Teacher's Guide to
Understanding Mathematics
Textbook for Class V

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Department of School Education
Ministry of Education and Skills Development
Royal Government of Bhutan

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FOREWORD

Provision of quality education for our children is a cornerstone policy of the Royal Government of Bhutan. Quality education in mathematics includes attention to many aspects of educating our children. One is providing opportunities and believing in our children's ability to understand and contribute to the advancement of science and technology within our culture, history and tradition. To accomplish this, we need to cater to children's mental, emotional and psychological phases of development, enabling, encouraging and supporting them in exploring, discovering and realizing their own potential. We also must promote and further our values of compassion, hard work, honesty, helpfulness, perseverance, responsibility, *thadamtsi* (for instance being grateful to what I would like to call '*Pham Kha Nga'*, consisting of parents, teachers, His Majesty the King, the country and the Bhutanese people, for all the goodness received from them and the wish to reciprocate these in equal measure) and *ley-ju-drey* — the understanding and appreciation of the natural law of cause and effect. At the same time, we wish to develop positive attitudes, skills, competencies, and values to support our children as they mature and engage in the professions they will ultimately pursue in life, either by choice or necessity.

While education recognizes that certain values for our children as individuals and as citizens of the country and of the world at large, do not change, requirements in the work place advance as a result of scientific, technological, and even political advancement in the world. These include expectations for more advanced interpersonal skills and skills in communications, reasoning, problem solving, and decision-making. Therefore, the type of education we provide to our children must reflect the current trends and requirements, and be relevant and appropriate. Its quality and standard should stem out of collective wisdom, experience, research, and thoughtful deliberations.

Mathematics, without dispute, is a beautiful and profound subject, but it also has immense utility to offer in our lives. The school mathematics curriculum is being changed to reflect research from around the world that shows how to help students better understand the beauty of mathematics as well as its utility.

The development of this textbook series for our schools, *Understanding Mathematics,* is based on and organized as per the new School Mathematics Curriculum Framework that the Ministry of Education has developed recently, taking into consideration the changing needs of our country and international trends. We are also incorporating within the textbooks appropriate teaching methodologies including assessment practices which are reflective of international best practices. The *Teacher's Guides* provided with the textbooks are a resource for teachers to support them, and will definitely go a long way in assisting our teachers in improving their efficacy, especially during the initial years of teaching the new curriculum, which demands a shift in the approach to teaching and learning of Mathematics. However, the teachers are strongly encouraged to go beyond the initial ideas presented in the Guides to access other relevant resources and, more importantly to try out their own innovations, creativity and resourcefulness based on their experiences, reflections, insights and professional discussions.

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The Ministry of Education is committed to providing quality education to our children, which is relevant and adaptive to the changing times and needs as per the policy of the Royal Government of Bhutan and the wish of our beloved King.

I would like to commend and congratulate all those involved in the School Mathematics Reform Project and in the development of these textbooks.

I would like to wish our teachers and students a very enjoyable and worthwhile experience in teaching, learning and understanding mathematics with the support of these books. As the ones actually using these books over a sustained period of time in a systematic manner, we would like to strongly encourage you to scrutinize the contents of these books and send feedback and comments to the Curriculum and Professional Support Division (CAPSD) for improvement with the future editions. On the part of the students, they can and should be enthusiastic, critical, venturesome, and communicative of their views on the contents discussed in the books with their teachers and friends rather than being passive recipients of knowledge.

Trashi Delek!

Thinley Gyamtsho

MINISTER

Ministry of Education

October of 2007

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INTRODUCTION

HOW MATHEMATICS HAS CHANGED

Mathematics is a subject with a long history. Although newer mathematical ideas are always being created, much of what your students will be learning is mathematics that has been known for hundreds of years, if not longer.

There are some changes in the content that you will teach. It may be that the content is new to your class, but not to your curriculum. Or, it may be new to your curriculum. For example, work on isometric drawings in geometry is new.

What you may notice most is a change in the approach to mathematics. Worldwide, there is now a greater emphasis on the need for students to understand the mathematics they learn rather than to memorize rote procedures. There are many reasons for this.

Worldwide, there is now a greater emphasis on the need for students to understand the mathematics they learn rather than to memorize rote procedures.

- In the long run, it is very unlikely that students will remember the mathematics they learn unless it is meaningful. It is much harder to memorize "nonsense" than something that relates to what they already know.
- Some approaches to mathematics have not been successful; there are many adults who are not comfortable with mathematics even though they were successful in school.

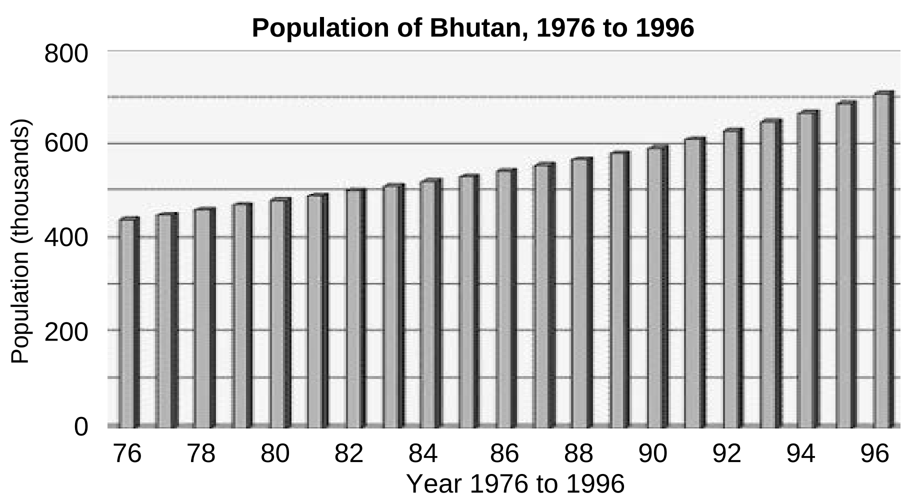
In this new program, you will find many ways to make mathematics more meaningful.

- We will always talk about why something is true, not simply that it is true. For example, the reason why 20×30 is the same as 2×3 but with two zeros added to the end ($20 \times 30 = 600$) is explained and not just stated.
- Mathematics should be taught using contexts that are meaningful to the students. They can be mathematical contexts or real-world contexts. These contexts will help students see and appreciate the value of mathematics

It is important always to talk about why something is true, not simply that it is true.

For example:

- In Unit 1 (Whole Number Computation), a task with a real-world context involves how much a family pays for rent for their home.
- A task with a broader context in Unit 7 (Data and Probability) involves looking at changes in Bhutan's population.



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- When discussing mathematical ideas, we expect students to use the processes of problem solving, communication, reasoning, making connections (connecting mathematics to the real world and connecting mathematical topics to each other) and representation (representing mathematical ideas in different ways, such as manipulatives, graphs, and tables). For example, students use grids to represent decimal thousandths. This will help them visualize the relationship between thousandths, hundredths, and tenths.
- There is an increased emphasis on problem solving because the reason we learn mathematics is to solve problems. Once students are adults, they are not told when to factor or when to multiply; they need to know how to apply those skills to solve certain problems. Students will be given opportunities to make decisions about which concepts and skills they need in certain situations.

We expect students to use the processes of problem solving, communication, reasoning, making connections, and representation.

A significant amount of research evidence has shown that these more meaningful approaches work. Scores on international tests are higher when the emphasis is on higher level thinking and not only on the application of skills.

Scores on international tests are higher when the emphasis is on higher level thinking and not only on the application of skills.

THE DESIGN OF THE STUDENT TEXTBOOK

Each unit of the textbook has the following features:

- a *Getting Started* to review prerequisite content
- two or three chapters, which cluster the content of the unit into sections that contain related content
- regular lessons and at least one *Explore* lesson
- a *Game*
- at least one *Connections* feature
- a *Unit Revision*

Getting Started

There are two parts to the *Getting Started*. They are designed to help you know whether the students are missing critical prerequisites and to remind students of knowledge and terminology they have already learned that will be useful in the unit.

- The *Use What You Know* section is an activity that takes 20 to 30 minutes. Students are expected to work in pairs or in small groups. Its purpose is consistent with the rest of the text's approach, that is, to engage students in learning by working through a problem or task rather than being told what to do and then just carrying it through.
- The *Skills You Will Need* section is a more straightforward review of required prerequisite skills for the unit. This should usually take about 20 to 30 minutes.

The Getting Started is designed to help you know whether the students are missing critical prerequisites and to remind students of knowledge and terminology they will need for the unit.

Regular Lessons

- Each lesson might be completed in one or two hours (i.e., one or two class periods), although some are shorter. The time is suggested in this *Teacher's Guide*, but it is ultimately at your discretion.
- Lessons are numbered #.#.#. The first number tells the unit, the second number the chapter, and the third number the lesson within the chapter. For example, Lesson 4.2.1 is Unit 4, Chapter 2, Lesson 1.

Lessons are numbered #.#.#. The first number tells the unit, the second number the chapter, and the third number the lesson within the chapter.

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• Each lesson is divided into five parts:

- A *Try This* task or problem
- The exposition (the main points of the lesson)
- A question that revisits the *Try This* task, called *Revisiting the Try This* in this guide
- one or more *Examples*
- *Practising and Applying* questions

Try This

• The *Try This* task is in a shaded box, like the one below from lesson 1.1.1 on page 2.

Try This

A dragonfly flaps its wings between 20 times and 40 times each second.

A. How many times does it flap in 1 min? (Remember: 1 min = 60 s)

• The *Try This* is a brief task or problem that students complete in pairs or small groups. It serves to motivate new learning. Students can do the *Try This* without the new concepts or skills that are the focus of the lesson, but the problem is related to the new learning. It should be completed in 5 to 10 minutes. The reason to start with a *Try This* is that we believe students should do some mathematics independently before you intervene.

The Try This is a brief task or problem that students complete in pairs or small groups to motivate new learning.

• The answers to the *Try This* questions are not found in the back of the student book (but they are in this *Teacher's Guide*).

The Exposition

• The exposition presents the main concepts and skills of the lesson. Examples are often included to clarify the points being made.

The exposition presents the main concepts and skills of the lesson and is taught in different ways, as suggested in this guide.

• You will help the students through the exposition in different ways (as suggested in this *Teacher's Guide*). Sometimes you will present the ideas first, using the exposition as a reference. Other times students will work through the exposition independently, in pairs, or in small groups.

• Key mathematical terms are introduced and described in the exposition. When a key term first appears in a unit of the textbook, it is highlighted in **bold type** to indicate that it is found in the glossary (at the back of the student textbook).

• Students are not expected to copy the exposition into their notebooks either directly from the book or from your recitation.

Revisiting the Try This

• The *Revisiting the Try This* question follows the exposition and appears in a shaded lozenge, like this example from lesson 1.1.1 on page 3.

B. What rectangles could you draw to multiply the number of wing flaps for the dragonfly in **part A**?

• The *Revisiting the Try This* question links the *Try This* task or problem to the new ideas presented in the exposition. This is designed to build a stronger connection between the new learning and what students already understand.

The Revisiting the Try This question links the Try This task to the new ideas presented in the exposition.

Examples

• The *Examples* are designed to provide additional instruction by modelling how to approach some of the questions students will meet in *Practising and Applying*. Each example is a bit different from the others so that students have multiple models from which to work.

The Examples model how to approach some of the questions students will meet in Practising and Applying

• The *Examples* show not only the formal mathematical work (in the left hand *Solution* column), but also student reasoning (in the right hand *Thinking* column). This model should help students learn to think and communicate mathematically. Photographs of students are used to further reinforce this notion.

• Some of the *Examples* present two different solutions. The example below, from lesson 1.1.3 on page 10 shows two possible ways to approach the task, *Solution 1* and *Solution 2*.

Example 3 Solving a Problem Involving Multiplication	
428 students in a school each brought Nu 25 to help other people who needed money. How much money was collected altogether?	
Solution 1 ~~1~~ ~~14~~ 428 × 25 2140 + 8560 10,700 Nu 10,700 was collected.	**Thinking**  • I knew I had to multiply 428 × 25. • I multiplied each part of 428 by 5 first. • When I finished that, I crossed out the regrouping numbers so I wouldn't use them when I multiplied 428 by 20. • I multiplied 428 × 20 by multiplying by 2 and then by 10. I knew that, to multiply by 10, you just add a 0 to the end of 856.
Solution 2 428 × 25 428 ÷ 4 = 107 107 × 100 = 10,700 Nu 10,700 was collected.	**Thinking**  • I knew I had to multiply 428 × 25. • Multiplying by 25 is the same as dividing by 4 and multiplying by 100 because 25 = 100 ÷ 4.

The Examples show the formal mathematical work in the Solution column and model student reasoning in the Thinking column. This model should help students learn to think and communicate mathematically.

• The treatment of *Examples* varies and is discussed in the *Teacher's Guide*. Sometimes the students work through these independently, sometimes they work in pairs or small groups, and sometimes you are asked to lead them through some or all of the examples.

• A number of the questions in the *Practising and Applying* section are modelled in the *Examples* to make it more likely that students will be successful.

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Practising and Applying

• Students work on the *Practising and Applying* questions independently, with a partner, or in a group, using the exposition and *Examples* as references.

• The questions usually start like the work in the *Examples* and get progressively more conceptual, with more explanations and more problem solving required later in the exercise set.

• The last question in the section always brings closure to the lesson by asking students to summarize the main learning points. This question could be done as a whole class.

Students work on the Practising and Applying questions independently, with a partner, or in a group, using the exposition and Examples as references.

Explore Lessons

• *Explore* lessons provide an opportunity for students to work in pairs or small groups to investigate some mathematics in a less directed way. Often, but not always, the content is revisited more formally in a regular lesson immediately before or after the *Explore* lesson. The *Teacher's Guide* indicates whether the *Explore* lesson is optional or core.

• There is no exposition or teacher lecture in an *Explore* lesson, so the parts of the regular lesson are not there. Instead, a problem or task is posed and students work through it by following a sequence of questions or instructions that direct their investigation.

• The answers for these lessons are not found in the back of the textbook, but are found in this *Teacher's Guide*.

Explore lessons provide an opportunity for students to work with a partner or in small groups to investigate some mathematics in a less directed way.

Connections

• The *Connections* is an optional feature that relates the content of the unit to something else.

• There are always one or more *Connections* features in a unit. The placement of a *Connections* feature in a unit is not fixed; it depends on the content knowledge required. Sometimes it will be early in the unit and sometimes later.

• The *Connections* feature always gives students something to do beyond simply reading it.

• Students usually work in pairs or small groups to complete these activities.

The Connections feature takes many forms. Sometimes it is a relevant and interesting historical note. Sometimes it relates the mathematical content of a unit to the content of a different unit. Other times it relates the mathematical content to a real world application.

Game

• There is at least one *Game* per unit.

• The *Game* provides an enjoyable way to practise skills and concepts introduced in the unit.

• Its placement in the unit is based on where it makes most sense in terms of the content required to play the *Game*.

The Game provides an enjoyable way to practise skills and concepts introduced in the unit.

• In most *Games* students work in pairs or small groups, as indicated in the instructions.

• The required materials and rules are listed in the student book. Usually, there is a sample shown to make sure that students understand the rules.

• Most *Games* require 15 to 20 minutes, but students can often benefit from playing them more than once.

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Unit Revision

- The *Unit Revision* provides an opportunity for review for students and for you to gather informal assessment data. *Unit Revisions* review all lesson content except the *Getting Started* feature, which is based on previous class content. There is always a mixture of skill, concept, and problem solving questions.
- The order of the questions in the *Unit Revision* generally follows the order of the lessons in the unit. Sometimes, if a question reflects more than one lesson, it is placed where questions from the later lesson would appear.
- Students can work in pairs or on their own, as you prefer.
- The *Unit Revision*, if done in one sitting, requires one or more hours. If you wish, you might break it up and assign some questions earlier in the unit and some questions later in the unit.

The Unit Revision provides an opportunity for review for students and allows you to gather informal assessment data.

Glossary

- At the end of the student textbook, there is a glossary of key mathematical terminology introduced in the units. When new terms are introduced in the units, they are in **bold** type. All of these terms are found in the glossary.
- The glossary also contains important mathematical terms from previous classes that students might need to refer to.
- In addition, there is a set of instructional terms commonly used in the *Practising and Applying* questions (for example, explain, predict,…) along with descriptions of what those terms require the student to do.

The glossary contains key mathematical terminology introduced in the units, important terms from previous classes, and a set of instructional terms.

Answers

- Answers to most numbered questions are provided in the back of the student textbook. In most cases, only the final answers are shown, not full solutions and explanations. For example, if students are asked to solve a problem and then "Show your work" or "Explain your thinking", only the final answer to the problem will be included, not the work or the reasoning.
- There is often more than one possible answer. This is indicated by the phrase *Sample Response*.
- Full solutions to the questions and explanations that show reasoning are provided in this *Teacher's Guide*, as are the answers to the lettered questions (such as A or B) in the *Try This* and the *Explore* lessons. Note that, when an answer or any part of an answer is enclosed in square brackets, it indicates that it has been omitted from the answers at the back of the student textbook.

The answers to most of the numbered questions are found in the back of the student book. This Teacher's Guide contains a full set of answers.

THE DESIGN OF THE TEACHER'S GUIDE

The *Teacher's Guide* is designed to complement and support the use of the student textbook.

- The sequencing of material in the guide is identical to the sequencing in the student textbook.
- The elements in the *Teacher's Guide* for each unit include:
  - a *Unit Planning Chart*
  - *Math Background* for the unit
  - a *Rationale for Teaching Approach*
  - support for each lesson
  - a *Unit Test*
  - a *Performance Task*
  - an *Assessment Interview* (Units 2, 4 and 6)

The Teacher's Guide is designed to complement and support the use of the student textbook.

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The support for each lesson includes:

- *Curriculum outcomes* covered in that lesson
- *Outcome relevance* (*Lesson relevance* in the case of optional *Explore* lessons)
- *Pacing* in terms of minutes or hours
- *Materials* required to teach the lesson
- *Prerequisites* that the lesson assumes students possess
- *Main Points to be Raised* explicitly in the lesson
- suggestions for working through the parts of the lesson
- *Suggested assessment* for the lesson
- *Common errors* to be alert for
- *Answers*, often with more complete solutions than are found in the student text
- suggestions for *Supporting Students* who are struggling and/or for enrichment

Unit Planning Chart

This chart provides an overview of the unit and indicates, for each lesson, which curriculum outcomes are being covered, the pacing, the materials required, and suggestions for which questions to use for formative assessment.

The Unit Planning Chart provides an overview of the unit.

Math Background and Rationale for Teaching Approach

This explains the organization of the unit or provides some background for you that you might not have, particularly in the case of less familiar content. In addition, there is an indication of why the material is approached the way it is.

This section provides information about the math behind the unit, and an explanation of why the math is approached the way it is.

Regular Lesson Support

Regular lesson support includes grouping and instructional strategies, alerts for common errors, suggestions for assessment, and teaching tips.

• Suggestions for grouping and instructional strategies are offered under the headings *Try This, Revisiting the Try This, The Exposition — Presenting the Main Ideas, Using the Examples,* and *Practising and Applying — Teaching Tips.*

• *Common errors* are sometimes included. If you are alert for these and apply some of the suggested remediation, it is less likely that students will leave the lessons with mathematical misunderstandings.

• A number of *Suggested assessment questions* are listed for each lesson. This is to emphasize the need to collect data about different aspects of the students' performance — sometimes the ability to apply skills, sometimes the ability to solve problems or communicate mathematical understanding, and sometimes the ability to show conceptual understanding.

• It is not necessary to assign every *Practising and Applying* question to each student, but they are all useful. You should go through the questions and decide where your emphasis will be. It is important to include a balance of skills, concepts, and problem solving. You might use the *Suggested assessment questions* as a guide for choosing questions to assign.

• You may decide to use the last *Practising and Applying* question to focus a class discussion that revisits the main ideas of the lesson and to bring closure to the lesson.

Explore Lesson Support

• As with regular lessons, for *Explore* lessons there is an indication of the curriculum outcomes being covered, the relevance of the lesson, and whether the lesson is optional or essential.

• Because the style of the lesson is different, the support provided is different than for the regular lessons. There are suggestions for grouping the students for the exploration, a list of *Observe and assess* questions to guide your informal formative assessment, and *Share and reflect* ideas on how to consolidate and bring closure to the exploration.

Because the style of an Explore lesson is different than a regular lesson, the support provided in the Teacher's Guide is also different.

Unit Test

A pencil-and-paper unit test is provided for each unit. It is similar to the unit revision, but not as long. If it seems that the test might be too long (for example, if students would require more than one class period to complete it) some questions may be omitted. It is important to balance the items selected for the test to include questions involving skills, concepts, and problem solving, and to include at least one question requiring mathematical communication.

If the test seems too long, some questions may be omitted but it is important to ensure a balance of questions involving skills, concepts, problem solving, and communication.

Performance Task

• The *Performance Task* is designed as a summative assessment task. Performance on the task can be combined with performance on a *Unit Test* to give a mark for a student on a particular unit.

• The task requires students to use both problem solving and communication skills to complete it. Students have to make mathematical decisions to complete the task.

• It is not appropriate to mark a performance task using percentages or numerical grades. For that reason, a rubric is provided to guide assessment. There are four levels of performance that can be used to describe each student's work on the task. A level is assigned for each different aspect of the task and, if desired, an overall profile can be assigned. For example, if a student's performance is level 2 on most of the aspects of the task, but level 3 on one aspect, an overall profile of level 2 might be assigned.

• A sample solution is provided for each task.

The Performance Task is designed as a summative assessment task that can be combined with a student's performance on the Unit Test to give an overall mark.

Unit Assessment Interviews

• Selected units (2, 4, and 6) provide a structure for an interview that can be used with one or several students to determine their understanding of the outcomes.

• Interviews are a good way to collect information about students because they allow you to interact with the students and to follow up if necessary. Interviews are particularly appropriate for students whose performance is inconsistent or who do not perform well on written tests, but who you feel really do understand the content.

• You may use the data you collect in combination with class work or even a unit test mark.

Interviews are a good way to collect information about students, since interviews allow you to interact with the students and to follow up if necessary.

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ASSESSING MATHEMATICAL PERFORMANCE

Forms of Assessment

It is important to consider both formative (continuous) and summative assessment.

Formative

- Formative assessment is observation to guide further instruction. For example, if you observe that a student does not understand an idea, you may choose to re-teach that idea to that student using a different approach.

Formative assessment is observation to guide further instruction.

- Formative assessment opportunities are provided through
  - prerequisite or diagnostic assessment in the *Getting Started*
  - suggestions for assessment questions in each regular lesson
  - questions that might be asked while students work on the *Try This* or during an *Explore* lesson
  - the *Unit Revision*
  - the unit *Assessment Interview* (for the units with interviews)
- Formative assessment can be supplemented by
  - everyday observation of students' mathematical performance
  - formal or informal interviews to reveal students' understanding
  - journals in which students comment on their mathematical learning
  - short quizzes
  - projects
  - a portfolio of work so students can see their progress over time, for example, in problem solving or mathematical communication (see *Portfolios* below)

Summative

- Summative assessment is used to see what students have learned and is often used to determine a mark or grade.

Summative assessment is used to see what students have learned and is often used to determine a mark.

- Summative assessment opportunities are provided through
  - the *Unit Test*
  - the *Performance Task*
  - the *Assessment Interview*
- Summative assessment can be supplemented with
  - short quizzes
  - projects
  - a portfolio that is assessed with respect to progress in, for example, problem solving or communication

Portfolios

One of the advantages of using portfolios for assessment is that students can observe their own growth over time, leading to greater confidence. In each unit, the section on math background identifies items that pertain to the various mathematical processes. The portfolio could be made up of student work items related to one of the mathematical processes: problem solving, communication, reasoning, or representation.

One of the advantages of using portfolios for assessment is that students can observe their own growth over time, leading to greater confidence.

Assessment Criteria

- It is right and fair to inform students about what will be assessed and how it will be assessed. For example, students should know whether the intent of a particular assessment task is to focus on application or on problem solving.

It is right and fair to inform students about what will be assessed and how it will be assessed.

• A student's mark and all assessments should reflect the curriculum outcomes for Class V. The proportions of the mark assigned for each unit should reflect both the time spent on the unit and the importance of the unit. The modes of assessment used for a particular unit should be appropriate for the content. For example, if the unit focuses mostly on skills, the main assessment might be a paper-and-pencil test or quizzes. If the unit focuses on concepts and application, more of the student's mark should come from activities like performance tasks.

The modes of assessment used for a particular unit should be appropriate for the content.

• The focus of this curriculum is not on procedures for their own sake but on a conceptual understanding of mathematics so that it can be applied to solve problems. Procedures are important too, but only in the context of solving problems. All assessment should balance procedural, conceptual, and problem solving items, although the proportions will vary in different situations.

All assessment should balance procedural, conceptual, and problem solving items, although the proportions may vary in different situations.

• Students should be informed whether a test is being marked numerically, using letter grades, or with a rubric. If a rubric is being used, then it should be shared with students before they begin the task to which it is being applied.

Determining a Mark

• In determining a student's mark, you can use the tools described above along with other information, such as work on a project or poster. It is important to remember that the mark should, as closely as possible, reflect student competence with the mathematical outcomes of the course. The mark should not reflect behaviour, neatness, participation, and other non-mathematical aspects of the student's learning. These are important to assess as well, but not as part of the mathematics mark.

It is important to remember that the mark should reflect student competence with the mathematical outcomes of the course and not behaviour, neatness, participation, and so on.

• In looking at a student's mathematics performance, the most recent data might be weighted more heavily. For example, suppose a student does poorly on some of the quizzes given early in Unit 1, but you later observe that he or she has a better understanding of the material in the unit. You might choose to give the early quizzes less weight in determining the student's mark for the unit.

• At present, you are required to produce a numerical mark for a student, but that should not preclude your use of rubrics to assess some mathematical performance. One of the values of rubrics is their reliability. For example, if a student performs at level 2 on a particular task one day, he or she is very likely to perform at the same level on that task another day. On the other hand, a student who receives a test mark of 45 one day might have received a mark of 60 on a different day if one question on the test had changed or if he or she had read an item more carefully.

• You can combine numerical and rubric data using your own judgment. For example, if a student's marks on tests average 50%, but the rubric performance is higher, for example, level 3, it is fair and appropriate to use a higher average for that student's class mark.

THE CLASSROOM ENVIRONMENT

This new curriculum requires a change in the classroom environment to include more pair and group work and an increased emphasis on communication. It is only in this way that students will really become engaged in mathematical thinking instead of being spectators.

This new curriculum requires a change in the classroom environment to include more pair and group work and an increased emphasis on communication.

• In every lesson, students should be engaged in some pair or small group work (for the *Try This*, selected *Practising and Applying* questions, or during an *Explore* lesson).

• Students should be encouraged to communicate with each other to share responses that are different from those offered by other students or from the responses you might expect. Communication involves not only talking and writing, but also listening and reading.

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Pair and Group Work

• There are many reasons why students should be working in pairs or groups, including

- to ensure that students have more opportunities to communicate mathematically (instead of competing with the whole class for a turn to talk)
- to make it easier for them to take the risk of giving an answer they are not sure of (rather than being embarrassed in front of so many other people if they are incorrect)
- to see the different mathematical viewpoints of other students
- to share materials more easily

• Sometimes students can work with the students who sit near them, but other times you might want to form the groups so that students who are struggling are working together. Then you can help them while the other students move forward. Students who need enrichment can also work together so that you can provide an extra challenge for them all at once.

• For students who are not used to working in pairs or groups, you should set down rules of behaviour that require them to attend to the task and to participate fully. You need to avoid a situation where four students are working together, but only one of them is really doing the work. You might post Rules for Group Work, as shown here to the right.

• Once students are used to working in groups, you might sometimes be able to base assessment on group performance rather than on individual performance.

Rules for Group Work

- Make sure you understand all of the work produced by the group.
- If you have a question, ask your group members first, before asking your teacher.
- Find a way to work out disagreements without arguing.
- Listen to and help others.
- Make sure everyone is included and encouraged.
- Speak just loudly enough to be heard.

Communication

Students should be communicating regularly about their mathematical thinking. It is through communication that they clarify their own thinking as well as show you and their classmates what they do or do not understand. When they give an answer to a question, you can always be asking questions like, *How did you get that? How do you know? Why did you do that next?*

It is through communication that students clarify their own thinking as well as show you and their classmates what they do or do not understand.

• Communication is practised in small group settings, but is also appropriate when the whole class is working together.

• Students will be reluctant to communicate unless the environment is risk-free. In other words, if students believe that they will be reprimanded or made to feel badly if they say the wrong thing, they will be reluctant to communicate. Instead, show your students that good thinking grows out of clarifying muddled thinking. It is reasonable for students to have some errors in their thinking and it is your job to help shape that thinking. If a student answers incorrectly, it is your job to ask follow-up questions that will help the student clarify his or her own thinking.
For example, suppose a problem requires a student to tell about a situation when 1,000,000 would be a lot. The student hesitates or answers inappropriately.
Follow up by asking questions like the following:

Students will be reluctant to communicate unless the environment is risk-free.

- *How do you read the number 1,000,000?*
- *What do you know about the number 1,000,000?*
- *How does a million compare to a thousand?*
- *Why should you be thinking about something that happens a lot?*

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• Many of the questions in the textbook require students to explain their thinking. The sample *Thinking* in the *Examples* is designed to provide a model for mathematical communication.

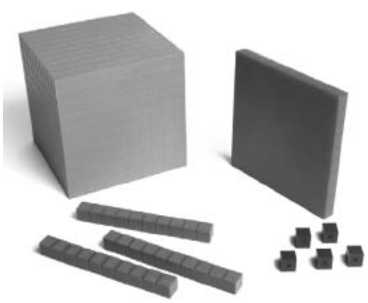
The sample Thinking in the Examples is designed to provide a model for mathematical communication.

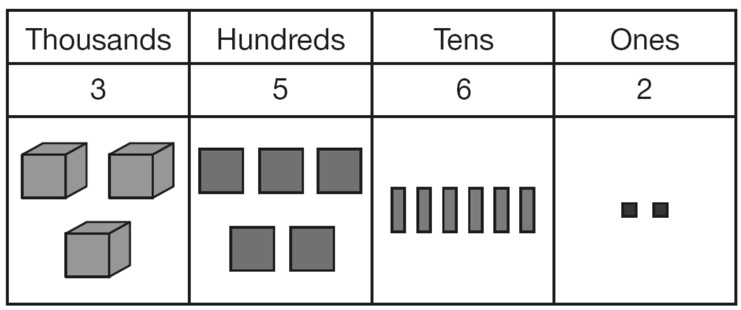
• One of the ways students communicate mathematically is by describing how they know an answer is right. Even when a question does not ask students to check their work, you should encourage them to think about whether their answer makes sense. When they check their work, they should usually check using a different way than the way they used to find their answer so that they do not make the same reasoning error twice. This will enhance their mathematical flexibility.

MATHEMATICAL TOOLS

Manipulatives

There is great value in using manipulative materials in mathematics instruction. Sometimes, it is essential. For example, Chapter 3 in Unit 2 cannot be completed without using interlocking or connecting cubes. Other times, for example, in Unit 1, some students will be successful without manipulative materials, but all students will benefit from using base ten blocks and place value charts. Students will start to see not only how to perform arithmetic calculations, but why they are done the way they are.





Modelling the number 3562 with base ten blocks on a place value mat

THE STUDENT NOTEBOOK

It is valuable for students to have a well-organized, neat notebook to look back at to review the main mathematical ideas they have learned. However, it is also important for students to feel comfortable doing rough work in that notebook or doing rough work elsewhere without having to waste time copying it neatly into their notebooks. In addition to the things you tell students to include in their notebooks, they should be allowed to make some of their own decisions about what to include in their notebooks.

Students should be allowed to make some of their own decisions about what to include in their notebooks.

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CLASS V CURRICULUM

STRAND A: NUMBER

KSO Number *By the end of **Class 6** students should*

♦ *have strong number sense with respect to whole numbers and decimals, and be able to draw on a wide variety of relationships and strategies within number to solve problems in new situations*

♦ *have a strong sense of the base ten system to millions and thousandths, and use place value patterns to understand new ideas and apply reasoning to computational problems and mental mathematics within mathematics itself and in real world situations*

♦ *efficiently select and apply appropriate estimation strategies, to answer real life questions and check for reasonableness of answers in calculation*

♦ *understand fractions and decimals to thousandths, and the relationship between them, and move freely from one form of representation to another, as might be appropriate in a given situation, to provide a strong foundation for higher level fractional ideas and computation*

♦ *understand meanings and appropriate application of integers, ratios, and percent in real world situations*

♦ *apply number theory concepts in relevant situations as a way to solve problems*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-A1 Meaning of Fractions: using and relating different meanings
• relating fraction meanings: part of a region, part of a group, part of a length, and as division
• develop the relationship between fractions and division
• change an improper fraction to a mixed number
• link concrete materials and/or pictorial representations to symbols to develop understanding

5-A2 Rename Fractions: with and without models
• develop an understanding of renaming fractions using concrete materials and/or pictorial representations first and then link to the symbolic
• understand equivalent fractions as the same region or group partitioned in different ways
• understand the relationship between the numerator and the denominator of a fraction

5-A3 Compare and Order Fractions
• develop and use benchmarks to compare fractions
• compare fractions with the same denominator
• compare fractions with the same numerator
• compare improper fractions as mixed numbers

5-A4 Thousandths: model and record
• develop decimal and fractional benchmarks (e.g., 0.432 m is a little less than half a metre)
• place decimal numbers on a number line and justify their placements
• read the quantitative value of each digit in decimals (e.g., 16.5 is "sixteen and 5 tenths" or "sixteen and a half")

5-A5 Thousandths: compare and order numbers to thousandths
• compare whole number parts of decimals first
• understand that decimals do not need the same number of places after the decimal to be compared (e.g., 0.7 > 0.423)
• understand that the number of decimal places after the decimal point does not indicate size

5-A6 Millions: interpret
• develop a sense of how big a million is
• interpret millions in different ways and justify the interpretation (e.g., 1 500 000 = $1\frac{1}{2}$ million = 1.5 million)

5-A7 Place Value: whole numbers to 7 digits
• read and represent whole numbers to 7 digits
• generalize place-value patterns as groups of three digits called periods

5-A8 Comparing: order 7-digit whole numbers
• compare and order numbers up to 7 digits
- in standard form (e.g., 3,256,876 > 3,255,996)
- in decimal form (e.g., 3.25 million > 34.3 million)
- in standard and decimal notation (e.g., 3,256,876 < 3.2 million)
- with different place value (e.g., 3420 thousand > 3,325,146 and 342 thousand < 2 million

5-A9 Ratio and Rate: exploring informally
• understand ratio as a multiplicative comparison of two numbers or quantities of the same type
• understand rate as a multiplicative comparison of two quantities described in different units
• explore ratio and rate in geometric, numerical, and measurement situations

STRAND B: OPERATIONS

KSO Operations *By the end of **Class 6** students should*

♦ *model and solve computational problems involving whole numbers and decimals by selecting appropriate operations and procedures for computation, estimation, and mental math*

♦ *choose appropriate method of computation in given situations (including pencil/paper, mental math, estimation)*

♦ *model and solve problems involving the addition and subtraction of simple fractions and be able to justify answers through reasoning*

♦ *informally explore simple algebraic situations*

♦ *demonstrate flexibility in procedures chosen to solve computational problems*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-B1 Estimate Products: 2 digits × 2 digits
• use a variety of strategies to estimate products

5-B2 2-digit × 2-digit and 2-digit × 3-digit Multiplication: with and without regrouping
• relate models or diagrams to algorithms
• develop personal and standard algorithms
• continue estimating to check

5-B3 4-digit × 1-digit Multiplication: with and without grouping
• extend 3-digit × 1-digit multiplication using similar strategies
• develop personal and standard algorithms
• continue estimating to check

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5-B4 Multiply Mentally: to 4 digits × 1 digit
• understand the difference between estimation and mental math
• understand that estimation strategies can often be used to calculate mentally
• develop efficiency with multiplying mentally by 10, 100, 1000
• apply associative principle (e.g., $25 \times 30 = 25 \times 3 \times 10 = 750$)
• apply double/half strategy (e.g., $50 \times 16 = 100 \times 8$)
• apply front-end strategy (e.g., $3 \times 325 = 900 + 60 + 15 = 975$)
• choose appropriate strategy (depending on numbers being calculated)

5-B5 4-Digit ÷ 1-Digit: with and without regrouping
• focus on the whole number (rather than the digits)
• link concrete models to algorithms
• express remainders as fractions where appropriate
• continue estimating to check

5-B6 4-Digit ÷ 2-Digit: introduce
• explore divisors which are multiples of 10 only (10, 20, 30, …)

5-B7 Divide Mentally
• use prior knowledge of basic facts
• divide by 10, 100, 1000
• link to place value

5-B8 Addition and Subtraction: simple fractions with common denominators
• link concrete models such as fraction strips to symbols
• use less formal language to build understanding (e.g., 2 fourths + 1 fourth = 3 fourths)

5-B9 Addition and Subtraction of Whole Numbers and Decimals: 5 digits to 1000ths
• perform addition and subtraction presented horizontally and vertically
• choose an appropriate method for computation: mentally, pictorially, or symbolically
• continue estimating in computation

5-B10 Decimals × Whole Numbers: simple products
• link concrete models to algorithm
• estimate (e.g., 4×2.45 as 4×2 or 4×3)

5-B11 Mentally Multiply: whole numbers by 0.1, 0.01, 0.001
• multiply by 0.1, 0.01, 0.001
• link to place value

5-B12 Open Number Sentences: applying number sense
• explore numerical situations which are always, sometimes, or never true (e.g., $324 + \square > 300$ is always true, assuming $\square$ is a whole number)
• work with open number sentences involving the four basic operations and a combination of operations
• understand that $\square$ can also be expressed as a letter variable or another shape or symbol

STRAND C: PATTERNS AND RELATIONSHIPS

KSO Patterns and Relationships *By the end of **Class 6** students should*

♦ *describe, extend, and create patterns to solve problems in real world situations and mathematical contexts (number, geometry, measurement)*

♦ *use patterns to generalize mathematical situations to aid in solving problems and understanding relationships*

♦ *explore and generalize how a change in one quantity in a relationship affects another, in order to efficiently solve similar (but new) problems*

♦ *represent mathematical patterns and relationships in a variety of ways (charts, tables, graphs, numerically)*

♦ *use patterns to assist in mental math strategies*

♦ *informally (through reasoning) solve linear equations via open sentences*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-C1 Open Sentences: patterns in addition, subtraction, multiplication, and division
• generate rules about how a change in one variable affects the result (e.g., □ × 10 : as □ increases by 1 the product increases by 10)

5-C2 Multiplication Computation Patterns: how a change in either factor affects the computation
• rearrange factors to simplify computation (e.g., 28 × 250 is more difficult than 7 × 1000)
• understand that dividing one factor by an amount and multiplying the other by the same amount produces no change in the final result

5-C3 Equivalent Fractions: multiplicative relationship
• investigate the multiplicative relationship between the numerators and denominators of equivalent fractions
• explore equivalent fractions by subdividing equally (e.g., for $\frac{3}{4}$, subdivide each fourth into 3 equal parts to result in $\frac{9}{12}$)
• explore equivalent fractions by grouping equally the fractional parts that make up the whole (e.g., group 4 sixths in groups of 2 to result in 2 thirds)
• investigate the results when the numerators of equivalent fractions differ by a constant amount

5-C4 Area and Perimeter: changing rectangle dimensions
• use models to discover patterns (e.g., same perimeter → longer length → shorter width)
• conclude, through investigation, that rectangles of the same area can have different perimeters
• connect models to symbols: if one dimension is multiplied by a factor, the other must be divided by that factor (e.g., 24 × 5 = 12 × 10)

5-C5 Place Value Patterns: base ten system to millions
• recognize the patterns in periods

5-C6 SI Measurement: pattern in changing units
• understand that a smaller measurement unit increases the number of those units and that a larger measurement unit decreases the number of those units
• apply the above relationship to reason through conversions

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STRAND D: MEASUREMENT

KSO Measurement *By the end of **Class 6** students should*

♦ *understand relationships among common SI units and choose appropriate units to solve measurement problems in given situations*

♦ *move freely among common SI units to communicate measurement ideas effectively, appropriate to a given measurement situation*

♦ *estimate effectively using a variety of strategies to solve measurement problems and understand when estimation is close enough*

♦ *use relationships and reasoning to develop and apply procedures for measuring in real situations and mathematical contexts*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-D1 Perimeter: polygons
- understand perimeter as the total distance around a figure
- develop generalizations for the perimeter of regular polygons (e.g., for equilateral triangles, the perimeter is 3 times the side length, square is 4 times)

5-D2 Perimeter and Area: rectangles and squares
- develop from concrete to symbolic
- develop formulas meaningfully
- understand that all squares with the same perimeter have the same area and vice versa
- understand that rectangles with the same perimeter can have different areas
- understand that rectangles with the same area can have different perimeters

5-D3 Area: composite shapes, estimate and measure
- use grids to measure the area of composite shapes (include squares and half squares)
- break up shapes into rectangles to area

5-D4 Angles: estimate and measure
- explore angle measurement in non-standard units (as wedges)
- understand that using a smaller wedge (unit) means using more wedges
- link wedges to degrees (degree is just a very small wedge)
- create and use an improvised protractor for 45, 90, 135, 180 degrees

5-D5 Angles: estimate size
- estimate angles relative to common referents: 45, 90, 180 degrees (about the same as, more than, less than)

5-D6 Volume and Capacity: solve simple problems
- understand volume as the amount of space an object occupies or how much it takes to build it
- develop a sense of size and referents for a cubic centimetre, cubic millimetre, cubic metre
- understand capacity as how much a container is capable of holding
- discover, through investigation, that 1 cm^3 holds 1 mL, and 1 dm^3 holds 1 L

5-D7 SI Units: reinforce relationships among various SI units
- apply relationships among kilometres/ hectometres/ decametres/ metres/ decimetres/ centimetres/ millimetres, litres/ millilitres, and kilograms/ grams
- use relationships to rename measures
- apply referents for various measurement standards (e.g., 30 cm is like a ruler, 1 dm is about a small hand span)

STRAND E: GEOMETRY

KSO Geometry *By the end of **Class 6** students should*

♦ *identify, draw, compare, and build physical models of 2-D and 3-D shapes to focus on their attributes and understand how they affect everyday life*

♦ *predict and verify results of transforming, combining, and subdividing shapes to understand other shapes and explain other geometrical ideas*

♦ *use geometric relationships and spatial reasoning to solve problems and understand everyday events and objects, as well as higher geometrical ideas*

♦ *appreciate the importance of geometry in understanding mathematical ideas and the world around*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-E1 Triangles: explore equilateral, isosceles, scalene triangles, and right, obtuse, and acute triangles
- discover properties of right, obtuse, and acute triangles
- sort and classify triangles by angle size and side lengths
- develop a personal referent for 90° (right) angles

5-E2 Combine Triangles: spatial sense and visualization
- use visualization to predict the results of combining triangles
- develop spatial sense by combining
  - two congruent equilateral triangles
  - two congruent isosceles triangles
  - two congruent isosceles right triangles
  - two congruent right triangles
  - two congruent acute triangles
  - two congruent obtuse triangles
  - two different isosceles triangles with congruent bases

5-E3 Diagonal Properties: squares and other rectangles
- develop generalizations for diagonals of squares and rectangles of each type below:
  - bisect each other (squares and rectangles)
  - intersect to form four right angles and four right isosceles triangles (squares)
  - intersect to form two pairs of congruent isosceles triangles (rectangles)
  - intersect to form two pairs of equal opposite angles (rectangles)
  - form two congruent angles with a sum of 90° at each vertex (squares)
  - form two non-congruent angles with a sum of 90° at each vertex (non-square rectangles)

5-E4 Parallelism and Perpendicularity: lines and line segments
- construct the following pairs of lines/line segments and use appropriate mathematical terminology:
  - parallel
  - intersecting
  - perpendicular at an end point
  - bisecting another line segment but not perpendicular
  - bisecting each other and perpendicular

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5-E5 Translations and Reflections using horizontal and vertical reflection lines: generalize and apply properties

• understand that the translation image of a shape is congruent to the original shape and is oriented the same way
• understand that corresponding sides of the original shape and the translated image are always parallel
• understand that parallel sides of the original shape are always parallel in the translation image
• understand that the reflection image of a shape is congruent to the original shape but faces the opposite way
• understand that corresponding points of a shape and its reflected image are equidistant from the reflection line
• understand that the line segment joining a point to its reflected image is perpendicular to the line of reflection
• understand that a refection line bisects all line segments joining corresponding points at right angles
• understand that corresponding sides of the original shape and the reflection image are not always parallel

5-E6 Rotations: quarter, half, and three-quarter rotations about the vertex of a shape

• predict, apply, and identify quarter ($\frac{1}{4}$), half ($\frac{1}{2}$), and three-quarter ($\frac{3}{4}$) rotations
• explore the results using a variety of turn centres
• understand that each point remains the same distance from the turn centre and the turn centre does not move

5-E7 Nets: prisms and pyramids

• create and interpret nets for various prisms and pyramids

5-E8 Isometric Drawings

• make and interpret drawings of structures made from cubes

STRAND F: DATA MANAGEMENT

KSO Data Management *By the end of* ***Class 6*** *students should*

♦ *collect, record, organize, and describe data in multiple ways to draw conclusions about everyday issues*

♦ *construct a variety of data displays and choose the most appropriate*

♦ *predict, read, interpret, and modify predictions for a variety of data displays, including interpolation and extrapolation (draw conclusions about things not specifically represented by the data)*

♦ *develop and apply measures of central tendency to data reflecting relevant situations, in order to draw conclusions and make decisions*

♦ *design and implement strategies for the collection of data*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-F1 Mean: effect of change in data

• understand the mean as a balance through concrete materials and pictorial representations
• understand that the mean of a set of data increases if any piece of data increases
• understand that the mean of a set of data decreases if any piece of data decreases
• understand that the mean increases if a piece of data below the mean is removed
• understand that the mean decreases if a piece of data above the mean is removed

5-F2 Collect, Organize, and Describe Data

• choose an appropriate display for data
• interpret displays/presentations of data to draw conclusions about real world issues

5-F3 Double Bar Graphs: create and interpret

• interpret displays/presentations of data to draw conclusions about real world issues
• construct and interpret simultaneous displays for two sets of data from the same population (e.g., data collected at different times)

5-F4 Coordinate Graphs: create and interpret

• use coordinate graphs for purposes of location
• create coordinate graphs using appropriate labels and scales

STRAND G: PROBABILITY

KSO Probability *By the end of* ***Class 6*** *students should*

♦ *explore, interpret, and make predictions for everyday events by estimating and conducting experiments*

♦ *understand the difference between theoretical and experimental probability and when each is relevant*

♦ *begin to conduct simulations to understand real-life probability situations*

♦ *understand the relationship between the numerical representations of probability and the events they represent*

Toward this, students in **Class 5** will be expected to master the following **SO** (Specific Outcomes):

5-G1 Experiments
• conduct simple experiments with coins, slips of paper, and dice to determine experimental probability
• use common language to describe probability (e.g., for a probability of 15/20, I picked red "15 out of 20 times")
• record results in charts
• predict and record experimental results as fractions and decimals
• understand that theoretical probability is the number of favourable outcomes divided by the number of possible outcomes
• understand that experimental probability is the number of times the favourable outcome occurs divided by the number of trials in the experiment

5-G2 Describe Probability
• understand that experimental probability is determined by performing experiments
• understand that theoretical probability is what you would expect to happen after considering the possible outcomes
• use fractions and decimals to describe theoretical probability and experimental probability

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UNIT 1 WHOLE NUMBER COMPUTATION

UNIT 1 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 1** **TG p. 5**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• Base ten blocks or Base Ten Models (BLM) • Dice	All questions
Chapter 1 Multiplication				
1.1.1 Multiplying Multiples of Ten **SB p. 2** **TG p. 8**	**5-B2 2-Digit × 2-Digit Multiplication: with and without regrouping** • relate models or diagrams to algorithms • develop personal and standard algorithms • continue to estimate to check **5-C1 Open Sentences: patterns in addition, subtraction, multiplication, and division** • generate rules about how a change in one variable affects the result (e.g., □ × 10 : as □ increases by 1 the product increases by 10) **5-C2 Multiplication Computation Patterns: how a change in either factor affects the computation**	1 h	• Base ten hundred blocks or Base Ten Models (BLM) or twenty 10 cm-by-10 cm squares	Q2, 3, 7
1.1.2 Estimating Products **SB p. 4** **TG p. 11**	**5-B1 Estimate Products: 2 digits × 2 digits** • use a variety of strategies to estimate products	1 h	None	Q4, 5, 6
1.1.3 Multiplying 2-digit Numbers by 3-Digit Numbers **SB p. 7** **TG p. 14**	**5-B2 2-Digit × 2-Digit and 2-Digit × 3-Digit Multiplication: with and without regrouping** • relate models or diagrams to algorithms • develop personal and standard algorithms • continue to estimate to check	2 h	• Base ten blocks or Base Ten Models (BLM)	Q1, 3, 9, 11
1.1.4 Multiplying 4-digit Numbers by 1-digit Numbers **SB p. 12** **TG p. 18**	**5-B3 4-digit × 1-digit Multiplication: with and without grouping** • extend 3-digit × 1-digit multiplication using similar strategies • develop personal and standard algorithms • continue to estimate to check	1 h	• Base ten blocks or Base Ten Models (BLM) • Place Value Charts I (BLM)	Q 1, 4, 11
1.1.5 EXPLORE: Mental Multiplication (Essential) **SB p. 15** **TG p. 23**	**5-B4 Multiply Mentally: to 4 digits × 1 digit** • understand the difference between estimation and mental math • understand that estimation strategies can often be used to calculate mentally • develop efficiency with multiplying mentally by 10, 100, 1000 • apply associative principle (e.g., 25 × 30 = 25 × 3 × 10 = 750) • apply double/half strategy (e.g., 50 × 16 = 100 × 8) • apply front-end strategy (e.g., 3 × 325 = 900 + 60 + 15 = 975) • choose appropriate strategy (depending on numbers being calculated)	1 h	None	Observe and Assess questions

UNIT 1 PLANNING CHART [Cont'd]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
[Cont'd] **1.1.5** EXPLORE: Mental Multiplication (Essential) **SB p. 15** **TG p. 23**	**5-C2 Multiplication Computation Patterns: how a change in either factor affects the computation** • rearrange factors to simplify computation (e.g., 28 × 250 is more difficult than 7 × 1000) • understand that dividing one factor by an amount and multiplying the other by the same amount produces no change in the final result			
GAME: Greatest Product (Optional) **SB p. 16** **TG p. 25**	Practise 4-digit by 1-digit multiplication in a game situation	25 min	• Dice	N/A
CONNECTIONS: Egyptian Multiplication (Optional) **SB p. 16** **TG p. 25**	Make a connection between current and historical approaches to multiplication	15 min	None	N/A
Chapter 2 Division				
1.2.1 Estimating Quotients **SB p. 17** **TG p. 26**	**5-B5 4-Digit ÷ 1-Digit: with and without regrouping** • continue estimating to check **5-B12 Open Number Sentences: applying number sense** • explore numerical situations which are always, sometimes, or never true (e.g., 324 + ■ > 300 is always true, assuming ■ is a whole number) • work with open number sentences involving the four basic operations and a combination of operations	1 h	None	Q1, 4, 7
1.2.2 Dividing 4-digit Numbers by 1-digit Numbers **SB p. 20** **TG p. 29**	**5-B5 4-Digit ÷ 1-Digit : with and without regrouping** • focus on the whole number (rather than the digits) • link concrete models to algorithms • continue estimating as a first step	2 h	• Base ten blocks or Base Ten Models (BLM)	Q1, 4, 5
GAME: Target 2000 (Optional) **SB p. 24** **TG p. 32**	Practise dividing 4-digit numbers by 1-digit numbers in a game situation	20 min	• Dice	N/A
1.2.3 EXPLORE: Mental Division (Essential) **SB p. 25** **TG p. 33**	**5-B7 Divide Mentally** • use prior knowledge of basic facts • divide by 10, 100, 1000 • link to place value	40 min	None	Observe and Assess Questions

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	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
1.2.4 Dividing 4-digit Numbers by Multiples of Ten **SB p. 26** **TG p. 36**	**5-B6 4-Digit ÷ 2-Digit: introduce** • explore divisors which are multiples of 10 only (10, 20, 30, …)	1.5 h	• 40 small items such as pencils or erasers	Q1, 3, 7
UNIT 1 Revision **SB p. 29** **TG p. 39**	Review the concepts and skills in the unit	2 h	• Base ten blocks or Base Ten Models (BLM)	All questions
UNIT 1 Test **TG p. 42**	Assess the concepts and skills in the unit	1 h	• Base ten blocks or Base Ten Models (BLM)	All questions
UNIT 1 Performance Task **TG p. 45**	Assess concepts and skills in the unit	1 h	• Base ten blocks or Base Ten Models (BLM) (optional)	Rubric provided
UNIT 1 Blackline Masters **TG p. 48**	BLM 1 Place Value Charts I (Ten thousands to Ones) BLM 2A Base Ten Models (Hundreds, Tens, and Ones) BLM 2B Base Ten Models (Thousands)			

Math Background

• This number unit is a way to gently move students into Class V. Some familiar content is extended and there are many new ideas.

• The focus of the unit is on being able to multiply and divide whole numbers, but also to understand why we use the approaches we do. Students learn why those approaches make sense.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 10** in **lesson 1.1.3**, where they have to figure out which two consecutive numbers result in a certain product, in **question 3** in **lesson 1.1.4** and **question 7** in **lesson 1.2.2**, where they figure out which digits are missing, and in **question 9** in **lesson 1.1.4**, where they look for numbers to meet a certain condition.

• Students use communication frequently as they explain their thinking in answering questions, for example, in **question 5b** in **lesson 1.1.2**, where they compare estimating choices, and in **question 6** in **lesson 1.2.4**, where they consider how two calculations are related. The last question in most lessons usually requires an element of communication in bringing closure to the lesson.

• Students use reasoning in answering questions such as **question 1** in **lesson 1.1.2** and **question 5** in **lesson 1.2.1**, where they use estimation to determine whether a calculation is reasonable, and in **question 8** in **lesson 1.2.2**, where they figure out which remainders are possible in a given situation.

• Students consider representation in **lesson 1.1.1** and in **lesson 1.1.3**, where they represent a product as the area of a rectangle, and in **lesson 1.2.3**, where they rename a number using an alternate place value representation to make mental division easier.

• Students use visualization skills in **lesson 1.1.1**, where they figure out a product of two multiples of ten by building a rectangle made up of hundred blocks (or paper models), and in **lesson 1.1.3**, where they relate the product of two-digit numbers to a rectangle made up of four parts.

• Students make connections in situations like those in **question 8** in **lesson 1.1.3**, where they link computations to probability ideas, and in **question 3** in **lesson 1.2.1**, where they link calculation skills with algebra skills. They also make connections in the mental math lessons, **lesson 1.1.5** and **lesson 1.2.3**, where more difficult calculations are related to simpler calculations. There are also many real world connections, for example, **question 4** in **lesson 1.2.4**.

Rationale for Teaching Approach

• This unit is divided into two chapters. **Chapter 1** focuses on multiplication. **Chapter 2** focuses on division. Because of the important connection between the two operations, multiplication is revisited during the division chapter.

• There are two **Explore** lessons focusing on mental calculation. There are no “rules” for how to calculate mentally, so it makes sense to approach these ideas as explorations.

• The **Connections** section provides students with an interesting alternate way to multiply; it strengthens their understanding that there are always many ways to perform a calculation. It also provides a historical link.

• The two **Games** provide opportunities to apply and practise multiplication and division in a pleasant way.

• Throughout the unit, it is important to encourage flexibility in computation and to accept a variety of approaches from students. You should invite students to be efficient and not record every step. Do not discourage this.

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Getting Started

Curriculum Outcomes	Outcome relevance
4 Multiplication Meanings **4** Multiplication Properties **4** Multiplication Facts **4** 3-digit × 1-digit Multiplication: with and without regrouping **4** Division Meanings **4** Multiplication and division facts: relate through properties **4** 2- and 3-Digit ÷ 1-Digit: with and without regrouping **4** Multiply Mentally: by 10 or 100	Students will find the work in the unit easier after they review the concepts of multiplication and division they encountered in Class IV.

Pacing	Materials	Prerequisites
1 h	• Base ten blocks or Base Ten Models A and B (BLM) • Dice	• multiplication and division facts • multiplication and division of 3-digit numbers by 1-digit numbers • familiarity with the terms *factor, product, multiple, quotient, dividend,* and *divisor*

Main Points to be Raised

Use What You Know

• You can multiply a 3-digit number by a 1-digit number by multiplying each part of the 3-digit number (hundreds, tens, and ones) by the 1-digit number and adding the products together.

• You must regroup 10 ones as 1 ten, 10 tens as 1 hundred, and 10 hundreds as 1 thousand to represent a number appropriately.

Skills You Will Need

• You should use mental calculation for multiplication or division facts.

• You can divide a 3-digit number by a 1-digit number by finding how many groups of the 1-digit number you can create out of the 3-digit number.

• The three parts of a multiplication equation are called the factors (there are two) and the product. The product is a multiple of each factor.

For example, for 2 × 7 = 14, 2 and 7 are the factors and 14 is the product. 14 is a multiple of 2 and 7.

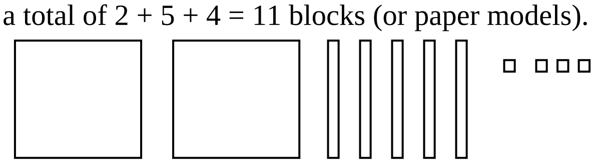
• The three parts of a division equation are called the dividend, the divisor, and the quotient.

For example, for 14 ÷ 2 = 7, 14 is the dividend, 2 is the divisor, and 7 is the quotient.

Use What You Know — Introducing the Unit

• Before assigning the activity, you might review the representation of numbers as hundreds, tens, and ones.

For example, you might show 254 represented as 2 hundreds, 5 tens, and 4 ones.
This uses a total of 2 + 5 + 4 = 11 blocks (or paper models).



• Model for your students how to play the game, i.e., how to choose a number made up of 7, 8, or 9 blocks (or paper models), how to choose a factor by rolling a die, and how to count the number of blocks needed to represent the product.

For example, if 314 (which uses 8 blocks) is multiplied by 4 (the roll of your die), the product is 1256.
You need 1 + 2 + 5 + 6 = 14 blocks to represent this number.

• Students can work in pairs to complete the activity.

Observe students as they work. As they work, you might ask questions such as the following:

• *How do you know it takes 7 blocks (or paper models) to represent 115?* (I need 1 hundred block, 1 ten block, and 5 one blocks and 1 + 1 + 5 = 7.)

• *Did you estimate when you multiplied by 4 to make sure your answer is reasonable?* (I know that 4 × 115 is more than 4 × 100, but not a lot more, so my answer of 460 made sense.)

• *Show me how you multiplied 224 by 6. Did you use pencil and paper or did you use mental math?* (I multiplied 200 by 6 to get 1200, 20 by 6 to get 120, and 4 by 6 to get 24. Then I added them using pencil and paper.)

• *Sonam represented her product using just 5 blocks (or paper models). Could you end up with fewer blocks when you multiply your roll by 162?* (If I multiply by 1, I would need 9 blocks. If I multiply by 2, I would need 4 blocks just for the ones. If I multiply by 3, I would need 6 blocks just for the ones. If I multiply by 4, I would need 8 blocks just for the ones. If I multiply by 5 or 6, it would be over 600, so I would need too many blocks.)

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions.

• You may have to review the terms in **question 5** for some students. You might suggest students refer to the glossaries at the back of the book.

• Students can work individually.

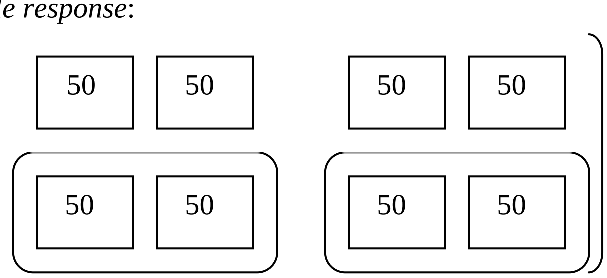
Answers

Sample play:
Choose these 9 blocks (or paper models): 3 hundreds, 5 tens, 1 one
Roll 3
3 × 351 = 1053, uses 9 blocks, so get 9 points.

1. a) 56 **b)** 36 **c)** 40 **d)** 49
e) 6 **f)** 8 **g)** 9 **h)** 7

2. a) 1012 **b)** 3890 **c)** 4872 **d)** 6093
e) 61 **f)** 64 **g)** 130 R 5 **h)** 126 R 1

[**3.** *Sample response*:
4 × 50
2 × 100



4. a) 18 students **b)** 432 flags

5. *Sample responses*:
a), b), and c)
4 × 5 = 20
the factors are 4 and 5;
the product is 20;
20 is also a multiple of 4 and of 5
d), e), and f)
20 ÷ 5 = 4
the quotient is 4;
the dividend is 20;
the divisor is 5

Supporting Students

Struggling students

• If students are struggling with choosing numbers to represent with 7, 8, or 9 blocks (or paper models), make it clear that they simply choose any 7, 8, or 9 blocks out of a pile of hundred, ten, and one blocks. To name the number, they begin by counting how many hundreds they have, then how many tens, and then how many ones.

• Some students may have trouble with the multiplication. You may have to pull aside a small group of students and re-teach how to multiply the hundreds, tens, and ones and then add the parts together. Others may have trouble with division. You may wish to re-teach the sharing algorithm using base ten blocks (or paper models).

For example, for 366 ÷ 6, you share 3 hundred blocks, 6 ten blocks, and 6 one blocks in 6 equal piles. Since the 3 hundreds cannot be shared equally by 6, they must be traded for 30 tens before they can be shared.

• Encourage students who struggle with **question 3** to think of 4 × 50 as 4 groups of 50. Point out that they must show it can be rearranged as 2 groups of 100.

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Enrichment

• You might encourage students to create and play their own game involving multiplication or division of a 3-digit number by a 1-digit number.

For example, a game might require you to choose 4 blocks (or paper models) and multiply by the amount rolled on a die. The person closest to a certain value, for example, 1000, wins a point.

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Chapter 1 Multiplication

1.1.1 Multiplying Multiples of Ten

Curriculum Outcomes	Outcome relevance
5-B2 2-Digit × 2-Digit Multiplication: with and without regrouping • relate models or diagrams to algorithms **5-C1 Open Sentences: patterns in addition, subtraction, multiplication, and division** • generate rules about how a change in one variable affects the result (e.g., □ × 10 : as □ increases by 1 the product increases by 10) **5-C2 Multiplication Computation Patterns: how a change in either factor affects the computation**	By seeing the relationship between multiplying 1-digit numbers and multiplying multiples of ten, for example, 20 × 30 and 2 × 3, students will find it easier to multiply 2-digit by 2-digit numbers later.

Pacing	Materials	Prerequisites
1 h	• Base ten hundred blocks or Base Ten Models A (BLM) or twenty 10 cm-by-10 cm squares	• multiplication facts • knowing that 10 × 10 = 100 • relating multiplication to calculating the area of a rectangle • writing a number in the form xx00 as xx hundreds, e.g., 2300 = 23 hundreds

Main Points to be Raised

• To multiply two numbers, you can create a rectangle where one number is the length and the other number is the width. The product is the area of the rectangle.

• To multiply two multiples of ten, you can arrange 10-by-10 squares into a rectangle. The number of squares is the product of the two multiples of ten.

For example, you can model 20 × 30 using 6 squares because 2 tens × 3 tens = 6 hundreds.

• You should use mental math to multiply multiples of ten that are less than 100.

• Multiplications are related.

For example, 20 × 3 is 10 times as much as 2 × 3, and 20 × 30 is 100 times as much as 2 × 3.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *Why might you use 30 as the number of flaps each second?* (30 is between 20 and 40.)

• *What other numbers besides 30 might you have used?* (Any value between 20 and 40.)

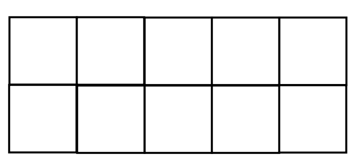
• *How do you know that the dragonfly flaps more than 100 times a minute?* (If it were 20 times each second, it would flap 100 times in 5 s, and 1 min is a lot more than that.)

• *Why might you add 30 ten times to calculate the number of flaps in 10 s?* (If there were 10 s and 30 flaps each second, you would add 30 for each of those seconds.)

The Exposition — Presenting the Main Ideas

• On the board, draw a 4-by-5 rectangle. Show how it can be divided into 4 rows of 5 columns and that the area is 4 × 5, the product of the two dimensions. Ask students to confirm this with another pair of dimensions, for example, a 3-by-7 rectangle.

• Put together ten of your 10 cm-by-10 cm squares into an array like this, so students can see it.



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Ask why each small square has an area of 100 (10 by 10) and what the dimensions of the rectangle are (50 by 20). Ask why the total area is 5 × 2 = 10 hundreds. Ask how to write that amount (10 hundreds = 1000).

• Have students draw a sketch to show a similar rectangle made up of 3 columns and 3 rows. Ask them to calculate the area (3 × 3 hundreds = 9 hundreds = 900).

• Talk with the students about why the answer had to be of the form []00 or [][]00 since there is an exact number of hundreds.

• Go through the exposition with students.

- Some students may notice that 3 tens × 2 tens can be thought of as (3 × 2) × (ten × ten).

- Draw attention to the relationship between multiplication facts and calculating products of multiples of ten. In other words, if you know a multiplication fact like 4 × 7, you also know 40 × 70.

Revisiting the Try This

B. This question allows an opportunity to make a formal connection between what was done in **part A** and the main ideas presented in the exposition. Make sure students recognize that the number of flaps could have been anywhere between 20 and 40, so they might use 20, 30, or 40 as one of the dimensions. Some students might even want to use a number like 21 as one of the dimensions. Although this has not been taught, it is all right for students to come up with their own strategies.

Using the Examples

Work through the example with the students to make sure they understand it. Point out that if they were answering the question, they would be expected to write down the work, much like what they see on the left (under **Solution**), but they might be thinking what they read on the right (under **Thinking**).

Practising and Applying

Teaching points and tips

Q 1: Encourage students to skip count by tens to determine each side length.

For example, for **part a)**, they could count 10, 20, 30 to find the side length of either dimension.

Q 2: Make sure students realize that they can sketch the rectangles and they need not draw to scale. Encourage them to think in groups of ten as they did in **question 1**.

Q 3: Observe whether students are doing these questions mentally or drawing diagrams. Either is acceptable at this point.

Q 4: Students should recognize that they only need to compare the product of the factors to 40.

For example, for **part A**, they could compare 6 × 8 to 40. If you wish, you can create extra practice questions like **question 3**, using other multiples of ten.

Q 5: Some students might want to use a multiplication table to see where they observe the products 49 and 36.

Q 6: Students can compare by commenting on the digits that make up the products; they do not have to subtract to compare.

Common errors

• Some students are likely to use only one zero rather than two zeros to show the product of multiples of ten.

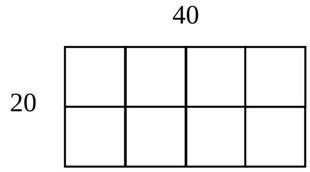
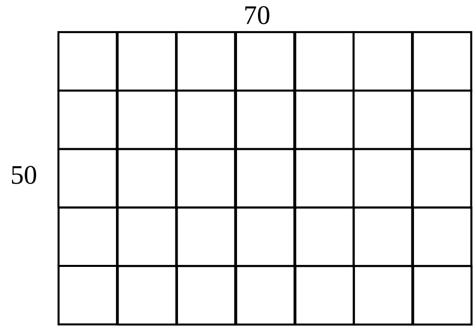
For example, students might write 20 × 30 = 60; because they only see one zero in the factors, they write only one zero in the product.

Encourage students to compare it to another calculation. For 20 × 30, ask them to calculate 2 × 30. Then ask what each of 2 × 30 and 20 × 30 means and whether they can have the same product.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can use a diagram to calculate the product of multiples of ten
Question 3	to see if students can calculate the product of multiples of ten
Question 7	to see if students can communicate about the relationship between the product of two multiples of ten and the product of the factors

Answers

A. Between 1200 and 2400 times.	**B.** 20 by 60 and 40 by 60
1. a) 30 × 30 = 900 **b)** 50 × 40 = 2000 **2. a)** 800  **b)** 3500  **3. a)** 3000 **b)** 1600 **c)** 3200 **d)** 5400	**4.** A **5.** *Sample response*: **a)** 70 × 70 **b)** 40 × 90 **6. a)** 3 × 20 = 60 and 30 × 20 is 600; 30 **×** 20 = 600 is 10 times 3 × 20 = 60 [*Sample response*: When you use 3 tens instead of 3 ones, the product is 10 times as much.] **b)** 4 × 60 = 240, but 40 × 60 is 2400; 40 **×** 60 = 2400 is 10 times 4 × 60 = 240 [*Sample response*: When you use 4 tens instead of 4 ones, the product is 10 times as much.] [**7.** *Sample response*: 70 × 30 = 7 tens × 3 tens, so you can multiply 7 × 3 and then change your answer to hundreds.]

Supporting Students

Struggling students

• Some students might benefit from using 10 cm-by-10 cm squares to create many rectangles and, for each, to write the products that are being represented. They might use base ten hundred blocks (or paper models) if they are available.

• Some students might need to review their multiplication facts to successfully complete this lesson.

Enrichment

• You might extend **question 5** to ask students to come up with as many factors as possible that result in a particular product that is a certain number of hundreds.

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1.1.2 Estimating Products

Curriculum Outcomes	Outcome relevance
5-B1 Estimate Products: 2 digits × 2 digits • use a variety of strategies to estimate products	In everyday life, estimation is often a more important skill than calculation. It is very important to develop students' estimation skills.

Pacing	Materials	Prerequisites
1 h	None	• multiplication facts • multiplication of multiples of 10

Main Points to be Raised

• Estimation is appropriate to answer some questions where exact calculations are not required.

• It is often easy to estimate using the nearest multiple of ten, but sometimes you might choose other numbers that are easy to work with. Always choose numbers that you can use mental math to calculate.

• It is useful to consider whether an estimate is high or low so that you will have a better idea of the exact answer.

• There is no one way of estimating that is the most correct.

• You can use estimation to check a calculation.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *Why might you use 20 and 40 instead of 19 and 42?* (They are close, but easier to calculate with mentally.)

• *Is it easy to tell if your estimate is too high or too low?* (Not really, since I increased one number (19 to 20), but I decreased the other (42 to 40).)

The Exposition — Presenting the Main Ideas

• Ask several students to measure the distance they can cover with 8 average-sized steps. Then ask what distance they could cover in 12 steps. Discuss why it might be appropriate to use an estimate rather than an exact calculation, since the distance itself was an estimate — maybe the next time they take 8 steps, they will cover a slightly different distance.

• Remind students that many problems in our everyday lives can be solved appropriately by using an estimate.

For example, you might use an estimate to decide how long it takes to walk from one place to another.

• Present the question 28 × 46 and ask students whether they think the product is closer to 100, 1000, or 10,000. Ask how they might decide without calculating exactly. Then invite students to suggest ways they might estimate the product even more closely, but still without calculating, using only mental math. Once students have had an opportunity to make some suggestions, point out the estimate in the exposition on **page 4**.

• Discuss why you might want to know whether an estimate is too high or too low, so you could adjust it to be closer to the exact answer.

Revisiting the Try This

B. Make sure students recognize that when both numbers are increased or decreased, it is very easy to determine whether an estimate is too high or too low. It is much less obvious when one is increased and the other is decreased, unless the change is much greater for one than for the other.

Using the Examples

Place students in pairs. Have one student in each pair become the "expert" on **example 1** and the other the "expert" on **example 2**. Each should then explain his or her example to the other student.

Practising and Applying

Teaching points and tips

Q 1: When students estimate with a number that has a 5 in the ones place, they can choose whether to round up or down. If they round in the opposite direction to what was done with the other number, the estimate will be closer to the exact value.

For example, you might estimate 28 × 35 as 30 × 30 (since you rounded 28 up, you might round 35 down), but you might estimate 22 × 35 as 20 × 40 (since you rounded 22 down, you might round 35 up).

Q 2: Students are only required to indicate which answer is incorrect. You may wish to have them speak about how they know the other two answers are reasonable. Encourage students to estimate regularly to check their calculations.

Q 3: Make sure students understand that these pictures are not drawn to scale, so they should not use the diagrams to help them relate the estimates in one part to the next part.

Q 5 and Q 8: These questions are designed to encourage students to recognize that there are many ways to estimate appropriately. No estimate is wrong, but some may be closer to the actual product than others.

Q 6: This question is more challenging than most on the page. Students will need not only to estimate the number of sticks required, but also how many 120s make up that number. Many students benefit by first estimating the number of boxes if there were 100 sticks in a box. Then help them see that for every five boxes of 120 sticks, they could remove of the boxes they thought they would need if there were only 100 sticks in a box.

Q 7: This question is designed to encourage students to create problems as well as to solve them.

Common errors

• Some students are uncomfortable with the idea that there are many correct ways to estimate. They may ask you to give them rules for rounding to estimate. It is not wise to give such rules because the best estimate to use in a given situation may depend on the details of that situation.

Suggested assessment questions from Practising and Applying

Question 4	to see if students can choose appropriate estimating values
Question 5	to see if students can use reasoning to select an estimate
Question 6	to see if students can solve a problem requiring the use of estimates

Answers

A. *Sample response*: About 20 × 40 = 800 students

B. i) *Sample response*:
20 and 40 because 19 is close to 20 and 42 is close to 40, but 20 × 40 is an easier mental calculation
ii) No, since I estimated one number by going up and the other number by going down.

1. *Sample responses*:
a) about 1500 [30 × 50] **b)** about 1500 [30 × 50]
c) about 1800 [60 × 30] **d)** about 5400 [90 × 60]

2. C is incorrect [since 30 × 50 is about 1500, not 150.]
The others seem reasonable:
[A. 37 × 48 is about 40 × 50 = 2000, which is close to 1776.
B. 69 × 63 is about 70 × 60 = 4200, which is close to 4347.]

3. *Sample responses*:
a) about 600 **b)** about 1200 **c)** about 1000

4. *Sample responses*:
a) about Nu 800 **b)** about Nu 2400 **c)** about Nu 800

[**5.** *Sample responses*:
a) 25 is halfway between 20 and 30 so 20 and 30 are both good estimates for 25
65 is halfway between 60 and 70 so 60 and 70 are both good estimates for 65.]

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[**5. b)** If I use a high estimate for one number, it would be better to use a low estimate for the other number so my answer will be closer to the actual answer; 30 × 70 rounds both numbers up, so the estimate will be high.] **6.** About 26 boxes **7.** *Sample responses*: **a)** There are 39 students and each has 60 sheets of paper in a notebook. About how many sheets of paper are there altogether? **b)** A truck travels 78 km every day. About how far does it travel in a month? **c)** The bank has a roll of 58 Nu 50 notes. Estimate how much the roll is worth.	**8.** 49 × 71; [*Sample response*: • 49 × 71 is probably closer than 48 × 72, since 49 is closer than 48 to 50 and 71 is closer than 72 to 70. • 50 × 73 is probably farther away than 48 × 72 from 50 × 70, since even though the 50 is exact, 73 is quite far from 70; there would be 3 extra groups of 50.] [**9.** *Sample response*: Estimating helps you make sure you did not make a careless mistake when you calculated.]

Supporting Students

Struggling students

• Some students might have difficulty with **question 6**. After they estimate 36 × 83 as, for example, 40 × 80 = 3200, they might find it easier to divide by 100 than by 120. Help them see that this is still a reasonable estimate, but because there are 20 extra sticks in each box, they will not need as many boxes. For some students, you might encourage them to figure out how many fewer boxes are needed, but you may choose not to go this far with struggling students. (Note that each five extra 20s is 100, so an answer of 32 boxes contains about 6 extra boxes since 32 ÷ 5 is a bit more than 6.)

• Some students might have difficulty creating problems as in **question 7**. You may want to provide samples of problems that they can adapt.

1.1.3 Multiplying 2-digit Numbers by 3-digit Numbers

Curriculum Outcomes	Outcome relevance
5-B2 2-Digit × 2-Digit or 2-Digit × 3-Digit Multiplication: with and without regrouping • relate models or diagrams to algorithms • develop personal and standard algorithms • continue to estimate to check	The ability to multiply by 2-digit numbers is a life skill. It is important that students understand why the procedures work and that they not just apply rules without understanding.

Pacing	Materials	Prerequisites
2 h	• Base ten blocks or Base Ten Models A (BLM)	• multiplication by multiples of ten • understanding that multiplication applies when groups of equal size are put together

Main Points to be Raised

• To multiply two numbers, you can determine the area of a rectangle with the dimensions of those two numbers.

• To multiply two 2-digit numbers mentally, you can set up a large rectangle made up of four parts that represent the four partial products to add to get the total product.

• The four partial products when you multiply two 2-digit numbers involve hundreds, tens (from two different parts of the large rectangle), and ones.

• It is equally acceptable to multiply from the left as from the right.

• To multiply a 2-digit by a 3-digit number, you can multiply the number in parts and add the parts.

• When you record the product of a 2-digit by 3-digit number using regrouping numbers, it is important not to use the regrouping numbers from one calculation when you perform the other calculation.

Try This — Introducing the Lesson

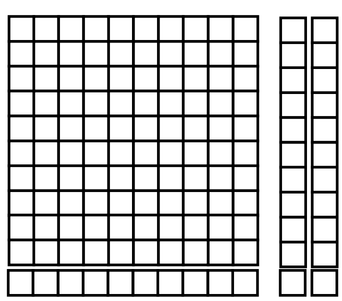
A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *Why might you use 20 and 40 instead of 22 and 42?* (They are close, but easier to calculate mentally.)

• *Is it easy to tell if your estimate is too high or too low?* (Yes; It will be low because I decreased both numbers.)

The Exposition — Presenting the Main Ideas

• Bring out base ten blocks (or base ten models made from paper). Arrange them as shown here:



Ask students what number you have represented (132). Then ask if they see 11 rows with 12 in each row.

Discuss why you could write 11 × 12 = 132 to describe the diagram. (There are 11 rows of 12 and that is what 11 × 12 means. The value is 132.)

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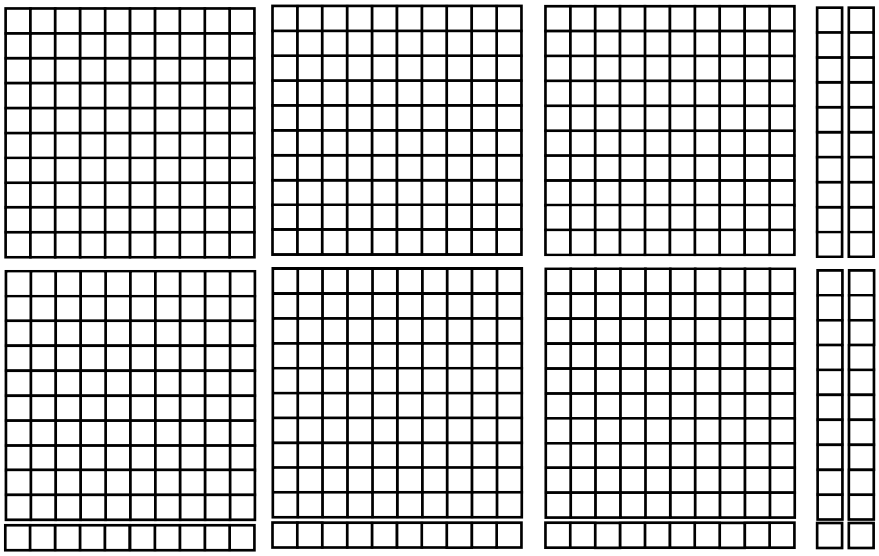
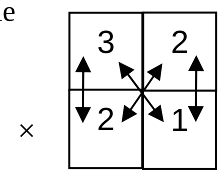
Point out that the 12 is made up of 10 + 2 and the 11 is made up of 10 + 1. So the whole rectangle is made up of four parts: 10 × 10, 10 × 2, 1 × 10, and 1 × 2. Then point out how you are multiplying each part of the 11 by each part of the 12 and putting the partial products together.

• Repeat the process with another calculation, such as 21 × 32:



Point out the four parts — the part showing the hundred blocks (20 × 30), the two parts representing the ten blocks (20 × 2 + 30 × 1) and the part representing the one blocks (2 × 1).

You may want to relate the four parts of the rectangle to these four products in the computation.



• Now present the question 124 × 35. Ask students how they might approach this question. Let them discuss some possibilities.

For example, they could add 100 × 35 to 24 × 35; both of these are calculations they already know how to do. Or they could add 124 × 30 to 124 × 5. For the first calculation, they multiply 124 × 3 and then multiply the answer by 10. All of these are calculations they already know how to do.

• Bring the exposition on **pages 7 and 8** to the students' attention:

- Make sure they understand that the sketches on the page match the base ten block blocks (or paper models) you have shown them. You may wish to model 23 × 34 with blocks so that students see the relationship directly.

- Have students discuss why multiplying the tens has to result in hundreds. Show how to record the areas as four partial products added together. Allow students to choose how to record their work.

For example, 32 × 21 could be shown in either of the ways described below:

32	32
× 21	× 21
600	2
40	30
30	40
+ 2	+ 600
672	672

• Draw attention to the 2-digit by 3-digit multiplication at the bottom right of **page 8**. Make sure students understand why the small 11 was crossed off (before the multiplication by 20 was performed).

Revisiting the Try This

B. Students can now use a more formal approach to record the product that they calculated informally in **part A.**

Using the Examples

Present the problems in the three examples to the students. Ask each student to choose two problems to solve. Then ask the students to compare their work to what is shown in the matching example. Suggest that they might want to read through the remaining example.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to sketch the diagram rather than to draw it exactly. Some students may benefit from continuing to use base ten blocks (or paper models).

Q 2: It is not an error that there is a missing calculation in one block. Students should be able to determine the missing values by using the rest of the diagram.

Q 3: Some students may benefit from using blocks (or paper models) for **parts a) to d)**, whereas others will need only to sketch. Although some students may wish to do the questions abstractly, it is a good idea to encourage them to continue to use a picture or a concrete model a bit longer.

Q 4: You may have to remind students that there are 31 days in March.

Q 5: Observe whether some students simplify the calculation by rearranging 16 rows of 25 squares into 4 rows of 100 squares and then adding the extra 2 rows of 25 squares.

Q 10: Students should use estimates to get them started on the solution to this question.

Q 11: Some students might benefit from actually rearranging small cards with the digits printed on them.

Common errors

• If the multiplication calculation is presented vertically, many students use only two of the four partial products. For example, for 35 × 48, they multiply 30 × 40 to get 1200 and 5 × 8 to get 40 and get an answer of 1240. Continue to encourage these students to use the blocks (or paper models) to see all four partial products.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can model a multiplication involving 2-digit numbers
Question 3	to see if students can calculate products of 2-digit numbers
Question 9	to see if students can solve a real-world problem involving the products of 2-digit numbers
Question 11	to see if students can use reasoning to solve a puzzle involving multiplication of 2-digit numbers

Answers

A. *Sample response*: About 800

B. i)

	40	+ 2
20	20 × 40 = 800	20 × 2 = 40
+ 2	2 × 40 = 80	2 × 2 = 4

Add the four parts: 800 + 80 + 40 + 4 = 924

B. ii)

$$\begin{array}{r} 42 \\ \times\ 22 \\ \hline 800 \\ 40 \\ 80 \\ +\ 4 \\ \hline 924 \end{array}$$

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1. a)

	40	+ 3
30	30 × 40 = 1200	30 × 3 = 90
+ 7	7 × 40 = 280	7 × 3 = 21

1200 + 280 + 90 + 21 = 1591

b)

	60	+ 2
30	30 × 60 = 1800	30 × 2 = 60
+ 9	9 × 60 = 540	9 × 2 = 18

1800 + 540 + 60 + 18 = 2418

c)

	50	+ 1
20	20 × 50 = 1000	20 × 1 = 20
+ 8	8 × 50 = 400	8 × 1 = 8

1000 + 400 + 20 + 8 = 1428

d)

	30	+ 4
30	30 × 30 = 900	30 × 4 = 120
+ 3	3 × 30 = 90	3 × 4 = 12

900 + 90 + 120 + 12 = 1122

2. a) 17 × 62 = 1054 **b)** 31 × 32 = 992

3. a) 2688 **b)** 705 **c)** 1344
d) 2706 **e)** 17,712 **f)** 6354

4. 31 × 45 = 1395 min

5. 2950 squares

6. 490 cm (or 4.9 m)

7. a) 10 × 11 = 110 **b)** 98 × 99 = 9702

8. A product greater than 5000; [*Sample response*: Since 3-digit numbers go from 100 to 990, and 200 × 38 is already over 5000, any number greater than 200 would also result in a product greater than 5000.]

9. a) Nu 675 **b)** Nu 1920

10. 44 and 45

11. 96 × 87 = 8352

12. *Sample response*:
A truck carries 368 kg of vegetables. Each kilogram sells for Nu 45. How much will the farmer get if she sells the entire load?

[**13.** *Sample response*:
To multiply 10 × 490, just put a 0 at the end of the 490; you do not have to add parts.
To multiply 12 × 49, you would add 10 × 49 to 2 × 49.
And, to multiply 32 × 46, you would add 30 × 40 + 2 × 40 + 30 × 6 + 2 × 6.]

Supporting Students

Struggling students

• Struggling students will benefit from continuing to use base ten blocks (or paper models) to model these calculations.

Enrichment

• Encourage students who might be interested to create other questions like **question 8**.

For example, if you multiply 71 by a 2-digit number, which is more likely, a product greater than 1000 or a product less than 1000?

1.1.4 Multiplying 4-digit Numbers by 1-digit Numbers

Curriculum Outcomes	Outcome relevance
5-B3 4-digit × 1-digit Multiplication: with and without grouping • extend 3-digit × 1-digit multiplication using similar strategies • develop personal and standard algorithms • continue to estimate to check	The ability to multiply 4-digit numbers is a life skill. The work on this outcome extends what students have already learned about working with 3-digit numbers.

Pacing	Materials	Prerequisites
1 h	• Base ten blocks or Base Ten Models A and B (BLM) • Place Value Charts I (BLM)	• representing numbers as thousands + hundreds + tens + ones • multiplication of multiples of 10, 100, and 1000 by a 1-digit number

Main Points to be Raised

• To multiply a multiple-digit number by a single-digit number, you can multiply each part of the multiple-digit number and put together the parts.

• When you represent a product, you must follow place value conventions.

For example, 12 hundreds must be renamed as 1 thousand + 2 hundreds.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *Why did you add 14,400 three times?* (I figured out the total for 2 months, and there are three sets of 2 months in six months.)

• *How could you know that the total is more than Nu 40,000?* (If it had been Nu 7000 for six months, the total would be Nu 42,000. Nu 7200 is more so the total must be higher.)

• *How could you know that the total is less than Nu 48,000?* (If it had been Nu 8000 for six months, the total would be Nu 48,000. Nu 7200 is less than Nu 8000.)

The Exposition — Presenting the Main Ideas

• Sketch a place value chart on the board.

• Represent a 4-digit number such as 2145 on the chart. Make sure students can interpret the value of the number.

Ten thousands	Thousands	Hundreds	Tens	Ones
	2	1	4	5

• Ask students why you might model twice 2145 by doubling each place value. Then, perform the doubling. Ask students why it is important to trade the 10 ones for 1 ten. (Otherwise, the answer might be read as 42,810.)

• Now multiply the number on the chart by a different 1-digit number, for example, 4. Again, show how you can multiply each place value by 4. Demonstrate again that it is important to take care of any required trading.

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4 × 2145

Ten thousands	Thousands	Hundreds	Tens	Ones
	4 × 2	4 × 1	4 × 4	4 × 5
	8	4	16	**20**
	8	4	**18**	0
	8	**5**	8	0

Notice that the 20 ones must be traded for 2 tens.
Then 18 tens must be traded for 1 hundred and 8 tens.

Show how the calculation can be recorded:

$$\begin{array}{r} 2145 \\ \underline{\times\quad 4} \\ 8000 \\ 400 \\ 160 \\ \underline{+\quad 20} \\ 8580 \end{array} \quad \text{or} \quad \begin{array}{r} 2145 \\ \underline{\times\quad 4} \\ 20 \\ 160 \\ 400 \\ \underline{+\ 8000} \\ 8580 \end{array}$$

Some students may wish to shortcut the recording as shown in **example 1, solution 2**.

$$\begin{array}{r} {\scriptstyle 12} \\ 2145 \\ \underline{\times\ 4} \\ 8580 \end{array}$$

You might also wish to model the same question using base ten blocks or paper models (as in **example 1**, **solution 1**).

• Students can use the information in the exposition for reference if they wish.

Revisiting the Try This

B. Encourage students to use a place value chart to answer the question.

Using the Examples

• Ask pairs of students to read through **solutions 1 and 2** of **example 1**. Ask them to choose which solution most closely matches what they would have done and to explain why.

Practising and Applying

Teaching points and tips

Q 1: Students can use place value charts, base ten blocks (or paper models), or sketch base ten blocks.

Q 3: You might want to model how a question like this is solved. Help students see where they might begin.

For example, in **question 3 c)**, they might think about what they might multiply 3 thousands by to get about 29 thousands. Make sure they know that, if a symbol is used twice, the same digit replaces it both times.
If different symbols are used, they represent different digits.

Q 5: Remind students that there are 60 s in 1 min and 60 min in 1 h.

Q 7: Encourage students to consider the place value chart shown in the exposition. The chart might look like this:

Thousands	Hundreds	Tens	Ones
__ × ?	__ × ?	__ × ?	__ × ?
6	12	18	30

[Continued]

Q 7 (cont'd): Each of the original number of thousands, hundreds, tens, and ones was multiplied by the same amount to get 6, 12, 18, and 30. They see that the multiplier has to be a factor of all four numbers, and so must be 3 or 6. It cannot be 3 since there could not have been 10 ones in the original number.

Q 9: Encourage students to think about which multiplication facts result in product with a ones digit of 4, for example, 7 × 2 = 14 or 8 × 8 = 64. Remind students that sometimes there is regrouping, so a fact with a product with a ones digit of, for example, 3 might still be possible.

Q 10: Encourage students to consider extreme cases, that is, the lowest and highest possibilities.

Common errors

• Some students who use place value charts may begin their trading from the left rather than from the right. This will not present a problem as long as students go back for a second look through.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can multiply a 4-digit number by a 1-digit number
Question 4	to see if students can solve a real-world problem involving multiplication
Question 11	to see if students can communicate about the multiplication process

Answers

A. Nu 43,200 **B.** Nu 43,350

1. 14,808; *Sample response*:

a)

Ten thousands	Thousands	Hundreds	Tens	Ones
	12	28	0	8

Ten thousands	Thousands	Hundreds	Tens	Ones
1	4	8	0	8

Product is 14,808.

b) 15,595; *Sample response*:

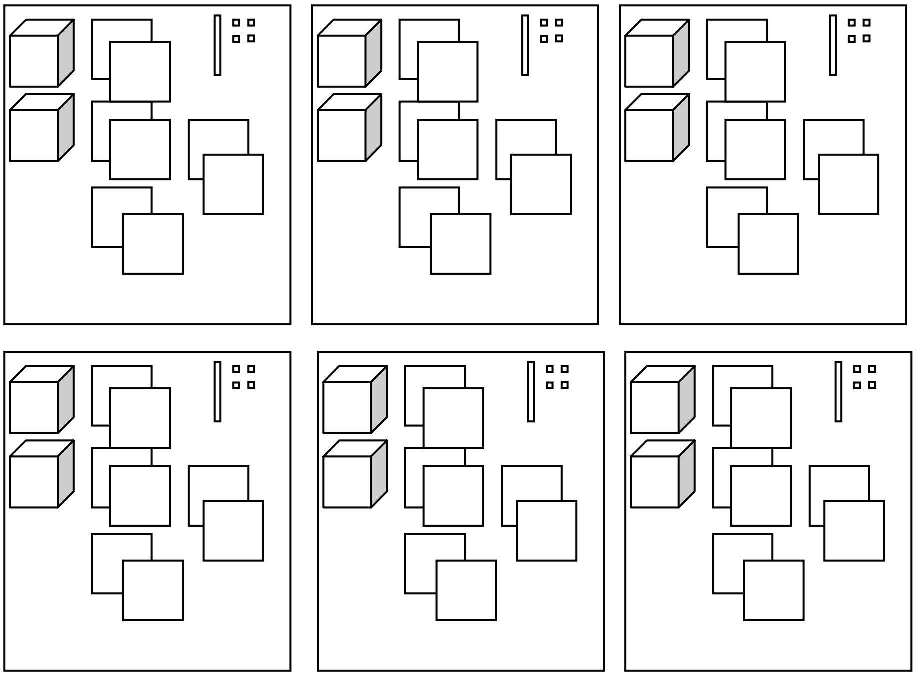
Ten thousands	Thousands	Hundreds	Tens	Ones
	15	5	5	45

Ten thousands	Thousands	Hundreds	Tens	Ones
1	5	5	9	5

Product is 15,595.

Reprint 2023

1. c) 16,884; *Sample response*:



12 thousands + 48 hundreds + 6 tens + 24 ones
= 16 thousands + 8 hundreds + 8 tens + 4 ones = 16,884

d) 31,392; [*Sample response*:

$$\left.\begin{array}{r} 3924 \\ \underline{\times \quad 8} \\ 24{,}000 \\ 7200 \\ 160 \\ \underline{+ \quad 32} \\ 31{,}392 \end{array}\right]$$

e) 21,686; [*Sample response*:

$$\left.\begin{array}{r} 3098 \\ \underline{\times \quad 7} \\ 21{,}000 \\ 630 \\ \underline{+ \quad 56} \\ 21{,}686 \end{array}\right]$$

f) 42,552; [*Sample response*:

$$\left.\begin{array}{r} 4728 \\ \underline{\times \quad 9} \\ 36{,}000 \\ 6\ 300 \\ 180 \\ \underline{+ \quad 72} \\ 42{,}552 \end{array}\right]$$

2. 19,072 km

3. a) ☐ = 4 (in both spots)
b) ◇ = 6 (in both spots) and ☐ = 2
c) ◇ = 8 and ☐ = 2

4. a) Nu 49,800 **b)** Nu 31,140 **c)** Nu 61,650

5. 3600 s

6. a) 36,960 feet **b)** 47,520 feet

7. *Sample response*:
6 × 1235; [All of the numbers 6, 12, 18 and 30 are multiples of 6.]

8. *Sample responses*:
a) 3500 **b)** 8 × 3500 = 28,000

9. *Sample response*: 4 × 1136 = 4544

[**10.** The least possible value is 1 × 1000 = 1000, which has 4 digits.
The greatest possible value is 9 × 9999 = 89,991 which has 5 digits.
So, the only possibilities are 4 or 5 digits.]

[**11.** *Sample response*:
You can multiply the same way. You have to multiply thousands and then regroup thousands to ten thousands if the product is more than 9999.]

Supporting Students

Struggling students

• Struggling students may have difficulty with **questions 8 and 9**. These questions are particularly suitable for strong students.

Enrichment

• Encourage students to create other puzzles like those in **questions 3, 7, and 9**.

For example:

For **question 7**, they might say the product is 8 thousands, 12 hundreds, 16 tens, and 20 ones.

For **question 9**, they might ask fellow students to find a product where the digit 5 is repeated many times (for example, 3×1185).

• You may want to give some students 4-digit by 2-digit calculations to complete. They can use the same pattern of thinking as in the previous lesson with 3-digit by 2-digit calculations.

Reprint 2023

1.1.5 EXPLORE: Mental Multiplication

Curriculum Outcomes	Outcome Relevance
5-B4 Multiply Mentally: to 4 digits × 1 digit • understand the difference between estimation and mental math • understand that estimation strategies can often be used to calculate mentally • develop efficiency with multiplying mentally by 10, 100, 1000 • apply associative principle (e.g., 25 × 30 = 25 × 3 × 10 = 750) • apply double/half strategy (e.g., 50 × 16 = 100 × 8) • apply front-end strategy (e.g., 3 × 325 = 900 + 60 + 15 = 975) • choose appropriate strategy (depending on numbers being calculated) **5-C2 Multiplication Computation Patterns: how a change in either factor affects the computation** • rearrange factors to simplify computation (e.g., 28 × 250 is more difficult than 7 × 1000) • understand that dividing one factor by an amount and multiplying the other by the same amount produces no change in the final result	• This essential exploration of multiplication using mental math is designed to build number sense. • It is important for students to distinguish between the different strategies that are useful for different types of calculations.

Pacing	Materials	Prerequisites
1 h	None	• place value: ones through ten thousands • the distributive principle of multiplication, that is, you can multiply parts of a number together and then add or subtract the parts • multiplication and division facts

Exploration

• Work through the introduction (the white box) with the students. For the second part of the introduction, you may want to talk about why it made sense to write 4999 as 5000 – 1. It is a lot of work to multiply and regroup when multiplying by 4999; it is much less work to multiply by 5000. Ask how they might multiply 4 × 2999.

• As students work through the exploration, they will encounter mental math strategies for

- multiplying by place values (100, 1000, etc.),
- multiplying by a number near a multiple of 100,
- multiplying a multiple of 4 by 25 or 250, and
- multiplying an even number by 5 or 50.

• Students will see that the product of two factors does not change if one factor is multiplied by the same amount that the other is divided by. It is a useful strategy if one factor can be transformed into a number that is a multiple of 10 or 100.

Observe while students work. While they work, you might ask questions such as the following:

• *Why did you rewrite 599 as 600 – 1?* (I can multiply 8 by 600 in my head and I can also subtract 8 in my head.)

• *Why is (24 ÷ 3) × (25 × 3) = 24 × 25?* (If you have 24 groups of 25 (24 × 25), you can put groups together 3 at a time. You will have only 24 ÷ 3 groups, but each group will have 3 × 25 items in it.)

• *Why is it convenient to divide one factor by 4 if you are multiplying by 25?* (If you multiply 25 by 4, you get 100, which is easy to multiply using mental math.)

• *How could you rewrite 5 × 64 as an equivalent multiplication that is easier to do using mental math?* (I would want to change the 5 to a 10, so I would multiply 5 by 2 and divide 64 by 2.)

Observe and Assess

As students work, notice the following:

• Do students make wise choices about how to use an equivalent multiplication to replace a given multiplication?

• Do they multiply and divide correctly?

• Do they recognize there is often more than one way to simplify a calculation to complete it using mental math?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and discuss questions such as these.

• *Which multiplications were easiest to do using mental math?*

• *Why would 44 × 25 (a multiple of 4 times 25) be easier to solve using mental math than 85 × 25 (an odd number times 25)?*

• *How would you use an equivalent multiplication for 50 × 66 to simplify mental computation?*

• *Do you see that you could simplify a calculation like 35 × 20 or like 28 × 64 using the same idea?*
35 × 20 = 7 × 100 (divide one factor by 5 and multiply the other by 5). 28 × 64 = 14 × 128 = 7 × 256 (divide one factor by 2 and multiply the other by 2 twice — now you are multiplying by a single digit number).

Answers

A. i) 35,000 **ii)** 67,000 **iii)** 21,000

B. i) Subtract 8 × 1 = 8 from 4800 to get 4792.
ii) Subtract 8 × 4 = 32 from 4800 to get 4768.
iii) Add 8 × 2 = 16 to 4800 to get 4816.
iv) Add 8 × 5 = 40 to 4800 to get 4840.

C. i)

	24	24 ÷ 2 = **12**	24 ÷ 3 = **8**	24 ÷ 4 = **6**
	25	25 × 2 = **50**	25 × 3 = **75**	25 × 4 = **100**
Product	**600**	**12 × 50 = 600**	**8 × 75 = 600**	**6 × 100 = 600**

ii) They all equal 600.
iii) Because you are multiplying by 100, you can simply put two zeros after the quotient.
iv)

	48	48 ÷ 2 = 24	48 ÷ 3 = 16	48 ÷ 4 = 12
	50	50 × 2 = 100	50 × 3 = 150	50 × 4 = 200
Product	2400	24 × 100 = 2400	16 × 150 = 2400	12 × 200 = 2400

The easiest column to use would be the first column: 24 × 100, because multiplying anything by 100 is easy.

D. *Sample responses*:
i) 10 × 32 = 320 **ii)** 100 × 43 = 4300 **iii)** 100 × 21 = 2100 **iv)** 1000 × 11 = 11,000

E. *Sample responses*:
a) Method 1: 5 × 492 = 10 × 246 = 2460
Method 2: 5 × 492 = 5 × 400 + 5 × 90 + 5 × 2 = 2460
b) Method 1: 25 × 484 = 100 × 121 = 12,100
Method 2: 25 × 484 = 50 × 242 = 5 × 2420 = 10 × 1210 = 12,100

Supporting Students

Struggling students

• If students are struggling with mental math, you may wish to simplify the questions so that students are working with easier numbers.
For example, they could multiply by 100s instead of by 1000s for **part A** or they could multiply only by 5 and 50 rather than by 25 and 250 in **parts C to E**.

• You may wish to model the calculations with materials like base ten blocks (or paper models) to show how the calculations make sense.

For example, to show 8 × 599, make 8 groups of 6 hundreds (using base ten hundred blocks) and then cover up one in each group. Students should see that there are now 8 groups of 599, but that they could have started with 8 × 600 and subtracted 8.

GAME: Greatest Product

• This game is designed to allow students to practise 4-digit by 1-digit multiplication.

• Students will need to consider probability ideas as well as place value concepts.

For example, if a student rolls a 5, he or she might decide to use it as the thousands digit in the 4-digit number since the probability of rolling another high number is not very high. He or she will also have to know the place value concept that the digits most to the left are the most important in determining the size of a number.

• If students do not think about the probability or the place value ideas when they start the game, these ideas might develop as they continue to play.

CONNECTIONS: Egyptian Multiplication

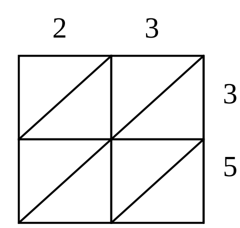
• This method of multiplication has historical roots. One of the interesting things about the method is that it only requires students to be able to double; they do not have to know any other multiplication facts.

• The method works because there is always a unique way of representing a number as the sum of the powers of 2 (1, 2, 4, 8, 16, 32, 64, …).

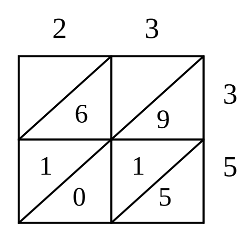
1. a) 3400 1 × 85 2 × 170 4 × 340 8 × 680 √ 16 × 1360 32 × 2720 √ 680 + 2720 = 3400	**b)** 2340	**c)** 7360

• You may also want to introduce lattice multiplication.

For example, to multiply two numbers like 23 and 35, set up a lattice like this:



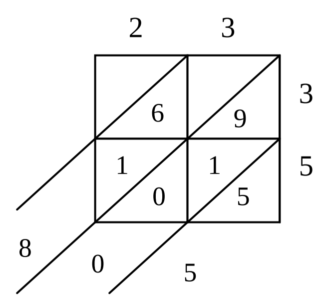
- In each square, put in the product of the numbers at the top and side.



- Add down the diagonals.

- The product is 805.

This strategy works because it automatically places the digits in the product in the correct place value column, whether ones, tens, hundreds, or thousands.



1.2.1 Estimating Quotients

Curriculum Outcomes	Outcome relevance
5-B5 4-Digit ÷ 1-Digit: with and without regrouping • continue estimating to check **5-B12 Open Number Sentences: applying number sense** • explore numerical situations which are always, sometimes, or never true (e.g., 324 + ■ > 300 is always true, assuming ■ is a whole number) • work with open number sentences involving the four basic operations and a combination of operations	• Students need to extend their understanding of how to divide 3-digit numbers so that they can divide 4-digit numbers to solve real-world problems. • Work with open number sentences will help support later work in algebra.

Pacing	Materials	Prerequisites
1 h	None	• division facts • division of a power of ten by a single digit divisor

Main Points to be Raised

• Although it is usually very helpful to round to the nearest ten, hundred, or thousand to estimate the product of two numbers, it is more useful to estimate quotients using values that are products of the divisor.

For example, to estimate 3100 ÷ 7, it is easier to use 2800 than to use 3000 to estimate the dividend.

• If you round the divisor, you can get an estimate that is farther from the actual quotient than if you round the dividend.

For example, 3912 ÷ 6 is about 3600 ÷ 6 = 600 is a better estimate that 4000 ÷ 5 = 800.

• Estimates can be high or low. If only the dividend is estimated, it is easy to predict whether the estimate is high or low. If the divisor is also estimated, it can be harder to predict.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *How do you know it was less than 1 m a day?* (If it were 1 m a day, it would be 6000 mm, but it was less than that.)

• *Why did you use 4200 as your estimated value instead of 4000 or 5000?* (It is easier to divide 4200 by 6 than to divide 4000 or 5000 by 6.)

• *How high is your stool? How do you know?* (I know how long 1 m is since we learned about that before, and I can see that my stool height is pretty close to 1 m.)

Some students might be curious about how rainfall is measured. It actually is measured in millimetres, even though we usually think of measuring liquids in millilitres.

The Exposition — Presenting the Main Ideas

• Ask students what 3500 ÷ 7 is. Then ask why 3512 ÷ 7 would be pretty close. Talk about how you might estimate 3512 ÷ 7 using 3500 ÷ 7 because it is a calculation that is easy to do using mental math.

• Have students suggest a way to estimate 2448 ÷ 5. Once they have shared some suggestions, point out how an estimate of 2500 ÷ 5 = 500 is actually closer to the actual value than an estimate of, for example, 2400 ÷ 4 = 600 or 2400 ÷ 6 = 400. Make sure students understand that these other estimates make sense, but that they are simply not as close.

• Read through the exposition with students. Make sure students realize they might have chosen a different value, for example, 5600 rather than 4800—it is a choice.

• Provide an opportunity for students to ask questions if they do not understand.

Revisiting the Try This

B. Encourage students to consider the multiplication facts for 6.

Using the Examples

• Work through **example 1** with the students. Read it to them to make sure they are comfortable with the symbols that are used.

For example, read the third line of text in **solution 2** as *"Since 3740 is greater than 3600, but also less than 4500"* and add *"then if you divide all three numbers by 9, the order should not change"*.

Then point out how this is exactly what happens in the next line.

• Make sure students understand that □ represents 3740 ÷ 9 in the first solution. Help students see that **solution 2** is very close to **solution 1**. The only difference is that the multiplication relationship was not specifically mentioned in **solution 2**.

• Present the problem in **example 2**. Let students work through it and then check their work against the work in the text.

Practising and Applying

Teaching points and tips

Q 1 and 2: Some students might benefit by having a multiplication table available.

Q 3: You may need to read through one of the parts to make sure students can interpret what they say.

For example, the first one says *"I divide a number by 9 and the answer is greater than 500. What could it be?"*

Q 4: Students should be encouraged to think through which estimate might be closest without actually performing the calculations.

Q 5: Point out to students that checking answers using estimation is something they should always do.

Q 6: Because 92 is beyond the facts for 6, students might think of 92 as 60 + 32 or as 30 + 30 + 32. Or, they might estimate very high, using 12 000.

Q 7: Point out to students that even though 2575 rounds to 2600, it might be better to estimate with 2500 in this situation because the number is being divided by 5.

Q 8: Students who have difficulty coming up with an idea might copy one of the problems on the page.

Common errors

• Students are more likely to struggle in situations where the rounded value is not a product of a multiplication fact and a power of ten.

For example, estimating 9200 ÷ 6, would be much more difficult for some students than, say, 4800 ÷ 6.

In these cases, encourage students to break up the greater value into values with which they are more comfortable or allow them to estimate very high or very low so they can use multiplication facts they know.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can estimate quotients of the division of a 4-digit number by a 1-digit number
Question 4	to see if students can recognize alternative approaches to estimating and communicate about their relative usefulness
Question 7	to see if students can apply estimation techniques to solve a real-world problem

Answers

A. *Sample responses*: **i)** About 760 mm, since 4653 is more than halfway between 4200 ÷ 6 = 700 and 4800 ÷ 6 = 800. **ii)** Yes, since 750 mm is almost as high as a metre stick.	**B.** *Sample responses*: **i)** 4200 ÷ 6 **ii)** 4800 ÷ 6

Answers [Cont'd]

1. *Sample responses*:
a) about 1000 [6000 ÷ 6]
b) about 800 [4000 ÷ 5]
c) about 130 [1300 ÷ 10]
d) about 500 [2000 ÷ 4]

2. *Sample responses*:
a) High estimate: about 2000 [6000 ÷ 3]
Low estimate: about 1000 [3000 ÷ 3]
b) High estimate: about 2000 [10,000 ÷ 5]
Low estimate: about 1000 [5000 ÷ 5]
c) High estimate: about 2000 [16,000 ÷ 8]
Low estimate: about 1000 [8000 ÷ 8]
d) High estimate: about 900 [6300 ÷ 7]
Low estimate: about 800 [5600 ÷ 7]

3. a) Any number greater than 4500
b) Any number less than 420
c) Any number greater than 5000

4. [*Sample responses*:
a) 6412 rounds to 6000; reducing 7 to 6 makes sense since I reduced 6412 to 6000
6412 is close to 6300;
6400 is very close to 6412 and 8 is close to 7]
b) 6300 ÷ 7; [since it changes only one number (the dividend) and not by too much.]

5. B; [*Sample response*:
7 × 199 is almost 7 × 200 = 1400, so 7 × 199 is almost 1400. 1272 is not close enough to 1400 to be called almost 1400.]

6. About 1500 tests; [9000 ÷ 6 is halfway between 6000 ÷ 6 = 1000 and 12,000 ÷ 6 = 2000, so it is about 1500.]

7. About 500 people; [2575 ÷ 5 is about 2500 ÷ 5 = 500]

[**8.** *Sample response*:
If it rained 4550 mm over 5 days. I might want to know how much it rained each day, if it rained the same amount each day.]

[**9.** *Sample response*:
It is a way to check that you did not make a careless mistake.]

Supporting Students

Struggling students

• Struggling students may need help to create situations where estimation is useful (**question 8)**. You might want to provide some starting ideas for these students.

Enrichment

• Ask students to create open sentences with particular solutions like those in **question 3**.

For example, you might ask them to create an open sentence where the solution are numbers greater than 800.

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1.2.2 Dividing 4-digit Numbers by 1-digit Numbers

Curriculum Outcomes	Outcome relevance
5-B5 4-Digit ÷ 1-Digit: with and without regrouping • focus on the whole number (rather than the digits) • link concrete models to algorithms • express remainders as fractions where appropriate • continue estimating as a first step **5-B12 Open Number Sentences: applying number sense** • explore numerical situations which are always, sometimes, or never true (e.g., 324 + ■ > 300 is always true, assuming ■ is a whole number) • work with open number sentences involving the four basic operations and a combination of operations • understand that ■ can also be expressed as a letter variable or another shape or symbol	The ability to divide a 4-digit number is an everyday life skill. The work on this outcome extends what students have already learned about working with 3-digit numbers.

Pacing	Materials	Prerequisites
2 h	• Base ten blocks or Base Ten Models A and B (BLM)	• the sharing meaning and the grouping meaning of division • division facts • multiplication of 4-digit by 1-digit numbers

Main Points to be Raised

• There are different ways you can divide. One method can best be explained using sharing language. Another method can best be explained using "how many groups of" language.

• When you use the subtractive ("how many groups of") division algorithm, the goal is to create as many groups of the divisor as is possible. But students have a choice as to how many groups they form at a time.

• What do you do with a remainder after dividing? It depends on the situation. Sometimes it is appropriate to write the remainder as a decimal or a fraction. Sometimes it must be ignored, and other times the quotient must be rounded up to the next whole.

• The remainder must always be less than the divisor.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. As they work, you might ask questions such as the following:

• *Why did you decide to work with 1252?* (There were 4 m of the 1256 m left over, so only 1252 m were used.)

• *How do you know the side length is more than 300 m?* (If the side lengths were 300 m, then four sides would use 1200 m, but the farmer used 1252 m.)

• *Why might you divide 52 by 4 to help figure out the side length?* (If I divide 52 by 4 and add that to 300, I would get the side length.)

The Exposition — Presenting the Main Ideas

• This lesson focuses on two division algorithms. It is important for students to learn both. Even though either algorithm can be used for any division problem, the more traditional division algorithm makes the most sense for "sharing" situations. These are situations where a certain amount is divided up into a known number of groups and you want to know the size of the group. This is the situation described in the exposition.

• The second algorithm, sometimes called subtractive division, is appropriate in situations where a certain amount is divided up into many groups of a known size and you want to know how many groups can be made. This is shown in the example.

• Ask students to recall what they know about dividing by having them divide, for example, 512 by 3. Discuss how they proceed and how the remainder is handled.

• If possible, provide base ten blocks (or paper models) for students. Ask:

If I divide 1425 into 3 equal groups, how much is in each group?

Let students work on this on their own to see what strategies they come up with. Then have them open their texts and lead them through steps 1, 2, and 3 in the exposition. As you work through the calculation, you may wish to cover up the digits in the dividend that are not being used in the earlier steps (for example, cover up the 25 in 1425 when you divide 1400 by 3). It is very important to emphasize the words to say with the calculations. It is this language that makes the algorithm make sense.

Revisiting the Try This

B. Students should not only perform the calculation, but should be prepared to use language like that in the exposition to explain their steps.

Using the Examples

• **Example 1** is designed to focus students on the subtractive algorithm for division. There are several reasons why it is important for students to learn this process. One reason is that this algorithm is better to use when the size of each group is known but the number of groups is not known. Another reason is that when they use the traditional algorithm, students calculate exactly how many of each place value should be used to create the equal shares. When they use the subtractive algorithm, students can choose how many groups to create at a time.

• Lead students through the first part of **example 1**. Make sure they understand that the use of a group of 400 oranges was a choice. A different student might have started with 500 oranges, 600 oranges, or even 20 oranges. The choice is up to the student. Have students work through the rest of the example in pairs. Ask if there are any questions.

• Then ask students to read the problems posed in **example 2** without reading the solution and work them out. You might wish to write the problems on the board rather than having the students read them from the text. Then have the students compare their thinking with the thinking provided in the example. It is important for students to know that sometimes we ignore the remainder, sometimes we write it as a fraction (or decimal), and sometimes we round the quotient up because there is a remainder.

Practising and Applying

Teaching points and tips

Q 1: Some students will use only one algorithm and others will use more than one. Either way is acceptable, but be careful to allow students to make the choice of which algorithm to use.

Q 2: Students will need to calculate each quotient and subtract.

Q 3 and 4: These are both sharing situations, so the more traditional division algorithm is most appropriate to use. The other algorithm can also be used.

Q 5: Students' answers to **part a)** will depend on whether or not they think of using chhetrums. If they do, each person will get Nu 1051.50. If not, they will think that two people will each get an extra ngultrum, so two will get Nu 1051 and the other two will get Nu 1052. For **part b)**, students can use either decimals or fractions in their answers.

Q 6: Encourage students to create problems that make sense.

Q 7: In **part b)**, students will need to fill in the missing digits both on the right (counting the number of groups) and on the left (within the division work).

For example, they might first choose 6 to fill in []00 if they notice that they multiplied 8 by something to get 4800. They would know the number below 4800 must be 525, which means the dividend must be 5325.

Other students might start at the bottom and work up.

Q 9: Students can use any multiple of 9 greater than 2700 if they add 4 to it.

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Common errors

• If a student underestimates the value for a quotient using the traditional algorithm, they may have difficulty completing the questions.

For example, for $\begin{array}{r} 4\square\square \\ 4\overline{)2310} \end{array}$, if they use 4 instead of 5 for the number of hundreds and get a remainder of 7 when they subtract, they will not know how to continue.

• You might either allow access to a multiplication table to make sure students always choose the greatest possible value for the quotient or you might encourage them to use the subtractive algorithm, where underestimating is all right.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can divide 4-digit numbers by 1-digit numbers
Question 4	to see if students can solve a real-world division problem
Question 5	to see if students can communicate about how to interpret a remainder

Answers

A. *Sample response*: Since there are 4 m left over, I would subtract 4 from 1256 to get 1252. Then I would divide 1252 by 4.	**B.** $\begin{array}{r} 313 \\ 4\overline{)1253} \\ -\underline{12} \\ 5 \\ -\underline{4} \\ 12 \\ -\underline{12} \\ 0 \end{array}$ 1252 ÷ 4 = 313
1. a) 1231 **b)** 1193 **c)** 589 **d)** 1248 R 3 **e)** 111 R 6 **f)** 516 R 2 **2. a)** 1 **b)** 120 **c)** 47 **3.** Two payments were Nu 2833 and one payment was Nu 2834. **4. a)** Nu 300 **b)** Nu 500 **c)** Nu 375 **5. a)** Nu 1051 R 2; give each person 50 Ch more. **b)** 5 R 10; you cannot use the remainder to buy more chocolate bars. **c)** 107 m^2 R 2; add an extra 0.5 m^2 to each section. **6.** *Sample response*: A table that costs Nu 3006 is paid for in 8 equal payments. How much is each payment?	**7. a)** $\begin{array}{r} 1206 \\ 6\overline{)7236} \\ [-\underline{6} \\ 12 \\ -\underline{12} \\ 036 \\ -\underline{36} \\ 0] \end{array}$ **b)** $\begin{array}{rr} 8\overline{)5325} & \\ -\underline{4800} & \mathbf{6}00 \\ \mathbf{5}25 & \\ -\underline{480} & \mathbf{6}0 \\ \mathbf{45} & \\ -\underline{40} & \underline{+\ 5} \\ \mathbf{5} & \mathbf{66}5 \end{array}$ [**8.** If you have 9 left, you know that you can fit one more group of 7 into the dividend, with only 2 left over.] **9.** *Sample response*: 2713, 2722, or 2731 [**10.** *Sample response*: When you divide, you can multiply to check your answer. For example, to divide 5213 by 3, you can multiply 3 × 1000 and 3 × 2000 to estimate that the answer should be between 1000 and 2000.]

Supporting Students

Struggling students

• Struggling students may have difficulty with **questions 7 to 9**. These questions are particularly suitable for strong students.

GAME: Target 2000

- This game is designed to allow students to practise dividing a 4-digit number by a 1-digit number.
- As with the earlier game in the unit, students might consider probability ideas while they play the game.

For example, if a student has already decided that the divisor is 2, and then he or she rolls a 5, he or she must decide whether to place the 5 as the thousands digit, considering how likely it would be to roll a 4 afterwards.

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1.2.3 EXPLORE: Mental Division

Curriculum Outcomes	Lesson relevance
5-B7 Divide Mentally • use prior knowledge of basic facts • divide by 10, 100, 1000 • link to place value	In everyday life, we often use mental calculation for buying and selling. This required exploration deals with mental division by 10, 100, or 1000.

Pacing	Materials	Prerequisites
40 min	None	• renaming numbers using place value, e.g., renaming 3 hundreds as 30 tens

Exploration

• Remind students of the required place value ideas.

For example, 320 is 3 hundreds and 2 tens, but it is also 32 tens, or 32 groups of ten.

Ask students to tell you how many groups of ten there are in 510, in 260, and in 450.

Then ask how many groups of one hundred there are in 400, in 800, and in 1200 (not 2 hundreds, but actually 12 hundreds). Make sure students distinguish between the digit in the tens place and the number of tens in a number.

For example, for 480, the number in the tens place is 8 but there are 48 groups of 10 in 480.

• Students can work through the questions on the page alone or in pairs. Observe while students work. You might ask questions such as the following:

• *How do you know that 3460 is 346 tens? Why does writing it that way help you easily divide by 10?* (Each thousand is 100 tens, so 3000 is 300 tens. Each hundred is 10 tens, so 400 is 40 tens. Altogether, there are 346 tens. When you divide by ten, you are asking how many tens, and you already know that there are 346.)

• *How do you know that 3400 ÷ 100 = 34? How can knowing 3400 ÷ 100 = 34 help you calculate 3400 ÷ 10?* (3400 ÷ 100 is asking how many hundreds are in 3400 and, since 3400 is 34 hundreds, the answer is 34. Since there are 10 tens in 1 hundred, there are 10 times as many tens in 3400 as there are hundreds. So, 3400 ÷ 10 is 10 × 34 = 340.)

Observe and Assess

As students work, notice the following:

• Do students explain clearly in **parts B ii) and C ii)**?

• Do they easily complete the chart?

• Do they seem to generalize to create rules for dividing by 10, 100, and 1000?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and discuss questions such as these.

• *Why is it easy to divide 5000 by either 10, 100, or 1000?*

• *Why is it useful to know place value ideas to divide by 10, 100, or 1000?*

• *If you know that a number divided by 100 is 43, what is the number? What is that number divided by 10?*

• *How would you describe a "rule" for dividing by 10, 100, or 1000?*

• *What would you do if you were dividing, for example, 531 by 10 instead of 530 by 10?*

Answers

A. i)

Number	80	410	3460	5000
Number of groups of ten	**8**	**41**	**346**	**500**
Number ÷ 10				

ii) *Sample response*:
15 tens is 15 groups of 1 ten.
15 tens ÷ 1 ten means how many groups of 1 ten are in 15 tens? The answer is 15.

iii)

Number	80	410	3460	5000
Number of groups of ten	8	41	346	500
Number ÷ 10	**8**	**41**	**346**	**500**

B. i)

Number	800	4100	3400	5000
Number of groups of one hundred	**8**	**41**	**34**	**50**
Number ÷ 100				

ii) *Sample response*:
12 hundreds is 12 groups of 1 hundred.
12 hundreds ÷ 1 hundred means how many groups of 1 hundred are in 12 hundreds? The answer is 12.

iii)

Number	800	4100	3400	5000
Number of groups of one hundred	8	41	34	50
Number ÷ 100	**8**	**41**	**34**	**50**

C. i)

Number	8000	12,000	20,000	45,000
Number of groups of one thousand	**8**	**12**	**20**	**45**
Number ÷ 1000				

ii) *Sample response*:
12 thousands is 12 groups of 1 thousand.
12 thousands ÷ 1 thousand means how many groups of 1 thousand are in 12 thousands? The answer is 12.

iii)

Number	8000	12,000	20,000	45,000
Number of groups of one thousand	**8**	**12**	**20**	**45**
Number ÷ 1000	**8**	**12**	**20**	**45**

D. i) 42 **ii)** 65 **iii)** 560 **iv)** 56

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Supporting Students

Struggling students

• Struggling students may need help with place value. Use a place value chart to help them regroup.

For example, show why 310 could be shown either of these ways:

Thousands	Hundreds	Tens	Ones
	3	1	0

Thousands	Hundreds	Tens	Ones
		31	0

Talk about why the second form makes it easier to divide by 10.

• Some students may find it easier to understand if you start with the second chart, group 10 tens into hundreds and then go back to the first chart.

Enrichment

• Ask students to create rules for dividing by 1000 or by 10,000.

1.2.4 Dividing 4-digit Numbers by Multiples of Ten

Curriculum Outcomes	Outcome relevance
5-B6 4-Digit ÷ 2-Digit: introduce • explore divisors which are multiples of 10 only (10, 20, 30, …)	Dividing by a multiple-digit number is much more difficult for students than dividing by a single-digit number. Yet there are many real-world situations where that is required. If students can learn to divide by multiples of ten by combining what they know about dividing by a single-digit number and by ten, they can reasonably estimate a quotient when there is a 2-digit divisor.

Pacing	Materials	Prerequisites
1.5 h	• 40 small items such as pencils or erasers	• dividing by 1-digit numbers • dividing by 10

Main Points to be Raised

• You can divide a number in parts.

For example, to divide by 6, you can first divide in half and then divide each half in 3 (the parts of 6 are 2 and 3, since 6 = 2 × 3).

• To divide by a multiple of ten, e.g. 5 × 10, you can either divide by ten and then divide by the other factor of the multiple of ten, or you can divide by that other factor and then by ten.

• When you divide x tens by y tens, the quotient is the same as $x \div y$.

• To determine the remainder when you divide by a multiple of ten, it is best to get the quotient first, then multiply the quotient by the multiple of ten, and then subtract from the dividend.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. You might ask questions such as the following:

• *How many Nu 20 notes would be in a pile worth Nu 100?* (5 notes.)

• *How do you know an answer of 1000 is too much?* (If there were 1000 notes, they would be worth Nu 20,000. Nu 5200 is a lot less. It should be about one fourth as many.)

• *How many notes would be in a pile of Nu 200? Nu 5200? How do you know?* (10 notes and 260 notes. I figured out how many hundreds there were and multiplied by 5.)

The Exposition — Presenting the Main Ideas

• Begin by displaying a pile of 40 small items, for example, 40 pencils. Invite four students, two boys and two girls, to the front of the class. Tell them that you want to give each student the same number of pencils.

- Model how you could divide by 4 by giving each of the 4 students a pencil until all 40 pencils are gone.

- Collect the pencils. Show how you could give half the pencils to the girls and half to the boys (which is like dividing 40 by 2). Then ask the girls to share their pencils (dividing their half by 2) and ask the boys to share their pencils (dividing their half by 2).

- Have students notice that each student still has 10 pencils.

- Point out that sometimes we divide “in parts” like this.

• Ask students to suggest a way to divide, for example, 600 pencils into 30 groups. Allow them time to offer some suggestions. Wait for someone to suggest the idea of dividing up the pencils into 10 groups of 60 and then dividing each group of 60 into three smaller groups. Altogether, there will still be 30 groups (three small groups ten times), but that the calculations might be easier to do.

• Have students look at the exposition on **page 26** to see how 40 shares are created by first dividing by 4 and then dividing each fourth into 10. Make sure they know you could divide first by 4 and then by 10 or you could divide first by 10 and then by 4.

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Revisiting the Try This

B. Students will notice that they were dividing by 20, so they could divide first by 2 and then by 10 or they could divide first by 10 and then by 2.

Using the Examples

• Pose the two questions in **examples 1 and 2** on the board. Allow students to work on them and then compare their results with the results in the text.

• Make sure students understand that whether dividing a multiple of ten or a non-multiple of ten by a multiple of ten, they can divide in parts. Note that the remainder when a part is divided is not necessarily the remainder when divide by the whole divisor. It is for this reason that the best strategy is to multiply the quotient by the divisor and subtract from the dividend to get the remainder.

For example, to calculate 450 ÷ 20, you can divide 45 by 2 to get a quotient of 22 with a remainder of 1. But the remainder of 1 is not the remainder when you divide 450 by 20. In that case, 22 × 20 = 440, so the remainder is actually 10. Notice that 10 is half of 20, just like 1 is half of 2.

• Then ask students to read through **example 3**. Help them understand that guessing and testing is an acceptable and useful strategy to use to solve problems.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to calculate in two steps as was done in the exposition and first two examples.

Q 2: Some students might use questions from **example 3** or other exercises to give them ideas for word problems.

Q 3: Encourage students to estimate before calculating so that they can check that the answer is reasonable.

Q 6: Students should not do the calculations. Instead, they should think about what division means. Dividing something into 30 parts means each person gets much less than if the same amount were divided into only 3 parts.

Q 8: You may have to lead some students to consider what the lowest and highest possible 4-digit numbers are. By using those values and dividing by 30, they will know the least and greatest possible quotients.

Q 9: It might be easier for students to read the question to them like this:

There are 5 twenties in 100. How does that help you know how many twenties are in 4000?

Q 10: Some students might argue that it is not easier to divide by 20 than by 21. For example, it is easier to divide 210 by 21 than by 20. But for most numbers, dividing by 20 is easier since they find it easy to divide by 2 and by 10 using mental math.

Common errors

• Many students will not recalculate the remainder from the single-digit divisor to consider the remainder for the two-digit divisor. You might encourage them to check that their answer is correct by multiplying the quotient by the divisor and adding their remainder to see if they get the right dividend.

For example, to calculate 412 ÷ 50, some students might divide by 10 to get 41 R 2 and then divide 41 by 5 to get 8 R 1. They will report the answer as 8 R 2, or 8 R 1, or even 8 R 3 (adding the two remainders). In fact, the result should be 8 R 12. If they use, for example, 8 R 1, when they multiply to check they will see that 8 × 50 + 1 = 401 and not 412.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can divide a 4-digit number by a multiple of ten
Question 3	to see if students can solve a real-word problem that requires dividing by a multiple of ten
Question 7	to see if students can communicate about different ways to divide by a 2-digit multiple of ten

Answers

A. 264 notes

B. *Sample response*:
No. I counted up by 20s to 1000. Then I multiplied by 5 and added 14.

1. a) 187 R 24 **b)** 84 R 19
c) 22 R 20 **d)** 251

2. *Sample response*:
Part d), I might have a roll of Nu 20 notes worth Nu 5020 and I want to know how many notes there are.

3. *Sample responses*:
a) About 100 h
b) About 60 h
c) About 200 h

4. a) 60 h **b)** 75 h, 20 min
c) 142 h, 10 min **d)** 150 h

5. 175 min

[**6.** *Sample response*:
If you are making 30 groups instead of only 3 groups, then each group can contain only $\frac{1}{10}$ as many items.]

7. Yes; [*Sample response*:
If you divide by 5, you have created 5 equal groups. If each of those groups is divided into 6 equal groups, there will be 30 equal groups altogether. If you divide by 10, you have created 10 equal groups. If each of those groups is divided into 3 equal groups, there will also be 30 equal groups. For example:
3000 ÷ 5 = 600 and 600 ÷ 6 = 100
3000 ÷ 10 = 300 and 300 ÷ 3 = 100]

[**8.** *Sample response*:
4-digit numbers go from 1000 to 9999.
Since 30 × 100 = 3000, if you divide anything less than 3000 by 30, the answer is less than 100, which is a 2-digit number. If you divide anything greater than 3000 by 30, the answer is more than 100, which is a 3-digit number. Since most of the 4-digit numbers are greater than 3000, the answer will usually have 3 digits.]

[**9.** *Sample response*:
Since there are 5 groups of 20 in 100, you figure out how many 100s are in 4000 and then multiply by 5.]

[**10.** *Sample response*:
To divide by 20, you can divide by 2 and then divide by 10. Both of those you can usually do in your head. There is no easy way to figure out how many groups of 21 are in something.]

Supporting Students

Struggling students

• Struggling students may need to compare to the subtractive algorithm. This will help convince them that dividing in parts works.

For example, for 480 ÷ 20, they could calculate:

$$\begin{array}{rl} 20\overline{)480} & \\ -400 & \text{20 groups} \\ 80 & \\ \underline{-80} & \underline{\text{ 4 groups}} \\ 0 & \text{24 groups} \end{array}$$

Then they compare to 480 ÷ 10 = 48 and 48 ÷ 2 = 24.

Enrichment

• Ask students to figure out how to divide by multiples of 100, for example, by 300 or by 400.

UNIT 1 Revision

Pacing	Materials
2 h	• Base ten blocks or Base Ten Models A and B (BLM) (optional)

Question(s)	Related Lesson(s)
1 – 3	Lesson 1.1.1
4 – 8	Lesson 1.1.2
9 – 13	Lesson 1.1.3
14 – 16	Lesson 1.1.4
17	Lesson 1.1.5
18 and 19	Lesson 1.2.1
20 and 21	Lesson 1.2.2
22	Lesson 1.2.3
23 – 25	Lesson 1.2.4

Revision Tips

Q 4: You might encourage students to say whether the estimate is high or low, or whether it is too hard to tell.

Q 8: There is no definite answer to this question, but normally it is good to lower one number and raise the other to get an estimate close to the exact value.

Q 9: You might provide base ten blocks (or paper models) for students to use for this question.

Q 15: Students who solve for the missing digits using guess and test will get a lot of practice with multiplication.

Q 16: Remind students there are 60 s in 1 min and 60 min in 1 h.

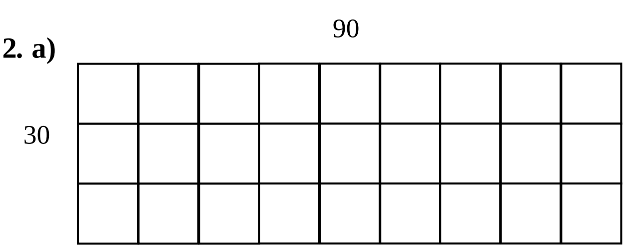
Q 18: Make sure students understand that they do not have to round, but that they can estimate with any convenient numbers, for example, 2800 for **part b)**.

Q 20: Students can use the division algorithm of their choice.

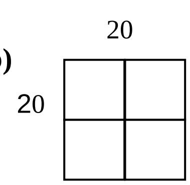
Answers

1. 30 × 60 = 1800

2. a)



b)



3. D

4. *Sample responses*:
a) About 2800 [40 × 70] **b)** About 1200 [30 × 40]
c) About 1600 [80 × 20] **d)** About 3200 [80 × 40]

5. *Sample response*: About 2100 m^2 [70 × 30]

6. A; [*Sample response*:
The answer should be about 50 × 60 = 3000, and 3961 is way too high.]

7. *Sample responses*:
a) About Nu 2000 [40 × 50]
b) About Nu 3200 [40 × 80]
c) About Nu 1400 [70 × 20]

8. *Sample responses*:
a) 30 × 80 or 40 × 70
[**b)** I think they are both about the same because each time I rounded one number up and the other number down by the same amount.]

9. a) 1008

	40 +	2
20 +	20 × 40	20 × 2
4	4 × 40	4 × 2

Answers [cont'd]

9. b) 1431

	50 +	3
20 +	20 × 50	20 × 3
7	7 × 50	7 × 3

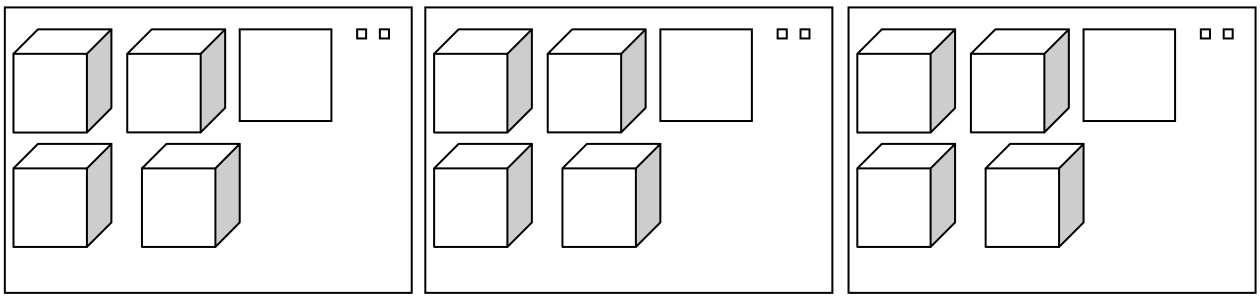
10. a) 28,944 **b)** 18,177

11. 864 squares

12. Nu 3195

13. *Sample response*:
I drove 25 km at an average speed of 38 km each hour. For how long did I travel?

14. a) 12,306
Sample response:



12 thousands + 3 hundreds + 6 ones = 12,306

b) 11,324; *Sample response*:

Thousands	Hundreds	Tens	Ones
10	12	12	4

Ten thousands	Thousands	Hundreds	Tens	Ones
1	1	3	2	4

11,324

c) 7190; *Sample response*:

Thousands	Hundreds	Tens	Ones
5	20	15	40

Thousands	Hundreds	Tens	Ones
7	1	9	0

7190

d) 20,251; *Sample response*:

Thousands	Hundreds	Tens	Ones
14	56	63	21

Ten thousands	Thousands	Hundreds	Tens	Ones
2	0	2	5	1

20,251

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15. $\begin{array}{r} 21\mathbf{4}3 \\ \underline{\times \quad 7} \\ 15{,}0\mathbf{0}1 \end{array}$ or $\begin{array}{r} 21\mathbf{5}3 \\ \underline{\times \quad 7} \\ 15{,}0\mathbf{7}1 \end{array}$

16. 14,400 s

17. a) 3400; [*Sample response*:
Put three zeros at the end to get 34,000.]
b) 1100; [*Sample response*:
Divide 44 by 4 and multiply 25 by 4 to change it to 11 groups of 100, or 1100.]
c) 15,600; [*Sample response*:
Divide 312 by 2 and multiply 50 by 2 to change it to 100 groups of 156, or 15,600.]

18. *Sample response*:
a) About 2000 [6000 ÷ 3]
b) About 500 [3500 ÷ 7]
c) About500 [2500 ÷ 5]
d) About 400 [3600 ÷ 9]

19. *Sample response*:
5 people are sharing the cost of a Nu 3015 purchase. About how much does each person pay?

20. a) 846 **b)** 809 R 6
c) 369 R 4 **d)** 443 R 5

21. Nu 1840

22. a) 36 **b)** 45 **c)** 280 **d)** 13

23. a) 134 R 10 **b)** 72 R 17
c) 31 R 10 **d)** 246 R 19

24. a) 150 h **b)** 21 h
c) 116 h, 40 min **d)** 141 h, 30 min

25. *Sample response*:
Divide by 100 and then multiply by 5.
Divide by 2 and then divide by 10.
Divide by 10 and then divide by 2.

UNIT 1 Whole Number Computation Test

1. a) How much greater is 30×40 than 20×30?

b) How much greater is 80×90 than 30×50?

2. Draw pictures to show how 2×3 and 20×30 are related. Explain how your pictures show this.

3. Estimate each.

a) 52×39

b) 67×48

c) the total cost of 52 items that each cost Nu 80

d) the total cost of 64 items that each cost Nu 90

4. a) Draw a picture to show what 41×23 represents.

b) Draw a picture to show what 43×21 represents.

c) Use your pictures in **parts a) and b)** to show why 41×23 is not equal to 43×21.

5. Calculate.

a) 52×46

b) 63×28

c) 7×5123

d) 3×7892

6. Show how you could use $3 \times 589 = 1767$ to calculate each. Explain your thinking.

a) 3×2589

b) 3×5890

c) 3×5891

7. From Paro to Delhi is 1164 km by air. A plane flew that distance 4 times.

a) How do you know the total distance the plane flew was about 4500 km?

b) What is the exact total distance flown?

8. Suppose you had to calculate this mentally.

$$\square \times 52\square\square = ?$$

a) What digits would you put in the blanks? Explain why you chose those digits.

b) What is the product?

9. Estimate each. Show what you did.

a) $4123 \div 4$

b) $4895 \div 4$

10. a) Use the digits 3, 4, 5, 6, and 0 in the blanks to get the greatest possible quotient.

$$[\ ]\overline{)[\ ][\ ][\ ][\ ]}$$

b) Calculate the quotient.

11. a) How much greater is $5232 \div 2$ than $6855 \div 5$?

b) How much greater is $9231 \div 3$ than $7038 \div 6$?

c) How much greater is $6400 \div 10$ than $5800 \div 100$?

12. Write a word problem that could be solved by dividing 4260 by 3. Solve your problem.

13. Describe two different ways to divide $4840 \div 40$.

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UNIT 1 Test

Pacing	Materials
1 h	• Base ten blocks or Base Ten Models A and B (BLM)

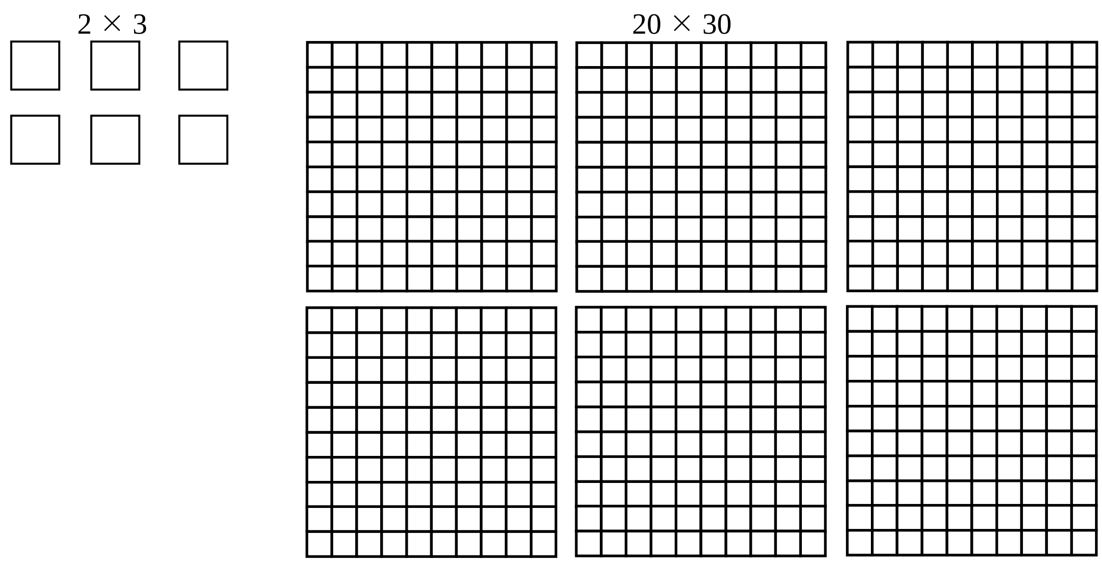
Question(s)	Related Lesson(s)
1 and 2	Lesson 1.1.1
3	Lesson 1.1.2
4 and 5	Lesson 1.1.3
5 – 7	Lesson 1.1.4
8	Lesson 1.1.5
9	Lesson 1.2.1
10 – 12	Lesson 1.2.2
13	Lesson 1.2.4

Select questions to assign according to the time available.

Answers

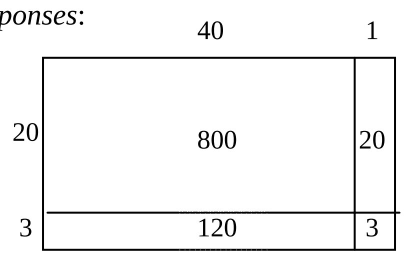
1. a) 600 **b)** 5700

2. *Sample response*:
2 × 3 is 2 rows of 3 objects.
20 × 30 is also 2 rows of 3 objects but each object is worth 100 instead of 1.

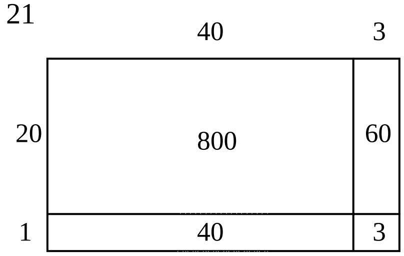


3. *Sample responses*:
a) 50 × 40 = 2000 **b)** 70 × 50 = 3500
c) 50 × 80 = 4000, so the total cost is Nu 4000.
d) 60 × 90 = 5400, so the total cost is Nu 5400.

4. *Sample responses*:
a) 41 × 23



b) 43 × 21



c) Both rectangles have parts that have a value of 800 and of 3, but for 41 × 23, the other parts have a value of 20 + 120 = 140 and for 43 × 21, the other parts have a value of 40 + 60 = 100.

Answers [Continued]

5. a) 2392 **b)** 1764
c) 35,861 **d)** 23,676

6. *Sample responses*:
a) Since 2589 = 2000 + 2589, add 3 × 2000 = 6000 to 1767 to get 7767
b) Since 5890 = 10 × 589, multiply 10 × 1767 to get 17,670.
c) Since 5891 = 10 × 589 + 1, multiply 10 × 1767 and then add 3 × 1 to get 17,673.

7. a) *Sample response*:
Since 4 × 1125 = 4500 (because 4 × 1100 = 4400 and 4 × 25 = 100, so 4 × 1125 = 4400 + 100 = 4500), then 4 × 1164 must be more than 4500.
b) 4656 km

8. *Sample responses*:
a) **5** × 52**00**, so I could multiply mentally 5 × 5000 = 25,000 and add it to 5 × 200 = 1000.
b) 26,000

9. *Sample responses*:
a) 4000 ÷ 4 = 1000 **b)** 5000 ÷ 4 = 1250

10. a) $3\overline{)6540}$ **b)** 2180

11. a) 1245 **b)** 1904 **c)** 582

12. *Sample response*:
Three people were sharing the cost of something worth Nu 4260. How much does each person pay?
(Answer: Nu 1420)

13. *Sample response*:
• Divide 4840 ÷ 10 = 484 and then divide 484 ÷ 4 = 121.
• Divide 4840 ÷ 2 = 2420 and then divide 2420 ÷ 2 = 1210. Finally, divide 2420 ÷ 10 = 121.

UNIT 1 Performance Task — Typing Speed

Dorji can type 38 words in a minute.
Yangchen can type 43 words in a minute.

A. i) Use mental math. Calculate how many words Dorji can type in 15 min. Explain what you did.

ii) Use a model. Figure out how many words Yangchen can type in 15 min. Show your work.

B. Estimate how many words each student can type in 3 h. Show what you did to estimate.

C. Estimate how many hours it would take for each student to type 20,000 words. Show what you did to estimate.

D. i) Choose a book with 100 or fewer pages. Estimate the number of words in the book.

ii) Estimate how long it would take Dorji or Yangchen to type all the words in the book. Show what you did to estimate.

E. Create two problems about typing using the information about Dorji and Yangchen. Solve each problem and show your work.

i) Use multiplication in the first problem.

ii) Use division in the second problem.

UNIT 1 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
5-B1 Estimate Products: 2 digits × 2 digits **5-B2** 2-Digit × 2-Digit Multiplication: with and without regrouping **5-B3** 4-digit × 1-digit Multiplication: with and without grouping **5-B4** Multiply Mentally: to 4 digits × 1 digit **5-B5** 4-Digit ÷ 1-Digit: with and without regrouping **5-B6** 4-Digit ÷ 2-Digit: introduce **5-B7** Divide Mentally	1 h	• Base ten blocks or Base Ten Models A and B (BLM) (optional)

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test.

• It could also be used as enrichment material for some students.

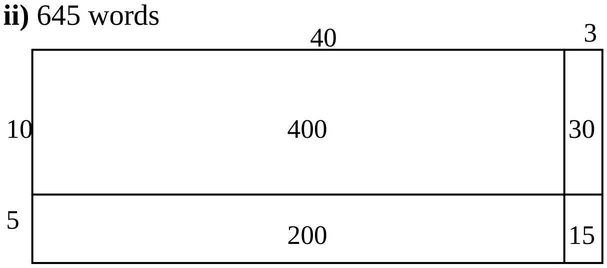
• You can assess performance on the task using the rubric on the next page.

• The task may take longer than the time you wish to allow for it. In that case, you can cut out parts of it, for example, **part C or part E**.

Sample Solution

A. i) 570 words
15 = 10 + 5
In 10 min: 10 × 38 = 380 words
In 5 min: half of 10 min: 380 ÷ 2 = 38 tens ÷ 2 = 19 tens or 190
In 15 min: 380 + 190 = 380 + 200 – 10 = 580 – 10 = 570

ii) 645 words



400 + 200 + 30 + 15 = 645

B. Dorji: 6840 words; Yangchen: 7740 words.
There are four 15 min in 1 h.
If Dorji can type 570 words in 15 min, he can type 4 × 570 = 2280 words in 1 h.
If he can type 2280 in 1 h, he can type 3 × 2280 = 6840 words in 3 h.
If Yangchen can type 645 words in 15 min, she can type 4 × 645 = 2580 words in 1 h.
If she can type 2580 in 1 h, she can type 3 × 2580 = 7740 words in 3 h.

C. If Dorji can type about 7000 words in 3 h, he could type 21,000 words in 3 × 3 = 9 h.
He could type 20,000 words in about 9 h.
If Yangchen can type about 8000 words in 3 h, she could type 16,000 words in 2 × 3 = 6 h.
She could type 4000 words in one fourth that time, or $1\frac{1}{2}$ h.
She could type 20,000 words in about $6 + 1\frac{1}{2} = 7\frac{1}{2}$ h.

D. i) The book had 78 pages and I counted 239 words on one page.
The book has about 80 × 230 words (I rounded high for number of pages and low for number of words on a page).
80 × 230 = 8 × 10 × 230 = 8 × 2300 = 18,400.
There are about 18,400 words in the book.

ii) Each student types about 40 words a minute.
18,400 ÷ 40 = 184 hundreds ÷ 40 is between 4 hundred and 5 hundred, so I used 450 min.
There are 60 min in 1 h. 450 ÷ 60 = 7 with 30 min left over.
I estimate 7.5 h.

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E. Multiplication problem: How many words can Yangchen type in 35 min? (Answer: 1535 words) *Solution* $35 \times 43 = 30 \times 40 + 5 \times 40 + 40 \times 3 + 5 \times 3$ $30 \times 40 = 1200$ $5 \times 40 = 200$ $40 \times 3 = 120$ $5 \times 3 = 15$ $35 \times 43 = 1200 + 200 + 120 + 15 = 1535$ words	Division problem About how long would it take Dorji to type 5200 words? (Answer: 2 h 20 min) *Solution* $5200 \div 38$ is about $5200 \div 40 = 5200 \div 10 \div 4 = 520 \div 4 = 130$. It would take more than 130 min, which is 2 h 10 min, so I estimate 2 h 20 min.

UNIT 1 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Multiplies whole numbers	Efficiently and accurately models 2-digit by 2-digit multiplication, efficiently and accurately performs mental multiplication and estimation, and accurately calculates products	Accurately models 2-digit by 2-digit multiplication, accurately performs mental multiplication and estimation, and accurately calculates products	Correctly models 2-digit by 2-digit multiplication, correctly performs mental multiplication and estimation, and generally calculates products correctly	Has difficulty with at least one of modelling 2-digit by 2-digit multiplication, performing mental multiplication and estimation, or calculating products correctly
Divides whole numbers	Efficiently and accurately estimates quotients, efficiently and accurately performs mental division, and accurately calculates quotients	Appropriately estimates quotients, accurately performs mental division, and accurately calculates quotients	Correctly estimates some quotients, performs some mental divisions, and generally calculates quotients correctly	Has difficulty performing mental divisions, estimating quotients, and calculating quotients
Recognizes multiplication and division situations	Creates clear and appropriate problems that are solved using multiplication and division, respectively, and applies the appropriate operations to solve the given problems	Creates appropriate problems that are solved using multiplication and division, respectively, and applies the appropriate operation to solve the given problems	Creates at least one appropriate problem that is solved using multiplication or division and generally applies the appropriate operation to solve the given problem(s)	Has difficulty creating problems that are solved using multiplication and division and does not apply the appropriate operation to solve the given problem(s)

BLM 1 Place Value Charts I

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

Ten thousands	Thousands	Hundreds	Tens	Ones

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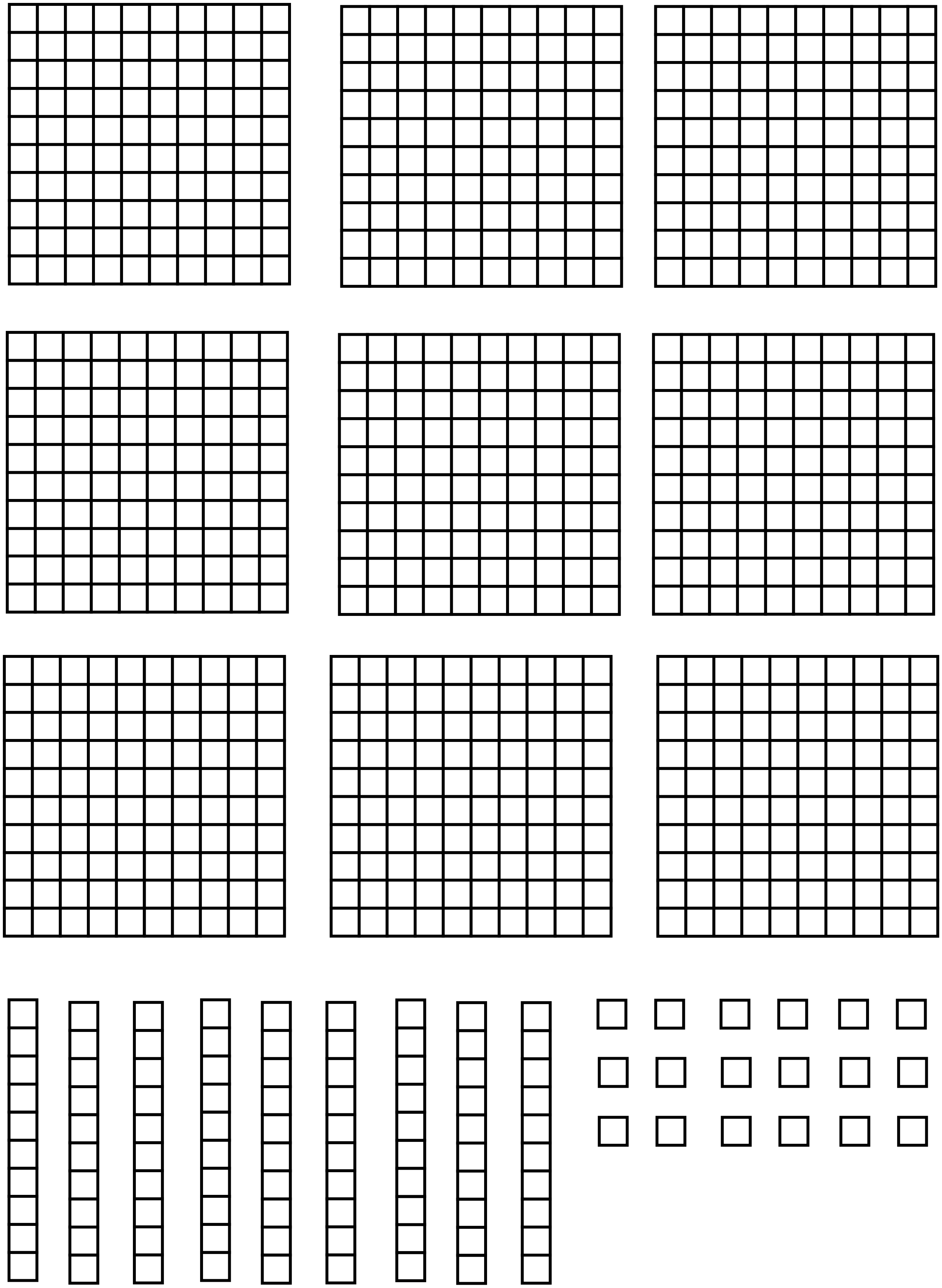
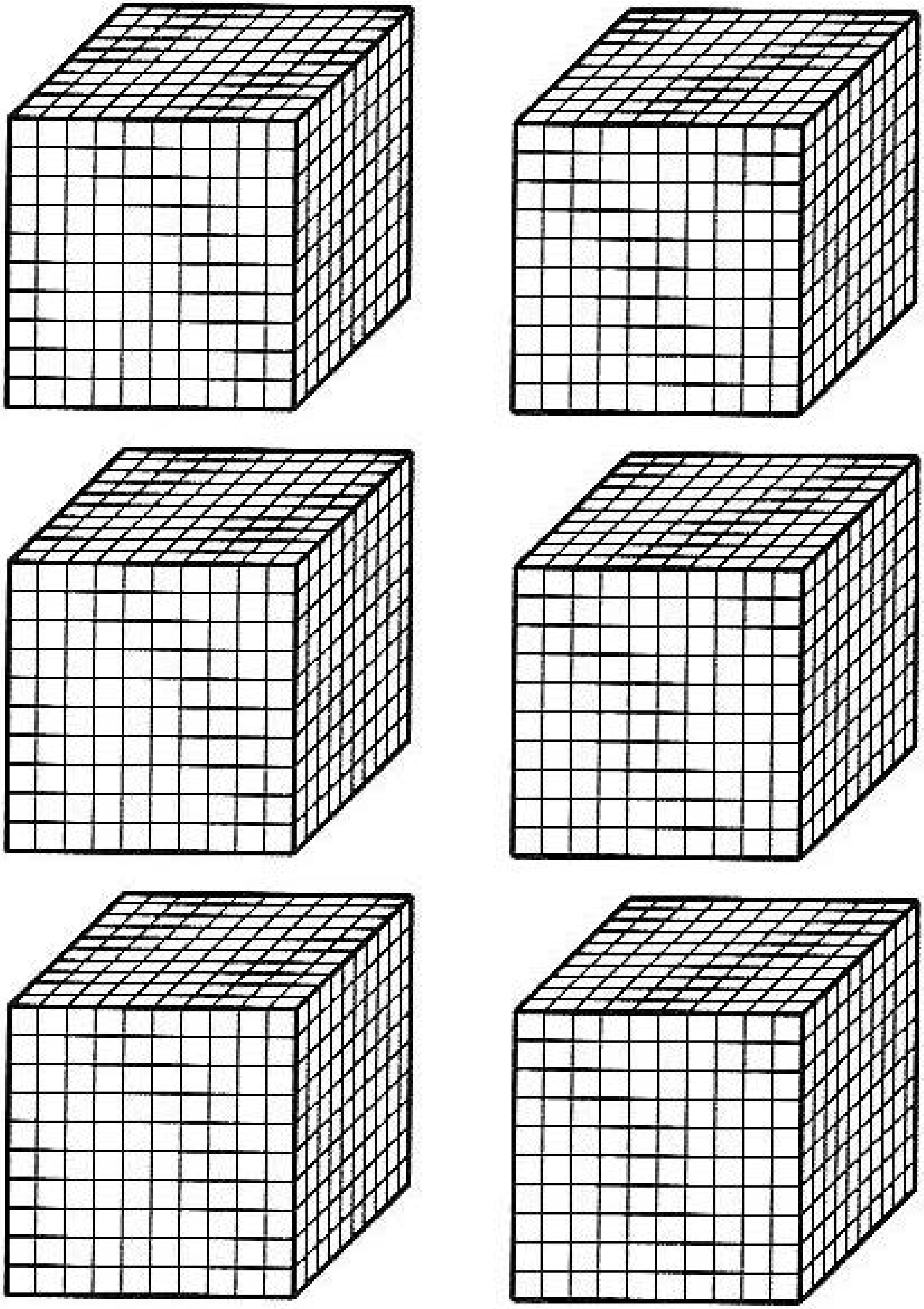
BLM 2A Base Ten Models



BLM 2B Base Ten Models



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UNIT 2 GEOMETRY

UNIT 2 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 31** **TG p. 56**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• Special Rectangles (BLM) or 104 mm-by-120 mm paper rectangles • Linking cubes (8 per group) • Sample Net of Cube (BLM) (optional)	All questions
Chapter 1 Triangles and Quadrilaterals				
2.1.1 Classifying Triangles by Side Length **SB p. 33** **TG p. 59**	**5-E1 Triangles: explore equilateral, isosceles, scalene triangles, and right, obtuse, and acute triangles** • discover properties of equilateral, isosceles, and scalene triangles through concrete experiences • sort and classify triangles by side lengths	1 h	• Sticks of three sizes (three or more of each length per student or pair) • Rulers	Q1, 2, 3, 6
2.1.2 Classifying Triangles by Angle **SB p. 37** **TG p. 62**	**5-E1 Triangles: explore equilateral, isosceles, scalene triangles, and right, obtuse, and acute triangles** • discover properties of right, obtuse, and acute triangles • sort and classify triangles by angle size and side lengths • develop a personal referent for 90° (right) angles	40 min	• Grid paper or Small Grid Paper (BLM)	Q1, 2, 3, 4
GAME: Triangle Dominoes (Optional) **SB p. 40** **TG p. 64**	Apply and practise triangle classification in a game situation	30 min	• Triangle Dominoes Game Cards (BLM)	N/A
2.1.3 EXPLORE: Combining Triangles (Essential) **SB p. 41** **TG p. 65**	**5-E2 Combine Triangles: spatial sense and visualization** • use visualization to predict the results of combining triangles • develop spatial sense by combining - two congruent equilateral triangles - two congruent isosceles triangles - two congruent isosceles right triangles - two congruent right triangles - two congruent acute triangles - two congruent obtuse triangles - two different isosceles triangles with congruent bases	1.5 h	• Combining Triangles (BLM) (optional) • Scissors	Observe and Assess questions

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UNIT 2 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
2.1.4 EXPLORE: Properties of Rectangles (Essential) **SB p. 43** **TG p. 68**	**5-E3 Diagonal Properties: squares and other rectangles** • develop generalizations for diagonals of squares and rectangles of each type below: - bisect each other (squares and rectangles) - intersect to form four right angles and four right isosceles triangles (squares) - intersect to form two pairs of congruent isosceles triangles (rectangles) - intersect to form two pairs of equal opposite angles (rectangles) - form two congruent angles with a sum of 90° at each vertex (squares) - form two non-congruent angles with a sum of 90° at each vertex (non-square rectangles)	1 h	• Paper • Scissors	Observe and Assess questions
Chapter 2 Transformations				
2.2.1 Properties of Translations **SB p. 45** **TG p. 70**	**5-E5 Translations and Reflections using horizontal and vertical reflection lines: generalize and apply properties** • understand that the translation image of a shape is congruent to the original shape and is oriented the same way	1 h	• Grid paper or Small Grid Paper (BLM) • Rulers	Q1, 2, 3, 6
2.2.2 Properties of Reflections **SB p. 48** **TG p. 73**	**5-E5 Translations and Reflections using horizontal and vertical reflection lines: generalize and apply properties** • understand that the reflection image of a shape is congruent to the original shape but faces the opposite way • understand that corresponding points of a shape and its reflected image are equidistant from the reflection line • understand that the line segment joining a point to its reflected image is perpendicular to the line of reflection • understand that a refection line bisects all line segments joining corresponding points at right angles	1 h	• Grid paper or Small Grid Paper (BLM) • Rulers	Q1, 2, 4
2.2.3 Parallel and Intersecting Lines **SB p. 51** **TG p. 76**	**5-E4 Parallelism and Perpendicularity: lines and line segments** • construct the following pairs of lines/line segments and use appropriate mathematical terminology: - parallel - intersecting - perpendicular at an endpoint - bisecting another line segment but not perpendicular - bisecting each other and perpendicular **5-E5 Translations and Reflections using horizontal and vertical reflection lines: generalize and apply properties** • understand that corresponding sides of the original shape and the translated image are always parallel	1.5 h	• Grid paper or Small Grid Paper (BLM) • Parallel and Intersecting Lines (BLM) • Rulers	Q2, 5, 6, 8

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	• understand that corresponding sides of the original shape and the reflection image are not always parallel • understand that parallel sides of the original shape are always parallel in the translation image			
2.2.4 Properties of Rotations **SB p. 56** **TG p. 80**	**5-E6 Rotations: quarter, half, and three-quarter rotations about the vertex of a shape** • predict, apply, and identify quarter ($\frac{1}{4}$), half ($\frac{1}{2}$), and three-quarter ($\frac{3}{4}$) rotations • explore the results using a variety of turn centres • understand that each point remains the same distance from the turn centre and the turn centre does not move	1.5 h	• A large cardboard circle, two congruent cardboard trapezoids with long side equal to circle radius, and a pushpin (optional) • Grid paper or Small Grid Paper (BLM) • Rulers	Q1, 3, 4, 8
CONNECTIONS: Kaleidoscope Images (Optional) **SB p. 60** **TG p. 84**	Make a connection between a real world toy and transformations	25 min	• Kaleidoscope Images (BLM) (optional)	N/A
Chapter 3 3-D Representations				
2.3.1 Prism and Pyramid Nets **SB p. 61** **TG p. 85**	**5-E7 Nets: prisms and pyramids** • create and interpret nets for various prisms and pyramids	1.5 h	• Grid Paper (1 cm by 1 cm) (BLM) • Sample Nets (BLMs): - Triangle-based Pyramid - Square-based Pyramid - Right Triangle-based Prism - Rectangle-based Prism - Regular Hexagon-based Prism	Q3, 4, 5
CONNECTIONS: Euler's Rule (Optional) **SB p. 65** **TG p. 88**	Make a connection between different aspects of 3-D shapes	25 min	Optional nets (BLMs): - Triangle-based Pyramid - Square-based Pyramid - Right Triangle-based Prism - Rectangle-based Prism - Regular - Pentagon-based Prism - Regular Hexagon-based Prism	N/A

UNIT 2 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
2.3.2 Interpreting Isometric Drawings **SB p. 66** **TG p. 89**	**5-E8 Isometric Drawings** • make and interpret drawings of structures made from cubes	1.5 h	• Linking cubes • Sample Net of Cube (BLM) (optional)	Q2, 4, 5
2.3.3 Creating Isometric Drawings **SB p. 70** **TG p. 92**	**5-E8 Isometric Drawings** • make and interpret drawings of structures made from cubes	1.5 h	• Isometric Dot Paper (BLM) • Linking cubes • Sample Net of Cube (BLM) (optional)	Q1, 4
UNIT 2 Revision **SB p. 73** **TG p. 95**	Review the concepts and skills in the unit	2 h	• Grid paper or Small Grid Paper (BLM) • Rulers • Sample Net of Square-based Pyramid (BLM) (optional) • Linking cubes (12 per student) • Sample Net of Cube (BLM) (optional)	All questions
UNIT 2 Test **TG p. 97**	Assess the concepts and skills in the unit	1 h	• Rulers • Grid Paper (1 cm by 1 cm) (BLM) • Linking cubes (10 per student) • Sample Net of Cube (BLM) (optional)	All questions
UNIT 2 Assessment Interview **TG p. 100**	Assess concepts and skills in the unit	15 min	See **p. 100**	All questions
UNIT 2 Performance Task **TG p. 101**	Assess concepts and skills in the unit	1 h	• Grid paper • Scissors • Isometric Dot Paper (BLM) (optional)	Rubric provided
UNIT 2 Blackline Masters **TG p. 104**	BLM 1 Small Grid Paper BLM 2 Grid Paper (1 cm by 1 cm) BLM 3 Special Rectangles (for **Getting Started**) BLM 4 Sample Net of Cube BLM 5 Triangle Dominoes Game Cards BLM 6 Combining Triangles (for **lesson 1.2.3**) BLM 7 Kaleidoscope Images (for **Connections**) BLM 8 Parallel and Intersecting Lines (for **lesson 2.2.3**) BLM 9 Sample Net of Triangle-based Pyramid BLM 10 Sample Net of Square-based Pyramid BLM 11 Sample Net of Right Triangle-based Prism BLM 12 Sample Net of Rectangle-based Prism BLM 13 Sample Net of Regular Pentagon-based Prism BLM 14 Sample Net of Regular Hexagon-based Prism BLM 15 Isometric Dot Paper			

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Math Background

• In this geometry unit, material students have studied in previous years is presented in greater depth.

• The focus of the unit is on discovering and relating properties of triangles and quadrilaterals, investigating properties of transformations, and representing 3-D objects in two dimensions.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 6** in **lesson 2.1.1**, where they classify a triangle based on limited information, in **question 5** in **lesson 2.2.1**, where they figure out how one translation rule relates to another given only other relationships, and in **question 6** in **lesson 2.2.4**, where they figure out which transformation moves a shape from one location to another.

• They use communication in **question 7** in **lesson 2.1.1**, where they explain why they agree or disagree with the triangle classifications shown, in **question 7** in **lesson 2.2.3**, where they explain how they know certain properties of transformations are true, and in **question 4** in **lesson 2.2.4**, where they describe examples of perpendicular and parallel line segments in a rachu design.

• They use reasoning in answering questions such as **question 6** in **lesson 2.2.1**, where they figure out which image might be a translation image, and in **question 5** in **lesson 2.3.2**, where they consider the number and location of hidden cubes in an isometric drawing.

• They consider representation in all of **chapter 3**. In **lesson 2.3.1**, 3-D prisms and pyramids are represented by 2-D nets. In **lessons 2.3.2 and 2.3.3**, cube structures are represented by isometric drawings. Students also consider representation in **lesson 2.2.1**, where they look at alternate representations of a translation.

• Students use visualization skills throughout **chapter 1**, where they classify triangles at first visually, and later using properties. In **chapter 2**, they consider the action of translations, reflections, and rotations. In **question 4** in **lesson 2.2.2**, they visualize what the reflection image will look like. They also use visualization skills throughout **chapter 3** as they picture what a net will look like when it is folded and when they compare isometric drawings to each other and to cube structures.

• They make connections in situations like those in **lesson 2.1.4**, where they use the classification of triangles from earlier in the chapter to discover the properties of the diagonals of quadrilaterals. In **questions 5 and 6 b)** in **lesson 2.2.3**, they connect parallel and perpendicular line segments to properties of reflections and translations. In addition, there are numerous real-world connections, such as **question 5** in **lesson 2.1.1** (textiles), **question 4** in **lesson 2.1.2** (roof shapes), **question 3** in **lesson 2.2.3** (classroom), and **question 4 a)** in **lesson 2.2.3** (rachu design).

Rationale for Teaching Approach

• This unit is divided into three chapters.

Chapter 1 is about the properties of triangles and quadrilaterals.

Chapter 2 focuses on the properties of transformations.

Chapter 3 examines various ways to represent a 3-D structure in two dimensions.

• There are two **Explore** lessons about polygons. The first focuses on visualization as students predict the result of combining various kinds of triangles. The second uses the properties of triangles students have already studied in the chapter to discover properties of quadrilaterals. Both of these topics are handled as explorations because this is the only effective way for students to learn these ideas.

• This unit has two **Connections**, both historical in nature. One relates transformations to a toy that is still popular today. The other highlights an interesting fact about prisms and pyramids that was discovered long ago.

• The **Game** provides an opportunity to apply and practise classifying triangles. It highlights the two different ways that triangles can be classified.

• Throughout the unit, it is important to encourage students to develop their visualization skills.

Getting Started

Curriculum Outcomes	Outcome relevance
4 Angles: (meaning) amount of turn **4** Isometric Drawings **4** Triangles: discover properties, name, construct (concretely) **4** Prisms, Pyramids, Cones, Cylinders **4** Slides, Flips, Turns (half, quarter): predict and confirm results for 2-D shapes **4** Angles: acute, obtuse **4** Congruence: polygons	Students will find the work in the unit easier after they review the concepts and skills related to geometry they learned in Class IV.

Pacing	Materials	Prerequisites
1 h	• Special Rectangles (BLM) or 104 mm-by-120 mm paper rectangles • Linking cubes (8 per group) • Sample Net of Cube (BLM) (optional)	• familiarity with the terms *obtuse angle, right angle, acute angle, equilateral triangle, isosceles triangle, scalene triangle, congruent shapes, flip, turn,* and *slide* • familiarity with the names of prisms and pyramids • recognizing a net for a given rectangular prism • building cube structures given a drawing

Main Points to be Raised

Use What You Know

• We can name angles for their size:

- Right means the angle is a square corner.

- Acute means the angle is smaller than a right angle.

- Obtuse means the angle is bigger than a right angle.

• Congruent means "the same".

• We can name triangles for the number of congruent sides:

- Equilateral means all sides are congruent.

- Isosceles means two sides are congruent.

- Scalene means no sides are congruent.

• Turn, flip, and slide are words to describe the actions that move one congruent shape to another.

Skills You Will Need

• Slides, flips, and turns do not change the size of a shape.

• A net is a 2-D pattern that can be folded to make a 3-D object.

• By examining the faces of prisms and pyramids, you can figure out its name.

Use What You Know — Introducing the Unit

• Students can work in pairs or small groups.

• First review the terms *obtuse angle, acute angle, right angle, congruent, equilateral triangle, isosceles triangle, scalene triangle, slide, flip,* and *turn* to make sure students can successfully interpret **part E**. Refer students to the glossary at the back of the book.

• Distribute the Special Rectangles (from the BLM) or 104 mm-by-120 mm rectangles, one per student. Ask students to work through **parts A to E**.

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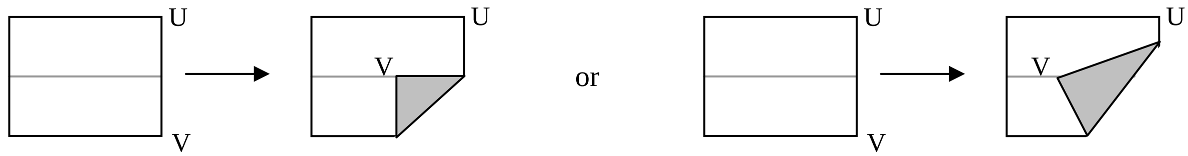
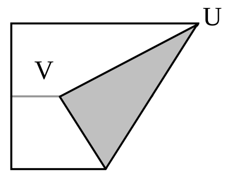
• Students may need help with **part B**. Some students may try to make a different fold, such as:



Incorrect fold

Incorrect fold

Be sure to stress that V must be on the fold line and the triangle must have U as a vertex:



Correct fold

• For **part E**, encourage students to mark their answers right on their rectangle. If they run out of room, they can use the other side as well.

Observe students as they work. As they work, you might ask questions such as the following:

• *How could you tell this angle was obtuse?* (I compared it to the corner of the rectangle and it was bigger than the corner.)

• *How can you tell that these side lengths are the same?* (I folded one side on top of the other so I could compare.)

• *How did you know this was a flip and not a slide or a turn?* (I could picture flipping it over, but not sliding it along or turning it.)

Skills You Will Need

• Students can work individually.

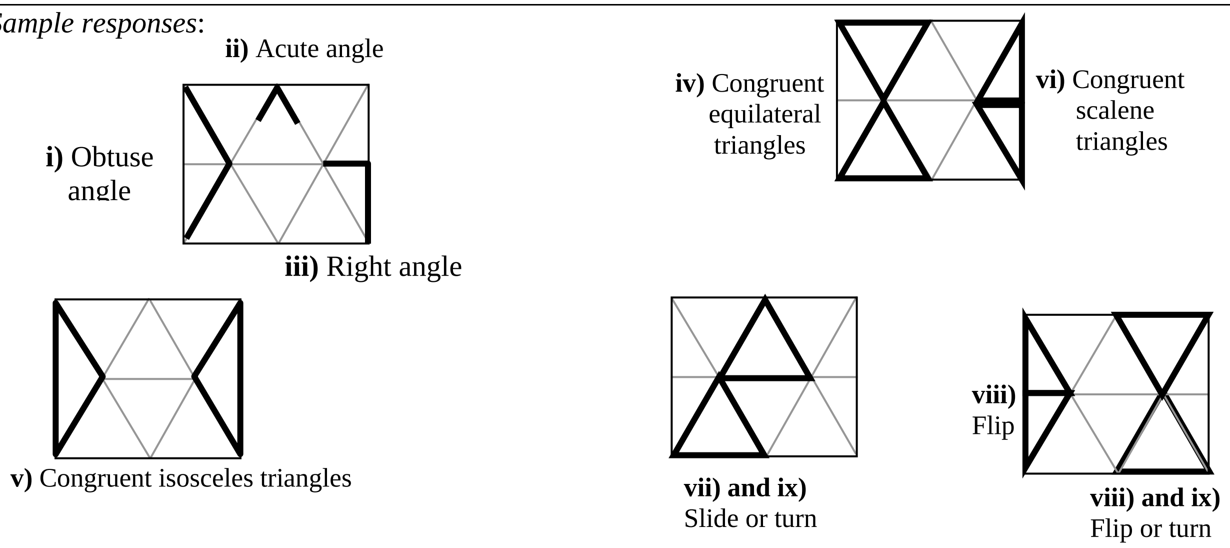
• To ensure students have the required skills for this unit, assign these questions.

• First review the terms *rectangular prism, prism, and pyramid* to make sure students can successfully interpret **questions 3 and 4**. Refer students to the glossary at the back of the book.

• Students may use solid cubes or cubes made from the Sample Net of Cube (BLM) to answer **question 5**. They will not be able to record their answers, but they may compare their structures to those of a partner to check their work.

Answers

E. *Sample responses*:



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Answers [Continued]

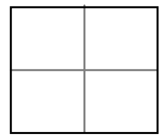
1. A and D **2. a)** C and N; D and F **b)** H and M **c)** G and E; G and K; E and K; F and L; D and L	**3.** A **4. a)** Triangle-based prism (or triangular prism) **b)** Pentagon-based pyramid (or pentagonal pyramid) **c)** Hexagon-based prism (or hexagonal prism) **d)** Square-based pyramid (or square pyramid)

Supporting Students

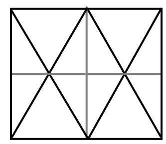
Struggling students

• If students are struggling with the folding in **parts B and C**, you might suggest that they draw the Xs onto the paper.

For example, have them fold the paper in half the other way:



They can then use a ruler to draw the fold lines:



• If students have difficulty with **question 3**, you might have them trace the nets, then cut them out and fold to check.

Enrichment

• For **part E**, you might challenge students to find as many answers as possible for each.

2.1.1 Classifying Triangles by Side Length

Curriculum Outcomes	Outcome relevance
5-E1 Triangles: explore equilateral, isosceles, scalene triangles, and right, obtuse, and acute triangles • discover properties of equilateral, isosceles, and scalene triangles through concrete experiences • sort and classify triangles by side lengths	• The abundance of triangles in the real world makes the study of their properties relevant and important. • Understanding why the properties that describe triangles are true makes it easier to remember the properties and to apply knowledge in problem-solving situations.

Pacing	Materials	Prerequisites
1 h	• Sticks of three sizes (three or more of each length per student or pair) • Rulers	• familiarity with the terms *congruent*, *line of symmetry*, *equilateral triangle*, *isosceles triangle*, and *scalene triangle*

Main Points to be Raised

• Every triangle can be classified as equilateral, isosceles, or scalene by comparing its side lengths.

• There is a relationship between the number of congruent sides, the number of congruent angles, and the number of lines of symmetry in a triangle.

• You can use the relationships among side lengths, angle measures, or the number of lines of symmetry to identify the type of triangle:

- An equilateral triangle has three congruent sides, three congruent angles, and three lines of symmetry.

- An isosceles triangle has two congruent sides and two congruent angles, but only one line of symmetry.

- A scalene triangle has no congruent sides, no congruent angles, and no lines of symmetry.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. If only three of each length of stick are available, have students trace each triangle as they make it. Encourage them to make different triangles by using different combinations of stick lengths.

B. Observe while students work. While they work, you might ask questions such as the following:

• *How are these triangles alike?* (They both use two sticks of the same length.)

• *What do you notice about the shapes of the triangles that have all three sides made with sticks of the same length?* (The shape is always the same but the size is different.) *Is that true for the triangles where only two sticks have the same length or all different lengths?* (No. Those triangles have different shapes.)

If students group the triangles using criteria other than side length, you might point out that the lengths of the sticks are important for this activity.

The Exposition — Presenting the Main Ideas

• Show the class a large equilateral triangle made of paper and tell them that all of its sides are the same length. Ask how they could verify this (by measuring or by folding the paper so the sides line up).

• Then tell the class that all the angles are the same size. Again, ask how they verify this (by folding the paper so the angles line up or by tracing an angle and comparing the others to it).

• Have the students look at the exposition on **pages 33 and 34**.

- Discuss the meaning of congruent sides and angles and how they are marked. Ask a volunteer to come to the board to sketch the paper triangle and mark the congruent sides and angles.

- Next, point out the definitions of equilateral, isosceles, and scalene triangles. Tell students that the paper triangle they have already seen is equilateral. Show similar examples of large paper isosceles and scalene triangles and sketch them on the board.

[Continued]

- Mark the lines of symmetry on the paper triangles. As you work through the exposition with the class, demonstrate how to fold along the lines of symmetry. Ask students to come to the board to mark the sides and angles to show congruence on the sketches of the isosceles and scalene triangles.

• Draw students’ attention to the chart on **page 34** to help them summarize what they have learned.

Revisiting the Try This

B. This question allows students to make a formal connection between what was done in **part A** and the main ideas presented in the exposition.

Using the Examples

• Have students work in pairs. One of the pair should become an expert on **example 1** and the other should become an expert on **example 2**. They should each explain their example to the other student.

• Point out that if they were the ones answering the question, they would be expected to write down the work, like what they see on the left under **Solution**, but would be thinking what they read on the right under **Thinking**.

Practising and Applying

Teaching points and tips

Q 1: This question provides a summary of the main ideas of this lesson. Students can refer to it while work they through the exercises. It may also be helpful to make, or have students make, a large copy of the chart to display in the classroom.

Q2: You might encourage students first to predict the answers. Then they can sketch the triangles and label the side lengths as they measure.

Q 3: Students can check their answers by copying the triangles and then folding to compare the angles, or by tracing one angle to compare it to another.

Q 4: For this question, students can check their answers by measuring the side lengths or by tracing the triangles and folding to check for congruent sides.

Q 5: This question provides an interesting link to everyday life. Some of the triangles are not exactly equilateral, but this can be accounted for by stretch in the fabric. Approximate answers are acceptable.

Q 7: Use this last question as a closure question. It is a way to highlight some of the important ideas of the lesson.

Common errors

• Some students will think that the triangle in **question 2 c)** is isosceles rather than scalene. To address this, you might encourage students to measure carefully. It can also help to turn the page so that the shortest side of the triangle is closest to they student, making the lack of symmetry more apparent.

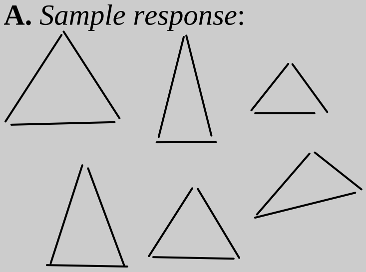
• Some students may miss the significance of the word “exactly” in **question 4 c)** and in **question 6**. To help avoid this, discuss these questions briefly, emphasizing the word “exactly”.

Suggested assessment questions from Practising and Applying

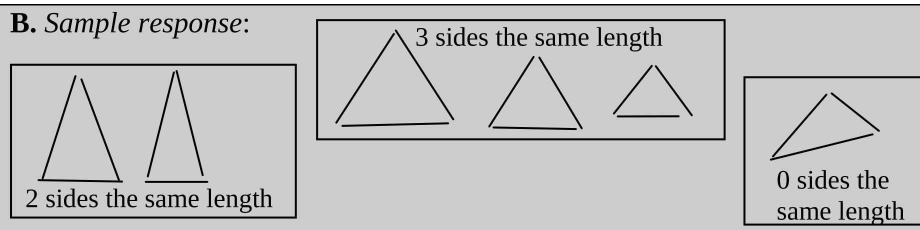
Question 1	to see if students recognize the properties of equilateral, isosceles, and scalene triangles
Question 2	to see if students can apply the definitions of equilateral, isosceles, and scalene
Question 3	to see if students understand the relationship between congruent sides and congruent angles
Question 6	to see if students can apply what they have learned to solve a mathematical problem

Answers

A. *Sample response*:



B. *Sample response*:



C. *Sample response*:
3 were equilateral; 2 were isosceles; 1 was scalene.

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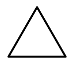
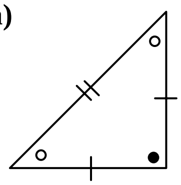
Answers [Continued]

1.

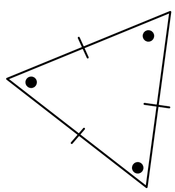
	Equilateral triangle	**Isosceles triangle**	**Scalene triangle**
Number of congruent sides	3	2	0
Number of congruent angles	3	2	0
Number of lines of symmetry	3	1	0
Sketch of example			

2. a) Isosceles **b)** Scalene **c)** Scalene

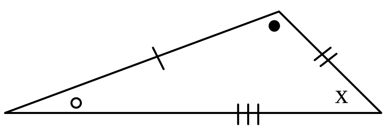
3. a)



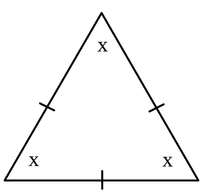
b)



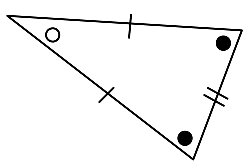
c)



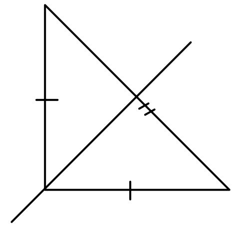
4. a)



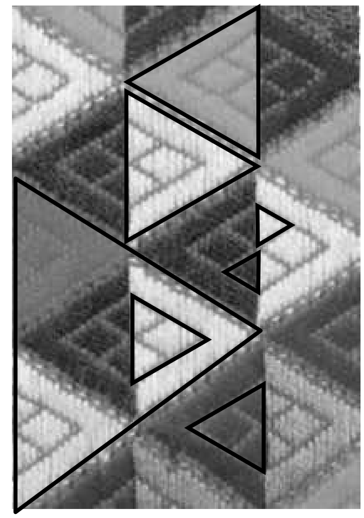
b)



4. c)

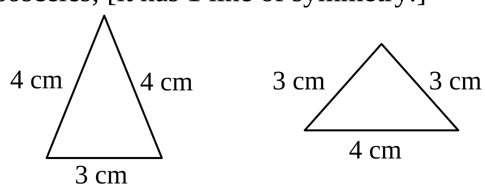


5. Equilateral (of different sizes)



6. a) Isosceles; [it has 1 line of symmetry.]

b)



7. a) Yes; [the side lengths are all different.]
b) Yes; [all 3 sides have the same length.]
c) No; [only 2 sides have the same length, so the triangle is isosceles.]

Supporting Students

Struggling students

• Students who are struggling with the concepts of this lesson may find it helpful to keep a "Triangle Journal". Have them write the titles "Equilateral", "Isosceles", and "Scalene" at the top of three pages. They can trace examples onto the appropriate pages as they work through the exercises with a partner. Ask them to mark congruent sides, congruent angles, and lines of symmetry.

Enrichment

• For **question 5**, you might challenge students to find other shapes in the pattern (trapezoids, hexagons, rhombuses, and parallelograms). This will provide a link to **lesson 2.1.3**, where different shapes are made by combining two triangles.

• Students can work in pairs. They each create their own version of **question 7**, then trade with a partner to solve.

2.1.2 Classifying Triangles by Angle

Curriculum Outcomes	Outcome relevance
5-E1 Triangles: explore equilateral, isosceles, scalene triangles, and right, obtuse, and acute triangles • discover properties of right, obtuse, and acute triangles • sort and classify triangles by angle size and side lengths • develop a personal referent for 90° (right) angles	The abundance of triangles in the real world makes the study of their properties not only relevant but important. This lesson builds on the previous lesson by investigating another way that triangles can be classified.

Pacing	Materials	Prerequisites
40 min	• Grid paper or Small Grid Paper (BLM)	• familiarity with the terms *acute angle*, *right angle*, and *obtuse angle*

Main Points to be Raised

• Classifying by angle is another way to sort triangles that is different from classifying by side length.

• Since a triangle can have at most one right angle or one obtuse angle, two angles in every triangle must be acute.

• To classify triangles by angle, you only have to look at the greatest angle.

• You can compare the greatest angle in a triangle to a right angle (like the corner of a page) to determine whether it is right, obtuse, or acute.

Try This — Introducing the Lesson

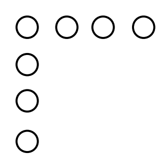
A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *How does the grid help you draw right, acute, and obtuse angles?* (You can tell whether the angle is greater or less than a right angle by comparing it to the grid lines.)

• *How can you tell what the other angles in the triangle are?* (Sometimes you can use the grid lines to help you see. Other times, the angle is not lined up with the grid in a way that helps you. When that happens, I first look at it and predict. Then I use the corner of another sheet of paper to check my prediction.)

If students have difficulty drawing the initial angles, you might encourage them to draw one arm of the angle on the grid, as shown in the text.

The Exposition — Presenting the Main Ideas



• Have seven students stand up to form a right angle (position them as shown by the circles to the right). Explain that they represent an angle in a triangle. Challenge three more students to meet the others at a right angle to try to form the third side of a triangle. Ask them why this cannot be done (the sides will not meet to form a triangle).

• Repeat the exercise for one right angle and one obtuse angle, then for two obtuse angles. Again, ask students to explain why it is not possible (the sides would not meet).

• Talk through the exposition on **page 37** together. Draw attention to the fact that you only need to know the greatest angle to classify a triangle by angle. Every triangle has two acute angles and one other angle, which can be obtuse, acute, or right. It is this other angle that gives the name to the triangle.

Revisiting the Try This

B. Students should realize that each triangle has two acute angles and one other angle (another acute angle, a right angle, or an obtuse angle).

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Using the Examples

• Work through **example 1** with the students to make sure they understand it.

• Ask pairs of students to read through **solutions 1 and 2** of **example 2**. Ask them to choose the solution that most closely matches what they would have done and tell why they would have chosen to do it that way.

Practising and Applying

Teaching points and tips

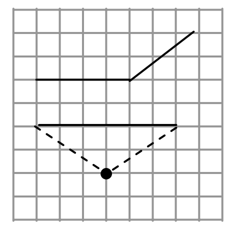
Q 1: This question provides a summary of the main ideas of this lesson. Students can refer to it while they work through the exercises. It may also be helpful to make, or have students make, a large copy of the chart to display in the classroom. Students need to realize that line 1 requires them to write a type of angle, not a measurement.

Q 4: You might encourage students to draw the triangles oriented the way a roof would be to make it easier for them to reach the appropriate conclusions.

Q 5: You may wish to assign this question only to selected students. Suggest that the students who attempt it begin with the angle and then draw the side lengths.

Common errors

• Many students will draw the obtuse angle in **question 5 b)** with one arm on the grid, as was modelled in the **Try This**. This makes it difficult to use the grid paper to make the other arm the same length. You might direct them instead to draw the non-congruent side on a gridline and make it an even number of units long. They can choose a point for the opposite vertex on the gridline that passes through the centre of that side.

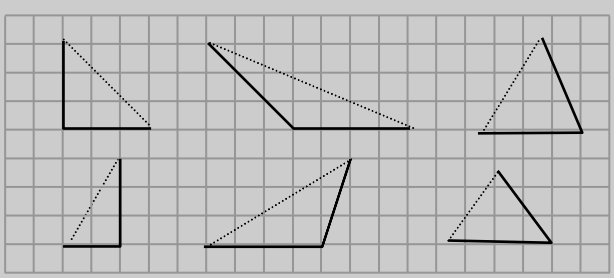


Suggested assessment questions from Practising and Applying

Question 1	to see if students recognize the properties of obtuse, right, and acute triangles
Question 2	to see if students can apply the definitions of obtuse, right, and acute triangles
Question 3	to see if students understand side length classification, angle classification, and the differences between them
Question 4	to see if students can relate their understanding to a real-world situation

Answers

A. i) *Sample response*:

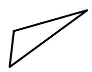
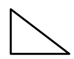


Right angle Obtuse acute Acute angle

ii) All other angles were acute.

B. *Sample response*:
For acute triangles, all of the angles are acute. Triangles can have only one right angle or one obtuse angle for the three sides to make a closed shape. Once the right angle or obtuse angle has been drawn, the other angles have to be acute.

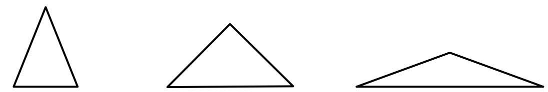
1.

	Obtuse triangle	**Right triangle**	**Acute triangle**
Greatest angle	Obtuse	Right	Acute
Number of obtuse angles	1	0	0
Number of right angles	0	1	0
Number of acute angles	2	2	3
Sketch of example			

2. a) Obtuse **b)** Right **c)** Acute

3. a) Acute, equilateral **b)** Acute, isosceles
c) Right, scalene **d)** Obtuse, isosceles

4. An acute triangle; [*Sample response*:
in an isosceles triangle roof, a right or obtuse angle has to be at the peak because there can be only 1 right or obtuse angle in the triangle. An acute angle at the peak makes the steepest sides.]

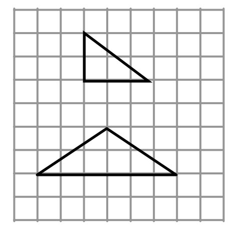


Acute Right Obtuse

5. *Sample responses*:

a)

b)



[**6.** *Sample response*:
An obtuse triangle has one obtuse angle and two acute angles. An isosceles triangle has two congruent angles. Since there is only one obtuse angle, the congruent angles must be acute.]

Supporting Students

Struggling students

• If students are struggling with **question 2**, you might reinforce that they should look only at the greatest angle in the triangle. Comparing it to the corner of a ruler or a paper can help them decide whether it is a right angle, greater than a right angle, or smaller than a right angle. A memory aid can help them remember the meanings of the words obtuse and acute.

For example, *obtuse* is a bigger word than *acute*, just as an obtuse angle is greater than an acute angle.

Enrichment

• You might challenge students to take **question 5** a step further and investigate different combinations of angle and side length classifications. They can use sticks, as in the **Try This**, or grid paper to try to create all possible combinations. (There are only seven combinations because an equilateral triangle must be acute).

GAME: Triangle Dominoes

• This optional game allows students to practise classifying triangles by side length and by angle.

• Encourage students to consider carefully all of their cards and both types of classification before deciding to draw a card from the deck.

• Here is a variation on the game:

- Deal out all of the cards to two, three, or four players.
- Each player arranges his or her cards in a separate row, trying to match triangles as in the original game.
- Score as follows: 2 points for triangles that match by side length and by angle, 1 point for triangles that match by only one of side length or angle, 0 points for triangles that do not match in either way.
- The player with the greatest score wins.

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2.1.3 EXPLORE: Combining Triangles

Curriculum Outcomes	Outcome Relevance
5-E2 Combine Triangles: spatial sense and visualization • use visualization to predict the results of combining triangles • develop spatial sense by combining - two congruent equilateral triangles - two congruent isosceles triangles - two congruent isosceles right triangles - two congruent right triangles - two congruent acute triangles - two congruent obtuse triangles - two different isosceles triangles with congruent bases	This essential exploration builds on the previous two lessons. It is designed to enhance spatial sense and visualization skills.

Pacing	Materials	Prerequisites
1.5 h	• Combining Triangles (BLM) (optional) • Scissors	• classifying triangles by side length and by angle • identifying congruent shapes

Exploration

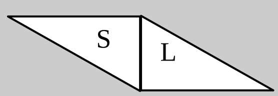
• Read through the introduction (in white) with the students. Make sure that they understand that each triangle has a matching congruent triangle. Point out an example, such as triangles L and S, and ask the class to give more examples of congruent pairs.

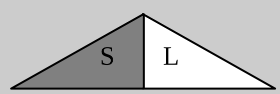
• Have students work in pairs or in small groups. Distribute scissors and a copy of Combining Triangles (BLM) to each group for students to complete **part A** (or have students trace the triangles in the text).

• Discuss **part B** with the students to make sure they understand what to do. You may wish to give them an example of the different ways to combine congruent triangles.

For example, each pair of congruent sides can be matched in two possible ways.

Both triangles "face up"





One triangle "face down"

• Be sure to emphasize that **part B** asks for as many different (that is, non-congruent) shapes as possible. Because of this, it might be a good idea to divide the work amongst the group members.

Observe while students work. While they work, you might ask questions such as the following:

• *How did you combine the triangles to make a new shape?* (I matched sides that were the same length.)

• *If you flip both triangles face down, will you get a new shape?* (No. I get a shape that is congruent to shape with both triangles face up.)

• *Does flipping a triangle face down always give a new shape?* (No. Equilateral and isosceles triangles look the same whether they are face up or face down.)

Observe and Assess

As students work, notice the following:

• Do they label the triangles correctly?

• Do they understand the various ways to combine the triangles?

• Do they recognize when answers are congruent shapes?

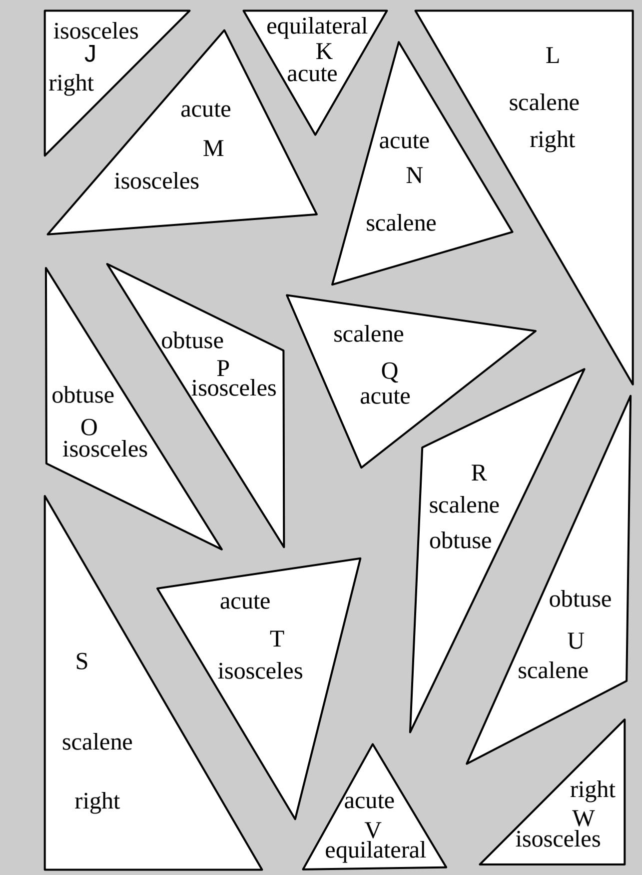
Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to compare and discuss their solutions and to answer these questions.

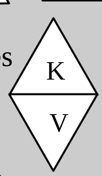
- *How do you know how to label the triangles?*
- *Which types of triangles made the fewest different shapes?* (Equilateral) *Which types made the most?* (Scalene)
- *Can you combine a pair of triangles to form any shape other than a triangle or a quadrilateral?* (No)

Answers

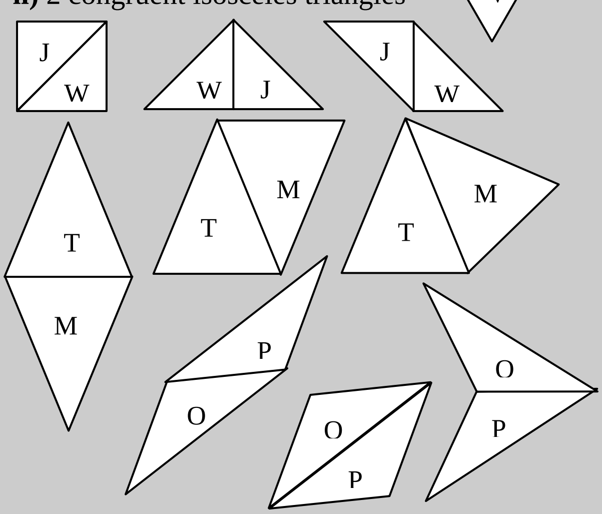
A. ii) and iii)



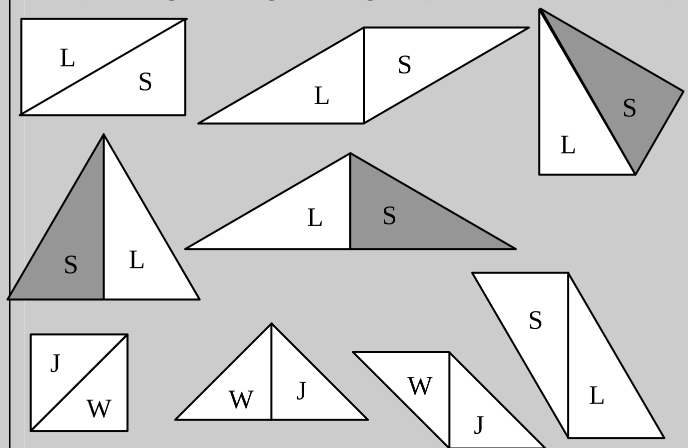
B. i) 2 congruent equilateral triangles



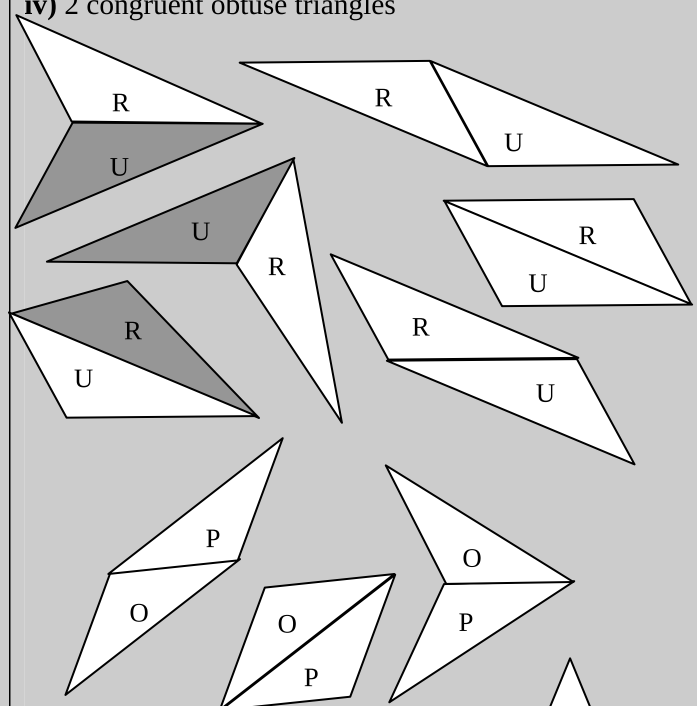
ii) 2 congruent isosceles triangles



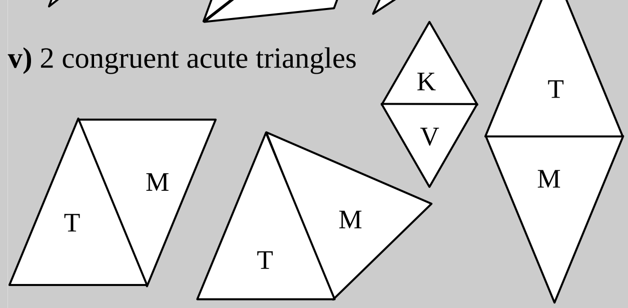
B. iii) 2 congruent right triangles (face down is shaded)



iv) 2 congruent obtuse triangles



v) 2 congruent acute triangles



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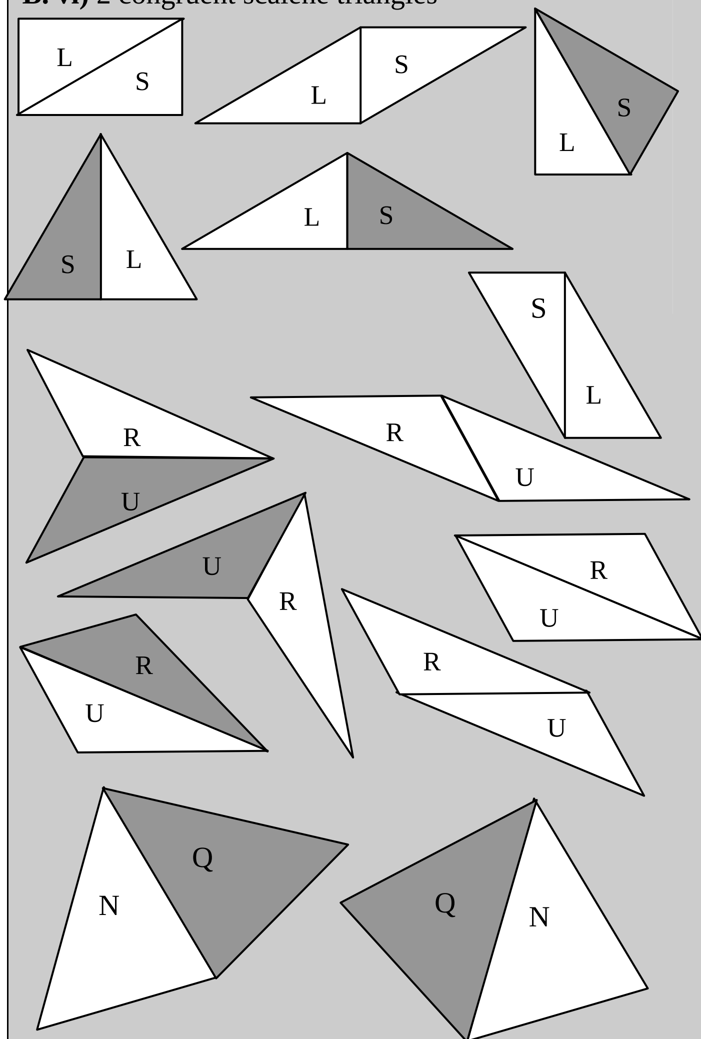
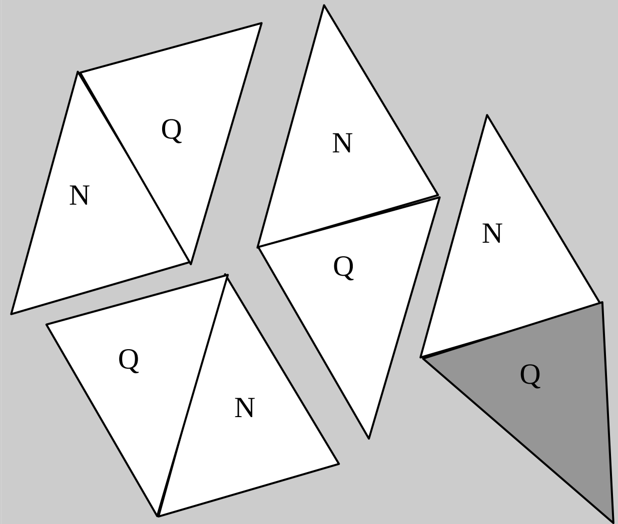
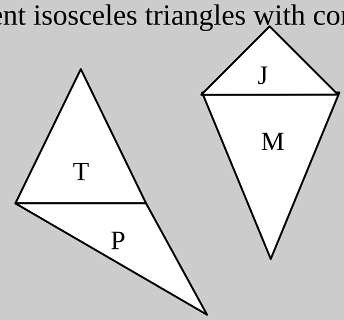
B. vi) 2 congruent scalene triangles





2 different isosceles triangles with congruent bases



C. i) Right isosceles
ii) Right scalene or right isosceles (a square is a rectangle)
iii) All except different isosceles triangles with congruent bases
iv) Right

Supporting Students

Struggling students

• For **part A**, some students might benefit from using charts that show the different side length classifications and angle classifications for triangles. If necessary, remind them that they can fold the triangles to see whether sides are congruent. They can also compare any angle with the right angle at the corner of a page or a ruler.

• If students are struggling to keep track of the ways they have combined the triangles in **part B**, you might suggest colouring each side according to its length. Students can work through the possibilities colour by colour, matching congruent sides one way, then flipping one of the triangles over for another way. It is acceptable for them to find most, but not all, of the combinations.

• Some students may have trouble sketching the shapes in **part B**. Encourage them to trace the shapes if necessary.

Enrichment

• Ask students who enjoy a challenge to answer this question:

Why can only right triangles be combined to make squares or other rectangles? (Two of the quadrilateral angles are also angles of the triangles used; squares and rectangles have only right angles.)

2.1.4 EXPLORE: Properties of Rectangles

Curriculum Outcomes	Outcome Relevance
5-E3 Diagonal Properties: squares and other rectangles • develop generalizations for diagonals of squares and rectangles of each type below: - bisect each other (squares and rectangles) - intersect to form four right angles and four right isosceles triangles (squares) - intersect to form two pairs of congruent isosceles triangles (rectangles) - intersect to form two pairs of equal opposite angles (rectangles) - form two congruent angles with a sum of 90° at each vertex (squares) - form two non-congruent angles with a sum of 90° at each vertex (non-square rectangles)	This essential exploration builds on the previous lessons. Rectangles (including squares) are divided along their diagonals into triangles. The properties of the triangles are then used to discover properties of the rectangles.

Pacing	Materials	Prerequisites
1 h	• Paper • Scissors	• recognizing congruent triangles • classifying triangles by side length and by angle

Exploration

• Read the introduction (in white) with the students. If necessary, discuss with students how to identify obtuse, right, and acute angles. Also discuss how to classify triangles by side length and by angle.

• Have students work alone, in pairs, or in small groups for **parts A to D**.

Observe while students work. While they work, you might ask questions such as the following:

• *How can you tell that these triangles are congruent?* (I put one on top of the other and I could see that they were exactly the same shape.)

• *How do you know the diagonals of the square make right angles where they cross?* (I know the triangles are right triangles from part B. I can also compare the angles to the corner of my ruler or page.)

• *How did the triangles change when you worked with the non-square rectangle?* (Instead of getting four triangles that were the same, I got two pairs of congruent triangles.)

Discuss **parts A to D** with the students to make sure they understand what to do. Then ask them to complete the rest of the exploration.

Observe and Assess

As students work, notice the following:

• Do they successfully classify the triangles by side length and by angle?

• Do they use the properties of the triangles to answer **parts C and D**?

• Do they recognize the similarities and differences in their answer to **part F**?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and answer these questions.

• *How do the properties of the triangles help describe the diagonals of the squares and rectangles?*

• *Why do the two angles at the corners always make a right angle?*

• *Do you think the results would be the same for any square or non-square rectangle?*

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Answers

Square **B. i)** They are all isosceles. **ii)** They are all right triangles. **iii)** They are congruent. **C. ii)** They form a right angle and they are congruent. **D. i)** At the centre. The triangles are congruent isosceles so the sides that are inside the square are all the same length. **ii)** Right. I measured against the corner of my paper.	**F.** Different: • Squares have four congruent right isosceles triangles but non-square rectangles have two pairs of congruent isosceles triangles (one pair are acute and the other pair are obtuse). • For a non-square rectangle, the two angles at each corner are not congruent but for a square they are congruent. Same: • The diagonals of squares and rectangles intersect at their centres. • The two angles at each corner of the rectangle and square form a right angle.
E. *Non-square Rectangle* **B. i)** They are all isosceles. **ii)** a and c are obtuse; d and b are acute. **iii)** Two pairs of congruent triangles. **C. ii)** They form a right angle. **D. i)** At the centre. The triangles are congruent isosceles. **ii)** Acute and obtuse. I can see that the triangles are acute and obtuse.	

Supporting Students

Struggling students

• If students are struggling to classify the triangles in **part B**, you might suggest that they fold them to compare side lengths and angles. It may also be helpful to mark congruent angles and sides and then label each triangle with its side length and angle classification.

• Some students may have trouble summarizing their results in **part F**. It may be helpful to have the class work together to create a chart with the headings "Same" and "Different". You may wish to put up the chart in the classroom.

Enrichment

• For an extra challenge you might ask students to repeat **parts A to D** for other quadrilaterals, such as kites, parallelograms, rhombuses, and trapezoids.

2.2.1 Properties of Translations

Curriculum Outcomes	Outcome relevance
5-E5 Translations and Reflections using horizontal and vertical reflection lines: generalize and apply properties • understand that the translation image of a shape is congruent to the original shape and is oriented the same way	Transformations such as translations and reflections are relevant because they are found everywhere in the world around us.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM) • Rulers	• familiarity with the terms *congruent* and *slide* • recognizing congruent shapes

Main Points to be Raised

• A slide is a type of transformation. It is also known as a translation.

• A translation moves each point in a shape in exactly the same way.

• The image of a translation is congruent to the original shape. It points the same way.

• A translation rule tells how far left or right and how far up or down the original shape moves.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Be sure to emphasize that they must describe how *each* vertex moves. Observe while students work. While they work, you might ask questions such as the following:

• *What do you notice about your answers for each vertex?* (They are all the same.)

• *Can you use one vertex to describe how the whole shape moves?* (Yes. If you know how one vertex moves, you know how they all move so you know how the whole shape moves.)

If students have difficulty describing the slide, you might suggest that they think of Shape W moving first to one of the other white congruent shapes and then to Shape X.

The Exposition — Presenting the Main Ideas

• Point out to students that the slides shown in the blanket are real-world examples of mathematics. Ask why it might be useful to be able to describe a slide precisely (for example, so someone can make the same blanket with the same pattern). Ask for other examples of slides in the real world (for example, other textile patterns, painted wall or building decorations, or games like Carrom if you think of the position of the striker before and after a shot).

• Read through the exposition with students. Make sure they realize that the word "unit" is optional and that they may omit it from translation rules. If either number of units is 0, they can also omit that part of the description.

• If it comes up, or you feel that your students have a strong understanding of the material, you might discuss that it is not incorrect to list the movement in reverse order — up or down, then left or right. Explain that we usually show the movement as left or right, then up or down because this format corresponds to work they will do in future years.

• Provide an opportunity for students to ask questions if they do not understand.

Revisiting the Try This

B. Students apply the idea that each point moves the same amount left or right and up or down to determine that each point moves the same distance. They can measure the line segments to verify the answer.

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Using the Examples

• Ask pairs of students to read through the examples together. For **solutions 1 and 2** of **example 2**, ask them to choose which solution more closely matches what they would have done and to explain why.

Practising and Applying

Teaching points and tips

Q 1: Remind students to describe translations in two steps — How far left or right? How far up or down?

Q 2: Some students may have difficulty drawing the shapes accurately on grid paper. You might suggest that they draw diagonal lines one square at a time, going corner to corner in each square.

Q 4: You may wish to assign this question only to selected students. If students need a hint to get started, suggest that they begin by drawing vertex A.

Q 5: You might encourage students to use an example to help them understand the translations described.

Q 6: You might ask students what it means for shapes to be congruent and how a translation moves each vertex.

Common errors

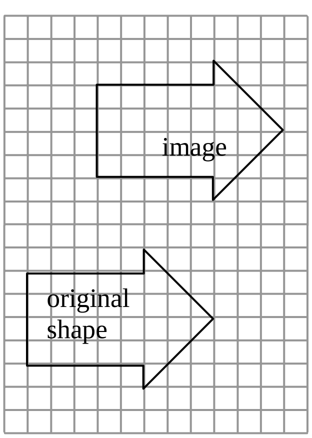
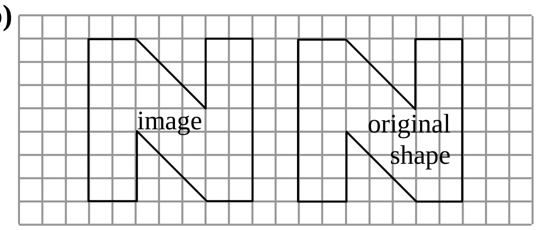
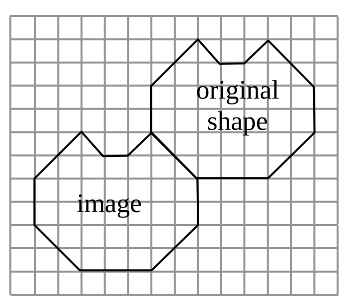
• Many students will place the original shape in a poor position on the grid paper in **questions 2, 3, and 4**. To make sure they have room on the grid paper to include the image, you might discuss this problem before assigning the questions. Encourage students to think ahead before drawing the original shape.

Suggested assessment questions from Practising and Applying

Question 1	to see if students understand how to write a translation rule
Question 2	to see if students can perform a translation given the translation rule
Question 3	to see if students can perform and describe multiple translations
Question 6	to see if students can explain how to recognize translation images

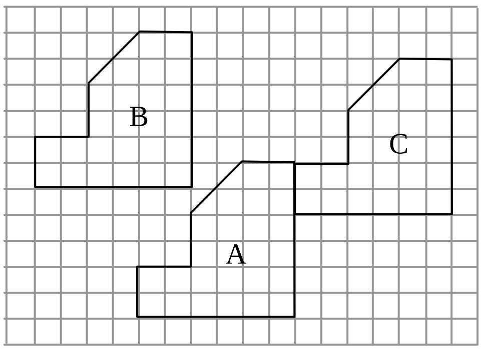
Answers

• *Each translation rule is written as the number of units left or right followed by the number of units up or down. It is not incorrect to list them in the opposite order.*
• *The use of the word "unit" when describing a translation rule is optional. For example, "5 right, 3 up" and "5 units right, 3 units up" are both correct.*

A. *Sample response*: 7 right and 6 down	**B.** Yes. Each point moves the same distance and direction.
1. a) 7 right and 3 down **b)** 1 right and 8 down **c)** 6 left and 5 down **d)** 1 left and 8 up **2. a)** 	**2. b)**  **c)** 

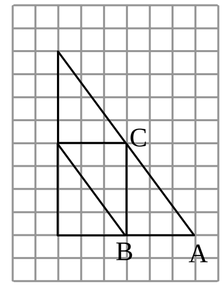
Answers [Continued]

3. a) and b)



c) 6 left and 4 down

4. a), b), and c)



d) 4

5. Choki’s rule doubles Nima's numbers and goes in the opposite direction.
[*Sample response*:
If Nima's rule were 7 left and 3 down, Choki’s rule would be 14 right and 6 up.]

6. D; [*Sample response*:
B and C point the wrong way. E is not congruent.]

Supporting Students

Struggling students

• For **questions 2 and 3**, some students may benefit from using a cut-out copy of the original shape to slide and trace.
• For **question 6**, some students may benefit from using a chart that lists the identifying properties of a translation:

- The image points the same way as the original shape.
- Each point moves in exactly the same way.

Enrichment

• For **question 4**, you might ask students to explain how they know the middle triangle is congruent and how they know it is not a translated image. Challenge them to translate a different shape to create a similar design.

2.2.2 Properties of Reflections

Curriculum Outcomes	Outcome relevance
5-E5 Translations and Reflections using horizontal and vertical reflection lines: generalize and apply properties • understand that the reflection image of a shape is congruent to the original shape but faces the opposite way • understand that corresponding points of a shape and its reflected image are equidistant from the reflection line • understand that the line segment joining a point to its reflected image is perpendicular to the line of reflection • understand that a refection line bisects all line segments joining corresponding points at right angles	This lesson extends the previous lesson by looking at reflections, another type of transformation.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM) • Rulers	• familiarity with the terms *scalene triangle*, *congruent*, *flip*, *line*, and *line segment* • drawing a right angle

Main Points to be Raised

• A flip is another type of transformation. It is also known as a reflection.

• How far a point moves in a reflection depends on how far it is from the reflection line; the closer it is to the reflection line, the less it moves.

• A reflection has three important properties:

- The image of a reflection is congruent to the original shape, but it points the other way.

- The line segment joining a point to its image is at right angles to the reflection line.

- This line segment is cut in half by the reflection line.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *How do you know the image triangle is congruent to the original triangle?* (It has been traced, so it has to be exactly the same size and shape.)

• *How do you know this is not a translation?* (The image triangle is not pointing the same way as the original.)

• *Does each vertex move the same distance like it did in a translation?* (No. For my triangle, two of the vertices moved the same distance but the third vertex did not.)

The Exposition — Presenting the Main Ideas

• Begin by drawing the following on the board:

$\longleftrightarrow$ Line ——— Line segment

Tell students that a line is straight and the arrows indicate that it goes on forever. A line segment is part of a line. You know it does not go on forever because there are no arrows. Explain that lines and line segments will help them describe the properties of flips, or reflections.

• Draw students' attention to the exposition on **page 48**. Read through the first section together, then ask them to identify the reflection line in the **Try This** activity (it is the fold line).

Discuss different examples of reflections in everyday life (looking in mirrors, looking in windows or water from certain angles, textile patterns, painted decorations on walls or buildings).

• Work through the rest of the exposition with the class. Draw one or two reflections on the board so students can see more examples. Allow ample time for students to ask any questions they have.

Revisiting the Try This

B. Students should connect the fold line with the idea of a reflection line and use the properties of reflections to justify their answer.

Using the Examples

• Assign students to pairs. Have one student in each pair become the expert on **example 1** and the other become the expert on **example 2**. Each student should then explain his or her example to the other.

Practising and Applying

Teaching points and tips

Q 2: If students draw lines that are not perfectly horizontal or vertical, have them turn the page to make them so.

Q 4: Some students might have difficulty visualizing the reflections. Remind them of the properties of reflections and emphasize that where the reflection is located is important. You might encourage them to perform the reflections to check their answers.

Q 5: You may wish to assign this question only to selected students. It builds on the ideas of **question 4** but is more challenging because it involves creating rather than recognizing.

Q 6: Observe whether students easily visualize transformations or whether they need to test by performing them.

Common errors

• It might be helpful to remind students before they begin the exercises that they should think ahead to where the image will be before they draw the original shape to ensure that the image will fit on the page.

• Some students may have difficulty with **question 5**. If the first attempt does not work, you might encourage students to modify the shape from the first attempt rather than start over.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can reflect a shape on a grid
Question 2	to see if students can reflect a shape not on a grid
Question 4	to see if students understand the properties of reflected images

Answers

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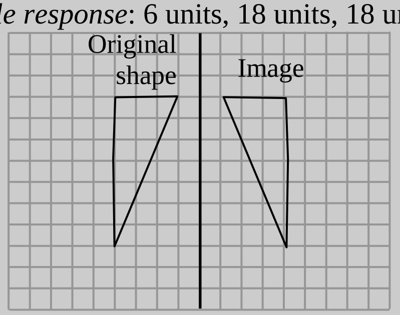
A. *Sample response*: 6 units, 18 units, 18 units



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<td>

B. The points farthest from the fold line moved the most. The point closest to the fold line moved the least.
I could have predicted this by looking at how close the points were to the reflection line (the fold line). The closest moves the least; the farthest moves the most.

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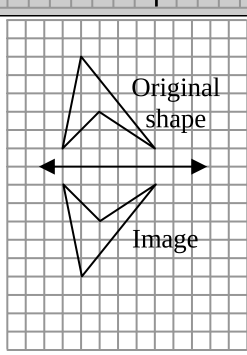
1. a)



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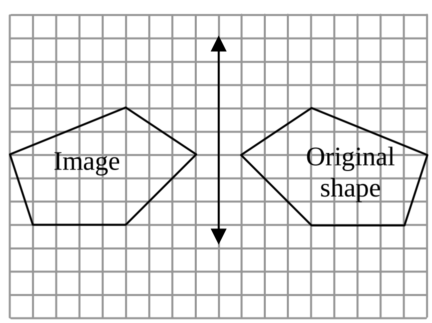
1. b)



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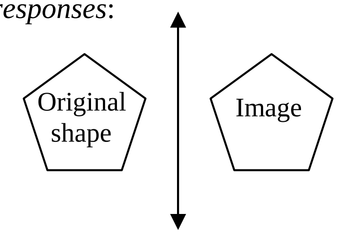
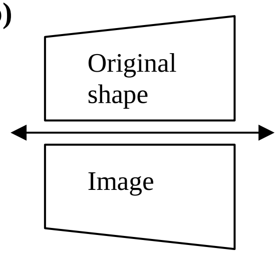
2. *Sample responses*:

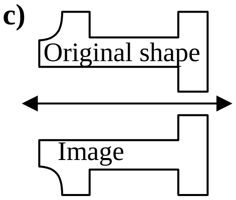
a)



b)

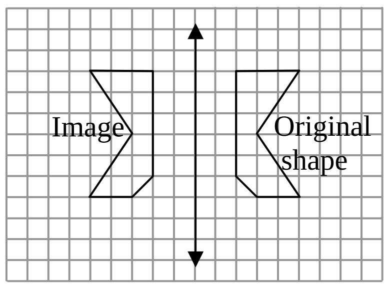


c)



3. *Sample responses*:

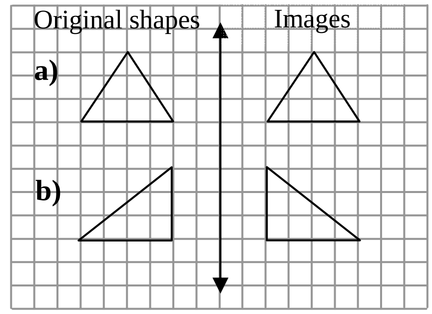
a) and b)



3. c) They are congruent and they are the same distance from the reflection/fold line.

4. The triangle, octagon, and pentagon

5. *Sample responses*:



6. a) Shape C is the reflected image. [The vertices moved by different amounts. It looks like a mirror image.]
b) Shape B is the translated image. [Each vertex moved 7 units right. The shapes are facing the same way.]

Supporting Students

Struggling students

• Some students may benefit from using a small mirror placed on its side on the reflection line to check their work. If what they see in the mirror looks the same as what they see when they take the mirror away, the reflection is correct.

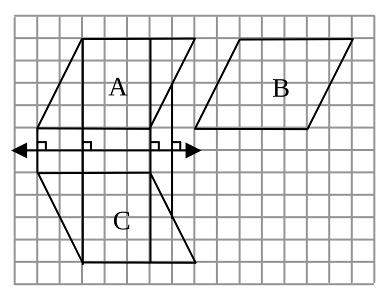
• For **questions 2 and 3**, some students may benefit from using a cut-out copy of the original shape. Have them flip the shape over the line and position it so that it is the same distance as the original from the reflection line.

• If students are struggling with **question 6**, you might add these identifying properties of reflections to the chart about translations from the previous lesson:

- The image points the opposite way to the original shape.

- It is possible to add a reflection line halfway between the two shapes that is at right angles to any line segment that joins a point to its image.

For example, in **question 6**, Shape A and Shape C point in opposite directions. You can add a reflection line and see from sample points that it is at right angles to line segments joining a point to its image. Some line segment joining sample points are shown to the right.



Enrichment

• In **question 6**, Shape A has been reflected to Shape C. Challenge students to reflect Shape C in a vertical line to create Shape D. Then ask them to think about these questions:

- *What single transformation takes Shape A to Shape D?* (Translation)

- *Is it always true that a horizontal reflection followed by a vertical reflection has the same image as a translation?* (No. I tried reflecting other shapes. It did not work for some of them.)

2.2.3 Parallel and Intersecting Lines

Curriculum Outcomes	Outcome relevance
5-E4 Parallelism and Perpendicularity: lines and line segments • construct the following pairs of lines/line segments and use appropriate mathematical terminology: - parallel - intersecting - perpendicular at an endpoint - bisecting another line segment but not perpendicular - bisecting each other and perpendicular **5-E5 Translations and Reflections using horizontal and vertical reflection lines: generalize and apply properties** • understand that corresponding sides of the original shape and the translated image are always parallel • understand that corresponding sides of the original shape and the reflection image are not always parallel • understand that parallel sides of the original shape are always parallel in the translation image	Parallel and perpendicular lines and line segments are found in the world around us. The new language in this unit allows students to form more concise descriptions of the properties of translations and reflections they have already studied, as well as of the rotations they will study in the next lesson.

Pacing	Materials	Prerequisites
1.5 h	• Grid paper or Small Grid Paper (BLM) • Parallel and Intersecting Lines (BLM) • Rulers	• familiarity with the terms *line*, *line segment*, *intersect*, *right scalene triangle*, and *trapezoid* • translating and reflecting a shape on grid paper

Main Points to be Raised

• Parallel line segments never meet.

• Perpendicular line segments meet at right angles. This can happen at an endpoint of either or both line segments, at a centre point of either or both line segments, or at other points.

• There are line segments that are not parallel or perpendicular.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Give each student or pair a copy of Parallel and Intersecting Lines (BLM) to record their answers. Observe while students work. While they work, you might ask questions such as the following:

• *What clues did you use to help you find line segments at right angles?* (The corners of the design looked like grid lines and I know they meet at right angles.)

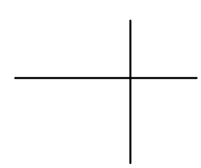
• *Why might it be easier to check that line segments meet at right angles than to check that they point in the same direction?* (I can use the corner of my page or a ruler to check that the angle is a right angle, but when they point in the same direction I can only use my eyes.)

The Exposition — Presenting the Main Ideas

• Ask a volunteer to come to the board to draw a pair of line segments that point in the same direction. Write "Parallel line segments" beside the line segments.

Parallel line segments

Then ask another volunteer to come to the board to draw a pair of line segments that meet at right angles. Write "Perpendicular line segments" beside these line segments.



Perpendicular line segments

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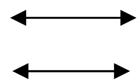
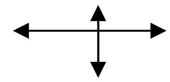
Explain that lines can also be parallel or perpendicular. Draw examples on the board.



Parallel lines



Perpendicular lines

• Draw students' attention to the exposition. As you read through it together, ask them to point out other examples of parallel and perpendicular line segments.

• After reading through the entire exposition, you may wish to ask the class whether it makes sense for perpendicular lines to meet at endpoints or at centre points (No, because the lines go on forever. There are no endpoints or centre points).

Revisiting the Try This

B. This question allows students to focus on the new language introduced in the lesson.

Using the Examples

• Have students work in pairs. One student in the pair should become an expert on **example 1** and the other should become an expert on **example 2**. Each student should then explain his or her example to the other.

• Work through **example 3** with the students to make sure they understand it.

• It is important to emphasize the last statements of the solutions for **example 3 a) and b)** — in a translation, every original side and its image are parallel, but this is not true for a reflection.

Practising and Applying

Teaching points and tips

Q 2: You might encourage students to use grid paper to answer this question. This will make **part d)**, in particular, much easier.

Q 3: This question highlights the important connection between parallel and perpendicular lines in the real world.

Q 5: This question provides a context that links this lesson to the previous lesson. Students apply the terms parallel and perpendicular to describe the properties of reflections.

Q 6: If necessary, remind students what a trapezoid is. This question is similar to **question 5**, but it relates to translations. Although there are only three examples, students should be convinced that translations preserve parallel line segments.

Q 7: Some students may need to be reminded that they can use specific examples to help explain their answers.

Common errors

• Students who have trouble performing reflections or translations will have difficulty with **questions 5 and 6**. It may be helpful to have them work with a partner to answer these questions. Refer also to the ideas presented for struggling students in **lessons 2.2.1 and 2.2.2**.

• Some students may answer **question 8 d)** with descriptions about location, such as, it is below or to the left. You might gently remind them that the lesson is about parallel and perpendicular lines. Encourage them to think about the new vocabulary they have learned.

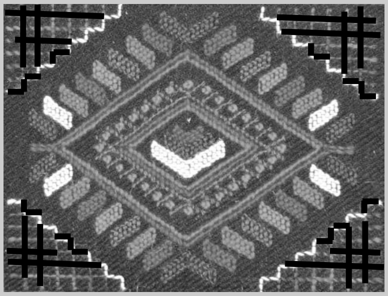
Suggested assessment questions from Practising and Applying

Question 2	to see if students can draw parallel line segments and perpendicular line segments
Question 5	to see if students can apply the ideas of parallel and perpendicular in the context of a reflection
Question 6	to see if students can identify how parallel lines relate to translations
Question 8	to see if students can recognize the relationship between parallel and perpendicular

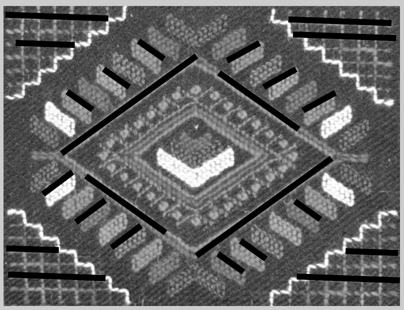
Answers

A. *Sample responses*:

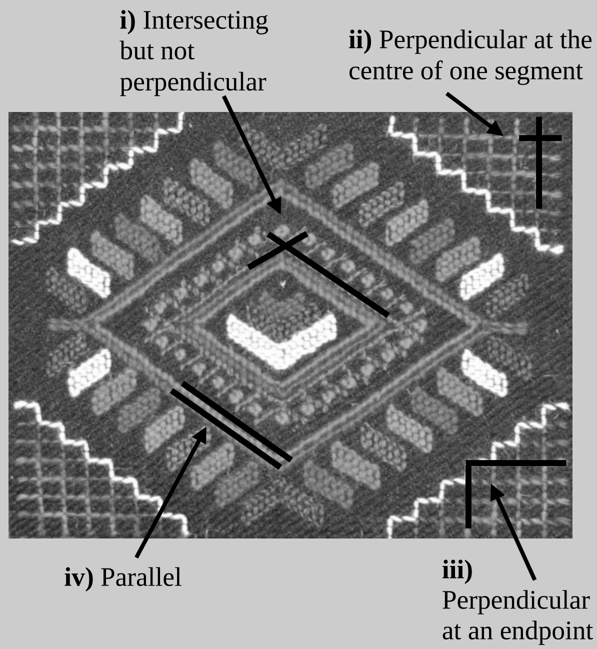
i)



ii)

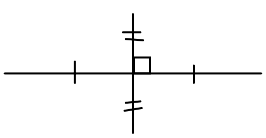


B. *Sample responses*:



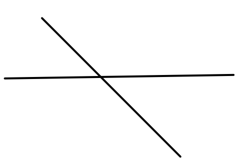
1. a) Perpendicular at an endpoint of one segment
b) Intersecting but not perpendicular
c) Parallel
d) Perpendicular at the centre points

2. *Sample responses*:
a)

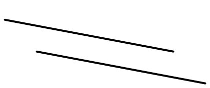


b)

c)

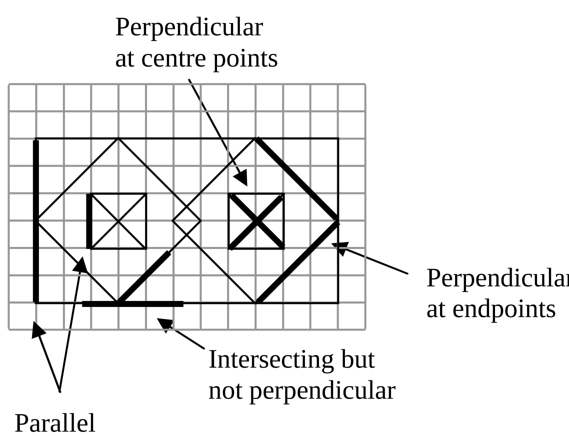


d)



3. *Sample responses*:
a) Opposite sides of a window, board, table, or doorway.
b) Window panes, the corner of a wall, board, table.
c) Two sides of an equilateral triangle on the board.

4. *Sample responses*:
a) and b)



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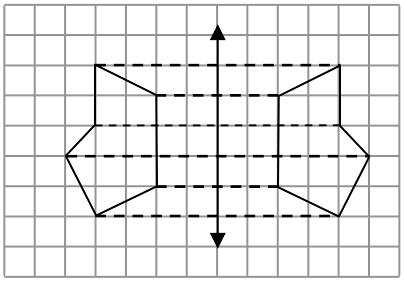
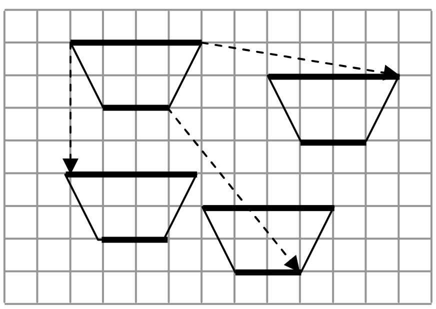
5. a), b), and c)



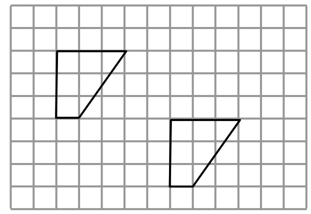
d)
- The lines connecting vertices to their images are all parallel.
- The lines connecting vertices to their images are perpendicular to the reflection line.
- The vertical sides of the original shape and their images are all parallel.

6. a) and b) *Sample responses*:
Translation rules are: 6 right, 1 down; 0 right, 4 down; and 4 right, 5 down.

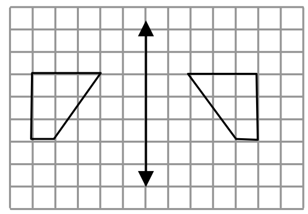


c) Yes, the parallel sides of the trapezoid are still parallel in the translation images.

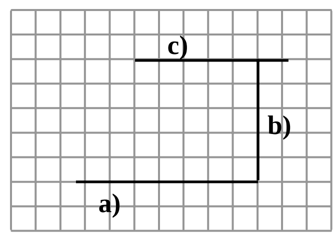
[**7.** *Sample responses*:
a) When you translate a shape, every point moves the same distance in the same direction, so sides face the same direction. You can see this on the grid.



b) When you reflect a shape, any side that is not parallel or perpendicular to the reflection line is flipped so its image goes in a different direction.
You can see this on the grid.]



8. *Sample responses*:
a), b), and c)



d) c) and a) are parallel

[**9.** *Sample response*:
Parallel lines never meet and perpendicular lines meet.]

Supporting Students

Struggling students

• If students confuse the meanings of words perpendicular and parallel, you might offer a mnemonic device or ask them to come up with a memory aid of their own.

For example, the double letter Ls in “parallel” are parallel to each other (when you use lowercase letters.)

Enrichment

• For **question 7**, you might challenge students to identify the criteria that would make all original sides and their images parallel for a reflection (A shape that has side lengths that are all parallel to each other and perpendicular to the reflection line).

2.2.4 Properties of Rotations

Curriculum Outcomes	Outcome relevance
5-E6 Rotations: quarter, half, and three-quarter rotations about the vertex of a shape • predict, apply, and identify quarter ($\frac{1}{4}$), half ($\frac{1}{2}$), and three-quarter ($\frac{3}{4}$) rotations • explore the results using a variety of turn centres • understand that each point remains the same distance from the turn centre and the turn centre doesn't move	Rotations are another type of transformation that we frequently see in the world around us. By studying the properties of rotations, students will be better able to analyse those situations.

Pacing	Materials	Prerequisites
1.5 h	• A large cardboard circle, two congruent cardboard trapezoids with long side equal to circle radius, and a pushpin (optional) • Grid paper or Small Grid Paper (BLM) • Rulers	• fractions ($\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{4}{4}$) • line segments • right angles

Main Points to be Raised

• A turn is a third type of transformation. It is also known as a rotation.

• Every rotation has a turn centre that does not move during the rotation.

• Every rotation can be described by its turn centre, size (angle or fraction of a full turn), and direction, clockwise or counterclockwise.

• We often describe the size of a rotation as a fraction of a whole turn.

• $\frac{1}{4}$ turns and $\frac{3}{4}$ turns create right angles when a point and its image are connected to the turn centre. $\frac{1}{2}$ turns create straight line segments.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. As they work, try to have them think of the action of the hour hand rather than just the position. You might ask questions such as the following:

• *How would an hour hand turn to finish here?* (It would move around the clock face.)

• *Is there another way it could have finished here?* (Yes. The hand could move the opposite way from how a clock hand usually moves; it could go around the other side of the circle.)

The Exposition — Presenting the Main Ideas

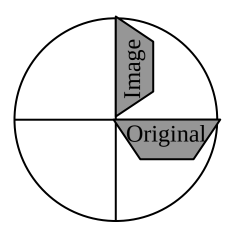
• Briefly review transformations with the class. You might ask questions such as:

- *What are the two types of transformations we have studied?* (Translations and reflections)

- *How can you describe the action of a translation?* (It is a slide. The points all move in exactly the same way.)

- *How can you describe the action of a reflection?* (It is a flip. Each point stays the same distance from the reflection line but moves to the other side of the line.)

• Explain to the class that this lesson is about rotations, which are the third type of transformation.



• If possible, you can model the information in the exposition with a large cardboard circle with quarters marked, two congruent trapezoids with the long side equal to the radius of the circle, and a pushpin to join the two at the turn centre. You can mark one trapezoid “Original” and the other “Image”. An example is shown to the right.

Alternately, you can draw the circle, the original trapezoid, and its image on the board. In this case you should model the action by hand movement and further indicate it with arrows.

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• As you work through the exposition, you may raise these points to support the ideas presented:

- The distance from a point to the turn centre does not change when the point is rotated because the size of the trapezoid does not change. You can relate this to a clock. When the clock hand moves, does it get shorter or longer? (No)

- The sizes of the rotation for the two directions possible (cw or ccw) must add to one whole, or $\frac{4}{4}$.

• It may be helpful to display a chart on the board or on paper that shows the rotation relationships explicitly. An example is shown to the right.

Be sure to mention that these are just a few examples. A rotation of any size is possible and the two corresponding sizes for cw and ccw will always add to one whole.

$\frac{1}{4}$ cw = $\frac{3}{4}$ ccw

$\frac{1}{2}$ cw = $\frac{1}{2}$ ccw

$\frac{3}{4}$ cw = $\frac{1}{4}$ ccw

• If time allows, you might finish by asking students to think of real-world examples of rotations (designs in textiles, wheels or tires on vehicles and bicycles, the Wheel of Life, clocks).

Revisiting the Try This

B. and C. A word description or a picture is an acceptable answer.

Using the Examples

• Copy the questions from **example 1** and **example 2** on the board. Ask students to work through them. They should then compare their solutions to those in the text.

Practising and Applying

Teaching points and tips

Q 2: You might suggest that students follow **example 2** for this question.

Q 5: **Part f)** has several correct answers. Be sure to read student answers carefully.

Q 6: This highlights an important fact: while translations, reflections, and rotations are different, in some cases it is impossible to tell the images apart.

Q 7: This question extends the ideas of **question 6** to non-symmetric shapes and shows that the results of **question 6** are not typical.

Q 8: This question synthesizes all the transformations studied in this chapter.

Common errors

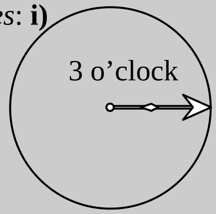
• Many students will rotate the original triangle in **question 5 b) and c)** rather than rotating the image triangles. This method yields the same design, although it will be created in a different order. Regardless, you might encourage students to read carefully because this type of error can often lead to an incorrect result.

Suggested assessment questions from Practising and Applying

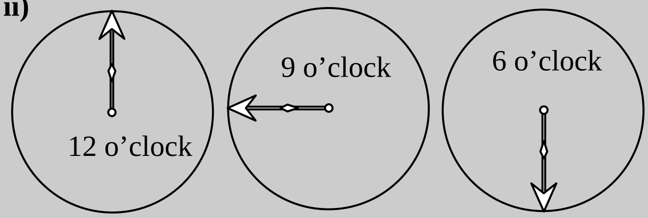
Question 1	to see if students can describe a rotation
Question 3	to see if students can rotate a shape a $\frac{1}{4}$ or $\frac{3}{4}$ turn
Question 4	to see if students can rotate a shape a $\frac{1}{2}$ turn
Question 8	to see if students can explain the differences between translations, reflections, and rotations

Answers

A. *Sample responses*: **i)**



ii)

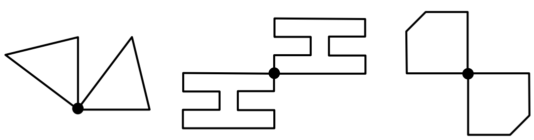


B. The turn centre is in the middle, where the clock hand attaches to the clock face.

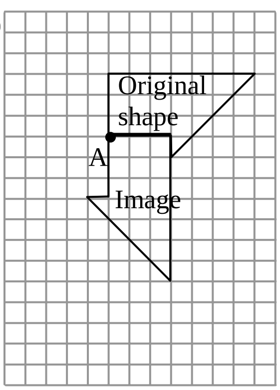
C. i) Starting at 3 o'clock and going forward to 12 o'clock is a $\frac{3}{4}$ turn clockwise.
ii) Starting at 3 o'clock and going backward to 9 o'clock is a $\frac{1}{2}$ turn counterclockwise.
iii) Starting at 3 o'clock and going forward to 6 o'clock is a $\frac{1}{4}$ turn clockwise.

1. a) A $\frac{1}{4}$ turn counterclockwise or a $\frac{3}{4}$ turn clockwise about the bottom left vertex of the grey shape.
b) A $\frac{1}{2}$ turn clockwise or counterclockwise about the bottom left vertex of the grey shape.
c) A $\frac{1}{2}$ turn clockwise or counterclockwise about the bottom right vertex of the grey shape

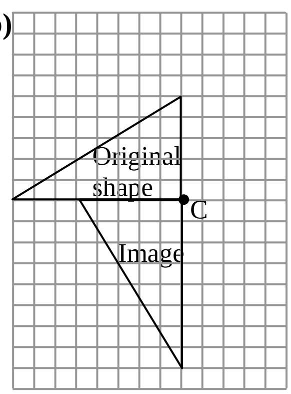
2.



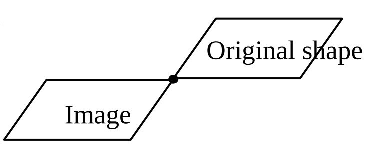
3. a)



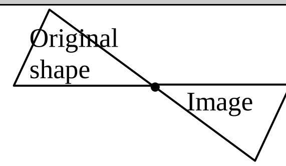
b)



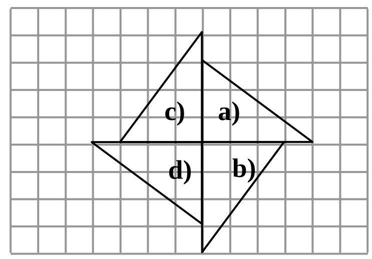
4. a)



4. b)



5.

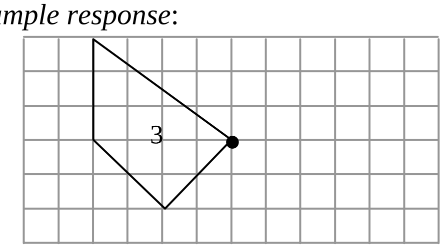


e) *Sample response*:
The triangles fit together around the turn centre to make a single shape with 8 sides.

f) *Sample response*: A $\frac{1}{4}$ turn clockwise three times.

6. a) Rotate a $\frac{1}{2}$ turn clockwise or counterclockwise around the dot.
b) The reflection line would be the vertical line through the dot.
c) Move 4 right.

7. a) *Sample response*:



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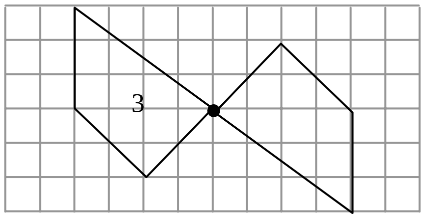
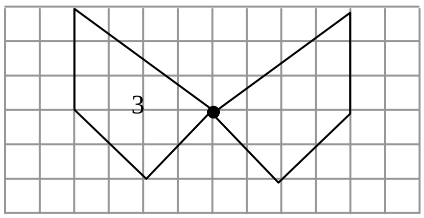
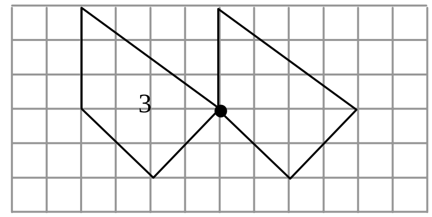
7. b) Rotation:



Reflection:



Translation:



[**c)** Only the translation image is facing the same way.]

[**8.** *Sample response*:
A rotation turns the shape, a reflection flips it, and a translation does not really change its look at all – it just moves the shape.]

Supporting Students

Struggling students

• For **questions 1 and 3**, some students may benefit from using a cut-out copy of the original shape. Have them turn the shape, keeping the turn centre in the same spot, and then trace it.

• If students are struggling with rotations in general, you might add these identifying properties of rotations to the chart about translations and reflections from previous lessons:

- We describe a rotation by the location of the turn centre, size (a fraction of a whole turn or angle), and direction (cw or ccw).

- A $\frac{1}{2}$ turn makes a straight line segment; $\frac{1}{4}$ and $\frac{3}{4}$ turns make right angles

- $\frac{1}{2}$ cw = $\frac{1}{2}$ ccw, $\frac{1}{4}$ cw = $\frac{3}{4}$ ccw, $\frac{3}{4}$ cw = $\frac{1}{4}$ ccw

- The image looks turned when you compare it to the original.

• You may choose not to assign **question 7** to struggling students because it extends the work of the lesson.

Enrichment

• For **question 5 f)**, you might challenge students to find as many answers as possible.

• Have students revisit the suggested enrichment for **lesson 2.2.2**. Ask them to reflect a shape in a horizontal line, then reflect that image in a vertical line. Then ask them to consider these questions:

- *What single transformation describes the move from the first shape to the last?* (A rotation)

- *Do you think this will always be true?* (Yes. I tried many shapes and it was always so.)

CONNECTIONS: Kaleidoscope Images

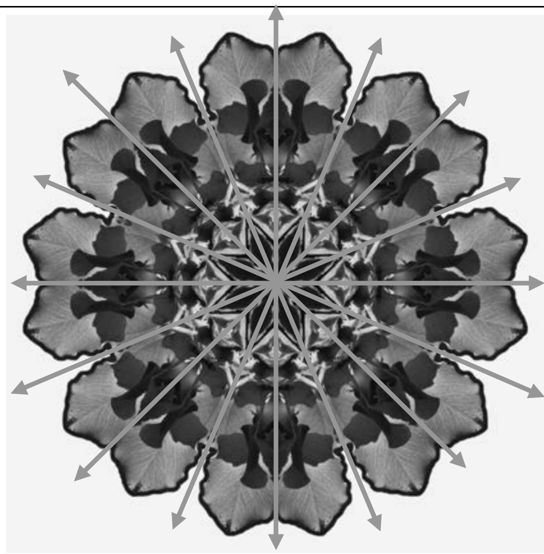
• This optional connection has historical roots. The kaleidoscope was invented in 1816 by David Brewster, a child prodigy from Scotland who attended university at the age of 12. Kaleidoscopes were a social phenomenon enjoyed by all economic and intellectual classes in a vast geographical region. Their popularity is still evident today.

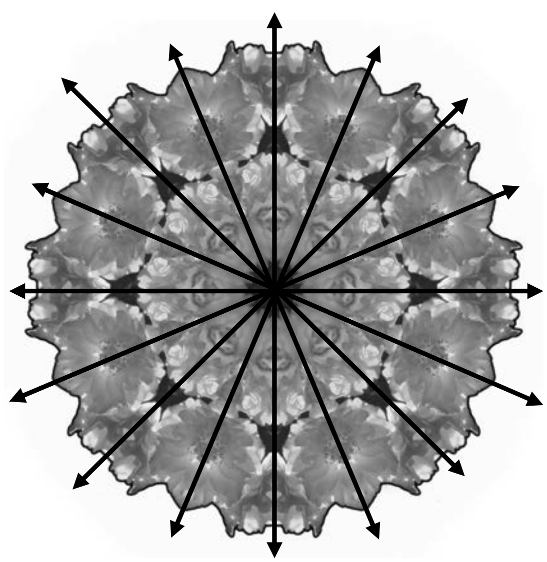
• Students may trace the hexagon shape or use a copy of it from the Kaleidoscope Images (BLM).

• You may wish to point out to students that the image in the last triangle reflects onto the original design. This is a property of the equilateral triangle discovered by David Brewster. Only two other types of triangles share this property — 30°- 60°- 90° triangles and 45°- 45°- 90° triangles. Interested students can make kaleidoscope designs with these types of triangles using the Kaleidoscope Images (BLM).

Answers

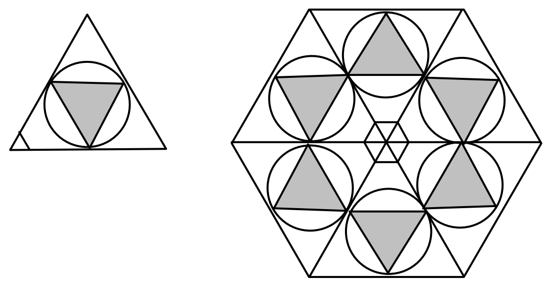
1.





2. $\frac{1}{4}$ turns, $\frac{1}{2}$ turns, and $\frac{3}{4}$ turns, either clockwise or counterclockwise around the centre (where the reflection lines intersect), as marked.

3. *Sample response*:



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Chapter 3 3-D Representations

2.3.1 Prism and Pyramid Nets

Curriculum Outcomes	Outcome relevance
5-E7 Nets: prisms and pyramids • create and interpret nets for various prisms and pyramids	We live in a 3-D world, but paper and other media are 2-D. It is therefore important to study various ways to represent 3-D objects. Nets are one way to represent prisms and pyramids two-dimensionally.

Pacing	Materials	Prerequisites
1.5 h	• Grid Paper (1 cm by 1 cm) (BLM) • Sample Net of Triangle-based Pyramid (BLM) • Sample Net of Square-based Pyramid (BLM) • Sample Net of Right Triangle-based Prism (BLM) • Sample Net of Rectangle-based Prism (BLM) • Sample Net of Regular Hexagon-based Prism (BLM)	• identifying prisms and pyramids that have triangle, rectangle, square, pentagon, hexagon, or octagon bases • familiarity with the terms *polygon* and *congruent*

Main Points to be Raised

• A net is a way to represent a 3-D shape on 2-D paper.

• The polygons in a net represent faces in the 3-D shape.

• Different nets may fold to make the same 3-D shape.

• A prism has two congruent bases and a rectangle for each side of the base shape.

• A pyramid has one base and a triangle for each side of the base shape.

• Prisms and pyramids are named for their base shape.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. As they work, try to get them to think about the polygons that make up the net. You might ask questions such as the following:

• *How many shapes are in the net?* (A: 4, B: 5, C: 5)

• *How many of the shapes are congruent?* (A: all, B: 2 congruent rectangles and 2 congruent triangles, C: 4 congruent triangles)

The Exposition — Presenting the Main Ideas

• Have students turn to the exposition on **page 61**. Mention that a 2-D shape with straight edges is known as a polygon.

• As you read through the exposition together, be sure to discuss these points:

- The bases of prisms and pyramids are also considered to be faces.

- The base of a prism is not necessarily the bottom face, but the face that could be a shape other than a rectangle.

- The base of a pyramid is not necessarily the bottom face, but the face that could be a shape other than a triangle.

• The sample nets (BLMs) can be used as further examples. Have students name the 3-D shape each will make using the polygons in the nets for clues. Then have them fold the nets to see if they were correct.

• Leave ample time for students to ask questions if they do not understand something.

Revisiting the Try This

B. Encourage students to use the language of *prism* and *pyramid* in their answer to this question.

Using the Examples

• Before students read **example 1**, model how you can roll a shape like a pyramid or prism so that a new face is flat on the table. Then ask pairs of students to read through **solutions 1 and 2** of **example 1**. Ask them to choose which solution most closely matches what they would have done and to explain why.
• Next, work through **example 2** with the students to make sure they understand it. Point out that this is only one way to draw the net on grid paper. Any face could be drawn first and the faces could be attached to each other in different ways.

Practising and Applying

Teaching points and tips

Q 2: Some students may benefit from using a model to trace, as in **example 1**. They may also sketch the nets freehand, which will require them to use visualization skills and to have a good understanding of 3-D shapes.

Q 3: This question can be done following the techniques of **example 2**. You may wish to discuss with the class why the altitudes shown meet the rectangle base at the centre point (the triangle faces are composed of two congruent triangles and the sides that meet the rectangle are congruent sides).

Q 4: In **part c)** students are asked to draw two nets for the hexagon-based pyramid. Suggest that they try this without tracing a model (no BLM is provided). This should not prove too difficult because one net for a hexagon-based pyramid is given in **question 1**. Other nets for pyramids shown throughout the chapter may help them think of a second net.

Q 5: You may wish to remind students to consider not only which shapes are part of the net but also the appearance and size of each shape.

Q 7: Use this question to make sure students can distinguish between nets for prisms and for pyramids.

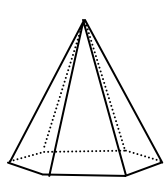
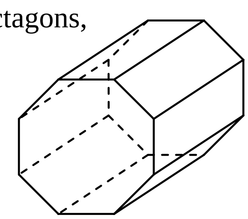
Common errors

• Many students will recognize that the net shown in **question 5** makes a triangle-based prism and will mistakenly think it that makes the triangle-based prism shown. You might suggest that they carefully compare the polygons in the net to the faces of the shape.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can create a net on grid paper given a 3-D shape and measurements
Question 4	to see if students can interpret and create nets
Question 5	to see if students can recognize the differences between a given shape and the shape a net will make

Answers

A. *Predictions may vary. Correct answers*: Net A makes a triangle-based pyramid. Net B makes a triangle-based prism. Net C makes a square-based pyramid.	**B.** *Sample response*: Net A had only triangles, so it had to be a triangle-based pyramid. Net B had 3 rectangles and 2 congruent triangles, so it had to be a triangle-based prism. Net C had 1 square and 4 triangles, so it had to be a square-based pyramid.
1. a) Hexagon-based pyramid; [• There are 6 triangles and 1 other polygon, so it is a pyramid. • The other polygon is a hexagon, so it is a hexagon-based pyramid.] 	**b)** Octagon-based prism; [• There are eight rectangles and two congruent polygons, so it is a prism. • The congruent polygons are octagons, so it is an octagon-based prism.] 

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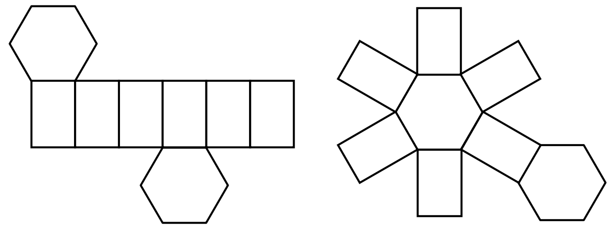
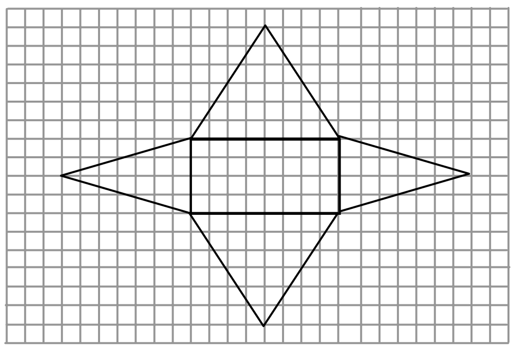
2. Hexagon-based prism; *Sample response*:



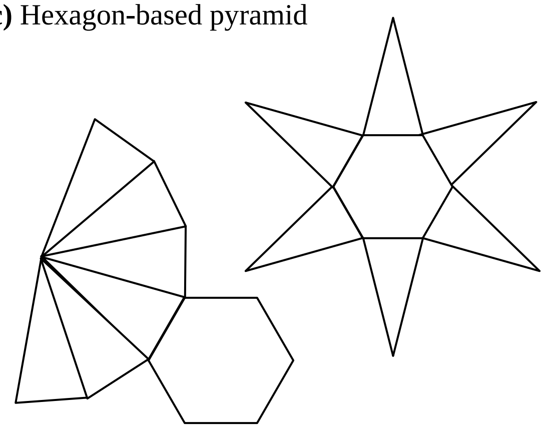
3. a) *Sample response*:



4. a) i) Hexagon-based prism
ii) Square-based pyramid
iii) Triangle-based prism
iv) Hexagon-based pyramid

b) A makes **i)** hexagon-based prism
B makes **ii)** square-based pyramid
C makes **iii)** triangle-based prism

4. c) Hexagon-based pyramid



5. No; [the triangular faces in the prism are right triangles. The triangles in the net are not right triangles.]

6. Yes, [they all form a square-based pyramid.
• Each has 1 square and 4 congruent triangles.
• The triangles in all the nets are congruent.
• The squares in all of the nets are congruent too.]

7. a) Prism
[• There are rectangles instead of triangles, so it must be a prism and not a pyramid.
• There are 2 congruent pentagons. A pyramid would have only 1 pentagon.]
b) Pentagon-based prism

Supporting Students

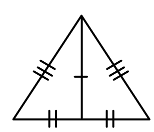
Struggling students

• If students are struggling with **questions 1 and 4 b)**, you might encourage them first to make a prediction and then to trace the nets, cut them out, and fold to check their prediction.

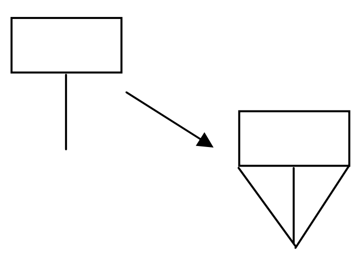
• Some students may be able to draw the rectangle face but have trouble accurately drawing the triangle faces in **question 3**. Remind them about the congruent triangles that make up the triangle faces and ask questions about what that tells them.

For example, you might ask questions such as these:

- *Which sides are congruent in the triangles that form the front face?* (As marked; students may point to the diagram in the student book rather than draw this.)



- *Where will the 6 cm length be located in the net?* (In the middle of a long side, as shown.)



- *Once you have drawn that line, how can you finish the triangle?* (I can connect the end of the 6 cm line segment to either end of the rectangle side.)

CONNECTIONS: Euler's Rule

• This optional connection is commonly known as Euler's Theorem (Euler is pronounced "oil-er"). It is a fundamental result in a branch of mathematics called topology. Usually presented as V – E + F = 2, the equation has been rearranged to avoid negative numbers.

• Euler's Theorem is true for all polyhedra without holes, not just for prisms and pyramids.

• Students may work together in pairs or small groups.

• You may wish to provide models of the shapes (BLMs of nets are provided) to make the activity more hands-on. Alternatively, students can see that all but the triangle-based pyramid and the pentagon-based prism are represented visually in the student book.

• Encourage students who like a challenge to think about the structure of prisms and pyramids to relate the answers in the V, E, and F columns to the properties of the base.

For example:

- When the shape is a prism, V is double the number of vertices in the base, E is triple the number of edges in the base, and F is the number of edges in the base plus 2.

- When the shape is a pyramid, V is the number of vertices in the base plus 1, E is double the number of edges in the base, and F is the number of edges in the base plus 1.

Answers

1.

3-D Shape	**V** (number of vertices)	**F** (number of faces)	**E** (number of edges)	**Euler's rule V + F – E = ?**
Triangle-based prism	6	5	9	2
Triangle-based pyramid	4	4	6	2
Rectangle-based prism	8	6	12	2
Square-based pyramid	5	5	8	2
Pentagon-based prism	10	7	15	2
Hexagon-based prism	12	8	18	2

2. No; [V + F – E = 12 + 6 – 15 = 3, but it must equal 2 for a prism.]

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2.3.2 Interpreting Isometric Drawings

Curriculum Outcomes	Outcome relevance
5-E8 Isometric Drawings • make and interpret drawings of structures made from cubes	Representing 3-D objects in two dimensions is relevant to real-world applications. Isometric drawings are one way to represent cube structures two-dimensionally. This lesson focuses on interpretation of these drawings.

Pacing	Materials	Prerequisites
1.5 h	• Linking cubes • Sample Net of Cube (BLM) (optional)	• building cube structures • familiarity with rhombuses

Main Points to be Raised

• Isometric drawings are 2-D representations of 3-D objects. Parallel lines and lengths are preserved. They are called isometric because each dot on the grid is equally distant from all adjacent dots.

• There may be cubes hidden from view in an isometric drawing. Different views can help to give a complete picture of the cube structure.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. As they work, try to get them to think about the features of the structures. You might ask questions such as the following:

• *How many cubes can you see in each picture?* (A: 6, B: 5, C: 6, D: 7)

• *How can you describe the structures?* (B has two towers of two cubes side by side with two single cubes attached to one of the bottom cubes; C has two towers of two cubes but they are not side by side.)

The Exposition — Presenting the Main Ideas

• Explain to students that the dot paper on which the cube structures are drawn is called isometric dot paper. Point out the origins of the word isometric:

iso means equal   *metric* means distance

Indicate that the dots are all the same distance apart, just like all the sides of a cube are the same length. This is why cube structures are often drawn on isometric dot paper.

• Draw students' attention to the exposition on **page 66**. As you read through the material together, you might ask questions about the isometric drawings presented in the **Try This** section:

- *Where do you see a rhombus in the isometric drawings?*

- *Where could cubes be hidden from view?*

• Emphasize that one isometric drawing may not tell you all the information you need to know to build a cube structure. There could be several structures with cubes hidden from view that match a given isometric drawing.

Revisiting the Try This

B. This question allows students to communicate about how they interpreted the isometric drawings.

Using the Examples

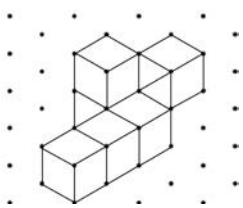
• Distribute cubes to pairs of student. One of the pair should become an expert on **example 1** and the other should become an expert on **example 2**. Each student should then explain his or her example to the other.

Practising and Applying

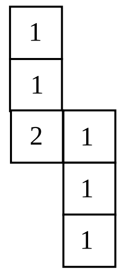
Teaching points and tips

Q 1 to Q 4: Some students may choose to do these questions without cubes. Many will enjoy using cubes. Both methods are acceptable.

Q 3: This isometric drawing creates an optical illusion, so students may have difficulty visualizing the structure. For many, it may appear as if the drawing is incorrect. This view from the top will help clarify things (each number indicates the number of cubes in each "tower").



Front

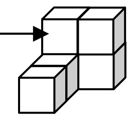


Front

Q 5: This question is an important next step. Ask students to try to answer this question using their visualization skills (i.e., without cubes).

For **b)**, emphasize that this is asking about cubes that are hidden from view but that are necessary for the structure to work.

For example:



For **c)**, make sure students know that they only need to indicate a general location, not an exact location or a specific number of cubes.

Q 6: Use this last question to draw attention to the fact that a single isometric drawing may not give the entire picture because cubes could be hidden.

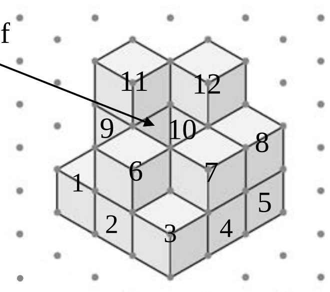
Common errors

• In **question 2**, Drawing A has 5 visible cubes and Drawing B has 4 visible cubes. Both drawings could have hidden cubes. This may lead some students to think incorrectly that there could be more than 5 cubes in the structure. You might suggest that they build the structure in Drawing A. They can try to add a cube and turn the structure to see if it corresponds to Drawing B.

• Some students might count the other part of cube 9 (shown to the right) as another cube in **question 5 a)**.

To avoid this, encourage students to visualize the cube structure and not just do a quick count.

This is part of cube 9.



Suggested assessment questions from Practising and Applying

Question 2	to see if students can compare isometric drawings
Question 4	to see if students recognize an isometric drawing with a given face view
Question 5	to see if students can interpret an isometric drawing

Answers

A. A and D	**B.** *Sample response*: I looked for a tower three cubes tall with two cubes on either side. A and D both had that, although you could not see one of the cubes in A because of the tower.
1. A, B, and D represent the same structure. [*Sample response*: I know because I made A out of cubes and turned it to try to see the other views. The only views I could see were B and D.]	**2.** No. [If there were a sixth cube, it would have to be hidden behind the tower in A that is two cubes tall. But then I would be able to see it in view B and I do not see it there.]

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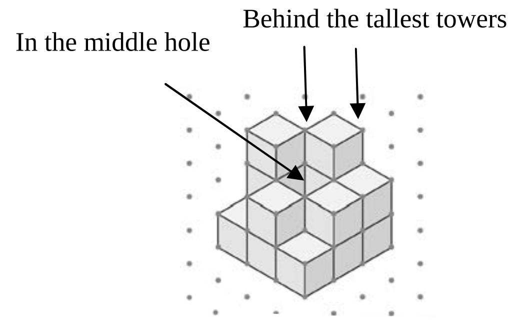
3. I can be sure there are seven cubes. I cannot be sure if there are eight cubes because there could be a hidden cube behind the tower. **4.** B is a drawing of the cube structure. [I looked for a row of three cubes in each structure that had a row of two cubes on the upper right when viewed from above. Drawing A had the two cubes on the upper left.] **5. a)** 12 cubes **b)** Two cubes; at the bottom of the two towers at the back.	**5. c)** You could hide one in the middle hole or more behind the two tallest towers.  [**6.** There may be hidden cubes that cannot be seen from a certain view.]

Supporting Students

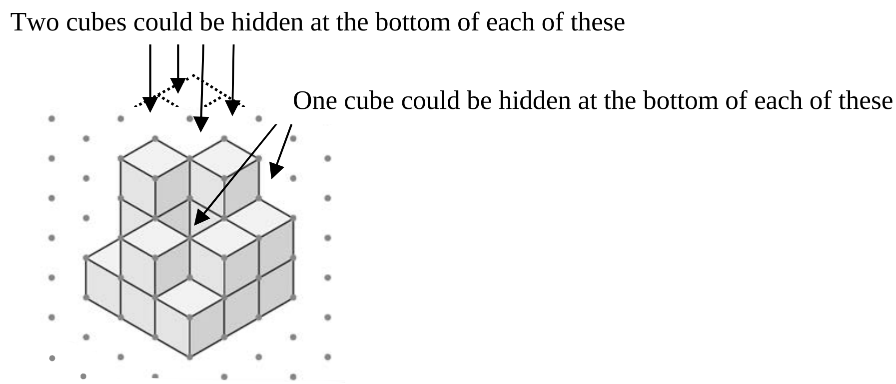
Struggling students

• If students are struggling with **question 5**, let them build the structure with cubes and view it from different angles. You might encourage them first to predict the answer and then to use the cubes to check their answer.

Enrichment

• For **question 5 c)**, you might challenge students to find where ten cubes could be hidden.

For example:



2.3.3 Creating Isometric Drawings

Curriculum Outcomes	Outcome relevance
5-E8 Isometric Drawings • make and interpret drawings of structures made from cubes	Representing 3-D objects in two dimensions is relevant to real-world applications. Isometric drawings are one way to represent cube structures two-dimensionally. This lesson focuses on creating isometric drawings.

Pacing	Materials	Prerequisites
1.5 h	• Isometric Dot Paper (BLM) • Linking cubes • Sample Net of Cube (BLM) (optional)	• building cube structures • familiarity with the terms *rectangular prism* and *rhombus*

Main Points to be Raised

• It is easier to draw 3-D cube structures on 2-D paper if you use isometric dot paper.

• A rhombus represents a cube face in an isometric drawing. There are three ways to draw a rhombus on isometric dot paper.

• To create an isometric drawing of a cube structure, begin with a cube that has three faces visible. Add faces as required to complete the structure.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *What makes it difficult to draw the cube structure?* (It is hard to make cubes that look exactly right.)

• *How is drawing a cube different from drawing a square?* (I can draw a square accurately because it is flat. Cubes are not flat. It is hard to draw a physical object because you have to show that it takes up space.)

If students' drawings are weak, you might tell them that drawing in 3-D can be difficult but that they will improve with practice.

The Exposition — Presenting the Main Ideas

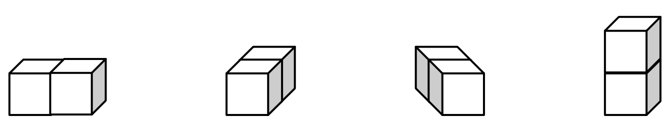
• Some students may have had difficulty producing a good sketch of the prism in the **Try This** section. If necessary, take a moment to reassure them that this is common and that isometric dot paper can make it much easier.

• Distribute cubes (two per student or pair) and isometric dot paper (one sheet per student) to the class. Work through the exposition together as a class. Be sure students have ample time to practice drawing:

- the three different types of rhombuses
- a single cube
- two cubes side by side

Encourage students who finish these tasks quickly to try drawing the two-cube structure from different views.

For example:



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Revisiting the Try This

B. This question allows students to compare their original freehand drawing to the isometric drawing; it should highlight the usefulness of isometric drawings.

Using the Examples

• Present the cube structure in the example to the students. Have each student or pair of students build a copy of the structure. Ask each student to make an isometric drawing of the structure. Then the students should compare their work to what is shown in the matching example. If some students finish before others, or if you feel they need more practice, encourage them to build and draw another simple structure.

Practising and Applying

Teaching points and tips

Q 1 to 4: You may need to remind students to begin with a cube that has three visible faces.

Q 1: The isometric drawings can be made directly from the photographs. The structures in **parts a), b), and c)** are shown from a view that corresponds to an isometric drawing to facilitate this. **Part d)** shows a slightly different view, but the structure is fairly simple.

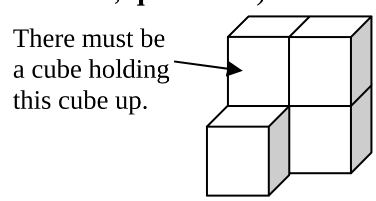
Q 5: Use this question to draw attention to the fact that several isometric drawings can represent the same cube structure.

Common errors

• In **question 4**, some students will confuse the idea of cubes that are simply hidden from view with the idea that there may be hidden cubes that must be there for the structure to work (see **lesson 2.3.2**, **question 5**).

For example, their answer to **part a)** may be "one cube" because one cube must be behind the front cube to hold up the cube above it.

To address this, reinforce that this time the number of cubes is known. In **lesson 2.3.2**, the number of cubes was not known. Remind students to consider that cubes that are hidden from view may be optional or they may be essential to the structure.

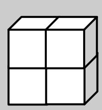


Suggested assessment questions from Practising and Applying

Question 1	to see if students can create an isometric drawing of a simple cube structure
Question 4	to see if students understand hidden cubes in an isometric drawing

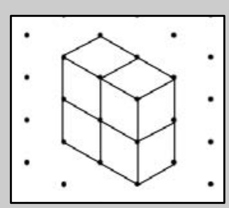
Answers

A. *Sample response*: Anything resembling this:



B. *Sample responses*:

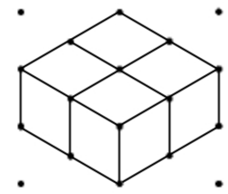
i)



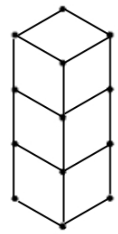
ii) The dots made it easy to draw the parallel and congruent edges.

Answers [Continued]

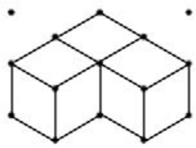
1. a)



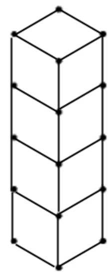
b)



c)

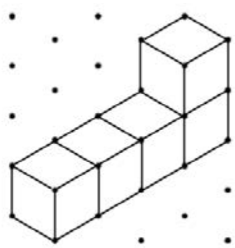


d)

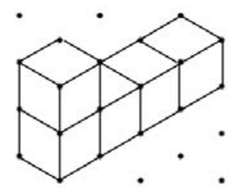


2. *Sample responses*:

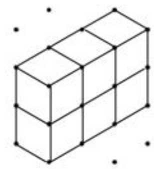
b)



c)

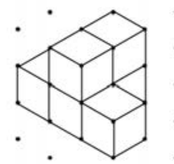


3. *Sample response*:



4. a) 2 cubes

c) *Sample response*:



[**5.** They could be looking at different views of the same structure.]

Supporting Students

Struggling students

• If students are struggling with the visualization skills required in **questions 1 and 4**, have them first build the structure. For **question 1**, you might encourage them to try only the first two or three parts using cubes.

• Some students might benefit from more practice. Have them repeat **question 2** using different structures. Encourage them to keep the structures simple. If necessary, remind them to create the drawing one cube face at a time, beginning with a cube that has three faces visible.

Enrichment

• You might challenge students to make isometric drawings of more complicated structures. Ask them to create isometric drawings from enough different views to show all of the cubes.

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UNIT 2 Revision

Pacing	Materials
2 h	• Grid paper or Small Grid Paper (BLM) • Rulers • Sample Net of Square-based Pyramid (BLM) (optional) • Linking cubes (12 per student) • Sample Net of Cube (BLM) (optional)

Question(s)	Related Lesson(s)
1 and 2	Lessons 2.1.1 and 2.1.2
3	Lesson 2.2.1
4 and 5	Lessons 2.2.1 and 2.2.2
6 and 7	Lesson 2.2.4
8	Lessons 2.2.1, 2.2.2, and 2.2.4
9	Lesson 2.2.3
10 – 12	Lesson 2.3.1
13	Lessons 2.3.2 and 2.3.3

Revision Tips

Q 2: Encourage students to use their answers from **question 1** to answer this question.

Q 5: This question provides a link between **chapter 1** and **chapter 2**. Encourage students to think about how the original shape compares to the image when a shape is translated or reflected.

Q 8: Students may wish to include an example in the answer to help them explain their thinking.

Q 10: You might provide a model of a square-based pyramid for students to use for this question.

Q 11: Some students may answer incorrectly because the nets make the same type of shape. You may wish to point out the word “exactly”.

Q 13: You might encourage students to build the structure with cubes and test for places where cubes could be hidden.

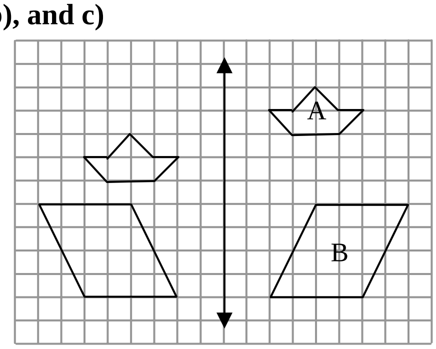
Answers

1. a) Isosceles, right
b) Scalene, obtuse
c) Equilateral, acute

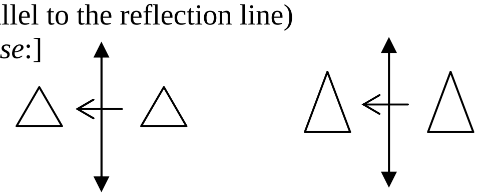
2. a)
a) 1 line of symmetry
b) 0 lines of symmetry
c) 3 lines of symmetry
b)
a) 2 acute angles
b) 2 acute angles
c) 3 acute angles

3. 7 units right and 1 unit up; [*Sample response*: I counted squares to see how the point at the tip of the arrow moved. I know all the points move the same way.]

4. a), b), and c)



5. Equilateral or isosceles triangle [(with a line of symmetry parallel to the reflection line)
Sample response:]

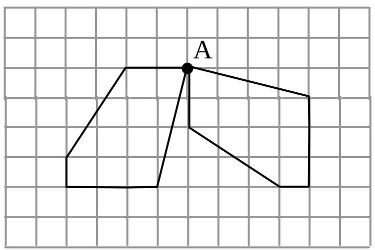


Answers [Continued]

6. a) A $\frac{1}{2}$ turn clockwise around vertex D or a $\frac{1}{2}$ turn counterclockwise around vertex D.

b) A $\frac{1}{2}$ turn clockwise around vertex D or a $\frac{1}{2}$ turn counterclockwise around vertex D.

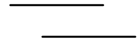
7. a)



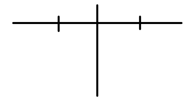
b) A $\frac{3}{4}$ turn clockwise around A.

[**8.** *Sample response*:
• A translation moves every point the same distance in the same direction, but a reflection and a rotation do not.
• In a translation, the shape ends up facing the same way. In a reflection, the image faces the opposite way. In a rotation, the shape faces a different way (unless it is a full rotation).
• A reflection has a reflection line, a transformation has a translation rule, and a rotation has a turn centre.]

9. a)



b)



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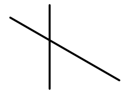
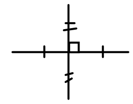
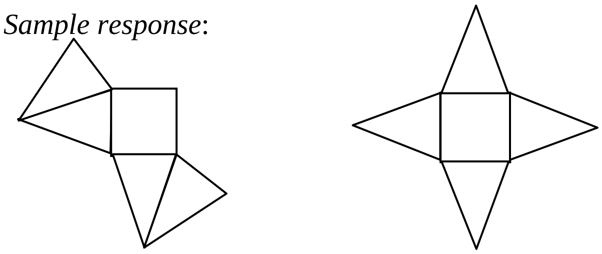


d)



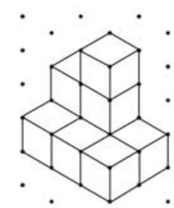
10. *Sample response*:



11. No; [They are both triangle-based prisms, but the rectangles are different sizes so the prisms will be different lengths.]

12. Hexagon-based prism. [There are 2 congruent hexagons and rectangles.]

13. a) 9; 11
b)



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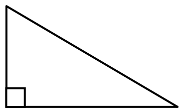
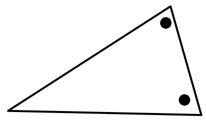
UNIT 2 Geometry Test

1. Classify each by side length and by angle.

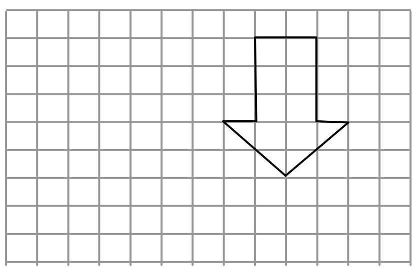
a)



b)

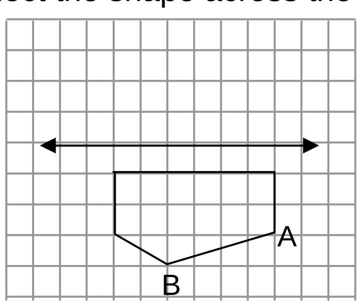


2. a) Translate 4 units left and 3 units down.



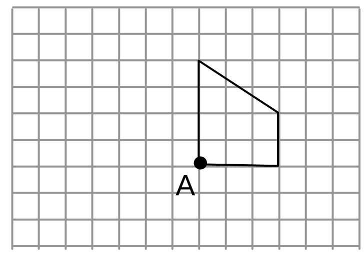
b) What translation would return the image to its original position?

3. a) Reflect the shape across the line.



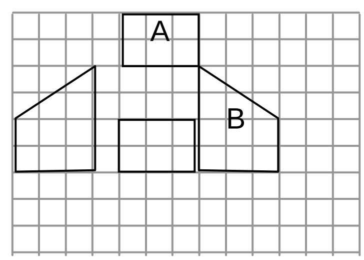
b) Join vertices A and B to their images. What do you notice about the 2 line segments you just drew?

4. a) Rotate the shape $\frac{3}{4}$ turn cw around A.



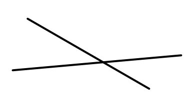
b) Which other rotation would result in the same image?

5. You can be certain which transformation moved B to its image, but you cannot be certain which transformation moved A to its image? Why?

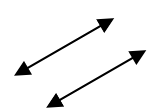


6. Describe each using words such as intersecting, parallel, perpendicular, centre point, and endpoint.

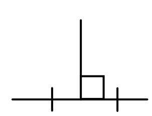
a)



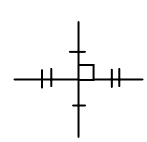
b)



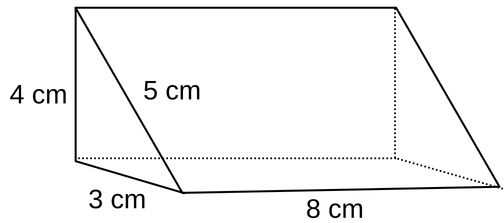
c)



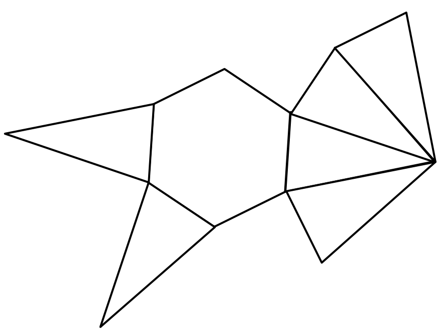
d)



7. Draw a net for this prism on grid paper. The bases are right triangles.



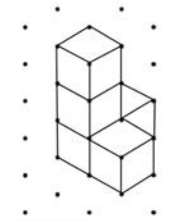
8. a) Is this a net for a pyramid or for a prism? What clues did you use to decide?



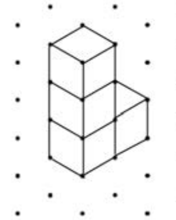
b) Draw another net for the same shape.

9. Could these drawings represent the same structure? How do you know?

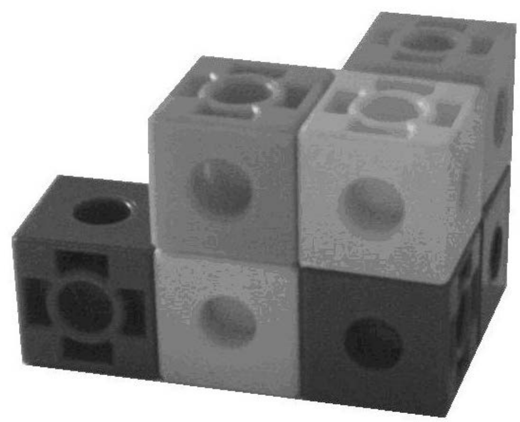
A.



B.



10. a) How many cubes could be hidden in this cube structure?



b) Create an isometric drawing of the structure.

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UNIT 2 Test

Pacing	Materials
1 h	• Rulers • Grid Paper (1 cm by 1 cm) (BLM) • Linking cubes (10 per student) • Sample Net of Cube (BLM) (optional)

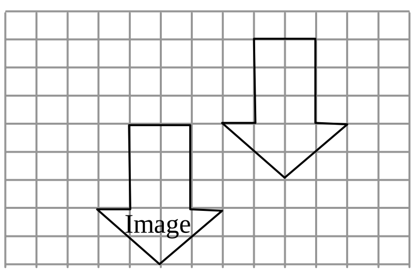
Question(s)	Related Lesson(s)
1	Lessons 2.1.1 and 2.1.2
2	Lesson 2.2.1
3	Lessons 2.2.2 and 2.2.3
4	Lesson 2.2.4
5	Lesson 2.2.2 to 2.2.4
6	Lesson 2.2.3
7 and 8	Lesson 2.3.1
9	Lesson 2.3.2
10	Lessons 2.3.2 and 2.3.3

Select questions to assign according to the time available.

Answers

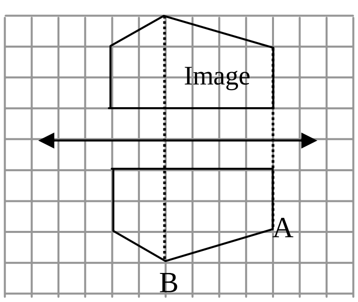
1. a) Scalene, right
b) Isosceles, acute

2. a)



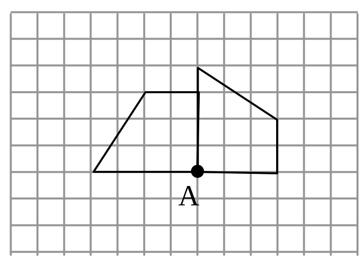
b) 4 units right, 3 units up

3. a) and b)



The 2 line segments are perpendicular to the line of reflection and parallel to each other.

4. a)

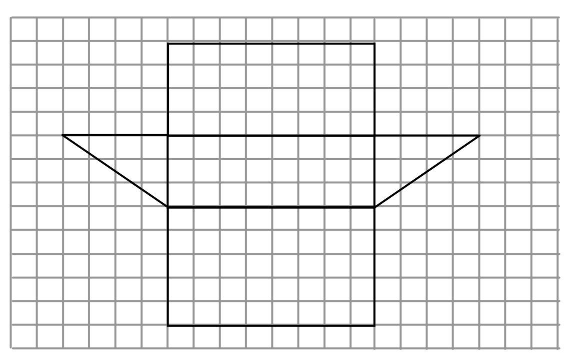


b) A $\frac{1}{4}$ turn counterclockwise around A

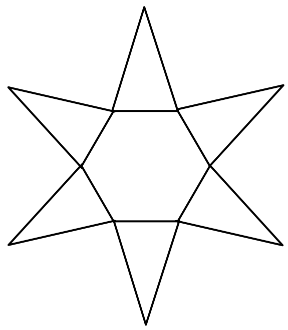
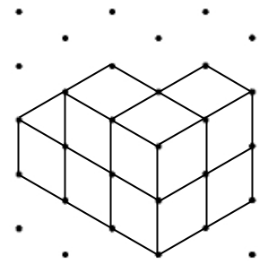
5. A could be translated or reflected, but B can only be reflected because of its shape.

6. a) Intersecting but not perpendicular
b) Parallel
c) Perpendicular at one centre point
d) Perpendicular at both centre points

7. *Sample response*:



Answers [Continued]

8. a) Pyramid; There are many triangles and only one other shape, so that shape has to be the base **b)** *Sample response*: 	**9.** No; the single cube at the back in A would show at the front left in B, but it is not there. **10. a)** 2 cubes **b)** 

UNIT 2 Assessment Interview

You may wish to take the opportunity to interview selected students to assess their understanding of the work of this unit. Interviews are most effective when done with individual students, although pair and small group interviews are sometimes appropriate. The results can be used as formative assessment or as summative assessment data. As the students work, ask them to explain their thinking.

Show the student a set of paper triangles: an equilateral triangle, a right scalene triangle, and an obtuse isosceles triangle. Also provide grid or square dot grid paper, isometric dot paper, and linking cubes. Ask the student the following questions:

• *Which triangle is obtuse? How do you know?*

• *How else could you classify that triangle? Why?*

• *Copy the right triangle onto the grid. Label its vertices A, B, and C.*
Show me how you would do each of these:

- *translate the triangle 3 units to the right and 4 units down*
- *reflect the triangle using a vertical line*
- *rotate the triangle a quarter turn clockwise around the right angle vertex*

• *Use five linking cubes to build a structure. Make an isometric drawing of it.*

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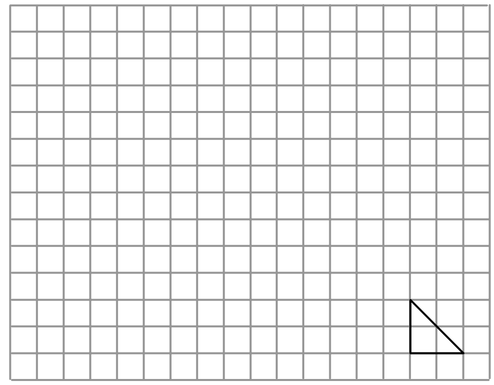
UNIT 2 Performance Task — Transforming a Triangle

A. You need grid paper that is 14 squares tall and 18 squares wide.

i) Draw a right isosceles triangle with congruent sides 2 units long on the grid in the bottom right corner as shown here.

ii) Reflect your triangle across its vertical side to make a larger triangle.

iii) What type of triangle is the larger triangle? How do you know?



B. i) Mark a turn centre at the top vertex of the larger triangle from **part A iii)**.

Transform your double triangle:

- Rotate it a $\frac{3}{4}$ turn cw.
- Rotate the image a $\frac{1}{2}$ turn cw.
- Rotate that image a $\frac{1}{4}$ turn cw.

ii) What shape have you made?

C. i) Erase all the lines inside the shape you created in **part B**.

ii) Translate your shape on the grid 5 times to create a net for a cube.

iii) Write directions for translating a square to make a net for a cube.

D. Identify 1 pair of line segments in your net for each.

i) parallel lines

ii) line segments that are perpendicular at both centre points

iii) line segments that are perpendicular at both endpoints

iv) line segments that are perpendicular at 1 endpoint and 1 centre point

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UNIT 2 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
5-E1 Triangles: explore equilateral, isosceles, scalene triangles, and right, obtuse, and acute triangles **5-E4** Parallelism and Perpendicularity: lines and line segments **5-E5** Translations and Reflections using horizontal and vertical reflection lines: generalize and apply properties **5-E6** Rotations: quarter, half, and three-quarter rotations about the vertex of a shape **5-E7** Nets: prisms and pyramids	1 h	• Grid paper • Scissors • Isometric Dot Paper (BLM) (optional)

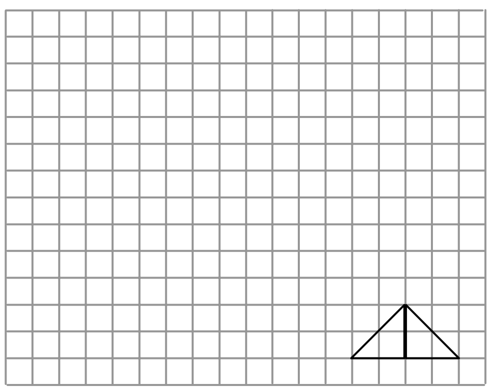
How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. You can assess performance on the task using the rubric provided on the next page.

• This task could also be used as enrichment material for some students. For additional enrichment after they have completed the task, have students work together in small groups to build a structure with their cubes. Then ask them to each make an isometric drawing of the structure.

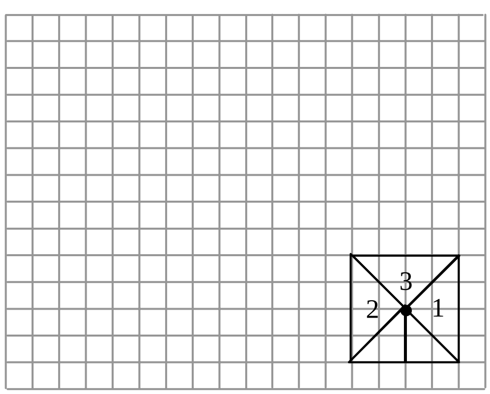
Sample Solution

A. i) and ii)



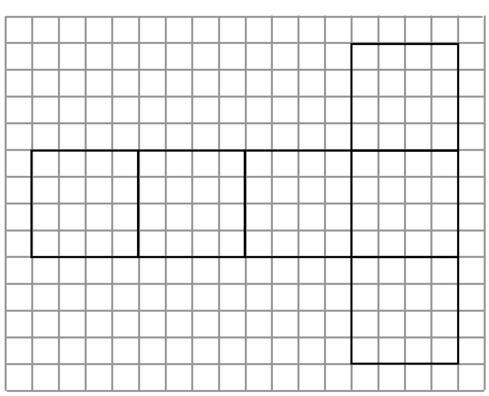
A. iii)
• I know it is a right triangle because I compared the top angle in the larger triangle to the corner of my ruler and it was the same angle.
• I know it is isosceles because I know the original triangle and its image are congruent.

B. i)



B. ii) Square

C. i) and ii)



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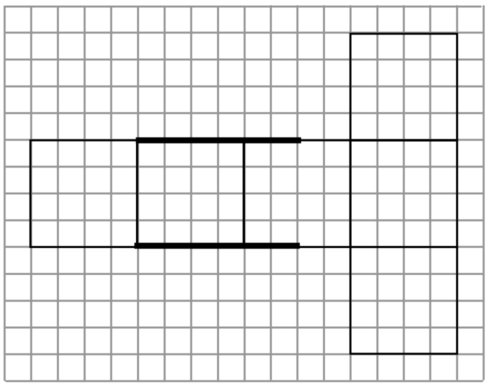
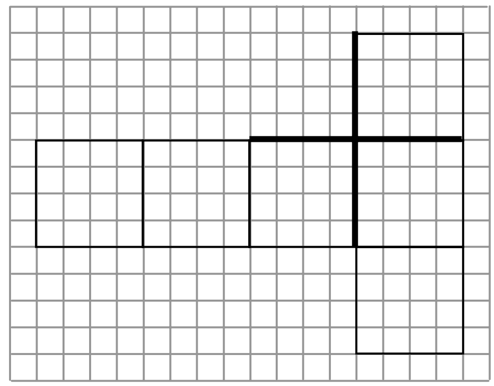
iii)
Sample response:
Translate the square and then each image following these translation rules:
- 4 units up
- 4 units up
- 4 units left, 4 units down
- 4 units left
- 4 units left

D. *Sample responses*:

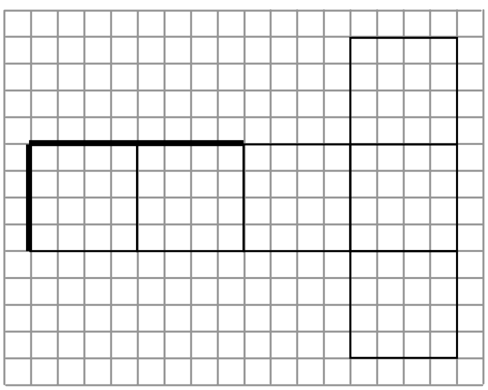
i)



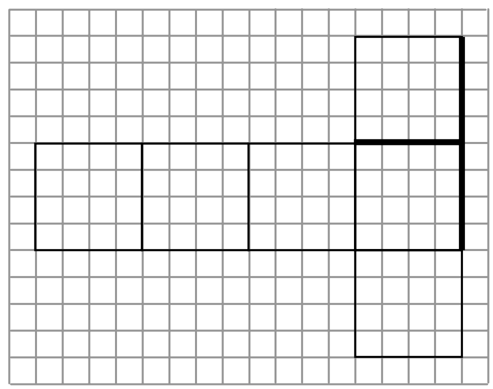
ii)



iii)



iv)



UNIT 2 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Classifies triangle and explains thinking	Classifies accurately and provides thorough explanations	Classifies accurately and provides reasonable explanations	Classifies accurately but with minimal explanation	Shows major flaws in classification or explanation
Performs transformations	Performs completely accurate transformations	Performs reasonably accurate transformations (errors do not suggest misconceptions)	Performs reasonably accurate transformations for most of the design	Shows major errors in transformations
Describes translations	Provides complete and accurate descriptions of all the transformations	Provides reasonably complete descriptions for all the transformations (errors or missing items do not suggest misconceptions)	Provides reasonably complete descriptions for most of the transformations	Shows major errors in descriptions
Identifies parallel and perpendicular line segments	Identifies completely and accurately	Identifies reasonably accurately with no major errors	Identifies reasonably accurately with some errors	Shows major flaws in identification

BLM 1 Small Grid Paper

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BLM 2 Grid Paper (1 cm by 1 cm)

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BLM 3 Special Rectangles

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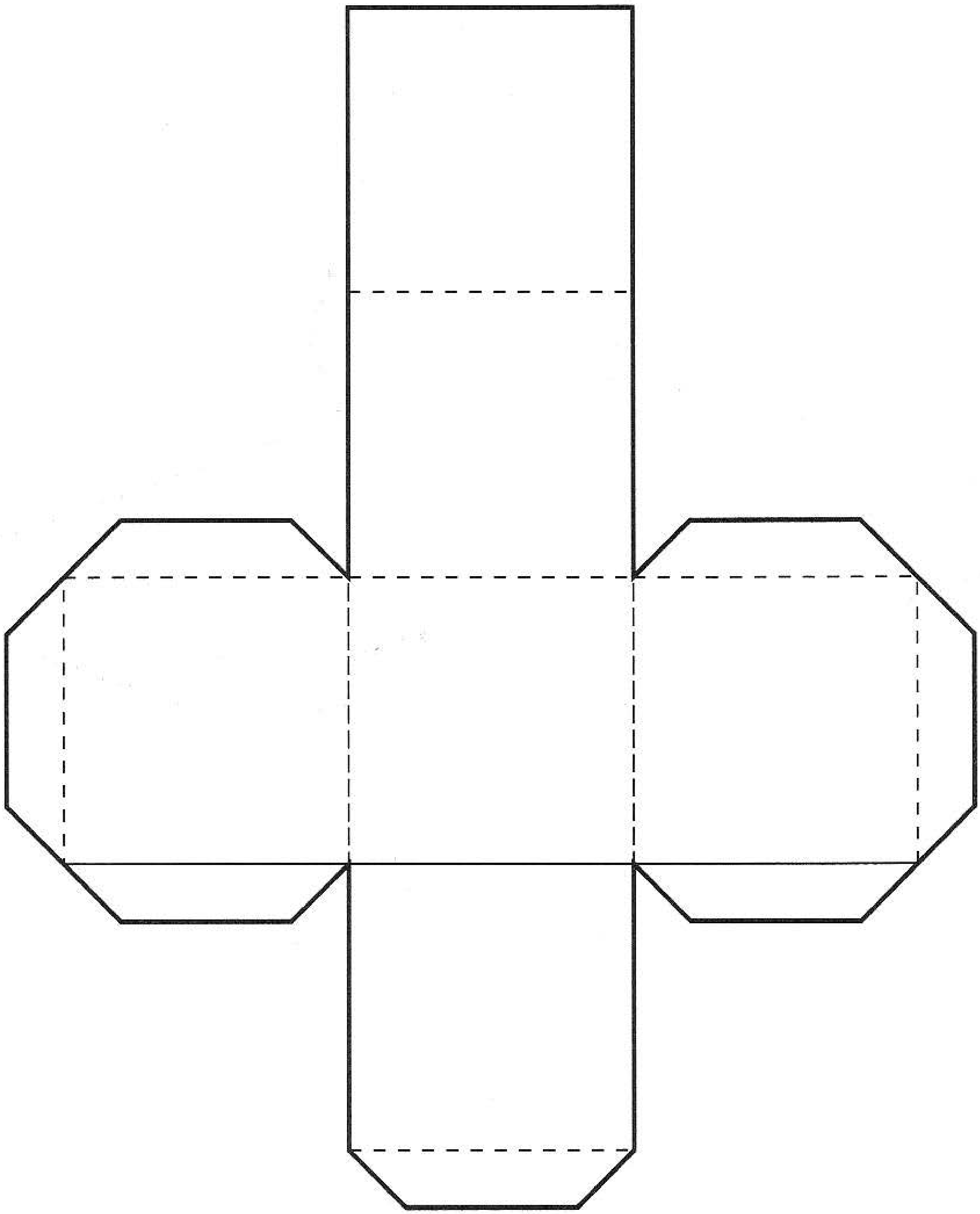
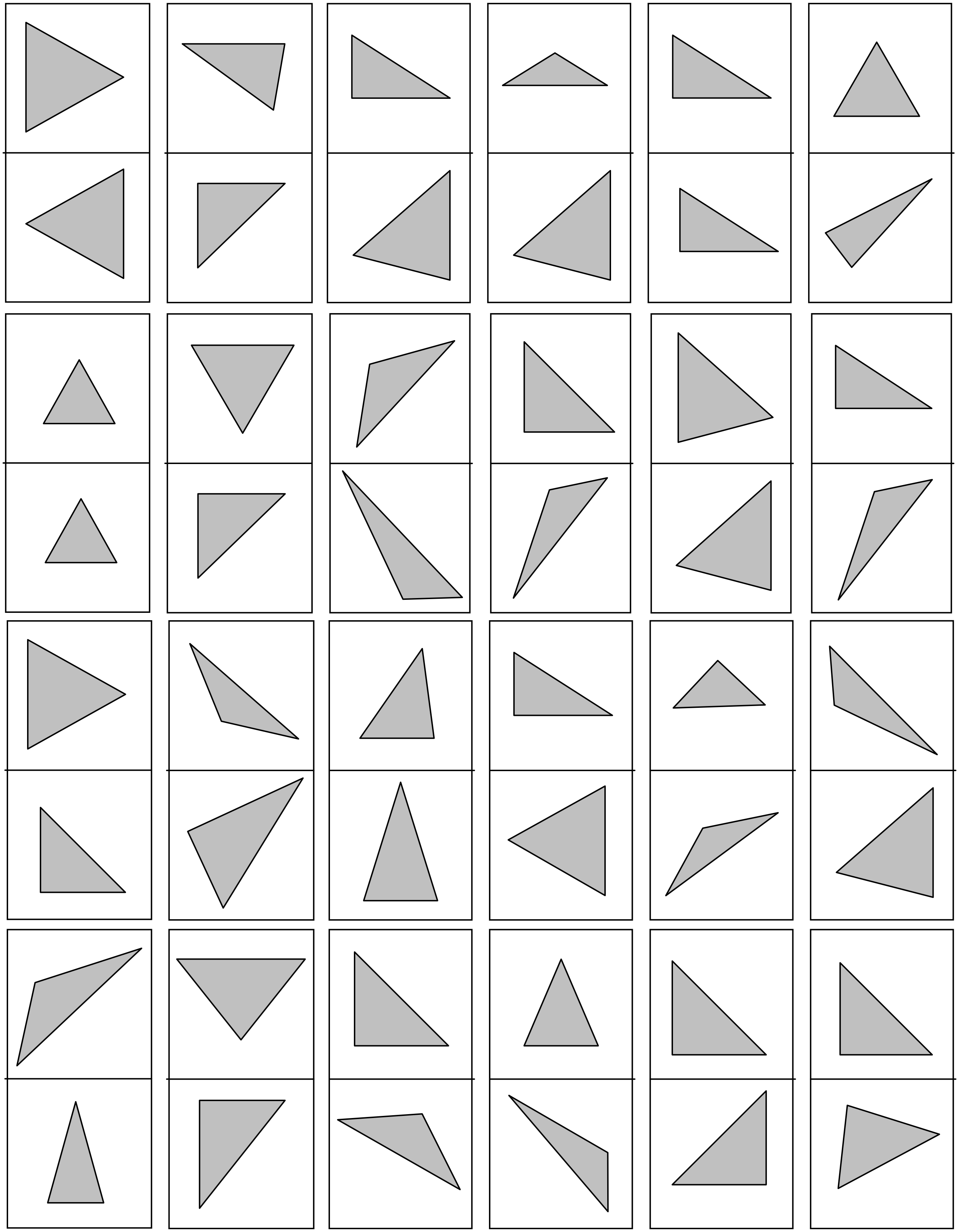
BLM 4 Sample Net of Cube



BLM 5 Triangle Dominoes Game Cards



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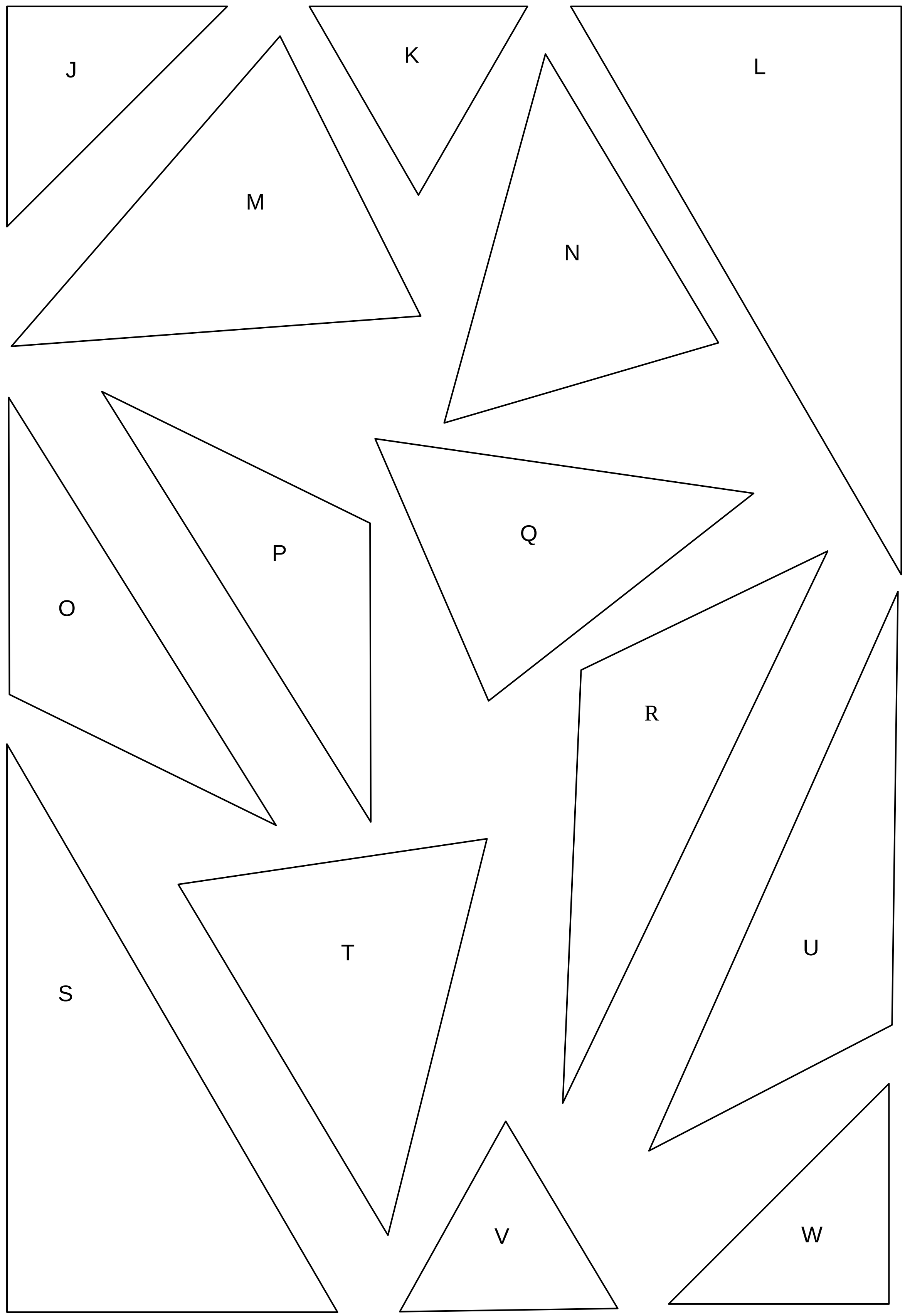
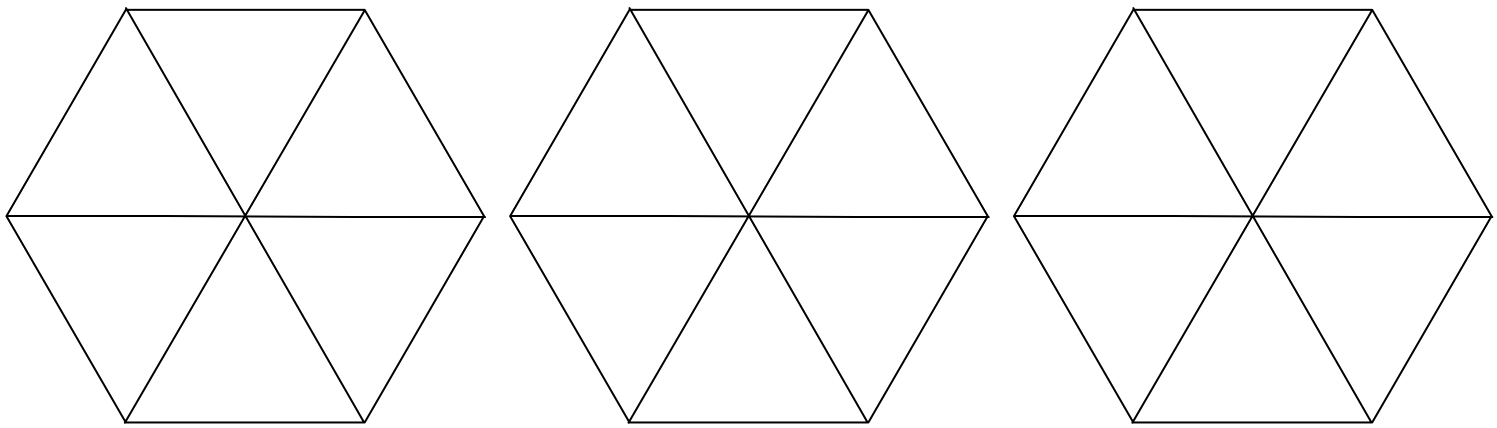
BLM 6 Combining Triangles

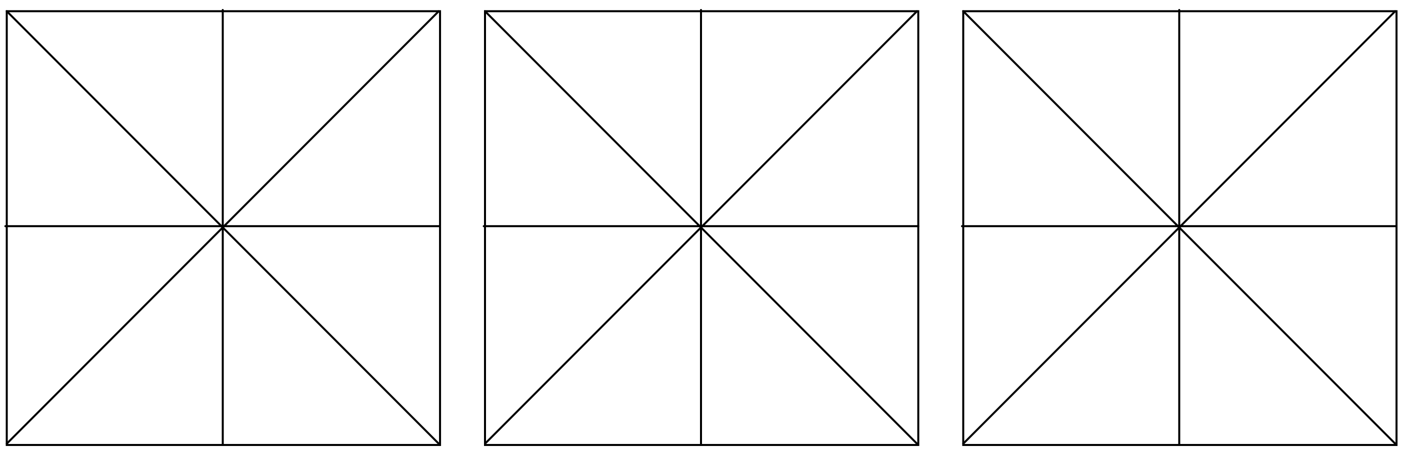


BLM 7 Kaleidoscope Images

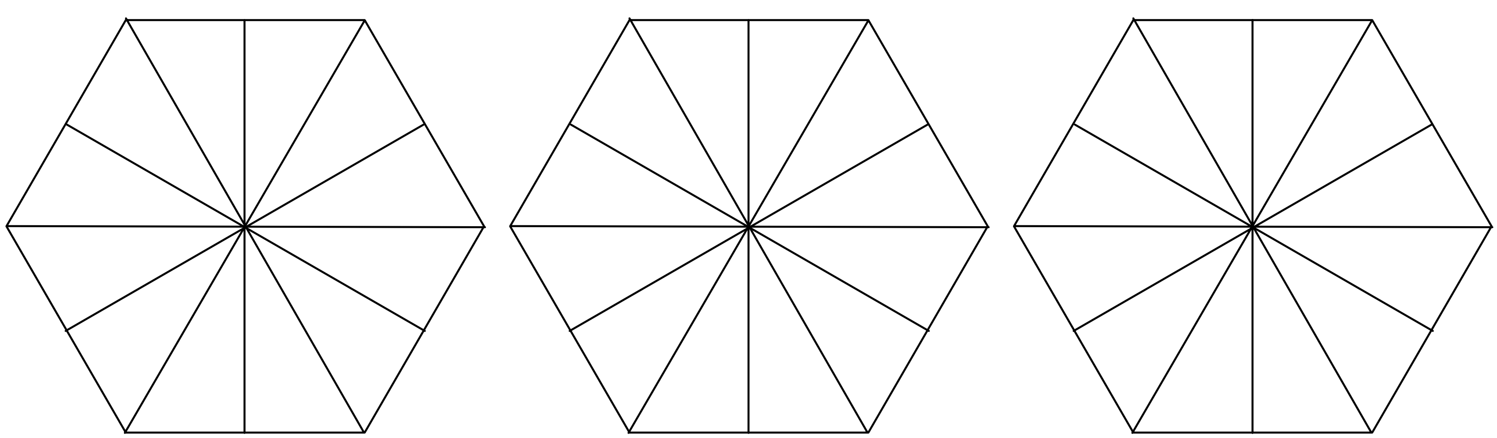
60°- 60°- 60°



45°- 45°- 90°



30°- 60°- 90°



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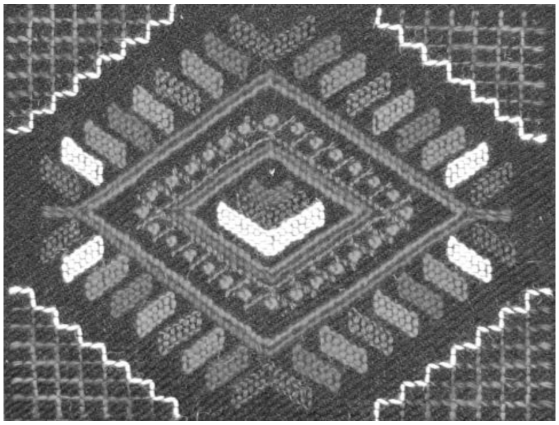
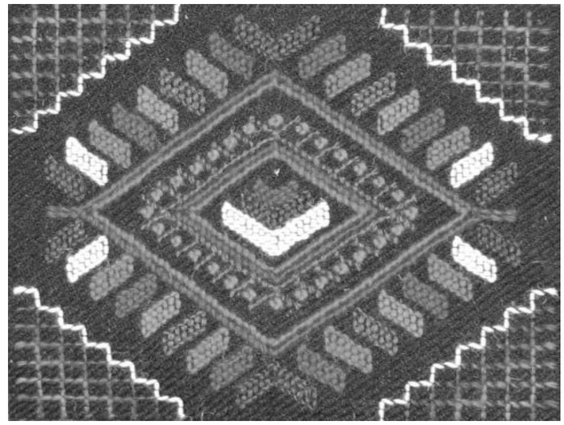
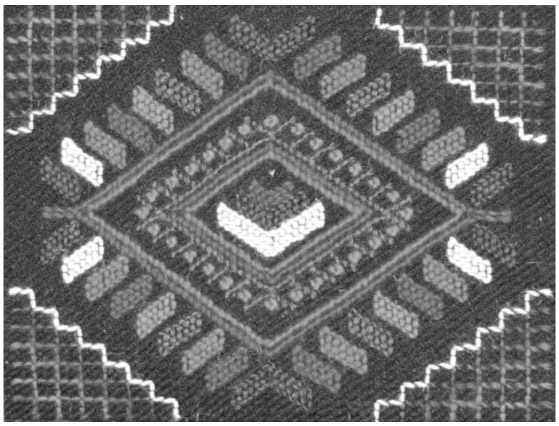
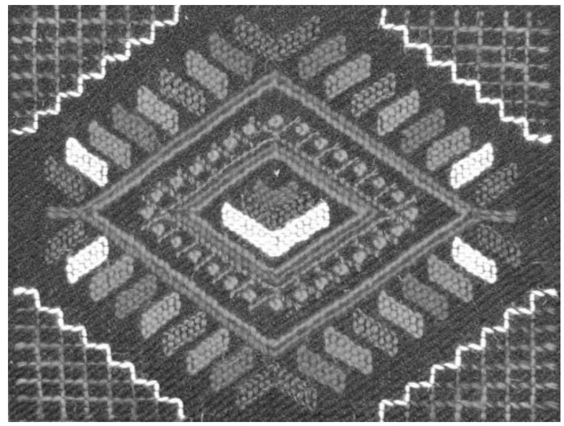
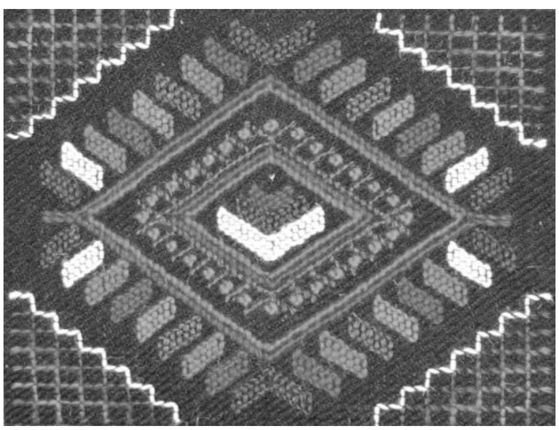
BLM 8 Parallel and Intersecting Lines

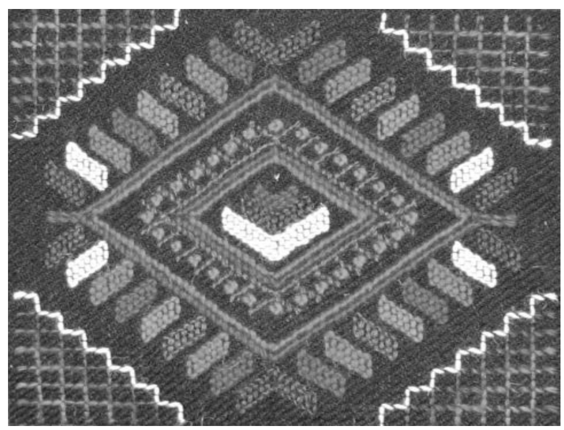


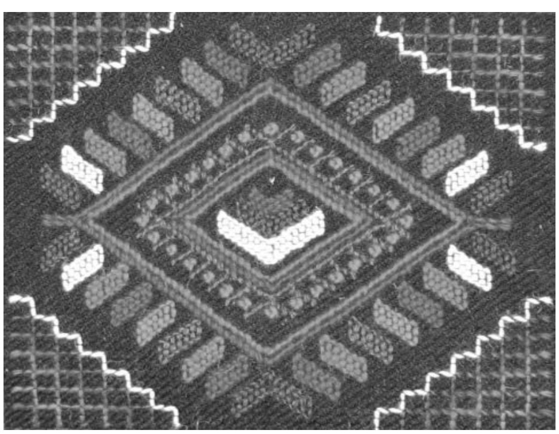


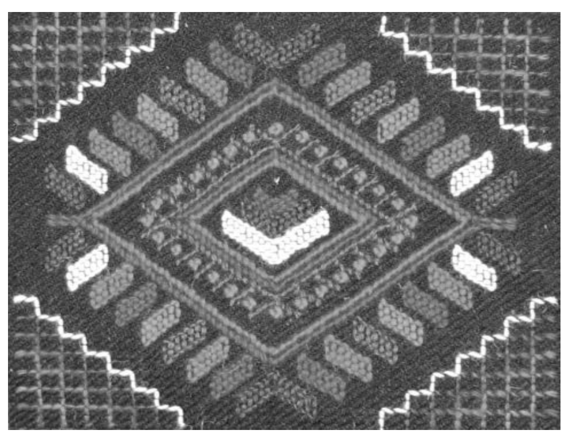




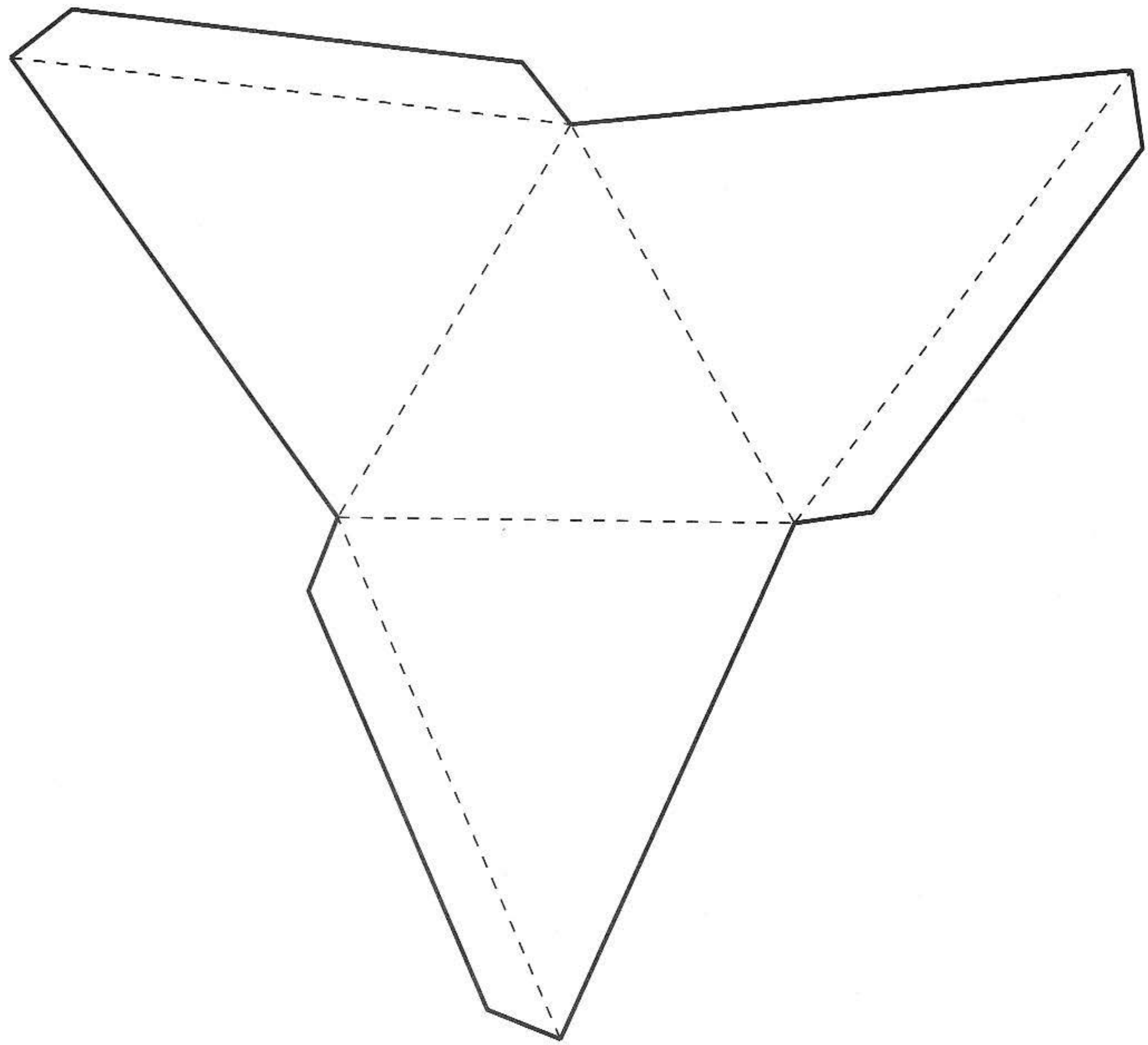








BLM 9 Sample Net of Triangle-based Pyramid



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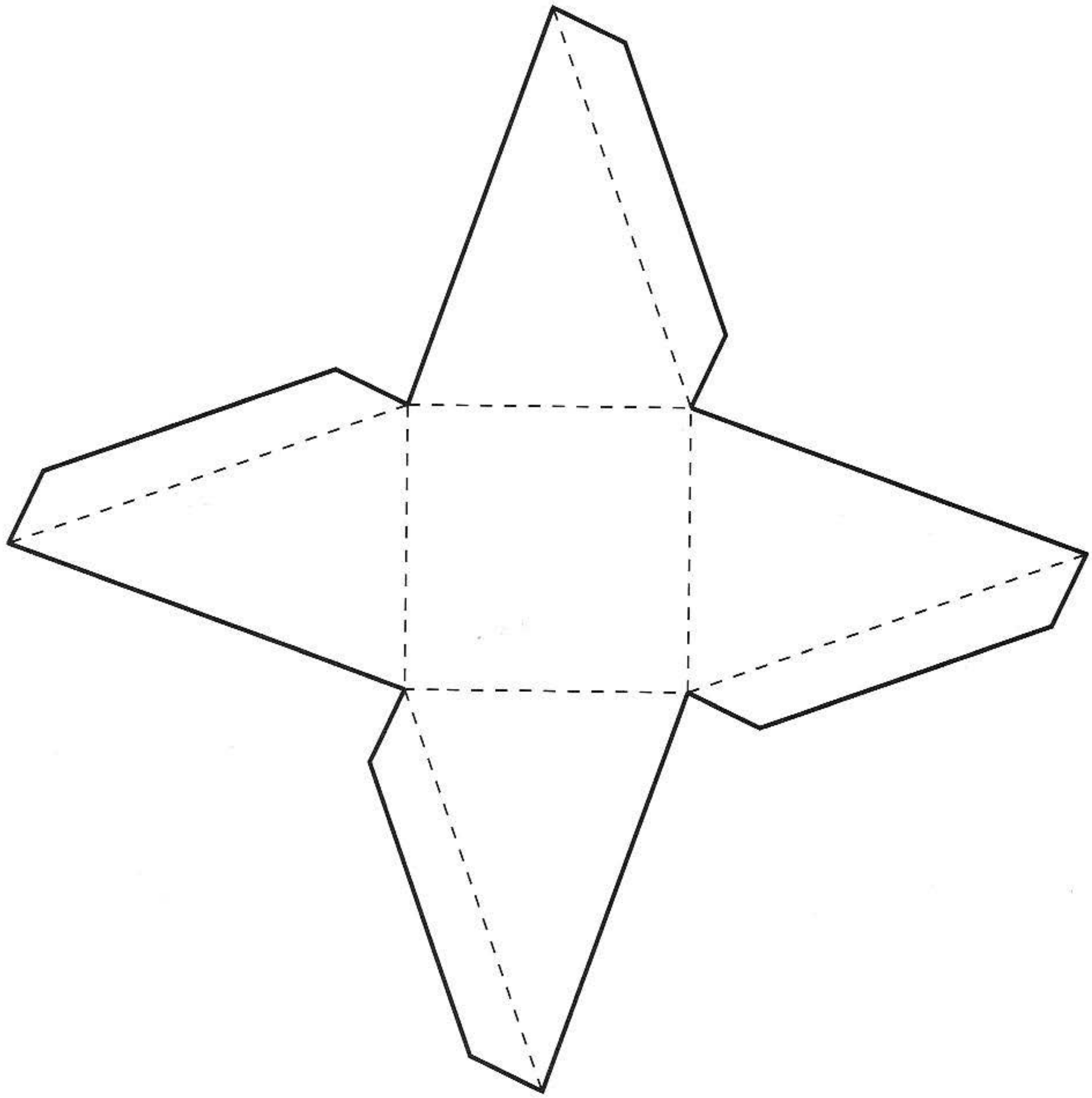
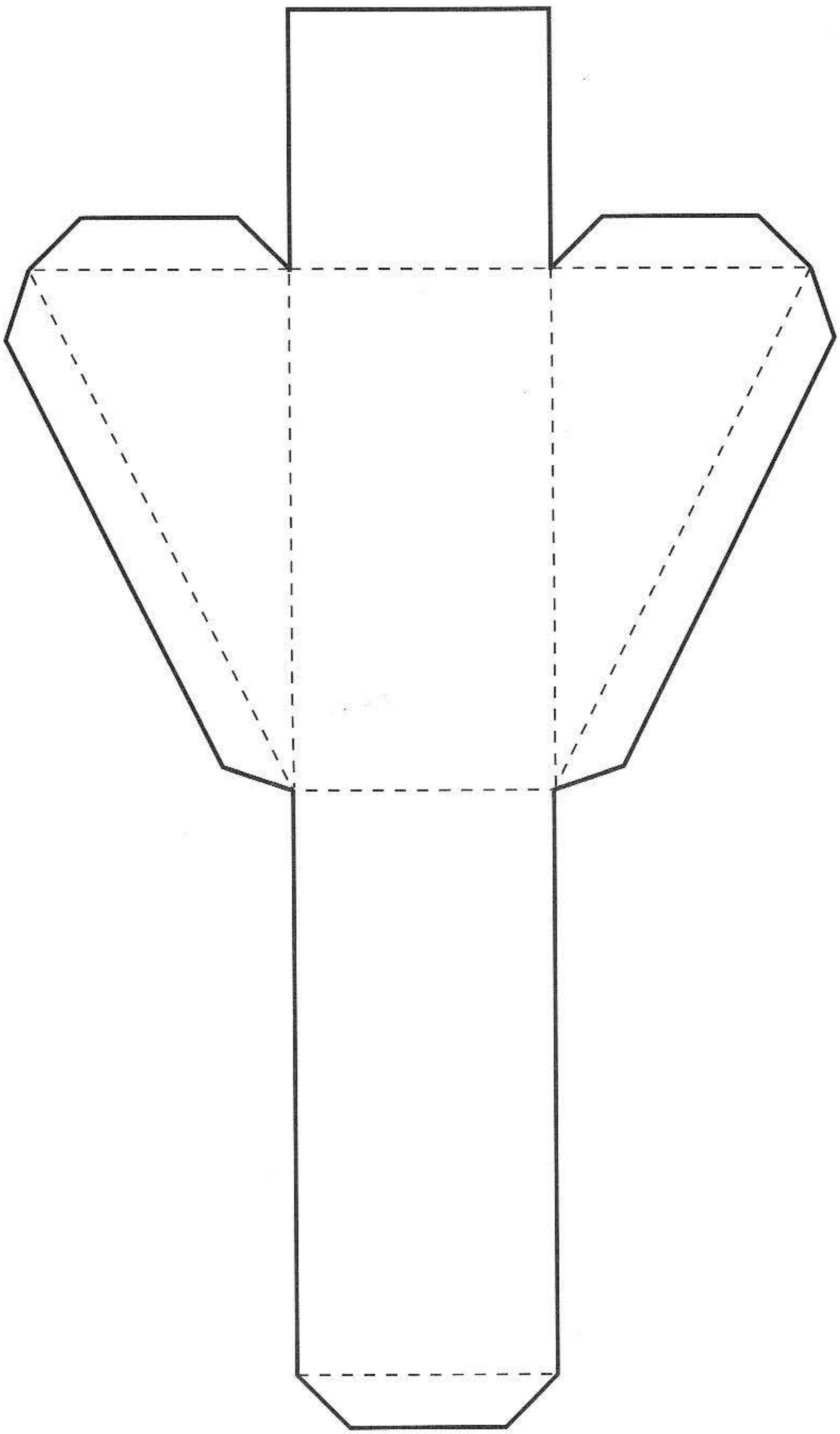
BLM 10 Sample Net of Square-based Pyramid



BLM 11 Sample Net of Right Triangle-based Prism



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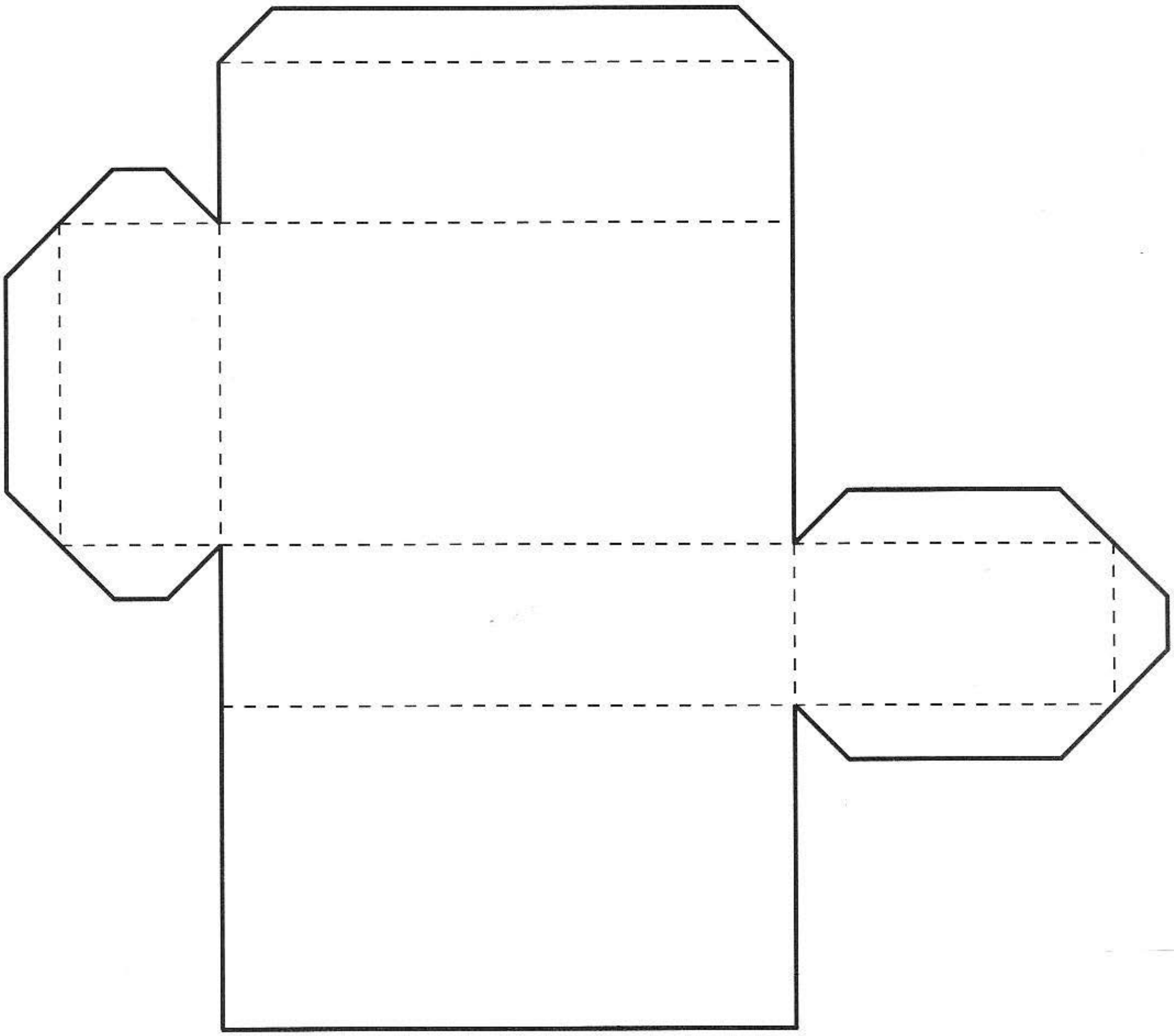
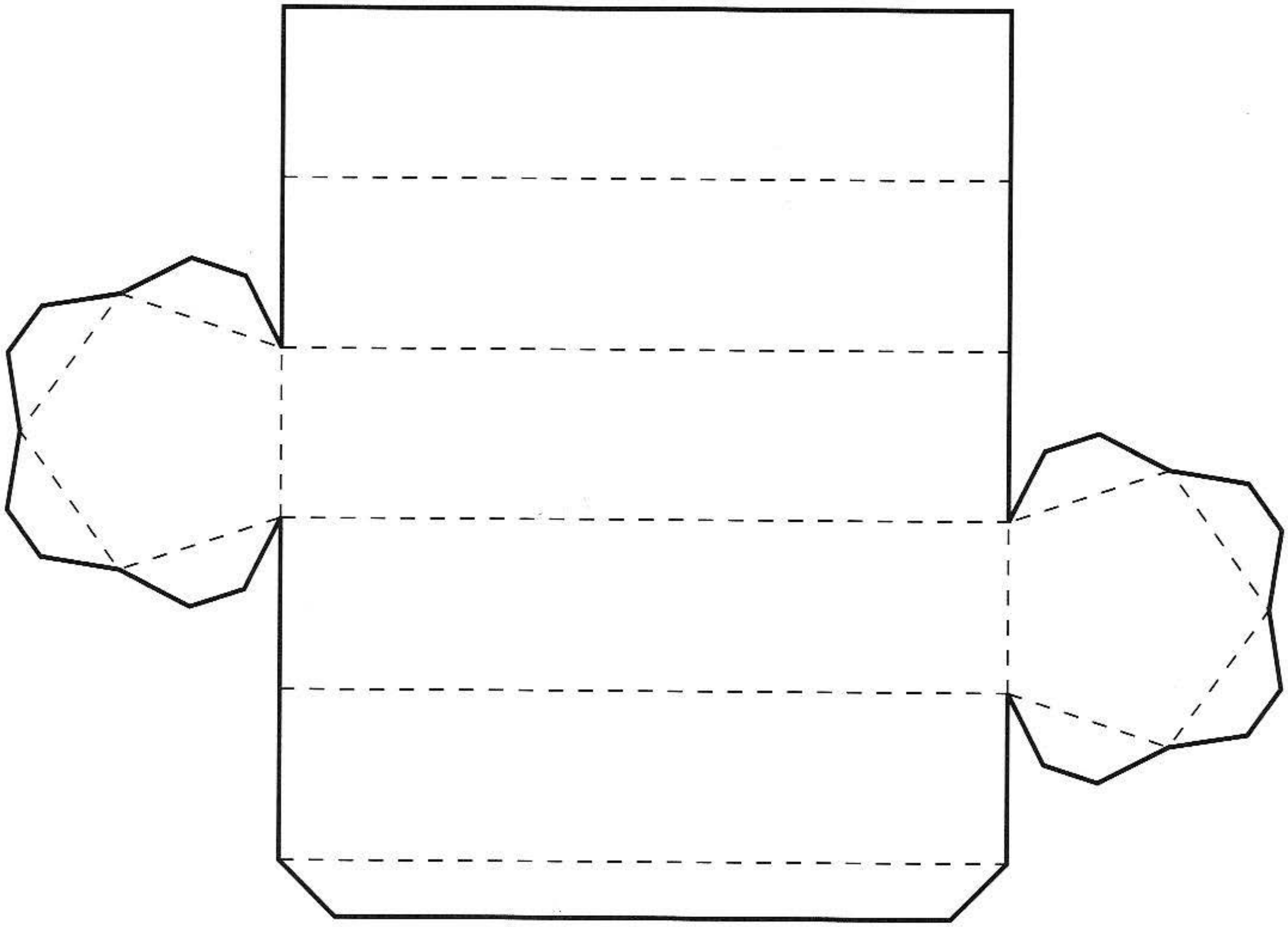
BLM 12 Sample Net of Rectangle-based Prism



BLM 13 Sample Net of Regular Pentagon-based Prism



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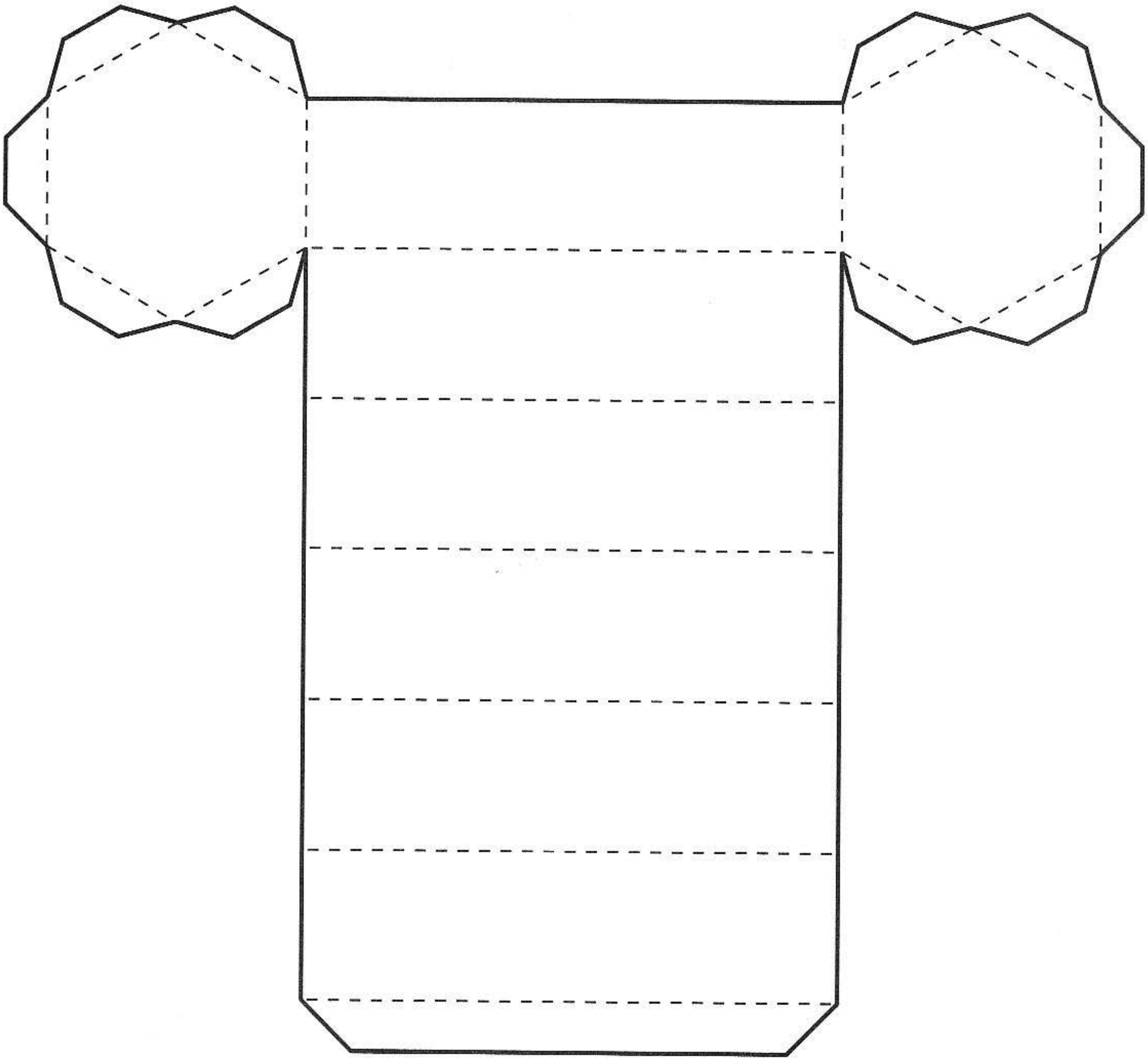
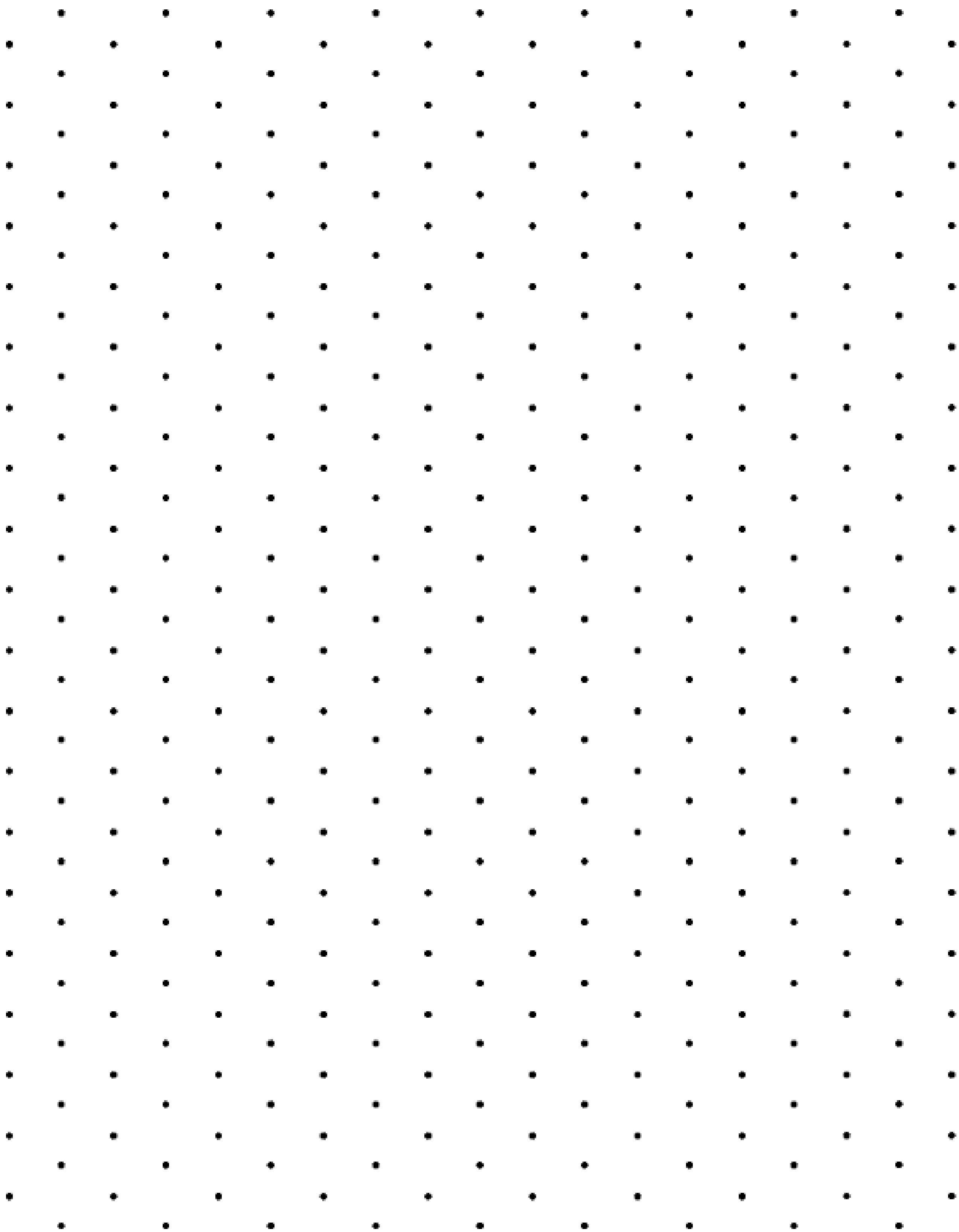
BLM 14 Sample Net of Regular Hexagon-based Prism



BLM 15 Isometric Dot Paper



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UNIT 3 FRACTIONS AND DECIMALS

UNIT 3 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 75** **TG p. 123**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• Paper squares	All questions
Chapter 1 Fractions				
3.1.1 EXPLORE: Meanings of Fractions (Essential) **SB p. 77** **TG p. 126**	**5-A1 Meaning of Fractions: using and relating different meanings** • relating fraction meanings: part of a region, part of a group, part of a length, and as division • develop the relationship between fractions and division	1 h	• Rulers	Observe and Assess questions
3.1.2 Fractions as Division **SB p. 79** **TG p. 129**	**5-A1 Meaning of Fractions: using and relating different meanings** • develop the relationship between fractions and division • change an improper fraction to a mixed number • link concrete materials and/or pictorial representations to symbols to develop understanding	1 h	• Paper circles • Fraction pieces (optional)	Q1, 4, 9
3.1.3 Equivalent Fractions **SB p. 82** **TG p. 132**	**5-A2 Rename Fractions: with and without models** • develop an understanding of renaming fractions using concrete materials and/or pictorial representations first and then link to the symbolic • understand equivalent fractions as the same region or group partitioned in different ways • understand the relationship between the numerator and the denominator of a fraction **5-C3 Equivalent Fractions: multiplicative relationship** • investigate the multiplicative relationship between the numerators and denominators of equivalent fractions • explore equivalent fractions by subdividing equally (e.g., for $\frac{3}{4}$, subdivide each fourth into 3 equal parts to result in $\frac{9}{12}$) • explore equivalent fractions by grouping equally the fractional parts that make up the whole (e.g., group 4 sixths in groups of 2 to result in 2 thirds) • investigate the results when the numerators of equivalent fractions differ by a constant amount	2 h	• Paper rectangles	Q2, 4, 5

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UNIT 3 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
CONNECTIONS: Fractions and Geometry (Optional) **SB p. 86** **TG p. 136**	Make a connection between mathematical strands: numbers (fractions) and geometry	20 min	None	N/A
3.1.4 Comparing and Ordering Fractions **SB p. 87** **TG p. 137**	**5-A3 Compare and Order Fractions** • develop and use benchmarks to compare fractions • compare fractions with the same denominator • compare fractions with the same numerator • compare improper fractions as mixed numbers	1 h	• Fraction Strips (BLM) (optional)	Q2, 4, 7
GAME: So Many Equivalents (Optional) **SB p. 90** **TG p. 139**	Practise creating equivalent fractions in a game situation	20 min	• Playing cards	N/A
3.1.5 EXPLORE: Adding and Subtracting Fractions (Essential) **SB p. 91** **TG p. 140**	**5-B8 Addition and Subtraction: simple fractions with common denominators** • link concrete models such as fraction strips to symbols • use less formal language to build understanding (e.g., 2 fourths + 1 fourth = 3 fourths)	1 h	• Fraction Strips (BLM)	Observe and Assess questions
Chapter 2 Decimals				
3.2.1 Decimal Thousandths **SB p. 93** **TG p. 143**	**5-A4 Thousandths: model and record** • develop decimal and fractional benchmarks (e.g., 0.432 m is a little less than half a metre)	1.5 h	• Square Thousandths Grids (BLM)	Q1, 3, 7, 10
3.2.2 Decimal Place Value **SB p. 97** **TG p. 147**	**5-A4 Thousandths: model and record** • develop decimal and fractional benchmarks (e.g., 0.432 m is a little less than half a metre) • place decimal numbers on a number line and justify their placements • read the quantitative value of each digit in decimals (e.g., 16.5 is "sixteen and 5 tenths" or "sixteen and a half")	1.5 h	• Place Value Charts II (BLM) • Square Thousandths Grids (BLM) (optional)	Q1, 4, 7
3.2.3 Comparing and Ordering Decimals **SB p. 101** **TG p. 150**	**5-A5 Thousandths: compare and order numbers to thousandths** • compare whole number parts of decimals first • understand that decimals do not need the same number of places after the decimal to be compared (e.g., 0.7 > 0.423) • understand that the number of decimal places after the decimal point does not indicate size	1 h	• Place Value Charts II (BLM)	Q1, 4, 7
GAME: In the Middle (Optional) **SB p. 104** **TG p. 152**	Practise decimal comparisons in a game situation	15 min	• Digit cards	N/A

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UNIT 3 Revision **SB p. 105** **TG p. 153**	Review the concepts and skills in the unit	2 h	• Fraction Strips (BLM) • Square Thousandths Grids (BLM)	All questions
UNIT 3 Test **TG p. 155**	Assess the concepts and skills in the unit	1 h	• Fraction Strips (BLM) • Square Thousandths Grids (BLM)	All questions
UNIT 3 Performance Task **TG p. 158**	Assess concepts and skills in the unit	1 h	None	Rubric provided
UNIT 3 Blackline Masters **TG p. 161**	BLM 1 Fraction Circles (Halves to Tenths) BLM 2 Fraction Strips (Whole to Twelfths) BLM 3 Square Thousandths Grids BLM 4 Place Value Charts II (Thousands to Thousandths)			

Math Background

• This unit extends students' knowledge about fractions, particularly the meanings of fractions and early ideas about adding and subtracting fractions with equal denominators. It also introduces decimal thousandths. The focus is on how decimals are related to fractions. Operations with decimals is taught in Unit 4.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in the **Try This** in **lesson 3.1.3**, where they solve a real-world problem, in **question 10** in **lesson 3.2.1**, where they rewrite a poem in a new context, and in **question 8** in **lesson 3.2.2**, where they create decimals to meet specific conditions.

• They use communication in **question 8** in **lesson 3.1.3**, where they explain why a procedure leads to an incorrect result, in **question 12** in **lesson 3.2.1**, where they discuss precision, and in **question 5** in **lesson 3.2.3**, where they explain decimal comparisons.

• They use reasoning in **question 9** in **lesson 3.1.2**, when they analyse what could have caused an error in thinking, in **question 10** in **lesson 3.1.3**, where they reason abut the number of possible equivalent fractions in a given range, and in **question 5** in **lesson 3.1.4**, where they solve an open sentence involving fractions.

• They consider representations in **question 1** in **lesson 3.1.2**, when they interpret diagrams showing sharing as fractions, and in **lesson 3.1.5**, where they use strips to represent fraction sums and differences.

• Students use visualization skills in **question 7** in **lesson 3.1.2**, where they visualize an alternate way to share objects to show division, and in **question 3** in **lesson 3.1.3**, where they draw a picture to show an equivalent fraction. They visualize to compare fractions in **question 2** in **lesson 3.1.4** and to create the whole, given a part, in the **Connections**.

• They make connections in **lessons 3.1.1 and 3.1.2**, where they connect various meanings of fractions, and in **question 3** in **lesson 3.2.2**, where they relate fractions to decimals.

Rationale for Teaching Approach

• This unit is divided into two chapters.

Chapter 1 focuses on fractions.

Chapter 2 focuses on decimals. Because of the important connection between the two operations, fractions are also included in **Chapter 2** as they relate to decimals.

• There are two **Explore** lessons. The first focuses on how the various meanings of fractions are related. The other is the students' first chance to add and subtract fractions, although it is informal.

• The **Game** provides an opportunity to compare and order decimal thousandths.

• Throughout the unit, you should emphasize the meaning of fractions and decimals, rather than focussing on rules for working with them. The rules will be more helpful once the students fully understand the meaning behind them.

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Getting Started

Curriculum Outcomes	Outcome relevance
4 Mixed Numbers: modeling **4** Renaming Fractions **4** Compare and Order Fractions **4** Hundredths: model and record **4** Hundredths: compare and order	Students will find the work in the unit easier after they have reviewed the concepts of fractions and decimals they encountered in Class IV.

Pacing	Materials	Prerequisites
1 h	• Paper squares	• representing and interpreting fractions as parts of wholes and as parts of groups or sets • representing and interpreting mixed numbers • ordering simple fractions • showing equivalence of fractions using pictures • representing and interpreting decimal tenths and hundredths • ordering decimal tenths and hundredths

Main Points to be Raised

Use What You Know

• The denominator of a fraction tells the total number of items in a set. The numerator tells the number of items being considered.

• Whenever a fraction or a mixed number is used, the whole must be specified.

• If two fractions have the same denominator, the greater fraction is the fraction with the greater numerator.

• To compare fractions with each other, it is sometimes useful to compare the fractions to 0, $\frac{1}{2}$, or 1.

Skills You Will Need

• Two different fractions might describe the same part of a whole. These are called equivalent fractions.

• You add the whole number and fraction parts of a mixed number to describe what is being represented.

• A decimal with one digit represents tenths and a decimal with two digits represents hundredths.

• To represent hundredths, you can use a grid with 100 sections.

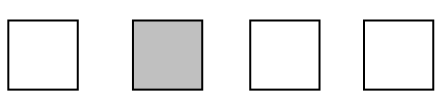
• To compare two decimal hundredths, compare how many hundredths each represents. The greater number of hundredths is the greater amount.

Use What You Know — Introducing the Unit

• Before assigning the activity, you may want to review the use of fractions to describe parts of a whole and parts of a set.

For example:

- Draw three empty squares and one shaded square on the board.



- Ask students what fraction describes the empty squares and what fraction describes the shaded square.

- Discuss why $\frac{4}{4}$ represents the entire group of squares.

• Students can work in pairs to complete the activity.

Observe students as they work. As they work, you might ask questions such as the following:

- *Why did you say that* $\frac{4}{12}$ *of the seats were full?* (There are 12 seats and people were sitting in 4 of them.)
- *Why were all of your denominators 12?* (There are 12 seats in the room.)
- *How do you know that* $\frac{2}{12}$ *is less than* $\frac{5}{12}$*?* (Because 2 out of 12 is not as many as 5 out of 12.)
- *How do you know that* $\frac{4}{12}$ *is closer to* $\frac{1}{4}$ *than* $\frac{7}{12}$ *is?* (If I draw a picture of each fraction, I can see that $\frac{4}{12}$ is only 4 chairs and $\frac{1}{4}$ is only 3 chairs, but $\frac{7}{12}$ is 7 chairs. 4 is closer to 3 than 7 is.)

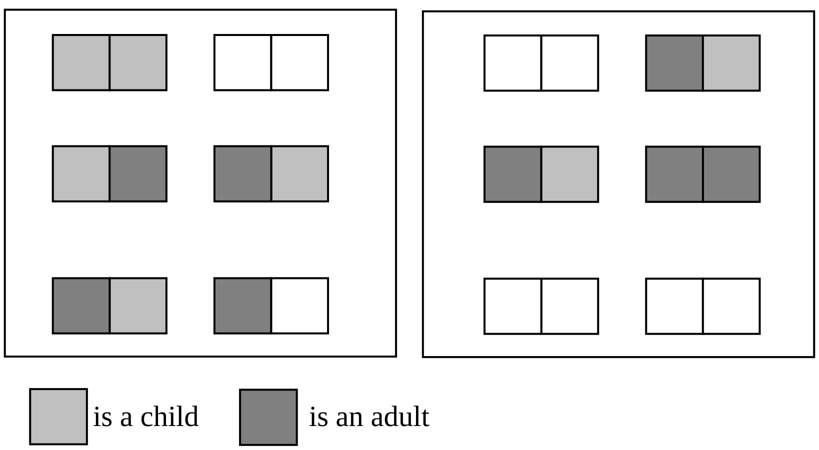
Skills You Will Need

- To ensure students have the required skills for this unit, assign these questions.
- Students can work individually.

Answers

A. *Sample responses*:

i) and ii)



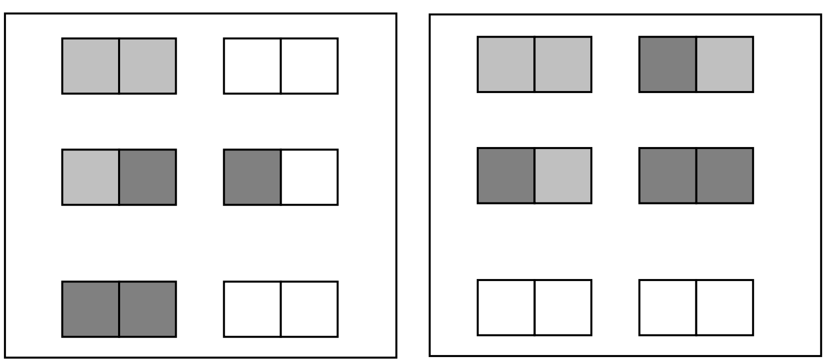
iii) $\frac{9}{12}$ and $\frac{6}{12}$ chairs are filled.

iv) $\frac{5}{12}$ and $\frac{2}{12}$ are filled with children.

v) $\frac{2}{12}$, $\frac{5}{12}$, $\frac{6}{12}$, $\frac{9}{12}$

B. *Sample responses*:

i) and ii)



iii) $\frac{7}{12}$ and $\frac{8}{12}$ chairs are filled.

iv) $\frac{3}{12}$ and $\frac{4}{12}$ are filled with children.

v) $\frac{3}{12}$, $\frac{4}{12}$, $\frac{7}{12}$, $\frac{8}{12}$

C. i) $\frac{9}{12}$ **ii)** $\frac{6}{12}$ **iii)** $\frac{3}{12}$

D. *Sample response:* It tells the fraction of chairs that are filled in the two rooms put together.

E. 4; *Sample response*:

If you arrange the 12 chairs into 3 rows, $\frac{1}{3}$ of the chairs form 1 row and there are 4 chairs in each row.

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1. *Sample responses*:

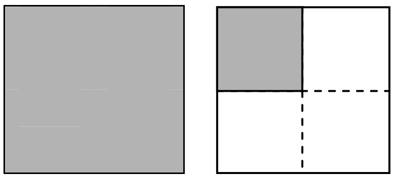
a) $\frac{1}{2}$ and $\frac{2}{4}$ **b)** $\frac{2}{8}$ and $\frac{1}{4}$ **c)** $\frac{1}{3}$ and $\frac{3}{9}$

2. a) $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{5}{6}$ **b)** $\frac{3}{8}, \frac{4}{8}, \frac{6}{8}, \frac{7}{8}$

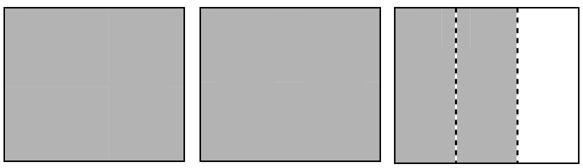
c) $\frac{1}{10}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}$ **d)** $\frac{4}{15}, \frac{4}{9}, \frac{4}{6}, \frac{4}{5}$

3. *Sample responses*:

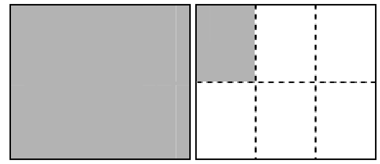
a)



b)



3. c)



4. A matches **ii)**, **B** matches **iii)**, **C** matches **i)**

5. a) $\frac{2}{10}$ **b)** $\frac{45}{100}$

c) $\frac{9}{10}$ **d)** $\frac{5}{100}$

6. a) 0.17, 0.23, 0.29, 0.45
b) 0.17, 0.3, 0.45, 0.5

Supporting Students

Struggling students

• If students are struggling with working with twelfths, you might suggest they use simpler fractions.

For example:
You could have just one room with 8 chairs in it. There could be only 6 people, some adults and some children.
They could compare only two fractions (rather than four) and they could compare only to 0, $\frac{1}{2}$, and 1 (not to $\frac{1}{4}$).

Enrichment

• Students might create other situations with different numbers of seats in a room, different numbers of people in the chairs, and, if they wish, additional rooms.

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Chapter 1 Fractions

1.1.1 EXPLORE: Meanings of Fractions

Curriculum Outcomes	Outcome relevance
5-A1 Meaning of Fractions: using and relating different meanings • relating fraction meanings: part of a region, part of a group, part of a length, and as division • develop the relationship between fractions and division	This lesson helps students relate the different ways we use fractions. It is confusing to use the same fraction to represent completely different situations unless the relationship between those situations is explained.

Pacing	Materials	Prerequisites
1 h	• Rulers	• meanings of fractions as part of a whole (a shape), part of a group, and part of a length

Main Points to be Raised

• By putting each object in a group into the same size section of a whole, you can see how the part-of-a-whole meaning and the part-of-a-group meaning of fractions are related.

• By putting each object in a group above the same size part of a length, you can see how the part-of-a-group meaning and part-of-a-length meaning of fractions are related.

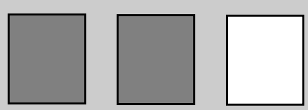
• By representing the numerator of a fraction as a number of objects and sharing those objects equally among the number the denominator represents, you can see how the division meaning of fraction and the part-of-a-whole meaning are related.

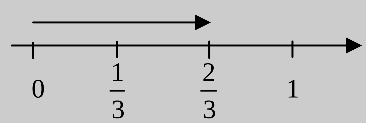
Exploration

• Remind students, if you did not do so for the **Getting Started** section, what a fraction means with respect to a whole, a group, and a length.

For example:

You might divide a shape into 3 equal parts and colour 2 of the parts, you might show 2 dark shapes and 1 light shape, and you might draw a number line divided into 3 sections and move to the end of the second section. All of these show the same fraction, $\frac{2}{3}$.





Point out that the same idea is shown in the text for the fraction $\frac{3}{4}$.

Although there is no picture, there is also an indication that $\frac{3}{4}$ is 3 ÷ 4. Draw a picture of a whole divided into 4 equal parts and colour one of them. Help students see that this picture clearly shows 1 whole being divided into 4 equal parts, where each part is $\frac{1}{4}$. This shows that $1 \div 4 = \frac{1}{4}$, and it relates fractions to division.

• Students can work through the activity with a partner.

• Observe while students work. While they work, you might ask questions such as the following:

- *How does the picture show that $\frac{3}{4}$ of the students are boys?* (There are 4 equal sections in the square and 3 sections are filled with boys.)

- *Why did you draw a rectangle with five sections?* (The denominator is 5 so I need five equal sections.)

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- *Where are the four sections of the number line? How could you show three of them?* (The four sections are the parts of the line between 0 and $\frac{1}{4}$, between $\frac{1}{4}$ and $\frac{2}{4}$, between $\frac{2}{4}$ and $\frac{3}{4}$, and between $\frac{3}{4}$ and 1. Crossing three sections would be like going from 0 to $\frac{3}{4}$.)

- *Why might it be helpful to divide each of the three squares into four equal sections to share them? How many sections would there be in all? How many would each person get? What part of one whole square is that?* (I could share by giving each person one of the four sections of each whole. There would be 12 sections altogether and each person would get three. Since each section is one fourth of a whole, each person would get three fourths, which is $\frac{3}{4}$ of a whole.)

Observe and Assess

As students work, notice the following:

• Do students draw appropriate diagrams as they work through the questions?

• Do they distinguish between the different meanings of the same fraction?

• Are their explanations clear?

• Do they use correct mathematical language as they work?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and discuss questions such as these.

• *How did you show* $\frac{5}{6}$ *as a part of a group?*

• *How did you show* $\frac{5}{6}$ *as part of a length?*

• *How can you show the relationship between* $\frac{5}{6}$ *as a part of a group and as part of a length?*

• *To show* $\frac{5}{6}$ *as division, how many objects did you draw? Why? How did you share them?*

Answers

A. i) *Sample response*:

The square is divided into 4 equal parts and $\frac{3}{4}$ of the parts have pictures of boys.

Or, the group of pictures has 4 pictures in it and $\frac{3}{4}$ of the pictures are of boys.

ii) *Sample response*:

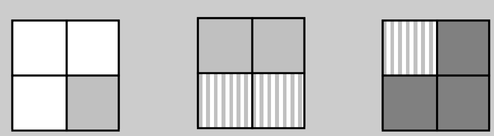
Cat	Cat	Dog	Dog	Dog

iii) The number line is divided into 4 equal parts and the distance from 0 to $\frac{3}{4}$ is 3 parts of the line. If a shape were divided into 4 equal parts, $\frac{3}{4}$ would be 3 parts of the shape.

iv) I could put the pictures of 3 boys and 1 girl above each of the 4 parts of the number line, with the pictures of boys above the first 3 parts.

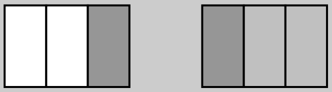
B. i) Each person would get $\frac{3}{4}$ of a square.

Sample response:
If each square were divided into 4 equal parts, there would be 12 parts altogether. Each of the 4 people sharing would get 3 parts.



Answers [Continued]

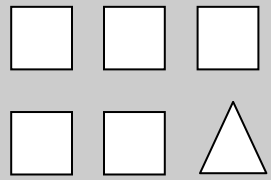
B. ii) *Sample response*:



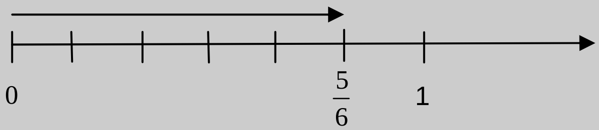
C. i) *Sample response*:

Shape

Group



Length (Number line)



Division

$5 \div 6 = \frac{5}{6}$ (each share is $\frac{1}{6}$ of each of 5 wholes, which is $\frac{5}{6}$ of a single whole)

1	2	3
4	5	6

1	2	3
4	5	6

1	2	3
4	5	6

1	2	3
4	5	6

1	2	3
4	5	6

ii) *Sample response*:
- If you put the squares in the group on the 5 grey parts of the shape and put the triangle on the white part, you show how the shape and group meanings are related.
- Or, if you put the 5 squares in the group on the first 5 parts of the number line and put the triangle on the last part, you show how the group and length (number line) meanings are related.

Supporting Students

Struggling students

- If students are struggling with relating meanings of fractions, you may wish to pull them aside and model the meanings by showing them a few more fractions.

For example, you might show $\frac{3}{5}$ and $\frac{3}{8}$ using all four meanings (part of a whole, part of a group, part of a length, and division) to show them how the numerator and denominator tell you what picture to draw.

It is important to show the relationship between the meanings. A particularly important relationship is that if you divide 3 objects into 8 equal shares (division), each share is the same amount as $\frac{3}{8}$ of one object (part of a whole).

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3.1.2 Fractions as Division

Curriculum Outcomes	Outcome relevance
5-A1 Meaning of Fractions: using and relating different meanings • develop the relationship between fractions and division • change an improper fraction to a mixed number • link concrete materials and/or pictorial representations to symbols to develop understanding	In higher mathematics, it is important that students understand why a fraction like $\frac{a}{b}$ is another way of writing $a \div b$. This is used in Class VII when students convert fractions to decimals. It is also used in later secondary classes.

Pacing	Materials	Prerequisites
1 h	• Paper circles • Fraction pieces (optional)	• representing fractions of a whole • mixed numbers

Main Points to be Raised

• One meaning of fraction is division. The fraction $\frac{a}{b}$ is another way to express $a \div b$.

• To show that $a \div b = \frac{a}{b}$ when $a > b$, it is useful to use the "how many groups" meaning for division. For example, to ask how many groups of 5 are in 12, you write $\frac{12}{5} = 12 \div 5 = 2\frac{2}{5}$. This is how you change an improper fraction to a mixed number.

• To show that $a \div b = \frac{a}{b}$ when $a < b$, it is useful to use the sharing meaning for division, i.e., *a* objects are divided into *b* equal groups. (For $\frac{2}{5}$, if 2 of something are shared among 5 groups, how much does each group get?)

• To show *a* objects being divided into *b* equal groups, you might divide each object into *b* parts. Then there are a total of $a \times b$ parts to be shared among *b* groups. Each group gets *a* parts. The parts are of size $\frac{1}{b}$, so the amount in each part is $\frac{a}{b}$ of one whole.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *How many times would she stir it in 1 h? How do you know?* (4; There are 4 quarter hours in 1 hour.)

• *How could you write the total cooking time as an improper fraction?* ($\frac{4}{4}+\frac{1}{4} = \frac{5}{4}$)

The Exposition — Presenting the Main Ideas

• Present the calculation $12 \div 3$. Ask students how to solve it. Make sure that they know that one way to calculate is to start with 12 objects and make 3 groups of equal size. The quotient, 4, is the size of each of the equal groups.

• Hold up 2 large paper circles. Invite 4 students to the front of the room. Ask how the 2 circles could be shared among the 4 students. Let them figure it out. Most students will realize that each student gets $\frac{1}{2}$ of a circle. Write $2 \div 4$ to show that you have shared 2 circles among 4 people. Let them see that they have shown that $2 \div 4 = \frac{1}{2}$. Another name for $\frac{1}{2}$ is $\frac{2}{4}$, so $2 \div 4 = \frac{2}{4}$.

• Model another example.

For instance, hold up 2 paper circles and say that you want to divide them among 5 students. Ask students to think about how much of a circle each student will get. Show how each circle can each be divided into 5 pieces. There would be 10 pieces, so each gets 2 pieces. Each piece is $\frac{1}{5}$ of a circle, so each share is $\frac{2}{5}$, just as $2 \div 5 = \frac{2}{5}$.

• Ask students to open their books to the exposition. Make sure they understand that each new colour in the diagram represents a different student. Help them see that the 6 students each get 5 pieces and that each piece is $\frac{1}{6}$, so each student's share is $\frac{5}{6}$.

• Point out that the grouping meaning of division, "How many groups of 2 are in 5?", is used to write the improper fraction as a mixed number. This meaning of division will be familiar to most students.

Revisiting the Try This

B. Although students are starting with the mixed number rather than with the improper fraction, it is the division meaning of fraction that allows students to relate $\frac{5}{4}$ to $1\frac{1}{4}$.

Using the Examples

• Write the questions in the two examples on the board. Ask students to answer the questions and then to check their work against the student solution in the text.

Practising and Applying

Teaching points and tips

Q 1: For **part a)**, make sure students realize that 3 objects are being shared and that each new colour represents a new share. In **part b)**, 4 objects are being shared.

Q 2: Observe whether students use the sharing meaning or the grouping meaning of division to solve these questions. Usually the grouping meaning makes it easier for students, but either approach is acceptable.

Q 3: This question encourages students to think about both meanings of division — sharing and grouping.

Q 5: Students should write the remainders as fractions.

Q 7: This question shows sharing in another way.

For example, rather than dividing the 3 items into fourths, counting 12 fourths, and then dividing 12 by 4 to see that each share is 3 fourths, students divide the 3 items into fourths, giving each person one of those fourths. Then the 3 fourths for each person are collected to show that it is 3 fourths of one whole.

Some students might find this way of sharing more natural than the method used in the exposition, but others may not.

Q 8: Students will need to think about equivalent fractions to answer this question.

Common errors

• **Question 9** points out one of the common errors students make in working with fractions as division.

For example, with a fraction like $\frac{2}{9}$, students will divide 9 by 2 rather than dividing 2 by 9 to interpret the fraction.

It is important for students to recognize whether the fraction is greater than 1 or less than 1 so that they can see if their answer makes sense.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can interpret a division question using fractions
Question 4	to see if students can solve a real-world division question with a fraction answer
Question 9	to see if students can correct poor reasoning relating division to fraction ideas

Answers

A. 5 times

B. *Sample response*:
You use division to go from an improper fraction to a mixed number. When you write the mixed number in **part A** as an improper fraction, the numerator gives the answer to **part A**.

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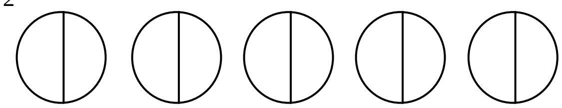
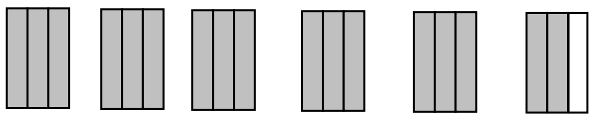
1. a) $3 \div 4 = \frac{3}{4}$ **b)** $4 \div 7 = \frac{4}{7}$

2. *Sample responses*:

a) $\frac{10}{2} = 5$



b) $\frac{17}{3} = 5\frac{2}{3}$



3. a) $2\frac{4}{5}$; [*One way*: Divide 14 by 5 to create a mixed number, using the quotient as the whole number part and the remainder as the number of fifths for the fraction part.
Another way: Draw rectangles, each divided into 5 equal parts. Each part represents $\frac{1}{5}$. Keep drawing rectangles until you have 14 parts to see how many wholes and how many leftover fifths there are.]

b) $11\frac{1}{2}$; [*One way*: Divide 23 by 2 to create a mixed number, using the quotient as the whole number part and the remainder as the number of halves for the fraction part.
Another way: Draw a rectangle and divide it into halves. Put together 23 halves to make wholes. Count how many wholes and how many halves you can make.]

4. $\frac{4}{6}$ of a pumpkin

5. a) $5\frac{1}{3}$ **b)** $4\frac{1}{6}$ **c)** 2 **d)** 7

6. 4 full packages and $\frac{2}{4}$ of another

7. a) Yes
b) Divide each of the 6 items into 9 parts. Each person gets 1 of the 9 parts from each item. So, each person gets 6 parts, or $\frac{6}{9}$ of an item.

8. a) *Sample response*: $\frac{4}{1}$, $\frac{8}{2}$, $\frac{12}{3}$, and $\frac{16}{4}$
b) Yes; [*Sample response*: $\frac{3}{2}$, $\frac{6}{4}$, and $\frac{12}{8}$]

9. a) No; [He divided 25 by 3 instead of dividing 3 by 25.]
[**b)** *Sample response*:
I would point out that 3 out of 25 is not even close to a whole, or even to a half, so an answer that is more than 8 does not make sense.]

[**10.** *Sample response*:
When you use fractions, you divide a whole into parts, so you use division.]

Supporting Students

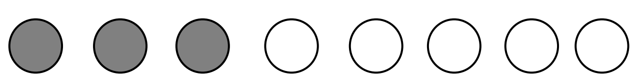
Struggling students

• Some students will have difficulty interpreting the sharing diagrams. It may be more useful to use concrete materials.

For example, provide them with small paper rectangles that they can actually cut up and divide to share them.

• Other students will have difficulty switching between the different meanings of division. It is acceptable to allow them to use only the sharing meaning if this helps them. Or, they can use only the grouping meaning, but this is usually more difficult.

For example, to show $3 \div 8 = \frac{3}{8}$ using the grouping meaning of division, show that 3 is $\frac{3}{8}$ of a group of 8.



Enrichment

• Students might use the relationship between fractions and division to show why one way to divide two numbers is to multiply both numbers by the same amount.

For example, they might solve $415 \div 5$ using $830 \div 10$, which is easier to calculate mentally. This works because $415 \div 5 = \frac{415}{5}$, which is equivalent to $\frac{830}{10}$, which is the same as $830 \div 10$.

3.1.3 Equivalent Fractions

Curriculum Outcomes	Outcome relevance
5-A2 Rename Fractions: with and without models • develop an understanding of renaming fractions using concrete materials and/or pictorial representations first and then link to the symbolic • understand equivalent fractions as the same region or group partitioned in different ways • understand the relationship between the numerator and the denominator of a fraction **5-C3 Equivalent Fractions: multiplicative relationship** • investigate the multiplicative relationship between the numerators and denominators of equivalent fractions • explore equivalent fractions by subdividing equally (e.g., for $\frac{3}{4}$, subdivide each fourth into 3 equal parts to result in $\frac{9}{12}$) • explore equivalent fractions by grouping equally the fractional parts that make up the whole (e.g., group 4 sixths in groups of 2 to result in 2 thirds) • investigate the results when the numerators of equivalent fractions differ by a constant amount	• Students must understand equivalent fractions in order to make sense of fractions involving larger numbers. For example, few of us have a good feel for what $\frac{26}{52}$ means until we realize it is equivalent to $\frac{1}{2}$. • This skill is also important for further algebraic development.

Pacing	Materials	Prerequisites
2 h	• Paper rectangles	• representing and interpreting fractions of a whole

Main Points to be Raised

• Equivalent fractions are fractions that represent the same amount of a whole.

• You can create an equivalent fraction by multiplying or dividing the numerator and denominator of a fraction by the same amount (as long as it is not zero). This can be modelled by subdividing each section of a fraction into equal fractions (multiplying numerator and denominator) or by grouping together equal numbers of sections of a fraction (dividing numerator and denominator).

• You usually do not get an equivalent fraction if you add the same amount to the numerator or denominator. (There are exceptions such as $\frac{3}{3} = \frac{3+4}{3+4} = \frac{7}{7}$.)

• When two fractions are equivalent, each denominator is the same multiple of its numerator.

For example, $\frac{3}{9} = \frac{2}{6}$ because each denominator is triple its numerator.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *How do you know that the amount to write each day is only a fraction of a page?* (If she wrote a whole page each day, she would write 9 pages. She needs only 6 pages.)

• *How do you know the amount for each day is more than half a page?* (If it were half a page, she would write only $4\frac{1}{2}$ pages in 9 days, not 6 pages.)

• *How did you get* $\frac{6}{9}$*?* (I knew that if she had only a 1-page report, she could write $\frac{1}{9}$ each day. She would have to write 6 of those each day to get 6 pages, and that is $\frac{6}{9}$.)

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The Exposition — Presenting the Main Ideas

• Fold a piece of paper in half lengthwise and widthwise. Open it up and mark the two left sections.

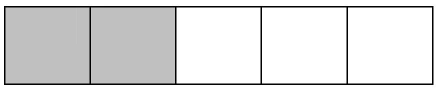
Ask students why you might call the marked amount either $\frac{2}{4}$ or $\frac{1}{2}$.

Remind students that $\frac{2}{4}$ and $\frac{1}{2}$ are called equivalent fractions because they describe the same amount.

• Ask students to fold paper to show another equivalent fraction for $\frac{1}{2}$.

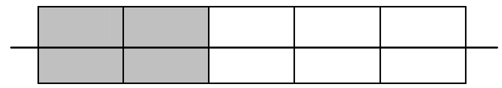
• On the board, draw a rectangle divided into 5 like this:

Colour 2 sections.



$\frac{2}{5}$

Draw a line through the middle.

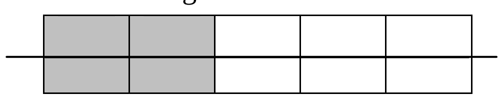


$\frac{4}{10}$

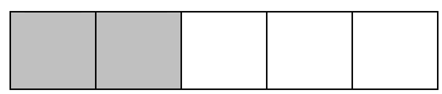
Point out that the same amount is coloured, but now there are twice as many sections (each old section is divided into two new sections) and twice as many small sections are coloured.

$\frac{4}{10} = \frac{2 \times 2}{5 \times 2}$ because both the numerator and denominator are multiplied by 2 since each old section was doubled.

• Note that you could also work backwards. If you start with $\frac{4}{10}$, you could group each pair of sections in a column into one large section.



$\frac{4}{10}$



$\frac{2}{5}$

$\frac{2}{5} = \frac{4 \div 2}{10 \div 2}$ because both numerator and denominator were divided by 2 when each group of two sections became one section.

• Show students that when you divide to get an equivalent fraction, you are allowed to divide by anything, but it is usually only helpful to do so if the numerator and denominator have a factor in common.

For example, $\frac{12}{15}$ might be renamed as $\frac{12 \div 3}{15 \div 3} = \frac{4}{5}$ because 12 and 15 have the factor 3 in common.

• Ask students not to use symbolic rules for equivalent fractions until they have enough experience using objects and pictures.

• Some students might wonder if you could add or subtract the same amount to get an equivalent fraction.

Point out that this cannot be done. For example, $\frac{1}{2} \neq \frac{1+1}{2+1} = \frac{2}{3}$.

• Ask students to work through the exposition in pairs, asking any questions they might have.

Make sure they understand the last picture in the exposition, where the $\frac{1}{4}$ represents 1 column out of 4.

• Avoid using the term “reducing” for fractions. We want students to understand that the fractions are equal when they are equivalent; to use the word “reducing” might make them think that one is less than the other.

Revisiting the Try This

B. Students can now apply what they have learned about creating equivalent fractions to their fraction answer from **part A**. You might point out that if they write, for example, $\frac{6}{9} = \frac{12}{18}$, it makes sense that it would take 18 days to write a 12-page report at the same pace.

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Using the Examples

• Present the problem in **example 1** to the students. Have them work on it on their own and then check their thinking against the thinking in the textbook. Discuss their work when they have finished. Bring out the idea that they might have thought the following:

- "I should use 2 × 40 in the denominator if 40 is the numerator because the denominator is twice the numerator."

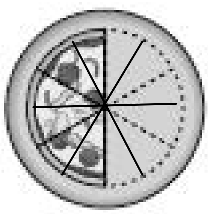
- "I should use 20 in the numerator if 40 is in the denominator for the same reason."

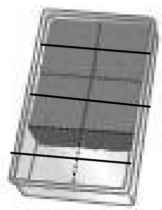
• Have the students read through **example 2**.

Practising and Applying

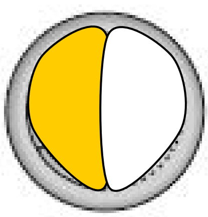
Teaching points and tips

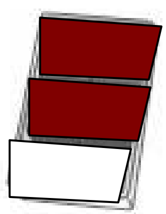
Q 1: Some students might choose to divide the pieces that are already there into more pieces.





Others will group pieces into fewer pieces.





For the drinks, students could pour them into a greater number of smaller cups or they could pour the amount from two cups into one larger cup.

Q 2: To show how the fractions are equivalent, students should either use dark lines to show the fewer, bigger sections they have created, or use additional lines to show the additional sections they have created.

Q 3: It is important that students use the same size whole for both pictures. This is because, for example, $\frac{1}{4}$ of a large object might be equal in size to $\frac{1}{2}$ of a small object even though the fractions are not equivalent.

Q 5: Students could use the 20 for either the numerator or the denominator in **parts a) and b)** but only for the denominator in **part c)** or for the numerator in **part d)**.

Q 7: This question is designed to help students think about the relationship between numerators and denominators when exploring equivalent fractions rather than always considering the relationships between two numerators and two denominators.

Q 9: This question is designed to help students see that two fractions can be equivalent even when one numerator is not a whole number multiple of another. In this case, both fractions have numerators and denominators that were based on multiplying the numerator and denominator of $\frac{2}{3}$ by the same amount. But to get directly from $\frac{6}{9}$ to $\frac{20}{30}$, you have to multiply 6 and 9 by $3\frac{1}{3}$.

Common errors

• Some students will still add to or subtract from the numerator and denominator by the same amount and believe that the new fraction is equivalent. They need to represent the fractions to see that they are not equivalent.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can represent an equivalent fraction using pictures
Question 4	to see if students can create equivalent fractions symbolically
Question 5	to see if students can solve a problem involving equivalent fractions

Answers

A. $\frac{6}{9}$	**B.** *Sample response*: $\frac{2}{3}$ or $\frac{4}{6}$

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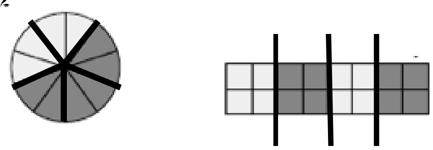
1. *Sample responses*:

a) $\frac{3}{6}$, $\frac{1}{2}$ **b)** $\frac{4}{6}$, $\frac{2}{3}$ (or $\frac{2}{6}$, $\frac{1}{3}$) **c)** $\frac{6}{12}$, $\frac{1}{2}$

2. a) *Sample response*:

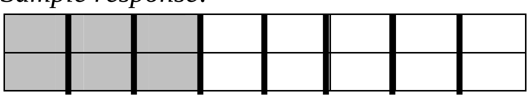
Circle: $\frac{6}{10}$ and $\frac{3}{5}$ Rectangle: $\frac{8}{16}$ and $\frac{2}{4}$

b)

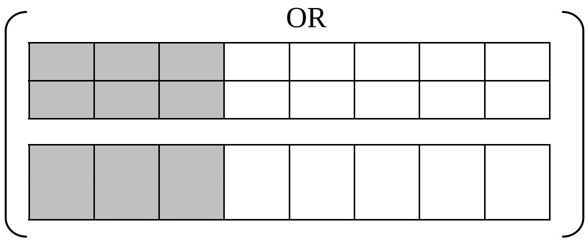


There are 5 big sections in the circle and only 3 instead of 6 are dark.
There are 4 big sections in the rectangle and only 2 instead of 8 are dark.

3. *Sample response*:



OR



4. *Sample responses*:

a) $\frac{10}{16}$, $\frac{15}{24}$, $\frac{150}{240}$, $\frac{1500}{2400}$ **b)** $\frac{5}{6}$, $\frac{50}{60}$, $\frac{10}{12}$, $\frac{100}{120}$

4. c) $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$ **d)** $\frac{100}{400}$, $\frac{1}{4}$, $\frac{2}{8}$, $\frac{3}{12}$

5. *Sample responses*:

a) $\frac{2}{20}$ **b)** $\frac{20}{50}$ **c)** $\frac{15}{20}$ **d)** $\frac{20}{60}$

6. $\frac{4}{5}$

7. *Sample responses*:

a) $\frac{2}{6}$, $\frac{3}{9}$, $\frac{10}{30}$

b) $6 \div 2 = 3$; $9 \div 3 = 3$; $30 \div 3 = 10$

c) $\frac{4}{6}$, $\frac{6}{9}$, $\frac{20}{30}$

8. No; [*Sample response*:

$\frac{8}{9}$ is only $\frac{1}{9}$ away from 1 and $\frac{6}{7}$ is $\frac{1}{7}$ away.

Since $\frac{1}{9}$ is less than $\frac{1}{7}$, $\frac{8}{9}$ is closer to 1, so they are not equal.]

9. *Sample response*: $\frac{3}{9} = \frac{10}{30}$

[**10.** You can multiply the numerator and denominator by any number to get an equivalent fraction, and there are more than 1000 numbers.]

Supporting Students

Struggling students

• Struggling students might continue to use concrete models to see how one fraction can be renamed.

For example, to rename $\frac{3}{4}$ as $\frac{6}{8}$, they can take an object that is divided into 4 equal parts with 3 parts coloured and then divide each of the 4 sections in 2, for a total of 8 parts (since $8 = 4 \times 2$). They will see that the 3 coloured parts are also divided in 2, so there are 6 coloured parts.

Enrichment

• Students might create problems involving equivalent fractions.

For example, they could determine the number of possible non-equivalent fractions that can be created with four numbers that can be used as numerators or denominators.

For example, with the numbers 2, 5, 8, and 11 there are 13 non-equivalent fractions: $\frac{2}{2}$, $\frac{2}{5}$, $\frac{2}{8}$, $\frac{2}{11}$, $\frac{5}{2}$, $\frac{5}{8}$, $\frac{5}{11}$, $\frac{8}{2}$, $\frac{8}{5}$, $\frac{8}{11}$, $\frac{11}{2}$, $\frac{11}{5}$, and $\frac{11}{8}$. ($\frac{5}{5}$, $\frac{8}{8}$ and $\frac{11}{11}$ are not used because they are equivalent to $\frac{2}{2}$.)

But if the numbers are 2, 4, 3, and 6, there are only 9 non-equivalent fractions: $\frac{2}{2}$, $\frac{2}{3}$, $\frac{2}{4}$, $\frac{2}{6}$, $\frac{4}{2}$, $\frac{4}{3}$, $\frac{3}{2}$, $\frac{3}{4}$, and $\frac{6}{2}$.

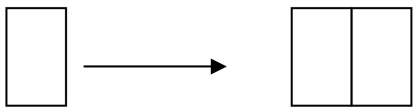
(Fractions like $\frac{6}{4}$ and $\frac{4}{6}$ are not used because they are equivalent to $\frac{3}{2}$ and $\frac{2}{3}$.)

CONNECTIONS: Fractions and Geometry

Students have many experiences with showing the fraction of a whole, but it is also important to give them experiences where they are given a fraction and have to create the whole. This connection also allows students to see how larger geometric shapes are made up of smaller shapes.

Answers

1. *Sample response*:



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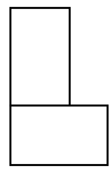
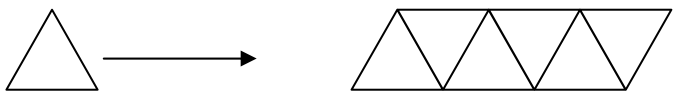


2. *Sample response*:



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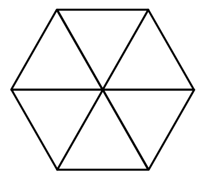

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3.1.4 Comparing and Ordering Fractions

Curriculum Outcomes	Outcome relevance
5-A3 Compare and Order Fractions • develop and use benchmarks to compare fractions • compare fractions with the same denominator • compare fractions with the same numerator • compare improper fractions as mixed numbers	When students work in Classes VI to VIII with concepts of ratio and percent, it is important that they be able to compare fractions. Many different strategies are shown because it is easier sometimes to compare fractions one way and other times to use another way.

Pacing	Materials	Prerequisites
1 h	• Fraction strips (BLM) (optional)	• comparing fractions with the same denominator • comparing two fractions with a numerator of 1 • relating the fraction $\frac{1}{2}$ to 0 and to 1 • equivalent fractions

Main Points to be Raised

• If two fractions have the same denominator, the fraction with the greater numerator is greater. This is because there are more pieces of the same size.

• If two fractions have the same numerator, the fraction with the lesser denominator is greater. This is because each has the same number of pieces, but the pieces are larger when the denominator is lower.

• Sometimes, you can compare two fractions by comparing each to 0, $\frac{1}{2}$, or 1. A fraction that is closer to 0 is less than a fraction that is closer to $\frac{1}{2}$ or to 1.

• If two fractions are greater than 1, you can sometimes compare their whole number parts.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. You might want to provide fraction models for students who might wish to use them. Observe while students work. While they work, you might ask questions such as the following:

• *Which of the fractions is less than $\frac{1}{2}$? How do you know?* ($\frac{1}{3}$ is less than $\frac{1}{2}$ and so is $\frac{1}{20}$. That is because the whole is divided into more, smaller parts.)

• *Why do you think $\frac{3}{5}$ is greatest?* (I modeled $\frac{3}{5}$ and I could see it was more than $\frac{1}{3}$. I know $\frac{1}{20}$ is very small.)

• *Do these fractions make sense to you?* (Yes, because when I go outside, I see that there are very few very old people but there are lots of children and adults.)

The Exposition — Presenting the Main Ideas

• Ask students to sketch a picture to model fifths. Ask how they would model $\frac{4}{5}$ and $\frac{2}{5}$. Ask how their pictures help them see that $\frac{4}{5} > \frac{2}{5}$. Then ask which of $\frac{3}{7}$ and $\frac{5}{7}$ they think is greater and why. After they have a chance to respond, point out that each time the piece sizes (or denominators) are the same, having more pieces (a greater numerator) means the fraction is greater.

• Now tell the students that you have $\frac{2}{3}$ of a chocolate bar and your friend has $\frac{2}{5}$ of a chocolate bar of the same size. Ask who has more. Encourage them to draw a picture to show their thinking. Help them see that 2 big pieces ($\frac{2}{3}$) is more than 2 small pieces of the same thing ($\frac{2}{5}$). Then ask which of $\frac{3}{7}$ and $\frac{3}{8}$ they think is greater and why. After they have a chance to respond, point out that each time the number of pieces (or numerators) are the same, having bigger pieces (a lower denominator) means the fraction is greater.

• Ask students how they know that $\frac{7}{8} > \frac{1}{20}$. See if they realize that $\frac{7}{8}$ is almost 1 whole, whereas $\frac{1}{20}$ is just a bit more than 0. For that reason, $\frac{7}{8}$ is greater.

• Finally, ask how they know that $\frac{21}{4} > \frac{16}{5}$. See if they realize that $\frac{21}{4}$ is a bit more than 5, whereas $\frac{16}{5}$ is just a bit more than 3.

• Lead students through the exposition, drawing out the relationship between what they are reading and the questions they have just answered.

In the case of comparing $\frac{4}{7}$ to $\frac{5}{6}$, ask how students know that $\frac{4}{7}$ is just a bit more than $\frac{1}{2}$ (because 4 is a bit more than half of 7) and that $\frac{5}{6}$ is almost 1 (because it is just $\frac{1}{6}$ short of 1). Note that the strategy of comparing one fraction to $\frac{1}{2}$ and the other to 1 was useful, but that it would not have been as useful for comparing $\frac{4}{7}$ and $\frac{5}{8}$ because both of these fractions are close to $\frac{1}{2}$.

Revisiting the Try This

B. Most students will compare $\frac{1}{3}$ and $\frac{3}{5}$ by noting that one is more than $\frac{1}{2}$ and one is less than $\frac{1}{2}$. Other students might choose to write an equivalent fraction for $\frac{1}{3}$, namely, $\frac{3}{15}$, and then compare that fraction to $\frac{3}{5}$, because they have the same numerator. Some students will draw pictures, but others will use words or symbols to explain their thinking.

Using the Examples

• Ask pairs of students to read through **examples 1 and 2**.

• Ask them if they have any questions. You might point out for **example 2** that other possible comparisons could have been mentioned, such as $\frac{3}{5} < \frac{2}{3}$ and $\frac{1}{3} < \frac{2}{5}$.

Practising and Applying

Teaching points and tips

Q 1: Encourage students who find this easy to make more than one comparison.

Q 2: Remind students that it is important that the wholes be the same size when two fractions are compared.

Q 3: Observe whether students use a variety of strategies. This is desirable.

Q 4: Students should be using different strategies for different comparisons.

For example, they might immediately recognize that $\frac{9}{4}$ and $\frac{19}{3}$ are both greater than 1, and so they are greater than the other fractions. To compare the two improper fractions, the might write each as a mixed number.

They might compare $\frac{2}{8}$ and $\frac{2}{5}$ using common numerators, but compare $\frac{2}{5}$ and $\frac{3}{5}$ using common denominators. They might compare $\frac{3}{5}$ and $\frac{8}{9}$ by deciding which is closer to 1.

Q 5: You might have to read this to students so that it is clear what is being asked. The third line says that a whole is divided into [] pieces and 5 pieces are coloured. Another identical whole is divided into 10 pieces and 5 pieces are coloured. The question is how many pieces might it have been divided into if more of the whole is coloured.

Q 6: Students could change the numerator or the denominator or both in each case.

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Common errors

• Some students who compare fractions with the same numerator think that the fraction with the greater denominator is greater (rather than less). Encourage them to draw a picture of each fraction before they decide which fraction is greater.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can draw a picture to compare two fractions
Question 4	to see if students can order a group of fractions using a variety of strategies
Question 7	to see if students can communicate about useful strategies for comparing fractions

Answers

A. Age 15 to 64; *Sample response*:

$\frac{3}{5} > \frac{1}{2}$, and $\frac{1}{3} < \frac{1}{2}$, and $\frac{1}{20}$ is really small, smaller than both $\frac{3}{5}$ and $\frac{1}{2}$.

B. i) I would compare them both to $\frac{1}{2}$; $\frac{3}{5}$ is a little more than $\frac{1}{2}$ and $\frac{1}{3}$ is less than $\frac{1}{2}$, so $\frac{1}{3}$ is less than $\frac{3}{5}$.

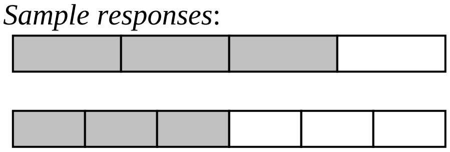
ii) They have the same numerator, so the fraction with the lower denominator is greater, so $\frac{1}{20}$ is less than $\frac{1}{3}$.

1. *Sample responses*:

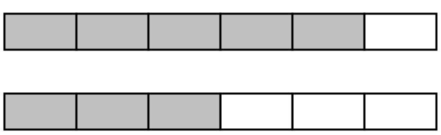
a) $\frac{1}{4} < \frac{1}{3}$ **b)** $\frac{5}{6} > \frac{2}{4}$

2. *Sample responses*:

a)



b)



3. a) $\frac{1}{2}$ **b)** $\frac{8}{9}$ **c)** $\frac{25}{26}$ **d)** $\frac{16}{3}$

4. $\frac{2}{8}, \frac{2}{5}, \frac{3}{5}, \frac{8}{9}, \frac{9}{4}, \frac{19}{3}$

5. a) 1, 2, 3, 4, 5, 6, 7, 8, 9

[**b)** *Sample response*:
It could be any number greater than 10, and numbers go on forever.]

6. *Sample responses*:

a) $\frac{2}{6}, \frac{1}{5}, \frac{0}{5}$ **b)** $\frac{3}{5}, \frac{4}{5}, \frac{5}{6}$

[**7.** *Sample response*:
The first two fractions have the same numerator so it is easy to compare their denominators, while the other fractions have the same denominator, so it is easy to compare their numerators.]

Supporting Students

Struggling students

• Struggling students may have difficulty with **question 5**. You need not assign that question to these students. You might also want to help them with **question 6** by suggesting they use denominator changes for **part a)** and numerator changes for **part b)**.

GAME: So Many Equivalents

• This game requires students to create equivalent fractions using either digit cards or a deck of cards with the face cards removed.

• Students will discover that if they choose two of the same card they win the most points possible, which is 18 points.

For example, if they choose 4 and 4, the equivalents are $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{5}{5}, \frac{6}{6}, \ldots, \frac{19}{19}$.

3.1.5 EXPLORE: Adding and Subtracting Fractions

Curriculum Outcomes	Lesson Relevance
5-B8 Addition and Subtraction: simple fractions with common denominators • link concrete models such as fraction strips to symbols • use less formal language to build understanding (e.g., 2 fourths + 1 fourth = 3 fourths)	• This essential exploration gives students their first experience with adding and subtracting fractions. It is done concretely and only with fractions with the same denominator. • Work on adding and subtracting fractions will continue through Class VIII.

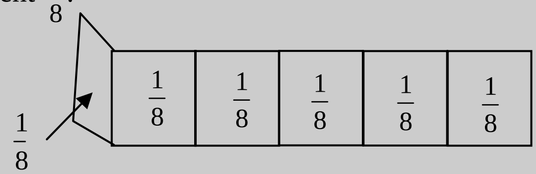
Pacing	Materials	Prerequisites
1 h	• Fraction Strips (BLM)	• representing fractions as parts of rectangles

Exploration

• Provide students with the fraction strips and ask them to notice how each strip or row is made up of more pieces than the strip above, but that the pieces are smaller. The total value of each strip is 1.

• Model how you could cut out the strip showing $\frac{1}{8}$s to represent $\frac{6}{8}$.

- Fold it to show how you could model $\frac{1}{8}+\frac{5}{8}$ as shown here:



They can see that $\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$.

- To show how much greater $\frac{5}{8}$ is than $\frac{1}{8}$, cut off $\frac{1}{8}$ from the $\frac{6}{8}$ strip and compare its length to the remaining $\frac{5}{8}$ strip. They will see that $\frac{5}{8}$ is $\frac{4}{8}$ longer than the $\frac{1}{8}$ strip, so $\frac{5}{8} – \frac{4}{8} = \frac{1}{8}$.

• They might read through **pages 91 and 92** to make sure they understand the directions.

• Observe while students work. While they work, you might ask questions such as the following:

- *What are your two fractions?* (I used $\frac{2}{6}$ and $\frac{4}{6}$.)

- *How did you add and subtract your fractions?* (I counted the parts from both of them to add them. I counted how much longer one strip was than the other to subtract them.)

- *How do you know your denominator will be 6?* (If I combine sixths and sixths or compare the lengths of sixths and sixths, the total number of strips or the difference in length is also sixths.)

- *How could you add* $\frac{4}{15}$ *and* $\frac{7}{15}$ *without using the strips?* (I would use 15 as the denominator and I would add 4 and 7 for the numerator.)

- *Can you subtract* $\frac{7}{15} – \frac{4}{15}$ *without using the strips?* (I would use 15 as the denominator and 7 – 4 for the numerator.)

Observe and Assess

As students work, notice the following:

• Do students recognize how to predict the numerator and the denominator for the sum and difference?

• Do they manipulate their strips correctly to show addition and subtraction?

• Can they clearly explain why it is easy to add and subtract fractions with the same denominator?

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Share and Reflect

After students have had sufficient time to work through the exploration, see if they have generalized what they have learned. Have them form small groups to discuss their observations and discuss questions such as these questions that ask for specific sums and differences.

- *What is* $\frac{3}{5} + \frac{1}{5}$*? What is* $\frac{3}{5} - \frac{1}{5}$*?*
- *If you add something to* $\frac{3}{5}$ *and then result is 1, what did you add?*
- *The difference between two fractions is* $\frac{1}{8}$*. What could the fractions be?*

Answers

A. *Sample responses*:

i) $\frac{3}{6}$ and $\frac{3}{6}$

ii)

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\frac{3}{6} + \frac{3}{6} = \frac{6}{6}$$

iii)

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\frac{3}{6} - \frac{3}{6} = \frac{0}{6}$$

B. *Sample responses*:

i) $\frac{5}{6}$ and $\frac{2}{6}$

ii)

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\frac{5}{6} + \frac{2}{6} = \frac{7}{6} = 1\frac{1}{6}$$

iii)

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\frac{1}{6}$	$\frac{1}{6}$

$$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$$

C. *Sample responses*:

(part A) i) $\frac{4}{8}$ and $\frac{3}{8}$

ii)

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$$\frac{4}{8} + \frac{3}{8} = \frac{7}{8}$$

Answers [Continued]

iii)

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$$\frac{4}{8} - \frac{3}{8} = \frac{1}{8}$$

(part B) i) $\frac{7}{8}$ and $\frac{2}{8}$

ii)

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$$\frac{7}{8} + \frac{2}{8} = \frac{9}{8}$$

iii)

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$\frac{1}{8}$	$\frac{1}{8}$

$$\frac{7}{8} - \frac{2}{8} = \frac{5}{8}$$

D. i) It is always the denominator for that row.
ii) Add the numerators to add the fractions or subtract the numerators to subtract the fractions.

E. i) No;
Sample response:
If I am finding out how much longer one strip is than another strip and they are both from the same row, the difference has to be less than the length of the row, which is 1.
ii) When the sum of the numerators was greater than the denominator.

F. *Sample response*:
The denominator does not change, so you can just add or subtract the whole number numerators.

Supporting Students

Struggling students

• Some students will forget to relate to the whole.

For example, if they combine the $\frac{3}{6}$ and $\frac{4}{6}$ strips to get 7 pieces, they might think they have $\frac{7}{7}$ rather than $\frac{7}{6}$.

Encourage them to pay attention to the labels on the strips. Make sure they understand that the label tells the denominator and that the numerator is based on how many sections they use.

Enrichment

• Some students might want to put together strips from different rows.

For example, by combining the $\frac{5}{6}$ and $\frac{1}{3}$ strips, they can see the total length is the same as $1\frac{1}{6}$.

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3.2.1 Decimal Thousandths

Curriculum Outcomes	Outcome relevance
5-A4 Thousandths: model and record • develop decimal and fractional benchmarks (e.g., 0.432 m is a little less than half a metre)	Students extend their understanding of place value to the thousandths place. This is important for interpreting smaller quantities, particularly measurements.

Pacing	Materials	Prerequisites
1.5 h	• Square Thousandths Grids (BLM)	• decimal tenths and hundredths • recognizing that 10 × 100 = 1000

Main Points to be Raised

• The third digit to the right of a decimal is called the thousandths place. 1 tenth = 10 hundredths and 1 hundredth = 10 thousandths.

• A number like 12 thousandths can be represented as a fraction or a decimal, for example, as $\frac{12}{1000}$ or as 0.012.

• The number of tiny rectangles shaded tells how many thousandths there are.

- Each column represents 100 thousandths, 0.100.
- Each square represents 10 thousandths, 0.010.
- Each small part represents 1 thousandth, 0.001.

• Decimal thousandths are often used to describe measurements.

For example, 1 m is written as 0.001 km.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. You may have to explain the situation.

For example, if there were 40 children in a class and half were boys, there would be 20 boys. If there were only 38 children and half were boys, there would be 19 boys. We say that the proportion of boys is the same.

We know Bhutan has many more than 1000 people, but in this problem we want use the proportions that really exist. It is like saying that for every 1000 people in Bhutan, 50 are from Gasa.

• Observe while students work. You might ask questions such as the following:

- *Why did you add the three numbers?* (To see how many people would be in those three dzongkhags. Then I could figure out how many did not live in them.)
- *Why did you subtract from 1000?* (Because we were saying that the given numbers were out of 1000.)
- *Why did you use a denominator of 1000?* (Because it is that many people out of 1000. I used the idea of a fraction as part of a group, and so the number of items in the whole group is the denominator.)

The Exposition — Presenting the Main Ideas

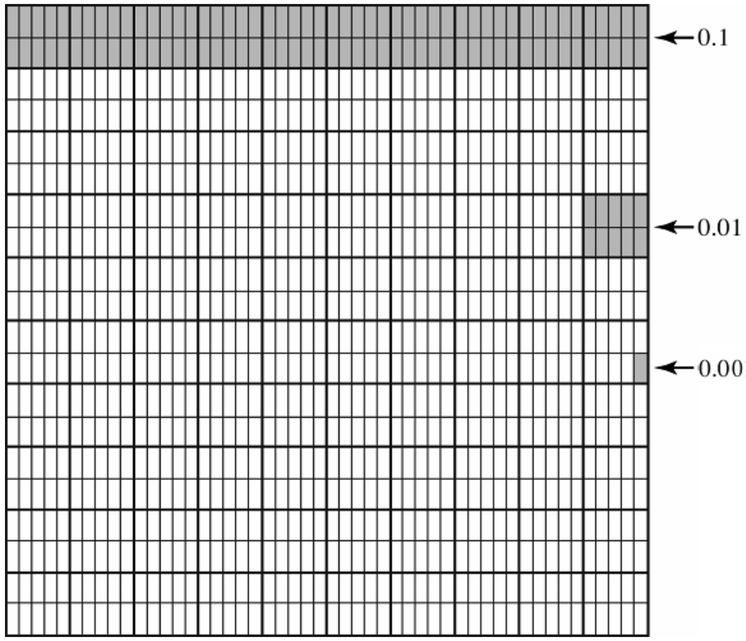
• Remind students of the 10-by-10 grid they have previously used to represent 100ths. Point out how each row or column represents $\frac{10}{100}$ or $\frac{1}{10}$ and each small square represents $\frac{1}{100}$. Then distribute thousandths grids.

• Show how each small square of the 10-by-10 grid is now divided into 10 tiny rectangles. Help students see that there are a total of 10 × 100 = 1000 tiny rectangles. Since 1000 parts make up a whole, each part is called $\frac{1}{1000}$.

• Tell them that $\frac{1}{1000}$ is 0.001 and is pronounced "one thousandth".

• Point out that the third digit to the right of the decimal point is called the thousandths place.

• On the thousandths grid, point out the parts that represent 0.1, 0.01, and 0.001.



• Note that the top row can be called 0.1 or 0.10 or 0.100 since it is one tenth of the whole grid. It can also be called 10 hundredths or 100 thousandths of the grid.

• Have students read through the exposition and ask any questions they might have. Make sure they understand the last idea — since 1000 mm = 1 m, each 1 mm = 0.001 m.

Revisiting the Try This

B. Students might want to write the amounts as fractions before changing them into decimals, but this is not required.

Using the Examples

• Write the questions from **example 1** and **example 3** on the board. Ask students to try to solve them with a partner or on their own. They can then compare their work to the solutions and student thinking in the text.

• Discuss **example 2** with the whole class. Make sure students understand that 0.7, 0.70, and 0.700 are equivalent decimals. Point out how this makes sense from what they know about equivalent fractions, that is,

$\frac{7}{10} = \frac{70}{100} = \frac{700}{1000}$, since the numerator and denominator are each multiplied by 10 each time.

• Point out that $0.7 \neq 0.07$ and $0.07 \neq 0.007$.

Practising and Applying

Teaching points and tips

Q 3: Students can model each decimal on a separate grid or they can use one grid for all three decimals. In this case, they should avoid overlapping the parts.

Q 5: Remind students that equivalent decimals are decimals that look different, but that represent the same amount.

Q 6: Watch that students write the 2-digit numbers as thousandths, for example, as 0.055, not 0.55.

Q 7: Encourage students to shade the grid or to imagine shading the grid to answer these questions.

Q 10: Remind students in this situation to think only about the small squares, and not about the tiny rectangles within the squares on their grids. For example, since 0.584 Asians is represented by 58 small squares plus 4 tiny rectangles, the number that would be used for Asians is 0.58.

Q 12: Students will now begin to think about the notion that using more decimal places allows us to be more precise. This will be followed up in later classes, but even now students can see, by comparing the results in **questions 6 and 10**, that decimals allow us to be more precise (or exact).

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Common errors

• Students often use an incorrect expression for decimal thousandths less than 0.100.

For example, they might write 23 thousandths as 0.230. Remind students to think about what the grid looks like to see if their decimal expression makes sense.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can write a fraction thousandth as a decimal
Question 3	to see if students can model decimal thousandths
Question 7	to see if students can relate thousandths to common fractions
Question 10	to see if students can relate decimal thousandths to decimal hundredths

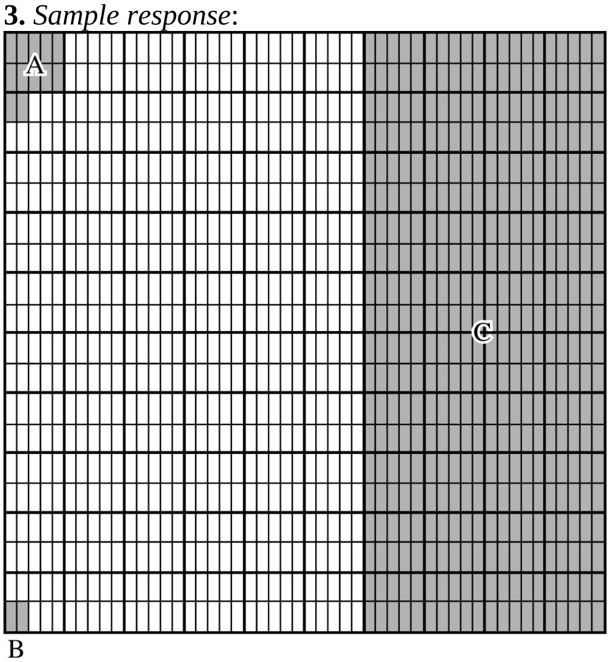
Answers

A. i) 678 **ii)** $\frac{678}{1000}$

1. a) 0.142 **b)** 0.057 **c)** 0.002

2. a) $\frac{8}{1000}$ **b)** $\frac{34}{1000}$ **c)** $\frac{398}{1000}$

3. *Sample response*:

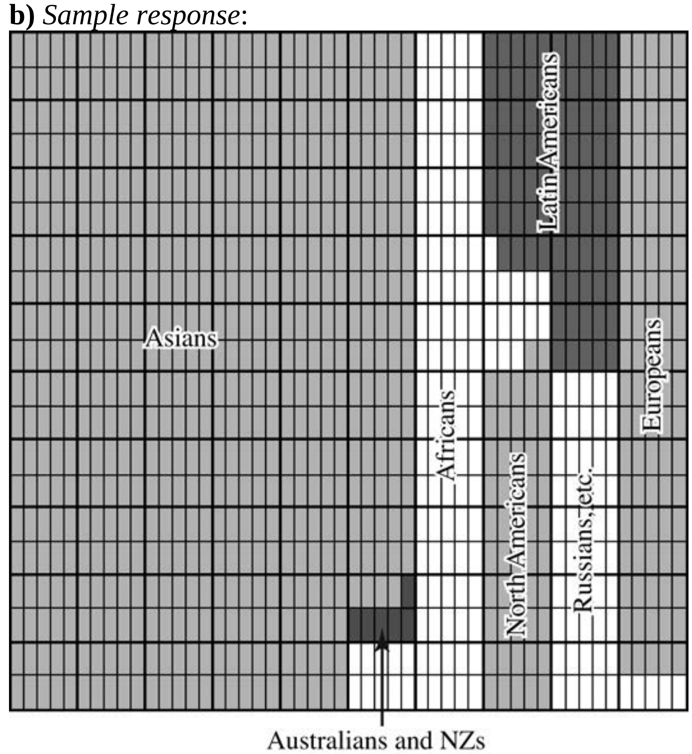


4. a) 0.312 km **b)** 0.068 km **c)** 0.002 km

5. 0.3 or 0.30

B. i) Gasa: 0.050, Chhukha: 0.117, Thimphu: 0.155
ii) 0.678

6. a) 0.584; 0.124; 0.095; 0.084; 0.055; 0.052; 0.006
b) *Sample response*:



7. a) 0.584 (Asians)
b) 0.095 (Europeans) or 0.124 (Africans)
c) 0.052 (North Americans) or 0.055 (Russians, etc.)

[**8.** *Sample response*:
Because 0.52 is more than half, so the number of North Americans would have to be more than 500, but it is only a little more than 50.]

9. 0.028; 0.010 or 0.01; 0.060 or 0.06; 0.330; 0.070 or 0.07; 0.007; 0.001

Answers [Continued]

10. a) 0.58 [**b)** *Sample response*: In a grid of 100 squares, think of the 584 as filling 10 squares in each of the first 5 columns, 8 squares in the next column, and 4 small parts. Since 4 small parts make up less than half another square, this is about 58 squares out of the 100 squares on the grid.]	[**11.** *Sample response*: Same: Both are 1 out of something Different: one is out of 100 and the other is out of 1000, so 0.01 is more than 0.001.] [**12.** *Sample response*: If you only use hundredths, you would probably think there is nothing between, for example, 0.48 and 0.49, since 49 hundredths comes right after 48 hundredths. But if you use thousandths, you realize there are numbers in between, e.g., 0.482.]

Supporting Students

Struggling students

• Provide additional grids if students need them to model decimals. You might want to display a grid with labels showing 0.1, 0.01, and 0.001.

• Struggling students may need support on **questions 9 and 10**. You might model the thinking for one part and then let them complete the other parts.

Enrichment

• Students might create designs on a thousandths grid.

For example, they might be asked to draw a letter that uses 0.235 of the grid.

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3.2.2 Decimal Place Value

Curriculum Outcomes	Outcome relevance
5-A4 Thousandths: model and record • develop decimal and fractional benchmarks (e.g., 0.432 m is a little less than half a metre) • place decimal numbers on a number line and justify their placements • read the quantitative value of each digit in decimals (e.g., 16.5 is “sixteen and 5 tenths” or “sixteen and a half”)	Students extend their understanding of place value to the thousandths place. This is important for interpreting smaller quantities, particularly measurements.

Pacing	Materials	Prerequisites
1.5 h	• Place Value Charts II (BLM) • Square Thousandths Grids (BLM) (optional)	• place value for hundredths through thousands • using a number line to model numbers

Main Points to be Raised

• The thousandths column is immediately to the right of the hundredths column on a place value chart.

• There is symmetry in the place value chart — the thousandths place is three places to the right of the ones place. This is the same number of places that the thousands place is to the left of the ones place.

• Decimal numbers can be read in more than one way, just like whole numbers can.

For example, 0.123 can be read as "one hundred twenty-three thousandths" or as "one tenth and twenty-three thousandths" or as "twelve hundredths and three thousandths."

• Decimal numbers can be placed on a number line by focusing first on the tenths digit, then on the hundredths digit, and finally on the thousandths digit.

For example, 0.235 is between 0.2 and 0.3, between 0.23 and 0.24, and between 0.234 and 0.236.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *Why did you multiply by 10?* (There are 10 tens in 100.)

• *How do you know that 10 groups of 0.07 is 0.7?* (If I colour 10 sets of 7 hundredths on a 10-by-10 grid, I have coloured 70 hundredths. I can write that as 7 tenths or as 0.7.)

The Exposition — Presenting the Main Ideas

• Use a large place value chart to remind students of the location of the thousands, hundreds, tens, ones, tenths, and hundredths places. Leave an empty column to the right. Tell students you will call this the thousandths place. Ask them to think about why you might do this.

• Some students will realize that it makes sense that 10 thousandths make 1 hundredth just like 10 hundredths make 1 tenth. Others may notice that it should go backwards from what happens on the other side of the decimal point. You may even show how the fraction $\frac{10}{1000} = \frac{1}{100}$.

• Record the number 234 on the place value chart and ask students to read it as "two hundred thirty-four." Ask them why you could also read it as "Twenty-three tens and four ones."

- Repeat the above with the number 23.4. Note that this can be read as "twenty-three and four tenths" or as "two hundred thirty-four tenths." Discuss why (2 tens is 20 ones, which is 200 tenths; 3 ones is 30 tenths).

- Repeat the above with the number 0.234. Note that this can be read as "two hundred thirty-four thousandths" or as "twenty-three hundredths and four thousandths" (23 full squares on the grid and 4 more small sections) or as "two tenths and thirty-four thousandths" (2 columns on the grid and another 34 small sections).

• Have students open their books to **pages 97 and 98**. Lead them through the first part of the exposition to review the idea you have just discussed with them. Stop when you get to the middle of **page 98**.

• Have them look at how the number 0.237 is shown between 0.2 and 0.3 (top row and bottom row) and between 0.23 and 0.24 (second and fourth rows). Relate this to the thousandths grid. 237 thousandths covers more than 2 columns (0.2), but not 3 columns (0.3). It covers more than 23 squares (0.23), but less than 24 squares (0.24).

• Now have students look at how 0.237 is shown on a number line, between 0.2 and 0.3 and between 0.23 and 0.24. Make sure they realize that 0.23 and 0.230 are at the same location since they are equivalent decimals. The same is true for 0.24 and 0.240.

Revisiting the Try This

B. Now that students can think of one thousandth as one tenth of one hundredth, they can answer the question about the width of one hair.

Using the Examples

• Assign pairs of students to work through the two examples. One student should read **example 1** while the other reads **example 2**. Then they can teach each other what they have learned. **Example 2** is a bit more difficult, so you may want to walk around and help students who are working on that example.

Practising and Applying

Teaching points and tips

Q 1: Some students will include unnecessary zeros, for example, in the tens place for **part i)**, but this is okay. You may need to remind some students that $\frac{1}{2} = 0.5$.

Q 2: Students might use end points of 0.4 (0.400) and 0.7 (or 0.700) or they can use lower or higher values for the end points.

Q 3: Some students may still benefit from using the thousandths grid.

Q 5: Students do not need to complete formal addition or subtraction to answer these questions. They should just be thinking about how the digits in particular columns on the place value chart change.

For example, 1 thousandth more means the digit in the thousandths place increases by 1.

Q 6: Students can use fractions with denominators of 1000. To compare to $\frac{1}{3}$, they would estimate by visualizing a thousandths grid or by using an equivalent fraction with a denominator close to 1000, like 999.

Q 8: Students need to realize that the three sections must add to 1.000. There is more than one possibility — it could be 0.124 + 0.248 (double the 0.124) and the remainder of the grid, or 0.124 and 0.062 (half the 0.124) and the remainder of the grid, or it could be 0.292 and 0.584 and 0.124. The latter design comes from dividing the rest of the grid into three equal sections after 0.124 is coloured, and combining two of them.

Q 9: Students need not use formal subtraction methods, but they can use the thousands grid as visual support.

Common errors

• Some students have trouble writing decimals when they hear them spoken.

For example, they write 200.037 for "two hundred thirty seven thousandths."

It is helpful to avoid using the word “and” when you read numbers, except for reading the decimal point.

For example, read 0.237 as "two hundred thirty-seven thousandths," not as "two hundred and thirty-seven thousandths" (which is 200.037).

Suggested assessment questions from Practising and Applying

Question 1	to see if students can record a decimal on a place value chart
Question 4	to see if students can describe a decimal amount in more than one way
Question 7	to see if students can communicate about how to compare two decimals

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Answers

A. 0.7 cm

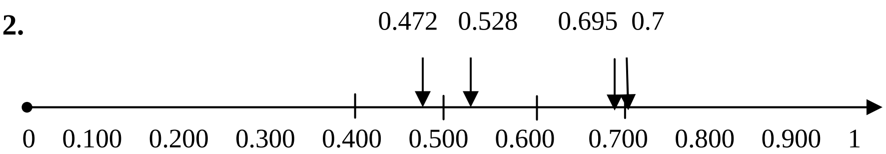
B. i) *Sample response*:
100 is 10 × 10.
10 hairs use 7 hundredths, so 100 hairs use 70 hundredths.
70 hundredths is 7 tenths.
ii) 0.007 cm

1. a) *Sample response*:

	Tens	Ones	Tenths	Hundredths	Thousandths
i)		0	0	0	1
ii)	1	0	3	4	0
iii)		0	5	1	0
iv)		0	2	4	5

b) i) 1 thousandth
ii) 10 and 34 hundredths
iii) 510 thousandths
iv) 245 thousandths

2.



3. *Sample responses*:
a) 0.996 **b)** 0.749 **c)** 0.899

4. a) 35 hundredths and 1 thousandth
b) 89 hundredths and 2 thousandths
c) 20 hundredths and 0 thousandths
d) 2 hundredths and 5 thousandths

5. a) 2.479 **b)** 3.119 **c)** 2.11

6. a) $\frac{352}{1000}$, $\frac{144}{1000}$, $\frac{174}{1000}$
b) 0.352; 0.144; 0.174
c) Pacific

[**7.** *Sample response*:
0.45 is 45 hundredths
0.455 is 45 hundredths + 5 thousandths
0.46 is 46 hundredths, which is 45 hundredths + 10 thousandths]

8. *Sample response*: 0.124, 0.248, and 0.628

9. *Sample responses*:
a) 0.234, 0.432, 0.324 **b)** 0.324

[**10.** *Sample response*:
Each column to the right is $\frac{1}{10}$ times the value.
Since 10 thousandths make 1 hundredth,
1 thousandth is $\frac{1}{10}$ of a hundredth, so it makes sense that the thousandth column is right beside the hundredth column.]

Supporting Students

Struggling students

• Struggling students may have difficulty with **questions 6 c), 8, and 9 b)**. These questions are particularly suitable for strong students.

Enrichment

• Some students might enjoy creating other problems like **question 8** for other students to solve. They might also come up with riddles.

For example, using each digit from 0 to 9 only once, they must provide clues for three decimals that another student can guess.

- one decimal is 1 thousandth less than 0.179 (0.178)
- one decimal is 12 tenths greater than 1.334 (2.534)
- one decimal is 3 tenths greater than 6.6 (6.9)

3.2.3 Comparing and Ordering Decimals

Curriculum Outcomes	Outcome relevance
5-A5 Thousandths: compare and order numbers to thousandths • compare whole number parts of decimals first • understand that decimals do not need the same number of places after the decimal to be compared (e.g., 0.7 > 0.423) • understand that the number of decimal places after the decimal point does not indicate size	Students extend their ability to compare decimal tenths and hundredths by comparing decimal thousandths. This skill is needed for later work in mathematics, particularly with scientific notation.

Pacing	Materials	Prerequisites
1 h	• Place Value Charts II (BLM)	• comparing whole numbers and decimal tenths and hundredths

Main Points to be Raised

• To compare two numbers involving decimal digits, start at the leftmost place value. As soon as you find one number with a digit that is greater in a particular place value (from the left), that number is a greater number.

• You cannot use the number of digits in a decimal number to decide whether it is greater or less than another decimal number.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *Why did you not put the 3 first?* (If I put 3 first, it would be less than 5.)

• *Could the first two digits be 5 and then 3?* (No, because 5.3 < 5.6.)

• *Could there be another solution?* (Yes. If I start with 7, I could rearrange the 5 and the 3.)

The Exposition — Presenting the Main Ideas

• Ask students which number in each pair is greater and why:

12.53 or 15.3

1.253 or 12.3

1.23 or 1.32

1.13 or 1.1

• Have students discuss their method.

• Ask students to read the first part of the exposition. Make sure they understand that working with thousandths does not change any of the processes with which they are already familiar for comparing tenths or hundredths.

• Point out that although we can compare 1256 to 237 by comparing the number of digits in the numbers, this is not possible for decimals. Ask why 0.23 < 0.4 even though it has more digits.

Revisiting the Try This

B. Students will have no difficulty seeing how this connects to what they did in **part A**. Ask them to explain why there are more possible answers in **part B** than in **part A**.

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Using the Examples

• Ask students to read through **example 1**. Make sure they understand that the solutions are two different approaches to the very same questions and remind them that there is often more than one way to approach a question in mathematics. Point out how **solution 2** is based on comparing using the same units, much like we do when we compare measurements.

For example, to compare 3 m to 159 cm, we rewrite the metres as centimetres.

• Ask students to work through **example 2** in pairs so that they can help each other. Point out that once again it was important to use the same units in order to compare.

Practising and Applying

Teaching points and tips

Q 2: This notation is unusual, but students should be able to relate this type of notation to the comparison of more standard types of numbers.

Q 3: If students need help, encourage them to draw a line that begins at 1.100 and ends at 1.500.

Q 4: Students will not need to do any unit conversions, but they will see the need to consider the units when comparing.

For example, 2.108 > 1.314 but because metres are so much smaller than kilometres, 1.314 km is greater.

Q 5: Students might find it difficult to come up with the words for the explanation. They need to think that the first number is more than 2 and the second number is less than 2.

Q 6: There are 10 possible solutions because the decimals allowed are limited to thousandths.

Common errors

• Many students will assume that a decimal with more digits is greater.

For example, they think that 0.234 > 0.79 since 234 > 79.

Remind students to read the decimals carefully, for example, 0.234 as "two hundred thirty-four thousandths" but 0.79 as "seventy-nine hundredths." It might also be helpful to use a thousandths grid to model both numbers.

Suggested assessment questions from Practising and Applying

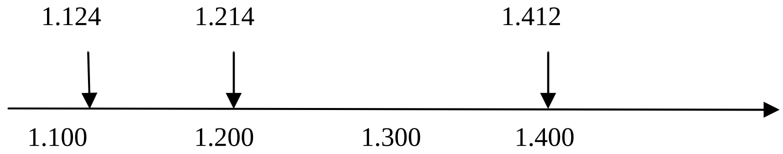
Question 1	to see if students can order decimal thousandths
Question 4	to see if students recognize the need to consider units to compare measurements involving decimals
Question 7	to see if students can communicate about how to compare decimals

Answers

A. Any of 5.73, 7.35, and 7.53	**B. i)** 5.703, 5.730, 7.035, 7.053, 7.305, 7.350, 7.503, 7.530 **ii)** *Sample response*: Same: You could still compare the whole number parts of the decimal for some numbers, like 7.[][][] > 5.[][][]. Different: There were more possible answers with decimal thousandths.
1. a) 0.035; 0.305; 1.024; 1.204 [**b)** *Sample response*: I knew 0.035 and 0.305 were least because they are less than 1. To compare them, I compared the digits in the tenths place. 0.**3**05 had more tenths than 0.**0**35. For the two greater numbers, 1.024 and 1.204, I compared the tenths.1.**2**04 had more tenths than 1.**0**24.]	**2.** 3:12.987; 3:14.175; 3:14.5; 4:1.122

Answers [Continued]

3.



4. a) 2.108 km **b)** 1.314 km **c)** 2.108 m

[**5.** *Sample response*:
Even with extra hundredths and thousandths, 1.9[][] is not as much as 2, and 2.[][][] is at least as much as 2.]

6. 3.451, 3.452, 3.453, 3.454, 3.455, 3.456, 3.457, 3.458, 3.459, 3.460

[**7.** *Sample response*:
I would tell them to use the same rules as for comparing hundredths. Start by comparing the whole numbers. If one of the whole numbers is greater, that is the greater number. If they are the same, then compare the digits in the tenths place. If one of the tenths is greater, that is the greater number. If they are the same, then compare the hundredths. If one of the hundredths is greater, that is the greater number. If they are the same, compare the thousandths. If one of the thousandths is greater, that is the greater number.]

Supporting Students

Struggling students

• Struggling students may benefit from continuing to use thousandths grid to compare the decimal parts of numbers.

Enrichment

• Students might create decimal comparisons with a particular number of solutions.

For example, if there have to be 35 solutions, then the number sentence might be 3.123 < [].[][][] < 3.159 or 2.992 < [].[][][] < 3.029.

GAME: In the Middle

Students need to play this game in a group with an odd number of people so that one value can be in the middle.

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UNIT 3 Revision

Pacing	Materials
2 h	• Fraction Strips (BLM) • Square Thousandths Grids (BLM)

Question(s)	Related Lesson(s)
1	Lesson 3.1.1
2 – 5	Lesson 3.1.2
6 – 8	Lesson 3.1.3
9 and 10	Lesson 3.1.4
11 and 12	Lesson 3.1.5
13 and 14	Lesson 3.2.1
15 – 17	Lesson 3.2.2
18 – 20	Lesson 3.2.3

Revision Tips

Q 2: Students should show 3 objects being shared among 5.

Q 5: Students can use the sharing meaning of division (sharing 13 among 5) or the "how many groups" meaning of division (how many 5s make up 13).

Q 6: It is essential that the wholes for both fractions be identical.

Q 9: For **part c)**, students might compare each fraction to $\frac{1}{2}$ or to 1. For **part d)**, it is useful to write each improper fraction as a mixed number.

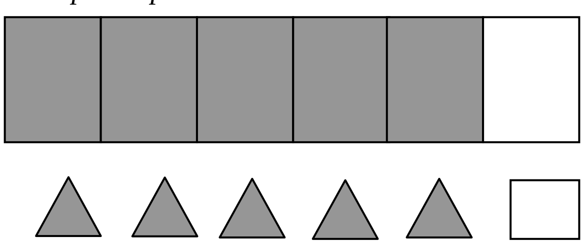
Q 11 and 12: Provide the fraction strip tower for this question.

Q 14: Remind students that there are 1000 mL in 1 L.

Q 16: Some students might benefit from thinking of the thousandths grid for **part a)**. If 99 thousandths are coloured, 1 more is 100 thousandths, or 0.1.

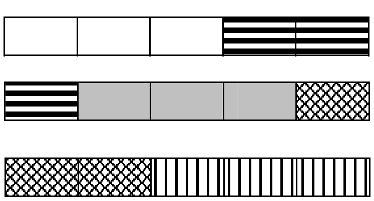
Answers

1. *Sample response*:



2. *Sample response*:

$3 \div 5 = \frac{3}{5}$

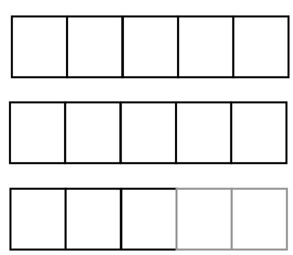


3. $2 \div 8 = \frac{2}{8}$

4. $\frac{3}{8}$

5. a) $\frac{13}{5} = 2\frac{3}{5}$

b) *Sample response*:

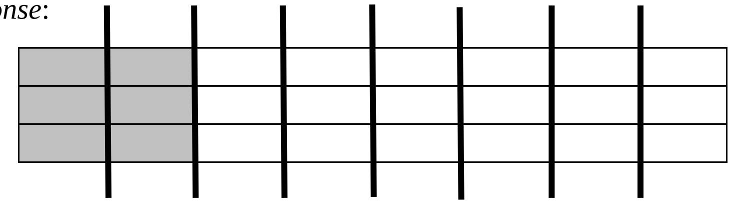


13 fifths = 2 wholes and 3 fifths

Answers [Continued]

6. *Sample response*:

$\frac{2}{8} = \frac{6}{24}$



7. a) $\frac{4}{16}$ or $\frac{16}{64}$ **b)** $\frac{4}{16}$ or $\frac{16}{64}$

8. *Sample response*: $\frac{1}{50}$

9. a) $\frac{3}{7}$ **b)** $\frac{7}{9}$ **c)** $\frac{10}{11}$ **d)** $\frac{33}{4}$

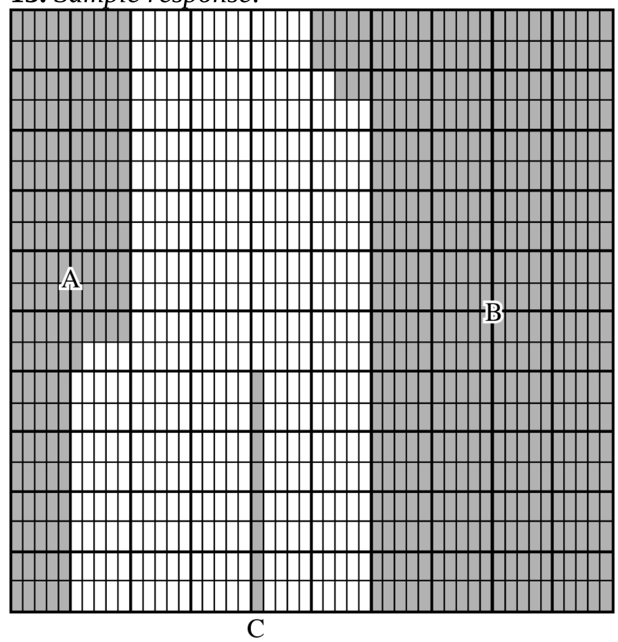
10. *Sample responses*:

a) $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{50}$ **b)** $\frac{49}{50}$, $\frac{9}{10}$, $\frac{8}{10}$

11. a) $\frac{10}{10}$ or 1 **b)** $\frac{4}{10}$

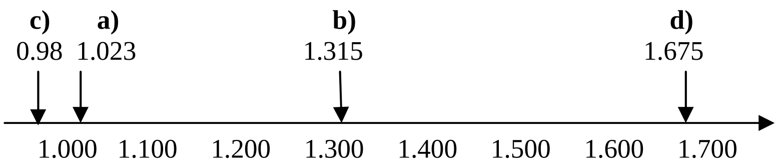
12. $\frac{3}{6}$ and $\frac{2}{6}$

13. *Sample response*:



14. a) 0.080 L **b)** 0.003 L

15.



16. a) 0.100 **b)** 0.999

17. a) 0.213 **b)** 0.222 **c)** 0.248

18. *Sample response*: 0.51 and 0.52

19. a) 2.004, 2.040, 3.09, 3.1

[**b)** The numbers that are less than 3 come first, 2.004 and 2.040.
2.004 is only 4 thousandths more than 2, but 2.040 is 40 thousandths more, 2.004 < 2.040, so 2.004 is least.
For the numbers greater than 3, 3.09 < 3.1 since 3.09 has a 0 in the tenths place and 3.1 has 1 tenth.]

20. *Sample response*: 1.450, 1.482, 1.500, 1.550, 1.600

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UNIT 3 Fractions and Decimals Test

1. Draw a picture to show each. Explain your picture.

a) $\frac{2}{3} = 2 \div 3$

b) $\frac{2}{3}$

2. a) If 5 boys share 3 large biscuits equally, what fraction of a biscuit will each boy get?

b) Draw a diagram to show how your answer to **part a)** is correct.

3. a) Write $\frac{17}{4}$ as a mixed number.

b) Draw a picture to show how your answer to **part a)** is correct.

4. Draw a diagram to show two fractions equivalent to $\frac{6}{8}$. Name the fractions.

5. a) What is the missing value in each?

i) $\frac{8}{10} = \frac{80}{[\ ]}$

ii) $\frac{8}{10} = \frac{[\ ]}{80}$

b) List three other fractions equivalent to $\frac{8}{10}$.

6. Dechen says that $\frac{15}{40} = \frac{1}{3}$.
How do you know she made an error?

7. In a class of 40 students, 23 are girls.
In another class of 35 students, 18 are girls.
Which class has a bigger fraction of girls?
How do you know?

8. Which fraction is less in each pair?
How do you know?

a) $\frac{3}{5}$ or $\frac{6}{11}$

b) $\frac{7}{8}$ or $\frac{13}{15}$

9. List three fractions for each.

a) less than $\frac{2}{9}$

b) more than $\frac{8}{9}$

10. Draw a picture to show why each is true. Explain your picture.

a) $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

b) $\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

11. Show what 0.215 looks like on a thousandths grid.

12. How could you use a thousandths grid to show why each statement is true?

a) 0.485 is more than 0.385

b) 0.485 is close to $\frac{1}{2}$

c) 0.485 is between 0.48 and 0.49

UNIT 3 Test

Pacing	Materials
1 h	• Fraction Strips (BLM) • Square Thousandths Grids (BLM)

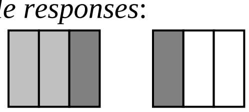
Question(s)	Related Lesson(s)
1	Lesson 3.1.1
2 and 3	Lesson 3.1.2
4 – 6	Lesson 3.1.3
7 – 9	Lesson 3.1.4
10	Lesson 3.1.5
11	Lesson 3.2.1
12	Lessons 3.2.2 and 3.2.3

Select questions to assign according to the time available.

Answers

1. *Sample responses*:

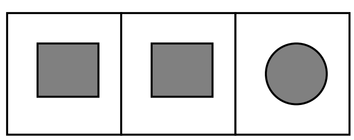
a)



There are 2 wholes in 3 parts: light grey, dark grey, and white. 2 parts are shaded, which is $\frac{2}{3}$.

If 2 wholes are divided by 3, ach share is $\frac{2}{3}$ of 1 whole.

b)

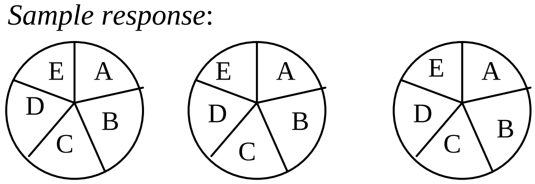


2 out of 3 parts, or $\frac{2}{3}$ of the rectangle, have squares;

2 out of 3 shapes, or $\frac{2}{3}$ of the group, are squares.

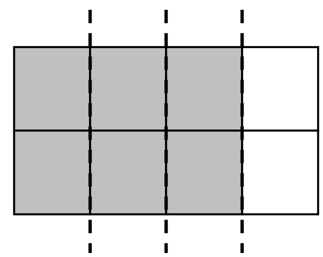
2. a) $\frac{3}{5}$

b) *Sample response*:

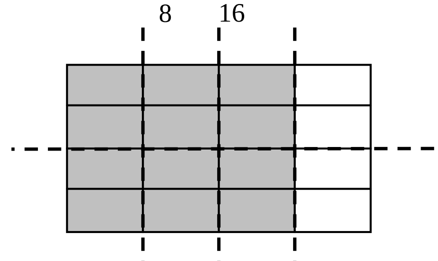


Each biscuit has 5 fifths.
Each letter represents a different boy.
Each boy gets 3 fifths.

4. $\frac{6}{8} = \frac{3}{4}$



$\frac{6}{8} = \frac{12}{16}$



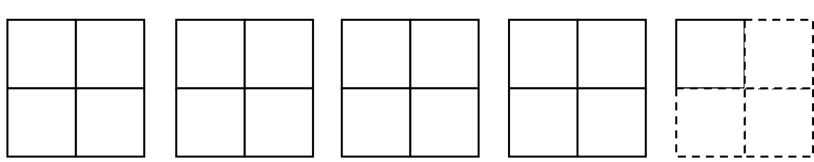
5. a) i) 100 **ii)** 64

b) *Sample response*: $\frac{4}{5}$, $\frac{40}{50}$, $\frac{400}{500}$

6. *Sample response*:
1 × 15 = 15 but 3 × 15 is not equal to 40.

3. a) $4\frac{1}{4}$

b) *Sample response*:



$\frac{17}{4} = 4\frac{1}{4}$

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7. The first class (of 40 students) has a bigger fraction of girls.
Sample response:
There are 17 out of 40 boys in the first class and 17 out of 35 boys in the second class.
That means boys are a bigger fraction of the second class, and so they are a smaller fraction of the first class.

8. a) $\frac{6}{11}$; *Sample response*:

$\frac{3}{5}$ is the same as $\frac{6}{10}$ and $\frac{6}{11} < \frac{6}{10}$ (elevenths are smaller than tenths, so 6 elevenths < 6 tenths)

b) $\frac{13}{15}$; *Sample response*:

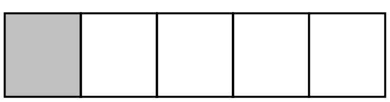
$\frac{7}{8} = \frac{105}{120}$ and $\frac{13}{15} = \frac{104}{120}$ and $\frac{104}{120} < \frac{105}{120}$

9. *Sample responses*:

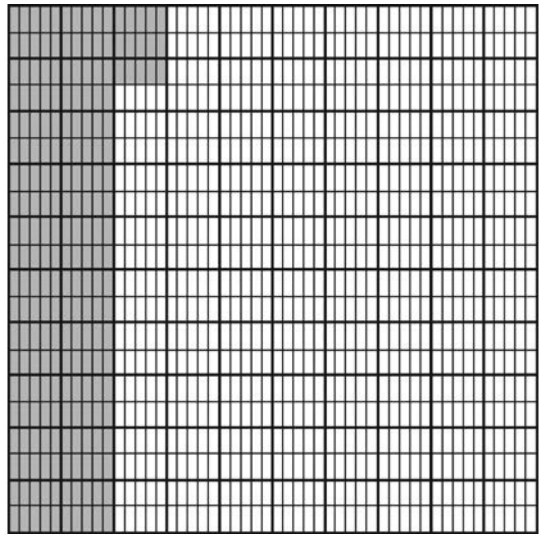
a) $\frac{2}{10}, \frac{2}{11}, \frac{1}{9}$ **b)** $\frac{17}{18}, \frac{25}{27}, \frac{26}{27}$

10. *Sample response*:
3 fifths + 1 fifth = 4 fifths

3 fifths is 2 fifths more than 1 fifth, so
3 fifths – 1 fifth = 2 fifths.



11. 0.215: 2 columns with 100 parts each + 15 parts



12. *Sample responses*:
a) 0.385 is less than 4 columns (each with 100 parts), but 0.485 is more than 4 columns.
b) 0.485 is 48 squares (each with 10 parts) + half a square; $\frac{1}{2}$ the grid is 50 squares.
c) 0.48 is 48 squares (each with 10 parts) and 0.49 is 49 squares.
0.485 is 48 squares + half of 1 square, so 0.485 is more than 48 but less than 49.

UNIT 3 Performance Task — Fractions in Your Life

You can describe many activities in your life using fractions.

For example:

- If you sleep 9 h a day, you sleep $\frac{9}{24}$ of a day.
- If you brush your teeth for 2 min, you brush them for $\frac{2}{60}$ h.
- If you go to school 6 days a week, you go for $\frac{6}{7}$ of a week.

A. i) Make a list of 6 or more activities that you do. Describe how much time you spend on each in seconds, minutes, hours, or days.

ii) Create a chart like this. Express each amount of time as a fraction. You can write one or more fractions for each activity. Put your fraction in the column that makes the most sense.

	Minute	Hour	Day	Year
Sleep			$\frac{9}{24}$	
Brush teeth		$\frac{2}{60}$		
Go to school			$\frac{6}{7}$	

B. Find an equivalent fraction for each fraction in your chart**.**

C. Choose two or more fractions in your chart that it makes sense to compare. Order them from least to greatest. What does the order of the fractions tell you? If there are no fractions that you can compare, explain why.

D. Choose two fractions in your chart that it makes sense to add. What is the sum? Why does it make sense to add them? If there are no fractions that can be added, explain why.

E. Draw three pictures to represent three of the fractions in your chart.

You might find this chart helpful

1 minute (1 min)	1 hour (1 h)	1 day	1 year
60 seconds (60 s)	60 minutes (60 min)	24 hours (24 h)	365 days

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UNIT 3 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
5-A1 Meaning of Fractions: using and relating different meanings **5-A2** Rename Fractions: with and without models **5-A3** Compare and Order Fractions **5-B8** Addition and Subtraction: simple fractions with common denominators **5-C3** Equivalent Fractions: multiplicative relationship	1 h	None

How to Use This Performance Task

- You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit, or it could replace or supplement the unit test.
- It could also be used as enrichment material for some students.
- You can assess performance on the task using the rubric on the next page.

Sample Solution

A. i)

- I sleep for 9 h each day.
- I can hold my breath for 50 s.
- I go to school for 6 h each day.
- I listen to music for 30 min at a time.
- I play with my brother for 2 h each day.
- The longest amount of time I was ever on a bus was 10 h.
- I went on a 10-day trip.

ii)

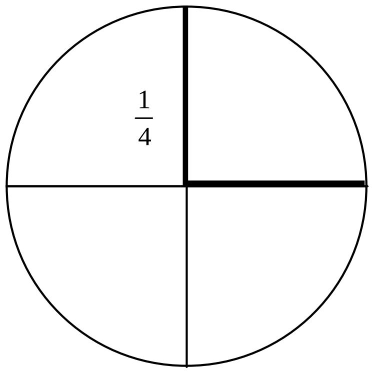
	Minute	Hour	Day	Years
Sleep			$\frac{9}{24}$	
Hold my breath	$\frac{50}{60}$			
Go to school			$\frac{6}{24}$	
Listen to music		$\frac{1}{2}$		
Play with brother			$\frac{2}{24}$	
Longest time on a bus			$\frac{10}{24}$	
Trip				$\frac{10}{365}$

B. $\frac{9}{24} = \frac{18}{48}$; $\frac{50}{60} = \frac{5}{6}$; $\frac{6}{24} = \frac{1}{4}$; $\frac{1}{2} = \frac{2}{4}$; $\frac{2}{24} = \frac{1}{12}$; $\frac{10}{24} = \frac{5}{12}$; $\frac{10}{365} = \frac{2}{73}$

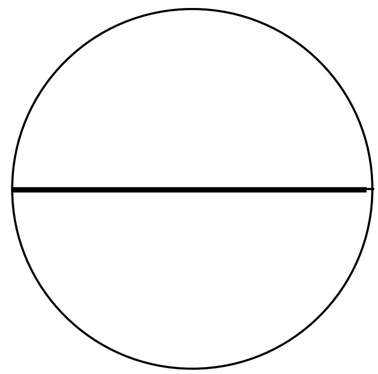
C. $\frac{2}{24} < \frac{6}{24} < \frac{9}{24}$; It makes sense to compare these because they all describe the fraction of a day I spend on different activities; The order tells me that I spend a larger fraction of my day sleeping than going to school or playing with my brother.

D. I could add $\frac{6}{24}$ and $\frac{2}{24}$ since both are about how much of the day I use for activities. The sum is $\frac{8}{24}$ or $\frac{1}{3}$ of the day.

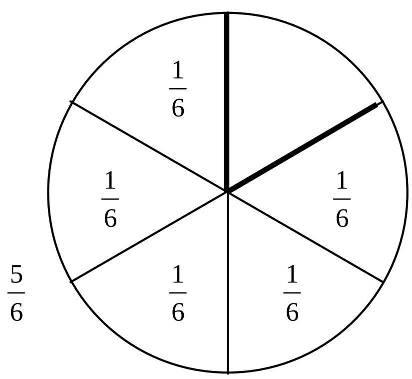
E. This shows the fraction of the day I am in school, $\frac{6}{24} = \frac{1}{4}$.



This shows the fraction of an hour I listen to music, $\frac{1}{2}$.



This is the fraction of a minute I can hold my breath, $\frac{50}{60} = \frac{5}{6}$.



UNIT 3 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Represents fractions	Insightfully represents fractions using symbols, equivalent fractions, and pictures	Appropriately represents fractions using symbols, equivalent fractions, and pictures	Correctly represents fractions using some of these approaches: symbols, equivalent fractions, and pictures	Has difficulty representing fractions using symbols, equivalent fractions, and/or pictures
Relates fractions	Insightfully compares and orders fractions and represents sums	Correctly compares and orders fractions and represents sums	Generally compares and orders fractions and represents sums correctly	Has difficulty comparing and ordering fractions and representing a fraction sum
Recognizes applications of fractions in everyday life	Creatively and insightfully recognizes how fractions describe everyday situations	Appropriately recognizes how fractions describe everyday situations	Sometimes recognizes how fractions describe everyday situations	Has difficulty recognizing how fractions describe everyday situations

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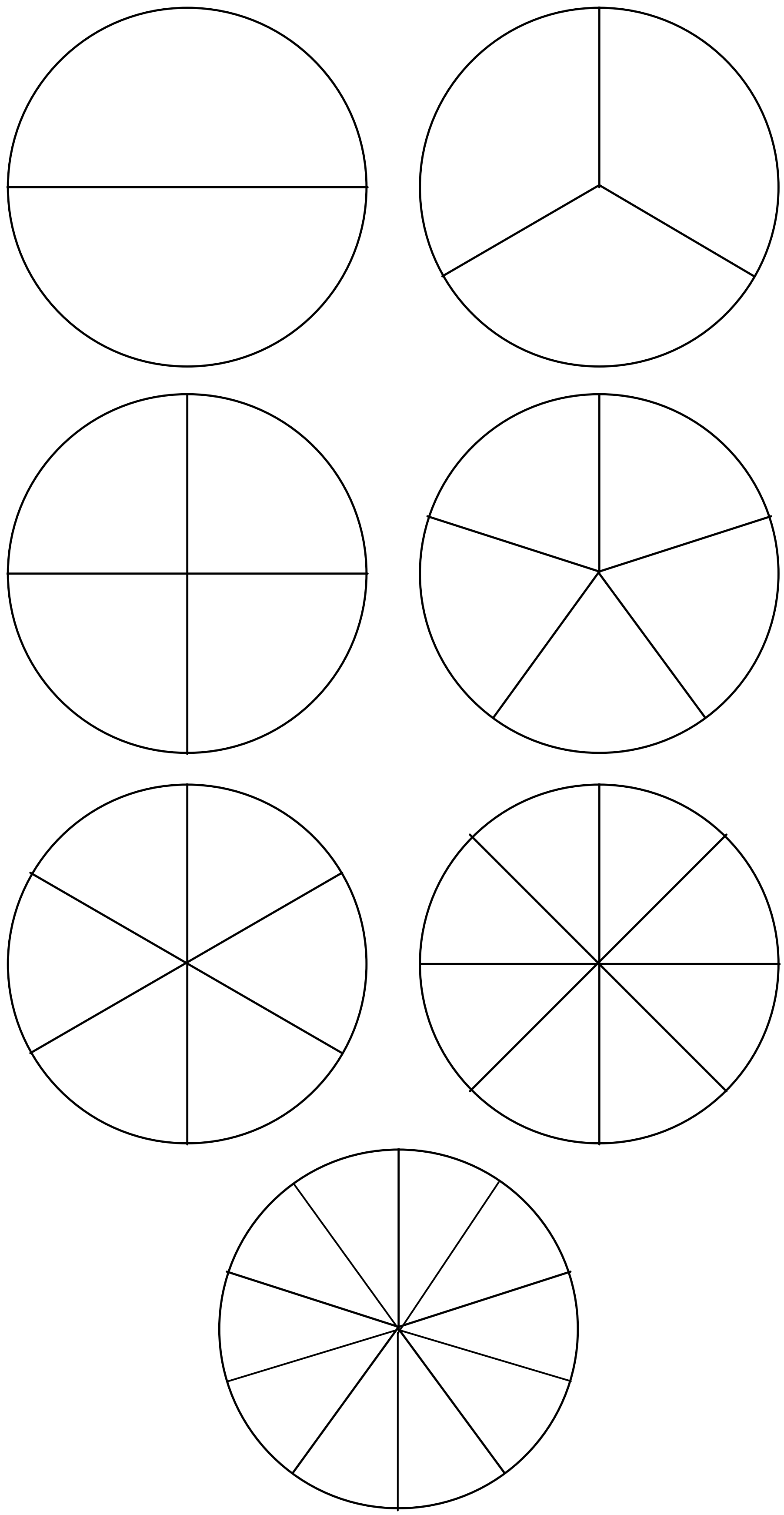
BLM 1 Fraction Circles



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BLM 2 Fraction Strips

1											
$\frac{1}{2}$	$\frac{1}{2}$										
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$									
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$								
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$							
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$						
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$				
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$			
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$		
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

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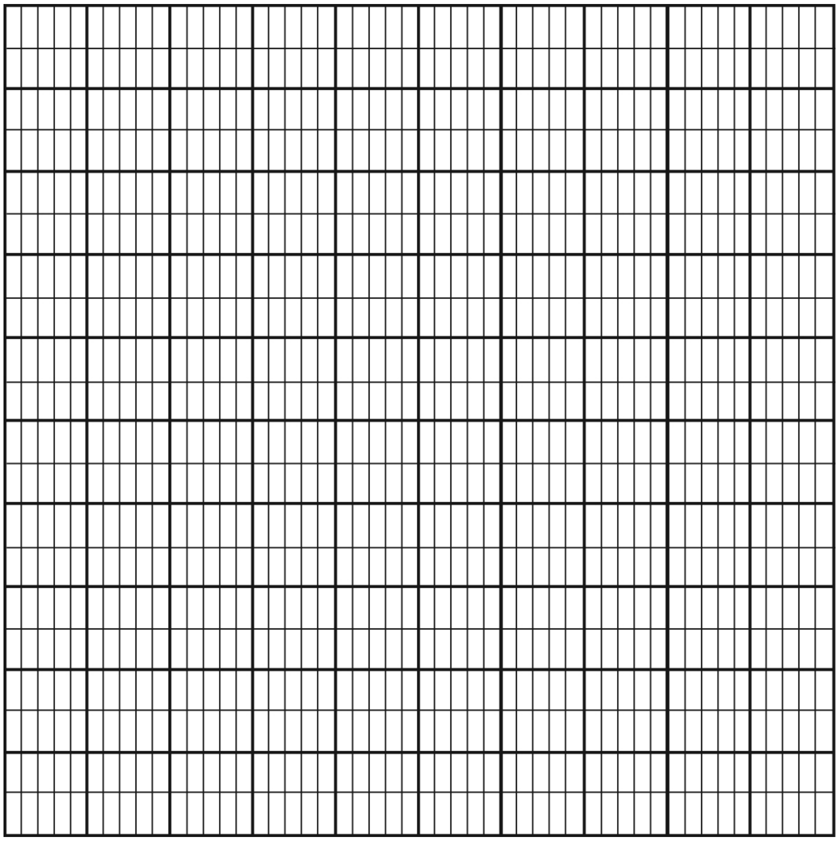
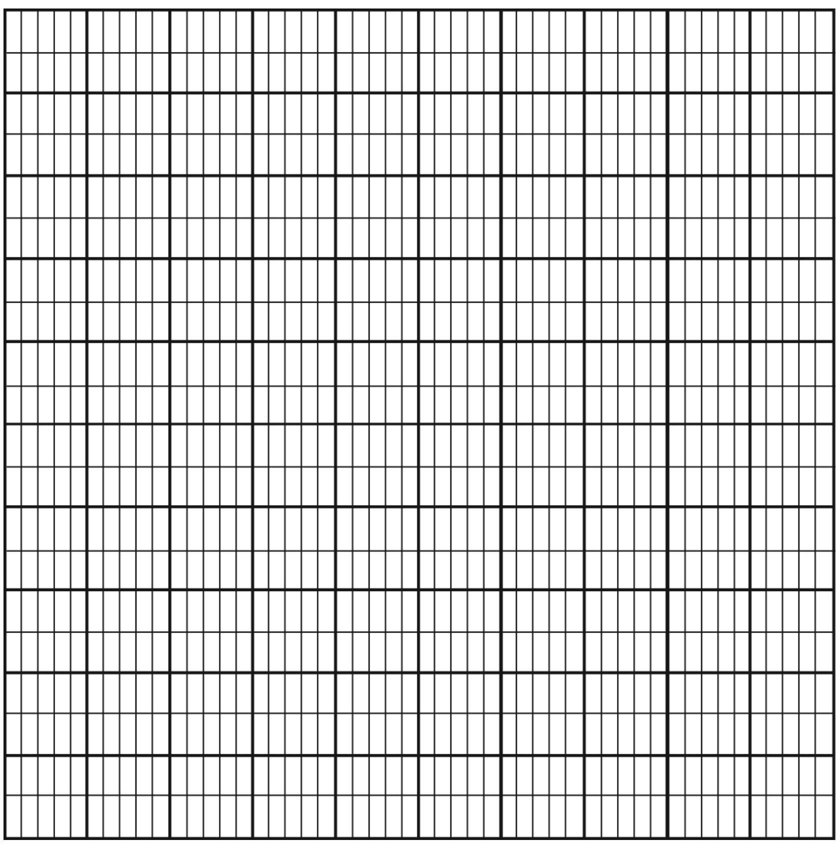
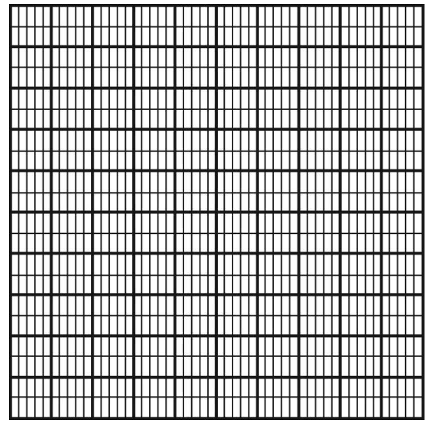
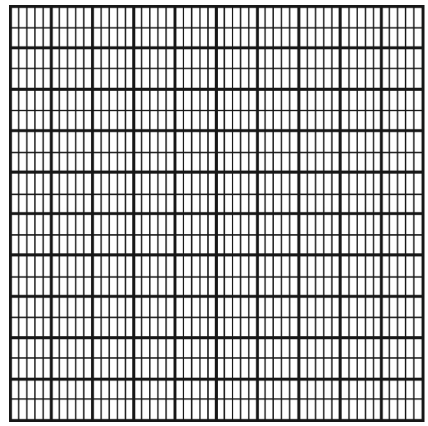
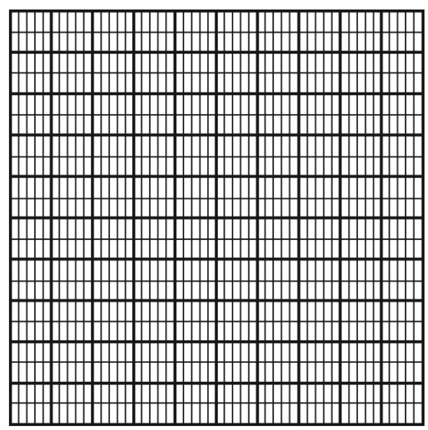
BLM 3 Square Thousandths Grids

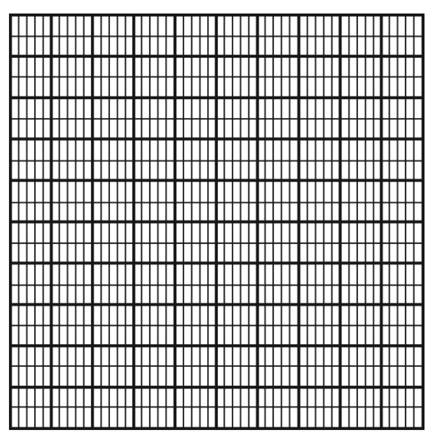












BLM 4 Place Value Charts II

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

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UNIT 4 DECIMAL COMPUTATION

UNIT 4 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 107** **TG p. 168**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• Hundredths Grids (BLM)	All questions
Chapter 1 Adding and Subtracting Decimals				
4.1.1 EXPLORE: Adding and Subtracting Decimals (Optional) **SB p. 109** **TG p. 171**	**5-B9 Addition and Subtraction of Whole Numbers and Decimals: 5 digits to 1000ths** • perform addition and subtraction presented horizontally and vertically • choose an appropriate method for computation: mentally, pictorially, or symbolically	1.5 h	• Base ten blocks or Base Ten Models (BLM): hundredths, tenths, and ones	Observe and Assess questions
4.1.2 Adding Decimal Thousandths **SB p. 111** **TG p. 174**	**5-B9 Addition and Subtraction of Whole Numbers and Decimals: 5 digits to 1000ths** • perform addition and subtraction presented horizontally and vertically • choose an appropriate method for computation: mentally, pictorially, or symbolically • continue estimating in computation **5-B12 Open Sentences: applying number sense** • work with open number sentences involving the four basic operations	1.5 h	• Square Thousandths Grids (BLM) • Place Value Charts II (BLM)	Q 2, 3, 8
4.1.3 Subtracting Decimal Thousandths **SB p. 116** **TG p. 178**	**5-B9 Addition and Subtraction of Whole Numbers and Decimals: 5 digits to 1000ths** • perform addition and subtraction presented horizontally and vertically • choose an appropriate method for computation: mentally, pictorially, or symbolically • continue estimating in computation **5-B12 Open Sentences: applying number sense** • work with open number sentences involving the four basic operations	1.5 h	• Square Thousandths Grids (BLM) • Place Value Charts II (BLM)	Q2, 4, 8, 10
GAME: Big Sum, Little Difference (Optional) **SB p. 121** **TG p. 182**	Practise adding and subtracting decimals and estimation in a game situation.	15 min	• Digit cards or a Deck of cards	N/A
Chapter 2 Multiplying Decimals				
4.2.1 Estimating Products **SB p. 122** **TG p. 183**	**5-B10 Decimals × Whole Numbers: simple products** • estimate (e.g., 4 × 2.45 as 4 × 2 or 4 × 3)	40 min	None	Q1, 3, 9

UNIT 4 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
4.2.2 Multiplying a Decimal by a Whole Number **SB p. 124** **TG p. 185**	**5-B10 Decimals × Whole Numbers: simple products** • link concrete models to algorithm • estimate (e.g., 4 × 2.45 as 4 × 2 or 4 × 3)	1 h	• Base ten blocks or Base Ten Models (BLM): hundredths, tenths, and ones	Q1, 4, 7
4.2.3 Multiplying by 0.1, 0.01, and 0.001 **SB p. 128** **TG p. 189**	**5-B11 Mentally Multiply: whole numbers by 0.1, 0.01, 0.001** • multiply by 0.1, 0.01, 0.001 • link to place value **5-C1 Open Sentences: patterns in addition, subtraction, multiplication, and division** • generate rules about how a change in one variable affects the result (e.g., for □ × 10, as □ increases by 1 the product increases by 10)	40 min	• Place Value Charts II (BLM)	Q1, 3, 5
CONNECTIONS: Telescopes and Binoculars (Optional) **SB p. 130** **TG p. 191**	Make a connection between multiplying by numbers like 0.1 and 0.01 and real world situations	10 min	None	N/A
UNIT 4 Revision **SB p. 131** **TG p. 192**	Review the concepts and skills in the unit	1.5 h	• Base ten blocks or Base Ten Models (BLM): hundredths, tenths, and ones • Square Thousandths Grids (BLM) • Place Value Charts II (BLM)	All questions
UNIT 4 Test **TG p. 194**	Assess the concepts and skills in the unit	1 h	• Base ten blocks or Base Ten Models (BLM): hundredths, tenths, and ones • Square Thousandths Grids (BLM) • Place Value Charts II (BLM)	All questions
UNIT 4 Performance Task **TG p. 197**	Assess concepts and skills in the unit	1 h	• Square Thousandths Grids (BLM) (optional) • Place Value Charts II (BLM) (optional)	Rubric provided
UNIT 4 Assessment Interview **TG p. 199**	Assess concepts and skills in the unit	15 min	See **p. 199**.	All questions
UNIT 4 Blackline Masters **TG p. 200**	BLM 1 Hundredths Grids Base Ten Models on **page 49** in UNIT 1 (used as decimal models: ones, tenths, and hundredths) Square Thousandths Grids on **page 163** in UNIT 3 Place Value Charts II on **page 164** in UNIT 3			

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Math Background

• This decimal computation unit follows the work students did in Unit 3 on identifying the meaning of decimal thousandths. It also follows Class IV on adding and subtracting decimal tenths and hundredths.

• The focus of the unit is on being able to add and subtract decimal thousandths, and to multiply decimals by whole numbers. There is a special emphasis on using mental math to multiply by decimal powers of 10 (0.1, 0.01, and 0.001).

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **part E** in **lesson 4.1.1**, where they have to create subtraction questions where each number, including the answer, must be represented by a given number of models, in the **Try This** in **lesson 4.1.2**, where they have to set up and solve a problem involving distances, and in **question 8** in **lesson 4.2.2**, where they solve a real world problem about growth.

• They use communication in **question 10** in **lesson 4.1.3**, where they explain how to subtract decimals, in **question 9** in **lesson 4.2.2**, where they consider possible results when multiplying decimals, and in **question 6** in **lesson 4.2.3**, where they explain multiplication by 0.1, 0.01, or 0.001.

• They use reasoning in answering questions such as **question 9** in **lesson 4.1.2**, where they consider how to estimate, and in **question 6** in **lesson 4.2.2**, where they reason which digits are missing in a computation

• They consider representation in **lesson 4.1.1**, where they model decimal hundredths using models, in **question 1** in **lesson 4.1.3**, where they use models to represent decimal thousandths, and in **lesson 4.2.3**, where they use a place value chart to make sense of multiplication by 0.1, 0.01, and 0.001.

• Students use visualization skills in **question 6** in **lesson 4.1.2**, where they visualize containers holding a particular amount of water, and in **lesson 4.2.2**, where they use models to represent multiplication of decimals.

• They make connections in **lesson 4.1.3**, where they connect a subtraction situation to a related addition situation or to a related subtraction question, in **question 8** in **lesson 4.1.3**, where they connect number work to pre-algebra work, in **question 5** in **lesson 4.2.1**, where they relate decimal work to measurement work, and in **lesson 4.2.3**, where they connect multiplication by decimal powers of 10 to division by whole number powers of 10.

Rationale for Teaching Approach

• This unit is divided into two chapters.

Chapter 1 addresses adding and subtracting.

Chapter 2 addresses multiplication.

• The **Explore** lesson focuses on using models to interpret decimal addition and subtraction.

• The **Connections** section contains a real-world application of multiplication by decimal powers of 10.

• The **Game** provides an opportunity to practice decimal addition and subtraction.

• Throughout the unit, it is important to encourage flexibility in computation and to accept a variety of approaches from students.

Getting Started

Curriculum Outcomes	Outcome relevance
4 Hundredths: model and record **4** Hundredths: compare and order **4** Addition and Subtraction of Decimals and Wholes: 10ths and 100ths **4** Multiply Mentally: by 10 or 100 **4** 3-digit × 1-digit Multiplication with/without regrouping	Students will find the work in the unit easier after they review what they know about decimal hundredths, mental multiplication by 10 and by 100, and multiplication by a 1-digit number from Class IV.

Pacing	Materials	Prerequisites
1 h	• Hundredths Grids (BLM)	• adding and subtracting decimal tenths and hundredths • comparing decimal tenths and hundredths • multiplying mentally by 10 or 100 • multiplying a 2-digit numbers by a 1-digit number

Main Points to be Raised

Use What You Know

• You can add and subtract decimals by combining parts, that is, wholes to wholes, tenths to tenths, and hundredths to hundredths.

• You can estimate decimal sums and differences by relating them to nearby whole numbers.

Skills You Will Need

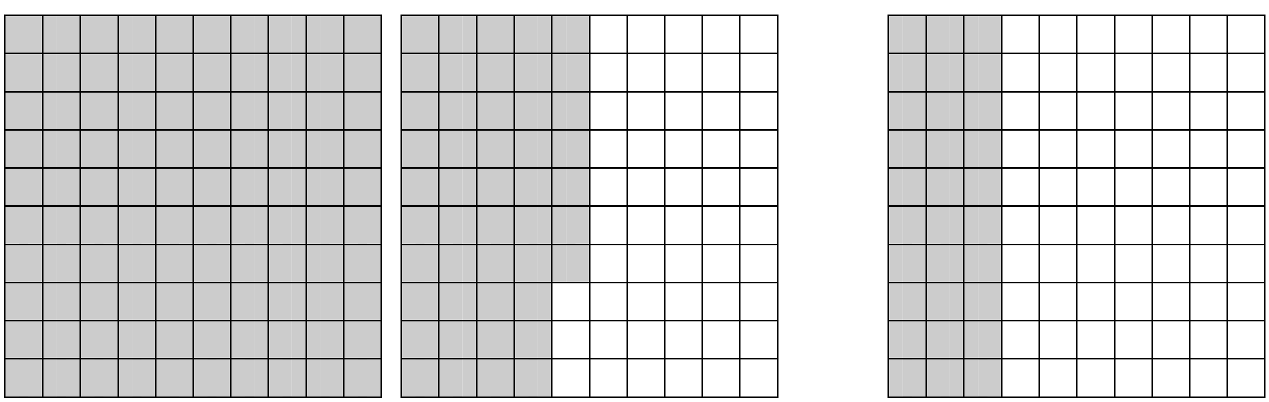
• You can use a 10-by-10 grid to model decimal hundredths. You show addition by combining parts and you show subtraction by taking away parts.

• You can multiply by 10 by moving digits one space to the left and using an extra 0 in the ones place. You can multiply by 100 by moving digits two spaces to the left and using two extra 0s, in the ones and tens places.

• You can multiply a 2-digit number by a 1-digit number by multiplying the tens and the ones and adding them.

Use What You Know — Introducing the Unit

• Before assigning the activity, you may wish to review the meaning of numbers like 1.47 or 0.3 using 10-by-10 decimal grids.



1.47

0.3

• You may wish to remind students that adding means putting together sections and that subtracting could mean taking away from a section.

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• Make sure students can interpret the bridges diagram on **page 107** of the text. They should understand that the value under each curved section represents the distance on the shore between bridges (on either side of the river) and that each bridge is 0.3 km long.

• Assign the activity to students either alone or in pairs.

• Observe students as they work. As they work, you might ask questions such as the following:

- *Why did you estimate using 1.5 + 2 + 0.3?* (1.47 is close to 1.50, which is 1.5; 1.98 is close to 2.00, which is 2. Then I had to add the distance across the bridge.)

- *Why could you compare 1.5 + 2 to 1.2 + 3.2 to compare the two paths?* (They both crossed the bridge, so I did not need to compare those parts. I only needed to compare the distances along the shore.)

- *Once you had a path for 8.15 km, why was it easy to find a path for 8.45 km?* (I needed to add 0.3 km and that is one bridge crossing.)

- *Why was it easy to draw a path for 8.3 km once you had a path for 4.15 km?* (I needed to do that path twice since 4.15 + 4.15 = 8.30, which is the same as 8.3.)

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions.

• Most questions require exact calculation, although **question 2 b)** can be solved using estimation.

• Although **question 5** does not involve decimals, it is included in this revision because students will need to multiply whole numbers in this unit.

• Students can work individually.

Answers

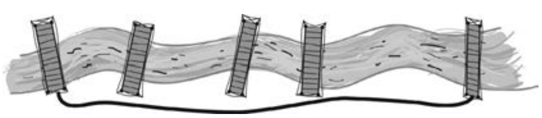
A. i) *Sample response*: About 4 km; 3.75 km **ii)** 4.7 km **iii)** Dechen; 0.95 km farther.

B. 2.93 km

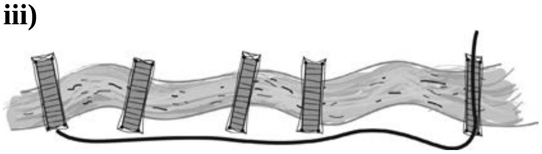
C. *Sample responses*:

i)

ii)



iii)



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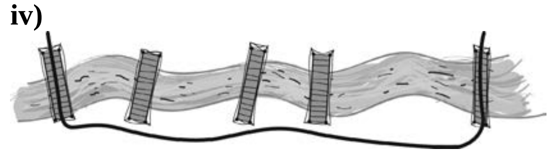
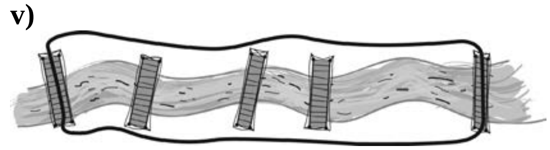
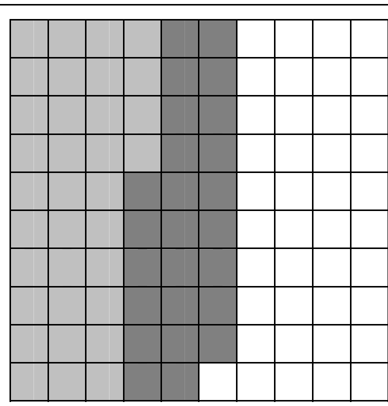


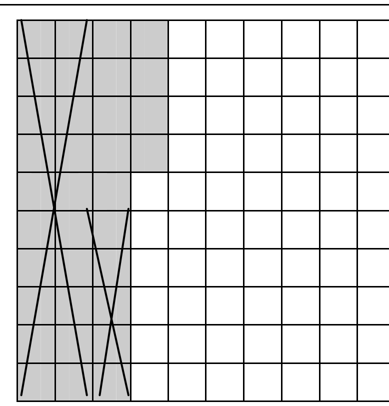
v)



1. a) 0.59
Sample response:



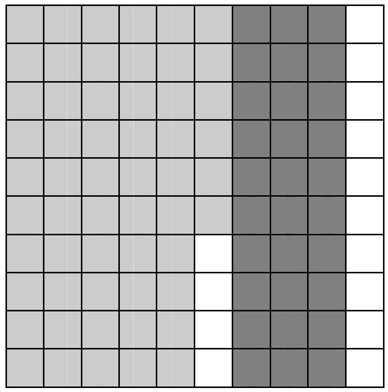
b) 0.09
Sample response:



Answers [Continued]

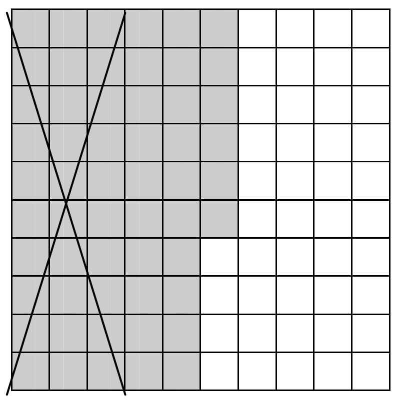
c) 0.86

Sample response:



d) 0.26

Sample response:



2. a) 1.25 + 3.4 **b)** 5.25 – 2.9

3. a) 5.2 **b)** 10.13
c) 2.37 **d)** 1.58

4. a) 120 **b)** 1370
c) 2500 **d)** 34,600

5. a) 40 **b)** 42
c) 36 **d)** 56
e) 96 **f)** 175
g) 252 **h)** 268

Supporting Students

Struggling students

• As they work on the first activity, students might label each part of the path and estimate total distance combinations to help them answer **part C**.

For example, if they have estimated the different sections as 1.5 km, 2 km, 1 km, and 3 km, they can look for combinations that are close to 5 km or to 8 km.

• Encourage students who are struggling to use hundredths grids for adding and subtracting the decimal parts of the number. They can add the whole parts separately.

For example, for 3.21 + 1.19, they could use the grid for 0.21 + 0.19 and add the 3 + 1 separately.

• Encourage students to use mental calculations, for example, to add 1.98 by adding 2 and then subtracting 0.02.

Enrichment

• You might encourage students to make up their own distances and create problems for classmates to solve using these distances. They could also vary the bridge lengths if they wish.

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4.1.1 EXPLORE: Adding and Subtracting Decimals

Curriculum Outcomes	Lesson Relevance
5-B9 Addition and Subtraction of Whole Numbers and Decimals: 5 digits to 1000ths • perform addition and subtraction presented horizontally and vertically • choose an appropriate method for computation: mentally, pictorially, or symbolically	This optional exploration will help students recall the representation of the sums and differences of decimal hundredths in a problem-solving context.

Pacing	Materials	Prerequisites
1.5 h	• Base ten blocks or Base Ten Models (BLM): hundredths, tenths, and ones	• representing addition as a total and subtraction as take away • representing decimal hundredths on a hundredths grid • understanding the relationship between hundredths and tenths and between tenths and ones

Exploration

• Show students a decimal hundredths grid. Ask students how to model various decimals, for example, 0.23 (as 2 columns and 3 more squares) or 0.7 (as 7 columns). Then show how you can use a small square to represent 0.01, a strip (column) to represent 0.1, and a full grid to represent a whole number.

• Model, for example, 2.31 by using two large 10-by-10 grids, three strips that are 10-by-1 and one small 1-by-1 square.

• Then model a decimal sum, such as 1.42 + 0.83, as a combination of (1 + 0) large 10-by-10 grids, (4 + 8) strips that are 10-by-1, and (2 + 3) small 1-by-1 squares. Point out that to get the final sum, ten of the (4 + 8) strips could be traded for one more whole.

• Model a decimal difference, such as 1.21 – 0.43, by starting with one 10-by-10 grid, two 10-by-1 strips, and one 1-by-1 grid. To remove four strips, you could trade the 10-by-10 grid for 10 strips and then remove four of them. To remove three small squares, you could trade one of the 10-by-1 strips for ten small squares, and then remove three of them.

• Ask students to work in pairs.

• Observe while students work. While they work, you might ask questions such as the following:

- *How could you predict that the model for 1.23 + 0.56 would include nine small squares?* (I have three small squares in the first number and six in the second number, so there are nine altogether.)

- *Why do you not really have to add to see that 3.84 + 2.7 = 4.04 + 2.5?* (If I move two strips from 2.7 to 3.84 and regroup the ten strips as another whole, I am combining the same amount.)

- *Why is it easier to remove 0.41 from 0.99 than to remove 0.45 from 1.03?* (I have nine strips and nine small squares so it is easy to remove four strips and one small square. But if I wanted to remove five small squares and I had only three, I would have to do some regrouping.)

- *Why can you add the digits to find out how many models you need?* (Each digit tells me how many of a certain type of model I need, so when I add the digits, I find out how many of each type of model I need.)

- *When you regroup, why are there always nine more or fewer models than you had before?* (Each model is worth 10 smaller models. So, for example, if I trade one strip for ten small squares, I lose one strip but I add ten other models, and 10 – 1 = 9.)

Observe and Assess

As students work, notice the following:

• Do students model decimal numbers correctly?

• Do they regroup appropriately when they model sums and differences?

• Do they recognize when reasoning, rather than calculating, is all that is required to answer a question?

[Continued]

• Do they recognize situations that simplify calculations?
• Do they recognize the effect of regrouping on the number of models required to model a number?
• Do they persevere to find solutions to questions that require the use of a fixed number of models?

Share and Reflect

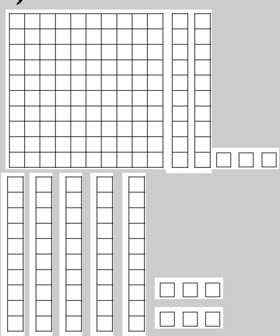
After students have had sufficient time to work through the exploration, they could form small groups to discuss their observations and discuss questions such as these.

• *Why might you add 2.99 + 3.85 by moving 0.01 from the 3.85 over to the 2.99 before you combine the numbers?*
• *Why is it easier to use models to calculate 2.98 – 1.45 than to calculate 3.0 – 1.45?*
• *In what situations do you use fewer models to represent a sum than the number of models you start with?*

Answers

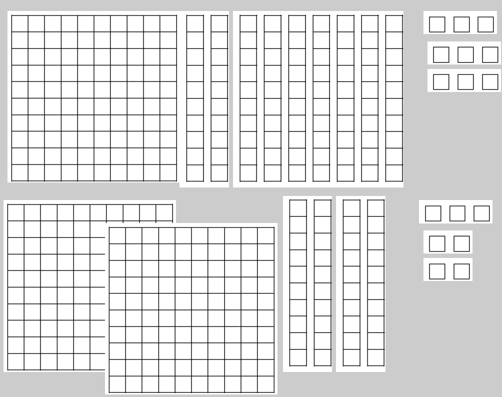
A. *Sample responses*:

i) 1.23 + 0.56 = 1.79



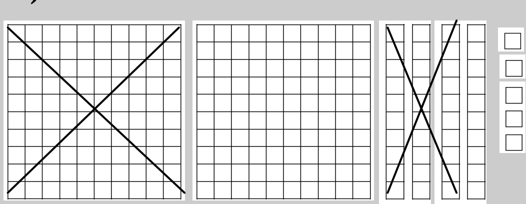
1 one + 7 tenths + 9 hundredths = 1.79.

ii) 1.99 + 2.47 = 4.46



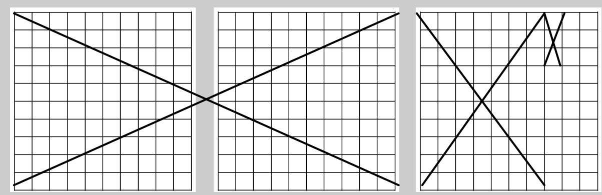
3 ones + 13 tenths + 16 hundredths;
Regroup 10 hundredths as 1 tenth and 10 tenths as 1;
4 ones + 4 tenths + 6 hundredths = 4.46.

iii) 2.45 – 1.3 = 1.15



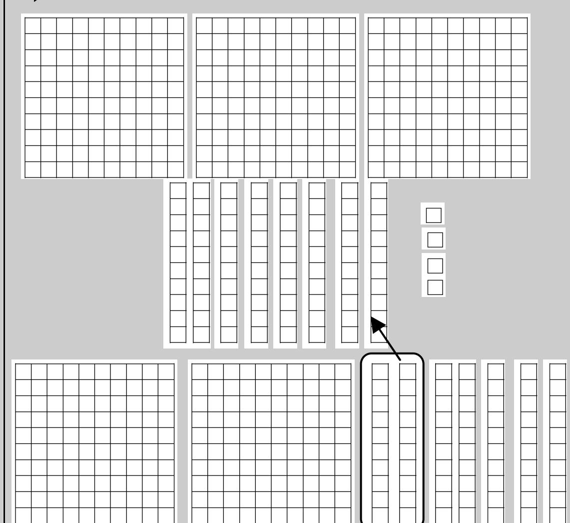
1 one + 1 tenth + 5 hundredths = 1.15 left.

iv) 3 – 2.73 = 0.27



0 ones + 2 tenths + 7 hundredths = 0.27 left.

v) 3.84 + 2.7 = 4.04 + 2.5



3.84 is 3 ones + 8 tenths + 4 hundredths and
2.7 is 2 ones + 7 tenths.
If you move 2 tenths from 2.7 to 3.84, you end up with 3 ones + 10 tenths + 4 hundredths, which is 4 ones + 0 tenths + 4 hundredths, or 4.04 and 2 ones + 5 tenths, which is 2.5.

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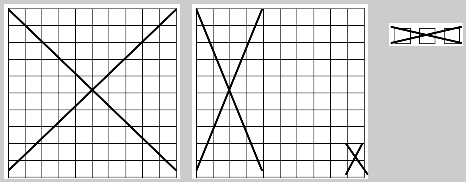
vi) 2.03 – 1.45 = 1.99 – 1.41



2.03 = 2 ones + 3 hundredths
I have to take away 1 one + 4 tenths + 5 hundredths.
If there had been only 1.99 to start with, I would take away only 1 one + 4 tenths + 1 hundredth to have the same amount left.

B. *Sample response*:
• For adding: For both whole numbers and decimals, you put together the same kinds of pieces and regroup 10 of one size for the next size if necessary.
• For subtracting: For both whole numbers and decimals, you model the greater number and then you remove pieces of the right size based on their place value positions.

C. i) I would not have to regroup after I added, so it would be easier to do.
ii) I would not have to do any regrouping before I subtracted, so it would be easier to do.

D. *Sample responses*:
i) 1.33 + 1.33 = 2.66 (14 models)
2.66 uses 2 ones + 6 tenths + 6 hundredths
ii) 1.15 + 1.15 = 2.30 (5 models)
2.30 uses 2 ones + 3 tenths
iii) If there are 10 or more of any type of model after you add them, you have to regroup 10 models of one type for 1 model of another type so you end up with fewer models than you started with.

E. i) 1.33 – 1.33 = 0 (0 models)
1.6 – 1.33 = 0.27 (9 models)
7 – 0.07 = 6.93 (18 models)
ii) *Sample response*:
Sometimes, before you can subtract, you have to regroup. When you regroup, you trade 1 model of one type for 10 models of another so you end up with more models than you started with.

F. *Sample responses*:
i) 1.11 + 1.11; 9.99 – 1.11
ii) 7.99 + 1.23; 3.11 – 1.87

Supporting Students

Struggling students

• **Parts D and E**, where students must create questions that use certain numbers of models, may be more difficult for some students than some of the other questions. For these students, you may want to simply let them choose to add or subtract numbers with given numbers of models and see what happens.

Enrichment

• Some students might want to investigate to determine all possible solutions, in terms of the number of models required, when adding or subtracting numbers with a given number of models.

For example, when you add a number made up of 6 models and a number made up of 6 models, the result will always be a number that requires any of 3 (e.g., 1.5 + 0.6) or 12 (e.g., 1.5 + 5.1) models. This is because 6 + 6 = 12 and 12 – 9 = 3.

4.1.2 Adding Decimal Thousandths

Curriculum Outcomes	Outcome relevance
5-B9 Addition and Subtraction of Whole Numbers and Decimals: 5 digits to 1000ths • perform addition and subtraction presented horizontally and vertically • choose an appropriate method for computation: mentally, pictorially, or symbolically • continue estimating in computation **5-B12 Open Sentences: applying number sense** • work with open number sentences involving the four basic operations	• Adding and subtracting decimals meaningfully is important both in practical settings and in higher mathematics. Students will be more successful if they are aware that mental computation is sometimes more useful. • Working with open sentences will help students develop their pre-algebra skills. This should make later work in algebra in higher classes easier for them.

Pacing	Materials	Prerequisites
1.5 h	• Square Thousandths Grids (BLM) • Place Value Charts (BLM)	• adding decimal hundredths • representing decimal thousandths

Main Points to be Raised

• When you add, you put together like values, for example, hundredths with hundredths, thousandths with thousandths, and so on.

• You can add either horizontally or vertically. If you add horizontally, you must still combine like values.

• It is sometimes convenient to use place value charts to remind you of the value each digit represents. Other times, it is convenient to use a grid to add decimals, especially for decimals under 1.

• Sometimes, you can use mental math to add decimals by considering one digit at a time. Other times, it is more efficient to rename both addends, moving a certain amount from one addend to the other to make the addition easy to perform mentally.

For example, 2.999 + 3.14 could be rewritten as 3 + 3.139 to make the calculation easier.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *How do you know that Rinzin walked less than 2 km?* (If Rinzin had walked 2 km, then Dorji would have walked 4 km. If I add them, I get 6 km and that is too far.)

• *Why might you divide 4.26 into approximately 3 equal sections to answer the question?* (One section could represent how far Rinzin walked and 2 sections could represent how far Dorji walked, since he walked twice as far.)

• *Why might a distance for Rinzin of about 1.4 km be reasonable?* (If Rinzin walked 1.4 km, then Dorji walked 2.8 km. When you add the distances, you get 4.2 km, which is close to 4.26 km.)

The Exposition — Presenting the Main Ideas

• Present the question 1.452 + 1.234 and ask pairs of students what they think the answer might be and why.

• Repeat the above with the question 1.452 + 1.2 and see whether students recognize that the 0.2 must be combined with the 0.4.

• Have students share the different strategies they used to solve the question. If students do not mention it, show them how a thousandths grid could be used to add the 0.452 + 0.234 (or 0.2) parts.

• Then present a question like 1.452 + 1.685. Students will have to decide how to regroup. After hearing their strategies, you might re-introduce the place value chart to show how the regrouping can be shown on the chart.

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• Finally, present the question 2.999 + 3.124 and ask students how they could solve it mentally. Lead the discussion to make sure students recognize that moving 0.001 from the 3.124 to the 2.999 might make the question easier to complete (as 3 + 3.123).

• Have students read through the exposition and ask if they have any questions.

Revisiting the Try This

B. Observe whether some students realize that they can add 0.001 to Rinzin's distance and 0.002 to Dorji's distance from the answers they got from **part A** or whether they start all over.

Using the Examples

• Assign students to pairs. Ask one student in the pair to become the "expert" on **examples 1 and 2** and the other to become the expert on **example 3**. They might want to try the questions first, before reading through the solutions, and then compare their thinking to what they see in the text. Each student in the pair should then his or her example to the other student.

Practising and Applying

Teaching points and tips

Q 1: Students need to recognize that they should not overlap the two sections representing the two addends.

Q 2: The reason that the solution box is on the left for two of the parts is to reinforce for students that this is possible. This will be important for later work in algebra.

Q 3: Students should be encouraged to do **part c)** in several steps, using mental math for each step.

Q 4: Encourage students to combine things in such a way that mental math can be used.

For example, it would be smart to combine 3.099 and 2.001 to get 5.1 before adding 15.237.

Q 5: Students need to understand that different digits can be used in the different boxes.

Q 6: Students need to recognize that L stands for litres.

Q 7: Make sure students realize that a lower value means a faster reaction time.

Q 10: Some teachers and students believe it is best to put in the extra zeros so that you are always adding the same number of decimal digits. It is correct to do this, but it is not required.

Common errors

• When students add decimals with different numbers of decimal places they sometimes incorrectly line up the values to be added.

For example, to add 2.3 + 3.145, they might write:

$$\begin{array}{l} 3.145 \\ \underline{+\ \ 2.3} \\ 3.168 \end{array}$$

It is important to encourage estimation, which might show the answer to be incorrect. Estimation will not always reveal the error. In any situation, students should use language like, "3 ones and 2 ones is 5 ones; 1 tenth and 3 tenths is 4 tenths" so they will be less likely to make an error in combining amounts that do not go together.

Suggested assessment questions from Practising and Applying

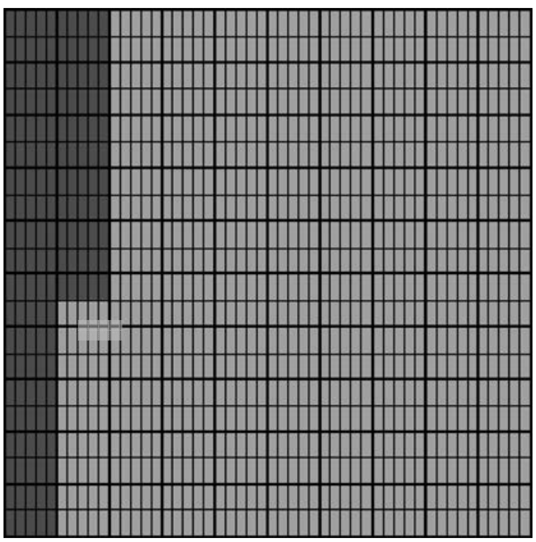
Question 2	to see if students can add decimal thousandths
Question 3	to see if students can use mental math to add decimal thousandths
Question 8	to see if students can solve a problem requiring the addition of decimal thousandths

Answers

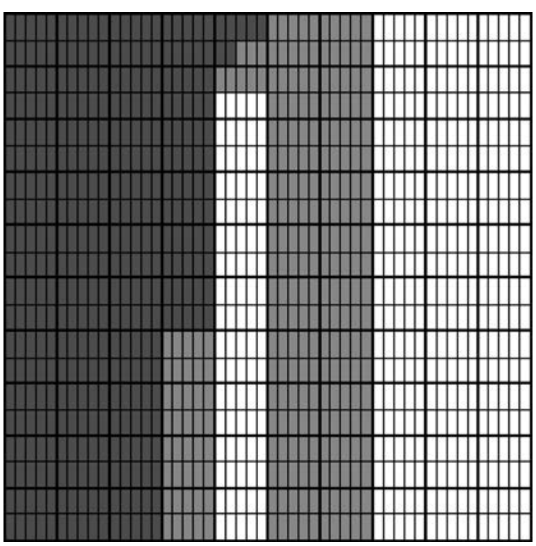
A. *Sample response*:
Dorji must walk between 2 km and 3 km. Then Rinzin would walk between 1 km and 1.5 km, and the total would be between 3 km and 4.5 km. Dorji's distance must be closer to 3 km because 4.26 is closer to 4.5 than to 3.

B. Rinzin walks 2.842 km and Dorji walks 1.421 km.
Sample response:
I knew Rinzin's distance was close to 3, so I tried 2.90. Half is 1.45.
2.90 + 1.45 = 4.35; this is too high.
I tried 2.80 + 1.40 = 4.20; this is too low.
I noticed the thousandths place was 3 so I figured I needed to add 2 thousandths + 1 thousandth.
The hundredths place was 6 so I figured I needed to add 4 hundredths + 2 hundredths.
So I tried 2.842 and 1.421 and it worked. 2.842 + 1.421 = 4.263.

1. a) 0.155 + 0.845 = 1



b) 0.367 + 0.248 = 0.615



2. a) 4.586 **b)** 0.444
c) 28.093 **d)** 25.399

3. a) 4.247; [*Sample response*: 0.010 is 1 hundredth, so add 1 hundredth to the 3 hundredths in 4.237 to get 4.247.]
b) 4.238; [*Sample response*: 0.001 is 1 thousandth, so add 1 thousandth to the 7 thousandths in 4.237 to get 4.238.]
c) 4.348; [*Sample response*: 0.111 is 1 tenth, 1 hundredth, 1 thousandth, so add one to each of the decimal digits in 4.237 to get 4.348.]

4. a) 20.337; [*Sample response*: Add the 0.001 to the 0.099 in 3.099 to get 3.1; then add 2 + 15.237 = 20.337.]
b) 19.426; [*Sample response*: Add 15 and 4 and then add 0.001 to 0.299 to get 0.3 and then add the other 0.126 = 19.426.]

5. **a)**

	3.527
+	**4**.267
	7.794

b)

	8.09**9**
+	**3.46**8
	11.567

6. a) 5.402 L **b)** 3.245 L and 1.262 L

7. a) Right hand: 1.8 s; Left hand: 1.93 s
b) Right hand

8. a) 0.192 m **b)** 192 mm

[**9.** *Sample response*:
The thousandths do not matter because they are so small compared to almost 4 and about 15.]

10. No; [*Sample response*:
You can add the 9 ones and 6 tenths to the 4 ones and 2 tenths and then just write down the hundredth and thousandth digits from 4.235. You do not really need the zeros.]

Supporting Students

Struggling students

• **Questions 7 and 9** may be difficult for some students. It is acceptable to not assign these questions to struggling students.

• You may wish to review adding with decimal hundredths with students for whom thousandths are difficult.

Enrichment

• You may provide students with a set of calculations where each digit from 0 to 9 is missing twice. They must figure out where the digits go.

For example:

```
  1.234      [].[]2[]      7.528
+ [].078   + 6.4[]9    +  [].13[]
  6.31[]     16.132      []5.66[]
```

• Students can then make up their own set of calculations for others to complete.

4.1.3 Subtracting Decimal Thousandths

Curriculum Outcomes	Outcome relevance
5-B9 Addition and Subtraction of Whole Numbers and Decimals: 5 digits to 1000ths • perform addition and subtraction presented horizontally and vertically • choose an appropriate method for computation: mentally, pictorially, or symbolically • continue estimating in computation **5-B12 Open Sentences: applying number sense** • work with open sentences involving the four basic operations	• Adding and subtracting decimals meaningfully is important both in practical settings as well as in higher mathematics. Students will be more successful if they are aware that sometimes mental computation is more useful. • Work with open sentences will help students develop their pre-algebra skills. This should make algebra in higher classes easier for them.

Pacing	Materials	Prerequisites
1.5 h	• Square Thousandths Grids (BLM) • Place Value Charts (BLM)	• subtracting decimal hundredths • representing decimal thousandths

Main Points to be Raised

• Every subtraction can be thought of as a "missing addition". This means that you are looking for the amount to add to one number to get the other.

• You can use a number line as a tool to show the amounts you add to one number to make another number. You may choose to add in steps.

• When you are adding up to get from one decimal number to another, it is often convenient to go to the next hundredth, then to the next tenth, and finally to the next whole.
For example, to calculate 4.1 – 3.872, go from 3.872 to 3.88, from 3.88 to 3.9, from 3.9 to 4, and then from 4 to 4.1. You only need to be able to subtract from 10 to do this: from 3.872 to 3.88 is 0.008 since 10 – 2 (from the 3.87**2**) is 8.

• You need not use a number line that has been marked into even intervals. You can simply mark your own number line to show the calculations without worrying about the interval sizes.

• You can subtract thousandths just like you subtract tenths and hundredths (and whole numbers) using regrouping when necessary.

• You can use the take-away meaning on a thousandths grid to subtract decimal thousandths.

• Sometimes it is easy to subtract mentally, for example, subtracting only tenths, only hundredths, or only thousandths. You might also subtract by using a related question.

For example, to subtract 3.98, you might subtract 4 (which is easier) and then add back the extra 0.02 you took away.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *Why can you add the same amount to both the numbers in the subtraction without changing the answer?* (If you think of the difference as how far apart they are on the number line, then when you shift both of them the same amount by adding the same to both numbers, they remain the same distance apart.)

• *Why is subtracting 2 easier than subtracting 1.99?* (There is no regrouping — you just count back 2 from the whole number part.)

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The Exposition — Presenting the Main Ideas

• Many students find subtraction much more difficult than addition. For this reason, one of the strategies shown for subtraction is called missing addend — you look for the number to add to one value to get another value.

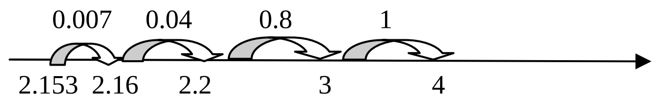
For example, 4 – 2.153 can be calculated by figuring out what to add to 2.153 to get to 4.

- It is often easier to do this in separate steps, which are then accumulated. It is also useful to show the steps on a number line.

- To get from 2.153 to 4, you can add 0.007 to get to 2.16, then add 0.04 to get to 2.2, then add 0.8 to get to 3 and finally add 1 more to get to 4.

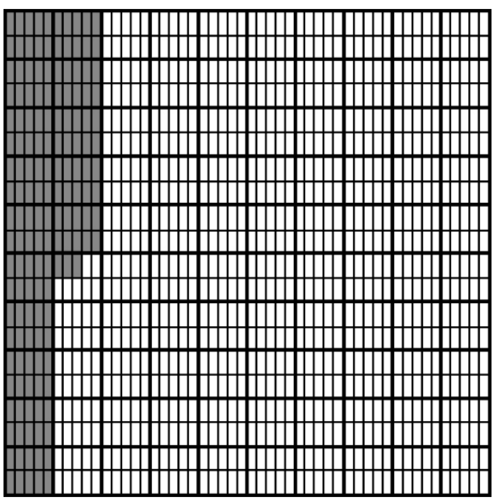
- Model the missing addend method on a number line. This provides some visual support to help students see what is happening. To determine the values to add, they should always think of getting to 10 of the least place value with which they are working.

For example, if you are at 3 thousandths, you need to add 10 – 3 = 7 thousandths. If you are at 6 hundredths, you need to add 10 – 6 = 4 hundredths, etc.



The total added is 1.847 (going backwards from the whole number addition makes it easier for students to see).

An alternative is to model the lower value on a thousandths grid (in this case starting at 0.153) and to see what must be added to fill the grid.



4 – 0.153
0.007 must be added to fill up the 6th square in the second column.
0.04 must be added to fill up the second column.
0.8 must be added to fill up the grid.
0.007 + 0.04 + 0.8 = 0.847
So 1 – 0.153 is 0.847.

Therefore 4 – 0.153 = 3.847
and
4 – 2.153 = 3.847 – 2 = 1.847.

• You can model the same subtraction using standard regrouping. You rewrite 4 as 4000 thousandths and regroup to create 3990 thousandths (or 399 hundredths) + 10 thousandths.

$$\begin{array}{r} {\scriptstyle 3\ 99\ 10} \\ \cancel{4.000} \\ -\ \underline{2.153} \\ 1.847 \end{array}$$

• Present the question 3.425 – 0.002 and ask why it is easy to answer using mental math. (take away 2 thousandths from the 5 thousands and nothing else changes). Ask students to suggest other decimal subtractions that are easy to do mentally (for example, 3.425 – 0.005 or 3.468 – 2.999) and others that are more difficult to do using mental math (for example, 3.425 – 1.678).

• You may decide to have students read through the exposition or you may simply ask them to use it as a reference.

Revisiting the Try This

B. Encourage students to think about what they have learned about mental subtraction of decimals to answer this question.

Using the Examples

• Ask the question in **example 1** of the whole class. Allow students to suggest how they would approach the question. Then assign the problem in **example 2**, suggesting that students draw a diagram to help them. Students should then compare their work with the solution and thinking in **example 2**. They can use **example 1** for reference if they wish.

Practising and Applying

Teaching points and tips

Q 1: Make sure that students recognize that 0.4 is 4 tenths (4 full columns) and not 4 thousandths (4 small parts of the grid).

Q 2: Students should be allowed to use any strategy they wish for calculation.

For example, they could use a grid, a number line, a place value chart, or symbolic calculations.

Q 3: You may have to help some students see that 0.003 was added to both numbers. You may also have to ask them why subtracting 2.1 might be seen as easier than subtracting 2.097.

Q 5: Some students might use the steepness of the sections of the graph to help them answer the question, whereas other students might perform the four different subtractions.

Q 6: You may have to help some students understand why the higher value is actually a worse performance (more time for the same distance is not as fast).

Q 7: Some students will add up from 1.555 to 1.7 by adding 0.005 to get to 1.56, then 0.04 to get to 1.6, and then 0.1 to get to 1.7. Others will subtract directly.

Q 8: It may helpful to translate the symbolic statements into words.

For example, **part b)** could be read, "Is it true that if you add a number to 2.6, the result is less than if you subtract the same number from 2.6?"

Q 9: Students might use a guess and test strategy.

For example, for **part b)**, they know the numbers have to add to about 8.5, but they have to be about 2.5 apart. They might start with 4 and 4.5, spread the numbers out by going back to 3 and up to 5.5, and then adjust so the answer is actually 8.464 and not 8.5.

Q 10: Students can use whichever approach for subtraction they wish in the explanation.

Common errors

• Many students struggle to subtract a value with more decimal digits from a value with fewer decimal digits. For these students, adding up may be a more helpful strategy than regrouping.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can calculate differences with decimal thousandths
Question 4	to see if students can communicate about mental math processes that are effective for subtracting decimal thousandths
Question 8	to see if students can reason about decimal subtraction using algebraic thinking
Question 10	to see if students can describe how to subtract decimal thousandths

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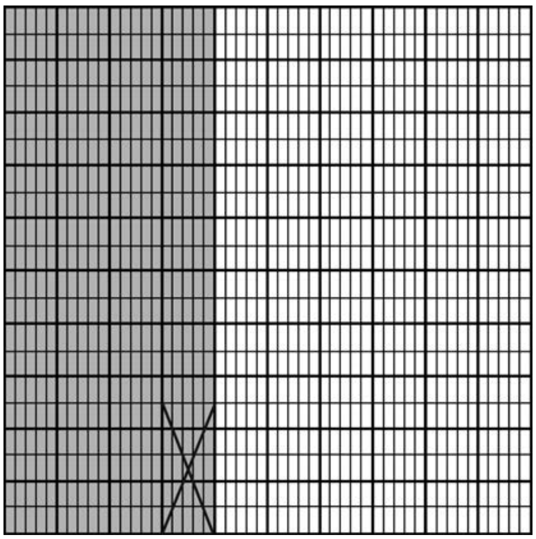
Answers

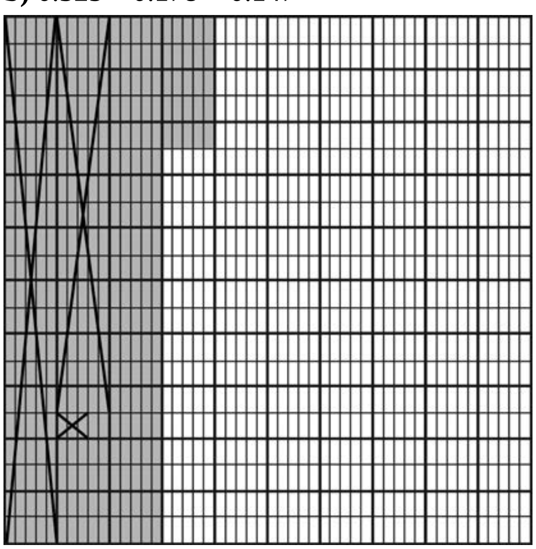
A. i) 3.5 – 2 = 1.5; *Sample response*:
If both numbers are on a number line, you are moving them both the same distance up the number line. They stay the same distance apart.
ii) *Sample response*:
If you add 0.01 to both numbers, the second number becomes 2, which is easy to subtract mentally:
3.2 – 1.99 = 3.21 – 2 = 1.21.

B. *Sample response*:
3 – 1.999 = 3.001 – 2 = 1.001
4 – 3.999 = 4.001 – 4 = 0.001

1. *Sample responses*:
a) 0.4 – 0.025 = 0.375



b) 0.325 – 0.178 = 0.147



2. a) 3.111 **b)** 1.268
c) 2.113 **d)** 11.044

3. *Sample responses*:
[**a)** It is easier to subtract 2.1 than 2.097 using mental math.]
b) 4.2 – 1.999 and 3.5 – 1.298

4. a) 3.544; [*Sample response:*
Take 1 thousandth from the 5 thousandths in 3.545 to get 3.544.]
b) 7.815; [*Sample response:*
Take 1 hundredth from the 2 hundredths in 7.825 to get 7.815.]
c) 5.904; [*Sample response:*
Take 1 tenth from the 60 tenths (6.0) in 6.004 is to get 5.904.]
d) 2.999; [*Sample response:*
Think of 3 as 3000 thousandths and then take away 1 thousandth to get 2999 thousandths, which is 2.999.]

5. Birth to 3 months; [the gain is over 2 kg (5.005 – 2.879 > 2) and the other periods all have gains of less than 2 kg.]

6. 0.014 s

7. a) 0.145 m; [*Sample response*:
Add 0.005 to 1.555 to get to 1.56. Next, add 0.04 to 1.56 to get to 1.6. Finally, add 0.1 to 1.6 to get to 1.7. Since I added 0.1 + 0.04 + 0.005 = 0.145, that is the difference.]

8. a) Always **b)** Never **c)** Always

9. *Sample responses*:
a) 4.2 and 4.264 **b)** 3.2 and 5.264

[**10.** *Sample response*:
I would tell him or her to add on from 1.895 to 4.3:
• Add 0.005 to 1.895 to get to 1.9. Next, add 0.1 to 1.9 to get to 2. Finally, add 2.3 to 2 to get to 4.3.
• Add together all the parts that were added on.
The total is 2.405.]

Supporting Students

Struggling students

• Struggling students will benefit from continuing to use number line models and thousandths grids to perform decimal subtraction. You may wish to work more on decimal hundredth subtractions before moving these students to subtractions involving decimal thousandths.

Enrichment

• Invite students who seem comfortable with decimal thousandth subtraction to invent a game for their classmates to play that requires them to practice these computations.

GAME: Big Sum, Little Difference

• This game allows students to practice decimal thousandth addition and subtraction as well as decimal thousandth comparisons.

• Some students will use strategies, but others will not.

For example, they might try for the greatest sum by using the greatest possible digits as whole numbers. Or they might try for the least difference by choosing whole numbers that are close together or that are the same.

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Chapter 2 Multiplying Decimals

4.2.1 Estimating Products

Curriculum Outcomes	Outcome relevance
5-B10 Decimals × Whole Numbers: simple products • estimate (e.g., 4 × 2.45 as 4 × 2 or 4 × 3)	It is useful for students to be able to estimate products with decimals in order to check their calculations when they multiply.

Pacing	Materials	Prerequisites
40 min	None	• multiplying whole numbers

Main Points to be Raised

• You can multiply a decimal number by a whole number by multiplying each of its parts and adding the parts together.

• You can estimate the product of a whole number and a decimal or the product of two decimals by rounding each to the nearest whole number and multiplying the whole numbers.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *How much fabric would be needed if each piece were 1 m long?* (42 m)

• *How much longer is 1.2 m than 1 m?* (20 cm longer)

• *How much fabric do you need for 5 students? How do you know?* (I would need 5 m for each student, plus another 5 × 20 cm, which is one more metre, so I would need 6 m in total.)

• *How does knowing the number of metres for 5 students help you answer the question?* (If 6 m are needed for 5 students, 6 × 8 m would be needed for 5 × 8 students. Even more fabric would be needed for 42 students than for 40 students.)

The Exposition — Presenting the Main Ideas

• Ask students why they think 4 × 2.2 might be not much more than 8. Encourage them to recognize that in a decimal number, the whole number part is the most significant amount. In other words, 2.2 is very close to 2, so 4 × 2.2 is very close to 4 × 2.

• Have the students explain why 4 × 2.2 is greater than 4 × 2 but less than 4 × 3.

• Ask whether they would estimate 4 × 2.9 using 4 × 2 or 4 × 3 and why.

• Encourage students to look at the exposition later for reference.

Revisiting the Try This

B. Students need to think of *a* × *b* as *a* sets of *b* in order to answer this question.

Using the Examples

• Ask students to read through the two examples and ask any questions they might have.

Practising and Applying

Teaching points and tips

Q 1: Although we usually round to the nearest whole number, it is not incorrect for students to round to another whole number that is nearby but not the nearest.

For example, 4 × 2.9 is better estimated as 4 × 3, but 4 × 2 is not incorrect as an estimate.

Q 2: Students are likely to estimate only the decimal, and not the whole number, but they could estimate the whole number as well.

For example, they could estimate **part d)** as 9 × 10. They should recognize that if they increase the decimal (or the whole number), the estimate is high and if they decrease the decimal (or the whole number), the estimate is low.

Q 3: Some might estimate 3.8 as 4 and their result would be 32 m. Others might estimate 3.8 as 3.5 and their result would be 28 m.

Q 4: The answer to this question assumes a six-day school week.

Q 6: Students might estimate 28.5 as 28, as 29, or as 30.

Q 7: Students must realize that 8 × 4 = 32, so a value greater than 4 in the whole digit position would be best.

Q 9: There are many correct answers for these questions. Students need to think of whole numbers that result in the correct product and then adjust these values to create decimal numbers.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can estimate the product of a whole numbers and a decimal
Question 3	to see if students can use estimation to solve everyday problems involving the product of a whole number and a decimal
Question 9	to see if students can solve a simple mathematical problem involving estimation of products of whole numbers and decimals

Answers

A. *Sample response*: 42 > 40 and 1.2 > 1, so 42 × 1.2 > 40 × 1 = 40.	**B.** 42 × 1.2
1. a) 4 × 3 **b)** 7 × 8 **c)** 10 × 6 **d)** 9 × 4 **2.** *Sample responses*: **a)** about 77; Lower **b)** about 48; Lower **c)** about 25; Higher **d)** about 81; Higher **3.** *Sample response*: A bit less than 32 m **4.** *Sample response*: Less than 18 km **5.** *Sample response*: More than 12 m	**6.** *Sample response*: Less than 1800 cm [**7.** *Sample response*: 4.2; 8 × 4 is 32, so I think it is a bit more than 4, but not a lot more.] **8.** B is greatest; [*Sample response*: B is more than 16 and the others are less than 16.] **9.** *Sample responses*: **a)** 4 × 5.01 **b)** 6 × 4.99 **c)** 20 × 2.03

Supporting Students

Struggling students

• Struggling students may need to review multiplication facts before they can be successful with this lesson.

Enrichment

• Ask students to write and solve word problems that require estimating products of whole numbers and decimals. You might even give a condition for the problem such as the estimate should be between 12 and 15.

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4.2.2 Multiplying a Decimal by a Whole Number

Curriculum Outcomes	Outcome relevance
5-B10 Decimals × Whole Numbers: simple products • link concrete models to algorithm • estimate (e.g., 4 × 2.45 as 4 × 2 or 4 × 3)	The ability to multiply a decimal by a whole number is a life skill. This outcome precedes work with multiplying a decimal by a decimal. It is important for students to develop estimation skills in order to check answers and solve problems meaningfully.

Pacing	Materials	Prerequisites
1 h	• Base ten blocks or Base Ten Models (BLM): hundredths, tenths, and ones	• dividing by 2

Main Points to be Raised

• When you multiply a decimal number by a whole number, you can think of it as repeated addition.

For example, 3 × 2.4 means 2.4 + 2.4 + 2.4.

• You can multiply a decimal number by a whole number using the same procedures you use to multiply two whole numbers.

• You can multiply starting at the left or at the right.

• You should estimate to check your product.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *Why did you decide to estimate using 12 s for the time?* (12.4 s is close to 12 s)

• *Why did you multiply by 3?* (300 is 3 times as much as 100.)

• *How did you multiply 12 by 3 using mental math?* (I multiplied 10 by 3 and 2 by 3. Then I added the parts together.)

The Exposition — Presenting the Main Ideas

• Present the question 6 × 3.2 to your students. Before you show them how, ask them how they might figure out the product. Provide base ten ones, tenths, and hundredths blocks or models they can use. If this is not possible, make sure they know how to represent 3.2 in a diagram as 3 wholes and 2 tenths.

• Collect student suggestions for how this multiplication can be done. Make sure that more than one method is shown (you can use the idea in the exposition to help you with this).

• Then present the question 2 × 1.34 and, again, ask students how they would proceed. Encourage them to use diagrams or models. No matter how they do the question, whether with models, diagrams, or symbolically, they must explain not only what they do, but why they do it.

For example, if they say that they multiplied 2 by 4, ask why. Ask whether the result shows wholes, tenths, or hundredths, and why.

• Model another way to show 2 × 1.34 by writing 1.34 as 134 hundredths (1 = 100 hundredths and 0.3 = 30 hundredths, so 1.34 = 100 + 30 + 4 hundredths). Then show that 2 × 134 hundredths is 268 hundredths. If they divide 268 by 100, they realize this is 2.68.

• Encourage the students to look at the exposition later for reference.

Revisiting the Try This

B. Students can use symbols, diagrams, or models to answer this question.

Using the Examples

• Work through **example 1** with the students. Ask them which method they prefer and why. It is to be expected that some students will prefer one method and others will prefer a different method.

• Present to the class the problem in **example 2**. Ask them to solve it and then check their answer against the solution and thinking in the text.

Practising and Applying

Teaching points and tips

Q 1: Students can draw diagrams of the models if models are not available.

Q 2: Students can use models, diagrams, or symbols for this question.

Q 3: Some students will estimate **A** and **B** as equal since the whole number parts are 7 and 3 in both. Others in **part B** will round 7.82 to 8 and will have different estimates.

Q 4: Some students may solve this mentally by adding $7 \times 2 = 14$ to $7 \times 0.6 = 4.2$.

Q 5: Some students will need to complete the calculations to answer the question whereas others may do only part of the calculation. The latter approach may be more efficient, but it can lead to problems in **part B** where regrouping affects the digit in the ones place.

Q 6: Make sure students understand that the missing digits within each question may be different.

Q 7: Observe whether students need to write out each calculation to get to the 20th equation or whether they use more advanced thinking to solve the question efficiently.

Q 8: Encourage students to estimate to check their answers.

Common errors

• Some students multiply the numbers and forget to put the decimal points in the answers. Requiring students to estimate the products can help with this.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can model the meaning of the product of a whole number and a decimal and use the model to calculate the product
Question 4	to see if students can solve a real-world problem involving the multiplication of a whole number and a decimal
Question 7	to see if students can use a pattern to find a product involving decimal multiplication

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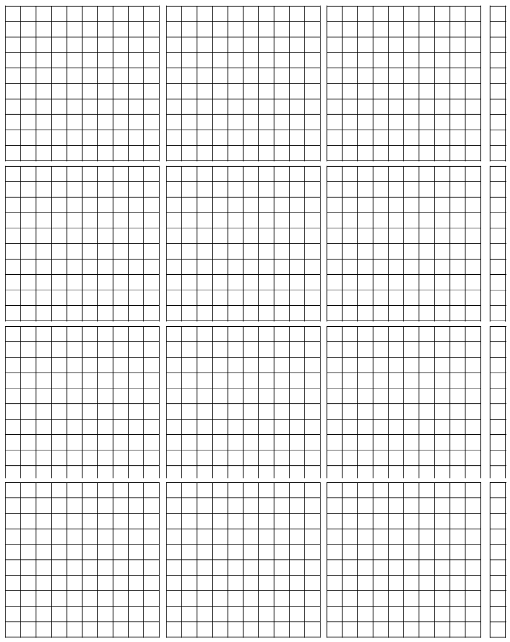
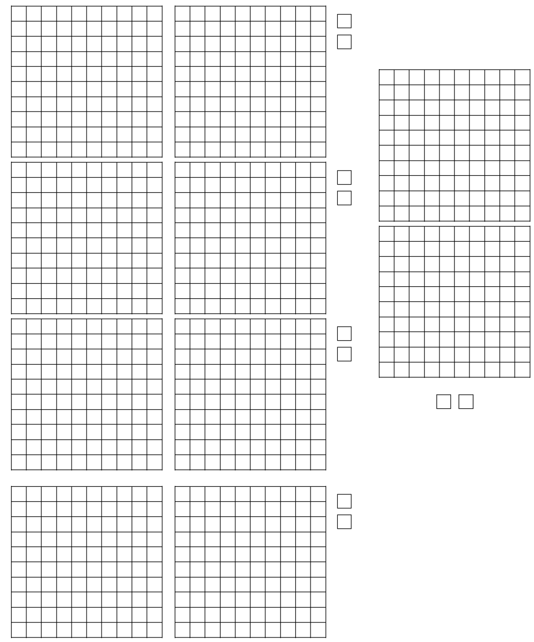
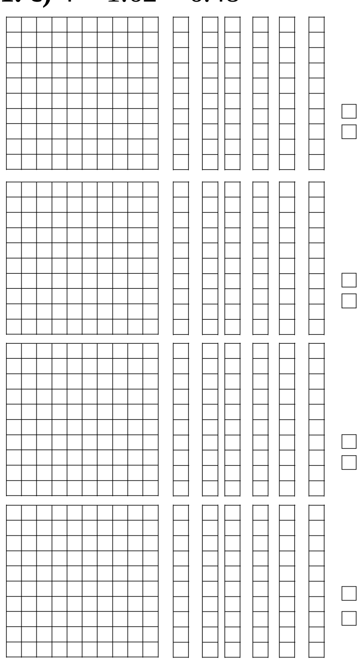
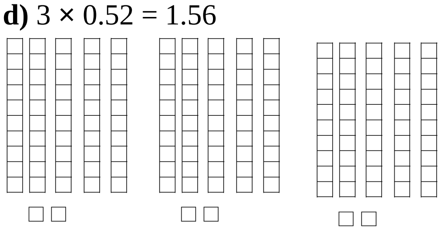
Answers

A. *Sample response*: About 36 s	**B**. 3 × 12.4 = 37.2 s
1. a) 4 × 3.1 = 12.4 	**1. b)** 5 × 2.02 = 10.1 
1. c) 4 × 1.62 = 6.48  **d)** 3 × 0.52 = 1.56  **2. a)** 32.4 **b)** 74.56 **c)** 110.7 **d)** 12.81	**3.** B is greatest; [*Sample response*: It is close to 24 and the others are closer to 21, 20, and 18.] A. 7 × 3.12 = 21.84 B. 3 × 7.82 = 23.46 C. 5 × 4.09 = 20.45 D. 6 × 3.15 = 18.9 **4.** 18.2 km **5.** B and C are true. [A is not true because the digit in the hundredths place is 5. B is true because the product is 19.68. C is true because the product is 15.68.] **6. a)** 7.8 **b)** 12.**8**3 × 9 × 7 70.**2** **8**9.81 **7. a)** 17, 17.5, and 18 **b)** 5 × 3.7 = 18.5 5 × 3.8 = 19 5 × 3.9 = 19.5 **c)** 5 × 5.3 = 26.5 **8.** 137.1 cm **9.** They are both right, [e.g., 4 × 5.43 = 21.72 but 4 × 5.25 = 21 (or 21.00).]

Supporting Students

Struggling students

• It is important to encourage struggling students to use models and diagrams before they work symbolically. You might also use a place value chart.

For example, for 4 × 3.72:

Tens	Ones	Tenths	Hundredths
	4 × 3	4 × 7	4 × 2
	12	28	8
	14	8	8
1	4	8	8

Enrichment

• Some students might be ready to multiply decimal thousandths by whole numbers. Or, they might want to create questions that meet certain conditions, such as when you multiply the digit in the tenths place is 4.

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4.2.3 Multiplying by 0.1, 0.01, and 0.001

Curriculum Outcomes	Lesson relevance
5-B11 Mentally Multiply: whole numbers by 0.1, 0.01, 0.001 • multiply by 0.1, 0.01, 0.001 • link to place value **5-B12 Open Sentences: applying number sense** • explore numerical situations which are always, sometimes, and never true (e.g., 324 + □ > 300 is always true if □ is a whole number) • include the four basic operations **5-C1 Open Sentences: patterns in addition, subtraction, multiplication, and division** • generate rules about how a change in one variable affects the result (e.g., for □ × 10, as □ increases by 1 the product increases by 10)	To prepare for later work with multiplication of decimals, students need to be able to multiply mentally by decimal powers of 10.

Pacing	Materials	Prerequisites
40 min	• Place Value Charts (BLM)	• place value up to decimal thousandths • the meaning of multiplication as how many groups

Main Points to be Raised

• When you multiply by 0.1, the digits move one place to the right. This is because each digit is now worth one tenth as much as before.

• When you multiply by 0.01, the digits move two places to the right.

• When you multiply by 0.001, the digits move three places to the right.

• Multiplying by 0.1, 0.01, or 0.001 is equivalent to dividing by 10, 100, or 1000, respectively.

Try This — Introducing the Lesson

A. and B. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *Which if bigger, a centimetre or a millimetre?* (A centimetre)

• *How many millimetres are in 1 cm?* (10) *Centimetres in 1 m?* (100) *Metres in 1 km?* (1000)

• *When you write centimetres as millimetres, will the number of millimetres be greater or less than the number of centimetres?* (It will be greater because millimetres are smaller and it takes more of them to measure the same distance.)

• *Why did you multiply instead of dividing?* (The numbers have to get bigger.)

The Exposition — Presenting the Main Ideas

• Remind students that 4 × 10 means 4 groups of 10 and 4 × 2 means 4 groups of 2. Ask them what they think 4 × 0.1 might mean. Talk about how it means 4 groups of one tenth, or 4 tenths. Ask how it is written (0.4).

• Repeat the above, asking about 6 × 0.1, 12 × 0.1, and 230 × 0.1. Point out that when we write 0.6, 1.2, and 2.30, we see that the digits have moved one place to the right on a place value chart.

Tens	Ones	Tenths	Hundredths
	6		
		6	

• Point out how this is the same as dividing by 10, since 6 ÷ 10 = 0.6.

• Repeat the same process, but multiply by 0.01 and by 0.001.

• Ask students why it makes sense that, for example, 78 × 0.1 = 7.8, but 78 × 0.01 = 0.78.

• Suggest that students read through the exposition to summarize this thinking.

Revisiting the Try This

C. Students should be thinking that dividing by 10, 100, or 1000 is just like multiplying by 0.1, 0.01, or 0.001.

Using the Examples

• Put the question from **example 1** on the board and ask students to work on it. When they have finished, they should check their thinking against the two solutions shown in the text.

Practising and Applying

Teaching points and tips

Q 2: Some students will answer **part a)** by dividing 17.2 by 10. Others will multiply 17.2 by 0.1. Yet other students may solve the equation to find the missing value (172) and then multiply 172 by 0.01.

Q 3: Many students will be able to solve this without completing the multiplication. They just need to realize how many places the digits move.

For example, for **part a)**, they think that since the digits move two places, it is the digit in the tens place that moves to the tenths place.

Q 4: Students need to recognize that 1 m = 0.001 km to answer this question.

Q 5: Help students by asking questions.

For example:

• *Will the real distances be greater or less than the map distances?*

• *By what will you multiply the map distances?*

• *Why would 1 cm on the map be 10 m in actual distance?*

Common errors

• Many students will move the digits in the wrong direction. They need to recognize that multiplying by 0.1, 0.01, or 0.001 reduces the size of a number; their products should show this.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can multiply by 0.1, 0.01, and 0.001
Question 3	to see if students recognize how the digits move when multiplying by 0.1, 0.01, and 0.001
Question 5	to see if students can solve a problem a real-world problem involving multiplication by 0.001

Answers

A. 440 mm; Each centimetre is 10 mm.

B. i) 32 cm = 320 mm
46 m = 4600 cm
23 km = 23,000 m
ii) 10 for cm to mm since there are 10 mm in 1 cm.
100 for m to cm since there are 100 cm in 1 m.
1000 for km to m since there are 1000 m in 1 km.

C. i) 32 mm = 3.2 cm
46 cm = 0.46 m
23 m = 0.023 km
ii) You could divide 32 by 10 or multiply by 0.1 to change mm to cm.
You could divide 46 by 100 or multiply by 0.01 to change cm to m.
You could divide 23 by 1000 or multiply by 0.001 to change m to km.

1. a) 0.6 **b)** 0.72 **c)** 0.045
d) 0.382 **e)** 0.019 **f)** 0.123

2. a) 1.72 **b)** 0.172

3. a) 2 **b)** 1 **c)** 1 **d)** 1

[**4.** *Sample response*:
3.45 km = 3450 m because 1000 m = 1 km.
So 3.45 km × 0.001 = 3450 m × 0.001 = 3.450 m, or 3.45 m.]

5. a) 2400 cm or 24 m **b)** 9200 cm or 92 m

[**6.** *Sample response*:
When you multiply a number by 0.01, it is like finding $\frac{1}{100}$ of the number. To find $\frac{1}{100}$ of a number, you can find $\frac{1}{100}$ of the value of each digit. This means each digit moves two places to the right, since each place has a value that is $\frac{1}{10}$ of the place to its left.]

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Supporting Students

Struggling students

• Struggling students should continue to use place value charts for these multiplications. They need to focus on the concept that multiplying by 0.1 moves the ones digit to the tenths place, multiplying by 0.01 moves the ones digit to the hundredths place, and multiplying by 0.001 moves the ones digit to the thousandths place.

Enrichment

• Ask students to create other questions like **question 5** for their classmates to solve.

CONNECTIONS: Telescopes and Binoculars

• This connections provides students with a real world context for multiplying by decimal powers of 10.

• It would be helpful to show students a pair of binoculars if some are available.

Answers

1. [**a)** *Sample response*:
The image you are seeing in the binoculars has been magnified or multiplied by 10 by the lenses, so to find the size without the binoculars, you need to divide by 10, which is the same as multiplying by 0.1.]
b) 2.5 × 0.1 = 0.25 cm

2. 2.5 × 0.01 = 0.025 cm

UNIT 4 Revision

Pacing	Materials
1.5 h	• Base ten blocks or Base Ten Models (BLM): hundredths, tenths, and ones • Square Thousandths Grids (BLM) • Place Value Charts (BLM)

Question(s)	Related Lesson(s)
1	Lesson 4.1.1
2 – 5a), 10a) and c)	Lesson 4.1.2
5 b) – 9 and 10b)	Lesson 4.1.3
11 and 12	Lesson 4.2.1
13 – 15	Lesson 4.2.2
16 and 17	Lesson 4.2.3

Revision Tips

Q 1: Students can use either models or diagrams.

Q 2: Some students may still benefit from using thousandths grids or place value charts.

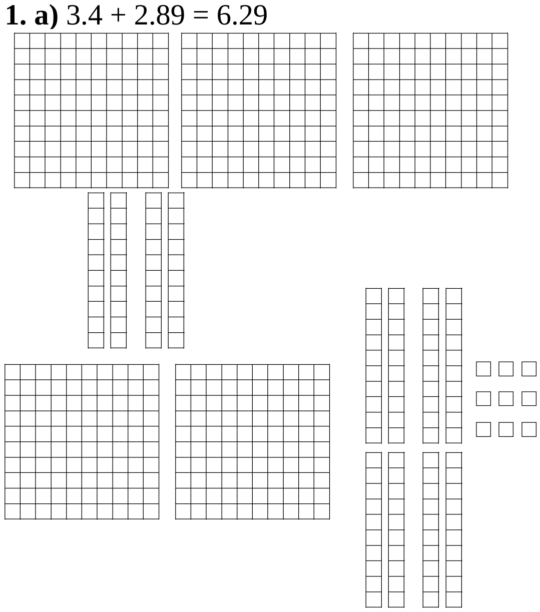
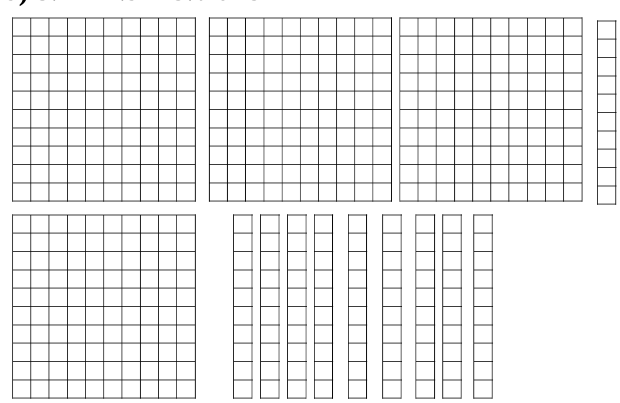
Q 6: For **part c)**, students should use several steps. For **part d)**, they might add up 0.002 to 2 and then add 3 more. Or, they might subtract 2 and then add back 0.002.

Q 13: You may want to provide models for some students to use to calculate. You may also choose to provide place value charts.

Q 14: Make sure students know that the same digit need not be used for all the blanks.

Q 17: Observe whether students realize that if the number is less, the unit must be greater in size.

Answers

1. a) 3.4 + 2.89 = 6.29  5 ones + 12 tenths + 9 hundredths Regroup 10 tenths as 1 one. 6 ones + 2 tenths + 9 hundredths = 6.29	**b)** 3.1 + 1.9 = 5.0 or 5  4 ones + 10 tenths Regroup 10 tenths as 1 one. 5 ones = 5

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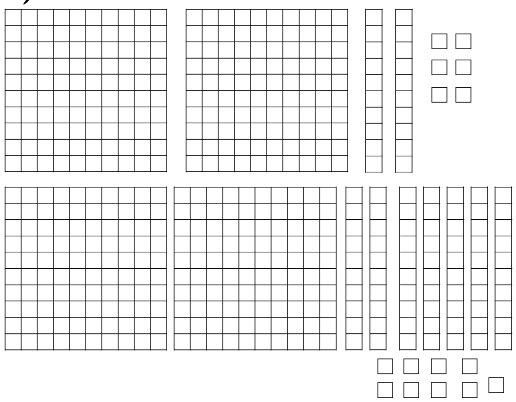
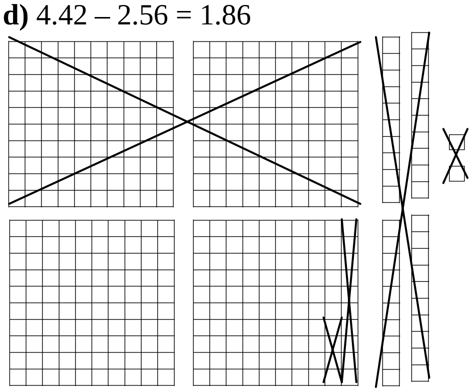
c) 2.26 + 2.79 = 5.05



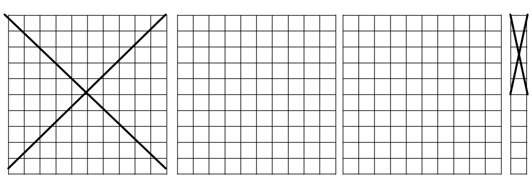
4 ones + 9 tenths + 15 hundredths.
Regroup 10 hundredths as 1 tenth.
Regroup 10 tenths as 1 one.
5 ones + 0 tenths + 5 hundredths = 5.05.

d) 4.42 – 2.56 = 1.86



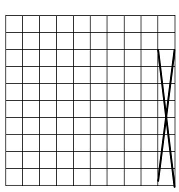
1 one + 8 tenths + 6 hundredths are left = 1.86.

e) 3.1 – 1.05 = 2.05



2 ones + 0 tenths + 5 hundredths are left = 2.05.

f) 1 – 0.08 = 0.92



92 hundredths are left = 0.92.

2. a) 4.767 **b)** 4.462 **c)** 24.465 **d)** 9.394

3. a) 5.287; [*Sample response*:
Add the 2 hundredths from 0.02 to the 6 hundredths in 5.267 to get 5.287 + 0 = 5.287.]
b) 10.458; [*Sample response*:
Add the 8 ones from 8.1 to 2.358 to get 10.358 and then add the 1 tenth from 8.1 to the 3 tenths in 10.358 to get 10.458.]
c) 6.04; [*Sample response*:
Add the 2 thousandths from 2.042 to 3.998 to get 4 and then add 2.04 to 4 to get 6.04.]

4. 146.1 cm

5. a) 13.56 m **b)** 1.466 m

6. a) 4.467; [*Sample response*:
Subtract 1 tenth from the 5 tenths in 4.567 to get 4.467.]
b) 4.566; [*Sample response*:
Subtract 1 thousandth from the 7 thousandths in 4.567 to get 4.566.]
c) 2.118; [*Sample response*:
Subtract 1 one from the 3 ones in 3.248 to get 2 ones, subtract 1 tenth from the 2 tenths in 3.248 to get 1 tenth, and subtract 3 hundredths from the 4 hundredths in 3.248 to get 1 hundredth. The result is 2.118.]
d) 3.002; [*Sample response*:
Add up from 1.998 to 5: add 0.002 to get to 2 and then add 3 more to get to 5, so the total difference is 3.002.]

7. a) 0.099 s **b)** 0.417 s and 0.420 s; 0.003 s

8. 3.752g

[**9.** So that you subtract tenths from tenths, hundredths from hundredths, and thousandths from thousandths.]

10. a) 3.353 L
b) 0.284 L
c) 1.381 L

11. *Sample responses*:
a) about 78; High [because 78 = 13 × 6 > 12.8 × 6]
b) about 24; Low [because 24 = 4 × 6 < 4 × 6.12]
c) about 42; High [because 42 = 7 × 6 > 7 × 5.79]
d) about 64; High [because 64 = 8 × 8 > 7.64 × 8]

12. *Sample response*: About 9 L

13. a) 33.2 **b)** 43.08
c) 46.41 **d)** 23.13

14. a)
$$\begin{array}{r} 3.8 \\ \underline{\times\ 7} \\ 26.6 \end{array}$$
b)
$$\begin{array}{r} \mathbf{17.5}3 \\ \underline{\times\ \ 8} \\ \mathbf{1}40.2\mathbf{4} \end{array}$$

15. 1.71 m

16. a) 0.8 **b)** 1.22 **c)** 0.069
d) 0.113 **e)** 0.172

17. 0.523 km

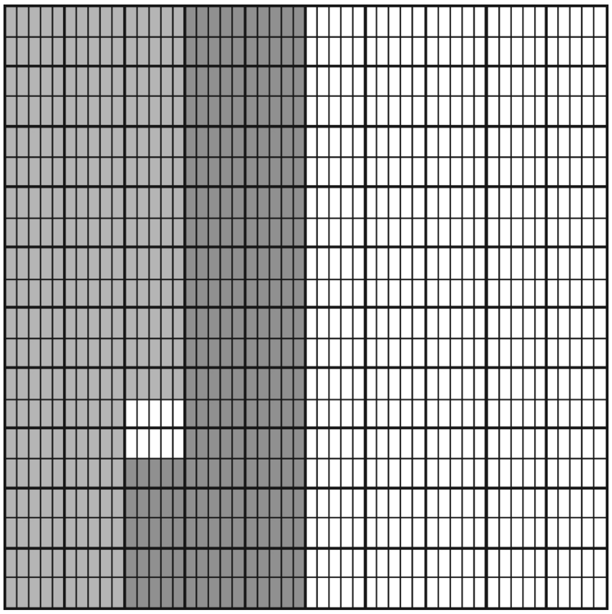
UNIT 4 Decimal Computation Test

1. Use models to calculate each.

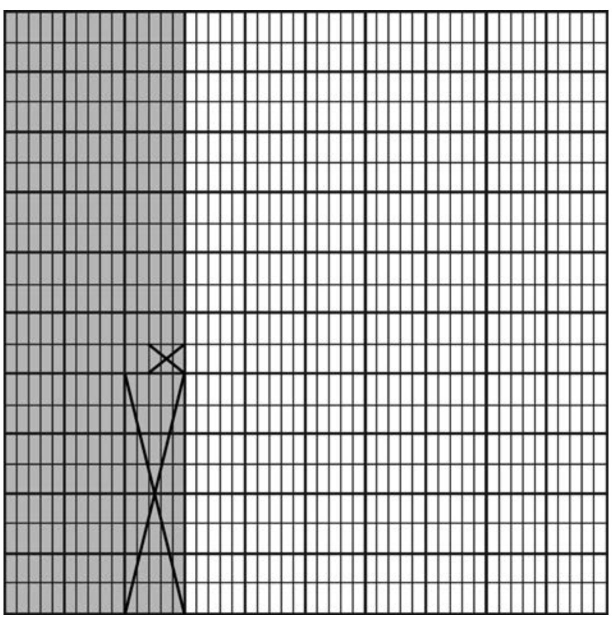
a) 2.87 + 1.89 **b)** 3.02 – 1.49

2. What calculation does each grid show?

a)



b)



3. Solve each using mental math. Describe how you calcuated.

a) 4.6 + 3.992 **b)** 5.1 – 3.998

c) 6.874 + 0.11 **d)** 16.345 – 1.22

4. Calculate each. Estimate to show that your answer makes sense.

a) 3.752 + 8.679 **b)** 12.3 – 3.765

5. Two packets of rice have masses of 1.752 kg and 2.78 kg.

a) What is the total mass?

b) How much heavier is the heavier packet?

c) How much more rice would be needed to have a total of 5 kg of rice?

6. Estimate and then calculate each.

a) 5 × 3.4 **b)** 4 × 7.93

c) 6 × 3.12 **d)** 8 × 8.45

7. Draw a model to show what 2 × 1.52 means. Calculate the product.

8. Each side of an equilateral triangle is 4.52 m long.

a) How do you know the perimeter is more than 12 m?

b) What is the perimeter?

9. One side of a rectangle is 3.42 m long. The other side is 4 times as long. How long is the other side?

10. For which of these products is 8 in the hundredths place? How do you know?

A. 880 × 0.01 **B.** 84 × 0.01

C. 48 × 0.01 **D.** 684 × 0.001

11. You want to divide a number by 10. What might you multiply by instead? Explain your thinking.

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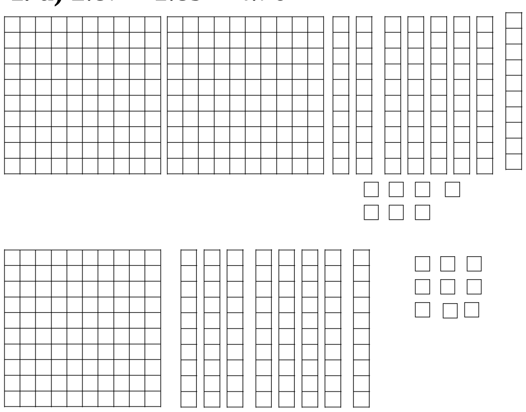
UNIT 4 Test

Pacing	Materials
1.5 h	• Base ten blocks or Base Ten Models (BLM): hundredths, tenths, ones • Square Thousandths Grids (BLM) • Place Value Charts (BLM)

Question(s)	Related Lesson(s)
1	Lesson 4.1.1
2 – 5	Lessons 4.1.2 and 4.1.3
6	Lessons 4.2.1 and 4.2.2
7 – 9	Lesson 4.2.2
10 and 11	Lesson 4.2.3

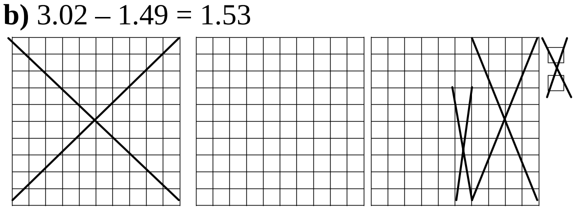
Answers

1. a) 2.87 + 1.89 = 4.76



There are 3 ones + 16 tenths + 16 hundredths.
Regroup 10 hundredths as 1 tenth.
Regroup 10 tenths as 1 one.
There are 4 ones + 7 tenths + 6 hundredths.

b) 3.02 – 1.49 = 1.53



1 one + 5 tenths + 3 hundredths are left = 1.53.

2. a) 0.265 + 0.225 = 0.49 **b)** 0.3 – 0.043 = 0.257

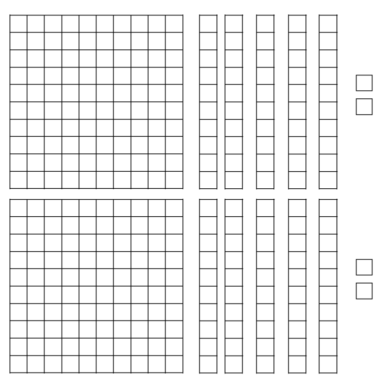
3. *Sample responses*:
a) Add 4 to 4.6 and then take away 8 thousandths.
3.6 + 4 = 8.6
8.6 – 0.008 = 8.592
b) Subtract 4 from 5.1 and then add back 2 thousandths.
5.1 – 4 = 1.1
1.1 + 0.002 = 1.102
c) Add 1 tenth to the 8 tenths and add 1 hundredth to the 7 hundredths.
6.874 + 0.1 = 6.974 and 6.974 + 0.01 = 6.984
d) Subtract 1, then subtract 2 tenths and then subtract 2 hundredths.
16.345 – 1 = 15.345
15.345 – 0.2 = 15.145
15.145 – 0.02 = 15.125

4. a) 12.431; *Sample response*:
This seems reasonable because the answer should be less than 4 + 9 = 13, but not a lot less.
b) 8.535; *Sample response*:
This seems reasonable because the answer should be about 12 – 4 = 8 and it is.

5. a) 4.532 kg **b)** 1.028 kg **c)** 0.468 kg

6. a) Estimate: about 15; Calculation: 17
b) Estimate: about 32; Calculation: 31.72
c) Estimate: about 18; Calculation: 18.72
d) Estimate: about 64; Calculation: 67.6

7. 2 × 1.52 = 3.04



Answers [Continued]

8. a) *Sample response*: 4.52 > 4. You have to multiply 4.52 by 3, so the perimeter is more than 4 × 3 = 12. **b)** 13.56 m **9.** 13.68 m	**10.** C and D *Sample response*: If you multiply by 0.01, the digit that moves to the hundredths place is the digit from the ones place and that only happened in C, not in A or B. If you multiply by 0.001, the digit that moves to the hundredths place is in the tens place and that happened in D. **11.** You could multiply by 0.1. 0.1 is one tenth. If you multiply by one tenth, you divide the whole into 10 parts and take one part. That is what dividing by 10 means.

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UNIT 4 Performance Task — Gold Coins

Gold coins are sometimes created to celebrate special events.
Bhutan created a gold coin to celebrate competing in the 1996 Olympics in Atlanta.

Coins can be measured in troy ounces and in kilograms. The Olympic coin shown here has a weight of 0.250 troy ounces and a mass of 0.008 kg. Its value is 5 Sertrum.

This chart shows the weight in troy ounces and the mass in kilograms of a set of gold coins.

	Coin A	Coin B	Coin C	Coin D	Coin E
Weight in troy ounces	1.00	0.50	0.25	0.10	0.05
Mass in kilograms	0.031	0.016	0.008	0.003	0.002

A. i) What is the combined weight (in troy ounces) of the five coins?
ii) What is the combined mass (in kilograms) of the five coins?

B. i) How much greater is the weight of the heaviest coin than of the lightest coin?
ii) How much greater is the mass of the heaviest coin than of the lightest coin?

C. i) What would be the weight of four sets of all five coins?
ii) What would be the mass of four sets of all five coins?

D. By what would you multiply the weight of the B coin to describe the weight of the E coin? How do you know?

E. Five gold coins have a total weight of 0.90 troy ounces. There may be more than one of the same coin.
i) Which coins are they? How do you know?
ii) Are there other solutions? Explain your thinking.
iii) How can you be sure you have all the solutions?

F. Write and solve your own word problem about a gold coin collection.

UNIT 4 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
5-B9 Addition and Subtraction of Whole Numbers and Decimals: 5 digits to 1000ths **5-B10** Decimals × Whole Numbers: simple products **5-B11** Mentally Multiply: whole numbers by 0.1, 0.01, 0.001	1 h	• Square Thousandths Grids (BLM) (optional) • Place Value Charts (BLM) (optional)

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. It could also be used as enrichment material for some students.

• You can assess performance on the task using the rubric on the next page. Make sure students realize that mass is used in these questions to describe the kilogram measures but that weight is used to describe the troy ounce measures.

Sample Solution

A. i) 1.9 troy ounces **ii)** 0.06 kg

B. i) 0.95 troy ounces **ii)** 0.029 kg

C. i) 4 × 1.9 = 7.6 troy ounces **ii)** 4 × 0.06 kg = 0.24 kg

D. 0.1; 0.05 is one tenth of 0.5, so you multiply by 0.1.

E. i) E, D, C, C, C
I needed five numbers that add to 0.9.
I noticed that the total weight of the B, C, D, and E coins was 0.9. But that was only four coins. I had to replace one coin with two other coins that have the same total weight. I replaced the B coin with two C coins.
ii) I found two other solutions: E, E, E, B, C and B, D, D, D, D.
iii) I used a system to make sure I had thought of everything.
- I started with five E coins. That would be too light.
- Then I tried four E coins. I would need another 0.7 troy ounces but that is not possible.
- Then I tried three E coins. I would need another 0.75 troy ounces using 2 coins. The only possibility was B and C but I had already found that answer (E, E, E, B, C).
- Then I tried two E coins. I would need another 0.8 troy ounces using three coins. If I used D, then I would need 0.7 troy ounces with two coins but that is not possible.
- Then I tried one E coin. I would need another 0.85 troy ounces using four coins. If I used D, I would need another 0.75 troy ounces using three coins but I had already found that answer (E, D, C, C, C).
- If I used no E coins, I realized I could use one B coin and I would need four other coins to add to 0.4 troy ounces. The only way to do that is with four D coins but I had already found that answer (B, D, D, D, D).
- I concluded that there are no other combinations.

F. Problem:
Two coins have a total mass that is 0.001 kg less than the total mass of three other coins.
What are the three coins?
Solution:
C, C, and D; The total mass of B and E is 0.018 kg. The total mass of C, C, and D is 0.019. 0.018 is 0.001 less than 0.019.

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UNIT 4 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Relates addition, subtraction, and multiplication to appropriate situations	Consistently and correctly relates situations to the appropriate operation to be performed	Usually relates situations to the appropriate operation to be performed	Sometimes relates situations to the appropriate operation to be performed	Rarely relates situations to the appropriate operation to be performed
Calculates decimal sums and differences	Correctly and efficiently calculates decimal sums and differences	Correctly calculates most decimal sums and differences	Correctly calculates some decimal sums and differences	Has difficulty calculating decimal sums and differences
Calculates products of whole numbers and decimals	Correctly and efficiently calculates products of whole numbers and decimals	Correctly calculates most products of whole numbers and decimals	Correctly calculates some products of whole numbers and decimals	Has difficulty calculating products of whole numbers and decimals
Creates and solves problems involving decimals	Creatively and insightfully creates and solves problems involving decimals	Correctly creates and solves problems involving decimals	Correctly solves most problems involving decimals	Has difficulty solving problems involving decimals

UNIT 4 Assessment Interview

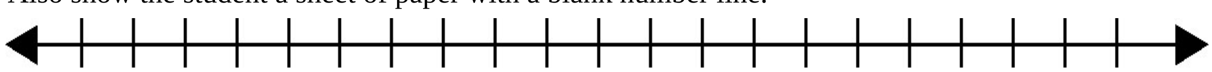
You may want to take the opportunity to interview selected students to assess their understanding of the work of this unit. Interviews are most effective when done with individual students, although pair and small group interviews are sometimes appropriate. The results can be used as formative assessment or as summative assessment data. As the students work, ask them to explain their thinking.

PART 1

• Allow the student to see a place value mat like the one in **lesson 4.1.2**.

Tens	Ones	Tenths	Hundredths	Thousandths

Also show the student a sheet of paper with a blank number line.



Ask the student to do the following:

• *Calculate* 4.154 + 3.087. *You can use either the place value chart or number line if you wish.*

• *Create your own decimal addition*: ■ . ■ ■ ■ + ■ . ■ ■ ■.
How might you use the chart or number line to find the sum?

• *Calculate* 4.021 – 1.857, *using either the place value chart or number line if you wish.*

• *Create your own decimal subtraction*: ■ . ■ ■ ■ – ■ . ■ ■ ■.
How might you use the chart or number line to find the difference?

PART 2

Ask the student to do the following:

• *Describe a problem that could be solved by calculating* 4 × 3.82.

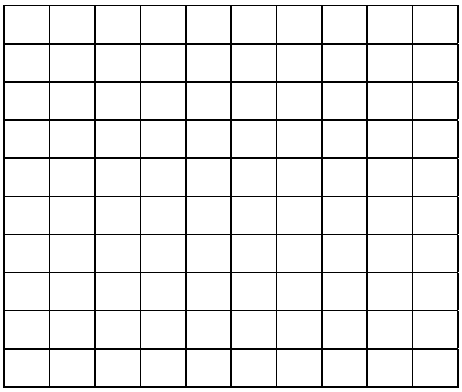
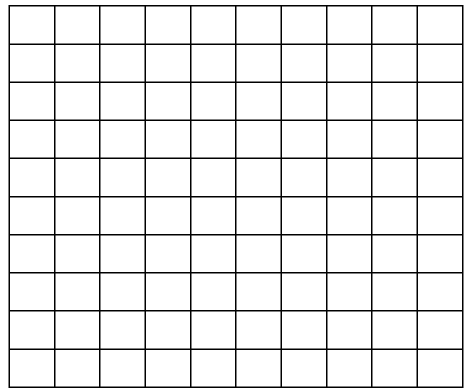
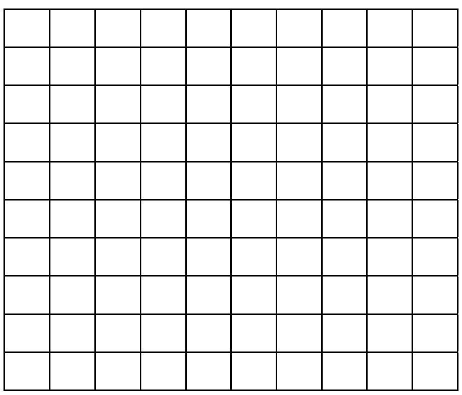
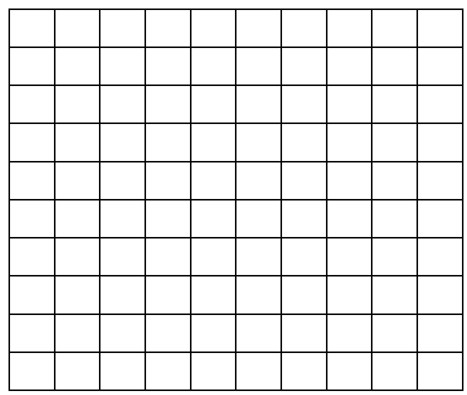
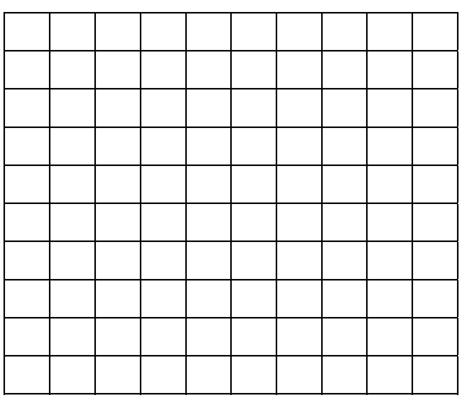
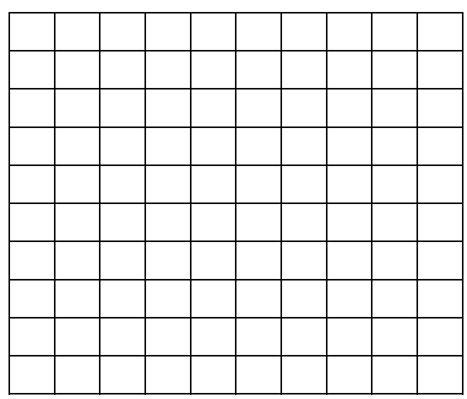
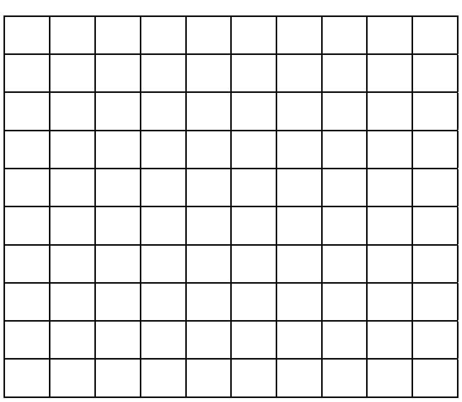
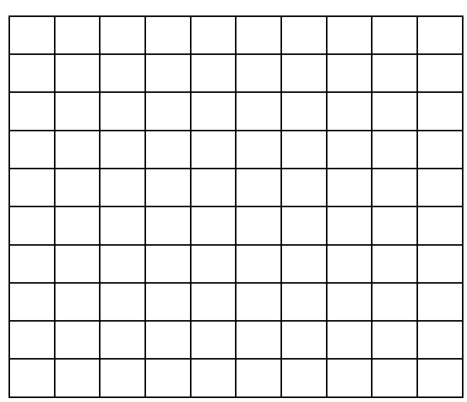
• *Estimate the product and then calculate it. Explain your thinking each time.*

PART 3

Ask the student:

• *Explain how you know that the products of* 5 × 3.62 *and* 5 × 3.22 *are 2 apart.* (5 × 3.62 = 18.1; 5 × 3.22 = 16.1)

BLM 1 Hundredths Grids

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UNIT 5 MEASUREMENT

UNIT 5 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 133** **TG p. 205**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• 12 crayons (for each pair of students) • 10 paper squares, each 10 cm by 10 cm (for each pair of students) • Rulers	All questions
Chapter 1 2-D Shapes				
5.1.1 EXPLORE: Polygons with the Same Perimeter (Essential) **SB p. 135** **TG p. 208**	**5-D1 Perimeter: polygons** • understand perimeter as the total distance around a figure • develop generalizations for the perimeter of regular polygons (e.g., for equilateral triangles, the perimeter is 3 times the side length, square is 4 times)	1 h	• Pieces of twine or string 48 cm long • Rulers	Observe and assess questions
5.1.2 EXPLORE: Perimeter of Rectangles (Optional) **SB p. 136** **TG p. 210**	**5-C4 Area and Perimeter: changing rectangle dimensions** • use models to discover patterns (e.g., same perimeter → longer length → shorter width) **5-D2 Perimeter and Area: rectangles and squares** • develop formulas meaningfully • understand that all squares with the same perimeter have the same area and vice versa • understand that rectangles with the same perimeter can have different areas	40 min	• Rulers • Grid Paper (1 cm by 1 cm) (BLM) (optional)	Observe and assess questions
5.1.3 EXPLORE: Area on a Grid (Essential) **SB p. 137** **TG p. 213**	**5-D3 Area: composite shapes, estimate and measure** • measure the area of composite shapes (include squares and half squares) on grids	1 h	• Square Dot Grid Paper (BLM)	Observe and assess questions
5.1.4 Area and Perimeter Relationships **SB p. 139** **TG p. 216**	**5-C4 Area and Perimeter: changing rectangle dimensions** • conclude, through investigation, that rectangles of the same area can have different perimeters • connect models to symbols: if one dimension is multiplied by a factor, the other must be divided by that factor (e.g., 24 × 5 = 12 × 10) **5-D2 Perimeter and Area: rectangles and squares** • develop from concrete to symbolic • develop formulas meaningfully • understand that rectangles with the same area can have different perimeters	1.5 h	• Rulers or metre sticks • Grid Paper (1 cm by 1 cm) (BLM) (optional)	Q1, 3, 5, 9
GAME: Cover the Grid (Optional) **SB p. 144** **TG p. 218**	Practise using the formula for the area of a rectangle in a game situation	20 min	• Grid Paper (1 cm by 1 cm) (BLM) • Coloured pencils or markers	N/A

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UNIT 5 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggeste d Pacing	Materials	Suggested Assessment
5.1.5 Area of Composite Shapes **SB p. 145** **TG p. 219**	**5-D3 Area: composite shapes, estimate and measure** • use grids to measure the area of composite shapes (include squares and half squares) • break up shapes into rectangles to area	1.5 h	None	Q1, 2, 3
CONNECTIONS: Unusual Ways to Measure Area (Optional) **SB p. 149** **TG p. 223**	Make a connection between probability and measurement and between areas of related shapes	30 min	• Small uniform pebbles or rice	N/A
Chapter 2 Angles				
5.2.1 EXPLORE: Measuring Angles (Essential) **SB p. 150** **TG p. 224**	**5-D4 Angles: estimate and measure** • explore angle measurement in non-standard units (as wedges) • understand that using a smaller wedge (unit) means using more wedges • link wedges to degrees (degree is just a very small wedge) • create and use an improvised protractor for 45, 90, 135, 180 degrees	1 h	• Fraction Circle Angle Units (BLM) • Paper strips (2 long and 2 short) • Scissors	Observe and assess questions
5.2.2 Comparing Angles to Special Angles **SB p. 153** **TG p. 227**	**5-D4 Angles: estimate and measure** • link wedges to degrees (degree is just a very small wedge) • create and use an improvised protractor for 45, 90, 135, 180 degrees **5-D5 Angles: estimate size** • estimate angles relative to common referents: 45, 90, 180 degrees (about the same as, more than, less than)	1.5 h	• Thin (tracing) paper or transparencies • Cut-out semicircle • Alphabet Letters (BLM)	Q1, 3, 7
Chapter 3 3-D Shapes and Metric Units				
5.3.1 Volume **SB p. 158** **TG p. 230**	**5-D6 Volume and Capacity: solve simple problems** • understand volume as the amount of space an object occupies or how much it takes to build it • develop a sense of size and referents for a cubic centimetre, cubic millimetre, cubic metre	1 h	• Metre sticks • Centimetre cubes	Q1, 2, 4, 5
5.3.2 Capacity **SB p. 161** **TG p. 233**	**5-D6 Volume and Capacity: solve simple problems** • understand capacity as how much a container is capable of holding • discover, through investigation, that 1 cm^3 holds 1 mL, and 1 dm^3 holds 1 L	1.5 h	• Calibrated measuring cups • Spoons of different sizes, cups and glasses of different sizes, another container such as a bucket • Water • Grid Paper (1 cm by 1 cm) (BLM) (optional) • Scissors	Q1, 3, 7

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5.3.3 Metric Units **SB p. 164** **TG p. 236**	**5-D7 SI Units: reinforce relationships among various SI units** • apply relationships among kilometres/ hectometres/ decametres/ metres/ decimetres/ centimetres/ millimetres, litres/ millilitres, and kilograms/ grams • use relationships to rename measures • apply referents for various measurement standards (e.g., 30 cm is like a ruler, 1 dm is about a small hand span) **5-C6 SI Measurement: pattern in changing units** • understand that a smaller measurement unit increases the number of those units and that a larger measurement unit decreases the number of those units • apply the above relationship to reason through conversions	1 h	• Place Value Chart II (BLM)	Q1, 3, 7
UNIT 5 Revision **SB p. 169** **TG p. 239**	Review the concepts and skills in the unit	2 h	• Rulers • Square Dot Grid Paper (BLM) • Improvised protractors • Place Value Chart II (BLM) (optional)	All questions
UNIT 5 Test **TG p. 241**	Assess the concepts and skills in the unit	1 h	• Rulers • Square Dot Grid Paper (BLM)	All questions
UNIT 5 Performance Task **TG p. 244**	Assess concepts and skills in the unit	1 h	• Rulers • Improvised protractors • Centimetre cubes	Rubric provided
UNIT 5 Blackline Masters **TG p. 246**	BLM 1 Square Dot Grid Paper BLM 2 Alphabet Letters (for **lesson 5.2.2**) BLM 3 Fraction Circle Angle Units Grid Paper (1 cm by 1 cm) on **page 105** in UNIT 2 Place Value Charts II on **page 164** in UNIT 3			

Math Background

• This measurement unit builds on students' previous experience with metric units, with perimeter and area, and with volume and capacity.

• Students will explore the connection between the area and the perimeter of simple shapes, learn to calculate the area of more complex shapes using a variety of methods, learn to measure angles using a variety of units, and explore volume and capacity and the relationship between metric units.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **lesson 5.1.3**, where they create shapes with given areas from other shapes, in the **Try This** in **lesson 5.1.5**, where they calculate the area of a complex shape, and in many of the exercises in **lesson 5.1.5**, where they use what they know about some measurements to calculate other measurements.

• They use communication in **question 9** in **lesson 5.1.4**, where they discuss what they need to know to calculate the perimeter and area of a rectangle, in **question 8** in **lesson 5.2.2**, where they reason about the lengths of the arms of an angle in terms of its size, and in **question 8** in **lesson 5.3.3**, where they describe how to change from one metric unit to another.

• They use reasoning in **question 4** in **lesson 5.1.4**, where they think about what operations are necessary for certain measurement calculations, and in **questions 2 and 5** in **lesson 5.3.3**, where they predict how the number of units required will change when a different measurement unit is used.

• They consider representation in **question 2** in **lesson 5.3.1**, where they represent a volume using different shapes, in **question 5** in **lesson 5.3.3**, where they figure out which unit to use to fit a certain measurement condition, and in **lesson 5.3.3**, where they relate metric prefixes to our place value system using a place value chart.

• Students use visualization skills in **lesson 5.1.1**, where they observe how the sides of a shape with a given perimeter get shorter when there are more sides, in **lesson 5.1.2** and **lesson 5.1.4,** where they learn that a shorter, fatter shape has more area than a longer, thinner shape with the same perimeter, in **lesson 5.2.2**, where they relate angles to benchmark angles, and in **question 7** in **lesson 5.3.2**, where they visualize the capacity of a box by looking at its net

• They make connections in **lesson 5.1.4,** where they learn how different formulas for the perimeter of a shape are connected, in the **Connections** feature, where they relate probability to measurement, in **lesson 5.2.1**, where they relate rotations to angle measurements using non-standard units, and in **questions 5 and 6** in **lesson 5.3.1**, where they relate different volume units and relate volume to area

Rationale for Teaching Approach

• This unit is divided into three chapters.

Chapter 1 addresses perimeter and area.

Chapter 2 focuses on angle measurement**.**

Chapter 3 addresses volume and capacity and the relationship between metric units.

• There are many **Explore** lessons in this unit because the best way to learn about measurement is to measure.

• The **Connections** section shows students a number of interesting ways to estimate the area of unusual shapes; several of the ways connect concepts of measurement to concepts of probability.

• The **Game** provides an opportunity to practice using the concept of area.

• Although there is some formula work in this unit, students should continue to use concrete materials to work with measurement at this level.

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Getting Started

Curriculum Outcomes	Outcome relevance
4 Millimetres, Centimetres, Decimetres, Metres, and Kilometres: estimate and measure **4** Volume: estimate and measure **4** Volume (rectangular prisms): estimate and measure with centimetres cubes (1 cm^3) **4** Area: estimate and measure (using cm^2 and m^2 symbols)	Students will find the work in the unit easier after they review what they know about measurement from Class IV.

Pacing	Materials	Prerequisites
1 h	• 12 crayons (for each pair of students) • 10 paper squares, each 10 cm by 10 cm (for each pair of students) • Rulers	• measuring length • measuring perimeter • measuring area with square units • describing the volume of a cube structure • millimetres, centimetres, decimetres, metres, and kilometres

Main Points to be Raised

Use What You Know

• The perimeter of a shape is the total distance around the shape.

• When you represent the perimeter of a shape as a single distance and then rearrange the shape using the same total distance, the new shape has the same perimeter as before, but it may have a different area.

• If two rectangles have the same perimeter, the rectangle that is most like a square has the greater area.

Skills You Will Need

• The area of a shape tells the number of square units the shape takes up.

• The volume of an object tells the number of cubes required to make it.

• It is useful to have referents for familiar metric units like the millimetre, centimetre, decimetre, metre, and kilometre.

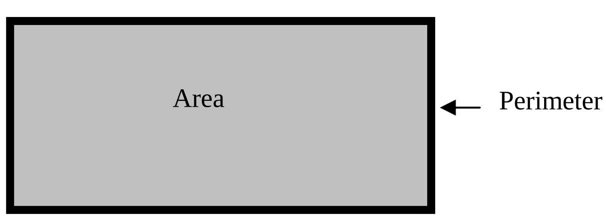
Use What You Know — Introducing the Unit

• Assign students to pairs. Before assigning the activity, make sure each pair of students has 12 crayons of similar length. The activity assumes 8 cm crayons, but any length is fine as long as the crayons are a uniform length.

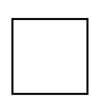
• Review the words *perimeter* and *area* so students recall them.

- The perimeter, or distance around, the rectangle below is the total length of the thick line segments.

- The area is the amount of grey space inside the rectangle, usually measured in square centimetres.



• You might remind students what a square centimetre looks like.


1 square centimetre

• The activity is designed to help students see directly how a long thin rectangle has less area than a square-like rectangle with the same perimeter.

• Make sure students realize that a square is a special type of rectangle.

• Tell students know that they can fold their paper squares in halves or in fourths to help them determine the area of the rectangle.

For example, the long rectangle shown below could be covered by about two half grey square units or by one whole grey square unit.

Observe students as they work. As they work, you might ask questions such as the following:

• *How did you know the perimeter was the same?* (Perimeter is distance around. The same crayons marked the distance around.)

• *Suppose you could instead use 24 half-crayons and arrange them into a rectangle that was 11 half-crayons long and 1 half-crayon wide. Do you think the area would be more or less than the areas you already found? Why?* (It would be less because it would be even longer and thinner. Longer, thinner rectangles had less area.)

• *What did you predict for* ***part C****? Why did you make that prediction?* (I predicted a greater area because it seems like I can fit more in when I make the rectangle wider.)

• *How would you describe to someone how to use a piece of string to outline a rectangle with a large area?* (Make it as much like a square as you can.)

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions.

• If necessary, review the term *volume* to describe the number of cubes it takes to build something. Review the metric abbreviations for millimetre, centimetre, decimetre, metre, and kilometre. You may also wish to show students or describe for them items of length 1 mm, 1 cm, 1 dm, 1 m, and 1 km.

• Students can work individually.

Answers

A. *Sample responses*:
i) 96 cm (assuming 1 crayon is 8 cm long — different crayon lengths will produce different answers)
ii) 96 cm; The perimeter is the distance around the outside of the shape, which is the length of the 12 crayons I measured in **part i)**.
iii) About 2 squares

B. *Sample responses*:
i) 96 cm; I used the same 12 crayons from **part A** and they are 96 cm long altogether.
ii) More; About 5 squares

C. *Sample response*:
Yes; A square that has 3 crayons on each side holds about 6 paper squares.

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1. a) 8 cm **b)** 6 cm **c)** 12 cm **d)** 14 cm **2.** *Sample responses*: **a)** About 11 cm^2 **b)** About 4 cm^2	**3. a)** 10 cubes **b)** 32 cubes **4.** *Sample responses*: **a)** Half the length of my fingernail **b)** The length of my thumb **c)** The length of my hair **d)** The length of this room **e)** The distance I walk to and from school each day

Supporting Students

Struggling students

• Most students will be successful with this concrete activity. You may have to make sure that students' shapes really are rectangles and that the squares do not overlap or leave gaps when students use their paper squares to measure area.

Enrichment

• Students might investigate arranging crayons into shapes other than rectangles to see that a longer, thinner shape always has less area than a more square or circular shape with the same perimeter, no matter what the shape.

5.1.1 EXPLORE: Polygons with the Same Perimeter

Curriculum Outcomes	Lesson Relevance
5-D1 Perimeter: polygons • understand perimeter as the total distance around a figure • develop generalizations for the perimeter of regular polygons (e.g., for equilateral triangles, the perimeter is 3 times the side length, square is 4 times)	This essential exploration will help students understand what the perimeter tells you about a shape. Perimeter is an important mathematical concept that is addressed in many classes beyond Class V.

Pacing	Materials	Prerequisites
1 h	• Pieces of twine or string 48 cm long • Rulers	• meaning of perimeter • concept of fractions • familiarity with the terms *equilateral triangle, hexagon,* and *circle*

Exploration

• Review with students the information provided in the box at the top of **page 135**. Make sure they understand what *perimeter* is. Review the terms *equilateral triangle* and *hexagon*. Make sure students understand why the perimeter of the square is 8 cm (2 cm + 2 cm + 2 cm + 2 cm). Using a piece of twine or string, demonstrate how you can bend the twine into different shapes with the same perimeter.

• Ask students to work on the exploration in pairs. Observe while students work. You might ask questions such as the following:

- *How many sides does your equilateral triangle have? How does knowing that number help you figure out the side length?* (Three sides; I have to divide 48 by 3 to get each side length.)

- *Why is dividing by 3 like finding* $\frac{1}{3}$ *?* ($\frac{1}{3}$ means that you take a whole length (the 48 cm), divide it into 3 equal parts, and take 1 of those parts.)

- *How did you make sure that only two sides were 12 cm?* (I subtracted 2 × 12 from 48. That left 24 cm. I decided to divide the 24 cm into 3 equal pieces of 8 cm each so there would be no more pieces that were 12 cm long.)

- *How did you arrange for one side to be 12 cm longer than another?* (I chose for one side to be 6 cm. Then I bent the twine and measured 18 cm so it would be 12 cm longer. I just bent the rest to make a shape.)

- *How did you make sure your hexagon was not regular?* (First I made a regular hexagon and then I shifted one vertex to make one side longer and another side shorter.)

- *How do you know that other shapes could be made?* (I know I could start with any one of the shapes I had already made and change it a little bit.)

Observe and Assess

As students work, notice the following:

• Do students recognize how fractions describe parts of the perimeter?

• Do they correctly relate the perimeter to the side length of regular (or equilateral) shapes?

• Do they measure correctly when they describe side lengths?

• Do they efficiently create the shapes that are required in **part D?**

• Do they show good reasoning when they talk about whether they have made all the possible shapes?

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Share and Reflect

After students have had sufficient time to work through the exploration, ask them:

• *For which parts do you think we all created the same shape? How do you know?*

• *For which parts do you think our shapes might be different? How do you know?*

• *If you know the perimeter of a shape and you know the shape is regular, can you find all the side lengths without measuring?*

• *If you know the perimeter of a shape and the shape is not regular, can you find all the side lengths without measuring?*

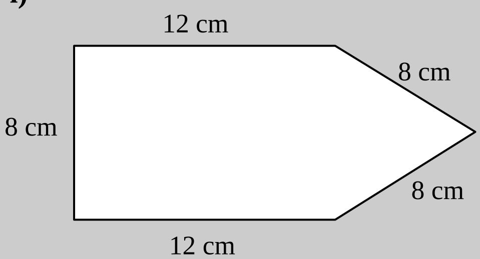
Answers

B. i) 16 cm **ii)** $\frac{1}{3}$

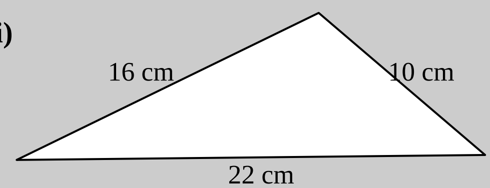
C. i) 8 cm **ii)** $\frac{1}{6}$

D. *Sample responses*:

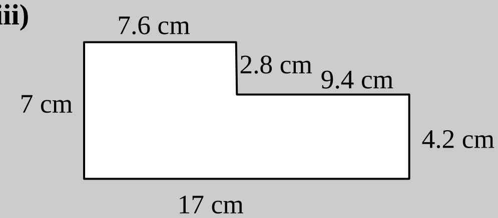
i)



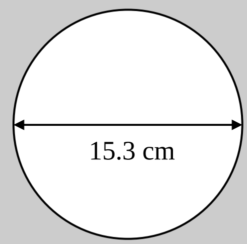
ii)



iii)



iv)



E. No; *Sample response*:
I could bend the shape many other ways to get as many shapes as I want.

Supporting Students

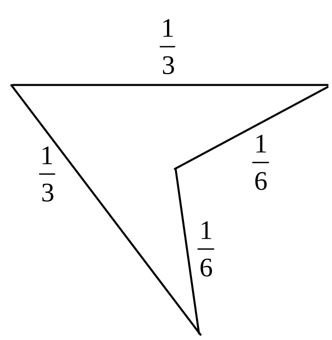
Struggling students

• **Part D i), ii), and iii)** may be difficult for some students. You could simplify the question by asking them to create other 4-sided, 6-sided, and circle shapes using 48 cm as the perimeter.

Enrichment

• Some students may wish to create shapes with a 48 cm perimeter for which they can use different fractions to describe the side lengths.

For example, they might be asked to create a 4-sided shape where two side lengths are each $\frac{1}{3}$ of the perimeter and then describe the other side lengths as fractions of the perimeter. One example is shown below.



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5.1.2 EXPLORE: Perimeter of Rectangles

Curriculum Outcomes	Lesson Relevance
5-C4 Area and Perimeter: changing rectangle dimensions • use models to discover patterns (e.g., same perimeter → longer length → shorter width) **5-D2 Perimeter and Area: rectangles and squares** • develop formulas meaningfully • understand that all squares with the same perimeter have the same area and vice versa • understand that rectangles with the same perimeter can have different areas	This optional exploration will help students realize that the perimeter of a rectangle is not tied to its area. The independence of linear and area measures is important both in solving real-world problems and in higher mathematics.

Pacing	Materials	Prerequisites
40 min	• Rulers • Grid Paper (1 cm by 1 cm) (BLM) (optional)	• measuring perimeter and area of rectangles • drawing line segments of given lengths

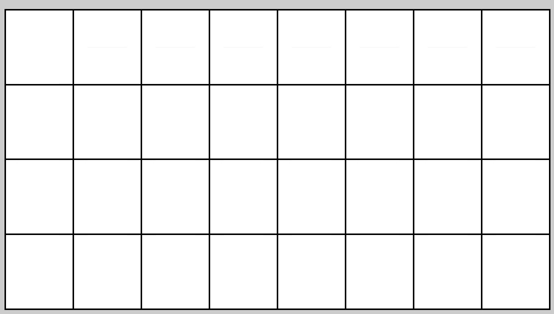
Exploration

• Model how to draw a rectangle with given dimensions, for example, a 4 cm-by-8 cm rectangle.

• Review the meaning of the terms *area* and *perimeter*.

- Demonstrate how you can find the perimeter of a 4 cm-by-8 cm rectangle by adding 4 + 8 + 4 + 8.

- Demonstrate how you can find the area by drawing the rectangle on centimetre grid paper (4 rows of 8 squares in the rectangle) or by creating 4 rows of 8 squares of size 1 cm by 1 cm inside the rectangle.



• Make sure students know that the term *dimensions* (of a rectangle) refers to its length and width.

• Ask students to work in pairs.

Observe while students work. While they work, you might ask questions such as the following:

• *How do you know that one dimension of the longer, thinner rectangle is more than 10?* (It has to be longer than the 10 cm-by-4 cm rectangle.)

• *How do you know the other dimension is less than 4?* (If it were not, the perimeter could not be the same.)

• *What dimensions did you use?* (I used 12 cm by 2 cm.)

• *How did you calculate the perimeter?* (I added the length twice and the width twice.)

• *How did you calculate the area?* (I drew 1 cm squares inside of it and counted how many rows and how many were in each row. Then I multiplied.)

• *How do you know that you need to know only the length and width to find the perimeter or area?* (To get the perimeter, I need the sum of four sides. Two sides are the same as the length and the other two sides are the same as the width. To get the area, I need to know how many rows there are and how many squares are in a row. That is what the length and width tell me.)

• *How will you create squares with a given perimeter?* (I chose a perimeter of 16. I knew that I had to divide by 4 to get the side length of the square.)

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Observe and Assess

As students work, notice the following:

• Do students correctly calculate perimeter and area?

• Do they understand the role of the length and width in determining perimeter and area?

• Can they generalize about the relationship between perimeter and area for either squares or rectangles with the same perimeter?

Share and Reflect

After students have had sufficient time to work through the exploration, discuss their results with them by asking these questions.

• *Did all your rectangles with the same perimeter have the same area?*

• *Which rectangles had the greatest area?*

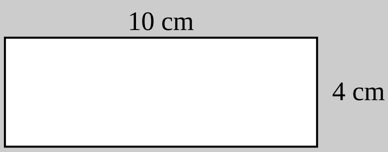
• *Can a square and a non-square rectangle have the same perimeter?*

• *Suppose two squares each have a perimeter of 20 cm. What are the side lengths of each of the squares? How do you know? What are the areas of each of those squares?*

• *Suppose two rectangles each have a perimeter of 20 cm. Do you know the side lengths of each rectangle?*

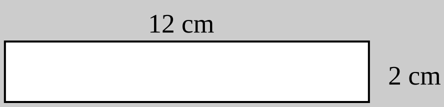
Answers

A. i)



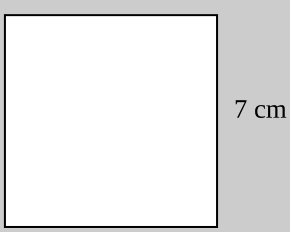
Perimeter: 10 + 4 + 10 + 4 = 28 cm
Because the opposite sides are the same lengths,
I know the two missing lengths are 10 cm and 4 cm.

ii) *Sample response*:



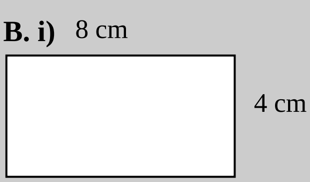
It is 2 cm longer and 2 cm narrower (less wide).

iii)



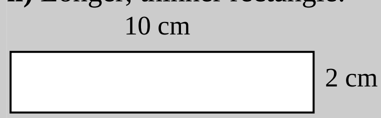
It is 3 cm shorter and 3 cm wider.
iv) 40 cm^2, 24 cm^2, and 49 cm^2
Rectangle B, Rectangle A, Square

B. i)

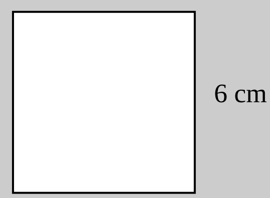


Perimeter: 8 + 4 + 8 + 4 = 24 cm

ii) Longer, thinner rectangle:



It is 2 cm longer and 2 cm narrower.
iii) Square



It is 2 cm thinner and 2 cm wider.
iv) 32 cm^2, 20 cm^2, and 36 cm^2
10 cm-by-2 cm rectangle, 8 cm-by-4 cm rectangle, square

C. i) *Sample response*:
The opposite sides are equal, so when you know 2 of the sides, you really know all 4. You have to add the 4 sides to get the perimeter.
ii) *Sample response*:
Add the length and width and multiply by 2.
iii) Multiply the side length by 4.

D. Only the second statement is true.
Sample response:
• The second statement is true. I saw in parts A, B, and C that rectangles that are not squares can have the same perimeter but different areas.
• The first statement is not true. If the perimeters of two squares are equal, then the side lengths are equal (because the side length is the perimeter divided by 4). If the side lengths are equal, then the squares are identical so the areas are the same.

Supporting Students

Struggling students

• Some students may need you to give more specific directions.

For example, instead of saying a longer, thinner rectangle in **part A ii)**, you might tell them to make the length 12 cm and the width 2 cm. The students should recognize that the perimeter has not changed.

• You may also suggest that students draw their rectangles on grid paper so that it is easier to calculate the area and the perimeter.

• For **part D**, you may need to be specific by suggesting that students use a particular perimeter, such as 40 cm.

Enrichment

• Students could investigate different rectangles with the same area to figure out how the shape of the rectangle affects the perimeter.

For example, by comparing 4 × 5, 2 × 10, and 1 × 20 rectangles, each with area 20, they can learn that a longer, thinner shapes has a greater perimeter.

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5.1.3 EXPLORE: Area on a Grid

Curriculum Outcomes	Lesson Relevance
5-D3 Area: composite shapes, estimate and measure • measure the area of composite shapes (include squares and half squares) on grids	This essential exploration will help students understand that they can determine the area of a shape even if the shape is not made up of full squares.

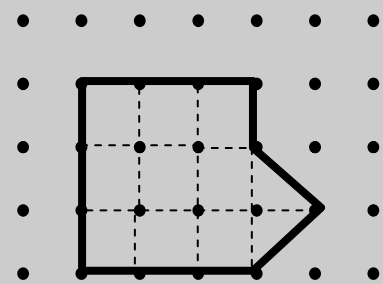
Pacing	Materials	Prerequisites
1 h	• Square Dot Grid Paper (BLM)	• area • recognizing halves of rectangles

Exploration

• Draw a 3-by-3 square on square dot grid paper and ask students what the area is (9 square units).

• Then move one vertex out as shown to the right:

Show that 2 extra half square units have been added to the original area of 9 to make an area of 10.



• Have students look at the drawing in the text on **page 137** and explain how they know the area is 4 square units.

• Ask students to work alone or in pairs.

Observe while students work. While they work, you might ask questions such as the following:

• *How did you know that the first area in* ***part A*** *was greater than 9?* (I saw a 3-by-3 square inside the shape.)

• *How did you know that the triangle in the top right corner added only* $\frac{1}{2}$ *square unit to the shape but that the triangle on the top left added 1 square unit to the shape?* (The triangle on the left is made up of two half squares, which is a whole square altogether. The triangle on the right is just one half square.)

• *How did you know how much area you wanted to add?* (It was already $10\frac{1}{2}$ square units, so I wanted to add only $\frac{1}{2}$ square unit.)

• *Why did you move that vertex inward to get* $9\frac{1}{2}$ *square units?* (I needed less area.)

• *Why did you start with a 4-by-4 square to get* $15\frac{1}{2}$ *square units in area?* (I knew it would be easy to take away a half square at the corner.)

Observe and Assess

As students work, notice the following:

• Do students correctly calculate the areas, using both whole and half squares?

• Do they correctly predict whether the vertex should be moved out or in to change the areas?

• Do they recognize when reasoning, rather than calculation, is all that is required to answer a question?

• Do they use more than one strategy for calculating areas?

• Are they efficient in calculating areas, for example, using symmetry when it is possible?

Share and Reflect

After students have had sufficient time to work through the exploration, discuss their work with them by asking these questions.

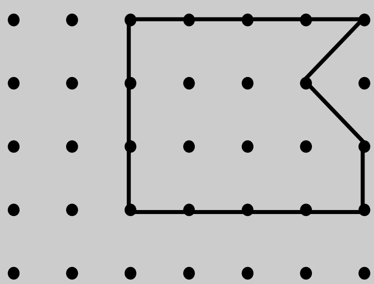
- *Which areas were easiest for you to find? Why?*
- *Why would it be easy to create a shape with an area of* $9\frac{1}{2}$ *square units?*
- *Why do you sometimes add only a half square unit and sometimes more when you move one vertex of a shape?*

Answers

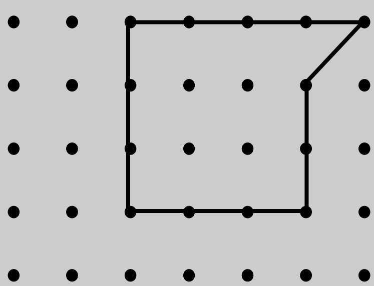
A. i) $10\frac{1}{2}$ square units
ii) 9 square units
iii) 9 square units

B. *Sample responses*:

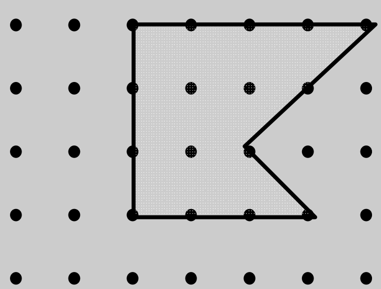
i)



ii)

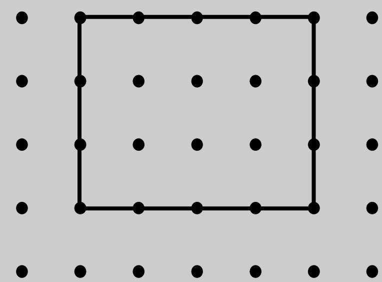


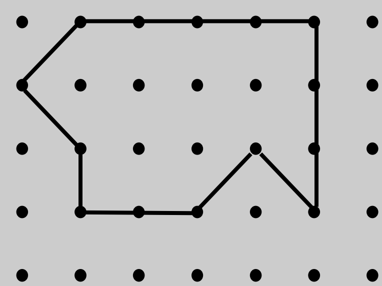
iii)



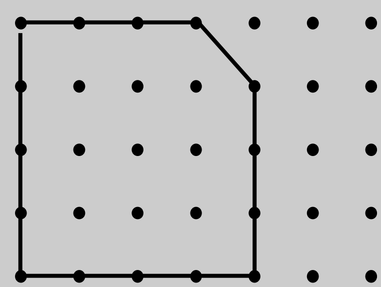
C. *Sample responses*:

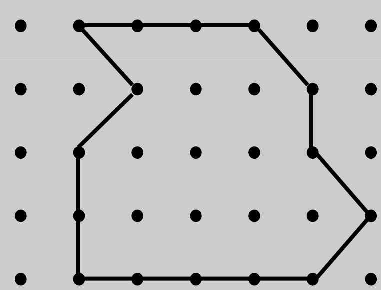
i)





ii)





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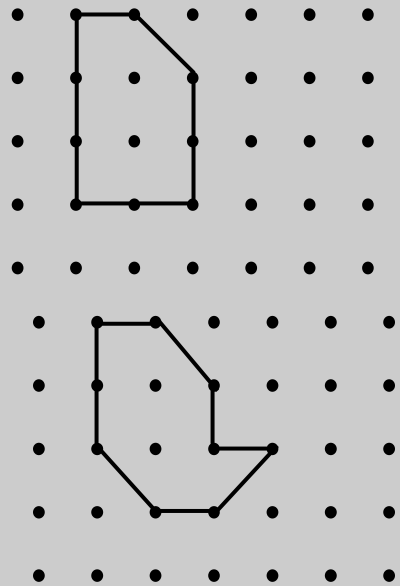
C. iii)



D. *Sample response*:

First way:

I counted $5\frac{1}{2}$ squares on the left half of the shape.
The shape is the same on the left as on the right so
I added $5\frac{1}{2} + 5\frac{1}{2}$ to get 10 + 1 = 11 square units.

Second way:

I imagined a 3-by-4 rectangle, which has an area of 12 square units, with two corners missing.
Each corner is a half a square unit.
So the area is $12 - \frac{1}{2} - \frac{1}{2} = 11$ square units.

Supporting Students

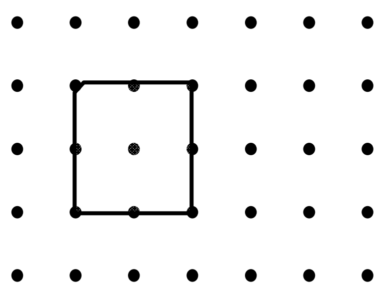
Struggling students

• If students are struggling with creating shapes of a certain area, you might simply have them create any shape they wish that uses some half squares and some whole squares. They should find the areas of those shapes.

Enrichment

• Some students might try to find all the possible sizes of shapes that result from moving only one vertex of a given shape.

For example, if you start with the shape below and move only one vertex, you can make shapes with areas of 2, 5, 6, $6\frac{1}{2}$, 7, $7\frac{1}{2}$, 8, and so on.



5.1.4 Area and Perimeter Relationships

Curriculum Outcomes	Outcome relevance
5-C4 Area and Perimeter: changing rectangle dimensions • conclude, through investigation, that rectangles of the same area can have different perimeters • connect models to symbols: if one dimension is multiplied by a factor, the other must be divided by that factor (e.g., $24 \times 5 = 12 \times 10$) **5-D2 Perimeter and Area: rectangles and squares** • develop from concrete to symbolic • develop formulas meaningfully • understand that rectangles with the same area can have different perimeters	Many real-world problems require students to be able to calculate the area and/or the perimeter of a rectangle. It is important that students learn how to use the dimensions of the rectangle to find these other values.

Pacing	Materials	Prerequisites
1.5 h	• Rulers or metre sticks • Grid Paper (1 cm by 1 cm) (BLM) (optional)	• perimeter and area

Main Points to be Raised

• The area of a rectangle is the number of square units that cover it.

• Rectangles with the same area can have different perimeters.

• The formula for the perimeter of a rectangle is $P = (l + w) \times 2$. There are other equivalent forms of this formula.

• The formula for the area of a rectangle is $P = l \times w$.

• Two rectangles can only have the same perimeter if the sum of the length and the width of one is the same as the sum of the length and the width of the other.

• Two rectangles can only have the same area if the product of the length and the width of one is the same as the product of the length and the width of the other.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. You might mention that the height of a doorknob is usually about 1 m and that the height of an average adult is between 1.5 m and 1.8 m. Observe while students work. You might ask questions such as the following:

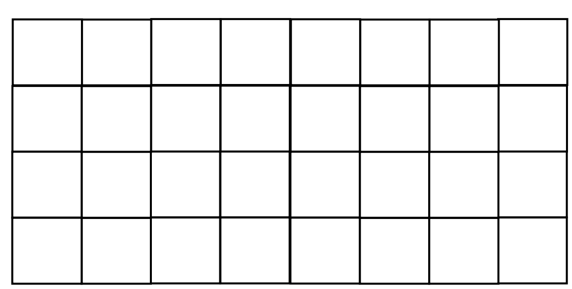
• *Why do you think the door is more than 2 m high?* (It looks like the door is more than double the height from the floor to the doorknob. The doorknob is 1 m from the floor.)

• *How wide do you think the door is? Why?* (It looks like it is about 1 m wide; it is about as wide as the height from the floor to the doorknob.)

• *What values will you use to calculate the perimeter?* (I have to add the height of the door twice and the width of the door twice. I think that is about 2 m + 2 m + 1 m + 1 m = 6 m.)

The Exposition — Presenting the Main Ideas

• Draw a rectangle on the board. Mark it as 4 units by 8 units. Draw lines across and down to separate it into 32 unit squares and mark the length and width (or breadth).



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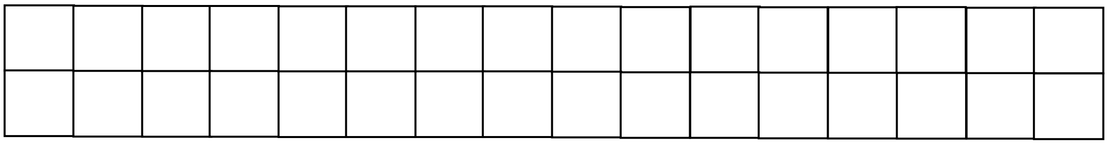
• Ask students what the area of the rectangle is (32 square units) and what the perimeter is (24 units).

• Ask how you could rearrange the 32 squares into a different rectangle.

For example, you could make a 16-by-2 rectangle:



• Ask why the area is still 32 square units. Ask for the perimeter of this new rectangle (36 units). Talk about how the perimeter changed even though the area did not change. This also happened in **lesson 5.1.2**.

• Tell students you are thinking about another rectangle that is 5 cm by 9 cm, but do not draw it. Ask them to figure out how they would calculate the perimeter and the area. After they offer suggestions, make sure they observe that to calculate the perimeter, they must add the length and width twice, but to calculate the area, they multiply the length and the width.

• Talk about why these operations are performed.

<u>To calculate the perimeter</u>, you add the four sides. Two sides are the length and two sides are the width. You can add them in any order. You could add both widths ($2w$) and then both lengths ($2l$) to get $2l + 2w$. Or, you could add one length and one width and double it to get $2(l + w)$.

[Note: Although the perimeter formula is often written as $P = 2 \times (l + w)$, it may be better to write it as $(l + w) \times 2$ because this is the order in which students perform the calculation.]

<u>To calculate the area</u>, you multiply the length and width. The number of rows of unit squares inside is the width and the number of squares in each row is the length. The area is $l \times w$.

• Tell students you are looking for a rectangle with area 40 cm^2. Ask them what its dimensions might be.

For example, it could be 8 cm by 5 cm or 4 cm by 10 cm.

Then tell them you are looking for a rectangle with perimeter 40 cm. Ask what its dimensions might be.

For example, it could be 8 cm by 12 cm, since 8 + 12 = 20.

• Lead students through the exposition to review the concepts you have taught.

Revisiting the Try This

B. Make sure students record the length and the width of the door before they apply the formulas.

Using the Examples

• Describe the problem in the example to the students. Ask them to solve it and then to check their work with the solution and thinking in the student text.

Practising and Applying

Teaching points and tips

Q 2: Make sure students recognize that the length and width must add to 50 cm, but that the area cannot be 400 cm^2.

Q 3: Make sure students recognize that the area must be 400 cm^2, but the perimeter cannot be 50 cm.

Q 4: Some students may suggest that you also multiply with perimeter, since you double the sum of the length and width. This is true, but point out that addition is also involved.

Q 5: It would be helpful to provide a referent for students for 1 square metre, perhaps the bottom half of the classroom door or a particular section of the floor.

Q 6: This item may not be familiar to students, but you can introduce it as an item that is often sold to tourists in Bhutan.

Q 7 and 8: These questions are designed to allow students to summarize what they have learned about the relationship between area and perimeter.

Q 9: This question helps students understand that the value of a formula is that it allows you to generalize.

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Common errors

• Many students forget to double after they calculate the perimeter. Or, if they are given the perimeter, they find a length and width that total to the entire perimeter rather than to half of it. Encourage students to sketch the shapes to test their results.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can calculate the perimeter and area of a rectangle
Question 3	to see if students realize that a more square rectangle has less perimeter for a given area
Question 5	to see if students can estimate area in square metres
Question 9	to see if students can communicate about finding perimeters and areas of rectangles

Answers

A. *Sample responses*: **i)** About 220 cm by 100 cm **ii)** More than 5 m because 220 + 220 + 100 + 100 = 640 cm. That is over 6 m because 100 cm = 1 m.	**B.** *Sample responses*: **i)** 220 cm × 100 cm = 22,000 cm^2 **ii)** 220 + 100 = 320 and 2 × 320 = 640 cm
1. a) Area = 400 cm^2 Perimeter = 100 cm **b)** Area = 440 cm^2 Perimeter = 84 cm **2.** *Sample response*: 25 cm by 25 cm **3.** *Sample response*: 10 cm by 44 cm [**4.** *Sample response*: The perimeter is the total length, so you add all the side lengths. The area is how many rows of squares are in a rectangle. Since you add the same number of rows many times, you are multiplying.] **5.** *Sample response*: About 24 m^2 **6. a)** 1200 cm^2 **b)** *Sample response*: 30 cm by 40 cm	**7.** The square; [*Sample response*: I tried two possible pairs of rectangles and squares with the same area. I used a 4 cm-by-4 cm square and a 2 cm-by-8 cm rectangle. The square had a shorter perimeter. I did it again with a 10 cm-by-10 cm square and a 4 cm-by-25 cm rectangle. The square had a shorter perimeter again.] **8.** The rectangle; [*Sample response*: I tried a two possible pairs of rectangles and squares with the same perimeter. I used a 4 cm-by-4 cm square and a 7 cm-by-1 cm rectangle. The rectangle had less area. I used a 10 cm-by-10 cm square and a 15 cm-by-5 cm rectangle. The rectangle had less area again.] [**9.** I can use formulas to calculate the perimeter and the area.]

Supporting Students

Struggling students

• Struggling students might find **question 4** difficult because it is abstract. You might simplify the question by asking what operations students perform to calculate the perimeter and area of a rectangle and why.

GAME: Cover the Grid

• This game is designed to allow students to practise calculating areas of rectangles.

• After students have played the game at least once, talk about possible strategies for winning.

For example, creating rectangles that will obstruct their opponent from being able to create rectangles, by breaking the available area into many small rectangles.

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5.1.5 Area of Composite Shapes

Curriculum Outcomes	Outcome relevance
5-D3 Area: composite shapes, estimate and measure • use grids to measure the area of composite shapes (include squares and half squares) • break up shapes into rectangles to area	An important aspect of spatial awareness is seeing how a shape can be broken up into simpler shapes. In this case, this skill is used to simplify the calculation of areas.

Pacing	Materials	Prerequisites
1.5 h	None	• formula for the area of a rectangle • interpreting marks to show congruent sides of a shape

Main Points to be Raised

• A composite shape is a shape that can be broken up into simpler shapes. You can find the area of the composite shape by adding the areas of the simpler shapes.

• Sometimes you have to use given measurements of a shape to determine measurements that are not given. You can do this by using properties of the shapes.

For example, you can use the concept that opposite sides of a rectangle are equal in length.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *What would the area be if it were a complete rectangle?* (54 cm × 75 cm = 4050 cm^2)

• *Do you think the area you must find is more than half or less than half of the rectangle? Why?* (More than half; If I drew a diagonal of the rectangle, everything below the diagonal and some more above it would be included in the shape.)

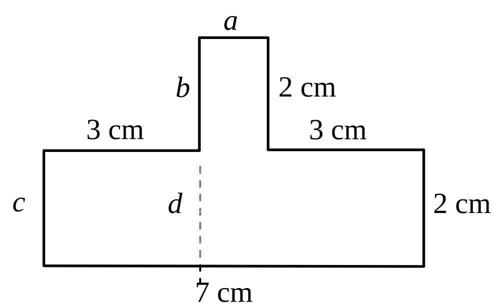
• *How did you divide the shape to make it easier to find the area?* (I divided it into 3 rectangles, one on the left, one in the middle, and one on the right. I decided that each was one third of the whole 75 cm width.)

The Exposition — Presenting the Main Ideas

• On the board draw a shape like the first shape in the exposition. Ask students if they can see the two rectangles that make up the shape. Ask them to describe the length and width of each rectangle. Discuss how you could calculate the area of the full shape by adding the areas of the two rectangles.

• Draw the second shape in the exposition. Ask students to figure out the length of each part of the perimeter of the shape that is not already marked, i.e., *a*, *b*, *c*, and *d* in the diagram below. Discuss that *c* must be 2 cm because the bottom shape is a rectangle, *b* must be 2 cm because the top part is a rectangle, and *a* must be 7 – 3 – 3 = 1 cm.

• Then show how you can divide the shape into three rectangles and calculate the area of each. The height of the middle rectangle is 2 cm + 2 cm = 4 cm.



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Revisiting the Try This

B. Encourage students to break up the staircase into three rectangles and combine the areas of the three rectangles.

Using the Examples

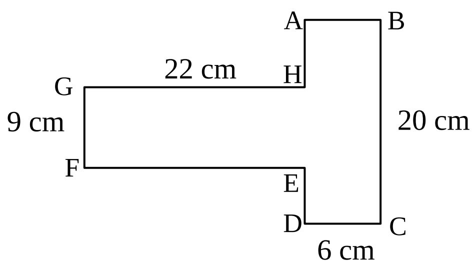
• Work through **example 1** with the students. Then ask pairs of students to read through **example 2** to make sure they understand how to calculate the area of a composite shape when only some of the dimensions are given.

Practising and Applying

Teaching points and tips

Q 1: You might have students work in pairs on these shapes because they are a bit difficult. You might also suggest that students label the vertices and refer to the side lengths using the names.

For example, you might label **part a)** as shown below.



The given lengths are GH = 22, BC = 20, and CD = 6.

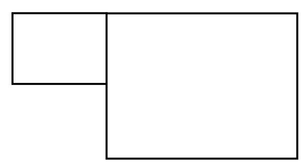
AB must be 6 since ABCD is a rectangle. DE + AH = 11 since 20 – 9. FE = GH, so FE must be 22 cm.

For **part d)**, remind students that the tick marks indicate equal side lengths. Encourage them to build a rectangle around the given shape and to take away any excess area.

Q 2: It is important for students to understand that the area of a shape does not change when the pieces of the shape are rearranged. In the diagram here, the small square can be on the left bottom or on the centre top of the other square, but the area is be the same.

Q 3: Students might benefit by recording some of the multiplication facts they know so they can combine them.

For example, to make a shape with a combined area of 30, they could think, "3 × 2 = 6 and 4 × 6 = 24, so I need to put those two rectangles together".



Q4: It might be useful for students to label the vertices in order to more easily refer to the side lengths.

Common errors

• Students may have difficulty determining the missing side lengths. Encourage them to mark everything they know, in sequence, when they begin. Help them focus on finding the parts they must add to determine a longer length or on finding the parts they must subtract to determine a shorter length.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can divide a composite shape appropriately and figure out missing side lengths to calculate its area
Question 2	to see if students recognize that area does not change when the parts of a shape are rearranged
Question 3	to see if students can create a composite shape with a given area

Answers

A. *Sample response*:
About 2200 cm^2
It looked like it was a bit more than half of a 54 cm-by-75 cm rectangle.
I took half of 54 × 75 = 4050, which is 2025.
Since it was a bit more than half, I estimated 2200 cm^2.

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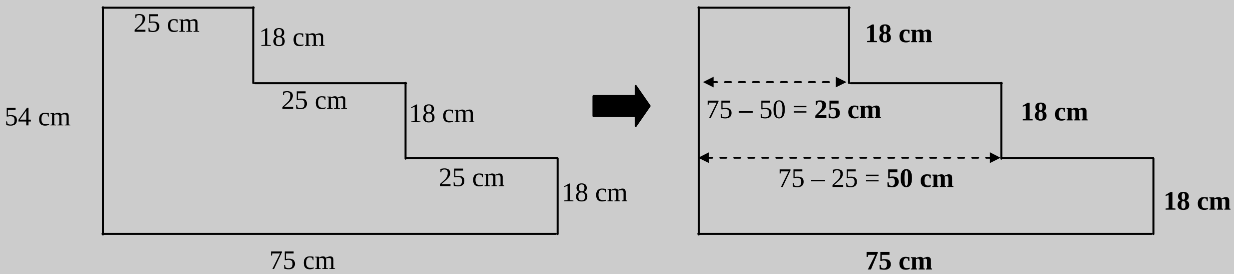
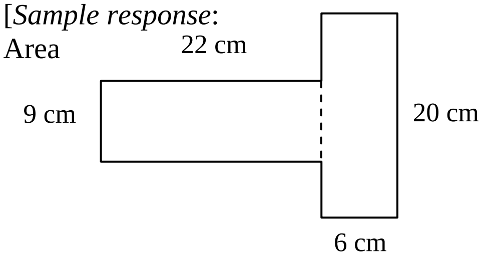
B. 75 ÷ 3 = 25 cm, so the top of each step is 25 cm.
54 ÷ 3 = 18 cm, so each riser is 18 cm.



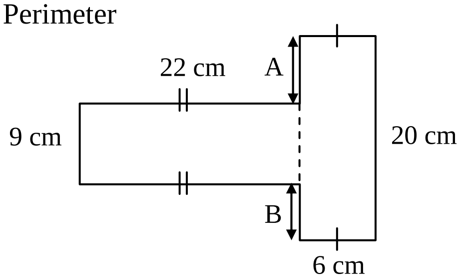
Area = 25 × 18 + 50 × 18 + 75 × 18
= 450 + 900 + 1350 = 2700 cm^2

1. a) Area = 318 cm^2 Perimeter = 96 cm
[*Sample response*:
Area



Area = 22 × 9 + 20 × 6 = 318 cm^2

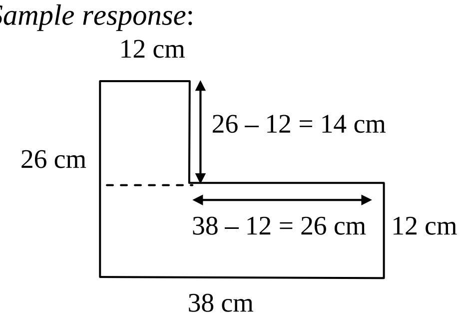
Perimeter



A + B = 20 – 9 = 11 cm

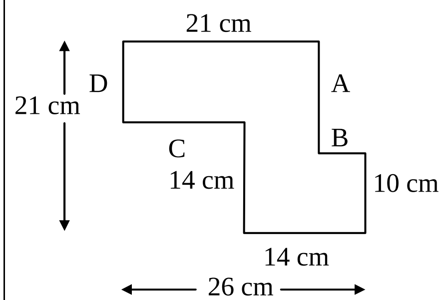
Perimeter = 20 + 6 + 11 + 22 + 9 + 22 + 6
= 96 cm]

b) Area = 624 cm^2 Perimeter = 128 cm
[*Sample response*:



Area = 12 × 14 + 12 × 38 = 12 × 52 = 624 cm^2
Perimeter = 12 + 14 + 26 + 12 + 38 + 26
= 128 cm]

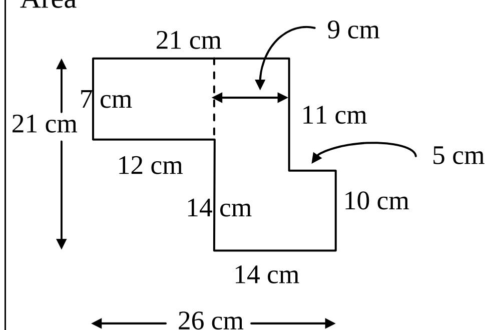
c) Area = 323 cm^2 Perimeter = 94 cm
[*Sample response*:
Perimeter
I figured out the missing dimensions in the order A, B, C, D.



A is 21 – 10 = 11 cm
B is 26 – 21 = 5 cm
C is 26 – 14 = 12 cm
D is 21 – 14 = 7 cm

Perimeter = 21 + 11 + 5 + 10 + 14 + 14 + 12 + 7 = 94 cm

Area



9 cm =
21 cm – 12 cm

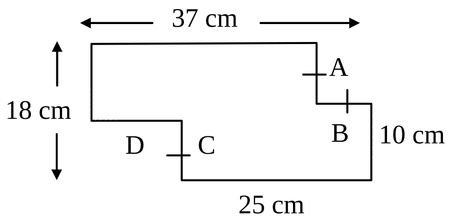
Area = 7 × 12 + 21 × 9 + 5 × 10 = 323 cm^2]

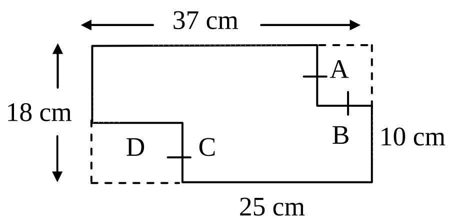
Answers [Continued]

1. d) Area = 506 cm^2 Perimeter = 118 cm

[*Sample response*:

A is 18 – 10 = 8 cm, so B and C = 8 cm
D is 37 – 25 = 12 cm



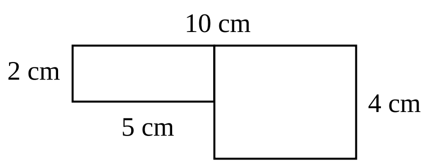


Area = 37 × 18 – 8 × 8 – 8 × 12 = 506 cm^2
Perimeter = 29 + 8 + 8 + 10 + 25 + 8 + 12 + 10
= 110 cm]

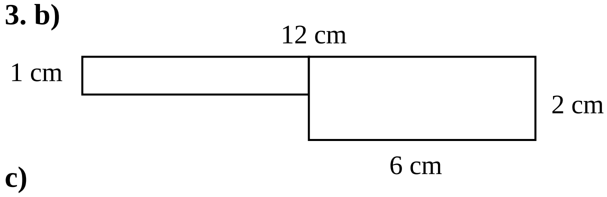
[**2.** Each shape is made up of the same two rectangles but they are put together different ways.]

3. *Sample responses*:

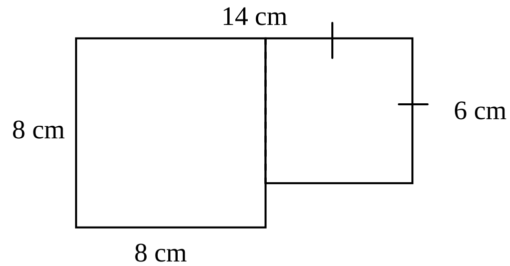
a)



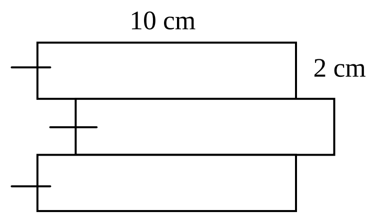
3. b)



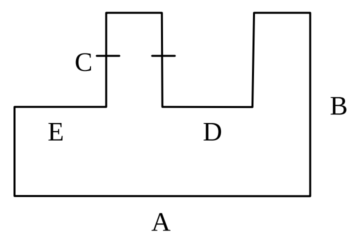
c)



d)



4. *Sample response*:



[Area =
Area of A × B – Area of C × D – Area of C × E]

Supporting Students

Struggling students

• You may wish to pair struggling students with stronger students to work on **question 1**. **Parts a), c), and d)**, are more difficult than the other questions.

Enrichment

• Invite students to create composite shapes indicating only some of the dimensions and let them challenge other students to figure out the missing dimensions, the perimeter, and the area. They should make the shapes by joining rectangles.

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CONNECTIONS: Unusual Ways to Measure Area

• Strategy 1 and strategy 2 are linked to probability. Both strategies are based on the assumption that, in a random irregular shape, the number of squares that are more than half covered will balance out against the number of squares that are less than half covered.

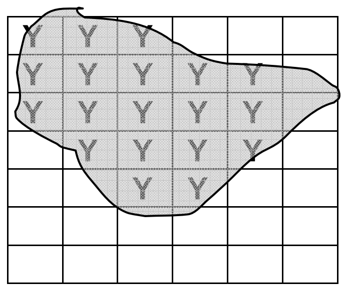
• Strategy 2 is a variation of strategy 1, where the mean is used to balance the number of partly covered squares with the entirely covered squares so that the estimated area is close to the actual area.

• Strategy 3 is a different idea and more concrete. It is based on students' previous experience that the same area can be rearranged into a different shape without changing the value of the area.

Answers

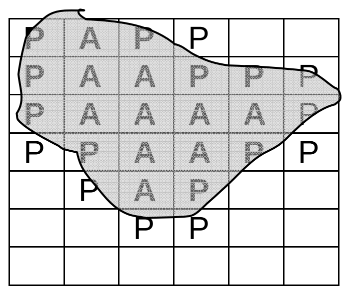
1. *Strategy 1*

Area = 19 square units



Strategy 2

Area = (17 + 10) ÷ 2 = 13.5 square units



Strategy 3

I covered the shape evenly with grains of rice. I made sure the rice was just one layer. Then I moved them onto a grid to form a rectangle. Again, I did it evenly and in one layer. It looked like about 16 square units.

5.2.1 EXPLORE: Measuring Angles

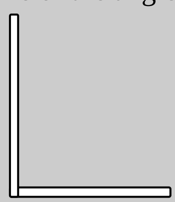
Curriculum Outcomes	Lesson Relevance
5-D4 Angles: estimate and measure • explore angle measurement in non-standard units (as wedges) • understand that using a smaller wedge (unit) means using more wedges • link wedges to degrees (degree is just a very small wedge) • create and use an improvised protractor for 45, 90, 135, 180 degrees	This essential exploration gives students insight into angle measurement. In order for measurement in degrees to be meaningful, students must understand that different units could be used to measure angles.

Pacing	Materials	Prerequisites
1 h	• Fraction Circle Angle Units (BLM) • Paper strips (2 long and 2 short) • Scissors	• meaning of fractions of a whole

Exploration

• Hold up two long strips of paper with one on top of the other. Show students how you form an angle by turning one strip at the vertex. Explain that the size of the angle is described by how much you turn.

For example, show a quarter of a circle turn.



Then show a turn that is less than a quarter turn and another that is one more than a quarter turn.

• Create the same angles again using short strips. Explain to students that the size of the angle tells you how much you turn and not the lengths of the strips. An angle with short arms can have the same measure as an angle with long arms. Draw two such angles on the board, for example:

• Remind students that one way to measure the length of an object is to lay many copies of a small length (a unit) against it. Tell them the same is true with angles; angles can be measured with units. Have students open the text to **page 150** near the bottom of the exposition. Help them see that Angle C measures 2 larger units (in the middle picture), but 4 smaller units (in the right picture).

• Distribute the Fraction Circle Angle Units (BLM) and scissors. If scissors are not available, students will need to trace the circle section angles.

• Demonstrate how to fit the sections into an angle to measure it.

• Ask students to work in pairs on the exploration.

Observe while students work. While they work, you might ask questions such as the following:

• *Why did you think you could fit in more of the O pieces than the N pieces?* (The O pieces are smaller so more can fit in.)

• *Why do you think you can fit in twice as many N pieces as M pieces?* (The N piece is half the size of the M piece. If it takes two Ns to make an M and you need a certain number of M pieces to make an angle, you would need twice as many N pieces to make the same angle.)

• *Which unit would you choose to measure the angle in* ***part B iii)****? Why?* (I would use the M angle so I would not have to use so many units.)

• *Why can you not know how big an angle is if someone tells you it has a measure of 4 units?* (Units can be different sizes. If the unit is big, an angle of 4 units would be much bigger angle than if the unit is small.)

Observe and Assess

As students work, notice:

• Do students carefully fit in the angles at the vertex?

• Do students make good predictions about how many times a unit angle will fit in?

• Do they understand why different angle units lead to different angle measures?

Share and Reflect

After students have had sufficient time to work through the exploration, discuss their work with them by asking:

• *When is the number of units small?*

• *When is the number of units double the number of another unit?*

• *Could the number of units be 1? How?*

Answers

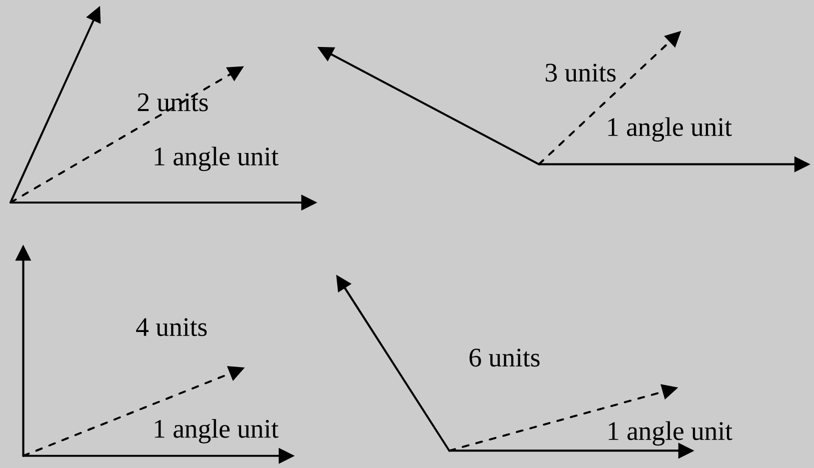
B. i) 1 of the $\frac{1}{3}$ circle angle units; 2 of the $\frac{1}{6}$ circle angle units; About $2\frac{1}{2}$ of the $\frac{1}{8}$ circle angle units; About $3\frac{1}{4}$ of the $\frac{1}{10}$ circle angle units; 4 of the $\frac{1}{12}$ circle angle units.	**iii)** $1\frac{1}{2}$ of the $\frac{1}{3}$ circle angle units; 3 of the $\frac{1}{6}$ circle angle units; 4 of the $\frac{1}{8}$ circle angle units; 5 of the $\frac{1}{10}$ circle angle units; 6 of the $\frac{1}{12}$ circle angle units.
ii) $\frac{1}{2}$ of the $\frac{1}{3}$ circle angle units; 1 of the $\frac{1}{6}$ circle angle units; About $1\frac{1}{4}$ of the $\frac{1}{8}$ circle angle units; About $1\frac{3}{4}$ of the $\frac{1}{10}$ circle angle units; 2 of the $\frac{1}{12}$ circle angle units.	**iv)** About $1\frac{1}{4}$ of the $\frac{1}{3}$ circle angle units; Almost $2\frac{1}{2}$ of the $\frac{1}{6}$ circle angle units; About $3\frac{1}{4}$ of the $\frac{1}{8}$ circle angle units; About 4 of the $\frac{1}{10}$ circle angle units; Almost 5 of the $\frac{1}{12}$ circle angle units.

Answers [Continued]

C. i) There were twice as many $\frac{1}{6}$ circle angle units as $\frac{1}{3}$ circle angle units; If the unit angle is half as big, twice as many will fit.

ii) There were four times as many $\frac{1}{12}$ circle angle units as $\frac{1}{3}$ circle angle units; If the unit angle is one-fourth the size, four times as many will fit.

D. i) *Students might show each angle unit (as in the example in the text) or one angle unit (as shown below):*



E. *Sample response*:
I first imagined dividing the angle in half. Then I divided the lower half angle into 3 equal parts and used 1 of those parts.

F. The measurement using unit X will be smaller than the measurement using unit Y.

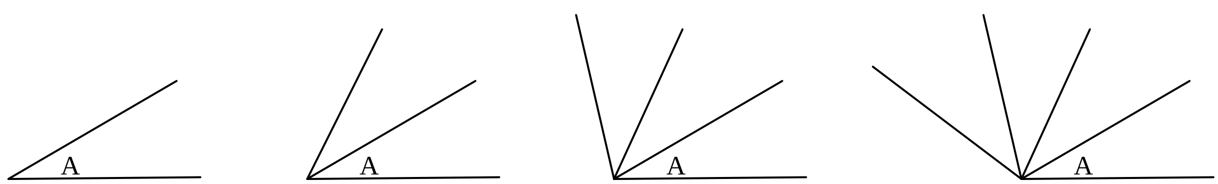
G. *Sample response*:
The same angle can have different measurements, depending on the size of the angle unit. So unless you know what the angle unit is, you cannot tell how big the angle is.

Supporting Students

Struggling students

• You might suggest that struggling students cut out various angles and make 1, 2, 3, or 4 copies of them. This will give them a better feel for how angles are related.

For example, using the angle A below, students can see an angle that is made up of 1, 2, 3, or 4 A units.



They could repeat this with other angles.

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5.2.2 Comparing Angles to Special Angles

Curriculum Outcomes	Outcome relevance
5-D4 Angles: estimate and measure • link wedges to degrees (degree is just a very small wedge) • create and use an improvised protractor for 45, 90, 135, 180 degrees **5-D5 Angles: estimate size** • estimate angles relative to common referents: 45, 90, 180 degrees (about the same as, more than, less than)	In all measurement situations, including the measuring of angles, students benefit by comparing to benchmarks.

Pacing	Materials	Prerequisites
1.5 h	• Thin (tracing) paper or transparencies • Cut-out semicircle • Alphabet Letters (BLM)	• dividing by 2

Main Points to be Raised

• Right angles result from quarter turns.

• A standard unit for angle measure is a degree. A degree is quite small. It takes 90 degrees to make a right angle.

• To estimate the size of angles, it is helpful to use benchmarks of 45°, 90°, 135°, and 180°.

• You can make a protractor by using a half-circle and dividing it into halves and quarters.

Try This — Introducing the Lesson

A. Remind students of what right angles, acute angles, and obtuse angles are.

• Allow students to try this alone or with a partner. Observe while students work. You might ask questions such as the following:

- *What sorts of angles do you see in the diamond designs on the bottom of the roof?* (Right angles)
- *What sort of angle do you see where the building meets the ground?* (Right angle)
- *What sort of angles do you see where the windows fit in the wall?* (Right angles)
- *What sort of angles do you see at the corners of the roof?* (Acute angles)

The Exposition — Presenting the Main Ideas

• Draw two angles on the board. Ask students which they think is the greater angle and why.

• Tell students that one way to compare the angles is to describe each angle in terms of the same unit. You know that an object that is 10 cm long is longer than an object that is 2 cm long because both are measured with the same unit of 1 centimetre. In the same way, if you measure one angle as 10 units and another angle as 2 units, the 10-unit angle is bigger, as long as the units are the same.

• Tell students that the unit people have chosen for measuring angles is called a degree. Draw a 90° angle and tell them that it is made up of 90 small units, each of 1°. Show them how the degree sign is written.

• Hold up a semicircle of paper. Show how you could fold it in half to make a 90° angle. Ask what measure they would give the angle if you folded it in half again (45°). Show how this is done.

• Unfold the paper to show what an angle made up of 90° + 45° (or 135°) looks like. Tell students that when you completely unfold the semicircle, you can see 180° (the two 90° angles side by side).

• Tell them that this device is called a protractor.

• Lead students through the exposition so that they can see the various benchmark angles and the improvised protractor.

Revisiting the Try This

B. Students need to recall that right angles are 90° and realize that acute angles are between 0° and 90°.

Using the Examples

• Assign pairs of students to work through the examples. One student in each pair should be responsible for **example 1** and the other for **example 2.** Ask a student who has worked on **example 1** to come to the front and explain the idea to the other students. Do the same for **example 2**.

Practising and Applying

Teaching points and tips

Q 1: Make sure students refer to the benchmark angles at the top of the page. They should do this question by visualizing, without using their protractors.

Q 3: Students should complete this question by visualizing. They can then use protractors to test how accurately they drew the angles.

Q 4: Students might answer this question by observing the folds in an improvised protractor. Alternatively, they might draw three 45° angles next to each other and then use a protractor or the benchmark angles on the page.

Q 5: Some students might suggest that all five angles look like they might be about 120°. This estimate is a bit high for some of the angles, but it is not unreasonable.

Q 6: Half the 45° angle is actually 22.5°, but 22° is an appropriate estimate.

Q 7: Provide the Alphabet Letters BLM for students to use or write the letters neatly on the board for students to copy.

Q 8: This question is designed to remind students which aspect of an angle matters in its size and which aspects do not matter.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can estimate the size of given angles
Question 3	to see if students can draw reasonable estimates for angles when the size is given
Question 7	to see if students can apply what they have learned about angle size in a practical situation

Answers

A. *Sample response*:
i) Right angles are where the walls meet the ground, at the corner of the walkway, and at the corners of the windows.
ii) Acute angles are where the bottom of the roof meets the side edge of the roof.

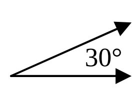
B. i) The right angles
ii) *Sample response*: Closer to 45°

1. *Sample responses*:
a) About 75° **b)** About 140°
c) About 40° **d)** About 100°

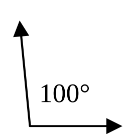
2. *Sample response*: The estimates are reasonable.

3. *Sample responses*:

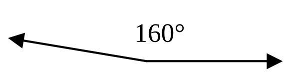
a)



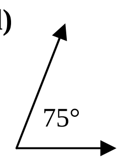
b)



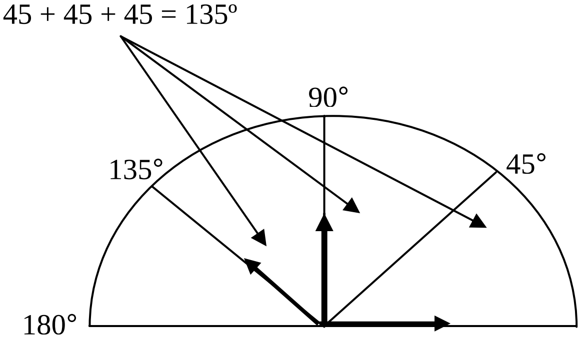
c)



d)



4. 45 + 45 + 45 = 135º



5. B, C, and E; [They are between 90º and 135º but closer to 135°, so they could be 120°. (Some students might think all the angles are about 120°.)]

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[**6.** Fold the 45° line on top of the 0° line. The fold line is at half of 45, which is just a bit more than 22°.] **7.** The angles closest to 30° were found in M, N, V, and W, but the angles in K, X, and Z were also close to 30°; [*Sample response*: I wrote down all the letters that have angles. A E F H I K L M N T V W X Y Z I did not try E, F, H, L, or T because all their angles are right angles. I measured the angles with my protractor.]	**8.** No; [An angle measure is about the amount of a circle that is turned. It does not matter if it is a big circle or a small circle. The turn amount is the same for both.]

Supporting Students

Struggling students

• You may need to provide improvised protractors or help students construct improvised protractors. Some students may wish to trace the benchmark angles to compare them with other angles.

Enrichment

• Ask students to make shapes with certain combinations of angles.

For example, you might ask them to make a shape with two angles about 135° and two angles about 45°.

5.3.1 Volume

Curriculum Outcomes	Outcome relevance
5-D6 Volume and Capacity: solve simple problems • understand volume as the amount of space an object occupies or how much it takes to build it • develop a sense of size and referents for a cubic centimetre, cubic millimetre, cubic metre	Many practical problems involve calculating volume. Students need to extend their idea of volume beyond finding the number of cubes needed to make something. Now they will learn to measure volume using standard metric measures.

Pacing	Materials	Prerequisites
1 h	• Metre sticks • Centimetre cubes	• length units: centimetre, metre, millimetre

Main Points to be Raised

• Volume is a measure of the three-dimensional space that an object takes up. It is based on the length, width, and height of the object.

• A cubic centimetre is a commonly-used volume unit. It is the space taken up by a cube that measures 1 cm on each edge.

• A cubic metre is a commonly-used volume unit for larger items. It is the space taken up by a cube that measures 1 m on each edge.

• A cubic millimetre is a commonly-used volume unit for very small objects. It is the space taken up by a cube that measures 1 mm on each edge.

Try This — Introducing the Lesson

A. Students need to imagine the box because only the top is pictured. They should imagine the 4 cm depth by relating it to the 12 cm width of the top that they can see.

• Allow students to try this alone or with a partner. Observe while students work. You might ask questions such as the following:

- *How many cubes would you need to model just the top?* (240 cubes to cover 12 cm by 20 cm)

- *Why would you need more than one layer of cubes?* (The box is 4 cm deep and since the cubes are only 1 cm deep, you would need 4 layers of cubes.)

The Exposition — Presenting the Main Ideas

• The exposition is designed to introduce three common volume units: the cubic centimetre, the cubic metre, and the cubic millimetre.

• If possible, model 1 cm^3 and 1 m^3 for the students.

- A cubic centimetre is about the size of a very small sweet.

- A cubic metre can be modelled by using a large box of that size or by extending five metre sticks (to represent edges of a 1 m by 1 m by 1 m cube) out from a corner of the room.

- A cubic millimetre is so small it is hard to model.

• Model for the students a structure made of 20 1-cm cubes. Help them see that the volume is 20 cm^3. Rearrange the structure in different ways, reinforcing the idea that the volume remains the same.

• Students can read through the exposition if they wish.

Revisiting the Try This

B. You may wish to provide centimetre cubes for students to get a feel for the size of the box.

Using the Examples

• Provide centimetre cubes to pairs of students. Ask them to solve the problem in **example 1** before having them look at the solution in the text.

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Practising and Applying

Teaching points and tips

Q 1 a): Students might benefit by noticing the symmetry of the shape. Then they only have to count the cubes in half the shape and double that number. Some students will benefit from building the shape.

Q 1 b): Students might calculate separately the volume of each part of the animal: the head, the tail, the legs, the body, and the neck.

Q 1 c): You may have to tell students that this is a 10 cm-by-10 cm square that is 1 cm thick.

Q 1 e): Some students will consider the possibility of hidden cubes. This should be accepted.

Q 3: Some students might think about hidden cubes, while others will not. Either approach should be accepted.

Q 4: You may need to provide a cell phone for students to look at. If none is available, you should either accept a greater range of answers or omit this question.

Q 5: Many students assume that 1 cm^3 = 10 mm^3, since 1 cm = 10 mm. This is not the case.

Q 6: It is important for students to recognize that in any measurement situation, we are looking at size and comparing one object to another benchmark object.

Common errors

• Many students will think that 1 m^3 = 100 cm^3, since 1 m = 100 cm. This is not the case as 1 m^3 = 1,000,000 cm^3. You can model this by beginning to lay out 100 cm^3. Students will see that it is only 10 cm × 10 cm × 1 cm, which is not even close to 1 m × 1 m × 1 m.

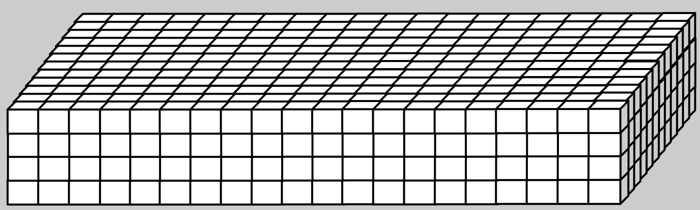
Suggested assessment questions from Practising and Applying

Question 1	to see if students can determine the volume of a cube structure
Question 2	to see if students can create a structure with a particular volume
Question 4	to see if students can estimate the volume of a real-world object
Question 5	to see if students have a sense of the various metric volume units

Answers

A. i) 960 cubes

ii)

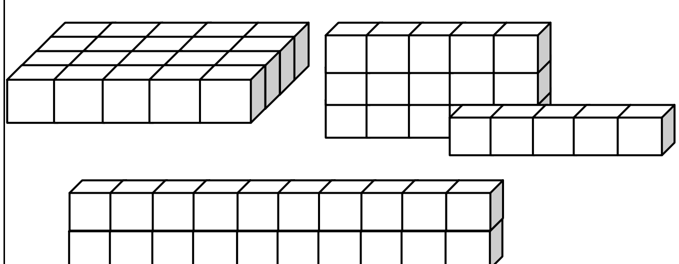


B. 960 cm^3

1. a) 32 cm^3 **b)** 30 cm^3

c) 48 cm^3 **d)** 36 cm^3 **e)** 24 cm^3

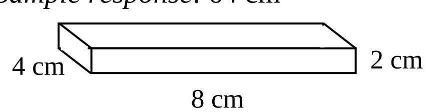
2. a) *Sample response*:



b) Yes; *Sample response*:
5 cm by 4 cm by 1 cm or 1 cm by 10 cm by 2 cm

3. a) 11 cm^3 **b)** 21 cm^3 [8 + 7 + 6]

4. *Sample response*: 64 cm^3



Answers [Continued]

5. 1000 times as big; [*Sample response:* 1 cm^3 has a base of 100 square millimetres (because it is 10 mm by 10 mm), and there are 10 layers like the base.]	[**6.** *Sample response*: For area and volume, you measure lengths and widths. For volume, you also measure heights. For both, you figure out how many times a unit fits into the shape.]

Supporting Students

Struggling students

• Hands-on work is critical for understanding volume concepts. You may wish to provide struggling students with opportunities to build whatever they wish to make with centimetre cubes. Each time, indicate the volume of what they have built. This background experience will help them when they are asked to determine the volumes of objects someone else has made and when they are asked to create an object with a given volume.

• It may be appropriate to refrain from any discussion of cubic metres until after students have a better understanding of cubic centimetres.

Enrichment

• Some students may wish to experiment to see how many different structures they can create with a particular volume.

• Other students may wish to use cubic metres to estimate the volumes of large objects, like the classroom. These students will likely need to use metre sticks.

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5.3.2 Capacity

Curriculum Outcomes	Lesson relevance
5-D1 Volume and Capacity: solve simple problems • understand capacity as how much a container is capable of holding • discover, through investigation, that 1 cm^3 holds 1 mL, and 1 dm^3 holds 1 L	Many practical problems involve calculating capacity. Students need to be familiar with a variety of metric capacity units. When they understand the relationship between volume and capacity, they will be able to solve a broader range of volume and capacity problems.

Pacing	Materials	Prerequisites
1.5 h	• Calibrated measuring cups • Spoons of different sizes, cups and glasses of different sizes, another container such as a bucket • Water • Grid Paper (1 cm by 1 cm) (BLM) • Scissors	• 1 cm^3 and 1 m^3

Main Points to be Raised

• Capacity tells how much an object can hold.

• Capacity is often measured in litres (L) or in millilitres (mL). Litres are usually used to measure bigger amounts, and milliliters measure smaller amounts.

• 1 L = 1000 mL.

• It is helpful to know some benchmark capacity measurements.

For example:

- a spoon holds between 5 mL and 15 mL, depending on the size of the spoon
- a drinking glass holds about 300 mL
- a water jug might hold about 2 L.

• A 1 cm^3 container holds 1 mL of water.

• A 1 dm^3 container holds 1 L of water.

• You can calculate volume by using water displacement. This is because of the relationship between volume and capacity units.

Try This — Introducing the Lesson

A. Students can do this alone, but should compare their results with those of other classmates. Observe while students work. While they work, you might ask questions such as the following:

• *When you counted the glasses, what did you have to think about?* (I thought about whether the glasses were the same size and how full the glasses were.)

• *Do you think you drink the same number of glasses of water each day?* (I do not think so. Some days I am too busy to drink water.)

• *Why might it be important to measure how much water you drink?* (Drinking lots of water keeps you healthy, so it is important to know if you are drinking enough.)

The Exposition — Presenting the Main Ideas

• Tell students what capacity means. Introduce the millilitre (mL) and its abbreviation. Model some objects in terms of how many millilitres they might hold, such as a measuring cup, a spoon, and a drinking glass.

• Then show students a container that might hold 1 L.

For example, you could build a 10 cm cube out of paper.

• Demonstrate a few larger items such as buckets or jugs and have students estimate how many litres they might hold. If it is possible, test their estimates by filling the containers using 1 L container units.

• Point out that it takes 1000 mL to make 1 L just like it takes 1000 mm to make 1 m.

• Discuss with students that volume tells how much space a solid object takes up and capacity talks about how much an object can hold, but that these are related. Show them a 1 cm × 1 cm × 1 cm cube and tell them that if it were hollow, it would hold 1 mL. Show them a 10 cm-by-10 cm-by-10 cm cube and tell them that if it were hollow, it would hold 1 L.

• Model the procedure shown at the end of the exposition where the volume of a small object is determined by immersing it in water and seeing how the mL reading changes.

For example, if the water level goes from 250 mL to 320 mL, the object had a volume of 320 – 250 = 70 cm^3.

You might tell students that this principle was discovered in ancient times by a Greek named Archimedes.

Revisiting the Try This

B. Students should relate their answer to the fact that a full drinking glass might hold 300 mL of water.

You may wish to share with them that drinking 1.5 L to 2 L of fluids a day is considered a healthy thing to do.

Using the Examples

• Work through the **example** with the students. It is fairly straightforward, but it requires them to think about the relationship between millilitres and litres.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to rewrite the litre measures as millilitres.

Q 3: It would be helpful to provide water and the pictured items, if possible. You can also substitute other available items for those pictured in the book.

Q 5: You may wish to substitute another item for a teacup if no teacup is available.

Q 6: If not enough measuring cups are available, you might have to do this as a demonstration or have small groups of students take turns.

Q 7: Observe whether students are able to visualize the dimensions of the box after the square corners are cut out.

Common errors

• Some students will not distinguish between millilitres and litres. It is important to remind them that millilitres are usually used for small amounts and litres are used for larger amounts.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can relate millilitres to litres
Question 3	to see if students can estimate the capacity of various everyday items
Question 7	to see if students can solve a problem relating capacity to volume

Answers

A. *Sample response*: 6 glasses of water	**B.** *Sample responses*: **i)** 1.8 L **ii)** 1800 mL
1. a) 3 L **b)** 1 L **c)** 1025 mL **2.** *Sample response*: about 1200 mL **3. a)** Litres **b)** Millilitres **c)** Millilitres **d)** Litres	**4. a)** 20 **b)** 10 **c)** 5 (or between 5 and 6) **d)** 2 (or between 2 and 3) **5.** *Sample response*: about 150 mL

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6. *Sample response*: I used 10 coins. The water went up 50 mL, so I knew the volume of 1 coin was 5 cm^3. **7. a)** 32 cm cubes; 32 mL **b)** 24 cm cubes; 24 mL [**8.** *Sample response*: Volume and capacity both tell how "big" an object is, but capacity measures how much the object holds and volume tells how much space the object takes up.]	[**9.** *Sample response*: Millilitres are useful for measuring very small amounts. You would have to use fractions or decimals to describe capacity if you could use only litres to measure small amounts.]

Supporting Students

Struggling students

• Struggling students should continue to fill containers and measure the contents using measuring cups. This will give them the experience they need so they can refer to benchmarks to make more sense of capacity measures they read or hear about.

Enrichment

• Ask students to create questions like **question 7** for other students to solve. They might begin with rectangles of different sizes to see how they might create boxes that have the same capacity.

5.3.3 Metric Units

Curriculum Outcomes	Outcome relevance
5-D7 SI Units: reinforce relationships among various SI units • apply relationships among kilometres/ hectometres/ decametres/ metres/ decimetres/ centimetres/ millimetres, litres/ millilitres, and kilograms/ grams • use relationships to rename measures • apply referents for various measurement standards (e.g., 30 cm is like a ruler, 1 dm is about a small hand span) **5-C6 SI Measurement: pattern in changing units** • understand that a smaller measurement unit increases the number of those units and that a larger measurement unit decreases the number of those units • apply the above relationship to reason through conversions	Understanding the relationship between metric prefixes will help students in many measurement situations.

Pacing	Materials	Prerequisites
1 h	• Place Value Chart (BLM)	• place value, thousands to thousandths

Main Points to be Raised

• We use standard units so that we can understand each other when we describe measurements.

• The metric system is useful because you can use what you already know about place value to relate different units. It is also useful because it applies to different kinds of measurement, like length, capacity, volume, and so on.

• The metric prefixes have defined meanings.

For example, centi means 0.01 of the base unit, milli means 0.001, and kilo means 1000, etc.

• You can use the relationship between metric units, in combination with a place value chart, to make it easy to change from one unit to another.

Try This — Introducing the Lesson

A. and B. Allow students to try this alone or with a partner. Observe while students work. You might ask questions such as the following:

• *How tall are you?* (About 1.6 m)

• *Why is the distance more than 35 times your height?* (35 × 2 = 70, but my height is less than 2, so even more of me would fit into that distance)

• *Why might it be better to talk about how many of your height fit into that distance instead of just saying the distance is 68 m more than your height?* (I can imagine it better by thinking of fitting in 35 or 40 of me.)

The Exposition — Presenting the Main Ideas

• Ask students to think about times when they need to measure things, for example, measuring food to cook or measuring how tall they are. Talk about how it is important that people know about measurement in order to make sure that things fit where they should, to make things fair, or to build things.

• Ask students to recall what they know about centimetres, metres, and millimetres.

Record some of the relationships:

100 cm = 1 m   1000 mm = 1 m   10 mm = 1 cm

• Ask students to describe 3 m in centimetres and then in millimetres. Talk about the operations they use.

• Then ask them to describe 3.2 m in centimetres and then in millimetres. Show them how to use a place value chart to convert, as is shown in the exposition.

• Tell students your height in centimetres. Ask them how to write it in metres. Again, show them how to use a place value chart to do the conversion.

• Introduce the other metric prefixes: *hecto*, *deca*, and *deci*. Explain that these are used less frequently.

• Lead the students through the examples in the exposition.

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Revisiting the Try This

C. Ask students how they could have predicted that the result would involve only the digits 7 and 0 and would be a number greater than 70.

Using the Examples

• Ask the students the question in **example 1**. Encourage them to think of as many possibilities as they can.

• Write the questions in **example 2** on the board and ask students to try them using a place value chart. They should then check their results against the solution in the student text.

Practising and Applying

Teaching points and tips

Q 1: Make sure students know that each of the numbers on the right fits in only one blank. This might help them use reasoning to eliminate answers.

Q 3: Encourage students to use place value charts.

Q 4: Students are free to convert either unit in the pair to the other unit.

Q 5: It might be helpful for students to make a list of the possible units.

Q 6: Help students understand that it is not possible for any digit other than 6, 2, 1, 3, or 0 to appear because the conversion only requires them to move the digits on a place value chart.

Common errors

• Many students go the wrong way when they perform a conversion.

For example, to convert from metres to millimetres, they divide by 1000 instead of multiplying, or vice versa. They should always to visualize the unit to help them see if the value should go up or down.

Suggested assessment questions from Practising and Applying

Question 1	to see if students are familiar with basic metric measurement units
Question 3	to see if students can convert from one metric measurement unit to another
Question 7	to see if students understand and can communicate about the convenience of the metric measurement system

Answers

A. *Sample response*: It is about 40 times as long as my height.	**B.** *Sample response*: To make the game fair, everyone should have to shoot at the target from the same distance. **C.** 70 m = 70,000 mm; I multiplied 70 by 1000 because I know 1000 mm = 1 m.

1. a) 65 km **b)** 30 cm **c)** 2 m **d)** 2 mm
e) 10 cm **f)** 3 km **g)** 17 cm **h)** 1 cm

2. a) Less, [since metres are bigger.]
b) Less, [since metres are bigger.]
c) Less, [since kilograms are bigger.]
d) More, [since decimetres are smaller.]

3. a) 340 **b)** 5120 **c)** 41,600
d) 0.41 **e)** 0.131

4. a) 512 mm; [*Sample response*: i
It is 51.2 cm and 51.2 > 32.4.]
b) 1 km; [*Sample response*:
22,000 cm = 220 m and 1 km = 1000 m]
c) 3.1 kg; [*Sample response*:
2120 g = 2.120 kg and 2.120 < 3.1]

5. a) m **b)** km **c)** g **d)** mL

6. a) Only 6, 2, 1, 3, and 0
[**b)** *Sample response*:
When you multiply and divide by 10, you might need extra zeros to put digits in the right place value column, but the digits just move, they do not change.]

[**7. a)** 1 kg = 1000 g just like 1 km = 1000 m.
b) 1 kg = 1000 g just like 1 m = 1000 mm.]

[**8.** *Sample response*:
Because all you have to do is figure out how many places to move the digits over. You do not calculate anything.]

Supporting Students

Struggling students

• Struggling students should continue to use place value charts for these conversions. You may choose to emphasize converting larger units to smaller units, where multiplication is used, rather than converting smaller units to larger units, where division is used.

Enrichment

• Students might enjoy exploring metric prefixes by answering questions like this:

- *How long is a kilosecond?*
- *How long is a kilominute?*
- *How long is a deciday?*

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UNIT 5 Revision

Pacing	Materials
2 h	• Rulers • Square Dot Grid Paper (BLM) • Improvised protractors • Place Value Chart (BLM) (optional)

Question(s)	Related Lesson(s)
1 – 3	Lesson 5.1.1
4 and 5	Lessons 5.1.2 and 5.1.3
6 and 7	Lesson 5.1.3
11	Lesson 5.1.5
12 – 14	Lesson 5.2.2
15 and 16	Lesson 5.3.1
17 – 19	Lesson 5.3.2
20 – 22	Lesson 5.3.3

Revision Tips

Q 2: Make sure students realize the shapes do not have to be rectangles.

Q 3: Students might pick a small value for one side, double the value for another side, and then put in other sides to use up the full 20 cm perimeter.

Q 7: Provide square dot grid paper, if possible. Otherwise, have students draw their own grid in their exercise books.

Q 8: Encourage students to do the calculation without drawing the rectangles.

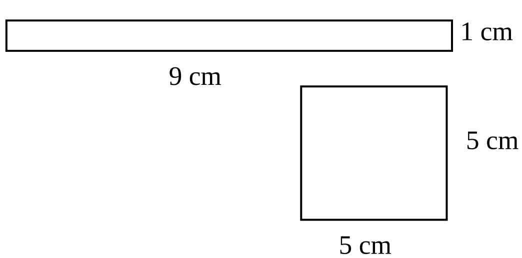
Q 11: You can tell students that the two parts of the shape are both rectangles.

Q 16: Students should assume there are no hidden cubes.

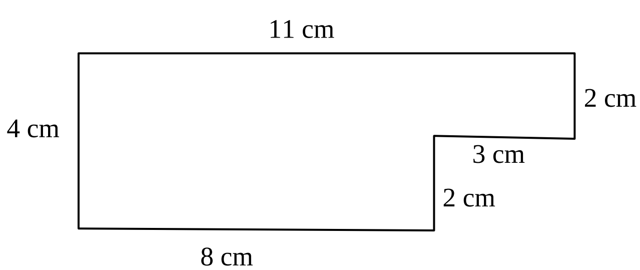
Answers

1. a) 30 cm **b)** 28 cm **c)** 30 cm

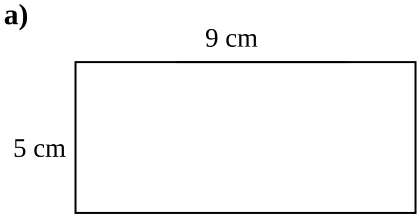
2. *Sample response*:



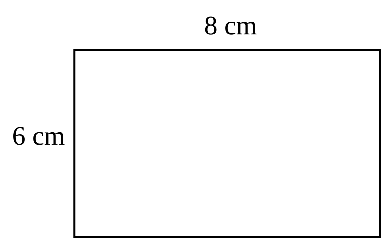
3. *Sample response*:



4. a)



b) *Sample response*:



[**c)** *Sample response*:
The second rectangle has a greater area. Its area is 48 cm^2 (6 cm × 8 cm) compared to 45 cm^2 (5 cm × 9 cm).]

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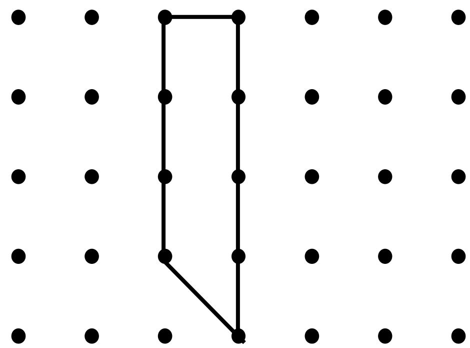
Answers [Continued]

5. The shorter rectangle is 2 cm wider than the longer rectangle.

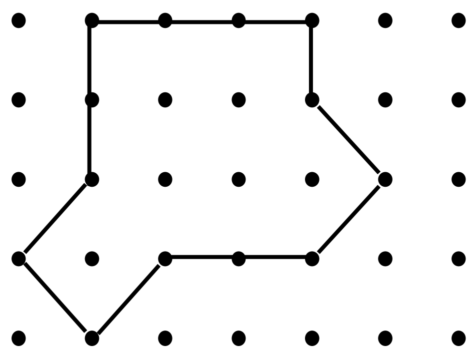
6. a) $10\frac{1}{2}$ square units **b)** $7\frac{1}{2}$ square units

7. *Sample responses*:

a)



b)



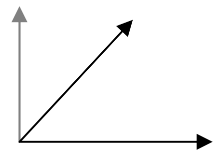
8. a) Perimeter = 22 cm Area = 24 cm^2
b) Perimeter = 40 cm Area = 96 cm^2

9. *Sample responses*:
a) 8 cm by 5 cm; 20 cm by 2 cm; 10 cm by 4 cm.
b) Least perimeter: 8 cm by 5 cm

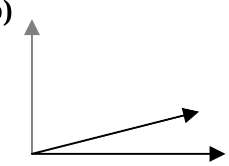
10. *Sample responses*:
a) 20 cm by 20 cm; 30 cm by 10 cm; 35 cm by 5 cm.
b) Greatest area: 20 cm by 20 cm

11. a) Area = 36 cm^2 Perimeter = 32 cm
b) Area = 9 cm^2 Perimeter = 16 cm

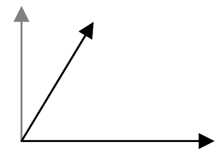
12. a)



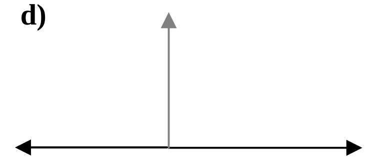
b)



c)



d)



13. *Sample responses*:
a) about 20° **b)** about 120°
c) about 5° **d)** about 165°

14. *Sample responses*:
a) about 10° **b)** about 100°
c) about 50° **d)** about 100°

15. *Sample response*: 3 cm by 5 cm by 2 cm

16. a) 9 cubes, or 9 cubic units
b) 22 cubes, or 22 cubic units

17. 1.2 L, 1500 mL, 2.8 L, 3200 mL, 4L

18. a) Litres
b) *Sample response*: a bucket and a sink

19. *Sample responses*:
a) A sink **b)** A spoon

20. a) 0.352 m **b)** 4200 g
c) 533 m **d)** 10,000 dm

21. a) m **b)** mm **c)** m

22. Move the digits 4 places to the left;
[*Sample response*:
Because 1 km = 10,000 dm, you have to multiply the number of kilometres by 10,000 to get the number of decimetres. You do that by moving the digits 4 places to the left.]

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UNIT 5 Measurement Test

1. Each shape listed below has a side length of 4 cm. What is the perimeter of each?

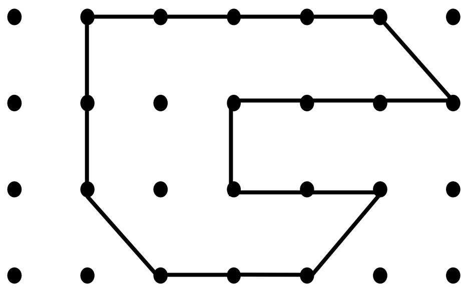
a) equilateral triangle

b) square

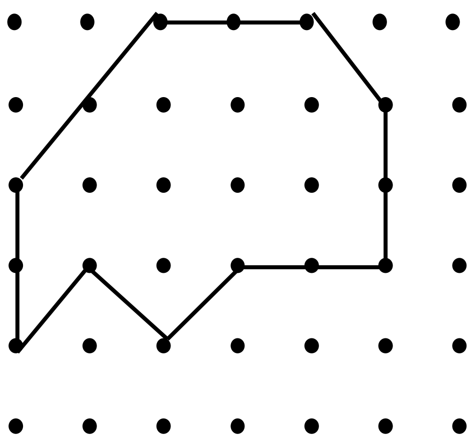
c) regular hexagon

2. What is the area of each shape?

a)



b)



3. On a grid, draw a shape with an area of $4\frac{1}{2}$ square units.

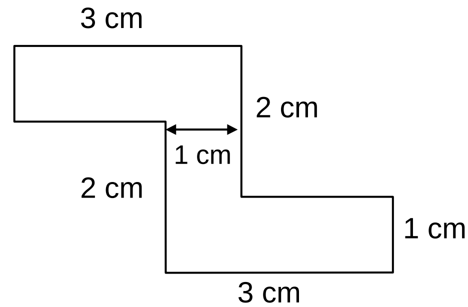
4. Sketch two rectangles for each. Label all dimensions.

a) an area of 50 cm^2

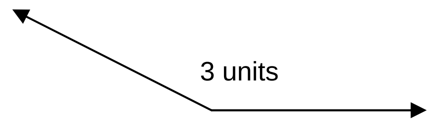
b) a perimeter of 40 cm

5. Sketch a shape that would have a small area but a large perimeter. Explain your strategy.

6. What is the area of this shape? Use a diagram to explain how you found the area..

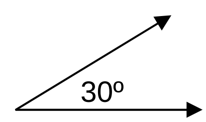


7. Trace the angle and then sketch the angle unit that would result in this angle measure.

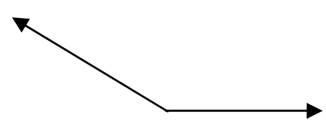


8. What does the measurement of an angle tell about the angle?

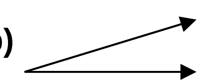
9. Use this 30° angle to estimate the size of each angle.



a)



b)



10. a) Sketch or describe a prism with a volume of 9 cubes.

b) Sketch or describe a different 3-D shape with the same volume.

11. A container has a capacity of about 200 mL. What might the container be?

12. Complete.

a) 2000 mL = ___ L **b)** 3 L = ___ mL

c) 1.4 kg = ___ g **d)** 3.2 dm = ___ mm

e) 52 cm = 520 ___ **f)** 0.320 km = 320 ___

UNIT 5 Test

Pacing	Materials
1 h	• Rulers • Grid Paper (1 cm by 1 cm) (BLM) • Centimetre cubes

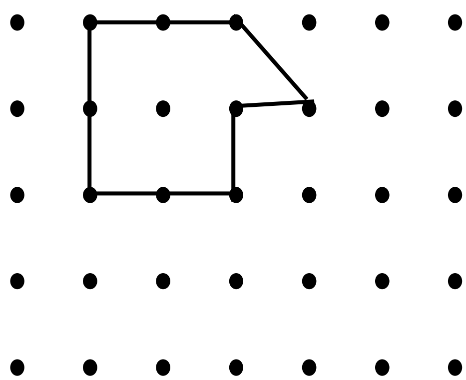
Question(s)	Related Lesson(s)
1	Lesson 5.1.1
2 and 3	Lesson 5.1.3
4 and 5	Lesson 5.1.4
6	Lesson 5.1.5
7	Lesson 5.2.1
8 and 9	Lesson 5.2.2
10	Lesson 5.3.1
11	Lesson 5.3.2
12	Lesson 5.3.3

Answers

1. a) 12 cm **b)** 16 cm **c)** 24 cm

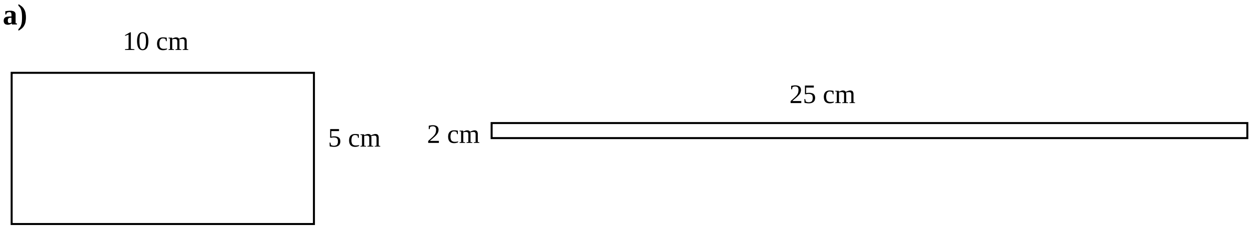
2. a) $9\frac{1}{2}$ square units **b)** 14 square units

3. *Sample response*:

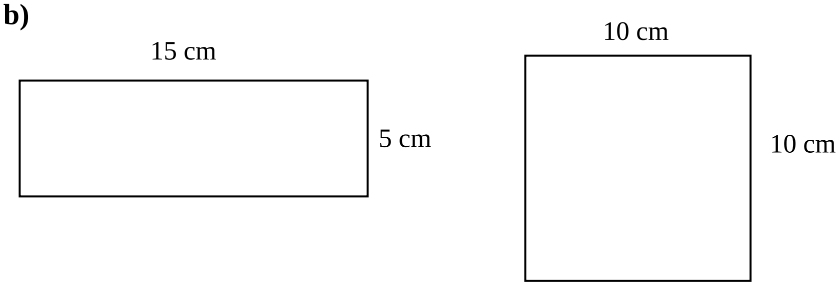


4. *Sample responses*:

a)



b)



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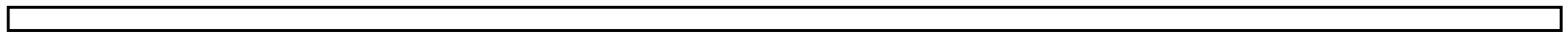
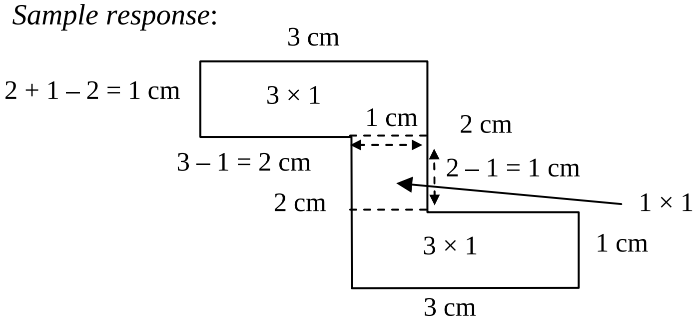
5. *Sample response*:



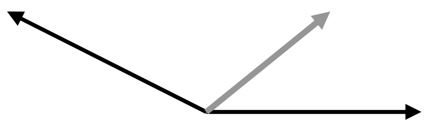
I needed long side lengths for a long perimeter. I had to make it very narrow so it would not have much area.

6. 7 cm^2

Sample response:



7.



8. It tells the amount one arm turned to get to the position of the other arm.

9. *Sample responses*:

a) 150° **b)** 15°

10. *Sample responses*:

a) One row of 9 cubes

b) Three layers, where each layer is a row of 3 cubes

11. *Sample response*:

It might be a small drinking glass, like a juice glass.

12. a) 2 **b)** 3000 **c)** 1400

d) 320 **e)** mm **f)** m

UNIT 5 Performance Task — A Clock Tower

A. Which do you think might be the height of the clock tower in Thimphu, from the base to the top of the roof?

140 dam or 140 dm or 140 mm

Explain your choice.

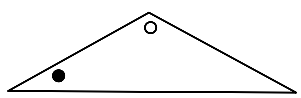
B. The square clock face in the photograph has a side length of about 12 mm. Create a square with a side length that is 10 times as long.

i) What is the perimeter of the square in centimetres?

ii) Draw the square on a centimetre grid. What is its area?

C. Sketch and label a different rectangle with the same area as the square in **part B**, but a greater perimeter. What is its perimeter?

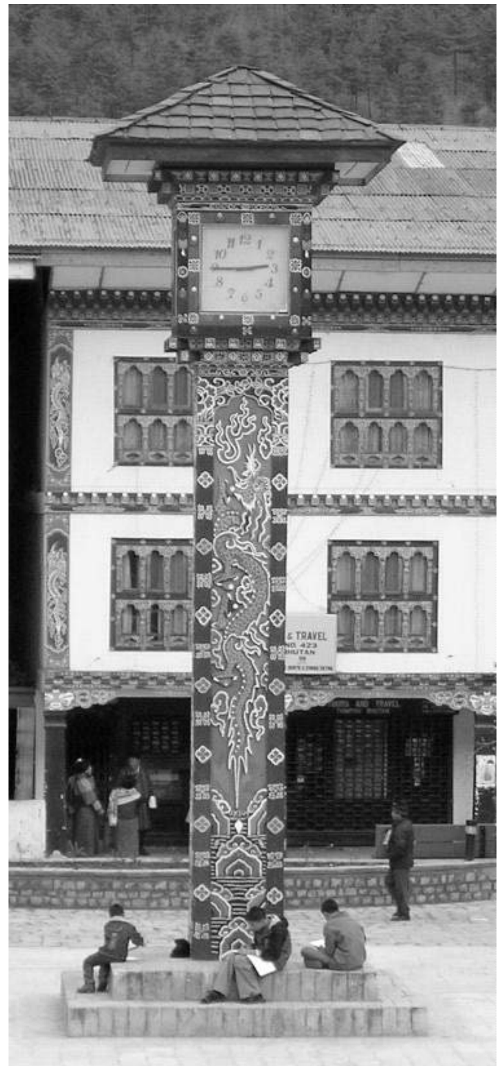
D. Look at the roof of the clock tower. Estimate the size of the two angles shown below. Use your improvised protractor to check.



Roof of clock tower

E. **i)** Use 40 centimetre cubes to build a rectangular prism that has the same shape as the tower under the clock. What is the volume of your prism in cubic centimetres?

ii) Suppose your prism is a container that can be filled with water. Estimate its capacity.



The clock tower in Thimphu

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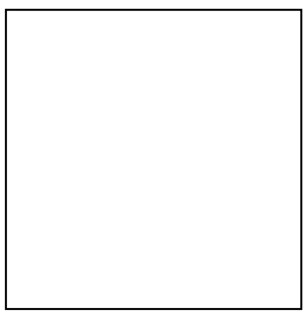
UNIT 5 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
5-D1 Perimeter: polygons **5-D2** Perimeter and Area: rectangles and squares **5-C4** Area and Perimeter: changing rectangle dimensions **5-D5** Angles: estimate size **5-D6** Volume and Capacity: solve simple problems **5-D7** SI Units: reinforce relationships among various SI units	1 h	• Rulers • Improvised protractors • Centimetre cubes

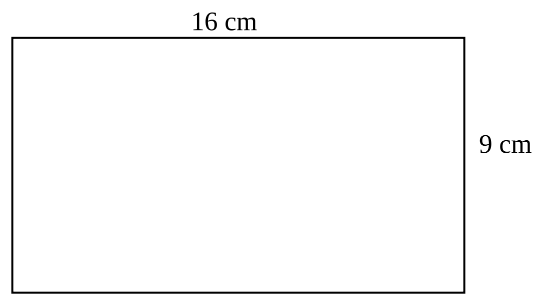
Sample Solution

A. 140 dm; 140 dm = 14 m. It looks like the tower is as high as 7 or 8 people. If each person were between 1.5 m and 2 m tall, then 14 m makes sense.
140 dam = 1400 m, which is way too tall, and 140 mm is only 14 cm, which is not nearly tall enough.

B. Perimeter = 48 cm; Area = 144 cm^2



C.



Perimeter = 50 cm

D. The bottom angle is about 30° and the top angle is about 120°.

E. i) I built a square-based prism with 10 layers of cubes. Each layer was 2 cubes by 2 cubes and its volume is 40 cm^3.
ii) about 40 mL because its volume is 40 cm^3 and 40 cm^3 of water = 40 mL of water

UNIT 5 Performance Task Assessment Rubric

The student	Level 4	Level 3	Level 2	Level 1
Takes measurements	Easily, correctly, and accurately measures perimeter, area, and volume	Correctly measures perimeter, area, and volume	Correctly measures at least two of perimeter, area, and volume	Has difficulty measuring perimeter, area, and volume
Estimates measurements	Easily and reasonably estimates capacity, angle measures, and heights	Reasonably estimates capacity, angle measures, and heights	Reasonably estimates at least two of capacity, angle measures, and heights	Has difficulty estimating at least two of capacity, angle measures, and heights
Relates measurements	Easily and reasonably relates perimeter and area of rectangles, and volume and capacity of solids	Reasonably relates perimeter and area of rectangles, and volume and capacity of solids	Reasonably relates perimeter and area of rectangles, or volume and capacity of solids, but not both	Has difficulty relating perimeter and area of rectangles, and volume and capacity of solids

BLM 1 Square Dot Grid Paper

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BLM 2 Alphabet Letters

A B C D E

F G H I J

K L M N O

P Q R S T

U V W X Y

Z

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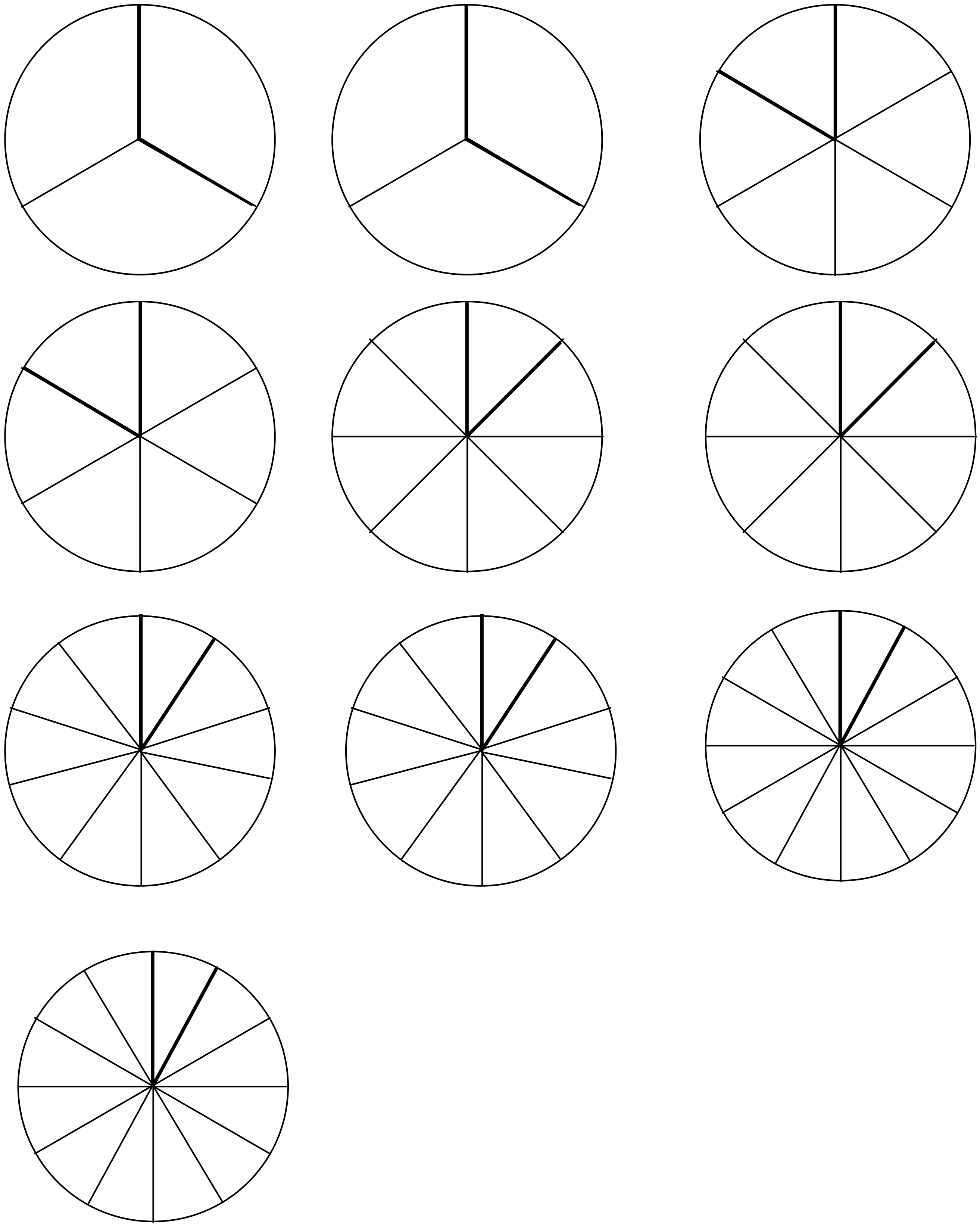
BLM 3 Fraction Circle Angle Units



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UNIT 6 NUMBER

UNIT 6 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 171** **TG p. 252**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	None	All questions
Chapter 1 Reading and Writing Numbers				
6.1.1 EXPLORE: How Much is a Million? (Essential) **SB p. 173** **TG p. 255**	**5-A6 Millions: interpret** • develop a sense of how big a million is **5-A9 Ratio and Rate: exploring informally** • understand ratio as a multiplicative comparison of two numbers or quantities of the same type	40 min	• Rulers • Pencils • Books	Observe and assess questions
CONNECTIONS: One Million (Optional) **SB p. 174** **TG p. 257**	Make a connection between different representations of one million	15 min	None	N/A
6.1.2 Whole Number Place Value **SB p. 175** **TG p. 258**	**5-A7 Place Value: whole numbers to 7 digits** • read and represent whole numbers to 7 digits • generalize place-value patterns as groups of three digits called periods **5-C5 Place Value Patterns: base ten system to millions** • recognize the patterns in periods	1 h	• Place Value Charts III (BLM)	Q2, 3, 4, 5
6.1.3 Renaming Numbers **SB p. 178** **TG p. 261**	**5-A6 Millions: interpret** • interpret millions in different ways and justify the interpretation (e.g., 1,500,000 = $1\frac{1}{2}$ million = 1.5 million) **5-A7 Place Value: whole numbers to 7 digits** • read and represent whole numbers to 7 digits • generalize place value patterns as groups of three digits called periods	1 h	• Place Value Charts III (BLM)	Q2, 6, 7
6.1.4 Comparing and Ordering Numbers **SB p. 181** **TG p. 264**	**5-A8 Comparing: order 7-digit whole numbers** • compare and order numbers up to 7 digits - in standard form (e.g., 3,256,876 > 3,255,996) - in decimal form (e.g., 3.25 million > 34.3 million) - in standard and decimal form (e.g., 3,256,876 < 3.2 million) - with different place value (e.g., 3420 thousand > 3,325,146 and 342 thousand < 2 million)	1 h	• One die	Q2, 4, 7, 8
GAME: Target 7 (Optional) **SB p. 185** **TG p. 267**	Practise comparison of large numbers in a game situation	20 min	• 30 Digit cards (0 to 9)	N/A

UNIT 6 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Chapter 2 Number Relationships				
6.2.1 Renaming Numbers Using Multiplication **SB p. 186** **TG p. 268**	**5-A9 Ratio and Rate: exploring informally** • understand ratio as a multiplicative comparison of two numbers or quantities of the same type • understand rate as a multiplicative comparison of two quantities described in different units • explore ratio and rate in geometric, numerical, and measurement situations	1 h	None	Q2, 4, 5
6.2.2 Using Number Sentences **SB p. 189** **TG p. 271**	**5-B12 Open Number Sentences: applying number sense** • explore numerical situations which are always, sometimes, or never true (e.g., 324 + □ > 300 is always true, assuming □ is a whole number) • work with open number sentences involving the four basic operations and a combination of operations • understand that □ can also be expressed as a letter variable or another shape or symbol	1 h	None	Q2, 4, 5
UNIT 6 Revision **SB p. 192** **TG p. 274**	Review the concepts and skills in the unit	1 h	• Place Value Charts III (BLM)	All questions
UNIT 6 Test **TG p. 276**	Assess the concepts and skills in the unit	1 h	• Place Value Charts III (BLM)	All questions
UNIT 6 Performance Task **TG p. 279**	Assess concepts and skills in the unit	1 h	None	Rubric provided
UNIT 6 Assessment Interview **TG p. 281**	Assess concepts and skills in the unit	10 min	See **p. 281**	All questions
UNIT 6 Blackline Masters **TG p. 282**	BLM 1 Place Value Charts III (One Millions Period to the Ones Period)			

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Math Background

• This number unit allows students to explore large numbers (to millions). Students consider how numbers are related by multiplication (for example, 1,000,000 is 1000 thousands) and how open number sentences describe relationships between numbers (for example, 5 + [ ] < 20 is a way to say that numbers between 0 and 15 are at least 5 less than 20). We focus on the place value system as students learn how the millions period relates to the thousands period.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 6** in **lesson 6.1.2** and in the **Try This** in **lesson 6.1.4**, where they must create a number to meet certain conditions, and
in **question 3** in **lesson 6.2.1**, where they solve a real-world problem.

• They use communication in **question 9** in **lesson 6.1.2**, where they describe features of the place value system, and in **question 7** in **lesson 6.2.1**, where they talk about situations where it is useful to rename a number.

• They use reasoning in **question 9** in **lesson 6.1.4**, where they must consider what values would make a number sentence true, and in **question 6** in **lesson 6.2.2**, where they create number sentences with specific solutions.

• They consider representation in **lesson 6.1.2** and in **lesson 6.1.3**, where they use alternate ways to represent a number, in **lesson 6.1.4**, where they use number lines for comparing numbers, and in **lesson 6.2.1**, where they look at how one number can be expressed in a different way by relating it to another number.

• Students use visualization skills in **part C** in **lesson 6.1.1**, where they imagine how much room 1 million exercise book pages would take up, and in **lesson 6.1.3**, where they use a place value chart as a tool for renaming numbers.

• They make connections in **part G** of **lesson 6.1.1,** where they think of when they might meet the value of one million in everyday life, in **questions 5 and 6** in **lesson 6.1.3**, where they relate large numbers to our world, and in **question 4** in **lesson 6.2.1**, where they use one piece of information to figure out other facts.

Rationale for Teaching Approach

• This unit is divided into two chapters.

Chapter 1 addresses describing and comparing large numbers.

Chapter 2 focuses on number relationships as expressed through multiplicative number sentences and open number sentences.

• There is one **Explore** lesson to give students a sense of how much a million is through some everyday experiences.

• The **Connections** section provides some history of the term *one million* and additional practice using the term properly.

• The **Game** provides an opportunity to practice large number comparisons.

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Getting Started

Curriculum Outcomes	Outcome relevance
4 Place Value: model whole numbers to 5 places **4** Compare and Order Whole Numbers to 5-Digits	Students will find the work in the unit easier once they review what they know about number, particularly place value, from Class IV.

Pacing	Materials	Prerequisites
1 h	None	• reading, writing, and comparing numbers using place value to the ten thousands place • using standard form and expanded form for numbers up to 100,000 • decimals to the thousandths

Main Points to be Raised

Use What You Know

• We read whole numbers based on periods of three digits. The periods for numbers of up to five digits are the ones (or units), consisting of hundreds, tens, and ones, and the thousands period, including ten thousands and thousands.

• To compare numbers, start at the greatest place value.

For example, you compare two 5-digit numbers by first comparing the ten thousands digit.

Skills You Will Need

• When a number is written as a set of digits side by side, the number is in standard form. You have to know the place value system to read or interpret the number.

• You can compare decimal numbers just like you compare whole numbers.

Use What You Know — Introducing the Unit

• To begin the year in a positive way and reintroduce students to numbers in the thousands, you may wish to start with a Guess my Number game. Think of a 4-digit number and tell students to ask questions so they can guess the number. The rule is that you have to be able to answer every question they ask with *yes* or *no*.

For example, they can ask “Is it even?” or “Is it greater than 3000?”, but they cannot ask “What digit is in the thousands place?” The goal is for students to guess the number as quickly as possible.

It might go like this if you think of the number 4123.

Is it even? (No)

Is it less than 5000? (Yes)

Is it more than 3000? (Yes)

Is it more than 4000? (Yes)

Is it more than 4500? (No)   and so on …

You might play the game several times to give many students an opportunity to participate.

• After playing the game a few times, write a 4-digit number and a 5-digit number on the board and ask students to read them. Have students tell the group how they decided the correct way to read the numbers. Make sure they do not say the word “and” when they read a number unless it is used to separate a whole number part from a decimal part.

For example, for 3014 we say "three thousand fourteen", not "three thousand and fourteen". But for 3.04 we say "three and four hundredths".

Point out that a comma is used to separate the thousands period from the ones period, but the comma can be omitted with four digit numbers.

• Review the place value chart with the students before they begin the activity. Ask students which digit is in the thousands place, which is in the hundreds place, and so on, using the 4-digit and 5-digit numbers on the board.

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• Briefly review how students might compare, for example, 13,258 and 14,567, by asking which is greater and how they know.

• Assign the activity to pairs of students.

• Observe students as they work. You might ask questions such as the following:

- *Why is 19,379 > 10,000 even though they have the same ten thousands digit?* (19,379 has 9 more thousands than 10,000, but 10,000 does not have anything extra.)

- *How did you decide the population of 19,379 was not the population of Bhutan?* (I think that I read that the population of Thimphu is 60,000, so the population of all of Bhutan has to be more than that.)

- *How did you know that the second digit of the first number was either 1 or 2?* (It could be in the 51 thousands or the 52 thousands, but it could not be as high as 53,000.)

- *Which digits could be in each place for the first number?* (The ones digit has to be 8. The thousands digit could be 1 or 2. The hundreds digit could be 5, 6, 7, 8, or 9 since that digit is at least 5 more than the tens digit. The tens digit could be 0, 1, 2, 3, or 4. It cannot be higher because then the hundreds digit would be too high.)

- *Are any of your clues not necessary?* (I did not need to say that the tens digit was even because I had already said it was 2.)

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions.

• You may wish to review the terms *standard form* and *expanded form*, as well as the tenths and hundredths places in the place value chart before assigning these exercises.

• Students can work individually.

Answers

A. *Sample responses*:

Ten thousands	Thousands	Hundreds	Tens	Ones
1	9	3	7	9

i) The ten thousands digit is 1.
ii) There are 9 thousands and 3 hundreds.
iii) There are more than 19 thousands, and 19 > 3.

B. A dzongkhag; I know Bhutan has more people than Thimphu and Thimphu has over 50,000 people

C. *Sample response:*
First number: 52,948; Second number: 31,067

D. *Sample responses*:
i) 12,097 and 58,643
ii) Clues for first number
• It has 9 more tens than hundreds.
• It has 5 fewer thousands than ones.
• It is less than 20,000.
Clues for second number
• It is between 58,000 and 59,000.
• It has 2 more hundreds than tens.
• It is an odd number.

1. a) 4 **b)** 0 **c)** 2

2. a) 23,473 **b)** 12,320 **c)** 40,208

3. a) iii **b)** ii **c)** i

4. a) 10,203 **b)** 6720 **c)** 12,320

5. a) 2007; 3427; 10,003; 12,300
b) 3420; 17,999; 20,007; 32,300

6. *Sample responses*:
a) 90,929; 92,000; 93,000
b) 93,000; 94,000; 95,000
c) 11,000; 12,000; 13,000

7. a) Yes; [*Sample response*: 0.1 = $\frac{1}{10}$; 0.01 = $\frac{1}{100}$; 100 pieces of a whole are smaller than only 10 pieces of that whole, so 0.01 < 0.1.]

b) Yes; [*Sample response*: 0.18 = $\frac{18}{100}$, but 0.08 is only $\frac{8}{100}$.]

8. a) *Sample response*: 83,923
b) 83,929
c) 83,020

Supporting Students

Struggling students

• Some students might have trouble making up clues for their numbers in the **Use What You Know** activity. You might provide clue "starters" they can use.

For example, they could use these ideas for their clues:

- The digit in the ___ place is ___ less than the digit in the ___ place.
- The number is between ___ and ____.
- The number is ____ than 20,000.

Enrichment

• Students might try to create two 5-digit numbers using all ten digits. Their three clues for each of the two numbers must make it impossible for any other 5-digit numbers to be chosen.

For example, the clues could be:

Clues for First Number

• It is between 42,000 and 44,000.

• The digit in the ones place is 8 more than the digit in the hundreds place.

•There are 8 tens.

Clues for Second Number

• It is between 30,000 and 31,000.

• The digit in the tens place is 1 greater than the digit in the ones place.

• The number is even.

The only possible solution is:

First Number: 42,189

Second Number: 30,576

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Chapter 1 Reading and Writing Numbers

6.1.1 EXPLORE: How Much is a Million?

Curriculum Outcomes	Lesson Relevance
5-A6 Millions: interpret • develop a sense of how big a million is **5-A9 Ratio and Rate: exploring informally** • understand ratio as a multiplicative comparison of two numbers or quantities of the same type	This essential exploration will help students get a sense of how much one million actually is. This is best accomplished by relating the number one million to other numbers that are meaningful to the students.

Pacing	Materials	Prerequisites
40 min	• Rulers • Pencils • Books	• 100 chhertum = Nu 1 • the number of millilitres in a glass • familiarity with the terms *equilateral triangle*, *hexagon*, and *circle*

Exploration

• Write the number 1,000,000 and read it as "one million". Tell students that 1 million is 1000 thousands. There is no need to introduce the millions place value period until the next lesson.

• Explain to students that one million is a very large number and that you want to help them understand how large it is by relating it to things they already know about.

• Ask students to work on the exploration in pairs. Make sure they realize that they need to complete only two of **parts A to D** and one of **part E or F**, but that they must do **part G**. If students finish early, invite them to do more parts.

• While you observe students at work, you might ask questions such as the following:

- *Is 1,000,000 chhertum a lot of money?* (It is a lot because it would pay our rent for almost 3 months, but at the same time it is not a lot because it would pay the rent for only 3 months, which is not even a whole year.)

- *Is 1,000,000 mm far?* (We figured out that 1,000,000 mm = 1000 m = 1 km, so it is not really very far. We can easily walk that far in about 10 minutes.)

- *How did you figure out how much space 1,000,000 exercise book pages take up?* (We counted that there were about 100 pages in one exercise book. 1000 pages would be only 10 books, so 1,000,000 pages would come from 10,000 books. 10 books fit easily in the classroom. If we made a stack of 100 books, it would not be as high as the classroom. If we made 100 stacks, that would be 10,000 books.)

- *How many millilitres of water fit in one glass? How many glasses are needed for 1000 mL?* (One glass holds about 300 mL, so 1000 mL fill 3 or 4 glasses.)

- *How did you estimate the number of lines for 1,000,000 letters?* (We decided not to use a math book; instead we used an English book because it has more words. We counted the number of letters in five different lines and used the middle number. Then we estimated how many lines would contain 1000 letters and we multiplied by 1000 to find the number of lines that contain 1,000,000 letters.)

- *Why is it helpful to figure out about 1000 students first before figuring out about 1,000,000 students?* (Because 1,000,000 is 1000 thousands, so we could multiply my answer by 1000.)

• Note that answers for **part F** may vary quite a bit, depending on the size of your school.

Observe and Assess

As students work, notice the following:

• Do they use the knowledge that 1,000,000 is 1000 thousands to solve some of the problems?

• Do they make good predictions about how much 1,000,000 is in the situations they explore?

• Do they make accurate and efficient calculations?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and discuss question such as these:

• *Which questions were easiest to solve? Why is that?*

• *Which were more difficult? Why is that?*

• *What other ways can you think of to help someone get an idea of how much one million is?*

Answers

A. i) Nu 10; *Sample response*: 100 Ch = Nu 1, so 1000 Ch = Nu 10 **ii)** Nu 10,000; *Sample response*: 1,000,000 = 1000 thousand, so 1,000,000 Ch = Nu 10 thousand, or Nu 10,000. **B. i)** Yes; *Sample response*: 1000 mm = 1 m and I can walk that in a couple of steps. **ii)** Yes; *Sample response*: 1,000,000 mm = 1000 sets of 1000 mm = 1000 m (since 1000 mm = 1m) and 1000 m = 1 km and I can easily walk that far. **C. i)** Yes; *Sample response*: Each exercise book has about 100 pages, so it would only take 10 exercise books. There are 40 students in the class, each with an exercise book, so 10 books do not take up much room. **ii)** Yes; *Sample response*: 1,000,000 is 1000 thousands, so it would take 1000 sets of 10 exercise books to have 1,000,000 pages. 1000 sets of 10 books is the same as 100 sets of 100 exercise books. We could make a stack of 100 exercise books and it would not go to the ceiling. We could make 100 stacks by making 10 rows of 10 stacks. They would fit on the floor.	**D. i)** Yes; *Sample response*: 1000 mL = 1L. That is about 4 cups of water. I drink more water than that every day. **ii)** No; *Sample response*: 1,000,000 mL = 1000 L. That is about 4000 glasses of water, which is too much for one day. **E.** *Sample responses*: **i)** 20 lines; I counted about 50 letters in a line, so 2 lines would have 100 letters and 20 lines would have 1000 letters. **ii)** 20,000 lines; if 1000, or 1 thousand letters is 20 lines, then 1000 thousand = 20,000 lines. **F.** *Sample responses*: **i)** 5 schools; In our school, there are 200 students, so 5 schools would hold 1000 students. **ii)** 5000 schools; 1,000,000 = 1000 thousands, so 1000 × 5 schools would be 5000 schools. **G.** *Sample responses*: **i)** 1,000,000 is a lot of countries. **ii)** 1,000,0000 is not a lot of grains of sand.

Supporting Students

Struggling students

• Students need to use multiplication relationships to get a sense of one million. Some students may find this difficult. You could make it simpler by letting them pick their own small object and finding out about 10, 100, 1000, and then 1,000,000 of them. It would be easiest for them to only have to deal with one dimension.

For example, they might estimate how long a line of 1000 or 1,000,000 insects might be or they might estimate how long 1,000,000 minutes might be.

Enrichment

• Students could create their own referents for 1,000,000.

For example, they might figure out about how much space 1,000,000 people would take up, about how much space 1,000,000 crayons or pieces of chalk would take up, or about how heavy 1,000,000 dogs would be compared to an equivalent weight in elephants.

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CONNECTIONS: One Million

This connection is designed to provide some history of the number 1,000,000 and to help students relate 1,000,000 to other mathematical concepts they know.

Answers

1. Yes; [*Sample response*:
A number that ends in 0 is even. I also know that 1,000,000 = 500,000 + 500,000. Since there are two groups of equal size making up the number, the number is even.]

2. Yes; [*Sample response*:
A number that ends in 0 is made up of groups of 5; you can divide it by 5 without a remainder.]

3. No; [*Sample response*:
1 megagram = 1,000,000 g
1000 g = 1 kg, so 1,000,000 g = 1000 kg, so 1 megagram = 1000 kg.
I can only lift about 20 kg.]

6.1.2 Whole Number Place Value

Curriculum Outcomes	Outcome relevance
5-A7 Place Value: whole numbers to 7 digits • read and represent whole numbers to 7 digits • generalize place-value patterns as groups of three digits called periods **5-C5 Place Value Patterns: base ten system to millions** • recognize the patterns in periods	Students need to learn how to read, write, and interpret large numbers that they encounter in their reading.

Pacing	Materials	Prerequisites
1 h	• Place Value Charts III (BLM)	• place value to the ten thousands place

Main Points to be Raised

• The digits of a number are written in groups of three called periods. The periods help us read the number.

• For whole numbers, the period with the least value is the ones (or units), followed by the thousands and then the millions. Each period allows for hundreds, tens, and ones of the period's unit value (whether ones, thousands, or millions, although we only go to one million in Class V).

• We read the numbers in each period separately.

• When you write numbers in expanded form, you can use words or symbols.

For example, you can write 1,000,002 as 1 million + 2 ones or as 1 × 1,000,000 + 2 × 1. Expanded form shows the non-zero digits, but not the zero digits.

• A place value chart can help with reading and writing numbers.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Why would you not write 1100099?* (It should only be a 5-digit number since it is not much more than 10,000)

• *Which place values change if there is one more athlete? Why?* (The ones become 0 because 9 + 1 = 10 and you regroup; the 10s become 0 because 9 tens + 1 ten = 10 tens, which you regroup to 100, and then the hundreds become 1)

• *How would the number change if there were 1001 more athletes?* (The number would change just like it did if there was 1 more athlete but the thousands digit would become 2.)

The Exposition — Presenting the Main Ideas

• Draw a place value chart without labelling the columns. Ask students to tell you what they know about the place value columns. Review the naming of the ones, tens, hundreds, thousands, and ten thousands places. Talk about the fact that each set of three places or columns is called a period. The hundreds, tens, and ones make up the ones period. Although it is possible to call the ones period the units period, it is better not to do so because you might also think of thousands as the unit for the thousands period, of millions as the unit for the millions period, and so on.

• Write a 5-digit number such as 42,416 and ask students to read it. Have them notice that they read the 42 just as they would if it were in the ones period, but that they add the word "thousand" at the end. Ask them how they think one might read 142,416. Then introduce the hundred thousands place. Point out how the thousands period has hundreds, tens, and ones, just as the ones period does.

• Inform the students that there is another period to the left of the thousands called the millions. Introduce the one millions place. Remind students that they learned about one million in the previous lesson. Show how it is written as 1,000,000 (with six zeros).

• Remind students how they have written 5-digit numbers in both standard and expanded form. Tell them 7-digit numbers can also be written in this way.

For example, 1,142,034 could be written as 1 million + 1 hundred thousand + 4 ten thousands + 2 (one) thousands + 3 tens + 4 ones. Indicate that the word "one" in "2 one thousands" is optional.

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• Encourage students to read the charts in the exposition along with the phrases below them to make sure the notion of expanded form is clear to them.

Revisiting the Try This

B. Show students that because the number of athletes was less than seven digits but more than three digits, they would only use the thousands period and the ones period.

Using the Examples

• Write the questions for both examples on the board. Ask students to work on them and then to check their work against the solutions and thinking in the text.

Practising and Applying

Teaching points and tips

Q 1: It may help for students to use the place value chart and then regroup.

For example, for 10 ten thousands, they could write the number 10 in the ten thousands column and do any required regrouping.

Q 4: Expanded form could involve both numbers and words, or only numbers.

For example, 3,422,006 could be 3 millions + 4 hundred thousands + 2 ten thousands + 2 (one) thousands + 6 ones or it could be $3 \times 1{,}000{,}000 + 4 \times 100{,}000 + 2 \times 10{,}000 + 2 \times 1000 + 6 \times 1$ (or + 6).

Q 6: Make sure that students use all seven of the digits from 0 to 6. They should realize that there are many possibilities.

Q 7: Students could use a place value chart. They should realize, for example, that a number 100 times as great requires them to move the digits of the number 2 places to the left.

Q 9: The purpose of this question is to draw students' attention to the convenience of the place value system we use.

Common errors

• Some students write numbers as they hear them.

For example, for "two hundred thousand eight," they would write 200,0008.

Help them understand which parts of what they hear translate into digits and which parts simply tell about the placement of the digits.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can identify the various place value columns
Question 3	to see if students can write the standard form of numbers described in words
Question 4	to see if students can write numbers in expanded form
Question 5	to see if students can use reasoning about place value

Answers

A. i) 11,099 **ii)** 11,100	**B.** Thousands and ones
1. a) Thousands **b)** Hundred thousands **c)** Hundred thousands **d)** Hundred thousands **2. a)** 4 **b)** 2 **c)** 3 **3. a)** 1,020,000 **b)** 404,020 **c)** 70,212 **d)** 4,200,000	**4. a)** 3 (one) millions + 4 hundred thousands + 2 ten thousands + 2 (one) thousands + 6 ones **b)** 8 (one) millions + 2 ones **c)** 3 hundred thousands + 4 ten thousands + 2 (one) thousands + 1 hundred **d)** 6 (one) millions + 2 hundreds + 3 ones

Answers [Continued]

5. 5; [Each part in expanded form represents a non-zero digit so if there are 2 parts and 7 digits, 5 must be zero.] **6.** *Sample responses*: **a)** 4,123,560 **b)** 3,124,560 **c)** 5,624,130	**7. a)** 2,000,000; 2 million **b)** 3,000,000; 3 million **8.** *Sample response*: 1,420,000 or 2,570,234 [**9.** *Sample response*: Every time we have more than ten, we regroup using a different place value column and so we do not need any more digits.]

Supporting Students

Struggling students

• You may want to use a more specific example in **question 5** for struggling students.

For example, ask about 2 millions + 3 thousands or 5 hundred thousands + 2 ten thousands.

• You may choose to lead a discussion that addresses **question 9** with the whole group rather than assigning it to individual students.

Enrichment

• Invite students to write 7-digit numbers and create clues for other students to guess their numbers.

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6.1.3 Renaming Numbers

Curriculum Outcomes	Outcome relevance
5-A6 Millions: interpret • interpret millions in different ways and justify the interpretation (e.g., 1,500,000 = $1\frac{1}{2}$ million = 1.5 million) **5-A7 Place Value: whole numbers to 7 digits** • read and represent whole numbers to 7 digits • generalize place value patterns as groups of three digits called periods	It is important for students to be able to rename numbers in order to use multiplicative relationships that become the basis for proportional reasoning. For example, by knowing that one million is 10 hundred thousands, students can more easily compare populations.

Pacing	Materials	Prerequisites
1 h	• Place Value Charts III (BLM)	• standard and expanded forms • representing place value to the millions • decimal tenths, hundredths, and thousands

Main Points to be Raised

• A number can be named many ways by changing the unit; the name does not change the number's value.

For example, you can write 2 hundreds as 20 tens or as 0.2 thousands.

• One way to rename numbers is to use a place value chart. Write the number in standard form and then place a decimal point at the appropriate place value in order to read it.

For example, to write 13,500 as tens, place a decimal point just to the right of the tens place and read 1350 tens. To read 13,500 as hundred thousands, place a decimal point just to the right of the hundred thousands place to see that it is 0.135 hundred thousands.

• To change into standard form a number written using a unit other than one, you place the digits on the place value chart based on the given unit and then fill in any necessary zeros.

For example, to write 2.4 ten thousands in standard form, place a 2 in the ten thousands place, a 4 immediately to the right (in the thousands place) and then fill in the zeros for hundreds, tens, and ones.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. You may want to put a place value chart up on the board.

• While you observe students at work, you might ask questions such as the following:

- *How did you know 5000 was more than 100 tens?* (100 tens is 10 groups of 100, which is only 1000. 5000 is more than that.)

- *Why did you multiply the number of thousands by 10 to get hundreds?* (There are 10 hundreds in 1000.)

- *How could you use a place value chart to show how many hundreds 2 ten thousands is?* (I would write 2 in the ten thousands column and put zeros to the right. I would then cover the zeros to the right of the hundreds to read the number. It would be 200.)

The Exposition — Presenting the Main Ideas

• Work through the exposition with the students. Help them see how to place a decimal point just to the right of the place value of interest to read the number. This is a way to think of that number as the unit. Show them that, for example, 1,400,000 can be read as 14 hundred thousands or as 1.4 millions.

• Write the number 345,200 on a place value chart. Have students read it many ways:

- 345.2 thousands (by placing the decimal point just to the right of the thousands place)

- 3452 hundreds (by placing the decimal point just to the right of the hundreds place)

- 3.452 hundred thousands (by placing the decimal point just to the right of the hundred thousands place)

- 34.52 ten thousands (by placing the decimal point just to the right of the ten thousands place)

- 0.3452 millions (by placing the decimal point just to the right of the millions place)

[Continued]

Note that students will not yet be able to read 0.3452 as "three thousand four hundred fifty-two ten thousandths of a million", but they can say "decimal (or point) three four five two."

• Now write a number using a different unit, for example, 2.3 ten thousands. Have students figure out how to place the digits on the place value chart to write the number in standard form. They would place the 2 in the ten thousands column, the 3 immediately to the right, and then they would add zeros until they get to the ones place.

Thousands			Ones		
Hundred	Ten	One	Hundred	Ten	One
	2	3	0	0	0

• Repeat with another value.

For example, write 0.23 millions in standard form by putting 0 in the millions place, 2 in the column to the right, 3 in the column to the right of that, and zeros in the places from the thousands place to the ones place.

Revisiting the Try This

B. Encourage students to explain their thinking to a partner.

C. You may need to support students by giving them examples.

For instance, ask why it might be easier to interpret 2.1 million than to interpret 2,100,000.

Using the Examples

• Write the questions in **examples 1 and 2** on the board. Have students work on the questions alone or in pairs and then check their work against the solutions in the student text.

Practising and Applying

Teaching points and tips

Q 1: Observe whether students realize that in each case, if the numbers are written on a place value chart, the decimal point goes immediately to the right of the millions place. Also observe whether they consider which numbers will be greater than 1 million and which will be less than 1 million as a way of checking their own work.

Q 2: Observe whether the students realize that the whole number digit (either 0 or 1 in these questions) is written in the millions column in each case.

Q 3: Students can use expanded form with words or symbols.

For example, they could write 0.2 million as 2 hundred thousands or as 2 × 100,000.

Q 5: Make sure students understand that the boxed entry is the headline.

Q 6: Make sure students understand that each number is to be written in the form “___ million”.

Q 8: Encourage students to think of at least three or four ways to rewrite the value.

Common errors

• Students may place the decimal point to the left instead of to the right of the place value column they should be using to read a set of values. Have them look at their answers to see if they make sense.

For example, to write 2.13 hundred thousand in standard form, they should be looking for a result that has 2 in the hundred thousands column.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can write a number written in a unit other than ones in standard form
Question 6	to see if students can write a number in standard form using unit other than ones
Question 7	to see if students can reason about predicting an alternate form for a number

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Answers

A. ii) 30 **iii)** 500 **iv)** 200 **B.** *Sample response*: For **part ii)**, if I wrote a 4 in the hundreds place and looked left from the tens place, I would see that it was 40 tens.	**C.** It is useful for recognizing numbers more quickly because you do not have to count all the zeros. It is also useful for adding numbers in different units. For example, 300 000 + 2,450,000 could be 2.45 million + 0.3 million = 2.75 million.
1. a) 0.1 **b)** 0.01 **c)** 0.5 **d)** 0.03 **2. a)** 900,000 **b)** 40,000 **c)** 1,100,000 **3. a)** 1 million + 8 ten thousands **b)** 3 million + 2 ten thousands **c)** 2 hundred thousands **d)** 6 hundred thousands **4. a)** 2,010,000 **b) i)** 1 **ii)** 0 **iii)** 2 **iv)** 0 **v)** 0 **vi)** 0	**5.** A, B, and C **6. a)** 0.1 million **b)** 0.02 million **c)** 0.4 million **d)** 0.04 million **e)** 0.3 million **f)** 1.4 million [**7.** *Sample response:* 0.3 million is an exact number of hundred thousands, so there are 0 ten thousands, 0 thousands, 0 hundreds, 0 tens, and 0 ones; that is 5 zeros.] **8.** *Sample response*: 2.1 million, 21 hundred thousands, and 210 ten thousands

Supporting Students

Struggling students

• You may wish to display on a poster or on the board examples of moving from standard form to a form with a different unit, and vice versa. Students can follow these as they work through the exercises.

For example, you might show how 0.23 ten thousands is rewritten as 2300 and how 120,000 can be written as 120 thousands.

• Struggling students might find **question 7** difficult. You may choose to allow them to simply show what 0.3 million is rather than to explain how they could have predicted, in advance, how many zeros there would be.

Enrichment

• Invite students to describe a variety of numbers that meet a certain condition related to renaming.

For example, you might ask students to write five numbers in standard form that meet each of these conditions:

- they can be written as 0.0[ ][ ] millions (for example, 22,000 or 35,000, etc.)

- they have a 0 in the hundredths place when they are written as ten thousands (for example, 1.201 ten thousands would be 12,011; they would always have a 0 in the hundreds places)

- they can be written as 1.[ ]1 hundred thousands (for example 131,000)

6.1.4 Comparing and Ordering Numbers

Curriculum Outcomes	Outcome relevance
5-A8 Comparing: order 7-digit whole numbers • compare and order numbers up to 7 digits - in standard form (e.g., 3,256,876 > 3,255,996) - in decimal form (e.g., 3.25 million > 34.3 million) - in standard and decimal form (e.g., 3,256,876 < 3.2 million) - with different place value (e.g., 3420 thousand > 3,325,146 and 342 thousand < 2 million)	Students need to extend their ability to compare smaller numbers so that they will also be able to compare greater values.

Pacing	Materials	Prerequisites
1 h	• One die	• recognizing which value is greater on a number line • comparing numbers with five or fewer digits • expanded form • decimal tenths and hundredths

Main Points to be Raised

• A number that is farther to the right on a number line is greater.

• To place numbers on a number line, it is convenient to use benchmarks.

• To compare two whole numbers, start at the column on the left and begin comparing digits in that column; work toward the right until a digit one number is greater than the corresponding digit in the other number.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know the first digit on the right cannot be 9?* (The number would then be more than 99 thousand. A number that is around 82 thousand is less than 99 thousand, not more.)

• *Why would you not put a 0 as the digit on the right?* (When we write whole numbers, we do not put zeros on the left side of them.)

• *Why could the digit in **part ii)** be either 8 or 9?* (Because 82 thousand is less than both 89 thousand and 99 thousand.)

• *Why does the digit on the left not matter?* (Because I need to compare only the thousands; once I have compared those, the hundreds do not matter.)

The Exposition — Presenting the Main Ideas

• Begin by playing a game. Have students each mark seven blanks on a piece of paper. Tell them you will roll a die and call the digit rolled. They must place the digit on one of the blanks. They cannot move it later. Repeat this until seven digits have been called. The student with the greatest value wins the game.

- You might play again. This time indicate that the student with the greatest value between 3 million and 4.1 million will win.

- This game will allow students to consider number comparisons informally before you present the concepts formally.

• Draw an unmarked number line on the board. Tell students that you will be placing numbers between 1 million and 10 million on the number line. Ask them how they would mark the line to make it easier to place the numbers (for example, place marks at 1 million, 2 million, 3 million, …, 9 million, 10 million). Tell students that these helping numbers are called *benchmarks*. You use benchmarks to make sense of the other numbers.

- Write the number 3,122,000 on the board. Ask students where to place the number 3,122,000 on the number line and see if they realize that it should be fairly close to 3 million, but slightly to the right.

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- Then write the number 4,250,000 and ask where to place it on the number line. See if they realize it is about one quarter of the way between 4 million and 5 million.

- Discuss how they know which of the two values is greater. Make sure they realize they could either have pointed out that 4,250,000 was farther to the right on the number line, or they could have compared the numbers by observing that a number with 4 millions must be greater than a number with only 3 millions.

• Repeat the activity above. This time tell them you will be placing numbers between 3 hundred thousand and 4 hundred thousand. Ask how the number line might be marked (e.g., at 300,000, at 310,000, at 320,000, etc., up to 400,000). Have them place the numbers 312,400 and 345,600 and then compare them.

• Finally, ask pairs of students to think about how they would compare numbers like 2.1 hundred thousand and 0.4 million. See if they realize that it would be more convenient either to write both as hundred thousands (2.1 hundred thousand and 4 hundred thousand) or to write both as millions (0.21 millions and 0.4 millions) in order to compare them. Some students may choose to write both in standard form. That is also acceptable, although less efficient.

• Tell students that these ideas they have discussed are described in the exposition on **pages 181 and 182**. They can read these pages to make sure they understand.

Revisiting the Try This

B. Observe how students mark the number line they are using. A useful number line might be marked at each ten thousand.

Using the Examples

• Present the problems from **example 1** and **example 2** on the board for students to try. They should then compare their work with the solutions in the text. Make sure that they realize that to order the numbers in **example 2**, they must consider all the numbers.

Practising and Applying

Teaching points and tips

Q 2: Notice whether students are always using the same strategy to compare, for example, only the place value chart, and encourage them to consider other strategies.

Q 3: If students choose to write each value in standard form to compare them, this is acceptable. However, you might encourage them to see if they can compare the numbers by writing each value using the same unit, whether millions or some other unit.

Q 6: The numbers to list can be any values between 2,100,000 and 2,153,197.

Q 7: Some students might choose to write all three numbers in standard form to compare them. Others will write all as millions (e.g., 3.2 million, 3.1 million, 3.19 million, and 3.20001 million). Many students will struggle to decide whether 3.19 million or 3.20001 million is closer to 3.2 million. It might help to realize that **C** is only 10 away, whereas **B** is 10,000 away from 3.2 million.

Q 8: Encourage students who are not struggling to think of as many possibilities as they can.

Q 9: You may want to deal with this question as a large group. You could begin by asking about these pairs of numbers:

1.2 million vs. 1.2 hundred thousand

3.8 million vs. 3.8 hundred thousand

0.1 million vs. 0.1 hundred thousand

0.1 million vs. 0.3 hundred thousand

0.1 million vs. 4.2 hundred thousand

Common errors

• Some students will not pay attention to the units; they will just compare the numerical values.

For example, when they see 78 ten thousands and 2 million, they think 78 > 2, so 78 ten thousands are more. You might have them think about why 2 km is longer than 30 cm even though 2 < 30. Then point out that to use different number units is the same idea. It is not only the number that matters, but the unit also matters.

Suggested assessment questions from Practising and Applying

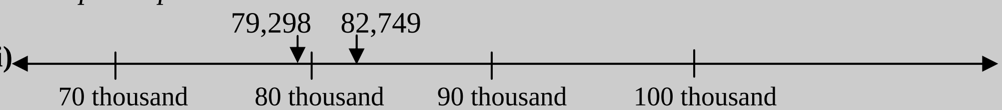
Question 2	to see if students can order numbers given in standard form
Question 4	to see if students can order numbers written with place value units other than ones
Question 7	to see if students can reason about number comparisons
Question 8	to see if students can solve a problem involving number comparisons

Answers

A. i) 1, 2, 3, 4, 5, 6, or 7 **ii)** 8 or 9

B. *Sample responses*:

i)



ii)

Millions			Thousands			Ones		
		One	**Hundred**	**Ten**	**One**	**Hundred**	**Ten**	**One**
				8	2	9	4	9
				9	9	2	9	8

9 > 8 so 99,298 > 82,949

1. a) 3.4 million
b) 15 hundred thousand
c) 5,213,478

2. a) 899,789 < 3,487,799 < 6,000,000
b) 213,867 < 762,813 < 2,013,687

3. a) 2.1 million
b) 0.4 million
c) 275 ten thousand

4. a) 78 ten thousand < 14 hundred thousand < 2 million
b) 2.3 hundred thousand < 0.6 million < 3,150,000

5. *Sample response*:

Millions		Thousands			Ones		
	One	Hundred	Ten	One	Hundred	Ten	One
	4	2	0	0	0	0	0
	5	0	0	0	0	0	0

4 million < 5 million, so 42 hundred thousand < 5 million

6. *Sample response*:
2,100,001; 2,100,002; 2,100,003

7. C

8. a) and b) *Sample response*:
0 and 4; 9 and 4
[The second missing digit could be 4, 5, 6, 7, 8, or 9 no matter what the first missing digit is.
If the first missing digit is 0 or 1 or 2, the second missing digit could also be 3.]

9. *Sample response*:
a) 4.1 million > 4.0 hundred thousand
b) 0.4 million = 4.0 hundred thousand
c) 0.9 million < 4.0 hundred thousand

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Supporting Students

Struggling students

• Struggling students might have difficulty with **questions 7, 8, and 9**.

- You may wish to work through **question 9** as a full class.

- You may choose not to assign **question 7** to struggling students.

- You may encourage them to work through **question 8** by modelling different ways to include digits. They might find only one pair rather than two pairs.

Enrichment

• Invite students to create other questions like **questions 7** and **8** for fellow students to solve.

GAME: Target 7

• Before students play this game, you or the students need to prepare sets of digit cards using slips of paper. You need three sets of the ten cards shown. Alternatively, you can use decks of playing cards. Use cards from only three suits and remove the face cards. The 10s could be used to represent 0s and the Aces could be used to represent 1s.

• Players must make their predictions before any digits of the third card are displayed. The reason that the third number is displayed from right to left is to add suspense to the game. It is not until the last card is turned over that each student will know whether his or her prediction is correct.

• Provide place value charts for students to use to check their work.

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Chapter 2 Number Relationships

6.2.1 Renaming Numbers Using Multiplication

Curriculum Outcomes	Outcome relevance
5-A9 Ratio and Rate: exploring informally • understand ratio as a multiplicative comparison of two numbers or quantities of the same type • understand rate as a multiplicative comparison of two quantities described in different units • explore ratio and rate in geometric, numerical, and measurement situations	Concepts of ratio and rate are critical to activities of everyday life, such as comparing prices and calculating percentages. Work in this unit sets the stage for later work with ratios and rates.

Pacing	Materials	Prerequisites
1 h	None	• writing a product in different ways

Main Points to be Raised

• You can write a product as an equivalent product to make calculations easier.

For example, it is easier to calculate 2 × 100 than to calculate 4 × 50, but the products are equal.

Similarly, it is easier to calculate quotients by writing the dividend as an appropriate product, e.g., for 200 ÷ 4, write 200 as 50 × 4.

• When one shape is made up of multiple copies of another shape, you can use a product to relate their area measurements.

• You can use multiplication relationships to help rename measurements.

For example, you can write kilometres as metres by multiplying by 1000.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How many dots are in the top group of dots? in the bottom group?* (4 and 8)

• *What do all three pictures have in common?* (They all show 4 in one group and 8 in the other group.)

• *What is the relationship between 4 and 8?* (8 is 4 more than 4, but it is also twice as much as 4.)

The Exposition — Presenting the Main Ideas

• Present the question 4 × 50. Ask students what this amount represents, not what the answer is (4 groups of 50). Ask how they would write the product if two of the groups were combined to make two large groups (2 × 100). Ask why 2 × 100 is very easy to calculate mentally.

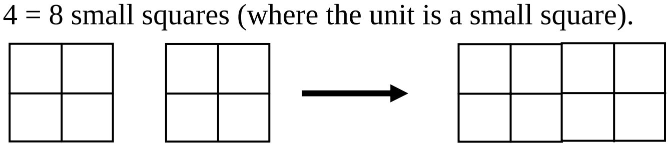
• Talk about how it is sometimes easier to calculate if you regroup items.

For example, ask students how they might regroup 12 groups of 25 (12 × 25) to make calculation of the total simpler. One possibility is 3 groups of 100, where 4 groups of 25 are combined to become 1 group of 100.

• Point out how this is like using a different unit, for example, 4 units of 50 is the same as 2 units of 100. Mention how this can be applied to measurement units. Ask how to write 3 km using a metre unit. Students know that 1 km = 1000 m, so 3 × 1 km = 3 × 1000 m = 3000 m.

• Show how these relationships can also be applied to shapes.

For example, draw a large square with four small squares inside it. Combine it with an identical shape. Point out that the area of the combined shape area can be described as either 2 large squares (where the unit is a large square) or 2 × 4 = 8 small squares (where the unit is a small square).



• Ask students to read through the exposition and ask any questions they might have.

Revisiting the Try This

B. Help students see the importance of considering 8 as 4×2 rather than as $4 + 4$ to answer this question.

Using the Examples

• Assign students to groups of three. Ask each student in each group to become an expert in one of the three examples. Each student must then explain his or her example to the other two students in the group.

Practising and Applying

Teaching points and tips

Q 2: Students may try to show the equality by showing that the products on both sides of the equality signs are the same. Encourage them instead to go back to what each expression means.

For example, 8×16 means 8 groups of 16. If the 8 groups are separated into 2 sections, there are 2 sections, each with 4 groups of 16, or $2 \times 4 \times 16$.

Q 4: Although students may want to calculate each product separately, they should be using the relationship to 6×7 each time.

Q 6 b): You may need to reassure students that each of the three L-shapes in **part b)** is identical to the L-shape in **part a)**.

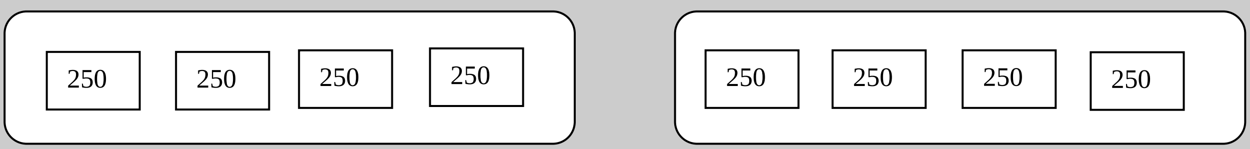
Suggested assessment questions from Practising and Applying

Question 2	to see if students can recognize why two products are equal
Question 4	to see if students can calculate one product using what they know about another product
Question 5	to see if students can use multiplication relationships to rename measurements

Answers

A. *Sample response:*
Each picture shows that 8 is two fours, or $8 = 2 \times 4$.

B. *Sample response*:
8×250 means 8 groups of 250.
Since $8 = 2 \times 4$, put 4 groups of 250 together at a time (which is 2 groups of 4 groups of 250) and you get 2 groups of 1000.

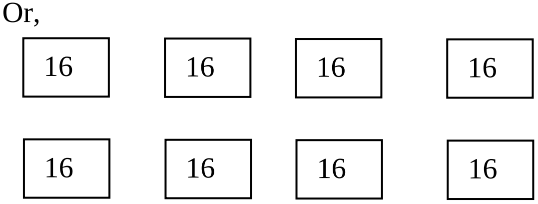


So $8 \times 250 = 2 \times 4 \times 250 = 2 \times 1000 = 2000$.

1. a) 10 **b)** 6, 9
c) 54, 108 **d)** 130, 13

[**2.** *Sample responses*:
a) $8 = 2 \times 4$ so $8 \times 16 = 2 \times 4 \times 16$.

Or,



There are 8 groups of 16, but there are 2 rows of 4 groups of 16.

2. b) 10 groups of 52 can be shown as 5 groups of 52 + 5 groups of 52. Since the groups are equal, half of 10 groups of 52 is 5 groups of 52.
c) $12 \times 100 = 12 \times (4 \times 25) = (12 \times 4) \times 25 = 48 \times 25$]

3. a) 70 km
b) 105 km
c) 175 km

4. a) $6 \times 7 = 42$
b) i) 84; [$12 \times 7 = 2 \times (6 \times 7) = 2 \times 42 = 84$]
ii) 420; [$6 \times 70 = (6 \times 7) \times 10 = 42 \times 10 = 420$]
iii) 420; [$60 \times 7 = 10 \times (6 \times 7) = 10 \times 42 = 420$]
iv) 840; [$12 \times 70 = 2 \times (6 \times 7) \times 10 = 2 \times 42 \times 10 = 840$]

Answers [Continued[

5. a) 100,000 **b) i)** 200,000; [Since 1 km = 100,000 cm, then 2 km = 2 × 100,000 cm = 200,000 cm.] **ii)** 1,000,000; [Since 1 km = 100,000 cm, then 10 km = 10 × 100,000 cm = 1,000,000 cm.] **iii)** 150,000; [Since 1 km = 100,000 cm, then 1.5 km = 100,000 cm + $\frac{1}{2}$ of 100,000 = 150,000 cm.]	**6. a)** 69 m^2 [**b)** Multiply 69 m^2 units by 3 to get 207 because there are 3 copies of the shape.] **7.** *Sample responses*: **a)** Rename 6 as 2 × 3 to calculate 6 × 25: **6** × 25 = **2 × 3** × 25 = 2 × 25 × 3 = 50 × 3 = 150 **b)** Rename 1 kg as 1000 g to find how many grams 500 kg is: 500 = 500 × 1, so 500 kg = 500 × **1 kg** = 500 × **1000 g** = 500,000 g.

Supporting Students

Struggling students

• A number of students will be able to answer questions by actually performing calculations, but will have difficulty using reasoning to predict how the calculations are related in the indicated ways. You may wish to allow these students to proceed in this way. By listening to other students' reasoning, they may learn to look at the deeper relationships.

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6.2.2 Using Number Sentences

Curriculum Outcomes	Outcome relevance
5-B12 Open Number Sentences: applying number sense • explore numerical situations which are always, sometimes, or never true (e.g., 324 + □ > 300 is always true, assuming □ is a whole number) • work with open number sentences involving the four basic operations and a combination of operations • understand that □ can also be expressed as a letter variable or another shape or symbol	Using and interpreting open number sentences is one of the early steps in thinking algebraically. Work in this unit builds a foundation for later success in algebra.

Pacing	Materials	Prerequisites
1 h	None	• operations with whole numbers

Main Points to be Raised

• An open number sentence is a sentence where a missing value must be determined. It can be true sometimes, always, or never, depending on the substituted values.

• An open number sentences can involve an equality or an inequality with one side greater or less than the other.

• You can represent the variable in an open number sentence with many different symbols.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How did you figure out the missing value?* (I divided 100 by 20.)

• *How do you know that the missing value must be greater than 4?* (216 is 4 more than 212, so I have to add more than 4.)

• *How many solutions are there?* (98 because there are 98 numbers less than 98 if you include 0.)

• *How do you know the missing value is greater than 45?* (I have to be able to take away something from it to get 45, so it has to be more than 45.)

The Exposition — Presenting the Main Ideas

• Write these number sentences on the board:

8 > 3   212 < 415   3 + 4 = 7

Tell students that each is called a number sentence. It describes a relationship between numbers and it can show a greater than, less than, or equal relationship.

• Now write these open number sentences on the board:

8 > [ ]   [ ] < 415   3 + [ ] = 7

Tell students that these are called *open number sentences* because there are unknown values. The solution of each sentence is the value or set of values that makes it true.

• Ask students to explain why there are more solutions for the first two number sentences than for the third number sentence. Point out that some open number sentences have no solutions.

For example, 3 + [ ] < 2 + [ ] is never true if the same number is used in the [ ]. Similarly, 4 ÷ 0 = [ ] is never true because you cannot divide by 0.

Have students verify this by substituting many different values for [ ]. Tell them they must use the same value on both sides of the equation.

• Remind students that some open number sentences have no solutions, some have one solution, and some have many solutions.

Revisiting the Try This

B. Some students might assume that equalities always have one solution, but inequalities have more. This is not always true, although in these particular examples it is true.

Using the Examples

• Work through the two examples with the students. Have them follow the thinking in the text along with you. Answer any questions they may have. Draw attention to the fact that even though a number sentence involves, for example, addition, you might be able to solve it using other operations, for example, subtraction.

Practising and Applying

Teaching points and tips

Q 1: You might have to prompt some students by suggesting which operations or which inequality sign they might use.

For example, suggest they write one sentence involving addition and one sentence involving a less-than sign for **part a)**.

Q 3: Some students will use particular values, for example, writing that 8 = 4 × 2, whereas others will use variables, for example, writing that [ ] = 4 × *n*

Q 4: You might help students talk through what each sentence says.

For example, for **part b)**, you might say, "I add a number to 42,100 to get 51,100. What do I add?"

Q 5: Some students might simply copy sentences from other questions, whereas others might create new sentences. For students who struggle with **part c)**, suggest they model their response on this number sentence: 42 – [ ] < 32 – [ ]

Q 6: Once students have one possible sentence, they can create others by adding the same value to both sides of the equation.

Common errors

• Students will sometimes perform the operation they see in a sentence even if it is not appropriate.

For example, for 23 + [ ] = 84, they might add 23 + 84. Suggest that they check to see if their solutions work.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can solve open number sentences
Question 4	to see if students can determine the number of solutions to open number sentences
Question 5	to see if students can create open number sentences to meet conditions

Answers

A. i) 5 **ii)** Any number 5 or greater **iii)** 5 **iv)** Any number 97 or less **v)** 50 **B. i), iii), and v)** had one answer but **ii) and iv)** had more than one answer	**C.** *Sample responses*: Parts **i)** and **iii)** have one solution each. Part **iv)** has 98 solutions, numbers 0, 1, 2, …, 97.
1. *Sample responses*: **a)** 18 + 162 = 180 180 – 18 = 162 **b)** 350 + 350 = 700 2 × 350 = 700 **c)** 10,000 – 8000 = 2000 10,000 > 2000 **d)** 1600 + 160 = 1760 1600 – 160 = 1440	**2. a)** 61; [*Sample response*: I would subtract 23 from 84.] **b)** 2000; [*Sample response*: I would find a number to subtract to get from 3012 to 1012.] **c)** 1000; [*Sample response*: I would find a number to multiply 3000 by to get to 3,000,000. **d)** 66; [*Sample response*: I would find the number of groups of 100 in 6600.]

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3. *Sample response*: 16 ÷ 4 = 4 4 × 3 = 12 4 × 25 = 100 **4. a)** One solution, 3002; [because there is only one number you can subtract from 6000 to get 2998.] **b)** One solution, 10,000; [because there is only one number you can add to 42,100 to get 51,100.] **c)** Many solutions, e.g., 1000, 2000, and 500; [because there are many numbers you can divide 2000 by to get a number less than 20.] **d)** No solution; [because there is no number you can multiply by 0 to get 48.]	**5.** *Sample responses*: **a)** 5 + [ ] = 8 **b)** 5 × [ ] > 20 **c)** [ ] = 20 ÷ 0 **6.** Yes; [*Sample response*: [ ] > 0] [**7.** *Sample response*: You could create a number sentence by subtracting to see how much greater one is than the other. You could create a number sentence by dividing to see how many times one number fits into the other.]

Supporting Students

Struggling students

• Some students may need more help to create open number sentences than to solve them. Help students see how to work backwards.

For example, to create an addition sentence to relate 35 and 47, students need to find a way to describe 47 in terms of 35, for example 47 = 35 + 12. They then replace the 12 with a variable.

Enrichment

• Students might be asked to create open number sentences with specific solutions.

For example, for a solution of 50, they might begin with a sentence like [ ] = 50 and then add, subtract, multiply, or divide both sides of the equation by the same amount to create other number sentences:

30 + [ ] = 80

2 × [ ] = 100

[ ] ÷ 5 = 10

UNIT 6 Revision

Pacing	Materials
1 h	• Place Value Charts III BLM)

Question(s)	Related Lesson(s)
1	Lesson 6.1.1
2 – 5	Lesson 6.1.2
6 and 7	Lesson 6.1.3
8 – 10	Lesson 6.1.4
11 – 13	Lesson 6.2.1
15 – 18	Lesson 6.2.2

Revision Tips

Q 1: Encourage students to relate the number one million to actual distances, times, etc. that they know rather than just using symbolic relationships.

Q 2: Students can use words and symbols or symbols alone to write numbers in expanded form.

Q 6: Emphasize that it is the form of the number that is of issue, not so much what kind of number might be written that way.

Q 9: This can be completed by writing both numbers in standard form, but encourage students to compare the numbers using other units, for example, hundred thousands or millions.

Q 15: Students do not need not to solve these questions; they must only explain how to solve them. This allows you to see whether they understand the process separately from whether they can carry it out.

Answers

1. *Sample response*:
It is 1000 thousands.
It is 500,000 + 500,000.
It is the amount of millilitres of water in about 3500 drinking glasses.

2.a)

Millions			Thousands			Ones		
		One	Hundred	Ten	One	Hundred	Ten	One
i)			3	1	4	2	1	4
ii)		1	0	0	3	4	1	2

b) i) 3 hundred thousands + 1 ten thousand + 4 (one) thousands + 2 hundreds + 1 ten + 4 ones

ii) 1 million + 3 (one) thousands + 4 hundreds + (one) ten + 2 ones

c) i) three hundred fourteen thousand, two hundred fourteen
ii) one million, three thousand, four hundred twelve

3. a) 234,005; *Sample response*:
2 hundred thousands + 3 ten thousands +
4 (one) thousands + 5 ones
b) 145,032; *Sample response*:
1 hundred thousand + 4 ten thousands +
5 (one) thousands + 3 tens + 2 ones
c) 2,030,003; *Sample response*:
2 millions + 3 ten thousands + 3 ones
d) 4,020,030; *Sample response*:
4 millions + 2 ten thousands + 3 tens

4. a) Millions **b)** Hundreds
c) Ones **d)** Hundred thousands

5. *Sample response*:
a) 1,420,000
b) 9,000,000
c) 1,180,213

[**6.** *Sample response*:
To compare with another number written in millions.]

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7. a) 2,100,000 **b)** 3,100,000
c) 300,000 **d)** 50,000

8. a) 87,146; [*Sample response*:
They are both whole numbers and 87,146 has fewer digits.]
b) 3,152,110; [*Sample response*:
It has the same number of millions but fewer hundred thousands.]
c) 417,000; [*Sample response*:
It has fewer hundred thousands.]
d) 345,789; [*Sample response*: It has fewer digits.]

9. a) 123,450 **b)** 2 million
c) 8 hundred thousands **d)** 0.02 million

[**10.** *Sample response*:
It could have fewer digits, for example, $99 < 111$.]

11. a) 10 **b)** 10,000
c) 61 **d)** 220 **e)** 90, 40

12. *Sample responses*:
a) 90
b) i) 900; [Since $30 = 3 \times 10$, then 30×30 is 10 sets of 3 groups of 10, or 10 groups of 90.]
ii) 180; [6×30 is 6 groups of 30. That is 2 sets of 3 groups of 30. $2 \times 90 = 180$.]
iii) 9000; [Since 3×3000 is 3 groups of 3000 and since 3000 is 3 thousands, this is 3 groups of 3 thousands. That is 9 thousands.]
iv) 360; [$6 \times 60 = 2 \times 3 \times 30 \times 2$; that is 2 groups of 90 twos; 90 twos = 180 and $2 \times 180 = 360$.]

13. a) 180 **b)** 600 **c)** 90

14. *Sample response*:
$70 = 210 \div 3$
$70 + 140 = 210$
$3 \times 70 = 210$

[**15.** *Sample responses:*
a) I would find a number to add to 56 to get 63.
b) I would find a number to add to 37 to get 94.
c) I would figure out how many tens 650 is.
d) I would divide 400 by 8.]

16. a) 7 **b)** 57
c) 65 **d)** 50

17. a) More than one
b) One
c) More than one
d) No solutions

18. *Sample response*: $100 + [\ ] > 50$

UNIT 6 Number Test

You can use this place value chart to help you.

Millions	Thousands			Ones		
One	**Hundred**	**Ten**	**One**	**Hundred**	**Ten**	**One**

1. a) Copy the place value chart and put each number on it.

i) 1,000,320
ii) 417,369
iii) 622 thousand
iv) 3 million, 16 thousand
v) 4 million, 200 thousand, 351

b) Write each number in expanded form.
c) Write the number word for each.

2. Which place value is each digit in the number 1,602,379?

a) 2 **b)** 1
c) 6 **d)** 0

3. Write a number in standard form for each.

a) 2 fewer hundred thousands than millions
b) 6 more millions than ten thousands

4. Write each in standard form and in expanded form.

a) 17 hundred thousand
b) 25 ten thousand
c) 0.4 million
d) 0.02 million

5. Which number in each pair is greater? How do you know?

a) 3,152,400 or 3,512,400
b) 124,317 or 142,317

6. Which number in each pair is less?

a) 0.5 million or 225,000
b) 17 hundred thousand or 2 million
c) 300 ten thousand or 7 hundred thousand

7. a) Calculate 6 × 50.

b) Use your answer for **part a)** to calculate each. Show your work.

i) 60 × 50 **ii)** 6 × 500

8. Copy and complete.

7 days = 1 week
__ days = 4 weeks
70 days = __ weeks

9. Create three number sentences that use the numbers 50 and 400.

10. a) How would you solve each?

i) 34 + × = 74
ii) 97 – ▲ = 35
iii) 4200 ÷ 10 = ■
iv) 6 × ∆ = 360

b) Solve each number sentence in **part a)**.

11. Does each number sentence have one solution, more than one solution, or no solutions? Explain.

a) 37 < ▲
b) 62 + × = 87
c) 45 + ■ = 30 + ■ + 5
d) 60 – ∆ > 55 – ∆

12. Create a subtraction number sentence that has more than one solution. Solve your number sentence.

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UNIT 6 Test

Pacing	Materials
1 h	• Place Value Charts III (BLM)

Question(s)	Related Lesson(s)
1 – 3	Lesson 6.1.2
4	Lesson 6.1.3
5 and 6	Lesson 6.1.4
7 and 8	Lesson 6.2.1
9 – 12	Lesson 6.2.2

Select questions to assign according to the time available.

Answers

1. a)

	Millions	Thousands			Ones		
	One	**Hundred**	**Ten**	**One**	**Hundred**	**Ten**	**One**
i)	1	0	0	0	3	2	0
ii)		4	1	7	3	6	9
iii)		6	2	2	0	0	0
iv)	3	0	1	6	0	0	0
v)	4	2	0	0	3	5	1

b) i) 1 (one) million + 3 hundreds + 2 tens
ii) 4 hundred thousands + 1 ten thousand + 7 (one) thousands + 3 hundreds + 6 tens + 9 ones
iii) 6 hundred thousands + 2 ten thousands + 2 (one) thousands
iv) 3 (one) millions + 1 ten thousand + 6 (one) thousands
v) 4 (one) millions + 2 hundred thousands + 3 hundreds + 5 tens + 1 one

c) i) One million, three hundred twenty
ii) Four hundred seventeen thousand, three hundred sixty-nine
iii) Six hundred twenty-two thousand
iv) Three million, sixteen thousand
v) Four million, two hundred thousand, three hundred fifty-one

2. a) Thousands place
b) Millions place
c) Hundred thousands place
d) Ten thousands place

3. *Sample responses*:
a) 4,200,000 **b)** 8,020,047

4. a) 1,700,000; 1 (one) million + 7 hundred thousands
b) 250,000; 2 hundred thousands + 5 ten thousands
c) 400,000; 4 hundred thousands
d) 20,000; 2 ten thousands

5. *Sample responses*:
a) 3,512,400; they have same number of millions but **3,5**12,400 has more hundred thousands than **3,1**52,400
b) 142,317; they have the same number of hundred thousands but **14**2,317 has more ten thousands than **12**4,317

6. a) 225,000
b) 17 hundred thousand
c) 7 hundred thousand

7. a) 300
b) i) 3000; *Sample response:*
Since 60 = 10 × 6, 60 × 50 = (10 × 6) × 50 = 10 × (6 × 50) = 10 × 300 = 3000
ii) 3000; *Sample response:*
6 × 500 = 6 × (5 × 100) = (6 × 5) × 100 = 30 × 100 = 3000

8. 28 days; 10 weeks

9. *Sample response:*
50 + 350 = 400
50 × 8 = 400
400 > 50

Answers [Continued]

10. a) i) Subtract 34 from 74 **ii)** Subtract 35 from 97 **iii)** Divide 4200 by 10 **iv)** Divide 360 by 6 **b) i)** 40 **ii)** 62 **iii)** 420 **iv)** 60	**11. a)** More than one solution; any number greater than 37 **b)** One solution; 25 **c)** No solution **d)** More than one solution; any (whole) number **12.** *Sample response:* $15 - x > 10$; $x = 0, 1, 2, 3, 4$

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UNIT 6 Performance Task — Numbers to Describe Our World

A. *How many grains of salt are there in a cup of salt?*

Use these clues to estimate:

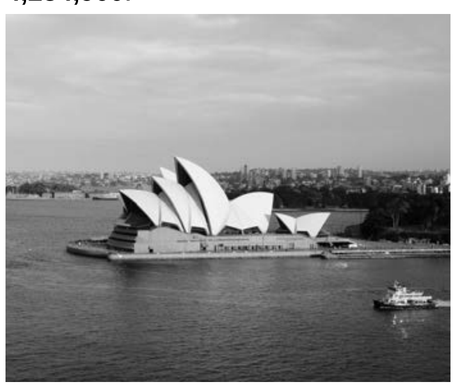
- It is one solution of ■ < 20 hundred thousand.
- It has only one non-zero digit in the thousands period.
- It can be renamed as a whole number of hundred thousands.
- If you add the digits, the sum is 3.

B. All the clues in **part A** are needed to figure out the number.
If one clue were missing, you might get a different number.
Find a different number for each set of clues.

i)
- It is one solution of ■ < 20 hundred thousand.
- It has one non-zero digit in the thousands period.
- It can be renamed as a whole number of hundred thousands.

ii)
- It is one solution of ■ < 20 hundred thousand.
- It has only one non-zero digit in the thousands period.
- If you add the digits, the sum is 3.

C. **i)** Choose one of the numbers in the shaded part of the chart.

110,000	4,254,900	7,393,538
A person has about **110,000** hairs on her head. 	The population of Sydney, Australia is **4,254,900**.  The Opera house in Sydney	The height of Mount Jumolhari is **7,393,538** mm. 

ii) Create a set of three or more clues to describe your number.
Your clues should include

- an open number sentence
- place value language
- renaming a number
- comparing your number with another number

iii) Are all of your clues necessary? Explain your thinking.

UNIT 6 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
5-A7 Place Value: whole numbers to 7 digits **5-A8** Comparing: order 7-digit whole numbers **5-A9** Ratio and Rate: exploring informally **5-B12** Open Number Sentences: applying number sense **5-C5** Place Value Patterns: base ten system to millions	1 h	None

How to Use This Performance Task

You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. It could also be used as enrichment material for some students. You can assess performance on the task using the rubric on the next page.

Sample Solution

A. 1,200,000 **B. i)** Without the last clue, the number could be 1,300,000. **ii)** Without the third clue, the number could be 1,100,001. **C. i)** *Sample response:* 4,254,900 **ii)** *Sample responses:* The number is a solution of [ ] < 5,000,000. The sum of the digits in the thousands period is 11. The number can be renamed as a whole number of hundreds. The sum of all of the digits is 24. The digit in the thousands place is 2 more than the digit in the hundred thousands place. The ten thousands digit is 5.	**C. iii)** Yes; • Without the first clue, the first digit could be more than 4. • Without the third clue, the last two digits might not be 0. • Without the second and fourth clues, there is no way to know that the digits in the millions and hundreds place are 9 and 4. • Without the last clue, the middle digits could be different, for example, 416 instead of 254.

UNIT 6 Performance Task Assessment Rubric

The student	Level 4	Level 3	Level 2	Level 1
Compares numbers	Creates one or more appropriate clues involving comparison and insightfully identifies which are essential	Creates one or more appropriate clues involving comparison and correctly identifies which are essential	Creates one or more appropriate clues involving comparison and/or correctly interprets a given number comparison	Incorrectly uses a clue involving comparison and/or incorrectly identifies whether it is essential
Uses place value concepts	Creates one or more appropriate clues involving place value, insightfully identifies which are essential, and uses place value relationships appropriately	Creates one or more appropriate clues involving place value, correctly identifies which are essential, and uses place value relationships appropriately	Creates one or more appropriate clues involving place value and/or correctly interprets given place value information	Incorrectly uses a clue involving place value and/or incorrectly identifies whether it is essential
Uses open sentences	Creates one or more appropriate clues involving an open sentence and insightfully identifies which are essential	Creates one or more appropriate clues involving an open sentence and correctly identifies which are essential	Creates one or more appropriate clues involving an open sentence and/or correctly solves a given open sentence	Incorrectly uses a clue involving open sentences and/or incorrectly identifies whether it is essential
Renames one number in terms of another	Creates one or more appropriate clues involving renaming one number in terms of another and insightfully identifies which are essential	Creates one or more appropriate clues involving renaming one number in terms of another and correctly identifies which are essential	Creates one or more appropriate clues involving renaming one number in terms of another and/or correctly renames a given number	Incorrectly uses a clue involving renaming one number in terms of another and/or incorrectly identifies whether it is essential

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UNIT 6 Assessment Interview

You may wish to take the opportunity to interview selected students to assess their understanding of the work of the first chapter of this unit. Interviews are most effective when done with individual students, although pair and small group interviews are sometimes appropriate. The results can be used as formative assessment or as summative assessment data. As the students work, ask them to explain their thinking.

Provide the student with a place value mat like the one in **lesson 6.1.2**.

Millions	Thousands			Ones		
One	**Hundred**	**Ten**	**One**	**Hundred**	**Ten**	**One**

• Ask the student to write two different 7-digit numbers and one 6-digit number, each containing three digits that are 0.

• Ask the following questions about each number:

- *Is it more or less than 4 million? How do you know?*
- *It is more or less than 212 ten thousands? How do you know?*
- *How would you write the number in expanded form?*
- *How do you read the number out loud?*
- *What number is 300,000 greater than your number?*
- *How could you write the number in the form ___ million? In the form ___ ten thousands?*

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BLM 1 Place Value Charts III

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One

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UNIT 7 DATA AND PROBABILITY

UNIT 7 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 195** **TG p. 286**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• Spinner • Paper clip • Grid paper or Small Grid Paper (BLM)	All questions
Chapter 1 Interpreting Data				
7.1.1 The Mean **SB p. 197** **TG p. 289**	**5-F1 Mean: effect of change in data** • understand the mean as a balance through concrete materials and pictorial representations	1.5 h	• Cubes or paper squares • Grid paper or Small Grid Paper (BLM) • Die	Q2, 3, 5, 7
7.1.2 EXPLORE: Effect of Data Changes on the Mean (Essential) **SB p. 201** **TG p. 294**	**5-F1 Mean: effect of change in data** • understand that the mean of a set of data increases if any piece of data increases • understand that the mean of a set of data decreases if any piece of data decreases • understand that the mean increases if a piece of data below the mean is removed • understand that the mean decreases if a piece of data above the mean is removed	40 min	None	Observe and assess questions
GAME: Target Mean (Optional) **SB p. 202** **TG p. 295**	Practise calculating the mean of a set of numbers in a game situation	20 min	• Die	N/A
Chapter 2 Graphing Data				
7.2.1 Choosing a Graph **SB p. 203** **TG p. 296**	**5-F2 Collect, Organize, and Describe Data** • choose an appropriate display for data • interpret displays/presentations of data to draw conclusions about real world issues	1.5 h	• Grid paper or Small Grid Paper (BLM)	Q3, 4, 7
7.2.2 Double Bar Graphs **SB p. 208** **TG p. 300**	**5-F3 Double Bar Graphs: create and interpret** • interpret displays/presentations of data to draw conclusions about real world issues • construct and interpret simultaneous displays for two sets of data from the same population (e.g., data collected at different times)	1.5 h	• Grid paper or Small Grid Paper (BLM)	Q1, 2, 5
7.2.3 Coordinate Graphs **SB p. 213** **TG p. 304**	**5-F4 Coordinate Graphs: create and interpret** • use coordinate graphs for purposes of location • create coordinate graphs using appropriate labels and scales	1.5 h	• Grid paper, Small Grid Paper (BLM), or Coordinate Grids (BLM) • Rulers	Q2, 3, 5, 7

UNIT 7 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Chapter 3 Probability				
7.3.1 Describing Probability **SB p. 217** **TG p. 308**	**5-G1 Experiments** • conduct simple experiments with coins, slips of paper, and dice to determine experimental probability • use common language to describe probability (e.g., for a probability of 15/20, I picked red "15 out of 20 times") • record results in charts	1.5 h	• Nu 1 coin • Dice • Spinners	Q2, 4, 8
CONNECTIONS: Magic Tricks with Dice (Optional) **SB p. 220** **TG p. 311**	An interesting activity with dice	15 min	• 3 dice	N/A
7.3.2 Using Numbers to Describe Probability **SB p. 221** **TG p. 312**	**5-G1 Experiments** • conduct simple experiments with coins and dice to determine experimental probability • predict and record experimental results as fractions and decimals • understand that theoretical probability is the number of favourable outcomes divided by the number of possible outcomes • understand that experimental probability is the number of times the favourable outcome occurs divided by the number of trials in the experiment • record results in charts **5-G2 Describe Probability** • understand that experimental probability is determined by performing experiments • understand that theoretical probability is what you would expect to happen after considering the possible outcomes • use fractions and decimals to describe theoretical probability and experimental probability	1.5 h	• Dice • Nu 1 coin • Numbered slips of paper	Q3, 4, 5
UNIT 7 Revision **SB p. 225** **TG p. 316**	Assess the concepts and skills in the unit	2 h	• Grid paper or Small Grid Paper (BLM) • Dice	All questions
UNIT 7 Test **TG p. 320**	Assess the concepts and skills in the unit	1 h	• Grid paper or Small Grid Paper (BLM) • Dice	All questions
UNIT 7 Performance Task **TG p. 324**	Assess concepts and skills in the unit	1 h	• Tape measures or rulers and string	Rubric provided
UNIT 7 Blackline Masters **TG p. 327**	BLM 1 Coordinate Grids Small Grid Paper BLM on **page 104** in UNIT 2			

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Math Background

• This data and probability unit includes a look at statistics, in particular how the mean of a set of data changes with changes in the data. It examines different types of graphs, specifically, bar graphs, pictographs, double bar graphs, and coordinate graphs. The unit also deals with describing experimental and theoretical probability using words, ratios, fractions, and decimals.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in the **Try This** in **lesson 7.2.3**, where they independently find a way to locate a point, in **question 7** in **lesson 7.2.3**, where they figure out what the missing coordinates of a parallelogram must be when they know some of the coordinates, and in **question 5** in **lesson 7.3.2**, where they create a situation to match a particular probability.

• They use communication in **question 8** in **lesson 7.2.1**, where they talk about the value of a graph as a way to communicate information, in **question 6** in **lesson 7.2.2**, where they respond to an alternate form of a double bar graph, and in **question 5** in **lesson 7.2.3**, where they explain why the order of coordinates matters.

• They use reasoning in **question 5** in **lesson 7.1.1**, where they select a set of data with a given mean, in **question 8** in **lesson 7.1.1**, where they determine whether a particular generalization about calculating means is true, and in **question 6** in **lesson 7.2.1**, where they reason about what a graph without labels or titles might represent.

• They consider representation in **lesson 7.1.1**, where they recognize that the mean is the value that results from physically equalizing a set of data and that it is also represented by the point where the data that exceeds it balances the missing amount from the data that is below it, in **lesson 7.3.1**, where they represent probabilities on a probability line, and in **question 4** in **lesson 7.3.1**, where they represent experimental results on a probability line

• Students use visualization skills in **question 1** in **lesson 7.1.1**, where they visualize where the mean must be on a graph, in **lesson 7.2.1**, where they interpret graphs, in **question 4** in **lesson 7.2.3**, where they observe a relationship among points they plot, and in **question 7** in **lesson 7.2.3**, where they figure out what the missing coordinates of a parallelogram must be when they know some of the coordinates.

• They make connections between means of different but related sets of data in **lesson 7.1.2**, and between bar graphs and pictographs in **lesson 7.2.1.** They also make connections between everyday events and probability language in **lesson 7.3.1**.

Rationale for Teaching Approach

• This unit is divided into three chapters.

Chapter 1 helps students learn what the mean of a set of data actually represents and how it is affected by changes in the data.

Chapter 2 focuses students on thinking about different types of graphs and choosing the appropriate type for a situation.

Chapter 3 deals with describing probabilities.

• There is one **Explore** lesson, where students can see how changes in a data set affect the mean of the data. This activity is more meaningful if the students do the investigation themselves rather than being told about it.

• The **Connections** section shows students an interesting trick that will help them notice how dice are constructed.

• The **Game** provides an opportunity to practice calculating means.

Getting Started

Curriculum Outcomes	Outcome relevance
4 Bar Graphs and Pictographs: construct and interpret **4** Ordered Pairs: position on a grid **4** Mean **4** Simple outcomes: more/less likely **4** Predict probability: near 0, near 1, near $\frac{1}{2}$ **4** Experiments: predict and record results (concrete materials) **4** Describe Results as fractions	Students will find the work in the unit easier once they review what they know about data and probability from Class IV.

Pacing	Materials	Prerequisites
1 h	• Spinner • Paper clip • Grid paper or Small Grid Paper (BLM)	• bar graphs • pictographs • using ordered pairs on a grid • definition of mean • some probability language • ability to use simple fractions and decimals • products of single-digit numbers

Main Points to be Raised

Use What You Know

• Bar graphs are useful for showing frequencies of experimental outcomes.

• The scale of a bar graph should be chosen to ensure that the graph is a reasonable size.

• A bar graph provides insight into the situation it describes.

• When a probability experiment is repeated, the results are not always the same each time.

Skills You Will Need

• A bar graph is used to describe frequencies.

• The mean of a set of numbers describes a balance point. It is calculated by dividing the total amount of data into equal shares.

• If the probability of an event is 0, the event will never happen; if the probability is 1, the event will certainly happen; and if the probability is $\frac{1}{2}$, the event is as likely to happen as not to happen.

• Experimental probability usually reflects theoretical probability.

Use What You Know — Introducing the Unit

• Ask students to open the text to **page 195** and to prepare a spinner with sections representing about the same fractions shown on the spinner. Students might trace the spinner in the text or they might just estimate; either way is acceptable. They can create a spinner by putting the point of a pencil through a paper clip to hold it at the centre of the circle and then spinning the paper clip by flicking it with a finger.

• Ask students to recall what a bar graph is. You might show a quick example by sketching a bar graph that compares the number of girls in the class to the number of boys. Ask students how they would decide on the size of each bar. Remind students that they can create a bar graph by drawing the bars either horizontally or vertically.

• To prepare for the activity, ask students to take out lined paper or grid paper they could use to create a bar graph.

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• As you observe students at work, you might ask questions such as the following:

- *Why do you think you got more 2s than 1s?* (The 2 section is bigger than the 1 section.)
- *Which number did you spin the most? How many times? Did that result surprise you?* (I spun the 3 the most — 15 times. I was not surprised that I got a lot of 3s, but I was surprised I got so many more 3s than 2s.)
- *How did you decide on a scale to use?* (My numbers of spins were 15, 5, 10, and 10, so it seemed logical to use the height of one line to represent 5 spins. I can show 15 as 3 heights and 10 as 2 heights.)
- *What does the graph tell you about the 1 section and the 3 section on the spinner?* (It shows that the 1 section is smaller than the 3 section because there were a lot more 3s than 1s.)
- *Does it make sense that your classmate's bar for 1 is so much lower than yours?* (Yes, because when you spin, you can never be sure what will happen.)

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions.

• You may wish to review the meaning of the terms *pictograph*, *mean*, and *probability* before students begin.

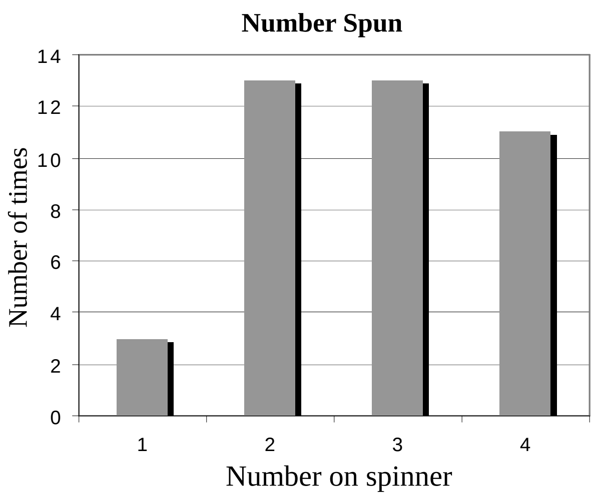
• Students can work individually.

Answers

A. *Sample response*:

3	2	3	3	1	4	4	3	4	2
2	4	2	2	4	3	3	2	3	4
3	3	2	4	2	1	4	2	3	2
4	3	3	2	4	2	1	2	4	3

B. *Sample response*:



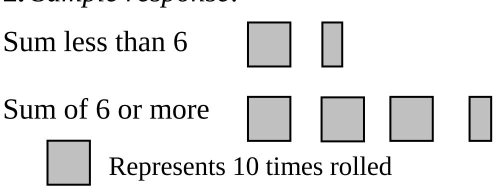
C. *Sample response*:
I chose 2 because I did not want bars that were too tall and used up a lot of space.

D. *Sample response*:
• It looks like the spinner is more likely to land on 2, 3, or 4 than on 1.
• The spinner is equally likely to land on 2 and 3.
• If you spin it many times, you will probable spin 4 about $\frac{1}{3}$ of the time.

E. *Sample responses*:
i) We both got mostly 2s, 3s, and 4s.
ii) I got the same number of 2s as 3s, but Karma did not; When you spin a spinner, you can never be sure what is going to happen.

1. *Sample response*:
A sum of 6 or more is more likely to be rolled than a sum less than 6.

2. *Sample response*:



3. Yes; [*Sample response*:
There is one number 1 less than 11 and one number 1 more than 11, so they balance each other. The other numbers are both 11.]

4. a) 15 **b)** 18

5. a) 0 **b)** $\frac{1}{2}$ **c)** 0 **d)** 1

6. a) Equally likely
b) Getting a product greater than 10
c) I will eat rice tomorrow

7. *Sample responses*: **a)** 1 **b)** 8 **c)** 3

Supporting Students

Struggling students

• Some students may have difficulty choosing a scale for the bar graph. You may suggest a scale, for example, 1 square represents 2 spins.

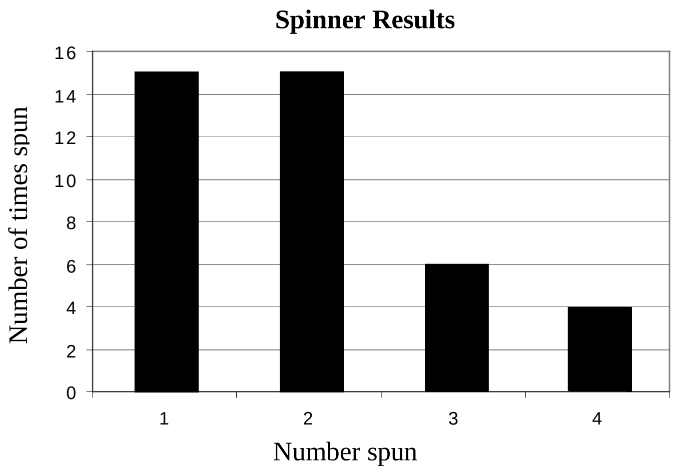
• If students have trouble using a graph to tell about the spinner, you might suggest that they just tell three things about the graph. You can then help them relate what they say to the spinner.

For example, they might say, “I spun more 3s than 1s,” and you could reply, “That is probably because the 3 section on the spinner is bigger”.

Enrichment

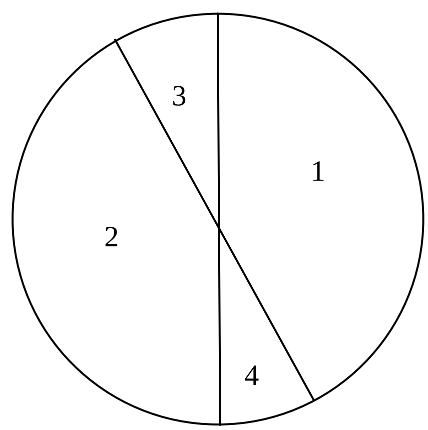
• Students might try to create a spinner that will match a pre-made graph and then test to see how close the experimental data is to the predicted data.

For example, say that you spun a spinner 40 times and the graph of the results looked like this:



The students must try to create a spinner that gets results that match the graph. In this case, it might be like this:

They must then test the spinner by spinning 40 times and comparing the results to the data in the bar graph.



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7.1.1 The Mean

Curriculum Outcomes	Outcome Relevance
5-F1 Mean: effect of change in data • understand the mean as a balance through concrete materials and pictorial representations	Many students can calculate a mean, but they need to understand what the mean represents for calculation to be meaningful.

Pacing	Materials	Prerequisites
1 h	• Cubes or paper squares • Grid paper or Small Grid Paper (BLM) • Die	• dividing by 1-digit numbers • creating a bar graph

Main Points to be Raised

• The mean summarizes a set of data.

• The mean tells what the value of each piece of data would be if the total were shared equally. You calculate the mean by adding all the data values and then dividing the total by the number of pieces of data.

• The total by which some data values exceed the mean is equal to the total by which other data values fall short of the mean. You can see this on a bar graph by counting the squares that are above the mean and the squares that are missing below the mean.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. As you observe students at work, you might ask questions such as the following:

• *Why would you not use 78 to represent all the data?* (I think it is too high to represent most of the data.)

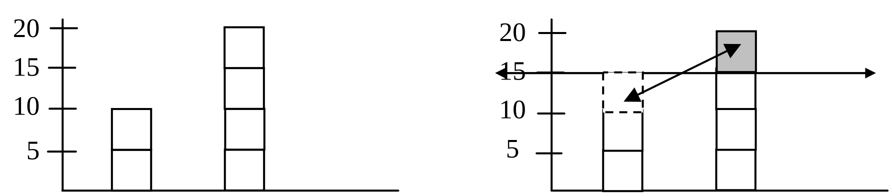
• *Why might you use 71 to represent the data?* (It is in the middle; there are numbers above 71 as well as numbers below 71.)

• *Are there other numbers you might use?* (I think a number like 72 or 73 might also be okay; these are also between the high and low values.)

The Exposition — Presenting the Main Ideas

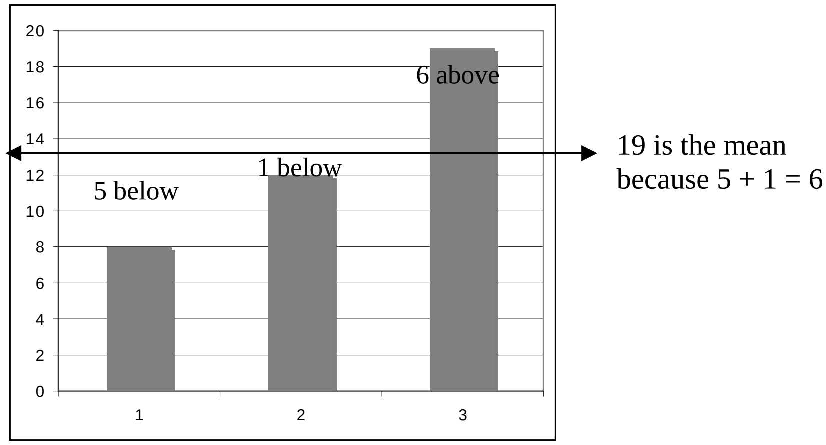
• Call two students to the front of the room. Give one student 20 crayons and the other 10 crayons. Ask whether it is fair. Have students suggest how to make the arrangement fair (by moving 5 crayons from the student holding 20 to the student holding 10). Tell students that the value 15 is called the mean because it is the amount each person gets if the total is shared fairly. Point out that it could be calculated by adding 20 and 10 (to get the total) and dividing by 2 (the number of people) to get each fair share.

• Draw a bar graph with a scale of 5 (see below) to show the original 10 and 20 crayons. Show how to draw a line at the mean, 15. Now show students that the amount above the mean is the same as the amount that is missing below the mean.



Explain that this is always the case with the mean. There will always be the same amount of data above the mean as there is missing data below the mean.

• Use another example. Write the numbers 8, 12, and 19 on the board. Ask what the mean is.
- Some students might add the values and divide by 3 to get 13, that is, (8 + 12 + 19) ÷ 3 = 13.
- Others might sketch or visualize a graph and look for a line where the amount above the line balances the amount below.



- Still other students might guess the mean, for example, 14, and then adjust it when they discover that the amount below their guess does not balance the amount above.

8 is 6 below 14 and 12 is 2 below 14, so the total below is 8. 19 is 5 above 14, so the total above is 5. There is too much below, so the mean should be less.

• Read through the exposition with students to make sure they follow the examples that explain the mean. Make sure they notice that shading is used on **page 197** to help them see how the original pieces of data were rearranged to even them out. Also make sure they understand on **page 198** that it is a coincidence that the total below the mean is actually the sum of the three data values that are below the mean. This did not happen with the example of 8, 12, and 19 that you worked through with the students.

Revisiting the Try This

B. Students are not expected to draw individual squares to represent the data, nor are they expected to use a formula to calculate the mean. Rather, they might guess a reasonable mean and then guess and test to adjust it to the correct value. Students will not be able to answer the second part of this question without trying other data sets. Encourage them to do so.

Using the Examples

• Have pairs of students work together through **example 1** and **example 2**. Make sure that students understand the examples by calling on a few students to explain the examples to the rest of the class.

• As they work through **example 2**, students should realize that if two data sets have the same mean, their totals are also equal if they contain the same number of pieces of data. This is not necessarily true if the two data sets have different numbers of values in them.

For example, the set 1, 2, 3, 4, 5 has a mean of 3 and a total of 15. The data set 3, 3, 3 also has a mean of 3, but the total of the data is 9, not 15. If a third data set with 5 values had a mean of 3, its total would have to be 15.

Practising and Applying

Teaching points and tips

Q 1: Students should be encouraged to move a pencil or another thin object vertically from right to left until the number of white squares to the left of the pencil balances the number of grey squares to its right.

Q 2: Students might draw a graph, but they might also rearrange squares or cubes to make equal shares.

Q 3: Students can solve this question visually, symbolically or by using guess and test.

Q 4: Observe whether students realize that the totals of the sets are equal for **parts a) and b)**, but not for **part c)**.

Q 5: Some students might calculate the mean for **part a)** and simply repeat that piece of data for **part b)**. Others might realize that they can add any amount they wish to one piece of data and subtract the same amount from another piece of data. Other students will try different combinations of numbers.

Q 6: See if students realize that the sum of the numbers must be 100.

Q 8: Make sure students do not generalize from just one example. Algebraically, the statements are true because if the data values are *a*, *b*, *c,* and *d*, then

$$\frac{\frac{a+b}{2}+\frac{c+d}{2}}{2} = \frac{a+b+c+d}{4}.$$

Q 9: Most students will predict 3 or 4 because these are the middle numbers. Although this is a reasonable prediction, in any single experiment these numbers may not be the mean.

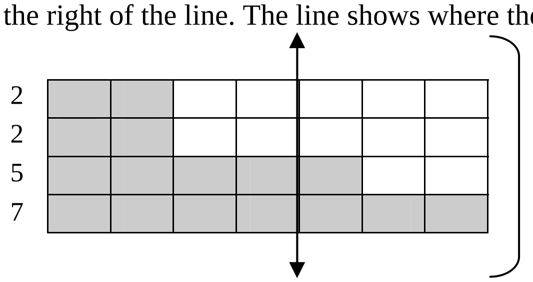
Common errors

• In using a graph to calculate the mean, some students may try to create the balance by counting the squares that represent the data below the mean rather than counting the amount by which each data value is less than the mean. Have them consider, for example, the data set 2, 3, 4. They know the mean is 3, but they see that 2 (left of the mean) is not equal to either 3 (the value to the right of the mean) or 1 (the value by which 3 is greater than the mean).

Suggested assessment questions from Practising and Applying

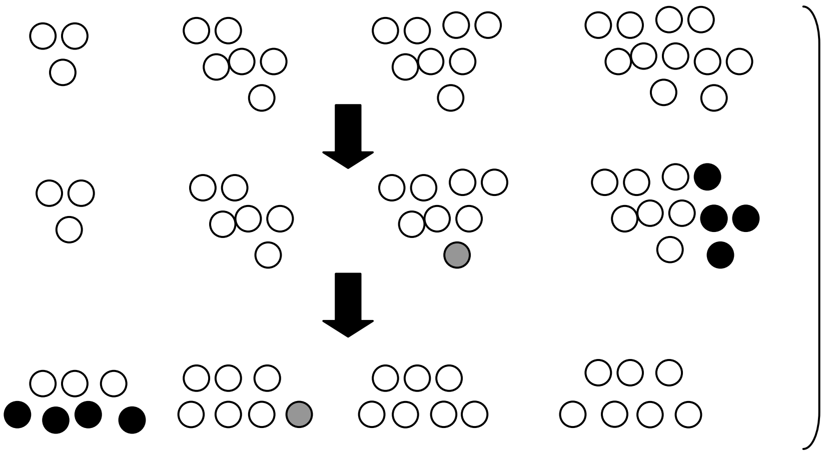
Question 2	to see if students can represent a mean visually
Question 3	to see if students can calculate a mean
Question 5	to see if students understand how two different sets of data can have the same mean
Question 7	to see if students can solve a real-world problem about means

Answers

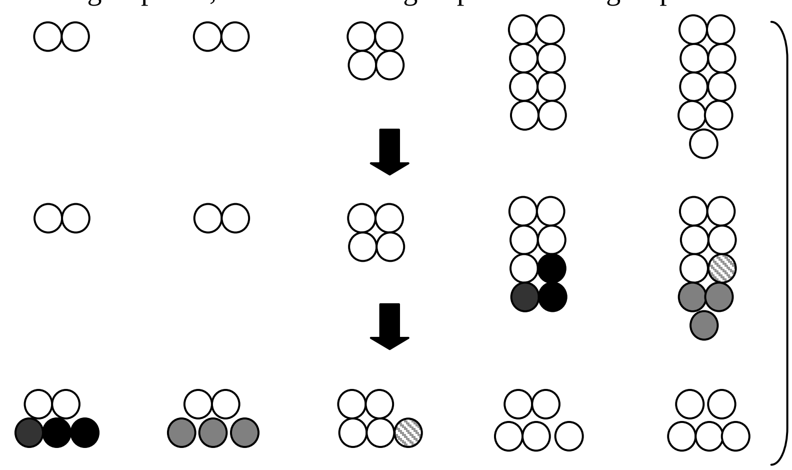
<table>
<tr><td>A. Sample response:
71 [Two numbers are less than 71, but they are just a bit less. One number is greater but it is a lot greater.]</td><td>B. i) 71
ii) No; Sample response:
The mean would be the same if the values had been 67, 70, 79, and 68. None of those is 71.</td></tr>
<tr><td colspan="2">1. 4; [Sample response:
I drew a line so the number of squares missing to the left of the line is the same as the number of extra squares to the right of the line. The line shows where the mean is, at 4.

The mean is 4.</td></tr>
</table>

Answers [Continued]

2. a) 7; [*Sample response*:
I made each group equal by moving 4 from the group of 11 to the group of 3 and 1 from the group of 8 to the group of 6. There are 7 in each group, so the mean is 7.



b) 5; [*Sample response*:
I made each group equal by moving 3 from the group of 8 to the first group of 2, 3 from the group of 9 to the second group of 2, and 1 from the group of 9 to the group of 4. There are 5 in each group, so the mean is 5.



3. a) 18 **b)** 45 **c)** 25

4. a) 8 **b)** 17 **c)** 6

5. a) 68
b) *Sample response*: 68, 68, 68, 68, 68

6. *Sample response*:
20, 20, 20, 20, 20; 20, 10, 30, 20, 20; 1, 5, 8, 20, 66

7. 13 km

8. Yes; [*Sample response*:
If the numbers are 1, 3, 3, and 21, the mean is 7.
The mean of 1 and 3 is 2.
The mean of 3 and 21 is 12.
The mean of 2 and 12 is 7.]

9. *Sample response*: $3\frac{1}{2}$
I rolled 12 times and got 2, 1, 4, 5, 1, 3, 6, 2, 1, 5, 4, 3.
The mean was just a bit over 3, so I was close.

[**10.** *Sample response*:
The mean is the number that has as much above it as is missing below it. If it were lower than all the numbers, everything would be above it, so that would not work. If it were higher than all the numbers, everything would be below it, so that would not work either.]

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Supporting Students

Struggling students

• Some students will be less comfortable using the graph to guess the mean than rearranging squares or counters or using guess and test. If they find these methods much easier, do not require them to do **question 1**.

Enrichment

• Students could create a variety of sets of data that meet given conditions with given means.

For example, you might ask them to create a set of data that has five different numbers with:

- a mean of 30 (e.g., 24, 25, 30, 35, 36),
- a mean that is 10 less than the greatest value (e.g., 20, 30, 30, 30, 40), or
- a mean that is 5 more than the least value (e.g., 12, 12, 17, 22, 22).

7.1.2 EXPLORE: Effect of Data Changes on the Mean

Curriculum Outcomes	Lesson relevance
5-F1 Mean: effect of change in data • understand that the mean of a set of data increases if any piece of data increases • understand that the mean of a set of data decreases if any piece of data decreases • understand that the mean increases if a piece of data below the mean is removed • understand that the mean decreases if a piece of data above the mean is removed	• By focusing on the idea of the mean as a balance point, students can predict, without calculating, how a mean will change when the data values are altered in certain ways. • This exploration will make it easier for students to predict means for new sets of data.

Pacing	Materials	Prerequisites
40 min	None	• calculating a mean

Exploration

• Write the numbers 10, 11, and 12 on the board. Ask students to explain why the mean is 11. Tell students that you will increase each number by 3 and ask them to calculate the new mean (i.e., the mean of 13, 14, and 15). Students should notice that the mean also increased by 3.

• Tell students that they will explore how the mean changes by changing other data sets.

• Observe while students work. While they work, you might ask questions such as the following:

- *Why will the mean increase when you change 70 to 78?* (The total is greater, so if you share it, each share is greater.)

- *Why will the mean decrease when you change 70 to 62?* (The amount greater than the mean balanced the amount missing from below the mean, so if you decrease one of the numbers, you have to move the mean down to keep the balance.)

- *Why would Bhagi not want to include a low test score?* (To raise the mean.)

- *What do you think would happen to the mean if you multiplied all the values by 2?* (I think the mean would also be multiplied by 2.)

Observe and Assess

As students work, notice:

• Do they use reasoning or do they re-calculate to determine the mean for the altered data?

• Do they verify their predictions in **part E** by using more data sets?

• Are their explanations clear?

Share and Reflect

After students have had sufficient time to work through the exploration, ask them:

• *What if you had changed the 56 instead of the 70? Would your answers have remained the same?*

• *Why did increasing the 70 to 78 increase the mean by 1? Would that also have been true if there had been 10 pieces of data?*

• *What would happen to the mean if the data value you removed from a set of the data was the mean value?*

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Answers

A. i) 64 **ii)** *Sample response*: It is how much each test score would be if you added up all the scores and then shared them equally among the 8 tests. **B. i)** It will increase. *Sample response*: The total will be greater, so the value when you divide by 8 will also be greater. **ii)** The new mean is 65; yes. **C. i)** It will decrease. *Sample response*: The total will be less so the value when you divide by 8 will also be less. **ii)** The new mean is 63; yes.	**D. i)** 54 because it is the lowest score and it brings down the mean. **ii)** The mean goes up; *Sample response*: This number was a lot lower than the mean so it does not take as much from above the mean to balance the values below the mean. The mean can move up. **iii)** It would decrease. **E. i)** Increases; If one of the data value increases, the total also increases, so there is more to share. **ii)** Decreases; If one of the data value decreases, the total also decreases, so there is less to share. **iii)** Decreases; There would be less above the old mean to balance what is missing below the old mean, so the new mean has to be less. **iv)** Increases; There would be too much above the old mean to balance what is missing below the old mean, so the new mean would have to be more.

Supporting Students

Enrichment

• Students might be asked to create a variety of data sets where the mean changes in predicted ways.

For example, they might have to change the data set 1, 3, 5, 7, 9 so the mean becomes 14.

GAME: Target Mean

This game is designed to provide students with practice in predicting what a mean might be and in calculating the mean.

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7.2.1 Choosing a Graph

Curriculum Outcomes	Outcome relevance
5-F2 Collect, Organize, and Describe Data • choose an appropriate display for data • interpret displays/presentations of data to draw conclusions about real world issues	Not only is it important for students to be able to construct a graph, it is also important that they make a thoughtful choice about the type of graph that is most appropriate in a given situation.

Pacing	Materials	Prerequisites
1.5 h	• Grid paper or Small Grid Paper (BLM)	• creating bar graphs and pictographs • writing a product in different ways • writing a number as a multiple of another number

Main Points to be Raised

• Bar graphs and pictographs are both useful ways to show the frequencies of different categories of data.

• It is often appropriate to use a scale for a bar graph or a pictograph so that the number of squares or symbols is not too great.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. As you observe students at work, you might ask questions such as the following:

• *Why did you choose a bar graph?* (It lets me show the number in each category.)

• *Could you have also used a pictograph?* (Yes, I could use a symbol like a circle to represent 2 games.)

• *How did you decide on your scale?* (I thought that 2 would be best for 12, 10, 18, and 7. If I had used 3 or 4, there would have been too many partial symbols.)

The Exposition — Presenting the Main Ideas

• Ask students to open the text to **page 203**. Point out the data values described in the middle of the page. Have students explain how the graph at the bottom of the page was constructed. Ask students to explain why someone would say that the scale was 10 students for one square.

• Ask how the graph would change if a scale of 5 were used instead. Next, ask about a scale of 20. Have students look at the graph with a scale of 20 on **page 204**. Ask why the pork bar extends only halfway across a square.

• Next, ask students to examine the pictograph on **page 204**. Ask them to explain how the pictograph shows exactly the same information as the bar graph on the same page. Ask students which graph they prefer and why.

Revisiting the Try This

B. Students may have varying opinions on the value of a bar graph compared to the value of a pictograph.

Using the Examples

• Write the data from **example 1** on the board: when a pair of dice was rolled 500 times, a sum of less than 5 occurred 70 times and a sum of 5 or more occurred 430 times. Ask students what scale they would use to create a bar graph and why. There are many reasonable answers. Not only would it make sense to use a scale of 70 as shown in the text, but it would also make sense to use a scale of 50 or even 40 or 100.

• Ask students to look at **example 1**, **solution 2**. Have them discuss why it makes sense to use a die for the symbol. Have the students explain how the pictograph is like the bar graph.

• Have students turn to the graph in **example 2**. Ask them to suggest things that are true about the bar graph before they read through the solution.

Practising and Applying

Teaching points and tips

Q 1: It might be helpful to write all the students' names on the board for students to look at. To describe the graph, students could state the size of a category, compare two categories, state the size of a combination of categories, or talk about categories that do not appear on the graph.

Q 2 and 3: There is no single correct way to choose a scale, but students should refer to the size or type of data values when they select the scale.

For example, for data values like 4, 8, 4, 2, it makes sense to use a scale of 4 because most of these numbers are multiples of 4. It is easy to show 2 as half of 4.

Q 4: The scales for the pictograph and bar graph need not be identical, although usually they would be to make the creation of the second graph easier.

Q 5: Point out that this graph could be changed into a line graph by connecting the top centre points of all the bars. This is a rare example of a bar graph that describes a trend.

Q 7: Students might simply state the areas of the countries. Encourage them to make comparisons.

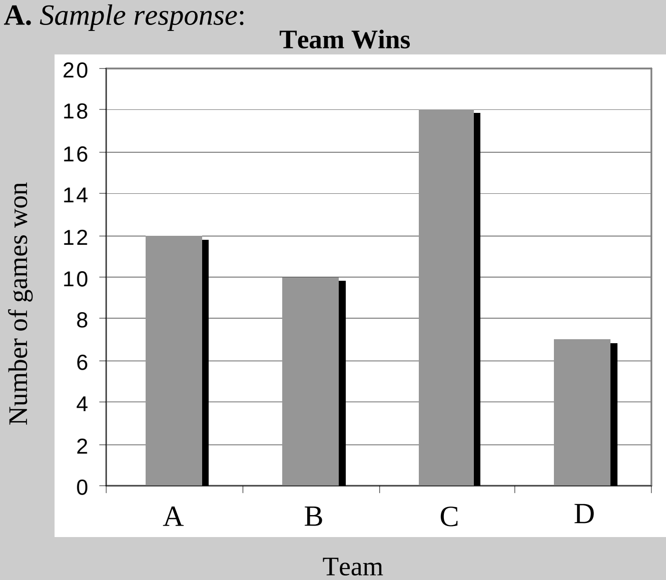
For example, they might note that the areas of El Salvador and Fiji are close or they might observe that the area of the Netherlands is about twice the area of El Salvador.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can use good judgment in selecting a scale for a graph
Question 4	to see if students can create a pictograph and a bar graph
Question 7	to see if students can use graphs to gather information

Answers

A. *Sample response*:



B. i) *Sample response*: Bar graph

ii) It makes sense to use a scale of 2 because the data values are not very high. I needed to show 18 but I did not want to use 18 squares. It worked well to use 9 squares.

iii) *Sample response*:

Not really; you can use either one. You might use a bar graph if you have grid paper and a pictograph otherwise.

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Answers [Continued]

Sample names for question 1:

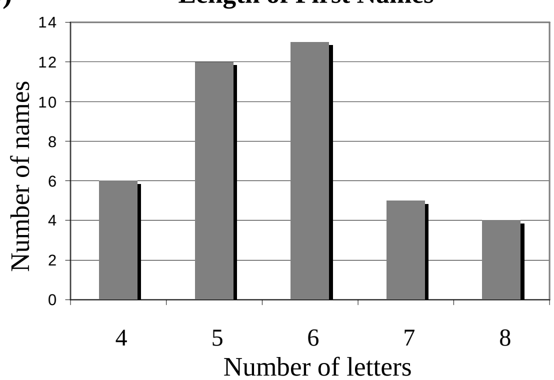
Dorji Penjor	Bandha Tamang	Dawa Penjor	Zangmo Dolma
Kinya Loday	Sushma Rai	Samdrup Gyelpo	Ugyen Lhamo
Pema Thinley	Deki Pelden	Kuenzang Namgay	Singye Wangmo
Wangchuk Namgay	Chimi Wangmo	Dorji Phumbho	Pema Choden
Sonam Jordhen	Jamyang Choden	Kailash Gurung	Tshering Pelden
Sithar Phuntsho	Sangay Tempa	Phumbo Choden	Karma Choki
Tandin Tshering	Jigme Tshewang	Pema Deki	Kelzang Lhamo
Phurba Thinley	Dupchula	Chimi Lhaden	Tshendu Wangmo
Sonam Gyeltshen	Loday Gyeltshen	Dawa Dem	Tshomo
Peldhen Dorji	Tashi Wangchuk	Sangay Bidha	Yoezer Lhamo

1. *Sample responses*:

a)

Number of letters	Frequency
4	6
5	12
6	13
7	5
8	4

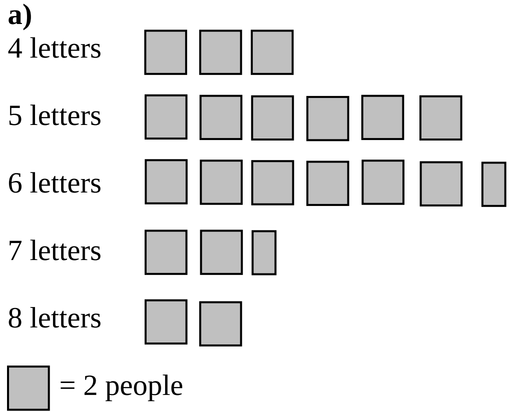
b) **Length of First Names**



c) Most names had 5 or 6 letters.
No names had fewer than 4 letters.
No name had more than 8 letters.

2. *Sample responses:*

a)

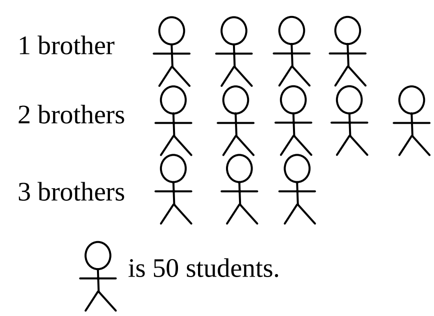


[**b)** I chose a scale of 2 because most of the numbers were even, but not too big.]

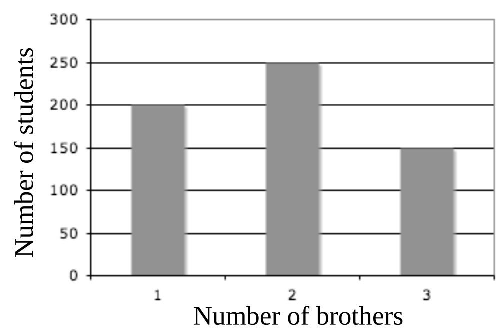
3. *Sample response*:
A. 50; [I can use 4, 5, and 3 symbols and that is not too many.]
B. 80; [I can use 5, 6, and 3 symbols and that is not too many.]
C. 50; [I can use 5, 6, and 4 and a bit more symbols. The partial symbols will not be easy to make, but any other way would need too many symbols.]

4. *Sample response*:

How Many Brothers We Have



How Many Brothers We Have



5. *Sample response*:
• It shows that the population has increased each year.
• It shows the population for each year from 1976 to 1996.

6. *Sample response*:
It might be about the number of Class V students whose favourite colour is brown, black, orange, or blue.

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7. *Sample response*: • Fiji has the least area of the five countries in the graph. • The area of Bhutan is greater than the areas of Armenia, El Salvador, Fiji, or the Netherlands. • The area of Bhutan is between two and three times as large as the area of Fiji.	[**8.** *Sample response*: With a graph, you can see and compare a lot of information quickly without doing a lot of reading.]

Supporting Students

Struggling students

• Some students will have difficulty paying attention to factors when choosing a scale. You may want to help with an example. Show how for the data values 25, 50, 75, it is very easy to use a scale of 25 because each value can be thought of as a group of 25s. On the other hand, for the values 25, 42, 57, a scale of 25 is not easy to use because it is hard to show the 42 and 57 properly. Only a scale of 1 or 2 is very easy for this data, but a scale of 5 would work if each square or symbol were divided into fifths. That way 42 would be $8\frac{2}{5}$ squares and 57 would be $11\frac{2}{5}$ squares or symbols.

Enrichment

• Encourage students to create graphs about topics of interest to them related to the numbers from 1 to 500. For example, they might draw graphs to compare the even numbers to numbers that are multiples of 3 or of 4.

7.2.2 Double Bar Graphs

Curriculum Outcomes	Outcome relevance
5-F3 Double Bar Graphs: create and interpret • interpret displays/presentations of data to draw conclusions about real world issues • construct and interpret simultaneous displays for two sets of data from the same population (e.g., data collected at different times)	Double bar graphs are useful for comparing related data sets. Students should know how to interpret these graphs when they see them in newspapers or magazines.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM)	• creating bar graphs

Main Points to be Raised

• A double bar graph shows two related sets of data at the same time. You can make comparisons within each group as well as between the two groups.

• You should only use a double bar graph when the data sets are comparable.

For example, both sets of values should be able to be shown using the same scale, the amount of data should be similar, and the categories need to be the same for both sets of data.

• Use a legend to make it clear which sets of bars represent which data.

• It is helpful if you draw the graphs consistently, with the bars in each category for one group always to the left of (or above) the bars for the other group. It is also helpful to use one colour or shading for the bars of one group and a different colour or shading for the bars of the other group.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Encourage them to make as many comparisons as possible.

• As you observe students at work, you might ask questions such as the following:

- *How many students are in each class? How do you know?* (There are 40 students in Class II and 41 students in Class V. I added the numbers in each column to find out.)

- *About how many times as many students like bananas as apples?* (About 3 times as many; I compared 20 + 18 = 38 to 8 + 5 = 13. Because 3 × 13 = 39, which is close to 38, I thought there were 3 times as many.)

- *How are the classes alike?* (Students in both classes like apples the least.)

- *How are the classes different?* (In Class V, mangos are as popular as bananas, but in Class II they are not.)

The Exposition — Presenting the Main Ideas

• Write the words *blue*, *orange*, *red*, and *other* on the board. Ask only the boys to raise their hands to show whether their favourite colour is blue, orange, red, or some other colour. Record the data. Repeat this with the girls and make two charts, one for the boys and one for the girls.

• Ask students to suggest how this information might be graphed. Most students will suggest creating two similar graphs, although it might be interesting to see whether any students independently come up with the idea of a double bar graph.

• Model how to show the information as a double bar graph (as is done in the text on **page 208**). Tell students this is called a *double bar graph* because there are two bars for each category. Make sure they notice that you are consistent, always drawing the bar for one group on the left and the bar for the other group on the right, for example, the boys on the left and the girls on the right. If the graph were horizontal, the bars of one group would always be above and the bars for the other group would always be below. Point out that the shading for the two groups should be clearly different to make it easy to see which group is which. Tell students that we use a *legend* to record which colour or shading is used for which group.

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• Point out that it makes sense to use a double bar graph because you can use it to compare the colours chosen either by boys or by girls, and you can use it to compare the choices of boys to the choices of girls, all at one time. Ask the students to use the graph you created to draw some conclusions about the favourite colours of the boys, of the girls, and of the boys compared to the girls in the class.

• Explain that it only makes sense to use double bar graphs when the data values being collected are about the same thing, the values can be shown using the same scale on the graph, and the sizes of the groups are not too different. Provide one or two other examples of when a double bar graph might be used.

For example, you could use double bar graphs to compare the number of siblings of students in Class V to the siblings of students in Class VI, to compare students' choices of favourite subject at the start of the school year and at the end of the school year, or to compare the populations of two countries in different years.

• Tell students they can use the exposition for later reference.

Revisiting the Try This

B. Before students draw their double bar graphs, ask them what scale they plan to use and why.

Using the Examples

• Assign pairs of students to read through **example 1** and **example 2**. Ask one student in each pair to become the expert on one of the examples and explain it to the other. For **example 1**, make sure students understand that although the data values about people who were unhappy are missing from the graph, this information is not needed to interpret or discuss the graph.

Practising and Applying

Teaching points and tips

Q 1: Students can choose whether or not, to copy and include the data from **part a)** as they add the new information from **part b)**.

Q 2: Ask students why it makes sense to use a double bar graph to show this type of data.

Q 3: Encourage students to compare the choices within Class A, within Class B, and between the two classes.

Q 4: Some students may be uncomfortable comparing, thinking that the glasses could be of different sizes. They may choose instead to estimate with millilitres.

Q 5: This type of bar graph is produced by the Government of Bhutan and is included in census information. It is used both for aesthetic reasons and because it uses space wisely.

Q 6: Some students will not think about the appropriateness of a double bar graph in a particular situation.

For example, if they were comparing areas and populations of countries, the values are usually so different that a double bar graph does not make sense.

Common errors

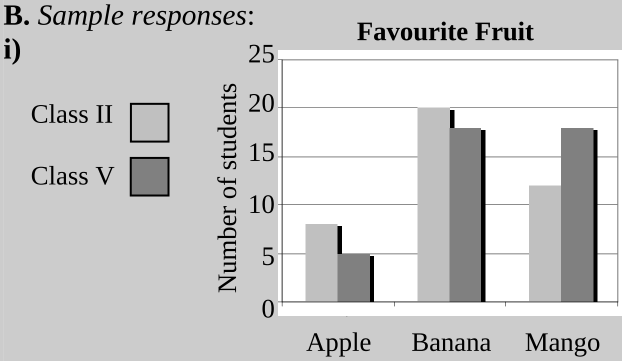
• Some students will not be consistent about the shading they use to distinguish the two groups being graphed and others will not be consistent about where to place the two bars relative to each other. Although these errors do not make the graphs incorrect, they do make the graphs much harder to read. Make sure students understand that theses details are important because we use graphs so that the information can be seen quickly.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can interpret a double bar graph
Question 2	to see if students can create a double bar graph
Question 6	to see if students can communicate about when it is most appropriate to use a double bar graph

Answers

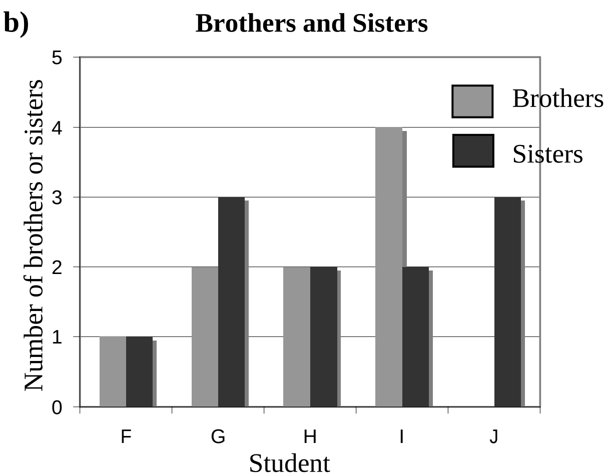
A. *Sample response*: Apples are the least favourite fruit for both classes. The older students seem to like mangos more than the younger students do. Both groups like bananas best, although the Class V students like mangos as much as they like bananas.	**B.** *Sample responses*: **i)**



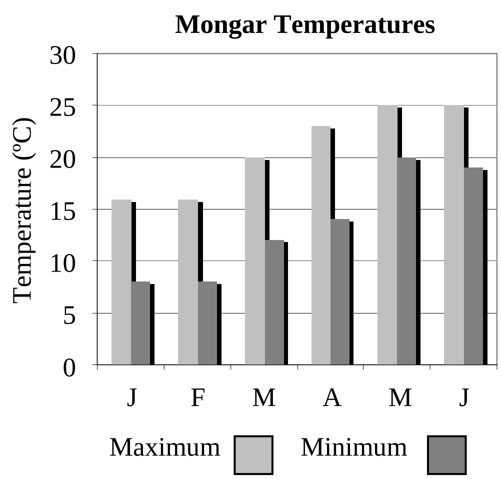
ii) I think the graph is easier to understand quickly because you can compare by just looking to see which bar is taller.

1. a) *Sample response*:
- Two students have more brothers than sisters.
- Two students have more sisters than brothers.
- One student has no brothers.
- One student has as many brothers as sisters.
- No one has more than 2 brothers or 2 sisters.
- There is the same number of students with 1 brother as with 2 brothers.

b)

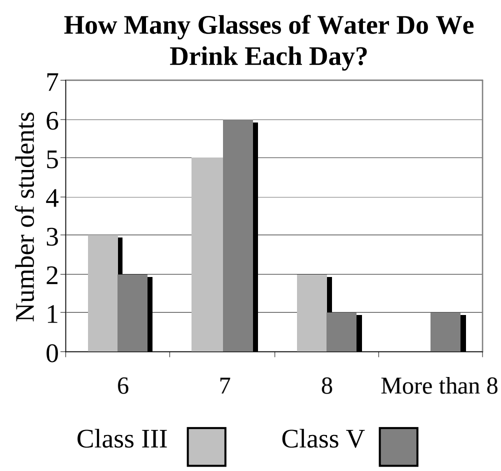


2.



3. *Sample response*:
- The United States are most interesting to one class, but India and Australia are more interesting to the other class.
- Both classes have an equal interest in China.
- India, Nepal, Australia, and the U.S. are of the most interest to students in general.

4. *Sample response*:



5. a) It compares two groups of data (males and females) in different categories (age groups) on the same graph.
[**b)** *Sample response*:
The population totals for males (or females) in the different age groups are easier to compare [because they are lined up at the same starting point. I would have to use a ruler to compare males and females in the same age group.]

6. *Sample response*:
The number of minutes of exercise that boys and girls in different classes get each week.

Reprint 2023

Supporting Students

Struggling students

• Some students may find it easier to stick with only horizontal or only vertical double bar graphs until they get comfortable with them.

• For **question 6**, rather than asking students to suggest topics, you might suggest topics and have students simply respond to whether or not a double bar graph would be appropriate and why or why not.

Enrichment

• Students might look for examples of double bar graphs in the newspaper, on a computer, or in magazines.

• They might also make a list of topics for which double bar graphs would be appropriate and a list of topics for which one might first think that such graphs would be appropriate, but where they might not actually be appropriate.

For example, a double bar graph that compares income and rent costs might seem appropriate, but it is unlikely that the same scale would suit both sets of data.

Reprint 2023

7.2.3 Coordinate Graphs

Curriculum Outcomes	Outcome relevance
5-F4 Coordinate Graphs: create and interpret • use coordinate graphs for purposes of location • create coordinate graphs using appropriate labels and scales	Students will use coordinate graphing as an important tool as they move up into higher mathematics. This lesson starts that process.

Pacing	Materials	Prerequisites
1.5 h	• Grid paper, Small Grid Paper (BLM), or Coordinate Grids (BLM) • Rulers	• recognizing multiples of numbers • properties of a parallelogram

Main Points to be Raised

• Two numbers are needed to locate a unique point on a plane (flat surface).

• A coordinate system is built by measuring distances horizontally and vertically from a point called the origin. The axes are perpendicular lines running horizontally and vertically that intersect at the origin.

• The two numbers that describe a location are called an ordered pair. The first number describes the horizontal distance from the origin and the second number describes the vertical distance from the origin.

• The order of the numbers in an ordered pair is important. If the order is changed, the location is usually different.

• Sometimes a scale is used on a coordinate grid if the distances are expressed with greater numbers.

Try This — Introducing the Lesson

A. Allow students to try this with a partner. As you observe students at work, you might ask questions such as the following:

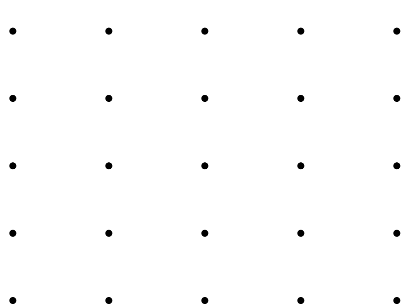
• *Why did you use steps as your unit of measure?* (I thought using steps would be easier than using a ruler.)

• *Why can you not just say that your partner is five steps away?* (Because there are many locations that are five steps away from me — she could be in front of me, behind me, or to the side.)

• *How many numbers did you use to describe her location? Why did you need that many?* (Two; By using both numbers, I could say how far forward she was and then how far to the side.)

The Exposition — Presenting the Main Ideas

• Draw a grid like this on the board.



Choose one point to focus on, but do not tell the students which point you have chosen.

Allow students to ask you yes-or-no questions to guess the location of the point.

For example, they might ask:

- *Is it in the top row?*
- *Is it in row 3?*
- *Is it in column 3?*

• Help them see that only two numbers are needed to locate the point — the row number and the column number. Also help them see that there has to be agreement on how to number the rows and the columns because it would otherwise be confusing to locate the point.

For example, they need to know if the second row means second from the top or second from the bottom.

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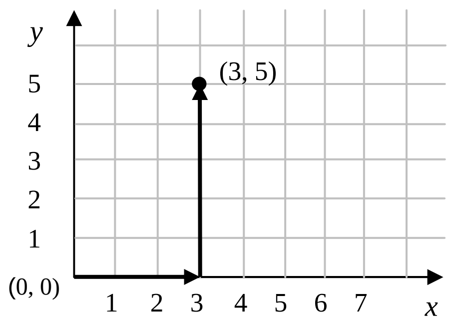
• Reintroduce the coordinate grid system students met in Class IV. Point out the *x-axis*, the *y-axis*, and the *origin*. Although we refer to the axes as lines, they are actually parts of lines called *rays* that go on indefinitely in only one direction. (In Class VI, the axes will also go in the negative direction and become lines.) Explain that we label the axes in this way so there will be no confusion about whether a horizontal move of 3 means right or left, or whether a vertical move means up or down.

• Locate a point on the grid and show students how to get there by first moving horizontally and then moving vertically. Point out that you must count how many spaces you move.



• Have students open the text to **page 214**.

Point out that a scale was used on the last grid in the exposition because moving 25, 30, or 50 spaces would use up too much room. Have them notice that the scales on the two axes can be different and that the scale is chosen according to the coordinates that are involved.

For example, if the *x*-values are all multiples of 5, it might make sense to use a scale of 5. If the values are all multiples of 20, it might make sense to use a scale of 20.

Revisiting the Try This

B. Discuss with students why it would make sense to call the position of the person who is giving the directions the origin.

Using the Examples

• Ask students to read through the two examples and ask any questions they might have. Reinforce the idea that you count the spaces, not the tick marks on the axes.

For example, although the places where the *x*-coordinate is 2 are on the third vertical line, you move right only two spaces to get from the origin to (2, 0), so the *x*-coordinate is 2.

Practising and Applying

Teaching points and tips

Q 1: Make sure students label the axes and the origin.

Q 2: Observe the scale that students choose. Notice that a scale of 5 or 10 would make more sense for the *x*-axis than for the *y*-axis. Discuss the reason for this with the students.

Q 5: Students can show their understanding visually by showing where the two points would be or they can use words.

Q 6: Students may have different interpretations of “far to the right” or “very high above”, but as long as they can explain their thinking and it makes sense, these differences should be allowed.

Q 7: Some students may choose to make the two given coordinates the coordinates on the same side of the parallelogram.

For example, if the vertices on one side of a parallelogram are (2, 6) and (5, 8), the other vertices could be (3, 3) and (6, 6).

Others will make the given coordinates opposite corners.

Q 8: Students might focus on the physical pattern (the points are on a line) or on the numerical pattern (the coordinates add to 10 each time).

Q 9: Students might use a ruler or they might use reasoning to answer this question.

Common errors

• Some students will not pay attention to the order of the coordinates. Others will count lines rather than spaces. For example, they would name the point that should be (3, 1) as (4, 2). Remind them of the rules.

Suggested assessment questions from Practising and Applying

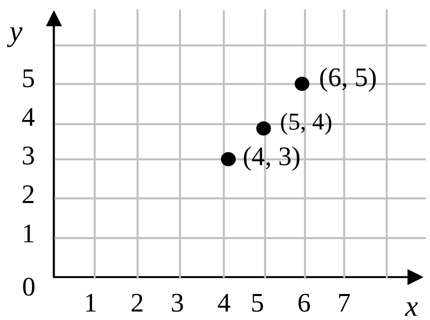
Question 2	to see if students can locate points on a coordinate grid
Question 3	to see if students can name points on a coordinate grid
Question 5	to see if students can explain why the order of the coordinates matters
Question 7	to see if students can solve a problem involving coordinates

Answers

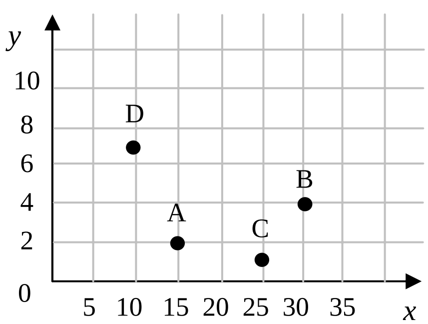
A. *Sample response*:
My partner is 2 m ahead of me and then 1 m to the right.

B. *Sample responses*:
i) Two
ii) I could have been the origin, (0, 0), and called 1 m one unit. Then my partner would have been at (1, 2).

1. a) and b)



2. [*Sample response*:
I used a scale of 5 on the *x*-axis so that I could show 30 with just 6 vertical lines.
I used a scale of 2 on the *y*-axis so I did not have to draw 7 horizontal lines.]



3. A(0, 25), B(10, 20), C(4, 30), D(8, 0)

4. *Sample responses:*
a) They are all in a line.
b) (5, 15) and (10, 30)

[**5.** *Sample response*:
The first number tells how far to go right and the second number tells how far to go up. Because the first numbers are different, I went farther to the right for (3, 2) than for (2, 3). The points cannot be at the same position.]

6. *Sample responses*:
a) (1, 1); [This point is only one space to the right and 1 space up from the origin.]
b) (20, 0); [This point is 20 spaces to the right of the origin.]
c) (0, 50); [This point is 50 spaces above the origin.]

7. *Sample response*:
(3, 8) and (4, 6)

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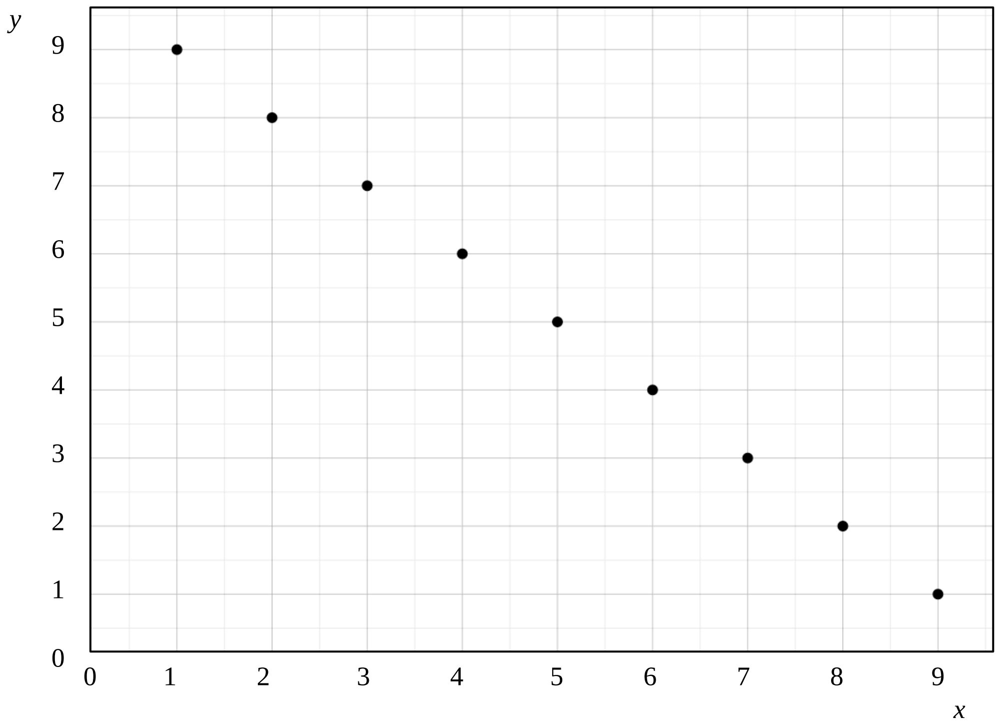
8. a)



b) *Sample response*:
The *x*-coordinate and the *y*-coordinate add to 10.

9. a) No; [*Sample response*:
I plotted (3, 3) and it was farther than 3 units away from the origin.]
b) (3, 0) or (0, 3)

10. a) It depends on the value of [ ];
[*Sample response*:
If [ ] = 5, the points are the same; if not, they are not the same.]
b) No; [they are the same distance from the origin.]

Supporting Students

Struggling students

• Rather than having students create their own grids, you might have them use grid paper.

Enrichment

• Invite students to draw a simple picture on grid paper. They can then write a list of the coordinates that form the picture, in order, and give them to another student to figure out what it is a picture of.

Chapter 3 Probability

7.3.1 Describing Probability

Curriculum Outcomes	Outcome relevance
5-G1 Experiments • conduct simple experiments with coins, slips of paper, and dice to determine experimental probability • use common language to describe probability (e.g., for a probability of 15/20, I picked red “15 out of 20 times”) • record results in charts	Students need to be able to describe probability using words and ratios before they begin to use fractions, decimals, or percents. The skills taught in this unit will give them a way to check the reasonableness of their fraction and decimal descriptions

Pacing	Materials	Prerequisites
1.5 h	• Nu 1 coin • Dice • Spinners	None

Main Points to be Raised

• Probability lets you predict whether or not something is probably going to happen, or how often it might happen.

• The terms *certain, impossible,* and *likely* are useful ways to describe probability.

• A probability line is a good way to represent how likely events are. The line goes from *impossible* to *certain*; the half closest to *impossible* is labelled *unlikely* and the half closest to *certain* is labelled *likely*.

• It is useful to describe probability using words, but it is also useful to count, for example, to tell how many times something happened out of a number of chances.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe while students work. While they work, you might ask questions such as the following:

• *Why is that unlikely?* (I do not think anyone in my class has travelled outside of Bhutan this year, so it is unlikely that most Class V students will leave the country this year.)

• *What else might also definitely be true?* (That they all finished Class IV.)

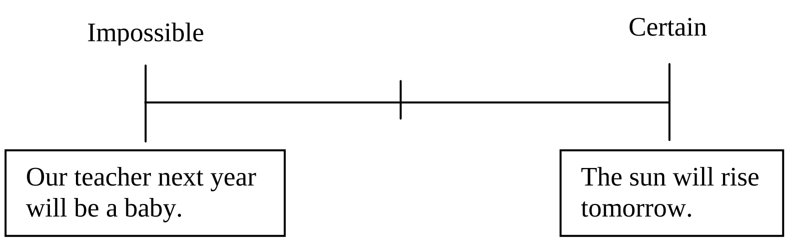
• *What do you think “likely” means?* (I think *likely* means that it happens more than it does not happen.)

The Exposition — Presenting the Main Ideas

• Ask students to think of something that they are absolutely sure will happen and something that they are absolutely sure cannot happen.

• Label a line with the word *impossible* at one end and the word *certain* at the other end. Write the events that the students suggest in the appropriate spots.

For example:



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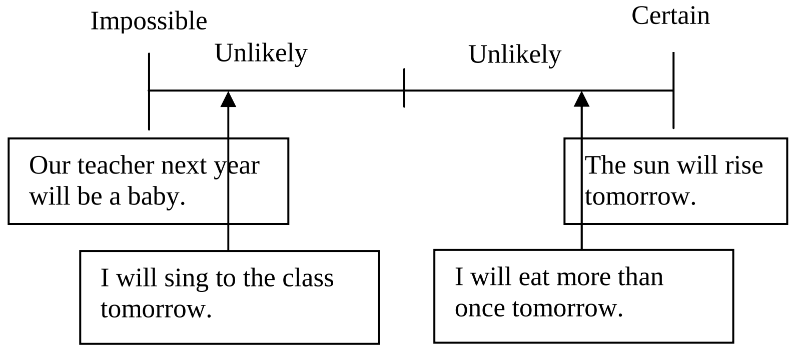
• Label the right half of the probability line with the term *likely* and the left half with the term *unlikely* as is shown in the text. Tell students that this is called a *probability line*.

Then ask for something that is likely, but not certain and something that is unlikely, but not impossible. Place those events on the line.



• Tell them that you rolled a die 10 times. A number greater than 1 came up 7 times. Ask them if they would say that it is *likely* or *unlikely* that a number greater than 1 will be rolled next. Ask where they would place that event on a probability line. Point out that the farther to the right you place an item, the more likely you think it is.

• Inform students that they can use the exposition for reference if they wish as they work through the exercises.

Revisiting the Try This

B. The placement of events that are likely or unlikely can vary. Some students may place them quite close to the ends of the line and others may place them more centrally in their respective areas.

Using the Examples

• Ask students to read through the two examples and make sure they understand them. If many students have difficulty reading, you may wish to read the examples aloud to the class.

Practising and Applying

Teaching points and tips

Q 1: Make sure students realize that there is not always a single correct answer because the answer depends on a particular student's situation.

For example, some students might say that they are unlikely to eat fruit because they rarely do, but others might say they are very likely to eat fruit. Other times, all students will answer in the same way, for example, to say whether or not a tree will talk.

Q 2: Allow some latitude in the placement of the events on the probability line for likely (but not certain) or unlikely (but not impossible) events, as long as they are placed in the correct half of the line.

Q 4 and 5: The placement of the events should be based on the experimental results, not on theoretical probability.

Q 7: Encourage students to spin the spinner at least 20 times.

Q 8: Students' interpretation of the term *likely* will vary. Technically, anything more than 15 times out of 30 is likely, but some students may want a higher number before they use that term.

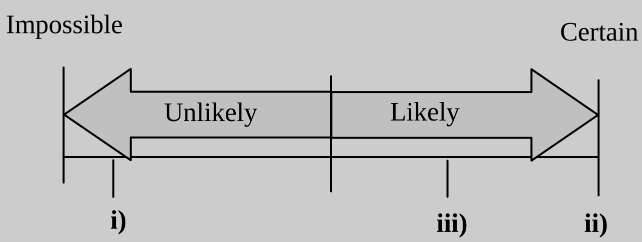
Suggested assessment questions from Practising and Applying

Question 2	to see if students can use a probability line
Question 4	to see if students can interpret experimental results using appropriate probability language
Question 8	to see if students can communicate clearly using probability language

Answers

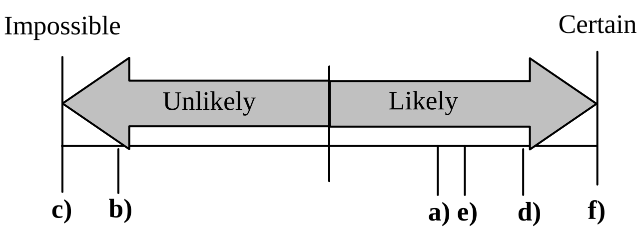
A. *Sample responses*:
i) The student is 4 years old.
ii) The student is learning about probability in math.
iii) The student is between 9 and 12 years old.

B.



1. *Sample responses*:
a) Likely
b) Very unlikely
c) Impossible
d) Very likely
e) Likely
f) Certain

2.



3. *Sample responses*:
a) I will go home after school; [It is certain that I will go home after school because I always do.]
b) I will have a snack when I get home.
c) I will play with my brother.
d) I will eat by myself; [It is very unlikely I will eat by myself because we usually eat together, but I sometimes eat supper alone when others in my household are away.]
e) I will lift up my house.

4. *Sample responses*:

6	10	2	9	4	7	7	5	11	6	8	3	7	6	8
9	5	8	6	7	12	7	11	10	5	8	9	6	4	7

a) 0 out of 30; Impossible
b) 2 out of 30; Very unlikely
c) 4 out of 30; Unlikely
d) 30 out of 30; Certain

5. *Sample responses*:

K	T	K	T	T	T	K	T	K	K	T	K	K	T	K
T	K	T	T	K	K	T	K	T	K					

a) As likely to happen as not to happen; [I tossed Khorlo 13 times and Tashi Ta-gye 12 times. 13 and 12 are very close, and the number of tosses of each could have been switched around, so I think it is as likely to get a Khorlo as not.]
b) As likely to happen as not to happen; [I tossed Khorlo 13 times and Tashi Ta-gye 12 times. 13 and 12 are very close, and the number of tosses of each could have switched around, so I think it is as likely to get a Tashi Ta-gye as not.]
c) Very unlikely; [It happened only 3 times in all my tries.]
d) Very unlikely; [It never happened in my tries, but I do not think it is impossible.]

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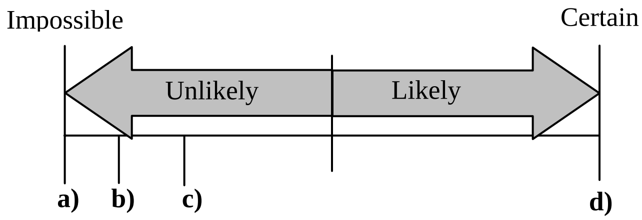
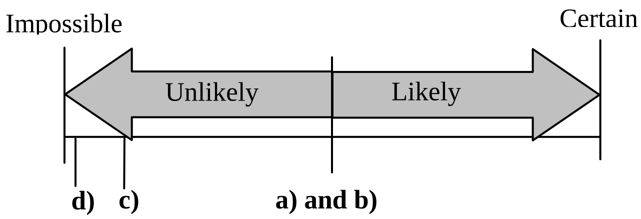
6. a)



b)



7. a) i) Spinning a number less than 4
ii) Spinning an odd number
iii) Spinning a 2
iv) Spinning a 4
[**b)** Spinning a number less than 4 is certain because every number on the spinner is less than 4.
Spinning a 4 is impossible because there is no 4 on the spinner.]

8. *Sample response:*
I would say 25 times would make it very likely and 5 times would make it not very likely. [If the number of times were close to 15, it might actually be as likely as not, but it just happens to have come up more one way than the other.]

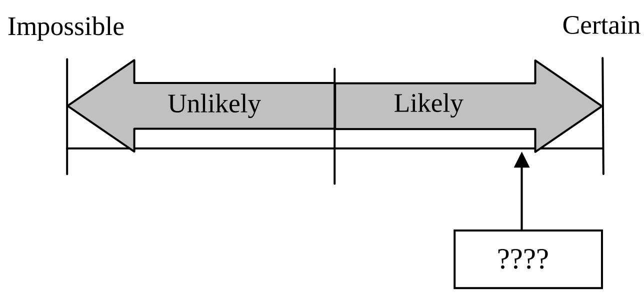
[**9.** *Sample response:*
It is a way of seeing the information quickly and easily.]

Supporting Students

Enrichment

• Students might wish to create events to fit particular points on a probability line.

For example, they might be asked to come up with an event that belongs in the location shown below.

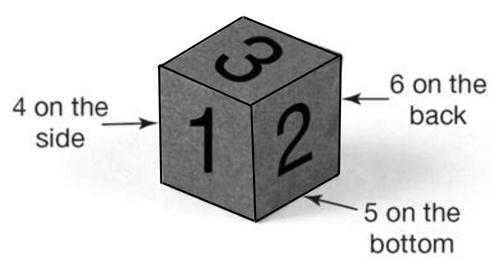


CONNECTIONS: Magic Tricks with Dice

Many students will find this very interesting. The reasoning is based on the fact that dice are created so that opposite faces add to 7. If this were not true, the trick would not work.

Answers

1. The numbers on opposite faces of a die add to 7, so the two pairs of opposite faces in the stack add to 14. If one of the numbers is 2, the rest of the numbers must add to 14 – 2 = 12.



2. a) 16
b) Subtract the top number from 21 since there are 3 pairs of opposite faces so the total is 21.

3. Subtract the top number from 28.

7.3.2 Using Numbers to Describe Probability

Curriculum Outcomes	Outcome relevance
5-G1 Experiments • conduct simple experiments with coins and dice to determine experimental probability • predict and record experimental results as fractions and decimals • understand that theoretical probability is the number of favourable outcomes divided by the number of possible outcomes • understand that experimental probability is the number of times the favourable outcome occurs divided by the number of trials in the experiment • record results in charts **5-G2 Describe Probability** • understand that experimental probability is determined by performing experiments • understand that theoretical probability is what you would expect to happen after considering the possible outcomes • use fractions and decimals to describe theoretical probability and experimental probability	Probability is described mathematically as a fraction. For this reason students need to begin using fractions and decimals to describe probability.

Pacing	Materials	Prerequisites
1.5 h	• Dice • Nu 1 coin • Numbered slips of paper	• meanings of fractions • simple fraction/decimal equivalence

Main Points to be Raised

• Experimental probability is defined as $\frac{\text{number of favourable results}}{\text{number of trials}}$.

• Theoretical probability is defined as $\frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$.

• Probabilities range from 0 to 1.

• An event that is likely to happen has a probability greater than $\frac{1}{2}$. An event that is unlikely to happen has a probability less than $\frac{1}{2}$. An event that is certain to happen has a probability of 1. An event that is impossible has a probability of 0.

• Experimental and theoretical probability can be different, but with many trials they are usually close.

• Experimental probability can change with more trials, but theoretical probability never changes.

Try This — Introducing the Lesson

A. Distribute dice and allow students to try this alone or with a partner. As you observe students at work, you might ask questions such as the following:

• *Which do you think you will roll more often: numbers greater than 2 or numbers that are 2 or lower? Why?* (Numbers greater than 2 because there are more numbers greater than 2 than there are numbers that are 2 or lower.)

• *How did you make your prediction?* (I listed the numbers from 1 to 6 over and over until I had written 30 numbers. Then I counted how many times a number greater than 2 appeared.)

• *You predicted 20 times, but it was 19 times. Do you think your prediction was a good one?* (Yes. When I roll, I cannot be sure what will happen. 19 is close to 20.)

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The Exposition — Presenting the Main Ideas

• Roll a die 10 times and report the results to your students. For example, you might roll 1, 4, 5, 4, 2, 6, 1, 3, 1, 5. Ask students why you might say the probability of getting a 1 is $\frac{3}{10}$ or 0.3. Discuss that there were 3 rolls of 1 out of a total of 10 rolls. Then ask students why they might have predicted the probability as $\frac{1}{6}$ before you rolled (there are 6 possible outcomes, but only 1 of them is a 1). Tell them that that the predicted probability is called *theoretical*; it is what you expect when you compare the number of *favourable outcomes* (outcomes that are desired) to the number of possible outcomes. The first probability is called *experimental probability*; it is what actually happened in the experiment.

• Write on the board:

theoretical probability = $\frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$ experimental probability = $\frac{\text{number of favourable results}}{\text{number of trials}}$

Show students how 3 represented the number of favourable results and 10 represented the number of trials and how 1 represented the number of favourable outcomes and 6 represented the number of possible outcomes.

• Ask students about the experimental and theoretical probability of rolling a number less than 7. They should see that the values are $\frac{10}{10}$ or $\frac{6}{6}$ = 1. The probability of an event that is certain to happen is always 1.

• Ask about the experimental and theoretical probability of rolling a 10. They should see that the values are $\frac{0}{10}$ or $\frac{0}{6}$ = 0. The probability of an impossible event is 0.

• Redraw a probability line with 0 and 1 as the end points.

• Ask students to describe events with probability $\frac{1}{2}$.

• Tell students that they can read the exposition for reference if they wish.

Revisiting the Try This

B. Ask students why the number of rolls does not matter in calculating the theoretical probability; it only matters for the experimental probability.

Using the Examples

• Provide dice to the students. Ask them to carry out the experiment in **example 1**, calculating their own probabilities. They should then consider the solution in **example 1**. Discuss **example 2** together with the class. Make sure they understand why three slips of paper were used and a 1 was written on two of them. Ask what slips of paper they would use if they want the theoretical probability of drawing a 1 to be $\frac{3}{4}$.

Practising and Applying

Teaching points and tips

Q 1: Students should understand that different students might obtain different results.

Q 2: Ask students why the denominator for all the theoretical probability fractions is 6.

Q 3: You may have to explain the game. A student rolls, for example, 2 and 3 to create 23. Then he or she rolls twice more to make another 2-digit number, but before doing so, he or she must predict whether the sum of 23 and the new number will be greater than 66. Then the student rolls to test.

Q 4: Ask why the denominator for each experimental probability is 25, but for each theoretical probability the denominator is 4.

Common errors

• Students may confuse theoretical and experimental probability.

• Some students may have difficulty writing probability as a decimal if the number of trials is not 10. You may suggest that these students perform the experiments 10 times rather than 20 or 25 times.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can make reasonable predictions about probability
Question 4	to see if students can calculate experimental and theoretical probability
Question 5	to see if students can create a situation to match a probability

Answers

Note: Fractions are sometimes shown in two forms, the likely response and in lowest terms because students may write the probabilities in lowest terms, but they should not be required to do so.

A. *Sample responses*:
My results were:

4	3	2	5	1	6	4	6	5	2
4	1	6	3	6	3	5	3	6	1
2	5	4	3	3	1	6	2	3	4

i) 20 times
ii) 22 times out of 30 I rolled a number greater than 2, so I was close.

B. *Sample responses:*
i) $\frac{22}{30} = \frac{11}{15}$
ii) $\frac{4}{6} = \frac{2}{3}$

1. *Sample responses*:
a)

T	T	K	T	T	T	T	T	K	K
T	K	T	K	K	K	T	K	T	T

b) i) $\frac{8}{20} = \frac{2}{5}$ **ii)** $\frac{12}{20} = \frac{3}{5}$ **iii)** 1

2. a) *Sample response*:

Number rolled	1	2	3	4	5	6
Number of times	3	4	2	5	3	3

b) *Sample responses*:
i) $\frac{4}{20} = \frac{20}{100} = 0.20$ **ii)** $\frac{9}{20} = \frac{45}{100} = 0.45$ **iii)** $\frac{8}{20} = \frac{40}{100} = 0.40$

c) i) $\frac{1}{6}$ **ii)** $\frac{3}{6} = \frac{1}{2}$ **iii)** $\frac{3}{6} = \frac{1}{2}$

d) *Sample responses*:
i) Rolling a number less than 4
ii) Rolling a 1

3. b) *Sample response*: Correct: 12 times, Incorrect: 8 times
$\frac{12}{20} = \frac{6}{10} = 0.6$

4. a) *Sample response*:

3	1	5	4	5	1	3	5	1	4	3	5	1	4	3
4	3	5	1	1	3	4	4	5	5					

b) *Sample responses*:
i) 0.24 **ii)** 0.24 **iii)** 0.48
c) i) 0.25 **ii)** 0.25 **iii)** 0.5

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5. *Sample responses*: **a)** 1, 1, 2, 3 **b)** 1, 3, 3, 3 **c)** 1, 1, 1, 1 **d)** 2, 4, 6, 8	[**6.** *Sample response*: The number of possible outcomes and favourable outcomes are based on what *can* happen and not on what *does* happen, so the number of possibilities cannot change. But if you do the experiment many times, you can never be sure exactly what will happen each time, so the fractions or decimals for the experimental probability can be different each time you do the experiment.]

Supporting Students

Struggling students

• Some students will find it easier to write probabilities for experimental results than for theoretical results. To express theoretical probability, students have to analyse what could happen instead of what did happen. Encourage them to list all the possible outcomes in an organized way to help them determine the denominator for a theoretical probability.

Enrichment

• Students might enjoy creating more questions like those in **question 5**, where they create a situation to match a probability.

UNIT 7 Revision

Pacing	Materials
2 h	• Grid paper or Small Grid Paper (BLM) • Dice

Question(s)	Related Lesson(s)
1 and 2	Lesson 7.1.1
3	Lesson 7.1.2
4 – 6	Lesson 7.2.1
7 – 9	Lesson 7.2.2
10 – 13	Lesson 7.2.3
14	Lesson 7.3.1
15 and 16	Lesson 7.3.2

Revision Tips

Q 1: Students could either draw a set of items rearranged into equal shares or they could use a bar graph to show that the missing amounts below the mean balance the extra amounts above the mean.

Q 2: Students should realize that the sum of the five new scores must be the same as the sum of the five original scores.

Q 6 and 8: Encourage students not to just read off values, but to make comparisons and combine information in different ways.

Q 10: Remind students that the scales on the two axes can be different.

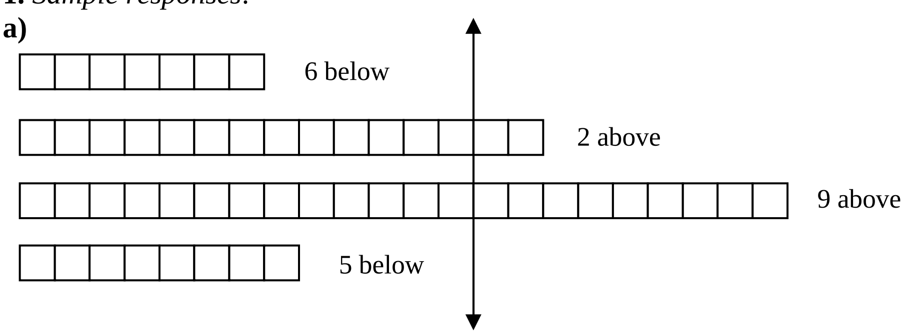
Q 12: The two coordinate pairs given can describe points that are on the same side or on opposite sides of the square.

Q 14: Students can use the words *likely, unlikely, certain*, and *impossible.*

Answers

1. *Sample responses*:

a)



Mean = 13

[I started with a vertical line where I thought the mean might be. I moved it to the right until there were as many squares to the right of it as there were missing squares to the left of it. 6 + 5 are below and 2 + 9 are above; these are equal.]

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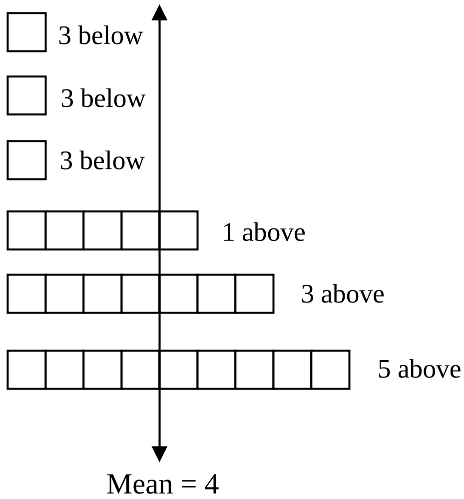
b)

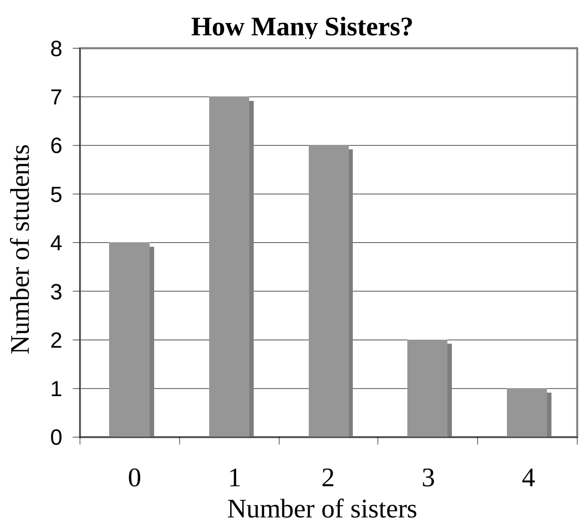


[I started with a vertical line where I thought the mean might be. I moved it to the right until there were as many squares to the right of it as there were missing squares to the left of it. 3 + 3 + 3 = 9 are missing on the left and 1 + 3 + 5 = 9 are extra on the right.]

2. a) 68 **b)** *Sample response*: 58, 78, 68, 68, 68

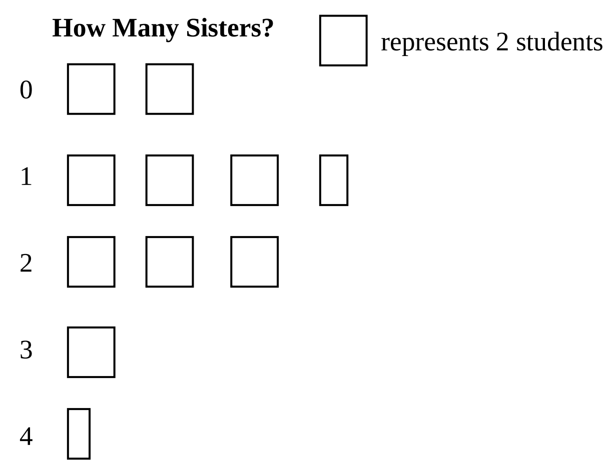
3. *Sample responses*:
a) Predicted : 2, 3, 5, 18
[The total is greater and both totals are divided by the same amount, 4.]
b) Predicted: They are the same.
[One number is less by 1 and one is greater by 1, so the totals are the same. They are both divided by the same amount, 4.]
c) Predicted: 3, 5, 10
[A low amount is removed, so the amount missing below the mean is less. That means the mean must move up so there is not too much above it.]
d) Predicted: 2, 3, 5, 10
[A high amount is removed, so the amount above the mean is less. That means the mean must move down so there is not too much below it.]

4. a)



4. b) *Sample response*:
- More people have 1 sister than any other number of sisters.
- More people have no sisters than 3 sisters.
- There are twice as many people with no sisters as 3 sisters.

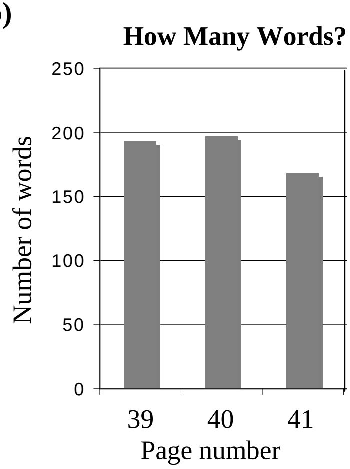
c) *Sample response*:



5. *Sample responses*:

a) 193, 197, 168

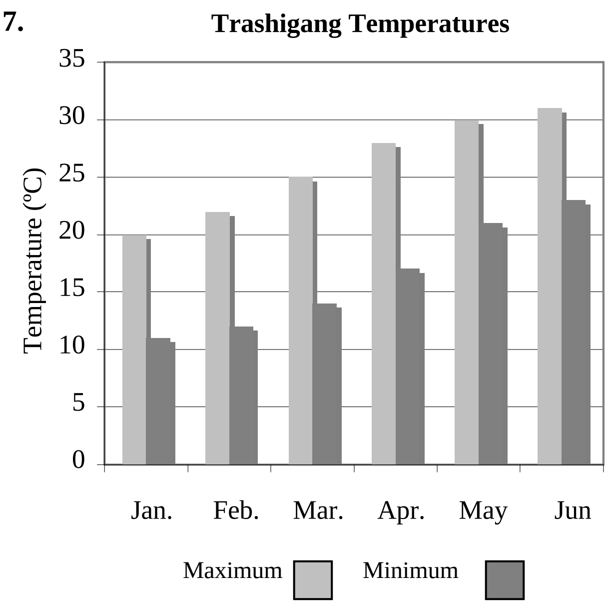
b)



c) I used a scale of 50 [because I had only a small piece of grid paper and the bars had to be less than 5 units tall.]

6. *Sample response*:
- Tashi watched more TV on Friday than on any other day.
- He watched almost as much TV on Monday as on Friday.
- He watched the least amount of TV on Wednesday.

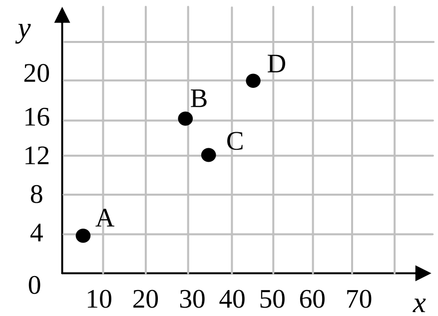
7.



8. *Sample response*:
- Both Class I and Class V students like chocolate bars the most.
- More Class I students than Class V students like chocolate bars best.
- The least favourite treat was either chips or momos.

9. Yes; [*Sample response*:
He can compare how much homework students do in each subject near the beginning of the school year and near the end of the year. He can also compare the amount of time they spent on each subject.]

10.

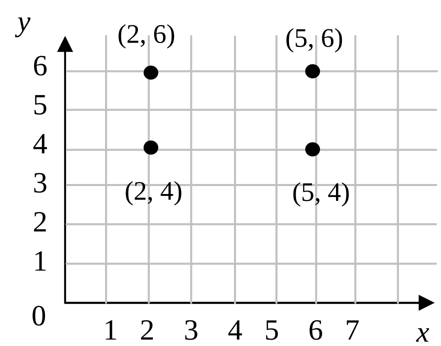


Sample response:
- I let one unit represent 10 on the *x*-axis [so I could easily get to 45.]
- I let one unit represent 4 on the *y*-axis [so it would be easy to show 4, 12, 16, and 20, which are all multiples of 4.]

11. A (10, 60), B (40, 40), C (20, 20), D (0, 30)

12. *Sample response*: (2, 8) and (5, 5)

13. a)



b) They form a rectangle.

14. *Sample responses*:

1	0	2	1	1	3	2	4	3	1
2	2	5	0	2	1	0	3	1	4
5	0	1	2	4					

a) Certain
b) 6 out of 25; Unlikely
c) 7 out of 25; Unlikely
d) 8 out of 25; Unlikely

15. a) *Sample response*:

4	2	4	5	3	2	1	6	4	2
3	1	1	4	1	6	5	2	3	5

b) *Sample responses*:
i) 0.4 **ii)** 0.1

c) i) $\frac{2}{6} = \frac{1}{3}$ **ii)** $\frac{1}{6}$

15. d) *Sample responses*:
i) Rolling a number greater than 1
ii) Rolling a 1

16. *Sample responses*:
a) 1, 2, 3
b) 2, 4, 6, 8, 9
c) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

UNIT 7 Data and Probability Test

1. a) Calculate the mean of 8, 19, 22, and 23.

b) Draw a picture to show why that is the mean.

2. a) Create a set of four different numbers with a mean of 25.

b) Show how to change those numbers to increase the mean to 30.

3. A die is rolled 50 times. The results are shown.

What is rolled	1	2	3	4	5	6
Number of times	8	8	10	7	11	6

a) Draw a bar graph to show the results.

b) Draw a pictograph to show the same results. Use a different scale.

c) Explain how you chose your scale either for **part a)** or for **part b)**.

d) Tell three things about one of your graphs.

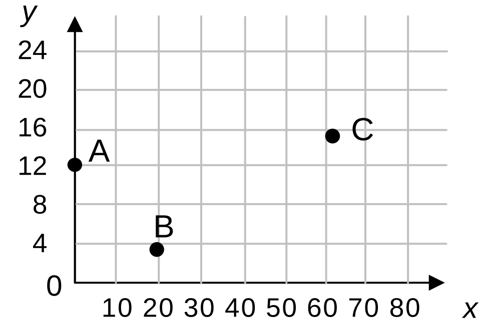
4. a) Create a double bar graph to compare the rolls of the die in **question 3** to these 50 other rolls.

What is rolled	1	2	3	4	5	6
Number of times	9	7	9	8	10	7

b) Tell three things about your double bar graph.

c) Explain why a double bar graph is a good graph to use for this data.

5. a) List the coordinates for point A, point B, and point C.



b) Copy the grid above and plot these points on it:

D(30, 16) E(50, 0) F(45, 14)

6. Two vertices of a rectangle are at (4, 7) and (6, 9). List possible coordinates for the other two vertices.

7. Name an event to match each probability description.

a) Certain **b)** Impossible

8. a) Roll 2 dice and subtract the lower number from the higher number rolled. Repeat the experiment 20 times altogether. Record your data.

b) Use your data. Mark on a probability line how likely it is that the difference will be 0 and how likely it is that the difference will be 3.

c) Write the experimental probability of a difference of 0 as a fraction and as a decimal.

9. a) Four slips of paper, marked 1, 1, 3, 5, and 6 are put in a bag and one slip of paper is drawn without looking. What is the theoretical probability of drawing a 1?

b) What slips of paper would you put in the bag for the probability of drawing a 1 to be $\frac{3}{7}$?

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UNIT 6 Test

Pacing	Materials
1 h	• Grid paper or Small Grid Paper (BLM) • Dice

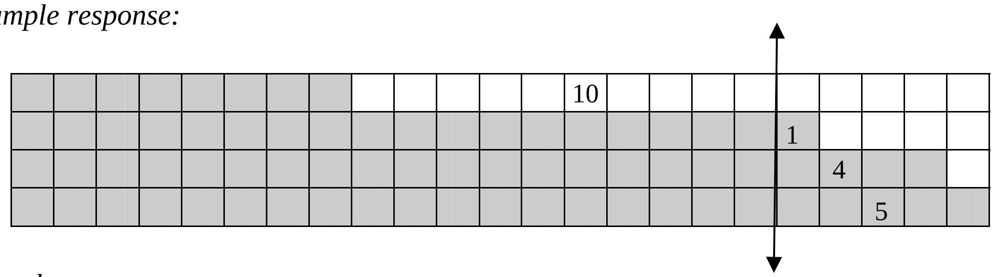
Question(s)	Related Lesson(s)
1	Lesson 7.1.1
2	Lesson 7.1.2
3	Lesson 7.2.1
4	Lesson 7.2.2
5 and 6	Lesson 7.2.3
7	Lesson 7.3.1
8 and 9	Lesson 7.3.2

Select questions to assign according to the time available.

Answers

1. a) 18

b) *Sample response:*



2. *Sample responses:*

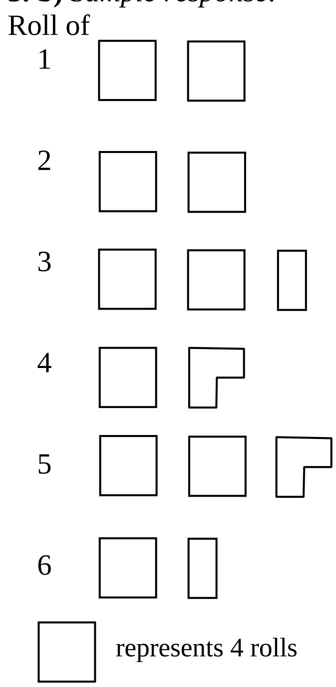
a) 20, 21, 29, 30

b) Add 5 to each number: 25, 26, 34, 35

3. a)

Answers [Continued]

3. b) *Sample response:*

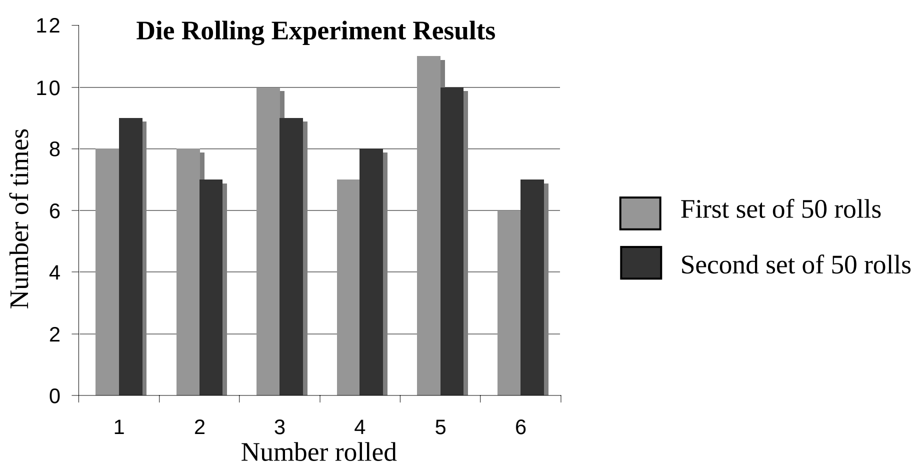


c) I chose the scale for the bar graph to be 2 because it was easy to show all the numbers clearly and the graph was not too high.

d) *Sample response:*

There were more rolls of 5 than of any other number.
The number that came up least often was 6.
There were as many 1s as 2s.

4. a)



b) *Sample response:*

The results for the two sets of rolls are similar.
There were fewer 5s on the second set of rolls.
There were more 4s on the second set of rolls.

c) It makes sense to use a double bar graph because there were the same number of rolls, the scales for showing both sets of data are similar, and it makes sense to compare the two sets of data.

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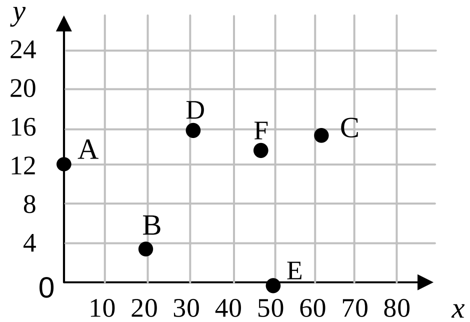
5. a) A (0, 12), B(20, 4), C(60, 16)

b)



6. *Sample response:* (4, 9) and (6, 7)

7. *Sample responses:*

a) The sun will set tonight.

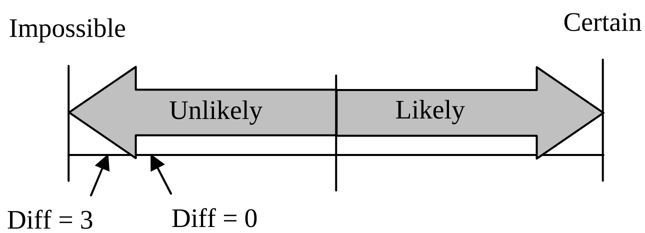
b) An elephant will pass a math test.

8. *Sample responses:*

a)

1	2	2	0	1	3	0	5	2	1	0	4	3	0	4	1	2	3	2	1

b)



c) $\frac{4}{20} = 0.2$

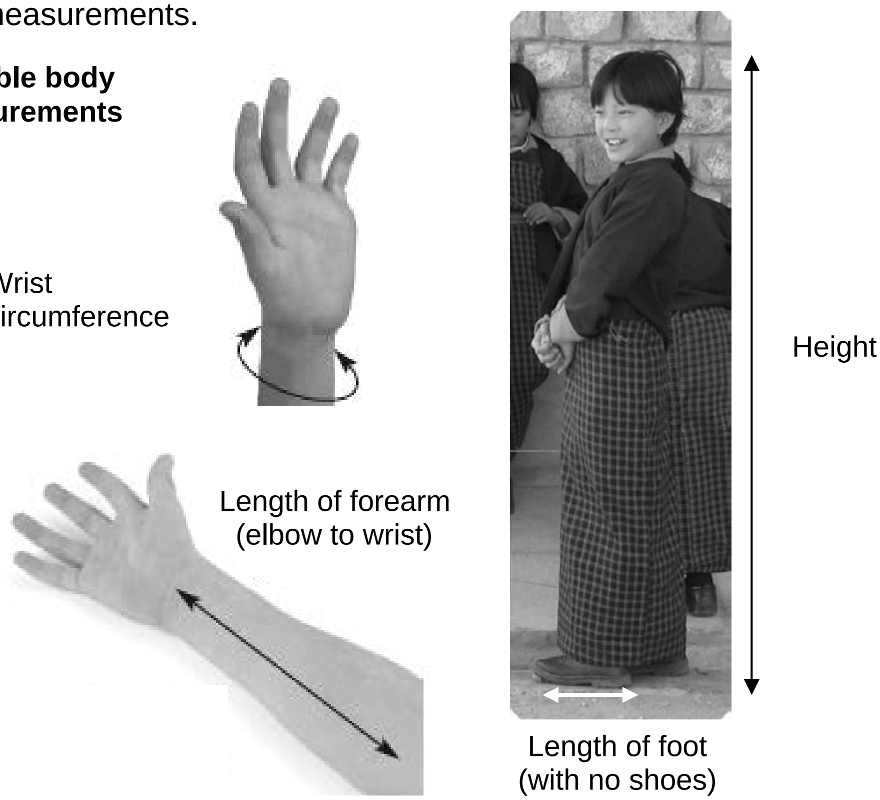
9. a) $\frac{2}{5}$

b) *Sample response:* 1, 1, 1, 2, 3, 4, 5

UNIT 7 Performance Task — Investigating Body Relationships

People who make clothes pay attention to how the measurements of different body parts relate to each other. For example, they know that a tall person likely has a greater wrist circumference than a short person. They might need to measure only a couple of parts of a person. They can use what they know about body relationships to predict the person's other measurements.

Possible body measurements



A. Choose two body measurements that you think might be related.

• Put the names of all the students in your class in a bangchung and draw out 10 names (do not include your own name).

• Collect information about the two body measurements for those 10 students.

B. Create a double bar graph that compares the two body measurements for the 10 students.

C. i) Determine the mean of each group of 10 measurements. Explain how you did it.

ii) Replace the lowest value in each group with your own body measurements. Calculate the mean of each group again.

iii) Tell how the means changed. How could you have predicted those changes?

D. Suppose you put all 20 measurements from **part A** on slips of paper in a bangchung and drew out one slip. Would each prediction below be true? How do you know?

i) It is likely that the measurement drawn is less than both means from **part C i)**.

ii) The probability is greater than $\frac{1}{2}$ that the measurement drawn is more than both means.

How could you test your predictions?

E. Is there a relationship between the two body measurements? Explain your thinking.

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UNIT 7 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
5-F1 Mean: effect of change in data **5-F2** Collect, Organize, and Describe Data **5-F3** Double Bar Graphs: create and interpret **5-G2** Describe Probability	1 h	• Tape measures or rulers and string

How to Use This Performance Task

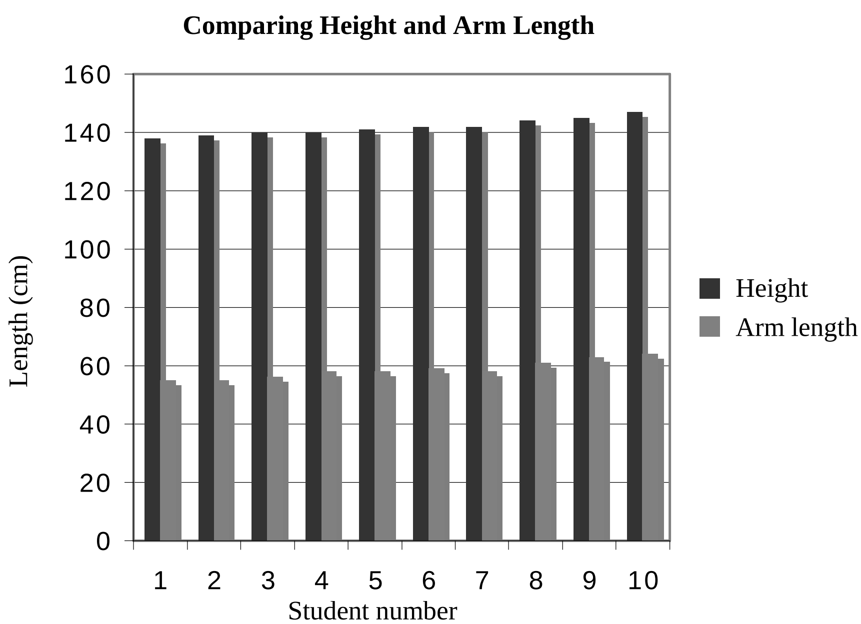
You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. It could also be used as enrichment material for some students. You can assess performance on the task using the rubric on the next page.

Sample Solution

A. I decided to measure height and arm span (the distance from fingertip to fingertip across outstretched arms).

Height (cm)	138	139	140	140	141	142	142	144	145	147
Arm length (cm)	55	55	56	58	58	59	58	61	63	64

B.



C. i)

											Mean
Height (cm)	138	139	140	140	141	142	142	144	145	147	141.8
Arm length (cm)	55	55	56	58	58	59	58	61	63	64	58.7

To get the mean, I added all the values and divided by 10.

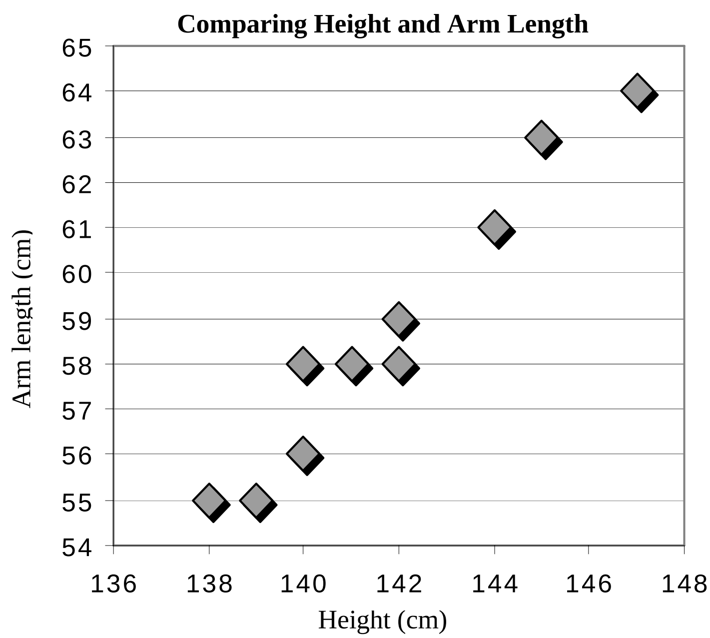
ii)

											Mean
Height (cm)	144	139	140	140	141	142	142	144	145	147	142.4
Arm length (cm)	62	55	56	58	58	59	58	61	63	64	59.4

iii) The mean of the heights increased to 142.4 cm. The mean of arm lengths increased to 59.4 cm.
I could have predicted the changes because the mean goes up when you replace a value with a higher value.

D. i) Not true;
The lesser mean is 58.7 so any measurement less than 58.7 would be favourable. Only
5 numbers out of the 20 are less than 58.7, so it is not likely that one of these numbers would be chosen.
ii) Not true;
The greater mean is 141.8 so any measurement more than 141.9 would be favourable. Only
5 numbers out of the 20 are more than 141.8, so the probability of drawing one of these numbers is $\frac{5}{20}$,
which is less than half.
I can do an experiment to test my predictions. I would put all 20 slips of paper in a bangchung. I would pull one slip out, record the value, and then return the slip to the bangchung. I would do this 20 times. Each time, I would compare the value to 58.7 for **part i)** and to 141.8 for **part ii)**.

E. It looks like there is a relationship between arm length and height because the people with longer arm lengths were also the taller people. I drew a coordinate graph where the *x*-coordinate is the height and the *y*-coordinate is the arm length to see if they formed a pattern. It looked like they did.



UNIT 7 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Creates and interprets a graph	Easily and correctly creates and interprets double bar graphs and coordinate graphs.	Correctly creates and interprets double bar graphs and coordinate graphs.	Has difficulty either creating or interpreting the double bar graph or the coordinate graph.	Has difficulty creating and interpreting the double bar graph.
Calculates a mean and predicts how a mean will change	Easily calculates both means, and correctly predicts and effectively explains how the mean will change.	Easily calculates both means, and correctly predicts and explains how the mean will change.	Correctly calculates both means and predicts how they will change.	Has difficulty calculating means and predicting how they will change.
Interprets probability	Effectively predicts both probabilities and clearly explains how to test those predictions.	Correctly predicts both probabilities and explains how to test those predictions.	Correctly predicts both probabilities.	Does not correctly predict both probabilities.

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BLM 1 Coordinate Grids