Understanding Mathematics

Textbook for Class VI

ཤེས་རིག
Department of School Education
Ministry of Education and Skills Development
Royal Government of Bhutan

Published by Department of School Education(DSE), Ministry of Education and Skills Development (MoESSD), Royal Government of Bhutan,Thimphu.

Tel: +975-8-271226 Fax: +975-8-271991

ACKNOWLEDGEMENTS

Advisors
Dasho Dr. Pema Thinley, Secretary, Ministry of Education
Tshewang Tandin, Director, Department of School Education, Ministry of Education
Yangka, Director for Academic Affairs, Royal University of Bhutan
Karma Yeshey, Chief Curriculum Officer, CAPSD

Research, Writing, and Editing
One, Two, ..., Infinity Ltd., Canada

Authors
Marian Small
Wendi Morrison

Reviewers
Tara Small

Editors
Jackie Williams
Carolyn Wagner

Bhutanese Reviewers

Sonam Dorji M	Bjishong MSS, Gasa
Dorji Penjor	Logodama PS, Punakha
Padam P Kafley	Tsaphel LSS, Haa
Kuenga Loday	Umling CPS, Sarpang
Dorji Wangdi	Panbang LSS, Zhemgang
Pelden Dorji	Moshi CPS, Trashigang
Radhika Chettri	Tencholing PS, Wangdue
R.K. Chettri	Tencholing PS, Wangdue
Devika Gurung	Mongar LSS, Mongar
Kinley Wangchuk	Norbugang CPS, Pemagatsel
Namgyel Dhendup	Patala PS, Tsirang
Karchung Dorji	Tangmachu PS, Lhuntse
Tshering Yangzom	RinchenKunphen PS, Thimphu
Rupak Sharma	Khasadrapchu MSS, Thimphu
Mindu Gyeltshen	EMSSD, Thimphu
Ugyen Lhadon	Gaupel LSS, Paro
Arjun Chettri	PCE, Paro
Lobzang Dorji	CAPSD, Paro

Coordination
Karma Yeshey and Lobzang Dorji, Curriculum Officers, CAPSD

The Ministry of Education wishes to thank
- all teachers in the field who have given support and feedback on this project
- the World Bank, for ongoing support for School Mathematics Reform in Bhutan
- the students at Drugyal LSS and Lango LSS for their photos
- Nelson Publishing Canada, for its publishing expertise and assistance

1st edition 2008
Reprint 2023

ISBN: 99936-0-321-X

CONTENTS

ROYAL GOVERNMENT OF BHUTAN
ཤེས་རིག་ལྷན་ཁག།
MINISTRY OF EDUCATION
THIMPHU :BHUTAN
Cultivating the Grace of Our Mind

MINISTER

December 15, 2008

Foreword

I am at once awed and fascinated by the magic and potency of numbers. I am amazed at the marvel of the human mind that conceived of fantastic ways of visualizing quantities and investing them with enormous powers of representation and symbolism. As my simple mind struggles to make sense of the complexities that the play of numbers and formulae presents, I begin to realize, albeit ever so slowly, that, after all, all mathematics, as indeed all music, is a function of forming and following patterns and processes. It is a supreme achievement of the human mind as it seeks to reduce apparent anomalies and to discover underlying unity and coherence.

Abstraction and generalization are, therefore, at the heart of meaning-making in Mathematics. We agreed, propped up as by convention, that a certain figure, a sign, or a symbol, would carry the same meaning and value for us in our attempt to make intelligible a certain mass or weight or measure. We decided that for all our calculations, we would allow the signifier and the signified to yield whatever value would result from the tension between the quantities brought together by the nature of their interaction.

One can often imagine a mathematical way of ordering our surrounding and our circumstance that is actually finding a pattern that replicates the pattern of the universe – of its solid and its liquid and its gas. The ability to engage in this pattern-discovering and pattern-making and the inventiveness of the human mind to anticipate the consequence of marshalling the power of numbers give individuals and systems tremendous privilege to the same degree which the lack of this facility deprives them of.

Small systems such as ours cannot afford to miss and squander the immense power and privilege the ability to exploit and engage the resources of Mathematics have to present. From the simplest act of adding two quantities to the most complex churning of data, the facility of calculation can equip our people with special advantage and power. How intelligent a use we make of the power of numbers and the precision of our calculations will determine, to a large extent, our standing as a nation.

I commend the good work done by our colleagues and consultants on our new Mathematics curriculum. It looks current in content and learner-friendly in presentation. It is my hope that this initiative will give the young men and women of our country the much-needed intellectual challenge and prepares them for life beyond school. The integrity of the curriculum, the power of its delivery, and the absorptive inclination of the learner are the eternal triangle of any curriculum. Welcome to Mathematics.

Imagine the world without numbers! Without the facility of calculation!

Tashi Delek.

Thakur S Powdyel.

Telephone : (00975) - 2 - 323825 / 325431 Fax : (00975) - 2 - 326424

INTRODUCTION

HOW MATHEMATICS HAS CHANGED

In Class VI this year, you will learn some new mathematics that Class VI students before you did not learn. Some things are the same, but many things are different. For example, many of the topics you will learn about in geometry and data are new to Class VI students.

You will learn mathematics differently this year. Instead of memorizing and following rules, you will do much more explaining and making sense of the mathematics. When you understand the mathematics, you will find it more interesting and easier to learn.

Your new textbook lets you work on problems about everyday life as well as on problems about Bhutan and the world around you. These problems help you see the value of math.

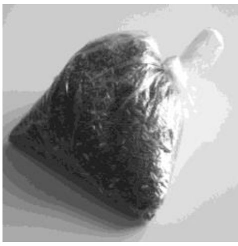
For example:

• One problem in Unit 5 asks you to compare teas made with different amounts of ingredients.

> Dechen and Sonam make their butter tea in different ways:
> • Dechen adds 1 tablespoon of black tea and 2 tablespoons of butter to a cup of milk.
> • Sonam adds 1 tablespoon of black tea and 1 tablespoon of butter to a cup of milk.
> Whose tea has a higher proportion of butter to tea?

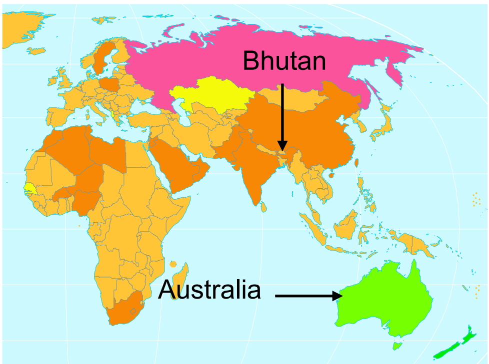
• In Unit 6, you will solve estimation problems involving large numbers:

> Estimate the number of grains of rice in 1 kg.



• In another lesson in Unit 6, you will compare Bhutan to Australia.

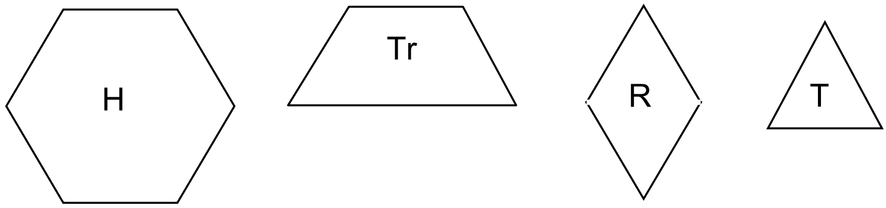
> The area of Bhutan is about 0.0061 of the area of Australia.
>
> The population of Bhutan is about 0.0369 of the population of Australia.
>
> **a)** Which decimal is greater?
>
> **b)** What does that tell you?



Your textbook will often ask you to use objects to learn the math.

For example:

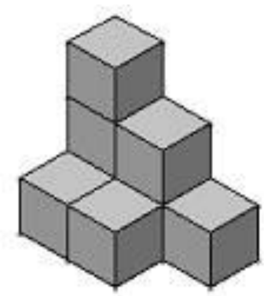
• You will use these shapes to study fractions.



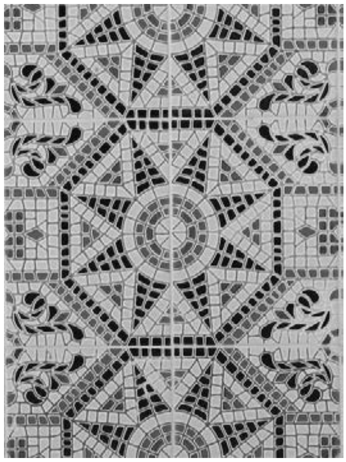
Fraction pieces are used in Unit 1

• You will look for angles in real-world objects and designs.

• You will use linking cubes to measure and estimate volume. You will also use linking cubes to work with isometric drawings



A structure made of linking cubes



Mosaic design

Your textbook will also ask you to explain *why* things are true. It will not be enough if you just say that they are true. For example, you will not only calculate the answer to 4.2 × 3.9, but you will also explain why the answer has to have two decimal places.

You will solve many types of problems and you will be encouraged to use your own way of thinking to solve them.

USING YOUR TEXTBOOK

Each unit has

• a *Getting Started* section

• two or three chapters

• regular lessons and at least one *Explore* lesson

• a *Game*

• a *Connections* activity

• a *Unit Revision*

Reprint 2023

Getting Started

There are two parts to the *Getting Started*. You will complete a *Use What You Know* activity and then you will answer *Skills You Will Need* questions. Both remind you of things you already know that will help you in the unit.

• The *Use What You Know* activity is done with a partner or in a group.

• The *Skills You Will Need* questions help you review skills you will use in the unit. You will usually do these by yourself.

Regular Lessons

• Lessons are numbered #.#.# — the first number tells the unit, the second number is the chapter, and the third number is the lesson in the chapter.

For example, Lesson 4.2.1 is Unit 4, Chapter 2, Lesson 1.

• Each regular lesson is divided into five parts:

- A *Try This* problem or task
- A box that explains the main ideas of the lesson; it is called the exposition
- A question that asks you to think about the *Try This* problem again, using what you have learned in the exposition
- One or more *Examples*
- *Practising and Applying* questions

Try This

• The *Try This* is in a grey box, like this one from Unit 3, lesson 3.1.2 on page 73.

Try This

Lobzang can run 100 m in 12.4 s.

A. About how long would it take him to run 300 m at that speed? Explain how you estimated.

You will solve the *Try This* problem with a partner or in a small group. The math you learn later in the lesson will relate back to this problem.

The Exposition

• The exposition comes after the *Try This*.

• It presents and explains the main ideas of the lesson.

• Important math words are in **bold** text. You will find the definitions of these words in the glossary at the back of this book.

• You are not expected to copy the exposition into your notebook.

Going Back to the Try This

• After the exposition, there is always a question that asks you to think again about the *Try This* problem. You can use the new ideas presented in the exposition to help you answer this question. The example below, from Unit 3, lesson 3.1.2 on page 74, follows an exposition that shows different ways to multiply decimals by whole numbers. In the *Try This*, you were asked to estimate the product of a decimal and whole number (see the Try This shown on page xi). Now you can find an exact product using what you learned in the exposition.

> **B. i)** Exactly how long would it take Lobzang to run 300 m at a speed of 100 m in 12.4 s?
>
> **ii)** How does your exact answer compare to your estimate from **part A**?

Examples

• The *Examples* prepare you for the *Practising and Applying* questions. Each example is a bit different from the others so that you can refer to many models.

• You will work through the examples sometimes on your own, sometimes with another student, and sometimes with your teacher.

• The *Solutions* column shows you what you should write when you solve a problem. The *Thinking* column shows you what you might be thinking as you solve the problem.

• Some examples show you two different solutions to the same problem. The example below from lesson 3.1.3 on page 79 shows two possible ways to calculate 2.2 × 4.15, *Solution 1* and *Solution 2*.

Example 3 Multiplying Decimals in Parts	
Calculate 2.2 × 4.15.	
Solution 1 2.2 = 2 + 0.2 2.2 × 4.15 = (2 × 4.15) + (0.2 × 4.15) 2 × 4.15 = 8.30 0.2 × 4.15 = 0.1 × (2 × 4.15) = 0.1 × 8.30 = 0.830 8.30 + 0.830 = 9.130 2.2 × 4.15 = 9.130	**Thinking**  • I knew that 2.2 groups of 4.15 was 2 groups of 4.15 plus another 0.2 of a group of 4.15, so I calculated them separately and then added them together.

Solution 2	Thinking
1 ~~1~~ 415 × 22 830 + 8300 9130 2.2 × 4.14 is about 2 × 4 = 8. 2.2 × 4.15 = 9.130	 • I multiplied 415 by 22 and then estimated to figure out where the decimal point would be — because 2 × 4 = 8, the decimal must be after the 9 in 9130.

Practising and Applying

• You might work on the *Practising and Applying* questions by yourself, with a partner, or in a group. You can use the exposition and examples to help you.

• The first few questions are similar to the questions in the *Examples* and the exposition.

• The last question helps you think about the most important ideas you have learned in the lesson.

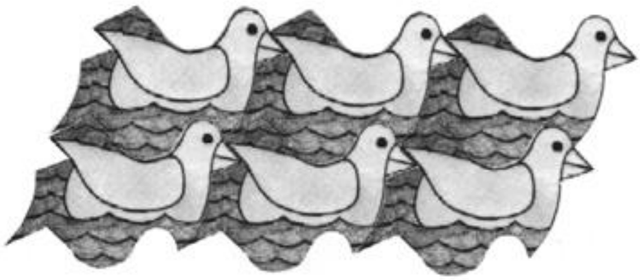
Explore Lessons

• An *Explore* lesson lets you work with a partner or in a small group to investigate some math.

• Your teacher does not tell you about the math in an *Explore* lesson. Instead, you work through the questions and learn your own way.

Connections Activity

• The *Connections* activity is usually something interesting that relates to the math you are learning. For example, in Unit 2, the *Connections* on page 44 is about relating geometric transformations to the creation of art.



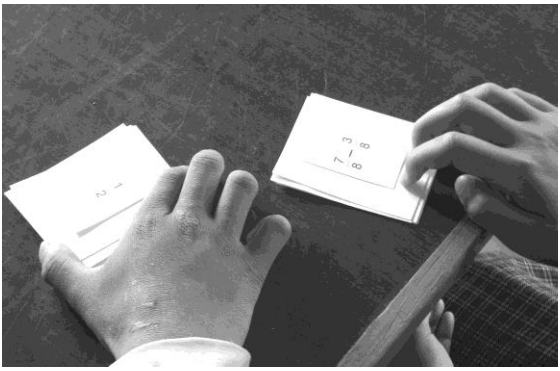
Creating tessellations using transformations

• Every unit has a *Connections* activity.

• You will usually work in pairs or small groups to complete the task or answer the question(s).

Game

• Each unit has at least one *Game*.



• The *Game* is a way to practise skills and concepts from the unit with a partner or in small group.

• The materials you need and the rules for the game are listed in the textbook. Usually the textbook shows a sample game to help you understand the rules.

Fraction Match game in UNIT 1

Unit Revision

• The *Unit Revision* helps you review the lessons in the unit.

• The order of the questions in the *Unit Revision* is usually the same as the order of the lessons in the unit.

• You can work with a partner or by yourself, as your teacher suggests.

Glossary

• At the end of this textbook you will find a glossary of new math words and their definitions. The glossary also contains other important math words from Class V that you need to remember.

• The glossary also has definitions of instructional words such as "explain", "predict", and "estimate". These will help you understand what you are expected to do.

Answers

• You will find answers to most of the numbered questions at the back of the textbook. Answers to questions that ask for explanations (Explain your thinking or How do you know?) are not included in your textbook. Your teacher has those answers.

• Questions with capital letters, such as A or B, do not have answers in the back of the textbook. Your teacher has the answers to these questions.

• If there could be more than one correct answer to a question, the answer will start with *Sample Response*. Even if your answer is different than the answer at the back of the textbook, it may still be correct.

ASSESSING YOUR MATHEMATICAL PERFORMANCE

Forms of Assessment

Your teacher will be checking to see how you are doing in your math learning. Sometimes your teacher will collect information about what you understand or do not understand in order to change the way you are taught. Other times your teacher will collect information in order to give you a mark.

Assessment Criteria

• Your teacher should tell you about what she or he will be checking and how it will be checked.

• The amount of the mark assigned for each unit should relate to the time the class spent on the unit and the importance of the unit.

• Your mark should show how you are doing on skills, applications, concepts, and problem solving.

• Your teacher should tell you whether the mark for a test will be a number such as a percent, a letter grade such as A, B, or C, or a level on a rubric (level 1, 2, 3, or 4). A rubric is a chart that describes criteria for your work, usually in four levels of performance. If a rubric is used, your teacher should let you see the rubric before you start to work on the task.

Determining a Mark or Grade

In determining your overall mark in mathematics, your teacher might use a combination of tests, assignments, projects, performance tasks, exams, interviews, observations, and homework.

THE CLASSROOM ENVIRONMENT

In almost every lesson you will have a chance to work with other students. You should always share your responses, even if they are different from the answers offered by other students. It is only in this way that you will really be engaged in the mathematical thinking.

Pair and Group Work

• There are many reasons why you should work in pairs or groups:

- to have more opportunities to communicate mathematically
- to make it easier for you to discuss an answer you are not sure of
- to see the different mathematical ideas of other students
- to share materials more easily

• Sometimes you might work with the person next to you, but at other times you might be asked to work with particular students.

• When you work in a group, it is important to contribute and to follow your teacher's rules for working in groups. Some sample rules are shown here.

Rules for Group Work

- Make sure you understand all the work produced by the group.
- If you have a question, ask your group members first, before asking your teacher.
- Find a way to work out disagreements without arguing.
- Listen to and help others.
- Make sure everyone is included and encouraged.
- Speak just loudly enough to be heard.

Communication

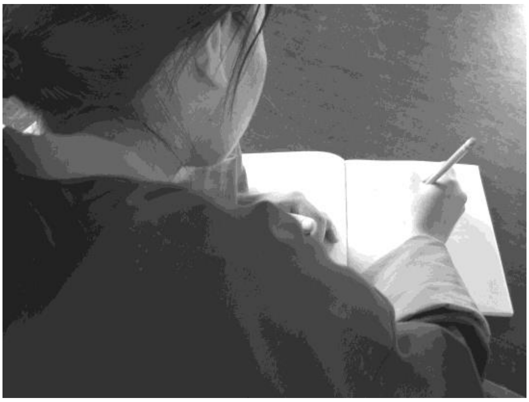
• Many of the questions in the textbook ask you to explain your thinking. Look for instructions like these:

- Explain your thinking.
- Show how you know.
- How do you know?
- How do you know you are right?
- Explain your prediction.
- Explain your estimate.

• The sample *Thinking* in the *Examples* provides a model for mathematical communication.

• One of the ways you communicate mathematically to yourself is by checking your work. Even when a question does not ask you to check your work, you should think about whether your answer makes sense. When you check your work, you should check using a different way than the way you used to find your answer so that you do not make the same error twice.

YOUR NOTEBOOK



• It is valuable for you to have a well-organized, neat notebook to look back at to review the main ideas you have learned. You should do your rough work in this same notebook. Do not do your rough work elsewhere and then waste valuable time copying it neatly into your notebook.

• Your teacher will sometimes show you important points to write down in your notebook. You should also make your own decisions about which ideas to include in your notebook.

UNIT 1 NUMBER RELATIONSHIPS

Getting Started

Use What You Know

Use these digits to answer the questions below.

0	5	2	1	9	8	3

You might find this place value chart helpful.

Millions	Thousands			Ones		
One	**Hundred**	**Ten**	**One**	**Hundred**	**Ten**	**One**

A. i) Make the greatest 7-digit number you can.

ii) How do you know it is not the greatest possible 7-digit number?

B. i) Make the least 7-digit number you can.

ii) Why did you not use 0 as the first digit?

C. i) Make two numbers that are about 6 million apart.

ii) Make two numbers that are about 3000 apart.

D. Create and solve a new problem using the digits above.

Skills You Will Need

1. What digit is in each place of 6,170,209?

a) the millions place

b) the ten thousands place

c) the hundred thousands place

2. What is each missing value?

a) 1 million = ____ thousand

b) 1 million = ____ hundred thousand

3. Write each number in standard form and in expanded form.

a) two hundred thousand, forty-five

b) three million, eight hundred three thousand, fifty-six

c) thirteen hundred thousand, eight hundred seventy

4. Order the numbers from **question 3** from least to greatest.

5. Write the number words for each.

a) 3,140,020 **b)** 309,045

6. Complete each.

a) 4,200,000 = ___ million

b) 3,140,000 = ___ hundred thousand

c) 6,200,000 = 620 ____________

d) 4.5 hundred thousand = ___ million

You might find this place value chart helpful for questions 7 to 9.

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

7. What digit is in each place in 1.234?

a) the hundredths place

b) the thousandths place

8. Write the number words for each.

a) 0.005 **b)** 22.05 **c)** 8.125

9. You can read the number 1.2 in three different ways:
"one and 2 tenths" or "one and 20 hundredths" or "one and 200 thousandths"
How do you know all three ways are correct?

10. List five multiples of each.

a) 5 **b)** 10 **c)** 16

11. List three factors of each.

a) 100 **b)** 55 **c)** 280

Chapter 1 Large Whole Numbers

1.1.1 EXPLORE: Solving Problems With Large Numbers

Fermi problems are named after a famous Italian physicist, Enrico Fermi. They involve large numbers and estimation. It is difficult or impossible to find an exact answer to a Fermi problem.

Here is an example:

Estimate the total time that Class VIII students in Bhutan will spend studying for examinations this year.

First write down the assumptions you need to make

- There are about 9000 students in Class VIII in Bhutan.
- Each student studies for exams twice a year.
- For each exam, students study about 1 h a day for about 70 days, or 70 h.

Solve the problem

70 h × 2 times a year = 140 h

9000 students at 140 h each is 9000 × 140 = 1,260,000 h

Change 1,260,000 h to an amount of time that is easy to understand

There are 24 h in a day, which is about 25 h.

There are 365 days in a year, which is about 350 days.

25 × 350 = 50 × 175, which is about 100 × 87 = 8700 h in 1 year

1,260,000 h ÷ 8700 h is about (900,000 + 450,000) ÷ 9000

(900,000 + 450,000) ÷ 9000 = 100 + 50 = 150 years

Class VIII students in Bhutan will spend about 150 years studying for their exams this year.

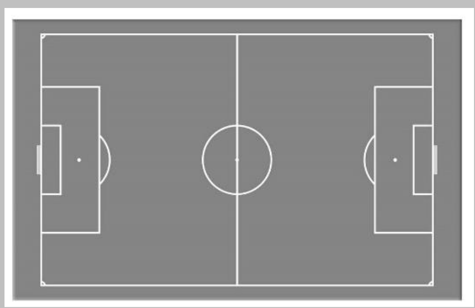
Solve each Fermi problem. Write down your assumptions.

A. Estimate how many kilometres you would walk in 10 million steps.

B. Estimate the number of grains of rice in 1 kg.

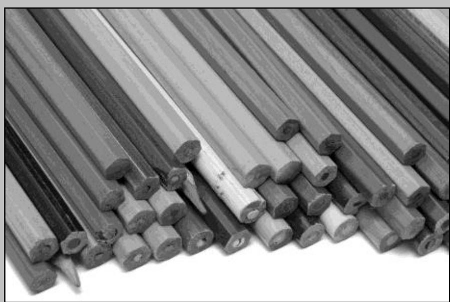
C. Estimate the number of Nu 1 coins it would take to cover this football field.

100 m



60 m

D. Estimate the number of pencils used by all the students in Bhutan in one school year.



1.1.2 Place Value with Large Whole Numbers

Try This

A. There are about 86 thousand seconds in a day. Estimate the number of seconds there are in each amount of time.

i) a week **ii)** a month **iii)** a year

• Numbers are written in groups of three digits to make the numbers easier to understand and read. Each group of three digits is called a **period**.

For example: 1,234,567 is easier to understand than 1234567.

• This place value chart shows the ones, thousands, and millions periods.

Millions period			Thousands period			Ones period		
H	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**
1	2	3	0	1	0	4	2	3

Millions period		
Hundred	**T**en	**O**ne
1	2	3

In the millions period, for example, there are
• hundred millions (H),
• ten millions (T), and
• one millions (O).

The number in the chart above is written 123,010,423. You read it aloud as
"one hundred twenty three million, ten thousand, four hundred twenty-three".

• The column to the left of the hundred millions place is the one billions place.

Billions	Millions			Thousands			Ones		
O	**H**	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**
1	2	0	0	1	0	4	0	3	2

- 1 billion is 1,000,000,000. It is also 1000 million.
- The number in the chart above is written 1,200,104,032. You read it as "one billion, two hundred million, one hundred four thousand, thirty-two".

• A number like 1,130,100,030 is in **standard form**.
• You can write 1,130,100,030 in **expanded form** in different ways:
- 1 billion + 1 hundred million + 3 ten million + 1 hundred thousand + 3 tens
- 1 × 1,000,000,000 + 1 × 100,000,000 + 3 × 10,000,000 + 1 × 100,000 + 3 × 10

• You compare large numbers in the same way that you compare small numbers.

For example,

3,245,100,200 > 123,456,789 because **<u>3</u>**,245,100,200 has billions and 123,456,789 has no billions.

<u>3</u>,245,100,200
123,456,789

3,245,100,200 > 3,235,999,999 because both numbers have 3 billions and 2 hundred millions, but 3,2**<u>4</u>**5,100,200 has **<u>4</u>** ten millions and 3,2**<u>3</u>**5,999,999 has only **<u>3</u>** ten millions.

3,2**<u>4</u>**5,100,200
3,2**<u>3</u>**5,999,999

B. Express each value from **part A** in expanded form in two ways.

Examples

Example 1 Writing Numbers in Standard Form

Write each in standard form.

a) three hundred twenty million, four hundred thousand

b) 3 × 1 hundred million + 5 × 1 hundred thousand + 3 × 1 hundred + 4 ones

c) 4 billion + 3 million + 2 thousand + 5 hundred

Solution

Thinking

I used a place value chart to help me figure out what each number was in standard form.

a) 320,400,000

a) I wrote 320 in the millions period, 400 in the thousands period, and zeros in the ones period.

Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O
3	2	0	4	0	0	0	0	0

b) 300,500,304

b) I knew a 3 went in the hundred millions place,
a 5 went in the hundred thousands place,
a 3 went in the hundreds place of the ones period, and
a 4 went in the ones place of the ones period.

Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O
3	0	0	5	0	0	3	0	4

c) 4,003,002,500	**c)** For 4 billions + 3 millions + 2 thousands + 5 hundreds, I put 4, 3, 2, and 5 where they belonged in the chart and zeros in the other places.

Billions	Millions			Thousands			Ones		
O	**H**	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**
4	0	0	3	0	0	2	5	0	0

Example 2 Writing Numbers in Expanded Form

Write 2 billion, six hundred ten million, twenty thousand, forty in expanded form.

Solution

2 × 1,000,000,000 +
6 × 100,000,000 +
1 × 10,000,000 +
2 × 10,000 +
4 × 10

Thinking

In a place value chart, I wrote
- 2 in the billions period,
- 610 in the millions period,
- 20 in the thousands period, and
- 40 in the ones period.

Billions	Millions			Thousands			Ones		
O	**H**	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**
2	**6**	**1**	0	0	**2**	0	0	**4**	0

Then, for each non-zero digit, I wrote a part of the number in expanded form.

Example 3 Writing Numbers in Standard Form

Write an equivalent expression for each place value expression.

a) 10 ten million **b)** 10,000 hundred

Solution

a)

1 hundred million or 100,000,000

Thinking

- I used a place value chart. There was a 10 in the ten millions place, so I traded it for 1 hundred million.

Millions			Thousands			Ones		
H	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**
	10	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0

[Continued]

Example 3 Writing Numbers in Standard Form [Continued]

Solution

b) 1,000,000
or 1 million

Thinking

b) I used a place value chart.

• I started with 10,000 hundreds and kept trading until I got to 1,000,000, which is 1 million.

Millions			Thousands			Ones		
H	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**
						10,000	0	0
					1000	0	0	0
				100	0	0	0	0
			10	0	0	0	0	0
		1	0	0	0	0	0	0

Practising and Applying

1. Write each in standard form.

a) three hundred two million, fifty-four thousand

b) two billion, fifty three million, eighty-nine

c) six billion, four hundred thousand, five

2. Write each in expanded form in two different ways.

a) 3,045,100,000

b) 1,203,500,000

3. Write each in standard form.

a) 1000 million

b) 10,000 ten thousand

c) 1,000,000 thousand

4. Complete each using a place value position.

a) 1,000,000 is 1000 _______.

b) 1,000,000,000 is 10,000 ______.

5. Order from least to greatest.

- 3.2 billion
- 8,840,230
- 4,235,100,023

6. Which number is greater?

21,243,567 or 21,342,899

Describe how you compared the numbers.

7. Arrange these words in different ways to make five different numbers. Write each number in standard form.

six | million | hundred thousand | two | fifty | four

8. Why is our number system called a base ten place value system?

Reprint 2023

1.1.3 Renaming Numbers

Try This

The population of Thailand was recently reported as about 68 million.

A. i) Write the number 68 million in standard form and in expanded form.
ii) Do you think the population could be exactly 68 million? Explain your thinking.

- In Class V, you learned how to **rename** a number like 3,200,000.
The place value chart below shows why each name makes sense.

3,200,000 = 3.2 million
= 32 hundred thousand
= 320 ten thousand

Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O
		3	2	0	0	0	0	0

- You can also rename numbers that are even larger.

For example, if you write 1,200,000,000 in a place value chart, you can see different ways to rename it.

Billions	Millions			Thousands			Ones		
O	H	T	O	H	T	O	H	T	O
1	2	0	0	0	0	0	0	0	0

1,200,000,000 can be renamed 120 ten million.

1,200,000,000 can be renamed 12 hundred million.

1,200,000,000 can be renamed 1.2 billion.

- Note that we usually rename a large number as a decimal when the number ends in many zeros.

For example:

You might rename 3,200,000,000 as 3.2 billion, but you are not likely to rename 3,200,345,023 as a decimal billion because there would be too many decimal places and it would be hard to read (3.200345023 billion).

B. Why do you think Thailand's population was reported in **part A** as 68 million instead of as a number in standard form?

Examples

Example 1 Renaming a Number from Standard Form

Rename 4,400,000,000 in each form.

a) billions **b)** hundred millions **c)** millions

Solution

a) 4.4 billon

b) 44 hundred million

c) 4400 million

Thinking

• I used a place value chart.

• I placed an arrow to the right of the place value I used to rename the number each time.

Billions	Millions			Thousands			Ones		
O	H	T	O	H	T	O	H	T	O
4	4	0	0	0	0	0	0	0	0

a) b) c)

Example 2 Changing the Place Value Unit for a Number

Rename each.

a) 310 million as billions

b) 4.2 billion as millions

Solution

a) 310 million
= 0.31 billion

Thinking

a) I wrote 310 million in a place value chart.

• I put an arrow to the right of the billions period. I could see the decimal was 0.31.

Billions	Millions			Thousands			Ones		
O	H	T	O	H	T	O	H	T	O
	3	**1**	0	0	0	0	0	0	0

b) 4.2 billion
= 4200 million

b) I wrote 4.2 billion in the chart.

• I put an arrow to the right of the one millions place. I could see that it was 4200 million.

Billions	Millions			Thousands			Ones		
O	H	T	O	H	T	O	H	T	O
4	**2**	**0**	**0**	0	0	0	0	0	0

Example 3 Comparing and Ordering Numbers

Order from least to greatest.

3.4 billion 23 million 252,000,040

Solution

23 million

< 252,000,040

< 3.4 billion

Thinking

- I used a place value chart.
- I wrote each number in standard form and then compared them.

Billions	Millions			Thousands			Ones		
O	H	T	O	H	T	O	H	T	O
3	4	0	0	0	0	0	0	0	0
		2	3	0	0	0	0	0	0
	2	5	2	0	0	0	0	4	0

Practising and Applying

1. Rename 3,450,000,000 as each.

a) _____ billion

b) _____ million

c) _____ ten million

2. Write each in standard form.

a) 4.2 billion

b) 3.14 ten million

c) 58 hundred million

d) 123 ten thousand

3. Order from least to greatest.

- 3.2 billion
- 123 ten million
- 3134 million
- 58 hundred million

4. Complete in three or more ways.

31.2 ________ = 312 ________

5. Karma writes the number 0.34 million in expanded form. How many parts does he add? How do you know?

6. Computer memory can be reported in different ways:

- in kilobytes, KB (1000 bytes)
- in megabytes, MB (1 million bytes)
- in gigabytes, GB (1 billion bytes)

Copy and complete.

a) 32 GB = ____ MB

b) 1412 MB = ____ GB

c) 68.2 GB = ____ KB

Computer hard drive

7. About how many whole numbers are between 0.38 billion and 384 million?

8. Why might it be useful to rename 430,000,000 as a decimal billion?

Chapter 2 Decimals and Integers

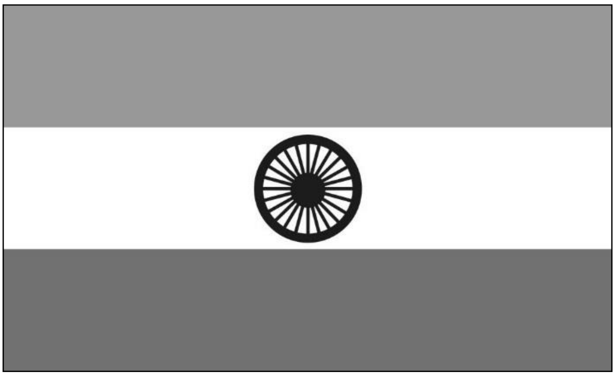
1.2.1 Place Value with Decimals

Try This

The population of India is about 1.3 billion. The population of the city of Kolkata is about 5.1 million.

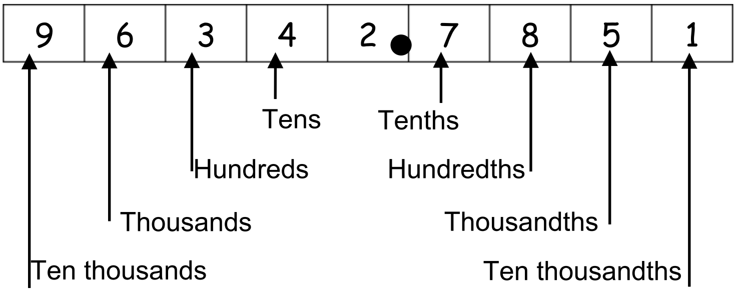
A. i) Find 0.001 (or $\frac{1}{1000}$) of the population of India.

ii) Compare that number to the population of Kolkata.



• The number 96,342.7851 has two parts, separated by a decimal point. The whole number part is 96,342 and the decimal part is 7851 ten thousandths.

• If you think of the ones place and decimal point together as a mirror, you can see that each place to the right matches a place to the left:



• Each place has $\frac{1}{10}$ the value of the place to its left.

• You can read a decimal more easily if you use an equivalent fraction.

For example:

$1.0003 = 1\frac{3}{10{,}000}$ = 1 and 3 ten thousandths, which is read as "one and three ten thousandths".

$1.3424 = 1\frac{3424}{10{,}000}$ = 1 and 3424 ten thousandths, which is read as "one and three thousand, four hundred, twenty-four ten thousandths".

B. A city in India has about 0.0001 (or $\frac{1}{10{,}000}$) of the population of India. What is the city's population?

Examples

Example Reading Decimals

Write each as a fraction or mixed number. Tell how you would read it.

a) 0.0235 **b)** 4.005

Solution

a) $0.0235 = \frac{235}{10,000}$ = 235 ten thousandths, which is read as "two hundred thirty-five ten thousandths".

b) $4.005 = 4\frac{5}{1000}$ = 4 and 5 thousandths, which is read as "four and five thousandths".

Thinking

a) I knew that 4 decimal places meant ten thousandths.

• The equivalent fraction helped me read the number.

b) I knew that 3 decimal places meant thousandths.

• The equivalent mixed number helped me read the number.

Practising and Applying

You might find this decimal place value chart helpful.

Tens	Ones	Tenths	Hundredths	Thousandths	Ten thousandths

1. Which digit is in each place of 3.1245?

a) the thousandths place

b) the ten thousandths place

2. Write each as a decimal.

a) 60 ten thousandths

b) 33 ten thousandths

c) 4203 ten thousandths

3. Copy and complete.

a) 0.0001 = _____ hundredths

b) 0.1 = _____ ten thousandths

4. a) Does 0.800 = 0.8000? How do you know?

b) How does knowing that 0.800 = 0.8000 help you read 0.800 in two different ways?

5. Write how you would read each decimal. If there is more than one way to write it, write all the ways.

a) 1.2300 **b)** 0.4356

c) 1.9802 **d)** 12.001

6. **a)** Nine people in Thimphu are about 0.0001 of its population. Estimate Thimphu's population.

b) Nine people in Haa are about 0.0008 of its population. How does this show that the population of Haa is less than the population of Thimphu?

7. What place do you think is to the right of the ten thousandths place in a place value chart? Why does this make sense?

1.2.2 Comparing and Ordering Decimals

Try This

Each day is about 0.003 of a year.

Each hour is about 0.0001 of a year.

A. Which decimal represents the greater amount of time?
How do you know?

You can compare decimals in different ways.

• If two decimals have whole number parts, compare the whole numbers to decide which is greater.

For example: **3**.25 > **1**.98 since 3 > 1

• If the whole number parts are equal or zero, begin by comparing the tenths place, and then compare the places to the right if necessary.

For example:

0.**2**15 > 0.**1**49, since **2** tenths > **1** tenth

0.2**1**5 > 0.2**0**5, since both have 2 tenths, and **1** hundredth > **0** hundredths

• You can write both decimals using the same place value unit and then compare them. A place value chart can help with this.

For example, this chart compares 0.2134 and 0.147.

Ones	Tenths	Hundredths	Thousandths	Ten thousandths
0	2	1	3	4
0	1	4	7	0

0.2134 is 2134 ten thousandths

0.147 = 0.1470, which is 1470 ten thousandths

2134 ten thousandths > 1470 ten thousandths since 2134 > 1470

So, 0.2134 > 0.147

B. Use place value to show which decimal in **part A** is greater and why. You can use a place value chart to help you.

Examples

Example Ordering Decimals	
Order from greatest to least. 3.1456 3.21 0.8568 0.9342	
Solution 3.21 > 3.1456 > 0.9342 > 0.8568	**Thinking** • **3**.21 and **3**.1456 are both greater than **0**.9342 and **0**.8568 because **3** > **0**. • **3.2**1 > **3.1**456 since **3** and **2** tenths > **3** and **1** tenth. • 0.**9**342 > 0.**8**568 because 0.**9**342 is greater than **9** tenths and 0.**8**568 is less than **9** tenths.

Practising and Applying

1. Order from least to greatest.

a) 0.1234; 1.2398; 0.3578; 0.92

b) 3.5764; 3.21514; 3.33; 3.14578

2. List five decimals that are less than 1 but greater than 0.9971.

3. List five decimals that are less than 0.0021.

4. Is 0.1234 > 0.0034? Use two different ways to explain how you know.

5. Order from least to greatest.
- 43 hundredths
- 26 ten thousandths
- 512 thousandths

6. Explain how you know that 0.04□□ > 0.012□ no matter what digit is in each □.

7. The area of Macau is about 0.001 of the area of India.

The area of Bhutan is about 0.0143 of the area of India.

a) Is Macau or Bhutan bigger?

b) About how many times as big?

The flag of Macau

8. How is comparing decimals like comparing whole numbers? How is it different?

1.2.3 Introducing Integers

Try This

A. Which temperature is colder, –4º or –6º? Explain your thinking.

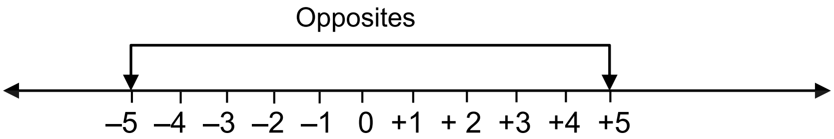
• Sometimes a temperature is reported using a **negative** number, for example, "negative 5 degrees". A negative temperature is below 0 degrees.

• A negative number is below or less than zero. Negative numbers belong to the set of numbers called **integers**, which also includes whole numbers.

• Each negative integer is the **opposite** of a whole number. The whole numbers, not including zero, are called the **positive** integers.

For example:

The positive integer 5, or +5, is opposite to the negative integer 5, or –5.



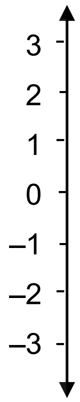
Opposites are equally far from 0 on either side of 0.

• Positive integers can be written with or without the "+" sign.

For example: +5 = 5

• Zero is its own opposite. There is no –0 because –0 = +0.

• Integers are often modelled on a **horizontal** number line like the line shown above, but a number line can also be **vertical**, as shown on the right.



• Integers that are farther left on a horizontal number line or lower down on a vertical number line are less.

For example:

–5 < –3 since –5 is left of –3.

OR

–5 < –3 since –5 is below –3.

B. How can you use a number line to figure out which temperature in **part A** is less?

Examples

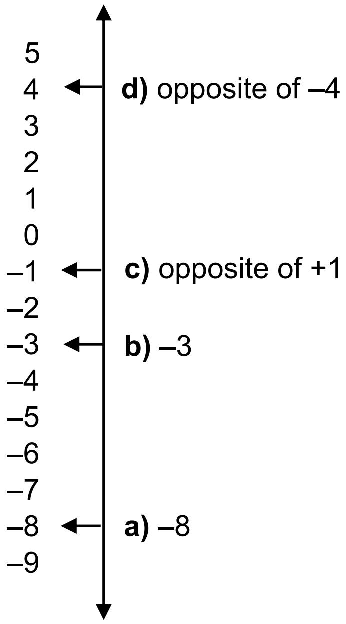
Example 1 Locating an Integer on a Number Line

Place each integer on a number line.

a) –8 **b)** –3

c) the opposite of +1 **d)** the opposite of –4

Solution



Thinking

• I sketched a number line from +5 to –9 because the greatest number was +4 (the opposite of –4) and the least number was –8.

• I knew the numbers went in the opposite order on either side of 0.

• I also knew that

a) –8 is 8 units down from 0.

b) –3 is 3 units down from 0.

c) the opposite of +1 is –1 and –1 is 1 unit down from 0.

d) the opposite of –4 is +4, or 4, and 4 is 4 units up from 0.

Example 2 Describing Situations Involving Integers

What integer can you use to describe each situation?

a) The temperature has fallen 8 degrees from 0°.

b) Bhagi has a debt of Nu 400.

c) A village in the South Pacific is 30 km below sea level.

d) It is 3 min before lunch time.

Solution	Thinking
a) –8	**a)** The temperature is 8 below zero, or –8.
b) –400	**b)** A debt means you owe money, which is like having less than Nu 0, or a negative amount of money.
c) –30	**c)** Sea level is 0 km so 30 km below sea level is negative 30.
d) –3	**d)** If you think of lunch time as 0, the time before lunch is negative.

Practising and Applying

1. Sketch a number line and mark each integer.

a) –6

b) –12

c) +2

d) –15

2. Name the integer that is opposite to each.

a) +3

b) –2

c) –5

d) 0

3. Two different integers are each 16 units away from 0 on a number line. What are the integers?

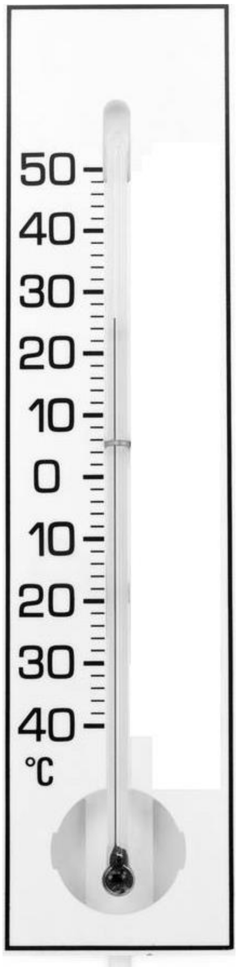
4. Describe three different things that –4 might represent.

5. The temperature was 0º and then it changed as described below. List the temperature for each. You might sketch a vertical number line to help you.

a) It went down 2°.

b) Then it went down another 1°.

c) Then it went up 4°.



6. Name two different integers that are 4 units away from –3.

7. An integer is less than –4 but greater than –8. What could it be?

8. Why is there the same number of negative integers as positive integers?

Reprint 2023

Chapter 3 Number Theory

1.3.1 Prime Numbers

Try This

A. i) How many different rectangles with each area can you draw on grid paper?

2 square units	4 square units
3 square units	6 square units
5 square units	8 square units
7 square units	9 square units

ii) How are your answers for 2, 3, 5, and 7 square units different from your answers for 4, 6, 8, and 9 square units?

• Some whole numbers can be written as a product of **factors** in more than one way.

For example:

6 = 1 × 6 and 6 = 2 × 3, so 1, 2, 3, and 6 are all factors of 6. This means that 6 is a **multiple** of 1, 2, 3, and 6.

• Some whole numbers can only be written as a product of two factors, 1 and themselves.

For example:

13 = 1 × 13, so only 1 and 13 are factors of 13. This means 13 is a multiple of only 1 and 13.

• A number that is multiple of only 1 and itself, like 13, is called a **prime number**. The first four prime numbers are 2, 3, 5, and 7.

• You can form every non-prime whole number by multiplying prime numbers, or **prime factors**.

For example:

To find the prime factors that make up 60, start by finding factors you know, like 3 × 20, and then break up those factors until you have only factors that are prime numbers.

60 = 3 × 20

3 × 4 × 5

3 × 2 × 2 × 5

60 = 3 × 2 × 2 × 5

• A prime number may be a small number like 2 or 3, or a large number like 101 or 45,533.

• 1 is not a prime number because, even though it is only a product of 1 and itself, the two factors are the same.

B. i) Which areas in **part A** are prime numbers?

ii) How did making rectangles for each area in **part A** help you determine if the area was a prime number?

Examples

Example 1 Testing Numbers to See if They are Prime

Which of these numbers is a prime number?

420 287 415 89

Solution

89

Thinking

- **420** = 10 × 42, so it can't be prime.
- I thought of **287** as 280 + 7. Both 280 and 7 are multiples of 7, so 287 is a multiple of 7. It can't be a prime number.
- Numbers that end in 5 are multiples of 5, so **415** can't be a prime number.
- I tried dividing **89** by different numbers to see if I could find factors other than 1 and 89 but I couldn't.

Example 2 Using a Geometric Model to See if a Number is Prime

Which numbers are prime numbers? Explain your thinking.

47 49 51 53

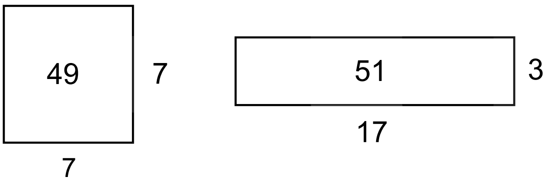
Solution

47 and 53 are prime numbers since the only possible rectangles are 1 by 47 and 1 by 53.

49 and 51 are not prime numbers since there was more than one rectangle for each.

Thinking

- I thought of each number as the area of a rectangle with a whole number length and width.
- I knew that if only one rectangle could represent the area, the number was a prime number.
- I knew each area had a rectangle with a width of 1, so I looked for other rectangles.
- I could only make one rectangle each for 47 and 53, so I knew they were prime numbers.
- I made a 7-by-7 square for 49 and a 3-by-17 rectangle for 51, so I knew 49 and 51 weren't prime numbers.



Practising and Applying

1. List all the prime numbers between 0 and 100.

2. Why is 2 the only even prime number?

3. Why is 5 the only prime number that is a multiple of 5?

4. How close together can two prime numbers be if they are both greater than 5? Why can they not be closer?

5. Both 13 and 31 are prime numbers. Find another pair of prime numbers that have reversed digits.

6. Choose four numbers that are not prime numbers. Show how each can be written as a product of prime factors.

7. How can you test to see if 423 is a prime number?

CONNECTIONS: The Sieve of Eratosthenes

The Greek mathematician Eratosthenes found a way to figure out which numbers are prime numbers. You can use a 100 chart to find them.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

He crossed off every multiple of 2, but not 2.
He crossed off every multiple of 3, but not 3.
He crossed off every multiple of 5, but not 5.
He crossed off every multiple of 7, but not 7.
He said the prime numbers were the numbers that were not crossed off.

1. Test Eratosthenes' method. Are the resulting numbers prime numbers?

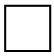
2. Why did Eratosthenes stop at multiples of 7?

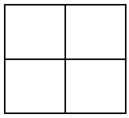
3. How can you find the prime numbers up to 200?

Reprint 2023

1.3.2 EXPLORE: Square and Triangular Numbers

• A number that can be modelled as a square is called a **square number**.





X X X
X X X
X X X

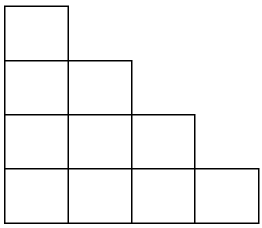
X X X X
X X X X
X X X X
X X X X

1, 4, 9, and 16 are the first four square numbers.

4 is the 2nd square number since there are 2 rows in the square.

16 is the 4th square number since there are 4 rows in the square.

• A number that can be modelled as a triangle where each row is one more than the row above it, starting with 1, is called a **triangular number**.



X
X X
X X X
X X X X
X X X X X
X X X X X X
X X X X X X X

10 and 28 are triangular numbers

10 is the 4th triangular number since there are 4 rows in the triangle.

28 is the 7th triangular number since there are 7 rows in the triangle.

A. i) List all the square numbers from 1 to 200.

ii) Order the numbers from **part i)**. What do you notice about how far apart square numbers are?

B. How do you know the product of 15 × 15 is a square number?

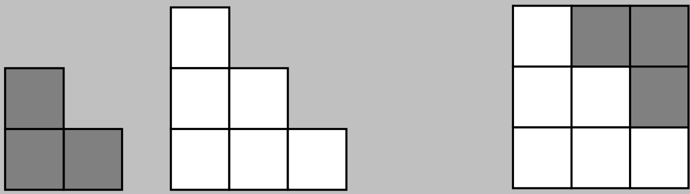
C. List all the triangular numbers from 1 to 100.

D. i) Draw a diagram to show the 10th triangular number.

ii) How can you tell from the diagram in **part i)** that the 10th triangular number is equal to 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10?

E. The 2nd and 3rd triangular numbers fit together to make a rectangle.

For example:



3   6   3 + 6 = 9

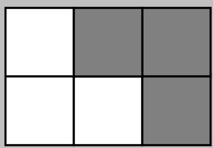
i) Name three other pairs of triangular numbers that can be put together to make rectangles.

ii) Use the idea above to figure out the sum of the 19th and 20th triangular numbers.

F. If you double a triangular number, you get a rectangle that has a width and length that are 1 unit apart.

For example:

3 + 3 = 6



The rectangle is 2 by 3.

i) Show this using two other triangular numbers.

ii) How does the diagram above show why the 2nd triangular number is half of 2 × 3?

iii) Use the idea in **part ii)** to figure out the value of the 20th triangular number.

CONNECTIONS: Triangular Numbers as Products

Look at the pattern below:

1st triangular number	$1 = 1 \times 1$
2nd triangular number	$3 = 1 \times 3$
3rd triangular number	$6 = 2 \times 3$
4th triangular number	$10 = 2 \times 5$
5th triangular number	$15 = 3 \times 5$
6th triangular number	$21 = 3 \times 7$
7th triangular number	$28 = 4 \times 7$

Use the pattern to predict the 20th triangular number. Explain your prediction.

1.3.3 EXPLORE: Factors

• A factor of a whole number is a whole number that divides into another whole number with no remainder.

For example:

4 is a factor of 12 because 12 ÷ 4 = 3 with no remainder.

4 is not a factor of 13 because 13 ÷ 4 = 3 R 1.

• Since 4 is a factor of 12, you can show 12 items in equal groups of 4. You cannot show 13 items in equal groups of 4.

X X X X X X X X X X X X	12 ÷ 4 = 3	X X X X X X X X X X X X X	13 ÷ 4 = 3 R 1

• You can also describe factors by using multiples. If a number is a multiple of another number, the second number is a factor of the first number.

For example:

36 is a multiple of 18 since 36 = 2 × 18, so 18 is a factor of 36.

• To find all the factors of a number, begin dividing the number by 1, then by 2, then by 3, and so on, to see which factors divide without a remainder.

A. i) Use whole numbers to complete each as many ways as you can.

45 = ☐ × ☐ 36 = ☐ × ☐ 60 = ☐ × ☐

ii) All the numbers you used in **part i)** are factors of 45, 36, or 60. How do you know this is true?

iii) What are the least and greatest factors of 45? of 36? of 60?

iv) Predict the least and greatest factors of 80.

B. i) The second greatest factor of 60 is ☐ in 2 × ☐ = 60. Why is that?

ii) The second greatest factor of 45 is ☐ in 3 × ☐ = 45, not 2 × ☐ = 45. How is 45 different from 60?

iii) Predict the second greatest factor of each. Explain your prediction and then test it.

120 40 125

C. Does a greater number have more factors than a lesser number? Explain your thinking. Use examples to help you explain.

Reprint 2023

GAME: Down to Prime

This game is for 2 to 4 players. You need two dice.

Play several rounds. For each round, take turns.

Do this on your turn:

• Roll the dice and create a 2-digit number.

• Subtract any of the number's factors except the number itself.

• Use the difference as the number to start with on your next turn.

The player who gets a difference that is a prime number in fewer subtractions wins 1 point.

• Throw the dice again for each round.

The winner is the first player to get 5 points.

Here is a sample round:

Player A

First turn

He rolls a 3 and 4, and creates the number 34.

He can subtract any factor of 34: 1, 2, or 17, but not 34.

He chooses to subtract 34 – 17 = 17 and ends up with the prime number 17 in one turn.

Player B

First turn

She rolls a 6 and 1, and creates the number 16.

She can subtract any factor of 16: 1, 2, 4, or 8, but not 16.

She chooses to subtract 16 – 8 = 8.

Second turn

She starts with 8 from the last turn.

She can subtract any factor of 8: 1, 2, or 4, but not 8.

She subtracts 8 – 1 = 7 and ends up with the prime number 7 in two turns.

Player A gets 1 point for the round because he got a prime number. in fewer turns.

1.3.4 Common Factors

Try This

Yuden and Samten have a piece of cloth that measures 90 cm by 60 cm. They want to cut congruent squares from the cloth, and have no cloth left over.

A. What size squares can they cut?

• If Number A is a factor of Number B, you can write Number B as a product of Number A and another number.

For example, since 4 is a factor of 8, then 8 = 4 × 2.

• If a number is a factor of two or more other numbers, it is called a **common factor** of those numbers.

For example, 18 and 12 have the common factors 1, 2, 3, and 6.

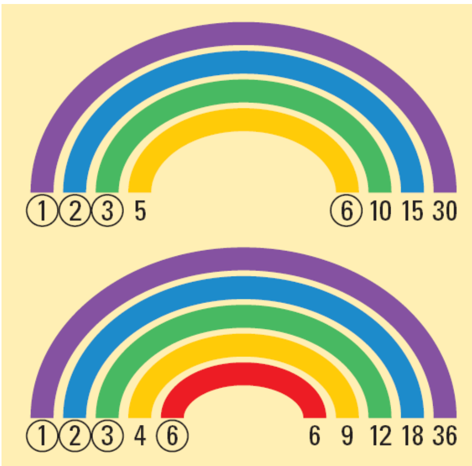
• One way to determine the common factor of two numbers is to make a list of the factors for each and then look for the same numbers on both lists.

• You can organize a list of factors by using a factor rainbow.

For example, for 30, you start at each end, 1 and 30, and work inwards going back and forth: 2 and 15, then 3 and 10, and finally 5 and 6.

Factors of 30:

Factors of 36:



The rainbow curves connect matching factors. This helps list the factors in an organized way.

Once you have both lists, you can circle the factors that are on both lists.

The numbers that appear on both lists are 1, 2, 3, and 6. These are the common factors of 30 and 36.

B. How is the problem you solved in **part A** related to common factors?

Examples

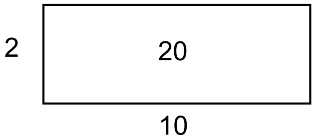
Example 1 Using a Geometric Model to Determine Common Factors

Find the common factors of 12 and 20. Explain what you did.

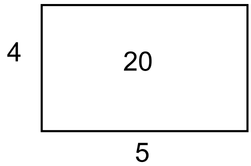
Solution

Factors of 20

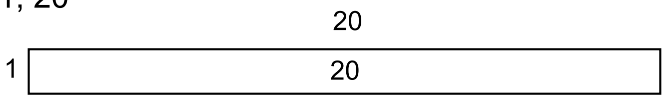
2, 10



4, 5



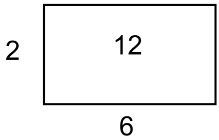
1, 20



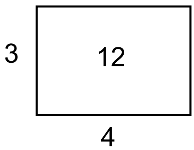
• I could not make any more rectangles for 20.

Factors of 12

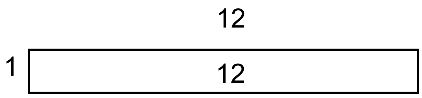
2, 6



3, 4



1, 12



• I could not make any more rectangles for 12.

The common side lengths were 1, 2, and 4.
The common factors are 1, 2, and 4.

Thinking

• I knew I could find factors of 20 by making rectangles with an area of 20 and whole number side lengths. The side lengths are factors.

• I did the same thing to find factors of 12.

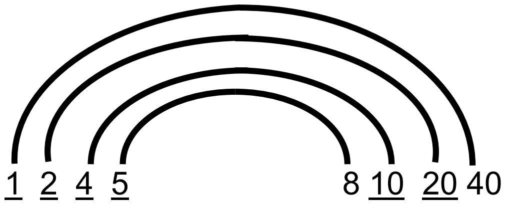
• To find common factors of 12 and 20, I looked for rectangles that had the same side length.

Example 2 Listing Factors to Find Common Factors

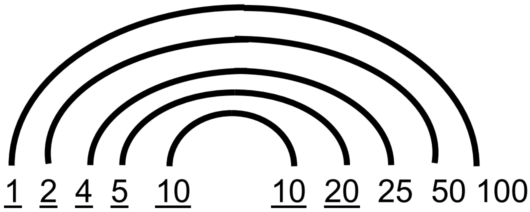
Find the common factors of 40 and 100. Show your work.

Solution

Factors of 40: 1, 2, 4, 5, 8, 10, 20, and 40



Factors of 100: 1, 2, 4, 5, 10, 20, 25, 50, and 100



Common factors of 40 and 100: 1, 2, 4, 5, 10, 20

Thinking

- I used factor rainbows to show the factors of 40 and 100.
- I checked to see if each factor of 40 was also a factor of 100.
- I underlined the factors that were common to both lists.

Practising and Applying

1. Name a common factor that is greater than 1 for each pair.

a) 18 and 20 **b)** 20 and 40

c) 36 and 40 **d)** 15 and 21

2. Use rectangles to show that 3 is a common factor of 39 and 42.

3. How do you know that 2 is not a common factor of 53 and 58?

4. 3 and 12 are common factors of the numbers □ and ∆. What other numbers also have to be common factors of □ and ∆?

5. 48 students sat in 54 chairs arranged in equal rows. The same number of students sat in each row. How might the chairs have been arranged?

6. A rectangular prism block measures 48-by-16-by-24. It is cut into congruent cubes with no wood left over. What are the possible edge lengths of each cube?

7. Decide whether each statement is true or false and explain why.

a) Common factors of two even numbers are always even.

b) Common factors of two odd numbers are always odd.

8. 3 is not a common factor of a pair of numbers. What other numbers cannot be common factors for the pair? Explain your thinking.

9. Is it possible for a pair of low numbers to have more common factors than a pair of greater numbers? Explain using examples.

UNIT 1 Revision

1. Write each in standard form.

a) six billion, twenty-two million, four hundred three thousand

b) three hundred eight million, eighty-seven thousand, eighty-six

c) two billion, one hundred three million, seventeen

2. Write each in expanded form in two ways.

a) 4,200,146,100

b) 356,100,200

3. Rearrange these words to make three different numbers. Write each number in standard form.

five	thousand
hundred	two
twenty	million

4. Write each in standard form.

a) 0.8 billion

b) 23.2 hundred million

c) 62 ten thousand

d) 57 hundred million

5. Order from least to greatest.

- 0.9 billion
- 28 ten million
- 1001 million
- 1,002,003 thousand

6. Which digit is in each place of the number 9.0234?

a) the thousandths place

b) the tenths place

c) the ten thousandths place

7. Write each as a decimal.

a) 54 ten thousandths

b) 65 thousandths

c) 650 ten thousandths

d) 324 ten thousandths

8. Write a number to match each description.

a) 6 in the thousandths place and 1 in the ten thousandths place

b) 8 in the ten thousandths place and 2 in the hundredths place

9. Describe two different ways to read each decimal.

a) 3.012

b) 4.123

c) 4.1

d) 3.0040

10. Does 50 cm = 0.0005 km? How do you know?

11. Is 1.2345 < 1.236 890? Use two different ways to explain how you know.

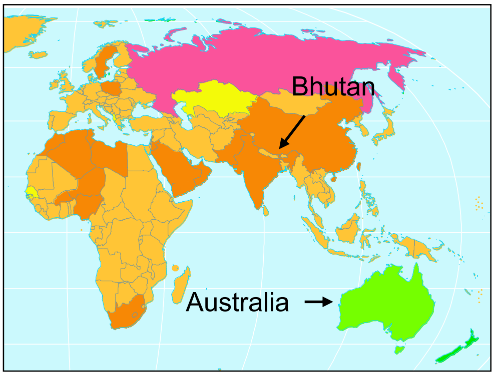
Reprint 2023

12. The area of Bhutan is about 0.0061 of the area of Australia.

The population of Bhutan is about 0.0369 of the population of Australia.

a) Which decimal is greater?

b) What does that tell you?



13. Order from least to greatest.

- 1234 thousandths
- 891 ten thousandths
- 36 hundredths

14. Light travels about 3000 km in 0.01 seconds. How far does light travel in 0.0001 seconds?

15. Draw a number line and mark each integer on the line.

a) –2

b) –8

c) +7

d) –5

16. What integer is opposite to each?

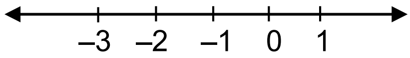
a) –6

b) +12

c) –9

d) +8

17. Name an integer that is twice as far from –3 as it is from –1.



18. An integer has an opposite that is between –12 and –9. What could the integer be?

19. The number ▲ is a multiple of 4. How do you know that a number that is 20 greater (▲ + 20) is not a prime number?

20. Use examples to show that each statement is true.

a) The sum of two square numbers is sometimes but not always a square number.

b) The sum of two triangular numbers is sometimes but not always a square number.

21. Find one common factor other than 1 for each pair of numbers.

a) 16 and 26 **b)** 18 and 36

c) 35 and 25 **d)** 150 and 400

22. A quilt that is 180 cm long and 80 cm wide is made up of congruent squares. How big could the squares be?

23. How many common factors do a number and its triple have? Explain your thinking.

UNIT 2 FRACTIONS AND DECIMALS

Getting Started

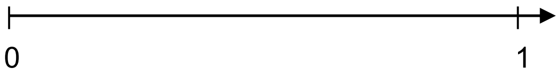
Use What You Know

A. What fraction of this group of four boys is younger than 4 years old?

B. Sketch three pictures that show the fraction in **part A**.

C. Copy the rectangle below. Use it to draw a picture of the fraction in **part A**. Explain how your picture shows the fraction.

D. Copy the number line below. Mark the fraction in **part A** on the number line.



E. Find two or more examples of the fraction in **part A** in your classroom. Describe each example.

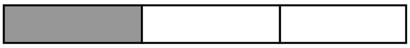
Skills You Will Need

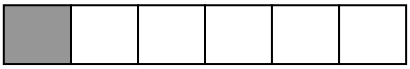
1. Which fraction pairs are equivalent?

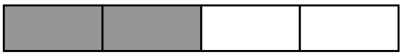
A. $\frac{1}{2}$ and $\frac{4}{8}$ **B.** $\frac{2}{3}$ and $\frac{4}{6}$ **C.** $\frac{3}{5}$ and $\frac{3}{8}$

2. a) What fraction does each picture show?

i) 

ii) 

iii) 

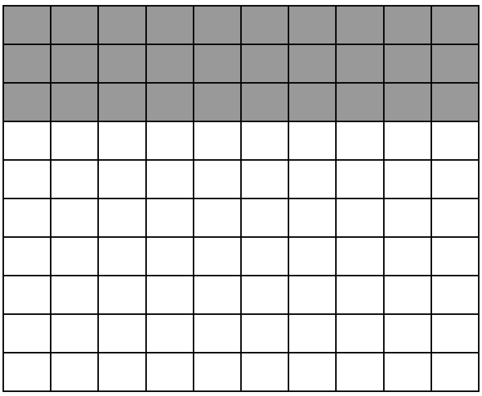
iv) 

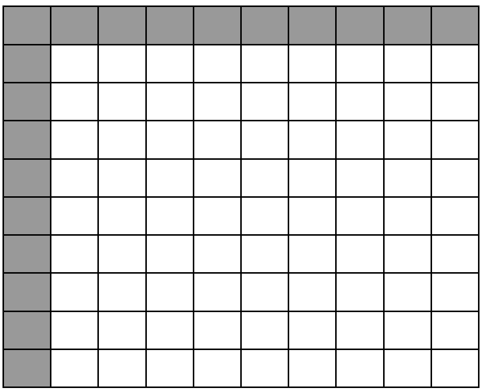
b) Order the four fractions in **part a)** from least to greatest.

3. Draw a picture to show each mixed number.

a) $3\frac{1}{3}$ **b)** $2\frac{1}{4}$ **c)** $1\frac{3}{5}$

4. What decimal does each grid show?

a) 

b) 

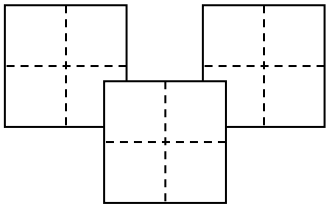
Chapter 1 Relating Fractions

2.1.1 Relating Mixed Numbers to Improper Fractions

Try This

A. Cut three congruent squares of paper into fourths as shown to the right. Use some of the fourths to create two different shapes for each fraction. Sketch each shape.

i) 6 fourths **ii)** 9 fourths **iii)** 11 fourths



• The **denominator** of a fraction tells how many equal parts make a whole. The **numerator** tells how many parts of the whole the fraction represents.

For example:

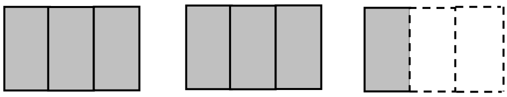
The fraction $\frac{2}{3}$, which is read “two thirds”, means 2 out of 3 parts.

• When the numerator of a fraction is greater than the denominator, it is an **improper fraction**.

For example:

The fraction $\frac{7}{3}$, which is read “seven thirds”, is an improper fraction.

• Since each group of 3 thirds makes a whole, or 1, you can also write $\frac{7}{3}$ as the **mixed number** $2\frac{1}{3}$.



3 thirds 3 thirds 1 third

$$\frac{7}{3} = 2\frac{1}{3}$$

• To write an improper fraction as a mixed number, you can divide the numerator by the denominator. The **quotient** of this division is the number of wholes in the mixed number. The remainder is the fraction.

For example:

$$\frac{7}{3} \rightarrow 7 \div 3 = 2 \text{ R } 1 \rightarrow 2\frac{1}{3}$$

B. For the shapes you created in **part A**, one whole is 4 fourths. Represent each shape as an improper fraction and as a mixed number.

Reprint 2023

Examples

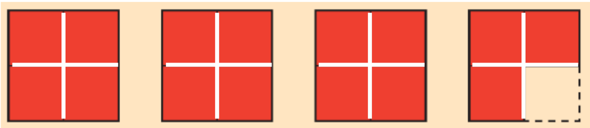
Example 1 Changing Improper Fractions to Mixed Numbers

Write each improper fraction as a mixed number. Show your work.

a) $\frac{15}{4}$ **b)** $\frac{13}{3}$

Solution

a) $\frac{15}{4}$ is 15 fourths



4 fourths + 4 fourths + 4 fourths + 3 fourths

$$\frac{15}{4} = 3\frac{3}{4}$$

b) $13 \div 3 = 4$ R 1, so $\frac{13}{3} = 4\frac{1}{3}$

Thinking

a) I needed to use 4 fourths to make a whole, or 1.

b) 13 thirds made 4 wholes with 1 third left over.

Example 2 Changing Mixed Numbers to Improper Fractions

Write each mixed number as an improper fraction. Show your work.

a) $4\frac{2}{5}$ **b)** $3\frac{1}{6}$

Solution

a) $4\frac{2}{5}$

$= \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{2}{5}$

$= \frac{22}{5}$

b) $3\frac{1}{6} = \frac{3 \times 6 + 1}{6} = \frac{19}{6}$

Thinking

a) The fraction was fifths and I know 5 fifths make a whole.

b) The fraction part was sixths, so I multiplied 3 × 6 to get the number of sixths in 3 wholes. Then I added 1 more sixth.

Example 3 Finding the Whole Number Part of a Mixed Number

Which improper fractions below can you write as mixed numbers where the whole number part is 4? Show your work.

A. $\frac{20}{6}$ **B.** $\frac{9}{2}$ **C.** $\frac{14}{3}$ **D.** $\frac{15}{6}$

Solution

A. $\frac{20}{6} \rightarrow 20 \div 6 = 3$ R 2

B. $\frac{9}{2} \rightarrow 9 \div 2 = \underline{\mathbf{4}}$ R 1

C. $\frac{14}{3} \rightarrow 14 \div 3 = \underline{\mathbf{4}}$ R 2

D. $\frac{15}{6} \rightarrow 15 \div 6 = 2$ R 3

B and C have a whole number part of 4.

Thinking

- I changed each improper fraction to a mixed number by dividing the numerator by the denominator.

Practising and Applying

1. Write each improper fraction as a mixed number.

a) $\frac{13}{6}$ **b)** $\frac{17}{2}$ **c)** $\frac{23}{3}$

2. Write each mixed number as an improper fraction.

a) $3\frac{1}{2}$ **b)** $4\frac{3}{4}$ **c)** $6\frac{2}{5}$

3. a) Each improper fraction below can be written as a mixed number where the whole number part is 5. What could each numerator be?

i) $\frac{?}{5}$ **ii)** $\frac{?}{8}$ **iii)** $\frac{?}{10}$

b) Show your work for one of the fractions in **part a)**.

4. Which number is greater in each pair? How do you know?

a) $\frac{21}{4}$ or $5\frac{3}{4}$ **b)** $3\frac{2}{3}$ or $\frac{24}{6}$

5. A full plate has 6 momos on it.

a) Write an improper fraction that describes 4 full plates and a plate with fewer than 6 momos on it.
Find more than one answer.

b) Write a mixed number that describes several full plates and a plate with 4 momos on it.
Find more than one answer.

6. An improper fraction is written as $4\frac{?}{3}$. What could the numerator be? Explain your thinking.

7. Why does writing an improper fraction as a mixed number help you understand the size of the number?

2.1.2 Comparing and Ordering Fractions

Try This

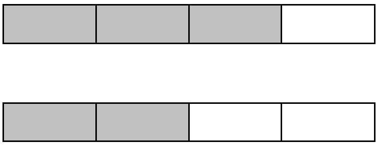
Sonam answered 20 questions correctly out of 30 on one quiz. Then he answered 6 questions correctly out of 10 on a second quiz.

A. i) On which quiz did he get a better mark?
ii) How do you know?

• Fractions that have the same denominator or the same numerator can easily be compared and ordered. For example:

Same denominator

$\frac{3}{4} > \frac{2}{4}$ since 3 fourths of a whole is more than 2 fourths of the same whole.



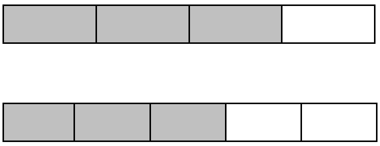
$\frac{3}{4}$

$\frac{2}{4}$

When the denominator is the same, the wholes are divided into the same number of parts. A greater numerator means more parts. So we are comparing 3 parts to 2 parts of the same size.

Same numerator

$\frac{3}{4} > \frac{3}{5}$ since 3 fourths of a whole is more than 3 fifths of the same whole.



$\frac{3}{4}$

$\frac{3}{5}$

When the denominator is less, the whole is divided into fewer parts, so each part is bigger. That means we are comparing 3 big parts to 3 smaller parts.

• You can also compare and order fractions by relating them to 0, $\frac{1}{2}$, or 1.

For example, to compare $\frac{1}{6}$ and $\frac{5}{7}$:

$\frac{1}{6}$ is less than $\frac{1}{2}$ (since 1 small part is less than 1 bigger part).

$\frac{5}{7}$ is more than $\frac{1}{2}$ (since 5 out of 7 is more than half of 7).

That means $\frac{5}{7} > \frac{1}{6}$.

• Another way to compare and order fractions is with **equivalent decimals**. This makes sense when the denominators are 10, 100, or numbers like 4, 5, 20, or 50 that fit evenly into 10 or 100.

For example, to compare $\frac{7}{10}$ and $\frac{13}{100}$: $\frac{7}{10}$ = 0.7 and $\frac{13}{100}$ = 0.13

0.<u>**7**</u> has 7 tenths, but 0.<u>**1**</u>3 has only 1 tenth, so 0.7 > 0.13 and $\frac{7}{10} > \frac{13}{100}$.

• Finally, you can compare and order fractions using **equivalent fractions** that have the same numerator or the same denominator.

For example, here is how to order $\frac{2}{3}$, $\frac{5}{8}$, and $\frac{3}{4}$:

Create equivalent fractions with a denominator of 24:

$\frac{2}{3} = \frac{8 \times 2}{8 \times 3} = \frac{16}{24}$ $\frac{5}{8} = \frac{3 \times 5}{3 \times 8} = \frac{15}{24}$ $\frac{3}{4} = \frac{6 \times 3}{6 \times 4} = \frac{18}{24}$

18 > 16 > 15, so $\frac{18}{24} > \frac{16}{24} > \frac{15}{24}$, and $\frac{3}{4} > \frac{2}{3} > \frac{5}{8}$.

B. i) What two fractions represent Sonam's quiz results in **part A**?
ii) Describe how you would compare the fractions.

Examples

Example 1 Using Equivalent Fractions With Same Numerators

Order from least to greatest. $\frac{3}{7}$ $\frac{9}{10}$ $\frac{6}{8}$ $\frac{2}{3}$

Solution	Thinking
$\frac{3}{7} = \frac{6 \times 3}{6 \times 7} = \frac{18}{42}$ $\frac{9}{10} = \frac{2 \times 9}{2 \times 10} = \frac{18}{20}$ $\frac{6}{8} = \frac{3 \times 6}{3 \times 8} = \frac{18}{24}$ $\frac{2}{3} = \frac{9 \times 2}{9 \times 3} = \frac{18}{27}$ $\frac{18}{42} < \frac{18}{27} < \frac{18}{24} < \frac{18}{20}$ $\frac{3}{7} < \frac{2}{3} < \frac{6}{8} < \frac{9}{10}$	 • The fractions didn't have the same numerator or denominator, so I created equivalent fractions. • I created equivalent fractions with the same numerator. (If I had used the same denominator, the denominator would be very big.) • 3, 9, 6, and 2 all go evenly into 18, so I used a numerator of 18. • Since the fractions all had the same numerator, the greater the denominator, the smaller the fraction.

Example 2 Comparing Fractions Using Equivalent Decimals

Which fraction is greater, $\frac{7}{10}$ or $\frac{3}{4}$? How do you know?

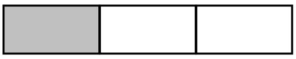
Solution	Thinking
$\frac{7}{10} = 0.7 = 0.70$ $\frac{3}{4} = \frac{25 \times 3}{25 \times 4} = \frac{75}{100} = 0.75$ $0.75 > 0.70$ so $\frac{3}{4} > \frac{7}{10}$.	• I wrote both fractions as decimals because 10 and 4 both go evenly into 100. 

Practising and Applying

1. Draw a picture to show why each is true.

a) $\frac{2}{3} > \frac{2}{5}$ **b)** $\frac{3}{6} < \frac{5}{6}$

2. Change the picture below to show why $\frac{1}{3} < \frac{5}{12}$.



3. Order from least to greatest.

a) $\frac{3}{8}, \frac{3}{10}, \frac{3}{4}$

b) $\frac{3}{9}, \frac{8}{9}, \frac{1}{9}$

c) $\frac{1}{8}, \frac{11}{12}, \frac{9}{20}$

d) $\frac{3}{4}, \frac{4}{10}, \frac{5}{8}$

4. Which task took more hours (h) to complete? How do you know?

$\frac{4}{5}$ h washing clothes

or

$\frac{3}{4}$ h sweeping

5. Two students are filling baskets with oranges.

• One basket is $\frac{5}{8}$ full.

• The other basket is $\frac{3}{5}$ full.

Which basket has more oranges? How do you know?

6. What is the greatest value you can use to make each true?

a) $\frac{?}{5} < \frac{3}{4}$

b) $4\frac{3}{8} > ?\frac{2}{3}$

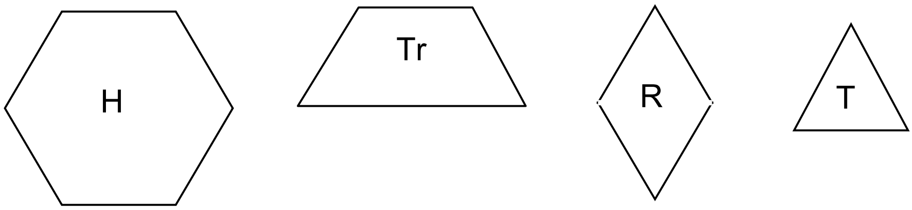
c) $\frac{?}{6} < \frac{9}{10}$

d) $2\frac{3}{7} > 2\frac{?}{9}$

7. Why is it useful to create equivalent fractions with the same denominator to compare them?

2.1.3 EXPLORE: Adding and Subtracting Fractions

You need fraction pieces like this for this activity. You will use them to model fraction addition and subtraction.



A. The H piece is a whole or 1. What fraction of the whole is each other piece?

B. Place pieces on the H piece as described below. Then cover those pieces with T pieces to see what fraction of the total area you covered. How many T pieces did you use for each?

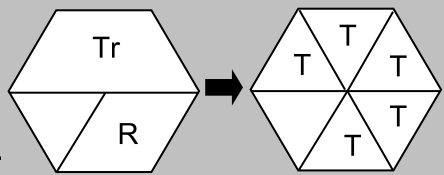
i) Place one Tr and one R on H.

ii) Place one Tr and one T on H.

iii) Place one R and one T on H.

iv) Place one R, one R, and one T on H.

For example, **part i)** is shown below:



C. i) Which model in **part B** represents $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$?

ii) What fraction sum does each other model in **part B** represent?

D. Place pieces as described below. Then place T pieces on the *uncovered* area to see what fraction of the total area is not covered. How many T pieces did you use for each?

i) Place one Tr on H.

ii) Place one R on Tr.

iii) Place one R and one T on H.

E. i) Which model in **part D** represents $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$?

ii) What fraction difference does each other model in **part D** represent?

Reprint 2023

2.1.4 Adding Fractions

Try This

Radhika is making Kewa Datshi for her family. She uses 4 potatoes, $\frac{1}{3}$ cup cheese, $\frac{1}{4}$ cup of red onions, and $\frac{1}{2}$ teaspoon chili powder.

A. i) If Radhika mixes the cheese and onions, how much will she have?

a bit less than $\frac{1}{2}$ cup or $\frac{1}{2}$ cup or a bit more than $\frac{1}{2}$ cup

How do you know?

ii) About how much will Radhika have if she mixes $\frac{1}{3}$ cup cheese and $\frac{1}{2}$ teaspoon chilli powder? Explain your estimate.

- To add two fractions, each must be part of the same size whole.

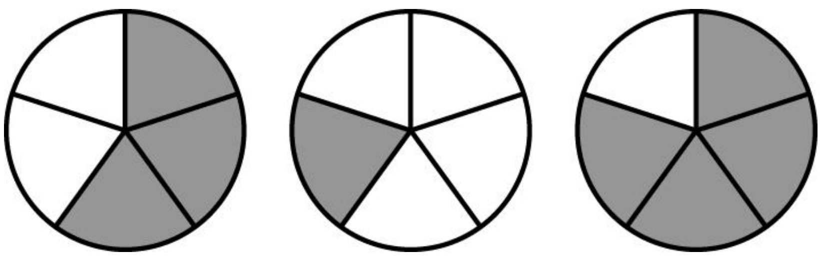
For example:

$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$ because, when you add a piece that is $\frac{3}{5}$ of a whole to a piece that is $\frac{1}{5}$ of the same whole, you will have $\frac{4}{5}$ of the whole.

- It does not matter what the whole looks like.

For example, the whole can be a shape like a rectangle or a circle:

$$\frac{3}{5} \quad + \quad \frac{1}{5} \quad = \quad \frac{4}{5}$$



$$\frac{3}{5} \quad + \quad \frac{1}{5} \quad = \quad \frac{4}{5}$$

- It makes sense that 3 parts and 1 part of the same thing make 4 parts.

In this case, 3 fifths + 1 fifth = 4 fifths.

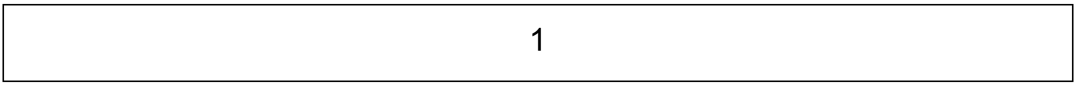
Reprint 2023

• When you are not adding fractions with the same denominator, the sum is not always obvious. You can use a model to add the fractions.

For example:

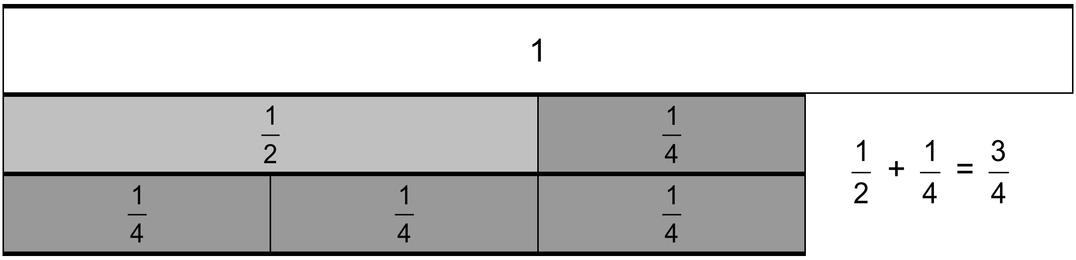
You can model $\frac{1}{2} + \frac{1}{4}$ by placing fraction strips below a whole.

First look at the whole, or 1



Then place the fractions you are adding together end to end

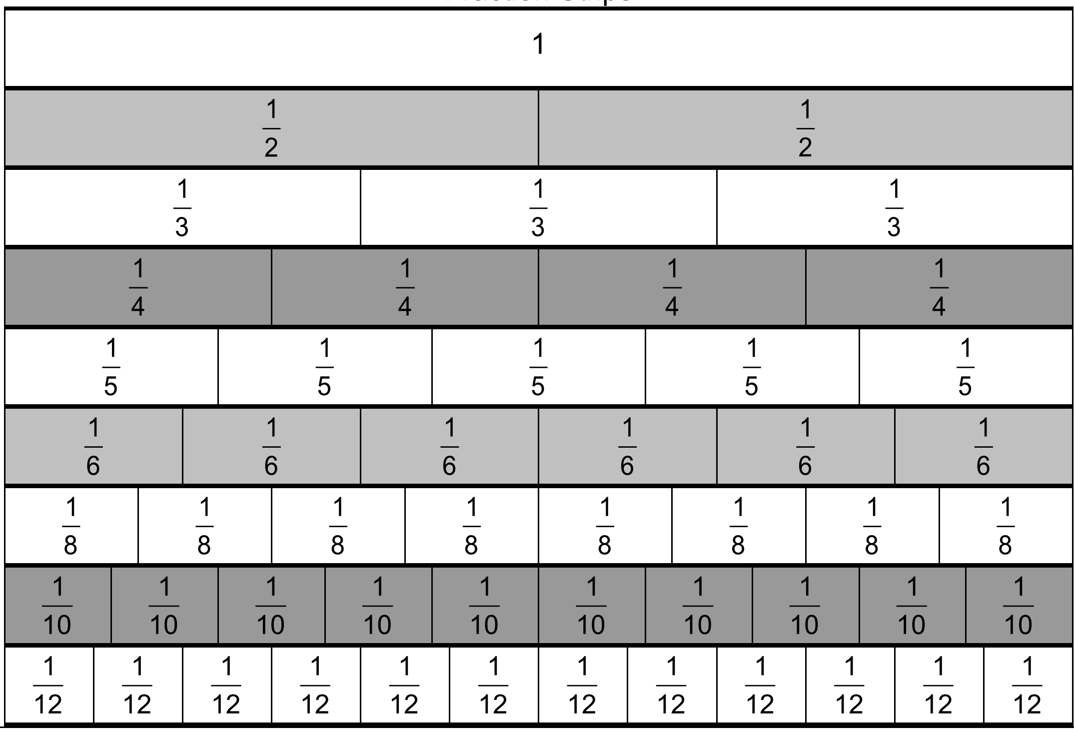
Use other strips that are all the same to see how much of the whole your fractions take up



$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

• You can make fraction strips like these to add other fractions:

Fraction Strips



B. Think about the fractions in **part A** and about the fraction strips.

i) How do you know $\frac{1}{3} + \frac{1}{4} > \frac{2}{4}$? How do you know $\frac{1}{3} + \frac{1}{4} < \frac{2}{3}$?

ii) Does it make sense that $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$?

C. If $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$, why is $\frac{1}{3}$ cup + $\frac{1}{2}$ teaspoon *not* equal to $\frac{5}{6}$?

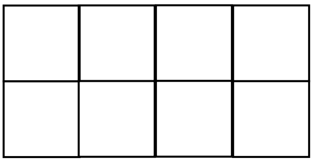
Examples

Example 1 Adding Fractions with the Same Denominator

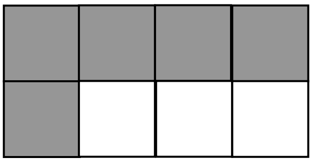
Add $\frac{5}{8}$ and $\frac{2}{8}$.

Solution 1

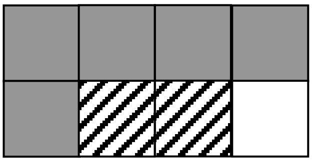
The whole is 8 eighths.



$\frac{5}{8}$



$\frac{5}{8} + \frac{2}{8}$



$\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$

Thinking

- I drew a picture to represent the whole. I divided it into eighths.
- To represent $\frac{5}{8}$, I shaded 5 eighths of the whole.
- To add $\frac{2}{8}$, I shaded 2 more eighths.

Solution 2

5 eighths + 2 eighths = 7 eighths

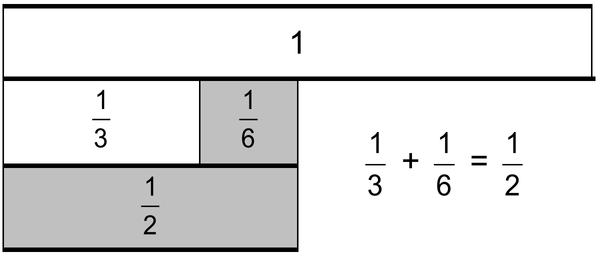
So $\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$.

Thinking

- It makes sense that 5 parts of something and 2 equal parts of the same thing make 7 parts altogether.

Example 2 Adding Fractions with Different Denominators

Add $\frac{1}{3}$ and $\frac{1}{6}$.

Solution	Thinking
$\frac{1}{3} + \frac{1}{6} = ?$  $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$	 • I used fraction strips to model the whole and the fractions I was adding. • I could see from the fraction strip that the two fractions together were half of the whole.

Practising and Applying

1. What is each sum?

a) $\frac{3}{8} + \frac{2}{8}$ **b)** $\frac{5}{8} + \frac{1}{8}$

c) $\frac{3}{10} + \frac{7}{10}$ **d)** $\frac{2}{5} + \frac{2}{5}$

2. Use fraction strips to add. Sketch pictures that show two of these.

a) $\frac{3}{4} + \frac{1}{8}$ **b)** $\frac{5}{12} + \frac{1}{6}$

c) $\frac{2}{3} + \frac{1}{4}$ **d)** $\frac{1}{3} + \frac{3}{6}$

3. What fractions are being added in each? What is the sum?

a)

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$

b)

$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$

c)

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$

4. The sum of two fractions is $\frac{5}{6}$.

What could the two fractions be? List five possible pairs of fractions.

5. Use an example to show why each is sometimes true.

a) eighths + fourths = halves

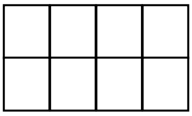
b) thirds + halves = sixths

c) thirds + fourths = twelfths

d) halves + fourths = fourths

6. Can you add $\frac{2}{3}$ of the population of Bhutan to $\frac{1}{3}$ of the population of India and get the whole population of either country? Explain your thinking.

7. You can use counters on this grid or colour the grid to represent fractions.



How would you represent each using the grid?

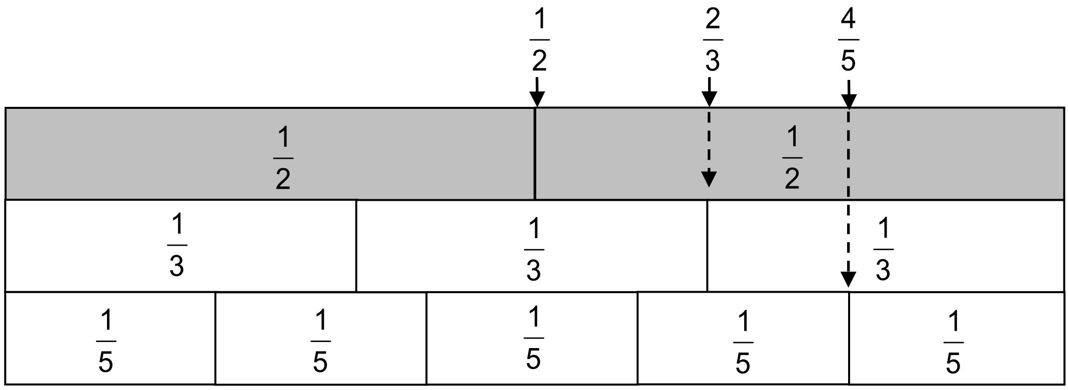
a) $\frac{1}{4}$ **b)** $\frac{3}{8}$ **c)** $\frac{1}{4} + \frac{3}{8}$

8. Why is it easier to add fractions when the denominators are the same?

CONNECTIONS: Fractions Between Fractions

Padam used fraction strips for halves, thirds, and fifths to figure out that $\frac{2}{3}$ is between $\frac{1}{2}$ and $\frac{4}{5}$.

$\frac{2}{3}$ is between $\frac{1}{2}$ and $\frac{4}{5}$.



When Padam looked at the three fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{5}$, he noticed that

- the numerator 2 is between 1 and 4 (the other numerators) and
- the denominator 3 is between 2 and 5 (the other denominators).

Padam discovered a strategy for finding in-between fractions. He created a fraction between two fractions by using a numerator that is between their numerators and a denominator that is between their denominators.

1. Use Padam's strategy to create each.

a) two fractions between $\frac{1}{2}$ and $\frac{9}{10}$

b) two fractions between $\frac{1}{2}$ and one of your fractions from **part a)**

2. Show that Padam's strategy does not *always* work. Use his strategy to create two fractions that are *not* between $\frac{1}{2}$ and $\frac{9}{10}$. How do you know you are right?

2.1.5 Subtracting Fractions

Try This

A. i) In this set of 5 pieces of fruit (3 bananas, 1 apple, and 1 mango), which is greater?

The fraction that is bananas

or

The fraction that is apples

ii) How much greater is it?

• When you subtract two fractions, you are finding the **difference** between
them — how much greater one is than the other.

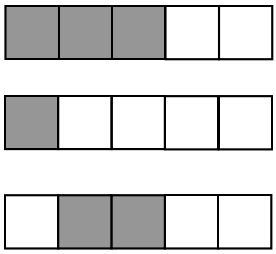
For example:

$\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$. If you compare a piece that is $\frac{3}{5}$ of a whole to a piece that is $\frac{1}{5}$ of the same whole, you see that $\frac{3}{5}$ is $\frac{2}{5}$ more than $\frac{1}{5}$.

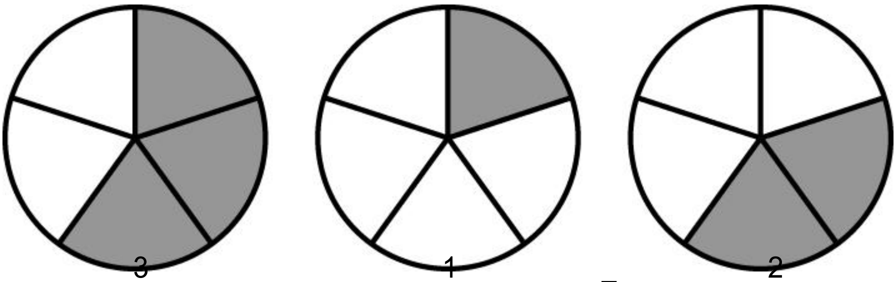
$\frac{3}{5}$

$\frac{1}{5}$

$\frac{2}{5}$



$$\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$$



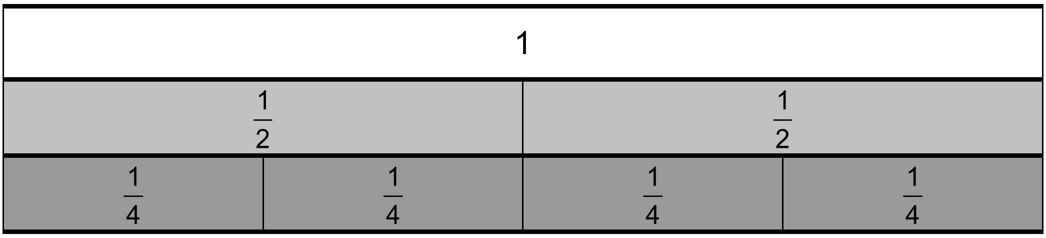
$$\frac{3}{5} \quad - \quad \frac{1}{5} \quad = \quad \frac{2}{5}$$

• It makes sense that 3 parts of something is 2 parts more than 1 part of the same thing. In this case, 3 fifths – 1 fifth = 2 fifths.

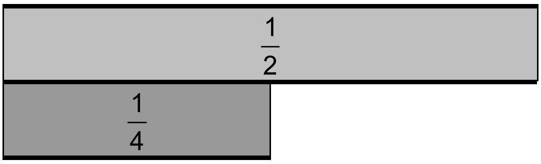
• When you subtract fractions that do not have the same denominator, the difference is not always obvious. You can use a model to subtract.

For example:

You can model $\frac{1}{2} - \frac{1}{4}$ using fraction strips:

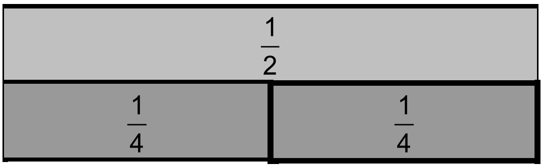


Compare the fraction strips to see how much longer one is than the other



$\frac{1}{2} - \frac{1}{4} = ?$

Figure out how much longer it is by using other strips



$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

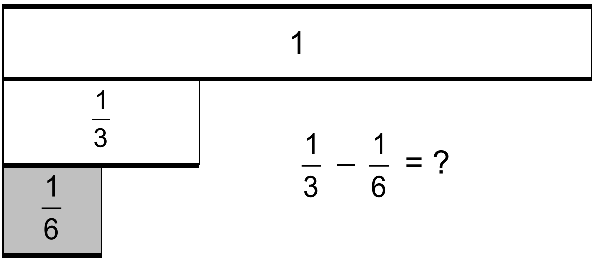
B. What fractions did you subtract to solve the problem in **part A**?

Examples

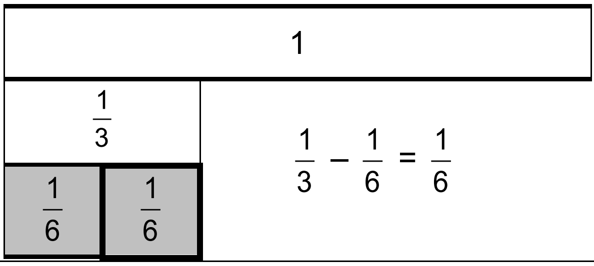
Example 1 Subtracting Fractions with Different Denominators

Subtract $\frac{1}{6}$ from $\frac{1}{3}$.

Solution



$\frac{1}{3} - \frac{1}{6} = ?$



$\frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

Thinking

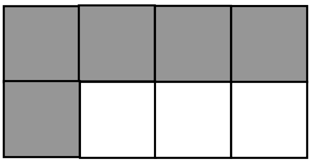
- I compared the length of the $\frac{1}{3}$ fraction strip to the length of the $\frac{1}{6}$ fraction strip.
- I saw that $\frac{1}{6}$ would fit to make up the difference.

Example 2 Subtracting Fractions with the Same Denominator

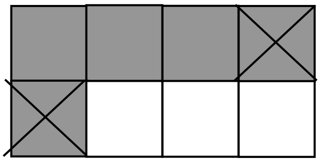
Subtract $\frac{2}{8}$ from $\frac{5}{8}$.

Solution 1

$\frac{5}{8}$



$\frac{5}{8} - \frac{2}{8}$



$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

Thinking

- I drew a picture to represent the whole. I divided it into eighths.
- Then I shaded 5 eighths to show $\frac{5}{8}$.
- To subtract $\frac{2}{8}$, I took away 2 eighths.
- There were 3 eighths left.

Solution 2

5 eighths – 2 eighths = 3 eighths

So $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$.

Thinking

- It makes sense that if you take 2 of something away from 5 of the same thing, there will be 3 left.

Practising and Applying

1. What is each difference?

a) $\frac{3}{8} - \frac{2}{8}$ **b)** $\frac{5}{8} - \frac{1}{8}$

c) $\frac{7}{10} - \frac{3}{10}$ **d)** $\frac{4}{5} - \frac{2}{5}$

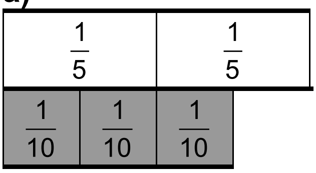
2. Use fraction strips to subtract. Sketch pictures that show two of these.

a) $\frac{3}{4} - \frac{1}{8}$ **b)** $\frac{5}{12} - \frac{1}{6}$

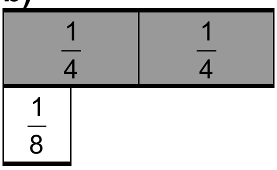
c) $\frac{2}{3} - \frac{1}{4}$ **d)** $\frac{3}{6} - \frac{1}{3}$

3. What fractions are being subtracted in each? What is each difference?

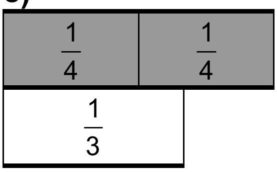
a)



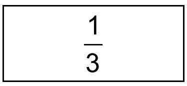
b)



c)



4. The difference between two fractions is $\frac{1}{3}$. What could the two fractions be? List five possible pairs.



5. Use an example to show why each is sometimes true.

a) thirds – fourths = twelfths

b) thirds – fourths = sixths

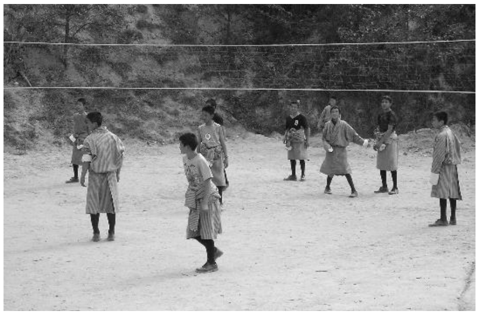
c) halves – fourths = fourths

d) thirds – twelfths = sixths

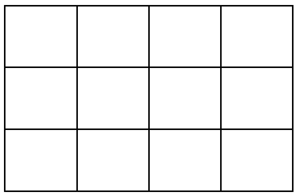
6. In Kuenga's school, $\frac{2}{3}$ of the students play sports.

In Ugyen's school, $\frac{1}{3}$ of the students play sports.

Is it possible to tell which school has more students who play sports? Explain your thinking.



7. You can use counters on this grid or colour the grid to represent fractions.



How could you represent each using the grid?

a) $\frac{1}{4}$ **b)** $\frac{1}{6}$ **c)** $\frac{1}{4} - \frac{1}{6}$

8. Why is it helpful to use fraction strips to solve a subtraction question when the denominators are not the same?

Chapter 2 Relating Fractions to Decimals

2.2.1 Naming Decimals as Fractions

Try This

Pelden, Dorji, and Devika are practising high jump.

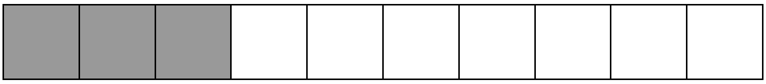
A. Write each jump height in metres using a decimal.

	Height (m and cm)	**Height** (m)
Pelden	1 m and 38 cm	? m
Dorji	95 cm	? m
Devika	1 m and 2 cm	? m

• Decimals with one decimal place is fraction tenths. You can represent them using a fraction strip in tenths.

For example:

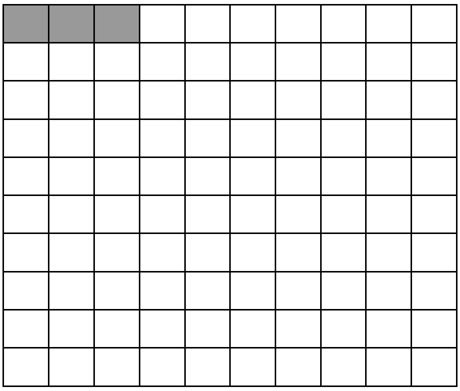
0.3 is $\frac{3}{10}$.



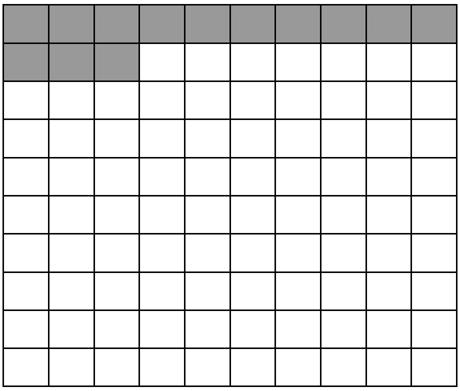
• Decimals with two decimal places are fraction hundredths. You can represent them using a hundredths grid.

For example:

0.03 is $\frac{3}{100}$.



0.13 is $\frac{13}{100}$.



• You can **rename** a decimal like 0.13 as $\frac{13}{100}$ or as $\frac{1}{10} + \frac{3}{100}$

(because each row of the grid is $\frac{1}{10}$

and there is an additional $\frac{3}{100}$

in the second row).

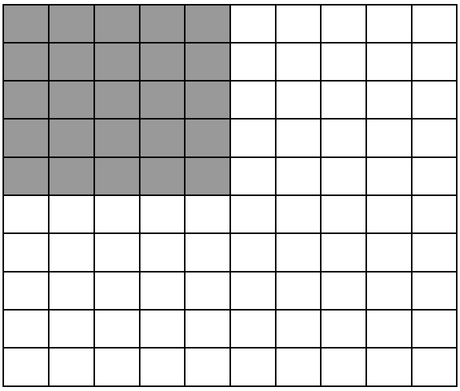
• Sometimes you can write a decimal as a fraction and then **simplify** it.

For example:

$0.25 = \frac{25}{100}$

25 squares of a 100 grid is $\frac{1}{4}$ of the grid.

So $0.25 = \frac{1}{4}$.



B. Write each decimal from **part A** as a fraction.

Examples

Example Writing Decimals as Fractions

Write each decimal as a fraction or as a fraction sum in two ways.

a) 0.05 **b)** 0.23 **c)** 1.4

Solution	Thinking
a) $0.05 = \frac{5}{100} = \frac{1}{20}$	**a)** I knew $\frac{5}{100}$ was half a row in a hundredths grid and there would be 20 half-rows in the grid. 
b) $0.23 = \frac{23}{100}$ $= \frac{2}{10} + \frac{3}{100}$	**b)** Since there were two decimal places, I knew the denominator was 100. • Since $\frac{23}{100}$ covered 2 rows of a grid and 3 more squares, I separated it into 2 tenths and 3 hundredths.
c) $1.4 = 1\frac{4}{10}$ $= \frac{14}{10}$ $= \frac{7}{5}$	**c)** I knew that a whole number and a decimal with one decimal place would be a mixed number with a fraction tenth. • I renamed it as an improper fraction. • I noticed that 14 and 10 were both multiples of 2, so I renamed the fraction.

Reprint 2023

Practising and Applying

1. Write each decimal as a fraction.

a) 0.8 **b)** 0.08

c) 2.3 **d)** 3.5

2. Which decimal is greater, 1.2 or 1.02? How do you know?

3. Which decimal is greater than $\frac{1}{2}$?

A. 0.46 **B.** 0.21 **C.** 0.8

4. The decimal 0.2■ is written as a fraction. Is it between $\frac{19}{100}$ and $\frac{3}{10}$? How do you know?

5. What do the fractions for these decimals have in common?

1.3 2.3 0.3 0.4

6. Draw a picture to show why 0.3 = 0.30. Explain how your picture shows they are equal.

7. The decimal 0.■ ■ is written as a fraction.

a) What could the decimal be if the denominator is 2?

b) What other number could be the denominator?

8. Why is it easy to write a decimal with one or two decimal places as a fraction?

GAME: Fraction Match

Play this game with a partner. You need the Fraction Match Game Cards.

How to play:

- Shuffle the cards and deal them all out.
- Both players shuffle their cards and arrange them in a stack face down.
- Players turn over their top cards at the same time. If the cards are of equal value, the first person to say “Match” wins both cards. If the cards do not match, players return their cards to the bottom of their stacks.

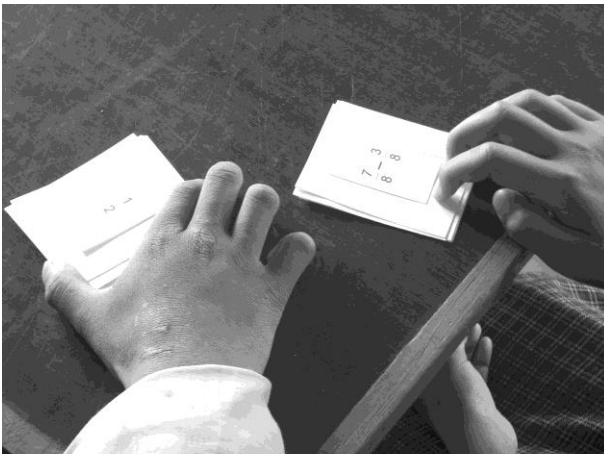
Keep playing until one player has no more cards.

The player who wins all the cards wins the game.

"Match"

$$\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$$

so $\frac{1}{2}$ matches $\frac{7}{8} - \frac{3}{8}$



2.2.2 Naming Fractions as Decimals

Try This

Choki read that 0.23 of Bhutanese households have piped water indoors. She asked the 40 students in her class and 8 of them had piped water indoors.

A. Which is greater?

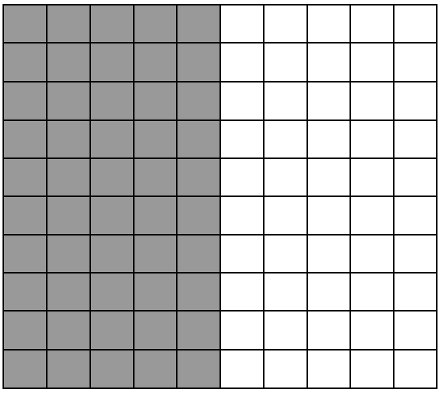
The fraction of Choki's classmates that have piped water indoors

or

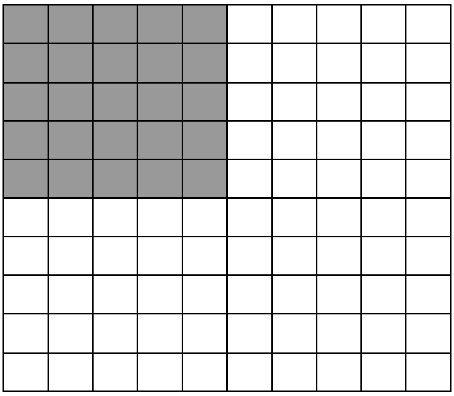
The fraction of Bhutanese households that have piped water indoors

- You can use a hundredths grid to help you rename some fractions as decimals.

For example:



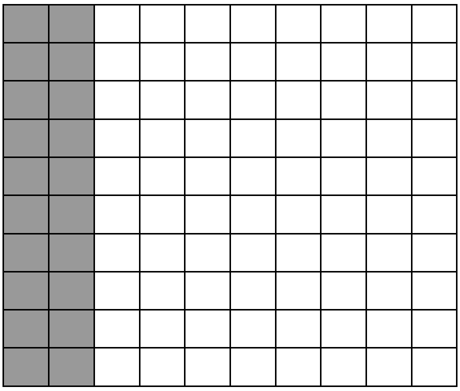
$\frac{1}{2} = \frac{50}{100} = 0.50$ or 0.5



$\frac{1}{4} = \frac{25}{100} = 0.25$

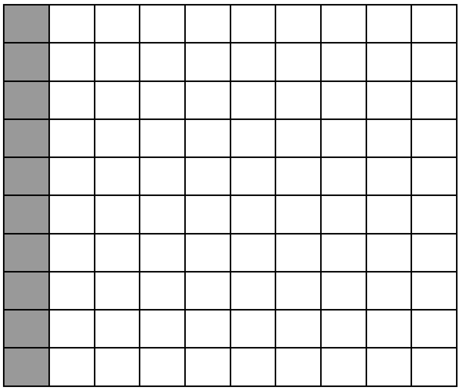
Since $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 0.25 + 0.25 + 0.25$,

then $\frac{3}{4} = 0.75$.



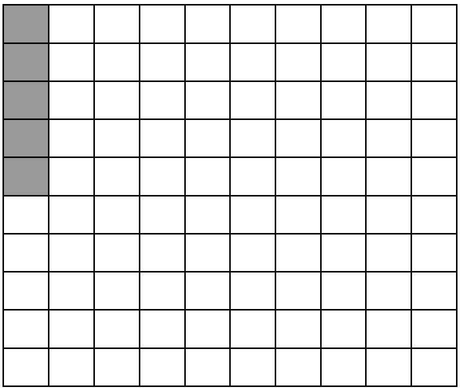
$\frac{1}{5}$ of the grid is 2 columns

so $\frac{1}{5}$ = 0.2 or 0.20.



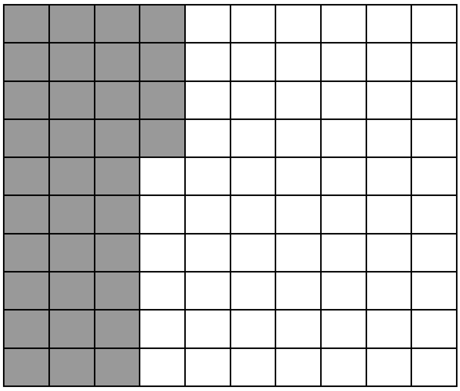
$\frac{1}{10}$ of the grid is 1 column

so $\frac{1}{10}$ = 0.1.



$\frac{1}{20}$ of the grid is 5 squares

so $\frac{1}{20}$ = 0.05.



$\frac{1}{3}$ of the grid is about 33 squares

so $\frac{1}{3}$ is about 0.33.

B. Write a decimal and a fraction for each.
i) the fraction of Bhutanese households that have piped water indoors
ii) the fraction of Choki's classmates that have piped water indoors

Reprint 2023

Examples

Example Writing Fractions as Decimals

Write each fraction as a decimal. Show your work.

a) $\frac{3}{5}$ **b)** $\frac{18}{100}$ **c)** $\frac{12}{50}$ **d)** $\frac{13}{20}$

Solution	Thinking
a) $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ $= 0.2 + 0.2 + 0.2 = 0.6$	**a)** I knew that $\frac{1}{5}$ was 0.2.
b) $\frac{18}{100} = 0.18$	**b)** Since the denominator was 100, I knew there were 2 decimal places and the 2 digits were the digits in the numerator.
c) $\frac{12}{50} = \frac{24}{100} = 0.24$	**c)** I first created an equivalent fraction that had a denominator of 100.
d) $\frac{13}{20}$ = 13 × 5 hundredths = 65 hundredths = 0.65	**d)** Each $\frac{1}{20}$ is 0.05 or 5 hundredths, so $\frac{13}{20}$ is 13 × 5 hundredths.

Practising and Applying

1. Write each fraction as a decimal.

a) $\frac{8}{10}$ **b)** $\frac{8}{100}$

c) $\frac{3}{50}$ **d)** $\frac{2}{4}$

2. Which has a greater fraction of blue?

a) $\frac{3}{10}$ blue or $\frac{1}{5}$ blue

b) $\frac{3}{4}$ blue or $\frac{3}{10}$ blue

3. Show how you can use $\frac{1}{5} = 0.2$ to help you write each fraction as a decimal.

$\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$

4. How do you know that $\frac{1}{3}$ is close to 0.33, but not exactly 0.33?

5. Write each as a decimal.

a) $\frac{120}{200}$ **b)** $\frac{50}{500}$

6. a) Write five fractions, each with a different denominator, that are easy to write as decimals. Tell why they are easy to write as decimals.

b) Write three fractions, each with a different denominator, that are more difficult to write as decimals. Tell why they are more difficult to write as decimals.

UNIT 2 Revision

1. Write each improper fraction as a mixed number.

a) $\frac{17}{3}$ **b)** $\frac{12}{5}$ **c)** $\frac{14}{4}$

2. Write each mixed number as an improper fraction.

a) $2\frac{1}{2}$ **b)** $5\frac{1}{4}$ **c)** $1\frac{7}{10}$

3. A store sells plates of 4 pakoras.

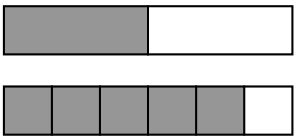
a) How many plates would you need to buy to get 32 pakoras?

b) What mixed number would describe several full plates of pakoras and one plate with only 1 pakora on it?

4. What fraction comparison does each picture show?

a)

b)



5. Order from least to greatest.

a) $\frac{4}{7}$, $\frac{4}{9}$, $\frac{1}{3}$, $\frac{2}{9}$

b) $\frac{7}{9}$, $\frac{2}{5}$, $\frac{4}{9}$, $\frac{14}{20}$

6. Which fraction in each pair is greater?

a) $\frac{3}{8}$ or $\frac{3}{10}$ **b)** $\frac{2}{7}$ or $\frac{5}{21}$

c) $\frac{1}{60}$ or $\frac{49}{50}$ **d)** $\frac{22}{100}$ or $\frac{2}{10}$

7. Kinley, Yuden, and Mindu each ate 2 servings of Ema Datshi.

Kinley's servings came from a pot that served 5.

Yuden's from a pot that served 8.

Mindu's from a pot that served 10.

Who ate about half a pot?

8. Use fraction strips to add each.

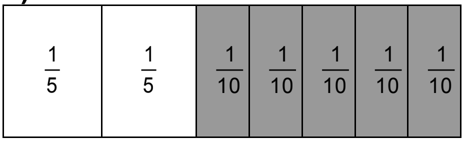
a) $\frac{5}{6} + \frac{1}{12}$ **b)** $\frac{1}{4} + \frac{1}{6}$

c) $\frac{2}{3} + \frac{1}{12}$ **d)** $\frac{1}{3} + \frac{5}{12}$

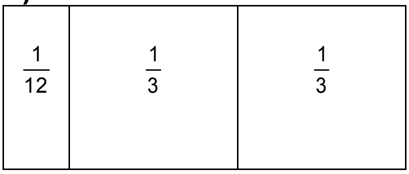
Reprint 2023

9. What fractions are being added in each picture? What is each sum?

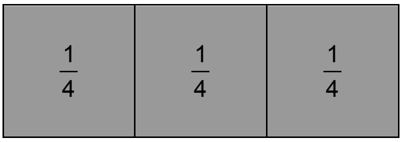
a)



b)



10. The sum of two fractions is $\frac{3}{4}$.



What could the two fractions be? List three possible pairs.

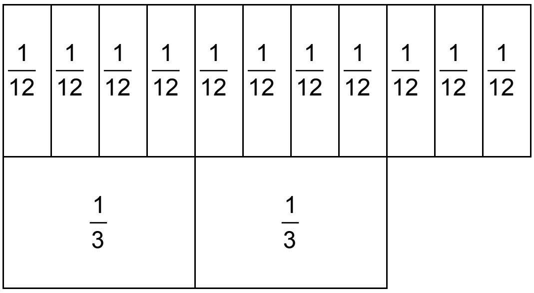
11. Use fraction strips to subtract.

a) $\frac{5}{6} - \frac{1}{12}$ **b)** $\frac{1}{4} - \frac{1}{6}$

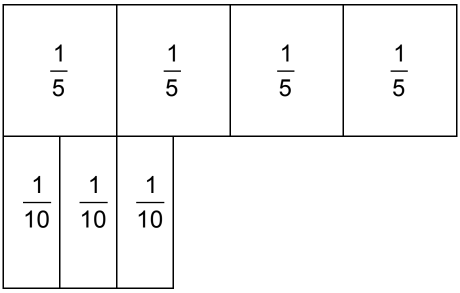
c) $\frac{2}{3} - \frac{1}{12}$ **d)** $\frac{5}{12} - \frac{1}{3}$

12. What fractions are being subtracted in each picture? What is each difference?

a)



b)



13. Write each as a fraction.

a) 0.4 **b)** 0.26

c) 2.8 **d)** 1.75

14. How are 3.05 and 3.5 the same? How are they different?

15. Which decimal below is greater than $\frac{3}{4}$?

A. 0.9 **B.** 0.35 **C.** 0.43

16. Write each fraction as a decimal.

a) $\frac{21}{100}$ **b)** $\frac{6}{10}$

c) $\frac{7}{20}$ **d)** $\frac{4}{5}$

17. Show four or more fractions with a numerator of 3 that can be written as a decimal.

UNIT 3 DECIMAL COMPUTATION

Getting Started

Use What You Know

- A tailor buys 72 m of fabric to make ghos.
- The fabric costs Nu 150 for each metre.
- It takes about 4 m of fabric to make one gho.
- The lining for a gho costs Nu 200.

A. How much does it cost to make each gho?

B. How many ghos can the tailor make with 72 m of fabric?

C. How much does 72 m of fabric cost?

D. The tailor sells each gho for Nu 1000.

i) How much is she being paid for her work on each gho?

ii) How much is she being paid for her work on all the ghos?

E. Write a word problem about the cost of making or selling something. Your problem must involve multiplying and dividing. Solve your problem.

Skills You Will Need

1. Estimate each. Show your work.

a) 38 × 45 **b)** 82 × 94 **c)** 3112 ÷ 6 **d)** 489 ÷ 8

2. Use mental math to calculate each.

a) 60 × 30 **b)** 42 × 100 **c)** 3000 ÷ 5 **d)** 5400 ÷ 6

3. Sketch a rectangle to show the parts you add to calculate each.

a) 47 × 22

b) 31 × 31

For example: 23 × 42

	40 +	2
20 +	20 × 40	20 × 2
3	3 × 40	3 × 2

23 × 42 = 800 + 120 + 40 + 6

Reprint 2023

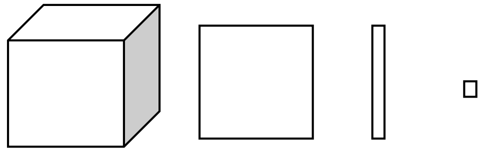
4. Calculate each product.

a) 32 × 57 **b)** 48 × 39 **c)** 99 × 31 **d)** 71 × 19

5. Use a place value chart or sketch base ten blocks to model each. Then find each answer.

a) 4 × 3112
b) 3 × 4008
c) 1234 ÷ 3
d) 2120 ÷ 4

Base Ten Blocks



1 thousand 1 hundred 1 ten 1 one

Place Value Chart

Ten thousands	Thousands	Hundreds	Tens	Ones

6. Multiply.

a) 6 × 3112 **b)** 4 × 3921 **c)** 5 × 8930 **d)** 9 × 1218

7. Which multiplications have a product of about 10?

A. 7 × 1.4 **B.** 6 × 3.47 **C.** 9 × 1.24

8. Calculate each quotient mentally.

a) 540 ÷ 10 **b)** 3600 ÷ 100 **c)** 42,000 ÷ 100

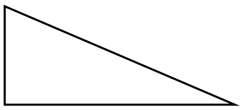
9. Divide.

a) $6\overline{)5234}$ **b)** $3\overline{)2985}$ **c)** $2\overline{)1428}$ **d)** $8\overline{)3724}$

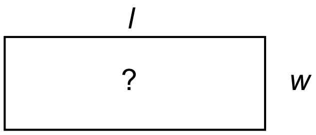
e) $30\overline{)4800}$ **f)** $20\overline{)3640}$ **g)** $10\overline{)5420}$ **h)** $60\overline{)3824}$

10. What operation do you use to calculate each?

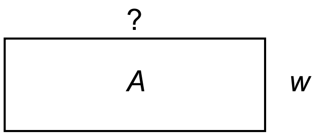
a) the perimeter of a triangle with three different side lengths



b) the area of a rectangle, if you know the length and the width



c) the length of a rectangle, if you know the area and the width



Reprint 2023

Chapter 1 Multiplication

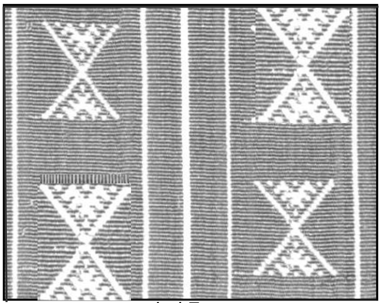
3.1.1 Estimating a Product

Try This

A. A piece of fabric hanging on Kinley's wall is 1.15 m wide and 1.1 m long.

i) Estimate the area of the fabric in square metres.

ii) Estimate the area in square centimetres. (Hint: 1 m = 100 cm)



• If you do not need an exact answer to a problem, you can estimate.

For example, an exact answer is not necessary to solve this problem:

Is an 11-year-old closer to 1000 days old, 5000 days old, or 10,000 days old?

• To estimate, use numbers that are easy to work with, like 1000s, 100s, or 10s. You can round up or down, depending on the numbers.

For example, to estimate how many days are in 11 years:

365 days each year for 11 years

365 is about 400. 11 is about 10. 10 × 400 = 4000	OR	365 is about 300. 11 is about 10. 10 × 300 = 3000
One number was rounded up and the other was rounded down, so the estimate should be close.		Both numbers were rounded down, so the estimate will be low.

Both estimates show that 5000 days is the answer to the problem.

• To estimate a decimal **product**, use whole numbers or simpler decimals.

For example:

Estimate the area of a piece of fabric that is 2.4 m long and 0.9 m wide.

Using whole numbers: 2.4 m × 0.9 m → 2 × 1 = 2 m^2

Using a simpler decimal and a whole number:

2.4 m × 0.9 m → 2.5 × 1 = 2.5 m^2

B. What numbers did you use to estimate each area in **part A**?

i) in square metres **ii)** in square centimetres

Examples

Example 1 Estimating a Product to Solve a Problem

Sonam walks 3526 m to and from school each school day.
About how many kilometres does she walk to and from school in a month?

Solution 1

3526 m each day for 27 days:

3526 × 27 is about 3000 × 20.

3000 × 20 = 60,000

60,000 m = 60 km

She walks more than 60 km.

Thinking

- I counted 27 school days in a month.
- I rounded both numbers down a lot so I knew it would be a very low estimate.

Solution 2

3526 m = 3.526 km

3.526 km each day for 27 days:

3.526 × 27 is about 3 × 30.

3 × 30 = 9

She walks about 90 km.

Thinking

- I first changed metres to kilometres.
- I rounded one number down and one number up so my estimate was closer to the exact answer than if I had rounded both up or both down.

Practising and Applying

1. Buthri just turned 8 years old. Estimate her age in each. Show your work.

a) weeks **b)** days **c)** hours

2. Estimate the area of each room. Show your work.

	Length (m)	**Width** (m)	**Area** (m^2)
a)	8.2	4.5	
b)	11.8	6.2	
c)	3.1	2.8	

3. List three pairs of decimals whose product you could estimate using 4 × 5.

4. Arjun estimated a product:

2■ × 3■ is about 600

What digits could go in the blanks to fit each case described below?

a) the estimate of 600 is close to the actual product

b) the estimate of 600 is low

5. Manju has Nu 340.50 in her bank account. Her brother has twice as much money. About how much does her brother have?

6. Estimate to decide which of these calculations are incorrect.

A. 2456 × 28 = 86,768

B. 3.2 × 4.2 = 13.44

C. 19.23 × 3500 = 57,305

7. Describe a situation where you might estimate a product instead of calculating an exact answer.

3.1.2 Multiplying a Decimal by a Whole Number

Try This

Lobzang can run 100 m in 12.4 s.

A. About how long would it take him to run 300 m at that speed? Explain how you estimated.

• You can use a place value chart to multiply a decimal by a whole number.

For example, to multiply 3 × 4.125:

Represent the decimal on the chart

Tens	Ones	Tenths	Hundredths	Thousandths
	4	1	2	5

Multiply each part of the number by 3

Tens	Ones	Tenths	Hundredths	Thousandths
	3 × 4 = 12	3 × 1 = 3	3 × 2 = 6	3 × 5 = 15

Regroup if there are 10 or more in one place

Tens	Ones	Tenths	Hundredths	Thousandths
	12	3	6	15
1	2	3	7	5

3 × 4.125 = 12.375

• Another way to multiply is shown on the right.

$$\begin{array}{r} {\scriptstyle 1} \\ 4.125 \\ \times 3 \\ \hline 12.375 \end{array}$$

• You should always estimate to make sure an answer makes sense.

For example:

Since 3 × 4.125 is about 3 × 4 = 12, the answer 12.375 makes sense.

• To multiply a decimal by 10, you can think about place value.
For example, to multiply 10 × 5.123:

Represent the decimal

Tens	Ones	Tenths	Hundredths	Thousandths
	5	1	2	3

Multiply each part by 10

Tens	Ones	Tenths	Hundredths	Thousandths
	10 × 5 = 50	10 × 1 = 10	10 × 2 = 20	10 × 3 = 30

Regroup

Tens	Ones	Tenths	Hundredths	Thousandths
	50	10	20	30
5	1	2	3	0

10 × 5.123 = 51.230 or 51.23

Notice what happened to the digits of 5.123 when it was multiplied by 10:

Tens	Ones	Tenths	Hundredths	Thousandths
	5	1	2	3
5	1	2	3	0

Each digit moved one place to the left. This makes sense because the value of each digit increased 10 times.

• To multiply a decimal by 100 or by 1000, you can use the same pattern.
- When you multiply by 100, each digit moves 2 places to the left.

For example: 100 × 5.123 = 512**.**3

- When you multiply by 1000, each digit moves 3 places to the left.

For example: 1000 × 5.123 = 5123. or 5123

B. i) Exactly how long would it take Lobzang to run 300 m at a speed of 100 m in 12.4 s?
ii) How does your exact answer compare to your estimate from **part A**?

Examples

Example 1 Using Equivalent Products

Solve each by finding the missing number.

a) $50 \times 4.562 = 5 \times$ ■ **b)** $800 \times 9.125 = 8 \times$ ■

Solution

a)

$50 \times 4.562 = 5 \times \underline{10 \times 4.562}$

So ■ $= 10 \times 4.562$.

$10 \times 4.562 = 45.62$

$50 \times 4.562 = 5 \times \underline{45.62}$

b)

$800 \times 9.125 = 8 \times \underline{100 \times 9.125}$

So ■ $= 100 \times 9.125$.

$100 \times 9.125 = 912.5$

$800 \times 9.125 = 8 \times \underline{912.5}$

Thinking

a) Multiplying by 50 is the same as multiplying by 5×10.

- I multiplied 10×4.562 using mental math.

b) Multiplying by 800 is the same as multiplying by 8×100.

- I multiplied 100×9.125 using mental math.

Example 2 Multiplying a Decimal by a Multiple of Ten or One Hundred

Calculate each.

a) 20×75.3 **b)** 400×8.234

Solution

a) $20 \times 75.3 = 2 \times 10 \times 75.3$

$10 \times 75.3 = 753$

$2 \times 753 = 1506$

$20 \times 75.3 = 1506$

b) $400 \times 8.234 = 4 \times 100 \times 8.234$

$100 \times 8.234 = 823.4$

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 823.4 \\ \underline{\times \quad 4} \\ 3293.6 \end{array}$$

$400 \times 8.234 = 3293.6$

Thinking

a) I changed 20 to 2×10 so I could multiply 75.3 by 10.

- To multiply by 10, I moved the digits one place to the left.
- Then I multiplied the product by 2.

b) Multiplying by 400 is the same as multiplying by 100×4.

- To multiply by 100, I moved the digits two places to the left.

Reprint 2023

Practising and Applying

1. a) Multiply each.

i) 9×4.126 **ii)** 5×23.89

iii) 6×125.3 **iv)** 8×52.42

b) Estimate to check two or more of your answers. Show how you estimated.

2. Use a place value chart to show why each is true.

a) $4 \times 38.125 = 152.5$

b) $9 \times 53.191 = 478.719$

3. Solve each.

a) $30 \times 6.08 = 3 \times ■$

b) $500 \times 4.955 = 5 \times ■$

4. Which digit will be in the tenths place of each product?

a) 2.314×10

b) 2.314×100

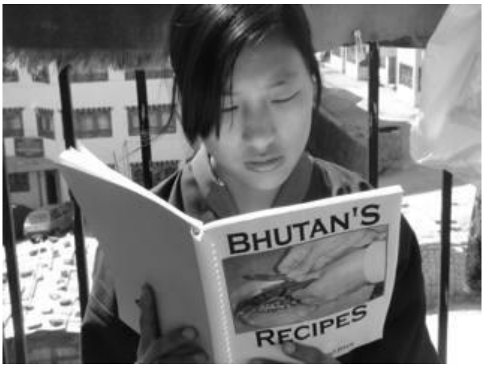
c) 2.314×1000

5. Multiply.

a) 10×5.123 **b)** 100×3.041

c) 30×18.72 **d)** 200×8.99

6. A recipe that serves 12 people uses 1.4 kg of meat. How much meat would be needed to serve 480 people?



7. Kachap can run 200 m in 25.7 s. How long would it take him to run each distance at that speed?

a) 1000 m

b) 20,000 m

8. a) What is the perimeter of a square with side length 8.23 m?

8.23 m


b) What would the perimeter be if the side length were 10 times longer?

c) How do the perimeters of the two squares in **parts a) and b)** compare?

9. Write a word problem that could be solved using $5 \times ■ = 2.3$. Solve your problem.

10. How is multiplying a whole number by a decimal different from multiplying two whole numbers?

Reprint 2023

3.1.3 Multiplying Decimals

Try This

Eden copied a photograph of a tiger and then reduced it.

The original photo was 6.7 cm by 4.4 cm. Eden's reduced photo is 0.6 times as long and 0.6 times as wide.


Original photo

A. Estimate the length and width of Eden's photo. Explain how you estimated.

• Multiplying two decimals is like multiplying two whole numbers.

For example:

3 times a number means 3 of that number, so 0.3 times a number means 0.3 or 3 tenths of that number.

To multiply 0.3 × 30:

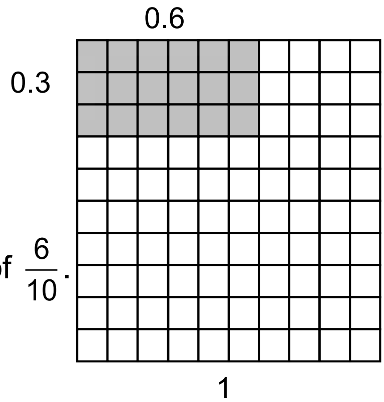
- Find 0.3 or 3 tenths of 30 by dividing 30 into 10 equal parts.
- Then count how many are in 3 parts.

30 in 10 parts is 30 ÷ 10 = 3. So each part has 3.
That means 3 parts have 3 × 3 = 9. So 0.3 × 30 = 9.

• Since we use multiplication to find the area of a rectangle, you can multiply decimal tenths by creating a rectangle on a hundredths grid.

For example:

To multiply 0.3 × 0.6, create a rectangle that is 0.3 by 0.6.



The area is 0.18 of the whole area of 1, so 0.3 × 0.6 = 0.18.

The grid also shows that 0.3 × 0.6 means $\frac{3}{10}$ of $\frac{6}{10}$.

$\frac{6}{10}$ of the grid is 6 full columns.

$\frac{3}{10}$ of these 6 columns is the shaded area shown.

0.3 × 0.6 = 3 tenths × 6 tenths = (3 × 6) hundredths = 18 hundredths or 0.18.

B. Multiply to calculate the exact dimensions of Eden's photo in **part A**.

Examples

Example 1 Multiplying Decimals

A car drives up a very steep hill at 15.5 km each hour. How far will the car travel in each amount of time at that speed?

a) 30 min **b)** 0.6 h

Solution 1

a) 30 min = 0.5 h

0.5 × 15.5

= 0.1 × 5 × 15.5

5 × 15.5 = 77.5

0.1 × 77.5 = 7.75

It will travel 7.75 km.

b) 0.6 × 15.5

= 6 tenths × 155 tenths

= 6 × 155 hundredths

= 930 hundredths

= 9.30

It will travel 9.30 km.

Thinking

a) Since 0.5 is 1 tenth of 5, I knew I could write 0.5 as 0.1 × 5 and then multiply 15.5 × 5.

• Then I multiplied the product by 0.1. When you multiply by 1 tenth, each digit moves one place to the right because its value is one tenth as much.

Tens	Ones	Tenths	Hundredths
7	7	5	
	7	7	5

7.75 km makes sense because 30 min is half an hour and 15.5 ÷ 2 is about 7.

b) I wrote 0.6 as 6 tenths and 15.5 as 155 tenths.

• I knew tenths × tenths = hundredths.

• I knew 930 hundredths = 9.30

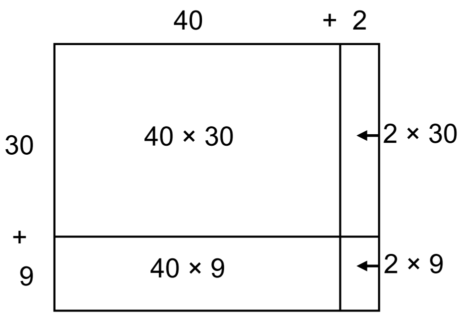
Tens	Ones	Tenths	Hundredths
	9	3	0

Example 2 Using an Estimate to Place the Decimal Point

Calculate 4.2 × 3.9.

Solution

42 × 39



Thinking

• I multiplied the decimals as if they were whole numbers.

• I drew a rectangle to multiply the numbers.

42 × 39 = 40 × 30 + 40 × 9 + 2 × 30 + 2 × 9 = 1200 + 360 + 60 + 18 = 1638 4.2 × 3.9 is about 4 × 4 = 16, so 4.2 × 3.9 = 16.38.	• I estimated the product and that helped me place the decimal.

Example 3 Multiplying Decimals in Parts	
Calculate 2.2 × 4.15.	
Solution 1 2.2 = 2 + 0.2 2.2 × 4.15 = (2 × 4.15) + (0.2 × 4.15) 2 × 4.15 = 8.30 0.2 × 4.15 = 0.1 × (2 × 4.15) = 0.1 × 8.30 = 0.830 8.30 + 0.830 = 9.130 2.2 × 4.15 = 9.130	**Thinking**  • I knew that 2.2 groups of 4.15 was 2 groups of 4.15 plus another 0.2 of a group, of 4.15, so I calculated them separately and then added them together.
Solution 2 $\begin{array}{r} 1 \\ \cancel{1} \\ 415 \\ \times\ \underline{22} \\ 830 \\ +\ \underline{8300} \\ 9130 \end{array}$ 2.2 × 4.14 is about 2 × 4 = 8. 2.2 × 4.15 = 9.130	**Thinking**  • I multiplied 415 by 22 and then estimated to figure out where the decimal point would be — because 2 × 4 = 8, the decimal must be after the 9 in 9130.

Practising and Applying

1. Use a hundredths grid to model each and then find each product.

a) 0.4 × 0.8

b) 0.2 × 0.9

c) 0.7 × 0.7

d) 0.7 × 0.2

2. a) Describe two ways to multiply each.

i) 0.5 × 0.8

ii) 0.6 × 0.9

iii) 0.2 × 1.4

iv) 0.6 × 1.0

b) Find each product.

3. Calculate.

a) 0.1 × 0.9

b) 0.1 × 13.5

c) 2.9 × 4.1

d) 2.9 × 4.13

e) 3.4 × 5.17

4. Why is it easy to multiply mentally by 0.1?

5. Estimate to decide where to put the decimal in each product.

a) 3.5 × 7.8 = 2730

b) 2.9 × 13.6 = 3944

c) 11.4 × 9.9 = 11286

d) 25.6 × 12.23 = 313088

6. Explain why 0.5 × 12.4 is easier to multiply mentally than 0.8 × 12.4.

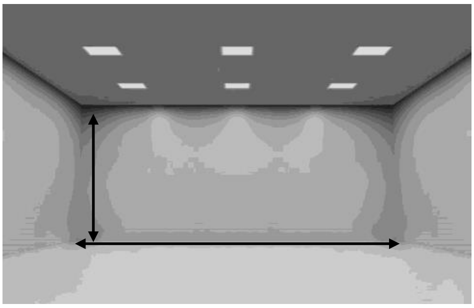
7. Place the digits 6, 7, and 8 so the product is close to 50.

6, 7, 8 → ■■ × 0.■ is about 50

8. Calculate the distance a car would travel in 3.5 h if it was travelling at a speed of 32.5 km each hour.

9. A Japanese train travelled at a speed of 317.5 km each hour for 3.25 h. About how far did it travel?

10. A wall is 3.2 m high and 5.1 m wide. What is the area of the wall?



11. An adult male is about 1.19 times the height he was at age 12. Predict the adult height of a 12-year-old boy who is 1.4 m tall.

12. a) Explain how you know that 0.■ × 0.■ is always less than 1.

b) Explain how you know that 5.■ × 4.■ is always between 20 and 30.

GAME: Target 10

Two to four players can play. You need a die.

• Each player draws four boxes with decimals and a multiplication sign:

$$\square . \square \times \square . \square$$

• To play one round, each player rolls a die four times to fill in his or her boxes with digits.

• You can wait until you have done all your rolls before you decide which digit to put in which box. Once you have recorded a digit in a box, you cannot move it.

• The player with the product closest to 10 wins 1 point for the round.

• The winner is the player with the most points after five rounds.

For example:

Player 1

2.1 × 4.5 = 9.45

About 0.5 away from 10

Player 2

6.3 × 1.4 = 8.82

More than 1 away from 10.

Player 1's product is closest to 10 so he wins 1 point for the round.

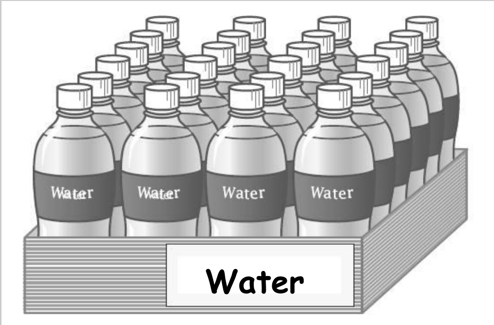
Chapter 2 Division

3.2.1 Estimating a Quotient

Try This

The organizers of an archery competition are bringing water to sell. They expect about 750 people to buy 2 bottles of water each. The water comes in cases of 24 bottles.

A. Estimate the number of cases the organizers should bring.



• When you do not need an exact answer to a problem, you can estimate.

For example, an exact answer is not necessary to solve this problem:

Each day, 8 students are selected for special responsibilities. After how many school days will each of the 338 students in the school have been selected?

• To estimate, round each number to a **multiple** of 10 or of 100 so the numbers are easy to divide. Or, round one number so it is a multiple of the other number.

For example, to solve the problem above:

Round both numbers to multiples of 10

338 ÷ 8 is about 350 ÷ 10 = 35, so it would take about 35 days.

OR

*Round the **dividend** to a multiple of 8 that is close to 338*

338 ÷ 8 is about 320 ÷ 8 = 40, so it would take about 40 days.

Both estimates, 35 days or 40 days, are reasonable solutions to the problem.

• To estimate a decimal quotient, use whole numbers or simpler decimals.

For example:

Karma travelled 72.4 km in 2.3 h. At about what speed was he travelling?

72.4 ÷ 2.3 is about 72 ÷ 2 = 36 km/h.

B. i) What division did you estimate in **part A**?

ii) What values did you use to estimate? Why?

Examples

Example Estimating a Quotient to Solve a Problem	
The perimeter of a hexagonal room is 22.7 m. All the walls are the same length. About how long is each wall?	
Solution 22.7 ÷ 6 = ■ 18 ÷ 6 = 3 24 ÷ 6 = 4 22.7 is closer to 24 than to 18. Each wall is a bit shorter than 4 m.	**Thinking** • I estimated using multiples of 6 that were close to 22.7. • I decided which multiple of 6 was closest to 22.7. 

Practising and Applying

1. Estimate each speed. Show your work.

	Distance (km)	**Time** (h)	**Speed** (kilometres in 1 hour)
a)	20.5	0.8	
b)	46.8	1.4	
c)	152.75	6.3	

2. The area and length of three different rooms are given. Estimate the width of each room.

	Area (m^2)	**Length** (m)	**Width** (m)
a)	9.8	3.6	
b)	39.2	7.4	
c)	84	11.3	

3. Which calculation would you use to estimate 12.5 ÷ 8.3? Why?

A. 13 ÷ 9

B. 13 ÷ 8

C. 12 ÷ 8

4. Fill in the blanks with two decimal numbers.

■ ÷ ■ is about 20 ÷ 5

Find two more possible pairs of decimals.

5. Lobzang estimated:

10■ ÷ 2■ is about 4.

a) Fill in the blanks with two digits to show a number situation where his estimate is reasonable.

b) Fill in the blanks with two digits to show a number situation where his estimate is not reasonable. Explain your thinking.

6. You need to estimate 3012 ÷ 11. Which is best? Explain your choice.

• increase both values

• decrease both values

• increase one, decrease the other

7. Estimate to decide which calculations are probably correct.

A. 312 ÷ 7.1 = 34.94

B. 390.7 ÷ 8 = 48.84

C. 2500 ÷ 9.12 = 374.12

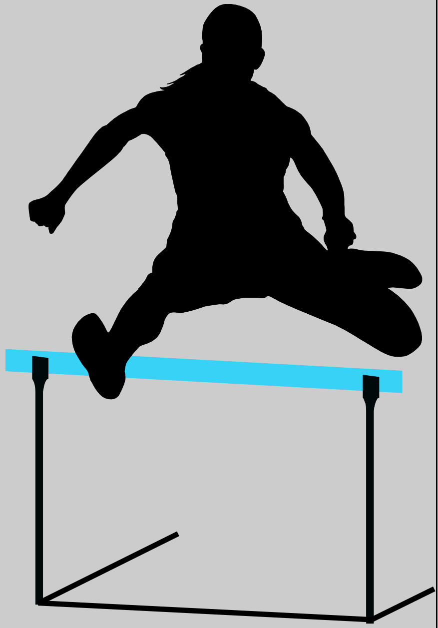
D. 96.4 ÷ 8.2 = 11.76

8. Why might you use multiplication to estimate a quotient?

3.2.2 Dividing a Decimal by a Whole Number

Try This

Pelden was in the hurdles event at a track meet. In the 100 m hurdle, runners run 13.0 m to the first hurdle. They jump over 10 hurdles that are equally spaced. The last hurdle is placed 10.5 m from the finish line.



A. i) Sketch a picture of the track.
ii) What is the distance between the hurdles?

There are several ways to divide a decimal by a whole number.

• You can use a related whole number division.

For example, to divide 36.6 ÷ 6:

Divide 366 by 6. Then insert the decimal point where it belongs.

$$\begin{array}{r} 61 \\ 6\overline{)366} \end{array} \qquad \begin{array}{r} 6.1 \\ 6\overline{)36.6} \end{array}$$

Estimate to place the decimal: Since 36 ÷ 6 = 6, the quotient should be close to 6.

36.6 ÷ 6 = 6.1

• You can use place value to rename the dividend.

For example, to divide 36.6 ÷ 6:

Think of 36.6 as a whole number of tenths or hundredths.

Tens	Ones	Tenths
3	6	6

36.6 can be renamed as 366 tenths.

36.6 ÷ 6 = 366 tenths ÷ 6 = 61 tenths = 6.1, so 36.6 ÷ 6 = 6.1.

• Another way to divide is shown on the right.

This time, 36.6 is being divided by 5.

Notice that 36.6 was renamed as 36.60.
This is because there was a remainder after the tenths were divided by 5.

```
     7.32
 5 ) 36.60
   – 35
      16
    – 15
       10
     – 10
        0
```

• You can divide a decimal by 10 or 100 just like you divide a whole number. Each digit moves one or two places to the right because the place value at right is one tenth as much.

For example: 34.6 ÷ 10 = 3.46
358.2 ÷ 100 = 3.582

B. What division describes what you did to solve the problem in **part A**?

Examples

Example Dividing a Decimal by a Multiple of Ten or Hundred

Solve each by dividing.

a) 310.2 ÷ 20 = ■ **b)** 428 ÷ 80 = ■

Solution

a) 310.2 ÷ 20 = (310.2 ÷ 2) ÷ 10

Divide by 2

```
     155.1
 2 ) 310.2
   – 2
     11
   – 10
      10
    – 10
       02
     – 2
        0
```

Divide the product by 10

155.1 ÷ 10 = 15.51

[Continued]

Thinking

a) To divide something into 20 groups, you can divide it into 2 groups and then divide each of those groups into 10 groups. So to divide by 20, I divided by 2 and then by 10.

• To divide by 10, I moved the digits one place to the right.

Example Dividing a Decimal by a Multiple of Ten or Hundred [Cont'd]

Solution

b) 428 ÷ 80 = (428 ÷ 10) ÷ 8

Divide by 10

428 ÷ 10 = 42.8

Divide by 8

$$\begin{array}{r} 5.3 \\ 8\overline{)42.8} \\ -\underline{40} \\ 28 \\ -\underline{24} \\ 4 \end{array} \qquad \begin{array}{r} 5.35 \\ 8\overline{)42.80} \\ -\underline{40} \\ 28 \\ -\underline{24} \\ 40 \\ -\underline{40} \\ 0 \end{array}$$

428 ÷ 80 = 5.35

Check:

8 × 5.35 = 42.80 = 42.8

Thinking

b) To divide by 80, I first divided by 10 (I moved the digits 2 places to the right) and then I divided by 8.

• When I divided, I had a remainder of 4 tenths. I renamed 42.8 as 42.80 so I could finish the division.

• I decided to check by multiplying.

Practising and Applying

1. a) Calculate each.

i) 25.05 ÷ 5 **ii)** 35.49 ÷ 7

iii) 94 ÷ 4 **iv)** 49.26 ÷ 6

b) Check two answers by multiplying.

2. Use mental math to calculate.

a) 4.12 ÷ 10 **b)** 389.2 ÷ 100

c) 56.7 ÷ 10 **d)** 56.7 ÷ 100

3. a) Calculate each.

i) 71.2 ÷ 20 **ii)** 452.7 ÷ 90

iii) 436.8 ÷ 80 **iv)** 486.9 ÷ 90

b) Check two answers by estimating.

4. Yangdon has 5.0 kg of rice. She keeps half for herself and divides the rest among 5 friends.

a) How much does Yangon keep?

b) How much does each friend get?

5. The perimeter of a hexagon with all equal sides is 9.0 m. How long is each side?

6. A park is 4.0 km by 4.6 km in area. It is divided into 8 equal sections. What is the area of each section?

7. A packet of rice is 821 g. It is divided into 4 equal portions. What is the mass of each portion?

8. Write a word problem that could be solved by dividing a decimal by a whole number. Solve your problem.

9. Give an example of when a 2-digit decimal divided by a 1-digit whole number results in a 3-digit decimal.

■.■ ÷ ■ = ■.■■

Reprint 2023

3.2.3 EXPLORE: Dividing by 0.1, 0.01, and 0.001

When you want to do a calculation you are not sure of, you can often relate it to a calculation you already know. Many times, you can use mental math to do that calculation.

A. i) Describe what happens to the dividend and the divisor in the divisions below. Do not complete the divisions.

4000 ÷	1000	= 4
4000 ÷	100	= ?
4000 ÷	10	= ?
4000 ÷	1	= ?
4000 ÷	0.1	= ?
4000 ÷	0.01	= ?
4000 ÷	0.001	= ?

ii) Copy and complete each division in **part i)**.

ii) What happens to the dividend when you divide by 0.1? by 0.01? by 0.001?

B. Use what you noticed in **part A** to calculate each.

i) 5000 ÷ 0.1 **ii)** 5000 ÷ 0.01 **iii)** 5000 ÷ 0.001

C. When you divide by 0.1, the quotient is 10 times the dividend. Why does that happen? (Hint: Dividing a number by 0.1 means finding how many 0.1s there are in the number.)

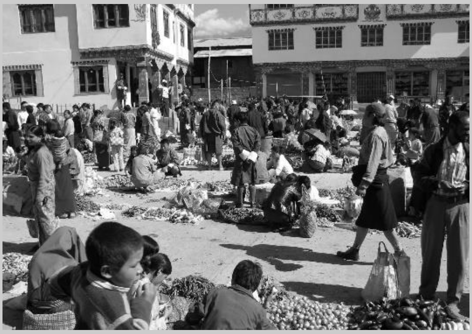
D. Describe a rule for mentally dividing each:

- a whole number by 0.1.
- a whole number by 0.01
- a whole number by 0.001

3.2.4 Dividing Decimals

Try This

At the market, you can buy packets of chilli powder that weigh 0.3 kg.



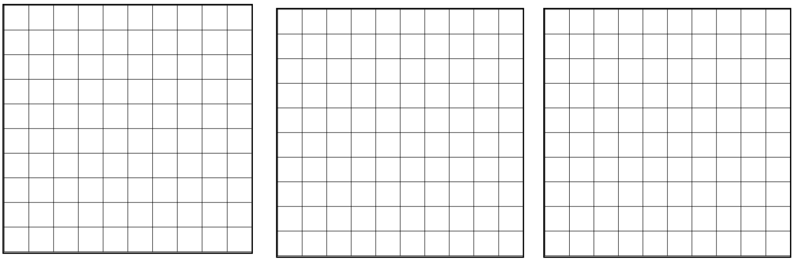
A. Estimate the number of packets that 3.6 kg of chilli powder can fill.

• To divide by a decimal, you can use hundredths grids.

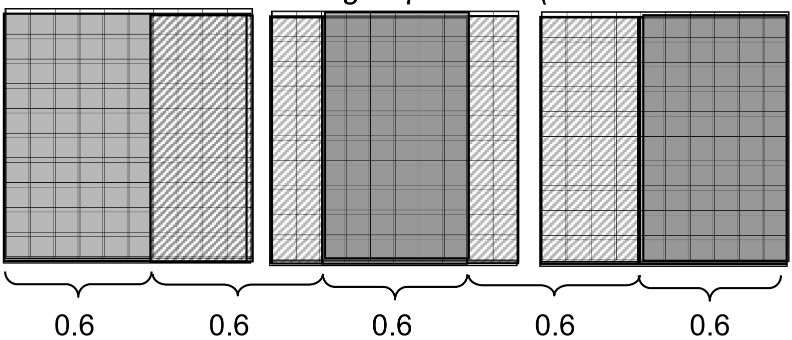
For example, to calculate 3 ÷ 0.6:

3 ÷ 0.6 means "How many groups of 0.6 are in 3?"

Represent the dividend 3 (which is 3 wholes)



Divide the 3 wholes into groups of 0.6 (0.6 is 6 tenths or 6 columns)



There are 5 groups of 0.6, so 3 ÷ 0.6 = 5.

You can rename 3 ÷ 0.6 as 30 tenths ÷ 6 tenths, which means "How many groups of 6 tenths are in 30 tenths?"

3 ÷ 0.6 = 30 tenths ÷ 6 tenths = 5

• Another way to calculate 3 ÷ 0.6 is to think of 0.6 as 0.1 × 6. You can divide by 0.1 and then divide the quotient by 6.

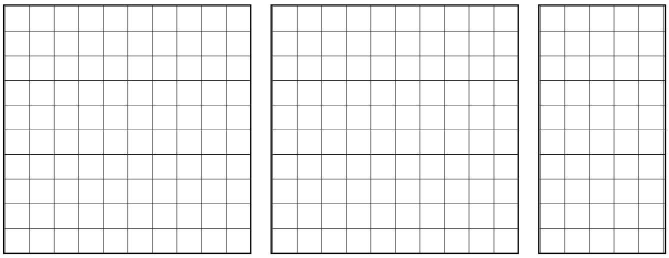
For example:

3 ÷ 0.6 → 3 ÷ 0.1 = 30 → 30 ÷ 6 = 5 So 3 ÷ 0.6 = 5.

For example, to divide 2.5 ÷ 0.45:

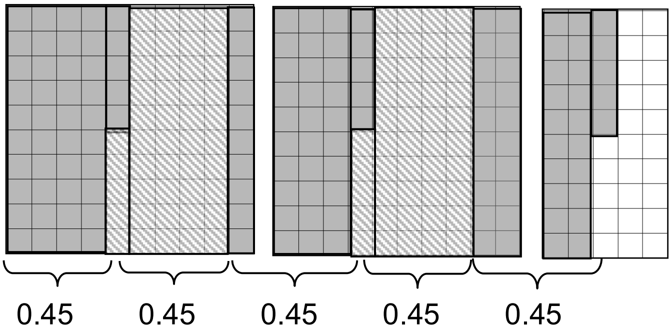
2.5 ÷ 0.45 means "How many groups of 0.45 are in 2.5?"

Represent the dividend, 2.5



2.5 is 2 whole grids and half a grid.

Divide 2.5 into groups of 0.45



There is 0.25 left over.

There are 5 groups of 0.45 and 25 small squares left over.

25 small squares is $\frac{25}{45}$ of a group of 0.45.

So 2.5 ÷ 0.45 is $5\frac{25}{45}$.

• Another way to divide by a decimal is to do an equivalent division using whole numbers. To create an equivalent division, you multiply both the dividend and the divisor by the same amount.

For example, for 4.5 ÷ 2.5:

4.5 × 10   2.5 × 10

$4.5 \div 2.5 = 45 \div 25 \quad \rightarrow \quad 45 \div 25 = 1\frac{20}{25} = 1\frac{80}{100} = 1.80$

B. i) Calculate an exact answer for the problem in **part A**.
ii) What is an equivalent whole number division?

Examples

Example 1 Dividing Decimals to Solve a Problem

Eden has 4.2 m of ribbon. How many pieces of each length can she cut?

a) 0.3 m **b)** 0.28 m

Solution

a) 4.2 ÷ 0.3

4.2 = 42 tenths and 0.3 = 3 tenths

4.2 ÷ 0.3 = 42 tenths ÷ 3 tenths = 14

She can make 14 pieces.

b) $4.2 \div 0.28 = 4.2 \times 100 \div 0.28 \times 100$

$= 420 \div 28$

$= 15$

She can make 15 pieces.

Thinking

a) I renamed 4.2 and 0.3 as tenths so I could divide whole numbers.

• 4.2 = 42 tenths. Each 1 is 10 tenths, so 4 is 40 tenths and 0.2 is 2 tenths.

b) I created an equivalent division using whole numbers by multiplying each number by 100.

Practising and Applying

1. Use hundredths grids to model and find each quotient.

a) 2.7 ÷ 0.9 **b)** 3.6 ÷ 1.8

c) 1.4 ÷ 0.2 **d)** 1.5 ÷ 0.4

2. Calculate each.

a) 2.7 ÷ 0.09 **b)** 4.9 ÷ 0.4

c) 14.4 ÷ 3.2 **d)** 1.8 ÷ 0.15

3. Tshering cuts 11.4 m of rope into equal pieces. How many pieces of each length can he make?

a) 80 cm **b)** 0.2 m

c) 0.6 m **d)** 1.25 m

4. A train travels 524.86 km in 5 h. What is its average speed (the number of kilometres in 1 h)?

5. How many 0.35 L glasses can you fill from a 1.75 L bottle of water?

6. Without calculating, explain how you know each is true.

a) 1.2 ÷ 0.3 = 12 ÷ 3

b) 1.2 ÷ 0.03 = 120 ÷ 3

c) 1.2 ÷ 0.03 = 10 × (1.2 ÷ 0.3)

7. Meghraj divided like this.

$$1.05\overline{)2.1} \rightarrow 105\overline{)210}$$

Do you agree with what he did? Explain your thinking.

8. Lhakpa divided like this.

$$\begin{array}{r} 3.2 \\ 0.6\overline{)2.0} \end{array}$$

Do you agree with what he did? Explain your thinking.

9. Explain what 3.4 ÷ 0.02 means. Describe how to calculate it.

Chapter 3 Combining Operations

3.3.1 Order of Operations

Try This

To enter some contests you have to answer a skill-testing mathematical question.

A. What is the answer to this skill-testing question?

$$3 \times 5.2 + 20.5 \div 0.5$$

• When a number expression involves a lot of computations, sometimes the order in which you do the computations matters and sometimes it does not.

For example:

To calculate 13.2 + 24.6 – 6.8, the order does not matter:

$$\left.\begin{array}{l} 13.2 + 24.6 - 6.8 = 37.8 - 6.8 = 31 \\ 24.6 - 6.8 + 13.2 = 17.8 + 13.2 = 31 \\ 13.2 - 6.8 + 24.6 = 6.4 + 24.6 = 31 \end{array}\right\} \text{You get the same result each time, 31.}$$

To calculate 2.5 × 3.1 + 2.9, the order does matter:

$$\left.\begin{array}{l} 2.5 \times 3.1 + 2.9 = 7.75 + 2.9 = 10.65 \\ 3.1 + 2.9 \times 2.5 = 6 \times 2.5 = 15 \end{array}\right\} \text{The results are not the same.}$$

• This is why there are rules that tell you the order to use. The rules are called the **order of operations**. We use them so everyone will get the same answer for the same calculation.

• Here are the rules:

Do any calculations inside the **Brackets** first.

Do **Multiplication** and **Division** next, in order, from left to right.

Do **Addition** and **Subtraction** last, in order, from left to right.

For example, to calculate 5.3 × (2.4 + 3.6) – 2.3:

Step 1: 2.4 + 3.6 = 6

Step 2: 5.3 × 6 = 31.8

Step 3: 31.8 – 2.3 = 29.5

$$5.3 \overset{2}{\times} (2.4 \overset{1}{+} 3.6) \overset{3}{-} 2.3$$

B. i) Why might someone get a different answer to **part A**?

ii) What is the correct answer?

Reprint 2023

Examples

Example Solving a Problem involving Order of Operations

a) Calculate 14 – 3.5 × (9.4 ÷ 4.7).

b) Remove the brackets and re-calculate. What do you notice?

c) Add brackets to 14 – 3.5 × 9.4 ÷ 4.7 to change its value.

Solution	**Thinking**
a) 14 – 3.5 × (9.4 ÷ 4.7) = 14 – 3.5 × (94 ÷ 47) = 14 – 3.5 × 2 = 14 – 7 = 7	 **a)** First I did what was in the brackets. Then I did the multiplication and finally the subtraction.
b) 14 – 3.5 × 9.4 ÷ 4.7 = 14 – 3.5 × 94 ÷ 47 = 14 – 329 ÷ 47 = 14 – 7 = 7	**b)** Even though I did the multiplication before the division, I got the same answer. I guess the brackets weren't necessary.
c) (14 – 3.5) × 9.4 ÷ 4.7 = 10.5 × 9.4 ÷ 4.7 = 98.7 ÷ 4.7 = 21	**c)** I put brackets around 14 – 3.5 so it would be done first instead of last. That's why I got a different answer.

Practising and Applying

1. Calculate each.

a) 3.2 × 1.5 + 4.3

b) 2.4 + 3 ÷ 1.5

c) 2.4 + (12 – 9) ÷ 1.5

d) 4.8 – 2.4 × 2 + 9.1 ÷ 1.3

2. Which calculations give the same result as 4.8 – 2.4 × 2 + 9.1 ÷ 1.3?

A. (4.8 – 2.4 × 2 + 9.1 ÷ 1.3)

B. 4.8 – (2.4 × 2) + (9.1 ÷ 1.3)

C. (4.8 – 2.4) × 2 + 9.1 ÷ 1.3

D. 4.8 – (2.4 × 2 + 9.1) ÷ 1.3

3. Write a calculation for this:

• Add 3.5 and 6.5.

• Divide the result by 0.2.

• Then add 4.2.

Calculate to find the answer.

4. Which expressions have unnecessary brackets?

A. 20.5 + 3.8 – 7.8 × (5.4 ÷ 9)

B. (20.5 + 3.8 – 7.8) × 5.4 ÷ 9

C. (20.5 + 3.8) – 7.8 × 5.4 ÷ 9

D. (20.5 + 3.8 – 7.8 × 5.4 ÷ 9)

5. Copy and complete each statement with two operation signs.

a) 1.2 □ 3 □ 2 = 0.2

b) 1 □ (3 × 3 □ 1) = 0.1

6. Suppose you ignore the order of operations.

a) Show how to get two different answers for each.

i) 6 + (12.5 + 5) × 4 ÷ 5

ii) 2.2 – 0.9 × 0.2 – 0.03

b) What is the correct answer for each? Show your work.

Reprint 2023

3.3.2 Solving a Problem Using All Four Operations

Try This

It takes 0.4 m of plastic to make a hair band and 0.25 m to make a bracelet.

A. How many more bracelets than hair bands can be made with 8 m?

• When you solve a problem that involves more than one operation, you have to decide which operations to use and in what order to do them.

For example:

Yangchen is 1.5 times as tall as her sister Yuden.
Yuden is 0.4 m taller than their sister Kamala.
If Kamala is 0.35 m tall, how tall is Yangchen?

First calculate Yuden’s height by adding because you know Kamala's height and you know that Yuden is 0.4 m taller:

0.35 + 0.4 = 0.75

Then multiply Yuden’s height by 1.5 because you now know Yuden's height and you know that Yangchen is 1.5 times taller:

1.5 × 0.75 = 1.125

Yangchen is 1.125 m tall.

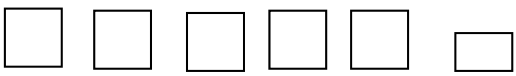
B. i) Which operations did you use in **part A**?
ii) In what order did you use them?

Examples

Example Solving a Decimal Problem With Multiple Operations

Six packets have a total mass of 4 kg. Five of the packets have the same mass. The sixth packet has a mass that is 0.2 kg less than each of the other five packets. What is the mass of the sixth packet?

Solution



If the sixth packet had a mass that was 0.2 kg heavier, there would be 6 packets with a total mass of 4 + 0.2 kg = 4.2 kg.

Each packet would be 4.2 ÷ 6 = 0.7 kg.
The lighter packet is 0.7 – 0.2 = 0.5 kg.

Thinking

• I drew a picture to help me figure out the problem. I used rectangles to represent the packets.

• I first solved a simpler related problem. That helped me solve the problem.

Reprint 2023

Practising and Applying

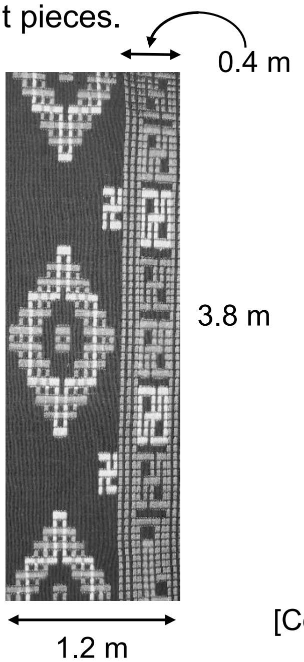
***Show your work each time*.**

1. A ball of wire has a mass of 57.3 g. An 8 cm piece of the wire is 3 g. About how long is the wire that forms the ball?

2. About how many more human babies would fit along the length of the blue whale than along the length of the sperm whale?

Animal	**Length** (m)
Largest blue whale	33.58
Largest sperm whale	20.7
Newborn human baby	0.5

3. A piece of fabric is 3.8 m long and 1.2 m wide. A 0.4 m strip is cut along the length to use as a border. The remaining fabric is divided into 8 congruent pieces.



[Cont'd]

a) What is the area of each piece?

b) What is the area of the border?

4. Kinley's bank account contained Nu 432.56. Six months later, after earning interest, it contains Nu 443.37. Estimate the amount of interest he earned each month.

5. A 67.2 km trek lasted 9 days. The trekkers travelled the same distance on each of the first 8 days. On the 9th day, they travelled 3 km less than on the other days. How many kilometres did they travel each day?
Show your work.

6. Write a word problem that you could solve using both addition and multiplication of decimals. Solve your problem.

7. Choose one question from **questions 1 to 5**. Tell how you knew which operations to perform and in what order.

CONNECTIONS: Decimal Magic Squares

In a Magic Square, the numbers in each row, column, and diagonal add to the same amount, called the magic sum.

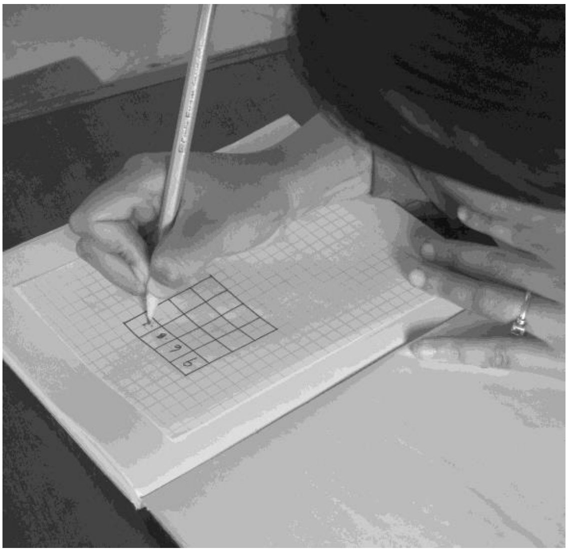
Here is an example of a Magic Square:

0.9	0.6	0.3	1.6
0.4	1.5	1	0.5
1.4	0.1	0.8	1.1
0.7	1.2	1.3	0.2

1. What is the magic sum for the Magic Square above?

2. Divide each number in the square by 0.1 to create a new Magic Square. Is it still a Magic Square? If so, what is the magic sum?

3. Multiply each number in the original square by 1.1 to create a new Magic Square. Is it still a Magic Square? If so, what is the magic sum?



UNIT 3 Revision

1. There are 154 h until Lemo's birthday. Estimate how long this is in each. Show your work.

a) days **b)** minutes

2. Estimate each distance. Show your work.

Time (h)	**Speed** (kilometres in 1 h)	**Distance** (km)
a) 3.2	22.5	
b) 8.1	28.4	
c) 5.9	3.14	

3. One or more answers is incorrect. Estimate to decide which one(s).

A. 1059 × 36 = 38,124

B. 5.19 × 3.7 = 29.203

C. 8.034 × 230 = 847.82

4. Use a place value chart to show how you know each is true.

a) 5 × 7.125 = 35.625

b) 8 × 12.219 = 97.752

5. Multiply.

a) 10 × 3.56 **b)** 100 × 17.204

c) 50 × 22.38 **d)** 300 × 16.24

6. A recipe that serves 5 people uses 0.625 kg of meat. How many kilograms of meat are needed to serve 25 people?

7. a) Calculate each.

i) 0.4 × 0.7 **ii)** 0.2 × 0.8

iii) 0.7× 1.7 **iv)** 0.8 × 8.4

b) Choose one calculation. Describe two ways to calculate it.

8. Estimate to decide where to place the decimal in each product.

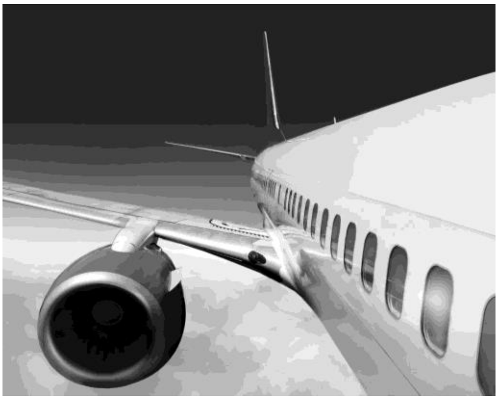
a) 4.2× 9.1 = 3822

b) 3.7 × 25 = 925

c) 3.6 × 9.7 = 3492

d) 18.2 × 15.6 = 28392

9. A plane flew at a speed of 555.5 km each hour for 2.4 h. Calculate the distance it travelled.



10. Copy and complete using the digits 4, 7, and 9.

■■ × 0.■ is about 30.

11. Estimate each speed.

Distance (km)	**Time** (h)	**Speed** (kilometres in 1 h)
a) 15.6	0.6	
b) 56.9	2.3	
c) 122.74	4.5	

12. Estimate to decide which calculations are probably correct.

A. 420 ÷ 1.4 = 30

B. 528.6 ÷ 6 = 88.1

C. 437.58 ÷ 7.8 = 156.1

D. 105.8 ÷ 9.2 = 11.5

13. Calculate each mentally.

a) 3.2 ÷ 10 **b)** 142.6 ÷ 100

c) 23.7 ÷ 100 **d)** 49.1 ÷ 100

14. a) Calculate each.

i) 43.2 ÷ 30 **ii)** 302.4 ÷ 50

iii) 177.1 ÷ 70 **iv)** 418.8 ÷ 60

b) Check two answers by multiplying.

15. A 617 g packet of rice is divided into 5 equal portions. How many grams are in each portion?

16. Explain how you know that 3.2 ÷ 0.1 = 10 × 3.2.

17. Calculate each.

a) 3.8 ÷ 0.02 **b)** 6.37 ÷ 0.7

c) 22.5 ÷ 1.5 **d)** 4.9 ÷ 0.25

18. About how many 0.35 L glasses can you fill from a 2.5 L bottle of water?

19. List three calculations using division that have the same quotient as 4.2 ÷ 0.7.

20. Calculate each.

a) 25 – 5.4 × 2.5

b) 6 × (4.3 + 2.7) ÷ 2

c) 3.1 × 3.1 – 2.8 × 2.8

21. Which expressions do not need the brackets?

A. (8 ÷ 4.6 + 3.9) × 4.3 × 5.2

B. 8 ÷ 4.6 + (3.9 × 4.3) × 5.2

C. (8 ÷ 4.6) + (3.9 × 4.3 × 5.2)

22. Copy and complete each statement with operation signs and brackets.

a) 13.5 ☐ 1.5 ☐ 2 = 30

b) 10 ☐ 2 × 1.2 ☐ 9 = 5.4

23. Car A travels 28 km each hour. Car B travels 32 km each hour. How much farther can Car B travel than Car A in 2.2 h?

24. At age 9, a man was 0.75 of his adult height. His adult height is 1.72 m. He was 0.53 m long at birth. How much did the man grow between birth and age 9? Show your work.

UNIT 4 RATIO, RATE, AND PERCENT

Getting Started

Use What You Know

A. i) Use grey squares and white squares to make a pattern.
Use twice as many grey squares as white squares. Sketch your pattern.

ii) How many grey squares did you use?

iii) How many white squares did you use?

iv) How many squares in total did you use?

B. Repeat **part A** two more times. Use a different total number of squares each time, but still use twice as many grey squares as white squares.

C. i) In **parts A and B**, what do you notice about the number of grey squares each time?

ii) What do you notice about the total number of squares each time?

D. Repeat **parts A to C**, but use three times as many grey squares as white squares.

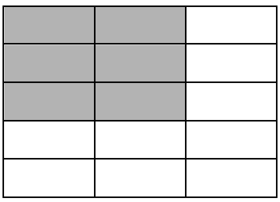
E. i) Use grey squares and white squares to make a pattern. $\frac{4}{5}$ of the squares should be grey. Sketch your pattern.

ii) How many grey squares are there for each white square? How do you know?

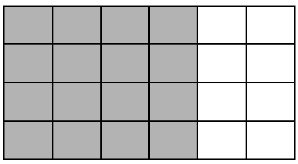
Skills You Will Need

1. What fraction is shaded in each?

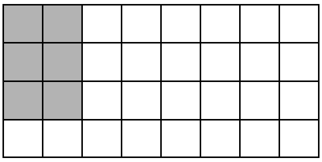
a)

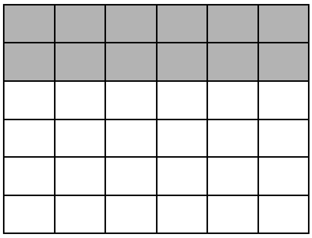


b)



Reprint 2023

c) 

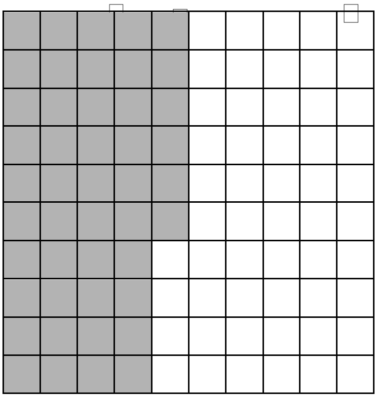
d) 

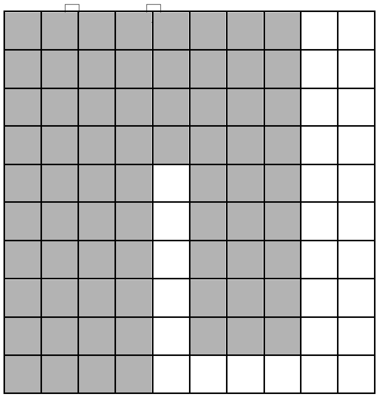
2. Draw a picture to show that $\frac{3}{4} = \frac{6}{8}$.

3. What is the missing number in each fraction?

a) $\frac{2}{3} = \frac{[]}{9}$ **b)** $\frac{3}{5} = \frac{12}{[]}$ **c)** $\frac{[]}{32} = \frac{7}{8}$ **d)** $\frac{[]}{50} = \frac{9}{10}$

4. What decimal does each hundredths grid show?

a) 

b) 

5. Use a hundredths grid to model each decimal.

a) 0.23 **b)** 0.56 **c)** 0.98 **d)** 0.03

6. Order these decimals from least to greatest.

0.43, 0.58, 0.45, 0.85

7. Rename each fraction as a decimal.

a) $\frac{7}{10}$ **b)** $\frac{1}{2}$ **c)** $\frac{8}{100}$ **d)** $\frac{6}{50}$

Reprint 2023

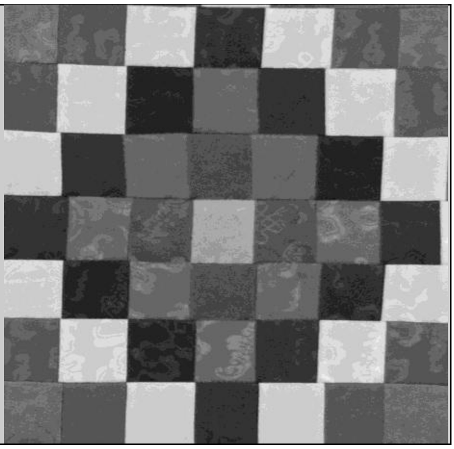
Chapter 1 Ratio and Rate

4.1.1 Introducing Ratios

Try This

There are 49 squares in this design.

A. What fractions can you use to describe the squares? Think about the colours of the squares and about rows and columns.



• A **ratio** is a way to compare numbers.

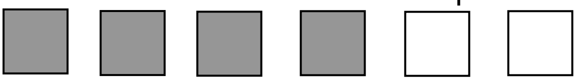
For example:

The ratio 4 : 2 compares the numbers 4 and 2. It is read as "four to two."

• There are part-to-part ratios and part-to-whole ratios.

For example:

There are four ratios that describe this set of squares.



Ratio	Type	Compares
4 : 2	part-to-part	grey squares to white squares
2 : 4	part-to-part	white squares to grey squares
4 : 6 or $\frac{4}{6}$	part-to-whole	grey squares to total squares
2 : 6 or $\frac{2}{6}$	part-to-whole	white squares to total squares

You can use a fraction to express a part-to-whole ratio because that is what a fraction is — a part-to-whole relationship.

• The parts of a ratio are called **terms**. It is important to know what each term represents.

For example:

The ratio 21 : 17 compares the number of boys to the number of girls in a class. To understand what the ratio tells you about the class, you need to know which term represents the boys and which term represents the girls.

Reprint 2023

• You can use a ratio to describe a comparison made with words.

For example:

Suppose a number is 2 times as much as another number. You can describe the relationship between these two numbers with the ratio 2 : 1.

• Ratios are part of our everyday lives.

For example:

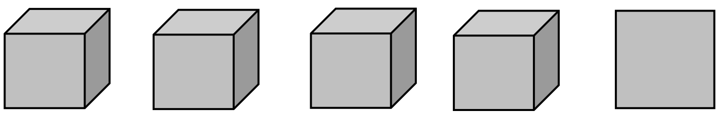
- If a recipe calls for 3 cups of flour for every 1 cup of sugar, then the ratio of flour to sugar is 3 : 1.
- If a rectangle is 2 times as long as it is wide, then the length-to-width ratio is 2 : 1.
- The ratio of males to females in Bhutan is reported to be 105 : 100. That means there are 105 males for every 100 females.
- In the air we breathe, the ratio of nitrogen to oxygen is about 4 : 1. That means there is 4 times as much nitrogen as oxygen in the air.

B. What ratios could you use to describe the squares in the design in **part A**?

Examples

Example Describing a Situation with Many Ratios

Describe this group of shapes using as many ratios as you can.



Solution	**Thinking**
Part-to-part ratios for the group Cubes to squares = 4 : 1 Squares to cubes = 1 : 4 *Part-to-whole ratios for the group* Cubes to all shapes = 4 : 5 Squares to all shapes = 1 : 5	 • I thought of the group of 5 shapes as the whole. Then I used part-to-part and part-to-whole ratios.
Part-to-part for each cube Edges to vertices: 12 : 8 *Part-to-part for the square* Sides to vertices = 4 : 4	• Next, I thought of the parts of each shape: - cubes: edges and vertices - squares: sides and vertices Then I created part-to-part ratios.

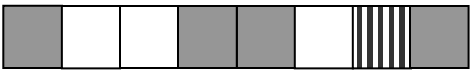
Practising and Applying

1. Write each ratio.

a) footballs to basketballs

b) footballs to total balls

2. Which squares (white, grey, and striped) are compared by each ratio?



a) 4 to 3

b) 1 : 4

c) $\frac{3}{8}$

d) 1 : 3

3. Sketch two different pictures that show the ratio 5 : 6.

4. The ratio 32 : 7 compares the residential students to the other students in a class.

a) Is this a part-to-part or a part-to-whole ratio? How do you know?

b) How many students are in the class?

5. Karma mixed different ratios of green and white paint to make four different shades of green. Which paint has the lightest shade? How do you know?

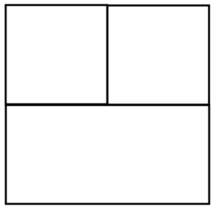
Shade of green paint	Cans of green	Cans of white
A	3	0
B	3	1
C	3	2
D	3	3

6. a) Draw a rectangle. Then draw another rectangle that is twice as long and twice as wide.

b) What ratio compares the length of the small rectangle to the length of the large rectangle?

c) What ratio compares the area of the small rectangle to the area of large rectangle?

7. a) What is the ratio of the number of small squares to the number of non-square rectangles in this picture?



b) What is the ratio of the area of one small square to the area of the non-square rectangle?

8. Duptho lives with his mother, his father, two sisters, and one brother. Use as many ratios as you can to describe his family.

9. a) Use three ratios to describe things or people in your class.

b) Name three other ratios that describe things or people in your life.

10. Can you always use more than one ratio to describe a situation that has two parts? Use an example to help you explain.

Reprint 2023

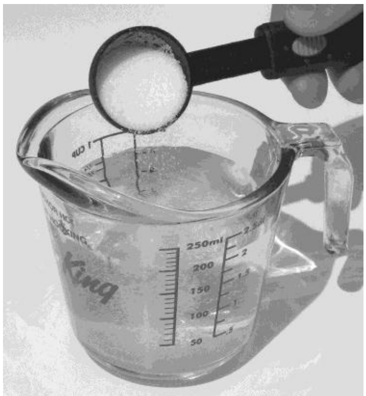
4.1.2 Equivalent Ratios

Try This

Tandin puts 15 mL of sugar into 200 mL of water to sweeten it.

A. i) If he puts 15 mL of sugar into 400 mL of water, will this water be as sweet as the 200 mL of water above?

ii) How many millilitres of sugar does he need to put into 400 mL of water for it to have the same sweetness as the 200 mL of water?



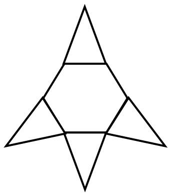
• A number can be renamed in different names.

For example: 200 = 2 × 100 200 = 4 × 50 200 = 300 – 100

A ratio can also have different names.

For example:

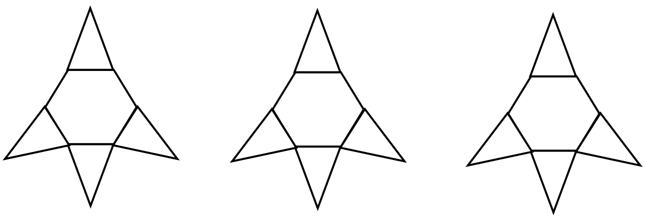
The ratio that compares the number of triangles to the number of hexagons in this design is 4 : 1.



4 triangles to 1 hexagon, or 4 : 1

4 : 1 means there are 4 times as many triangles as hexagons

In three copies of the design, the ratio of triangles to hexagons is 12 : 3, but there are still 4 times as many triangles as hexagons, so 4 : 1 also describes the design below.



12 triangles to 3 hexagons, or 12 : 3

There are still 4 times as many triangles as hexagons, so 12 : 3 = 4 : 1.

12 : 3 and 4 : 1 are **equivalent ratios** because both ratios describe the relationship between the triangles and the hexagons in the group of designs.

• Equivalent ratios such as 12 : 3 = 4 : 1 both name the same relationship, but in a different way. They are like equivalent fractions, such as $\frac{12}{3} = \frac{4}{1}$.

Reprint 2023

• It is often easier to understand a ratio when it is in **lower terms**. You can change a ratio to lower terms by dividing each term by the same value.

For example:

With the ratio 12 : 3, you can divide 12 by 3 and then divide 3 by 3 to get the ratio in lower terms, 4 : 1. Now it is easy to tell that there are 4 triangles for every 1 hexagon, or 4 times as many triangles as hexagons.

• To find an equivalent ratio, you can multiply or divide each term by the same amount.

For example:

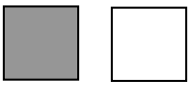
Some equivalent ratios for 6 : 8 are	3 : 4	6 ÷ 2 : 8 ÷ 2 = 3 : 4
	12 : 16	6 × 2 : 8 × 2 = 12 : 16
	600 : 800	6 × 100 : 8 × 100 = 600 : 800

You cannot make an equivalent ratio by adding or subtracting the same amount to each term; only multiplying or dividing will work.

For example:

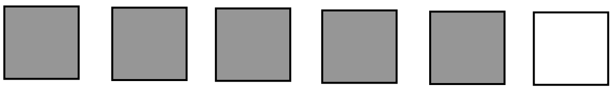
If the ratio of grey to all squares in one design is 1 : 2, this is a different ratio than 5 : 6 (1 + 4 : 2 + 4 → 5 : 6).

Ratio of 1 : 2



There are as many grey squares as white squares.

Ratio of 5 : 6



There are 5 times as many grey squares as white squares.

B. What equivalent ratios did you use to solve the problem in **part A ii)**?

Examples

Example 1 Creating Equivalent Ratios

A recipe for the dough for Haapai Hantue uses 1 cup of buckwheat flour and 2 cups of white flour.

a) What fractions describe the ratios for the flour?

b) If you use 3 cups of buckwheat flour, how much white flour do you need?

Solution	Thinking
a) Buckwheat flour to total flour is 1 : 3 or $\frac{1}{3}$. White flour to total flour is 2 : 3 or $\frac{2}{3}$. [Continued]	**a)** I knew I could use fractions to describe part-to-whole ratios. • The total amount of flour was 3 cups, so I knew the denominator was 3. 

Example 1 Creating Equivalent Ratios [Continued]

Solution	Thinking
b) $1 : 2 = 3 \times 1 : 3 \times 2$ $= 3 : 6$ 6 cups of white flour	**b)** I multiplied 1 cup of buckwheat flour by 3 to get 3 cups, so I also had to multiply 2 cups of white flour by 3 to make the ratio equivalent. 

Example 2 Creating an Equivalent Ratio to Meet a Condition

What is each missing term? **a)** $10 : 16 = 20 : \square$ **b)** $10 : 16 = 15 : \square$

Solution	Thinking
a) $10 : 16 = 10 \times 2 : 16 \times 2$ $= 20 : 32$ **b)** $10 : 16 = 10 \div 2 : 16 \div 2$ $= 5 : 8$ $5 : 8 = 5 \times 3 : 8 \times 3$ $= 15 : 24$	**a)** 20 is 2 × 10, so the second term has to be 2 × 16 for the ratios to be equivalent.  **b)** I didn't know what to multiply 10 by to get 15, so I wrote the ratio in lower terms. Then I saw that I could multiply both terms by 3, so I knew the second term was 24.

Practising and Applying

1. Which pairs are equivalent ratios?

A. 2 : 3 and 6 : 8

B. 6 : 7 and 12 : 14

C. 5 : 8 and 7 : 10

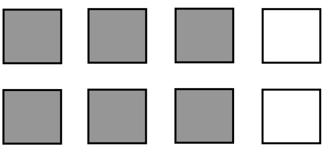
D. 8 : 10 and 4 : 5

2. What is each missing term?

a) $30 : 4 = 15 : \square$ **b)** $4 : 5 = \square : 20$

c) 1 to 1 = $\square$ to 8 **d)** $7 : \square = 4 : \square$

3. a) Does this picture show that 3 : 4 = 6 : 8? How do you know?



b) Change the picture to show two other equivalent ratios. Explain what you did and why.

4. Draw a picture to show 3 : 5 = 6 : 10. Explain how your picture shows equivalent ratios.

5. Tashi says you can add the same amount to each term in a ratio to find an equivalent ratio. Do you agree? Explain your thinking.

6. Write a ratio equivalent to each.

a) 1,000,000 : 1,000,000,000

b) 10 : 10,000

7. The ratio of the length of one side of a shape to its perimeter is 1 : 4. Sketch two possible shapes with this ratio and show their dimensions.

8. The difference between the terms of a ratio is 5. Can a ratio that is equivalent to it have a difference of 8 between the terms? How do you know?

9. How are equivalent ratios like equivalent fractions?

10. Why might a ratio in lower terms be easier to understand?

4.1.3 Comparing Ratios

Try This

There are 19 boys and 23 girls in Choki's class. A new student will soon join the class.

A. What will be the ratio of boys to girls in the class in each case?

i) if the new student is a boy **ii)** if the new student is a girl

• To compare two part-to-whole ratios, you can write them as fractions and then compare them.

For example:

One measure of fitness is to compare a person's body fat mass to his or her total mass. A low ratio of body fat mass to total mass is an indication of fitness.

A typical female has a ratio of body fat mass to total mass of 3 : 10.
A female athlete has a ratio of body fat mass to total mass of 4 : 25.

Who has a lower ratio of body fat mass to total mass?

- Write each as a fraction: $3 : 10 = \frac{3}{10}$ and $4 : 25 = \frac{4}{25}$

- Compare them using equivalent fractions: $\frac{3}{10} = \frac{30}{100}$ and $\frac{4}{25} = \frac{16}{100}$

$$\frac{16}{100} < \frac{30}{100}$$

Since $\frac{16}{100} < \frac{30}{100}$, then 4 : 25 < 3 : 10. The athlete has a lower ratio of body fat mass to total mass. We say that her **proportion** of body fat is lower.

• To compare two part-to-part ratios, you first write the related part-to-whole ratio for each as a fraction and then you compare the two fractions.

For example:

Two colours of paint have been mixed in these ratios:
Can 1 has a yellow-to-blue ratio of 3 : 2.
Can 2 has a yellow-to-blue ratio of 4 : 2.

Which can has the lower proportion of blue?

Can 1: a ratio of 3 yellow to 2 blue means $\frac{2}{5}$ is blue.

Can 2: a ratio of 4 yellow to 2 blue means $\frac{2}{6}$ is blue.

Since $\frac{2}{6} < \frac{2}{5}$, Can 2 has a lower proportion of blue than Can 1.

In the example below, none of the terms in the ratios are the same:
Two colours of paint have been mixed in these ratios:
Can 1 has a yellow-to-blue ratio of 1 : 1.
Can 2 has a yellow-to-blue ratio of 2 : 4.
Which can has the higher proportion of yellow?

Can 1: a ratio of 1 yellow to 1 blue means $\frac{1}{2}$ is yellow.

Can 2: a ratio of 2 yellow to 4 blue means $\frac{2}{6}$ or $\frac{1}{3}$ is yellow.

Since $\frac{1}{2} > \frac{1}{3}$, Can 1 has a higher proportion of yellow than Can 2.

• It only makes sense to compare ratios when they describe similar things.
For example, you might compare:
- the ratio of boys to girls in two different classes
- the ratio of flour to sugar in two different recipes
- the ratio of archers to football players in two different groups of athletes

B. Compare the original boy-to-girl ratio of 19 : 23 to the two ratios you created in **part A**. What do the comparisons tell you?

Examples

Example Comparing Situations Using Ratios

Dechen and Sonam make their butter tea in different ways:
- Dechen adds 1 tablespoon of black tea and 2 tablespoons of butter to a cup of milk.
- Sonam adds 1 tablespoon of black tea and 1 tablespoon of butter to a cup of milk.

Whose tea has a higher proportion of butter to tea?

Solution

Dechen

2 butter to 1 tea means $\frac{2}{3}$ is butter.

Sonam

1 butter to 1 tea means $\frac{1}{2}$ is butter.

$\frac{2}{3} > \frac{1}{2}$

Dechen's tea has a higher proportion of butter to tea.

Thinking

• The butter to tea ratios in the mixture were part-to-part ratios.
I changed them to part-to-whole ratios so I could compare them.

• The denominator of each part-to-whole ratio is the sum of the parts in each mixture.

Practising and Applying

1. In Class A, there are 18 boys and 22 girls. In Class B, there are 22 boys and 17 girls.

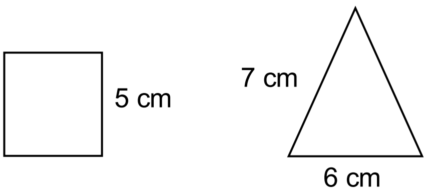
a) Which class has the higher ratio of boys to girls? What is the ratio?

b) Which class has the lower ratio of boys to girls? What is the ratio?

2. Paint is made using different proportions of green and white. Which paint is darkest? How do you know?

	Green part	White part
A	3	2
B	3	4
C	2	4
D	6	9

3. Which shape has a higher ratio of the length of the longest side to the perimeter? Show your work.



Square Isosceles triangle

4. In a group of 20 boys, 12 play sports. In a group of 15 girls, 9 play sports. Which group has the higher ratio of sports players? Show your work.

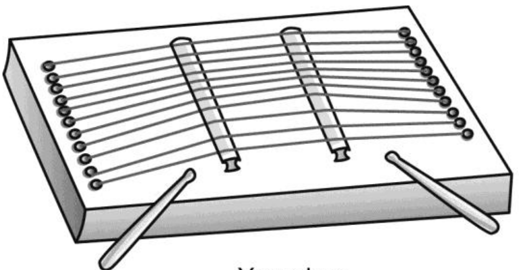
5. Tenzin's family sells nuts at the market.

• In Package A, there are 200 g of groundnuts and 150 g of cashews.

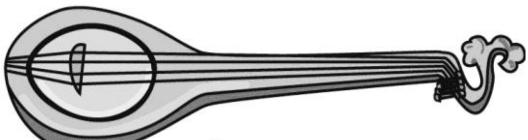
• In Package B, there are 30 g of groundnuts and 10 g of cashews.

Which package has the higher proportion of groundnuts? Show your thinking.

6. In one school music club, the ratio of dramnyen players to yangchen players is 32 : 8. In a second club, the ratio of dramnyen players to yangchen players is 25 : 5. Which music club has the higher proportion of dramnyen players? Show your work.



Yangchen



Dramnyen

7. When might it be useful to compare two ratios?

Reprint 2022

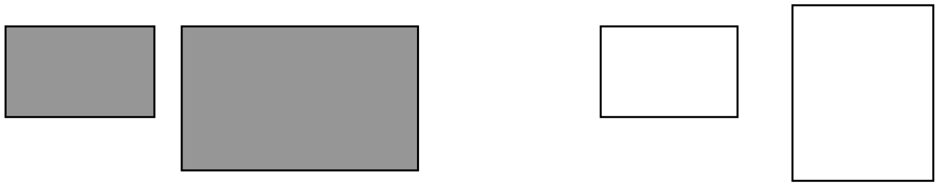
Reprint 2023

4.1.4 EXPLORE: Similarity

• Two shapes are **similar** if one shape looks like an **enlargement** or a **reduction** of the other shape.

For example:

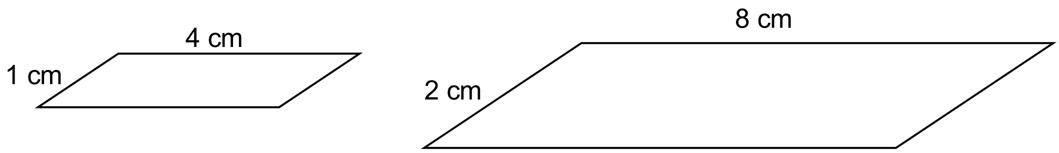
The grey rectangles are similar, but the white rectangles are not similar.



• When shapes are similar, the ratios of the lengths of their corresponding sides are equivalent.

For example:

These parallelograms are similar because 1 : 2 = 4 : 8.



A. i) Draw Rectangle A with dimensions 8 cm and 6 cm. Then draw a rectangle that is similar to Rectangle A. Call it Rectangle B.

ii) What is the ratio of the lengths of the corresponding sides in lower terms?

iii) Measure one diagonal in each rectangle. What is ratio of the lengths of the diagonals in lower terms?

iv) How does the ratio in **part iii)** compare to the ratio in **part ii)**?

B. i) Draw two similar rectangles, Rectangle C and Rectangle D. Rectangle D should have side lengths that are 3 times as long as the corresponding sides of Rectangle C.

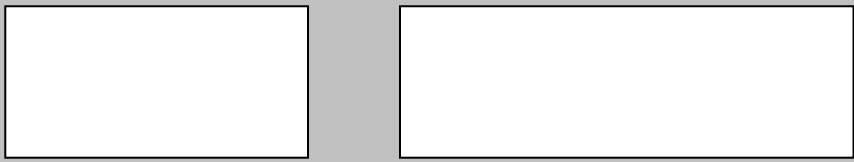
ii) What is the ratio of the lengths of the corresponding sides in lower terms?

iii) Measure and then calculate to find the perimeter of each shape. What is the ratio of the perimeters in lower terms?

iv) How does the ratio in **part ii)** compare to the ratio in **part iii)**?

C. i) Measure the side lengths of each rectangle.

Rectangle E Rectangle F

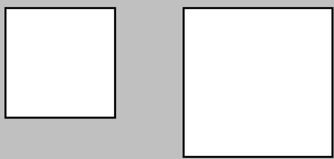


ii) What is the length-to-width ratio for each rectangle?

iii) Are Rectangles E and F similar? How can you tell from the ratios? How can you tell from looking at the rectangles?

D. Are these two rectangles similar? Explain your thinking.

E. Explain why this statement is true: All squares are similar.



F. How can equivalent ratios help you decide whether two rectangles are similar?

4.1.5 Introducing Rates

Try This

Chandra can run 5 km in 30 min. You can describe how fast she runs in different ways, if you assume she always runs at the same speed.

A. Complete each statement.

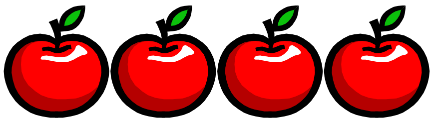
i) Chandra can run ___ km in 6 min.

ii) In ___ min, Chandra can run 10 km.

• A **rate** is like a ratio because it compares quantities. It is different from a ratio because the terms in a rate have different units.

For example:

4 apples for Nu 20



The different units in this rate are ngultrums and the number of apples.

• There are different ways to describe the same rate.

For example, if 4 apples cost Nu 20, the rate could be described in these ways:

- 4 apples for Nu 20, 4 apples **per** Nu 20, or 4 apples/Nu 20
- Nu 20 for 4 apples or Nu 20/4 apples

• Another way to describe a rate is using an **equivalent rate**. To find an equivalent rate, you multiply or divide both terms by the same amount.

For example:

- An equivalent rate for 4 apples/Nu 20 is 2 apples/Nu 10.
- An equivalent rate for Nu 20/4 apples is Nu 40/8 apples.

• An equivalent rate where the second term is 1 is called a **unit rate**.

For example:

The unit rate for Nu 20/4 apples is Nu 5/1 apple, or Nu 5/apple.

• You can use rates to describe many things in your life.

For example:

- the number of hours per week you are in school (hours per week)
- the number of students in each class (students per classroom)
- the speed at which you can read (words per minute)
- the speed of a car (kilometres per hour, or km/h)

• You can compare rates in different ways.

For example:

- In one store 4 apples cost Nu 20. In another store, 4 apples cost Nu 30. Because one term is the same for both rates (4 apples), you can compare the number of ngultrums:

Nu 30/4 apples > Nu 20/4 apples because 30 > 20.

- In one store 3 apples cost Nu 15. In another store, 4 apples cost Nu 24. To compare them, you can think of each as a unit rate.

$\left.\begin{array}{l}\text{Nu 15/3 apples = Nu 5/apple}\\ \text{Nu 24/4 apples = Nu 6/apple}\end{array}\right\}$ Nu 24/4 apples > Nu 15/3 apples

B. Why is the speed at which Chandra can run called a rate?

Examples

Example 1 Describing and Comparing Rates

A flight from Kolkata to Delhi takes 1 h 55 min. A train ride between the two cities takes 8 h 15 min. The distance is 1461 km.

a) What rates describe the speeds of the plane and the train?

b) Which is faster, the plane or the train? How do you know?

Solution

a) *Plane* 1 h 55 min = 115 min

1461 km/115 min

Train 8 h 15 min = 495 min

1461 km/495 min

b) The plane is faster since it covers the same distance in less time.

Thinking

a) Each rate compares the distance traveled to the time it takes to travel that distance.

b) 1461 km/115 min < 1461 km/495 min because 115 min < 495 min.

Example 2 Creating a Unit Rate

Tau types 344 words in 8 min. How many words can he type in 1 min?

Solution

÷ 8

344 words/8 min = 43 words /1 min

÷ 8

He can type 43 words/min.

Thinking

• The number of words he can type in 1 min is the unit rate.

• I had to divide 8 min by 8 to get 1 min, so I divided 344 words by 8 to find the other term.

Practising and Applying

1. Describe each as a rate.

a) A car travels 70 km in 1.5 h.

b) 2 kg of chicken costs Nu 170.

c) 1 dozen bananas cost Nu 20.

2. Which pairs of rates are equivalent?

A. 15 km/2 h and 30 km/3 h

B. 52 words in 2 min and 39 words in 90 s

C. 6 h per day and 40 h per week

D. 2 items for Nu 80 and 3 items for Nu 120

3. What is each missing term?

a) 300 m in 4 min = □ m in 2 min

b) 5 for Nu 90 = □ for Nu 450

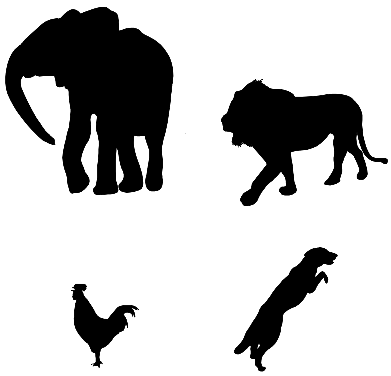
c) 18 h/ 3 days = 12 h/ □ days

4. The heart rates of different animals are shown in the chart.

Animal	Heart Rate
Large dog	200 beats in 2 min
Lion	40 beats in 1 min
Elephant	140 beats in 4 min
Chicken	120 beats in 30 s

a) Write each rate as a unit rate (number of beats per minute).

b) Order the animals from least to greatest heart rate.



5. Which boy travelled the fastest?

Boy	Travelling speed
Tandin	12 km in 3 h
Rinzin	10 km in 1 h
Karma	8 km in 30 min
Pema	10 km in 2 h

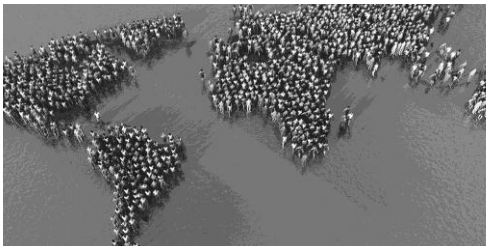
6. A car travels 18 km in 30 min.

a) Why can you also write the rate as 30 min/18 km?

b) Describe the rate in at least three other ways.

7. Yeshi's family pays Nu 4500 rent each month. Describe this rate in two other ways.

8. The world's population is increasing at a rate of about 900,000,000 people per year. Describe this rate in two other ways.



9. These facts have been reported about Bhutan.

- The birth rate each year is about 34 births/1000 people.
- The literacy rate is about 47 out of every 100 people can read and write.

How can you use the information above to find each?

a) the number of births in Bhutan each year

b) the number of Bhutanese who can read and write

Chapter 2 Percent

4.2.1 Introducing Percent

Try This

In 2003, the following was reported about Bhutan:

- The total length of the roads was about 4000 km.
- For every 100 km of road in Bhutan, 62 km were paved.

A. i) What ratio describes the information above?

ii) Estimate the number of kilometres of paved road in 2003.

• A **percent** (%) is a special part-to-whole ratio where the second term is always 100.

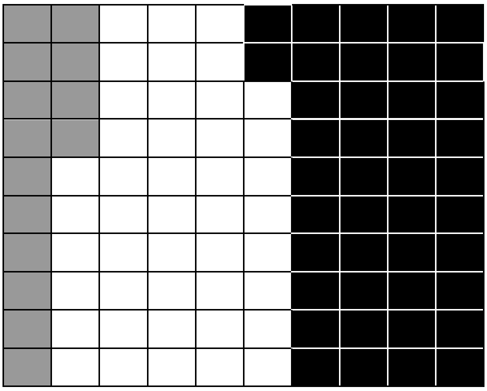
For example:

If you write a test and answer 60 out of 100 questions correctly, the ratio of correct answers to total answers is 60 : 100, $\frac{60}{100}$, or 60%.

• A good way to visualize percent is to use a 10-by-10 grid of 100 squares.

For example:

- The ratio of grey squares to total squares is 14 : 100, or 14%.
- The ratio of black squares to total squares is 42 : 100, or 42%.



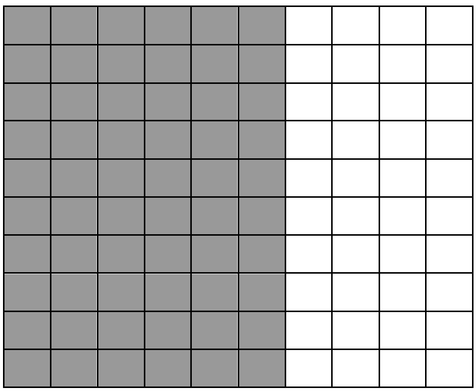
14% grey and 42% black

• If the second term of a ratio is not 100, you can still describe it as a percent by finding an equivalent ratio with a second term of 100.

For example:

$6 : 10 = 6 \times 10 : 10 \times 10 = 60 : 100 = 60\%$

The grid to the right shows why this makes sense. The ratio of grey columns to total columns is 6 : 10 and the ratio of grey squares to total squares is 60 : 100, which is 60%.



6 grey : 10 total = 60% grey

• Using percents to describe ratios makes it is easy to compare ratios because it is like having two ratios with the same second term (100).

For example:

Class A is 48% boys and Class B is 54% boys. It is easy to tell that Class B has a greater proportion of boys than Class A.

• Percents are used in many situations.

For example, percents can be used to describe these ratios:

- the part of the earth’s surface that is covered with water
- the part of the world's population that lives in Asia
- the part of Bhutan’s population that is over 50 years old
- the part of Bhutan’s population that goes to school

B. What percent of the roads in Bhutan from **part A** are paved?

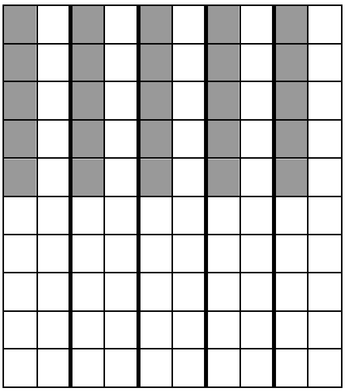
Examples

Example Describing a Ratio as a Percent

Kamala did a probability experiment. She rolled a die 20 times. She rolled the number 1 five of those times. What percent of the rolls were 1?

Solution 1

The ratio of 1s rolled to total rolls = 5 : 20



25 of the 100 squares are shaded, so 25% of the rolls were 1.

Thinking

• I wrote a ratio to describe the proportion of 1s she rolled.

• I shaded 5 squares in every group of 20 squares (2 columns) in a grid of 100 squares.

• Then I counted the total number of shaded squares there were out of 100.

Solution 2

Rolling five 1s in 20 rolls is $\frac{5}{20}$.

$$\frac{5}{20} = \frac{[]}{100} \rightarrow \frac{5}{20} \overset{\times 5}{\underset{\times 5}{=}} \frac{25}{100}$$

25% of the rolls were 1.

Thinking

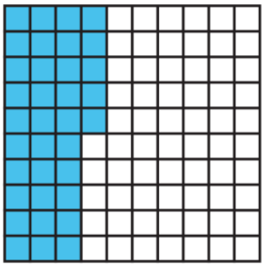
• I wrote the ratio of 1s rolled to total rolls as a fraction.

• Then I found an equivalent fraction with a denominator of 100.

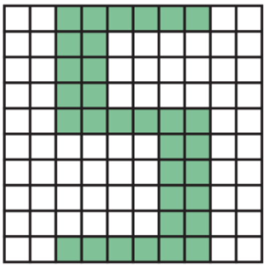
Practising and Applying

1. What percent of each grid is grey? What percent is white?

a)



b)



2. Shade a 10-by-10 grid to show each.

a) 51% **b)** 17% **c)** 83%

3. Copy and complete the chart.

	Ratio	Fraction	Percent
12 to 100			
$\frac{91}{100}$			
0.01			
50 out of 100			

4. Water covers about 70% of the earth's surface area.

- The Pacific Ocean covers about 32% of the earth's surface area.
- The Atlantic Ocean covers about half as much surface area as the Pacific Ocean.

Represent this information on one 10-by-10 grid.

5. What percent would you use to describe each?

a) all of something

b) almost all of something

c) none

d) almost none

6. What percent would make sense in each statement?

a) __% of people are female.

b) __% of people eat breakfast.

c) __% of books have words.

d) A good archer hits the target __% of the time.

7. Order from least to greatest.

16%, 1 out of 10, 22%, 2 out of 10

8. Where have you seen or used percents before?

9. a) Why does it makes sense to model a percent on a 10-by-10 grid?

b) Every time you show a percent on a grid, you actually show two percents. Explain why that happens.

4.2.2 Representing a Percent in Different Ways

Try This

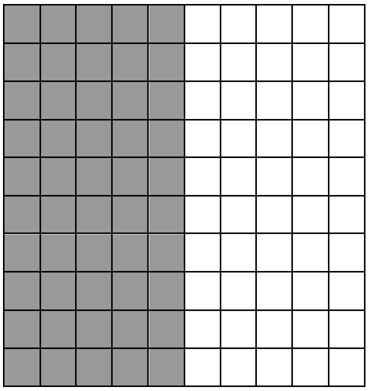
About 70% of Bhutan is covered in forests.

A. About what fraction of the land is forested?

• You can represent a percent in many ways.

For example, you can show 50% these ways:

- 50 squares shaded on a 10-by-10 grid
- a ratio such as 50 : 100 or 50 to 100
- a fraction, $\frac{50}{100}$, or an equivalent fraction, $\frac{1}{2}$
- a decimal, 0.50, or an equivalent decimal, 0.5



50%, 50 : 100, $\frac{50}{100}$, $\frac{1}{2}$, 0.50

• You are probably not surprised to learn that you can represent a percent as a decimal. This makes sense because you can represent decimal hundredths on the same
10-by-10 grid you use to represent percents.

• The form you choose, whether it is a fraction, decimal, ratio, or percent, depends on the situation.

For example:

$\frac{1}{2}$ of the students passed Test A. 58% passed Test B. *Did the marks improve?*

You can rename $\frac{1}{2}$ as a percent to compare it with 58%.

$\frac{1}{2} = \frac{50}{100} = 50\%$ $\qquad$ $50\% < 58\%$

The marks improved by 8%.

B. i) What decimal represents the proportion of forested land in Bhutan?

ii) How do you know that it is close to $\frac{2}{3}$ of the land?

Example Renaming Percents as Decimals and Fractions	
Write each percent as a fraction in lower terms and as a decimal. **a)** 25% **b)** 35%	
Solution	**Thinking**
a) $25\% = \frac{25}{100} = \frac{1}{4}$ $\frac{25}{100} = 0.25$ $\frac{1}{4}$ and 0.25	 **a)** Percent means "out of 100", so I used a fraction with the denominator 100. • I knew that $\frac{25}{100}$ was $\frac{1}{4}$. • I used the fraction out of 100 to write the decimal.
b) $35\% = \frac{35}{100} = \frac{7}{20}$ $\frac{35}{100} = 0.35$ $\frac{7}{20}$ and 0.35	**b)** I did the same thing as in **part a)**, except that to rename the fraction in lower terms, I divided the numerator and denominator by 5.

Practising and Applying

1. Rename each percent as a fraction and as a decimal.

a) 33% **b)** 80%

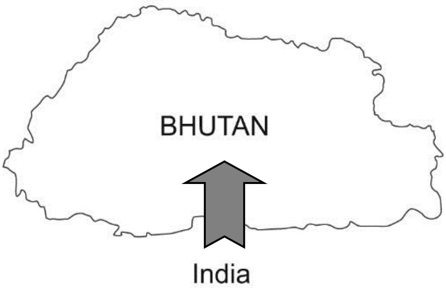
c) 15% **d)** 68%

2. Write each decimal as a percent.

a) 0.39 **b)** 0.18

3. 91% of Bhutanese students who enter school go to Class V or higher. Write that number as a fraction and as a decimal.

4. About 85% of Bhutan's imports come from India. Is this more or less than $\frac{3}{4}$ of its imports? How do you know?



5. What percent of the numbers from 1 to 100 are in the 5-times table? How do you know?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

6. The ratio of red to blue in a purple dye is 2 : 3.

a) What percent of the dye is blue?

b) If two parts remain red, how many parts of blue need to be added to change the dye to 80% blue? Show your work.

7. 50% of 42 students are girls. How many of the students are girls?

8. When you write a percent as a decimal, why is it in the form 0.[ ][ ]?

GAME: Ratio Match

This game is for 2 or 3 players.

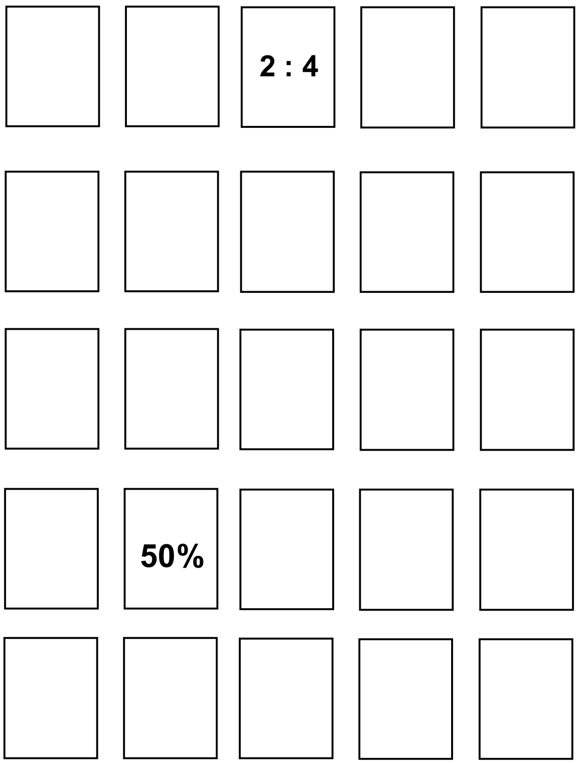
You need a set of Ratio Match Game Cards.

How to play:

- Shuffle the cards and place them face down in a 5-by- 6 array.
- Take turns. On your turn, flip over any two cards.
  - If the ratios, percents, fractions, or decimals shown are equivalent, pick up and keep those cards and take another turn.
  - If the cards are not equivalent, flip them face down. Your turn is over.
- Play until all the cards have been turned over and matched.

The player with the most cards at the end wins.

For example:



2 : 4 and 50% are equivalent.
The player keeps these cards
and takes another turn.

4.2.3 EXPLORE: Writing a Fraction as a Percent

A fraction with a denominator that is a **factor** of 100 is easy to write as a percent because factors of 100 divide into 100 with no remainder.

For example:

5 is a factor of 100 because 100 ÷ 5 = 20 and there is no remainder.

$\frac{1}{5}$ as a percent is 20% because $\frac{1}{5} = \frac{1 \times 20}{5 \times 20} = \frac{20}{100} = 20\%$.

A. i) Write $\frac{1}{4}$ as an equivalent fraction with denominator 100.

ii) Write $\frac{1}{4}$ as a percent and then model that percent on a grid to show that it is $\frac{1}{4}$ of the grid. What do the 1 and the 4 in your percent show?

B. Repeat **part A** with these fractions:

$$\frac{1}{2}, \frac{1}{5}, \frac{1}{10}, \frac{1}{20}, \frac{1}{25}, \text{ and } \frac{1}{50}$$

C. i) How does knowing the percent for $\frac{1}{4}$ help you find the percent for $\frac{3}{4}$?

ii) Use the percents you found in **part B** to write at least five other fractions as percents.

D. Explain why you cannot write $\frac{1}{3}$ as a whole number percent.

CONNECTIONS: Map Scales

Most maps use a **scale ratio** to show the relationship between the distances on the map and the actual distances.

For example:

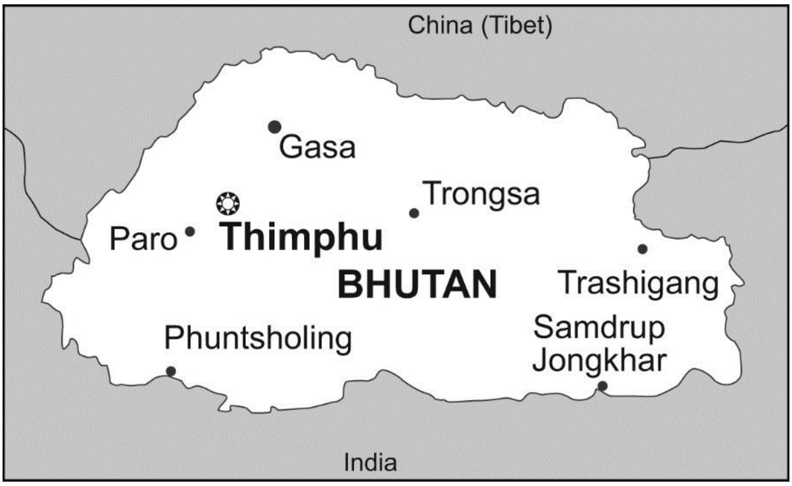
A scale ratio of 2 : 600,000 on a map means that 2 cm on the map represents an actual distance of 600,000 cm or 6 km.

1. a) If the scale ratio is 2 : 600,000, how many kilometres do 2 cm represent?

b) How many kilometres does 1 cm represent?

2. Estimate the scale for this map of Bhutan. Show your work.

(Hint: the width of Bhutan, measuring straight across from west to east at the widest point, is about 300 km).



UNIT 4 Revision

1. Which colours are compared by each ratio below?

a) 2 to 3 **b)** 2 : 2

c) $\frac{2}{7}$ **d)** 3 : 7

2. Sketch two different pictures to show the ratio 3 : 4. Explain how each picture shows the ratio.

3. You toss a die 30 times. Here are the results.

Number rolled	1	2	3	4	5	6
Number of times	5	4	6	5	8	2

Use three or more ratios to describe the information. Include a fraction as one of the ratios.

4. What are the missing terms?

a) 16 to 12 = 4 to ☐

b) 2 : 9 = ☐ : 27

c) 3 : 2 = ☐ : 8

d) 11 : ☐ = 3 : ☐

5. Sketch a picture for each.

a) show that 4 : 6 = 2 : 3

b) show that 4 : 6 ≠ 2 : 4

6. a) It takes 21 kg of milk to make 1 kg of butter. What is the ratio of milk to butter?

b) Make a chart to show the amount of milk needed to make 2 kg, 3 kg, and 4 kg of butter.

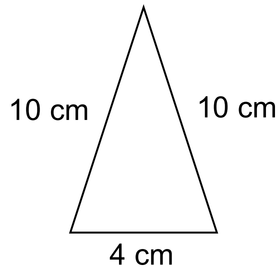
7. Different shades of paint were mixed in two cans.

- Can A used 2 parts of blue and 1 part of white.
- Can B used 6 parts of blue and 2 parts of white.

Which paint is darker, Can A or Can B? How do you know?

8. In a group of 50 teachers, 12 teach primary school. In another group of 30 teachers, 8 teach primary school. Which group has a greater ratio of primary teachers?

9. Describe the dimensions of two other triangles that are similar to the triangle shown below.



10. Which is the best price for the buyer? How do you know?

- 3 chocolate bars for Nu 250
- 5 chocolate bars for Nu 400
- 6 chocolate bars for Nu 450

11. How far would you expect to travel in $3\frac{1}{2}$ h if you drove at 35 km/h the whole time?

12. At one of the games at the Paro Tshechu, you get three chances to win a prize if you pay Nu 40. How many chances do you get if you pay Nu 400?

13. Which pair of rates are equivalent?

A. 56 km/2 h and 70 km/3 h

B. 111 words in 3 min and 54 words in 2 min

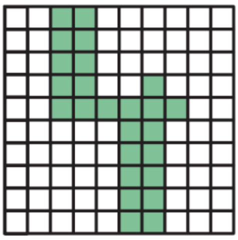
C. Nu 100 for 6 items and Nu 250 for 15 items

14. Write this rate in another way:

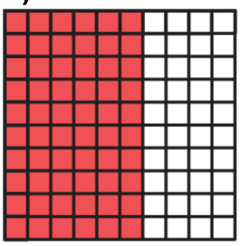
5 boxes for Nu 400

15. What percent of each grid is shaded? What percent is not shaded?

a)



b)



16. Shade a grid to show each.

a) 36% **b)** 64% **c)** 17%

17. a) Tell whether or not you think each statement is reasonable. Explain your thinking.

A. Close to 100% of the babies born in Bhutan are Bhutanese.

B. About 50% of all 11-year-olds are boys.

C. About 50% of the time, the sun sets in the west.

b) Write another reasonable statement that uses 100%.

18. Copy and complete.

	Ratio	Fraction	Percent
35 to 100			
$\frac{65}{100}$			
0.60			
82 out of 100			

19. A football team in a league won 75% of its games. Is this team likely to be one of the better teams or one of the worse teams in the league? Explain your thinking.

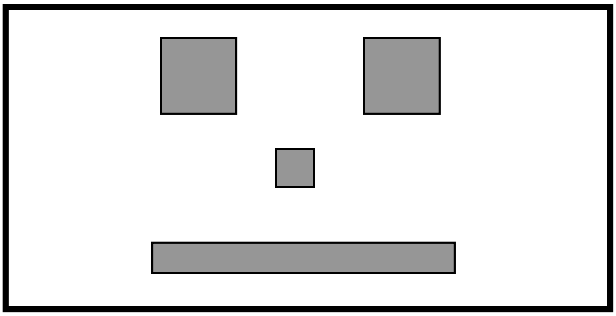
20. a) Write $\frac{7}{25}$ as an equivalent fraction with denominator 100.

b) Write $\frac{7}{25}$ as a percent. Shade the percent on a hundredths grid to show that it is $\frac{7}{25}$ of the grid.

UNIT 5 MEASUREMENT

Getting Started

Use What You Know



A. Estimate the area of the grey region in the face design above.

B. Estimate the area of the white region.

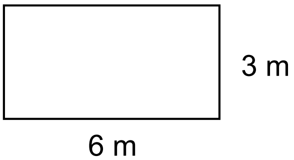
C. i) Sketch a rectangle with about the same area as the white region. Label it with its dimensions.

ii) Explain how you know your rectangle has about the same area as the white region in the face design.

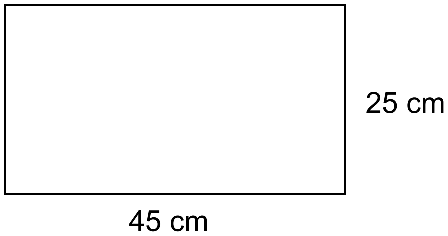
Skills You Will Need

1. What are the perimeter and the area of each?

a)



b)



2. Complete.

a) ____ cm = 1 m

b) ____ m = 1 km

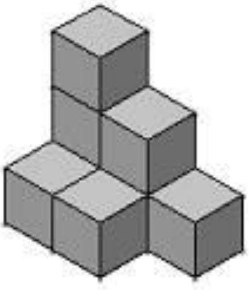
c) ____ mm = 1 cm

d) 1 L = ____ mL

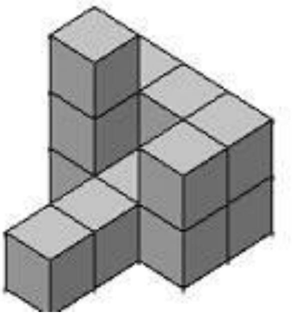
Reprint 2023

3. What is the volume of each structure in cubic units?
(Each cube is 1 cubic unit.)

a)



b)



4. A rectangular prism has a volume of 36 cm^3.
What could be its dimensions (length, width, and height)?
List two possible sets of dimensions.

5. Order these capacities from least to greatest.

325 mL 2.1 L 2300 mL 0.45 L

6. Name something that might have each capacity or volume.

a) 250 mL **b)** 16 cm^3 **c)** 27 m^3 **d)** 5 L

7. Sketch what a clock looks like at each time.

a) 10 minutes after 3

b) 20 minutes before 11

8. How long did each event take?

a) R. K. arrived at work at 9:05 a.m. and left work at 4:45 p.m.

b) Dechen started walking at 6:52 a.m. and got to school at 8:41 a.m.

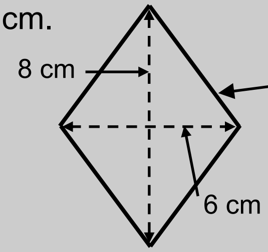
c) Rinzin went to sleep at 10:15 p.m. and woke up at 5:53 a.m.

Chapter 1 Area

5.1.1 Area of a Parallelogram

Try This

Suppose the diagonals in each rhombus in this fabric design are 6 cm and 8 cm.

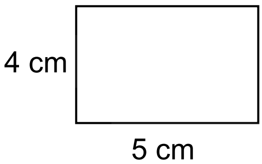


A. Estimate the area of one rhombus. Explain how you estimated.

- You already know the **formula** for the **area** of a rectangle:

Area of a rectangle = length × width $\rightarrow$ $A = l \times w$

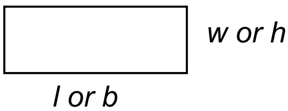
For example:



$A = 5 \times 4 = 20\ \text{cm}^2$

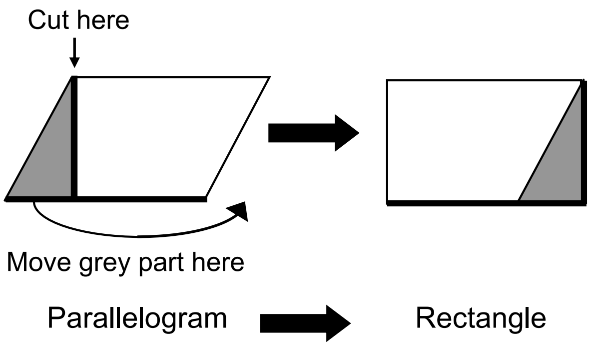
Because the length of a rectangle is its **base** and the width is its height, you can also write the formula as shown below:

$A = l \times w \rightarrow A = \text{base} \times \text{height} \rightarrow A = b \times h$



- You can use the rectangle formula, $A = b \times h$, to develop a formula for the area of a **parallelogram**.

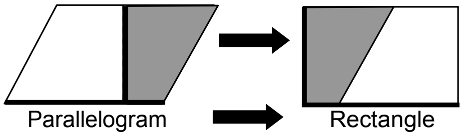
- You can change any parallelogram with base b and height h into a rectangle with base b and height h. There are many ways to do this.



Notice:

- The base of the rectangle is equal to the base of the parallelogram (the thick horizontal line).
- The height of the rectangle is equal to the height of the parallelogram (the thick vertical line).

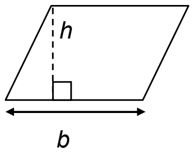
- No matter where you cut the parallelogram, you can make a rectangle with the same base and height as the parallelogram:



Notice that the area of the parallelogram is equal to the area of the rectangle (because they are made of the same two pieces).

- Because the bases, the heights, and the areas are the same, you can calculate the area of a parallelogram using the formula for the area of a rectangle:

Area of a parallelogram = $b \times h$



You can measure the height of a parallelogram using any line segment that is perpendicular to the base and goes to the side that is opposite the base. Note that the height is *not* the slanted side length.

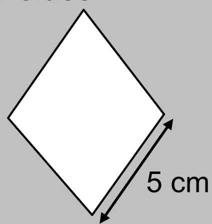
B. Recall that a rhombus is a parallelogram with four equal sides.

i) To calculate the area of the rhombus from **part A** using the area formula, what dimensions do you need to know?

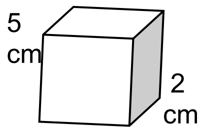
ii) One side length is 5 cm. How do you know the height is less than 5 cm?

iii) How do you know the area is less than 25 cm^2?

iv) Was your estimate in **part A** reasonable? Explain your thinking.



Examples

Example 1 Calculating the Area of a Parallelogram	
What is the area of this parallelogram? Show your work. 	
Solution	**Thinking**
$A = b \times h$ $b = 5$ cm $h = 1$ cm $A = 5 \times 1$ $= 5$ cm^2	• I knew that 5 cm was the length of the base, even though it was on top, because opposite sides of a parallelogram are the same length. • I could see from the diagram that the height was 1 cm. • I remembered to use square units because it was an area measure: cm × cm = cm^2.

Reprint 2023

Example 2 Calculating the Height of a Parallelogram

A parallelogram has a base of 8 m and an area of 12 m^2.
What is its height? Show your work.

Solution

$b \times h = A$

$8 \times h = 12$

$8 \times 2 = 16$ Too high

$8 \times 1 = 8$ Too low

$8 \times 1.5 = 12$

The height is 1.5 cm.

Thinking

- I knew the base times the height was equal to the area.
- I multiplied the base, 8, by different numbers until I got a product of 12.
- Since 12 was halfway between 8 and 16, I knew *h* was halfway between 1 and 2.

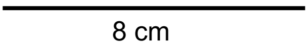
Example 3 Creating a Parallelogram with a Given Area

Draw three different parallelograms with an area of 16 cm^2.

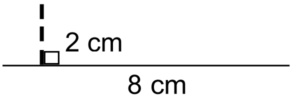
Solution

First Parallelogram

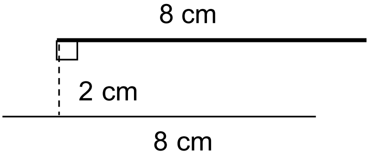
Step 1



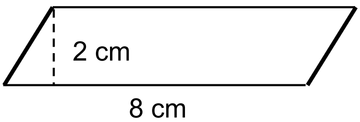
Step 2



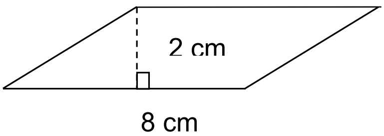
Step 3



Step 4



Second parallelogram



Third parallelogram (a rectangle)

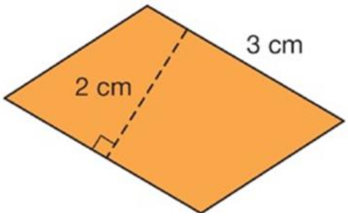
Thinking

- I knew the base and height had to multiply to 16, so I used a base of 8 cm and a height of 2 cm.
- I showed the steps I followed for my first parallelogram.
- I did the second parallelogram the same way as the first, except that I drew the top side farther to the right.
- I remembered that a rectangle was a parallelogram.

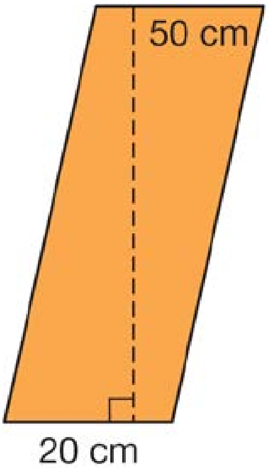
Practising and Applying

1. Calculate the area of each.

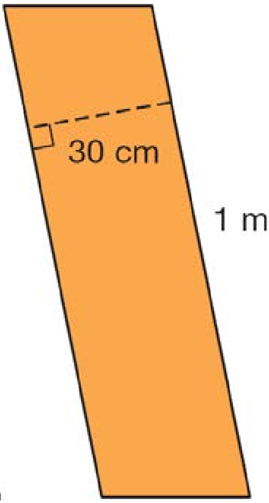
a)



b)



c)



2. a) A parallelogram has a height of 3 cm and an area of 24 cm^2. What is the length of its base?

b) Sketch two different parallelograms that fit the description in **part a)**. Label the base and height of each.

3. Sketch two different parallelograms each with an area of 18 cm^2. Label the base and height of each.

4. a) Carefully draw a parallelogram with a 40 mm base and a 30 mm height.

b) Calculate the area in square millimetres (mm^2).

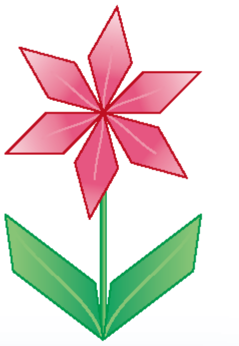
c) Turn the parallelogram so that the "slanted" side is the base. Measure the new base and height to the nearest millimetre.

d) Calculate the area (mm^2) using the base and height in **part c)**.

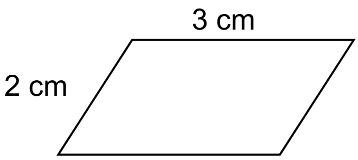
e) How do the answers to **parts b) and d)** compare? Why?

5. a) Sketch a flower like the one below. Make the area of each leaf twice the area of each petal. Label the base and height of each.

b) Explain how you created the flower.



6. Why do you need to use a ruler to find the area of this parallelogram?



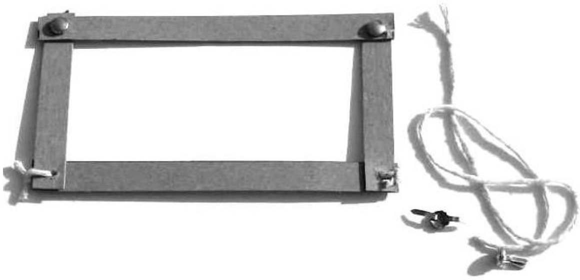
7. Why is there always more than one parallelogram with the same area and base?

Reprint 2023

CONNECTIONS: Changing a Parallelogram

In the previous lesson, you discovered that you could create different parallelograms with the same base, height, and area. In this activity, you will explore what happens to a parallelogram as you gradually reshape it into other parallelograms, leaving only the base the same.

1. Cut out two strips of cardboard of length 15 cm and two strips of length 8 cm. Fasten them together as shown below to make a rectangle.

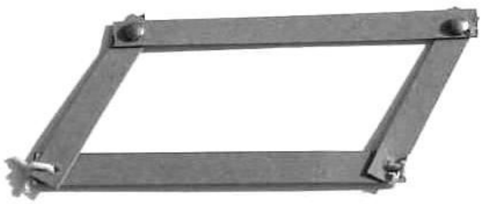


Attach the pieces with string or butterfly pins at so the angles at the vertices can change.

a) What are the length of the base and the height of the parallelogram? Measure the outside edges of the sides.

b) Use formulas to find the perimeter and area of the parallelogram.

2. Keep the base fixed and move the top a bit to the right.



a) What is the length of the base? What is the height of the parallelogram?

b) Use formulas to find the perimeter and area of the parallelogram.

3. Repeat **Step 2**, moving the top even farther to the right.

4. Look at the base, height, area, and perimeter measurements for the three different parallelograms.

a) Which measurements stayed the same?

b) Which measurements changed?

c) Which shape had the greatest area? Which shape had the least area?

5. What would happen to the area if you kept moving the top farther and farther to the right? Why does this happen?

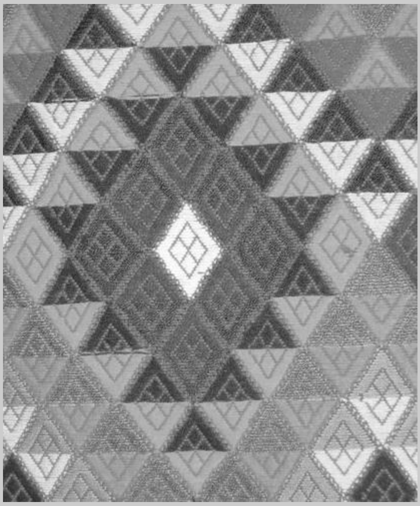
5.1.2 Area of a Triangle

Try This

Suppose the piece of fabric shown here measures 35 cm by 29 cm.

A. Estimate the area of the fabric.

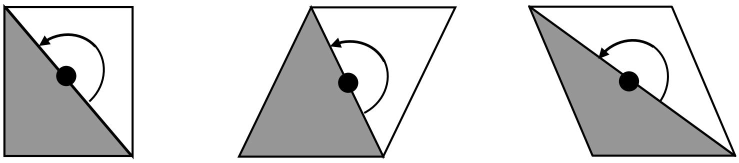
B. Estimate the area of this triangle. Explain how you estimated.



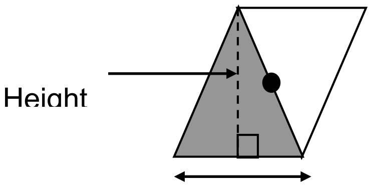
• Recall that the formula for the area of a rectangle was used to develop the formula for the area of a parallelogram.

• You can use the parallelogram formula, $A = b \times h$, to develop a formula for the area of a triangle:

- When you rotate any triangle a half turn clockwise or counterclockwise around the midpoint of one of its sides, the two triangles make the shape of a parallelogram. It does not matter what type of triangle you use.



Rotating right, acute, and obtuse triangles to create parallelograms



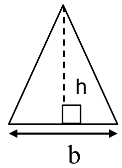
Notice:

• The base of the parallelogram is the same as the base of the triangle.

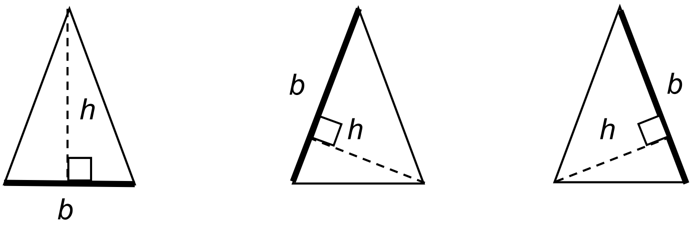
• The height of the parallelogram is the same as the height of the triangle.

• The triangle is half the area of the parallelogram.

- The formula for the area of a parallelogram is $A = b \times h$. The base and height of the parallelogram and triangle are the same, but the triangle has $\frac{1}{2}$ the area of the parallelogram, so the formula for the area of a triangle is *Area of a triangle* $= b \times h \div 2$



• You can use any side of the triangle as the base, as long as you measure the height using a line that is perpendicular to that base and goes to the vertex that is opposite the base.



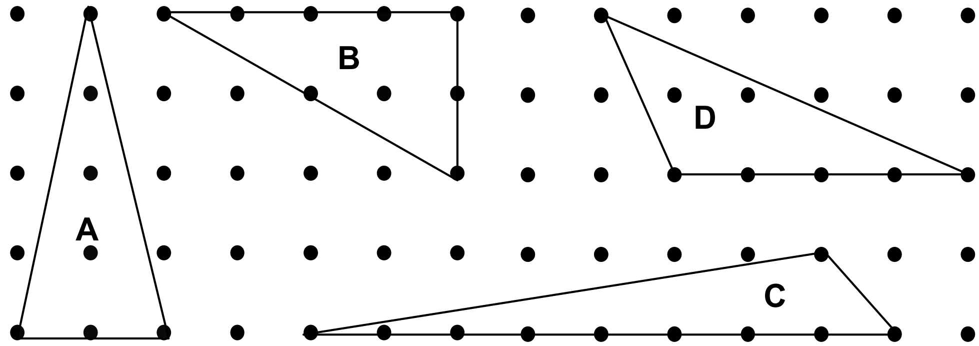
This triangle has three possible bases.

C. Use your estimate for the area of the triangle in **part B** to estimate its base and height. Explain your thinking.

Examples

Example 1 Comparing the Areas of Triangles

Which triangles have the same area?



Solution	Thinking
$A = b \times h \div 2$ *Triangle A* $b = 2$ units, $h = 4$ units $A = 2 \times 4 \div 2$ $= 4$ square units [Continued]	• I counted grid spaces to find the base and height for each triangle. • For Triangle A's height, I imagined a vertical line from the top vertex to the base at a right angle to the base. 

Example 1 Comparing the Areas of Triangles [Continued]

Solution

Triangle B

b = 4 units, h = 2 units

$A = 4 \times 2 \div 2 = 4$ square units

Triangle C

b = 8 units, h = 1 unit

$A = 8 \times 1 \div 2 = 4$ square units

Triangle D

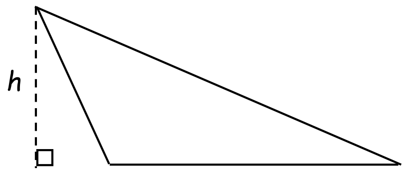
b = 4 units, h = 2 units

$A = 4 \times 2 \div 2 = 4$ square units

All four triangles have the same area.

Thinking

- For Triangle B's height, I used the vertical side on the right. For its base, I used the horizontal side along the top.
- For Triangle C's height, I imagined a vertical line that went straight down from the top vertex to the base at a right angle.
- For Triangle D's height, I imagined a vertical line that went straight down from the top vertex outside the shape.



Example 2 Finding the Base of a Triangle

The area of a triangle is 20 m^2. The height is 4 m. How long is the base?

Solution

$A = b \times h \div 2$

$A = 20$ m^2, $h = 4$ m

$20 = b \times 4 \div 2$

$20 = b \times 2$

$b = 20 \div 2 = 10$

The base is 10 m.

Thinking

- I put the values I knew into the formula for the area of a triangle.
- Since the base was multiplied by 2 to get 20, I divided 20 by 2 to find the base.

Example 3 Creating Triangles with a Given Area

List the dimensions of three different triangles, each with an area of 16 cm^2.

Solution

$A = b \times h \div 2$

$16 = b \times h \div 2$

$32 = b \times h$

Possible triangle dimensions

b = 16 cm, h = 2 cm

b = 4 cm, h = 8 cm

b = 10 cm, h = 3.2 cm

Thinking

- Since you divide the product of the base and height by 2 to get the area of 16, the product of the base and height has to be $2 \times 16 = 32$.
- I looked for numbers that multiplied to 32.

Practising and Applying

1. Calculate the area of each.

a)

3 cm

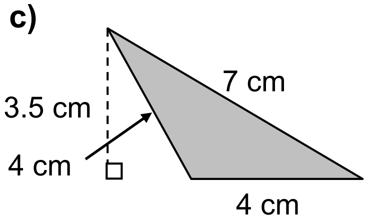
2 cm

b)

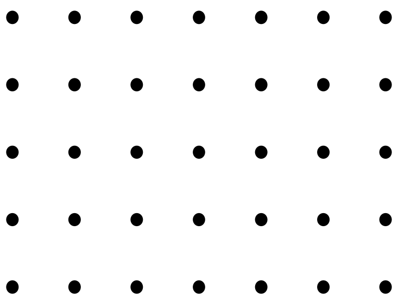
3m 2.7 m

6 m

c)



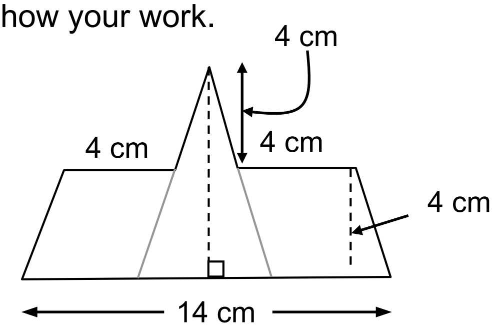
2. Draw two different triangles, each with an area of 6 square units, on a grid like this. The triangles can overlap if necessary.



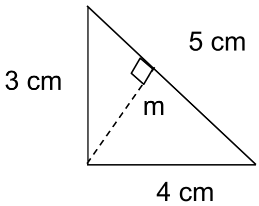
3. A triangle has an area of 40 cm^2 and a base of 1 m. What is its height?

4. A parallelogram and a triangle have the same base and area. Will their heights also be the same? How do you know? Use examples to help you explain.

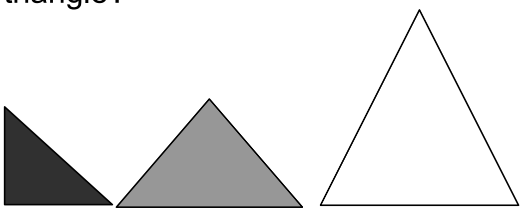
5. Calculate the area of this shape. Show your work.



6. Use what you know about the area of the triangle to figure out the value of *m*. Show your work.



7. How can you tell that the black triangle has $\frac{1}{2}$ the area of the grey triangle and $\frac{1}{4}$ the area of the white triangle?



8. Explain why the formula for the area of a triangle, $A = b \times h \div 2$, makes sense.

9. Two triangles have the same area, but one is tall and thin and the other is short and wide. How is that possible? Use examples to help you explain.

GAME: Grid Fill

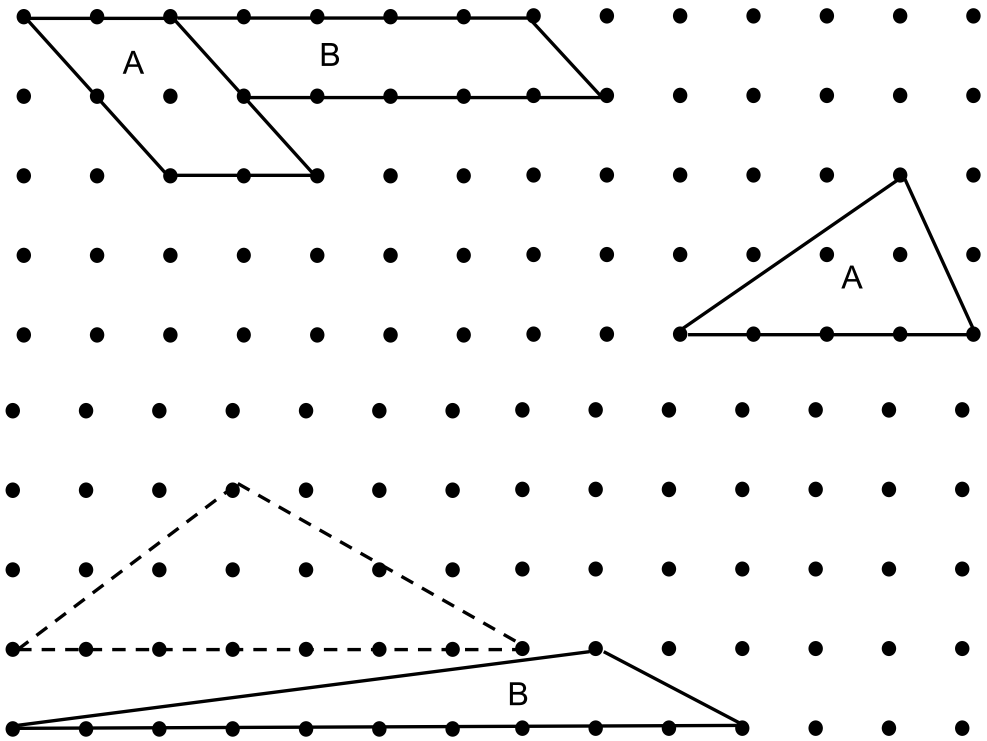
Play with a partner. Decide who will be Player A and who will be Player B.

You need a 10-by-14 grid to share. You can use dot paper (as shown below) or grid paper.

- Take turns rolling a die twice. Add, subtract, or multiply the numbers you roll to get a value. Sketch a parallelogram (but not a rectangle) or a triangle with an area of that value. Shapes cannot overlap.
- Record your letter, A or B, inside each shape you sketch.
- The last player to draw a shape that fits inside the grid wins the game.

For example:

Players A and B are playing the game below.
Each player has taken two turns.
It is now Player A's turn. If she rolls a 3 and a 4, she can create a triangle or a parallelogram with an area of 7 square units (3 + 4), 1 square unit (4 – 3), or 12 square units (3 × 4).
She decides to draw a triangle with a base of 7 and height of 2, as shown by the dashed lines.



Alternative rule: The winner is the player who covers the greater area.

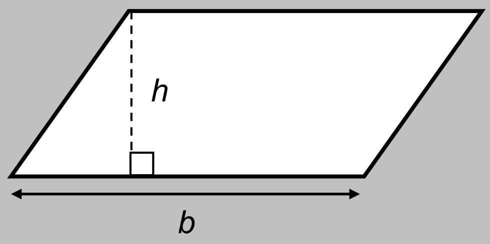
Reprint 2023

5.1.3 EXPLORE: Relating Areas

Sometimes you can use what you know about the area of a parallelogram or triangle to make predictions about the area of a different parallelogram or triangle.

Part 1

The formula for the area of a parallelogram is $A = b \times h$.



A. Choose three pairs of values for *b* and *h*. Calculate the area of each parallelogram. Record the information in a chart like this:

	Parallelogram A	Parallelogram B	Parallelogram C
b			
h			
A			

B. i) Double the value of each *b*, but do not change *h*. Create another chart to record the new values.

ii) How do the areas in **part B i)** compare to the areas in **part A**?

C. i) Leave the values of *b* as they are in your chart for **part B**, but double the value of each *h*. Create another chart to record the new values.

ii) How do the areas in **part C i)** compare to the areas in **part B i)**?

iii) How do the areas in **part C i)** compare to the areas in **part A**?

[Continued]

Reprint 2023

D. Answer each question below with a prediction. Then check your answer with an example. Explain why your answer will always be true.

How does the area of a parallelogram change if you do each?

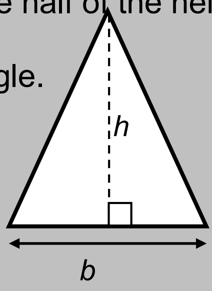
i) triple the base and do not change the height

ii) triple the base and triple the height

iii) double the base and triple the height

iv) double the base and take half of the height

E. Repeat **part D** for a triangle.



Part 2
You can also use what you know about the area of a parallelogram or triangle to describe the area using other units.

F. The area of a parallelogram is 1 m^2.

i) What might be its dimensions in metres?

ii) What might be its dimensions in centimetres?

iii) What is the area of the parallelogram in square centimetres?

iv) Why is the area *not* 100 cm^2, even though 100 cm = 1 m?

G. Repeat **part F i), ii), and iii)** for a triangle with an area of 1 m^2.

H. Another parallelogram has an area of 1 km^2.

i) What might be its dimensions in kilometres?

ii) What might be its dimensions in metres?

iii) What is the area of the parallelogram in square metres?

iv) Why is the area *not* 1000 m^2, even though 1000 m = 1 km?

I. Repeat **part H i), ii), and iii)** for a triangle with an area of 1 km^2.

Reprint 2023

Chapter 2 Volume

5.2.1 Volume of a Rectangular Prism

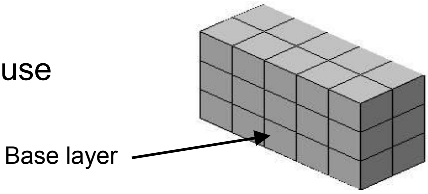
Try This

The car in the photo is about 1.5 m tall. The building beside the car has a shape that is close to a rectangular prism
(not including the roof). It is about 12 m long and about 7 m wide.

A. Use the height of the car to estimate the height of the building. Explain how you estimated.

- The **volume** of a 3-D shape is the amount of space it takes up.
- You can measure the volume of a **rectangular prism** by finding out how many centimetre cubes are needed to build it. A centimetre cube is a cube that is 1 cm on each edge (1 cm × 1 cm × 1 cm = 1 cm^3).

For example:
The volume of this prism is 30 cm^3 because it is made with 30 centimetre cubes.

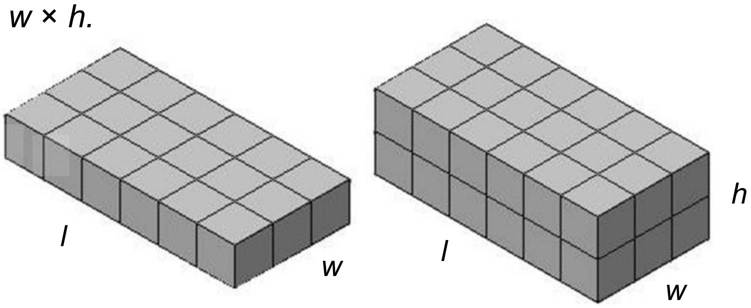


- The base layer is 5 rows of 2 cubes, so it is 5 cm long by 2 cm wide.
- There are 3 layers, so its height is 3 cm.
- The volume is 30 cm^3.

Notice that you can multiply the length, width, and height to get the volume:

$$5 \text{ cm} \times 2 \text{ cm} \times 3 \text{ cm} = 30 \text{ cm}^3$$

This makes sense if you think of the length, width, and height as l, w, and h. The base layer has $l \times w$ cubes, and there are h layers, so the volume is
$l \times w \times h$.



$V = l \times w \times h$

• You can use various measurements to calculate the volume of a rectangular prism.

- You can use all three **dimensions** (l, w, and h).
- Or, you use the area of the base ($l \times w$) and the height (h).

For example:

A rectangular prism has a base of 24 cm² and a height of 2 cm. What is the volume of the prism?

Since $l \times w = 24\text{ cm}^2$ and $h = 2$ cm, the volume is $24^2\text{ cm} \times 2\text{ cm} = 48\text{ cm}^3$.

• If you know the volume and two dimensions of a rectangular prism, you can find the third dimension.

For example:

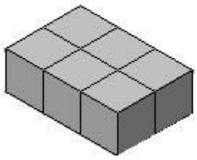
A rectangular prism has a volume of 48 cm³, a length of 6 cm, and a width of 2 cm. What is the height of the prism?

$l \times w \times h = V \rightarrow 6 \times 2 \times \blacksquare = 48 \rightarrow 6 \times 2 \times \underline{\mathbf{4}} = 48$ The height is 4 cm.

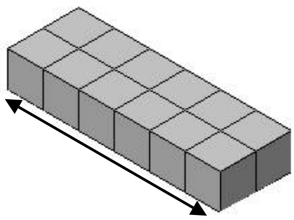
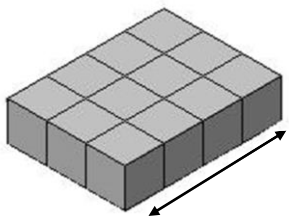
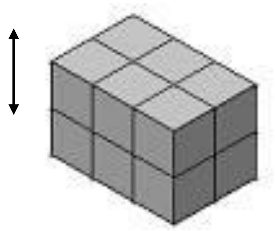
• If one dimension of a prism is doubled, the volume is doubled.

For example:

Start with rectangular prism that is 3 units long by 2 units wide by 1 unit high. Double each dimension in turn.



Volume = $3 \times 2 \times 1 = 6$

Double the length	Double the width	Double the height
		
Volume = $\underline{\mathbf{6}} \times 2 \times 1 = 12$	Volume = $3 \times \underline{\mathbf{4}} \times 1 = 12$	Volume = $3 \times 2 \times \underline{\mathbf{2}} = 12$

The pictures above show that when you double the length, the width, or the height, you always use two times the number of cubes.

• You can see from the three prisms shown above that different prisms can have the same volume. As long as the product of the length, width, and height is the same, the rectangular prisms have the same volume.

Reprint 2023

• Sometimes the different dimensions of a prism are in different units. It is important to describe them using the same unit before you calculate volume.

For example:

Suppose a rectangular prism has l = 1 m, w = 25 cm, and h = 10 cm.

$$V = 1 \text{ m} \times 25 \text{ cm} \times 10 \text{ cm}$$

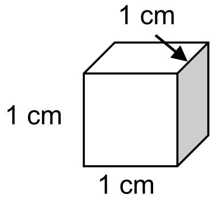
Since 1 m = 100 cm, $V = 100 \text{ cm} \times 25 \text{ cm} \times 10 \text{ cm}$

$$= 25{,}000 \text{ cm}^3$$

• The units commonly used for volume are **cubic millimetres** (mm^3), **cubic centimetres** (cm^3), and **cubic metres** (m^3), but other units are possible.

- 1 mm^3 is the volume of a cube that is 1 mm along each edge.
- 1 cm^3 is the volume of a cube that is 1 cm along each edge.
- 1 m^3 is the volume of a cube that is 1 m along each edge.

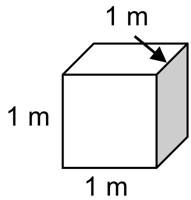
• You might think that 1 cm^3 = 10 mm^3 because 1 cm = 10 mm. This is not the case because there are *three* dimensions to consider:



1 cm^3 = 1000 mm^3 since $1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$

$$= 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm}$$
$$= 1000 \text{ mm}^3$$

You might think that 1 m^3 = 100 cm^3 because 1 m = 100 cm. This is not the case for the same reason:



1 m^3 = 1,000,000 cm^3 since $1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$

$$= 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm}$$
$$= 1{,}000{,}000 \text{ cm}^3$$

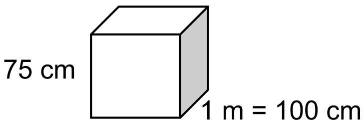
B. Use your estimated height from **part A**, the given length and width, and the volume formula to estimate the volume of the building (without the roof).

Examples

Example 1 Calculating the Volume of a Rectangular Prism

A box has a square base with an edge length of 1 m. Its height is 75 cm. What is its volume? Show your work.

Solution



V = 100 × 100 × 75 = 750,000 cm^3

The volume is 750,000 cm^3.

Thinking

• I sketched the prism to get a sense of what it looked like.

• I changed 1 m to 100 cm so the units were the same.

• I remembered to use cubic units because cm × cm × cm = cm^3.

Example 2 Calculating the Height of a Rectangular Prism

A rectangular prism has a volume of 300 cm^3. The base is 20 cm long and 5 cm wide. What is the height of the prism?

Solution

$V = l \times w \times h$

$300 = 20 \times 5 \times h$

$300 = 100 \times h$

$h = 3$

The prism is 3 cm high.

Thinking

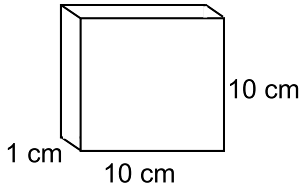
- I put the values I knew into the volume formula.
- I figured out what I had to multiply 100 by to get 300.

Example 3 Creating a Rectangular Prism with a Given Volume

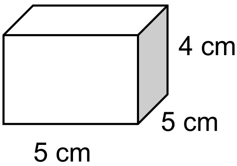
The volume of a rectangular prism is 100 cm^3. Sketch and label three different prisms with that volume.

Solution

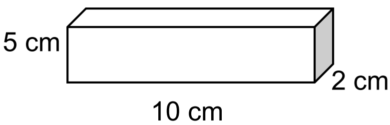
$100 = \underline{100} \times 1 = \underline{10 \times 10} \times 1$



$100 = \underline{25} \times 4 = \underline{5 \times 5} \times 4$



$100 = \underline{50} \times 2 = \underline{5 \times 10} \times 2$



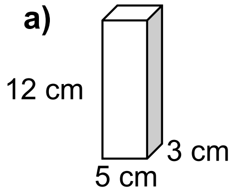
Thinking

- Since $l \times w \times h = 100$, I knew I needed three numbers that multiplied to 100.
- To do this, I first wrote 100 as a product of two numbers. Then I wrote one of those numbers as a product of two numbers.

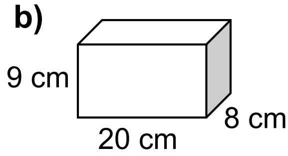
Practising and Applying

1. Calculate the volume of each.

a)



b)



c)

10 cm

8 cm

1.2 m

2. Each prism below has a volume of 144 cm^3.

a) Area of base = 12 cm^2
What is the height of the prism?

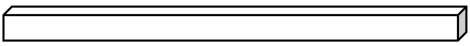
b) l = 6 and w = 4
What is the height of the prism?

c) l = 12 and h = 6
What is the width of the base?

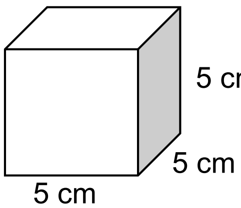
3. Sketch and label three different rectangular prisms, each with a volume of 60 cm^3.

4. Three different rectangular prisms each have a volume of 120 m^3. What might their dimensions be?

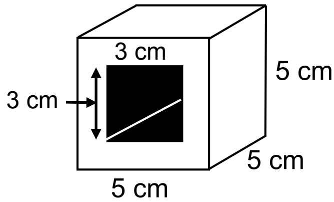
5. A rectangular prism is long, thin, and short. Its volume is 80 cm^3. What might its dimensions be?



6. Kinley made a rectangular prism with 36 centimetre cubes. The prism fits inside the box shown below. What are the dimensions of the prism?



7. A 3 cm square hole is cut all the way through a 5 cm cube of wood. Find the volume of the remaining wood.



8. Tashi has two suitcases. His other suitcase holds about half as much as the suitcase shown below. What could be the dimensions of his other suitcase?

9. a) Rectangular prism A has a volume of 2400 cm^3. Its base has an area of 300 cm^2. What is its height?

b) Use the information in **part a)** to find the dimensions of each prism.

i) Rectangular prism B is twice as long as A, but it has the same width and height.

ii) Rectangular prism C is twice as long and twice as high as A, but it has the same width.

iii) Rectangular prism D is twice as long and half as wide as A, but it has the same height.

10. Explain why 1 km^3 $\neq$ 1000 m^3 even though 1 km = 1000 m.

11. Use the volume formula to explain why changing the length, the width, or the height of a rectangular prism also changes its volume.

5.2.2 Relating Volume to Capacity

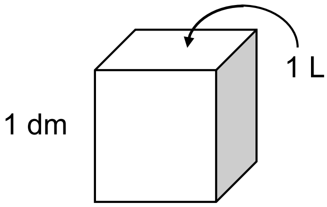
Try This

A. Estimate how many litres of water this bucket could hold. Explain how you estimated.
(Hint: You might visualize a square-based prism of the same height and about the same width.)

- **Capacity** is a measure of how much a 3-D shape could hold if it were a container filled to the top with a liquid.
- The **litre** (L), **millilitre** (mL), and **kilolitre** (kL) are units for measuring capacity. They are mostly used for liquids or for solids that you can pour, such as sand or salt.

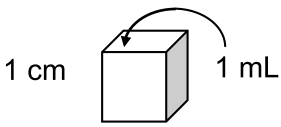
1 L = 1000 mL   1000 L = 1 kL

- A litre is the amount that fills a cube that is 1 dm along each edge.



- If a container holds a very small amount, you can use a smaller capacity measure: millilitres.

- Since 1 dm = 10 cm, the volume of the 1 dm cube in cubic centimetres is 10 cm × 10 cm × 10 cm = 1000 cm^3.
- 1 L = 1000 mL and 1 dm = 1000 cm^3, so 1000 mL fills 1000 cm^3.
- 1000 mL fills 1000 cm^3, so 1 mL fills 1 cm^3.

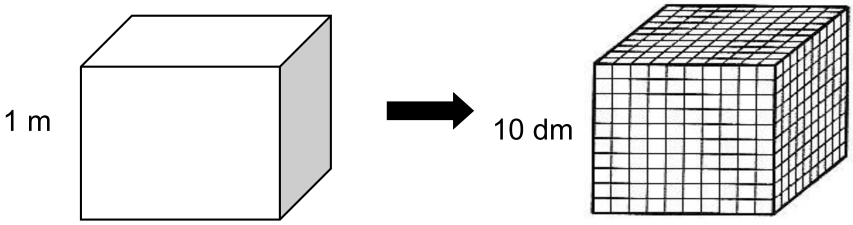


So, 1 mL is the amount that fills a cube that is 1 cm along each edge.

Reprint 2023

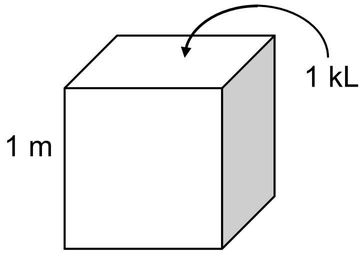
• If a container holds a very large amount, you can use a larger capacity measure: kilolitres.

- Consider a cube that is 1 m along each edge. Since 1 m = 10 dm, the cube measures 10 dm along one edge.



$1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^3$ $\qquad$ $10 \text{ dm} \times 10 \text{ dm} \times 10 \text{ dm} = 1000 \text{ dm}^3$

- 1 L fills each cubic decimetre (1 dm^3), so 1000 L, or 1 kL fills 1000 dm^3.
- 1000 dm^3 = 1 m^3, so 1 kL fills 1 m^3.

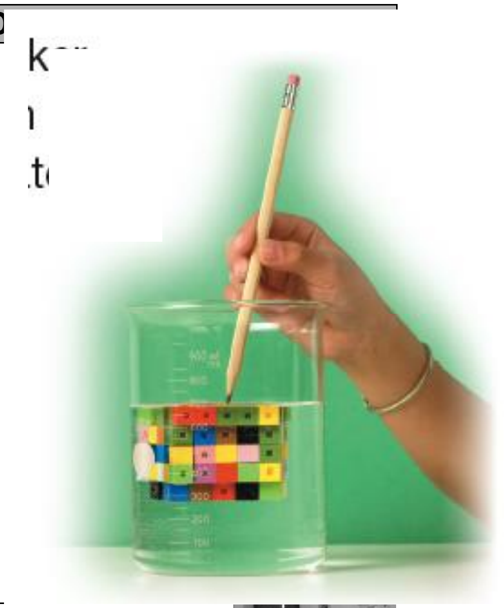


• You can use the relationships between volume and capacity to solve measurement problems.

B. Use your capacity estimate from **part A** to estimate the volume of a prism that is about the same size as the bucket in cubic centimetres.

Examples

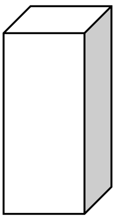
Example 1 Relating the Capacity of a Rectangular Prism to its Volume	
A rectangular prism container holds about 250 mL of water. What might be its dimensions?	
Solution 1 mL = 1 cm^3, so 250 mL = 250 cm^3. 250 = 25 × 10 = 5 × 5 × 10 One possible set of dimensions: length 10 cm width 5 cm height 5 cm	**Thinking**  • I changed from capacity units to volume units. • I found three numbers that multiplied to 250. • I could choose which number I wanted for length, width, or height

Example 2 Calculating Volume Using Water D	
Karma fills a beaker with 650 mL of water. She immerses a cube structure made of 1 cm^3 cubes in the water, causing the water level to rise to 700mL. How many 1 cm^3 cubes are in the structure?	
Solution	**Thinking**
700 mL – 650 mL = 50 mL 50 mL = 50 cm^3 The structure has a volume of 50 cm^3 so there are 50 centimetre cubes in the structure.	• I knew the amount that the water level rose in millilitres was the volume of the structure in cubic centimetres (1 mL = 1 cm^3). 

Practising and Applying

1. About how many millilitres of salt would each box hold?

a)

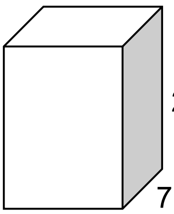


17 cm

5 cm

10 cm

b)



21 cm

7 cm

15 cm

2. Describe two possible rectangular prisms that hold each amount of water. For each, sketch and label the dimensions of one of the prisms.

a) 300 mL

b) 4 L

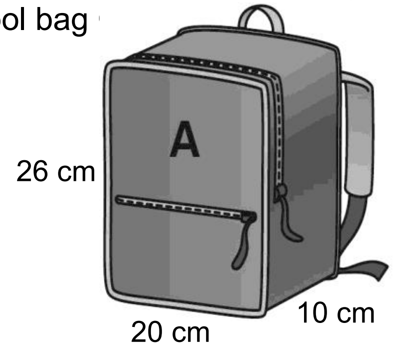
c) 5.2 L

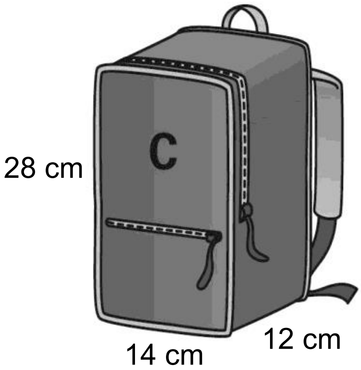
d) 2 kL

3. a) Sithar drops a handful of pebbles into 350 mL of water. The water level rises to 527 mL. What is the volume (in cm^3) of the pebbles?

b) What if the water level had risen to 1 L?

4. Which holds more, school bag A or school bag ?



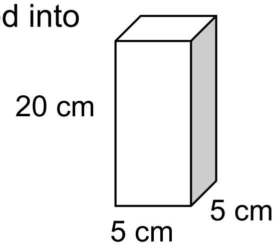


5. Two rectangular prism boxes each hold 500 mL of sugar. One box is twice as tall as the other.

a) What do you know about the area of the bases of the two boxes?

b) What do you know about the dimensions of the bases of the two boxes?

6. Water is poured into the container shown here. How deep will the water be for each amount of water below? Show your work.



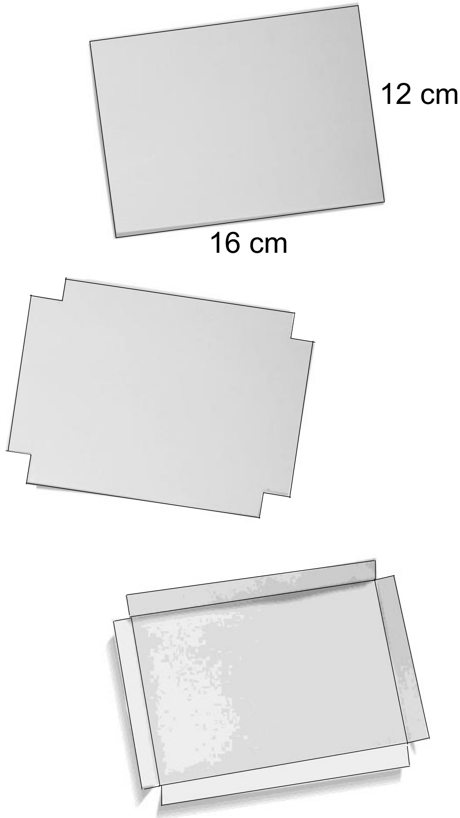
a) 250 mL **b)** 375 mL

7. A rectangular prism container holds about 1320 mL of water. Its length, width, and height are consecutive whole numbers. One of its dimensions is 10 cm. What are the dimensions of the prism?

8. Ugyen cuts identical squares with whole number dimensions out of the four corners of a sheet of paper as shown below. She folds up the sides to form a box with a capacity of 180 mL.

What size were the squares that Ugyen cut from each corner?

(Hint: You might draw a picture or use a model to help you solve this problem.)



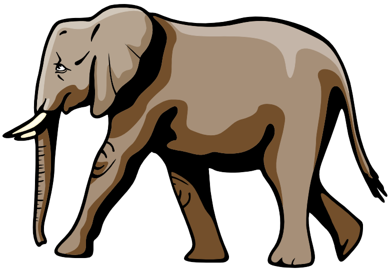
9. Why is it helpful to measure the volume of some objects by immersing them in water?

Chapter 3 Mass

5.3.1 The Tonne

Try This

A male Asian elephant has a mass of about 5400 kg.



There are about 370 groundnuts in 1 kg.

A. About how many groundnuts would balance an elephant on an enormous scale?
Explain how you estimated.

- We use either **kilograms** or **grams** to measure **mass** so that the numbers that describe the mass are not too big or too small.

For example:

If an object with a mass of 2500 kg were measured in grams, it would be 2,500,000 g. The number 2500 is easier to understand than 2,500,000.

- There is a unit bigger than a kilogram. It is the **tonne** (t) or **metric ton**.

$$1 \text{ tonne} = 1000 \text{ kg} \qquad 1 \text{ kg} = \frac{1}{1000} \text{ or } 0.001 \text{ t}$$

A 2500 kg object has a mass of 2.5 t. The number 2.5 is easier to understand than 2500.

- To change kilograms to tonnes, you divide by 1000 because 1 kg = $\frac{1}{1000}$ t.

For example:

315 kg = 0.315 t because 315 ÷ 1000 = 0.315.

- To change tonnes to kilograms, you multiply by 1000 because 1 t = 1000 kg.

For example:

73 t = 73,000 kg because 73 × 1000 = 73,000.

B. i) Write the mass of the elephant in tonnes.
ii) Why would you not write the mass of a groundnut in tonnes?

Reprint 2023

Example Comparing Mass Measurements

Which measurements are equal? Which is heaviest?

4 kg   4 t   0.4 t   0.04 t   40,000 g

Solution	Thinking
a) 4 kg = 0.004 t 40,000 g = 40 kg = 0.040 t 40,000 g = 0.04 t 4 t is the heaviest.	 • I wrote all the measurements in tonnes and then compared them. • 1 kg = 0.001 t since 1000 kg = 1 t. • I knew that 0.040 = 0.04. • All the measurements except 4 t were less than 1 t.

Practising and Applying

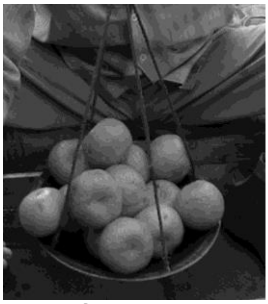
1. Match each mass to an object below.

60 g   4 kg   2t   500 g   12 kg   12 t

A

B

C



Oranges

D

E

F

2. Order from lightest to heaviest.

a) 350 g, 3.5 kg, 1.2 t, 1500 kg, 1.82 t

b) 2.03 t, 2300 kg, 2033 kg, 0.23 t, 23 kg

3. Write a mass in kilograms that is a bit lighter than 2.3 t.

4. In 2003, farmers in Bhutan produced 38,000 t of rice. How many kilograms is this?

5. An Asian wild buffalo is 909 kg. What is its mass in tonnes?

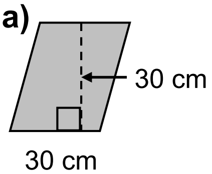
6. The Penden cement plant in the Samtse Dzongkhag produces about 300 t of cement per day. If the cement is packed into 100 kg bags, how many bags would that be?

7. Why might someone describe a tonne as a kilo-kilogram?

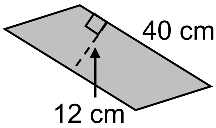
UNIT 5 Revision

1. Calculate the area of each shape.

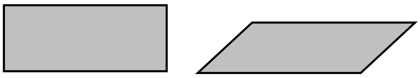
a)



b)



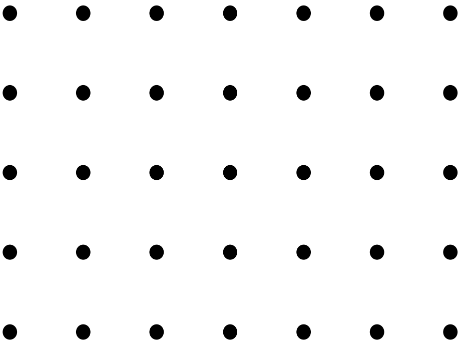
2. Which shape has a greater area? How do you know?



3. a) A parallelogram has an area of 60 cm^2. If its height is 10 cm, what is the length of the base?

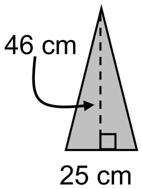
b) A different parallelogram is 60 cm^2. If its base is 11 cm greater than its height, what is the height?

4. Use a grid like this. Show why parallelograms with the same base, height, and area might not be congruent.

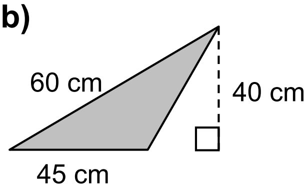


5. Calculate the area of each shape.

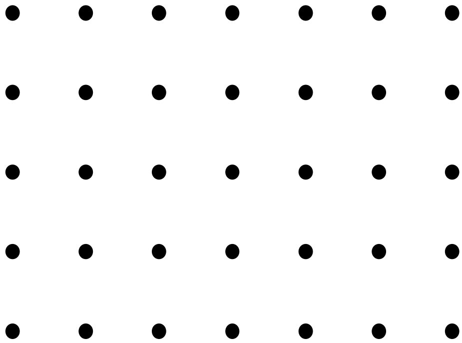
a)



b)



6. Use a grid like this. Show two triangles, each with an area of 4 square units but with different heights.



7. The area of a triangle is 30 m^2. The base is 2.5 m. What is its height?

8. Calculate the area of this shape.

80 cm 60 cm

30 cm

60 cm

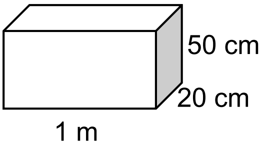
9. a) Parallelogram B has $\frac{1}{2}$ the height and 4 times the base length of Parallelogram A. How do the areas of Parallelograms A and B compare?

b) Triangle A has $\frac{1}{2}$ the height and half the base length of Triangle B. How do the areas of Triangles A and B compare?

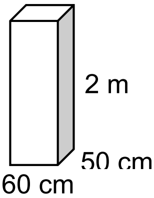
10. A triangle and a parallelogram have the same base, but the parallelogram is twice as high as the triangle. How do the areas of the triangle and parallelogram compare?

11. Calculate the volume of each.

a)



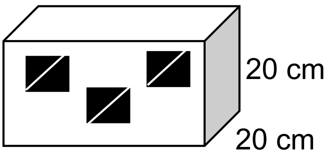
b)



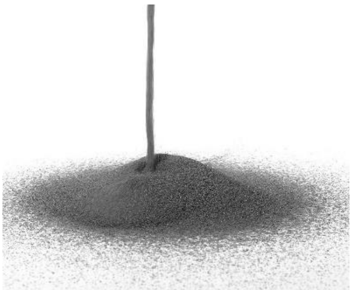
12. A rectangular prism has a volume of 200 cm^3. What could be its dimensions? List two possible sets of dimensions.

13. A rectangular prism with a volume of 3 m^3 has a base with an area of 5000 cm^2 What is the height of the prism?

14. Three 4 cm square holes are cut all the way through a block of wood. How much wood is left?



15. Suppose you poured sand into a rectangular prism container that is 50 cm by 20 cm by 30 cm. How many litres of sand would the container hold?

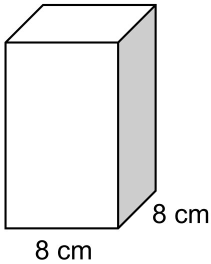


Sand pours like water.

16. A rectangular prism holds 2.5 L of water. What could its dimensions be? List two possible sets of dimensions.

17. An object is immersed in 1 L of water. The water level rises to 1.25 L. What is the volume of the object?

18. 512 mL of water is poured into this container. How deep will the water be?



19. Complete.

a) 23 t = ____ kg

b) 3.4 t = ____ kg

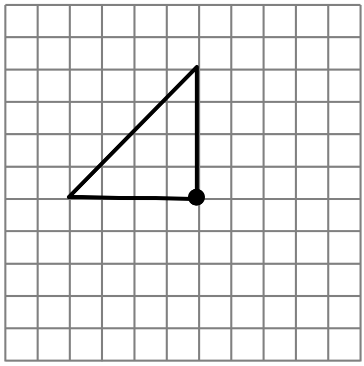
c) 1520 kg = ____ t

20. The mass of Object A is 0.1 of the mass of Object B. Object B has a mass of 25,000 kg. What is the mass (in tonnes) of Object A?

UNIT 6 GEOMETRY

Getting Started

Use What You Know



A. Follow these instructions to create a design using transformations.

i) Draw a triangle on grid paper, as shown.

ii) Reflect the triangle across the vertical side.

iii) Rotate the image from **part ii)** a $\frac{1}{4}$ turn clockwise (cw) around the marked point.

iv) Rotate the image from **part iii)** a $\frac{3}{4}$ turn counterclockwise (ccw) around the same point.

B. Identify these things in your design:

i) right angles

ii) perpendicular line segments that meet at their centre points

iii) lines of symmetry

C. Describe how the diagonals of a square intersect.

Skills You Will Need

1. Match each quadrilateral below with its name.

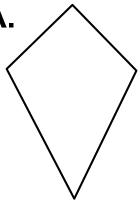
a) rhombus

b) trapezoid

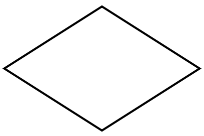
c) parallelogram

d) kite

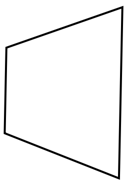
A.



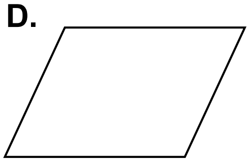
B.



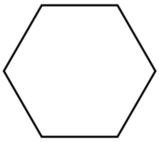
C.

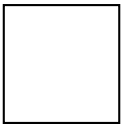


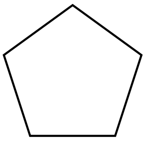
D.

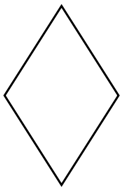


2. Which shapes are regular polygons? How do you know?

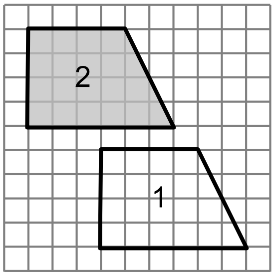
A. 

B. 

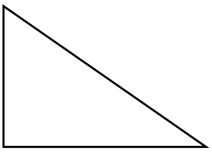
C. 

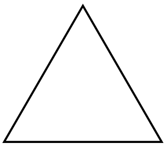
D. 

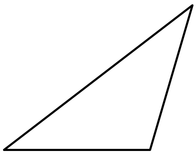
3. Describe the translation that moved Shape 1 to Shape 2.



4. Identify each triangle as acute, obtuse, or right.

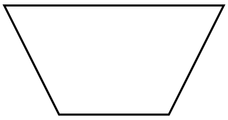
a) 

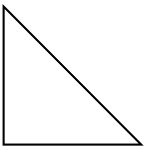
b) 

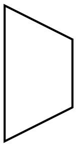
c) 

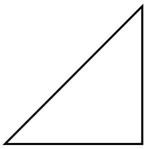
5. Identify each triangle from **question 4** as scalene, isosceles, or equilateral.

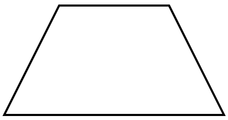
6. Which shapes are congruent?

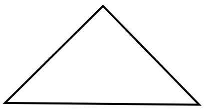
A. 

B. 

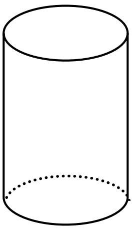
C. 

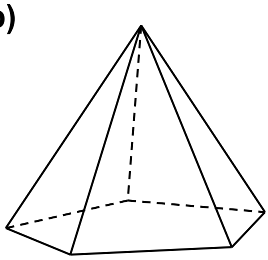
D. 

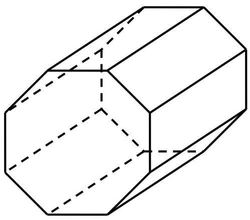
E. 

F. 

7. Name each 3-D shape.

a) 

b) 

c) 

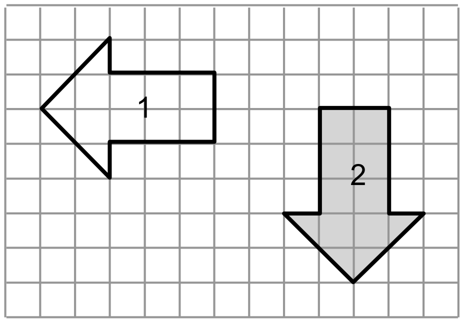
Chapter 1 2-D Geometry: Transformations

6.1.1 Rotations

Try This

Shape 1 has been transformed in one motion to create Shape 2.

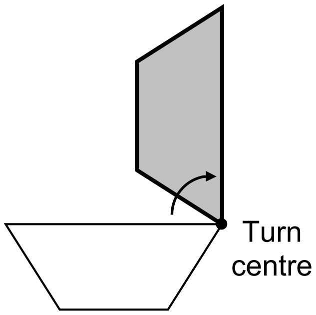
A. Why do you think the motion might be a rotation rather than a reflection or a translation?



• A **rotation** is a **transformation** that turns a shape around a point called the **turn centre**.

A rotation is described by three things:

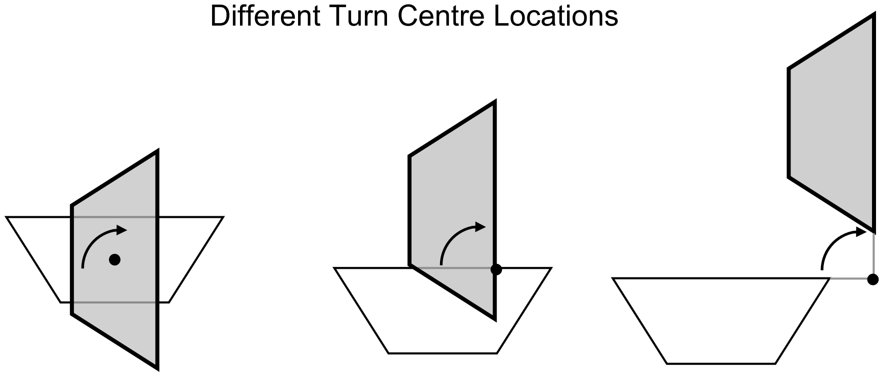
- Position of the turn centre
- Size of the rotation, often described with a fraction.
- Direction of the rotation, either clockwise(cw) or counterclockwise (ccw).



A $\frac{1}{4}$ cw rotation around the turn centre

• You have seen rotations where the turn centre is at a vertex of the shape. The turn centre can also be located in other places.

Different Turn Centre Locations



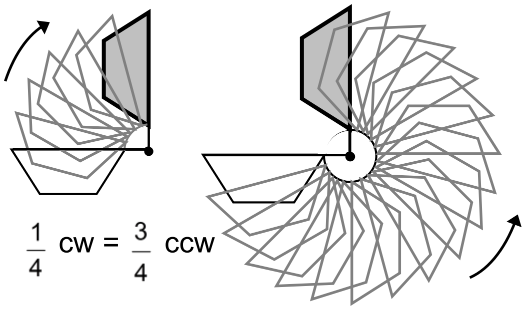
Inside the shape | On a side of the shape | Outside the shape

A $\frac{1}{4}$ cw rotation around three different turn centres

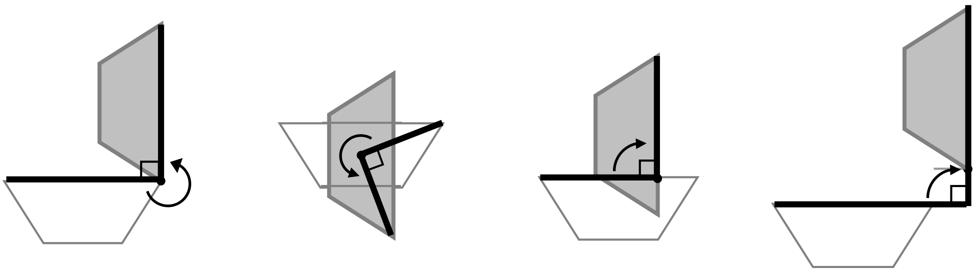
• The properties of a rotation are the same no matter where the turn centre is located.

- The distance from any point on the shape to the turn centre does not change when the shape is rotated. This means any point and its **image** point are the same distance from the turn centre.
- A rotation image is **congruent** to the **original shape**.

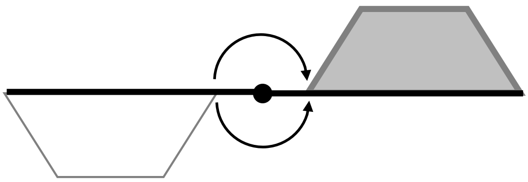
- A $\frac{1}{4}$ rotation in one direction results in the same image as a $\frac{3}{4}$ turn in the opposite direction.



- A $\frac{1}{2}$ turn cw results in the same image as a $\frac{1}{2}$ turn ccw.
- $\frac{1}{4}$ and $\frac{3}{4}$ turns create a right angle (90º) at the turn centre.



- A $\frac{1}{2}$ turn creates a **straight angle** (180º) at the turn centre.



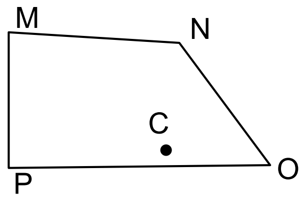
B. i) Sketch the arrows from **part A**. Show the turn centre for the rotation on your sketch. How did you find the turn centre?

ii) Describe the rotation in two different ways.

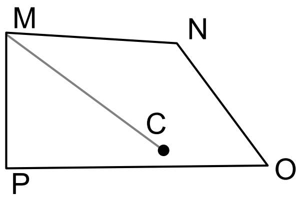
Examples

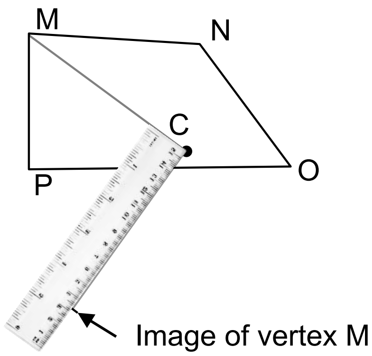
Example 1 Rotating a Shape a 3/4 turn

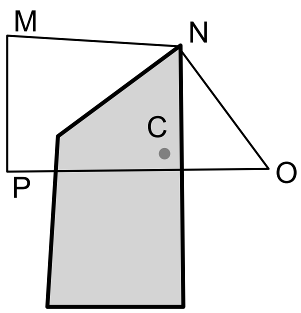
Rotate the quadrilateral a $\frac{3}{4}$ turn cw around point C.



Solution





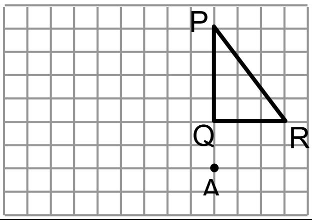


Thinking

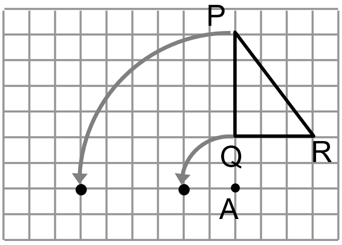
- I knew that a $\frac{3}{4}$ turn cw creates the same image as a $\frac{1}{4}$ turn ccw, so I instead used a $\frac{1}{4}$ turn ccw.
- I knew that a $\frac{1}{4}$ turn makes a right angle at the turn centre.
- This is how I located the image of M:
  - I drew a line segment from M to C and measured it.
  - I used the corner of my ruler to create a right angle at C. Then I marked the image of M the same distance from C as M was from C.
- I did the same thing for each vertex.
- I joined the image vertices to create the image.

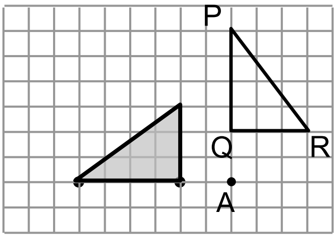
Example 2 Rotating a Shape on a Grid

Rotate the triangle a $\frac{1}{4}$ turn ccw around A.



Solution



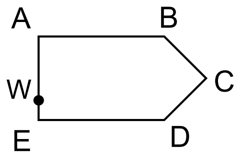


Thinking

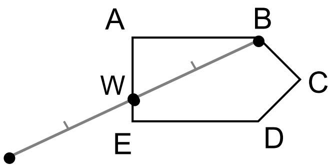
- I knew a $\frac{1}{4}$ turn would make a right angle at the turn centre.
- I used the right angles in the grid to locate the images of two vertices:
  - P is 6 units above A, so its image is 6 units left of A because the turn is ccw.
  - Q is 2 units above A, so its image is 2 units left of A because the turn is ccw.
- I used the two image vertices and the grid to draw a congruent image.
- Vertical lines become horizontal and horizontal lines become vertical.

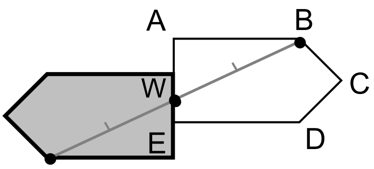
Example 3 Rotating a Shape a 1/2 turn

Rotate the shape a $\frac{1}{2}$ turn around W.



Solution





Thinking

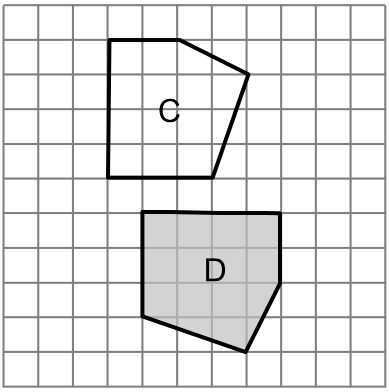
- I knew that a $\frac{1}{2}$ turn would make a straight line at the turn centre.
- I measured the distance from B to W. Then I drew a line segment from B through W that was twice as long. I marked the image of B at the end.
- I repeated this for all of the vertices.
- I joined the image vertices to create the image.

Practising and Applying

1. a) Copy shapes C and D below onto grid paper.

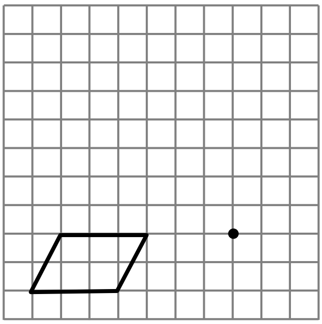
b) Identify the turn centre.

c) Describe two ways that shape C can be rotated to shape D.

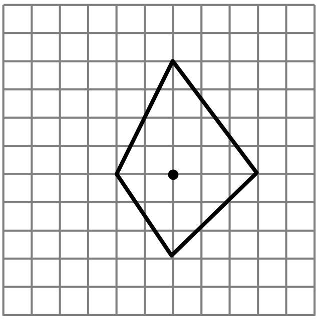


2. Copy each shape and turn centre below onto grid paper. Then rotate each shape as described.

a) $\frac{3}{4}$ turn ccw around the turn centre

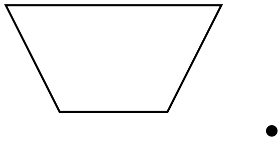


b) $\frac{1}{4}$ turn ccw around the turn centre

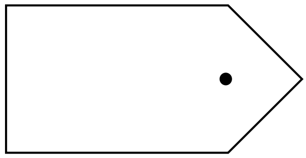


3. Trace each shape and turn centre. Rotate each shape a $\frac{1}{2}$ turn around its turn centre.

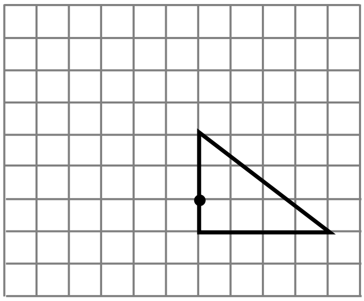
a)



b)



4. a) Copy the triangle and turn centre onto grid paper.



b) Rotate the triangle a $\frac{3}{4}$ turn cw around the turn centre.

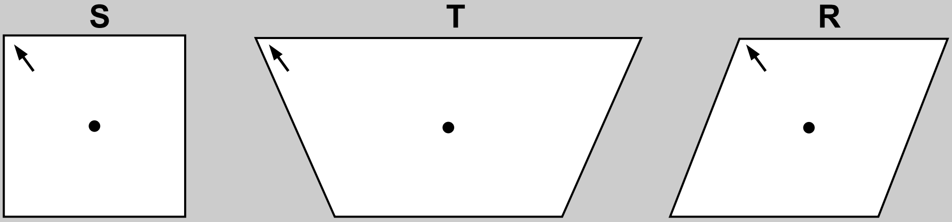
c) Mark a new turn centre on the grid inside the image.
Rotate the image a $\frac{1}{2}$ turn around the new turn centre.

5. In what ways is a rotation with the turn centre inside the shape similar to a rotation with the turn centre outside the shape? In what ways is it different?

6.1.2 Rotational Symmetry

Try This

When you rotate some shapes less than a full turn around a turn centre, they look like they are in their original position.



A. Trace shapes S, T, and R. Mark the turn centres and arrows before cutting them out. Place each traced shape over the original shape with the turn centres and arrows matching.

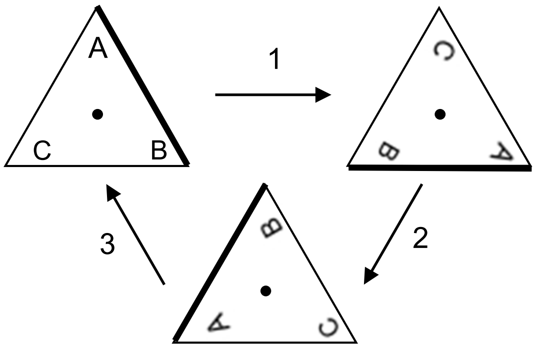
Hold the tip of a pencil on the turn centre of shape S. Turn the shape slowly one full turn cw or ccw. How many times does the traced shape line up with the original shape in one full turn? Repeat with shapes T and R.

A shape has **turn symmetry**, or **rotational symmetry** if it looks the same when it is rotated less than one full turn around a turn centre.

• The number of times the shape looks the same during one full turn is called the **order of turn symmetry**.

For example:

This triangle has turn symmetry of order 3 because it looks the same 3 times in one full turn.



• If a shape has no turn symmetry, it has turn symmetry of order 1 because it looks the same only once in one full turn — at the end of the turn.

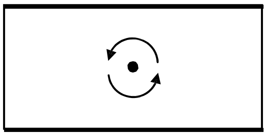
• To describe the turn symmetry of a shape, include two things:

- the turn centre
- the order of turn symmetry

• To predict whether a shape has turn symmetry, and if so, its order of turn symmetry by picturing the rotations in your mind. You can also look for congruent parts of the shape that could line up if the shape is turned.

For example:

A rectangle has pairs of congruent sides. A $\frac{1}{2}$ turn takes one long side to the other. A second $\frac{1}{2}$ turn brings the side back to its original position. Since it looks the same in two positions, the rectangle has turn symmetry of order 2.



• The order of turn symmetry for a **regular polygon** is equal to the number of its sides. You can turn it around the centre point so that one side lines up with each of the other sides, and then turn it back to its original position.

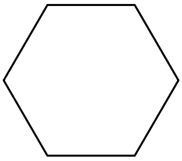
A. Which shapes in **part A** have turn symmetry? What is the order?

Examples

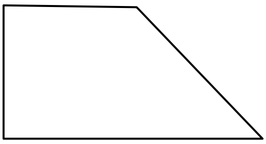
Example 1 Investigating Turn Symmetry in Shapes

What is the order of turn symmetry for each shape?

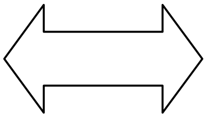
a)



b)

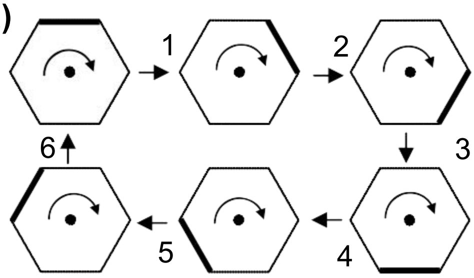


c)



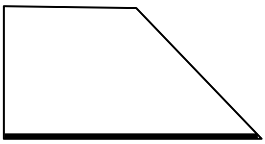
Solution

a)

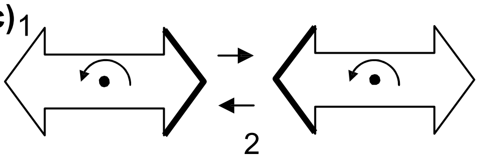


The regular hexagon has turn symmetry of order 6.

b) The quadrilateral has turn symmetry of order 1.



c)



This shape has turn symmetry of order 2.

Thinking

a) I used a turn centre in the middle of the regular hexagon.

• The top side is turned to each of the 5 other equal sides, then back to its original position.

• The hexagon looks the same 6 times within one turn.

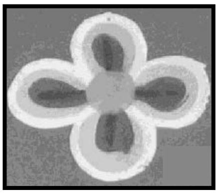
b) One side is longer than the others, so no matter where I put a turn centre, only a full turn will return the longest side to the same position.

• There is no turn symmetry, so the order is 1.

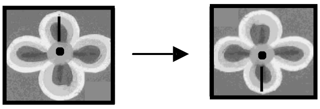
c) I was able to put a turn centre in the middle of the shape.

Example 2 Investigating Turn Symmetry in a Design

Describe the turn symmetry in this design.



Solution



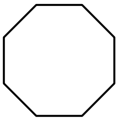
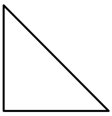
This design has turn symmetry of order 2.

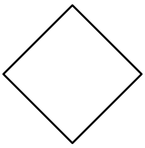
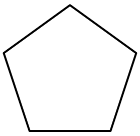
Thinking

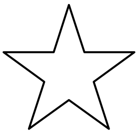
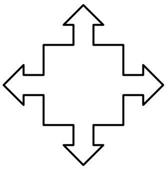
- The design fits in a rectangle that has turn symmetry of order 2.
- When I rotate the rectangle around its turn centre until it looks the same, the design looks the same too.

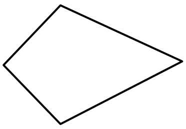
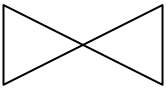
Practising and Applying

1. Predict whether each shape has turn symmetry. Explain your thinking.

a)  **b)** 

c)  **d)** 

e)  **f)** 

g)  **h)** 

2. For each shape in **question 1**, state the order of turn symmetry.

3. Describe the turn symmetry of each.

a) 

b) 

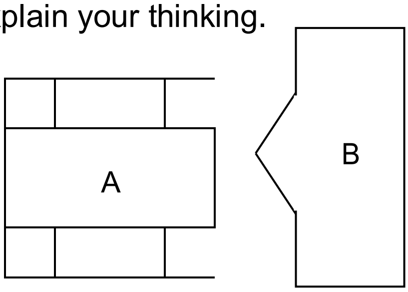
4. Describe three examples of turn symmetry in your classroom.

5. a) Draw a 5-by-5 square on grid paper. Describe the turn symmetry.

b) Add a 3-by-3 square to the outside of the middle of each side of the original square. Describe the turn symmetry.

c) Would there be turn symmetry if you had added only three new squares in **part b)**? Explain your thinking.

6. Namgyel says that only one of these shapes could have turn symmetry. Which shape is it? Explain your thinking.



7. Why is it often easier to predict the order of turn symmetry for a regular polygon than for another shape with turn symmetry?

6.1.3 Combining Transformations

Try This

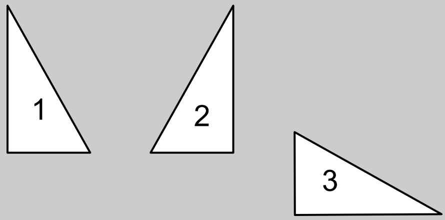
All of these triangles are congruent.

A. Describe each transformation:

i) Triangle 1 to Triangle 2

ii) Triangle 2 to Triangle 3

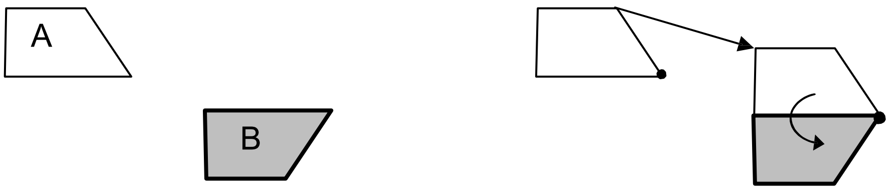
iii) Triangle 1 to Triangle 3



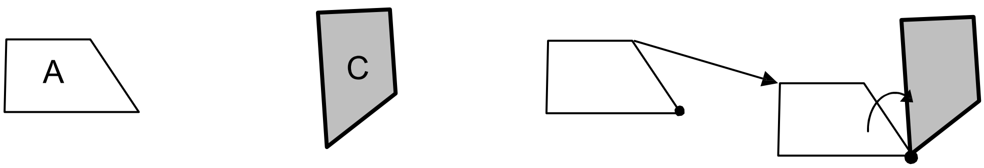
• Congruent shapes can always be transformed onto one another, but it may be necessary to combine transformations to do it.

For example:

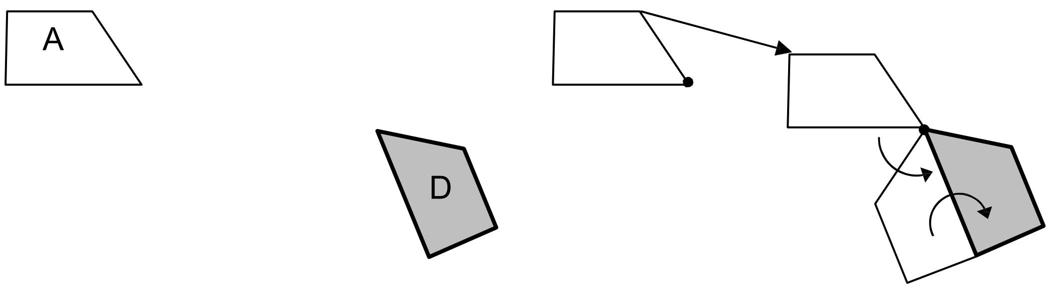
To transform A to B ... you can **translate** A so 2 pairs of **corresponding vertices** match, and then **reflect** the image.



To transform A to C ... you can translate A so 1 pair of corresponding vertices matches, and then rotate the image.



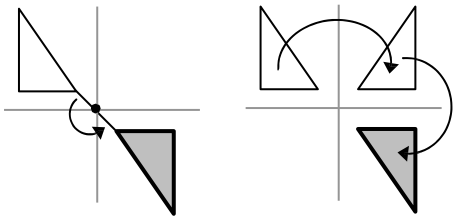
To transform A to D ... you can translate A so 1 pair of corresponding vertices matches, then rotate the image so 2 pairs of corresponding vertices match, and then finally reflect that image.



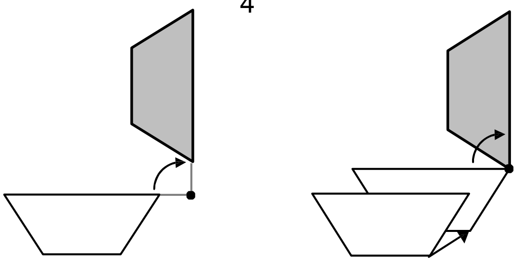
• Some combinations of transformations result in the same image as a single transformation.

For example:

- A $\frac{1}{2}$ turn rotation has the same image as a **horizontal reflection** followed by a **vertical reflection**.



- A $\frac{1}{4}$ cw rotation with the turn centre not at a vertex has the same image as a translation followed by a $\frac{1}{4}$ cw rotation around a vertex.



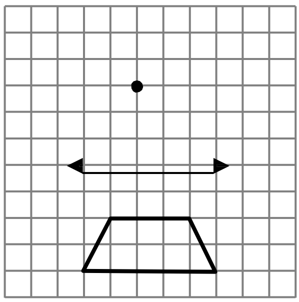
B. Describe another way to transform Triangle 1 to Triangle 3 from **part A**.

Examples

Example 1 Predicting the Image of Combined Transformations

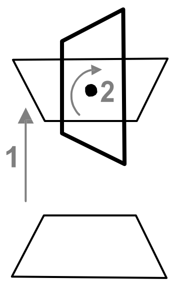
Karchung wants to reflect the trapezoid in the line and then rotate it a $\frac{1}{4}$ turn cw around the point.

Predict what the image will look like.
Transform the shape to check your prediction.



Solution

Predict

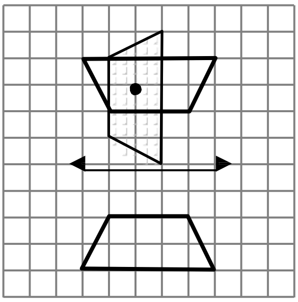


Thinking

• I knew the reflection would flip the trapezoid so the long side would be on top and the turn centre would be inside the shape.

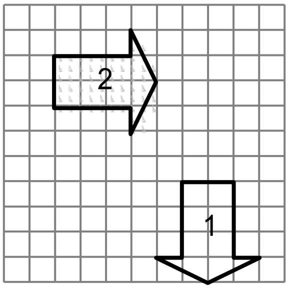
• I knew the rotation would turn the trapezoid so the long side would be vertical and on the right.

Reprint 2023

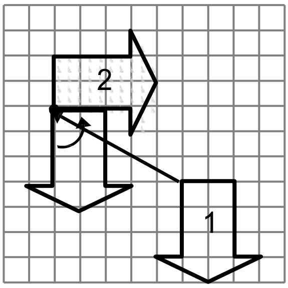
Check 	• For the reflection, I moved each vertex so it was the same distance from the line on the other side. Then I joined the image vertices to get the image. • For the rotation, the right angles at the turn centre didn't follow the gridlines, so I used my ruler to make a right angle and measure an equal distance to the turn centre.

Example 2 Describing Combined Transformations

Describe one way to transform Arrow 1 to Arrow 2.



Solution 1



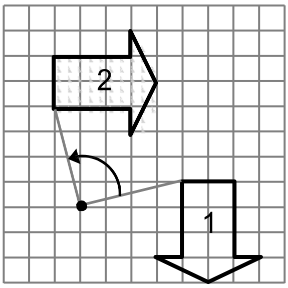
Thinking

• I could tell a $\frac{1}{4}$ turn ccw was involved because of the way the arrows pointed.

• I translated Arrow 1 so the top left vertex moved to its corresponding vertex on Arrow 2.

• I rotated the image a $\frac{1}{4}$ turn ccw around that vertex.

Solution 2



Thinking

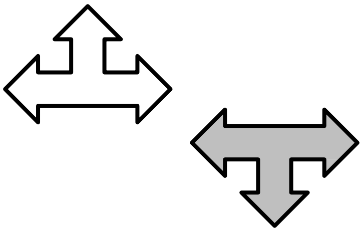
• I looked for a single $\frac{1}{4}$ turn ccw that would move Arrow 1 to Arrow 2.

• I tried different points for the turn centre until I found the point where each vertex and its image were the same distance away from the point.

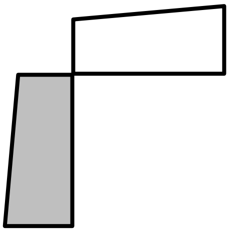
Practising and Applying

1. Each white shape was transformed twice to get the grey shape. Describe the transformations that might have been used.

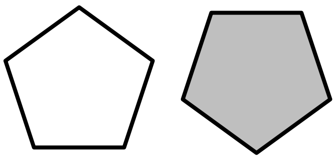
a)



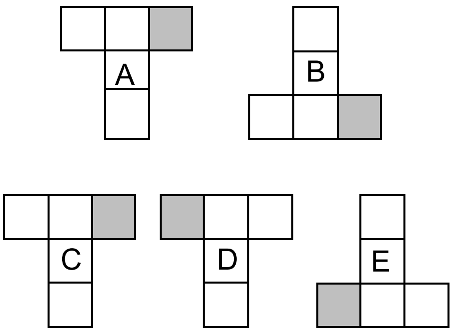
b)



c)

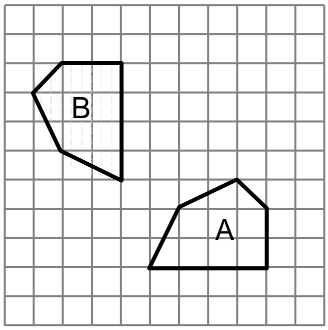


2. a) Which shapes are images of Shape A after a single transformation? What type of transformation is each?



b) Which shapes are images of Shape A after a combination of two transformations? Describe each combination of transformations.

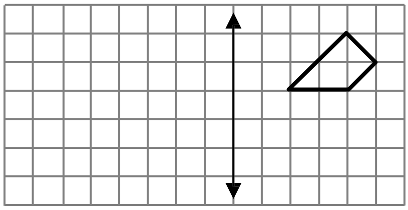
3. a) Predict a single transformation that will move Shape A to Shape B.



b) Predict a combination of two transformations that will move Shape A to Shape B.

c) Perform the transformations in **parts a) and b)** to check your predictions.

4. a) Predict what the image will be after the shape below has been translated 2 units left and 3 units down, and then reflected across the line. Explain your prediction.



b) Do you think the image will be the same if the transformations are done in the reverse order? Explain your thinking.

c) Copy the shape onto grid paper and transform it to check your answers to **parts a) and b)**.

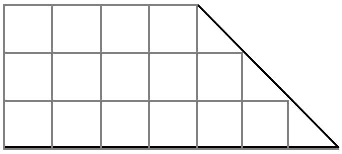
5. Tshering reflected a shape more than once. Rupak said it looked like a translation. How many reflections might Tshering have done? How do you know?

GAME: Transformation Challenge

Play this game with a partner. You need two pieces of grid paper.

In this game, you try to get as low a score as possible as you describe transformations.

• Draw two copies of this shape on grid paper and cut them out. Label one shape with one player's name or initial and the other shape with the other player's name or initial.



• Play the game on another sheet of grid paper.

Here are the rules:

• You each place your shape on the grid paper. Take turns going first.

• After you place the shapes, you each secretly write down a description of a transformation or a combination of transformations that would move Player A's shape to Player B's shape or Player B's shape to Player A's shape.

• You each get points for your description according to the chart.

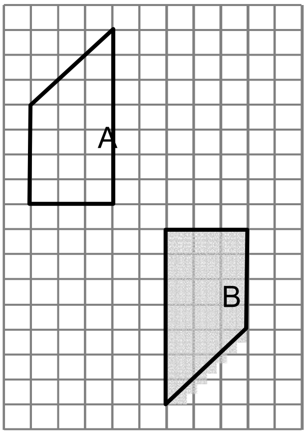
Transformation	Points
Translation	3
Reflection	2
Rotation	1

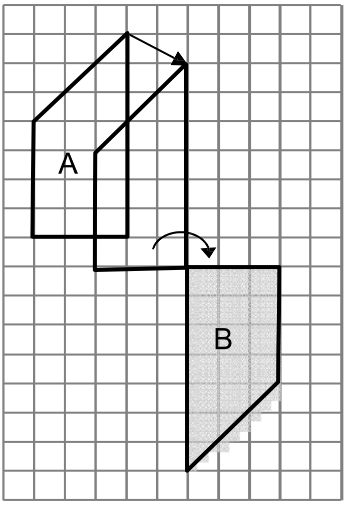
• Check each other's descriptions for accuracy.

• The winner is the player with the lower score after 10 rounds of play.

[Continued]

Sample round:





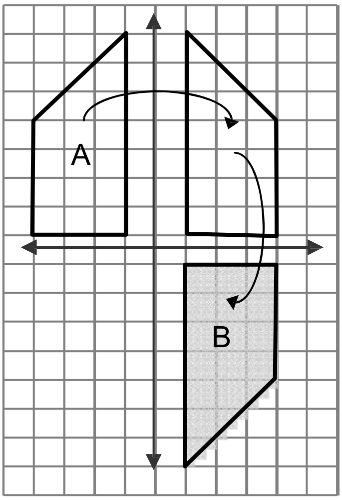
Player A

I could move Shape A to Shape B by translating it and then rotating it.

I scored 1 + 3 = 4 points.

Player B

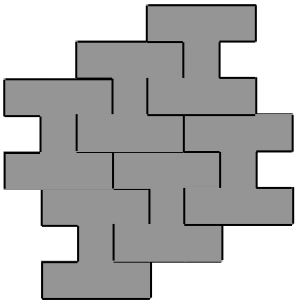
I could move Shape A to Shape B by reflecting it horizontally and then reflecting it vertically.
That's 2 + 2 = 4 points.



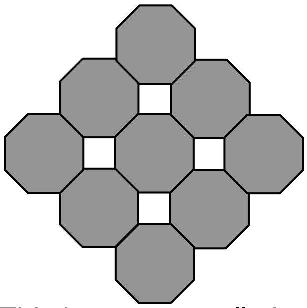
Reprint 2023

6.1.4 EXPLORE: Tessellations

A **tessellation** is an arrangement of congruent shapes that fit together without gaps and without overlapping. It can go on forever in all directions.

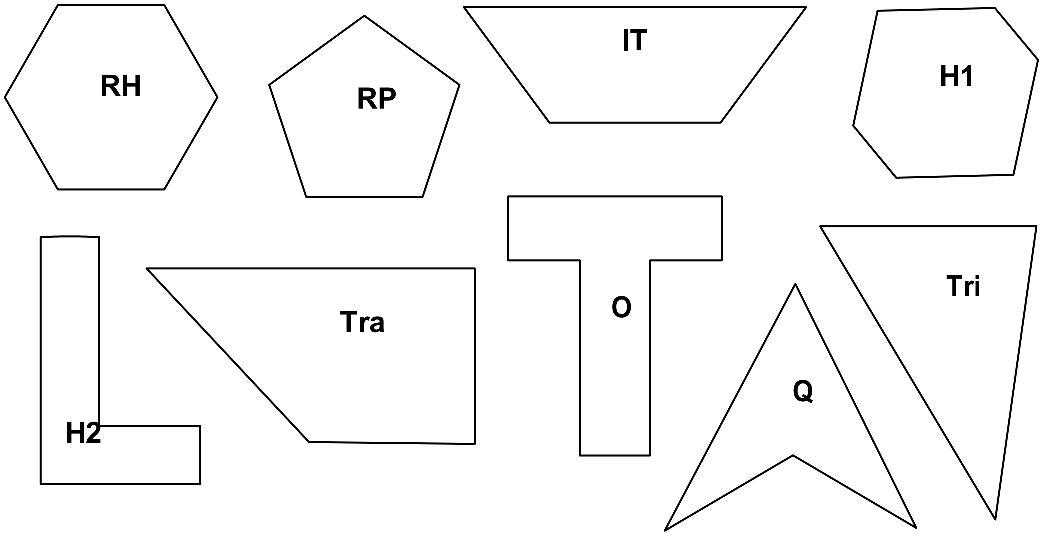


This is a tessellation.



This is not a tessellation.

Some of these shapes will fit together to make a tessellation. Some will not.



A. Predict which shapes will make a tessellation and which will not.

B. Trace several copies of one shape. Cut them out.
Try to fit the shapes together to create a tessellation.

C. Repeat **part B** for at least four other shapes.

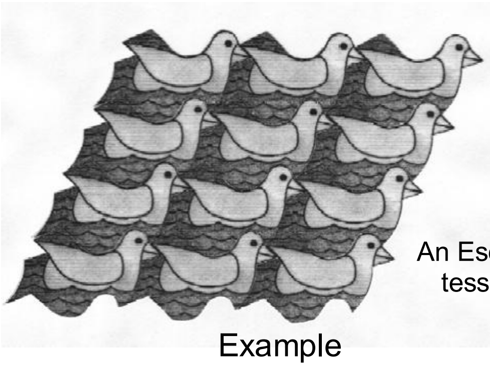
D. Were your predictions in **part A** correct?

E. How did you decide whether a shape could be used to make a tessellation?

CONNECTIONS: Escher-type Tessellations

M.C. Escher, a Dutch artist, is well known for the creative tessellations he made.

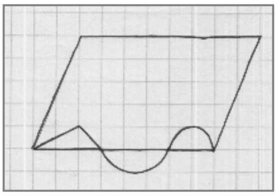
Follow these steps to make your own Escher-type tessellation.



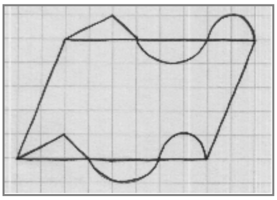
An Escher-type tessellation

Example

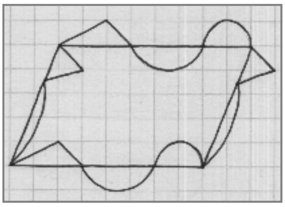
Step 1 Begin with a parallelogram. Modify one side.



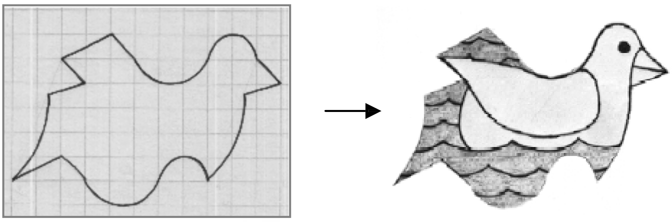
Step 2 Modify the parallel side in exactly the same way.



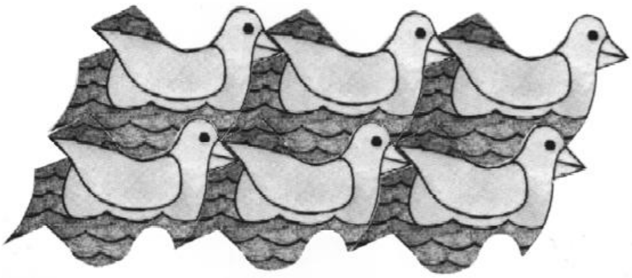
Step 3 Modify one of the other sides in any way. Then modify its parallel side in the same way.



Step 4 Erase the parallelogram lines to create your shape. Decorate your shape and cut it out.



Step 5 Make a tessellation by fitting copies of your shape together top-to-bottom and side-to-side, like a puzzle.

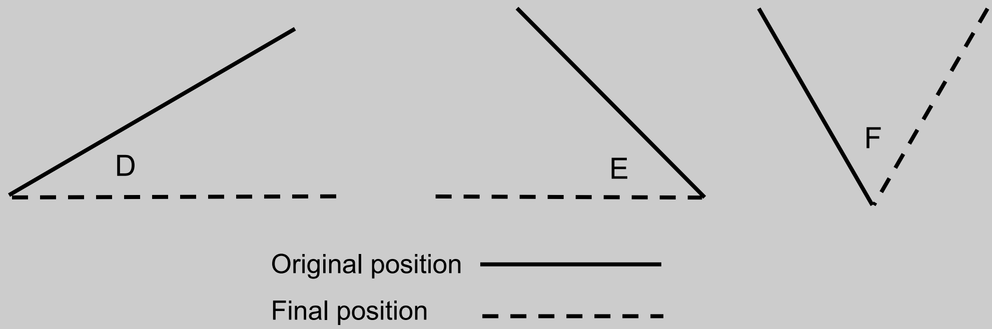


Reprint 2023

6.2.1 Measuring Angles

Try This

Each angle shows the original position and the final position of a line segment after a fraction of a full turn.

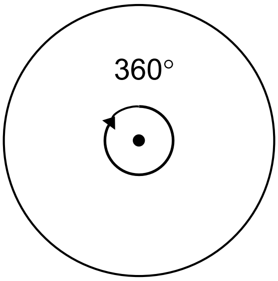


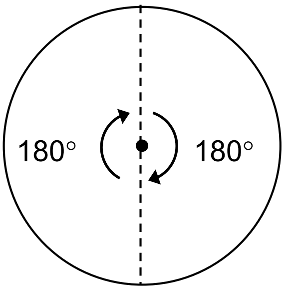
A. i) How many of each angle will fit together to make a full turn?
ii) What fraction of a whole turn does each angle represent?

- An **angle** measure can be expressed as a fraction of a full turn. It can also be measured in units called **degrees**.

- A degree (°) is $\frac{1}{360}$ of a full turn.

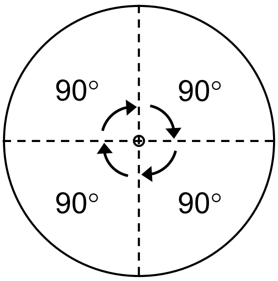
- A full turn measures 360° (360 degrees).





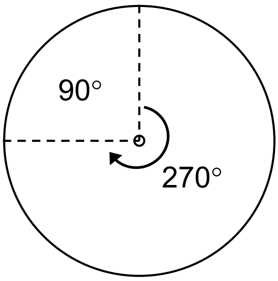
A $\frac{1}{2}$ turn is 180º.

360° ÷ 2 = 180°



A $\frac{1}{4}$ turn is 90º.

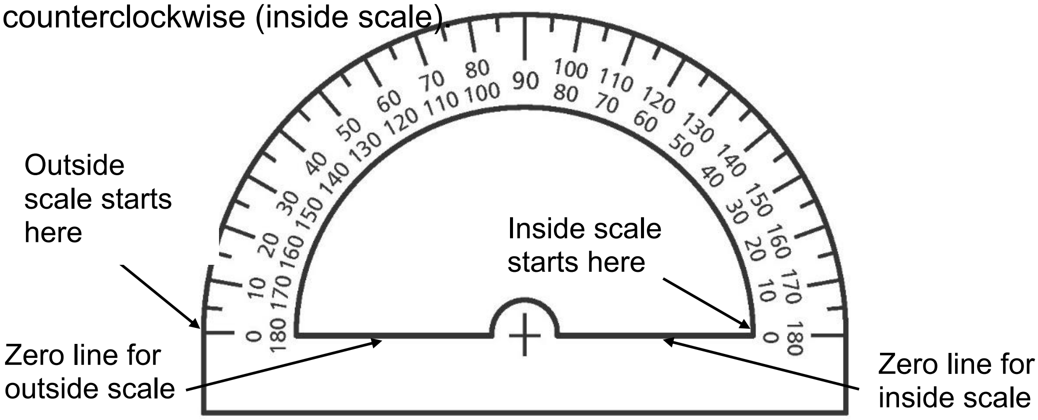
360° ÷ 4 = 90°



A $\frac{3}{4}$ turn is 270º.

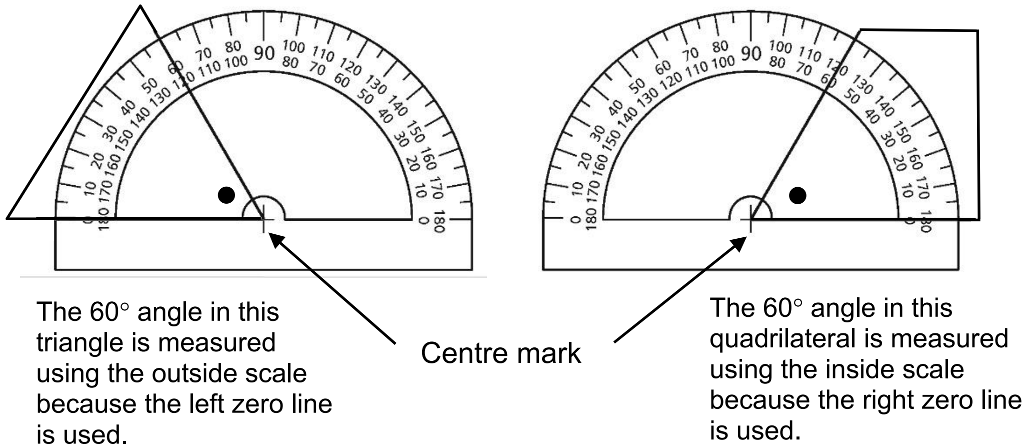
360° – 90° = 270°

• A **protractor** is a tool for measuring angles in degrees. Many protractors
have two scales from 0° to 180°, one clockwise (outside scale) and one counterclockwise (inside scale).



• This is how to measure an angle with a protractor:

- Place the centre mark of the protractor on the vertex of the angle.
- Move the protractor so that one of the zero lines is on top of one arm of the angle.
- Read the angle measure at the other arm. Use the scale (inside or outside) that matches the zero line you used.



B. i) Use a protractor to measure each angle in **part A**.

ii) Write each degree measure as a fraction with a denominator of 360.

iii) Compare the fractions in **part B ii)** to the fractions in **part A ii)**. What do you notice?

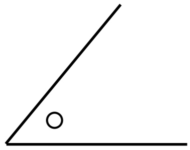
Reprint 2023

Examples

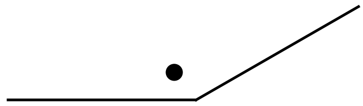
Example 1 Estimating and Measuring Angles

Estimate the size of each angle. Then measure each with a protractor.

a)

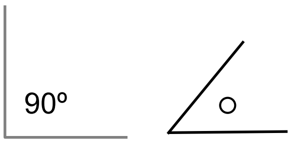


b)



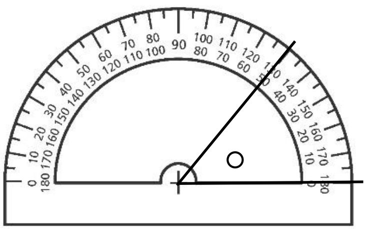
Solution

a) *Estimate*



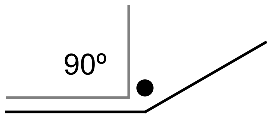
About 45°

Measure



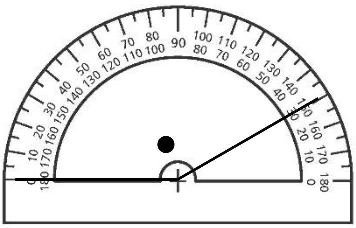
The angle is 50°.

b) *Estimate*



About 135°

Measure



The angle is 150°.

Thinking

a) I compared the angle to a right angle, which is 90°. It looked like it was about half the size.

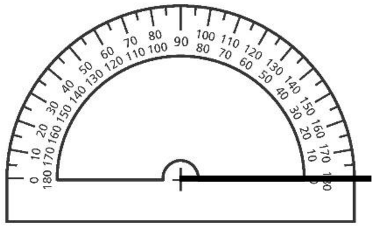
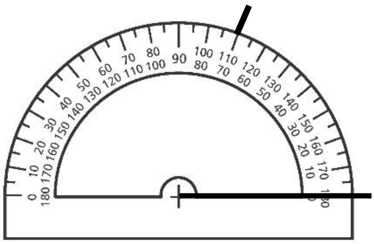
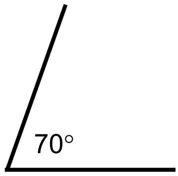
- I placed my protractor over the angle so that one arm was under the right zero line and the vertex was at the centre point.
- I read the scale on the inside because that is the scale that goes with the right zero line.

b) I compared the angle to a right angle, which is 90°. It looked like it was about 90° plus another half of that: 90° + half of 90° = 90° + 45° = 135°.

- I placed my protractor over the angle so that one arm was under the left zero line and the vertex was at the centre point.
- I read the scale on the outside because that is the scale that goes with the left zero line.

Example 2 Drawing an Angle of a Given Measure

Draw an angle that measures 70°.

Solution	**Thinking**
	 • I drew a line segment to be one arm of the angle. • I placed the protractor so that the line segment began at the centre mark and was under the right zero line.
	• Since I used the right zero line, I used the inside scale to find 70°. • I made a mark at 70°.
	• I used my ruler to connect the mark with the end of the line segment. Then I labelled the angle with its size.

Practising and Applying

1. State each angle measurement.

a)

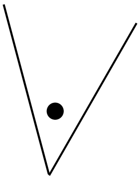
b)

c)

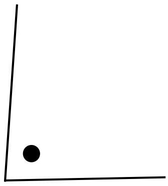
d)

2. Estimate the size of each angle. Then use a protractor to measure.

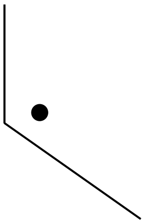
a)



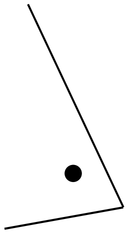
b)



c)

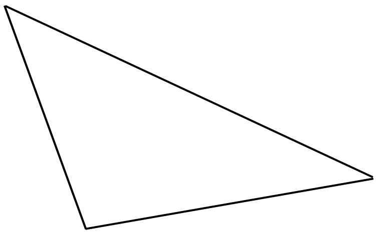


d)

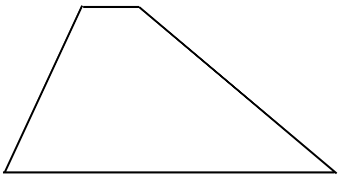


3. Sketch each polygon and label its angle measures.

a)



b)



4. a) Find three or more angles in your classroom. Estimate the size of each.

b) Measure each angle with a protractor and compare the measurements to your estimates.

5. a) Use only a ruler to draw angles that are about each size

- 170°
- 60°
- 120°
- 95°

b) Use a protractor to draw each angle.

c) How accurate were your estimated angles in **part a)**?

6. Look at the lines that divide the fields in the photo below.

a) Find angles that are about each size:

- 20°
- 140°
- 165°

b) Find an angle in the field that you think is different in size than the angles in **part a)**. Use a protractor to check. What size is it?

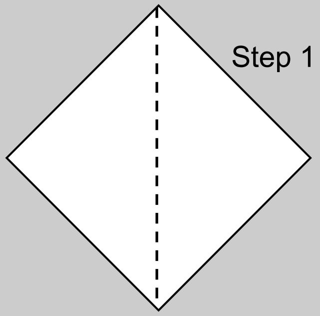
7. Tashi measured an angle in an acute triangle and found that it was 120°. What did he do wrong?

6.2.2 Bisectors

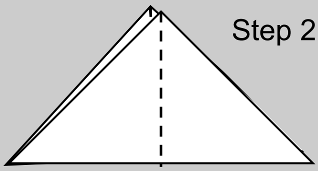
Try This

A. Follow these steps to fold a square into a kite:

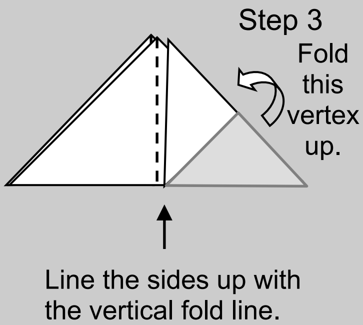
Step 1 Fold and then unfold along one diagonal of a square piece of paper.



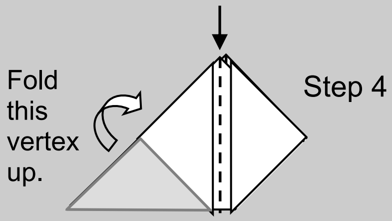
Step 2 Fold the bottom vertex up to the top vertex to form a right triangle.



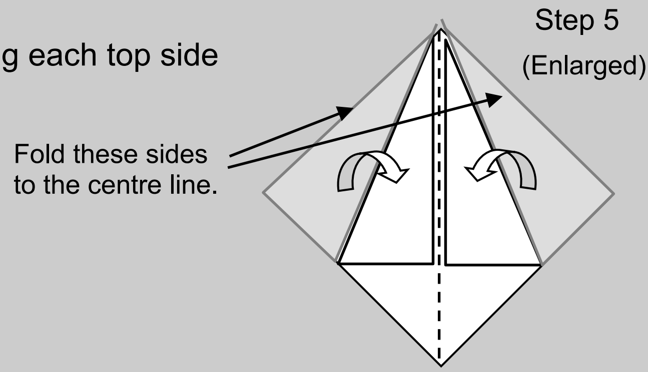
Step 3 Fold the right vertex of the triangle to the top vertex, lining the side up with the vertical fold line.



Step 4 Now fold the left vertex to the top vertex, lining up the side with the vertical fold line.



Step 5 Make a kite by folding each top side to the centre line.



B. Unfold the kite to make the original square. Look at the fold lines. Use a protractor and a ruler to help you identify these things:

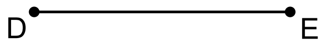
i) angles that are half of other angles

ii) line segments that are cut in half by other line segments

iii) line segments that meet or cross at right angles

• A **line segment** is a part of a line, so it has two **end points**.

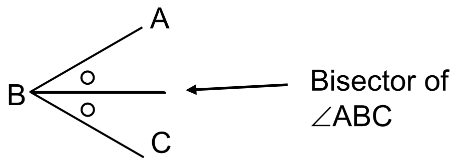
Note how the line segment is named by its two endpoints.



This is line segment DE.

• A **bisector** divides something into two equal parts.

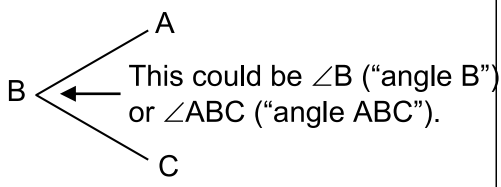
• An **angle bisector** divides an angle into two congruent angles.



Note

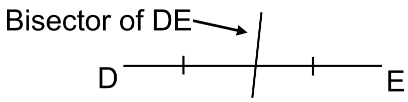
- how the angle is named by its vertices and the symbol ∠.

- the use of identical symbols to show that the two angles are congruent.

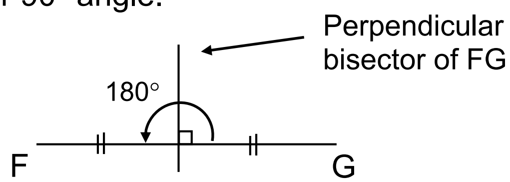


• The bisector of a line segment meets or crosses the line segment at its **centre point** at any angle.

Note the use of identical symbols to show that the two line segments are congruent.



• A **perpendicular bisector** meets or crosses a line segment at its centre point at a right, or 90º angle.



Note that a perpendicular bisector is also an angle bisector because it divides the 180° angle (straight angle) formed by the line segment into two 90° angles.

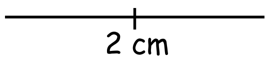
C. Describe examples of angles bisectors, perpendicular bisectors, and other line segment bisectors in your square from **part B**.

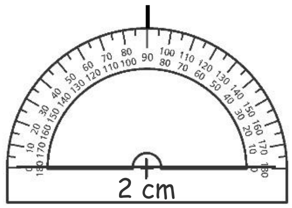
Examples

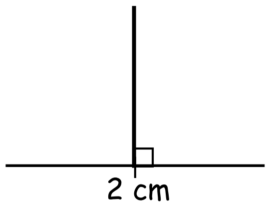
Example 1 Drawing a Perpendicular Bisector

Draw a perpendicular bisector for a line segment that is 4 cm long.

Solution







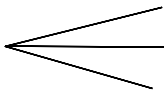
Thinking

- I drew a line segment 4 cm long and marked its centre at 2 cm.
- I placed the centre of my protractor at the 2 cm mark and made another mark to show 90°.
- I joined the marks and added a small square symbol to show the right angle.

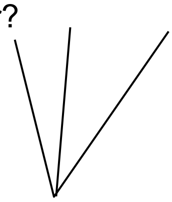
Example 2 Identifying Angle Bisectors

Which show an angle bisector?

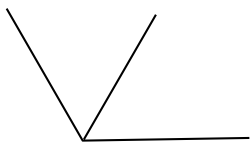
A



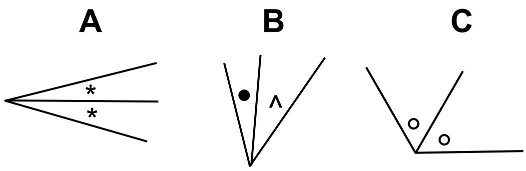
B



C



Solution



A and C show angle bisectors.

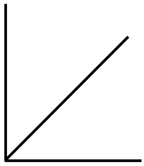
Thinking

- I measured each angle with a protractor.
- I marked congruent angles with the same symbol.

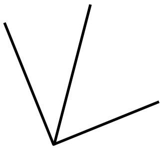
Practising and Applying

1. Which show an angle bisector? How do you know?

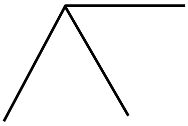
A.



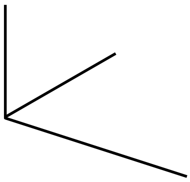
B.



C.

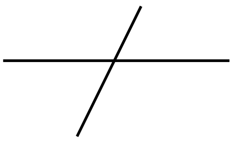


D.

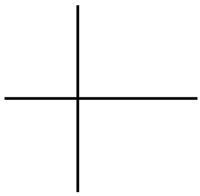


2. Which show a line segment bisector? How do you know?

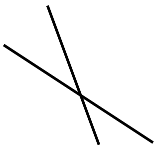
A.



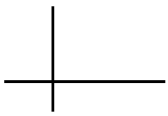
B.



C.



D.



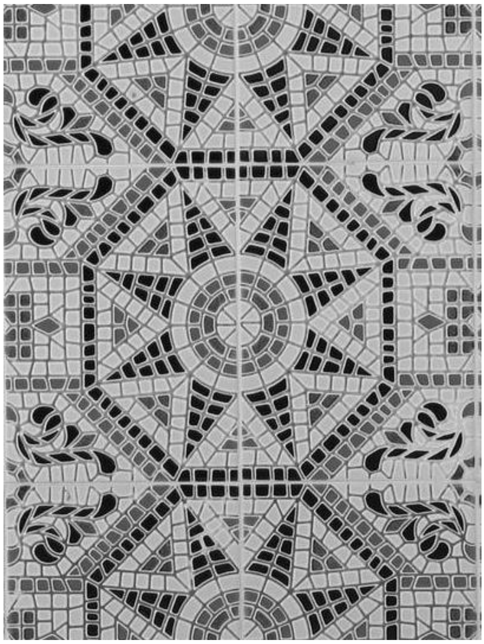
3. Which of the bisectors in **question 2** are perpendicular bisectors? How do you know?

4. Draw two line segments that are 6 cm long.

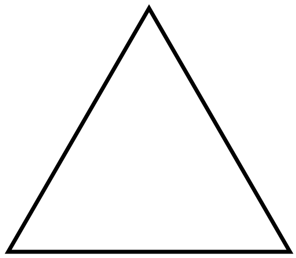
a) Draw a perpendicular bisector for one of the line segments.

b) Draw a bisector for the other line segment that is not perpendicular to the line.

5. Identify examples of angle bisectors, line bisectors, and perpendicular bisectors in this design. Measure with a ruler and protractor to check your answers.



6. Trace this equilateral triangle.



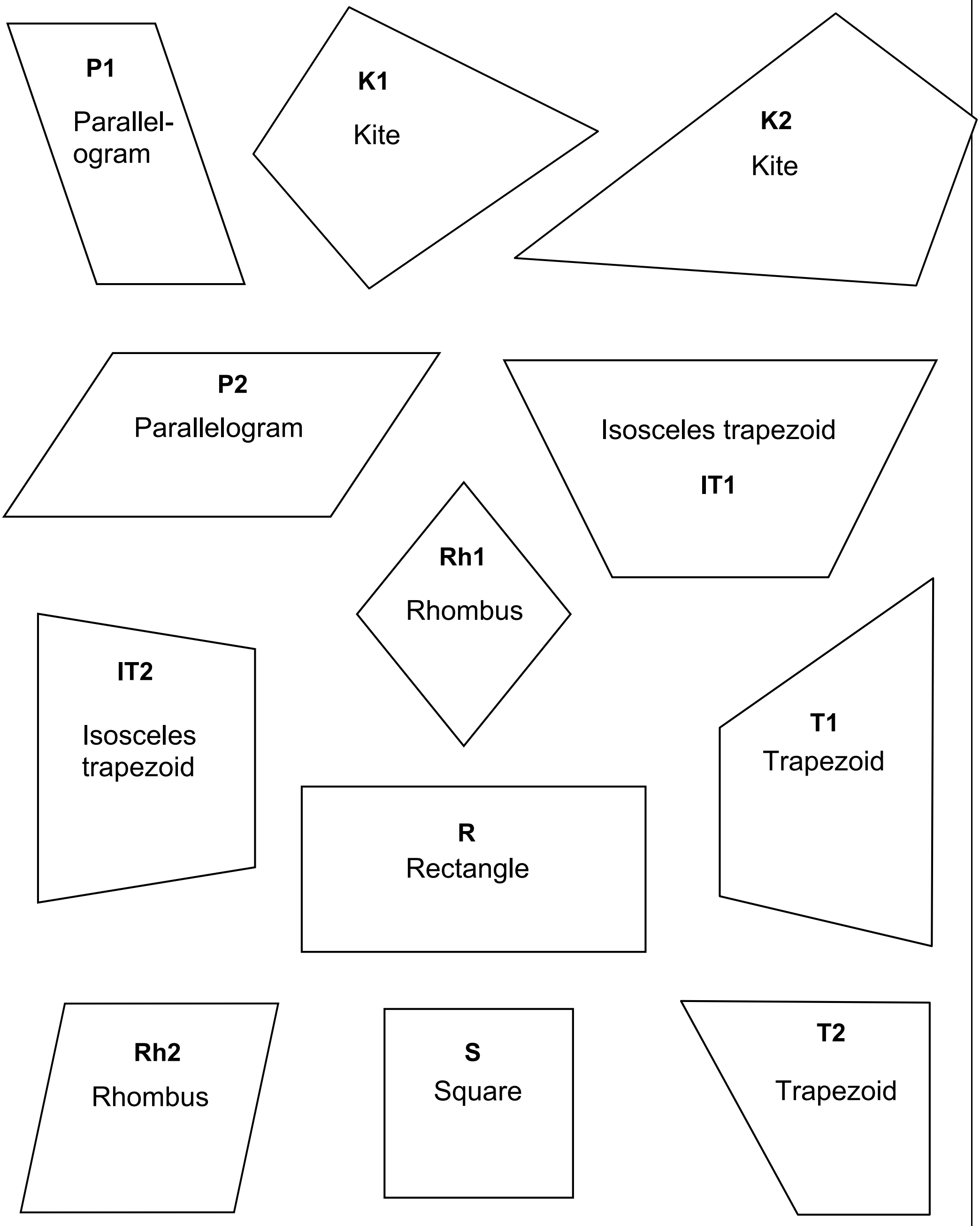
a) Inside the triangle, draw a perpendicular bisector for each side. Extend each bisector so it crosses the triangle to the opposite vertex.

b) Measure the angles at each vertex. Are the perpendicular bisectors of the sides also angle bisectors? How do you know?

7. How are angle bisectors and perpendicular bisectors the same? How are they different?

6.2.3 EXPLORE: Sorting Quadrilaterals

You have learned that the **diagonals** of rectangles have special properties. The diagonals of other **quadrilaterals** also have special properties.



A. Trace each quadrilateral on **page 54** and label it with its symbol. Use a ruler to draw the diagonals in each quadrilateral.

B. Copy the charts below. Write the letters of the quadrilaterals that match each description. Use a ruler and protractor to help you.

i)

The diagonals bisect each other	
One diagonal bisects the other	
Neither diagonal bisects the other	
The diagonals are perpendicular to each other	
Both diagonals bisect the angles of the quadrilateral	
One diagonal bisects the angles of the quadrilateral	
Both diagonals are lines of symmetry	
One diagonal is a line of symmetry	
Neither diagonal is a line of symmetry	

ii) The diagonals divide the quadrilateral into:

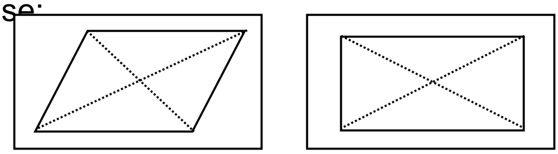
four congruent right scalene triangles	
four congruent right isosceles triangles	
two pairs of congruent triangles - one pair obtuse scalene - one pair acute scalene	
two pairs of congruent triangles - one pair obtuse isosceles - one pair acute isosceles	
two pairs of congruent right scalene triangles	
one pair of congruent acute scalene triangles and two non-congruent triangles	
four non-congruent triangles	

C. i) A shape has diagonals that divide it into 4 congruent triangles. What could the shape be?

ii) A shape has diagonals where one is a perpendicular bisector of the other. What could the shape be?

GAME: Go Fish

Play in a group of 2 to 4 players. You need a deck of Go Fish Game Cards like these:



In this game, you match quadrilaterals by their diagonal properties.

This is how to play:

• The dealer gives each player seven cards. The rest of the cards are placed face down in the middle in a pile called "the pond."

• Players take turns. On your turn, secretly choose one of your cards and think of a property about the diagonals.

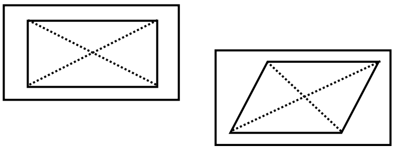
Next, fish for a matching card from any other player by asking a question that starts like this:

“Do you have a quadrilateral with diagonals that …?”

If the other player has a matching card, he or she must give it to you.

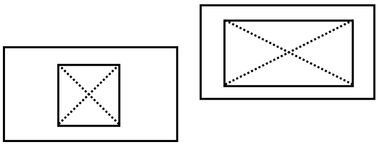
For example:

“Do you have a quadrilateral with diagonals that create two pairs of other?” congruent triangles?"



The rectangle and parallelogram match because the diagonals create two pairs of congruent triangles.

“Do you have a quadrilateral with diagonals that bisect each



The rectangle and square match because the diagonals bisect each other for both shapes.

If the other player does not have a match, he or she says, “Go Fish!” and you must draw a card from the pond.

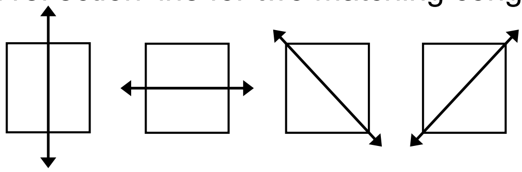
• Place all your matched cards, whether from a player or from the pond, face down in front of you.

• The game ends when one player has used all of his or her cards.
The winner is the player with the most matches in front of him or her.

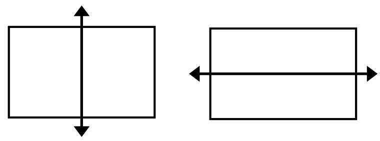
Chapter 3 3-D Geometry

6.3.1 EXPLORE: Planes of Symmetry

Some 2-D shapes have **mirror symmetry**. A **line of symmetry** acts like a reflection line for two matching congruent halves of the shape.

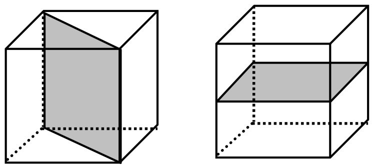


A square has four lines of symmetry.

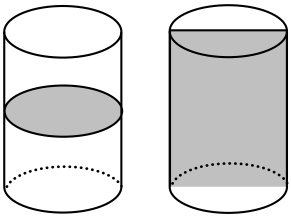


A rectangle has two lines of symmetry.

Some 3-D shapes have mirror symmetry. The imaginary surface that cuts a 3-D shape into congruent matching halves is called a **plane of symmetry**.



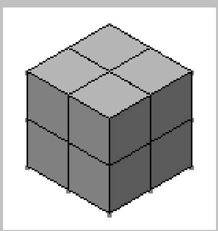
Some planes of symmetry of a cube.



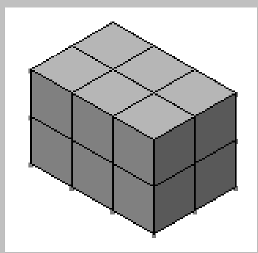
Some planes of symmetry of a cylinder.

A. Build these structures with cubes. Split each structure along a plane of symmetry in as many different ways as you can.

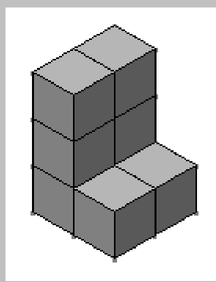
i)



ii)

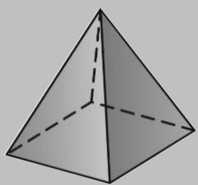


iii)



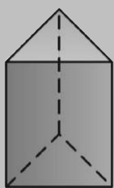
B. Sketch or describe the planes of symmetry of each 3-D shape.

i)



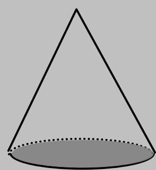
Square-based pyramid

ii)



Equilateral triangle-based prism

iii)



Cone

C. i) Describe the number of lines of symmetry of a circle.

ii) Describe the number of planes of symmetry of a cone or cylinder.

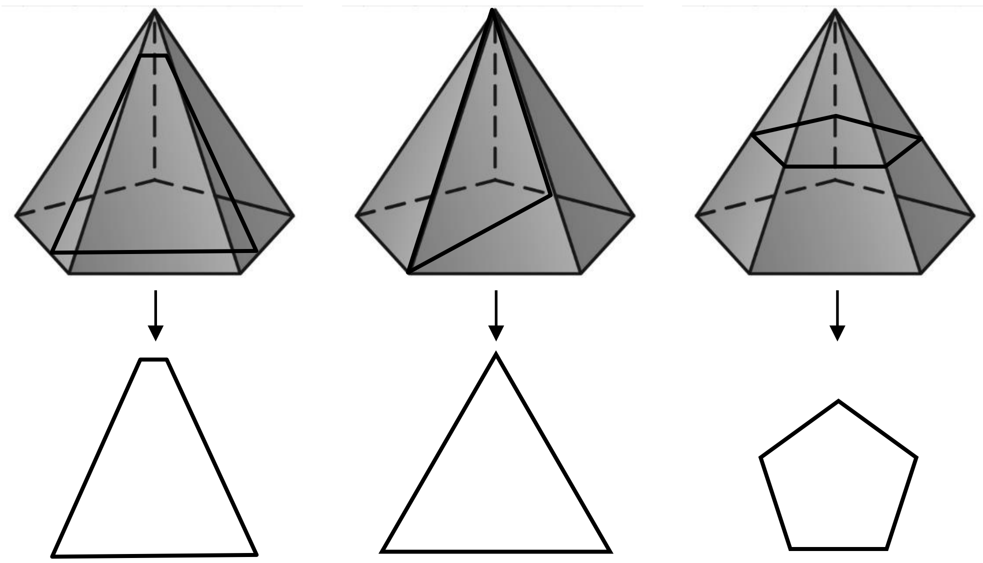
Reprint 2023

6.3.2 EXPLORE: Cross-sections

• When you make a straight cut through a 3-D shape, the 2-D shape of the cut surface that is exposed is called a **cross-section**.

• A cross-section can be made at any angle. In the examples below, each cross-section is parallel or perpendicular to the base.

Some cross-sections of a regular pentagon-based pyramid



Trapezoid cross-section | Triangle cross-section | Pentagon cross-section

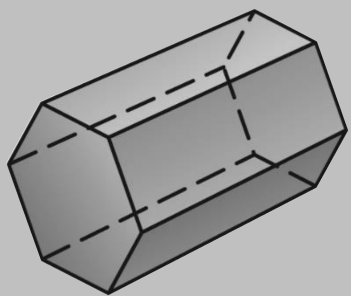
A. Make each shape below out of clay.

- cylinder
- cone
- square-based pyramid
- rectangle-based prism
- triangle-based prism

Use string or thin wire to make a straight cut that is parallel or perpendicular to any face. Sketch the cross-section.

Rebuild the 3-D shape and repeat until you have several cross-sections of different shapes and sizes.

B. Predict what the cross-sections of a hexagon-based prism will look like if they are made perpendicular or parallel to the base.

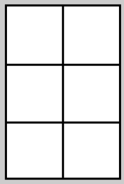


Reprint 2023

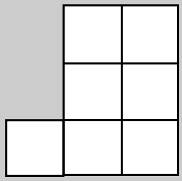
6.3.3 Interpreting Orthographic Drawings

Try This

Dorji looked at a cube structure straight on from different views. He made these drawings of what he saw.



View from the back



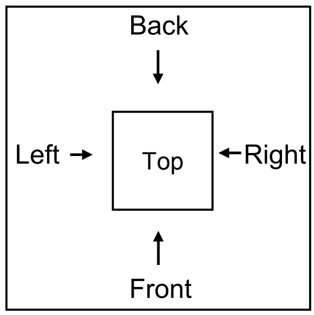
View from the right

A. Use linking cubes to build a structure that matches Dorji's drawings.

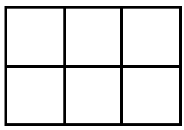
• **Orthographic drawings** are a set of 2-D drawings of a 3-D structure. Each drawing is called a **face view**. Each face view is made by looking at the structure straight on from a different direction.

• A set of orthographic drawings can help you see features of the structure that might be hidden in any single view.

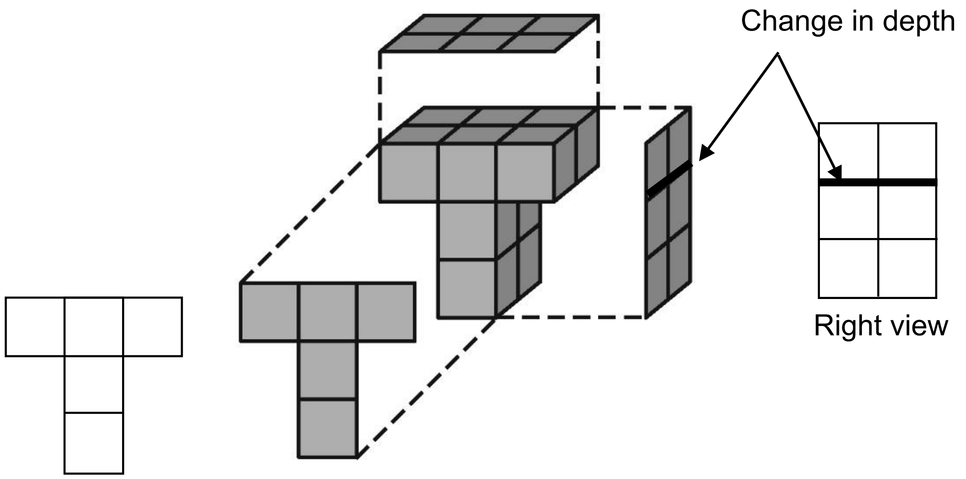
• The views that might be included in a set of orthographic drawings are top, front, back, left, and right.



• A thicker line is used to show a change in depth.



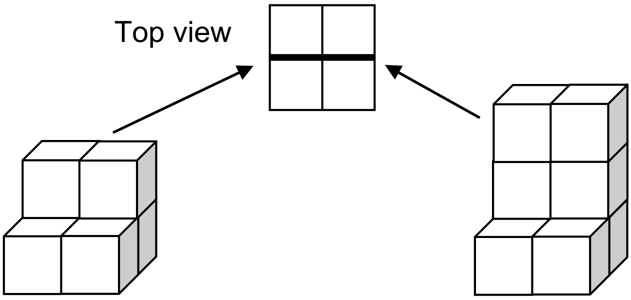
Top view



Front view

Right view

• A single face view is not enough to represent a 3-D structure.
For example:
These structures are different but they have the same top view:

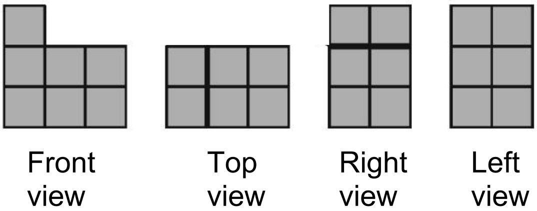


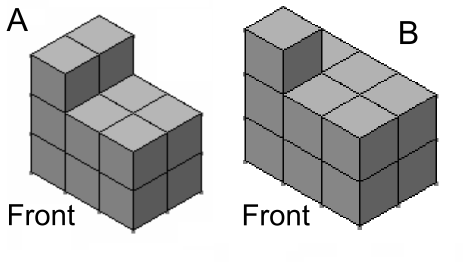
B. Build another cube structure to match Dorji's drawings in **part A**.

Examples

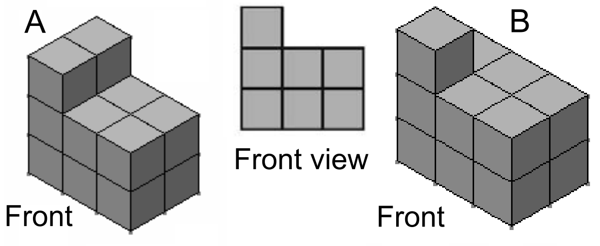
Example 1 Matching Face Views with Structures

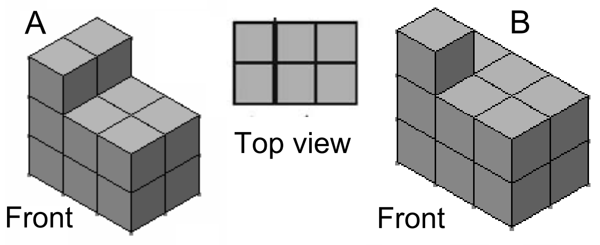
Which cube structure matches this set of face views?





Solution

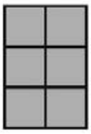




Thinking

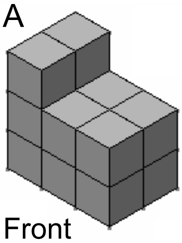
• First
I compared
the front views.
Both structures
matched.

• Next I compared the top views. Structure A matched. Structure B didn't match because it didn't show a change of depth outlining a single cube.

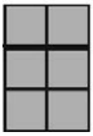


Left view

A



Front



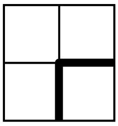
Right view

Structure A matches the four face views.

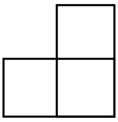
• I double-checked to make sure Structure A matched by comparing the right and left views to the structure.

Example 2 Building Structures to Match Face Views

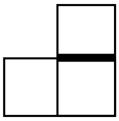
Build a cube structure to match these face views.



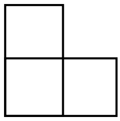
Top view



Front view

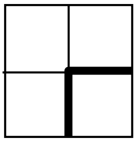


Left view

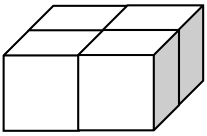


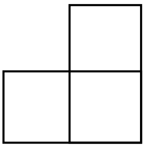
Right view

Solution

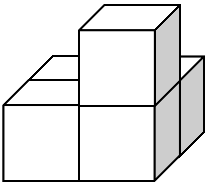


Top view



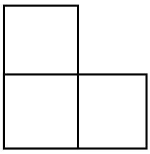


Front view

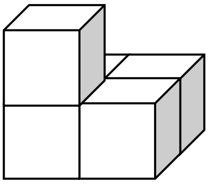


Right

Front

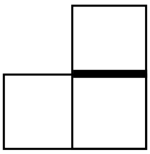


Right view

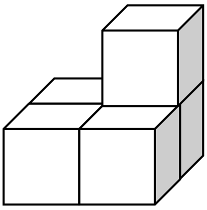


Back

Right



Left view



Front

Left

Thinking

• I started with the top view. I knew the base layer was 2 by 2 and there was a change in depth.

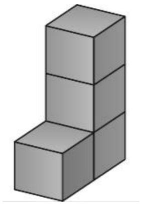
• The front view told me that there was only one more cube to add.

• I looked at the cube structure from the right and from the left to make sure those views matched what I had built.

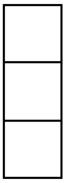
Practising and Applying

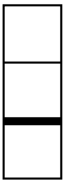
1. Identify each view of the structure below as top, front, back, left, or right.

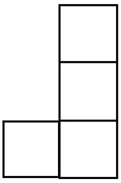
Structure

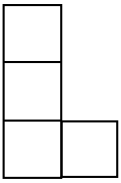


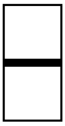
Front

a) 

b) 

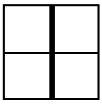
c) 

d) 

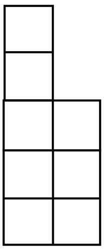
e) 

2. Build each cube structure with linking cubes.

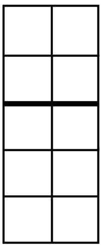
a)



Top view

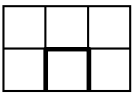


Front view

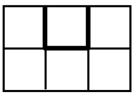


Right view

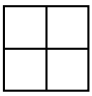
b)



Top view

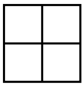


Front view

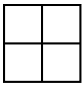


Right view

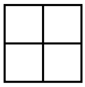
3. a) Use 8 linking cubes to build a structure with this set of face views.



Top view



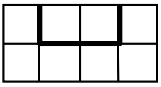
Front view



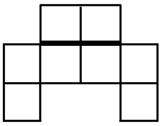
Left view

b) Build another structure with the same set of face views.
Use as many cubes as you need.

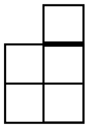
4. Build two different cube structures that have this set of face views.



Top view



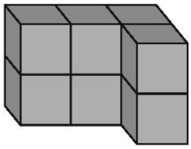
Front view



Right view

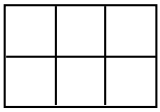
5. Sithar made this set of orthographic drawings of the structure below. What suggestions can you make to help Sithar improve each drawing?

Structure



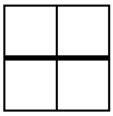
Front

a)



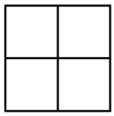
Front view

b)



Left view

c)



Right view

6. Why is it important to have more than one orthographic drawing or face view when you create a cube structure?

Reprint 2023

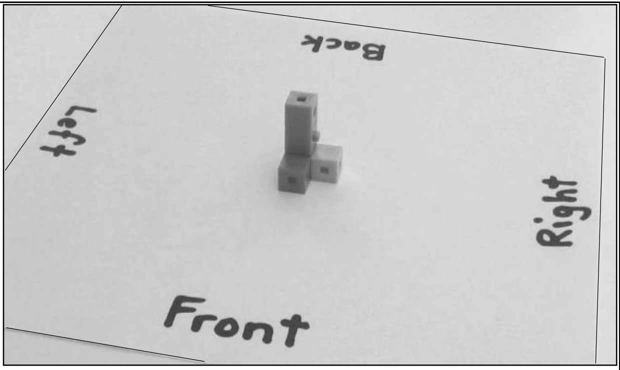
6.3.4 Creating Orthographic Drawings

Try This

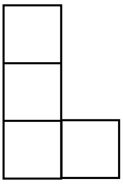
A. i) Build a cube structure using 7 or more cubes.
ii) Draw the top, left, right, back, and front face views of your structure.

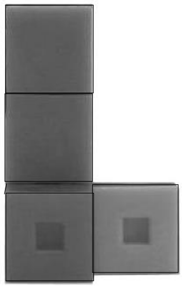
• When you create orthographic drawings, it is helpful to begin by placing the structure on a paper marked front, right, left, and back.

• You can also use grid paper
if the structure is built from cubes.



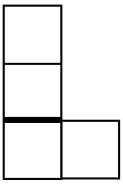
• For each view (front, right, left, and back), turn the paper to see each face straight on.

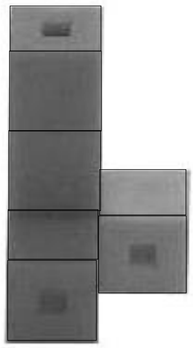

Front view



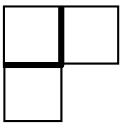
• Draw what you see.

• Look at the structure from a slightly different angle to see if there are changes in depth. Mark the changes in depth with a heavier line.


Front view



• To draw the top view, look straight down at the structure from above with the front of the structure closest to you.


Top view

Reprint 2023

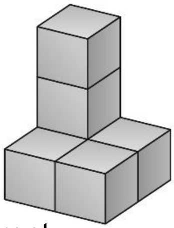
B. i) Add another cube to the structure you built in **part A**. Draw the new set of face views.

ii) Describe how the set of face views changed.

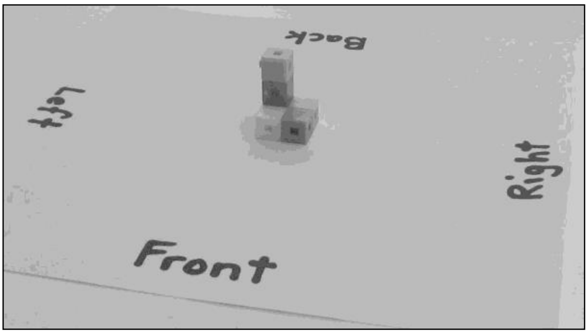
Examples

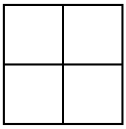
Example Creating Orthographic Drawings

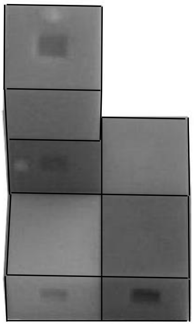
Draw a set of face views for this cube structure.

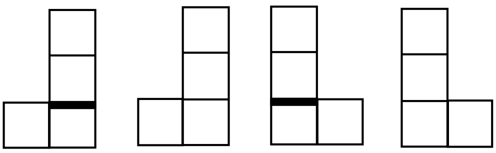


Solution









Right view Back view Front view Left view

Thinking

• First I built the structure using cubes and placed it on a sheet of paper. I marked front, right, left, and back on the paper.

• To see the top view, I looked down on the structure with the front facing me. I drew what I saw.

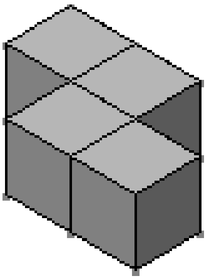
• Then I looked at the top from a different angle to see where the change in depth was. I added thicker lines to the face view to show it.

• I repeated this, looking at the structure from the front, back, right, and left to draw those views.

Practising and Applying

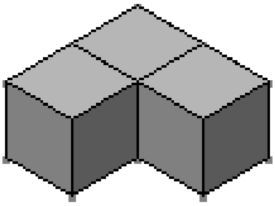
1. Build each cube structure. Draw the top, front, right, left, and back face views for each.

a)



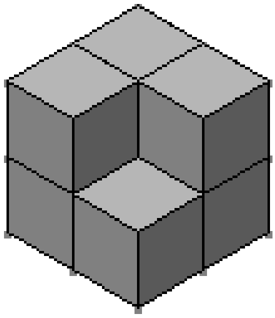
Front

b)



Front

c)

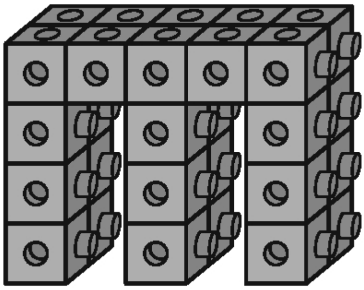


Front

2. a) Build a rectangular prism using cubes. Draw the top, front, left, and right views.

b) Remove any three cubes from your prism. Draw the new face views.

3. a) Draw the front and back face views of this cube structure.



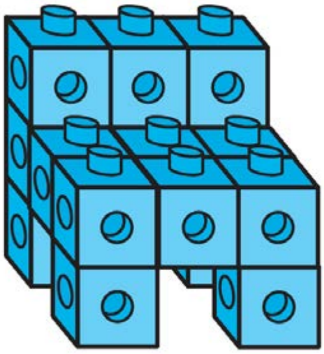
Front

b) Draw the left and right face views.

c) Draw the top face view.

d) If you could view the structure from below, what would the bottom view look like?

4. a) Build a model chair from cubes.



b) Draw the top face view.

c) Draw the front face view.

d) Draw another face view.

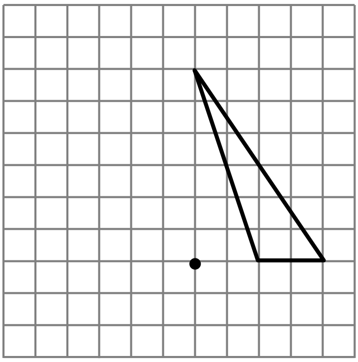
5. Build two different cube structures that have the same front face view, but different top and right face views. Draw the top, front, and right face views.

6. Is it possible for two different structures to have the same top, front, right, and left face views? Explain your thinking.

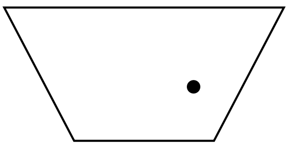
UNIT 6 Revision

1. Copy the diagrams below. Rotate each shape around the turn centre as described.

a) $\frac{1}{4}$ turn ccw



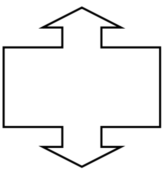
b) $\frac{3}{4}$ turn ccw



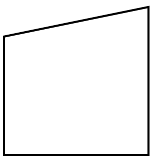
c) $\frac{1}{2}$ turn

2. Predict the order of rotational symmetry of each shape. Explain each prediction.

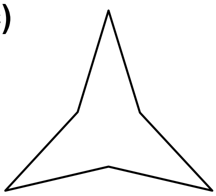
a)



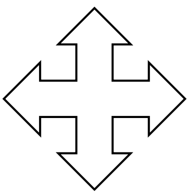
b)



c)



d)

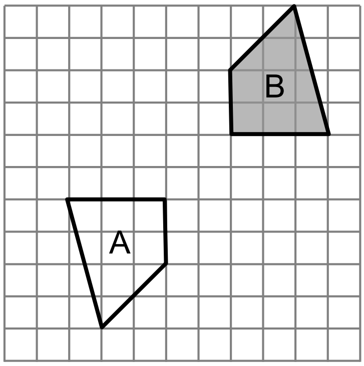


3. Describe the rotational symmetry of a regular hexagon. Explain your thinking.

4. Describe how you can transform Shape A to Shape B using each.

a) a combination of two transformations

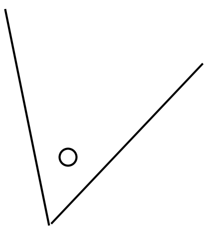
b) a single transformation



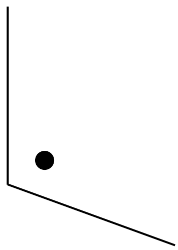
5. Can you make a tessellation with this triangle? Use grid paper to help you decide. Show your work.

6. Use a protractor to measure each angle.

a)



b)

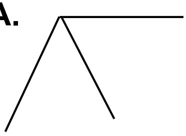


7. a) Draw a 120° angle.

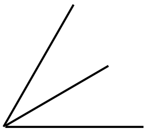
b) Draw a 65° angle.

8. Which are angle bisectors? How do you know?

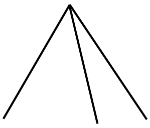
A.



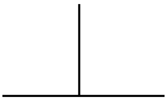
B.



C.

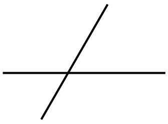


D.

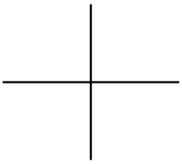


9. Which are line bisectors? How do you know?

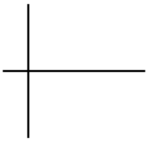
A.



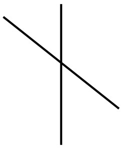
B.



C.



D.



10. Draw a line segment that is 4 cm long. Draw a perpendicular bisector of the line segment.

11. How are the diagonals of rectangles and kites alike? How are they different?

12. Describe the triangles formed by the diagonals of these quadrilaterals.

a) isosceles trapezoid

b) parallelogram

c) square

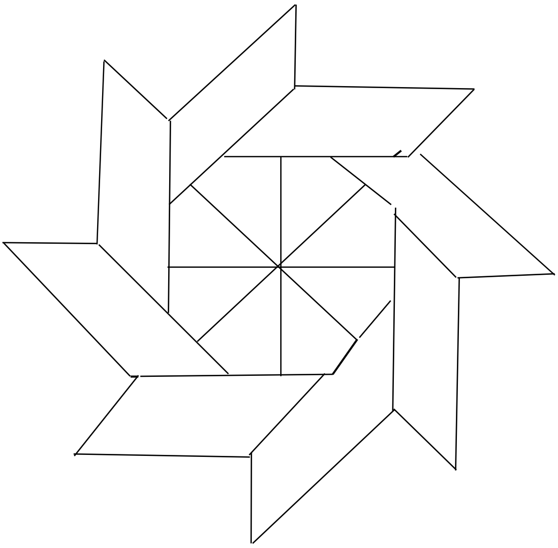
13. Identify these features in the folded paper flower:

a) angle bisectors

b) perpendicular bisectors

c) bisectors of a line that are not perpendicular bisectors

d) rotational symmetry



14. Examine this pentagon-based prism.

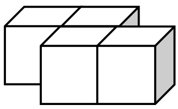
a) Describe or sketch the planes of symmetry.

b) Describe or sketch three possible cross-sections.

15. a) Describe the cross-sections that are perpendicular or parallel to the base of a square-based pyramid.

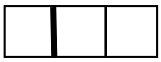
b) Which of the cross-sections could also be planes of symmetry?

16. Which orthographic drawing matches the cube structure?



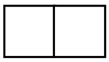
Front

A.



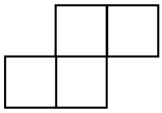
Front view

B.



Left view

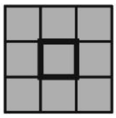
C.



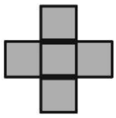
Top view

17. For each view that did not match in **question 16**, draw the correct view.

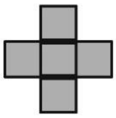
18. Which of the two cube structures below matches this set of face views? How do you know?



Top view

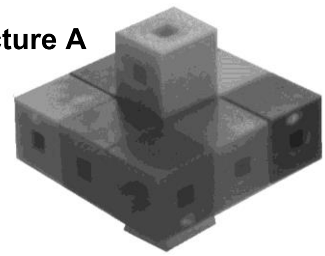


Front view

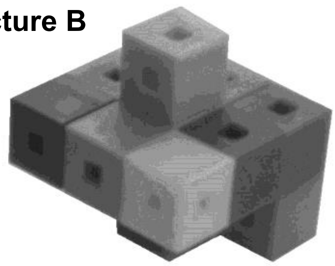


Left view

Structure A



Structure B

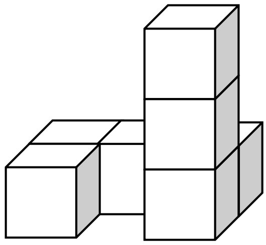


19. a) Build a cube structure with 5 cubes.

b) Draw the front, back, left, right, and top views of your structure.

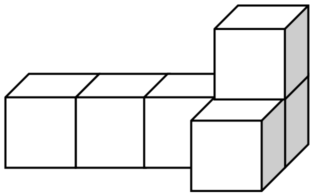
20. Build each cube structure shown. Draw the front, right, left, and top views of each.

a)



Front

b)



Front

UNIT 7 DATA AND PROBABILITY

Getting Started

Use What You Know

Lucky Seven Game Rules

1. Place 10 counters in a pile between two players.

2. Roll one die each and find the sum of the two dice:

- If the sum is 5, 6, 7, or 8, Player 1 gets a counter.
- If the sum is 2, 3, 4, 9, 10, 11, or 12, Player 2 gets a counter.

3. Repeat until there are no more counters left in the pile.

The winner is the player with more counters at the end of the game.

A. Play Lucky Seven with a partner. Decide before you start who will be Player 1 and who will be Player 2.

i) Do the first two rolls. Record who wins the counters in a chart like this.

Roll	Sum	Player 1	Player 2
1	11		√
2	6	√	
3			

ii) Which prediction do you think is true?

• Player 1 will win. • Player 2 will win. • It will end in a tie.

iii) Roll another 6 times. Record the winner each time.

iv) Do you want to change the prediction you made in **part ii)**? If so, what is your new prediction?

v) Finish the game to see who wins.

• What fraction describes the experimental probability that each player gets a counter on any roll?

• Was your prediction in **part ii) or iv)** correct?

B. Play the game two more times.

C. What prediction can you make about who will win the next time you play? Why?

Skills You Will Need

1. This chart compares the amount of time Kinley and Buthri spent on homework over 5 days. Create a double bar graph for the data.

Homework Time (in minutes)

	Day 1	Day 2	Day 3	Day 4	Day 5
Kinley	40	45	55	30	30
Buthri	55	60	70	35	45

2. Plot these points on a coordinate grid and connect them in order. What shape do they make?

A (2, 4)   B (2, 8)   C (6, 8)   D (6, 0)   E (5, 0)   F (5, 4)   G (2, 4)

3. a) Which statement below is true about every one of these points?

(3, 5)   (4, 7)   (5, 9)   (6, 11)

- The *y*-coordinate is 1 less than double the *x*-coordinate.

OR

- The *y*-coordinate is 1 more than double the *x*-coordinate.

b) Name two other points that fit this pattern.

c) Plot the six points from **parts a) and b)** on a coordinate grid and connect them. Describe the pattern they make.

4. a) Calculate the mean of this set of data: 3, 5, 7, 10, 15, 20

b) Predict how the mean will change in each case. Explain your prediction and then test it.

i) if 20 is changed to 26

ii) if 3 and 5 are changed to 1 and 1

iii) if 20 is removed from the data set

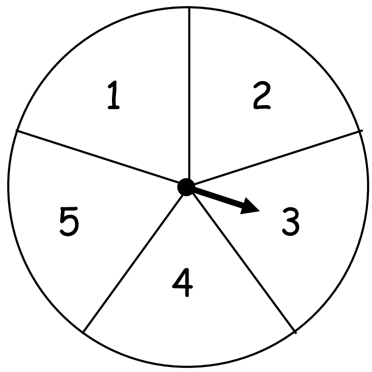
5. What is the probability of each? Write each as a fraction.

a) tossing a coin and getting Khorlo

b) rolling a 4 on a die

c) rolling a number greater than 2 on a die

d) spinning an odd number on this spinner



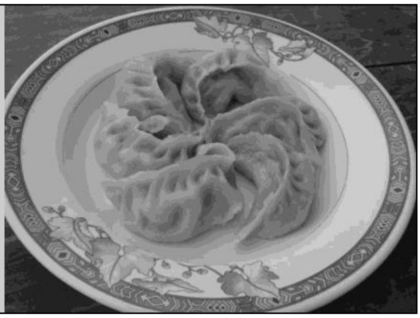
Chapter 1 Collecting Data

7.1.1 Choosing a Sample

Try This

You want to find out the favourite type of momo of students in your school but you do not have the time to ask everyone.

A. Who would you choose to ask?
Explain your choice.



• Sometimes you want to collect information from a group of people but you cannot ask everyone in the group. When this happens, you can **survey** a **sample** of the group.

• A sample is a small group within the whole group. The whole group is called the **population**.

• The sample you choose must represent the population. If it does not represent the whole population, the sample is **biased**.

For example:

If you want to collect information about how much time adults spend cooking, you should not ask only women; you should ask men and women. A sample of only women might be a biased sample.

• When you read reported information you should always consider what the population is and then think about the sample that was used to collect the information about that population.

For example:

Suppose you read that 3% of Bhutanese people are not very happy.

You should ask yourself questions like these before you make any conclusions:

- *What is the population being surveyed?*
- *Were adults and children included in the sample?*
- *Were males and females included?*
- *Were people with all levels of education included?*
- *Were poor people and wealthier people included?*
- *Were enough people asked?*

It is important to consider these things before you trust the data you read.

B. How would you change your sample in **part A** so it is not biased?

Reprint 2023

Examples

Example Avoiding Bias in a Survey Sample	
Suppose you want to find out if students in your school would like school to start later each day. How should you set up the survey to avoid bias?	
Solution *These are things I considered doing* • Getting to school early one day and asking the first 40 people I saw. • Asking all the students in my class since it is easier for me to ask them than to find other students to ask. • Putting all the names of the students in the school in a hat, drawing 40 names, and then phoning all of them. *This is what I decided to do* Put all the students' names in a bangchung, draw 40 names, and then survey them at school.	**Thinking**  • I knew I couldn't ask only people who come to school early because they might like being early. • I knew I couldn't ask only students in my class since they may think differently than younger students. • I knew I couldn't phone 40 students because not all of them have phones. • Putting the names of every student in the school in a bangchung and then drawing 40 names without looking gives everyone an equal chance of being selected. That way, the sample will probably not be biased.

Practising and Applying

1. Tell whether each sample below could be biased for collecting data for the survey in the **example** above. Explain your thinking.

a) 10 students, 10 teachers, and 10 parents

b) The first 100 students on the house master's list of all the students in the school

c) Only the students who walk to school

2. How would you avoid bias in a sample to find out the answers to these questions?

a) *How much time do people in Bhutan spend watching TV each night?*

b) *What is the favourite food of families in Bhutan?*

c) *What is the favourite sport for people in your community?*

d) *At what time do most people in Thimphu eat breakfast?*

3. You want to know how many hours adults in Bhutan spend walking to work each week.

a) Why would you use a sample instead of asking everyone in the population?

b) Why would you not just ask people in Thimphu?

4. You want to find out how people feel about voting in elections. How would you avoid a biased sample?

7.1.2 EXPLORE: Sample Size

If the size of the sample you use to collect information is too small, the results likely will not represent the whole population.

Try each of these three experiments to see what can happen when a small sample is used.

Experiment 1

You want to find out how many “P” words students in your class can write in one minute.

A. i) In one minute, write down as many different words as you can that start with the letter P.

ii) Share your results with two other classmates.
Find the mean number of words.

iii) Use your results from **part ii)** to predict the mean number of “P” words students in your class can write

B. How many “P” words did each student in your class write in one minute? Calculate the mean number of words.

C. Did your sample of three students give you enough information to make a good prediction about the whole class (the population)? Explain your thinking.

Experiment 2

You want to find out how long it takes each student in your class to write his or her name 15 times.

D. i) Time how long it takes you to write your name 15 times.

ii) Share your results with two other classmates.
Find the mean time.

iii) Use your results from **part ii)** to predict how long it will take for each student in your class to write his or her name 15 times.

[Continued]

E. How long did it take for each person in your class to write his or her name 15 times? Calculate the mean time for the class.

F. Did your sample of three give you enough information to make a good prediction about the whole class (the population)? Explain your thinking.

Experiment 3

You want to find out how often someone will roll a sum of 5 when rolling two dice.

G. i) Roll a pair of dice four times. Record the sum for each roll. What percent of the time did you roll a sum of 5?

ii) Use your results from **part i)** to predict the percent of the time the students in your class will roll a sum of 5.

H. What percent of the time did each person in your class roll a 5? Calculate the mean percent for the class.

I. Did your sample of four rolls give you enough information to make a good prediction about the whole class (the population)? Explain your thinking.

Making a conclusion

J. Is a sample size of three or four a good sample size for making a prediction about a whole class? Why do you think that?

Reprint 2023

Chapter 2 Graphing Data

7.2.1 Double Bar Graphs with Intervals

Try This

A. i) Start counting silently from 1 when your teacher says “Start”.
Stop when your teacher says “Stop” (after 30 seconds).
Write down the last number you counted.

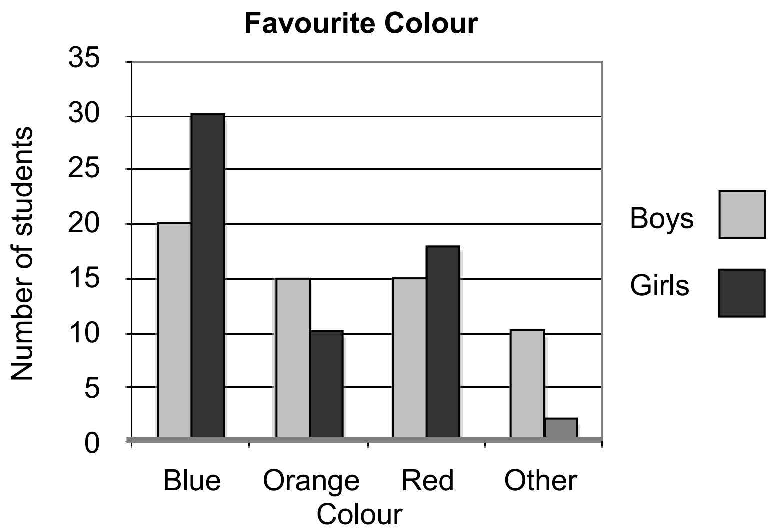
ii) Repeat **part i)**, but this time count silently down from 100.

B. Suppose you collected silent counting data from every student in your class. Why might it be a lot of work to graph all these data values?

• A **double bar graph** is a way to show two sets of data at the same time so that you can make many comparisons.

For example:

In the double bar graph below, you can compare
- which colours are more popular with boys
- which colours are more popular with girls
- how boys and girls compare in terms of favourite colours



Here are some things the graph tells you:
- More boys chose blue than orange, red, or other.
- Three times as many girls prefer blue as prefer orange.
- More girls than boys prefer red.

• Sometimes you might want to create a **bar graph** or a **double bar graph** where the information is organized into number **intervals**.

For example:

- A group of Class V students and a group of Class VI students were asked how many telephone calls they had made in the last three days.

Number of calls by Class V students

0	2	3	0	0	4	6	5	1	2
10	3	4	16	1	1	3	0	0	5

Number of calls by Class VI students

2	3	3	5	10	12	8	10	3	20
2	1	8	15	3	7	0	17	10	2

- You want to make a graph to show the different numbers of calls the students in each class made.

- The least number of calls is 0 and the greatest number of calls is 20 but you do not want a bar for each number of calls — that would be 40 bars.

- To make a graph with fewer bars, you can group the data into intervals:

Class V

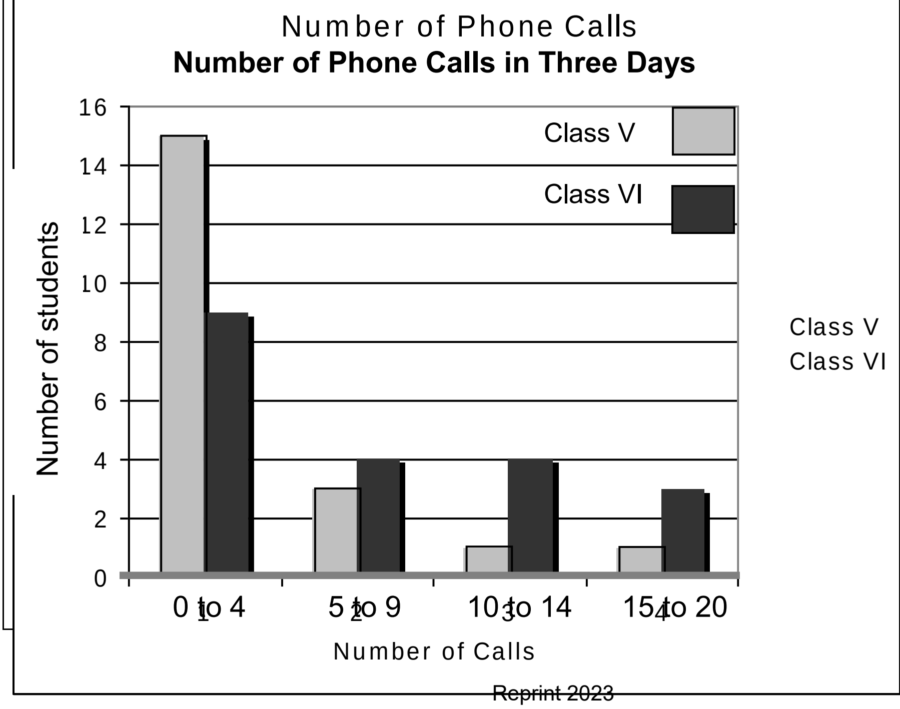
Number of calls	Number of students
0 to 4	15
5 to 9	3
10 to 14	1
15 to 20	1

Class VI

Number of calls	Number of students
0 to 4	9
5 to 9	4
10 to 14	4
15 to 20	3

Notice that the intervals were chosen so that they are almost equal.

- The graph of the data looks like this if a **scale** of 2 is used along



the tallest
o show
nts, a scale
= 2 students

h shows this:
classes, most
made fewer
lls.
ass VI
than Class V
made 5 or
s.

C. i) Why might it be a good idea to use intervals to graph the data for the whole class from **part A**?

ii) Sketch a double bar graph for the data using intervals**.**

iii) What interval size did you use for the horizontal axis? Why?

iv) What scale did you use for the vertical axis? Why?

Examples

Example Creating a Double Bar Graph

Kamala did a probability experiment. She rolled two dice 20 times.
She found the sum and the difference of the two numbers each time.

Here are the data values she collected. The first number listed in each pair is the first number rolled and the second is the second number rolled.
Draw a double bar graph of the data.

4, 5	1, 2	3, 6	4, 1	2, 2	5, 6	3, 2	1, 3	6, 1	5, 4
6, 3	1, 2	2, 2	4, 1	3, 5	6, 3	5, 1	4, 5	2, 2	1, 6

Solution

Sums

2	3	4	5	6	7	8	9	10	11
	//	////	///	/	//	/	~~////~~ /		/
	2	4	3	1	2	1	6		1

Differences

0	1	2	3	4	5
///	~~////~~ //	//	~~////~~	/	//
3	7	2	5	1	2

Sums

Sum	Number of times rolled
0 or 1	not possible
2 or 3	2
4 or 5	7
6 or 7	3
8 or 9	7
10 or 11	1

Differences

Difference	Number of times rolled
0 or 1	10
2 or 3	7
4 or 5	3
6 or 7	not possible
8 or 9	not possible
10 or 11	not possible

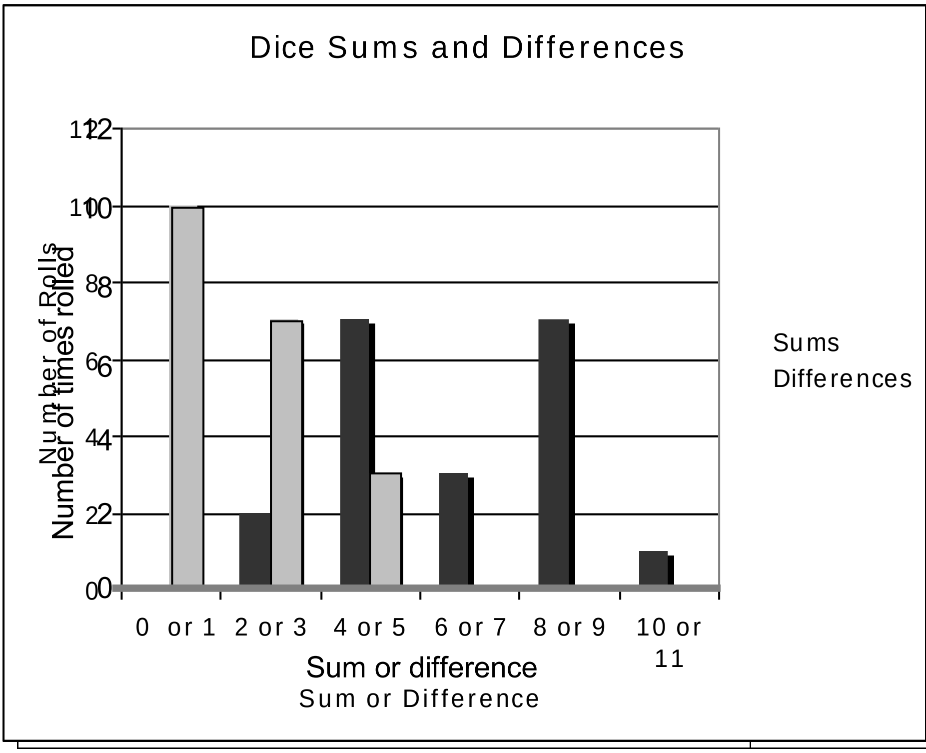
[Continued]

Thinking

• I made tally charts for the sums and differences to organize the data.

• I knew that if I made a bar for each sum or difference, I would need 12 bars (0 to 11). That would have been too many bars, so I used intervals of 2.

• I organized the data in intervals in charts.



d need a
was 10 units
ed a scale of
lls.

f 0 or 1,
o black bar.
nces of
9, and
ere were
s.

Practising and Applying

1. What does the double bar graph in the **example** above tell you about what sums and differences are likely when you roll two dice?

2. a) Repeat the experiment in the **example** above and create your own double bar graph using a different interval size.

b) Compare your results with the solution in the **example**.

3. a) Change the interval size you used in **question 2**. Draw the graph with the new interval size.

b) Does the new graph show the same things about the data? Explain your thinking.

4. A teacher is comparing her students' performance on an English exam (E) to their performance on a math exam (M). This chart shows the number of students that got marks in each range for each subject.

For example, in English, 14 students got a mark in the 60s (from 60 to 69).

	30s	40s	50s	60s	70s	80s
E	5	8	5	14	6	2
M	7	6	8	9	6	4

a) Create a double bar graph.

b) What does the graph show about how the students performed on the two exams?

5. Why does it make sense that if you use intervals on the horizontal axis of a bar graph, it is more likely that you will need to use a scale on the vertical axis?

7.2.2 Stem and Leaf Plots

Try This

A. i) Collect information about the height (in centimetres) of each student in your class.
ii) Describe two ways to graph the information. Sketch one graph.

• One way to organize and display data is to use a **stem and leaf plot**. A stem and leaf plot groups data into intervals that are based on place value.

For example:

- A teacher counted the number of oral questions his students answered in class during one month. The data values are shown below.

0	4	15	15	5	6	9	10	8	4
11	18	5	20	7	33	12	23	30	6
12	18	16	19	18	14	20	35	21	22
12	19	8	17	30	6	22	40	28	35

- You can write the tens digits of the data values in a column, in order, on the left. These are the **stems**.

Stems	
0	All data values from 0 to 9 will go here.
1	All data values from 10 to 19 will go here.
2	All data values from 20 to 29 will go here.
3	All data values from 30 to 39 will go here.
4	All data values from 40 to 49 will go here.

Each stem forms an interval. The intervals in this plot are 0 to 9, 10 to 19, 20 to 29, 30 to 39, and 40 to 49.

- Then you write the ones digits for each tens digit in a row, in order, on the right. These are the **leaves**.

Stems	Leaves
0	0 4 4 5 5 6 6 6 7 8 8 9
1	0 1 2 2 2 4 5 5 6 7 8 8 8 9 9
2	0 0 1 2 2 3 8
3	0 0 3 5 5 ⟵ The numbers in this row are: 30, 30, 33, 35, and 35.
4	0

• For the plot, you can tell that 5 people answered 30 to 39 questions. You can also tell how many questions they answered:

- Two students answered 30 questions.
- One student answered 33 questions.
- Two students answered 35 questions.

• If you shade the leaves to create bars, the stem and leaf plot looks like a horizontal bar graph with a scale of 1.

0	0 4 4 5 5 6 6 6 7 8 8 9
1	0 1 2 2 2 4 5 5 6 7 8 8 8 9 9
2	0 0 1 2 2 3 8
3	0 0 3 5 5
4	0

• All the numbers in the stem and leaf plot above are 1-digit and 2-digit numbers, so the place value used for the stems was the tens digit.
If a data set includes 3-digit numbers, you might use the hundreds digit as the stem and the tens and ones digits together as the leaves.

For example, consider this set of data with 2-digit and 3-digit numbers:

124, 225, 137, 228, 159, 426, 218, 551, 535, 127, 78, 235, 12

Stems	Leaves	
0	12 78	← The numbers in this row are 12 and 78.
1	24 27 37 59	
2	18 25 28 35	← These numbers are 218, 225, 228, and 235.
3		
4	26	
5	35 51	

Examples

Example Creating a Stem and Leaf Plot

Create a stem and leaf plot to show these temperatures (°C):

26, 24, 19, 28, 24, 27, 30, 32, 26, 28, 22, 25

Solution

1	9
2	2 4 4 5 6 6 7 8 8
3	0 2

Thinking

• The numbers are all in the 10s, 20s, or 30s, so I made the tens digits the stems.

• I put the numbers (leaves) in order for each stem.

Practising and Applying

1. List the data values in this stem and leaf plot in order from least to greatest.

0	7 8 8 9 9
1	
2	1 1 2 3 3 6
3	0 0 5 8

2. Sketch a bar graph of the data in each stem and leaf plot below. Use the same intervals as the stem and leaf plot.

a)

3	2 3 3 6
4	
5	0 2 2 8 9 9
6	1 1 3 5

b)

20	6
21	0 1 1 3 4 5
22	6 7 7 8 8
23	1

c)

4	12 25 26 42 50
5	03 10 17 22
6	00 11 13 78

3. **a)** List all of the multiples of 4 that are less than 70. Arrange the numbers in a stem and leaf plot.

b) What does the stem and leaf plot tell you about how many multiples of 4 there are in each interval of 10 numbers?

4. This stem and leaf plot shows the number of minutes 14 students spent on homework one night.

1	5
2	0 5 5 5
3	0 0 0 5
4	5
5	0 5
6	0 0

a) Create a bar graph to show the information. Use the same intervals as the stem and leaf plot.

b) Compare how the two graphs show the same data.

5. Suppose you rolled two dice 30 times and multiplied the numbers rolled each time.

a) Predict what the stem and leaf plot of the 30 products will look like. Explain your thinking.

b) Test your prediction by completing the rolls and creating the stem and leaf plot.

6. a) In which row does the mean of the data in this plot appear?

1	15 15
2	25 25
3	35 35

b) Create a stem and leaf plot with at least two rows of data where the mean is in the first row.

7. You can change a stem and leaf plot into a bar graph with intervals but you cannot change a bar graph with intervals into a stem and leaf plot. Why is that?

7.2.3 Line Graphs

Try This

The average monthly high temperatures (ºC) in Punakha are listed below.

J	F	M	A	M	J	J	A	S	O	N	D
16	20	21	24	27	31	32	31	30	28	22	15

A. Create a bar graph or a stem and leaf plot to display the temperatures.

• A line graph is often used to display the same sort of data that has been collected at different points in time.

For example:

The chart below shows the monthly precipitation, to the nearest millimetre, in Thimphu during one year.

Precipitation in Thimphu (in millimetres)

J	F	M	A	M	J	J	A	S	O	N	D
4	9	16	22	24	41	75	72	34	15	4	2

If this information is displayed in a **line graph**, you might be able to see a **trend** in the precipitation over the year. A trend is a pattern of change, usually over time.

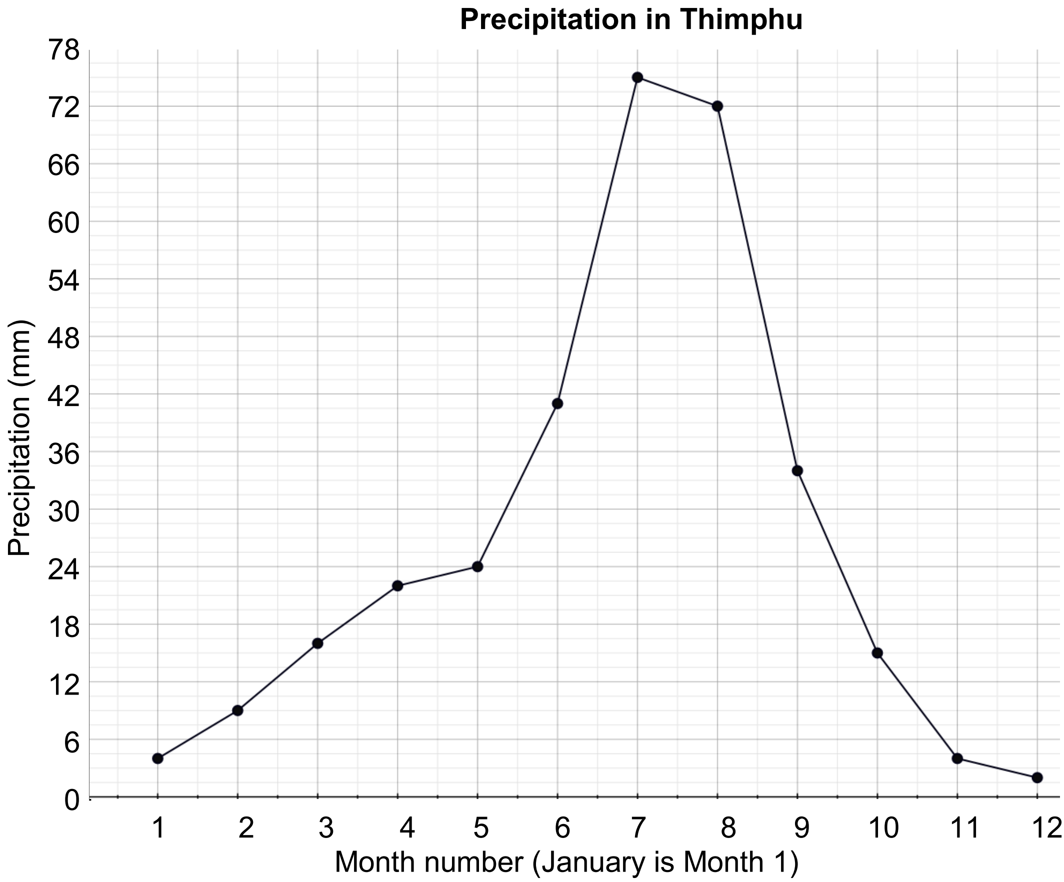
• This is how to create the graph (shown on the next page):

- Time (in months) goes on the **horizontal axis**.
- The amount of rain (in millimetres) goes on the **vertical axis**. Since the data values range from 2 mm to 75 mm, a scale of 1 unit to 10 mm makes sense.
- Plot each point in the chart: a point at (1, 4) for January (month 1), a point at (2, 9) for February (month 2), and so on.
- Connect the points, in order, with a line.

• Once you have created a graph, you can look for trends.

For example:

The precipitation increases each month from January until July and then it begins to decrease.



- You can use line graphs to look for trends in situations like these:
  - to show the change in temperature from month to month over a year
  - to show the change in the price of an item from week to week over a month
  - to show the change in the value of the ngultrum compared to the U.S. dollar from year to year over a decade
  - to show the change in population of a country from decade to decade over many years

B. i) Show the temperature data from **part A** using a line graph.

ii) Compare how the stem and leaf plot and the line graph show the same data. What are the advantages of each type of graph?

Examples

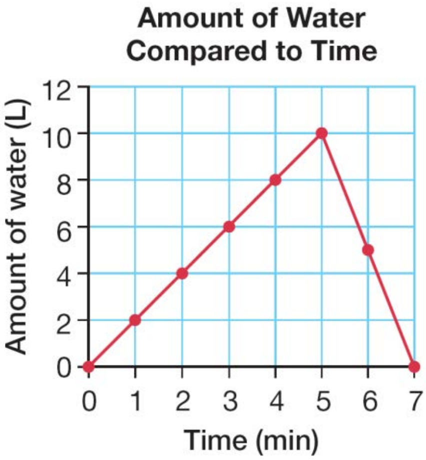
Example Interpreting a Line Graph

The line graph shows the amount of water in a 10 L container as it is being filled and then emptied.

a) Which part of the graph shows the container being filled? Explain your thinking.

b) Which part of the graph shows the container being emptied? Explain your thinking.

c) Which took longer, filling or emptying? Explain your thinking.



Solution

a) The part from 0 min to 5 min; The water went from 0 L to 10 L in the first 5 min, so the container was being filled.

b) The part from 5 min to 7 min. The water went from 10 L to 0 L in the last 2 min, so it was being emptied.

c) It took longer to fill than empty since it took 5 min to fill, from 0 L to 10 L, and it only took 2 min to empty, from 10 L to 0 L.

Thinking

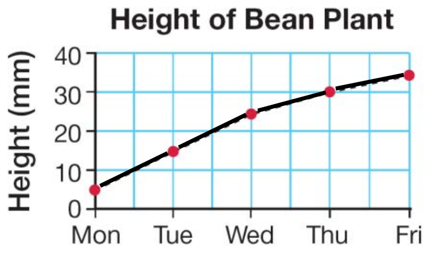
a) I knew that when the container was being filled, the amount of water was increasing. I could see that happened when the graph was going up.

b) I knew that when the container was being emptied, the amount of water was decreasing and the graph would go down.

c) I looked at the horizontal axis to figure out how long it took to fill and how long it took to empty.

Practising and Applying

1. Thinley measured the height of a plant over 5 days. What trends do you see in the graph?



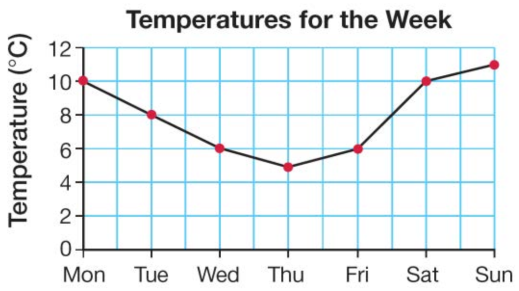
2. Which statement best describes the trend in the graph below?

A. The temperature increases.

B. The temperature decreases.

C. The temperature decreases and then increases.

D. The temperature increases and then decreases.



3. When she rides her bicycle, Nima turns the pedals 10 times to travel 20 m. Draw a graph to show the distance travelled for the total number of pedal turns up to 100 pedal turns.

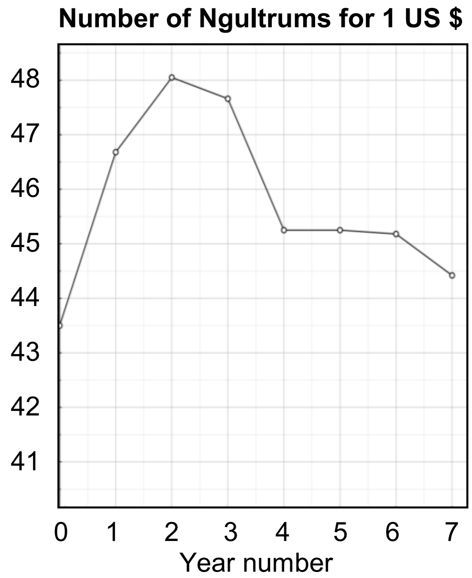
4. Bijoy recorded the high temperatures for one week in July. On Monday, the high temperature was 29°. Each day after that was 2°C warmer than the day before.

a) Draw a line graph to show the temperatures for that week.

b) What trends do you see?

c) Predict how the graph would change if the temperature had gone up 3°C each day instead of 2ºC.

5. A line graph shows the number of ngultrums that could be purchased with one U.S. dollar on January 1 of 8 consecutive years. What trend does the graph show?



6. When Jigme walks, he travels 50 m per minute. When he cycles, he travels 200 m per minute.

a) Draw two line graphs on the same axes to compare how far Jigme travels by foot and by bicycle in 20 minutes.

b) What trends do you observe?

7. Why does a line graph help you see a trend?

CONNECTIONS: Telling a Story about a Graph

Mindu and Karma each sketched a line graph to describe a trek in the mountains.

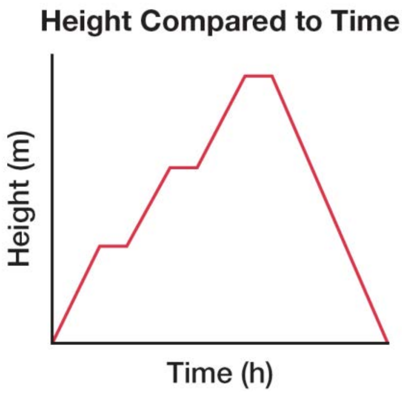
Here is how Mindu described his trek in words:

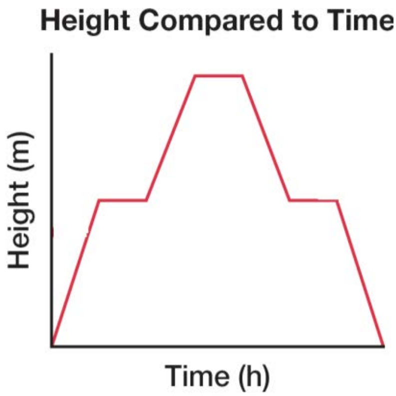
I climbed to the top and then back down again. I rested three times during the trek. The first two rests were on the climb up, and the last rest was at the top.

Here is how Karma described his trek in words:

I climbed to the top. There was a flat section about halfway up.
I rested at the top and then went back down the same way I came up.

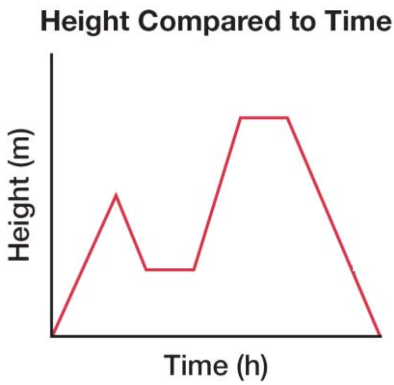
1. Which graph matches Mindu's description? Karma's description?





2. Explain why a flat part of the graph can show the trekker either taking a rest or walking on a flat section of the trail.

3. Write a story that describes this graph of a trek.



7.2.4 Coordinate Graphs

Try This

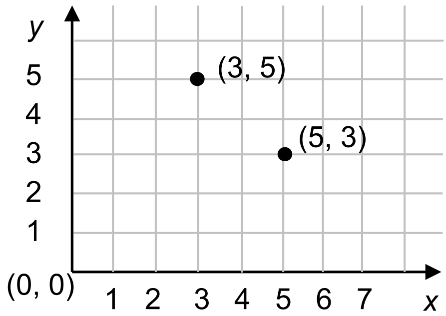
A. i) Plot these points on a coordinate grid:

(7, 5) (6, 4) (5, 3) (4, 2) (3, 1) (2, 0)

ii) What do you notice about all the points?

• You have already learned about plotting points on a **coordinate grid** to create a **coordinate graph**.

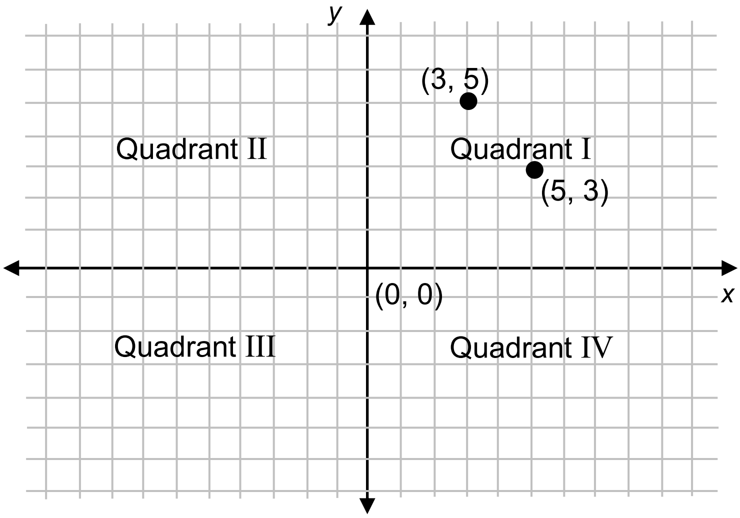
• A coordinate grid has a horizontal ***x*-axis** and a vertical ***y*-axis**. When you plot a point on the grid given its **coordinates**, the first number in the **ordered pair** (the ***x*-coordinate**) tells how far right to go from the **origin** (0, 0), and the second number (the ***y*-coordinate**) tells how far to go up.



The point (3, 5) is 3 units to the right of the *y*-axis and 5 units above the *x*-axis.

The point (5, 3) is 5 units to the right of the *y*-axis and 3 units above the *x*-axis.

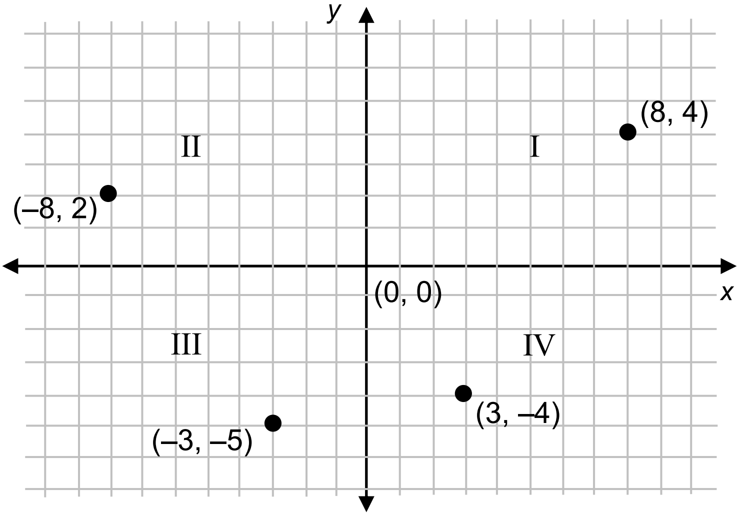
• The coordinate grid you have used until now is only one part of a larger grid that is divided into four **quadrants**. The points (3, 5) and (5, 3) shown above are in Quadrant I.



• Here are some general rules for deciding what quadrant a point belongs in:

- If both coordinates are positive, the point is in Quadrant I.
- If the *x*-coordinate is negative, the point is left of the *y*-axis, in Quadrant II or Quadrant III.
- If the *y*-coordinate is negative, the point is below the *x*-axis, in Quadrant III or Quadrant IV.

For example:



The coordinates in each quadrant can be described in this way:

Quadrant I (+ , +)
Quadrant II (– , +)
Quadrant III (– , –)
Quadrant IV (+ , –)

B. i) If you extend the pattern in **part A** to include negative coordinates, in which quadrant or quadrants does the pattern continue?

ii) List three pairs of coordinates involving negative coordinates that continue the pattern.

Examples

Example 1 Plotting Points on a Coordinate Grid

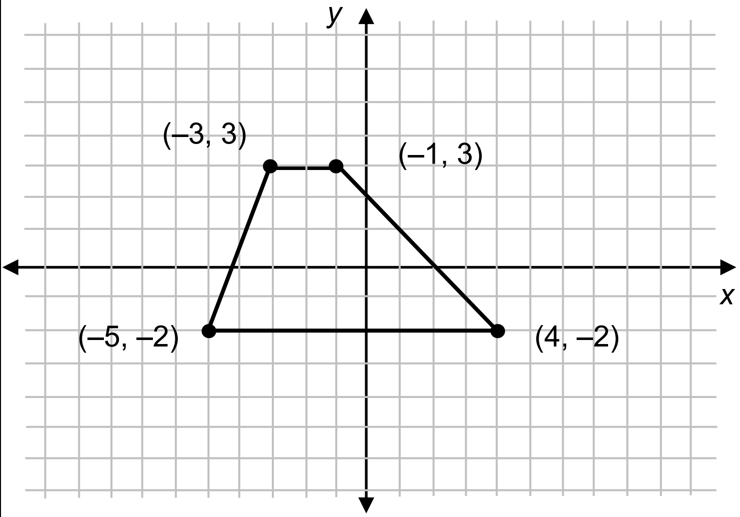
Plot these points on a coordinate grid:

(–1, 3) (–3, 3) (–5, –2) (4, –2)

Connect them in order and then connect the last point to the first point.

Describe the shape that you created.

Solution



The points form a trapezoid.

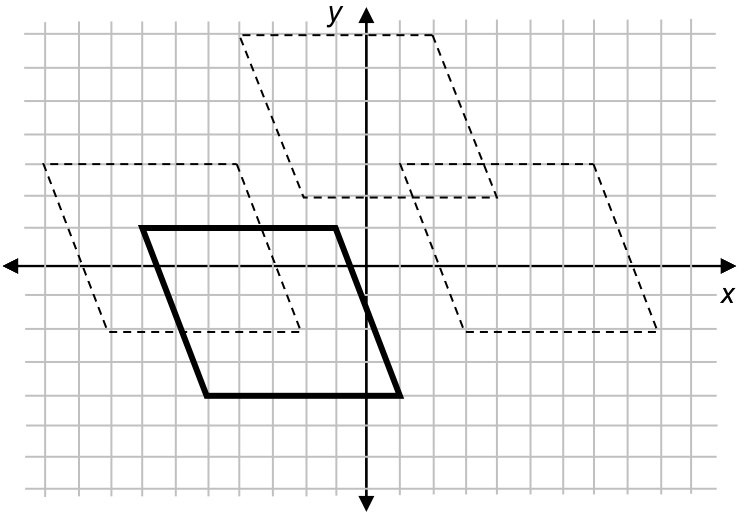
Thinking

- I knew that if the first coordinate was negative, it was left of the *y*-axis.
- I knew that if the second coordinate was negative, it was below the *x*-axis.

Example 2 Naming Coordinates to Fit Rules

A parallelogram has vertices in three of the four quadrants.
What might the coordinates of the vertices be?

Solution



The coordinates of the parallelogram could be (–1, 1), (1, –4), (–5, –4), and (–7, 1).

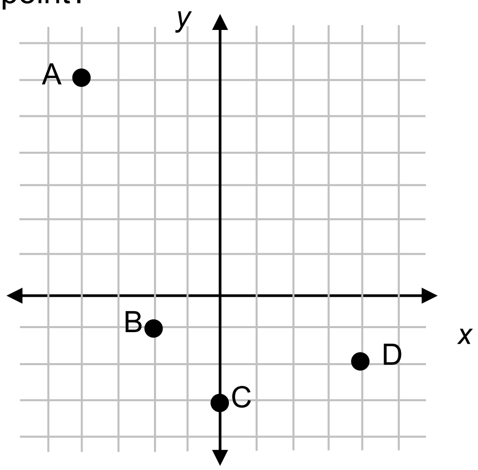
Thinking

- I sketched a parallelogram and then moved it around until the vertices were in three quadrants.

Reprint 2023

Practising and Applying

1. What are the coordinates of each point?



2. Plot each point on a coordinate grid.

a) (–3, 5) **b)** (7, –2)

c) (–1, –5) **d)** (–3, –2)

3. Plot these points:

(3, 2) (–3, 2) (–3, –2) (3, –2)

What do you notice?

4. Name the coordinates of a point that fit each description.

a) In Quadrant IV but close to (0, 0)

b) In Quadrant III and more than 6 units from (0, 0)

c) In Quadrant II and farther from the *x*-axis than the *y*-axis

d) 2 units left and 3 units down from (–6, –2)

e) 3 units right and 2 units down from (–6, –2)

f) 6 units left of (–6, –2)

5. What letter of the alphabet would these coordinates form if they were plotted?

(–2, –2), (–2, 4), (2, 4), (2, 1), (–2, 1)

6. Two vertices of a square are at (–5, 3) and (2, –4). Where might the other two vertices be?

7. Plot (2, –3) and (–4, –7).

a) Name a point that you think is between these two points.

b) Tell why you think it is between (2, –3) and (–4, –7).

8. a) Draw a line that goes through only two quadrants. Name two points on the line — one in each of the two quadrants.

b) Draw a line that goes through three quadrants. Name three points on the line — one in each of the three quadrants.

9. Which point is farthest from the origin?

(5, 0) (3, 4) (–3, –4) (0, –5)

10. Imagine drawing a coordinate graph on a piece of paper. Why can any point on the paper be described by only two coordinates?

Reprint 2023

GAME: Four in a Line

This game is for two players. You need grid paper.

This is how to get ready:

- Create a chart like this.

X	O

- Create a coordinate grid with all four quadrants.

This is how to play:

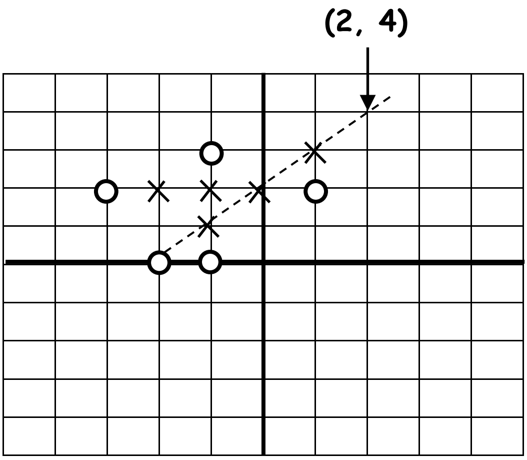
- Player A plots a point on the grid by making an X. He or she then writes down the coordinates of the point in the X column of the chart.
- Player B plots a point on the grid by making an O and writes down the coordinates in the O column of the chart.
- Players take turns plotting points and writing their coordinates in the chart.
- The first player to plot four Xs or four Os in a line, vertically, horizontally, or diagonally, wins the game.

For example:

Player A and Player B have each had five turns. It is now Player A's turn.

If Player A plots her next point at (2, 4), she can win the game because there will be four Xs in a line.

X	O
(–1, 2)	(–1, 3)
(0, 2)	(1, 2)
(–1, 1)	(–1, 0)
(–2, 2)	(–3, 2)
(1, 3)	(–2, 0)
(2, 4)	



7.3.1 Mean, Median, and Mode

Try This

The number of people who attended a tsechu on six different days were:

312, 325, 325, 218, 401, and 397

A. What single number do you think best describes the daily attendance at the tsechu?

• The **mean** of a set of data or numbers is a single number that describes the whole set. The mean describes each "share" when the total of all the data is equally shared among all the pieces of data.

For example:

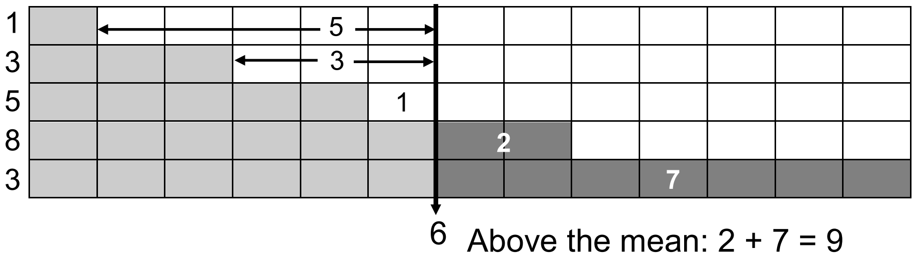
The mean of 8, 10, 10, 12 is 10 because the total is 40, and 40 shared equally among the four values is 10.

$$(8 + 10 + 10 + 12) \div 4 = 40 \div 4 = 10$$

• The mean is a "central number" for a set of data since there are values above and below the mean. The higher total of the data values greater than the mean is balanced by the lower total of the data values below the mean.

For example, the mean of 1, 3, 5, 8, 13 is 6:

Below the mean: 5 + 3 + 1 = 9



Above the mean: 2 + 7 = 9

• The **median** is another "central number" for a set of data. If the data values are ordered from least to greatest, the median is the middle number. There are the same number of data values above the median as below it.

The median of 1, 4, 7, 10, 15 is 7 because there are two values below 7 and two values above 7.

Median

1 4 <u>7</u> 10 15

Reprint 2023

• If a set of data has an even number of data values, the median is the mean of the middle two numbers.

For example, the median of 1, 5, 6, 8, 10, 12 is 7:

Median

1   5   **6 ↓ 8**   10   12

7

7 is the mean of 6 and 8. There are as many values below 7 (1, 5, and 6) as above 7 (8, 10, and 12).

• The **mode** is a third "central number" for a set of data. The mode is the value that occurs most often. There can be one mode, more than one mode, or no mode in a set of data.

For example:

The mode of 1, **5**, **5**, **5**, 8, 10, 10, 12 is 5.

The modes of **2**, **2**, **5**, **5**, 6 are 2 and 5.

1, 5, 8, 10, 12 has no mode.

• Although a mean and median only make sense for numerical data, the mode can be identified for a data set that is not numerical.

For example:

If a group of students have birthdays in these months: January, January, March, April, May, May, May, May, and June, the mode month is May.

• Statistics is a branch of mathematics that involves collecting, interpreting, and summarizing data.

B. What are the mean, median, and mode attendance for the data in **part A**?

Examples

Example 1 Comparing Means, Medians, and Modes

Five families compared the rents they paid. Their monthly rents were Nu 4500, Nu 6000, Nu 5800, Nu 6200, and Nu 6000. Which value is greatest: the mean rent, the median rent, or the mode rent?

Solution

The mean is Nu 5700:

$$\frac{4500+6000+5800+6200+6000}{5} = \text{Nu } 5700$$

The median is Nu 6000:

4500   5800   **6000**   6000   6200

The mode is Nu 6000:

4500   5800   **6000**   **6000**   6200

The median and mode are equal and greatest.

Thinking

• To find the mean, I shared the total equally among the 5 values.

• I ordered the numbers to find the median.

• The mode was easy to find with the numbers in order.

Example 2 Creating Data to Meet Conditions	
Create a set of six numbers where the median is less than the mean and the mean is less than the mode.	
Solution	**Thinking**
1, 2, 3, 4, 6, 6 Median = $3\frac{1}{2}$ Mean = $22 \div 6 = 3\frac{2}{3}$ Mode = 6	 • I created a set of numbers with a mode greater than the mean and median by using the greatest number twice. • The two 6s also helped make the mean greater than the median.

Practising and Applying

1. What are the mean, median, and mode of each set of numbers?

a) 8, 1, 2, 2, 3, 1, 5, 7, 7

b) 0, 5, 0, 2, 0, 6, 2, 1

c) 1, 2, 3, 3, 0, 15

d) 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3

2. Find the missing value in this set of data for each description below.

1, 4, 5, 5, ?, 1

a) The median is 3.

b) The only mode is 1.

c) The mean is 4.

3. Which is greatest in each set of data below: the median, the mean, or the mode?

a) 4, 5, 10, 2, 4

b) 17, 23, 19, 21, 20, 20

c) 8, 11, 2, 2, 2, 8, 2

4. Create a set of numbers for each. Do not use the same number for all the values of any data set.

a) 6 numbers with a median of 5

b) 4 numbers with a mode of 6

c) 5 numbers with a mean greater than the mode

5. Sonam is in Class VI. She lives with her two parents and one younger sister.

a) Is the median age in her family greater or less than Sonam's age? Explain your thinking.

b) Is the mean age for her family greater than or less than Sonam's age? Explain your thinking.

c) Could there be a mode age for her family? Explain your thinking.

6. The same number is missing from each set of data. Both sets have the same median. What is the missing value?

2, 3, 5, ▲ , 4 ▲, 1, 5, 2 ,5, 4

7. A set of data contains the masses of a cat, a dog, a tiger, and an elephant.

a) How does the median of the set of data compare to the mass of the tiger? Explain your thinking.

b) How does the mean compare to the mass of the tiger? Explain your thinking.

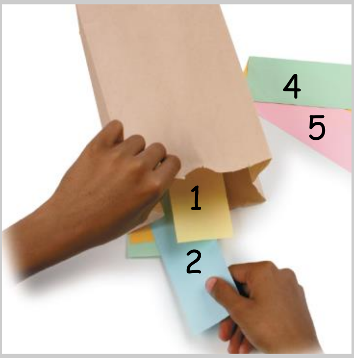
8. Why are the mean, the median, and the mode all possible ways to describe the average or typical number in a set of numbers?

7.3.2 Theoretical Probability

Try This

Imagine that you have 100 slips of paper.
You write the numbers from 1 to 100
on the slips and put them in a bag.
You draw one slip of paper from the bag.

A. What is the probability that the number you draw will be a multiple of 5? How do you know?



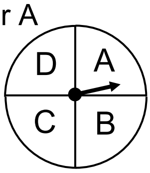
• The **theoretical probability** of an **event** is the fraction of the time you expect the event to happen if you repeat the event many times.

For example:
If you tossed a coin many times, you would expect to get a Khorlo half the time because the theoretical probability of tossing a Khorlo is $\frac{1}{2}$ (or 0.5 or 50%). This is because there is 1 **favourable outcome** (Khorlo) out of 2 **possible outcomes** (Khorlo and Tashi Ta-gye).

$$\text{Theoretical probability of an event} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

• If you are asked to create an event that has a probability of $\frac{1}{2}$, you can use the event described above or an event such as rolling a die and getting an even number. You can create events for other probabilities, too.

For example:

Probability	Event	Explanation
$\frac{1}{6}$	Rolling a die and getting a 1	There is 1 favourable outcome (1) out of 6 possible outcomes (1, 2, 3, 4, 5, 6).
$\frac{5}{6}$	Rolling a die and getting a 1, 2, 3, 4, or 5.	There are 5 favourable outcomes (1, 2, 3, 4, 5) out of 6 possible outcomes (1, 2, 3, 4, 5, 6).
$\frac{1}{4}$	Spinning the letter A on this spinner 	There is one favourable outcome (A) out of 4 possible outcomes (A, B, C, D).

Reprint 2023

B. Create another event with the same probability as the event in **part A**.

Examples

Example Creating an Event with a Particular Probability

Create an event with each theoretical probability. **a)** $\frac{2}{3}$ **b)** 0.4 **c)** 75%

Solution

a) $\frac{2}{3} = \frac{4}{6}$

Event: Rolling a number less than 5

4 favourable outcomes: 1, 2, 3, and 4

6 possible outcomes: 1, 2, 3, 4, 5, and 6

Probability of a number less than 5 = $\frac{4}{6} = \frac{2}{3}$.

b) $0.4 = \frac{4}{10} = \frac{2}{5}$

Event: Drawing a slip with the letter A on it from a bag with these 5 slips:

A	A	B	C	D

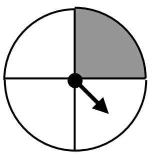
2 favourable outcomes: A and A

5 possible outcomes: A, A, B, C, and D

Probability of drawing a slip with A = $\frac{2}{5}$ = 0.4.

c) $75\% = \frac{3}{4}$

Event: Spinning white on this spinner:



3 favourable outcomes: white, white, white

4 possible outcomes: white, white, white, grey

Probability of spinning white = $\frac{3}{4}$ = 75%.

Thinking

a) I wrote $\frac{2}{3}$ as $\frac{4}{6}$ so I could create an event that used a die, which has 6 possible outcomes.

b) I knew I couldn't use a die or coin for tenths or fifths, so I thought of something where there were 5 possible things.

- I used slips of paper because you can use them for any probability; you just include as many slips as you need.

c) I changed the percent to a fraction because I knew the denominator would tell me how many possible events I needed and the numerator would tell me how many events should be favourable.

- Spinners are good for any probability but it's sometimes hard to make the sections exactly equal.

Reprint 2023

Practising and Applying

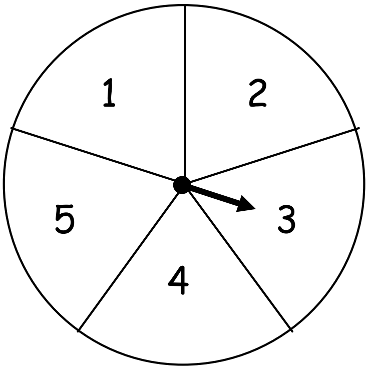
1. What is the theoretical probability of each?

a) rolling a 2 or 3 on a die

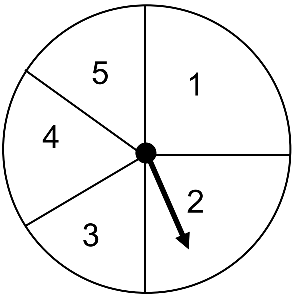
b) rolling an even number on a die

c) tossing Tashi Tag-ye on a coin

d) spinning 2 on this spinner



2. Karchung says that the probability of spinning 2 on this spinner is $\frac{1}{5}$ because there are 5 sections.
Do you agree?
Explain your thinking.



3. Each of these numbers is written on a slip of paper and put into a bag.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

What is the theoretical probability of drawing each from the bag?

a) an odd number

b) a number less than 31

c) a number greater than 42

d) a multiple of 6

4. There are slips of paper in a bag. Some slips have letters on them and some have numbers. The probability of drawing a slip with a letter is $\frac{5}{9}$.
How many slips could be in the bag?
What is written on the slips?

5. Create an event for each probability.

a) $\frac{2}{6}$ **b)** $\frac{2}{4}$

c) $\frac{3}{5}$ **d)** $\frac{3}{8}$

e) 25% **f)** 0.8

6. Which of the probability devices below could you use to create an event for any theoretical probability? Explain your thinking.

- rolling a die
- tossing a coin
- drawing slips from a bag
- spinning a spinner

UNIT 7 Revision

1. Suppose you want to collect data on how often people in your community get sick. You ask 30 people at the hospital or basic health unit.

a) Why might the results be biased?

b) What might be a better sample?

2. You want to know how much time people in Bhutan spend watching TV.

a) Why would you not ask only students in your school?

b) Why might you not ask only people who live in Zhemgang?

3. a) Roll two dice twice and find the sum each time.

b) Based on your sample of two rolls in **part a)**, predict the probability of rolling two dice and getting a sum greater than 7.

c) Roll the two dice another 23 times. Based on your sample of 25 rolls, what is the experimental probability of getting a sum greater than 7?

d) Was your sample of 2 rolls a good predictor of what happened with 25 rolls? Explain your thinking.

4. A teacher wants to compare how her students did on two tests to see if they improved. This chart shows the number of students that got a mark in each interval on each test.

For example, on Test 1, 12 students had a mark from 70 to 79.

	< 50	50s	60s	70s	80s	90s
1	5	9	9	12	4	1
2	3	10	8	15	4	0

a) Create a double bar graph of the data set.

b) What does the graph tell the teacher about whether the students improved?

5. a) Draw the graph from **question 4** again using these intervals:

< 50   50 to 69   70 or more

b) Compare how the two graphs show the same information.

6. What place value would you use for the stems to create a stem and leaf plot for each set of numbers?

a) 25, 87, 93, 45, 62, 8, 75

b) 102, 117, 237, 614, 512, 518, 413, 295, 303

c) 440, 423, 468, 500, 491, 437

7. Order the numbers in this stem and leaf plot from least to greatest.

3	1 2 3
4	0 1 1
5	1

8. Create a bar graph using the numbers from each stem and leaf plot.

a)

4	0 0 2 3
5	
6	0 0 1 1 4 5 6
7	0 1 1 2 3

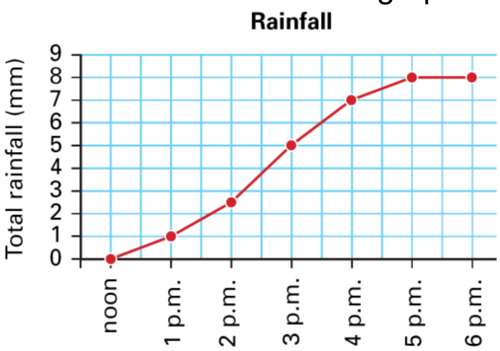
b)

1	20 35 45 57
2	00 40 65 85 85
3	02 12 15 18 28
4	00 05

9. Conduct this experiment:

Roll two dice 30 times. Each time, double the numbers you roll and then add them.
Create a stem and leaf plot of the data.

10. Which statement below best describes the trend in this graph?



A. rainfall increases steadily

B. rainfall increases more during some hours than others

C. rainfall decreases steadily

D. rainfall decreases more during some hours than others

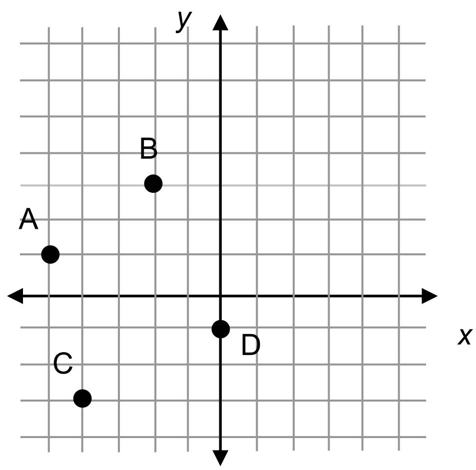
11. a) Create a line graph to show the number of minutes after 7 p.m. the sun set over 8 weeks.

Week	1	2	3	4
Minutes after 7	18	22	24	28

Week	5	6	7	8
Minutes after 7	27	23	22	20

b) What trend do you see in the graph?

12. What are the coordinates of each point?



13. Plot these points on the same coordinate grid.

a) (4, –1)

b) (–4, –2)

c) (–3, 0)

d) (–5, 6)

14. Plot four points on a coordinate grid. For each point, the *x*-coordinate should be the opposite of the *y*-coordinate. What do you notice about the four points?

15. a) Draw a triangle in quadrant II. Call it Triangle A.

b) Draw a reflection image of Triangle A using the *x*-axis as the reflection line. Call it Triangle B. What do you notice about the coordinates of the vertices of Triangles A and B?

c) Draw a rotation image of Triangle A that is a half-turn with the origin as the centre of rotation. What do you notice about the coordinates of the vertices of Triangles A and C? of Triangles B and C?

16. What value is missing from this set of data for each condition below to be true?

10, 2, 2, 6, ?, 1

a) the median is 2

b) the mode is 2

c) the mean is 5

17. In each set of data, which is least: the median, the mean, or the mode?

a) 1, 1, 1, 18, 19

b) 100, 104, 106, 108, 108

c) 5, 5, 10, 15, 2

18. Suppose you listed the maximum temperatures for the month of March for Samtse, Gasa, Thimphu, and Mongar.

a) Do you think the median temperature is greater than or less than the temperature for Gasa? Explain your thinking.

b) Do you think the mean temperature is greater than or less than the temperature for Samtse? Explain your thinking.

19. In each set of data, the median is less than the mean. What is each missing value?

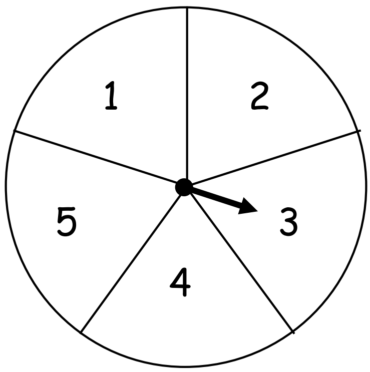
a) 2, 3, 5, ?, 4

b) ?, 1, 5, 2 , 5, 2

20. What is the theoretical probability of each?

a) rolling a 1 or a 6 on a die

b) spinning a number less than 4 on this spinner



21. Create an event to match each theoretical probability.

a) 10%   **b)** $\frac{2}{10}$

GLOSSARY

Instructional Terms

calculate: Figure out the number that answers a question; compute

classify: Put things into groups according to a rule and name the groups; e.g., classify triangles as right, acute, or obtuse by the size of their angles

compare: Look at two or more objects or numbers and describe how they are the same and how they are different and how they relate; e.g., compare the numbers 6.5 and 5.6; compare the lengths of the students' feet; compare two shapes

conclude: Judge or make a decision after looking at all the data

create: Make your own example or problem

describe: Tell, draw, or write about what something is or what something looks like; tell about a process in a step-by-step way

determine: Decide what the answer or result is for a calculation, a problem, or an experiment

draw: 1. Show something using a picture **2.** Take out an object without looking; e.g., draw a playing card from a deck of cards

estimate: Use what you know to make a sensible decision about an amount; e.g., estimate how long it takes to walk from your home to school; estimate how many leaves are on a tree; estimate the sum of 3210 + 789

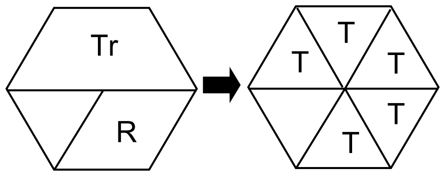
explain (your thinking): Tell what you did and why you did it; write about what you were thinking; show how you know you are right

explore: Investigate a problem by questioning and trying new ideas

measure: Use a tool to tell how much; e.g., use a ruler to measure a height or distance; use a protractor to measure an angle; use balance scales to measure mass; use a measuring cup to measure capacity; use a stopwatch to measure time

model: Show an idea using objects, pictures, words, and/or numbers; e.g., model fractions using pattern blocks or pattern block shapes

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$



Modelling fractions with pattern blocks

predict: Use what you know to figure out what is likely to happen; e.g., predict the number of times you will roll a number greater than 2 when you roll a die 30 times

relate: Describe how two or more objects, drawings, ideas, or numbers are similar

represent: Show information or an idea in a different way; e.g., represent a set of data in a stem and leaf plot

show (your work): Record all the calculations, drawings, numbers, words, or symbols that you used to calculate an answer or to solve a problem

simplify: Write a number in a simpler form; e.g., write an equivalent fraction with a lower numerator and denominator See *lower terms*

sketch: Make a quick drawing, usually free-hand without tools, to show your work; e.g., sketch a picture of a field with given dimensions

solution: The complete answer to a calculation or problem, showing all the work involved to get the answer

solve: Find an answer to a problem

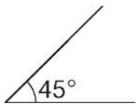
sort: Separate a set of objects, drawings, ideas, or numbers into groups according to an attribute; e.g., sort 2-D shapes by the number of sides

visualize: Form a picture in your head of what something is like; e.g., visualize the number 6 as 2 rows of 3 dots, like you would see on a die

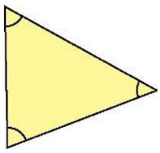
Definitions of Mathematical Terms

A

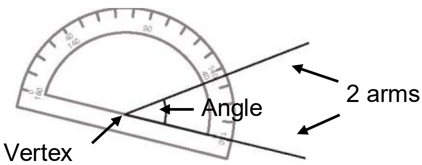
acute angle: An angle less than 90º; e.g.,



acute triangle: A triangle in which all angles are acute angles; e.g.,



angle: A figure formed by two arms with a shared endpoint, or vertex; the measure of an angle is the amount of turn between the two arms; angles are often measured in degrees (º)



angle bisector: A line through the vertex of an angle that separates the angle into two equal parts

anticlockwise: See *counterclockwise*

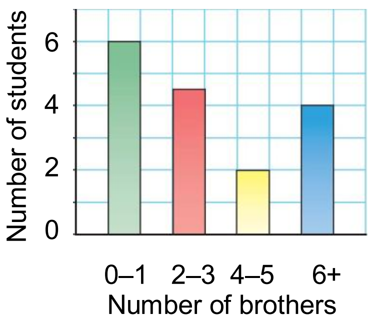
area: The number of square units (often in square centimetres or square metres) needed to cover a shape See *Measurement Reference* on **page 231**

average: Average is a term we sometimes use instead of the term mean See *mean*

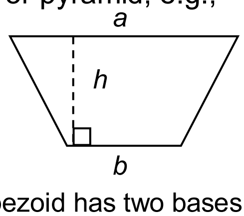
B

bar graph: A graph that compares the sizes of bars that each represent the number associated with a category or an interval in a set of data; e.g.,

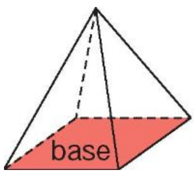
Number of Brothers



base: **1.** In a 2-D shape, the line segment(s) that is (are) perpendicular to the height **2.** In a 3-D shape, the face(s) that determines the name of a prism or pyramid; e.g.,



A trapezoid has two bases, a and b



A square-based pyramid

bias: When the results of data collection are affected or influenced, often as a result of a poorly-chosen sample; e.g., if a survey about colour preferences of school-age children involves only girls, the results are biased

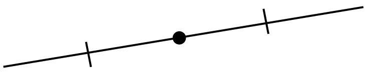
bisect: Divide something in half; e.g., an angle bisector divides an angle in half; if line segment AB passes through the centre point of line segment CD, AB bisects CD

bisector: See *bisect*

C

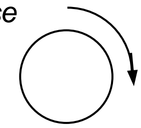
capacity: The amount that a container holds when full, measured in millilitres (mL), litres (L), or kilolitres (kL) See *Measurement Reference* on **page 231**

centre point: The point that divides a line segment in half; e.g.,



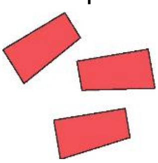
ccw: See *counterclockwise*

clockwise (cw): The direction that the hands of a clock move; describes one direction of a rotation – the other direction is counterclockwise
See *counterclockwise*



common factor: A whole number that divides into two or more other whole numbers with no remainder; e.g., 4 is a common factor of 8 and 12 because 8 ÷ 4 = 2 and 12 ÷ 4 = 3.

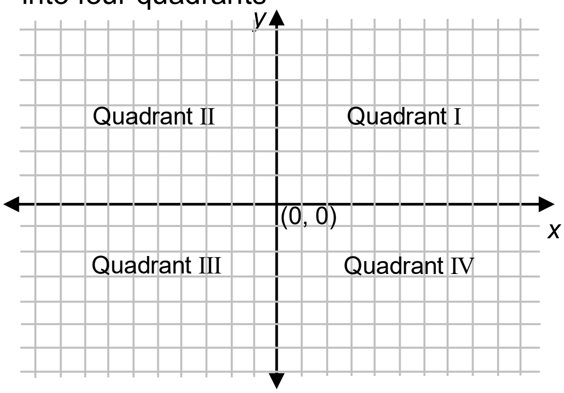
congruent: Identical in size and shape; shapes, line segments (e.g., side lengths), and angles can be congruent; e.g., these three shapes are congruent



Congruent shapes

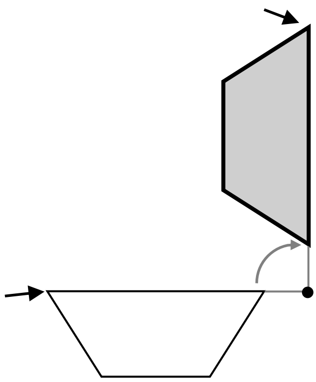
coordinate graph: A graph created by plotting points on a coordinate grid
See *coordinate grid*

coordinate grid: A 2-D graphing system that consists of a horizontal axis and a vertical axis that divide the grid into four quadrants

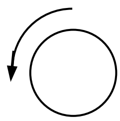


coordinates: See *x-coordinate* and *y-coordinate*

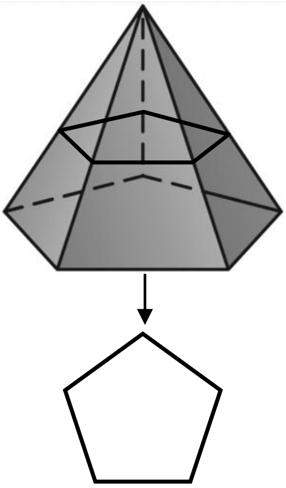
corresponding vertices: Matching vertices on an original shape and its transformational image; e.g., the two vertices below marked with black arrows are corresponding vertices



counterclockwise (ccw): The direction opposite to the direction the hands of a clock move; sometimes called anticlockwise; describes one direction of a rotation – the other direction is clockwise See *clockwise*

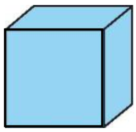


cross-section: The 2-D shape that results when you make a straight cut through a 3-D shape; e.g., if you cut through a pentagon-based pyramid as shown below, you get a pentagon cross-section



Pentagon
cross-section

cube: A 3-D shape that has six congruent square faces; e.g.,



cubic centimetre (cm^3): A standard unit of measure for volume that is equivalent to the amount of space taken up by a cube that is 1 cm along each edge

cubic metre (m^3): A standard unit of measure for volume that is equivalent to the amount of space taken up by a cube that is 1 m along each edge

cubic millimetre (mm^3): A standard unit of measure for volume that is equivalent to the amount of space taken up by a cube that is 1 mm along each edge

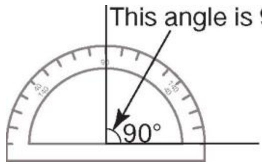
cuboid: Another name for a rectangular prism See *rectangular prism*

cw: See clockwise

D

data: Information collected in a survey, in an experiment, or by observing; the word data is plural, not singular; e.g., a set of data can be a list of students' names or it can be the numerical scores of a set of quiz marks

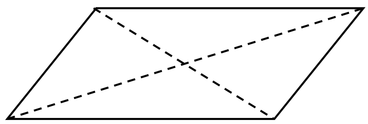
degree: A unit of measure for angle size; e.g.,



A 90 degree angle

denominator: The number in a fraction that represents the total number of parts in a whole set or the number of parts the whole has been divided into; e.g., in $\frac{4}{5}$, the denominator is 5

diagonal: A line segment that connects two vertices of a polygon that are not next to each other; e.g., the two dashed lines below are the diagonals of the parallelogram



difference: The result of a subtraction; e.g., in 45 – 5 = 40, the difference is 40

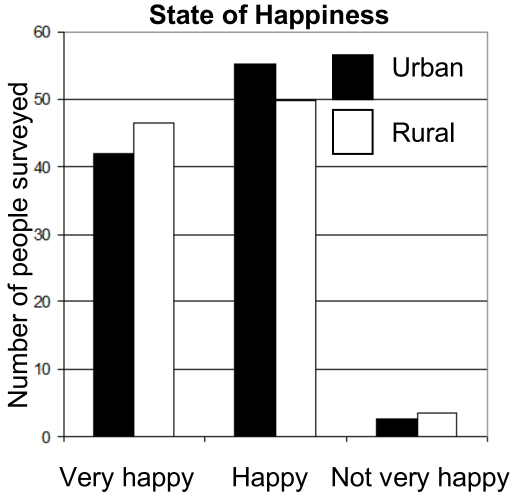
dimension: The size or measure of an object, usually length, width or breadth, depth, and height; e.g., the width and length of a rectangle are its dimensions

dividend: A number that is being divided; e.g., in 45 ÷ 5 = 9, the dividend is 45

divisor: The number by which another number is divided; e.g., in 45 ÷ 5 = 9, the divisor is 5

Reprint 2023

double bar graph: A special bar graph that shows two sets of data using the same categories or intervals; e.g.,

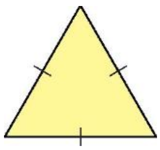


E

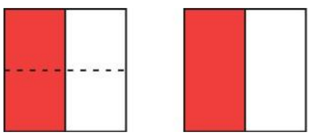
endpoint: The point where a line segment begins or ends

enlargement: See *similar*

equilateral triangle: A triangle with three sides of equal length (and with all angles equal and 60º)



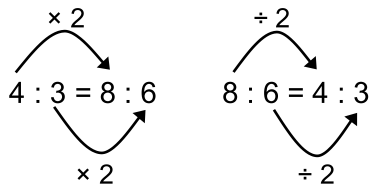
equivalent fractions: Fractions that represent the same part of a whole or set; e.g., $\frac{2}{4}$ is equivalent to $\frac{1}{2}$



equivalent decimal: A decimal that represents the same part of a whole or set; e.g., 0.5 is equivalent to 0.50; 0.5 is also equivalent to $\frac{1}{2}$

equivalent rate: Rates that describe the same relationship; you can find an equivalent rate by multiplying or dividing each term by the same non-zero number; e.g., a rate of 26 km in 2 days is equivalent to a rate of 52 km in 4 days or 13 km in 1 day

equivalent ratios: Ratios that make the same comparison; you can find an equivalent ratio by multiplying or dividing each term by the same non-zero number; e.g., 4 : 3 and 8 : 6 are equivalent ratios

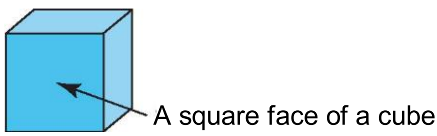


event: A set of outcomes for a probability experiment; e.g., if you roll a die with the numbers 1 to 6 on it, the event of rolling an even number is made up of the outcomes 2, 4, or 6

expanded form: A way of writing a number that shows the place value of each digit; e.g., 1209 in expanded form is $1 \times 1000 + 2 \times 100 + 9 \times 1$ or 1 thousand + 2 hundreds + 9 ones

F

face: A 2-D shape that forms a flat surface of a 3-D shape; e.g.,



face view: See *orthographic drawing*

factor: **1.** One of the numbers you multiply in a multiplication; e.g., 3 and 4 are the factors in $3 \times 4 = 12$ **2.** A whole number that divides into another whole number with no remainder; e.g., the factors of 24 are 1, 2, 3, 4, 6, 8, and 12

favourable outcome: The desired outcome when you calculate a theoretical probability; e.g., when you find the theoretical probability of rolling a number less than 3 on a die, rolls of 1 and 2 are favourable outcomes

formula: A general rule stated in mathematical language; e.g., the formula for the area of a rectangle is Area = length × width, or $A = l \times w$

fraction: A quotient of two integers written in the form of a numerator and a denominator; e.g., $\frac{4}{5}$ and $\frac{13}{5}$ are fractions

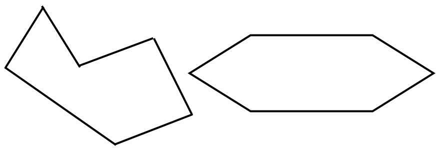
G

gram (g): A standard unit of measure used for mass See *mass*

graph: A picture of a set of data that can be used to understand the data; e.g., when you arrange data values in a stem and leaf plot, you create a picture that shows how the data set is shaped

H

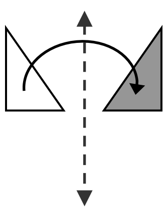
hexagon: A six-sided polygon; e.g.,



horizontal: A left-right or across direction as opposed to a vertical (up-down) or diagonal direction; e.g., a horizontal line segment

horizontal axis: See *x-axis*

horizontal reflection: A reflection across a vertical reflection line; e.g.,



I

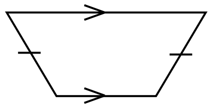
image: The new shape that you create when you apply a transformation to an original shape; e.g., after a rotation, the resulting shape is called the rotation image

improper fraction: A fraction in which the numerator is greater than or equal to the denominator; e.g., $\frac{5}{4}$ and $\frac{6}{6}$

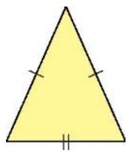
integers: The set of whole numbers and their opposites (zero is its own opposite): …, –2, –1, 0, 1, 2, …

interval: A range of values used to create a bar graph with intervals instead of categories See *bar graph*

isosceles trapezoid: A trapezoid with two congruent non-parallel sides; e.g.,



isosceles triangle: A triangle with two congruent sides



K

kilogram (kg): A standard unit of measure for mass See *mass*

kilolitre (kL): A standard unit of measure for capacity See *capacity*

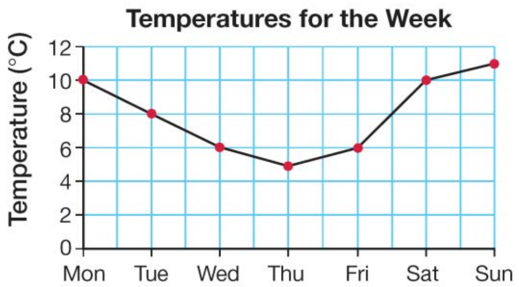
L

lateral face: The surface of a prism or pyramid that is not a base See *prism*

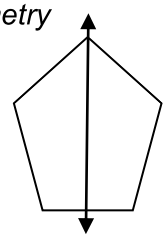
leaves: See *stem and leaf plot*

line: A set of points that form a straight path that goes on forever in both directions; e.g.,

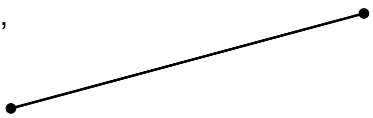
line graph: A graph that consists of points plotted and connected on a grid; line graphs are often used to see trends in data; e.g., this line graph shows the trend in the temperature over one week



line of symmetry: A line through a shape so that one side is a reflection or mirror image of the other side; e.g., this pentagon has one line of symmetry See *mirror symmetry*



line segment: A part of a line; it consists of two end points and all the points in between;
e.g.,



litre (L): A standard unit of measure for capacity See *capacity*

lower terms: 1. A fraction in lower terms is an equivalent fraction that has a lower numerator and denominator
2. A ratio in lower terms is an equivalent ratio that uses lower numbers;
e.g., 12 : 4 in lower terms is 6 : 2

M

mass: How light or heavy an object is; common units for measuring mass are grams and kilograms; tonnes are used for very heavy objects See *Measurement Reference* on **page 231**

mean: A single number that represents all the values in a data set; to calculate the mean, you add the values together and then divide the total by the number of values in the set; it is often called the average; e.g., the mean of 3, 4, 5, 6 is (3 + 4 + 5 + 6) ÷ 4 = 4.5

median: The middle value of a set of data arranged in order. If the set has an even number of values, the median is the mean of the two middle values; e.g., in the data set below, the median is 10:

1 7 $\underline{9\ \ 11}$ 11 13

The median is the mean of 9 and 11.

metre (m): A unit of measurement for length; e.g., 1 m is about the distance from a doorknob to the floor;
1000 mm = 1 m; 100 cm = 1 m;
1000 m = 1 km

metric system/prefixes: A standard system of units and prefixes for measuring and reporting length, area, mass, volume, capacity, and so on, where each unit is made up of ten of the next smallest unit See *Measurement Reference* on **page 231**

metric tonne: See *tonne*

millilitre (mL): A standard unit of measure for capacity See *capacity*

mirror symmetry: A property of a shape; when a 2-D shape is folded or reflected across a line (the line of symmetry), the two sides of the shape match; also called line or reflectional symmetry See *line of symmetry*

mixed number: A number made up of a whole number and a proper fraction; e.g., $5\frac{1}{7}$

mode: The piece(s) of data that occurs (occur) most often in a set of data; there can be more than one mode or there might be no mode; e.g., in the data set below, the modes are 12 and 14:

4 7 9 $\underline{12\ \ 12}$ 13 $\underline{14\ \ 14}$ 16

multiple: The product of a whole number and any other whole number; e.g., when you multiply 10 by the whole numbers 0, 1, 2, 3, 4, …, you get the multiples 0, 10, 20, 30, 40, …

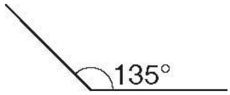
N

negative (number or integer): A number (or integer) less than zero See *integers*

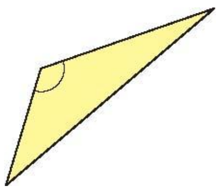
numerator: The number in a fraction that shows the number of parts of a given size the fraction represents; e.g., in $\frac{4}{5}$, the numerator is 4

O

obtuse angle: An angle greater than 90º and less than 180º; e.g.,

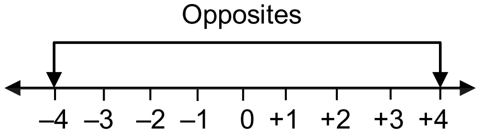


obtuse triangle: A triangle in which one of the angles is an obtuse angle; e.g.,



ones period: The cluster of three digits in a whole number that contains the hundreds digit, the tens digit, and the ones digit; e.g., in the number 123,456, the digits 456 make up the ones period

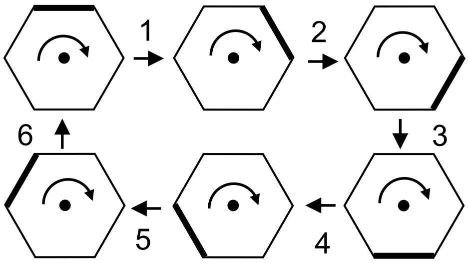
opposite integers: Two integers that are the same distance away from zero but in opposite directions; e.g., +4 and –4 are opposite integers



order of operations (rules): Rules that describe the sequence to use to calculate an expression to ensure everyone gets the same answer:

1 Do calculations inside brackets first
2 Divide and multiply from left to right
3 Add and subtract from left to right

order of turn symmetry: A measure of rotational symmetry; the number of times a shape looks the same during one full turn; a shape with an order of symmetry of greater than 1 has rotational symmetry; e.g., a regular hexagon has order of turn symmetry of 6 See *rotational symmetry*

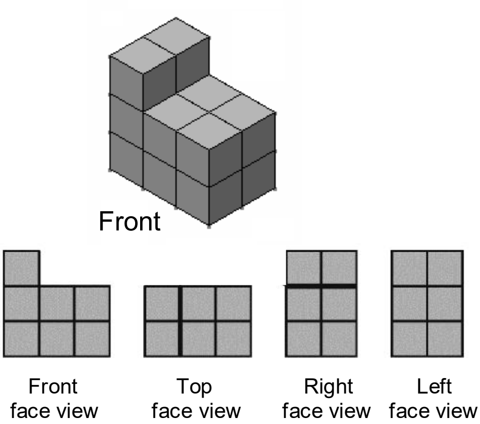


ordered pair: A pair of numbers in a particular order that describes the location of a point in a coordinate grid; e.g., the ordered pairs (3, 5) and (5, 3) describe the locations of two different points on the grid shown on **page 230** See *x-axis*

origin: The intersection of the axes in a coordinate grid, represented by the ordered pair (0, 0) See *x-axis*

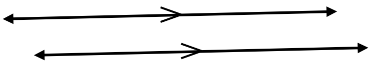
original shape: In a transformation, the shape that you start with is called the original shape and result of the transformation is called the image

orthographic drawings: A set of 2-D drawings of a 3-D shape; each drawing is called a face view; e.g., the four orthographic drawings below this cube structure are four of its face views

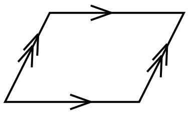


P

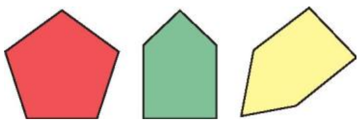
parallel lines or line segments: Lines or line segments that never meet, so they are always the same distance apart; e.g.,



parallelogram: A quadrilateral with pairs of opposite sides that are equal in length and parallel; e.g.,



pentagon: A polygon with five sides; a regular pentagon has five congruent sides and five congruent angles; e.g.,



These are all pentagons. The first shape is a regular pentagon.

per: See *rate*

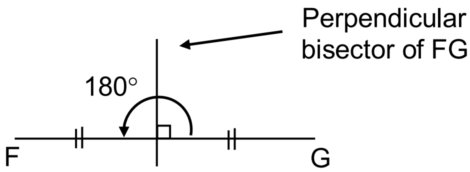
percent: A special ratio that compares a number to 100, using the symbol %; e.g., if 3 out of 4 students are girls, then 75% are girls because $\frac{3}{4} = \frac{75}{100}$ = 70%

perimeter: **1.** The boundary or outline of a 2-D shape **2.** The length of the boundary

period: A group of three digits in a number, often separated by a comma or a space; e.g., in the number 458,675, the thousands period is 458 and the ones period is 675

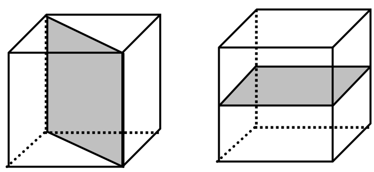
perpendicular: Meeting or crossing at a right angle

perpendicular bisector: A line or line segment that is at a right angle to another line segment and divides the line segment in half; e.g.,



place value: The value of a digit depends on its place in the number; e.g., in the number 123.4, the digit 3 has a value of 3 because it is in the ones place, the digit 2 has a value of 20 because it is in the tens place

plane of symmetry: An imaginary surface that cuts a 3-D shape into congruent halves where one half is the mirror image of the other half; e.g.,



Some planes of symmetry of a cube

plot (a point): Locate a point on a coordinate grid using its coordinates See *coordinate grid*

polygon: A closed 2-D shape with three or more sides; e.g., triangles, quadrilaterals, pentagons, and so on are polygons, but a circle is not

population: The entire group of subjects that you are interested in collecting data about; e.g., for collecting data about the favourite type of momo of students at a school, the population is all of the students in the school

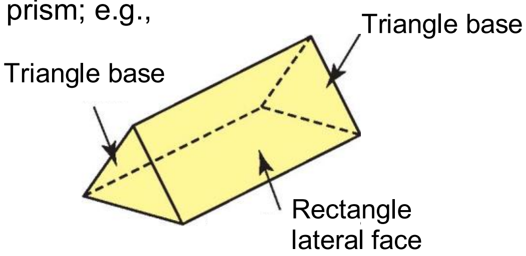
positive (number or integer): A number (or integer) greater than zero See *integers*

possible outcome: A thing that could happen in a probability situation; e.g., when you roll a die, there are six possible outcomes: 1, 2, 3, 4, 5, and 6

prime factors: The factors of a number that are prime numbers; usually written as a product; e.g., the prime factors of 24 are $2 \times 2 \times 2 \times 3$

prime numbers: A number that has exactly two factors, the number 1 and the number itself; e.g., 2 is a prime number because its factors are 1 and 2; other prime numbers include 3, 5, 7, and 11

prism: A 3-D shape with two parallel and opposite congruent bases; the other faces are parallelograms (usually rectangles); the shape of the bases determines the name of the prism; e.g.,



A triangle-based prism

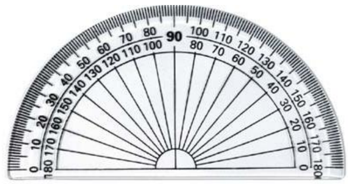
probability: A number from 0 (will never happen) to 1 (certain to happen) that tells how likely it is that an event will happen; it can be a decimal, a fraction, or an expression using words; sometimes it is called chance or likelihood

product: The result of multiplying two or more numbers; e.g., in $5 \times 6 = 30$, the product is 30

proper fraction: A fraction in which the denominator is greater than the numerator; e.g., $\frac{1}{7}$, $\frac{4}{5}$, $\frac{29}{40}$

proportion (ratio): A comparison of two ratios in fraction form; e.g., if $\frac{2}{6}$ of a group is boys and $\frac{2}{5}$ of another group is boys, the first group has a lower proportion of boys than the second group because $\frac{2}{6} < \frac{2}{5}$; a proportion can also be an equation with two equivalent ratios in fraction form; e.g., $\frac{2}{6} = \frac{1}{3}$.

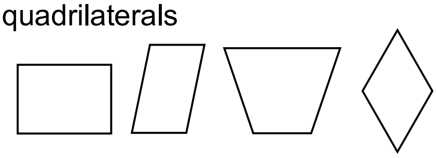
protractor: A tool used to measure the size of an angle



Q

quadrant: See *coordinate grid*

quadrilateral: A four-sided polygon; e.g., rectangles, parallelograms, trapezoids and rhombuses are all types of quadrilaterals

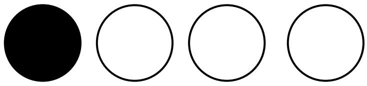


quotient: The result of dividing one number by another number; e.g., in $45 \div 5 = 9$, the quotient is 9

R

rate: A comparison of two quantities measured in different units; unlike a ratio, a rate includes the units because the units are different; e.g., 45 km in 1 hour = 45 km per hour or 45 km/h

ratio: A number or quantity compared with another, expressed in symbols as $a : b$ or $\frac{a}{b}$; no units are shown because the units are the same; it can be a part-to-part comparison or a part-to-whole comparison; e.g., all three ratios describe the set of counters below

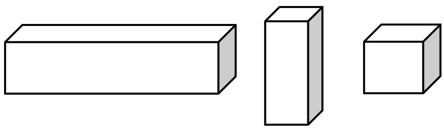


1 black counter to 3 white counters → 1 : 3

1 black counter to 4 counters → 1 : 4

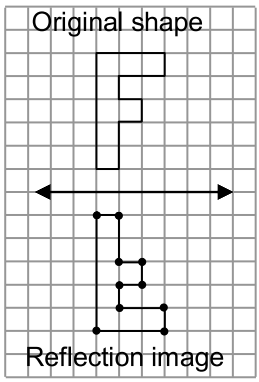
3 white counters to 4 counters → 3 : 4

rectangular prism: A prism with rectangle bases; a cube is a special rectangular prism; e.g.,



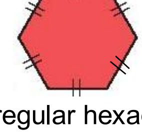
reduction: See *similar*

reflection: A transformation that produces a mirror image of a shape across a reflection line; also called a flip; e.g., this is a vertical reflection of the F-shape across a horizontal reflection line:



reflection line: See *reflection*

regular polygon: A polygon with sides and angles congruent;
e.g.,

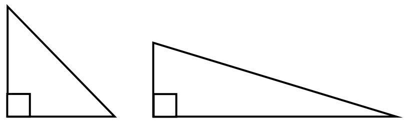


A regular hexagon

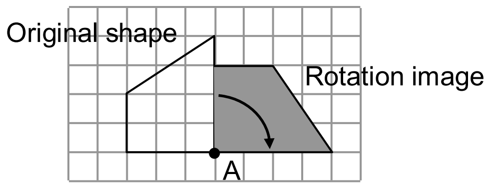
rename (a number): Change a number to another form to make it easier to calculate or compare, but without changing its value; e.g., you can rename 0.4 in many different ways: $\frac{4}{10}$, 4 tenths, 0.40, and 40 hundredths

right angle: An angle that measures 90º; sometimes called a square corner See the right angles in the *right triangles* below

right triangle: A triangle with one right angle; e.g.,



rotation: A transformation in which each point in a shape moves around a point (the turn centre) in the same way; you describe a rotation by the size of the turn (often a fraction of a full turn) and the direction of the turn (clockwise or counterclockwise e.g., this is a $\frac{1}{4}$ clockwise rotation of a trapezoid around turn centre A:

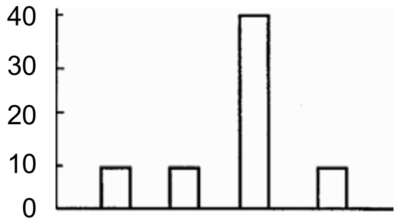


rotational symmetry: If a shape looks the same more than once during a complete rotation, it has rotational symmetry; also called turn symmetry See *order of turn symmetry*

S

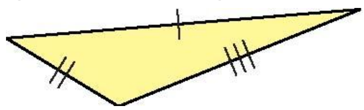
sample: If you cannot collect data from the entire population you are interested in, you can collect data from a carefully chosen sample; e.g., to collect data about the favourite type of momo of all the students at a school, a good sample might be five students chosen randomly from each classroom

scale (on a graph): The value of each interval on an axis; the scale tells how to interpret the graph; e.g., the scale on the vertical axis of this graph is 10



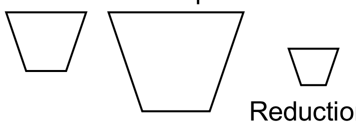
scale ratio: The ratio of the distance on a map to the actual distance;
e.g., a scale ratio of 1 : 2000 means that 1 cm on the map represents 2000 cm or 20 m in actual distance

scalene triangle: A triangle with no congruent sides; e.g.,



similar: Two shapes are similar if one shape looks like an enlargement or a reduction of the other shape; e.g.,

These three shapes are similar



Reduction

Enlargement

simplify: To simplify a fraction means to write it in lower terms or as a mixed number; e.g., you can simplify $\frac{18}{10}$ as $\frac{9}{5}$ and then as $1\frac{4}{5}$

speed: The rate at which a moving object changes position with time, often given as a unit rate; e.g., a sprinter who runs 100 m in 10 s has a speed of 10 m/s

square number: A number that can be modelled as a square; e.g., 4 and 9 are square numbers

4	9
X X	X X X
X X	X X X
	X X X

standard form (of a number): The usual way to write a number;
e.g., 23,650 is in standard form

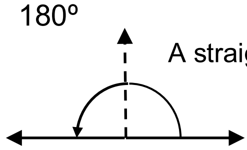
stems: See *stem and left plot*

stem and leaf plot: A graph of a set of data where the data is arranged in place value intervals called stems; e.g.,

Data set: 26, 24, 19, 28, 24, 27, 30, 32, 26, 28, 22, 25

Stems	Leaves
1	9
2	2 4 4 5 6 6 7 8 8
3	0 2

straight angle: An angle that measures 180º



A straight angle is 180°.

sum: The result of adding numbers;
e.g., in 5 + 4 + 7 = 16, the sum is 16

survey: A method of collecting data

T

term: 1. Each number or item in a pattern; e.g., in the pattern 1, 3, 5, 7, …, the third term is 5 **2.** The numbers in a ratio or rate; e.g., the ratio 2 : 3 has two terms

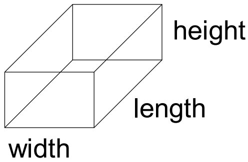
theoretical probability: A number from 0 to 1 that tells how likely an event is to occur; it is calculated using

$$\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}};$$

e.g., the theoretical probability of rolling a 4 on a die is $\frac{1}{6}$ because there are 6 possible outcomes (1, 2, 3, 4, 5, 6) and 1 of them is favourable (4)

thousands period: The group of three digits in a whole number that contains the hundreds thousands digit, the ten thousands digit, and the one thousands digit; e.g., in the number 123,456,
the digits 123 make up the thousands period

three-dimensional (3-D): A shape with three dimensions: length, width (or breadth or depth), and height; e.g.,

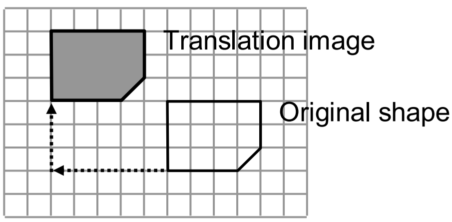


tesselation: An arrangement of congruent 2-D shapes that covers a surface (in all directions) without gaps or overlapping

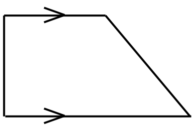
tonne (t): A standard unit of measure for mass; 1 t is equivalent to 1000 kg See *mass*

transformation: Changing a shape according to a rule; transformations include translations, rotations, and reflections See *translation*, *reflection*, and *rotation*

translation: A transformation in which each point of a shape moves the same distance and in the same direction; also called a slide; e.g., the pentagon below has been translated 5 units left and 3 units up



trapezoid: A quadrilateral that has one pair of opposite parallel sides; e.g.,



trend: See *line graph*

triangular number: The sum of consecutive whole numbers starting at 1; each triangular number can be modelled as a triangle; e.g., 3 and 6 are triangular numbers

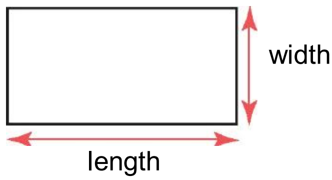
```
 3       6
X       X
X X     X X
        X X X
1 + 2   1 + 2 + 3
```

turn centre: The point around which all the points in a shape turn or rotate in a clockwise (cw) or counter-clockwise (ccw) direction during a rotation
See *rotation*

turn symmetry: See *rotational symmetry*

24-hour clock (system): A system of measuring and reporting time where the day starts at 00:00 (midnight), goes to 12:00 (noon), and ends at 23:59 (just before midnight) See *Measurement Reference* on **page 231**

two-dimensional (2-D): A shape with two dimensions: length and width (or breadth); e.g.,



U

unit rate: A rate with a second term of 1; e.g., 4 km/h is a unit rate because it means 4 km in 1 h

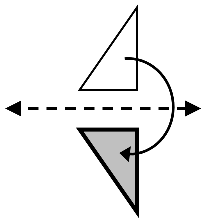
V

vertex (vertices): The point at the corner of an angle or shape where two or more sides or edges meet; e.g., a cube has eight vertices, a triangle has three vertices, and an angle has one vertex

vertical: An up-down direction as opposed to a horizontal (left-right) direction; e.g., a vertical line:

vertical axis: See *y-axis*

vertical reflection: A reflection across a horizontal reflection line;
e.g.,



volume: The amount of space occupied by an object; often measured in cubic centimetres (cm^3) or cubic metres (m^3) See *Measurement Reference* on **page 231**

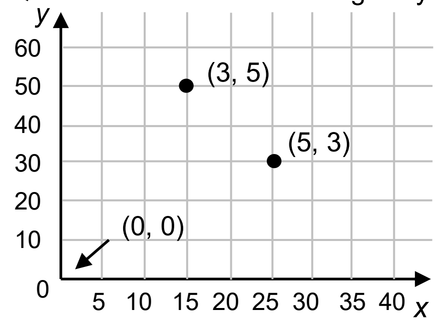
W

whole numbers: The set of numbers that begins at 0 and continues forever in this pattern: 0, 1, 2, 3, …

X

***x*-axis:** One of the two axes in a coordinate grid; sometimes called the horizontal axis; e.g., the *x*-axis below goes from 0 to 40 See *coordinate grid*

Quadrant I of the coordinate grid system



***x*-coordinate:** The first value in an ordered pair; it represents the distance along the *x*-axis from (0, 0) on a coordinate grid; e.g., in (5, 3), the *x*-coordinate is 5 See *x-axis*

Y

***y*-axis:** One of the two axes in a coordinate grid; sometimes called the vertical axis on a coordinate grid; e.g., the *y*-axis of the grid shown above goes from 0 to 60 See *x-axis*

***y*-coordinate:** The second value in an ordered pair; it represents the distance along the *y*-axis from (0, 0); e.g., in (5, 3), the *y*-coordinate is 3 See *y-axis*

MEASUREMENT REFERENCE

Measurement Abbreviations and Symbols

Time second minute hour	 s min h	**Capacity** millilitre litre kilolitre	 mL L kL
Length millimetre centimetre decimetre metre kilometre	 mm cm dm m km	**Volume** cubic centimetre cubic metre cubic millimetres	 cm^3 m^3 mm^3
Mass milligram gram kilogram tonne	 mg g kg t	**Area** square centimetre square metre	 cm^2 m^2

Metric Prefixes

Prefix	kilo × 1000	hecto × 100	deka × 10	unit 1	deci × 0.1 or $\frac{1}{10}$	centi × 0.01 or $\frac{1}{100}$	milli × 0.001 or $\frac{1}{1000}$
Example	*kilo*metre km	*hecto*metre hm	*deka*metre dam	metre m	*deci*metre dm	*centi*metre cm	*milli*metre mm
	1000 m	100 m	10 m	1 m	0.1 m	0.01 m	0.001 m

Measurement Formulas and Relationships

Perimeter rectangle $P = 2 \times (l + w)$ or $2 \times l + 2 \times w$ square $P = 4 \times s$	**Area** rectangle $A = l \times w$ square $A = s \times s$ parallelogram $A = b \times h$ triangle $A = b \times h \div 2$

Volume of a rectangular prism: $V = \textit{Area of base} \times \textit{height}$ or $V = l \times w \times h$

Relationship between the volume and capacity of water: 1 cm^3 = 1 mL

24-hour Clock System

Relating the 12-hour clock system to the 24-hour clock system:

Midnight	Morning	Noon	Afternoon	Evening
12:00 a.m.	6:00 a.m.	12:00 p.m.	6:00 p.m.	11:59 p.m
00:00	06:00	12:00	18:00	23:59

Reprint 2023

ANSWERS

UNIT 1 NUMBER RELATIONSHIPS pp. 1–30

Getting Started — Skills You Will Need pp. 1-2

1. a) 6 **b)** 7 **c)** 1

2. a) 1000 **b)** 10

3. a) 200,045; 2 hundred thousands + 4 tens + 5 ones
b) 3,803,056; 3 millions + 8 hundred thousands + 3 thousands + 5 tens + 6 ones
c) 1,300,870; 1 million + 3 hundred thousands + 8 hundreds + 7 tens

4. 200,045; 1,300,870; 3,803,056

5. a) Three million, one hundred forty thousand, twenty
b) Three hundred nine thousand, forty-five

6. a) 4.2 **b)** 31.4
c) Ten thousand **d)** 0.45

7. a) 3 **b)** 4

8. a) Five thousandths
b) Twenty-two and five hundredths
c) Eight and one hundred twenty-five thousandths

9. They are equivalent, 0.2 = 0.20 = 0.200 or $\frac{2}{10} = \frac{20}{100} = \frac{200}{1000}$.

10. *Sample responses*:
a) 10, 15, 20, 40, 80
b) 10, 20, 30, 40, 80
c) 16, 32, 48, 64, 80

11. *Sample responses*:
a) 1, 5, 10 **b)** 11, 5, 55 **c)** 2, 28, 140

1.1.2 Place Value With Large Whole Numbers p. 8

1. a) 302,054,000 **b)** 2,053,000,089
c) 6,000,400,005

2. a) 3 billions + 4 ten millions + 5 millions + 1 hundred thousand;
$3 \times 1{,}000{,}000{,}000 + 4 \times 10{,}000{,}000 + 5 \times 1{,}000{,}000 + 1 \times 100{,}000$
.
b) 1 billion + 2 hundred millions + 3 millions + 5 hundred thousands;
$1 \times 1{,}000{,}000{,}000 + 2 \times 100{,}000{,}000 + 3 \times 1{,}000{,}000 + 5 \times 100{,}000$.

3. a) 1,000,000,000 **b)** 100,000,000
c) 1,000,000,000

4. a) Thousand **b)** Hundred thousand

5. 8,840,230; 3.2 billion; 4,235,100,023

6. 21,342,899

7. *Sample response*:
6,200,054; 2,600,054; 56,200,004; 52,600,004; 56,400,002

1.1.3 Renaming Numbers p. 11

1. a) 3.45 **b)** 3450 **c)** 345

2. a) 4,200,000,000 **b)** 31,400,000
c) 5,800,000,000 **d)** 1,230,000

3. 123 ten million; 3134 million;
3.2 billion; 58 hundred million

4. *Sample response*:
31.2 ten million = 312 million
31.2 billion = 312 hundred million
31.2 hundred million = 312 ten million

5. Two; 0.34 has two non-zero digits.

6. a) 32,000 **b)** 1.412 **c)** 68,200,000
7. About 4 million

1.2.1 Place Value With Decimals — p. 13

1. a) 4 **b)** 5

2. a) 0.0060 or 0.006 **b)** 0.0033 **c)** 0.4203

3. a) 0.01 (or $\frac{1}{100}$) **b)** 1000

4. a) Yes

5. *Sample responses*:
a) One and two hundred thirty thousandths, or one and twenty-three hundred ten thousandths
b) Four thousand, three hundred, fifty-six ten thousandths
c) One and nine thousand, eight hundred, two ten thousandths
d) Twelve and one thousandth, or twelve and ten ten thousandths

7. a) About 90,000 **b)** Less

8. Hundred thousandths

1.2.2 Comparing and Ordering Decimals — p. 15

1. a) 0.1234; 0.3578; 0.92; 1.2398
b) 3.14578; 3.21514; 3.33; 3.5764

2. *Sample response*:
0.9981; 0.9991; 0.9992; 0.9993; 0.9994

3. *Sample response*:
0.0001; 0.0002; 0.0003; 0.0004; 0.0005

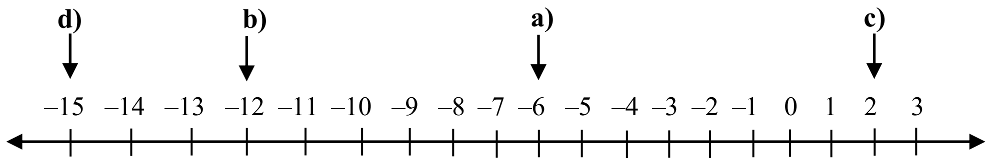
4. Yes

5. 26 ten thousandths; 43 hundredths; 512 thousandths

7. a) Bhutan
b) About 14 times as big

1.2.3 Introducing Integers — p. 18

1. *Number line could be vertical or horizontal*:



2. a) –3 **b)** +2 or 2
c) +5 or 5 **d)** 0

3. +16 or 16, and –16

4. *Sample response*:
4 km below sea level; a debt of Nu 4; a temperature 4° below zero.

5. a) –2° **b)** –3° **c)** +1° or 1°

6. –7, and +1 or `

7. –5, –6, or –7

1.3.1 Prime Numbers — p. 21

1. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

4. They can be 2 apart, like 11 and 13

5. *Sample response*: 17 and 71

6. *Sample response*:
10 = 2 × 5; 20 = 2 × 2 × 5;
70 = 7 × 2 × 5; 100 = 2 × 2 × 5 × 5

CONNECTIONS: The Sieve of Eratosthenes — p. 21

1. Yes, except for 1.

3. Create a chart that goes up to 200 instead of 100 and use the same technique.

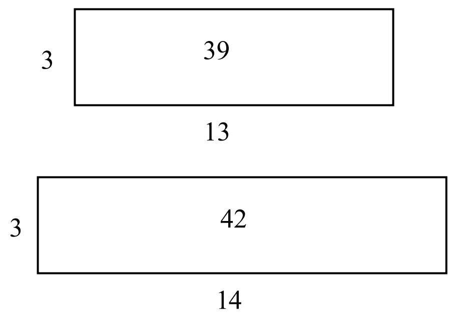
CONNECTIONS: Triangular Numbers as Products p. 23

210

1.3.4 Common Factors p. 28

1. *Sample responses*:
a) 2 **b)** 20 **c)** 4 **d)** 3

2.



4. 1, 2, 4, and 6

5. In rows of 1, 2, 3, or 6

6. 1 unit, 2 units, 4 units, or 8 units

7. a) False **b)** True

8. Any multiple of 3

9. Yes

UNIT 1 Revision pp. 29–30

1. a) 6,022,403,000 **b)** 308,087,086
c) 2,103,000,017

2. *Sample responses*:
a) 4 × 1,000,000,000 + 2 × 100,000,000 + 1 × 100,000 + 4 × 10,000 + 6 × 1000 + 1 × 100;
4 billions + 2 hundred millions + 1 hundred thousand + 4 ten thousands + 6 thousands + 1 hundred
b) 3 × 100,000,000 + 5 × 10,000,000 + 6 × 1,000,000 + 1 × 100,000 + 2 × 100;
3 hundred millions + 5 ten millions + 6 millions + 1 hundred thousand + 2 hundreds

3. *Sample response*:
22,500,000; 20,500,002; 20,502,000

4. a) 800,000,000 **b)** 2,320,000,000
c) 620,000 **d)** 5,700,000,000

5. 28 ten million, 0.9 billion, 1001 million, 1,002,003 thousand

6. a) 3 **b)** 0 **c)** 4

7. a) 0.0054 **b)** 0.065
c) 0.0650 **d)** 0.0324

8. *Sample responses*:
a) 0.1061 **b)** 0.1208

9. *Sample responses*:
a) 3 and 12 thousandths, or 3012 thousandths
b) 4 and 123 thousandths, or 4 and 1230 ten thousandths
c) 4 and 1 tenth, or 4 and 100 thousandths
d) 3 and 4 thousandths, or 3 and 40 ten thousandths

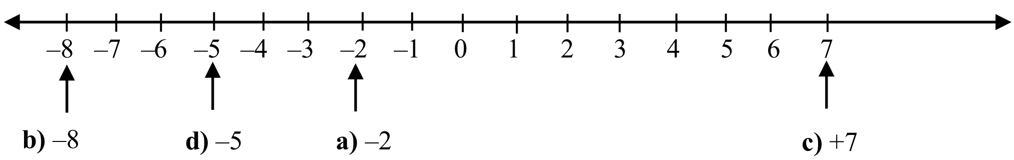
10. Yes **11.** Yes

12. a) 0.0369
b) *Sample response*:
The people in Australia are more spread out.

13. 891 ten thousandths, 36 hundredths, 1234 thousandths

14. About 30 km

15.



UNIT 1 Revision [Cont'd] pp. 29–30

16. a) +6 or 6 **b)** –12
c) +9 or 9 **d)** –8

17. +1 or 1

18. +10 (or 10) or +11 (or 11)

21. *Sample responses*:
a) 2 **b)** 18 **c)** 5 **d)** 50

22. The side lengths could be 1 cm, 2 cm, 4 cm, 5 cm, 10 cm, or 20 cm.

23. As many factors as the lower number has

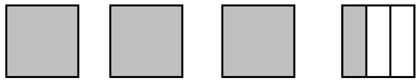
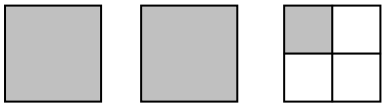
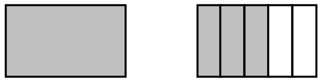
UNIT 2 FRACTIONS AND DECIMALS pp. 31–56

Getting Started — Skills You Will Need pp. 31–32

1. A and B

2. a) i) $\frac{3}{4}$ **ii)** $\frac{1}{3}$
iii) $\frac{1}{6}$ **iv)** $\frac{2}{4}$

b) $\frac{1}{6}, \frac{1}{3}, \frac{2}{4}, \frac{3}{4}$

3. *Sample responses*:
a)

b)

c)


4. a) 0.3 or 0.30 **b)** 0.19

2.1.1 Relating Mixed Numbers to Improper Fractions p. 35

1. a) $2\frac{1}{6}$ **b)** $8\frac{1}{2}$ **c)** $7\frac{2}{3}$

2. a) $\frac{7}{2}$ **b)** $\frac{19}{4}$ **c)** $\frac{32}{5}$

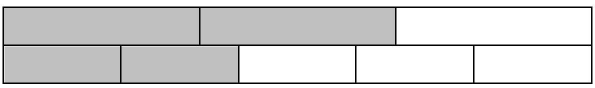
3. a) i) 26 to 29 **ii)** 41 to 47 **iii)** 51 to 59

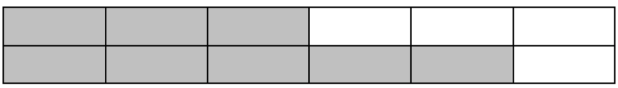
4. a) $5\frac{3}{4}$ **b)** $\frac{24}{6}$

5. *Sample responses*:
a) $\frac{25}{6}$ and $\frac{29}{6}$ **b)** $3\frac{4}{6}$ and $5\frac{4}{6}$

6. 1 or 2

2.1.2 Comparing and Ordering Fractions p. 38

1. *Sample responses*:
a)

$\frac{2}{3} > \frac{2}{5}$

b)

$\frac{3}{6} < \frac{5}{6}$

2. *Sample response*:

$\frac{1}{3} < \frac{5}{12}$

3. a) $\frac{3}{10}, \frac{3}{8}, \frac{3}{4}$ **b)** $\frac{1}{9}, \frac{3}{9}, \frac{8}{9}$

c) $\frac{1}{8}, \frac{9}{20}, \frac{11}{12}$ **d)** $\frac{4}{10}, \frac{5}{8}, \frac{3}{4}$

Reprint 2023

4. Washing clothes **5.** The basket that is $\frac{5}{8}$ full	**6. a)** 3 **b)** 3 **c)** 5 **d)** 3

2.1.4 Adding Fractions — p. 43

1. a) $\frac{5}{8}$ **b)** $\frac{6}{8}$ **c)** $\frac{10}{10}$ **d)** $\frac{4}{5}$

2. a) $\frac{7}{8}$

<table>
<tr><td colspan="2">1/4</td><td colspan="2">1/4</td><td colspan="2">1/4</td><td>1/8</td></tr>
<tr><td>1/8</td><td>1/8</td><td>1/8</td><td>1/8</td><td>1/8</td><td>1/8</td><td>1/8</td></tr>
</table>

b) $\frac{7}{12}$

<table>
<tr><td>1/12</td><td>1/12</td><td>1/12</td><td>1/12</td><td>1/12</td><td colspan="2">1/6</td></tr>
<tr><td>1/12</td><td>1/12</td><td>1/12</td><td>1/12</td><td>1/12</td><td>1/12</td><td>1/12</td></tr>
</table>

c) $\frac{11}{12}$ **d)** $\frac{5}{6}$

3. a) $\frac{3}{10} + \frac{2}{5} = \frac{7}{10}$

b) $\frac{1}{6} + \frac{2}{3} = \frac{5}{6}$

c) $\frac{2}{4} + \frac{1}{3} = \frac{5}{6}$

4. *Sample response*:

$\frac{1}{6} + \frac{4}{6}$; $\frac{2}{6} + \frac{3}{6}$; $\frac{5}{12} + \frac{5}{12}$; $\frac{1}{3} + \frac{1}{2}$; $\frac{1}{12} + \frac{3}{4}$

5. *Sample responses*:

a) $\frac{2}{8} + \frac{1}{4} = \frac{1}{2}$ **b)** $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

c) $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ **d)** $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

6. No

7. a) Place counters on any 2 squares (or colour 2 squares).
b) Place counters on any 3 squares (or colour 3 squares).
c) Place counters on any 5 squares (or colour 2 squares one colour and 3 squares another colour).

CONNECTIONS: Fractions Between Fractions — p. 44

1. *Sample responses*:

a) $\frac{3}{4}$ and $\frac{7}{8}$ **b)** $\frac{5}{6}$ and $\frac{4}{7}$ (fractions between $\frac{1}{2}$ and $\frac{7}{8}$)

2. *Sample response*:

$\frac{3}{9}$ and $\frac{6}{6}$

2.1.5 Subtracting Fractions — p. 48

3. a) $\frac{2}{5} - \frac{3}{10} = \frac{1}{10}$ **b)** $\frac{2}{4} - \frac{1}{8} = \frac{3}{8}$

c) $\frac{2}{4} - \frac{1}{3} = \frac{1}{6}$

4. *Sample response*:

$\frac{2}{3} - \frac{1}{3}$; $1 - \frac{2}{3}$; $\frac{11}{12} - \frac{7}{12}$; $\frac{10}{12} - \frac{1}{2}$; $\frac{3}{4} - \frac{5}{12}$

5. *Sample responses*:

a) $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$ **b)** $\frac{2}{3} - \frac{2}{4} = \frac{1}{6}$

c) $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ **d)** $\frac{1}{3} - \frac{2}{12} = \frac{1}{6}$

6. No

7. a) Place counters in any 3 squares.
b) Place counters in any 2 squares.
c) Place counters in any 3 squares (or colour 3 squares in the first row), and place counters in any 2 squares (or colour 2 squares in the second row). Compare the number of counters (or the number of coloured squares) in the two rows.
Or
Put counters in any 3 squares and then take away 2 counters.

2.2.1 Naming Decimals as Fractions p. 51

1. a) $\frac{8}{10}$ **b)** $\frac{8}{100}$

c) $\frac{23}{10}$ **d)** $\frac{358}{10}$

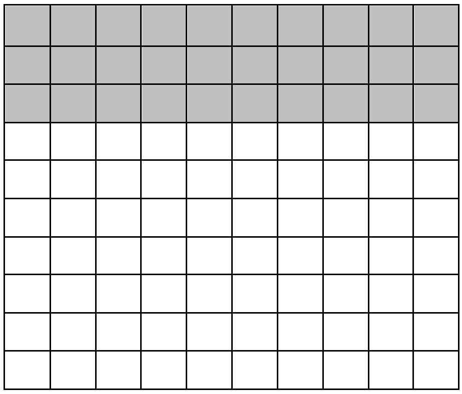
2. 1.2 is greater

3. C

4. Yes

5. *Sample response*: They are all tenths.

6. *Sample response*:



7. a) *Sample response*: 0.50
b) *Sample response*: 4, 5, 10, 20, 25, 50, 100

2.2.2 Naming Fractions as Decimals p. 54

1. a) 0.8 **b)** 0.08 **c)** 0.06 **d)** 0.5

2. a) $\frac{3}{10}$ **b)** $\frac{3}{4}$

3. *Sample response*:
$\frac{2}{5} = 0.2 + 0.2 = 0.4$; $\frac{3}{5} = 0.4 + 0.2 = 0.6$;
$\frac{4}{5} = 0.4 + 0.4 = 0.8$

5. a) 0.6 **b)** 0.1

6. *Sample responses*:
a) $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$

b) $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{9}$

UNIT 2 Revision pp. 55–56

1. a) $5\frac{2}{3}$ **b)** $2\frac{2}{5}$ **c)** $3\frac{2}{4}$

2. a) $\frac{5}{2}$ **b)** $\frac{21}{4}$ **c)** $\frac{17}{10}$

3. a) 8 **b)** *Sample response*: $3\frac{1}{4}$

4. a) $\frac{2}{3} > \frac{1}{6}$ **b)** $\frac{1}{2} < \frac{5}{6}$

5. a) $\frac{2}{9}$, $\frac{1}{3}$, $\frac{4}{9}$, $\frac{4}{7}$ **b)** $\frac{2}{5}$, $\frac{4}{9}$, $\frac{14}{20}$, $\frac{7}{9}$

6. a) $\frac{3}{8}$ **b)** $\frac{2}{7}$ **c)** $\frac{49}{50}$ **d)** $\frac{22}{100}$

7. Nima

8. a) $\frac{11}{12}$ **b)** $\frac{5}{12}$ **c)** $\frac{3}{4}$ **d)** $\frac{3}{4}$

9. a) $\frac{2}{5} + \frac{5}{10} = \frac{9}{10}$ **b)** $\frac{1}{12} + \frac{2}{3} = \frac{3}{4}$

10. *Sample response*:
$\frac{1}{4} + \frac{1}{2}$ $\frac{1}{12} + \frac{2}{3}$; $\frac{1}{3} + \frac{5}{12}$

11. a) $\frac{3}{4}$ **b)** $\frac{1}{12}$ **c)** $\frac{7}{12}$ **d)** $\frac{1}{12}$

12. a) $\frac{11}{12} - \frac{2}{3} = \frac{3}{12}$ or $\frac{1}{4}$
b) $\frac{4}{5} - \frac{3}{10} = \frac{5}{10}$ or $\frac{1}{2}$

13. a) $\frac{4}{10}$ (or $\frac{2}{5}$) **b)** $\frac{26}{100}$ (or $\frac{13}{50}$)
c) $\frac{28}{10}$ (or $\frac{14}{5}$) **d)** $\frac{175}{100}$ (or $\frac{7}{4}$)

14. *Sample response*:
• They are the same because both are between 3 and 4 and both have digits of 3 and 5.
• They are different because 3.5 > 3.05. 3.5 is 3 wholes and 5 tenths and 3.05 is 3 wholes and 5 hundredths.

15. A

16. a) 0.21 **b)** 0.6 **c)** 0.35 **d)** 0.8

17. *Sample response*:
$\frac{3}{100} = 0.03$, $\frac{3}{10} = 0.3$, $\frac{3}{5} = 0.6$, $\frac{3}{4} = 0.75$

UNIT 3 DECIMAL COMPUTATION pp. 57–86

Getting Started — Skills You Will Need pp. 57-58

1. *Sample responses*:
a) about 1600
b) about 7200
c) about 500
d) about 60

2. a) 1800 **b)** 4200
c) 600 **d)** 900

3. *Sample responses*: 47 × 22 = 800 + 140 + 80 + 14

a)

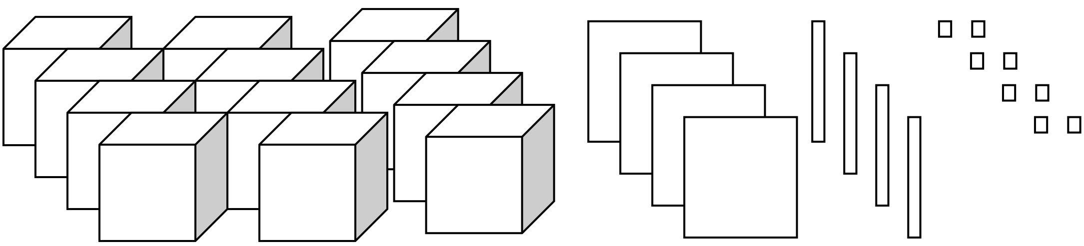
	40	+ 7
20	20 × 40 = **800**	20 × 7 = **140**
+ 2	2 × 40 = **80**	2 × 7 = **14**

3. b) 31 × 31 = 900 + 30 + 30 + 1

30 × 30 = **900**	30 × 1 = **30**
1 × 30 = **30**	1 × 1 = **1**

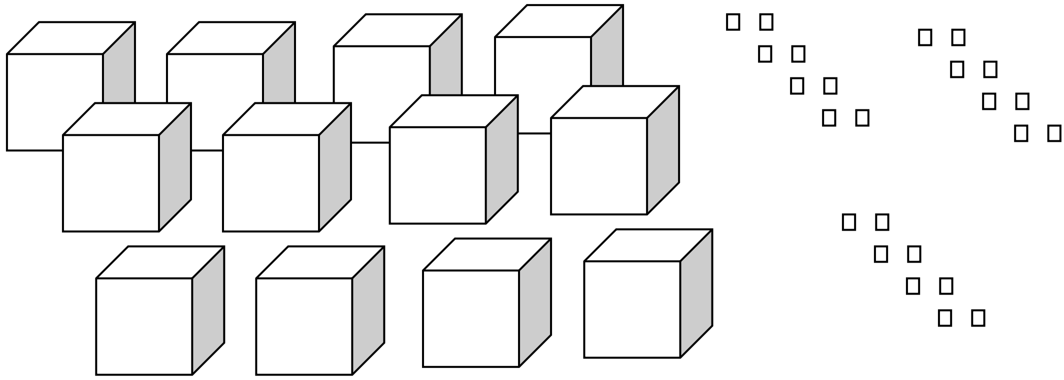
4. a) 1824
b) 1872
c) 3069
d) 1349

5. *Sample responses*:
a) 12,448



Getting Started — Skills You Will Need [Cont'd] pp. 57-58

b) 12,024



c) 411 R 1

Thousands	Hundreds	Tens	Ones
1	2	3	4
	12	3	4

÷ 3

	12 ÷ 3	3 ÷ 3	4 ÷ 3
	4	1	1 R 1

d) 530

Thousands	Hundreds	Tens	Ones
2	1	2	0
	20	12	0

÷ 4

	20 ÷ 4	12 ÷ 4	0 ÷ 4
	5	3	0

6. a) 18,672 **b)** 15,684 **c)** 44,650 **d)** 10,962

7. A and C

8. a) 54 **b)** 36 **c)** 420

9. a) 872 R 2 **b)** 995 **c)** 714 **d)** 465 R 4
e) 160 **f)** 182 **g)** 542 **h)** 63 R 44

10. a) Addition **b)** Multiplication **c)** Division

3.1.1 Estimating a Product p. 60

1. *Sample responses*:
a) about 400 weeks
b) about 3200 days
c) about 75,000 h

2. *Sample responses*:
a) about 40 m^2
b) about 72 m^2
c) about 9 m^2

3. *Sample response*:
3.9 × 4.9; 4.2 × 5.1; 3.8 × 5.1

4. *Sample responses*:
a) 1 and 2 **b)** 8 and 9

5. About Nu 680

6. A and C

7. *Sample response*:
If you want to know whether you have enough money to buy a number of items, you might estimate the total price, using a high estimate to be sure.

3.1.2 Multiplying a Decimal by a Whole Number p. 64

1. a) i) 37.134 **ii)** 119.45 **iii)** 751.8 **iv)** 419.36

2. *Sample responses*:

a)

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
	4 × 3	4 × 8	4 × 1	4 × 2	4 × 5
	12	32	4	8	20
	15	2	4	10	
1	5	2	5		

b)

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
	9 × 5	9 × 3	9 × 1	9 × 9	9 × 1
	45	27	9	81	9
	47	7	17	1	9
4	7	8	7	1	9

3. a) 60.8 **b)** 495.5

4. a) 1 **b)** 4 **c)** 0

5. a) 51.23 **b)** 304.1
c) 561.6 **d)** 1798

6. 56 kg

7. a) 128.5 s **b)** 2570 s

8. a) 32.92 m
b) 329.2 m
c) The second value is 10 times as great as t he first value.

9. *Sample response*:
I cut a 2.3 m rope into 5 equal pieces.
How long was each piece? (0.46 m)

3.1.3 Multiplying Decimals p. 68

2. b) i) 0.4 **ii)** 0.54 **iii)** 0.28 **iv)** 0.6

3. a) 0.09 **b)** 1.35 **c)** 11.89
d) 11.977 **e)** 17.578

5. a) 27.30 **b)** 39.44
c) 112.86 **d)** 313.088

7. *Sample response*: **87** × 0.**6**

8. 113.75 km

9. *Sample response*: about 1000 km

10. 16.32 m^2 **11.** 1.666 m

3.2.1 Estimating a Quotient p. 71

1. *Sample responses*:
a) about 25 kilometres in 1 h
b) about 30 kilometres in 1 h
c) about 25 kilometres in 1 h

2. *Sample responses*:
a) About 3 m
b) About 5 m
c) About 8 m

3. C

4. *Sample response*:
19.6 ÷ 4.9 or 20.5 ÷ 5.1 or 19.8 ÷ 5.1

5. *Sample responses*:
a) 10**0** ÷ 2**6** **b)** 10**0** ÷ 2**9**

7. B and D

3.2.2 Dividing a Decimal by a Whole Number — p. 74

1. a) i) 5.01 **ii)** 5.07 **iii)** 23.5 **iv)** 8.21

2. a) 0.412 **b)** 3.892 **c)** 5.67 **d)** 0.567

3. a) i) 3.56 **ii)** 5.03 **iii)** 5.46 **iv)** 5.41

4. a) 2.5 kg **b)** 0.5 kg

5. 1.5 m

6. 2.3 km^2

7. 205.25 g

8. *Sample response*:
4.2 kg of flour is equally divided into 4 containers. How much flour is in each container? (1.05 kg)

9. *Sample response*:
$4.2 \div 4 = 1.05$

3.2.4 Dividing Decimals — p. 78

1. a) 3 **b)** 2 **c)** 7 **d)** 3.75 (or $3\frac{3}{4}$)

2. a) 30 **b)** 12.25 **c)** 4.5 **d)** 12

3. a) 14 pieces (with some left over)
b) 57 **c)** 19
d) 9 pieces (with some left over)

4. 104.972 km in 1 h

5. 5 glasses

7. Yes

8. No

3.3.1 Order of Operations — p. 80

1. a) 9.1 **b)** 4.4 **c)** 4.4 **d)** 7

2. A and B

3. $(3.5 + 6.5) \div 0.2 + 4.2$; answer is 54.2.

4. A, C, and D

5. a) $1.2 \underline{\div} 3 \underline{\div} 2 = 0.2$
b) $1 \underline{\div} (3 \times 3 \underline{+} 1) = 0.1$

6. a) *Sample responses*:
i) 18.1; 13.4 **ii)** 0.23; 0.67
b) i) 20 **ii)** 1.99

3.3.2 Solving a Problem Using All Four Operations — p. 82

1. 152.8 cm

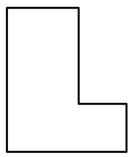
2. About 25 babies

3. a) 0.38 m^2 **b)** 1.52 m^2

4. About Nu 1.80

5. 7.8 km every day except the last day, when they travelled 4.8 km

6. *Sample response*:
A room is shaped like an L.
The small square on the end has an area of 3 m^2. The length and the width of the other part of the room are 3.5 m and 2.6 m.
What is the total area of the room? (12.1 m^2)



CONNECTIONS: Decimal Magic Squares — p. 83

1. 3.4 **2.** Yes; 34 **3.** Yes; 3.74

UNIT 3 Revision — pp. 84-85

1. *Sample responses*:
a) about 6 days
b) about 9000 min

2. *Sample responses*:
a) about 69 km **b)** about 250 km
c) about 18 km

3. B and C

4. a)

Tens	Ones	Tenths	Hundredths	Thousandths
	7	1	2	5
	5 × 7	5 × 1	5 × 2	5 × 5
	35	5	10	25
3	5	6	2	5

b)

Tens	Ones	Tenths	Hundredths	Thousandths
1	2	2	1	9
8 × 1	8 × 2	8 × 2	8 × 1	8 × 9
8	16	16	8	72
9	7	7	5	2

5. a) 35.6 **b)** 1720.4 **c)** 1119 **d)** 4872

6. 3.125 kg

7. a) i) 0.28 **ii)** 0.16 **iii)** 1.19 **iv)** 6.72

8. a) 38.22 **b)** 92.5 **c)** 34.92 **d)** 283.92

9. 1333.2 km

10. *Sample response*: **79** × 0.**4**

11. *Sample responses*:
a) about 32 kilometres in 1 h
b) about 28 kilometres in 1 h
c) about 25 32 kilometres in 1 h

12. B and D

13. a) 0.32 **b)** 1.426 **c)** 0.237 **d)** 0.491

14. a) i) 1.44 **ii)** 6.048 **iii)** 2.53 **iv)** 6.98

15. 123.4 g

17. a) 190 **b)** 9.1 **c)** 15

18. About 7

19. *Sample response*:
42 ÷ 7, 420 ÷ 70, 4200 ÷ 700

20. a) 11.5 **b)** 21 **c)** 1.77

21. B and C

22. a) (13.5 + 1.5) × 2 = 30
b) (10 + 2) × 1.2 − 9 = 5.4

23. 8.8 km

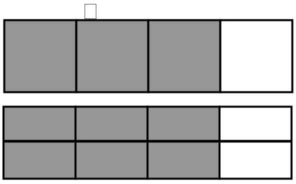
24. 0.76 m

UNIT 4 RATIO, RATE, AND PERCENT pp. 87–112

Getting Started — Skills You Will Need pp. 87-88

1. *Sample responses (equivalent fractions might be used.)*:
a) $\frac{6}{15}$ **b)** $\frac{16}{24}$ **c)** $\frac{6}{32}$ **d)** $\frac{12}{36}$

2. *Sample response*:



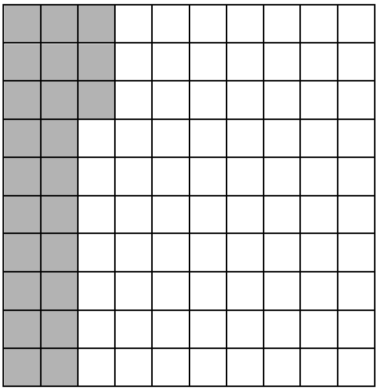
3. a) 6 **b)** 20 **c)** 28 **d)** 45

4. a) 0.46 **b)** 0.71

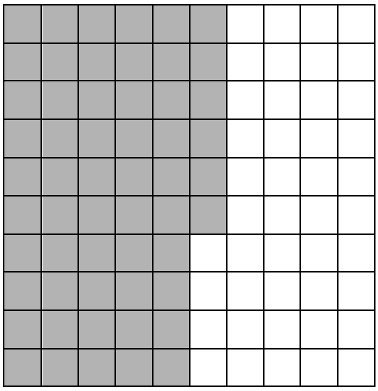
Getting Started — Skills You Will Need [Cont'd] pp. 87-88

5. *Sample responses*:

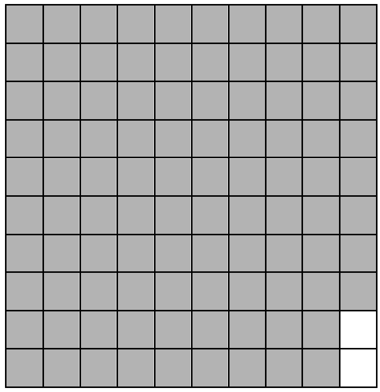
a)



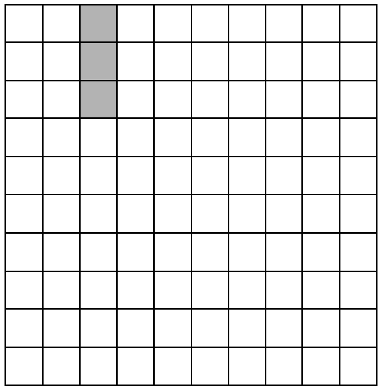
b)



5. c)



d)



6. 0.43, 0.45, 0.58, 0.85

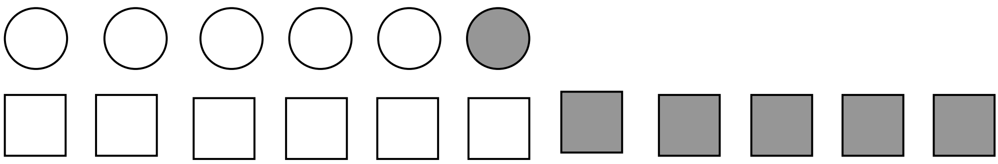
7. a) 0.7 **b)** 0.5
c) 0.08 **d)** 0.12

4.1.1 Introducing Ratios p. 91

1. a) 3 : 4 **b)** 3 : 7

2. a) Grey to white
b) Striped to grey
c) White to total
d) Striped to white

3. *Sample response*:

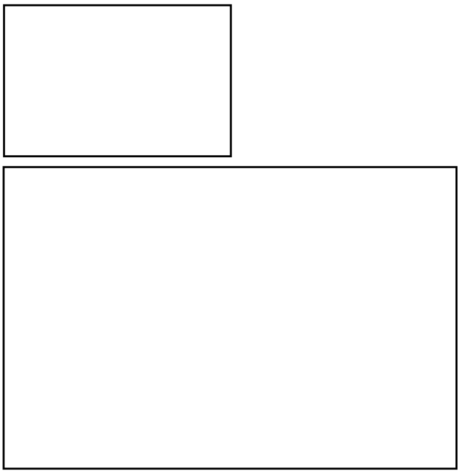


4. a) Part-to-part **b)** 39

5. D

6. *Sample responses*:

a)



b) 1 : 2 **c)** 1 : 4

7. a) 2 : 1 **b)** 1 : 2

8. *Sample response*:

Duptho to family = $\frac{1}{6}$

Adults to children = $\frac{2}{4}$

Adults to family = $\frac{2}{6}$

Duptho to brother = 1 : 1
Duptho to sisters = 1 : 2
Duptho to children = 1 : 4

9. *Sample responses*:
a) 22 : 18 to compare boys to girls
1 : 40 to compare teachers to students
6 : 1 to compare windows to doors
b) 7 : 17 to describe the number of hours I am in school compared to the hours I am not on a school day
6 : 7 to compare school days to total days in each week
3 : 1 to compare the other people in the family to me

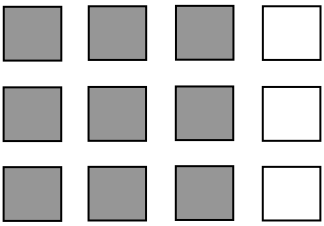
10. Yes

4.1.2 Equivalent Ratios p. 94

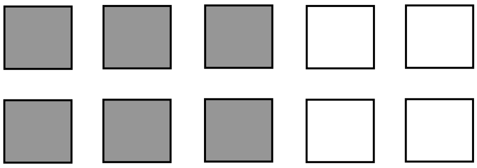
1. B and D

2. a) 2 **b)** 16
c) 8 **d)** *Sample response*: 7, 4

3. a) Yes
b) *Sample response*:



4. *Sample response*:

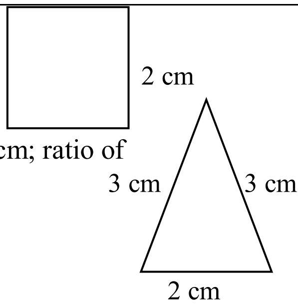


5. No

6. *Sample responses*:
a) 1 : 1000
b) 1 : 1000

7. *Sample response*:
A square with a 2 cm side and perimeter of 8 cm; ratio of side length to perimeter is 2 : 8 = 1 : 4.

A triangle with 2 cm, 3 cm, and 3 cm sides and perimeter of 8 cm; ratio of base to perimeter is
2 : 8 = 1 : 4.



8. No

4.1.3 Comparing Ratios p. 97

1. a) B; 22 : 17 **b)** A; 18 : 22

2. A **3.** Triangle

4. Both groups have the same ratio of sports players

5. Package B **6.** The second music club

7. *Sample response*:
To find out whether something will taste the way you expect based on the recipe if you change the amounts of some of the ingredients.

4.1.5 Introducing Rates p. 102

1. *Sample responses*:
a) 70 km/1.5 h
b) Nu 170/2 kg
c) Nu 20/12 bananas

2. B and D

3. a) 150 **b)** 25 **c)** 2

4. a)

Large dog	100 beats/1 min
Lion	40 beats/1 min
Elephant	35 beats/1 min
Chicken	240 beats/1 min

b) Elephant, lion, dog, chicken

5. Karma

6. b) *Sample response*:
36 km/h; 72 km/2 h; 9 km/15 min

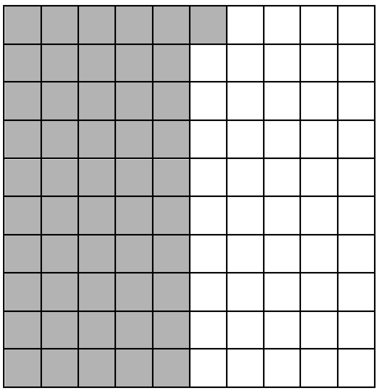
7. *Sample response*:
Nu 9000/2 months
Nu 27,000/6 months
Nu 54,000/year

8. *Sample response*:
1 year/900 million people
300 million people/4 months
450 million people/6 months

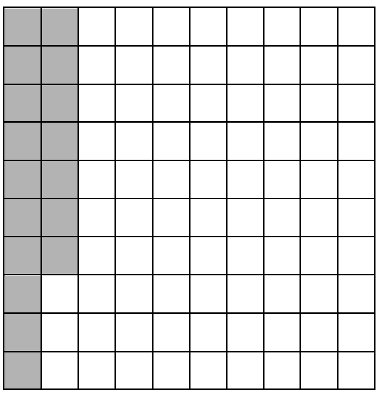
4.2.1 Introducing Percent p. 105

1. a) 35%; 65% **b)** 32%; 68%

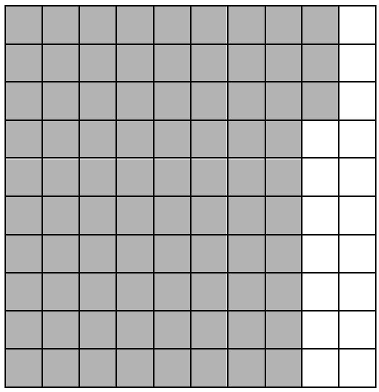
2. a)



b)



2. c)

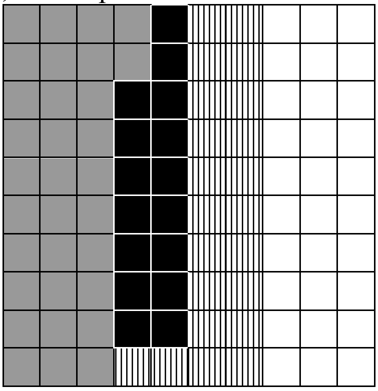


3.

	Ratio	Fraction	Percent
12 to 100	12 : 100	$\frac{12}{100}$	12%
$\frac{91}{100}$	91 : 100	$\frac{91}{100}$	91%
0.01	1 : 100	$\frac{1}{100}$	1%
50 out of 100	50 : 100	$\frac{50}{100}$	50%

Reprint 2023

4. *Sample response*:
Grey for Pacific Ocean, black for Atlantic Ocean, and striped for other water.



5. a) 100% **b)** *Sample response*: 99%
c) 0% **d)** *Sample response*: 2%

6. *Sample responses*:
a) 50% **b)** 70%
c) 90% **d)** 90%

7. 1 out of 10, 16%, 2 out of 10, 22%

8. *Sample response*:
Marks on tests; in the newspaper when it talked about the results of a survey; in a bank to show interest earned or charged.

4.2.2 Representing a Percent in Different Ways p. 107

1. a) $\frac{33}{100}$ and 0.33 **b)** $\frac{80}{100}$ and 0.80
c) $\frac{15}{100}$ and 0.15 **d)** $\frac{68}{100}$ and 0.68

2. a) 39% **b)** 18%

3. $\frac{91}{100}$ and 0.91

4. More **5.** 20%

6. a) 60% **7.** 21 girls
b) 5 parts

CONNECTIONS: Map Scales p. 110

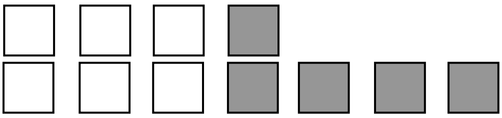
1. a) 6 km **b)** 3 km

2. 6 : 30,000,000

UNIT 4 Revision pp. 111–112

1. a) *Sample response*: Grey to white
b) *Sample response*: Striped to grey
c) *Sample response*: Grey to all squares
d) White to all squares

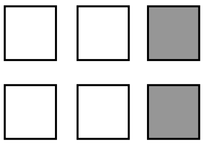
2. *Sample response*:



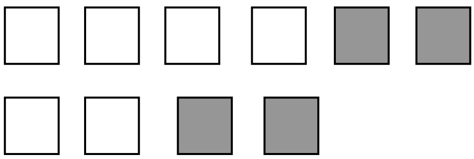
3. *Sample response*:
5 : 4 to compare rolls of 1 to rolls of 2.
5 : 30 to compare rolls of 1 to total rolls.
8 : 5 to compare rolls of 5 to rolls of 4.
$\frac{11}{30}$ of the rolls were even numbers.

4. a) 3 **b)** 6 **c)** 12
d) *Sample response*: 22 and 6

5. *Sample responses*:
a) 2 columns out of a total of 3 columns are white; 4 squares out of a total of 6 squares are white.



b) In the first row, more than half are white and the white : total ratio is 4 : 6.
In the second row, only half are white and the white : total ratio is 4 – 2 : 6 – 2 = 2 : 4, so it cannot be the same comparison as the first row.



6. a) 21 : 1

b)

Milk	21	42	63	84
Butter	1	2	3	4

7. Can B

8. The group of 30 teachers.

9. *Sample response*:
20 cm, 20 cm, and 8 cm or
5 cm, 5 cm, and 2 cm.

10. 6 chocolate bars for Nu 450

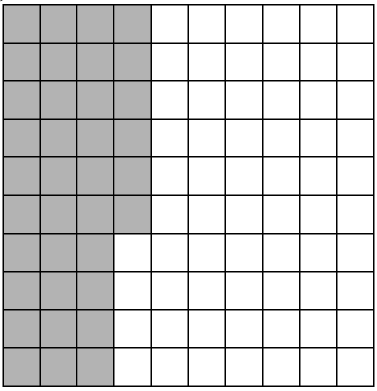
11. 122.5 km **12.** 30 chances

13. C

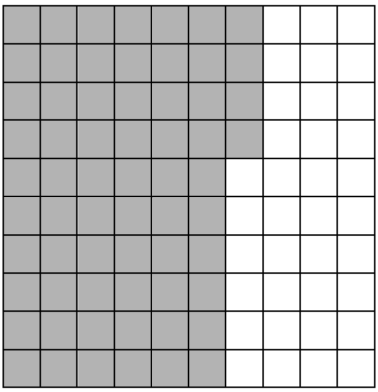
14. *Sample response*: 10 boxes for Nu 800

15. a) 25%; 75% **b)** 60%; 40%

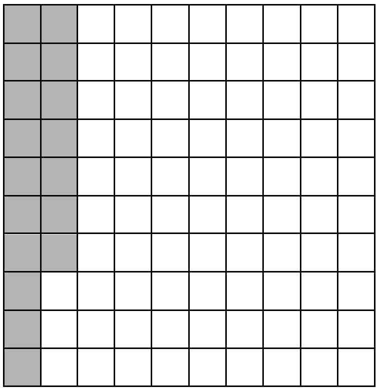
16. a)



b)



16. c)



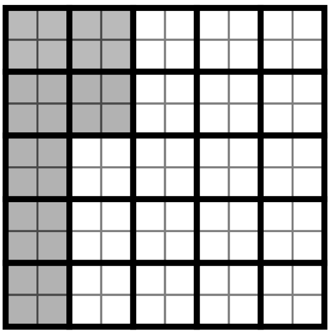
17. a) A is reasonable;
b) *Sample response*:
100% of my sisters are girls.

18.

	Ratio	**Fraction**	**Percent**
35 to 100	35 : 100	$\frac{35}{100}$	35%
$\frac{65}{100}$	65 : 100	$\frac{65}{100}$	65%
0.60	60 : 100	$\frac{60}{100}$	60%
82 out of 100	82 : 100	$\frac{82}{100}$	82%

19. One of the better teams

20. a) $\frac{28}{100}$
b) 28%



Getting Started — Skills You Will Need p. 107

1. a) Perimeter = 18 cm; Area = 18 cm^2
b) Perimeter = 140 cm; Area = 1125 cm^2

2. a) 100 **b)** 1000 **c)** 10 **d)** 1000

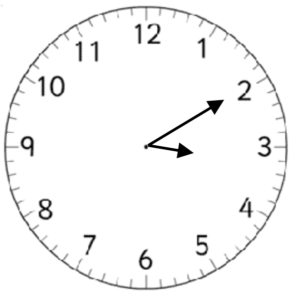
3. *Sample responses (assuming no hidden cubes)*:
a) 8 cubic units **b)** 14 cubic units

4. *Sample response*:
$l = 2$, $w = 2$, $h = 9$ or $l = 9$, $w = 4$, $h = 1$

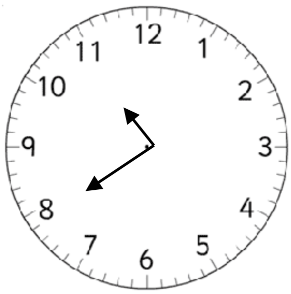
5. 325 mL, 0.45 L, 2.1 L, 2300 mL

6. *Sample responses*:
a) A drinking glass **b)** A pencil
c) A large truck **d)** A pail

7. a)



b)

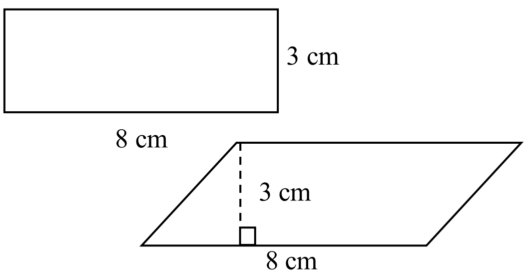


8. a) 7 h and 40 min **b)** 1 h and 49 min **c)** 7 h and 38 min

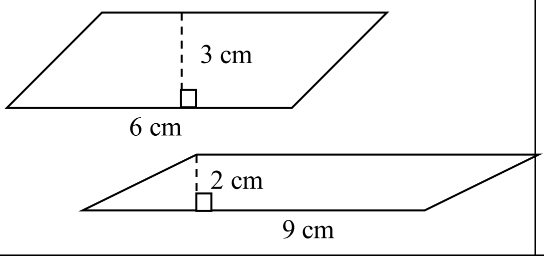
5.1.1 Area of a Parallelogram p. 118

1. a) 6 cm^2 **b)** 1000 cm^2 **c)** 3000 cm^2

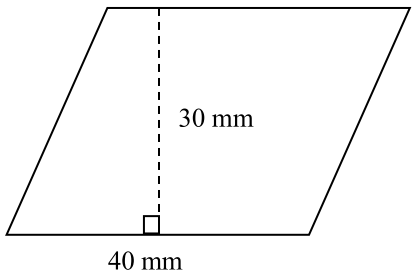
2. a) 8 cm
b) *Sample response*:



3. *Sample response*:



4. a)

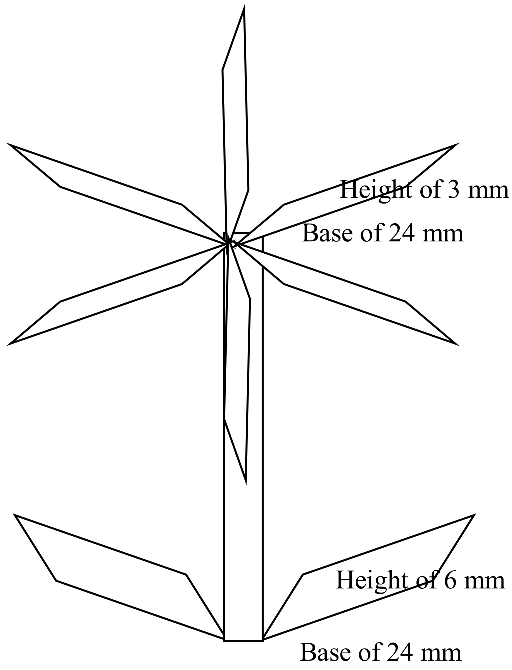


b) 1200 mm^2
c) Base is 33 mm, height is 37 mm
d) 1221 mm^2
e) *Sample response*:
They are almost the same.

5.1.1 Area of a Parallelogram [Continued] p. 118

5. *Sample response*

a)



CONNECTIONS: Changing a Parallelogram p. 119

1. a) Base = 15 cm; height = 8 cm
b) Perimeter = 46 cm; Area = 120 cm^2

2. *Sample responses*:
a) Base = 15 cm; height = 6 cm
b) Perimeter = 46 cm, Area = 90 cm^2

3. *Sample responses*:
a) Base = 15 cm; height = 5 cm

3. b) Perimeter = 46 cm, Area = 75 cm^2

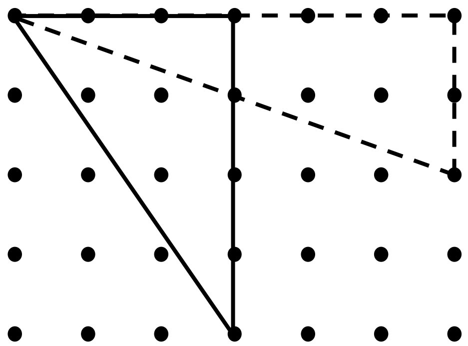
4. a) Base and perimeter
b) Height and area
c) The rectangle; the really slanted parallelogram

5. It would become smaller and smaller.

5.1.2 Area of a Triangle p. 123

1. a) 3 cm^2 **b)** 8.1 cm^2 **c)** 7 cm^2

2. *Sample response*:



3. 0.8 cm

4. No

5. 56 cm^2

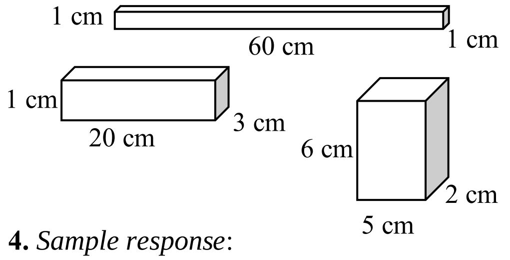
6. m = 2.4 cm

5.2.1 Volume of a Rectangular Prism p. 131

1. a) 180 cm^3 **b)** 1440 cm^3 **c)** 9600 cm^3

2. a) 12 cm **b)** 6 cm **c)** 2 cm

3. *Sample response*:



4. *Sample response*:
2 m by 6 m by 10 m
6 m by 4 m by 5 m
1 m by 1 m by 120 m

5. *Sample response*:
1 cm by 1 cm by 80 cm

6. *Sample response*:
4 cm by 3 cm by 3 cm

7. 80 cm^3

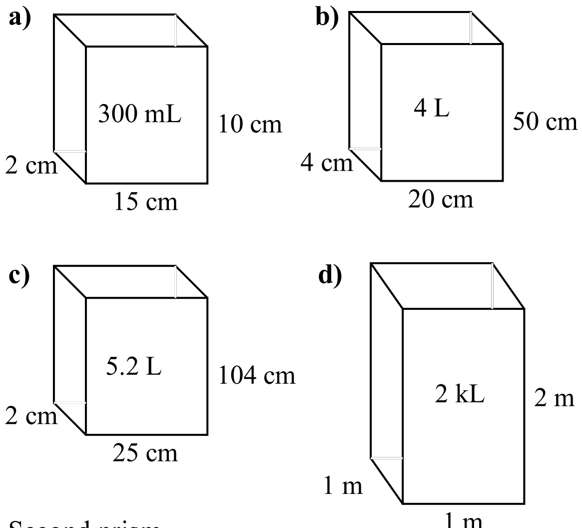
8. *Sample response*:
25 cm by 70 cm by 30 cm

9. a) 8 cm
b) i) 4800 cm^3
ii) 9600 cm^3
iii) 2400 cm^3

5.2.2 Relating Volume to Capacity pp. 134–135

1. a) About 850 mL **b)** About 2205mL

2. *Sample responses*:
First prism



Second prism
a) 2 cm by 30 cm by 5 cm
b) 16 cm by 25 cm by 10 cm
c) 2 cm by 50 cm by 102 cm
d) 2 m by 0.5 m by 2 m

3. a) 177 cm^3 **b)** 650 cm^3

4. A

5. a) The area of the base of the tall box is half the area of the other prism base.
b) The product of the length and width of the base of the tall box is half the product of the length and width of the shorter box.

6. a) 10 cm
b) 15 cm

7. 10 cm, 11 cm, and 12 cm

8. Each square was 3 cm by 3 cm.

5.3.1 The Tonne p. 137

1. A. 2 t **B.** 60 g **C.** 4 kg

D. 12 kg **E.** 12 t **F.** 500 g

2. a) 350 g, 3.5 kg, 1.2 t, 1500 kg, 1.82 t
b) 23 kg, 0.23 t, 2.03 t, 2033 kg, 2300 kg

3. *Sample response*: 2299 kg

4. 38,000,000 kg

5. 0.909 t

6. 3000 bags

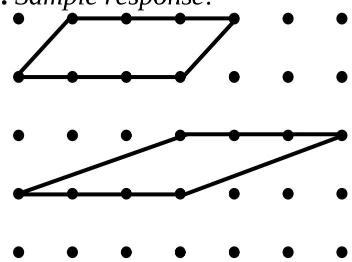
UNIT 5 Revision pp. 138–139

1. a) 900 cm^2 **b)** 480 cm^2

2. The rectangle

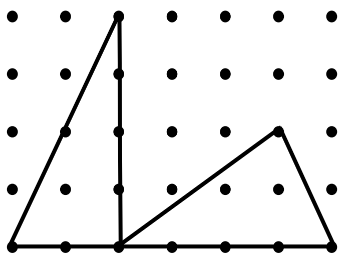
3. a) 6 cm **b)** 4 cm

4. *Sample response*:



5. a) 575 cm^2 **b)** 900 cm^2

6. *Sample response*:



7. 24 m

8. 3300 cm^2

9. a) Parallelogram B has twice the area of Parallelogram A.
b) Triangle A has one fourth the area of Triangle B.

10. The parallelogram has four times the area of the triangle.

11. a) 0.1 m^3 or 100,000 cm^3
b) 0.6 m^3 or 600,000 cm^3

12. *Sample response*:
10 cm by 10 cm by 2 cm or
5 cm by 20 cm by 2 cm

13. 600 cm or 6 m

14. 11,040 cm^3

15. about 30 L

16. *Sample response*:
About 25 cm by 10 cm by 10 cm or
25 cm by 50 cm by 2 cm

17. 250 cm^3

18. About 8 cm

19. a) 23,000
b) 3400
c) 1.520

20. 2.5 t

Getting Started — Skills You Will Need pp. 141-142

1. a) B **b)** C
c) D or B
d) A or B (A rhombus has congruent adjacent sides so it may also be classed as a kite.)

2. A, B, and C

3. 3 units left, 5 units up

4. a) Right **b)** Acute **c)** Obtuse

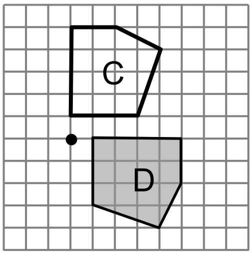
5. a) Scalene **b)** Equilateral **c)** Isosceles

6. A and E are congruent; D, B, and F are congruent.

7. a) Cylinder
b) *Sample response*: Pentagon-based pyramid
c) *Sample response*: Octagon-based prism

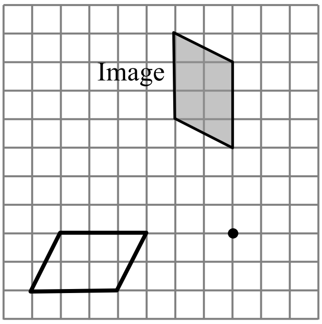
6.1.1 Rotations p. 147

1. a), b)

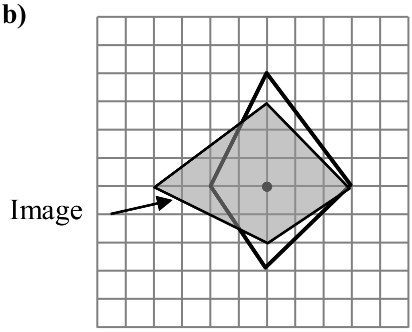


c) A $\frac{1}{4}$ turn cw around the turn centre or a $\frac{3}{4}$ turn ccw around the turn centre.

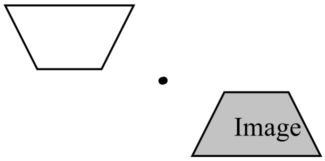
2. a)



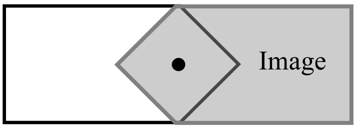
2. b)



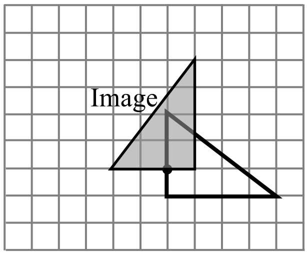
3. a)



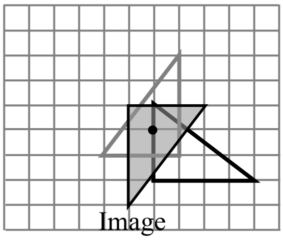
b)



4 a), b)



c) *Sample response*:



5. *Sample response*:
Similar: Each image vertex is always the same distance to the turn centre as the original vertex.
Different: When the turn centre is inside the shape, the image and the original shape overlap. When it is outside the shape, they might not overlap

6.1.2 Rotational Symmetry p. 150

1. *Sample response*:

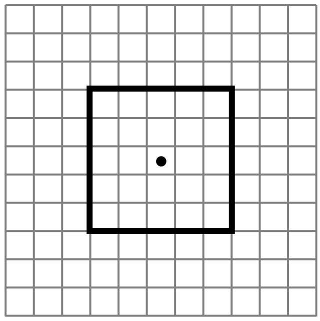
a) Yes **b)** No
c) Yes **d)** Yes
e) Yes **f)** Yes
g) No **h)** Yes

2. a) 8 **b)** 1 **c)** 4 **d)** 5
e) 5 **f)** 4 **g)** 1 **h)** 2

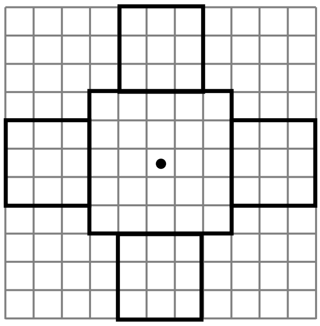
3. a) The design in the middle of the coin has turn symmetry of order 4; the turn centre is in the middle of the design.
b) Turn symmetry of order 2; the turn centre is in the middle of the centre flower.

4. *Sample response*:
- The rectangular chalkboard has turn symmetry of order 2.
- The design on the window frame has turn symmetry of order 4.
- The square desktop has turn symmetry of order 4.

5. a) Turn symmetry of order 4, turn centre is marked.



b) Turn symmetry of order 4, turn centre is marked.

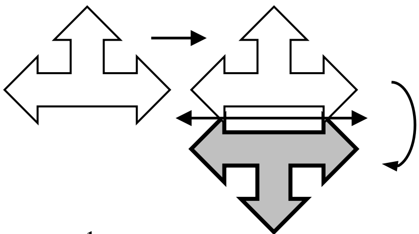


c) No

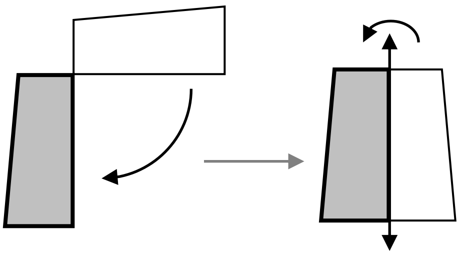
6. Shape A

6.1.3 Combining Transformations p. 154

1. *Sample responses*:
a) Translate right to line up with the grey shape then reflect across a horizontal line between the shapes.

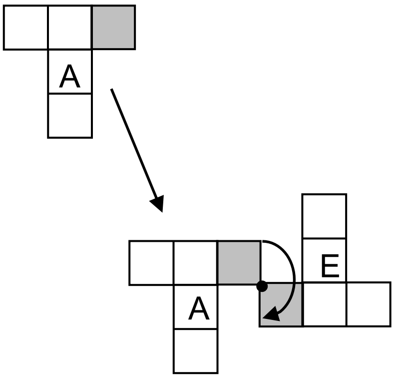


b) Rotate a $\frac{1}{4}$ turn cw around the vertex where the shapes touch, then reflect across the left side.



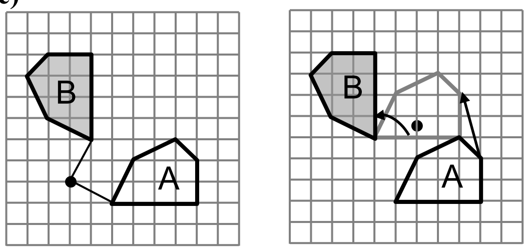
1. c) Rotate a $\frac{1}{2}$ turn around a point in the centre, then translate it right.

2. a) C is a translation image of A.
E is a rotation image of A.
b) *Sample response*:
• Reflect A across a horizontal line and then translate it to shape B.
• Reflect A across a vertical line and then translate it to shape D.
• Translate A so that the grey square touches E and then rotate it a $\frac{1}{2}$ turn around the point where they touch.

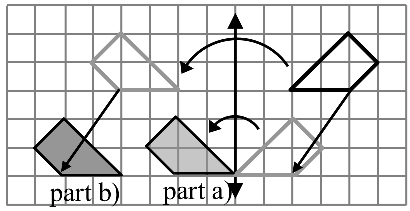


3. *Sample responses*:
a) Rotate A a $\frac{1}{4}$ turn ccw around a point below Shape B and to the left of Shape A.

3. b) Translate Shape A so the lower left vertex of Shape A touches the lower right vertex of Shape B, then rotate Shape A a $\frac{1}{4}$ turn cw around that vertex.
c)



4. a) The final image will be to the left of the line. It will be pointing in the opposite direction with its right vertex on the line.
b) No
c)



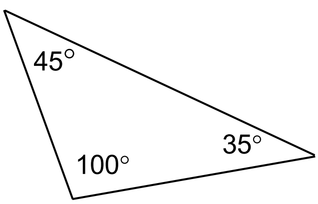
5. *Sample response*:
Tshering might have done two reflections.

6.2.1 Measuring Angles pp. 162-163

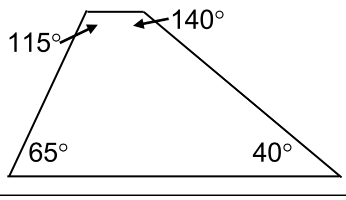
1. a) 120° **b)** 140°
c) 80° **d)** 25°

2. *Sample responses*:
a) About half of 90°; 45°
b) Just under 90°; 85°
c) Greater than 90°; 125°
d) Less than part b); 75°

3. a)

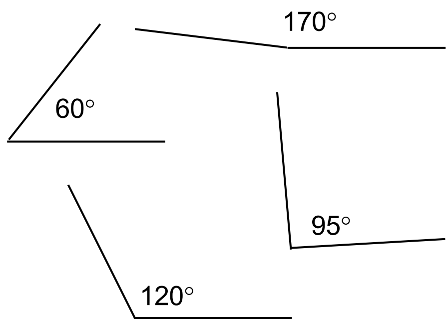


b)



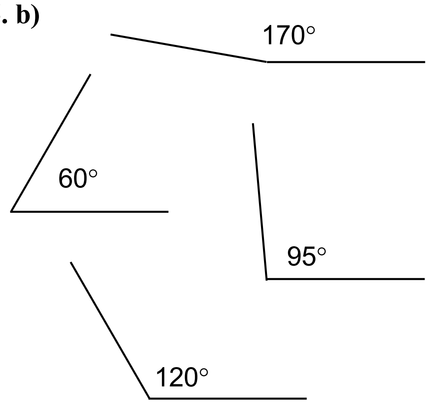
4. a) and b) *Sample responses*:
• Corner of window: estimate 90°, actual 90°.
• Angle between the bottom of the flag and the flagpole in a photo: estimate 45°, actual 40°.
• Angle that the bottom of my book makes with
the edge of the table: estimate 30°, actual 38°.

5. a) *Sample response*:



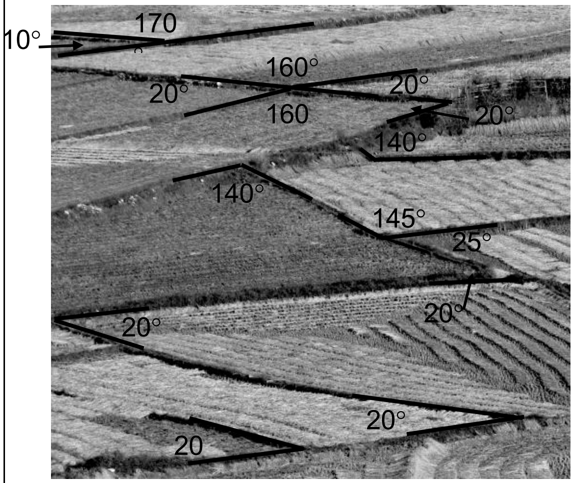
6.2.1 Measuring Angles [Continued] pp. 162-163

5. b)



c) *Sample response*:
Within a few degrees; I have to measure some of
the estimated angles to tell that they are not the same as the measured angles.

6. a) and b) *Measurements are estimates*:



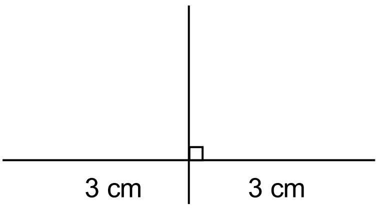
6.2.2 Bisectors p. 167

1. A and C

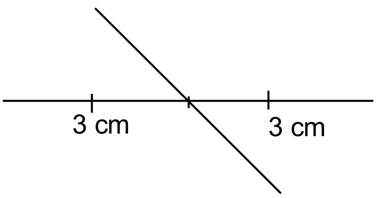
2. A, B, and C

3. B

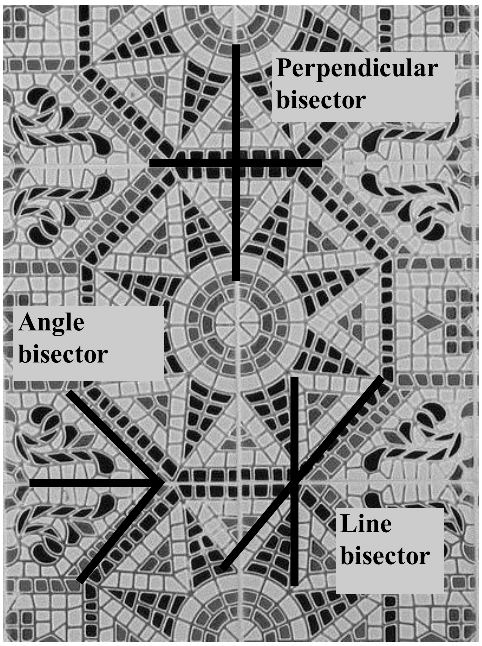
4. a)



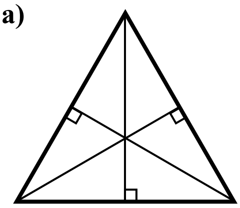
b) *Sample response*:



5. *Sample responses*:



6. a)



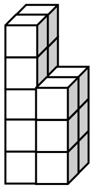
b) Yes

7. *Sample response*:
Same: They are the same because they both divide something in half.
Different: They are different because perpendicular bisectors divide both an angle and the line segment in half but other angle bisectors only divide the angle in half.

6.3.3 Interpreting Orthographic Drawings p. 176

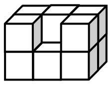
1. a) Back **b)** Front **c)** Right
d) Left **e)** Top

2. a)



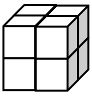
Front

b)



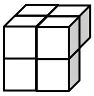
Front

3. a)



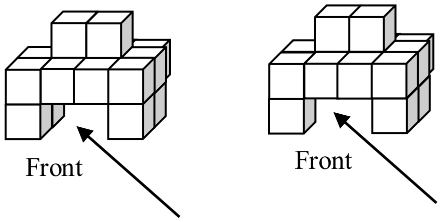
Front

b)



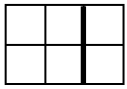
Front

4.



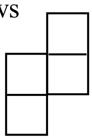
One extra cube in the left structure.

5. a) Add a heavy vertical line to show the change in depth.
b) The heavy line should be vertical, not horizontal.
c) This is a good drawing.

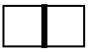


6.3.4 Creating Orthographic Drawings p. 179

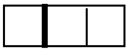
1. a) Views



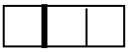
Top



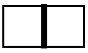
Front



Left

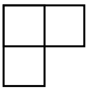


Right

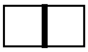


Back

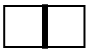
b) Views



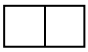
Top



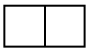
Front



Right

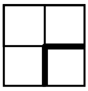


Left

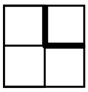


Back

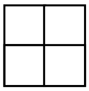
c) Views



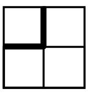
Top



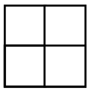
Front



Left

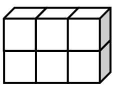


Right



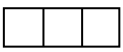
Back

2. *Sample responses*:
a) Structure

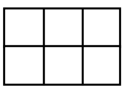


Front

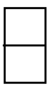
Views



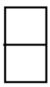
Top



Front



Left

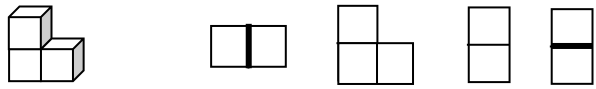


Right

6.3.4 Creating Orthographic Drawings {Cont'd} p. 179

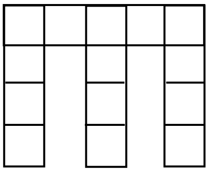
2. b)

Structure Views



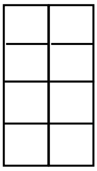
Front Top Front Left Right

3. a)



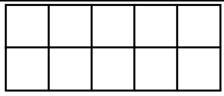
Front and back view

b)



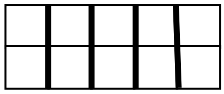
Left and right view

c)



Top view

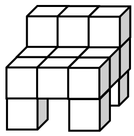
d)



Bottom view

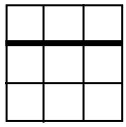
4. *Sample responses*:

a)



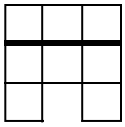
Front

b)



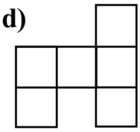
Top view

c)



Front view

d)

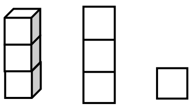


Right view

6.3.4 Creating Orthographic Drawings [Continued] p. 179

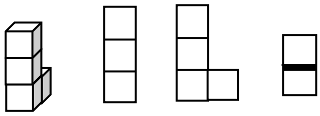
5. *Sample response*:

Structure Views



Front Front and right Top

Structure Views

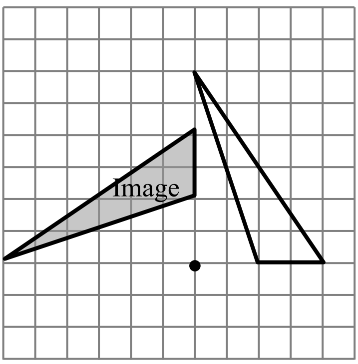


Front Front Right Top

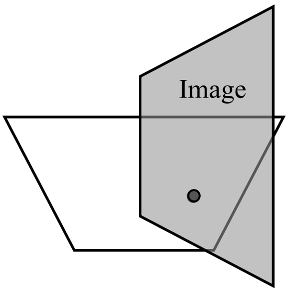
6. Yes.

UNIT 6 Revision pp. 180-182

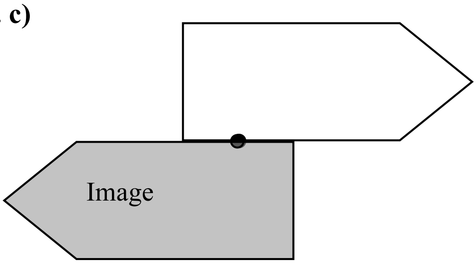
1. a)



1. b)



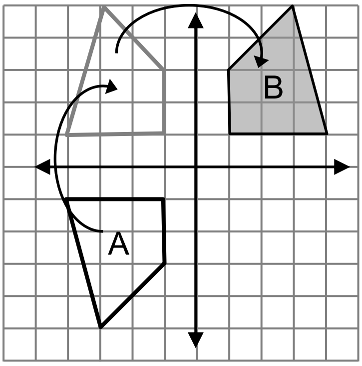
1. c)



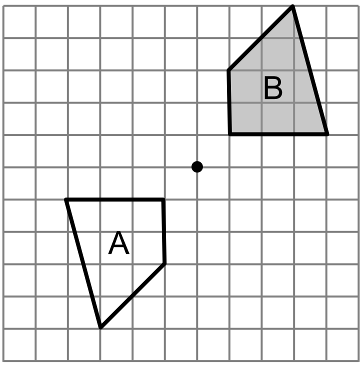
2. a) 2 **b)** 1 **c)** 3 **d)** 4

3. Turn symmetry of order 6

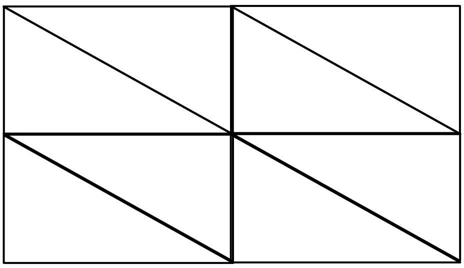
4. a) *Sample response*: Reflect horizontally and then vertically across the lines shown.



b) Rotate a $\frac{1}{2}$ turn cw or ccw around the point shown.

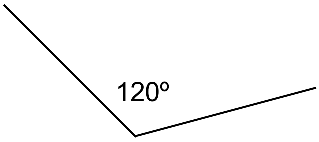


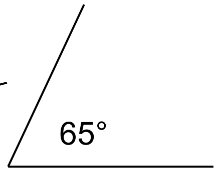
5. Yes; *Sample response*:



6. a) 55° **b)** 110°

7. a) **b)**

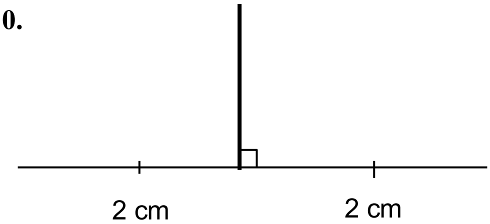




8. A, B, and D

9. B, C, and D

10.

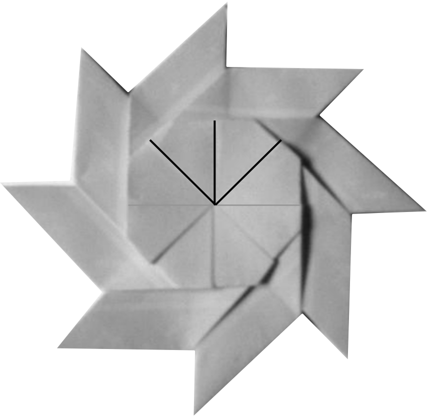


11. Alike:
• Both divide the shape into two pairs of congruent triangles.
Different:
• The triangles are right triangles for a kite; there is one pair of obtuse and one pair of acute triangles for a rectangle.
• Diagonals of rectangle bisect each other; in a kite, only one diagonal is bisected.

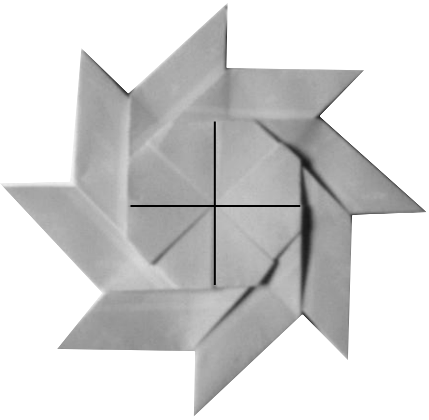
12. a) One pair of congruent triangles and one pair not congruent
b) Two pair of congruent triangles — one pair acute scalene, one pair obtuse scalene
c) Four congruent right triangles

UNIT 6 Revision [Continued] — pp. 180-182

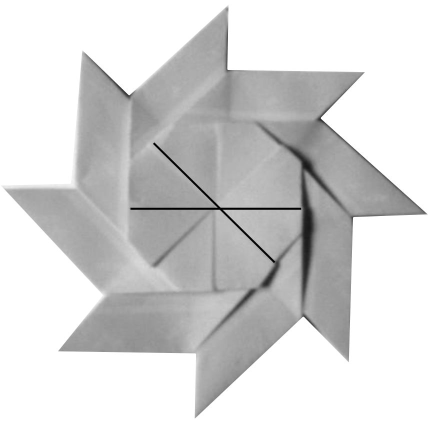
13. *Sample response*:
a) Angle bisector



b) Perpendicular bisectors

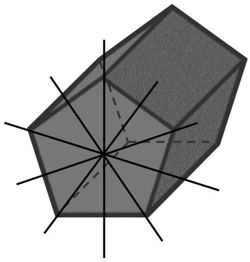


c) Non-perpendicular bisectors

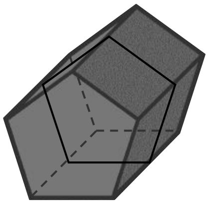


d) Rotational symmetry of order 8 with the turn centre where the bisectors intersect.

14.

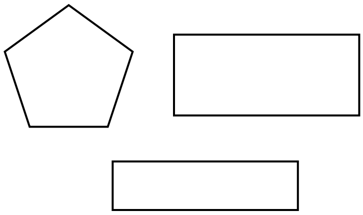


5 perpendicular
to the base



1 parallel
to the base

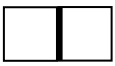
14. b) *Sample response*:



15. a) The triangular cross-sections are perpendicular to the base and run through the top vertex.
• The trapezoid cross-sections are perpendicular to the base but do not go through the top vertex.
• The cross-sections parallel to the base are all squares of different sizes.
b) Only the triangular cross-sections are also planes of symmetry.

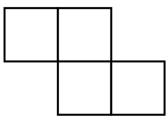
16. A

17. B



Left view

C

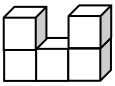


Top view

18. A

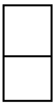
19. *Sample responses*:

a)

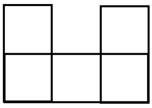


Front

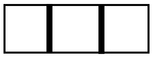
19. b)



Left and right view

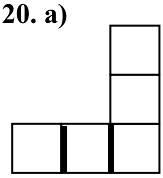


Front and back view

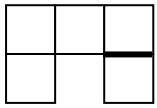


Top view

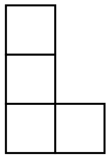
20. a)



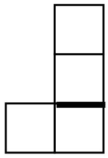
Front view



Top view

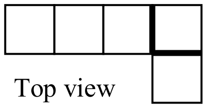


Right view

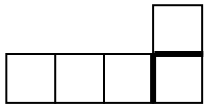


Left view

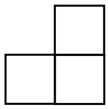
b)



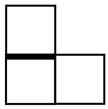
Top view



Front view



Right view

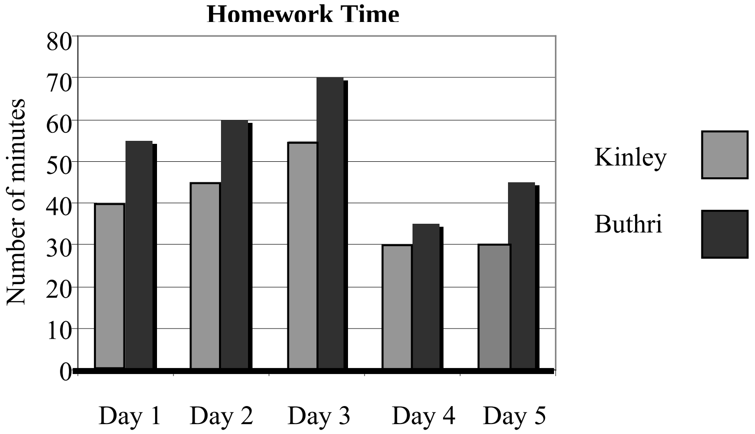


Left view

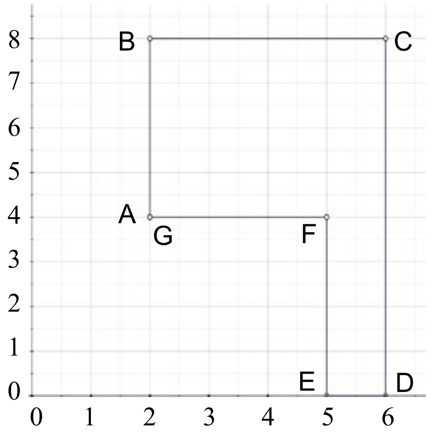
Getting Started — Skills You Will Need

p 183

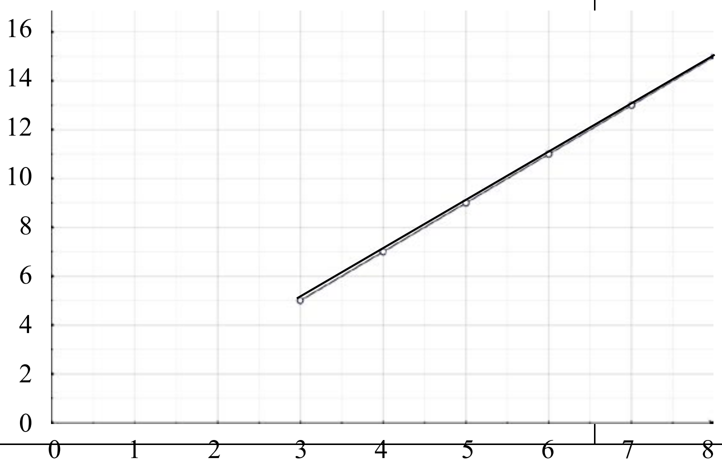
1.



2. A hexagon (with all right angles)



3. a) The *y*-coordinate is 1 less than double the *x*-coordinate.
b) *Sample response*: (7, 13) and (8, 15)
c) They form a line.



4. a) 10
b) i) The mean increases
ii) The mean decreases
iii) The mean decreases

5. a) $\frac{1}{2}$

b) $\frac{1}{6}$

c) $\frac{4}{6}$

d) $\frac{3}{5}$

7.1.1 Choosing a Sample p. 185

1. *Sample response*:

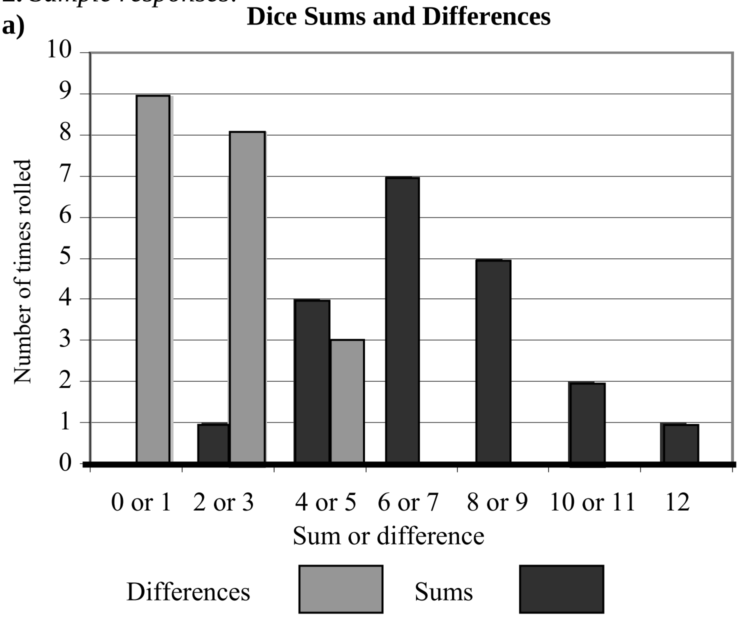
a) Biased **b)** Might not be biased **c)** Biased

7.2.1 Double Bar Graphs with Intervals p. 189

1. *Sample response*:
It is more likely to get a high number using the sum rather than the difference.

2. *Sample responses*:

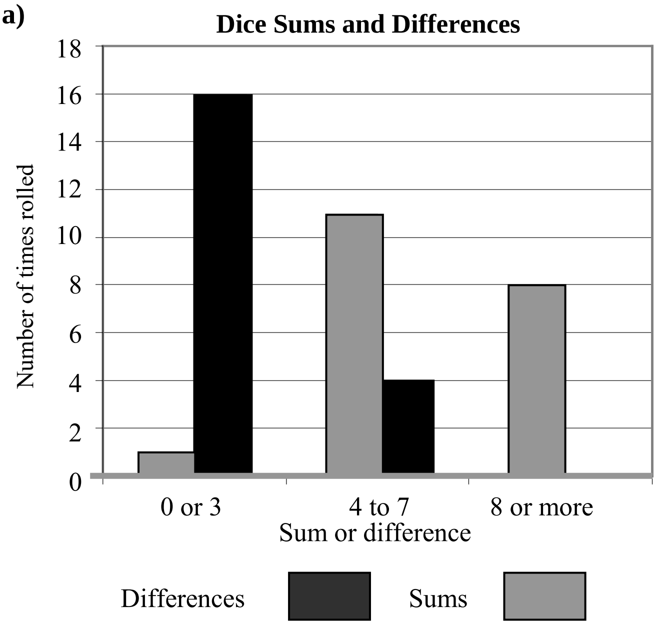
a)



b) My graph had a shape similar to the original graph and the same things are true about it. But the actual values were not always the same. For example, I only got a difference of 0 or 1 nine times, not ten times.

3. *Sample responses*:

a)

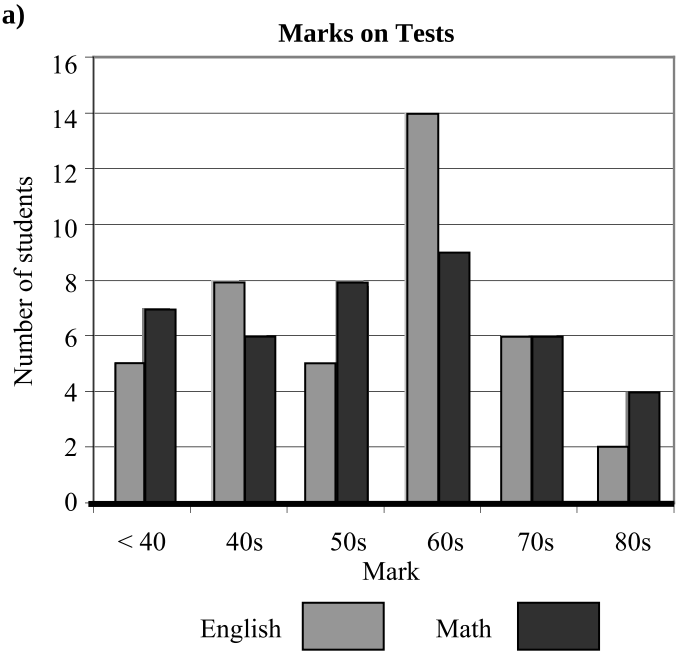


b) It shows some different things.

7.2.1 Double Bar Graphs with Intervals [Continued] p. 189

4. *Sample responses*:

a)

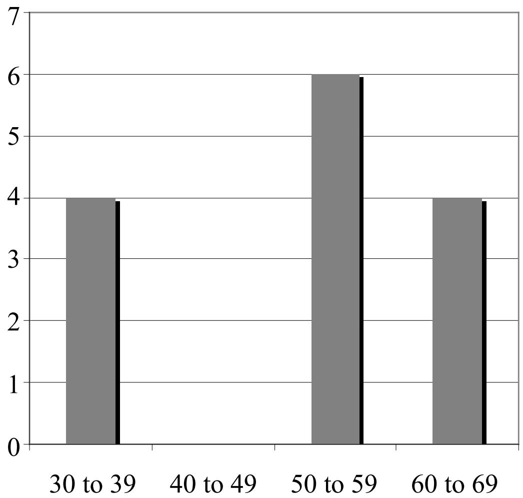


b) The graph shows that for both subjects, many students received marks in the 60s.
It also shows that in the 80s there were more math marks than English marks.

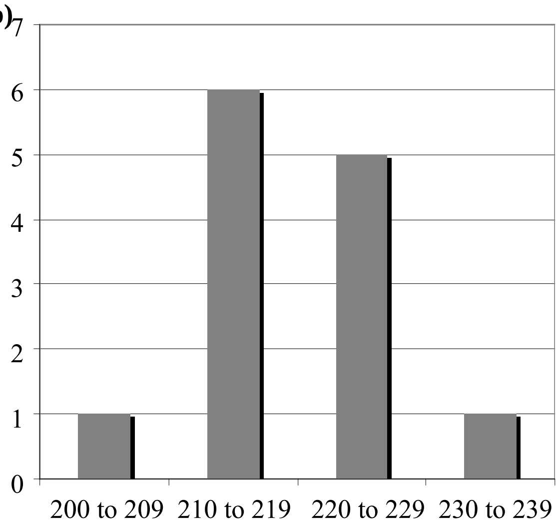
7.2.2 Stem and Leaf Plots p. 193

1. 7, 8, 8, 9, 9, 21, 21, 22, 23, 23, 26, 30, 30, 35, 38

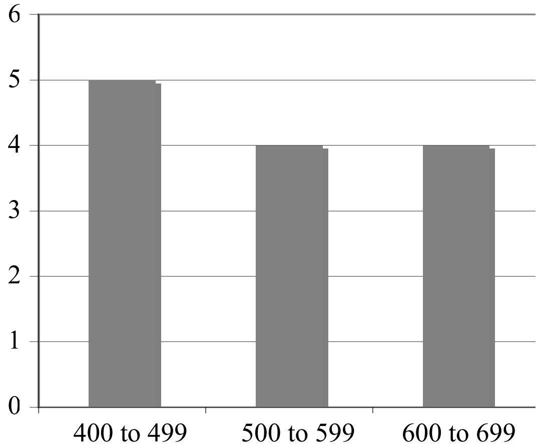
2. a)



2. b)



c)

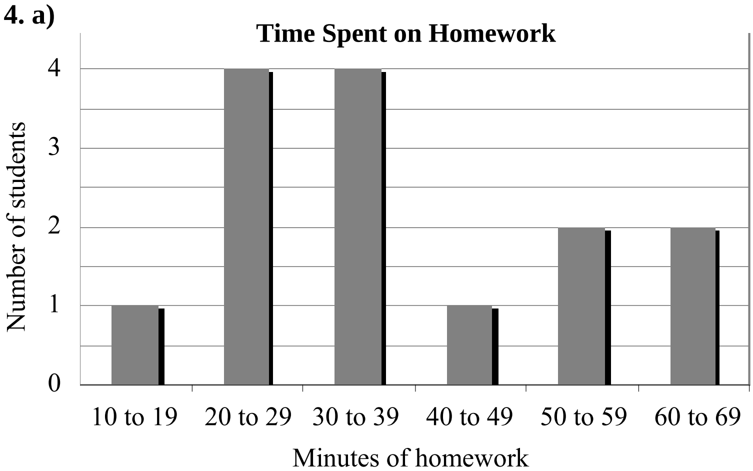


3. a)

0	0	4	8
1	2	6	
2	0	4	8
3	2	6	
4	0	4	8
5	2	6	
6	0	4	8

b) *Sample response*:
There are always either 2 or 3 multiples of 4 in every group of 10 numbers.

4. a)



b) *Sample response*:
Both graphs show the same information but the stem and leaf plot also includes the data values. For example, both graphs show that four students spent between 20 and 29 minutes on homework but the stem and leaf plot also shows that one student spent 20 minutes and three students spent 25 minutes.

5. *Sample responses*:
a) I predict a plot with a lot of numbers in the 0 to 9 and the 10 to 19 intervals and fewer numbers in the other rows

7.2.2 Stem and Leaf Plots [Continued] p. 195

b)

3, 3	5, 3	4, 5	1, 2	6, 4	2, 5	5, 6	3, 2	6, 6	1, 1	4, 4	3, 5	2, 5	5, 6	6, 2
5, 4	2, 1	3, 5	4, 3	1, 5	5, 1	5, 2	2, 2	3, 1	5, 3	1, 3	6, 4	6, 3	4, 1	1, 4

0	1	2	2	3	3	4	4	4	5	5	6	9
1	0	0	0	2	2	5	5	5	5	6	8	
2	0	0	4	4								
3	0	0	6									

My prediction was good.

6. a) In the second row since the mean is 225.

b) *Sample response*:
The mean is 19, which is in the first row.

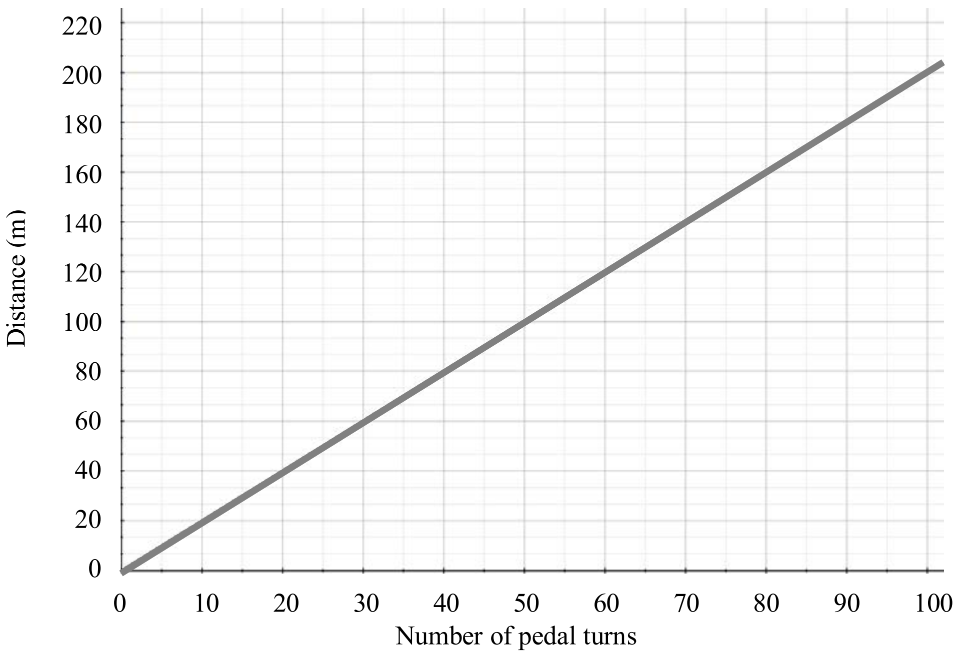
1	7	7	7	7	7	8	9
2							
3	0						

7.2.3 Line Graphs p. 199

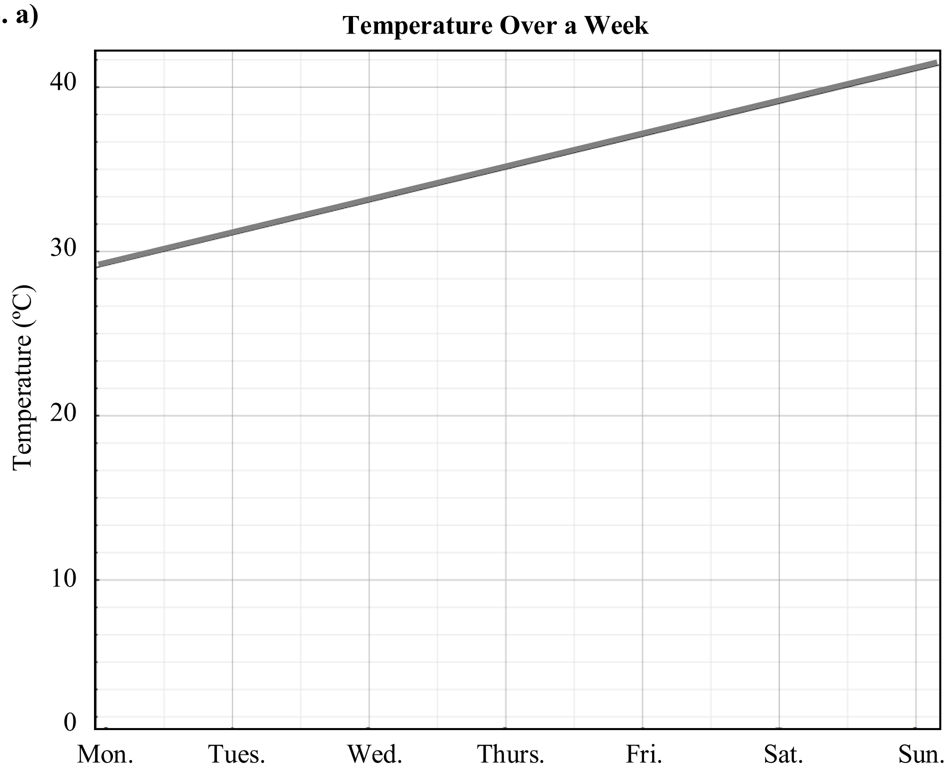
1. *Sample response*:
The plant grows a little bit every day. The growth was a bit faster from Monday to Wednesday than from Wednesday to Friday.

2. C

3. **Distance Travelled by Number of Pedal Turns**



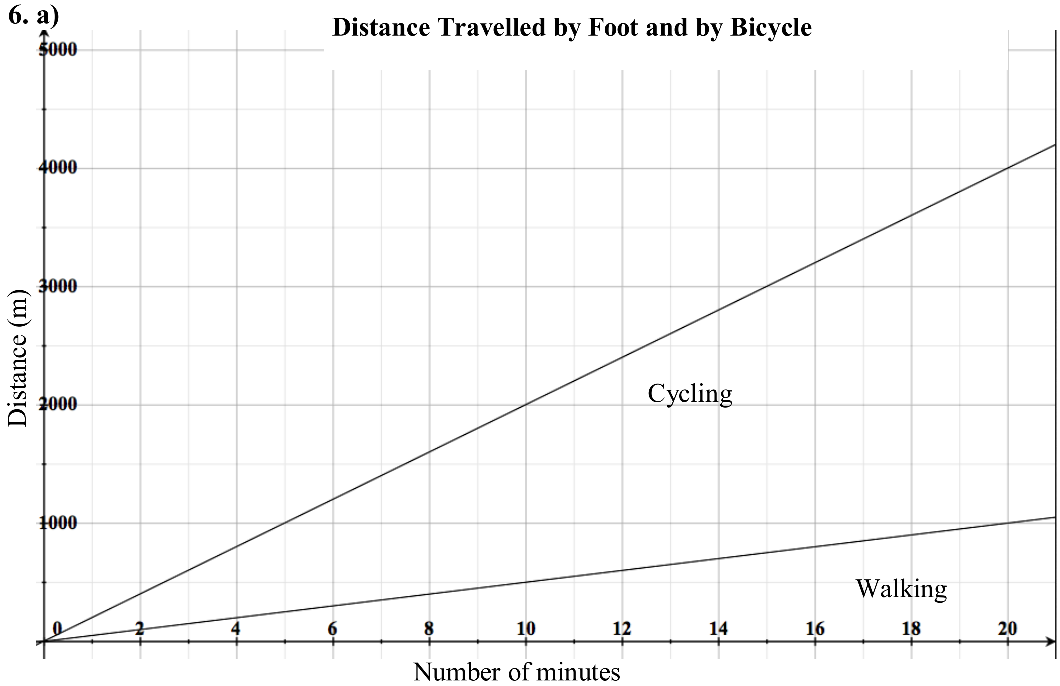
4. a)



b) *Sample response*: The temperature increased at a steady rate.
c) *Sample response*: The temperature would have increased faster so the line would be steeper.

5. *Sample response*: The graph shows that the number of ngultrums per dollar increased for a while, then it went down, then it stayed steady, and then it went down again.

6. a)



b) *Sample response:*
The difference between the distances becomes greater and greater as more time passes.

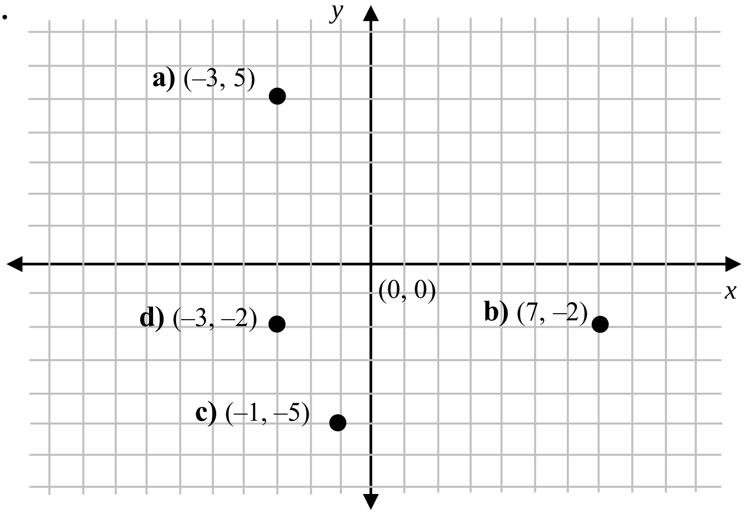
CONNECTIONS: Telling a Story about a Graph p. 200

1. The graph on the left matches Mindu's description; the graph on the right describes Karma's description.	**3.** *Sample response*: I started walking up a small hill and did not rest going up the hill. I started to come down and then walked on a flat section for a little while. Then I climbed a bigger hill, stopped at the top for a rest, and walked back down without resting.

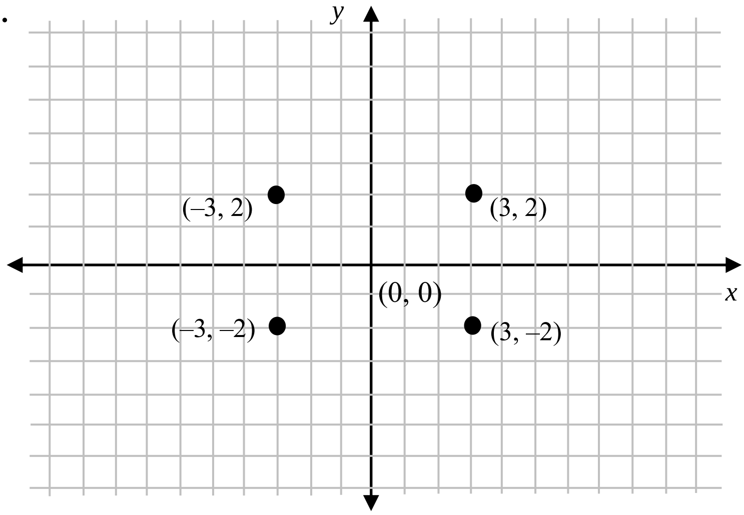
7.2.4 Coordinate Graphs p. 204

1. A(–4, 6)   B(–2, –1)   C(0, –3)   D(4, –2)

2.



3.



The points are the vertices of a rectangle. They are symmetric around the origin.

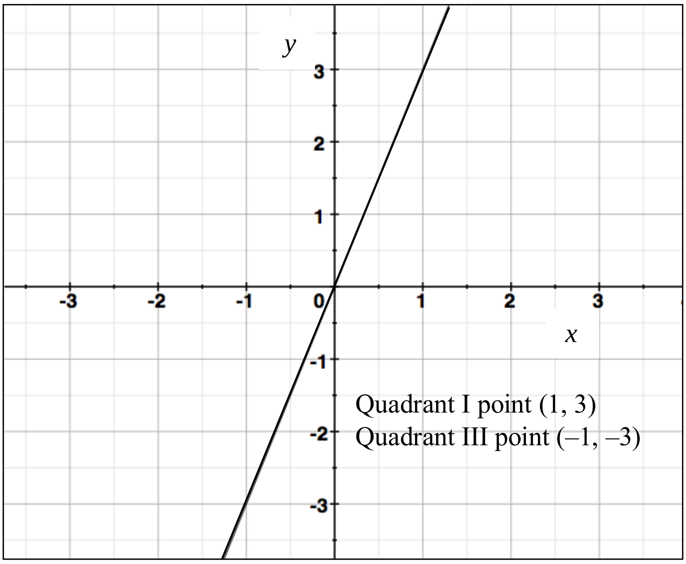
4. a) *Sample response*: (1, –1)   **b)** *Sample response*: (–20, –20)   **c)** *Sample response*: (–2, 20)
d) (–8, –5)   **e)** (–3, –4)   **f)** (–12, –2)

5. P   **6.** *Sample response*: (2, 3) and (–5, –4)

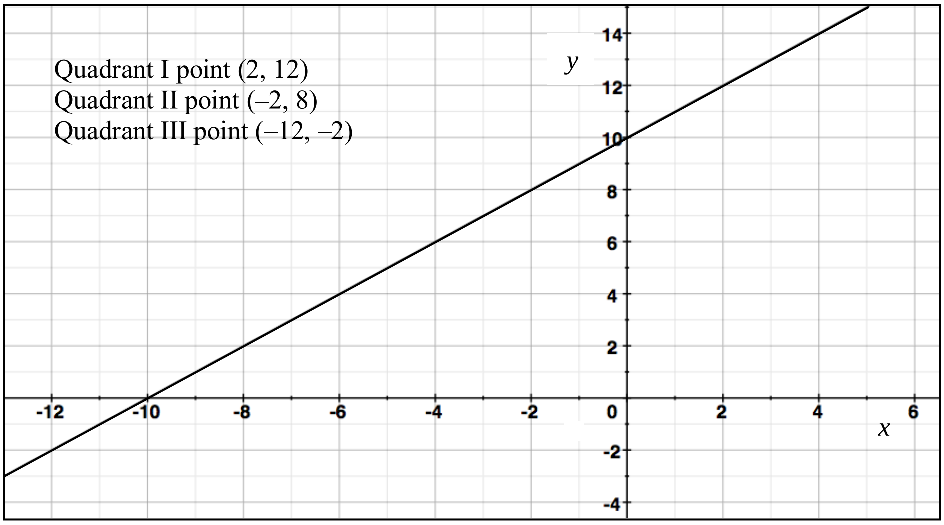
7. a) *Sample response*: (–1, –5)

8. *Sample responses*:

a)



b)



9. They are equally far from the origin.

7.3.1 Mean, Median, and Mode p. 208

1. a) Mean = 4, median = 3, modes = 1, 2, 7
b) Mean = 2, median = 1.5, mode = 0
c) Mean = 4, median = $2\frac{1}{2}$, mode = 3
d) Mean = 3, median = 3, mode = 3

2. a) 2 **b)** 1 **c)** 8

3. a) Mean **b)** All are equal **c)** Mean

4. *Sample responses*:
a) 2, 3, 4, 6, 8, 10
b) 3, 6, 6, 6
c) 1, 1, 3, 8, 10

5. a) Greater
b) Greater
c) Yes

6. *Sample response*:
4 for the first set of numbers and 4 for the second set.

7. a) The median is less than the tiger's mass;
b) The mean is greater than the tiger's mass;

7.3.2 Theoretical Probability p. 211

1. a) $\frac{2}{6}$ **b)** $\frac{3}{6}$ **c)** $\frac{1}{2}$ **d)** $\frac{1}{5}$

2. No

3. a) $\frac{50}{100}$ **b)** $\frac{30}{100}$ **c)** $\frac{58}{100}$ **d)** $\frac{16}{100}$

4. *Sample response*:
9 slips; 4 with numbers and 5 with letters.

5. *Sample responses*:
a) Rolling a number greater than 4 on a die.
b) Drawing a slip of paper with a * from a bag that contains 2 slips with a * and 2 other slips.
c) Spinning grey on a spinner with 5 equal sections: 3 grey sections and 2 white sections.
d) Choosing a slip of paper with a * from a bag that contains 8 slips: 3 slips with a * and 5 other slips.
e) Choosing a slip of paper with a * from a bag that contains 8 slips: 2 slips with a * and 6 other slips.
f) Spinning a number less than 5 using the spinner in question 1.

6. Drawing slips from a bag and spinning a spinner

UNIT 7 Revision pp. 212–214

1. a) People at the hospital are sicker than most.
b) *Sample response*: I could ask some doctors how many patients they have and how many have been sick this year.

2. *Sample responses*:
a) Students might watch a different amount of TV than adults.
b) Fewer people would have TVs in rural areas than in urban areas.

3. *Sample responses*:
a) Sums are 3 and 9. **b)** 50%
c) For a total of 25 rolls:

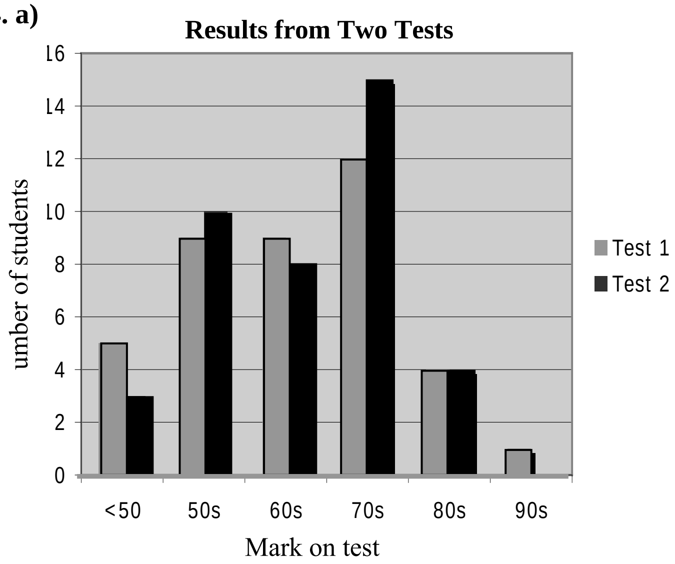
3	9	4	8	6	9	5	9	6
7	4	8	11	7	11	5	6	10
7	7	8	8	5	7	12	7	6

7 or less	Greater than 7
15	10

Experimental probability of a sum > 7: $\frac{10}{25} = 40\%$

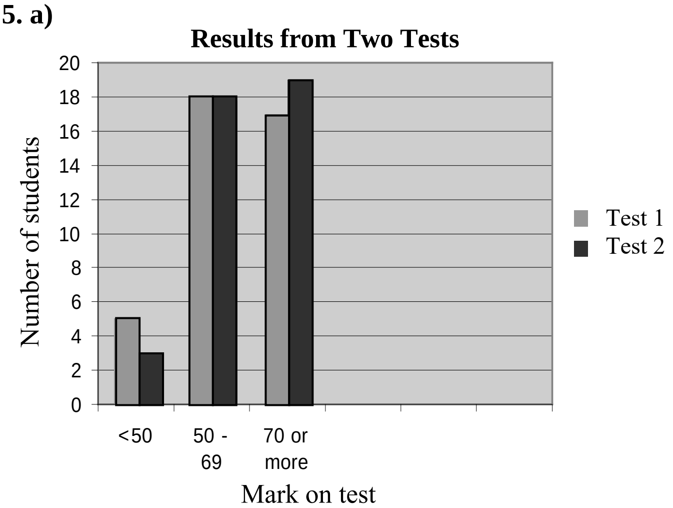
d) No

4. a)



b) *Sample response*:
Test 2 marks improved for marks in the 70s, stayed the same for marks in the 80s, but dropped for marks in the 90s. Marks lower than 50 dropped, which shows an improvement.

5. a)

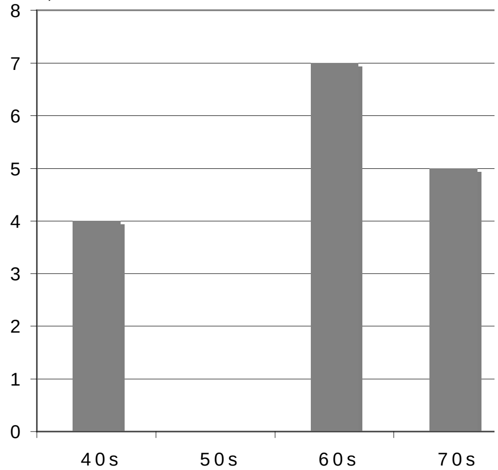


b) *Sample response*:
You can still see that there are fewer marks below 50, but you cannot see which category improved the most for marks greater than 70.

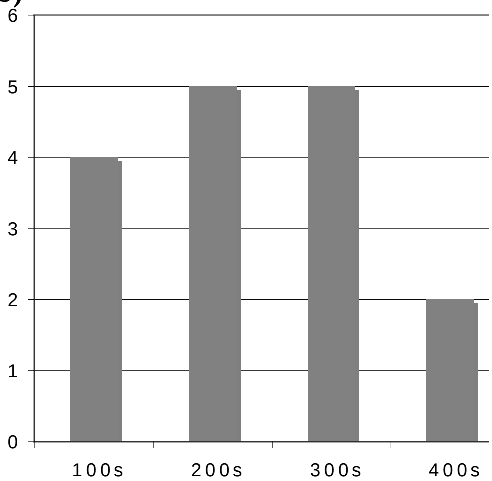
6. a) Tens digit
b) *Sample response*: Hundreds digit
c) *Sample response*: Hundreds digit

7. 31, 32, 33, 40, 41, 41, 51

8. a)



b)



9. *Sample response*:
Rolls:

3, 3	5, 3	4, 5	1, 2	6, 4	2, 5	5, 6	3, 2	6, 6	1, 1	4, 4	3, 5	2, 5	5, 6	6, 2
5, 4	2, 1	3, 5	4, 3	1, 5	5, 1	5, 2	2, 2	3, 1	5, 3	1, 3	6, 4	6, 3	4, 1	1, 4

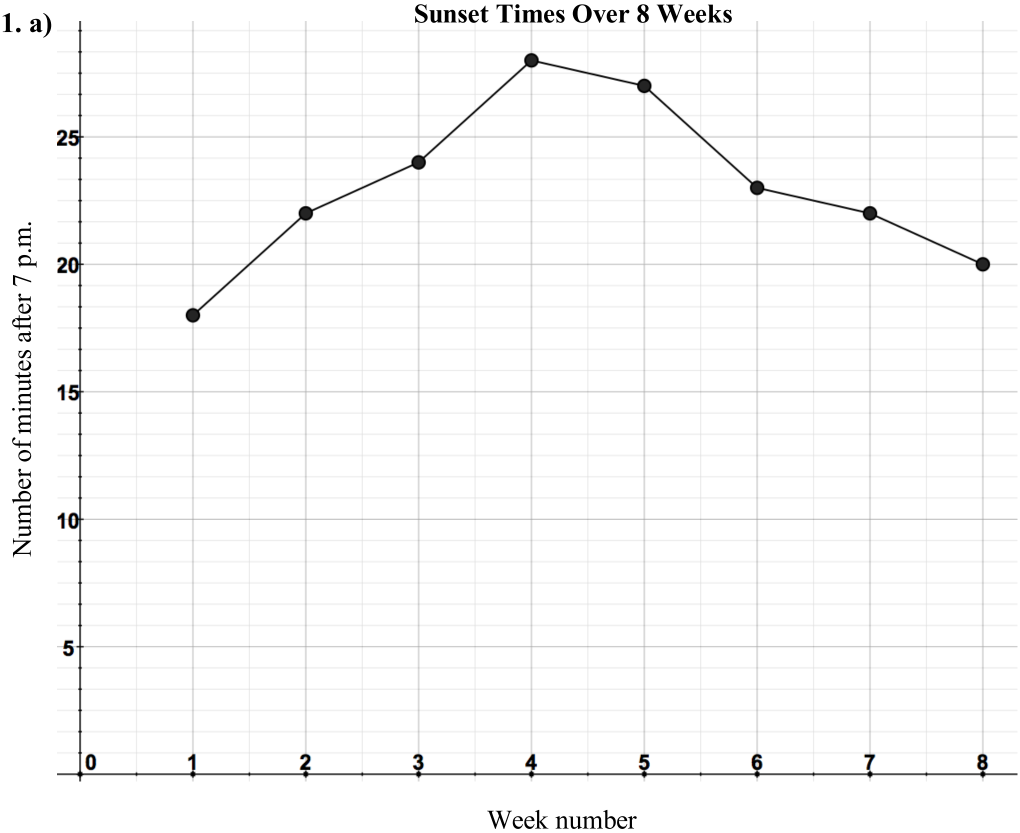
Values after I double and add:

12	16	18	6	20	14	22	10	24	4	16	16	14	22	16
18	6	16	14	12	12	14	8	8	16	8	20	18	10	10

0	4 6 6 8 8 8
1	0 0 0 2 2 2 4 4 4 4 6 6 6 6 6 6 8 8 8
2	0 0 2 2 4

10. B

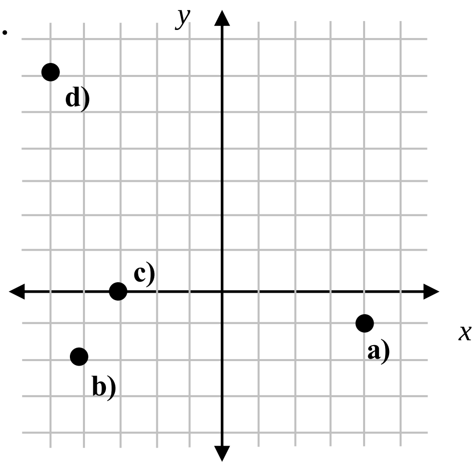
11. a)



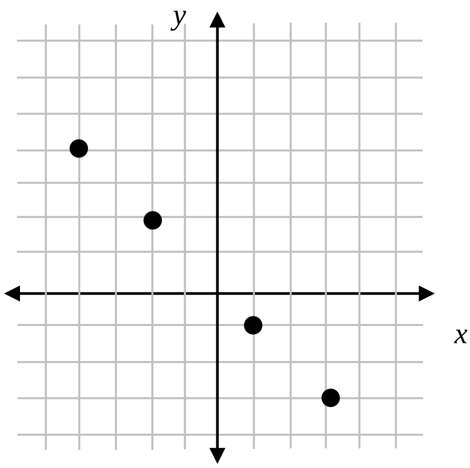
b) The sunset time was getting later for a while, but then it started to get earlier.

12. A(–5, 1), B(–2, 3), C(–4, –3), D(0, –1)

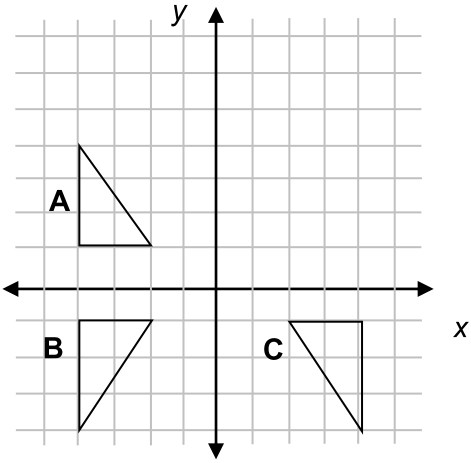
13.



14.



Sample response: The points are all in a line.

15. *Sample response*:  **b)** The *x*-coordinates are the same but the *y*-coordinates are opposites. **c)** The *x*- and *y*- coordinates are opposites; the *y*-coordinates are the same but the *x*-coordinates are opposites.	**16. a)** *Sample response*: 2 **b)** *Sample response*: 2 or any other number that is not already on the list **c)** 9 **17. a)** Mode or median **b)** Mean **c)** Median or mode **18. a)** Greater **b)** Less **19.** *Sample responses*: **a)** 20 **b)** 20 **20. a)** $\frac{2}{6}$ **b)** $\frac{3}{5}$ **21.** *Sample responses*: **a)** Spinning grey on a spinner with 10 sections where 1 section is grey **b)** Spinning grey on a spinner with 10 sections where 2 sections are grey