Understanding Mathematics

Textbook for Class V

ཤེས་རིག

Department of School Education
Ministry of Education and Skills Development
Royal Government of Bhutan

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MINISTER

ROYAL GOVERNMENT OF BHUTAN
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THIMPHU : BHUTAN

FOREWORD

Provision of quality education for our children is a cornerstone policy of the Royal Government of Bhutan. Quality education in mathematics includes attention to many aspects of educating our children. One is providing opportunities and believing in our children's ability to understand and contribute to the advancement of science and technology within our culture, history and tradition. To accomplish this, we need to cater to children's mental, emotional and psychological phases of development, enabling, encouraging and supporting them in exploring, discovering and realizing their own potential. We also must promote and further our values of compassion, hard work, honesty, helpfulness, perseverance, responsibility, *thadamtsi* (for instance being grateful to what I would like to call '*Pham Kha Nga',* consisting of parents, teachers, His Majesty the King, the country and the Bhutanese people, for all the goodness received from them and the wish to reciprocate these in equal measure) and *ley-ju-drey* — the understanding and appreciation of the natural law of cause and effect. At the same time, we wish to develop positive attitudes, skills, competencies, and values to support our children as they mature and engage in the professions they will ultimately pursue in life, either by choice or necessity.

While education recognizes that certain values for our children as individuals and as citizens of the country and of the world at large, do not change, requirements in the work place advance as a result of scientific, technological, and even political advancement in the world. These include expectations for more advanced interpersonal skills and skills in communications, reasoning, problem solving, and decision-making. Therefore, the type of education we provide to our children must reflect the current trends and requirements, and be relevant and appropriate. Its quality and standard should stem out of collective wisdom, experience, research, and thoughtful deliberations.

Mathematics, without dispute, is a beautiful and profound subject, but it also has immense utility to offer in our lives. The school mathematics curriculum is being changed to reflect research from around the world that shows how to help students better understand the beauty of mathematics as well as its utility.

The development of this textbook series for our schools, *Understanding Mathematics,* is based on and organized as per the new School Mathematics Curriculum Framework that the Ministry of Education has developed recently, taking into consideration the changing needs of our country and international trends. We are also incorporating within the textbooks appropriate teaching methodologies including assessment practices which are reflective of international best practices.

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The *Teacher's Guides* provided with the textbooks are a resource for teachers to support them, and will definitely go a long way in assisting our teachers in improving their efficacy, especially during the initial years of teaching the new curriculum, which demands a shift in the approach to teaching and learning of Mathematics. However, the teachers are strongly encouraged to go beyond the initial ideas presented in the Guides to access other relevant resources and, more importantly to try out their own innovations, creativity and resourcefulness based on their experiences, reflections, insights and professional discussions.

The Ministry of Education is committed to providing quality education to our children, which is relevant and adaptive to the changing times and needs as per the policy of the Royal Government of Bhutan and the wish of our beloved King.

I would like to commend and congratulate all those involved in the School Mathematics Reform Project and in the development of these textbooks.

I would like to wish our teachers and students a very enjoyable and worthwhile experience in teaching, learning and understanding mathematics with the support of these books. As the ones actually using these books over a sustained period of time in a systematic manner, we would like to strongly encourage you to scrutinize the contents of these books and send feedback and comments to the Curriculum and Professional Support Division (CAPSD) for improvement with the future editions. On the part of the students, they can and should be enthusiastic, critical, venturesome, and communicative of their views on the contents discussed in the books with their teachers and friends rather than being passive recipients of knowledge.

Trashi Delek!

Thinley Gyamtsho

MINISTER

Ministry of Education

October of 2007

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INTRODUCTION

HOW MATHEMATICS HAS CHANGED

In Class V this year, you will learn some new mathematics that Class V students before you did not learn. Some things are the same, but many things are different. For example, many of the topics you will learn about in geometry are new to all Class V students.

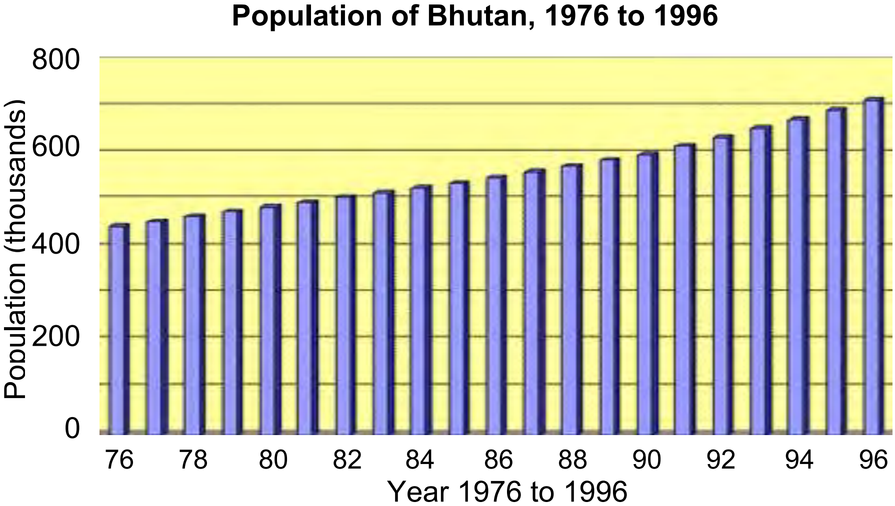
You will learn mathematics differently this year. Instead of memorizing and following rules, you will do much more explaining and making sense of the mathematics. When you understand the mathematics, you will find it more interesting and easier to learn.

Your new textbook lets you work on problems about everyday life as well as on problems about Bhutan and the world around you. These problems will help you see the value of math.

For example:

• One problem will ask you to calculate how much rent a family will pay for their home in 6 months.

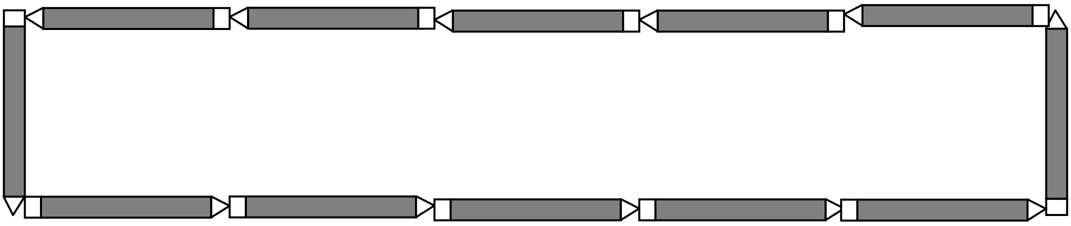
• In another lesson you will look at a graph of the population of Bhutan to see how it is changing.



Your textbook will often ask you to use objects to learn the math.
For example:

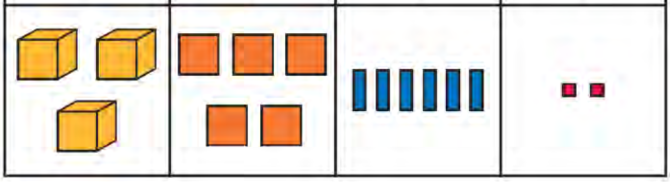
• You will build with cubes to learn about geometry.

• You will measure perimeter with everyday objects, like crayons.



• You will use base ten models of thousands, hundreds, tens, and ones to learn to multiply and divide.

Thousands	Hundreds	Tens	Ones
3	5	6	2



Modelling the number 3562 with base ten blocks on a place value mat

• You will roll dice to predict how likely it is that something will happen.

This textbook will also ask you to explain *why* things are true. It will not be enough if you just say that they are true. For example, you will not only calculate the answer to 20 × 30, but also explain why 20 × 30 = 600.

You will solve many types of problems and you will be encouraged to use your own way of thinking to solve them.

USING YOUR TEXTBOOK

Each unit has

- a *Getting Started* section
- two or three chapters
- regular lessons and at least one *Explore* lesson
- a *Game*
- a *Connections* activity
- a *Unit Revision*

Getting Started

There are two parts to the *Getting Started*. You will complete a *Use What You Know* activity and then you will answer *Skills You Will Need* questions. Both remind you of things you already know that will help you in the unit.

- The *Use What You Know* activity is done with a partner or in a group.
- The *Skills You Will Need* questions help you review skills you will use in the unit. You will usually do these by yourself.

Regular Lessons

- Lessons are numbered #.#.# — the first number tells the unit, the second number is the chapter, and the third number is the lesson in the chapter.

For example, Lesson 4.2.1 is Unit 4, Chapter 2, Lesson 1.

- Each regular lesson is divided into five parts:

- A *Try This* problem or task
- A box that explains the main ideas of the lesson; it is called the exposition
- A question that asks you to think about the *Try This* problem again, using what you have learned in the exposition
- one or more *Examples*
- *Practising and Applying* questions

Try This

- The *Try This* is in a grey box, like this one from lesson 1.1.1 on page 2.

Try This

A dragonfly flaps its wings between 20 times and 40 times each second.

A. How many times does it flap in 1 min?
(Remember: 1 min = 60 s)

You will solve the *Try This* problem with a partner or in a small group. The math you learn later in the lesson will relate back to this problem.

The Exposition

• The exposition comes after the *Try This*.

• It presents and explains the main ideas of the lesson.

• Important math words are in **bold** text. You will find the definitions of these words in the glossary at the back of the textbook.

• You are not expected to copy the exposition into your notebook.

Going Back to the Try This

• There is always a question after the exposition that asks you to think about the *Try This* problem again. You can use the new ideas presented in the exposition. In the example below from lesson 1.1.1 on page 3, the exposition shows how to use rectangle models to multiply. You can use these rectangles to solve the *Try This* problem again in a different way.

> **B.** What rectangles could you draw to multiply the number of wing flaps for the dragonfly in **part A**?

Examples

• The *Examples* prepare you for the *Practising and Applying* questions. Each example is a bit different from the others so that you can refer to many models.

• You will work through the examples sometimes on your own, sometimes with another student, and sometimes with your teacher.

• What is special about the examples is that the *Solutions* column shows you what you should write when you solve a problem, and the *Thinking* column shows you what you might be thinking as you solve the problem.

• Some examples show you two different solutions to the same problem. The example below from lesson 1.1.3 on page 9 shows two possible ways to answer the question, *Solution 1* and *Solution 2*.

<table>
<tr><th colspan="2">Example 3 Solving a Problem Involving Multiplication</th></tr>
<tr><td colspan="2">428 students in a school each brought Nu 25 to help other people who needed money. How much money was collected altogether?</td></tr>
<tr><td>Solution 1
1
~~14~~
428
× 25
2140
+ 8560
10,700

Nu 10,700 was collected.</td><td>Thinking

• I knew I had to multiply 428 × 25.
• I multiplied each part of 428 by 5 first.
• When I finished that, I crossed out the regrouping numbers so I wouldn't use them when I multiplied 428 by 20.
• I multiplied 428 × 20 by multiplying by 2 and then by 10. I knew that, to multiply by 10, you just add a 0 to the end of 856.</td></tr>
</table>

Solution 2	**Thinking**
428 × 25 428 ÷ 4 = 107 107 × 100 = 10,700 Nu 10,700 was collected.	• I knew I had to multiply 428 × 25. • Multiplying by 25 is the same as dividing by 4 and multiplying by 100 because 25 = 100 ÷ 4. 

Practising and Applying

• You might work on the *Practising and Applying* questions by yourself, with a partner, or in a group. You can use the exposition and examples to help you.

• The first few questions are similar to the questions in the *Examples* and the exposition.

• The last question helps you think about the most important ideas you have learned in the lesson.

Explore Lessons

• An *Explore* lesson lets you work with a partner or in a small group to investigate some math.

• Your teacher does not tell you about the math in an *Explore* lesson. Instead, you work through the questions and learn your own way.

Connections Activity

• The *Connections* activity is usually something interesting that relates to the math you are learning. For example, in Unit 2, the *Connections* on page 60 is about a toy called a kaleidoscope that is based on geometry ideas you are learning in the unit.

• Every unit has a *Connections* activity.

• You will usually work in pairs or small groups to complete the task or answer the question(s).

Game

• Each unit usually has at least one *Game*.

• The *Game* is a way to practise skills and concepts from the unit with a partner or in small group.

• The materials you need and the rules for the game are listed in the textbook. Usually the textbook shows a sample game to help you understand the rules.

Unit Revision

• The *Unit Revision* helps you review the lessons in the unit.

• The order of the questions in the *Unit Revision* is usually the same as the order of the lessons in the unit.

• You can work with a partner or by yourself, as your teacher suggests.

Glossary

• At the end of the textbook you will find a glossary of new math words and their definitions. The glossary also contains other important math words from Class IV that you need to remember.

• The glossary also has definitions of instructional words such as "explain", "predict", and "estimate". These will help you understand what you are expected to do.

Answers

• You will find answers to most of the numbered questions at the back of the textbook. Answers to questions that ask for explanations (Explain your thinking or How do you know?) are not included in your textbook. Your teacher has those answers.

• Questions with capital letters, such as A or B, do not have answers in the back of the textbook. Your teacher has the answers to these questions.

• If there could be more than one correct answer to a question, the answer will start with *Sample Response*. Even if your answer is different than the answer at the back of the textbook, it may still be correct.

ASSESSING YOUR MATHEMATICAL PERFORMANCE

Forms of Assessment

Your teacher will be checking to see how you are doing in your math learning. Sometimes your teacher will collect information about what you understand or do not understand in order to change the way you are taught. Other times your teacher will collect information in order to give you a mark.

Assessment Criteria

• Your teacher should tell you about what she or he will be checking and how it will be checked.

• The amount of the mark assigned for each unit should relate to the time the class spent on the unit and the importance of the unit.

• Your mark should show how you are doing on skills, applications, concepts, and problem solving.

• Your teacher should tell you whether the mark for a test will be a number such as a percent, a letter grade such as A, B, or C, or a level on a rubric (level 1, 2, 3, or 4). A rubric is a chart that describes criteria for your work, usually in four levels of performance. If a rubric is used, your teacher should let you see the rubric before you start to work on the task.

Determining a Mark or Grade

In determining your overall mark in mathematics, your teacher might use a combination of tests, assignments, projects, performance tasks, exams, interviews, observations, and homework.

THE CLASSROOM ENVIRONMENT

In almost every lesson you will have a chance to work with other students. You should always share your responses, even if they are different from the answers offered by other students. It is only in this way that you will really be engaged in the mathematical thinking.

Pair and Group Work

• There are many reasons why you should work in pairs or groups:

- to have more opportunities to communicate mathematically
- to make it easier for you to discuss an answer you are not sure of
- to see the different mathematical ideas of other students
- to share materials more easily

• Sometimes you might work with the person next to you, but at other times you might be asked to work with particular students.

• When you work in a group, it is important to contribute and to follow your teacher's rules for working in groups. Some sample rules are shown here.

Rules for Group Work

- Make sure you understand all of the work produced by the group.
- If you have a question, ask your group members first, before asking your teacher.
- Find a way to work out disagreements without arguing.
- Listen to and help others.
- Make sure everyone is included and encouraged.
- Speak just loudly enough to be heard.

Communication

• Many of the questions in the textbook ask you to explain your thinking. Look for instructions like these:

- Explain.
- Explain your thinking.
- Show how you know.
- How do you know?
- How do you know you are right?
- Explain your prediction.
- Explain your estimate.

• The sample *Thinking* in the *Examples* provides a model for mathematical communication.

• One of the ways you communicate mathematically to yourself is by checking your work. Even when a question does not ask you to check your work, you should think about whether your answer makes sense. When you check your work, you should check using a different way than the way you used to find your answer so that you do not make the same error twice.

YOUR NOTEBOOK

• It is valuable for you to have a well-organized, neat notebook to look back at to review the main ideas you have learned. You should do your rough work in this same notebook. Do not do your rough work elsewhere and then waste valuable time copying it neatly into your notebook.

• Your teacher will sometimes show you important points to write down in your notebook. You should also make your own decisions about which ideas to include in your notebook.

UNIT 1 NUMBER

Getting Started

Use What You Know

A. Copy the place value chart and write 19,379 in it.

Ten thousands	Thousands	Hundreds	Tens	Ones

Use the chart to help you explain why each is true about 19,379.

i) 19,379 is between 10,000 and 20,000.

ii) The digit in the thousands place of 19,379 is 6 more than the digit in the hundreds place.

iii) 19,379 is more than 3000.

B. Does the number 19,379 more likely describe the population of Bhutan or the population of a dzongkhag? How do you know?

C. i) Make two 5-digit numbers that match these clues.

Clues for First Number	Clues for Second Number
• It is between 51,000 and 53,000. • The digit in the hundreds place is 5 more than the digit in the tens place. • It has 8 ones.	• It has a 1 in the thousands place. • The digit in the tens place is 3 more than the digit in the ten thousands place. • It is an odd number.

D. i) Use all 10 digits to make two 5-digit numbers.

0	1	2	3	4	5	6	7	8	9

ii) Write clues for each number from **part i)**.

Clues for Your First Number	Clues for Your Second Number
• • •	• • •

iii) Test your clues by asking another student to read your clues and figure out the numbers.

Skills You Will Need

Use this place value chart to help you.

Ten thousands	Thousands	Hundreds	Tens	Ones

1. Which digit of 42,050 is in each place?

a) the ten thousands place

b) the hundreds place

c) the thousands place

2. The number "five thousand three" is written as 5003 in standard form. Write each number in standard form.

a) twenty three thousand, four hundred seventy-three

b) twelve thousand, three hundred twenty

c) forty thousand, two hundred eight

3. Match each number with how you would say it.

a) 32,146	**i)** "fourteen thousand, thirty"
b) 40,002	**ii)** "forty thousand, two"
c) 14,030	**iii)** "thirty two thousand, one hundred forty-six"

4. Write each in standard form.

a) 1 ten thousand + 2 hundreds + 3 ones

b) 6 thousands + 7 hundreds + 2 tens

c) 12 thousands + 32 tens

5. Order from least to greatest.

a) 3427; 12,300; 2007; 10,003

b) 3420; 32,300; 20,007; 17,999

6. List three numbers that are greater than each.

a) 89,929 **b)** 92,077 **c)** 10,004

7. Is each true? If so, how do you know it is true?

a) $0.01 < 0.1$ **b)** $0.18 > 0.08$

8. a) Write a 5-digit number that matches these clues:

• The digit 3 is in the thousands place.

• The digit 2 is in the tens place.

• The digit 8 is in the ten thousands place.

b) What is the greatest possible number for **part a)**?

c) What is the least possible number for **part a)**?

Chapter 1 Reading and Writing Numbers

1.1.1 EXPLORE: How Much is a Million?

1,000,000 is 1000 thousands. You read 1,000,000 as "one million."
These activities will help you understand how big the number 1,000,000 is.

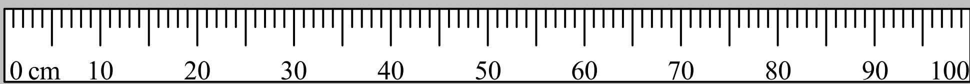
Choose two of parts A to D to complete. Explain your thinking.

A. Copy and complete each statement.

i) 1000 chhertum = Nu ___ **ii)** 1,000,000 chhertum = Nu ___

B. i) Could you walk 1000 mm in one hour?

ii) Could you walk 1,000,000 mm in one hour?



C. i) Would 1000 exercise book pages fit inside your classroom?

ii) Would 1,000,000 exercise book pages fit inside your classroom?

D. i) Could you drink 1000 mL of water in one day?

ii) Could you drink 1,000,000 mL of water in one day?

Choose part E or part F.

E. Choose a book. Count the number of letters in one line.

i) Estimate how many lines 1000 letters would be.

ii) Estimate how many lines 1,000,000 letters would be.

[Continued]

F. i) 1000 students want to go to school. Estimate how many schools are needed.

ii) Estimate how many schools are needed for 1,000,000 students.

G. i) Tell about a situation when 1,000,000 of something would be a lot.

ii) Tell about a situation when 1,000,000 of something would not be a lot.

CONNECTIONS: One Million

1,000,000

The name *million* is based on the Italian word *milla*, which means 1000.

People sometimes use the word *million* to mean any big number.

• A person with a lot of money is called a *millionaire*, even if he or she has more than one million dollars.

• People say, "I've said it a million times." to mean they have said something many times.

The number one million has a special prefix in the metric system — mega. For example, 1 megametre is 1,000,000 metres.

mega	------	-----	kilo	hecto	deca	unit
megametre						**metre**
1,000,000 m	**100,000 m**	**10,000 m**	**1000 m**	**100 m**	**10 m**	**1 m**

1. Is 1,000,000 an even number? How do you know?

2. Is 1,000,000 made up of groups of five? How do you know?

3. Do you think you could lift a megagram? How do you know?

1.1.2 Whole Number Place Value

Try This

Eleven thousand, ninety nine athletes participated in the 2004 Olympics.

A. i) Write the number of athletes using a numeral.

ii) How many athletes would there have been if there had been 1 more?

• The value of each digit in a number depends on its place in the number.

For example:

The 2 in **2**000 means 2 thousands, but in **2**0,000 it means 2 ten thousands.

• Digits are written in groups of three called **periods**.

This place value chart shows the **thousands period** and the **ones period**.

Thousands period | Ones period

Thousands			Ones		
Hundred	Ten	One	Hundred	Ten	One
1	2	3	4	5	6

- Each period has three digits: hundreds, tens, and ones
- You read the numbers in each period separately. The number above is read as "one hundred twenty-three thousand, four hundred fifty-six."
- You use commas to separate the periods: 123,456

• The one millions place is to the left of the hundred thousands place.

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One

• A place value chart can help you write numbers in **expanded form**.

For example:

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One
		1	3	0	4	7

13,047 = 1 ten thousand + 3 one thousands + 4 tens + 7 ones

The chart can also help you read 13,047 as "thirteen thousand, forty-seven".

[Continued]

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One
	2	1	3	0	4	7

213,047
= 2 hundred thousands + 1 ten thousand + 3 one thousands + 4 tens + 7 ones
213,047 is read as "two hundred thirteen thousand, forty-seven."

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One
1	2	1	3	0	4	7

1,213,047 = 1 one million + 2 hundred thousands + 1 ten thousand + 3 one thousands + 4 tens + 7 ones
1,213,047 is read as "one million, two hundred thirteen thousand, forty-seven."

• A number in expanded form shows the value of each non-zero digit.

• A number written with just digits and commas is in **standard form**.
For example, 1,213,047 is in standard form.

B. Which periods did you use to write the number in **part A**?

Examples

Example 1 Writing a Number in Standard Form

Write each in standard form. **a)** two million, three hundred thousand
b) two hundred thousand, three hundred two

Solution

a)

M	Thousands			Ones		
O	H	T	O	H	T	O
2	3	0	0	0	0	0

2,300,000

b)

M	Thousands			Ones		
O	H	T	O	H	T	O
	2	0	0	3	0	2

200,302

Thinking

I used a place value chart.

a) There were 2 one millions and 3 hundred thousands, so I put 2 and 3 in those places and a 0 everywhere else.

b) There were hundred thousands so I knew it was a 6-digit number.

• There were no ten thousands, no one thousands, and no tens, so I used three 0 digits.

• I put commas between the periods in the numbers in **part a) and b)**.

Example 2 Writing a Number in Expanded Form	
Write each number in expanded form. **a)** 982,102 **b)** 7,203,000	
Solution	**Thinking** 
a) 9 hundred thousands + 8 ten thousands + 2 one thousands + 1 hundred + 2 ones	**a)** I wrote the number of hundred thousands, ten thousands, one thousands, hundreds, and ones as an addition sentence. • I knew that 2 one thousands could have been written as 2 thousands instead.
b) 7 one millions + 2 hundred thousands + 3 one thousands	**b)** There were three things to add since there were three non-zero digits. (I know I could have written 7 one millions as 7 millions instead.)

Practising and Applying

1. Which place is worth each?
For example, the millions place is worth 1000 one thousands.

a) 10 hundreds

b) 10 ten thousands

c) 100 one thousands

d) 1000 hundreds

2. Which digit in 2,345,789 is in each place?

a) the ten thousands place

b) the millions place

c) the hundred thousands place

3. Write each in standard form.

a) one million, twenty thousand

b) four hundred four thousand, twenty

c) seventy thousand, two hundred twelve

d) forty-two hundred thousand

4. Write each in expanded form.

a) 3,422,006

b) 8,000,002

c) 342,100

d) 6,000,203

5. A 7-digit number in expanded form has only two parts added together. How many digits are zero? How do you know?

6. For each description below, use all of the digits 0, 1, 2, 3, 4, 5, and 6 to create a 7-digit number.

a) It is more than 230,000.

b) It is between 3 million and 4 million.

c) It is greater than 5,230,000.

7. Write each number described below in standard form and in expanded form.

a) 100 times as much as 20,000

b) 10 times as much as 300,000

8. Write two different numbers to match this description:
A number with a digit in the hundred thousands place that is 3 greater than the digit in the millions place

9. Why do we only need 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9) to write any number?

1.1.3 Renaming Numbers

Try This

A. Copy and complete each. The first one is done for you.

i) 4 hundreds = <u>40</u> tens

ii) 3 thousands = ___ hundreds

iii) 5 thousands = ___ tens

iv) 2 ten thousands = ___ hundreds

• There is often more than one way to write a number.

For example: 100 is 10 × 10, so 100 = 10 tens

100 is also $\frac{1}{10}$ of 1000, so 100 = 0.1 thousand

• When you write a number a different way, you are **renaming** the number.

• You can use a place value chart to help you rename numbers.

For example, this chart shows why **100** can be renamed **10 tens**.

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One
				1	0	0

The chart shows why **100** can be renamed **0.1 one thousand**.

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One
			0	1		

• You can rename millions in the same way.

For example, you can rename 1,400,000 different ways:

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One
1	4	0	0	0	0	0

If you look left from the hundred thousands place, you can see why **1,400,000** can be renamed **14 hundred thousand**.

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One
1	4	0	0	0	0	0

If you look left from the millions place, you can see why **1,400,000** can be renamed **1.4 million**.

B. How could you have used a place value chart to rename each number in **part A**?

C. Why might it be useful to rename numbers this way?

Examples

Example 1 Renaming Whole Numbers in Decimal Form

Write each number as a decimal number of millions.

a) 300,000 **b)** 70,000 **c)** 2,500,000

Solution	Thinking
a) 0.3 million	**a)** 100,000 = 0.1 million, so 300,000 = 0.3 million
b) 0.07 million	**b)** I wrote 70,000 in a place value chart. I compared the place of the digit 7 in 70,000 to the millions place. It was two places to the right, so I knew the decimal was 0.07.

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One
0	0	7	0	0	0	0

Solution	Thinking
c) 2.5 million	**c)** There were 2 millions and 500,000 is half (0.5) a million.

Example 2 Writing Decimal Millions in Standard Form

Write each as a whole number in standard form.

a) 0.2 million **b)** 1.04 million **c)** 2.1 million

Solution	Thinking
a) 200,000	**a)** 0.1 million = 100,000, so 0.2 million = 200,000
b) 1,040,000	**b)** I wrote 1.04 million in a place value chart to decide how many 0s to write at the end.

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One
1	0	4	0	0	0	0

Solution	Thinking
c) 2,100,000	**c)** 0.1 million is 100,000 to add to the 2 million.

Practising and Applying

1. Copy and complete each.

a) 100,000 = ____ million

b) 10,000 = ____ million

c) 500,000 = ____ million

d) 30,000 = ____ million

2. Write each in standard form.

a) 0.9 million

b) 0.04 million

c) 1.1 million

3. Write each in expanded form.

a) 1.08 million **b)** 3.02 million

c) 0.2 million **d)** 0.6 million

4. a) Write 2.01 million in standard form.

b) Which digit is in each place?

i) the ten thousands place

ii) the tens place

iii) the millions place

iv) the hundreds place

v) the hundred thousands place

vi) the thousands place

5. Look at this newspaper headline.

Population increases by 0.5 million

Choose all the statements that mean the same thing.

A. Population grew by half a million

B. Population increased by 500,000

C. Population grew by $\frac{1}{2}$ million

D. Population increased by 50,000

6. Write each underlined number as a decimal million.

a) A heart pumps about <u>100,000</u> times a day.

6. b) We take about <u>20,000</u> breaths a day.

c) The distance to the moon is about <u>400,000</u> km.

d) The area of Bhutan is about <u>40,000</u> km^2.

e) A computer file uses <u>300,000</u> bytes. (A byte is a unit of computer memory.)

f) Another computer file uses <u>1,400,000</u> bytes.

7. How do you know 0.3 million will end in five zeros when it is written in standard form?

8. What are some different ways to rename 2,100,000?

1.1.4 Comparing and Ordering Numbers

Try This

A. Find one digit you could use in both numbers to make each statement true. Find more than one answer for each.

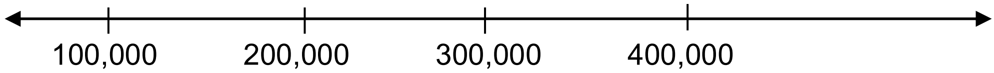
i) 82,__49 > __9,298　　　**ii)** 82,__49 < __9,298

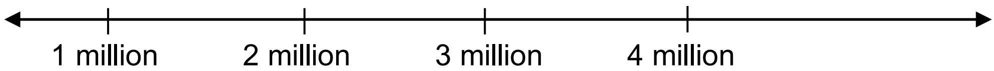
• You can use a number line to compare numbers.

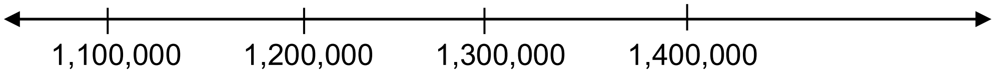
You begin by marking benchmark numbers on the number line. Benchmark numbers are numbers that are easy to work with.

If the numbers are less than 100, you might mark 10, 20, 30, …

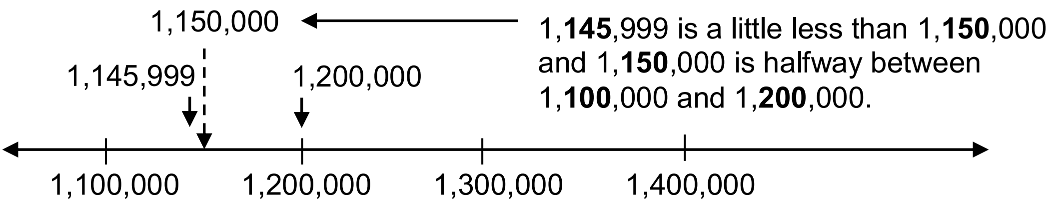
If the numbers are large, you might use these benchmark numbers:







For example, to compare 1,200,000 and 1,145,999, you can use benchmarks to mark them on the number line.



1,145,999 < 1,200,000 because 1,145,999 is farther to the left.

• You can also compare numbers using a place value chart.

For example, to compare 1,200,000 and 1,145,999, write them in the chart, one above the other.

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One
1	2	0	0	0	0	0
1	1	4	5	9	9	9

1,200,000 > 1,145,999 because the value of the first two digits of **1**,**2**00,000 is greater than the value of the first two digits of **1**,**1**45,999.

• You might compare two numbers that are in expanded or decimal form by writing them both in standard form.

For example, to compare 0.5 million and 8 hundred thousand:

0.5 million = 500,000 8 hundred thousand = 800,000

Since 500,000 < 800,000, you know 0.5 million < 8 hundred thousand.

• You might also rename one number like the other to compare them.

For example, to compare 0.07 million and 600,000:

Since 600,000 = 0.6 million and 0.07 < 0.6, then 0.07 million < 600,000.

B. i) Use a number line to compare your numbers from **part A i)**.

ii) Now use a place value chart to compare the numbers.

Examples

Example 1 Comparing Numbers in the Same Form

Which number is greater in each pair?

a) 3,210,000 or 899,972

b) 1.2 million or 2.1 million

c) 3,104,999 or 3,140,000

Solution

c) 3,210,000 > 899,972

Thinking

a) I used a place value chart. The first number had millions and the other didn't, so I knew the first number was greater.

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One
3	2	1	0	0	0	0
	8	9	9	9	7	2

b) 2.1 million > 1.2 million

b) Both numbers are in the same form so I just compared the first parts: 2.1 > 1.2

c) 3,140,000 > 3,104,999

c) They were both 7-digit whole numbers, so I just compared the value of the first three digits: **314** in **3,14**0,000 > **310** in **3,10**4,999.

Example 2 Ordering Numbers in Different Forms

Order from least to greatest.

a) 3,200,000; 3.14 million; 258,000

b) 80,000; 12.3 hundred thousand; 0.4 million

Solution

a)

3,200,000; 3.14 million; 258,000

3.14 million = 3,140,000

3,140,000 < 3,200,000

258,000 < 3,140,000 < 3,200,00
so
258,000 < 3.14 million < 3,200,000

b)

Thinking

a) Since two numbers were in standard form, I changed the third number to standard form.

• 3,140,000 < 3,200,000 since they both have 3 million but 3,**1**40,000 has **1** hundred thousand and 3,**2**00,000 has **2** hundred thousands.

• 258,000 < 1 million, so it was least.

b) I used a place value chart to write both 80,000 and 0.4 million as hundred thousands.

Millions	Thousands			Ones		
One	Hundred	Ten	One	Hundred	Ten	One
		8	0	0	0	0
1	2	3	0	0	0	0
	4	0	0	0	0	0

80,000 = 0.8 hundred thousand

0.4 million = 4 hundred thousand

0.8 hundred thousand
< 4 hundred thousand
< 12.3 hundred thousand

So

80,000 < 0.4 million
< 12.3 hundred thousand

• Then all I had to do was order 0.8, 12.3, and 4: 0.8 < 4 < 12.3

Practising and Applying

1. Which is greater?

a) 3.4 million
or
2.8 million

b) 15 hundred thousand
or
13.2 hundred thousand

c) 5,123,478
or
5,213,478

2. Order from least to greatest.

a) 6,000,000; 3,487,799; 899,789

b) 213,867; 2,013,687; 762,813

3. Which is greater?

a) 2.1 million
or
345,718

b) 0.4 million
or
3.8 hundred thousand

c) 275 ten thousand
or
2.1 million

4. Order from least to greatest.

a) 78 ten thousand
14 hundred thousand
2 million

b) 3,150,000
0.6 million
2.3 hundred thousand

5. Use a place value chart or a number line to show this is true:

42 hundred thousand < 5 million

6. List three numbers between 2.1 million and 2,153,197.

7. Which number below is closest to 3,200,000?

A. 3.1 million

B. 319 ten thousand

C. 3,200,010

8. The two missing digits below are different.

158,699 < 3__2,157 < __22,589

a) What might be the two digits?

b) Find another answer to **part a)**.

9. Complete each to make it true:

a) ☐.☐ million > ☐.☐ hundred thousand

b) ☐.☐ million = ☐.☐ hundred thousand

c) ☐.☐ million < ☐.☐ hundred thousand

GAME: Target 7

Create three sets of the following ten digit cards.

0	1	2	3	4	5	6	7	8	9

Shuffle the cards and set them face down in a deck.

Play with a partner.

• Players take turns taking the top card from the deck until they each have 7 cards. They lay their cards down in the order they took them to make a 7-digit number, starting at the millions place.

• Each player predicts whether his or her number will be greater than, less than, or equal to a third 7-digit number to be created from the deck.

• Players turn over the next seven cards to make a third 7-digit number, starting at the ones place, to compare their number to.

• Each player gets 1 point for a correct prediction.

• The cards are shuffled and the players play another round.

• The first player to get 10 points wins.

Example:

Player A's number is 2,395,688.

Player A's prediction: My number will be less than the third number.

Player B's number is 7,810,025.

Player B's prediction: My number will be greater than the third number.

The third number is 6,102,045.

2,395,688 < 6,102,045 and 7,810,025 > 6,102,045

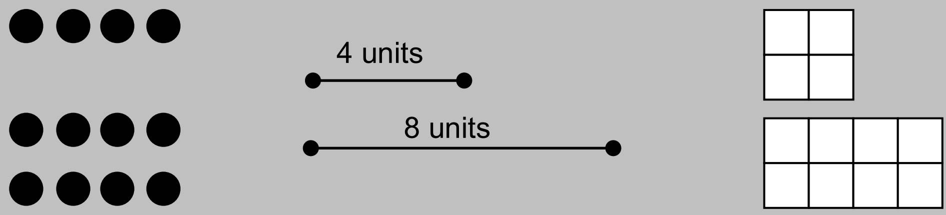
Both predictions are correct, so each player scores 1 point.

Chapter 2 Number Relationships

1.2.1 Renaming Numbers Using Multiplication

Try This

A. What do all three pictures tell you about how 4 and 8 are related?



• You can rename numbers using place value. You can also rename numbers using multiplication. This can make calculations easier.

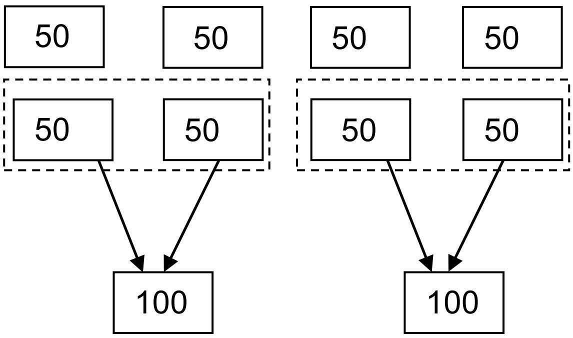
For example:

To multiply 4 × 50:

4 groups of 50 — 4 × 50

2 groups of 2 groups of 50 — 2 × (2 × 50)

2 groups of 100 — 2 × 100



4 × 50 = 2 × 100 = 200

In the picture above, you can see how 4 × 50 was calculated by renaming 4 as 2 × 2. This shows what it looks like using numbers:

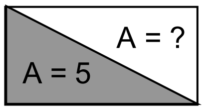
4 × 50 = **2 × 2** × 50 = 2 × (2 × 50) = 2 × 100 = 200

• Renaming numbers using multiplication can help with measurement too.

For example:

You know that a rectangle is made up of two identical triangles. You know that each triangle has area of 5 square units.

You can calculate the area of the rectangle by multiplying.



Area of the rectangle = 2 × 5 square units.

• You can also use multiplication to rename measurements.

For example:

To change 3000 m to __ km, you can rename 3000 as 3 × 1000, and use the fact that 1000 m = 1 km:

If **3000** m = **3 × 1000** m, then 3000 m = 3 × 1 km = 3 km.

B. How could you use the relationship between 4 and 8 in **part A** to calculate 8 × 250?

Examples

Example 1 Renaming Numbers to Divide

Use 100 = 25 × 4 to calculate 200 ÷ 4.

Solution

200 ÷ 4 = ___

200 = ___ × 4

200 = 2 × 100
= 2 × (25 × 4)
= (2 × 25) × 4
= 50 × 4

If 200 = 50 × 4, then 200 ÷ 4 = 50.

Thinking

• 200 ÷ 4 means "How many groups of 4 are in 200?" I knew I could figure it out by writing 200 as groups of 4.

• To do this, I renamed 200 as 2 × 100 and then renamed 100 as 25 × 4.

• Since 2 × 25 = 50, 2 × 25 = 4 = 50 × 4.

• 200 = 50 × 4 means 200 is 50 groups of 4.

Example 2 Renaming Measurements

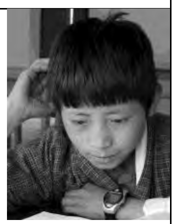
Use 1500 = 3 × 500 to complete 1500 m = ___ km.

Solution

1500 m = 3 × 500 m
= 500 m + 500 m + 500 m
= 1 km + $\frac{1}{2}$ km

1500 m = 1$\frac{1}{2}$ km

Thinking



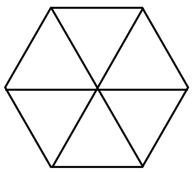
• 1000 m is 1 km so 500 m is half a kilometre.

• Since $\frac{1}{2}$ = 0.5, I could have written 1.5 km instead.

Example 3 Relating Shape Measurements

Every regular hexagon is made of 6 congruent equilateral triangles.

How many equilateral triangles could you make with 12 regular hexagons?



Example 3 Relating Shape Measurements [Continued]	
Solution 1 $12 \times 6 =$ ___ $(2 \times 6) \times 6 =$ ___ $2 \times (6 \times 6) =$ ___ $2 \times 36 = 72$ You could make 72 equilateral triangles.	**Thinking**  • 1 hexagon has 6 triangles so 12 hexagons have 12 × 6 triangles. • It's easy to multiply by 2, so I renamed 12 as 2 × 6 and then multiplied 6 × 6.
Solution 2 $12 \times 6 =$ ___ $(10 + 2) \times 6 =$ ___ $10 \times 6 + 2 \times 6 =$ ___ $60 + 12 = 72$ You could make 72 equilateral triangles.	**Thinking**  • Sometimes I find it easier to multiply numbers in parts, so I renamed 12 × 6 as 10 × 6 + 2 × 6.

Practising and Applying

1. Fill in the blanks.

a) 70 = ___ × 7

b) 36 = ___ × 6 or 36 = ___ × 4

c) 540 = ___ × 10 or 540 = ___ × 5

d) 1300 = ___ × 10 or 1300 = ___ × 100

2. Explain how you know each is true.

a) 8 × 16 = 2 × 4 × 16

b) 5 × 52 = half of 10 × 52

c) 12 × 100 = 48 × 25

3. A truck driver travels 35 km each hour. How far will he go in each amount of time?

a) 2 hours

b) 3 hours

c) 5 hours

4. a) Calculate 6 × 7.

b) Use your answer to **part a)** to calculate each. Show your work.

i) 12 × 7 **ii)** 6 × 70

iii) 60 × 7 **iv)** 12 × 70

5. a) Complete: 1 km = ___ cm

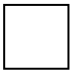
b) Use your answer to **part a)** to complete each. Show your work.

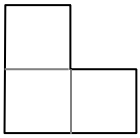
i) 2 km = ____ cm

ii) 10 km = ____ cm

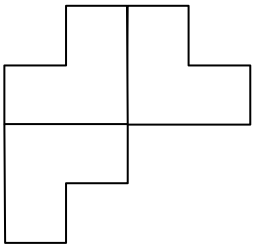
iii) 1.5 km = ____ cm

6. **a)** What is the area of this shape?

 is 23 m^2



b) How could you use your answer to **part a)** to find the area of this shape?



7. Give an example of how you would rename a number to do each.

a) calculate

b) rename a measurement

1.2.2 Using Number Sentences

Try This

A. For each number sentence, find one or more numbers that make it true.

i) 20 × __ = 100 **ii)** 212 + __ > 216 **iii)** 50 ÷ 10 = __

iv) __ < 98 **v)** __ – 5 = 45

B. What do you notice about the answers for **parts i), iii), and v)** compared to **parts ii) and iv)**?

• You can use number sentences to show how numbers are related.

For example:

The number sentence 8 > 3 shows how 8 and 3 compare: 8 is greater than 3

• Some number sentences are called **open number sentences**.
An open number sentence has at least one number you do not know yet.

• You **solve** an open sentence by figuring out what the unknown number is.

For example:

To solve 8 = 3 + ■, you find how much you have to add to 3 to get 8.

Since 8 = 3 + **5**, the **solution** to the open number sentence is ■ = 5.

You can now compare 8 and 3 by saying 8 is 5 more than 3.

- ■ in a number sentence means a number you do not know yet.
You can use another shape like Δ or a letter like x instead:

8 = 3 + ■ or 8 = 3 + Δ or 8 = 3 + x

• An open number sentence can have many solutions.

For example:

325 + x > 200 + x has many solutions, including x = 0, 1, 2, 3, and 4.

When x = 0, 325 + **0** > 200 + **0**, and you can say that 325 > 200.

When x = 4, 325 + **4** > 200 + **4**, and you can say that 329 > 204.

• Some open number sentences have no solutions.

For example:

200 ÷ 0 = ■ has no solution because you cannot divide by 0.

C. Choose three number sentences from **part A**. How many solutions does each have?

Examples

Example 1 Creating Number Sentences

Make up five different number sentences to compare 80 and 8.

Solution	Thinking
	I wrote one of the five number sentences using the greater than symbol and then I wrote each of the other four number sentences using one of the four operations: 
80 > 8	• I knew 80 was greater than 8.
8 + **72** = 80 80 – 8 = **72**	• For addition, I figured out how much more 80 was than 8. Then I used my addition sentence to write a subtraction sentence.
8 × **10** = 80 80 ÷ **10** = 8	• For multiplication, I figured out how many times greater 80 was than 8. Then I used my multiplication sentence to write a division sentence.

Example 2 Solving Open Number Sentences

Solve each number sentence.

a) 3200 = ▲ + 2200 **b)** 1,000,000 = x × 1000

c) 221 = ■ ÷ 1000 **d)** Δ – 5000 = 7200

Solution	Thinking
a) 3200 = ▲ + 2200 3200 = $\underline{1000}$ + 2200 3200 is 1000 more than 2200	 **a)** I knew I had to add something to 2200 to get 3200. The only thing different between **3**200 and **2**200 was the thousands, so I added 1 thousand, or 1000.
b) 1,000,000 = x × 1000 1,000,000 = $\underline{1000}$ × 1000 1,000,000 is 1000 times as much as 1000.	**b)** 1 million is 1000 thousands, or 1000 × 1000.
c) 221 = ■ ÷ 1000 221 = $\underline{221{,}000}$ ÷ 1000 221,000 is 1000 times as much as 221.	**c)** I knew if I divided ■ by 1000 to get 221, I could also multiply 221 by 1000 to find ■.
d) Δ – 5000 = 7200 $\underline{12{,}200}$ – 5000 = 7200 7200 is 5000 less than 12,200.	**d)** I found Δ by adding 7200 + 5000 to get 12,200.

Example 3 Counting Solutions to an Open Number Sentence

Does each number sentence have one solution, more than one solution, or no solution? How do you know?

a) 7235 = 5235 + Δ **b)** 375 – ■ > 200 **c)** 40 × x > 400

Solution	**Thinking**
a) There is only one solution because 2000 is the only number that works.	**a)** I knew that 7235 = 5235 + Δ meant, "What do you add to 5235 to get 7235?"
b) There is more than one solution. Any number less than 175 will work.	**b)** 375 – 175 = 200, so I knew that if I subtracted a number less than 175, the answer would be more than 200.
c) There is more than one solution. Any number greater than 10 will work.	**c)** 40 × 10 = 400, so I knew that if I multiplied 40 by a number greater than 10, the answer would be more than 400.

Practising and Applying

1. Create two number sentences that use the pair of numbers.

a) 18 and 180

b) 350 and 700

c) 2000 and 10,000

d) 160 and 1600

2. Solve each. Describe what you did.

a) 23 + ■ = 84

b) 3012 – ▲ = 1012

c) 3,000,000 = Δ × 3000

d) 6600 ÷ 100 = x

3. A number sentence shows that one number is 4 times as much as another.

• What could the number sentence be?

• Find two more possible number sentences.

4. For each number sentence below,

• tell if it has one, more than one, or no solution

• if it has one or more solutions, tell what the solution is

a) 6000 – ▲ = 2998

b) 42,100 + ■ = 51,100

c) 2000 ÷ x < 20

d) 0 × Δ = 48

5. Create a number sentence that has each.

a) only one solution

b) many solutions

c) no solutions

6. Is it possible to have an open number sentence where every counting number (1, 2, 3, 4, …) is a solution? Use an example to show how you know.

7. Why is there always more than one number sentence using any pair of numbers? Think about numbers like 9 and 90.

UNIT 1 Revision

1. Tell three things that are true about the value of the number one million.

1,000,000

2. a) Create a place value chart and put each number on it.

i) 314,214

ii) 1,003,412

b) Write each number in expanded form.

c) Write the number word for each.

3. Write each number in standard form and in expanded notation.

a) two hundred thirty-four thousand, five

b) one hundred forty-five thousand, thirty-two

c) two million, thirty thousand, three

d) four million, twenty thousand, thirty

4. Which place value is each digit in 2,314,056?

a) 2	**b)** 0
c) 6	**d)** 3

5. Write a number in standard notation for each description.

a) The hundred thousands digit is 3 more than the millions digit.

b) The millions digit is 9 more than the tens digit.

c) The ten thousands digit is 7 more than the hundred thousands digit.

6. Why might it be useful to write a number as a decimal million?

For example:

3,100,000 as 3.1 million

7. Write each in standard form.

a) 21 hundred thousand

b) 310 ten thousand

c) 0.3 million

d) 0.05 million

8. Which number in each pair is less? How do you know?

a) 145,200 or 87,146

b) 3,152,110 or 3,521,880

c) 417,000 or 714,112

d) 1,210,111 or 345,789

9. Which number in each pair is less?

a) 0.2 million or 123,450

b) 23 hundred thousand or 2 million

c) 102 ten thousand or 8 hundred thousand

d) 0.2 million or 0.02 million

10. A number with digits that are all 9 is less than a number with digits that are all 1. How is that possible? Use an example to help explain.

11. Copy and complete.

a) 1000 is ___ hundred

b) 1,000,000 is ___ hundred

c) 610 is ___ tens

d) 880 is ___ × 4

e) 360 is ___ × 4 or ____ × 9

12. a) Calculate 3 × 30.

b) Use your answer to **part a)** to calculate each. Show your work.

i) 30 × 30

ii) 6 × 30

iii) 3 × 3000

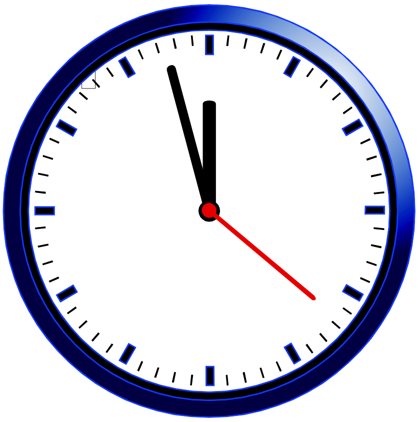
iv) 6 × 60

13. Copy and complete.

a) ___ min = 3 h

b) ___ min = 10 h

c) ___ min = $1\frac{1}{2}$ h



Recall that 60 min = 1 h.

14. Create three number sentences using the numbers 70 and 210.

15. How would you solve each?

a) 56 + x = 63

b) 37 + ▲ = 94

c) 650 ÷ 10 = ■

d) 8 × ∆ = 400

16. Solve each number sentence in **question 15**.

17. Does each number sentence have one, more than one, or no solutions?

a) 45 > ■

b) 45 + ■ = 90

c) 45 + ■ = 40 + ■ + 5

d) 45 – ■ > 55 – ■

18. Create a number sentence with more than one solution.

UNIT 2 WHOLE NUMBER COMPUTATION

Getting Started

Use What You Know

Play this game with a partner.

Take turns.

• Choose 7, 8, or 9 base ten models, including at least one of each type: hundreds, tens, and ones.

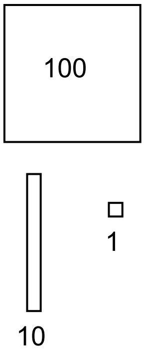
• Roll a die.

• Multiply the number on the die by the number of each type of block you chose. Add these to get a product.

• Represent the product using the least number of blocks possible. Use imaginary thousands blocks if necessary.

Each player scores as many points as the number of blocks he or she used.

Play five rounds. The player with the fewest points wins.



Skills You Will Need

1. Complete.

a) $7 \times 8 = \square$ **b)** $9 \times 4 = \square$ **c)** $8 \times 5 = \square$ **d)** $7 \times 7 = \square$

e) $48 \div 8 = \square$ **f)** $72 \div 9 = \square$ **g)** $63 \div 7 = \square$ **h)** $35 \div 5 = \square$

2. Calculate.

a) 4×253 **b)** 10×389 **c)** 6×812 **d)** 9×677

e) $366 \div 6$ **f)** $512 \div 8$ **g)** $915 \div 7$ **h)** $379 \div 3$

3. Explain how you know $4 \times 50 = 2 \times 100$ without calculating.

4. 144 students are divided into 8 equal teams.

a) How many students are on each team?

b) Each student holds 3 flags for a celebration. How many flags are there?

5. What does each word mean? You can use an example to explain.

a) factor **b)** product **c)** multiple

d) quotient **e)** dividend **f)** divisor

Chapter 1 Multiplication

2.1.1 Multiplying Multiples of Ten

Try This

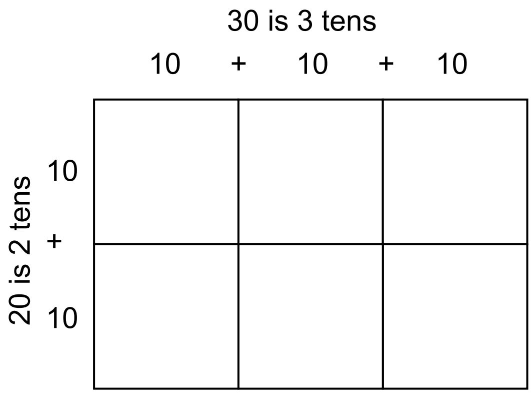
A dragonfly flaps its wings between
20 times and 40 times each second.

A. How many times does it flap in 1 min?
(Remember: 1 min = 60 s)

• Multiplying is like finding the area of a rectangle. The length and the width are the **factors** (the numbers you multiply). The area is the **product**.

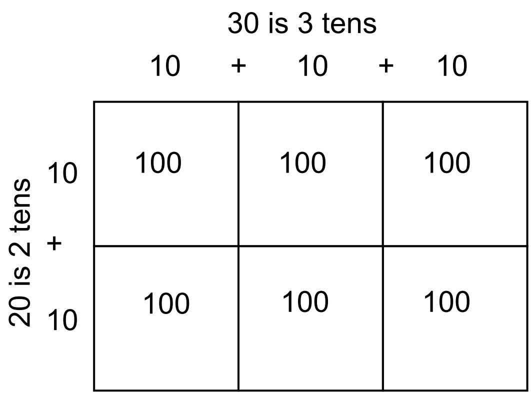
For example:

This rectangle shows 30 × 20:



10 × 10 = 100, so the area of each square inside the rectangle is 100.

30 × 20 = 3 tens × 2 tens
= 6 hundreds
= 600



3 tens × 2 tens means 3 × 2 sets of 10 × 10.

B. What rectangles could you draw to multiply the number of wing flaps for the dragonfly in **part A**?

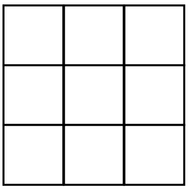
Examples

Example Comparing Products	
How much more is 30 × 70 than 50 × 40?	
Solution 30 × 70 = 3 tens × 7 tens = 21 hundreds = 2100 50 × 40 = 5 tens × 4 tens = 20 hundreds = 2000 2100 – 2000 = 100 30 × 70 is 100 more than 50 × 40.	**Thinking**  • I used multiplication facts I know: 3 × 7 = 21 and 5 × 4 = 20. • I knew that 20 hundreds was 2000 and 21 hundreds was 2100 since each set of 10 hundreds is 1000.

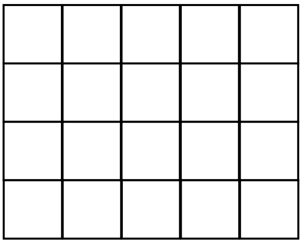
Practising and Applying

1. What multiplication equation does each diagram show? (Each small square is 10 × 10.)

a)



b)



2. Draw rectangles to show these. Calculate the products.

a) 20 × 40

b) 50 × 70

3. Calculate each product.

a) 50 × 60 **b)** 20 × 80

c) 80 × 40 **d)** 60 × 90

4. Which is greater than 4000?

A. 60 × 80 **B.** 80 × 30

C. 70 × 40 **D.** 10 × 60

5. What could be the missing values?

a) ☐ × ☐ = 4900

b) ☐ × ☐ = 3600

6. Compare each pair of products. Tell what you notice. Why do you think this happens?

a) 3 × 20 and 30 × 20

b) 4 × 60 and 40 × 60

7. Why is 70 × 30 just as easy to multiply as 7 × 3, even though the numbers are bigger?

2.1.2 Estimating Products

Try This

There are 19 classrooms in Sonam's school. Each class has about 42 students.

A. Estimate the number of students in the school.

• If you do not need an exact answer to a problem, you can **estimate**.

• To estimate, use values that you find easy to work with.

For example:

To estimate 28 × 46, you can use 30 × 50.

30 is a little more than 28. 50 is a little more than 46.

30 × 50 = 3 tens × 5 tens = 15 hundreds = 1500

So 28 × 46 is about 1500.

1500 is a high estimate because 30 is more than 28 and 50 is more than 46.

B. i) Which numbers would you multiply to estimate 19 × 38? Why?

ii) Can you be sure whether your estimate will be high or low? Explain your thinking.

Examples

Example 1   Estimating to Solve a Problem

A game uses sticks that come in boxes of 120.
There are 19 players and each player uses 39 sticks.
About how many boxes of sticks are needed?

Solution

19 × 39 is about 20 × 40 = 800.

800 is a bit high.

7 boxes would have 7 × 120 sticks:

$$\begin{aligned} 7 \times 120 &= 7 \times 100 + 7 \times 20 \\ &= 700 + 140 \\ &= 840 \end{aligned}$$

Fewer than 800 sticks are needed, so 7 boxes will be enough.

Thinking

- I estimated 19 × 39 using easy numbers that were both a bit high.
- I knew it would take 8 boxes of 100 to make 800; so I guessed only 7 boxes of 120 since 120 is more.
- I multiplied 7 × 120 by multiplying 7 × 1 hundred and 7 × 2 tens and then adding them.

Example 2   Estimating to Check a Calculation

Pelden says that 19 × 36 = 354. How can you use estimation to show Pelden that his answer is incorrect?

Solution

19 × 36 should be close to 20 × 40.

20 × 40 = 800

354 is much lower than 800, so Pelden's answer is incorrect.

Thinking

- I estimated using numbers that are easy to multiply.
- Since I rounded both numbers up to 20 × 40, I knew the exact answer would be lower, but not as low as 354.

Practising and Applying

Show your work for all estimates.

1. Estimate each product.

a) 29 × 47 **b)** 32 × 52

c) 57 × 35 **d)** 93 × 56

2. One of these answers is incorrect. Estimate to decide which one. Explain how you estimated.

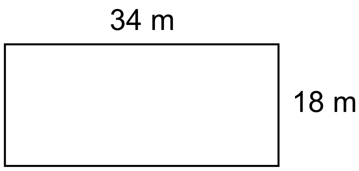
A. 37 × 48 = 1776

B. 69 × 63 = 4347

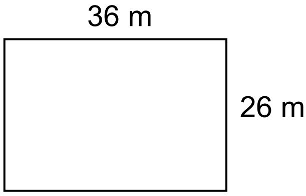
C. 31 × 54 = 154

3. Estimate the number of square metres in each area.

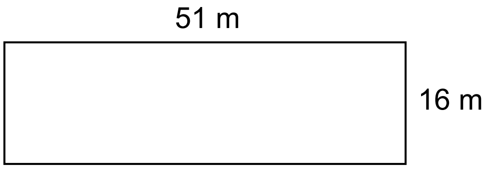
a)



b)



c)



4. Estimate each total cost.

a) 17 geometry sets, each costs Nu 40

b) 36 orange juices, each costs Nu 60

c) 81 erasers, each costs Nu 11

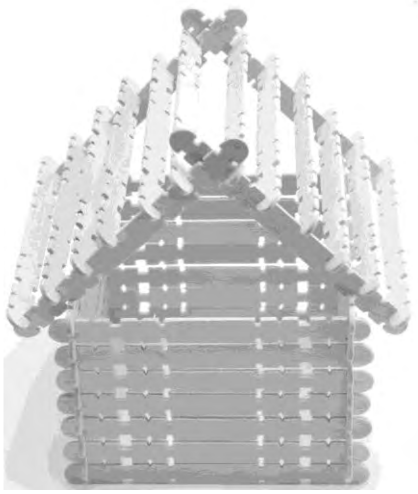
5. **a)** Explain why all of these are possible ways to estimate 25 × 65.

• 20 × 70 • 20 × 60

• 30 × 60 • 30 × 70

b) Why might it be better to use 20 × 70 instead of 30 × 70?

6. A class of students is building houses out of sticks. There are 36 students. Each student needs 83 sticks. The sticks come in boxes of 120. About how many boxes of sticks will the class need?



7. Write an estimation word problem that you could solve using each.

a) 40 × 60

b) 30 × 80

c) 60 × 50

8. 50 × 70 could be used to estimate each product:

• 49 × 71

• 48 × 72

• 50 × 73

For which product will 50 × 70 give an estimate that is closest to the actual answer? How do you know?

9. Why is it a good idea to check your answers using estimation when you multiply?

2.1.3 Multiplying 2-digit Numbers by 3-digit Numbers

Try This

Ugyen's teacher is making a chart to record the marks for the students.
There are 42 students in the class.
There are 22 columns for marks

	Quiz	Unit test	Notebook
Devika			
Lobzang			
Arjun			

A. Estimate the number of squares in the chart.

• As you saw in **lesson 1.1.1**, multiplying is like finding the area of a rectangle.

For example:

- The product of 23 × 34 is the area of a rectangle that is 23 units wide and 34 units long.

	34
23	23 × 34

- To find the product, it is helpful to divide the area into four parts.

	30	+ 4
20	20 × 30	20 × 4
+ 3	3 × 30	3 × 4

- You can then calculate the area of each part.

	30	+ 4
20	20 × 30 = **600**	20 × 4 = **80**
+ 3	3 × 30 = **90**	3 × 4 = **12**

- The product is the sum of the parts:

$23 \times 34 = 600 + 80 + 90 + 12$
$= 782$

	30	+ 4
20	**600**	**80**
+ 3	**90**	**12**

• You can show the product using an area diagram, as shown above. Or, you can calculate, as shown here.

Notice that the calculation to the right shows all four parts to be added.

$$\begin{array}{r} 34 \\ \times\ \underline{23} \\ 600 \\ 80 \\ 90 \\ \underline{+\ 12} \\ 782 \end{array}$$

• If you are multiplying a 2-digit number by a 3-digit number, you can also do it in parts.

For example, to multiply 234×23:

$234 = 200 + 34$

$234 \times 23 = 200 \times 23 + 34 \times 23$

$200 \times 23 = 2 \times 100 \times 23 = 2 \times 2300 = 4600$

$34 \times 23 = 782$ (from above)

So $234 \times 23 = 4600 + 782 = 5382$.

OR

$234 \times 23 = 20 \times 234 + 3 \times 234$

$20 \times 234 = 2 \times 10 \times 234 = 2 \times 2340 = 4680$

$3 \times 234 = 702$

$234 \times 23 = 4680 + 702 = 5382$

$$\begin{array}{rl} \cancel{11} & \\ 234 & \\ \underline{\times\ \ 23} & \\ 702 & [3 \times 234] \\ +\ \underline{4680} & [20 \times 234] \\ 5382 & \end{array}$$

B. i) Show how to use a rectangle diagram to calculate the number of squares in Ugyen's teacher's grid.

ii) Write the calculation. Show the four parts to be added.

Examples

Example 1 Multiplying Two 2-Digit Numbers

Which is greater, 35 × 48 or 38 × 45?

Solution

35 × 48

	40	+ 8
30	30 × 40 = 1200	30 × 8 = 240
+ 5	5 × 40 = 200	5 × 8 = 40

35 × 48 = 1200 + 200 + 240 + 40 = 1680

38 × 45

$$\begin{array}{rl} 45 & \\ \times\ 38 & \\ \hline 1200 & [30 \times 40] \\ 320 & [8 \times 40] \\ 150 & [30 \times 5] \\ +\ 40 & [8 \times 5] \\ \hline 1710 & \end{array}$$

1710 > 1680, so 38 × 45 > 35 × 48

Thinking

- For 35 x 48, I used a rectangle to multiply. It helped me figure out the parts to add.
- For 38 x 45, I multiplied in four parts:

- tens × tens

- tens × ones

- ones × tens

- ones × ones

It was just like when I used the rectangle to multiply.

Example 2 Solving a Puzzle

Two of the digits 3, 5, 8, and 9 go in the blanks. Which digit goes where?

△1 × 2☐ = 899

Solution

Possible digits: 3, 5, 8, and 9

30 × 30 = 900, so

△1 × 2☐ = 899 might be 31 × 29 = 899.

$$\begin{array}{rl} 31 & \\ \underline{\times\ \ 29} & \\ 600 & [20 \times 30] \\ 270 & [9 \times 30] \\ 20 & [20 \times 1] \\ \underline{+\ \ \ 9} & [9 \times 1] \\ 899 & \end{array}$$

Since **3**1 × 2**9** = 899, △ = 3 and ☐ = 9.

Thinking

- Since 899 is close to 900, I knew I needed two numbers that multiplied to about 900.
- I knew 30 × 30 = 900, so I tried numbers as close to 30 as I could.

Example 3 Solving a Problem Involving Multiplication

428 students in a school each brought Nu 25 to help other people who needed money. How much money was collected altogether?

Solution 1

$$\begin{array}{r} 1 \\ \cancel{14} \\ 428 \\ \underline{\times\ \ 25} \\ 2140 \\ +\ \ \underline{8560} \\ 10,700 \end{array}$$

Nu 10,700 was collected.

Thinking

- I knew I had to multiply 428 × 25.
- I multiplied each part of 428 by 5 first.
- When I finished that, I crossed out the regrouping numbers so I wouldn't use them when I multiplied 428 by 20.
- I multiplied 428 × 20 by multiplying by 2 and then by 10. I knew that, to multiply by 10, you just add a 0 to the end of 856.

Solution 2

428 × 25

428 ÷ 4 = 107

107 × 100 = 10,700

Nu 10,700 was collected.

Thinking

- I knew I had to multiply 428 × 25.
- Multiplying by 25 is the same as dividing by 4 and multiplying by 100 because 25 = 100 ÷ 4.

Practising and Applying

1. Draw a diagram to calculate each.

a) 43 × 37 **b)** 62 × 39

c) 51 × 28 **d)** 34 × 33

2. What multiplication equation does each diagram represent?

a)

10 × 60	10 × 2
7 × 60	

b)

30 × 30	30 × 2
	1 × 2

3. Calculate each.

a) 48 × 56 **b)** 15 × 47

c) 21 × 64 **d)** 33 × 82

e) 246 × 72 **f)** 353 × 18

4. Each day in March, Tshering did homework for 45 min. How many minutes of homework did she do in March?

5. A painting is made of 25 rows of 118 squares. How many squares are in the painting?

6. How tall is a stack of 35 boxes, if each box is 14 cm tall?

7. You multiply two different 2-digit numbers. Find each.

a) the least possible product

b) the greatest possible product

8. You multiply 38 by a 3-digit number. Which is more likely?

• a product greater than 5000

OR

• a product less than 5000

Explain your thinking.

9. 1 kg of potatoes costs Nu 15. What is the price of each?

a) 45 kg **b)** 128 kg

10. The product of two consecutive numbers is 1980. What are the two numbers?

11. Arrange the digits 6, 7, 8, and 9 in the boxes to make the greatest possible product.

☐ ☐ × ☐ ☐

12. Write a word problem that could be solved using 45 × 368.

13. Use examples to show that, when you multiply by a 2-digit number, you usually add parts together, but not always.

2.1.4 Multiplying 4-digit Numbers by 1-digit Numbers

Try This

Mindu's parents pay Nu 7200 each month in rent for their home.

A. How much rent would they pay in 6 months?

• You can multiply numbers greater than 1000 just like you multiply numbers less than 1000.

For example, to calculate 8 × 3154:

Think of 3154 in parts: 3 thousands + 1 hundred + 5 tens + 4 ones

Thousands	Hundreds	Tens	Ones
3	1	5	4

Since 8 × 3154 means 8 groups of 3154, find 8 groups of each part:

Thousands	Hundreds	Tens	Ones
8 × 3 = 24	8 × 1 = 8	8 x 5 = 40	8 x 4 = 32
24,000	800	400	32

Then, add the parts: 24,000 + 800 + 400 + 32 = 25,232

• You can write the multiplication like this:

$$\begin{array}{rl} 3154 & \\ \underline{\times \quad 8} & \\ 24{,}000 & [8 \times 3 \text{ thousands}] \\ 800 & [8 \times 1 \text{ hundred}] \\ 400 & [8 \times 5 \text{ tens}] \\ \underline{+ \quad 32} & [8 \times 4 \text{ ones}] \\ 25{,}232 & \end{array}$$

B. How much rent would Mindu's parents pay in 6 months if the rent were Nu 7225 each month?

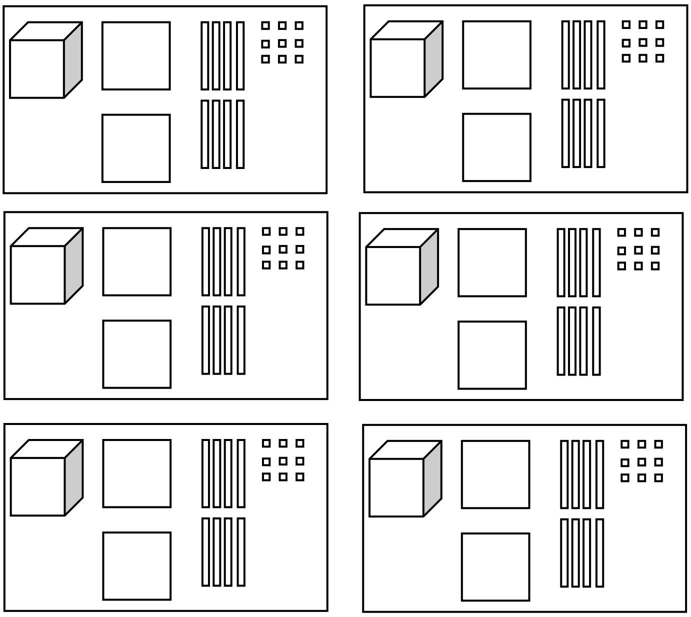
Examples

Example Multiplying 4-digit Numbers by 1-digit Numbers

Calculate 6 × 1289. Show your work.

Solution 1

Model 6 groups of 1289



Count each type of block

6 thousands + 12 hundreds + 48 tens + 54 ones

Write the sum

6000 + 1200 + 480 + 54 = 7734

Thinking

- I knew 6 × 1289 meant 6 groups of 1289.
- I knew 1289 was
  1 thousand
  + 2 hundreds
  + 8 tens
  + 9 ones

Solution 2

$$\begin{array}{r} {\scriptstyle 1\,5\,5\,\,} \\ 1289 \\ \times \quad 6 \\ \hline 7734 \end{array}$$

Thinking

- I used a place value chart.
- I multiplied each part by 6 —
I started with the ones and then did the tens, the hundreds, and the thousands. I regrouped each time.

Thousands	Hundreds	Tens	Ones
1	2	8	9
			6 × 9 = 54
		6 × 8 + 5 = 53	4
	6 × 2 + 5 = 17	3	4
6 × 1 + 1 = **7**	**7**	**3**	**4**

Practising and Applying

1. Use a place value chart or base ten models to find each product. Show your work.

a) 4 × 3702 **b)** 5 × 3119

c) 6 × 2814 **d)** 8 × 3924

e) 7 × 3098 **f)** 9 × 4728

2. By air, the distance from Bangkok to London is 9536 km. A plane flew from Bangkok to London and back. How far did it fly?

3. Copy and complete each.

In **part a)**, □ is the same digit.

In **part b)**, ◇is the same digit.

◇ and □ are different for each part.

a)

$$\begin{array}{r} 1\square 38 \\ \times\quad 9 \\ \hline 12{,}9\square 2 \end{array}$$

b)

$$\begin{array}{r} \square 538 \\ \times\quad 7 \\ \hline 17{,}7\diamond\diamond \end{array}$$

c)

$$\begin{array}{r} 37\square 8 \\ \times\quad \diamond \\ \hline 29{,}824 \end{array}$$

4. Calculate the rent.

a) Nu 6225 a month for 8 months

b) Nu 5190 a month for 6 months

c) Nu 6850 a month for 9 months

5. How many seconds are in 1 h?

6. In some countries, people measure using feet, yards, and miles. There are 5280 feet in a mile. How many feet are in each number of miles?

a) 7 miles **b)** 9 miles

7. Kinley multiplied a 1-digit number by a 4-digit number. The product was 6 thousands, 12 hundreds, 18 tens, and 30 ones.

□ × □ □ □ □ = 6 thousands, 12 hundreds, 18 tens, and 30 ones

Which numbers could he have multiplied? How do you know?

8. When you multiply a number by 8, the product is between 25,000 and 30,000.

? × 8 is between 25,000 and 30,000

a) What could the number be?

b) How do you know?

9. Notice that the product below has three digits that are 4.

$6 \times 8234 = \underline{4}9{,}\underline{4}0\underline{4}$

Which other 1-digit and 4-digit numbers could you multiply and get a product with three 4s?

10. The product of a 1-digit number and a 4-digit number can have 4 digits or 5 digits.

□ × □ □ □ □ = □ □ □ □

OR

□ × □ □ □ □ = □ □ □ □ □

Explain why this happens.

11. Why is multiplying a 4-digit number by a 1-digit number not much different than multiplying a 3-digit number by a 1-digit number?

2.1.5 EXPLORE: Mental Multiplication

Sometimes it is quicker to multiply in your head than on paper.
For example:

• *You can use place value.*

27 × 1000 = 27 × 1 thousand = 27 thousand or 27,000

• *You can multiply by breaking one factor into parts.*

Since 4999 = 5000 – 1,

8 × 4999 = 8 × 5000 – 8 × 1 = 40,000 – 8 = 39,992

A. Use mental math to calculate each.

i) 35 × 1000 **ii)** 67 × 1000 **iii)** 7 × 3000

B. Use 8 × 600 = 4800 to calculate each using mental math. Explain how you calculated.

i) 8 × 599 **ii)** 8 × 596 **iii)** 8 × 602 **iv)** 8 × 605

C. i) Copy and complete the chart. Complete rows 1 and 2 first. Multiply the values in each column to complete row 3. (The first column has been done for you.)

Column number		1	2	3	4
Row 1	24	24 ÷ 2 = **12**	24 ÷ 3 = ☐	24 ÷ 4 = ☐	24 ÷ 6 = ☐
Row 2	× 25	25 × 2 = **50**	25 × 3 = ☐	25 × 4 = ☐	25 × 6 = ☐
Product		**12 × 50 = 600**			

ii) What do you notice about the products?

iii) Why might you use column 3 instead of another column to multiply 24 × 25?

iv) Make a similar chart for 48 × 50. Which column would you use to multiply 48 × 50? Why?

D. Calculate each by multiplying the first factor by a number and dividing the second factor by the same number.

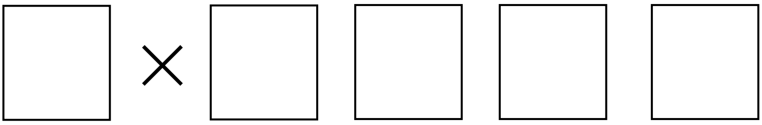
i) 5 × 64 **ii)** 50 × 86 **iii)** 25 × 84 **iv)** 250 × 44

E. Describe two ways to calculate each using mental math.

i) 5 × 492 **ii)** 25 × 484

GAME: Greatest Product

Play in groups of 2 to 4 players. You need at least one die.
Each player draws the multiplication boxes shown below.



Players take turns.

- Roll the die and write the digit in one of the boxes.
- Keep rolling until 5 digits are placed. You cannot move a digit after you have written it.
- Calculate the product.
- The player with the greatest product wins.

For example:
Suppose you roll a 6, a 2, a 3, a 1, and then a 6.
If you place the digits like this: 6 × 6321 = 37,926, you will get a greater product than if you place them like this: 1 × 2366 = 2366.

CONNECTIONS: Egyptian Multiplication

Here is an interesting way to multiply that requires only doubling and adding. It is called Egyptian multiplication.

- Make two columns, the first starting with 1 and the other starting with the second factor (78).
- Double to get each new row. Stop before the number in the first column is greater than the first factor (35).
- Look for numbers in the left column to add to get the first factor (35). Check off those rows.
- Add the checked numbers in the right column.

78 + 156 + 2496 = 2730, so 35 × 78 = 2730

35 ×	78	
1	78	√
2	156	√
4	312	
8	624	
16	1248	
32	2496	√

1 + 2 + 32 = 35
78 + 156 + 2496 = 2730
So 35 × 78 = 2730.

1. Use Egyptian multiplication to calculate each.
a) 40 × 85 **b)** 65 × 36 **c)** 80 × 92

Chapter 2 Division

2.2.1 Estimating Quotients

Try This

It rains a lot on Reunion, an island in the Indian Ocean. It once rained 4653 mm in 6 days.

A. i) Suppose it rained the same amount each day. About how many millimetres is that each day?

ii) Is that as high as your stool? How do you know?

• If you do not need an exact answer to a problem, you can estimate.

• To estimate, you can round to values that are easy to work with.

For example:

To estimate 5203 ÷ 8, you might round to 4800 ÷ 8.

4800 is easy to work with because 48 ÷ 8 = 6.

4800 ÷ 8 = 48 hundreds ÷ 8
= 6 hundreds
= 600

• Depending on what numbers you use, you could end up with a high estimate or a low estimate. In the example above, 600 is a low estimate because 5203 was rounded down to 4800 and 8 stayed the same.

• To estimate a **quotient**, it is usually better to round the **dividend** (as shown above) and not the **divisor**, but there are times when you might round the divisor.

For example:

To estimate 3892 ÷ 9, you might round to 4000 ÷ 10, since it is easier to divide by 10 than by 9.

4000 ÷ 10 = 400 tens ÷ 1 ten = 400

B. What numbers would you divide for each?

i) a low estimate for the amount of rain each day in **part A**

ii) a high estimate for the amount of rain each day in **part A**

Examples

Example 1 Using an Overestimate and an Underestimate	
How do you know that 3740 ÷ 9 is between 400 and 500?	
Solution 1 If 3740 ÷ 9 = ☐, then 9 × ☐ = 3740. 9 × 400 = 3600 9 × 500 = 4500 Since 3740 is between 3600 and 4500, ☐ is between 400 and 500. 3740 ÷ 9 is between 400 and 500.	**Thinking**  • I related division to multiplication. • I multiplied 9 by 400 and 9 by 500 to see if 3740 was between the two products.
Solution 2 3600 < **3740** < 4500 3600 ÷ 9 < **3740 ÷ 9** < 4500 ÷ 9 400 < **3740 ÷ 9** < 500 3740 ÷ 9 is between 400 and 500.	**Thinking**  • I rounded the dividend down to a value that was easy to divide by 9 to get a low estimate. • Then I rounded the dividend up to a value that was easy to divide by 9 to get a high estimate.

Example 2 Estimating to Solve a Problem	
5256 mm of rain once fell in Reunion over 8 days. About how many millimetres fell each day?	
Solution 5256 ÷ 8 is a bit less than 5600 ÷ 8. 5600 ÷ 8 = 56 hundred ÷ 8 = 7 hundred = 700 So 5256 ÷ 8 is a bit less than 700. About 700 mm of rain fell each day.	**Thinking**  • I rounded the dividend to a number that I could easily divide by 8.

Practising and Applying

Show your work for each estimate.

1. Estimate each quotient.

a) 6212 ÷ 6

b) 4002 ÷ 5

c) 1278 ÷ 9

d) 1928 ÷ 4

2. Calculate a high estimate and a low estimate for each.

a) 4889 ÷ 3

b) 8123 ÷ 5

c) 9125 ÷ 8

d) 6002 ÷ 7

3. For which values is each true?

a) ☐ ÷ 9 > 500

b) ☐ ÷ 7 < 60

c) ☐ ÷ 5 > 1000

4. **a)** Explain why all of these are possible ways to estimate 6412 ÷ 7.

• 6000 ÷ 6

• 6300 ÷ 7

• 6400 ÷ 8

b) Which estimate do you feel is closest to the actual answer? Why?

5. One of these answers is incorrect. Estimate to decide which one. Explain how you know you are right.

A. 1175 ÷ 5 = 235

B. 1272 ÷ 7 = 199

C. 4536 ÷ 7 = 648

6. 9200 tests are divided into 6 piles of equal height. About how many forms are in each pile?

7. 2575 passengers got on 5 trains in India. If the same number of passengers got on each train, about how many are on each?

8. Describe a situation when you might need to estimate using the calculation below:

4500 ÷ 5

9. Why is it a good idea to estimate, even when you are calculating an exact answer?

2.2.2 Dividing 4-digit Numbers by 1-digit Numbers

Try This

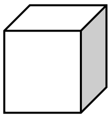
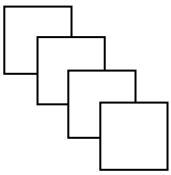
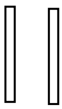
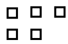
A farmer used 1256 m of fencing to fence a square field for growing peppers. The side lengths of the field are whole numbers. There were 4 m of fencing left over.

A. How would you figure out the side length of the field?

• You can divide by sharing.

For example:

To divide 1425 ÷ 3, imagine 3 people sharing 1425 represented like this:

Thousands	Hundreds	Tens	Ones
1	4	2	5
			

Step 1

1 thousand cannot be shared by 3 to give each person any thousands.

So, trade 1 thousand for 10 hundreds.

Now there are 14 hundreds.

Each person gets **4** hundreds and

2 hundreds are left over.

$$\begin{array}{r} \mathbf{4} \\ 3\overline{)1425} \\ -\underline{12} \\ \mathbf{2} \end{array}$$

Step 2

2 hundreds cannot be shared by 3 to give each person any hundreds.

So, trade 2 hundreds for 20 tens.

Now there are 22 tens.

Each person gets **7** tens and

1 ten is left over.

$$\begin{array}{r} 4\mathbf{7} \\ 3\overline{)1425} \\ -\underline{12} \\ 22 \\ -\underline{21} \\ \mathbf{1} \end{array}$$

Step 3 1 ten cannot be shared by 3 to give each person a ten. So, trade 1 ten for 10 ones. Now there are 15 ones. Each person gets **5** ones. 1425 ÷ 3 = 475	$\begin{array}{r} 47\mathbf{5} \\ 3\overline{)1425} \\ -\underline{12} \\ 22 \\ -\underline{21} \\ 15 \\ -\underline{15} \\ 0 \end{array}$

B. Use the division method above to calculate the answer to **part A**.

Examples

Example 1 Dividing a 4-Digit Number by Multiplying and Subtracting	
A school collected Nu 3424 to buy oranges for the students. An orange costs Nu 5. How many oranges can the school purchase?	
Solution 3424 ÷ 5	**Thinking** 
$\begin{array}{r\|l} 5\overline{)3424} & \\ -\underline{2000} & \text{400 oranges} \\ 1424 & \end{array}$	• I first bought 400 oranges, which cost Nu 5 × 400 = Nu 2000. • I subtracted Nu 2000 from Nu 3424 to see what was left.
$\begin{array}{r\|l} 5\overline{)3424} & \\ -\underline{2000} & \text{400 oranges} \\ 1424 & \\ -\underline{1000} & \text{200 oranges} \\ 424 & \end{array}$	• There were Nu 1424 left, so I bought 200 more oranges, which cost Nu 5 × 200 = Nu 1000. • I subtracted Nu 1000 from Nu 1424 to see what was left.
$\begin{array}{r\|r} 5\overline{)3424} & \\ -\underline{2000} & \text{400 oranges} \\ 1424 & \\ -\underline{1000} & \text{200 oranges} \\ 424 & \\ \underline{-400} & \text{80 oranges} \\ 24 & \\ -\underline{20} & \underline{+\quad\text{4 oranges}} \\ 4 & \text{684 oranges} \end{array}$	• I kept subtracting amounts for more oranges until there were Nu 4 left. (Nu 4 weren't enough to buy even 1 more orange.) • I added up all the oranges to see how many had been purchased.
The school can buy 684 oranges.	• There will be Nu 4 left over.

Example 2 Interpreting a Remainder

Calculate each and then decide how the remainder should be handled in each situation.

A. A square field has a perimeter of 1214 m. What is the side length?

B. 1214 students are divided into 4 groups. How many are in each group, if the groups are as equal as possible?

C. How many Nu 4 erasers can you buy with Nu 1214?

Solution	**Thinking**
1214 ÷ 4 = 303 R 2	 • Each situation involved dividing 1214 by 4.
Situation A The remainder is 2 m. You can divide 2 m among the four sides: 2 ÷ 4 = 0.5	• The side length is 303.5 m.
Situation B The remainder is 2 students. You cannot have half a student, so the 2 extra students could be added to one group or 1 extra student could be added to each of two groups.	• The group sizes could be 303, 303, 303, and 305 or 303, 303, 304, and 304.
Situation C The remainder is Nu 2. Nu 2 is not enough to purchase another eraser and you can not purchase a part of an eraser, so you ignore the remainder.	• 303 erasers could be purchased and there would be Nu 2 left over.

Practising and Applying

1. Calculate each.

a) 9848 ÷ 8 **b)** 8351 ÷ 7

c) 4712 ÷ 8 **d)** 4995 ÷ 4

e) 1005 ÷ 9 **f)** 3098 ÷ 6

2. How much greater is the first quotient than the second?

a) 3215 ÷ 5 than 2568 ÷ 4

b) 5216 ÷ 8 than 3192 ÷ 6

c) 2506 ÷ 7 than 933 ÷ 3

3. When Karchung bought a new TV, he paid Nu 8500 in three separate payments. How much was each payment, if the payments were as equal as possible?

4. The cost of a fancy table is Nu 1500. How much will each person pay if the cost is shared equally by each number of people?

a) 5 people

b) 3 people

c) 4 people

5. Calculate each and then decide how to handle the remainder.

a) Four people are sharing Nu 4206 equally. How much does each get?

b) A field with an area of 430 m^2 is divided into 4 equal sections. What is the area of each section?

6. Write a word problem that you could solve using 3006 ÷ 8.

7. Copy and complete one of the divisions below. You can put a different digit in each box.

a)
$$\begin{array}{r}
1206 \\
6\overline{)\,[\,][\,]3[\,]} \\
-\underline{6} \\
12 \\
-\underline{12} \\
036 \\
-\underline{36} \\
0
\end{array}$$

b)
$$\begin{array}{r|r}
8\overline{)53[\,]5} & \\
-\underline{4800} & [\,]00 \\
[\,]25 & \\
-\underline{480} & [\,]0 \\
[\,][\,] & \\
-\underline{40} & +\ \underline{5} \\
0 & [\,][\,]5
\end{array}$$

8. You are dividing a number by 7. How do you know the remainder will not be 9?

9. When you divide a number by 9

• the quotient is greater than 300

• the remainder is 4

List three possible dividends.

10. Why is it important to be able to multiply when you divide 4-digit numbers by 1-digit numbers? Use an example to help you explain.

GAME: Target 2000

Play in groups of 2 to 4 players. You need at least one die.

Each player draws the division boxes shown below.

$\square \; \square \; \square \; \square \div \square$

Players take turns.

- Roll the die and write the digit in one of the boxes.
- Keep rolling until all five digits are placed. You cannot move a digit after you have written it.
- Calculate the quotient.
- The player with the quotient closest to 2000 wins.

For example:

Suppose you roll these digits in this order: 6, 2, 3, 1, and then 6.

6 2 3 1 6

If you place the digits like this: 6126 ÷ 3, your quotient will be closer to 2000 than if you place them like this: 2366 ÷ 1.

6126 ÷ 3 = 2042 is closer to 2000 than 2366 ÷ 1 = 2366

2.2.3 EXPLORE: Mental Division

You can divide by 10, 100, and 1000 using place value ideas.

A. i) Complete Row 2 of the chart below.

Number	80	410	3460	5000
Number of groups of ten	8			
Number ÷ 10				

ii) Explain, using words or a diagram, why 15 tens ÷ 1 ten = 15.

iii) Use the idea in **part ii)** to complete Row 3 of the chart.

B. i) Complete Row 2 of the chart below.

Number	800	4100	3400	5000
Number of groups of one hundred	8			
Number ÷ 100				

ii) Explain, using words or a diagram, why 12 hundreds ÷ 1 hundred = 12.

iii) Use the idea in **part ii)** to complete Row 3 of the chart

C. i) Complete Row 2 of the chart below.

Number	8000	12,000	20,000	45,000
Number of groups of one thousand	8			
Number ÷ 1000				

ii) Explain, using words or a diagram, why 12 thousands ÷ 1 thousand = 12.

iii) Use the idea in **part ii)** to complete Row 3 of the chart

D. Use mental math strategies to calculate.

i) 4200 ÷ 100

ii) 6500 ÷ 100

iii) 5600 ÷ 10

iv) 56,000 ÷ 1000

2.2.4 Dividing 4-digit Numbers by Multiples of Ten

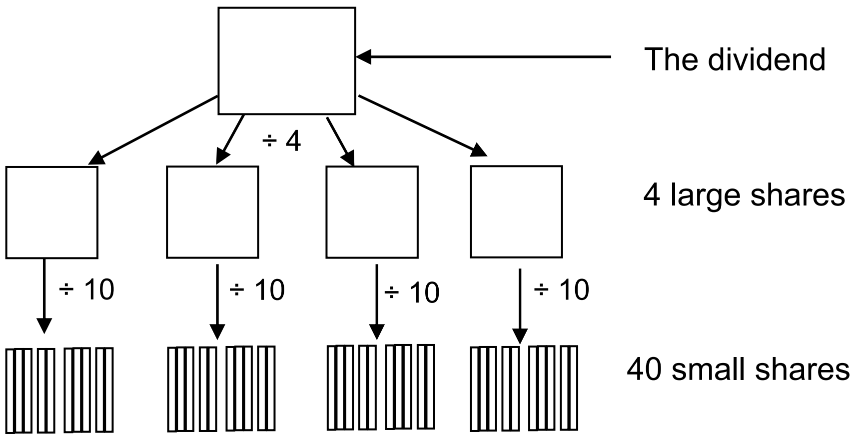
Try This

A. A pile of Nu 20 notes is worth Nu 5280. How many Nu 20 notes are in the pile?

• Dividing by 10, 20, 30, …, or 90 is as simple as dividing by 1, 2, 3, …, or 9.
For example:
3200 ÷ 40 is finding how many groups of 4 tens are in 320 tens.

$$\begin{aligned} 3200 \div 40 &= 320 \text{ tens} \div 4 \text{ tens} \\ &= 320 \div 4 \\ &= 80 \end{aligned}$$

• You can also divide by separating the divisor into parts.
For example:
To divide 3200 ÷ 40, divide 3200 ÷ 4 and then divide the result by 10.
3200 ÷ 40 → 3200 ÷ 4 = 800 → 800 ÷ 10 = 80
So 3200 ÷ 40 = 80.
This works because, to make 40 shares, you can first share 4 ways and then divide each of those shares 10 ways.



B. Was the strategy you used to calculate in **part A** similar to either of the two strategies shown above? Explain your thinking.

Examples

Example 1 Dividing Two Multiples of Ten

Calculate 3570 ÷ 80.

Solution

Divide to find the quotient (without remainder)

3570 = 357 tens and 80 = 8 tens

So, 3570 ÷ 80 = 357 tens ÷ 8 tens

= 357 ÷ 8 = 44 R ?

To find the remainder, multiply the quotient by the divisor and then subtract from the dividend

44 × 80 = 3520 and 3570 – 3520 = 50

3572 ÷ 80 = 44 R 50

Thinking

- I thought of both numbers as tens to make dividing easier.
- Since the remainder when dividing by 8 might be different than when dividing by 80, I had to figure out the remainder for the original question.

Example 2 Dividing by a Multiple of Ten

Calculate 4143 ÷ 60.

Solution

Estimate to find the quotient

4143 ÷ 60 → 4140 ÷ 60

4140 ÷ 10 = 414 → 414 ÷ 6 = 69

Multiply the quotient and then subtract to find the remainder

69 × 60 = 4140 and 4143 – 4140 = 3

4143 ÷ 60 = 69 R 3

Thinking

- I rounded 4143 down to the nearest multiple of 10.
- To divide 4140 by 60, I divided it by 10 and then by 6.
- I multiplied the quotient by the divisor and then subtracted from the dividend to get the remainder.

Example 3 Using Guess and Test and Multiplying to Divide

Snacks cost Nu 40 each. How many snacks can you buy with Nu 2200?

Solution 1

2200 ÷ 40 = ■, so ■ × 40 = 2200

10 × 40 = 400	10 is not enough.
20 × 40 = 800	20 is not enough.
50 × 40 = 2000	50 is close, but not enough.
55 × 40 = 2200	55 is the right number.

So, 2200 ÷ 40 = 55.

I can buy 55 snacks.

Thinking

- I changed the division to multiplication.
- I guessed the number of snacks until I got the right amount.
- After trying 50, I tried 55 because 5 × 40 = 200.

Practising and Applying

1. Calculate each.

a) 5634 ÷ 30

b) 4219 ÷ 50

c) 900 ÷ 40

d) 5020 ÷ 20

2. Choose one division from **question 1**. Write a word problem you could solve using that division.

3. A car travels 30 km each hour. About how many hours would it take to go each distance?

a) 3200 km

b) 1700 km

c) 5720 km

4. How many hours have passed after each number of minutes?

For example:

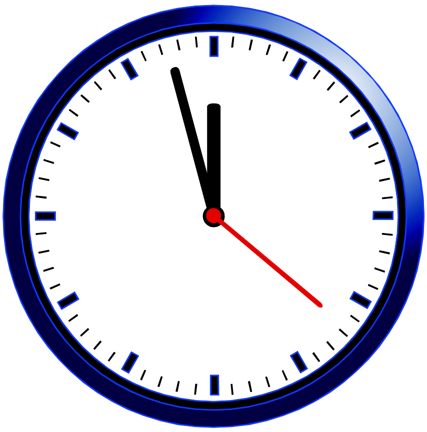
300 min = 5 h

a) 3600 min

b) 4520 min

c) 8530 min

d) 9000 min



There are 60 min in 1 h.

5. If you blink 20 times a minute, how many minutes will it take for 3500 blinks?

6. Explain why 5000 ÷ 30 is less than 5000 ÷ 3 and not more.

7. Radhika says that you can divide by 30 by

• dividing by 5 and then dividing the result by 6

OR

• dividing by 10 and then dividing the result by 3

Do you agree? Explain your thinking using an example.

8. Namgyel is dividing a 4-digit number by 30. There is no remainder. He says the quotient will have 3 digits. Explain his thinking.

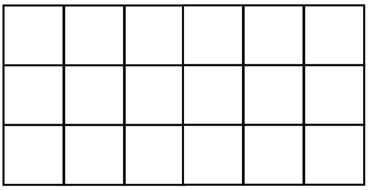
■ ■ ■ ■ ÷ 30 = ■ ■ ■

9. How does knowing 100 ÷ 20 = 5 help you calculate 4000 ÷ 20?

10. Why is it easier to divide by 20 than to divide by 21?

UNIT 2 Revision

1. Which multiplication equation does the diagram show? (Each small square is 10 by 10.)



2. Draw a rectangle diagram to show each multiplication.

a) 30 × 90 **b)** 20 × 20

3. Which is greater than 2500?

A. 30 × 70 **B.** 40 × 50

C. 80 × 10 **D.** 40 × 70

4. Estimate. Show your work.

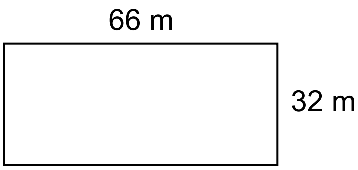
a) 39 × 67

b) 33 × 42

c) 77 × 25

d) 81 × 37

5. Estimate the area. Show your work.



6. One of these answers is incorrect. Estimate to decide which one. Explain your thinking.

A. 47 × 63 = 3961

B. 23 × 89 = 2047

C. 51 × 48 = 2448

7. Estimate each total cost. Show your work.

a) 37 books, each costs Nu 50

b) 43 pencil boxes, each costs Nu 80

c) 75 pens, each costs Nu 17

8. a) Describe two different ways to estimate 35 × 75.

b) Is one way better than the other? Explain your thinking.

9. Draw a diagram to help you calculate each. Then calculate.

a) 24 × 42

b) 53 × 27

10. Calculate each.

a) 67 × 432

b) 83 × 219

11. A blanket is made of 24 rows of 36 squares. How many squares are in the blanket?

12. 1 kg of potatoes costs Nu 15. How much would 213 kg cost?

13. Write a word problem that you could solve using 25 × 38.

14. Use a place value chart or base ten models to find each product. Show your work.

a) 3 × 4102

b) 2 × 5662

c) 5 × 1438

d) 7 × 2893

15. Copy and complete

$$\begin{array}{r} 21[\ ]3 \\ \times\quad 7 \\ \hline 15{,}0[\ ]1 \end{array}$$

16. How many seconds are in 4 h?

17. Calculate each using mental math. Describe how you calculated.

a) 34 × 1000

b) 44 × 25

c) 312 × 50

18. Estimate. Show your work.

a) 6245 ÷ 3

b) 3219 ÷ 7

c) 2560 ÷ 5

d) 3718 ÷ 9

19. Write an estimation word problem that you could solve using 3000 ÷ 5.

20. Calculate.

a) 4230 ÷ 5

b) 5669 ÷ 7

c) 2218 ÷ 6

d) 3992 ÷ 9

21. Padam paid Nu 9200 for a new TV in five equal payments. How much was each payment?

22. Calculate using mental math.

a) 3600 ÷ 100

b) 4500 ÷ 100

c) 2800 ÷ 10

d) 13,000 ÷ 1000

23. Calculate.

a) 4030 ÷ 30

b) 3617 ÷ 50

c) 1250 ÷ 40

d) 4939 ÷ 20

24. There are 60 min in 1 h. How many hours have passed after each number of minutes?

a) 9000

b) 1260

c) 7000

d) 8490

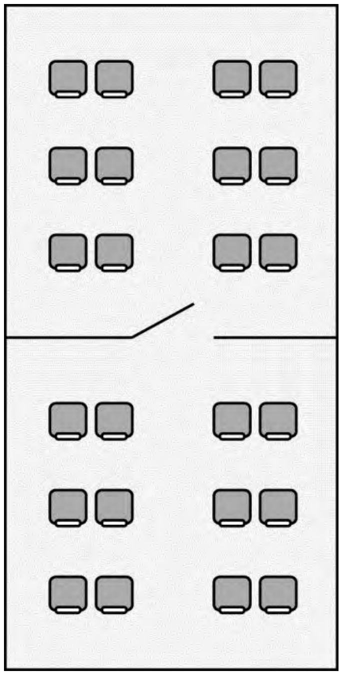
25. Describe two or more different ways to divide 5030 by 20.

UNIT 3 FRACTIONS AND DECIMALS

Getting Started

Use What You Know

There are 15 people in two rooms.
Some are children and some are adults.
Each room has 12 chairs.



A. i) Copy the diagram of the chairs in the two rooms.

ii) Draw 15 people sitting on the chairs. Use one colour for children and another colour for adults. Leave one or more chairs empty in each room.

iii) What fraction of the chairs in each room are filled?

iv) What fraction of the chairs in each room are filled with children?

v) Order the four fractions from **part iii)** and **part iv)** from least to greatest.

B. Repeat **part A** using a different total number of adults and children.

C. Which fraction in **parts A and B** is closest to each?

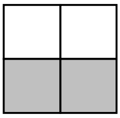
i) 1 **ii)** $\frac{1}{2}$ **iii)** $\frac{1}{4}$

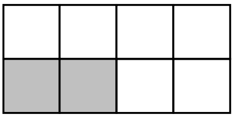
D. 12 people sat in the first room and 3 people sat in the second room. Explain why the fraction $1\frac{1}{4}$ can be used to describe the seating in the two rooms.

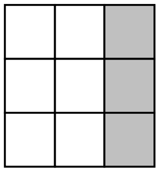
E. In one room, $\frac{1}{3}$ of the chairs are full. How many chairs is that? How do you know?

Skills You Will Need

1. Write two fractions for each picture.

a) 

b) 

c) 

2. Order from least to greatest.

a) $\frac{5}{6}, \frac{3}{6}, \frac{1}{6}, \frac{2}{6}$

b) $\frac{4}{8}, \frac{7}{8}, \frac{6}{8}, \frac{3}{8}$

c) $\frac{1}{6}, \frac{1}{4}, \frac{1}{10}, \frac{1}{3}$

d) $\frac{4}{5}, \frac{4}{9}, \frac{4}{6}, \frac{4}{15}$

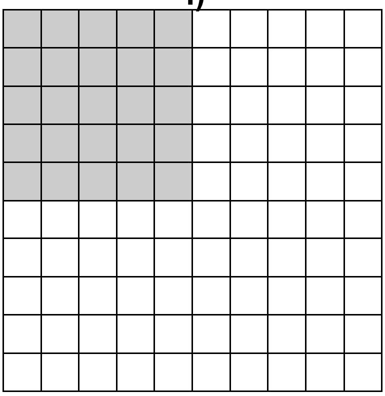
3. Represent each mixed number using squares of paper. To model the fraction part of each mixed number, you can fold the square. Sketch each model.

a) $1\frac{1}{4}$ **b)** $2\frac{2}{3}$ **c)** $1\frac{1}{6}$

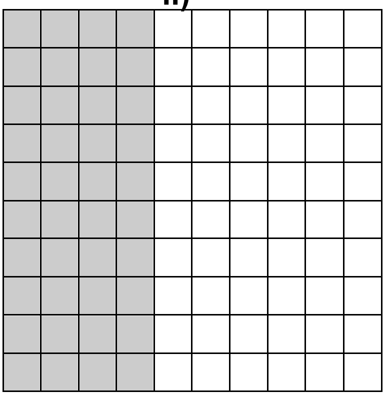
4. Match each decimal with a grid model.

A. 0.4 **B.** 0.04 **C.** 0.25

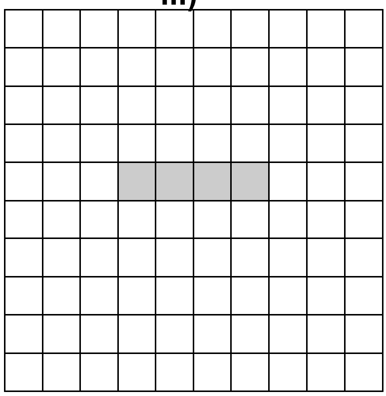
i)



ii)



iii)



5. Write each decimal as a fraction.

a) 0.2 **b)** 0.45 **c)** 0.9 **d)** 0.05

6. Order from least to greatest.

a) 0.23, 0.45, 0.17, 0.29

b) 0.17, 0.5, 0.45, 0.3

Chapter 1 Fractions

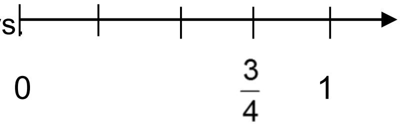
3.1.1 EXPLORE: Meanings of Fractions

Fractions can mean different things, but those different things are related.

The fraction $\frac{3}{4}$ can mean $\frac{3}{4}$ of a shape, $\frac{3}{4}$ of a group, $\frac{3}{4}$ of a length, or it can mean what you get when you divide 3 by 4.

$\frac{3}{4}$ of this square is grey.

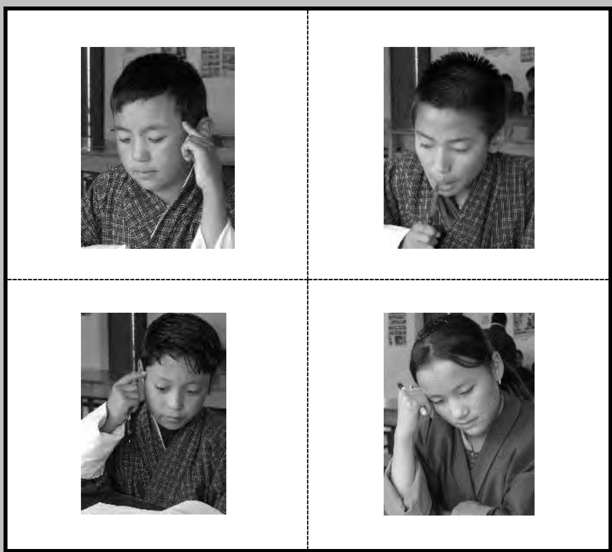
$\frac{3}{4}$ of this group of students are boys



$\frac{3}{4}$ of the distance from 0 to 1 on a number line

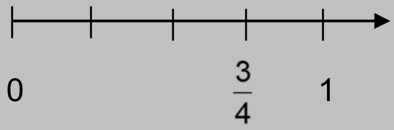
$\frac{3}{4} = 3 \div 4$

A. i) How does this picture show that $\frac{3}{4}$ of a shape is like $\frac{3}{4}$ of a group?



ii) Draw a picture to show how $\frac{2}{5}$ of a rectangle is like $\frac{2}{5}$ of a group of animals: 2 cats and 3 dogs.

iii) How is $\frac{3}{4}$ on the number line like $\frac{3}{4}$ of a shape?

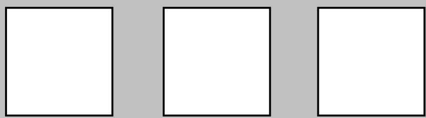


iv) How can you show that $\frac{3}{4}$ on a number line is like $\frac{3}{4}$ of a group?

B. The answer to 12 ÷ 4 tells how much each person gets if 12 whole things are shared equally among 4 people.

The answer to 3 ÷ 4 tells how much each person gets if 3 whole things are shared equally among 4 people.

i) Suppose these 3 squares are being shared equally by 4 people. How much would each person get? Explain your thinking.



ii) Draw a picture to show why $\frac{2}{3}$ = 2 ÷ 3.

C. **i)** Represent $\frac{5}{6}$ using a shape, a group, a length, and a division.

ii) Show how two of the meanings are related.

3.1.2 Fractions as Division

Try This

Bijoy cooked a pot of soup for $1\frac{1}{4}$ h.

She stirred it every $\frac{1}{4}$ h.

A. How many times did she stir the soup?

• One meaning of a fraction is division. A fraction such as $\frac{5}{6}$ can mean 5 ÷ 6.

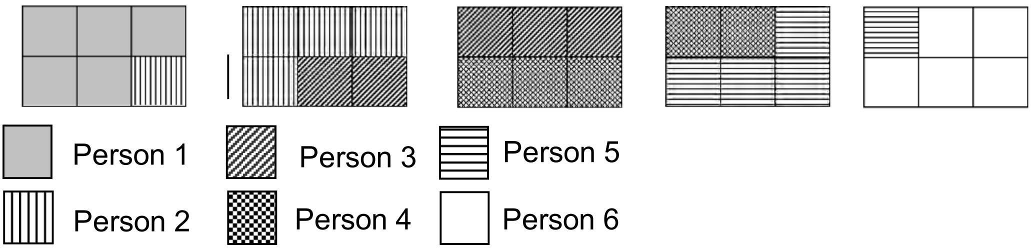
The answer to the division 5 ÷ 6 is $\frac{5}{6}$.

For example:

If 5 whole things are divided equally among 6 people, each person gets $\frac{5}{6}$.

In the picture below, there are 5 rectangles in sixths, or 30 sixths altogether.

When the 30 sixths are shared among 6 people, each gets 5 sixths or $\frac{5}{6}$.



The $\frac{5}{6}$ that each person gets do not have to be next to each other,

for example, each person might get $\frac{1}{6}$ of each of the 5 rectangles.

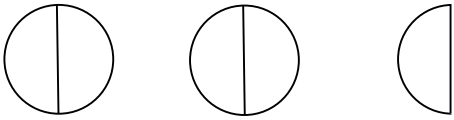
• You can use the division meaning of a fraction to write an **improper fraction** as a **mixed number**.

For example: $\frac{5}{2} = 5 \div 2 = 2\frac{1}{2}$

When 5 wholes are shared by 2 people, each person gets 2 wholes and a half.

[Continued]

When you use division to change $\frac{5}{2}$ to a mixed number, you are grouping 5 halves into groups of 2 to make 2 wholes and 1 half.



5 halves are 2 wholes and 1 half.

$\frac{5}{2} = 2\frac{1}{2}$

B. Explain how the problem in **part A** relates to the division meaning of a fraction.

Examples

Example 1 Renaming Fractions as Whole Numbers

Rename each fraction as a whole number. **a)** $\frac{15}{5}$ **b)** $\frac{20}{4}$

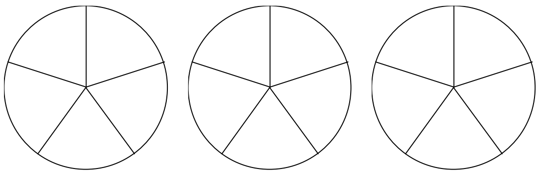
Solution

a) $\frac{15}{5} = 15 \div 5 = 3$

b) $\frac{20}{4} = 20 \div 4 = 5$

Thinking

a) It takes 5 fifths to make 1 whole, so I divided the 15 fifths by 5 to see how many wholes there were.



b) It takes 4 fourths to make 1 whole, so I divided the 20 fourths by 4 to see how many wholes there were.

Example 2 Renaming Fractions as Mixed Numbers

Rename each improper fraction as a mixed number. **a)** $\frac{13}{6}$ **b)** $\frac{50}{8}$

Solution

a) $\frac{12}{6} = 12 \div 6 = 2$

$\frac{13}{6} = 2 + \frac{1}{6} = 2\frac{1}{6}$

b) $50 \div 8 = 6 \text{ R } 2$

$\frac{50}{8} = 6\frac{2}{8}$

Thinking

a) I know that 12 sixths make 2 wholes, so I renamed 13 as 12 + 1.

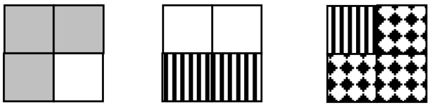
b) A quotient of 6 means there are 6 wholes.

• Because I divided by 8, I knew the remainder of 2 meant there were 2 eighths left over.

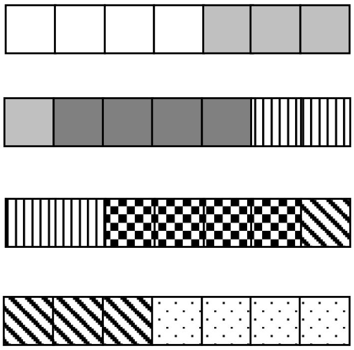
Practising and Applying

1. What division is being modelled in each picture?

a)



b)



2. Draw a picture to show why each is true.

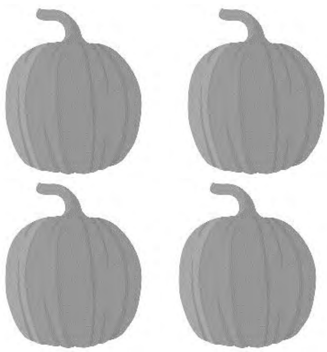
a) $\frac{10}{2} = 5$ **b)** $\frac{17}{3} = 5\frac{2}{3}$

3. Rename each improper fraction as a mixed number.

a) $\frac{14}{5}$ **b)** $\frac{23}{2}$

Describe two different ways to rename each.

4. If 6 families share 4 pumpkins, how much of a pumpkin will each family get?



5. Solve each.

a) $16 \div 3 = \square$ **b)** $25 \div 6 = \square$

c) $18 \div 9 = \square$ **d)** $35 \div 5 = \square$

6. Pads of paper are sold in packages of 4. Pema used 18 pads. How many full and part packages did he use?

7. Dorji showed that $3 \div 4 = \frac{3}{4}$ by doing this:

- He thought of $3 \div 4$ as 3 whole items shared among 4 people.
- He gave each person $\frac{1}{4}$ of each item, so each got $\frac{3}{4}$ of one item.

a) Do you agree with his method?

b) Use Dorji's method to show that $6 \div 9 = \frac{6}{9}$.

8. a) Four different fractions were all renamed as 4. What might they have been?

b) Could more than one fraction be renamed as $1\frac{1}{2}$? Explain your thinking.

9. Duptho wrote $\frac{3}{25} = 8\frac{1}{3}$.

a) Is he correct? How do you know?

b) If he is incorrect, how could you help him see that his answer was incorrect?

10. Why does it make sense that fractions are about division?

3.1.3 Equivalent Fractions

Try This

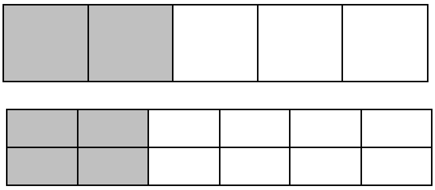
Yangdon must write a 6-page report over the next 9 days. She plans to write the same amount each day.

A. What fraction of a page must she write each day?

• Two fractions are equal, or **equivalent**, if they both describe the same amount of the same whole.

For example:

$\frac{2}{5} = \frac{4}{10}$



$\frac{2}{5}$ and $\frac{4}{10}$ are **equivalent fractions** since they both describe the same amount of the same whole rectangle.

• You can always create equivalent fractions by dividing fraction parts.

For example:

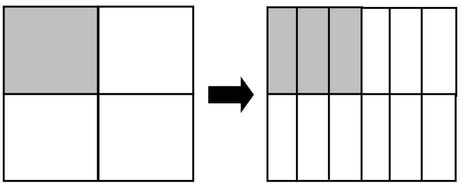
In the picture above, each part of $\frac{2}{5}$ was divided into 2 equal parts.

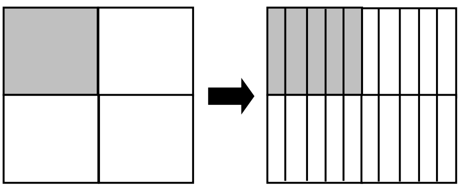
That meant there were 2 times as many parts altogether ($2 \times 5 = 10$) and 2 times as many grey parts ($2 \times 2 = 4$), so $\frac{2}{5}$ became $\frac{4}{10}$.

This explains why you can create an equivalent fraction by multiplying the **numerator** and **denominator** by the same number. It can be any number except 0.

The pictures below show why $\frac{1}{4} = \frac{3}{12}$ and $\frac{1}{4} = \frac{5}{20}$.

$\frac{1}{4} \rightarrow \frac{3 \times 1}{3 \times 4} \rightarrow \frac{3}{12}$ $\qquad$ $\frac{1}{4} \rightarrow \frac{5 \times 1}{5 \times 4} \rightarrow \frac{5}{20}$



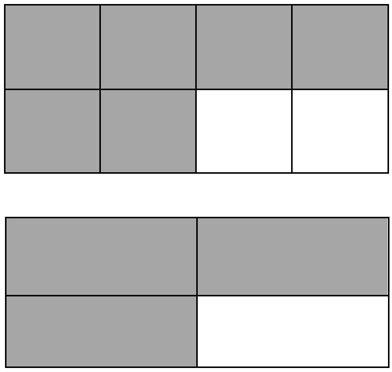


• You can sometimes create equivalent fractions by combining fraction parts.

For example:

$\frac{6}{8} = \frac{3}{4}$

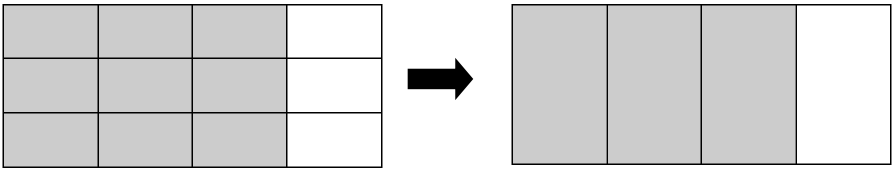
You combine 2 eighths to make 1 fourth, which means 6 eighths make 3 fourths.



> This explains why you can create an equivalent fraction by dividing the numerator and denominator by the same number.
> It can be any number except 0.

The picture below shows why $\frac{9}{12} = \frac{3}{4}$.

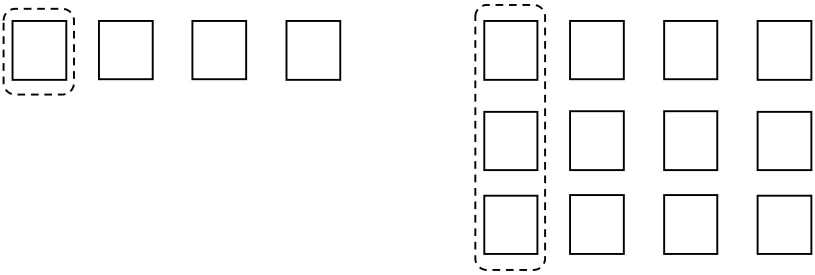
$$\frac{9}{12} \rightarrow \frac{9 \div 3}{12 \div 3} \rightarrow \frac{3}{4}$$



• When two fractions are equivalent, the numerator is the same fraction of the denominator.

For example:

Since $\frac{1}{4} = \frac{3}{12}$, then 1 is $\frac{1}{4}$ of 4, just like 3 is $\frac{1}{4}$ of 12.



Both times, you are counting the number of squares in 1 column out of 4.

B. Name two other fractions you could use to answer **part A**.

Examples

Example 1 Creating Equivalent Fractions	
a) List four fractions that are equivalent to $\frac{4}{8}$. **b)** Show that a fraction equivalent to $\frac{4}{8}$ could have 40 in the numerator or 40 in the denominator.	
Solution	**Thinking** 
a) $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$ $\frac{4}{8} = \frac{4 \div 2}{8 \div 2} = \frac{2}{4}$ $\frac{4}{8} = \frac{4 \times 3}{8 \times 3} = \frac{12}{24}$ $\frac{4}{8} = \frac{4 \times 2}{8 \times 2} = \frac{8}{16}$	**a)** I divided or multiplied the numerator and the denominator by the same amount each time. • I noticed that the denominator was always twice the numerator, so I think my answers are right.
b) $\frac{4}{8} = \frac{4 \times 10}{8 \times 10} = \frac{40}{80}$ $\frac{4}{8} = \frac{4 \times 5}{8 \times 5} = \frac{20}{40}$	**b)** Since 10 × 4 = 40, I multiplied the numerator and denominator by 10 to get an equivalent fraction with 40 in the numerator. • Since 5 × 8 = 40, I multiplied the numerator and denominator by 8 to get an equivalent fraction with 40 in the denominator.

Example 2 Non-equivalent Fractions	
You can multiply the numerator and denominator by the same number to create an equivalent fraction. Show why you cannot add the same number to each to create an equivalent fraction.	
Solution	**Thinking** 
$\frac{1}{2} \rightarrow \frac{1 \text{ and } 1}{2 \text{ and } 1} \rightarrow \frac{2}{3}$ $\frac{2}{3} > \frac{1}{2}$ so they are not equivalent.	• To show that something doesn't work, I need to find only one example.

Practising and Applying

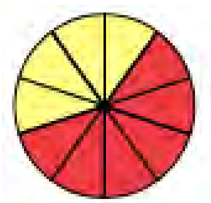
1. Write two equivalent fractions for each picture.

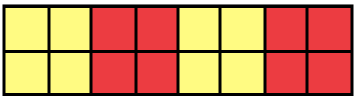
a)

b)

c)

2. Copy each diagram.





a) Write two equivalent fractions to describe the dark part of each.

b) Show how the fractions are equivalent.

3. Draw a picture to show that $\frac{3}{8} = \frac{6}{16}$.

4. List four fractions that are equivalent to each.

a) $\frac{5}{8}$ **b)** $\frac{15}{18}$

c) $\frac{25}{75}$ **d)** $\frac{10}{40}$

5. Use the number 20 in the numerator or denominator to make a fraction equivalent to each.

a) $\frac{1}{10}$ **b)** $\frac{2}{5}$

c) $\frac{3}{4}$ **d)** $\frac{40}{120}$

6. 25 people were asked if they were happy and 20 said yes. Write a fraction that describes the part of the group that was happy. Both the numerator and denominator must be less than 10.

7. a) The numerator of a fraction is $\frac{1}{3}$ of the denominator. List three possible fractions.

b) Show how you know the numerator is $\frac{1}{3}$ of the denominator.

c) Repeat **part a)** for a numerator that is $\frac{2}{3}$ of the denominator.

8. Tashi said that $\frac{8}{9} = \frac{6}{7}$ because he subtracted the same number from both the numerator and denominator. Do you agree? Explain your thinking.

9. $\frac{20}{30} = \frac{6}{9}$ even though you cannot divide 20 and 30 by a whole number and get 6 and 9. List another pair of equivalent fractions like this.

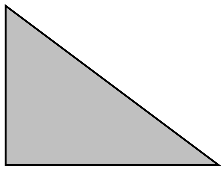
10. How you do you know that there are more than 1000 fractions equivalent to $\frac{2}{3}$?

CONNECTIONS: Fractions and Geometry

There is a natural connection between geometry and fractions. That is why you can use geometric ideas to make a whole when you know a part.

For example:

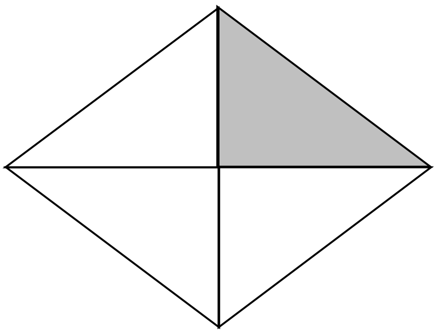
If you say the shape below is $\frac{1}{4}$, you can use symmetry and transformations to create the whole.

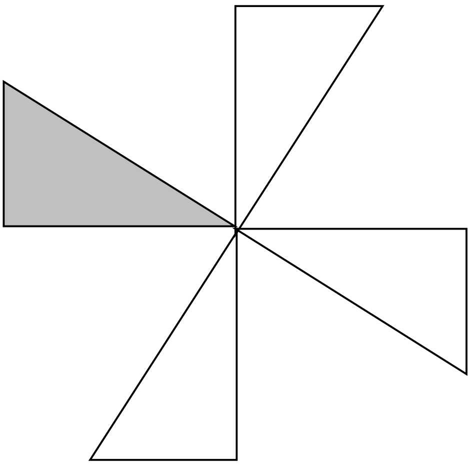


$\frac{1}{4}$ of a whole

There are many possible wholes. Here are two possibilities:

.





1. Use a shape to represent $\frac{1}{2}$. Show two or more different possible wholes.

2. Use a shape to represent $\frac{1}{6}$. Show two or more different possible wholes.

3.1.4 Comparing and Ordering Fractions

Try This

About $\frac{1}{3}$ of the population of Bhutan is between 0 and 14 years old.

About $\frac{3}{5}$ of the population is between 15 and 64 years old.

About $\frac{1}{20}$ of the population is over 65 years old.

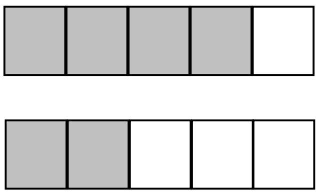
A. Which population age group is the largest? How do you know?

There are different ways to compare fractions.

• If the fractions have the same denominator, the fraction with the greater numerator is greater.

For example:

$\frac{4}{5} > \frac{2}{5}$ since 4 of anything (fifths) is greater than 2 of the same thing (fifths).

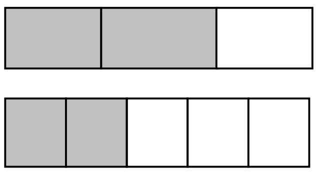


$\frac{4}{5}$ of a whole is more than $\frac{2}{5}$ of the same whole.

• If the fractions have the same numerator, the fraction with the lower denominator is greater.

For example:

1 third is bigger than 1 fifth, so $\frac{2}{3} > \frac{2}{5}$, since 2 thirds > 2 fifths



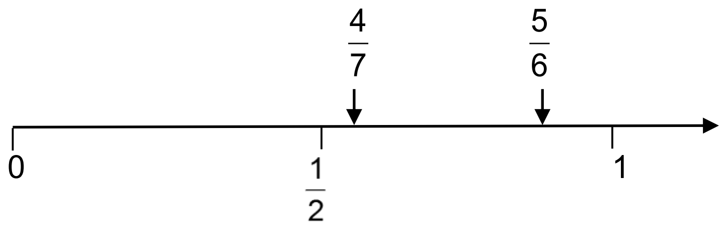
$\frac{2}{3}$ of a whole is more than $\frac{2}{5}$ of the same whole.

• You can sometimes compare fractions to each other by comparing them to 0, $\frac{1}{2}$, and 1.

For example, to compare $\frac{4}{7}$ and $\frac{5}{6}$:

$\frac{4}{7}$ is just a bit more than $\frac{4}{8}$ or $\frac{1}{2}$. $\frac{5}{6}$ is just a bit less than $\frac{6}{6}$ or 1.

So $\frac{4}{7} < \frac{5}{6}$.



• To compare improper fractions, you can sometimes divide the numerator by the denominator to get the whole number part of the mixed number. Then you can compare the whole numbers.

For example, to compare $\frac{11}{5}$ to $\frac{26}{8}$:

$11 \div 5 = 2$ R 1, so $\frac{11}{5}$ is a bit more than 2.

$26 \div 8 = 3$ R 2, so $\frac{26}{8}$ is a bit more than 3.

So $\frac{11}{5} < \frac{26}{8}$.

B. i) Which strategy would you use to compare $\frac{1}{3}$ and $\frac{3}{5}$ in **part A**?

ii) Which strategy would you use to compare $\frac{1}{3}$ and $\frac{1}{20}$?

Examples

Example 1 Using Different Strategies to Compare Fractions

Order these fractions from least to greatest.

$$\frac{3}{8} \qquad \frac{3}{5} \qquad \frac{4}{5} \qquad \frac{1}{20}$$

Solution

$$\frac{3}{5} < \frac{4}{5}$$

$$\frac{3}{8} < \frac{3}{5}$$

$$\frac{1}{20} < \frac{3}{8}$$

In order:

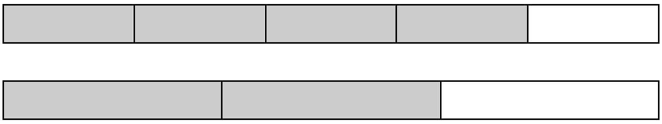
$$\frac{1}{20} < \frac{3}{8} < \frac{3}{5} < \frac{4}{5}$$

Thinking

- 3 fifths is less than 4 fifths.
- 3 parts of a whole divided into 8 parts is less than 3 parts of the same whole divided into 5 parts.
- $\frac{1}{20}$ is close to 0, since it's only 1 part out of 20, and $\frac{3}{8}$ is almost $\frac{4}{8}$ or $\frac{1}{2}$.

Example 2 Interpreting a Fraction Picture

Which fraction comparisons does this model show?



Solution

$$\frac{4}{5} > \frac{2}{3}$$

$$\frac{1}{3} > \frac{1}{5}$$

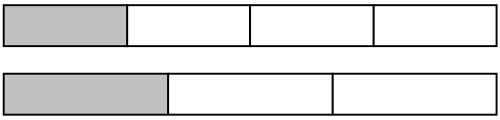
$$\frac{5}{5} = \frac{3}{3}$$

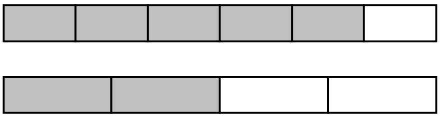
Thinking

- All the fractions are of the same whole so they can be compared.
- The top picture shows $\frac{4}{5}$ grey and $\frac{1}{5}$ white.
- The bottom picture shows $\frac{2}{3}$ grey and $\frac{1}{3}$ white.
- There is more grey in the top, but more white in the bottom.
- Both pictures show 1 can be $\frac{5}{5}$ or $\frac{3}{3}$.

Practising and Applying

1. Describe each strip using a fraction. Tell which fraction is greater in each pair of strips.

a) 

b) 

2. Draw a picture to show each.

a) $\frac{3}{4} > \frac{3}{6}$ **b)** $\frac{5}{6} > \frac{3}{6}$

3. Which fraction is greater?

a) $\frac{1}{2}$ or $\frac{1}{6}$ **b)** $\frac{4}{9}$ or $\frac{8}{9}$

c) $\frac{2}{30}$ or $\frac{25}{26}$ **d)** $\frac{16}{3}$ or $\frac{33}{10}$

4. Order from least to greatest.

$\frac{3}{5}$ $\frac{2}{8}$ $\frac{2}{5}$ $\frac{9}{4}$ $\frac{19}{3}$ $\frac{8}{9}$

5. a) Find all possible values for the missing number.

$$\frac{5}{\square} > \frac{5}{10}$$

b) Why would it be difficult to find all the missing values for $\frac{5}{\square} < \frac{5}{10}$?

6. List three fractions for each.

a) less than $\frac{2}{5}$ **b)** greater than $\frac{2}{5}$

7. Why might you use a different strategy to compare $\frac{3}{5}$ and $\frac{3}{9}$ than to compare $\frac{3}{5}$ and $\frac{6}{5}$?

GAME: So Many Equivalents

Play in groups of 2 to 4. You need a deck of cards. Use only the 1 to 9 cards (each worth the value on the card) and the Aces (Aces are worth 1).
Shuffle the cards and turn them over in a stack.

Players take turns. On your turn:

• Pick two cards and make a fraction.

• Write as many equivalent fractions as possible with a numerator and a denominator that are 20 or less.

• You get 1 point for each equivalent fraction.

Play 5 rounds. The player with the most points wins.

For example:

A player picks a 3 and a 5 and makes the fraction $\frac{3}{5}$.

The player then creates two equivalent fractions, $\frac{6}{10}$ and $\frac{9}{15}$, for 2 points.

3.1.5 EXPLORE: Adding and Subtracting Fractions

You can use fraction strips to add and subtract fractions.

1

$\frac{1}{2}$ | $\frac{1}{2}$

$\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$

$\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$

$\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$

$\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$

$\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$

$\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$

$\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$

• You can add two fractions by combining them.

For example, to add $\frac{1}{8} + \frac{5}{8}$:

$\longleftarrow \frac{1}{8} \longrightarrow \longleftarrow \frac{5}{8} \longrightarrow$

$\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$

1 eighth + 5 eighths is 6 eighths, so $\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$.

• Sometimes the sum is more than 1.

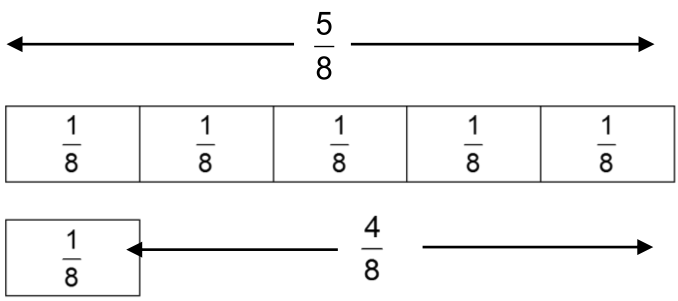
For example:

To add $\frac{5}{8} + \frac{5}{8}$, combine 5 eighths and 5 eighths to get 10 eighths, or $\frac{10}{8}$.

Since $\frac{8}{8}$ is a whole, $\frac{10}{8}$ is 1 whole and 2 eighths, or $1\frac{2}{8}$.

• You can subtract two fractions by seeing the difference between them —how much longer one is than the other.

For example, to subtract $\frac{5}{8} - \frac{1}{8}$:



5 eighths is 4 eighths longer than 1 eighth, so $\frac{5}{8} - \frac{1}{8} = \frac{4}{8}$.

A. i) Create two fractions using strips from the $\frac{1}{6}$ row.

ii) Add them. Write an equation to show what you did.

iii) Subtract them. Write an equation to show what you did.

B. Repeat **part A** using different strips from the $\frac{1}{6}$ row.

C. Repeat **parts A and B** using strips from the same row, but not the $\frac{1}{6}$ row.

D. i) What do you notice about the denominator of your sum and the denominator of your difference?

ii) How could you have predicted the numerator?

E. i) When you subtracted in **parts A and B**, did you ever get an answer greater than 1? Explain why that happens.

ii) When you added in **parts A and B**, when did you get an answer greater than 1?

F. Why is it easy to add and subtract fractions when the denominators are the same?

Chapter 2 Decimals

3.2.1 Decimal Thousandths

Try This

If Bhutan's population were represented by only 1000 people, there would be 50 people living in Gasa, 117 in Chhukha, and 155 in Thimphu.

Dzongkhag	Number of people
Gasa	50
Chhukha	117
Thimphu	155

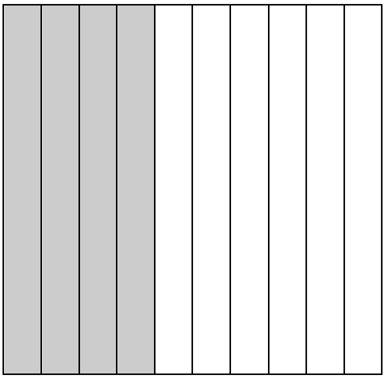
A. i) How many of the 1000 would not live in these three dzongkhags?

ii) What fraction is that out of 1000?

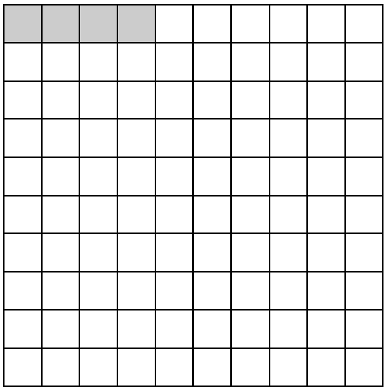
• You already have met decimal tenths and decimal hundredths.

For example:

0.4 means 4 out of 10, or $\frac{4}{10}$.



0.04 means 4 out of 100, or $\frac{4}{100}$.

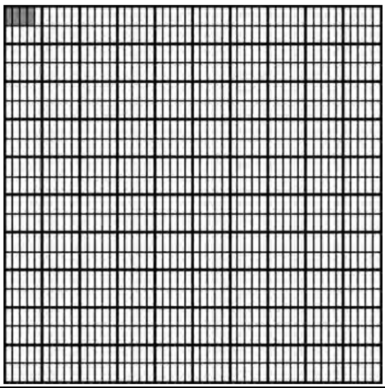


• Similarly, the decimal thousandth 0.004 means 4 out of 1000, or $\frac{4}{1000}$.

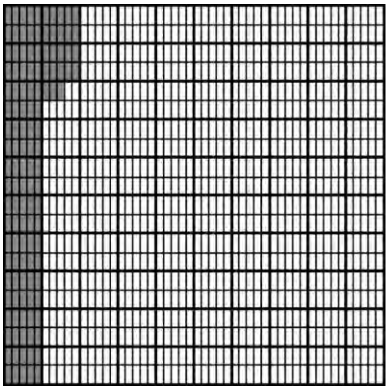
Here is model for $\frac{4}{1000}$ using a grid that is divided into 1000 parts.

4 of the 1000 parts have been shaded.

0.004 is read as "4 thousandths".



• This model for 0.123 shows 100 parts shaded in the first column and 23 parts shaded in the second column. You could model 0.123 by shading any 123 parts on the grid.



• One column of the grid is $\frac{1}{10}$ or 0.1 of the grid. It is also $\frac{100}{1000}$ or 0.100. This model shows why 0.123 is a bit more than $\frac{1}{10}$.

• Decimals thousandths are useful for working with measurements because some measurement units are $\frac{1}{1000}$ or 0.001 of other units.
For example, since 1 mm is 0.001 m, then 123 mm is 0.123 m.

B. i) Describe the population of each dzongkhag in the chart in **part A** as a decimal thousandth.
ii) What decimal describes your answer to **part A ii)**?

Examples

Example 1 Representing Measurements as Decimal Thousandths

Complete each. Show your work.
a) 312 mm = ____ m **b)** 510 m = ____ km **c)** 90 mL = ___ L

Solution

a) Since 1000 mm = 1 m, then
312 mm = $\frac{312}{1000}$ m = 0.312 mm.

b) Since 1000 m = 1 km, then
510 m = $\frac{510}{1000}$ km = 0.510 km.

c) Since 1000 mL = 1 L, then
90 mL = $\frac{90}{1000}$ L = 0.090 L.

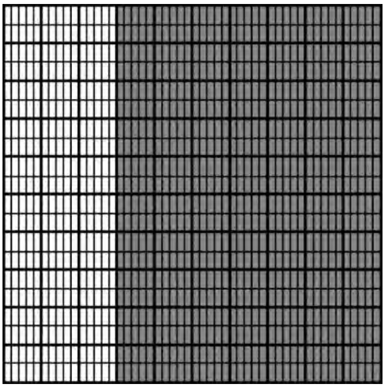
Thinking

• 1000 smaller units make 1 larger unit for each:
- millimetres and metres
- metres and kilometres
- millilitres and litres

• I wrote a fraction that compared each measurement to 1000 and then I wrote the decimal.

Example 2 Equivalent Decimals	
Write two decimals that represent the same amount as 0.700.	

Solution



0.700 = 0.70 = 0.7

Thinking

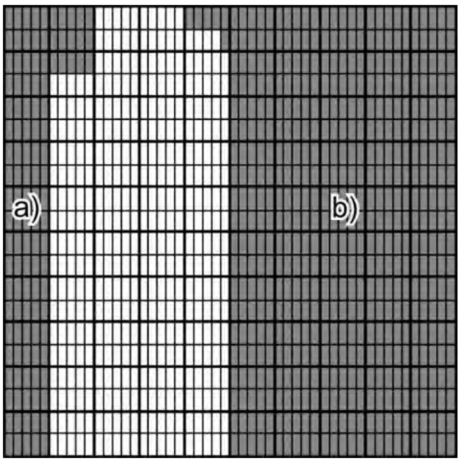
• I shaded 700 parts on the thousandths grid to model 0.700.

• I realized it was the same as shading 70 out of the 100 squares (0.70) or 7 out of the 10 columns (0.7).

Example 3 Relating, Fractions, and Decimals	
Which fraction, $\frac{1}{10}$, $\frac{1}{2}$, or –, is a good estimate for each decimal?	

a) 0.115 **b)** 0.506

Solution



a) $\frac{1}{10}$ is a good estimate for 0.115.

b) $\frac{1}{2}$ is a good estimate for 0.506.

Thinking

• I shaded each decimal on the grid. Even though I drew both decimals on the same grid, I always compared the decimal to the whole grid, not what was left.

• I looked to see if each amount was closer to $\frac{1}{10}$, $\frac{1}{2}$, or $\frac{1}{4}$ of the grid.

• 0.115 is just a little more than one column and one column is $\frac{1}{10}$.

• 0.506 is close to 5 columns, which is $\frac{1}{2}$ of the grid.

Practising and Applying

1. Write each as a decimal.

a) $\frac{142}{1000}$ **b)** $\frac{57}{1000}$ **c)** $\frac{2}{1000}$

2. Write each as a fraction.

a) 0.008 **b)** 0.034 **c)** 0.398

3. Use a different colour to model each on a single thousandths grid.

a) 0.012 **b)** 0.002 **c)** 0.4

4. Write each as a decimal of a kilometre.

a) 312 m **b)** 68 m **c)** 2 m

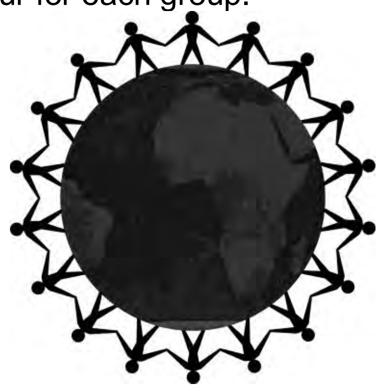
5. Write 0.300 using equivalent decimals.

6. The poem *If the World were a Village of 1000 People* says that, if the world only had 1000 people, there would be

• 584 Asians
• 124 Africans
• 95 Europeans
• 84 Latin Americans
• 55 Russians and former Soviet Republics
• 52 North Americans
• 6 Australians and New Zealanders

a) Write each as a decimal thousandth.

b) Colour a thousandths grid to model the world's population. Use a different colour for each group.



7. Which decimal thousandth in **question 6** could be estimated with each fraction?

a) $\frac{1}{2}$ **b)** $\frac{1}{10}$ **c)** $\frac{1}{20}$

8. Deki said the decimal for North Americans in **question 6** was 0.52. How do you know that does not make sense?

9. Write decimals for these facts about the global village of 1000 people:

• 28 babies are born each year
• 10 people die each year
• 60 people are over 65 years old
• 330 are children
• 70 people own an automobile
• 7 people are teachers
• 1 person is a doctor

10. Suppose the poem from **question 6** was *If the World were a Village of 100 People*.

a) What decimal would you use to describe the Asians?

b) How do you know?

11. How is the number 0.001 like the number 0.01? How is it different?

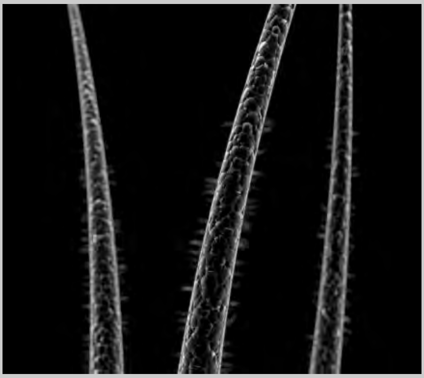
12. Why would someone say that you can be *more exact* by using a decimal thousandth than by using a decimal hundredth?

3.2.2 Decimal Place Value

Try This

10 human hairs, placed side by side, are about 0.07 cm wide.

A. How wide would 100 hairs be, placed side by side?



• The decimal 0.001 represents one thousandth.

Thousandths describe a **place value** on a place value chart.

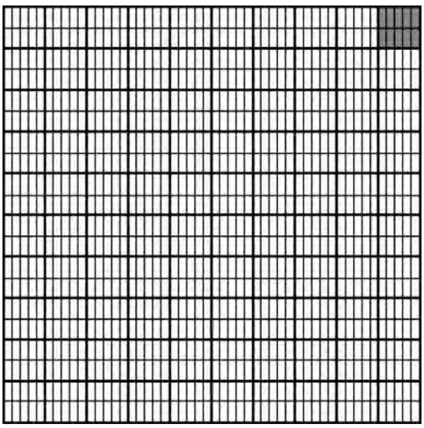
Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

The thousandths place is 3 places to the right of the ones place just like the thousands place is 3 places to the left of the ones place.

• It makes sense for thousandths to be to the right of hundredths.

In a place value chart, 10 of any value makes 1 of the value to the left, so 10 thousandths = 1 hundredth.

This is true since 10 small parts of the thousandths grid fill 1 of the 100 squares of the grid.



Ones	Tenths	Hundredths	Thousandths
10 tenths	10 hundredths	10 thousandths	

• The number 0.237 is read "two hundred thirty-seven thousandths". It can be renamed in different ways.

Ones	Tenths	Hundredths	Thousandths
0	2	3	7

0.237 is 2 tenths, 3 hundredths, 7 thousandths

Ones	Tenths	Hundredths	Thousandths
0		23	7

0.237 is 23 hundredths, 7 thousandths

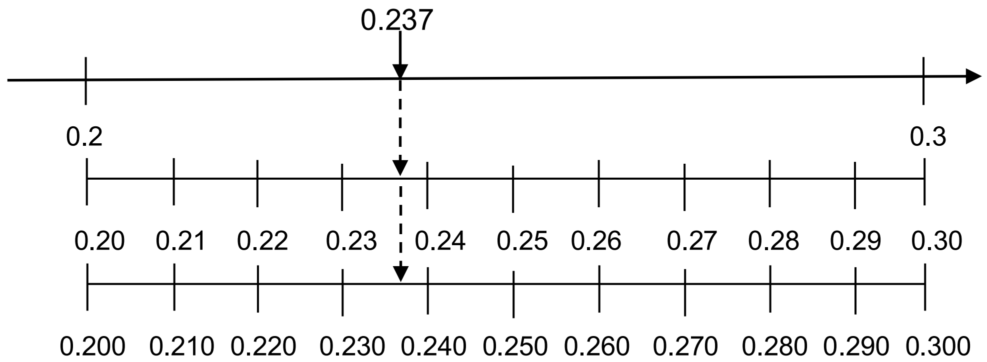
Ones	Tenths	Hundredths	Thousandths
0	2		37

0.237 is 2 tenths, 37 thousandths

• This place value chart shows how you can use place value to show that 0.237 is between 0.2 and 0.3 and between 0.23 and 0.24.

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
			0	2		
			0	2	3	
			0	**2**	**3**	**7**
			0	2	4	
			0	3		

These number lines also show that 0.237 is between 0.2 and 0.3 and between 0.23 and 0.24.



B. i) How does knowing where the thousandths place is on a place value chart help you answer **part A**?

ii) Using the information in **part A**, how wide is 1 hair?

Examples

Example 1 Renaming Decimals Using Place Value

Rename 479 thousandths in three different ways.

Solution

479 thousandths is

- 4 tenths, 7 hundredths, 9 thousandths

OR

- 47 hundredths, 9 thousandths

OR

- 4 tenths, 79 thousandths

Thinking

- 479 thousandths is 0.479:
  - 4 is in the tenths place,
  - 7 in the hundredths place, and
  - 9 in the thousandths place.
- I knew 4 tenths = 40 hundredths, so 4 tenths, 7 hundredths = 47 hundredths.
- I knew 7 hundredths = 70 thousandths, so 7 hundredths, 9 thousandths = 79 thousandths.

Example 2 Relating Decimals Using a Number Line

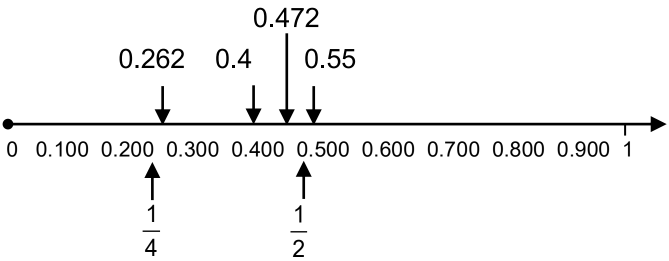
a) Place each decimal on a number line: 0.4, 0.55, 0.472, 0.262

b) Which decimal is closest to $\frac{1}{2}$? Which is closest to $-$?

Solution

a)

Ones	Tenths	Hundredths	Thousandths
0	4	0	0
0	5	5	0
0	4	7	2
0	2	6	2



b) 0.55 and 0.472 are both close to $\frac{1}{2}$, but 0.472 looks closer to $\frac{1}{2}$.

0.262 is closest to $-$.

Thinking

a) I wrote all four decimals in a place value chart and put zeros in the blank spaces to the left. This helped me place the decimals on the number line.

b) I know 0.5 is $\frac{5}{10} = \frac{1}{2}$ and 0.25 is $\frac{25}{100} = -$.

Practising and Applying

1. a) Write a decimal in a place value chart to match each description.

i) a decimal with 1 in the thousandths place and 0 everywhere else

ii) a decimal with 4 in the hundredths place, 1 in the tens place, 3 in the tenths place, and 0 everywhere else

iii) a decimal with three decimal digits that is slightly more than $\frac{1}{2}$

iv) a decimal between 0.236 and 0.34

b) How would each decimal in **part a)** be read?

2. Place each number on the same number line.

a) 0.472 **b)** 0.7

c) 0.528 **d)** 0.695

3. Complete each with a decimal thousandth.

a) 0.[ ][ ][ ] km is almost 1 km

b) 0.[ ][ ][ ] km is about $\frac{3}{4}$ km

c) 0.[ ][ ][ ] km is about $\frac{9}{10}$ km

4. Describe each in terms of hundredths and thousandths.

For example, 0.271 is 27 hundredths and 1 thousandth.

a) 0.351 **b)** 0.892

c) 0.2 **d)** 0.025

5. Write a decimal to match each description.

a) 1 thousandth more than 2.478

b) 1 thousandth less than 3.12

c) 11 thousandths more than 2.099

6. This chart shows the part of the earth covered by three oceans.

Ocean	Part of the earth
Pacific	352 thousandths
Indian	144 thousandths
Atlantic	174 thousandths

a) Write a fraction for each ocean.

b) Write a decimal for each ocean.

c) Which ocean covers about $\frac{1}{3}$ of the earth?

7. Explain how you know that 0.455 is between 0.45 and 0.46.

8. A thousandths grid is completely coloured in three sections:

• One section is 124 thousandths of the grid.

• One section is twice as large as another section.

What three decimals could describe the sections?

9. a) Place the three digits in the blanks to create three different numbers.

2, 3, and 4 → 0.[ ][ ][ ]

b) Which number is closest to 0.3?

10. Why does it make sense that the thousandths place is the next place to the right of the hundredths place?

3.2.3 Comparing and Ordering Decimals

Try This

A. Use the digits 3, 5, and 7 in the blanks to make this true.
Find one or more possible answers.

□.□□ > 5.64

• You can compare decimal thousandths just like you compare tenths and hundredths — you compare digits with the same place value.

For example, to compare 12.37 and 13.105, you compare the whole number part of the decimal.

Tens	Ones	Tenths	Hundredths	Thousandths
1	**2**	3	7	0
1	**3**	1	0	5

13.105 > **12**.37 since 13 > 12.

To compare 13.105 and 13.106, you compare the digits in the thousandths place since both have 13 ones, 1 tenth, and 0 hundredths.

Tens	Ones	Tenths	Hundredths	Thousandths
1	3	1	0	**5**
1	3	1	0	**6**

13.10**6** > 13.10**5**, since 6 thousandths > 5 thousandths

To compare 13.112 and 13.106, you compare the digits in the hundredths place since both have 13 ones and 1 tenth.

Tens	Ones	Tenths	Hundredths	Thousandths
1	3	1	**0**	6
1	3	1	**1**	2

13.1**1**2 > 13.1**0**6, since 1 hundredth > 0 hundredths

• You can use the number of digits in whole numbers to compare them because the number of digits is a clue to the size of a whole number.

For example:

1234 > 123 because a 4-digit whole number is always greater than a 3-digit whole number.

This is not the case for the decimal part of a decimal.

For example, 0.199 > 0.04 but 0.199 < 0.2.

B. i) Put the digits 0, 3, 5, and 7 in the blanks to make this true.
Find one or more possible answers.

□.□□□ > 5.642

ii) How is this the same as what you did in **part A**? How is it different?

Examples

Example 1 Ordering Decimals by Estimating

Order from least to greatest.

0.023 0.3 0.213 0.203

Solution 1

Ones	Tenths	Hundredths	Thousandths
0	0	**2**	3
0	3		
0	2	**1**	3
0	2	**0**	3

3 tenths > 2 tenths or 0 tenths, so 0.3 is greatest.

2 tenths > 0 tenths so 0.023 is least.

0.213 > 0.203 since 1 hundredth > 0 hundredths.

The numbers, from least to greatest, are
0.023, 0.203, 0.213, 0.3.

Thinking

- I used a place value chart to compare.
- I know that when a value in a column farther to the left is greater, the number is greater.

Solution 2

0.023 is about 0.02, or 2 hundredths
0.3 = 0.30, or 30 hundredths
0.213 is about 0.21, or 21 hundredths
0.203 is about 0.20, or 20 hundredths

Since 2 < 20 < 21 < 30, then
0.023 < 0.203 < 0.213 < 0.3

Thinking

- Since they all had a different number of hundredths, I didn't need to compare the thousandths. I estimated them as hundredths.

Example 2 Solving a Problem by Ordering Decimals

Deki has a choice of three bags of apples. The bags all cost the same and are of equal quality. Bag A has 1.2 kg of apples, Bag B has 990 g, and Bag C has 1.147 kg. Which bag should she buy?

Solution

Bag A: 1.2 kg

Bag B: 0.99 kg

Bag C: 1.147 kg

Thinking

• I know that 990 g is 0.990 kg, since there are 1000 g in a kilogram.

Ones	Tenths	Hundredths	Thousandths
1	2		
0	9	9	
1	1	4	7

0.99 is least since there are 0 ones.

1.**1**47 < 1.**2** since there are fewer tenths.

0.99 < 1.147 < 1.2

Deki should buy the 1.2 kg bag because it has more to eat.

• I used a place value chart to compare the decimals:

- 0.99 is less than 1
- both 1.2 and 1.147 are greater than 1
- 1.147 < 1.2 because 1 tenth < 2 tenths

Practising and Applying

1. a) Order from least to greatest.

1.024 0.305 1.204 0.035

b) Explain your thinking.

2. When people run marathons, the time is sometimes reported as a decimal thousandth of a minute.

For example, 3:21.175 means 3 h, 21.175 min.

Order these times from least to greatest.

3:14.5 3:12.987

3:14.175 4:1.122

3. Draw a number line and place these three numbers on it.

1.214 1.412 1.124

4. Which distance is greater?

a) 2.108 km or 1.314 km

b) 2.108 m or 1.314 km

c) 2.108 m or 1.314 m

5. Explain why 2.■■■ > 1.9■■, no matter what digits go in the blanks.

6. Copy and complete.

3.45 < ■.■■■ < 3.461

Find as many answers as possible.

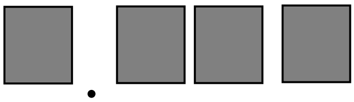
7. You are asked to explain to some classmates who missed school how to compare decimals thousandths. What would you say?

GAME: In the Middle

Play in groups of three.

You need a set of 36 digit cards (four of each of the digits from 1 to 9).

- The dealer shuffles the cards and gives each player four cards.
- Each player arranges his or her cards into a number in this form:



- The dealer puts the numbers in order. The player whose number is in the middle wins 1 point.

Play 10 rounds. Take turns being the dealer.

The player with the most points at the end wins.

For example:

Player A gets the digits 3, 5, 1, 2 and creates **3.521**.
Player B gets the digits 2, 5, 9, 1 and creates **5.129**.
Player C gets the digits 4, 3, 9, 1 and creates **4.913**.

$$3.521 < 4.913 < 5.129.$$

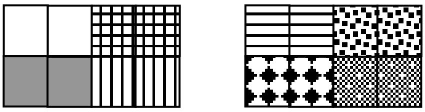
Player C is in the middle and wins 1 point.

UNIT 3 Revision

1. Draw pictures to show two different meanings for $\frac{5}{6}$.

2. Draw a picture to show why $\frac{3}{5} = 3 \div 5$.

3. Write the division sentence that this model represents.



$$\square \div \square = \frac{\square}{\square}$$

4. If 8 girls share 3 cakes, how much cake will each get?

5. a) Write $\frac{13}{5}$ as a mixed number.

b) Draw a picture to explain how you can change $\frac{13}{5}$ to a mixed number by dividing 13 by 5.

6. Draw a picture to show that $\frac{2}{8} = \frac{6}{24}$.

7. Write a fraction equivalent to each. Use the number 16 in either the numerator or the denominator.

a) $\frac{2}{8}$ **b)** $\frac{8}{32}$

8. Each year in Bhutan, there are about 20 births for every 1000 people. Write a fraction to describe that amount. Use a numerator less than 10.

9. Which fraction is greater?

a) $\frac{3}{7}$ or $\frac{3}{9}$ **b)** $\frac{2}{9}$ or $\frac{7}{9}$

c) $\frac{8}{15}$ or $\frac{10}{11}$ **d)** $\frac{33}{4}$ or $\frac{18}{5}$

10. List three fractions for each.

a) less than $\frac{6}{10}$

b) greater than $\frac{6}{10}$

11. Calculate.

a) $\frac{3}{10} + \frac{7}{10}$ **b)** $\frac{7}{10} - \frac{3}{10}$

12. Two fractions have a sum of $\frac{5}{6}$ and a difference of $\frac{1}{6}$. What are the fractions?

13. Use one thousandths grid to model these three decimals. Use a different colour for each decimal.

A. 0.156 **B.** 0.413 **C.** 0.008

14. Write each as a part of a litre.

a) 80 mL **b)** 3 mL

15. Place all four decimals on the same number line.

a) 1.023

b) 1.315

c) 0.98

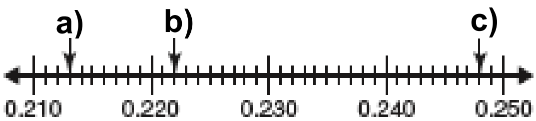
d) 1.675

16. Write a decimal to match each description.

a) a decimal that is 1 thousandth greater than 0.099

b) a decimal that is 3 thousandths less than 1.002

17. Write a decimal thousandth for each position on the number line.



18. Copy and complete.

■.■■ < 0.512 < ■.■■

19. a) Order from least to greatest.

2.004 3.09 3.1 2.040

b) Explain your thinking.

20. Copy and complete.

1.4 < ■.■■■ < 1.68

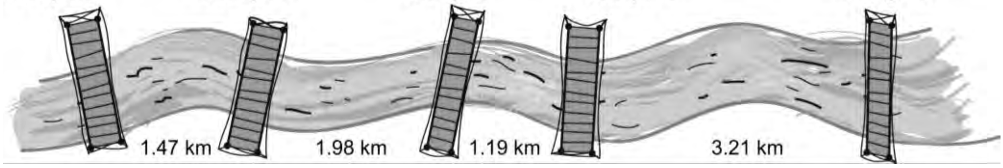
Find five different answers.

UNIT 4 DECIMAL COMPUTATION

Getting Started

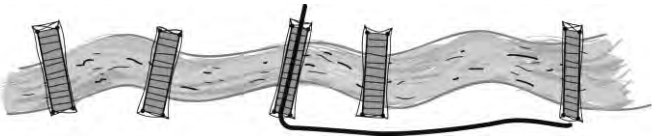
Use What You Know

Five bridges have been built across a river. The distances between the bridges are shown. Each bridge is 0.3 km across.



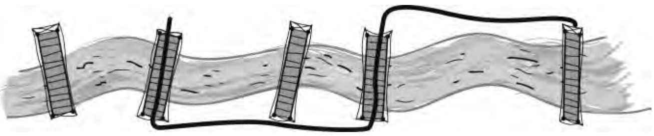
A. i) The picture below shows Sonam's path on the map above. Estimate how far she walked. Then calculate how far she walked.

ii) This picture shows Dechen's path. How far did Dechen walk?

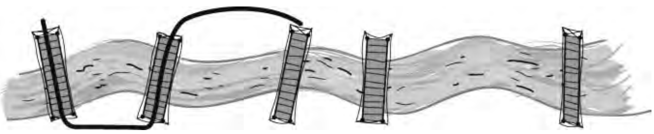


iii) Who walked farther, Sonam or Dechen? How much farther?

B. The distances on the north side of the river are about the same as the distances on the south side. How much longer is Path 1 than Path 2?



Path 1

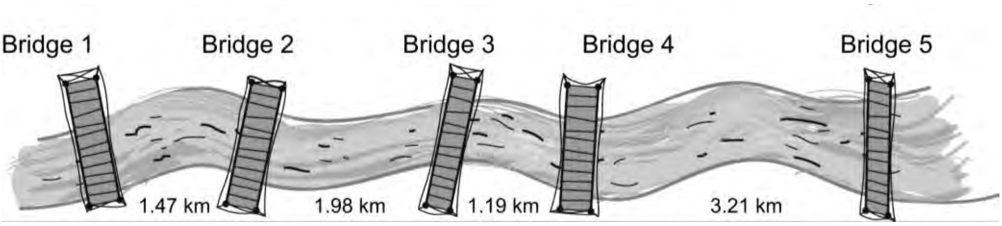


Path 2

[Continued]

C. Sketch the map below. Draw a path of each length on the map.

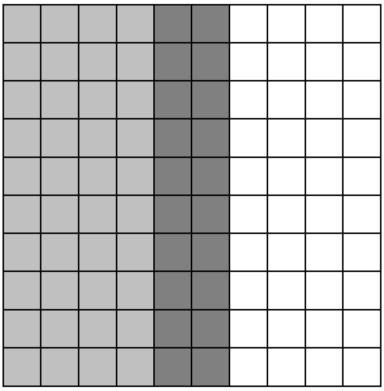
i) 4.64 km **ii)** 7.85 km **iii)** 8.15 km **iv)** 8.45 km **v)** 16.3 km



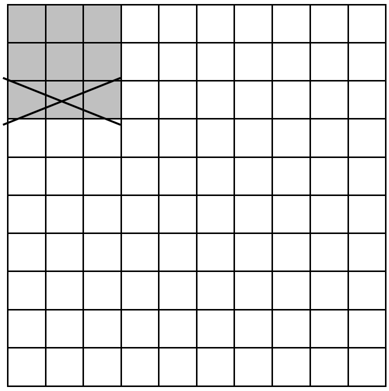
Skills You Will Need

1. You can use a hundredths grid to add and subtract decimals.

This grid shows 0.4 + 0.2 = 0.6.



This grid shows 0.09 – 0.03 = 0.06.



Use a hundredths grid to calculate each. Sketch your work.

a) 0.34 + 0.25 **b)** 0.34 – 0.25

c) 0.56 + 0.3 **d)** 0.56 – 0.3

2. Which is greater in each pair?

a) 1.25 + 3.4 or 1.35 + 2.4 **b)** 3.09 – 1.7 or 5.25 – 2.9

3. Add or subtract.

a) $\begin{array}{r} 3.4 \\ +\ \underline{1.8} \end{array}$ **b)** $\begin{array}{r} 1.79 \\ +\ \underline{8.34} \end{array}$ **c)** $\begin{array}{r} 4.12 \\ -\ \underline{1.75} \end{array}$ **d)** $\begin{array}{r} 3.05 \\ -\ \underline{1.47} \end{array}$

4. Multiply.

a) 12 × 10 **b)** 137 × 10 **c)** 25 × 100 **d)** 346 × 100

5. Multiply.

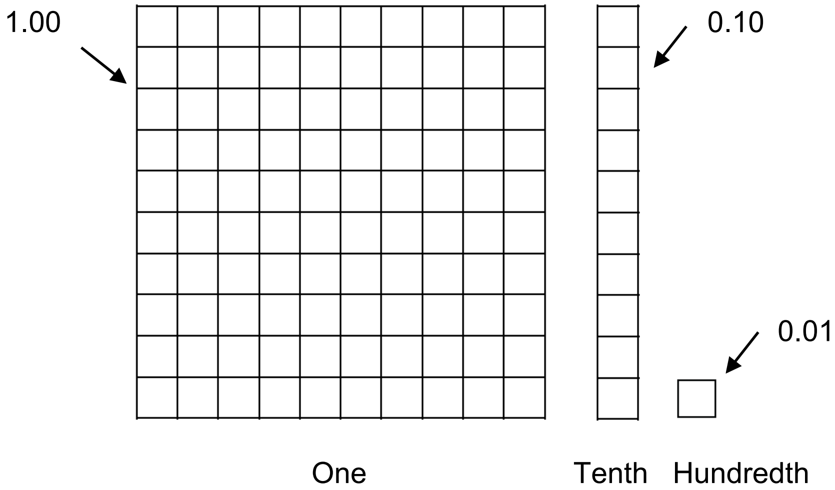
a) 5 × 8 **b)** 6 × 7 **c)** 4 × 9 **d)** 7 × 8

e) 8 × 12 **f)** 7 × 25 **g)** 9 × 28 **h)** 4 × 67

Chapter 1 Adding and Subtracting Decimals

4.1.1 EXPLORE: Adding and Subtracting Decimals

One way to add and subtract decimals is by using models.
Each model below represents a different value.



A. Use models to show that each is true.

i) 1.23 + 0.56 = 1.79 **ii)** 1.99 + 2.47 = 4.46

iii) 2.45 – 1.3 = 1.15 **iv)** 3 – 2.73 = 0.27

v) 3.84 + 2.7 = 4.04 + 2.5 **vi)** 2.03 – 1.45 = 1.99 – 1.41

B. How are adding and subtracting decimals like adding and subtracting whole numbers?

C. Look back at the equations in **part A v) and vi)**.

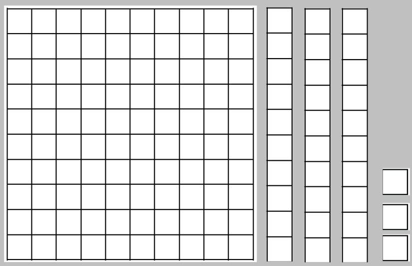
i) You know that 3.84 + 2.7 = 4.04 + 2.5. Why might you prefer to use 4.04 + 2.5 instead of 3.84 + 2.7 to find the sum?

ii) You know that 2.03 – 1.45 = 1.99 – 1.41. Why might you prefer to use 1.99 – 1.41 instead of 2.03 – 1.45 to find the difference?

[Continued]

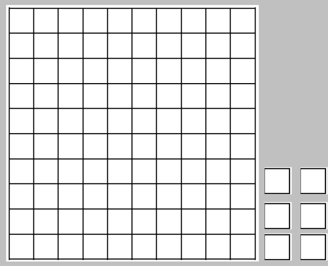
D. i) Create two decimals, each represented by 7 models.
Their sum must be represented by 14 models.

For example: 1.33 + 1.06 = 2.39



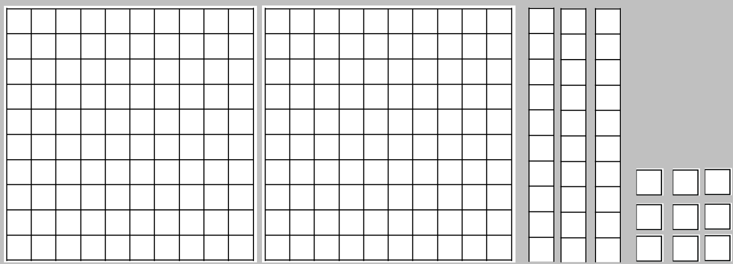
1.33 uses 7 models

+



1.06 uses 7 models

=



2.39 uses 14 models

ii) Create two decimals, each represented by 7 models, that have a sum that is represented by 5 models.

iii) Explain why you can add 7 models + 7 models but have only 5 models in the sum.

E. i) Create two decimals, each represented by 7 models, that have a difference that is represented by each number of models:

• 0 models • 9 models • 18 models

ii) Explain why you can subtract 7 models from 7 models but end up with 9 or 18 models.

F. i) Create a decimal addition and subtraction that would not need regrouping to calculate.

■ . ■ ■ + ■ . ■ ■ ■ . ■ ■ – ■ . ■ ■

ii) Create a decimal addition and subtraction that would need regrouping to calculate.

■ . ■ ■ + ■ . ■ ■ ■ . ■ ■ – ■ . ■ ■

4.1.2 Adding Decimal Thousandths

Try This

Each morning, Dorji walks to Rinzin's house and then they walk to school. Together, they walk 4.26 km. Dorji walks twice as far as Rinzin walks.

A. About how far does each boy walk? Explain how you estimated.

• Adding decimal thousandths is just like adding decimal tenths or hundredths. You combine amounts that represent the same place value.

For example, to add 1.452 + 1.234:

Tens	Ones	Tenths	Hundredths	Thousandths
	1	4	5	2
	1	2	3	4
	2	**6**	**8**	**6**

The addition can be written vertically or horizontally:

$$\begin{array}{r} 1.452 \\ +\ \underline{1.234} \\ 2.686 \end{array}$$

1.452 + 1.234 = 2.686

• You have to regroup if there are 10 or more in a place value column.

For example, to add 0.809 + 0.452:

Tens	Ones	Tenths	Hundredths	Thousandths
		8	0	9
		4	5	2
		12	5	11
	1	**2**	**6**	**1**

When you need to regroup, you might find it easier to start with the thousandths and move to the left.

When you add this way, you usually write the addition vertically to align the digits of the same place value.

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ } \\ 0.809 \\ +\ \underline{0.452} \\ 1.261 \end{array}$$

• Decimals with a different number of decimal digits can still be added.

For example, to add 3.8 + 1.467:

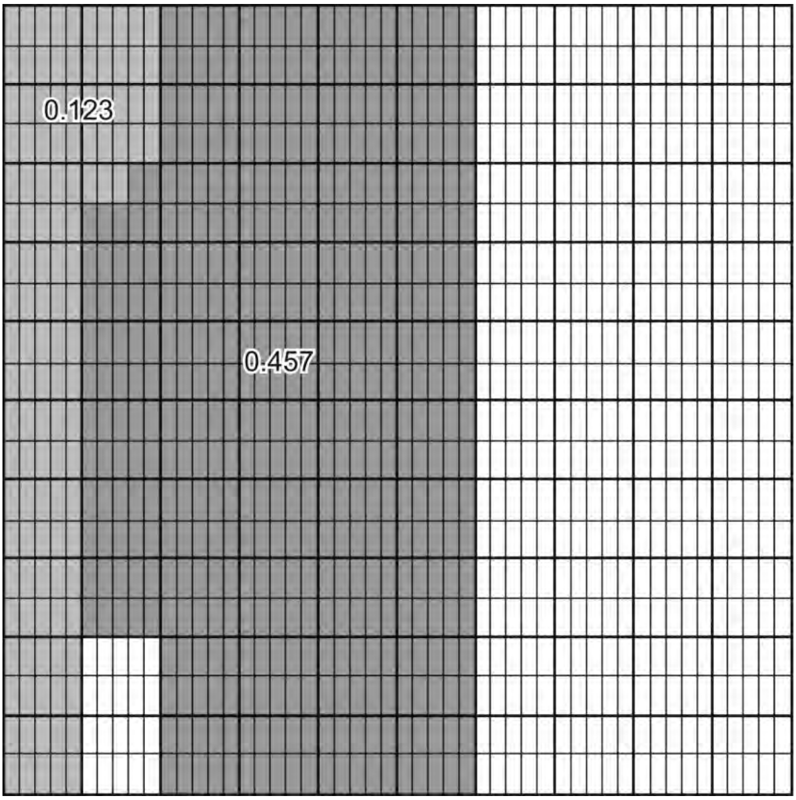
Tens	Ones	Tenths	Hundredths	Thousandths
	3	8		
	1	4	6	7
	4	12	6	7
	5	**2**	**6**	**7**

When you add without using a place value chart, align the digits so that you add digits with the same place value.

$$\begin{array}{r} {}^{1}\\ 3.8 \\ +\ 1.467 \\ \hline 5.267 \end{array}$$

• You can use a thousandths grid to help you add when the values are small and the sum is less than 1. Each column in the grid is 1 tenth, each square is 1 hundredth, and each small rectangle is 1 thousandth.

For example, to add 0.123 + 0.457:



0.123 + 0.457 covers

- 5 full columns, or 5 tenths
- 8 full squares, or 8 hundredths
- 0 small rectangles, or 0 thousandths

5 tenths, 8 hundredths, 0 thousandths = 0.580

So 0.123 + 0.457 = 0.580, or 0.58.

• You can use mental math to do some decimal additions.

For example, to add 1.035 + 3.005:

If you rename 3.005 as 3 and rename 1.035 as 1.040 by moving 0.005 from one number to the other, the numbers are easier to add mentally.

0.005 ⟵ Moving the 0.005 does not change the sum.

1.03**5** + 3.00**5** = 1.040 + 3 = 4.040

B. Suppose the total distance walked in **part A** was 4.263 km. How far did each boy actually walk? Explain how you found the answer.

Examples

Example 1 Estimating Decimal Sums

Which sum is greater? How do you know?

2.678 + 9.129 or 5.982 + 6.187

Solution	**Thinking**
2.678 + 9.129 is about 2.7 + 9.1 = 11.8 5.982 + 6.186 is about 6.0 + 6.2 = 12.2 5.982 + 6.187 is greater.	 • I estimated to decide which would be greater. • I didn't estimate with whole numbers because I thought the sums would be too close. • I estimated using decimal tenths.

Example 2 Adding Decimal Thousandths

Add 13.799 + 4.285.

Solution 1	**Thinking**
1 1 1 13.799 + <u>4.285</u> 18.084	 • I lined up the digits so that I added thousandths to thousandths, hundredths to hundreds, tenths to tenths, and ones to ones. • I started adding digits at the right and regrouped when I had to. • I knew the sum would be about 14 + 4 = 18 and it was.

Example 2 Adding Decimal Thousandths [Continued]

Solution 2

13.799 + 4.285
= 13.8 + 4.284

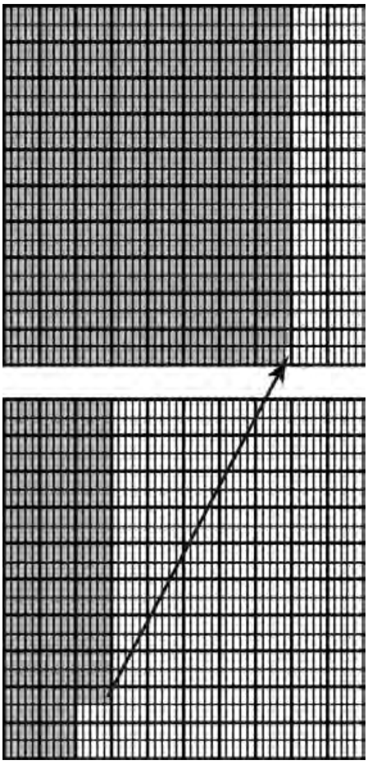
$$\begin{array}{r} {}^{1} \\ 13.8 \\ +\ \underline{\ \ 4.284} \\ 18.084 \end{array}$$

13.799 + 4.285
= 18.084

Thinking

• I renamed both numbers, without changing their sum, to make them easier to add.

• I pictured a thousandths grid to do it:



- The decimal part of 13.799 (0.799) was 7 columns, 9 squares, and 9 small rectangles.

- The decimal part of 4.285 (0.285) was 2 columns, 8 squares, and 5 small rectangles.

- I knew if I moved 1 small rectangle from 0.285 to 0.799, 0.799 would be 8 columns (0.8) and 0.285 would be 2 columns, 8 squares, and 4 small rectangles (0.284).

0.799 + 0.285
= 0.8 + 0.284

So, 13.799 + 4.285 = 13.8 + 3.284.

Example 3 Using Mental Math to Add Decimal Thousandths

One package of meat has a mass of 1.561 kg. Another package has a mass that is 0.425 kg greater. What is the mass of the heavier package?

Solution 1

1.561 + 0.425 = ?

0.425 is
4 tenths, 2 hundredths, 5 thousandths

1.561 + 4 tenths = 1.961

1.961 + 2 hundredths = 1.981

1.981 + 5 thousandths = 1.986

The heavier package is 1.986 kg.

Thinking

• The mass of the second package was 0.425 kg more than the mass of the first package, 1.561 kg.
I knew I had to add to find the mass of the second package.

• I added in parts mentally.

Practising and Applying

1. Use a thousandths grid to model and solve each addition.

a) 0.155 + 0.845 = ☐

b) 0.367 + 0.248 = ☐

2. Solve.

a) 4.35 + 0.236 = ☐

b) 0.246 + 0.198 = ☐

c) ☐ = 12.187 + 15.906

d) ☐ = 8.009 + 17.39

3. Add each using mental math. Explain how you added.

a) 4.237 + 0.010

b) 4.237 + 0.001

c) 4.237 + 0.111

4. Describe how you would add each. Then add.

a) 3.099 + 15.237 + 2.001

b) 15.127 + 4.299

5. What are the missing digits?

a)
$$\begin{array}{r} 3.5[]7 \\ +\ \underline{[].26[]} \\ 7.794 \end{array}$$

b)
$$\begin{array}{r} 8.09[] \\ +\ \underline{[].[][]8} \\ []1.567 \end{array}$$

6. The liquid in four identical large containers was measured.

3.245 L   1.262 L

1.283 L   2.157 L

a) If the contents of the two fullest containers were put together, how much liquid would there be?

b) Which two containers together have 4.507 L of liquid?

7. Chabilal measured his reaction time in thousandths of a second. Here are his results.

Using his right hand

0.420	0.360	0.360	0.301	0.359

Using his left hand

0.360	0.375	0.422	0.344	0.429

a) Find the total time for each hand.

b) Which hand reacts faster?

8. Dorji drew a triangle with side measurements of 0.032 m, 0.078 m, and 0.082 m.

a) What is the perimeter in metres?

b) What is the perimeter in millimetres? (Hint: 1000 mm = 1 m)

9. Dawa estimated that 3.904 + 15.008 is about 4 + 15 = 19. Why did he ignore the digits in the thousandths place?

10. Gembo added 4.235 + 9.6 by renaming 9.6 as 9.600.

$$\begin{array}{r} 4.235 \\ \underline{+\ 9.6} \end{array} \quad \Rightarrow \quad \begin{array}{r} 4.235 \\ +\ \underline{9.600} \\ 13.835 \end{array}$$

Was it necessary to rename 9.6 as 9.600? Explain your thinking.

4.1.3 Subtracting Decimal Thousandths

Try This

One way to subtract two numbers is to add the same amount to both.
For example: 5 – 3 = (5 + 1) – (3 + 1) = 6 – 4
Ugyen used this method to subtract decimals:
3.4 – 1.9 = (3.4 + 0.1) – (1.9 + 0.1)

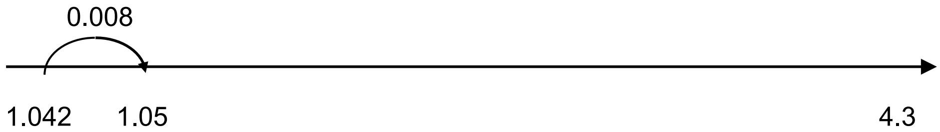
A. i) Finish Ugyen's calculation. Why does this method work?
ii) Why might this method be useful for subtracting 3.2 – 1.99?

Subtracting decimal thousandths is like subtracting tenths or hundredths.

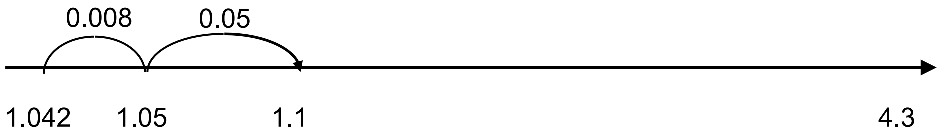
• One way to subtract is to add up using a number line model.
You use several convenient numbers along the way to get from the lower value to the higher value.

For example, to subtract 4.3 – 1.042, think 1.042 + **?** = 4.3:

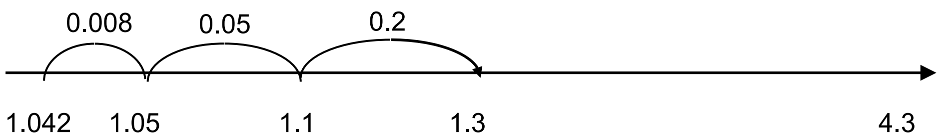
Start at 1.042 and add **0.008** to get to 1.05.



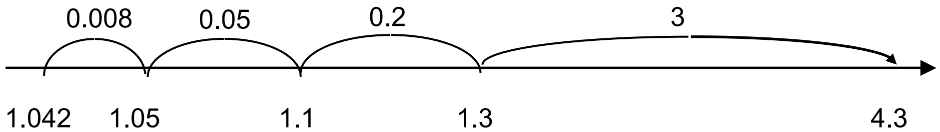
Add **0.05** to get to 1.1.



Add **0.2** to get to 1.3.



Add **3** to get to 4.3.



Altogether, **3** + **0.2** + **0.05** + **0.008** = **3.258** was added.
1.042 + **3.258** = 4.3, so 4.3 – 1.042 = **3.258**.

• Another method of subtracting decimals is to line up the digits in each place value and subtract, regrouping when necessary.

For example, to subtract 4.3 – 1.042:

Model the subtraction:

	Ones	Tenths	Hundredths	Thousandths
	4	**3**		
–	**1**	**0**	**4**	**2**

Regroup as necessary, from left to right:

	Ones	Tenths	Hundredths	Thousandths
	4	~~3~~ **2**	~~10~~ **9**	**10**
–	1	0	4	2

Subtract from right to left:

	Ones	Tenths	Hundredths	Thousandths
	4	~~3~~ 2	~~10~~ 9	10
–	1	0	4	2
	3	**2**	**5**	**8**

When we subtract this way, we usually write the subtraction vertically in order to line up the digits that are in the same place value column:

$$\begin{array}{r} {\scriptstyle 2\ 9\ 10} \\ 4.\cancel{300} \\ -\ \underline{1.042} \\ 3.258 \end{array}$$

Another way of regrouping 4.3 is to think of it as

4 ones + 30 hundredths + 0 thousandths. Then regroup it as

4 ones + 29 hundredths + 10 thousandths.

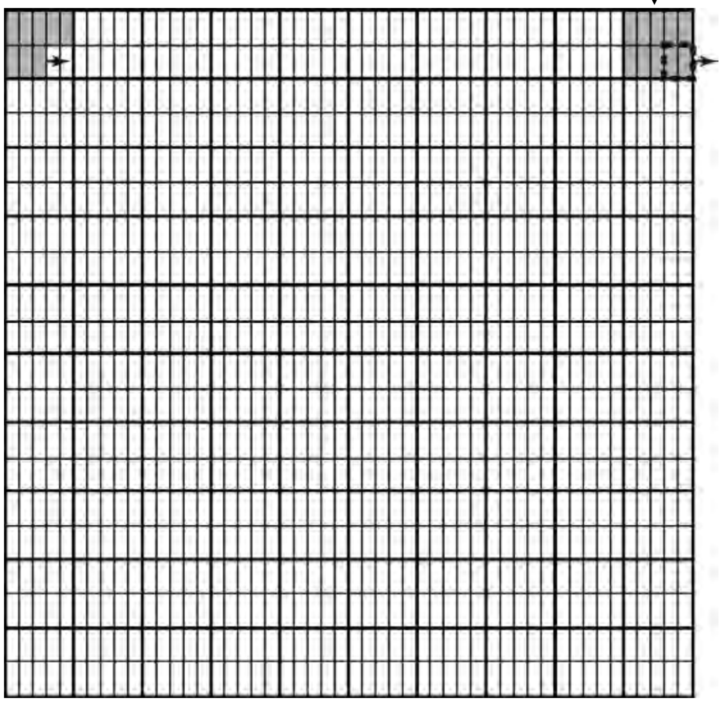
$$\begin{array}{r} {\scriptstyle 29} \\ 4.\cancel{30}0 \\ -\ \underline{1.042} \end{array} \quad \Rightarrow \quad \begin{array}{r} {\scriptstyle 29\ 10} \\ 4.\cancel{300} \\ -\ \underline{1.042} \\ 3.258 \end{array}$$

• You can also use a thousandths grid to subtract by adding up or by taking away.

For example, to subtract 0.01 – 0.008:

You could shade 8 thousandths and then add up to 1 hundredth:
0.008 + 0.002 = 0.010
So 0.01 – 0.008 = 0.002

Or, you could shade 1 hundredth and then take away 8 thousandths:
0.01 – 0.008 = 0.002



• You can sometimes use mental math strategies to subtract decimals.

For example, to subtract 4.823 – 1.21:

Since 1.21 = 1 + 0.2 + 0.01, subtract each amount separately.

4.823 – 1.21 → 4.823 – 1 = 3.823
3.823 – 0.2 = 3.623
3.623 – 0.01 = 3.613

For example, to subtract 4.1 – 2.99:

You might first subtract 3, which is a little too much, but easy to subtract. Then you can add back the 0.01 extra you subtracted.

4.1 – 2.99 → 4.1 – 3 = 1.1
1.1 + 0.01 = 1.11

B. Create and solve two subtractions involving decimal thousandths that you can calculate using the method from **part A**.

Examples

Example 1 Using Mental Math to Subtract Decimals

How can you solve 3 – 1.998 = ☐ using mental math?

Solution

3 – 1.998 = ☐ → 1.998 + ☐ = 3

0.998 + 0.002 = 1, so

1.998 + **0.002** = 2

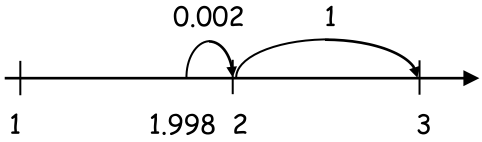
2 + **1** = 3, so

1.998 + 1.002 = 3

3 – 1.998 = 1.002

Thinking

- I knew 998 thousandths + 2 thousandths was 1000 thousandths.
- I visualized a number line to help me:



Example 2 Adding and Subtracting to Solve a Problem

Tandin started running at school. He ran 1852 m north, then another 2478 m north, and then 1482 m south. How far is he from the school, in kilometres?

Solution

Change metres to kilometres

1852 m = 1.852 km

2478 m = 2.478 km

1482 m = 1.482 km

Add to find how far he went north

```
 1 11
  1.852
+ 2.478
  4.330
```

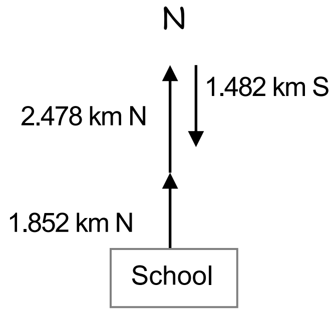
Subtract to find where he ended up

```
  3 12 12 10
  4.330
– 1.482
  2.848
```

Tandin is 2.848 km north of school.

Thinking

- First I wrote the metres as kilometres.
- I drew a picture to understand the problem. I needed to figure out whether to add or subtract.



- I saw that I had to add the two north distances and then subtract the south distance.

Practising and Applying

1. Use a thousandths grid to model and solve each.

a) 0.4 – 0.025 = ☐

b) 0.325 – 0.178 = ☐

2. Calculate.

a) 5.213 – 2.102 **b)** 3.162 – 1.894

c) 3.6 – 1.487 **d)** 12.05 – 1.006

3. Buthri subtracted 4.306 – 2.097 by writing 4.309 – 2.1.

a) Explain why she did that.

b) Write two other subtraction questions that you might calculate by increasing both numbers by the same amount.

4. Subtract each using mental math. Explain how you subtracted.

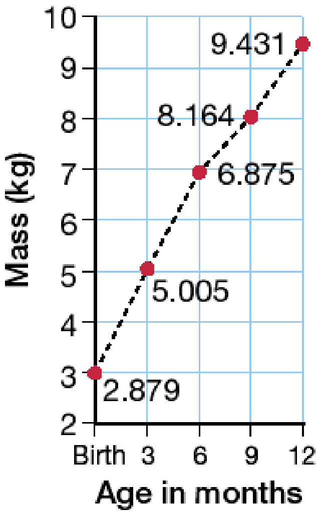
a) 3.545 – 0.001

b) 7.825 – 0.01

c) 6.004 – 0.1

d) 3 – 0.001

5. This graph shows how a baby gained weight in its first year. In which 3-month period did the baby gain the most weight? How do you know?



6. In the 2004 Olympics, the woman who won the gold medal in cycling finished her first race in 12.126 seconds and her second race in 12.14 s. How much faster was she in the first race?

7. Tshering's height was 1.555 m when he was 12 years old and 1.7 m when he was 13 years old.

a) How much did he grow?

b) Explain how you could use mental math to calculate **part a)**.

8. In each statement below, ☐ is a decimal thousandth between 0 and 1. Tell whether each statement is sometimes true, always true, or never true.

a) 3.4 – ☐ > 2.4 – ☐

b) 2.6 + ☐ < 2.6 – ☐

c) 1.284 – ☐ < 3.2 + ☐

9. Two numbers have a sum of 8.464.

a) What are the two numbers if their difference is less than 0.1?

b) What are the two numbers if their difference is between 2 and 3?

10. How would you explain to someone how to calculate 4.3 – 1.895?

GAME: Big Sum, Little Difference

Play in groups of 2 to 4. You need a set of 40 digit cards (four each of the digits 0 to 9) or a deck of cards (use only the number cards and Aces, Aces are 1 and tens are 0).

Shuffle the cards and turn them over in a stack.

How to play:

• Each player gets 10 cards.

• Each player arranges his or her 10 cards into two numbers of this form:

□□.□□□   □□.□□□

• Each player adds and then subtracts the two numbers.

• The player with the greatest sum gets 1 point.

• The player with the least difference gets 1 point.

The first player with 5 points wins.

For example:

Player A: 76.532 + 55.331 = 131.863   76.532 – 55.331 = 21.201

Player B: 54.430 + 66.500 = 120.93   66.500 – 54.430 = 12.07

Player A gets 1 point for the greatest sum.

Player B gets 1 point for the least difference.

Chapter 2 Multiplying Decimals

4.2.1 Estimating Products

Try This

All 42 children in Class III will be wearing special dress for an event.
Each piece of clothing is made from a piece of fabric that is 1.2 m long.

A. How do you know that more than 40 m of fabric will be needed to make all the costumes?

• Multiplying decimals is like multiplying whole numbers.

For example:

5 × 2 means 2 + 2 + 2 + 2 + 2

5 × 2.3 means 2.3 + 2.3 + 2.3 + 2.3 + 2.3

• To **estimate** the product of a decimal and a whole number, you can multiply whole numbers.

For example:

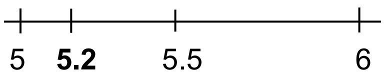
3 × 5.2 is about 3 × 5 = 15.

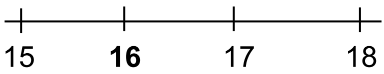
Since 5.2 > 5, you know 3 × 5.2 is *a bit more than* 15.

If you wish, you can get a closer estimate.

Since 3 × 5 = 15 and 3 × 6 = 18, you know 3 × 5.2 is between 15 and 18.

Since 5.2 is closer to 5 than to 6, 3 × 5.2 is closer to 15 than to 18.





That means 3 × 5.2 is about 16.

B. What decimal product were you estimating in **part A?**

Examples

Example 1 Comparing Estimates

Lobzang estimated 6 × 4.8 as 6 × 4 = 24.

Dorji estimated 6 × 4.8 as 6 × 5 = 30.

Which estimate is closer to the exact product? How do you know?

Solution	Thinking
4.8 is almost 5, so 6 × 4.8 is almost 6 × 5 = 30 Dorji's estimate is closer.	• I knew 4.8 was closer to 5 than to 4, since 4.8 is 2 tenths away from 5, but 8 tenths away from 4.

Example 2 Solving a Problem by Estimating a Product

A coin is 2.9 cm wide. Could a line of 30 coins be 1 m long?

Solution	Thinking
30 × 2.9 is almost 30 × 3 = 90 The line would be almost 90 cm long. Since 1 m = 100 cm, the line would not be 1 m long.	• I found the length of the line by multiplying 30 × 2.9, since it's like adding 2.9 thirty times. 

Practising and Applying

1. What whole number calculation could you use to estimate each?

a) 4 × 2.9 **b)** 7 × 8.04

c) 10 × 6.12 **d)** 8.6 × 4

2. Estimate. Tell whether your estimate is higher or lower than the exact product.

a) 11.3 × 7 **b)** 6 × 8.03

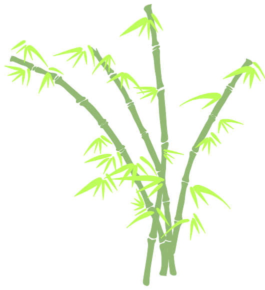
c) 5 × 4.85 **d)** 8.89 × 9

3. Yangchen bought fabric to make 8 ghos. She bought 3.8 m for each gho. Estimate how much fabric she bought.

4. Dorji walks 1.4 km to school each day and the same distance home. About how far does he walk to and from school in a 6-day school week?

5. A square platform has a side length of 3.12 m. Estimate the perimeter of the platform.

6. For 60 days, a bamboo plant grew about 28.5 cm each day. About how much did it grow altogether?



7. What might the decimal number be in the calculation below? Explain your thinking.

8 × □.□ is closer to 35 than to 30

8. Estimate to decide which is greatest. Explain your thinking.

A. 4 × 3.5

B. 3 × 5.5

C. 2 × 6.5

9. Give an example of a multiplication for each.

a) □ × □.□ is a bit more than 20

b) □ × □.□ □ is almost 30

c) □ □ × □.□ is a bit more than 40

4.2.2 Multiplying a Decimal by a Whole Number

Try This

Pema can run 100 m in 12.4 s.

A. Suppose he could run at that speed for 300 m. About how long will it take him to run 300 m?

• To multiply a decimal by a whole number, you can multiply in parts.
For example, to multiply 6 × 3.2:

First multiply the ones:

6 × 3 ones = 18

$$\begin{array}{r} 3.\square \\ \times\ \underline{\ \ 6} \\ 1\ 8 \end{array}$$

Then multiply the tenths and add:

6 × 2 tenths = 12 tenths

= 1.2

$$\begin{array}{r} 3.2\ \ \\ \times\ \underline{\ \ 6}\ \ \\ 1\ 8\ \ \\ +\ \underline{\ \ 1.2} \\ 19.2 \end{array}$$

• You can also multiply in parts the other way.
For example, to multiply 6 × 3.2:

First multiply the tenths:

6 × 2 tenths = 12 tenths

= 1 one + 2 tenths

$$\begin{array}{r} {\scriptstyle 1}\ \ \ \\ \square.2 \\ \times\ \underline{\ \ 6} \\ 2 \end{array}$$

Then multiply the ones and add the regrouped ones:

6 × 3 ones = 18 ones

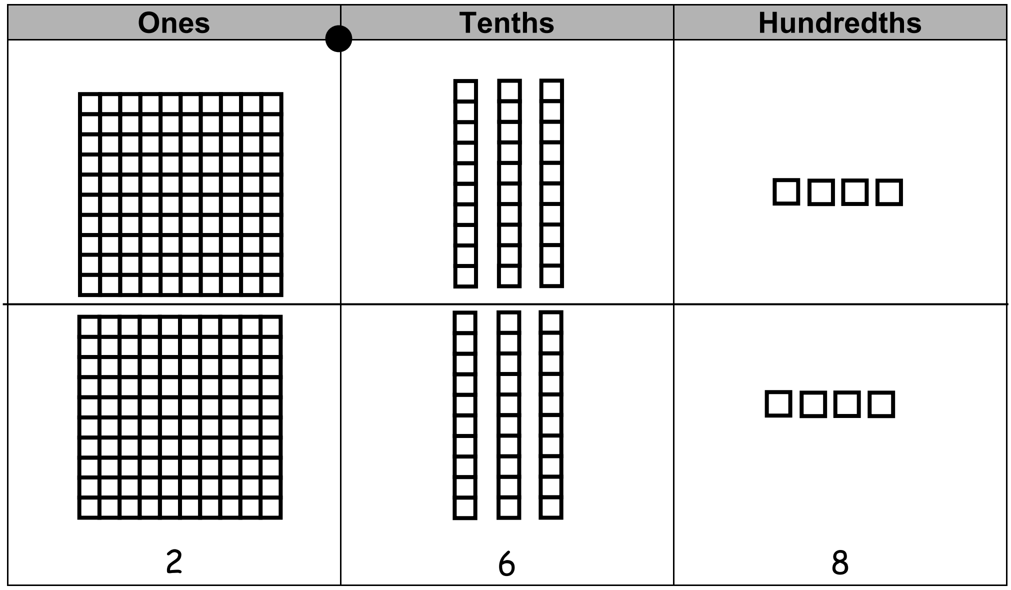
18 ones + 1 one = 19 ones

$$\begin{array}{r} {\scriptstyle 1}\ \ \ \\ 3.2 \\ \times\ \underline{\ \ 6} \\ 19.2 \end{array}$$

• You can also use models to help you multiply decimals by whole numbers.

For example, to multiply 2 × 1.34:

2 × 1.34 means 2 sets of models, each representing 1.34.



2 × 1.34 = 2.68

• Another way to multiply a decimal by a whole number is to write the decimal as a tenth or a hundredth and then multiply whole numbers.

For example:

2 × 6.1 = 2 × 61 tenths = 122 tenths = 12.2

2 × 1.34 = 2 × 134 hundredths = 268 hundredths = 2.68

• No matter what method you use to multiply, it is always a good idea to estimate to make sure your product has the right number of decimal places.

For example:

2 × 6.1 is about 2 × 6 = 12, so the product 12.2 seems reasonable.

2 × 1.34 is about 2 × 1 = 2, so the product 2.68 seems reasonable.

B. In **part A**, you estimated how long it would take Pema to run 300 m. Calculate to find an exact answer. Show your work.

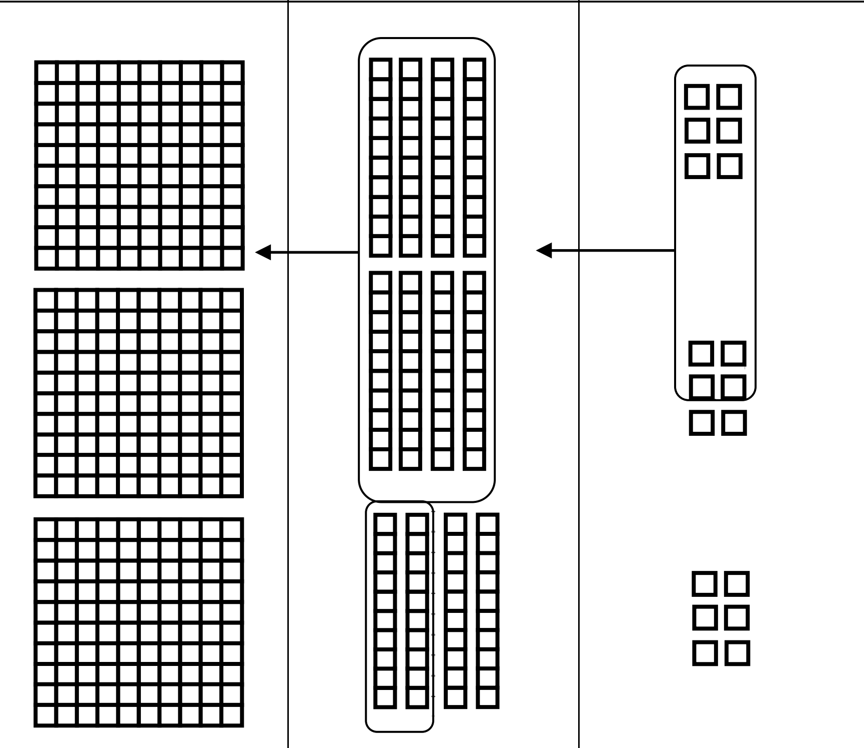
Examples

Example 1 Choosing a Multiplication Strategy

Calculate 3 × 1.46. Show your work.

Solution 1	Thinking
1.46 = 146 hundredths 146 hundredths × 3 300 120 + 18 438 hundredths → 4.38 3 × 1.46 = 4.38	 • I renamed 1.46 as 146 hundredths so I could multiply whole numbers. • I remembered to write the product as a decimal hundredth.

Solution 2	Thinking
Model 3 × 1.46	

Ones	Tenths	Hundredths
		
3 + 1 (regrouped) **4**	2 + 1 (regrouped) **3**	8 **8**

3 × 1.46 = 4.38	• I regrouped because there were 10 or more models in the hundredths place and the tenths place — 10 hundredths is 1 tenth and 10 tenths is 1 one.

Example 2 Solving a Problem by Multiplying Decimals

Choki ran 200 m in 35.7 s. Suppose she could run at that speed for 1000 m. How long will it take her to run 1000 m?

Solution

5 × 200 = 1000

If 200 m takes 35.7 s, then
1000 m takes 5 times as long.

$$\begin{array}{r} {}^{2\,3} \\ 35.7 \\ \times\quad 5 \\ \hline 178.5 \end{array}$$

It would take her 178.5 s to run 1000 m.

Thinking

- I figured out how many runs of 200 m were in 1000 m.
- I multiplied the number of 200 m runs by the time for 200 m.

Practising and Applying

1. Model and then find each product.

a) 4 × 3.1 **b)** 5 × 2.02

c) 4 × 1.62 **d)** 3 × 0.52

2. Calculate.

a) 6 × 5.4 **b)** 8 × 9.32

c) 9 × 12.3 **d)** 7 × 1.83

3. Estimate to decide which is greatest. Explain your thinking.

A. 7 × 3.12 **B.** 3 × 7.82

C. 5 × 4.09 **D.** 6 × 3.15

Check your answer by calculating.

4. Kamala walks 2.6 km each day. How far does she walk in 7 days?

5. Which of these statements are true? Explain your thinking.

A. The digit 7 is in the hundredths place of the product of 5 × 3.47.

B. The digit 9 is in the ones place of the product of 6 × 3.28.

C. The digit 6 is in the tenths place of the product of 4 × 3.92.

6. Copy and fill in the missing digits.

a)
$$\begin{array}{r} [\ ].8 \\ \times\quad 9 \\ \hline 70.[\ ] \end{array}$$

b)
$$\begin{array}{r} 12.[\ ]3 \\ \times\quad 7 \\ \hline [\ ]9.81 \end{array}$$

7. a) Calculate each product.

5 × 3.4 = ☐

5 × 3.5 = ☐

5 × 3.6 = ☐

b) Write the next three equations in the pattern above.

c) Predict the 20th equation.

8. Eden is 3 times as tall as she was when she was born. She was 45.7 cm at birth. How tall is she now?

9. Mindu says that the product of a whole number and a decimal has the same number of decimal places as the decimal. Tandin disagrees. Who is right? Use examples to explain your thinking.

4.2.3 Multiplying by 0.1, 0.01, and 0.001

Try This

Tshewang measured a length as 44 cm.

A. What is 44 cm in millimetres?
How do you know?

B. i) Complete each.

32 cm = ___ mm

46 m = ___ cm

23 km = ___ m

ii) What did you multiply by to perform each calculation? Why?

The decimals 0.1, 0.01, and 0.001 are special numbers to multiply by.

• When you multiply by 0.1, 0.01, and 0.001, you can think of 0.1 as 1 tenth, 0.01 as 1 hundredth, and 0.001 as 1 thousandth.

For example:

Since 0.1 is 1 tenth, 5 × 0.1 means 5 sets of 1 tenth, or 5 tenths.

5 × 0.1 = 5 tenths = 0.5

5 × 0.01 = 5 hundredths = 0.05

5 × 0.001 = 5 thousandths = 0.005

352 × 0.1 = 352 tenths = 35.2

352 × 0.01 = 352 hundredths = 3.52

352 × 0.001 = 352 thousandths = 0.352

• Notice above that the digits are the same but they move right 1, 2, or 3 places, depending on whether you are multiplying by 0.1, 0.01, or 0.001.

For example, 352 × 0.01 = 3.52:

Hundreds	Tens	Ones	Tenths	Hundredths
3	5	2		
		3	5	2

• Another way to think about multiplying by 0.1, 0.01, or 0.001 is to think about fractions. When you multiply by 0.1, 0.01, or 0.001, it means you want $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1000}$ of the amount you are multiplying, so it is like dividing by 10, 100, or 1000.

Since our place value system is built so that each column is $\frac{1}{10}$ of the value of the column to its right:

- multiplying by 0.1, or $\frac{1}{10}$ means moving each digit 1 place to the right.
- multiplying by 0.01, or $\frac{1}{100}$ means moving each digit 2 places to the right.
- multiplying by 0.001, or $\frac{1}{1000}$ means moving each digit 3 places to the right.

For example, 0.1 × 52.3 = 5.23:

Hundreds	Tens	Ones	Tenths	Hundredths
	5	2	3	
		5	2	3

5 tens become 5 ones, 2 ones become 2 tenths, and 3 tenths become 3 hundredths because you are taking $\frac{1}{10}$ of each.

C. i) Complete each.

32 mm = ___ cm   46 cm = ___ m   23 m = ___ km

ii) Why could you multiply or divide to complete **part i)**?

Examples

Example Comparing Products Involving 0.1, 0.01, and 0.001

Rinzin multiplied the same number, ■, by 0.1, 0.01, and 0.001.
If ■ × 0.01 = 3.42, what are ■ × 0.1 and ■ × 0.001?

Solution 1	Thinking
■ × 0.01 = ■ hundredths 3.42 = 342 hundredths ■ = 342 342 × 0.1 = 34.2 342 × 0.001 = 0.342	• First I figured out what number ■ was. • Then I multiplied the number by 0.01 and 0.001. 

Solution 2	Thinking
If ■ × 0.01 = 3.42, then ■ × 0.1 = 10 × 3.42 = 34.2 and ■ × 0.001 = 3.42 ÷ 10 = 0.342.	• 0.1 is 10 times 0.01 because each $\frac{1}{10} = \frac{10}{100}$. • 0.001 is $\frac{1}{10}$ as much as 0.01 since thousandths are $\frac{1}{10}$ the size of hundredths. 

Practising and Applying

1. Calculate.

a) 6 × 0.1　　**b)** 7.2 × 0.1

c) 4.5 × 0.01　　**d)** 38.2 × 0.01

e) 19 × 0.001　　**f)** 123 × 0.001

2. ■ × 0.1 = 17.2.

a) What is ■ × 0.01?

b) What is ■ × 0.001?

3. Which digit will be in the tenths place in each product?

a) 123.8 × 0.01　　**b)** 1.74 × 0.1

c) 123 × 0.001　　**d)** 410.2 × 0.01

4. How do you know that 3.45 km × 0.001 = 3.45 m?

5. Distances on a map are 0.001 of the distance they really area.

a) Two streets are 2.4 cm apart on the map. What is their real distance apart in centimetres? In metres?

b) What is their real distance apart if they are 9.2 cm apart on the map?

6. Explain why multiplying by 0.01 results in a number with the same digits, but each digit is two places to the right of where it started.

CONNECTIONS: Telescopes and Binoculars

Telescopes are used to look at objects that are very far away. Lenses inside the telescope might magnify objects to 100 times (100X) the size they appear to the naked eye.

Binoculars are like two very small telescopes. They might magnify objects to 10 times (10X).

1. a) An object as seen through a pair of 10X binoculars is 4 cm across. The size it appears without the binoculars is 4 × 0.1 cm. Why?

b) Suppose the image was 2.5 cm across. What decimal multiplication would describe its size without the binoculars?

2. An object as seen through a 100X telescope is 2.5 cm across. What decimal multiplication would describe its size without the telescope?

UNIT 4 Revision

1. Use models to find the sum or difference.

a) 3.4 + 2.89 **b)** 3.1 + 1.9

c) 2.26 + 2.79 **d)** 4.42 – 2.56

e) 3.1 – 1.05 **f)** 1 – 0.08

2. Solve.

a) 1.214 + 3.553 = ☐

b) 3.167 + 1.295 = ☐

c) ☐ = 16.281 + 8.184

d) ☐ = 5.318 + 4.076

3. Calculate each using mental math. Explain how you calculated.

a) 5.267 + 0.02

b) 8.1 + 2.358

c) 3.998 + 2.042

4. Phuntsho is 143.6 cm tall. Dawa is 2.5 cm taller than Phuntsho. How tall is Dawa?

5. A rectangle is 4.123 m long and 2.657 m wide.

a) What is the perimeter?

b) How much longer is the length than the width?

6. Calculate each using mental math. Explain how you calculated.

a) 4.567 – 0.1

b) 4.567 – 0.001

c) 3.248 – 1.13

d) 5 – 1.998

7. Some reaction times were measured in seconds. Five times are shown below.

0.321	0.417	0.389	0.420	0.381

a) What is the difference between the fastest time and the slowest time?

b) Which two times are closest? What is the difference between the times?

8. Sonam's gold ring has a mass of 12.634 g. Pema's gold ring has a mass of 16.386 g. How many more grams of gold are there in Pema's ring?

9. Why is it important to line up the decimal points if you are subtracting decimals in columns?

10. Deki drank 1.325 L milk on Monday, 0.872 L on Tuesday, and 1.156 L on Wednesday.

a) How much milk did Deki drink on all three days?

b) How much less did Deki drink on Tuesday than Wednesday?

c) Deki drank 0.225 L of milk more on Thursday than she did on Wednesday. How much milk did she drink on Thursday?

11. Estimate each. Predict whether your estimate is high or low and explain how you know.

a) 12.7 × 6

b) 4 × 6.12

c) 7 × 5.79

d) 7.64 × 8

12. Seven jugs of water each hold 1.3 L. About how many litres of water are there altogether?

13. Calculate.

a) 4 × 8.3

b) 6 × 7.18

c) 7 × 6.63

d) 9 × 2.57

14. Copy and complete.

a)
$$\begin{array}{r} [\,].8 \\ \underline{\times\ \ [\,]} \\ 26.6 \end{array}$$

b)
$$\begin{array}{r} 1[\,].[\,]3 \\ \underline{\times\ 8} \\ [\,]40.2[\,] \end{array}$$

15. Karma is 3 times as tall now as the last time he was measured. At that time, he was 0.57 m tall. How tall is he now?

16. Calculate.

a) 8 × 0.1

b) 12.2 × 0.1

c) 6.9 × 0.01

d) 11.3 × 0.01

e) 172 × 0.001

17. Kinley reported a measurement as 523 m. When he used a different unit, the number was 0.001 times as great. What unit of measurement did he change to?

UNIT 5 MEASUREMENT

Getting Started

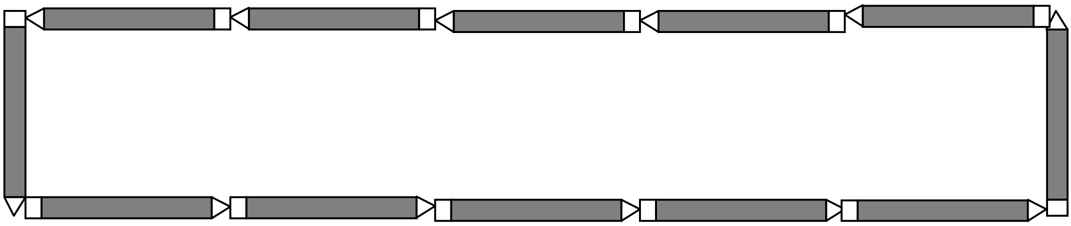
Use What You Know

Work in groups of two or three.

A. i) Lay out 12 crayons of the same length in a line. How long is the line?

ii) Arrange the same 12 crayons in a rectangle as shown here.

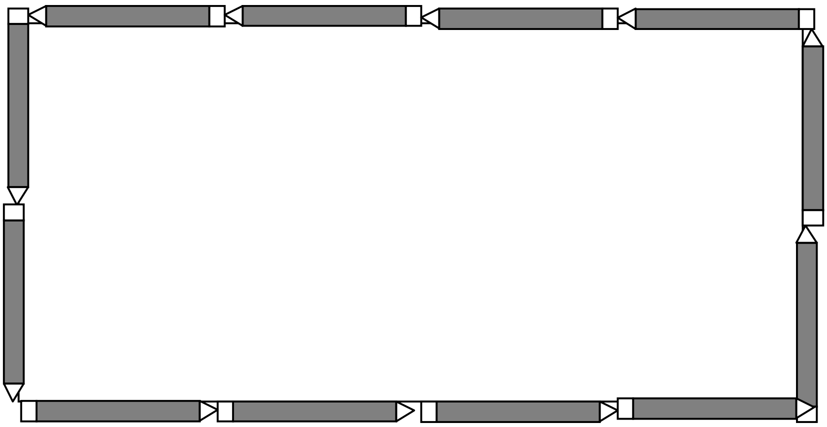
How do you know the perimeter of the rectangle without measuring it?



iii) Cut out about 10 paper squares, each 10 cm by 10 cm.

About how many paper squares will fit inside the rectangle without overlap or gaps?

B. i) Arrange the same 12 crayons in a rectangle as shown here.
What is the perimeter of this rectangle? How do you know?



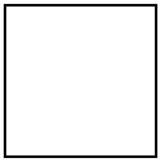
ii) Predict whether more or fewer paper squares will fit inside this rectangle than the rectangle in **part A**. Check your prediction.

C. Predict whether you can arrange the crayons to make a rectangle that will fit more paper squares than the rectangle in **part B**. Check your prediction.

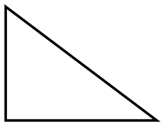
Skills You Will Need

1. What is the perimeter of each shape? Measure to the nearest centimetre.

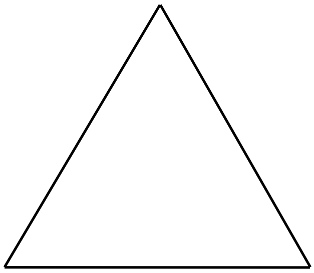
a)



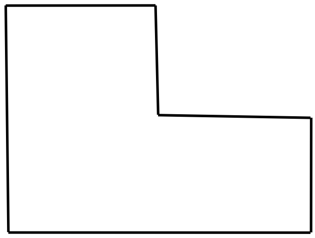
b)



c)

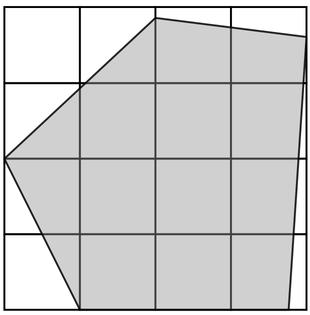


d)

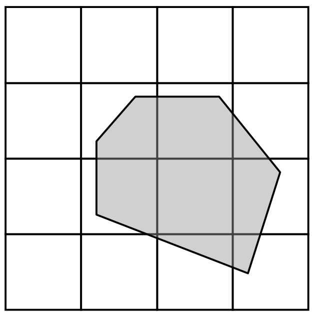


2. Estimate the area of each grey shape. Each grid square is 1 cm^2.

a)

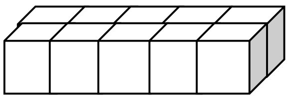


b)

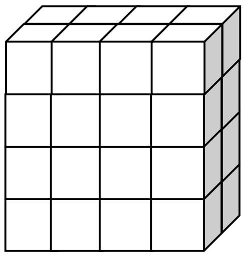


3. What is the volume of each prism in number of cubes?

a)



b)



4. What might be each length?

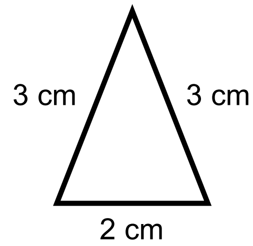
For example, 5 mm might be the thickness of a notebook.

a) 4 mm **b)** 4 cm **c)** 4 dm **d)** 4 m **e)** 4 km

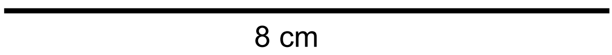
Chapter 1 2-D Shapes

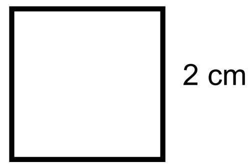
5.1.1 EXPLORE: Polygons with the Same Perimeter

• The **perimeter** of a shape is the distance around it.
For example:
The perimeter of this triangle is 3 + 3 + 2 = 8 cm.



• One way to make shapes with the same perimeter is to cut a string and then bend it into different shapes.
For example:
An 8 cm string could be bent into the triangle above or into the square shown here with side 2 cm long.





It could also be bent into other shapes, all with the same perimeter.

A. Cut a piece of string or twine to be 48 cm long.

B. Arrange the twine into an **equilateral triangle**.

i) How long is each side?

ii) What fraction of the whole perimeter is each side length?

C. Arrange the twine into a **regular hexagon** (all sides equal).

i) How long is each side?

ii) What fraction of the whole perimeter is each side length?

D. Make shapes, each with a perimeter of 48 cm, to match the rules below. Sketch each shape and label the side lengths.

i) Only two sides are 12 cm.

ii) One side is 12 cm longer than another side.

iii) It is a hexagon, but not all sides are equal length.

iv) It is a circle. (Instead of labelling the side lengths on your sketch, label how wide the circle is.)

E. In **part D**, did you make all possible shapes with a perimeter of 48 cm? Explain your thinking.

5.1.2 EXPLORE: Perimeter of Rectangles

To determine the perimeter of some shapes, you need to know all the **dimensions**. The dimensions of a 2-D shape are its side lengths.
For certain shapes, you do not need to know all the dimensions because some of the dimensions are related.

A. i) Draw a rectangle that is 10 cm by 4 cm. Call it Rectangle A.

• Calculate its perimeter. Show your work.

ii) Draw a longer, thinner Rectangle B with the same perimeter as Rectangle A.

• How do the dimensions of Rectangle B compare with the dimensions of Rectangle A?

iii) Draw a square with the same perimeter as Rectangle A.

• How do the dimensions of the square compare with the dimensions of Rectangle A?

iv) Calculate the areas of the three rectangles in **part A**. Then, order the rectangles from least to greatest areas.

B. Repeat **part A**, but start with a rectangle that is 8 cm by 4 cm.

C. i) Why did you only need to know two side lengths to calculate the perimeter of each rectangle in **parts A and B**?
ii) How can you calculate the perimeter of a rectangle if you know its length and width?
iii) How can you calculate the perimeter of a square if you know one side length?

D. Which statement below is true? How do you know?

• Two squares with the same perimeter can have different areas.

• Two rectangles with the same perimeter can have different areas.

5.1.3 EXPLORE: Area on a Grid

• **Area** describes the number of square units needed to cover a shape.
If a shape is on a grid, you can find its area in different ways.

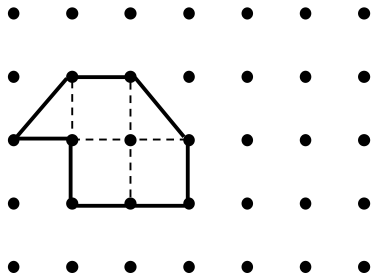
• To find the area of some shapes, you can count whole square units and part square units and then combine them.

For example:

The area of this shape is

3 whole square units + 2 half square units.

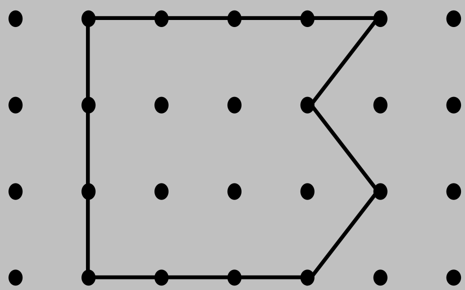
The total area is 4 square units.



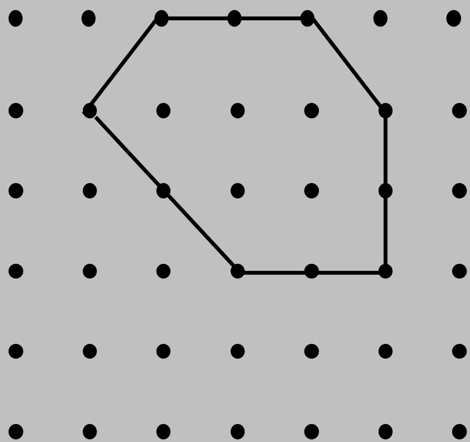
• As you do the exploration below, see if you can think of other ways to use the square units of the grid to find the area of different shapes.

A. What is the area of each shape in square units?

i)

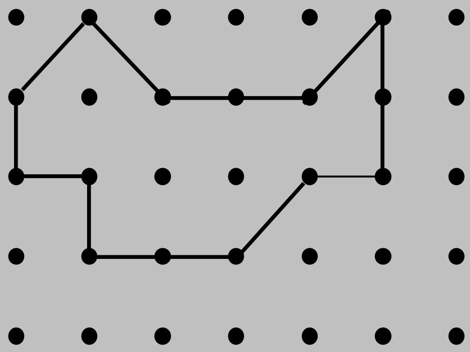


ii)



[Continued]

A. iii)



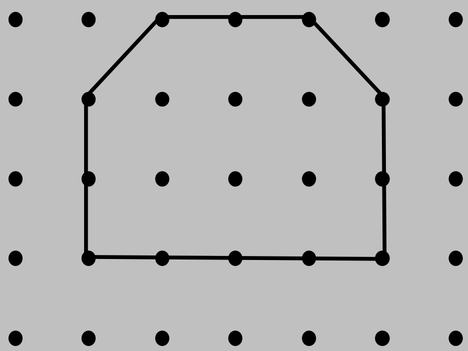
B. Move one vertex of the shape in **part i)** to change the area to each.

i) 11 square units **ii)** $9\frac{1}{2}$ square units **iii)** $8\frac{1}{2}$ square units

C. On a square dot grid, create two different shapes with each area.

i) 12 square units **ii)** $15\frac{1}{2}$ square units **iii)** $5\frac{1}{2}$ square units

D. The shape below has an area of 11 square units. Describe two different ways to show that this is true.



5.1.4 Area and Perimeter Relationships

Try This

A. i) Estimate the height and width of your classroom door, in centimetres. To do this, you might think about how tall and wide the door is compared to your body.

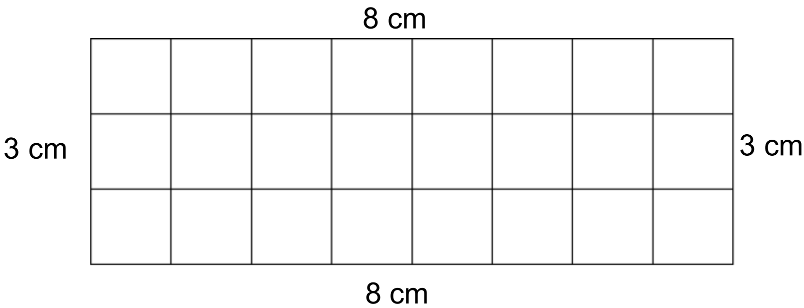
ii) Do you think the perimeter of the door is greater than 5 m or less than 5 m? Explain your thinking.

• The area of a rectangle is a measure of the number of units that cover it. The perimeter is a measure of the distance around it.

For example:

- The area of a 3 cm by 8 cm rectangle is 24 **square centimetres** (24 cm^2), since it is covered by 24 square units, each 1 cm by 1 cm.

- The perimeter is 8 cm + 3 cm + 8 cm + 3 cm = 22 cm.



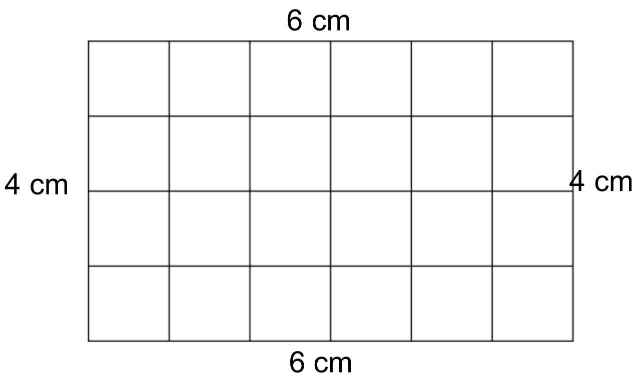
A = 24 cm^2

P = 22 cm

• It is possible to rearrange the 24 squares into a different rectangle.

- The area is still 24 cm^2, but the perimeter can be different.

- The perimeter of this rectangle is 6 cm + 4 cm + 6 cm + 4 cm = 20 cm.



A = 24 cm^2

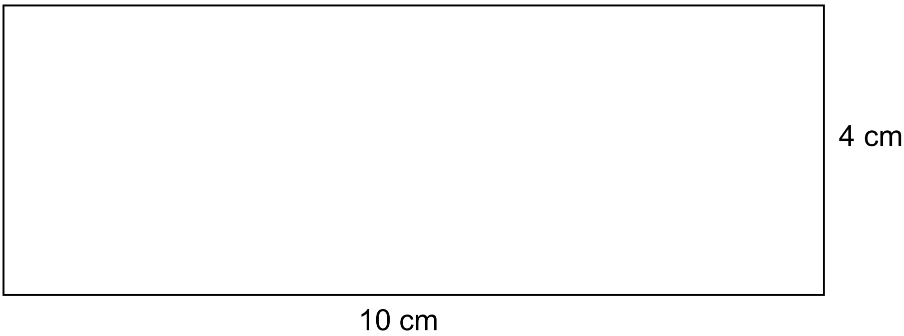
P = 20 cm

• To calculate the perimeter of a rectangle, you can use a rule or **formula**. Because there is more than one way to calculate the perimeter, there is more than one formula.

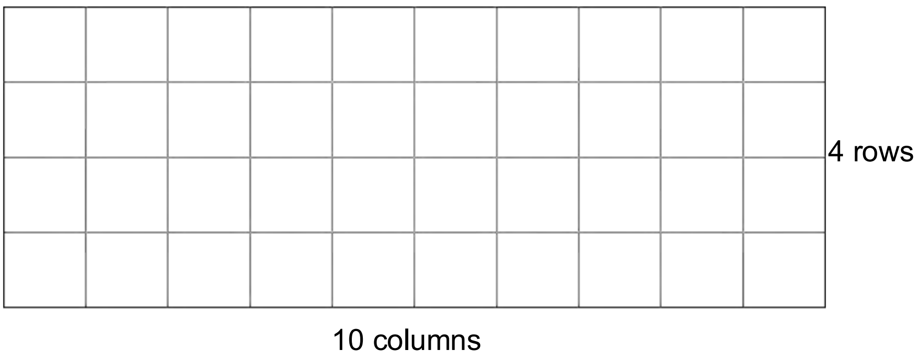
Perimeter of a rectangle

Perimeter = Length + Width + Length + Width
Perimeter = 2 × Length + 2 × Width
Perimeter = (Length + Width) × 2

• You can also calculate the area of a rectangle using a formula.
For example:
- This rectangle is 10 cm long and 4 cm wide.



- Imagine the rectangle covered in 1 cm-by-1 cm squares.



There are 4 rows of 10 squares, which is 40 squares altogether.
- The number of squares in each row is the length of the rectangle.
- The number of rows is the width.
- The area is the product of those numbers.

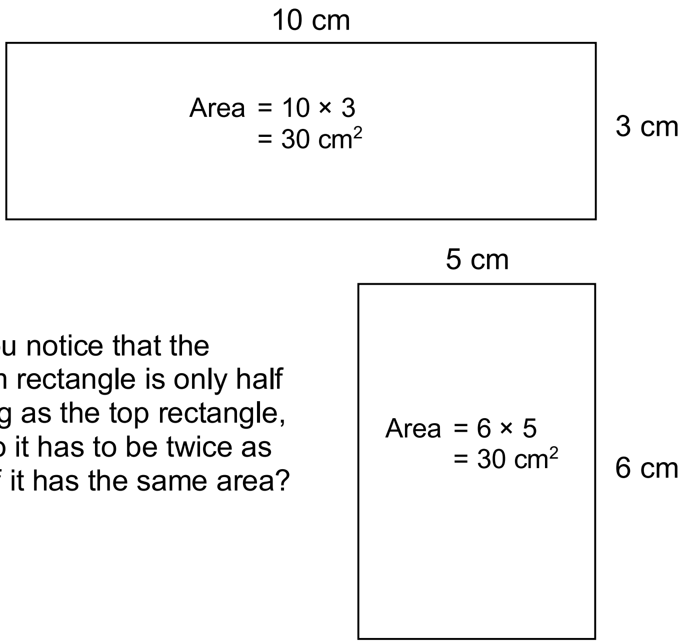
Area of a rectangle

Area = Length × Width

• For two rectangles to have the same area, the products of the lengths and widths have to be the same.

For example:

These rectangles have the same area because 10 × 3 = 30 and 6 × 5 = 30.



Did you notice that the bottom rectangle is only half as long as the top rectangle, and so it has to be twice as wide if it has the same area?

• Rectangles can have the same area but different perimeters.

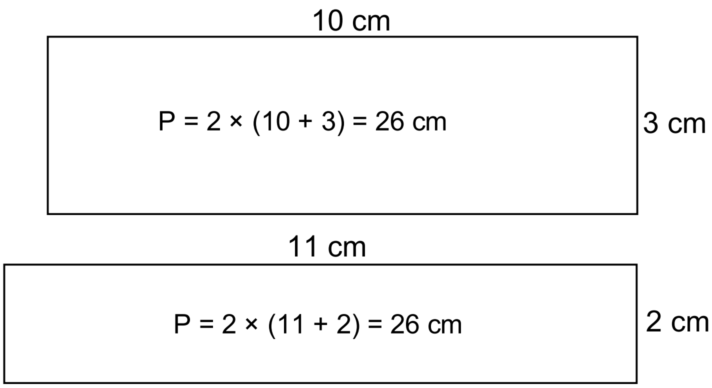
For example:

Both rectangles above have an area of 30 cm^2, but the perimeter of the top rectangle is (10 + 3) × 2 = 26 cm, and the perimeter of the bottom rectangle is (5 + 6) × 2 = 22 cm.

• For two rectangles to have the same perimeter, the length and width must have the same sum.

For example:

These rectangles have the same perimeter because their length and width have a sum of 13: 10 + 3 = 13 and 11 + 2 = 13



Did you notice that the bottom rectangle is longer than the top rectangle, and so it cannot be as wide if it has the same perimeter?

B. Use the dimensions you estimated in **part A** and the area and perimeter formulas to estimate each.

i) the area of the door **ii)** the perimeter of the door

Example Calculating Area and Perimeter

The dimensions of this number plate are 40 cm by 15 cm.

a) What is its area?

b) What is its perimeter?

c) What other rectangle has the same area?

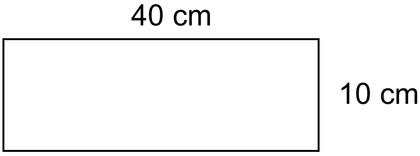
d) What other rectangle has the same perimeter?

Solution	Thinking
a) Area = 15 × 40 = 15 × 4 × 10 = 60 × 10 = 600 The area is 600 cm^2.	 **a)** I imagined the area covered by 15 rows with 40 squares in each.
b) Perimeter = (40 + 15) × 2 = 55 × 2 = 110 cm The perimeter is 110 cm.	**b)** To calculate the perimeter, I added the length and width and then doubled the sum.
c) 40 × 15 = 600 60 × 10 = 600 The rectangle could be 60 cm by 10 cm.	**c)** I knew that for a rectangle to have the same area, the length and width had to have the same product as 40 × 15 = 600.
d) 40 + 15 = 55 30 + 25 = 45 The rectangle could be 25 cm by 30 cm.	**d)** I knew that, for a rectangle to have the same perimeter, the length and width had to have the same sum as 40 + 15 = 55.

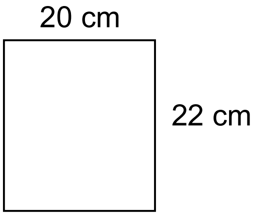
Practising and Applying

1. Calculate the area and perimeter of each.

a)



b)

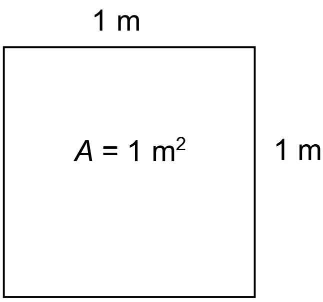


2. A rectangle has the same perimeter as the rectangle in **question 1a)**, but it has a greater area. What could be its dimensions?

3. A rectangle has the same area as the rectangle in **question 1b)**, but it has a greater perimeter. What could be its dimensions?

4. Explain why you add values to calculate the perimeter of a rectangle, but you multiply to calculate the area.

5. One square metre is the area of a square with a side length of 1 m. Estimate the area of your classroom floor, in square metres.



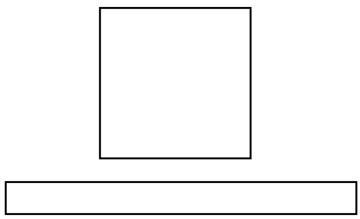
6. This wall hanging is sold in Bhutanese tourist shops. It is 60 cm by 20 cm.

a) What is its area?

b) A hanging of different dimensions has the same area. What might be its dimensions?

7. A square and a long thin rectangle have the same area. Which shape has less perimeter? How do you know?

8. These two shapes have the same perimeter. Which shape has less area? How do you know?



9. You know the length and width of a rectangle. Why do you not need to see the rectangle to figure out its area and perimeter?

GAME: Cover the Grid

Play in groups of 2 or 3. You need a 22 cm-by-18 cm grid and a pair of dice. Each player needs a different coloured pencil.

Take turns. On your turn:

• Roll two dice.

• The product of the numbers you roll is the area of a rectangle.

• Try to colour a rectangle with that area on the grid. If you cannot, you lose your turn. It cannot overlap any area already coloured.

The game is over after each player has had 5 turns or there is no area left to colour.

The player with the most coloured squares wins the game.

For example:

• On the grid below, Player A is using grey and Player B is using black.

• Player A has coloured a 20 cm^2 rectangle, a 2 cm^2 rectangle, and a 15 cm^2 rectangle.

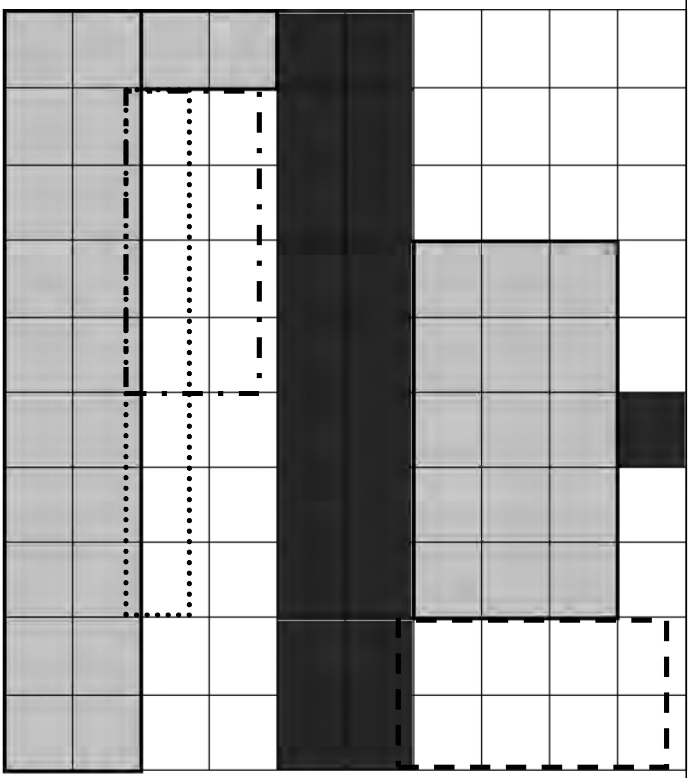
• Player B has coloured a 20 cm^2 rectangle and a 1 cm^2 rectangle.

It is Player B's turn and she has rolled a 2 and a 4.

Since 2 cm × 4 cm = 8 cm^2, she should look for rectangles of these dimensions:

- 2 rows by 4 columns
- 4 rows by 2 columns
- 1 row by 8 columns
- 8 rows by 1 column

Only three of the choices (2 by 4, 4 by 2, and 8 by 1, as shown by the dashed or dotted lines) are available.

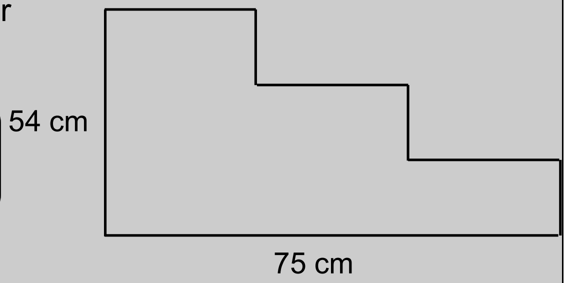


5.1.5 Area of Composite Shapes

Try This

This piece of wood is to be used for the side of a staircase.

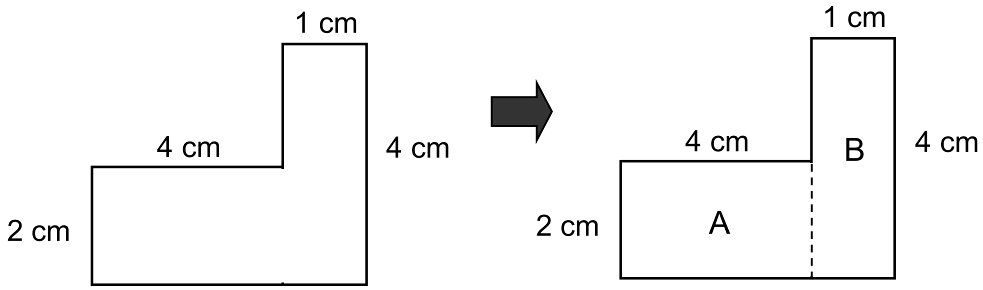
A. Estimate the area of the wood. Explain how you estimated.



• A **composite shape** is a shape that can be divided into familiar shapes. To find the area of a composite shape, you can divide it into familiar shapes, find the area of each, and then add the areas.

For example:

To find the area of the shape below, you could divide it into two rectangles (A and B), find the area of each, and then add the areas.

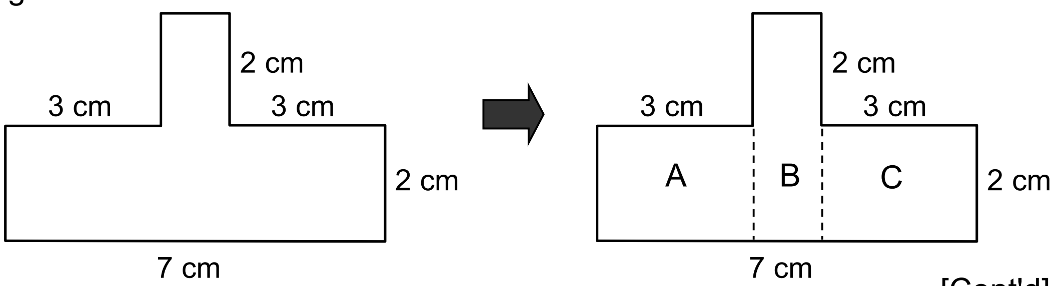


The area of Rectangle A is 4 × 2 = 8 cm^2.
The area of Rectangle B is 1 × 4 = 4 cm^2.
The total area is 8 + 4 = 12 cm^2.

• After you divide a shape into familiar shapes, you may have to figure out some missing information.

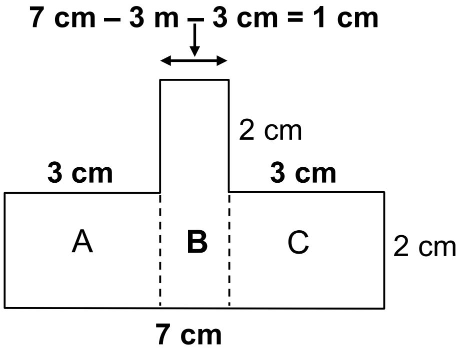
For example:

To find the area of the shape below, you could divide it into three rectangles (A, B, and C), but the dimensions of Rectangle B are not given.

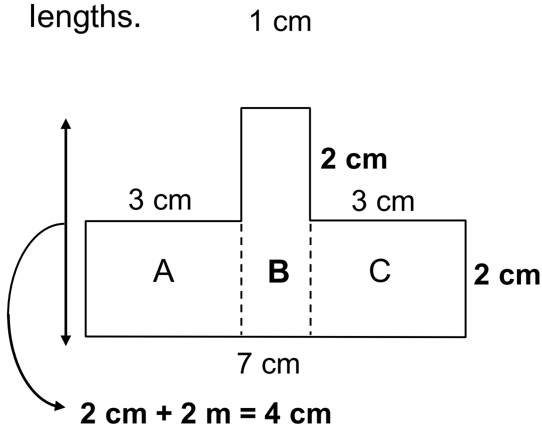


[Cont'd]

- To figure out the width of Rectangle B, subtract both 3 cm lengths from the total 7 cm length.



- To figure out the length of Rectangle B, add both 2 cm lengths.



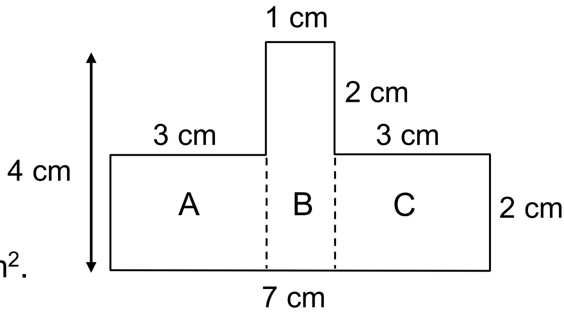
- To figure out the total area, add the three areas:

Rectangle A is 3 × 2 = 6 cm^2.

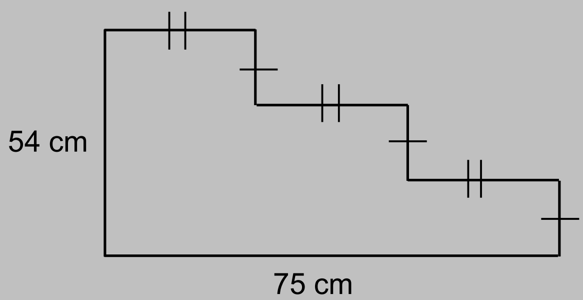
Rectangle B is 1 × 4 = 4 cm^2.

Rectangle C is 3 × 2 = 6 cm^2.

The total area is 6 + 4 + 6 = 16 cm^2.



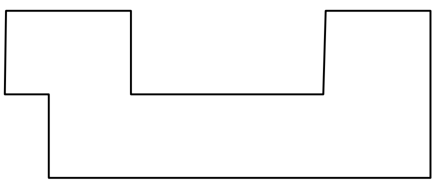
B. Calculate the area of the wood in **part A**. Show your work.



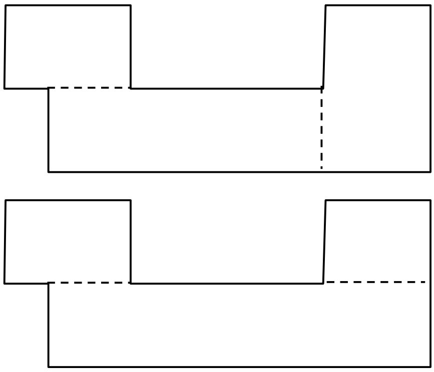
Examples

Example 1 Dividing a Composite Shape to Calculate Area

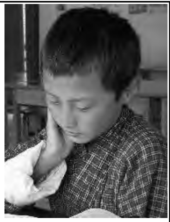
Describe two ways to divide this shape into rectangles.



Solution



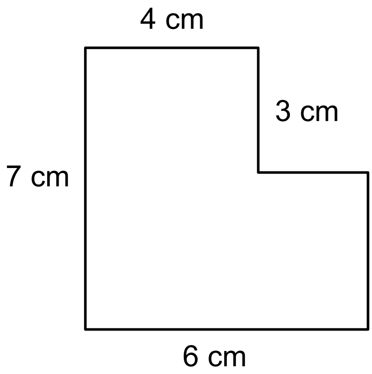
Thinking



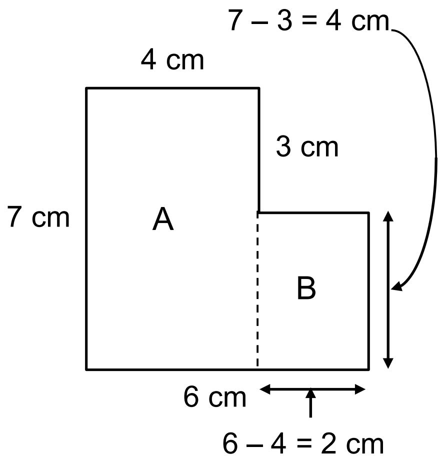
- I was able to divide it into three rectangles two different ways.

Example 2 Determining Missing Dimensions to Calculate Area

Calculate the area of this shape.



Solution 1



Total area $= 7 \times 4 + 4 \times 2$

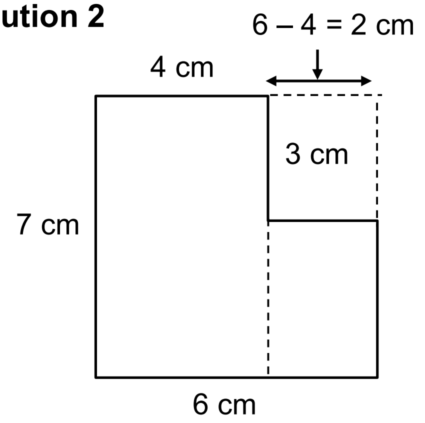
$= 28\text{ cm}^2 + 8\text{ cm}^2$

$= 36\text{ cm}^2$

The area is 36 cm^2.

Thinking

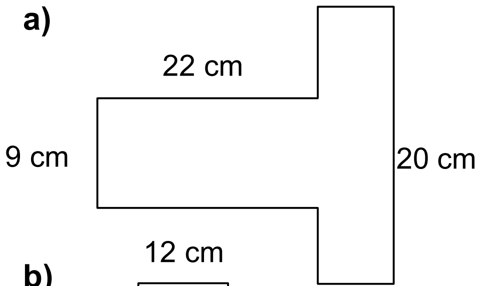
- I divided the shape into two rectangles.
- I divided the shape the way I did because I knew what the dimensions of Rectangle A were (7 cm by 4 cm), and I knew I could figure out the dimensions of Rectangle B.

Example 2 Determining Missing Dimensions to Calculate Area [Cont'd]	
Solution 2  Area = $7 \times 6 - 3 \times 2 = 36$ cm^2	**Thinking**  • I imagined a big rectangle that was 7 cm by 6 cm with a 3 cm-by-2 cm rectangle missing from the top right corner. • Instead of adding to the find the area, I subtracted.

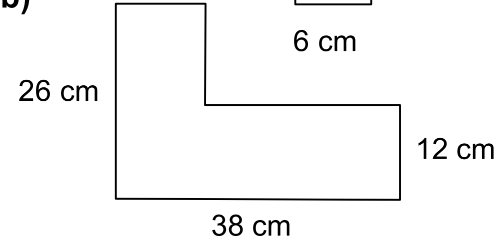
Practising and Applying

1. Calculate the area and perimeter of each shape. Show your work.

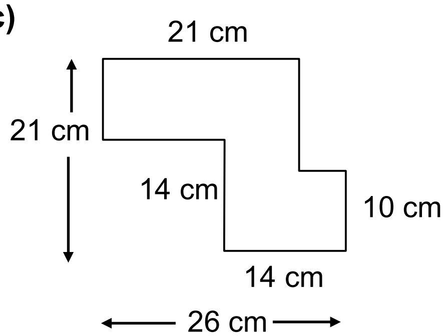
a)



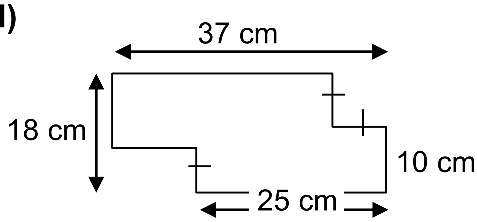
b)



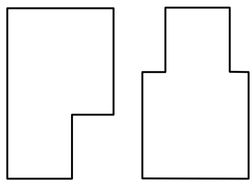
c)



d)



2. How do you know these shapes have the same area without calculating the areas?

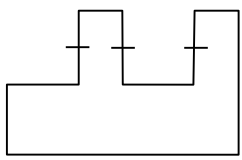


3. Combine two or three rectangles to create a shape with each area.

a) 30 cm^2 **b)** 18 cm^2

c) 100 cm^2 **d)** 60 cm^2

4. Sketch the shape below. Mark the side lengths that you would need to know to find its area. Explain your thinking.



CONNECTIONS: Unusual Ways to Measure Area

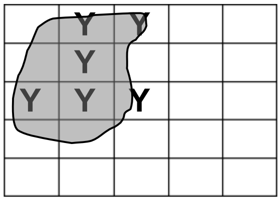
Some shapes cannot be divided into familiar shapes. For these shapes, you can use a centimetre grid and strategies like those described below.

Begin by tracing the shape onto the grid.

Strategy 1

• Count the squares that are more than half covered (marked Y for “yes”).

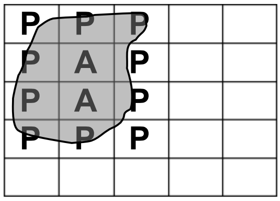
• The area of this shape is about 6 cm^2.



Strategy 2

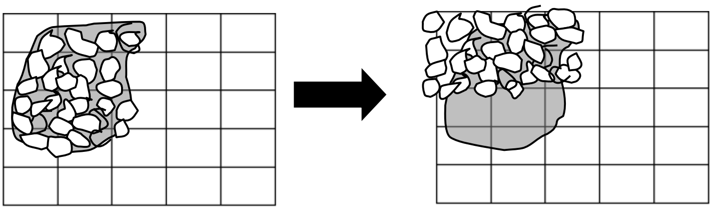
• Count the squares that are partly covered (P) and the squares that are all covered (A) and use the number in the middle.

• This shape has 2 A squares and 10 P squares. The middle of 2 and 10 is 6, so the area is about 6 cm^2.



Strategy 3

• Cover the shape with a thin layer of small rocks or rice and then move the rocks into a rectangle on the grid.



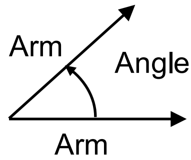
• The rocks cover a rectangle that is about 2 cm by 3 cm, so the area is about 6 cm^2.

1. Draw a shape like the one above on a grid. Try at least two of the strategies described above to estimate the area.

Chapter 2 Angles

5.2.1 EXPLORE: Measuring Angles

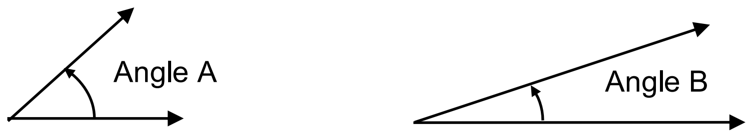
• The measure of an **angle** describes how much one **arm** has turned away from the other arm.



The more the arm has turned, the greater the measure of the angle.

For example:

Angle A is greater than Angle B.

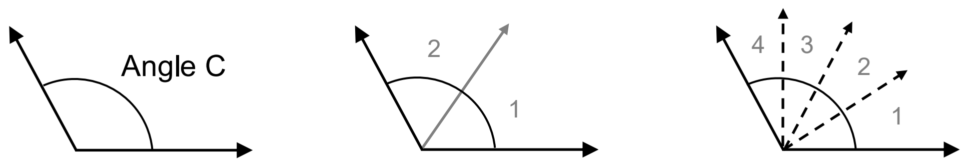


It does not matter that the arms of Angle B are longer than the arms of Angle A. Since the turn of Angle B is less, the measure of Angle B is less.

• One way to measure an angle is to see how many times a smaller angle fits inside the angle. The measure will depend on the size of the angle unit.

For example:

Angle C is 2 units if you use the grey angle unit but it is 4 units if you use the smaller dashed angle unit.



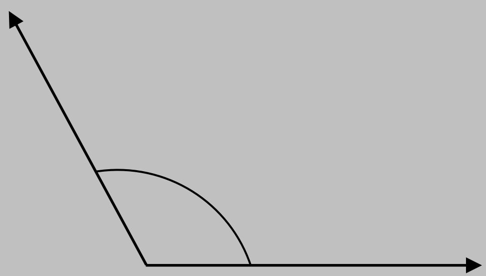
Angle C is 2 units. Angle C is 4 units.

A. Cut out sections of circles of each of size to use as angle units. Label them as M, N, O, P, and Q.

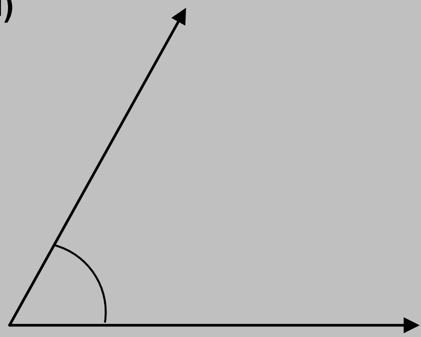
M is – N is – O is – P is — Q is —

B. About how many times does each angle unit from **part A** fit inside each angle? Measure to estimate to the nearest quarter of a unit.

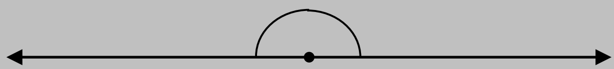
i)



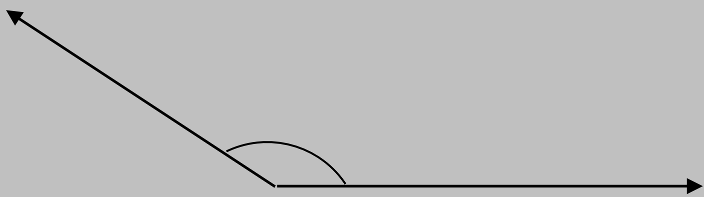
ii)



iii)



iv)



C. For each angle in **part B**, how do these angle measures compare?

i) the – angle unit and the – angle unit

Why might you have predicted that?

ii) the – angle unit and the — angle unit

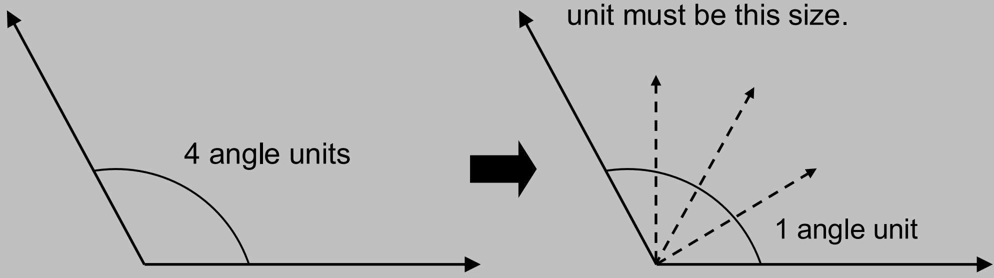
Why might you have predicted that?

[Continued]

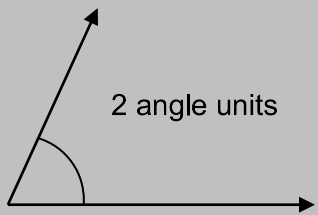
D. Trace each angle. Inside each angle, sketch the angle unit used to measure it.

For example:

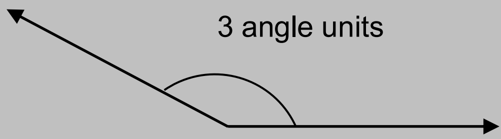
If this angle has a measure of 4 angle units, then each angle unit must be this size.



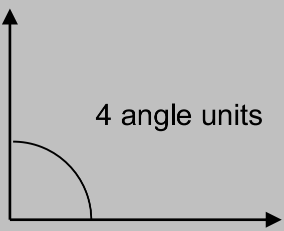
i)



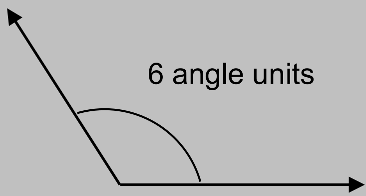
ii)



iii)



iv)



E. Describe the strategy you used to answer **part D iv)**.

F. Suppose you measure an angle twice, using angle unit X and then using angle unit Y. Angle unit X is bigger than angle unit Y. What do you know about the two measurements?

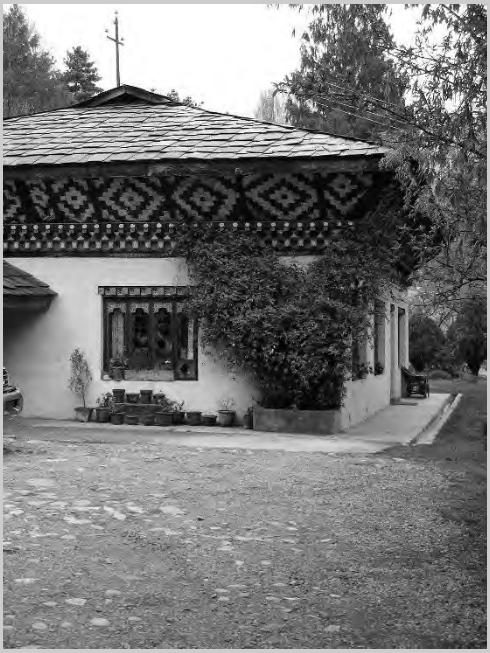
G. Why is it important to tell what angle unit was used when you report an angle measurement?

5.2.2 Comparing Angles to Special Angles

Try This

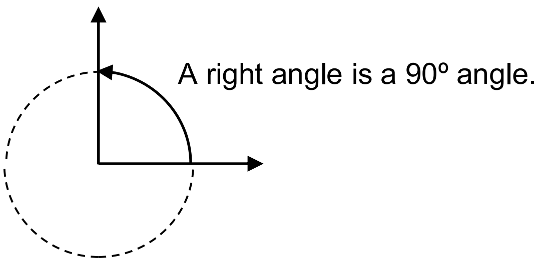
Look at this picture of a cottage.

A. **i)** Where do you see right angles?
ii) Where do you see acute angles?

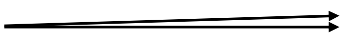


• The special angle that looks like the corner of a square is a **right angle**.

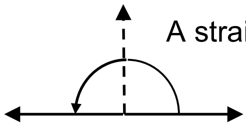
- It looks like one arm was turned $\frac{1}{4}$ of the way around a circle.
- A right angle is also described as a 90 **degree** (90°) angle.



- A 90° angle is made up of 90 very small 1° angles. The 1° angle is the angle unit.
- A 1° angle is so small it is hard to show. Even the tiny angle below is actually 2° or 3°.

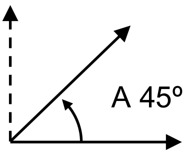


• Two 90° angles put together make a 180° angle. It looks like a straight line and is called a **straight angle**.

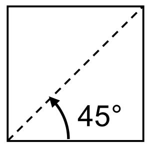


A straight angle is 180°.

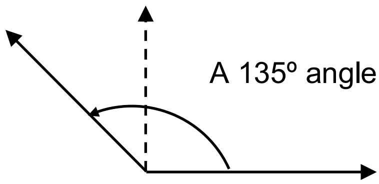
• Another special angle is the 45° angle. It is half of a right angle.
The angle between the diagonal of a square and the sides is a 45° angle.



A 45º angle is half a right angle.

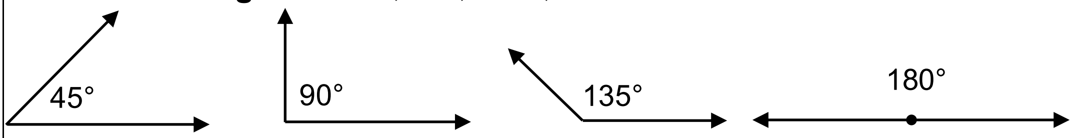


• If you put together a 90° angle and a 45° angle, you get a 135° angle.

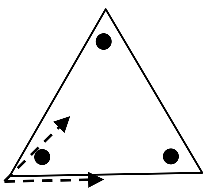


A 135º angle is a 90º angle + 45º angle.

• To estimate the size of an angle, you can compare it to these special **benchmark angles** of 45°, 90°, 135°, and 180°.

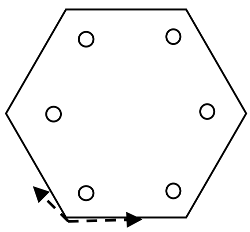


For example:



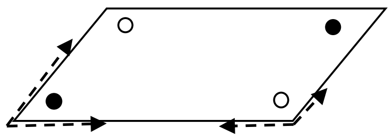
Using a 45º benchmark

The angles in the triangle are slightly more than 45°, so an estimate of 60º is reasonable.



Using a 135º benchmark

The angles in the hexagon are slightly less than 135°, so an estimate of 120º is reasonable.



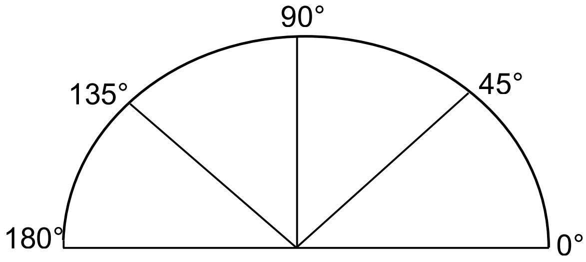
Using 45º and 135º benchmarks

The angles in the parallelogram appear to be about 45º and 135º.

• You can make your own tool, called an improvised **protractor**, for measuring and estimating angles.

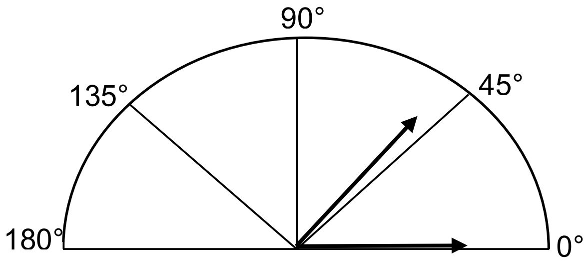
You mark the position of the benchmark angles and 0° on a half circle of transparent plastic or tracing paper.

To do this, fold a half circle in half to make the 90° mark and then fold it in half again to make the 45° and 135° marks.



• You can use your protractor to estimate angle measures.

For example, the angle below looks like it might be about 50°.



B. i) Which of the angles you identified in **part A** are 90° angles?

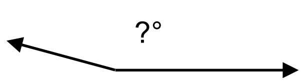
ii) Are the acute angles from **part A** closer to 45° or 90°?

Examples

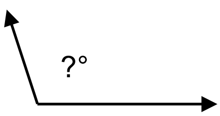
Example 1 Comparing Angles to Benchmarks

Estimate if each angle is closer to 45°, 90°, 135°, or 180°. Show your work.

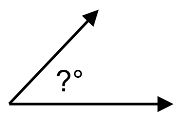
a)



b)

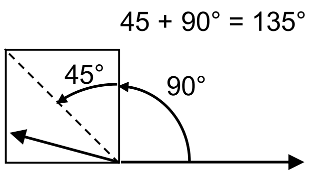


c)



Solution

a) closer to 180°



Thinking

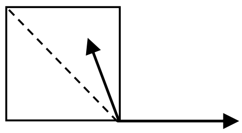
a) To see if the angle was closer to 135° or to 180°, I used a square (90°) and its diagonal (45°) to make a 135° angle.

• The angle was closer to a straight angle (180°) than to a 135° angle.

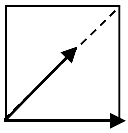
Example 1 Comparing Angles to Benchmarks [Continued]

Solution

b) as close to 90° as to 135°



c) closer to 45°



Thinking

b) I used a square and its diagonal to make a 135° angle.

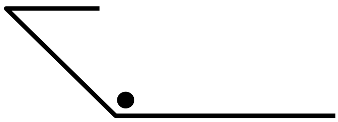
• The angle was exactly halfway between 90° and 135°.

c) I used the diagonal of a square to make a 45° angle.

• The angle seemed to be exactly 45°.

Example 2 Estimating Angle Sizes with a Protractor

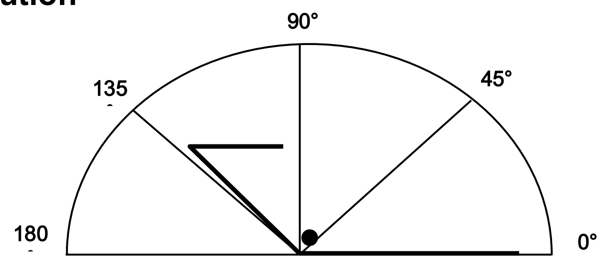
a) Estimate the size of this angle using your improvised protractor.



b) Finish drawing the sides of the shape so that the other bottom angle is about 60°.

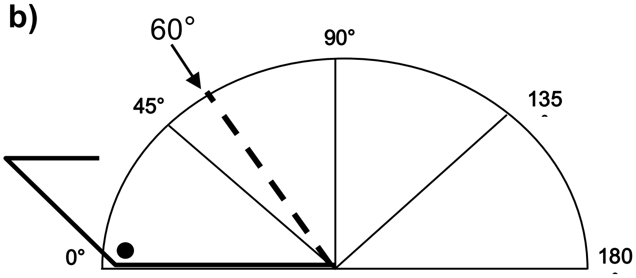
Solution

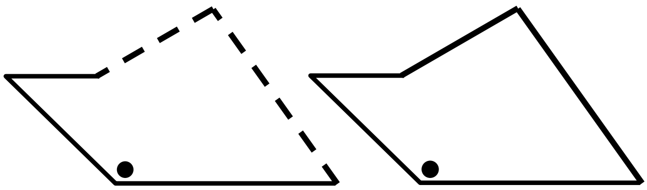
a)



The angle is almost 135°.

b)





Thinking

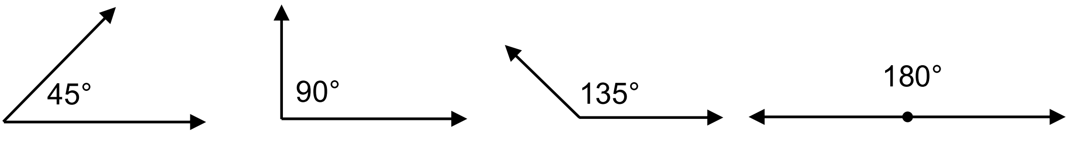
a) I could tell by looking that the angle was between 90° and 135° but closer to 135°.

b) I used my protractor to create a 60° angle at the bottom right — I had to flip the protractor over so that the angles went clockwise instead of counterclockwise.

• I finished the shape by adding 2 more sides.
(I knew that there were other possibilities.)

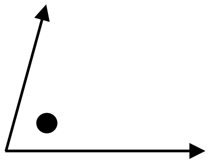
Practising and Applying

Here are some benchmark angles to help you.

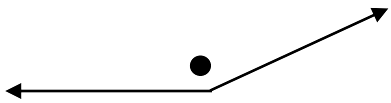


1. Estimate the size of each angle by comparing with benchmark angles. Do not use a protractor.

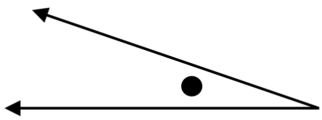
a)



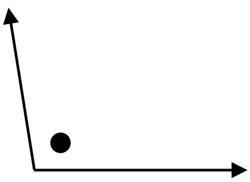
b)



c)



d)



2. Use your improvised protractor to check your estimates in **question 1**.

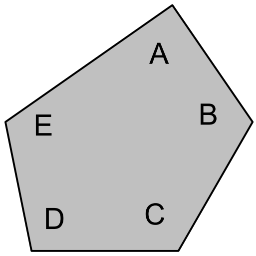
3. Draw an angle that is about each size. Use your protractor.

a) 30° **b)** 100°

c) 160° **d)** 75°

4. Use your protractor to show that a 135° angle is 3 times the size of a 45° angle.

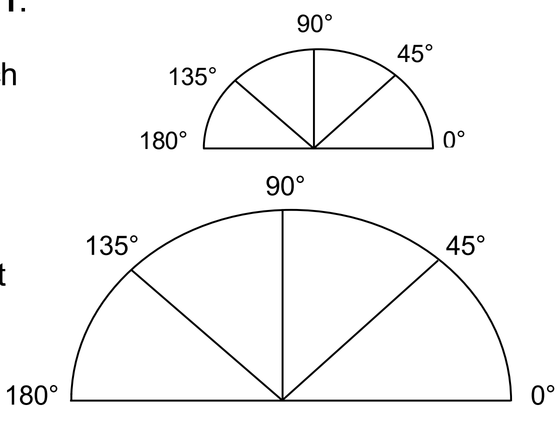
5. Which angles in this shape are about 120°? Explain how you know.



6. Explain how you could fold your protractor to estimate an angle of about 22°.

7. Some letters of the alphabet have angles you can measure. For example, L and F have 90º angles. Which letters have angles that are about 30°? Describe what you did.

8. Gembo made a protractor that was twice as big as Tandin's protractor. Will Gembo's angle measures be twice as big as Tandin's? Explain your thinking.



5.3.1 Volume

Try This

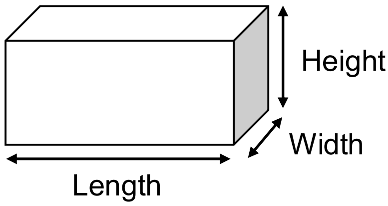
This picture shows the top of a box.

A. i) How many centimetre cubes would it take to build a model of the box?

ii) Build or sketch the model.

The top of this box is 20 cm by 12 cm.
The box is 4 cm deep.

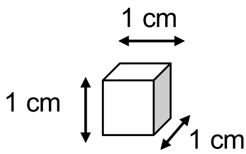
This box is called a **three-dimensional shape** or **3-D shape** because it has 3 dimensions: length, width, and height.



• **Volume** is a measure of the amount of space a 3-D shape takes up.

• If you build a shape out of centimetre cubes, the volume is the number of cubes you use and is described in **cubic centimetres** (**cm^3**).

• A cubic centimetre is the amount of space taken up by a cube that is 1 cm on each edge.

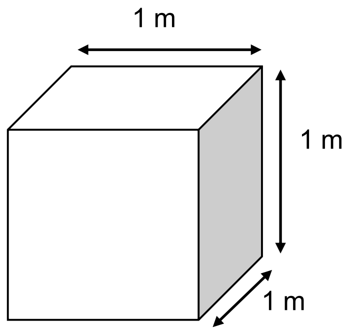


1 cubic centimetre
1 cm^3

For example:

This shape uses 16 centimetre cubes, so it has a volume of 16 cm^3.

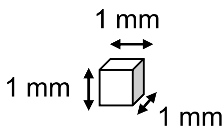
• Sometimes the volume of a 3-D shape is so large that it is measured in **cubic metres** (**m³**). A cubic metre is the amount of space taken up by a cube that is 1 m on each edge.



1 cubic metre
1 m^3

For example:
The volume of this building would be measured in cubic metres.

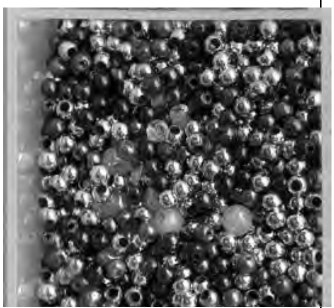
• Sometimes the volume of a 3-D shape is so small that it is measured in **cubic millimetres** (**mm³**). A cubic millimetre is the amount of space taken up by a cube that is 1 millimetre on each edge.



1 cubic millimetre
1 mm^3

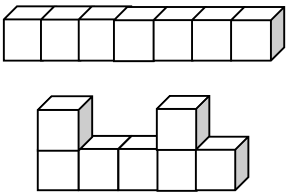
For example:

The volume of one of these beads might be measured in cubic millimetres.



B. What is the volume of the box in **part A** in cubic centimetres?

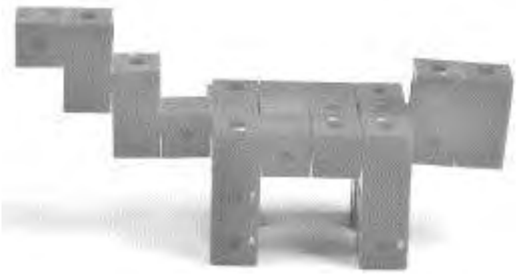
Examples

Example Building Different Shapes with the Same Volume	
Build two different shapes, each with a volume of 7 cm^3.	
Solution 	**Thinking**  • I used 7 centimetre cubes, each with a volume of 1 cm^3. • No matter how I arranged them, the volume was always 7 cm^3.

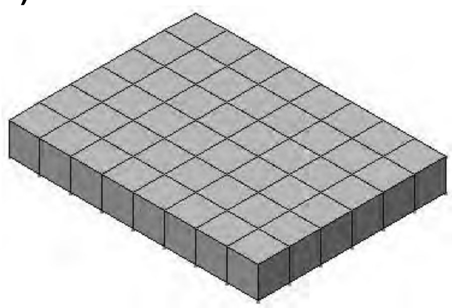
Practising and Applying

1. What is the volume of each? (Each cube is 1 cm^3.)

b)

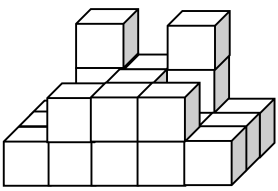


c)



d)

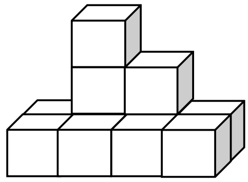
e)



2. a) Use cubes to build three different shapes, each with a volume of 20 cm^3. Sketch your shapes.

b) Is it possible to build a rectangular prism with a volume of 20 cm^3? If so, what are the dimensions?

3. The base of a shape is made of 8 centimetre cubes. Two smaller layers, one smaller than the other, are stacked on top, as shown here.



a) What is the volume of the shape?

b) What is the greatest volume a shape can have if follows the same rules as above: an 8 cm^3 base with two smaller layers stacked on top?

4. Use cubes to build a rectangular prism that is about the size of a cell phone. Sketch the prism. What is its volume?

5. How much bigger is 1 cm^3 than 1 mm^3? Explain your thinking.

6. How is measuring volume like measuring area? How is it different?

5.3.2 Capacity

Try This

It is important to drink enough water every day.

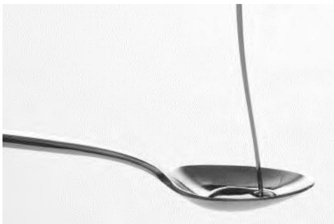
A. About how many glasses of water do you drink in one day?

- The **capacity** of a container tells how much it can hold.
- Capacity is usually measured in **litres** (**L**) or **millilitres** (**mL**).

1 L = 1000 mL   1 mL = 0.001 L

- Millilitres measure the capacity of containers that hold small amounts. Litres are for containers that hold greater amounts.

A small spoon has a capacity of about 5 mL.



A drinking glass has a capacity of about 300 mL.

A jug like this has a capacity of about 2 L.

An oil drum has a capacity of about 250 L.

• There is a special relationship between capacity and volume.

A container that is 10 cm × 10 cm × 10 cm, or 1000 cm^3, holds 1 L of water.

A container that is 1 cm × 1 cm × 1 cm, or 1 cm^3, holds 1 mL of water.

• You can use the relationship between 1 mL and 1 cm^3 of water to find the volume of objects with irregular shapes. To do this, you place the object completely in water and then measure how much the water rises.

For example:

- The cube structure to the right was put in 600 mL of water. A pencil was used to make sure the whole structure was under water.
- The water level rose from 600 mL to 660 mL, or 60 mL.
- Since a volume of 1 cm^3 raises the water level by 1 mL, a volume of 60 cm^3 would raise the water level by 60 mL.
- That means the cube structure has a volume of 60 cm^3. (Since each cube is 1 cm^3, there must be 60 cubes in the structure.)

B. i) About how many litres of water do you drink each day?

ii) How many millilitres of water is that?

Examples

Example Comparing Capacities	
Container A holds 1.5 L of water. Container B holds 1520 mL. Which container holds more?	
Solution 1.5 L = 1000 mL + 500 mL = 1500 mL 1500 mL < 1520 mL Container B holds more.	**Thinking** • I knew that 1 L was 1000 mL, so 0.5 L was 500 mL. 

Practising and Applying

1. Which size container holds more?

a) 750 mL or 3 L

b) 250 mL or 1 L

c) 1025 mL or 1 L

2. Estimate the size of a container that would hold four glasses of water.

3. Would you measure the capacity of each in litres or millilitres?

a) a bucket

b) a cupped hand

c) a soup bowl

d) a sink

4. A 2 L bottle is full of water. How many glasses of each size can be poured from the bottle?

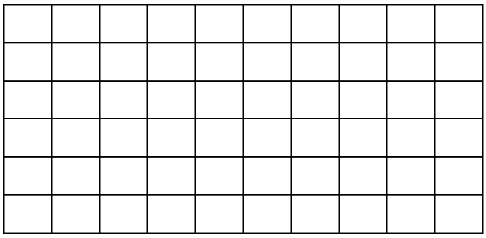
a) 100 mL **b)** 200 mL

c) 350 mL **d)** 750 mL

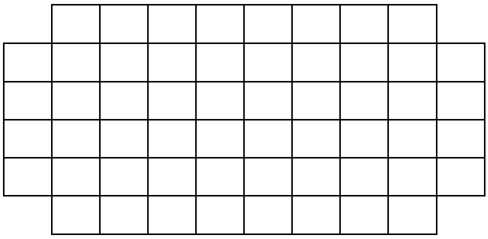
5. About how much water do you think a teacup holds? Test to check your estimate.

6. Put water in a measuring cup. Find the volume of a small object by putting it in the water and seeing how much the water level rises. Describe what you did and what you discovered.

7. a) Use centimetre grid paper. Cut out a 10 cm-by-6 cm rectangle.

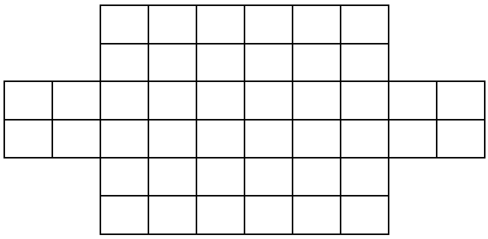


Then cut a square out of each corner.



Fold the paper to make a box with no top. How many centimetre cubes will the box hold? What is its capacity in millilitres?

b) Cut three more squares out of each corner and then fold to make another box with no top.



How many centimetre cubes will this box hold? What is its capacity in millilitres?

8. Describe how volume is different from capacity. Describe how they are the same.

9. Why is it useful to have both litre and millilitre units for measuring capacity?

5.3.3 Metric Units

Try This

In the Olympic games, an archery target is 70 m from the archer.

A. How does that distance compare to your height?

B. Why is it important to have the same distance for all competitors?

• Measuring is an important part of our everyday life. You can use the measurement of one item to estimate the measurements of other items.

For example:

If you know that your hand is about 15 cm long, you can estimate how long other objects are, such as the length of a table or someone's height.

• **Standard units** of measurement are used so that other people will understand.

For example:

If you say a table is 20 sticks long, other people will not know how long it is unless you show them the stick you used. If you say it is 45 cm long, they will know what it means because centimetres are a standard measure.

• One reason we use the **metric system** of measurement is to make it easier to convert measurements from one unit to another.

• Metric measurement relationships are based on multiplying and dividing by 10s, 100s, 100s, and so on. You can use place value to convert measurements.

• The chart below shows the prefixes for the basic metric units, along with the symbols normally used.

Thousands	**Hundreds**	**Tens**	**Ones**	**Tenths**	**Hundredths**	**Thousandths**
kilo (k)	**hecto (h)**	**deca (da)**		**deci (d)**	**centi (c)**	**milli (m)**
1000	100	10	1	0.1 or $\frac{1}{10}$	0.01 or $\frac{1}{100}$	0.001 or $\frac{1}{1000}$

For example:

- You use the prefix *deca* when you have 10 of a unit.

- You use the prefix *milli* when you have 0.001 or $\frac{1}{1000}$ of the unit.

• This chart shows the names of metric length units based on the metre.

The base unit (metre)

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
kilo	hecto	deca		deci	centi	milli
1 km	1 hm	1 dam	1 m	1 dm	1 cm	1 mm
1000 m	100 m	10 m	1 m	0.1 m	0.01 m	0.001 m

When a metre is the base unit, the other units are: kilometre, hectometre, decametre, decimetre, centimetre, and millimetre.

• You can use a place value chart and the relationships between units to go from a larger unit to a smaller unit or from a smaller unit to a larger unit.

- If 1 m = 100 cm, then 41 m = (41 × 100) cm.

You can multiply by 100 by moving each digit two places to the left.

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
		4	1			
4	1	0	0			

41 m = 4100 cm

- To go from 4100 cm to 41 m, you divide by 100 instead and move each digit of 4100 in the opposite direction, two places to the right. You can ignore the 0s after the decimal point in 41.00.

If 1 cm = 0.01 or $\frac{1}{100}$ m, then 4100 cm = (4100 ÷ 100) m.

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
4	1	0	0			
		4	1	0	0	

4100 cm = 41 m

- To change 2.2 kg to grams:

If 1 kg = 1000 g then 2.2 kg = (2.2 × 1000) g.

You can multiply by 1000 by moving each digit three places to the left.

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
			2	2		
2	2	0	0			

2.2 kg = 2200 g

It makes sense that 2.2 kg should be more than 2000 g, since 2 kg = 2000 g.

C. What is 70 m in millimetres? Did you multiply or divide? Why?

Examples

Example 1 Relating Measurements to Familiar Objects

Describe something that might have each measurement.

a) 30 cm **b)** 1 m **c)** 10 m

Solution	Thinking
a) My ruler is 15 cm long, so two rulers would be 30 cm.	I used objects in the classroom. **a)** The length is printed on my ruler.
b) 1 m is the distance from the floor to the door knob.	**b)** For 1 m, I looked for something that was as long as a metre stick.
c) 10 m is about the length of the classroom.	**c)** For 10 m, I imagined 10 metre sticks lying end to end.

Example 2 Relating Metric Measurements

Complete each. Show your work.

a) 52.3 dam = ___ m **b)** 64 dam = ___ dm

c) 412 m = ___ dm **d)** 65 cm = ____ m

Solution	Thinking
a) 52.3 dam = 523 m	**a)** I knew that if 1 dam = 10 m, then 52.3 dam = (52.3 × 10) m, so I moved the digits of 52.3 one place to the left.

1000s	100s	10s	1s	10ths	100ths	1000ths
		5	2	3		
	5	2	3			

Solution	Thinking
b) 64 dam = 6400 dm	**b)** I knew that if 1 dam = 100 dm, then 64 dam = (64 × 100) dm, so I moved the digits of 64 two places to the left and filled in the zeros that I needed.

1000s	100s	10s	1s	10ths	100ths	1000ths
		6	4			
6	4	0	0			

c) 412 m = 4120 dm

c) I knew that if 1 m = 10 dm, then 412 m = (412 × 10) dm, so I moved the digits of 412 one place to the left.

1000s	100s	10s	1s		10ths	100ths	1000ths
	4	1	2				
4	1	2	0				

d) 65 cm = 0.65 m

d) I knew that if 1 cm = 0.01 or $\frac{1}{100}$ m, then 65 cm = (65 ÷ 100) m, so I moved the digits of 65 two places to the right and filled in the zero that I needed in front of the decimal.

1000s	100s	10s	1s	10ths	100ths	1000ths
		6	5			
			0	6	5	

Practising and Applying

1. Fill in each blank below with the appropriate measurement from the list to the right.

a) The distance between Paro and Thimphu is ____.

b) A book is about ____ long.

c) A tall person is about ____ tall.

d) A fingernail is about ____ thick.

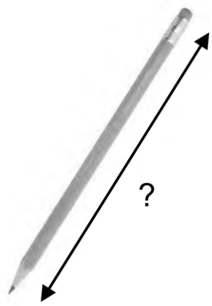
e) A hand is about ____ wide.

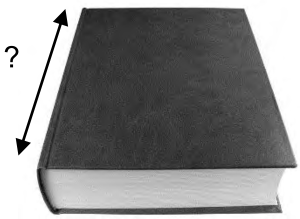
f) The distance I can walk in 40 min is ____.

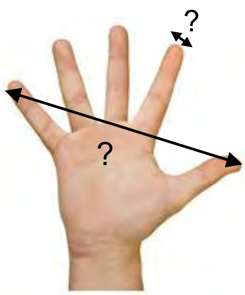
g) A new pencil is about ____ long.

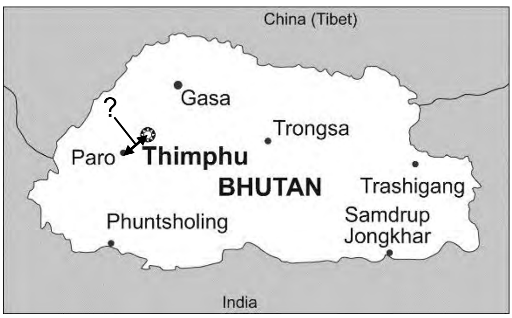
h) A fingertip is about ____ wide.

1 cm
10 cm
17 cm
3 km
2 mm
65 km
2 m
30 cm









Here is a metric unit chart to help you.

kilo	hect	deca		deci	centi	milli
k	h	da		d	c	m
1000	100	10	1	0.1 or $\frac{1}{10}$	0.01 or $\frac{1}{100}$	0.001 or $\frac{1}{1000}$

2. Tell whether the number value for each new measurement will be less or greater. Explain how you know.

	Going from	to
a)	centimetres	metres
b)	millimetres	metres
c)	grams	kilograms
d)	decametres	decimetres

3. Complete.

a) 34 cm = ___ mm

b) 51.2 hm = ___ m

c) 416 dm = ___ mm

d) 41 cm = ____m

e) 131 g = ___ kg

4. Which measurement is greater in each pair? Explain how you know.

a) 32 cm or 512 mm

b) 22,000 cm or 1 km

c) 3.1 kg or 2120 g

5. Complete using the correct unit.

a) 316 cm = 3.16 ___

b) 715 m = 0.715 ___

c) 52.1 kg = 52,100 ___

d) 5.21 L = 5210 ___

6. Suppose you changed 6.213 km to another metric length unit such as metres, centimetres, or millimetres.

a) Which digits will be in the new measurement?

b) Why are the digits you listed in **part a)** the only possible digits?

7. Explain how the relationship between grams and kilograms is the same as each relationship below.

a) the relationship between metres and kilometres

b) the relationship between millimetres and metres

8. Why is it simple to change from one metric unit to another?

5.4.1 The 24-hour Clock System

Try This

The Druk Air schedule shows there is a Wednesday flight that leaves Paro at 10:00 a.m. and arrives in Bangkok at 3:20 p.m. The Friday flight leaves at 8:40 am and arrives at 1:55 p.m.

A. Which flight is faster? Explain your thinking.

• Bus and airline schedules are based on a **24-hour clock** system. In this system, times start at midnight at 00:00 (0 hours, 0 minutes), go to noon, which is 12:00, and then continue through the afternoon and evening to 13:00, 14:00, 15:00, 16:00, …. until just before midnight, 23:59.

This is how the 12-hour clock system compares to the 24-hour clock system:

Midnight	Morning	Noon	Afternoon	Evening
12:00 a.m.	6:00 a.m.	12:00 p.m.	6:00 p.m.	11:59 p.m.
00:00	06:00	12:00	18:00	23:59

• You can think of time in the 24-hour clock system as the number of hours after midnight.

For example:

14:20 means that 14 hours and 20 minutes have passed since midnight.

• Notice that morning times for both systems are similar, but the times between noon and midnight are not. The hour number in the 24-hour clock system is 12 more than in the 12-hour clock system. The minutes and seconds are the same.

For example:

6:40 a.m. is 06:40 (note the "0"; also note that the “a.m.” is not written)

1:40 p.m. is 13:40 (1 + 12 = 13)

•The 24-hour clock system helps avoid misunderstandings about time.

For example:

In the 12-hour clock system, if you say "It is 1 o’clock", it is not clear which 1 o’clock you mean. In the 24-hour clock system, 1:00 a.m. is 01:00 and 1:00 p.m. is 13:00, so there is no confusion.

Examples

Example Interpreting Times Using the 24-Hour Clock

a) These times are all on the same day. Order them from earliest to latest.

16:32 2:30 a.m. 15:23 04:35 5:20 p.m.

b) How much time is there between the earliest time and the latest time?

Solution

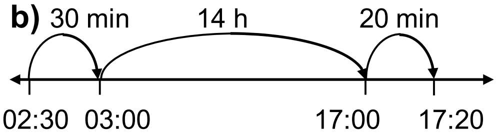
a) 2:30 a.m. = 02:30

5:20 p.m. = 17:20

The times in order are

2:30 a.m., 04:35, 15:23, 16:32, 5:20 p.m.

b)



14 h + 30 min + 20 min = 14 h 50 m

There are 14 h and 50 min between 02:30 and 17:20.

Thinking

a) I wrote the 12-hour times in 24-hour time.
I added 12 to the hour for the afternoon (p.m.) time.

• I ordered the hours just like I would order numbers.

b) I used a time line.

• I thought of the time in three sections:

- the time from 02:30 to 03:00,
- the time from 03:00 to 17:00, and
- the time from 17:20 to 17:20.

I added these three sections.

Practising and Applying

1. Karma's brother was born 13 min after 3 p.m. At what time was he born in 24-h clock time?

2. Write each time in 24-hour time.

a) 4:15 a.m. **b)** 6:23 p.m.

c) noon **d)** 9:34 p.m.

3. Write each time in 12-hour time.

a) 07:00 **b)** 18:25

c) 22:17 **d)** 11:33

4. How much time is there between each pair of times?

a) 3 a.m. and 13:30 the same day

b) 14:20 one day and 09:15 the next day

c) 15:15 one day and 07:00 the next day

5. Why are these times never used?

a) 12:69 **b)** 27:22

6. Explain how to change a 24-hour clock time to a 12-hour clock time.

UNIT 5 Revision

1. What is the perimeter of each?

a) a hexagon with each side length 5 cm

b) a square with each side length 7 cm

c) an equilateral triangle with each side length 10 cm

2. Draw two different shapes, each with a perimeter of 20 cm.

3. Draw a 6-sided shape. One side should be twice as long as another side and the perimeter should be 30 cm.

4. a) Draw a 5 cm-by-9 cm rectangle.

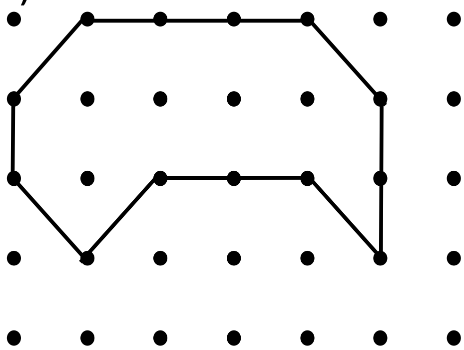
b) Draw another rectangle with the same perimeter.

c) Which rectangle has the greater area? How do you know?

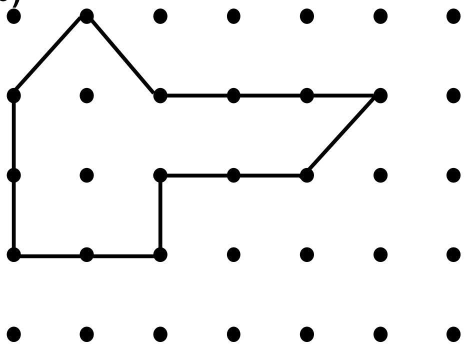
5. Two rectangles have the same perimeter but one rectangle is 2 cm longer. How do the widths of the two rectangles compare?

6. Find the area of each.

a)



b)



7. Draw a shape with each area on a grid.

a) $3\frac{1}{2}$ square units

b) $11\frac{1}{2}$ square units

8. What is the perimeter and area of each rectangle?

a) length 8 cm and width 3 cm

b) length 12 cm and width 8 cm

9. Three different rectangles have the same area, 40 cm^2.

a) List the dimensions of three possible rectangles.

b) Which has the least perimeter?

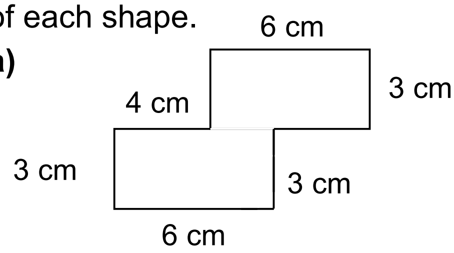
10. Three different rectangles have the same perimeter, 80 cm.

a) List the dimensions of three possible rectangles.

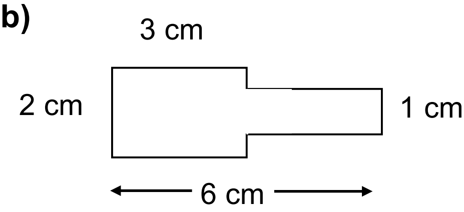
b) Which has the greatest area?

11. Calculate the area and perimeter of each shape.

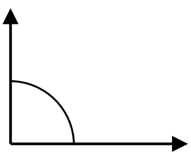
a)



b)



12. Copy the angle below four times. For each measurement listed, sketch the unit angle inside the angle.

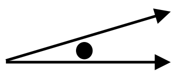


a) 2 units **b)** 4 units

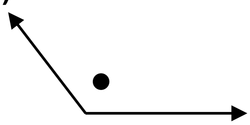
c) 1.5 units **d)** 0.5 units

13. Estimate the size of each angle using your improvised protractor.

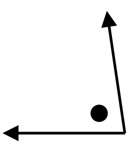
a)



b)



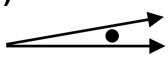
c)



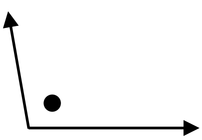
d)

14. Estimate the size of each angle without using your protractor.

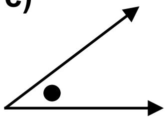
a)



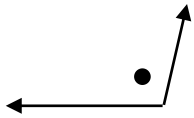
b)



c)



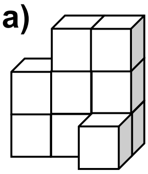
d)



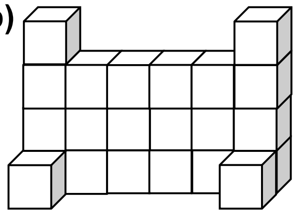
15. A rectangular prism has a volume of 30 cm^3. What might its dimensions be?

16. What is the volume of each? (Hint: there are no hidden cubes.)

a)



b)



17. Order from least to greatest:

3200 mL, 4 L, 2.8 L, 1500 mL, 1.2 L

18. a) Would you measure the capacity of a school bag in litres or in millilitres?

b) Describe two amounts that you would measure in litres instead of in millilitres.

19. Describe a container with each capacity.

a) about 10 L **b)** about 10 mL

20. Complete.

a) 352 mm = ____ m

b) 4.2 kg = ____ g

c) 53.3 dam = ____ m

d) 1 km = ____ dm

21. Complete.

a) 452 mm = 0.452 ____

b) 518 cm = 5180 ____

c) 42 dm = 4.2 ____

22. How do you change a kilometre measure to a decimetre measure? Explain why it works.

23. Write each in 24-hour clock time.
a) 1:23 p.m. **b)** midnight

24. Write each in 12-hour clock time.
a) 17:49 **b)** 06:17
c) 15:18 **d)** 18:15

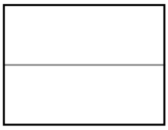
25. How much time is there between each pair of times?
a) 13:45 and 18:12 the same day
b) 19:30 one day and 03:15 the next day

UNIT 6 GEOMETRY

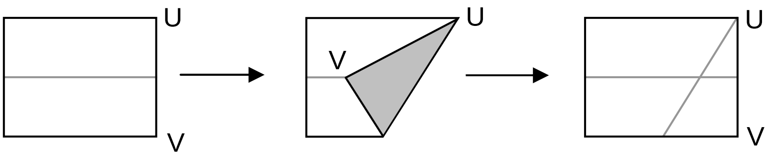
Getting Started

Use What You Know

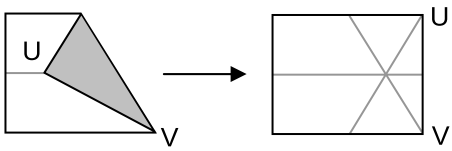
A. Fold a rectangular piece of paper in half lengthwise.
Make a crease across the middle, then unfold.



B. Slide vertex V along the crease until U and V are two vertices of a triangle. Make another crease, then unfold.



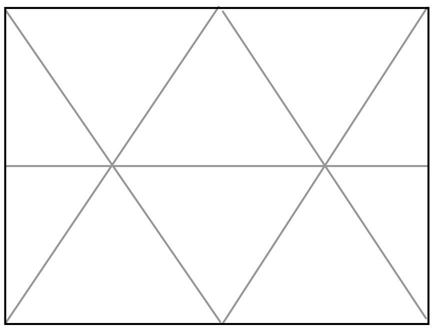
C. Slide vertex U along the first crease until U and V are two vertices of a triangle. Make another crease, then unfold.



D. Repeat for the two vertices at the other end of the rectangle.

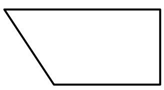
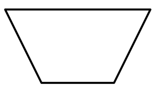
E. Identify these things on your rectangle:

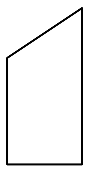
i) an obtuse angle
ii) an acute angle
iii) a right angle
iv) congruent equilateral triangles
v) congruent isosceles triangles
vi) congruent scalene triangles
vii) a slide
viii) a flip
ix) a turn



Skills You Will Need

1. Which shapes are congruent?

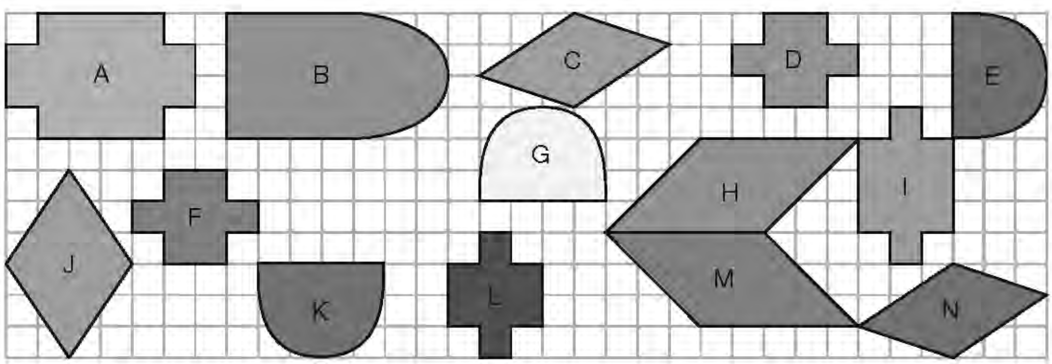
A

B

C

D


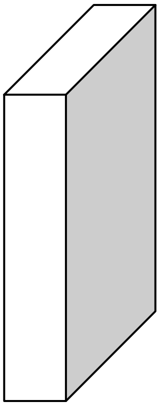
2. a) Which pairs of shapes below show a slide?

b) Which pair shows a flip?

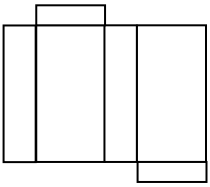
c) Which pairs show a turn?



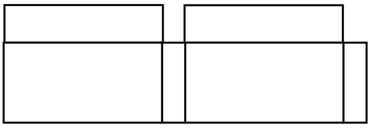
3. Which is a net for this rectangular prism?



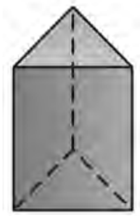
A.



B.

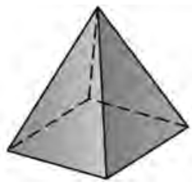


4. Name each 3-D shape.

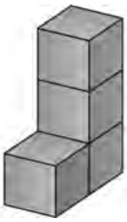
a) 

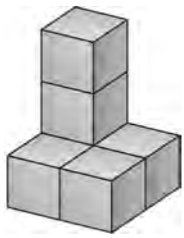
b) 

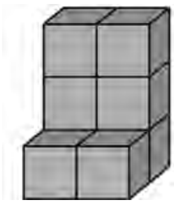
c) 

d) 

5. Use linking cubes to build each structure.

a) 

b) 

c) 

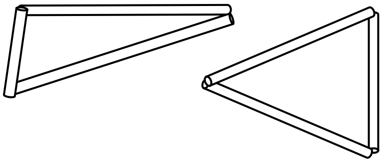
Chapter 1 Triangles and Quadrilaterals

6.1.1 Classifying Triangles by Side Length

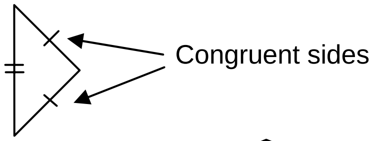
Try This

A. Use sticks to make six different triangles.

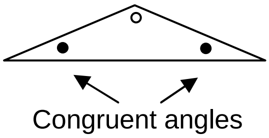
B. Sort your triangles into groups that have something in common.



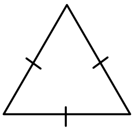
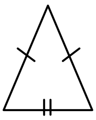
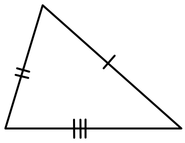
• Sides of equal length are called **congruent** sides. You can show that two sides are congruent by using the same number of marks.



• Angles of the same size are called congruent angles. You can use the same symbols to show angles are congruent.



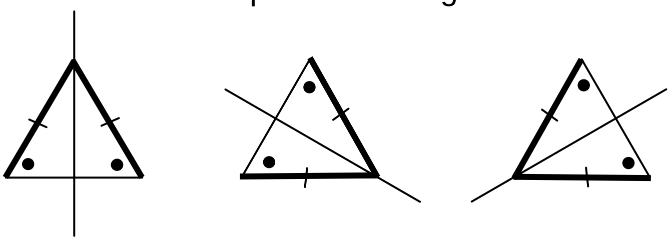
• You can group, or **classify**, triangles by the number of congruent sides.

A triangle with 3 congruent sides is an **equilateral triangle**.	A triangle with 2 congruent sides is an **isosceles triangle**.	A triangle with 0 congruent sides is a **scalene triangle**.
		

• Once you know the type of triangle, you can predict the number of congruent angles and the number of lines of symmetry.

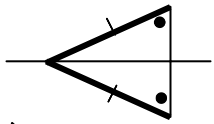
- When you fold a triangle along a **line of symmetry**, each side folds onto a congruent side and each angle folds onto a congruent angle.

- You can fold an equilateral triangle three ways to match congruent sides and angles. That means an equilateral triangle has three lines of symmetry.

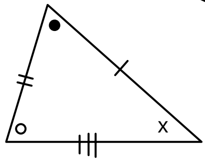


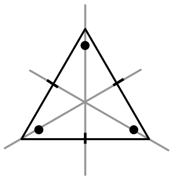
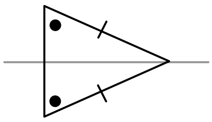
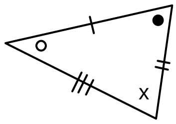
This also tells you that an equilateral triangle has 3 congruent angles.

- You can fold an isosceles triangle only 1 way to match congruent sides and angles. That means an isosceles triangle has exactly 1 line of symmetry and 2 congruent angles.



- You cannot fold a scalene triangle to match congruent sides and angles, so a scalene triangle has 0 lines of symmetry and 0 congruent angles.



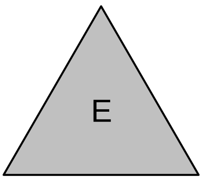
Equilateral triangle	Isosceles triangle	Scalene triangle
• 3 congruent sides • 3 congruent angles • 3 lines of symmetry 	• 2 congruent sides • 2 congruent angles • 1 line of symmetry 	• 0 congruent sides • 0 congruent angles • 0 lines of symmetry 

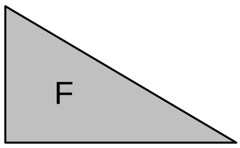
C. How many triangles in **part A** were isosceles? equilateral? scalene?

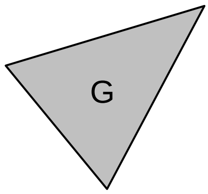
Examples

Example 1 Classifying Triangles by Measuring Side Lengths

What type of triangle is each?

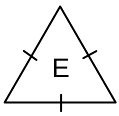




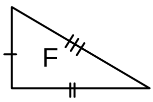


Solution

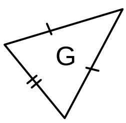
Triangle E is equilateral.



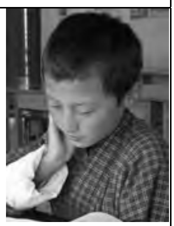
Triangle F is scalene.



Triangle G is isosceles.



Thinking

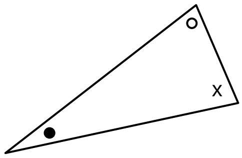


- I traced each triangle and measured the sides.
- On each triangle, I used the same number of marks on congruent sides.
- 3 congruent sides make a triangle equilateral.
- 2 congruent sides make a triangle isosceles.
- All sides different make a triangle scalene.

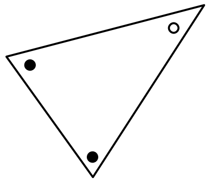
Example 2 Classifying Triangles Using Properties

Without measuring, classify each triangle by side length.

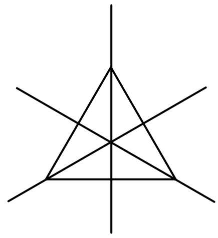
a)



b)



c)



Lines of symmetry

Solution

a) Scalene

b) Isosceles

c) Equilateral

Thinking

a) All the angles were different, so I knew all the side lengths were different too.

b) There were exactly 2 congruent angles, so I knew it must have exactly 2 congruent sides.

c) There are 3 lines of symmetry, so I knew it had 3 congruent sides and angles.

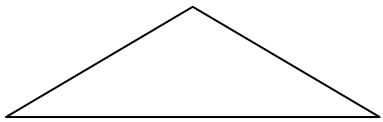
Practising and Applying

1. Copy and complete the chart.

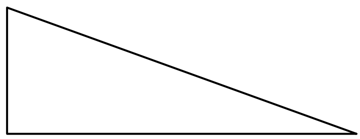
	Equilateral triangle	Isosceles triangle	Scalene triangle
Number of congruent sides			
Number of congruent angles			
Number of lines of symmetry			
Sketch of example			

2. For each triangle, measure the side lengths and classify.

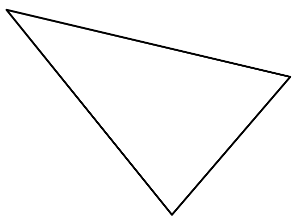
a)



b)

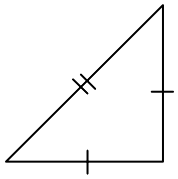


c)

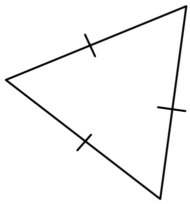


3. Sketch each triangle. Without measuring, mark the angles to show congruence.

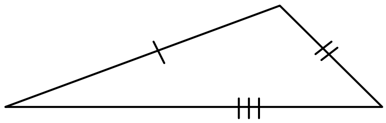
a)



b)

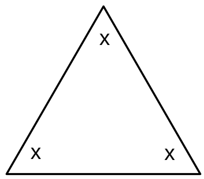


c)

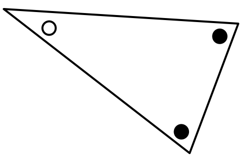


4. Sketch each triangle. Without measuring, mark the sides to show congruence.

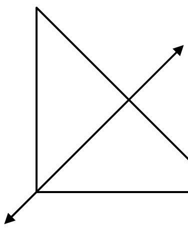
a)



b)

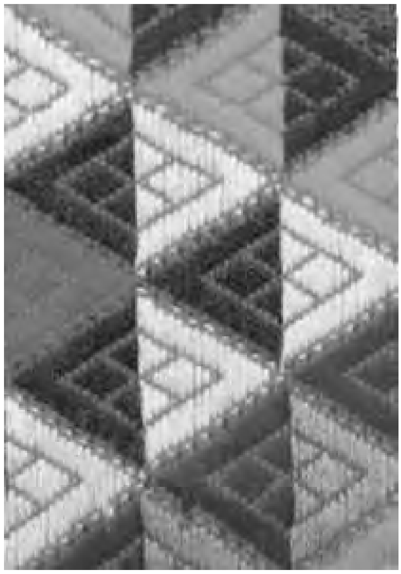


c)



This triangle has exactly 1 line of symmetry.

5. What type of triangles do you see in this fabric?



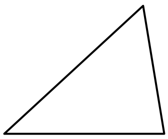
6. A triangle has exactly 1 line of symmetry. One side is 3 cm long and another is 4 cm long.

a) What kind of triangle is it? How do you know?

b) Draw two possible triangles.

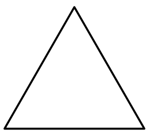
7. Have these been classified correctly? Explain your thinking.

a)



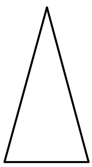
Scalene?

b)



Equilateral?

c)



Equilateral?

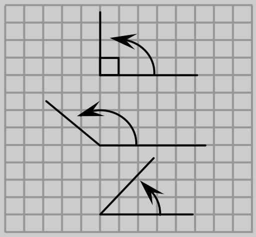
6.1.2 Classifying Triangles by Angle

Try This

A **right angle** is sometimes called a square corner. That is why it is usually marked with a square.

An **obtuse angle** is greater than a right angle.

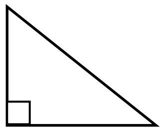
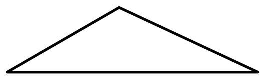
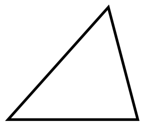
An **acute angle** is smaller than a right angle.



A. i) On grid paper, draw two right angles, two acute angles, and two obtuse angles. On each, draw a third side to create a triangle.

ii) Describe the other angles in each triangle as right, acute, or obtuse.

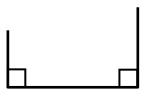
• In the previous lesson, you classified triangles by side length.
You can also classify triangles by angle:

When the greatest angle is a right angle, the triangle is a **right triangle**	When the greatest angle is obtuse, the triangle is an **obtuse triangle**	When the greatest angle is acute, the triangle is an **acute triangle**
		

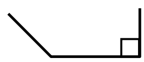
• Once you know the triangle type, you can predict whether the other angles are right, obtuse, or acute. This explains why:

- You can have only 1 obtuse or right angle in a triangle. If you have more than 1, the sides will not make a closed shape.

2 obtuse angles



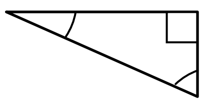
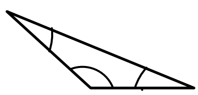
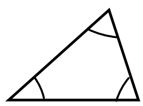
2 right angles



1 right angle and 1 obtuse angle

- This means that a right triangle has 1 right angle and 2 acute angles and an obtuse triangle has 1 obtuse angle and 2 acute angles.

- Since the greatest angle in an acute triangle is acute, an acute triangle must have 3 acute angles.

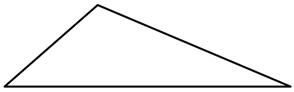
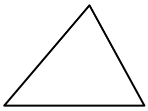
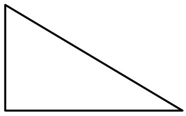
Right triangles ...	**Obtuse triangles ...**	**Acute triangles ...**
have 1 right angle and 2 acute angles 	have 1 obtuse angle and 2 acute angles 	have 3 acute angles 

B. Why were all the other angles you described in **part A ii)** acute?

Examples

Example 1 Classifying Triangles by Angle

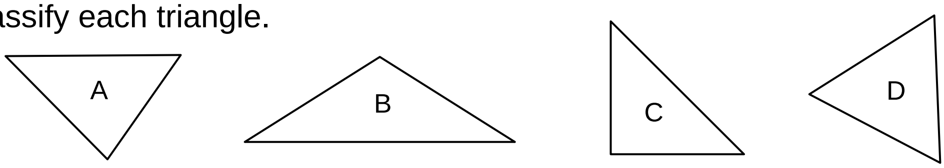
Classify each triangle by angle. Explain your thinking.

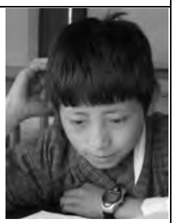
a)  **b)**  **c)** 

Solution	**Thinking**
a) This is an obtuse triangle because its greatest angle is obtuse. **b)** This is an acute triangle because all its angles are acute. **c)** This is a right triangle because its greatest angle is a right angle.	 • I compared the greatest angle in each to a right angle to see if it was right, obtuse, or acute. • I used the corner of a piece of paper as my right angle. **a)** The greatest angle is on top. It's larger than a right angle, so it must be an obtuse angle. **b)** All of the angles are smaller than a right angle, so they're all acute. **c)** The greatest angle is a right angle.

Example 2 Classifying Triangles by Side Length and Angle

Classify each triangle.



Solution 1	**Thinking**
Triangle A is scalene. Triangles B and C are isosceles. Triangle D is equilateral.	 • I measured the sides to see how many were the same length. • Then I classified by side length.
Solution 2	**Thinking**
Triangles A and D are acute. Triangle B is obtuse. Triangle C is right.	 • I compared the greatest angle in each to a right angle. • Then I classified by angle.

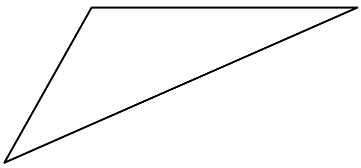
Practising and Applying

1. Copy and complete the table.

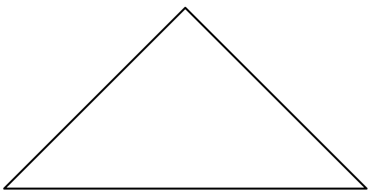
	Obtuse triangle	Right triangle	Acute triangle
Greatest angle			
Number of obtuse angles			
Number of right angles			
Number of acute angles			
Sketch of example			

2. Classify each by angle.

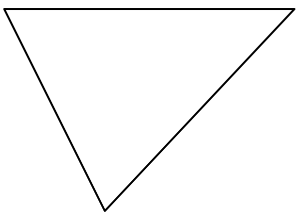
a)



b)

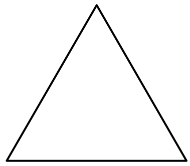


c)

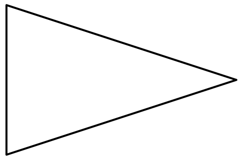


3. Classify each by angle and also by side length.

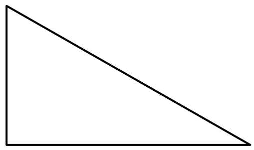
a)



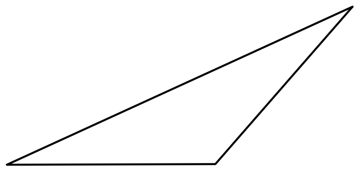
b)



c)



d)



4. Make three isosceles triangles:

• one with a right angle

• one with an obtuse angle

• one with only acute angles

Which would make the steepest roof? Why does that make sense?

5. Draw each triangle on grid paper.

a) a scalene right triangle

b) an isosceles obtuse triangle

6. Suppose an obtuse triangle is also isosceles. How do you know that its two congruent angles are acute angles?

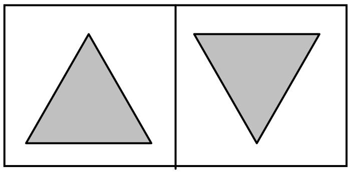
GAME: Triangle Dominoes

In this game, you use triangle cards as dominoes. Triangles match if they are the same kind of triangle.

Play in groups of two or three. You will need a deck of Triangle Dominoes Game Cards.

How to play

• Find a card with two congruent triangles. Place it face up to begin the row.

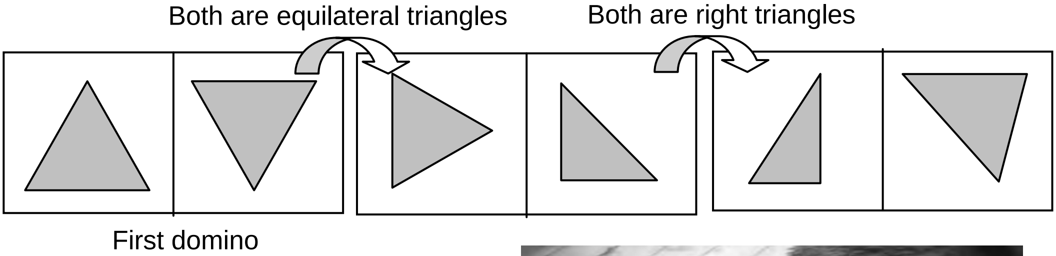


• Shuffle the rest of the deck and put it face down.

• Take turns drawing cards until each player has six cards.

• On your turn, add a card to the row of dominoes if it matches either one of the end triangles.

Triangles can match by side length classification (equilateral, isosceles, or scalene) or by angle classification (acute, obtuse, or right).

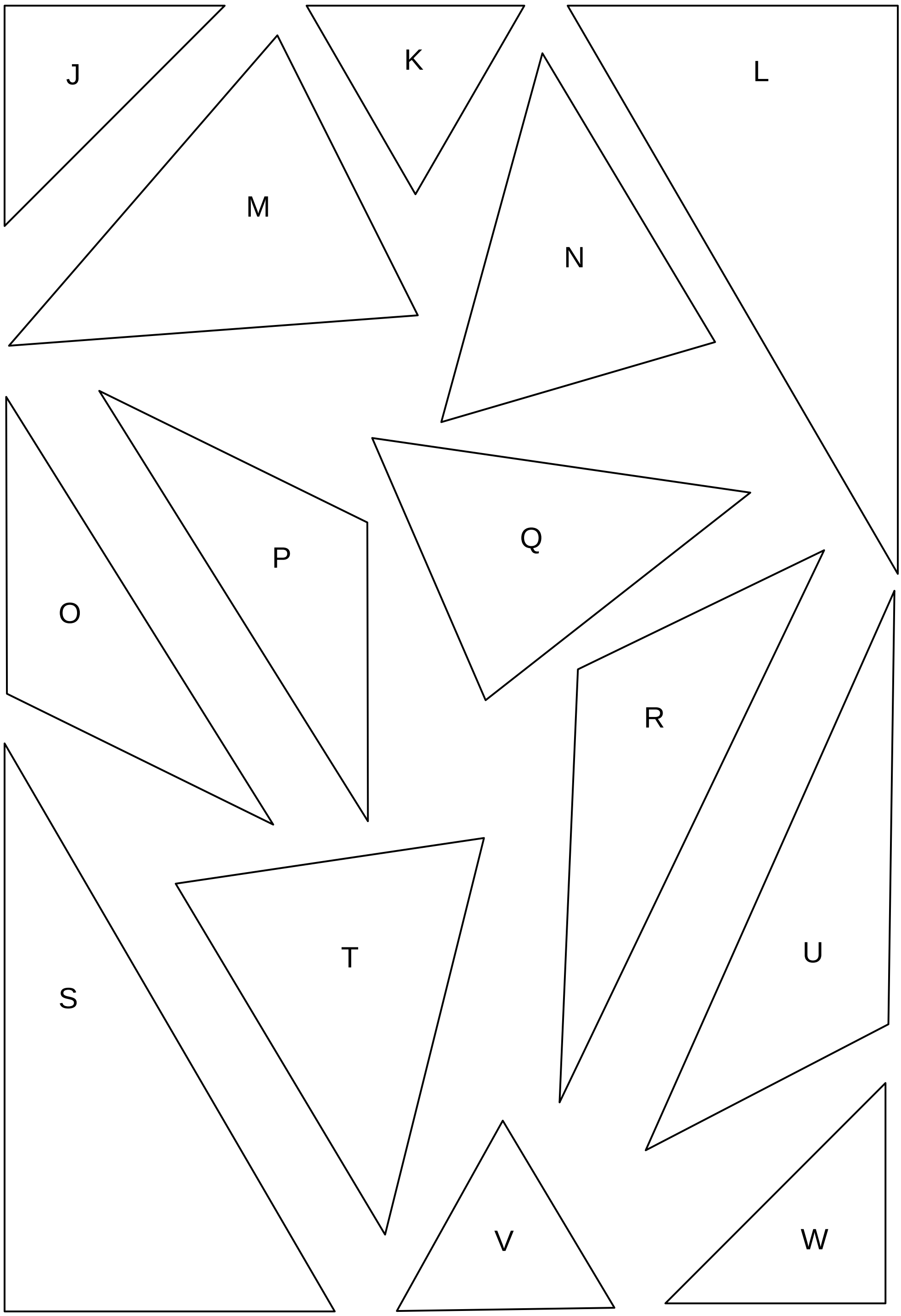


• If you do not have a matching card, draw a card from the deck. You must wait until your next turn to play again.

• The first player to play all his or her cards wins.

6.1.3 EXPLORE: Combining Triangles

Triangles can be combined to make other shapes.



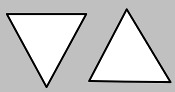
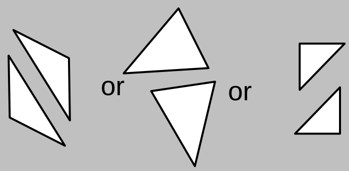
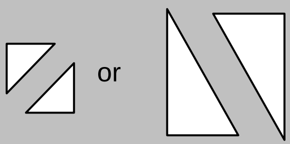
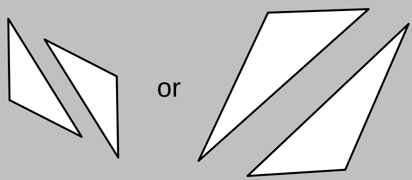
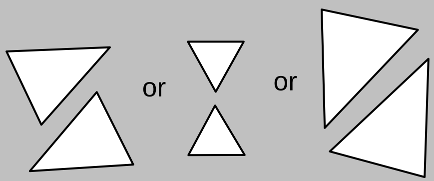
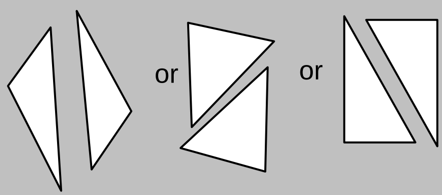
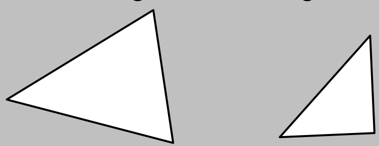
[Continued]

A. i) Trace the triangles on **page 41** and cut them out.
On each triangle, write its letter name.

ii) Label each triangle equilateral, isosceles, or scalene.

iii) Label each triangle obtuse, right, or acute.

B. Put together each pair of triangles to make a single shape. Make sure that one side of one triangle completely matches one side of the other triangle. Try to make as many shapes as possible. Sketch each shape you make.

i) 2 congruent equilateral triangles 	**ii)** 2 congruent isosceles triangles 
iii) 2 congruent right triangles 	**iv)** 2 congruent obtuse triangles 
v) 2 congruent acute triangles 	**vi)** 2 congruent scalene triangles 
vii) 2 different isosceles triangles with congruent bases 	

C. Which types of triangles from **part B** can be combined to make each shape?

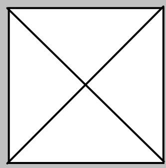
i) a square **ii)** a rectangle

iii) a parallelogram **iv)** a triangle

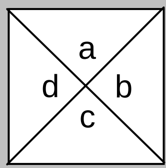
6.1.4 EXPLORE: Properties of Rectangles

If you know a shape is a rectangle, you can make predictions about its **diagonals**.

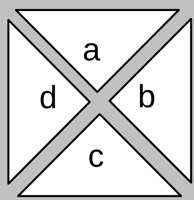
A. i) Make a square out of paper. Fold it to make creases on the diagonals and then unfold.



ii) Label the angles where the diagonals cross a, b, c, and d.



iii) Cut along the diagonals to create four triangles.

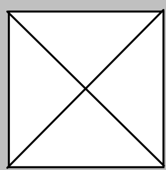


B. i) Classify each triangle by side length.

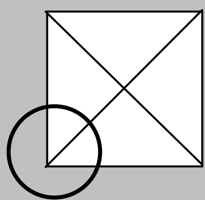
ii) Classify each triangle by angle.

iii) What do you notice about the sizes of the triangles?

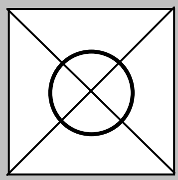
C. i) Put the triangles back together to make a square.



ii) What do you notice about the two angles formed by the diagonals at each corner of the square?

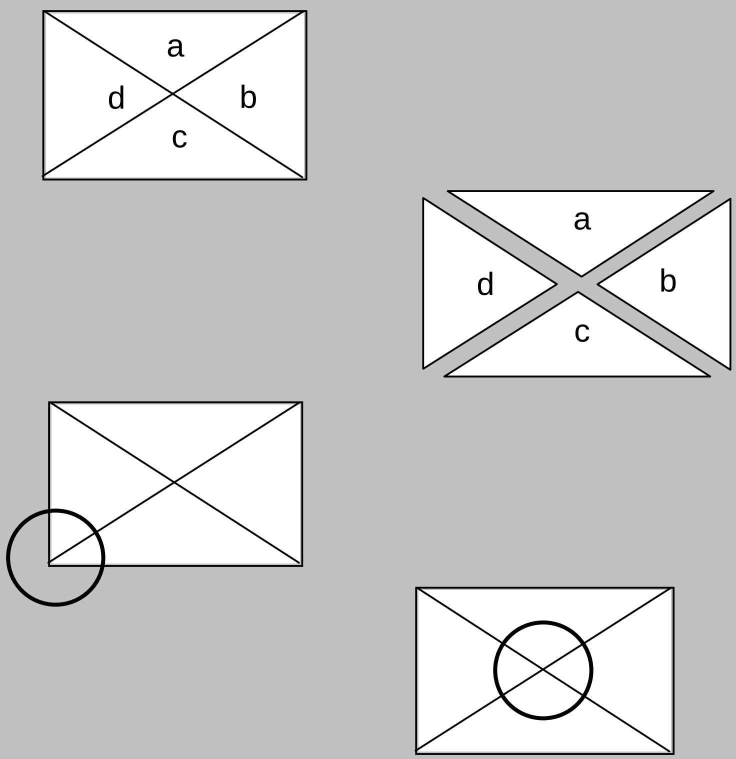


D. i) Where on each diagonal is the **intersection point** (the point where the diagonals cross)? How do you know?



ii) What type of angles do the diagonals make at their intersection point? How do you know?

E. Repeat **parts A to D** for a non-square rectangle.



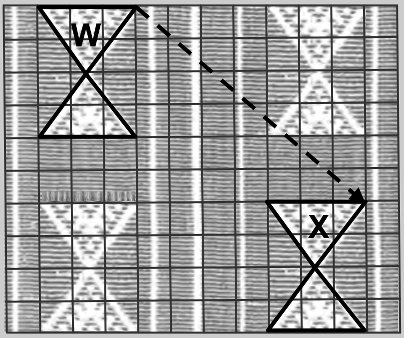
F. Compare your answers to **parts B to D** for the square and the non-square rectangle. How are they different? How are they the same?

Chapter 2 Transformations

6.2.1 Properties of Translations

Try This

This blanket is decorated with congruent shapes. A slide from Shape W to Shape X is shown.



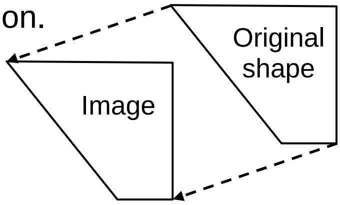
A. Describe how each vertex moves in the slide:

__ units right or __ units left

__ units up or __ units down

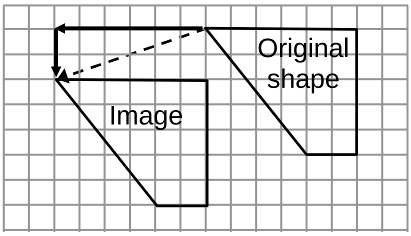
• A **transformation** moves a 2-D shape according to a rule.
It creates a new shape, called the **image**.

• A slide, or **translation**, is one type of transformation.
In a translation, because each vertex moves the same distance in the same direction, the image is congruent to the original shape and faces the same way.



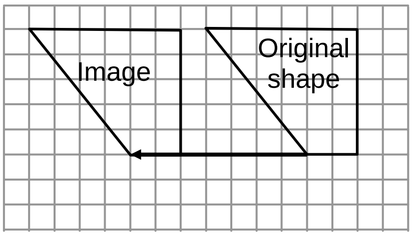
• To describe the arrow that shows the translation, you can state a **translation rule**.
It tells how the shape moved:
- the number of units left or right
- the number of units up or down

6 units left, 2 units down



• Sometimes a translation moves only left or right or only up or down.
For example, this shape moved 7 units left and 0 units up or down.

7 units left



• To draw a translation image, use the translation rule to locate the image of each vertex. Then connect them with line segments.

• If you know a transformation is a translation, then you know the following:
- Each vertex in the shape moves according to the same translation rule.
- The original shape and image are congruent and they face the same way.

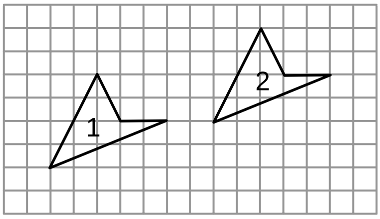
B. Would the line segments connecting each vertex to its image in **part A** be the same length? How do you know?

Examples

Example 1 Describing a Translation

a) Describe the translation rule that would move Shape 1 to Shape 2.

b) Describe the translation rule that would move Shape 2 to Shape 1.



Solution

a) 7 units right and 2 units up

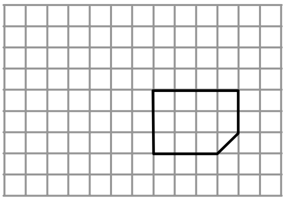
b) 7 units left and 2 units down

Thinking

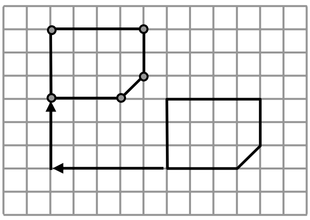
- Since every point moves the same way, I just counted units to see how one vertex moved.
- Since **part b)** is the reverse of **part a)**, I just changed right to left and up to down.

Example 2 Translating a Shape on a Grid

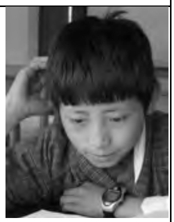
Translate the shape 5 units left and 3 units up.



Solution 1

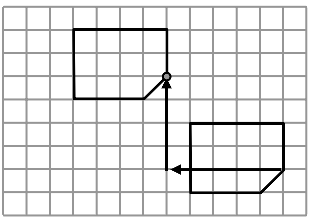


Thinking



- I translated each vertex by counting 5 units left and 3 units up.
- Then I connected the vertices.

Solution 2



Thinking

- I translated one vertex and then I drew a congruent shape.
- I used the grid to help me draw a congruent shape.

Practising and Applying

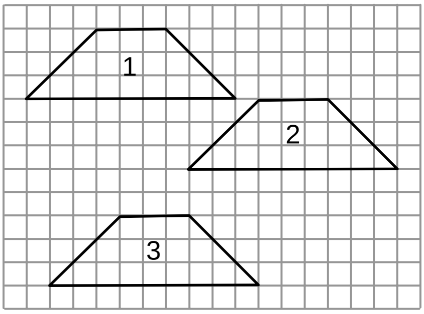
1. Describe each translation.

a) Shape 1 to Shape 2

b) Shape 1 to Shape 3

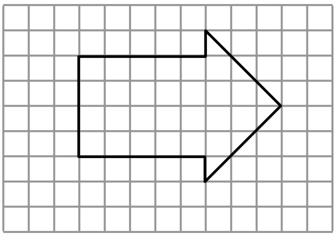
c) Shape 2 to Shape 3

d) Shape 3 to Shape 1

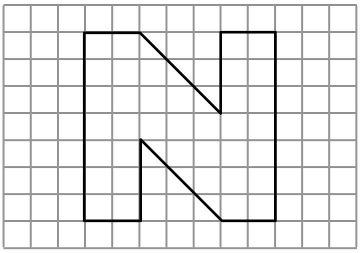


2. Copy each shape onto grid paper. Then translate it as described.

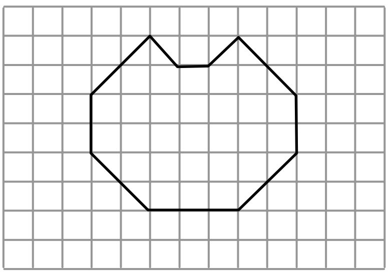
a) 3 units right, 8 units up



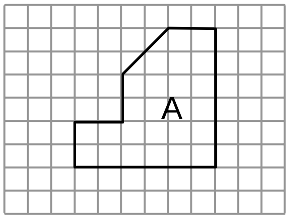
b) 9 units left



c) 5 units left, 4 units down



3. a) Copy Shape A onto grid paper. Translate it 4 units left and 5 units up to create Shape B.



b) Translate Shape B 10 units right and 1 unit down to create Shape C.

c) Describe a translation that moves Shape C to Shape A.

4. A triangle has three vertices, A, B, and C.

• B is 3 units left of A.

• C is 4 units above B.

a) Draw the vertices on grid paper and connect them.

b) Translate the triangle 3 units left. Draw it. Next, translate the image triangle 4 units up. Draw it.

c) How many congruent triangles do you see?

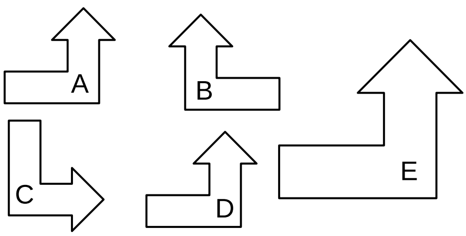
5. Nima translated Shape 1 to create Shape 2.

Eden translated Shape 2 using Nima's rule to create Shape 3.

Choki translated Shape 3 back to Shape 1.

Compare Choki’s translation rule to Nima’s rule.

6. Which is the translation image of Shape A? Explain your thinking.



6.2.2 Properties of Reflections

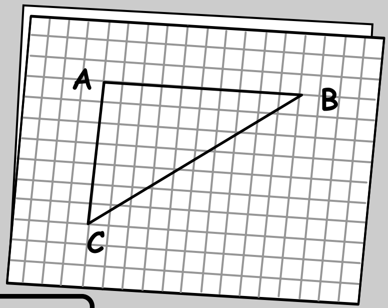
Try This

• Fold a piece of grid paper in half along a grid line with the grid on the outside.

• Draw a scalene triangle on one half.
Press hard so you can see its image on the other half of the paper.

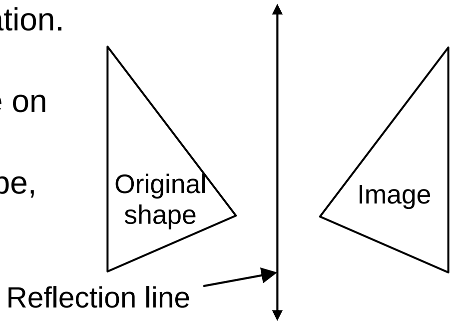
• Label its vertices. Then flip over the folded paper and trace the triangle on other side.

• Unfold the paper and compare the first triangle with its image.

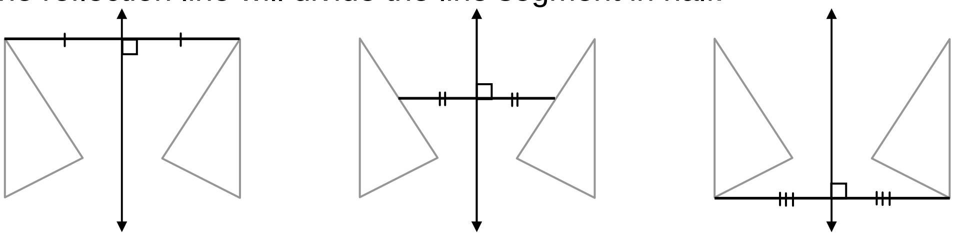


A. How far is each vertex from its image vertex?

• A **reflection** is another kind of transformation.
In a reflection, a shape is flipped across a **reflection line** to result in a mirror image on the other side of the line.
The image is congruent to the original shape, but it faces the opposite way.



• If a line segment is drawn from any point on the shape to its image point,
- the line segment will cross the reflection line at right angles.
- the reflection line will divide the line segment in half.



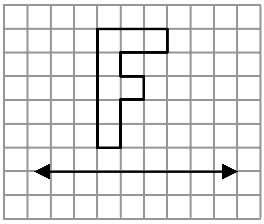
• If you know a transformation is a reflection, you know the following:
- The original shape and image are congruent but face the opposite way.
- The line segment joining a point on the shape to its image point crosses the reflection line at a right angle.
- The distance from a point on the shape to the reflection line is the same as the distance from its image point to the reflection line.

B. Which vertex in **part A** moved the farthest? Which moved the least? How could you have predicted that?

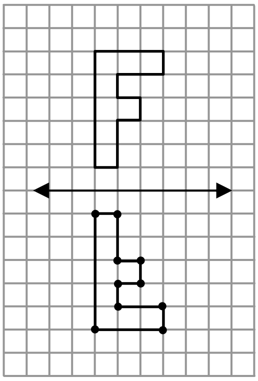
Examples

Example 1 Reflecting a Shape on a Grid

Reflect the shape across the reflection line.



Solution

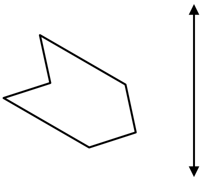


Thinking

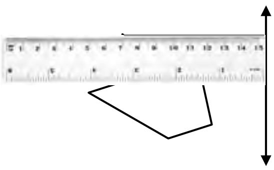
- For each vertex, I counted the number of units to the reflection line. Then I counted the same number of units on the other side of the reflection line and marked the image vertex.
- I connected the image vertices to complete the image.

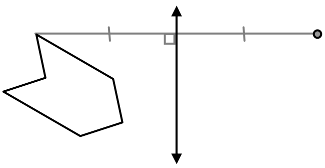
Example 2 Reflecting a Shape Using a Ruler

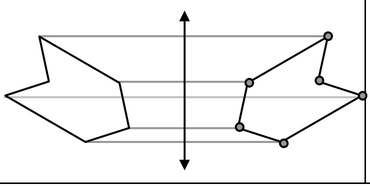
Reflect the shape across the line.



Solution







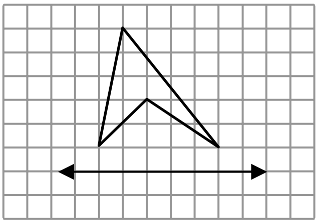
Thinking

- I used the corner of my ruler as a right angle to draw a line segment from one vertex to the reflection line.
- I extended the line segment the same distance on the other side of the reflection line and marked the image vertex.
- I did the same thing for the other vertices. Then I joined the image vertices to complete the image.

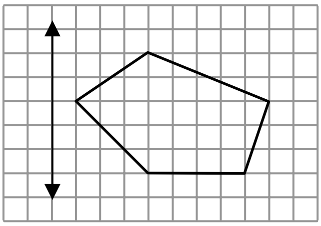
Practising and Applying

1. Copy each shape and reflection line onto grid paper. Draw the reflected image.

a)

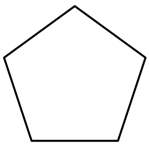


b)

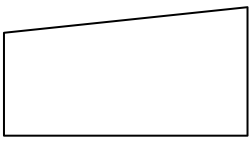


2. Trace each shape. Draw a horizontal or vertical reflection line beside, above, or below the shape. Then reflect the shape.

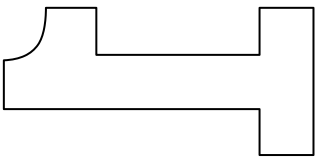
a)



b)



c)

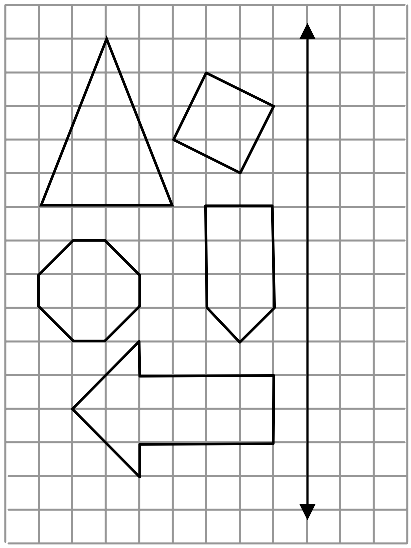


3. a) Draw a shape and a vertical reflection line on grid paper.

b) Draw the reflected image.

c) Fold the paper along the reflection line. What do you notice about the size and position of the image?

4. Which shapes will look like they face the same way after they are reflected across the reflection line?

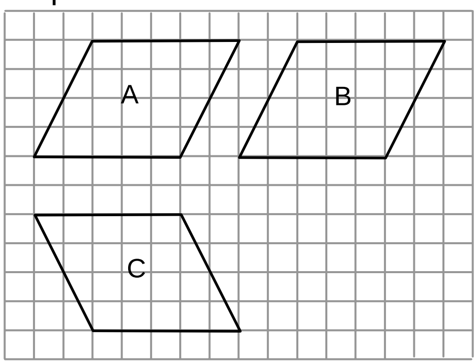


5. Draw a reflection line on grid paper.

a) Draw a shape that will look like it faces the same way after it is reflected. Test your prediction.

b) Draw a shape that will look like it faces the opposite way after it is reflected. Test your prediction.

6. Purna Bahadur transformed Shape A twice.



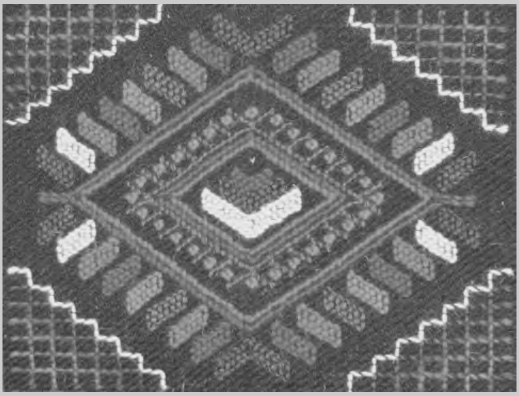
a) Which shape is a reflection? How do you know?

b) Which shape is a translation? How do you know?

6.2.3 Parallel and Intersecting Lines

Try This

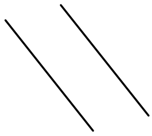
Examine this textile design.



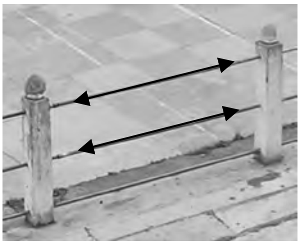
A. i) Identify line segments that intersect at right angles.

ii) Identify pairs of line segments that point in the same direction.

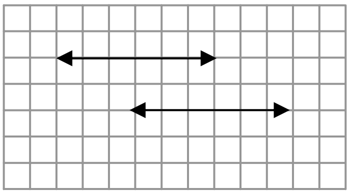
• Recall that **lines** go on forever and **line segments** are parts of lines. Lines or line segments that are always the same distance apart and that never intersect are **parallel**.



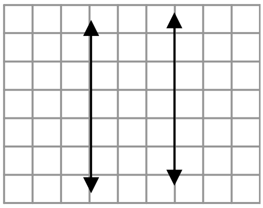
Parallel line segments



Parallel fence rails

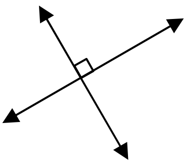


Parallel horizontal lines

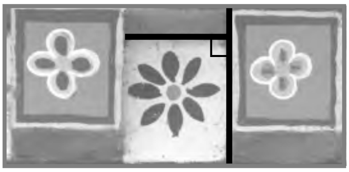


Parallel vertical lines

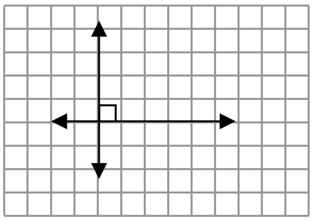
• Lines or line segments that intersect at right angles are **perpendicular**.



Perpendicular lines



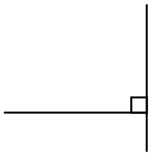
Perpendicular line segments in a design



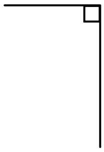
Perpendicular lines drawn on a grid

• Perpendicular line segments can intersect in different ways:

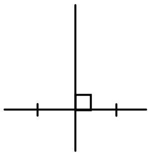
- at **endpoints**
- at the middle point, or **centre point**
- at other points



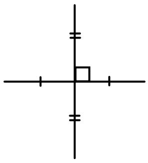
Perpendicular at an endpoint of one line segment



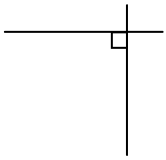
Perpendicular at an endpoint of each line segment



Perpendicular at a centre point

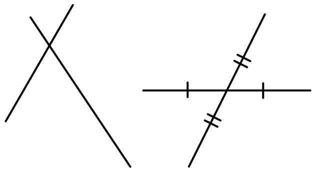


Perpendicular at two centre points



Perpendicular, but not at centre points or endpoints

• Line segments can also intersect without being perpendicular.



B. Examine the design from **part A**. Find pairs of line segments that are

i) intersecting, but not perpendicular

ii) perpendicular at the centre of one line segment but not the other

iii) perpendicular at an endpoint of both line segments

iv) parallel

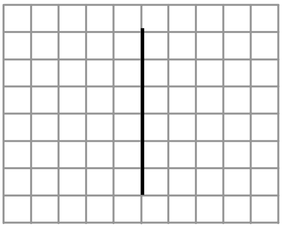
Examples

Example 1 Creating Parallel and Perpendicular Lines Using a Grid

a) Draw a line segment parallel to this line segment.

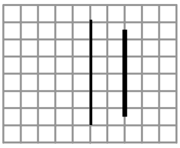
b) Draw a line segment perpendicular to this line segment at an endpoint.

c) Draw a line segment perpendicular to this line segment at its centre point.



Solution

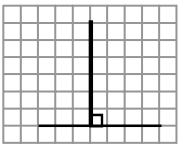
a)



Thinking

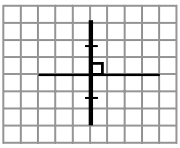
a) The line segment is vertical so I used a vertical grid line to draw the parallel line segment. (All vertical lines are parallel.)

b)



b) I used the horizontal grid line that touched the bottom endpoint. (Horizontal and vertical lines are always perpendicular.)

c)

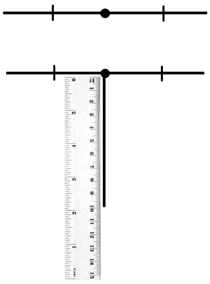


c) I found the centre of the line segment by counting units. Then I used a horizontal grid line to draw the perpendicular line segment.

Example 2 Drawing a Perpendicular Line Segment

Draw a line segment with an endpoint that is perpendicular to the centre point of another line segment.

Solution



Thinking

- I drew a line segment 10 cm long.
- Then I marked its centre point with a dot.
- I used the corner of my ruler as a right angle to draw a perpendicular line segment.

Example 3 Parallelism and Perpendicularity in Transformations

a) i) Draw a right scalene triangle on a grid. Mark one side.
Translate the triangle using two different translation rules.

ii) What do you notice about the marked side and its image?

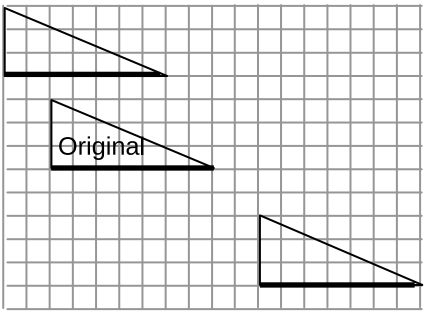
iii) Is it the same for every side?

b) Repeat **part a)** for a reflection. Use one reflection line.

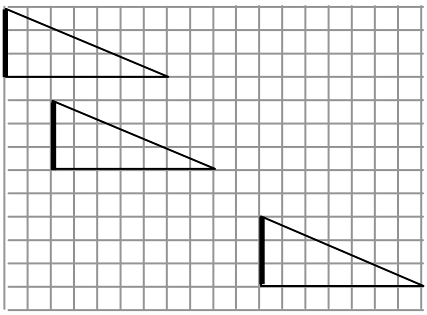
Solution

Translation

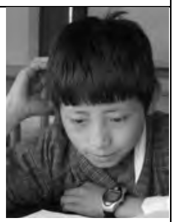
a) i) and ii) The horizontal side and its image are parallel.



iii) The vertical side and its image are parallel.



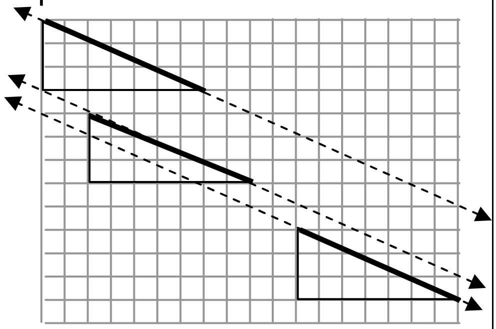
Thinking



- I marked the horizontal sides by making them darker.
- I knew the horizontal sides were parallel because they followed parallel grid lines.
- I knew the vertical sides were parallel because they followed parallel grid lines.

Example 3 Parallelism and Perpendicularity in Transformations [Cont'd]

iii) The slanted side and its image are parallel.



• I knew the slanted sides were parallel because I drew dashed lines from each and it looked like they would never meet.

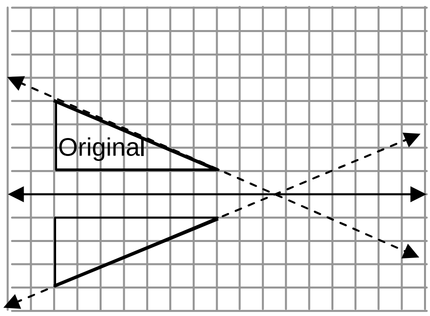
It is the same for every side.

In a translation, every original side and its image are parallel

Reflection

b) The horizontal and vertical sides and their images are parallel, but the slanted side and its image are not.

• I knew the horizontal sides were parallel and the vertical sides were parallel because they followed parallel grid lines.



• I knew the slanted sides weren't parallel because I drew dashed lines from each and they crossed at the reflection line.

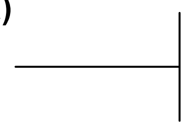
It is not the same for every side.

In a reflection, not every original side and its image are parallel.

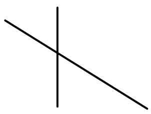
Practising and Applying

1. Describe each pair of line segments. Use words such as perpendicular, parallel, intersecting, endpoint, and centre point.

a)

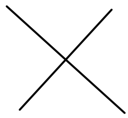


b)



c)

d)



2. Draw a pair of line segments for each.

a) perpendicular at both centres

b) perpendicular at an endpoint of each

c) intersecting, but not perpendicular

d) parallel

3. Describe examples of line segments in your classroom that are:

a) parallel

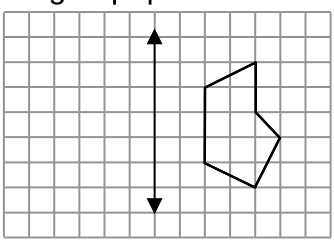
b) perpendicular

c) intersecting, but not perpendicular

4. a) Create a design for a rachu on grid paper. Include perpendicular and parallel line segments.

b) Label your design using words such as perpendicular, parallel, intersecting, endpoint, and centre point.

5. a) Copy the shape and reflection line onto grid paper.



b) Draw the reflection image.

5. c) Join each vertex to its image with a line segment.

d) Identify parallel and perpendicular line segments.

6. a) On a grid, draw a trapezoid and mark the parallel sides.

b) Use three different translation rules to translate the trapezoid.

c) Are the sides that were parallel in the original still parallel in the image?

7. Show how you know each is true.

a) When you translate a shape, each side and its image are always parallel.

b) When you reflect a shape, each side and its image may not be parallel.

8. a) Draw a line segment.

b) Draw a line segment that is perpendicular to the one in **part a)**.

c) Draw a line segment that is perpendicular to the one in **part b)**.

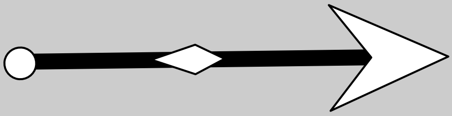
d) Describe how the line segment in **part c)** relates to the original line segment.

9. Why can two lines not be both perpendicular and parallel?

6.2.4 Properties of Rotations

Try This

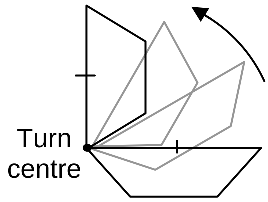
Chandra Maya made a design for the hour hand on a clock.



A. i) Sketch her hour hand on a clock face showing 3 o'clock.
ii) What will it look like at 12 o'clock? 9 o'clock? 6 o'clock?

• A **rotation** is a transformation that turns a shape around a point called the **turn centre**.

The distance from any point to the turn centre does not change when it is rotated, so the image is congruent to the original shape.



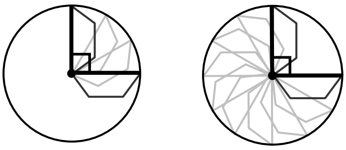
• Rotations are described by direction and size.

- The direction is **clockwise (cw)** or **counterclockwise (ccw)**.

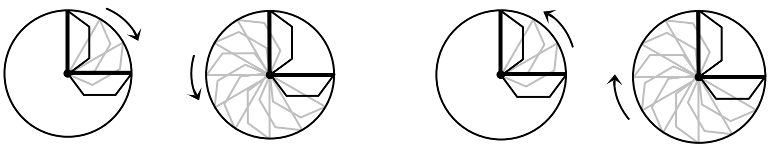
- The size of a rotation can be described with a fraction.

• A full turn ($\frac{4}{4}$ turn) returns the shape to its original position.

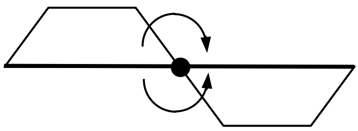
• $\frac{1}{4}$ turns and $\frac{3}{4}$ turns create right angles.



A $\frac{1}{4}$ turn in one direction results in the same image as a $\frac{3}{4}$ turn in the other direction.



• A $\frac{1}{2}$ turn creates a straight line segment.



A $\frac{1}{2}$ turn cw results in the same image as a $\frac{1}{2}$ turn ccw.

B. What is the turn centre for the rotations in **part A**?

C. Describe these rotations. Tell the direction and size of the turn:

i) from 3 o'clock forward to 12 o'clock

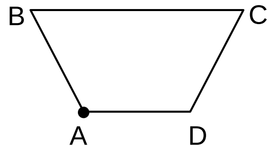
ii) from 3 o'clock backward to 9 o'clock

iii) from 3 o'clock forward to 6 o'clock

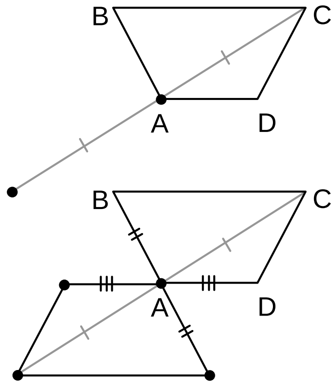
Examples

Example 1 Rotating a Shape Half a Turn

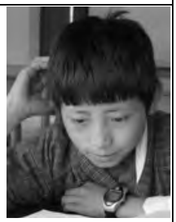
Rotate the shape a $\frac{1}{2}$ turn around A.



Solution



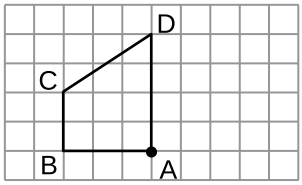
Thinking



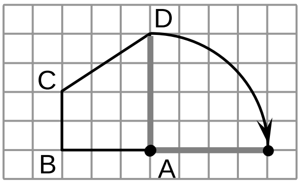
- I knew that $\frac{1}{2}$ turns created straight line segments.
- I decided to start with C.
I measured the distance from C to A. Then I drew a line segment twice as long that started at C and passed through A. I marked the image of C at the end.
- I repeated this for each vertex. Then I joined the image vertices to create the rotated image.

Example 2 Rotating a Shape on a Grid

Rotate the shape a $\frac{3}{4}$ turn ccw around A.



Solution



Thinking

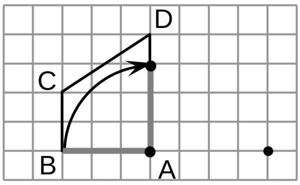
- I knew that

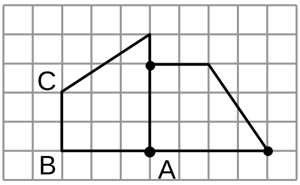
- a $\frac{1}{4}$ turn cw creates the same image as a $\frac{3}{4}$ turn ccw.

- $\frac{1}{4}$ turns make right angles.

Example 2 Rotating a Shape on a Grid [Continued]

Solution





Thinking

• I used the right angles in the grid to help create $\frac{1}{4}$ turns for locating 2 image vertices:

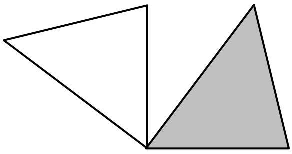
- D is 4 units above A, so its image is 4 units right of A.
- B is 3 units left of A, so its image is 3 units above A.

• I used these 2 image vertices and the grid to draw a congruent image.

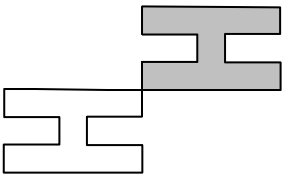
Practising and Applying

1. Describe two ways the grey shape can be rotated to the white shape.

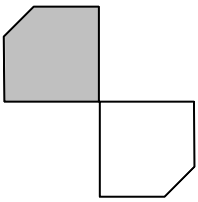
a)



b)



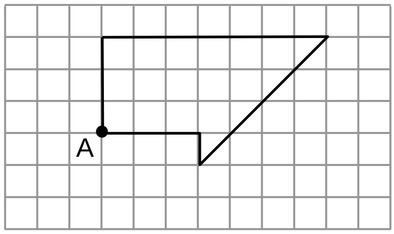
c)



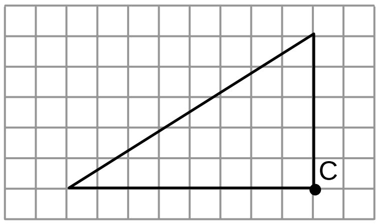
2. Trace the shapes from **question 1**. Mark each turn centre.

3. Copy each shape below onto grid paper. Rotate it as described.

a) a $\frac{1}{4}$ turn clockwise around A



b) a $\frac{3}{4}$ turn clockwise around C

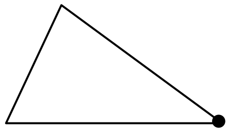


4. Trace each shape and turn centre.

Rotate the shape a $\frac{1}{2}$ turn.

a)

b)



5. a) Draw a right scalene triangle on grid paper. The right angle is the turn centre.

b) Rotate the triangle a $\frac{1}{4}$ turn cw.

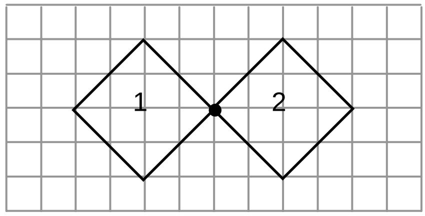
c) Rotate the image from **part b)** a $\frac{1}{2}$ turn.

d) Rotate the image from **part c)** a $\frac{1}{4}$ turn ccw.

e) Describe the design you created.

f) What other rotations would create the same design?

6. a) Describe a rotation that would move Shape 1 onto Shape 2.



b) Describe a reflection that would move Shape 1 onto Shape 2.

c) Describe a translation that would move Shape 1 onto Shape 2.

7. a) Change Shape 1 from **question 6** by changing the position of one vertex, but leaving the other three alone. Call it Shape 3.

b) Apply the transformations you described in **question 6** to Shape 3.

c) Do the translation, rotation, and reflection images of Shape 3 appear to be all facing the same way as Shape 3? Explain your thinking.

8. In what ways are a rotation, a reflection, and a translation different?

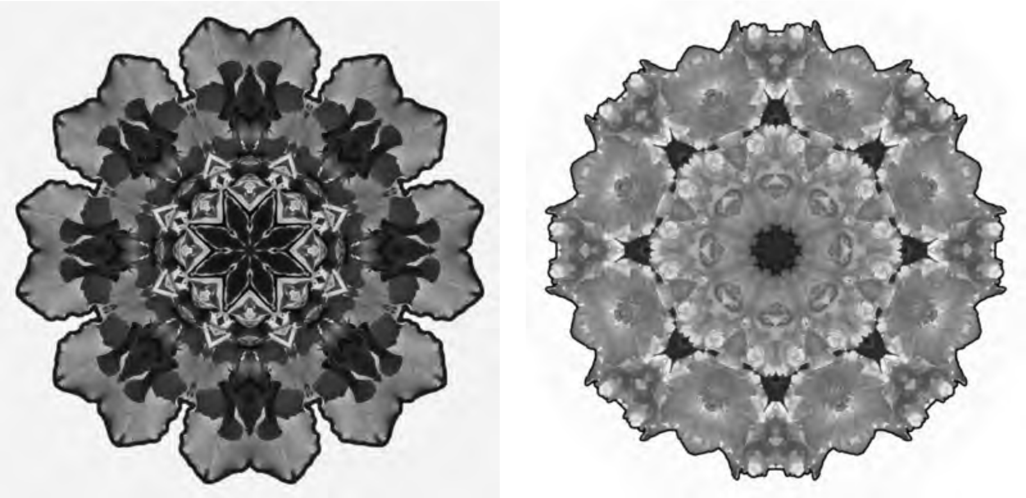
CONNECTIONS: Kaleidoscope Images

A kaleidoscope is a tube that contains bits of coloured glass or objects. The objects are reflected by mirrors inside the tube.

When you look through the tube, you see a design created by the objects and their reflections. When you turn the tube, the objects move and the design changes.

A kaleidoscope

Kaleidoscope Designs



1. Identify the reflection lines in the kaleidoscope designs.

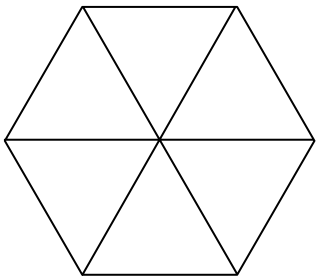
2. Describe the rotations in the designs. Where is the turn centre?

3. Create your own kaleidoscope design by following these instructions:

• Draw a design in one of the equilateral triangles in a copy of this hexagon.

• Use one side of the triangle as a line of reflection to reflect the design onto the next triangle.

• Repeat this until the hexagon is full of reflected images.



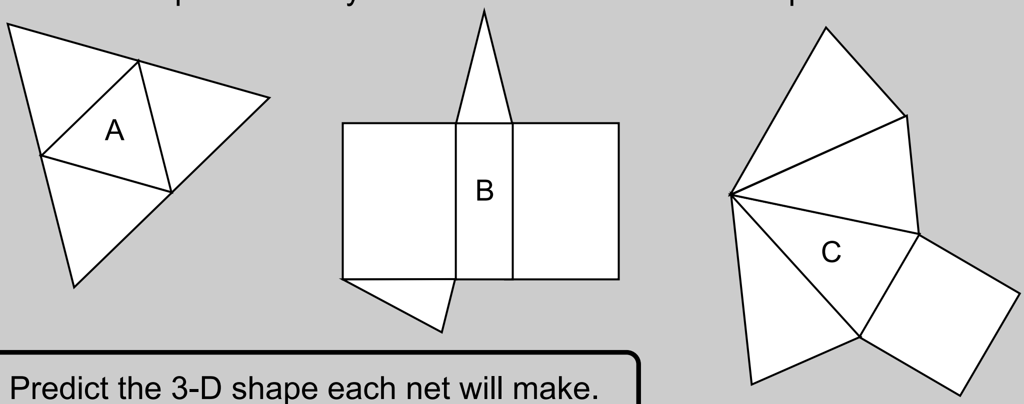
Reprint 2024

Chapter 3 3-D Representations

6.3.1 Prism and Pyramid Nets

Try This

A **net** is a 2-D pattern that you can fold to make a 3-D shape.



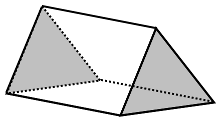
A. Predict the 3-D shape each net will make.

• Each **polygon** in a net represents a **face** of a 3-D shape.
The polygons give clues about the 3-D shape that the net will make.

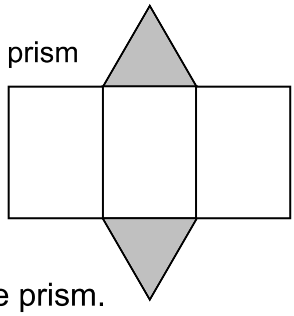
• An object with two congruent polygon **bases** connected by rectangle faces is called a **prism**. A net for that prism has 2 congruent polygons and a number of rectangles.

For example:

Triangle-based prism



Net for a triangle-based prism

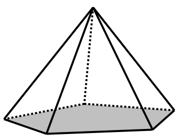


The 2 congruent polygon bases tell you the name of the prism.
There is 1 rectangle face for each side of the base polygon.

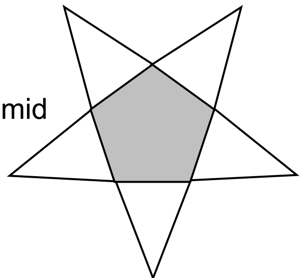
• A **pyramid** has 1 polygon base with triangle faces attached.
A net for a pyramid has triangles and 1 polygon, which could be a triangle or another shape.

For example:

Pentagon-based pyramid

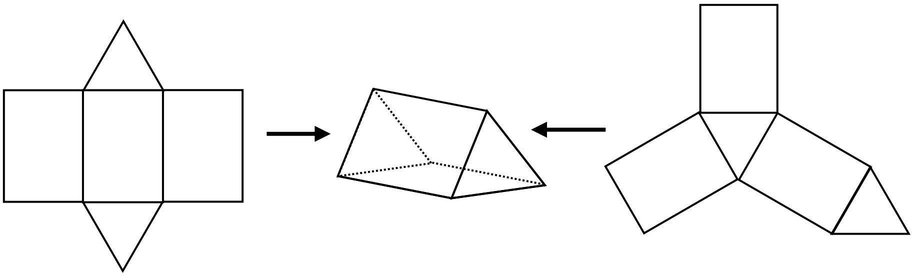


Net for a pentagon-based pyramid



The base polygon tells you the name of the pyramid.
There is 1 triangle face for each side of the base polygon.

• There are different ways to connect the faces to form a net for the same 3-D shape.



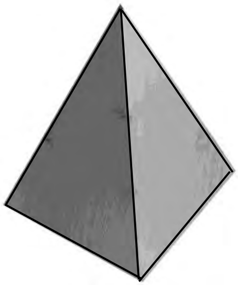
Both nets make the same 3-D shape.

B. What clues did you use to make your predictions for **part A**?

Examples

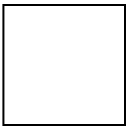
Example 1 Drawing a Net by Tracing

Draw a net for this square-based pyramid.

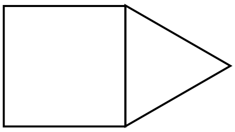


Solution 1

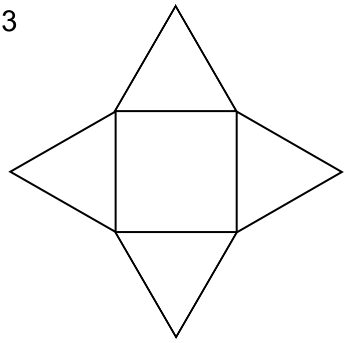
Step 1



Step 2



Step 3



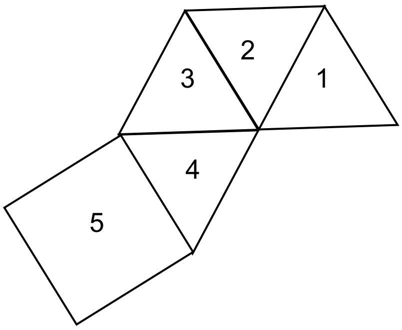
Thinking

Step 1: I placed the pyramid on my paper and traced the square base.

Step 2: I rolled the pyramid onto one of the triangle faces and traced it. Then I rolled it back onto the square base.

Step 3: I repeated Step 2 for each other triangle face.

Solution 2

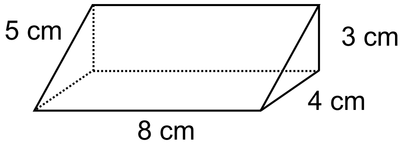


Thinking

- I put some tape on one triangle face and traced that face.
- I rolled the pyramid onto the next triangle face and traced it.
I repeated this until I got back to the face with the tape.
- I finished by rolling the pyramid onto the square base and tracing it.

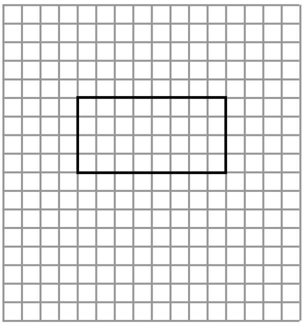
Example 2 Drawing a Net Using a Grid

Draw a net for the triangle-based prism. The base is a right triangle.

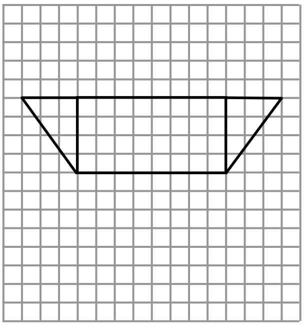


Solution

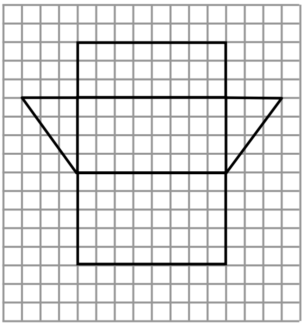
Step 1



Step 2



Step 3



Thinking

Step 1: I drew an 8 cm-by-4 cm rectangle on a centimetre grid to represent the bottom face.

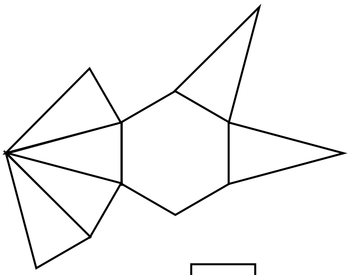
Step 2: I drew the 2 right triangle bases, each 3 cm tall.

Step 3: I drew the other rectangle faces:
- the 8 cm-by-3 cm face in the back
- the 8 cm-by-5 cm slanted face

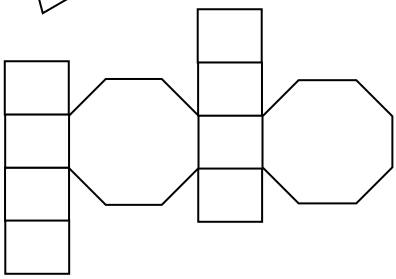
Practising and Applying

1. Identify the 3-D shape each net will make. What clues help you?

a)

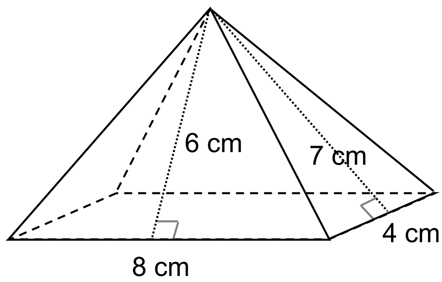


b)



2. Identify the 3-D shape below. Sketch 2 different nets for it.

3. This is a rectangle-based pyramid. Each triangle face is made of two congruent triangles.

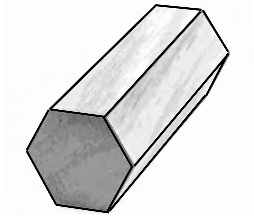


a) Draw its net on grid paper.

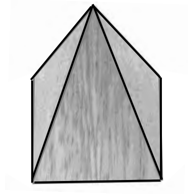
b) Cut out your net and fold it to check.

4. a) Identify each shape.

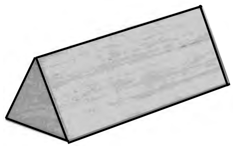
i)



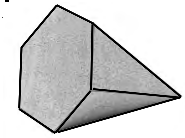
ii)



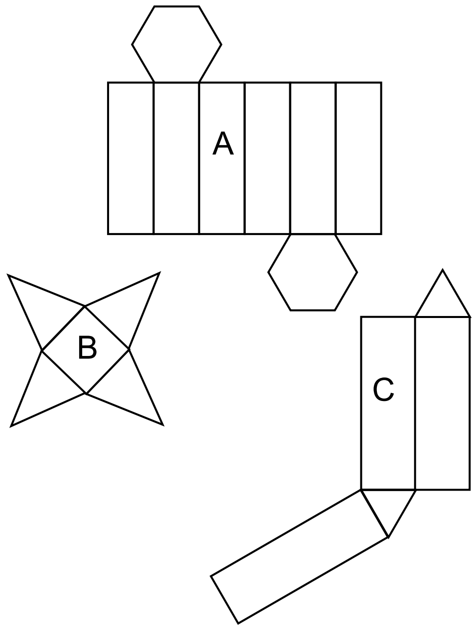
iii)



iv)

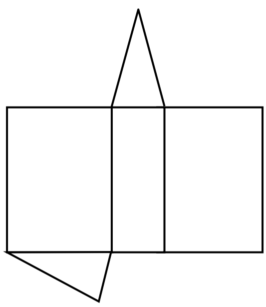


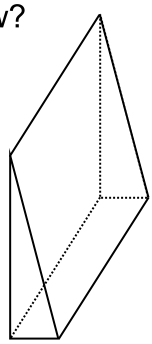
b) Match each net to a shape in **part a)**.



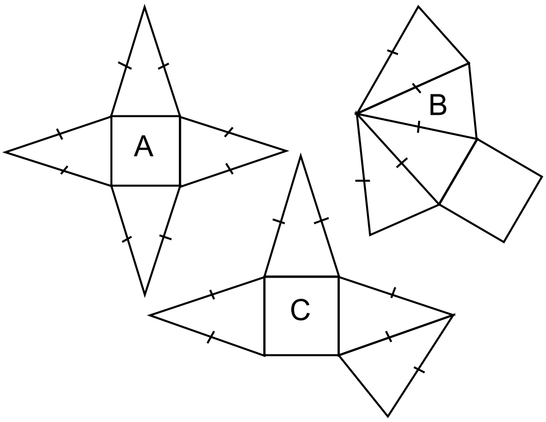
c) Draw two different nets for the shape in **part a)** that has no matching net.

5. Is this a net for the 3-D shape shown? How do you know?

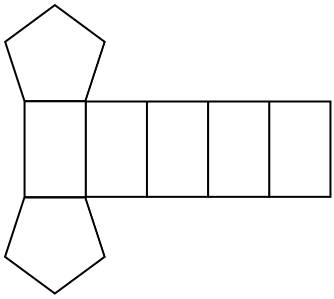




6. Do these nets all form the same shape? How do you know?



7. a) Does this net make a prism or a pyramid? State 2 clues you used to decide.



b) Name the shape the net makes.

CONNECTIONS: Euler's Rule

Leonhard Euler, a Swiss mathematician who lived in the 1700s, discovered a fact that is always true for pyramids and prisms.

1. Copy and complete the chart to discover Euler's Rule.

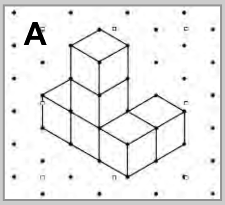
3-D Shape	**V** (Number of vertices)	**F** (Number of faces)	**E** (Number of edges)	**Euler's Rule V + F – E = ?**
Triangle-based prism				
Triangle-based pyramid				
Rectangle-based prism				
Square-based pyramid				
Pentagon-based prism				
Hexagon-based prism				

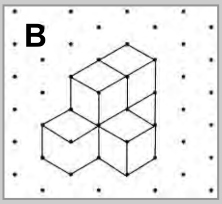
2. Could a prism have 12 vertices, 6 faces, and 15 edges? How do you know?

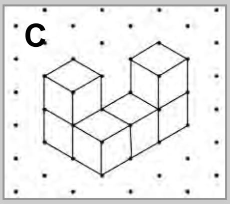
6.3.2 Interpreting Isometric Drawings

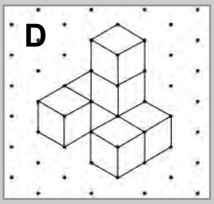
Try This

These are drawings of structures made of linking cubes.
Some of the drawings show the same structure from different views.



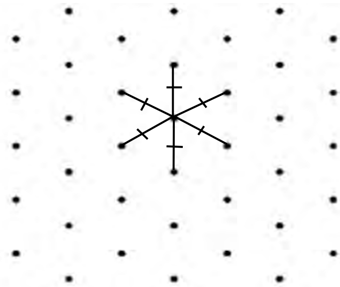






A. Which drawings are of the same structure?

Drawings of **three-dimensional** or **3-D** shapes are often done on isometric dot paper.



• The distance between the dots on isometric paper is the same, so lengths that are the same on the 3-D shape are the same in the **isometric drawing**. The dots on the paper look like they form rhombuses.

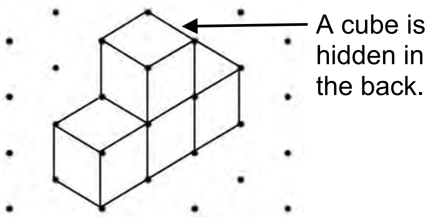
• There may be hidden cubes in an isometric drawing. Drawings from more than one view will give you a more complete picture.

For example, here is the same cube structure from two different views:

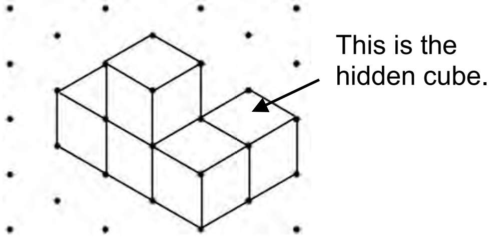
View from the front

View from the back

These are isometric drawings of the structure above from different views:



View from the front



View from the back

From the front, it looks like the structure has four cubes, but the back view shows the fifth cube.

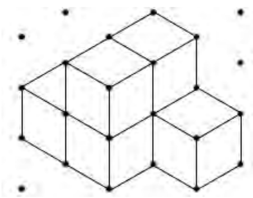
B. What clues did you use to answer **part A**?

Examples

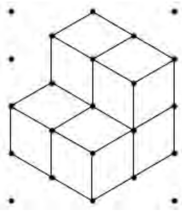
Example 1 Comparing Isometric Drawings

Could these isometric drawings both be of the cube structure shown here? Explain.

A

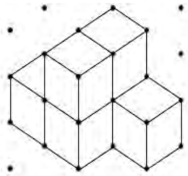


B

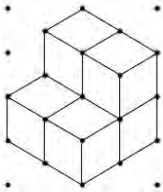


Solution

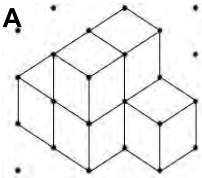
A



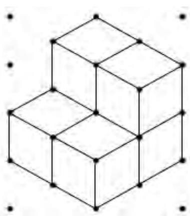
B



A



B



Both drawings
could be isometric drawings of the structure in the photo.

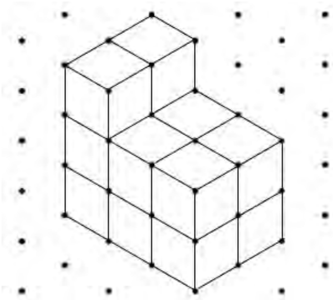
Thinking

- I compared Drawing A to the photo of the structure. They looked like they could match.
- Drawing B looks like it has fewer cubes than the structure in the photo, but some cubes could be hidden in the drawing.
- I built a structure like the one in the photo and then turned it different ways to see if I could get a view like Drawing A.
- I saw that Drawing A matched.
- I kept turning it until I found a view that matched Drawing B.
- I saw that Drawing B matched.

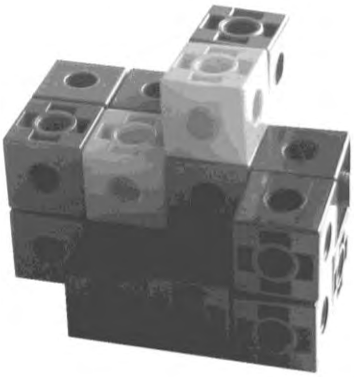
Example 2 Describing a Cube Structure

Bina Gurung made an isometric drawing of a cube structure.

How many cubes could be hidden?



Solution



There could be 6 hidden cubes in the drawing.

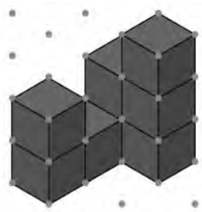
Thinking

- I built a cube structure like the one in the isometric drawing.
- I held it so it was the same view as the drawing.
- I added cubes to the back and side to see where they could be hidden.
- There could be as many as 6 hidden cubes.
- You could see the hidden cubes from the front if you looked at the structure from a bit higher.

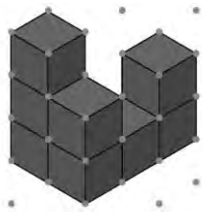
Practising and Applying

1. Which drawings below are of the same structure? How do you know? (Each structure has the same number of cubes.)

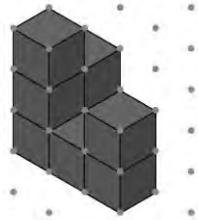
A



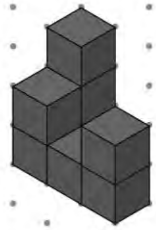
B



C

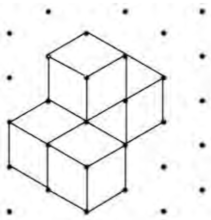


D

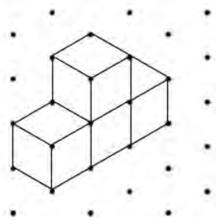


2. These are isometric drawings of the same structure. Could it have more than 5 cubes? Explain your thinking.

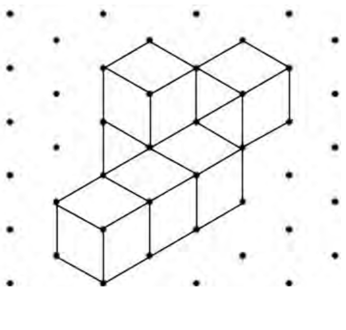
A



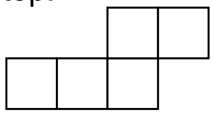
B



3. Look at this drawing. How many cubes are you sure the structure has? How many more could it have?

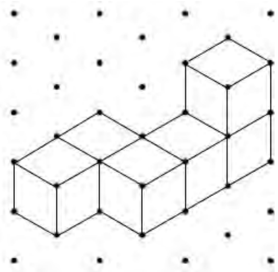


4. This is a view of a cube structure from the top.

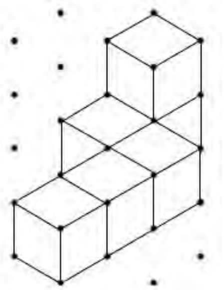


Which drawing below is of the same structure? How do you know?

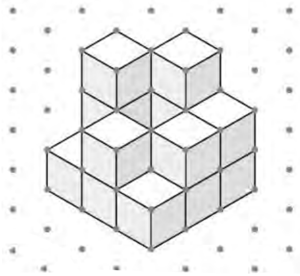
A



B



5. a) How many cubes can you see in this isometric drawing?



b) How many cubes do you know are there but you cannot see? Where are they?

c) Where might other cubes be hidden?

6. How is it possible for one isometric drawing to match two or more different cube structures?

6.3.3 Creating Isometric Drawings

Try This

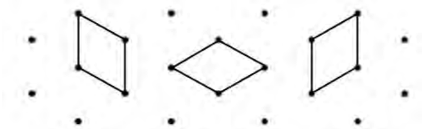
Yuden built a rectangular prism using cubes. It was 2 cubes wide, 2 cubes tall, and 1 cube long.

A. Sketch Yuden's prism. Show the cubes in the prism.

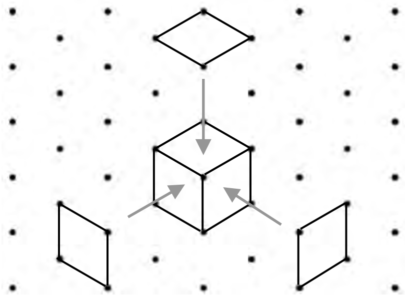
• You can draw a 3-D shape on isometric dot paper by connecting dots. The arrangement of the dots makes the drawing look three-dimensional.

• On isometric dot paper, a square face is represented by a rhombus. Sides of equal length on the square have equal length on the rhombus. Sides that are parallel on the square are parallel on the rhombus.

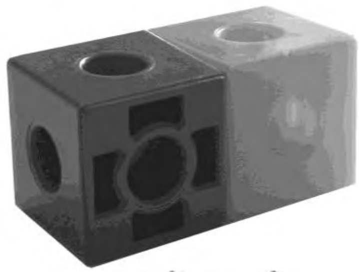
• There are three different ways to draw a rhombus to represent a square face.

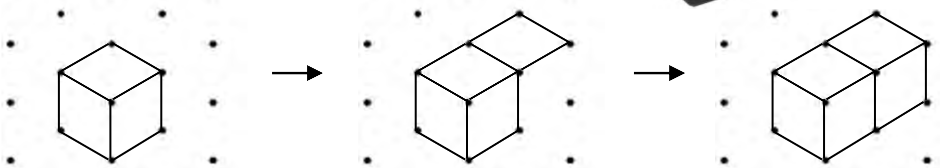


• To draw a cube, draw the three square faces you can see as three different rhombuses.



• To draw two cubes, draw one cube and then add rhombus faces to show the second cube.





• To draw more complicated structures, first draw the cubes with 3 faces showing. Then add rhombuses and lines to show the rest of the faces (see the **Example** on the next page).

B. i) Draw Yuden's prism on isometric dot paper. Compare it to your sketch from **part A**.
ii) How did the arrangement of the dots help you draw the prism?

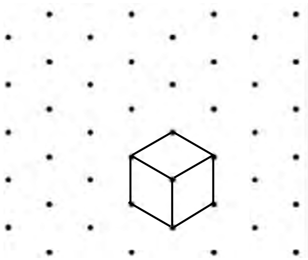
Examples

Example Creating an Isometric Drawing

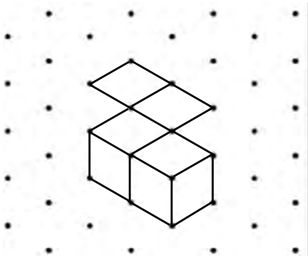
Draw this cube structure on isometric dot paper.

Solution

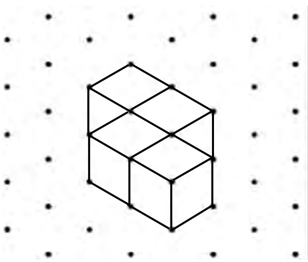
Step 1



Step 2

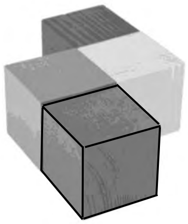


Step 3

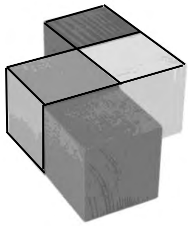


Thinking

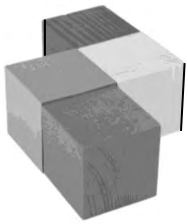
Step 1: The front cube had 3 faces showing, so I drew it first.



Step 2: I added rhombuses for the 4 other full faces that I could see.



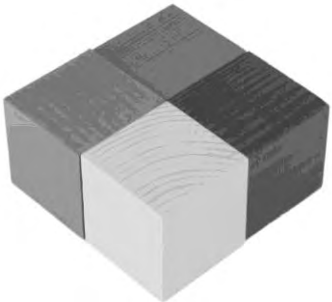
Step 3: For the 2 partly hidden faces, I drew a vertical line for the part I could see.



Practising and Applying

1. Create an isometric drawing of each.

a)



b)

c)

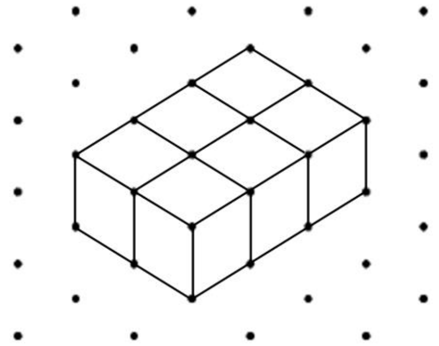
d)

2. a) Build a cube structure with five or more cubes.

b) Create an isometric drawing.

c) Create another isometric drawing from a different view.

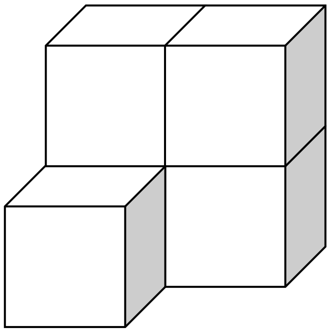
3. Dorji built a rectangular prism using cubes. He then created this isometric drawing.



Dorji's drawing

Build Dorji's prism. Create an isometric drawing from a different view.

4. This structure is made of 6 cubes.



a) How many cubes are hidden?

b) Build the cube structure.

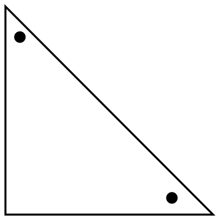
c) Create an isometric drawing that shows all 6 cubes.

5. Why might two people create different isometric drawings of the same structure?

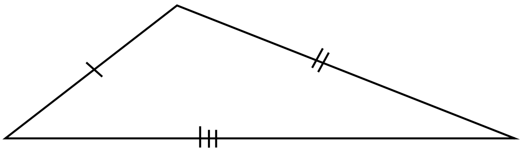
UNIT 6 Revision

1. Classify each triangle by side length and by angle.

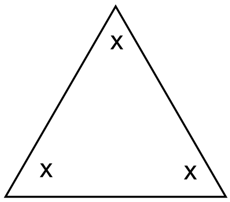
a)



b)



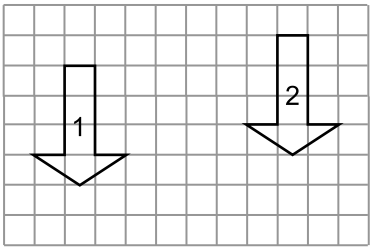
c)



2. a) How many lines of symmetry are there in each triangle in **question 1**?

b) How many acute angles are there in each triangle?

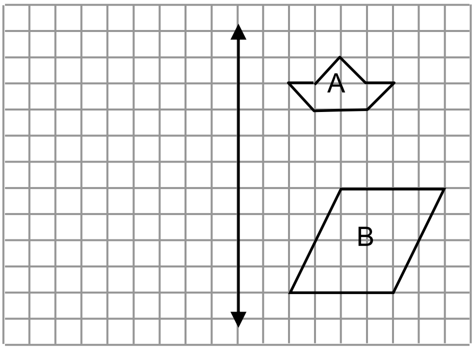
3. State the translation rule that would move Shape 1 to Shape 2. How did you figure out the answer?



4. a) Copy the shapes and reflection line onto grid paper.

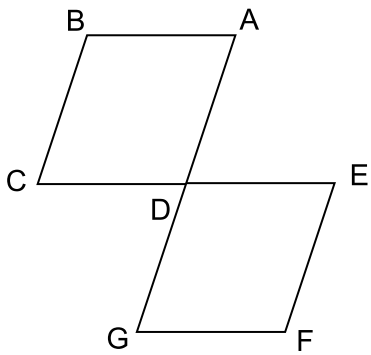
b) Translate Shape A left 8 units and down 2 units.

c) Reflect Shape B across the line.



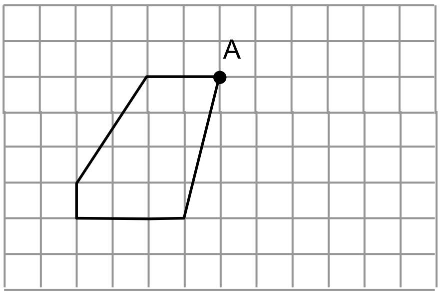
5. Chabilal reflected Triangle A across a reflection line to create Triangle B. He then translated Triangle B so it fit on top of Triangle A. What type of triangle is he using? Show your work.

6. a) Shape ABCD was rotated to become Shape DEFG. Tell the direction, size of rotation, and turn centre.



b) Repeat **part a)** for rotating Shape DEFG to Shape ABCD.

7. a) Rotate the shape a $\frac{1}{4}$ turn counterclockwise around Vertex A.



b) Which other rotation would create the same image?

8. Describe how these transformations are the same and different:

• translation

• reflection

• rotation

9. Draw a pair of line segments to match each description.

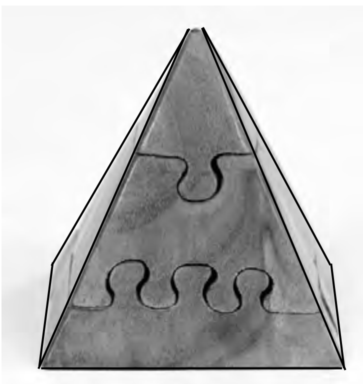
a) parallel

b) perpendicular at a centre point

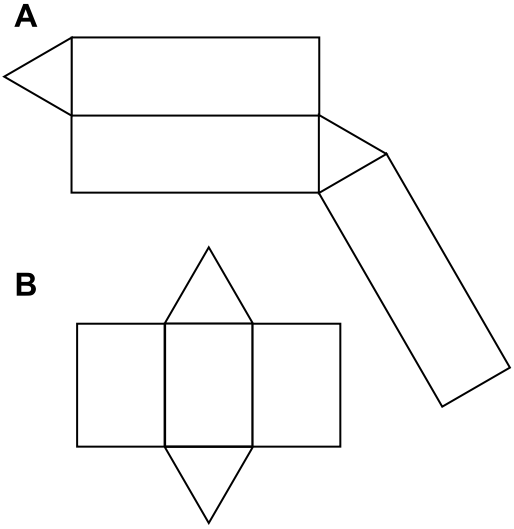
c) intersecting, not perpendicular

d) perpendicular at both centres

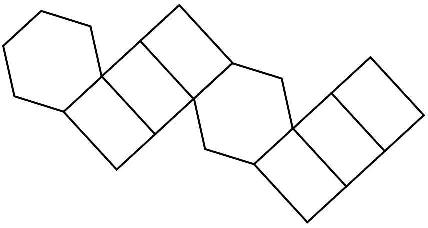
10. Rinzin wants to wrap this 3-D puzzle for a gift. Sketch two different nets for it.



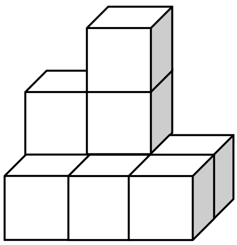
11. Will these nets fold to make exactly the same shape? How do you know?



12. Identify the shape this net will make. What clues did you use?



13. a) What is the least number of cubes this structure could have? What is the greatest number?

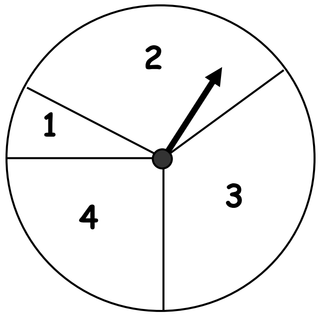


b) Create an isometric drawing of the structure.

UNIT 7 DATA AND PROBABILITY

Getting Started

Use What You Know



A. Use a spinner like this. Spin it 40 times.
Record your spins.

B. Draw a bar graph to show the number of times you spun each number. Use a scale of more than 1 on your graph.

C. What did you use for the scale of your graph? Why did you choose that value?

D. Tell three things your graph tells you about the spinner.

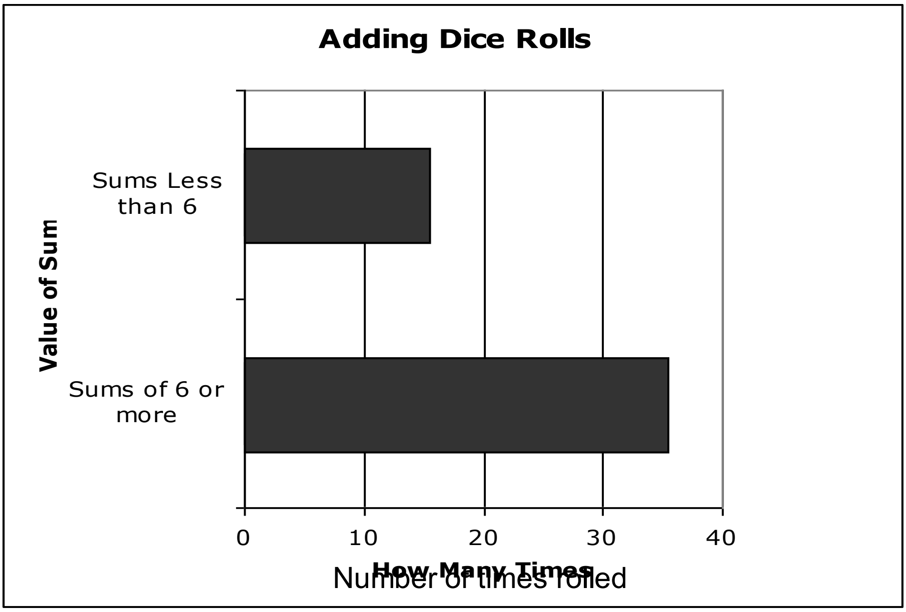
E. Compare your graph to a classmate's graph.

i) How are they the same?

ii) How are they different? Why are they different?

Skills You Will Need

1. A pair of dice was rolled 50 times. What does the bar graph below tell you about the results?



2. Show the information from **question 1** in a pictograph.

3. Is 11 the mean of this set of numbers? 10, 11, 11, 12
How do you know?

4. Calculate the mean of each set of data.

a) 16, 18, 24, 2 **b)** 17, 17, 17, 21

5. For each situation below, tell whether the probability of it happening is closer to 0, or closer to 1, or closer to $\frac{1}{2}$.

a) You roll a 7 when you roll a pair of dice.

b) A newborn baby is a boy.

c) If all the numbers from 1 to 100 are put on slips of paper and put in a hat, you will choose 1, 11, 21, 31, 41, 51, 61, 71, 81, or 91.

d) The ground is wet when it rains.

6. Which event is more likely in each pair or are they equally likely?

a) Rolling a 4 when you roll a die OR Rolling a 2 when you roll a die

b) Multiplying a one-digit number by a one-digit number and getting a product greater than 10

OR

Multiplying a one-digit number by a one-digit number and getting a product less than 10

c) You will eat a dog tomorrow OR You will eat rice tomorrow.

7. A bag has 10 coloured cubes in it (red, blue, green, and yellow).
You do not know how many there are of each colour.
You do an experiment by pulling out a cube 50 times without looking and returning it to the bag each time.

a) Suppose you pull out a red cube twice.
Predict how many red cubes are in the bag.

b) What if you pull out a red cube 40 times?
Predict how many red cubes there are.

c) What if you pull out a red cube 15 times?
Predict how many red cubes there are.

Chapter 1 Interpreting Data

7.1.1 The Mean

Try This

Karma's marks on his last four quizzes are shown below.

67 71 78 68

A. What single number would you use to describe how well Karma did on all four quizzes?

• The **mean** of a set of data or numbers is a single number that describes the whole set. It is a type of average.

For example, the mean of 6 represents this set of numbers: 1, 3, 5, 8, 13

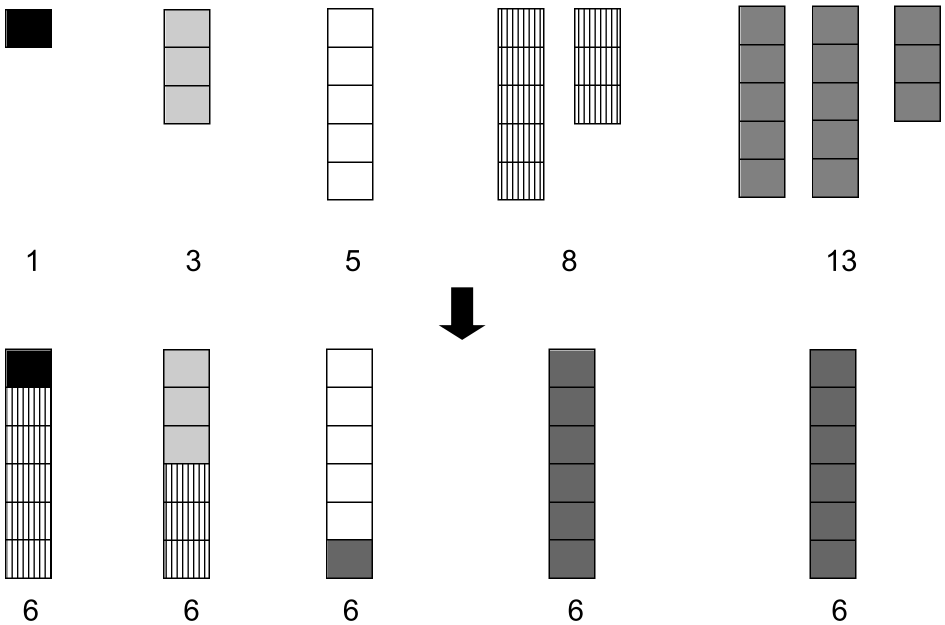
• The mean is calculated by totalling all the pieces of data in a set and then sharing it equally among the pieces of data.

For example, for the data set 1, 3, 5, 8, 13, the mean is 6 because the total of the data is 30 and there are 5 pieces of data:

$$(1 + 3 + 5 + 8 + 13) \div 5 = 30 \div 5 = 6$$

• You can think of finding the mean as representing each number with a stack of cubes and then rearranging the cubes to make equal stacks.

Set of data: 1, 3, 5, 8, 13



1, 3, 5, 8, and 13 are rearranged into 5 equal stacks with 6 in each stack.
So the mean of 1, 3, 5, 8, and 13 is 6.

• If you compare each piece of data to the mean, you will see that the total below the mean and the total above the mean are equal. That is why the mean is sometimes called the balance point.

For example, the data set 1, 3, 5, 8, 13 has a mean of 6:

1 is **5 below** the mean.
3 is **3 below** the mean.
5 is **1 below** the mean.

The **total below** the mean is 1 + 3 + 5 = **9**.

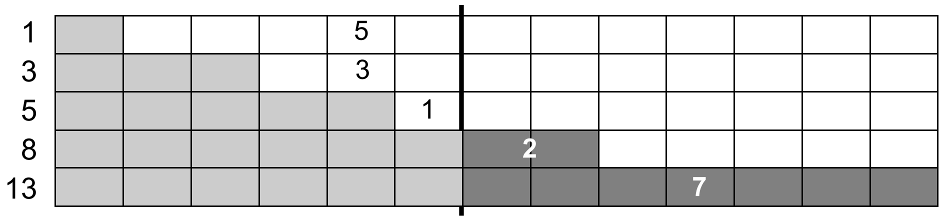
8 is **2 above** the mean.
13 is **7 above** the mean.

The **total above** the mean is 2 + 7 = **9**.

If you look at the picture on **page 197** of the stacks of cubes, you will see that the cubes from the 2 stacks above the mean (8 and 13) filled up the 3 stacks below the mean (1, 3, and 5) to make 5 equal stacks of 6.

• You can also see this relationship between the data below the mean, the data above the mean, and the mean using a bar graph:

Below the mean: 5 + 3 + 1 = 9 Mean = 6



Above the mean: 2 + 7 = 9

B. i) What is the mean of the set of data in **part A?**

ii) Is the mean of a set of data always one of the numbers in the set? Explain your thinking.

Examples

Example 1 Calculating the Mean By Estimating and Balancing	
Tenzin wanted to see how high he could count in 1 min. He tried it six times and he got to these numbers each time: 96, 100, 95, 101, 97, and 99. What is the mean number he got to?	
Solution Estimate for the mean: 99 *Below* 96 is 3 below, 95 is 4 below, and 97 is 2 below. There is a total of 9 below. *Above* 100 is 1 above and 101 is 2 above. There is a total of 3 above.	**Thinking**  • I needed a number with the same amount below it as above it, so I tried 99.

<table>
<tr><td>New estimate for mean: 98
Below
96 is 2 below, 95 is 3 below and 97 is 1 below.
There is a total of 6 below.
Above
100 is 2 above and 102 is 4 above
There is a total of 6 above.
The mean is 98.</td><td>• With a mean of 99, there was too much below 99 to balance what was above, so I tried 98 because it was lower.
• I knew 98 was correct because the totals above and below it were the same.</td></tr>
</table>

<table>
<tr><th colspan="2">Example 2 Solving a Problem Involving the Mean</th></tr>
<tr><td colspan="2">The mean mass of a group of 4 dogs is the same as the mean mass of a different group of 4 dogs. What is the unknown mass in Group 1?

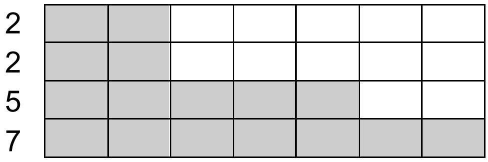
Group 1: 14 kg, 10 kg, 13 kg, ☐ kg
Group 2: 14 kg, 12 kg, 16 kg, 14 kg</td></tr>
<tr><td>Solution 1
The mean mass of the dogs in Group 2
(14 + 12 + 16 + 14) ÷ 4 = 56 ÷ 4
The mean mass of the dogs in Group 1
(14 + 10 + 13 + ☐) ÷ 4
So, 14 + 10 + 13 + ☐ = 56
37 + ☐ = 56
56 – 37 = ☐
56 – 37 = 19
The unknown mass is 19 kg.</td><td>Thinking

• Since the means were equal and both involved a number divided by 4, I knew that the total masses divided by 4 were equal.
• I used a number sentence to figure out the missing number.</td></tr>
<tr><td>Solution 2
The mean for Group 2 is 14 kg since there is a total of 2 below 14 (12 kg) and a total of 2 above 14 (16 kg).
So the mean in Group 1 must also be 14 kg.
Since 10 is 4 below 14 and 13 is 1 below 14, there is a total of 5 below 14.
The missing number has to be 5 above 14:
14 + 5 = 19
The unknown mass is 19 kg.</td><td>Thinking

• I figured out the mean for Group 2 by thinking about balancing the amounts above and below the mean.
• Then I used the same idea for the masses in Group 1.</td></tr>
</table>

Practising and Applying

1. Use the graph to find the mean of 2, 2, 5, and 7. Describe how you used the graph.



2. Find the mean of each set of data by drawing a picture. Explain how you used the picture.

a) 3, 6, 8, 11

b) 2, 2, 4, 8, 9

3. Calculate the mean of each set of data.

a) 12, 15, 20, 25

b) 38, 40, 43, 47, 57

c) 12, 16, 22, 28, 35, 37

4. The two sets of numbers in each pair have the same mean. Find the unknown number in each set.

a) 3, 8, 9, 12 and 2, 7, 15, ☐

b) 2, 4, 6, 6, 7 and 1, 1, 1, 5, ☐

c) 2, 5, 7, 12, 19 and 8, 10, 12, ☐

5. a) Calculate Yeshi's mean score for these test scores: 58, 62, 67, 75, 78

b) Jigme has the same mean score, but different test scores. What might be his test scores?

6. A set of five numbers has a mean of 20. One number is 20. List three possibilities for the other four numbers.

7. The mean distance from school to home for Kinley, Kachap, and Bal is 10 km. Kinley lives 5 km from school and Kachap lives 12 km from school. How far from school does Bal live?

8. Thinley says that you can find the mean of four numbers by following these steps:

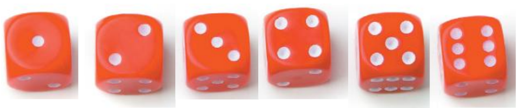
• Find the mean of the first two numbers.

• Then find the mean of the last two numbers.

• Then find the mean of the two means.

Do you agree with Thinley? Explain using an example.

9. Predict the mean number when a die is rolled 12 times. Do an experiment to test your prediction. Describe your experiment.



10. How do you know the mean of a set of numbers is always between the least number and the greatest number in the set?

7.1.2 EXPLORE: Effect of Data Changes on the Mean

Bhagi's test scores for the second half of the year are shown here:

62 68 54 70 56 63 69 70

A. i) Calculate Bhagi's mean test score.

ii) What does the mean represent?

B. Suppose the first 70 was incorrect and the score was really 78.

62 68 54 **70** 56 63 69 70

62 68 54 **78** 56 63 69 70

i) Predict whether the mean will increase, decrease, or stay the same. Explain your prediction.

ii) Calculate the new mean. Was your prediction correct?

C. Repeat **part B** but change the first 70 to 62 instead.

62 68 54 **70** 56 63 69 70

62 68 54 **62** 56 63 69 70

D. Bhagi's teacher said that he could choose one test score to not include in his mean score.

i) Which test score do you think Bhagi should not include? Why?

ii) How does the mean score change by not including that score? Explain your thinking.

iii) Would the mean score increase or decrease if the test score of 68 were not included?

62 68 54 70 56 63 69 70

62 54 70 56 63 69 70

E. Complete each sentence and explain your thinking.

i) If a piece of data in a set of data is increased, the mean ____.

ii) If a piece of data is decreased, the mean ____.

iii) If one piece of data above the mean is not included, the mean ____.

iv) If one piece of data below the mean is not included, the mean ____.

GAME: Target Mean

Play in a group of 2 to 4. You will need one die.

Decide together on a target mean between 11 and 66.

Take turns. On your turn:

- Create 3 two-digit numbers:
  - Roll a die and decide whether the number you rolled will be the ones digit or the tens digit.
  - Roll again to get the other digit.
  - Repeat this to create 2 other two-digit numbers.
- Calculate the mean of your 3 numbers.

The player closest to the target mean wins 1 point.

The game is over when someone has 4 points.

For example:

The players decide on a target mean of 40.

Tashi's turn

He rolls a 3 and decides to use it as the tens digit. Then he rolls 5.

First number: 35

He rolls a 6 and decides to use it as the ones digit. Then he rolls 2.

Second number: 26

He rolls a 1 and decides to use it as the ones digit. Then he rolls 6.

Third number: 61

Tashi's mean: (35 + 26 + 61) ÷ 3 = 122 ÷ 3 = 40 R 2

Buthri's turn

She rolls a 2 and decides to use it as the ones digit. Then she rolls 1.

First number: 12

She rolls a 4 and decides to use it as the tens digit. Then she rolls 2.

Second number: 42

She rolls a 6 and decides to use it as the tens digit. Then she rolls 1.

Third number: 61

Buthri's mean: (12 + 42 + 61) ÷ 3 = 115 ÷ 3 = 38 R 1

40 R 2 is closer to 40 than 38 R 1, so Tashi gets 1 point.

Chapter 2 Graphing Data

7.2.1 Choosing a Graph

Try This

Four football teams are comparing their success.

Team A has won 12 games.

Team B has won 10 games.

Team C has won 18 games.

Team D has won 7 games.

A. Sketch a graph that compares the teams.

• To show the amount of data in different groups, you can use a **bar graph** or a **pictograph**.

• If there is a lot of data, you can decide how much data to represent with each square in the bar graph or each symbol in the pictograph.

For example:

Suppose you collect data about the favourite momos of the students in your school. You want to display it so it is easy to interpret.

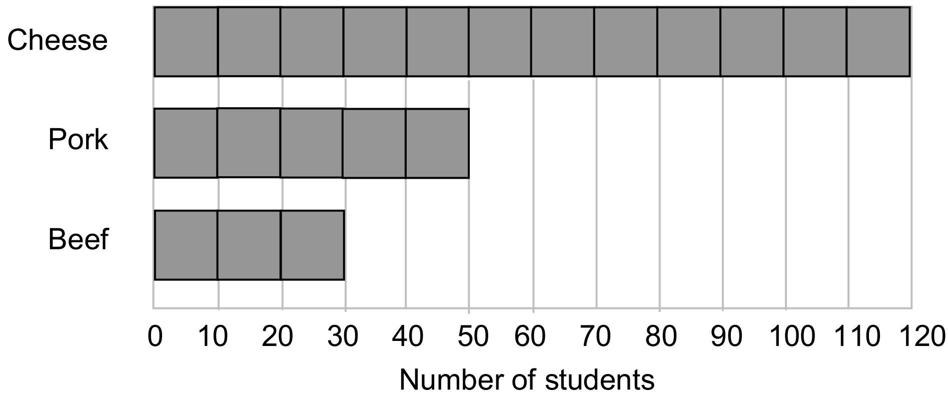
You find out that 120 students chose cheese

50 students chose pork

30 students chose beef

You could make a bar graph where each square represents 10 students. This makes it easier to draw the graph.

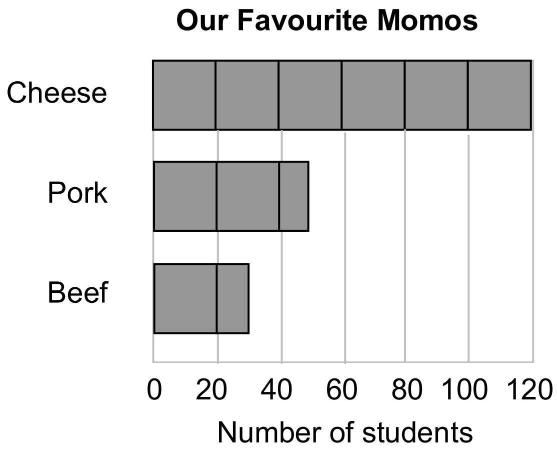
Our Favourite Momos



The numbers along the bottom show the **scale** — each square represents 10 students.

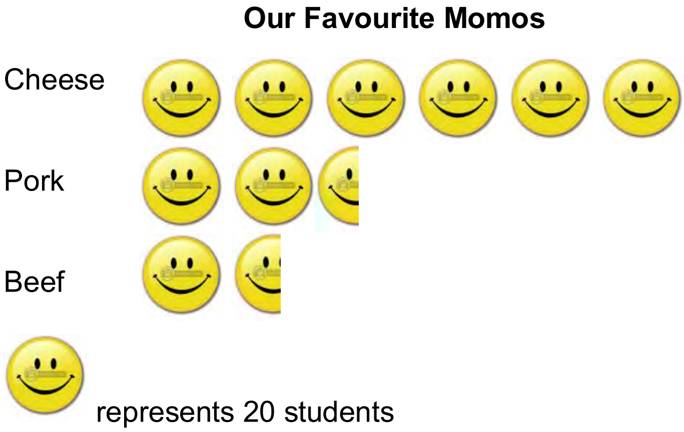
• You could have used a scale where each square represents 5 students, but the bars would have to be twice as long.

• You could make each square represent 20 students, but you would need two and a half squares to show 50 for pork and one and a half squares to show 30 for beef.



• If you use a pictograph to show the data, you need to make the same kinds of decisions about what scale to use. The scale tells what each symbol represents.

• With data values like 30, 50, and 120, it makes sense to use a scale where each symbol represents 10 or 20 students because you do not need too many symbols or too many partial symbols.



B. i) What type of graph did you use in **part A**?

ii) What scale might make sense for this set of data? Explain why.

iii) Is there any advantage to using a bar graph? a pictograph?

Examples

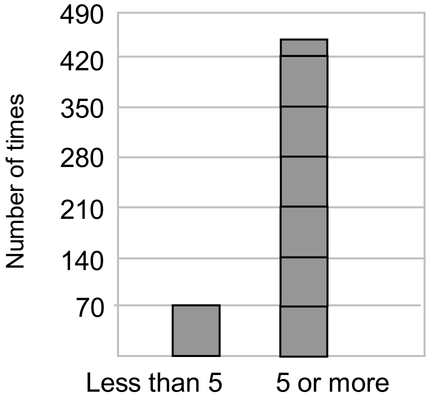
Example 1 Choosing a Scale for a Graph

Dechen's class did an experiment with two dice. On each roll, the numbers were added. The chart below shows the results of a total of 500 rolls. Draw a bar graph or a pictograph for the data.

Sum	Number of times
Less than 5	70
5 or more	430

Solution 1

Our Dice Roll Sums



Thinking

- I knew I needed a scale that didn't need a lot of squares to represent 430.
- Since 430 ÷ 70 is about 6, I knew that if 1 square represented 70, 430 would need a bit more than 6 squares.
- Because of the scale I used, someone reading the graph could only estimate the number of times 5 or more was rolled.

Solution 2

Our Dice Roll Sums

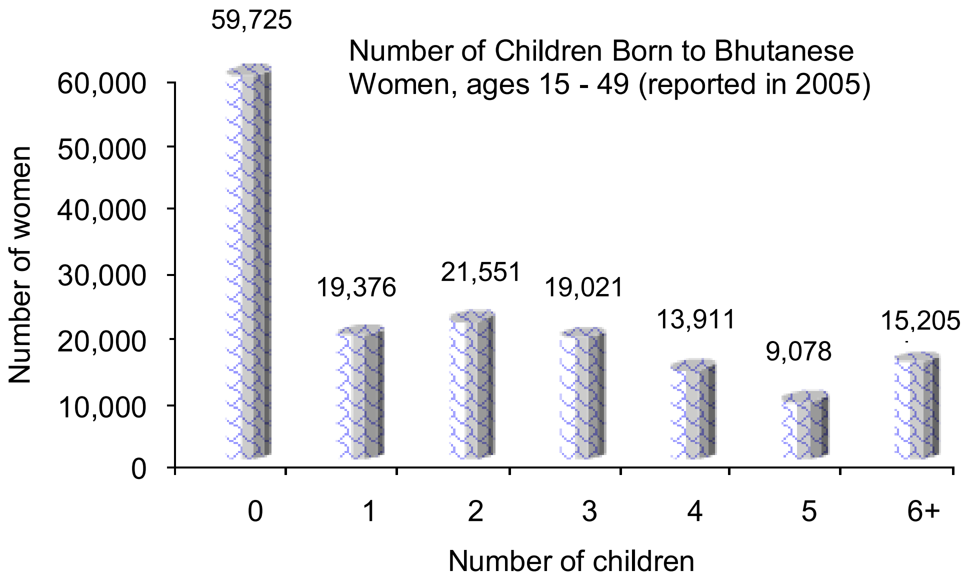
 represents 50 rolls.

Thinking

- I used a die for my pictograph symbol since the graph was about rolling dice.
- I let each die represent 50, even though it meant using partial dice.
- Sometimes when you use partial symbols on a pictograph, it's hard to tell exactly how much each partial symbol represents — maybe I should have used a different scale or a symbol that was a different shape, like a circle or square.

Example 2 Interpreting a Bar Graph

Describe five different things the bar graph tells you.



Solution

- About 60,000 women had no children.
- There were more women with 2 children than with 1 child or with 3 children.
- There were more women with 6 children than with 4 or 5 children.
- About 3 times as many women had no children as had 2 children.
- The numbers of women with 1, 2, or 3 children were not that different.

Thinking

- The data was grouped by the number of children women had.
- I saw how many women had different numbers of children.
- I also compared the sizes of the different groups.

Practising and Applying

1. a) Write down the names of all the students in your class. Count how many letters are in each first name. Record the data in a chart.

b) Create a bar graph to show how many first names have each number of letters.

c) Tell three things the graph shows.

2. a) Draw a pictograph where each symbol represents more than 1 to show the same information as in **question 1**.

b) How did you decide on the scale for the symbols?

3. Information was collected about the number of students who have 1 brother, 2 brothers, and 3 brothers in three different schools. What scale would you use to create pictographs for each school? Explain your choice.

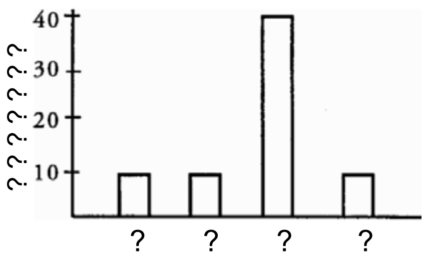
School A	Number of brothers		
	1	2	3
Number of students	200	250	150

School B	Number of brothers		
	1	2	3
Number of students	400	480	240

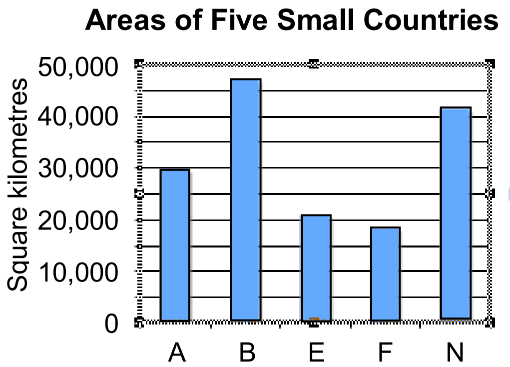
School C	Number of brothers		
	1	2	3
Number of students	250	300	110

4. Create a pictograph and a bar graph for School A in **question 3**.

6. What might this graph be about?



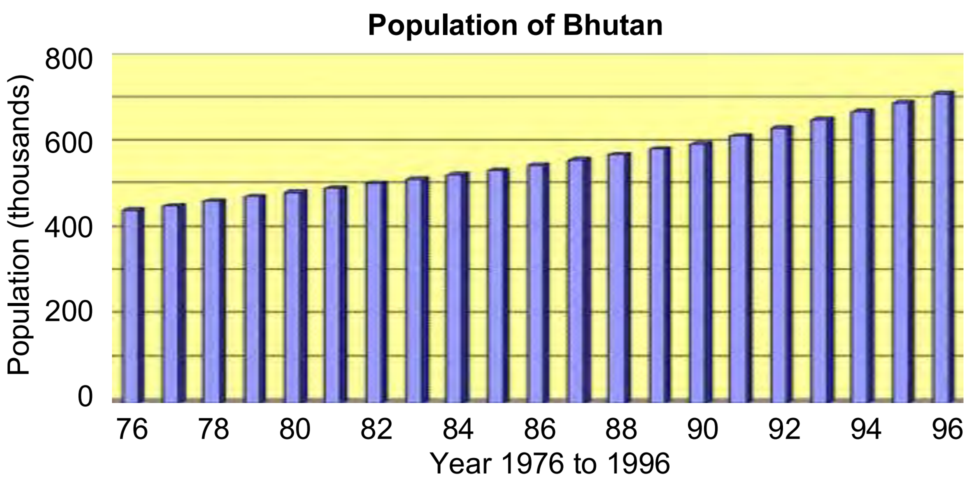
7. The areas of five small countries are shown on the graph below. List three different conclusions you can make by looking at the graph.



A is Armenia
B is Bhutan
E is El Salvador
F is Fiji
N is Netherlands

8. Why is a graph a good way to share information?

5. This graph shows the population of Bhutan from 1976 to 1996. Tell two things this graph shows you about the population.



7.2.2 Double Bar Graphs

Try This

Information was collected from the students in Class II and Class V about their favourite fruit.

A. How do the choices in the two classes compare?

Favourite Fruit

	Class II	Class V
Apple	8	5
Banana	20	18
Mango	12	18

• Sometimes the same type of information is collected at two different times, or the same type of information is collected from two different groups. It is helpful to draw a **double bar graph** to show both sets of data at the same time.

For example:

Suppose you ask 60 boys and 60 girls about their favourite colour and get these results.

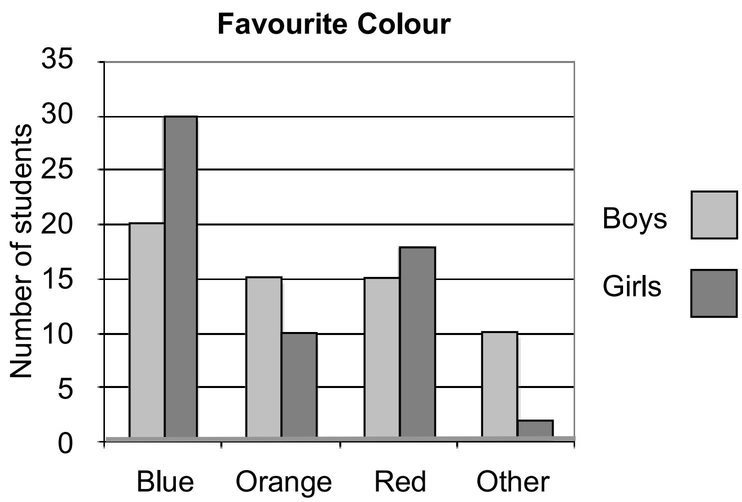
Boys' Favourite Colour

Blue	20
Orange	15
Red	15
Other	10

Girls' Favourite Colour

Blue	30
Orange	10
Red	18
Other	2

You can show both sets of data in a double bar graph. Then you can easily compare the favourite colour of boys and girls.



• Here are some things you can tell from the double bar graph on the previous page:

- Most boys and most girls chose blue.
- More girls than boys chose blue.
- More boys than girls chose a color other than blue, orange, or red.
- The same number of boys chose orange as chose red, but more girls chose red than chose orange.

• A double bar graph is only appropriate sometimes.

For example:

If you asked only 5 girls about their favourite colour, but you asked 60 boys, comparisons between the two groups would not be meaningful.

• Remember these important points about a double bar graph:

- You need different colours or markings for the bars that describe the two different groups. This makes it easy to read the graph.
- You need a **legend** that shows which colour goes with which group.
- The graph should have a title and labels.
- The bars for each group should always be on the same side.

For example, in the Favourite Colour graph, each bar for boys was to the left of the bar for girls.

B. i) Sketch a double bar graph to show the information in **part A**.

ii) Was it easier to draw conclusions from the chart or from the graph? Why?

Examples

Example 1 Interpreting a Double Bar Graph

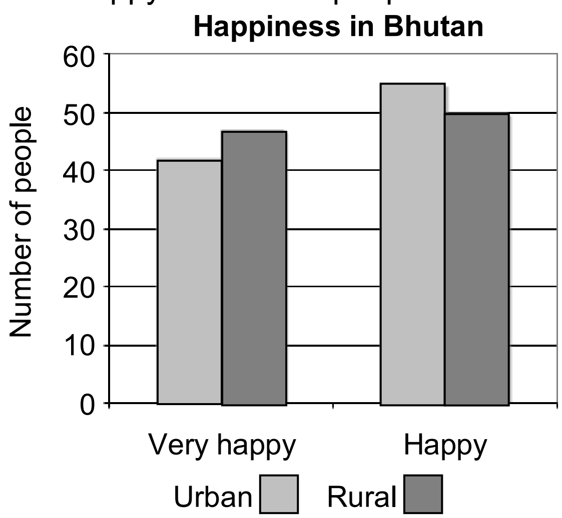
In 2005, a survey was done to see how happy Bhutanese people are.

The graph compares how urban and rural people answered.

It shows the number of people out of each 100 who reported being very happy or happy.

For example, out of each group of 100 urban people asked, 42 said they were very happy.

What information about the happiness of the Bhutanese can you learn from the graph?



Example 1 Interpreting a Double Bar Graph [Continued]	
Solution • Fewer urban people are very happy than are happy. The same is true for rural people. • About 10 more urban people out of every 100 are happy than are very happy. • About 5 more rural people out of every 100 are happy than are very happy. • A greater fraction of rural people than urban people are very happy.	**Thinking**  I knew I could make these comparisons: • urban people who are very happy compared to urban people who are happy • rural people who are very happy compared to rural people who are happy • very happy people who are urban compared to very happy people who are rural • happy people who are urban compared to happy people who are rural

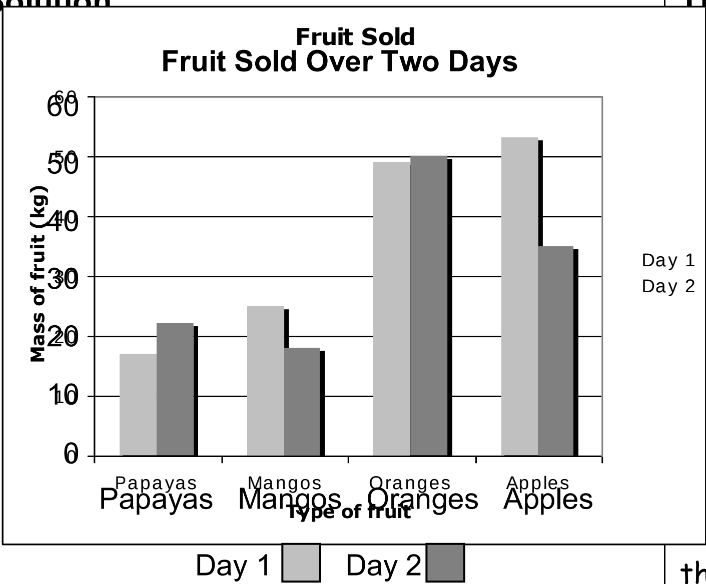
Example 2 Constructing a Double Bar Graph

Create a double bar graph about the amount of fruit sold at a market.

Fruit Sold (kg) Over Two Days

	Papayas	Mangos	Oranges	Apples
Day 1	17	25	49	53
Day 2	22	18	50	35

Solution



Thinking

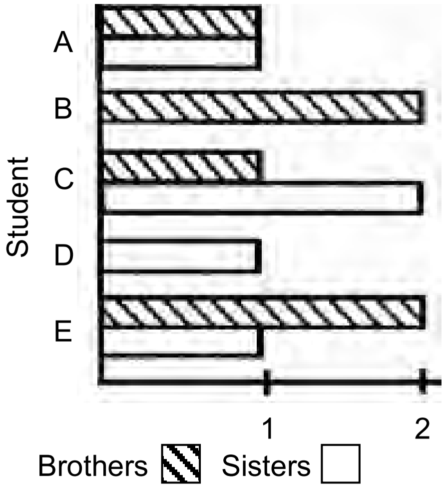
• A double bar aph helped e compare y 1 with y 2.

• I used a scale where one it of height represents) kg because the data ent up to 53 kg and I dn't want the bars to be o high.

• I put the Day 1 data on the left and the Day 2 data on the right for each kind of fruit.

Practising and Applying

1. a) List six or more things that this graph tells you about these five students.



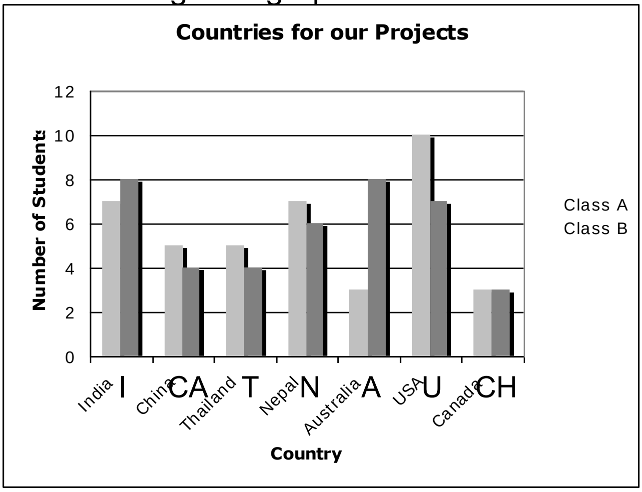
b) Create a new double bar graph using the information about these five students.

Student	Brothers	Sisters
F	1	1
G	2	3
H	2	2
I	4	2
J	0	3

2. The chart below shows the maximum and minimum monthly temperatures (°C) in Mongar from January to June. Create a double bar graph to show the information.

	J	F	M	A	M	J
Max	16	16	20	23	25	25
Min	8	8	12	14	20	19

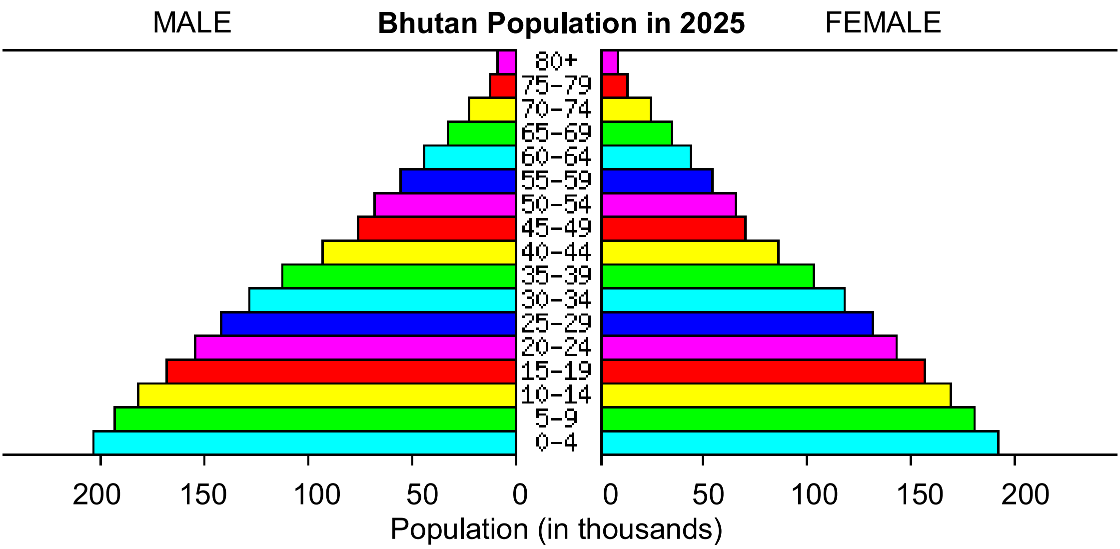
3. Students in two classes were asked to choose a country for their project. The graph below shows which countries they chose. Tell three things the graph shows.



I	India	CA	Canada
T	Thailand	N	Nepal
A	Australia	U	U.S.
CH	China		

4. Ask 10 students in your class and 10 students in another class how many glasses of water they drink in a day. Make a double graph to compare the data for the two classes.

5. This pyramid double bar graph displays data on two sides of a centre line. It shows the predicted population of males and females in Bhutan in 17 different age groups in 2025.



a) Why is it called a double bar graph?

b) Which is easier to compare?

• the population of males (or females) in the different age groups

OR

• the population in the same age group for males and females

Explain your thinking.

6. Give an example of data that would be appropriate to display using a double bar graph.

7.2.3 Coordinate Graphs

Try This

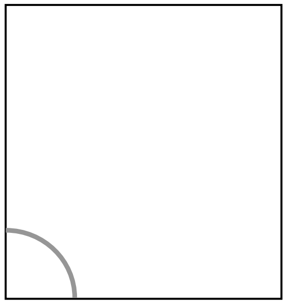
A. Work with a classmate. Ask him or her to stand somewhere away from you in the classroom. Describe exactly your partner's location in relation to where you are. Use as few measurements as possible.

• It takes two numbers to describe the location of something on a flat surface, like a piece of paper.

For example:

Suppose you said a point is 3 cm from the bottom left corner of a piece of paper. There are many different possible locations.

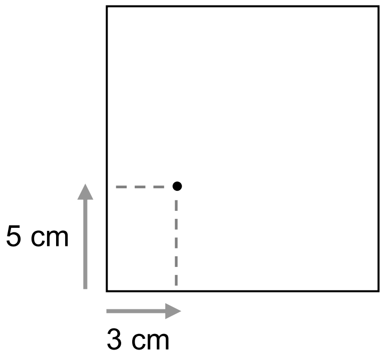
3 cm from the bottom left corner could be anywhere on the grey curve.



If you were to describe the position using two numbers, there is only one possible location.

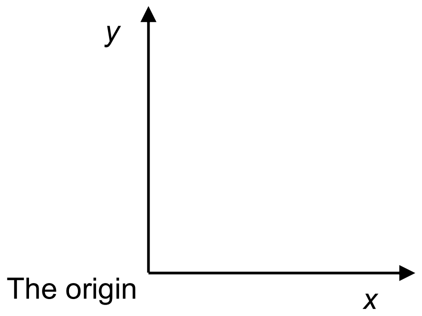
For example:

Suppose you said a point is 3 cm to the right of the bottom left corner of the paper and 5 cm above the bottom left corner. There is only one possible location, as shown here.



• People have invented a **coordinate grid** system to locate points.

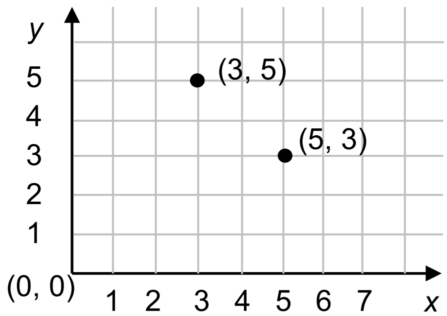
• A coordinate grid system has two perpendicular lines called **axes**.
The horizontal line is the ***x*-axis** and the vertical line is the ***y*-axis**.
The point where they meet is the **origin**.



• You can **plot** points on the grid using **ordered pairs**. The first number in the pair is the ***x*-coordinate**. It tells how far right to go from the origin. The second number is the ***y*-coordinate**. It tells how far up to go from the origin.

For example:

- The ordered pair (3, 5) means to go right 3 and up 5.
- The ordered pair (5, 3) means to go right 5 and up 3.
- The origin is (0, 0).



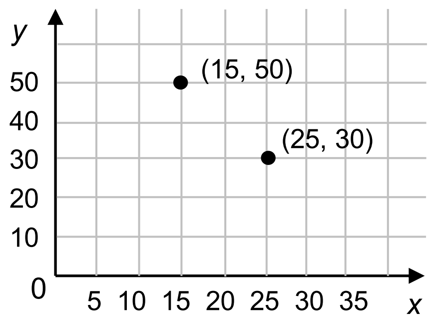
• If the **coordinates** are large numbers, you can use a scale on each axis.

For example:

Each grid space might represent 2, 5, or 10.

The units on one axis can be different from those on the other.

On this grid, the *x*-coordinates are plotted on a scale of 5.
The *y*-coordinates are plotted on a scale of 10.

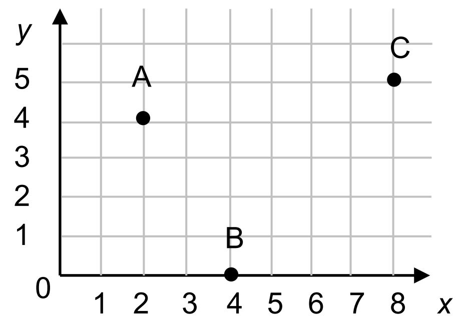


B. i) How many measurements did you use to describe your partner's location in **part A**?
ii) How could you have described his or her location using ordered pairs?

Examples

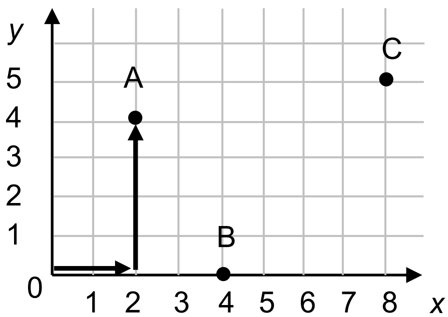
Example 1 Naming Points on a Grid

What are the coordinates of each point?
How do you know?



Solution

• To get to A, I went right 2 spaces and up 4 spaces, so the coordinates are (2, 4).



• To get to B, I went right 4, but I did not go up, so I went up 0. B is (4, 0).

• To get to C, I went right 8 and up 5. The coordinates are C (8, 5).

Thinking

• I started at the origin (0, 0) for each point.

• First I counted how many grid spaces right I had to go. That was the first number in the ordered pair.

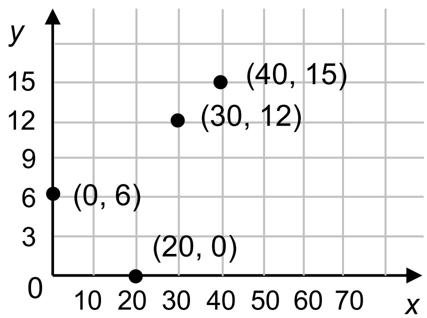
• Then I counted how many grid spaces up I had to go. That was the second number in the ordered pair.

Example 2 Plotting Points on a Grid

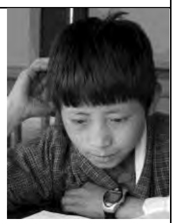
Plot these points on the same grid. Label each point with an ordered pair.

a) (30, 12) **b)** (20, 0) **c)** (0, 6) **d)** (40, 15)

Solution



Thinking



• I knew the first number was how far right along the *x*-axis from the origin and the second number was how far up along the *y*-axis.

• I wanted my grid to be a reasonable size:

- I used a scale of 10 on the *x*-axis because the *x*-coordinates were 0, 20, 30, and 40.

- I used a scale of 3 on the *y*-axis because the *y*-coordinates were 0, 6, 12, and 15.

Practising and Applying

1. a) Plot and label the points (6, 5) and (4, 3) on the same coordinate grid.

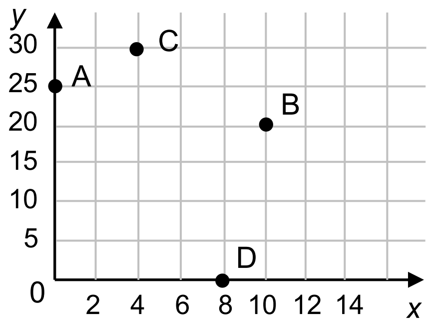
b) Plot and label a third point that is between the first two points.

2. Plot and label all the points below on the same grid using a scale on each axis. Describe each scale and why you chose it.

A. (15, 2) **B.** (30, 4)

C. (25, 1) **D.** (10, 7)

3. What are the coordinates of each lettered point?



4. a) What do you notice when you plot these points on the same grid?

(1, 3), (2, 6), (3, 9), and (4, 12)

b) List two other pairs of coordinates that would fit the pattern in **part a)**.

5. How would you explain to someone how you know that (2, 3) and (3, 2) are not at the same location on a coordinate grid?

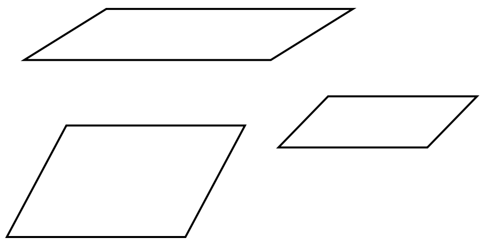
6. The locations of three points on a grid are described below. List possible coordinates for each point. Explain your thinking.

Point A is near the origin.

Point B is far to the right of the origin.

Point C is very high above the origin.

7. Two vertices of a parallelogram are located at (2, 6) and (5, 8) on a coordinate grid. What are possible coordinates for the other two vertices?



8. a) Plot (1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), and (9, 1) on the same coordinate grid.

b) What is true about all the coordinates?

9. A point is 3 units away from the origin.

a) Can its coordinates be (3, 3)? How do you know?

b) If not, what could be the coordinates?

10. a) Is it possible for (5, [ ]) and ([ ], 5) to be in the same location if [ ] is the same value? Explain your thinking.

b) If [ ] = 4 in both ordered pairs in **part a)**, is one point closer to the origin? How do you know?

Chapter 3 Probability

7.3.1 Describing Probability

Try This

A. Think about Class V students in Bhutan. Describe one thing for each.

i) Something that is unlikely to be true about a Class V student.

ii) Something that is definitely true about a Class V student.

ii) Something that is likely to be true about a Class V student.

• **Probability** is about describing how likely it is that an **event** will happen.

• You can use special words to describe the probability of an event.
Some common probability words are:

- certain
- likely and very likely
- unlikely and very unlikely
- impossible

• A **probability line**, like the one below, can help you choose probability words. You think about where an event would be placed on the line. Then you use the placement to decide on the best probability word.

For example:

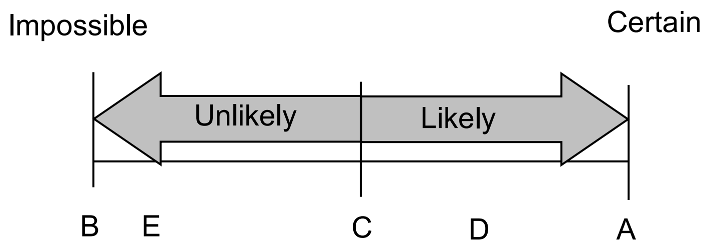
Event A: Your class has a teacher.

Event B: You will be in Class X next year.

Event C: A particular student in your class is a girl.

Event D: A student in your class is 10 to 12 years old.

Event E: A student in your class was not born in Bhutan.



Event A: Certain

Event B: Impossible

Event C: As likely to happen as not to happen

Event D: Likely

Event E: Very unlikely

• If you conduct a probability experiment, you can describe the probability that an event will occur by counting or by using probability words.

For example:

You roll a die 20 times and get these results:

1	3	6	1	2	5	1	4	5	5
1	4	2	6	3	4	2	2	1	4

You can organize the results into a chart like this:

Number rolled	1	2	3	4	5	6
Number of times	5	4	2	4	3	2

- A 6 was rolled "2 times out of 20" (counting), so rolling a 6 is unlikely (using probability words).
- An even number was rolled "10 times out of 20", so rolling an even number is as likely to happen as not to happen.
- It is impossible to roll a 7 because there is no 7 on a die.

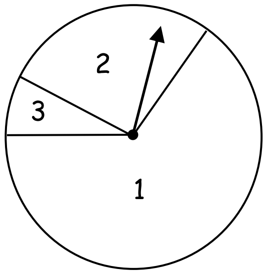
B. Draw a probability line. Place each event from **part A** on the line.

Examples

Example 1 Describing Experimental Results

You spin this spinner 30 times and get these results:

2	1	1	2	1	1	1	2	1	1
1	1	3	1	2	3	2	1	1	2
1	1	2	2	1	1	1	2	3	1



Describe the probability of spinning each number by counting and by using probability words.

a) 1 **b)** 2 **c)** 3 **d)** 4

Solution

a) I got 1 “18 times out of 30”, so 1 is likely.

b) I got 2 “9 times out of 30”, so 2 is unlikely.

c) I got 3 “3 times out of 30”, so 3 is very unlikely.

d) There is no 4 on the spinner, so 4 is impossible.

Thinking

• I counted how many times I spun each number and then compared it to the number of times I spun altogether, 30.

Example 2 Using a Probability Line

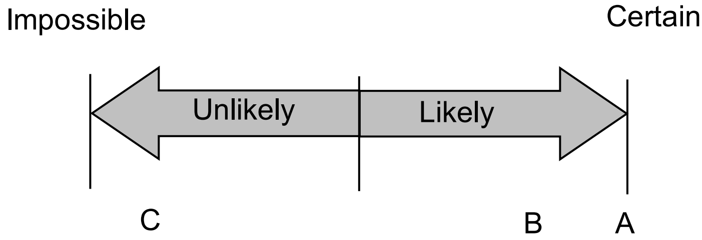
Use a probability line to show how likely each event is in your life.

A. Being in Class VI next year

B. Watching TV tomorrow

C. Seeing my grandmother this month

Solution



Thinking

A. I am sure I'll pass Class V because my marks are good.

B. I watch TV most days, but sometimes I don't, so I can't say it's certain.

C. We usually visit my grandmother twice a year, so it is very unlikely. But it's not impossible. If she became sick this month, we might go to help her.

Practising and Applying

1. Use probability words to describe the probability of each event.

a) I will have fruit for breakfast.

b) I will stay up past 10 p.m. tonight.

c) A tree will talk to me tomorrow.

d) We will have homework today.

e) I will play football next week.

f) I will go to a tsechu this year.

2. Draw a probability line. Label it with the letters of the events from **question 1**.

3. Name an event in your life that matches each probability word. Explain two of your choices.

a) certain **b)** likely

c) very likely **d)** very unlikely

e) impossible

4. Roll two dice 30 times. Record the sum each time. Describe the probability of each sum using counting and using probability words.

a) 1 **b)** 4

c) 8 **d)** less than 15

5. Toss a coin 25 times. Record the result each time. Describe each probability using words. Explain your choice of words.

a) Tossing a Khorlo

b) Tossing a Tashi Ta-gye

c) Tossing 2 Khorlos in a row

d) Tossing 5 Khorlos in a row

6. a) Use a probability line to show the probabilities from **question 4**.

b) Use a probability line to show the probabilities from **question 5**.

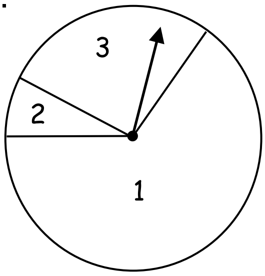
7. Use a spinner like the one below.

a) Name an event that matches each probability word.

i) certain **ii)** very likely

iii) very unlikely **iv)** impossible

b) Explain two of your choices in **part a)**.



8. Suppose you spun a spinner 30 times. How many times would you have to spin a particular number to say that it is very likely? not very likely? Explain your thinking for each.

9. Why is a probability line a good way to show probabilities?

CONNECTIONS: Magic Tricks With Dice

It is possible to predict numbers on the faces of a die without seeing them.

For example, you can figure out the total of the three hidden faces in this stack of two dice by subtracting the number at the top from 14.

1. How does the trick work? (Hint: Look at how the six numbers on the six faces die are arranged.)

2. a) What is the total of the five numbers on the five hidden faces in this stack of three dice?

b) How does the trick work for three dice?

3. Explain how the trick works for a stack of four dice.

7.3.2 Using Numbers to Describe Probability

Try This

A. i) Predict how many times you will roll a number greater than 2 if you roll a die 30 times.
ii) Roll a die 30 times to test your prediction.

• In addition to counting, for example, "15 out of 20", and using probability words such as "likely", you can also use fractions or **equivalent decimals** to describe probabilities.

For example:

If an event happened "5 out of 10 times", the probability of it happening could be described by the fraction $\frac{5}{10}$ or by the equivalent decimal, 0.5.

• The greatest possible probability is 1. The least possible probability is 0.

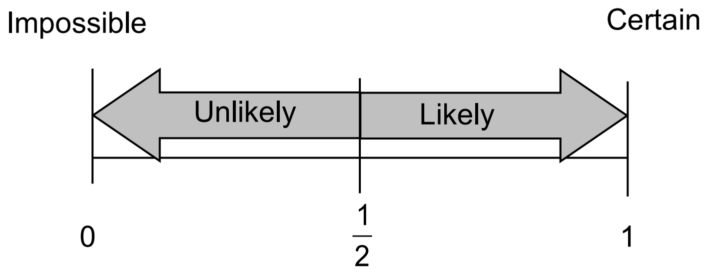
For example:

If you did an experiment 10 times,

- the most an event could happen is 10 times out of 10 or $\frac{10}{10}$, which is 1.
- the least it could happen is 0 times out of 10 or $\frac{0}{10}$, which is 0.

• If you look at a probability line, you will see this:

- If an event is impossible, the probability is 0.
- If an event is certain, the probability is 1.
- If an event is likely, the probability is greater than $\frac{1}{2}$.
- If an event is unlikely, the probability is less than $\frac{1}{2}$.



• There are two kinds of probability:

- **Experimental probability** is based on the results of repeating an experiment many times. Each time is called a **trial**.

- **Theoretical probability** is the probability you expect if you think about all of the possible **outcomes**.

For example:

You want to know the probability of tossing a coin and getting a Khorlo.

You could do an experiment to get the experimental probability.

Experimental probability

$= \frac{\text{number of favourable results}}{\text{number of trials}}$

If you toss a coin 40 times and get a Khorlo 22 times, there are 22 **favourable results** in 40 trials.

So the experimental probability of tossing a Khorlo is $\frac{22}{40}$.

You could think about outcomes to get the theoretical probability.

Theoretical probability

$= \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$

If you toss a coin, there is only 1 **favourable outcome** (Khorlo) out of 2 **possible outcomes** (Khorlo and Tashi Ta-gye).

So the theoretical probability of tossing a Khorlo is $\frac{1}{2}$.

A Khorlo is a favourable result or outcome because it is the event you want to find out about.

• Notice that the experimental probability and theoretical probability above are not exactly the same. They can be the same, but often they are not.

-The theoretical probability of any particular outcome never changes, but the experimental probability can change if you repeat the experiment.

- The experimental probability is usually close to the theoretical probability if you do the experiment many times.

B. i) Write as a fraction your experimental probability for rolling a number greater than 2 in **part A**.

ii) What is the theoretical probability?

Examples

Example 1 Calculating Experimental and Theoretical Probability

Find the experimental and theoretical probability of each event below. Express each as a fraction or decimal. For each experiment, use 25 trials.

a) rolling a 1 or 2 on a die

b) rolling an even number

c) rolling a number less than 2

Solution

Results of my experiment

Number rolled	1	2	3	4	5	6
Number of times	3	5	4	4	4	5

a) *Rolling a 1 or 2*

Experimental probability = $\frac{8}{25} = \frac{32}{100} = 0.32$

Theoretical probability = $\frac{2}{6}$

b) *Rolling an even number*

Experimental probability = $\frac{14}{25} = \frac{56}{100} = 0.56$

Theoretical probability = $\frac{3}{6}$

c) *Rolling a number less than 2*

Experimental probability = $\frac{3}{25} = \frac{12}{100} = 0.12$

Theoretical probability = $\frac{1}{6}$

Thinking

First I did the experiment and recorded the results in a chart.

a) Rolling a 1 or 2 means I have to combine the values for 1 and 2.

• 1 or 2 was rolled 8 times in 25 trials.

• There are 2 favourable (1, 2) outcomes out of 6 possible outcomes (1, 2, 3, 4, 5, 6).

b) An even number was rolled 14 times in 25 trials.

• There are 3 favourable outcomes (2, 4, and 6) out of 6 possible outcomes.

c) The only number less than 2 is 1 and it was rolled 3 times in 25 trials.

• There is 1 favourable outcome (1) out of 6 possible outcomes.

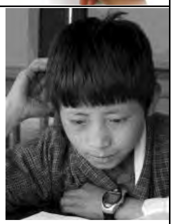
Example 2 Creating a Situation for a Probability

You are going to put slips of paper with numbers on them into a bag. You want the theoretical probability of drawing a slip with 1 on it to be $\frac{2}{3}$. What could you put in the bag?

Solution

1	1	3

Thinking



• I knew that if there were 3 outcomes and 2 were favourable, the probability would be $\frac{2}{3}$.

• Since the favourable outcome had to be 1, I needed two slips to have 1 on them. The other slip could have any other number.

Practising and Applying

1. a) Toss a Nu 1 coin 20 times. Record the results.

b) What is the experimental probability of tossing each?

i) a Khorlo

ii) a Tashi Ta-gye

iii) either a Khorlo or a Tashi Ta-gye

2. a) Roll a die 20 times. Record the results.

b) What is the experimental probability of rolling each, as a decimal?

i) a 2

ii) a number less than 4

iii) an odd number

c) What fraction shows the theoretical probability for each event in **part b)**?

d) Look at your experimental results from **part a)**.

i) Name an event with an experimental probability greater than $\frac{4}{20}$.

ii) Name an event with an experimental probability less than $\frac{4}{20}$.

3. a) Conduct an experiment:

• Roll two dice to create a two-digit number.

• Make a prediction — will the sum of this number and the next two-digit number you create be greater than 66?

• Roll again and record whether or not your prediction was correct.

• Repeat until you have rolled 20 times, predicted and checked 20 times.

b) What was your experimental probability for being correct? Write it as a decimal.

4. Place four slips of paper like this in a bag.

1	3
4	5

a) Draw a slip, record the number, and return the slip to the bag. Repeat this 25 times.

b) What is the experimental probability, as a decimal, of drawing each?

i) a 3

ii) an even number

iii) a number less than 4

c) What is the theoretical probability of each event in **part b)**?

5. What four number slips would you put in a bag to make each true?

a) the probability of drawing a 1 is $\frac{1}{2}$

b) the probability of drawing a 3 is $\frac{3}{4}$

c) the probability of drawing a 5 is 0

d) the probability of drawing an even number is 1

6. Why can the theoretical probability of an event never change but the experimental probability can change?

UNIT 7 Revision

1. Draw a picture to determine the mean of each set of data. Tell how you used the picture.

a) 7, 15, 22, 8

b) 1, 1, 1, 5, 7, 9

2. a) Calculate Sonam's mean score for these test scores: 52, 62, 74, 80, 72.

b) List a different set of five test scores that have the same mean.

3. Predict which set of data in each pair has the greater mean. Explain your thinking.

a) 2, 3, 5, 10 or 2, 3, 5, 18

b) 2, 3, 5, 10 or 1, 3, 5, 11

c) 2, 3, 5, 10 or 3, 5, 10

d) 2, 3, 5, 10 or 2, 3, 5

4. a) Create a bar graph to show the number of sisters of these 20 students.

Number of Sisters of 20 Students

0	1	1	2	1	0	3	2	2	1
1	2	2	2	4	0	1	1	0	3

b) Tell three things about your graph.

c) Make a pictograph to show the same information.

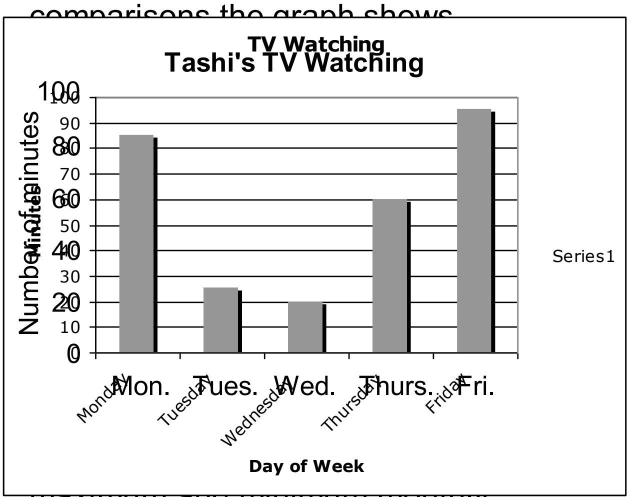
5. You can estimate how many words are on a page in a book.

a) Open a book to somewhere in the middle. Count the number of words on three pages.

b) Create a bar graph to show the number of words on the three pages.

c) What scale did you use? Why?

6. The number of minutes that Tashi watched TV is shown on the bar graph below. List three comparisons the graph shows.

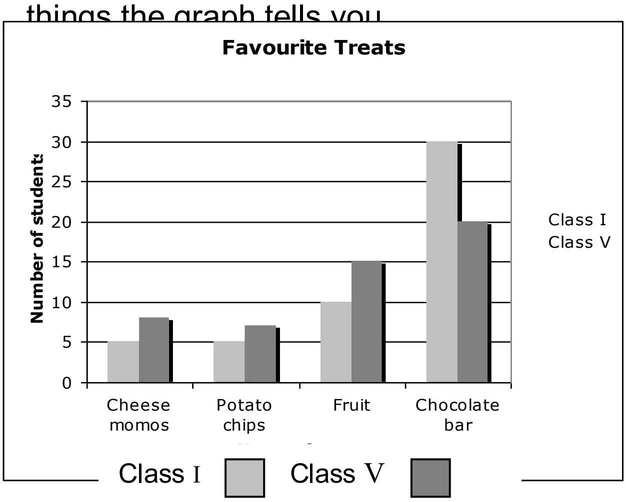


maximum and minimum monthly temperatures in Trashigang for January to June. Create a double bar graph to show the information.

Temperature in Trashigang (ºC)

	J	F	M	A	M	J
Max	20	22	25	28	30	31
Min	11	12	14	17	21	23

8. This graph shows the favourite treats of 50 students in Class I and 50 students in Class V. List three things the graph tells you.



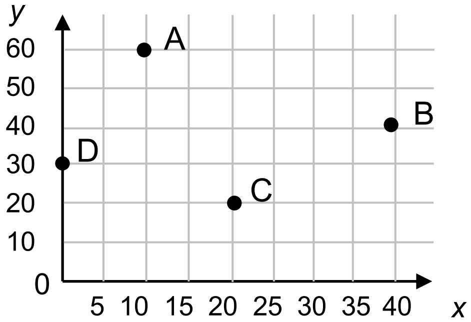
9. Sithar says that a double bar graph would be a good way to show the number of hours students spent doing homework in each subject in March and in November. Do you agree? Explain your thinking.

10. Plot the points below on the same grid. Describe the scale you chose and explain why you chose that scale.

A. (5, 4) **B.** (30, 16)

C. (35, 12) **D.** (45, 20)

11. Name the coordinates of each point.



12. Two vertices of a square are at (5, 8) and (2, 5). Where might the other vertices be?

13. a) Plot the points (2, 4), (2, 6), (5, 4), and (5, 6).

b) What shape do they form?

14. Roll a die 25 times. Record the difference between the numbers you roll each time. What words describe the probability of each difference?

a) 6 or less **b)** 2

c) 1 **d)** greater than 2

15. a) Roll a die 20 times. Record the results.

b) What is the experimental probability, as a decimal, of rolling each?

i) a 1 or a 2

ii) a number greater than 5

c) What is the theoretical probability, as a fraction, of each event in **part b)**?

d) Look at the results of your experiment.

i) Name an event with an experimental probability greater than 0.5.

ii) Name an event with an experimental probability less than 0.5.

16. What number slips would you put in a bag to make each theoretical probability true?

a) probability of drawing a 3 is $\frac{1}{3}$

b) probability of drawing an even number is $\frac{4}{5}$

c) probability of drawing a 2 is $\frac{1}{10}$

GLOSSARY

Instructional Terms

calculate: Figure out the number that answers a question; compute

classify: Put things into groups according to a rule and name the groups; e.g., classify triangles as right, acute, or obtuse by the size of their angles

compare: Look at two or more objects or numbers and identify how they are the same and how they are different; e.g., compare the numbers 6.5 and 5.6; compare the size of the students’ feet; compare two shapes

create: Make your own example or problem

describe: Tell, draw, or write about what something is or what something looks like; tell about a process in a step-by-step way

determine: Decide what the answer or result is for a calculation, a problem, or an experiment

draw: 1. Show something using a picture **2.** Take out an object without looking; e.g., draw a card from a deck

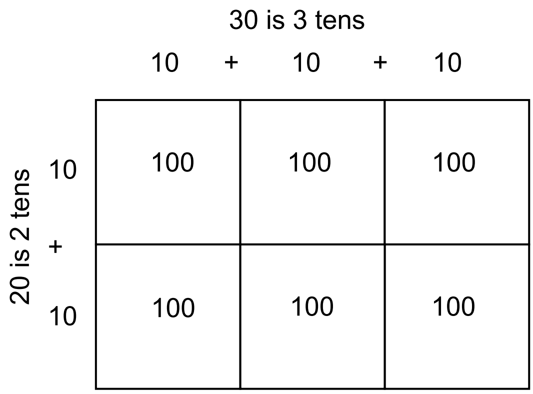
estimate: Use what you know to make a sensible decision about an amount; e.g., estimate how long it takes to walk from your home to school; estimate how many leaves are on a tree; estimate the sum of 3210 + 789

explain (your thinking): Tell what you did and why you did it; write about what you were thinking; show how you know you are right

explore: Investigate a problem by questioning and trying new ideas

measure: Use a tool tell how much; e.g., use a ruler to measure a height or distance; use a protractor to measure an angle; use balance scales to measure mass; use a measuring cup to measure capacity; use a stopwatch to measure elapsed time

model: Show an idea using objects, pictures, words, and/or numbers; e.g., you can model 30 × 20 using a rectangle model



20 × 30 = 2 tens × 2 tens = 6 hundreds = 600

predict: Use what you know to figure out what is likely to happen; e.g., predict the number of times you will roll a number greater than 2 when you roll a die 30 times

show (your work): Record all the calculations, drawings, numbers, words, or symbols that you used to calculate an answer or to solve a problem

sketch: Make a quick drawing to show your work; e.g., sketch a picture of a field with given dimensions

solve: 1. Find an answer to a problem **2.** Find a missing number in an open number sentence; e.g., you solve 3 + ■ = 7 by finding the value of ■, which is 4

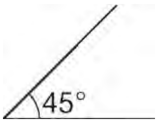
sort: Separate a set of objects, drawings, ideas, or numbers into groups according to an attribute; e.g., sort 2-D shapes by the number of sides

visualize: Form a picture in your head of what something is like; e.g., visualize the number 6 as 2 rows of 3 dots like you would see on a die

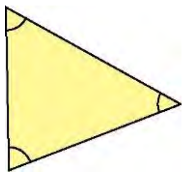
Definitions of Mathematical Terms

A

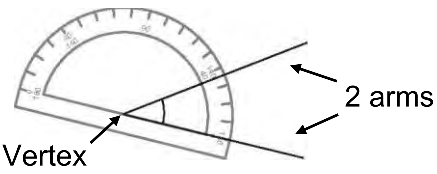
acute angle: An angle less than 90º; e.g.,



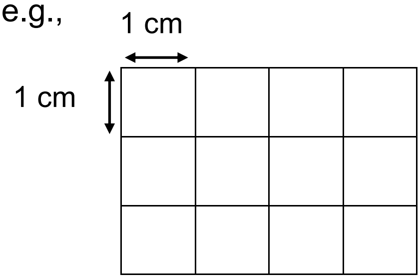
acute triangle: A triangle in which all angles are acute angles; e.g.,



angle: A figure formed by two arms with a shared endpoint, or vertex; the measure of an angle is the amount of turn between the two arms; angles are often measured in degrees



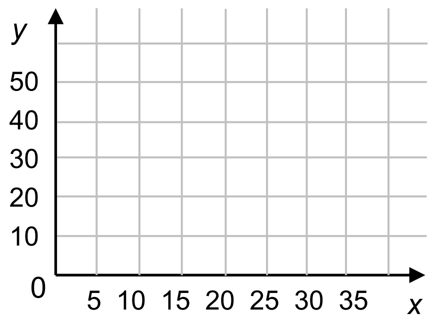
area: The number of square units (often square centimetres or square metres) needed to cover a shape; e.g.,



The area is 12 cm^2.

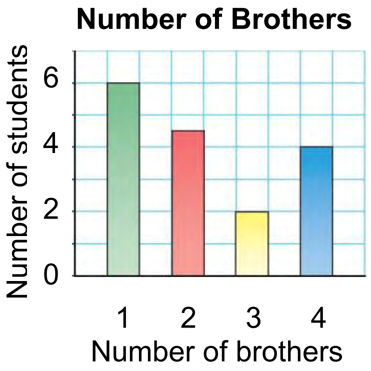
average: Average is a term we can use instead of the term mean
See *mean*

axis (axes): One of the two lines used to create a graph or to locate points in a coordinate grid; e.g.,

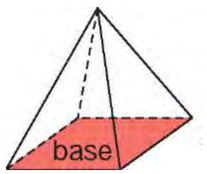


B

bar graph: A graph that compares the sizes of bars that each represent a category in a set of data; e.g.,

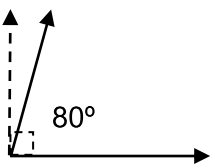


base: The face(s) that determines the name of a prism or pyramid; e.g.,



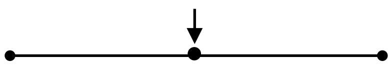
A square-based pyramid

benchmark angles: Special angles that you can use to estimate the size of other angles; these angles are usually 45º, 90º, 135º, and 180º; e.g., this angle is about 80º because it is a little less than a 90º angle



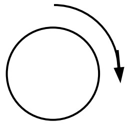
capacity: The amount that a container can hold, often measured in millilitres (mL) or litres (L)

centre point: A point on a line segment that divides the line segment in half; e.g.,

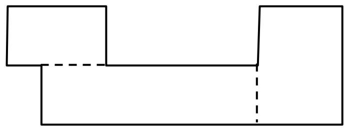


C

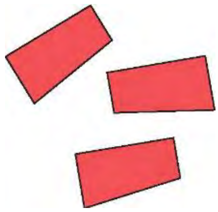
clockwise (cw): The direction that the hands of a clock move; used to describe the direction of a rotation



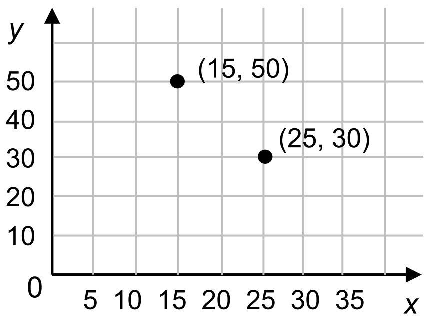
composite shape: A shape that is made up of several simple shapes; e.g., this composite shape can be divided into three rectangles



congruent: Identical in size and shape; shapes, side lengths, and angles can be congruent; e.g., these three shapes are congruent

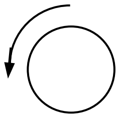


coordinate grid: A grid system based on the intersection of two axes; the *x*-axis is the horizontal axis and the *y*-axis is the vertical axis; the origin is the point of intersection of the two axes; e.g.,

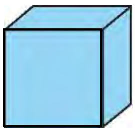


coordinates: A pair of numbers that tells a position on a coordinate grid, often written as ordered pairs; e.g., the coordinates of the two points on the grid at the bottom of **page 229** are (15, 50) and (25, 30)

counterclockwise (ccw): The direction opposite to the direction the hands of a clock move; used to describe the direction of a rotation



cube: A 3-D shape that has six congruent square faces

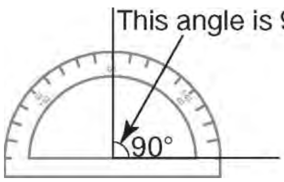


cuboid: Another name for a rectangular prism See *rectangular prism*

D

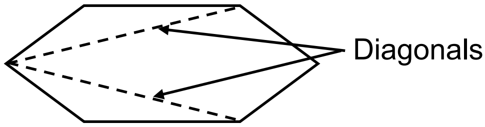
data: Information collected in a survey, in an experiment, or by observing; the word data is plural, not singular; e.g., a set of data can be a list of students' names and the numbers of their quiz marks

degree: A unit of measure for angle size; e.g.,



denominator: The number in a fraction that represents the total number of parts in a set or the number of parts the whole has been divided into; e.g., in $\frac{4}{5}$, the denominator is 5

diagonal: A line segment joining two vertices of a polygon that are not next to each other



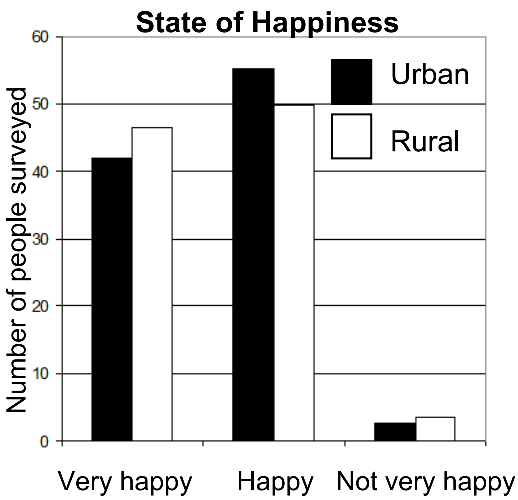
difference: The result of a subtraction; e.g., in 45 – 5 = 40, the difference is 40

dimension: The size or measure of an object, usually length; e.g., the width and length of a rectangle are its dimensions

dividend: A number that is being divided; e.g., in 45 ÷ 5 = 9, the dividend is 45

divisor: The number by which another number is divided; e.g., in 45 ÷ 5 = 9, the divisor is 5

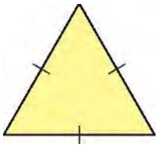
double bar graph: A special bar graph that shows two sets of data using the same categories; e.g.,



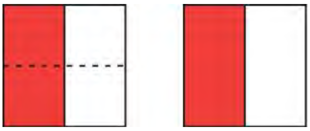
E

endpoint: The point where a line segment begins or ends

equilateral triangle: A triangle with three sides of equal length (and with all angles equal and 60º)



equivalent fractions: Fractions that represent the same part of a whole or set; e.g., $\frac{2}{4}$ is equivalent to $\frac{1}{2}$



equivalent decimals: Decimals that represent the same part of a whole or set; e.g., 0.5 is equivalent to 0.50

event: A set of outcomes for a probability experiment; e.g., if you roll a die with the numbers 1 to 6, the event of rolling an even number has the outcomes 2, 4, or 6

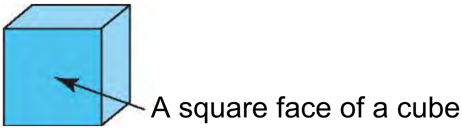
expanded form: A way of writing a number that shows the place value of each digit; e.g., 1209 in expanded form is 1 × 1000 + 2 × 100 + 9 × 1 or 1 thousand + 2 hundreds + 9 ones

experimental probability: The probability of an event based on the results of an experiment with many trials; it is calculated using this expression:

$$\frac{\text{Number of favourable results}}{\text{Number of trials}}$$

F

face: A 2-D shape that forms a flat surface of a 3-D shape; e.g.,



factor: One of the numbers you multiply in a multiplication operation; e.g., 3 and 4 are the factors in 3 × 4 = 12

favourable outcome: The desired outcome when you calculate a theoretical probability; e.g., when you find the theoretical probability of rolling a number less than 3 on a die, rolls of 1 and 2 are the favourable outcomes

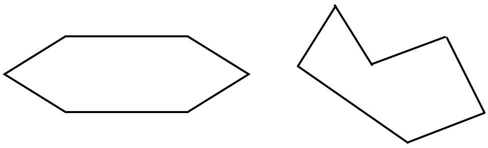
favourable result: The desired result when you calculate an experimental probability; e.g., when you find the experimental probability of rolling an even number on a die, rolls of 2, 4, and 6 are favourable results

flip: See *reflection*

formula: A general rule stated in mathematical language; e.g., the formula for the area of a rectangle is Area = length × width or $A = l \times w$

H

hexagon: A six-sided polygon; e.g.,

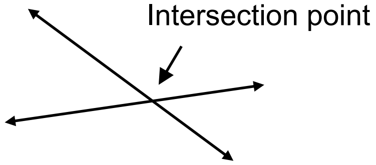


I

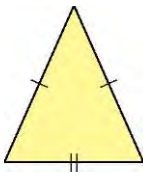
image: The new shape that you create when you apply a transformation to a shape; e.g., after a reflection the new shape is called the refection image

improper fraction: A fraction in which the numerator is greater than or equal to the denominator;
e.g., $\frac{5}{4}$ and $\frac{6}{6}$

intersection point: The point where two or more lines or line segments meet or cross; e.g.,

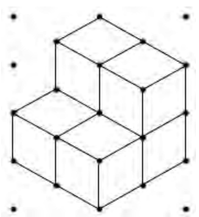


isosceles triangle: A triangle with two sides of equal length



isometric dot paper: A dot grid where each point on the grid is equally far from all nearby points
See *isometric drawing*

isometric drawing: A drawing of a 3-D shape, often done on isometric dot paper; e.g.,

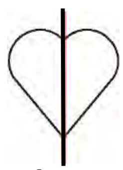


L

legend: The description on a graph that tells what the parts represent; e.g., the legend of the double bar graph on **page 230** tells you that the black bars represent the urban population and the white bars represent the rural population

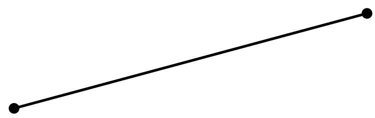
line: A set of points that form a straight path that goes on forever in each direction; e.g.,

line of symmetry: A line that divides a 2-D shape into halves that match when you fold the shape on the line of symmetry; e.g.,



Line of symmetry

line segment: A part of a line; it consists of two end points and all the points in between; e.g.,



M

mean: A single number that represents a data set; you calculate the mean by adding the numbers together and dividing the total by the number of elements in the set; it is often called the average;
e.g., the mean of 3, 4, 5, 6 is
(3 + 4 + 5 + 6) ÷ 2 = 4.5

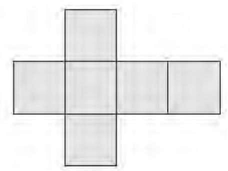
metre (m): A unit of measurement for length; e.g., 1 m is about the distance from a doorknob to the floor; 1000 mm = 1 m;
100 cm = 1 m; 1000 m = 1 km

metric system: A standard system of units for measuring length, area, mass, volume, capacity, and so on, where each unit is made up of ten of the next smallest unit
See *Measurement Reference* on **page 238**

mixed number: A number made up of a whole number and a fraction; e.g., $5\frac{1}{7}$

N

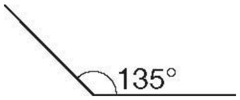
net: A 2-D pattern you can fold to create a 3-D shape; e.g., this is a net for a cube:



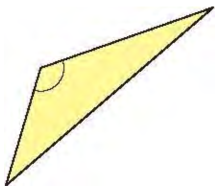
numerator: The number in a fraction that shows the number of parts of a given size the fraction represents; e.g., in $\frac{4}{5}$, the numerator is 4

O

obtuse angle: An angle greater than 90º and less than 180º; e.g.,



obtuse triangle: A triangle in which one of the angles is an obtuse angle; e.g.,



ones period: The cluster of three digits in a whole number that contains the hundreds digit, the tens digit, and the ones digit; e.g., in the number 123,456, the digits 456 make up the ones period

open number sentence: A number sentence with a missing term; e.g., □ + 5 = 8, 3 × □ = 24, 12 ÷ 6 = □, and □ – 7 = 3 are all open number sentences

ordered pair: A pair of numbers in a particular order that describe the location of a point in a coordinate grid; e.g., the ordered pairs (3, 5) and (5, 3) describe the location of two different points on a grid

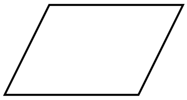
origin: The intersection of the axes in a coordinate grid, represented by the ordered pair (0, 0)

outcome: A single possibility in a probability situation; e.g., when you roll a die, there are six possible outcomes: 1, 2, 3, 4, 5, and 6

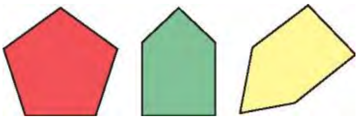
P

parallel lines or line segments: Lines or line segments that never meet, so they are always the same distance apart; e.g.,

parallelogram: A quadrilateral with pairs of opposite sides that are parallel; e.g.,



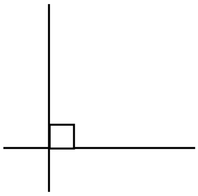
pentagon: A polygon with five sides; e.g.,



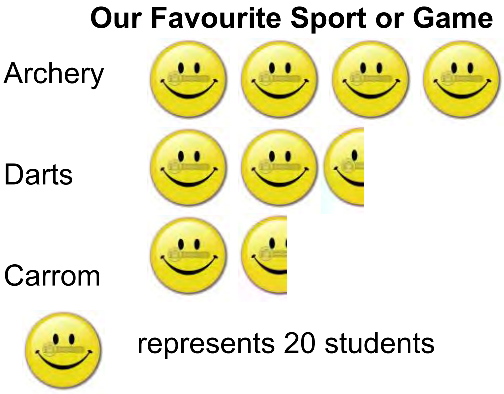
perimeter: **1.** The boundary or outline of a 2-D shape **2.** The length of the boundary

period: A group of three digits in a number, often separated by a comma or a space; e.g., in the number 458,675, the thousands period is 458 and the ones period is 675

perpendicular: At a right angle; e.g.,



pictograph: A graph that uses picture or symbols; e.g.,



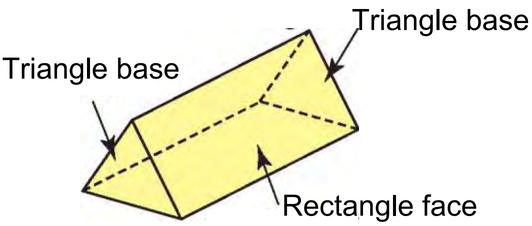
place value: The value of a digit depends on its place in the number; e.g., in the number 123.4, the digit 3 has a value of 3 because it is in the ones place, the digit 2 has a value of 20 because it is in the tens place, and so on

plot (a point): Locate a point on a coordinate grid using its coordinates

polygon: A closed 2-D shape with three or more sides; e.g., triangle, quadrilateral, pentagon, and so on

possible outcomes: See *outcome*

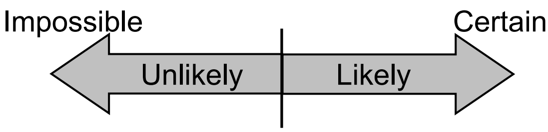
prism: A 3-D shape with two parallel and opposite congruent bases; the other faces are rectangles; the shape of the bases determines the name of the prism; e.g.,



A triangle-based prism

probability: A number from 0 (will never happen) to 1 (certain to happen) that tells how likely it is that an event will happen; it can be a decimal or fraction; sometimes it is called chance

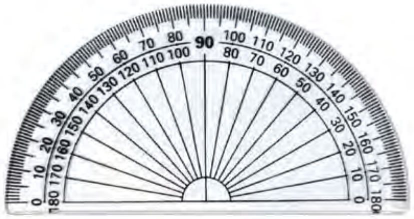
probability line: A number line from 0 to 1 used to compare probabilities



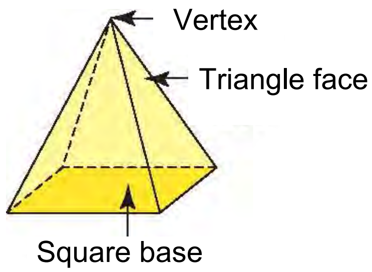
product: The result of multiplying numbers; e.g., in 5 × 6 = 30, the product is 30

proper fraction: A fraction in which the denominator is greater than the numerator; e.g., $\frac{1}{7}$, $\frac{4}{5}$, $\frac{29}{40}$

protractor: A tool used to measure the size of an angle



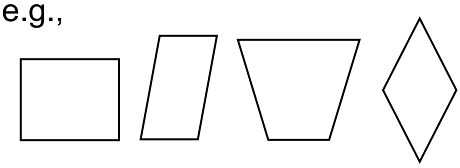
pyramid: A 3-D shape with a single polygon base; the other faces are triangles that meet at a single vertex.; the shape of the base determines the name of the pyramid; e.g., square-based pyramid



A square-based pyramid

Q

quadrilateral: A four-sided polygon; e.g.,

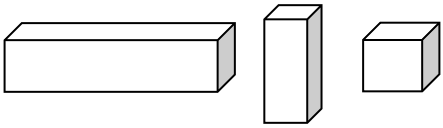


quotient: The result of dividing one number by another number;
e.g., in 45 ÷ 5 = 9, the quotient is 9

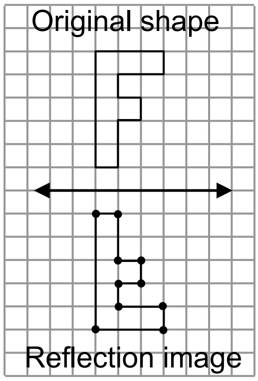
R

rectangle: A parallelogram with four right angles. A square is a special rectangle that four equal sides; e.g.,

rectangular prism: A prism with rectangle bases; e.g.,

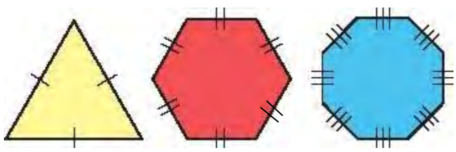


reflection: A transformation that produces a mirror image of a shape across a reflection line; also called a flip; e.g., this is a reflection of the F-shape across a horizontal refection line:



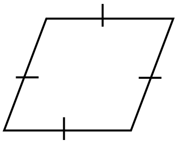
reflection line: See *reflection*

regular polygon: A polygon with all sides and angles congruent; e.g.,



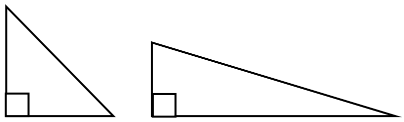
rename (a number): Change a number to another form to make it easier to calculate or compare; e.g., you can rename 1,400,000 as 14 hundred thousand or 1.4 million.

rhombus: A parallelogram with all sides equal; a square is a special rhombus that has four right angles; e.g.,

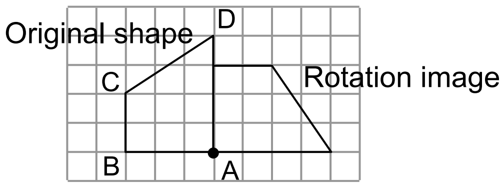


right angle: An angle that measures 90º; sometimes called a square corner See the right angles in the *right triangle*s below

right triangle: A triangle with one right angle; e.g.,

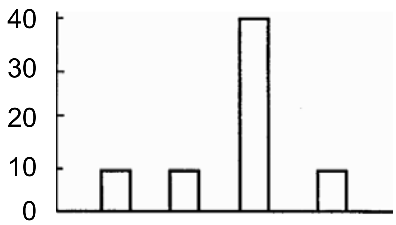


rotation: A transformation in which each point in a shape moves around a point (the turn centre) through the angle of rotation; e.g., this is a 90º cw rotation of trapezoid ABCD around vertex A:

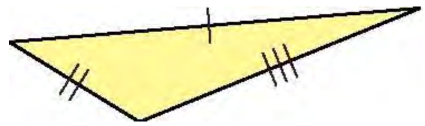


S

scale (on a graph): Numbers and marks at regular intervals on the axes of a graph; the value of each interval on an axis; the scale tells how to interpret a graph; e.g., the scale on the vertical axis of the graph below is 10



scalene triangle: A triangle with no congruent sides; e.g.,



slide: See *translation*

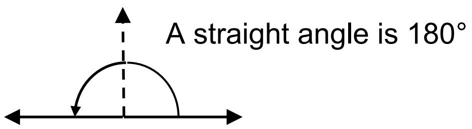
solution: **1.** The complete answer to a problem **2.** The value that makes an open sentence true; e.g., in ■ + 4 = 39, the solution is ■ = 35 because 35 + 4 = 39

square: A rectangle with all sides equal

standard form (of a number): The usual way to write numbers; e.g., 23,650 is in standard form

standard unit: A unit of measurement that is part of an accepted measurement system; e.g., metres, kilograms, litres, and square metres are all standard units

straight angle: An angle that measures 180º



sum: The result of adding numbers; e.g., in 5 + 4 + 7 = 16, the sum is 16

symmetry: Line or reflectional symmetry means that when a shape is folded or reflected across a line (the reflection line), the two sides of the shape match

T

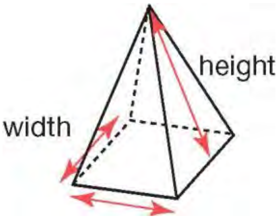
theoretical probability: A number from 0 to 1 that tells how likely an event is to occur; it is calculated using the expression:

$$\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

e.g., the theoretical probability of rolling a 4 on a six-sided die is $\frac{1}{6}$

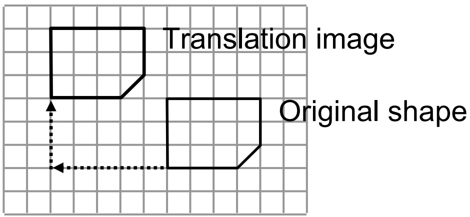
thousands period: The group of three digits in a whole number that contains the hundreds thousands digit, the ten thousands digit, and the one thousands digit; e.g., in the number 123,456, the digits 123 make up the thousands period

three-dimensional (3-D): A shape with three dimensions: length, width (or breadth or depth), and height; e.g.,



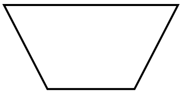
transformation: Changing a shape according to a rule; transformations include translations, rotations, and reflections

translation: A transformation in which each point of a shape moves the same distance and in the same direction; also called a *slide*; e.g., the pentagon has been translated 5 units left and 3 units up



translation rule: A rule that describes what happens to a shape when it is translated; it tells how many units right or left and how many units up or down

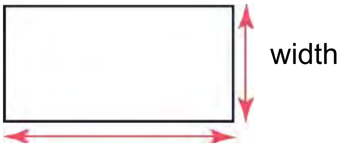
trapezoid: A quadrilateral in which one pair of opposite sides are parallel; e.g.,



triangle: A polygon with three sides

turn centre: The point around which all the points in a shape turn or rotate in a clockwise (cw) or counter-clockwise (ccw) direction
See *rotation*

two-dimensional (2-D): A shape with two dimensions: length and width (or breadth); e.g.,



vertex (vertices): The point at the corner of an angle or shape where two or more sides or edges meet; e.g., a cube has eight vertices, a triangle has three vertices, an angle has one vertex

volume: The amount of space occupied by an object; often measured in cubic centimetres or cubic metres

W

whole numbers: The set of numbers that begins at 0 and continues forever in this pattern: 0, 1, 2, 3, …

X

***x*-axis:** One of the two axes in a coordinate grid; sometimes called the horizontal axis See *coordinate grid*

***x*-coordinate:** The first value in an ordered pair; it represents the distance along the *x*-axis from (0, 0); e.g., in (15, 50), 15 is the *x*-coordinate See *coordinate grid*

Y

***y*-axis:** One of the two axes in a coordinate grid; sometimes called the vertical axis See *coordinate grid*

***y*-coordinate:** The second value in an ordered pair; it represents the distance along the *y*-axis from (0, 0); e.g., in (15, 50), 50 is the *y*-coordinate See *coordinate grid*

MEASUREMENT REFERENCE

Measurement Abbreviations and Symbols

Time		**Capacity**	
second	s	millilitre	mL
minute	min	litre	L
hour	h		
Length		**Volume**	
millimetre	mm	cubic centimetre	cm^3
centimetre	cm	cubic metre	m^3
metre	m	cubic millimetres	mm^3
kilometre	km		
Mass		**Area**	
gram	g	square centimetre	cm^2
kilogram	kg	square metre	m^2

Metric Prefixes

Prefix	kilo × 1000	hecto × 100	deka × 10	unit 1	deci $\times \frac{1}{10}$	centi $\times \frac{1}{100}$	milli $\times \frac{1}{1000}$
Example	*kilo*metre km	*hecto*metre hm	*deka*metre dam	metre m	*deci*metre dm	*centi*metre cm	*milli*metre mm
	1000 m	100 m	10 m	1 m	0.1 m	0.01 m	0.001 m

Measurement Formulas for a Rectangle

Perimeter = 2 × length + 2 × width

Area = length × width

l

$P = 2 \times l + 2 \times w$

$A = l \times w$

w

ANSWERS

UNIT 1 NUMBER pp. 1–24

Getting Started — Skills You Will Need p. 2

1. a) 4 **b)** 0 **c)** 2

2. a) 23,473 **b)** 12,320 **c)** 40,208

3. a) iii **b)** ii **c)** i

4. a) 10,203 **b)** 6720 **c)** 12,320

5. a) 2007; 3427; 10,003; 12,300
b) 3420; 17,999; 20,007; 32,300

6. *Sample responses*:
a) 90,929; 92,000; 93,000
b) 93,000; 94,000; 95,000
c) 11,000; 12,000; 13,000

7. a) Yes **b)** Yes

8. a) *Sample response*: 83,923
b) 83,929
c) 83,020

CONNECTIONS: One Million p. 4

1. Yes **2.** Yes **3.** No

1.1.2 Whole Number Place Value p. 7

1. a) Thousands **b)** Hundred thousands
c) Hundred thousands **d)** Hundred thousands

2. a) 4 **b)** 2 **c)** 3

3. a) 1,020,000 **b)** 404,020
c) 70,212 **d)** 4,200,000

4. a) 3 (one) millions + 4 hundred thousands + 2 ten thousands + 2 (one) thousands + 6 ones
b) 8 (one) millions + 2 ones
c) 3 hundred thousands + 4 ten thousands + 2 (one) thousands + 1 hundred
d) 6 (one) millions + 2 hundreds + 3 ones

5. 5

6. *Sample responses*:
a) 4,123,560 **b)** 3,124,560
c) 5,624,130

7. a) 2,000,000; 2 million
b) 3,000,000; 3 million

8. *Sample response*:
1,420,000 or 2,570,234

1.1.3 Renaming Numbers p. 10

1. a) 0.1 **b)** 0.01 **c)** 0.5 **d)** 0.03

2. a) 900,000 **b)** 40,000 **c)** 1,100,000

3. a) 1 million + 8 ten thousands
b) 3 million + 2 ten thousands
c) 2 hundred thousands
d) 6 hundred thousands

4. a) 2,010,000
b) i) 1 **ii)** 0 **iii)** 2
iv) 0 **v)** 0 **vi)** 0

5. A, B, and C

6. a) 0.1 million **b)** 0.02 million
c) 0.4 million **d)** 0.04 million
e) 0.3 million **f)** 1.4 million

8. *Sample response*:
2.1 million, 21 hundred thousands, and 210 ten thousands

1.1.4 Comparing and Ordering Numbers p. 14

1. a) 3.4 million
b) 15 hundred thousand
c) 5,213,478

2. a) 899,789 < 3,487,799 < 6,000,000
b) 213,867 < 762,813 < 2,013,687

3. a) 2.1 million
b) 0.4 million
c) 275 ten thousand

4. a) 78 ten thousand < 14 hundred thousand < 2 million
b) 2.3 hundred thousand < 0.6 million < 3,150,000

5. *Sample response*:

Millions		Thousands			Ones		
	One	Hundred	Ten	One	Hundred	Ten	One
	4	2	0	0	0	0	0
	5	0	0	0	0	0	0

4 million < 5 million, so 42 hundred thousand < 5 million

6. *Sample response*:
2,100,001; 2,100,002; 2,100,003

7. C

8. a) and b) *Sample response*:
0 and 4; 9 and 4

9. *Sample response*:
a) 4.1 million > 4.0 hundred thousand
b) 0.4 million = 4.0 hundred thousand
c) 0.9 million < 4.0 hundred thousand

1.2.1 Renaming Numbers Using Multiplication p. 18

1. a) 10 **b)** 6, 9
c) 54, 108 **d)** 130, 13

3. a) 70 km **b)** 105 km **c)** 175 km

4. a) 6 × 7 = 42
b) i) 84 **ii)** 420 **iii)** 420 **iv)** 840

5. a) 100,000
b) i) 200,000 **ii)** 1,000,000 **iii)** 150,000

6. a) 69 m^2

7. *Sample responses*:
a) Rename 6 as 2 **×** 3 to calculate 6 × 25:
6 × 25 = **2 × 3** × 25 = 2 × 25 × 3 = 50 × 3 = 150
b) Rename 1 kg as 1000 g to find how many grams 500 kg is:
500 = 500 × 1, so 500 kg = 500 × **1 kg** = 500 × **1000 g** = 500,000 g.

1.2.2 Using Number Sentences p. 21

1. *Sample responses*:
a) 18 + 162 = 180, 180 – 18 = 162
b) 350 + 350 = 700, 2 × 350 = 700
c) 10,000 – 8000 = 2000, 10,000 > 2000
d) 1600 + 160 = 1760, 1600 – 160 = 1440

2. a) 61 **b)** 2000 **c)** 1000 **d)** 66

3. *Sample response*:
16 ÷ 4 = 4, 4 × 3 = 12, 4 × 25 = 100

4. a) One solution, 3002
b) One solution, 10,000
c) Many solutions, e.g., 1000, 2000, 500
d) No solution

5. *Sample responses*:
a) 5 + [ ] = 8 **b)** 5 × [ ] > 20 **c)** [ ] = 20 ÷ 0

6. Yes

UNIT 1 Revision p.p. 22–23

1. *Sample response*:
It is 1000 thousands.
It is 500,000 + 500,000.
It is the amount of millilitres of water in about 3500 drinking glasses.

2.a)

Millions			**Thousands**			**Ones**		
		One	**Hundred**	**Ten**	**One**	**Hundred**	**Ten**	**One**
i)			3	1	4	2	1	4
ii)		1	0	0	3	4	1	2

b) i) 3 hundred thousands + 1 ten thousand + 4 (one) thousands + 2 hundreds + 1 ten + 4 ones
ii) 1 million + 3 (one) thousands + 4 hundreds + (one) ten + 2 ones

c) i) three hundred fourteen thousand, two hundred fourteen
ii) one million, three thousand, four hundred twelve

3. a) 234,005; *Sample response*:
2 hundred thousands + 3 ten thousands + 4 (one) thousands + 5 ones

b) 145,032; *Sample response*:
1 hundred thousand + 4 ten thousands + 5 (one) thousands + 3 tens + 2 ones

c) 2,030,003; *Sample response*:
2 millions + 3 ten thousands + 3 ones

d) 4,020,030; *Sample response*:
4 millions + 2 ten thousands + 3 tens

4. a) Millions **b)** Hundreds
c) Ones **d)** Hundred thousands

5. *Sample response*:
a) 1,420,000 **b)** 9,000,000 **c)** 1,180,213

7. a) 2,100,000 **b)** 3,100,000
c) 300,000 **d)** 50,000

8. a) 87,146 **b)** 3,152,110
c) 417,000 **d)** 345,789

9. a) 123,450 **b)** 2 million
c) 8 hundred thousands **d)** 0.02 million

11. a) 10 **b)** 10,000 **c)** 61
d) 220 **e)** 90, 40

12. *Sample responses*:
a) 90
b) i) 900 **ii)** 180
iii) 9000 **iv)** 360

13. a) 180 **b)** 600 **c)** 90

14. *Sample response*:
$70 = 210 \div 3$, $70 + 140 = 210$, $3 \times 70 = 210$

16. a) 7 **b)** 57 **c)** 65 **d)** 50

17. a) More than one
b) One
c) More than one
d) No solutions

18. *Sample response*: 100 + [ ] > 50

Getting Started — Skills You Will Need p. 25

1. a) 56 **b)** 36 **c)** 40 **d)** 49
e) 6 **f)** 8 **g)** 9 **h)** 7

2. a) 1012 **b)** 3890 **c)** 4872 **d)** 6093
e) 61 **f)** 64 **g)** 130 R 5 **h)** 126 R 1

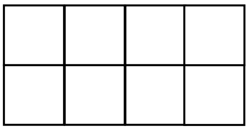
4. a) 18 students
b) 432 flags

5. *Sample responses*:
a), b), and c) 4 × 5 = 20
the factors are 4 and 5;
the product is 20;
20 is also a multiple of 4
and a multiple of 5
d), e), and f) 20 ÷ 5 = 4
the quotient is 4;
the dividend is 20;
the divisor is 5

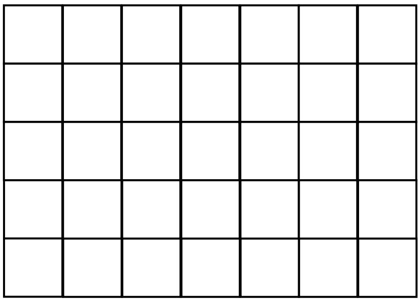
2.1.1 Multiplying Multiples of Ten p. 27

1. a) 30 × 30 = 900 **b)** 50 × 40 = 2000

2. a) 800



b) 3500



3. a) 3000
b) 1600
c) 3200
d) 5400

4. A

5. *Sample responses*:
a) 70 × 70
b) 40 × 90

6. a) 3 × 20 = 60 and 30 × 20 is 600;
30 × 20 = 600 is 10 times 3 × 20 = 60
b) 4 × 60 = 240, but 40 × 60 is 2400;
40 × 60 = 2400 is 10 times 4 × 60 = 240

2.1.2 Estimating Products p. 30

1. *Sample responses*:
a) about 1500
b) about 1500
c) about 1800
d) about 5400

2. C is incorrect. The others seem reasonable.

3. *Sample responses*:
a) about 600 **b)** about 1200
c) about 1000

4. *Sample responses*:
a) about Nu 800
b) about Nu 2400
c) about Nu 800

6. About 26 boxes

7. *Sample responses*:
a) There are 39 students and each has 60 sheets of paper in a notebook. About how many sheets of paper are there altogether?

b) A truck travels 78 km every day. About how far does it travel in a month?
c) The bank has a roll of 58 Nu 50 notes. Estimate how much the roll is worth.

8. 49 × 71

2.1.3 Multiplying 2-digit Numbers by 3-digit Numbers p. 35

1. a)

	40	+ 3
30	30 × 40 = 1200	30 × 3 = 90
+ 7	7 × 40 = 280	7 × 3 = 21

1200 + 280 + 90 + 21 = 1591

b)

	60	+ 2
30	30 × 60 = 1800	30 × 2 = 60
+ 9	9 × 60 = 540	9 × 2 = 18

1800 + 540 + 60 + 18 = 2418

c)

	50	+ 1
20	20 × 50 = 1000	20 × 1 = 20
+ 8	8 × 50 = 400	8 × 1 = 8

1000 + 400 + 20 + 8 = 1428

2. a) 17 × 62 = 1054
b) 31 × 32 = 992

3. a) 2688 **b)** 705 **c)** 1344
d) 2706 **e)** 17,712 **f)** 6354

4. 31 × 45 = 1395 min

5. 2950 squares

6. 490 cm (or 4.9 m)

7. a) 10 × 11 = 110 **b)** 98 × 99 = 9702

8. A product greater than 5000

9. a) Nu 675 **b)** Nu 1920

10. 44 and 45

11. 96 × 87 = 8352

12. *Sample response*: A truck carries 368 kg of vegetables. Each kilogram sells for Nu 45. How much will the farmer get if she sells the entire load?

2.1.4 Multiplying 4-digit Numbers by 1-digit Numbers p. 38

1. 14,808; *Sample response*:

a)

Ten thousands	Thousands	Hundreds	Tens	Ones
	12	28	0	8

↓

Ten thousands	Thousands	Hundreds	Tens	Ones
1	4	8	0	8

14,808

1. b) 15,595; *Sample response*:

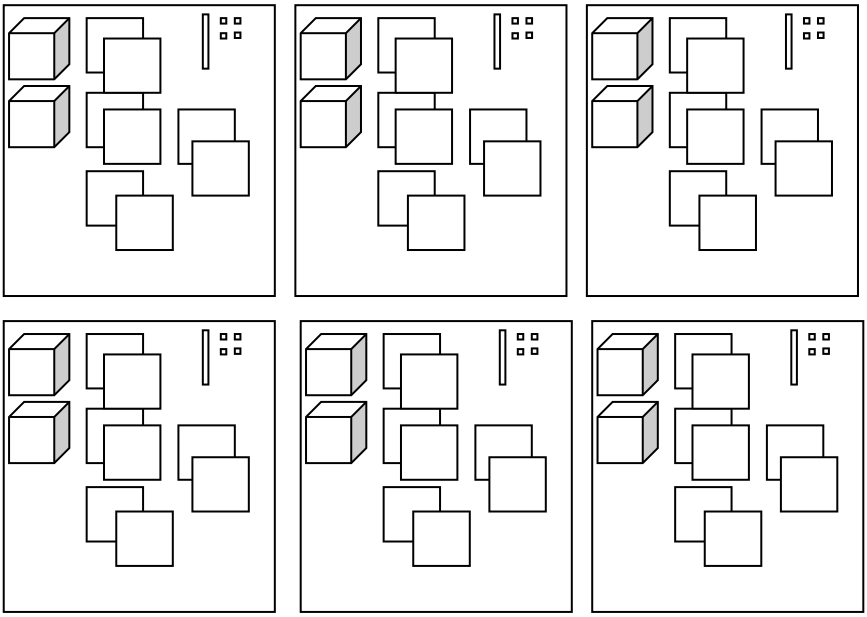
Ten thousands	Thousands	Hundreds	Tens	Ones
	15	5	5	45

↓

Ten thousands	Thousands	Hundreds	Tens	Ones
1	5	5	9	5

15,595

c) 16,884; *Sample response*:



12 thousands + 48 hundreds + 6 tens + 24 ones
= 16 thousands + 8 hundreds + 8 tens + 4 ones = 16,884

1. d) 31,392 **e)** 21,686 **f)** 42,552

2. 19,072 km

3. a) ☐ = 4 (in both spots)
b) ◇ = 6 (in both spots) and ☐ = 2
c) ◇ = 8 and ☐ = 2

4. a) Nu 49,800
b) Nu 31,140
c) Nu 61,650

5. 3600 s

6. a) 36,960 feet
b) 47,520 feet

7. *Sample response*: 6 × 1235

8. *Sample responses*:
a) 3500
b) 8 × 3500 = 28,000

9. *Sample response*: 4 × 1136 = 4544

CONNECTIONS: Egyptian Multiplication p. 40

1. a) 3400; **b)** 2340 **c)** 7360
1 × 85
2 × 170
4 × 340
8 × 680 √
16 × 1360
32 × 2720 √
680 + 2720 = 3400

2.2.1 Estimating Quotients p. 43

1. *Sample responses*:
a) about 1000 **b)** about 800
c) about 130 **d)** about 500

2. *Sample responses*:
a) High estimate: about 2000
Low estimate: about 1000

b) High estimate: about 2000
Low estimate: about 1000
c) High estimate: about 2000
Low estimate: about 1000
d) High estimate: about 900
Low estimate: about 800

3. a) Any number greater than 4500
b) Any number less than 420
c) Any number greater than 5000

4. b) 6300 ÷ 7

5. B

6. About 1500 tests

7. About 500 people

8. *Sample response*: If it rained 4550 mm over 5 days. I might want to know how much it rained each day, if it rained the same amount each day.

2.2.2 Dividing 4-digit Numbers by 1-digit Numbers p. 47

1. a) 1231 **b)** 1193
c) 589 **d)** 1248 R 3
e) 111 R 6 **f)** 516 R 2

2. a) 1 **b)** 120 **c)** 47

3. Two payments were Nu 2833 and one payment was Nu 2834.

4. a) Nu 300 **b)** Nu 500 **c)** Nu 375

5. a) Nu 1051 R 2; give each person 50 Ch more.
b) 5 R 10; you cannot use the remainder to buy more chocolate bars.
c) 107 m^2 R 2; add an extra 0.5 m^2 to each section.

6. *Sample response*: A table that costs Nu 3006 is paid for in 8 equal payments. How much is each payment?

7. a) $\begin{array}{r} 1206 \\ 6\overline{)7236} \end{array}$

b) $\begin{array}{rr} 8\overline{)5325} & \\ -\underline{4800} & \mathbf{6}00 \\ \mathbf{5}25 & \\ -\underline{480} & \mathbf{6}0 \\ \mathbf{45} & \\ -\underline{40} & \underline{+\ 5} \\ \mathbf{5} & \mathbf{66}5 \end{array}$

9. *Sample response*: 2713, 2722, or 2731

2.2.4 Dividing 4-digit Numbers by Multiples of Ten p. 52

1. a) 187 R 24 **b)** 84 R 19
c) 22 R 20 **d)** 251

2. *Sample response*: Part d), I might have a roll of Nu 20 notes worth Nu 5020 and I want to know how many notes there are.

3. *Sample responses*:
a) About 100 h
b) About 60 h
c) About 200 h

4. a) 60 h
b) 75 h, 20 min
c) 142 h, 10 min
d) 150 h

5. 175 min

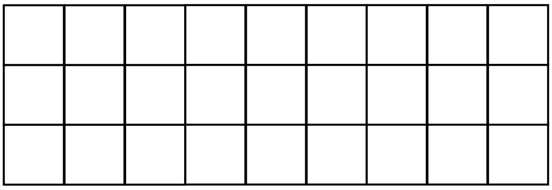
7. Yes

1. $30 \times 60 = 1800$

2. a)

90

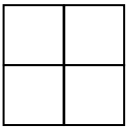
30



b)

20

20



3. D

4. *Sample responses*:
a) About 2800 **b)** About 1200
c) About 1600 **d)** About 3200

5. *Sample response*: About 2100 m^2

6. A

7. *Sample responses*:
a) About Nu 2000 **b)** About Nu 3200
c) About Nu 1400

8. *Sample response*:
a) 30×80 or 40×70

9. a) 1008

	40 +	2
20 +	20×40	20×2
4	4×40	4×2

b) 1431

	50 +	3
20 +	20×50	20×3
7	7×50	7×3

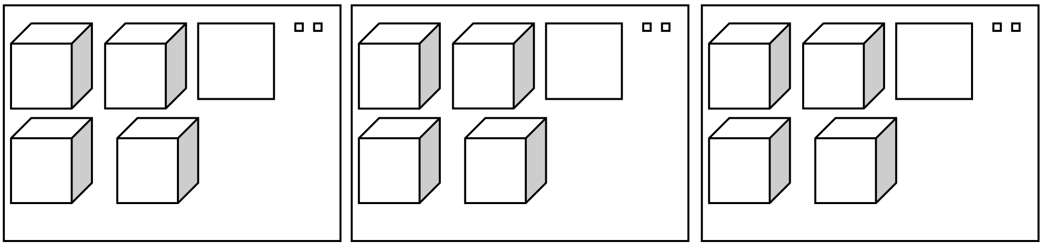
10. a) 28,944 **b)** 18,177

11. 864 squares

12. Nu 3195

13. *Sample response*: I drove 25 km at an average speed of 38 km each hour. For how long did I travel?

14. a) 12,306; *Sample response*:



12 thousands + 3 hundreds + 6 ones = 12,306

b) 11,324; *Sample response*:

Thousands	Hundreds	Tens	Ones
10	12	12	4

Ten thousands	Thousands	Hundreds	Tens	Ones
1	1	3	2	4

11,324

14. c) 7190; *Sample response*:

Thousands	Hundreds	Tens	Ones
5	20	15	40

Thousands	Hundreds	Tens	Ones
7	1	9	0

7190

d) 20,251; *Sample response*:

Thousands	Hundreds	Tens	Ones
14	56	63	21

Ten thousands	Thousands	Hundreds	Tens	Ones
2	0	2	5	1

20,251

15. $\begin{array}{r} 21\mathbf{4}3 \\ \times\quad 7 \\ \hline 15{,}0\mathbf{0}1 \end{array}$ or $\begin{array}{r} 21\mathbf{5}3 \\ \times\quad 7 \\ \hline 15{,}0\mathbf{7}1 \end{array}$

16. 14,400 s

17. *Sample responses*:
a) 34,000 **b)** 1100 **c)** 15,600

18. *Sample response*:
a) About 2000 **b)** About 500
c) About 500 **d)** About 400

19. *Sample response*: 5 people are sharing the cost of a Nu 3015 purchase. About how much does each person pay?

20. a) 846 **b)** 809 R 6
c) 369 R 4 **d)** 443 R 5

21. Nu 1840

22. a) 36 **b)** 45
c) 280 **d)** 13

23. a) 134 R 10 **b)** 72 R 17
c) 31 R 10 **d)** 246 R 19

24. a) 150 h **b)** 21 h
c) 116 h, 40 min **d)** 141 h, 30 min

25. *Sample response*:
Divide by 100 and then multiply by 5.
Divide by 2 and then divide by 10.
Divide by 10 and then divide by 2.

UNIT 3 FRACTIONS AND DECIMALS pp. 55-86

Getting Started — Skills You Will Need p. 56

1. *Sample responses*:

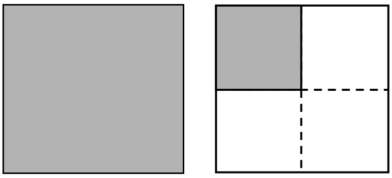
a) $\frac{1}{2}$ and $\frac{2}{4}$ **b)** $\frac{2}{8}$ and $\frac{1}{4}$ **c)** $\frac{1}{3}$ and $\frac{3}{9}$

2. a) $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{5}{6}$ **b)** $\frac{3}{8}, \frac{4}{8}, \frac{6}{8}, \frac{7}{8}$

c) $\frac{1}{10}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}$ **d)** $\frac{4}{15}, \frac{4}{9}, \frac{4}{6}, \frac{4}{5}$

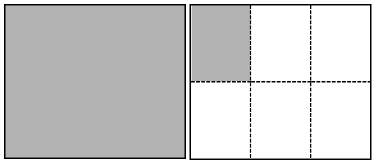
3. *Sample responses*:

a)



b).

3. c)



4. A matches **ii)**
B matches **iii)**
C matches **i)**

5. a) $\frac{2}{10}$ **b)** $\frac{45}{100}$

c) $\frac{9}{10}$ **d)** $\frac{5}{100}$

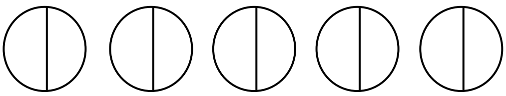
6. a) 0.17, 0.23, 0.29, 0.45
b) 0.17, 0.3, 0.45, 0.5

3.1.2 Fractions as Division p. 61

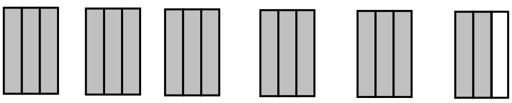
1. a) $3 \div 4 = \frac{3}{4}$ **b)** $4 \div 7 = \frac{4}{7}$

2. *Sample responses*:

a) $\frac{10}{2} = 5$



b) $\frac{17}{3} = 5\frac{2}{3}$



3. a) $2\frac{4}{5}$ **b)** $11\frac{1}{2}$

4. $\frac{4}{6}$ of a pumpkin

5. a) $5\frac{1}{3}$ **b)** $4\frac{1}{6}$ **c)** 2 **d)** 7

6. 4 full packages and $\frac{2}{4}$ of another

7. a) Yes

7. b) Divide each of the 6 items into 9 parts. Each person gets 1 of the 9 parts from each item. So, each person gets 6 parts, or $\frac{6}{9}$ of an item.

8. a) *Sample response*: $\frac{4}{1}, \frac{8}{2}, \frac{12}{3}$, and $\frac{16}{4}$

b) Yes

9. a) No.

3.1.3 Equivalent Fractions p. 65

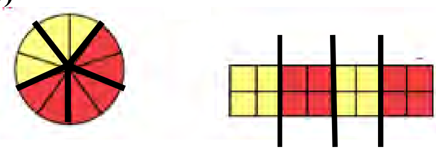
1. *Sample responses*:

a) $\frac{3}{6}, \frac{1}{2}$ **b)** $\frac{4}{6}, \frac{2}{3}$ (or $\frac{2}{6}, \frac{1}{3}$) **c)** $\frac{6}{12}, \frac{1}{2}$

2. a) *Sample response*:

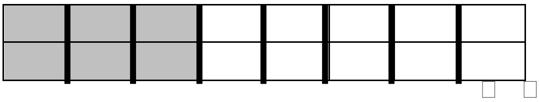
Circle: $\frac{6}{10}$ and $\frac{3}{5}$ Rectangle: $\frac{8}{16}$ and $\frac{2}{4}$

b)



There are 5 big sections in the circle and only 3 instead of 6 are dark.
There are 4 big sections in the rectangle and only 2 instead of 8 are dark.

3. *Sample response*:



4. *Sample responses*:

a) $\frac{10}{16}, \frac{15}{24}, \frac{150}{240}, \frac{1500}{2400}$ **b)** $\frac{5}{6}, \frac{50}{60}, \frac{10}{12}, \frac{100}{120}$

4. c) $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}$ **d)** $\frac{100}{400}, \frac{1}{4}, \frac{2}{8}, \frac{3}{12}$

5. *Sample responses*:

a) $\frac{2}{20}$ **b)** $\frac{20}{50}$ **c)** $\frac{15}{20}$ **d)** $\frac{20}{60}$

6. $\frac{4}{5}$

7. *Sample responses*:

a) $\frac{2}{6}, \frac{3}{9}, \frac{10}{30}$

b) $6 \div 2 = 3;\ 9 \div 3 = 3;\ 30 \div 3 = 10$

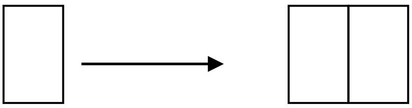
c) $\frac{4}{6}, \frac{6}{9}, \frac{20}{30}$

8. No

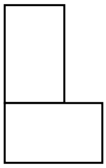
9. *Sample response*: $\frac{3}{9} = \frac{10}{30}$

CONNECTIONS: Fractions and Geometry p. 66

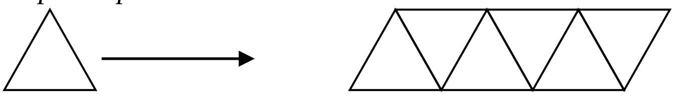
1. *Sample response*:



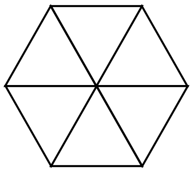
OR



2. *Sample response*:



OR



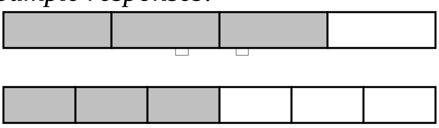
3.1.4 Comparing and Ordering Fractions p. 70

1. *Sample responses*:

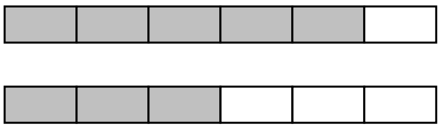
a) $\frac{1}{4} < \frac{1}{3}$ **b)** $\frac{5}{6} > \frac{2}{4}$

2. *Sample responses*:

a)



b)



3. a) $\frac{1}{2}$ **b)** $\frac{8}{9}$ **c)** $\frac{25}{26}$ **d)** $\frac{16}{3}$

4. $\frac{2}{8}, \frac{2}{5}, \frac{3}{5}, \frac{8}{9}, \frac{9}{4}, \frac{19}{3}$

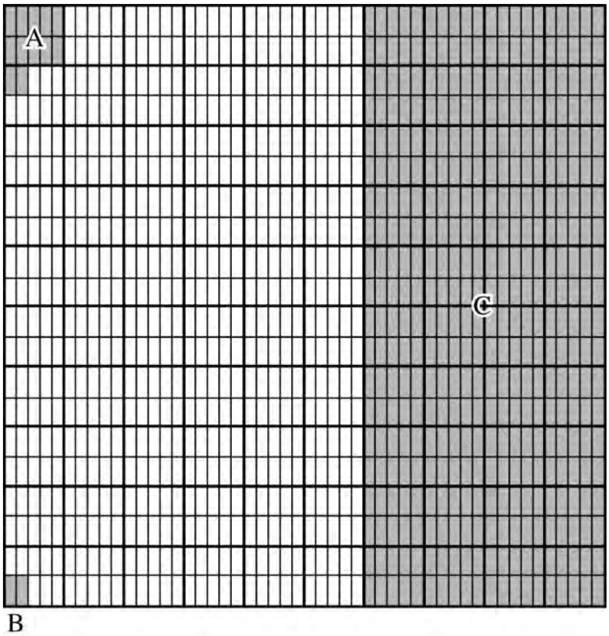
5. a) 1, 2, 3, 4, 5, 6, 7, 8, 9	**6.** *Sample responses*: **a)** $\frac{2}{6}, \frac{1}{5}, \frac{0}{5}$ **b)** $\frac{3}{5}, \frac{4}{5}, \frac{5}{6}$

3.2.1 Decimal Thousandths p. 76

1. a) 0.142 **b)** 0.057 **c)** 0.002

2. a) $\frac{8}{1000}$ **b)** $\frac{34}{1000}$ **c)** $\frac{398}{1000}$

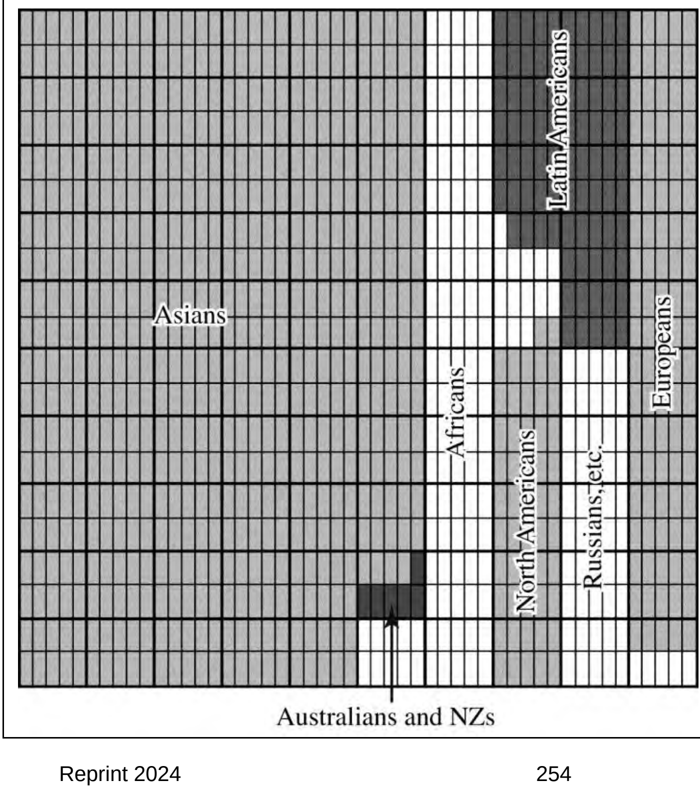
3. *Sample response*:



4. a) 0.312 km
b) 0.068 km
c) 0.002 km

5. 0.3 or 0.3

6. a) 0.584; 0.124; 0.095; 0.084; 0.055; 0.052; 0.006
b) *Sample response*:



7. a) 0.584 (Asians)
b) 0.095 (Europeans) or 0.124 (Africans)
c) 0.052 (North Americans) or 0.055 (Russians, etc.)

9. 0.028;
0.010 or 0.01;
0.060 or 0.06;
0.330;
0.070 or 0.07;
0.007;
0.001

10. a) 0.58

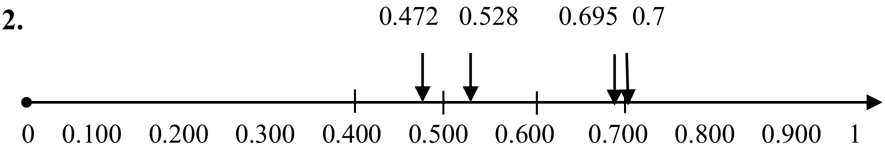
3.2.2 Decimal Place Value — p. 80

1. a) *Sample response*:

	Tens	Ones	Tenths	Hundredths	Thousandths
i)		0	0	0	1
ii)	1	0	3	4	0
iii)		0	5	1	0
iv)		0	2	4	5

b) i) 1 thousandth
ii) 10 and 34 hundredths
iii) 510 thousandths
iv) 245 thousandths

2.



3. *Sample responses*:
a) 0.996 **b)** 0.749 **c)** 0.899

4. a) 35 hundredths and 1 thousandth
b) 89 hundredths and 2 thousandths
c) 20 hundredths and 0 thousandths
d) 2 hundredths and 5 thousandths

5. a) 2.479 **b)** 3.119 **c)** 2.11

6. a) $\frac{352}{1000}$, $\frac{144}{1000}$, $\frac{174}{1000}$
b) 0.352; 0.144; 0.174
c) Pacific

8. *Sample response*: 0.124, 0.248, and 0.628

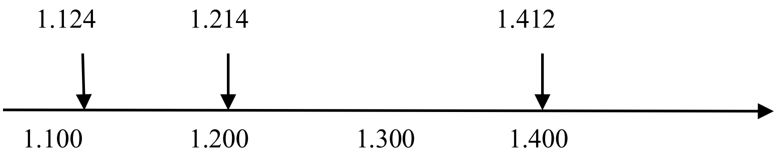
9. *Sample responses*:
a) 0.234, 0.432, 0.324
b) 0.324

3.2.3 Comparing and Ordering Decimals — p. 83

1. a) 0.035; 0.305; 1.024; 1.204

2. 3:12.987; 3:14.175; 3:14.5; 4:1.122

3.

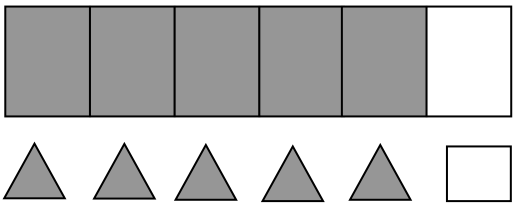


4. a) 2.108 km **b)** 1.314 km **c)** 2.108 m

6. 3.451, 3.452, 3.453, 3.454, 3.455, 3.456, 3.457, 3.458, 3.459, 3.460

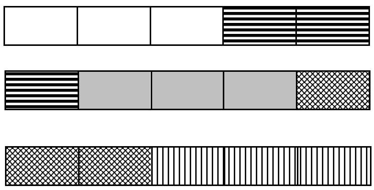
UNIT 3 Revision — pp. 85–86

1. *Sample response*:



2. *Sample response*:

$\frac{3}{5} = 3 \div 5$

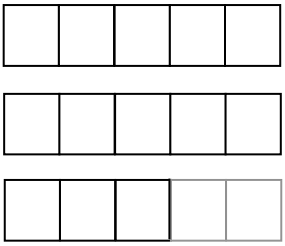


3. $2 \div 8 = \frac{2}{8}$

4. $\frac{3}{8}$

5. a) $\frac{13}{5} = 2\frac{3}{5}$

b) *Sample response*:



13 fifths = 2 wholes and 3 fifths

7. a) $\frac{4}{16}$ or $\frac{16}{64}$ **b)** $\frac{4}{16}$ or $\frac{16}{64}$

8. *Sample response*: $\frac{1}{50}$

9. a) $\frac{3}{7}$ **b)** $\frac{7}{9}$ **c)** $\frac{10}{11}$ **d)** $\frac{33}{4}$

10. *Sample responses*:

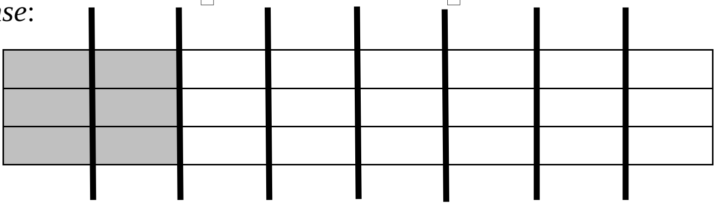
a) $\frac{3}{10}, \frac{1}{5}, \frac{1}{50}$ **b)** $\frac{49}{50}, \frac{9}{10}, \frac{8}{10}$

11. a) $\frac{10}{10}$ or 1 **b)** $\frac{4}{10}$

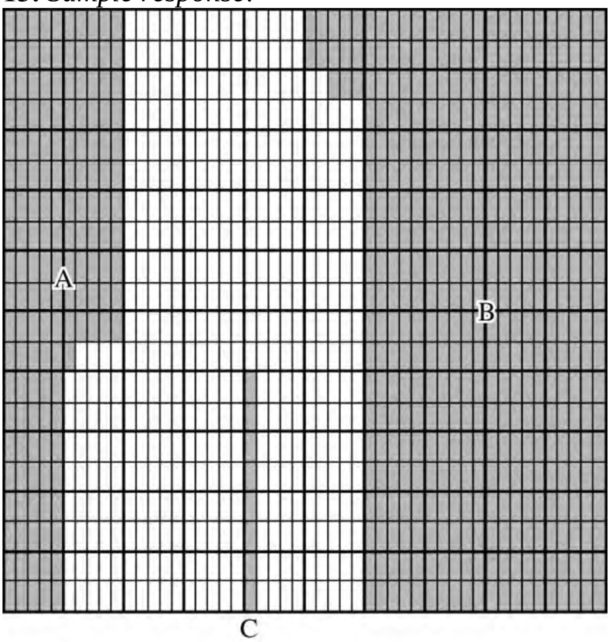
12. $\frac{3}{6}$ and $\frac{2}{6}$

6. *Sample response*:

$\frac{2}{8} = \frac{6}{24}$

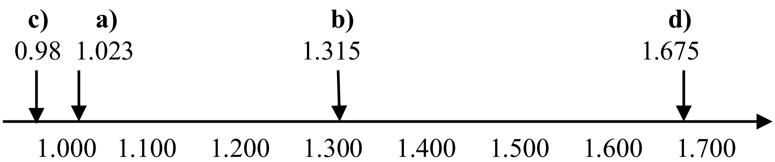


13. *Sample response*:



14. a) 0.080 L **b)** 0.003 L

15.



16. a) 0.100 **b)** 0.999

17. a) 0.213 **b)** 0.222 **c)** 0.248

18. *Sample response*: 0.51 and 0.52

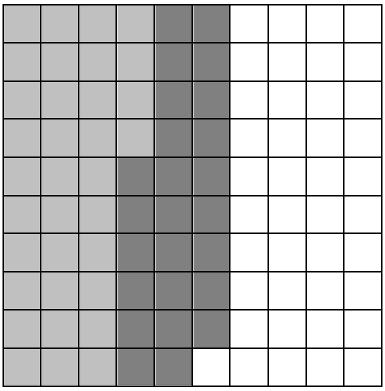
19. a) 2.004, 2.040, 3.09, 3.1

20. *Sample response*:
1.450, 1.482, 1.500, 1.550, 1.600

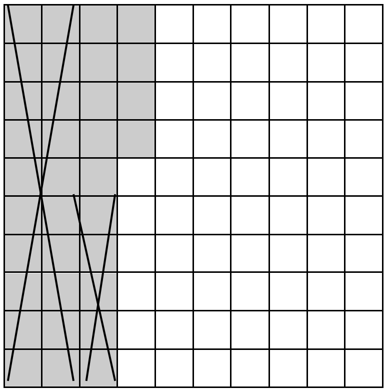
UNIT 4 DECIMAL COMPUTATION pp. 87–112

Getting Started — Skills You Will Need p. 88

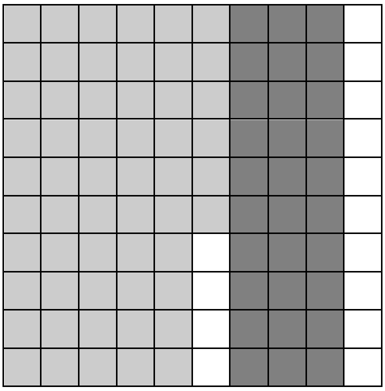
1. a) 0.59; *Sample response*:



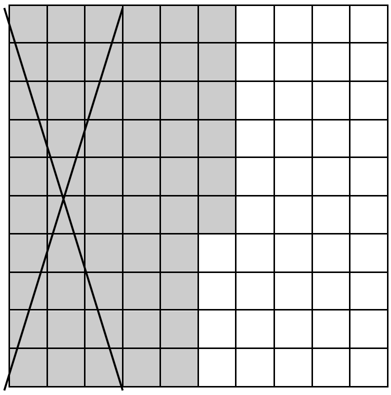
b) 0.09; *Sample response*:



1. c) 0.86; *Sample response*:



d) 0.26; *Sample response*:



2. a) 1.25 + 3.4
b) 5.25 – 2.9

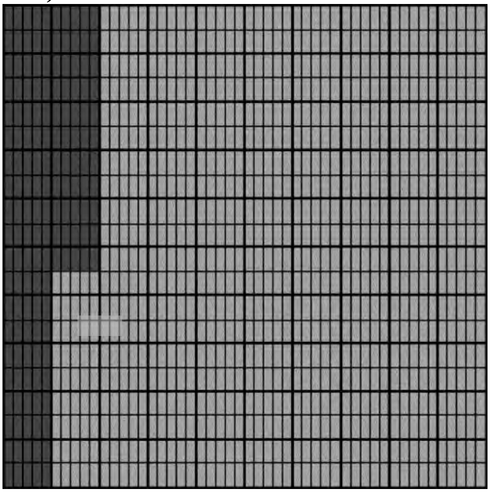
3. a) 5.2 **b)** 10.13
c) 2.37 **d)** 1.58

4. a) 120 **b)** 1370
c) 2500 **d)** 34,600

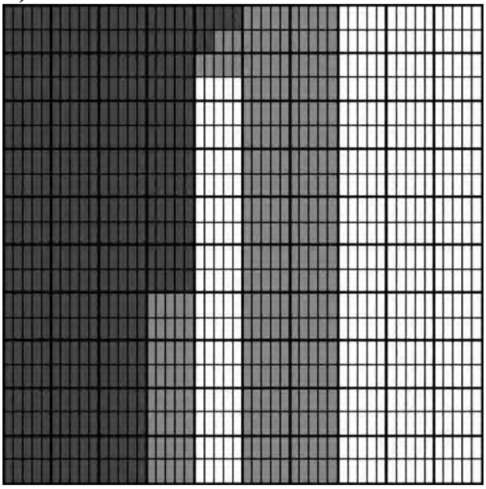
5. a) 40 **b)** 42 **c)** 36
d) 56 **e)** 96 **f)** 175
g) 252 **h)** 268

4.1.2 Adding Decimal Thousandths p. 95

1. a) 0.155 + 0.845 = 1



b) 0.367 + 0.248 = 0.615



2. a) 4.586 **b)** 0.444
c) 28.093 **d)** 25.399

3. a) 5.247 **b)** 4.238 **c)** 4.348

4. a) 20.337 **b)** 19.426

5.

a)

$$\begin{array}{r} \mathbf{3.527} \\ +\ \underline{\mathbf{4}.267} \\ 7.794 \end{array}$$

b)

$$\begin{array}{r} 8.09\mathbf{9} \\ +\ \underline{\mathbf{3.46}8} \\ \mathbf{1}1.567 \end{array}$$

6. a) 5.402 L
b) 3.245 L and 1.262 L

7. a) Right hand: 1.8 s; Left hand: 1.93 s
b) Right hand

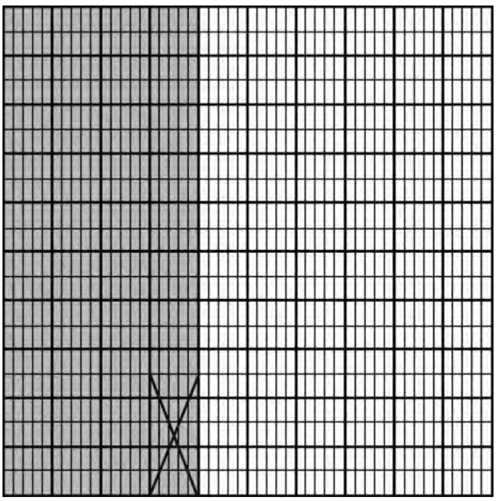
8. a) 0.192 m
b) 192 mm

10. No

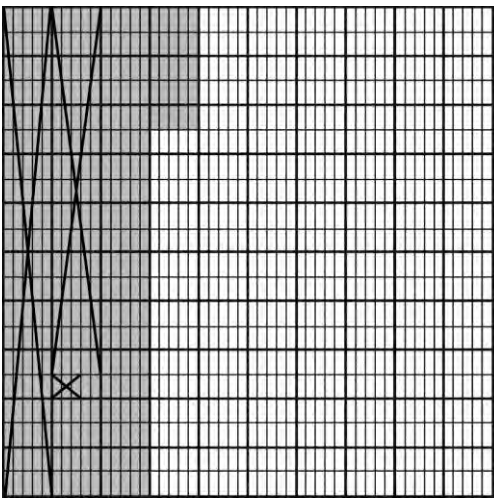
4.1.3 Subtracting Decimal Thousandths p. 100

1. *Sample responses*:

a) 0.4 – 0.025 = 0.375



b) 0.325 – 0.178 = 0.147



2. a) 3.111 **b)** 1.268
c) 2.113 **d)** 11.044

3. *Sample response*:
b) 4.2 – 1.999 and 3.5 – 1.298

4. a) 3.544 **b)** 7.815
c) 5.904 **d)** 2.999

5. Birth to 3 months

6. 0.014 s

7.a) 0.145 m

8. a) Always **b)** Never **c)** Always

9. *Sample responses*:
a) 4.2 and 4.264 **b)** 3.2 and 5.264

4.2.1 Estimating Products p. 103

1. a) 4 × 3 **b)** 7 × 8
c) 10 × 6 **d)** 9 × 4

2. *Sample responses*:
a) about 77; Lower
b) about 48; Lower
c) about 25; Higher
d) about 81; Higher

3. *Sample response*: A bit less than 32 m

4. *Sample response*: Less than 18 km

5. *Sample response*: More than 12 m

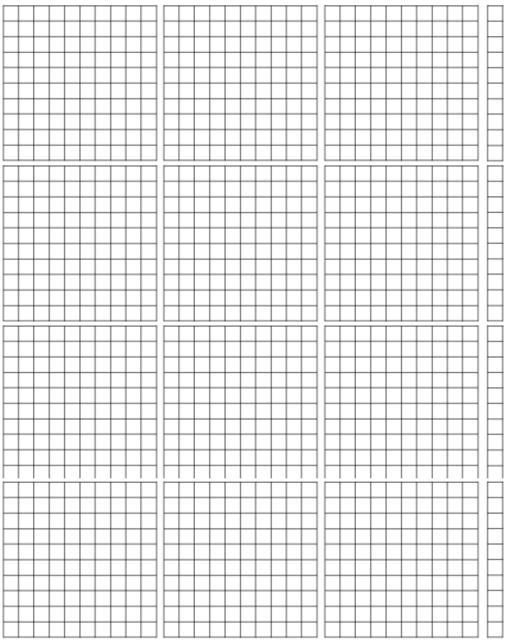
6. *Sample response*: Less than 1800 cm

8. B is greatest

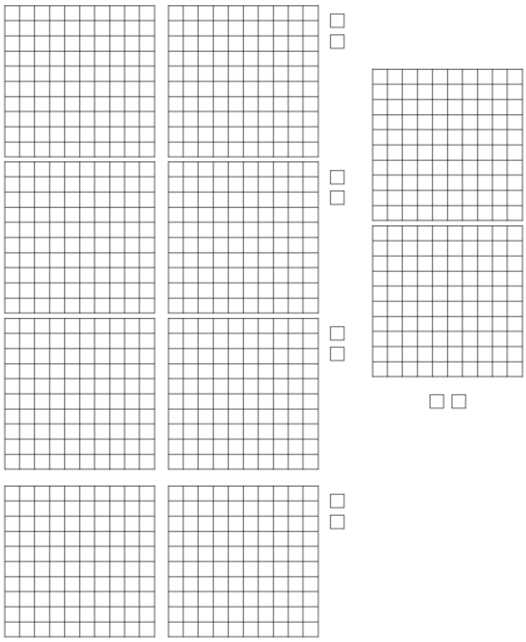
9. *Sample responses*:
a) 4 × 5.01
b) 6 × 4.99
c) 20 × 2.03

4.2.2 Multiplying a Decimal by a Whole Number p.107

1. a) 4 × 3.1 = 12.4

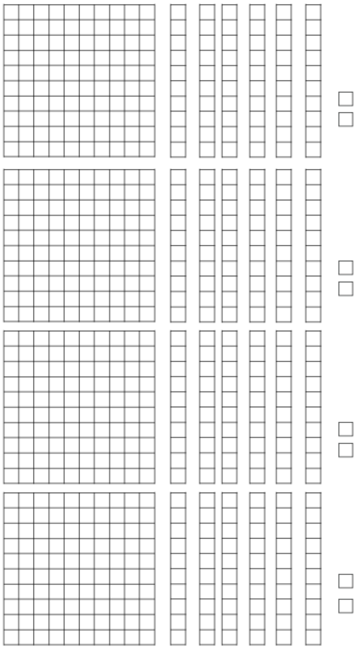


1. b) 5 × 2.02 = 10.1

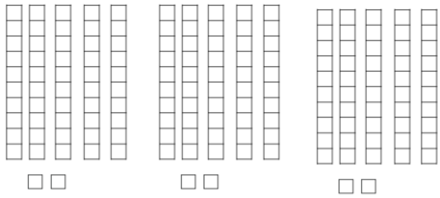


4.2.2 Multiplying a Decimal by a Whole Number [Cont'd] p.107

c) 4 × 1.62 = 6.48



d) 3 × 0.52 = 1.56



3. B is greatest

A. 7 × 3.12 = 21.84 B. 3 × 7.82 = 23.46

C. 5 × 4.09 = 20.45 D. 6 × 3.15 = 18.9

4. 18.2 km **5.** B and C are true.

6. a) $\begin{array}{r} 7.8 \\ \times\ 9 \\ \hline 70.\mathbf{2} \end{array}$ **b)** $\begin{array}{r} 12.\mathbf{8}3 \\ \times\ \ 7 \\ \hline \mathbf{8}9.81 \end{array}$

7. a) 17, 17.5, and 18

b) 5 × 3.7 = 18.5
5 × 3.8 = 19
5 × 3.9 = 19.5

c) 5 × 5.3 = 26.5

8. 137.1 cm **9.** They are both right

4.2.2 Multiplying By 0.1, 0.01, and 0.001 p.110

1. a) 0.6 **b)** 0.72 **c)** 0.045 **d)** 0.382 **e)** 0.019 **f)** 0.123

2. a) 1.72 **b)** 0.172

3. a) 2 **b)** 1 **c)** 1 **d)** 1

5. a) 2400 cm or 24 m

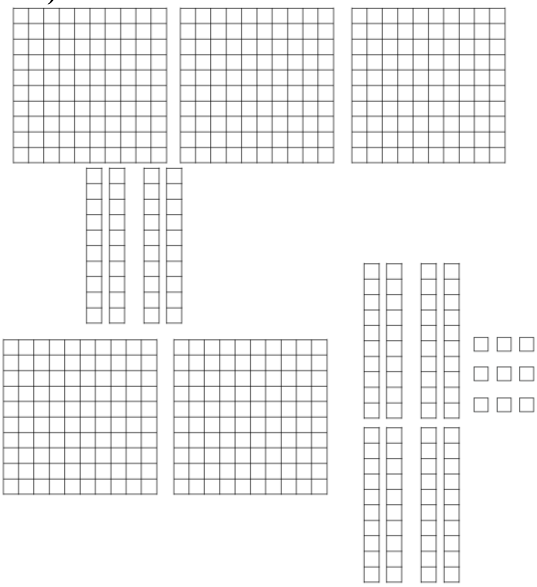
CONNECTIONS: Telescopes and Binoculars p.110

1. b) 2.5 × 0.1 = 0.25 cm

2. 2.5 × 0.01 = 0.025 cm

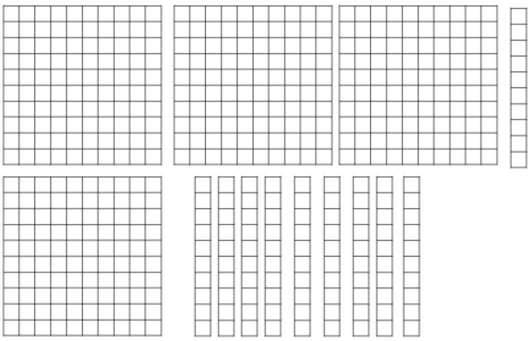
UNIT 4 Revision pp. 111–112

1. a) 3.4 + 2.89 = 6.29



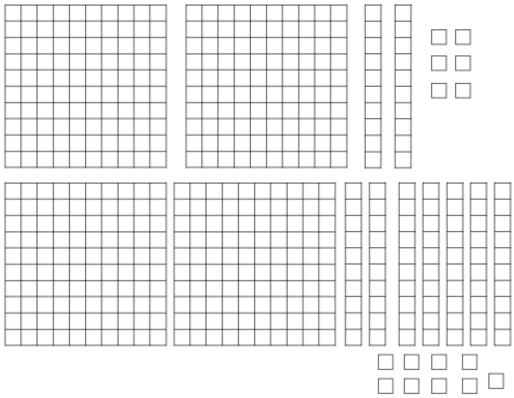
5 ones + 12 tenths + 9 hundredths
Regroup 10 tenths as 1 one.
6 ones + 2 tenths + 9 hundredths = 6.29

b) 3.1 + 1.9 = 5.0 or 5



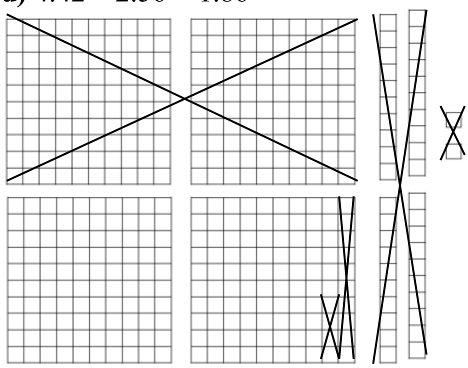
4 ones + 10 tenths
Regroup 10 tenths as 1 one.
5 ones = 5

c) 2.26 + 2.79 = 5.05



4 ones + 9 tenths + 15 hundredths.
Regroup 10 hundredths as 1 tenth.
Regroup 10 tenths as 1 one.
5 ones + 0 tenths + 5 hundredths = 5.05

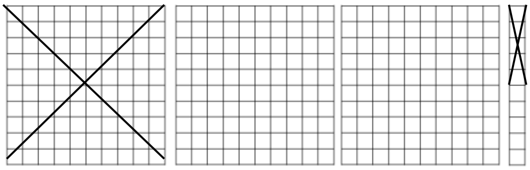
d) 4.42 – 2.56 = 1.86



1 one + 8 tenths + 6 hundredths are left = 1.86

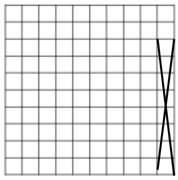
UNIT 4 Revision Cont’d — pp. 111–112

1. e) 3.1 – 1.05 = 2.05



2 ones + 0 tenths + 5 hundredths are left = 2.05

f) 1 – 0.08 = 0.92



92 hundredths are left = 0.92

2. a) 4.767 **b)** 4.462 **c)** 24.465 **d)** 9.394

3. a) 5.287 **b)** 10.458 **c)** 6.04

4. 146.1 cm

5. a) 13.56 m **b)** 1.466 m

6. a) 4.467 **b)** 4.566 **c)** 2.118 **d)** 3.002

7. a) 0.099 s **b)** 0.417 s and 0.420 s; 0.003 s

8. 3.752g

10. a) 3.353 L **b)** 0.284 L **c)** 1.381 L

11. *Sample responses*:
a) about 78; High **b)** about 24; Low
c) about 42; High **d)** about 64; High

12. *Sample response*: About 9 L

13. a) 33.2 **b)** 43.08 **c)** 46.41 **d)** 23.13

14. a)
$$\begin{array}{r} 3.8 \\ \times\ 7 \\ \hline 26.6 \end{array}$$
b)
$$\begin{array}{r} 17.53 \\ \times\ \ 8 \\ \hline 140.24 \end{array}$$

15. 1.71 m

16. a) 0.8 **b)** 1.22 **c)** 0.069 **d)** 0.113 **e)** 0.172

17. 0.523 km

UNIT 5 MEASUREMENT — pp. 113–154

Getting Started — Skills You Will Need — p. 114

1. a) 8 cm **b)** 6 cm
c) 12 cm **d)** 14 cm

2. *Sample responses*:
a) About 11 cm^2 **b)** About 4 cm^2

3. a) 10 cubes **b)** 32 cubes

4. *Sample responses*:
a) Half the length of my fingernail
b) The length of my thumb
c) The length of my hair
d) The length of this room
e) The distance I walk to and from school each day

5.1.4 Area and Perimeter Relationships — p. 123

1. a) Area = 400 cm^2 Perimeter = 100 cm
b) Area = 440 cm^2 Perimeter = 84 cm

2. *Sample response*: 25 cm by 25 cm

3. *Sample response*: 10 cm by 44 cm

5. *Sample response*: About 24 m^2

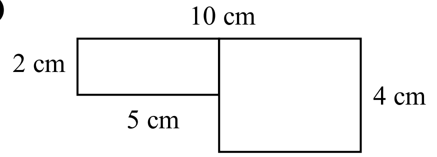
6. a) 1200 cm^2
b) *Sample response*: 30 cm by 40 cm

7. The square **8.** The rectangle

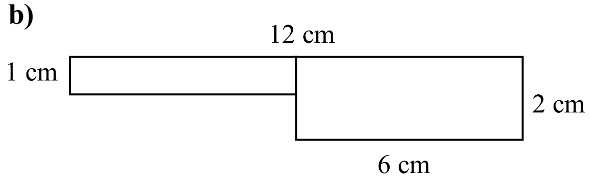
5.1.5 Area of Composite Shapes p. 128

1. a) Area = 318 cm^2
Perimeter = 96 cm
b) Area = 624 cm^2
Perimeter = 128 cm
c) Area = 323 cm^2
Perimeter = 94 cm
d) Area = 506 cm^2
Perimeter = 110 cm

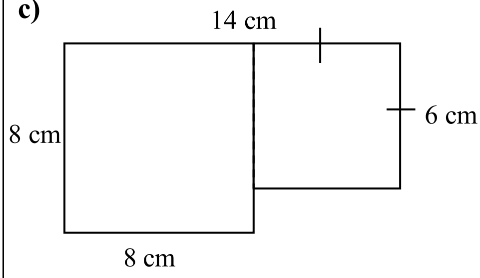
3. *Sample responses*:
a)



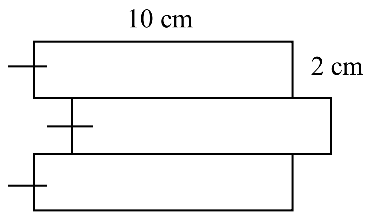
b)



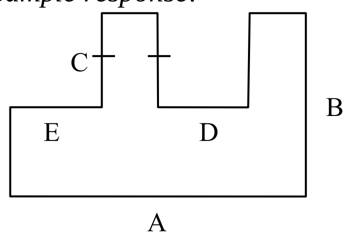
c)



d)

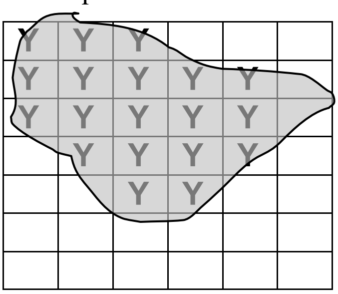


4. *Sample response*:

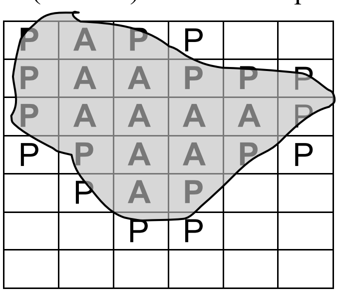


CONNECTIONS: Unusual Ways to Measure Area p. 129

1. *Strategy 1*
Area = 19 square units



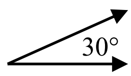
Strategy 2
Area = (17 + 10) ÷ 2 = 13.5 square units



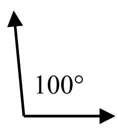
5.2.2 Comparing Angles to Special Angles p. 137

1. *Sample responses*:
a) About 75°
b) About 140°
c) About 40°
d) About 100°

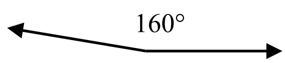
3. *Sample responses*:
a)



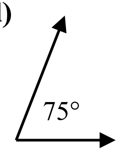
b)



3. c)

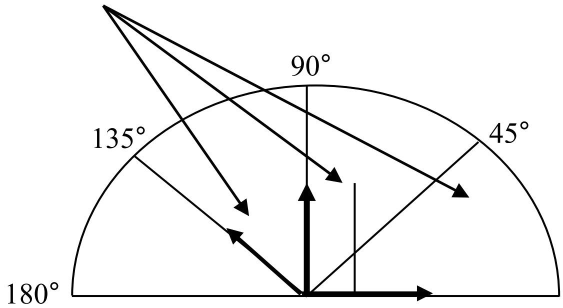


d)



5.2.2 Comparing Angles to Special Angles [Cont'd] p. 137

4. 45 + 45 + 45 = 135°



5. B, C, and E

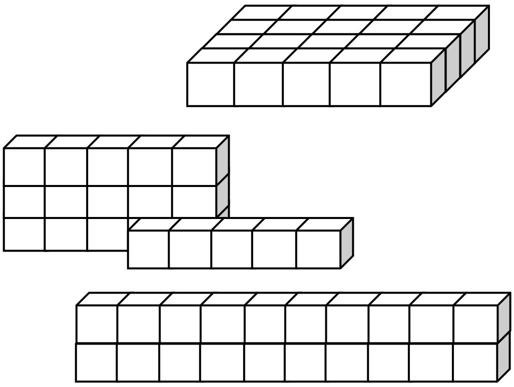
7. *Sample response*:
The angles closest to 30° were found in M, N, V, and W, but the angles in K, X, and Z were also close to 30°.

8. No

5.3.1 Volume p. 140

1. a) 32 cm^3 **b)** 30 cm^3 **c)** 48 cm^3
d) 36 cm^3 **e)** 24 cm^3

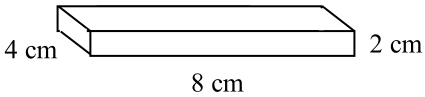
2. a) *Sample response*:



2. b) Yes; *Sample response*:
5 cm by 4 cm by 1 cm or
1 cm by 10 cm by 2 cm

3. a) 11 cm^3
b) 21 cm^3

4. *Sample response*: 64 cm^3



5. 1000 times as big

5.3.2 Capacity p. 142-143

1. a) 3 L **b)** 1 L **c)** 1025 mL

2. *Sample response*: about 1200 mL

3. a) Litres **b)** Millilitres
c) Millilitres **d)** Litres

4. a) 20 **b)** 10
c) 5 (or between 5 and 6)

4. d) 2 (or between 2 and 3)

5. *Sample response*: about 150 mL

6. *Sample response*:
I used 10 coins. The water went up 50 mL, so I knew the volume of 1 coin was 5 cm^3.

7. a) 32 cm cubes; 32 mL
b) 24 cm cubes; 24 mL

5.3.3 Metric Units pp. 147–148

1. a) 65 km **b)** 30 cm **c)** 2 m **d)** 2 mm
e) 10 cm **f)** 3 km **g)** 17 cm **h)** 1 cm

2. a) Less **c)** Less **d)** More

3. a) 340 **b)** 5120 **c)** 41,600
d) 0.41 **e)** 0.131

4. a) 512 mm **b)** 1 km **c)** 3.1 kg

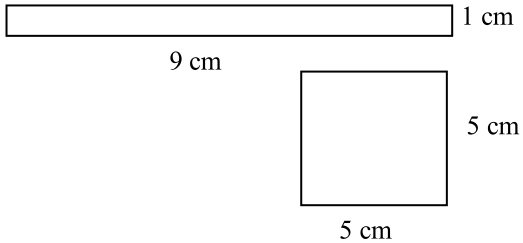
5. a) m **b)** km **c)** g **d)** mL	**6. a)** Only 6, 2, 1, 3, and 0

5.4.1 The 24-hour Clock System	**pp. 150**
1. 15:13 **2. a)** 04:15 **b)** 18:23 **c)** 12:00 **d)** 21:34 **3. a)** 7:00 a.m.	**3. b)** 6:25 p.m. **c)** 10:17 p.m. **4. a)** 10 h and 30 min **b)** 18 h and 55 min **c)** 15 h and 45 min

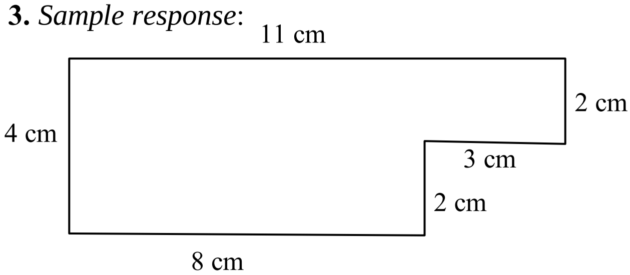
UNIT 5 Revision pp. 151-153

1. a) 30 cm **b)** 28 cm **c)** 30 cm

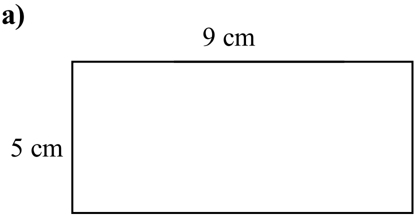
2. *Sample response*:



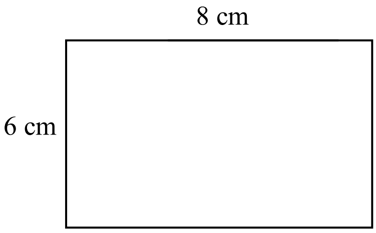
3. *Sample response*:



4. a)



b) *Sample response*:

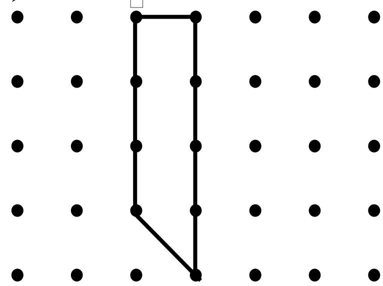


5. The shorter rectangle is 2 cm wider than the longer rectangle.

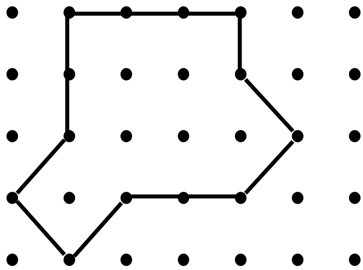
6. a) $10\frac{1}{2}$ units **b)** $7\frac{1}{2}$ units

7. *Sample responses*:

a)



b)



8. a) Perimeter = 22 cm
Area = 24 cm^2
b) Perimeter = 40 cm
Area = 96 cm^2

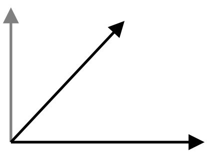
9. *Sample responses*:
a) 8 cm by 5 cm; 20 cm by 2 cm; 10 cm by 4 cm.
b) Least perimeter: 8 cm by 5 cm.

UNIT 5 Revision [Cont'd] **pp. 151-153**

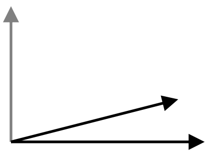
10. *Sample responses*:
a) 20 cm by 20 cm; 30 cm by 10 cm; 35 cm by 5 cm.
b) Greatest area: 20 cm by 20 cm

11. a) Area = 36 cm^2
Perimeter = 32 cm
b) Area = 9 cm^2
Perimeter = 16 cm

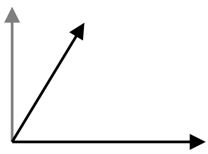
12. a)



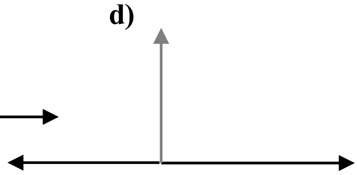
b)



c)



d)



13. *Sample responses*:
a) about 20° **b)** about 120°
c) about 75° **d)** about 165°

14. *Sample responses*:
a) about 10° **b)** about 100°
c) about 50° **d)** about 100°

15. *Sample response*:
3 cm by 5 cm by 2 cm

16. a) 9 cubes, or cubic units
b) 22 cubes, or cubic units

17. 1.2 L, 1500 mL, 2.8 L, 3200 mL, 4L

18. a) Litres
b) *Sample response*: a bucket and a sink

19. *Sample responses*:
a) A sink **b)** A spoon

20. a) 0.352 m **b)** 4200 g
c) 533 m **d)** 10,000 dm

21. a) m **b)** mm **c)** m

22. Move the digits 4 places to the left

23. a) 13:23
b) 00:00

24. a) 5:49 p.m.
b) 6:17 a.m.
c) 3:18 p.m.
d) 6:15 p.m.

25. a) 4 h and 27 min
b) 7 h and 45 min

UNIT 6 GEOMETRY pp. 155–198

Getting Started — Skills You Will Need p. 155-156

1. A and D

2. a) C and N; D and F
b) H and M
c) G and E; G and K; E and K; F and L; D and L

3. A

4. a) Triangle-based prism (or triangular prism)
b) Pentagon-based pyramid (or pentagonal pyramid)
c) Hexagon-based prism (or hexagonal prism)
d) Square-based pyramid (or square pyramid)

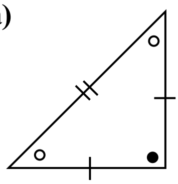
6.1.1 Classifying Triangles by Side Length pp. 159-160

1.

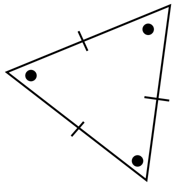
	Equilateral triangle	Isosceles triangle	Scalene triangle
Number of congruent sides	3	2	0
Number of congruent angles	3	2	0
Number of lines of symmetry	3	1	0
Sketch of example			

2. a) Isosceles
b) Scalene
c) Scalene

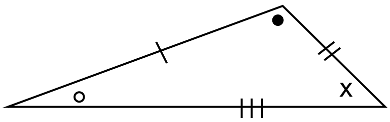
3. a)



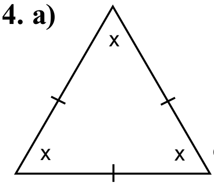
b)



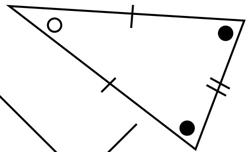
c)



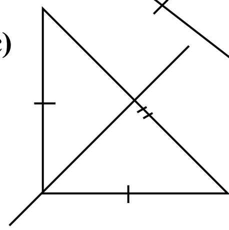
4. a)



b)

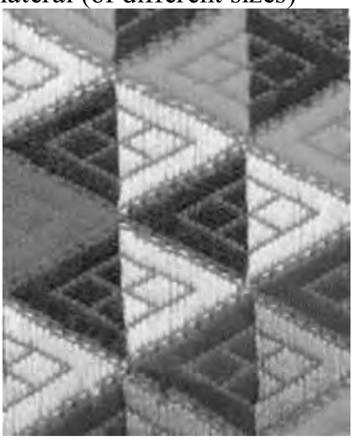


c)

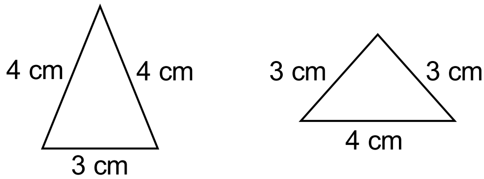


6.1.1 Classifying Triangles by Side Length [Cont'd] pp. 159-160

5. Equilateral (of different sizes)



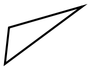
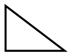
6. a) Isosceles
b)



7. a) Yes
b) Yes
c) No

6.1.2 Classifying Triangles by Angle pp. 163-164

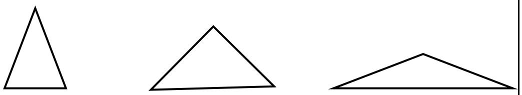
1.

	Obtuse triangle	**Right triangle**	**Acute triangle**
Greatest angle	Obtuse	Right	Acute
Number of obtuse angles	1	0	0
Number of right angles	0	1	0
Number of acute angles	2	2	3
Sketch of example			

2. a) Obtuse
b) Right
c) Acute

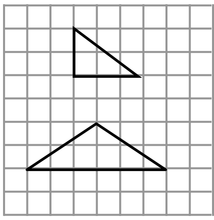
3. a) Acute, equilateral
b) Acute, isosceles
c) Right, scalene
d) Obtuse, isosceles

4. An acute triangle.



Acute Right Obtuse

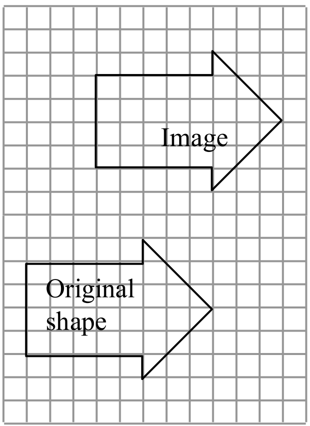
5. *Sample responses*:
a)
b)



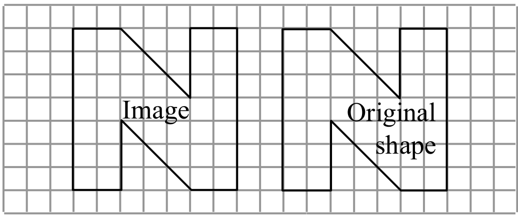
6.2.1 Properties of Translations p. 171

1. a) 7 right and 3 down
b) 1 right and 8 down
c) 6 left and 5 down
d) 1 left and 8 up

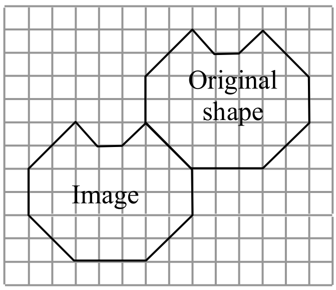
2. a)



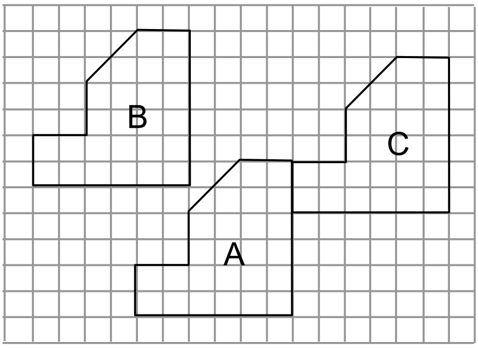
2. b)



c)

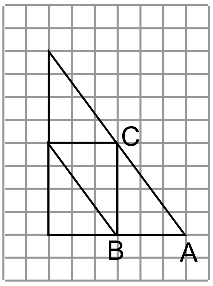


3. a) and b)



c) 6 left and 4 down

4. a), b), and c)



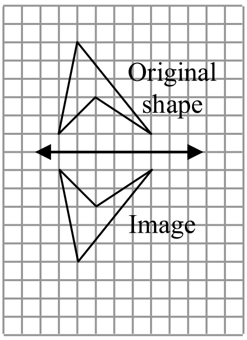
d) 4

5. Choki's rule doubles Nima's numbers and goes in the opposite direction.

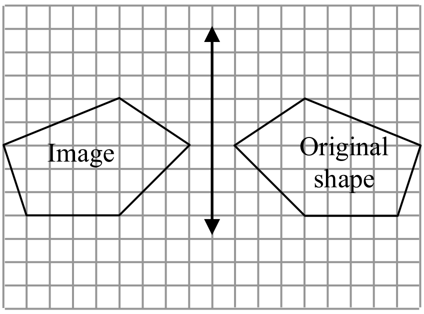
6. D

6.2.2 Properties of Reflections p. 174

1. a)

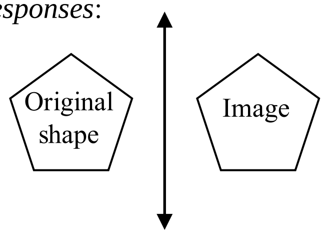


b)

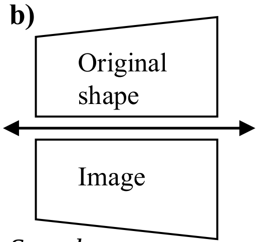


2. *Sample responses*:

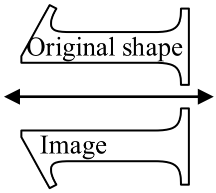
a)



2. b)

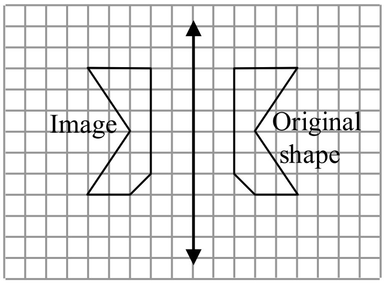


c)



3. *Sample responses*:

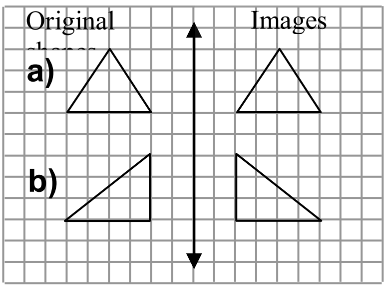
a) and b)



c) They are congruent and they are the same distance from the reflection/fold line.

4. The triangle, octagon, and pentagon.

5. *Sample responses*:



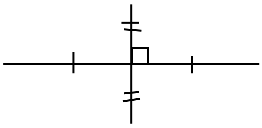
6.2.2 Properties of Reflections [Cont'd] p. 174

6. a) Shape C is the reflected image.
b) Shape B is the translated image.

6.2.3 Parallel and Intersecting Lines p. 179

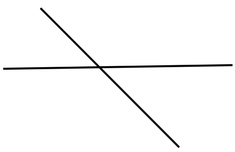
1. a) Perpendicular at an endpoint of one segment
b) Intersecting but not perpendicular
c) Parallel
d) Perpendicular at the centre points

2. *Sample responses*:
a)

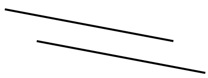


b)

c)

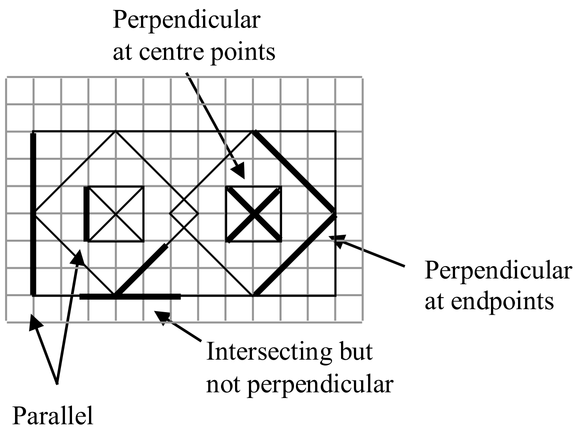


d)

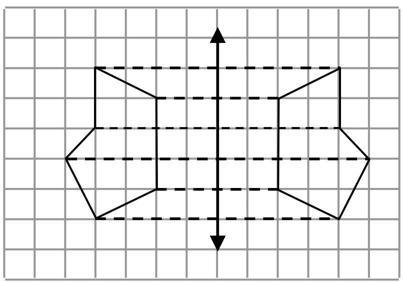


3. *Sample responses*:
a) Opposite sides of a window, board, table, or doorway
b) Window panes, the corner of a wall, board, table
c) Two sides of an equilateral triangle on the board

4. *Sample responses*:
a) and b)

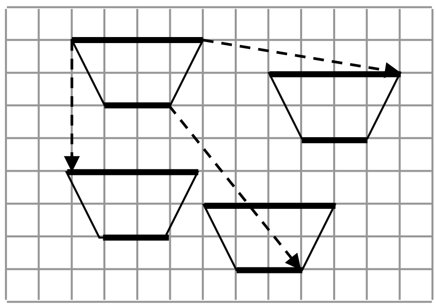


5. a), b), and c)



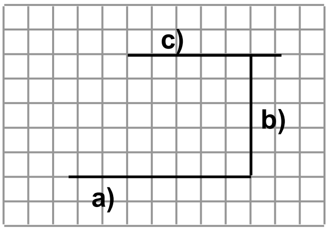
d)
- The lines connecting vertices to their images are all parallel.
- The lines connecting vertices to their images are perpendicular to the reflection line.
- The vertical sides of the original shape and their images are all parallel.

6. a) and b) *Sample responses*: Translation rules are: 6 right, 1 down; 0 right, 4 down; and 4 right, 5 down.



c) Yes, the parallel sides of the trapezoid are still parallel in the translation images.

8. *Sample responses*:
a), b), and c)

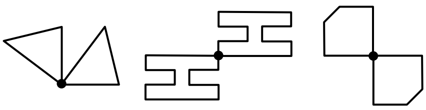


d) c) and a) are parallel

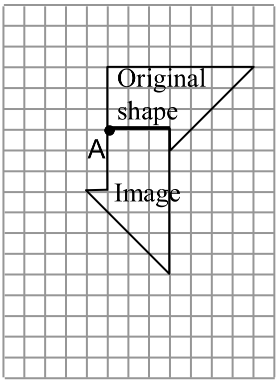
6.2.4 Properties of Rotations pp. 182–183

1. a) A $\frac{1}{4}$ turn counterclockwise or a $\frac{3}{4}$ turn clockwise about the bottom left vertex of the grey shape
b) A $\frac{1}{2}$ turn clockwise or counterclockwise about the bottom left vertex of the grey shape.
c) A $\frac{1}{2}$ turn clockwise or counterclockwise about the bottom right vertex of the grey shape

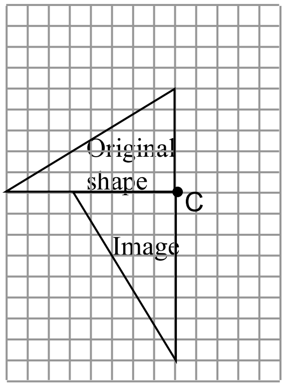
2.



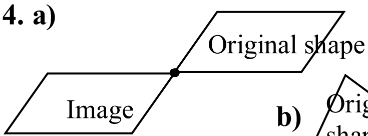
3. a)



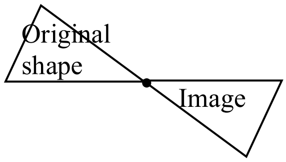
b)



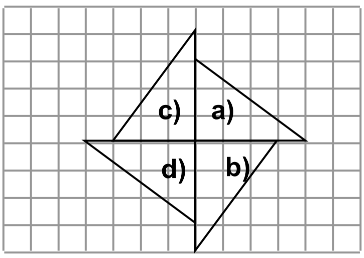
4. a)



b)



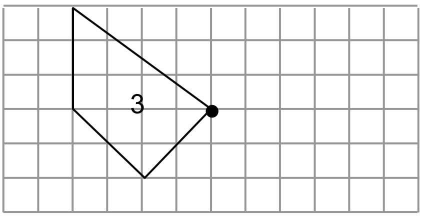
5.



e) *Sample response*: The triangles fit together around the turn centre to make a single shape with 8 sides.
f) *Sample response*: A $\frac{1}{4}$ turn clockwise three times

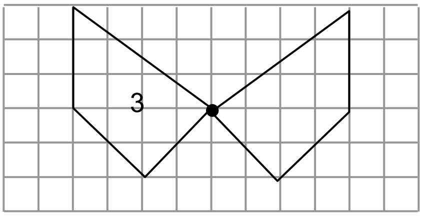
6. a) Rotate a $\frac{1}{2}$ turn clockwise or counter clockwise around the dot.
b) The reflection line would be the vertical line through the dot.
c) Move 4 right.

7. a) *Sample response*:

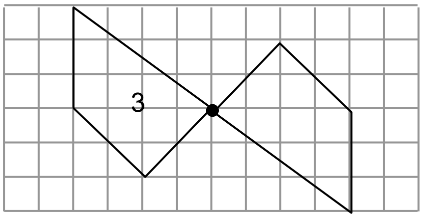


6.2.4 Properties of Rotations [Cont'd] pp. 182–183

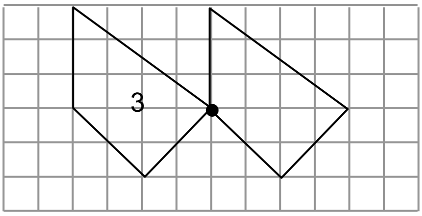
7. b) Rotation:



Reflection:

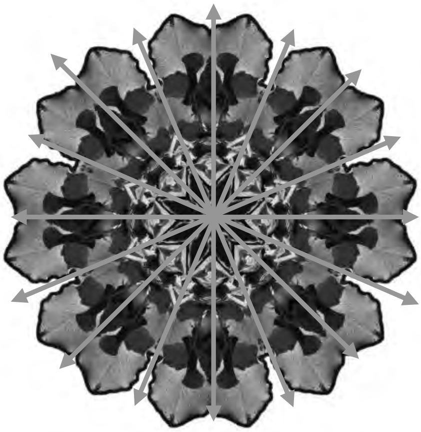


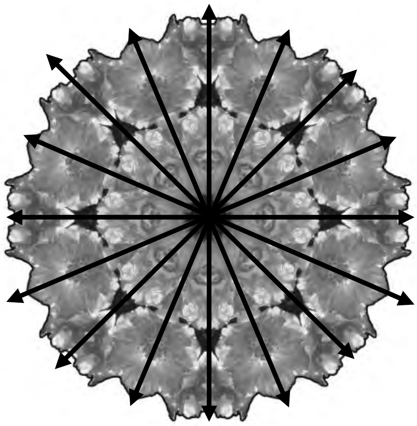
Translation:



CONNECTIONS: Kaleidoscope Images p. 184

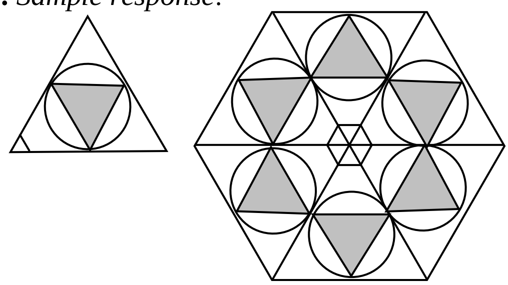
1.





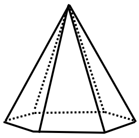
2. $\frac{1}{4}$ turns, $\frac{1}{2}$ turns, and $\frac{3}{4}$ turns, either clockwise or counterclockwise around the centre (where the reflection lines intersect), as marked

3. *Sample response*:

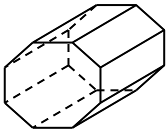


6.3.1 Prism and Pyramid Nets p. 188-189

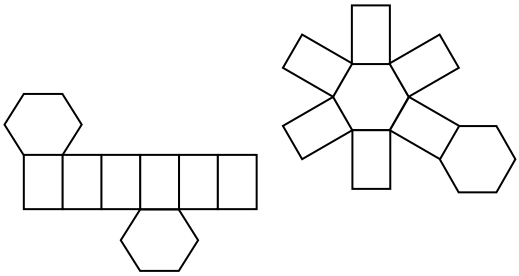
1. a) Hexagon-based pyramid



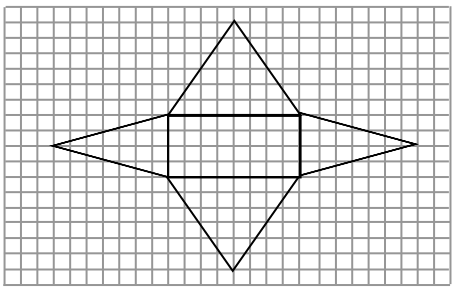
b) Octagon-based prism



2. Hexagon-based prism; *Sample response*:



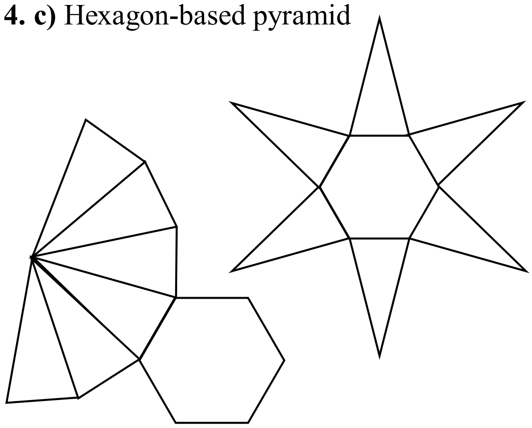
3. a) *Sample response*:



4. a) i) Hexagon-based prism
ii) Square-based pyramid
iii) Triangle-based prism
iv) Hexagon-based pyramid

b) A makes **i)** hexagon-based prism
B makes **ii)** square-based pyramid
C makes **iii)** triangle-based prism

4. c) Hexagon-based pyramid



5. No

6. Yes

7. a) Prism
b) Pentagon-based prism

CONNECTIONS: Euler's Rule p. 189

1.

3-D Shape	**V** (number of vertices)	**F** (number of faces)	**E** (number of edges)	**Euler's rule V + F – E = ?**
Triangle-based prism	6	5	9	2
Triangle-based pyramid	4	4	6	2
Rectangle-based prism	8	6	12	2
Square-based pyramid	5	5	8	2
Pentagon-based prism	10	7	15	2
Hexagon-based prism	12	8	18	2

2. No

6.3.2 Interpreting Isometric Drawings — p. 193

1. A, B, and D represent the same structure.

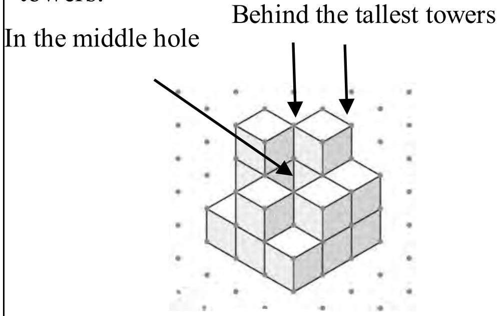
2. No

3. I can be sure there are seven cubes.
There could be eight cubes because there could be a hidden cube behind the tower.

4. B is a drawing of the cube structure.

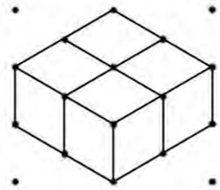
5. a) 12 cubes
b) Two cubes; at the bottom of the two towers at the back

5. c) You could hide one in the middle hole or more behind the two tallest towers.

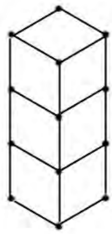


6.3.3 Creating Isometric Drawings — p. 196

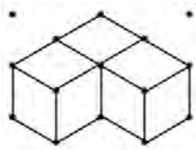
1. a)



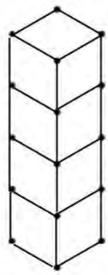
b)



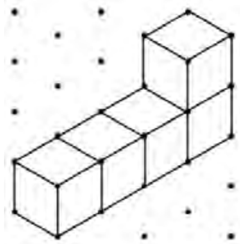
c)



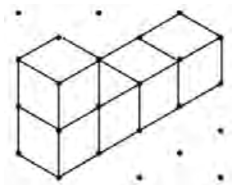
d)



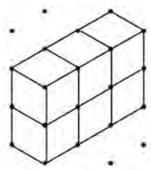
2. *Sample responses*:
b)



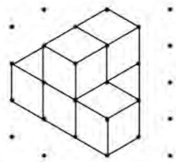
c)



3. *Sample response*:



4. a) 2 cubes
c) *Sample response*:



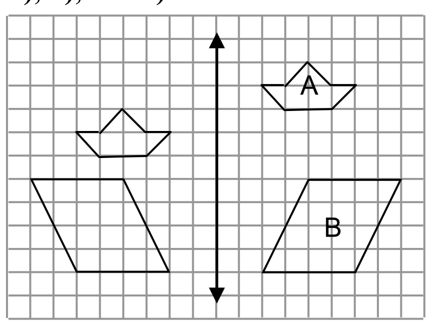
UNIT 6 Revision — pp. 197–198

1. a) Isosceles, right **b)** Scalene, obtuse
c) Equilateral, acute

2. a) a) 1 line of symmetry
b) 0 lines of symmetry
c) 3 lines of symmetry
b) a) 2 acute angles
b) 2 acute angles
c) 3 acute angles

3. 7 units right and 1 unit up

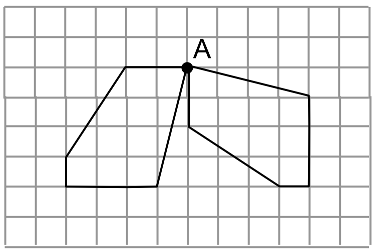
4. a), b), and c)



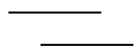
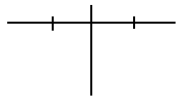
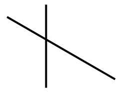
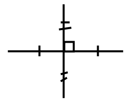
5. Equilateral or isosceles triangle

6. a) A $\frac{1}{2}$ turn cw around vertex D or
a $\frac{1}{2}$ turn ccw around vertex D
b) A $\frac{1}{2}$ turn cw around vertex D or
a $\frac{1}{2}$ turn ccw around vertex D

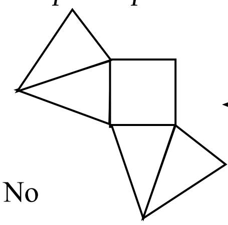
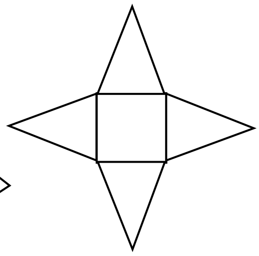
7. a)



b) A $\frac{3}{4}$ turn clockwise around A

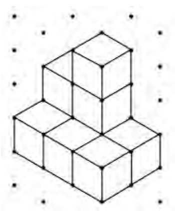
9. a) 
b) 
c) 
d) 

10. *Sample response*:

11. No

12. Hexagon-based prism

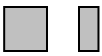
13. a) 9; 11
b) 

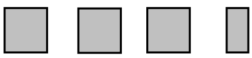
UNIT 7 DATA AND PROBABILITY pp. 199–230

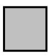
Getting Started — Skills Your Will Need p. 199-200

1. *Sample response*:
A sum of 6 or more is more likely to be rolled than a sum less than 6.

2. *Sample response*:

Sum less than 6 

Sum of 6 or more 

 Represents 10 times rolled

3. Yes

4. a) 15 **b)** 18

5. a) 0 **b)** $\frac{1}{2}$ **c)** 0 **d)** 1

6. a) Equally likely
b) Getting a product greater than 10
c) I will eat rice tomorrow

7. *Sample responses*: **a)** 1 **b)** 8 **c)** 3

7.1.1 The Mean p. 204

1. 4

2. a) 7 **b)** 5

3. a) 18 **b)** 45 **c)** 25

4. a) 8 **b)** 17 **c)** 6

5. a) 68
b) *Sample response*: 68, 68, 68, 68, 68

6. *Sample response*:
20, 20, 20, 20, 20;
20, 10, 30, 20, 20;
1, 5, 8, 20, 66

7. 13 km **8.** Yes

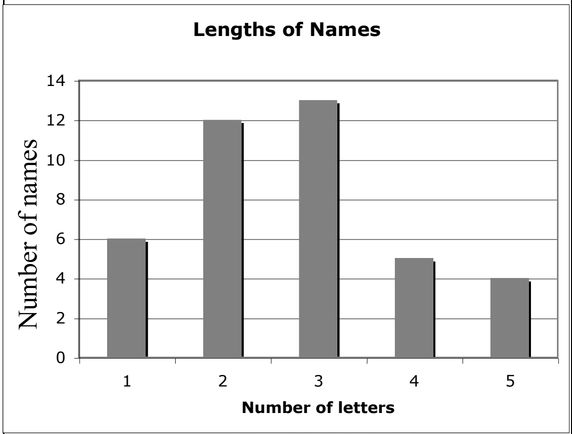
9. *Sample response*: $3\frac{1}{2}$
I rolled 12 times and got 2, 1, 4, 5, 1, 3, 6, 2, 1, 5, 4, 3. The mean was just a bit over 3, so I was close.

7.2.1 Choosing a Graph pp. 210–211

1. *Sample responses*:

a)

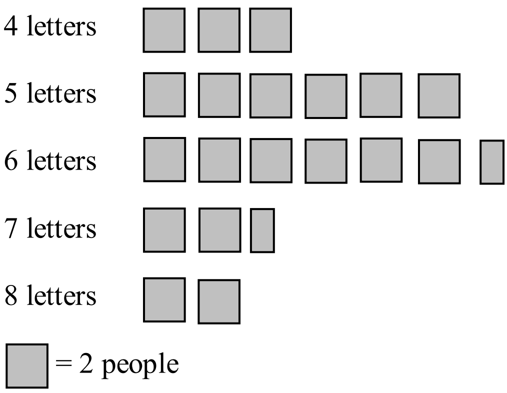
Number of letters	Frequency
4	6
5	12
6	13
7	5
8	4



c) Most names had 5 or 6 letters.
No names had fewer than 4 letters.
No name had more than 8 letters.

2. *Sample responses:*

a)

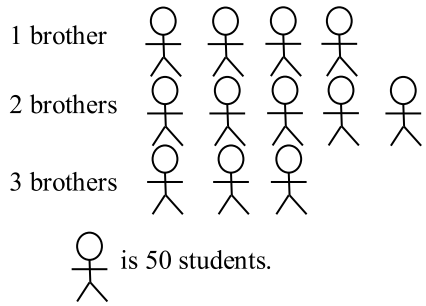


3. *Sample response*:

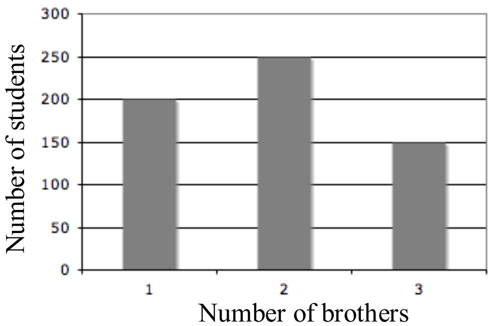
A 50
B 80
C 50

4. *Sample response*:

How Many Brothers We Have



How Many Brothers We Have



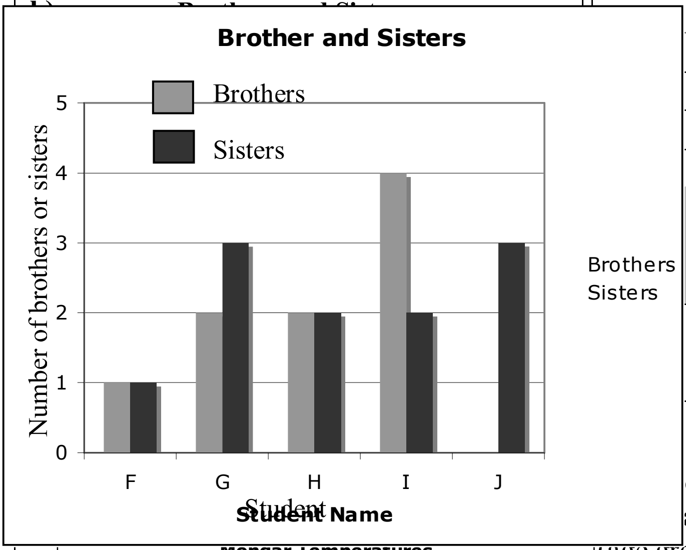
5. *Sample response*:
- It shows that the population has increased each year.
- It shows the population for each year from 1976 to 1996.

6. *Sample response*:
It might be about the number of Class V students whose favourite colour is brown, black, orange, or blue.

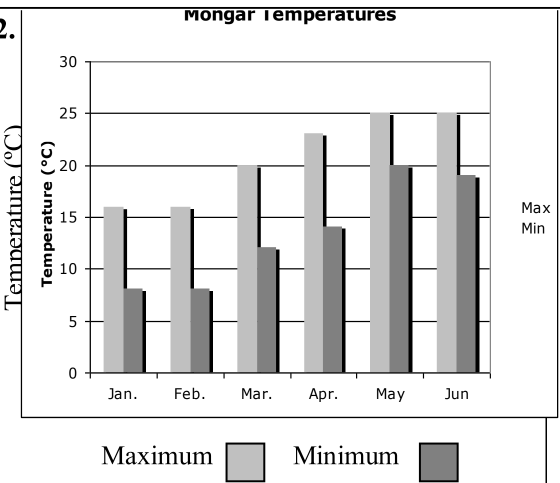
7. *Sample response*:
- Fiji has the least area of the five countries in the graph.
- The area of Bhutan is greater than the areas of Armenia, El Salvador, Fiji, or the Netherlands.
- The area of Bhutan is between two and three times as large as the area of Fiji.

1. a) *Sample response*:
- Two students have more brothers than sisters.
- Two students have more sisters than brothers.
- One student has no brothers.
- One student has as many brothers as sisters.
- No one has more than 2 brothers or 2 sisters.
- There is the same number of students with 1 brother as with 2 brothers.

b)

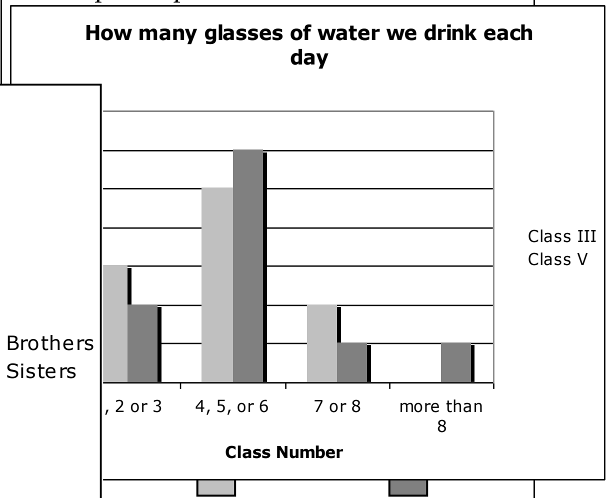


2.



3. *Sample response*:
- The United States are most interesting to one class, but India and Australia are more interesting to the other class.
- Both classes have an equal interest in China.
- India, Nepal, Australia, and the U.S. are of the most interest to students in general.

4. *Sample response*:

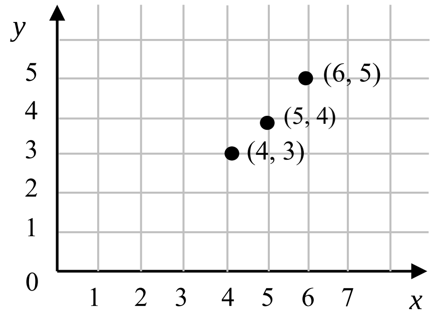


compares two groups of data
and females) in different categories
(age groups) on the same graph.

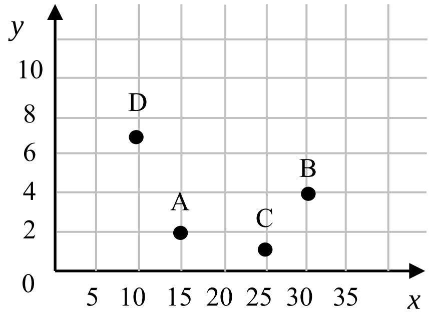
6. *Sample response*:
The number of minutes of exercise that boys and girls in different classes get each week.

7.2.3 Coordinate Graphs p. 220

1. a) and b)



2.



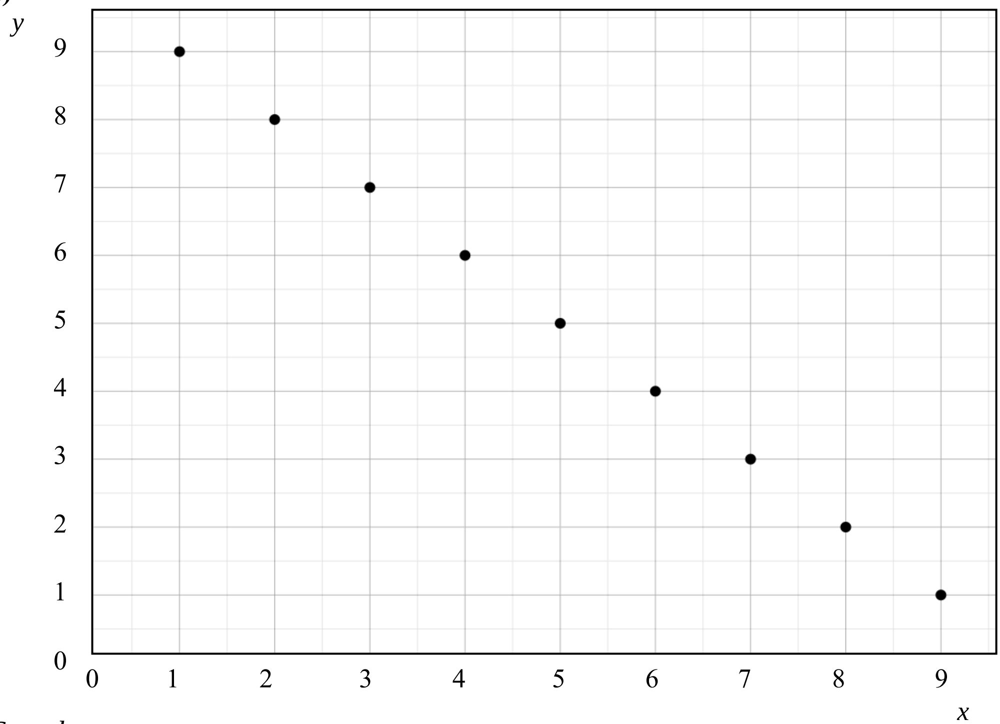
3. A(0, 25), B(10, 20), C(4, 30), D(8, 0)

4. *Sample responses:*
a) They are all in a line.
b) (5, 15) and (10, 30)

6. *Sample responses*:
a) (1, 1) **b)** (20, 0) **c)** (0, 50)

7. *Sample response*:
(3, 8) and (4, 6)

8. a)



b) *Sample response*:
The *x*-coordinate and the *y*-coordinate add to 10.

9. a) No
b) (3, 0) or (0, 3)

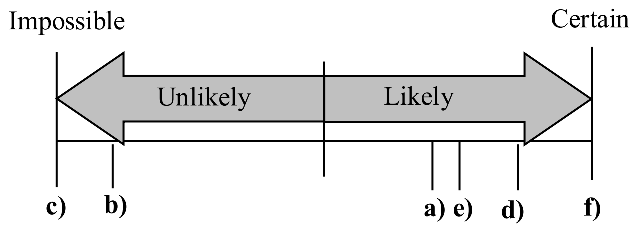
10. a) It depends on the value of [ ];
b) No

7.3.1 Describing Probability pp. 219–220

1. *Sample responses*:

a) Likely **b)** Very unlikely **c)** Impossible
d) Very likely **e)** Likely **f)** Certain

2.



3. *Sample responses*:

a) I will go home after school
b) I will have a snack when I get home.
c) I will play with my brother.
d) I will eat by myself
e) I will lift up my house.

4. *Sample responses*:

6	10	2	9	4	7	7	5	11	6	8	3	7	6	8
9	5	8	6	7	12	7	11	10	5	8	9	6	4	7

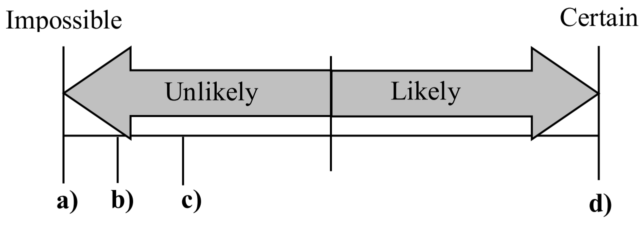
a) 0 out of 30; Impossible
b) 2 out of 30; Very unlikely
c) 4 out of 30; Unlikely
d) 30 out of 30; Certain

5. *Sample responses*:

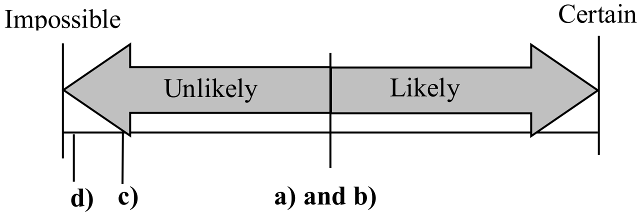
K	T	K	T	T	T	K	T	K	K	T	K	K
T	K	T	T	K	K	T	K	T	K	T	K	

a) As likely to happen as not to happen
b) As likely to happen as not to happen
c) Very unlikely
d) Very unlikely

6. a)



b)



7. a) i) Spinning a number less than 4
ii) Spinning an odd number
iii) Spinning a 2
iv) Spinning a 4

8. *Sample response:*
I would say 25 times would make it very likely and 5 times would make it not very likely.

CONNECTIONS: Magic Tricks With Dice — p. 220

1. The numbers on opposite faces of a die add to 7, so the two pairs of opposite faces in the stack add to 14. If one of the numbers is 2 (the one on top), then the rest of the numbers must add to 14 – 2 = 12.

2. a) 16
b) Subtract the top number from 21 since there are 3 pairs of opposite faces so the total is 21.

3. Subtract the top number from 28.

7.3.2 Using Numbers to Describe Probability — p. 224

1. *Sample responses*:
a)

T	T	K	T	T	T	T	T	K	K
T	K	T	K	K	K	T	K	T	T

b) i) $\frac{8}{20} = \frac{2}{5}$ **ii)** $\frac{12}{20} = \frac{3}{5}$ **iii)** 1

2. a) *Sample response*:

Number rolled	1	2	3	4	5	6
Number of times	3	4	2	5	3	3

b) *Sample responses*:
i) $\frac{4}{20} = \frac{20}{100} = 0.20$
ii) $\frac{9}{20} = \frac{45}{100} = 0.45$
iii) $\frac{8}{20} = \frac{40}{100} = 0.40$
c) i) $\frac{1}{6}$ **ii)** $\frac{3}{6} = \frac{1}{2}$ **iii)** $\frac{3}{6} = \frac{1}{2}$

2. d) *Sample responses*:
i) Rolling a number less than 4
ii) Rolling a 1 **ii)** 0.25 **iii)** 0.5

3. b) *Sample response*:
Correct: 12 times, Incorrect: 8 times.
$\frac{12}{20} = \frac{6}{10} = 0.6$

4. a) *Sample response*:

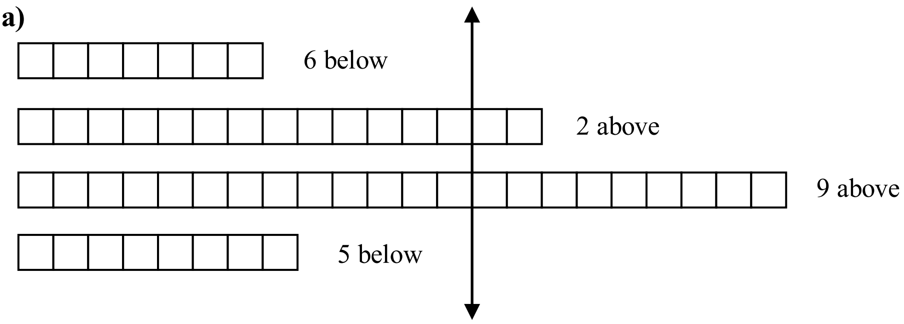
3	1	5	4	5	1	3	5	1	4	3	5	3
4	3	5	1	1	3	4	4	5	5	1	4	

b) *Sample responses*:
i) 0.24 **ii)** 0.24 **iii)** 0.48
c) i) 0.25

5. *Sample responses*:
a) 1, 1, 2, 3
b) 1, 3, 3, 3
c) 1, 1, 1, 1
d) 2, 4, 6, 8

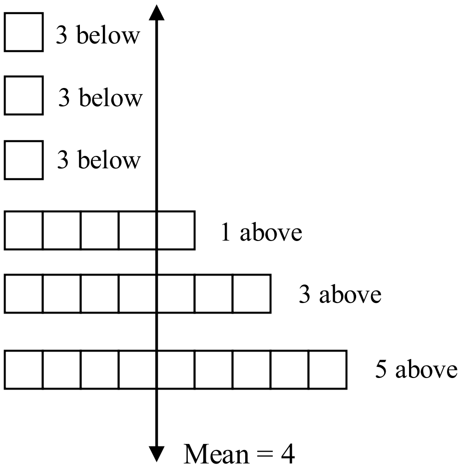
UNIT 7 Revision — pp. 225–226

1. *Sample responses*:
a)



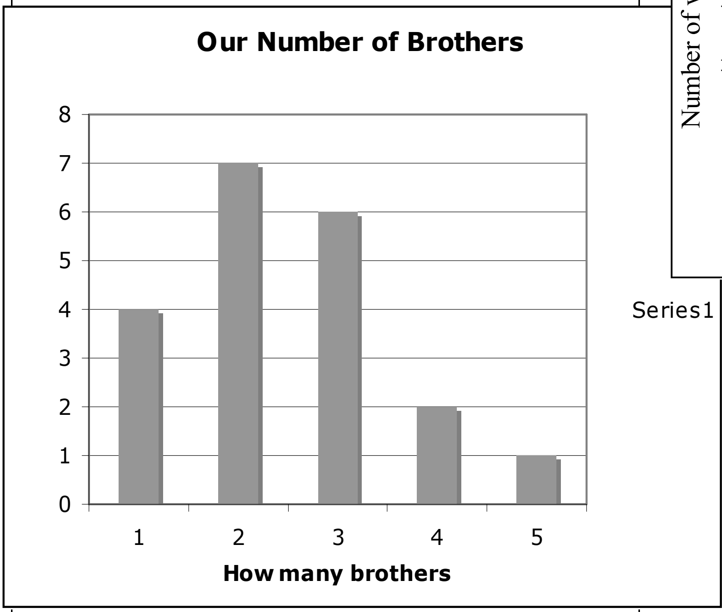
Mean = 13

1. b)



2. a) 68
b) *Sample response*: 58, 78, 68, 68, 68

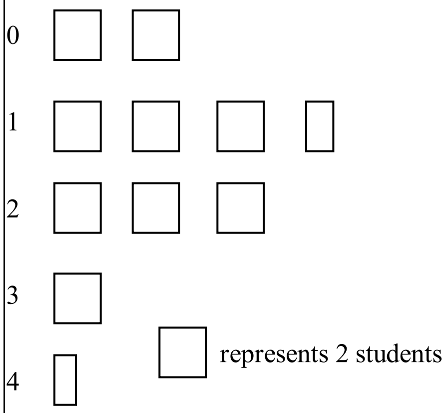
3. *Sample responses*:
a) Predicted : 2, 3, 5, 18
b) Predicted: They are the same.
c) Predicted: 3, 5, 10
d) Predicted: 2, 3, 5, 10



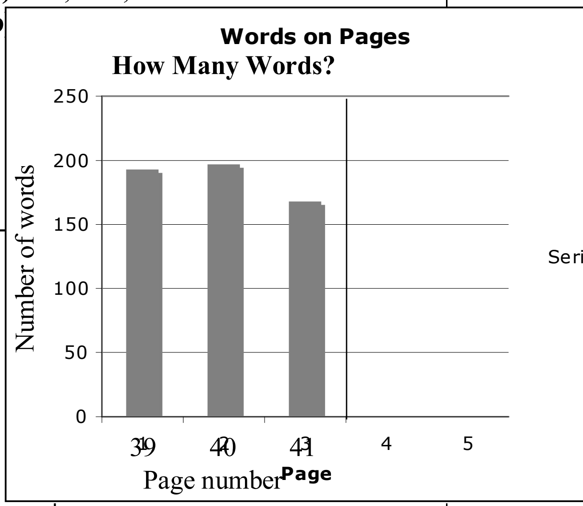
b) *Sample response*:
• More people have 1 sister than any other number of sisters.
• More people have no sisters than 3 sisters.
• There are twice as many people with no sisters as 3 sisters.

4. c) *Sample response*:

How Many Sisters?



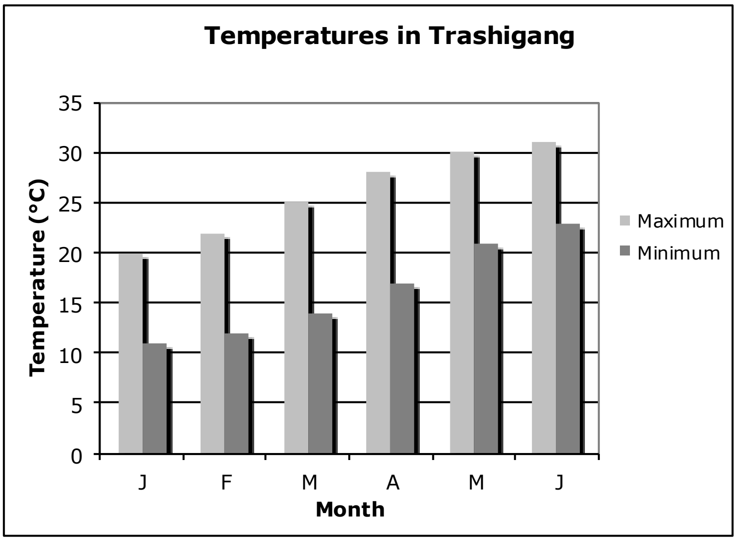
5. *Sample responses*:
a) 193, 197, 168
b



sed a scale of 50

nple response:
ni watched more TV on Friday
n any other day.
vatched almost as much TV on
ay as on Friday.
vatched the least amount of TV
ednesday.

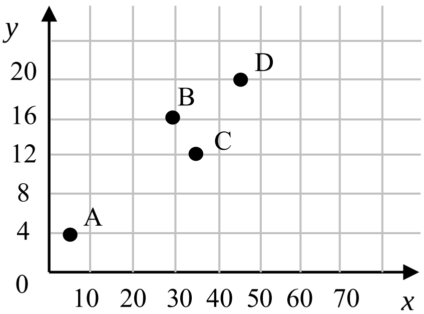
7.



8. *Sample response*:
• Both Class I and Class V students like chocolate bars the most.
• More Class I students than Class V students like chocolate bars best.
• The least favourite treat was either chips or momos.

9. Yes

10.

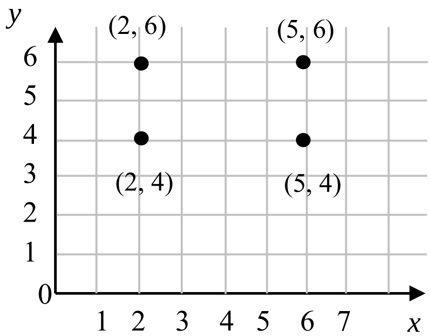


Sample response:
• I let one unit represent 10 on the *x*-axis.
• I let one unit represent 4 on the *y*-axis.

11. A (10, 60), B (40, 40),
C (20, 20), D (0, 30)

12. *Sample response*: (2, 8) and (5, 5)

13. a)



b) They form a rectangle.

14. *Sample responses*:

1	0	2	1	1	3	2	4
2	2	5	0	2	1	0	3
5	0	1	2	4	3	1	1
4							

a) Certain
b) 6 out of 25; Unlikely
c) 7 out of 25; Unlikely
d) 8 out of 25; Unlikely