equivalent fractions
• understand concretely that two or more fractions can have different names but the same value
• find number patterns in equivalent fractions using models

4-A5 Fractions: compare and order
• compare visually, in a variety of ways
• compare fractions with the same denominator
• compare fraction with the same numerator
• develop and use benchmark fractions to compare and order fractions

4-A6 Hundredths: model and record
• model decimal hundredths
• develop the concept of hundredths in our place-value system by continuing the pattern of dividing by 10
• explore the relationship between decimals and fractions

4-A7 Hundredths: compare and order
• compare whole number part first, decimal part second

STRAND B: OPERATIONS

KSO Operations *By the end of **Class 6** students should*
♦ *model and solve computational problems involving whole numbers and decimals by selecting appropriate operations and procedures for computation, estimation, and mental math*
♦ *choose appropriate method of computation in given situations (including pencil/paper, mental math, estimation)*
♦ *model and solve problems involving the addition and subtraction of simple fractions and be able to justify answers through reasoning*
♦ *informally explore simple algebraic situations*
♦ *demonstrate flexibility in procedures chosen to solve computational problems*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-B1 Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers
• apply familiar addition and subtraction strategies to numbers
• relate addition and subtraction of decimals to addition and subtraction of whole numbers
• continue estimating

4-B2 Add and Subtract Mentally: to four digits
• develop and use mental strategies: front end, compensation, counting on/back, compatible numbers
• determine when it is most suitable to use mental addition and subtraction

4-B3 Multiplication Meanings: explore
• explore various meanings of multiplication, focusing on multiplication as skip counting and repeated addition

4-B4 Multiplication Properties: explore
• explore the commutative, distributive, and associative properties
• explore multiplying by zero and 1

4-B5 Multiplication Facts: to 9 × 9
• develop facts to 9 × 9 through concrete and pictorial representations
• develop and practise strategies for fact recall
• recall facts to 9 × 9

4-B6 3-Digit by 1-Digit Multiplication: with and without regrouping
• develop alternative and standard algorithms (from understanding)
• use estimation to predict and verify products
• connect concrete models to symbolic recordings (e.g., relate rectangle area models to written algorithm)

4-B7 Division Meanings: small numbers
• understand division as grouping or sharing

4-B8 Division Properties: dividing by zero and 1
• dividing by zero and dividing by 1
• understand that order matters when dividing

4-B9 Multiplication and Division Facts: relate through properties
• understand multiplication and division as two ways of looking at the same situation
• use multiplication facts to recall division facts

4-B10 Divide 2-Digit and 3-Digit by 1-Digit: with and without regrouping
• develop alternative and standard algorithms (from understanding)
• connect concrete models to symbolic recordings
• understand remainders in context as a fraction, ignored, rounded, or addressed specifically
• continue estimating

4-B11 Multiply Mentally: by 10 or 100
• explore multiplication of 2-digit numbers by 10 and by 100
• develop visual images of whole numbers multiplied by 10 or 100 using base ten materials

STRAND C: PATTERNS AND RELATIONSHIPS

KSO Patterns and Relationships *By the end of **Class 6** students should*
♦ *describe, extend, and create patterns to solve problems in real world situations and mathematical contexts (number, geometry, measurement)*
♦ *use patterns to generalize mathematical situations to aid in solving problems and understanding relationships*
♦ *explore and generalize how a change in one quantity in a relationship affects another, in order to efficiently solve similar (but new) problems*
♦ *represent mathematical patterns and relationships in a variety of ways (charts, tables, graphs, numerically)*
♦ *use patterns to assist in mental math strategies*
♦ *informally (through reasoning) solve linear equations via open sentences*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-C1 Apply Patterns: solve computation problems
• explore and apply patterns to solve computation problems (e.g., multiplying by 8, 9, or 10)

4-C2 Open Sentences and Computation Patterns: multiplication and division
• generate rules about how a change in one factor affects the result (e.g., for □ × 10, as □ increases by 1 the product increases by 10)

4-C3 Multiplying by 10, by 100, by 1000: apply pattern visually and symbolically
• identify and continue patterns with increasing powers of ten

STRAND D: MEASUREMENT

KSO Measurement *By the end of **Class 6** students should*
♦ *understand relationships among common SI units and choose appropriate units to solve measurement problems in given situations*
♦ *move freely among common SI units to communicate measurement ideas effectively, appropriate to a given measurement situation*
♦ *estimate effectively using a variety of strategies to solve measurement problems and understand when estimation is close enough*
♦ *use relationships and reasoning to develop and apply procedures for measuring in real situations and mathematical contexts*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-D1 Linear Units: mm, cm, m, and km; estimate and measure
• estimate and measure in mm, cm, m and km
• investigate and develop unit relationships

4-D2 Volume: estimate and measure to develop concept of volume
• explore the meaning of volume through non-standard units by counting the number of units it takes to build a solid
• estimate and measure volume in non-standard units

4-D3 Volume of Rectangular Prisms: estimate and measure in cubic centimetres
• estimate then verify the volume of rectangular prisms using centimetre cubes
• determine the volume of a rectangular prism and build prisms with a specified volume
• connect volume to dimensions (dimensions of first layer × number of layers)

4-D4 Area: estimate and measure in square centimetres
• understand that area is expressed as the number of square units required to cover a given surface
• use square centimetre symbol, cm^2

4-D5 Constant Area and Different Perimeters: explore
• explore the concept concretely
• understand that-different shapes can have the same area
• understand that area and perimeter are generally independent of each other

4-D6 Dimensions and Area of a Rectangle: relate factors and product
• relate dimensions of rectangles to area (product) concretely
• develop a personal formula for area

4-D7 Angles: amount of turn
• develop the meaning of angle and measure of an angle concretely
• understand angle as a turn and the measure of angle as the amount of turn (i.e., smaller angle = smaller turn)
• investigate to discover that the lengths of the arms of an angle do not influence angle size
• differentiate and describe right, acute, and obtuse angles

STRAND E: GEOMETRY

KSO Geometry *By the end of **Class 6** students should*
♦ *identify, draw, compare, and build physical models of 2-D and 3-D shapes to focus on their attributes and understand how they affect everyday life*
♦ *predict and verify results of transforming, combining, and subdividing shapes to understand other shapes and explain other geometrical ideas*
♦ *use geometric relationships and spatial reasoning to solve problems and understand everyday events and objects, as well as higher geometrical ideas*
♦ *appreciate the importance of geometry in understanding mathematical ideas and the world around*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-E1 Isometric Drawings: interpret
• build shapes from isometric drawings, include shapes that have “hidden" cubes

4-E2 Quadrilaterals: discover properties (concretely)
• investigate a variety of quadrilaterals to discover properties (sides, angles, diagonals, and symmetry)

4-E3 Quadrilaterals: sort by properties (concretely) and make generalizations
• use properties to sort quadrilaterals (e.g. quadrilaterals with right angles)
• use properties to make generalizations; include properties that relate sides and those that relate angles

4-E4 Triangles: discover properties (concretely), name, and draw
• sort, identify, and draw equilateral, isosceles, and scalene triangles
• sort triangles by various properties (e.g., number of lines of symmetry or number of identical angles)

4-E5 Prisms, Pyramids, Cones, and Cylinders: describe and compare
• explore relationships concretely to identify properties (e.g., prisms: the number of vertices for any prisms is twice the number of vertices for the base (e.g., a triangle-based prism has 6 vertices)
• include relationships that deal with faces, edges, and vertices and understand why those relationships make sense
• examine the similarities and differences between any pair of 3-D shapes

4-E6 Composite Shapes: combining shapes
• find all possible composite shapes that can be made by combining a given set of shapes
• predict first, then verify by combining

4-E7 Nets: sketch for rectangular prisms
• sketch a variety of nets for rectangular prisms including square-based prisms and cubes

4-E8 Models: building for cylinders, cones, prisms, and pyramids
• build, from given nets, cylinders and cones
• build skeleton models for prisms and pyramids

4-E9 Slides, Flips, and Turns (half and quarter): predict and confirm results for 2-D shapes
• predict and confirm results for 2-D shapes under transformations

4-E10 Reflective Symmetry: generalize for properties of various quadrilaterals
• explore the symmetry of various quadrilaterals

4-E11 Congruence: polygons
• understand that congruent polygons are a perfect match because they are the same shape and size
• explore congruence through variety of materials (e.g., pattern blocks, tangrams, pictures of shapes) and methods (including tracing)

STRAND F: DATA MANAGEMENT

KSO Data Management *By the end of **Class 6** students should*
♦ *collect, record, organize, and describe data in multiple ways to draw conclusions about everyday issues*
♦ *construct a variety of data displays and choose the most appropriate*
♦ *predict, read, interpret, and modify predictions for a variety of data displays, including interpolation and extrapolation (draw conclusions about things not specifically represented by the data)*
♦ *develop and apply measures of central tendency to data reflecting relevant situations, in order to draw conclusions and make decisions*
♦ *design and implement strategies for the collection of data*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-F1 Collect, Organize, and Describe Data: real-world issues
• explore a variety of ways to collect data (e.g., asking an open question or a question with options to choose from)
• choose most appropriate method for collecting simple data
• make decisions about the format for presenting data (charts, tables, graphs)

4-F2 Bar Graphs and Pictographs: construct and interpret
• pictographs: choose an appropriate symbol and decide how much each represents (scale)
• bar graphs: decide value of each square (scale)
• include vertical and horizontal graphs
• interpret results and draw conclusions from data

4-F3 Ordered Pairs: position on a grid
• introduce the coordinate grid (quadrant I)
• explore the convention for naming points (ordered pairs) and why order is significant
• compare the use of a coordinate grid to the use of a block grid

4-F4 Mean
• introduce the mean as a summary statistic for a set of data that balances data by sharing it equally

4-F5 Describing Data
• determine the maximum and minimum data values given numerical data
• relate frequency to the heights of bar graphs

STRAND G: PROBABILITY

KSO Probability *By the end of **Class 6** students should*
♦ *explore, interpret, and make predictions for everyday events by estimating and conducting experiments*
♦ *understand the difference between theoretical and experimental probability and when each is relevant*
♦ *begin to conduct simulations to understand real-life probability situations*
♦ *understand the relationship between the numerical representations of probability and the events they represent*

Toward this, students in **Class 4** will be expected to master the following **SO** (Specific Outcomes):

4-G1 Simple Outcomes: more or less likely
• predict whether an outcome is more, equally or less likely than another by investigating with probability devices such as spinners, dice, coins, and coloured cubes

4-G2 Predict Probability: near 0, near 1, or near $\frac{1}{2}$
• determine whether a probability is closer to 0, 1, or $\frac{1}{2}$ using these ideas:
- a probability near 0: an event rarely occurs
- a probability near 1: an event almost always occurs
- a probability near $\frac{1}{2}$: event has an equal chance of occurring or not occurring

4-G3 Experiments: predict and record results (concrete materials)
• investigate concretely using probability devices such as dice, spinners, coloured cubes, and coins
• predict outcomes, verify by experiments, record outcomes, and compare findings with predictions
• devise ways to record experimental results
• compare results of a few trials with those of many
• use common language to describe probability results (e.g., “2 out of 5”)

4-G4 Describe Probability Results: as a fraction
• express simple experimental results as fractions (restrict the total number of possible events to simple numbers)

UNIT 1 NUMERATION, ADDITION, AND SUBTRACTION

UNIT 1 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 1** **TG p. 4**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• Teacher- or student-made digit cards or a deck of playing cards with the face cards removed	All questions
Chapter 1 Whole Number Place Value				
1.1.1 EXPLORE: Modelling 4-digit Numbers (Essential) **SB p. 3** **TG p. 6**	**4-A1 Place Value: model whole numbers to five places** • recognize value of each digit	1 h	• Base ten blocks and Base Ten Blocks (BLM)	Observe and Assess questions
1.1.2 EXPLORE: Describing 10,000 (Optional) **SB p. 5** **TG p. 9**	**4-A1 Place Value: model whole numbers to five places** • estimate the value of numbers	40 min	• Ruler, metre stick, or measuring tape	Observe and Assess questions
1.1.3 Place Value: 5-digit Numbers **SB p. 6** **TG p. 11**	**4-A1 Place Value: model whole numbers to five places** • recognize value of each digit • read numbers several ways and record numbers • write numbers in expanded form • estimate the value of numbers	1 h	• Base ten blocks and Base Ten Blocks (BLM) • Place value charts or Place Value Charts I (BLM)	Q1, 3, 5
1.1.4 Renaming Numbers **SB p. 11** **TG p. 15**	**4-A1 Place Value: model whole numbers to five places** • read numbers several ways and record numbers • write numbers in expanded form	1 h	• Place value charts or Place Value Charts I (BLM) • Base ten blocks or Base Ten Blocks (BLM) (optional)	Q2, 3, 5, 7
1.1.5 Comparing and Ordering Numbers **SB p. 15** **TG p. 18**	**4-A2 Compare and Order : whole numbers to five digits** • order two or more numbers and justify order • identify numbers greater or less than a given number and numbers between given numbers	1 h	None	Q2, 4, 6
GAME: As High as You Can **SB p. 18** **TG p. 20**	Practise comparing numbers in a game situation	20 min	• Dice	N/A
Chapter 2 Addition and Subtraction				
1.2.1 Adding and Subtracting Mentally **SB p. 19** **TG p. 21**	**4-B2 Add and Subtract Mentally: to four digits** • develop and use mental strategies: front end, compensation, counting on/back, compatible numbers • determine when it is most suitable to use mental addition and subtraction	1 h	None	Q1, 2, 3, 7

UNIT 1 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Chapter 2 Addition and Subtraction [Continued]				
GAME: Add High and Subtract Low **SB p. 23** **TG p. 24**	Practise mental addition and subtraction in a game situation	20 min	• Dice	N/A
1.2.2 Estimating Sums and Differences **SB p. 24** **TG p. 25**	**4-B1 Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers** • apply familiar addition and subtraction strategies to numbers with five or more digits • continue estimating	1 h	None	Q1, 2, 6
GAME: Estimating the Range **SB p. 26** **TG p. 27**	Practice estimating sums and differences in a game situation	30 min	• Decks of playing cards or teacher- or student-made digit cards	N/A
1.2.3 Adding 5-Digit Numbers **SB p. 27** **TG p. 28**	**4-B1 Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers** • apply familiar addition and subtraction strategies to numbers with five or more digits • continue estimating	1 h	• Place value charts or Place Value Charts I (BLM) (optional)	Q3, 4, 7
GAME: Give Me Thousands **SB p. 29** **TG p. 31**	Practice addition in a game situation	20 min	• Decks of playing cards or teacher- or student-made digit cards	N/A
1.2.4 Subtracting 5-Digit Numbers **SB p. 30** **TG p. 32**	**4-B1 Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers** • apply familiar addition and subtraction strategies to numbers with five or more digits • continue estimating	1 h	• Place value charts or Place Value Charts I (BLM) (optional)	Q1, 3, 5
CONNECTIONS: A Different Way to Subtract **SB p. 32** **TG p. 35**	Explore alternative subtraction procedures	15 min	None	N/A
UNIT 1 Revision **SB p. 33** **TG p. 36**	Review the concepts and skills in the unit	2 h	• Base ten blocks or Base Ten Blocks (BLM) • Place value charts or Place Value Charts I (BLM)	All questions
UNIT 1 Test **TG p. 38**	Assess the concepts and skills in the unit	1 h		All questions
UNIT 1 Performance Task **TG p. 40**	Assess concepts and skills in the unit	1 h	None	Rubric provided
UNIT 1 Blackline Masters **TG p. 42**	BLM 1A to 1D Base Ten Blocks (100s blocks; 10s and 1s blocks; 1000s blocks; 10,000s sticks) BLM 2 Place Value Charts I (Whole Numbers: ten thousands place to ones place)			

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Math Background

• The focus of this number unit is on helping students learn to represent, add, and subtract 5-digit numbers. First, they review work with 4-digit numbers. Students learn to use mental math to add and subtract 4-digit numbers and to estimate sums and differences of 5-digit numbers.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 6** in **lesson 1.1.3**, where they create numbers to meet a given condition, in **question 5** in **lesson 1.1.5** and **question 5** in **lesson 1.2.2**, where they create numbers to match specifications, in **question 4** in **lesson 1.2.3** and **question 3** in **lesson 1.2.4**, where they solve real-world problems, and in **question 7** in **lesson 1.2.3**, where they solve a mathematical puzzle involving addition.

• Students use communication frequently as they explain their thinking in answering **part B** in **lesson 1.1.2**, where they describe a method for finding out how many times they can cross their classroom in 10,000 steps, **question 7** in **lesson 1.1.3**, where they explain why there are more 5-digit numbers than 4-digit numbers, **question 7** in **lesson 1.2.2**, where they describe different ways to estimate, and **question 7** in **lesson 1.2.4**, where they explain why they would use a certain subtraction method for a given set of numbers.

• Students use reasoning in answering questions such as **part F** in **lesson 1.1.1**, where they reason about the greatest and least number of blocks that can represent a 4-digit number, in **question 6** in **lesson 1.1.5**, where they use characteristics of a number to draw conclusions about it, in **questions 1 and 2** in **lesson 1.2.1**, where they choose appropriate mental math strategies, and in **question 6** in **lesson 1.2.2**, where they reason about when it makes sense to estimate.

• Students consider representation in **part H** of **lesson 1.1.1**, where they represent a 4-digit number in different ways, in **question 1** in **lesson 1.1.3**, where they represent a 5-digit number in different ways, and throughout **lesson 1.1.4**, where they represent the same number in different ways.

• Students use visualization in **part B** of **lesson 1.1.1**, where they think of a thousand as 10 hundreds, in **lesson 1.1.5**, where they use a number line to help them compare numbers, and in **question 4** in **lesson 1.2.1**, where they use a number line to help them add up to solve a subtraction.

• Students make connections in **part D** in **lesson 1.1.2**, where they think of situations that relate to the number 10,000, in **question 5** in **lesson 1.1.3**, where they relate the base ten block and standard form representations of numbers, in **question 5** in **lesson 1.1.4,** where they relate unit conversion to place value concepts, and in **question 4** in **lesson 1.2.1**, where they connect addition and subtraction processes.

Rationale for Teaching Approach

• This unit is divided into two chapters.

Chapter 1 focuses on representing large whole numbers and on place value concepts.

Chapter 2 focuses on adding and subtracting large whole numbers.

• The **first Explore** lesson allows students to review ideas about 4-digit numbers in a concrete way to prepare them for the work of the unit. The **second Explore** lesson has students explore the 5-digit number 10,000 to give them a sense of size of the 5-digit numbers with which they will work throughout the unit.

• The **Connections** shows students an interesting way to subtract that they might not have seen before.

• There a number of games in the unit. The **Games** provide an opportunity to practise in a pleasant way. One game focuses on comparing larger numbers. Another game focuses on estimating sums and differences. A third game focuses on adding 5-digit numbers.

• Near the start of the unit, actual base ten blocks (ones, tens, hundreds, and thousands) are required. Students will not see the size relationships among the blocks if they only use the blackline masters (BLM) of the blocks. The thousands block must be 10 times the size of the hundreds block, which is not the case with the blackline master models. Once the relationships have been established, students can use the BLM models.

• Throughout the unit, it is important to encourage flexibility in computation and to accept a variety of approaches from students. Being efficient and not recording every step should be welcomed and not discouraged.

Getting Started

Curriculum Outcomes	Outcome Relevance
3 3-Digit Addition: whole numbers **3** 3-Digit Subtraction: with and without regrouping, models, symbols **3** Whole Numbers to 4 digits: reading in several ways **3** Whole Numbers and Base Ten: understand groupings to 1000s **3** Whole Number Place Value: to 4 places **3** Comparing and Ordering: 4-digit whole numbers	Students will find the work in the unit easier after they review the concepts of addition and subtraction they encountered in Class III.

Pacing	Materials	Prerequisites
1 h	• Teacher- or student-made digit cards or a deck of playing cards with the face cards removed	• adding, subtracting, and comparing 3-digit numbers • representing 4-digit numbers using base ten blocks and in standard and expanded form • comparing 4-digit numbers • renaming numbers using place value concepts

Main Points to be Raised

Use What You Know

• You can add 3-digit numbers by combining digits in the same column (i.e., with the same place value). Regroup 10 units for 1 unit in the column to the left where necessary.

• You can subtract two 3-digit numbers by considering digits in the same column (i.e., with the same place value). Regroup 1 unit for 10 units in the column to the right where necessary.

• One 3-digit whole number is greater than another if it has more hundreds. If the number of hundreds is the same, then you compare the tens. If the number of tens is the same, then you compare the ones.

Skills You Will Need

• You can represent a 4-digit whole number in expanded form as the sum of thousands, hundreds, tens and ones.

• When you write a number in expanded form that includes 0 as a digit, it is preferable to omit the value of the 0 digits.

For example, 6003 = 6 thousands + 3 ones.

• You can use regrouping to rename a 4-digit number.

For example, you can rename 3400 as 2 thousands and 14 hundreds by trading 1 thousand for 10 hundreds.

• The place value columns for 4-digit whole numbers (from right to left) are ones, tens, hundreds, and thousands.

Use What You Know — Introducing the Unit

• Before assigning the **Use What You Know** activity, you may wish to review addition, subtraction, and comparison procedures for 3-digit numbers.

• Students can work in groups of two or three to complete the **Use What You Know** activity.

While you observe students at work, you might ask questions such as:

• *Why did you write a 4 there?* (When I add 9 and 5, I get 14, so I write a 4 in this column and 1 in the next column.)

• *The sum was 15. Why did you write 5 and not 15 here?* (If I wrote the 15, I would read the number wrong. Since it is 15 tens, I trade 10 tens for 1 hundred and write only the 5 tens here.)

• *How do you know the hundreds digit will go up by 1 and not by 2?* (When I add 148, I am only adding 1 hundred. Even if I have a lot of tens, there can only be one extra hundred when I regroup.)

• *Why did you arrange the digits as 751 instead of 257?* (I want the greatest number possible so my sum is greater and I can win.)

• *Why did you subtract 236 rather than 632?* (I want the greatest result, so I want to subtract as little as possible.)

• *How do you know your number is greater?* (My number has more hundreds.)

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Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions.

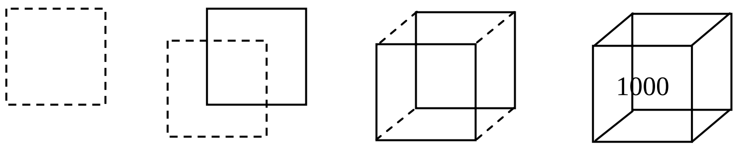
• Students can work individually.

• You may wish to display on the board a place value chart with headings to remind students of the place value columns.

Thousands	Hundreds	Tens	Ones

• You may also wish to suggest a quick way to sketch thousands blocks.

For example, a student could draw two squares, one behind and to the right of the other, and then join them as shown. Students should write the numeral 1000 on the sketch of each block.



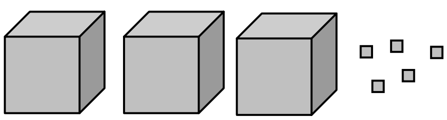
Answers

NOTE: *Read about* ***Answers*** *on* ***page xiv*** *in the* ***Introduction*** *to this Teacher's Guide.*

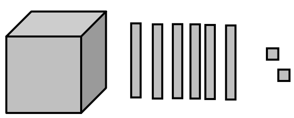
Sample round:

Player A	Player B	
Takes digits 9, 4, 7:	Takes digits 1, 7, 2:	
794 + 148 = 942	721 + 148 = 869	
Takes digits 1, 2, 0:	Takes digits 5, 5, 9:	
942 – 102 = 840	869 – 559 = 310	Since 840 > 310, Player A gets 1 point.

1. a)



b)



2. a) 4 **b)** 3

3. a) tens **b)** thousands
c) hundreds **d)** ones

4. a) *Sample response*: 5402 **b)** 5778

5. a) 5 thousands + 9 hundreds + 2 ones
b) 6 thousands + 8 ones

6. a) **<u>40</u>** hundreds + **<u>5</u>** tens + **<u>6</u>** ones
b) **<u>3</u>** thousands + **<u>10</u>** tens + **<u>8</u>** ones

7. a) 899, 1245, 4217
b) 1037, 4923, 5101, 9764

8. a) 899 **b)** 1402
c) 876 **d)** 1123

9. a) 211 **b)** 26
c) 34 **d)** 827

Supporting Students

Struggling students

• If students are having difficulty with adding, subtracting, or comparison procedures, form a small group of these students and revisit the processes they learned in Class III. You may wish to support their understanding using base ten block models.

Enrichment

• Some students may wish to invent their own game involving addition and/or subtraction of 4-digit numbers. Encourage students to write out their rules and play their created game with a partner.

Chapter 1 Whole Number Place Value

1.1.1 EXPLORE: Modelling 4-digit Numbers

Curriculum Outcomes	Outcome Relevance
4-A1 Place Value: model whole numbers to five places • recognize value of each digit	This essential exploration facilitates work with 5-digit numbers in later lessons by reminding students of processes they learned using 4-digit numbers. Students need to recognize that the meaning of each digit in a number is determined by its placement in the number.

Pacing	Materials	Prerequisites
1 h	• Base ten blocks and Base Ten Blocks (BLM)	• understanding the place value of 1- , 2- , 3- , and 4-digit numbers • comparing 4-digit numbers • recognizing that each thousand can be replaced by 10 hundreds, each hundred by 10 tens, and each ten by 10 ones • modelling tens, hundreds, and thousands

Main Points to be Raised

• Each thousand is 10 hundreds, each hundred is 10 tens, and each ten is 10 ones.

• You can think of the digits in a number as independent values. Each tells how many thousands, hundreds, tens, or ones are in the number.

• A number with more thousands is greater than a number with fewer thousands.

• The sum of the digits of a number tells the least number of base ten blocks needed to model it.

• You can rename a number by changing thousands to hundreds, hundreds to tens, and tens to ones.

Exploration

Note that actual base ten blocks (ones, tens, hundreds, and thousands) are required for this lesson. Students will not see the size relationships among the blocks if they only use the blackline master because the transition from the hundreds blocks to the thousands blocks in the BLM is not proportional.

• Show students a base ten thousands block. Help students understand that you could construct a thousands block using 1000 ones blocks, 100 tens blocks, or 10 hundreds blocks.

• Point out that one thousands block represents each 1000 can. For example, 4 thousands blocks show 4000.

• Ask students to work in pairs. If they need help interpreting the questions, provide assistance. Observe students at work. While they work, you might ask questions such as the following:

• *Why did you use more hundreds than thousands to model 2314?* (There are 2 thousands but 3 hundreds.)

• *How many blocks did you use for 2314? Why?* (I used 10 blocks since 10 = 2 + 3 + 1 + 4.)

• *How do you know how many thousands to use?* (I looked at the first digit.)

• *How did you choose which other 4-digit numbers to show with 10 blocks?* (I started with 4 thousands and I had to use 6 more blocks. Sometimes I used more hundreds and fewer tens and other times I did the opposite.)

• *How did you decide which number was greater?* (I first looked at the thousands place.)

• *Why did the total number of blocks go up by 9 when you traded?* (When I trade 1 thousand for 10 hundreds I lose 1 thousand but I gain 10 hundreds, so overall I gain 9 blocks.)

Observe and Assess

As students work, notice the following:

• Do students recognize that all base ten blocks other than the ones are built from 10 of the next smallest size?

• Do students use the correct number of each type of block to display a given number?

• Do students recognize that the total number of blocks is based on the sum of the digits?

• Do students regroup correctly to rename numbers?

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Share and Reflect

After students have had sufficient time to do the exploration, have them form groups to discuss the following questions. You might follow up with a class discussion.

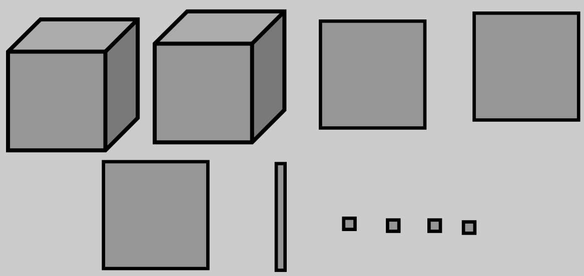
- *Why can you represent more numbers with 12 blocks than with 5 blocks?*
- *What is the least number you could represent with your number of blocks? How do you know?*
- *What is the greatest number you could represent with your number of blocks? How do you know?*

Answers

A. *Sample response*:
Alike: Edges are 10 units long except on the ones block and the short edge on the tens block.
Different: They are different sizes.

B. I can use 10 hundreds blocks to make 1 thousands block.

C.



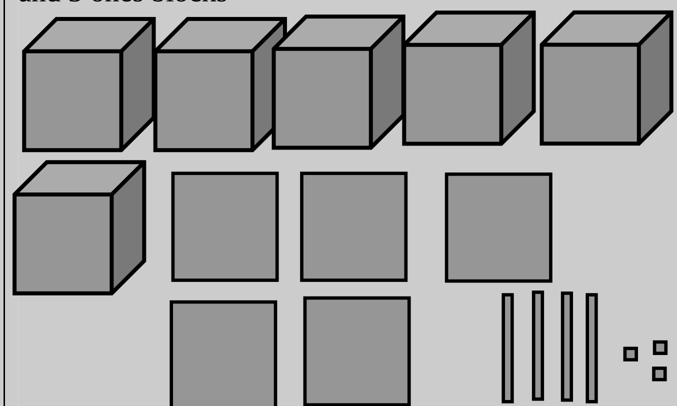
D. *Sample response*:
The number of blocks is the sum of the digits:
2 + 3 + 1 + 4 = 10 and 3 + 1 + 4 + 2 = 10.

E. 3142 needs 3 thousands, but 2314 needs only 2 thousands.

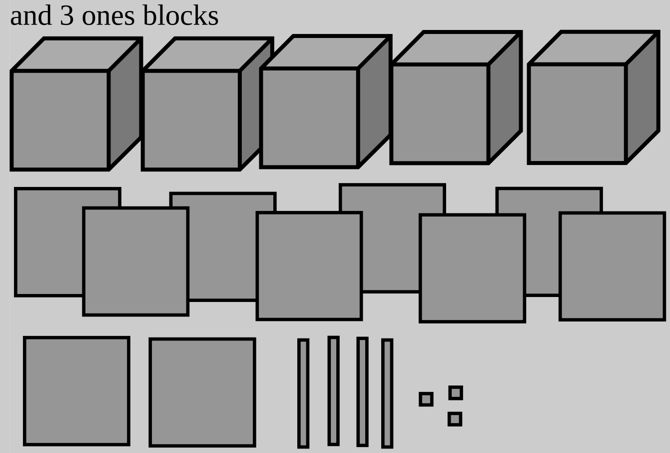
F. i) *Sample response*:
3214, 2143, 9100, 1900, and 8110
ii) *Sample response*: Greatest is 9100 and least is 2143.
iii) 3142 has 3 thousands, 1 hundreds, 4 tens, and 2 ones. If I trade 1 thousand for 10 hundreds, there are 2 thousands, 11 hundreds, 4 tens, and 2 ones.
iv) *Sample response*:
3142 ones; 1 thousands, 21 hundreds, 4 tens, and 2 ones; 314 tens and 2 ones; 31 hundreds and 42 ones
v) 3142 blocks; all ones blocks.
vi) 10 blocks; 3 thousands blocks, 1 hundreds block, 4 tens blocks, and 2 ones blocks.

G. *Sample response*: I chose to use 7 blocks.
7000 (7 thousands blocks)
6100 (6 thousands blocks, 1 hundreds block)
5110 (5 thousands blocks, 1 hundreds block, 1 tens block)
1213 (1 thousands block, 2 hundreds blocks, 1 tens block, 3 ones blocks)

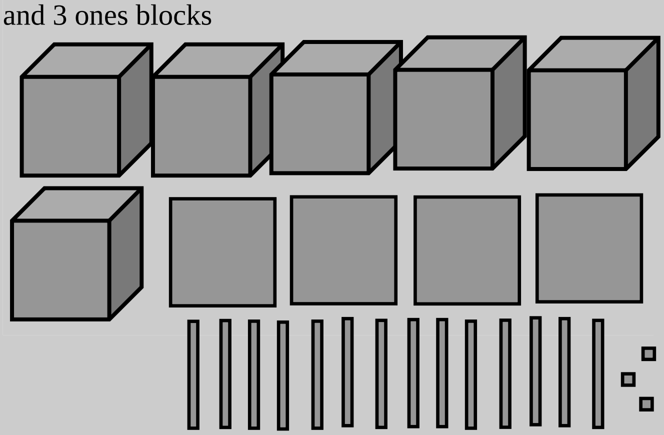
H. *Sample response*: 6543
6 thousands blocks, 5 hundreds blocks, 4 tens blocks, and 3 ones blocks



5 thousands blocks, 15 hundreds blocks, 4 tens blocks, and 3 ones blocks



6 thousands blocks, 4 hundreds blocks, 14 tens blocks, and 3 ones blocks



• Do students realize that the standard form of a number allows us to model it with the fewest blocks, and that the model with the most blocks is based on using all ones?

• Do students compare 4-digit numbers correctly?

Supporting Students

Struggling students

• If students have difficulty renaming 4-digit numbers, take time to show how trading is done.

For example, demonstrate how 3178 can be shown

as 3 thousands + 1 hundred + 7 tens + 8 ones, or

as 2 thousands + 11 hundreds + 7 tens + 8 ones, or

as 1 thousand + 21 hundreds + 7 tens + 8 ones, and so on.

Some students may also wish to trade hundreds for tens. Encourage them to do so.

• Some students may not come to the generalization that the sum of the digits of a number tells the least number of blocks required to model it. Allow them to simply model and count. Do not force them to make the generalization if they are not ready.

Enrichment

• Some students might enjoy the challenge of figuring out all the different possible numbers of blocks that could be used to represent a 4-digit number.

For example, 2125 can be represented by 10 blocks (2 + 1 + 2 + 5), 19 blocks, 28 blocks, 37 blocks, 46 blocks, 55 blocks, 64 blocks, and so on.

Students may tire of looking for all of the possibilities (there are 235 possibilities since the pattern that increases by 9 starting at 10 and goes up to 2125 has 235 values), so you might suggest that they find all the possibilities up to 200.

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1.1.2 EXPLORE: Describing 10,000

Curriculum Outcomes	Lesson Relevance
4-A1 Place Value: model whole numbers to five places • estimate the value of numbers	This optional exploration allows students to get a good sense of how much 10,000 actually is. It will make their work with large numbers more meaningful.

Pacing	Materials	Prerequisites
40 min	• Ruler, metre stick, or measuring tape	• familiarity with the number 10,000 • knowing that there are 365 days in a year • measuring in centimetres • adding tens and hundreds • recognizing simple additive patterns

Main Points to be Raised

• You can think of the number 10,000 in terms of smaller, more readily understood numbers, for example, in terms of distance and time.

• You can also think of the number 10,000 in terms of its relationship to other numbers in various patterns.

Exploration

• Begin by asking students what they know about the number 10,000. They might tell you the number of digits or that it is 10 thousands or perhaps what each digit in the number represents. Then ask students if they have some sense of how much 10,000 is. Listen to their suggestions.

For example, you could ask if there are 10,000 students in the school, or whether they think there might be 10,000 students in all the Class IVs in Bhutan.

• Suggest to them that one way of getting a sense of a number is to compare it to numbers you already know.

• Ask them how long they think 10,000 cm might be. Listen to what they suggest. Show them how long 100 cm is by showing a metre stick or the distance from the floor to the door knob. Ask how many centimetres tall each of them is. Ask why 10 students lying in a line would not make a line 10,000 cm long. Ask why 100 students lying in a line could make a line 10,000 cm long.

• Walk across the classroom and count your steps. Have students estimate how many times you could cross the classroom in 10,000 steps. Point out that they will soon have an opportunity to use their own steps.

• Write the pattern 1000, 2000, 3000, … on the board and ask why 10,000 is part of that pattern.

• Finally, ask students how many ones they would have to add to make 10,000 (10,000 ones). Ask how many thousands they would have to add (10 thousands). Ask why it makes sense that it takes 100 hundreds to make 10,000.

• Draw student attention to the introductory box on **page 5** of the student text. Point out that you have already discussed a number of the ideas raised in the box.

• Ask students to work in pairs. Observe students at work. While they work, you might ask questions such as:

• *How many days are in a year? Why would 3 years be about 1000 days?* (365 days is 1 year, so 3 years is 365 + 365 + 365 = 900 + 180 + 15, which is a bit more than 1000.)

• *I noticed you took 20 steps to cross the classroom. How many times could you cross if you took 200 steps? What if you took 1000 steps?* (10 times and 50 times)

• *How do you know your pattern contains 1000?* (I used all the thousands that were even and 10 is even.)

• *What else might happen a lot of times that you could count?* (Maybe I could see how long it would take to count to 10,000 or how much time until I blink 10,000 times.)

Observe and Assess

As students work, notice the following:

• Do students make reasonable estimates about the number of days in 30 years or about the distance that would be travelled in 10,000 steps?

• Do students use efficient and reasonable calculations to estimate or calculate?

• Do students create correct patterns to include 10,000?

• Do students come up with reasonable ideas for their own way to represent 10,000?

Share and Reflect

After students have had sufficient time to do the exploration, discuss the following questions as a class:

• *Do you think 10,000 is a big number or not?*

• *Would it seem like a big number if you were counting rice grains?*

• *Would it seem like a big number if you were counting people?*

• *Which of the ways that we thought about 10,000 in this lesson made it easiest for you to understand how much 10,000 is?*

Answers

A. *Sample response*: There are almost 400 days in 1 year, so there are almost 400 × 10 days in 10 years 400 × 10 = 400 tens = 4000. 30 years have less than 4000 + 4000 + 4000 = 12,000 days. I estimated high, so 10,000 seems about right. **B.** *Sample responses*: **i)** I would see how many times I could cross the room in 100 steps. Then I would compare that number to 10,000. **ii)** 100 steps was 8 times across. So 1000 steps would be 80 times across, and 10,000 steps would be 800 times across.	**C.** *Sample response*: 500, 1000, 1500, 2000, 2500, 3000, …; I know 10,000 is in the pattern because the pattern includes every 1000 number (1000, 2000, 3000, 4000, ...). **D.** *Sample response*: I could buy 200 snacks for Nu 10,000; My favourite snack costs Nu 50. I could buy 2 snacks for Nu 100. I could buy 20 snacks for 10 × 100 = Nu 1000. So I could buy 200 snacks for Nu 10,000.

Supporting Students

Struggling students

• If students are having difficulty understanding 10,000, you might suggest that they instead investigate ways to represent 1000. They might think about the following:

- how long it would take them to count to 1000,
- how long 1000 cm is, or
- how many pages in a book would contain 1000 words.

Enrichment

• Some students might investigate additional ways to describe 10,000.

For example, they might think about how long it would take to say 10,000 words, eat 10,000 meals, or write 10,000 words.

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1.1.3 Place Value: 5-digit Numbers

Curriculum Outcomes	Outcome Relevance
4-A1 Place Value: model whole numbers to five places • recognize value of each digit • read numbers several ways and record numbers • write numbers in expanded form • estimate the value of numbers	Students need to recognize that the meaning of each digit in a number is determined by its placement in the number. They also need to be able to read and write large numbers in various forms and estimate their value to interpret information involving large numbers meaningfully.

Pacing	Materials	Prerequisites
1 h	• Base ten blocks and Base Ten Blocks (BLM) • Place value charts or Place Value Charts I (BLM)	• understanding place value concepts for 4-digit numbers, including familiarity with the place value chart, standard form and expanded form • understanding what an odd number is

Main Points to be Raised

• The thousand, hundreds, tens, and ones blocks are different sizes. The sizes show that 10 ones make 1 ten, 10 tens make 1 hundred, and 10 hundreds make 1 thousand.

• You can use a stick of 10 thousands blocks to represent 10,000.

• We use a comma to separate the thousands digit and the hundreds digit when we write 5-digit numbers. This makes it easier to read the number quickly.

• A number that is written with only digits is in standard form. Numbers can also be written in expanded form to help interpret the value of each of the digits. Expanded form can use digits only or a combination of digits and words.

For example, you can write 32,105 in expanded form as
- 30,000 + 2000 + 100 + 5, or
- 3 ten thousands + 2 thousands + 1 hundred + 5 ones.

• A place value chart makes it easier to interpret a number quickly. You can use either numerals or counters in the spaces in the place value chart.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *What are some numbers that 90 thousand is greater than?* (1 thousand and 80 thousand)

• *What are some numbers that 90 thousand is less than?* (91 thousand and 92 thousand)

• *How many ten thousands are in 90 thousand?* (9 ten thousands)

• *Why do you think it might take more than 4 digits to write 90 thousand as a number?* (The greatest 4-digit number is 9999, and that is less than 10 thousand, which is a lot less than 90 thousand.)

The Exposition — Presenting the Main Ideas

Note that actual base ten blocks (ones, tens, hundreds, and thousands) are required for this lesson. Students will not see the size relationships among the blocks if they only use the blackline master (BLM) models because the transition from the hundreds blocks to the thousands blocks in the BLM is not proportional. Once the relationship has been established, students may use the blackline master blocks if there are not enough blocks to go around.

• Model base ten thousands, hundreds, tens, and ones blocks to review what each block represents.

• Ask students how they might represent 9 thousand (9 thousands blocks). Then ask how they might represent 10 thousand (10 thousands blocks).

• Show students how to line up 10 thousands blocks in a stick to model 1 ten thousand, or 10,000. Talk about how it is just like a line of 10 ones that makes 1 ten.

• Write the number 30,000 on the board and read it as "thirty thousand". Ask students to suggest how they might use 10,000 sticks to show 30,000.

• Write the number 32,105 on the board. Read it to students as “thirty two thousand, one hundred five”.
(Note: Avoid saying “and” before the word five, as “and” is reserved for reading the decimal point.).
Point out that the digit 3 represents 3 ten thousands, the 2 represents 2 thousands, the 1 represents 1 hundred, and the 5 represents 5 ones. Indicate that each “number” is actually a digit and that 32,105 is a 5-digit number.

• Point out the comma between the thousands and hundreds. Tell students that we use the comma to make it easier to read the number. You read the value to the left of the comma as one number (32) followed by the word “thousand”. Then you read the rest of the number as a single number.

For example, 32,105 is “32 thousand, 105”.

• Draw a place value chart on the board and show how to place the digits or counters on the chart.

Ten thousands	Thousands	Hundreds	Tens	Ones
3	2	1	0	5

• Mention that when we write a number like 32,105, the number is in what is called standard form using only numerals as digits. Write the term *standard form* on the board.

• Then write the term *expanded form* on the board and indicate that this is another way to write numbers. Using expanded form reminds us of the place value of each digit. Show two ways to write the number using expanded form, one with only numerals and one with words and numerals:

Numerals: 30,000 + 2000 + 100 + 5

Words and numerals: 3 ten thousands + 2 thousands + 1 hundred + 5 ones

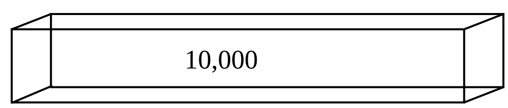
• Have students read several numbers, such as 32,100; 40,102; and 40,404. Talk about how the comma helps them read each number in two parts.

For example: 32,100 is “thirty-two thousand, one hundred”.

Indicate that even though there are three “4” digits in 40,404, each digit represents a different amount: one means 40,000, one means 400, and the other means 4.

• You may wish to review with students how to sketch the thousands block quickly (see **page 5** in this teacher’s guide). Suggest that they use the same method to sketch a 10,000 block:

- Draw a rectangle and an identical rectangle behind it.
- Connect the rectangles at the vertices.
- Label the block 10,000.



• Tell students that **pages 6 to 8** of the student text discusses the same ideas. Suggest that they read over these pages if they wish.

Revisiting the Try This

B. This question allows students to make a formal connection between their own descriptions of 90 thousand in **part A** and the more formal work on place value involving 5-digit numbers in the exposition.

Using the Examples

• Explain to students how the examples work. Each example shows a full solution of a question or problem.

- The left column shows what the student would write to solve the problem if they were asked to show their work or explain their thinking.
- The right column shows what the student might be thinking while he or she solves the problem.

• Make sure students understand the question in **example 1**. Read through the example with students to help them understand the thinking. Follow up by showing a different number with base ten blocks, e.g., 30,045, and have them repeat the thinking with the new example.

• Then read through **example 2** with students to make sure they understand the three types of models shown — the given standard form, the place value chart model with counters, and expanded form with numbers and words.

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Practising and Applying

Teaching points and tips

Q 1 a): Students can use either or both types of expanded form for this question.

Q 2: If there are not enough blocks, students can sketch pictures or use the base ten blocks paper models provided in the blackline master Base Ten Blocks.

Q 3: There are many correct answers for these questions.

Q 4: Students might be more successful if they assign the tens digit and the ones digit first.

Q 5: Any number where the digits add to 8 will work for this question.

Q 6: Students use the set of digits twice, once to create the first number and once for the second number. No two digits are 2 apart, so the ten thousands digits must be 1 apart and the thousands digits must be as close as possible to 9 apart, or the ten thousands digits could be 3 apart with the thousands digit of the smaller number as high as possible and the thousands digit of the higher number as low as possible.

Q 7: This question might be handled as a whole group discussion. Students should realize that the least and greatest 4-digit numbers, 1000 and 9999, are only about 9000 apart, but the least and greatest 5-digit numbers, 10,000 and 99,999, are about 90,000 apart.

Common errors

• Many students have difficulty working with numbers that have 0 digits. Include a number of these values when you discuss in the exposition how to represent 5-digit numbers.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can write 5-digit numbers in different forms
Question 3	to see if students understand place value
Question 5	to see if students recognize how base ten block models relate to the digits in 5-digit numbers

NOTE: Answers or parts of answers to numbered questions that are in square brackets throughout the Teacher's Guide are NOT found in the answers at the back of the student text. (See ***Answers*** *on* ***page xiv*** *in the* ***Introduction*** *to this Teacher's Guide.)*

Answers

A. *Sample response*:
It is 90 thousands.
It is 9 ten thousands.
It is 1 thousand more than 89 thousands.
It is 1 thousand less than 91 thousands.

B. i) 9 sticks of 10,000, where each stick is 10 thousands blocks
ii) *Sample response*: 9 ten thousands
iii) *Sample response*:

Ten thousands	Thousands	Hundreds	Tens	Ones
9	0	0	0	0

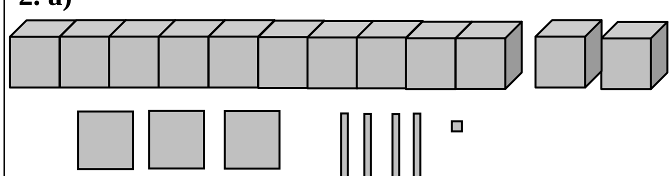
1. a) 3 ten thousands + 2 thousands + 1 hundred + 2 tens + 1 one
or
30,000 + 2000 + 100 + 20 + 1

b) *Sample response*:

Ten thousands	Thousands	Hundreds	Tens	Ones
3	2	1	2	1

c) 32,121

2. a)



b) *Sample response*:

Ten thousands	Thousands	Hundreds	Tens	Ones
1	2	3	4	1

c) 1 ten thousand + 2 thousands + 3 hundreds + 4 tens + 1 one
10,000 + 2000 + 300 + 40 + 1

Lesson 1.1.3 Answers [Continued]

3. *Sample responses*: **a)** 32,143 **b)** 10,432 **c)** 12,345 **4.** *Sample responses*: **a)** 49,823 **b)** 49,723 **5.** *Sample responses*: **a)** 21,113; 31,112; 11,213 **b)** 8 counters for each. **c)** The sum is 8 for each.	**6.** *Sample response*: 69,152 and 51,269 [**7.** *Sample response*: The 5-digit numbers go from 10,000 to 99,999. That is more than 89 thousand numbers. The 4-digit numbers only go from 1000 to 9999. That is less than 9000 numbers.]

Supporting Students

Struggling students

• Some students have trouble with one form for representing 5-digit numbers, but not with other forms. If students struggle, you might let them use the representation that is more meaningful to them.

• Students who are struggling might have difficulty with **question 4**. In place of that question, you might simply create numbers and have these students interpret them.

Enrichment

• Students might make up more questions like **questions 4 and 5** for classmates to solve.

1.1.4 Renaming Numbers

Curriculum Outcomes	Outcome Relevance
4-A1 Place Value: model whole numbers to five places • read numbers several ways and record numbers • write numbers in expanded form	As they understand different ways to name numbers, students become more flexible with numbers and develop a stronger number sense. This allows for more fluency and flexibility with calculations.

Pacing	Materials	Prerequisites
1 h	• Place value charts or Place Value Charts I (BLM) • Base ten blocks or Base Ten Blocks (BLM)	• modelling 4-digit numbers with base ten blocks • familiarity with the place value system patterns, i.e., each column represents a value 10 times as great as the column to the right • familiarity with the sequence of columns in the place value system

Main Points to be Raised

• You can find the number of thousands in a number by using the digit to the left of the hundreds place. You can find the number of hundreds in a number by using the digit to the left of the tens place. This strategy works since you can always trade one 10,000 for 10 thousands and one 1000 for 10 hundreds.

• You can rename a number by trading some or all of its parts for smaller parts, e.g., some or all the thousands for hundreds.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How many hundreds are in 200? How do you know?* (2, since it is 2 hundreds.)

• *How many hundreds are in 1000? 2000?* (There are 10 hundreds in 1000, so there are 20 hundreds in 2000.)

• *How do you read 98,676?* (“Ninety-eight thousand, six hundred seventy-six”)

• *How can that help you figure out how many groups of 1000 are in 98,676?* (Because I said 98 thousand, I know there are 98 groups of 1000.)

• *How can that help you figure out how many groups of 100 are in 98,676?* (98 groups of 1000 is 980 groups of 100 because there are 10 times as many hundreds as thousands.)

The Exposition — Presenting the Main Ideas

• Model the number 12,300 using 1 ten thousand base ten stick, 2 thousands blocks, and 3 hundreds blocks. Point out how to write the number on a place value chart with each digit in the correct place.

Ten thousands	Thousands	Hundreds	Tens	Ones
1	2	3	0	0

Show how the ten thousand stick can be traded for 10 thousands, so there would be 12 thousands + 3 hundreds. Write it on a place value chart like this:

Ten thousands	Thousands	Hundreds	Tens	Ones
	12	3	0	0

On the board write 12 thousands + 3 hundreds. This is an expanded form of the number.

Have students note that the 12 in the thousands column is the number made up of the original digit in the thousands column and all digits to its left. You might show this by using your hand to cover the digits to the right of the thousands, i.e., over the 3, 0, and 0 in the original place value chart.

• Now show how 12,300 can be renamed in terms of hundreds only.

Each of the 12 thousands is traded for 10 hundreds, which adds 120 hundreds to the existing 3 hundreds, for a total of 123 hundreds. On a place value chart, you could write:

Ten thousands	Thousands	Hundreds	Tens	Ones
		123	0	0

Again show students where the number 123 comes from. It is a combination of the values that appeared in the ten thousands, thousands, and hundreds columns in the original place value chart.

• Indicate that this last chart model shows that 12,300 is made up of 123 hundreds (since there are 0 tens and 0 ones). The previous chart showed that it was made up of more than 12 thousands and 3 hundreds, which is not quite 13 thousands.

• Indicate to students that a summary of these ideas is presented on **pages 11 and 12** in the student text so that they can refer back to them as they work through the examples and exercises.

Revisiting the Try This

B. This question allows students to make a formal connection between the number of groups they found in **part A** and the place value concepts they learned in the exposition.

Using the Examples

• Work through **example 1** with students. Help them understand how the student first traded 1 ten thousand for 10 thousands and then traded the rest of the ten thousands, all at once, for 50 thousands. Make sure they understand that each line in the chart represents the same number.

• Read the question in **example 2** as a whole class and then assign students to pairs before reading the solution. You may wish to write the question on the board with textbooks closed so that students do not look at the solution as they work. When they have finished, they can compare their work with the solution shown in the text.

• Make sure students understand that in **example 1** they are going from standard form to alternative forms, but in **example 2** they are going from an alternative form (expanded form using words) to standard form.

Practising and Applying

Teaching points and tips

Q 1: Students' sketches need not be detailed.

Q 2: Make sure students notice that they are not just writing in expanded form, but that they are also renaming the number.

Q 3: Students can use a place value chart.

Q 4: You may need to help some students see that this question asks them to rename each number in the form ___ hundreds. It might help if they use a place value chart.

Q 5 and 6: These questions show one of the most important uses of the concept of renaming numbers. To convert metric measurement units to other units, we use the strategies in this lesson.

For example:

1 m = 100 cm is parallel to 1 hundred = 100 ones

Q 7: Encourage students to talk about their answers with a partner.

Common errors

• Many students are more successful going from standard form to non-standard form than the other way around. You may wish to provide more experiences of the first sort and then have students look at their work, going backwards, before asking them to begin with non-standard forms.

• Many students, if asked how many thousands are in 22,145, will say 2 and not 22 since the digit 2 is in the thousands place. Be careful to distinguish between the questions "What digit is in the thousands place of 22,145?" (the answer is 2) and "How many thousands are in 22,145?" (the answer is 22).

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Suggested assessment questions from Practising and Applying

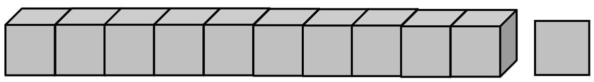
Question 2	to see if students can move from standard form to non-standard form
Question 3	to see if students can move from non-standard form to standard form
Question 5	to see if students can apply place value concepts to a practical situation
Question 7	to see if students can communicate about how to represent a number in different ways

Answers

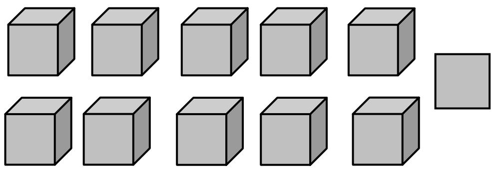
A. *Sample responses*:
i) About 99 groups
ii) About 990 groups

B. 98,676 = 98 thousands + 6 hundreds + 7 tens + 6 ones, so
98,676 is about 98 groups of 1000.
98,676 = 986 hundreds + 7 tens + 6 ones, so 98,676 is about 986 groups of 100.

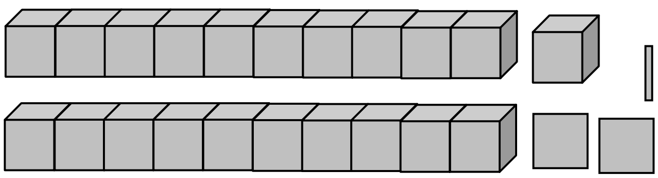
1. a)



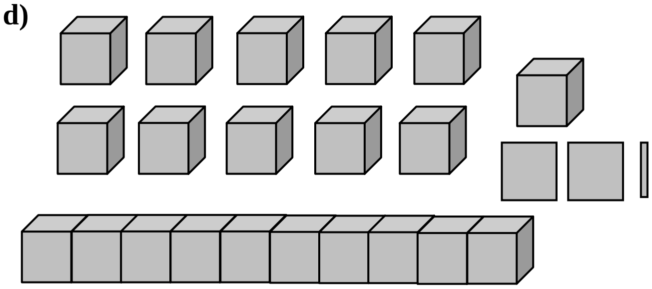
b)



c)



d)



2. a) **30** thousands + **4** hundreds + **7** tens
b) **124** hundreds + **8** tens
c) 3 ten thousands + **10** thousands +
2 hundreds + **81** ones

3. a) 42,003 **b)** 51,070 **c)** 17,025
d) 48,000 **e)** 15,208

4. a) 121 boxes
b) 150 boxes
c) 162 boxes (with 80 pencils left over)
d) 82 boxes (with 45 pencils left over)

5. a) 130 m
[**b)** *Sample response*:
It is like writing 13,000 as ___ hundreds.]

6. a) 27 km
[**b)** *Sample response*:
It is like writing 27,000 as ___ thousands.]

[**7.** *Sample response*:
• I would have him write 32,100 in expanded form:
3 ten thousands + 2 thousands + 1 hundred.
• Then I would tell him to trade the 3 ten thousands for 30 thousands:
32 thousands + 1 hundred
• Then, I would tell him to trade the 32 thousands for 320 hundreds:
321 hundreds.]

Supporting Students

Struggling students

• Some students struggle with renaming when they do not use the place value chart. Encourage them to use the chart. Make sure they understand that each time they remove 1 from any column, they add 10 to the column to its right, or vice versa.

• Some students may have difficulty with the applications in **questions 4, 5, and 6**. If so, partner them with other students who are not struggling.

1.1.5 Comparing and Ordering Numbers

Curriculum Outcomes	Outcome Relevance
4-A2 Compare and Order : whole numbers to five digits • order two or more numbers and justify order • identify numbers greater or less than a given number and numbers between given numbers	In everyday life we are frequently called on to decide which of two amounts is greater. For example, a town with a greater population than another receives more government services. Often these numbers are more than 10,000. It is important for students to be able to compare such numbers.

Pacing	Materials	Prerequisites
1 h	None	• representing numbers on a number line and recognizing that numbers farther to the right are greater • familiarity with the symbols < and >

Main Points to be Raised

• When two numbers have the same number of digits, the number that has a greater value in the leftmost place value or digit is the greater number. If those digits are the same, you keep moving to the right until the digits differ. The number with the greater digit in that place is the greater number.

• If one whole number has fewer digits than a second whole number, it is less than the second number.

• A number is between two other numbers if it is greater than one number but less than the other.

• If you use a number line to compare and order numbers, the number farthest to the right is the greatest.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you read the population of Ha? of Samtse?* (“Eleven thousand, six hundred forty-eight”; “Sixty thousand”)

• *How can reading the numbers out loud help you compare them?* (When I hear “11 thousand” compared to “60 thousand”, I know that 11 thousand is less since 11 is less than 60.)

• *Why did the digits in the ones place not matter?* (If one dzongkhag has more thousands of people than another dzongkhag, the larger dzongkhag has more people even if it has a smaller number in the ones place.)

The Exposition — Presenting the Main Ideas

• Ask students to create any two 5-digit numbers. Write them in a place value chart.

For example:

Ten thousands	Thousands	Hundreds	Tens	Ones
1	2	3	4	5
5	4	3	2	1

Ask students to say which number they think is greater and why. Encourage them to visualize how to model each number with blocks or counters. Help them see how the greater number would involve a model with more ten thousands. Point out how they can tell this from the standard form of the numbers — the digit in the ten thousands place for 54,321 is greater than the digit in the ten thousands column of 12,345. Draw a parallel to what they learned about comparing 2-digit, 3-digit and 4-digit numbers.

• Remind students that the < and > signs are used to compare numbers. The open end of the sign is next to the greater number. It points to the lesser number. So for the comparison above, they could write 12,345 < 54,321 or 54,321 > 12,345.

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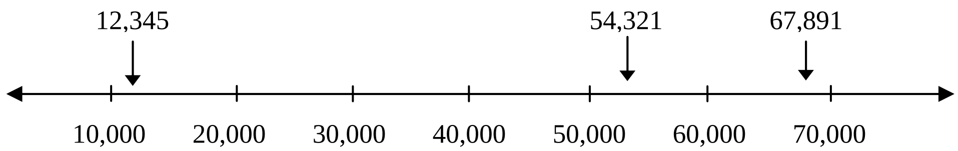
• Ask students to suggest an even greater number. Add it to the chart. For example:

Ten thousands	Thousands	Hundreds	Tens	Ones
1	2	3	4	5
5	4	3	2	1
6	7	8	9	1

Ask why they think the last number is greater.

• Demonstrate how to show the order of the three numbers by using each comparison separately or by showing both comparisons together.
For example: 12, 345 < 54,321 and 54,321 < 67,891 or 12,345 < 54,321 < 67,891

• Draw a number line and estimate the positions of the three values you are comparing.



Help students notice that the values farther to the right represent greater numbers.

• Indicate to students that a summary of these ideas is presented on **page 15** in the student text. They can refer back to the summary as they work through the examples and exercises.

Revisiting the Try This

B. This question allows students to compare the populations presented in **part A** more formally using number lines, place value charts, or comparing digits.

Using the Examples

• Have students work in pairs. One student should study **example 1** and become an “expert”. The other can study **example 2**. Ask students to explain their example to their partner. In this way, students will learn to communicate clearly about the concepts being presented.

• Have students close their texts. Write the question for **example 3** on the board. Students can work in pairs to answer the question and then compare their solutions and thinking with what is in the text.

Practising and Applying

Teaching points and tips

Q 1: Students can use place value charts, sketch number lines, or compare digits to answer this question.

Q 2: Encourage students to write the order in one line using the format [ ] < [ ] < [ ].

Q 3: Assure students that these data values about Bhutan are accurate.

Q 5: This question is more challenging than some of the others since the students must create the numbers. Some students might find it easier to write the digits on slips of paper and move the slips of paper around.

Q 6: Encourage students to begin by writing examples of possible numbers between 20,000 and 22,000.

Q 7: This question might be handled best in a whole class discussion.

Common errors

• Some students only look at the leftmost digits of numbers to compare them. This does not work if the numbers have a different number of digits. Remind students to think about place value as they compare digits.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can order 4-digit and 5-digit numbers
Question 4	to see if students understand how a number can be between two other numbers
Question 6	to see if students can reason about the sizes of numbers that fit a certain criterion and apply place value concepts to describe those numbers

Answers

A. Samtse; *Sample response*: 60 thousand > 11 thousand and 60 thousand > 13 thousand, since 60 > 11 and 60 > 13.	**B.** Trongsa is between Ha and Samtse; *Sample response*: 13,419 > 11,648 but 13,419 < 60,100
1. a) 42,978 **b)** 51,302 **c)** 82,135 **d)** 53,299 **2. a)** 10,003 < 13,287 < 15,149 **b)** 7820 < 28,147 < 32,875 **3.** The number of homes with 3 or 4 people; [*Sample response*: 39 thousand > 26 thousand] **4.** *Sample response*: 15,900; 16,000; 16,001	**5.** *Sample responses*: **a)** 70,124 **b)** 10,247 **c)** 24,107 **d)** 14,072 **6. a)** 2 (ten thousands) **b)** It could have 0 or 1 thousand; [*Sample response*: Both 2**0**,100 and 2**1**,100 would work.] [**7.** *Sample response*: You start from the left and compare digits with the same place value. If the left digits are the same, you move to the right. You do this no matter how many digits the numbers have.]

Supporting Students

Struggling students

• Some students will need a place value chart to make comparisons. They might find it difficult to place numbers on the number line. Allow and encourage those students to use a place value chart.

• It might be difficult for some students to create numbers to meet conditions as required in **questions 5 and 6**. You might choose instead to give them numbers and ask them questions about them.

For example, for the number 32,145 you might ask:

- *What three numbers are greater than 32,145?*
- *What three numbers are less than 32,145?*
- *What number is less than 32,145 but has the same ten thousands digit?*
- *What number is greater than 32,145 but has three digits the same?*

Enrichment

• Have students choose five different digits, with a 0 as one of the digits. Ask them to create ten different numbers using those digits and to order them from least to greatest. Have them include the greatest possible number and the least possible number that could be created.

GAME: As High as You Can

• This game allows students to practise comparing 5-digit numbers.

• Instead of rolling a die, students can put digit cards 1 to 6 in a bag and draw them randomly.

• If students are sharing one die, they will have to take turns completing their boxes. If multiple dice are available, students can complete their boxes at the same time.

• After students have played the game at least once, talk about:

- why they may wish to use the highest digits rolled in the leftmost boxes
- why they might not always end up with the greatest number possible for the digits rolled.

For example, a student may roll a 5 and record it in the leftmost box because she thinks she might not roll any more high digits. But then she rolls a 6 and can only record it the thousands place because the ten thousands place is already filled.

Chapter 2 Addition and Subtraction

1.2.1 Adding and Subtracting Mentally

Curriculum Outcomes	Outcome Relevance
4-B2 Add and Subtract Mentally: to four digits • develop and use mental strategies: front end, compensation, counting on/back, compatible numbers • determine when it is most suitable to use mental addition and subtraction	The ability to add and subtract mentally is a useful life skill. We are often in situations where it is not convenient to use a pencil and paper or a calculator.

Pacing	Materials	Prerequisites
1 h	None	• adding and subtracting 2-digit and 3-digit numbers in parts • knowing that you can add too much if you take the excess away • knowing that you can subtract too much if you add back the extra you subtracted • familiarity with place value addition and subtraction concepts, e.g., realizing that 3456 + 2 thousand changes the thousands digit by 2 • understanding the relationship between addition and subtraction

Main Points to be Raised

• You can perform mental addition by adding the second number in parts to the first number.

For example, first add the thousands, then the hundreds, then the tens, and finally the ones.

• You can perform mental addition by adding an easier, nearby number and then adjusting the answer to take into account either the extra you added or the missing amount you did not add.

• You can add two numbers by adding parts of one number to parts of the other number in more convenient ways. Make sure all the parts are included in the addition.

• You can perform mental subtraction by counting up in easy steps from the lower number to the higher number, using the relationship between addition and subtraction. A number line is a useful tool for this sort of thinking.

• You can perform mental subtraction by subtracting the second number in parts from the first number.

For example, first subtract the thousands, then the hundreds, then the tens, and finally the ones.

• You can subtract by subtracting an easier number that is a bit too much and then adding back the extra you subtracted.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know Tshering earned more than Nu 5850?* (Nu 5850 is Nu 3000 more than Nu 2850. She earned more than Nu 3000 in the second month.)

• *How do you know that Tshering earned less than Nu 6600?* (If I add Nu 3000 to Nu 3600, I get Nu 6600. The amount she earned in the first month was less than Nu 3000.)

• *How did you add?* (I added the thousands together, then the hundreds together, then the tens, and then the ones. Since I had 14 hundreds, I traded for 1 thousand and 4 hundreds.)

The Exposition — Presenting the Main Ideas

• Write 425 + 325 on the board. Ask students how they might add using mental math. There are several possibilities, but they might add the 400 + 300 to get 700, add the 25 + 25 to get 50, and then put the parts together to get 750.

• Then ask about these calculations: 378 + 399, 425 – 315, and 712 – 398.

In each case, discuss strategies students might use to perform each calculation using mental math.

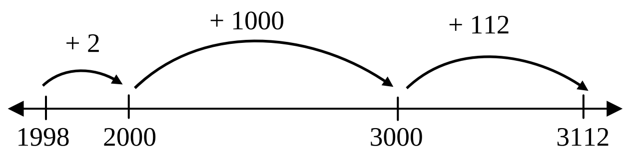
For example:

- For 378 + 399, they might add 400 and then subtract 1.
- For 425 – 315, they might first subtract the 300, then the 10, and finally the 5.
- For 712 – 398, they might subtract 400, since it is easier, and then add back the extra 2 they took away.

• Discuss how the same ideas work using 4-digit numbers. On the board, write 3875 + 4225. Ask students to work in pairs to come up with some mental math strategies. They can then share their ideas with the class. Do the same for 9125 – 3994. Make sure the various strategies mentioned on **pages 19 and 20** come up in the discussion.

• When you discuss the adding up strategy, help students see how a sketch of a number line model can be useful to record their thinking.

For example, for 3112 – 1998, you can show a small jump of +2 to get to 2000, then a large jump of + 1000 to get to 3000, and finally a jump of +112. Students have to add all the jumps to find the difference.



3112 – 1998 = 2 + 1000 + 112 = 1114

Try to space the numbers on the line so that the sizes of the jumps are reasonably proportional. Make sure students know that the number line model is simply a sketch and need not be exact.

• Tell students to refer to **pages 19 and 20** in the student text as they work through the examples and exercises.

• You might create a poster that summarizes some of the methods, or strategies. Students can refer to it.

Mental Addition Strategies	**Mental Subtraction Strategies**
• Add in parts. • Add too much and then take away the extra you added. • Add parts of one number to parts of the other and then combine.	• Subtract in parts. • Subtract too much and then add back the extra you subtracted. • Add up; you might use a number line.

Revisiting the Try This

B. This question helps students recognize how they can use the mental math strategies they learned in the exposition to simplify the calculation they did in **part A**.

Using the Examples

• Write the questions from **example 1** on the board. Ask students to close their texts. Have them work in pairs to discuss mental math strategies to perform the computations. They can then compare their ideas with those in the text.

• Write the questions from **example 2** on the board. Ask students to close their texts Have them work in pairs to discuss mental math strategies to perform the computations. They can then compare their ideas with those in the text. Make sure they understand that the strategy used in **part b)**, dealing with one set of 2 digits and then with the other set of 2 digits, only works if the numbers being subtracted have digits that are less for each set of 2 digits. For example, it works for 3825 – 1915, but not for 3825 – 1975 without some regrouping.

Practising and Applying

Teaching points and tips

Q 1: Students might refer to the poster you created to help them consider possible strategies.
For example, for **part a)**, they might add 3711 in parts or they might add 5289 by adding 5300 as 5000 + 300 and then take away the extra 11. For **part b)**, they are most likely to add 3000 and take away 7.

Q 2: For **part a)**, students are most likely to take away 3000 and add back 1, but you should accept other strategies as well. For **part b)**, some students might subtract 4000 from 9000, add the extra 4 from the 9004, add back 200, and then subtract 2.

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Q 4: Students need to understand fully the relationship between subtraction and addition to answer this question. For **part a)**, make sure they realize that they must use mental math to add the jump amounts to solve the subtraction.

Q 5 and 6: There are many correct answers to this question. Students can choose any numbers at all, as long as they can explain clearly how they would use mental math to calculate.

Q 7: This might be handled best as a group discussion where students talk about why certain calculations lend themselves more than others to mental math. Students should be aware that there are individuals who can handle any calculation with mental math.

Common errors

• Many students who add or subtract using an easier nearby value change their answers incorrectly.

For example, if they calculate 3000 – 1998 by subtracting 2000, they often subtract an extra 2 rather than adding an extra 2 at the end.

Have students focus on whether they have added too much (in which case they should subtract) or subtracted too much (in which case they need to add back).

• Some students may subtract the least digit from the greatest digit without keeping track of subtracting the least number from the greatest number.

For example, if the numbers were 9002 – 3804, students might subtract 2 from 4 because 2 < 4 without realizing that 3804 < 9002. These students are applying a rote procedure without thinking. Encourage them to estimate first, which will help them attend to the values of the two numbers involved in a more holistic way.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use mental math strategies to add
Question 2	to see if students can use mental math strategies to subtract
Question 3	to see if students can communicate about mental math strategies
Question 7	to see if students recognize when mental math strategies are most useful

Answers

A. Nu 6450	**B.** *Sample response*: Add 4000 instead of 3600 to 2850 to get 6850 and then subtract the extra 400 you added to get 6450.
1. a) 9000; [*Sample response*: 5289 + 3711 Start with 5289. Add the 3700 part of 3711 by adding 4000 instead of 3700: 5289 + 4000 = 9289. Subtract 300: 9289 – 300 = 8989. Add 11 by adding 10 and then 1: 8989 + 10 = 8999 → 8999 + 1 = 9000.] **b)** 6839; [*Sample response*: 3846 + 2993 Start with 3846. Add 3000 instead of 2993: 3846 + 3000 = 6846. Subtract the extra 7 I added by subtracting 6 and then 1: 6846 – 6 = 6840 → 6840 – 1 = 6839.] **2. a)** 4126; [*Sample response*: 7125 – 2999 Start with 7125. Subtract 3000 instead of 2999: 7125 – 3000 = 4125. Add back the 1 extra I subtracted: 4125 + 1 = 4126.]	**b)** 5202; [*Sample response*: 9004 – 3802 Subtract the thousands and hundreds first: 9000 – 3800 = 5200. Subtract the ones: 4 – 2 = 2. Add the parts: 5200 + 2 = 5202.] [**3.** *Sample responses*: **a)** You can subtract 4000 by changing the thousands digit and then adding the extra 1 you subtracted. **b)** You can add 4000 by changing the thousands digit and then subtracting the extra 1 you added.] **4. a)** 2100 – 1476 = 624 **b)** 1476 + 624 = 2100 **5.** *Sample response*: 3999; [I would only have to change the thousands digit and then subtract 1.]

Lesson 1.2.1 Answers [Continued]

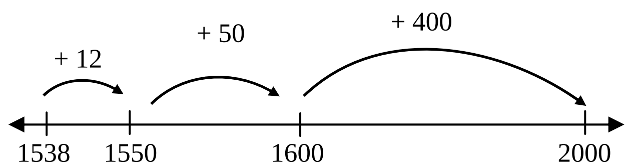
6. *Sample response*: 2999; [I could subtract 3000 by changing the thousands digit and then add back the extra 1 that I subtracted.]	[**7.** *Sample response*: • For 3075 + 2125, I know that 75 + 25 = 100. I also know 3000 + 2100 = 5100 (30 hundreds + 21 hundreds = 51 hundreds). I can add 100 + 5100 = 5200 using mental math. • For 3178 + 5767, I have to do additions like 78 + 67, which are more difficult to do in my head.]

Supporting Students

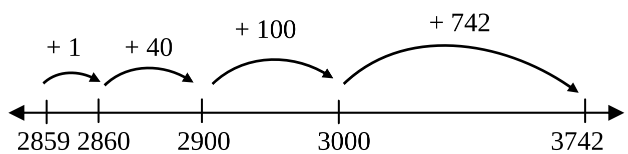
Struggling students

• Some students have difficulty putting themselves in the place of thinking like someone else. For them, **question 3** might be difficult. Instead, ask how they would calculate using mental math.

• Some students may need more examples of subtracting by adding up before they can deal with **question 4**. You might first provide examples like those shown below and then have them consider the question.



Subtraction: **2000 – 1538** = 12 + 50 + 400 **= 462**   Addition: 1538 + 12 + 50 + 400 = **1538 + 462 = 2000**



Subtraction: **3742 – 2859** = 1 + 40 + 100 + 742 **= 883**   Addition: 2859 + 1 + 40 + 100 + 742 = **2859 + 883 = 3742**

• Some students may have difficulty choosing numbers in **questions 5 and 6.** You may instead give them choices about which they would prefer.

For example, which calculation, 3812 + 3998; 3812 + 4756; or 3812 + 9999, would you choose to do with mental math? Why?

Enrichment

• Have students write a small "manual" to help a fellow student figure out how and when to add using mental math. They can choose their own examples and bring out whichever points they think are most important.

GAME: Add High and Subtract Low

• This game allows students to practise mental addition and subtraction.

• If students are sharing one die, they will have to take turns making their numbers. If multiple dice are available, students can make their numbers at the same time.

• After students have played the game at least once, talk about these things:

- Ask why they chose the values they did for the extra digits of their own choice.
- Then ask how they chose how to organize the digits for the greatest sum and least difference.

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1.2.2 Estimating Sums and Differences

Curriculum Outcomes	Outcome Relevance
4-B1 Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers • apply familiar addition and subtraction strategies to numbers with five or more digits • continue estimating	The ability to estimate is important both because sometimes an estimate is all that is required in a particular situation and because a good estimate is a way to check on the reasonableness of a calculated sum or difference.

Pacing	Materials	Prerequisites
1 h	None	• adding and subtracting 2-digit numbers • reading and interpreting 5-digit numbers

Main Points to be Raised

• An estimate tells *about* how many.

• To estimate a sum or difference, you should focus on the digits in the greater place values (e.g., the thousands and ten thousands place for 5-digit numbers).

• There is never one correct estimate. There are always different ways to estimate.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Why did you decide to add for **part i)**, but to subtract for **part ii)**?* (**Part i)** asks for a total but **part ii)** asks for a comparison, and that means I need to subtract.)

• *How do you know the total number is about 50,000?* (33 thousand is close to 35 thousand and 16 thousand is close to 15 thousand; 35 thousand + 15 thousand = 50 thousand)

• *How do you know the difference is around 16,000?* (16 thousand + 16 thousand = 32 thousand and 32 thousand is close to 33 thousand.)

The Exposition — Presenting the Main Ideas

• Write the numbers 22,179 and 35,812 on the board. Ask students to read the numbers. Then write them in a place value chart.

Ten thousands	Thousands	Hundreds	Tens	Ones
2	2	1	7	9
3	5	8	1	2

Ask why

- the first number is about 22 thousand and the second number is about 35 or 36 thousand.

- they think the sum might be about 5 or 6 ten thousands (2 ten thousands + 3 ten thousands = 5 ten thousands, although 35,812 is also close to 4 ten thousands).

- the sum might about 57 thousand or 58 thousand (22 thousand + 35 thousand = 57 thousand, although 35,812 is about 36 thousand).

• Use the word *estimate* to describe what they have done, i.e., told “about how much” the total is.

• Now ask students to estimate the difference between the two numbers, i.e., how far apart the two numbers are. They might think “3 ten thousands – 2 ten thousands = 1 ten thousand” or “35 thousands – 22 thousands = 13 thousands”. They might choose 36 thousand or 4 ten thousands for the greater number.

• If you feel it is needed, use another example, perhaps 42,315 + 53,495. Students should understand they can round one number to ten thousands and round the other number to thousands.

For example: 42,315 + 53,495 is about 40,000 + 54,000 = 94,000.

• Point out to students that they can find the information you have presented on **page 25** of the student text.

Revisiting the Try This

B. This question allows students to recognize that they can make choices when they estimate. They apply that concept to the problem they solved in **part A.**

Using the Examples

• Write the questions from **example 1** and **example 2** on the board. Ask students to work in pairs to answer the questions. Then discuss with the class the solutions provided in the text. Poll the class to ask how close their thinking was to the estimates shown in the solutions in the text. Encourage them if they come up with different approaches and estimates than the ones given in the text.

Practising and Applying

Teaching points and tips

Q 1 and 2: Some students use only the ten thousands digits and some use both the thousands and ten thousands. Some round up, e.g., 53,702 to 54,000, and others do not. For an estimate, any of these choices is acceptable. It is good for students to see that when they add, if they increase one value and decrease the other, the estimate will be better than if they increase both or decrease both.

Q 2: Students should realize that the estimate for a difference will be better if they increase both values or decrease both values, rather than increasing one and decreasing the other.

Q 3: Encourage students to think of 47,000 as 47 thousands and then to think of pairs of thousands that add to 47 thousands. For each of those values, they can use a nearby number with no zeros.

Q 4: Encourage students to think of 12,000 as 12 thousands. They can then look for two numbers with a difference of 12, both of which are thousands, e.g., 42 thousand and 30 thousand.

Q 5: This problem may be a challenge for some students. They might begin with a pair of numbers that add to 36 and see how far apart they are. They can then increase one number and decrease the other number until the difference is 8.

For example, if they start with 18 thousand and 18 thousand, the numbers are not 8 thousand apart. So they might increase the first 18 to 24 and decrease the other 18 to 12. The difference now is 12 thousand, which is too much.

Q 6: Some students might need prompting for this. Have them think about whether the population of Thimphu today is the same as it was a week ago.

Common errors

• Some students calculate exactly, round their result, and present it as their estimate. You must insist that students estimate each value independently and then calculate to get an estimate. It is this type of estimating that is most useful in everyday life.

• Some students might struggle with the language of “estimate”. This word mean the act of estimating (a verb, e.g., "Estimate the sum."). The same word also means the result (a noun, e.g., “The estimate is about 5000.”). In written form the words are exactly the same but they are pronounced differently. As a verb, it is pronounced “es-ti-mate”. As a noun, it is pronounced “es-ti-mit”. Model the correct use of the word both ways and explain to students that it can be used in both ways.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can estimate sums
Question 2	to see if students can estimate differences
Question 6	to see if students have a sense of when estimation is appropriate

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Answers

A. *Sample responses*: **i)** About 49,000 **ii)** About 17,000	**B.** *Sample responses*: **i)** About 49,000; 33,169 + 16,116 is about 33,000 + 16,000 = 49,000. About 46,000; 33,169 + 16,116 is about 30,000 + 16,000 = 46,000. My first estimate rounds both numbers to the nearest thousand, so this is a good estimate. My second estimate rounds one of the numbers to the nearest ten thousand so it is easier to add using mental math by increasing the ten thousands digit of 16,000 by 3. **ii)** About 17,000; 33,169 – 16,116 is about 33,000 – 16,000 = 17,000. About 10,000; 33,169 – 16,116 is about 30,000 – 20,000 = 10,000. My first estimate rounds to the nearest thousand, which is precise enough to give a good idea of the size difference between dzongkhags. My second estimate rounds the numbers to the nearest ten thousand so that the mental math is easier and I could get the result more quickly.
1. *Sample responses*: **a)** About 70,000; [30,000 + 40,000] **b)** About 70,000; [54,000 + 16,000] **c)** About 60,000; [30,000 + 30,000] **d)** About 58,000; [18,000 + 40,000] **2.** *Sample responses*: **a)** About 25,000 **b)** About 6000 **c)** About 35,000 **d)** About 60,000 **3.** *Sample response*: 16,999 and 29,999 **4.** *Sample response*: 41,999 and 30,121	**5.** *Sample response*: 21,999 and 14,111 [**6.** *Sample response*: The population changes every day so it will always be an estimate and not exact.] [**7.** *Sample response*: Estimate using only ten thousands: 33,295 + 18,492 is about 30,000 + 20,000 = 50,000, so 33,295 + 18,492 is about 50,000. Estimate using ten thousands and thousands: 33,295 + 18,492 is about 33,000 + 18,000 = 51,000, so 33,295 + 18,492 is about 51,000.]

Supporting Students

Struggling students

• Some students have difficulty making decisions about whether to estimate using only ten thousands or using both ten thousands and thousands. Although they should make the choice on their own, if they are struggling too much you might suggest that they always use just the ten thousands unless the numbers are less than 10,000. Or, suggest that they do it both ways and then decide which is the better estimate.

Enrichment

• Have students prepare other questions like **question 5** and trade with classmates. They can then solve each other's problems.

GAME: Estimating the Range

• This game allows students to practise estimating sums and differences of 5-digit numbers.

• After students have played the game at least once, talk about how they arranged the numbers and chose the operation to maximize their points.

• Discuss with students how this game does not require them to calculate exactly, as they can estimate the answer within a wide range.

• Instead of playing cards, students can use digit cards. They need four sets of 0 to 9 cards.

1.2.3 Adding 5-digit Numbers

Curriculum Outcomes	Outcome Relevance
4-B1 Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers • apply familiar addition and subtraction strategies to numbers with five or more digits • continue estimating	Although many people use calculators to add large numbers, there are still occasions when calculations are done by hand. It is important for students to be able to perform these calculations.

Pacing	Materials	Prerequisites
1 h	• Place value charts or Place Value Charts I (BLM) (optional)	• adding 4-digit numbers • estimating the sum of 5-digit numbers

Main Points to be Raised

• Adding 5-digit whole numbers is based on the same procedures used for adding 4-digit whole numbers.

• You always add place values that are the same, i.e., tens with tens, thousands with thousands, and so on.

• You can add from the right, regrouping as you go. If the sum digit in a column is greater than 10, you trade for 1 of the value in the column to the left.

• You can add from the left. Sometimes you have to go back and increase a digit because the sum to its right is greater than 10.

• You should always estimate to see if your sum is reasonable. You can estimate before or after you calculate.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know the answer is more than 59,000 km?* (That is what I would get if I added 29 thousand and 30 thousand. Both numbers are greater than that.)

• *How do you know the answer is less than 61,128 km?* (That is what I would get if I added 30 thousand to 31,128, but I am adding less than that.)

• *What would be a good estimate? Why?* (I think 60,000 km is good since it is between 59,000 and 61,128.)

The Exposition — Presenting the Main Ideas

• Write 38,145 + 46,285 on the board. Ask students how they might go about finding the sum. Follow their steps using a place value chart.

For example:

Ten thousands	Thousands	Hundreds	Tens	Ones
3	8	1	4	5
+ 4	6	2	8	5
8	4	4	3	0

Show students how you could start adding from the left, but sometimes digits must be changed as a consequence of moving to the right.

For example, the 7 ten thousands must be changed to 8 ten thousands when the thousands sum digit turns out to be 14. That is because 7 ten thousands + 14 thousands = 8 ten thousands + 4 thousands.

$$\begin{array}{r} 38{,}145 \\ +\ \underline{46{,}285} \\ 7 \end{array} \longrightarrow \begin{array}{r} 38{,}145 \\ +\ \underline{46{,}285} \\ 84 \end{array}$$

Students might be interested to know that the reason we often add from the right instead of from the left is to avoid having to make changes to a digit after recording it. However, starting at the left is more natural and gives a better sense of the size of the sum as you calculate.

• Some students might have other invented or personal procedures for adding. These should be regarded as acceptable as long as they always give a correct answer.

For example, some students might do this:

$$\begin{array}{r} 38{,}145 \\ +\ \underline{46{,}285} \\ 70{,}000 \\ 14{,}000 \\ 300 \\ 120 \\ \underline{+\ 10} \\ 84{,}430 \end{array}$$

• Encourage students to estimate to check that their sums seem reasonable. They might estimate before they calculate or after they calculate.

• Remind students they can turn to **pages 27 and 28** in the student text to see these ideas again.

Revisiting the Try This

B. This question encourages students to try out several methods of adding. They can confirm their addition using their estimate from **part A**.

Using the Examples

• Ask students to close their texts. Read the problem from the **example** to the students, writing the numbers on the board. Let them try the problem. They can then open their texts and compare their work with the solution in the text. Ask students whether they solved the question as shown in **Solution 1**, as shown in **Solution 2**, or in a different way. Have some of students share their different strategies.

Practising and Applying

Teaching points and tips

Q 1: Allow students to use any strategy they wish.

Q 2: To answer this question, students need to estimate to the nearest thousand rather than to the nearest ten thousand.

Q 5: Some students might begin with 27,396 + 20,000 and then, since the second number contains zeros, decrease it to 19,999 and increase the other number by 1 to 27,397. They might continue using a similar strategy. Other students might look for combinations in each column that add to the appropriate digit.

For example, they look for two numbers in the ten thousands column that add to 4, like 1 and 3 or 2 and 2. They then apply that strategy to each column.

Q 6: You might observe students to see if they make good decisions about which digits to use first.

For example, the ones digit in the very first number in **part a)** is a good place to start since it must be 3. It is less clear what the thousands digit in the second number in **part b)** will be, since the thousands digit in the sum is also unknown.

Q 7: This is a challenging question. Some students might realize that the numbers must all be between 23,000 and 24,000 since 23,000 + 23,000 + 23,000 = 69,000 and 24,000 + 24,000 + 24,000 = 72,000. Once they realize this, they might see that numbers around 23,500 are too high and then work down.

Q 8: Many students just add the left-most digits without lining the numbers up. This question is designed to bring that error to their attention.

Common errors

• Some students continue to make the same types of errors they made when adding 4-digit numbers, such as forgetting to regroup or regrouping improperly (e.g., for 3425 + 5907, they might add 5 ones + 7 ones, record the 1 in the sum, and then carry over the 2 to the tens place). Rarely are new types of errors introduced when working with these greater numbers.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can add 5-digit numbers in more than one way
Question 4	to see if students can solve a problem involving addition
Question 7	to see if students can solve a mathematical puzzle involving addition

Answers

A. *Sample response*: About 60,000 km

B. *Sample response*:

```
 1   1
 29,145
+31,128
 60,273

 29,145
+31,128
 ~~5~~
 60,2~~6~~     5 changes to 6 since 9 + 1 = 10
 60,273     6 changes to 7 since 5 + 8 = 13
```

1. a) 71,061 **b)** 64,467 **c)** 90,700

2. B

[*Sample response*:
B is about 43,000 since 18,112 is a just bit more than 18,000 and 24,875 is just a bit less than 25,000, and 18,000 + 25,000 = 43,000.
A is close to 44,000 since 31,286 is a bit more than 31,000 and 12,998 is almost 13,000, and 31,000 + 13,000 = 44,000.
C is more than 50,000 since 27,379 + 26,712 are both over 25,000.]

3. a) 61,127; [*Sample response*:
First way:

```
 11 11
 57,128
+  3999
 61,127
```

Second way:
57,128 + 4000 = 61,128
Then subtract 1 to get 61,127.]

b) 73,000; [*Sample response*:
First way:

```
 ~~1~~1 11
  62,418
+ 10,582
  73,000]
```

[**3. b)** [continued] *Second way*:
Add 10,000 to 62,418. Then add 500. Then add 82 by adding 100 and taking away 18.
10,582 = 10,000 + 500 + 82
62,418 + 10,000 = 72,418
72,418 + 500 = 72,918
72,918 + 82 = 72,918 + 100 – 18
72,918 + 100 = 73,018
73,018 – 18 = 73,000]

4. 97,568 insects

5. *Sample response*:
17,397 + 29,999
17,398 + 29,998
17,399 + 29,997

6. a) 78,**1**2**3** + 1**4**,2**5**1 = 9**2**,374 **b)** 17,3**2**6 + **34**,**5**91 = 5**1**,917

7. 23,345, 23,355, and 23,365

[**8.** *Sample response*:
The 3 in 38,125 is 3 ten thousands, but the 7 in 7829 is only thousands. You cannot add digits when they represent different place values.]

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Supporting Students

Struggling students

• Some students may need to use place value charts to ensure that they line up digits correctly. Alternatively, they might use grid paper (or lined paper turned sideways) to help line up the numbers .

• Some students struggle with questions like **question 5 and 7**. You may choose not to assign those questions to struggling students.

Enrichment

• Have students create questions like **question 6** for classmates to solve. The students can trade questions and solve each other's questions.

GAME: Give Me Thousands

• This game allows students to practise adding 5-digit numbers.

• Instead of playing cards, students can use digit cards. They need four sets of 0 to 9 cards.

• Make sure students know that they should spread the cards out between them so that they can draw cards from anywhere in the set of cards.

1.2.4 Subtracting 5-digit Numbers

Curriculum Outcomes	Outcome Relevance
4-B1 Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers • apply familiar addition and subtraction strategies to numbers with five or more digits • continue estimating	Although many people use calculators to subtract large numbers, there are still occasions when calculations are done by hand. It is important for students to be able to perform these calculations.

Pacing	Materials	Prerequisites
1 h	• Place value charts or Place Value Charts I (BLM) (optional)	• subtracting 4-digit numbers • adding 5-digit numbers • estimating the difference between 5-digit numbers

Main Points to be Raised

- Subtracting 5-digit whole numbers is based on the same procedures used for 4-digit whole numbers.
- You always subtract digits in place values that are the same: tens from tens, thousands from thousands, and so on.
- Sometimes to subtract you need to regroup. You can do all the regrouping at once at the start of the calculation or you can regroup one place at a time.
- You can add up to subtract, getting to convenient benchmark numbers along the way, and then totaling the various parts you added.
- You can subtract in parts.
- You should always estimate to see if a difference is reasonable.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

- *How do you know the answer is more than 6000 km?* (That is what I would get if I subtracted 18,000 from 24,000. I know that 18,040 and 24,510 are farther apart than that.)
- *How do you know the answer is less than 10,000 km?* (That is what I would get if I subtracted 15,000 km from 25,000 km. I know that 18,040 and 24,510 are closer together than that.)
- *What would be a good estimate? Why?* (6000 km is good since I used 18,000 and 24,000 for my estimates.)

The Exposition — Presenting the Main Ideas

- Write the subtraction 51,210 – 28,145 on the board. Ask students how they might go about calculating the difference. Follow their steps on the board.

Many students will start at the right and regroup. Others might start at the left and subtract in parts.

For example:

51,210 – 20,000 = 31,210

31,210 – 8000 = 31,210 – 10,000 + 2000 = 21,210 + 2000 = 23,210

23,210 – 100 = 23,110

23,110 – 45 = 23,110 – 10 – 35 = 23,100 – 35

23,100 – 35 = 23,100 – 30 – 5 = 23,070 – 5 = 23,065

- Take the time to hear from a number of students. If all students have used the same strategy, encourage them to come up with a different way to subtract. Then show them the alternatives in the student text.
- Have students turn to **page 30** in the student text to see how adding on can also be used to subtract.

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Revisiting the Try This

B. This question encourages students to try out several methods of subtracting. They can confirm their difference using their estimate from **part A**.

Using the Examples

• Ask students to close their texts. Read the problem from the **example** to the students, writing the numbers on the board. Let them try the problem. They can then open their texts and compare their work with the solution in the text. Ask students whether they solved the question as shown in **Solution 1**, as shown in **Solution 2**, or in a different way. Have some of those students share their other strategies.

• Liechtenstein is pronounced “LICK-ten-stine”. Grenada is pronounced “gre-NAY-da”.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to consider adding on, subtracting in parts, or using more traditional regrouping.

Q 2: Students should first realize that the number must be close to 20,000 and then work from there. They can get the 20,000 using their estimation skills.

Q 3: Encourage students to estimate to check.

Q 4: Students should consider which digits are easiest to fill in first. Some might make the mistake of filling in a 3 for the first digit of the difference in **part a)** without thinking about the need for regrouping. Watch for that sort of error and help students see why the digit could be 2 or 3, depending on what happens in the thousands column.

Q 6: This is a very challenging question. Most students will need to try many different calculations before they are ready to answer this. Remind them to consider

- a small 5-digit number subtracted from a small 5-digit number

- a small 5-digit number subtracted from a large 5-digit number

- a small 5-digit number subtracted from a medium sized 5-digit number

- a medium sized 5-digit number subtracted from a large 5-digit number, and

- a large 5-digit number subtracted from a larger 5-digit number

Q 7: There is no right answer to this question. Students can share their preferences and their rationale.

Common errors

• Some students continue to make the error of subtracting digits within a column without taking into account their order.

For example, for 30,256 – 14,812, they might subtract the 0 from the 4 rather than the 4 from 0 (or 10) in the thousands column.

Encouraging students to estimate will help with this. You might also encourage students to consider all the necessary regroupings before they do any subtracting.

For example, for 30,256 – 14,812, they might change the 30,256 to 2 ten thousands + 9 thousands + 12 hundreds + 5 tens + 6 ones before they do any subtracting.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can subtract 5-digit numbers in more than one way
Question 3	to see if students can apply subtraction of 5-digit numbers to solve a problem
Question 5	to see if students can estimate differences between 5-digit numbers

Answers

A. *Sample response*: 6500 m
I used 18,000 instead of 18,040 and I used 24,500 instead of 24,510.
Then I subtracted to get
6 thousands + 5 hundreds = 6500.

B. *Taking away and regrouping*:

$$\begin{array}{r} {\scriptstyle 4\ 11} \\ 24{,}\cancel{51}0 \\ -\ \underline{18{,}040} \\ 6\ 470 \end{array}$$

Adding up:
From 18,040 to18,100 is 60.
From 18,100 to 19,000 is 900.
From 19,000 to 20,000 is 1000.
From 20,000 to 24,510 is 4510.

60 + 900 + 1000 + 4510 is 4510 + 1000 = 5510
5510 + 900 = 6410
6410 + 60 = 6470

Subtracting in parts:
18,040 = 10,000 + 8000 + 40
24,510 – 10,000 = 14,510
14,510 – 8000 = 6510
6510 – 40 = 6470

1. a) 15,444; [*Sample response*:
Subtract in parts
14,812 = 14,000 + 800 + 12
30,256 – 14,812 is
30,256 – 14,000 = 16,256
16,256 – 800 = 15,456
15,456 – 12 = 15,444

Take away and regroup

$$\begin{array}{r} {\scriptstyle 2\ 9\ 12} \\ \cancel{30{,}25}6 \\ -\ \underline{14{,}812} \\ 15{,}444\ ] \end{array}$$

b) 13,178; [*Sample response*:
Take away and regroup

$$\begin{array}{r} {\scriptstyle 5\ 11\ \ 10\ 10\ 12} \\ \cancel{62{,}112} \\ -\ \underline{48{,}934} \\ 13{,}178 \end{array}$$

Subtract too much and change
62,112 – 50,000 = 12,112
Add back 1066 since I subtracted 1066 too much.
12,112 + 1066 = 13,178]

c) 38,835; [*Sample response*:
Take away and regroup

$$\begin{array}{r} {\scriptstyle 4\ 16\ 12\ 9\ 12} \\ \cancel{57{,}302} \\ -\ \underline{18{,}467} \\ 38{,}835 \end{array}$$

1. c) [continued] *Add up*
From 18,467 to 18,500 is 33.
From 18,500 to 19,000 is 500.
From 19,000 to 20,000 is 1000.
From 20,000 to 57,302 is 37,302.
33 + 500 + 1000 + 37,302 is
37,302 + 1000 = 38,302
38,302 + 500 = 38,802
38,802 + 33 = 38,835]

2. *Sample response*:
20,500; 20,501; 20,502

3. 36,659 km

4. a)

$$\begin{array}{r} 42{,}8\mathbf{1}6 \\ -\ \underline{15{,}\mathbf{3}78} \\ \mathbf{2}7{,}438 \end{array}$$

b)

$$\begin{array}{r} 30{,}\mathbf{0}41 \\ -\ \underline{\mathbf{17}{,}3\mathbf{8}5} \\ \mathbf{1}2{,}65\mathbf{6} \end{array}$$

5. C

6. a 5-digit difference
[*Sample response*:
To get a 4-digit difference, the smaller number must be between 1000 and 9999 smaller.
To get a 5-digit difference, the smaller number must be between 10,000 and 99,999 smaller.
There are more numbers between 10,000 and 99,999 than between 1000 and 9999.]

7. *Sample response*:
I would add up. [It is easy to get to 29,000, to 30,000, and then to 41,000 by adding in steps. I think it is easier than regrouping.]

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Supporting Students

Struggling students

• Struggling students may have difficulty with **question 6**. Do not assign this question to those students.

• If students are struggling too much with two different ways to perform subtractions, allow them to perform the subtraction in the way that is most comfortable for them.

• For **question 4**, you may suggest that students add the difference and the subtrahend (the number being subtracted). For some students, that might be easier than performing the subtraction.

Enrichment

• Have students create problems like **question 4** using a sequence of subtractions where each of the digits 0, 1, 2, ..., 9 is missing once. They can trade with another student and try each other's problems.

For example:

$$\begin{array}{r} 57{,}3?8 \\ -\ \underline{29{,}28?} \\ 26{,}?65 \end{array} \qquad \begin{array}{r} 30{,}?07 \\ -\ \underline{1?{,}?84} \\ 13{,}323 \end{array} \qquad \begin{array}{r} 86{,}?11 \\ -\ \underline{13{,}2??} \\ 73{,}2?2 \end{array}$$

CONNECTIONS: A Different Way to Subtract

• Students might be interested to know that this method of subtraction was invented by an 8-year-old child.

• The reason it works is based on integer work, so it will be difficult to explain to Class IV students why it works. For your own understanding, here is the explanation:

If you are subtracting, for example, 5 – 8, it is the same as – (8 – 5). So if you perform 8 – 5 instead of 5 – 8, you need to subtract the result. That is why the subtraction is performed after the initial result is obtained.

Answers

1. 2106;

$$\left[\begin{array}{r} 5003 \\ -\ \underline{2897} \\ 3\overline{894} \end{array}\right. \begin{array}{l} \\ \\ = 3000 - 800 - 90 - 4 \\ = 2200 - 94 \\ \quad 2200 - 94 = 2200 - 100 + 6 = 2106] \end{array}$$

2. 175;

$$\left[\begin{array}{r} 3121 \\ -\ \underline{2946} \\ 1\overline{825} \end{array}\right. \begin{array}{l} \\ \\ = 1000 - 800 - 25 \\ = 200 - 25 \\ = 175] \end{array}$$

3. 4363;

$$\left[\begin{array}{r} 8037 \\ -\ \underline{3674} \\ 5\overline{64}3 \end{array}\right. \begin{array}{l} \\ \\ = 5000 - 600 - 40 + 3 \\ = 4400 - 40 + 3 \\ = 4360 + 3 \\ = 4363] \end{array}$$

UNIT 1 Revision

Pacing	Materials
2 h	• Base ten blocks or Base Ten Blocks (BLM) (optional) • Place value charts or Place Value Charts I (BLM)

Question(s)	Related Lesson(s)
1	Lesson 1.1.1
2 – 4	Lesson 1.1.3
5 – 7	Lesson 1.1.4
8 – 11	Lesson 1.1.5
12	Lesson 1.2.1
13	Lesson 1.2.2
14 and 15	Lessons 1.2.3 and 1.2.4

Revision Tips

Q 1: For **part b)**, students should look for numbers with a digit sum of 14. For **part c)**, they should realize that one block must be traded for 10 of a smaller block.

Q 3: There are many correct answers to this question.

Q 4: Students need to notice the place value words on the right side. This question does not just ask them to write each number in expanded form.

Q 5: Encourage students to use place value notions rather than additions or subtractions.

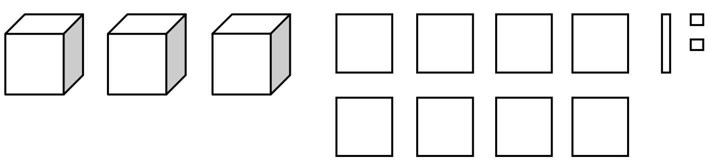
Q 7: If necessary, remind students that since there are 1000 m in 1 km, we want to know how many thousands there are in 34,216.

Q 8: Encourage students to test all possibilities.

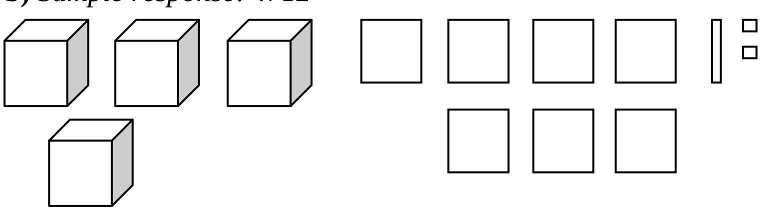
Q 13: Students need to estimate using both thousands and ten thousands.

Answers

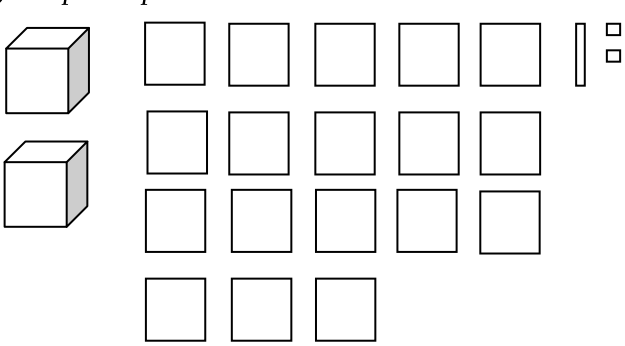
1. a)



b) *Sample response*: 4712



c) *Sample response*: 23 blocks



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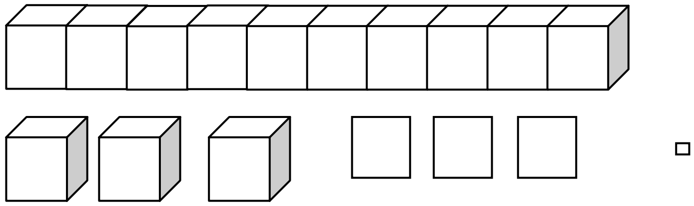
2. a)



b) 10,000 + 3000 + 300 + 1
c) 1 ten thousand + 3 thousands + 3 hundreds + 1 one
d)

Ten thousands	Thousands	Hundreds	Tens	Ones
1	3	3	0	1

3. *Sample responses*:
a) 40,504
b) 52,301
c) 94,121

4. a) **534** hundreds + **17** ones
b) **16** thousands + **7** ones
c) **2** ten thousands + **13** hundreds + **8** tens + **9** ones

5. *Sample response*:
578 hundreds + 1 ten
5 ten thousands + 78 hundreds + 1 ten

6. 412 full trips; [41,245 is 412 hundreds + more, but not a lot more.]

7. 34 km and a bit more

8. a) **4**4,217 > **3**8,217
b) **3**1,384 > **3**0,562 or **4**1,384 > **4**0,562 or **4**1,384 > **3**0,562

9. a) 8945; 23,179; 30,045
b) 8976; 16,127; 18,000; 99,434

[**10.** *Sample response*:
36,000 is 36 thousands.
29,243 is only 29 thousands and a bit more.]

11. *Sample response*:
23,219; 24,000; 24,100; 24,200; 24,500; 25,000

12. a) 4125 + 3897 = 8022; [*Sample response*:
Add 4000 and then subtract 100 and 3.
4125 + 4000 = 8125
8125 – 103 = 8022]

b) 6225 + 4875 = 11,100; [*Sample response*:
Add 25 and 75 to get 100; add 200 and 800 to get 1000; add 6000 and 4000 to get 10,000.
10,000 + 1000 + 100 = 11,100]

c) 8120 – 3798 = 4322; [*Sample response*:
Subtract 4000 and then add back 202.
8120 – 4000 = 4120
4120 + 202 = 4322]

d) 6245 – 3512 = 2733; [*Sample response*:
Subtract in parts.
6245 – 3000 = 3245
3245 – 500 = 2745
2745 – 10 = 2735
2735 – 2 = 2733]

13. D

14. a) 80,587 **b)** 59,504
c) 21,493 **d)** 32,832

15. a) 49,357 **b)** 12,913

UNIT 1 Numeration, Addition, and Subtraction Test

1. a) Model 14,032 with base ten blocks. Sketch your model.

b) Write 14,032 in expanded form in two different ways.

c) Use a place value chart to show 14,032.

2. Write a number for each.

a) 4 ten thousands + 3 hundreds + 2 tens

b) The digit in the ten thousands column is 3 more than the digit in the thousands column.

c) 512 hundreds + 3 tens + 2 ones

d) 29 thousands + 57 tens + 6 ones

3. Complete.

a) 37,104 = __ thousands + 1 hundred + __ ones

b) 37,112 = __ ten thousands + 71 _____ + 12 ones

4. Why can 22,137 be renamed each way?

a) 22 thousands + 1 hundred + 3 tens + 7 ones

b) 1 ten thousand + 12 thousands + 1 hundred + 3 tens + 7 ones

5. A city has 32,156 people.

a) If the people were grouped in 1000s, about how many groups would there be?

b) If the people were grouped in 100s, about how many groups would there be?

6. Order from least to greatest.

a) 37,135; 8979; 15,679; 9992

b) 42,378; 41,887; 56,002; 6987

7. Use the digits 1, 3, and 5 to make each true.

a) 4_,718 < 45,_37 < _1,713

b) _9,817 < 34,_17 < _7,605

8. Why are there more whole numbers between 10,000 and 11,000 than between 1000 and 1100?

9. Use mental math. Show your thinking.

a) 6136 + 2992

b) 8437 – 3512

10. Estimate each in two ways. Show your thinking.

a) 42,517 + 29,021

b) 57,212 – 19,678

11. One car has travelled 37,145 km. A second car has travelled 51,012 km.

a) How far have the two cars travelled altogether?

b) How much farther has the second car travelled?

12. Write two 5-digit numbers (with no zero digits) to make each true.

a) Their sum is 38,012.

b) Their difference is 14,372.

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UNIT 1 Test

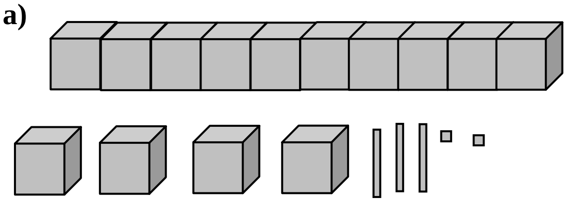
Pacing	Materials
1 h	• Base ten blocks or Base Ten Blocks (BLM) • Place value charts or Place Value Charts I (BLM) (optional)

Question(s)	Related Lesson(s)
1 and 2	Lesson 1.1.3
3 – 5	Lesson 1.1.4
6 – 8	Lesson 1.1.5
9	Lesson 1.2.1
10	Lesson 1.2.2
11 and 12	Lessons 1.2.3 and 1.2.4

Select questions based on time available.

Answers

1. a)



b) 1 ten thousand + 4 thousands + 3 tens + 2 ones;
10,000 + 4000 + 30 + 2

c)

Ten thousands	Thousands	Hundreds	Tens	Ones
1	4	0	3	2

2. a) 40,320 **b)** *Sample response*: 52,103
c) 51,232 **d)** 29,576

3. a) **37** thousands + 1 hundred + **4** ones
b) **3** ten thousands + 71 **hundreds** + 12 ones

4. *Sample responses*:
a) There are 2 ten thousands and each is 10 thousands, so that makes 20 thousands from the ten thousands and another 2 thousands. That is 22 thousands. The rest of the number is 137.
b) You can trade 1 of the ten thousands for 10 thousands. Then there are 10 + 2 = 12 thousands, but there is still 1 ten thousand.

5. a) 32 groups (with 156 people left over)
b) 321 groups (with 56 people left over)

6. a) 8979; 9992; 15,679; 37,135
b) 6987; 41,887; 42,378; 56,002

7. a) 4**1**,718 < 45,**3**37 < **5**1,713
b) *Sample response*: **1**9,817 < 24,**5**17 < **3**7,605

8. *Sample response*:
10,000 and 11,000 are 1000 apart, so there are 1000 numbers between them, but 1000 and 1100 are 100 apart, so there are only 100 numbers between them.

9. *Sample responses*:
a) Add 3000, which is 8 too much, and then take away 8 by taking away 10 and adding 2.
6136 + 3000 = 9136
9136 – 10 = 9126
9126 + 2 = 9128
b) Subtract in parts. First subtract 3000, then subtract 500 (by subtracting 1000 and then adding 500), then subtract 10 and 2.
8437 – 3000 = 5437
5437 – 500 = 4437 + 500 = 4937
4937 – 10 = 4927
4927 – 2 = 4925

10. *Sample responses*:
a) About 4 ten thousands + 3 ten thousands = 7 ten thousands, or 70,000;
About 43 thousands + 30 thousands = 73 thousands, or 73,000.
b) About 6 ten thousands – 2 ten thousands = 4 ten thousands, or 40,000;
About 57 thousands – 20 thousands = 37 thousands, or 37,000.

11. a) 88,157 km **b)** 13,867 km

12. *Sample responses*:
a) 13,891 + 24,121 **b)** 37,593 – 23,221

UNIT 1 Performance Task — Guess My Secret Number

Namgyel has created a game called Guess My Secret Number.
You use his clues to a guess a number.
His secret number is a 5-digit number. It describes the population of a town he is studying.

____ ____,____ ____ ____

Here are his clues:
Clue 1: If you group the number in hundreds, there are 313 groups and some left over.
Clue 2: The last digit is 9.
Clue 3: If you show the number with base ten blocks, you could use 24 blocks or more.
Clue 4: If you add the number to 47,689, the sum is about 79,000.

A. What is Namgyel's secret number? How do you know?

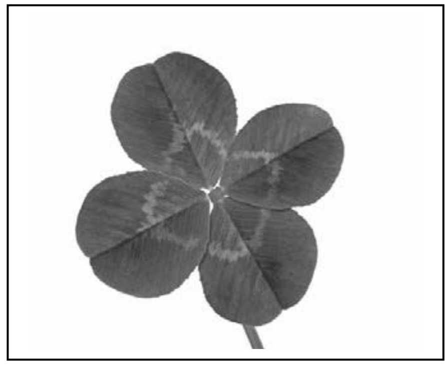
B. i) Choose a 5-digit number from this list:

The largest four-leaf clover collection	72,928
The diameter of the earth	12,756 km
The number of athletes at the 2000 Olympics	10,651
The largest milkshake ever made	22,712 L

ii) Make four or more clues for your number.
Each clue should involve one of these:

- place value or comparing
- estimation or mental math
- addition
- subtraction

None of the clues should tell you the exact answer.
Try to make a set of clues that fit only one number.
Test your clues to make sure they work.



Some people think that finding a four-leaf clover will bring you luck.

A milkshake is a cold drink made from milk and ice cream.

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UNIT 1 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
4-A1 Place Value: model whole numbers to five places **4-A2** Compare and Order : whole numbers to five digits **4-B1** Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers **4-B2** Add and Subtract Mentally: to four digits	1 h	None

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could supplement the unit test. It could also be used as enrichment material for some students.

• You can assess performance on the task using the rubric below.

Sample Solution

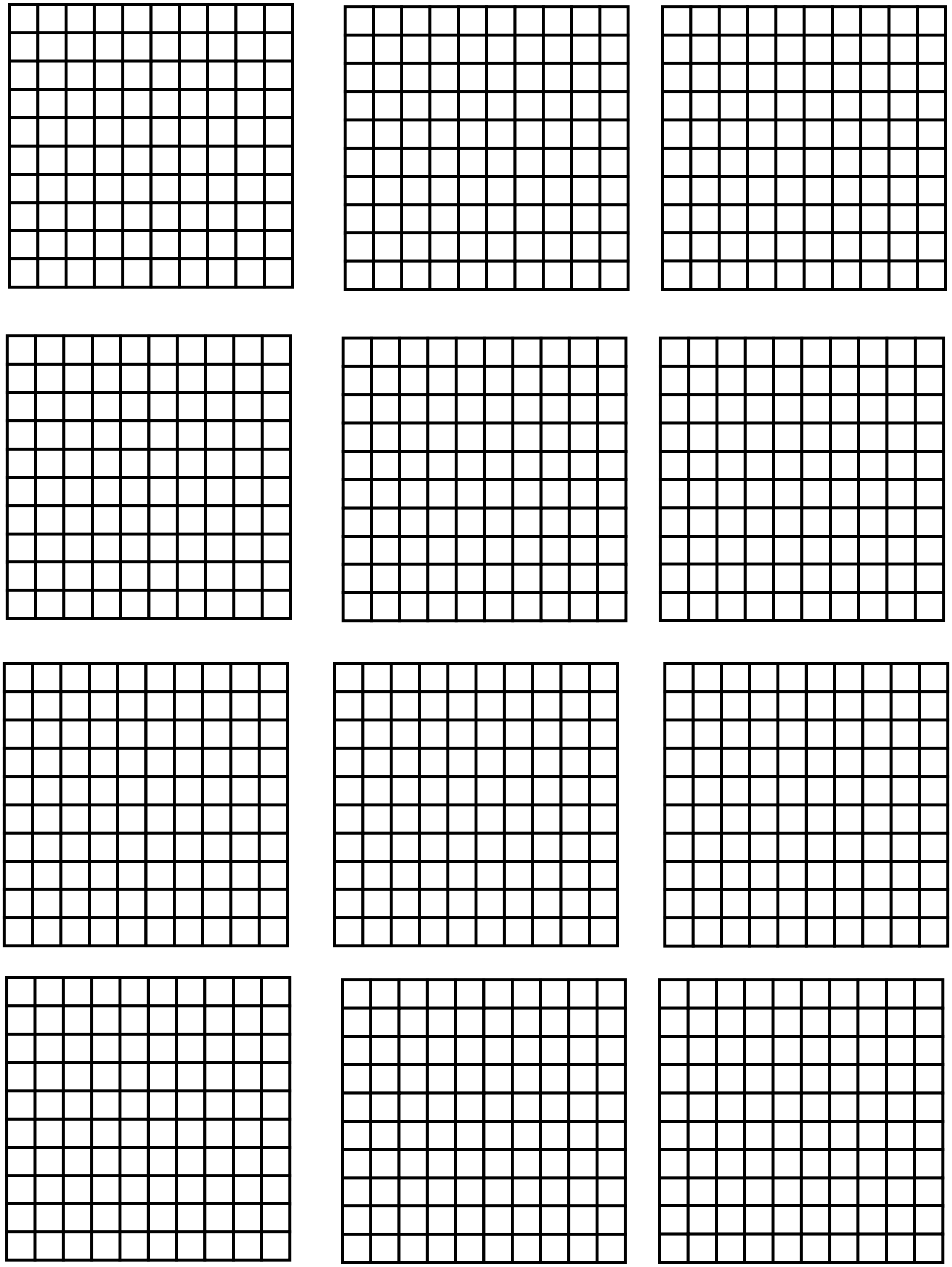
A. 31,389 Clue 1 told me the number started like this: 31,3__ __ Clue 2 told me that the last digit was 9, so I knew the number was 31,3__9. Clue 3 told me that the sum of the digits was 24. Since 3 + 1 + 3 + 9 = 16, the missing digit had to be 8 since 24 – 16 = 8. I checked to see if Clue 4 worked. 31,389 + 47,689 is about 31 thousands + 48 thousands = 79 thousands = 79,000	
B. For 72,928: - If you group the number in hundreds, there are 729 hundreds and a few more. - If you trade all the hundreds for tens, you have 92 tens altogether. - If you show the number with base ten blocks, you need 6 more ones blocks than thousands blocks. - If you add the number to 2222, the ones digit is 0.	For 22,712: - The number is between 22,000 and 23,000. - If you add the number to 37,897, the tens digit is 0. - You can show the number with 14 base ten blocks. - If you trade all the hundreds for tens, there will be 71 tens. - The ones digit is 5 less than the hundreds digit.

UNIT 1 Performance Task Assessment Rubric

The student	Level 4	Level 3	Level 2	Level 1
Uses place value information	Consistently and correctly uses place value concepts including comparison and renaming of numbers	Often correctly uses place value concepts including comparison and renaming of numbers	Sometimes correctly uses place value concepts including comparison and renaming of numbers	Rarely uses place value concepts
Adds and subtracts 5-digit numbers	Consistently, correctly, and efficiently adds and subtracts 5-digit numbers	Consistently and correctly adds and subtracts 5-digit numbers	Usually adds and subtracts 5-digit numbers correctly	Has difficulty adding and subtracting 5-digit numbers
Creates clues for his or her number	Creatively and insightfully creates clues for the number; creates a set of clues that fits only one number	Creates correct clues for the number; creates a set of clues that fits one or two numbers	Creates some correct clues for the number; creates a set of clues that fits many numbers	Rarely uses meaningful clues for the number

UNIT 1 Blackline Masters

BLM 1 A Base Ten Blocks (Hundreds Blocks)



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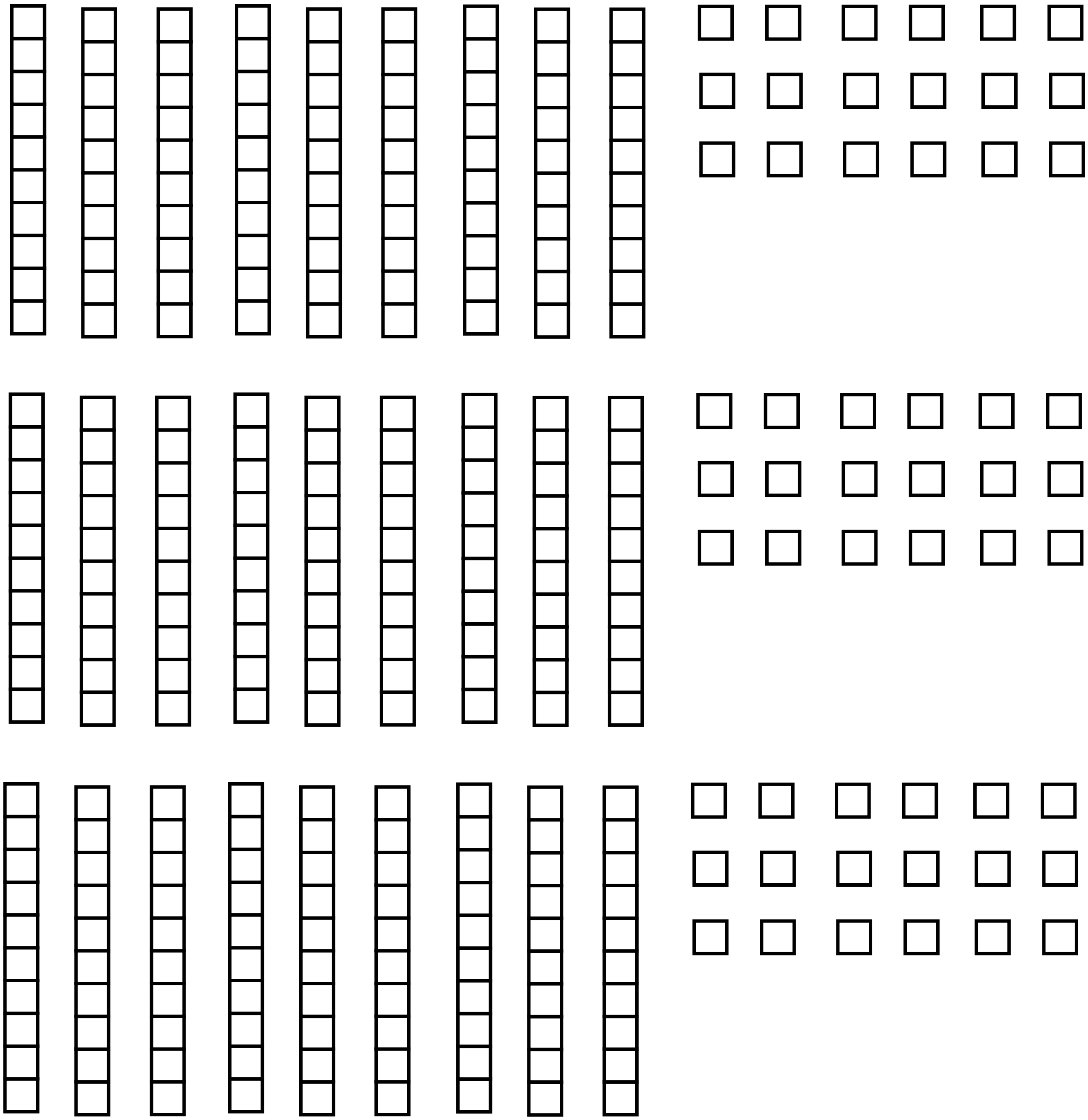
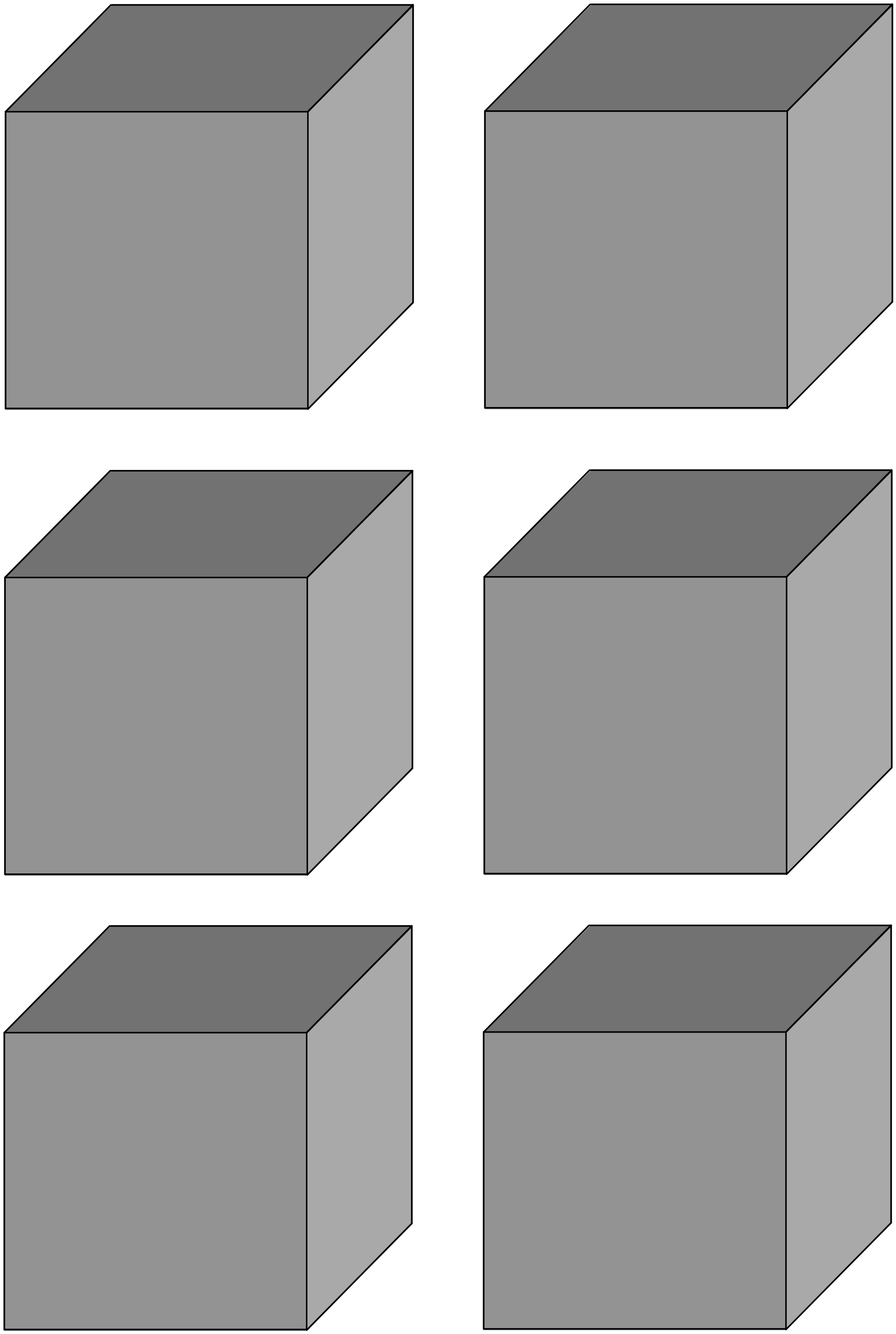
BLM 1B Base Ten Blocks (Tens Blocks and Ones Blocks)



BLM 1C Base Ten Blocks (Thousands Blocks)



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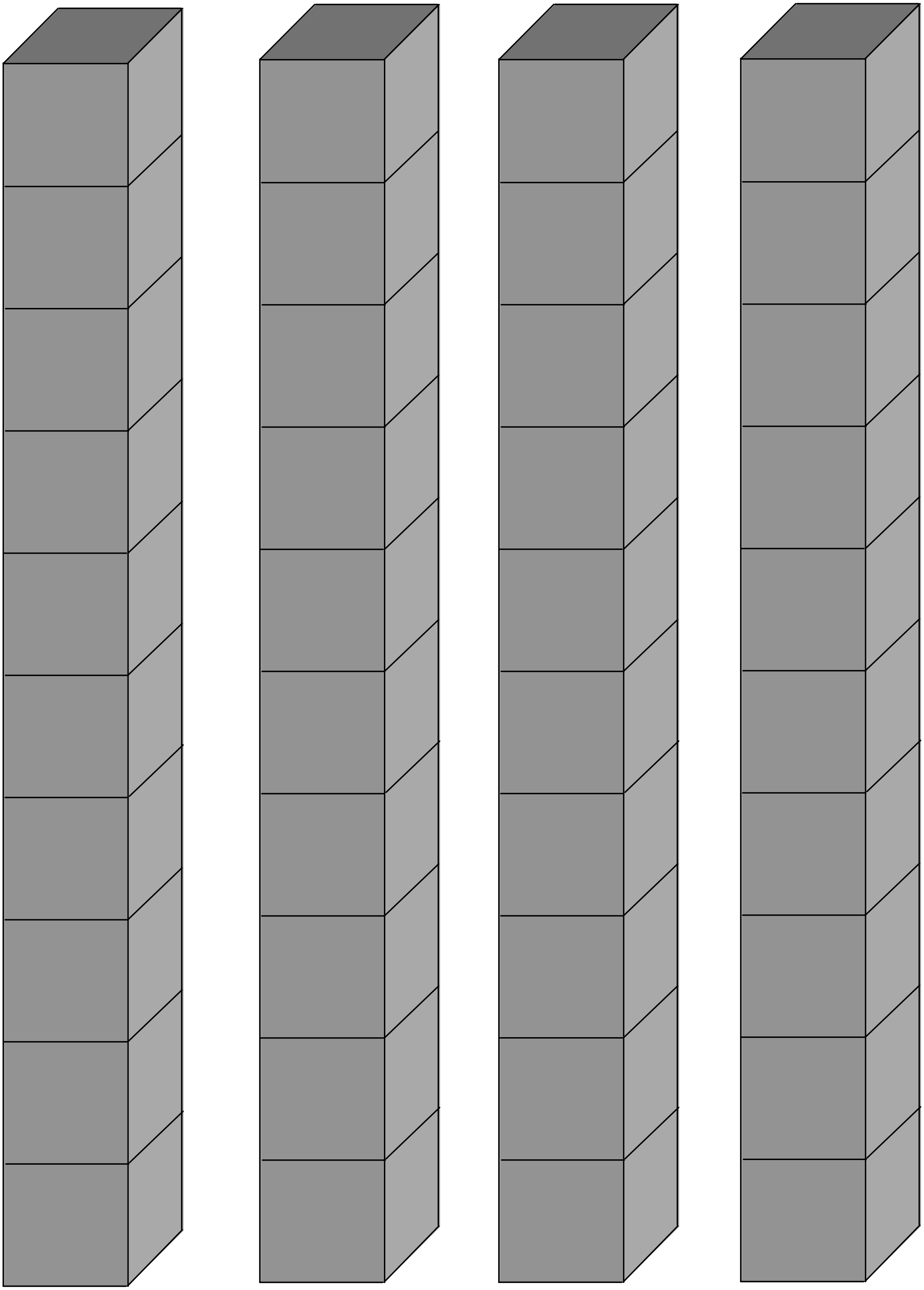
BLM 1D Base Ten Blocks (Ten Thousands Sticks)



BLM 2 Place Value Charts I

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

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UNIT 2 MULTIPLICATION AND DIVISION FACTS

UNIT 2 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 35** **TG p. 49**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• Teacher- or student-made number cards (7 per pair) • Counters (60 per pair)	All questions
Chapter 1 Multiplication				
2.1.1 Multiplying by Skip Counting **SB p. 37** **TG p. 52**	**4-B3 Multiplication Meanings: explore** • explore various meanings of multiplication, focusing on multiplication as skip counting and repeated addition **4-B5 Multiplication Facts: to 9 × 9** • develop facts to 9 × 9 through concrete and pictorial representations • develop and practise strategies for fact recall • recall facts to 9 × 9	1 h	None	Q1, 2, 4, 9
2.1.2 Multiplying Using Arrays **SB p. 41** **TG p. 55**	**4-B3 Multiplication Meanings: explore** • explore various meanings of multiplication, focusing on multiplication as skip counting and repeated addition **4-B4 Multiplication Properties: explore** • explore the commutative, distributive, and associative properties **4-B5 Multiplication Facts: to 9 × 9** • develop facts to 9 × 9 through concrete and pictorial representations • develop and practise strategies for fact recall • recall facts to 9 × 9	1 h	• Grid paper or Centimetre Grid Paper (BLM) (optional) • Counters (optional)	Q1, 4, 5
GAME: Array Fact Match **SB p. 44** **TG p. 58**	Practise the array meaning of multiplication in a game situation	20 min	• Array Fact Match Game Cards (BLM)	N/A
2.1.3 EXPLORE: Meanings of Multiplication (Essential) **SB p. 45** **TG p. 59**	**4-B3 Multiplication Meanings: explore** • explore various meanings of multiplication, focusing on multiplication as skip counting and repeated addition	1 h	None	Observe and Assess questions
2.1.4 Relating Facts by Doubling and Halving **SB p. 47** **TG p. 61**	**4-B4 Multiplication Properties: explore** • explore the commutative, distributive, and associative properties **4-B5 Multiplication Facts: to 9 × 9** • develop facts to 9 × 9 through concrete and pictorial representations • develop and practise strategies for fact recall • recall facts to 9 × 9	1 h	None	Q1, 2, 4
GAME: Matching Doubles **SB p. 50** **TG p. 63**	Practise relating facts that are doubles of one another in a game situation	15 min	• Two sets of Matching Doubles Game Cards (BLM)	N/A

UNIT 2 Planning Chart [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
2.1.5 Multiplying by 7, 8, and 9 **SB p. 51** **TG p. 64**	**4-B4 Multiplication Properties: explore** • explore the commutative, distributive, and associative properties **4-B5 Multiplication Facts: to 9 × 9** • develop facts to 9 × 9 through concrete and pictorial representations • develop and practise strategies for fact recall • recall facts to 9 × 9	1 h	• Grid paper or Centimetre Grid Paper (BLM) (optional)	Q1, 3, 7, 8
2.1.6 EXPLORE: Multiplication Table Patterns (optional) **SB p. 54** **TG p. 67**	**4-B4 Multiplication Properties: explore** • explore the commutative, distributive, and associative properties **4-B5 Multiplication Facts: to 9 × 9** • develop facts to 9 × 9 through concrete and pictorial representations • develop and practise strategies for fact recall • recall facts to 9 × 9	1 h	• Multiplication Tables (BLM) (optional)	Observe and Assess questions
CONNECTIONS: Multiplication Fact Digit Circles **SB p. 55** **TG p. 69**	Explore patterns when you multiply by a number	20 min	• Multiplication Fact Digit Circles (BLM) (optional)	N/A
CONNECTIONS: Finger Multiplication **SB p. 55** **TG p. 69**	Explore patterns when you multiply by 9	10 min	None	N/A
Chapter 2 Division				
2.2.1 Division as Sharing **SB p. 56** **TG p. 70**	**4-B7 Division Meanings: small numbers** • understand division as grouping or sharing	1 h	• 24 counters	Q1, 2, 6, 9
2.2.2 Division as Grouping **SB p. 59** **TG p. 73**	**4-B7 Division Meanings: small numbers** • understand division as grouping or sharing	1 h	• Counters	Q1, 3, 6, 7
2.2.3 Multiplication and Division Fact Families **SB p. 61** **TG p. 76**	**4-B5 Multiplication Facts: to 9 × 9** • develop facts to 9 × 9 through concrete and pictorial representations • develop and practise strategies for fact recall • recall facts to 9 × 9 **4-B8 Division Properties: explore** • understand that order matters when dividing **4-B9 Multiplication and Division Facts: relate through properties** • understand multiplication and division as two ways of looking at the same situation • use multiplication facts to recall division facts	1 h	None	Q2, 4, 7
2.2.4 EXPLORE: Multiplying and Dividing with 1 and 0 (Essential) **SB p. 63** **TG p. 78**	**4-B4 Multiplication Properties: explore** • explore multiplying by 0 and 1 **4-B8 Division Properties: explore** • dividing with 0 and 1	40 min	None	Observe and Assess questions

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UNIT 2 Revision **SB p. 64** **TG p. 80**	Review the concepts and skills in the unit	2 h	• Grid paper or Centimetre Grid Paper (BLM) (optional)	All questions
UNIT 2 Test **TG p. 82**	Assess the concepts and skills in the unit	1 h	• Grid paper or Centimetre Grid Paper (BLM) (optional)	All questions
UNIT 2 Performance Task **TG p. 85**	Assess concepts and skills in the unit	40 min	• Counters (optional)	Rubric provided
UNIT 2 Blackline Masters **TG p. 87**	BLM 1 Centimetre Grid Paper BLM 2A Array Match Game Cards BLM 2B Array Match Game Cards (continued) BLM 3 Matching Doubles Game Cards BLM 4 Multiplication Fact Digit Circles BLM 5 Multiplication Tables			

Math Background

• Although students were introduced to multiplication and division in earlier classes, in Class IV they should solidify their knowledge of multiplication facts and division facts, using a variety of strategies.

• The focus of this unit is on ensuring students have strategies to recall multiplication facts in preparation for later work involving dividing and multiplying greater numbers.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 5** in **lesson 2.1.4**, where they solve a real-world problem using fact strategies, in **question 8** in **lesson 2.2.1**, where they solve a problem involving sharing, and in **question 4** in **lesson 2.2.2**, where they solve a problem involving equal teams.

• Students use communication as they explain their thinking in **question 7** in **lesson 2.1.2**, where they explain a generalization about arrays with one column, in **question 8** in **lesson 2.1.5**, where they explain how knowing how to multiply by 10 helps to multiply by 9, 8, and 7, in **question 4** in **lesson 2.2.1**, where they explain a repeated subtraction process for dividing, and in **question 8** in **lesson 2.2.3**, where they explain whether and why they agree with a statement relating division and multiplication.

• Students use reasoning in answering questions such as **question 5** in **lesson 2.1.2**, where they reason about how arrays can be rearranged to simplify calculations, in **question 7** in **lesson 2.1.2**, where they generalize to draw a conclusion, in **questions 3** and **4** in **lesson 2.1.5**, where they come to generalizations by observing patterns, in **question 5** in **lesson 2.2.1**, where they figure out how to divide a line into 6 equal sections, in **question 5** in **lesson 2.2.3**, where they relate additions and subtractions to a multiplication and division fact family, and in **lesson 2.2.4**, where they develop and explain rules for multiplication and division involving 1 and 0.

• Students consider representation in **question 2** in **lesson 2.1.1**, where the use number lines to represent repeated addition, in **lesson 2.1.3**, where they consider different meanings of multiplication, in **question 1** in **lesson 2.2.2**, where they sketch a picture to show a division, and in **question 4** in **lesson 2.2.3**, where they figure out which multiplications and divisions are modelled by a number line diagram.

• Students use visualization skills in **question 3** in **lesson 2.1.1**, where they change a number line to model a new multiplication, in **question 4** in **lesson 2.1.4**, where they sketch pictures to describe numerical relationships, and in **question 5** in **lesson 2.2.1**, where they figure out how to divide a line into 6 equal sections.

• Students make connections in **question 6** in **lesson 2.1.1**, where they relate multiplication to a graphing situation, in **lessons 2.1.4** and **2.1.5**, where they relate different multiplication facts to each other, in **lesson 2.1.6**, where they relate patterns in the multiplication table to strategies they can use to calculate products, and in **question 6** in **lesson 2.2.2**, where they connect skip counting to division.

Rationale for Teaching Approach

• This unit is divided into two chapters.

Chapter 1 focuses on multiplication.

Chapter 2 focuses on division.

• There are three **Explore** lessons in the unit:

The **first Explore** is designed to help students see that a multiplication describes a number of situations that are distinct but related. They learn these meanings of multiplication: repeated addition, a count of the total of a number of equal groups, an array, a rate, the area of a rectangle, and the total when one set of items is combined with another set of items.

The **second Explore** lesson encourages students to observe patterns in the multiplication table. Students consider how those patterns can help them calculate products they do not already know.

The **third Explore** lesson allows students to apply the rules for multiplying and dividing with 0.

• There are two **Connections** in the unit:

The **first Connections** shows students how the units digits in each multiplication table behave as they look at related geometric patterns.

The **second Connections** shows students how to multiply by 9 using their fingers.

• The two **Games** provide an opportunity to practise multiplication facts in a game situation.

• Throughout the unit, it is important to encourage flexibility in approach to the mathematics.

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Getting Started

Curriculum Outcomes	Outcome relevance
3 Early Multiplication: repeated addition **3** Multiplication Meanings: arrays, groups, number lines **3** Multiplication Properties: commutative, associative, zero, 1 **3** Multiplication Facts: develop and practise strategies **3** Multiplication Facts: select appropriate strategies **3** Division Meanings: groups, shares (small numbers) **3** Multiplication and Division: relationship **3** Division Facts: relate to multiplication	Students will find the work in the unit easier after they review the concepts of multiplication and division they encountered in Class III.

Pacing	Materials	Prerequisites
1 h	• Teacher- or student-made number cards (7 per pair) • Counters (60 per pair)	• recognizing these meanings of multiplication: array, repeated addition, and equal groups • relating division facts to arrays • relating multiplication and division to real-life situations • using a number line to show repeated addition • knowing the format and meaning of multiplication and division facts • familiarity with multiplication facts

Main Points to be Raised

Use What You Know

• You can describe an array using both multiplication and division facts. The multiplication fact describes the product of the number of rows and number of columns. The division fact describes the number of rows if the total is divided into a given number of columns or vice versa.

• Sometimes you can arrange the same number of items in more than one array.

For example, you can arrange 24 counters as a 4-by-6 array or as an 8-by-3 array.

• You can use multiplication to describe the total of a number of equal groups; you can use division to describe a sharing or grouping situation.

Skills You Will Need

• Sometimes knowing one multiplication or division fact can help you figure out another fact.

For example, if you know that $4 \times 7 = 28$, you can add 7 to 28 to calculate 5×7 because it is one more group of 7.

Use What You Know — Introducing the Unit

• Before assigning the **Use What You Know** activity, make sure that students are comfortable with creating an array by modelling a 3-by-9 array for them. Show how you could write two multiplication facts and two division facts to describe the array. Review what each fact means:

x x x x x x x x x
x x x x x x x x x
x x x x x x x x x

$3 \times 9 = 27$ means there are 3 rows of 9 items for 27 altogether.

$9 \times 3 = 27$ means there are 9 columns of 3 items for 27 altogether.

$27 \div 3 = 9$ means that, if you take 27 items and arrange them in groups of 3, there are 9 groups.

$27 \div 9 = 3$ means that if you take 27 items and share them into 9 groups, there are 3 items in each group.

Students can work in pairs to complete the **Use What You Know** activity. While you observe students at work, you might ask questions such as:

• *Why did you write $5 \times 6 = 30$?* (There are 5 rows, 6 columns, and 30 counters.)

• *How did you decide how to rearrange the counters into a different array?* (I had 3 rows of 8, so I split each row in half and moved those counters down. Then I had 6 rows of 4.)

• *What would make the problem you write a multiplication problem?* (There have to be equal groups.)

• *What would make the problem you write a division problem?* (I need to make equal groups or equal shares.)

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions.

• Students can work individually.

• Notice the use of the word “multiplication” instead of “multiplication fact” in **question 1**. This is deliberate. A multiplication fact is an equation like 6 × 4 = 24, so 6 × 4 is not a multiplication fact. A similar situation arises in **question 3** with the use of the word “division” instead of “division fact”.

• For each part of **question 6**, make sure students do not simply write the answer to the second fact, but that they relate the product to the first number fact that is given.

• For **question 7**, draw students’ attention to the definitions of *product* and *quotient* on the page. Indicate that those terms will be used throughout the unit.

Answers

B. *Sample responses*:
i) 8 × 4 = 32, 4 × 8 = 32 **ii)** 32 ÷ 4 = 8, 32 ÷ 8 = 4

C. *Sample responses*:
i)
x x x x x x x
x x x x x x x
x x x x x x x
x x x x x x x
x x x x x x x
x x x x x x x

ii)
7 × 6 = 42, 6 × 7 = 42, 42 ÷ 6 = 7, 42 ÷ 7 = 6

D. *Sample response*:
Kuenga’s mother works 7 hours each day.
How many hours does she work in 6 days?

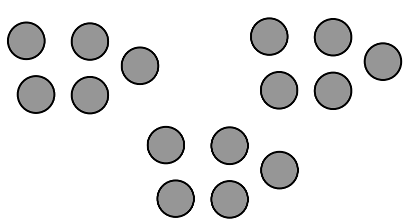
1. a) 4 + 4 + 4 + 4 + 4 + 4 **b)** 7 + 7 + 7
c) 9 + 9 **d)** 6 + 6 + 6 + 6

2. a) It is an array with 3 rows and 4 columns.
b) There are 4 groups of 3 diamonds.
c) There are 4 jumps of 3 spaces.

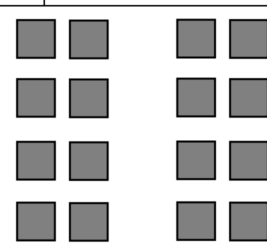
3. *Sample response*:
12 ÷ 3 = 4
[12 ÷ 3 asks how many 3s are in 12 and all the pictures show that.
12 squares in 3 rows = 4 columns
12 diamonds in groups of 3 = 4 groups
12 spaces in jumps of 3 = 4 jumps]

4. A, C, E, F

5. *Sample responses*:
a)



b)



6. a) 24;
[*Sample response*:
Added 4 to 20 since 6 × 4 is 1 more group of 4 than 5 × 4.]
b) 7;
[*Sample response*:
I knew that if there are four 7s in 28, then if you divide 28 into 4 groups, each group is 7.]
c) 24;
[*Sample response*:
I knew that if 3 groups of 8 is 24 and you split each group of 8 into 2 groups of 4, there are 6 groups of 4 but you still have 24.]

7. a) 42 **b)** 45 **c)** 6 **d)** 8

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Supporting Students

Struggling students

• If students are struggling with creating problems for multiplication and division in **part D**, you might suggest some problems and then have students tell what multiplication or division they relate to and why.

For example, you might assign a problem like either of these:

• *Mindu bought 3 bags of momos. There were 8 momos in each bag. How many momos were there altogether?*

• *Mindu's mother was sharing 24 momos so that each child got the same amount. How many could each get?*

Notice that in the second problem students can choose the number of children.

Enrichment

• You might encourage students to create a game that involves relating multiplication and division to array models. They could then try out their game by having other students play it.

Chapter 1 Multiplication

2.1.1 Multiplying by Skip Counting

Curriculum Outcomes	Outcome relevance
4-B3 Multiplication Meanings: explore • explore various meanings of multiplication, focusing on multiplication as skip counting and repeated addition **4-B5 Multiplication Facts: to 9 × 9** • develop facts to 9 × 9 through concrete and pictorial representations • develop and practise strategies for fact recall • recall facts to 9 × 9	Skip counting is not only one of the basic meanings of multiplication, but it is a useful strategy to relate one multiplication fact to other facts. For example, skip counting by 4s to 24 relates 6 × 4 to 8 × 4.

Pacing	Materials	Prerequisites
1 h	None	• using number lines to show addition • applying the commutative principle of multiplication (i.e., the order of the factors does not matter) • familiarity with pictographs with a scale (one symbol represents multiple items)

Main Points to be Raised

• Repeated addition means that you add the same number over and over. Multiplication is a way to describe repeated addition:

- The first factor tells how many times you add the number.

- The second factor tells the number that you add repeatedly.

NOTE: Because of the commutative property of multiplication, the factors could be reversed. However, it is less confusing for students to follow the convention described above for the meaning of each factor.

• One way to model repeated addition is by showing equal jumps on a number line.

• Skip counting is counting in a regular pattern where you say only certain numbers, e.g., every fourth or fifth number. It is called skip counting because you skip the numbers that you do not say. You can use skip counting to multiply.

For example, 3 × 7 means that, starting at 0, you say every 7th number 3 times to get to 21.
The last number said is the product, in this case 21.

• Skip counting can be related to jumping equal distances on a number line.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

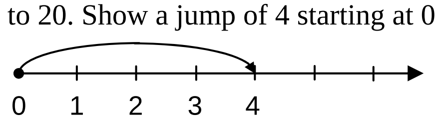
• *How did you figure out the number for Class IV A?* (I counted 5, 10, 15, so it was 15.)

• *How do you know there must be 10 more cats in the Class IV C than in Class IV B?* (There are two extra cat pictures and each cat picture means 5 cats, so there are 5 + 5 = 10 extra cats.)

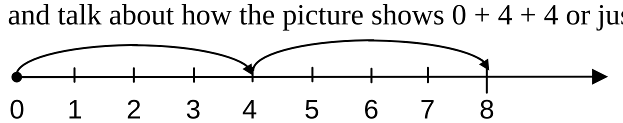
• *How did you get the total number of cats?* (I counted by 5s from 5 to 45.)

The Exposition — Presenting the Main Ideas

• Draw a number line from 0 to 20. Show a jump of 4 starting at 0 and talk about why the picture shows 0 + 4.



Then add a jump past 4 to 8 and talk about how the picture shows 0 + 4 + 4 or just 4 + 4. Talk about how it can also be 2 × 4.



Now ask students how to show 4 + 4 + 4 + 4 + 4 on the number line. Have them recall why this can also be written as 5 × 4.

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• Have students observe that when you jump like this on the number line, the arrows skip over the numbers 1, 2, 3, 4 to land on 5, then skip over 6, 7, 8, 9 to land on 10, then skip over 11, 12, 13, 14 to land on 15, and finally skip over 16, 17, 18, 19 to land on 20. Talk about why this is called *skip counting* (because you skip some numbers). Point out that the pattern for counting is regular — you say every 5th number.

• Help students see how you can use either a number line or skip counting to find a product.

For example, 3 × 8 = 8 + 8 + 8 is making 3 jumps of 8 or starting at 0 and saying every 8th number 3 times.

• Let students know that they can examine **pages 37 and 38** of the student text for a review of these ideas.

Revisiting the Try This

B. This question allows students to make a formal connection between skip counting, repeated addition, multiplication, and the pictograph problem described in **part A**.

Using the Examples

• Read the problem from the **example** with students. Then ask them to work in pairs to look at the three solutions provided in the student text. Have them discuss which solution they like best and why. Ask several pairs of students to share their preferences.

Practising and Applying

Teaching points and tips

Q 1: If students reverse the factors to skip count, the result will still be correct, so allow them to do that.

For example, for 4 × 6, they could say 6, 12, 18, and 24 or they could say 4, 8, 12, 16, 20, and 24.

Q 2 b): Most students will record 6 × 3, which is what is intended. If students use 3 × 6, indicate that this is correct but make them aware of the convention that the second number tells what is added.

Q 3: Students can either sketch the model and change it or describe how they might change it. Note that some students may simply model the multiplications indicated instead of describing how to change the models in **question 2**. Accept that work, but ask students to redo at least one of the responses so that they start with the model in the previous question. This will help them later as they relate multiplication facts.

Q 4: Remind students to count Ugyen as well as her friends, so there are 5 students.

Q 5: Although students are not required to write the multiplication fact, you might ask them to tell you what multiplication they did to solve the problem.

Q 6: A pictograph where each symbol represents multiple items is a good application of multiplication.

Q 7: Students should consider all possibilities.

Q 9: This closure question might be discussed with the class as a whole. Students likely will indicate that the second number is the one to count by and the first number is how many times you skip count (i.e. when to stop skip counting). There may be students, however, that reverse the factors. This is entirely acceptable as along as they can explain that, for example, 5 × 9 = 9 × 5.

Common errors

• When skip counting, students may count incorrectly.

For example, to skip count by 8, they skip 8 numbers instead of skipping 7. So to calculate 5 × 8, they might count 1, 2, 3, 4, 5, 6, 7, 8, **9**, 10, 11, 12, 13, 14, 15, 16, 17, **18** and say “9, 18, 27, ...”.
Make sure students realize that in this situation the 8th number in the group is they number they say aloud, rather than saying 8 numbers silently.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can skip count to solve multiplications
Question 2	to see if students can relate repeated addition to multiplication
Question 4	to see if students can solve a real-world multiplication problem
Question 9	to see if students understand and can communicate about skip counting

Answers

A. 45 pet cats **B. i)** Class IV A: 15; I counted 5, 10, 15. Class IV B: 10: I counted 5, 10. Class IV C: 20: I counted 5, 10, 15, 20.	**ii)** 45: I counted 5, 10, 15, 20, 25, 30, 35, 40, 45. **C. i)** Class IV A: 3 × 5 = 15 **ii)** 9 × 5 = 45 Class IV B: 2 × 5 = 10 Class IV C: 4 × 5 = 20
1. a) 24; [*Sample response*: 6, 12, 18, 24] **b)** 15; [*Sample response*: 3, 6, 9, 12, 15] **c)** 18; [*Sample response*: 9, 18] **d)** 24; [*Sample response*: 3, 6, 9, 12, 15, 18, 21, 24] **2. a)** 7 × 2 = 14 **b)** 6 × 3 = 18 **c)** 4 × 6 = 24 [**3. a)** Show another 2 jumps of 6. **b)** Divide each jump of 6 into two jumps of 3. **c)** Divide each jump of 6 into two jumps of 3 and add another jump of 3.] **4.** 25 books	**5.** 48 legs **6. a)** Dorji: 9 apples, Tashi: 6 apples, Tenzin: 9 apples **b)** 24 apples **7.** He might have skip counted by 2s (or 3s, 4s, or 6s). [**8.** 6 × 8 is 2 more groups of 8 than 4 × 8 = 32, so I would start at 32 and skip count by 8s two more times: 32, 40, 48.] **9.** *Sample response*: I choose to count by the second number. I stop when I have said as many numbers as the first number. [For 5 × 9, I skip count by 9s and say five numbers: 9, 18, 27, 36, 45.]

Supporting Students

Struggling students

• Some students may need support in translating a symbolic multiplication phrase like 4 × 6 into a repeated addition number line model. If these students are more successful by simply adding, e.g., 6 + 6 + 6 + 6, without the number line, allow them to do so.

Enrichment

• You might give students a product with a lot of factors, like 18, 20, or 24, ask them to figure out all the repeated additions or skip counting situations that could result in that number.

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2.1.2 Multiplying Using Arrays

Curriculum Outcomes	Outcome relevance
4-B3 Multiplication Meanings: explore • explore various meanings of multiplication, focusing on multiplication as skip counting and repeated addition **4-B4 Multiplication Properties: explore** • explore the commutative, distributive, and associative properties **4-B5 Multiplication Facts: to 9 × 9** • develop facts to 9 × 9 through concrete and pictorial representations • develop and practise strategies for fact recall • recall facts to 9 × 9	When students recognize the relationship between multiplication and arrays, they will have useful understandings for later work with areas of rectangles. They will also understand why and how to use the commutative, distributive, and associative principles to relate a new product to a known product.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Centimetre Grid Paper (BLM) (optional) • Counters (optional)	• understanding the multiplication meaning of adding equal groups

Main Points to be Raised

• An array can model a multiplication:

- The number of rows represents one factor.
- The number of columns represents the other factor.
- The total number of items represents the product.

• You can relate arrays to repeated addition by using repeated addition to add the number of items in each row or by skip counting.

• Using an array makes it clear why factors can switch order and result in the same product. You can think about turning the array.
For example, you can turn a 3 row-by-8 column array (3 × 8) to model an 8 row-by-3 column array (8 × 3). This shows that 3 × 8 = 8 × 3.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. You might choose to provide 32 counters to pairs of students to work with. While you observe students at work, you might ask questions such as the following:

• *Could there be one row? How many students would be in that row?* (It could be one row of 32 students.)

• *Could there be five rows of the same length?* (No, it would be possible for 30 students or 35, but not for 32.)

• *How can you rearrange this array to create another array?* (I could move the bottom half up to the right side. Then there would be half as many rows but twice as many columns.)

The Exposition — Presenting the Main Ideas

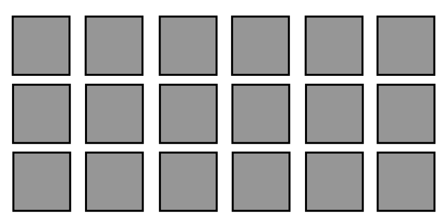
• On the board draw an array, such as a 3-by-6 array. You might use dots, squares, or Xs. Point out which are *rows* and which are *columns*. Use the term *product* to describe the total number of items in the array.

• Show how we write the number of rows multiplied by the number of columns equals the product, e.g., 3 × 6 = 18, to describe the multiplication.

• Show the relationship between the array and repeated addition by showing how the numbers in each row are equal so the total is the sum of the number in each row, e.g., 6 + 6 + 6 = 18.

• Also show how you can find the total by skip counting 6, 12, 18. You count one row at a time.

3 rows



6 columns

3 × 6 = 18

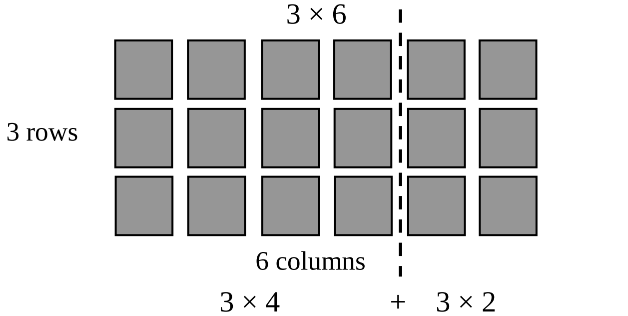
6 + 6 + 6 = 18

6, 12, 18

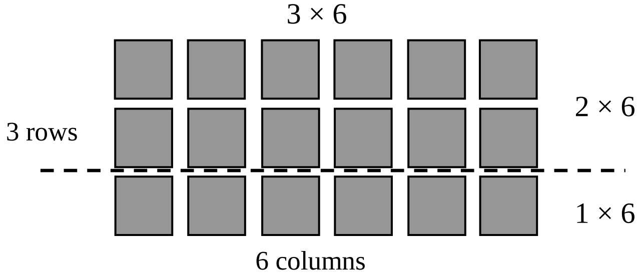
• Have students turn their heads sideways to view the same array as 6 × 3. This shows them why the results of the two multiplications 3 × 6 and 6 × 3 must be equal.

• Show students how the array to divide the array into parts. Calculate the sum of the parts to determine the number of items in the full array.

For example: By dividing vertically, students can see that 3 × 6 = 3 × 4 + 3 × 2.



By dividing horizontally, students can see that 3 × 6 = 2 × 6 + 1 × 6.



• If you feel students need another example, repeat the thinking above with another array, e.g., a 5-by-7 array.

• Make sure students know they can refer to **page 41** of the student text to review these ideas.

• Students can use grid paper to make it easier to draw arrays.

Revisiting the Try This

B. This question allows students to make a connection between the informal ways they arranged the students in the array to solve the problem in **part A** and the array's relationship to multiplication.

Using the Examples

• Work through **example 1** with students. Make sure they understand that in each case the idea is to use known facts to solve unknown facts.

• Have students turn to **page 43** in the student text to look at the picture in **example 2**. Help them notice the array formed by the first 7 rows. Discuss how the solution shows how the student used the array and then added 2 to determine the total number of signs. Ask if there would be another way to use the array.

For example, discuss how the student could have used an 8-by-4 array and subtracted 2 or the student could have used two arrays: an 8-by-2 array and a 7-by-2 array.

Practising and Applying

Teaching points and tips

Q 1: Remind students that the first factor in the multiplication fact should be the number of rows.

Q 2: You might choose to have students draw the array on a piece of paper and then turn the paper.

Q 4: Encourage students to think about where else they see arrays in their everyday lives.

Q 5: The arrays can be cut horizontally or vertically. Make sure students understand that the best way to cut an array is so the smaller arrays represent known facts.

Q 6: You may wish to provide 24 counters for students to use to help them answer this question.

Q 7: Some students might also generalize to situations where 1 is the first factor.

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Suggested assessment questions from Practising and Applying

Question 1	to see if students can write a multiplication fact to match a given array
Question 4	to see if students can recognize an array in a real-world situations and relate it to multiplication
Question 5	to see if students see how an array can be reorganized to make calculating a product simpler

Answers

A. *Sample response*:
8 rows and 4 columns; 4 rows and 8 columns

B. *Sample response*:
8 × 4 and 4 × 8

1. a) 3 × 6 = 18
b) 4 × 3 = 12
c) 3 × 8 = 24

2. a) 6 × 3 = 18 for a)
3 × 4 = 12 for b)
8 × 3 = 24 for c)
[**b)** *Sample response*: 6 × 3, 4 × 3, and 8 × 3 because I find it easy to count by 3s.]

3. a)

X X X X X X X X
X X X X X X X X
X X X X X X X X
X X X X X X X X

b)

X X X X X X X
X X X X X X X
X X X X X X X

c)

X X X X X X
X X X X X X

d)

X X X X X X X X
X X X X X X X X
X X X X X X X X
X X X X X X X X
X X X X X X X X

4. 7 × 4 = 28 or 4 × 7 = 28

5. *Sample responses*:
a)

X X X X X X X X
X X X X X X X X 3 × 8 = 24
X X X X X X X X
- - - - - - - - - - - - - - - -
X X X X X X X X
X X X X X X X X 3 × 8 = 24
X X X X X X X X

So 6 × 8 = 24 + 24 = 48.

b)

X X X X X X X | X X
X X X X X X X | X X
X X X X X X X | X X
X X X X X X X | X X

4 × 7 + 4 × 2
28 8

So 4 × 9 = 28 + 8 = 36.

6. *Sample responses*:
a)

X X X X X X
X X X X X X
X X X X X X
X X X X X X

X X X X X X X X
X X X X X X X X
X X X X X X X X

b) 4 × 6 = 24 and 3 × 8 = 24

7. a) They all have only one column.
[**b)** It shows that all I have to do is know how many rows there are to know how many are in the array.]

[**8.** An array of 5 rows and 7 columns has 5 equal rows or groups of 7, which is what 5 × 7 means.]

Supporting Students

Struggling students

• Some students may have difficulty knowing how to cut an array into smaller arrays to simplify a calculation. They might make horizontal or vertical cuts that do not allow them to use known multiplication facts. Clarify that the whole point of cutting the array is to be able to build from facts that students already know. Point out how a student can make multiple cuts as below, if necessary, and then add the pieces.

For example, to multiply 7 × 9:

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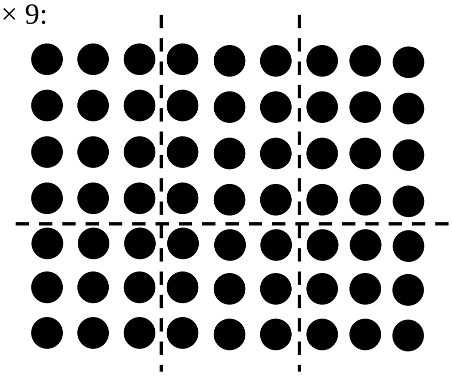

$$7 \times 9 = 4 \times 3 + 4 \times 3 + 4 \times 3 + 3 \times 3 + 3 \times 3 + 3 \times 3$$

Enrichment

• Students might explore which arrays can be rearranged to create other arrays (that do not use the same factors) and which cannot.

For example, a 2 × 6 array can be rearranged into a 3 × 4 array and a 1 × 9 array can be rearranged into a 3 × 3 array, but a 1 × 7 array cannot be rearranged at all.

GAME: Array Fact Match

• This very simple game is designed to allow students to practise making the connection between arrays and multiplication.

• If they wish, students can create their own array and multiplication fact cards to make up a different version of the game.

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2.1.3 EXPLORE: Meanings of Multiplication

Curriculum Outcomes	Outcome Relevance
4-B3 Multiplication Meanings: explore • explore various meanings of multiplication, focusing on multiplication as skip counting and repeated addition	This essential exploration helps students see that multiplication is used in a variety of different but related problem situations. Being aware of these relationships is essential for students to use multiplication effectively to solve a broad variety of real-world problems.

Pacing	Materials	Prerequisites
1 h	None	• familiarity with the symbolism for multiplication

Main Points to be Raised

• The different meanings of multiplication are distinct but related. They include:

- repeated addition (e.g., **I.** the Tshering's earnings problem)

- an array (e.g., **II.** the array of leaves problem)

- a set of equal groups (e.g., **III.** the groups of hearts problem)

- a rate (e.g., **IV.** the coin problem)

- the area of a rectangle (e.g., **V.** the area of a rectangle)

- the number of combinations if all the items of one group are matched with all the items of another group (e.g., **VI.** the boy-girl pairs problem)

• Students are not expected to know the name of each meaning. They can refer to the meaning by citing the problem number or the problem content.

For example, to indicate the combinations meaning, they might either say "problem VI" or "the boy-girl problem".

Exploration

• Discuss with students the six situations on **page 45** of the student text. Make sure they see that the solution to each problem is 12. For the last situation (**VI.** the boy-girl pairs problem), you may need to draw students' attention to the 12 lines that match the boys with the girls. Although students have already met the repeated addition, array, and equal groups meanings of multiplication, the other three meanings will be new to them.

• Assign students to work in pairs. While you observe students at work, you might ask questions such as the following:

• *Why might the phrase "4 times as far" in* ***part B*** *make you think about the coin problem?* (That problem talks about 4 times as many.)

• *How did you decide how to sketch the picture for* ***part C****?* (I made it look like the boy-girl pairs problem. I drew 5 colours of tiles on the first row and I drew 3 different patterns in the second row. Then I matched them. I noticed that Dechen drew 5 rows, each row with 3 patterns in one of the colours, and I can see that that works too.)

• *Why did you decide that* ***part D i)*** *was an equal groups problem?* (The groups are the feet and the number of toes tells how many in each group.)

• *Which three meanings did you decide to use for your problems?* (**II.** (array), **III.** (equal groups), and **VI.** (combinations))

Observe and Assess

As students work, notice the following:

• Do students make distinctions between the problem types or do they find the distinctions difficult to see?

• Do students recognize why all the different situations represent multiplication?

• Do students create problems for **part E** that exhibit different meanings of multiplication?

• Are students' created problems meaningful?

• Do students solve their problems correctly?

Share and Reflect

After students have had sufficient time to do the exploration, have them form groups to compare their problems. Afterwards, you may have a class discussion and pose these questions:

- *Who wrote a problem like the coin problem? What made it like the coin problem?*
- *Who wrote a problem like the rectangle area problem? What was your problem?*
- *How are Devika's and Arjun's problems alike? How are they different?*

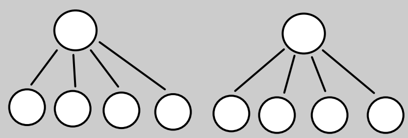
Answers

A. *Sample response*:
Tshering's earnings, the leaves, the hearts, and the rectangle each show 4 groups or rows of 3.
Since Mindu has 3 coins and Arjun has 4 times as many, I could show Arjun's coins as 4 groups of 3 coins.
There are 4 groups of 3 lines going from the boys to the girls.

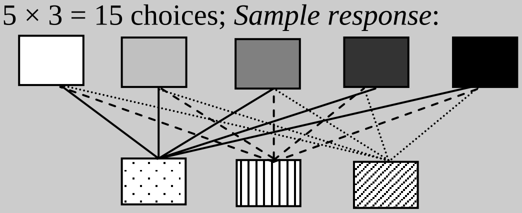
B. i) It is most like the coin problem (**IV**).
ii) 2 × 4 = 8
Sample response: Each counter means 1 km.

Man

Friend

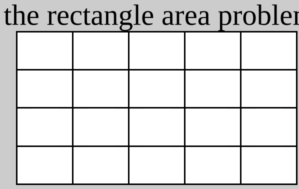


C. i) It is most like the boy-girl pairs problem (**VI**).
ii) 5 × 3 = 15 choices; *Sample response*:

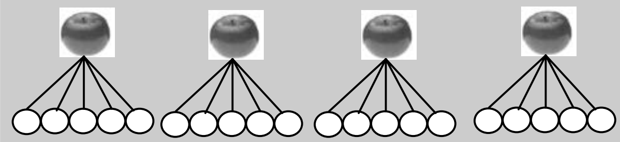


D. *Sample responses*:
i) It is like the hearts problem (**III**); 4 × 5 = 20

ii) It is like the rectangle area problem (**V**); 4 × 5 = 20

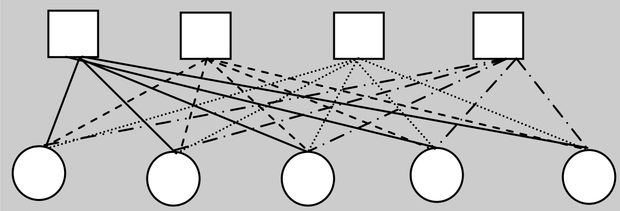


iii) It is like the coins problem (**IV**); 5 × 4 = 20

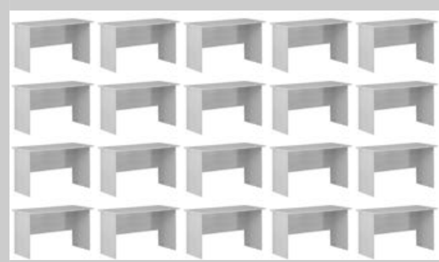


iv) It is like the earnings problem (**I**);
4 × 5 = 20; 5, 10, 15, 20

v) It is like the boy-girl pairs problem (**VI**); 4 × 5 = 20



vi) It is like the leaves problem (**II**); 4 × 5 = 20



E. *Sample response*:
- What is the sum if you add 8 to itself 5 times?
8 + 8 + 8 + 8 + 8 = 5 × 8 = 40
- There are 7 families, each with 3 children. How many children are there altogether? 7 × 3 = 21 children
- My brother does 1 hour of homework each night. I do 2 times as much. How many hours of homework do I do each night? 2 × 1 = 2 hours

Supporting Students

Struggling students

- Some students may have trouble with the three new meanings of multiplication (rate, area of a rectangle, and combinations). It is less important that they can identify the differences between these meanings and the known meanings. But they must learn to recognize when they should apply multiplication to problem situations that are related to the meanings.

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2.1.4 Relating Facts by Doubling and Halving

Curriculum Outcomes	Outcome relevance
4-B4 Multiplication Properties: explore • explore the commutative, distributive, and associative properties **B5 Multiplication Facts: to 9 × 9** • develop facts to 9 × 9 through concrete and pictorial representations • develop and practise strategies for fact recall • recall facts to 9 × 9	There are many independent facts for students to learn. It is helpful if they learn strategies that relate known facts to unknown facts. Relating facts to doubled or halved facts is one of these strategies.

Pacing	Materials	Prerequisites
1 h	None	• familiarity with some simple multiplication facts • halving or doubling single digit numbers

Main Points to be Raised

• The product of two numbers is double the product you would get if you multiplied one number by half of the other number.

For example, you can find 5 × 6 using 5 × 3 and then doubling. You can find 5 × 6 by taking half of 10 × 6.

• To multiply by 5, it is often easier to use doubling/ halving, i.e., multiply by 10 and then take half.

• If is often easier to multiply by an even number by halving/doubling, i.e., taking half of the even number and then doubling the result.

For example, to find 7 × 8, double 7 × 4.

[Although you do not need to use the following language with students, it is useful for you to know that these strategies are an application of the associative principle of multiplication.

That principle states that
(a × b) × c = a × (b × c),
so if b = 2, then (2a) × c = a × (2c) describes the strategy of halving, then doubling.]

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

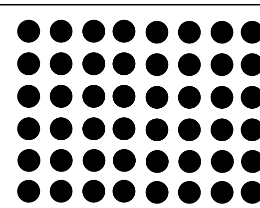
• *What do the two numbers in your fact tell you?* (number of groups and number of children in each group)

• *How do you know that 3 × 4 = 12?* (3 × 4 = 4 + 4 + 4 = 12)

• *Why does it make sense that the second product is twice as much as the first product?* (There are twice as many children in each group.)

The Exposition — Presenting the Main Ideas

• Draw a 6-by-8 array on the board. Divide it in half vertically to help students see why 6 × 8 is double 6 × 4. Point out how, if you know what 6 × 4 is, this makes it possible to figure out 6 × 8.



• Then take the original grid and this time divide it in half horizontally to see why 6 × 8 is double 3 × 8. Point out how if you already know 3 × 8, this helps you find 6 × 8.

• Present another example, e.g., 7 × 8. Ask students which number they might take half of in order to use a simpler fact to figure 7 × 8 out (take half of 8 so 7 × 8 → 7 × 4 = 28 and 28 × 2 = 56). Repeat this with 4 × 8, where students might use 2 × 8 or 4 × 4 and then double.

• Discuss how you can also use this idea if you know the doubled fact and want to find the “undoubled” fact. This is especially useful when you multiply by 5 because it is easy to multiply by 10.
For example: 6 × 5 → 6 × 10 = 60 and half of 60 is 30.

You might sketch a 6-by-10 array to show how a 6-by-5 array is half of it.

• Present another example, e.g., 8 × 5, and ask students how they could use 8 × 10 to figure out 8 × 5.

• Make sure students know they can refer to **pages 47 and 48** of the student text to review these ideas.

Revisiting the Try This

B. This question allows students to recognize that the notion of doubling and halving could have helped them solve the second problem about the boys dancing in **part A** after they had solved the first problem about girls dancing.

Using the Examples

• Assign students to work in pairs. One of the pair should become an expert by reading through **example 1** and the other should become an expert by reading through **example 2**. Then they should teach what they learned to their partner.

• Make sure students understand that they could double and then take half, or take half and then double no matter what numbers are used. The choice should be based on using a fact they know in order to figure out a fact that they do not know.

Practising and Applying

Teaching points and tips

Q 1: Although it is not incorrect to take half of the odd number, it is only taking half of the even number that will help students complete the calculation.

Q 2: Although it is not incorrect to double the 9 rather than the 5, it will not make the problem easier.

Q 3: This particular strategy will be useful later as students work with greater numbers.

For example, you can multiply 8 × 50 using 4 × 100 because 8 × 50 = 4 × 100.

Q 6: Even with greater numbers, students will be able to multiply by 8 by doubling 3 times.

Q 8: There are many patterns in the multiplication table that help students relate facts to one another. This exercise focuses on facts that are double one another.

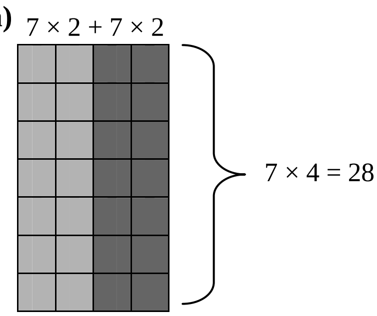
For example, since 4 times a number is twice 2 times a number, every number in the 4 row is double the corresponding number in the 2 row.

Note that **EXPLORE lesson 2.1.6** extends this notion to more patterns in the table.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use halving, then doubling to simplify a calculation
Question 2	to see if students can use doubling, then halving to simplify a calculation
Question 4	to see if students can draw a picture to explain the halving and doubling strategies

Answers

A. A. i) 3 × 4 = 12 **ii)** 3 × 8 = 24 **iii)** 24 is double 12	**B.** I could double the number of girls because there is double the number of boys dancing in each group.
1. a) 54; [6 × 9 = ? If 3 × 9 = 27, 6 × 9 = 27 + 27 = 54.] **b)** 28; [4 × 7 = ? If 2 × 7 = 14, 4 × 7 = 14 + 14= 28.] **c)** 72; [8 × 9 = ? If 4 × 9 = 36, 8 × 9 = 36 + 36 = 72.] **d)** 56; [7 × 8 = ? If 7 × 4 = 28, 7 × 8 = 28 + 28 = 56.] **2. a)** 45; [9 × 5 = ? 9 × 5 is half of 9 × 10 = 90. Half of 90 = 45, so 9 × 5 = 45.] **b)** 25; [5 × 5 = ? 5 × 5 is half of 5 × 10 = 50. Half of 50 = 25, so 5 × 5 = 25.] **3. a) i)** 3 × 8 = 24 **ii)** 3 × 10 = 30 **b) i)** 3 × 8 → 6 × 4 = 24; [3 × 8 = 6 × 4] **ii)** 3 × 10 → 6 × 5 = 30; [3 × 10 = 6 × 5]	**4.** *Sample responses*: **a)** 

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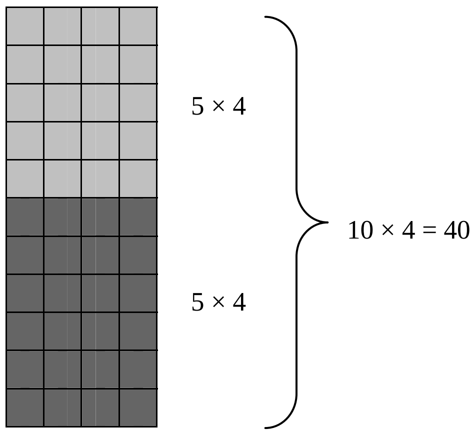
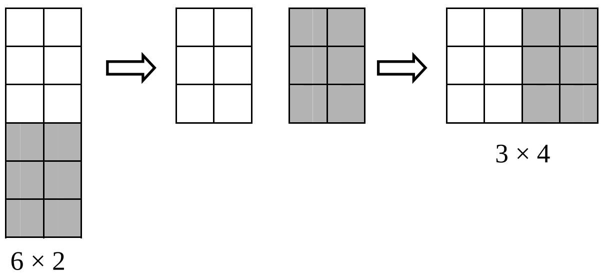
4. b)

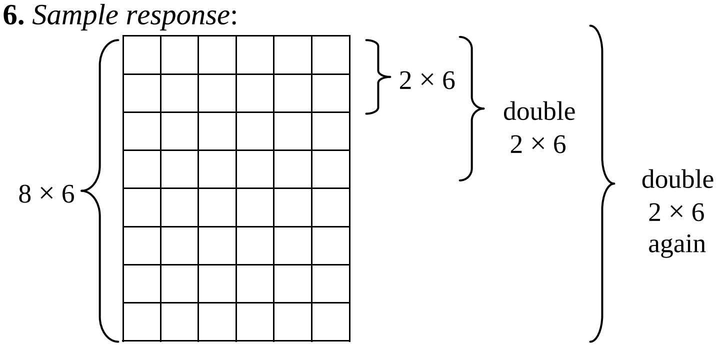


c)



5. a) 6 children **b)** 12 children and 24 children

6. *Sample response*:



[**7.** *Sample response*:
I chose 7.
7 × 3 = 21
7 × 6 = 21 + 21 = 42]

8. The row for 2 is double the row for 1.
The row for 4 is double the row for 2.
The row for 6 is double the row for 3.
The row for 8 is double the row for 4.
[*Sample response*:
2 groups is double 1 group.
4 groups is double 2 groups.
6 groups is double 3 groups.
8 groups is double 4 groups.]

Supporting Students

Struggling students

• Some students may be able to use the halving/doubling or doubling/halving strategy, but they may have difficulty explaining why it works. Those students may struggle with **question 4**. Instead of having them sketch pictures, you might sketch the pictures and have students explain how the pictures show the relationships.

Enrichment

• Students might explore the strategy of tripling and then taking one third or taking one third and then tripling.

For example, for 9 × 2, multiply one third of 9 by 2 (3 × 2) and then multiply the result (6) by 3 (6 × 3 = 18).

GAME: Matching Doubles

• This game is designed to help students focus on the relationship between facts where one is double the other.

• Students could create another game where they match facts that are equal because one factor is doubled and the other is halved.

For example, 4 × 8 and 2 × 16 match, and so do 6 × 2 and 3 × 4.

2.1.5 Multiplying by 7, 8, and 9

Curriculum Outcomes	Outcome relevance
4-B4 Multiplication Properties: explore • explore the commutative, distributive, and associative properties **4-B5 Multiplication Facts: to 9 × 9** • develop facts to 9 × 9 through concrete and pictorial representations • develop and practise strategies for fact recall • recall facts to 9 × 9	There are many independent facts for students to learn. It is helpful if they learn strategies that relate known facts to unknown facts. Relating facts to multiples of 10 simplifies multiplication by 7, 8, and 9.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Centimetre Grid Paper (BLM) (optional)	• multiplying 1-digit numbers by 10 • subtracting 1-digit numbers from 2-digit numbers • relating arrays to multiplication

Main Points to be Raised

• To multiply a number by 9, you can multiply by 10 and then subtract the number you are multiplying by.

• To multiply a number by 8, you can multiply by 10 and then subtract double the number you are multiplying by. Another strategy for multiplying by 8 is to double three times because 8 = 2 × 2 × 2.

• To multiply a number by 7, you can multiply by 10 and then subtract triple the number you are multiplying by. Another strategy for multiplying by 7 is to multiply by 5 and add double the number you are multiplying by.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How does the calendar show you how many days are in a week?* (1 row for each week and 7 days in a row.)

• *What addition could you do to tell how far away Karma's visit is?* (7 + 7 + 7 + 7 + 7 + 7 + 7 + 7)

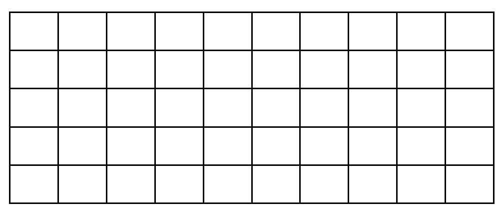
• *How can you write the addition as a multiplication?* (8 × 7)

• *How can you calculate the answer if you do not remember it?* (I could use 7 × 7 = 49 and then add 49 + 7 = 56.)

The Exposition — Presenting the Main Ideas

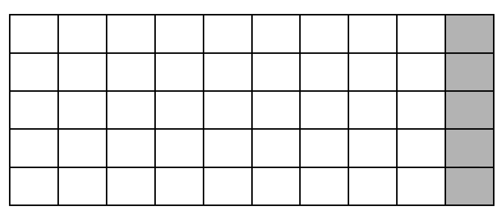
• Draw a 5-by-10 grid on the board. Ask students how they know the total number of squares in the grid is 50. Talk about why it is easy to know how much 5 × 10 is (5 × 10 is 5 tens, which is 50 since that is what 5 tens means).

5 × 10 = 50



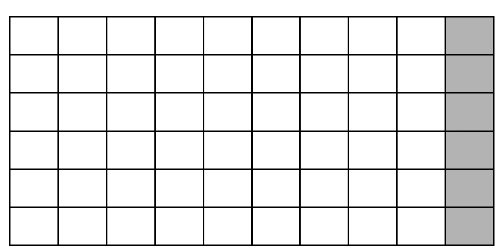
• Shade the final column of 5 within the grid. Ask why the unshaded part of the grid is a 5-by-9 grid. Ask why the number of squares in the unshaded part must be 50 – 5 = 45.

5 × 9 = 50 – 5



• Add another row to the original grid to show a 6-by-10 grid. Ask how students could use it to help them calculate 6 × 9.

6 × 9 = 60 – 6



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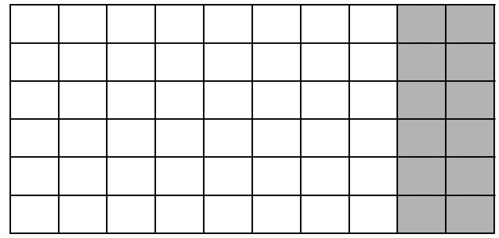
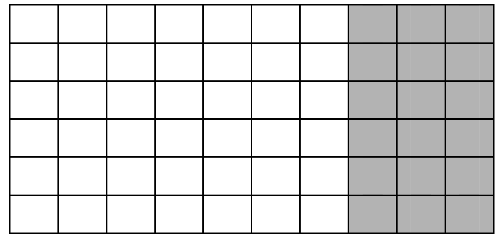
• Shade an additional column to the right. Ask students why the unshaded part is now 6-by-8. Discuss why the number of squares in that part is 60 – 2 × 6.

6 × 8 = 60 – 2 × 6



• Ask students how to change the grid to show 6 × 7. Again, ask how the original grid could be used to find the amount for 6 × 7.

6 × 7 = 60 – 3 × 6



• Let students know that they can review this material on **pages 51 and 52** of the student text.

Revisiting the Try This

B. This question allows students to apply the strategy for multiplying by 7 to answer the question in **part A**. Some students may have used that strategy already in **part A**.

Using the Examples

• With texts closed, ask students to use a strategy to multiply 7 × 8 by relating it to other facts. Then ask them which strategies they used. Encourage students to share different approaches.

For example, to multiply 7 × 8:

- double 7 three times
- add a group of 7 to 7 × 7
- multiply 7 by 10 and then subtract 2 × 7
- multiply 8 by 10 and then subtract 3 × 8
- add 2 × 8 to 5 × 8

Go over **example 1** with students to see three of the possible strategies that might have been used.

• Work through **example 2** with students so they see how to multiply by 4. They can also multiply 4 by 5 and 4 by 2 and then add the two amounts.

Practising and Applying

Teaching points and tips

Q 1: Students might choose to use a similar strategy repeatedly or they might vary the strategies they use.

Q 3: There are many patterns students might notice. For example:
• the sum of the digits is 9
• the ones digits go down by 1 while the tens digits go up by 1

Q 5 b): The calculation suggested in this question is called “calculation of a digital sum”. Digital sums are studied in a variety of mathematical situations. For example, digital sums are used to test for divisibility by 3 and by 9.

Q 6: Students might double 3 times, or subtract 6 from 30, or subtract 3 from 30 and then 3 from 27. Or, they might use another strategy.

Q 7: Encourage students to write a problem that is reasonable (the numbers should make sense for the situation).

Q 8: This question might work well as a class discussion.

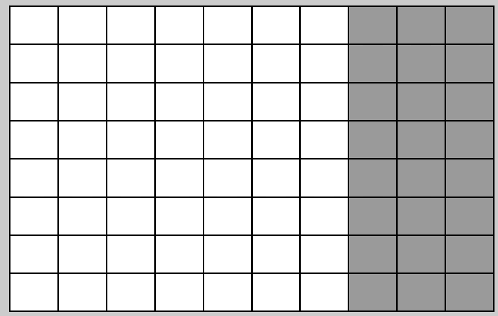
Suggested assessment questions from Practising and Applying

Question 1	to see if students can use a variety of strategies to multiply by 8 and by 9
Question 3	to see if students can use a multiplication pattern to predict another value in the pattern
Question 7	to see if students can create a meaningful multiplication problem to match a symbolic expression
Question 8	to see if students understand and can communicate about multiplication strategies

Answers

A. i) 7 days
ii) 56 days; 8 × 7 = 56

B. 8 × 7 = 8 × 10 – 8 × 3 = 80 – 24 = 56



1. a) 63; [7 × 9 = 7 × 10 – 7 × 1 = 70 – 7 = 63]
b) 54; [6 × 9 = 6 × 10 – 6 × 1 = 60 – 6 = 54]
c) 48; [*Sample response*:
8 × 6 = 10 × 6 – 2 × 6 = 60 – 12 = 48]
d) 64; [*Sample response*:
8 × 8 = ?
Double 8 is 16.
Double 16 is 16 + 16 = 32.
Double 32 is 32 + 32 = 64.
8 × 8 = 64.]

2. a) 9 **b)** 18 **c)** 27
d) 36 **e)** 45 **f)** 54

3. *Sample responses*:
a) I notice that the ones digit goes down by 1 while the tens digit goes up by 1.
b) 8 × 9 = 72; [I would go down 2 ones and up 2 tens from 54 (6 × 9 = 54, so 7 × 9 = 63 and 8 × 9 = 72).]

[**4.** He noticed the sum of the digits had to be 9.
If it was not 9, he would have made a mistake.]

5. a) 8, 16, 24, 32, 40, 48, 56, 64
b) 8, 7, 6, 5, 4, 3, 2, 1
c) The numbers decrease by 1 each time.

6. 24 bananas

7. *Sample response*:
The teacher grouped our class into groups of 5.
There are 8 groups. How many students are there?
(8 × 5 = 40)

[**8.** *Sample response*:
It is easy to multiply by 10 and by 2 and 3. It is also easy to subtract numbers like 6, 12, and 18.
To multiply 6 by 9, you can multiply 6 by 10 and then subtract 6.
To multiply 6 by 8, you can multiply 6 by 10 and then subtract 2 × 6 = 12. You could subtract 12 in parts by subtracting 10 and then subtracting 2 more.
To multiply 6 by 7, you can multiply 6 by 10 and then subtract 3 × 6 = 18. You could subtract 18 by subtracting 20 and then adding back 2.]

Supporting Students

Struggling students

• Some students may prefer to memorize multiplications and not to use strategies. Encourage them to learn the strategies because they will be useful for multiplications involving greater numbers. Try to emphasize the visual rather than the symbolic explanations with these students. Use simple and appropriate language.

For example, if you have 5 groups of 8, there are 2 fewer in each group than if you had 5 groups of 10, so 5 × 8 = 5 × 10 – 5 × 2.

Enrichment

• Students can develop strategies to relate multiplying 1-digit numbers by 11 or 12 to multiplying by 10.

For example, 5 × 12 = 5 × 10 + 5 × 2.

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2.1.6 EXPLORE: Multiplication Table Patterns

Curriculum Outcomes	Lesson Relevance
4-B4 Multiplication Properties: explore • explore the commutative, distributive, and associative properties **4-B5 Multiplication Facts: to 9 × 9** • develop facts to 9 × 9 through concrete and pictorial representations • develop and practise strategies for fact recall • recall facts to 9 × 9	This optional exploration allows students to relate many of the multiplication strategies they have learned in the previous lessons to the multiplication table. They may also notice patterns that provide insight for other strategies they can use in the future to calculate or check products.

Pacing	Materials	Prerequisites
1 h	• Multiplication Tables (BLM) (optional)	• adding and subtracting involving numbers less than 100

Main Points to be Raised

• The values in a row or column headed by any particular digit are always half of the values headed by the row or column headed by the double of that digit.

• The values in a row or column headed by any particular digit go up by the amount of that digit.

• The numbers on the main diagonal going from top left to bottom right increase by increasing odd numbers.

• There are many other patterns in the table. Some patterns involve a single row or column and other patterns involve more rows and columns.

For example:

• Within any 2-by-2 square in the table, the sums of the diagonals are 1 apart.

• The numbers in every other row or column alternate between even and odd. The numbers in the alternate rows or columns are all even.

Exploration

• Introduce the multiplication table to students. You might provide a copy of the table for each pair of students (see **page 92** of this teacher's guide for a blackline master) or have them refer to **page 54** in the student text.

• Talk about and model a consistent way to refer to the rows and columns in the table. For example, for the products of the multiplication facts for 3, refer to the row as "the ×3 row" and to the column as "the ×3 column".

• Review the idea of what a pattern is by providing some examples:

- The example in the book shows how the numbers in the ×1 row can be added to the numbers in the ×2 row to get the numbers in the ×3 row.

- Another pattern you might point out is how the numbers in any particular column increase in a constant way.

• Ask students to work with a partner. While you observe students at work, you might ask questions such as the following:

• *Why would the numbers in the ×4 row be half the numbers in the ×8 row?* (If I have 4 groups of something, it is only half as many as 8 groups of that amount.)

• *Why do the numbers in the ×9 column go up by 9?* (If I have one more group of 9, the amount is 9 more.)

• *How could you use the pattern of numbers in the ×9 column going up by 9 to help you multiply?* (If I want to find 6 × 9, I would use 5 × 9 and then add 9 more.)

• *Look at the numbers on the diagonals of any 2-by-2 square within the table. What do you notice?* (The sums are 1 apart, for example 20 + 30 = 50 and 24 + 25 = 49.)

20	25
24	30

• *What do you notice about the numbers on this diagonal*: *0, 1, 4, 9, 16, 25, 36, 49, ...? How could that help you figure out 8 × 8?* (They go up by 1, 3, 5, 7, 9, 11, 13, …, so 8 × 8 is 15 more than 7 × 7 = 49. It is 64.)

Observe and Assess

As students work, notice the following:

- Do students observe patterns with ease?
- Do students use the meanings of multiplication to explain patterns they observe?

For example, the sum of the ×2 row and the ×3 row = the ×5 row because 2 groups of a number plus 3 groups of a number is 5 groups of that number.

- Do students show creativity in finding patterns?
- Do students recognize how some of the patterns they found could be useful in multiplication situations?

Share and Reflect

After students have had sufficient time to do the exploration, discuss some of the patterns they observed and pose these additional questions:

- *Who found a pattern that would help you find 8 × 8? How would it help?*
- *Who found a pattern that would help you find 7 × 6? How would it help?*
- *What patterns did you find that involved more than one row or column? How could those patterns be useful when you multiply?*

Answers

A. i) The numbers in the ×2 row are double the ×1 row, the numbers in the ×4 row are double the ×2 row, and the numbers in the ×6 row are double the ×3 row. **ii)** The ×4 row and the ×8 row. **B.** The columns for ×1 and ×2, for ×2 and ×4, for ×3 and ×6, and for ×4 and ×8. *Sample response*: If there are twice as many in each group but the same number of groups, the total amount is double. **C. i)** They increase by 6s since there is 1 more in each of the 6 groups, so that is 6 more each time. **ii)** They increase by 8s since there is 1 more in each of the 8 groups, so that is 8 more each time. **D. i)** They increase by 3, then by 5, by 7, and by 9. **ii)** Yes; They increase by 11, 13, 15, and 17.	**E.** *Sample response*: Pattern 1: The numbers in the ×0 row and the ×0 column are all 0. Pattern 2: The numbers in the ×1 row and the ×1 column repeat the row and column names. Pattern 3: You can add the numbers in the ×2 row and the ×4 row to get the ×6 row. Pattern 4: In every other row, the numbers are even; in the other rows, they alternate between even and odd. Pattern 5: The digits of the products in the ×9 row and in the ×9 column add to 9. **F.** *Sample response*: Pattern 1 helps me multiply by 0. Pattern 3 helps me multiply 6 by 9. I can multiply 2 by 9 and 4 by 9 and add. Pattern 5 helps me check the answer when I multiply by 9; I have to make sure the digits add to 9.

Supporting Students

Struggling students

- Some students may need suggestions for patterns. Other students may find patterns but have difficulty explaining how they could be useful for multiplying. You may wish to make a list of a few patterns and a separate list of calculations that are easier if you know those patterns. Ask students to match the items in the two lists.

For example, they can match each pattern from the left with a fact on the right:

- the pattern in the ×0 row	What is 9 × 6?
- the pattern in the ×1 column	What is 0 × 7?
- the pattern of even/odd numbers in each row	Does 8 × 5 = 39?
- the pattern of digit sums in the ×9 row	Does 5 × 9 = 55?
- the pattern of ones digits in the ×8 column	What is 7 × 8?

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CONNECTIONS: Multiplication Fact Digit Circles

• This connection focuses students on exploring the pattern of the ones digits for the multiplication facts for each number by relating the pattern to a shape embedded in a circle.

• It also helps students see that the patterns for some digits relate to the patterns of other digits:

You get a pentagon for 4, 6, and 8.

You get a 10-sided shape for 7 and 9.

You get a vertical line for 5.

• Students can draw their own circles or you can provide copies of the blackline master Multiplication Fact Digit circles from **page 91** of this teacher's guide.

CONNECTIONS: Finger Multiplication

• This connection provides an interesting way for students to multiply 1-digit numbers by 9.

• Here is an explanation of how and why finger multiplication works:

9 × a number = 10 × the number – the number

If the number is a 1-digit number, then the number being subtracted is less than 10.

That means that the tens digit is 1 less than the tens digit would be for 10 × the number.

In that case, the tens digit is the number being multiplied.

For example, for 9 × 5:

If the 5th finger is turned down, there are 4 fingers to the left of it (1 less than 5).

Because the sum of the digits of the product is 9, that means the ones digit must be the rest of the fingers (9 fingers are raised and the tens digit fingers are to the left of the finger folded down).

Chapter 2 Division

2.2.1 Division as Sharing

Curriculum Outcomes	Outcome relevance
4-B7 Division Meanings: small numbers • understand division as grouping or sharing	Students need to know when to apply the operation of division. One situation that requires division is a problem where an amount is equally shared.

Pacing	Materials	Prerequisites
1 h	• 24 counters	• familiarity with multiplication facts to 9 × 9 • using an array to describe multiplication

Main Points to be Raised

• One of the meanings of division is sharing.
The quotient is the result of dividing. It tells the share size if you know the total number of items and the number of sharers (or groups).

• You can model sharing with an array. The array makes it clear how division is related to multiplication

• You can check division by using multiplication.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know each student will get more than 5 biscuits?* (That would be 3 students × 5 biscuits each, which is only 15. There are 18 biscuits.)

• *How do you know each student will get fewer than 8 biscuits?* (That would be 3 students × 8 biscuits each, which is 24. There are only 18 biscuits.)

• *How did you figure out your answer?* (I shared 18 counters into 3 groups to see how many each group got.)

The Exposition — Presenting the Main Ideas

• Hold 20 counters in your hand and tell the class you have 20 counters. Invite 5 students up to the front and model how to share the counters equally by giving 1 to each student repeatedly until all the counters have been shared. Have each student show the 4 counters he or she got. Write 20 ÷ 5 = 4 to describe that process.

• Sketch the process on the board using an array, where each column represents one of the five students and each row shows one of the counters in the student's share. Write 20 ÷ 5 = 4 to describe the process. Read the fact to them like this: "If 20 counters and shared by 5 students, each student gets 4 counters". Have students observe that the array also shows 4 × 5= 20.

Student

1	2	3	4	5
x	x	x	x	x
x	x	x	x	x
x	x	x	x	x
x	x	x	x	x

20 ÷ 5 = 4
4 × 5 = 20

• Now hold 24 counters. Bring up 4 students to share them equally. Ask students to sketch an array to predict the resulting share size and to describe the related division and multiplication facts. Check their answers by performing the sharing. The facts should be 24 ÷ 4 = 6 and 6 × 4 = 24.

• Refer to students to **page 56** of the student text for a review of what you have done in this lesson.

Student

1	2	3	4
x	x	x	x
x	x	x	x
x	x	x	x
x	x	x	x
x	x	x	x
x	x	x	x

24 ÷ 4 = 6
6 × 4 = 24

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Revisiting the Try This

B. This question allows students to make a formal connection between the sharing problem they solved in **part A** and multiplication and division facts.

Using the Examples

• Present the problem from the **example** to students. Have them come up with their own strategy for solving the problem. Then work through the example with students. Help them see why each solution is an appropriate way to solve the problem. Ask if any students used a different approach. If so, have them share it with the other students.

Practising and Applying

Teaching points and tips

Q 1: Suggest that students first predict the size of the share and then use their picture to check.

Q 2: Students might be given 35 counters to share into 7 piles or they might sketch an array model using 7 columns.

Q 3: Ask students why the two related multiplication facts use the same three values: 4, 5, and 20.
Ask why the quotient for **part b)** has to be less than the answer to **part a)**.

Q 4: Although repeated subtraction most obviously models counting the number of groups of a given size in a total rather than modeling a share size, you can also think of repeated subtraction in terms of the sharing model.

Students can think of creating the groups and then giving one item to each group until all the items are gone. The subtraction describes the result of giving one item to each group.

Q 5: Encourage students first to divide the line into 2 or 3 equal parts and to then divide each of those parts.

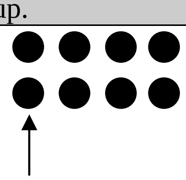
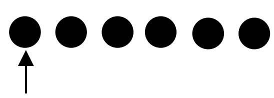
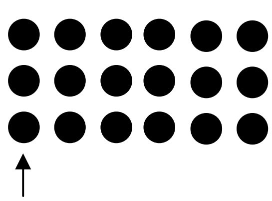
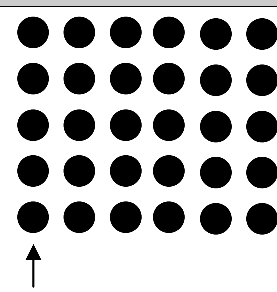
Q 7: Students need to realize that the number of items must be a number in the 6 times table (a multiple of 6).

Q 8: Because the number of tins is divisible both by 3 and by 4, it must also be divisible by 12. Any multiple of 12 could be an answer.

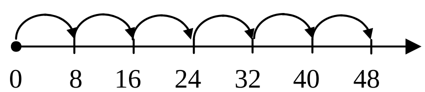
Suggested assessment questions from Practising and Applying

Question 1	to see if students can write a division fact to solve a division problem
Question 2	to see if students can solve a real-world division problem
Question 6	to see if students can create a problem for a given division fact
Question 9	to see if students can communicate about how to divide

Answers

A. 6 biscuits; 18 shared in 6 groups means there will be 3 in each group.	**B.** 18 ÷ 6 = 3 and 3 × 6 = 18
1. a) 8 ÷ 4 = 2  one share is 2 **b)** 6 ÷ 6 = 1  one share is 1 **c)** 18 ÷ 6 = 3  one share is 3	**d)** 30 ÷ 5 = 6  one share is 6 **2.** 5 students **3. a)** 5 toys **b)** 4 toys

Lesson 2.2.1 Answers [Continued]

[**4. a)** *Sample response*: Subtracting 7s is like giving 1 thing to each of 7 people. If you keep doing that until you have no more things (when you get to 0), and then you count how many times you did it, you are figuring out the size of each share. **b)** 35 – 5 = 30 30 – 5 = 25 25 – 5 = 20 20 – 5 = 15 15 – 5 = 10 10 – 5 = 5 5 – 5 = 0 I subtracted 7 times, so 35 ÷ 5 = 7.] **5. a)**  [*Sample response*: First I divided the line in half: from 0 to 24 and from 24 to 48. Then I divided each half into 3 equal sections. I knew 3 × 8 = 24, so I knew each section was 8.] **b)** 8; [*Sample response*: There are 6 equal sections of 8 in 48.]	**6.** *Sample response*: There were 30 momos and 5 people eating them. How many momos should each person get if they each get the same amount? **7.** *Sample response*: 6, 12, 18, or 24 items **8.** *Sample response*: 12, 24, or 36 tins **9.** *Sample response*: Sharing 36 counters into 9 equal groups Finding what to multiply 9 by to get 36: 9 × ? = 36

Supporting Students

Struggling students

• Some students may need actual counters to act out the sharing. Other students might draw Xs to act out the sharing. Acting out the sharing, one way or the other, is appropriate at this stage. Students should not be hurried out of this stage if they are not ready.

Enrichment

• Students might be given other problems like **question 7** to try, but this time suggest that there are, for example, two items left over. Students will need to use more sophisticated strategies to solve these problems.

Reprint 2023

2.2.2 Division as Grouping

Curriculum Outcomes	Outcome relevance
4-B7 Division Meanings: small numbers • understand division as grouping or sharing	Students need to know when to apply the operation of division, Another situation that requires division is a problem where you want to know how many equal groups you can make.

Pacing	Materials	Prerequisites
1 h	• Counters	• familiarity with multiplication facts to 9 × 9 • familiarity with division as sharing • using an array to describe multiplication

Main Points to be Raised

• One of the meanings of division is grouping. The quotient tells how many groups of a given size you can make from a total amount.

• The sharing meaning of division can be related to the grouping meaning since sharing involves creating groups.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How many are in a pair?* (Two students make a pair.)

• *How do you know the number of pairs is less than 10?* (If there were 20 students, there would be 10 pairs. There are fewer than 20 students.)

• *How might you solve this?* (Sketch 16 circles, make groups of 2, and count the number of groups of 2.)

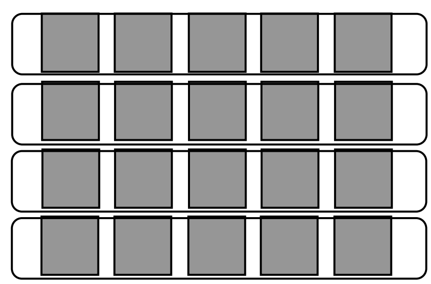
The Exposition — Presenting the Main Ideas

• On the board draw 20 squares in a 4-by-5 array. Tell students the squares represent 20 biscuits. Ask students how many plates of biscuits they could make if there are to be 5 biscuits on each plate. Model their thinking by circling each row of 5 squares.

• Point out how the picture on the board looks just like the picture they used to show sharing 20 among 5 groups. Mention that this time when they write the fact 20 ÷ 5 = 4, they read it as "If you put 20 items into groups of 5, there are 4 groups".

• If you feel students need another grouping example, sketch 16 circles and group them in 8s to show 16 ÷ 8 = 2.

• Refer to students to **page 59** of the student text for a review of what you have done.



20 items ÷ groups of 5 = 4 in each group
20 ÷ 5 = 4

Revisiting the Try This

B. This question allows students to make a formal connection between the pairing problem they solved in **part A** and the process of division as grouping.

Using the Examples

• Present the problem from the **example** to students. Ask students to come up with their own strategies for solving the problem. Then work through the example with students. Help them see why each solution is an appropriate way to solve the problem. Ask if any students used a different approach. If so, have them share it with the class.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to label their pictures so it is clear what each number in the problem refers to.

Q 3: Some students might benefit from first imagining the book is 30 or 36 pages and then adjusting the answer.

Q 4: Some students might benefit from using counters or sketching a picture of 30 items. They could then try different possible groupings, beginning at 2.

Q 5: Students can use their knowledge of multiplication to help them answer this question.

For example, knowing that 7 × 8 = 56 tells them they could write the fact 56 ÷ 7 = 8.

Q 6: Encourage students to see that skip counting involves forming groups, in this case groups of 6.

Q 7: Some students will realize that every sharing problem involves grouping and every grouping problem involves sharing, but the way the question is posed differs.

For example, if you share 30 among 6, you could be creating groups of 6 or you could be sharing among 6 groups.

Suggested assessment questions from Practising and Applying

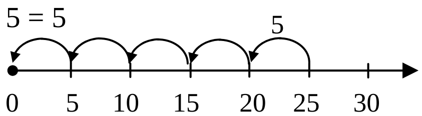
Question 1	to see if students can write a division fact to model and solve a problem
Question 3	to see if students can solve a real-world division problem
Question 6	to see if students can relate skip counting to division
Question 7	to see if students can create division problems to match a mathematical expression

Answers

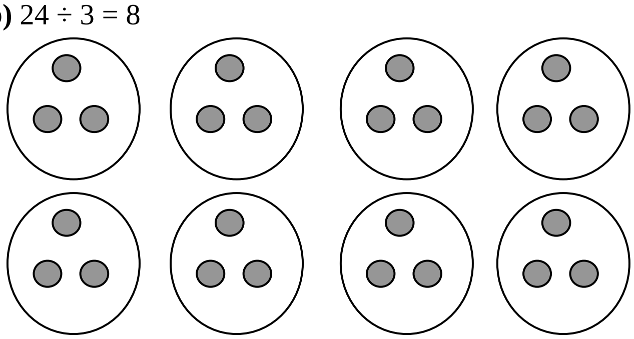
A. 8 pairs

B. i) 16 ÷ 2 = 8
ii) *Sample response*:
I am finding out how many groups of 2 are in 16, so it is a grouping problem.

1. *Sample responses*:
a) 25 ÷ 5 = 5



b) 24 ÷ 3 = 8



c) 36 ÷ 9 = 4

Group 1	X	X	X	X	X	X	X	X	X
Group 2	X	X	X	X	X	X	X	X	X
Group 3	X	X	X	X	X	X	X	X	X
Group 4	X	X	X	X	X	X	X	X	X

2. 4 cars

3. 7 nights

4. *Sample response*:
6 teams of 5, 5 teams of 6, 3 teams of 10, 10 teams of 3

5. 32 ÷ 8 = 4, 56 ÷ 7 = 8, 24 ÷ 6 = 4

[**6.** *Sample response*:
a) When she skip counts by 6s, she makes groups of 6 numbers. If she says 7 numbers, that means she made 7 groups of 6, and that is what 42 ÷ 6 = 7 means.
b) 7, 14, 21, 28, 35, 42, 49, 56; That is 8 numbers, so 56 ÷ 7 = 8.]

7. *Sample response*:
• 72 students are divided up into groups of 9.
How many groups are there? (8 groups)
• 72 students are formed into 9 groups.
How many students are in each group? (8 students)

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Supporting Students

Struggling students

• Some students may need actual counters to act out the grouping. Other students might sketch a model.

• Some students will struggle with distinguishing grouping situations from sharing situations. Make sure they realize that both situations relate to division. They do not need to not focus on the differences.

Enrichment

Ask students to create the following game for division that they can play with classmates:

• They create a slip of paper that reads: "I have 8." on one side and "Who has 42 ÷ 7?" on the other side.
• The next slip of paper they make has the answer to the previous question on one side (6 for "Who has 42÷ 7?") and a new division question on the other side, for example, "Who has 72 ÷ 9?"

• The next slip of paper they make has the answer to the previous question on one side (8 for "Who has 72÷ 9?") and a new division question on the other side.

• This continues until the last slip of paper, which has a division question where the quotient is 8, e.g., "Who has 64 ÷ 8?" to link back to the first side of the first slip, "I have 8."

• They mix up the slips of paper each player takes one slip.

• The first player begins by asking his or her question. A player with the answer responds, turns over his or her card that has the answer, and then asks his or her question.

• The game is over when each person has had one chance to answer and ask a question.

2.2.3 Multiplication and Division Fact Families

Curriculum Outcomes	Outcome relevance
4-B5 Multiplication Facts: to 9 × 9 • develop facts to 9 × 9 through concrete and pictorial representations • develop and practise strategies for fact recall • recall facts to 9 × 9 **4-B8 Division Properties: explore** • understand that order matters when dividing **4-B9 Multiplication and Division Facts: relate through properties** • understand multiplication and division as two ways of looking at the same situation • use multiplication facts to recall division facts	When students think of division as “reverse” multiplication, they can recall division facts using many of the strategies they learned for recalling multiplication facts.

Pacing	Materials	Prerequisites
1 h	None	• using an array to represent multiplication

Main Points to be Raised

• Any array represents both multiplication and division facts. The two or four facts that describe a given array are called a fact family.

• To find a quotient, you can use a related multiplication fact.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How can you figure out the number of squares in each row?* (Put an X in each of 5 rows until I have drawn 40 Xs.)

• *What multiplication fact describes the situation?* (5 × ? = 40)

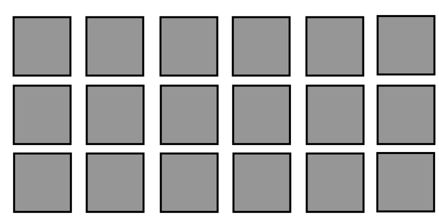
• *How would knowing your multiplication facts help you figure out the number of squares in each row?* (I would be thinking about 5 × 8 = 40, so 40 ÷ 5 would be 8.)

The Exposition — Presenting the Main Ideas

• On the board draw 18 squares in a 3-by-6 array. Ask students to use two multiplication facts to describe the array. Then ask what division facts describe the array and why. Students might see that the 18 is shared into 6 groups, so 18 ÷ 6 = 3 makes sense. 18 is also shown in groups of 3 and there are 6 groups of 3, so 18 ÷ 3 = 6 also makes sense.

• Write the four facts on the board.

Tell students these are called a *fact family*.



3 × 6 = 18   6 × 3 = 18   18 ÷ 6 = 3   18 ÷ 3 = 6

• Ask students for the fact family that involves 4, 6, and 24 and the fact family that involves 4, 4, and 16. Have students come to the board to sketch arrays for these families. Point out that the first family has four facts but the second family only has two facts because the same factor is used twice.

X X X X X X
X X X X X X
X X X X X X
X X X X X X

4 × 6 = 24   6 × 4 = 24
24 ÷ 6 = 4   24 ÷ 4 = 6

X X X X
X X X X
X X X X
X X X X

4 × 4 = 14
16 ÷ 4 = 4

• Refer to students to **page 61** of the student text for a review of what you have done.

Revisiting the Try This

B. This question allows students to make a formal connection between the array problem they solved in **part A** and the concept of a fact family.

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Using the Examples

• Present the problem from the **example** to students. Ask them to use a related multiplication to solve the problem and then check their work against the solution.

Practising and Applying

Teaching points and tips

Q 3: Students should notice that there are only two facts when the number of rows is the same as the number of columns. Otherwise, there are four facts.

Q 4: Some students may also wish to write the addition and subtraction facts for these.

Q 5: Students should relate the repeated additions and subtractions to either multiplication or division and then relate those multiplications or divisions to the other operation.

Q 7: Students do not need to use the word fact family in their explanations.

Q 8: This question might best be handled in a full class discussion.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can relate an array to a fact family
Question 4	to see if students realize that multiplication and division are related to repeated addition
Question 7	to see if students can use a multiplication fact to help them solve division questions

Answers

A. 8 squares; *Sample response*: 5 × __ = 40, 5 × <u>8</u> = 40	**B. i)** 40 ÷ 8 = 5 40 ÷ 5 = 8 5 × 8 = 40 8 × 5 = 40 **ii)** To solve 40 ÷ 5, I can use 8 × 5 = 40, so 40 ÷ 5 = 8.
1. 9 × 7 = 63 7 × 9 = 63 63 ÷ 7 = 9 63 ÷ 9 = 7 **2. a)** 5 × 5 = 25 and 25 ÷ 5 = 5 **b)** 4 × 8 = 32 8 × 4 = 32 32 ÷ 4 = 8 32 ÷ 8 = 4 [**3.** Both numbers I multiply by or divide by are 5. If I change the order of the numbers, I am repeating the same facts.] **4. a)** 9 × 4 = 36 and 36 ÷ 4 = 9 **b)** 5 × 5 = 25 and 25 ÷ 5 = 5	**5. a)** 5 × 9 = 45 and 45 ÷ 9 = 5 **b)** 42 ÷ 6 = 7 and 7 × 6 = 42 [**6. a)** Find out what to multiply 4 by to get 32. **b)** Divide 32 by 4.] [**7.** *Sample response*: I know 5, 6, and 30 are the numbers in the fact family so for **part a)** the answer is the missing number, 5. For **part b)** the answer is the missing number, 6.] **8.** Yes; [*Sample response*: If I know all the multiplication facts, then I know the three numbers in each fact family. When I see two of the numbers in a division, I know that the answer is the missing third number.]

Supporting Students

Struggling students

• Students usually have no difficulty understanding what a fact family is. Some students may have difficulty relating the fact family to the additions and subtractions in **questions 4 and 5**. You may need to remind them of how a repeated addition model represents multiplication and how a repeated subtraction model represents division. They can then translate the symbolic statements in **question 5** to a picture, as in **question 4**.

Enrichment

• Ask students to find all the fact families that include a particular product, such as 24 or 18.

2.2.4 EXPLORE: Multiplying and Dividing with 1 and 0

Curriculum Outcomes	Outcome Relevance
4-B4 Multiplication Properties: explore • explore multiplying by 0 and 1 **4-B8 Division Properties: explore** • dividing with 0 and 1	This essential exploration focuses on the properties of multiplying by 0 and 1, dividing by 1, and dividing 0 by other numbers. Knowing these properties will help students simplify the number of individual multiplication and division facts they must memorize.

Pacing	Materials	Prerequisites
40 min	None	• interpreting multiplication as the number in an array • interpreting division as sharing or grouping

Main Points to be Raised

• The product of 1 and any number is the number.

• The product of 0 and any number is 0.

• The quotient of a number divided by 1 is the number.

• The quotient of 0 divided by another number is 0.

• Each of the generalizations involving multiplication and division by 1, division of 0, and multiplication by 0 can be explained using known meanings of multiplication and division.

Exploration

• Tell students that they will be focusing on multiplication and division involving 1 and 0.

• Ask students to work in pairs. While you observe students at work, you might ask questions such as the following:

• *How many rows do your arrays have?* (1 row)

• *Why is it easy to count the total number if you know the number of columns?* (Since there is only 1 row, the number of columns is the total number.)

• *What divisions do your arrays show?* (3 ÷ 3 = 1 and 3 ÷ 1 = 3; 4 ÷ 4 = 1 and 4 ÷ 1 = 4; 5 ÷ 1 = 5 and 5 ÷ 5 = 1.)

• *How did your pattern help you figure out 0 × 8?* (The numbers keep going down by 8 so it had to be 8 – 8 = 0.)

• *Why does it make sense that 0 × 5 = 0?* (If I have no groups, there is nothing in the total.)

• *Why does it make sense that 0 ÷ 5 = 0?* (Once I know that 0 × 5 = 0, I use the division fact in the same fact family. Or I think that 0 items shared by 5 people means each person gets 0.)

Observe and Assess

As students work, notice the following:

• Do students draw arrays correctly?

• Do students form appropriate conclusions?

• Do students apply their conclusions correctly?

• Can students make sense of why 1 and 0 have the properties they observe?

• Can students clearly communicate rules about multiplying and dividing with 1 and 0?

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Share and Reflect

After students have had sufficient time to do the exploration, have each group of students share their answers to **part E** with another group of students. You may wish to show how the pattern in **part B** could also help show why a number multiplied by 1 is that number. Then ask:

• *Knowing what multiplication means, why does it make sense that 1 × 5 = 5?*

• *Knowing what division means, why does it make sense that 5 ÷ 1 = 5?*

• *Knowing what division means, why does it make sense that 0 ÷ 5 = 0?*

• *Knowing what multiplication means, why does it make sense that 0 × 5 = 0?*

Answers

A. i) 1 × 6

X X X X X X

6 × 1

X
X
X
X
X
X

1 × 8

X X X X X X X X

8 × 1

X
X
X
X
X
X
X
X

1 × 4

X X X X

4 × 1

X
X
X
X

ii) 6, 8, and 4

iii) 6 ÷ 1 = 6 and 6 ÷ 6 = 1
8 ÷ 1 = 8 and 8 ÷ 8 = 1
4 ÷ 1 = 4 and 4 ÷ 4 = 1

iv) Since the product is the number of columns or rows, then 1 × 9 = 9 and 9 × 1 = 9.
Since each array showed [ ] ÷ 1 = [ ], then 9 ÷ 1 = 9.

B. i) 4 × 8 = 32
3 × 8 = 24
2 × 8 = 16
1 × 8 = 8

ii) 0 × 8 = 0; The pattern goes down by 8s and 8 – 8 = 0.

C. i) If I have 0 groups of 6, I do not have anything, and that means 0.

ii) If I have 6 groups but nothing in them, I still have nothing, and that means 0.

D. 0 ÷ 4 means there are 0 items to be shared into 4 groups so there is nothing in each group.
0 ÷ 4 = 0

E. i) If you multiply by 0, the product is 0.

ii) If you divide 0 by a number (but not 0), the quotient is 0.

iii) If you multiply a number by 1, the product is the number.

iv) If you divide a number by 1, the quotient is the number.

Supporting Students

Struggling students

• Some students may generalize the rules for 1 and 0 but have difficulty explaining them. Help students by encouraging them to use whatever meanings of multiplication and division make the most sense to them.

For example:

For 9 × 1, students who are not clear on how the array model helps them see why the product is 9 might think about adding 1 nine times: 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1. They can see that the total is the number of 1s, or 9.

For 9 ÷ 1, they might find it easier to think of a grouping model, i.e., "How many 1s are in 9?" instead of a sharing model.

Enrichment

• Some students might be interested in knowing by you cannot divide by 0. For example, to divide 15 ÷ 5 you can subtract 5 three times from 15 until you get to 0, so 15 ÷ 5 = 3. So, to divide 15 ÷ 0, you must determine how many times you can subtract 0 from 15 before you get to 0. There is no answer because you will never get to 0. Therefore, 15 ÷ 0 is undefined. For 0 ÷ 0, you can subtract 0 any number of times from 0 to get to 0, so the answer could be 1, 2, 3, or any number. Since there are too many answers, 0 ÷ 0 is indeterminate.

UNIT 2 Revision

Pacing	Materials
2 h	• Grid paper or Centimetre Grid Paper (BLM) (optional)

Question(s)	Related Lesson(s)
1 – 3	Lesson 2.1.1
4 – 7	Lesson 2.1.2
8	Lesson 2.1.3
9 and 10	Lesson 2.1.4
11 and 12	Lesson 2.1.5
13	Lesson 2.2.1
14 – 17	Lesson 2.2.2
18	Lesson 2.2.3
19	Lesson 2.2.4

Revision Tips

Q 2: You might ask students how this could be solved with multiplication as well as how it could be solved with addition.

Q 5: Students might write 3 × 9 or 9 × 3 or both.

Q 7: Discuss with students why it makes sense to divide the array in such a way that they already know the products for the two parts.

Q 9: Make sure students use a sketch along with a verbal explanation, not just a numerical answer.

Q 10 a): Students could halve or double either factor.

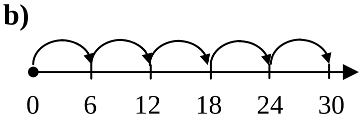
Q 12: There are many possible correct answers.

For example, 7 × 10 and 7 × 2; 7 × 9 and 7 × 1; 10 × 8 and 3 × 8.

Answers

1. a) 4 × 6 = 24

b)



2. 40 books

3. [**a)** He would start at 21 and then skip count by 7s two times: 21, 28, 35.]
b) 5 × 7 = 35

4. 2 × 5 = 10

5. 3 × 9 = 27 or 9 × 3 = 27

6. a)

X X X X X X X X X
X X X X X X X X X
X X X X X X X X X
X X X X X X X X X
X X X X X X X X X

5 × 9 = 45

b)

X X X X X X
X X X X X X
X X X X X X
X X X X X X
X X X X X X
X X X X X X

6 × 6 = 36

7. a) and b) *Sample responses*:

X X X X X X | X X X
X X X X X X | X X X
X X X X X X | X X X
X X X X X X | X X X
X X X X X X | X X X

5 × 6 = 30   5 × 3 = 15
5 × 9 = 30 + 15 = 45

X X X X X X
X X X X X X
X X X X X X
- - - - - - - - - - - -
X X X X X X
X X X X X X
X X X X X X

3 × 6 = 18
3 × 6 = 18
6 × 6 = 18 + 18 = 36

[**8.** *Sample responses*:
a) There are 7 days in 1 week, so there are 7 + 7 + 7 days in 3 weeks and that is what 3 × 7 can mean.
b) If I covered the area of a rectangle with squares arranged in 3 rows of 7 squares, I would have a 3 row-by-7 column array. That is what 3 × 7 can mean.]

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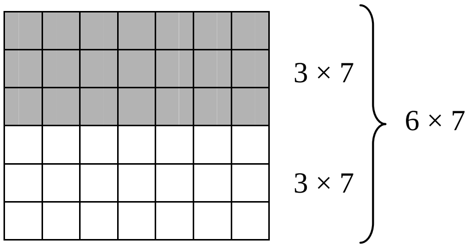
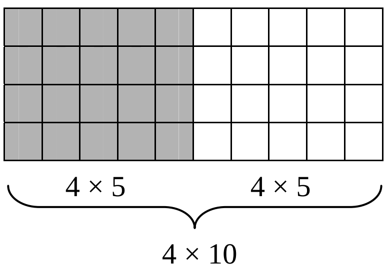
[**8. c)** I can draw 7 coins for my friend and then draw 3 coins for each of my friend's coins because I have 3 times as many. I would have 3 groups of 7 coins. That is what 3 × 7 can mean.]

9. *Sample responses*:

a)



b)



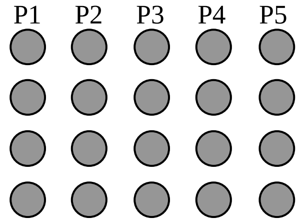
10. a) 48; [3 × 8 = 24, so 6 × 8 = 24 + 24 = 48.]
b) 40; [5 × 4 = 20, so 5 × 8 = 20 + 20 = 40.]

[**11. a)** Subtract 5 from 50 to get 45.
b) Subtract 2 × 5 from 50 to get 40.]

12. *Sample responses*:
a) I could use 7 × 4 and double it [since 7 × 8 is double 7 × 4.]
b) 56; [7 × 8 → 7 × 2 → 14 × 2 = 28 × 2 = 56]

13. *Sample responses*:

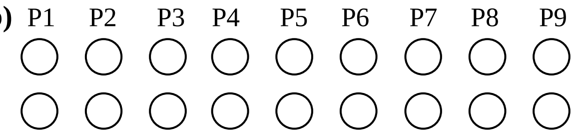
a)



20 ÷ 5 = 4

P = person

b)



18 ÷ 9 = 2

14. 18 ÷ 2 = 9 pairs

15. *Sample responses*:
a) Six students are sharing 30 sweets. How many candies can each student have?
b) The teacher is dividing 30 students into groups of 6. How many groups can the teacher make?

16. 9 triangles

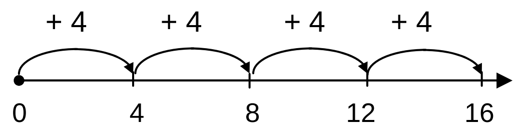
17. 5 days

18. 2 × 8 = 16
8 × 2 = 16
16 ÷ 2 = 8
16 ÷ 8 = 2

[**19. a)** The product is always 0 no matter what the number is.
b) The quotient is always the number you start with.]

UNIT 2 Multiplication and Division Facts Test

1. a) Which multiplication fact does this model show?



b) Sketch a number line model for 4 × 7.

2. Sketch an array to show each and write the multiplication fact.

a) 3 × 7 **b)** 5 × 5

3. a) Describe three different things that 6 × 7 could mean.

b) Show how all three meanings relate to 6 groups of 7.

4. Choose four parts below to complete.

Sketch a picture to show why each is true.

Use your picture to write a multiplication fact.

a) 6 × 5 is double 3 × 5

b) 6 × 5 = 2 × 5 + 4 × 5

c) 8 × 5 is half of 8 × 10

d) 8 × 8 = 8 × 10 – 8 × 2

e) 8 × 5 is double the double of 2 × 5

f) 4 × 9 = 4 × 10 – 4 × 1

5. Tell three different ways you could use other facts to multiply 5 × 9.

6. Solve each.

a) 6 students are sharing 42 momos. How many can each get if they share equally?

b) How many 5 cm paperclips can you line up to make 30 cm?

c) How many squares that are not touching can you make with 36 sticks of equal length?

7. a) 35 items are put into groups of 7. How many items are in each group?

b) 35 items are shared by 7 people. How many items does each person get?

c) Explain why the answers to **part a) and part b)** are the same. Use words or a picture.

8. Write the fact family that includes each set of numbers.

a) 7 and 21

b) 6 and 9

c) 8 and 56

9. What multiplication fact can you use to help you divide 28 by 4? Why?

10. Which is greater in each pair?

a) 5 × 0 or 4 × 1

b) 0 ÷ 8 or 7 ÷ 1

11. Explain why 0 ÷ 7 = 0.

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UNIT 2 Test

Pacing	Materials
1 h	• Grid paper or Centimetre Grid Paper (BLM) (optional)

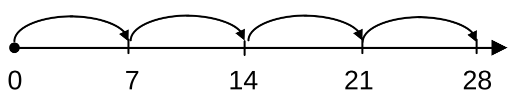
Question(s)	Related Lesson(s)
1	Lesson 2.1.1
2	Lesson 2.1.2
3	Lesson 2.1.3
4	Lessons 2.1.4 and 2.1.5
5	Lessons 2.1.5 and 2.1.6
6 and 7	Lessons 2.2.1 and 2.2.2
8 and 9	Lesson 2.2.3
10 and 11	Lesson 2.2.4

Select questions based on time available.

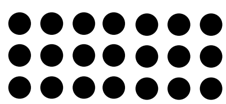
Answers

1. a) $4 \times 4 = 16$

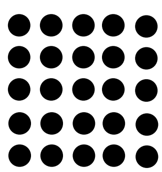
b)



2. a) $3 \times 7 = 21$



b) $5 \times 5 = 25$



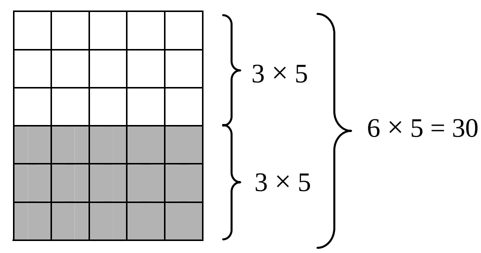
3. *Sample responses*:

a) 6 groups of 7 things;
An array that is 6 rows by 7 columns;
The number of boy-girl pairs if each of 6 boys is paired with each of 7 girls.

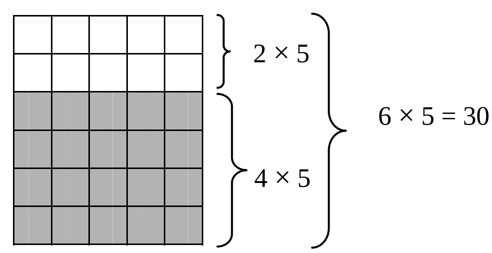
b) The first meaning is 6 groups of 7 things.
The array has 6 groups of 7 if you think of each row as a group.
Each boy is matched with 7 girls, so that is a group of 7. There are 6 groups since there are 6 boys.

4. *Sample responses*:

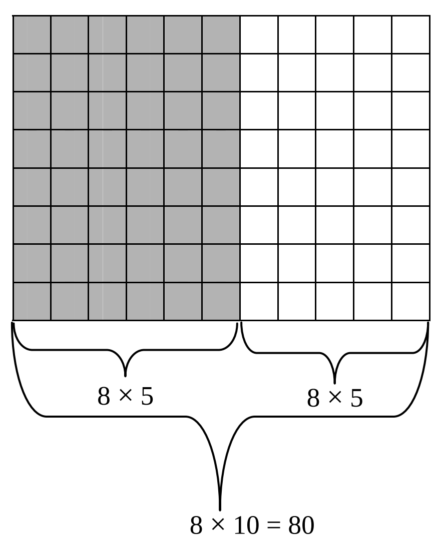
a)



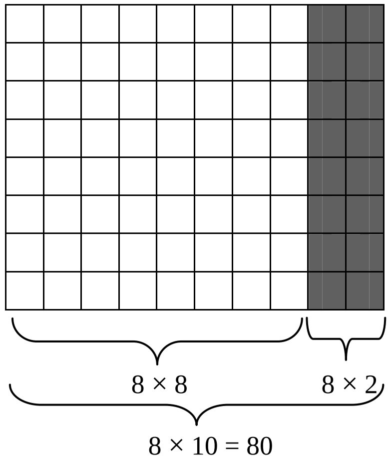
b)



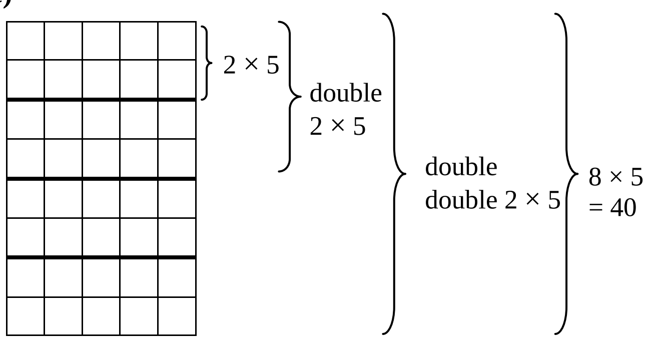
c)



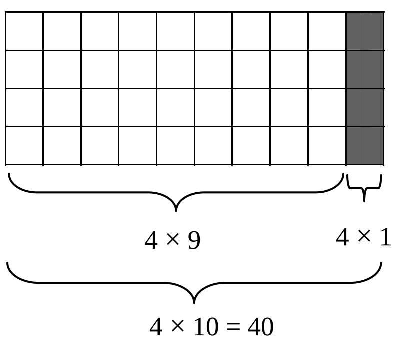
4. d)



e)



f)



5. *Sample response*:
• I could multiply 5 × 10 = 50 and then subtract 5 × 1;
50 – 5 = 45.
• I could double 5 three times and then add 5:
5 × 9 → 5 × 8 = 5 × 2 × 2 × 2 = 40; 40 + 5 = 45.
• I could multiply 4 × 9 = 36 and then add 9;
36 + 9 = 45.

6. a) 7 momos
b) 6 paperclips
c) 9 squares

7. a) 5
b) 5
c) *Sample response*:
Sharing 35 items with 7 people is like making groups of 7. If you share 35 items among 7 people, you give each person 1 item. That is like making 1 group of 7. Then you do it again and that is like making 2 groups of 7. You do this until there are no items left. That is like making 5 groups of 7.

8. a) 7 × 3 = 21
3 × 7 = 21
21 ÷ 3 = 7
21 ÷ 7 = 3

b) 6 × 9 = 54
9 × 6 = 54
54 ÷ 6 = 9
54 ÷ 9 = 6

c) 7 × 8 = 56
8 × 7 = 56
56 ÷ 8 = 7
56 ÷ 7 = 8

9. *Sample response*:
4 × 7 = 28 since it is in the same fact family.

10. a) 4 × 1 **b)** 7 ÷ 1

11. *Sample response*:
If you have nothing to share, then the share size is nothing.

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UNIT 2 Performance Task — Stick Puzzle Game

The object of the Stick Puzzle Game is to distribute a number of sticks equally among several players.

Here are the rules for the game:

• You must use a total of 60 to 85 sticks.

• No player can have more than 9 sticks.

• All players get the same number of sticks. No sticks can be left over.

A. i) If there were 6 players, why would each player get 10 or more sticks?

ii) Find the possible number of players and possible number of sticks for each player. Two solutions are already provided. Find four other solutions. Explain how you found your solutions.

	Possible number of players	**Possible number of sticks per player**	**Total number of sticks used**
Solution 1	7	9 sticks	63
Solution 2	8	8 sticks	64
Solution 3			
Solution 4			
Solution 5			
Solution 6			

B. i) Change the rules for the total number of sticks and the number of sticks that players can have so that there are at least six solutions.

• You must use a total of __ to __ sticks.

• No player can have more than __ sticks.

• All players get the same number of sticks. No sticks can be left over.

ii) Use your new rules to find the possible number of players and possible number of sticks for each player. Find at least six solutions. Explain how you found your solutions.

UNIT 2 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
4-B3 Multiplication Meanings: explore **4-B5** Multiplication Facts: to 9 × 9 **4-B7** Division Meanings: small numbers **4-B9** Multiplication and Division Facts: relate through properties	40 min	• Counters (optional)

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could supplement the unit test. It could also be used as enrichment material for some students.

• You can assess performance on the task using the rubric below.

Sample Solution

A. i) 60 is 6 tens. There are 60 or more sticks. If there are 6 players, each would get 10 or more sticks.
The rules only allow each player to get 9 sticks.
ii)

	Possible number of players	Possible number of sticks for each player	Total number of sticks used
Solution 1	7	9	63
Solution 2	8	8	64
Solution 3	8	9	72
Solution 4	9	7	63
Solution 5	9	8	72
Solution 6	9	9	81

I multiplied 7 by 7, 8, and 9, but 7 × 7 and 7 × 8 were too low (too few sticks were used).
I multiplied 8 by 7, 8, and 9 but 8 × 7 was too low.
I multiplied 9 by 7, 8, and 9 and they all worked.

B. i) You must use a total of **<u>35</u>** to **<u>50</u>** sticks.
No player can have more than **<u>7</u>** sticks.
ii)

	Possible number of players	Possible number of sticks for each player	Total number of sticks used
Solution 1	6	6	36
Solution 2	6	7	42
Solution 3	7	6	42
Solution 4	8	5	40
Solution 5	8	6	48
Solution 6	9	4	36

I started by multiplying different numbers by 6.
If I went below 6, it was too low, and if I went above 9, it was too high.
I did the same thing for 7, 8, and 9. I made sure the products were from 35 to 50.

UNIT 2 Performance Task Assessment Rubric

The student	Level 4	Level 3	Level 2	Level 1
Communicates about how he or she found a variety of solutions	Communicates clearly, completely, and meaningfully about the possible number of players and sticks	Communicates clearly and meaningfully about the possible number of players and sticks	Communicates reasonably meaningfully about the possible number of players and sticks	Has difficulty communicating about the possible number of players and sticks
Multiplies and/or divides	Consistently, correctly, and efficiently solves necessary multiplication and division questions to solve the problem	Solves necessary multiplication and division questions to solve the problem correctly	Solves most of the multiplication and division questions required for the problem	Makes errors in a number of the multiplication and division questions required for the problem
Creates rules to meet conditions for part B	Insightfully creates new rules for the game	Creates new rules for the game that work,	Creates new rules for the game that almost work, but not quite	Has difficulty creating new rules for the game

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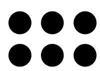
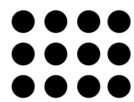
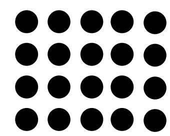
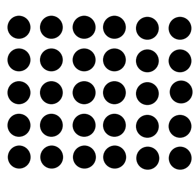
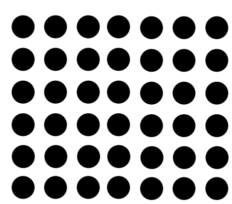
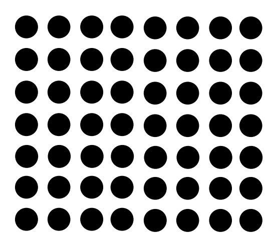
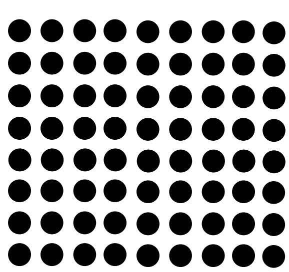
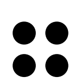
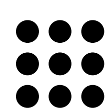
BLM 1 Centimetre Grid Paper

$2 \times 3 = 6$	$3 \times 4 = 12$	$4 \times 5 = 20$
		
$5 \times 6 = 30$	$6 \times 7 = 42$	$7 \times 8 = 56$
		
$8 \times 9 = 72$	$2 \times 2 = 4$	$3 \times 3 = 9$
		

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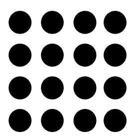
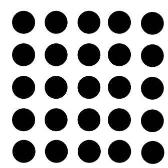
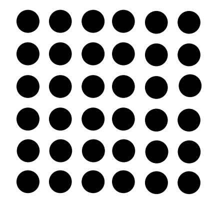
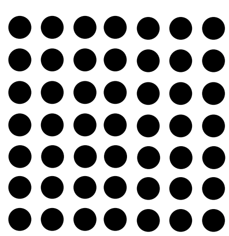
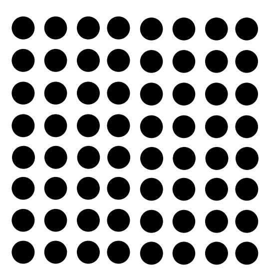
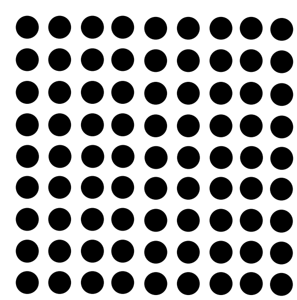
$4 \times 4 = 16$	$5 \times 5 = 25$	$6 \times 6 = 36$
		
$7 \times 7 = 49$	$8 \times 8 = 64$	$9 \times 9 = 81$
		

BLM 3 Matching Doubles Game Cards

3×5	6×5	4×8	4×4
6×9	3×9	8×8	4×8
2×5	2×10	2×5	4×5
3×10	3×5	3×10	3×5
3×5	6×5	4×8	4×4
6×9	3×9	8×8	4×8
2×5	2×10	2×5	4×5

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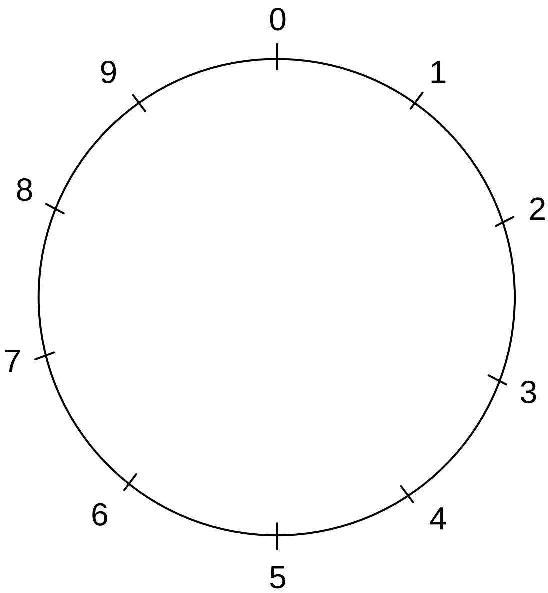
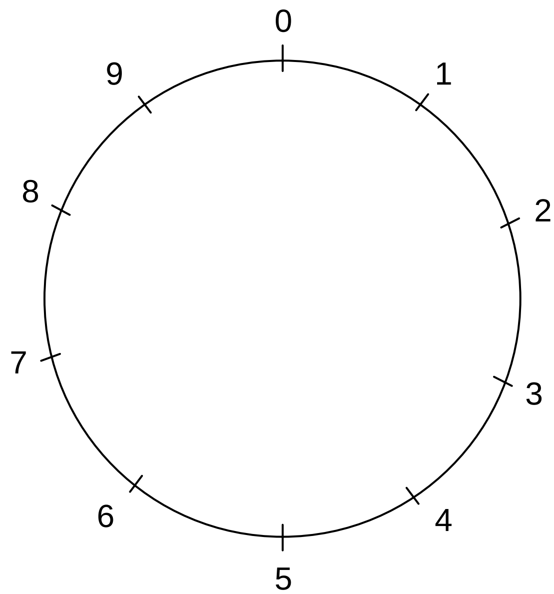
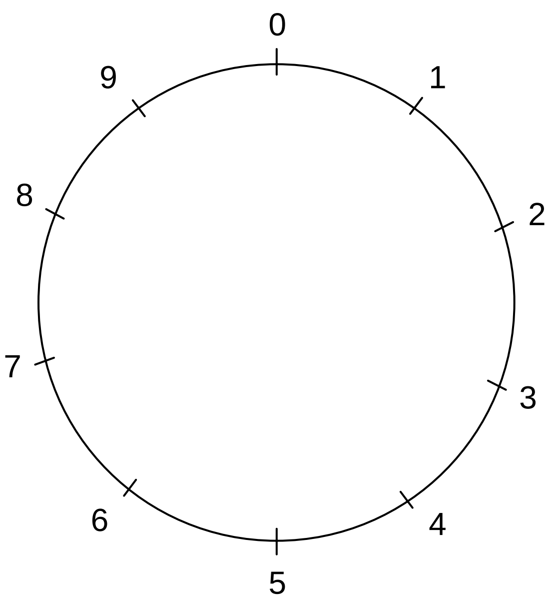
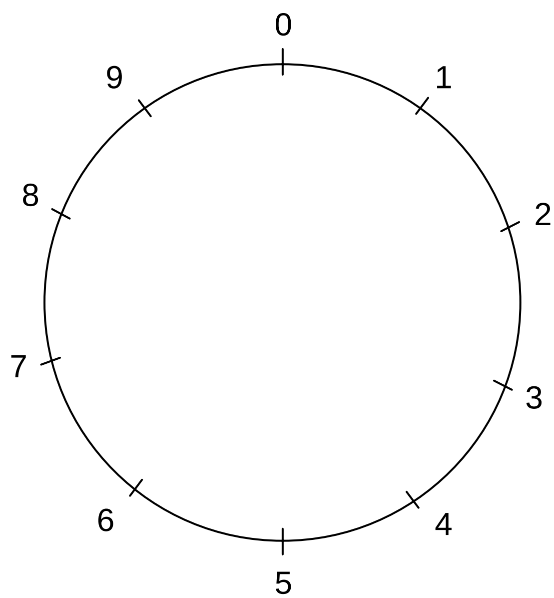
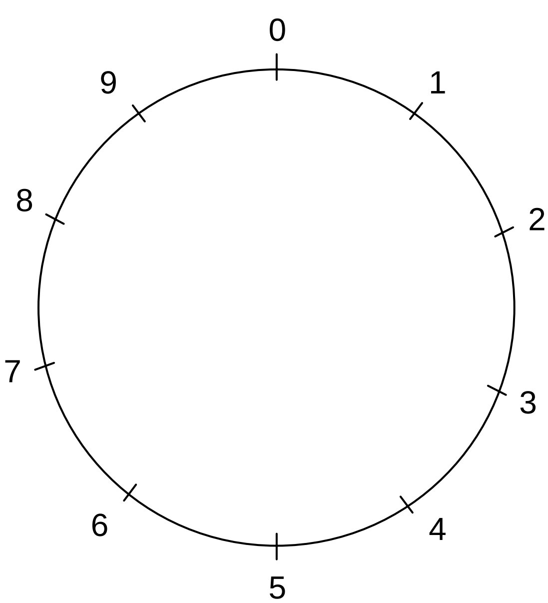
BLM 4 Multiplication Fact Digit Circles

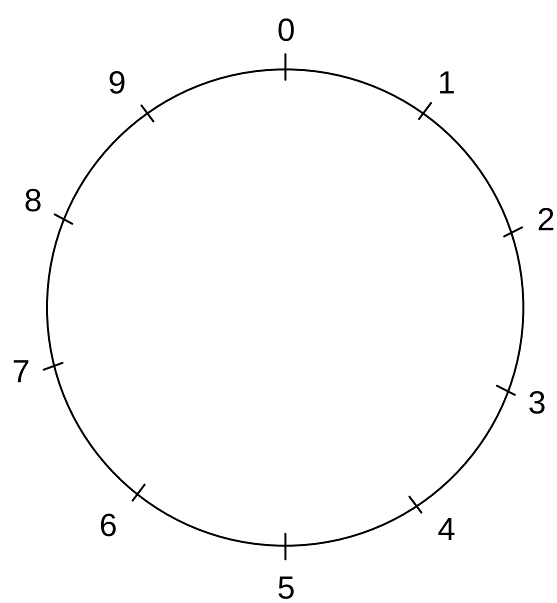












BLM 5 Multiplication Tables

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
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8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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UNIT 3 MEASUREMENT

UNIT 3 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 67** **TG p. 98**	Review prerequisite concepts, skills, and terminology, and pre-assessment	1 h	• Square centimetre tiles or Centimetre Grid Paper (BLM) cut into squares • Rulers	All questions
Chapter 1 Length and Area				
3.1.1 Introducing Millimetres **SB p. 69** **TG p. 100**	**4-D1 Linear Units: mm, cm, m, km; estimate and measure** • estimate and measure: mm, cm, m, km • investigate and develop unit relationships	1 h	• Millimetre rulers	Q1, 3, 4, 6
3.1.2 Estimating and Measuring Area **SB p. 72** **TG p. 103**	**4-D4 Area: estimate and measure in square centimetres** • understand that area is expressed as the number of square units required to cover a given surface • use the square centimetre symbol, cm^2	1 h	• Centimetre Grid Paper (BLM)	Q1, 3, 5
3.1.3 Relating the Area of a Rectangle to Multiplying **SB p. 76** **TG p. 106**	**4-D6 Dimensions and Area of a Rectangle: relate factors and product** • relate dimensions (factors) of rectangles to area (product) concretely • develop a personal formula for area	1 h	• Centimetre Grid Paper (BLM), centimetre square tiles, or Centimetre Grid Paper (BLM) cut into squares	Q2, 3, 4, 6
3.1.4 EXPLORE: Rectangle Perimeters with a Given Area (Essential) **SB p. 78** **TG p. 110**	**4-D5 Constant Area and Different Perimeters: explore** • explore the concept concretely • understand that different shapes can have the same area • understand that area and perimeter are generally independent of each other	40 min	• Rulers or Centimetre Grid Paper (BLM)	Observe and Assess questions
GAME: Filling a Grid (Optional) **SB p. 79** **TG p. 111**	To practise area calculations in a game situation.	20 min	• Centimetre Grid Paper (BLM) • Dice	N/A
CONNECTIONS: Relating Perimeter and Area (Optional) **SB p. 80** **TG p. 112**	To make a connection between mathematical knowledge of area and perimeter and real-world applications	20 min	• Rulers • String	N/A

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UNIT 3 PLANNING CHART [Continued]

Chapter 2 Angles				
3.2.1 Describing Angles **SB p. 81** **TG p. 113**	**4-D7 Angles: amount of turn** • develop the meaning of angle and the measure of an angle concretely • understand angle as a turn and the measure of angle as the amount of turn (i.e., a smaller angle means a smaller turn) • investigate to discover that the lengths of the arms of an angle do not influence angle size	1 h	• A large pair of cardboard strips joined by a paper fastener or tack/pin (optional) • Tracing paper	Q3, 4, 6, 7
3.2.2 Classifying Angles **SB p. 84** **TG p. 116**	**4-D7 Angles: amount of turn** • differentiate among and describe right, acute, and obtuse angles	40 min	None	Q1, 2, 5
Chapter 3 Volume				
3.3.1 Measuring Volume Using Cubes **SB p. 86** **TG p. 118**	**4-D2 Volume: estimate and measure to develop concept of volume** • explore the meaning of volume through non-standard units by counting the number of units it takes to build a solid • estimate and measure volume in non-standard units	40 min	• Linking cubes	Q1, 2, 4
3.3.2 EXPLORE: Volume of Rectangle-based Prisms (Essential) **SB p. 88** **TG p. 121**	**4-D3 Volume of Rectangle-based prisms: estimate and measure in non-standard units** • calculate the volume of rectangle-based prisms using cubes • determine the volume of a rectangle-based prism and build prisms with a specified volume • connect volume to dimensions (dimensions of first layer × number of layers)	40 min	• Linking cubes	Observe and Assess questions
GAME: Building Bigger Boxes (Optional) **SB p. 89** **TG p. 122**	To practise calculating volumes of rectangle-based prisms in a game situation.	25 min	• Linking cubes • Dice • Spinners	N/A
UNIT 3 Revision **SB p. 90** **TG p. 123**	Review the concepts and skills in the unit	2 h	• Millimetre rulers • Centimetre Grid Paper (BLM) • Tracing paper • Linking cubes	All questions
UNIT 3 Test **TG p. 126**	Assess the concepts and skills in the unit	1 h	• Millimetre rulers • Centimetre Grid Paper (BLM) • Tracing paper • Linking cubes	All questions
UNIT 3 Performance Task **TG p. 128**	Assess concepts and skills in the unit	20 min	• Linking cubes	Rubric provided
UNIT 3 Blackline Masters **TG p. 130**	BLM 1 Fraction Circle in Sixths (for GAME: Building Bigger Boxes) Centimetre Grid Paper on **page 87** in UNIT 2			

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Math Background

• This measurement unit allows students to explore concepts of length, area, angle and volume. There is some formal work using new units like millimetres and using rules for calculating areas of rectangles and volumes of rectangle-based prisms. There is also informal work in comparing angles and comparing volumes using non-standard units. Students explore how area and perimeter are independent measures.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 3** in **lesson 3.1.2**, where they must draw shapes with a given area, in **question 3** in **lesson 3.1.3**, where they solve a real-world problem involving area, and in **part F** in **lesson 3.3.2**, where they create prisms with particular volumes.

• Students use communication frequently as they explain their thinking in **question 7** in **lesson 3.1.1**, where they describe why they would use a particular unit in a particular measurement situation, in **question 7** in **lesson 3.1.3**, where they communicate about the impact of knowing the formula for calculating the area of a rectangle, in **part D** in **lesson 3.1.4**, where they synthesize what they know about how area and perimeter relate to make a summary statement about how knowing the area of a shape does not let them predict its perimeter, and in **question 5** in **lesson 3.2.1**, where they apply the definition of an angle.

• Students use reasoning in **question 5** in **lesson 3.1.1**, where they decide whether or not measurement statements are true, in **question 6** in **lesson 3.1.3**, where they reason about how the value of an area affects the number of possible rectangles with that area, in **question 4** in **lesson 3.2.2**, where they figure out what can happen if they put together two angles of a certain type, and in **question 4** in **lesson 3.3.1**, where they recognize that information about a shape's volume is not sufficient to describe the shape fully.

• Students consider representation in **lesson 3.1.3**, where they represent a rectangle's area as an array and therefore as a product, and in **lesson 3.3.2**, where they think of a rectangle-based prism as being made up of layers.

• Students use visualization skills in **question 4** in **lesson 3.1.1**, where they visualize to predict what a line of a certain length might look like, in **questions 2 and 4** in **lesson 3.1.2**, where they estimate areas, in **lesson 3.1.4**, where they see how rectangles that are longer and thinner must have greater perimeters, in **questions 2 and 4** in **lesson 3.2.2**, where they create angles to fit certain conditions, and in **question 2** in **lesson 3.3.1**, where they imagine an object with a particular volume.

• They make connections in **question 4** in **lesson 3.1.2**, where they estimate areas of real-world objects, and in **question 5** in **lesson 3.2.2**, where they look for angles in their environment.

Rationale for Teaching Approach

• This unit is divided into three chapters.

Chapter 1 focuses on length and area measurements.

Chapter 2 focuses on informal angle measures and helps students see that the measure of an angle is a measure of turn.

Chapter 3 focuses on volume, particularly of rectangle-based prisms.

• There are two **Explore** lessons. The first lesson allows students to see how perimeter and area are independent measures. The second lesson helps students make sense of the formula for the volume of a rectangle-based prism.

• The **Connections** helps students see how the knowledge that shapes with the same area can have different perimeters can be useful in real life.

• There are two **Games**. The first game provides an opportunity to practise using a rule for calculating the area of rectangles. The second game provides practice using a rule for calculating the volume of a rectangle-based prism.

• Throughout the unit, it is important to encourage flexibility and to accept a variety of approaches from students.

Getting Started

Curriculum Outcomes	Outcome relevance
3 Area: estimate and measure in square centimetres, concretely **3** Arrays and Dimensions of Rectangles: develop the relationship (concretely) **3** Perimeter: standard units **3** Angles: right angle, and less than and greater than a right angle	Students will find the work in the unit easier after they review measurement concepts and skills from Class III.

Pacing	Materials	Prerequisites
1 h	• Square centimetre tiles or Centimetre Grid Paper (BLM) cut into squares • Rulers	• estimating and measuring area in square centimetres • calculating the perimeters of a rectangle • relating angle size to a right angle

Main Points to be Raised

Use What You Know

• You can measure the area of a shape by covering it with units such as square centimetres.

• A square centimetre is the area of a square that is 1 cm on each side.

Skills You Will Need

• The perimeter of a polygon is the sum of its side lengths.

• The area of a shape is the number of square units you need to use to cover it.

• You can calculate the area of a rectangle by forming an array of square units on top of the rectangle and then multiplying.

• You can compare angles to a right angle visually.

Use What You Know — Introducing the Unit

• Before assigning the **Use What You Know** activity, you may wish to review what a square centimetre is and how you can cover shapes with square centimetres to find their area.

• Students can work in pairs to complete the **Use What You Know** activity.

Observe students as they work. You might ask:

• *Why did you think Shape A has more area?*

• *Do you have to turn Shape A to compare the areas?*

• *How could you have compared the areas without measuring them?*

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions. You may wish to remind students what perimeter means before they begin **question 1**.

• Students can work individually.

Answers

A. *Sample response*: A **B.** Both have an area of 9 square centimetres. **C.** *Sample response*: No; Both squares have the same area.	**D.** *Sample response*: The two shapes are exactly the same but one is turned and each shape has something different around it. That tricks my eyes into thinking they look different.

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1. *Sample responses:* **a**) 16 cm **b)** 20 cm **c)** 22 cm **2.** *Sample responses:* **a)** 12 square centimetres **b)** 24 square centimetres **c)** 24 square centimetres	**3. a)** 2 rows **b)** 6 columns **c)** Yes; [If I multiply 2 by 6 I get 12, which is the area. This works because the tiles cover the rectangle and they are in an array.] **4. a)** A, C **b)** B, D

Supporting Students

Struggling students

• If students have difficulty handling the individual square tiles or pieces of paper to measure area, encourage them to trace the squares onto centimetre grid paper.

• Some students may believe that you can only calculate area if a shape is vertical or horizontal. Encourage these students to trace Shape A and then re-orient the paper to calculate its area. Then have them cover the shape in its original orientation with square centimetre tiles to see that it did not have to be moved; the area was the same.

Enrichment

• You might encourage students to create their own optical illusion where two shapes of the same size appear to be of different sizes.

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3.1.1 Introducing Millimetres

Curriculum Outcomes	Outcome relevance
4-D1 Linear Units: mm, cm, m, km; estimate and measure • estimate and measure: mm, cm, m, km • investigate and develop unit relationships	Measuring and estimating lengths using a variety of units are important life skills. Students need to be able to estimate and measure using a variety of units, depending on available tools. They also need to know relationships between units to understand measurements made by others.

Pacing	Materials	Prerequisites
1 h	• Millimetre rulers	• measuring length in centimetres • familiarity with decimal measurements

Main Points to be Raised

• Millimetres are smaller units than centimetres. They are useful when a measurement is less than 1 cm or if it falls between two centimetre markings.

• 1 mm = 0.1 cm; 1 cm = 10 mm.

• You can use a combination of centimetres and millimetres to measure a distance, e.g., 3 cm, 2 mm, or you can write the millimetres as tenths of centimetres, e.g., 3.2 cm, or you can write the centimetres as 10 mm, e.g., 32 mm.

• There are 1000 mm in 1 m.

Try This — Introducing the Lesson

A. If it is possible, you might create a rain gauge to show what one might look like. You can use an empty bottle or a graduated cylinder with a small funnel on top to help collect the rain. If there are no markings on the side of the container, you can drop a ruler into the water to measure its depth.

While you observe students at work, you might ask questions such as the following:

• *How do you know that more than 2 cm of rain fell?* (The height of the water is above the 2 cm mark.)

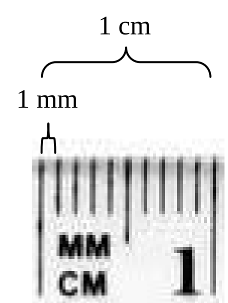
• *How do you know that less than 3 cm fell?* (The height of the water is below the 3 cm mark.)

• *How many marks are there between the 2 and the 3?* (10 marks)

• *Why might you call the height 2.3 cm?* (0.3 is 3 tenths and the water went up to the 3rd mark out of 10.)

The Exposition — Presenting the Main Ideas

• Have students choose an item to measure whose length falls between two centimetre markings on the ruler, for example, a pencil. Ask them how they would describe the length of their item.



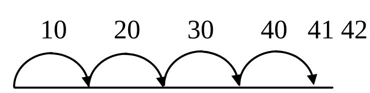
For example, they might say:

about 10 cm a little more than 10 cm between 10 and 11 cm

• Have them look at their rulers. Point out that their rulers have 10 small markings that make up each centimetre. Indicate that each mark represents one millimetre. Write the abbreviated form, 1 mm. Have students notice that their fingernail might be about 1 mm thick. This gives them a referent for 1 mm.

• Make sure students understand that 1 cm = 10 mm. To measure in millimetres they might skip count by 10s and then add the extra millimetres.

For example, the line to the right is 4.2 cm long. To measure it in millimetres using a ruler, they might skip count:



Then discuss why this measurement could be recorded each way.

42 mm 4.2 cm 4 cm and 2 mm (or 4 cm, 2 mm)

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Take time to explain the 4.2 cm measurement. Remind students that since 10 mm make up 1 cm, each 1 mm is $\frac{1}{10}$ cm or 0.1 cm. Talk about why 4.2 means 4 cm + another 0.2 cm, or 4 cm + 2 mm.

• Have students practise measuring in millimetres by working with a partner to measure another item whose measurement falls between two centimetre measurements. They should record the measurement both in millimetres and in centimetres.

• Asking students why a measure of, for example, 7 cm, 6 mm is written as 7.6 cm.

• Ask students to recall how many centimetres make up a metre (100 cm = 1 m). Then ask how many millimetres would make up a metre (1000 mm = 1 m) and why it would be that number (if there are 10 mm in 1 cm and 100 cm in 1 m, then 10 × 100 = 1000 mm in 1 m).

• Tell students that they can examine **page 69** in the student text for a review of these ideas.

Revisiting the Try This

B. This question allows students to make a formal connection between measuring in millimetres and the problem described in **part A**.

Using the Examples

• Have students read through **example 1** with a partner. Provide time for them to ask questions.

• Challenge students to draw a line 51 mm long. They can then check their approach by seeing what is shown in **example 2**.

Practising and Applying

Teaching points and tips

Q 1: Ask students why they will use decimals in **part b)** but not in **part a)**.

Q 2: Encourage students to predict which measurement will be greatest before they actually measure.

Q 3: Ask students how they know that all the measurements listed are less than 12 cm.

Q 4: Ask students why it might be easier for them to estimate if they think of the 45 mm as being between 4 cm and 5 cm.

Q 5: Ask students how they thought about choice **A.** Did they think about 2000 mm as 200 cm or as 2 m? For choice **B**, ask if 5 mm is more or less than 1 cm. Then ask whether that knowledge makes it easier to understand **B.**

Q 7: Make sure students understand that there are no rules for what units to use. It is simply practical to use large units for long distances and small units for short distances so that the numbers are meaningful. However, it is not incorrect to say that a distance is many thousands of millimetres or that a length is a decimal number of metres.

Common errors

• Some students may write the centimetres and millimetres in the wrong order when reporting joint measurements. Remind them to record the centimetres first so that the first thing that they read gives them a good idea of how long the item is. This is much like how the first digit in a multi-digit number is the most meaningful.

For example, 421 is about 400; the 4 matters more than the 1 in the ones place.

If they write joint measures with the centimetre measure first, it makes the transition to decimal measures easier.

For example, 4 cm, 2 mm → 4.2 cm

Suggested assessment questions from Practising and Applying

Question 1	to see if students can measure in millimetres and in centimetres to the nearest millimetre
Question 3	to see if students can draw an item of a particular length in millimetres
Question 4	to see if students can estimate lengths in millimetres
Question 6	to see if students can switch between metric units

Answers

A. *Sample response*: About 2 cm

B. *Sample responses*:
i) 23 mm
ii) It is 3 mm more.

1. a) 31 mm **b)** 3.1 cm **c)** 3 cm, 1 mm

2. a) 15 mm **b)** 182 mm **c)** 240 mm

3. a)
b)
c)
d)

4. *Sample responses*:
a)
b) My line was 2 mm longer than 45 mm.

5. A and B

6. a) Longer; *Sample response*: My ruler is 15 cm long, which is 150 mm.
b) 56 cm
c) Half a metre is 50 cm and 56 cm > 50 cm.
d) *Sample response*: The width of my desk.

7. *Sample response*:
a) The width of a button, the thickness of my notebook, the width of a pencil.
b) The length of a desk, the height of a chair, the width of a book.
c) The length of a room, the length of a hallway, the distance down a street.

[**8.** *Sample response*: A short distance like the width of my fingernail would be a very small number of kilometres, so the number would be too small to understand. A long distance, like the width of the classroom would be a lot of millimetres, so the number would be too large to understand.]

[**9.** There are 1000 mm in 1 m.]

Supporting Students

Struggling students

• Some students may have difficulty using decimals. Allow these students to use millimetres only, such as 102 mm, combined measurements like 10 cm, 2 mm, or fractions like $10\frac{2}{10}$ cm.

Enrichment

• Ask students to create more statements involving millimetres, like those in **question 5**, to which other students could respond.

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3.1.2 Estimating and Measuring Area

Curriculum Outcomes	Outcome relevance
4-D4 Area: estimate and measure in square centimetres • understand that area is expressed as the number of square units required to cover a given surface • use the square centimetre symbol, cm^2	It is just as important for students to be able to estimate area as it is for them to be able measure area to solve real-world problems they will face as older students and as adults.

Pacing	Materials	Prerequisites
1 h	• Centimetre Grid Paper (BLM)	• combining halves to make a whole

Main Points to be Raised

• Area is the amount of space an object takes up.
It is measured in 2-D units, often square centimetres.

• A square centimetre unit is the area of a square that is 1 cm on each side, but it is possible to use other area units as long as the units do not leave gaps.

• One square centimetre is written as 1 cm^2. Shapes other than a square can also have an area of 1 cm^2.

• Shapes that look different can have the same area.

• You can measure area either by covering a shape with area units or by drawing the shape on centimetre grid paper and counting squares.

• Sometimes a shape uses only parts of the squares on a centimetre grid. You can combine the parts to find the total area.

Try This — Introducing the Lesson

A. Encourage students to estimate before measuring. Students might trace both rectangles and try to fit one on top of the other by cutting it up and rearranging it. Or, they might trace one rectangle and superimpose it on the other to compare them directly, although this method will not meet with much success unless students have excellent spatial sense.

While you observe students at work, you might ask questions such as the following:

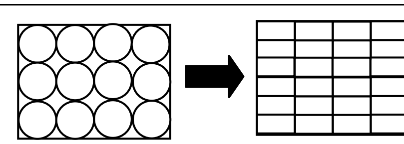
• *Why do you think that the rectangle on the right has more area?* (It is longer.)

• *Why do you think that the rectangle on the left has more area?* (It is wider.)

• *How could you test which has more area?* (Cover each with counters to see which holds more counters.)

• *Why is it difficult to compare them by putting one rectangle on top of the other?* (One is longer but narrower and the other is wider but shorter so you cannot compare either dimension to decide.)

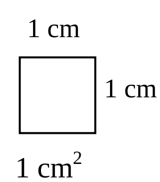
The Exposition — Presenting the Main Ideas



• Draw a rectangle-based shape on the board. Fill it with congruent circles. Ask students why it does not make sense to say that the area of the rectangle is, for example, 12 circles (parts of the rectangle are not covered by the circles so its area is more than 12 circles). Then divide the rectangle into smaller rectangles of equal size and ask students to tell how many smaller rectangles cover the original rectangle. Have students discuss why rectangles make better area units than circles.

• Mention that a square is a special type of rectangle that is often used as an area unit. Indicate that one of the most common units to measure area is the square centimetre. Tell students that the abbreviation is cm^2.

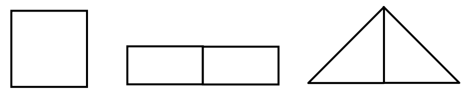
• Have students use their rulers to draw a square that is 1 cm on each side so they will feel the size of 1 cm^2. They might notice that it is about the size of their fingernail.



• Provide copies of centimetre grid paper to pairs of students. Ask them to draw an outline around 2 squares. Ask why the shape they have drawn has an area of 2 cm^2. Repeat, but this time have them enclose a shape made up of 4 squares. Ask what the area is (4 cm^2).

• Have students cut out 1 cm^2 and then cut the square into two small pieces and rearrange the two pieces to make another shape. Discuss how this shows why a shape that is not a square can still have an area of 1 cm^2.

For example, all these shapes have an area of 1 cm^2:



Each shape has an area of 1 cm^2.

• Have students turn to **page 73** of the student text, toward the end of the exposition, to see how to estimate and calculate more complex. Go through the material with them.

• Give students time to review the material on **pages 72 and 73** of the student text if they wish.

Revisiting the Try This

B. This question allows students to verify their predictions by correctly measuring the areas of the rectangles in **part A**.

Using the Examples

• Assign students to pairs students to read through the two **examples**. One student should be responsible for **example 1** and the other for **example 2**. Each student should then lead his or her partner through the example he or she mastered.

Practising and Applying

Teaching points and tips

Q 1: For **part c)**, encourage students to recognize that they can put two halves together to get an exact measurement.

Q 2: Students can trace the shapes onto centimetre grid paper.

Q 3: Students should realize there are a number of shapes they could draw with this area. The only condition is that they use 6 square units (whether full squares or part squares).

Q 4: Students might use the referent that the area of their fingernail is about 1 cm^2 to answer these questions or they might refer to other shapes they have drawn recently.

Q 6: Make sure students understand that it is not just areas that occupy only full squares that can be measured exactly; areas that occupy half squares can be combined and measured exactly, for example, **question 1 c)**, and **questions 2 b) and c)**.

Common errors

• Some students may have difficulty counting the area when part squares are involved.

For example, they might simply count each part square as a full square.

Remind them that the area is the number of square centimetres that would cover the shape, no matter what the shape is.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can measure the areas of simple shapes on centimetre grid
Question 3	to see if students can create shapes with a given area
Question 5	to see if students can estimate areas

Answers

A. *Sample response*: The rectangle on the left	**B. ii)** They have the same area; *Sample response*: One rectangle is taller, but the other is wider.

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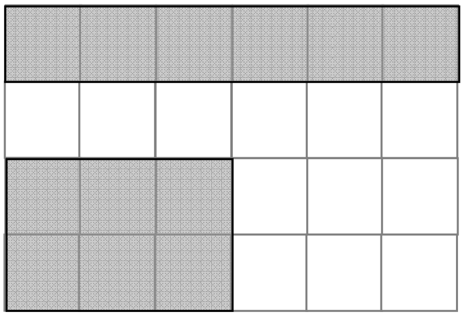
1. a) 6 cm^2 **b)** 7 cm^2 **c)** 3 cm^2

2. a) About 5 cm^2; 6 cm^2.
b) About 5 cm^2; 4.5 cm^2.
c) About 9 cm^2; 11 cm^2.

3. *Sample response*:



[I knew I needed a shape that covered 6 square centimetres, so I used a centimetre grid. I drew a shape that was 1 row of 6 squares. Then I drew a shape that was 2 rows of 3 squares.]

4. A

5. *Sample responses*:
a) and b)
My eraser: about 10 cm^2; 12 cm^2
A calculator: about 70 cm^2; 81 cm^2
My ruler: about 45 cm^2; 48 cm^2

[**6.** *Sample response*:
I cannot measure an exact area if the shape uses part squares that are not exactly halves.]

Supporting Students

Struggling students

• Some students may need to work with shapes that occupy full squares before working with shapes that do not.

• Some students may need more experience measuring shapes before they are comfortable making estimates. Allow them to draw as many shapes as they wish to get a good sense of how much space, for example, 5 cm^2, 10cm^2, 20 cm^2, and 40 cm^2 take up before having them estimate other areas.

Enrichment

• Ask students to hunt for objects with areas to fit particular requirements.

For example:

Can you find something that has an area between 10 cm^2 and 20 cm^2? between 50 cm^2 and 60 cm^2?

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3.1.3 Relating the Area of a Rectangle to Multiplying

Curriculum Outcomes	Outcome relevance
4-D6 Dimensions and Area of a Rectangle: relate factors and product • relate dimensions (factors) of rectangles to area (product) concretely • develop a personal formula for area	Because rectangles are prevalent in our world, it is useful for students to have quick techniques for calculating their areas.

Pacing	Materials	Prerequisites
1 h	• Centimetre Grid Paper (BLM), centimetre square tiles, or Centimetre Grid Paper (BLM) cut into squares	• relating multiplication to an array model • familiarity with multiplication facts

Main Points to be Raised

• The area of a rectangle is the product of its dimensions. This is because you can form an array of congruent squares on the rectangle where the number of squares in each row is one dimension of the rectangle and the number of rows is the other dimension.

• Since an array is a way to show multiplication, it makes sense that the product of the dimensions is the area of a rectangle.

Try This — Introducing the Lesson

A. Have students work alone or in pairs on this problem. While you observe students at work, you might ask questions such as the following:

• *How could Devika draw a rectangle with area of 12 cm^2?* (It could be 3 units by 4 units.)

• *Would it be possible to add squares to her rectangle to get a rectangle with area 14 cm^2?* (No. To make a rectangle I would have to add either 3 squares or 4 squares, not 2 squares.)

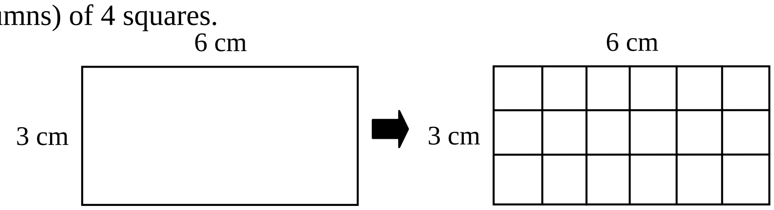
• *How could Devika make the rectangle?* (She could take 14 square cm tiles and arrange them into a rectangle.)

The Exposition — Presenting the Main Ideas

• Have students turn to **page 76** in the student text to read the exposition. They will see how a 4 cm-by-5 cm rectangle is covered by an array that is 4 square centimetre units by 5 square centimetre units. Help them see why this is $4 \times 5 = 20$ cm^2 (because there are 4 groups (rows) of 5 square centimetres, you can find the total number of squares by multiplying 4×5).

• Make sure students understand that they could also have found the area using 5×4, which has the same result. This makes sense since there are 5 groups (columns) of 4 squares.

• To make sure students understand, ask them to tell what the area of a 3 cm-by-6 cm rectangle would be and to explain why. Reinforce the result by sketching the array of squares inside such a rectangle on the board.



3 rows of 6 squares
Area is $3 \times 6 = 18$

• Although at this grade level we use only examples with whole number dimensions, the formula for a rectangle applies no matter what the side lengths are.

Revisiting the Try This

B. This question allows students to recognize how knowing multiplication facts can help them find the dimensions of a rectangle with a given area.

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Using the Examples

• Ask students to close their texts. Present the question in the example. After they have created their own solutions, students can read the **example** in the text.

Practising and Applying

Teaching points and tips

Q 1: Allow students either to multiply, to use square centimetre tiles to cover the rectangle, or to copy onto grid and count grid squares.

Q 2: Students will probably calculate the area by multiplying rather than by using a grid, but they still need to verify the calculated area using a grid or tiles.

Q 3: Students must not only calculate each area, but must also combine the areas in different ways to solve the problem.

Q 4: Some students may consider only the multiplication facts and think of 1-digit dimensions. This thinking will allow them to create two rectangles for 36 cm^2 but not for 15 cm^2. Remind them that dimensions can be longer than 10.

Q 5: Students may list a variety of products of small single digit numbers like 6 (3 × 2) and 9 (3 × 3), and then look for combinations that make 20. Others will write 20 as a sum, e.g., 12 + 8, and then work backwards.

Q 6: Students should not be misled into believing that there are more rectangles with area 36 than with area 23; that is not true. But there are more rectangles with area 36 that have whole number unit side lengths.

Q 7: This is one of the most important ideas about measurement formulas. We create formulas so we can use values that are easier to measure to determine values that are harder to measure. It is easier to use a ruler to measure lengths than to use a grid to measure an area since a ruler is a readily accessible tool.

Suggested assessment questions from Practising and Applying

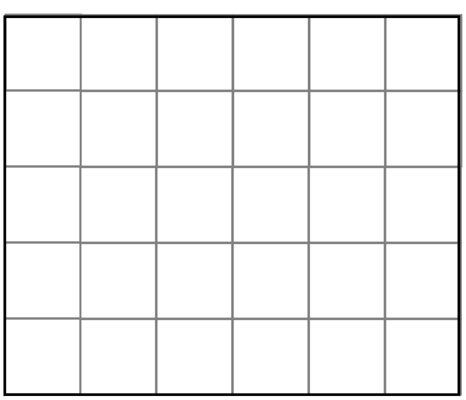
Question 2	to see if students can calculate the area of a rectangle using a ruler and verify using a grid
Question 3	to see if students can solve a real-world problem that involves calculating areas of rectangles
Question 4	to see if students can create a rectangle with a given area
Question 6	to see if students can reason and communicate about the relationship of the number of possible rectangles with whole number side lengths with a given area to the value of the area

Answers

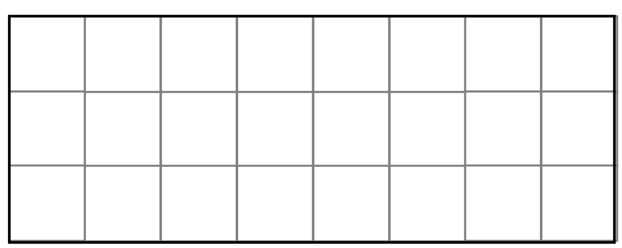
A. *Sample response*:	**B.** *Sample response*:
No; I could make a rectangle with 1 row of 14 square centimeters. It would have an area of 14 cm^2.	Since 14 = 2 × 7, a rectangle that is 2 cm by 7 cm would have an area of 14 cm^2.

1. a) 10 cm^2 **b)** 9 cm^2

2. a) 30 cm^2

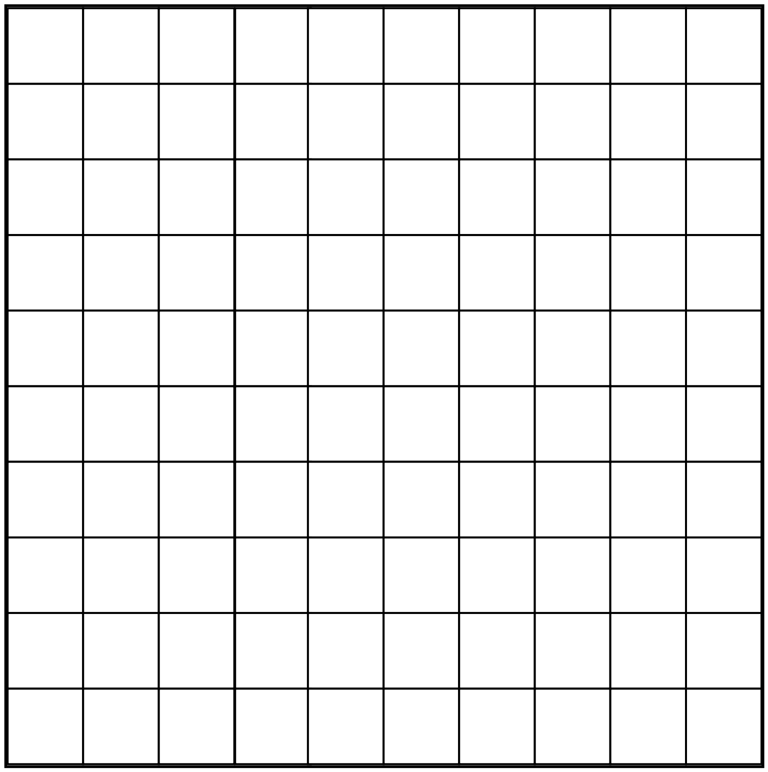


b) 24 cm^2



Lesson 3.1.3 Answers [Cont'd]

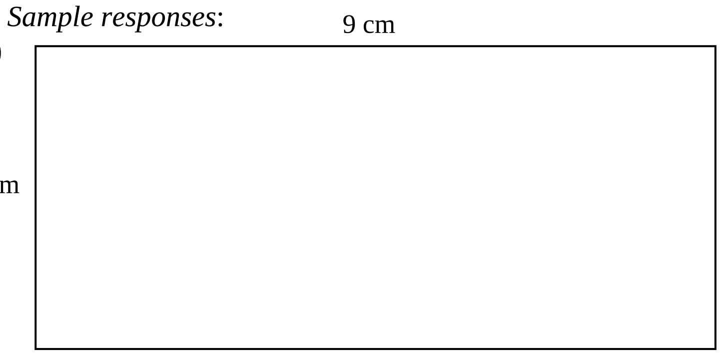
c) 100 cm^2

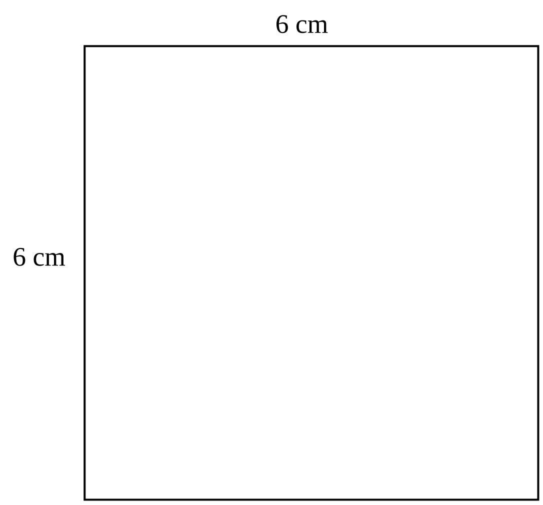


3. Rectangles A and C; [The total area of A and C is 21 cm^2 + 25 cm^2 = 46 cm^2, which is less than 50 cm^2. If I add A + B or B + C, the total area is more than 50 cm^2.]

4. *Sample responses*:

a)





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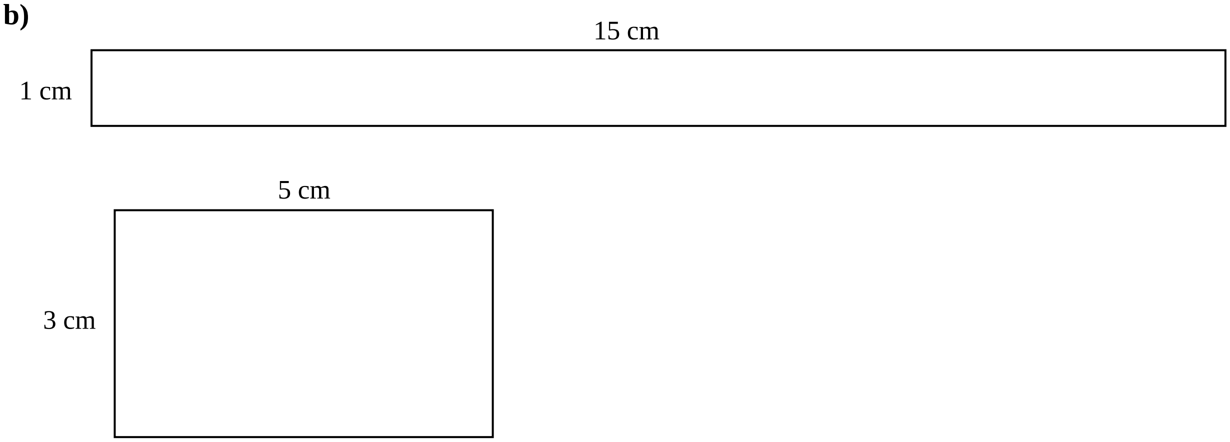
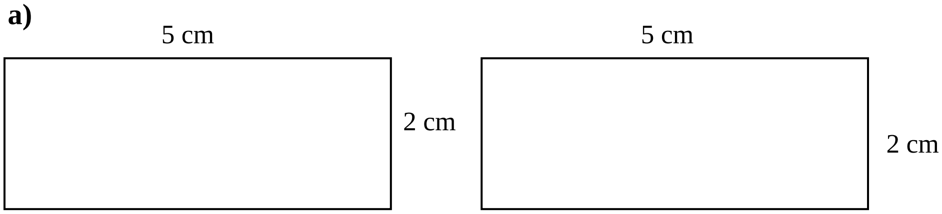
Lesson 3.1.3 Answers [Cont'd]

b)

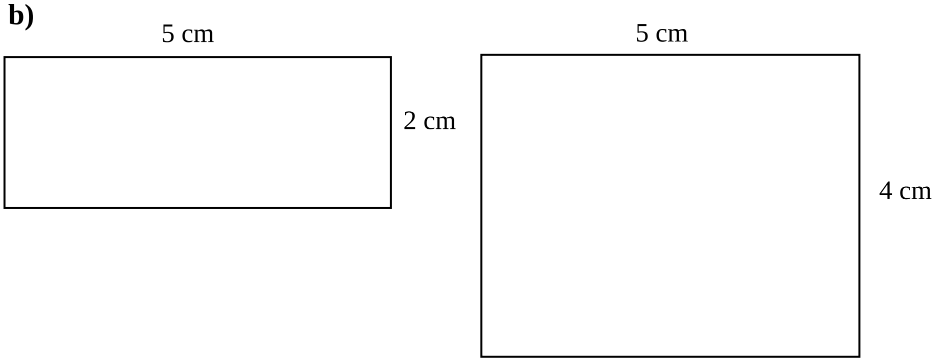


5. *Sample responses*:

a)



b)



[**6.** *Sample response*:
There are more ways to multiply whole numbers to get 36 than to get 23.]

[**7.** *Sample response*:
I can use the ruler to measure the length and the width. Then I can multiply those two numbers to find the area.]

Supporting Students

Struggling students

• Most students have little difficulty multiplying the dimensions of a rectangle to calculate its area. However, students who are not fluent with multiplication facts may have more trouble calculating dimensions given the area. Encourage these students to use centimetre grid paper, counting squares to form rectangles, or, if possible, provide square centimetre tiles that they can move around and rearrange into rectangles to go from an area to the linear dimensions.

Enrichment

• Some students may wish to solve the problem of finding the area value between 0 and 100 for which they can form the most possible rectangles with whole number side lengths.

3.1.4 EXPLORE: Rectangle Perimeters with a Given Area

Curriculum Outcomes	Outcome Relevance
4-D5 Constant Area and Different Perimeters: explore • explore the concept concretely • understand that-different shapes can have the same area • understand that area and perimeter are generally independent of each other	This essential exploration ensures that students fully understand the difference between area and perimeter. These different measures are used for different purposes in real-world situations, so the distinction is important

Pacing	Materials	Prerequisites
40 min	• Rulers or Centimetre Grid Paper (BLM)	• familiarity with the meaning of perimeter and area

Main Points to be Raised

• Rectangles with the same area can have different perimeters. Therefore, knowing the area of a rectangle does not tell you exactly what it looks like or what its perimeter is.

Exploration

Ask students to work on the exploration in pairs. Observe while students work. You might ask questions such as the following:

• *How did you choose your rectangles?* (I looked for numbers that multiplied to 9.)

• *How did you calculate the perimeter?* (I added the length twice and the width twice.)

• *When did the rectangle usually have the greater perimeter?* (When it was longer and thinner.)
Why does that make sense? (I am adding big numbers when the lengths are really long.)

For **part D**, some students might say that if the rectangle is a square, then it is possible to tell the perimeter if you know the area.

Observe and Assess

As students work, notice the following:

• Do students easily create rectangles with the correct areas?

• Do students calculate perimeters correctly?

• Do students generalize from the examples they have seen to notice that longer, thinner rectangles have greater perimeters?

• Do students realize that knowing that different rectangles can have the same area means that they cannot predict a perimeter by knowing an area?

Share and Reflect

After students have had sufficient time to do the exploration, you may have a class discussion and pose these questions:

• *How does knowing the area of a rectangle help you draw the rectangle?*

• *If you know the area of a rectangle, what kind of rectangle would you draw to have the same area but a greater perimeter? Why?*

• *If I tell you that a shape has an area of 20 cm^2, what do you know about its perimeter?*

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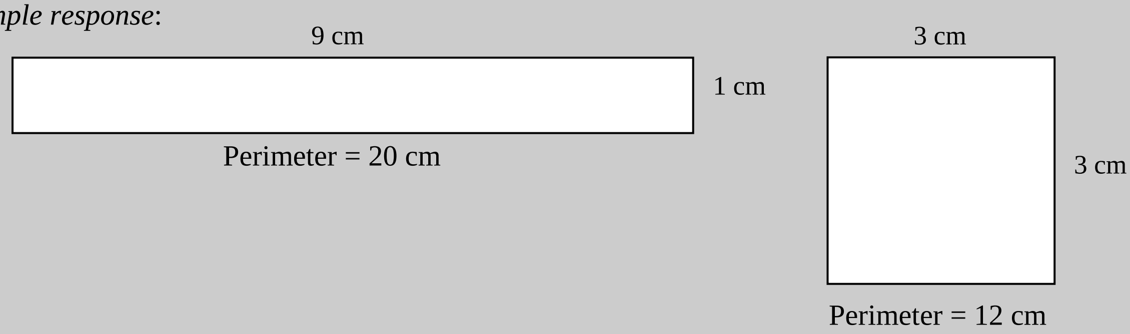
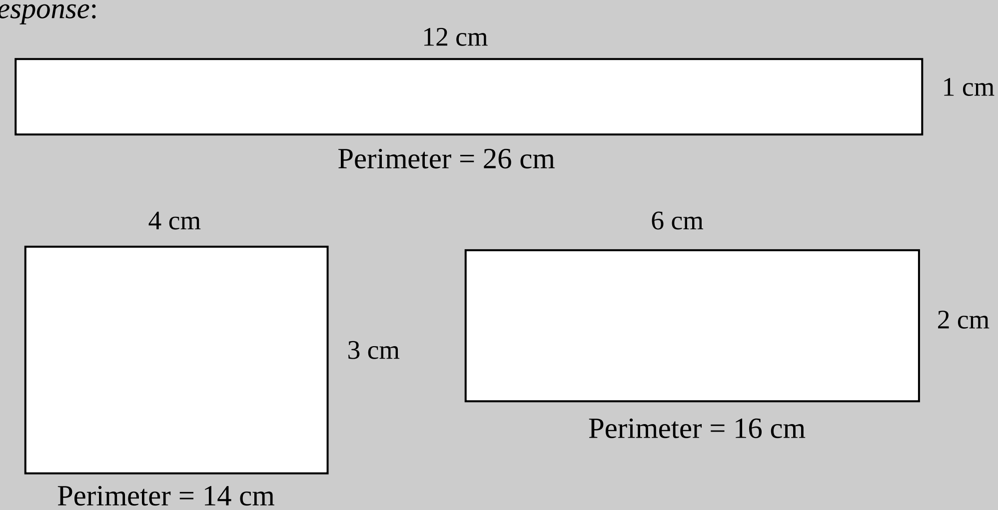
Answers

A. *Sample response*:



B. *Sample response*:



C. *Sample responses*:
i) The first shape has the greater perimeter; In parts A and B, I noticed that when rectangles have the same area, the long, thin rectangle has a greater perimeter.
ii) I was right; the first shape has a perimeter of 22 cm and the second shape has a perimeter of 14 cm.

D. No; Shapes with the same area can have different perimeters.

Supporting Students

Struggling students

- If students are struggling to create rectangles with given areas, you may wish to give them the rectangles and let them explore the perimeters.

Enrichment

- Some students may wish to investigate whether rectangles with the same perimeter can have different areas.

GAME: Filling a Grid

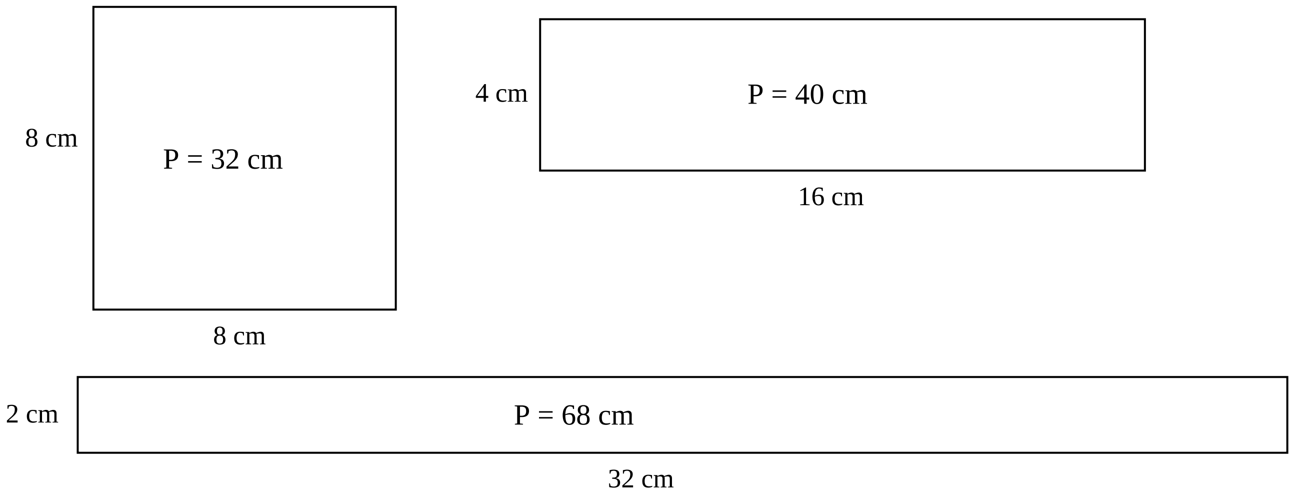
- This game is designed to allow students to use what they learned about calculating areas of rectangles. However, they do not actually have to use that skill in the game if they choose not to create rectangles.
- Students will notice that rolling greater numbers is useful early in the game, but later in the game it is not.
- Students can check each other’s area calculations.

CONNECTIONS: Relating Perimeter and Area

This connection provides a practical purpose for the previous lesson. It also extends students' knowledge informally. It is only in a much later class that students will learn formally that the square is the rectangle with the least perimeter for a given area and that the circle is the shape with the least perimeter for a given area.

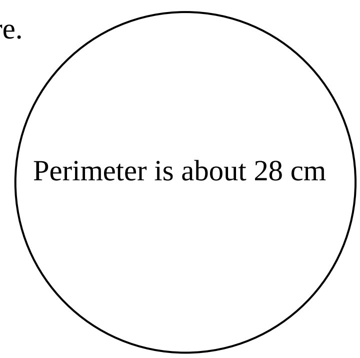
Answers

1. *Sample response*:



The square has the shortest perimeter.

2. Yes; It is even shorter than the square.



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Chapter 2 Angles

3.2.1 Describing Angles

Curriculum Outcomes	Outcome relevance
4-D7 Angles: amount of turn • develop the meaning of angle and the measure of an angle concretely • understand angle as a turn and the measure of angle as the amount of turn (i.e., a smaller angle means a smaller turn) • investigate to discover that the lengths of the arms of an angle do not influence angle size	It is important that students work with angles and understand what the measurements of angles signify so they will understand geometric concepts and principles from Class IV onward.

Pacing	Materials	Prerequisites
1 h	• A large pair of cardboard strips joined by a paper fastener or a tack/pin (optional) • Tracing paper	• familiarity with the concept of a right angle as a square corner

Main Points to be Raised

• An angle is made of two arms (parts of lines or line segments) that meet at an end point called the vertex.

• The measure or size of an angle tells how much one arm is turned away from the other arm.

• The size of an angle is not related to the length of its arms.

• We often describe the size of an angle as part of a full turn of one arm around the other arm.

For example, a right angle is a $\frac{1}{4}$ turn.

• An angle is sometimes named by a letter assigned to its vertex.

Try This — Introducing the Lesson

A. Have students work in pairs on this problem. Each student can spread two fingers (not a thumb) to create angles. They might begin by comparing their angles to see who was able to spread their fingers more. This may help them associate a greater spread with a greater number.

While you observe students at work, you might ask questions such as the following:

• *Look at the right angle on* ***page 81*** *of your text. What makes it a special angle?* (It looks like the corner of a square.)

• *How do you know that you have spread your fingers more than Kuenzang's?* (We put one of my fingers next to one of his and saw that my other finger was open more than his was.)

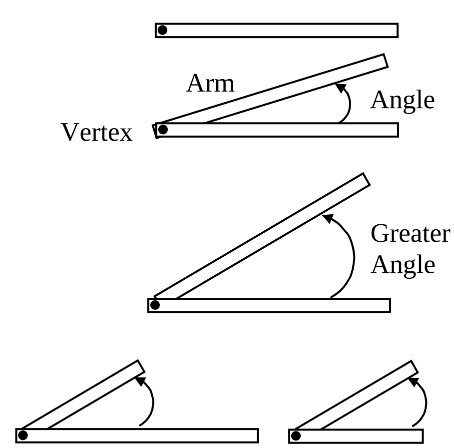
• *How do you know that your fingers did not open as far as a right angle?* (A right angle would make a square corner where my fingers meet and it was smaller than that.)

The Exposition — Presenting the Main Ideas

• Fasten together two long thin paper strips using a paper fastener or a tack. Show students how you can hold one strip fixed and turn the other strip to make angles of different sizes. Do not always hold one arm horizontal.

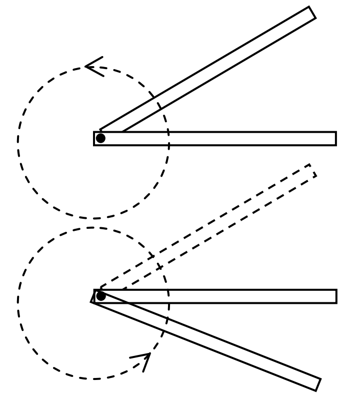
• Point out that an angle is made up of two sides called *arms* that meet at a single point called a *vertex*. Tell students that one angle is greater than another angle if the second arm has opened more. Model this with the paper strips.

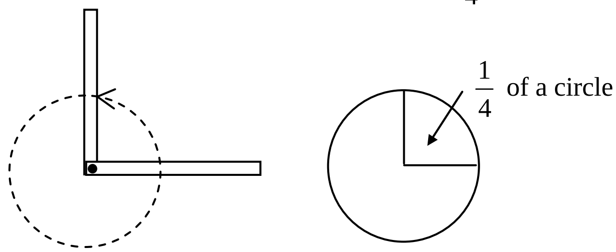
• Make an angle with the strips. Then fold one of the strips back to shorten it. Point out that the amount that strips were open did not change, so the angle size is the same even though one arm is much shorter than the other arm. Then fold the other arm back so there are two short arms. Emphasize again that the angle size did not change when the arm length changed.



• Model what would happen if you turned one arm all the way around until it came back to the first arm again. Point out how you created a circle as you went around.

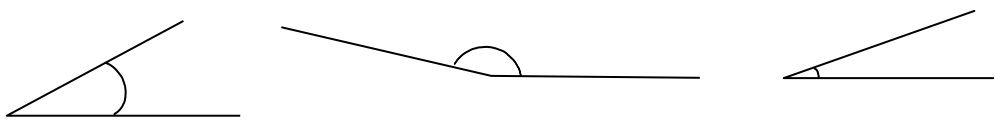
• Have students look on **page 82** of the student text to see the circle displayed in the exposition. Have them note that a right angle is always $\frac{1}{4}$ of a turn around that circle.





A right angle is $\frac{1}{4}$ turn around a circle.

• Show how arcs, which are sections of the circumference of a circle (without arrows), are used to denote angles.



• Let students know that they can review the information about angles on **pages 81 and 82** of the student text.

Revisiting the Try This

B. This question allows students to formally connect the angle they made with their fingers in **part A** to the formal language of arms and vertex.

Using the Examples

• Work with students through the **example**. Point out that when you have to compare angles that are facing in different directions, you usually have to reorient one of the angles if the angles are close in size. It makes sense to trace one angle and then superimpose it on top of the other angle. Make sure students understand the thinking modelled in the example.

Practising and Applying

Teaching points and tips

Q 1: Make sure that students focus on the amount of turn and not on the arm length when they create greater and smaller angles.

Q 2: Allow students to use whatever notation is comfortable for them to indicate where the arms and vertex are.

Q 3: Encourage students to justify their answers. Provide tracing paper if students' paper is not thin enough to trace the angles on the page.

Q 4: Rough estimates are sufficient for this question. Students can use a dashed line to show how a right angle would compare to the angle they drew for each.

Q 6: One of the most important reasons for studying angles is their use in geometry. This question begins to set up this connection.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can compare the size of two angles
Question 4	to see if students can construct angles relative to the sizes of other angles
Question 6	to see if students can identify angles within shapes
Question 7	to see if students can communicate about the features of an angle that affect its size

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Answers

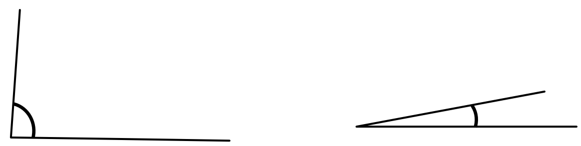
A. *Sample response*: $\frac{1}{2}$ a right angle

B. The arms are the fingers. The vertex is the middle point in the space between the fingers

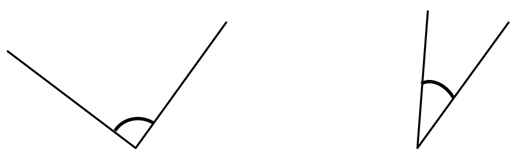
1. *Sample responses*:

a) Greater angle Smaller angle

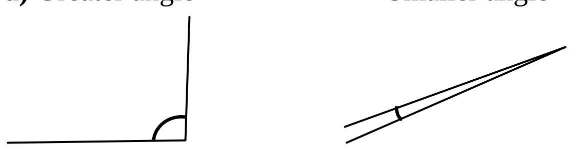
b) Greater angle Smaller angle



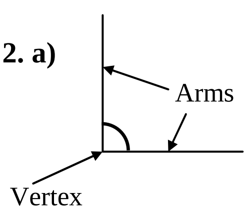
c) Greater angle Smaller angle



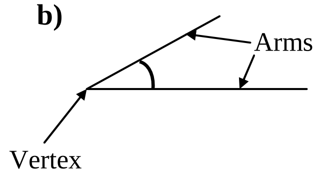
d) Greater angle Smaller angle



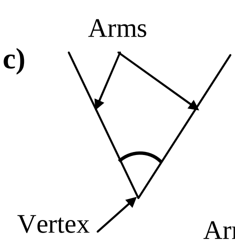
2. a)



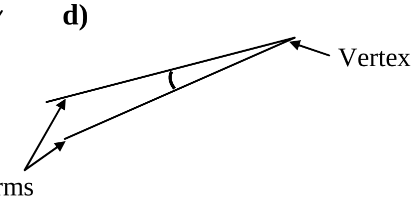
b)



c)



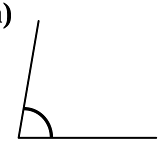
d)



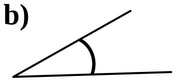
3. a) B **b)** D

4. *Sample responses*:

a)



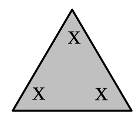
b)



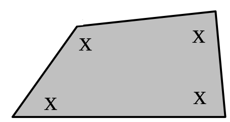
c)

[**5.** The arms do not meet at a vertex.]

6. a) 3 angles; 3 sides



b) 4 angles; 4 sides



c) 4 angles; 4 sides

[**7.** *Sample responses*:
a) He probably thinks that because the arms are longer, the angle is greater. That is not true.
b) I would have him trace angle B and then put the tracing on top pf angle A, with the bottom arm of angle B on top of the bottom arm of angle A.
He will see that the top arm of angle B was turned farther away from the bottom arm than the top arm of angle A.]

Supporting Students

Struggling students

• Some students may not take into account the need to match one arm of each angle in order to compare the sizes of two angles. Help those students by first having them compare angles that are oriented the same way. Once they are comfortable with this, you might use two angles that they have already compared. Trace and then re-orient one of the angles. Emphasize that the size of an angle does not change just because it is moved; this is much like how the length of a line segment does not change if it moves.

3.2.2 Classifying Angles

Curriculum Outcomes	Outcome relevance
4-D7 Angles: amount of turn • differentiate among and describe right, acute, and obtuse angles	Knowing about angle types will support students in their study of geometry in Classes IV to VIII.

Pacing	Materials	Prerequisites
40 min	None	• knowing what a right angle is

Main Points to be Raised

- An angle that is less than a right angle is called an acute angle.
- An angle that is greater than a right angle is called an obtuse angle.
- An angle that is made up of two right angles with a shared arm is called a straight angle. It represents a half turn around a circle with a centre at the vertex.

Try This — Introducing the Lesson

A. and B. Have students work in pairs.

While you observe students at work, you might ask questions such as the following:

- *How could you compare the angles?* (I could trace one angle and put it on top of the other.)
- *Which angle looks greater? How do you know?* (Angle B looks greater; I turned the picture of it in my head and the second arm was more to the left.)
- *How did you create a greater angle?* (I traced angle B and moved the short arm on the right so that the angle opened up more.)

The Exposition — Presenting the Main Ideas

- Have students turn to **page 84** in the student text to see the pictures of right, acute, straight, and obtuse angles. You may wish to write the vocabulary terms on the board for students to refer to.
- To make sure students understand what the terms mean, draw several of each type of angle on the board and ask students to classify them. Have a volunteer come to the board to draw a dashed line to show how a right angle would compare in each case.

Revisiting the Try This

C. and D. This question allows students to use the new vocabulary to name the angles in **parts A and B.** It also helps them see why Angle B was greater than Angle A and why the angle they drew was obtuse.

Using the Examples

- Present the task of drawing both an acute and an obtuse angle to the students. They can each draw the angles and have a partner check their work. Then ask them to read through the **example** to see another model.

Practising and Applying

Teaching points and tips

Q 2: Let students compare their work to see that acute angles (or obtuse) angles can be very different in size, but right angles and straight angles cannot.

Q 3: Some students may think this angle is obtuse since it is more than a right angle. However, it is actually a reflex angle (which students do not have to know at this point). Make sure students notice the location of the arc. Talk about how that shows which angle they are supposed to look at.

Q 4: This question allows students to test a conjecture. This is an important step in their mathematical growth.

Q 6: You may follow up by asking students for other examples where a classification provides enough information so that measurement is not required.

For example, an obtuse angle is always greater than a right angle.

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Common errors

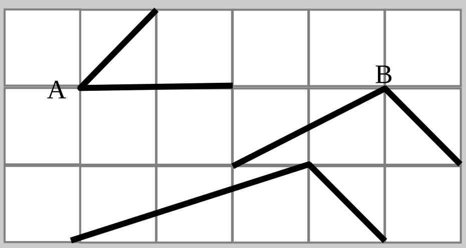
• Some students may have difficulty classifying angles when one of the arms is not in a horizontal position. Help those students by suggesting that they turn their heads (rather than turning the angles) to visualize the angles in standard position. They can also trace the angles and reoriented them.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can classify angles
Question 2	to see if students can draw an angle to fit a classification
Question 5	to see if students can make connections between angle classification and their environment

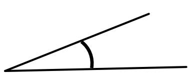
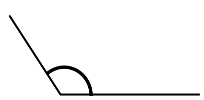
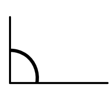
Answers

<table>
<tr>
<td>A. Sample response:
Angle B is greater; I turned it in my mind and the arm moved farther than for A.

B. Sample response:


I checked that my angle was greater by tracing it and comparing it to angle A and angle B.</td>
<td>C. i) Acute
ii) Obtuse

D. i) Obtuse
ii) No; My angle had to be greater than angle B. Angle B is obtuse, so my angle had to be greater than a right angle and greater than an acute angle.</td>
</tr>
<tr>
<td>1. a) Acute   b) Right
c) Obtuse   d) Obtuse

2. Sample responses:
a)    b) 
c)    d) 

[3. It is greater than a straight angle, so it is not an obtuse angle.]</td>
<td>4. No; [If the acute angles are small, the two angles together could still be acute.
If the acute angles are of a particular size, the two angles together could be a right angle.
If the acute angles are large, the two angles together could be obtuse.]

5. Sample responses:
Acute: the angle made by the door when it is open a bit
Right: the angle formed by the walls in each corner of the classroom
Obtuse: The angle formed by the door when it is open all the way

6. Yes; [An acute angle is always less than a right angle and an obtuse angle is always greater than a right angle. That means an obtuse angle is always greater than an acute angle.]</td>
</tr>
</table>

Supporting Students

Struggling students

• Some students may have difficulty with **question 4**. You might support them by drawing three acute angles of different sizes and asking them to classify the angles made up of two of each acute angle.

Enrichment

• Some students may wish to make other conjectures like the conjecture in **question 4**.

For example, they could consider what would happen if they put together three acute angles, or two or three obtuse angles.

Chapter 3 Volume

3.3.1 Measuring Volume Using Cubes

Curriculum Outcomes	Outcome relevance
4-D2 Volume: estimate and measure to develop concept of volume • explore the meaning of volume through non-standard units by counting the number of units it takes to build a solid • estimate and measure volume in non-standard units	Before students work with standard units, it is important to introduce the concept of volume using non-standard units. This allows students to focus on what volume is without becoming confused with the actual units. It is also important that students develop the habit of estimating to check their measurement calculations.

Pacing	Materials	Prerequisites
40 min	• Linking cubes	N/A

Main Points to be Raised

• A three-dimensional (3-D) object is not flat; a two-dimensional (2-D) object is flat. A 3-D object has length, width, and height.

• The volume of a 3-D object tells how much space it occupies (or how much material is needed to build it).

• To measure the volume of an object, you can build a cube model of the object that is the same size as the object. The number of cubes required to build the model is the volume of the object.

• If you want to compare the volume of two objects, you must consider the size of the units. If two objects are made up of the same number of units but the units are of a different size, the object made up of larger units has more volume.

• Different objects can have the same volume.

• The volume of an object does not change if you move the object.

Try This — Introducing the Lesson

A. Have students work in pairs on this problem.

While you observe students at work, you might ask questions such as the following:

• *How do you know this book would use up more space?* (It is a lot bigger.)

• *These books are really close in size. Which of these books would use up more space?* (I am not sure. This one is wider, but it is not as tall, so they are really close in size.)

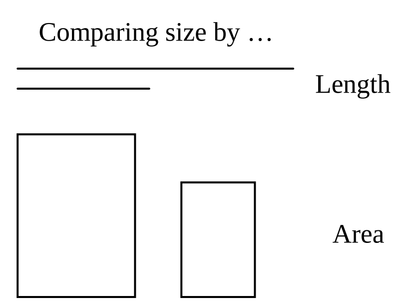
• *Why can you not just measure the books with a ruler to decide which takes up more space?* (The length of a book tells a little about it, but I have to also think about how thick it is and how tall it is.)

The Exposition — Presenting the Main Ideas

• Hold up a flat piece of paper and a book. Ask students to describe how the objects are the same (e.g., they are both made of paper, they both have a length and a width, they both might have words on them, and so on) and how they are different. Focus students on the fact that the book has height and the paper does not. Introduce the language three-dimensional (3-D) to describe this. Tell students that we say the book is 3-D because there are three "distances" associated with it — its length, its width, and its height. Have students notice that there are only two "distances" associated with the piece of paper — its length and width; it has no height. (In reality, the paper is 3-D but because its height is extremely thin it can represent a 2-D shape. True 2-D shapes are found on 3-D objects. For example, the face of a cube is a 2-D square.)

• Draw two lines on the board, one longer than the other. Mention that we use the word "length" to compare the sizes of the lines.

Then hold up two pieces of paper, one much larger than the other. Mention that we use the word "area" to compare the sizes of the papers.



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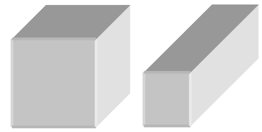
Then hold up two 3-D objects. Indicate that we need a word to compare the sizes of these 3-D objects. The word we use is *volume*.

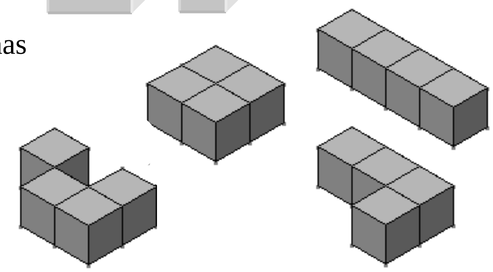


Comparing size by volume

• Bring out linking cubes. Tell them each cube is one unit.

Put together four cubes and tell students than an object of that size has a volume of four units because it takes four cubes to make it. Re-arrange the four cubes to create a different object. Indicate that the volume is still four units.

• Let students know they can read the material on **page 86** of the student text to review these ideas.



Each object has a volume of 4 units.

Revisiting the Try This

B. This question helps students make a formal connection between the informal thinking they used in **part A** and the formal mathematical language they have learned.

Using the Examples

• Have students read through the **example** to clarify their understanding of what volume is.

Practising and Applying

Teaching points and tips

Q 1: Provide linking cubes. Students might work in pairs. Each partner can make sure the other modelled the structure correctly.

Q 2: Encourage students to build objects where all three dimensions are greater than 1 unit.

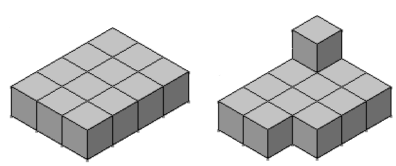
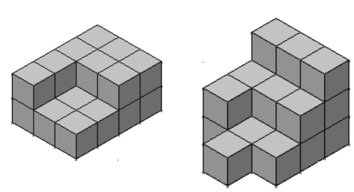
Q 3: The intent of this question is to focus students on the notion that the number of units and the size of the unit are both relevant in describing volume.

Q 4: This might be handled as a group discussion.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can determine the volume of a linking cube structure
Question 2	to see if students can build an object with a particular volume
Question 4	to see if students recognize that different objects can have the same volume

Answers

A. *Sample response*: My math book would take up more room than my notebook.	**B.** *Sample answer*: The book that takes up more space has a greater volume.
1. a) 20 cubes **b)** 12 cubes **c)** 16 cubes	**2.** *Sample responses*: **a)**  **b)** 

Lesson 3.3.1 Answers [Continued]

3. No; [The cubes used for the object on the right are bigger, so four of the big cubes have a greater volume than four of the small cubes.]	**4.** No; [*Sample response*: The box could be very wide or narrow or it could be tall or shallow and still be 60 cubes.]

Supporting Students

Struggling students

• Some students may have difficulty with **question 3**. These students should build objects using the same number of both large and small cubes to see how the size of the unit cube influences the size of the final object.

Enrichment

• Some students may wish to see how many different objects they can build with a given small volume, e.g., 4 cubes or 5 cubes.

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3.3.2 EXPLORE: Volume of Rectangle-based Prisms

Curriculum Outcomes	Outcome Relevance
4-D3 Volume of Rectangle-based prisms: estimate and measure in non-standard units • calculate the volume of rectangle-based prisms using cubes • determine the volume of a rectangle-based prism and build prisms with a specified volume • connect volume to dimensions (dimensions of first layer × number of layers)	This essential exploration forms the basis for more formal work in later classes on the formulas for volume of all prisms.

Pacing	Materials	Prerequisites
40 min	• Linking cubes	• finding the area of a rectangle in nonstandard units

Main Points to be Raised

• To find the volume of a rectangle-based prism made of linking cubes, you multiply the area of the base by the height (the number of layers).

• To find the area of the base of a rectangle-based prism, you multiply its length by its width.

Exploration

Ask students to work on the exploration in pairs. The exploration is designed to help students see why you multiply the area of the base by the height (in this case, the number of layers) to find the volume of a rectangular prism.

Observe while students work. You might ask questions such as the following:

• *How do you measure the volume of the prism?* (Count the number of cubes used.)

• *Why is the volume doubled when you double the height?* (There are two layers of cubes instead of one layer.)

• *Why is the volume tripled if the height is tripled?* (There are three layers instead of one layer.)

• *Do you think the volume changes if you switch the length and the height?* (No. I could turn the prism around and find the volume and it would be the same.)

• *Why is it helpful to know your multiplication facts to find the dimensions of a prism with a volume of 20?* (I need to find two numbers that multiply to 20, e.g. 2 × 10, to find the area of the base and height, so it is useful to know facts with a product of 20. Then I need to find two numbers that multiply to, e.g., 10, to find the dimensions of the base, so it is useful to know facts with a product of 10.)

• *You have built a prism and now you want to find its volume. How do you know which face is the base?* (It does not matter. I can choose any layer to be the bottom layer and I will always get the same volume.)

Observe and Assess

As students work, notice the following:

• Do students count cubes correctly to determine the volume?

• Do students see the relationship between the area of the base, the height (in number of layers), and the volume of the prism?

• Do students use the dimensions of a prism to calculate the area of the base and then multiply the number of layers to find the volume quickly?

• Can students work backwards from the volume of a prism to determine its dimensions?

• Do students realize that any outside layer of a rectangle-based prism made of cubes can be used as the base?

Share and Reflect

After students have had sufficient time to do the exploration, you may have a class discussion and ask:

• *Why can you not determine the volume of a rectangle-based prism if you know only two of its dimensions?*

• *If you know all three dimensions, do you need to know which dimension is the height? Why or why not?*

• *Could rectangle-based prisms with different dimensions have the same volume?*

Answers

A. i) The volume is how many cubes are used and I used 6 cubes. **ii)** 6; 1 **iii)** I can multiply 6 × 1 and get 6, which is the volume. **B. i)** 12 cubes **ii)** I can multiply 6 × 2 and get 12, which is the volume. **C. i)** 18 cubes **ii)** I can multiply 6 × 3 and get 18, which is the volume.	**D.** 12 cubes; 12 × 1 = 12. 24 cubes; 12 × 2 = 24. 36 cubes; 12 × 3 = 36. **E. i)** 36 cubes; 3 × 6 = 18 and 18 × 2 = 36. **ii)** 32 cubes; 2 × 4 = 8 and 4 × 8 = 32. **iii)** 75 cubes; 5 × 5 = 25 and 25 × 3 = 75. **F.** *Sample responses*: **i)** 2 cubes by 5 cubes by 2 cubes **ii)** 2 cubes by 5 cubes by 3 cubes **iii)** 4 cubes by 5 cubes by 2 cubes

Supporting Students

Struggling students

• Some students may need additional experience calculating the volumes of prisms before tackling **part F**, where they go from the volume to the linear dimensions.

• If students struggle with **part F**, you might suggest that they first break up the volume into two factors and then break up one of those two factors into two other factors. Some students might benefit by assuming there is only one layer, determining the area of the base, and then later breaking up the base into equal parts they can stack to make layers.

Enrichment

• Some students may wish to explore all the possible prisms they could create with a given volume, for example, 20 cubes or 30 cubes.

GAME: Building Bigger Boxes

• Students should use linking cubes to build their boxes.

• You can create a spinner using a fraction circle, a paper clip, and a pencil or pen. There is a BLM of a fraction circle in sixths on **page 130** of this teacher's guide that you can use to create the spinners. Students will need to sketch the pictures of the base layers on the spinners.

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UNIT 3 Revision

Pacing	Materials
2 h	• Millimetre rulers • Centimetre Grid Paper (BLM) • Tracing paper • Linking cubes

Question(s)	Related Lesson(s)
1 – 4	Lesson 3.1.1
5 – 8	Lesson 3.1.2
9 and 10	Lesson 3.1.3
11	Lesson 3.2.1
12 – 14	Lesson 3.2.2
15 – 17	Lesson 3.3.1
18	Lesson 3.3.2

Revision Tips

Q 6: Students might estimate by using the area of their fingernail as a 1 cm^2 referent.

Q 9: Observe whether students use the rule for multiplying dimensions to find the area of a rectangle or if they divide the rectangle into grid squares and skip count or multiply.

Q 10: Ask students to explain how they could have predicted which of their rectangles has a greater perimeter.

Q 11: Provide tracing paper, if needed.

Q 17: Provide linking cubes.

Answers

1. a) 38 mm **b)** 52 mm

2. a)

b)

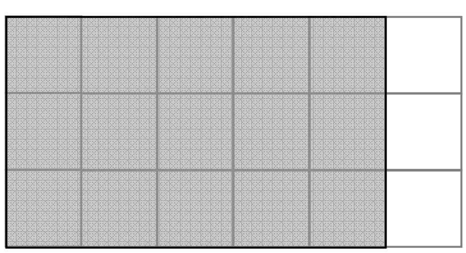
3. 40 mm; 1 cm = 10 mm, so 4 cm = 40 mm.

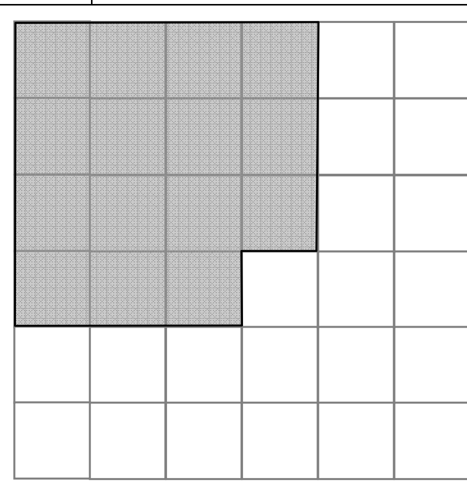
4.

5. a) 6 cm^2 ; [I counted 6 whole square centimetres.]
b) 5.5 cm^2; [I counted 4 whole square centimetres and 3 half square centimetres.
$4 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 5\frac{1}{2} = 5.5$.]

6. a) *Sample response*: About 16 cm^2;
Area: 20 cm^2
b) *Sample response*: About 10 cm^2;
Area: 8 cm^2

7. *Sample responses*:
a)



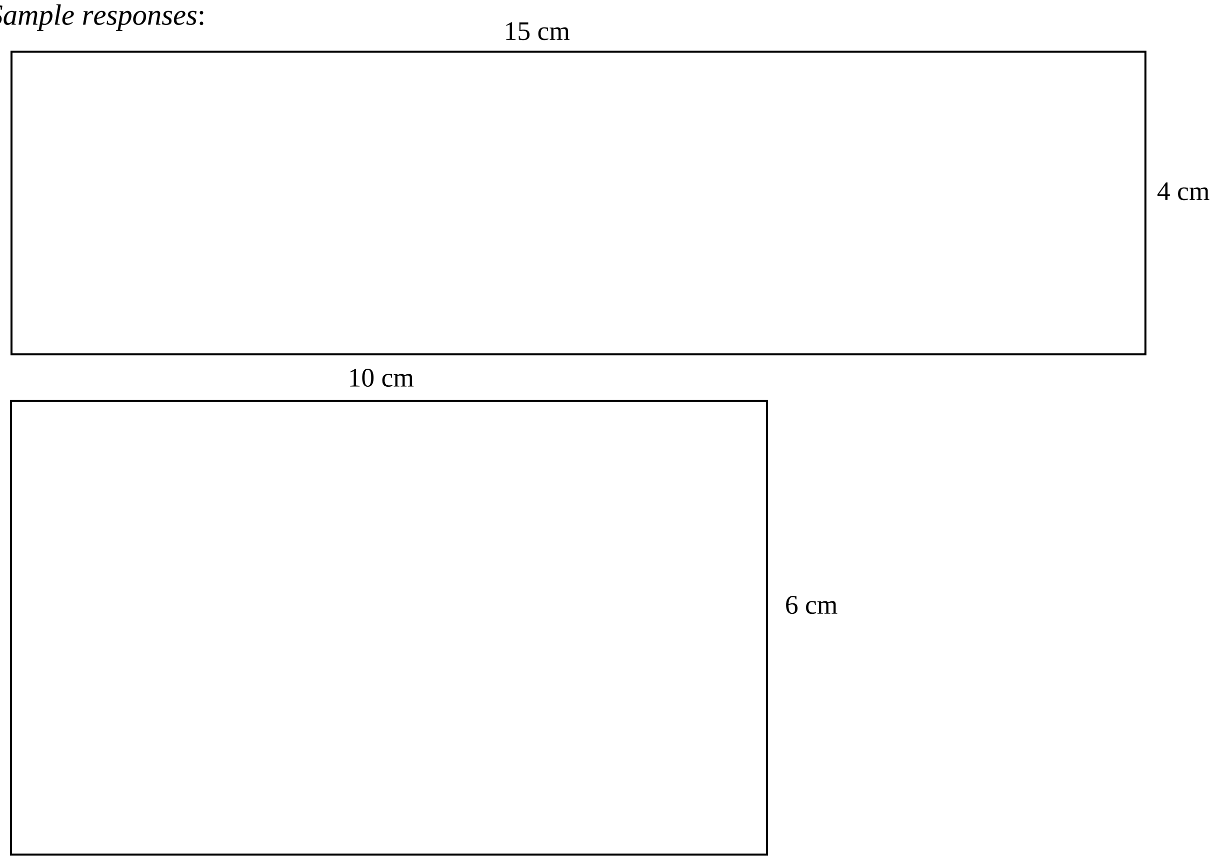


[**7. b)** I made both shapes by tracing around 15 squares on a centimetre grid.]

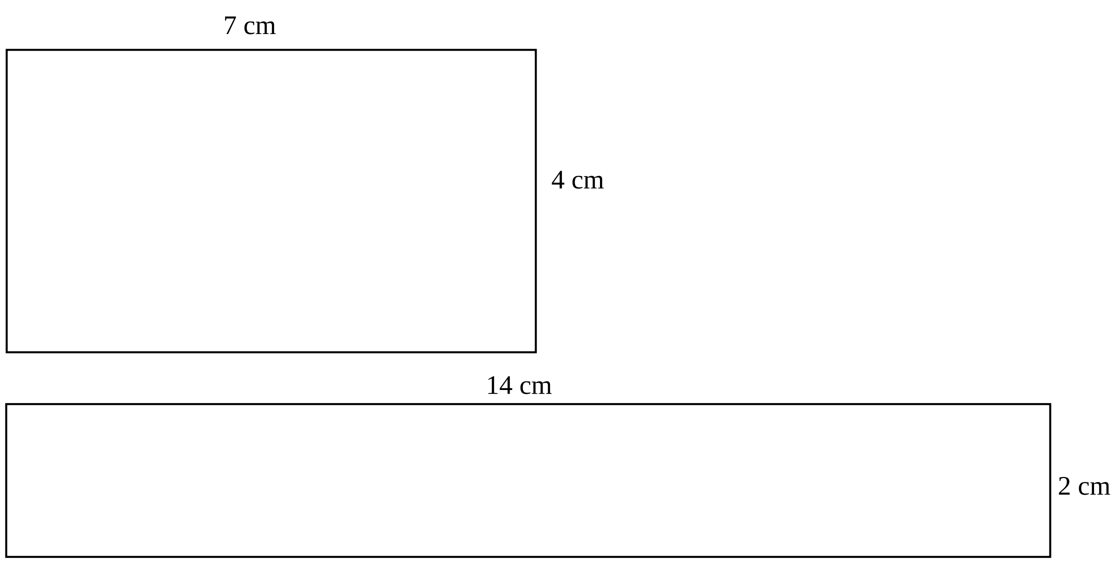
8. *Sample responses*: **a**) Geometry box **b)** Chocolate bar

9. *Sample responses*:

a)



b)

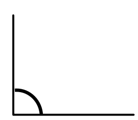


10. 4 cm-by-4 cm rectangle; Perimeter = 16 cm.
8 cm-by-2 cm rectangle; Perimeter = 20 cm.
1 cm-by-16 cm rectangle; Perimeter = 34 cm.

11. a) Angle B **b)** Angle C **c)** Angle E

12. *Sample responses*:

a) A right angle



b) An acute angle

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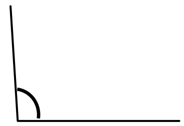
Revision Answers [Cont'd]

13. a) B **b)** A, D, and E

14. a) *Sample response*:

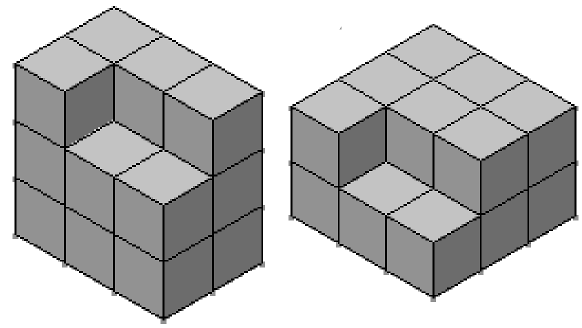


[**b)** It is greater than a right angle but less than a straight angle.]

15. No; *Sample response*: A taller prism could have a much smaller base. For example, if the base is only 1 cube and the prism is 10 cubes tall, it has less volume than if the base were 3 cubes by 2 units and the prism had only 2 layers.

16. *Assuming no hidden cubes or gaps*:
a) 12 cubes
b) 18 cubes

17. *Sample response*:



18. a) 12 blocks; 3 × 2 = 6 and 6 × 2 = 12.
[**b)** *Sample response*:
Length = 6 blocks, width = 1 block, height = 2 blocks; 6 × 1 = 6 and 6 × 2 = 12. I know I am right because I multiply the 3 dimensions to give the volume. For both rectangle-based prisms, the dimensions multiplied to give 12.]

UNIT 3 Measurement Test

1. Draw a line of each length.

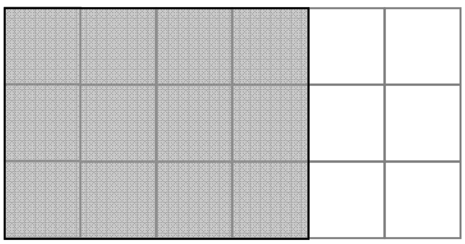
a) 29 mm **b)** 6 cm, 8 mm

2. An object is 60 mm long. How many centimetres long is it? How do you know?

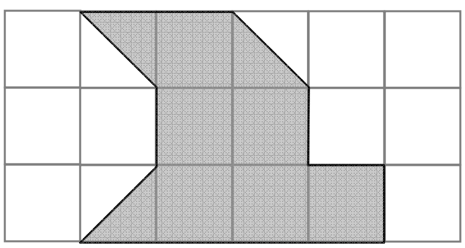
3. When is it useful to use millimetres to measure?

4. Each shape is on centimetre grid paper. What is the area of each? Explain how you got your answer.

a)



b)

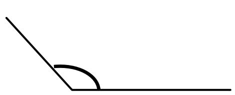


5. Draw two shapes on grid paper, each with an area of 14 cm^2.

6. a) Sketch two rectangles that each have an area of 40 cm^2.

b) Can a rectangle with an area of 40 cm^2 have a perimeter of 26 cm? Explain your thinking.

7. a) Draw an angle that is less than this angle. What type of angle is your angle?



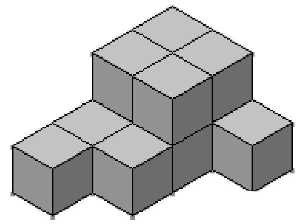
b) Draw an angle that is greater than the angle shown in **part a)**, but less than a straight angle. What type of angle is it?

8. a) Draw an angle that is made up of three acute angles joined together.

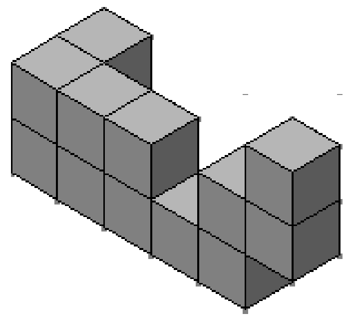
b) Does the combined angle have to be an obtuse angle? Explain your thinking.

9. What is the volume of each?

a)



b)



10. Build two objects, each with a volume of 10 cubes.

11. a) What is the volume of a rectangle-based prism with these measurements? Show your work.

length = 4 cubes

width = 2 cubes

height = 3 cubes

b) Describe the length, width, and height of a different rectangle-based prism with the same volume. How do you know you are right?

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UNIT 3 Test

Pacing	Materials
1 h	• Millimetre rulers • Centimetre Grid Paper (BLM) • Tracing paper • Linking cubes

Question(s)	Related Lesson(s)
1 – 3	Lesson 3.1.1
4 and 5	Lesson 3.1.2
6	Lesson 3.1.3
7 and 8	Lesson 3.2.2
9 and 10	Lesson 3.3.1
11	Lesson 3.3.2

Assign questions according to the time available.

Answers

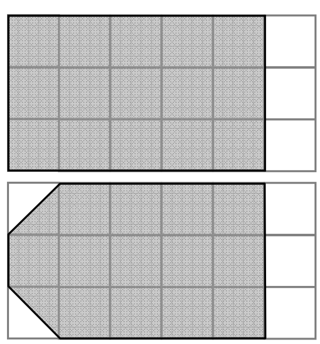
1. a)
b)

2. 6 cm; Each 10 mm = 1 cm.

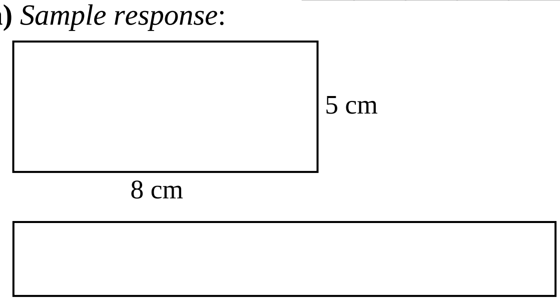
3. *Sample response*:
When a measurement is less than 1 cm or when it falls between centimetre markings on a ruler.

4. a) 12 cm^2; *Sample response*:
I skip counted whole squares: 4, 8, 12.
b) 7.5 cm^2; I counted 6 whole squares, 2 half squares that made 1 full square, and then 1 more half square.

5. *Sample response*:

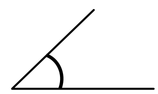


6. a) *Sample response*:



b) Yes; If the dimensions are 5 cm by 8 cm, the perimeter is 26 cm and the area is 40 cm^2.

7. *Sample responses*:
a)

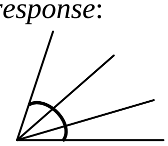


Acute

b)

Obtuse

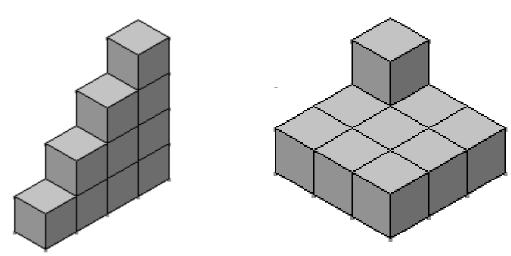
8. a) *Sample response*:



b) No; I can use three small acute angles and the combined angle could still be acute.

9. *Assuming no hidden cubes and no gaps*:
a) 12 cubes **b)** 12 cubes

10. *Sample response*:



11. a) 24 cubes
b) *Sample response*:
length = 6 cubes
width = 2 cubes
height = 2 cubes
If I multiply 6 × 2 × 2, I get 24, which is the same as 4 × 2 × 3 = 24.

UNIT 3 Performance Task — Building Prisms

A. i) Use linking cubes to build a rectangle-based prism with a volume of 30 cubes.

ii) What is the area of the base?

iii) What is the perimeter of the base?

iv) What is the height in millimetres?

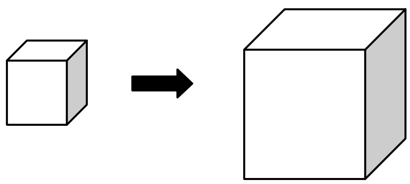
B. i) Rearrange the 30 cubes to build another rectangle-based prism. Its base should have the same area as the base of prism in **part A** but a different perimeter.

ii) How can the perimeter of the base be different even though the area of the base stayed the same?

iii) Did the height of the prism change? Explain.

iv) Did the volume change? Why or why not?

C. Suppose each linking cube in your prism from **part B** was twice as long on each edge.



i) How would the height of the prism change?

ii) How would the area of the base change?

D. Build a shape that is not a prism with the same volume as your prism in **part A**.

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UNIT 3 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
4-D1 Linear units: mm, cm, m, km; estimate and measure **4-D2** Volume: estimate and measure to develop concept of volume **4-D3** Volume of rectangle-based prisms: estimate and measure in non-standard units **4-D5** Constant area and different perimeters: explore **4-D6** Dimensions and area of a rectangle: relate factors and product	20 min	• Linking cubes

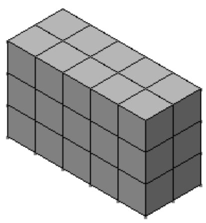
How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could supplement the unit test. It could also be used as enrichment material for some students.

• You can assess performance on the task using the rubric below.

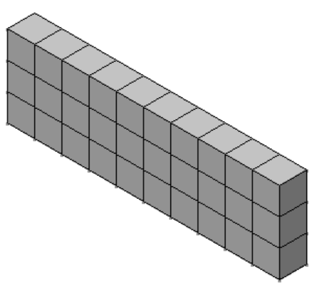
Sample Solution

A. i)



ii) For 2 cm cubes: 40 cm^2
For 1 cm cubes: 10 cm^2
iii) For 2 cm cubes: 28 cm
For 1 cm cubes: 14 cm
iv) For 2 cm cubes: 60 mm
For 1 cm cubes: 30 mm

B. i)

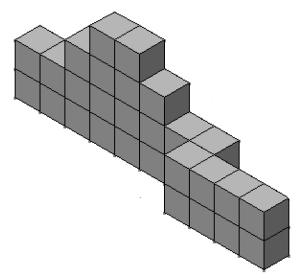


ii) For 2 cm cubes: The perimeter is greater because the base is a longer, thinner rectangle — it is 44 cm instead of only 28 cm.
For 1 cm cubes: The perimeter is greater because the base is a longer thinner, rectangle — it is 22 cm instead of only 14 cm.
iii) For 2 cm cubes: No, it is still 60 mm.
For 1 cm cubes: No, it is still 30 mm.

iv) No; It uses the same number and type of cubes, so it has the same volume.

C. i) For 2 cm cubes: The height would be 120 mm instead of 60 mm.
For 1 cm cubes: The height would be 60 mm instead of 30 mm.
ii) For 2 cm cubes: The area of the base would be 160 cm^2 instead of 40 cm^2.
For 1 cm cubes: The area of the base would be 40 cm^2 instead of 10 cm^2.

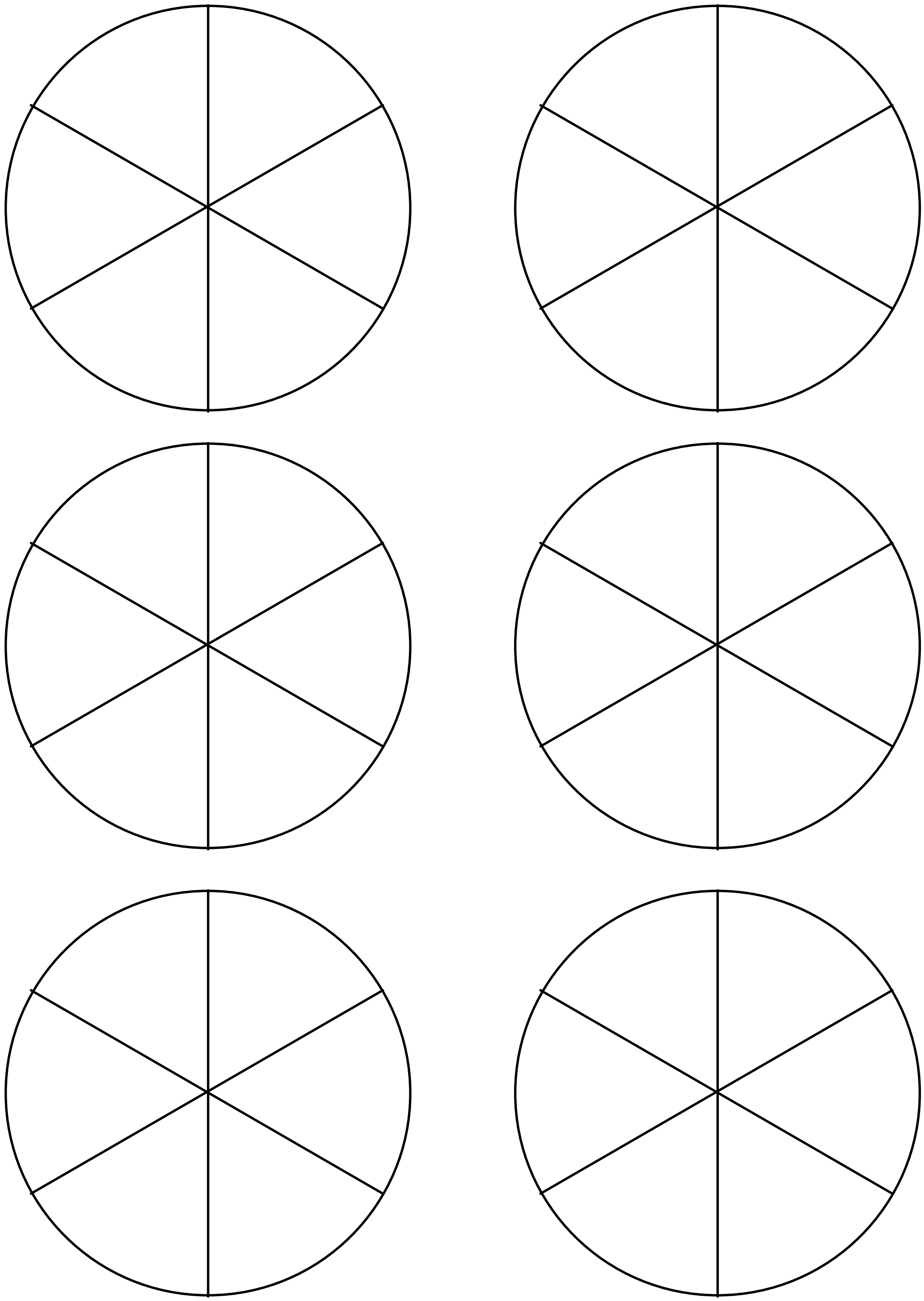
D.



UNIT 3 Performance Task Assessment Rubric

The student	Level 4	Level 3	Level 2	Level 1
Relates a volume to a 3-D object	Efficiently builds a first shape with a given volume and transforms it into another shape with the same volume	Builds two shapes with a given volume, but creates them independently	Correctly builds one shape with a given volume	Has difficulty building a shape with a given volume
Determines area	Correctly and efficiently measures the area of each base and easily transforms a base into another base with the same area but a different perimeter using an analysis of what the dimensions could be	Correctly measures the area of each base and correctly transforms a base into another base with the same area but a different perimeter by analysing what the dimensions could be	Correctly measures the area of at least one base and, through trial and error rather than by analysis, transforms one base into another base with the same area but a different perimeter	Has difficulty calculating areas and is unable to create a base with the same area but a different perimeter
Communicates reasoning	Explains parts B and C insightfully and completely	Explains parts B and C correctly and almost completely	Explains answers to at least one of parts B and C correctly	Has difficulty explaining reasoning for parts B and C

BLM 1 Fraction Circles in Sixths (for GAME: Building Bigger Boxes)



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UNIT 4 MULTIPLICATION AND DIVISION WITH GREATER NUMBERS

UNIT 4 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 93** **TG p. 135**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• Base ten blocks or Base Ten Blocks (BLM)	All questions
Chapter 1 Multiplication				
4.1.1 Multiplying by Tens and Hundreds **SB p. 95** **TG p. 137**	**4-B11 Multiply Mentally: by 10 or 100** • explore multiplication of 2-digit numbers by 10 and by 100 • develop visual images of whole numbers multiplied by 10 or 100 using base ten materials • read numbers different ways (e.g., 5300 can be read as 53 hundred or as 5 thousand, 3 hundred) **4-C1 Apply Patterns in Computations** • explore and apply patterns to solve computation problems (e.g., multiplying by 8, 9, or 10) **4-C2 Open Sentences and Computation Patterns: multiplication and division** • generate rules about how a change in one factor affects the result (e.g., for □ × 10, as □ increases by 1 the product increases by 10) **4-C3 Multiplying by 10, by 100, by 1000: apply pattern visually and symbolically** • identify and continue patterns with increasing powers of ten	1 h	• Base ten blocks or Base Ten Blocks (BLM) • Whole Number Place Value Charts (BLM)	Q1, 4, 7
4.1.2 Estimating Products **SB p. 100** **TG p. 141**	**4-B6 3-Digit by 1-Digit Multiplication: with and without regrouping** • use estimation to predict and verify multiplications	1 h	None	Q3, 4, 6
4.1.3 Multiplying Using Rectangles **SB p. 102** **TG p. 144**	**4-B6 3-Digit by 1-Digit Multiplication: with and without regrouping** • develop alternative and standard algorithms (from understanding) • connect concrete models to symbolic recordings (e.g., relate rectangle area models to written algorithm) • use estimation to predict and verify multiplications	1 h	• Base ten blocks or Base Ten Blocks (BLM)	Q1, 3, 5
4.1.4 Multiplying a 3-digit Number by a 1-digit Number **SB p. 106** **TG p. 147**	**4-B6 3-Digit by 1-Digit Multiplication: with and without regrouping** • develop alternative and standard algorithms (from understanding) • connect concrete models to symbolic recordings (e.g., relate rectangle area models to written algorithm) • use estimation to predict and verify multiplications	2 h	• Base ten blocks or Base Ten Blocks (BLM)	Q2, 3, 7, 10

UNIT 4 PLANNING CHART [Continued]

GAME: Lots of Tens (Optional) **SB p. 110** **TG p. 150**	To practise multiplication in a game situation	20 min	• Dice	N/A
4.1.5 EXPLORE: Multiplication Patterns (Essential) **SB p. 111** **TG p. 151**	**4-C1 Apply Patterns in Computations** • explore and apply patterns to solve computation problems (e.g., multiplying by 8, 9, or 10) **4-C2 Open Sentences and Computation Patterns: multiplication and division** • generate rules about how a change in one factor affects the result (e.g., for □ × 10, as □ increases by 1 the product increases by 10)	40 min	None	Observe and Assess questions
Chapter 2 Division				
4.2.1 Dividing Tens and Hundreds **SB p. 112** **TG p. 153**	**4-B10 2, 3-Digit by 1-Digit Division: with and without regrouping** • develop alternative and standard algorithms (from understanding) • connect concrete models to symbolic recordings • continue estimating	1 h	• Base ten blocks or Base Ten Blocks (BLM)	Q2, 4, 5
4.2.2 Estimating Quotients **SB p. 115** **TG p. 156**	**4-B10 2, 3-Digit by 1-Digit Division: with and without regrouping** • continue estimating	1 h	None	Q2, 4, 6
4.2.3 Dividing by Multiplying and Subtracting **SB p. 117** **TG p. 158**	**4-B10 2, 3-Digit by 1-Digit Division: with and without regrouping** • develop alternative and standard algorithms (from understanding) • connect concrete models to symbolic recordings • continue estimating	1 h	None	Q2, 4, 7
4.2.4 Dividing in Parts **SB p. 119** **TG p. 161**	**4-B10 2, 3-Digit by 1-Digit Division: with and without regrouping** • develop alternative and standard algorithms (from understanding) • connect concrete models to symbolic recordings • continue estimating	1 h	• Base ten blocks or Base Ten Blocks (BLM)	Q3, 4, 6
GAME: Two Hundred Plus (Optional) **SB p. 121** **TG p. 163**	To practise division in a game situation	25 min	• Dice	N/A
4.2.5 Dividing by Sharing **SB p. 122** **TG p. 164**	**4-B10 2, 3-Digit by 1-Digit Division: with and without regrouping** • develop alternative and standard algorithms (from understanding) • connect concrete models to symbolic recordings • understand remainders in context as a fraction, ignored, rounded, or addressed specifically • continue estimating	2 h	• Base ten blocks or Base Ten Blocks (BLM)	Q3, 5, 6

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CONNECTIONS: When do Remainders Change? (Optional) **SB p. 126** **TG p. 167**	To use division to explore remainder patterns	20 min	None	N/A
UNIT 4 Revision **SB p. 127** **TG p. 168**	Review the concepts and skills in the unit	2 h	• Base ten blocks or Base Ten Blocks (BLM)	All questions
UNIT 4 Test **TG p. 170**	Assess the concepts and skills in the unit	1 h	• Base ten blocks or Base Ten Blocks (BLM)	All questions
UNIT 4 Performance Task **TG p. 172**	Assess concepts and skills in the unit	1 h	None	Rubric provided
UNIT 4 Blackline Masters **TG p. 174**	BLM 1 Whole Number Place Value Charts Base Ten Blocks (BLM) from **pages 42 to 44** in UNIT 1			

Math Background

• This number unit builds on students' knowledge of multiplication and division facts and their ability to multiply and divide 2-digit numbers by 1-digit numbers. Here they multiply and divide using greater numbers.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 4** in **lesson 4.1.1**, where they solve a problem about a natural phenomenon using multiplication by tens and hundreds, in **question 7** in **lesson 4.1.4**, where they create a problem to fit a symbolic expression, in **question 4** in **lesson 4.2.3**, where they solve a problem about money, and in **question 1** in **lesson 4.2.5**, where they solve a measurement problem that requires division.

• Students use communication in **question 8** in **lesson 4.1.1**, where they explain the effect of multiplying by hundreds, in **part C** in **lesson 4.1.5**, where they describe and explain patterns, in **question 8** in **lesson 4.2.1**, where they explain why certain divisions are easier to perform than others, and in **question 3** in **lesson 4.2.5**, where they describe how to handle a remainder.

• Students use reasoning in **question 5** in **lesson 4.1.2**, where they consider what numbers to multiply to result in an estimated product, in **question 9** in **lesson 4.1.4**, where they reason about what digit might be missing in a calculation, in **question 1** in **lesson 4.2.2**, where they decide which estimate would be best, in **question 7** in **lesson 4.2.3**, when they reason about how they know that a result is incorrect, and in **question 2** in **lesson 4.2.4**, where they decide how best to rename dividends.

• Students consider representation in **question 7** in **lesson 4.1.1**, where they use pictures to explain the relationship between products, in **question 1** in **lesson 4.1.3**, where they draw a rectangle model to represent a product, and in **question 5** in **lesson 4.2.1**, where they draw a picture to describe a division relationship.

• Students use visualization in **question 4** in **lesson 4.1.2**, where they place numbers on a number line, in **question 2** in **lesson 4.1.3**, where they go from a rectangle model to the related product, in **question 2** in **lesson 4.1.4**, where they relate a base ten block model to the associated symbolic product, in **question 2** in **lesson 4.2.1**, where they relate a base ten block model to the associated symbolic quotient, and in **question 2** in **lesson 4.2.5**, where they use base ten block models to help them divide.

• Students make connections in **lesson 4.1.1**, where they connect place value ideas to multiplication by 10 and 100, in **question 5** in **lesson 4.1.4**, where they relate different meanings of multiplication (the rate meaning to the equal groups meaning in this case), in **question 4** in **lesson 4.2.2**, where they use the relationship between multiplication and division to solve problems, and in **question 9** in **lesson 4.2.5**, where they connect their thinking about fractions to work in division.

Rationale for Teaching Approach

• This unit is divided into two chapters.

Chapter 1 focuses on multiplying a 3-digit number by a 1-digit number. A variety of strategies are presented to provide both teachers and students with the opportunity to choose models that make the most sense to them. It is not necessary for students to use all the algorithms provided, but you can present them all to provide students with a choice. Estimation is also considered so students can predict and consider the reasonableness of calculated products.

Chapter 2 focuses on dividing a 3-digit number by a 1-digit number. A variety of strategies are presented to provide both teachers and students with the opportunity to choose models that make the most sense to them. It is not necessary for students to use all the algorithms provided, but you can present them all to provide students with a choice. Estimation is also considered so students can predict and consider the reasonableness of calculated quotients.

• There is one **Explore** lesson. It allows students to explore multiplication patterns to see the effect on the product of changing a factor.

• The **Connections** provides some preliminary insight into divisibility rules (which will be studied in a later class) in a situation that involves practising division skills.

• There are two **Games**. The first game provides an opportunity to practise multiplying a 3-digit number by a 1-digit number. The second game provides an opportunity to practise division.

• Throughout the unit, it is important to encourage flexibility in computation and to accept a variety of approaches from students. Being efficient and not recording every step should be welcomed and not discouraged.

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Getting Started

Curriculum Outcomes	Outcome relevance
3 2-Digit × 1-Digit Multiplication: concretely, symbolically **3** Multiplication and Division: relationship **4** Multiplication Properties: explore **4** Multiplication Facts: to 9 × 9 **4** Division Meanings: small numbers **4** Division Properties: explore **4** Multiplication and Division Facts: relate through properties	Students will find the work in the unit easier after they review multiplication and division concepts from Class III and from earlier in Class IV

Pacing	Materials	Prerequisites
1 h	• Base ten blocks or Base Ten Blocks (BLM)	• multiplying by 10 • familiarity with multiplication and division facts • understanding division as sharing • multiplying and dividing a 2-digit number by a 1-digit number • writing number sentences for multiplication and division • familiarity with base ten blocks to model numbers • familiarity with the relationship between multiplication and division

Main Points to be Raised

Use What You Know

• You can estimate a product using convenient multiples of 10.

• You can share base ten blocks to represent a division.

• You can write either a multiplication sentence or a division sentence to describe a grouping situation.

Skills You Will Need

• You can multiply by showing equal groups.

• You can estimate a product using convenient multiples of 10.

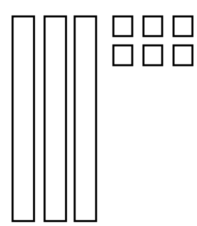
• You can relate any division to a multiplication.

• You can model division as sharing or equal groups.

Use What You Know — Introducing the Unit

• Before assigning the **Use What You Know**, review the use of base ten blocks to model 2-digit numbers.

For example, ask students to model 36 (3 rods and 6 ones). If blocks are unavailable, you can use the Base Ten Blocks (BLM) or students can draw blocks. You can suggest that students sketch the blocks using sticks and dots for tens and ones as shown to the right.



Sketch of 3 rods and 6 ones

• You might also review how to record a multiplication and division sentence to represent a situation.

For example, show 28 ÷ 7 as 28 counters in groups of 7. Ask students what two multiplication sentences it shows (4 × 7 = 28 or 7 × 4 = 28) and what two division sentences it shows (28 ÷ 4 = 7 or 28 ÷ 7 = 4).

• You may choose to review how to divide a 2-digit number by a 1-digit number, but it might be better to let students work on their own and review those skills after you see how they do.

• Students can work in pairs to complete the **Use What You Know** activity.

Observe students as they work. You might ask:

• *How many pages would Dorji read in 3 nights if he reads 10 pages each night?*

• *Why did you divide 48 by 3?*

• *Why might you think of 48 as 30 + 18 to do the division?*

• *Why is the result less when you divide by 4 than when you divide by 3?*
Why is the result greater when you divide 97 by 3 rather than dividing 48 by 3?

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions.

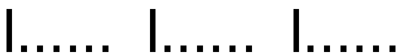
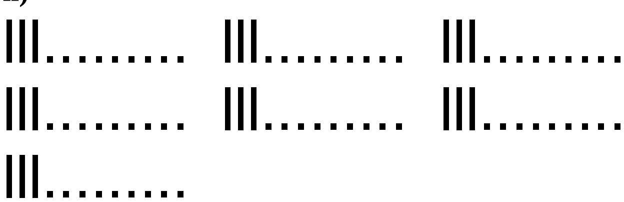
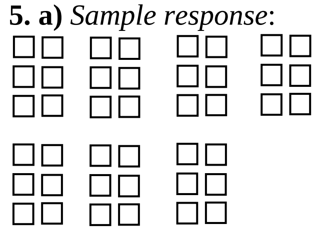
• You may wish to model how to answer **question 4**.

For example, show how you can solve 6 ÷ 2 = [ ] by thinking about the multiplication sentence 3 × 2 = 6.

• Observe to see how comfortable students are with their multiplication and division facts. A solid knowledge of these facts will greatly assist their work in this unit.

• Students can work individually. Students can sketch base ten rods and ones as sticks and dots, if they wish.

Answers

A. i) If he reads 10 pages each night, he reads only 30 pages altogether. **ii)** If he reads 20 pages a night, he reads 60 pages altogether. **iii)** 16 pages **B. i)** *Sample response*: 48 ÷ 3 = 16 **ii)** 	**C. i)** 12 pages **ii)** *Sample response*: 9 pages on 2 nights and 10 pages on 3 nights **iii)** *Sample response*: 32 pages on 2 nights and 33 pages on 1 night **iv)** *Sample response*: 19 pages on 3 nights and 20 pages on 2 nights **D.** *Sample response*: You are dividing Nu 42 among 3 children. How much money does each child get? (Nu 14)
1. a) 0 **b)** 7 **c)** 36 **d)** 30 **e**) 32 **f)** 36 **g)** 0 **h)** 25 **2. a) i)** 78 **ii)** 273 **iii)** 210 **b)** *Sample responses*: **i)**  **ii)**  **iii)**  **3.** A	**4. a)** 5 × [ ] = 50 **b)** 4 × [ ] = 49 **c)** 9 × [ ] = 98 **5. a)** *Sample response*:  [**b)** I showed 42 in 7 groups. There are 6 in each group.] **6. a)** 4 **b)** 8 **c)** 0 **d)** 1 **e)** 9 **f)** 7 **g)** 9 **h)** 8 **7. a)** 8 × 6 = 48 **b)** 48 ÷ 8 = 6

Supporting Students

Struggling students

• If students are having difficulty representing division with base ten blocks, you may wish to re-teach the process of sharing when it involves regrouping or trading.

For example, to show 42 ÷ 3, display 4 tens and 2 ones blocks. Use 3 pieces of paper to represent the shares. Share by placing 1 ten on each paper. Trade the last ten for 10 ones. Combine those 10 ones with the other 2 ones and share them by placing 4 ones on each paper.

Enrichment

• You might ask students to create a set of five division questions where the quotient is about 24 as well as five multiplication questions where the product is about 78.

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Chapter 1 Multiplication

4.1.1 Multiplying by Tens and Hundreds

Curriculum Outcomes	Outcome relevance
4-B11 Multiply Mentally: by 10 or 100 • explore multiplication of 2-digit numbers by 10 and by 100 • develop visual images of whole numbers multiplied by 10 or 100 using base ten materials • read numbers different ways (e.g., 5300 can be read as 53 hundred or as 5 thousand, 3 hundred) **4-C1 Apply Patterns in Computations** • explore and apply patterns to solve computation problems (e.g., multiplying by 9, 11, or 10) **4-C2 Open Sentences and Computation Patterns: multiplication and division** • generate rules about how a change in one factor affects the result (e.g., for $\square \times 10$, as $\square$ increases by 1 the product increases by 10) **4-C3 Multiplying by 10, by 100, by 1000: apply pattern visually and symbolically** • identify and continue patterns with increasing powers of ten	In order to multiply 2-digit and 3-digit numbers, it is necessary that students be able to multiply by tens and hundreds. For example, to multiply 6×23, students need to know how to calculate 6×20.

Pacing	Materials	Prerequisites
1 h	• Base ten blocks or Base Ten Blocks (BLM) • Whole Number Place Value Charts (BLM)	• familiarity with place value for 3-digit numbers • familiarity with multiplication representing a set of equal groups

Main Points to be Raised

• When you multiply by 10, the digits move one place to the left and there are always 0 ones. This pattern makes it easy to perform these computations using mental math.

• When you multiply by 100, the digits move two places to the left and there are always 0 tens and 0 ones. This pattern makes it easy to perform these computations using mental math.

• When you multiply a 1-digit number by a multiple of ten or by a multiple of hundred, you can use place value to rename the multiplication and then use what you know about multiplying by 10 or by 100.

For example:

$4 \times 30 = 4 \times 3$ tens $= 12$ tens $= 12 \times 10 = 120$.

$4 \times 300 = 4 \times 3$ hundreds $= 12$ hundreds $= 12 \times 100 = 1200$.

Try This — Introducing the Lesson

A. Assign the question to pairs of students. While you observe students at work, you might ask questions such as:

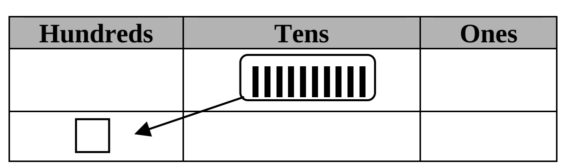
• *How do you know she will need more red beads than blue beads?* (There are more red beads on each bracelet.)

• *Why might you count by 100s?* (There are 100 red beads on each bracelet, so I say, “100, 200, 300, 400, 500”.)

• *How can you figure out the number of blue beads she needs?* (100 beads for the first two bracelets, 100 beads for the next two, and then another 50.)

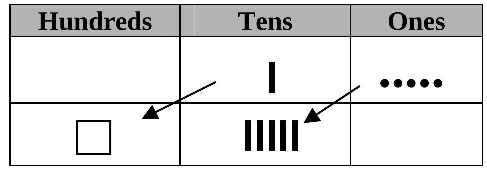
The Exposition — Presenting the Main Ideas

• Remind students why $2 \times 10 = 20$ by modelling it with 2 tens blocks ($2 \times 10 = 20$ because 2×10 is 2 tens). Ask how much 3×10 and 4×10 are (30, or 3 tens and 40, or 4 tens). Then ask how much 10×10 is. Students might say 10 tens or 100. Model how 10 tens = 1 hundred = 100 using a place value chart.



$10 \times 10 = 10$ tens $= 1$ hundred $= 100$

• Now ask students how much 15 × 10 (15 tens) is.
Place 1 tens block and 5 ones blocks on a place value chart. Discuss how multiplying by 10 results in trading a 1 for a 10, a 10 for 100, or a 100 for 1000 by referring to the type of grouping shown previously.



15 × 10 = 1 hundred, 5 tens = 150

Using a place value chart, have students notice that when 15 is multiplied by 10, the digits 1 and 5 each move one place to the left on the chart. The digit 0 is used to indicate that there are no ones (since there are only tens or groups of tens).

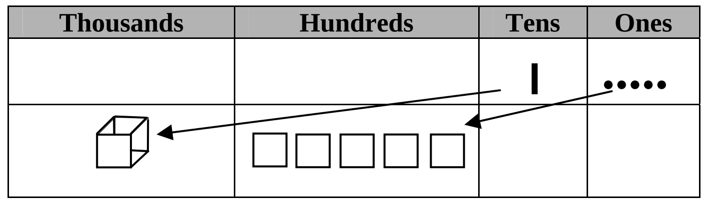
Hundreds	Tens	Ones
	1	5
1	5	0

15 × 10 = 150

• Repeat this with 100s.

Show 2 × 100 = 200 (because 2 × 100 is 2 hundreds = 200), 3 × 100 = 300 (because 3 × 100 is 3 hundreds = 300), that 4 × 100 = 400 (because 4 × 100 is 4 hundreds = 400), and so on.

Then show why 15 × 100 = 1500 because 1 ten and 5 ones become 1 thousand and 5 hundreds since 100 tens = 10 hundreds, or 1000 and 5 hundreds = 500.
Again, note that the digits of 15 moved over, but this time they moved over two places. We use zeros to show that there are no tens and no ones, since there are only hundreds or groups of 100.



15 × 100 = 1 thousand, 5 hundreds = 1500

Thousands	Hundreds	Tens	Ones
		1	5
1	5	0	0

15 × 100 = 1500

• Now ask students to consider what 2 × 30 might be. Allow students to make suggestions to say they know the result is 60. Show that 2 × 30 can be written as 2 × 3 tens, which 6 tens, or 6 × 10 = 60. Note that students can also solve these multiplications mentally by using place value language. For example:

2 × 30 = 2 × 3 tens = 6 tens, and 6 tens = 60.

• Similarly, ask what 8 × 300 might be. Allow students to suggest responses. Make sure students see that 8 × 300 can be written as 8 × 3 hundreds, which is 24 hundreds, or 8 × 300 = 2400. Note that students can also solve these multiplications mentally by using place value language. For example:

8 × 300 = 8 × 3 hundreds = 24 hundreds = 2400.

• Work through the exposition on **pages 95 to 97** of the student text with students to review these ideas.

Revisiting the Try This

B. This question allows students to make a formal connection between their informal methods and using place value to multiply by 10s and 100s to solve **part A**.

Using the Examples

• Ask students to work in pairs. One student in the pair should become an expert on **example 1**. The other should become an expert on **example 2**. Each student can then teach his or her partner the example he or she has studied.

• Provide an opportunity for students to ask for help from you as they study their examples.

• Make sure that students are comfortable with the ideas in the examples by checking their ability to solve these two problems:

- *There are 16 classes with 40 students in each class. How many students are there altogether?*
- *What number is missing in this number sentence?* 5400 = 900 × [ ]

Practising and Applying

Teaching points and tips

Q 1: Encourage students to use place value charts if they find them helpful. Also encourage students to rewrite the multiples of 10 and 100 in words, e.g., 40 as 4 tens or 500 as 5 hundreds.

Q 3: For **parts e) and f)**, encourage students to see that they can multiply three numbers in any order. They might also find it helpful to write 750 as 75 tens and 1200 as 12 hundreds.

Q 4: This question is designed to stretch students' thinking. They can think of the question as 200 × 2 tens, or 400 tens, or as 20 × 2 hundreds, or 40 hundreds.

Q 5: There are solutions to these questions that do not involve multiples of 10 or 100, but you should encourage students to search for these simpler values.

Q 6: Observe whether students realize how the answers to the various parts of the question are related.

For example, observe whether students realize that the answer to **part b)** must be twice the answer to **part a)** and the answer to **part d)** must be four times the answer to **part b)**.

Q 7: For **parts d) and e)**, students might draw pictures like those in **example 2**.

Q 9: You may ask students to share their answers with a partner for this question.

Common errors

• Sometimes teachers tell students to add one or two zeros when multiplying by 10 or by 100. Some students interpret this by literally adding 0, which does not change a number, e.g., for 3 × 100 they write 3 + 0 + 0 = 3. Be careful of the language you use in describing the process for multiplying by 10 or by 100.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can multiply by multiples of 10 or 100 using mental math
Question 4	to see if students can solve a simple problem involving multiplication by tens or hundreds
Question 7	to see if students can communicate about relationships involving multiplication by tens or hundreds

Answers

A. i) 500 red beads
ii) 250 blue beads

B. *Sample response*:
To find the number of red beads, I multiply 5 × 100. I can do that mentally by multiplying 5 × 1 hundred = 5 hundreds = 500.
To find the number of blue beads, I multiply 5 × 50. I can do that mentally by multiplying 5 by 5 and then by 10 because 5 × 50 = 5 × 5 × 10.

1. a) 70 **b)** 230 **c)** 800 **d)** 5300
e) 200 **f)** 360 **g)** 2800 **h)** 8000

2. 20 trips

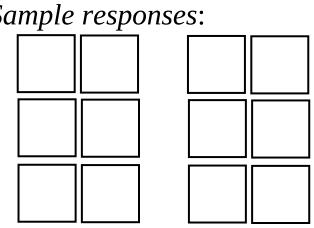
3. a) 16 **b)** 15 **c)** 12
d) 4 **e)** 25 **f)** 100

4. 4000 times

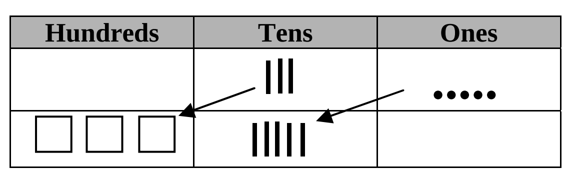
5. *Sample responses*:
a) 8 × 100 or 4 × 200 **b)** 10 × 56 or 20 × 28

6. a) 400 times **b)** 800 times
c) 1200 times **d)** 3200 times

7. *Sample responses*:
a)

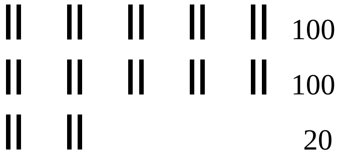


[2 groups of 600 is 12 hundreds = 1200.]
b)



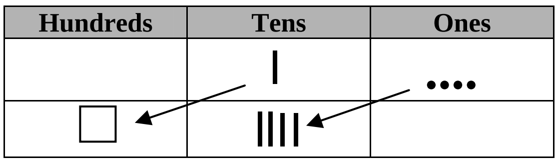
[7 × 50 = 7 × 5 tens = 35 tens; 35 × 10 = 350.]

c)

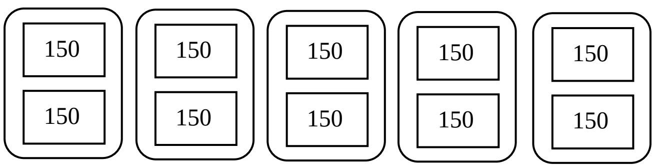


[12 groups of 20 is 100 + 100 + 40 = 240.]

d) $7 \times 20 = 7 \times 2$ tens
$= 14$ tens
$= 14 \times 10$



e)



[10 groups of 150 make 5 groups of 300 if the groups of 150 are paired up.]

[**8.** *Sample response*:
If there are 51 hundreds, all the blocks are hundreds blocks, so there are 0 tens and 0 ones blocks.]

[**9.** *Sample response*:
For 8×30, I would tell my friend to think of 30 as 3 tens. That makes 8 groups of 3 tens, which is 24 tens = 240.
For 7×300, I would tell my friend to think of 300 as 3 hundreds. That makes 7 groups of 3 hundreds, which is 21 hundreds = 2100.]

Supporting Students

Struggling students

• Some students may be able to multiply by tens or hundreds but have difficulty explaining their thinking. Encourage these students to use base ten blocks and/or place value charts to model what the multiplication means. This should help them come up with words they can use for their explanations.

Enrichment

• You might ask students to create a variety of word problems that require multiplication by tens or hundreds for other students to solve.

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4.1.2 Estimating Products

Curriculum Outcomes	Outcome relevance
4-B6 3-Digit by 1-Digit Multiplication: with and without regrouping • use estimation to predict and verify multiplications	In order to make sure that their products are reasonable, it is important that students be able to estimate using what they know about multiplying by tens and hundreds.

Pacing	Materials	Prerequisites
1 h	None	• multiplying by tens and hundreds • comparing 3-digit numbers

Main Points to be Raised

• To estimate a product, you can multiply using nearby tens or hundreds since they are easy to multiply using mental math.

• There is more than one way to estimate a product.

For example, you might estimate 7 × 462 as 7 × 500, as 8 × 400, or as 10 × 400.

• You can sometimes tell if an estimate is high or low (if you increase only one number or both numbers or if you decrease only one number or both numbers), but sometimes you might not be sure (if you increase one number and decrease the other).

• Sometimes an estimate is all that is required to solve a problem.

Try This — Introducing the Lesson

A. Assign the question to pairs of students. While you observe students at work, you might ask questions such as

• *Why do they not need 9 × 242 sticks?* (Each person does not need 9 sticks because they play in pairs.)

• *About how many pairs of students will there be?* (About 120 pairs) *How do you know?* (242 is about 24 tens and half of that is the number of pairs. Half of 24 tens is 12 tens, which is 120.)

• *How did you estimate the number of sticks?* (9 is close to 10, so I multiplied 120 by 10 to get 1200. I knew that was high, so I estimated it to be about 1100 sticks.)

The Exposition — Presenting the Main Ideas

• Ask students to consider why 38 + 9 is close to 40 + 10. Then ask why 38 × 9 is close to 40 × 10. Ask how an estimate like this might help them predict whether or not a calculated answer is reasonable.

• Present the calculation 6 × 532. Ask students how they might estimate the product using easier numbers. Make sure they realize that there are many possibilities.

For example: 6 × 532 is about 6 × 500, or 5 × 600, or 6 × 530, or 10 × 400, and so on.

Ask how they know that 6 × 500 and 6 × 530 will be low estimates (532 is rounded down but 6 stays the same), but why they are not sure whether 5 × 600 and 10 × 400 are high or low (one number is rounded down and the other number is rounded up in each case).

Discuss why these calculations are easier to do using mental math than the original 6 × 532.

• Discuss situations where an estimate might be appropriate, i.e., if you do not need an exact answer.

• Check student understanding by asking them how they might estimate the amount of money they would need to buy 8 items that each cost Nu 160.

• Let students know that they can refer to **page 100** in the student text for a review of these ideas.

Revisiting the Try This

B. Encourage students to think of many different ways to estimate to solve the problem in **part A**. Show how both the 9 and the 242 ÷ 2 can be estimated.

Using the Examples

• Present the problem in the **example** for students to try. Once they have completed the problem, have them read through the example to see whether their solution was closer to **solution 1** or to **solution 2**, or whether they used a different approach.

Practising and Applying

Teaching points and tips

Q 1: Make sure students realize they can increase one or both numbers.

Q 2: Make sure students realize they can decrease one or both numbers.

Q 3: There is no best estimate. Which estimate students prefer is personal and depends on the situation.

Q 4: For **part a)**, students round 421, not 7; they need to realize that 421 is between 400 and 500. For **part b)**, students round 9 and not 627. For **part c)**, they might round to nearest ten instead of the nearest hundred, since $8 \times 400 > 3000$.

Q 5: Encourage students who are struggling first to think of an easy number pair that multiples to 4200, such as 10×420. They can then estimate either or both values.

Q 6: You may need to point out to students that problems involving estimation often use the word "about", e.g., About how many...?

Q 7: This question might be handled best in a class discussion.

Common errors

• Sometimes students do not realize that if one number is increased and the other number is decreased, you can sometimes, but not always, decide whether or not the estimate is far from the actual product.

For example, if you estimate 7×322 as 10×300, you can be fairly certain your estimate is far from the product. In fact, it is about 3×300, or 900, away from the actual value. But if you estimate 7×322 as 8×300, you are less sure how far you are from the actual value.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can estimate in more than one way and evaluate their estimates
Question 4	to see if students can use reasoning to relate a product to a given estimate
Question 6	to see if students recognize when estimation is useful

Answers

A. *Sample response*: About 900 sticks	**B.** *Sample response*: 242 is about 200 and 200 ÷ 2 = 100, so there are about 100 pairs. 100 × 9 = 900 242 is about 250 and 250 ÷ 2 = 125, so there are about 125 pairs. 125 × 10 = 1250
1. *Sample responses*: **a)** About 1600; [4 × 400 = 1600] **b)** About 3600; [9 × 400 = 3600] **c)** About 2500; [10 × 250 = 2500] **d)** About 2500; [5 × 500 = 2500] **2.** *Sample responses*: **a)** About 4000; [8 × 500 = 4000] **b)** About 1400; [7 × 200 = 1400]	**2. c)** About 4500; [5 × 900 = 4500] **d)** About 900; [9 × 100 = 900] **3.** *Sample responses*: **a)** 5 × 600 = 3000 6 × 500 = 3000 [The estimates are the same, so they are equally good.]

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3. b) 9 × 700 = 6300 10 × 650 = 6500 [6300 is better than 6500 because I changed only one number and not by much.] **4.** *Sample responses*: **a)** 421 is between 400 and 500. 7 × 400 = 2800 and 7 × 500 = 3500, so 7 × 421 is between 2800 and 3500. **b)** 9 < 10, so 9 × 627 < 10 × 627 = 6270. **c)** 8 × 300 = 2400 and 8 × 400 = 3200. 352 is about halfway between 300 and 400, so 8 × 352 is about halfway between 2400 and 3200. The halfway mark is less than 3000.	**5.** *Sample response*: 42 × 100 42 × 102 42 × 104 **6.** *Sample response*: There are 6 schools, each with about 525 students. About how many students are there in total? [**7.** *Sample response*: If I estimate a product, I can check my calculation to see if I have made a mistake.]

Supporting Students

Struggling students

• Some students may be able to estimate but have difficulty deciding whether an estimate is high or low. You might allow these students to estimate without making the decision about whether the estimate is high or low.

• For **question 4**, you may wish to point out that 2800 = 7 × 400 and 3500 = 7 × 500 to start students off.

Enrichment

• Some students might make up other questions like those in **question 4** for partners to solve.

For example, they might use statements such as:

- 8 × 512 is between 4000 and 4800.
- 8 × 512 is more than 4000.
- 8 × 512 is less than 5120.

4.1.3 Multiplying Using Rectangles

Curriculum Outcomes	Outcome relevance
4-B6 3-Digit by 1-Digit Multiplication: with and without regrouping • develop alternative and standard algorithms (from understanding) • connect concrete models to symbolic recordings (e.g., relate rectangle area models to written algorithm) • use estimation to predict and verify multiplications	Using the rectangle model allows students to make sense of multiplication procedures. It also supports later work with multiplication in algebraic situations.

Pacing	Materials	Prerequisites
1 h	• Base ten blocks or Base Ten Blocks (BLM)	• understanding that multiplication is about equal groups • multiplying by tens • knowing that the area of a rectangle is length × width

Main Points to be Raised

• You can model multiplication as a rectangle because the area of a rectangle is the product of its dimensions.

• When you create a rectangle to multiply, it is useful to separate the parts of a 2-digit number into the tens part and ones part or the parts of a 3-digit number into the hundreds, tens, and ones parts. You can then multiply each part separately using what you know about multiplying by ones, tens, and hundreds.

Try This — Introducing the Lesson

A. Assign the question to pairs of students. While you observe students at work, you might ask questions such as

• *How do you know that there are more than 80 tiles?* (8 rows of 10 is 80 tiles and there are 8 rows of 12.)

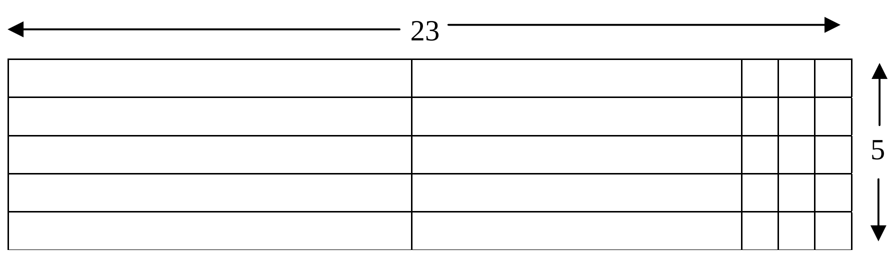
• *How do you know there are less than 120 tiles?* (10 rows of 12 is 120 tiles and there are only 8 rows of 12.)

• *How do you know 8 rows of 12 is equal to 4 rows of 24 tiles?* (If I grouped tiles by 2 rows of 12 at a time, there would be 4 groups of 24 tiles.)

• *How did you figure out how many tiles there were?* (I drew a picture. I divided the floor into 8 rows of 6 tiles and another 8 rows of 6. I know that 8 × 6 is 48, so I added 48 + 48 to get 96.)

The Exposition — Presenting the Main Ideas

• Use base ten blocks to create a rectangle that is 5 by 23. Have students recreate this array with their own base ten blocks. Then have students consider the part of the rectangle made up of tens blocks separately from the part made up of ones blocks.



Lead students to see that breaking the array into these parts shows how you can calculate 5 × 23 by calculating 5 × 20 + 5 × 3.

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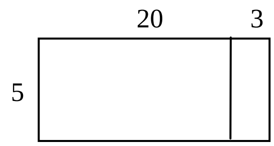
• Help students see how the rectangle at the bottom of **page 102** of the student text represents the base ten block rectangle shown in the middle of the page. Talk about how the total value of the blocks in the rectangle represents the product of its dimensions, which are 5 by 23.

Show how to create a rectangle model like this:

Sketch a rectangle.
Label its dimensions.

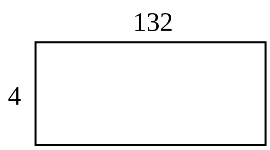
Break up the rectangle into parts.
Label the dimensions of each part.



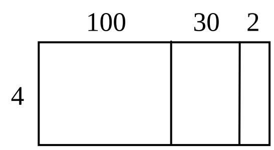
Talk about how this shows that 5 × 23 = 5 × 20 + 5 × 3. Discuss how students can calculate the product using what they already know about multiplying 1-digit numbers and multiplying by tens to multiply each part of the rectangle.

• Ask students how they might multiply 4 × 132 using an array. Lead them to see that they could draw a picture like the picture at the top of **page 103** by sketching a rectangle, labelling the dimensions and then breaking up the 3-digit dimension into its parts. Notice that using a rectangle model to model multiplication does not allow students to use hundreds blocks since they cannot use the blocks to form a rectangle with equal rows. Instead students must use 10 tens blocks to show 100 as the linear dimension, as shown on **page 103**.

Sketch a rectangle.
Label its dimensions.



Break up the rectangle into parts.
Label the dimensions of each part.



Discuss how this is convenient since they can use mental math to find the areas of each part.

Discuss how to record the partial products and add to find the total product.

Revisiting the Try This

B. Some students may have solved **part A** using a rectangular array and breaking it into parts, but for those who did not, this question provides an opportunity to see the usefulness of an array model for multiplying.

Using the Examples

• Present the problems in the **examples** for students to try. Once they have completed the problems, have them read through the examples to make sure they used an appropriate method.

Practising and Applying

Teaching points and tips

Q 1: Although students can break up the rectangles in any way they wish, encourage them to separate the ones, tens, and hundreds to make the mental calculation easier.

Q 3: Many students will benefit from drawing sketches.

Q 5: Students might benefit from drawing sketches to support their thinking.

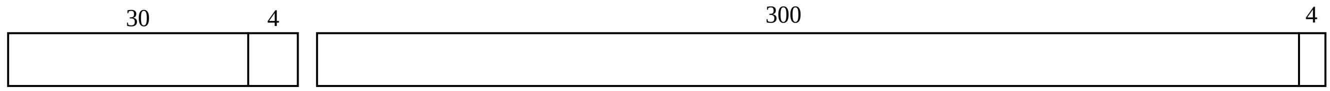
Q 6: Students can add pairs of products separately to compare vegetables and flowers. Or, they might estimate to determine an answer.

Common errors

• Sometimes students divide up the arrays in ways that do not really help them simplify the calculations. Help those students see the benefit of dividing up the arrays into hundreds, tens, and ones.

• If they are multiplying by a number like 304, sometimes students forget that the 3 is hundreds and not tens. Encourage them first to write out the number in expanded form, before drawing the rectangles.

• Although it is not important that rectangles be drawn to an exact scale, it may be useful to use a rough scale. For example, the rectangles for 5 × 34 versus 5 × 304 may vary as much as shown below.



Suggested assessment questions from Practising and Applying

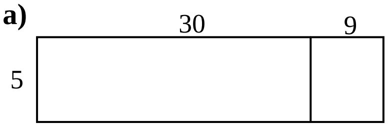
Question 1	to see if students can draw an array to match a product
Question 3	to see if students can solve problems involving multiplication by 2-digit and 3-digit numbers
Question 5	to see if students can multiply 2-digit and 3-digit numbers using a strategy of their own choice

Answers

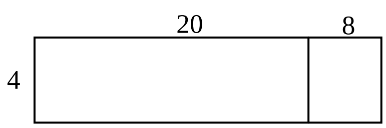
A. 96 tiles

B. *Sample response*:
I can make an array that is 8 by 12 and then break it up into two parts: 8 × 10 and 8 × 2.
Then I can add the parts:
8 × 12 = 8 × 10 + 8 × 2 = 80 + 16 = 96.

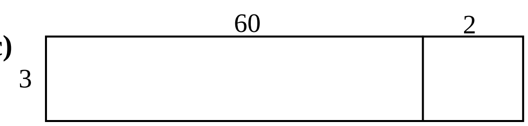
1. a)



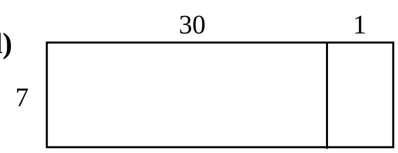
b)



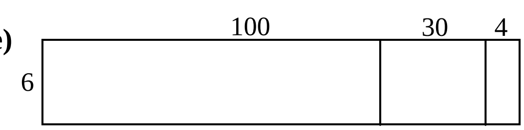
c)



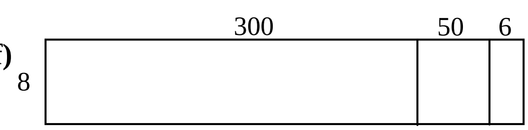
d)



e)



f)



2. a) 318; [300 + 18 = 318]
b) 185; [150 + 35 = 185]
c) 1704; [1600 + 80 + 24 = 1704]

3. a) 140 **b)** 108 **c)** 942 **d)** 528

4. a) 160; 20
184

b) 360; 9
72; 9
432

c) 560
49; 7
609

d) 1500; 300
300
1810

5. a) 111 **b)** 464 **c)** 465
d) 336 **e)** 3078 **f)** 1264

6. There are more flowers;
[8 × 38 = 8 × 30 + 8 × 8 = 240 + 64 = 304.
3 × 46 = 3 × 40 + 3 × 6 = 120 + 8 = 138.
There are 304 + 138 = 442 vegetables.
4 × 26 = 4 × 20 + 4 × 6 = 80 + 24 = 104.
7 × 57 = 7 × 50 + 7 × 7 = 350 + 49 = 399.
There are 104 + 399 = 503 flowers.
503 > 442.]

[**7.** *Sample response*: I can multiply the parts mentally and then add the parts mentally.]

Supporting Students

Struggling students

• Some students may struggle with **question 4**.

For example, for **part a)**, they might write a 2 rather than a 20 in the blank space on the right. Help students by reading the first question orally with them, "Eight twenty-threes is eight ___ plus eight threes". Help them see that it is 8 twenties, and not 3 twos that are missing.

• For **question 6**, you may need to help students with the steps they must go through. They must consider the sum of the two rows of vegetables and compare it to the sum of the two rows of flowers.

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4.1.4 Multiplying a 3-digit Number by a 1-digit Number

Curriculum Outcomes	Outcome relevance
4-B6 3-Digit by 1-Digit Multiplication: with and without regrouping • develop alternative and standard algorithms (from understanding) • connect concrete models to symbolic recordings (e.g., relate rectangle area models to written algorithm) • use estimation to predict and verify multiplications	Multiplying a 3-digit number by a 1-digit number is an important life skill. Students should see alternative ways to perform these calculations so that they can find the way(s) that make most sense to them. Visualizing with base ten blocks helps students make sense of the traditional algorithm.

Pacing	Materials	Prerequisites
2 h	• Base ten blocks or Base Ten Blocks (BLM)	• understanding that multiplication is about equal groups • multiplying by tens • knowing that the area of a rectangle is length × width

Main Points to be Raised

• You can think of multiplication as calculating the total in a number of equal groups.

• By modelling equal groups of hundreds, tens, and ones on a place value chart, you can see why you can find the product of a 1-digit number and a 3-digit number by multiplying the hundreds, tens, and ones separately by the number of groups and then adding the partial products.

• You can start multiplying at the right or at the left.

- You can start at the right or at the left, calculating and recording each partial product, and then adding them at the end.

- If you start at the right, you can regroup the ones as tens, the tens as hundreds, and the hundreds as thousands as you move to the left.

Try This — Introducing the Lesson

A. Assign the question to pairs of students. While you observe students at work, you might ask questions such as

• *How do you know the principal will need more than 600 pieces of paper?* (If there were only 200 copies to make, she would need 600 pieces of paper, but there are more copies than that.)

• *How do you know she will need less than 900 pieces of paper?* (If there were 300 copies to make, she would need 900 pieces of paper, but she needs fewer copies than that.)

• *How might you estimate the number of pieces of paper?* (It is halfway between 600 and 900, so it is about 750.)

• *How do you know it is more than 750?* (253 is more than halfway between 200 and 300, but not a lot more.)

The Exposition — Presenting the Main Ideas

• Use base ten blocks or draw a picture of base ten blocks to model the number 132 on a place value mat. Then ask, *How much is 4 × 132?* Have students describe what the model should look like. Then draw 4 groups of 132.

• Students should realize that when they multiply 4 × 132 they are putting together 4 groups of 100 (4 × 100), 4 groups of 30 (4 × 30), and 4 groups of 2 (4 × 2). You might show how to think of this 4 × 132 as 4 × [1 hundred + 3 tens + 2 ones]. Have students work through that calculation with you on the board (as it is shown on **page 106** of the student text).

• The difference between this method and the method shown on **page 107**, which also starts from the right, is that in the method on **page 106** the regrouping is not done as you go and the partial products are added at the end.

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• Model the multiplication shown on **page 107** to show how to do the regrouping and record it.

• Check student understanding by having them try multiplying 6 × 382 the way they learned in the previous lesson as well as in the two new ways they learned in this lesson.

Multiplying in parts:

From the left		From the right:
382		382
× 6		× 6
1800	OR	12
480		480
+ 12		+ 1800
2292		2292

Starting on the right and regrouping as you go:

$$\begin{array}{r} {}^{4\,1} \\ 382 \\ \times\ \ \underline{\ \ 6} \\ 2292 \end{array}$$

Revisiting the Try This

B. Students can choose to write the calculation from **part A** multiplying either from the left or from the right. Allow both choices. You might also encourage students to show the base ten block model for the calculation.

Using the Examples

• Work through **example 1** with students. Make sure they understand why the missing digit had to be 3 or 8 by going through the multiplication facts for 8 with them and noticing that the only facts that have a 4 in the ones digit (as required since the product is 2304) are 8 × 3 and 8 × 8.

• Ask students to close their texts. Pose the problem in **example 2** and have students try it. Then they can check their work against the solutions in the text. Make sure students see that the student who used **solution 1** chose to multiply in parts, regrouping at the end, while the student who used **solution 2** chose to regroup as she went. Reinforce that either approach is correct and acceptable. In fact, students could also multiply from the left, as they learned in the previous lesson.

Practising and Applying

Teaching points and tips

Q 2: Students need to count the number of equal groups and the size of each of those groups.

Q 3: Some students might need to be told-that in each case the calculations were begun from the right.

Q 4: Students should use their estimation skills to realize that the product **A** must be about 1500, **B** must be more than 5400, and **C** must be about 1800. Estimating for **D** might be harder, so students might actually calculate that one.

Q 5: Students need to realize that the notion of "3 times as much" means multiplying by 3. The same concept is used in **question 8.**

Q 6: Some students will calculate all four products to find the answer for **part a)**. Others will estimate to choose which to calculate. Since **B** and **D** are about 3000 apart, these seem like good candidates to try.

Q 7: Some students might model problems like those in **questions 5 and 8**.

Q 9: Students might use an estimation strategy for **part c)**, but they will need a different strategy for **part a)**. For example, they might create a simpler problem. They might reason that 9 × 300 = 2700, leaving 9 × [ ]5 = 2[ ]5. They can then try different digits: 9 × 15 = 135, and then 9 × 25 = 225.

Q 10: Many students will realize that they might try 8 × 642 or 6 × 842 since both products are greater than 4800. They should recognize that the first product is 8 × 42 greater than 4800 and the second product is only 6 × 42 greater than 4800.

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Common errors

• When students multiply from the right and have to regroup twice (ones as tens and tens as hundreds), they sometimes use the wrong value for the second regrouping.

For example, they reuse the 3 (from the regrouping of 30 ones as 3 tens) when multiplying the 4 by 500. Encourage them to cross out a regrouped number once it has been used.

$$\begin{array}{r} 13 \\ 538 \\ \times\ \underline{\ \ 4} \\ 52 \end{array} \qquad \begin{array}{r} 1\cancel{3} \\ 538 \\ \times\ \underline{\ \ 4} \\ 52 \end{array}$$

Suggested assessment questions from Practising and Applying

Question 2	to see if students can relate a visual model for multiplication to a symbolic expression
Question 3	to see if students can add partial products to create a total product
Question 7	to see if students can relate a product to a situation
Question 10	to see if students can solve a mathematical problem involving multiplication

Answers

A. 759 pages	**B.** *Sample response*: $\begin{array}{r} 253 \\ \underline{\times\ 3} \\ 600 \\ 150 \\ \underline{+\ 9} \\ 759 \end{array}$
1. a) 2436 **b)** 1820 **2. a)** 5 × 228 = 1140 **b)** 4 × 318 = 1272 **3. a)** $\begin{array}{r} 300 + 20 + 8 \\ \underline{\times\ 6} \\ \underline{\mathbf{48}} \\ 120 \\ \underline{+\ 1800} \\ \mathbf{1968} \end{array}$ **b)** $\begin{array}{r} 400 + 30 + 9 \\ \underline{\times\ \ 9} \\ \underline{\mathbf{81}} \\ 270 \\ \underline{+\ \mathbf{36}00} \\ \mathbf{3951} \end{array}$	**4. A.** 1476 **C.** 1794 **D.** 2720 **5.** Nu 720 **6. a)** B and D **b)** B **7.** *Sample response*: There were 382 students. Each had 7 pebbles. How many pebbles were there altogether? (2674 pebbles) **8.** 524 people **9. a)** 2 **b)** 4 **c)** 3 **10.** 8 × 642 [**11.** *Sample response*: For both, I multiply the hundreds, tens, and ones separately and then I add the products.]

Supporting Students

Struggling students

• Some students find one algorithm (starting from the left or from the right) more accessible than the other. Allow them to use the algorithm that makes more sense to them.

• Some students will struggle when there is multiple regrouping, especially if they multiply from the right and regroup as they go. Encourage these students to use the other method, writing down the three partial products and then adding.

• If students have difficulty creating a problem for **question 7**, encourage them to base their problem on one of the given problems on the page.

Enrichment

• Some students might enjoy creating other puzzles like those in **question 9** for fellow students to complete.

GAME: Lots of Tens

This game allows students to practise multiplication. Since it is the tens digit (and not the overall number) that must be greatest, students cannot estimate. They must calculate.

For example, using 3, 1, 5, and 4, a student might realize that 4×315 is better than 4×351 since there is a 6 in the tens place for 4×315, but a 0 in the tens place for 4×351. Students must consider the product of the multiplier and the tens digit as well as any regrouping from the ones place.

Reprint 2023

4.1.5 EXPLORE: Multiplication Patterns

Curriculum Outcomes	Outcome Relevance
4-C1 Apply Patterns in Computations • explore and apply patterns to solve computation problems (e.g., multiplying by 9 or 10) **4-C2 Open Sentences and Computation Patterns: multiplication and division** • generate rules about how a change in one factor affects the result (e.g., for □ × 10, as □ increases by 1 the product increases by 10)	This essential exploration helps students recognize that it is helpful to use patterns in mathematical situations. In particular, by thinking about what multiplication means, they can see how changing one factor affects a product.

Pacing	Materials	Prerequisites
40 min	None	• multiplying a 1-digit number by a 3-digit number

Main Points to be Raised

• When one factor increases by 10, the product increases by the product of the multiplier and 10.

• When one factor increases by 100, the product increases by the product of the multiplier and 100.

• Some computational patterns break down when regrouping is required.

For example, the pattern when multiplying numbers by 111 breaks down once the multiplier is greater than 9.

Exploration

Students can work on the exploration in pairs. Observe while students work. You might ask questions such as

• *Which digits changed? Why?* (When the 3-digit number increased by 10, the ones digit did not change; it was always 0. The tens digit went down by 1 and the hundreds digit went up by 1. This is because I am adding 9 more tens, or 90. Adding 90 is like adding 100 and subtracting 10.)

• *By how much did the digits increase or decrease?* (The thousands digit increased by 1 whether I was multiplying by 8 or by 9, but the hundreds digit went down by 2 when I multiplied by 8 and only by 1 when I multiplied by 9. That is because 8 is 2 less than 10, but 9 is only 1 less than 10. When I added 800, I had 1 more thousand but 2 fewer hundreds; when I added 900, I had 1 more thousand but 1 fewer hundred.)

• *Why is it useful to be able to add mentally to understand the pattern?* (If I add 800 by adding 1 thousand and subtracting 2 hundreds, then I understand why the thousands digit goes up by 1 and the hundreds digit goes down by 2.)

• *Why do you think the pattern will stop?* (Once I get to 10 × 111, I do not just add 1 to each digit since I have to regroup.)

Observe and Assess

As students work, notice the following:

• Do students calculate correctly?

• Do students use the meaning of multiplication to see how products are related and predict new products based on previously-calculated products?

• Can students explain clearly what the patterns are?

• Can students explain clearly why the patterns occur?

• Do students recognize when or why a pattern might break down?

Share and Reflect

After students have had enough time to do the exploration, you may have a class discussion with these questions:

• *Why is it useful to relate a new product to a product you have already figured out?*

• *Why do the products increase more slowly when you multiply by 8 than when you multiply by 9?*

• *How does understanding regrouping help you explain multiplication patterns?*

• *Why is it useful to use mental math to explain the patterns?*

Answers

A.

i)	ii)	iii)
i) 9 × 120 = 1080 9 × 220 = 1980 9 × 320 = 2880 *Sample response*: The values go up by 900 each time. There are 9 more groups of 100 each time.	**ii)** 9 × 120 = 1080 9 × 130 = 1170 9 × 140 = 1260 *Sample response*: The tens digit goes down by 1 and the hundreds digit goes up by 1 each time. There are 9 more groups of 10, or 90, each time. When I add 90, I can add 100 and subtract 10. So, the hundreds digit goes up by 1 and the tens digit goes down by 1.	**iii)** 9 × 120 = 1080 9 × 230 = 2070 9 × 340 = 3060 *Sample response*: The thousands digit goes up by 1 and the tens digit goes down by 1 each time. There are 9 more groups of 1000 and 9 more groups of 10 each time. When I add 990, I can add 1000 and subtract 10, so the thousands digit goes up by 1 and the tens digit goes down by 1.

B.

i)	ii)	iii)
i) 8 × 120 = 960 8 × 220 = 1760 8 × 320 = 2560 *Sample response*: The values go up by 800 each time. There are 8 more groups of 100 each time.	**ii)** 8 × 120 = 960 8 × 130 = 1040 8 × 140 = 1120 *Sample response*: I am adding 8 × 10, or 80. Since 80 = 100 – 20, the tens digit goes down by 2 and the hundreds digit goes up by 1.	**iii)** 8 × 120 = 960 8 × 230 = 1840 8 × 340 = 2720 *Sample response*: I am adding 8 × 110, or 880. 880 = 1000 – 100 – 20, so the thousands digit goes up by 1, the hundreds digit goes down by 1, and the tens digit goes down by 2.

C.	D.
C. *Sample response*: For **parts i)**, for × 8 they went up by 800 but for × 9 they went up by 900. That makes sense since × 8 is 8 more 100s and × 9 is 9 more 100s. For **parts ii)**, for × 8 they went up by 80 but for × 9 they went up by 90. That makes sense since × 8 is 8 more 10s and × 9 is 9 more 10s. For **parts iii)**, for × 8, three digits changed each time but for × 9, only two digits changed. That makes sense because adding 880 each time is the same as adding 1000 and subtracting 120 (880 = 1000 – 100 – 20). So the thousands, hundreds, and tens digits all change. But since 990 = 1000 – 10, only the thousands and tens digits change.	**D.** 3 × 111 = 333 4 × 111 = 444 5 × 111 = 555 *Sample response*: The same digit repeats for each product and the digits go up by 1 each time. No. When I get to 10 × 111, there will be a 0 at the end and so the same digit cannot repeat.

Supporting Students

Struggling students

• If students have difficulty observing the patterns, draw their attention to individual digits (hundreds, tens, or ones). If they are successful at observing patterns but not at explaining them, partner them with other students so they can work together to come up with explanations.

Enrichment

• Students might explore other multiplication patterns, for example, multiplying 101 or 201 by 1-digit numbers.

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Chapter 2 Division

4.2.1 Dividing Tens and Hundreds

Curriculum Outcomes	Outcome relevance
4-B10 2, 3-Digit by 1-Digit Division: with and without regrouping • develop alternative and standard algorithms (from understanding) • connect concrete models to symbolic recordings • continue estimating	Division is an important skill in everyday life. Students should develop a variety of strategies to perform these calculations meaningfully.

Pacing	Materials	Prerequisites
1 h	• Base ten blocks or Base Ten Blocks (BLM)	• familiarity with division facts • multiplying tens and hundreds by 1-digit numbers

Main Points to be Raised

• You can use place value ideas to divide tens and hundreds by 1-digit numbers.

For example, 150 ÷ 5 = 15 tens ÷ 5 = 3 tens, or 30.

• You can think of a related multiplication of tens or hundreds to check a division of tens and hundreds.

For example, if you divide 3600 ÷ 4 = 900, you can check by multiplying 4 × 900 to get 3600.

Try This — Introducing the Lesson

A. Assign the question to pairs of students. You may wish to explain that a recipe is a list of ingredients and instructions for cooking or baking a specific dish. While you observe students at work, you might ask questions such as

• *How do you know Ugyen will use less onion than meat in each serving?* (There is more meat altogether, so there will be more meat in each serving.)

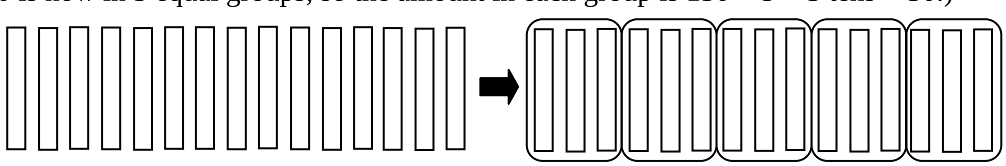
• *Why did you divide by 6?* (There are 6 servings, so I divide by 6 to find the amount in each serving.)

• *Why is it easy to divide 60 by 6?* (I know that 60 is 6 tens, and 6 tens ÷ 6 = 1 ten = 10.)

• *Suppose you first divided 240 by 2. What would you do next?* (After I divide by 2, I can divide each quotient by 3. Altogether, I would make 6 servings. Dividing by 2 and then by 3 is like dividing by 6.)

The Exposition — Presenting the Main Ideas

• Model the number 150 using 15 rods. Ask students how they know the number you are modelling is 150 (15 tens = 150). Form the blocks into 5 equal groups of three rods. Ask students how the model shows 150 ÷ 5. (The 150 is now in 5 equal groups, so the amount in each group is 150 ÷ 5 = 3 tens = 30.)



150 is 15 tens

150 divided into 5 groups is 3 tens in each group.
150 ÷ 5 = 30

• Ask students to suggest how you might model 3600 ÷ 9. Encourage them to see that you could start with 36 flats and then form 9 equal groups with them (as shown on **page 113** of the student text). Since there are 4 hundreds blocks in each group, 3600 ÷ 9 = 4 hundreds = 400.

• Draw students' attention to the fact that they only need to know the result of 36 ÷ 9 and the fact that the units are hundreds to figure out the quotient: 3600 ÷ 9 = 36 hundreds ÷ 9 = 4 hundreds = 400.

• Check student understanding by asking them to model 210 ÷ 3 and 3500 ÷ 5.

• If necessary, do a quick review of multiplication and division facts.

Revisiting the Try This

B. Students could write 240 as 24 tens and 60 as 6 tens to make the calculations easier in **part A**.

Using the Examples

• Ask students to close their texts. Present the question in the **example** and let them try the question. Then suggest that they read through the solution on **page 114** with a partner to see if their thinking was similar.

Practising and Applying

Teaching points and tips

Q 2: Make sure students understand that each stick represents 10 and each square represents 100.

Q 3: Rather than using a rule, encourage students to think of 540 as 54 tens, 420 as 42 tens, and so on.

Q 5: Students can sketch 12 flats, each worth 100, and then group them in 3 groups (1200 ÷ 3 = 400). Note that this model also shows 12 blocks divided into 3 groups (12 ÷ 3 = 4).

Q 6: By writing 40 as 4 tens, students might understand better that they are looking for numbers of the form [ ] tens ÷ [ ], where the quotient of the two missing values is 4.

Q 7: Remind students that dividing by 6 means to form 6 equal groups and dividing by 3 means to form 3 equal groups.

Q 8: This question is designed to draw attention to the fact that although it is almost always easier to add, subtract, or multiply numbers like 400 than numbers like 450, this is not the cause with division. Here it is simpler to use numbers that are associated with division facts.

For example, 450 ÷ 9 is easier than 400 ÷ 9 because 45 ÷ 9 is a division fact that students know. Similarly, 320 ÷ 8 is easier than 300 ÷ 8.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can relate a visual model for division to a symbolic expression
Question 4	to see if students can divide tens or hundreds by 1-digit numbers
Question 5	to see if students can explain how dividing by tens or hundreds works

Answers

A. i) 40 g **ii)** 10 g	**B. i)** I can write 240 as 24 tens and divide 24 tens ÷ 6 = 4 tens = 40. I can write 60 as 6 tens and divide 6 tens ÷ 6 = 1 ten = 10.

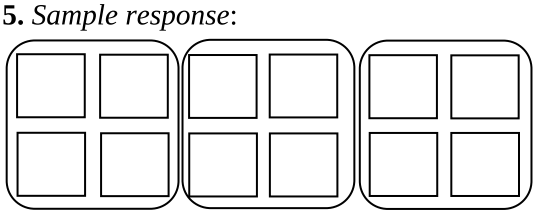
1. 70

2. a) 300 ÷ 6 = 50 **b)** 2000 ÷ 5 = 400
c) 3200 ÷ 4 = 800 **d)** 480 ÷ 8 = 60

3. a) They are equal. **b)** 280 ÷ 4 is greater.

4. a) 40 **b)** 900 **c)** 80 **d)** 700

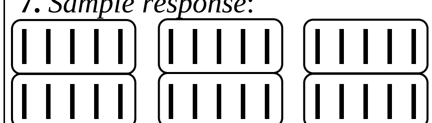
5. *Sample response*:



[12 items divided into 3 groups is 12 ÷ 3.
For 1200 ÷ 3, the items are hundreds: 12 hundreds ÷ 3.]

6. *Sample response*: 240 ÷ 6; 200 ÷ 5; 280 ÷ 7

7. *Sample response*:



[The top row shows 15 tens ÷ 3 = 5 tens (150 ÷ 3 = 50).
Both rows together show 30 tens ÷ 6 = 5 tens (300 ÷ 6 = 50).
Since 150 ÷ 3 = 50 and 300 ÷ 6 = 50,
150 ÷ 3 = 300 ÷ 6.]

[**8.** *Sample response*:
45 ÷ 9 = 5, so you can think of 450 ÷ 9 as 45 tens ÷ 9 and use 45 ÷ 9 = 5 to solve it.
But 400 ÷ 9 = 40 tens ÷ 9 and there is no division fact for 40 ÷ 9.]

Supporting Students

Struggling students

• Some students will struggle with **questions 5 and 7**, where they are asked to sketch a picture to explain an idea. You may wish to provide diagrams and ask students how the diagrams explain the ideas.

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Enrichment

• Students might create a game involving dividing tens and hundreds by 1-digit numbers and play the game with other students.

For example, a student might invent a game where the goal is to get a quotient as close as possible to 30.

A player rolls a die twice. The first number is the number to divide by. The second number is one of the digits in a 3-digit number where the final digit is 0. The player chooses the third digit.

The player whose quotient is closest to 30 wins.

For example, if a player rolls a 6 and then a 4, he or she might include the digit 2 to create 240 ÷ 6 = 40.

4.2.2 Estimating Quotients

Curriculum Outcomes	Outcome relevance
4-B10 2, 3-Digit by 1-Digit Division: with and without regrouping • continue estimating	To make sure that their quotients are reasonable, it is important that students be able to estimate using what they know about dividing tens and hundreds.

Pacing	Materials	Prerequisites
1 h	• None	• dividing tens and hundreds by 1-digit numbers • comparing 3-digit numbers

Main Points to be Raised

• To estimate the quotient of a 3-digit number divided by a 1-digit number, it makes sense to use a multiple of 10 or 100 that is divisible by the 1-digit divisor rather than using the nearest multiple of 100.

For example, to divide 417 ÷ 6, you can think of 417 as about 42 tens, rather than as about 4 hundreds.

• You can sometimes tell whether an estimate is high (if you increase only the dividend or decrease only the divisor) or low (if you decrease only the dividend or increase only the divisor).

Try This — Introducing the Lesson

A. Assign the question to pairs of students. While you observe students at work, you might ask questions such as

• *How do you know that the number in each smaller group is more than 25?* (If there were 100 kangaroos there would be 25 in each group, but there are more than 100 kangaroos.)

• *How do you know that the answer is about half of 60?* (114 is about 120. If the kangaroos form 2 groups, each group is 120 ÷ 2 = 60. If they form 4 groups, each group is half of 60, which is 30.)

• *Why might you estimate the number as 27?* (If there were 100 kangaroos, there would be 25 in each group. If there were 120 kangaroos, there would be 30 in each group. 114 is between 110 and 120, and 27 is between 25 and 30.)

The Exposition — Presenting the Main Ideas

• Ask students what 500 ÷ 5 is and how they know. (500 ÷ 5 = 5 hundreds ÷ 5 = 1 hundred) Then ask why they might use 500 ÷ 5 to estimate 460 ÷ 5. (It is easy to do mentally and it would give a reasonable estimate.)

• Ask what 450 ÷ 5 is and how they know (45 tens ÷ 5 = 9 tens = 90). Then ask why they might also use 450 ÷ 5 to estimate 460 ÷ 5. Discuss why 450 ÷ 5 is easier to figure out than 460 ÷ 5 since 45 ÷ 5 is a fact students already know.

• Ask students why 500 ÷ 5 is a high estimate for 460 ÷ 5 and why 450 ÷ 5 is a low estimate for 460 ÷ 5.

• Ask students how they would estimate 582 ÷ 3 (e.g., 600 ÷ 3 or 570 ÷ 3) and then 582 ÷ 7 (e.g., 560 ÷ 7).

• After they volunteer their approaches, work through the information on **page 115** of the student text to make sure students understand the ideas you have been discussing with them.

Revisiting the Try This

B. Students should realize that 120 is a good estimate for 114 because 12 is divisible by 4. But some students who are comfortable with the notion that 100 = 4 × 25 might prefer to use an estimate of 100.

Using the Examples

• Work through the **example** with students. You can check their understanding by proposing a similar question for them to try.

For example, [ ]53 ÷ 8 is about 60. What is the missing digit?

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Practising and Applying

Teaching points and tips

Q 1 and 2: Students might benefit by writing out the multiplication facts for the 4, 5, 6, and 7 times.

Q 3: This question depends on the grouping meaning of division rather than on the sharing meaning. But students should realize that they can still think about how to share 216 into 6 equal groups to solve the problem. The number in each group tells how many plates can be made.

Q 4: Students could make the connection between multiplication and division to complete this question.

Q 7: This question requires students to recognize that dividing by 9 tells the number of groups of 9. If one number is 90 greater than another, there are 10 more groups of 9. You might not assign this question to struggling students.

Q 8: Students might use the problem in the **Try This** or **questions 3 or 6** as models for problems they might create.

Q 9: You might have students discuss this question in small groups.

Common errors

• Some students cannot get past the notion that multiples of 100 are always easier to work with. These students need to use base ten blocks to see that no trading is done (which makes the computations easier) when the estimate is based on a multiplication fact involving the divisor.

For example, using 600 ÷ 7 to estimate 582 ÷ 7 is not helpful because 60 (tens) is not a multiple of 7. However, 560 ÷ 7 is helpful because 56 tens ÷ 7 = 8 tens = 80.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can make good decisions to estimate a quotient
Question 4	to see if students can relate division to multiplication
Question 6	to see if students can solve a real-world problem that requires estimating a quotient

Answers

A. *Sample response*: 30 kangaroos	**B.** *Sample response*: 120; 120 = 12 tens and it is easy to divide 12 by 4.
1. a) *Sample response*: 280; [28 is easy to divide by 4.] **b)** About 70 birds **2.** *Sample responses*: **a)** About 100; [400 ÷ 4 = 100] **b)** About 80; [400 ÷ 5 = 80] **c)** About 90; [540 ÷ 6 = 90] **d)** About 20; [140 ÷ 7 = 20] **e)** About 90; [360 ÷ 4 = 90] **f)** About 70; [490 ÷ 7 = 70] **3.** *Sample response*: About 35 plates; [If she made 30 plates, there would be 6 × 30 = 180 momos. If she made 40 plates, there would be 6 × 40 = 240 momos. 216 is between 180 and 240, so I used 35 because it is between 30 and 40.] **4. a)** 8 or 8 **b)** 6 or 7 **c)** 2 **d)** 3 **e)** 6 **f)** 2 **5.** No; [8 × 200 = 1600, which has 4 digits, not 3 digits.]	**6.** *Sample response*: About 18 fish; [I knew it was a bit less than 20 since 8 × 20 = 160, which is too many fish.] [**7.** *Sample response*: 517 is about 520. 520 is 90 more than 430. Since dividing by 9 tells how many groups of 9, if there are 90 more, then there are 10 more groups of 9.] **8.** *Sample response*: There are 3 people sharing Nu 257. I want to know about how much money each person will get. [**9.** *Sample response*: I would round 422 to 420 to divide by 6 because 42 ÷ 6 = 7, but I would round 422 to 400 to divide by 5 because 40 ÷ 5 = 8.]

Supporting Students

Struggling students

• For some students, you will need to suggest that they use the relationship between multiplication and division to answer **question 4**.

• You might choose not to assign **questions 5, 7, and 8** to struggling students.

4.2.3 Dividing by Multiplying and Subtracting

Curriculum Outcomes	Outcome relevance
4-B10 2, 3-Digit by 1-Digit Division: with and without regrouping • develop alternative and standard algorithms (from understanding) • connect concrete models to symbolic recordings • continue estimating	Students should have a choice of calculation strategies to maximize their chance of success. It is for this reason that they should be exposed to a variety of division algorithms.

Pacing	Materials	Prerequisites
1 h	None	• multiplying tens and hundreds by 1-digit numbers • subtracting 3-digit numbers

Main Points to be Raised

• One way to divide is to find how many groups of one number are in another number.

• The number of groups can be determined in steps.

For example, first you decide how many hundreds of groups there are, then how many tens of groups e, and finally how many more groups can be created.

• There are always many ways to determine the number of groups when dividing a 3-digit number by a 1-digit number. It is a matter of the individual choosing numbers that feel comfortable.

Try This — Introducing the Lesson

A. Assign the question to pairs of students. While you observe students at work, you might ask questions such as

• *What nearby value did you use to estimate 139?* (150)

• *Why did you use that value?* (I know 150 ÷ 3 = 50 and 139 is not that far away from 150.)

• *Why did you not use 140?* (I cannot divide 14 ÷ 3 without a remainder, so it is easier to use 12 tens.)

The Exposition — Presenting the Main Ideas

• Ask students to imagine that there are 216 sticks with which to make hexagons. Ask how many hexagons can be made if each stick is part of only one hexagon. Allow students to work in pairs to come up with an approach and a solution. After students share their approaches, tell them that when they found out how many hexagons they could make, they were actually determining the number of groups of 6 in 216.

• Have students explain how they know the result must be between 30 and 40 (6 × 30 = 180 and 6 × 40 = 240).

• Model the strategy from **page 117** in the student text, where groups of 6 are subtracted from 216. Make sure students understand that the two models shown are both appropriate ways to count the number of groups and that there are other ways to count the number of groups.

• Ask students what would happen if the number of sticks had been 219 instead of 216. They should understand that the number of groups does not change, but that there is a remainder. There are 3 extra sticks, which is not enough to form another hexagon.

• Although this algorithm can eventually be used for any division situation, whether it be a grouping or sharing problem, the steps will not be meaningful to students when they are first learning it unless the situation is a grouping situation. Once students understand this algorithm well, they can also apply it to sharing problems.

Revisiting the Try This

B. Students can divide 139 by 3 formally by finding the number of groups (or number of plates). They might realize right away that 40 groups of 3 make 120 and then the other 19 biscuits can be placed on 6 additional plates, with a remainder of 1 biscuit. Some students will feel that the leftover biscuit could be added to one of the plates (or be eaten by the person who is arranging the biscuits on the plates).

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Using the Examples

• Present the problem in the **example** to students to try. They can compare their solutions to the solution in the student text.

Practising and Applying

Teaching points and tips

Q 1: Ask students how they know that the value for **part b)** must be less than the value for **part a)**.

Q 3: Students should realize that the remainder is extra; they cannot form part of another window. The number of windows is actually the value of 164 ÷ 4.

Q 4: Some students might not recognize this immediately as a division question. Help them by asking why they want to know the numbers of 5s in 605.

Q 7: Students might benefit by thinking about the relationship between multiplication and division to answer this question.

Q 8: This question is designed to alert students to the importance of proficiency in multiplication and subtraction in order to be successful in division.

Common errors

• Some students do not calculate the number of groups efficiently.

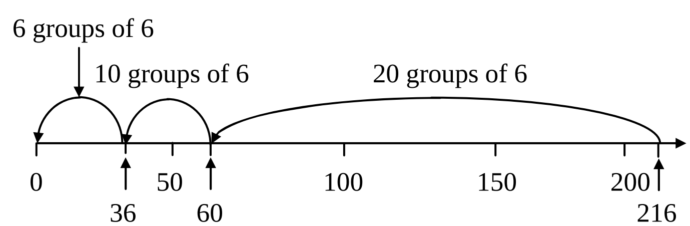
For example, they might take out 2, 5, or perhaps 10 groups at a time, but not 100 groups, or 30 groups.

This is not incorrect, but it can be very time-consuming. Help students see the advantage of taking out more groups at a time by comparing the number of steps it takes to complete the division. At the same time, reassure students who need to use many steps that what they are doing is correct.

• Students can lose track of what they have subtracted so far and what is left to be subtracted. Some students find it helpful to sketch a visual model like a number line.

For example:

$$
\begin{array}{r|l}
6\overline{)216} & \\
-\underline{120} & 20 \text{ groups} \\
96 & \\
-\underline{60} & 10 \text{ groups} \\
36 & \\
-\underline{36} & \underline{\ \ 6 \text{ groups}} \\
0 & 36 \text{ groups}
\end{array}
$$



Suggested assessment questions from Practising and Applying

Question 2	to see if students can divide by subtracting
Question 4	to see if students can solve a real-world problem requiring division
Question 7	to see if students can reason about an error in dividing

Answers

A. *Sample response*: About 45 plates; 150 ÷ 3 = 15 tens ÷ 3 = 50 120 ÷ 3 = 12 tens ÷ 3 = 40 139 is between 120 and 150, so 139 ÷ 3 is about 45.	**B. i)** 46 R 1 means 46 plates; *Sample answer*: $\begin{array}{r\|l} 3\overline{)139} & \\ -\underline{90} & 30 \text{ groups} \\ 49 & \\ -\underline{45} & 15 \text{ groups} \\ 4 & \\ -\underline{\ \ 3} & \underline{\ \ 1 \text{ group}} \\ 1 & 46 \text{ groups} \end{array}$ **ii)** The remainder means there is 1 extra biscuit so there might be one plate with 4 biscuits.

Lesson 4.2.3 Answers [Continued]

1. a) 28 teams **b)** 23 teams

2. *Sample responses*:

a) 31

$8\overline{)248}$	
$-\underline{240}$	30 groups
8	
$\underline{-8}$	$\underline{1 \text{ group}}$
0	31 groups

b) 111

$6\overline{)666}$	
$-\underline{600}$	100 groups
66	
$-\underline{60}$	10 groups
6	
$\underline{-6}$	$\underline{1 \text{ group}}$
0	111 groups

c) 128

$4\overline{)512}$	
$-\underline{400}$	100 groups
112	
$-\underline{80}$	20 groups
32	
$-\underline{32}$	$\underline{8 \text{ groups}}$
0	128 groups

d) 77 R 1

$8\overline{)617}$	
$-\underline{560}$	70 groups
57	
$-\underline{48}$	6 groups
9	
$\underline{-\ 8}$	$\underline{1 \text{ group}}$
1	77 groups

e) 228

$4\overline{)912}$	
$-\underline{800}$	200 groups
112	
$-\underline{100}$	25 groups
12	
$\underline{-12}$	$\underline{3 \text{ groups}}$
0	228 groups

f) 74 R 2

$5\overline{)372}$	
$-\underline{350}$	70 groups
22	
$-\ \underline{20}$	$\underline{4 \text{ groups}}$
2	74 groups

3. a) 41 R 2 windows
b) There are 2 window panes left over.

4. 121 notes

5. 93 squares

6. 84 groups

[**7.** *Sample response*:
5 × 81 is more than 400. You cannot divide 382, which is less than 400, by 5 and get a quotient more than 80.]

[**8.** *Sample response*:
When you form groups, you have to multiply in order to know how many items you used. Then you have to subtract to know how many more items have to be put into groups.]

Supporting Students

Struggling students

• Some students have difficulty interpreting a remainder. Make sure they think of the context of the problem to decide what to do with the remainder, e.g., whether or not an extra group needs to be formed, even if it is not the same size as the other groups.

• Some students may need to use a number line to keep track. (See *Common Errors*.)

Enrichment

• Students might create problems involving division by subtracting that meet particular conditions.

For example:

- The quotient is 65, the remainder is 3.
- The quotient is 57 more than the remainder.

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4.2.4 Dividing in Parts

Curriculum Outcomes	Outcome relevance
4-B10 2, 3-Digit by 1-Digit Division: with and without regrouping • develop alternative and standard algorithms (from understanding) • connect concrete models to symbolic recordings • continue estimating	Students should have a choice of approaches to any computation to maximize their chance of success. It is for this reason that they should be exposed to a variety of division algorithms.

Pacing	Materials	Prerequisites
1 h	• Base ten blocks or Base Ten Blocks (BLM)	• familiarity with multiplication facts • decomposing a number into addends • dividing tens and hundreds by 1-digit numbers

Main Points to be Raised

• One way to divide is to break up the dividend into parts that are more easily divided by the divisor.

For example, for 317 ÷ 4, you might write 317 as 280 + 36 + 1, since 280 and 36 are easily divided by 4. Divide each part by 4 and add the partial quotients.

• To help you choose the parts, you can record the facts for the divisor and use the products or 10 or 100 times those products.

For example, to divide 718 by 3, you might write the multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, and 27, and then write 718 = 6 hundred + 9 tens + 27 + 1.

Try This — Introducing the Lesson

A. Assign the question to pairs of students. While you observe students at work, you might ask questions such as

• *How do you know you have to divide 345 by 5?* (If I know Namgyel has 5 times as many stamps as Chencho, I can divide 345 into 5 equal groups, and one of the groups will be the number of stamps Chencho has.)

• *How do you know that Chencho has fewer than 100 stamps?* (Namgyel would have to have 500 stamps for that to be true.)

• *Why might you round 345 to 350?* (I could write 350 as 35 tens and divide 35 tens by 5 to get 70 tens.)

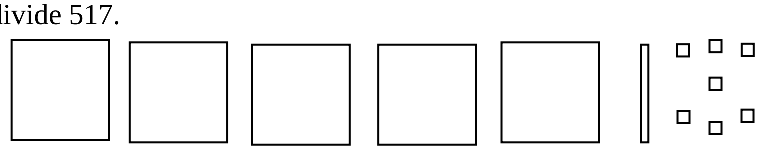
• *How do you know Chencho has 1 fewer stamp than 70?* (345 is 5 less than 350, so there is 1 fewer in each of the 5 groups.)

The Exposition — Presenting the Main Ideas

• Ask students to compare how easy it is to divide 360 ÷ 4 versus 517 ÷ 4. Help students see that dividing 360 by 4 is easier because 36 is one of the products in the 4 times table.

• Then help them use that idea to divide 517.

Model 517 with base ten blocks.



Help them see that to create 4 groups (divide by 4) you can first break up the 517 into parts and then put the appropriate amounts from each part into the 4 groups.

For example:
517 ÷ 4 = (400 + 100 + 17) ÷ 5
517 ÷ 4 = (400 + 80 + 20 + 17) ÷ 5   [100 was broken into 80 + 20 because both divide easily by 4.]
517 ÷ 4 = (400 ÷ 4) + (80 ÷ 4) + (20 ÷ 4) + (17 ÷ 4)
= 100 + 20 + 5 + 4 R 1
= 129 R 1

Show students how to record this:

$$\begin{array}{r} 129\text{ R }1 \\ 100+20+\;\;5+4+0 \\ 4\overline{)400+80+20+16+1} \end{array}$$

• Ask students to think of another way they might have broken up the 517 into parts that are easy to divide by 4 (e.g., 517 = 400 + 80 + 36 + 1).
• Examine with students the example on **page 120** of the student text. Discuss why it makes sense to use the parts that were used.

Revisiting the Try This

B. Students might write 345 as 300 + 45 or perhaps as 200 + 100 + 25 + 20. There are many other possible ways to break up 345. Assure students that they can use any multiples of the divisor that they find easy to use.

Using the Examples

• Remind students of what perimeter means. Then pose the question in the **example**. Encourage students to solve the problem using a sharing model. Students can then compare their solution to the solution in the text.

Practising and Applying

Teaching points and tips

Q 1: Ask students why it makes sense to divide 524 ÷ 4.

Q 2: Allow for alternative suggestions for this question. Students should rename in whatever way is most helpful to them.

Q 4: Observe how students handle the remainder. Although it is acceptable to just call it a remainder, it makes sense in this context to add 0.5 g to each portion.

Q 5: This question extends the idea in the exposition. Most students will not have difficulty with the extension.

Q 7: This question might be handled as a group discussion.

Common errors

• Some students may have difficulty with a situation like the one in **question 5** if there is a remainder.

For example, to divide 358 ÷ 4, students might write 358 = 360 – 2 and then some will write the quotient as 90 – 2 = 88 instead of as 90 – 0.5 (recognizing that the 2 is 2 out of 4). At this stage, those students might be better off using only sums.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can divide by breaking up the dividend into parts
Question 4	to see if students can solve a real-world problem requiring division
Question 6	to see if students can show flexibility in their calculations

Answers

A. 69 stamps	**B.** *Sample response*: $\begin{array}{r} 69 \\ 60+\;\;9 \\ 5\overline{)300+45} \end{array}$
1. 131 stamps **2.** *Sample responses*: **a)** 360 + 18 **b)** 140 + 35 + 3 **c)** 500 + 100 + 45 + 3 **d)** 600 + 210 + 2 **e)** 630 + 81 + 4 **f)** 400 + 160 + 4 + 2	**3. a)** 63 **b)** 25 R 3 **c)** 129 R 3 **d)** 270 R 2 **e)** 79 R 4 **f)** 141 R 2 **4.** *Sample response*: 92 g with 2 g of meat left over.

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5. Yes; *Sample response*: There is 1 fewer group of 3 in 297 than in 300 since 300 – 3 = 297. **6.** *Sample response*: 594 = 600 – 6; 594 = 300 + 270 + 24	[**7.** *Sample response*: When I divide by subtracting, I break the number into parts and divide each part. For example, if I divide 312 ÷ 3 by taking 100 groups of 3 and then 4 groups of 3, I write 312 = 300 + 12.]

Supporting Students

Struggling students

• Some students may have difficulty performing the mental addition and subtraction that make this method efficient. If that is the case, you might simplify the dividends so that mental addition and subtraction are easier.

For example, instead of dividing 648 ÷ 5, suggest that struggling students divide 553 ÷ 5.

GAME: Two Hundred Plus

• Students should use any method that they prefer to calculate their quotients.

• Most students will find it difficult to predict what the remainder will be before they divide, although they should be able to estimate whether the quotient is 200 or more.

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4.2.5 Dividing by Sharing

Curriculum Outcomes	Outcome relevance
4-B10 2, 3-Digit by 1-Digit Division: with and without regrouping • develop alternative and standard algorithms (from understanding) • connect concrete models to symbolic recordings • understand remainders in context as a fraction, ignored, rounded, or addressed specifically • continue estimating	Students should have a choice of approaches or algorithms to any computation to maximize their chance of success. The method presented in this lesson is just one of many included in this unit.

Pacing	Materials	Prerequisites
2 h	• Base ten blocks or Base Ten Blocks (BLM)	• knowing that division can represent a sharing situation • familiarity with the place value system • multiplying tens and hundreds by 1-digit numbers • subtracting 3-digit numbers

Main Points to be Raised

• One way to solve a division is to model the dividend and then share it equally into the number of groups the divisor indicates.

• To do that, it makes sense to start sharing with the larger blocks (the hundreds) and trade larger blocks for smaller blocks if it is not possible to give each group the same number of each type of block.

• Sometimes there is a remainder when you divide. In those cases, you have to consider what it makes sense to do with that remainder:

- Sometimes you can treat it as a fraction or a decimal.
- Sometimes you can ignore it.
- Sometimes it changes the answer.

Try This — Introducing the Lesson

A. Assign the question to pairs of students. Some students might like work with base ten blocks.

While you observe students at work, you might ask questions such as

• *Why did you rename 420 as 300 + 120 to solve the problem?* (So I could divide in parts: $\begin{array}{r} 100 + \ \ 40 \\ 3\overline{)300 + 120} \end{array}$ → 140.)

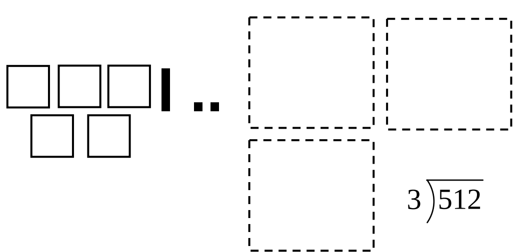
• *How could you estimate to check?* (420 is between 300 and 600, so the amount for each brother is between Nu 100 (300 ÷ 3) and Nu 200 (600 ÷ 3).)

• *I notice that you formed groups of 3 with the blocks. Why did you do that instead of sharing equally among 3 groups, or brothers?* (420 ÷ 3 could mean 420 in groups of 3 or 420 in 3 groups. Forming equal groups of 3 was easier for me.)

The Exposition — Presenting the Main Ideas

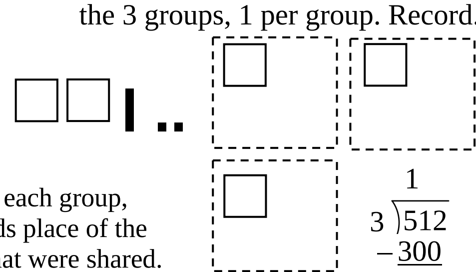
• Tell students that a family has Nu 512 that they need to divide into 3 equal amounts to have enough food money for the next 3 weeks. Ask students to review the methods they have already learned to solve the problem (dividing by subtracting and dividing in parts). Indicate that you will show them another way to divide that is based on sharing.

• Model 512 with base ten blocks. Make 3 boxes or groups (one for each of the 3 weeks) and begin recording the division as shown below.



$3\overline{)512}$

- Share the 5 hundreds blocks among the 3 groups, 1 per group. Record.



As you put 1 hundred in each group, record a 1 in the hundreds place of the quotient. Subtract 300 that were shared. Now 212 are left to be shared.

$$\begin{array}{r} 1 \\ 3\overline{)512} \\ -\underline{300} \\ 212 \end{array}$$

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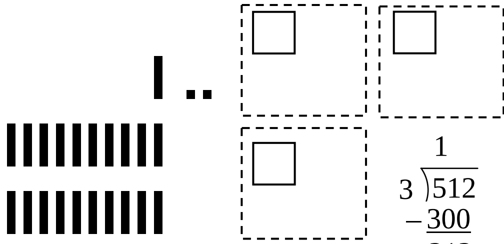
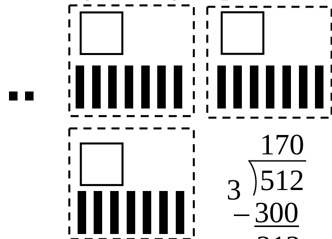
- Since you cannot share the last 2 hundreds among 3 groups, trade them for 20 tens.



$$\begin{array}{r} 1 \\ 3\,\overline{)\,512} \\ -\underline{300} \\ 212 \end{array}$$

- Share the 21 tens among the 3 groups and record.



$$\begin{array}{r} 170 \\ 3\,\overline{)\,512} \\ -\underline{300} \\ 212 \\ -\underline{210} \\ 2 \end{array}$$

As you put 7 tens in each group, record a 7 in the tens place of the quotient.
Subtract the 210 that were shared. Now 2 are left to be shared.
Since you cannot share the 2 among 3, you record a 0 in the ones place of the quotient.
2 becomes the remainder.

- Each group gets 170 and the remainder is 2.

• Discuss the choices about how to handle the remainder. Point out that if the problem were about sharing Nu 512 into 3 groups, the remainder of Nu 2 could be traded for chhetrum and shared further. However, if the situation were 512 students being divided up into 3 groups, the remaining 2 students would need to be added to the existing groups, making the groups slightly unequal.

• Let students work in pairs to apply the procedure to share 338 among 6 groups. They will realize that because there are 3 hundreds and 6 groups, they must trade all the hundreds for tens right away, leaving a blank in the hundreds place of the quotient.

• Lead students through the explanation in the exposition on **pages 122 and 123** of the student text, where 315 is divided by 4.

• Although this algorithm can eventually be used for any division situation, whether it be a grouping or sharing problem, the steps will not be meaningful to students when first learn it unless the situation is a sharing situation. Once students understand this algorithm, they can apply it to grouping problems.

Revisiting the Try This

B. Encourage students to share their base ten blocks into three groups to model 420 ÷ 3.

Using the Examples

• Pose the question in the **example**. Encourage students to use their base ten blocks to solve the problem. Then they can compare their solution to the solution in the text.

Practising and Applying

Teaching points and tips

Q 1: Students might draw a sketch to confirm why they are dividing to solve the problems.

Q 2: Provide base ten blocks for students to use.

Q 4: Remind students of the other methods they know about dividing by subtracting and dividing in parts.

Q 7: Students might draw a sketch to help them see why they would divide 500 by 6.

Q 8: Students must recognize that they must add the two amounts and then divide by 2. Some students might try shifting fish from one group to the other repeatedly until the groups are equal.

Q 9: For the remainder to be written as a fraction, students must realize that the numbers must represent continuous measures.

Q 10: Students should use the relationship between multiplication and division to help them answer this question.

Common errors

• Students may have difficulty if the digit in the tens place of the quotient is 0.

For example, for 416÷ 4, students often write the answer as 14 instead of 104.

$$\begin{array}{r} 14 \\ 4\overline{)416} \\ -\,400 \\ 16 \\ -\ \underline{16} \\ 0 \end{array} \quad \text{instead of} \quad \begin{array}{r} 104 \\ 4\overline{)416} \\ -\,400 \\ 16 \\ -\ \underline{16} \\ 0 \end{array}$$

Remind them to check their answers by estimating. You might also remind them to ensure that each digit in the quotient is above the proper digit in the dividend (with the same place value).

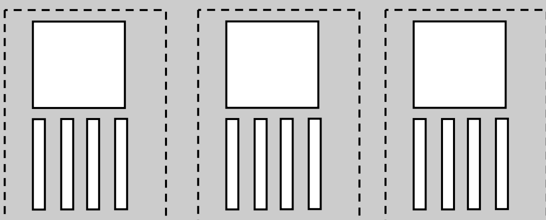
Suggested assessment questions from Practising and Applying

Question 3	to see if students can solve a division problem and interpret the remainder
Question 5	to see if students can divide
Question 6	to see if students can solve a real-world problem requiring division

Answers

A. Nu 140

B. i)



ii)

$$\begin{array}{r} 140 \\ 3\overline{)420} \\ -\,\underline{300} \\ 120 \\ -\,\underline{120} \\ 0 \end{array}$$

1. a) 40 m **b)** 50 m **c)** 25 m

2. a) 67 R 1 **b)** 57
c) 73 R 4 **d)** 205 R 2

3. a) *Sample response*: About 30 sha balay
b) 29 R 5
c) *Sample response*:
I would give 1 extra sha balay to 5 of the families.

4. *Sample response*:

$$\begin{array}{r} 154 \text{ R } 1 \\ 100+\ \ 50+\ \ 4+0 \\ 4\overline{)400+200+16+1} \end{array}$$

$$\begin{array}{r} 154 \text{ R } 1 \\ 4\overline{)617} \\ -\,\underline{400} \\ 217 \\ -\,\underline{200} \\ 17 \\ -\ \ \underline{16} \\ 1 \end{array}$$

5. a) 70 R 7 **b)** 79 R 1 **c)** 56 R 3 **d)** 67 R1

6. a) Her sister's friends
b) Tshering's friends: 35 momos
Her sister's friends: 39 momos

7. 83 people

8. 259 fish

9. *Sample response*:
a) 100 g of meat is divided into 3 packages.
How many grams of meat are in each package?
b) 100 students are divided into groups of 3.
How many groups are there?

10. *Any of the following numbers*:
213, 220, 227, 234, 241, 248, 255, 262, 269, 276

[**11. a)** *Sample response*:
I cannot share 2 hundreds blocks among 5 people, but if I trade them for 20 tens blocks I can share.
b) I would share the 20 tens, giving each person 4 tens. There would still be 15 to share, so each person would get 3 more, or 43 altogether.]

Supporting Students

Struggling students

• Some students might focus on performing the division with the blocks rather than trying to record all of the steps until they become more comfortable with the process.

• You might choose not to assign **questions 8, 9, and 10** to students who are struggling.

Enrichment

• Students might solve this puzzle:

Where should you place the digits 2, 3, 6, 7, 8, and 9 to make this true? $\begin{array}{r} [\,][\,] \\ [\,]\overline{)[\,][\,][\,]} \end{array}$ (267 ÷ 3 = 89)

CONNECTIONS: When Do Remainders Change?

• Students might share the work of dividing in **question 1** to make the work take less time.

• This Connections is a preview of the divisibility rules for 3 and 9: the sum of the digits tells you whether a number is divisible by 3 or 9 and it tells you the remainder when dividing by 3 and 9. This is not generally true when you divide by other numbers.

Answers

1. a) By 2: The remainder changes. By 3: The remainder is always 1. **b)** By 2: The remainder changes. By 3: The remainder is always 2. **c)** By 2: The remainder changes. By 3: The remainder is always 0. **2.** *Sample responses*: **a)** 417 ÷ 9 = 46 R 3 **b)** 471 ÷ 9 = 52 R 3; No. **c)** 714 ÷ 9 = 79 R 3; No.	**3.** *Sample responses*: **a)** 424 ÷ 4 = 106 R 0; 442 ÷ 4 = 110 R 2; the remainder changes. **b)** 35 ÷ 5 = 7 R 0; 53 ÷ 5 = 10 R 3; the remainder changes. **c)** 522 ÷ 6 = 87 R 0; 225 ÷ 6 = 37 R 3; the remainder changes.

UNIT 4 Revision

Pacing	Materials
2 h	• Base ten blocks or Base Ten Blocks (BLM)

Question(s)	Related Lesson(s)
1 and 2	Lesson 4.1.1
3	Lesson 4.1.2
4 – 6	Lesson 4.1.3
7 – 10	Lesson 4.1.4
11 – 13	Lesson 4.2.1
14 – 16	Lesson 4.2.2
17	Lesson 4.2.3
18 and 19	Lesson 4.2.4
20 – 23	Lesson 4.2.5

Revision Tips

Q 2: Students might benefit from renaming the numbers using place value. For example, they could rename 3200 as 32 hundreds and 480 as 48 tens.

Q 3: There is not one right answer for each of these estimates. Students might round up, round down, or use values in between.

Q 8: Have students focus on the number of equal groups and on the size of each group.

Q 9: Students should estimate to help them answer this question.

Q 10: Students can estimate to solve **parts a) and c)**, but not **part b)**.

Q 11: Remind students that the sticks are tens and the squares are hundreds.

Q 18: Encourage students to write out the multiplication facts if that helps them.

Q 23: Students can use the relationship between multiplication and division to help them answer this question.

Answers

1. a) 40 **b)** 360 **c)** 600 **d)** 3600
e) 90 **f)** 480 **g)** 1400 **h)** 4600

2. a) 32 **b)** 48 **c)** 8 **d)** 5

3. *Sample responses*:
a) About 1800; [6 × 300] **b)** About 3600; [9 × 400]
c) About 3500; [5 × 700] **d)** About 5600; [8 × 700]

4. a) 4 × 33 = 132; 120, 12
b) 5 × 226 = 1130; 1000, 100, 30

5. a) 4 × 28 = 80 + 32 = 112

	20	8
4	80	32

b) 6 × 59 = 300 + 54 = 354

	50	9
6	300	54

c) 5 × 63 = 300 + 15 = 315

	60	3
	300	15

d) 9 × 71 = 630 + 9 = 639

	70	1
9	630	9

6. a) 8, 10, 8
b) 1600;
80, 8, 10;
56, 8;
1736

7. a) 168 **b)** 222 **c)** 201
d) 1512 **e)** 1032 **f)** 4696

8. a) 4 × 218 = 872 **b)** 3 × 352 = 1056

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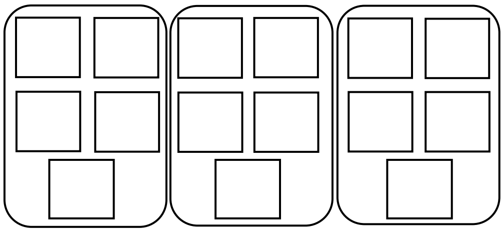
9. a) B and D **b)** D

10. a) 4 **b)** 6 **c)** 3

11. a) 320 ÷ 8 = 40 **b)** 1800 ÷ 6 = 300
c) 2100 ÷ 7 = 300 **d)** 180 ÷ 2 = 90

12. a) 40 **b)** 600 **c)** 40 **d)** 500

13. *Sample response*:



[15 items divided into groups of 3 is 15 ÷ 3.
For 1500 ÷ 3, the items are hundreds: 15 hundreds ÷ 3.]

14. *Sample responses*:
a) About 300; [600 ÷ 2 = 300]
b) About 90; [810 ÷ 9 = 90]
c) About 130; [910 ÷ 7 = 130]
d) About 40; [240 ÷ 6 = 40]
e) About 200; [600 ÷ 3 = 200]
f) About 50; [450 ÷ 9 = 50]

15. *Sample response*: About 30 packages

16. *Sample responses*:
a) 5 **b)** 8 **c)** 2 **d)** 2 **e)** 2 **f)** 1

17. 163 notes

18. *Sample responses*:
a) 360 + 16 + 2 **b)** 700 + 70 + 42 + 4
c) 500 + 300 + 20 + 3 **d)** 600 + 120 + 21 + 2
e) 360 + 42 + 2 **f)** 360 + 27 + 4

19. *Sample response*:
815 = 500 + 300 + 15; 815 = 500 + 200 + 100 + 15

20. a) 200 R 2 **b)** 87 R 3
c) 55 R 3 **d)** 38 R2

21. 64 people

22. a) 99 **b)** 129 **c)** 76 R 1 **d)** 29

23. *Any of the following numbers*:
322, 326, 330, 334, 338, 342, 346, 350, 354, 358

UNIT 4 Multiplication and Division With Greater Numbers Test

1. Multiply.

a) 7 × 10 **b)** 48 × 100

c) 9 × 500 **d)** 17 × 300

2. What number is missing in each?

a) 6400 = [ ] × 100 **b)** 270 = [ ] × 90

3. Estimate each.

a) 6 × 175 **b)** 4 × 279

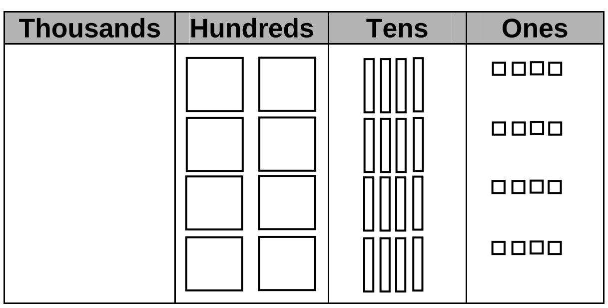
c) 8 × 412 **d)** 5 × 372

4. Multiply.

a) 2 × 78 = [ ] **b)** [ ] = 5 × 89

c) 6 × 159 = [ ] **d)** [ ] = 9 × 146

5. What multiplication sentence does this set of blocks model?



6. Which two products have a sum of 4456?

A. 7 × 338 **B.** 9 × 483

C. 5 × 418 **D.** 6 × 629

7. The same digit is missing in both places. What is the missing digit?

[ ] × 6[ ]7 = 2588

8. Explain why you can use 18 ÷ 3 to find 180 ÷ 3. If you use a picture to explain, tell how your picture shows it.

9. 418 students are arranged in groups of 5. About how many groups are there?

10. What is one possible value for each missing number?

a) 1247 ÷ [ ] is about 300

b) 2511 ÷ [ ] is about 300

11. Show two ways to rename 534 so you can divide it in parts by 7.

12. Calculate each.

a) 276 ÷ 3 **b)** 732 ÷ 4

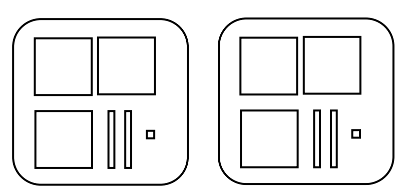
c) 345 ÷ 6 **d)** 615 ÷ 8

13. What could the 3-digit number be?

[ ][ ][ ] ÷ 6 is between 70 and 80.

The remainder is 5.

14. What division sentence does this set of base ten blocks model?



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UNIT 4 Test

Pacing	Materials
1 h	• Base ten blocks or Base Ten Blocks (BLM) (optional)

Question(s)	Related Lesson(s)
1 and 2	Lesson 4.1.1
3	Lesson 4.1.2
4 – 7	Lessons 4.1.3 and 4.1.4
8	Lesson 4.2.1
9 and 10	Lesson 4.2.2
11	Lesson 4.2.4
12 and 13	Lessons 4.2.3, 4.2.4, and 4.2.5
14	Lesson 4.2.5

Assign questions according to the time available.

Answers

1. a) 70 **b)** 4800 **c)** 4500 **d)** 5100

2. a) 64 **b)** 3

3. *Sample responses*:
a) About 1100 **b)** About 1200
c) About 3200 **d)** About 1800

4. a) 156 **b)** 445 **c)** 954 **d)** 1314

5. 4 × 244 = 976

6. A and C

7. 4

8. *Sample response*:

The picture shows 18 tens in 3 groups, which is 180 ÷ 3. If you think of each ten as a stick, it shows 18 sticks divided into 3 groups, which is 18 ÷ 3.

9. *Sample response*: About 80 groups

10. a) 4 **b)** 8

11. *Sample response*:
534 = 490 + 42 + 2
or
534 = 420 + 70 + 28 + 14 + 2

12. a) 92 **b)** 183 **c)** 57 R 3 **d)** 76 R 7

13. *Sample response*: 437

14. 642 ÷ 2 = 321

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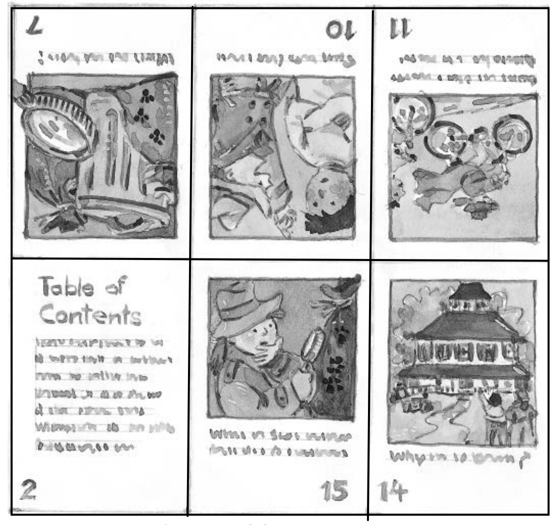
When a book is printed, the pages are put on large pieces of paper called forms.

Each form for a book contains the same number of pages.

Every page in a book must be on a form.

After the forms are printed, the forms are cut apart so the pages of the book can be put together.



A form with 6 pages

Show your work and explain your thinking for each part.

A. A book is being printed. There are 6 pages on each form. How many pages are there in a book with each number of forms?

i) 25 forms

ii) 52 forms

iii) 112 forms

B. How many forms are needed to print a book with each number of pages?

i) 612 pages

ii) 426 pages

iii) 132 pages

C. A book with 212 pages is to be printed. None of the pages can be blank.

i) Why can there not be 5 pages on a form?

ii) Could there be 4 pages on a form?

D. A book with 336 pages is to be printed.

There are fewer than 10 pages on each form.

None of the pages can be blank.

How many pages might there be on each form?

Find more than one answer.

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UNIT 4 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
4-B6 3-Digit by 1-Digit Multiplication: with and without regrouping **4-B10** 2, 3-Digit by 1-Digit Division: with and without regrouping	1 h	None

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could be used to supplement the unit test.

• This task could also be used as enrichment material for some students.

• You can assess performance on the task using the rubric on the next page.

Sample Solution

A. i) 150 pages since 6 × 25 = 150 **ii)** 312 pages since 6 × 52 = 312 **iii)** 672 pages since 6 × 112 = 672 **B. i)** 102 forms since 612 ÷ 2 = 102 **ii)** 71 forms since 426 ÷ 6 = 71 **iii)** 22 forms since 132 ÷ 6 = 22	**C. i)** If you divide 212 pages into forms of 5, there is a remainder of 2 pages. Every page of the book must be on a form. **ii)** Yes; there could be 53 forms since 212 ÷ 4 = 53. **D.** 1, 2, 3, 4, 6, 7, or 8 pages; When I divide 336 by each number of pages, there is no remainder. 336 ÷ 1 = 336 336 ÷ 2 = 168 336 ÷ 3 = 112 336 ÷ 4 = 84 336 ÷ 6 = 56 336 ÷ 7 = 48 336 ÷ 8 = 42

UNIT 34Performance Task Assessment Rubric

The student	Level 4	Level 3	Level 2	Level 1
Communicates about the multiplication and division processes	Uses detailed explanations that clearly show a full understanding of the processes used to multiply and divide a 3-digit number by a 1-digit number	Shows a good understanding of the processes used to multiply and divide a 3-digit number by a 1-digit number	Shows some understanding of the processes used to multiply and divide a 3-digit number by a 1-digit number	Shows misconceptions about how the multiplication or division process works
Reasons about the multiplication and division processes	Shows insight by explaining clearly in **part C** why particular numbers of pages are or are not possible on a form; includes multiple possibilities and good reasoning in **part D**	Explains why particular numbers of page are or are not possible on a form for **part C**; includes one possibility for **part D** with a reasonable explanation	Explains one of **parts C and D** correctly	Has difficulty explaining the reasoning involved in **parts C and D**

BLM 1 Whole Number Place Value Charts

Thousands	Hundreds	Tens	Ones

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UNIT 5 GEOMETRY

UNIT 5 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 129** **TG p. 179**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h, 15 min	• BLM 1 Getting Started Triangles (optional) • Scissors	All questions
Chapter 1 Triangles and Quadrilaterals				
5.1.1 Sorting and Drawing Triangles **SB p. 131** **TG p. 181**	**4-E4 Triangles: discover properties (concretely), name, and draw** • sort, identify, and draw equilateral, isosceles, and scalene triangles	1 h, 15 min	• BLM 2 Sorting Triangles (optional) • Scissors and rulers • BLM 3 Triangle Types • Sticks or straws for concrete models (optional)	Q1, 3, 4, 6
5.1.2 EXPLORE: Properties of Triangles (Essential) **SB p. 136** **TG p. 185**	**4-E4 Triangles: discover properties (concretely), name, and draw** • sort triangles by various properties (e.g., number of lines of symmetry or number of congruent angles)	1 h, 15 min	• BLM 4 Properties of Triangles (optional) • Scissors and rulers	Observe and Assess questions
5.1.3 Sorting Quadrilaterals **SB p. 138** **TG p. 187**	**4-E2 Quadrilaterals: discover properties (concretely)** • investigate a variety of quadrilaterals to discover properties (sides, angles, diagonals, and symmetry) **4-E3 Quadrilaterals: sort by properties (concretely) and make generalizations** • use properties to sort quadrilaterals (e.g., quadrilaterals with right angles) • use properties to make generalizations; include properties that relate sides and those that relate angles	1 h, 15 min	• BLM 5 Sorting Quadrilaterals (optional) • Scissors and rulers	Q1, 5, 6, 7
5.1.4 EXPLORE: Diagonals and Symmetry (Essential) **SB p. 142** **TG p. 190**	**4-E2 Quadrilaterals: discover properties (concretely)** • investigate a variety of quadrilaterals to discover properties (sides, angles, diagonals, and symmetry) **4-E10 Reflective Symmetry: generalize for properties of various quadrilaterals** • explore the symmetry of various quadrilaterals	1 h, 15 min	• BLM 6 Diagonals and Symmetry (optional) • Scissors	Observe and Assess questions
Chapter 2 Polygons and Transformations				
5.2.1 EXPLORE: Congruent Polygons (Essential) **SB p. 144** **TG p. 192**	**4-E11 Congruence: polygons** • understand that congruent polygons are a perfect match because they are the same shape and size • explore congruence through a variety of materials (e.g., pattern blocks, tangrams, pictures of shapes) and methods (including tracing)	1 h	• BLM 7 Congruent Polygons (optional) • BLM 8 Optical Illusions (optional)	Observe and Assess questions

UNIT 5 PLANNING CHART [Continued]

5.2.2 EXPLORE: Combining Polygons (Essential) **SB p. 146** **TG p. 194**	**4-E6 Composite Shapes: combining shapes** • find all possible composite shapes that can be made by combining a given set of shapes • predict first, then verify by combining **4-E11 Congruence: polygons** • understand that congruent polygons are a perfect match because they are the same shape and size • explore congruence through a variety of materials (e.g., pattern blocks, tangrams, pictures of shapes) and methods (including tracing)	1 h, 15 min	• BLM 9 Combining Polygons • Scissors	Observe and Assess questions
GAME: Shape Puzzles (Optional) **SB p. 147** **TG p. 196**	Practise combining polygons in a game situation	30 min	• BLM 10 Puzzle Cards and Shapes • Scissors	N/A
CONNECTIONS: Tangrams (Optional) **SB p. 148** **TG p. 197**	Make a connection between combining polygons, congruent shapes, and a commonly found historical puzzle	30 min	• BLM 11 Tangrams • Scissors	N/A
5.2.3 Slides and Flips **SB p. 149** **TG p. 199**	**4-E9 Slides, Flips, and Turns (half and quarter): predict and confirm results for 2-D shapes** • predict and confirm results for 2-D shapes under transformations	1 h	• BLM 12 Sides and Flips (optional) • BLM 13 Textile Design (optional) • Grid paper	Q2, 4, 5, 7
5.2.4 Turns **SB p. 153** **TG p. 203**	**4-E9 Slides, Flips, and Turns (half and quarter): predict and confirm results for 2-D shapes** • predict and confirm results for 2-D shapes under transformations	1 h	• Grid paper • Scissors • Large cardboard isosceles trapezoid and small trapezoids (optional)	Q3, 4, 6
CONNECTIONS: Logos (Optional) **SB p. 157** **TG p. 206**	Make a connection between transformations and car logos	30 min	• BLM 14 Logos (optional)	N/A
Chapter 3 3-D Geometry				
5.3.1 EXPLORE: Building Shapes from Drawings (Essential) **SB p. 158** **TG p. 207**	**4-E1 Isometric Drawings: interpret** • build simple cube structures from isometric drawings, include shapes that have "hidden" cubes	1 h	• Linking cubes • BLM 15 Sample Net of Cube (optional) • BLM 26 Isometric Dot Paper (optional)	Observe and Assess questions
5.3.2 Describing and Comparing 3-D Shapes **SB p. 160** **TG p. 209**	**4-E5 Prisms, Pyramids, Cones, and Cylinders: describe and compare** • explore relationships concretely to identify properties (e.g., prisms: the number of vertices for any prism is twice the number of vertices for the base, e.g., a triangle-based prism has 6 vertices) • include relationships that deal with faces, edges, and vertices and understand why those relationships make sense • examine the similarities and differences between any pair of 3-D shapes	1 h, min	• Linking cubes • BLMs 15 to 25 Sample Nets • Scissors	Q2, 3, 4, 5

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5.3.3 Folding and Making Nets **SB p. 164** **TG p. 212**	**4-E7 Nets: sketch for rectangular prisms** • sketch a variety of nets for rectangular prisms including square-based prisms and cubes **4-E8 Models: building for cylinders, cones, prisms, and pyramids** • build, from given nets, cylinders and cones • build skeleton models for prisms and pyramids	1 h, 15 min	• Centimetre grid paper • BLMs 15 to 25 Sample Nets • Scissors	Q1, 3, 4, 6
5.3.4 EXPLORE: Building Skeletons (Essential) **SB p. 168** **TG p. 215**	**4-E5 Prisms, Pyramids, Cones, and Cylinders: describe and compare** • explore relationships concretely to identify properties (e.g., prisms: the number of vertices for any prism is twice the number of vertices for the base, e.g., a triangle-based prism has 6 vertices) • include relationships that deal with faces, edges, and vertices and understand why those relationships make sense • examine the similarities and differences between any pair of 3-D shapes **4-E8 Models: building for cylinders, cones, prisms, and pyramids** • build, from given nets, cylinders and cones • build skeleton models for prisms and pyramids	1 h, 15 min	• Sticks of two different sizes • Clay, gum, or flour dough	Observe and Assess questions
UNIT 5 Revision **SB p. 170** **TG p. 217**	Review the concepts and skills in the unit	2 h	• Rulers • Scissors (optional) • Linking cubes • Centimetre grid paper • BLMs 15 to 19, 21, 24, 25 Sample Nets	All questions
UNIT 5 Test **TG p. 220**	Assess the concepts and skills in the unit	1 h	• Rulers • Scissors (optional) • Linking cubes • Centimetre grid paper • BLMs 15 to 18, and 24 Sample Nets	All questions
UNIT 5 Performance Task **TG p. 225**	Assess concepts and skills in the unit	1 h, 15 min	• Linking cubes • BLM 15 Sample Net (optional) • Centimetre grid paper	Rubric provided
UNIT 5 Assessment Interview **TG p. 228**	Assess concepts and skills in the unit	30 min	• Centimetre grid paper • BLM 22 Sample Net • Sticks and balls of clay	All questions
UNIT 5 Blackline Masters **TG p. 229**	• BLM 1 Getting Started Triangles • BLM 2 Sorting Triangles • BLM 3 Triangle Types • BLM 4 Properties of Triangles • BLM 5 Sorting Quadrilaterals • BLM 6 Diagonals and Symmetry • BLM 7 Congruent Polygons • BLM 8 Optical Illusions • BLM 9 Combining Polygons • BLM 10A Puzzle Cards • BLM 10B Puzzle Shapes • BLM 11 Tangrams • BLM 12 Slides and Flips • BLM 13 Textile Design	• BLM 14 Logos • BLM 15 Sample Net of Cube • BLM 16 Sample Net of Cone • BLM 17 Sample Net of Cylinder • BLM 18 Sample Net of Triangle-based Pyramid • BLM 19 Sample Net of Square-based Pyramid • BLM 20 Sample Net of Pentagon-based Pyramid • BLM 21 Sample Net of Hexagon-based Pyramid • BLM 22 Sample Net of Triangle-based Prism • BLM 23 Sample Net of Rectangle-based Prism • BLM 24 Sample Net of Pentagon-based Prism • BLM 25 Sample Net of Hexagon-based Prism • BLM 26 Isometric Dot Paper • Centimetre grid paper on **page 87** in UNIT 2			

Math Background

• In this geometry unit students learn about 2-D shapes (with a focus on polygons), motion geometry (slides, flips, and turns), and 3-D shapes (with a focus on polyhedra).

• The focus of the unit is on triangles, quadrilaterals, congruent polygons, and transformations, as well as various ways to represent and compare 3-D shapes.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections. For example:

• Students use problem solving in **question 2** in **lesson 5.1.1**, where they modify a scalene triangle on isometric dot paper to make it equilateral, in **question 5** in **lesson 5.1.1**, where they investigate equilateral, isosceles, and scalene triangles of a given perimeter, in **question 2** in **lesson 5.3.3**, where they decide which of three arrangements of shapes is a net, and in **question 5 b)** in **lesson 5.3.3**, where they rearrange the shapes in a net to create a new net.

• Students use communication in **question 1** in **lesson 5.1.1**, where they explain how they checked their predictions, in **question 1** in **lesson 5.2.3**, where they explain how they know a given motion is a slide or a flip, in **question 2** in **lesson 5.3.3**, where they explain how they know a given net will form a rectangle-based prism, in the **Try This** in **lessons 5.1.1 and 5.1.3**, where they describe their sorting rules, and throughout **lesson 5.3.2**, where they explain how 3-D shapes are alike and how they are different. The last question in most lessons requires an element of communication in bringing closure to the lesson.

• Students use reasoning in the **Try This** in **lessons 5.1.1 and 5.1.3**, where they sort triangles and quadrilaterals by their properties, in **question 4 b)** in **lesson 5.2.3**, where they determine the slide that will return an image to its original position, throughout **lesson 5.3.2**, where they compare and contrast various 3-D shapes, and in **lesson 5.3.4**, where they relate the number of vertices, edges, and faces in prisms and pyramids to the number of sides in the base.

• Students consider representation throughout **Chapter 3**. In **lesson 5.3.1**, they represent cube structures using isometric drawings, in **lesson 5.3.3**, they represent 3-D shapes using 2-D nets, and in **lesson 5.3.4**, they use skeleton models to represent 3-D shapes.

• Students use visualization in **Chapter 1**, where they predict whether triangles are equilateral, isosceles, or scalene, in **Chapter 2**, where they predict slide, flip, and turn images of a shape, in **lesson 5.2.1**, where they examine optical illusions for congruent polygons, in **lesson 5.2.2**, where they predict shapes that can be made by combining given polygons, and in **lesson 5.3.3**, where they visualize the results of folding nets.

• Students make connections in **question 4** in **lesson 5.1.1**, where they identify triangles in a photograph (of the front grille of a truck), in **question 2** in **lesson 5.2.3**, where they identify slides and flips in a textile pattern, and in the two **Connections**, where they identify slides, flips, and turns in a familiar logos and in an historical puzzle.

Rationale for Teaching Approach

• This unit is divided into three chapters.

Chapter 1 is about the properties of triangles and quadrilaterals.

Chapter 2 focuses on combining polygons, identifying congruent polygons, and performing transformations.

Chapter 3 examines various ways to build, describe, and compare 3-D shapes.

• This unit has two **Explore** lessons in each chapter.

The **Chapter 1 Explore** lessons focus on sorting triangles and quadrilaterals into groups in different ways.

The **Chapter 2 Explore** lessons focus on polygons. In the first exploration, students investigate polygons for congruence. In the second exploration, students combine polygons to make different shapes.

The **Chapter 3 Explore** lessons focuses on building cube structures from isometric drawings and building skeleton models of prisms and pyramids.

The above topics are handled as explorations because this is the most effective way for students to learn these ideas.

• This unit has two **Connections**. The first is historical and deals with Tangrams, an ancient but still-popular puzzle from China. The second provides a real-world connection using common logos found on cars in Bhutan.

• The **Game** provides an opportunity to apply and practise combining polygons to make other shapes.

• Throughout the unit, it is important to encourage students to develop their visualization skills.

• You will notice that in some of the lessons the teacher can choose whether to have students trace shapes from the textbook or to use the provided blackline masters. Your choice will depend on the availability of a photocopier and paper. (Note that many of the blackline masters have been set up to minimize the cutting involved in order to reduce the amount of time students spend cutting.)

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Getting Started

Curriculum Outcomes	Outcome relevance
2 Slides and Flips: 2-D shapes **3** Squares and Rectangles: relating **3** Parallelograms: name, describe, and represent **3** Prisms and Pyramids: name, describe, and generalize **3** Nets: cut and assemble for prisms and pyramids (pentagonal, hexagonal) **3** Turns: 2-D shapes **3** Angles: right angles and less/more than right angles **3** Reflective Symmetry: find various lines of reflection in polygons	Students will find the work in the unit easier after they review the prerequisite concepts and skills from Class II and Class III.

Pacing	Materials	Prerequisites
1 h, 15 min	• BLM 1 Getting Started Triangles (optional) • Scissors	• identifying right angles • identifying congruence • identifying quadrilaterals: squares, rectangles, parallelograms, kites

Main Points to be Raised

Use What You Know

• Triangles are the same when they are the same size and shape even if they are oriented differently.

• You can combine triangles in different ways to make four-sided shapes.

• You can sometimes combine triangles to form a larger triangle.

Skills You Will Need

• Slide, flip, and turn are words to describe actions that move one shape to an identical shape.

• By examining the faces of a prism or pyramid, you can figure out its name.

• An arrangement of 2-D shapes is a net if it can be folded to make a 3-D shape.

• You can think of a line of symmetry as a fold line that matches two identical halves.

Use What You Know — Introducing the Unit

• Students can work in pairs or small groups. Provide each pair or group with a copy of BLM 1 Getting Started Triangles and a pair of scissors. Alternatively, students can trace the shapes on **page 129** of the student text, label them, and cut them out.

• Review the terms *square, rectangle*, and *parallelogram* to make sure students can interpret **part B**. You might note that **part ii)** says non-square rectangle, since squares are rectangles. Refer students to the glossary at the back of the student text.

• Encourage students to consider sliding, flipping, and turning the triangles in **part A** to make the larger shapes.

• Observe students as they work. While they work, you might ask questions such as the following:

- *How can you tell if triangles are the same?* (I can put one triangle on top of another, turning or flipping them if necessary, to see if they match.)

- *How do you know that the rectangle you made is not a square?* (Not all the sides are the same length.)

- *Why is the parallelogram you made not a rectangle?* (The angles are not square corners.)

Skills You Will Need

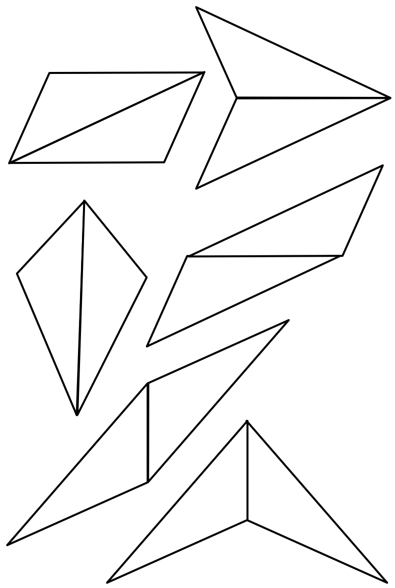
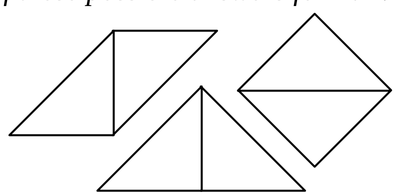
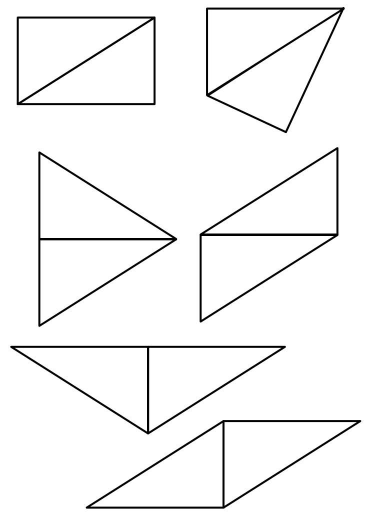
• Students can work individually.

• To ensure students have the required skills for this unit, assign these questions.

• Before students begin, review the terms *net, line of symmetry, ____-based prism*, and *perimeter* to make sure students can interpret **questions 3 to 5**. Refer students to the glossary at the back of the student text.

• Encourage students to visualize folding the potential nets to answer **question 3**. Similarly, they can visualize folding along the potential line of symmetry to see if the two halves match in **question 4**.

Answers

A. i) A/C, B/E, D/F **ii)** *Any two of these possible answers for* A/C:  *Any two of these possible answers for* B/E: 	*Any two of these possible answers for* D/F:  **B. i)** B/E **ii)** D/F **iii)** A/C, B/E, and D/F **C. i)** B/E and D/F **ii)** They each have a right angle.
1. a) Shape C **b)** Shape D **c)** Shape B **2. a)** Shape B **b)** Shape C **c)** Shape D **d)** Shape A	**3.** C **4.** A and B **5. a)** 32 cm; [10 + 10 + 6 + 6 = 32] **b)** 12 m; [5 + 4 + 3 = 12] **c)** 16 cm; [5 + 5 + 3 + 3 = 16]

Supporting Students

Struggling students

• Some students might find it difficult to combine the triangles into larger shapes in **part A**. You might suggest that they colour-code the sides and match same colours.

• If students have trouble identifying the non-net in **question 3**, you might have them look for pairs of opposite sides that match.

• Some students might benefit from tracing the shapes in **question 4** and then folding to see if the given lines are lines of symmetry.

Enrichment

• For **part A**, you might challenge students to find all possible ways to combine each pair of matching triangles.

• For **question 4**, students can look for all lines of symmetry for each shape.

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Chapter 1 Triangles and Quadrilaterals

5.1.1 Sorting and Drawing Triangles

Curriculum Outcomes	Outcome relevance
4-E4 Triangles: discover properties (concretely), name, and draw • sort, identify, and draw equilateral, isosceles, and scalene triangles	The abundance of triangles in the real world makes the study of their properties important.

Pacing	Materials	Prerequisites
1 h, 15 min	• BLM 2 Sorting Triangles (optional) • Scissors and rulers • BLM 3 Triangle Types • Sticks or straws for concrete models (optional)	• understanding the congruence of sides of shapes • measuring line segments

Main Points to be Raised

• Triangles can be classified according to the number of congruent side lengths:

- Equilateral triangles have three congruent sides.

- Isosceles triangles have two congruent sides.

- Scalene triangles have no congruent sides.

• You can compare side lengths by doing any of the following:

- measuring

- holding a copy of the triangle next to the original

- folding a copy of the triangle so that sides line up

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Distribute scissors and copies of BLM 2 Sorting Triangles to each student or pair. Alternatively, have them trace the triangles on **page 131** of the student text, label them, and cut them out.

Observe while students work. Take note of the various sorting rules students use. As students work, you might ask questions such as the following:

• *How did you choose your groups?* (I looked at the shapes of the triangles, *or* I looked at the angles.)

• *How do you know that this triangle belongs in this group?* (It was long and skinny so it belongs in this group, *or* It had a right angle, so it belongs in the group with a right angle.)

• *What do you notice about the sides in Triangle B?* (It looks like two of the sides are the same length.)

• Encourage students to use all different types of sorting rules. If students sort the triangles by angles or other criteria, you might suggest that they also think about the side lengths for this activity.

The Exposition — Presenting the Main Ideas

• Ask the class to share the different ways that they sorted their triangles. Record the sorting rules on the board. If any groups sorted the triangles by side length, have those groups go last.

• Have students look at the exposition on **pages 131 and 132** of the student text.

- Point out the meaning of *congruent* sides.

- Discuss the definitions of *equilateral*, *isosceles*, and *scalene*. For each type, ask students to identify a triangle from the **Try This** that looks like it might be an example. Have them set aside those triangles.

- Next, model each of the different methods of comparing side lengths. For each method, choose one of the triangles you have set aside. Work together as a class using that method to verify whether the triangle is equilateral, isosceles, or scalene.

- After you have explored all the methods, ask students to say which method they prefer and why. Explain that all methods are acceptable. Also point out that even if it seems obvious what type a triangle is, it is a good idea to verify its type using one of the methods.

Revisiting the Try This

B. This question allows students to make a formal connection between what was done in **part A** and the main ideas presented in the exposition. In this case, they use their new vocabulary and apply the methods of comparing side lengths while sorting and classifying the triangles by side length.

Using the Examples

• Present the question in **example 1** for students to answer on their own. As they read through the solutions, each student can compare his or her work to what is shown in the student text.

• To draw scalene and isosceles triangles following the technique shown in **example 2**, you begin by drawing two sides of the triangle. For the equilateral triangle, you start with a concrete model. If any students notice this difference, you might tell them that you may need many attempts to draw an equilateral triangle if you start by drawing two sides. Using a model makes it much quicker.

Practising and Applying

Teaching points and tips

Q 2: Some students will be able to use visualization to answer this question. Others may benefit from a guess-and-check approach.

Q 3: Have students follow **example 2** to help them answer this question. (In particular, it can be difficult to draw an equilateral triangle using other methods.) Encourage students to use the side length measurements to check the accuracy of their answers.

Q 4: This question provides a link to everyday life. The photo is of a portion of the front grille of a TATA truck. If students have difficulty locating the non-equilateral triangles, point out the shapes in the background. Students can record their answers on copies of BLM 3 Triangle Types.

Q 5: This question might be assigned only to selected students. It is more challenging because it combines the additional concept of perimeter with the concept of triangle classification. No previous models of how to approach it have been provided. Because there are many possible answers for **parts b) and c)**, it is important that you read student answers carefully.

Common errors

• Many students will not recognize that the triangle in **question 1**, **part c)** is isosceles because of its orientation. You might encourage students to view the triangles from different perspectives.

• Given the number of dots shown, students will have difficulty with **question 2** if they try to move any vertex other than the vertex on the right.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can recognize equilateral, isosceles, and scalene triangles
Question 3	to see if students can draw equilateral, isosceles, and scalene triangles
Question 4	to see if students can identify equilateral, isosceles, and scalene triangles in a real-world context
Question 6	to see if students understand the techniques for deciding whether sides are congruent

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Answers

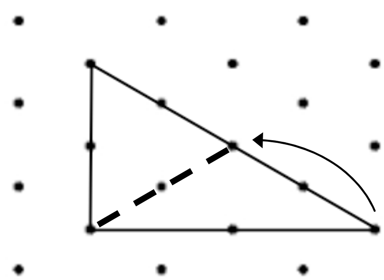
A. *Sample response*:
By angles:
- A, H (have a right angle)
- B, C (have an angle bigger than a right angle)
- D, E, F, G (all angles are smaller than a right angle)

Or by appearance:
- A, B, F, G, H (straight relative to the page)
- C, D, E (crooked relative to page)

B. i) D, G
ii) A, B, F
iii) C, E, H

1. a) Scalene; [*Sample response*:
I measured to see that each of the sides was a different length.]
b) Equilateral; [*Sample response*:
I traced the triangle and folded to see that all of the sides were the same length.]
c) Isosceles; [*Sample response*:
I traced the triangle and compared its sides to the triangle in the book. Two sides in the triangle are the same length.]

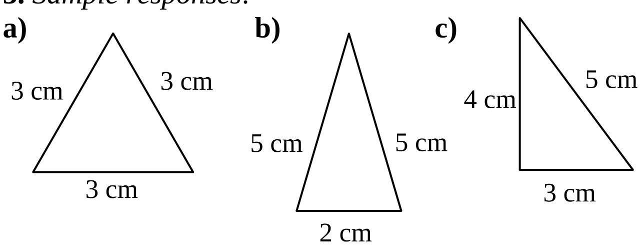
2.



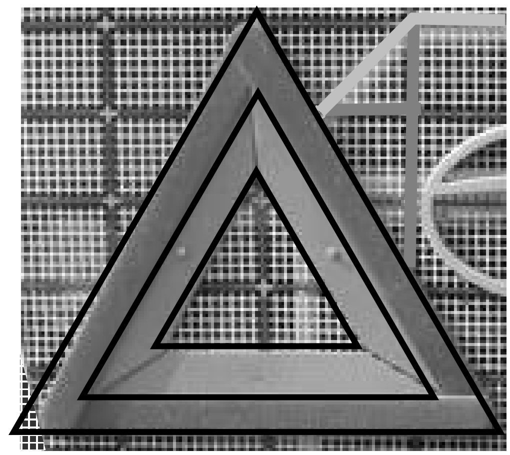
Each side length is 2 grid units long so I know I am right.

3. *Sample responses*:

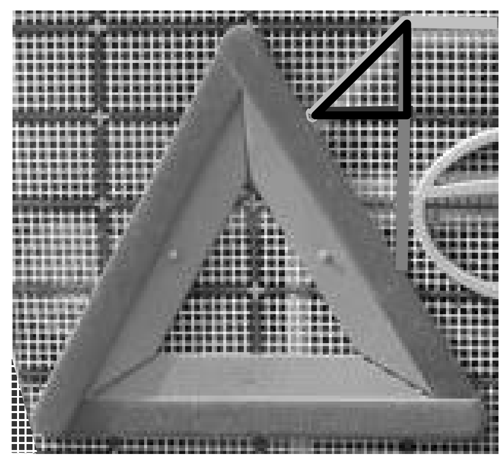
a) **b)** **c)**



4. Equilateral:

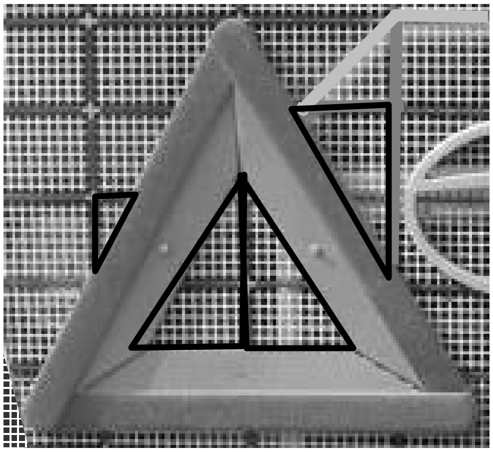


Isosceles:

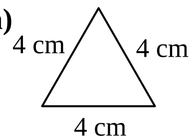


Lesson 5.1.1 Answers [Continued]

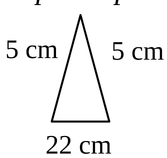
Scalene:



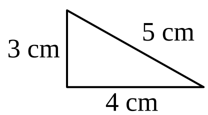
5. a)



b) *Sample response*:



c) *Sample response*:



[**d)** There is only one answer for **part a)**. There can only be one equilateral triangle with a perimeter of 12 cm because each side must be the same, 4 cm.

There are other answers for **part b)** because I can make some sides shorter and others longer, for example, 3 cm, 4.5 cm, and 4.5 cm. The perimeter will still be 12 cm. I need to make sure two sides are congruent.

There are other answers for **part c)** because I can make some sides shorter and others longer, for example, 2 cm, 4.5 cm, and 5.5 cm. The perimeter of 12 cm. I need to make sure none of the sides are congruent.]

[**6.** *Sample response*:
I would measure because it is easier to keep track of the sides. With the other methods, I have trouble remembering which sides I have compared and which sides I still have to do.]

Supporting Students

Struggling students

• If students struggle to draw the isosceles and scalene triangles in **question 3**, you might have them model the triangles concretely as they did for the equilateral triangle.

For example, students can model an isosceles triangle with two sticks (or straws) of the same length and a third stick of a different length. They can mark the vertices and join them with straight lines. For a scalene triangle, they can use three sticks of different lengths.

• If students are having trouble remembering the new vocabulary in this lesson, it may be helpful to keep a Triangle Journal. Have them write the terms *equilateral*, *isosceles*, and *scalene* at the top of three pages. They can copy the definition for each term and trace several examples for each type of triangle onto the appropriate pages.

Enrichment

• You might challenge students to repeat **question 5** with different perimeters, such as 15 cm or 18 cm.

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5.1.2 EXPLORE: Properties of Triangles

Curriculum Outcomes	Outcome Relevance
4-E4 Triangles: discover properties (concretely), name, and draw • sort triangles by various properties (e.g., number of lines of symmetry or number of congruent angles)	An important part of mathematical thinking is to recognize how knowing some information about one shape (in this case, how the side lengths compare) gives you other information about the shape (in this case, about its symmetry or the number of congruent angles).

Pacing	Materials	Prerequisites
1 h, 15 min	• BLM 4 Properties of Triangles (optional) • Scissors and rulers	• identifying lines of symmetry

Exploration

• Begin by reviewing the terms *equilateral, isosceles,* and *scalene* from **lesson 5.1.1**. Draw, or ask a volunteer to draw, an example of each type of triangle on the board.

• Have students work in pairs or in small groups for **parts A to D**. Distribute BLM 4 Properties of Triangles and scissors to each pair or group. Alternatively, have students trace the triangles on **page 136** of the student text, label them, and cut them out.

• Discuss **part C** with students to make sure they know what to do. Be sure to point out the definition of congruent angles. You may wish to work together as a class to compare the angles of Triangle C, as shown in the student text. Students can compare their cut-out triangles to the triangles in their books. Make sure they understand that they should compare different pairs of angles.

Observe while students work. While they are working, you might ask questions such as the following:

• *How does folding help you identify lines of symmetry?* (If the two sides of the triangle line up exactly, then I know the fold is a line of symmetry.)

• *Is it possible for a line of symmetry not to go through a vertex of a triangle?* (No. If a line does not go through a vertex, then there will be two vertices on one side of the line and one vertex on the other side. It cannot be a line of symmetry because it does not divide the triangle into two identical halves.)

• *Why do you only have to compare two sets of angles for some triangles?* (If I know two angles are congruent, then I only have to compare one of them to the third angle.)

Observe and Assess

As students work, notice the following:

• Do they identify the lines of symmetry?

• Do they identify the congruent angles?

• Do they recognize the relationship between the type of triangle and the number of lines of symmetry and the number of congruent angles in the table for **part D**?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and answer these questions.

• *How did you decide the type of triangle when you measured?*

• *How do you decide what type of triangle it is if you know the number of lines of symmetry?*

• *How do you decide what type of triangle it is if you know the number of congruent angles?*

Lesson 5.1.2 Answers

A. to C.

Triangle	Type of triangle: equilateral, isosceles, or scalene	Number of lines of symmetry	Number of congruent angles
A	scalene	0	0
B	equilateral	3	3
C	isosceles	1	2
D	scalene	0	0
E	equilateral	3	3
F	isosceles	1	2
G	scalene	0	0
H	equilateral	3	3
I	isosceles	1	2

D. Equilateral triangles have 3 lines of symmetry and 3 congruent angles.
Isosceles triangles have 1 line of symmetry and 2 congruent angles.
Scalene triangles have 0 lines of symmetry and 0 congruent angles.

Supporting Students

Struggling students

• Some students may have trouble keeping track of the angles they have compared in **part C**. You might encourage them to colour-code the angles as they work.

For example, they can put a blue dot on angle 7 and angle 9 since they are congruent. They can then put a red dot on angle 8 since it is not congruent to angle 7 or 9.

• If students have trouble identifying the relationships in the table for **part D**, they might benefit from rewriting the results in three separate tables, one for each type of triangle. Alternatively, they can lightly shade each row of the table according to the type of triangle.

For example, all equilateral rows can be one colour, all isosceles rows can be a second colour, and all scalene rows can be a third colour.

Enrichment

• You might challenge students to think about the connection between lines of symmetry and congruent angles by answering this question:

Why does each line of symmetry in a triangle show a pair of congruent angles? (A line of symmetry means the triangle is divided into two congruent halves. For each angle on one half, there is a matching congruent angle on the other half.)

Reprint 2023

5.1.3 Sorting Quadrilaterals

Curriculum Outcomes	Outcome relevance
4-E2 Quadrilaterals: discover properties (concretely) • investigate a variety of quadrilaterals to discover properties (sides, angles, diagonals, and symmetry) **4-E3 Quadrilaterals: sort by properties (concretely) and make generalizations** • use properties to sort quadrilaterals (e.g., quadrilaterals with right angles) • use properties to make generalizations; include properties that relate sides and those that relate angles	The abundance of quadrilaterals in the real world makes the study of their properties important.

Pacing	Materials	Prerequisites
1 h, 15 min	• BLM 5 Sorting Quadrilaterals (optional) • Scissors and rulers	• understanding congruent shapes • familiarity with various quadrilaterals • identifying lines of symmetry • identifying isosceles and equilateral triangles

Main Points to be Raised

• Any four-sided shape is called a quadrilateral.

• Quadrilaterals are often named for the number of parallel sides and the number of congruent sides:

- A kite has no parallel sides and two pairs of congruent sides.

- A trapezoid has one pair of parallel sides. An isosceles trapezoid is a special trapezoid that also has a pair of congruent sides.

- A parallelogram has two pairs of parallel, congruent sides.

• Some quadrilaterals have more than one name:

- A rectangle is a parallelogram with right angles.

- A rhombus is a parallelogram with all sides congruent.

- A square can be thought of as a rectangle with all sides congruent or as a rhombus with all angles congruent.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Distribute scissors and copies of BLM 5 Sorting Quadrilaterals to each student or pair. Alternatively, have them trace the quadrilaterals on **page 138** of the student text, label them, and cut them out.

Observe while students work. As they work, you might ask questions such as the following:

• *What sorting rule did you use?* (I looked to see whether or not a shape had a right angle.)

• *How might you use side lengths to sort the shapes?* (I could see how many of the sides were the same length.)

• *How else might you use sides to sort the shapes?* (I could see if any of the sides went in the same direction.)

• While it is important to encourage all different types of sorting rules, gently guide students to focus on sides.

The Exposition — Presenting the Main Ideas

• Have students turn to the exposition on **page 138** of the student text. Read through this page together.

- Ask students to identify examples of parallel sides in the quadrilaterals from the **Try This**. Ask them to look for examples of parallel lines in the classroom (e.g., opposite sides of the board, a window, a door, or a desk).

- When you discuss the parallelogram and the trapezoid on **page 138**, be sure to point out the congruent sides.

• Work through the rest of the exposition with students.

- As you encounter each new term, write it on the board. Ask a student volunteer to draw an example next to each term. Alternatively, write the names and draw the examples on large pieces of paper that can be put up on the wall of the classroom for easy reference.

- Some students may find it confusing that a quadrilateral such as a square can have many correct names (e.g. rectangle, rhombus, and parallelogram). You may wish to use the most specific name consistently, while accepting any other appropriate name as correct. You can emphasize that many quadrilaterals can have more than one correct name by asking questions such as, “What else could this be called?” or “What other name is also correct for this quadrilateral?”

• Allow ample time for students to ask any questions they have.

Revisiting the Try This

B. This question allows students to make a formal connection between what was done in **part A** and the main ideas presented in the exposition. In this case, they examine the shapes in the **Try This** to determine the number of parallel and congruent sides in order to name them using the new vocabulary.

Using the Examples

• Present the question in **example 1** for students to answer. As they read through the solutions, each student can compare his or her work to what is shown. Ask each student to choose the solution that most closely matches what he or she did.

Practising and Applying

Teaching points and tips

Q 2: Encourage students to follow **example 1** for this question. This can be especially helpful if they are having trouble thinking of other ways to sort the quadrilaterals.

Q 3 and Q 6: Shapes A, B, C, E, and L all have more than one correct name. You should accept all correct answers. When a student offers a less specific name (for example, when a square is called a rectangle), you can lead a class discussion to highlight the fact that many names are possible.

Q 4 and Q 5: It might be helpful to suggest that students think of the quadrilateral names. This will get them thinking of the number of parallel sides and congruent sides, which in turn gives them tools for comparing.

Q 7: Use this last question as a closure question. It is a way to highlight the important ideas students have learned in the lesson. It can also help you identify students who need extra help to understand the naming system for quadrilaterals.

Common errors

• In **question 1**, some students may not recognize that E is a square because of its orientation. You might point out that parallel sides do not need to be lined up with the sides of the page (i.e., vertical or horizontal). Encourage them to turn the page to view the shapes from different perspectives.

• In **question 1**, students may have difficulty recognizing that B is a rhombus because it can be difficult to compare side lengths visually. Remind them of the three methods for comparing side lengths.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use properties to sort quadrilaterals
Question 5	to see if students can apply what they know about properties to compare quadrilaterals
Question 6	to see if students can name various quadrilaterals
Question 7	to see if students can recognize and explain why a quadrilateral can have more than one name

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Answers

A. i) *Sample response:* They all have four sides. **ii)** *Sample response*: - A, E, F (1 or more right angles.) - B, C, D, G (0 right angles)	**B. i)** C and E **ii)** A, C, D, and E **iii)** F and G
1. and 2. *Sample responses*: By number of right angles: 4 right angles: A, E 1 right angle: D 0 right angles: B, C By parallel sides: 2 pairs of parallel sides: A, B, E 1 pair of parallel sides: C No parallel sides: D By congruent sides: 4 congruent sides: B, E 2 pairs of congruent sides: A, D 1 pair of congruent sides: C **3.** A: Rectangle (or parallelogram) B: Rhombus (or parallelogram) C: Isosceles trapezoid (or trapezoid) D: Kite E: Square (or rectangle, or rhombus, or parallelogram)	**4.** Alike: Both have two pairs of congruent sides. Different: A rectangle has two pairs of parallel sides but a kite has no parallel sides. **5. a)** K and L have a right angle, but M does not. **b)** L and M have two pairs of parallel sides, but K has only one pair of parallel sides. **6.** K: Trapezoid L: Parallelogram or rectangle M: Parallelogram **7.** They are all correct. [A square is a special rhombus (that has right angles) and a rhombus is a special parallelogram (that has right angles and all sides equal).]

Supporting Students

Struggling Students

• If students are struggling with the quadrilateral names in **questions 3 and 6**, you might suggest they keep a Quadrilateral Journal similar to the Triangle Journal from **lesson 5.1.1**. Have them write the terms *Kite*, *Trapezoid*, and *Parallelogram* at the top of three pages. They can trace several examples for each type of quadrilateral onto the appropriate pages and label each with its most specific name.

• Some students may have trouble with the comparisons in **questions 4 and 5**. To assist in identifying properties, you might have them trace each shape twice. On one copy they can mark parallel sides in the same colour. On the other copy they can mark all congruent sides in the same colour.

Enrichment

• You might ask students who enjoy a challenge to come up with a complete list of all possible names for each quadrilateral in **questions 3 and 6**.

• Ask students to consider why there might be shapes called isosceles trapezoids but not isosceles parallelograms.

5.1.4 EXPLORE: Diagonals and Symmetry

Curriculum Outcomes	Outcome Relevance
4-E2 Quadrilaterals: discover properties (concretely) • investigate a variety of quadrilaterals to discover properties (sides, angles, diagonals, and symmetry) **4-E10 Reflective Symmetry: generalize for properties of various quadrilaterals** • explore the symmetry of various quadrilaterals	An important part of mathematical thinking is to recognize how knowing some information about one shape (in this case, the type of quadrilateral it is) gives you other information about the shape (in this case, whether or not its diagonals are lines of symmetry).

Pacing	Materials	Prerequisites
1 h, 15 min	• BLM 6 Diagonals and Symmetry (optional) • Scissors	• identifying lines of symmetry

Exploration

• Work through the introduction (in white) with students. Make sure that they understand that every quadrilateral has exactly two diagonals.

As an example, draw a kite on the board and ask a student volunteer to add the diagonals. Repeat with other types of quadrilaterals until you feel students understand.

Review the definition of *line of symmetry* (e.g., if you fold a shape along a line of symmetry, the fold divides the shape in half and the halves match exactly). Tell the class that some diagonals are lines of symmetry and some are not.

• Have students work in pairs or in small groups for **parts A to E**. Distribute BLM 6 Diagonals and Symmetry and scissors to each pair or group. Alternatively, have students trace the quadrilaterals on **page 142** of the student text, label them, and cut them out.

Observe while students work. While they are working, you might ask questions such as the following:

• *How do you know that Shape D is a rhombus?* (It has four congruent sides and two pairs of sides going in the same direction.)

• *What other name could a rhombus have?* (A parallelogram.)

• *Why is this diagonal a line of symmetry?* (I can fold the quadrilateral along the diagonal and the two halves match exactly.)

Observe and Assess

As students work, notice the following:

• Do they name the quadrilaterals successfully?

• Do they identify the lines of symmetry correctly?

• Do they identify the diagonals correctly?

• Can they distinguish between a line of symmetry and a diagonal?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and answer these questions.

• *What are the names of the quadrilaterals? Are those the only names?*

• *How do you identify lines of symmetry? diagonals?*

• *Which quadrilaterals have lines of symmetry?*

• *Which quadrilaterals have diagonals that are lines of symmetry?*

Reprint 2023

Answers

A. to C. (*Sample responses for* **A.**)

Quadrilateral	Name	Number of lines of symmetry	Number of diagonals that are lines of symmetry
A	Square	4	2
B	Rectangle	2	0
C	Isosceles trapezoid	1	0
D	Rhombus	2	2
E	Trapezoid	0	0
F	Parallelogram	0	0
G	Kite	1	1

D. i) A (square), B (rectangle), C (isosceles trapezoid), D (rhombus), and G (kite)
ii) A (square), B (rectangle), and D (rhombus)

E. i) A (square) and D(rhombus)
ii) G (kite)
iii) A (square), B (rectangle), and C (isosceles trapezoid)

Supporting Students

Struggling students

• If students are struggling with identifying the lines of symmetry in **part B**, you might consider modifying the activity slightly.

For example, you could have them draw the diagonals in **part C** and then check by folding to see if they are also lines of symmetry.

• Some students may identify a diagonal incorrectly as a line of symmetry when it divides the quadrilateral into two congruent halves that are not reflections of each other, as with a parallelogram. Remind them that for it to be a line of symmetry, the two identical halves must match when you fold along the line.

Enrichment

• Ask students who enjoy a challenge to draw a kite accurately using what they know about lines of symmetry. If necessary, suggest that they begin with a scalene triangle and think about folding. They can check their work by measuring side lengths.

5.2.1 EXPLORE: Congruent Polygons

Curriculum Outcomes	Outcome Relevance
4-E11 Congruence: polygons • understand that congruent polygons are a perfect match because they are the same shape and size • explore congruence through a variety of materials (e.g., pattern blocks, tangrams, pictures of shapes) and methods (including tracing)	This essential exploration introduces the idea of congruent polygons. It prepares students to study transformations, where the image of a transformation is congruent to the original shape.

Pacing	Materials	Prerequisites
1 h	• BLM 7 Congruent Polygons (optional) • BLM 8 Optical Illusions (optional)	• understanding an optical illusion (Unit 3) • identifying congruent shapes

Exploration

• With students, work through the introduction (in white) on **page 144** of the student text. Discuss the definition of *polygon*. Make sure they also understand what it means when two polygons are congruent.

As an example, have students trace one of the congruent triangles in the introduction and hold it over the other triangle to see that it is an exact match. Then have them trace the pentagon on the left and hold it over the other pentagon to see that it is not an exact match.

Ask students to identify another pair of congruent shapes in the introduction. (The pentagon in the top row of shapes is congruent to the pentagon in the second row.)

• Discuss the term *optical illusion* with students. Refer back to **Unit 3 page 67** in the student text if necessary. Explain to students that they will investigate some optical illusions created with congruent polygons. Have students work in pairs or small groups for **parts A to C**.

Observe while students work. While they are working, you might ask questions such as the following:

• *How do you know these parallelograms are congruent?* (When I held the tracing of this parallelogram over the other parallelogram, it was an exact match.)

• *Why do you think Square g looks bigger than Square h?* (Square g is taller than Square h, so it is bigger.)

• *Why do you think the white hexagon in Shape i looks smaller than the white hexagon in Shape l?* (The black hexagon around the outside of the white hexagon in Shape l makes it seem bigger.)

If time permits, students can create their own optical illusions. See *Enrichment*.

Observe and Assess

As students work, notice the following:

• Do they identify the grey squares in **part B** and the white hexagons in **part C**?

• Do they understand what it means for two shapes to be congruent?

• Do they recognize that shapes can be congruent even when they appear not to be congruent?

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Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and answer these questions.

• *What is a polygon?* (A closed shape with only straight sides)

• *How do you know that two polygons are congruent?* (They are the same size and shape. I can trace one and hold it over the other and they are a perfect match.)

• *When is it possible for two polygons to look like they are not congruent even when they are congruent?* (When a polygon is turned it can look like it is a different size or shape. Also, when a polygon has another polygon inside or outside it, it can look like it is not congruent to another polygon.)

Answers

A. They are all congruent. **B.** The grey squares in d, e, and f are congruent. Grey squares g and h are congruent.	**C.** The white hexagons in i, j, and l are congruent. The white hexagons in m and k are congruent.

Supporting Students

Struggling students

• If students are struggling with the traced polygons in **parts A, B, and C**, you might distribute BLM 7 Congruent Polygons, BLM 8 Optical Illusions, and scissors. The polygons on these BLMs are the same size as those in the student book. Students can cut out a polygon from the BLM and hold it over a polygon in the student book to see if it matches.

• For **part A**, you may need to suggest that students pick up the parallelograms (cut-out or traced) and flip them over. Be sure to tell the class that any action that does not change the size or shape of a polygon is permissible.

Enrichment

• For **part A**, you might challenge students to create their own optical illusions based on the same principle. They can draw a polygon and then trace it in various orientations.

• To create an optical illusion like those in **parts B and C**, follow these steps:

- Cut out four rectangles: two 4 cm-by-6 cm, one 6 cm-by-8 cm, and one 2 cm-by-4 cm.
- Shade the largest and the smallest rectangles.
- Glue the small rectangle so that it is centred inside one of the unshaded rectangles.
- Glue the other unshaded rectangle so that it is centred inside the largest rectangle.

• For **parts B and C**, you can also challenge students to look for congruent polygons that are different in colour.

- *Can you identify congruent white and black shapes?* (The white hexagons in Shape m and Shape k and the black hexagon in Shape i are congruent. The black hexagon in Shape m and the white hexagons in Shapes i, j, and l are congruent.)

- *Can you identify congruent grey and black shape*s? (The black square in Shape d and the grey squares in Shapes g and h are congruent.)

5.2.2 EXPLORE: Combining Polygons

Curriculum Outcomes	Outcome Relevance
4-E6 Composite Shapes: combining shapes • find all possible composite shapes that can be made by combining a given set of shapes • predict first, then verify by combining **4-E11 Congruence: polygons** • understand that congruent polygons are a perfect match because they are the same shape and size • explore congruence through a variety of materials (e.g., pattern blocks, tangrams, pictures of shapes) and methods (including tracing)	This essential exploration builds on the concepts of congruent polygons developed in **lesson 5.2.1**. Polygons can be combined in different ways to create congruent shapes. Thinking about how shapes can be dissected and combined is useful in studying transformations, as in **lessons 5.2.3 and 5.2.4**, and in later work with measurement formulas and describing attributes and properties of shapes.

Pacing	Materials	Prerequisites
1 h, 15 min	• BLM 9 Combining Polygons • Scissors	• familiarity with names of various quadrilaterals • identifying congruent shapes

Exploration

• Work through the introduction (in white) with students. Make sure they understand that the polygons are combined by matching whole sides.

• Because of the variety of predictions and correct answers possible, it is best for student to work together in pairs or in small groups. Distribute scissors and BLM 9 Combining Polygons to each pair or group to complete **parts A to C**. Alternatively, have students trace the polygons on **page 146** of the student text and cut them out. Tell them that they can use as many of each shape as they wish.

Observe while students work. While they are working, you might ask questions such as the following:

• *Could you make a parallelogram in any other way?* (Yes. I could use four triangles instead of a trapezoid and one triangle.)

• *Could you make a hexagon in any other way?* (No. I would need more triangles or trapezoids to do that.)

• *How do you know this shape is congruent to the other shape you made?* (I can trace this shape and place the other polygons for the new shape over the tracing. They line up exactly.)

Since predicting can be inaccurate, be sure to allow for all types of answers. The real value in making a prediction can be what you learn from your inaccuracies.

Observe and Assess

As students work, notice the following:

• Do they predict accurately the shapes that can be made from the given polygons?

• Do they identify successfully the shapes they made?

• Do they understand when the different shapes they made are congruent?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and answer these questions.

• *Which shapes could you not make?*

• *Which shapes could you make in only one way?*

• *Which shapes could you make in many different ways? How many different ways could you make those shapes?*

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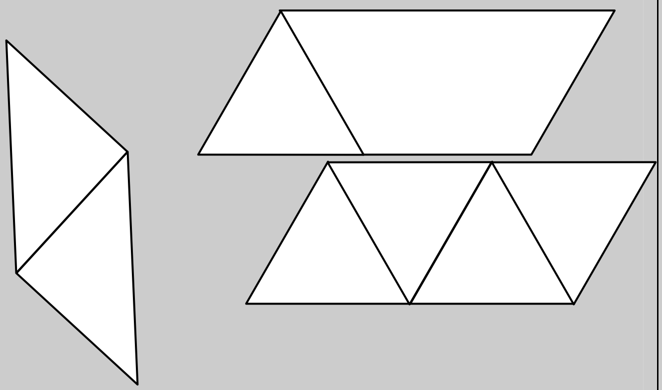
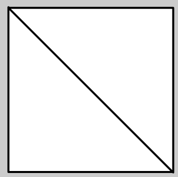
Answers

A. i) and ii) *Sample responses*:
I predict that I can make all of the shapes.

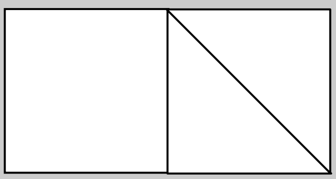
Parallelogram:



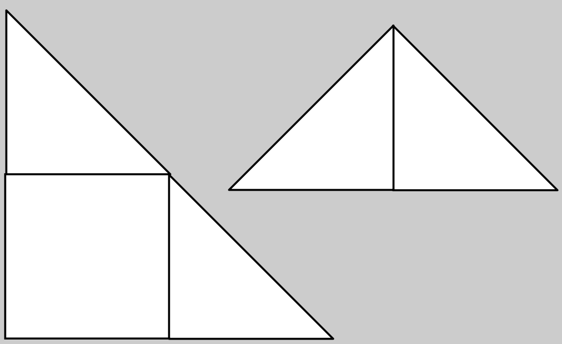
Square:



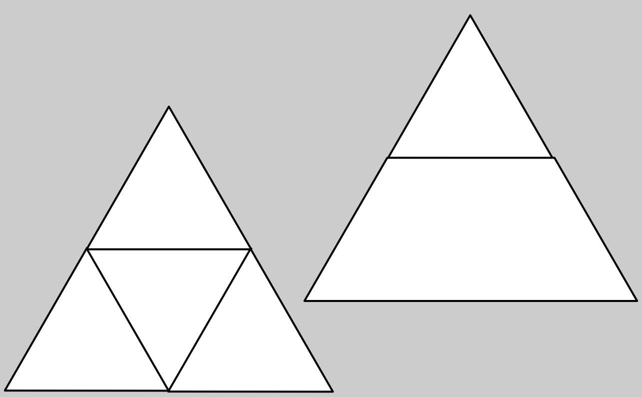
Rectangle:



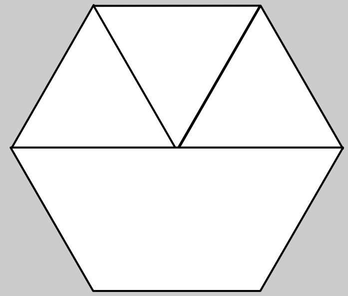
Isosceles triangle:



Equilateral triangle:

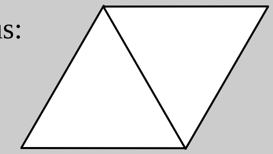


Hexagon:

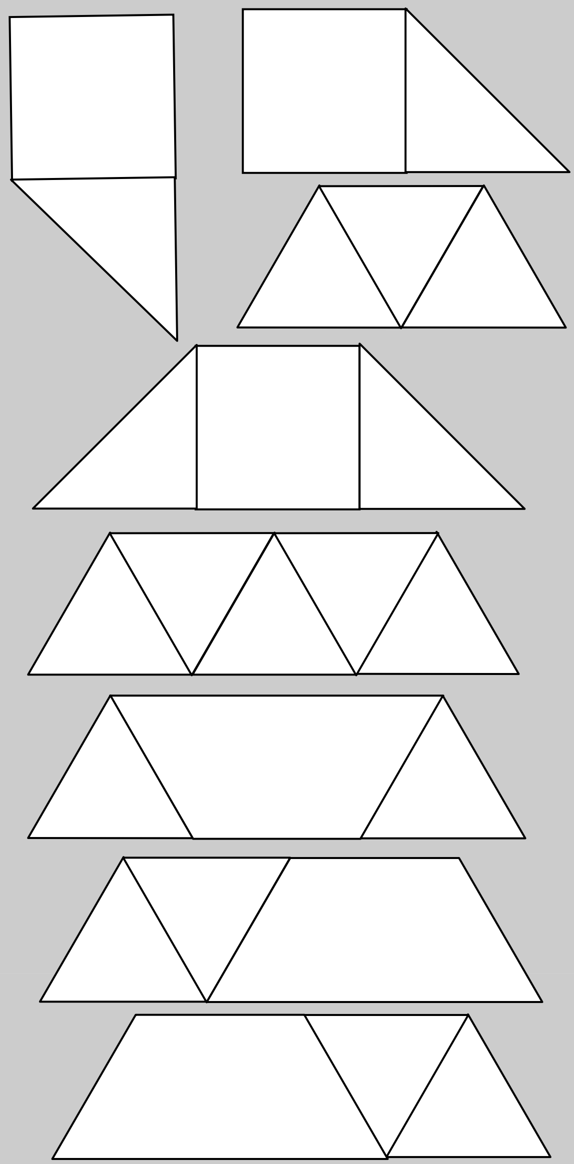


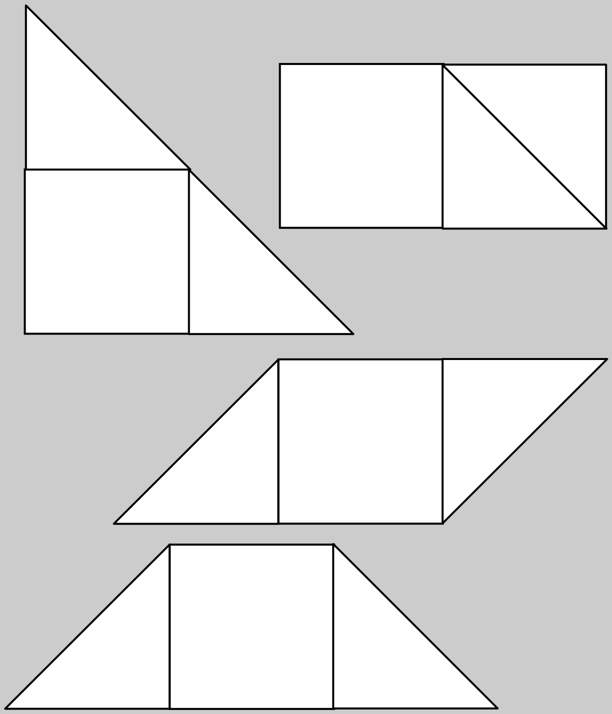
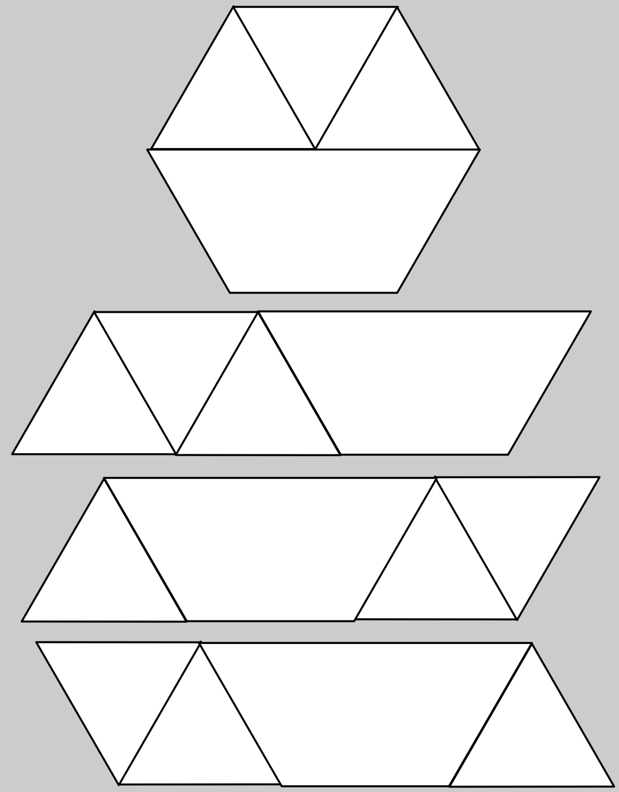
Kite: I could not make a kite with the shapes.

Rhombus:



Trapezoid:



B. *Sample response*: I chose two triangles and a square. I think I will be able to make squares and rectangles.  I could make a rectangle, a triangle, a trapezoid, and a parallelogram. I could not make the square using all three shapes.	**C.** *Sample response*: I chose three triangles and a trapezoid. I think I will be able to make a hexagon and parallelograms. 

Supporting Students

Struggling students

• If students are struggling to predict the shapes that can be made in **parts B or C**, you might suggest that they begin with only two polygons. After they have made and tested their predictions for those two polygons, they can add one more polygon to the two they already have. Encourage students to think about the shapes they can make with two polygons and how adding the new polygon to the arrangement will change it. Similarly, for **part C**, they can add another shape to the three shapes they already have.

Enrichment

• For **part A**, you might challenge students to make as many different congruent shapes as possible.

• If students need a further challenge, have them repeat **parts B and C** using five of the given polygons.

GAME: Shape Puzzles

• This optional game allows students to practise combining polygons.

• Begin by cutting out the Puzzle Cards and Puzzle Shapes from BLM 10A and BLM 10B.

• Here is a variation on the game:

- Place a book upright between the players so each can make his or her shapes without being seen.
- Players take turns drawing a Puzzle Card. They place it where both players can see it.
- Both players try to make the shape shown on the Puzzle Card in as many ways as possible.

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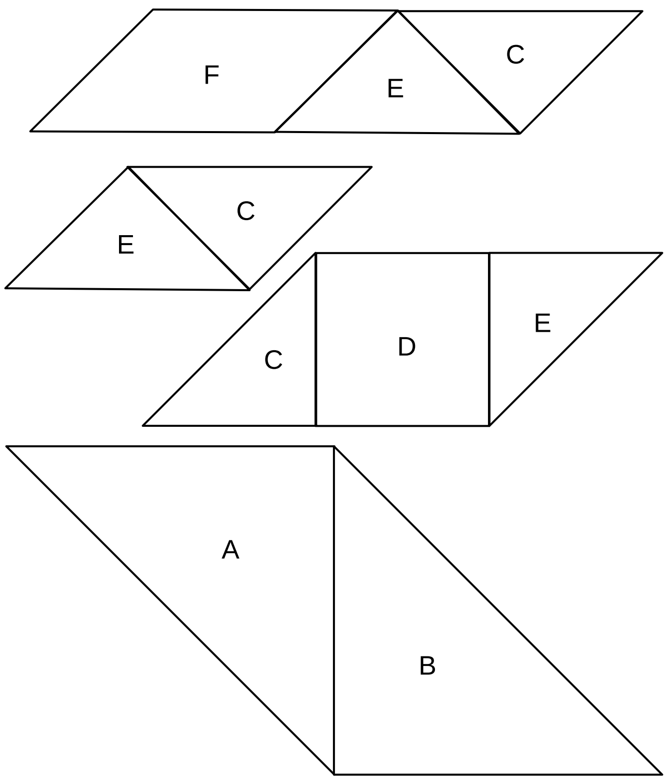
CONNECTIONS: Tangrams

- This optional connection has historical roots and is still very popular today. It provides an interesting context to practise combining polygons.
- Students can use BLM 11 Tangrams to help them figure out their answers.

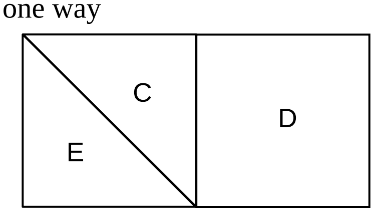
Answers

1. A and B, C and E

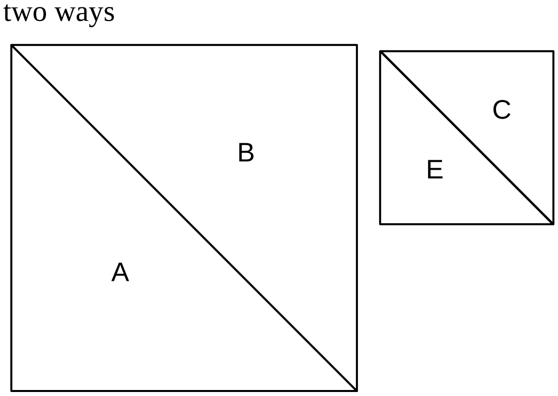
2. Parallelogram: four ways



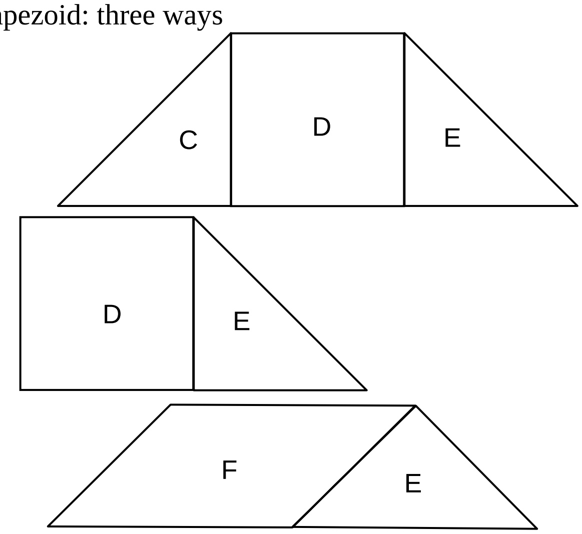
Rectangle: one way



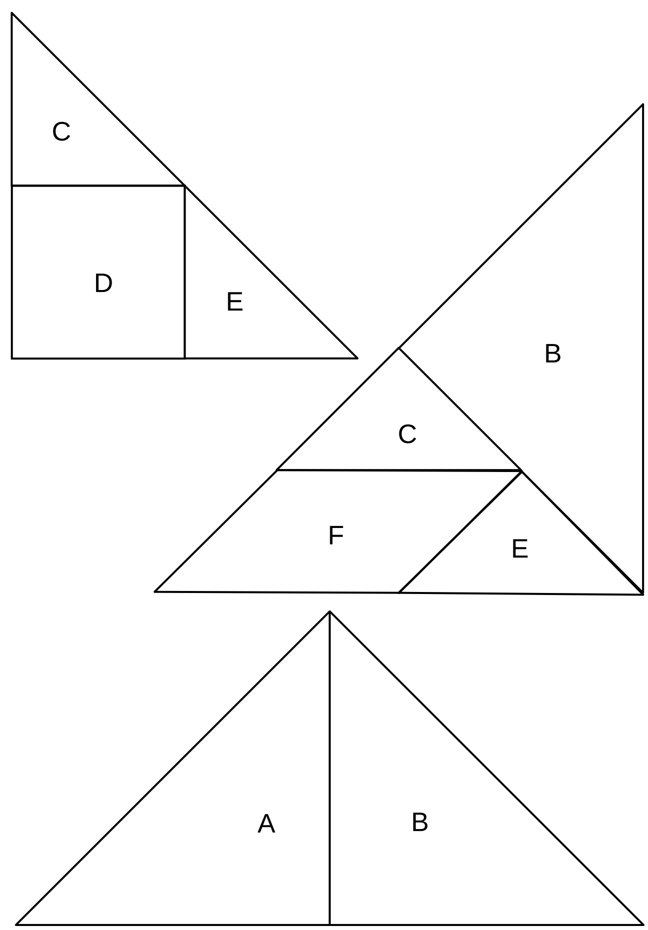
Square: two ways



Trapezoid: three ways



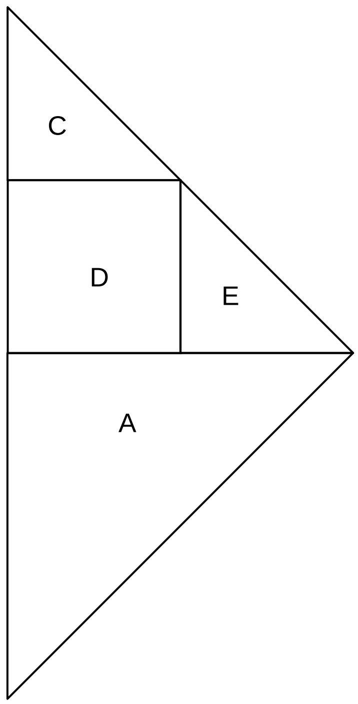
Isosceles triangle: six ways

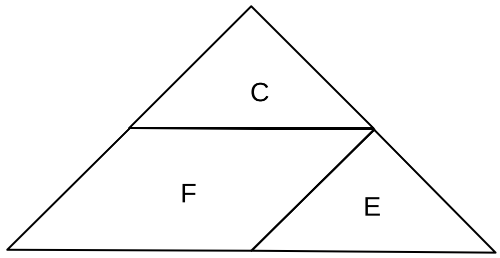


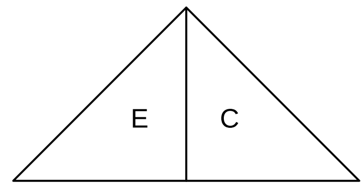
[Continued]

CONNECTIONS Answers [Continued]

Isosceles triangle [Cont'd]:







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5.2.3 Slides and Flips

Curriculum Outcomes	Outcome relevance
4-E9 Slides, Flips, and Turns (half and quarter): predict and confirm results for 2-D shapes • predict and confirm results for 2-D shapes under transformations	Transformations such as slides and flips are found everywhere in the world around us.

Pacing	Materials	Prerequisites
1 h	• BLM 12 Slides and Flips (optional) • BLM 13 Textile Design (for question 2; optional) • Grid paper	• identifying slides and flips • identifying congruent shapes

Main Points to be Raised

• Slides and flips make a copy of a shape in a new position. This copy is called the image.

• The image of a slide or flip is congruent to the original shape.

• The image of a slide points in the same direction as the original shape.

• The image of a flip points in the opposite direction of the original shape.

• A slide is one motion, but it can be described on a grid in two parts as how far left or right and then how far up or down.

Try This — Introducing the Lesson

A. Allow students to try this in pairs or small groups. Distribute scissors and copies of BLM 12 Slides and Flips to each pair or group. Alternatively, have them trace and cut out four copies of the parallelogram at the top of **page 149** of the student text.

Observe while students work. As they work, you might ask questions such as the following:

• *What did you do to the parallelograms to create the shape on the left?* (I moved one over and up.)

• *What did you do to the parallelograms to create the shape on the right?* (I flipped one over to the other side.)

• *Could you make the shape on the right without flipping a parallelogram?* (No. I would not be able to get the bottom part of the shape pointing out to the right.)

• If students have difficulty creating the shapes, you might tell them that each is made from two copies of the parallelogram. Suggest that they begin with one copy on top of another, oriented in the same way as the parallelogram was before they cut it out. It may also be helpful to ask them to think about what actions they are doing to create the shapes.

The Exposition — Presenting the Main Ideas

• Draw students' attention to the exposition on **page 149** of the student text. Read through the first part together. Be sure to use the terms *original shape* and *image* consistently when speaking with students to help them become comfortable with this new terminology.

• A common misconception about diagonal slides is that they are made up of two motions, one motion to the right or left followed by a motion up or down. This misconception is promoted by the way diagonal slides on a grid are described in two parts, for example, 2 units right and 4 units up. To help students understand this, suggest that they place their finger on one the vertices of the original shape and then slide their finger in two motions to determine the location of the image vertex. Then have them slide a tracing of the shape in one motion so that the original vertex and image vertex match.

• When reading about flips, be sure to point out the mirror lines. Explain that a mirror line is like a fold line that matches the original shape to the image. Have students trace an original shape, flip image, and mirror line from **page 149** and fold along the mirror line to verify this.

Revisiting the Try This

B. Students make the connection between the new terminology (slides and flips) and the actions they performed to create the shapes in **part A**.

Using the Examples

• Assign students to pairs. Have one student in the pair become the expert on **solution 1** of **example 1** and have the other student become the expert on **Solution 2**. Each student can then explain his or her example to the other student.

• Work through **example 2** with the class to make sure they understand it.

Practising and Applying

Teaching points and tips

Q 1: You might encourage students to look for as many examples as possible for each type of transformation. Although there is only one flip, it will help for students to rule out other possibilities.

Q 2: Students can record their answers on BLM 13 Textile Design. They can show each mirror line by making a fold in the paper that aligns the two shapes.

Q 6: This question might be assigned only to selected students. It touches on ideas that will be studied in greater detail in Class V. It may be necessary to remind students that in **part b)**, the single motion is described in two parts: left or right, then up or down.

Q 8: Use this last question as a closure question. It is a way to highlight the important ideas students have learned in the lesson.

Common errors

• Some students will look only at the orientation and not at the size of the shapes in **question 1**. Briefly discuss congruence and remind them that the image of a slide or flip must be congruent to the original shape.

• If students struggle to identify the mirror lines in **question 3 part b)**, suggest that they trace the original shape and the image onto a separate sheet of paper. They can fold to see where the mirror line is located then hold the paper over the book to see where on the grid the mirror line falls.

They can also draw the shapes directly on grid paper, which they can fold to locate the mirror line. It may help to point out that the mirror line reflects the grid as well as the shape, so the number of spaces from the original shape to the mirror line is the same as the number of spaces from the flip image to the mirror line.

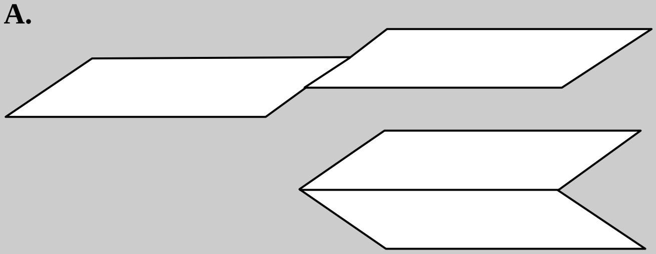
• Students may have difficulty predicting what the images will look like in **questions 4 and 5**. It may help to provide them with cut-out copies of the shapes to physically slide and flip.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can identify slides and flips in a real-world context
Question 4	to see if students can predict and describe the image of a slide
Question 5	to see if students can predict and describe the image of a flip
Question 7	to see if students can explain the differences between a slide image and a flip image

Answers

A.



B. The top shape shows a slide.
The bottom shape shows a flip.

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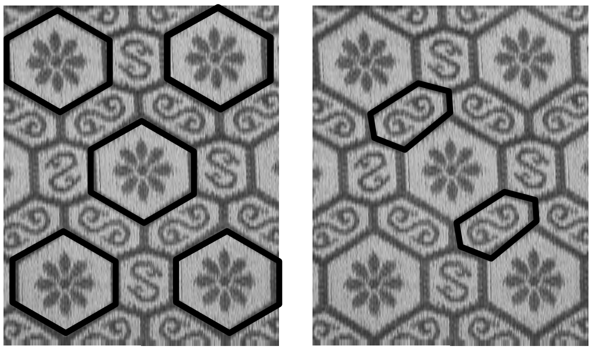
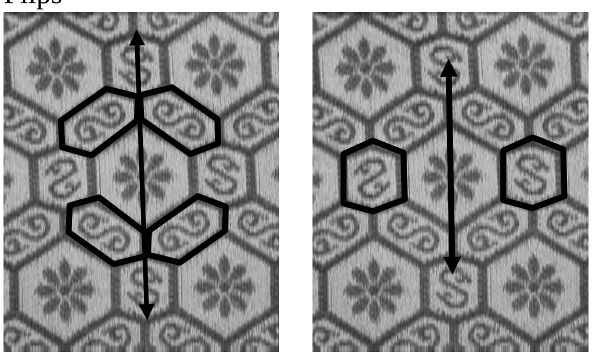
1. a) A and D, C and F; [They are the congruent and they face the same way.]
b) D and E; [They are congruent, face in opposite directions, and I can visualize a horizontal mirror line between them.]

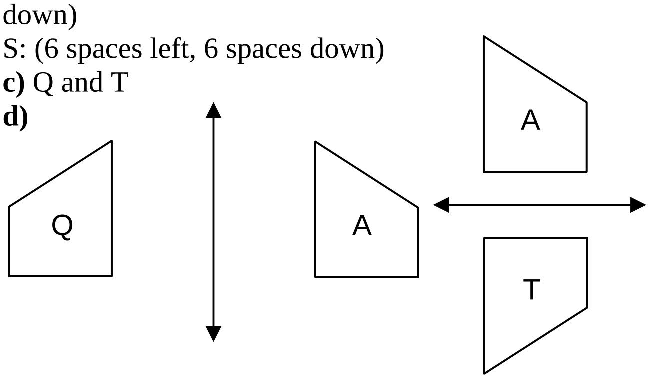
2. *Sample responses*:
Slides



Flips

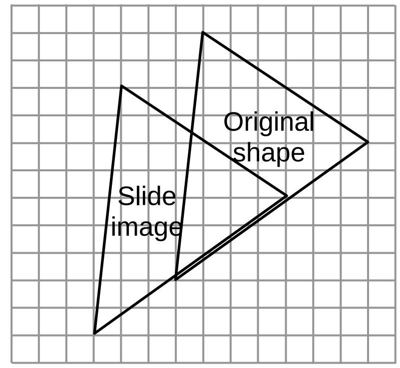


3. a) R and S
b) R: (5 spaces left) or (5 spaces left, 0 spaces up or down)
S: (6 spaces left, 6 spaces down)
c) Q and T
d)



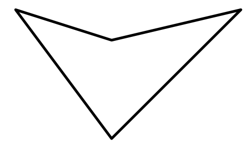
4. a) *Sample responses*:
It will be congruent and face the same way, but it will be farther down and to the left.

b)

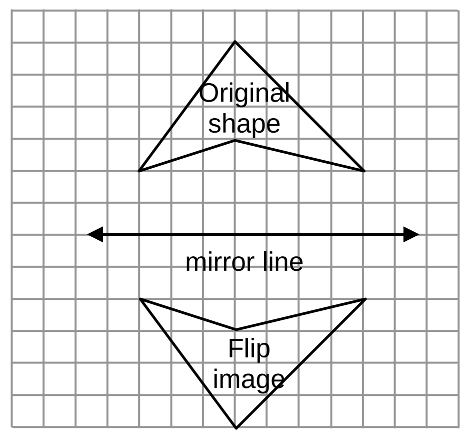


c) (3 spaces right, 2 spaces up)

5. a) *Sample responses*:
It will be congruent, but it will face the opposite way (it will point down instead of up) and it will be below the mirror line.



b)



Sample response: My prediction was correct.

6. a) The top vertex moved 12 squares (down).
The left and right vertices moved 4 squares (down).
The bottom middle vertex moved 6 squares (down).
b) Each vertex moved (3 spaces left, 2 spaces down).
c) *Sample response*: The vertices all move the same way for slides, but they move differently for flips.

[**7.** Different:
A slide image faces the same way as the original shape.
A flip image faces the opposite way.
The vertices all move the same way for a slide, but they move differently for a flip.
Alike:
Slides and flips both have images that are congruent to the original shape.]

8. They are both right. [When you flip a rectangle it looks the same as when you slide it because it is symmetrical.]

Supporting Students

Struggling students

• If students are struggling with the ideas in this lesson, you might make charts for the classroom wall that list the properties of each transformation.

For example:

Slides

- A slide image is congruent to the original shape.
- A slide image points the <u>same</u> way as the original shape.
- To describe a slide on a grid you tell how far left or right and how far up or down.

Flips

- A flip image is congruent to the original shape.
- A flip image points the <u>opposite</u> way as the original shape.
- To describe a flip you show the mirror line.

Include a visual reference for each type of transformation, such as those shown on **page 149** in the student text (with or without the grid).

Enrichment

• For a challenge, have students create their own shape on grid paper. Ask them to write instructions to transform the shape using slides and flips. They can exchange instructions with a partner and attempt to follow the instructions.

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5.2.4 Turns

Curriculum Outcomes	Outcome relevance
4-E9 Slides, Flips, and Turns (half and quarter): predict and confirm results for 2-D shapes • predict and confirm results for 2-D shapes under transformations	Turns are a difficult transformation for students to understand, yet they are important in many mathematical situations. By starting with half and quarter turns, students get a gradual introduction into the concept.

Pacing	Materials	Prerequisites
1 h	• Grid paper • Scissors • Large cardboard isosceles trapezoid and small trapezoids (optional)	• identifying and performing slides and flips • identifying congruent shapes • understanding clockwise and counterclockwise directions

Main Points to be Raised

• A turn image is congruent to the original shape.

• A turn is described by three things:

- the turn centre

- the direction of the turn

- the size of the turn

• The direction of a turn can be clockwise (cw) or counterclockwise (ccw).

• The size of a turn can be described as a fraction of a whole turn, or full circle.

• In the image of a $\frac{1}{4}$ turn, horizontal parts of a shape become vertical and vertical parts become horizontal.

• In the image of a $\frac{1}{2}$ turn, horizontal parts of a shape stay horizontal and vertical parts stay vertical.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Distribute two sheets of grid paper and scissors to each student or pair. They can use one sheet to create the cut-out kite and the other sheet to complete the activity. Observe while students work. As they work, you might ask questions such as the following:

• *Which ways could you turn the kite?* (I could turn it to the left or to the right.)

• *Could you turn the kite in the other direction and have it end up in the same spot?* (Yes, but I would have to turn it farther.)

• *What would eventually happen if you kept turning the kite?* (It would end up back where it started.)

• If students have difficulty drawing the kite accurately, you might suggest that they first draw the vertices on grid paper, making sure they are placed exactly as shown in the student text, and then connect them.

• Some students may need guidance in order to place the kite on the grid in such a way that both tracings stay entirely on the paper.

• While it is not incorrect to turn the kite a $\frac{1}{2}$ turn or a $\frac{3}{4}$ turn, encourage students who do so to stop turning the first time one of the diagonals lines up with a different gridline.

The Exposition — Presenting the Main Ideas

Begin by reviewing slides and flips briefly. It may help to refer to the charts you have placed on the class wall. You might ask questions such as:

• *How can you describe the single motion of a slide?* (How many spaces left or right and up or down the shape moves.)

• *What do you have to show to describe a flip?* (The mirror line.)

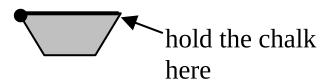
• *What do you know about slide images and flip images?* (They are congruent to the original shape.)

Tell the class that turns are like slides and flips because the image is congruent to the original shape.

• If possible, use a large cardboard isosceles trapezoid to model the actions as you work through the exposition with the class. Students can work in pairs to model the actions using small cut-out trapezoids at their desks.

• Trace the trapezoid on the board in its position before the turn and label it “Original”. Ask a student volunteer to hold the turn centre fixed as you turn the trapezoid. Have the student trace the trapezoid in its final position and label it “Turn image”.

• When you discuss the size of a turn, you can hold chalk at the vertex shown to trace out part of the circle as you turn the trapezoid.



Emphasize the idea that for a $\frac{1}{4}$ turn, horizontal sides become vertical and vertical sides become horizontal, while for a $\frac{1}{2}$ turn, horizontal sides stay horizontal and vertical sides stay vertical. Explain that this will be useful as they draw the images for turns.

Revisiting the Try This

B. This question allows students to make a formal connection between what was done in **part A** and the main ideas presented in the exposition. In this case, they think of the turn from **part A** in terms of the three relevant features: the turn centre, the direction of the turn, and the size of the turn. If students forget which kite was the original, have them label either kite as “Original” and the other as “Turn image”.

Using the Examples

• Ask students to read through the two examples with a partner. Have them duplicate the motions on grid paper. Provide time for students to ask about anything that might have confused them.

Practising and Applying

Teaching points and tips

Q 1: You might encourage students to look at two shapes at a time. If necessary, suggest that they first look for possible turn centres. While a turn centre can be located anywhere, you should direct them to look at vertices, as shown in the exposition.

Q 3 and 4: Predictions will vary. Keep in mind that they do not have to be correct predictions to be valid answers. Incorrect predictions can lead to deeper learning. Encourage students to follow **examples 1 and 2** when they draw the turn images.

Q 5: This question brings out the idea that a $\frac{1}{2}$ turn in one direction results in the same image as a $\frac{1}{2}$ turn in the other direction. Some students may have noticed this. Be sure to discuss how this is only true for $\frac{1}{2}$ turns.

Q 6: Use this last question as a closure question. It is a way to highlight the important ideas students have learned in this lesson and the previous one.

Common errors

• Many students will mistakenly think that Q and S show a turn in **question 1**. You might suggest that they trace Q onto paper, hold the paper fixed at the turn centre with the tip of their pencil, and turn the paper to see if the traced shape and S look the same. They need to realize that if a turn moves Q to S, the turn centre is the single point that does not move.

• Students may turn the trapezoid in the wrong direction in **question 4**. Since the image is the same, it may be difficult to notice in their completed work. If you see them turning the trapezoid in the wrong direction, be sure to point it out to make sure they understand the difference between cw and ccw.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can make and confirm a prediction about a $\frac{1}{4}$ turn
Question 4	to see if students can make and confirm a prediction about a $\frac{1}{2}$ turn
Question 6	to see if students can distinguish between a slide, a flip, and a turn

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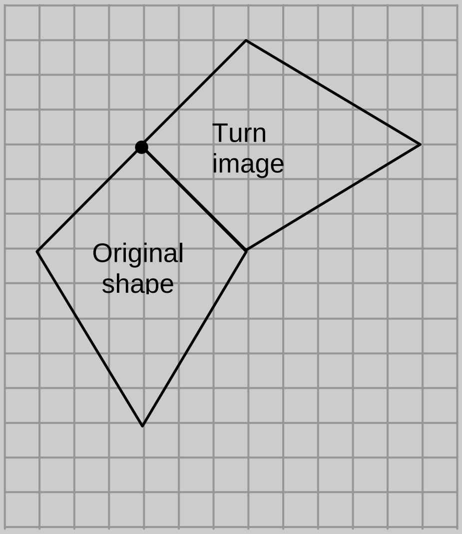
Answers

A. *Sample response*:

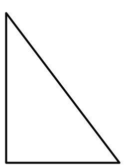


B. *Sample response*:
$\frac{1}{4}$ turn ccw around the vertex at the right angle.

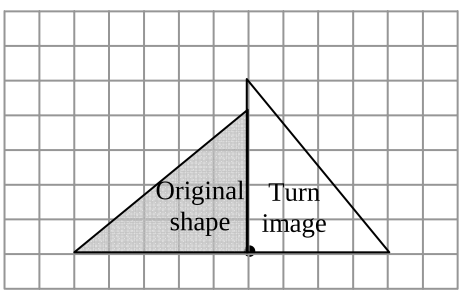
1. U and S, V and T; [*Sample response*:
I pictured turning one shape around the vertex where they touch. The shapes would match perfectly.]

2. $\frac{1}{4}$ turn ccw around the point where the two shapes touch.

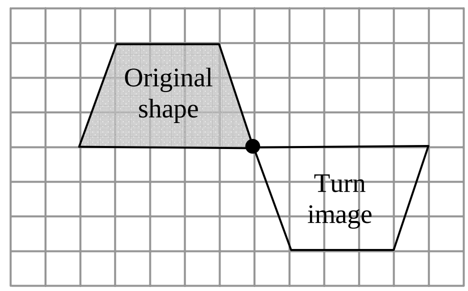
3. a)



b)



4. a) It will be congruent, but it will be upside down.
b)



5. They are both right. [If you go halfway around a circle one direction, you end up in the same place as if you went halfway around the circle in the other direction.]

6. a) C; [It is congruent and it faces down instead of sideways, as if A has been turned cw $\frac{1}{4}$ around its lower left vertex.]
b) B; [It is congruent and it faces the opposite way.]
c) D; [It is congruent and it faces the same way.]

Supporting Students

Struggling students

• Create a chart for the classroom wall to go with the charts for slides and flips:

Turns

- A turn image is congruent to the original shape.

- To describe a turn you tell about the turn centre, the direction of the turn, and the size of the turn.

• Some students might benefit from the use of cut-outs to turn and trace for **questions 3 and 4**.

Enrichment

• For **question 1**, you might challenge students to identify which pairs show slides and flips.

• Have students revisit the enrichment from **lesson 5.2.3**. They can create their own shape on grid paper and write instructions to transform it using turns as well as slides and flips. They can exchange instructions with a partner and attempt to follow the instructions.

CONNECTIONS: Logos

• This optional connection provides students with an opportunity to see transformations in the world around them.

• Before assigning the questions, have students look for other slides and flips in the TATA logo and for a flip in the Mercedes logo.

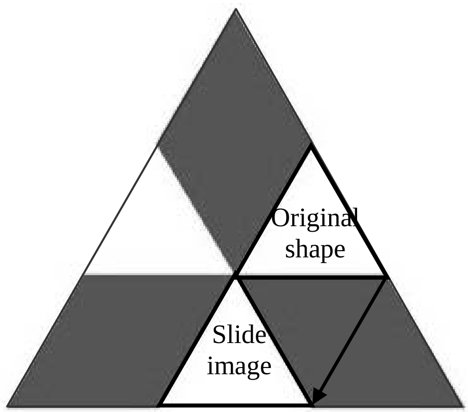
For example, in the TATA logo, the Ts not only show a slide but also a flip across a vertical flip line. The As also show a flip and a slide (this is because the Ts and As are vertically symmetrical).
In the Mercedes logo, there is a flip across a vertical line down the centre as well as a flip across a diagonal line through the centre.

• Students can record their answers on BLM 14 Logos, or trace the logos to record their answers as suggested in the student book.

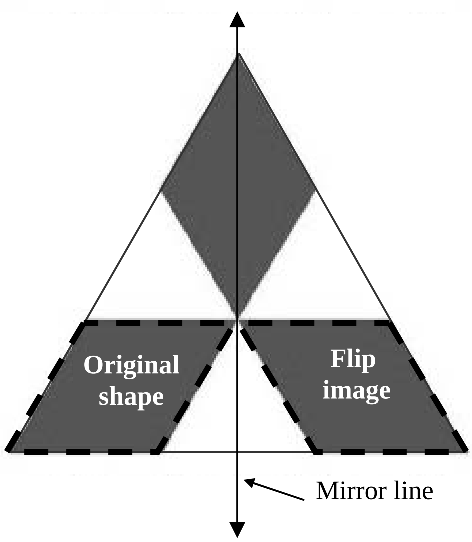
Answers

Sample responses:

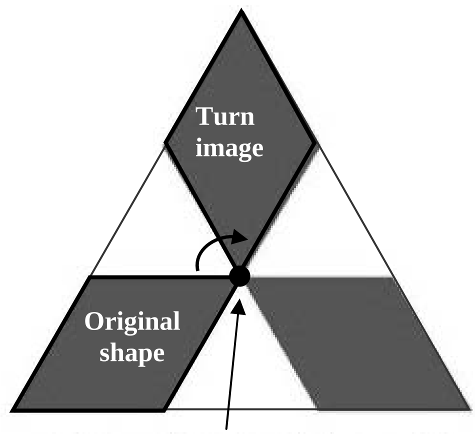
1. a)



b)



c)



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Chapter 3 3-D Geometry

5.3.1 EXPLORE: Building Shapes from Drawings

Curriculum Outcomes	Outcome Relevance
4-E1 Isometric Drawings: interpret • build simple cube structures from isometric drawings, include shapes that have "hidden" cubes	This essential exploration serves as an introduction to isometric drawings, which will be used further in Class V. Students develop visualization and spatial skills they interpret the isometric drawings.

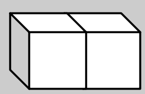
Pacing	Materials	Prerequisites
1 h	• Linking cubes • BLM 15 Sample Net of Cube (optional) • BLM 26 Isometric Dot Paper (optional)	• visualizing simple 3-D shapes • familiarity with the terms *front, overhead*

Exploration

• Assign students to pairs or small groups. Distribute seven or more linking cubes to each pair or group. With students, work through the introduction (in white) on **page 158** of the student text.

• You may wish to explain that *iso* means "equal" and *metric* means "distance". The dots in an isometric drawing are all the same distance apart. Contrast this to a square grid, where the diagonal distances are longer than the vertical and horizontal distances. BLM 26 Isometric Dot Paper can be used to create more examples of isometric drawings.

• Make sure that students understand the idea of a *front view*. Have students build this cube structure using linking cubes and place it on a piece of paper with front marked nearest to them.

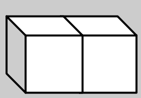


Front

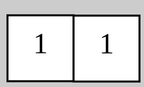
Have them turn the paper so that they can view the structure from different perspectives to see how its appearance changes.

• Explain that isometric drawings do not tell everything about a structure because they show only one view. Describing a structure in words can give more details. An overhead view that tells the number of cubes in each stack of cubes can also be useful.

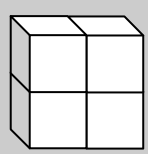
For example, these overhead views correspond to the cube structures shown:



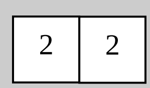
Front



Overhead view



Front



Overhead view

Ask students to build these cube structures and look at them from above. Then have them sketch the overhead view of each structure.

Observe while students work through the exploration. While they are working, you might ask questions such as the following:

• *How many cubes do you see in isometric drawing A?* (Two.)

• *Could an extra cube be hidden behind Structure A?* (No. I would be able to see it in the isometric drawing.)

• *How did you know where to add the extra cube behind Structure B?* (I added it at the back, where the cubes in front might hide it.)

Observe and Assess

As students work, notice the following:

• Do they build the cube structures to match the isometric drawings?

• Do they understand that different structures can match the same isometric drawing?

• Do they recognize where cubes can be hidden and where they cannot?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form groups to discuss their observations and answer these questions.

• *Why does an isometric drawing not always tell you enough to build a cube structure?*

• *How can two different structures match the same isometric drawing?*

• *How do you know where a cube might be located if you cannot see it in the isometric drawing?*

Answers

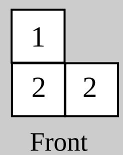
A. i) 2 cubes
ii) *Sample response*:
I cannot add any more cubes and still have it match the picture.

B. Structure B:
i) 3 cubes
ii) *Sample response*:
I can add one more cube behind the single cube so it is hiding behind the two-cube tower.

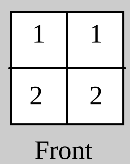
Structure C:
i) 4 cubes
ii) *Sample response*:
I can add one more cube behind the front single cube so it is hiding behind the two-cube tower.

Structure D:
i) 3 cubes
ii) *Sample response*:
I cannot add any more cubes and still have it match the picture.

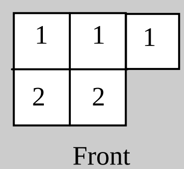
C. (*Overhead views showing number of cubes in each position*)
i)



ii) Behind the right two-cube tower;



iii) Yes, at the back on the right;



D. i) You cannot always see all the cubes in a drawing. Some cubes might be hidden. You could put those cubes in your structure or leave them out and still have it match the picture.
ii) I need to know the number of cubes used. It would also help to know if there are hidden cubes.

Supporting Students

Struggling students

• If students are struggling to figure out where to add the extra cubes in **part C**, you might suggest they use a trial and error approach.

For example, they could add a sixth cube anywhere and check to see if it changes the view from the front. If it does, they can move it to another position and try again.

Enrichment

• Challenge students to build four different cubes structures that look the same from the front view.

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5.3.2 Describing and Comparing 3-D Shapes

Curriculum Outcomes	Outcome relevance
4-E5 Prisms, Pyramids, Cones, and Cylinders: describe and compare • explore relationships concretely to identify properties (e.g., prisms: the number of vertices for any prism is twice the number of vertices for the base, e.g., a triangle-based prism has 6 vertices) • include relationships that deal with faces, edges, and vertices and understand why those relationships make sense • examine the similarities and differences between any pair of 3-D shapes	Because we live in a 3-D world, it is important to understand, describe, and compare 3-D shapes. Although students will have met these shapes in earlier grades, the focus here is on properties.

Pacing	Materials	Prerequisites
1 h, 15 min	• Linking cubes • BLMs 15 to 25 Sample Nets for: - Cube - Cone - Cylinder - Triangle-based Pyramid - Square-based Pyramid - Pentagon-based Pyramid - Hexagon-based Pyramid - Triangle-based Prism - Rectangle-based Prism - Pentagon-based Prism - Hexagon-based Prism • Scissors	• familiarity with prisms and pyramids

Main Points to be Raised

• A 3-D shape has different parts:

- A face is a 2-D shape that forms a flat surface on the 3-D shape.

- An edge is where two faces meet.

- A vertex is a point where three or more edges meet.

• A prism has two congruent polygon bases connected by parallelogram or rectangle faces. (In this lesson, we will only use rectangle faces.)

• A pyramid has one polygon base and triangle faces that meet at a vertex opposite the base.

• A cone has a circular base connected by a curved surface to a point called the apex.

• A cylinder has two congruent circular bases connected by a curved surface.

Try This — Introducing the Lesson

A. Allow students to try this in small groups. Distribute a square-based pyramid, a hexagon-based pyramid, and a hexagon-based prism to each group. (You can make these from BLMs 19, 21, and 25.)

Observe while students work. As they work, you might ask questions such as the following:

• *What features did you look at when you compared the shapes?* (I looked at the kinds of polygons I could see on the faces of each shape.)

• *What polygons do you see in the 3-D shapes?* (I see rectangles, triangles, hexagons, and a square.)

• *How else might you compare the 3-D shapes?* (I could look at the number of each kind of polygon in each shape or I could look at whether or not it has a pointy end.)

The Exposition — Presenting the Main Ideas

Have students turn to the exposition on **page 160** of the student text. If possible, distribute to the class linking cubes (or a cube made from BLM 15 Sample Net of Cube) and models of cones, cylinders, pentagon-based pyramids, and triangle-based prisms (BLMs 16, 17, 20, and 22).

As you work through the exposition together, be sure to discuss these points:

- A base is a special kind of face that helps you tell the name of a prism or pyramid.
- You can place a 3-D shape in any position so that the base is not always on the bottom.
- The apex of a cone is like a vertex, but edges do not meet there so it has a special name.
- When you have more than one vertex, they are called *vertices.*

• Be sure to emphasize the list of features to compare at the end of the exposition. Students will find it helpful to consult this list as they work through the example and the exercises in **Practising and Applying**.

• The other sample nets (BLMs) will be useful as further examples and for the exercises in **Practising and Applying**.

• Leave ample time for students to ask questions if they do not understand something.

Revisiting the Try This

B. Students apply what they have learned about vertices, edges, faces, and naming 3-D shapes to the shapes in **part A**.

Using the Examples

• Assign students to pairs or small groups. Distribute models of a cone, a triangle-based pyramid, and a triangle-based prism (BLMs 17, 18, and 22) to each pair or group. Copy the question from the example onto the board. Have each group answer the question. Then have them compare their solution to another group's solution and also to the solutions in the student text.

Practising and Applying

Teaching points and tips

Q 2: If necessary, encourage students to think about the shapes of the faces when they compare these two rectangular prisms.

Q 3: Remind students to consult the list of features at the end of the exposition as well as **example 1** to help them make their comparisons.

Q 4: This question is designed to reinforce the notion that a shape can rest on a side other than its base.

Q 5: Use this last question as a closure question. It is a way to highlight the new concepts (cones and cylinders) students have learned in the lesson.

Common errors

• Some students may be inclined to call the apex of the cone a vertex in **question 1**. This is incorrect. Remind them that a vertex is a point where edges meet and that in a cone, no edges meet at the apex.

• Students may have difficulty with **question 1 parts a) and b)**, and **question 4** because the shapes are not resting on their bases. It may help to show models of the shapes in various positions.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can recognize the similarities and differences between two rectangular prisms
Question 3	to see if students can compare and contrast a variety of 3-D shapes
Question 4	to see if students can recognize a prism in a different orientation
Question 5	to see if students understand the features of cones and cylinders

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Answers

A. *Sample response*: Alike: They are all 3-D shapes. They all have polygon shapes on them. Different: i) and iii) have points, but ii) does not. ii) and iii) have a hexagon, but i) does not.	**B. i)** 5 faces (4 triangle faces and 1 square base), 8 edges, and 5 vertices; Square-based pyramid. **ii)** 8 faces (6 rectangle faces and 2 hexagon bases), 18 edges, and 12 vertices; Hexagon-based prism. **iii)** 7 faces (6 triangle faces and 1 hexagon base), 12 edges, and 7 vertices; Hexagon-based pyramid.
1. a) Cone; 1 circle base, 1 curved surface, no vertex (has an apex), 1 curved edge. **b)** Pentagon-based prism; 7 faces (2 pentagon bases and 5 rectangle faces), 10 vertices, 15 edges. **c)** Pentagon-based pyramid; 6 faces (1 pentagon base and 5 triangle faces), 6 vertices, 10 edges. **2.** *Sample response*: Alike: - Both are prisms. - Both have 4 rectangle faces, 2 square bases, 8 vertices, and 12 edges. Different: - One can be called a cube and the other a square-based prism. - The cube has all faces congruent and the square-based prism has 2 congruent bases and 4 congruent side faces. - The cube has all square faces and the square-based prism has 2 square faces and 4 non-square rectangle faces. **3.** *Sample responses*: **a)** Alike: - Both have circle bases and one curved surface. - Both have one or more curved edges. - Neither has any vertices. Different: - The cylinder has two bases and the cone has one base. - The cylinder has two curved edges and the cone has one curved edge. - The cone has an apex but the cylinder does not.	**b)** Alike: - Both have two bases. Different: - The cylinder has circle bases and the prism has triangle bases. - The cylinder has a curved surface but the prism does not. - The cylinder has two curved edges and the prism has none. - The prism has some rectangle faces and the cylinder has none. **c)** Alike: - Both have one base. - Both have a point opposite the base. Different: - The cone has a circle base while the pyramid has a pentagon base. - The cone has a curved surface but pyramid does not. - The cone has a curved edge but pyramid does not. - The pyramid has some triangle faces but the cone has none. - The cone has an apex, while the pyramid has a vertex. **4.** No, he is wrong. [*Sample response*: A prism has two congruent bases joined by rectangles. The two congruent bases in this prism are triangles. This is a triangle-based prism.] [**5.** A cone is like a pyramid because they both have one base with a point opposite to it (a vertex for the pyramid and an apex for the cone). A cone is like a cylinder because they both have one curved surface and one or more curved edges.]

Supporting Students

Struggling students

• If students are struggling with the comparisons in **questions 2 and 3**, you might consider making a classroom chart to put on the wall listing the features for comparison:

- the number of faces
- the shape of the faces
- the number of edges
- the number of vertices
- the shape of the bases
- whether it has a curved surface
- whether it has a curved edge

Students may also find it helpful to keep a journal of the 3-D shapes they have encountered. Each page can have a picture of a 3-D shape copied from the student text with all of the relevant features labelled.

5.3.3 Folding and Making Nets

Curriculum Outcomes	Outcome relevance
4-E7 Nets: sketch for rectangular prisms • sketch a variety of nets for rectangular prisms including square-based prisms and cubes **4-E8 Models: building for cylinders, cones, prisms, and pyramids** • build, from given nets, cylinders and cones • build skeleton models for prisms and pyramids	A net is a convenient way to represent a 3-D shape in two dimensions and is particularly useful for studying the faces.

Pacing	Materials	Prerequisites
1 h, 15 min	• Centimetre grid paper • BLMs 15 to 25 Sample Nets for: - Cube - Cone - Cylinder - Triangle-based Pyramid - Square-based Pyramid - Pentagon-based Pyramid - Hexagon-based Pyramid - Triangle-based Prism - Rectangle-based Prism - Pentagon-based Prism - Hexagon-based Prism • Scissors	• familiarity with 3-D shape names • assembling nets for rectangle-based prisms

Main Points to be Raised

• A net is a 2-D shape that can be folded to make a 3-D shape. The shapes in a net correspond to the faces of the 3-D shape.

• The arrangement of shapes in a net is important; if the shapes are arranged improperly, they might not fold to make a 3-D shape.

• Nets for cones and cylinders are easy to recognize, A net for a cone has one circle while a net for a cylinder has two circles.

• A net is useful for studying the number, shapes, and arrangement of faces in a 3-D shape.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. As they work, you might ask questions such as the following:

• *How many shapes are in each net?* [**i)** six, **ii)** six, **iii)** three]

• *How many of the shapes in each net are congruent?* [**i)** two squares, four rectangles, **ii)** six squares, **iii)** two circles]

• *When you fold the net, what part of the 3-D shape will each square, rectangle, or circle be?* (One of the faces.)

• Provide models of the various 3-D shapes studied in **lesson 5.3.2** to help students visualize the form each folded net will take.

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The Exposition — Presenting the Main Ideas

• Talk through the exposition with the class. If possible, use large models to demonstrate how to fold the two 2-D arrangements of shapes on **page 164** of the student text. Talk about the role of the tabs for connecting the faces and how they are not actually part of the net.

• When you read about drawing a net for a rectangle-based prism, explain that the example goes into more detail.

• Assign students to pairs. Distribute scissors and copies of BLM 16 Sample Net for a Cone and BLM 17 Sample Net for a Cylinder to each pair. After discussing the nets for cones and cylinders on **page 165**, have students cut out and fold each net. You might ask them questions such as:

• *Which shapes in the nets became curved surfaces?* (In the cylinder it is the rectangle, and in the cone it is the wedge of the circle.)

• *What makes the net for a cone easy to recognize?* (It has one circle.)

• *What makes the net for a cylinder easy to recognize?* (It has two congruent circles.)

Be sure to point out that, like nets for other 3-D shapes, the nets for cones and cylinders do not all look the same. In particular, the placement and size(s) of the circle(s) and the size of the rectangle or wedge may vary.

• Allow ample time for students to ask questions to ensure that they understand.

Revisiting the Try This

B. Students revisit and formalize the thought processes they used in **part A**.

Using the Examples

• Assign students to groups. Give each group 12 linking cubes and centimetre grid paper. Work through the example with students to make sure they understand it. Model each step. Emphasize that it is important to roll the prism in order to maintain the correct arrangement of shapes in the net.

Practising and Applying

Teaching points and tips

Q 3: Some students may choose to sketch the nets for the cube freehand. This will require them to use visualization skills and have a good understanding of 3-D shapes. Other students may prefer to use a linking cube to create the nets using the techniques of **example 1**. In this case, encourage them to roll the cube differently to create the different nets.

Q 4: Remind students to follow the method shown in **example 1**.

Q 5: Encourage students to look at the six faces to help them decide what the 3-D shape could be. They could consider which shapes have those faces and then which of those shapes are congruent and which are not.

Q 6: Use this last question as a closure question. It is a way to highlight the ideas students have learned in the lesson. You might encourage them to think of the ways in which nets are useful: to study the number of faces, the shapes of the faces, and the way the faces are arranged.

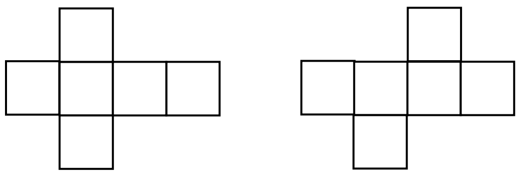
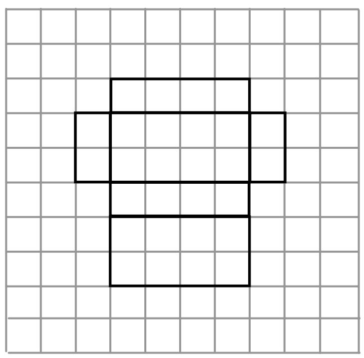
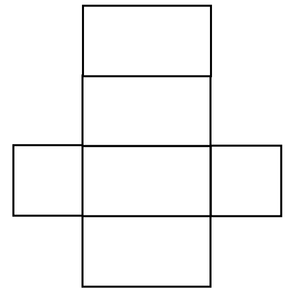
Common errors

• Some students may not recognize the net in **question 1 part a)** as a net for a cone because the wedge part is a different shape than in the cone in the net they made from BLM 17. Remind them that nets for the same shape can look different. Encourage them to visualize folding the net to make the 3-D shape. They may also benefit from tracing the net and cutting it out to see the shape it makes.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can identify nets for a cone and a cylinder
Question 3	to see if students can draw a net for a cube given in an isometric drawing
Question 4	to see if students can use a model to create a net for a rectangle-based prism
Question 6	to see if students can visualize and compare nets for different 3-D shapes

Answers

A. i) Square-based prism **ii)** Cube **iii)** Cylinder	**B.** *Sample response*: I looked at the shapes to see what the bases might be. I could also tell if the 3-D shape had one base or two.
1. a) Cone **b)** Cylinder **2.** B; [There are matching sides and a matching top and bottom.] **3.** *Sample response*:  **4.** *Sample response*: 	**5. a)** Square-based prism **b)** *Sample response*:  [**6.** *Sample response*: Alike: - Both will be made of 6 rectangles. Different: - The net for the cube will have all squares, while the net for the rectangle-based prism will have six non-square rectangles. - The squares in the net for the cube will all be congruent, but the net for the prism will have three pairs of congruent shapes.]

Supporting Students

Struggling students

• If students have difficulty with **question 4**, you might suggest putting a small piece of tape on each face of the prism as they trace it to help them keep track of what they have done.

• If students need to improve their visualization skills, encourage them first to make a prediction for **questions 1, 2, and 5** and then to trace the nets, cut them out, and fold to check their work.

Enrichment

• For **question 1**, you might challenge students to trace the shapes in the net in a different arrangement to create new nets for the cone and cylinder. Have them cut out the nets and fold to check.

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5.3.4 EXPLORE: Building Skeletons

Curriculum Outcomes	Outcome Relevance
4-E5 Prisms, Pyramids, Cones, and Cylinders: describe and compare • explore relationships concretely to identify properties (e.g., prisms: the number of vertices for any prism is twice the number of vertices for the base, e.g., a triangle-based prism has 6 vertices) • include relationships that deal with faces, edges, and vertices and understand why those relationships make sense • examine the similarities and differences between any pair of 3-D shapes **4-E8 Models: building for cylinders, cones, prisms, and pyramids** • build, from given nets, cylinders and cones • build skeleton models for prisms and pyramids	This essential exploration extends the ideas of the previous two lessons. Skeleton models are another way to represent and investigate 3-D shapes. They are particularly useful for studying vertices and edges.

Pacing	Materials	Prerequisites
1 h, 15 min	• Sticks of two different sizes • Clay, or gum, or flour dough	• familiarity with pyramids and prisms

Exploration

• With students, work through the introduction (in white) on **page 168** of the student text. Make sure they understand that there are no faces on the skeleton; the skeleton is a frame, like a body's skeleton, and the faces are like a skin that would cover the frame. The edges define where the faces would be.

• Have students work in small groups for **parts A to E**. Give each group 24 short sticks, 48 longs sticks, and a quantity of clay, gum, or dough. (If you do not have these things, you may consider splitting the shapes in **part A** between two groups and having the groups share their skeletons to fill in the charts for **part B**.)

Observe while students work. While they are working, you might ask questions such as the following:

• *What part of the skeleton represents the vertices?* (The clay balls.)

• *What part of the skeleton represents the edges?* (The sticks.)

• *How did you count the number of faces?* (I looked at the shapes that the sticks made.)

Be sure students have completed **part B** correctly before they proceed with the rest of the exploration.

Observe and Assess

As students work, notice the following:

• Do they build the skeletons successfully?

• Do they complete the charts in **part B** correctly?

• Do they recognize the relationship between the base and the number of vertices, edges, and faces?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and answer these questions.

• *What is the relationship between the number of sides of the base and each?*

- the number of vertices

- the number of edges

- the number of faces

• *How can you use these relationships to make predictions about other shapes without building their skeletons?*

Answers

B.

Name of prism	Number of sides of the base	V Number of vertices	E Number of edges	F Number of faces
Rectangle-based prism	4	8	12	6
Cube	4	8	12	6
Pentagon-based prism	5	10	15	7
Triangle-based prism	3	6	9	5

Name of pyramid	Number of sides of the base	V Number of vertices	E Number of edges	F Number of faces
Triangle-based pyramid	3	4	6	4
Square-based pyramid	4	5	8	5
Pentagon-based pyramid	5	6	10	6

C. i) There are twice as many vertices as the number of sides in the base; There are two bases, each with the same number of sides.
ii) There are three times as many edges as the number of sides in the base; There are two bases, each with the same number sides connected by the same number of edges.
iii) There are two more faces than the number of sides in the base; There are two bases, plus faces on each side of the base.

D. i) There is one more vertex than the number of sides of the base; There are the same number of vertices in the base as the number of sides in the base and there is one vertex across from the base.
ii) There are twice as many edges as the number of sides in the base; There is one base connected to the vertex by the same number of edges as the number of vertices in the base (which is the same as the number of sides).
iii) There is one more face than the number of sides in the base; There is a face attached to each side of the base, plus the base itself.

E. i) Vertices: 12
Edges: 18
Faces: 8
ii) Vertices: 7
Edges: 12
Faces: 7

Supporting Students

Struggling students

• If students are struggling to count the number of vertices, edges, and faces in **part B**, you might consider giving guidance as to how to count.

For example:

Have them follow these steps when working with a pyramid:

- Count the vertices in the base, then count the opposite vertex.
- Count the edges in the base, then count the edges that meet at a single point.
- Count the face that is the base, then count the triangle faces.

Have them follow these steps when working with a prism:

- Count the vertices in one base, then count the vertices in the other base.
- Count the edges in one base, then count the edges in the other base, then count the edges connecting the two bases.
- Count the faces that are the bases, then count the rectangle faces.

It may also be helpful for students to work in pairs so that one student can count the bottom part and the other can count the top part, and so on.

Enrichment

• Challenge students to visualize or build a shape with a given number of vertices or edges, e.g. a shape with 16 vertices.

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UNIT 5 Revision

Pacing	Materials
2 h	• Rulers • Scissors (optional) • Linking cubes • Centimetre grid paper • BLMs 15 to 19, 21, 24, and 25 (Sample Nets)

Question	Related Lesson(s)
1	Lesson 5.1.1
2	Lesson 5.1.2
3	Lessons 5.1.3 and 5.1.4
4	Lesson 5.1.4
5	Lesson 5.2.1
6	Lesson 5.2.2
7	Lessons 5.2.3 and 5.2.4
8 and 9	Lesson 5.2.3
10	Lesson 5.2.4
11	Lesson 5.3.1
12 and 15	Lesson 5.3.2
13	Lesson 5.3.2
14	Lesson 5.3.3
16	Lessons 5.3.3 and 5.3.4

Revision Tips

Q 1: Some students may choose to measure side lengths, while others may wish to make copies of the triangles to fold or compare to the originals. Both of these methods are acceptable.

Q 3: Allow students to use any of the possible appropriate names for the quadrilaterals.

Q 6: You might encourage students to trace the shapes on a separate sheet of paper, cut them out, and arrange them. They can then make sketches of the arrangements. Alternatively, they can trace the shapes directly into an arrangement on their page.

Q 11: The nature of this question makes it difficult to record an answer. You might suggest that students include a word description of their structure or make an overhead view with each square labelled to show the number of cubes it represents.

Q 12 and 13: Some students may find it helpful to use models of the 3-D shapes for making the descriptions and comparisons. Provide copies of BLM 15 to 18, 20, 23, and 24 as required.

Q 14: Encourage students to measure to see which shapes are congruent to help them decide which faces of the 3-D shape are congruent.

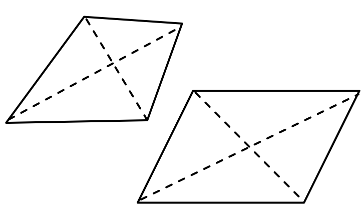
Answers

1. A is equilateral, B is isosceles, and C is scalene.
[*Sample response*:
I measured the side lengths to look for congruent sides.]

2. *Sample responses*:
a)
- Has a right angle: A, E
- Does not have a right angle: B, C, D
b)
- Scalene: A, D
- Isosceles: B, E
- Equilateral: C
c) A: 0
B: 1
C: 3
D: 0
E: 1
d) A: 0
B: 2
C: 3
D: 0
E: 2

3. a) A: trapezoid or isosceles trapezoid
B: rectangle or parallelogram
C: parallelogram, rhombus, rectangle, or square
D: parallelogram or rhombus
E: trapezoid
F: kite
G: parallelogram
b) *Sample response*:
- Has parallel sides: A, B, C, D, E, G
- Does not have parallel sides: F
c) *Sample response*:
- Has one pair of congruent sides: A
- Has two pairs of congruent sides: B, C, D, F, G
- Has no congruent sides: E

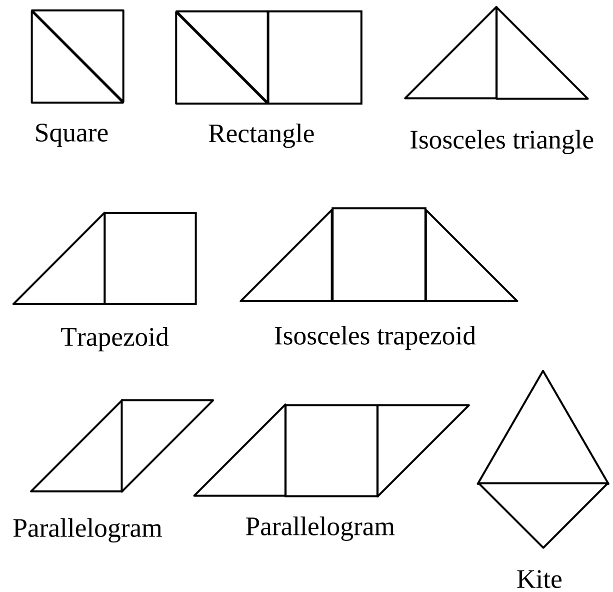
4. a)



b) *Sample response*:
Alike:
- They both have two pairs of congruent sides
Different:
- The congruent sides are across from each other in the parallelogram and beside each other in the kite.
- The parallelogram has parallel sides. The kite does not.
- The kite has a diagonal that is a line of symmetry. The parallelogram does not.

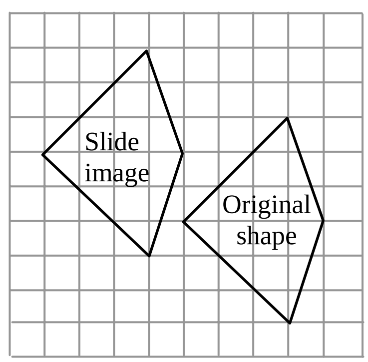
5. a) A and D, B and E
[**b)** They are exactly the same size and shape.
I traced one shape in each pair and put the tracing over the other shape to compare.]

6. Eight shapes

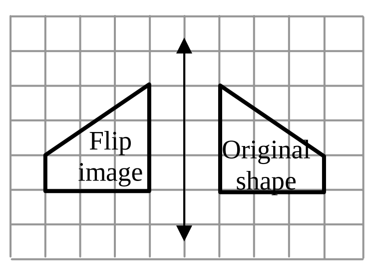


7. Slide: B and D
Flip: A and B
Turn: A and C

8.



9.



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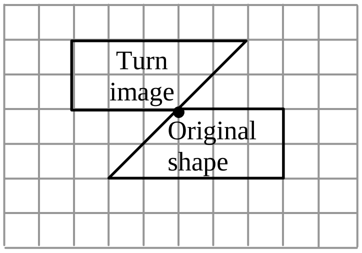
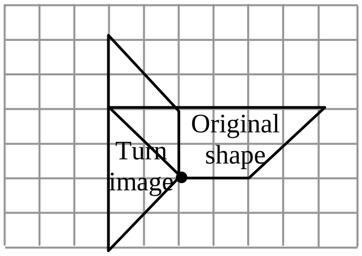
10. a)



b)



11. a) *Sample response*:
The two possible structures look like this when viewed from above; the numbers indicate the number of cubes in each position:

1	2	1

Front

or

		1
1	2	1

Front

b) Yes; [There could be cube at the back that you cannot see in the isometric drawing.]

12. a) Hexagon-based pyramid; 7 faces (1 hexagon base and 6 triangle side faces), 12 edges, 7 vertices.
b) Hexagon-based prism; 8 faces (2 hexagon bases and 6 rectangle side faces), 18 edges, 12 vertices.
c) Triangle-based pyramid; 4 faces (1 triangle base and 3 triangle side faces), 6 edges, 4 vertices.

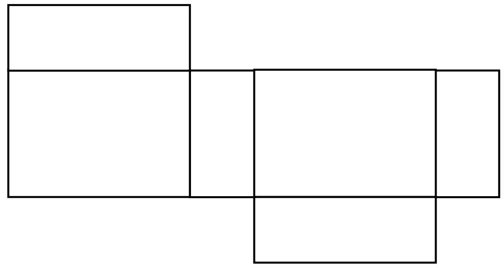
13. *Sample responses*:
a) Alike:
- They both have two bases.

Different:
- The cylinder has circle bases but the prism has pentagon bases.
- The cylinder has a curved surface and curved edges. The pentagon-based prism does not.

The prism has vertices but the cylinder does not.
b) Alike:
- They both have one base.
- Each has a point across from the base.

Different:
- The cone has a circle base, while the pyramid has a square base.
- The cone has a curved surface and a curved edge. The square-based pyramid does not.

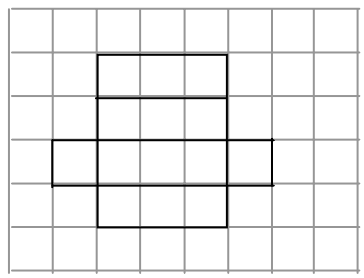
14. a) *Sample response*:

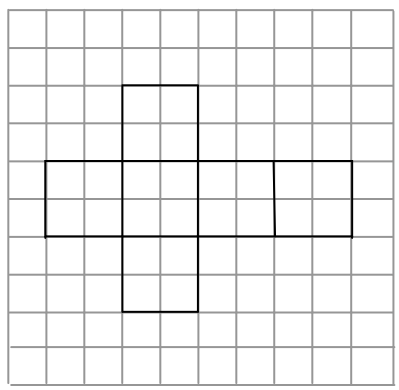


b) Rectangle-based prism

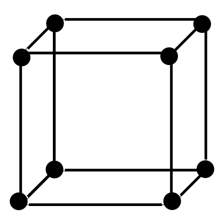
[**15.** *Sample response*:
A cube has square faces. A rectangle-based prism has rectangle faces. Squares are special rectangles, so a cube is a special rectangle-based prism.]

16. *Sample responses*:
a)





b)



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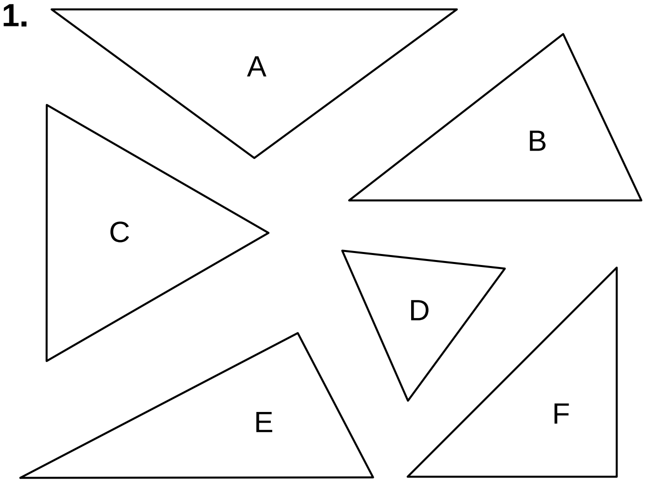
UNIT 5 Geometry Test

1.



a) Determine whether each triangle above is equilateral, isosceles, or scalene.

A is ______________________________

B is ______________________________

C is ______________________________

D is ______________________________

E is ______________________________

F is ______________________________

b) Tell what you did in **part a)** to figure out what type each triangle was.

c) How many lines of symmetry does each triangle have?

A _____ C _____ E _____

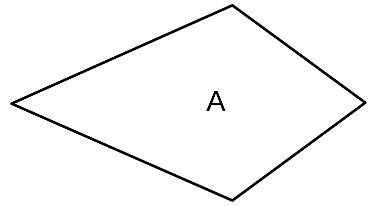
B _____ D _____ F _____

d) How many congruent angles does each triangle have?

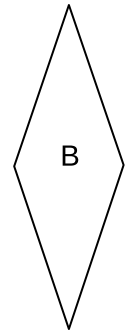
A _____ C _____ E _____

B _____ D _____ F _____

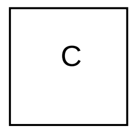
2. a) Name each quadrilateral.



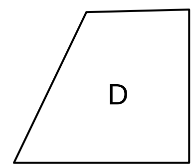
A __________________



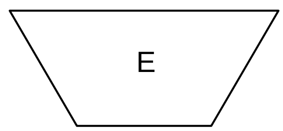
B __________________



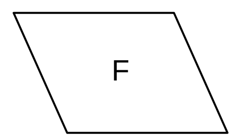
C __________________



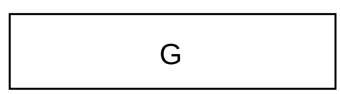
D__________________



E __________________



F__________________



G __________________

b) Sort the quadrilaterals in **part a)** into two or more groups. Tell your sorting rule.

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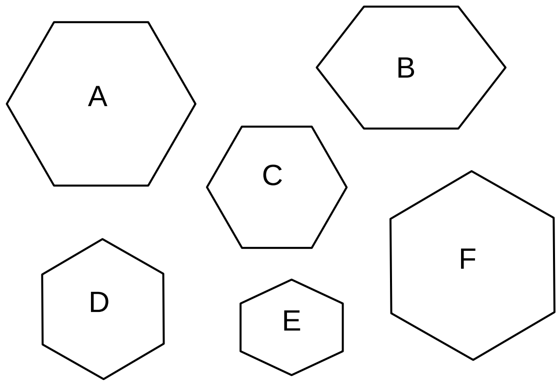
3.

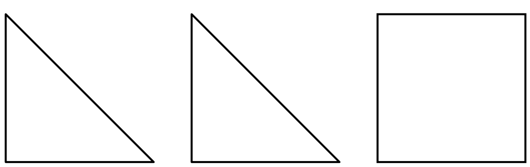


a) Which hexagons above are congruent?

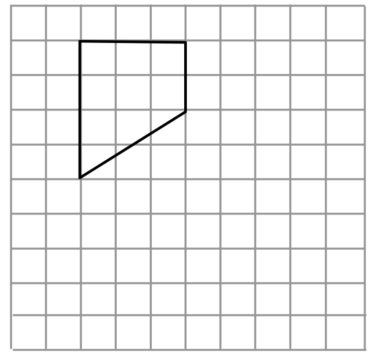
b) Tell what you did in **part a)** to find the congruent hexagons.

4. Create different shapes by combining these polygons along whole sides.

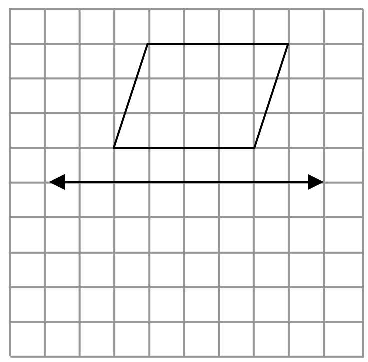
- How many shapes did you make? Name each shape.
- Sketch each shape, showing the polygons you used.



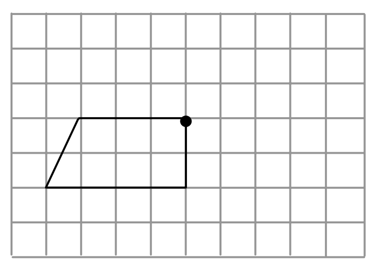
5. Draw the image of this trapezoid after a slide that is (3 units right, 4 units down).



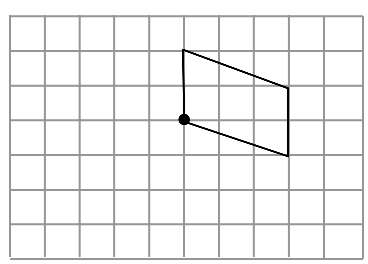
6. Draw the flip image of this parallelogram.



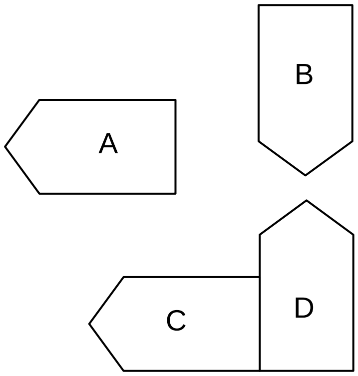
7. a) Draw the image of this shape after a $\frac{1}{2}$ turn ccw around the turn centre shown.



b) Draw the image of this shape after a $\frac{1}{4}$ turn cw around the turn centre shown.



8.

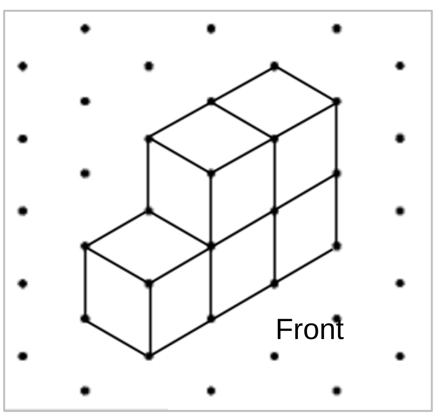


Which shapes show a slide? ________

Which shapes show a flip? ________

Which shapes show a turn? ________

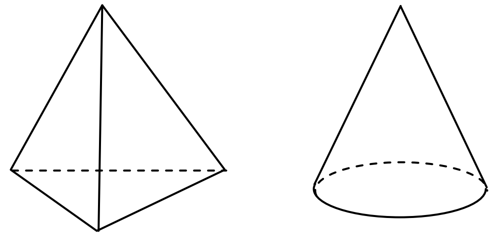
9. Build two different cube structures that match this isometric drawing.



Describe your structures using words or pictures.

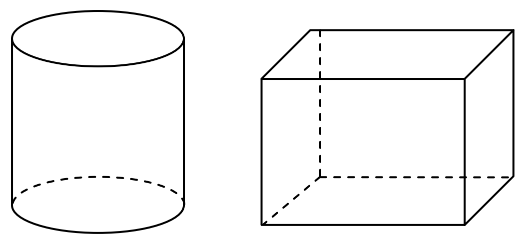
10. Name each shape. Then compare each pair of shapes by telling how they are alike and how they are different.

a)



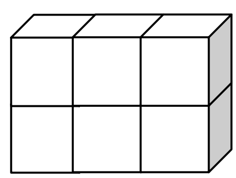
______________ ______________

b)



______________ ______________

11. This prism is made from six linking cubes. Create a net for it on grid paper.



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UNIT 5 Test

Pacing	Materials
1 h	• Rulers • Scissors (optional) • Linking cubes • Centimetre grid paper • BLMs 15 to 18 and 24 Sample Nets

Question	Related Lesson(s)
1	Lessons 5.1.1 and 5.1.2
2	Lessons 5.1.3 and 5.1.4
3	Lesson 5.2.1
4	Lesson 5.2.2
5	Lesson 5.2.3
6	Lesson 5.2.3
7	Lesson 5.2.4
8	Lessons 5.2.3 and 5.2.4
9	Lesson 5.3.1
10	Lesson 5.3.2
11	Lesson 5.3.3

Select questions to assign according to the time available.

Answers

1. a) A: isosceles, B: scalene, C: equilateral, D: equilateral, E: scalene, F: isosceles
b) *Sample response*:
I measured the sides in each triangle to see how many were the same length.
c) A: 1, B: 0, C: 3, D: 3, E: 0, F: 1
d) A: 2, B: 0, C: 3, D: 3, E: 0, F: 2

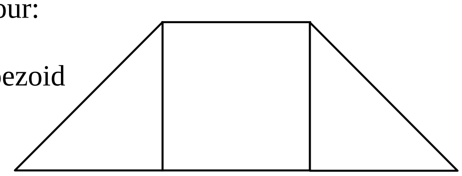
2. a) *Sample response*:
A: kite, B: rhombus, C: square, D: trapezoid,
E: isosceles trapezoid, F: parallelogram, G: rectangle
b) *Sample response*:
- No parallel sides: A and B
- One pair of parallel sides: D and E
- Two pairs of parallel sides: C, F, and G

or:
- No diagonals are lines of symmetry: D, E, F, G
- One diagonal is a line of symmetry: A
- Both diagonals are lines of symmetry: B, C

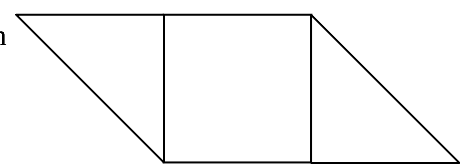
3. a) A and F; C and D
b) *Sample response*:
I traced each hexagon and put the tracing over the other hexagons to see if they matched.

4. Four:

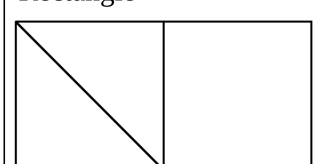
Trapezoid



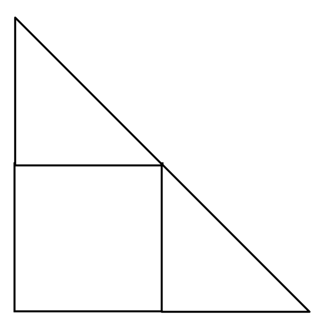
Parallelogram



Rectangle



Triangle



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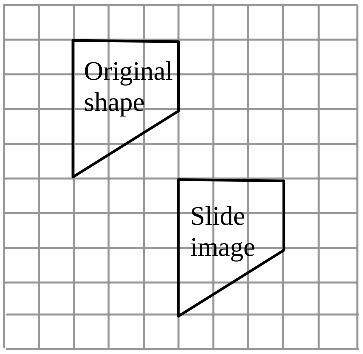
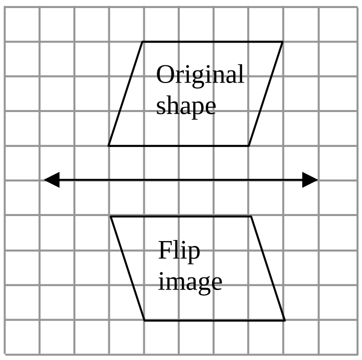
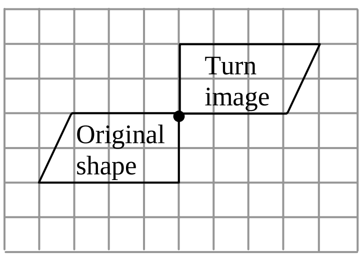
5.



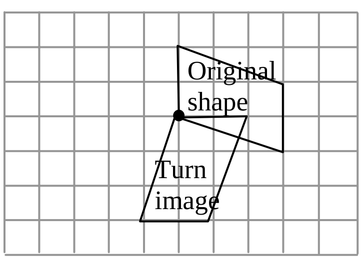
6.



7. a)

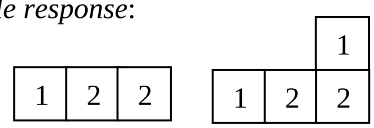


b)



8. Slide: A and C; flip: B and D; turn: C and D

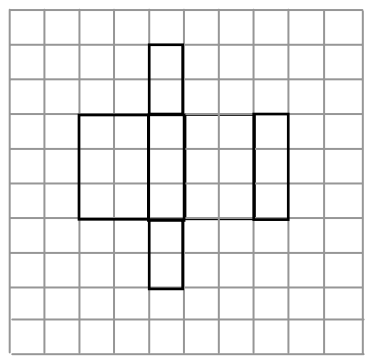
9. *Sample response*:



10. a) Triangle-based pyramid, cone
Sample response:
Alike:
- Both have one base with a point across from it.
Different:
- The prism has four vertices but the cone has none.
- The cone has an apex but the prism does not.
- The cone has a curved edge and a curved surface but the prism does not.

b) Cylinder, rectangle-based prism
Sample response:
Alike:
- Both have two congruent bases.
Different:
- The cylinder has no vertices; the prism has eight vertices.
- The cylinder has circle faces; the prism has only rectangle and square faces.
- The cylinder has a curved surface; the prism does not.

11. *Sample response:*



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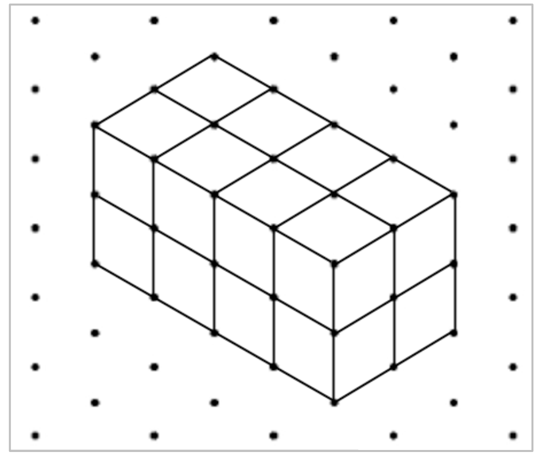
UNIT 5 Performance Task — Building and Examining a Structure

A. i) This structure is made of 16 centimetre cubes. Build a cube structure to match it.



ii) What type of 3-D shape is it? How many faces, vertices, and edges does it have?

B. i) Draw a net for your structure on centimetre grid paper.

ii) Draw the diagonals for each quadrilateral face in your net.

C. Find an example of each shape below in your net and trace it.

i)	scalene triangle	**ii)**	isosceles triangle
iii)	parallelogram that is not a rectangle	**iv)**	kite
v)	trapezoid	**vi)**	isosceles trapezoid
vii)	rhombus that is not a square	**viii)**	hexagon

D. i) Compare the triangles from **part C**. How many lines of symmetry does each triangle have? How many congruent angles does each triangle have?

ii) Sort the quadrilaterals from **part B** into two or more groups. Tell your sorting rule.

E. Look for an example of a slide, a flip, and a turn in your net.
Trace each and label the original shape and the image.

i) For the slide, describe how the shape moved.

ii) For the flip, draw the mirror line.

iii) For the turn, mark the turn centre and describe the size of the turn and the direction of the turn (cw or ccw).

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UNIT 5 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
4-E1 Isometric Drawings: interpret **4-E3** Quadrilaterals: sort by properties (concretely) and make generalizations **4-E4** Triangles: discover properties (concretely), name, and draw **4-E7** Nets: sketch for rectangular prisms **4-E9** Slides, Flips, and Turns (half and quarter): predict and confirm results for 2-D shapes	1 h	• Linking cubes • BLM 15 Sample Net of Cube (optional) • Centimetre grid paper

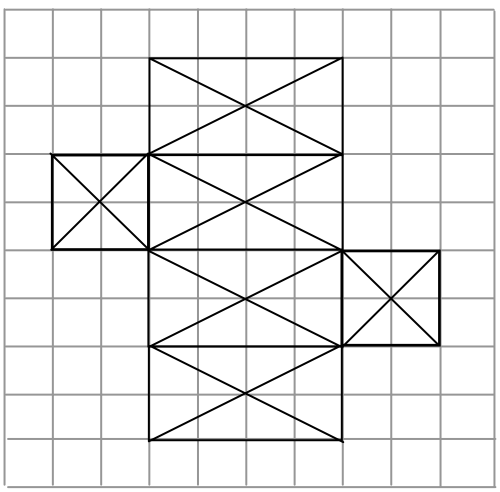
How to Use This Performance Task

You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. It could also be used as enrichment material for some students. You can assess performance on the task using the rubric provided on the next page.

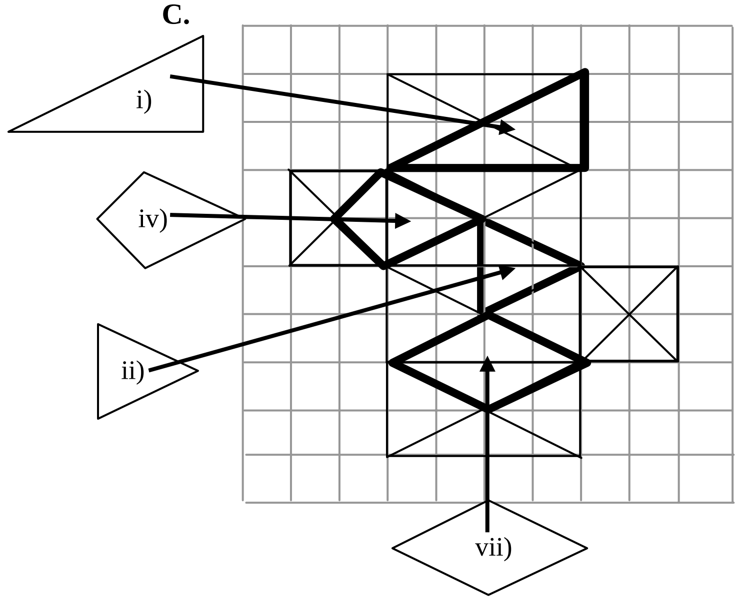
Sample Solution

A. ii) Square-based prism; 6 faces, 8 vertices, 12 edges.

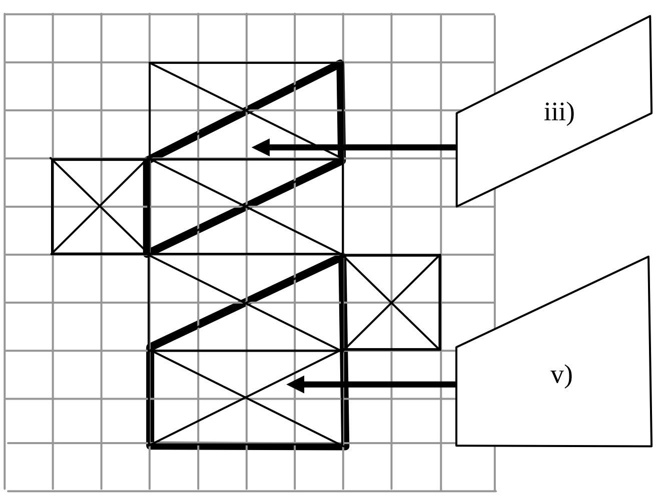
B. i) and ii)

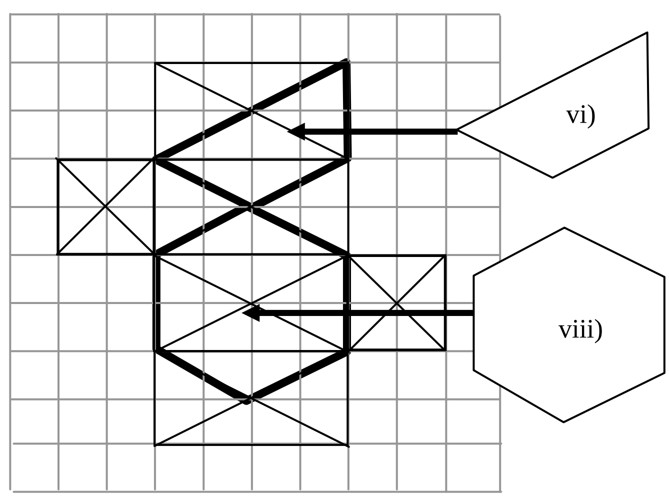


C.



C. [continued]





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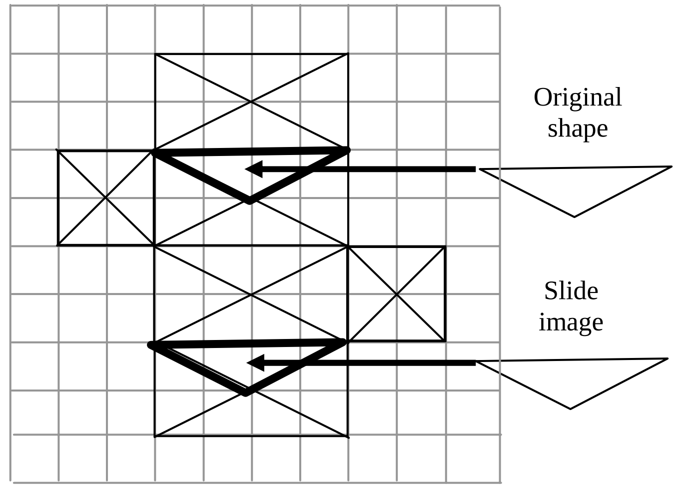
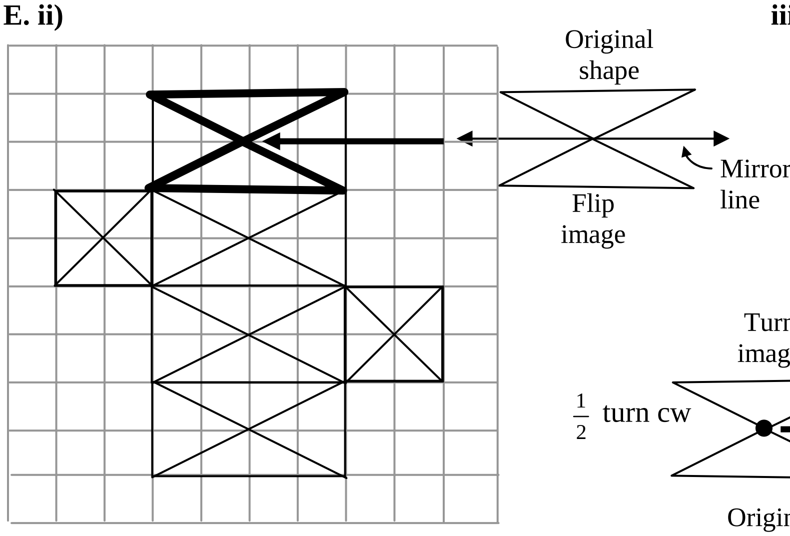
D. i) Triangle i) has a right angle but no congruent angles or lines of symmetry.
Triangle ii) has no right angles, but two congruent angles and one line of symmetry.
ii) Two or more congruent sides: iii), iv), vi), vii)
No congruent sides: v)

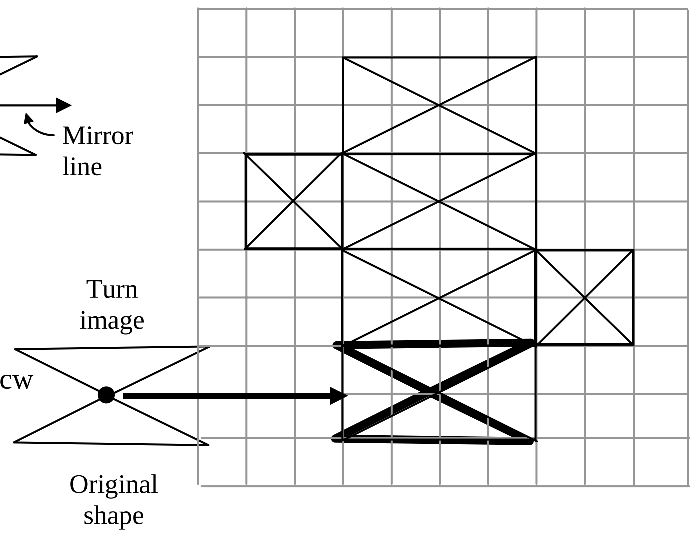
E. i) The shape moved (0 units left or right, 4 units down).



E. ii)



iii)



UNIT 5 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Interprets an isometric drawing to build a cube structure	Interprets the isometric drawing to build a completely accurate structure	Interprets the isometric drawing to build a reasonably accurate structure	Interprets the isometric drawing to build a somewhat accurate structure	Misinterprets the isometric drawing to build an inaccurate structure
Draws a net for a rectangle-based prism	Draws a completely accurate net	Draws a reasonably accurate net with only minor flaws	Draws a somewhat accurate net with some flaws	Draws an arrangement that is not a net
Identifies triangles and their properties	Identifies triangles completely accurately	Identifies triangles reasonably accurately with no major errors	Identifies triangles reasonably accurately with some errors	Shows major errors in identifying triangles
Identifies quadrilaterals and sorts them by their properties	Identifies and sorts quadrilaterals completely and accurately, with a clearly-stated sorting rule	Identifies and sorts quadrilaterals reasonably accurately, with a stated sorting rule (errors do not suggest misconceptions)	Identifies and sorts quadrilaterals reasonably accurately for most of the quadrilaterals (some misconceptions may be present)	Shows major errors in identifying and sorting quadrilaterals
Identifies and describes examples of slides, flips, and turns	Identifies accurately and provides complete descriptions of all transformations	Identifies and describes transformations reasonably accurately with no major errors or omissions	Identifies and describes transformations reasonably accurately with some errors or omissions	Shows major flaws in identifying or describing transformations

UNIT 5 Assessment Interview

• You may wish to take the opportunity to interview selected students to assess their understanding of the work of this unit.

• Interviews are most effective when done with individual students, although pair and small group interviews are sometimes appropriate.

• The results can be used as formative assessment or as summative assessment data.

• As the student works, ask him or her to explain his or her thinking.

Part 1

• On grid paper, ask the student to draw two triangles that would have different names. In each case, one of the sides must be on a grid line. Ask what makes the two triangles different.

• Ask the student to choose one of the triangles and slide it to a final position that is (5 units right, 2 units down).

• Ask the student to flip the new triangle (the slide image) using a vertical flip line. Ask how many grid units each of the vertices moved.

• Ask the student to take the other triangle and turn it $\frac{1}{4}$ turn cw around one of the vertices.

Part 2

• Remove the title from BLM 22 Sample Net of a Triangle-based Prism. Show the BLM to the student and ask the following questions:

- Will the resulting shape will be a prism, a pyramid, a cone, or a cylinder? How do you know?
- How many faces, vertices and edges will the resulting 3-D shape have? How do you know?

• Provide sticks and balls of clay and ask the student to build a skeleton of the shape in BLM 22.

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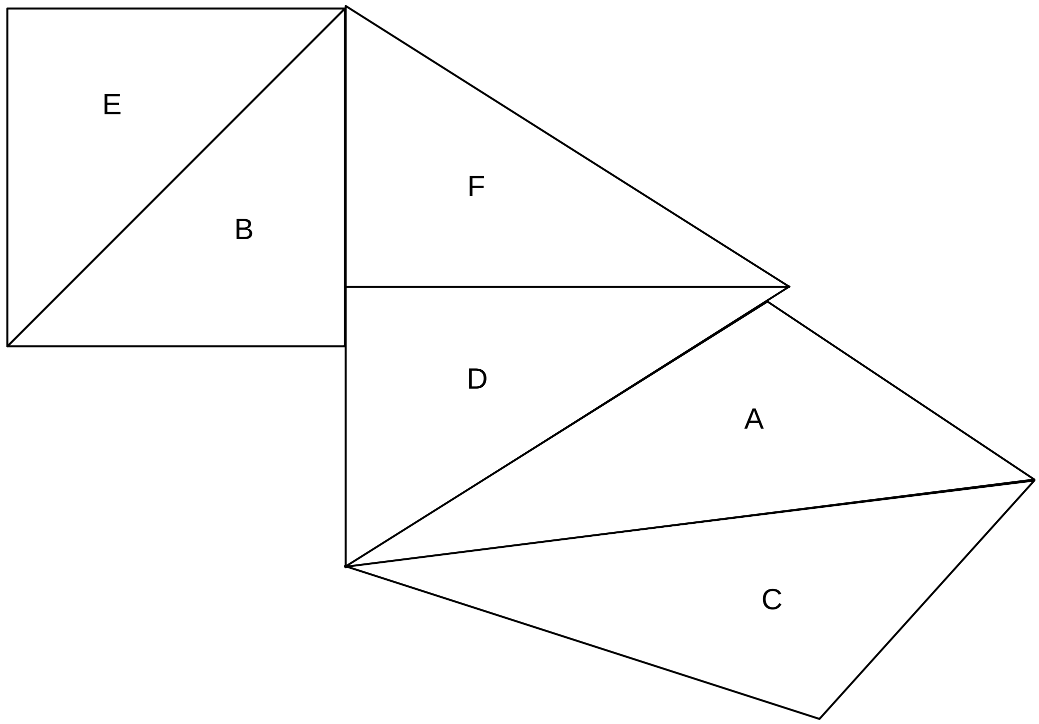
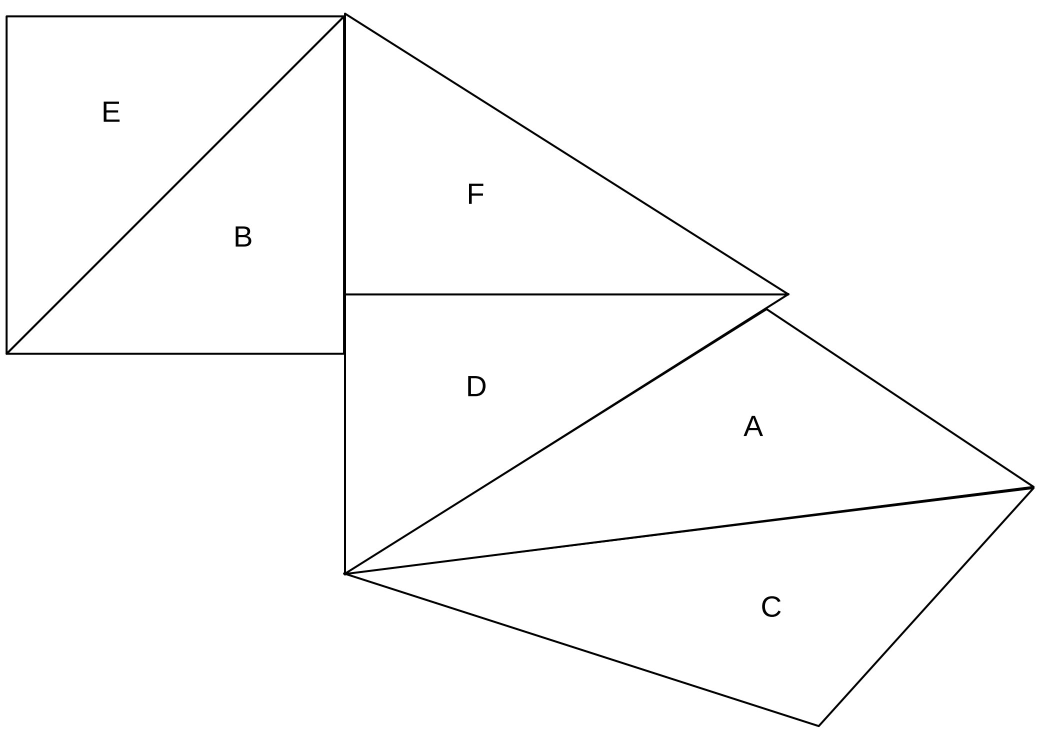
BLM 1 Getting Started Triangles





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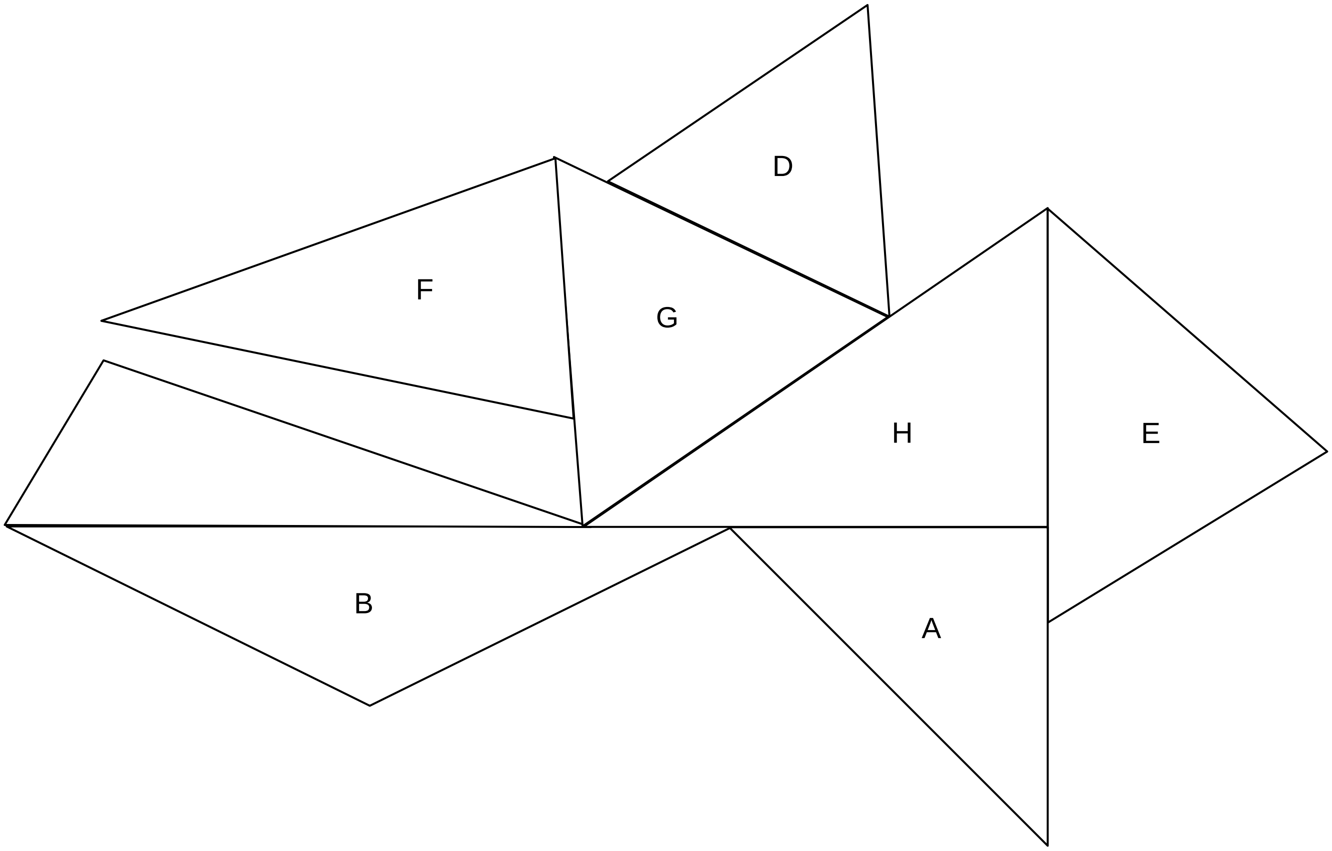

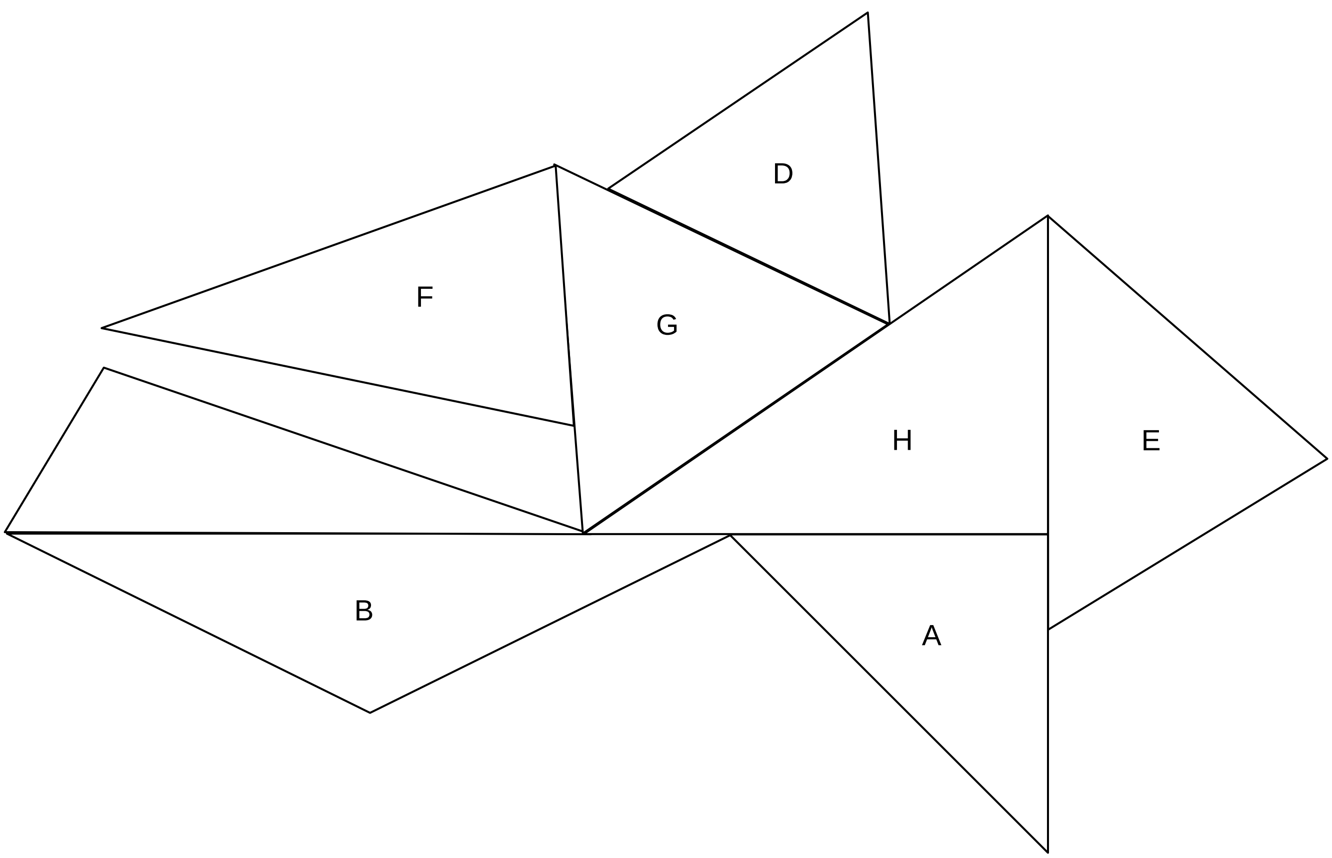

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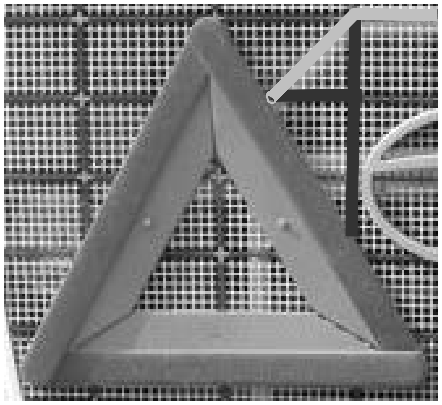
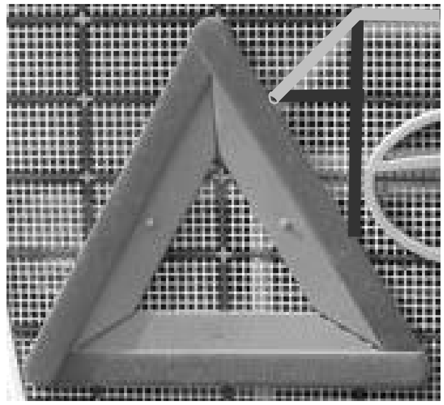
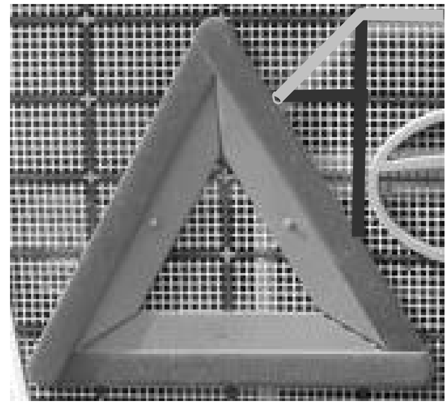
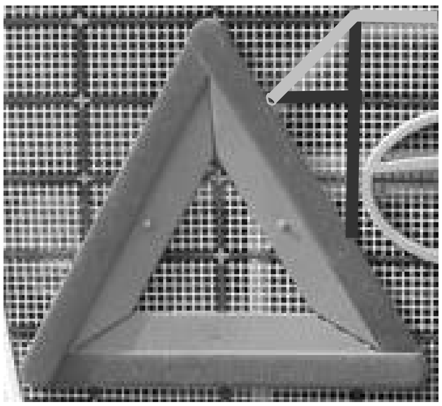
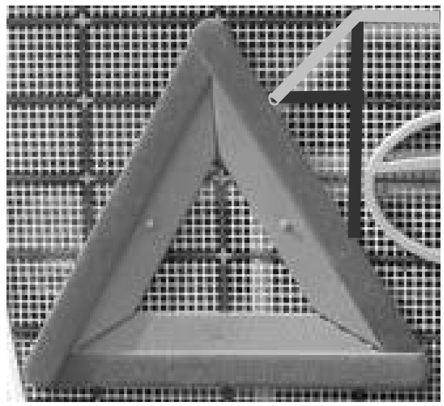
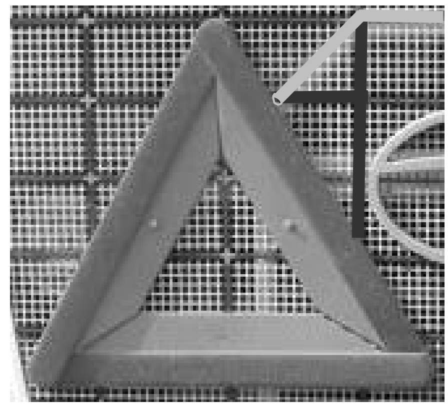
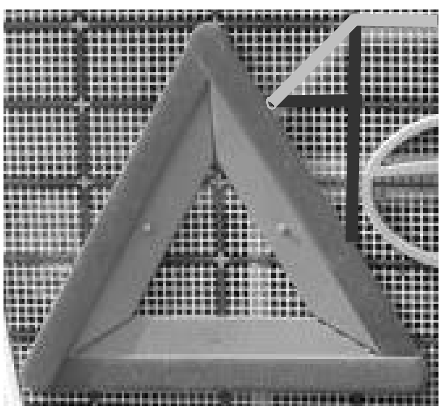
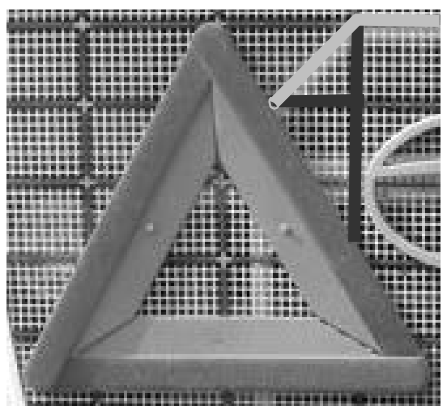
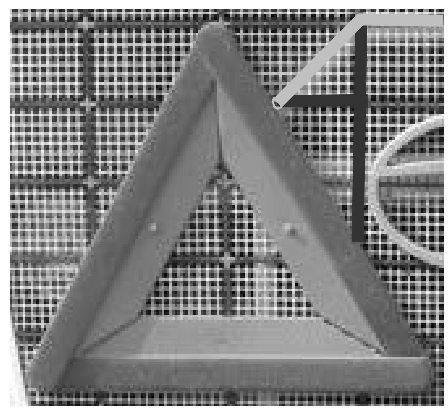
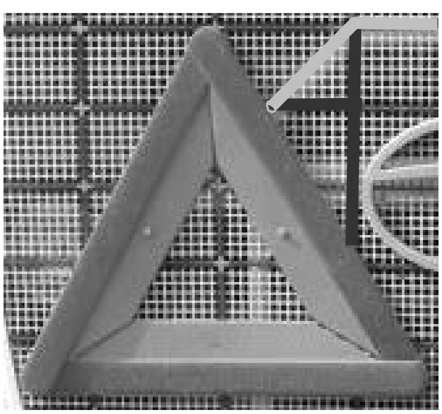
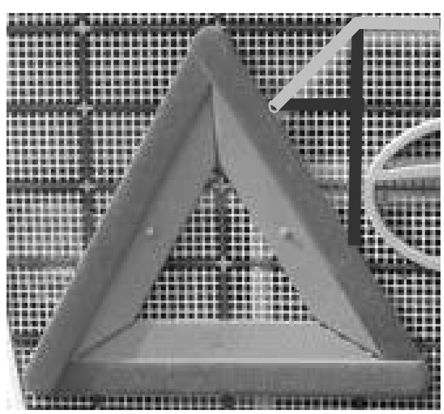
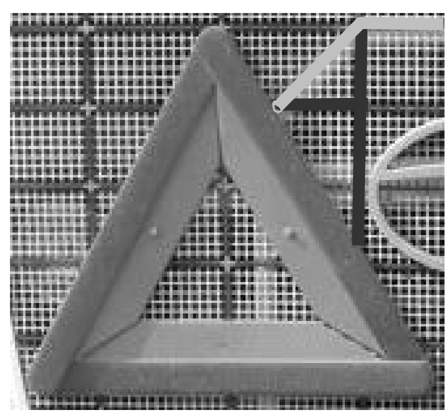
BLM 3 Triangle Types

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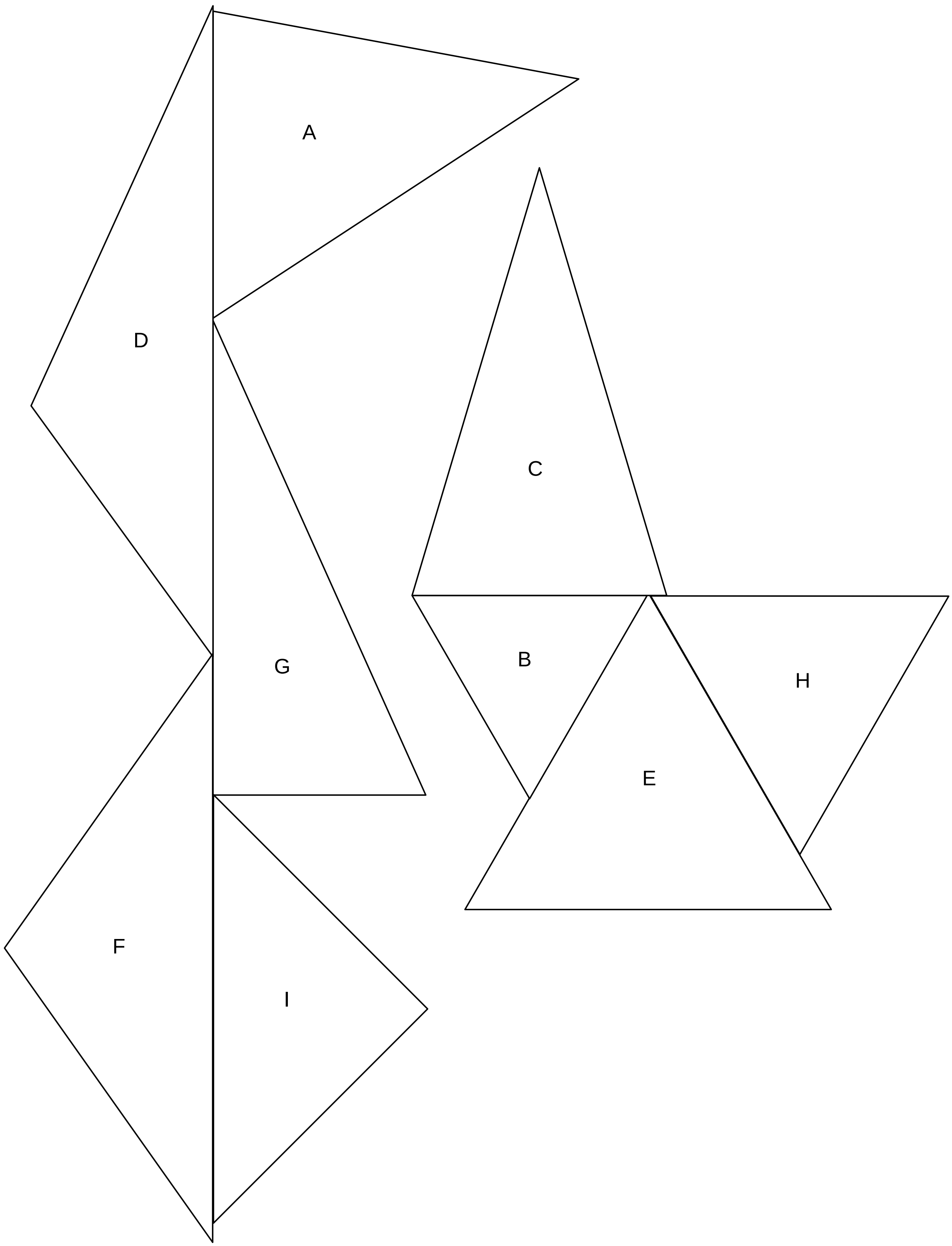
BLM 4 Properties of Triangles



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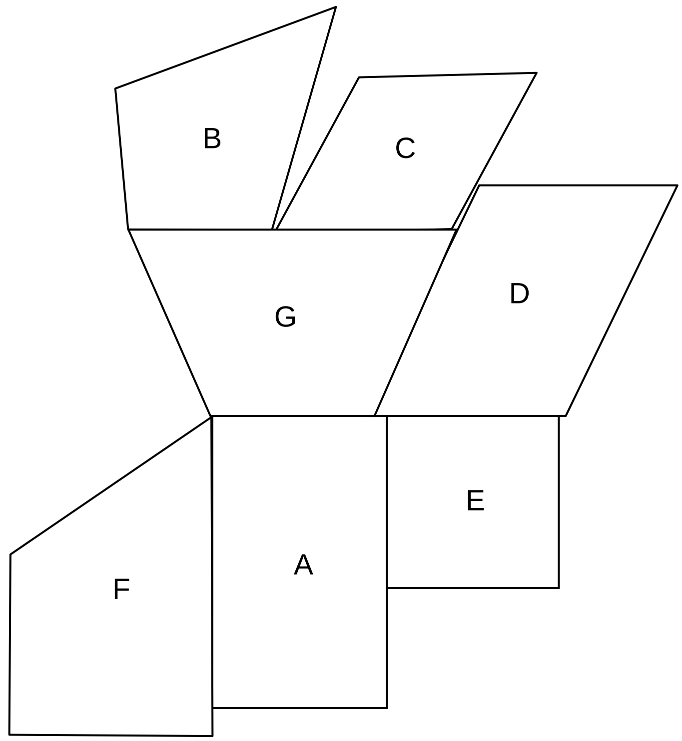
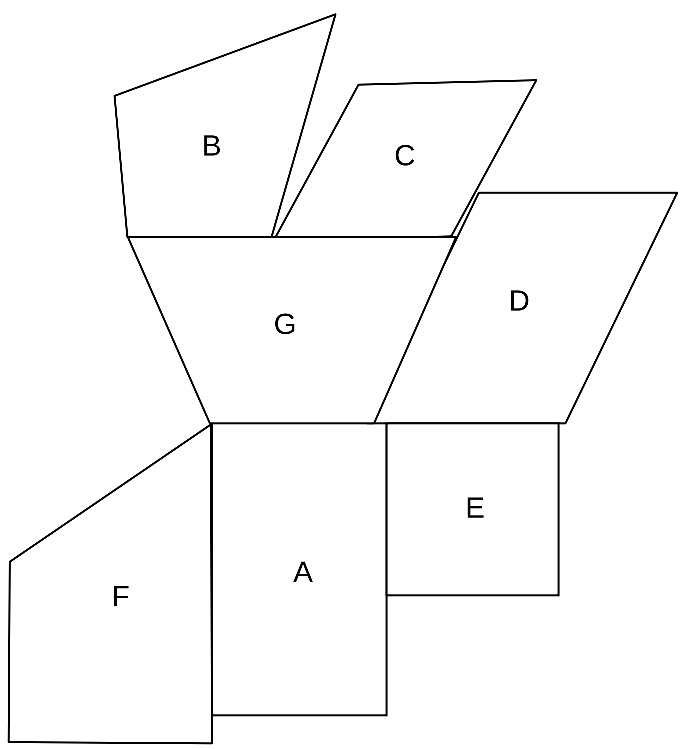
BLM 5 Sorting Quadrilaterals





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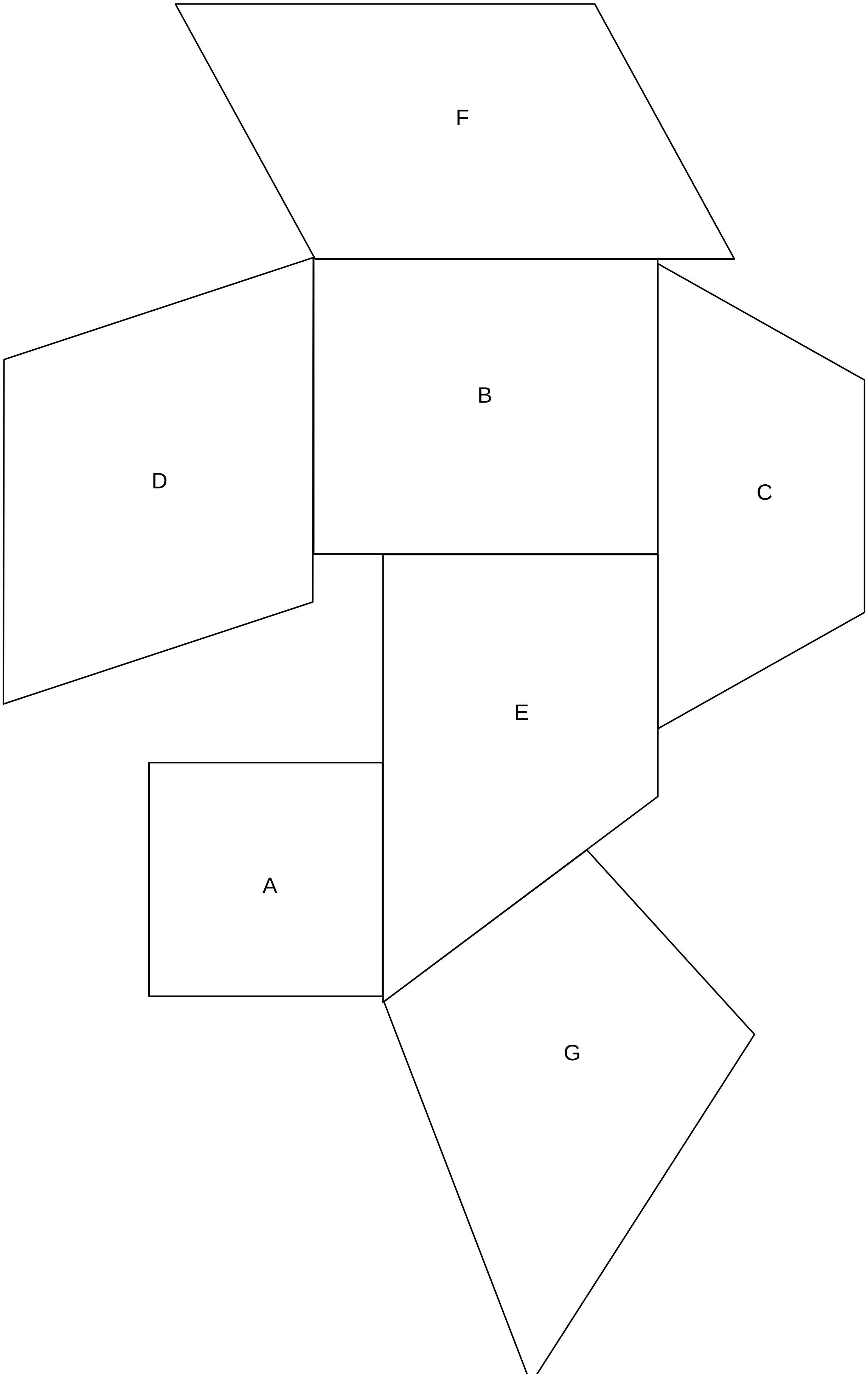
BLM 6 Diagonals and Symmetry



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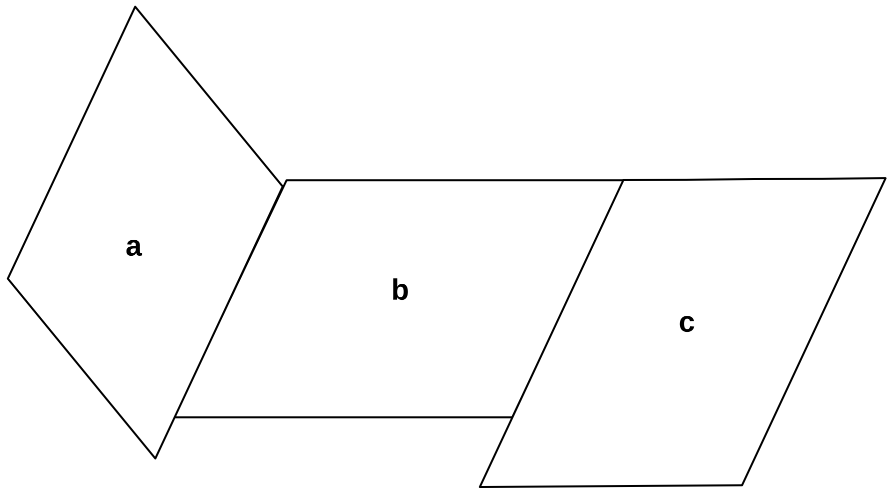
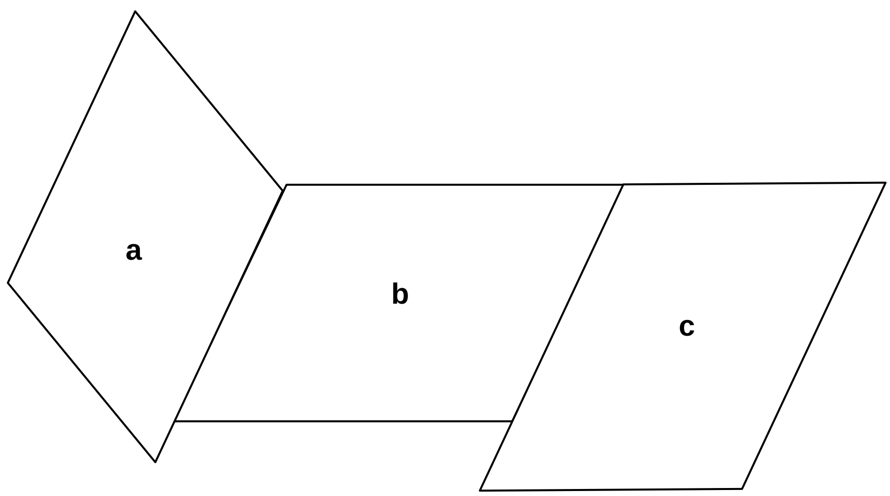
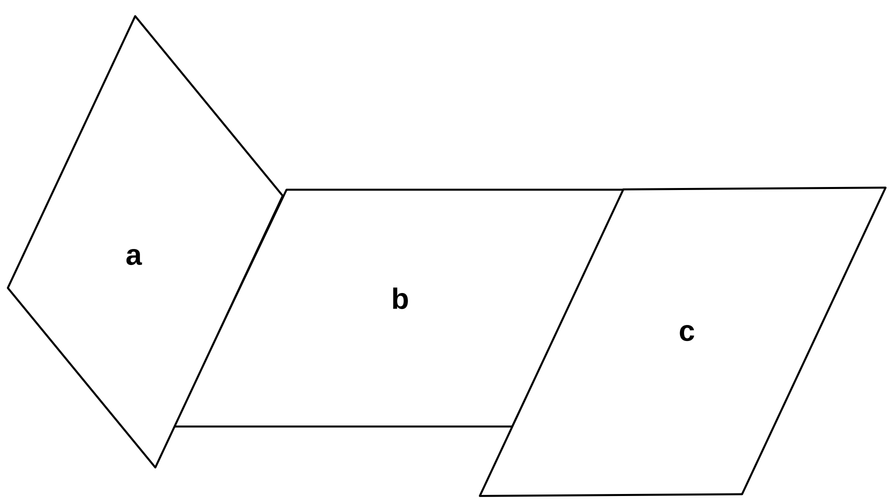
BLM 7 Congruent Polygons







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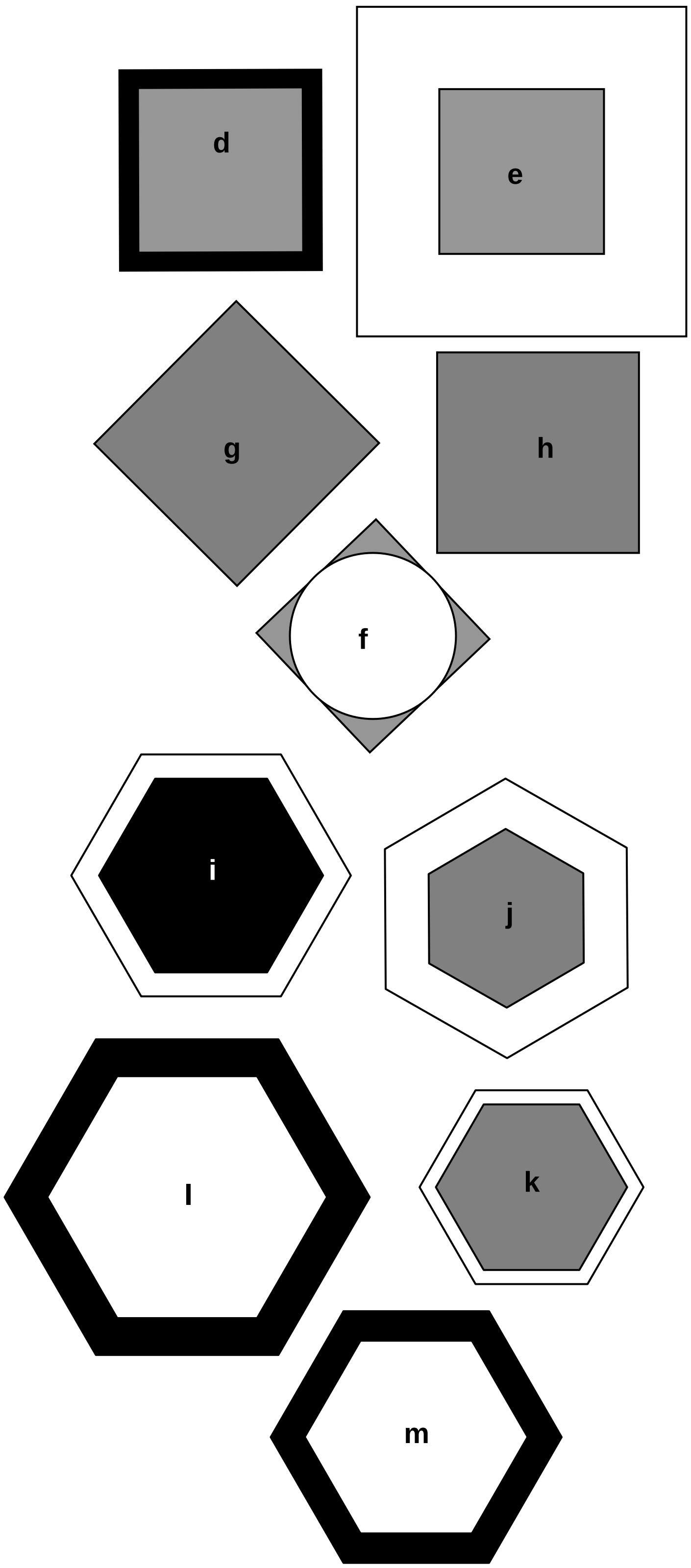
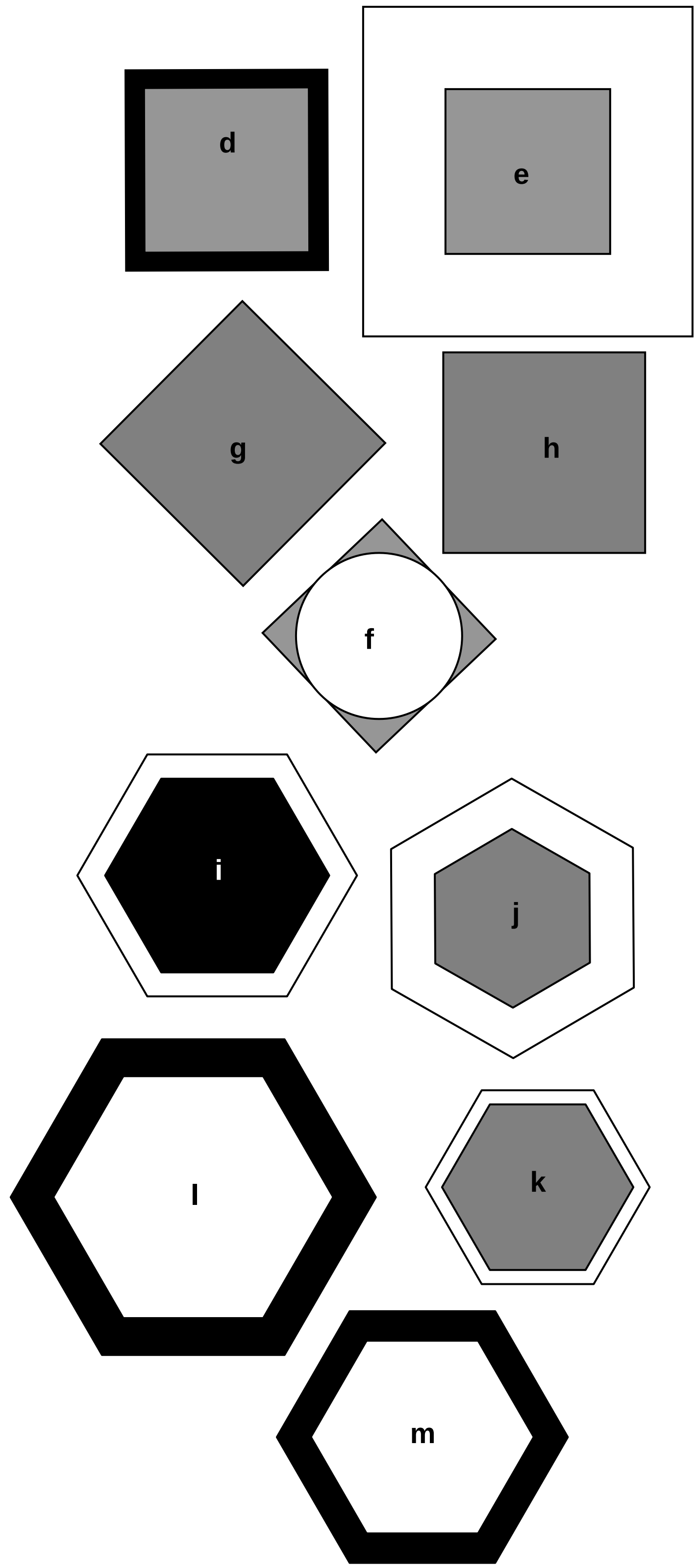
BLM 8 Optical Illusions





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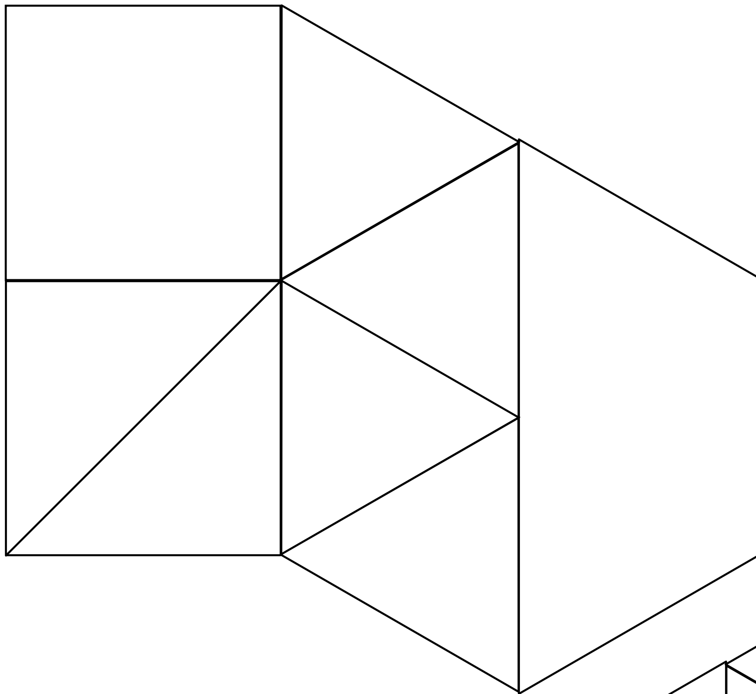
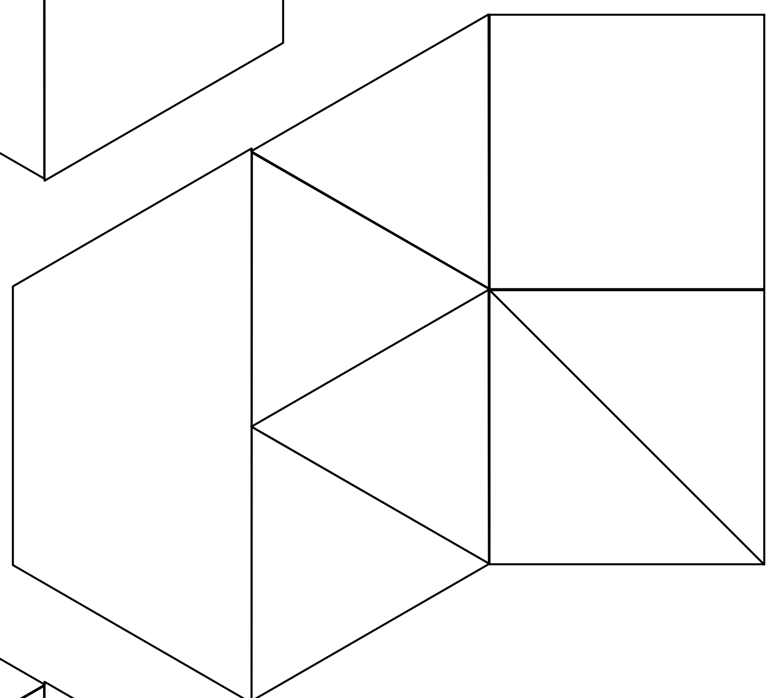
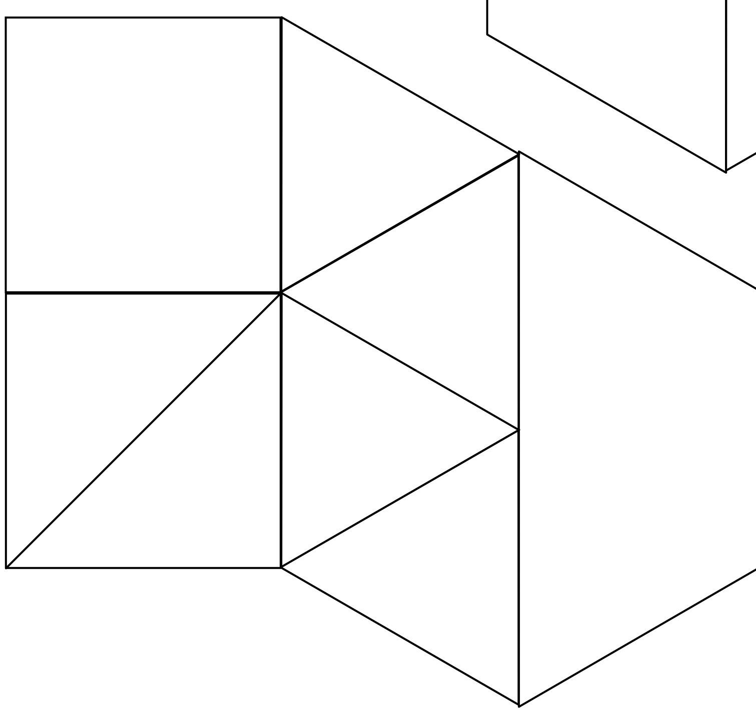
BLM 9 Combining Polygons







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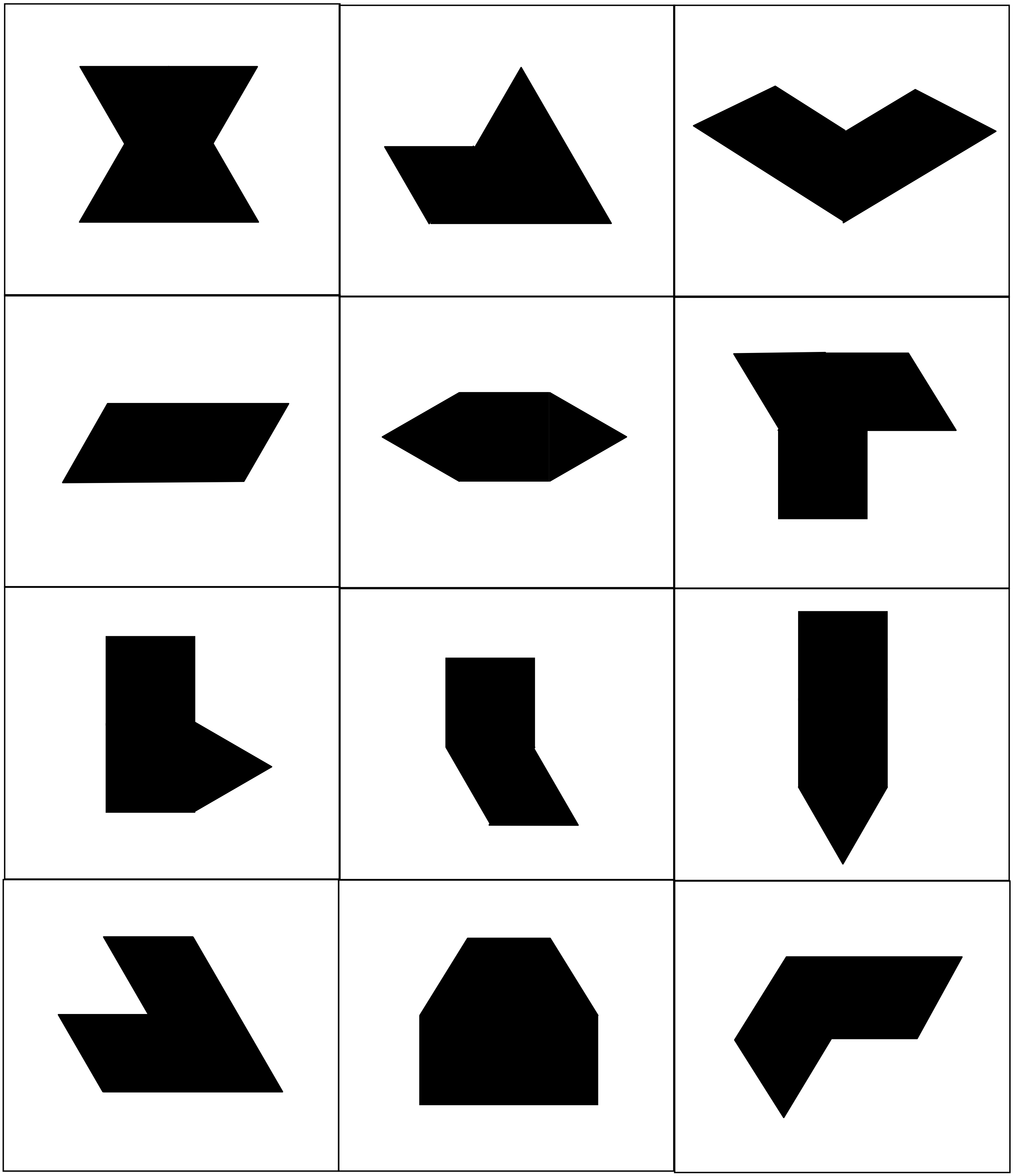
BLM 10A Puzzle Cards



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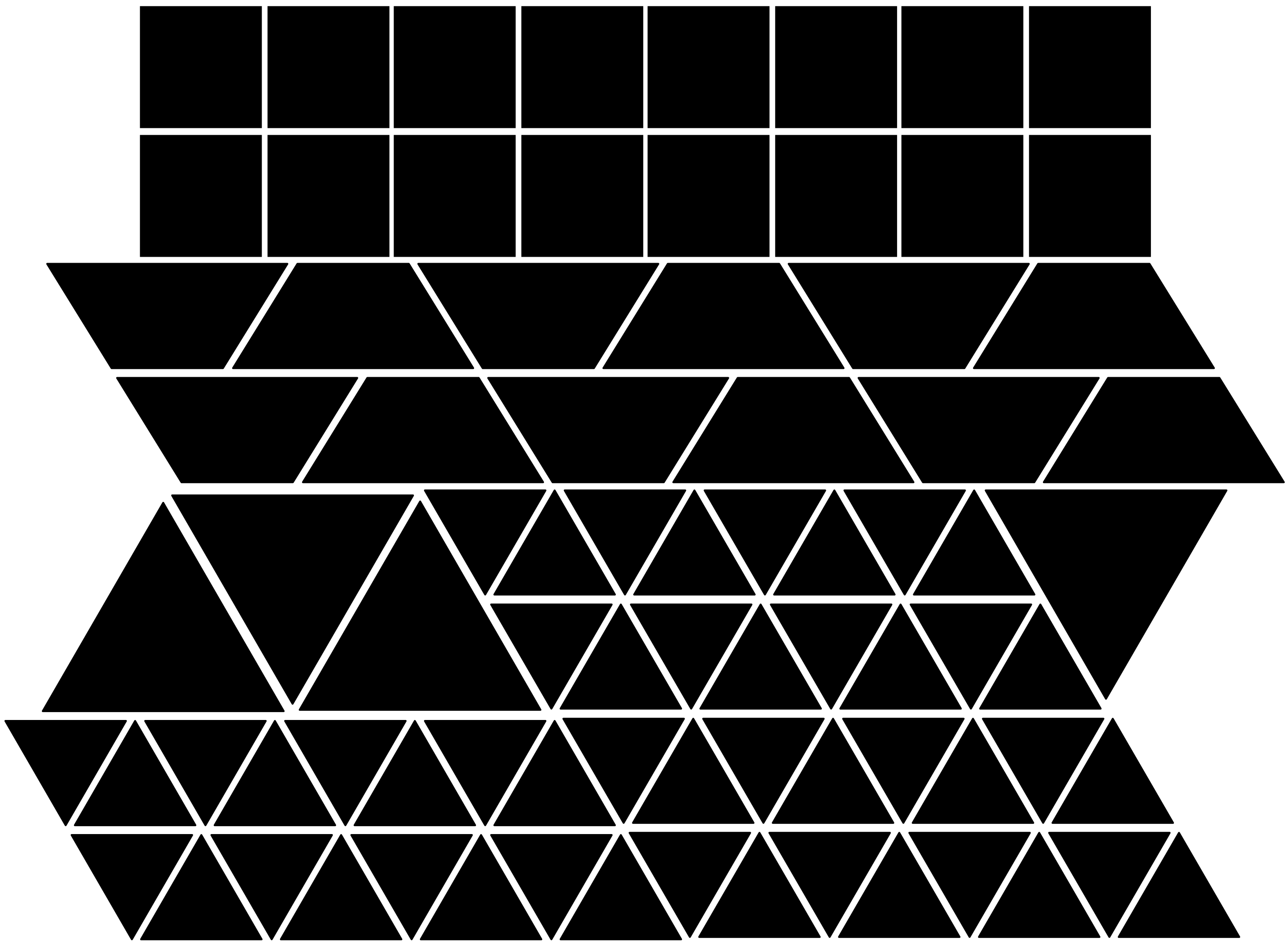
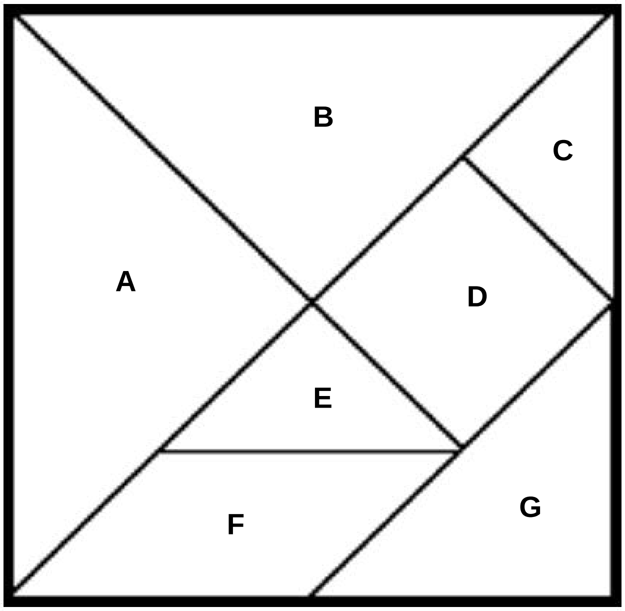
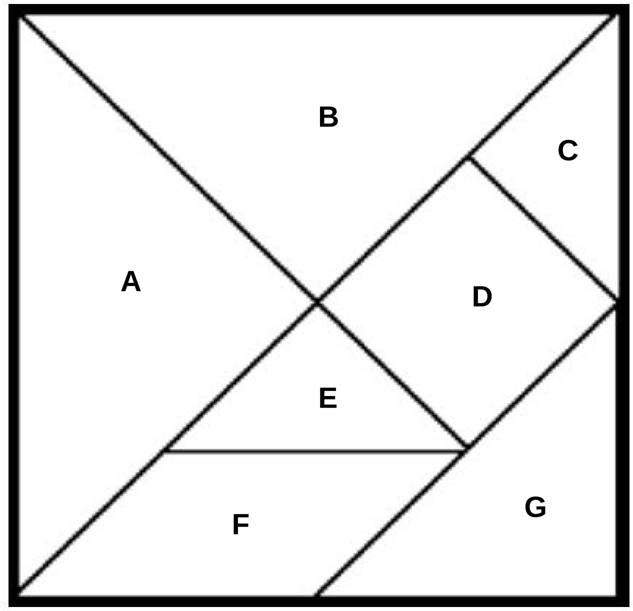
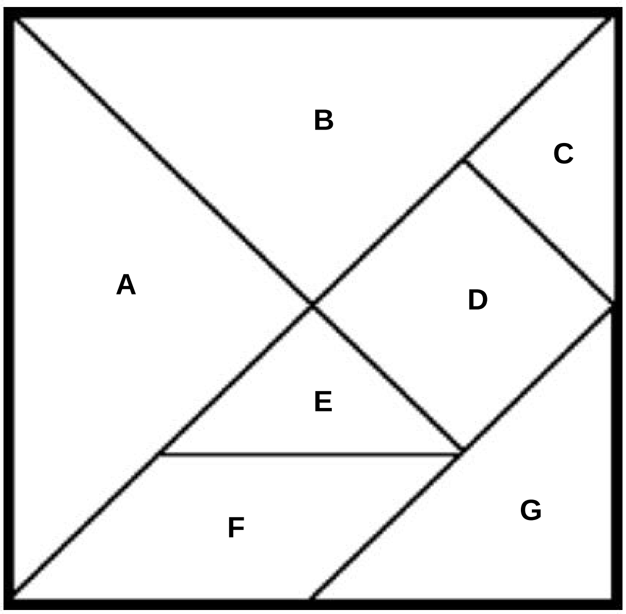
BLM 10B Puzzle Shapes

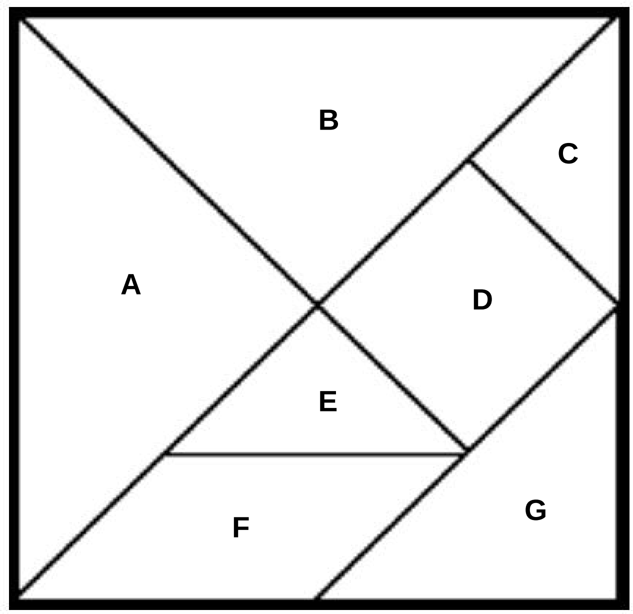


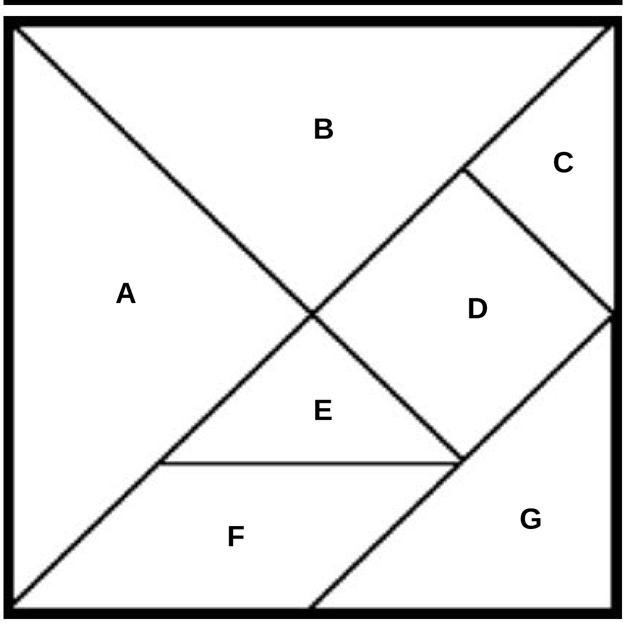
BLM 11 Tangrams

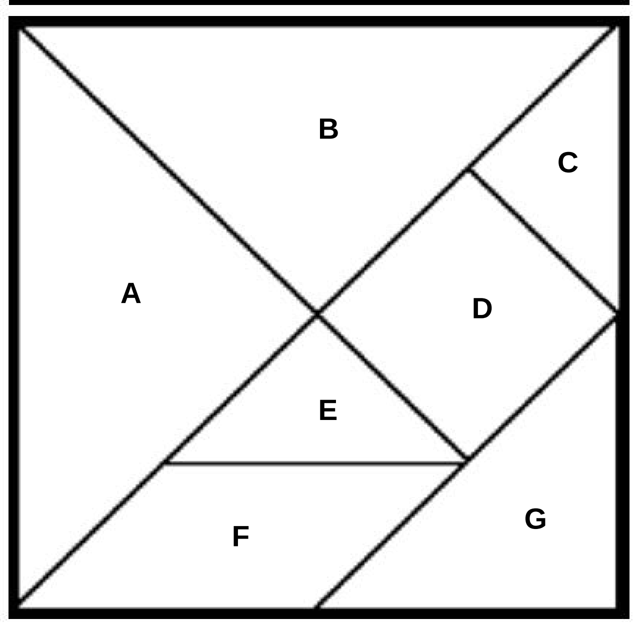




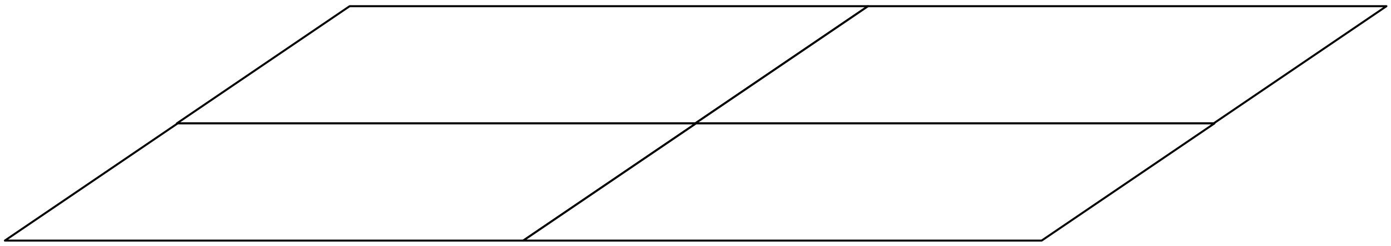


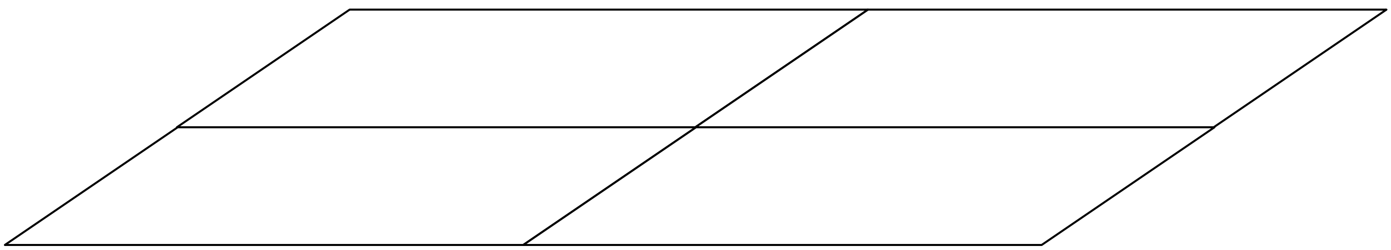


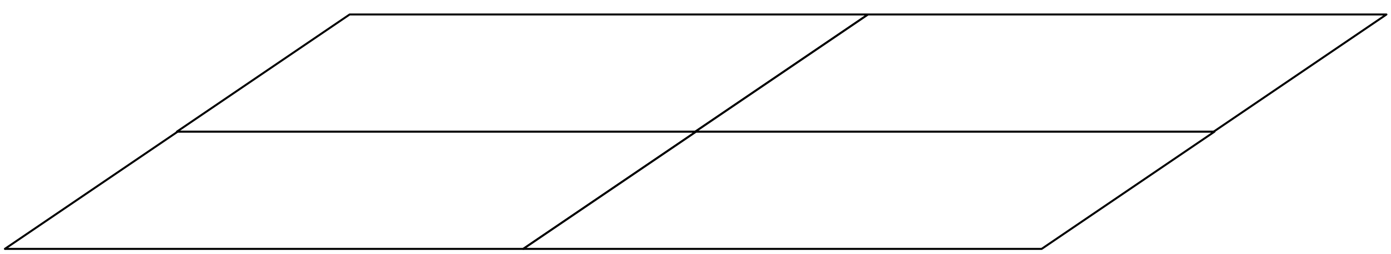


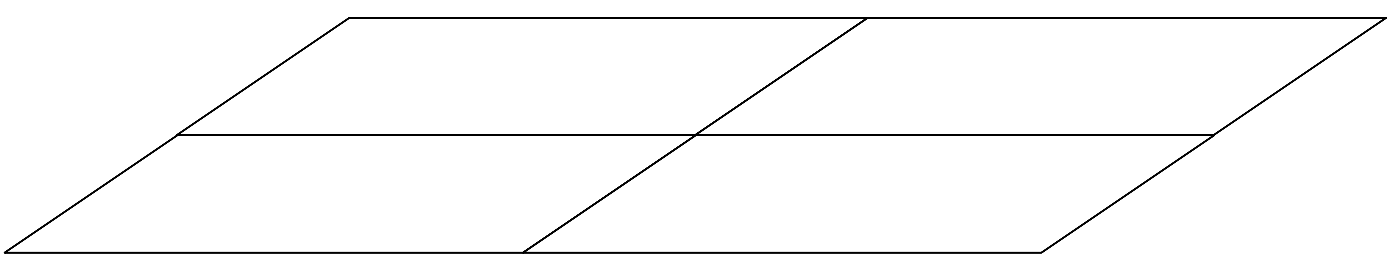


BLM 12 Slides and Flips

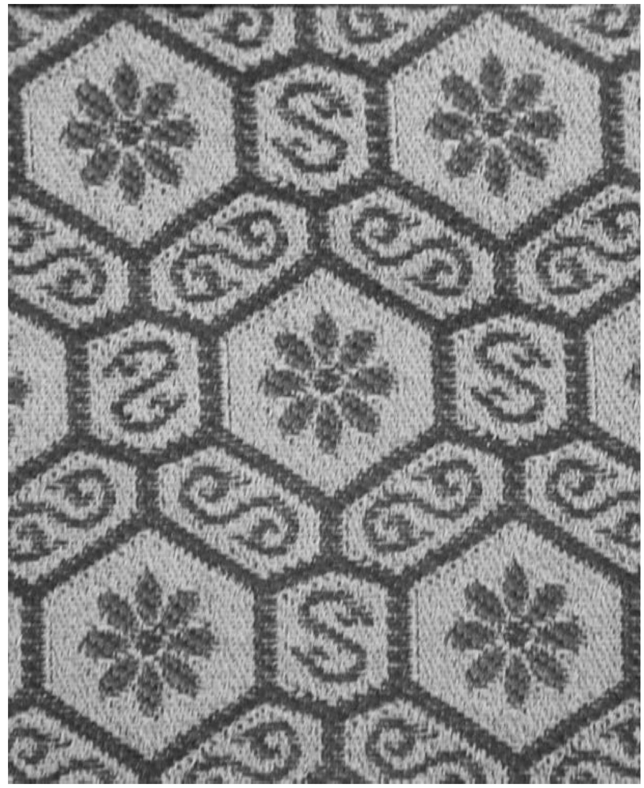








BLM 13 Textile Design



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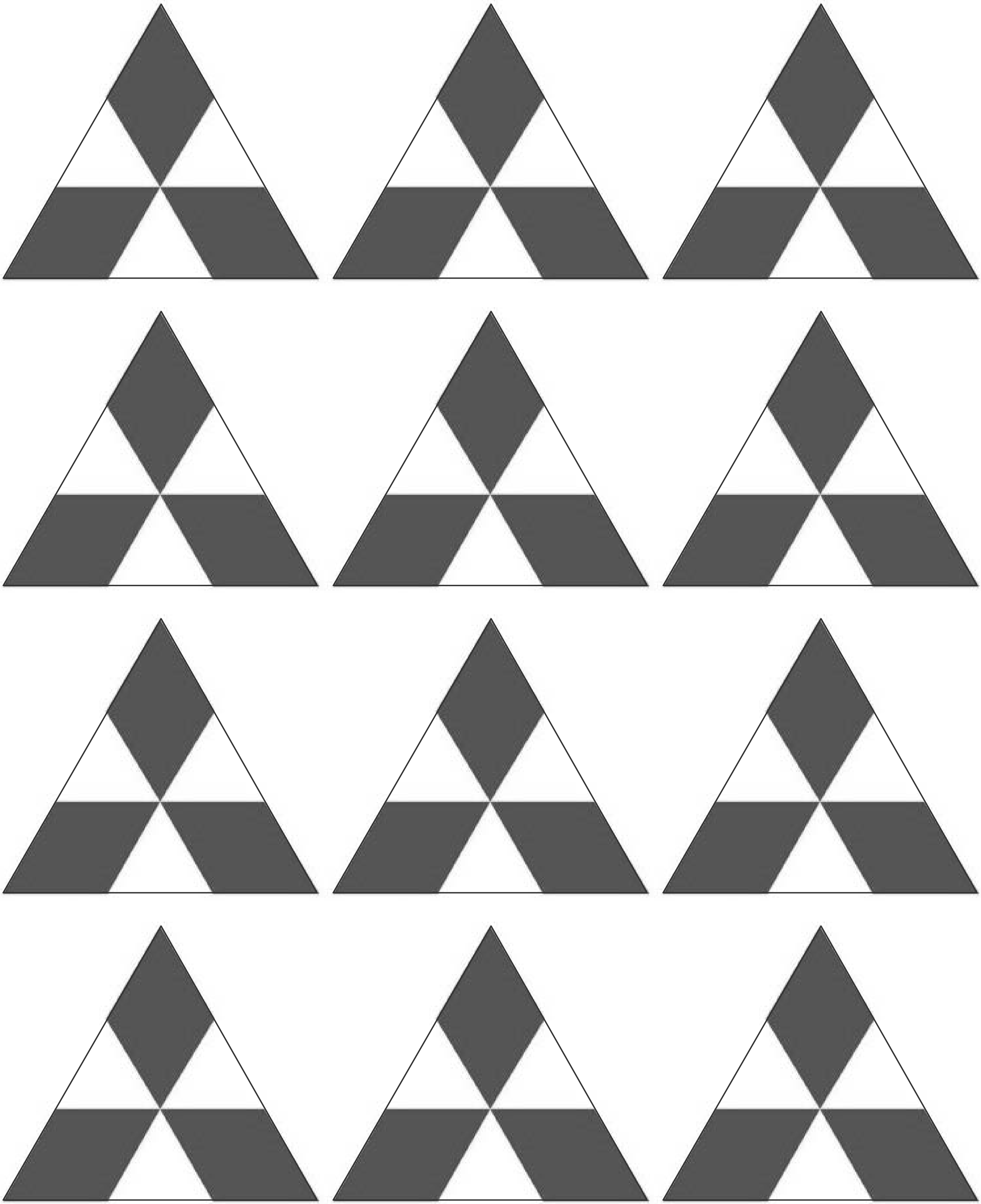
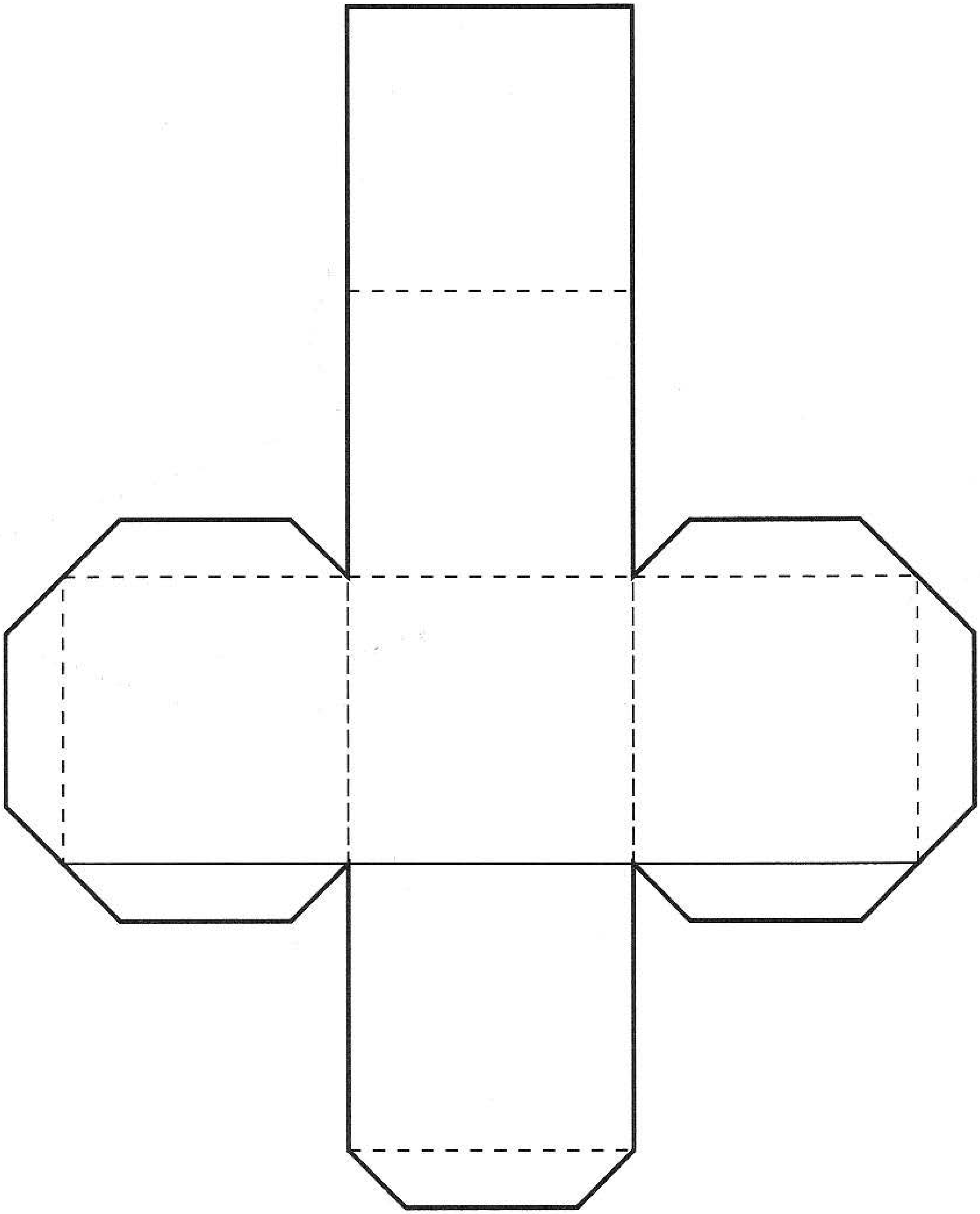
BLM 14 Logos



BLM 15 Sample Net of Cube



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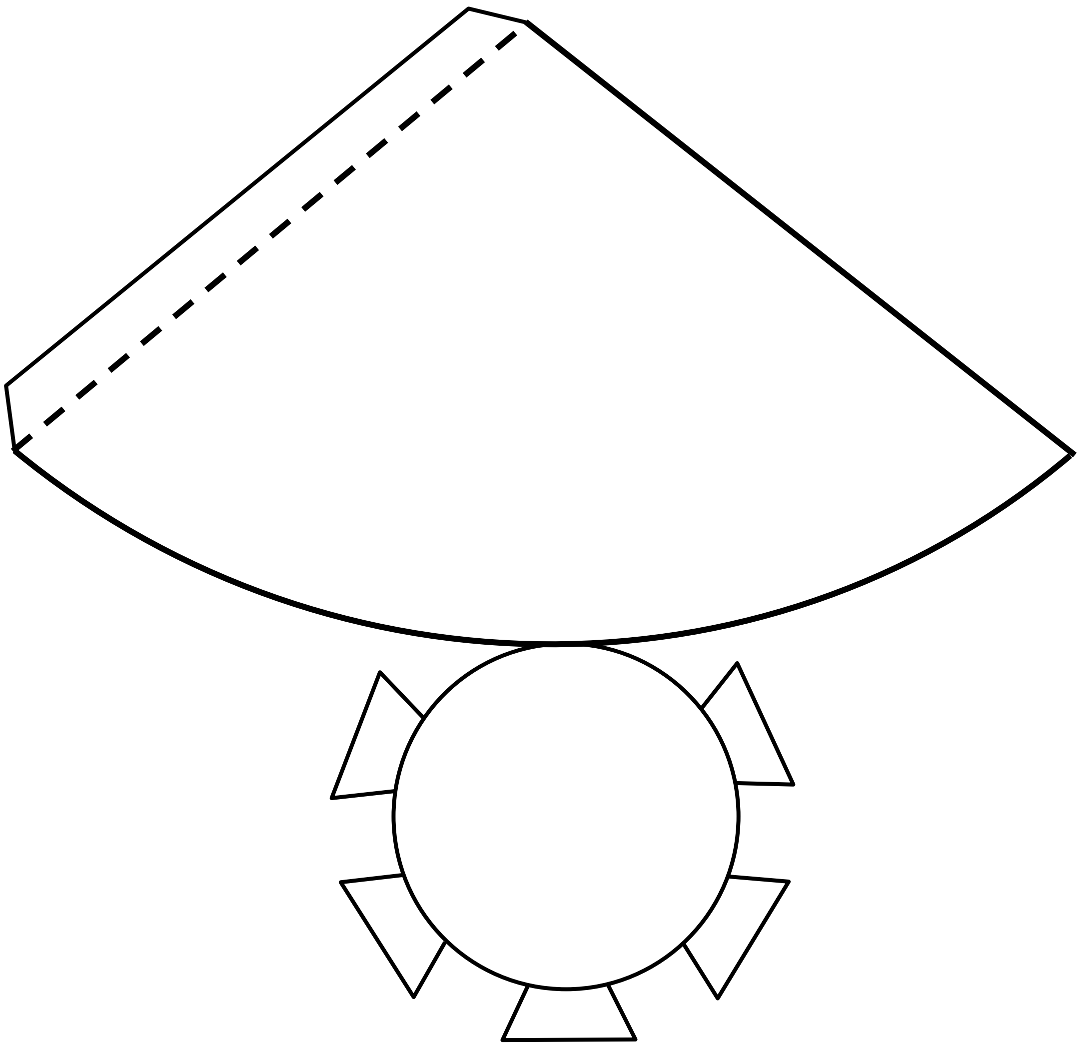
BLM 16 Sample Net of Cone



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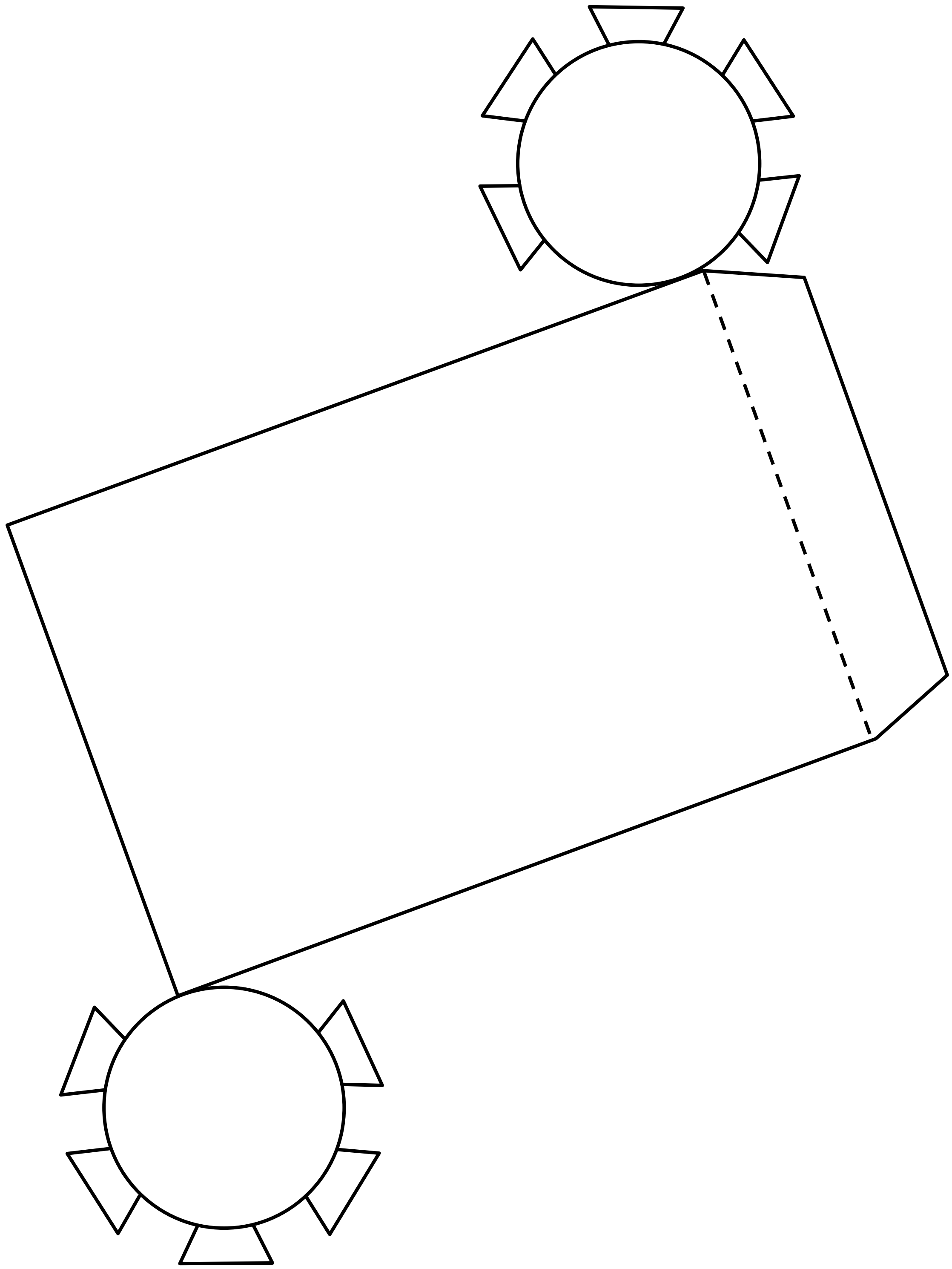
BLM 17 Sample Net of Cylinder



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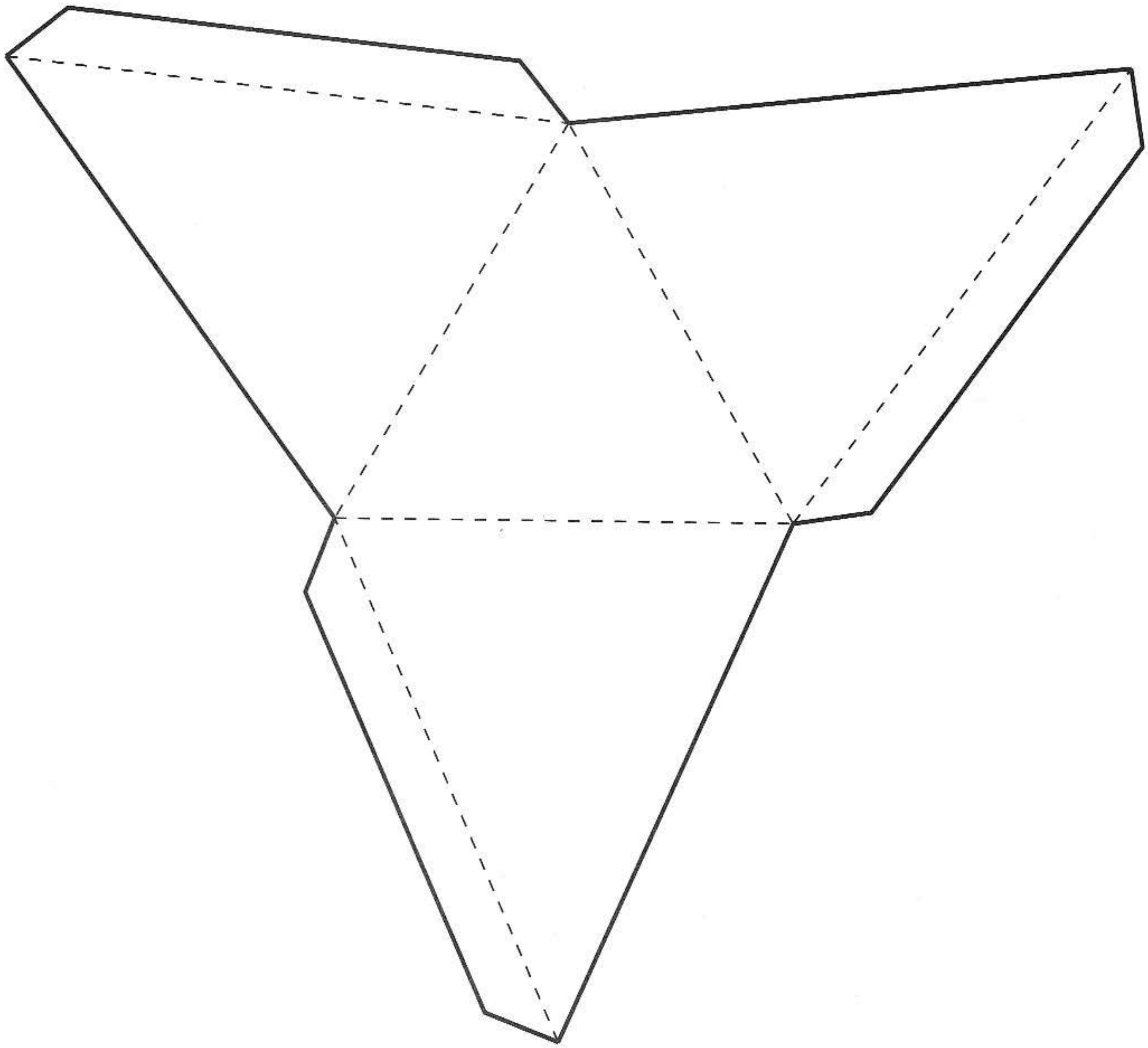
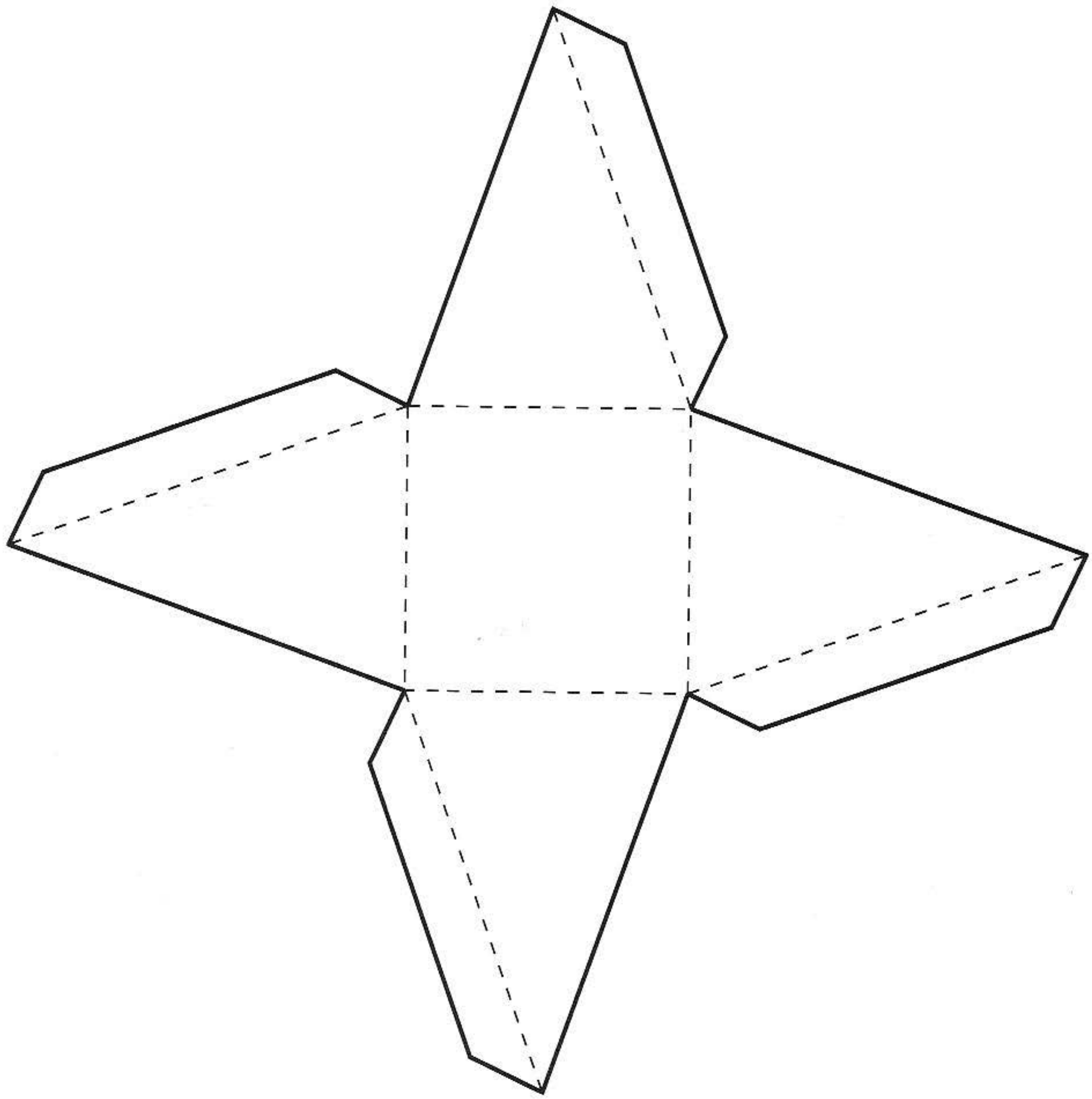
BLM 18 Sample Net of Triangle-based Pyramid



BLM 19 Sample Net of Square-based Pyramid



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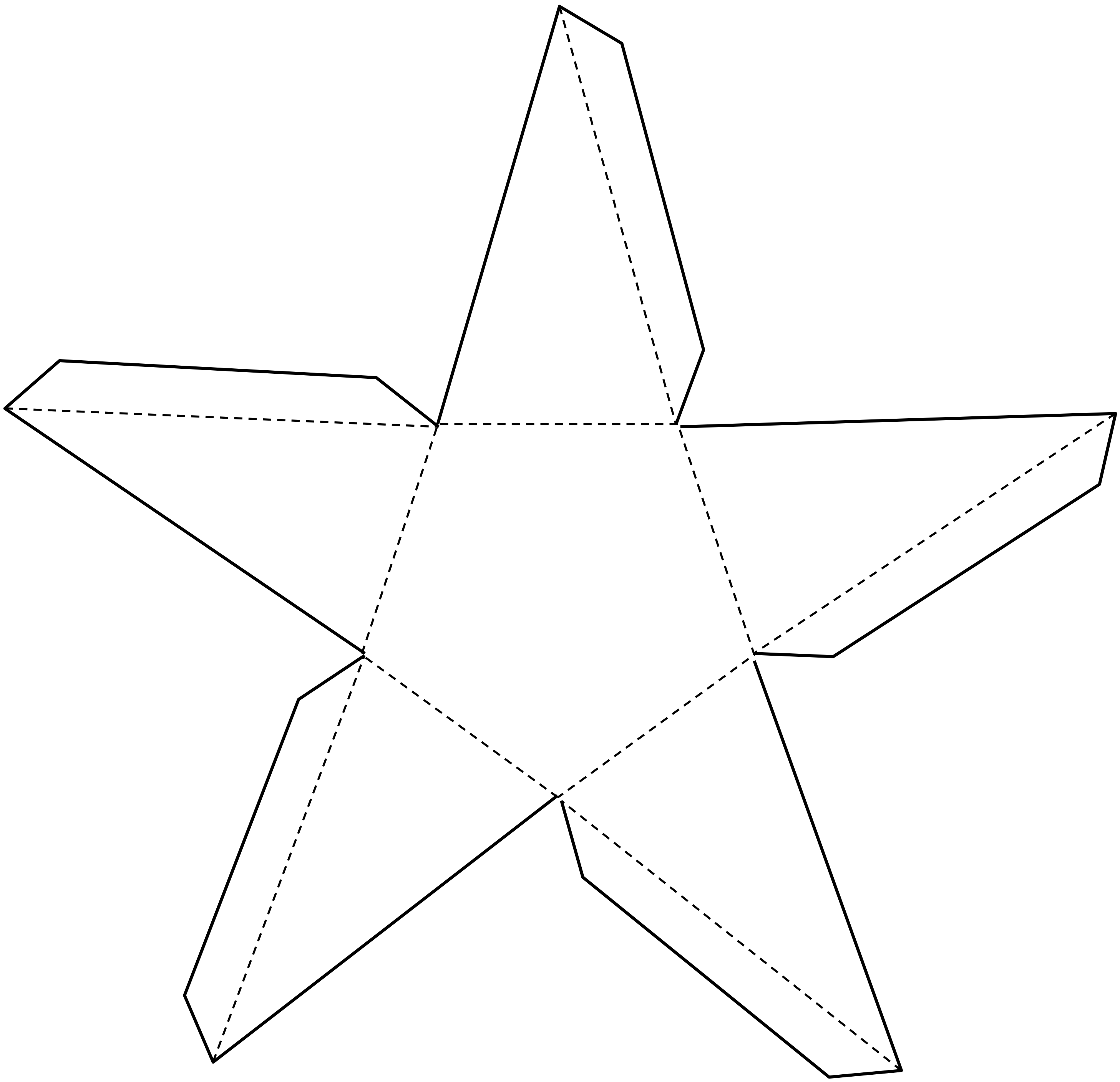
BLM 20 Sample Net of Pentagon-based Pyramid



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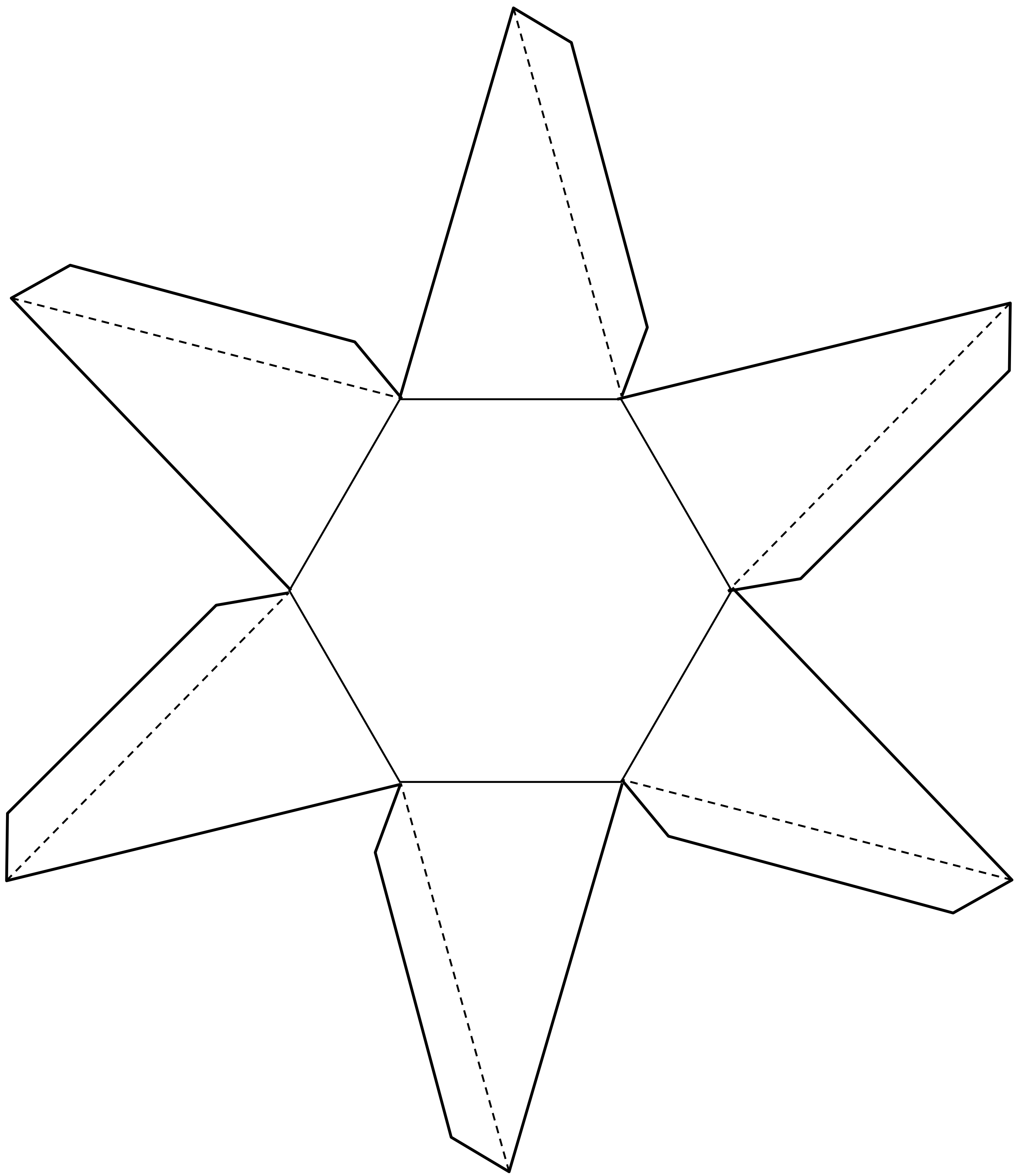
BLM 21 Sample Net of Hexagon-based Pyramid



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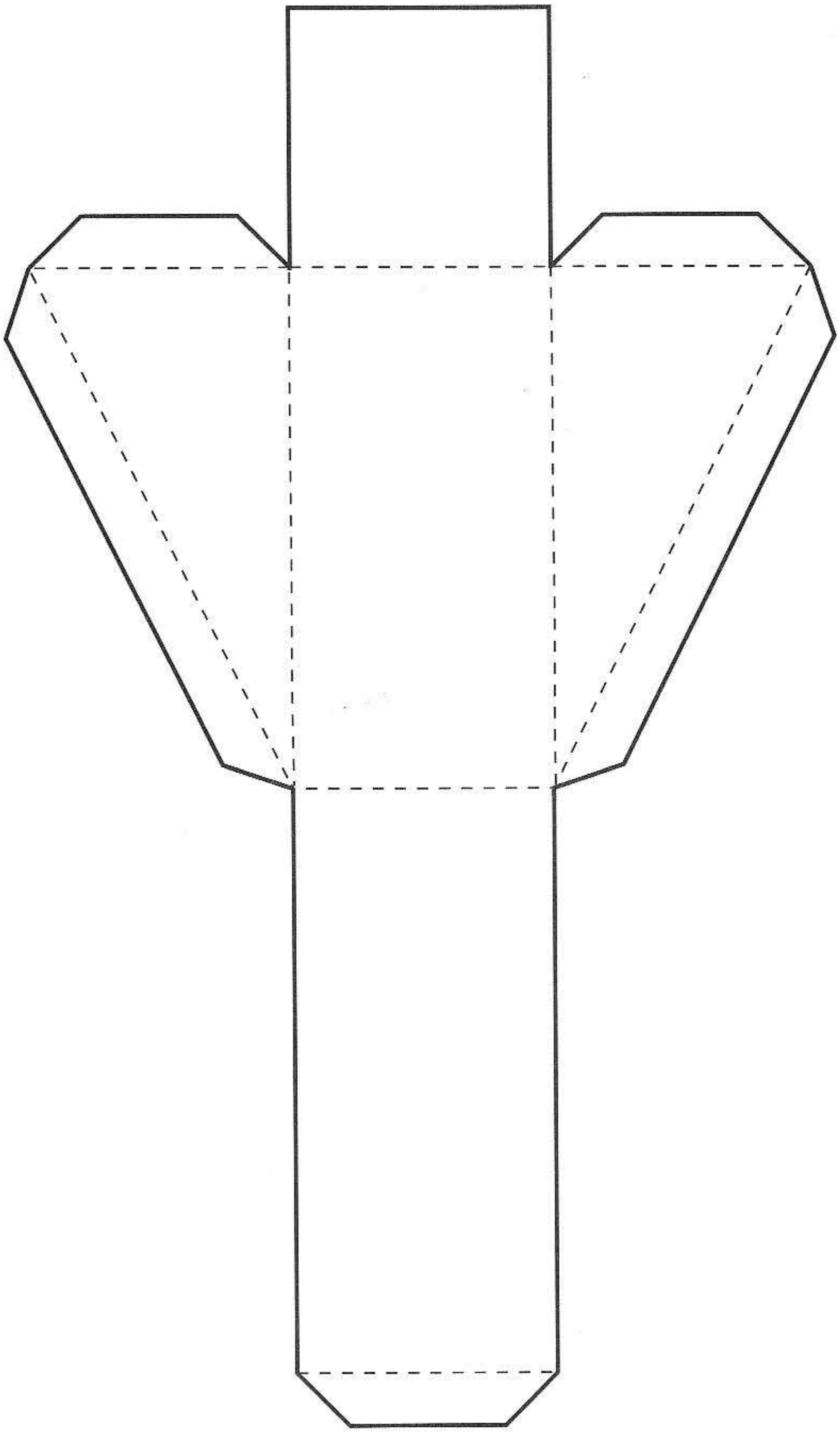
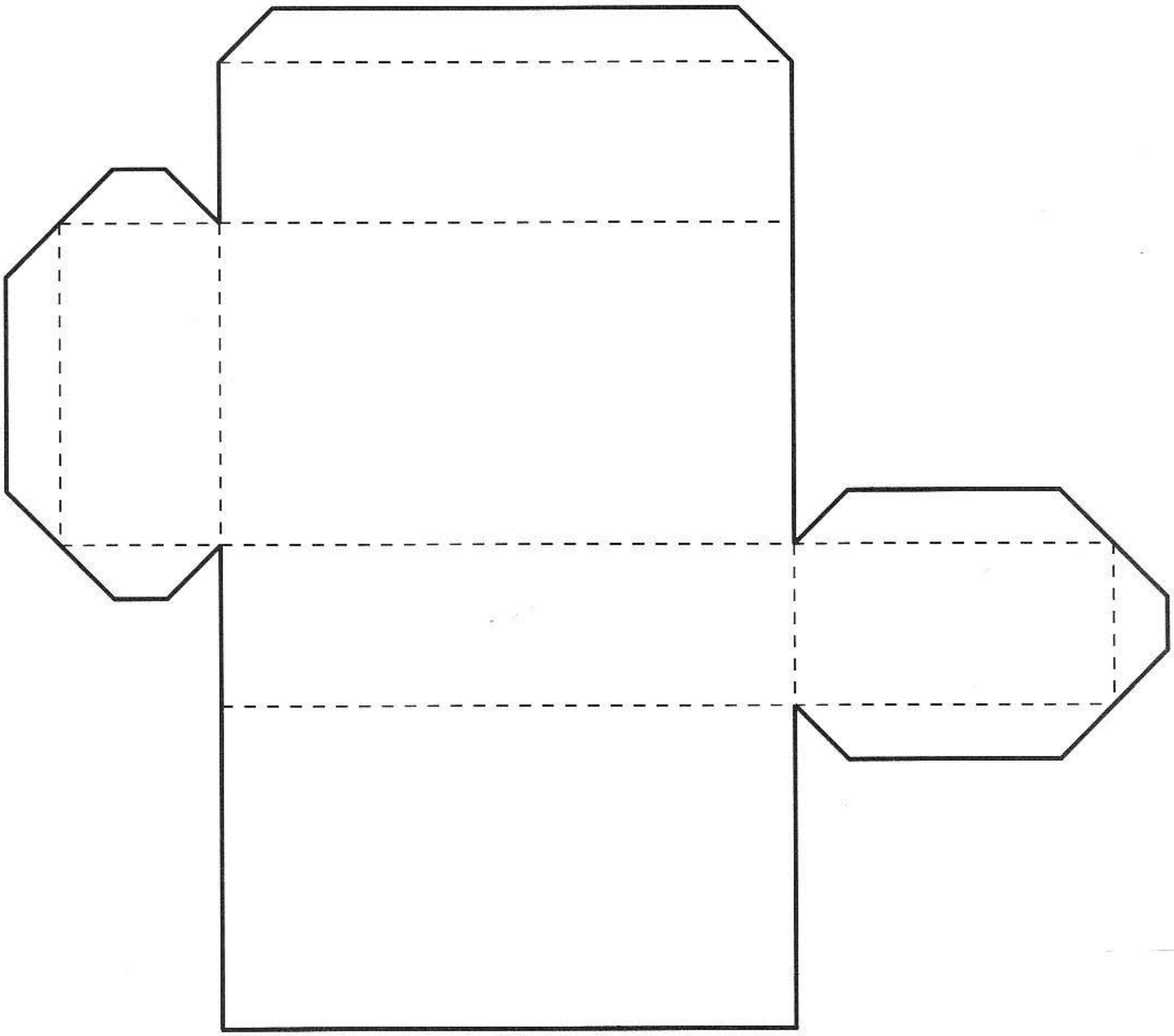
BLM 22 Sample Net of Triangle-based Prism



BLM 23 Sample Net of Rectangle-based Prism



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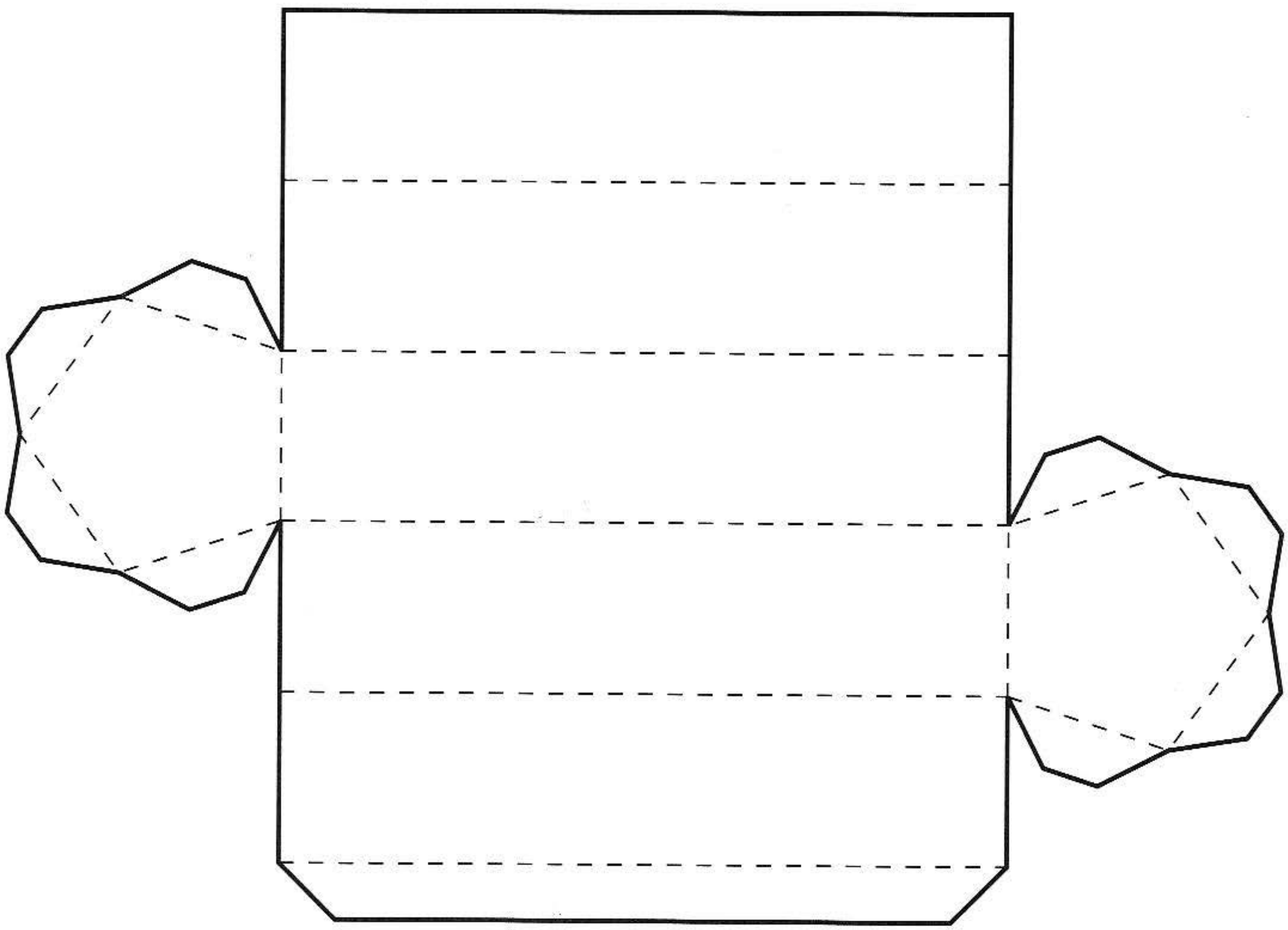
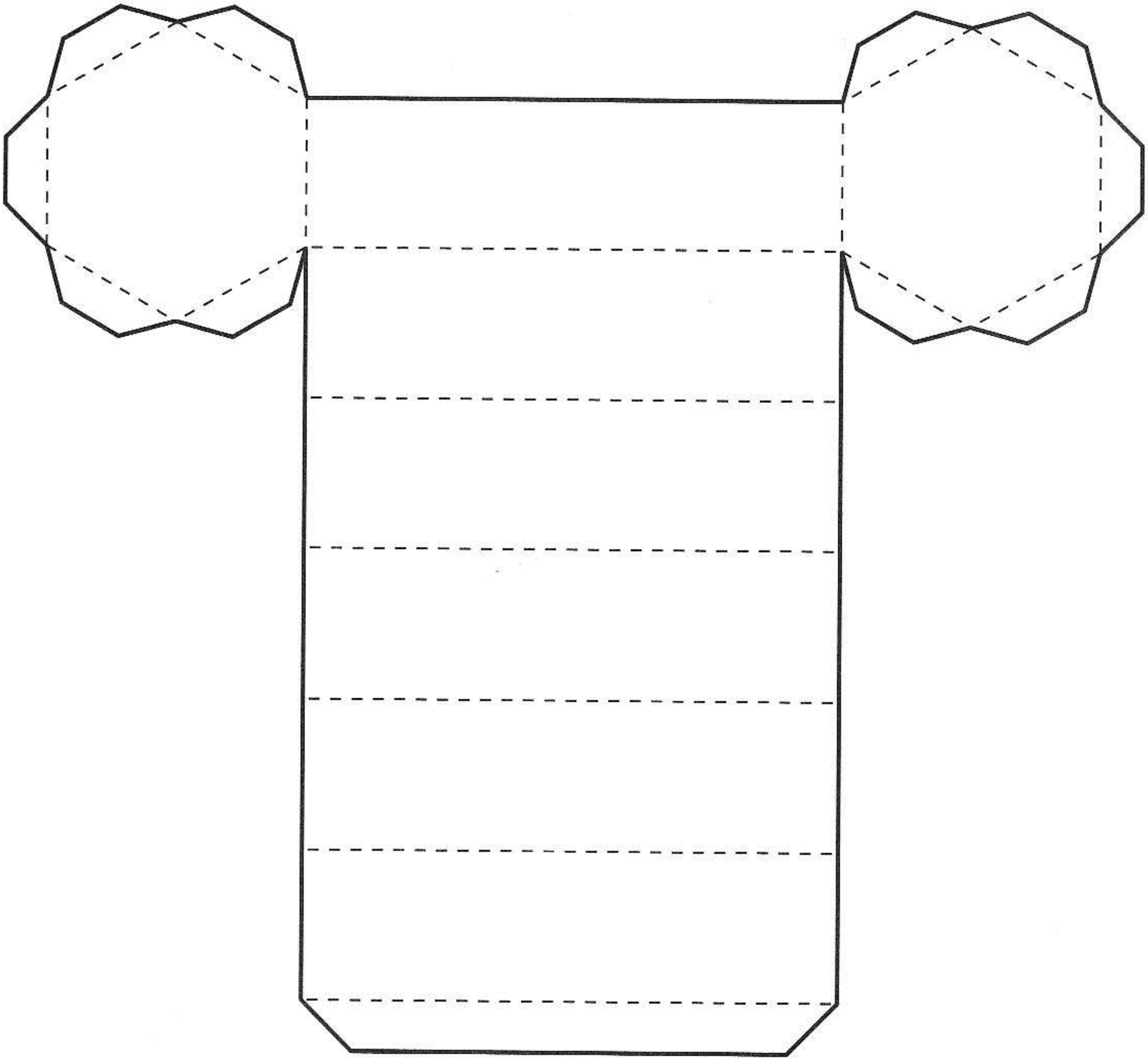
BLM 24 Sample Net of Pentagon-based Prism



BLM 25 Sample Net of Hexagon-based Prism



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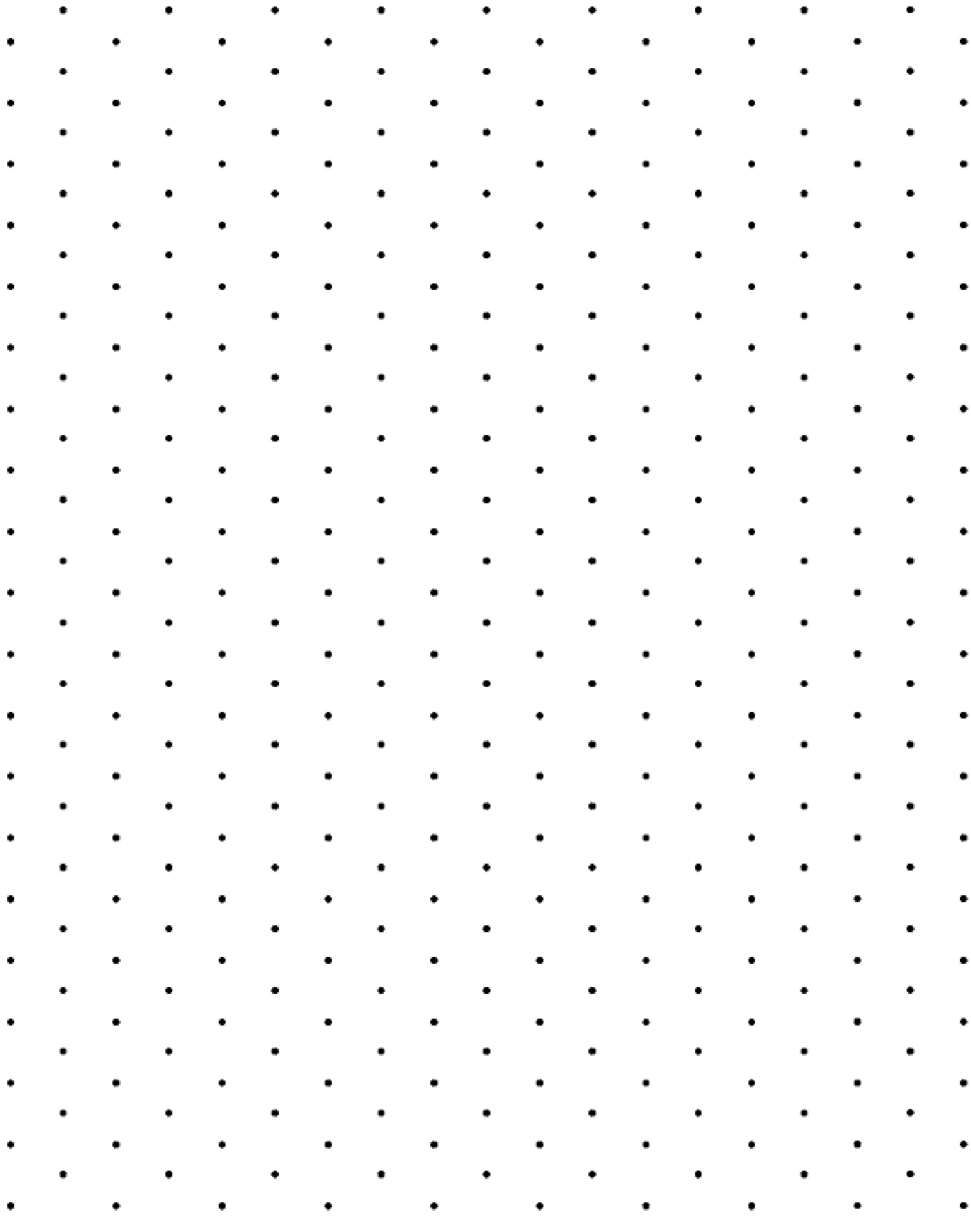
BLM 26 Isometric Dot Paper



UNIT 6 FRACTIONS AND DECIMALS

UNIT 6 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 173** **TG p. 260**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	None	All questions
Chapter 1 Fractions				
6.1.1 EXPLORE: Renaming Fractions (Optional) **SB p. 175** **TG p. 262**	**4-A4 Renaming Fractions: equivalent fractions** • understand concretely that two or more fractions can have different names but the same value	40 min	• Rectangle Fraction Pieces (BLM)	Observe and Assess questions
6.1.2 Equivalent Fractions **SB p. 176** **TG p. 264**	**4-A4 Renaming Fractions: equivalent fractions** • understand concretely that two or more fractions can have different names but the same value • find number patterns in equivalent fractions using models	1 h	• Scissors (optional)	Q1, 3, 6
6.1.3 Comparing and Ordering Fractions **SB p. 179** **TG p. 269**	**4-A5 Fractions: compare and order** • compare visually, in a variety of ways • compare fractions with the same denominator • compare fraction with the same numerator • develop and use benchmark fractions to compare and order fractions	1 h	None	Q2, 3, 5
GAME: Closer to 1 (Optional) **SB p. 182** **TG p. 272**	Practise comparing fractions in a game situation	25 min	• Sets of number cards • Rectangle Fraction Pieces (BLM) (optional)	N/A
6.1.4 Modelling Mixed Numbers **SB p. 183** **TG p. 273**	**4-A3 Fractions and Mixed Numbers: model** • develop visual images for fractions and mixed numbers through concrete materials • use contexts which include part of a whole and part of a group	1 h	• Pattern blocks or Pattern Block Pieces (BLM)	Q1, 2, 4, 7
Chapter 2 Representing Decimals				
6.2.1 Modelling Hundredths **SB p. 186** **TG p. 276**	**4-A6 Hundredths: model and record** • model decimal hundredths • develop the concept of hundredths in our place-value system by continuing the pattern of dividing by 10 • explore the relationship between decimals and fractions	1 h	• Hundredths Grids (BLM)	Q1, 5, 6, 7
6.2.2 Comparing and Ordering Decimals **SB p. 189** **TG p. 279**	**4-A7 Hundredths: compare and order** • compare whole number part first, decimal part second	1 h	• Hundredths Grids (BLM)	Q2, 4, 5

Chapter 3 Decimal Addition and Subtraction				
6.3.1 Adding Decimals **SB p. 192** **TG p. 282**	**4-B1 Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers** • apply familiar addition and subtraction strategies to numbers • relate addition and subtraction of decimals to addition and subtraction of whole numbers • continue estimating	1 h	• Place Value Charts (BLM)	Q1, 3, 7
6.3.2 Subtracting Decimals **SB p. 195** **TG p. 285**	**4-B1 Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers** • apply familiar addition and subtraction strategies to numbers • relate addition and subtraction of decimals to addition and subtraction of whole numbers • continue estimating	1 h	• Place Value Charts (BLM)	Q1, 3, 6
CONNECTIONS: Decimals from Whole Numbers (Optional) **SB p. 198** **TG p. 287**	Make a connection between related decimal subtractions	10 min	None	N/A
GAME: Aim for 5 (Optional) **SB p. 198** **TG p. 287**	Practise decimal addition and subtraction in a game situation	20 min	• Dice	N/A
UNIT 6 Revision **SB p. 199** **TG p. 288**	Review the concepts and skills in the unit	2 h	• Hundredths Grids (BLM) • Place Value Charts (BLM)	All questions
UNIT 6 Test **TG p. 290**	Assess the concepts and skills in the unit	1 h	• Hundredths Grids (BLM) • Place Value Charts (BLM)	All questions
UNIT 6 Performance Task **TG p. 292**	Assess concepts and skills in the unit	30 min	• Hundredths Grids (BLM)	Rubric provided
UNIT 6 Blackline Masters **TG p. 295**	BLM 1 Rectangle Fraction Pieces BLM 2 Pattern Block Pieces BLM 3 Hundredths Grids BLM 4 Place Value Charts			

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Math Background

• This number unit allows students to explore fraction and decimal concepts. Students explore fraction equivalence and comparison, and are introduced to mixed numbers. They also represent decimal hundredths and learn decimal addition and subtraction.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 3** in **lesson 6.1.4**, where they think about the number of wholes they can cover with various fractional pieces, in **question 3** in **lesson 6.3.1**, where they solve a real-world problem involving decimal addition, in **question 7** in **lesson 6.3.1**, where they create a problem to match a computation, and in **question 9** in **lesson 6.3.2**, where they solve a mathematical problem involving decimal subtraction.

• Students use communication in **question 6** in **lesson 6.1.3**, where they talk about why one pair of fractions might be easier to compare than another pair of fractions, in **question 9** in **lesson 6.1.4**, where they explain how or whether two mixed numbers can be equivalent, in **question 6** in **lesson 6.2.1**, where they explain the extension of the place value system to hundredths, and in **question 9** in **lesson 6.3.1**, where they discuss the process of adding decimals.

• Students use reasoning in **question 6** in **lesson 6.1.2**, where they reason about why two fractions can or cannot be equivalent, in **question 2** in **lesson 6.1.3**, where they consider which of two fractions is greater, in **question 5** in **lesson 6.2.2**, where they create decimals to fit descriptions, in **question 7** in **lesson 6.2.2**, where they figure out why a particular substitution yields in one result and a different substitution yields a different result, and in **question 4** in **lesson 6.3.2**, where they consider different subtractions that result in the same value.

• Students consider representation in **lesson 6.1.1**, where they realize that the same whole can be represented by more than one fraction, in **question 1** in **lesson 6.1.2**, where they use diagrams to determine whether two fractions are equivalent, and in **lesson 6.3.2**, where they choose a subtraction model.

• Students use visualization skills in **lesson 6.1.1**, where they use different size fraction pieces to show the same amount, in **question 2** in **lesson 6.1.2**, where they draw pictures to show whether or not two fractions are equivalent, in **question 1** in **lesson 6.1.3**, where they use pictures of fractions to decide which is greater, and in **question 4** in **lesson 6.1.4**, where they visualize a situation to match a mixed number.

• Students make connections in **question 4** in **lesson 6.2.1**, where they connect decimal tenths and hundredths representations, in **question 5** in **lesson 6.2.1**, where they relate decimals to fractions, in **question 8** in **lesson 6.2.2**, where they relate decimal and whole number comparisons, and in **question 10** in **lesson 6.3.2**, where they connect decimal subtraction and whole number subtraction.

Rationale for Teaching Approach

• This unit is divided into three chapters.

Chapter 1 focuses on fraction equivalence and comparison and introduces mixed numbers.

Chapter 2 focuses on modelling and representing decimals up to the hundredths place. This extends student knowledge about decimal tenths.

Chapter 3 focuses on adding and subtracting decimal tenths and hundredths.

• There is one **Explore** lesson. It lets students explore equivalence of fractions concretely before they use equivalence concepts more formally.

• The **Connections** helps students see how to transform a decimal subtraction into an equivalent but easier subtraction.

• There are two **Games**. The first game allows students to practise comparing fractions. In the second game students practise adding and subtracting decimals, as well as estimating.

• Throughout the unit, it is important to encourage flexibility and to accept a variety of approaches from students.

Getting Started

Curriculum Outcomes	Outcome relevance
3 Simple Fractions: real contexts **3** Decimal Tenths: model and record **3** Decimal Tenths: compare and order numbers to tenths	Students will find the work in the unit easier after they review the concepts of fractions and decimals they encountered in Class III.

Pacing	Materials	Prerequisites
1 h	None	• using a fraction to describe a part of a whole • using a fraction to describe a part of a set • recognizing that a fraction describes equal parts • familiarity with terms numerator, denominator, and fraction • using a decimal tenth to describe a part of a whole • ordering decimal tenths

Main Points to be Raised

Use What You Know

• You can use a fraction to describe a part of a set or a part of a whole.

• The denominator of a fraction tells the total number of equal parts. The numerator of a fraction tells how many parts are being considered.

• Whenever you describe a fraction of a whole, you can automatically use another fraction to describe the rest of the whole.

Skills You Will Need

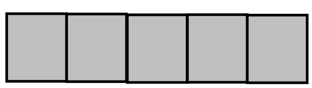
• You can use a decimal to describe tenths. The digit after the decimal point tells how many tenths there are.

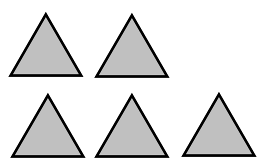
• To order decimal tenths, you compare the digits in the tenths place.

Use What You Know — Introducing the Unit

• Before assigning the **Use What You Know** activity, you may wish to review what a fraction is and what the numerator and denominator of a fraction represent; use both the “part of a whole” and “part of a set” meanings.

For example, show each of these pictures and ask students what $\frac{2}{5}$ of each would be.





• Students can work in pairs to complete the activity. Observe students as they work. You might ask:

- *How did you know the denominator would be 4?*

- *Why could* $\frac{1}{4}$ *be used to describe more than one thing about the set of shapes?*

- *Why does your picture have 5 items in it?*

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions. You may wish to remind students what a decimal like 0.2 or 0.3 means by drawing a picture of a whole divided into ten parts.

• Students can work individually.

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Answers

A. i) $\frac{2}{4}$

ii) *Sample response*: The round shapes

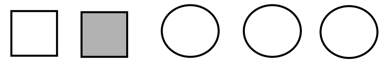
iii) *Sample response*:
The shapes that are triangles.
The shapes that are white.
The shapes that have four sides.

iv) The whole group of four shapes.

B. i) $\frac{1}{3}$ **ii)** The part that is grey.

iii) No; The three parts are not all the same size.

C. *Sample response*:



$\frac{2}{5}$ tells about the shapes that are squares.

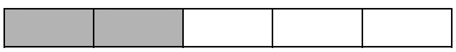
$\frac{3}{5}$ tells about the shapes that are circles.

$\frac{1}{5}$ tells about the shapes that are shaded.

1. *Sample response*: $\frac{2}{3}$ boys and $\frac{1}{3}$ girls

2. a) $\frac{2}{5}$

b) *Sample response*:



3. *Sample responses*:

a) About $\frac{1}{2}$ **b)** About $\frac{1}{8}$

c) About $\frac{5}{6}$ **d)** $\frac{1}{1}$ (or 1) if the glass looks full

or $\frac{0}{1}$ (or 0) if the glass looks empty

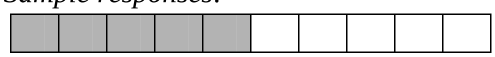
4. A and B

5. $\frac{2}{10}$; 0.2

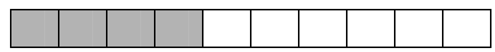
6. 0.8 is shaded or 0.2 is white

7. *Sample responses*:

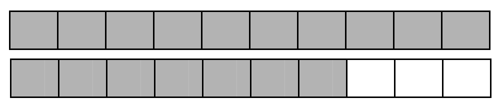
a)



b)



c)



8. 0.3, 0.8, 0.9, 1.0, 2.5

Supporting Students

Struggling students

• Some students may struggle more to understand a fraction as part of a set than with a fraction as part of a whole. Many students think that all the objects in a set must be identical if you want to use fractions to describe them. Make sure that students realize that although the parts of a whole must be equal (in area) to use a fraction to describe them, the same is not true for the parts of a set.

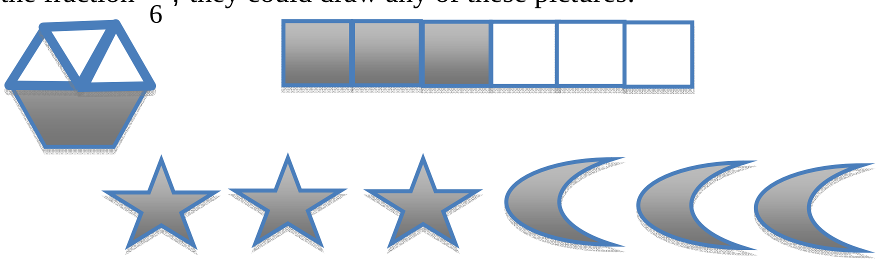
For example, in a group of 2 children and 1 adult, you can use the fraction $\frac{2}{3}$ to describe the children even though the children are much smaller than the adult.

• Some students may be less comfortable with decimals greater than 1 than with decimals less than 1. You may have to remind students that, for example, 2.5 means 2 + 0.5.

Enrichment

• Students might enjoy choosing a favourite fraction and representing it in many different ways.

For example, for the fraction $\frac{3}{6}$, they could draw any of these pictures:



6.1.1 EXPLORE: Renaming Fractions

Curriculum Outcomes	Lesson Relevance
4-A4 Renaming Fractions: equivalent fractions • understand concretely that two or more fractions can have different names but the same value	This optional exploration is an informal concrete reminder for students about what equivalent fractions are and what makes fractions equivalent. It is helpful for students to explore why we can multiply numerator and denominator by the same amount to achieve an equivalent fraction before they apply that strategy formally.

Pacing	Materials	Prerequisites
40 min	• Rectangle Fraction Pieces (BLM)	• naming fractions of a whole

Main Points to be Raised

• If two fractions both name the same part of the same whole, the fractions are equal (or equivalent).

Exploration

• With students, read through the white paragraph at the top of **page 175** of the student text. Make sure they understand these things:

- It takes three $\frac{1}{3}$ pieces to cover the whole rectangle.

- It takes two $\frac{1}{6}$ pieces (sixth pieces) to cover the $\frac{1}{3}$ piece.

• Ask students to work in pairs on the exploration. Have them cut the fraction strips into 10 strips. Then they can either fold them or cut them up into individual fraction pieces. Folding is probably more convenient, if it does not confuse them.

• Observe while students work. You might ask questions such as the following:

- *Why do you think you needed more sixth pieces than fourth pieces to cover $\frac{1}{2}$?* (Sixth pieces are smaller so it takes more of them.)

- *What happened when you tried to cover $\frac{1}{2}$ with fifths?* (Two fifths were not enough and three fifths were too much, so I could not use fifths.)

- *What denominators could you use when you tried to cover $\frac{1}{2}$ with other pieces?* (All the even-number denominators)

- *What were the denominators when you tried to cover $\frac{1}{3}$ with smaller pieces?* (They were numbers that are three times whole numbers, like 6, 9, and 12.)

- *What did you notice about the numerators for the fractions for $\frac{1}{3}$?* (They were also all three times whole numbers.)

Observe and Assess

As students work, notice the following:

• Do students make sure that the pieces cover exactly before they decide that fractions are equivalent?

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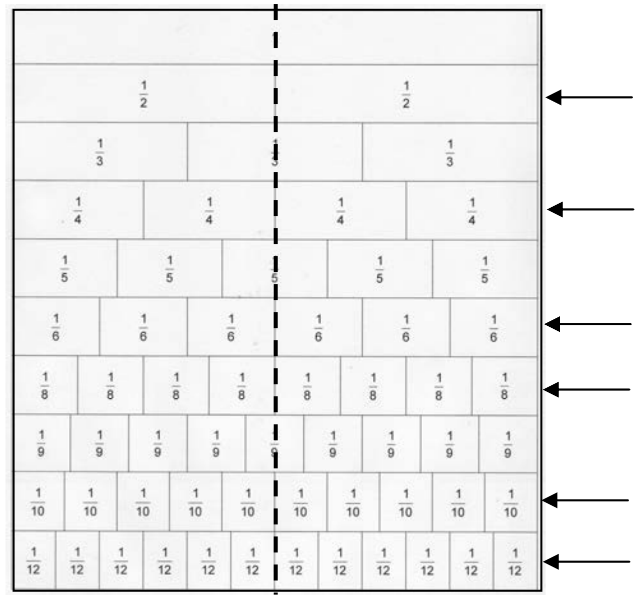
• Do students predict equivalence even before they cut out the pieces, by imagining a vertical line and visualizing the pieces that end at that line?

For example, do they see equivalents for $\frac{1}{2}$ like this?



• Do students try many possibilities or do they quit after trying just one possibility?

• Do students write the appropriate equations to describe what they did?

• Do students show insight in deciding which pieces to try?

Share and Reflect

After students have had sufficient time to complete the exploration, you may have a class discussion and pose these questions:

• *When the denominator increased, what happened to the numerator?*

• *When the numerator increased, what happened to the denominator?*

• *Which fractions had the most names? Why do you think that happened?*

Answers

A. $\frac{1}{2} = \frac{2}{4}$ **B.** $1 = \frac{2}{2}$ or $\frac{3}{3}$ or $\frac{4}{4}$ or $\frac{5}{5}$ or $\frac{6}{6}$ or $\frac{8}{8}$ or $\frac{9}{9}$ or $\frac{10}{10}$ or $\frac{12}{12}$ $\frac{1}{2} = \frac{2}{4}$ or $\frac{3}{6}$ or $\frac{4}{8}$ or $\frac{5}{10}$ or $\frac{6}{12}$	$\frac{1}{3} = \frac{2}{6}$ or $\frac{3}{9}$ or $\frac{4}{12}$ $\frac{1}{4} = \frac{2}{8}$ or $\frac{3}{12}$ $\frac{1}{5} = \frac{2}{10}$ $\frac{1}{6} = \frac{2}{12}$

Supporting Students

Struggling students

• Few students should struggle with this activity because of its concrete nature. However, some students may need to be reminded to count the number of pieces they are using and to use that number as the numerator.

For example, if they use four $\frac{1}{8}$ pieces to make $\frac{1}{2}$, they should write the fraction $\frac{4}{8}$.

Enrichment

• Some students may use what they observed to predict other names for other fractions, such as $\frac{2}{16}$ for $\frac{1}{8}$ or $\frac{3}{30}$ for $\frac{1}{10}$. They can explain their predictions.

6.1.2 Equivalent Fractions

Curriculum Outcomes	Outcome relevance
4-A4 Renaming Fractions: equivalent fractions • understand concretely that two or more fractions can have different names but the same value • find number patterns in equivalent fractions using models	Most operation work with fractions in higher classes requires students to use an equivalent form of a fraction. To have a sense of the size of a fraction such as $\frac{7}{12}$, students need to recognize that since another name for $\frac{6}{12}$ is $\frac{1}{2}$, then $\frac{7}{12}$ is about $\frac{1}{2}$.

Pacing	Materials	Prerequisites
1 h	• Scissors (optional)	• naming fractions of wholes in a picture • familiarity with multiplication facts

Main Points to be Raised

• There is always more than one name for a number. This is also true of a fraction.

• When two fractions describe the same part of the same whole, they are called equivalent.

• You can multiply the numerator and denominator of a fraction by the same amount (other than 0) to get an equivalent fraction.

Try This — Introducing the Lesson

A. Students can work alone or in pairs. While you observe students at work, you might ask questions such as

• *How did you decide how to show* $\frac{6}{9}$ *?* (There are 9 small squares and I had to colour 6 of them.)

• *How did you decide how to show* $\frac{2}{3}$ *?* (I noticed there were 3 rows, so I coloured 2 of them.)

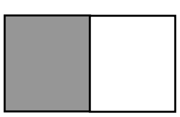
• *How many small squares did you cover each time?* (6 squares)

• *Does it matter which 6 small squares you colour?* (No. I can colour any 6 out of 9.)

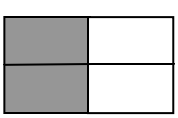
The Exposition — Presenting the Main Ideas

• Ask students why 4 + 1 and 10 ÷ 2 are both other names for the number 5. Follow up by asking for even more names (e.g., 3 + 2, 12 – 7, 20 ÷ 4). Mention that just as whole numbers like 5 have different names, so do fractions.

• On the board, draw a rectangle and shade $\frac{1}{2}$ of it.



Then, draw a line across and show why $\frac{2}{4}$ is another name for the same amount.



Point out that when you divide each original part into two smaller parts, there are twice as many parts that are shaded. Show how you could record this by writing $\frac{1}{2} = \frac{2 \times 1}{2 \times 2}$ or $\frac{2}{4}$.

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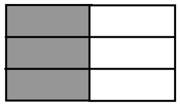
Show how instead you could have changed each of the original parts into three smaller parts. Now three times as many parts are shaded.



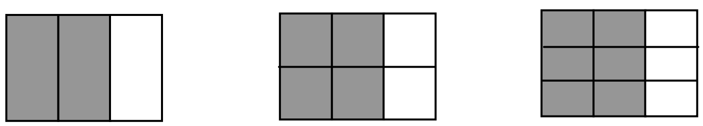
You could write $\frac{1}{2} = \frac{3\times 1}{3\times 2}$, or $\frac{3}{6}$.

• Tell students that when two fractions represent the same part of the whole, we say they are *equivalent* fractions.

Ask students to suggest how to draw a picture to show two other fractions equivalent to $\frac{1}{2}$. Encourage them to see that you only need to subdivide the original parts into more smaller parts.

• Repeat the activity beginning with the fraction $\frac{2}{3}$ so that students can see at least two other equivalent fractions for $\frac{2}{3}$.

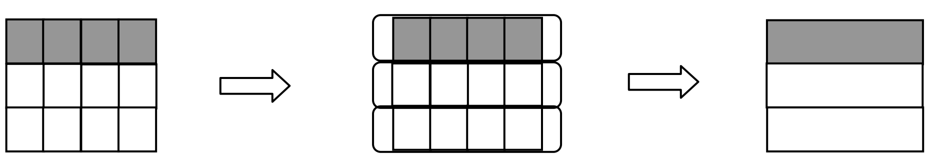
For example: $\frac{2}{3}$ $\qquad$ $\frac{2\times 2}{2\times 3}$ or $\frac{4}{6}$ $\qquad$ $\frac{3\times 2}{3\times 3}$ or $\frac{6}{9}$



• Have students note that they are multiplying the numerator and denominator by the same amount. Assure them that this is always possible (if the multiplication is not by 0).

• Show how you can sometimes divide the numerator and denominator by the same amount to create an equivalent fraction.

For example, begin with a drawing for $\frac{4}{12}$. Show how to combine groups of four parts to show that the amount is also $\frac{1}{3}$. Note how this is the reverse of what was done above.



$$\frac{4}{12} \quad = \quad \frac{4\div 3}{12\div 3} \quad = \quad \frac{1}{3}$$

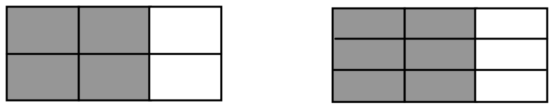
• Provide an opportunity for students to begin with a fraction of their own choice. Ask them to create diagrams to show two equivalents for that fraction. Each student can ask a partner to check his or her work.

Revisiting the Try This

B. This question allows students to make a formal connection between using multiplication and division to create equivalent fractions and the problem they solved in **part A**.

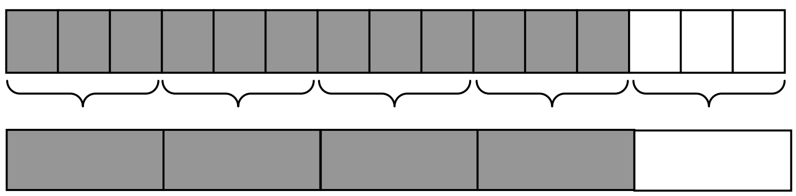
Using the Examples

• Work through **examples 1 and 2** with students. Point out why the student in **example 1** used a 30 cm length (since it is easy to draw fifteen 2-cm parts or ten 3-cm parts). Ask what length rectangle they might use to show that $\frac{4}{6} = \frac{6}{9}$. Possibilities are 18 cm, 36 cm, 54 cm, and so on. Other values can be used, but they are less convenient. Point out how the equivalence could also be shown using an array of rows and columns, such as three columns and either two or three rows.

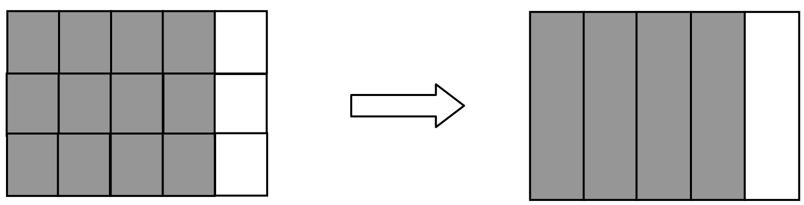


• In discussing **example 2**, help students to see that you can always subdivide parts or multiply the numerator and denominator by the same amount to get an equivalent fraction. However, it is only helpful to combine parts or to divide the numerator and denominator by the same amount if both are multiples of the same number, other than 1.

For example, for $\frac{12}{15}$, you might divide both numerator and denominator by 3 to get an equivalent fraction, $\frac{4}{5}$.



or



It is not convenient to merge parts for some fractions, e.g., $\frac{4}{9}$. If you try to group two parts at a time, the ninth part is left out. If you try to group three parts at a time, the fourth shaded part is left out.

Practising and Applying

Teaching points and tips

Q 1: Students have yet not worked with shapes that look like those in **part a) or d)**, but it does not matter because they are simply looking to see if the same amount of the same whole is shaded. For **part d)**, they may wish to trace and cut out the shape on the left and rotate it to test the equivalence with the shape on the right.

Q 2: If they wish, students can draw rectangles; any shapes for the whole are acceptable. Encourage students to use shapes that are easy to create.

For example, for **part a)** they might use rectangles than are 10 cm wide and for **part c)** they might use rectangles that are 40 cm wide.

Some students will know whether or not fractions are equivalent before they draw pictures.

For example, for **part c)**, they might realize that equivalence is not possible since there are four pieces (the numerators) both times, but the piece sizes (the denominators) are different.

Q 3: Students can work symbolically by multiplying or dividing numerator and denominator by the same amount, or they can draw pictures.

Q 4: It might help to talk about how shoes come in pairs with students who do not see this right away.

Q 6: Some students will draw pictures to show that the fractions are not equivalent. Others will use reasoning. For example, $\frac{1}{10}$ is a very small part of the whole, whereas $\frac{5}{6}$ is most of the whole.

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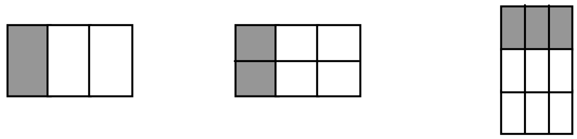
Common errors

• Some students may not recognize that two fractions are equivalent if the shaded portions are not in exactly the same position.

For example, they might realize that the first two fractions below are equivalent, but not that the third fraction is equivalent to both of the others.



Make sure students have opportunities to cut out and trace pictures and turn them if necessary to test equivalence.

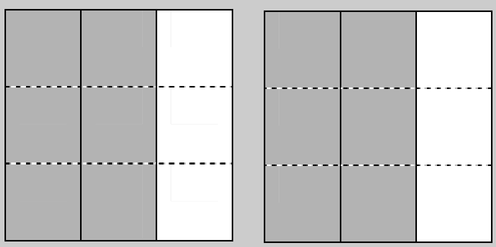
Suggested assessment questions from Practising and Applying

Question 1	to see if students can identify equivalent fractions from diagrams
Question 3	to see if students can create equivalent fractions
Question 6	to see if students can use reasoning to see why two fractions can or cannot be equivalent

Answers

A. *Sample responses*:

i)



ii) They are both made up of 6 small squares.

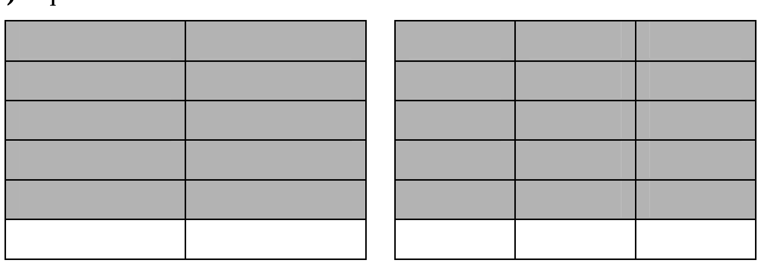
B. *Sample response*:
They show the same amount of the same whole. If I start with $\frac{2}{3}$ and divide each of the 3 parts into 3 equal smaller parts, I get $\frac{6}{9}$.

1. a) $\frac{2}{3}$ and $\frac{4}{6}$; Equivalent **b)** $\frac{3}{5}$ and $\frac{8}{10}$; Not equivalent

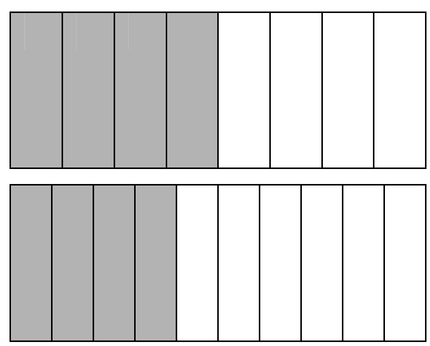
c) $\frac{1}{4}$ and $\frac{2}{8}$; Equivalent **d)** $\frac{1}{3}$ and $\frac{2}{6}$; Equivalent

2. a) Equivalent

b) Equivalent

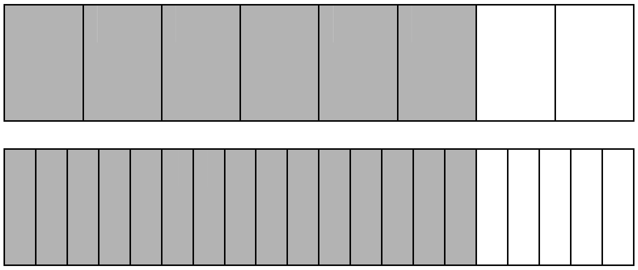


c) Not equivalent



Lesson 6.1.2 Answers [Continued]

2. d) Equivalent



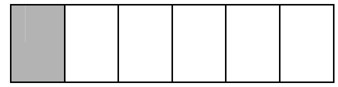
3. *Sample responses*:

a) $\frac{8}{10}$ and $\frac{16}{20}$ **b)** $\frac{6}{16}$ and $\frac{9}{24}$

c) $\frac{1}{3}$ and $\frac{4}{12}$ **d)** $\frac{2}{2}$ and $\frac{8}{8}$

[**4.** *Sample response*:
If you count each pair of shoes, 2 out of 3 pairs are muddy, or $\frac{2}{3}$. If you count each shoe, 4 out of 6 shoes are muddy, or $\frac{4}{6}$.]

5. *Sample response*:

$\frac{1}{6}$ 

$\frac{2}{12}$ 

[If I start with $\frac{1}{6}$ and divide each of the 6 parts into 2 equal parts I get $\frac{2}{12}$, which is like multiplying the numerator and denominator by 2. I end up with an equivalent fraction because the same amount is shaded.]

[**6.** *Sample response*:
$\frac{1}{10}$ is only a small part of a whole and $\frac{5}{6}$ is most of the same whole, so the fractions cannot be equivalent.]

Supporting Students

Struggling students

• Some students may have more difficulty creating equivalent fractions by dividing (or combining parts) than by multiplying (or creating additional parts). Help these students by showing them how the whole they have created with more parts could be the beginning point and how they can work back to the original fraction with which they began.

Enrichment

• Ask students to create equivalent fractions to meet particular conditions, e.g.,

- a fraction equivalent to $\frac{2}{3}$ with a denominator of 21

- a fraction equivalent to $\frac{8}{14}$ with a denominator of 21, or

- two fractions equivalent to $\frac{3}{15}$ where either the numerator or the denominator is 30.

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6.1.3 Comparing and Ordering Fractions

Curriculum Outcomes	Outcome relevance
4-A5 Fractions: compare and order • compare visually, in a variety of ways • compare fractions with the same denominator • compare fraction with the same numerator • develop and use benchmark fractions to compare and order fractions	In higher classes, students will need to be able to compare fractions in a variety of computational and problem solving situations. Fractions are easiest to compare when they have the same denominator, the same numerator, or when they can be compared to familiar amounts like 0, $\frac{1}{2}$, or 1. This is a good beginning point for students.

Pacing	Materials	Prerequisites
1 h	None	• recognizing fractions • using a number line to compare values

Main Points to be Raised

• One fraction is greater than another fraction if it is more of the same whole.

• If the denominators of two fractions are the same, the fraction with the greater numerator is greater.

• If the numerators of two fractions are the same, the fraction with the greater denominator is less.

• You can sometimes compare fractions by comparing how close they are to 0, $\frac{1}{2}$, or 1. A fraction closer to 0 is less; a fraction closer to 1 is greater.

Try This — Introducing the Lesson

A. and B. Students can work alone or in pairs. Students may wish to model the loaf of bread using a folded piece of paper. While you observe students at work, you might ask questions such as the following:

• *How did you show* $\frac{1}{2}$*?* (I divided the paper into two equal parts.)

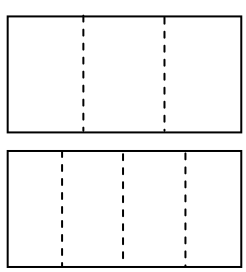
• *How did you show* $\frac{1}{3}$*?* (I divided the paper into three equal parts.)

• *How do you know they are different amounts?* (When I divide something into two parts, the parts are different sizes than if I divide the same thing into three parts.)

• *Why did you need to know that the loaves were the same size?* (If I have $\frac{1}{3}$ of something very big and $\frac{1}{2}$ of something very small, the $\frac{1}{3}$ could be larger.)

The Exposition — Presenting the Main Ideas

• Fold one piece of paper into thirds and an identical paper into fourths. Line up the papers so that students can compare them.



Help students see how this arrangement shows that $\frac{1}{3} > \frac{1}{4}$ since a larger part of the paper is in the $\frac{1}{3}$ part than in the $\frac{1}{4}$ part. It also shows that $\frac{2}{4} < \frac{2}{3}$. Ask students what other comparisons it shows (e.g., $\frac{3}{4} > \frac{2}{3}$).

• Ask students to imagine two identical pieces of paper that you have folded into eight pieces each. Tell them to imagine that you have coloured four parts on one paper and five parts on the other paper. Ask whether they can tell which paper has more colouring without seeing the papers. Discuss how they know that $\frac{5}{8} > \frac{4}{8}$.

Suggest another fraction comparison based on folding paper, for example, $\frac{3}{12}$ compared to $\frac{8}{12}$. Ask students to explain why they do not need to see models to know which is greater. Help them understand that just like eight toys are more than three toys, or eight children are more than three children, eight twelfths must be greater than three twelfths. Eight of anything is greater than three of that same thing. Help students deduce that when the denominators are equal, they can compare fractions by comparing the numerators.

• Have students look at the pictures on **page 179** in the student text to see another fraction comparison, in this case $\frac{4}{5}$ compared to $\frac{3}{5}$.

• Have students turn to **page 180** to see the pictures of $\frac{2}{3}$ and $\frac{2}{8}$. Ask them how the picture shows that 2 third parts take up more space than 2 eighth parts. Then ask:

How do you know that $\frac{3}{5} > \frac{3}{10}$ *?* Make sure students understand why fifths are larger than tenths of the same whole. For that reason, three large parts are more than three small parts. Help students deduce that when the numerators are equal, they can compare fractions by comparing the denominators. In this case, the greater denominator implies a smaller fraction because the pieces are smaller.

• Have students look at the number line on **page 180** of the student text. Talk about why $\frac{8}{9}$ and $\frac{1}{3}$ are placed where they are on the number line.

- For $\frac{8}{9}$, they must imagine dividing the line between 0 and 1 into 9 equal parts. They go up to the end of the 8th part.
- For $\frac{1}{3}$, they must imagine dividing the same line into 3 equal parts. They go up to the end of the 1st part.

You may wish to draw an 18 cm line to model this.

Point out that since $\frac{8}{9}$ is closer than $\frac{1}{3}$ to the 1, then $\frac{8}{9}$ must be greater. Help students see that they could have compared the numerators and denominators within each fraction to see that $\frac{8}{9}$ is almost 1 and $\frac{1}{3}$ is less than $\frac{1}{2}$, so $\frac{8}{9}$ must be greater.

Revisiting the Try This

C. This question allows students to use the formal comparison techniques they learned to see why $\frac{1}{2} > \frac{1}{3}$.

Using the Examples

• Assign students to pairs and ask them to read through the two examples. One student should be responsible for **example 1** and the other for **example 2**. Each student can then lead his or her partner through the example he or she mastered.

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Practising and Applying

Teaching points and tips

Q 1 and 2: Have students describe the strategies they use, i.e.,

- Do they compare numerators when the denominators are the same?

- Do they compare denominators when the numerators are the same?

- Do they compare each fraction to 0 or to 1?

Q 3: There are many possible answers to each part. Allow students to choose a strategy for each.

Q 4: This question allows students to apply in a new situation what they learned in the lesson.

Q 6: Use this closure question with the whole group as a way of summarizing the lesson.

Common errors

• Some students think that fractions are equal if the numerator and denominator are the same distance apart. For example, they think that $\frac{2}{3}$ is the same as $\frac{3}{4}$ since $3 – 2 = 4 – 3$.

Help students to overcome these misconceptions by using picture models.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can compare fractions in symbolic form
Question 3	to see if students can describe fractions to meet various conditions
Question 5	to see if students can compare fractions in a problem situation

Answers

A. Sangay's family; *Sample response*:

I drew a picture of $\frac{1}{2}$ and $\frac{1}{3}$ of the same whole and saw that $\frac{1}{2}$ was more than $\frac{1}{3}$.

B. *Sample response*:

$\frac{1}{3}$ could be more than $\frac{1}{2}$, if the $\frac{1}{3}$ was of a bigger whole than what the $\frac{1}{2}$ was.

C. Comparing fractions with the same numerator.

1. a) $\frac{2}{7}$ and $\frac{3}{7}$; $\frac{3}{7}$ is greater. **b)** $\frac{5}{6}$ and $\frac{5}{8}$; $\frac{5}{6}$ is greater.

c) $\frac{1}{3}$ and $\frac{1}{2}$; $\frac{1}{2}$ is greater. **d)** $\frac{2}{5}$ and $\frac{2}{8}$; $\frac{2}{5}$ is greater.

2. a) $\frac{7}{10}$ **b)** $\frac{6}{9}$ **c)** $\frac{3}{8}$

d) $\frac{7}{10}$ **e)** $\frac{3}{5}$ **f)** $\frac{11}{15}$

3. *Sample responses*:

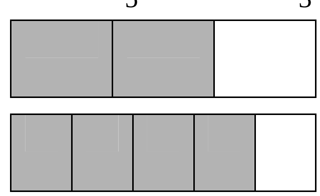
a) $\frac{4}{4}$ **b)** $\frac{1}{10}$ **c)** $\frac{4}{10}$

d) $\frac{3}{5}$ **e)** $\frac{7}{9}$

4. a) 0, 1, 2, or 3
b) 4 or 5 [Some students may use 1, 2, or 3; these are correct, but not expected.]
c) 0, 1, 2, 3, 4
d) 5 [4 and 7 are also acceptable]

5. Yanka; [*Sample response*: $\frac{8}{20}$ is less than $\frac{1}{2}$.

Both $\frac{4}{5}$ and $\frac{2}{3}$ are more than $\frac{1}{2}$ and almost 1. I drew a picture to compare $\frac{4}{5}$ (Yank) and $\frac{2}{3}$ (Ushering).



$\frac{4}{5} > \frac{2}{3}$

6. *Sample responses*:

a) $\frac{2}{3}$ and $\frac{1}{3}$; [The denominators are the same so I just compare the numerators]

b) $\frac{5}{7}$ and $\frac{6}{10}$; [The numerators are not the same, the denominators are not the same, and they are both between $\frac{1}{2}$ and 1.]

Supporting Students

Struggling students

• Although the goal is for students to begin to abstract some of the rules for comparing fractions so they can compare fractions without using diagrams, some students will benefit from continuing to use diagrams until the comparison principles begin to make sense to them. They can either colour or shade fraction parts or they can use folded paper. It is critical that they always use the same whole to compare pairs of fractions.

Enrichment

• Ask students to create fractions to meet various conditions, such as,

- a fraction greater than $\frac{3}{4}$ but less than $\frac{9}{10}$

- less than half of, for example, $\frac{7}{9}$

You might also ask questions like this:

One fraction is as close to 0 as another fraction is to 1. What could the two fractions be?

GAME: Closer to 1

This game is designed to allow students to practice comparing fractions and creating equivalent fractions. If possible, have fraction strips available for students to use to compare their fractions visually.

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6.1.4 Modelling Mixed Numbers

Curriculum Outcomes	Outcome relevance
4-A3 Fractions and Mixed Numbers: model • develop visual images for fractions and mixed numbers through concrete materials • use contexts which include part of a whole and part of a group	Mixed numbers are commonly used to describe amounts that include both wholes and portions. Students get a sense of these numbers by using models.

Pacing	Materials	Prerequisites
1 h	• Pattern blocks or Pattern Block Pieces (BLM)	• representing parts of wholes and parts of sets with fractions • using a number line to compare values

Main Points to be Raised

• You use a mixed number to describe an amount that is greater than 1 but that includes a fractional part.

• You can think of the whole number and fraction part of a mixed number as being added.

For example, $1\frac{2}{3}$ means $1 + \frac{2}{3}$.

• The fraction part of a mixed number is always less than 1.

• A mixed number can be used to describe groups and parts of a group or wholes and parts of a whole.

Try This — Introducing the Lesson

A. Have students work alone or in pairs on this problem. While you observe students at work, you might ask questions such as the following:

• *How do you know that fewer than 7 hexagons will be covered?* (There are 7 trapezoid blocks and they are smaller than hexagon blocks.)

• *How many hexagon blocks can Yeti cover with 2 trapezoids?* (1 hexagon block.)

• *How many full hexagon blocks can Yeti cover with 7 trapezoids?* (Only 3 blocks, using 6 trapezoids. To cover another full hexagon he needs 2 more trapezoids but he has only 1 more.)

• *What could he do with the last trapezoid?* (He could cover half of a hexagon.)

The Exposition — Presenting the Main Ideas

• Have students turn to **page 183** in the student text to examine the exposition. They will see pictures of $2\frac{1}{2}$ and $1\frac{1}{3}$. Introduce the term *mixed number* to describe this combination of a whole number and a fraction. Point out that the wholes are all the same size and the fraction is a part of that same whole.

• Note that the fraction part of a mixed number must always be a part of a whole, but the whole could be a region or a set.

• Confirm students' understanding by asking them to create a model for the mixed number $2\frac{2}{3}$.

Revisiting the Try This

B. This question allows students to use a mixed number to describe the geometric situation presented in **part A.**

Using the Examples

• Present the questions in the three examples. Students can try them alone or in pairs and then compare their answers to those in the text. You may need to explain the solution to **example 3**. In this comparison, the fraction in the lower number (in this case, $\frac{2}{3}$) is actually greater than the fraction in the greater number (in this case, $\frac{1}{3}$).

Practising and Applying

Teaching points and tips

Q 1: Some students might need to draw lines from the centre of the hexagon to the vertices to figure out what parts of each hexagon are being shown.

Q 2: Students can use pattern blocks or Pattern Block Pieces (BLM).

Q 3: Students need to realize that each triangle piece is a $\frac{1}{6}$ piece, each rhombus piece is a $\frac{1}{3}$ piece, and each trapezoid piece is a $\frac{1}{2}$ piece.

Q 5: Students might draw a picture to help them answer this question.

Q 6 and 7: Some students will deduce that to compare mixed numbers, you can often just compare the whole number parts, ignoring the fraction parts.

Q 8 shows why sometimes you need to consider the fraction parts as well.

Q 9: Allow students to come up with their own definition of equivalent mixed numbers. They will likely realize that the fractional parts must be equivalent and the whole number parts must be identical.

Suggested assessment questions from Practising and Applying

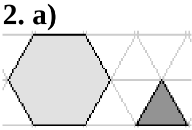
Question 1	to see if students can describe a pictorial representation of a mixed number
Question 2	to see if students can create a representation of a mixed number
Question 4	to see if students can interpret a mixed number in a real-world situation
Question 7	to see if students can compare mixed numbers

Answers

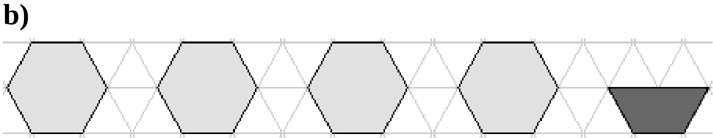
<table>
<tr><td>A. Sample response:
Three and half of a fourth.</td><td>B. 9 trapezoids</td></tr>
<tr><td>

1. a) $3\frac{1}{3}$ **b)** $1\frac{1}{2}$ **c)** $2\frac{2}{6}$

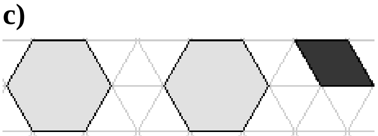
2. a)



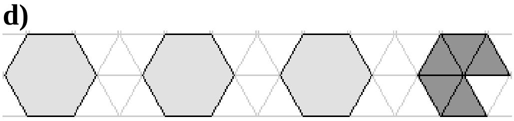
b)



c)



d)



</td><td>

3. a) $1\frac{3}{6}$ **b)** $1\frac{2}{3}$ **c)** $3\frac{1}{2}$

4. There is room for 5 whole classes and half of a sixth class.

5. $3\frac{1}{2}$

6. a) $5\frac{1}{3}$ **b)** $2\frac{1}{3}$

7. $10\frac{5}{6}$; [*Sample response*:

$10\frac{5}{6} > 10$ but $7\frac{1}{3} < 8$, and $8 < 10$.]

[**8.** *Sample response*:

$4\frac{2}{5} = 4 + \frac{2}{5}$ and $\frac{2}{5} < 1$, so $4\frac{2}{5}$ is more than 4 but less than $4 + 1 = 5$.]

</td></tr>
</table>

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9. Yes; [*Sample response*: $2\frac{1}{2} = 2\frac{2}{4}$ because $\frac{1}{2} = \frac{2}{4}$.]	**10.** *Sample response*: If there were 4 cakes at a party and 1 was half eaten, I could use a mixed number to tell how many cakes are left ($3\frac{1}{2}$).

Supporting Students

Struggling students

• Most students will readily recognize mixed numbers, but may have more difficulty using them to interpret real-world situations. Think of other examples to make mixed numbers meaningful to students.

For example, talk about $1\frac{1}{2}$ dozen eggs or $2\frac{1}{2}$ cakes.

6.2.1 Modelling Hundredths

Curriculum Outcomes	Outcome relevance
4-A6 Hundredths: model and record • model decimal hundredths • develop the concept of hundredths in our place-value system by continuing the pattern of dividing by 10 • explore the relationship between decimals and fractions	Students work with hundredths frequently in everyday life, whether with money or measurements. It is important that they be able interpret these amounts meaningfully. It is helpful for them to understand why these values are represented in the way they are.

Pacing	Materials	Prerequisites
1 h	• Hundredths Grids (BLM)	• representing tenths and hundredths of a whole • interpreting fractions

Main Points to be Raised

• Decimal tenths are represented by placing a digit immediately to the right of the decimal point.

• Decimal hundredths are represented by placing two digits in the two decimal places immediately to the right of the decimal point.

• The decimal 0.*ab* is $\frac{ab}{100}$.

• The reason the decimal places to the right of the decimal point are tenths and hundredths is to keep the pattern of the place value system going. Each place represents a value that is $\frac{1}{10}$ of the place to its left.

• There is symmetry around the ones place (along with the decimal point). The tens on the left match the tenths on the right, and the hundreds on the left match the hundredths on the right.

Try This — Introducing the Lesson

A. Have students work alone or in pairs on this problem. While you observe students at work, you might ask questions such as the following:

• *Will more or less than half the grid is covered?* (Less than half, since 33 are less than 50.)

• *How do you know that you cannot colour exactly 8 squares using one colour?* (The other colour would be either 16 squares or 4 squares. When I add 8 to 16 or 8 to 4, I do not get 33.)

• *How did you figure out how many squares to use for each colour?* (I tried numbers to see which worked. When I tried 10 red squares and 20 blue squares I was close to 33 but too low, so I changed to 11 and 22.)

The Exposition — Presenting the Main Ideas

• Display a hundredths grid for students. Make sure they realize that since there are 10 rows and 10 columns, there are 100 equal parts.

• Ask them how to cover $\frac{1}{100}$ of the grid and make sure they realize this is just one small square. Tell them that this is written 0.01 and read as "one hundredth". Remind them that they learned in Class III that 0.1 means $\frac{1}{10}$.

They should notice that one decimal place to the right of the decimal point represents tenths and two decimal places to the right represent hundredths.

• Ask different students how to model each of these decimals on the grid: 0.04, 0.12, 0.50 and 0.6. Make sure they notice that 0.6 is actually 60 hundredths since it is 6 tenths, so 60 small squares are shaded.

• Have students look at the models for 0.07 and 0.38 on **page 186** of the student text to clarify their understanding of the representations.

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• Then ask students to look at the place value chart on **page 187** of the student text. Have them notice that the new columns behave just like the other columns — 10 of an item in one column can be traded for one item in the column to the left. Discuss why this makes sense, i.e.,
$\frac{10}{10}$ (or 10 tenths) = 1 and $\frac{10}{100}$ (10 hundredths) is equivalent to $\frac{1}{10}$.

Revisiting the Try This

B. This question allows students to use decimals to describe how the they coloured the grid in **part A.**

Using the Examples

• Ask pairs of students to read through the two examples to make sure they understand both how to name a shaded area and to shade a given area.

Practising and Applying

Teaching points and tips

Q 2: Encourage students first to show the 0.49 to make sure they can fit all of the decimals on one grid. If they need a second grid, that is acceptable.

Q 4: Students should be thinking about both tenths and hundredths to answer this question.

Q 5: At this point, student are not doing formal fraction/decimal conversions, but rather exploring relationships informally. They can use visual clues rather than numerical relationships to answer these questions.

Q 7: Students might discuss this question in pairs.

Common errors

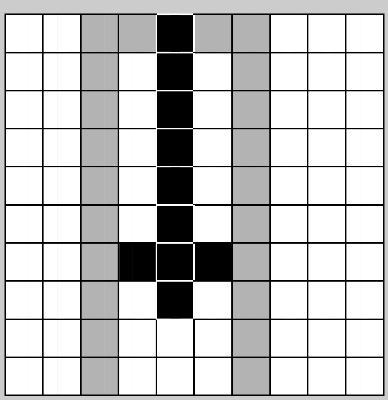
• Many students mistakenly write the decimal for fractions like $\frac{5}{100}$ as 0.5 rather than as 0.05. Help students by asking them to write the decimal for $\frac{1}{2}$ to see that it is 0.5. Then ask them to shade a hundredths grid to see that it is actually $\frac{50}{100}$, not $\frac{5}{100}$.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use a decimal to describe a grid model
Question 5	to see if students can relate decimals to fractions
Question 6	to see if students can make sense of the place value positions to the right of the decimal point
Question 7	to see if students can diagnose a common error involved in writing decimals

Answers

A. *Sample response*:



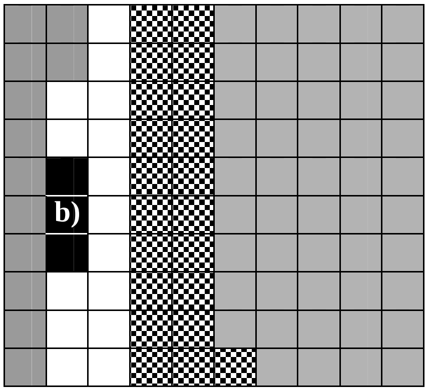
B. *Sample response*: 0.22 and 0.11

Lesson 6.2.1 Answers [Continued]

1. a) 0.08 **b)** 0.12 **c)** 0.40

2. *Sample responses*:

a) **c)** **d)**



3. a) 0.09 **b)** 0.64
c) 0.80 or 0.8 **d)** 1.00 or 1.0 or 1

4. 0.60 and 0.6

5. a) 0.50 or 0.5 **b)** 0.25
c) 0.10 or 0.1 **d)** 1.00 or 1.0 or 1

[**6.** *Sample response*:
As you go right on a place value chart, the amounts get smaller—from 1000 to 100 to 10 to 1 and then from 0.1 to 0.01. Since 0.01 is less than 0.1, it makes sense that it is to the right.]

[**7.** *Sample response*:
You need to include a 0 before the 3 if you want it to say 3 hundredths instead of 3 tenths.
0.3 is 3 tenths.
3 hundredths is 0 tenths + 3 hundredths = 0.03.]

Supporting Students

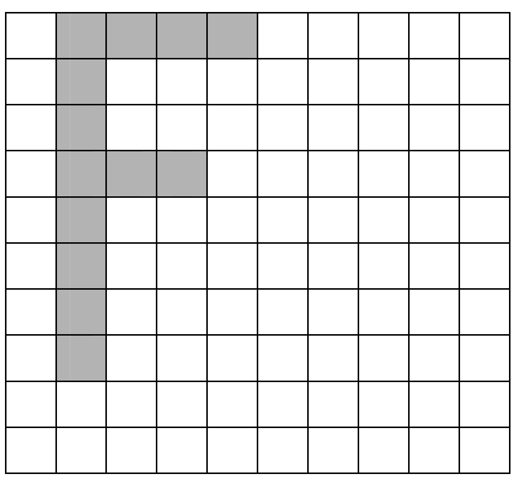
Struggling students

• Most students will be able to represent decimal hundredths on a grid or vice versa. They may have more difficulty with questions where they have to explain, such as **questions 6 and 7**. Have them partner with other students where the other student does the explanation and they react to it, describing what makes sense to them and what does not.

Enrichment

• Some students might enjoy creating pictures on the grid that look like a particular thing but are represented by a certain decimal.

For example, they might be asked to create a letter of the alphabet with a decimal representation of 0.13.



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6.2.2 Comparing and Ordering Decimals

Curriculum Outcomes	Outcome relevance
4-A7 Hundredths: compare and order • compare whole number part first, decimal part second	As students continue to work with decimals in real-world and mathematical contexts, they need to be able to decide which decimal is greater, for example, to compare measurements.

Pacing	Materials	Prerequisites
1 h	• Hundredths Grids (BLM)	• understanding decimal tenths and hundredths

Main Points to be Raised

• If two decimal values are greater than 1, you can sometimes compare them by comparing only the whole number parts.

• You cannot compare the size of two decimal values by counting the total number of digits in each number and comparing those values.

• If two decimal values have the same whole number part, you can compare them by comparing the parts that are less than 1. You can write each of those parts as a fraction and compare the numerators if the denominators are the same.

• You can order decimal numbers by comparing them two at a time.

Try This — Introducing the Lesson

A. Begin this activity by having students talk about their own experiences with the long jump. If students do not know what the long jump is, you might describe it and have a student demonstrate. Talk about how they could measure the distance to the nearest hundredth of a meter (or the nearest centimeter) if they used a ruler marked with centimeters. For students not familiar with the Olympics, explain that athletes from countries around the world compete in these events every four years. (The long jump is part of the summer Olympics. Winter Olympics are also held every four years and feature sports involving snow and ice.) Discuss the importance of measuring accurately to determine a winner in these kinds of events.

Have students work alone or in pairs on this problem. While you observe students at work, you might ask questions such as the following:

• *About how long is 8 m?* (I think it is longer than our classroom.)

• *About how far do you think you could jump?* (I think I could jump 3 m.)

• *How do you know that C. Tomlinson jumped farther than Y. Lamella?* (Lamella's number is less than 8 and Tomlinson’s number is greater than 8.)

• *How did you decide who jumped the farthest?* (I used the three values that were greater than 8 and I rewrote the decimals as fractions.)

The Exposition — Presenting the Main Ideas

• Ask students to describe what 2.1 and 1.47 mean in terms of decimal grids. Make sure they understand that the whole number tells how many full grids to colour and the decimal tells the part of another grid to colour. Ask which value they think is greater and why.

• Have them look at the grids on **page 189** of the student text to see the values represented. They can verify why 2.1 is greater than 1.47. Point out that even though 147 > 21, 1.47 is not greater than 2.1. When they compare values, they have to consider the fact that one decimal is in hundredths and the other is in tenths.

• Let students look at the comparison of 2.1 and 2.24 on **page 190** to see how to compare decimals with the same whole number part.

• Ask students to predict the order from least to greatest for these decimals: 0.23, 2.43, 1.46, 1.5
Use decimal grids to help students verify their predictions.

• Students might then read the information on **page 190** in the exposition about ordering decimals.

Revisiting the Try This

B. and C. Students can use a visual model to help them compare the distances they compared more informally in **part A.** They should understand that their answer to **part C** could be less than 8, exactly 8, or more than 8.

Using the Examples

• Present the tasks in each example and have students work in pairs to try them. They can then compare their solutions to those in the examples. Ask students whether their approaches were the same as those in the examples.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to use grids to compare the decimal parts of the numbers.

Q 2: Observe whether students first use the whole number values to compare. Observe whether they focus on the number of digits in the provided values (a common error).

Q 3: Many students mistakenly write 3.09, 3.010, 3.011, and 3.012 and then they do not know how to handle 3.12. You might suggest that they read the decimals meaningfully, e.g., “three and seven hundredths, three and eight hundredths”, and so on, before they write them.

Q 4: Students might try out their explanations with other students before recording them.

Q 5: Encourage students to write several possible values where possible.

Q 7: This question is a more complicated problem than some of the others. You might model an example.

For example, for **part a)**, if you chose the digit 8, the decimals would be, in order:

0.48, 3.44, 3.48, 3.84, 8.34, and 8.43.

But if the digit were 4, the decimals would be:

0.44, 3.44, (3.44, 3.44) , 4.34, and 4.43.

Q 8: You may wish to handle this question as a class discussion.

Common errors

• Many students have difficulty making the transition from, for example, 2.09 to 2.10. They write 2.010 to follow 2.09. Help students by colouring grids to see why the value after 9 hundredths, or 0.09, is 10 hundredths, or 0.10, and not 0.010.

• Some students ignore the decimal point to compare decimals.

For example, to compare 3.45 and 8.9, they assume that since 345 > 89, then 3.45 > 8.9.

Encourage them to read and interpret the decimals.

For example, 3.45 are 3 and a part of 1, but 8.9 is 8 and a part of 1.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can order decimals
Question 4	to see if students can explain a decimal comparison
Question 5	to see if students can create decimals according to given criteria

Answers

A. C. Tomlinson jumped farthest.
Sample response:
I only looked at 8.25, 8.21, and 8.03 because they are all more than 8. I knew that 7.98 is less than 8.
8.25 are 8 and 25 hundredths, 8.21 are 8 and 21 hundredths, and 8.03 are 8 and 3 hundredths.
8.25 > 8.21 > 8.03 because
25 hundredths > 21 hundredths > 3 hundredths.
8.25 m is greatest.

B. *Sample response*:
Since I am only looking at 8.25, 8.21, and 8.03, and they all have the same whole number part, I can use hundredths grids to show the decimal part of each decimal (0.25, 0.21, and 0.03) and then compare them.

C. *Sample response*: 8.00 m or 8 m

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1. a) 1.2 < 1.37 **b)** 1.3 > 1.28
c) 3.04 < 3.40 **d)** 2.10 > 2.01
e) 4.1 = 4.10

2. a) 0.89, 1.25, 1.28, 3.02, 3.1
b) 2.4, 2.49, 3.71, 3.87, 4.92
c) 0.01, 0.11, 1.01, 1.10

3. a) 3.09, 3.10, 3.11 **b)** 4.00, 4.01

[**4.** *Sample response*:
0.99 is $\frac{99}{100}$, which is not quite 1.
1.2 is 1 + $\frac{20}{100}$, which is more than 1.
That is why 0.99 < 1.2.

99 are 99 ones and 12 are 12 ones.
That is why 99 > 12.]

5. *Sample responses*:
a) 4 **b)** 3.9 **c)** 2.54 **d)** 2.10

6. Nu 4.21

7. a) *Sample response using the digit 9*:
0.49, 3.44, 3.49, 3.94, 9.34, 9.43
b) 0.34, 0.40, 0.43, 3.04, 3.40, 3.44
[**c)** *Sample response*:
When I use 0, the last two decimals are less than the middle two decimals, but if I use any number from 4 to 9, the last two decimals are greater.]

[**8.** *Sample response*:
a) When I compare decimals, I start with the digits with the greatest value and then look at the next digits, just like with whole numbers, e.g.,
5.43 > 5.23 because 5 = 5 but 0.4 > 0.2.
543 > 523 because 500 = 500 but 40 > 20.
b) To compare whole numbers I can compare the number of digits; I cannot do that with decimals, e.g.,
21 < 179 because 21 has 2 digits and 179 has 3 digits, but 2.1 > 1.79.]

Supporting Students

Struggling students

• Some students may have difficulty with **question 7**. You might suggest that they replace the brackets with each digit from 0 to 9 and see what happens. They might choose not to answer **part c)** if they are struggling.

Enrichment

• Students might enjoy creating riddles that involve decimal comparisons for other students to solve.

For example, clues to figure out the number 2.35 might be:

- It is greater than 2.3.
- It is less than 2.39.
- It is closer to 2.39 than 2.3.
- If you model it with hundredths grids, you cover all half or full columns.

Chapter 3 Adding and Subtracting Decimals

6.3.1 Adding Decimals

Curriculum Outcomes	**Outcome relevance**
4-B1 Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers • apply familiar addition and subtraction strategies to numbers • relate addition and subtraction of decimals to addition and subtraction of whole numbers • continue estimating	Students need to add decimals in many real-world situations, particularly those involving money and measurements.

Pacing	**Materials**	**Prerequisites**
1 hr	• Place Value Charts (BLM)	• familiarity with place value concepts • adding whole numbers

Main Points to be Raised

• When you add decimals, you add tenths to tenths and hundredths to hundredths.

• To add two decimals greater than one, you can add the whole number part and the decimal parts separately. You can use grids for the decimal parts, if desired, and then combine the totals.

• You add decimals just like you add whole numbers. After you add values in the same place value column, you trade 10 of those for 1 in the next place value column to the left.

Try This — Introducing the Lesson

A. Have students work alone or in pairs on this problem. While you observe students at work, you might ask questions such as the following:

• *Why do you think she spent more than Nu 20?* (Nu 17.25 is more than Nu 17, and Nu 6.20 is more than Nu 6, and 17 + 6 = 23.)

• *Why might it help to think of this amount in chetrums?* (Then I can add 1725 chetrum's and 620 chetrum's. I know how to add whole numbers.)

• *How did you calculate the sum?* (I added the ngultrum parts and the chetrum parts separately.)

The Exposition — Presenting the Main Ideas

• Create a whole number place value chart on the board. Remind students of how to add 135 + 48 by trading 10 of the ones for 1 ten.

• Extend the place value chart to include decimal tenths and hundredths. Work with students to add 1.35 + 4.8. Make sure they understand why the 8 tenths in 4.8 is placed under the 3 tenths in 1.35 rather than lining up digits from the left or the right when the problem is presented vertically.

• Go through the process of decimal addition.

• Reinforce why it makes sense to use decimal grids to add the decimal parts of 0.35 and 0.8. They will see that altogether, 1 full grid and 0.15 of another grid will be coloured. This is in addition to the **1** and **4** full grids that would be coloured to model **1**.35 and **4**.8.

• Help students see why the answer makes sense, i.e., a number around 1 is added to a number between 4 and 5, so the answer should be around 5 or 6, and it is.

• Have students try to add another pair of decimals, e.g., 12.37 and 1.49. Make sure they line up hundredths with hundredths and regroup when necessary.

• Suggest that students refer to **page 192** in the student text for later reference, if they wish.

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Revisiting the Try This

B. and C. This question allows students to apply what they have learned about adding decimal values to the problem they solved in **part A**.

Using the Examples

• Assign pairs of students to become experts, one on **example 1** and the other on **example 2**. Ask each student to teach the other student in the pair about the example he or she has mastered.

Practising and Applying

Teaching points and tips

Q 1: Provide place value mats for students to use if they wish.

Q 2: Encourage students to estimate before calculating to check the reasonableness of their answer.

Q 3: Ask students what whole number calculation this problem relates to.

Q 4: Ask students why they might consider only the whole number parts to make estimates except in **part b)**.

Q 5: Make sure students realize that they may have to add more 0.13s than are shown in the text.

Q 6: Make sure students realize that a different digit can be used in each box in each equation.

Q 7: If students struggle for ideas, suggest money and measurement situations.

Q 8: Some students will benefit from using a hundredths grid to answer this question.

Q 9: Students might respond individually to this question and then you might take a class poll to see how many students gave each different response.

Common errors

• The most common error students make is not lining up the decimals properly before adding. They often end up adding, for example, ones to tenths or tenths to hundredths. If you ask students to make sure they add ones to ones, all of the other place values will be lined up for column addition. Students could use grid paper or lined paper turned sideways to record their additions.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can add decimals
Question 3	to see if students can solve a real-world problem involving decimals
Question 7	to see if students can relate a decimal addition to a real-world situation

Answers

A. Nu 23.45 **B.** *Sample response*: 6.20 + 17.25 is about 6 + 17 = 23, so the sum 23.45 makes sense.	**C.** If I line the numbers up on the left, I would add ones to tens, tenths to ones, and hundredths to tenths.
1. a) 19.07 **b)** 4.22 **c)** 192.09 **d)** 34.03 **2.** 8.31 m **3.** 0.80 square metres **4.** *Sample responses*: **a)** 43 **b)** 0.67 **c)** 24 **d)** 200	**5.** 7 times [1. and 2. 0.13 + 0.13 = 0.26 3. 0.26 + 0.13 = 0.39 4. 0.39 + 0.13 = 0.52 5. 0.52 + 0.13 = 0.65 6. 0.65 + 0.13 = 0.78 7. 0.78 + 0.13 = 0.91 0.91 + 0.13 = 1.04 OVER 1]

Lesson 6.3.1 Answers [Continued]

6. a) 36.**<u>1</u>**4 + 8.2**<u>8</u>** = **<u>4</u>**4.42 **b)** 1.**<u>9</u>**8 +3.5**<u>7</u>** = 5.55 **c)** **<u>1</u>**1**<u>5</u>**.83 + 74.**<u>8</u>** = 190.63 **7.** *Sample response*: I measured a crooked wall. There was 4.12 m before the wall turned. Then there was another 5.89 m after the turn. How long was the wall? (10.01 m)	**8.** 50 pairs; [0.01 + 0.99, 0.02 + 0.98, 0.03 + 0.97, …, 0.48 + 0.52, 0.49 + 0.51, 0.50 + 0.50; Since the first numbers go from 0.01 to 0.50, which is like 1 to 50, I knew there were 50 pairs.] [**9.** *Sample response*: I must line up the digits by their place value; This will make sure tenths are added to tenths and hundredths are added to hundredths.]

Supporting Students

Struggling students

• **Questions 5, 6, and 8** may be difficult for struggling students. These questions are not essential for full understanding and so could be omitted. Alternatively, you could pair these students with other students and have them work on these questions together.

Enrichment

• Students might create additional questions like **question 6** for other students to solve.

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6.3.2 Subtracting Decimals

Curriculum Outcomes	Outcome relevance
4-B1 Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers • apply familiar addition and subtraction strategies to numbers • relate addition and subtraction of decimals to addition and subtraction of whole numbers • continue estimating	Students need to subtract decimals in many real-world situations, particularly those involving money and measurements.

Pacing	Materials	Prerequisites
1 hr	• Place Value Charts (BLM)	• familiarity with place value concepts • subtracting whole numbers

Main Points to be Raised

• When you subtract decimals, you subtract tenths from tenths and hundredths from hundredths.

• You can subtract the whole number part and the decimal parts of decimals separately. You may need to regroup one whole to perform the decimal part of the subtraction.

• You subtract decimals just like you subtract whole numbers. To subtract values in the same place value column, you may need to trade 1 in the next place value to the left for 10 in the place value column you are subtracting.

• You can also subtract decimals by adding up. You can model this on a number line. The difference is the amount you have to add to get from one value to the other value.

Try This — Introducing the Lesson

A. Have students work alone or in pairs on this problem. While you observe students at work, you might ask questions such as the following:

• *Why would you subtract?* (To find out how much more one amount is than another, I subtract. For example, 5 – 4 = 1 since 5 is 1 more than 4.)

• *Why might you add?* (I need to figure out what to add to 0.47 to get to 1.12.)

• *How did you calculate the difference?* (I added 0.3 to get to 0.50, then I added 0.50 to get to 1, and then I added another 0.12. That is a total of 0.65.)

The Exposition — Presenting the Main Ideas

• On the board create a whole number place value chart. Remind students how to subtract 46 from 132 by trading the 100 from 10 tens.

• Ask students to suggest how they might calculate 4.6 – 1.32.

• Some students are likely to suggest working with a place value chart. If they do, extend the place value chart to include decimal tenths and hundredths. Work with students to calculate 4.6 – 1.32. Discuss why the 3 tenths in 1.32 are placed beneath the 6 in 4.6. Discuss how they then proceed as they would with whole numbers.

• Some students might suggest adding up. If they do, have them examine the number line model on **page 196** of the student text. Discuss why the jumps that are shown are convenient jumps and why these jumps must be added up to determine the difference. Discuss alternative jump sizes, e.g., add 0.6 to get from 1.32 to 1.92, then add another 0.08 to get to 2, and then add another 2.06.

• Have students consider the value of estimating to check an answer.

For example, if they calculate 4.6 – 1.32 by first estimating that the result should be about 4 – 1 = 3, this will help them avoid misaligning place values when they subtract.

• Indicate that for reference students can use **pages 195 and 196** in the student text.

Revisiting the Try This

B. This question reminds students that they can choose among alternate strategies for performing decimal subtractions.

Using the Examples

• Present the questions in **examples 1 and 2** for students to try. They can then compare their processes to those shown in the student text. Ask students who solved the problems correctly in different ways to describe how they approached the problems.

Practising and Applying

Teaching points and tips

Q 1: Observe whether students simply subtract the whole numbers, e.g., 4 – 2 in **part a)** or whether they round, e.g., subtracting 4 – 3 in **part a)**. Either approach is acceptable.

Q 3: Encourage students to estimate to check the reasonableness of their differences.

Q 4: Students might use simple values like 3.45 – 2.00 or they may choose more complex values. If they use only simple values, challenge them to use at least one more complex pair of values.

Q 5: Students need not predict. They can subtract repeatedly. Some students will notice the pattern and recognize how many times the 0.09 must be subtracted.

Q 7: If some students need an idea, direct them to consider **question 2**.

Q 8: Ask students which digit they figured out first and why they chose to determine that digit first.

Q 9: Students might estimate to realize that if the total is 5 and the difference is 1, the numbers are about 3 and 2.

Q 10: This question might be handled in a group discussion.

Common errors

• As with addition, the most common error students make is attempting to subtract using a vertical algorithm and not subtracting the correct digits from each other. Students could use grid paper or lined paper turned sideways to record their subtractions.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can estimate a difference
Question 3	to see if students can calculate a decimal difference
Question 6	to see if students can solve a real-world problem involving decimal subtraction

Answers

A. 0.65 m (or 65 cm)	**B.** *Sample response*: I would add up from 0.47 to 0.50, then to 1, and finally to 1.12. Then I would add the numbers 0.03 + 0.5 + 0.12 = 0.5 + 0.12 + 0.3 = 0.65.
1. *Sample responses*: **a)** About 1; [4 – 3 = 1] **b)** About 60; [100 – 40 = 60 **c)** About 1; [8 – 7 = 1] **d)** About 78; [84 – 6 = 78] **2.** 1.06 m	**3. a)** 1.63 **b)** 0.42 **c)** 162.85 **d)** 19.12 **4.** *Sample responses*: 2.46 – 1.01 = 1.45 2.35 – 0.9 = 1.45 2.78 – 1.33 = 1.45

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5. 10 times; [1. 1 – 0.09 = 0.91 2. 0.91 – 0.09 = 0.82 3. 0.82 – 0.09 = 0.73 4. 0.73 – 0.09 = 0.64 5. 0.64 – 0.09 = 0.55 6. 0.55 – 0.09 = 0.46 7. 0.46 – 0.09 = 0.37 8. 0.37 – 0.09 = 0.28 9. 0.28 – 0.09 = 0.19 10. 0.19 – 0.09 = 0.10] **6.** 3.5 km	**7.** *Sample response*: I must walk 5 km to school. I have already walked 3.12 km. How much farther do I have to walk? (1.88 km) **8. a)** 3.1<ins>**4**</ins> – <ins>**1**</ins>.<ins>**4**</ins>9 = 1.65 **b)** 12.<ins>**0**</ins>4 – <ins>**8**</ins>.8<ins>**7**</ins> = 3.17 **c)** 14.<ins>**0**</ins>2 – <ins>**7**</ins>.8<ins>**9**</ins> = 6.1<ins>**3**</ins> **9.** 3.14 and 1.98 **10.** *Sample responses*: **a)** I can use the same strategies that I use to subtract whole numbers. **b)** There might be fewer digits in the greater number than in the lesser number.

Supporting Students

Struggling students

• Some students may have difficulty with **question 9**. Encourage them to use estimation to determine the whole number parts of the answers and to consider the hundredth digits to decide what the hundredths digits could be for the two numbers. If students still find the question too difficult, you might supply some of the digits and let them figure out the remaining digits.

For example:

3.[ ]4 – [ ].9[ ] = 1.16

3.[ ]4 + [ ].9[ ] = 5.12

Enrichment

• Ask students to create other questions like **question 4 or 9** for their classmates to solve.

CONNECTIONS: Decimals from Whole Numbers

This connection provides a strategy that is particularly useful for subtracting a decimal from a whole number using a vertical, regrouping algorithm. This strategy removes the need to do a great deal of the regrouping.

Answers

1. a) 10 – 3.86 = 9.99 – 3.85 = 6.14 **b)** 7 – 4.38 = 6.99 – 4.37 = 2.62 **c)** 8 – 1.27 = 7.99 – 1.26 = 6.73

GAME: Aim for Five

Students should not change the placement of any digit once they have put it in a particular position. In this way, students are forced to estimate as they go. They get to think about both sums and differences.

UNIT 6 Revision

Pacing	Materials
2 h	• Hundredths Grids (BLM) • Place Value Charts (BLM)

Question(s)	Related Lesson(s)
1 – 3	Lesson 6.1.2
4 – 6	Lesson 6.1.3
7 – 10	Lesson 6.1.4
11 and 12	Lesson 6.2.1
13 – 16	Lesson 6.2.2
17 and 18	Lesson 6.3.1
19 – 23	Lesson 6.3.2

Revision Tips

Q 2: Some students might benefit from using rectangle fraction pieces to review what makes two fractions equivalent before using these more complex numerators and denominators.

Q 6: **Part c)** may be difficult for some struggling students and could be omitted for them.

Q 8: You may need to help students see that the number in one group must be less than 5 in order to result in a mixed number.

Q 12: If students require an additional grid because they do not place their decimals appropriately, allow this.

Q 19: Observe whether students round the numbers or whether they use only the whole number portions. Either approach is legitimate for estimating.

Q 22: Make sure students understand that the decimals for the two parts are not the same.

Answers

1. a) $\frac{6}{8}$ and $\frac{3}{4}$; Equivalent.

b) $\frac{1}{4}$ and $\frac{3}{12}$; Equivalent.

2. A and D

3. *Sample responses*:

a) $\frac{4}{10}, \frac{6}{15}$ **b)** $\frac{4}{5}, \frac{16}{20}$

c) $\frac{8}{18}, \frac{12}{27}$ **d)** $\frac{6}{16}, \frac{9}{24}$

4. a) $\frac{6}{7}$ **b)** $\frac{4}{5}$ **c)** $\frac{9}{10}$ **d)** $\frac{8}{9}$

5. *Sample responses*:

a) $\frac{4}{10}$ **b)** $\frac{4}{9}$ **c)** $\frac{3}{5}$ **d)** $\frac{3}{10}$

6. a) 0 or 1 **b)** 1 to 9 **c)** 2 (or 1)

7. a) $3\frac{1}{4}$ **b)** $2\frac{1}{2}$ **c)** $1\frac{2}{3}$

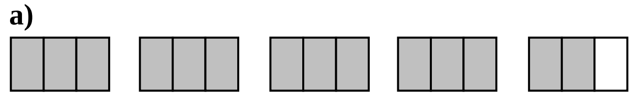
8. a) 3

b) 2

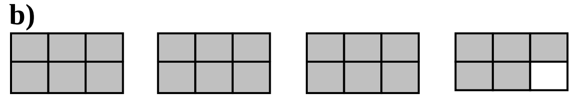
c) $1\frac{1}{4}$; One group of 4 plus 1 more person is $1\frac{1}{4}$.

9. *Sample responses*:

a)



b)



10. *Sample responses*:

a) $8\frac{1}{4}$ **b)** $3\frac{1}{2}$

11. a) 0.06 **b)** 0.20 **c)** 0.42

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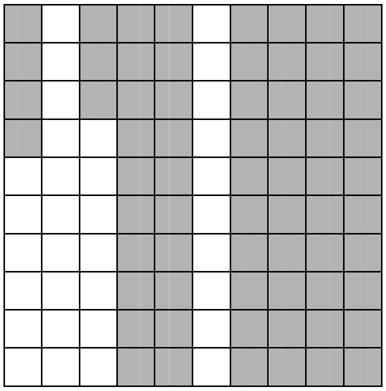
12. a) **b)** **c)**



13. *Sample responses*:
a) 0.13 **b)** 0.89 **c)** 0.84

14. 18 squares

15. a) 0.8, 0.92, 1.47, 3.0
b) 0.88, 8.08, 8.80
c) 2.02, 2.22, 3.14, 3.41

16. *Sample response*:
1.24 < 3.5; 1.24 is less than 2 and 3.5 is greater than 2.

17. a) 12.17 **b)** 6.75 **c)** 3.41 **d)** 19.11

18. 8.10 km

19. *Sample responses*:
a) About 23; [20 + 3 = 23]
b) About 1.2; [3 tenths + 9 tenths = 12 tenths = 1.2]
c) About 7; [12 – 5 = 7]
d) About 86; [93 – 7 = 86]

20. a) 3.51 **b)** 10.27 **c)** 2.77 **d)** 3.47

21. 0.96 m

22. *Sample responses*:
a) 2.1 + 2.02 **b)** 4.93 – 3.1

23. 7

UNIT 6 Fractions and Decimals Test

1. Draw a picture to show that $\frac{3}{6}$ and $\frac{4}{8}$ are equivalent fractions.

2. List three fractions that are equivalent to $\frac{5}{15}$.

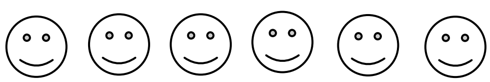
3. Order from least to greatest.

$\frac{3}{4}$ $\frac{3}{7}$ $\frac{6}{7}$ $\frac{3}{10}$ $\frac{3}{50}$

4. List three fractions that are between $\frac{1}{5}$ and $\frac{1}{2}$.

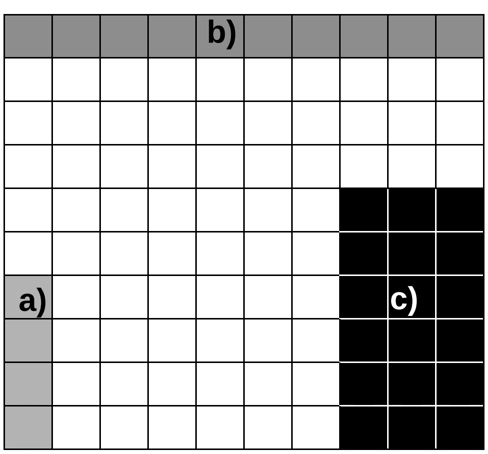
5. Draw a picture to show $4\frac{1}{3}$.

6. The mixed number $1\frac{1}{2}$ is used to describe these 6 faces.



How many faces make 1?
Explain your thinking.

7. Write a decimal for each shaded part.



d) Shade another part to show 0.18.

8. Explain why 0.9 is greater than 0.09.

9. Order from least to greatest:

3.4 3.04 4.30 0.34 0.43 4.03

10. What digit might be missing?

[ ].23 < 5.1

Find more than one answer.

11. Add or subtract.

a) 5.9 + 4.27 **b)** 5.58 + 11.27

c) 2.4 – 1.68 **d)** 14.02 – 6.65

12. Chime walked 2.9 km.
Keenan walked 7.1 km.

a) How far did they walk altogether?

b) How much farther did Chime walk than Keenan?

13. The sum of two numbers is 12.33.
Their difference is 4.07.
What are the numbers?

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UNIT 6 Test

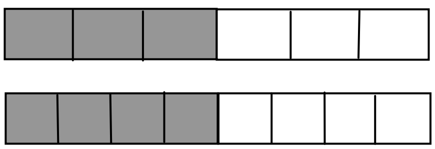
Pacing	Materials
1 h	• Hundredths Grids (BLM) • Place Value Charts (BLM)

Question(s)	Related Lesson(s)
1 and 2	Lesson 6.1.2
3 and 4	Lesson 6.1.3
5 and 6	Lesson 6.1.4
7	Lesson 6.2.1
8 – 10	Lesson 6.2.2
11 – 13	Lessons 6.3.1 and 6.3.2

Assign questions according to the time available.

Answers

1. *Sample response*:

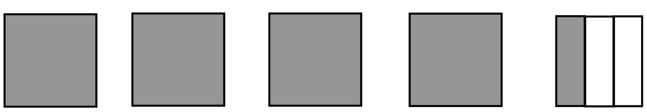


2. *Sample response*:
$\frac{1}{3}, \frac{2}{6}, \frac{10}{30}$

3. $\frac{3}{50}$ $\frac{3}{10}$ $\frac{3}{7}$ $\frac{3}{4}$ $\frac{6}{7}$

4. *Sample response*:
$\frac{1}{4}, \frac{1}{3}, \frac{2}{5}$

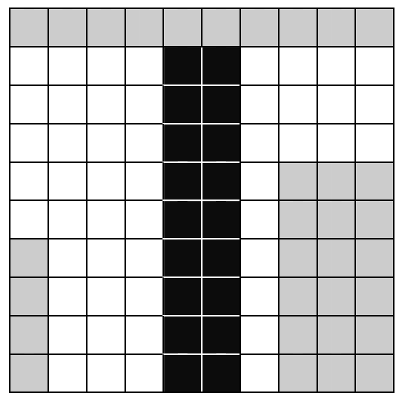
5. *Sample response*:



6. 4 faces; *Sample response*:

4 faces make 1 and then 2 more faces make $\frac{2}{4}$, which is $\frac{1}{2}$.

7. a) 0.04 **b)** 0.1 or 0.10 **c)** 0.18
d) 18 squares are coloured; *Sample response*:



8. *Sample response*:

0.9 is $\frac{9}{10}$, which is almost 1.

0.09 is only $\frac{9}{100}$, which is a lot less than 1.
Both are 9 pieces but tenths are much bigger than hundredths.

9. 0.34 0.43 3.04 3.4 4.03 4.30

10. 0, 1, 2, 3, or 4.

11. a) 10.17 **b)** 16.85
c) 0.72 **d)** 7.37

12. a) 10.0 km (or 10 km) **b)** 4.2 km

13. 4.13 and 8.2

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UNIT 6 Performance Task — A Walk to School

Each morning Pelden walks 0.7 km to meet his friend Thinley. They walk together another 1.9 km to get to school.

A. Use a hundredths grid to represent the 0.7 km that Pelden walks alone. Explain why you shaded the number of squares that you did.

B. i) Use a mixed number to describe the distance Pelden and Thinley walk together.

ii) Sketch a picture to show the mixed number.

C. How far does Pelden walk altogether to school and home again? Show your work.

D. Another friend, Rinchin, walks farther than Thinley but not as far as Pelden.

i) How far from school might Rinchin live? Explain your thinking.

ii) How much farther does Rinchin walk in 6 days than Thinley? How do you know?

iii) Describe a fraction or mixed number with a denominator of 4 that is close to your answer to **part ii)**. Explain your thinking.

E. Write your own word problem that uses decimals and fractions or mixed numbers. Solve your problem.

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UNIT 6 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
4-A3 Fractions and Mixed Numbers: model **4-A4** Renaming Fractions: equivalent fractions **4-A5** Fractions: compare and order **4-A6** Hundredths: model and record **4-A7** Hundredths: compare and order **4-B1** Add and Subtract Decimals and Whole Numbers: 10ths and 100ths and larger whole numbers	30 min	• Hundredths Grids (BLM)

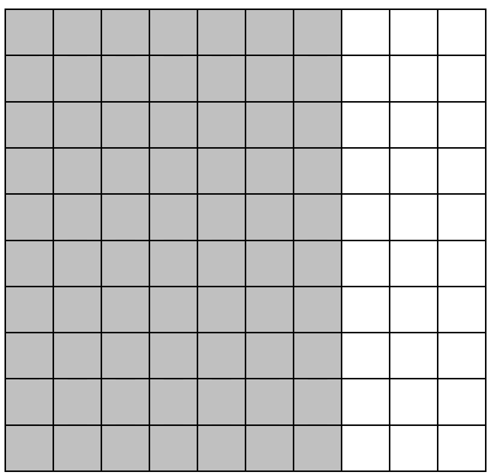
How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could supplement the unit test. It could also be used as enrichment material for some students.

• You can assess performance on the task using the rubric below.

Sample Solution

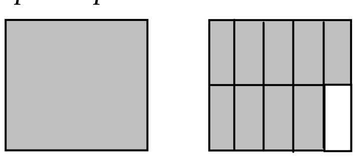
A.



I shaded 70 squares because 0.7 is $\frac{7}{10} = \frac{70}{100}$.

B. i) $1\frac{9}{10}$

ii) *Sample response*:



C. 5.2 km;
1.9 + 0.7 = 2.6
2.6 + 2.6 = 5.2

D. i) 2.0 km;
2.0 > 1.9, but 2.0 < 2.6.
ii) 1.2 km;
2.0 – 1.9 = 0.1
0.1 + 0.1 = 0.2
0.2 + 0.2 + 0.2 + 0.2 + 0.2 + 0.2 = 1.2

iii) $1\frac{1}{4}$;

$0.2 = \frac{2}{10}$, which is 2 columns of a hundredths grid.

That is almost, but not quite, $\frac{1}{4}$ of the grid.

E. I walk $2\frac{1}{2}$ km to school.
My friend Bijoy walks 3.1 km to school.
Who walks farther? How much farther?
(Bijoy walks 0.6 km more each way.)

UNIT 6 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Represents fractions, mixed numbers, and decimals	Models fractions, mixed numbers, and decimals correctly and efficiently	Models fractions, mixed numbers, and decimals correctly	Models some fractions, mixed numbers, and decimals correctly	Has difficulty modelling fractions, mixed numbers, and decimals
Compares fractions and decimals	Identifies a fraction, decimal, or mixed number correctly and efficiently to meet a given condition	Identifies a fraction, decimal, or mixed number correctly to meet a given condition	Identifies only some fraction, decimals, or mixed numbers correctly to meet given conditions	Has difficulty identifying fractions, decimals, or mixed numbers to meet given conditions
Creates and solves problems involving decimals	Is creative and insightful in creating and solving problems involving decimals	Creates and solves problems involving decimals correctly	Solves most problems involving decimals correctly	Has difficulty solving problems involving decimals
Communicates thinking	Explains answers to parts A, D, and F insightfully and completely	Explains answers to parts A, D, and F correctly and almost completely	Explains answers to at least two parts of parts A, D, and F correctly	Has difficulty explaining reasoning for parts A, D, and F

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UNIT 6 Blackline Masters

BLM 1 Rectangle Fraction Pieces

1											
$\frac{1}{2}$	$\frac{1}{2}$										
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$									
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$								
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$							
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$						
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$				
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$			
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$		
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

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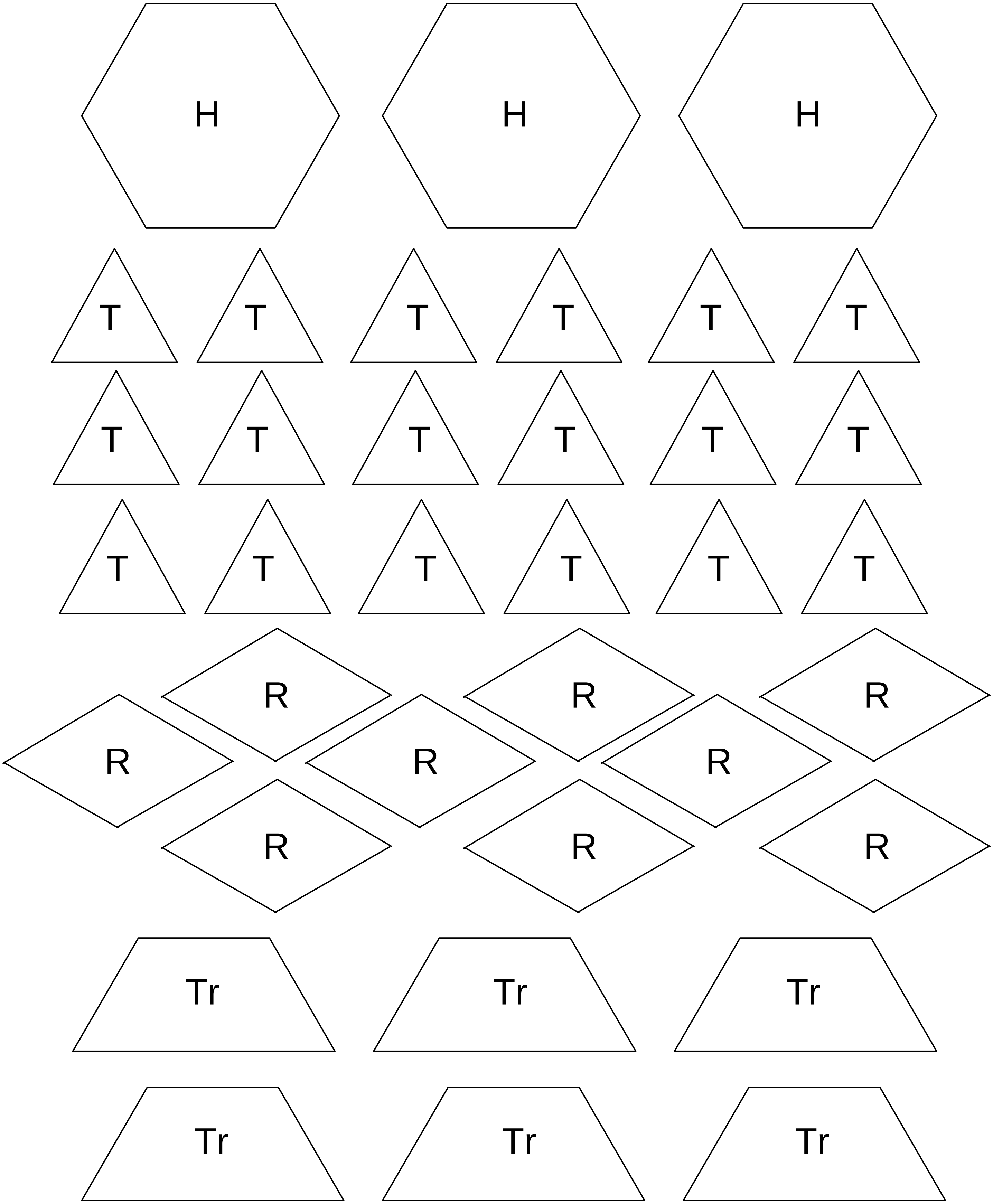
BLM 2 Pattern Block Pieces



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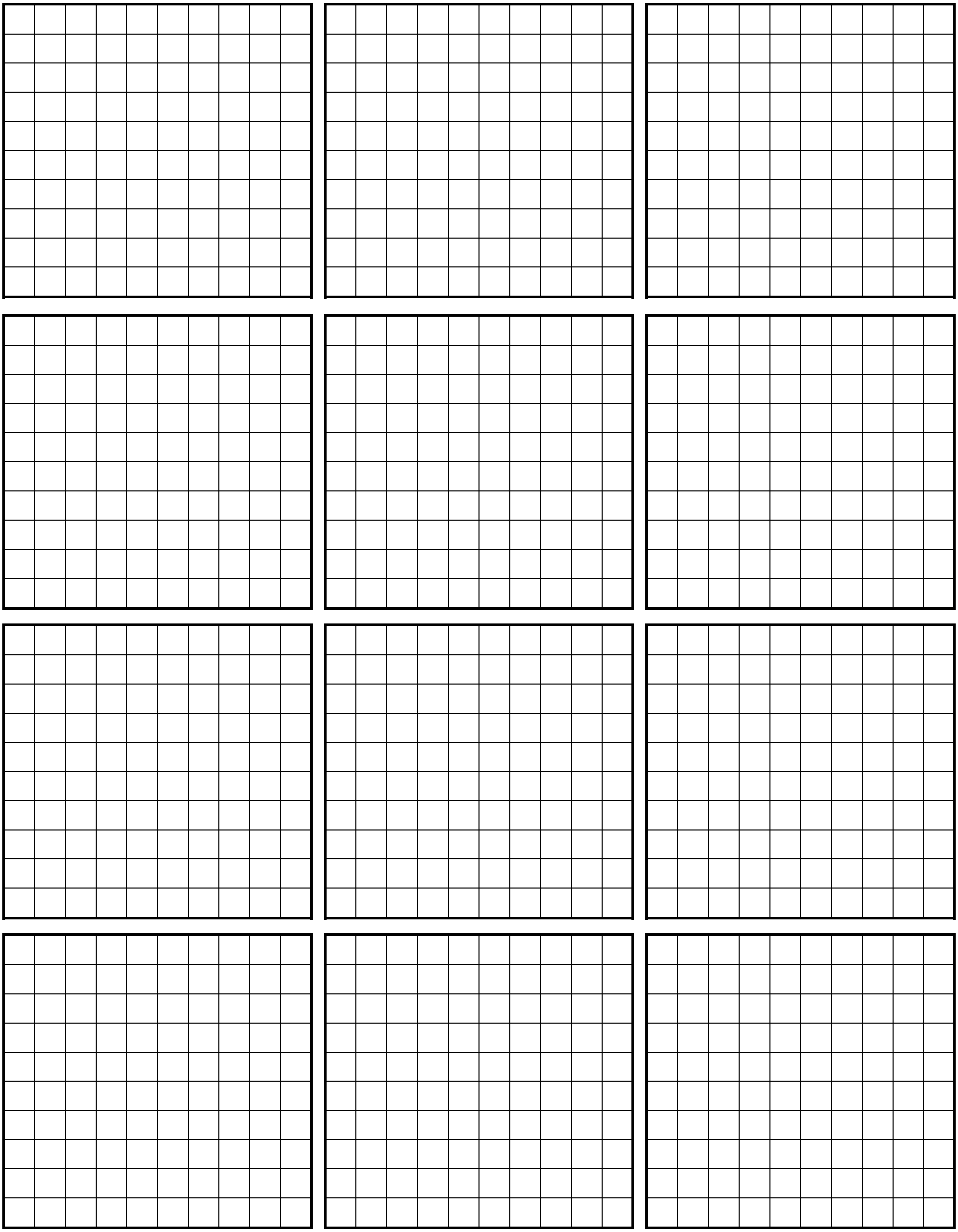
BLM 3 Hundredths Grids



BLM 4 Place Value Charts

Hundreds	Tens	Ones	Tenths	Hundredths

Hundreds	Tens	Ones	Tenths	Hundredths

Hundreds	Tens	Ones	Tenths	Hundredths

Hundreds	Tens	Ones	Tenths	Hundredths

Hundreds	Tens	Ones	Tenths	Hundredths

Hundreds	Tens	Ones	Tenths	Hundredths

Hundreds	Tens	Ones	Tenths	Hundredths

Hundreds	Tens	Ones	Tenths	Hundredths

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UNIT 7 DATA AND PROBABILITY

UNIT 7 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 201** **TG p. 303**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• One die for each pair of students • Lined or grid paper (optional)	All questions
Chapter 1 Collecting and Displaying Data				
7.1.1 Interpreting and Creating Pictographs **SB p. 203** **TG p. 305**	**4-F2 Bar Graphs and Pictographs: construct and interpret** • pictographs: choose an appropriate symbol and decide how much each represents (scale) • interpret results and draw conclusions from data	1 h	• Dice • Paper circles with happy faces • Lined or grid paper (optional)	Q1, 2, 4, 8
7.1.2 Interpreting and Creating Bar Graphs **SB p. 208** **TG p. 309**	**4-F2 Bar Graphs and Pictographs: construct and interpret** • bar graphs: decide value of each square (scale) • include vertical and horizontal graphs • interpret results and draw conclusions from data **4-F5 Describing Data** • determine the maximum and minimum data values given numerical data • relate frequency to the heights of bar graphs	1 h	• Lined or grid paper	Q2, 3, 7
7.1.3 Using a Coordinate Grid **SB p. 213** **TG p. 313**	**4-F3 Ordered Pairs: position on a grid** • introduce the coordinate grid (quadrant I) • explore the convention for naming points (ordered pairs) and why order is significant • compare the use of a coordinate grid to the use of a block grid	1 h	• Grid paper	Q1, 2, 5
GAME: Three in a Row **SB p. 216** **TG p. 316**	Practise plotting points for ordered pairs in a game situation	20 min	• Grid paper • Dice	N/A
7.1.4 EXPLORE: Collecting Data (Essential) **SB p. 217** **TG p. 317**	**4-F1 Collect, Organize, and Describe Data: real-world issues** • explore a variety of ways to collect data (e.g., asking an open question or a question with options to choose from) • choose most appropriate method for collecting simple data • make decisions about the format for presenting data (charts, tables, graphs)	1 h	• Lined or grid paper (optional)	Observe and Assess questions
7.1.5 EXPLORE: Interpreting the Mean (Essential) **SB p. 218** **TG p. 319**	**4-F4 Mean** • introduce the mean as a summary statistic for a set of data that balances data by sharing it equally	1 h	• Coloured counters (optional)	Observe and Assess questions

UNIT 7 PLANNING CHART [Continued]

Chapter 2 Probability				
7.2.1 EXPLORE: Conducting Experiments **SB p. 219** **TG p. 321**	**4-G3 Experiments: predict and record results (concrete materials)** • investigate concretely using probability devices such as dice, spinners, coloured cubes, and coins • predict outcomes, verify by experiments, record outcomes, and compare findings with predictions • devise ways to record experimental results • compare results of a few trials with those of many • use common language to describe probability results (e.g., “2 out of 5”)	1 h	• Dice	Observe and Assess questions
7.2.2 Predicting Likelihood **SB p. 220** **TG p. 323**	**4-G1 Simple Outcomes: more or less likely** • predict whether an outcome is more, equally or less likely than another by investigating with probability devices such as spinners, dice, coins, and coloured cubes	1 h	• Slips of paper (10 slips per pair) • Bangchung (or alternative containers) • Dice (1 pair) • Fraction Circles for Spinners (BLM) • Paperclips (for spinners)	Q1, 2, 5
7.2.3 Using Fractions to Describe Probability **SB p. 224** **TG p. 326**	**4-G2 Predict Probability: near 0, near 1, or near $\frac{1}{2}$** • determine whether a probability is closer to 0, 1, or $\frac{1}{2}$ using these ideas: - a probability near 0: an event rarely occurs - a probability near 1: an event almost always occurs - a probability near $\frac{1}{2}$: event has an equal chance of occurring or not occurring **4-G4 Describe Probability Results: as a fraction** • express simple experimental results as fractions (restrict the total number of possible events to simple numbers)	1 h	• Red and blue cubes and a bag • Dice • Coin (optional)	Q1, 5, 8
CONNECTIONS: Predicting Probability Runs **SB p. 226** **TG p. 328**	Make a connection between individual and combined probability experiments	20 min	• Nu 1 coin	N/A
UNIT 7 Revision **SB p. 227** **TG p. 329**	Review the concepts and skills in the unit	2 h	• Grid paper (BLM) • Counters (optional) • Red and blue cubes and a bag • Fraction Circles for Spinners (BLM) • Paper clips (for spinners) • Dice	All questions

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UNIT 7 Test **TG p. 331**	Assess the concepts and skills in the unit	1 h	• Grid paper • Red and blue cubes and a bag • Dice	All questions
UNIT 7 Performance Task **TG p. 334**	Assess concepts and skills in the unit	1 h	• Slips of paper • Container or bag • Grid paper (optional)	Rubric provided
UNIT 7 Assessment Interview **TG p. 336**	Assess concepts and skills in the unit for selected students	15 min	• Dice • Grid paper	All questions
UNIT 7 Blackline Masters **TG p. 337**	BLM 1 Fraction Circles for Spinners Centimetre Grid Paper (BLM) on **page 87** in UNIT 2			

Math Background

• This data and probability unit allows students to explore a variety of statistical concepts, including data display, data collection, data description, and theoretical probability. At this stage, probability is usually taught using first-hand data because the description of experimental probability is based on data collected in an experiment.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 3** in **lesson 7.1.3**, where they use properties of squares to figure out where two missing vertices might be, in **part F** in **lesson 7.1.5**, where they look for a missing data value when they know some values and the mean, and in **question 5** in **lesson 7.2.2**, where they create a spinner to meet a probability condition.

• Students use communication to explain their thinking in **question 7** in **lesson 7.1.2**, where they describe a choice of scale, in **question 6** in **lesson 7.1.3**, where they explain the importance of the order in an ordered pair, in **lesson 7.1.4**, where they talk about their data collection methods, and in **question 6** in **lesson 7.2.2**, where they discuss what makes a good prediction.

• Students use reasoning in **question 2** in **lesson 7.1.1** and in **question 3** in **lesson 7.1.2**, where they make conclusions from graphs, in **part C** in **lesson 7.1.5**, where they reason about why different sets of data have the same mean, in **lesson 7.2.1**, where they make probability predictions, in **question 3** in **lesson 7.2.2**, where they reason about which outcome is more likely, and in **question 7** in **lesson 7.2.3**, where they reason about the likelihood of results with a particular spinner.

• Students consider representation in **question 3** in **lesson 7.1.1**, where they recognize the importance of representing the symbols properly within a pictograph, in **question 7** in **lesson 7.1.1**, where they reason about an appropriate symbol in a pictograph, in **lesson 7.1.5**, where they use a visual representation of a mean to create sets with a given mean, and in **lesson 7.2.3**, where they use a number line to describe likelihood.

• Students use visualization skills in **question 1** in **lesson 7.1.1**, where they read and interpret a graph, in **question 5** in **lesson 7.1.2**, where they use a visual tool to help make number comparisons, and in **lesson 7.2.2**, where they use the appearance of a spinner to make probability predictions.

• Students make connections in **question 3** in **lesson 7.1.3**, where they connect geometric knowledge to its use in representing data visually, and in **lesson 7.2.3**, where they connect fractions and word descriptions of probability.

Rationale for Teaching Approach

• This unit is divided into two chapters.

Chapter 1 focuses on data displays using bar graphs, pictographs, and coordinate grids, data collection, and some data description, specifically the mean. In considering graphs and data collection, the focus is on making good choices about graph scale and survey questions. There is also some attention to ensuring that data displays are clear and easy to interpret. The mean is only introduced; no mechanical procedures for calculating the mean are developed at this level.

Chapter 2 focuses on probability. The use of fractions to describe probability is introduced for the first time at the end of the chapter. Probabilities are still experimental, although students make predictions based on what objects look like; this is a precursor to theoretical probability.

• There are three **Explore** lessons. The only meaningful way to learn about collecting data is to actually do it, so the data collection lesson is an explore lesson. The introduction of the concept of mean is also an exploration so that students can get a feel for what the mean is. The last explore lesson allows students to practise the skill of recording and describing collected data in a probability experiment.

• The **Connections** presents a curious situation — how one can predict not only individual outcomes but sequences of outcomes.

• There is one **Game**. It allows students to practise plotting points on a coordinate grid.

• Throughout the unit, it is important to encourage flexibility and to accept a variety of approaches from students.

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Getting Started

Curriculum Outcomes	Outcome relevance
3 Collect, Organize, and Describe Data: choose strategies **3** Collecting Data: design and implement **3** Pictographs: interpret and construct **3** Bar Graphs: interpret and construct **3** High and Low Probabilities: relate to real-life situations **3** Probability Experiments: predict and record results	Students will find the work in the unit easier after they review the concepts of data and probability they encountered in Class III.

Pacing	Materials	Prerequisites
1 h	• One die for each pair of students • Lined or grid paper (optional)	• using and interpreting a bar graph with a simple scale • using and interpreting a pictograph with a simple scale • skip counting by 2s and 5s • familiarity with simple data collection strategies • familiarity with the concept of likelihood

Main Points to be Raised

Use What You Know

• When you ask a question to collect data, it is important to think about an appropriate sample, an appropriate question, and appropriate timing.

• When you create a bar graph or a pictograph, it is important to line up symbols or squares properly. It is also important to choose a scale that fits the collected data.

Skills You Will Need

• A bar graph is different from a pictograph in that the symbols used to describe data frequency are always connected squares rather than discrete (separate) symbols.

• You can read data from a bar graph or a pictograph both by considering the data within a single row or column and by comparing data in different rows or columns.

• The scale of a bar graph tells how many units each square in the graph represents.

• You can predict whether an event is likely or unlikely to happen by considering what has happened in the past.

Use What You Know — Introducing the Unit

• Before assigning the **Use What You Know** activity, you may wish to review how to record collected data in a chart. You may also wish to review students' previous experience with bar graphs and pictographs.

For example, you might ask students, "How many of you have fewer than two brothers? How many of you have two or more brothers?" Together, you might record the data first in a tally chart and then in a frequency chart. Then create both a simple bar graph and a simple pictograph to describe the same information, perhaps with a scale of 2 for both. You could ask students how each graph shows the collected information. Also ask why it makes sense to use a scale of 2 rather than no scale or a scale of, say, 3 or 4.

• Students can work in pairs to complete the activity. Make sure they understand that the chart shown in **part A** is only a sample. If they ask their question differently, the chart will be different.

Observe students as they work. You might ask:
• *How can you use the numbers in your chart to check to see if you asked 20 students?*
• *Why would you not ask "How old are you?" or "How many children are in your family?"*
• *Why did you use four rows in your chart?*
• *Why did you choose to create a bar graph?*
• *How did you choose your scale?*

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions.
• Students can work individually.

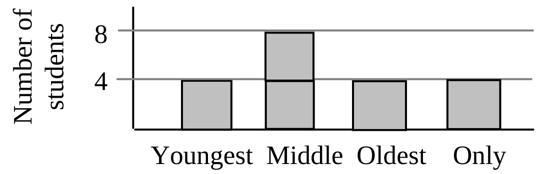
Answers

A. *Sample responses*:
i) I would ask the students in my class; There are 40 students and that is probably enough people to ask.
ii) Probably not; Whether a child is youngest, oldest, or in the middle does not change over the year unless a baby is born during the year.
iii) What is your place in the children in your family? Choose one of these:
- *I am the youngest child.*
- *I have one or more older siblings and one or more younger siblings.*
- *I am the oldest child.*
- *I am the only child.*

iv)

Place in family	Number of students
Youngest	4
In the middle	8
Oldest	4
Only child	4

B. *Sample responses*:
i) **Our Places in Our Families**

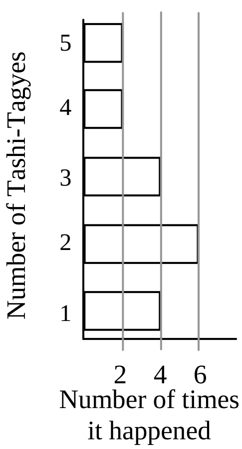


ii) I used a scale of 4 because 4 and 8 are both groups of 4.

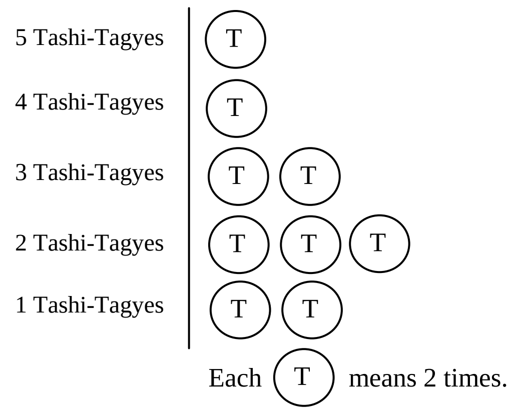
1. A and C

2. *Sample responses*:
a) 12 students had 5 letters in their name.
4 more students had 5 letters in their name than had 6 letters in their name.
b) 20 people had 6 letters in their name.
More people had 5 letters in their name than had 6 letters or 7 letters.

3. a) **Flipping Five Coins**



b) **Flipping Five Coins**



4. *Sample responses*:
a) The sun will rise tomorrow morning.
I will be in Bhutan tomorrow.
b) I will be in Australia tomorrow.
I will buy a car tomorrow.

5. *Sample responses*:
a) 6, 1, 4, 6, 1, 5, 3, 1, 2, 3, 6, 5, 5, 2, 2, 4, 3, 6, 3, 5
b) I rolled a five **4** out of **20** times.

Supporting Students

Struggling students

- Some students may struggle with using a scale for a bar graph or a pictograph. Allow these students at first to use a square or symbol to represent 1 (i.e., no scale or a scale of 1). Then show them how to double up the squares or symbols to change the scale to 2. Help them understand that this strategy also works if they wish to combine other numbers of squares or symbols, as long as they are consistent.

Enrichment

- Students might devise their own question of interest like the question in the **Use What You Know** section. They could then gather the data for their question and display the results in both a chart and a graph of their choice.

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Chapter 1 Collecting and Displaying Data

7.1.1 Interpreting and Creating Pictographs

Curriculum Outcomes	Outcome Relevance
4-F2 Bar Graphs and Pictographs: construct and interpret • pictographs: choose an appropriate symbol and decide how much each represents (scale) • interpret results and draw conclusions from data	It is important for students to be able to interpret graphs they encounter and to construct their own simple graphs. In the case of pictographs, they must recognize that a good choice of symbol can make the construction of the graph easier when the values to be represented cannot all be represented with whole symbols.

Pacing	Materials	Prerequisites
1 h	• Dice • Paper circles with happy faces • Lined or grid paper (optional)	• modelling numbers in groups of 2, 3, 4, 5, and 10 • multiplying by 2, 3, 4, 5, and 10 • familiarity with pictographs • familiarity with even and odd numbers

Main Points to be Raised

• In a pictograph, symbols represent data in order to show how many data values are in each category.

• The categories in a pictograph cannot overlap; in this way, it is clear in which category each piece of data belongs.

• The symbols in a pictograph can be arranged horizontally or vertically; the symbols should be lined up to match from row to row, or from column to column.

• The symbol used should make sense for the topic of the graph. Or, it can be a simple shape.

• A simple shape like a circle or square is easy to use to show and interpret partial symbols.

• The same symbol must be used throughout the pictograph, no matter what the category.

• The symbol used can represent 1 unit or it can represent multiple units, but it represents the same amount throughout the pictograph.

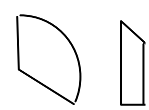
• A pictograph should include a statement telling how much each symbol represents. It should also include a title and category labels.

• To determine an appropriate scale for a pictograph, you should examine the values of the data you are graphing. Often a scale is chosen so that whole numbers of symbols can be used for as many categories as possible.

For example, a scale of 3 would work for this set of data: 3, 6, 3, 10.

• It is helpful if your symbol can be divided into easily recognizable parts in situations where you need to use partial symbols in a graph.

For example, it is easier to see $\frac{1}{3}$ using the partial symbol on the left than using the partial symbol on the right.



• You can read or interpret a pictograph by considering data in only one row (or column).

• You can make conclusions about the data in a pictograph by combining the data from several rows (or columns) or by comparing the data in two or more rows (or columns).

Try This — Introducing the Lesson

A. Students can work alone or in pairs. While you observe students at work, you might ask questions such as

• *How does your chart show that you rolled the dice 30 times?* (The total of the numbers for even and odd is 30.)

• *Which value is greater?* (The number of evens.)

• *How will you be able to tell from the pictograph that there were more evens?* (The row or column will be longer.)

• *What will be the title and labels on your pictograph?* (My title will be “Even and Odd Dice Rolls”; my labels will be “Even” and “Odd”.)

• *Why was it important to line up your symbols on the two rows?* (So I could easily see which number was greater.)

The Exposition — Presenting the Main Ideas

• Review pictographs by having a group of six boys and two girls come to the front of the room.

- Ask the other students how they could make a pictograph to show the number of boys and girls in the group. Tell them that you will be using paper circles with happy faces as the symbol, taping them to the board to make the pictograph.

- Follow students' directions to create a pictograph on the board. Make sure they realize that the same symbol is used for each row or column, that the pictograph can be horizontal or vertical, and that the symbols must be lined up.

- Talk about how the happy face symbol is appropriate for this data (each symbol represents a person) and ask what other symbols you might have used (e.g., stick people).

- Prompt students to think of a title for the pictograph.

• Now tell students that you want to make the graph smaller by using fewer symbols (each symbol now represents 2 students, not just 1).

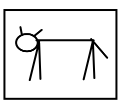
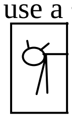
- Have students instruct you in changing the graph (e.g., instead of removing half the symbols, you might double up the symbols by taping one on top of another to show concretely how each now represents 2).

- Talk about how it is important to have a statement saying how much each symbol represents (otherwise, the reader assumes that only half as many people are represented).

- Ask students how the graph shows the total number of students in each category. Ask how it shows how many more boys than girls were graphed.

• Discuss what you would have done differently if there had been seven boys and two girls. Point out that it is useful if the symbol is easy to divide in half.

• Students might realize that you can divide any symbol if it is enclosed in a box or circle such as the stick figure in a box shown below, but it is helpful to use a symbol that is easy to divide even when it stands alone.

e.g., If  represents 2 cows, then  represents 1 cow.

• Read through the exposition on **pages 201 and 202** in the student text.

• Make sure students understand that scales are normally used when the data values are high.

• Provide some examples of how you might choose a scale for different sets of data.

- Talk about why you might use a scale of 3 for this set of data: 6, 9, 12, 6; but a scale of 4 for this set of data: 4, 12, 16, 8.

- Discuss why, for a data set like 8, 8, 12, 10, you might use a scale of 4, using $2\frac{1}{2}$ symbols for 10, if there is not enough room for a scale of 2; otherwise, a scale of 2 might be better.

• You might suggest that students use lined or grid paper to make their pictographs to help them line up their symbols. This is optional.

Revisiting the Try This

B. and C. These questions allow students to practise using both the concepts of choice of scale and the reading and interpretation of pictographs as they relate to the problem they addressed in **part A.**

Using the Examples

• Work through **example 1** with students. Make sure they understand that to describe the birthdays in each season they should look at the faces in the columns, not in the rows. Point out that the pictograph has a title, a label, and a statement that shows that each face means 2 children. If you wish, you can call the statement about how much each symbol represents a *legend*, but that is not required.

• After reading through the solution for **example 1**, ask students to suggest some other information they could read or other conclusions they could make (for example, 12 children were born in summer and autumn).

• Pose the question in **example 2** for students to work on in pairs or small groups. They can then check their work against the solution in the student text.

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Practising and Applying

Teaching points and tips

Q 1: Make sure students understand that they can combine information or make comparisons for **part d)**.

Q 2 b): Students need to understand that there cannot be a student whose data appears in more than one line, so finding the total value of all the symbols tells the total number of students who were asked.

Q 3: This question should focus students on the importance of lining up symbols and including a legend.

Q 4: The pictographs can be vertical or horizontal. Remind students to use a title, label the categories, show the scale, and line up symbols. Observe whether they use a meaningful symbol or a plain circle or square.

Q 5: Encourage students to be creative with their symbol choice, but to use a symbol that is easy to show parts of. Remind them that they can embed their symbol in a circle or a square and then divide the circle or square to show partial symbols.

Q 6: It is important for students to realize that when a symbol represents a greater amount, then fewer symbols are needed. Students might choose either graph in **part c)**, as long as they justify their choice.

Q 8: This question might be handled in a class discussion to conclude the lesson.

Common errors

• Many students have difficulty showing partial symbols. You may wish to have them practise strategies for this.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can read and interpret a pictograph with no partial symbols
Question 2	to see if students can read and interpret a pictograph with partial symbols
Question 4	to see if students can create a pictograph and justify the choice of a scale
Question 8	to see if students can identify the critical features of a pictograph

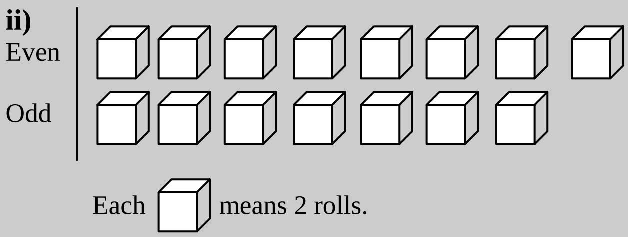
Answers

A. *Sample responses*:

i)

Number rolled	Number of times
Even	16
Odd	14

ii)



B. *Sample responses*:
i) 2
ii) Both numbers were even numbers and I thought 8 symbols would not make the pictograph too big.

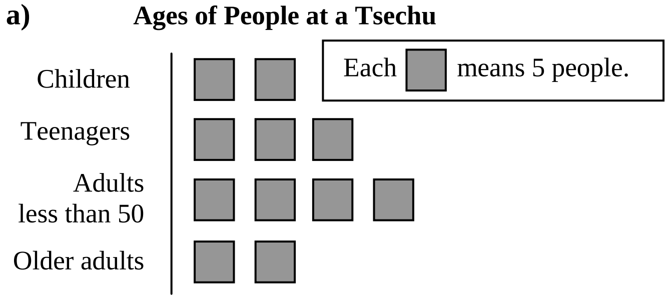
C. *Sample response*:
I can read that there were 16 evens and 14 odds.
I can make these conclusions:
• I rolled almost as many odds as evens.
• The total number of rolls was 30.

1. [**a)** *Sample response*:
All the numbers were even and were not very big.]
b) Sheep: 6 hours Rabbit: 8 hours
Human: 8 hours Mouse: 12 hours
c) *Sample response*:
Sheep sleep 6 hours each day, rabbits sleep 8 hours each day, and mice sleep 12 hours each day.
d) *Sample response*:
Mice sleep more than sheep, rabbits, or humans.
Mice sleep twice as long as sheep.

2. a) 25 students; [There are two whole faces; each face means 10 students. There is also one half face, which means 5 students. 10 + 10 + 5 = 25.]
b) 130 students; [There are 12 whole faces (12 × 10 = 120) and two half faces (5 + 5 = 10). 120 + 5 + 5 = 130.]
c) *Sample response*: More students have sisters than do not have sisters. There are 10 more students with no sisters than with three or four sisters.

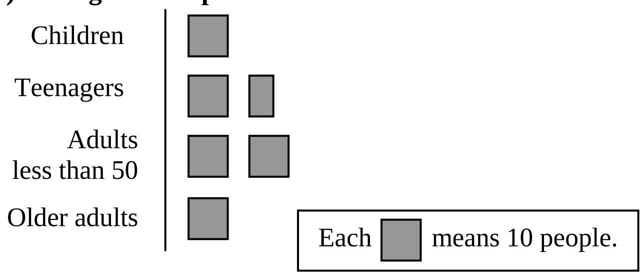
3. *Sample response*:
The squares are not lined up. The scale is not given.

4. *Sample responses*:
a) **Ages of People at a Tsechu**

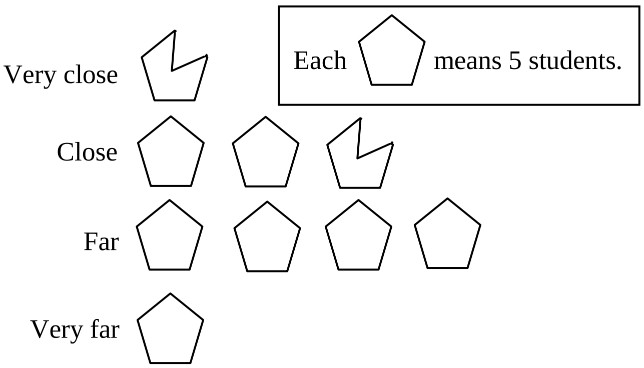


[**b)** I chose 5 because the numbers are all groups of 5.]
c) I chose a scale of 10.
d) There will be half as many squares in each line. There will be some half-squares. I predict this because 10 is twice as much as 5 and some of the numbers are groups of 5, but only half a group of 10.

e) **Ages of People at a Tsechu**



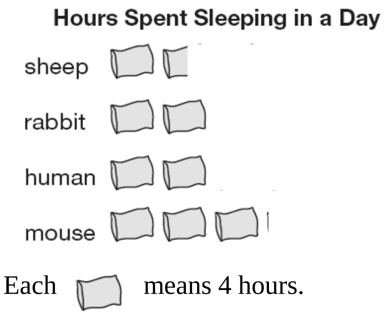
5. *Sample responses*:
a) **How Far We Walk To School**



[**b)** I chose a scale of 5 because two of the numbers were groups of 5.]

[**c)** I used a shape with 5 sides that looked a bit like a house because the data set is about walking to school from home. To show amounts that were only 4, I used 4 of the sides.]

6. a)



b) Yes. [You can still count the number of hours in both graphs. The numbers are the same.]
c) *Sample response*:
The graph with the 2 hour scale [because there are no half-pillows.]

[**7.** *Sample response*:
You can divide those shapes easily into halves and quarters and people will know what they mean.]

8. *Sample response*:
- Use a scale that does not make the graph too big or small and where I do not have to use a lot of part symbols.
- Line up the symbols.
- Use the right number of symbols in each row.
- Use a symbol that is easy to divide into parts if necessary.

Supporting Students

Struggling students

• Some students will find it easier to read a pictograph than to make conclusions. One way to help students who struggle with making conclusions is to provide fill-in-the-blank formats to which they can respond.

For example:

There were ____ more in the category _____ than in the category ____.

There were _____ times as many in the category _____ than in the category _____.

There were _____ altogether.

• To help students who struggle with creating pictographs, here are some strategies:

- Ask them always to use a rectangle for a symbol (e.g., if the scale is 4, the rectangle could be 4 cm long so it is easy to show partial symbols for 1, 2, and 3).
- Suggest that they work on grid paper to make it easy to line up the symbols properly.
- Provide a suitable scale or give them a choice of suitable scales, rather than asking them to select a scale.

Enrichment

• Students who find it easy and enjoyable to create pictographs might think about topics of interest to students in the school. They could conduct surveys and display the data in pictographs in locations where other students will see the results.

Reprint 2023

7.1.2 Interpreting and Creating Bar Graphs

Curriculum Outcomes	Outcome relevance
4-F2 Bar Graphs and Pictographs: construct and interpret • bar graphs: decide value of each square (scale) • include vertical and horizontal graphs • interpret results and draw conclusions from data **4-F5 Describing Data** • determine the maximum and minimum data values and the range given numerical data • relate frequency to the heights of bar graphs	Students need to be able to interpret graphs they encounter and to construct their own simple graphs. Bar graphs are among the most commonly-used graphs, so it is very important to be comfortable with them. Learning how to describe data using graphs is an important connection students must make to help them see the value of representing information visually.

Pacing	Materials	Prerequisites
1 h	• Lined or grid paper	• modelling numbers as groups of 2, 3, 4, 5, and 10 • multiplying by 2, 3, 4, 5, and 10 • familiarity with pictographs • familiarity with even and odd numbers

Main Points to be Raised

• A bar graph is like a pictograph except that it uses connected squares rather than individual symbols to represent data frequency.

• The connected squares form bars. The bars can be horizontal or vertical. The length of a bar tells how many or how much is in that category.

• A bar graph is generally used to describe a data set whose categories are discrete and do not overlap.

• You can choose the scale on a bar graph to make the graph fit a desired space. People often choose a scale so that they can use whole numbers of squares for as many bars as possible.

• A bar graph needs a title, labelled categories, and a labelled scale.

• The minimum value of a set of data is represented by the length of the shortest bar; the maximum value is represented by the length of the longest bar.

• As with a pictograph, the viewer of a bar graph can read data or can draw conclusions by comparing or combining data.

Try This — Introducing the Lesson

A. Students can work alone or in pairs. They can choose any page in the book that they wish. Although they do not have to be exact, encourage them to be reasonably careful as they count the letters.

While you observe students at work, you might ask questions such as

• *Which letter did you find the most of?* (A)

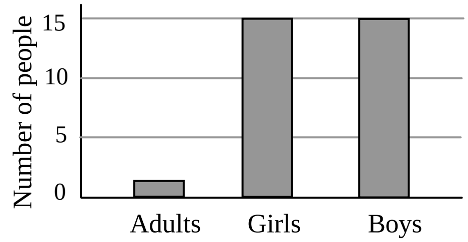
• *How can you tell that from looking at your graph?* (The bar for A was the longest.)

• *How did you use the number of letters you found to make your graph?* (I looked for the greatest amount and knew that I had to make room for a bar that long. I drew a vertical line and numbered every 5 lines using 5, 10, 15, 20…. Then I made three separate bars, one for each letter. I drew each bar up to the line number for that piece of data.)

The Exposition — Presenting the Main Ideas

• Draw a simple bar graph on the board.

For example, it could be a graph with three bars: a bar of height 1 labelled Adults, a bar of height 15 labelled Girls, and a bar of height 15 labelled Boys. Be sure to use a grid or lines for the graph.



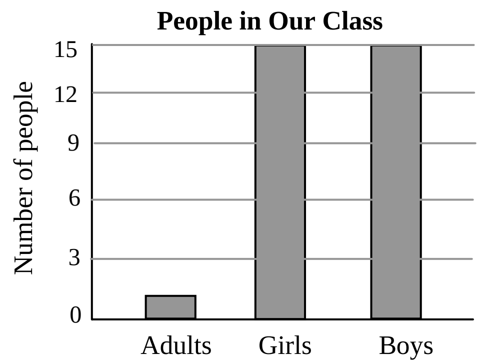
Ask students what this graph might describe about a classroom. Ask for a suitable title.

• Point out the important features of the graph:
  - The lengths of the bars represent the number in each category.
  - The bars are equally spaced.
  - The scale is labelled and the scale increments are spaced equally.
  - The graph has a title and category labels.

• Ask students to predict how the graph would change if these things happened:
  - Only the students in the class were being represented (only two bars, both the same, 15 units each).
  - The graph displayed the data in two categories: adults and children (one bar of 1 unit and one bar of 30 units).

• Re-label the scale on the graph so each horizontal line represents 3 people. Change the length of the bars accordingly.

Discuss with students how the scale has changed and how it affects the graph (longer bars are needed). Discuss why a scale of 3 or 5 make sense for the values 1, 15, and 15 (two of the data values, 15, can be grouped in 3s or in 5s). Ask what scale they would use if there was 1 adult, 22 girls, and 18 boys (e.g., a scale of 4).



• Introduce the terms *maximum* and *minimum* to describe the greatest and least values in a set of data.

• Talk about how you read and interpret a bar graph much like you read and interpret a pictograph.

• With students, work through the bar graph example on **page 208** of the student text. Make sure they can read and interpret the graph. Discuss how they can tell that the scale is 2.

• Discuss other aspects of the exposition on **page 209**, making sure students understand how to estimate the lengths of the bars when it is difficult to be exact. Make sure they notice that the graphs shown are sometimes vertical and sometimes horizontal.

Revisiting the Try This

B. This question allows students to make a formal connection between their work in **part A** and what they have learned about choosing a scale and making conclusions about the data in a bar graph.

Using the Examples

• Assign students to work in pairs. Have one partner become an expert on **example 1** and the other become an expert on **example 2**. Have each then lead their partner through their example so that both understand both examples.

• Provide time for students to ask any questions they might have about the two examples.

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Practising and Applying

Teaching points and tips

Q 1: For **part d)**, remind students that they can compare or combine data for the four categories.

Q 3: If students inadvertently create a vertical bar graph, do not require them to redo the graph.

Q 4: This question is designed to help students see the connections between bar graphs and pictographs.

Q 5: Some reasonable scales students might use are 10, 20, 30, or even 40 or 50.

Q 6: This question is designed to help students see the importance of proper spacing, both horizontal and vertical, in creating a graph that is easy to interpret. Technically, the given graph provides the required information, but it is misleading.

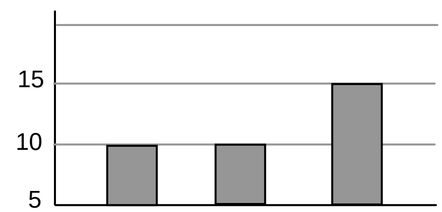
Q 8: You might have students talk about this question in small groups and then share with the class.

Common errors

• Some students count lines rather than spaces.

For example:

If the scale is 5, to show the data 10, 10, 15, they might draw this:
Help them understand that the base line must always be 0.



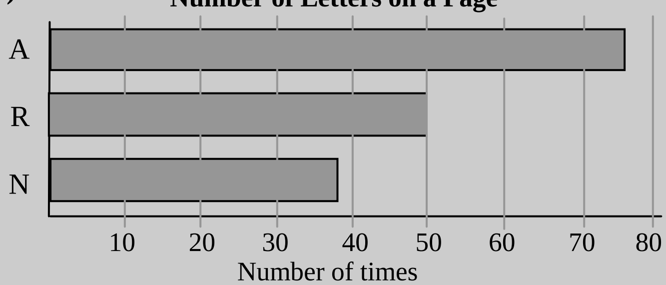
Suggested assessment questions from Practising and Applying

Question 2	to see if students can read and make conclusions about the data in simple bar graph
Question 3	to see if students can create a simple bar graph and choose a scale
Question 7	to see if students can communicate their reasoning about choosing a scale for a bar graph

Answers

A. *Sample responses*:
i) A: 76, R: 50, N: 38
ii)

Number of Letters on a Page



B. *Sample responses*:
i) 10
ii) The numbers were big, so I knew that if I used a smaller scale the graph would take a lot of room.

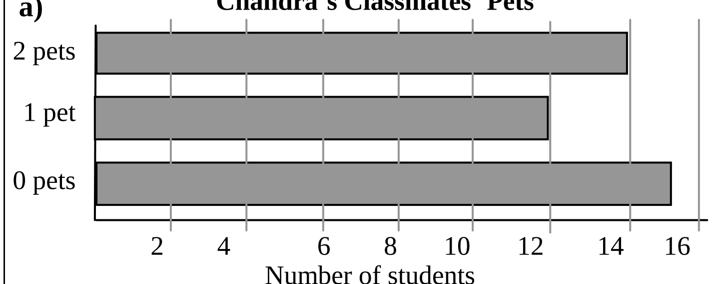
C. *Sample response*:
A appears more often than R or N.
A appears about twice as much as N.

1. a) 16 hours
b) Red fox and chimpanzee
c) 20 hours and 4 hours
d) *Sample response*:
Brown bats sleep most of the day.
Chimpanzees sleep over twice as long each day as giraffes.
e) 5
f) *Sample response*: A scale of 4; [The bar for the giraffe would be exactly 1 square, the bars for the chimpanzee and red fox would be 2 ½ squares, and the bar for the brown bat would be 5 squares.]

2. a) 2 **b)** 10 hours **c)** 1 hour
d) *Sample response*: Dechen slept the most. Karma slept 1 hour less than Eden.

3. *Sample responses*:
a)

Chandra's Classmates' Pets

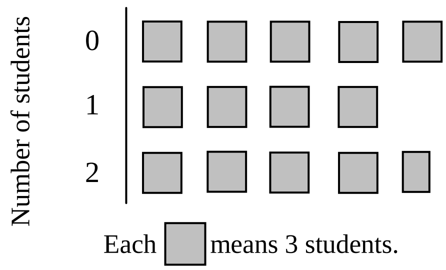


b) 2; [The numbers were small and more were even than odd.]
c) The bar for 1 pet went as far as the sixth line. Each line means 2, and 6 × 2 = 12.
d) 41 students were asked about their pets.
More students have no pets than have 1 pet or 2 pets.
There are three more students with 0 pets than with 1 pet.

Answers Lesson 7.1.2 [Continued]

4. *Sample responses*:

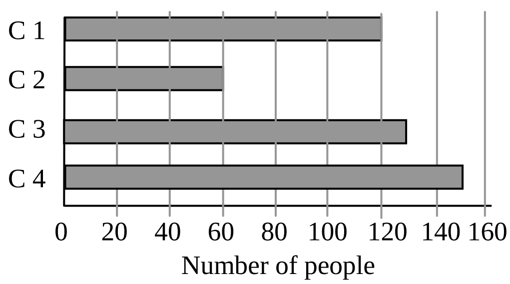
a) **Chandra's Classmates' Pets**



b) Both show the data in the same way.
They both show
• the longest line for 0 pets
• the shortest line for 1 pet
• an in-between line for 2 pets

5. *Sample responses*:

a) **Archery Competition Attendance**



b) 20; [I chose 20 so the graph would not be too big and because all the numbers are groups of 10 so they are easy to show with a scale of 20.]
[**c)** The bar for Competitions 1 and 3 are about twice as long as the bar for Competition 2.]

6. *Sample response*:
The lines for the scale numbers 0, 2, 4, and 6 should be equally spaced. The bars should be the same width and they should be equally spaced.

[**7.** *Sample response*:
A scale of 2 would be good for Group 1 since all the numbers are even but it would not be good for Group 2 because all the numbers are odd.]

8. *Sample response*:
I could use a bar graph to show how many students ate different kinds of food for supper last night.

Supporting Students

Struggling students

• Some students have difficulty dealing with partial units.

For example, if the scale is 10, they have difficulty estimating how high to go for 14.

Encourage students to use data values that are 20 or less and a scale of 2 until they are more comfortable using scales greater than 2.

Enrichment

• Students might look in magazines or newspapers for bar graphs and put together a poster showing “Bar Graphs Around Us”.

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7.1.3 Using a Coordinate Grid

Curriculum Outcomes	Outcome relevance
4-F3 Ordered Pairs: position on a grid • introduce the coordinate grid (quadrant I) • explore the convention for naming points (ordered pairs) and why order is significant • compare the use of a coordinate grid to the use of a block grid	Students' future work in mathematics involves significant use of coordinate grids, particularly in algebra. Coordinate grids are a useful way to show not only location, but also relationships.

Pacing	Materials	Prerequisites
1 h	• Grid paper	• reading a city (block grid) map • familiarity with the term vertices • familiarity with basic properties of a square

Main Points to be Raised

• Although block grid maps are useful for locating a place, they can be ambiguous because there are many positions within each block. The use of a coordinate grid eliminates this problem.

• Locating a point is called *plotting* the point.

• Each point on a coordinate grid is named by an ordered pair of numbers.

- The first number tells how far the point is to the right of the origin, or (0, 0).

- The second number tells how far up the point is.

• The order of the numbers matters.

For example, (3, 2) is not the same as (2, 3).

Try This — Introducing the Lesson

A. Students can work in pairs or in small groups. Make sure they understand that their map should approximately reflect the distances in the room.

While you observe students at work, you might ask questions such as the following:

• *Why did you say that the desk was at the right? Why might that be confusing?* (From where I was standing it is at the right, but if I were standing somewhere different it would not be at the right. So if I told a different person the teacher's desk was at the right, he or she might not understand what I meant.)

• *How did you show about how far away the desk was from you?* (In my mind, I thought of the distance from me to the student desk in front of me as 1 unit. Then I counted units by estimating.)

• *Why might you use words like "about as far as"?* (To help someone understand how far away the teacher's desk is from the door, I might compare it to a different distance they are more likely to know.)

The Exposition — Presenting the Main Ideas

• Have students look at the map on **page 213** of the student text. Ask why you might help someone find Thimphu by saying it is in section B3. Ask students to use similar coding to describe several other locations, for example, Paro and Samdrup Jongkhar. Ask whether there are other locations in B3. Talk about the idea that B3 is a space that is quite big and so it is not a specific location.

• Introduce the concept of a coordinate grid by drawing a 5-by-5 grid on the board. Help students see that you first locate the point (0, 0) and call it the *origin*. Then show how to use horizontal and vertical lines to the right and up from (0, 0) to plot points at the intersections of these lines, e.g., (3, 2) and (0, 4). Show how to count to the right and then up to locate other points. Count one space at a time to emphasize how you move. Be sure to move consistently first to the right and then up, to ensure that students use the correct order.

• Write the terms *coordinate grid* and *ordered pair* on the board for students to refer to.

• Show how the point (1, 4) is located closer to the left and higher up than the point (4, 1).

• Help students see that there is only one point that describes any ordered pair.

• If a student asks what to do between intersection points, assure them that fractions and decimals can be used to further describe locations, but that they will not do that in this lesson.

• You may be interested to know that although this topic is sometimes included in a geometry chapter, we have placed it in a chapter on data for two reasons:

- Location is one kind of data, and a coordinate graph is a visual way to display data much like a bar graph is.
- Learning how to name and locate points on a coordinate grid prepares students for later work with line graphs.

Revisiting the Try This

B. This question allows students to use a coordinate grid to approach the problem from **part A** in a different way. They should see that it is more effective to describe a location using a coordinate grid.

Using the Examples

• Have students read through **example 1**. Allow time for any questions they might have.

• Work through **example 2** together with students. Make sure they understand that they must use what they know about squares, i.e., that all four sides are the same length and that, in this case, the side lengths are on horizontal and vertical grid lines.

Practising and Applying

Teaching points and tips

Q 1: Observe to make sure students go first to the right.

Q 2: You may choose to provide grids rather than having students sketch them.

Q 3: Students need to recognize that these two vertices could be diagonally opposite one another (this is different than what they saw in the example). Some students might use slanted side lengths and put the two given vertices on the same side of the square; this is also correct.

Q 4: This is students' first opportunity to see how a coordinate grid can be used to show a relationship. This will be a very important use of ordered pairs and coordinate grids in higher classes.

Some students will focus on the geometry to help them find other points that fit the relationship. Others will use the numerical values to help them.

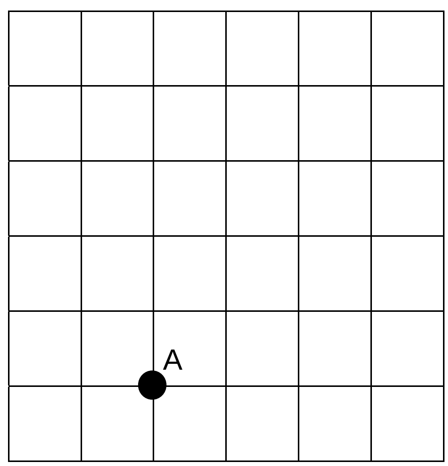
Q 5: This problem is a bit more challenging than some of the others. You may choose not to assign it to struggling students.

Common errors

• Some students count lines rather than spaces.

For example, they call point A below (3, 2) because A is on the third vertical line, starting at the left, and the second horizontal line, starting at the bottom.

Help these students by insisting that they label each graph starting at (0, 0). A few students label points starting at the top left rather than at the bottom left, but this is a less common error.



• Other students confuse the order of numbers in the ordered pair. Proper and consistent modelling should minimize this problem.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use ordered pairs to describe plotted points
Question 2	to see if students can plot points on a coordinate grid
Question 3	to see if students can solve a problem involving a coordinate grid

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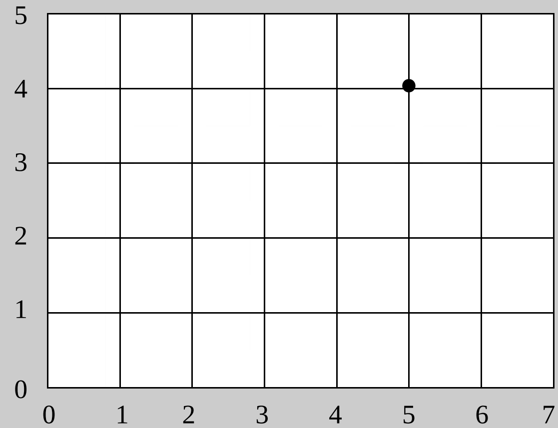
Answers

A. *Sample response*:

The teacher's desk is near the top toward the right side.

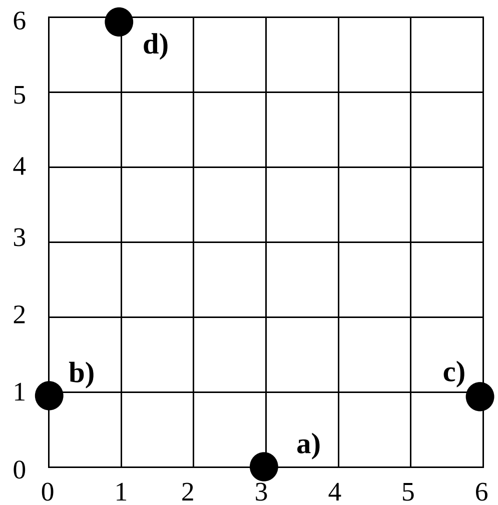
B. *Sample response*:



The teacher's desk is at (5, 4).

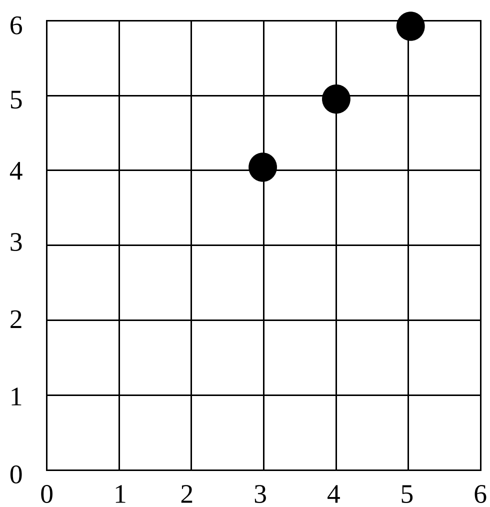
1. A(0, 4), B(3, 5), C(4, 4), D(6, 3), E(5, 0), F(2, 1)

2.



3. *Sample response*: (3, 6) and (5, 4)

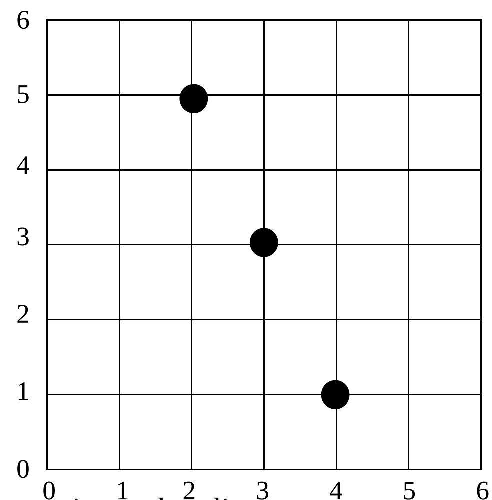
4. a) i)



The three points make a line.

ii) *Sample response*: (2, 3)

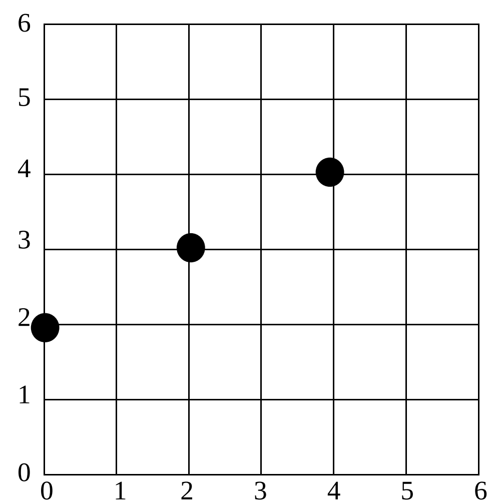
b) i)



The three points make a line.

ii) *Sample response*: (1, 7)

c) i)



The three points make a line.

ii) (6, 5)

5. *Sample response*: (2, 2) and (4, 2)

Lesson 7.1.3 Answers [Continued[

[**6.** The first number tells how far to go in one direction and the second number tells how far to go in the other direction. For example, (2, 4) means only 2 to the right and 4 up but (4, 2) means 4 to the right and only 2 up. You end up in two different places.]

Supporting Students

Struggling students

• Most students will not have much difficulty reading a coordinate grid if it is labelled, but they might have more difficulty plotting points correctly. Students might begin by plotting points on graphs that are not labelled except for the origin, which forces them to count lines rather than just locating the correct lines and then finding where they intersect.

Enrichment

• Ask students to create a picture on a coordinate grid. They can provide a partner with the coordinates of points which, if joined in the order given, re-create their picture.

GAME: Three in a Row

• Discuss the rules with students and either provide 6-by-6 grids or have students draw and label their grids. Students can choose whether or not to number the axes of the grids.

• Although any two symbols could be used, X and O are suggested.

• Ensure that students understand that if they roll a 2 and a 3, they could plot either (2, 3) or (3, 2), whichever point is to their advantage. If both possible points are already occupied, the player loses that turn.

Reprint 2023

7.1.4 EXPLORE: Collecting Data

Curriculum Outcomes	Outcome Relevance
4-F1 Collect, Organize, and Describe Data: real-world issues • explore a variety of ways to collect data (e.g., asking an open question or a question with options to choose from) • choose most appropriate method for collecting simple data • make decisions about the format for presenting data (charts, tables, graphs)	Data collection and presentation are important topics not only in mathematics, but in other subjects such as social studies as well as in everyday life. Students need many experiences, first simpler and then more complex, to gather the skills for collecting and presenting data accurately and effectively. In each class, greater sophistication is developed.

Pacing	Materials	Prerequisites
1 h	• Lined or grid paper (optional)	• creating a bar graph or a pictograph with a scale

Main Points to be Raised

• When you collect data, it is important to ask the right question to collect the information you really want.

• When you display data, it is important to consider which display is most effective, meaningful, and easy to interpret.

Exploration

• With students, read through the white paragraph at the top of **page 217** of the student text. Make sure they understand that the goal of the survey is to find out about favourite colours. Explain that they should first consider some choices Bijoy might give and then try doing the survey themselves.

• Students might survey 20 students in their own class or in another class.

Observe while students work. You might ask questions such as the following:

• *Why did you not just ask them to name their favourite colour?* (They might say so many different colours that it would be hard to make a graph.)

• *Why did you decide to use three choices and "other"?* (I thought that most people would pick red, blue, or green, but I wanted a possibility if they did not like those colours. I did not want a graph with more than four categories because it would take too long to draw and it would be hard to interpret.)

• *How did you keep track of the answers students gave?* (I made a tally chart with a row for each colour choice.)

• *What does your pictograph show about colour choices? How does it show it?* (Red is the most popular colour; I know because the red bar is the longest.)

Observe and Assess

As students work, notice the following:

• Do students make good choices about the number and type of colour categories they use to collect their data? Do they explain their thinking clearly?

• Do students carefully organize the data they collect?

• Are students' displays accurate and easy to interpret?

• Is students' choice of scale reasonable?

• Do students explain clearly and effectively their thinking about using a graph, drawing conclusions, and making choices about display?

Share and Reflect

After students have had sufficient time to complete the exploration, you may have a class discussion and ask

- *How can you tell from your graph that 20 students were surveyed?*
- *Why might you have chosen to use different colour categories?*
- *What is the advantage of using a graph to show your results instead of using the tally chart?*
- *Why might it be a good idea to use a bar graph instead of a pictograph or vice versa?*

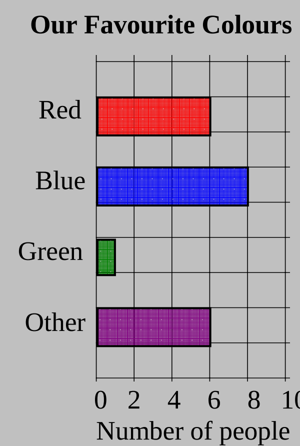
Answers

A. *Sample responses*:
i) If she did not give choices, she might get too many different answers. It would be hard to figure out how to organize the information.
ii) 4 choices; If she used fewer than 4 colours, then some colours people like would not be included. If she used more than 5 colours, the graph would be too complicated.
iii) She should choose red, blue, and green, plus a category called "other". Red, blue and green are popular and "other" would give people a choice if they did not like red, green, or blue.

B. *Sample responses*:
i) Before she starts, she should make up a chart with columns for red, blue, green, and other. When she asks each person, she should make a tally mark to show each choice.
ii)

Red	Blue	Green	Other
𝍸 𝍷	𝍸 𝍷𝍷𝍷	𝍷	𝍸 𝍷

C. i) With a graph, it is easier to see quickly how the numbers in the categories compare.
ii) *Sample response*:



D. *Sample response*:
More people like blue than red or green.
The number of people who like blue is about the same as the combined numbers of those who like red and green.
Green is not a very popular colour.
There are a lot of people who do not choose red, green, or blue as their favourite colour.

E. *Sample response*:
I would include a category for purple since a lot of people chose "other" because purple was their favourite colour.

Supporting Students

Struggling students

- Some students may have trouble explaining their thinking. Have them work with a partner so that they can talk over their explanations together. They can use point form to list their ideas and choose the ideas that they think might be more clear or convincing than other ideas.

Enrichment

- Students who enjoy collecting and displaying data might choose a different topic of current interest to students in the school. They could collect and display data on that topic.

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7.1.5 EXPLORE: Interpreting the Mean

Curriculum Outcomes	Outcome Relevance
4-F4 Mean • introduce the mean as a summary statistic for a set of data that balances data by sharing it equally	The concept of the mean is developed through a number of classes as a useful way to summarize data. Students need to begin with a concrete exploration of the concept.

Pacing	Materials	Prerequisites
1 h	• Coloured counters (optional)	• representing whole numbers

Main Points to be Raised

• The mean of a set of numbers is the value of each share if the total of the data values is shared equally.

• Two different sets of data can have the same mean.

• The mean is a value between the maximum and minimum values of a data set.

• If you know the mean and all but one piece of data, you can determine the missing data value.

Exploration

• Ask 15 students to come to the front of the room. Arrange 7 in one group, 6 in a second group, and 2 in a third group.

- Ask the 15 students to rearrange themselves so that all three groups have the same number of students.

- Help students see that the group of 2 increased in size and the groups of 6 and 7 decreased in size.

- Indicate that the number 5, which represents the equal size of the three groups, is called the mean. Write the word *mean* on the board for students to see.

• At this stage of development, the goal is not to provide a formula for the mean, but simply to develop the concept of the mean as “an average”.

• With students, read through the example on **page 218** in the student text. They can see that the picture at the top represents groups of 2, 10, and 18 and the picture at the bottom represents the final result if the original groups are rearranged to be equal in size. Help students locate the original 2 in the first row (look for the striped squares), the original 10 still in the middle row, and the original 18 split among the bottom and top rows (look for the black squares). If possible, provide 30 counters in three colours so students can manipulate them to create equal groups.

• Have students notice that when you find a mean, sometimes all the group sizes change (as happened with the students at the front of the room) and sometimes only some groups change size (as on **page 218**).

Observe while students work. You might ask questions such as the following:

• *How do you show that the mean of 5, 6, and 7 is 6?* (I moved 1 from the group of 7 to the group of 5 so all three groups had 6.)

• *What is the total for 12, 15, 18, and 11? How many groups are there?* (56; 4 groups.)

• *What is the total for 13, 15, 18, and 10? How many groups are there?* (56; 4 groups.)

• *Why did the means have to be the same in **part B**?* (I am sharing the same total amount into the same number of groups.)

• *What was your first set of five data values with a mean of 15 in **part D**?* (15, 15, 15, 15, 15) *How did you get your next set of data values?* (I moved 1 from one group of 15 to another group and used 14, 16, 15, 15, and 15.)

Observe and Assess

As students work, notice the following:

• Do students use an appropriate procedure to determine the mean?

• Do students show good reasoning to predict whether related data sets might have the same mean?

• Can students both find the mean for a set of data and create a set of data for a given mean?

• Can students predict a missing data value when they know the mean and the other values?

Share and Reflect

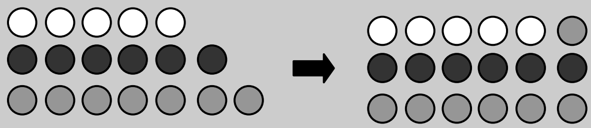
After students have had sufficient time to complete the exploration, you may have a class discussion and ask

• *Why is the mean between the maximum and minimum data values?*

• *Why is the mean a good number to use to describe a set of different data values?*

Answers

A. The mean is 6. *Sample response*:
I used counters to show 5, 6, and 7. I took 1 from the 7 and gave it to the 5 so all the values would be equal.



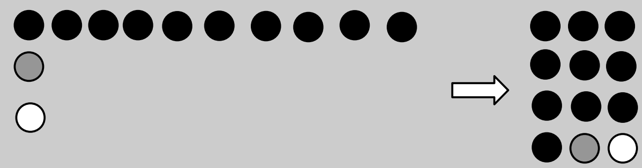
B. i) 14 **ii)** 14 **iii)** 14

C. Each set of data has the same mean;
Each time the total is the same and the number of data values was the same, so when I share the same amount among the same number of groups, I get the same amount for each.

D. *Sample response*:
15, 15, 15, 15, 15
14, 15, 15, 15, 16
10, 10, 10, 10, 35

E. It is always in between the maximum and minimum data values. [It would be the minimum and maximum value only if all the data values were the same.]

F. i) 1; *Sample response*:
A mean of 4 for three data values means that all the counters can make three groups of 4, so there must be 12 counters altogether.
So, 1, 10, and 1:



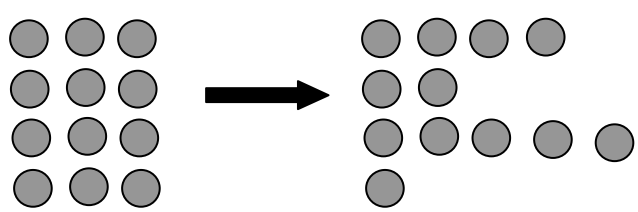
ii) 4; *Sample response*:
A mean of 5 for three data values means that all the counters can make three groups of 5, so there must be 15 counters altogether.
So, 1, 10, and 4:

Supporting Students

Struggling students

• Some students will be able to calculate a mean but they will have trouble creating a data set with a given mean. Help them by showing how you could start with many equal sets (each representing the mean) and move counters from one set to another to create other data sets with the same mean.

For example, to create a set of four data values with a mean of 3, you might do this:



Have them work backwards to see that the new values do indeed have the desired mean.

Enrichment

• Some students might enjoy creating puzzles involving the mean.

For example, a puzzle might be:

The mean of five numbers is 5. Two of the numbers are doubles of two of the other numbers.
What could the numbers be? (e.g., 1, 3, 5, 6, 10)

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7.2.1 EXPLORE: Conducting Experiments

Curriculum Outcomes	Outcome relevance
4-G3 Experiments: predict and record results (concrete materials) • investigate concretely using probability devices such as dice, spinners, coloured cubes, and coins • predict outcomes, verify by experiments, record outcomes, and compare findings with predictions • devise ways to record experimental results • compare results of a few trials with those of many • use common language to describe probability results (e.g., "2 out of 5")	This essential exploration allows students to use experimental data to describe probability. This is a first step in understanding what a probability statement means.

Pacing	Materials	Prerequisites
1 h	• Dice	• familiarity with the term even number • familiarity with the term half

Main Points to be Raised

• You can describe the results of a probability experiment using language such as " ___ out of ____".

• If you repeat an experiment a certain number of times, you can use those results to predict what will happen in another experiment, but you can never be sure what will happen.

Exploration

• Hold up a single die. Ask students to predict how many times you will roll the number four in 12 rolls. Each student can write down his or her prediction. Roll the die 12 times. Ask students to tell you whether they predicted correctly.

• Tell students you will now repeat the experiment. Ask them to predict how many rolls will be a four. Help students see that they could base their prediction on the results or they can use logical thinking.

For example:

If 5 fours were rolled the first time, they might predict 5 fours this time.

Or, they might reason that since there are six possible results and rolling a four is just one result, they would predict 2 fours this time.

• Carry out the experiment. Show students how you can record the results of the experiment, i.e.,

___ out of 12 rolls were a four.

Notice that the use of fractions is delayed until a later lesson. Students find this word format easier to begin with.

• Assign students to work in pairs. Make sure each pair has a single die.

Observe while students work. You might ask questions such as the following:

• *Why do you think you will roll either a four, five, or six as often as you roll a one, two, or three?* (On a die, there is 1 four, 1 five, and 1 six, which make up 3 results, and there is 1 one, 1 two, and 1 three, which is also 3 results.)

• *Why is it not surprising that you predicted 6 rolls of four, five, or six but only got 5?* (I can never be sure what will happen with dice. 5 is pretty close to 6.)

• *Were your predictions closer with 24 rolls than with 12 rolls?* (A bit closer.)

• *Was it easier to predict for one of the experiments than for another?* (I predicted fairly well for all of them.)

• *How did you use your results from 12 rolls to predict the results for 24 rolls*? (I doubled the amount.)

Observe and Assess

As students work, notice the following:

- Do students make reasonable predictions for 12 rolls?
- Do students use the information from 12 rolls to make reasonable predictions for 24 rolls?
- Do students record the results of their experiments correctly?
- Do students make clear comparisons when they compare results to predictions?

Share and Reflect

After students have had sufficient time to complete the exploration, you may have a class discussion and ask

- *Did your predictions get better when you did more experiments?*
- *Were your predictions closer when you rolled 24 times than when you rolled only 12 times?*
- *Why can you never be sure that your prediction will happen when you roll dice?*

Answers

A. *Sample response*:
On a die, there are three numbers that are 4, 5, or 6 and three numbers that are not.

B. *Sample responses*:
i) 6 times
ii)

Number rolled	Number of times
1, 2, 3	|||| 4
4, 5, 6	~~||||~~ ||| 8

iii) 8 out of 12 rolls were a 4, 5, or 6.
iv) I rolled more 4s, 5s, and 6s than I predicted; They were close because I rolled only 2 more than I predicted.

C. *Sample responses*:
i) 12 times
ii)

Number rolled	Number of times
1, 2, 3	~~||||~~ ~~||||~~ ||| 13
4, 5, 6	~~||||~~ ~~||||~~ | 11

iii) 11 out of 24 rolls were a 4, 5, or 6.
iv) I rolled fewer 4s, 5s, and 6s than I predicted; They were close because I rolled only 1 fewer than I predicted.

D. Yes; 11 out of 24 is closer to half than 8 out of 12.

E. *Sample response*:

Experiment I
I predict 12 times.

Number rolled	Number of times
Even (2, 4, 6)	~~||||~~ ~~||||~~ || 12
Odd (1, 3, 5)	~~||||~~ ~~||||~~ || 12

My prediction was exactly right.

Experiment II
I predict 8 times.

Number rolled	Number of times
5 or 6	~~||||~~ |||| 9
1, 2, 3, or 4	~~||||~~ ~~||||~~ ~~||||~~ 15

My prediction was close, but not exactly right.

Experiment III
I predict 12 times.

Number rolled	Number of times
1, 2, or 3	~~||||~~ ~~||||~~ ||| 13
4, 5, or 6	~~||||~~ ~~||||~~ | 11

My prediction was close, but not exactly right.

Supporting Students

Struggling students

- Students will rarely have difficulty conducting the experiments, but they may have difficulty recording their results in charts that are easy to read. You may wish to suggest a chart format for each of the experiments in **part E.**

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7.2.2 Predicting Likelihood

Curriculum Outcomes	Outcome relevance
4-G1 Simple Outcomes: more or less likely • predict whether an outcome is more, equally or less likely than another by investigating with probability devices such as spinners, dice, coins, and coloured cubes	Before they start using fractions to describe probability, it is helpful for students to use verbal descriptions of likelihood. In that way they can link the fractions to meaningful descriptions of likelihood.

Pacing	Materials	Prerequisites
1 h	• Slips of paper (10 slips per pair) • Bangchung (or alternative containers) • Dice (1 pair) • Fraction Circles for Spinners (BLM) • Paperclips (for spinners)	• creating tally charts

Main Points to be Raised

• You can never be certain what will happen in a probability experiment, but you can analyse a situation to make a reasonable prediction.

• If you repeat an experiment more times, it is more likely that you will be able to predict the overall results.

• You can use the relative areas of the sections in a spinner to predict the number of times each number will be spun.

Try This — Introducing the Lesson

A. Have students prepare for the activity by providing or asking them to cut up 10 slips of paper. Provide a bangchung or another container to each pair. The slips of paper should be about the same size, but they do not have to be completely identical.

Have students work alone or in pairs on this problem. While you observe students at work, you might ask questions such as the following:

• *Why did you predict you would take out more blank slips than slips with a name?* (More than half the slips were blank.)

• *Why is it important to put back the slip you chose each time?* (If I do not replace it, I know that I will take out 4 slips with names in 10 tries so there is no point in doing the experiment.)

• *Do you think you would get exactly the same results if you did this again?* (No. You can never predict what is going to happen. I know the results do not have to be the same next time I do it, since Nidup and Chandra got different results than we did and they were doing the same experiment.)

The Exposition — Presenting the Main Ideas

• Tell students you will roll two dice 20 times and add the two numbers rolled each time. Ask them to predict whether you will get more rolls with a sum of 2 or more rolls with a sum of 8. Ask students how they predicted. Help them see that there are not as many ways to get a sum 2 as to get a sum of 8, so it might be more likely that you will roll a sum of 8:

Sum of 2: 1 + 1

Sum of 8: 2 + 6, 6 + 2, 3 + 5, 5 + 3, 4 + 4

• Roll the dice to test students' predictions. Have a volunteer record the sum of each roll.

For example: 2, 5, 4, 8, 9, 3, 6, 7, 7, 8, 5, 8, 7, 9, 2, 8, 7, 6, 8, 5

• Have students turn to **page 220** in the student text. Ask them to look at each spinner and the results provided for 10 spins. Ask students whether the results are what they would have expected and why. Then ask then if the results for 20 spins are closer to what they would have predicted and why.

• If possible, create similar spinners and repeat the experiment with your own spinners. Show students how to create a spinner by using a pencil to hold a paper clip down at the centre and then spinning the paperclip.

Revisiting the Try This

B. Students can work in pairs to repeat the experiment in **part A** using 30 trials. Talk about whether the results are about the same or whether they are different and why that might be.

Using the Examples

• On the board, draw a spinner like the spinner in **example 1** and pose the problem in the example. Ask students to work in small groups to make the spinner. Have them test their prediction using 20 spins. They can then compare their results to the solution in the text.

• Work through **example 2** with students. After you present the question, have them decide whether they agree with Bhagi or with Yeshi before you read through the solutions. After you work through the solutions with students, ask them which of the two solutions they prefer and why. Make sure that they understand that, even if they choose Spinner X because the total amount labelled A is greater than on Spinner Y, there is no guarantee that more As will be spun with that spinner.

Practising and Applying

Teaching points and tips

Provide Spinners P and Q to pairs of students.

Q 1: Ask students to justify their predictions based on the number of relevant sections in the spinner.

Q 2: Make sure students know that if their prediction did not happen, it does not necessarily mean it was a poor prediction.

Q 3: Observe whether students' experience with **questions 1 and 2** helps them make better predictions.

Q 5: Help students see that there are many possibilities for this spinner.

For example:

- It could be a spinner with two equal sections, one labelled 3 and one labelled 6.

- It could be a spinner with four equal sections, labelled 1, 3, 8, and 10.

There are many other possibilities.

Q 6: This question might suit a small group discussion.

Common errors

• Some students base their predictions on subjective judgment, such as their favourite numbers or a number they think might be lucky. Encourage students to use only the sizes of the spinner sections to make their predictions.

• Some students assume that numbers you spin follow a pattern.

For example, if they spin 3, 5, 6, and then 3 again, they assume they will then spin 5 and 6 again.

These students must have many experiences to show them that this is not what happens.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can make reasonable predictions
Question 2	to see if students can carry out an experiment to verify predictions
Question 5	to see if students can solve a problem involving probability

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Answers

A. *Sample response*:
i) A blank slip; There are more slips that are blank than with my name, so I am more likely to pick a blank slip.
ii) I took out a slip with my name 6 times and a blank slip 4 times, so my prediction was wrong.

B. I took out a slip with my name 10 times and a blank slip 20 times, so my prediction was right.

1. a) Less than 4; [Three sections are less than 4 but only two sections are greater than 6.]
b) About the same; [There are as many even numbers as odd numbers.]
c) Less than 4; [Three sections are less than 4 but only one section is 4.]

2. *Sample responses*:
a) My prediction was correct.

Less than 4	7
Greater than 6	4

[Note that spins of 4, 5, or 6 are not included in the chart so the total of the spins is not 20.]

b) My prediction was not correct, but it was close.

Even	11
Odd	9

c) My prediction was correct.

4	2
Less than 4	8

[Note that spins of 5 to 8 are not included in the chart so the total of the spins is not 20.]

3. a) Greater than 2
b) About the same; [Two sections are greater than 5 and two sections are less than 5.]

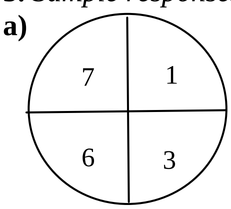
4. *Sample responses*:
a) My prediction was correct.

Greater than 2	12
2 or 4	8

b) My prediction was correct.

Greater than 5	10
Less than 5	10

5. *Sample responses*:
a)



[**b)** I made four sections all the same size. I wrote numbers less than 5 in two sections and numbers more than 5 in the other two sections. That way the part of the circle with numbers less than 5 was equal to the part of the circle with numbers more than 5.]
c) My prediction was correct.

Less than 5	10
Greater than 5	10

6. No; [*Sample response*:
You can never be sure what will happen, so even a bad prediction will seem to be correct some of the time. Four spins is too few spins to be a good test of a prediction.]

Supporting Students

Struggling students

• Most students will be comfortable predicting, even if their predictions are not correct. However, they might have difficulty with questions like **question 5** that have a problem solving element. If they struggle with this, you might pose a simpler problem.

For example: Draw two spinners, where you are more likely to spin a 5 on one spinner than on the other. (You might provide students with two fraction spinners in fourths.)

Enrichment

• Students might create spinners that they think will lead to certain results and then test their predictions.

For example, they might try to create and test a spinner that will usually result in 3 fours in 10 spins.

7.2.3 Using Fractions to Describe Probability

Curriculum Outcomes	Outcome relevance
4-G2 Predict Probability: near 0, near 1, or near $\frac{1}{2}$ • determine whether a probability is closer to 0, 1, or $\frac{1}{2}$ using these ideas: - a probability near 0: an event rarely occurs - a probability near 1: an event almost always occurs - a probability near $\frac{1}{2}$: event has an equal chance of occurring or not occurring **4-G4 Describe Probability Results: as a fraction** • express simple experimental results as fractions (restrict the total number of possible events to simple numbers)	As students read about probability in the media or as they move into higher classes, they will need to become familiar with probabilities described as fractions.

Pacing	Materials	Prerequisites
1 hr	• Red and blue cubes and a bag • Dice • Coin (optional)	• familiarity with fractions less than 1

Main Points to be Raised

• Fractions between 0 and 1 can be used to describe probabilities. Greater fractions are used to describe events that are more likely to occur.

• A probability of 1 is used to describe an event that always happens or is certain. A probability of 0 is used to describe an event that never happens or is impossible.

- An event with a probability near 1 is very likely to happen.
- An event with a probability near 0 is very unlikely to happen.
- An event with a probability near $\frac{1}{2}$ but more than $\frac{1}{2}$ is likely to happen.
- An event with a probability near $\frac{1}{2}$ but less than $\frac{1}{2}$ is unlikely to happen.
- An event with a probability of $\frac{1}{2}$ is as likely to happen as not to happen.

Try This — Introducing the Lesson

A. Provide pairs of students with a bag, 4 red cubes, and 2 blue cubes. Have them work in pairs on this problem.

While you observe students at work, you might ask questions such as the following:

• *Why do you think you will take out a red cube more than once?* (There are more reds than blues, so I think I will take out red more than half the time.)

• *Is it possible you will take out only one red?* (It is possible, but it is not very likely.)

• *How many reds did you predict you would take out? Why?* (I predicted 8 reds. If I took out 6 cubes, I would predict 4 reds since there are 4 reds and 6 cubes. But since I am taking out twice as many cubes, I expect to take out twice as many reds.)

The Exposition — Presenting the Main Ideas

• Ask students to describe an event that is very likely but not certain when flipping five coins.

For example, it is very likely that you will get one or more Tashi-Tagyes when you flip five coins.

- Toss five coins to see whether their prediction is correct. Repeat the experiment for a total of 10 times.
- Record the results using words like “9 times out of 10” and then write it as a fraction, e.g., $\frac{9}{10}$, explaining that the fraction tells that the result occurred 9 times in 10 tries.
- Point out that a fraction close to 1, like $\frac{9}{10}$, shows that you were very likely to get one or more Tashi-Tagyes.

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• Draw a number line on the board with endpoints 0 and 1, and with $\frac{1}{2}$ in the middle. Below 0, record “Never happens” and below 1, record “Always happens”. Discuss why events that are likely to happen all appear on the right side of the line and events that are unlikely to happen appear on the left side of the line.

• Ask students where they would put $\frac{9}{10}$ on the number line. Ask what that means about the probability (very likely). Talk about how the farther you go to the right, the more likely it is that the event will occur, and the farther you go to the left, the more unlikely it is that the event will occur.

• Suggest that students can use **page 224** in the student text for later reference, if they wish.

Revisiting the Try This

B. This question allows students to practise what they have learned about writing a probability as a fraction using the problem they solved in **part A.**

Using the Examples

• Assign pairs of students to work through the **example** together. Allow them to ask any questions they might have.

Practising and Applying

Teaching points and tips

Q 1 to 4: Ask students why it is necessary to count the number of flips to answer **part a)**.

Q 5: Encourage students to keep their fractions with a denominator of 12, rather than using simpler forms, so that they can be easily compared.

Q 7: Students do not have to spin the spinner. They should make a judgment based only on comparing the sizes of the sections.

Q 8: Students might first try this alone, but a group discussion of this question will be useful.

Common errors

• Sometime students use the numerator of a fraction to describe how often a desired outcome occurs, but use the denominator, rather than the number of trials, to describe how often the outcome does not occur.

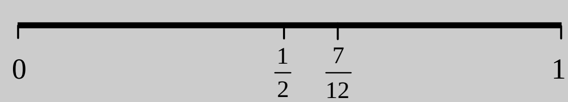
For example, if in 20 tosses of a coin, the result Tashi-Tagye occurs 9 times and Khorlo occurs 11 times, they use the fraction $\frac{9}{11}$ instead of $\frac{9}{20}$.

You might help these students by writing on the board: Probability = $\frac{\text{number of times something happened}}{\text{total number of times you did experiment}}$

Suggested assessment questions from Practising and Applying

Question 1	to see if students can estimate whether a probability is about 0, 1, or $\frac{1}{2}$
Question 5	to see if students can describe experimental results using a fractional probability
Question 8	to see if students can communicate about using fractions to describe probabilities

Answers

A. *Sample responses*: **i)** 8 times out of 12 **ii)** I took out a red cube 7 times out of 12; My prediction was close.	**B. i)** $\frac{7}{12}$; Likely **ii)** Closer to $\frac{1}{2}$ 

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Answers for Lesson 7.2.3 [Continued]

1. a) $\frac{6}{12}$; It is exactly $\frac{1}{2}$. **b)** As likely to happen as not to happen. **2. a)** $\frac{7}{12}$; It is closer to $\frac{1}{2}$ but a bit greater than $\frac{1}{2}$. **b)** Likely. **3. a)** $\frac{5}{10}$; It is exactly $\frac{1}{2}$. **b)** As likely to happen as not to happen. **4. a)** $\frac{5}{12}$; Closer to $\frac{1}{2}$ but a bit less than $\frac{1}{2}$. **b)** Not likely.	**5.** *Sample responses*: 3, 2, 6, 3, 2, 4, 1, 1, 4, 6, 5, 2 **a)** $\frac{2}{12}$ **b)** $\frac{7}{12}$ **c)** $\frac{7}{12}$ **6. a)** Closer to 0; Not very likely. **b)** Closer to $\frac{1}{2}$ but greater than $\frac{1}{2}$; Likely. **c)** Closer to $\frac{1}{2}$ but greater than $\frac{1}{2}$; Likely. **7.** *Sample responses*: **a)** 3 **b)** 1 or 2 **c)** 1 [**8**. *Sample response*: If you write __ times out of __, the first number is the numerator of the fraction and the second number is the denominator.]

Supporting Students

Struggling students

• Some students might be able to write the fraction for a number, but have difficulty positioning it on the number line. You may wish to provide the fraction strips from Unit 6 to help students position numbers on the number line.

Enrichment

• Students might create situations to fit given probabilities.

For example:

Ask them to name an event that is very unlikely.

Ask them to name an event with a probability of $\frac{7}{10}$.

CONNECTIONS: Predicting Probability Runs

Answers

1. *Sample responses*: **a)** KK<u>TTTT</u>KKKTKTTKKKTTTT The longest run was four Ts. **b)** Two students had a run of four and the other student's longest run was two.	**2.** *Sample response*: I predict a run of four Ts or four Ks; My results show a run of six Ks: KTTKKTKTTKTKTKTTTTTKTTTT<u>KKKKKK</u>TTKTTKKKKK.

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UNIT 7 Revision

Pacing	Materials
2 h	• Grid paper (BLM) • Counters (optional) • Red and blue cubes and a bag • Fraction Circles for Spinners (BLM) • Paper clips (for spinners) • Dice

Question(s)	Related Lesson(s)
1 – 3	Lesson 7.1.1
4 – 6	Lesson 7.1.2
7 – 9	Lesson 7.1.3
10	Lesson 7.1.5
11	Lesson 7.2.1
12 and 13	Lesson 7.2.2
14 and 15	Lesson 7.2.3

Revision Tips

Q 1: Make sure students notice the legend describing how many people each face represents. Remind students that, to draw conclusions, they can compare data within the graph or combine data from the graph.

Q 2: If students struggle, you might have them use counters to represent the faces in the graph in **question 1** and then make stacks of two counters to change the scale to 10 for **question 2**.

Q 3: Students are most likely to choose a scale of 2, but other scales can be used if students estimate.

Q 6: Students should understand that a bar graph might still be interpreted correctly if the lines are not the same distance apart, but it is not as effective in conveying the information visually.

Q 9: To simplify the task, encourage students to use a rectangle with horizontal and vertical grid lines.

Q 10: If possible, provide students with counters.

Q11: Provide red and blue cubes for this question.

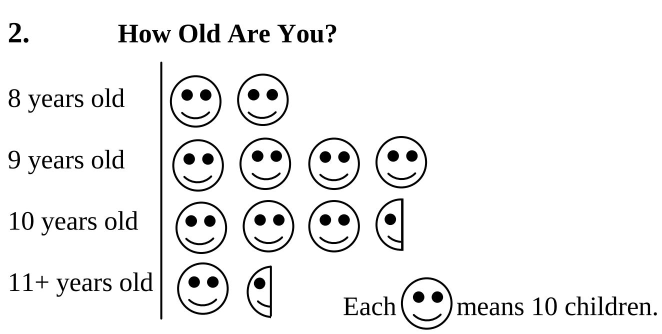
Q 12 and 13: Provide materials for spinners.

Q 14: Provide a die for this question. You might draw a number line on the board to which students can refer.

Answers

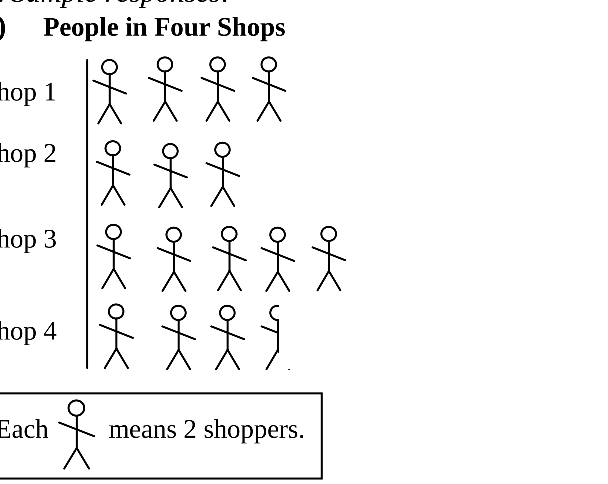
1. a) 20 children
b) 110 children
c) *Sample response*:
More children are 9 years old than any other age. There are fewer children that are 11+ years old than any other age.

2.



3. *Sample responses*:

a)



b) I chose a scale of 2 because the numbers were small and mostly even.

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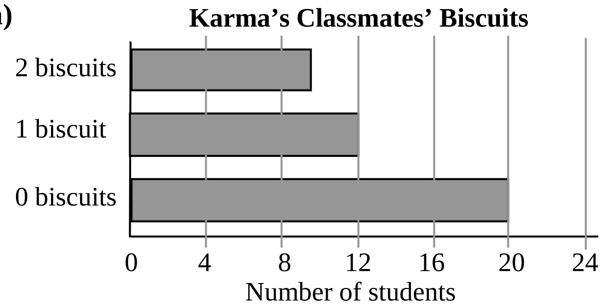
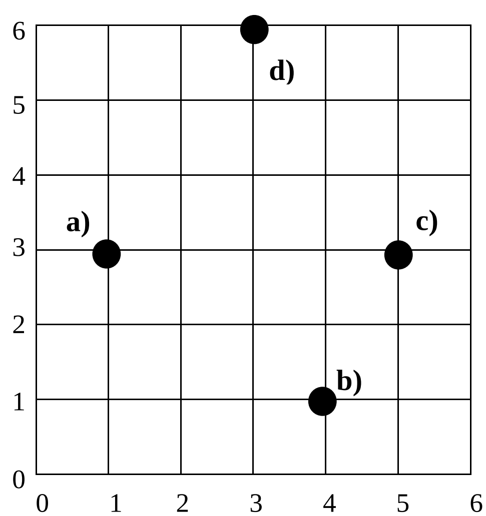
4. a) 5
b) *Sample response*:
47 children chose summer and 26 chose spring.
c) *Sample response*:
Autumn is the least popular season.
Over 70 children chose spring or summer.

5. *Sample responses*:
a)


b) 4; Two of the data values, 12 and 20, can be grouped in 4s.
c) About half the class did not have a biscuit.
More students ate 1 biscuit than ate 2 biscuits.
There are 41 students in the class.

6. *Sample response*:
If they are not evenly spaced, it is hard to compare the categories. One category might look like it is double the size of another when it really is not.

7. A (1, 4) B (3, 5) C (6, 6)
D (0, 1) E (2, 3) F (4, 2)

8.


9. *Sample response*: (2, 5), (5, 3), and (5, 5)

10. a) 11 **b)** 15 **c)** 8

11. *Sample responses*:
a) I predict that I will take out more blues.
b) I took out blue 6 times out of 10, so my prediction was correct.

12. *Sample response*:
I predict that I will spin 1 most often.

Section	1	2	3	4	5
Number of spins	6	5	2	3	4

I did spin 1 the most.

13. *Sample responses*:
a) Spinner Y; [The three section is a bigger part of Spinner Y than of Spinner X.]
b) I spun each spinner 20 times.
On Spinner X, I spun three 2 times out of 20.
On Spinner Y, I spun three 8 times out of 20.
My prediction was correct.

Spinner X

Number	**1**	**2**	**3**	**4**	**5**
Number of spins	7	5	2	3	3

Spinner Y

Number	**1**	**2**	**3**
Number of spins	11	1	8

14. *Sample responses*:

Number rolled	**1**	**2**	**3**	**4**	**5**	**6**
Number of rolls	2	1	1	2	3	1

a) $\frac{3}{10}$

b) Closer to $\frac{1}{2}$

c) Not very likely

15. a) $\frac{4}{5}$

[**b)** *Sample response*:
$\frac{4}{5}$ is closer to 1 than to 0 so it is likely. Because it is closer to 1 than to $\frac{1}{2}$, it is very likely.]

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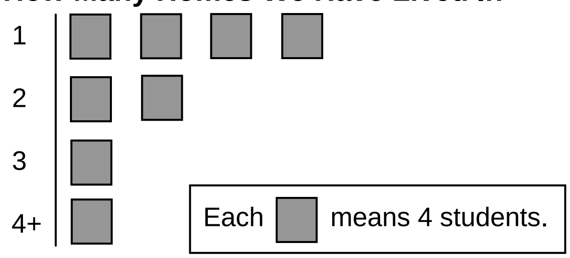
UNIT 7 Data and Probability Test

1. This pictograph shows information about the number of homes students have lived in.

How Many Homes We Have Lived In



a) How many students have lived in more than two homes?

b) How many students were asked about the number of homes they have lived in?

c) Make two conclusions about the data in the graph.

2. Graph the data shown in **question 1** in a pictograph that uses a scale of 8 instead of 4.

3. a) Graph the data shown in **question 1** in a bar graph that uses a scale of 2.

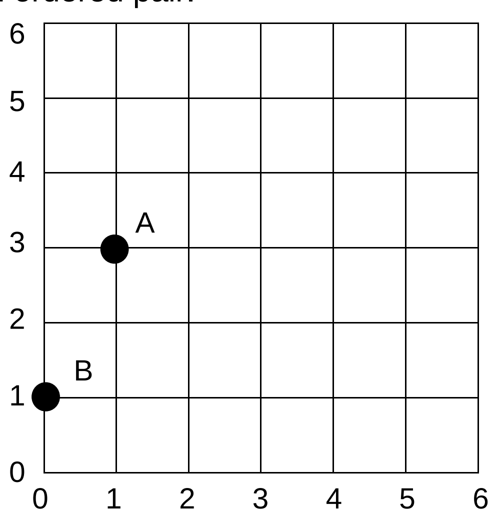
b) How are the pictograph in **question 1** and the bar graph in **question 3 a)** alike?

4. a) Roll a die 15 times. Record the results.

b) Draw a bar graph that uses a scale to show your results.

c) Tell why you chose the scale you used.

5. Name each point on the grid using an ordered pair.



6. Use the grid in **question 5**. Draw a rectangle. One side length must go through point A and another side length must go through point B. What are the vertices of the rectangle as ordered pairs?

7. There are 2 red cubes and 6 blue cubes in a bag. You take out a cube 12 times and put it back each time.

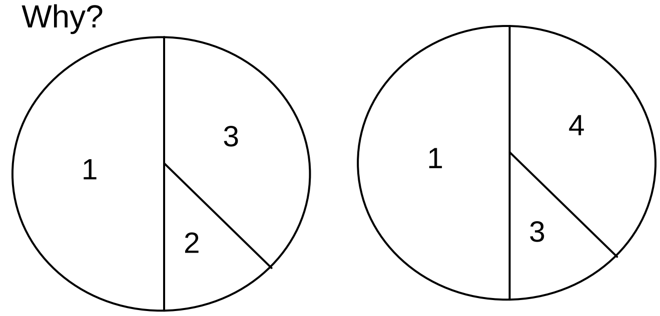
a) Predict whether you will get more red cubes or more blue cubes.

b) Do an experiment to test your prediction. Record your results. What happened?

8. a) Draw a picture that shows how to find the mean of 5, 6, and 10. Tell what the mean is.

b) Create a set of four numbers with the same mean as the numbers in **part a)**. Show how you got your set of numbers.

9. a) If you wanted to spin an odd number, would you choose Spinner X or Spinner Y? Why?



Spinner X Spinner Y

b) Test your prediction. What happened?

10. Roll a die 12 times. Record your results.

a) Use a fraction to describe the probability of rolling a 3.

b) What other die rolling event might have the same probability as rolling a 3?

c) Is the probability closer to 0, to $\frac{1}{2}$, or to 1?

11. Use a fraction to describe each probability.

a) an event that is very likely

b) an event that is only a bit likely

UNIT 7 Test

Pacing	Materials
1 h	• Grid paper • Red and blue cubes and a bag • Dice

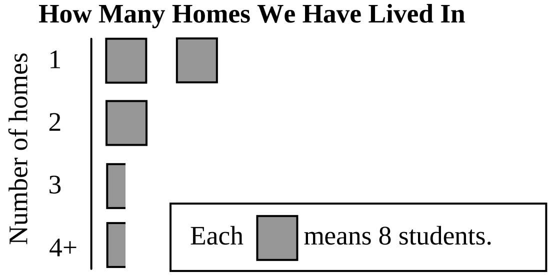
Question(s)	Related Lesson(s)
1 and 2	Lesson 7.1.1
3 and 4	Lesson 7.1.2
5 and 6	Lesson 7.1.3
7	Lessons 7.1.4 and 7.2.1
8	Lesson 7.1.5
9	Lesson 7.2.2
10 and 11	Lesson 7.2.3

Assign questions according to the time available.

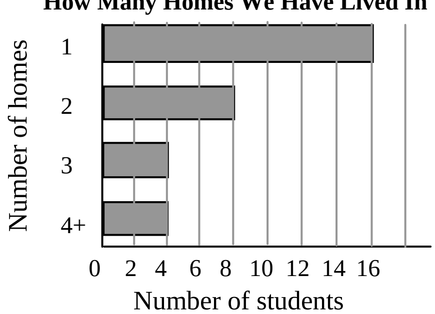
Answers

1.a) 8 students
b) 28 students
c) *Sample response*:
More students have lived in 1 home than in 3 or more homes.
Twice as many students have lived in 1 home as in 2 homes.

2. **How Many Homes We Have Lived In**



3. a) **How Many Homes We Have Lived In**



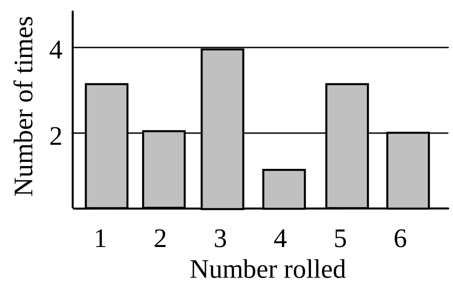
b) *Sample response*:
For both graphs,
the row that shows the students who lived in 1 home is the longest,
the rows for 3 and 4 homes are equal and the shortest,
and the row for 2 homes is in the middle.

4. *Sample responses*:

a)

Number rolled	1	2	3	4	5	6
Number of times	3	2	4	1	3	2

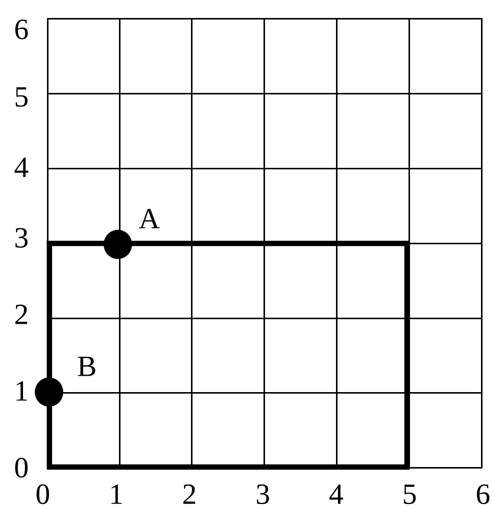
b)



c) I chose a scale of 2 because the numbers were small.

5. A (1, 3); B (0, 1)

6. *Sample response*: (0, 0), (5, 0), (5, 3), (0, 3)



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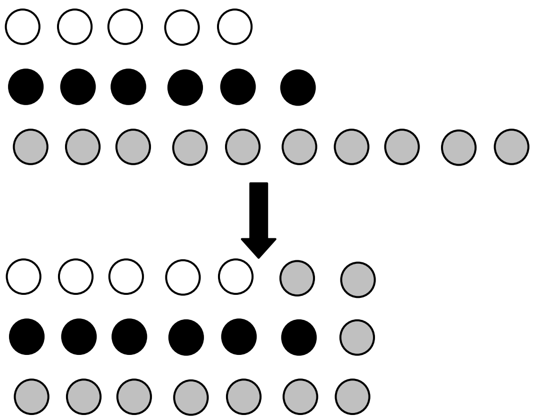
7. *Sample responses*:
a) I predict I will take out more blues.
b)

	Red	Blue
Number of times taken	5	7

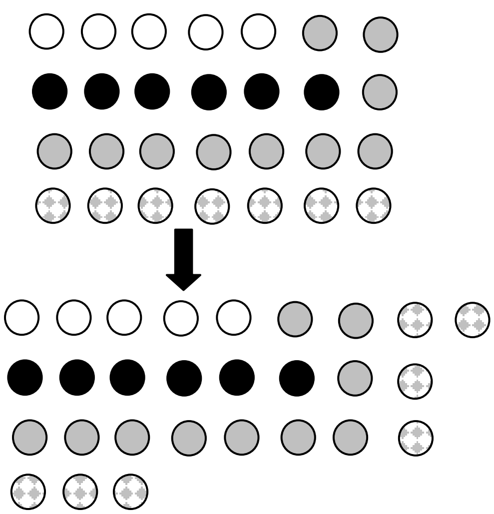
I got more blues, like I predicted.

8. *Sample responses*:
a)



The mean is 7.

b) The numbers are 9, 8, 8 and 3.
I used my picture from part a). I added another row and then rearranged the counters.



9. *Sample responses*:
a) I would choose Spinner X; The section that is odd in Spinner X is bigger than the section that is odd in Spinner Y.
b) I spun each spinner 10 times. I got an odd number 7 times with Spinner X and 8 times with Spinner Y. My prediction was wrong.

10. *Sample responses*: 5, 6, 3, 1, 6, 2, 4, 5, 2, 5, 3, 4
a) $\frac{2}{12}$
b) Rolling a 2
c) Closer to 0

11. *Sample responses*:
a) $\frac{9}{10}$ **b)** $\frac{11}{20}$

UNIT 7 Performance Task — Finding Out About Us

PART I

A. i) Think of something you want to find out about your classmates.

For example:
Favourite subject
Favourite animal
Favourite activity

ii) Prepare a question to ask them to find out what you want to know. There should be 2, 3, or 4 answer choices.

B. Survey 20 students. Record your data.

C. How do you know that the mean of your data set is greater than 6?

D. Draw a bar graph or a pictograph that uses a scale to show your data.

Part II

E. i) Use 20 slips of paper. Write the 20 answers you got in your survey on the slips. Put the slips into a bag.

ii) Suppose you take out a slip of paper 10 times, record what it says, and return it to the bag each time. Predict which answer you will get most often. Explain your prediction.

iii) Do the experiment to test your prediction. Record your results. Describe what happened.

Part III

F. i) Use a fraction to describe the probability of each answer from **part E**.

ii) Draw a number line from 0 to 1 and place each fraction on it.

iii) Which probability is closest to 0? How do you know?

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UNIT 7 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
4-F1 Collect, Organize, and Describe Data: real-world issues **4-F2** Bar Graphs and Pictographs: construct and interpret **4-F4** Mean **4-G1** Simple Outcomes: more or less likely **4-G2** Predict Probability: near 0, near 1, or near $\frac{1}{2}$ **4-G3** Experiments: predict and record results (concrete materials)	1 h	• Slips of paper • Container or bag • Grid paper (optional)

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could supplement the unit test. It could also be used as enrichment material for some students.

• You can assess performance on the task using the rubric below.

Sample Solution

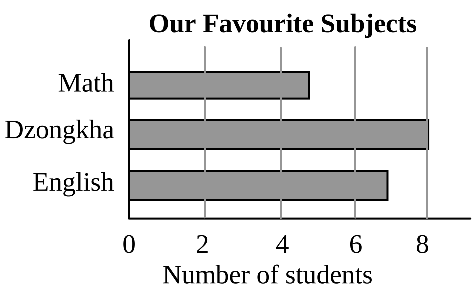
A. i) Their favourite subject in school.
ii) Which is your favourite subject?
Math
Dzongkha
English

B.

Math	Dzongkha	English
5	8	7

C. If the mean were 6, I could show 3 groups of 6 counters when I rearrange the numbers. But the total number of counters would only be 18. I have 20 answers, so I know the mean must be more than 6.

D.



E. ii) Dzongkha; Since the most students chose Dzongkha, there are more slips for it, so I think I am more likely to take it from the bag.

iii)

Math	Dzongkha	English
$\vert\vert\vert$	$\cancel{\vert\vert\vert\vert}$	$\vert\vert$

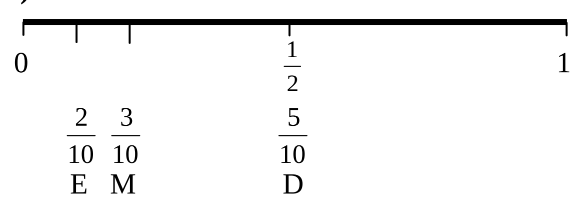
I took out Dzongkha most often, as I predicted.

F. i) Math $\frac{3}{10}$

Dzongkha $\frac{5}{10}$

English $\frac{2}{10}$

ii)



iii) $\frac{2}{10}$ is closest to 0; 2 times out of 10 is less than 3 times out of 10 or 5 times out of 10.

UNIT 7 Performance Task Assessment Rubric

The student	Level 4	Level 3	Level 2	Level 1
Collects data/carries out an experiment	Carefully, completely, and correctly collects required data through a survey or by conducting an experiment; chooses a good question for the survey	Collects most of the required data through a survey or by conducting an experiment; chooses a reasonable question for the survey	Collects some of the required data through a survey or by conducting an experiment; chooses an acceptable question for the survey	Has difficulty collecting the required data through a survey or by conducting an experiment; has difficulty choosing an appropriate survey question
Represents data	Uses graphs and charts insightfully to display data; places probabilities correctly on the number line	Uses graphs and charts correctly to display data; places probabilities correctly on the number line	Uses graphs and charts mostly correctly to display data; places some probabilities correctly on the number line	Has difficulty displaying data using graphs and charts; has difficulty placing probabilities on the number line
Communicates thinking	Explains insightfully and completely the size of the mean, the rationale for the probability prediction, and which probability is closest to 0	Explains reasonably correctly the size of the mean, the rationale for the probability prediction, and which probability is closest to 0	Correctly explains some of: the size of the mean, the rationale for the probability prediction, and which probability is closest to 0	Has difficulty explaining the size of the mean, the rationale for the probability prediction, and which probability is closest to 0

UNIT 7 Assessment Interview

- You may wish to interview selected students to assess their understanding of the work of this unit.
- Interviews are most effective when done with individual students, although pair and small group interviews are sometimes appropriate.
- The results can be used as formative assessment or as summative assessment data.
- As students work, ask them to explain their thinking.

Part 1

Ask the student to roll a die 20 times and record the results. Then ask:

- *Which event had a probability closest to 0, rolling a 1, 2, 3, 4, 5, or 6?*

- *Which event had a probability closest to* $\frac{1}{2}$ *?*

- *Predict how many 4s there will be if you roll the die 10 more times. Explain your prediction.*

- Have the student create a bar graph or a pictograph to show the number of times he or she rolled each of the possible values, 1 to 6. The graph should use a scale of 2.
- Ask the student to roll the die 5 times and use cubes to help calculate the mean of the 5 numbers.

Part 2

- On a centimetre grid with pre-drawn axes, ask the student to draw a dot at the point (3, 5) and another dot at the point (5, 3).
- Have the student draw a line that goes through both points. Ask the student to name two other points on that line using ordered pairs.

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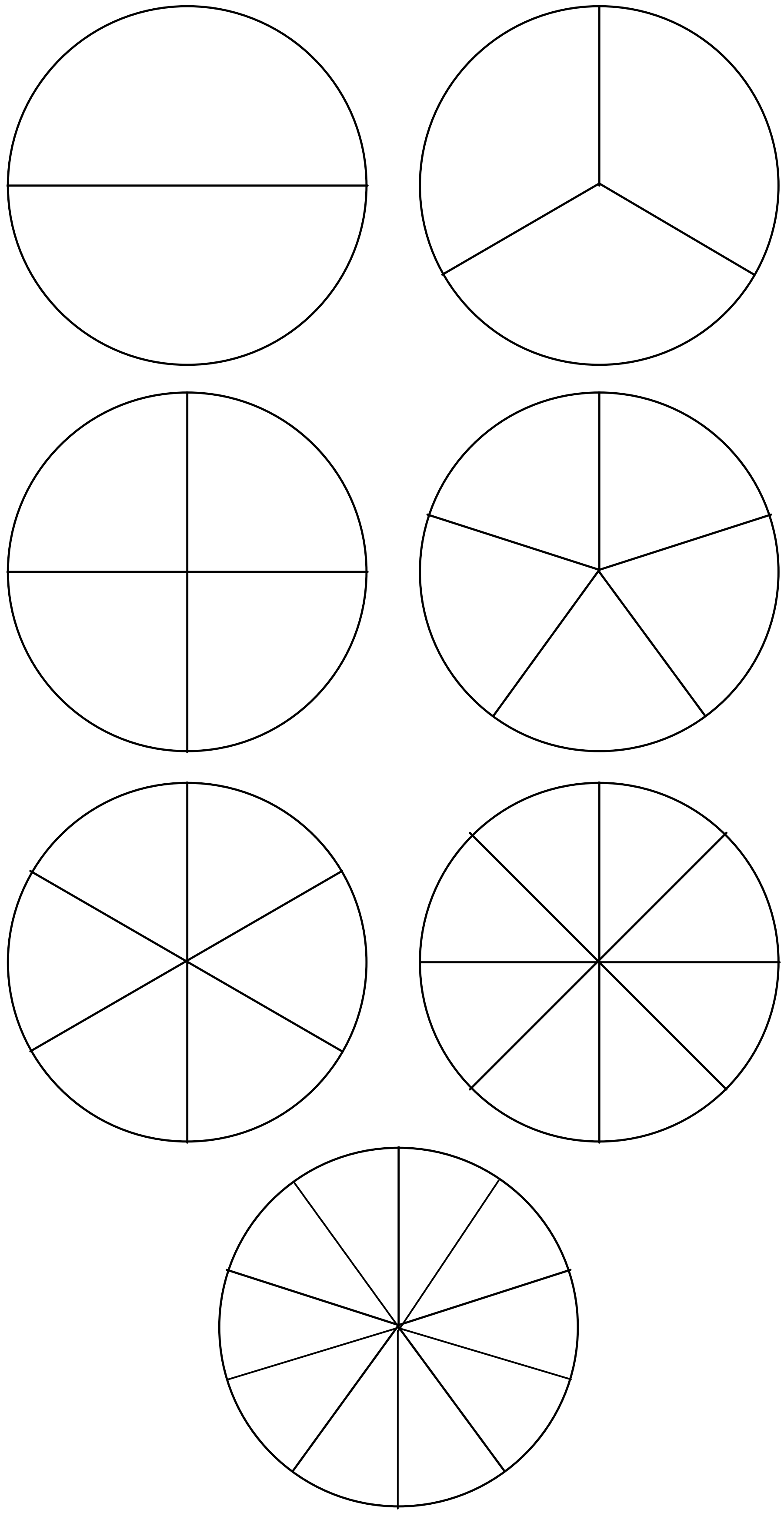
UNIT 7 Blackline Masters

BLM 1 Fraction Circles for Spinners



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