Teacher's Guide to

Understanding Mathematics

Textbook for Class VII

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Department of School Education

Ministry of Education and Skills Development

Royal Government of Bhutan

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ROYAL GOVERNMENT OF BHUTAN
MINISTRY OF EDUCATION
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MINISTER

FOREWORD

Provision of quality education for our children is a cornerstone policy of the Royal Government of Bhutan. Quality education in mathematics includes attention to many aspects of educating our children. One is providing opportunities and believing in our children's ability to understand and contribute to the advancement of science and technology within our culture, history and tradition. To accomplish this, we need to cater to children's mental, emotional and psychological phases of development, enabling, encouraging and supporting them in exploring, discovering and realizing their own potential. We also must promote and further our values of compassion, hard work, honesty, helpfulness, perseverance, responsibility, *thadamtsi* (for instance being grateful to what I would like to call '*Pham Kha Nga',* consisting of parents, teachers, His Majesty the King, the country and the Bhutanese people, for all the goodness received from them and the wish to reciprocate these in equal measure) and *ley-ju-drey* — the understanding and appreciation of the natural law of cause and effect. At the same time, we wish to develop positive attitudes, skills, competencies, and values to support our children as they mature and engage in the professions they will ultimately pursue in life, either by choice or necessity.

While education recognizes that certain values for our children as individuals and as citizens of the country and of the world at large, do not change, requirements in the work place advance as a result of scientific, technological, and even political advancement in the world. These include expectations for more advanced interpersonal skills and skills in communications, reasoning, problem solving, and decision-making. Therefore, the type of education we provide to our children must reflect the current trends and requirements, and be relevant and appropriate. Its quality and standard should stem out of collective wisdom, experience, research, and thoughtful deliberations.

Mathematics, without dispute, is a beautiful and profound subject, but it also has immense utility to offer in our lives. The school mathematics curriculum is being changed to reflect research from around the world that shows how to help students better understand the beauty of mathematics as well as its utility.

The development of this textbook series for our schools, *Understanding Mathematics,* is based on and organized as per the new School Mathematics Curriculum Framework that the Ministry of Education has developed recently, taking into consideration the changing needs of our country and international trends. We are also incorporating within the textbooks appropriate teaching methodologies including assessment practices which are reflective of international best practices. The *Teacher's Guides* provided with the textbooks are a resource for teachers to support them, and will definitely go a long way in assisting our teachers in improving their efficacy, especially during the initial years of teaching the

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new curriculum, which demands a shift in the approach to teaching and learning of Mathematics. However, the teachers are strongly encouraged to go beyond the initial ideas presented in the Guides to access other relevant resources and, more importantly to try out their own innovations, creativity and resourcefulness based on their experiences, reflections, insights and professional discussions.

The Ministry of Education is committed to providing quality education to our children, which is relevant and adaptive to the changing times and needs as per the policy of the Royal Government of Bhutan and the wish of our beloved King.

I would like to commend and congratulate all those involved in the School Mathematics Reform Project and in the development of these textbooks.

I would like to wish our teachers and students a very enjoyable and worthwhile experience in teaching, learning and understanding mathematics with the support of these books. As the ones actually using these books over a sustained period of time in a systematic manner, we would like to strongly encourage you to scrutinize the contents of these books and send feedback and comments to the Curriculum and Professional Support Division (CAPSD) for improvement with the future editions. On the part of the students, they can and should be enthusiastic, critical, venturesome, and communicative of their views on the contents discussed in the books with their teachers and friends rather than being passive recipients of knowledge.

Trashi Delek!

Thinley Gyamtsho

MINISTER Ministry of Education

October of 2007

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INTRODUCTION

HOW MATHEMATICS HAS CHANGED

Mathematics is a subject with a long history. Although newer mathematical ideas are always being created, much of what your students will be learning is mathematics that has been known for hundreds of years, if not longer.

The learning of mathematics helps a person to solve problems. While solving problems, skills related to, representation of mathematical ideas, making connections with other topics in mathematics and connections with the real world, providing reasoning and proof, and communicating mathematically would be required. The textbook is designed to promote the development of these process skills.

Worldwide, there is now a greater emphasis on the need for students to understand the mathematics they learn rather than to memorize rote procedures.

Worldwide, there is now a greater emphasis on the need for students to understand the mathematics they learn rather than to memorize rote procedures. There are many reasons for this.

• In the long run, it is very unlikely that students will remember the mathematics they learn unless it is meaningful. It is much harder to memorize “nonsense” than something that relates to what they already know.

• Some approaches to mathematics have not been successful; there are many adults who are not comfortable with mathematics even though they were successful in school.

In the student textbooks, many ways are shown to make mathematics more meaningful.

It is important always to talk about why something is true, not simply that it is true.

• We will always talk about why something is true, not simply that it is true. For example, the reason why a number with a digit sum of 18 is divisible by 9 is demonstrated and not just stated.

• Mathematics should be taught using contexts that are meaningful to the students. They can be mathematical contexts or real-world contexts. These contexts will help students see and appreciate the value of mathematics.

For example:

In Unit 2 (Fractions), a task with a real-world context involves fractions and cooking.

> Kamala has $\frac{7}{2}$ cups of rice. Does she have enough to make a meal that needs $3\frac{2}{3}$ cups of rice? Show your work.

Unit 3 (Ratio) has a task with a broader context involving the speed of various animals.

Animal	Distance (m)	Time (s)
Cheetah	200	6.4
Bear	500	36.0
Zebra	250	14.0
Elephant	20	1.8
Tortoise	10	120.0
Rabbit	300	20.0
Lion	400	16.0

a) Which animal runs at each speed?
i) about 11 m/s **ii)** about 25 m/s
b) Which animal could travel each?
i) 900 m in 1 min **ii)** about 5 m in 1 min
c) Which is fastest? How do you know?
d) Which is slowest? How do you know?

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• When discussing mathematical ideas, we expect students to use the processes of problem solving, communication, reasoning, making connections (connecting mathematics to the everyday world and connecting mathematical topics to each other) and representation (representing mathematical ideas in different ways, such as manipulatives, graphs, and tables). For example, students represent integers using counters and number lines to help them see how the rules for adding and subtracting make sense.

We expect students to use the processes of problem solving, communication, reasoning, making connections, and representation.

• There is an increased emphasis on problem solving because the reason we learn mathematics is to solve problems. Once students are adults, they are not told when to factor or when to multiply; they need to know how to apply those skills to solve certain problems. Students will be given opportunities to make decisions about which concepts and skills they need in certain situations.

A significant amount of research evidence has shown that these more meaningful approaches work. Scores on international tests are higher when the emphasis is on higher level thinking and not only on the application of skills.

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THE DESIGN OF THE STUDENT TEXTBOOK

Each unit of the textbook has the following features:

• a *Getting Started* to review prerequisite content

• two or three chapters, which cluster the content of the unit into sections that contain related content

• regular lessons and at least one *Explore* lesson

• a *Game*

• at least one *Connections* feature

• a *Unit Revision*

Getting Started

There are two parts to the *Getting Started*. They are designed to help you know whether the students are missing critical prerequisites and to remind students of knowledge and terminology they have already learned that will be useful in the unit.

The Getting Started is designed to help you know whether the students are missing critical prerequisites and to remind students of knowledge and terminology they will need for the unit.

• The *Use What You Know* section is an activity that takes 20 to 30 minutes. Students are expected to work in pairs or in small groups. Its purpose is consistent with the rest of the text's approach, that is, to engage students in learning by working through a problem or task rather than being told what to do and then just carrying it through.

• The *Skills You Will Need* section is a more straightforward review of required prerequisite skills for the unit. This should usually take about 20 to 30 minutes.

Regular Lessons

Lessons are numbered #.#.#. The first number tells the unit, the second number the chapter, and the third number the lesson within the chapter.

• Each lesson might be completed in one or two hours (i.e., one or two class periods), although some are shorter. The time is suggested in this *Teacher's Guide*, but it is ultimately at your discretion.

• Lessons are numbered #.#.#. The first number tells the unit, the second number the chapter, and the third number the lesson within the chapter. For example, Lesson 4.2.1 is Unit 4, Chapter 2, Lesson 1.

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• Each lesson is divided into five parts:
- A *Try This* task or problem
- The exposition (the main points of the lesson)
- A question that revisits the *Try This* task, called *Revisiting the Try This* in this guide
- one or more *Examples*
- *Practising and Applying* questions

Try This

• The *Try This* task is in a shaded box, like the one below from lesson 1.1.2 on page 5.

Try This

> Yuden bought 9 kg of chicken. Each kilogram cost Nu 85.
> The shopkeeper said that Yuden owed a total of Nu 755.
>
> **A.** Describe two or more ways Yuden could have known the total was incorrect.

• The *Try This* is a brief task or problem that students complete in pairs or small groups. It serves to motivate new learning. Students can do the *Try This* without the new concepts or skills that are the focus of the lesson, but the problem is related to the new learning. It should be completed in 5 to 10 minutes. The reason to start with a *Try This* is that we believe students should do some mathematics independently before you intervene.

The Try This is a brief task or problem that students complete in pairs or small groups to motivate new learning.

• The answers to the *Try This* questions are not found in the back of the student book (but they are in this *Teacher's Guide*).

The Exposition

• The exposition presents the main concepts and skills of the lesson. Examples are often included to clarify the points being made.

The exposition presents the main concepts and skills of the lesson and is taught in different ways, as suggested in this guide.

• You will help the students through the exposition in different ways (as suggested in this *Teacher's Guide*). Sometimes you will present the ideas first, using the exposition as a reference. Other times students will work through the exposition independently, in pairs, or in small groups.

• Key mathematical terms are introduced and described in the exposition. When a key term first appears in a unit of the textbook, it is highlighted in **bold** type to indicate that it is found in the glossary (at the back of the student textbook).

• Students are not expected to copy the exposition into their notebooks either directly from the book or from your recitation.

Revisiting the Try This

• The *Revisiting the Try This* question follows the exposition and appears in a shaded lozenge, like this example from lesson 1.1.2 on page 6.

> **B.** How could Yuden have used a divisibility test to know the total was incorrect?

The Revisiting the Try This question links the Try This task to the new ideas presented in the exposition.

• The *Revisiting the Try This* question links the *Try This* task or problem to the new ideas presented in the exposition. This is designed to build a stronger connection between the new learning and what students already understand.

Examples

• The *Examples* are designed to provide additional instruction by modelling how to approach some of the questions students will meet in *Practising and Applying*. Each example is a bit different from the others so that students have multiple models from which to work.

• The *Examples* show not only the formal mathematical work (in the left hand *Solution* column), but also student reasoning (in the right hand *Thinking* column). This model should help students learn to think and communicate mathematically. Photographs of students are used to further reinforce this notion.

• Some of the *Examples* present two different solutions. The example below, from lesson 1.1.4 on page 14, shows two possible ways to approach the task, *Solution 1* and *Solution 2*.

The Examples model how to approach some of the questions students will meet in Practising and Applying

The Examples show the formal mathematical work in the Solution column and model student reasoning in the Thinking column. This model should help students learn to think and communicate mathematically.

Examples

Example 1 Calculating the GCF of Three Numbers

Calculate GCF (50, 70, 140).

Solution 1

50 → **5** × 5 × **2**

70 → **5** × 7 × **2**

140 → 7 × 10 × 2 → 7 × **2** × **5** × 2

The common factors are 2 and 5.

GCF (50, 70, 140) = 2 × 5 = 10

Thinking

• I used a factor tree to find the factors of each.

• I looked for prime factors that were common to all three.

• I multiplied the common factors to get the GCF.

Solution 2

Factors of 50

1 2 5 **10** 25 50

Factors of 70

1 2 5 7 **10** 14 35 70

Factors of 140

1 2 4 5 7 **10** 14 20 28 35 70 140

The greatest number in all three lists is 10.

GCF (50, 70, 140) = 10

Thinking

• I used a factor rainbow to find factor pairs for each number.

• I divided each number by 1, then by 2, then by 3, and so on until I had a list of all possible different factors.

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• The treatment of *Examples* varies and is discussed in the *Teacher's Guide*. Sometimes the students work through these independently, sometimes they work in pairs or small groups, and sometimes you are asked to lead them through some or all of the examples.

• A number of the questions in the *Practising and Applying* section are modelled in the *Examples* to make it more likely that students will be successful.

Practising and Applying

• Students work on the *Practising and Applying* questions independently, with a partner, or in a group, using the exposition and *Examples* as references.

Students work on the Practising and Applying questions independently, with a partner, or in a group, using the exposition and Examples as references.

• The questions usually start like the work in the *Examples* and get progressively more conceptual, with more explanations and more problem solving required later in the exercise set.

• The last question in the section always brings closure to the lesson by asking students to summarize the main learning points. This question could be done as a whole class.

Explore Lessons

• *Explore* lessons provide an opportunity for students to work in pairs or small groups to investigate some mathematics in a less directed way. Often, but not always, the content is revisited more formally in a regular lesson immediately before or after the *Explore* lesson. The *Teacher's Guide* indicates whether the *Explore* lesson is optional or core.

Explore lessons provide an opportunity for students to work with a partner or in small groups to investigate some mathematics in a less directed way.

• There is no exposition or teacher lecture in an *Explore* lesson, so the parts of the regular lesson are not there. Instead, a problem or task is posed and students work through it by following a sequence of questions or instructions that direct their investigation.

• The answers for these lessons are not found in the back of the textbook, but are found in this *Teacher's Guide*.

Connections

• The *Connections* is an optional feature that relates the content to something else.

The Connections feature takes many forms. Sometimes it is a relevant and interesting historical note. Sometimes it relates the mathematical content of a unit to the content of a different unit. Other times it relates the mathematical content to a real-world application.

• There are always one or more *Connections* features in a unit. The placement of a *Connections* feature in a unit is not fixed; it depends on the content knowledge required. Sometimes it will be early in the unit and sometimes later.

• The *Connections* feature always gives students something to do beyond simply reading it.

• Students usually work in pairs or small groups to complete these activities.

Game

• There is at least one *Game* per unit.

• The *Game* provides an enjoyable way to practise skills and concepts introduced in the unit.

The Game provides an enjoyable way to practise skills and concepts introduced in the unit.

• Its placement in the unit is based on where it makes most sense in terms of the content required to play the *Game*.

• In most *Games* students work in pairs or small groups, as indicated in the instructions.

• The required materials and rules are listed in the student book. Usually, there is a sample shown to make sure that students understand the rules.

• Most *Games* require 15 to 20 minutes, but students can often benefit from playing them more than once.

Unit Revision

• The *Unit Revision* provides an opportunity for review for students and for you to gather informal assessment data. *Unit Revisions* review all lesson content except the *Getting Started* feature, which is based on previous class content. There is always a mixture of skill, concept, and problem solving questions.

The Unit Revision provides an opportunity for review for students and allows you to gather informal assessment data.

• The order of the questions in the *Unit Revision* generally follows the order of the lessons in the unit. Sometimes, if a question reflects more than one lesson, it is placed where questions from the later lesson would appear.

• Students can work in pairs or on their own, as you prefer.

• The *Unit Revision*, if done in one sitting, requires one or more hours. If you wish, you might break it up and assign some questions earlier in the unit and some questions later in the unit.

Glossary

• At the end of the student textbook, there is a glossary of key mathematical terminology introduced in the units. When new terms are introduced in the units, they are in **bold** type. All of these terms are found in the glossary.

The glossary contains key mathematical terminology introduced in the units, important terms from previous classes, and a set of instructional terms.

• The glossary also contains important mathematical terms from previous classes that students might need to refer to.

• In addition, there is a set of instructional terms commonly used in the *Practising and Applying* questions (for example, explain, predict, …) along with descriptions of what those terms require the student to do.

Answers

• Answers to most numbered questions are provided in the back of the student textbook. In most cases, only the final answers are shown, not full solutions and explanations. For example, if students are asked to solve a problem and then "Show your work" or "Explain your thinking", only the final answer to the problem will be included, not the work or the reasoning.

The answers to most of the numbered questions are found in the back of the student book. This Teacher's Guide contains a full set of answers.

• There is often more than one possible answer. This is indicated by the phrase *Sample Response*.

• Full solutions to the questions and explanations that show reasoning are provided in this *Teacher's Guide*, as are the answers to the lettered questions (such as A or B) in the *Try This* and the *Explore* lessons. Note that when an answer or any part of an answer is enclosed in square brackets, this indicates that it has been omitted from the answers at the back of the student textbook.

THE DESIGN OF THE TEACHER'S GUIDE

The *Teacher's Guide* is designed to complement and support the use of the student textbook.

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• The sequencing of material in the guide is identical to the sequencing in the student textbook.

• The elements in the *Teacher's Guide* for each unit include:

  - a *Unit Planning Chart*
  - *Math Background* for the unit
  - a *Rationale for Teaching Approach*
  - support for each lesson
  - a *Unit Test*
  - a *Performance Task*
  - an *Assessment Interview* (Units 2 and 5)

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The support for each lesson includes:

- *Curriculum outcomes* covered in that lesson
- *Outcome relevance* (*Lesson relevance* in the case of optional *Explore* lessons)
- *Pacing* in terms of minutes and hours
- *Materials* required to teach the lesson
- *Prerequisites* that the lesson assumes students possess
- *Main Points to be Raised* explicitly in the lesson
- suggestions for working through the parts of the lesson
- *Suggested assessment* for the lesson
- *Common errors* to be alert for
- *Answers*, often with more complete solutions than are found in the student text
- suggestions for *Supporting Students* who are struggling and/or for enrichment

Unit Planning Chart

This chart provides an overview of the unit and indicates, for each lesson, which curriculum outcomes are being covered, the pacing, the materials required, and suggestions for which questions to use for formative assessment.

The Unit Planning Chart provides an overview of the unit.

Math Background and Rationale for Teaching Approach

This explains the organization of the unit or provides some background for you that you might not have, particularly in the case of less familiar content. In addition, there is an indication of why the material is approached the way it is.

This section provides information about the critical math behind the unit, and an explanation of why the math is approached the way it is.

Regular Lesson Support

Regular lesson support includes grouping and instructional strategies, alerts for common errors, suggestions for assessment, and teaching tips.

• Suggestions for grouping and instructional strategies are offered under the headings *Try This, Revisiting the Try This, The Exposition — Presenting the Main Ideas, Using the Examples,* and *Practising and Applying — Teaching Tips.*

• *Common errors* are sometimes included. If you are alert for these and apply some of the suggested remediation, it is less likely that students will leave the lessons with mathematical misunderstandings.

• A number of *Suggested assessment questions* are listed for each lesson. This is to emphasize the need to collect data about different aspects of the student's performance — sometimes the ability to apply skills, sometimes the ability to solve problems or communicate mathematical understanding, and sometimes the ability to show conceptual understanding.

• It is not necessary to assign every *Practising and Applying* question to each student, but they are all useful. You should go through the questions and decide where your emphasis will be. It is important to include a balance of skills, concepts, and problem solving. You might use the *Suggested assessment questions* as a guide for choosing questions to assign.

• You may decide to use the last *Practising and Applying* question to focus a class discussion that revisits the main ideas of the lesson and to bring closure to the lesson.

Explore Lesson Support

• As with regular lessons, for *Explore* lessons there is an indication of the curriculum outcomes being covered, the relevance of the lesson, and whether the lesson is optional or essential.

• Because the style of the lesson is different, the support provided is different than for the regular lessons. There are suggestions for grouping the students for the exploration, a list of *Observe and assess* questions to guide your informal formative assessment, and *Share and reflect* ideas on how to consolidate and bring closure to the exploration.

Because the style of an Explore lesson is different than a regular lesson, the support provided in the Teacher's Guide is also different.

Unit Test

A pencil-and-paper unit test is provided for each unit. It is similar to the unit revision, but not as long. If it seems that the test might be too long (for example, if students would require more than one class period to complete it) some questions may be omitted. It is important to balance the items selected for the test to include questions involving skills, concepts, and problem solving, and to include at least one question requiring mathematical communication.

If the test seems too long, some questions may be omitted but it is important to ensure a balance of questions involving skills, concepts, problem solving, and communication.

Performance Task

• The *Performance Task* is designed as a summative assessment task. Performance on the task can be combined with performance on a *Unit Test* to give a mark for a student on a particular unit.

• The task requires students to use both problem solving and communication skills to complete it. Students have to make mathematical decisions to complete the task.

• It is not appropriate to mark a performance task using percentages or numerical grades. For that reason, a rubric is provided to guide assessment. There are four levels of performance that can be used to describe each student's work on the task. A level is assigned for each different aspect of the task and, if desired, an overall profile can be assigned. For example, if a student's performance is level 2 on most of the aspects of the task, but level 3 on one aspect, an overall profile of level 2 might be assigned.

• A sample solution is provided for each task.

The Performance Task is designed as a summative assessment task that can be combined with a student's performance on the Unit Test to give an overall mark.

Unit Assessment Interviews

• Selected units (2 and 5) provide a structure for an interview that can be used with one or several students to determine their understanding of the outcomes.

• Interviews are a good way to collect information about students because they allow you to interact with the students and to follow up if necessary. Interviews are particularly appropriate for students whose performance is inconsistent or who do not perform well on written tests, but who you feel really do understand the content.

• You may use the data you collect in combination with class work or even a unit test mark.

Interviews are a good way to collect information about students, since interviews allow you to interact with the students and to follow up if necessary.

ASSESSING MATHEMATICAL PERFORMANCE

Forms of Assessment

It is important to consider both formative (continuous) and summative assessment.

Formative

• Formative assessment is observation to guide further instruction. For example, if you observe that a student does not understand an idea, you may choose to re-teach that idea to that student using a different approach.

Formative assessment is observation to guide further instruction.

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- Formative assessment opportunities are provided through
  - prerequisite or diagnostic assessment in the *Getting Started*
  - suggestions for assessment questions in each regular lesson
  - questions that might be asked while students work on the *Try This* or during an *Explore* lesson
  - the *Unit Revision*
  - the unit *Assessment Interview* (for the units with interviews)
- Formative assessment can be supplemented by
  - everyday observation of students' mathematical performance
  - formal or informal interviews to reveal students' understanding
  - journals in which students comment on their mathematical learning
  - short quizzes
  - projects
  - a portfolio of work so students can see their progress over time, for example, in problem solving or mathematical communication (see *Portfolios* below)

Summative

- Summative assessment is used to see what students have learned and is often used to determine a mark or grade.
- Summative assessment opportunities are provided through
  - the *Unit Test*
  - the *Performance Task*
  - the *Assessment Interview*
- Summative assessment can be supplemented with
  - short quizzes
  - projects
  - a portfolio that is assessed with respect to progress in, for example, problem solving or communication

Summative assessment is used to see what students have learned and is often used to determine a mark.

Portfolios

One of the advantages of using portfolios for assessment is that students can observe their own growth over time, leading to greater confidence. In each unit, the section on math background identifies items that pertain to the various mathematical processes. The portfolio could be made up of student work items related to one of the mathematical processes: problem solving, communication, reasoning, or representation.

One of the advantages of using portfolios for assessment is that students can observe their own growth over time, leading to greater confidence.

Assessment Criteria

- It is right and fair to inform students about what will be assessed and how it will be assessed. For example, students should know whether the intent of a particular assessment task is to focus on application or on problem solving.
- A student's mark and all assessments should reflect the curriculum outcomes for Class VII. The proportions of the mark assigned for each unit should reflect both the time spent on the unit and the importance of the unit. The modes of assessment used for a particular unit should be appropriate for the content. For example, if the unit focuses mostly on skills, the main assessment might be a paper-and-pencil test or quizzes. If the unit focuses on concepts and application, more of the student's mark should come from activities like performance tasks.
- The focus of this curriculum is not on procedures for their own sake but on a conceptual understanding of mathematics so that it can be applied to solve problems. Procedures are important too, but only in the context of solving

It is right and fair to inform students about what will be assessed and how it will be assessed.

The modes of assessment used for a particular unit should be appropriate for the content.

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problems. All assessment should balance procedural, conceptual, and problem solving items, although the proportions will vary in different situations.

• Students should be informed whether a test is being marked numerically, using letter grades, or with a rubric. If a rubric is being used, then it should be shared with students before they begin the task to which it is being applied.

All assessment should balance procedural, conceptual, and problem solving items, although the proportions may vary in different situations.

Determining a Mark

• In determining a student's mark, you can use the tools described above along with other information such as work on a project or poster. It is important to remember that the mark should, as closely as possible, reflect student competence with the mathematical outcomes of the course. The mark should not reflect behaviour, neatness, participation, and other non-mathematical aspects of the student's learning. These are important to assess as well, but not as part of the mathematics mark.

It is important to remember that the mark should reflect student competence with the mathematical outcomes of the course and not behaviour, neatness, participation, and so on.

• In looking at a student's mathematics performance, the most recent data might be weighted more heavily. For example, suppose a student does poorly on some of the quizzes given early in Unit 1, but later you observe that he or she has a better understanding of the material in the unit. You might choose to give the early quizzes less weight in determining a student's mark for the unit.

• At present, you are required to produce a numerical mark for a student, but that should not preclude your use of rubrics to assess some mathematical performance. One of the values of rubrics is their reliability. For example, if a student performs at level 2 on a particular task one day, he or she is very likely to perform at the same level on that task another day. On the other hand, a student who receives a test mark of 45 one day might have received a mark of 60 on a different day if one question had changed on the test or if he or she had read an item more carefully.

• You can combine numerical and rubric data using your own judgment. For example, if a student's marks on tests average 50%, but the rubric performance is higher, for example, level 3, it is fair and appropriate to use a higher average for that student's class mark.

THE CLASSROOM ENVIRONMENT

This new curriculum requires a change in the classroom environment to include more pair and group work and an increased emphasis on communication. It is only in this way that students will really become engaged in mathematical thinking instead of being spectators.

This curriculum requires a change in the classroom environment to include more pair and group work and an increased emphasis on communication.

• In every lesson, students should be engaged in some pair or small group work (for the *Try This*, selected *Practising and Applying* questions, or during an *Explore* lesson).

• Students should be encouraged to communicate with each other to share responses that are different from those offered by other students or from the responses you might expect. Communication involves not only talking and writing but also listening and reading.

It is through communication that students clarify their own thinking as well as show you and their classmates what they do or do not understand.

Pair and Group Work

• There are many reasons why students should be working in pairs or groups, including

- to ensure that students have more opportunities to communicate mathematically (instead of competing with the whole class for a turn to talk)

- to make it easier for them to take the risk of giving an answer they are not sure of (rather than being embarrassed in front of so many other people if they are incorrect)

- to see the different mathematical viewpoints of other students

- to share materials more easily

• Sometimes students can work with the students who sit near them, but other times you might want to form the groups so that students who are struggling are working together. Then you can help them while the other students move forward. Students who need enrichment can also work together so that you can provide an extra challenge for them all at once.

• For students who are not used to working in pairs or groups, you should set down rules of behaviour that require them to attend to the task and to participate fully. You need to avoid a situation where four students are working together, but only one of them is really doing the work. You might post Rules for Group Work, as shown here to the right.

• Once students are used to working in groups, you might sometimes be able to base assessment on group performance rather than on individual performance. For example, suppose a problem requires a student to explain why $-3 + (+4) = +1$. The student hesitates or answers inappropriately. Follow up by asking questions like the following:

- *How would you show* –3?

- *What does it mean to add +4?*

- *How could you use counters to model the sum?*

- *Why can you get rid of some of the counters?*

• Many of the questions in the textbook require students to explain their thinking. The sample *Thinking* in the *Examples* is designed to provide a model for mathematical communication.

The sample Thinking in the Examples is designed to provide a model for mathematical communication.

• One of the ways students communicate mathematically is by describing how they know an answer is right. Even when a question does not ask students to check their work, you should encourage them to think about whether their answer makes sense. When they check their work, they should usually check using a different way than the way they used to find their answer so that they do not make the same reasoning error twice. This will enhance their mathematical flexibility.

Communication

Students should be communicating regularly about their mathematical thinking. It is through communication that they clarify their own thinking as well as show you and their classmates what they do or do not understand. When they give an answer to a question, you can always be asking questions like, *How did you get that? How do you know? Why did you do that next?*

• Communication is practised in small group settings, but is also appropriate when the whole class is working together.

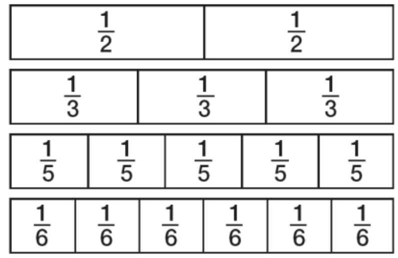
Students will be reluctant to communicate unless the environment is risk-free.

• Students will be reluctant to communicate unless the environment is risk-free. In other words, if students believe that they will be reprimanded or made to feel badly if they say the wrong thing, they will be reluctant to communicate. Instead, show your students that good thinking grows out of clarifying muddled thinking. It is reasonable for students to have some errors in their thinking and it is your job to help shape that thinking. If a student answers incorrectly, it is your job to ask follow-up questions that will help the student clarify his or her own thinking.

MATHEMATICAL TOOLS

Manipulatives

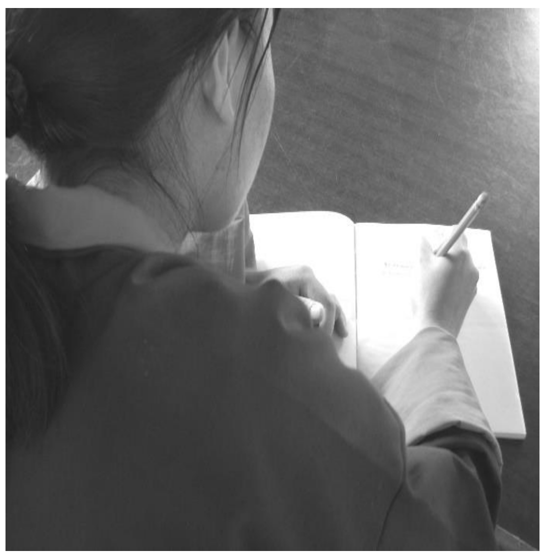
• There is great value in using manipulative materials in mathematics instruction; sometimes, it is essential. For example, the work in Chapter 2 will be greatly enhanced if students have access to fraction strips, grids, and counters. Unit 4 cannot be completed without using interlocking cubes. Other times, for example, in Unit 1, some students can be successful without manipulative materials, but all students will benefit from using them. Students will start to see not only how to perform arithmetic calculations, but why they are done the way they are.



Fraction strip

THE STUDENT NOTEBOOK

It is valuable for students to have a well-organized, neat notebook to look back at to review the main mathematical ideas they have learned. However, it is also important for students to feel comfortable doing rough work in that notebook or doing rough work elsewhere without having to waste time copying it neatly into their notebooks. In addition to the things you tell students to include in their notebooks, they should be allowed to make some of their own decisions about what to include in their notebooks.



Students should be allowed to make some of their own decisions about what to include in their notebooks.

Reprint 2023

UNIT 1 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 1** **TG p. 5**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• 100 Charts (BLM)	All questions
Chapter 1 Whole Numbers and Decimals				
1.1.1 EXPLORE: Divisibility by 3 and 9 (Optional) **SB p. 3** **TG p. 9**	**7-A1 Divisibility: develop and apply rules for 3, 4, 6, 9** • develop meaningful divisibility rules through exploration and models	1 h	• Base ten blocks or Base Ten Models (BLM)	Observe and Assess questions
1.1.2 Divisibility Tests **SB p. 5** **TG p. 13**	**7-A1 Divisibility: develop and apply rules for 3, 4, 6, 9** • develop meaningful divisibility rules through exploration and models	1 h	None	Q2, 5, 10, 14
CONNECTIONS: Casting Out Nines (Optional) **SB p. 9** **TG p. 17**	Make a connection between using divisibility tests and arithmetic computation	15 min	None	N/A
GAME: Divisibility Spin (Optional) **SB p. 9** **TG p. 17**	Practise use of divisibility tests in a game situation	20 min	• Digit cards • Fraction Circle Spinners (BLM) and paper clips	N/A
1.1.3 Lowest Common Multiple **SB p. 10** **TG p. 18**	**7-A2 Common Multiples: use common multiples and least common multiples (LCM) to solve problems** • use various methods to calculate LCM: prime factorisation and listing of multiples **7-A1 Divisibility: develop and apply rules for 3, 4, 6, 9** • understand the usefulness of divisibility rules for mental computations	1 h	None	Q1, 3, 6, 7
1.1.4 Greatest Common Factor **SB p. 13** **TG p. 21**	**7-A3 Common Factors: use common factors and greatest common factor (GCF) to solve problems** • understand that common factors and GCF are helpful to rename fractions in lowest terms • use prime factorisation and the listing of factors in developing GCF **7-A1 Divisibility: develop and apply rules for 3, 4, 6, 9** • understand the usefulness of divisibility rules for mental computations	1 h	None	Q1, 5, 7
CONNECTIONS: Carrom Math (Optional) **SB p. 16** **TG p. 24**	Make a connection between numerical and geometric situations using the greatest common factor	20 min	• Rulers (optional) • Grid paper or Small Grid Paper (BLM) (optional)	N/A

UNIT 1 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Chapter 2 Powers				
1.2.1 Introducing Powers **SB p. 17** **TG p. 25**	**7-A4 Large Numbers: model** • develop models using powers, bases, and exponents to represent repeated multiplication • understand exponents as a means of expressing factors in a compact form • understand terms "squared" and "cubed" to describe powers of two and powers of three • relate "squared" with a 2-D object and "cubed" with a 3-D object	1 h	None	Q3, 4, 8
GAME: Rolling Powers (Optional) **SB p. 19** **TG p. 27**	Practise estimating sizes of powers in a game situation	20 min	• Dice	N/A
1.2.2 Expanded, Standard, and Exponential Forms **SB p. 20** **TG p. 28**	**7-A4 Large Numbers: rename** • investigate exponential, expanded, and standard forms • use expanded forms of numbers to demonstrate understanding of place value as well as exponents	1 h	• Place Value Charts (BLM) (optional)	Q1, 2, 4
Chapter 3 Decimal Operations				
1.3.1 Multiplying Decimals **SB p. 23** **TG p. 31**	**7-B1 Add, Subtract, Multiply, Divide: whole numbers and decimals** • choose an appropriate method (pencil, mental, estimation) for a given situation	1 h	• Small Grid Paper (BLM) (optional) • Ten Thousandths Grid (BLM) (optional)	Q1, 6, 9, 10
1.3.2 Dividing Decimals **SB p. 27** **TG p. 35**	**7-B1 Add, Subtract, Multiply, Divide: whole numbers and decimals** • choose an appropriate method (pencil, mental, estimation) for a given situation	1 h	None	Q3, 5, 6, 10
1.3.3 EXPLORE: Mental Math with Decimals (Optional) **SB p. 30** **TG p. 37**	**7-B2 Properties of Operations: decimals and integers** • apply distributive, associative, and commutative properties in mental computation **7-B8 Add and Subtract Integers and Decimals Mentally: develop and use strategies** • develop and use mental strategies: front-end, compatible numbers, and working by parts	40 min	None	Observe and Assess questions
1.3.4 Order of Operations **SB p. 31** **TG p. 40**	**7-B4 Order of Operations: whole numbers and decimals** • understand why order is important and what the conventional order is (brackets, exponents, division/multiplication, and addition/subtraction)	1 h	None	Q1, 4, 5

UNIT 1 Revision **SB p. 33** **TG p. 42**	Review the concepts and skills in the unit.	2 h	• Base ten blocks or Base Ten Models (BLM) (optional)	All questions
UNIT 1 Test **TG p. 44**	Assess the concepts and skills in the unit.	1 h	• Small Grid Paper (BLM) (optional)	All questions
UNIT 1 Performance Task **TG p. 47**	Assess concepts and skills in the unit.	1 h	• Rulers • Protractors	Rubric provided
UNIT 1 Blackline Masters **TG p. 49**	BLM 1 100 Charts BLM 2 Base Ten Models (Hundreds, Tens, and Ones) BLM 3 Fraction Circle Spinners (for the **Game**) BLM 4 Place Value Charts (Billions to Ones; Periods and Powers) BLM 5 Small Grid Paper BLM 6 Ten Thousandths Grid			

Math Background

• This number unit builds on some of the more familiar content from Class VI.

• The unit focusses on work with whole numbers and decimals. Students will explore divisibility of whole numbers, express multiplication using exponential notation, and multiply and divide decimals.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 5** in **lesson 1.1.2**, where they use divisibility to help solve a real-world problem, in the **Try This** in **lesson 1.1.3**, where they must figure out a number based on clues, in **question 10** in **lesson 1.3.1**, where they apply decimal multiplication to solve a contextual problem, and in **question 5** in **lesson 1.3.4**, where they have to figure out how a calculation was done incorrectly.

• Students use communication frequently as they explain their thinking, such as in **question 14** in **lesson 1.1.2**, where they explain which digits in a number are relevant when testing for divisibility, in **question 8** in **lesson 1.1.4**, where they generalize about the greatest common factor of prime numbers, in **question 11** in **lesson 1.2.1**, where they discuss how exponentiation is like multiplication, in **question 3** in **lesson 1.2.2**, where they compare two numbers in exponential form, and in **question 12** in **lesson 1.3.1**, where they explain the relationship between multiplying whole numbers and decimals.

• Students use reasoning in answering questions such as the **Try This** in **lesson 1.1.2**, where they figure out why a calculation is incorrect, in **question 11** in **lesson 1.1.2**, where they use known information to determine the validity of given statements concerning divisibility, in **question 3** in **lesson 1.1.3**, where they have to work backwards to find numbers with a certain LCM, in **question 5** in **lesson 1.3.2**, where they must provide examples to support their responses concerning claims about decimal division, in **lesson 1.3.3**, where they reason about what calculations make sense to perform mentally, and in **question 2** in **lesson 1.3.4**, where they decide whether the brackets in an expression are actually necessary to describe a calculation.

• Students consider representation in **lesson 1.1.2**, where they use the base ten representation of a number to determine divisibility, in the **Try This** in **lesson 1.1.4**, where they develop a geometric interpretation of greatest common factor, in **lesson 1.2.1**, where they realize that it is more efficient to use powers to represent repeated multiplication, in **lesson 1.2.2**, where they represent whole numbers in different forms, in **lesson 1.3.1**, where they represent a product as the area of a rectangle, and in **lesson 1.3.2**, where they make sense of division by rewriting decimals in different units.

• Students use visualization skills in **lesson 1.1.1**, where they use base ten block models for numbers to explain the divisibility rules, and in the **Try This** in **lesson 1.3.1**, where they create an area model to conceptualise decimal multiplication.

• Students make connections in **lesson 1.1.2**, where they relate divisibility tests to each other, in **lesson 1.1.4**, where they relate GCF and LCM, in the first **Connections** feature, where they relate the game of Carrom to mathematical concepts, and in **lesson 1.2.1**, where they relate squaring and cubing of numbers to their geometric meanings.

Rationale for Teaching Approach

• This unit is divided into three chapters.

Chapter 1 focuses on whole number concepts: divisibility tests, common multiples, including lowest common multiple and common factors, including lowest common factor.

Chapter 2 focuses on powers and forms (exponential, expanded, and standard) of representing whole numbers.

Chapter 3 has students examine multiplication, division, and the order of operations with decimals.

• The two **Explore lessons** allow students to develop rules for divisibility and strategies for mental computation.

• Two **Connections** features appear in **chapter 1**. The first illustrates a useful application of divisibility tests to calculations and the other relates mathematical concepts to a game.

• The unit's **Game** provides an opportunity to apply and practise the divisibility tests.

• Throughout the unit, it is important to encourage flexibility and to accept a variety of approaches from students.

Getting Started

Curriculum Outcomes	Outcome relevance
6 Common Factors: whole numbers **6** Prime Numbers: distinguish from composites **6** Large Numbers: reading and writing **6** Multiply Decimals by Whole Numbers: pictorially, symbolically **6** Multiply Decimals by Decimals: concretely, symbolically **6** Whole Numbers and Decimals: Single-digit Division **6** Estimation Strategies for Multiplication and Division: Whole Numbers and Decimals **6** Divide Decimals by Decimals: estimating and developing algorithms through reasoning **6** Divide Mentally: whole numbers by 0.1, 0.01, 0.001	Students will find the work in the unit easier after they review factors and multiples, prime numbers, place value, and calculations with decimals.

Pacing	Materials	Prerequisites
1 h	• 100 Charts (BLM)	• familiarity with the terms *factor*, *multiple*, *common factor*, and *prime number* • place value from billions through thousandths • multiplying and dividing by powers of 10 • multiplying and dividing by simple decimals

Main Points to be Raised

Use What You Know

• The multiples of a number form patterns.

• A number can be a multiple of many numbers.

• If *a* is a factor of *b*, then *b* is a multiple of *a*.

Skills You Will Need

• A common factor of two numbers is a factor of both numbers.

• To write a number in expanded form, you must consider each digit and its placement in the number.

• You can think of multiplication as determining an area. You can think of division as showing how many groups of one number make up another number.

• Calculations with decimals are just like calculations with whole numbers. The final step involves a place value decision.

• When you multiply or divide by a power of 10, you consider only what happens to the place value of the digits of the original number.

• Rounding involves place value considerations.

Use What You Know — Introducing the Unit

• Before assigning the activity, you may wish to review the meaning of some of the terms that will come up in the activity, particularly *factor* and *multiple*. You could do this using a game format.

For example, you could say "I am thinking of a 2-digit multiple of 4. Try to guess the number by asking me yes-no questions such as ‘Is it also a multiple of 8?’” (A yes-no question can be answered by saying yes or no.) Play the game a few times, encouraging the students to guess the number in as few questions as possible.

• Once you feel confident that students recall the concepts of factor and multiple, students can work alone or in pairs to complete the activity.

• Provide copies of a 100 chart for students to mark on. If they use a pencil, they can erase and reuse their charts many times.

While you observe students at work, you might ask questions such as the following:

• *How did you know that you should mark all the numbers in that column? Did you have to test them all?* (They all end in 2, so they are even numbers. That means 2 is a factor of each.)

• *How did you know those numbers were the tens?* (Because every ten has 2 and 5 as factors.)

• *How many multiples of 4 are in each row? Why is that?* (2 or 3 because there are 10 numbers in a row. 10 ÷ 4 = 2 R 2 and 20 ÷ 4 = 5. That is, every row has at least two multiples of 4 and any two successive rows have exactly 5 multiples of 4. Alternate rows have 2 or 3 multiples of 4.

• *If your clues involve multiples of a big number, like 22, why will it be easier to guess?* (There are only a few multiples to try.)

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions.

• First review the terms *common factor, expanded form, product, quotient, place value column*, and *round* to ensure students can interpret the questions. Refer students to the glossary at the back of the book.

• For **question 3**, note that this mathematics series assumes 1,000,000,000 is 1 billion, whereas others consider 1,000,000,000,000 to be 1 billion

• Encourage students to use mental math to answer **questions 8 and 9**.

• Students can work individually.

Answers

NOTE: *Read about* ***Answers*** *in the student textbook on* ***page xvi*** *in the* ***Introduction*** *to this Teacher's Guide.*

A. It has a factor of 2.

B. i) Numbers in these columns have 2 as a factor

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	85	87	88	89	90
91	92	83	94	95	96	97	98	99	100

Numbers in these columns have 5 as a factor

ii) 60; 60 is a multiple of 4 and has 2, 3, and 5 as factors.

C *Sample responses*:
i) 84;
My secret number has 2, 3, and 7 as factors.
It is also a multiple of 4.
ii) The only numbers in the 100 chart with 7 as a factor are 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98.
Among those, the only numbers with 2 as a factor are 14, 28, 42, 56, 70, 84 and 98.
Among those, the only numbers with 3 as a factor are 42 and 84.
4 is not a factor of 42, so 84 is the only possibility.

1. a) 1, 2, 4 **b)** 1, 3 **c)** 1, 2

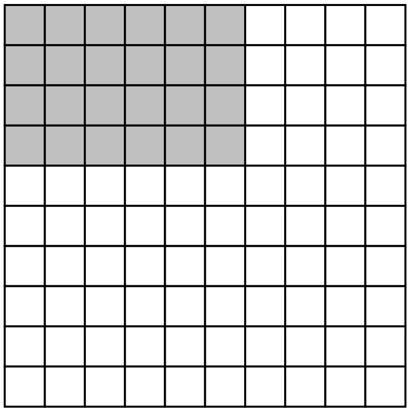
2. 23, 17

3. a) 4 × 100,000 + 1 × 10,000 + 2 × 1000 + 1 × 100 + 5 × 10
4 hundred thousands + 1 ten thousand + 2 thousands + 1 hundred + 5 tens
b) 3 × 100,000 + 6 × 10,000 + 5 × 1000 + 1 × 100 + 2 × 10 + 4 × 1
3 hundred thousands + 6 ten thousands + 5 thousands + 1 hundred + 2 tens + 4 ones
c) 1 × 1,000,000 + 3 × 1000 + 1 × 10
1 million + 3 thousands + 1 ten
d) 1 × 1,000,000,000 + 9 × 100,000 + 1 × 1000 + 1 × 100 + 4 × 10 + 2 × 1
1 billion + 9 hundred thousands + 1 thousand + 1 hundred + 4 tens + 2 ones

4. 0.24; *Sample response*:

0.6

0.4



5. a) 157.0 **b)** 534.24 **c)** 0.84
d) 103.5 **e)** 0.21 **f)** 0.96
g) 0.019 **h)** 0.23

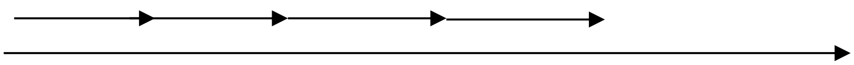
7. a) 105.2 **b)** 22.3
c) 20.8 **d)** 111.5
e) 2 **f)** 7.5
g) 6.75 **h)** 25.5

8. a) 340 **b)** 24.5
c) 0.328 **d)** 0.0234

9. a) Hundredths place
b) Thousandths place
c) Tens place
d) Hundreds place

10. a) 0.2 and 0.24 (0.23 is also acceptable)
b) 14.9 and 14.92
c) 2.0 and 2.00

6. 4; *Sample response*:



0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

Supporting Students

Struggling students

• If students are struggling with the concept of factors and multiples, write some multiplication equations for them and point out the factors and multiples.

For example, for 2 × 4 = 8, point out that 2 and 4 are factors of 8 and that 8 is a multiple of 2 and 4. You can then ask students to find multiples of, say, 5 by writing multiplication equations where 5 is a factor, such as 5 × 1 = ?, 5 × 2 = ?, and so on.

• Some students may have trouble interpreting the clues in **part B** because the sentences are complex. Have them break down the sentences.

For example, for the fourth clue in **part B**, they might think:

First I think about only the numbers that are shaded because of the third clue. For each of those numbers, I have to figure out whether 3 is a factor. If it is, then the number is a multiple of 3. I could start with 3 and see if each number is a multiple of 3. I could think of something I could multiply 3 by to get the number, or I could notice that every third number is a multiple of 3 because the shaded numbers are spaced equally. Once I have found one number that is a multiple of 3, it is easy to find other multiples of 3.

• Some students might benefit from the use of a place value chart for **question 3**. If necessary, remind them of the place value columns.

Billion	Millions			Thousands			Ones		
	H	T	O	H	T	O	H	T	O

• For **questions 4 and 6**, some students may be able to perform the calculation, but have difficulty drawing the picture. The intent of the picture is to ensure that students understand what the operations really mean. You might encourage those students to think first about what they would draw for whole numbers.

For example, they might start with pictures for 4 × 6 or for 8 ÷ 2 and then consider how to change those pictures to include decimals.

Enrichment

• You might challenge students to predict why the secret number in **Use What You Know** was a multiple of 2, 3, 4, and 5 and why there was only one possible answer. (It had 2 as a factor, 3 as a factor, 4 as a factor, and 5 as a factor. The next multiple of all four numbers is 120, which is beyond the chart.)

For example, have them figure out why the number they found was $3 \times 4 \times 5$ and not $2 \times 3 \times 4 \times 5$. (Because it was a multiple of 4, it was automatically a multiple of 2.)

Ask them whether the same thing would happen if they had instead used 2, 3, 8, and 5 and why. (The secret number would have been $3 \times 8 \times 5$, which is not on the chart.)

Chapter 1 Whole Numbers and Decimals

1.1.1 EXPLORE: Divisibility by 3 and 9

Curriculum Outcomes	Outcome Relevance
7-A1 Divisibility: develop and apply rules for 3, 4, 6, 9 • develop meaningful divisibility rules through exploration and models	This optional exploration of the divisibility tests for 3 and 9 provides a conceptual foundation for students to make sense of a broader group of divisibility rules in the next lesson.

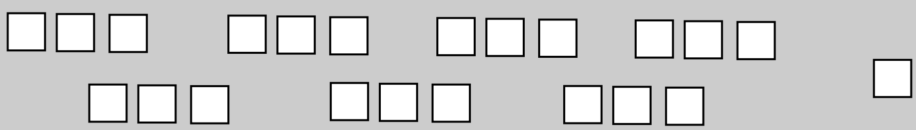
Pacing	Materials	Prerequisites
1 h	• Base ten blocks or Base Ten Models (BLM)	• meaning of quotient and remainder with whole number division • dividing a 2-digit to 4-digit number by a 1-digit number

Exploration

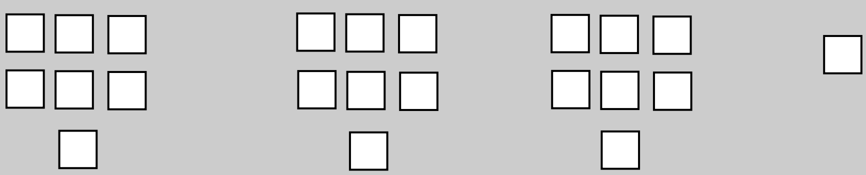
• Work through the introduction (in white) with the students. Make sure that they understand that the remainder must be zero when we say that one number is divisible by another. Also make sure that they understand that we say a number is divisible by, say, 3 if it can be modelled as groups of 3 with none left over or as 3 equal groups with a whole number amount in each group. Show an example.

For example, 22 is not divisible by 3 because when you group 22 in 3s, there is 1 left over. Or, if you make 3 equal groups, there is 1 left over.

22 is not a multiple of 3 because …



there are 7 equal groups of 3 and 1 left over.



there are 3 equal groups of 7 and 1 left over.

• Have students work alone or in pairs for **parts A to H**. You may wish to demonstrate how to complete a row of the chart for **part A**.

For example, if the number were 300, the row would show 300, 3, and 1.

Ask them to use five different 3-digit numbers that they are certain are multiples of 3.

While you observe students at work, you might ask questions such as the following:

• *Could any of your numbers have been in the 200s? How do you know?* (They could have been in the 200s because the multiples of 3 happen every 3rd number, and 3 before 300 is 297.)

• *How did the number in the third column show that the sum of the digits was a multiple of 3?* (There was no remainder when I divided by 3, so I know the number is a multiple of 3.)

• Discuss **parts A to H** with the students to make sure they are proceeding successfully.

• Distribute base ten blocks or Base Ten Models (BLM) for students to complete **parts I to L**.

Observe and Assess

As students work, notice the following:

• Do they successfully choose to use numbers that are multiples of 3 and 9 in their charts?

• Do they understand how to check whether the sum of the digits is a multiple of 3 or 9?

• Do they understand, in **part F**, that each hundred and each ten can be made up of 3s with 1 left over?

• Do they see how the leftovers from the hundreds and tens can be combined with the ones, and do they understand that if these leftovers can be grouped into 3s (or 9s), the number is divisible by 3 (or 9)?

• Do they successfully generalize what they have learned to apply to numbers with other numbers of digits?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and these questions.

• *How do you know that 156 is a multiple of 3, but 157 is not?*

• *Why is 156 a multiple of 3 but not a multiple of 9?*

• *What is the lowest number greater than 156 that is a multiple of 9? How do you know?*

• *How would you explain the strategy of adding digits to show that 414 is a multiple of 9?*

Answers

A. i) to iv)
Sample responses:

Number	Sum of digits	Sum of digits ÷ 3
300	3	1
600	6	2
900	9	3
315	9	3
633	12	4

v) Yes

B. i) No; 6 + 5 = 11; 11 ÷ 3 = 3 R 2
ii) *Sample response*:
301 sum of digits = 4 4 ÷ 3 = 1 R 1
500 sum of digits = 5 5 ÷ 3 = 1 R 2
625 sum of digits = 13 13 ÷ 3 = 4 R 1
998 sum of digits = 26 26 ÷ 3 = 8 R 2

C. i) to iv) *Sample responses*:

Number	Sum of digits	Sum of digits ÷ 9
270	9	1
279	18	2
360	9	1
900	9	1
999	27	3

v) Yes

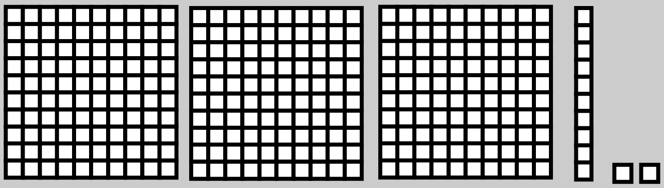
D i) 9 + 1 = 10 and 10 ÷ 9 = 1 R 1

ii) *Sample response*:

800	sum of digits = 8	8 ÷ 9 = 0 R 8
400	sum of digits = 4	4 ÷ 9 = 0 R 4
850	sum of digits = 13	13 ÷ 9 = 1 R 4
660	sum of digits = 12	12 ÷ 9 = 1 R 3

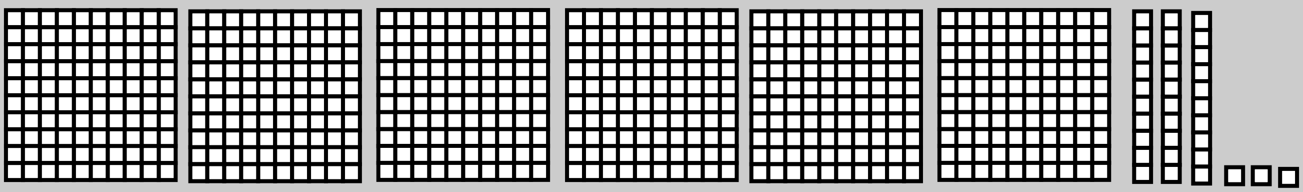
E. i) If the sum of the digits of a number is a multiple of 3 (or is divisible by 3), the number is divisible by 3.
ii) If the sum of the digits of a number is a multiple of 9 (or is divisible by 9), the number is divisible by 9.

F. i) 312



ii) 1 each
iii) 6; the number of ones is the same as the sum of the digits.
iv) Only the 6 ones need to be grouped in 3s and they can.

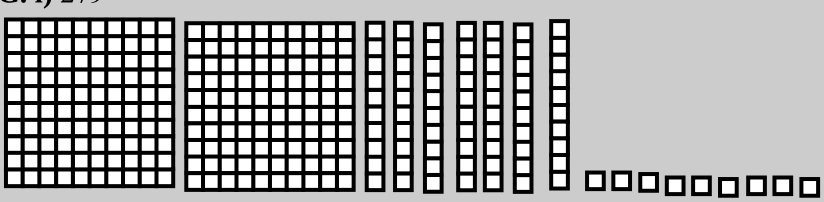
v) *Sample response*:
633



After dividing each hundred and ten model into groups of 3, there is 1 one left over from each.
There are 12 ones altogether; the number of ones is the same as the sum of the digits.
You can divide the ones into groups of 3.

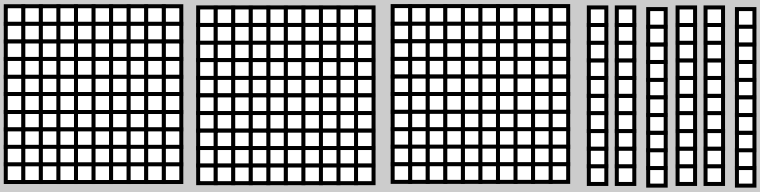
vi) *Sample response*:
For 633:
633 has 6 hundreds, 3 tens, and 3 ones.
When you group each of the 6 hundreds and 3 tens into groups of 3, there is 1 one left over from each because 100 = 99 + 1 and 99 ÷ 3 = 33 and 10 = 9 + 1 and 9 ÷ 3 = 3. That makes 9 ones left over from the hundreds and tens. That leaves you with 12 ones altogether (which is the sum of the digits), which can be divided into groups of 3.

G. i) 279



ii) After dividing each hundred and ten into groups of 9, there is 1 one left over from each
iii) There are 18 ones altogether; the number of ones is the same as the sum of the digits.
iv) You can divide the ones into groups of 9, so after dividing the hundred and ten models into groups of 9 and then dividing the ones into groups of 9, there is nothing left over.

v) *Sample response*:
360



After dividing each hundred and ten into groups of 9, there is 1 one left over from each
There are 9 ones altogether; the number of ones is the same as the sum of the digits
You can divide the ones into groups of 9, so after dividing the hundred and ten models into groups of 9 and then dividing the ones into groups of 9, there is nothing left over.

vi) For 279:
279 has 2 hundreds, 7 tens, and 9 ones.
When you group each of the 2 hundreds and 7 tens into groups of 9, there is 1 one left over from each because $100 = 99 + 1$ and $99 \div 9 = 11$ and $10 = 9 + 1$ and $9 \div 9 = 1$. That makes 9 ones.
That leaves you with 18 ones altogether (which is the sum of the digits), which can be divided into groups of 9.

H. If you modelled a 4-digit number like 4005 (which is divisible by 3), it would have 4 models for thousands and 5 models for ones. Each thousand model, when divided into groups of 3, will have 1 left over because $1000 = 999 + 1$ and $999 \div 3 = 333$. So there would be 4 ones left over plus the 5 ones, which is 9 ones. You can divide 9 ones into groups of 3 or 9.
If you modelled a 5-digit number like 59,130 (which is divisible by 3), it would have 5 models for ten thousands, 9 models for thousands, 1 model for hundreds, and 3 models for tens. Each ten thousand, thousand, hundred, and ten model, when divided into groups of 3, will have 1 left over because
$10{,}000 = 9999 + 1$ and $9999 \div 3 = 3333$; $1000 = 999 + 1$ and $999 \div 3 = 333$; $100 = 99 + 1$ and $99 \div 3 = 33$; and $10 = 9 + 1$ and $9 \div 3 = 3$. So there would be 18 ones left over. You can divide 18 ones into groups of 3.

Supporting Students

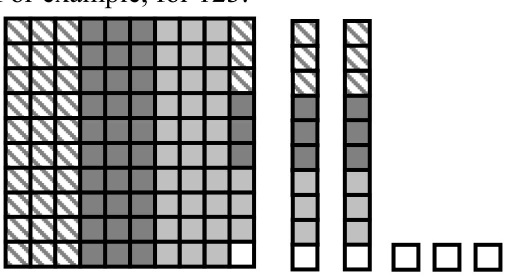
Struggling students

• If students are struggling with selecting numbers that are multiples of 3 (or 9), suggest they multiply 3 (or 9) by various multipliers greater than 40.

For example, they could try $3 \times 55 = 165$.

• If students have difficulty seeing why each hundred and each ten has one left over when you group the numbers in 3s, provide a grid model. They can draw groups of 3 to see the leftovers.

For example, for 123:



Enrichment

• You might challenge students to adapt the ideas in the exploration to create a way to test whether a number is divisible by 4. This will preview what they will learn in the next lesson.

1.1.2 Divisibility Tests

Curriculum Outcomes	Outcome relevance
7-A1 Divisibility: develop and apply rules for 3, 4, 6, 9 • develop meaningful divisibility rules through exploration and models	Students will be able to factor numbers more easily if they use divisibility rules. This skill will simplify calculations of greatest common factors and least common multiples. It will also make work with fractions easier and, in Class VIII, will help with calculating square roots.

Pacing	Materials	Prerequisites
1 h	None	• dividing multi-digit numbers by 1-digit numbers • place value • multiplication facts for 2, 3, 4, 5, 9, and 10 • expressing a number in expanded form

Main Points to be Raised

• A divisibility test is a shortcut to determine whether one number is a multiple of another number.

• When you say that one number is divisible by another number, it is the same as saying that the first number is a multiple of the second number or that the second number is a factor of the first number.

• Divisibility tests are based on understanding that the digit in a particular place value column represents that many tens, hundreds, thousands, etc.

For example, 3 in the hundreds column means 3 × 100, whereas 3 in the ones column means 3 × 1.

• If one number is a multiple of another, then any multiple of the first number is also a multiple of the second number.

For example, because 100 is a multiple of 4, any multiple of 100 is also a multiple of 4.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How might you use addition (or multiplication, or division) to solve the problem?* (You can add nine sets of 85; you can multiply 85 by 9; you can divide 755 by 9.)

• *How do you know the price is between Nu 720 and Nu 810?* (Nu 85 is between Nu 80 and Nu 90; 9 × 80 = 720 and 9 × 90 = 810, so 9 × 85 should be in between.)

• *How can you calculate 9 × 85 using mental math?* (I could add 9 × 5 to 9 × 80.)

• *Does it make sense that the ones digit is 5?* (Yes, if I multiply by 5, the ones digit has to be 5 or 0.)

If students incorrectly multiply 9 × 85, you might suggest they start with 10 × 85 and subtract 1 × 85.

The Exposition — Presenting the Main Ideas

• Present the problem in the second paragraph of the exposition:

- *Can 404 kg of rice be divided into 4 kg packages with no rice left over?*

Ask students to work in pairs to come up with a solution. When they have finished, ask for their strategies. Some students may say that 404 kg can go into 100 packages of 4 kg plus 1 more package, for a total of 101 packages.

• Help students see that a good way to test divisibility is to relate a number to another number they are sure about. Try out this strategy by asking these questions:

- *Is 301 divisible by 2? Is it divisible by 5? Is it divisible by 10? Is it divisible by 3?*

- *Is 998 divisible by 3? Is it divisible by 9? Is it divisible by 4?*

After they have thought about the questions for a while, point out how comparing the 301 to 300 or the 998 to 999 or 1000 makes it easier for them to determine whether 301 and 998 are divisible by 3.

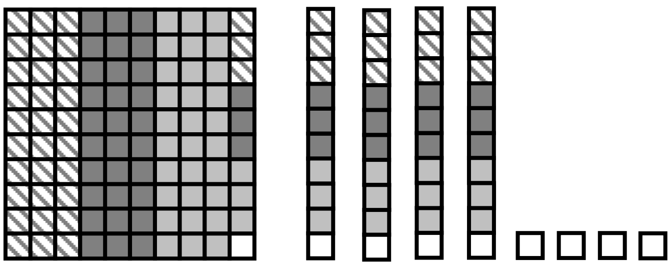
• Introduce the term *divisibility test* to describe a shortcut to determine whether a particular number is divisible by another. If students did the previous **Explore** lesson, remind them that they created divisibility tests for 3 and 9. Ask them to recall those tests.

• Draw attention to the chart of divisibility tests on **page 5**. To make sure students understand them, have them apply each test to a particular number, for example, 351.

• Encourage students to read through the exposition, where the tests are applied to the number 360.

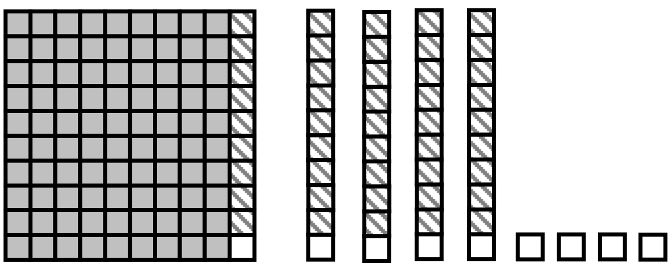
• Have them turn to **page 6**, where the tests for 3, 9, and 4 are explained using base ten models. Suggest that they read through these and explain to a partner what they have learned.

• Test the students' understanding by asking them to explain what would happen if they grouped 144 into 3s, 9s, and 4s.



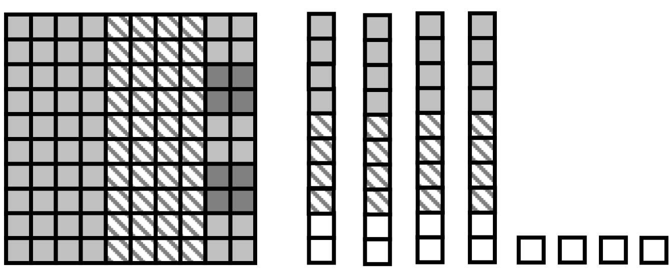
144 grouped in 3s

9 leftovers ÷ 3 = 3



144 grouped in 9s

8 leftovers ÷ 9



144 grouped in 4s

12 leftovers ÷ 4 = 3

Revisiting the Try This

B. This question allows students to make a formal connection between what was done in **part A** and the main ideas presented in the exposition. In this case, students need to apply the divisibility test for 9.

Using the Examples

• Have students work in pairs. One student should become an expert on **example 1** and the other should be the expert on **example 2**. Each student should then explain his or her example to the other student.

• Make sure students understand that when you add the digits (or double the tens digit in the case of 4) using a divisibility test for 3, 9, or 4, the remainder indicates how much more or less than a multiple of 3, 9, or 4 the number is.

For example, for the number 965, 9 + 6 + 5 = 20, which is 2 more than a multiple of 3 or of 9. That means that 965 – 2 = 963 is divisible by 3 or 9. Or, for the number 4, 2 × 6 + 5 = 17, which is 1 more than a multiple of 4. That means 365 – 1 = 364 is divisible by 4.

Practising and Applying

Teaching points and tips

Q 1 to 3: Some students may choose to perform the divisions. Although this is not wrong, encourage them to try the divisibility tests as well.

Q 4 b): If the remainder is 1 after you double 5, add 7, and divide by 4, you know that 3057 – 1 = 3056 is divisible by 4. It also means that when you divide 3057 by 4, the remainder is 1.

Q 5: Many students will not know where to begin. If they are stuck, ask them how they might test 185, 34, and 69 for divisibility by 2, 5, and 3 to solve the problem. Then ask them which of those numbers are not divisible by 2 (185 and 69) and which are not divisible by 3 (185 and 34).

Q 6: Remind students to list all possible digits. When they have finished, encourage them to think about how many answers they got and why that number of answers makes sense. For example, there are three answers for **part a)** because there are three (or four) multiples of 3 in every 10 numbers.

Q 7: Some students may not recognize this as a divisibility question. Suggest they draw a picture to help them see that they are trying to decide whether 987 can be grouped in 9s.

Q 10: You might encourage students to make an organized list of numbers and to eliminate possibilities.

Q 11: This is an important generalization of divisibility. If a number is divisible by *a* and *b*, and if *a* and *b* have no common factors, the number is divisible by their product. This is not the case if *a* and *b* have factors in common.

Q 15: Use the last question of each exercise set as a closure question. It is a way to highlight the most important ideas students have learned in the lesson.

Common errors

• Many students generalize the workings of one divisibility test to apply to other tests.

For example, students will generalize the test for divisibility for 3 and add the digits to see if a number is divisible by 2, 5, 10, or 4. Help them see that this does not work — although the sum of the digits of 13 is even, 13 is not an even number and is not divisible by 2.

• Some students have difficulty remembering how many digits they must consider when they apply a divisibility test. For example, the tests for 2, 5, and 10 require you to look at only one digit, the test for 4 requires you to consider two digits, and the tests for 3 and 9 require you to consider all the digits. Students should focus on place value (the thousands, hundreds, or tens) in terms of what they are dividing by to decide whether that place value has to be considered. For example, you must consider the hundreds to test for divisibility by 3 because 100 is not a multiple of 3. But for divisibility by 4, you do not need to consider the hundreds because 100 is a multiple of 4.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can apply a divisibility test
Question 5	to see if students recognize that using a divisibility test can help them solve a real-world problem
Question 10	to see if students can use divisibility ideas to solve a mathematical problem
Question 14	to see if students can explain a divisibility test

Answers

A. *Sample response*:
• 9 × 85 = 9 × 80 + 9 × 5 = 720 + 45 = 765, so 755 is wrong.
• 85 + 85 + 85 + 85 + 85 + 85 + 85 + 85 + 85 = 170 + 170 + 170 + 170 + 85 = 340 + 340 + 85 = 680 + 85 = 765, so 755 is wrong.
• 755 ÷ 9 = 84 R 8. If 755 were right, the quotient would be 85, but it is not.

B. The total has to be a multiple of 9 because it is 9 of the same amount (9 × Nu 85).
She could have used the divisibility test for 9. She would have known that 755 is not a multiple of 9 because 7 + 5 + 5 = 17 and 17 is not a multiple of 9.

NOTE: Answers and parts of answers that are in square brackets throughout the Teacher's Guide are NOT found in the answers in the student textbook.

1. a) No **b)** Yes **c)** Yes

2. a) No **b)** No **c)** Yes

3. a) Yes **b)** No **c)** Yes

4. a) 5 **b)** 1 **c)** 1 **d)** 1

5. a) Item B **b)** Item C **c)** Item A

6. a) 2, 5, or 8
b) 0, 3, 6, or 9
c) 0, 2, 4, 6, or 8
d) 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9
e) 3
f) 8

7. No; [987 is not divisible by 9.]

8. Yes; [1219 is not divisible by 3.]

9. 32,154

10. *Sample response*: 1206, 2106, 4266

11. A is true [because if a number has both 3 and 4 as factors, it has to be 3 × 4 × ■ = 12 × ■, which is a multiple of 12.]
B is false [because, for example, 18 is divisible by 3 and 9 and not by 27.]

12. 97,864

13. 108

[**14.** *Sample response*:
4376 is 4 thousands, 3 hundreds, 7 tens, and 6 ones:
4 thousands = 4 × (4 × 250), so it is divisible by 4.
3 hundreds = 3 × (4 × 25), so it is divisible by 4.
All you have left to look at are the tens and ones to see if they are divisible by 4.]

[**15.** *Sample response*:
To find the sum of the digits, you only have to add two or more 1-digit numbers. Even for a 5-digit number, the sum cannot be any higher than 45 (9 + 9 + 9 + 9 + 9), so you can quickly recognize whether it is a multiple of 9.]

Supporting Students

Struggling students

• **Questions 8, 10, 11, 12, and 13** may be less suitable for students who struggle with more abstract mathematical thinking. Although these are valuable questions, you may choose not to assign them for certain students.

• If students have difficulty recalling the divisibility tests, you may wish to encourage them to create their own chart to summarize the tests for reference.

• Some students may be able to apply the tests but have difficulty relating the results of a divisibility test to the remainder when they divide a number by 2, 3, 4, 5, 9, or 10. These students may need further work with concrete materials so they can see that relationship.

Enrichment

• You might challenge students to create a divisibility test for 8 that parallels the tests for 2 and 4.

• Students might be interested in finding out that there are divisibility tests for every number.

For 7, the test goes like this:

Write down the number of interest. Remove the ones digit, double the removed digit, and subtract that double from the rest of the number. Use the resulting difference and repeat until you can tell whether or not the difference is divisible by 7. If it is, the original number is divisible by 7.

For example, for 3549:

3549 → 354   354 – 18 = 336 → 336 → 33   33 – 12 = 21

9 × 2 = 18   6 × 2 = 12

21 is divisible by 7,
so 3549 is divisible by 7.

CONNECTIONS: Casting Out Nines

Provide examples to show that casting out nines is useful to show an incorrect calculation, but that this method may mislead you into thinking a calculation is correct when it is not. This occurs only if the incorrect answer is a multiple of 9 more or 9 less than the correct answer.

For example:

• Consider 412 + 397 = 809, which is correct:

Add digits:	7 (which is 4 + 1 + 2)	19 (which is 3 + 9 + 7)	17 (which is 8 + 0 + 9)
Cast out 9s:	7 – 0 = 7	19 – 18 = 1	17 – 9 = 8
Add leftovers:		7 + 1 = 8	

• Consider 412 + 397 = 818, which is incorrect (but 818 is a multiple of 9):

Add digits:	7 (which is 4 + 1 + 2)	19 (which is 3 + 9 + 7)	17 (which is 8 + 1 + 8)
Cast out 9s:	7 – 0 = 7	19 – 18 = 1	17 – 9 = 8
Add leftovers:		7 + 1 = 8	

• If the error were not a multiple of 9 and the incorrect sum were written as, say, 810, the sum of the digits would not be 8 (it would be 9) and the answer would be clearly incorrect.

Answers

1. 3489 + 2379 = 5868 Check: 3 + 4 + 8 + 9 = 24; 24 – 18 = 6 2 + 3 + 7 + 9 = 21; 21 – 18 = 3 6 + 3 = 9; 9 – 9 = 0 5 + 8 + 6 + 8 = 27; 27 – 27 = 0 It works.	**2.** 1425 – 387 = 1047 Check: 1 + 4 + 2 + 5 = 12; 12 – 9 = 3 3 + 8 + 7 = 18; 18 – 18 = 0 3 – 0 = 3 1 + 0 + 4 + 7 = 12; 12 – 9 = 3 It works.	**3.** 25 × 38 = 950 Check: 2 + 5 = 7 3 + 8 = 11; 11 – 9 = 2 7 × 2 = 14; 14 – 9 = 5 9 + 5 + 0 = 14; 14 – 9 = 5 It works.

GAME: Divisibility Spin

• This game provides a lot of practice with the divisibility tests for 2, 3, 4, 5, 9, and 10.

• Students need to recognize that all numbers are divisible by 1 and that a number is divisible by 6 when it is divisible by 2 and by 3.

• To play the game, students need to draw a circle and divide it into four equal sections. They then need to divide each section in half. The numbers in the eight sections should duplicate the numbers in the book (that is, all the numbers from 1 to 10 except 7 and 8).

• When students arrange their cards to get a 3-digit number, there are always six possible numbers if the digits are different. If two of the digits are the same, there are three possible numbers. If all the digits are the same, there is only one number.

• Students could record the results in a chart like this:

Round	Score	Total

• When the game is over, you might ask students to indicate which number combinations led to the most points.

• For a variation to the game, students could draw four cards and create 4-digit numbers. Because there could be as many as 24 numbers, they may simply write down 6 numbers that are possible.

1.1.3 Lowest Common Multiple

Curriculum Outcomes	Outcome relevance
7-A2 Common Multiples: use common multiples and least common multiples (LCM) to solve problems • use various methods to calculate LCM: prime factorisation and listing of multiples **7-A1 Divisibility: develop and apply rules for 3, 4, 6, 9** • understand the usefulness of divisibility rules for mental computations	It is useful for students to recognize the lowest common multiple when they work with fractions (to find common denominators). This skill is also helpful for solving certain real-world problems.

Pacing	Materials	Prerequisites
1 h	None	• factoring into primes • listing multiples of a number

Main Points to be Raised

• A common multiple is a number that is a multiple of two or more other numbers.

For example, 18 is a common multiple of 6 and 9 because it is a multiple of both 6 and 9.

Two or more numbers have an infinite number of common multiples.

• We refer to the lowest common multiple as the LCM. It is the least of the set of positive common multiples. There is only one lowest common multiple for each group of two or more numbers.

• You can find the lowest common multiple of two numbers by listing the positive multiples of both numbers and looking for the least number that appears on both lists.

• You can also find the lowest common multiple of two numbers by listing the prime factors of each number and creating a new number using all of the prime factors, without repeating factors already listed.

For example, for $2 \times 2 \times 7$ and $2 \times 3 \times 7 \times 7$, you first list the factors $2 \times 2 \times 7$ and then add only $3 \times 7 \times 7$ from the other number because the 2 is repeated. You end up with the factors $2 \times 2 \times 3 \times 7 \times 7$.

• The lowest common multiple of two numbers is their product if they have no factors other than 1 in common.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know that there are not exactly 40 students in the class?* (If there were 40 students, there would be students left over when they made groups of 9 or 12.)

• *What are the possible numbers if you know that there are none left over with groups of 9?* (Multiples of 9 are 9, 18, 27, 36, 45, 54, … .The only likely numbers for a class are 27, 36, 45, or maybe 54.)

• *Which of those make groups of 12 without leftovers? How do you know?* (Only 36, because $36 = 3 \times 12$.)

The Exposition — Presenting the Main Ideas

• Write the fractions $\frac{1}{6}$ and $\frac{2}{9}$ on the board. Tell the students you want to write the two as equivalent fractions with the same denominator. Ask how to do it.

Once students have suggested that the denominator could be 18, ask how they know. If they do not suggest it, show two ways to prove it:

- First, list the multiples of each number and show that 18 is on both lists (do not include 0).

- Then, write each denominator as the product of prime factors ($6 = 2 \times 3$ and $9 = 3 \times 3$) and notice that the factorisation $2 \times 3 \times 3$ includes both 2×3 and 3×3.

Point out that 18 is a common multiple of 6 and 9. Write the words *common multiple* on the board.
Tell the students that this means it is a multiple of both numbers.

Ask students what other numbers are common multiples of 6 and 9 and how they know. Ask why 18 is the least of those numbers. Tell students it is called the *lowest common multiple* because it is the least number. (Some people use the term *least common multiple* instead of *lowest common multiple*.)

• Have students open their texts to **page 10**. Point out that the exposition shows that 18 is also a common multiple of 2 and 3, although it is not the lowest common multiple.

• Point out that the LCM of 14 and 35 is calculated in the text much like they calculated the LCM of 6 and 9.

Revisiting the Try This

B. This question allows students to see how the concept of LCM is useful in solving a real-world problem about grouping.

Using the Examples

• List the three problems in the examples on the board. Ask students to work in pairs to solve the problems and then compare their answers to the solutions in the text. In **example 1**, make sure they understand which factors from the three prime factorisations are included in the lowest common multiple and which are not.

Practising and Applying

Teaching points and tips

Q 1: You can direct students to review **example 1** to help them see how to deal with three numbers instead of two. Suggest that one approach is to find some common multiples for the first two numbers and then for the second two numbers. Next, they can look at the two lists to find multiples in common.

Q 2: Some students will simply calculate the values for each side of the equation and then see if the values are equal. Other students will use reasoning.

For example, in **part b)** a student might reason that the common multiples of 5 and 8 are even because 8 is even. That means the same numbers are also common multiples of 10 and 8.

Q 3: Students must work backwards. They might notice that the factors of 45 are $3 \times 3 \times 5$ and create two numbers using different combinations of these factors, such as 3×3 and 5, or 3×3 and 5×3, or $3 \times 3 \times 5$ and 1.

Q 4: You might provide examples for students to consider. For example:

The lowest common multiple of 3, 5, and 15 is 15 and the lowest common multiple of 4, 6, and 12 is 12.

This may help them notice that one of the numbers must be a multiple of the other two numbers.

Q 6: In order for a number to be a common multiple of two other numbers, its prime factorisation must include all the factors of each of the two numbers, and possibly some more factors. Because the LCM includes all those factors, each of the other numbers is a multiple of it.

For example, the LCM of $2 \times 3 \times 5$ and $3 \times 3 \times 5 \times 7$ is $2 \times 3 \times 3 \times 5 \times 7$. Every other common multiple includes those factors and possibly others. For that reason, they are multiples of the LCM.

Q 7: To answer this question, students must realize they are looking for the LCM of 2 and 3.

Q 8: Students should realize that the product of two numbers is always a common multiple of those two numbers. It is the LCM only if the only common factor of the two numbers is 1.

Common errors

• When they use the prime factorisation method for calculating LCMs, many students will duplicate factors that are not necessary.

For example, for the LCM of $3 \times 2 \times 2$ and $3 \times 2 \times 5$, they will write $3 \times 3 \times 2 \times 2 \times 2 \times 5$. This is a common multiple, but it is not the LCM.

Once students have calculated the LCM, encourage them to check by listing the multiples of both numbers to see if they find a lower common multiple.

• Some students will not factor all the way down to primes and will therefore calculate the wrong LCM.

For example, to calculate the LCM of 12 and 18, a student might write $12 = 4 \times 3$ and $18 = 2 \times 3 \times 3$ and use a common multiple of $4 \times 3 \times 2 \times 3$, not realizing that there are two 2s buried in the 4.

Encourage them to always check that all of their factors are primes when they use the prime factorisation method.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can calculate a LCM
Question 3	to see if students can work backwards to see why a particular number is the LCM of other numbers
Question 6	to see if students can use mathematical reasoning to explain how common multiples of two numbers are related
Question 7	to see if students can use the LCM to solve a real-world problem

Answers

A. 36 students	**B.** 36 is LCM (9, 12).
1. a) 140; [5 × 2 × 2 × 7 = 140] **b)** 32; [2 × 2 × 2 × 2 × 2 = 32] **c)** 114; [2 × 19 × 3 = 114] **d)** 1210; [5 × 2 × 11 × 11 = 1210] **2. a)** True [*Sample response*: LCM (7, 18) = 7 × 2 × 3 × 3. Because 14 = 7 × 2 and the 2 is already in 18, you use the same factors for LCM (14, 18).] **b)** True [*Sample response*: LCM (5, 8) = 5 × 2 × 2 × 2. Because 10 = 5 × 2 and the 2 is already in 8, you use the same factors for LCM (10, 8).] **c)** False [*Sample response*: LCM (6, 11) = 2 × 3 × 11, but 12 = 2 × 3 × 2, so you need an extra 2 for LCM (12, 11).] **3.** *Sample response*: 1 and 45, 3 and 45, 15 and 9	[**4.** *Sample response*: The other numbers are factors of the number that is the LCM. The largest of the three numbers is a multiple of each of the other two numbers. For example, if the numbers were 5, 9, and 45, the LCM would be 45.] **5.** No, [because it has to be a multiple of both numbers. The lowest multiple of a number (other than 0) is the number itself.] **6. a)** 30 [**b)** Common multiples of 30 must have 2 × 3 = 6 and 2 × 5 = 10 as factors. That means the number must be 2 × 3 × 5 × ■ = 30 × ■, which is a multiple of 30.] **7.** 5 times; [the LCM of 2 and 3 is 6, so he does both tasks every 6 days in 30 days, which is 5 times.] **8.** No; [*Sample response*: It might work for LCM (8, 15), but it does not always work, e.g., LCM (2, 4) = 4, not 2 × 4.]

Supporting Students

Struggling students

• Struggling students might have difficulty with **questions 4, 5, and 6**. You may choose either not to assign these to struggling students or to have them work with a non-struggling partner on these questions.

• You might encourage students who struggle with prime factorisation to use strategy 1 from the exposition for calculating the LCM.

Enrichment

• You might ask students to figure out why the procedure below is a way to calculate the LCM of Number 1 and Number 2:

- Use many copies of squares of side length Number 1 to form bigger squares.

- Do the same with Number 2.

- Find the side length of the smallest square that is common to both groups.

For example, for the LCM of 4 and 6, the squares for 4 would be 4 by 4, 8 by 8, 12 by 12, and so on. For 6, the squares would be 6 by 6, 12 by 12, 18 by 18, and so on. The smallest square found in both groups is 12 by 12. Therefore 12 is the LCM of 4 and 6.

1.1.4 Greatest Common Factor

Curriculum Outcomes	Outcome relevance
7-A3 Common Factors: use common factors and greatest common factor (GCF) to solve problems • understand that common factors and GCF are helpful to rename fractions in lowest terms • use prime factorisation and the listing of factors in developing GCF **7-A3 Divisibility: develop and apply rules for 3, 4, 6, 9** • understand the usefulness of divisibility rules for mental computations	It is helpful for students to recognize the greatest common factor when they work with fractions (to find equivalent fractions in lower terms) and when they solve certain real-world problems.

Pacing	Materials	Prerequisites
1 h	None	• factoring into primes • listing multiples of a number

Main Points to be Raised

• A common factor is a number that is a factor of a group of two or more other numbers.

For example, 3 is a common factor of 6 and 9 because it is a factor of both numbers.

• We refer to the greatest common factor as the GCF. It is the greatest number in the set of common factors. There is only one GCF for two or more numbers, even though there may be other common factors.

• You can find the GCF of two numbers by listing the factors of each number and looking for a number that appears on both lists.

• You can also find the GCF of two numbers by listing the prime factors of each number and creating a new number using all of the prime factors that appear on both lists. Because 1 is always a common factor of two numbers, 1 is the GCF if there are no other factors in common.

• You can simplify a fraction by dividing the numerator and the denominator by the greatest common factor.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know that the square cannot be 6 cm by 6 cm?* (A 6 cm square would not fit against a 135 cm length. There would be 3 cm left over at the end.)

• *How do you know that the students could put together 1–by-1 squares to make the rectangle?* (The students could use 120 rows of 135 squares to fill it.)

• *Why does the size of the square have to be a factor of 120 and a factor of 135?* (If it were not a factor, it would not fit in exactly.)

• *How could you use divisibility tests to help you figure out some possible square sizes?* (Because the side length has to be a factor, I would need to know by what numbers 135 and 120 are divisible. I can use divisibility tests to help with that.

The Exposition — Presenting the Main Ideas

• Write the fraction $\frac{16}{20}$ on the board. Ask students for an equivalent fraction in lower terms. They might suggest $\frac{8}{10}$ or $\frac{4}{5}$. Ask how they got the fractions (probably by dividing both the numerator and the denominator by a number).

Ask why the number they divided by had to be a factor of both 16 and 20 (so that the new terms are both whole numbers). Write the term *common factor* on the board. Explain that in writing the equivalent fractions, students were actually finding common factors of 16 and 20.

• Ask if there is a fraction with lower terms than $\frac{4}{5}$. Ask what the numerator and denominator of $\frac{16}{20}$ were divided by to get $\frac{4}{5}$. When students realize the value is 4, explain that 4 is the *greatest common factor* of 16 and 20. (This is sometimes also called the *highest common factor*, although the text does not use that term.) Because there is no greater common factor, there is no way to write the fraction in lower terms.

• Suggest that students examine **page 13** in the student text to see how to determine the greatest common factor. They can choose to list the factors, usually in order, starting at 1, and look for numbers on both lists. Or, they can factor both numbers into prime factors and look for primes that are factors of both numbers. (The term *factor* is used rather than *factorise*, but both are correct.)

• Point out that because 4 and 5 have no common factors other than 1, there is no way to write $\frac{4}{5}$ in lower terms. Make sure students realize that any two numbers always have 1 as a common factor. Also make sure they realize that the term *common factor* only applies when you are working with whole numbers.

Revisiting the Try This

B. This question encourages students to recognize a geometric interpretation of greatest common factor.

Using the Examples

• On the board, list the two problems in the examples. Ask students to work individually or in pairs to solve the problems and then compare their answers to the solutions in the text. Make sure students realize that **example 2** describes a general rule: the product of the GCF and the LCM of two numbers is always equal to their product of the two numbers. The reason for this is that when you write both original numbers in prime factored form, you create the LCM by multiplying all the factors without any unnecessary repetitions. The GCF includes only the factors that would have been repeated. Together, the two sets of factors include both complete sets of factors of the original number.

For example,

$40 = 2 \times 2 \times 2 \times 5$

$36 = 2 \times 2 \times 3 \times 3$

LCM = $2 \times 2 \times 2 \times 5 \times 3 \times 3$ (notice that the 2s in 36 are not used because they are already there from 40)

GCF = 2×2 (the repeated factors that were not included)

LCM × GCF = $2 \times 2 \times 2 \times 5 \times 3 \times 3 \times 2 \times 2 = (2 \times 2 \times 2 \times 5) \times (2 \times 2 \times 3 \times 3) = 40 \times 36$

Practising and Applying

Teaching points and tips

Q 1 d): Students may not be comfortable calculating the GCF of three numbers. Help students see how to generalize the process they used for two numbers.

Q 3: Some students may recognize right away that they need common factors of 56 and 64. Others will simply try different values.

Q 4: The purpose of this question is to help students see that the GCF of two numbers must always be a factor of the LCM. This is because the GCF is a factor of each of the two numbers and the LCM must include all the factors of both numbers in it.

Q 6: Students need to work backwards. They might use the idea that the LCM × GCF = the product of the numbers, so the two numbers must multiply to $10 \times 300 = 3000$. Any combination of numbers with that product would work as long as they both include 2×5 as factors and have no other factors in common. Possibilities are 2×5 and $2 \times 5 \times 30$ or $2 \times 5 \times 3$ and $2 \times 5 \times 10$. Once they have written 2×5 and $2 \times 5 \times 30$, they just need to rearrange the 30 into factors and put some factors with the first number and some with the second number.

Q 7: It may not be obvious to students that because 2 is the GCF of 4 and 6, they could choose to calculate the cost of 2 cakes at each store to compare.

Q 8: This question emphasizes that many numbers have a GCF of 1.

Common errors

• Many students have difficulty with the idea that if there are no common factors, we still write that the common factor is 1.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can calculate a GCF
Question 5	to see if students can communicate about a valuable application of GCF
Question 7	to see if students can recognize how to use the GCF to solve a real-world problem

Answers

<table>
<tr><td>A. 15 cm by 15 cm</td><td>B. 15 is GCF (120, 135).</td></tr>
<tr><td>1. a) 2 b) 7 c) 24 d) 2

2. a) 1 b) 4

3. Sample response:
8; [If there were 8 rows of 8 chairs and 7 students sat in each of the 8 rows, that would be 56 students.]

4. No; [The LCM of any number is a multiple of the GCF, and 50 is not a multiple of 6.]

[5. Sample response:
If you divide 30 and 40 by the GCF, you get an equivalent fraction in lowest terms.]

6. Sample response:
10 and 300
20 and 150</td><td>7. a) The price is less in Store B; [Sample response:
GCF (4, 6) = 2, so you could compare the price of 2 cakes at both stores.
In Store A, 2 cakes cost Nu 180, but in Store B, 2 cakes cost Nu 170. The price is less in Store B.]
b) The price is less in Store B; [Sample response:
LCM (6, 4) = 12, so you could compare the price of 12 cakes at both stores.
In Store A, 12 cakes cost Nu 1080, but in Store B, 12 cakes cost Nu 1020. The price is less in Store B.]

8. a) 1; [Each prime number is 1 multiplied by itself. If the primes are different, the other factor will be different, so 1 is the only common factor.]
b) 1; [If numbers are not 2 apart, 3 apart, or 4 apart and so on, they cannot both be multiples of 2, 3, or 4 and so on. That means the only factor they can have in common is 1.]

[9. a) 1 is a factor of every number, so it is a common factor of any two numbers.
b) When one of the numbers is a multiple of the other.]</td></tr>
</table>

Supporting Students

Struggling students

• Remind students who have difficulty calculating the GCF to use strategies they learned earlier for finding factors of numbers by using the factor rainbow or divisibility tests.

• Some students may find **questions 4, 8, and 9** difficult. You may choose not to assign these to struggling students.

Enrichment

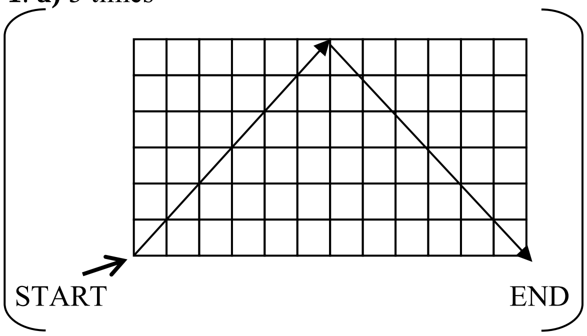
• Ask students to create other problems like **question 7** that are solved using the greatest common factor.

CONNECTIONS: Carrom Math

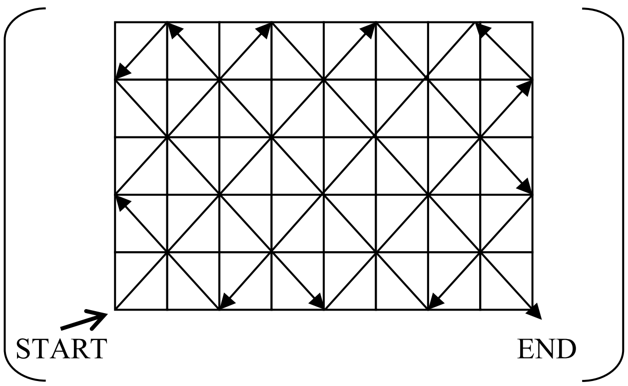
This connection provides a very interesting application of greatest common factor and least common multiple.

Answers

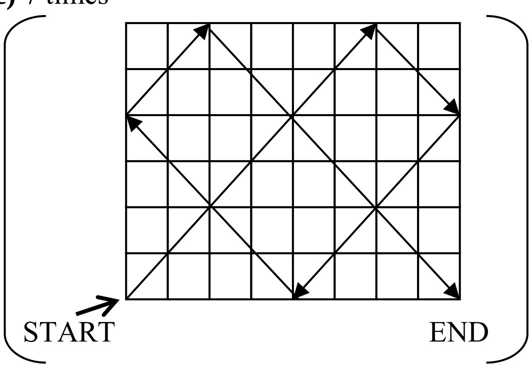
1. **a)** 3 times



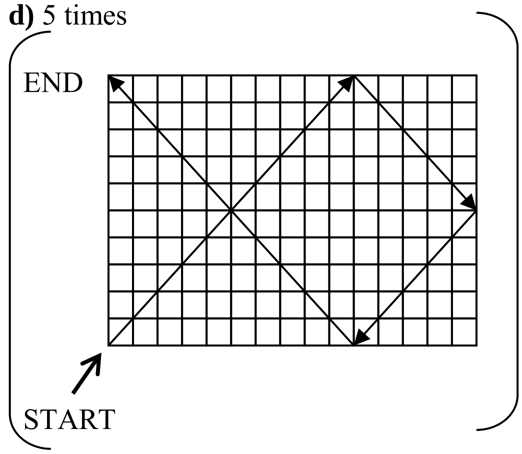
b) 13 times



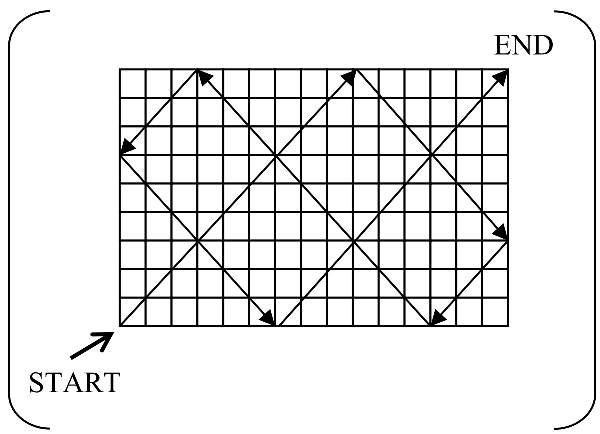
c) 7 times



d) 5 times



2. [**a)** Add the length and width and divide by their greatest common factor.]
b) 8; [I predict 8 because (9 + 15) ÷ 3 = 8; Yes]



Chapter 2 Powers

1.2.1 Introducing Powers

Curriculum Outcomes	Outcome relevance
7-A4 Large Numbers: model • develop models using powers, bases, and exponents to represent repeated multiplication • understand exponents as a means of expressing factors in a compact form • understand terms “squared” and “cubed” to describe powers of two and powers of three • relate “squared” with a 2-D object and “cubed” with a 3-D object	As students move into higher classes in mathematics, they will be expected to use and interpret exponential notation. It is important that they learn why this notation is useful and why we use some of the language we do to describe powers.

Pacing	Materials	Prerequisites
1 h	None	• ability to multiply whole numbers • formula for the area of a square • formula for the area of a cube

Main Points to be Raised

• You can use a base and an exponent to write a power as a shortcut to writing out a long repeated multiplication. The exponent tells how many times the base is multiplied by itself. Note that the word *power* refers to the full amount (the base raised to the exponent, not just the exponent). Powers are only used when the same number is being multiplied repeatedly.

• You can read 3^5 as "three to the fifth" or "three raised to the fifth power" or "3 raised to the power of 5."

• The reason we use the term *squared* to describe raising a number to the second power or *cubed* to describe raising a number to the third power relates to area and volume. The area of a square is found by multiplying the side length by itself twice and the volume of a cube by multiplying the edge length by itself three times, so we use the terms *squared* and *cubed*.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do the values change from one day to the next?* (They double each day.)

• *Why is the value on Day 8 not double the value from Day 4?* (The amount on Day 5 is double the value of Day 4 and then it just keeps growing, so it would be much more than that by Day 8)

• *Why might someone say that this pattern of rice grains is growing very quickly?* (The increase in the number of grains gets to be more and more. For example, the increase is only 1 grain from Day 1 to Day 2, but the increase is 32 grains from Day 6 to Day 7.)

The Exposition — Presenting the Main Ideas

• Read through the exposition with the students. Draw their attention to the power in the middle and make sure they understand what the base and exponent are and that the entire amount is called a power.

• Make sure students recall the formula for the area of a square and for the volume of a cube so they can make sense of the last part of the exposition.

Revisiting the Try This

B. This question allows students to use the exponential notation that was introduced in the exposition.

Using the Examples

• Assign pairs of students to read through the two examples. One student in each pair should be responsible for **example 1** and the other for **example 2**. They should then teach each other what they have learned.

Practising and Applying

Teaching points and tips

Q 2: You may ask students how writing 8^7 or 4^9 is different from writing 7^8 or 9^4.

Q 3: Make sure students realize they do not have to find the value of each of these powers.

Q 4: It is important for students know how to represent what a power means and not to just calculate it.

Q 6: This question is designed to reinforce the notion that order matters when you create powers.

Q 8: You might ask students why powers could not be used to describe the number of small squares if each medium-sized square contained, for example, 9 small squares rather than 4 small squares, but there were still four medium-sized squares.

Q 9: Some students will need some hints to get them thinking about 0 or 1.

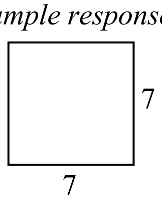
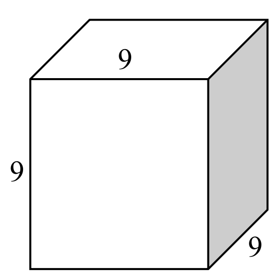
Common errors

• Some students confuse the base and the exponent. Remind them what each represents. You might write a note on the board showing that, for example, $3^5 = 3 \times 3 \times 3 \times 3 \times 3$ (and not $5 \times 5 \times 5$).

Suggested assessment questions from Practising and Applying

Question 3	to see if students can represent a repeated multiplication as a power
Question 4	to see if students can interpret the meaning of a power
Question 8	to see if students can apply the use of powers to a problem

Answers

A. i) 128 **ii)** 512	**B.** 2^7 and 2^9
1. a) Base = 3, Exponent = 6 **b)** Base = 4, Exponent = 10 **c)** Base = 1, Exponent = 2 **d)** Base = 0, Exponent = 4 **2. a)** $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$ **b)** $9 \times 9 \times 9 \times$ 9 **b)** $9 \times 9 \times 9 \times 9$ **3. a)** 6^7 **b)** 8^6 **c)** 2^8 **4.** *Sample responses*: **a)**  **b)** 	**5.** *Sample responses:* **a)** Seven squared **b)** Nine cubed **6. a)** 3^2; by 1 **b)** 5^3; by 25 **c)** Same value **d)** 3^5; by 179 **7.** 6; [$2^1 = 2$ and $2^6 = 64$, so 2^2, 2^3, 2^4, and 2^5 must be in between. Since $2^7 = 128$, there are 6 powers altogether.] **8.** 4^3 **9.** No; [*Sample response*: $1^2 = 1^3$] **10.** No; [It is not the same number being multiplied by itself four times.] [**11.** *Sample response*: An exponent tells you the number of times the base appears in the product.]

Supporting Students

Struggling students

• Most students will not struggle with the notation for powers. They should realize that writing 3^5 as $3 \times 3 \times 3 \times 3 \times 3$ is a similar idea to writing 3×5 as $3 + 3 + 3 + 3 + 3$; it is a shortcut.

GAME: Rolling Powers

• In this game, a student who does not know the base before predicting (in other words, if he or she were to predict before either die was rolled), would be just as likely to get a value below 28 as to get a value above 28.

This is because there are an equal number of values above 28 and below 28.

These values are less than 28:

$1^1, 1^2, 1^3, 1^4, 1^5, 1^6, 2^1, 2^2, 2^3, 2^4, 3^1, 3^2, 3^3, 4^1, 4^2, 5^1, 5^2, 6^1$

And these values are greater than 28:

$2^5, 2^6, 3^4, 3^5, 3^6, 4^3, 4^4, 4^5, 4^6, 5^3, 5^4, 5^5, 5^6, 6^2, 6^3, 6^4, 6^5, 6^6$

• Once they know the base, the chances of predicting correctly increase.

For example, if the base is 1, you know you should choose less than 28 and you will always be right.
If the base is 4, you have a better chance of being right if you choose greater than 28.

1.2.2 Expanded, Standard, and Exponential Forms

Curriculum Outcomes	Outcome relevance
7-A4 Large Numbers: rename • investigate exponential, expanded, and standard forms • use expanded forms of numbers to demonstrate understanding of place value as well as exponents	To prepare for later work using scientific notation, students need to become comfortable with writing numbers not only in standard and expanded form, but also in exponential form.

Pacing	Materials	Prerequisites
1 h	• Place Value Charts (BLM) (optional)	• familiarity with standard and expanded forms of numbers through the billions

Main Points to be Raised

• We can represent the place value columns by powers of 10. The exponent decreases by one each time you move to the next column to the right.

• You can write a whole number using standard form, expanded form, or exponential form.

For example, in standard form 3,210,000 is
3 × 1,000,000 + 2 × 100,000 + 1 × 10,000.
In expanded form it is
3 millions + 2 hundred thousands + 1 ten thousand.
In exponential form it is $3 \times 10^6 + 2 \times 10^5 + 1 \times 10^4$.

• When you raise 10 to a power, the exponent tells how many zeros follow the digit 1 in the standard form of the number.

For example, 10^8 is 100,000,000.

• You can think of the place value columns in periods of three to make it easier to read numbers. The periods students in Class VII need to use are the ones period, the thousands period, the millions period, and the billions period. Each period consists of hundreds, tens, and ones of the unit for that period.

NOTES:

• A *lakh* is a unit in the Indian numbering system. One lakh is equal to one hundred thousand. When describing lakhs, the comma is not placed the way it is for other numbers.

For example, 3 million (30 lakh) would be written as 30,00,000 instead of as 3,000,000.

• This mathematics series assumes 1,000,000,000 or 10^9 to be 1 billion, whereas others consider 1,000,000,000,000 or 10^{12} to be 1 billion.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Why was there no part including thousands in the expanded form?* (There was a 0 in the thousands place.)

• *Does it matter what the non-zero digits are when you are predicting how many parts there will be in the expanded form of a number?* (No. It does not matter if there are, for example, 2 thousands or 3 thousands. You will still need a part for thousands.)

• *Which digits do you need to pay attention to so you can predict the number of parts in the expanded form?* (You need to count only the non-zero digits.)

• *Can a greater number be written with fewer parts in expanded form?* (Yes. For example, 1,000,000 can be written with only one part, but 342, which is much less, requires three parts.)

The Exposition — Presenting the Main Ideas

• Ask students to determine the value of 10^2 and of 10^3. Ask students to predict how much 10^4 and 10^5 are.

• Write the number 1,002,300,040 on the board and ask students to write it in expanded form. Talk about how it is in standard form when it is written as 1,002,300,040.

• On the board draw a place value chart like the one on **page 20**, leaving the row with the powers of 10 blank, and write the number 1,002,300,040 on the chart.

- Label the hundreds column with 10^2, the thousands column with 10^3, and the ten thousands column with 10^4.

- Ask students to predict what powers of 10 to write to the left of ten thousands. Then ask what power to write in the tens column. Point out that the exponents increase by 1 as you go left and decrease by 1 as you go right. • Show students how to write 1,002,300,040 in exponential form. Then ask them to write 2,324,010 in both expanded form and exponential form. • Encourage students to read through the exposition and ask any questions they might have. Make sure they understand the idea that the exponent for 10 tells the number of zeros after the 1 when the number is written in standard form. Help them notice that the place value periods are defined by 10^3, 10^6, 10^9, and so on, that is, exponents that are multiples of 3.

Revisiting the Try This

B. Students have the opportunity to notice the close link between the expanded form and the exponential form of a number.

Using the Examples

• Write the problem from the example on the board. Ask students to solve it and then compare their solutions with the two solutions in the text. Take a poll to find out how many students solved it as in **solution 1**, how many as in **solution 2**, and how many in a different way.

Practising and Applying

Teaching points and tips

Q 2: Students can use either the expanded form using words or the expanded form using only symbols.

Q 4 and 5: These questions are designed to help students realize that the greatest power of 10 is related to the first part of the exponential notation and defines the number of digits a number has.

For example, a number with a first part of 10^6 has 7 digits.

Q 7: This question will assess whether students see the connection between the powers of 10 that describe a place value column and the fact that we regroup when there are 10 in any place value column.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can represent a number using exponential form
Question 2	to see if students can interpret a number expressed in exponential form
Question 4	to see if students can reason about the relationship between exponential form and the number of digits in a whole number

Answers

A. i), ii), and iii) Three *Sample response*: Each number has three non-zero digits and there is one part of the expanded form for each non-zero digit.	**B. i)** $1 \times 10^7 + 2 \times 10^3 + 3$ $1 \times 10^9 + 3 \times 10^8 + 2 \times 10^4$ $2 \times 10^8 + 3 \times 10^1 + 2$ **ii)** They each have three parts added together because they each have three non-zero digits.
1. a) $3 \times 10^7 + 4 \times 10^6 + 2 \times 10^2$ **b)** $3 \times 10^6 + 4 \times 10^3 + 5 \times 10^2 + 2$ **c)** $6 \times 10^8 + 2 \times 10^7 + 3 \times 10^5 + 5 \times 10^4$ **d)** $1 \times 10^8 + 1 \times 10^7 + 8 \times 10^6 + 3 \times 10^2 + 4 \times 10^1 + 2$ **e)** $2 \times 10^{10} + 2 \times 10^9 + 3 \times 10^8 + 4 \times 10^6 + 2 \times 10^5 + 5 \times 10^3 + 3 \times 10^1 + 2$	**2. a)** 4,050,006,000; 4 (one) billions + 5 ten millions + 6 thousands **b)** 30,005,000,636; 3 ten billions + 5 one millions + 6 hundreds + 3 tens + 6 ones **c)** 700,404,209; 7 hundred millions + 4 hundred thousands + 4 one thousands + 2 hundreds + 9 ones **d)** 506,800,802,306; 5 hundred billions + 6 (one) billions + 8 hundred millions + 8 hundred thousands + 2 (one) thousands + 3 hundreds + 6 ones

3. *Sample response*: Alike: • In standard form, they both have one digit that is 3 and the other digits are all 0. [• They are both read as “30 something", namely 30 thousand and 30 million.] Different: • One number is greater than the other because 30 million is more than 30 thousand. [• One number has six digits and the other has nine digits.] **4. a)** 9; [*Sample response*: Because the exponent is 8, the number is in the hundred millions and there are 8 places to the right of the 4. OR The exponent of a power of 10 tells you how many zeros are after the one, so $4 \times 10^8 =$ $4 \times 100{,}000{,}000 = 400{,}000{,}000$, which has 9 digits.] **b)** *Sample response*: • It is less than 5 hundred million. • It is more than 4 thousand.	**5.** The power with the greatest exponent tells the most. [*Sample response*: The farther you go to the left in a place value chart, the greater the value of the digit and the greater the exponent for its power of 10. The place value of the power of 10 with the greatest value will give you a good idea of the size of the number.] **6.** 5; [$10^5 = 10 \times 10 \times 10 \times 10 \times 10$ Every time you multiply by 10, you add an extra zero to the number. If you start with one 10 and multiply by four other 10s, you will have 5 zeros.] [**7.** Each time a place value column is filled up with 10 or more units, we have to add a higher place value column for those tens. Each time another place for tens is added, it becomes the next power of 10.]

Supporting Students

Struggling students

• Some students may find **question 5** difficult to interpret. You may choose not to assign this to struggling students.

Enrichment

• Students might create numbers to follow more complicated clues.

For example, they could be asked to write two numbers in expanded form where one is $3 \times 10^6 + 2 \times 10^4$ greater than another (for example, 5,210,040 and 2,190,040).

Chapter 3 Decimal Operations

1.3.1 Multiplying Decimals

Curriculum Outcomes	Outcome relevance
7-B1 Add, Subtract, Multiply, Divide: whole numbers and decimals • choose an appropriate method (pencil, mental, estimation) for a given situation	Many everyday calculations require the ability to multiply decimals.

Pacing	Materials	Prerequisites
1 h	• Small Grid Paper (BLM) (optional) • Thousandths Grids (BLM) (optional)	• representing a product as the area of a rectangle • renaming decimal tenths as a whole number of tenths

Main Points to be Raised

• You can represent a product as the area of a rectangle whose dimensions are the two factors. You can use this area on a grid to help make sense of the product of two decimals.

• Multiplying decimals is related to multiplying the associated whole numbers. This becomes clear when you rename the decimals using other units.

For example, because 2.2 × 4.5 is 22 tenths × 45 tenths, the product 2.2 × 4.5 is related to the product of 22 and 45.

• To calculate the number of decimal places in the decimal product starting from the whole number product, you count the total number of decimal places in the two factors. You then move that many decimal places in from the right.

For example, since 3 × 57 = 171, then 0.3 × 5.7 = 1.71 (2 decimal places in from the right).

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *What numbers would you multiply to get the 1200? Why do those numbers work to estimate the area?* (40 by 30 because 42.3 is close to 40 and 26.2 is close to 30.)

• *What did your rectangle look like?* (It was 42.3 units long and 26.2 units wide.)

• *Where is the 42 m by 0.2 m section?* (It is in the bottom left corner.)

• *How is this picture like a picture you would draw to calculate 42 × 26?* (I would also break it into four parts — a 40-by-20 part, a 2-by-20 part, a 6-by-40 part and a 6-by-2 part.)

• *Why is it easier to calculate the area when you use these parts?* (I already know how to do each of those multiplications.)

The Exposition — Presenting the Main Ideas

• Write the expression 2.2 × 1.5 on the board. Ask students why it can be rewritten as 22 tenths × 15 tenths. Then have students open the text to **page 23**. Ask how the picture shows 22 tenths × 15 tenths. Point out that the number of grey squares is 22 × 15; ask why each small grey square represents only 0.01, or $\frac{1}{100}$. Show students that the product of 330 is really 330 hundredths and ask why it is written as 3.30, with two decimal places.

• Ask students how a diagram for 2.3 × 1.6 would be different than the previous diagram. Talk about how there would now be 23 × 16 squares, each of size 0.01. Make sure students see that it was because you multiplied tenths by tenths that you got an answer in the hundredths.

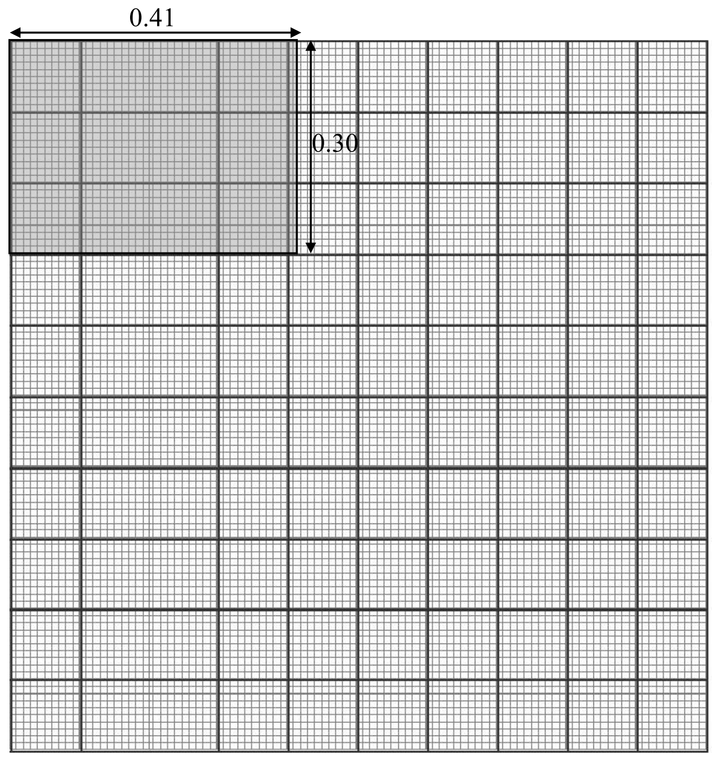
• Then write the expression 0.80 × 0.62 on the board. Have students look at the diagram on **page 24** of the text. Help them see that each of the very small squares is 0.0001 because the grid is marked into 10 sections of 10, or sections of 0.01 on each side. They can then see that there are 62 × 80 very small squares. Because there are 4960 squares of size 0.0001, the total area is less than 1; the area is 0.4960.

• Now model a diagram like this one using a Ten Thousandths Grid (BLM). Students can see that the grid is made up of 10,000 tiny squares, so each tiny square is 0.0001 and each small square (of 10-by-10 very tiny squares) is 0.01. They should see that the shaded area is 0.30 rows by 0.41 columns, which is 30 × 41 very small squares, each of size 0.0001. That means the product of 0.30 × 0.41 is 0.1230 (30 hundredths × 41 hundredths = 1230 ten thousandths).

30 hundredths × 41 hundredths
= 1230 ten thousandths

0.30 × 0.41
= 0.1230

• Show students how to generalize that in each case that they saw, the decimal point in the product was placed to represent the total number of decimal places in the factors.



Revisiting the Try This

B. Students have the opportunity to apply the rule they learned during the lesson. You may wish to ask them how their diagram supports the rule they are using.

Using the Examples

• Write the questions from **example 1** and **example 2** on the board. Ask students to work alone or in pairs to complete the questions and then check their work against the solutions in the text.

Practising and Applying

Teaching points and tips

Q 1: Students should realize that they do not have to recalculate; they simply place the decimal point. If students are calculating each question separately, you may wish to intervene to ensure adequate time is available for other questions to be completed.

Q 3: Students might draw a sketch to help them see that they are calculating the area of a rectangle.

Q 4: Students must recognize that this question asks them to multiply 1.5 by 1.07. They do not need to calculate the product, only to estimate it.

Q 5: This question is designed to provide yet another reason for the rule about the placement of the decimal point.

Q 6: If some students can think of only one method, you might draw their attention to the fact that $1.5 = 1\frac{1}{2}$.

Q 8: Some students may need prompting. You might suggest a few multiplications, such as 1.2 × 3.4, 1.2 × 0.1, and 3 × 1.5. Ask which they would calculate using mental math and why.

Q 9: This question allows students some flexibility in answering, but they are still applying the main rule they learned in the lesson.

Q 10: There are many correct estimates, for example, 15 × 2, 15 × 2.5, 14 × 2.5, etc.

Common errors

• Many students place a decimal point by counting from the left rather than from the right.

For example, for 3.2×2.8, they multiply 32 by 28 to get 896 and write the result as 89.6 rather than as 8.96. It is important to have students estimate to place the decimal point.

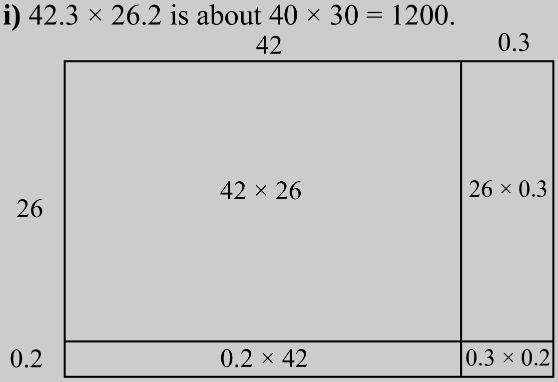
Suggested assessment questions from Practising and Applying

Question 1	to see if students can recognize how to place the decimal point when they multiply decimal numbers
Question 6	to see if students are flexible about procedures for decimal multiplication
Question 9	to see if students can use the rules for decimal placement in a product in a problem-solving situation
Question 10	to see if students can apply decimal multiplication to solve a real-world problem

Answers

A. i) 42.3×26.2 is about $40 \times 30 = 1200$.

ii)



iii) 1108.26 m^2

B. The two factors have a total of two decimal places, so the product has two decimal places.

1. a) 3241.68 **b)** 324.168
c) 3.24168 **d)** 3.24168

[**2.** *Students need to answer only two parts.*
Sample responses:
a) $80 \times 40 = 3200$, so 3241.68 makes sense.
b) $8 \times 40 = 320$, so 324.168 makes sense.
c) $1 \times 4 = 4$, so 3.24168 makes sense.
d) $0.08 \times 40 = 3.2$, so 3.24168 makes sense.]

3. 0.0048 km^2 **4.** *Sample response:* 1.6 m

5. Yes; [*Sample response*:
He knew that it was correct to rename each number as a whole number multiplied by a power of 10 and then multiply the factors in a different order.]

6. *Sample response*:
Method 1
1.5 is one and a half, so add 4.048 to half of 4.048.
$4.048 + 2.024 = 6.072$
[*Method 2*
Multiply 15×4048 and then put four decimal places in the product.
$15 \times 4048 = 10 \times 4048 + 5 \times 4048 = 40{,}480 + 20{,}240$
$= 60{,}720$
60,720 ten thousandths = 6.0720 = 6.072]

7. 3.57 cm^2

8. *Sample response*:
• 3.45×0.01; [Move the decimal point of 3.45 two places to the left to get 0.0345.]
• 0.5×40.444; [Divide 40.444 by 2 to get 20.222.]

9. *Sample responses*:
a) 20.4×5.06
b) $.204 \times .065$ (or 0.204×0.065)
c) 40.6×5.20
d) $.502 \times .604$ (or 0.502×0.604)

10. *Sample response*:
About 34 km; [14.8×2.25 is about $15 \times 2 + \frac{1}{4}$ of 16, which is about 34.]

11. No; [*Sample response*:
The product of 0.5×2.0 is 1.00 but Sonam dropped the zeros because 1.00 = 1.]

[**12.** *Sample response*:
You can ignore the decimal places and then multiply but you have to remember where to put the decimal point in the product. You can use the rule for placing the decimal or you can estimate.]

Supporting Students

Struggling students

• Students who have difficulty with whole number multiplication involving multi-digit numbers will have difficulty with this lesson. They may first need additional practice with whole number products.

• Students who are struggling may be asked to show just one method in **question 6** and might need you to give them some examples as a starting point in **question 8**. You might choose not to assign **question 9** and **question 11**, which require more abstract thinking.

Enrichment

• Students might create questions that meet certain clues or conditions.

For example, they might create a question where the result has 3 decimal digits with a 2 in the tenths place.

• Students might create questions in the style of **question 9** with different numbers and requirements.

1.3.2 Dividing Decimals

Curriculum Outcomes	Outcome relevance
7-B1 Add, Subtract, Multiply, Divide: whole numbers and decimals • choose an appropriate method (pencil, mental, estimation) for a given situation	Many everyday calculations require the ability to divide decimals.

Pacing	Materials	Prerequisites
1 h	None	• dividing whole numbers • renaming decimal tenths as hundredths, hundredths as tenths, hundredths as thousandths, etc.

Main Points to be Raised

• When the quotient and divisor are written in the same units, you can divide the number of units in each to get the quotient.

For example, 3.2 m ÷ 2 m = 1.6, just like 3.2 cm ÷ 2 cm = 1.6.

You can rename decimal numbers using different units to make calculations easier.

For example, you can rewrite 3.2 ÷ 0.2 (which is 3.2 ones ÷ 0.2 ones) as 32 tenths ÷ 2 tenths.

• If decimals have different numbers of decimal places, you may wish to rename one of them to get the same units before you divide.

For example, 3.24 ÷ 0.2 = 324 hundredths ÷ 2 tenths = 32.4 tenths ÷ 2 tenths = 32.4 ÷ 2.

• You can show the process of getting the same units by showing how the digits move in relation to the decimal point for both the dividend and the divisor. You can also think of this as moving the decimal point.

For example, you can change 0.2 to 2 by thinking of moving either the decimal one place to the right or the digit one place to the left.

• When you divide a decimal by a whole number, you sometimes rename the decimal by adding place values at the right so that the quotient can be more exact.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

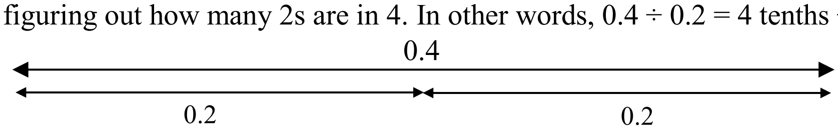
• *About how many tails would fit into 1 m? How do you know?* (About 6. I knew that 2 tails would make 0.3 m, so 6 tails would make 0.9 m, which is almost 1 m.)

• *Why would knowing how many tails fit in 1 m help solve the problem?* (I could multiply that number by 1.7.)

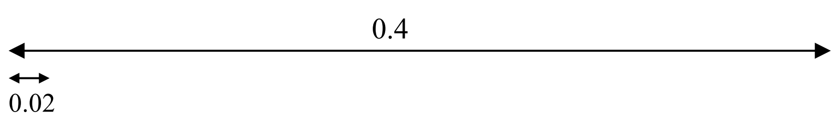
• *Why might you first estimate 1.5 ÷ 0.15?* (It is easy to know that there are 10 groups of 15 hundredths in 15 tenths. Then I could add one more tail to get up to 1.65, which is close to 1.7.)

The Exposition — Presenting the Main Ideas

• On the board, draw a picture that is 0.4 m long. Break it up into sections of 0.2 m. Ask how the picture shows how many 2 tenths are in 4 tenths. Also ask how the picture shows 0.4 ÷ 0.2 = 2. Point out that all that really mattered was figuring out how many 2s are in 4. In other words, 0.4 ÷ 0.2 = 4 tenths ÷ 2 tenths = 4 ÷ 2 = 2.



• Ask students how much of the length of 0.4 m would be taken up by 0.02 m. It is considerably less.



Point out that there would be ten sets of 0.02 in each 0.2, so there are 2 × 10 = 20 sets of 0.02 in 0.4.

Help students see that if you think of 0.4 as 0.40 or 40 hundredths, it makes sense that you could fit in 20 sets of 2 hundredths. Write 0.4 ÷ 0.02 = 4 tenths ÷ 2 hundredths = 40 hundredths ÷ 2 hundredths = 40 ÷ 2 = 20.

• Have students find the symbolism for both calculations (0.4 ÷ 0.2 and 0.4 ÷ 0.02) on **page 27** of the student text to see how you can use arrows to show that the units change into equivalent units. • Work through the example of 0.8 ÷ 0.3 on **page 28** to show students how 8 was renamed as 8.00 so that the quotient could be more precise. • Have students try to work out 0.6 ÷ 0.4 in a similar way.

Revisiting the Try This

B. This question provides an opportunity for students to calculate a quotient involving a decimal divisor.

Using the Examples

• Write the question from **example 1** on the board and allow students to try it. Discuss their answers with the class. Inform the students that they can later read through the solution in the text for reference. Then read through **example 2** with the class. Make sure they understand why it was acceptable to include the extra zeros at the end of 620 and why it resulted in a more accurate calculation.

Practising and Applying

Teaching points and tips

Q 1: Observe whether students realize there is no need to perform the calculations; all they need to do is to place the decimal point in 3125.

Q 2: Encourage students to talk through the calculations.

For example, for 5 ÷ 0.16, they can say to themselves, *How many groups of 16 hundredths are in 5?* This should encourage them to think of 5 as 500 hundredths.

Q 4: Students need to realize that they must divide the area by the base to get the height.

Q 5: Remind students that the example that is given is not necessarily the example they would use for their own explanations.

Q 7: You may have to remind students that there are 60 s in 1 min and 60 min in 1 h.

Q 8: Make sure that students realize that they must replace the black box with a single digit.

Q 9: This question forces students to think about the meaning of the division. They are dividing 412 hundredths by 3 hundredths, so a remainder of 1 makes no sense; the remainder has to be less than 3 hundredths.

Q 10: Students should refer to the meaning of the operations.

For example, 0.8 ÷ 0.4 means how many groups of 4 tenths are in 8 tenths.

Common errors

• Many students are not careful about moving the digits (or decimal point) the same number of places, especially if they need to use extra zeros to make it work.

For example, to divide 3.1 by 0.02, they need to use 3.10 to move the digits in the same way in the dividend as in the divisor.

Students should estimate to see if their answers are reasonable.

• Some students will struggle with interpreting the remainders if the division does not work out evenly. They need to think about what the division means.

For example, for 0.3 ÷ 0.2, the equivalent division is 3 ÷ 2 and the remainder of 1 should be thought of as one half of the divisor. Thus the number of times 0.2 fits into 0.3 is $1\frac{1}{2}$ times.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can divide using a decimal divisor
Question 5	to see if students can use reasoning to generalize about division by decimals
Question 6	to see if students can solve a real-world problem involving division by decimals
Question 10	to see if students understand the reason why the process for decimal division works

Answers

A. *Sample response*:
About 11 or 12 times; 12 × 0.15 = 1.8, which is close to 1.7 OR 170 cm ÷ 15 cm is about 165 ÷ 15 = 11.

B. 11.333 times

1. a) 312.5 **b)** 312.5
c) 3125 **d)** 31.25

[**2.** *Students need to answer only two parts.*
Sample responses:
a) 500 ÷ 1.6 = 312.5
500 ÷ 1.6 is a bit less than halfway between
500 ÷ 1 = 500 and 500 ÷ 2 = 250.
312.5 is a bit less than 375, which is halfway between 250 and 500.
b) 50 ÷ 0.16 = 312.5
50 ÷ 0.16 = 5000 ÷ 16 which is a bit more than
4800 ÷ 16 = 300.
312.5 is a bit more than 300.
c) 50 ÷ 0.016 = 3125
50 ÷ 0.016 = 50,000 ÷ 16 which is a bit less than
50,000 ÷ 10 = 5000 and 50,000 ÷ 20 = 2500.
3125 is a bit less than 3750, which is halfway between 2500 and 5000.
d) 5 ÷ 0.16 = 31.25
5 ÷ 0.16 = 500 ÷ 16 which is a bit less than halfway between 500 ÷ 10 = 50 and 500 ÷ 20 = 25.
31.25 is a bit less than 37.5, which is halfway between 25 and 50.]

3. a) 9.78 **b)** 6248.00 **c)** 150.08

4. 3.5 cm

5. *Sample responses*:
a) Yes; [0.4 ÷ 0.2 = 2]
b) Yes; [0.5 ÷ 0.2 = 2.5]
c) Yes; [0.5 ÷ 0.8 = 0.625]

6. a) 108 km/h **b)** 90 km/h

7. *Sample response*:
About 4000 h;
[174 km are about 170,000 m.
174,000 ÷ 0.013 is about 150,000 ÷ 0.01.
150,000 ÷ 0.01 = 15,000,000 s
There are 3600 s in 1 h.
15,000,000 ÷ 3600 = 150,000 ÷ 36
= 150 thousands ÷ 36
≈ 160 thousands ÷ 40
= 4 thousands
= 4000]

8. a) 4.667 (rounded to nearest thousandth)
b) 42

9. a) The remainder is $\frac{1}{3}$ or 0.333, not 1

[**b)** *Sample response*:

$$0.03\overline{)4.12} \quad \rightarrow \quad \begin{array}{r} 137 \\ 3\overline{)412} \\ \underline{-3} \\ 11 \\ \underline{-9} \\ 22 \\ \underline{-21} \\ 1 \end{array} \qquad 412 \div 3 = 137 \text{ R } \frac{1}{3}$$
]

[**10.** *Sample response*:
8 ÷ 4 means how many sets of 4 are in 8 sets of the same thing, or 8 ÷ 4.
• 0.8 ÷ 0.4 = 8 tenths ÷ 4 tenths, or how many sets of 4 tenths are in 8 tenths, or 8 ÷ 4.
• 0.08 ÷ 0.04 = 8 hundredths ÷ 4 hundredths, or how many sets of 4 hundredths are in 8 hundredths, or 8 ÷ 4.]

Supporting Students

Struggling students

• Division by decimals is often difficult for students. It is essential that they describe the question meaningfully, not just using symbols.

For example, they would say 4.12 ÷ 0.3 as, "How many 3 tenths are in 4.12?"

• It might be important to have students first work with questions where the answer is exact, rather than requiring rounding.

• You might pair up struggling students with other students for **question 5** and **question 9**, which are more abstract.

1.3.3 EXPLORE: Mental Math with Decimals

Curriculum Outcomes	Lesson Relevance
7-B2 Properties of Operations: decimals and integers • apply distributive, associative, and commutative properties in mental computation **7-B8 Add and Subtract Integers and Decimals Mentally: develop and use strategies** • develop and use mental strategies - front-end - compatible numbers - working by parts	Mental math is an important tool for student success in everyday mathematics. If students think about when to use mental math, they will be more likely to use it.

Pacing	Materials	Prerequisites
1 h	None	• adding, subtracting, multiplying, and dividing decimals

Exploration

• Write the calculation 4.02 ÷ 0.1 on the board. Ask students why they would not need a pencil to figure out the answer. If necessary, help them to see that to know how many tenths are in 4.02, you can think of 4.02 as 40.2 tenths, and the answer has to be 40.2. Some students will say that you multiply the number by 10 to find out how many tenths are in it.

• Then write the calculation 4.67 + 1.11 on the board. Ask students how they could do the addition in their heads. Make sure they realize they could simply add 1 to each of the one, tenths, and hundredths digits.

• Ask students to suggest one other decimal calculation they might complete mentally.

Suggest that students work in pairs. While you observe students at work, you might ask questions such as the following:

• *Why is it easy to add 0.001 to 3.099?* (There are 99 thousandths, so adding 1 thousandth makes 100 thousandths.)

• *What other possible values can you think of to subtract using mental math?* (I could subtract 0.000. That would be really easy because the number does not change. It is also easy to subtract 0.100 because it is the same as 0.1. I do not think it is difficult to subtract 0.001 because I can think of 0.1 as 100 thousandths, and subtracting 1 thousandth is not difficult.)

• *Why is it easy to multiply an even number by 0.5?* (You just take half of the number.)

• *Why is it easy to divide a number by 0.5?* (You just double the number.)

Observe and Assess

As students work, notice the following:

• Do they choose reasonable numbers to add, subtract, multiply, and divide?

• Are their explanations for how to perform the mental calculations clear and complete?

• Can they justify why mental math would be appropriate for these situations?

• Do they calculate correctly using mental math?

Share and Reflect

After students have had sufficient time to work through the exploration, discuss the numbers students chose and their explanations for calculating.

• *Why did you choose 0.001 instead of 0.158?*

• *Why is it useful to think of 4.1 as 4.100 for that calculation?*

• *Why might you instead use 0.250 to multiply or divide by?*

Answers

A. *Sample response*:
0.001;
3.099 is 3 ones and 99 thousandths.
0.001 is 1 thousandth.
3.099 + 0.001 = 3 ones and 100 thousandths
= 3.100 or 3.1

0.101;
3.099 + 0.101 is 1 tenth more than 3.099 + 0.001 = 3.1.
$3.099 + 0.101 = 3.2$

0.201;
3.099 + 0.201 is 1 tenth more than 3.099 + 0.101 = 3.2.
$3.099 + 0.201 = 3.3$

B. *Sample response*:
0.001;
4.1 is 4 ones and 100 thousandths.
0.001 is 1 thousandth.
4.1 – 0.001 = 4 ones and 99 thousandths
= 4.099
0.101;
4.1 – 0.101 is subtracting 1 more tenth from 4.1 than
$4.1 - 0.001 = 4.099$
4.1 – 0.101 = 4.099 – 1 tenth
= 40 tenths and 99 thousandths – 1 tenth
= 39 tenths and 99 thousandths
= 3.999
0.201;
4.1 – 0.201 is subtracting 1 more tenth from 4.1 than
$4.1 - 0.101 = 3.999$
$4.1 - 0.201 = 3.899$

C. *Sample response*:
0.5;
0.5 is one half so $2.48 \times 0.5 = 2.48 \div 2 = 1.24$.

0.02;
$0.02 = 0.01 \times 2$
$2.48 \times 0.01 \times 2 = 0.248 \times 2 = 0.496$

0.05;
$0.05 = 0.5 \times 0.1$
$$\begin{aligned} 2.48 \times 0.05 &= 2.48 \times 0.5 \times 0.1 \\ &= 1.24 \times 0.01 \\ &= 0.124 \end{aligned}$$

D. *Sample response*:
0.2;
Dividing by 0.2 is like ÷ 2 and then ÷ 0.1.
$4.2 \div 2 = 2.1$
$2.1 \div 0.1 = 2.1 \times 10 = 21$

0.4;
$4.2 \div 0.4$ is half of $4.2 \div 0.2 = 21$.
$4.2 \div 0.4 = 10.5$

0.02;
Dividing by 0.2 is like ÷ 2 and then ÷ 0.01.
$4.2 \div 2 = 2.1$
$2.1 \div 0.01 = 2.1 \times 100 = 210$

Supporting Students

Struggling students

• Some students prefer to write out calculations rather than performing them mentally because this gives them more confidence in the answer. You may wish to encourage those students by showing them how much easier it can be to perform a calculation mentally.

• You may wish to start off some students by providing some choices and asking which they would solve using mental math.

For example, you might ask them to choose among 3.099 + 4.856 or 3.099 + 3.001 or 3.099 + 6.978.

1.3.4 Order of Operations

Curriculum Outcomes	Outcome relevance
7-B4 Order of Operations: whole numbers and decimals • understand why order is important and what the conventional order is (brackets, exponents, division/multiplication, and addition/subtraction)	In many circumstances involving decimals, students need to know the rules for order of operations to calculate correctly.

Pacing	Materials	Prerequisites
1 h	None	• operations with decimals

Main Points to be Raised

• Without rules for the order of operations, different people might get different answers for a written calculation.

• The appropriate order of operations is this:

- Calculate anything inside brackets (or parentheses) first.

- Apply exponents next.

- Divide and multiply numbers next to each other, in order from left to right.

- Add and subtract numbers next to each other, in order from left to right.

• When there are brackets inside brackets, you first calculate that which is in the innermost brackets.

• When you read an expression, you might talk about whether the bracket comes before or after an expression.

For example, to read 3.2 + (5.3 – 1.4), you might say, "three and two tenths plus, open bracket, five and three tenths minus one and four tenths, close bracket".

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Make sure they understand that the white area around the garden represents the wooden platform. While you observe students at work, you might ask questions such as the following:

• *Why did you calculate the areas of both rectangles?* (You need to subtract the area of the garden from the area of the big rectangle.)

• *How did Yeshi get the 1.6?* (He subtracted 2.8 – 1.2)

• *Does Yeshi's calculation make sense to you?* (I do not think it is right. When I calculated 3.8 × 2.8 and then subtracted 1.6 × 0.9, I got 9.2, but when I did Yeshi's calculation, I got 5.472.)

The Exposition — Presenting the Main Ideas

• Write the calculation 3.5 + 6.8 + 2 × 6.4 on the board. Ask students for the answer to see if they all perform the operations in the same order. Whether or not they do, make sure students understand that the rule is to multiply 2 by 6.4 first before adding the numbers.

• Remind students that they have already learned about order of operations for whole numbers and point out that the same rules apply now.

• List the order of operations rules shown on **page 31** or ask students to look at the page in the student text. Ask them to apply the order to the expression 3.5 × (2.8 + 1.4 × 1.5) – 2.1.

• You might choose to list the rules for order of operations on a poster for students to refer to. Many people use the invented word BEDMAS to help them remember the correct order: **B**rackets, **E**xponents, **D**ivide and **M**ultiply (in order from left to right), **A**dd and **S**ubtract (in order from left to right).

Revisiting the Try This

B. This question allows students to explain the need for rules for order of operations and also to see how the use of brackets can lead to fewer errors.

Using the Examples

• Present the question in the example. Ask students to think about how they would write it symbolically. They should then compare their solutions and thinking to the solution and thinking in the text.

Practising and Applying

Teaching points and tips

Q 1: Remind students that they should apply the exponent in **part b)** before performing the divisions and multiplications.

Q 2: Encourage students to experiment by trying to insert and remove brackets in different places.

Q 4: This question is a good reminder for students of the importance of translating English phrases into mathematical symbolism.

Q 5: Students will have to do some problem solving to answer this question.

Common errors

• Students might struggle with questions like in the example where there are brackets inside brackets. Sometimes it helps to use a different shape for the pairs of brackets.

For example, you might write $((4.1 \times 5)^3 - 2) \div 4$ as $[(4.1 \times 5)^3 - 2] \div 4$. This helps students match the opening and closing brackets.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use the order of operations
Question 4	to see if students can symbolize calculation instructions, taking order of operation rules into account
Question 5	to see if students can reason out how a student might have incorrectly applied the rules for order of operations

Answers

A. 9.56 m^2 **B.** No; he did not subtract the area of the hole from the area of the piece of wood.	**C. i)** He did not follow the order of operations. He subtracted 1.2 from 2.8, but he should have multiplied 3.8 × 2.8 and then 1.2 × 0.9 and then subtracted the second product from the first. **ii)** The brackets might have helped him but they are not necessary because the order of operations rules say that you do all multiplications before any subtraction.
1. a) 0.7 **b)** 8.08 **c)** 8.2 **2. A** and **B** **3. a)** Not correct; 14.4 **b)** Not correct; 34.26 **c)** Correct **d)** Correct **4. a)** $(8 \div 0.1 + 12) \times 3 - 2$ **b)** $[(4.2 + 3.5) \times 3]^2 - 4$ **c)** $[(6.2 \times 2 + 5.6)^2 + 3] \div 2$ **5. a)** 8.192; [Subtracted 0.2 from 1.8 before cubing instead of cubing 0.2 and then subtracting from 1.8. The correct answer is 8.192.]	**5. b)** 34.1; [Calculated everything from left to right in order instead of doing the multiplications before the additions and subtractions. The correct answer is 34.1.] **c)** 29; [Divided (30 – 4.2 ÷ 0.2 + 8) by 0.4 instead of dividing only 8 by 0.4. The quotient would be added finally to the result obtained when 4.2 ÷ 0.2 is first subtracted from 30. The correct answer is 29.] [**6.** If there were no rules, different people might get different answers for the same calculation. *Sample response*: 3 + 7 × 8 would be 80 if 3 + 7 were calculated first. The proper value is 59 because 7 × 8 must be calculated first.]

Supporting Students

Struggling students

• Struggling students might focus on how to calculate correctly rather than on explaining errors others might have made, as is required in **question 5**.

UNIT 1 Revision

Pacing	Materials
2 h	• Base ten blocks or Base Ten Models (BLM) (optional)

Question(s)	Related Lesson(s)
1	Lessons 1.1.1 and 1.1.2
2 – 5	Lesson 1.1.2
6 – 8	Lesson 1.1.3
9	Lesson 1.1.4
10	Lessons 1.1.3 and 1.1.4
11	Lesson 1.1.4
12 – 15	Lesson 1.2.1
16 – 18	Lesson 1.2.2
19 – 22	Lesson 1.3.1
23 – 25	Lesson 1.3.2
26	Lessons 1.3.1 and 1.3.3
27 and 28	Lesson 1.3.4

Revision Tips

Q 3: Students need to recall that when you divide the sum of the digits by 3, the amount that is left over is the remainder when you divide the number by 3.
The remainder when you divide by 4 is the remainder when you divide the last two digits (as a number) by 4. Or, it is the remainder when you divide the sum of the ones digit and double the tens digit by 4.

Q 4: To be divisible by 15, a number must be divisible both by 5 and by 3.

Q 11: Students need to recognize the connection to GCF.

Q 16: Students might use a pattern to help them answer this question.

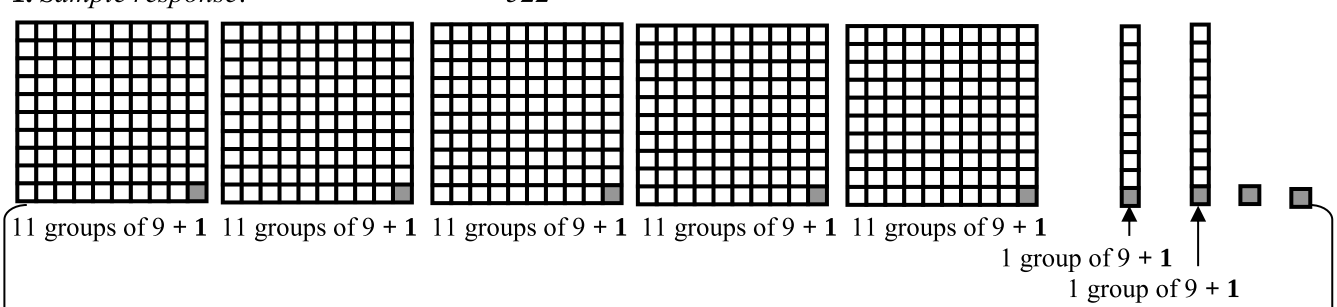
Q 20: Encourage students to find more than one answer.

Q 23: Encourage students to reason why this is true rather than simply calculating both answers and seeing that they are equal.

Q 26: Some students might use decimals like 12.0 × 4.0. Although this is not incorrect, you should encourage them to use decimals that do not have zeros in the decimal places.

Answers

1. *Sample response*: 522



Altogether there are 57 groups of 9 and 5 + 2 + 2 = 9 ones.
Since you can make another group of 9 with the ones, 522 can be grouped into groups of 9. It is divisible by 9.

2. a) Divisible by 2, 4, 5, and 10 **b)** Divisible by 3 and 9 **c)** Divisible by 3, 5, and 9 **d)** Divisible by 2 and 4	**3. a)** By 3: remainder is 2; by 4: remainder is 2. **b)** By 3: remainder is 2; by 4: remainder is 1. **c)** By 3: remainder is 1; by 4: remainder is 2. **d)** By 3: remainder is 1; by 4: remainder is 3.

Answers [Continued]

4. 1485 is divisible by 15; [*Sample response*:
If a number is divisible by 15 then it is divisible both by 5 and by 3, so you can use the tests for 3 and 5.
1 + 4 + 8 + 5 = 18; because 18 is a multiple of 3, so is 1485.
1485 ends in 5, so it is divisible by 5.
So 1485 is divisible by 15.]

5. a) 2, 5, or 8 **b)** 4 **c)** Any digit 0 to 9

6. a) 336 **b)** 90 **c)** 210

7. 3, 21, 105

8. a) No; [90 is not a multiple of 4.]
b) Yes; [*Sample response*: 1 and 90]
c) *Sample response*: 1 and 90 or 30 and 45

9. a) 10 **b)** 5 **c)** 5

10. *Sample response*: 30 and 600 or 150 and 120

11. a) 4 ways:
[1 row with 64 squares,
2 rows with 32 squares in each,
4 rows with 16 squares in each, or
8 rows with 8 squares in each.]
b) 5 ways:
[1 row with 36 squares,
2 rows with 18 squares in each,
3 rows with 12 squares in each,
4 rows with 9 squares in each,
6 rows with 6 squares in each.]
c) 1 row, 2 rows, or 4 rows; [1, 2 and 4 are common factors of 36 and 64.]

12. a) $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$ **b)** $11 \times 11 \times 11$

13. a) 9^6 **b)** 3^8

[**14.** 2^{10} is the product of 10 twos.
2^9 is the product of 9 twos.
There is one extra multiplication by 2 in 2^{10}, so it is twice as much.]

15. $3^2 + 3^3 + 3^4 + 3^5 = 360$

[**16.** You are multiplying together many 5s. The ones place will always be 5 because 5 x 5 = 25, which is regrouped as 2 tens and 5 ones.
OR
$5 \times 5 = 25$
$5 \times 5 \times 5 = 125$
$5 \times 5 \times 5 \times 5 = 625$
All powers of 5 end in the digit 5 so 5^{30} will too.]

17. a) $3 \times 10^6 + 1 \times 10^5 + 2 \times 10^4 + 3$
b) $3 \times 10^9 + 1 \times 10^8 + 2 \times 10^7 + 3 \times 10^3 + 4 \times 10^2$

18. a) 300,020,308;
3 hundred millions + 2 ten thousands + 3 hundreds + 8 ones
OR
$3 \times 100{,}000{,}000 + 2 \times 10{,}000 + 3 \times 100 + 8$
b) 600,070,003,205;
6 hundred billions + 7 ten millions + 3 thousands + 2 hundreds + 5 ones
OR
$6 \times 100{,}000{,}000{,}000 + 7 \times 10{,}000{,}000 + 3 \times 1000 + 2 \times 100 + 5$

19. a) 380.16 **b)** 3.8016
c) 0.38016 **d)** 3.8016

20. *Sample response*:
1 decimal place and 4 decimal places [because the sum is 5.]

21. **4.3** × **1.2** = **5**.16

22. *Sample responses*:
a) 10 cm base and 15.5 cm height
b) 20 cm base and 7.75 cm height

[**23.** *Sample response*:
32.5 ÷ 0.5 means how many 5 tenths are in 325 tenths, and that is 325 ÷ 5.]

24. a) 850 **b)** 61.5

25. a) 7.29 **b)** 1.33

26. *Sample responses*:
a) 43.5 × 0.1, 8.4 × 0.5, 3.0 × 3.1
[**b)** 43.5 × 0.1: move the digits of 43.5 one place right to get 4.35.
8.4 × 0.5: multiplying by 0.5 is the same as doubling and 8.4 × 0.5 = 8.4 × 2 = 16.8.
3.1 × 3.0: multiply 3 × 3 ones = 9 and 3 × 1 tenth = 0.3 and add them together to get 9.3.]

27. a) 2.16 **b)** 72.6 **c)** 12.3

28. a) Not necessary; [you add 5.2 + 3.6 anyway after the product has been calculated and before subtracting.]
b) Not necessary; [the order that 4.5 and 3.6 and 0.1 are multiplied together does not change the product]
c) Necessary; [otherwise you would cube 5 instead of cubing 3.2 + 5.]

UNIT 1 Number Test

1. List all possible digits that make each true.

a) 1■,234 is divisible by 3

b) 10,2■3 is divisible by 9

c) 512■ is divisible by 4

d) 234,5■2 is divisible by 6

2. Calculate.

a) LCM (20, 12) **b)** GCF (20, 12)

c) LCM (3, 15, 9) **d)** GCF (3, 15, 9)

3. What value will make each true?

a) LCM (3, ■) = 18

b) GCF (24, ■) = 6

4. Without calculating the value of each power, explain how you know each is true.

a) 3^4 is one third of 3^5

b) 2^6 is 4 times 2^4

5. Write 9^4 as the product of each.

a) 4 numbers

b) 5 numbers

6. Write each in standard form.

a) $1 \times 10^{10} + 6 \times 10^7 + 8 \times 10^6$

b) $6 \times 10^5 + 8 \times 10^3 + 3 \times 10^2 + 2$

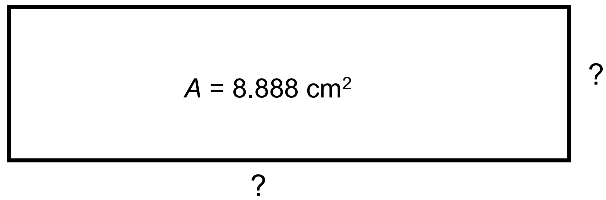
7. A number has 11 digits.

a) What do you know about the first part of the number when it is in expanded form?

b) What do you know about the first part of the number when it is in exponential form?

8. Sketch or use grid paper to draw a picture to show why 2.1 × 4.2 = 8.82.

9. The area of a rectangle is 8.888 cm^2. The length and width have non-zero decimal digits. List two possible pairs of numbers for the length and width.



10. Describe two ways to multiply 0.2 × 9.5.

11. a) Without calculating, predict which is greatest. Explain your prediction.

A 3.4 ÷ 0.2 **B** 7.1 ÷ 0.001

C 12.6 ÷ 6 **D** 10.3 ÷ 5

b) Calculate the quotient you chose in **part a)**.

12. The product of two numbers has four decimal places. The quotient is a whole number. What could the numbers be?

13. Write a decimal multiplication you could do mentally. Explain how you would calculate.

14. Calculate: $4.8 \div 0.4 + (3 - 2.1)^2$

UNIT 1 Test

Pacing	Materials
1 h	• Small Grid Paper (BLM) (optional)

Question(s)	Related Lesson(s)
1	Lessons 1.1.1 and 1.1.2
2 and 3	Lessons 1.1.3 and 1.1.4
4 and 5	Lesson 1.2.1
6 and 7	Lesson 1.2.2
8 – 10	Lesson 1.3.1
11	Lesson 1.3.2
12	Lessons 1.3.1 and 1.3.2
13	Lessons 1.3.1 and 1.3.3
14	Lesson 1.3.4

Select questions to assign according to the time available.

Answers

1. a) 2, 5, 8 **b)** 3 **c)** 0, 4, 8 **d)** 2, 5, 8

2. a) 60 **b)** 4 **c)** 45 **d)** 3

3. a) 18 **b)** *Sample response*: 18

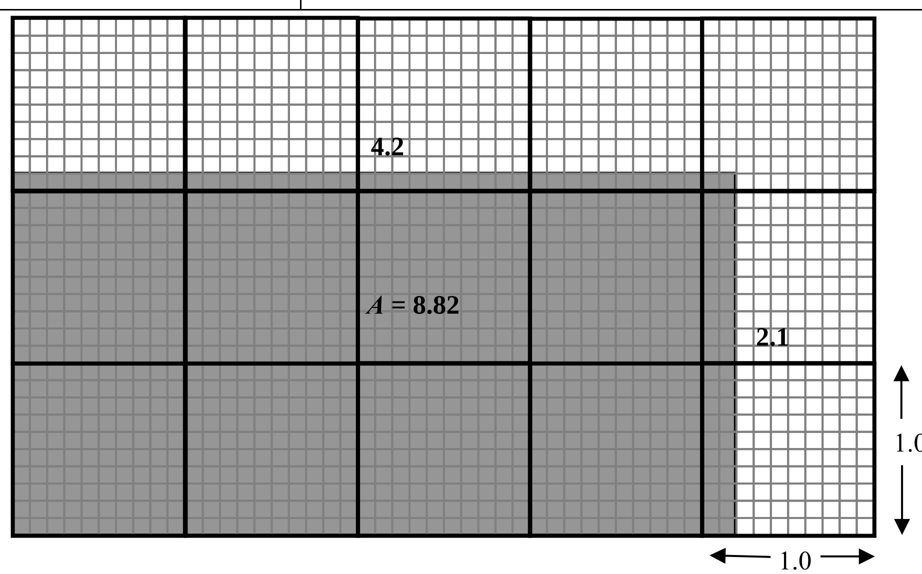
4. a) $3^5 = 3 \times 3 \times 3 \times 3 \times 3$ and $3^4 = 3 \times 3 \times 3 \times 3$.
So $3^5 = 3 \times 3^4$ and that means 3^4 is one third of 3^5.
b) $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$ and $2^4 = 2 \times 2 \times 2 \times 2$.
So $2^6 = 2^4 \times 2 \times 2 = 2^4 \times 4$.

5. a) $9 \times 9 \times 9 \times 9$
b) *Sample response*: $3 \times 3 \times 9 \times 9 \times 9$

6. a) 10,068,000,000 **b)** 608,302

7. a) It is at least 10 billion, or at least $1 \times 10{,}000{,}000{,}000$.
b) It is at least 1×10^{10}.

8. *Sample response*: 2.1×4.2



9. *Sample response*:
11.11 cm × 0.8 cm or 111.1 cm × 0.08 cm

10. *Sample response*:
- $0.2 \times 9.5 = 2 \times 0.1 \times 9.5 = 2 \times 0.95 = 1.9$
- $0.2 \times 9.5 = 0.2 \times 10 - 0.2 \times 0.5 = 2 - 0.1 = 1.9$

11. a) B; there are more than 7000 thousandths in 7. None of the other quotients will be as large.
b) 7100

12. *Sample response*: 0.04 and 0.02

13. *Sample response*:
0.8×0.4
Multiply $8 \times 4 = 32$ (hundredths)
Because there are two decimal places altogether in 0.8 and 0.4, there are also two decimal places in 32, so the answer is 0.32.

14. 12.81

UNIT 1 Performance Task — Creating a Shape

In this task you will create two hexagons. A hexagon is a shape with six sides and six angles.

A. i) Calculate each value in the chart to determine four of the six side lengths and three of the six angles of a hexagon.

Side lengths (cm)	Angles (°)
LCM (3, 8, 12)	GCF (45, 135)
$3^4 \div 2^4$	This number is divisible by 2, 3, 4, and 5.
$3.4 \times 2.1 + 1.8 \div 0.6$	$4.5 \div 0.05$
The number of digits in the number in standard form: $3 \times 10^8 + 5 \times 10^2 + 6 \times 10^1$	

ii) Draw the hexagon.

B. i) Draw your own hexagon.

ii) Express four of the side lengths and three of the angles using each idea below at least once.
- Lowest Common Multiple
- Greatest Common Factor
- Divisibility by 3 and 4
- Exponential form
- Multiplication of decimals
- Division of decimals
- An expression where you need to use the order of operations rules

Write your ideas in chart form:

Side lengths (cm)	Angles (°)

UNIT 1 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
7-A1 GCF: using common factors and greatest common factors to solve problems **7-A2** LCM: using common multiples and least common multiples to solve problems **7-A3** Divisibility: develop and apply rules for 3, 4, 6, 9 **7-A4** Large Numbers: model **7-A5** Large Numbers: rename **7-B1** Add, Subtract, Multiply, Divide: whole numbers and decimals **7-B3** Order of Operations: whole numbers and decimals	1 h	• Rulers • Protractors

How to Use This Performance Task

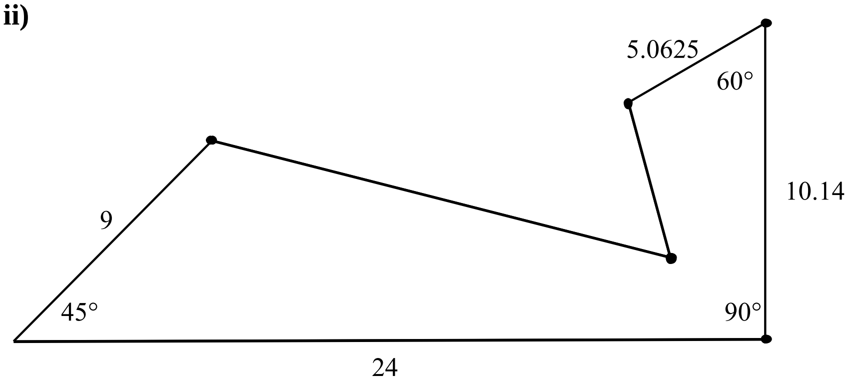
You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. It could also be used as enrichment material for some students. You can assess performance on the task using the rubric on the next page.

Sample Solution

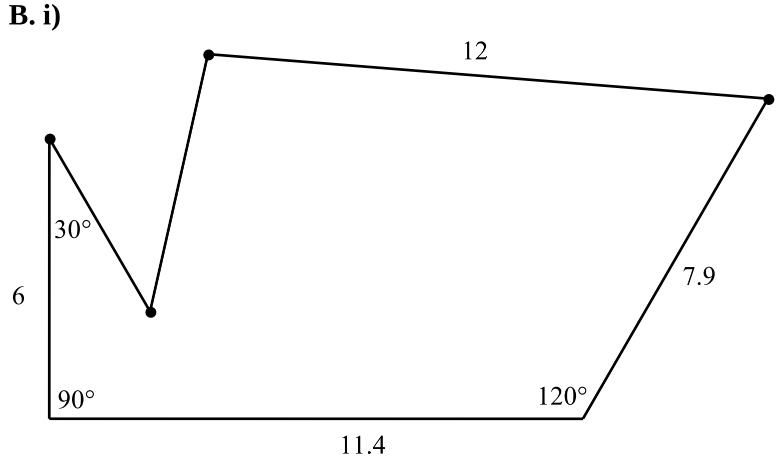
A. i)

Side lengths (cm)	Angles (º)
24	45
5.0625	e.g., 60
10.14	90
9	

ii)



B. i)



Sample Solution [Continued]

ii)

Side lengths (cm)	Angles (º)
GCF (96, 150) = 6	LCM (18, 15) = 90
The number of digits in a number that has 7×10^{11} as its greatest part when in exponential form = 12.	The second number after 100 that is divisible both by 3 and by 4 = 120.
7.6 × 1.5 = 11.4	0.6 ÷ 0.02 = 30
3.1 × 4.8 – 4.4 × 2.6 + 4.46 = 7.9	

UNIT 1 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Calculates correctly	Calculates given and created expressions correctly	Calculates given and created expressions correctly for the most part	Calculates given and created expressions with several errors	Makes many calculation errors
Creates expressions	Uses all required concepts correctly and in a varied way	Uses all required concepts correctly	Uses most of the required concepts correctly	Uses some of the required concepts correctly
Provides appropriate models	Provides two models that meet stated requirements with reasonably accurate measurements	Provides two models that mostly meet stated requirements with reasonably accurate measurements	Provides at least one model that mostly meets stated requirements with reasonably accurate measurements	Has significant flaws in the two required models

BLM 1 100 Charts

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

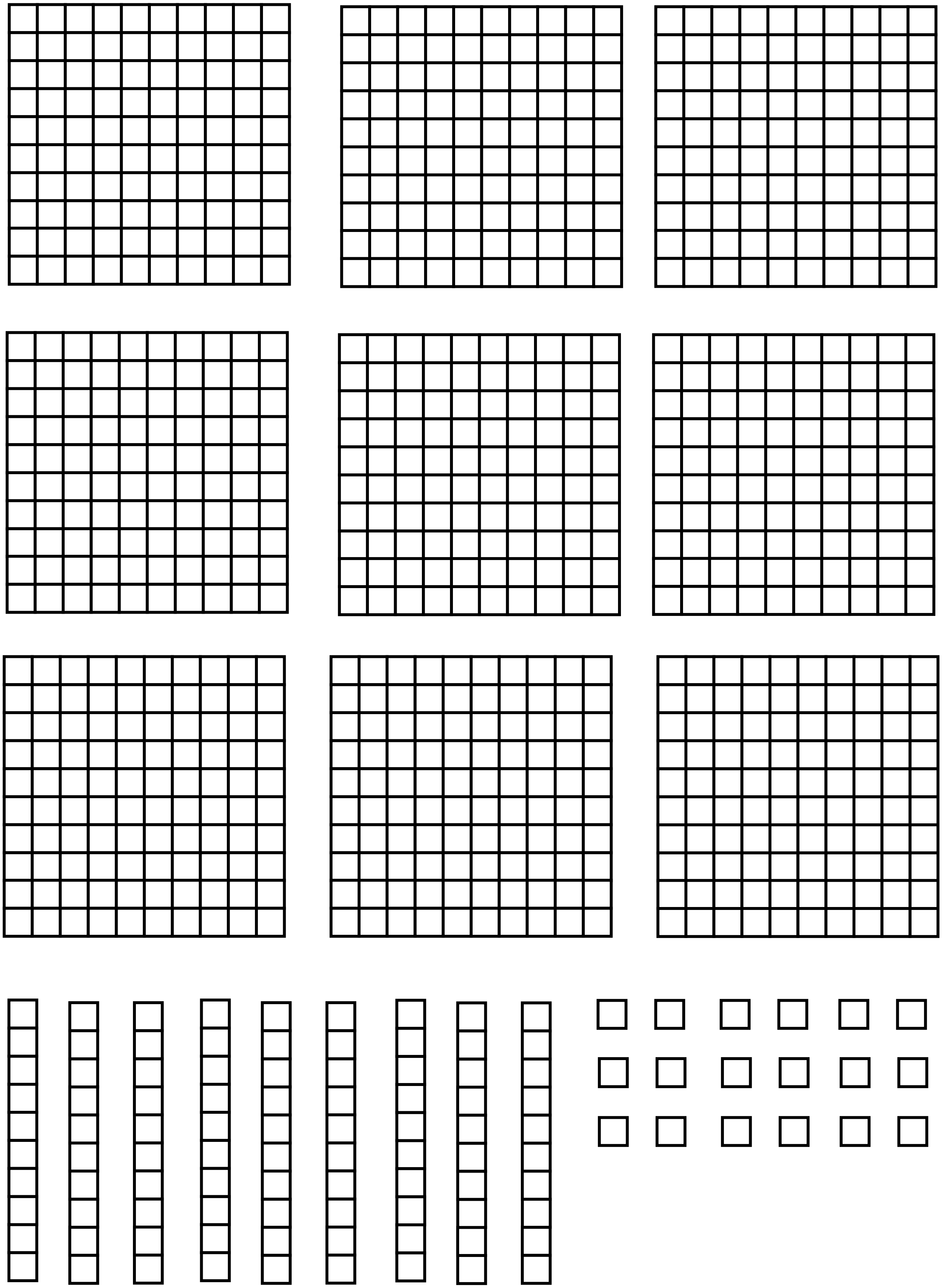
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

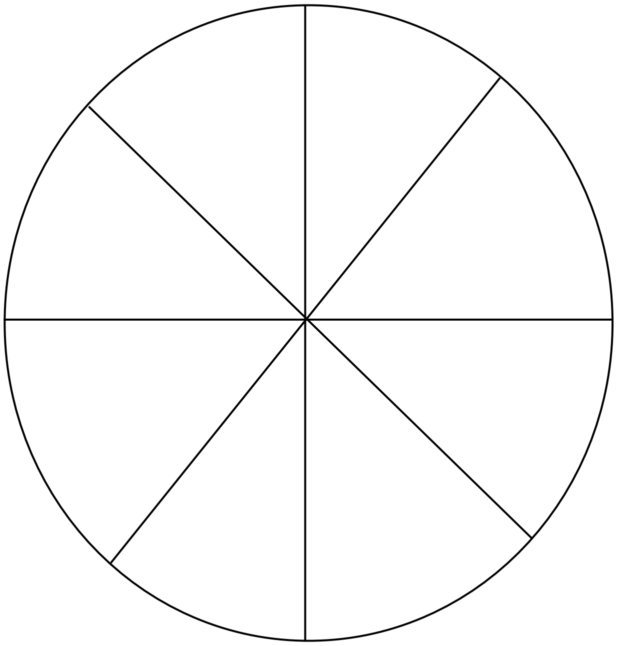
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

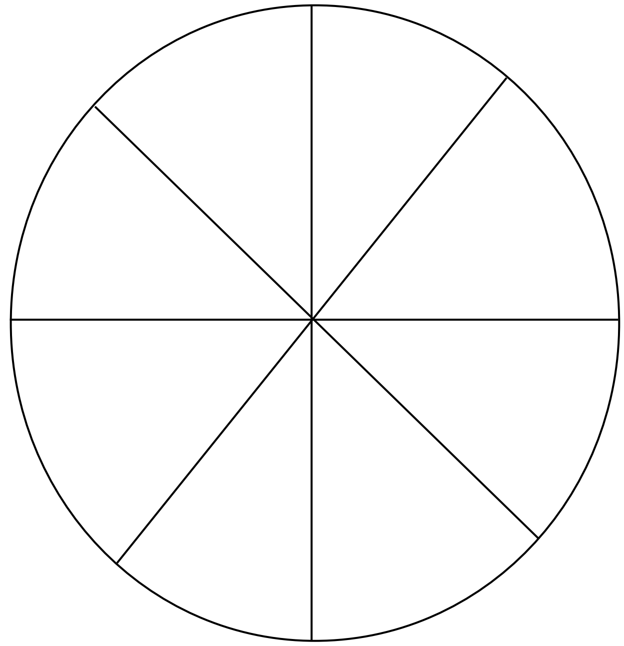
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

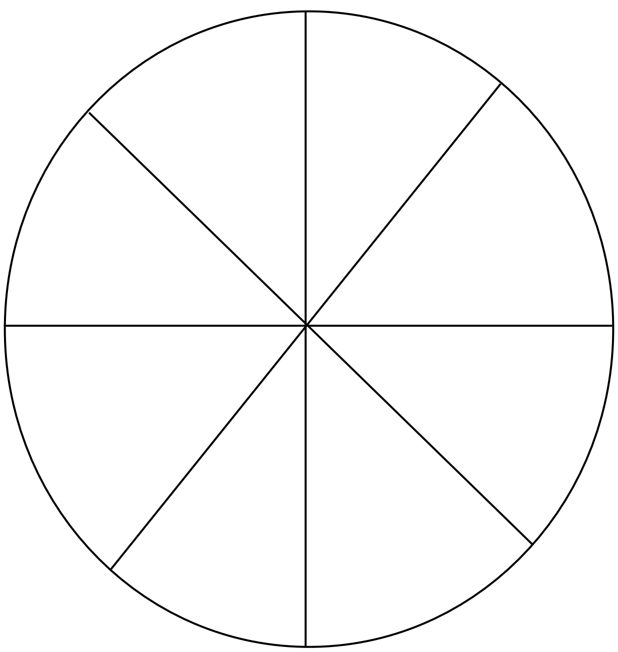
BLM 2 Base Ten Models

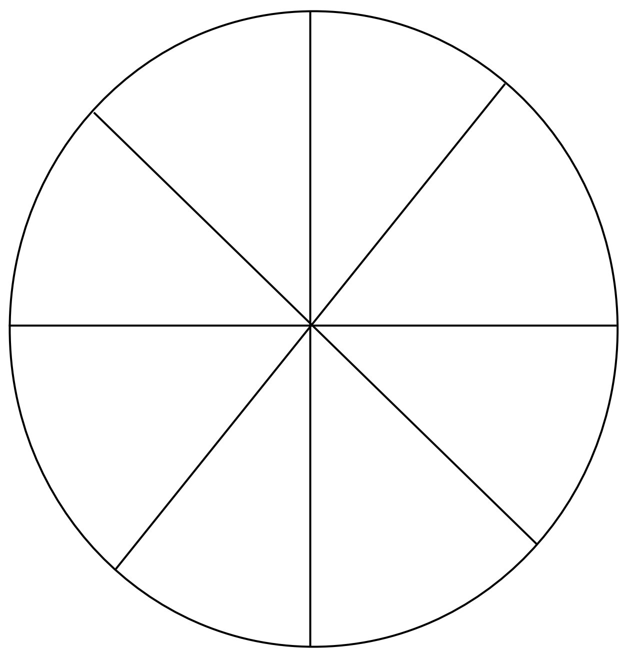


BLM 3 Fraction Circle Spinners









BLM 4 Place Value Charts

Billions	Millions			Thousands			Ones		
Ones	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	1

Billions	Millions			Thousands			Ones		
Ones	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	1

Billions	Millions			Thousands			Ones		
Ones	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	1

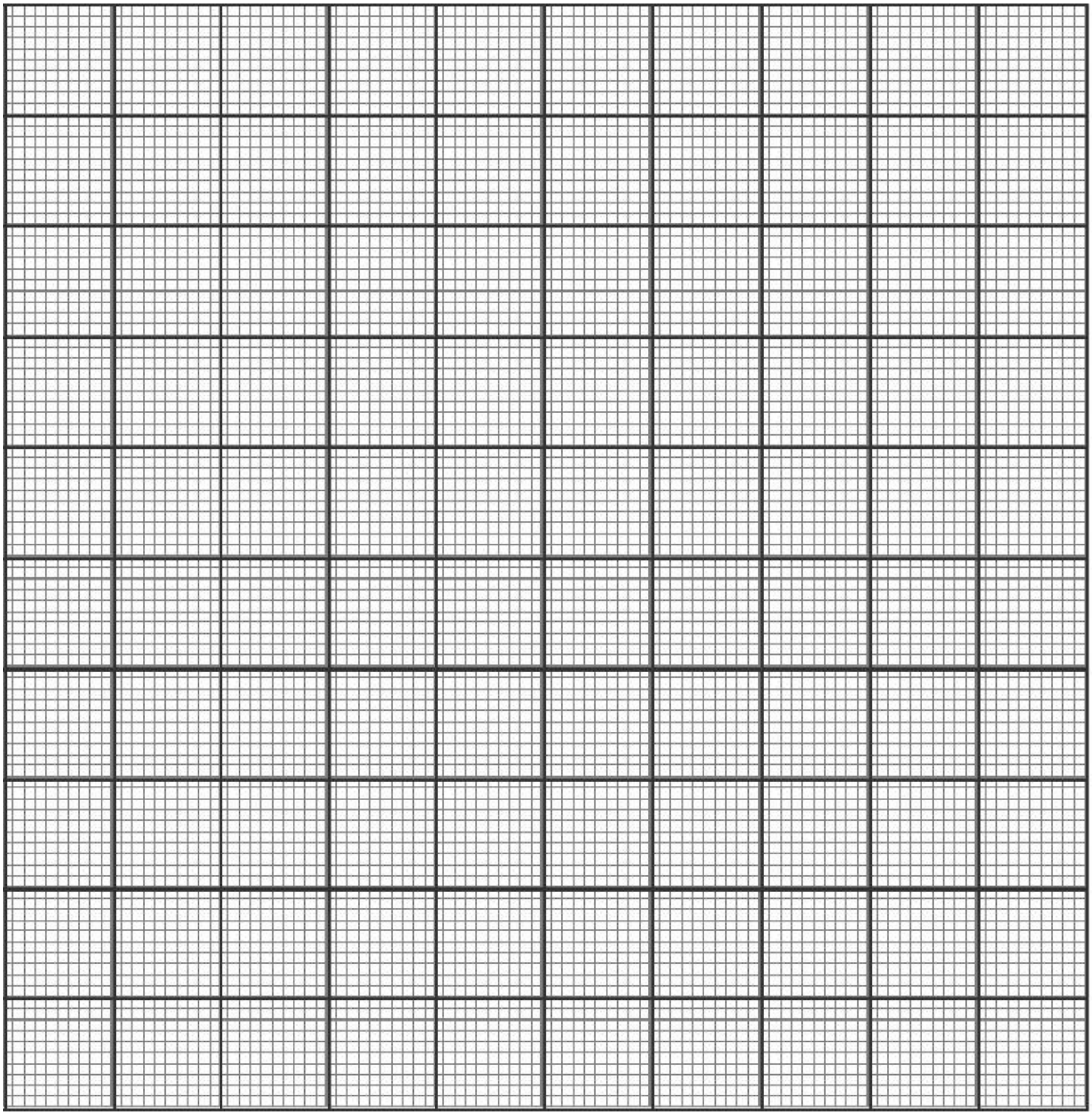
Billions	Millions			Thousands			Ones		
Ones	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	1

Billions	Millions			Thousands			Ones		
Ones	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	1

Billions	Millions			Thousands			Ones		
Ones	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	1

BLM 5 Small Grid Paper

BLM 6 Ten Thousandths Grid



UNIT 2 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 35** **TG p. 58**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	None	All questions
Chapter 1 Fraction Addition and Subtraction				
2.1.1 Comparing and Ordering Fractions **SB p. 37** **TG p. 60**	**7-A6 Compare and Order: decimals, proper/improper fractions, and mixed numbers** • order fractions on a number line • compare fractions using a variety of strategies including benchmarks, common denominator, common numerator, decimal equivalents	1 h	• Fraction Number Lines (BLM)	Q2, 4, 9
2.1.2 Adding Fractions Using Models **SB p. 42** **TG p. 63**	**7-B5 Add and Subtract: simple fractions of various denominators** • develop algorithm pictorially • estimate the sum or difference of fractions	1 h	• Fraction Strips (BLM) • Counters	Q3, 5, 7, 8
2.1.3 Adding Fractions and Mixed Numbers Symbolically **SB p. 48** **TG p. 66**	**7-B5 Add and Subtract: simple fractions and mixed numbers of various denominators** • develop algorithm symbolically • estimate the sum or difference of fractions and mixed numbers **7-A2 Common Multiplies: use common multiples and least common multiples (LCM) to solve problems** • use the LCM to add and subtract fractions	1 h	None	Q1, 3, 7, 8
GAME: A "Whole" in One (Optional) **SB p. 52** **TG p. 68**	Practise adding simple fractions with different denominators in a game situation	30 min	• Slips of paper with numbers 2 to 10 on them	N/A
2.1.4 Subtracting Fractions and Mixed Numbers **SB p. 53** **TG p. 69**	**7-B5 Add and Subtract: simple fractions and mixed numbers of various denominators** • develop algorithm pictorially and symbolically • estimate the sum or difference of fractions and mixed numbers	1 h	• Fraction Strips (BLM) • Counters	Q1, 2, 5, 10
2.1.5 Subtracting Mixed Numbers in Different Ways **SB p. 58** **TG p. 72**	**7-B5 Add and Subtract: simple fractions and mixed numbers of various denominators** • develop algorithm pictorially and symbolically • estimate the sum or difference of fractions and mixed numbers	1 h	• Fraction Number Lines (BLM)	Q2, 5, 7, 9
Chapter 2 Fraction Multiplication and Division				
2.2.1 Multiplying a Fraction by a Whole Number **SB p. 62** **TG p. 75**	**7-B6 Multiply and Divide: fraction by a whole number** • develop and apply strategies necessary for calculation of fractions • use concrete models and pictorial representations	1 h	None	Q2, 3, 6

UNIT 2 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
2.2.2 Dividing a Fraction by a Whole Number **SB p. 65** **TG p. 78**	**7-B6 Multiply and Divide: fraction by a whole number** • develop and apply strategies necessary for calculation of fractions • use concrete models and pictorial representations	1 h	• Fraction Strips (BLM)	Q2, 4, 6, 7
Chapter 3 Relating Fractions and Decimals				
2.3.1 Naming Fractions and Mixed Numbers as Decimals **SB p. 68** **TG p. 81**	**7-A7 Rename: Mixed Numbers and Fractions** • rename fractions and mixed numbers as decimals • use pictorial models to represent mixed numbers and fractions • introduce the terminology "repeating" and "period" as well as notation to show that a decimal repeats • explore patterns in various fractions, especially sevenths	1 h	None	Q3, 4, 7
2.3.2 EXPLORE: Relating Repeating Decimals and Fractions (Essential) **SB p. 72** **TG p. 85**	**7-A8 Rename: Repeating Decimals to Fractions** • explore 1- and 2-digit repeating decimals • use patterns to rename and make predictions	1 h	None	Observe and Assess questions
CONNECTIONS: Repeating-Decimal Graphs (Optional) **SB p. 73** **TG p. 87**	Make a connection between fractions, decimals, and graphing	25 min	• Grid paper or Small Grid Paper (BLM)	N/A
UNIT 2 Revision **SB p. 74** **TG p. 88**	Review the concepts and skills in the unit	2 h	• Fraction Strips (BLM) • Fraction Number Lines (BLM) • Counters	All questions
UNIT 2 Test **TG p. 90**	Assess the concepts and skills in the unit	1 h	• Fraction Strips (BLM) (optional) • Fraction Number Lines (BLM) (optional) • Counters	All questions
UNIT 2 Performance Task **TG p. 92**	Assess concepts and skills in the unit	1 h	• Fraction strips (BLM) or Fraction Number Lines (BLM) (optional)	Rubric provided
UNIT 2 Assessment Interview **TG p. 94**	Assess concepts and skills in the unit	15 min	See **p. 94**	All questions
UNIT 2 Blackline Masters **TG p. 95**	BLM 1 Fraction Strips (1 whole to twelfths) BLM 2 Fraction Number Lines (1 whole to twelfths) Small Grid Paper on **page 53** in UNIT 1			

Math Background

• This unit builds on comparing, adding, and subtracting simple fractions as well as on the fraction/decimal relationships the students learned in Class VI. Multiplication and division of fractions are among the new ideas introduced in this unit.

• The focus of the unit is on fraction computation and the relationship between fraction and decimal number representations.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 5** in **lesson 2.1.1**, where they figure out which digits are missing, and in **question 8** in **lesson 2.1.3**, **question 10** in **lesson 2.1.4**, **question 9** in **lesson 2.1.5**, and **question 9** in **lesson 2.2.2**, where they look for numbers to meet a certain condition.

• They use communication frequently as they explain their thinking in answering questions, such as in **question 2** in **lesson 2.1.1**, where they describe strategies for ordering fractions, **question 6** in **lesson 2.1.2**, where they consider what fractions can make up a whole, and **question 5** in **lesson 2.1.4**, where they describe estimation strategies. The last question in most lessons usually requires an element of communication in bringing closure to the lesson.

• They use reasoning in answering questions such as **question 6** in **lesson 2.1.1**, where they create and test a conjecture, **question 3** in **lesson 2.1.3** and **question 1** in **lesson 2.1.4**, where they use estimation to show that a calculation is reasonable, and **question 10** in **lesson 2.3.1**, where they determine what remainders are possible in a given situation and how that relates to repeating decimals.

• They consider representation in **lesson 2.1.2**, where they choose among fraction circles, fraction strips, and grids to represent the sum of various fractions, in **question 9** in **lesson 2.2.1**, where they use models to explain the relationship between addition and multiplication, and in **lesson 2.3.1**, where they use a model to represent a fraction as a division.

• Students use visualization skills in **lesson 2.1.5**, where they calculate a difference using steps on a number line, in **lesson 2.2.1**, where they find the product of a fraction and a whole number using rectangular regions and number lines, and in **lesson 2.2.2**, where they see division of a fraction by a whole number as the “sharing” of rectangular regions.

• They make connections in situations like those in **question 2** in **lesson 2.2.1**, where they link addition of fractions with multiplication, and in **question 2** in **lesson 2.3.1** and all of **lesson 2.3.2**, where they link the representation of one fraction as a decimal to the representation of others. There are also many real-world connections, for example, **question 7** in **lesson 2.1.3** and **questions 3 and 4** in **lesson 2.1.4**.

Rationale for Teaching Approach

• This unit is divided into three chapters.

Chapter 1 is about comparing, adding, and subtracting fractions.

Chapter 2 focuses on multiplying and dividing fractions.

Chapter 3 examines the relationship between fractions and decimals.

• The **Explore lesson** allows students to explore 1-digit and 2-digit repeating decimals and their corresponding fraction representations.

• The **Connections** section provides students with a visual representation of repeating decimal patterns using a graphing technique.

• The **Game** provides an opportunity to apply and practise addition of fractions in a pleasant way.

• Throughout the unit, it is important to encourage estimation as a way of determining whether the results of computations make sense, to encourage flexibility in computation, and to accept a variety of approaches from students.

Getting Started

Curriculum Outcomes	Outcome relevance
5 Rename fractions: with and without models (conceptual) **6** Rename Mixed Numbers and Improper Fractions **6** Comparing Fractions **6** Renaming: simple fractions to decimals **6** Addition and Subtraction: simple fractions	Students will find the work in the unit easier after they review the concepts and skills related to fractions and decimals they learned in Class VI.

Pacing	Materials	Prerequisites
1 h	None	• familiarity with the terms *fraction, improper fraction, mixed number,* and *decimal* • representations of fractions

Main Points to be Raised

Use What You Know

• You can represent a fraction as equal parts of a whole.

• To figure out what the fraction is in a fraction model, you determine how many equal parts make up the whole and then see how many are shaded.

• A piece can represent different fractions, depending on what you are using to represent the whole.

For example, in the puzzle, piece B is $\frac{1}{2}$ of piece A, but $\frac{1}{8}$ of the whole puzzle.

Skills You Will Need

• You can rename any mixed number as an improper fraction by renaming each whole as a fraction in the form $\frac{a}{a}$ and adding the pieces. You can do the reverse to go from an improper fraction to a mixed number.

• You can use benchmarks, common denominators, or common numerators to compare fractions.

• You can represent any fraction as an equivalent fraction by multiplying or dividing the numerator and the denominator by the same value.

• A decimal is a representation for a fraction with a denominator of 10, 100, 1000, and so on.

Use What You Know — Introducing the Unit

Introduce the task on **page 35** by discussing with students why the diagram might be viewed as a puzzle. The puzzle they are asked to solve here is to figure out what fraction of the whole each piece represents.

Students can work in pairs or small groups. While you observe students at work, you might ask questions such as the following:

• *How did you know that A is $\frac{1}{4}$ of the whole puzzle?* (It looks like there would be 4 large squares like A in the whole puzzle, so A would be $\frac{1}{4}$.)

• *How do you know that piece B and piece F are the same fraction of the whole puzzle even though they are different shapes?* (It would take 8 pieces shaped like piece B to fill in the whole puzzle and it would take 8 pieces shaped like piece F to fill in the whole puzzle, so each of them is $\frac{1}{8}$ of the puzzle.)

• *How did you order the fractions from least to greatest?* (The numerator for each piece is 1, so I looked only at the denominators — the greater the denominator, the less the fraction.)

Skills You Will Need

- Students can work individually.
- To ensure students have the required skills for this unit, assign these questions.
- First, review the terms *improper fraction* and *mixed number* to make sure students can successfully interpret **questions 1 and 3**. Refer students to the glossary at the back of the student text.
- Encourage students to use different strategies for comparison when answering **question 5**.

Answers

A. i) $\frac{1}{4}$ **ii)** $\frac{1}{2}$ **iii)** $\frac{1}{8}$ **iv)** $\frac{1}{2}$

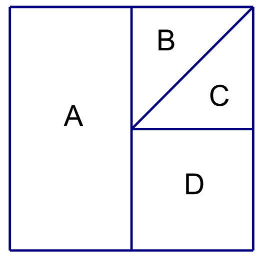
v) *Sample response*:
F and B have the same area, so they represent the same fraction of the whole area ($\frac{1}{8}$).

B. i) A: $\frac{1}{4}$; B: $\frac{1}{8}$; C: $\frac{1}{16}$; D: $\frac{1}{32}$; E: $\frac{1}{32}$; F: $\frac{1}{8}$; G: $\frac{1}{16}$; H: $\frac{1}{16}$; I: $\frac{1}{16}$; J: $\frac{1}{16}$; K, L, M, and N are all $\frac{1}{32}$

ii) B and F;
C, G, H, I, and J;
D, E, K, L, M, and N

C. $\frac{1}{32}, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}$

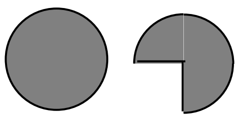
D. *Sample response*:



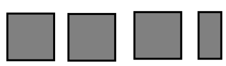
1. a) $\frac{19}{8}, 2\frac{3}{8}$ **b)** $\frac{5}{4}, 1\frac{1}{4}$ **c)** $\frac{7}{2}, 3\frac{1}{2}$

2. *Sample responses:*

a)

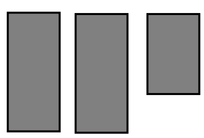


b)



c)

d)



3. a) $\frac{17}{5}$ **b)** $\frac{13}{2}$ **c)** $\frac{13}{3}$ **d)** $\frac{19}{8}$

4. A and C

5. a) < **b)** = **c)** >

6. $1\frac{1}{4}$ h **7.** $\frac{1}{4}$

8. a) 0.3 **b)** 0.27 **c)** 0.5 **d)** 0.6

Supporting Students

Struggling students

- If students are struggling with **question B**, you might encourage them to start with a smaller whole.

For example, they might see that piece D is $\frac{1}{2}$ of piece C and piece C is $\frac{1}{4}$ of piece A. Because it would take 4 of piece A to fill the puzzle, it would take $4 \times 4 = 16$ of piece C and $2 \times 16 = 32$ of piece D to fill the puzzle. That means piece D is $\frac{1}{32}$ of the puzzle.

- Some students may have trouble with **parts c) and d)** of **question 8**. Encourage them first to find equivalent fractions for $\frac{1}{2}$ and $\frac{3}{5}$ that have a denominator of 10 and then to write the decimal.

Enrichment

- For **question D**, you might challenge students to create a fraction puzzle that contains at least four pieces of different shapes. They should make sure to draw the puzzle in such a way that each piece's fraction of the puzzle can be determined by relating it either to the whole or to another piece.

2.1.1 Comparing and Ordering Fractions

Curriculum Outcomes	Outcome relevance
7-A6 Compare and Order: decimals, proper/improper fractions, and mixed numbers • order fractions on a number line • compare fractions using a variety of strategies including benchmarks, common denominator, common numerator, decimal equivalents	When they know several strategies for comparing fractions, students will find it easier to add and subtract fractions.

Pacing	Materials	Prerequisites
1 h	• Fraction Number Lines (BLM)	• finding a common denominator • rewriting improper fractions as mixed numbers and vice versa

Main Points to be Raised

- You can compare and order fractions and mixed numbers using a variety of strategies, such as a number line, a common denominator, and a common numerator.
- On a number line, the fraction farthest to the right is greatest.
- When two fractions have the same denominator, the fraction with the greater numerator is greater than the fraction with the lower numerator.
- When two fractions have the same numerator, the fraction with the greater denominator is less than the fraction with the lower denominator.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know the fraction for Kachap's house will be greater than the fraction for Jigme's house?* (Kachap's house is farther from 0 and closer to 1 than Jigme's house.)

• *How can you tell what denominator to use for the fractions when you mark Sithar's, Jigme's, and Kachap's houses on the number line?* (I need to divide the number line from 0 to 1 into 4 equal parts, so I can use 4 for the denominator of the fraction.)

If students answer **question B ii) and iii)** incorrectly, continue to divide the distance to each previous location in half so students can see how to divide the number line from 0 to 1 into equal parts.

The Exposition — Presenting the Main Ideas

• On the board, draw two number lines from 0 to 2, one above the other. Mark and label one line in fourths using mixed numbers. Mark and label the other line in thirds using improper fractions. Choose two fractions that can be easily modelled with those lines and show students how to compare the two fractions using the number line (e.g., $\frac{5}{3} > 1\frac{1}{4}$ because $\frac{5}{3}$ is farther right on the number line).

• Choose two fractions that have a denominator other than 3 or 4 (e.g., $\frac{5}{3}$ and $\frac{11}{6}$). Find equivalent fractions with a common denominator (e.g., $\frac{5}{3} = \frac{10}{6}$). Show students that to compare two fractions with the same denominator, you can just compare the numerators (e.g., $\frac{10}{6} < \frac{11}{6}$, so $\frac{5}{3} < \frac{11}{6}$).

• Work through the exposition with the students. Reinforce the idea that you can compare fractions using a common numerator as well as a common denominator. Although we usually use a common denominator to compare and order fractions, for some fractions it is easier to find a common numerator.

For example, it is easier to find a common numerator than a common denominator for $\frac{3}{17}$ and $\frac{2}{9}$ because it is much easier to find a common multiple of 2 and 3 than to find a common multiple of 17 and 9.

Revisiting the Try This

B. In **part A** students used a number line to locate fractions, and they are now asked to use the strategies of common denominator and common numerator to do so.

Using the Examples

• Present the problems in the three examples to the students. Ask each student to choose two of the problems to solve. Then the student can compare his or her work to what is shown in the matching example. Suggest that they then read through the other example.

Practising and Applying

Teaching points and tips

Q 1: Make sure students realize that it is often easier to compare improper fractions by writing them as mixed numbers.

For example, in **part a)**, by writing $\frac{29}{5}$ as $5\frac{4}{5}$, they can tell immediately that $\frac{29}{5} < 6\frac{3}{10}$ because 5 < 6.

Q 2: Encourage students to examine the question for clues as to which method might be most helpful.

For example, in **part a)**, since $1\frac{3}{4} = \frac{7}{4}$, all three fractions have a numerator of 7, so it is easiest to use a common numerator for ordering the fractions.

Q 4: Students might benefit from using more than one strategy for comparison.

For example, since $\frac{11}{6} < 2$, and $\frac{17}{8} > 2$, and $\frac{10}{4} > 2$, $\frac{11}{6}$ is the least number. Then you only have to compare $\frac{17}{8}$ and $\frac{10}{4}$ to find the greatest number.

Q 5: In **part b)**, you may wish to restrict ? to a 1-digit number (to avoid students giving extreme answers like ? = 1 or # = 50). In **part c)**, students should recognize that as long as 2 × ■ is greater than 9, the inequality will be true.

Q 6 d): Encourage students to use an example with proper fractions and an example with improper fractions to test the generalization more fully.

Q 9: Use this last question to highlight the value of using different strategies to compare fractions.

Common errors

• In **question 2 a)**, where the fractions will have a common numerator, many students will think the fraction with the least denominator is the least fraction. Encourage the students to think of the fraction as a sharing situation.

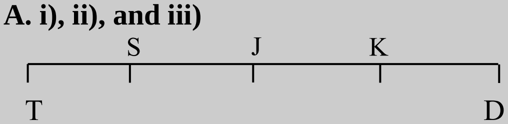
For example, if 7 things are shared among 6 students, each student gets less than if 7 things are shared among 4 students or among 3 students.

Suggested assessment questions from Practising and Applying

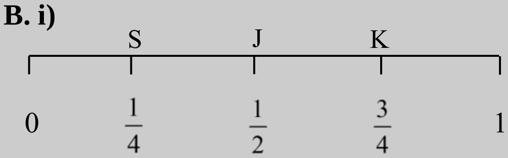
Question 2	to see if students can use different strategies for ordering fractions
Question 4	to see if students can order improper fractions to solve a real-world problem
Question 9	to see if students can communicate about when to use various strategies for comparing fractions

Answers

A. i), ii), and iii)



B. i)



ii) $\frac{7}{8}$ **iii)** $\frac{1}{8}$

C. *Sample responses*:

i) Jigme's house is $\frac{1}{2} = \frac{2}{4}$, Devika's house is $1 = \frac{4}{4}$, and Kachap's house is $\frac{3}{4}$. $\frac{3}{4}$ is between $\frac{2}{4}$ and $\frac{4}{4}$.

ii) Jigme's house is $\frac{1}{2} = \frac{3}{6}$, Devika's house is $1 = \frac{3}{3}$, and Kachap's house is $\frac{3}{4}$. $\frac{3}{4}$ is between $\frac{3}{6}$ and $\frac{3}{3}$.

Answers [Continued]

NOTE: Answers or parts of answers that are in square brackets throughout the Teacher's Guide are NOT found in the answers in the student textbook.

1. a) < **b)** = **c)** < **d)** =

2. a) $\frac{7}{6}$, $1\frac{3}{4}$, $\frac{7}{3}$; [*Sample response*:
I wrote $1\frac{3}{4}$ as $\frac{7}{4}$ and used a common numerator.]

b) $2\frac{1}{3}$, $\frac{11}{4}$, $\frac{9}{2}$; [*Sample response*:
I wrote $2\frac{1}{3}$ as $\frac{7}{3}$ and found equivalent fractions with a common denominator (12).]

c) $1\frac{5}{9}$, $\frac{21}{12}$, $\frac{11}{6}$; [*Sample response*:
I made three number lines from 1 to 2, one in ninths, one in twelfths, and one in sixths. I located each fraction and looked for the one farthest to the right.]

3. Pelden

4. a) Yuden; Rupak
[**b)** *Sample response*:
I found equivalent fractions with a common denominator (24) and compared them.]

5. a) Any value from 1 to 36.
b) *Sample response*:

?	#
1	3
2	6
3	9

c) Any number greater than 4.

6. a) $\frac{1}{2}$, $\frac{3}{5}$, $\frac{2}{3}$; The new fraction is in the middle.
b) i) $\frac{12}{9}$
ii) $\frac{5}{4}$, $\frac{12}{9}$, $\frac{7}{5}$; The new fraction is in the middle.
c) *Sample responses*:
i) $\frac{1}{2}$ and $\frac{3}{4}$ to create $\frac{4}{6}$
ii) $\frac{1}{2}$, $\frac{4}{6}$, $\frac{3}{4}$; The new fraction is in the middle.
[**d)** *Sample response*:
It looks like the fraction formed by adding the numerators and adding the denominators of two fractions will always be between the two fractions.
I tried it again and it was the same:
$\frac{1}{3}$ and $\frac{1}{2}$ makes $\frac{2}{5}$ and $\frac{1}{3} < \frac{2}{5} < \frac{1}{2}$]

7. *Sample response*:
Dechen served two cakes of the same size to her guests. $\frac{2}{3}$ of the first cake and $\frac{3}{5}$ of the second cake were left over. Which cake had more left over?

8. a) $\frac{7}{3}$, $\frac{8}{3}$
b) $\frac{9}{4}$, $\frac{10}{4}$, $\frac{11}{4}$
c) $\frac{11}{5}$, $\frac{12}{5}$, $\frac{13}{5}$, $\frac{14}{5}$
d) No; [*Sample response*:
You can keep finding more and more fractions between 2 and 3 by making the denominator greater and greater. For a denominator of 6 there are 5 fractions between 2 and 3, for a denominator of 7 there are 6 fractions between 2 and 3, and so on.]

[**9.** *Sample response*:
$\frac{3}{10}$ and $\frac{3}{17}$ can easily be compared using a common numerator. For $\frac{2}{3}$ and $\frac{11}{12}$, it would be easier to find equivalent fractions with a common denominator.]

Supporting Students

Struggling students

• Some students might benefit from using marked and labelled number lines for **question 8**.

For example, by writing 2 as $\frac{6}{3}$, $\frac{8}{4}$, $\frac{10}{5}$ and 3 as $\frac{9}{3}$, $\frac{12}{4}$, $\frac{15}{5}$ for **parts a), b), and c)**, students can more easily determine all the fractions with the required denominators that are between the two numbers.

Enrichment

• For **question 6**, you might challenge students to extend what they have discovered to show a quick way to find three fractions that they know will be between two given fractions.

2.1.2 Adding Fractions Using Models

Curriculum Outcomes	Outcome relevance
7-B5 Add and Subtract: simple fractions of various denominators • develop algorithm pictorially • estimate the sum or difference of fractions	By using models to visualize the addition of fractions, students will find it easier to make sense of the algorithm for adding fractions.

Pacing	Materials	Prerequisites
1 h	• Fraction Strips (BLM) • Counters	• writing a fraction given a model of the fraction • drawing a model for a given fraction • naming an equivalent to a fraction

Main Points to be Raised

• You can use fraction strips and grids to add fractions.

• To add fractions with strips, place one strip at the end of the other and look for another strip that has the same total length. It is sometimes easier to visualize if both strips are cut up into the same size pieces (they have the same denominator).

• To add fractions with a grid, create a grid that has the same number of rows as the denominator of one fraction and the same number of columns as the denominator of the other fraction. In this way, you can represent both fractions easily.

• It is sometimes easier to interpret a fraction if it is written in its lowest terms.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How might you use **Model iii)** to help you decide what the white part of **Model i)** is?* (In **Model iii)**, you can see that the white part is 1 of 6 equal parts, or $\frac{1}{6}$. It is the same in **Model i)**.)

• *How do you know the dark grey part of **Model ii)** is $\frac{2}{4}$?* (I pictured the whole circle being divided into pieces of that size, and could see that there would be 4 of those pieces.)

• *Does it make sense that the dark grey part represents $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{6}$?* (Yes, they are all equivalent fractions.)

The Exposition — Presenting the Main Ideas

• Have the students examine the fraction strips and find, for example, the different ways to represent $\frac{1}{3}$ using equal size strips (one $\frac{1}{3}$ strip, two $\frac{1}{6}$ strips, three $\frac{1}{9}$ strips, four $\frac{1}{12}$ strips). Write the corresponding equivalent fractions for $\frac{1}{3}$ ($\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$).

• Work through the part of the exposition on **page 42** with the students. Then draw attention to the grid model on **page 43**. Discuss with the students how the grid was created (i.e., the denominator of the first fraction tells you how many rows the grid will have and the denominator of the second fraction tells you how many columns the grid will have). Practise setting up blank grids for two or three sample addition problems.

• Work through the rest of the exposition with students. Ensure they understand why the counters were moved to add the $\frac{1}{5}$

• You may wish to do an additional example using the grid model for adding fractions, to ensure that students are aware that when they "fill in" the columns for the second fraction, they will first have to move enough counters to clear the number of columns they need to fill.

Revisiting the Try This

B. Students should view the three fraction circle illustrations as models of addition of fractions.

Using the Examples

• Work through **example 1** and **example 3** with the students to make sure they understand them.

• Ask pairs of students to read through **solutions 1 and 2** of **example 2**. Ask them to choose which solution most closely matches what they would have done and why.

Practising and Applying

Teaching points and tips

Q 2 c): If students answer 54, encourage them to try to find a lower number that could be used as the common denominator.

Q 3: You might have students share and compare the strategies they used for estimating the sums.

Q 4: This question might be assigned only to selected students.

Q 6: In **A**, students should recognize that because $\frac{3}{5}$ is greater than $\frac{1}{2}$, $\frac{1}{2}$, and $\frac{3}{5}$ together would be more than one whole cake and this is not possible.

Q 7 b): Observe whether students organize their approach to this question or just randomly put the numbers into the blanks.

Q 8: Suggest that students think about what fraction of the jug was filled when $\frac{1}{2}$ cup of juice was added (i.e., the jug went from $\frac{1}{2}$ full to $\frac{3}{4}$ full).

Common errors

• Some students might add both the numerators and the denominators when they add fractions. Make sure students first estimate the sum and then check to see if their answer makes sense.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can estimate fraction sums and add fractions using different models
Question 5	to see if students recognize that they can use a model for addition of fractions to solve a real-world problem
Question 7	to see if students can solve a problem using addition of fractions
Question 8	to see if students can explain their reasoning in solving a problem that involves the addition of fractions

Answers

A. i) $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$

ii) $\frac{2}{4}, \frac{1}{3}, \frac{1}{6}$

iii) $\frac{3}{6}, \frac{2}{6}, \frac{1}{6}$

B. *Sample response*:

$\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$

$\frac{1}{8}, \frac{1}{8}, \frac{3}{4}$

C. i) $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$

$\frac{2}{4} + \frac{1}{3} + \frac{1}{6} = 1$

$\frac{3}{6} + \frac{2}{6} + \frac{1}{6} = 1$

Sample response:

$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$

$\frac{1}{8} + \frac{1}{8} + \frac{3}{4} = 1$

ii) *Sample response*:
I would line up fraction strips for each fraction and show that they match the 1 strip.

$\frac{3}{5}$

1. a) $\frac{3}{4}$ **b)** $\frac{5}{8}$ **c)** **d)** $\frac{3}{5}$

2. *Sample responses*:
a) 12 **b)** 10 **c)** 18 **d)** 15

3. *Sample responses for estimates*:

a) Estimate: about 2; $\frac{9}{6}$ or $1\frac{1}{2}$

b) Estimate: about $1\frac{1}{2}$; $\frac{11}{8}$ or $1\frac{3}{8}$

c) Estimate: bout 1; $\frac{11}{10}$ or $1\frac{1}{10}$

d) Estimate: about $\frac{1}{2}$; $\frac{4}{9}$

e) Estimate: about $\frac{3}{4}$; $\frac{11}{15}$

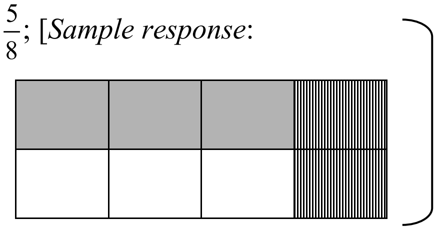
f) Estimate: about 2; $\frac{17}{12}$ or $1\frac{5}{12}$

4. *Sample responses*:

a) $\frac{3}{9} + \frac{5}{12}$

b) $\frac{2}{2} + \frac{1}{2}$; $\frac{3}{4} + \frac{3}{4}$; $\frac{1}{4} + \frac{5}{4}$; $\frac{1}{8} + \frac{11}{8}$; $\frac{5}{8} + \frac{7}{8}$, and so on.

5. $\frac{5}{8}$; [*Sample response*:



6. B; [*Sample response*:
The fractions in B add up to less than one whole, while the fractions in A add up to more than one whole.]

7. a) and b) *Sample responses*:

$\frac{3}{4} + \frac{5}{6} = \frac{19}{12}$ or $1\frac{7}{12}$;

$\frac{3}{4} + \frac{6}{5} = \frac{39}{20}$ or $1\frac{19}{20}$;

$\frac{3}{5} + \frac{4}{6} = \frac{19}{15}$ or $1\frac{4}{15}$;

$\frac{3}{6} + \frac{4}{5} = \frac{13}{10}$ or $1\frac{3}{10}$;

$\frac{3}{6} + \frac{5}{4} = \frac{7}{4}$ or $1\frac{3}{4}$.

8. 2 cups; [*Sample response*:
$\frac{1}{2}$ cup is $\frac{1}{4}$ of the jug, so the whole jug is
$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$ cups.]

[**9.** *Sample response*:
When the denominators are the same, you just have to add the numerators because the size of the fraction pieces is the same. $\frac{4}{6} + \frac{1}{6} = \frac{5}{6}$ because 4 sixths + 1 sixth is 5 sixths.]

[**10.** *Sample response*:
When you use a grid, you automatically get the equivalent fractions with a common denominator. When you add $\frac{1}{5} + \frac{2}{3}$ on a grid, 1 out of 5 rows is 3 out of 15 squares, $\frac{3}{15}$, and 2 out of 3 columns is 10 out of 15 squares, $\frac{10}{15}$. With strips, it does not always show the equivalent fractions.]

Supporting Students

Struggling students

• Struggling students may have difficulty with **question 4**. This question is particularly suitable for strong students.

Enrichment

• Encourage students to create and answer questions like **question 7** using different sets of digits.

2.1.3 Adding Fractions and Mixed Numbers Symbolically

Curriculum Outcomes	Outcome relevance
7-B5 Add and Subtract: simple fractions and mixed numbers of various denominators • develop algorithm symbolically • estimate the sum or difference of fractions and mixed numbers **7-A2 Common Multiplies: use common multiples and least common multiples (LCM) to solve problems** • use the LCM to add and subtract fractions	Being able to add fractions and mixed numbers is an important skill both for everyday life and for higher classes in mathematics. It is important that students understand why the procedures work and not just apply rules without understanding.

Pacing	Materials	Prerequisites
1 h	None	• naming a fraction in an equivalent form • finding a lowest common multiple • writing fractions in lowest terms • renaming improper fractions as mixed numbers

Main Points to be Raised

- To add fractions with the same denominator, you add the numerators.
- To add fractions with unlike denominators, you find equivalent fractions that have the same denominator and then add the numerators.
- To find a common denominator for fractions, you find a common multiple of the denominators (preferably the lowest common multiple).
- You can simplify sums by writing them as mixed numbers and/or in lowest terms.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Which two fractions show the runner's total running time?* ($\frac{1}{2}$ and $\frac{1}{6}$)

• *Can you answer the question without using addition?* (Yes. If he walks for $\frac{1}{3}$ of his training time and runs for the rest of it, then the fraction for running is $\frac{2}{3}$ because $\frac{2}{3}$ and $\frac{1}{3}$ combined make the whole time.)

The Exposition — Presenting the Main Ideas

• Write the example $\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$ on the board. Make a drawing like the one in the student text that shows combining 1 fifth and 3 fifths to make 4 fifths. Then do the same thing for $\frac{1}{7}+\frac{3}{7}=\frac{4}{7}$ and $\frac{1}{9}+\frac{3}{9}=\frac{4}{9}$.

Ask what is the same and what is different in these examples. Lead students to see that the denominator tells them the size of the piece and the numerator tells them how many pieces they have. If the denominators are the same, they only need to add the numerators to find the sum.

• Review the term *multiple* with the students. Write the number 5 on the board and ask students to tell you the first several multiples. Write the multiples on the board.

• Lead students through the exposition.

• Some students may notice that you can always find a common denominator by multiplying the denominators together. Remind them that it is usually best to use the lowest common denominator.

Revisiting the Try This

B. Students apply what they have learned about finding a common denominator to adding $\frac{1}{2} + \frac{1}{6}$ from **part A**.

Using the Examples

• Have students work in pairs. One student should become an expert on **example 1** and they should become an expert on **example 2**. Each student should then explain his or her example to the other.

Practising and Applying

Teaching points and tips

Q 1 and 3: Although the questions ask students to choose only one question to estimate, encourage students to estimate first whenever they are adding fractions.

Q 4: You might encourage students to use fraction strips to help explore possible answers for this question.

Q 5: Remind students that they can add the whole numbers and fractions separately, then put them together to record the sum.

Q 7: Make sure students are aware that the two different kinds of sugar listed in the recipe need to be added to find the total amount of sugar. For **part b)**, you might ask students to share their explanations with a partner.

Q 8: Encourage students to use what they know about fractions to help them with this question — the greater the denominator, the smaller the fraction, and the greater the numerator, the greater the fraction.

Q 9: This question highlights the importance of using a common denominator to add fractions.

Common errors

• Some students will forget to use three different denominators in **question 4 a)**. To address this, provide fraction strips (BLM) and remind them that they need to use three strips of different sizes.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can estimate fraction sums and add proper fractions by finding a common denominator
Question 3	to see if students can calculate the sum of two mixed numbers and write answers in lowest terms when necessary
Question 7	to see if students can solve a real-world problem by adding fractions and mixed numbers, and explain their reasoning
Question 8	to see if students can solve a problem using addition of fractions

Answers

A) $\frac{4}{6}$ or $\frac{2}{3}$	**B.** *Sample response*: $\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$ or $\frac{2}{3}$
1. a) i) $\frac{7}{9}$ **ii)** $\frac{7}{8}$ **iii)** $\frac{22}{15} = 1\frac{7}{15}$ **iv)** $\frac{29}{20} = 1\frac{9}{20}$ [**b)** *Sample response*: $\frac{3}{8} + \frac{1}{2}$; $\frac{3}{8}$ is close to $\frac{1}{2}$, so $\frac{3}{8} + \frac{1}{2}$ is close to 1. My answer of $\frac{7}{8}$ is close to 1, so the answer is reasonable.]	**2. a)** $\frac{15}{8} = 1\frac{7}{8}$ **b)** $\frac{25}{18} = 1\frac{7}{18}$ **3. a) i)** $3\frac{4}{6} = 3\frac{2}{3}$ **ii)** $2\frac{9}{10}$ **iii)** $9\frac{11}{15}$ **iv)** $7\frac{11}{12}$

Answers [Continued]

[**3. b)** *Sample response*:

$4\frac{2}{3}$ is a bit less than 5 and $3\frac{1}{4}$ is a bit more than 4 so $4\frac{2}{3} + 3\frac{1}{4}$ is about 8. My answer of $7\frac{11}{12}$ is about 8 so the answer is reasonable.]

4. *Sample responses*:

a) $\frac{1}{2} + \frac{2}{5} + \frac{1}{10}$

[**b)** I knew $\frac{1}{2} = \frac{5}{10}$ and $\frac{2}{5} = \frac{4}{10}$, so I needed $\frac{1}{10}$ more to make 1.]

5. $4\frac{7}{12}$ cups

6. a) $\frac{8}{15}$ **b)** $\frac{13}{15}$

7. a) $\frac{17}{24}$ cup

b) Yes; [*Sample response*:

The ingredients added together make $3\frac{5}{24}$ cups.

$3\frac{1}{2} = 3\frac{12}{24}$

Since $3\frac{12}{24} > 3\frac{5}{24}$, the bowl is big enough.]

8. a) $\frac{2}{5} + \frac{1}{4} = \frac{13}{20}$

b) $\frac{5}{1} + \frac{4}{2} = 7$

c) $\frac{1}{2} + \frac{3}{5} = \frac{11}{10}$, or $\frac{2}{4} + \frac{3}{5} = \frac{11}{10}$

[**9.** *Sample response*:
When fractions have the same denominator, you only need to add the numerators to get the new numerator. You use the denominator you already know.]

Supporting Students

Struggling students

• If students are struggling with any part of **question 8**, you may wish to ask leading questions to help them. For example, in **question 8 b)**, you might ask students to name the greatest fraction that can be made with the given digits ($\frac{5}{1}$ or 5). Next, have them consider the remaining digits to get the next greatest fraction.

Enrichment

• For **question 4**, you might challenge students to find many combinations of three fractions with different denominators that add to 1.

GAME: A "Whole" in One

• This game is designed to allow students to practice addition of fractions with unlike denominators.

• If students ask, tell them the name of the game is a play on words that relates to an expression used by people who play golf. A "hole in one" happens when a player's ball goes into the hole on his or her first shot.

• Students may realize that if they draw two numbers that are doubles of the other two, they can always make 1.

• Students can change the target sum if they wish.

For example, instead of 1, the target sum could be $\frac{1}{2}$ or 2.

2.1.4 Subtracting Fractions and Mixed Numbers

Curriculum Outcomes	Outcome relevance
7-B5 Add and Subtract: simple fractions and mixed numbers of various denominators • develop algorithm pictorially and symbolically • estimate the sum or difference of fractions and mixed numbers	Students need to extend their understanding of addition of fractions and mixed numbers to subtraction of fractions and mixed numbers so they can deal with a greater variety of real-world problems.

Pacing	Materials	Prerequisites
1 h	• Fraction Strips (BLM) • Counters	• using equivalent fractions • knowledge of lowest common multiple and its relationship to common denominators • renaming improper fractions as mixed numbers and vice versa

Main Points to be Raised

• To subtract fractions using fraction strips, you line up the strips. A strip that represents how much longer one strip is than the other is the difference.

• It is sometimes easier to figure out what strip to use to represent the difference if you use equivalent fractions with the same denominator.

• You can use a grid to subtract fractions. Create a grid using the two denominators as the number of rows and number of columns. Represent the greater fraction on the grid with counters. Remove counters that represent the other fraction of the grid.

• To subtract fractions symbolically, use equivalent fractions with the same denominator. Subtract the numerators to create the numerator for the difference. Use the common denominator as the denominator for the difference.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How would you find Tshering's time on the project if Meghraj worked on it for* $\frac{3}{4}$ *h?* (Tshering worked $\frac{1}{4}$ h longer than Meghraj, so I would add $\frac{1}{4}$ to $\frac{3}{4}$. Tshering worked on the project for 1 h if Meghraj worked for $\frac{3}{4}$ h.)

• *How can you find other answers that will work?* (If I choose a time for Meghraj and then add $\frac{1}{4}$ h to it, I will have another answer for the question.)

• *Would it make sense for Tshering to have spent* $\frac{1}{4}$ *h on the project?* (No, because that would mean that Meghraj did not work on the project at all.)

The Exposition — Presenting the Main Ideas

• Ask students to suggest how they would use fraction strips to compare two fractions such as $\frac{4}{5}$ and $\frac{2}{5}$. Have them note that in this example, it is easy to see the length of the strip that makes up the difference (it is the length of two $\frac{1}{5}$ strips, or $\frac{2}{5}$).

• Read through the first example in the exposition on **page 53** with the students. Note that the hardest part about using fraction strip models is determining the size of the strip that makes up the difference.

• Work through the grid model example in the exposition with the students. Make sure students recognize that the subtraction can be thought of as subtracting squares in the grid and not just as rows and columns.

• Finish working through the exposition with students, pointing out how this procedure is similar to addition (finding equivalent fractions with a common denominator, then subtracting the numerators).

• Provide an opportunity for students to ask questions if they do not understand.

Revisiting the Try This

B. Encourage students to consider the answer to a subtraction sentence as the difference between two fractions. They may notice that you can begin with any fraction and then add $\frac{1}{4}$ to get a possible pair. They may also realize that once they have one pair, they can simply add the same amount to both values to get another pair.

Using the Examples

• Write the problems in the three examples on the board. Ask each student to choose two of the problems to solve. Students should then compare their work to what is shown in the matching example in the student text. Suggest that they also read through the example they did not solve.

Practising and Applying

Teaching points and tips

Q 1: You might encourage students to estimate before they begin and to use the estimates to see if their answers make sense.

Q 4: Some students may not recognize that this problem can be solved in different ways.

For example, they may subtract $\frac{3}{4} - \frac{1}{8}$ and compare the result to $\frac{1}{2}$ or they may subtract $\frac{3}{4} - \frac{1}{2}$ and compare the result to $\frac{1}{8}$.

Q 5: Remind students that they can use a model or estimate to answer this question without having to do all of the subtractions.

Q 6: You might have students note the similarity of this question to the question in the **Try This**.

Q 7 c): Students need to recognize that two steps are needed to solve this part. They must add the two given fractions and then subtract from 1. Or, they must subtract the two fractions, one at a time.

Q 10: Encourage students to use what they know about fractions to help them with this question: the greater the denominator, the smaller the fraction; the greater the numerator, the greater the fraction.

Common errors

• Many students will subtract instead of adding in **question 8 b)**. Have students read the question carefully.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can estimate differences and subtract proper fractions
Question 2	to see if students can extend what they know about subtracting fractions to mixed numbers
Question 5	to see if students can explain their estimation strategies for subtraction
Question 10	to see if students can solve a problem using subtraction of fractions

Answers

A. *Sample response*:
Tshering
$\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{6}{4}$

Meghraj
$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}$

B. *Sample response*:
Yes;
You could start at $\frac{1}{2}$ and keep adding $\frac{1}{2}$ to each value:
$\frac{1}{2} - \frac{1}{4}, 1 - \frac{3}{4}, 1\frac{1}{2} - 1\frac{1}{4}, \ldots$

1. a) i) $\frac{1}{8}$ **ii)** $\frac{2}{9}$ **iii)** $\frac{9}{20}$ **iv)** $\frac{2}{15}$

[**b)** *Sample response*:
$\frac{5}{8} - \frac{1}{2}$; $\frac{5}{8}$ is a bit more than $\frac{1}{2}$, so $\frac{5}{8} - \frac{1}{2}$ will be a bit more than 0, so $\frac{1}{8}$ is a reasonable answer.]

2. a) $1\frac{1}{5}$ **b)** $1\frac{3}{6}$ or $1\frac{1}{2}$

c) $3\frac{2}{7}$ **d)** $2\frac{1}{10}$

3. Red rice; $\frac{1}{12}$ cup more

4. a) More; [*Sample response*:
If $\frac{1}{2}$ of a tank was used, there would be $\frac{1}{4}$ of a tank left ($\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$), and there is only $\frac{1}{8}$ of a tank left.]

b) $\frac{1}{8}$ more than $\frac{1}{2}$ of a tank

[**5.** *Sample response*:
The difference is less than $\frac{1}{2}$ for both because the number being subtracted is $\frac{1}{2}$ or greater and the numbers they are subtracted from are less than 1.]

6. *Sample response*: $\frac{1}{1} - \frac{1}{4}$; $\frac{5}{4} - \frac{1}{2}$; $\frac{3}{2} - \frac{3}{4}$

7. a) Red

b) $\frac{1}{15}$

c) $\frac{4}{15}$; [*Sample response*:
$\frac{2}{5} + \frac{1}{3} = \frac{11}{15}$, so $1 - \frac{11}{15}$ or $\frac{4}{15}$ did not vote]

8. The other fraction is between $\frac{1}{2}$ and $\frac{3}{4}$;
[*Sample response*
The lesser fraction is between 0 and $\frac{1}{4}$, and the difference is $\frac{1}{2}$, so the other fraction has to be $\frac{1}{2}$ more than the lesser fraction, or between $\frac{1}{2}$ and $\frac{3}{4}$.]

9. a) $\frac{17}{40}$ **b)** $\frac{5}{20}$ or $\frac{1}{4}$ **c)** More; $\frac{1}{8}$

10. a) $\frac{5}{2} - \frac{3}{4} = \frac{7}{4}$

b) $\frac{3}{5} - \frac{2}{4} = \frac{2}{20}$ or $\frac{1}{10}$

c) $\frac{5}{4} - \frac{2}{3} = \frac{7}{12}$

[**11.** *Sample response*:
I like finding equivalent fractions with a common denominator. Here are some examples:
$\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$ and $\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$]

Supporting Students

Struggling students

• Struggling students may need help with estimation strategies (**questions 1 and 5**). You might remind them of valuable benchmarks to use with fractions: close to 0, close to 1, and close to $\frac{1}{2}$.

• If students are struggling with any part of **question 10**, you may wish to ask leading questions to help them. For example, in **question 10 a)**, you might ask students to name the greatest fraction that can be made with the given digits ($\frac{5}{2}$) and then have them consider the remaining digits to get the smallest fraction to subtract from it.

Enrichment

• You might ask students to create and solve other questions like **question 10** using different digits.

2.1.5 Subtracting Mixed Numbers in Different Ways

Curriculum Outcomes	Outcome relevance
7-B5 Add and Subtract: simple fractions and mixed numbers of various denominators • develop algorithm pictorially and symbolically • estimate the sum or difference of fractions and mixed numbers	Being able to subtract mixed numbers is a skill for everyday life. The work on this outcome extends what students have already learned about subtracting fractions.

Pacing	Materials	Prerequisites
1 h	• Fraction Number Lines (BLM)	• renaming mixed numbers and improper fractions in various ways. • subtracting proper fractions • renaming fractions as equivalent ones

Main Points to be Raised

• To use a number line as a model for subtracting fractions, you find the distance between two numbers on the number line. You might do it in steps, first going from a fraction or mixed number to the nearest whole number, and then doing the rest. You might also jump a whole number of steps and then add or subtract enough to land where you wish.

• To subtract mixed numbers where the fraction of the greater number is less than the fraction of the lower number, you can use a number line model.

• To subtract mixed numbers, it is sometimes helpful to rename them as improper fractions, or to rename the greater mixed number, before subtracting.

For example, you might rename $2\frac{1}{5}$ as $\frac{11}{5}$ or $1\frac{6}{5}$.

• To subtract mixed numbers and fractions with unlike denominators, you can use equivalent fractions with
a common denominator.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know Ngawang jumped at least 1 m farther than Jamyang?* (Jamyang jumped less than 4 m and Ngawang jumped 5 m.)

• *How much farther would Jamyang have to jump to get to 4 m?* (He would have to jump $\frac{1}{4}$ m more.)

The Exposition — Presenting the Main Ideas

• On the board, draw a number line marked with the whole numbers from 0 to 10. Ask students to find on the number line, for example, the distance from 2 to 5. When students have answered a few questions with whole numbers, add marks for $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, and so on to the number line. Ask students to find distances such as
the distance from $1\frac{1}{2}$ to 3, and so on.

• Have students look at the exposition on **page 58** to see to use the number line for subtracting a fraction or a mixed number from a whole number.

• Have students recall how to rewrite a mixed number as an improper fraction.

• Go through the rest of the exposition with students. You might ask some students to share the method they prefer for subtracting mixed numbers and to tell why they prefer it.

• Provide an opportunity for students to ask questions if they do not understand.

Revisiting the Try This

B. Students can use any strategy presented in the exposition to find the difference between two mixed numbers.

Using the Examples

• Ask pairs of students to read through **solutions 1 and 2** of **example 1**. Ask them to choose which solution most closely matches what they would have done and to tell why they chose it. Do the same thing for **example 2**.

Practising and Applying

Teaching points and tips

Q 1: You might encourage students to try different methods.

For example, they might use the number line for some and rename the whole number for others.

Q 2: If some students rewrite both mixed numbers as improper fractions for each exercise, you might encourage them to try one exercise by renaming only the greater mixed number and to compare the results. Point out how estimating is easier when the numbers are left as mixed numbers.

Q 3: You might point out to students that this question is very similar to what they did in the **Try This**.

Q 8: You might choose to assign this question only to selected students.

Q 9: Some students might not be familiar with how Magic Squares work. Remind them that in a Magic Square each row, column, and diagonal has the same sum.

Q 10: You might have students discuss the different ways of renaming a mixed number.

For example, $3\frac{1}{4}$ could be renamed as $1\frac{9}{4}$, or it could be renamed as $2\frac{5}{4}$.

Common errors

• In **question 2**, some students might find equivalent fractions with a common denominator and then subtract the lesser fraction from the greater fraction regardless of whether the greater fraction is in the minuend or the subtrahend. You might have them model the question on a number line so they can see that this does not work.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can apply what they know about subtracting fractions to subtracting mixed numbers
Question 5	to see if students can rename mixed numbers in different ways
Question 7	to see if students can subtract mixed numbers to solve a real-world problem
Question 9	to see if students can both add and subtract mixed numbers to solve a problem

Answers

A. $1\frac{1}{4}$ m	**B.** $4\frac{3}{4}$ m; *Sample response*: $8\frac{1}{2} - 3\frac{3}{4} = 7\frac{3}{2} - 3\frac{3}{4} = 7\frac{6}{4} - 3\frac{3}{4} = 4\frac{3}{4}$
1. a) $3\frac{2}{5}$ **b)** $5\frac{4}{7}$ **c)** $2\frac{1}{6}$ **d)** $4\frac{5}{9}$ **2. a)** $1\frac{31}{40}$ **b)** $3\frac{8}{9}$ **c)** $1\frac{3}{4}$ **d)** $2\frac{17}{18}$ **3.** $2\frac{3}{8}$ laps	**4.** $1\frac{1}{4}$ h **5. a)** $3\frac{5}{6} = 2\frac{11}{6}$ **b)** $7\frac{3}{10} = 6\frac{13}{10}$ **6.** $2\frac{3}{4}$ h longer **7.** $1\frac{3}{4}$ fewer laps

Answers [Continued]

8. a) $1\frac{5}{8}$ m from one and $\frac{3}{4}$ m from the other.

b) No; [She would have $2\frac{3}{8}$ m, but she needs $3\frac{1}{2}$ m.]

9. a)

$1\frac{4}{5}$	$3\frac{9}{10}$	$2\frac{2}{5}$
$3\frac{3}{10}$	$2\frac{7}{10}$	$2\frac{1}{10}$
3	$1\frac{1}{2}$	$3\frac{3}{5}$

b) The magic sum is $8\frac{1}{10}$

[**10.** *Sample response*:
If you are subtracting $4\frac{1}{8} - 2\frac{3}{8}$, you can subtract 2 from 4 but you cannot subtract $\frac{3}{8}$ from $\frac{1}{8}$ without using negatives. If you regroup $4\frac{1}{8}$ as $3\frac{9}{8}$, you can subtract the whole numbers ($4 - 2 = 2$) and the fractions ($\frac{9}{8} - \frac{3}{8} = \frac{6}{8}$) to get $1\frac{6}{8}$, or $1\frac{3}{4}$.]

Supporting Students

Struggling students

• If students are struggling with **question 8 b)**, explain that they need to add the two answers they obtained in **part a)** and then compare that result to $3\frac{1}{2}$ m.

• Some students may have trouble determining where to start in **question 9**. You might calculate the magic sum as a class and then ask students which square they might try to fill in next (there are two options). You may choose not to assign **question 9** to struggling students.

Enrichment

• For **question 9**, you might challenge students to design their own Magic Squares. Remind them that they need to provide enough information so that the magic sum can be determined and that there must be a starting point for filling in the other squares.

Students might enjoy constructing Magic Squares using the following method:

1. Write the number $1\frac{1}{2}$ in the centre cell on the top row.
2. Move one cell up and one cell to the right. (To do this, you have to assume that the top row "wraps around" to the bottom row and that the right column "wraps around" to the left column.)
3. If this cell is empty, write in the next highest number in the sequence.
4. If this cell is not empty, move down one cell within the same column, "wrapping around" from the bottom row to the top row if necessary.
5. Repeat steps 2 to 4 until you have filled all the cells. The largest number in the sequence should be in the middle of the bottom row. If this is not the case, then you have made a mistake somewhere.

$17\frac{1}{2}$	$24\frac{1}{2}$	$1\frac{1}{2}$	$8\frac{1}{2}$	$15\frac{1}{2}$
$23\frac{1}{2}$	$5\frac{1}{2}$	$7\frac{1}{2}$	$14\frac{1}{2}$	$16\frac{1}{2}$
$4\frac{1}{2}$	$6\frac{1}{2}$	$13\frac{1}{2}$	$20\frac{1}{2}$	$22\frac{1}{2}$
$10\frac{1}{2}$	$12\frac{1}{2}$	$19\frac{1}{2}$	$21\frac{1}{2}$	$3\frac{1}{2}$
$11\frac{1}{2}$	$18\frac{1}{2}$	$25\frac{1}{2}$	$2\frac{1}{2}$	$9\frac{1}{2}$

Chapter 2 Fraction Multiplication and Division

2.2.1 Multiplying a Fraction by a Whole Number

Curriculum Outcomes	Outcome relevance
7-B6 Multiply and Divide: fraction by a whole number • develop and apply strategies necessary for calculation of fractions • use concrete models and pictorial representations	By seeing the relationship between multiplying a whole number by a fraction and fraction addition, students will find it easier to make sense of multiplying fractions later on.

Pacing	Materials	Prerequisites
1 h	None	• multiplying whole numbers • adding fractions with common denominators

Main Points to be Raised

• Multiplying a fraction by a whole number is similar to multiplying two whole numbers. You can show this by representing the multiplication as repeated addition of the fraction.

• You can model multiplication of a fraction by a whole number by using an area model for a fraction or by using jumps on a number line.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How might you find how much time Kuenga spends walking to and from school in 2 days?* (Add $\frac{3}{4}$ h + $\frac{3}{4}$ h.)

• *How can you find out how much time Kuenga spends walking to and from school in 3 days? 4 days?* (I could continue to add another $\frac{3}{4}$ h for each day to get the total for any number of days.)

• *Why does it make sense that the total time he spends walking to and from school in 6 days will be less than 6 h?* (He spends less than 1 h each day walking to and from school, so in 6 days his total time will be less than 6 h.)

The Exposition — Presenting the Main Ideas

• Write a multiplication sentence such as 3 × 5 on the board. Remind students that they can think of multiplication as repeated addition; you can think of 3 × 5 as 5 + 5 + 5 (three fives) = 15.

• Ask students what repeated addition they might use for a multiplication sentence such as $3 \times \frac{1}{2}$. Students should see that they can represent this as 3 groups of $\frac{1}{2}$, or $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1\frac{1}{2}$ (or $\frac{3}{2}$).

• Work through the exposition with students.

- For the section showing fractions as parts of a whole area, you might show how all that happened on the right is putting the four grey pieces together. The pieces are shown on top of one whole to make it clear what fraction the total represents. You might also write 4 × 1 fifth = (4 × 1) fifths.

- For the section showing the number line, make sure students see that there are five jumps because of the 5 in the equation and that each jump is $\frac{2}{3}$ long.

• Provide an opportunity for students to ask questions if they do not understand.

Revisiting the Try This

B. Students apply multiplication of a fraction by a whole number to their own time getting to and from school, and use the repeated addition model to diagram it. They will observe that repeated addition only works as a model for multiplication if all the addends (numbers being added) are the same.

Using the Examples

• Work through the example with the students to make sure they understand it.

Practising and Applying

Teaching points and tips

Q 2: This is an important generalization. Students should recognize that repeated addition can only be used as a model for multiplication when all of the addends are the same.

Q 3: Students may need to review writing improper fractions as mixed numbers.

Q 4: Encourage students to generalize a statement from their observations in this question — when the whole number a fraction is being multiplied by is equal to the denominator of the fraction, the product will be equal to the numerator of the fraction.

Q 6: Students might observe that because the answer would be 3 h for 4 trees, it has to be 7 times as much for 28 trees.

Q 7: Many students will notice that the product is always calculated using only the numerators if the denominator is not changed.

Q 9: Students can choose whichever model they prefer, perhaps even a model of their own creation.

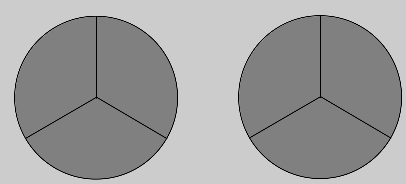
Common errors

• Some students may have difficulty showing more than one model in **question 9.** Remind students that they can use a region or rectangle model, or a number line model.

Suggested assessment questions from Practising and Applying

Question 2	to see if students realize that repeated additions can only be a model for multiplication if the addends are the same
Question 3	to see if students can find the product of a fraction multiplied by a whole number and write the answer as a mixed number
Question 6	to see if students recognize that they can multiply a fraction by a whole number to solve a real-world problem

Answers

A. $4\frac{1}{2}$ h **B.** *Sample response*: $\frac{1}{3}$ h **i)** $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ or $6 \times \frac{1}{3} = \frac{6}{3} = 2$ h	**ii)**  **iii)** *Sample response*: No, not everyone takes the same amount of time to get to and from school each day, so the fractions would not all be the same. You would have to add.
1. a) $5 \times \frac{1}{3} = \frac{5}{3}$ or $1\frac{2}{3}$ **b)** $3 \times \frac{7}{10} = \frac{21}{10}$ or $2\frac{1}{10}$ **c)** $6 \times \frac{2}{9} = \frac{12}{9}$ or $\frac{4}{3}$ or $1\frac{3}{9}$ or $1\frac{1}{3}$ **2.** A ($5 \times \frac{3}{8}$) and C ($7 \times \frac{5}{3}$); [*Sample response*: B cannot be written as a multiplication because the fractions being added are not equivalent.]	**3. a)** $2\frac{2}{5}$ **b)** $5\frac{1}{2}$ **c)** $3\frac{6}{8}$ or $3\frac{3}{4}$ **d)** $1\frac{5}{10}$ or $1\frac{1}{2}$ **4. a) i)** 5 **ii)** 7 **iii)** 3 [**b)** *Sample response*: When the whole number is the same as the denominator of the fraction, the answer is a whole number and it's the numerator of the fraction.]

5. $\frac{14}{3}$ or $4\frac{2}{3}$ apples

6. 21 h

7. a) $\frac{21}{10}$ and $\frac{21}{10}$

[**b)** *Sample response*:
The whole number and the numerator of the fraction are switched around but the product is the same.]

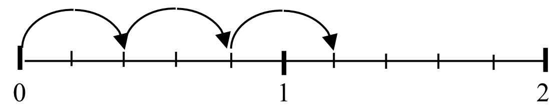
c) *Sample response*:

$3 \times \frac{2}{5} = \frac{6}{5}$ and $2 \times \frac{3}{5} = \frac{6}{5}$

8. $\frac{8}{3}$ or $2\frac{2}{3}$ cups of walnuts

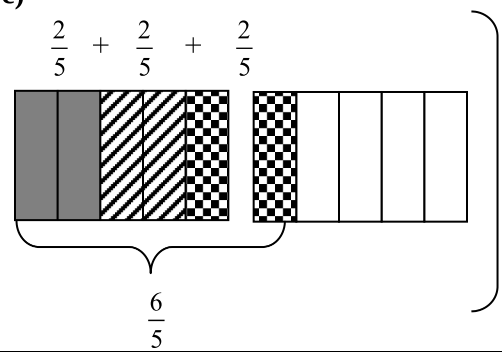
[**9.** *Sample responses*:

a)



b) The number line shows 3 jumps of $\frac{2}{5}$, or $3 \times \frac{2}{5} = \frac{6}{5}$.

c)



Supporting Students

Struggling students

• Some students may have trouble simplifying the answer in **question 6**. You might have those students solve simpler problems and look for a pattern.

For example, it would take 3 h to pick the peaches from 4 trees, 6 h for 8 trees, 9 h for 12 trees, and so on.

Enrichment

• You might ask students to create and solve their own word problems involving multiplying a fraction by a whole number.

2.2.2 Dividing a Fraction by a Whole Number

Curriculum Outcomes	Outcome relevance
7-B6 Multiply and Divide: fraction by a whole number • develop and apply strategies necessary for calculation of fractions • use concrete models and pictorial representations	By being able to model the division of a fraction by a whole number, students will find it easier to make sense of dividing fractions later on.

Pacing	Materials	Prerequisites
1 h	• Fraction Strips (BLM)	• dividing whole numbers • familiarity with the area/region model for fractions

Main Points to be Raised

• Dividing a fraction by a whole number is similar to dividing two whole numbers. Just as with whole numbers, it means sharing.

• You can illustrate dividing a fraction by a whole number using an area/region model of a fraction. You represent a fraction and then split it into the number of equal pieces required by the divisor.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How might you find out how much rice is needed for 2 servings?* (If $\frac{2}{3}$ cup is needed for 4 servings, then I know $\frac{1}{3}$ cup would make 2 servings because $\frac{1}{3}$ for 2 servings + $\frac{1}{3}$ for 2 servings is $\frac{2}{3}$ for 4 servings.)

• *How can you use the information for 2 servings to find out what you need for 1 serving?* (I know $\frac{1}{3}$ cup is needed for 2 servings, so half of that would make 1 serving. I guessed the amount would be $\frac{1}{6}$ cup and checked it by seeing if two $\frac{1}{6}$ smade $\frac{1}{3}$ and it did.)

The Exposition — Presenting the Main Ideas

• Write a division of whole numbers example such as 6 ÷ 3 on the board. Ask students to come up with a sharing question that this expression could answer.

For example, if 6 apples are shared equally by 3 people, how many apples will each person get? (6 ÷ 3 = 2, so each person would get 2 apples.)

• Draw a picture of $\frac{1}{2}$ a cake. Ask how much of the cake each person would get if 2 people shared the $\frac{1}{2}$ cake. (You might draw a dotted line to show the $\frac{1}{2}$ cake being cut into two pieces.) Students should recognize that each piece is $\frac{1}{4}$ of the cake.

• Work through the exposition with the students. In the last part of the exposition, make sure students understand how to determine that one third of $\frac{1}{5}$ is $\frac{1}{15}$.

• Provide an opportunity for students to ask questions if they do not understand.

Revisiting the Try This

B. Students use what they learned in the exposition to draw a picture that models the division they did in **part A.**

Using the Examples

• Present the problem in the example. Ask pairs of students to solve it and then read through **solutions 1 and 2**. Ask them which solution most closely matches what they did.

Practising and Applying

Teaching points and tips

Q 1: Remind students to think in terms of sharing when they draw the picture for each division.

Q 2: Some students may choose to find equivalent fractions where the numerator divides evenly by the whole number.

For example, for **part b)**, they might rename $\frac{5}{8}$ as $\frac{10}{16}$. In **part c)**, they might rename $\frac{4}{5}$ as $\frac{12}{15}$.

Q 5: Remind students to solve the problem they create.

Q 6: Encourage students to organize the division expressions they write. They should be able to explain how they know they have written all the different division expressions possible.

Q 7: This is an important generalization. Students should realize that, when the numerator is divisible by the whole number divisor, the denominator stays the same; when the numerator is not divisible by the divisor, the denominator changes to the product of the denominator and the divisor.

Common errors

• Some students will have difficulty with **question 2 d), g), and h)**. You might remind them how to generate a list of equivalent fractions, continuing the list until they have a fraction with a numerator that can be divided evenly by the whole number in the division.

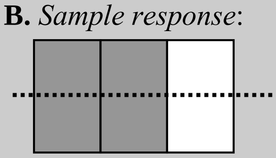
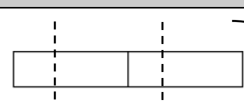
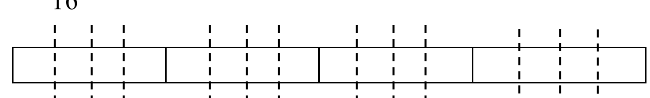
For example, for $\frac{7}{2} \div 4$, the equivalent fractions they generate might be $\frac{7}{2} = \frac{14}{4} = 2\frac{1}{6} = \frac{28}{8}$.

• Some students will divide both the numerator and the denominator by the divisor. Have students estimate to see if their answers are reasonable.

Suggested assessment questions from Practising and Applying

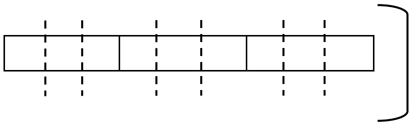
Question 2	to see if students can find the quotient for a fraction divided by a whole number
Question 4	to see if students recognize that they can divide a fraction by a whole number to solve a real-world problem
Question 6	to see if students can solve a problem by dividing a fraction by a whole number
Question 7	to see if students can explain what happens in division of a fraction by a whole number when the numerator of the fraction is (or is not) divisible by the whole number

Answers

A. $\frac{1}{6}$ cup	**B.** *Sample response*: 
1. a) $\frac{1}{4}$; [*Sample response*: ]	**b)** $\frac{1}{16}$; [*Sample response*: ]

Answers [Continued]

1. c) $\frac{2}{9}$; [*Sample response*:



2. a) $\frac{1}{15}$ **b)** $\frac{5}{16}$ **c)** $\frac{4}{15}$

d) $\frac{7}{8}$ **e)** $\frac{2}{3}$ **f)** $\frac{3}{4}$

g) $\frac{3}{10}$ **h)** $\frac{3}{28}$ **i)** $1\frac{1}{10}$

3. $\frac{5}{12}$ h

4. $\frac{3}{4}$ cup

5. *Sample response*:

Five students share $\frac{1}{2}$ of a cake equally. What fraction of the whole cake does each student get? ($\frac{1}{10}$)

6. a) $\frac{2}{4} \div 6 = \frac{1}{12}$; $\frac{4}{2} \div 6 = \frac{1}{3}$; $\frac{2}{6} \div 4 = \frac{1}{12}$;
$\frac{6}{2} \div 4 = \frac{3}{4}$; $\frac{4}{6} \div 2 = \frac{1}{3}$; $\frac{6}{4} \div 2 = \frac{3}{4}$

b) $\frac{6}{2} \div 4$ and $\frac{6}{4} \div 2$

c) $\frac{2}{4} \div 6$ and $\frac{2}{6} \div 4$

d) $\frac{6}{2} \div 4$ and $\frac{6}{4} \div 2$; $\frac{2}{4} \div 6$ and $\frac{2}{6} \div 4$;
$\frac{4}{2} \div 6$ and $\frac{4}{6} \div 2$

[**7.** *Sample response*:

If 2 fifths ($\frac{2}{5}$) are shared by 2 people, they can be shared equally as 1 fifth ($\frac{1}{5}$) each.

If 3 fifths ($\frac{3}{5}$) are shared by 2 people, they cannot be shared equally unless the fifths are divided into tenths. Each person then gets 3 tenths ($\frac{3}{10}$).]

Supporting Students

Struggling students

• If students are having trouble listing all the possible division expressions in **question 6 a)**, you might have them organize the expressions.

For example, what are all the expressions you can write with 2 as the divisor? 4 as the divisor? 6 as the divisor?

Enrichment

• For **question 6**, you might challenge students to create and solve their own problem using different digits.

Chapter 3 Relating Fractions and Decimals

2.3.1 Naming Fractions and Mixed Numbers as Decimals

Curriculum Outcomes	Outcome relevance
7-A7 Rename: Mixed Numbers and Fractions • rename fractions and mixed numbers as decimals • use pictorial models to represent mixed numbers and fractions • introduce the terminology "repeating" and "period" as well as notation to show that a decimal repeats • explore patterns in various fractions, especially sevenths	Recognizing the relationship between fractions and decimals is important for making sense of concepts and operations.

Pacing	Materials	Prerequisites
1 h	None	• rewriting fractions with denominators of 10, 100, 1000, and so on as decimals • identifying factors of 10, 100, 1000, and so on • recognizing that a fraction represents a division • renaming a fraction as an equivalent one • dividing a decimal by a whole number

Main Points to be Raised

- You can find a decimal equivalent to any fraction and a fraction equivalent to any decimal.
- A fraction that you can write as an equivalent fraction with a denominator of 10, 100, 1000, and so on has a decimal equivalent called a terminating decimal.
- You can write a decimal equivalent for a fraction by using the division meaning of a fraction.
- A fraction that you cannot write as an equivalent fraction with a denominator of 10, 100, 1000, and so on has a decimal equivalent called a repeating decimal.
- The length of the repeating part of a decimal is called its period.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *What fraction of a jug would each person get if the 5 people shared 1 jug of water?* ($\frac{1}{5}$ of a jug)

• *If there were more water, how much more would each person get for each additional jug?* (Each person would get another $\frac{1}{5}$ of a jug for each additional jug of water.)

• *For your answer, why did you find an equivalent fraction that has a denominator of 10 or 100?*
($\frac{8}{10}$ is an equivalent fraction for $\frac{4}{5}$ and it is easy to write as a decimal.)

The Exposition — Presenting the Main Ideas

• Write fractions such as $\frac{7}{10}$ and $\frac{13}{100}$ on the board. Ask students to recall how to write fractions with denominators of 10, 100, 1000, and so on as decimals.

• It might be helpful for students if you note that:

$10 = 2 \times 5$ $\quad$ $100 = 2 \times 2 \times 5 \times 5$ $\quad$ $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$ $\quad$ and so on

If a fraction has a denominator that you can write as a product of only 2s and/or 5s, you can find an equivalent fraction that has a denominator of 10, 100, 1000, and so on.

• Work through the exposition with the students.

• Make sure that students understand the picture on **page 68** that shows why $\frac{3}{8} = 3 \div 8$. Remind students that we use division to show sharing, which is what each of the 3 wholes divided into 8 sections shows (on the left). Then point out that the share is rearranged and placed on top of one whole to show why it is $\frac{3}{8}$ of a whole.

• Work through the division of 3 by 8 with the students. Discuss why you write the 3 as 3.000 in order to express the quotient to three decimal places.

• To explain why the division of 1 by 3 goes on forever, you may want to use language like this:

- Think of 1 or 1.0 as 10 tenths. If 3 people share 10 tenths, each gets 3 tenths, but there is 1 tenth left over.
- Think of the leftover 1 tenth as 10 hundredths. If 3 people share 10 hundredths, each gets 3 hundredths, but there is 1 hundredth left over.
- Think of the leftover 1 hundredth as 10 thousandths. If 3 people share 10 thousandths, each gets 3 thousandths, but there is 1 thousandth left over.

Students will soon see the pattern. The sharing will never end.

• Make sure students can describe the difference between a terminating decimal and a repeating decimal. Note that any fraction, when written as a decimal, will be either a terminating decimal or a repeating decimal.

Revisiting the Try This

B. Once students recognize that the number of people sharing the jugs of water is the denominator of the fraction, they can use equivalent fractions or fractions as division to write fractions as decimals. They should consider properties of fractions that relate to recognizing which are terminating and which are repeating decimals.

Using the Examples

• Present the questions in both examples to students. Have them try the questions with a partner. They should then compare their solutions with those solutions in the student text. In each case, they might observe which solution most closely matches what they did.

Practising and Applying

Teaching points and tips

Q 1 f): Some students may not recognize eighths as a fraction that can be written as an equivalent fraction with a denominator of 10, 100, 1000, and so on. You might have them write the denominator as $2 \times 2 \times 2$ and ask what each 2 has to be multiplied by to make 10.

Q 2: This question should help students realize that once they know the decimal representation for a fraction, they can write the decimal representation for any whole number multiple of the fraction by multiplying the decimal by that number.

For example, since $\frac{1}{5}$ (1 fifth) is 0.2, then $\frac{3}{5}$ (3 fifths) is $3 \times 0.2 = 0.6$.

Q 3: Remind students to examine the denominator to determine in advance whether the decimal will be a terminating decimal or a repeating decimal.

Q 4 c): Students may need some guidance to recognize the pattern for sevenths.

Q 7: You might encourage students to discuss why it is easier to compare the decimal representations than the fraction representations.

Common errors

• Some students will divide the denominator by the numerator instead of the numerator by the denominator in **question 3**. Ask students whether the decimal for each fraction will be less than or greater than 1.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can apply either the equivalent fraction method or division meaning method to rewrite fractions as decimals
Question 4	to see if students recognize patterns in the decimal equivalents for fractions
Question 7	to see if students can recognize when it might be easier to use decimal equivalents to compare fractions

Answers

A. $\frac{4}{5}$; 0.8

B. i) 2 **ii)** 1.333… **iii)** 1
iv) 0.8 **v)** 0.666…
vi) 0.571428571428… **vii)** 0.5

C. 2.0, 1, 0.8, 0.5; *Sample response*:
Only the denominators 2, 4, 5, and 8 can be written as tenths, hundredths or thousandths. The other denominators had factors like 3 and 7 that do not divide evenly into powers of 10.

1. a) $\frac{5}{10} = 0.5$ **b)** $\frac{6}{10} = 0.6$
c) $\frac{75}{100} = 0.75$ **d)** $\frac{35}{100} = 0.35$
e) $\frac{44}{100} = 0.44$ **f)** $\frac{875}{1000} = 0.875$

2. 0.2
a) 0.6 **b)** 0.8 **c)** 1.4
d) 1.6 **e)** 2.2 **f)** 2.4

3. a) 0.272727… **b)** 0.625 **c)** 0.222…
d) 0.48 **e)** 0.8333… **f)** 0.58333…

4. a) [0.11…, 0.22…, 0.33…
The repeating part of the decimal is the same as the numerator of the fraction.]
0.11…, 0.22…, 0.33…, 0.44…, 0.55…, 0.66…, 0.77…, 0.88…
b) [0.0909…, 0.1818…, 0.2727…
The repeating part of the decimal is the numerator × 9 with a period of 2 (09, 18, 27, and so on).]
0.0909…, 0.1818…, 0.2727…, 0.3636…, 0.4545…, 0.5454…, 0.6363…, 0.7272…, 0.8181…, 0.9090…
c) [0.142857142857…,
0.285714285714…,
0.428571428571…
The repeating part of the decimal is 142857.
The pattern for $\frac{1}{7}$ begins with 1. For each fraction after $\frac{1}{7}$, the decimal begins with the next greater digit (2, 4, 5, 7, and finally 8).]
0.142857142857…,
0.285714285714…,
0.428571428571…,
0.571428571428…,
0.714285714285…,
0.857142857142….

5. a) i) $\frac{1}{3}$ **ii)** $\frac{1}{3}$ **iii)** $\frac{1}{3}$
[*Sample response*:
0.3, 0.33, 0.333 all terminate, so they all have zeros from that point on, e.g., 0.3000, 0.3300, 0.3330. But $\frac{1}{3}$ is greater because it has threes repeating forever, 0.3333….]
b) i) $\frac{3}{10} < \frac{3}{9}$ **ii)** $\frac{33}{100} < \frac{33}{99}$ **iii)** $\frac{333}{1000} < \frac{333}{999}$

6. a) 0.48 > 0.46 **b)** 0.875 < 0.88

7. a) 0.875 > 0.833…, so $\frac{7}{8} > \frac{5}{6}$.
b) 0.166… > 0.16, so $\frac{1}{6} > \frac{4}{25}$.
c) 0.22 < 0.222…, so $\frac{11}{50} < \frac{2}{9}$.
d) 0.3636… > 0.36, so $\frac{11}{4} > \frac{18}{50}$.

8. 3 digits; 3.14

9. a) 0, 1, 2, 3, 4, 5, and 6
[**b)** *Sample response*:
To find the decimal for $\frac{1}{7}$, you divide 1 by 7.
You cannot get a remainder of 0 since the decimal is a repeating one. You might get all the possible remainders, 1 through 6, but at some point you will get a remainder you have had before, and from that point on the decimal will repeat.]

[**10.** *Sample response*:
For fractions like $\frac{4}{11}$ and $\frac{9}{25}$, it would not be easy to find a common denominator to compare them, but you can quickly compare the decimal equivalents (0.363636… and 0.36).]

Supporting Students

Struggling students

• Some students may have trouble with **parts e) and f)** of **question 3** because the decimal for these fractions does not repeat until the second or third digit. You might need to show students how to write these decimals so that it is clear which part of the decimal is repeating.

Enrichment

• For **question 9**, you might encourage students to generalize for other fractions.

For example, would a fraction with a denominator of 17 have to repeat? How do you know? How long could the period be for fraction 17ths?

Note that the decimal representation for 17ths only repeats after 16 digits. If a computer or calculator is available, students might enjoy exploring other fractions.

For example, 23rds result in a repeating decimal with a period of 22, but 37ths result in a repeating decimal with a period of only 3.

2.3.2 EXPLORE: Relating Repeating Decimals and Fractions

Curriculum Outcomes	Outcome Relevance
7-A8 Rename: Repeating Decimals to Fractions • explore 1- and 2-digit repeating decimals • use patterns to rename and make predictions	This essential exploration provides a strategy for writing fractions for repeating decimals.

Pacing	Materials	Prerequisites
1 h	None	• multiplying a decimals by a whole number • dividing a decimal by a whole number

Exploration

• Work through the introduction (in white) with the students. Make sure that they understand that you can divide a repeating decimal by a whole number in the same way that you can divide a terminating decimal by a whole number — the division just has to be carried out far enough for the repeating pattern to become clear.

• Have students work alone, in pairs, or in small groups for **parts A to F**. You may wish to give them an example of how to recognize what number a repeating decimal has been multiplied (or divided) by.

For example, they can recognize that 0.555… is 0.111… × 5.

While you observe students at work, you might ask questions such as the following:

• *How did you know what to multiply 0.010101… by in* ***part C i)****?* (Since $\frac{13}{99}$ is 13 × $\frac{1}{99}$, I knew I had to multiply 0.010101… by 13.)

• *How did you know that 13 × 0.010101… is 0.131313…?* (I multiplied each group of 01 by 13.)

• *In* ***part D iv)****, could you write a simpler fraction for the decimal?* ($\frac{44}{99}$ is the same as $\frac{4}{9}$.)

• Discuss **parts A to F** with the students to make sure they are proceeding successfully.

Observe and Assess

As students work, notice the following:

• Do they recognize how to use the decimal for $\frac{1}{9}$ to write a fraction with a denominator of 99 as a decimal?

• Do they successfully multiply and divide a repeating decimal by a whole number?

• Do they recognize that they can use the patterns they notice both to write repeating decimals for other fractions with the same denominator, and to write other fractions for repeating decimals that follow similar patterns?

Share and Reflect

After students have had sufficient time to work through the exploration, they could form small groups to discuss their observations and answer these questions. Or, you could discuss them with the whole class.

• *How are the patterns in* ***part A*** *and* ***part C*** *the same? How are they different?*

• *What pattern could you use to determine that 0.272727… is equal to* $\frac{27}{99}$*? What pattern could you use to determine that 0.272727… is equal to* $\frac{3}{11}$*?*

Answers

A. i) $\frac{2}{9}$ **ii)** $\frac{5}{9}$ **iii)** $\frac{7}{9}$ **B.** $\frac{1}{9}$, or 1 ninth = 0.111… so 9 ninths is 0.99999…, but $\frac{9}{9} = 1$ **C. i)** 0.131313… **ii)** 0.373737… **iii)** 0.515151… **iv)** 0.747474… **D. i)** $\frac{41}{99}$ **ii)** $\frac{42}{99}$ or $\frac{14}{33}$ **iii)** $\frac{43}{99}$ **iv)** $\frac{44}{99}$ or $\frac{4}{9}$	**E. i)** $\frac{6}{9} = \frac{2}{3}$ **ii)** $\frac{4}{9}$ **iii)** $\frac{27}{99} = \frac{3}{11}$ **iv)** $\frac{81}{99} = \frac{9}{11}$ **v)** $\frac{15}{99} = \frac{5}{33}$ **vi)** $\frac{6}{99} = \frac{2}{33}$ **F. i)** Write the repeating digit as the numerator of a fraction with the denominator 9. Then, if necessary, write the fraction in lowest terms. **ii)** Write the two repeating digits as the numerator of a fraction with the denominator 99. Then, if necessary, write the fraction in lowest terms.

Supporting Students

Struggling students

• If students are struggling with **part F**, you might have them make two columns on their paper:

- In column 1, they should write $\frac{1}{9}$ = 0.111…, $\frac{2}{9}$ = 0.222…

- In column 2, they should write $\frac{1}{99}$ = 0.010101…, $\frac{2}{99}$ = 0.020202…

Ask the students to continue both patterns for a few more numbers and then use what they observe to help them answer **part F**.

Enrichment

• You might challenge students to come up with a way of finding and writing decimal equivalents for repeating decimals with a period of 3.

CONNECTIONS: Repeating Decimal Graphs

• This optional connection can be used as enrichment for some students.

• Make sure students understand why the y in one row appears as the x in the next row. (Each pair of digits is listed, so for 0.0769, the pair starting with 0 appears in the row with 0 and 7, the pair starting with 7 appears in the row with 7 and 6, etc.)

• Students may have difficulty understanding the phrase "the fraction thirteenth family". You might spend some time explaining fraction families.

For example,:

The fraction fifth family is $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$ and the fraction ninth family is $\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{6}{9}$, $\frac{7}{9}$, and $\frac{8}{9}$.

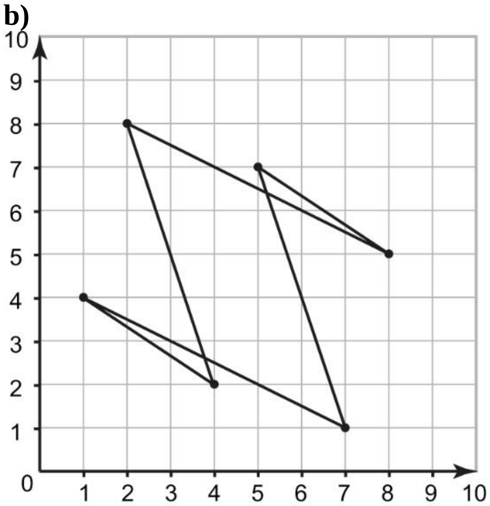
• If students become interested in exploring such graphs further, the fraction 17ths family provides an excellent challenge, while the fraction 37ths family, which is much easier to graph, provides some very interesting variations.

Answers

<table>
<tr>
<td>

1. a) 0.142857, 0.285714, 0.428571, 0.571428, 0.714285, 0.857142

b)

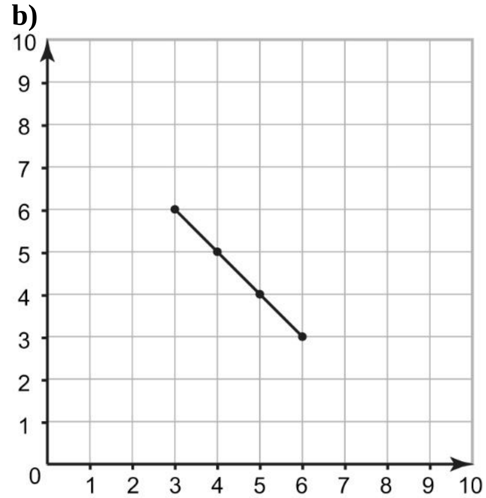


c) One shape

</td>
<td>

2. a) *Sample response*:
A straight line

b)



</td>
</tr>
</table>

UNIT 2 Revision

Pacing	Materials
2 h	• Fraction Strips (BLM) (optional) • Fraction Number Lines (BLM) (optional) • Counters

Question	Related Lesson(s)
1 – 3	Lesson 2.1.1
4	Lesson 2.1.2
5	Lesson 2.1.3
6	Lesson 2.1.4
7 and 8	Lesson 2.1.5
9	Lesson 2.2.1
10	Lesson 2.2.2
11	Lessons 2.2.1 and 2.2.2
12 and 13	Lesson 2.3.1
14	Lesson 2.3.2

Revision Tips

Q 1 and Q 2: Some students may choose to use fraction strips or a number line; others will use equivalent fractions with common denominators.

Q 3: Students can either rewrite 85 months in years or rewrite $7\frac{2}{3}$ years in months.

Q 7: For **part a)**, encourage students to consider which values for ■ will make the fraction greater than $\frac{1}{2}$

Similarly, for **part b)**, they should consider which values for ■ would make the fraction less than $\frac{1}{2}$.

Q 8 c): Encourage students to think about what the sum of the three fractions has to be, and why.

Q 13: You might encourage students to explain their strategies for comparing the fractions.

Q 14: Ensure that students notice that they are asked to write each fraction in lowest terms.

Answers

1. a) < **b)** > **c)** < **d)** =

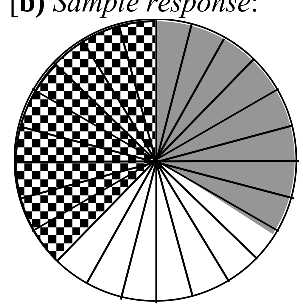
2. a) $\frac{13}{8}$, $\frac{7}{4}$, $1\frac{7}{8}$; [*Sample response*:
I used a common denominator.]

b) $3\frac{1}{5}$, $3\frac{3}{10}$, $\frac{21}{6}$; [*Sample response*:
I used a number line.]

c) $1\frac{3}{9}$, $\frac{9}{6}$, $\frac{27}{15}$; [*Sample response*:
I first wrote the fractions in lowest terms, and then used a common denominator.]

3. Rinzin
[*Sample response*:
85 months is $7\frac{1}{12}$ years, and $7\frac{2}{3} > 7\frac{1}{12}$.]

4. a) $\frac{17}{24}$ of the cake was eaten. [**b)** *Sample response*:



$\frac{1}{3} + \frac{3}{8} = \frac{8}{24} + \frac{9}{24} = \frac{17}{24}$

5. a) $\frac{11}{20}$

b) $\frac{41}{24}$ or $1\frac{17}{24}$

c) $6\frac{8}{10}$ or $6\frac{4}{5}$

6. a) $\frac{11}{24}$ **b)** $\frac{5}{24}$ **c)** $7\frac{1}{8}$

d) $2\frac{2}{5}$ **e)** $4\frac{7}{12}$ **f)** $1\frac{3}{6}$ or $1\frac{1}{2}$

7. a) 3 or 4; [*Sample response*:

If ■ were 3 or 4, then $\frac{■}{5}$ would be greater than $\frac{1}{2}$ and Dorji would be able to subtract without regrouping.]

b) 1 or 2; [*Sample response*:

If ■ were 1 or 2, then $\frac{■}{5}$ would be less than $\frac{1}{2}$ and Dorji would not be able to subtract without regrouping.]

c) *Sample response*: $2\frac{7}{10}$; [$6\frac{1}{5} - 3\frac{1}{2} = \frac{31}{5} - \frac{7}{2}$

$= \frac{62}{10} - \frac{35}{10}$

$= \frac{27}{10} = 2\frac{7}{10}$]

8. a) Archery

b) $\frac{7}{20}$

c) $\frac{3}{20}$; [*Sample response*:

$\frac{3}{5} + \frac{1}{4} = \frac{17}{20}$, and $1 - \frac{17}{20} = \frac{3}{20}$, so $\frac{3}{20}$ of the class did not vote for archery or for football.]

9. a) 4

b) $\frac{21}{5}$ or $4\frac{1}{5}$

c) $\frac{25}{3}$ or $8\frac{1}{3}$

10. a) $\frac{2}{9}$ **b)** $\frac{11}{10}$ or $1\frac{1}{10}$ **c)** $\frac{5}{18}$

11. a) $\frac{21}{4}$ or $5\frac{1}{4}$ h **b)** $\frac{3}{8}$ h

12. 0.125

a) 0.25 **b)** 0.375 **c)** 0.625

d) 0.875 **e)** 1.375

13. a) i) $\frac{4}{9}$ is greater **ii)** $\frac{4}{9}$ is greater **iii)** $\frac{4}{9}$ is greater

[*Sample response*:

0.4 , 0.44 , 0.444 are all terminating decimals, so they all have zeros from some point on, but $\frac{4}{9}$ is a repeating decimal with fours repeating forever.]

b) i) $\frac{4}{9} > \frac{4}{10}$

ii) $\frac{4}{9} = \frac{44}{99}$ and $\frac{44}{99} > \frac{44}{100}$

iii) $\frac{4}{9} = \frac{444}{999}$ and $\frac{444}{999} > \frac{444}{1000}$

14. a) $\frac{6}{11}$ **b)** $\frac{14}{33}$ **c)** $\frac{7}{9}$

UNIT 2 Fractions Test

Express all fractions in lowest terms. Write all improper fractions as mixed numbers.

1. Order from least to greatest.

a) $1\frac{3}{5}$, $\frac{13}{8}$, $1\frac{7}{10}$ **b)** $\frac{12}{5}$, $2\frac{1}{3}$, $\frac{19}{8}$

2. In 2007, Saturdays and Sundays made up $\frac{2}{7}$ of February, $\frac{3}{10}$ of June, $\frac{1}{3}$ of September, and $\frac{4}{15}$ of November.

a) Which month had the greatest fraction of weekend time?

b) Which month had the least?

3. To add $\frac{1}{4} + \frac{2}{3}$, Pema says that grid with 4 rows and-3 columns is a good model.

a) Do you agree? Explain.

b) Which equivalent fractions for $\frac{1}{4} + \frac{2}{3}$ will the grid model show?

c) Which other model would work? Explain how you would use the model to add $\frac{1}{4} + \frac{2}{3}$.

4. Add.

a) $\frac{4}{9} + \frac{1}{6}$ **b)** $\frac{3}{4} + \frac{5}{6}$

c) $3\frac{1}{2} + 2\frac{1}{3}$ **d)** $3\frac{5}{8} + 6\frac{7}{12}$

5. Subtract.

a) $\frac{5}{6} - \frac{2}{9}$ **b)** $5\frac{1}{2} - 3\frac{3}{8}$

c) $7 - 2\frac{3}{5}$ **d)** $4\frac{4}{9} - 1\frac{7}{12}$

6. Choki spends $\frac{2}{3}$ h each day on homework.

a) How many hours does Choki spend on homework in one week?

b) Choki spends an equal amount of time each day on four subjects. How many hours does she spend on each subject in one day?

7. Multiply.

a) $7 \times \frac{4}{7}$ **b)** $15 \times \frac{5}{6}$

c) $5 \times \frac{5}{9}$ **d)** $9 \times \frac{3}{5}$

8. Divide.

a) $\frac{2}{3} \div 6$ **b)** $\frac{35}{4} \div 7$

c) $\frac{5}{9} \div 3$ **d)** $\frac{4}{3} \div 8$

9. Write each as a decimal.

a) $\frac{1}{4}$ **b)** $\frac{5}{8}$

c) $\frac{3}{11}$ **d)** $\frac{2}{7}$

e) $1\frac{2}{9}$

10. Write each as a fraction.

a) 0.333…

b) 0.181818…

c) 0.575757…

d) 0.484848…

UNIT 2 Test

Pacing	Materials
1 h	• Fraction Strips (BLM) (optional) • Fraction Number Lines (BLM) (optional) • Counters

Question	Related Lesson(s)
1 and 2	Lesson 2.1.1
3	Lesson 2.1.2
4	Lesson 2.1.3
5	Lessons 2.1.4 and 2.1.5
6	Lessons 2.2.1 and 2.2.2
7	Lesson 2.2.1
8	Lesson 2.2.2
9	Lesson 2.3.1
10	Lesson 2.3.2

Select questions to assign according to the time available.

Answers

1. a) $1\frac{3}{5}$, $\frac{13}{8}$, $1\frac{7}{10}$ **b)** $2\frac{1}{3}$, $\frac{19}{8}$, $\frac{12}{5}$

2. a) September **b)** November

3. a) *Sample response*:

Yes, 1 of the 4 rows can be used to model $\frac{1}{4}$, and 2 of the 3 columns can be used to model $\frac{2}{3}$.

b) $\frac{3}{12}$ and $\frac{8}{12}$

c) *Sample response*:

Fraction strips could be used:

<table>
<tr><td colspan="3">$\frac{1}{4}$</td><td colspan="4">$\frac{1}{3}$</td><td colspan="4">$\frac{1}{3}$</td></tr>
<tr><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td></tr>
</table>

$\frac{1}{4} + \frac{2}{3} = \frac{11}{12}$

4. a) $\frac{11}{18}$ **b)** $\frac{19}{12} = 1\frac{7}{12}$

c) $5\frac{5}{6}$ **d)** $10\frac{5}{24}$

5. a) $\frac{11}{18}$ **b)** $2\frac{1}{8}$

c) $4\frac{2}{5}$ **d)** $2\frac{21}{36}$

6. a) $\frac{14}{3}$ (Students may multiply by 6, thinking she does homework only on school days, so $\frac{12}{3}$ or 4 is also an acceptable answer.)

b) $\frac{1}{6}$ h

7. a) 4 **b)** $\frac{25}{2} = 12\frac{1}{2}$

c) $\frac{25}{9} = 2\frac{7}{9}$ **d)** $\frac{27}{5} = 5\frac{2}{5}$

8. a) $\frac{1}{9}$ **b)** $\frac{5}{4} = 1\frac{1}{4}$

c) $\frac{5}{27}$ **d)** $\frac{1}{6}$

9. a) 0.25 **b)** 0.625
c) 0.272727… **d)** 0.285714285714…
e) 1.222…

10. a) $\frac{1}{3}$ **b)** $\frac{2}{11}$

c) $\frac{19}{33}$ **d)** $\frac{16}{33}$

UNIT 2 Performance Task — Describing a Garden Plot

Deki has a garden plot for growing vegetables.

- Deki plants $\frac{1}{8}$ of the garden plot with radishes.
- The area for potatoes is 4 times as large as the area for radishes.
- Deki plants $\frac{3}{4}$ of the plot with potatoes and chillies.
- Beans take up the rest of the plot.

A. i) What fraction of the garden plot is used for potatoes?

ii) What fraction is used for chillies? How does this compare with the fraction used for potatoes?

iii) What fraction of the plot is used for beans?

B. Deki plants three kinds of chillies. He uses an equal area for each kind. What fraction of the plot is used for each kind of chilli?

C. Use what you have learned about fractions to create a description of a garden plot like the plot above.

Your description should use:

- four or more different types of crops
- two or more fractions with different denominators
- fraction addition, subtraction, multiplication, or division

You should describe the entire area of the garden plot.

If you were to give your description to a classmate, he or she ought to be able to figure out what fraction of the garden is planted with each crop.

UNIT 2 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
7-A6 Compare fractions using a variety of strategies **7-B5** Add and subtract simple fractions of various denominators **7-B6** Multiply and divide a fraction by a whole number	40 min	• Fraction Strips (BLM) or Fraction Number Lines (BLM) (optional)

How to Use This Performance Task

You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. It could also be used as enrichment material for some students. You can assess performance on the task using the rubric provided on the next page.

Sample Solution

A. i) $\frac{1}{2}$ **ii)** $\frac{1}{4}$; less **iii)** $\frac{1}{8}$

R	B	P	P
C	C	P	P

B. $\frac{1}{12}$

C.

- $\frac{1}{10}$ of my garden plot is planted with onions.
- The area for growing potatoes is 3 times as large as the area for onions.
- $\frac{3}{10}$ of the plot is planted with onions and chillies.
- Radishes and beans each take up the same area. Together, they take up $\frac{2}{5}$ of the plot.
- The rest of the garden plot is planted in turnips.

What fraction of the garden plot does each vegetable use?

Answer:

O	P	P	P	B
C	C	R	R	B

UNIT 2 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Calculates correctly	Shows completely correct calculations for adding, subtracting, multiplying, and dividing with fractions	Shows mostly correct calculations, with minor errors in one or two of the operations	Shows many correct calculations, but with some errors, and does not meet some of the specifications	Shows errors in most calculations, and does not meet the specifications
Calculates creatively	Uses a wide variety of strategies in performing the calculations	Uses a number of strategies in performing the calculations	Uses a few strategies repetitively in calculations	Makes no obvious use of strategies in calculations
Uses fraction concepts and notation properly	Makes consistently correct use of fraction concepts and notation	Usually makes correct use of fraction concepts and notation	Makes correct use of fraction notation; does not understand some fraction concepts (e.g. that sum of all fractions used must equal 1)	Makes some correct use of fraction concepts and notations; does not understand many fraction concepts
Creates a proper garden plot	Effectively uses all four fraction operations; provides complete and clear information	Uses most fraction operations; provides complete and clear information	Uses only two fraction operations; may provide incomplete information	Shows little understanding of fraction concepts and operations; provides incomplete information (i.e., diagram of garden plot cannot be produced from information given)

UNIT 2 Assessment Interview

You may wish to take the opportunity to interview selected students to assess their understanding of the work of this unit. Interviews are most effective when done with individual students, although it is sometimes appropriate to interview students in pairs or small groups. The results can be used as formative assessment or as a piece of summative assessment data. As the students work, ask them to explain their thinking.

Have available a set of fraction strips and fraction number lines that students can use if they wish. Make it clear that they can decide whether or not to use the materials; there is no penalty or benefit to them either way.
Ask the student to explain each:

- why $\frac{17}{8} < 3\frac{4}{9}$
- why $\frac{3}{4} + \frac{5}{6} = 1\frac{7}{12}$
- why $\frac{5}{6} - \frac{3}{4} = \frac{1}{12}$
- why $4 \times \frac{3}{4} = 3$
- why $\frac{5}{6} \div 2 = \frac{5}{12}$
- why $\frac{7}{8} = 0.875$
- why $\frac{5}{9} = 0.5555....$

BLM 1 Fraction Strips

1

$\frac{1}{2}$ $\frac{1}{2}$

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$

$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

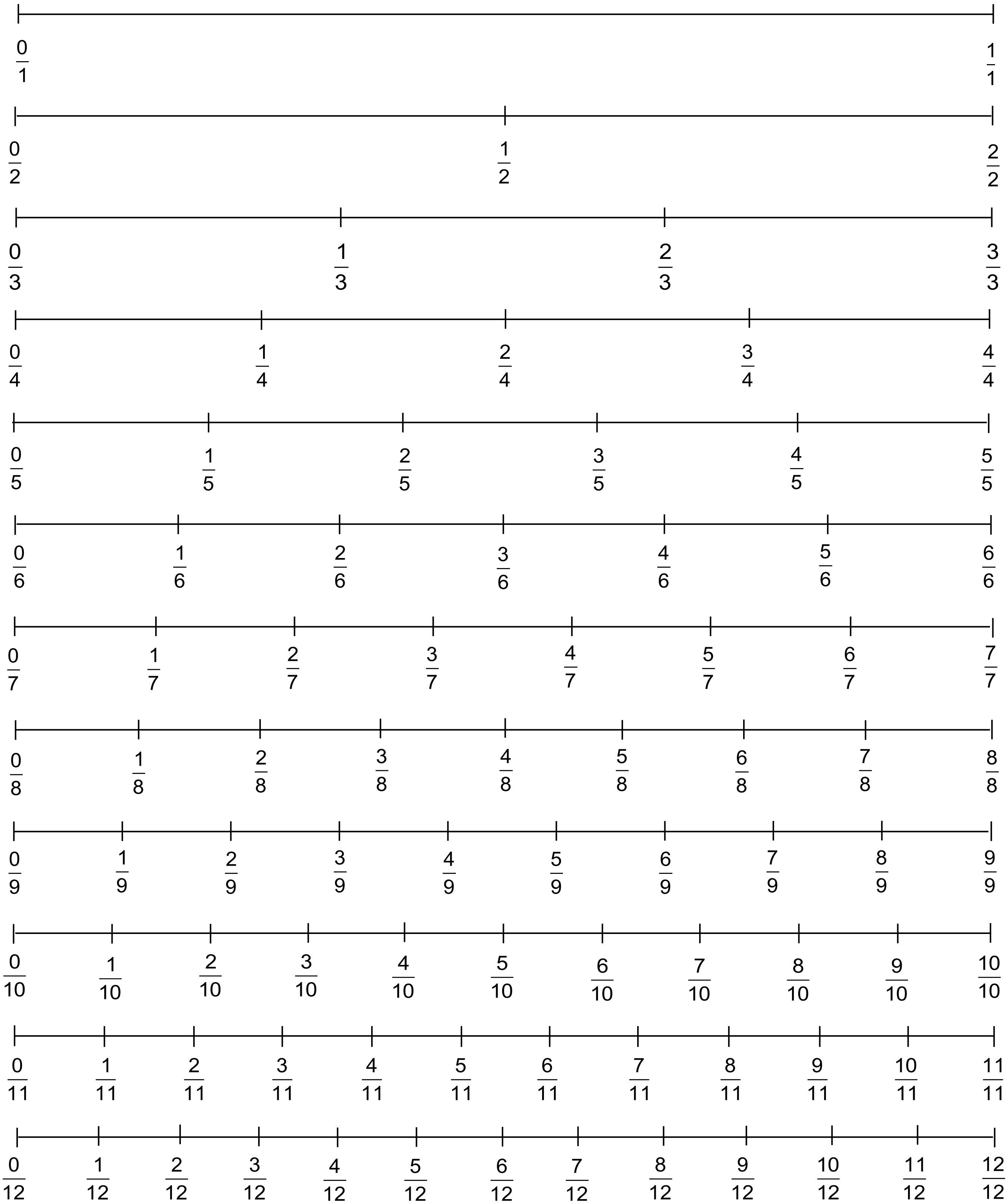
$\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$

$\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$

$\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$ $\frac{1}{10}$

$\frac{1}{12}$ $\frac{1}{12}$ $\frac{1}{12}$ $\frac{1}{12}$ $\frac{1}{12}$ $\frac{1}{12}$ $\frac{1}{12}$ $\frac{1}{12}$ $\frac{1}{12}$ $\frac{1}{12}$ $\frac{1}{12}$ $\frac{1}{12}$

BLM 2 Fraction Number Lines



UNIT 3 RATIO, RATE, AND PERCENT

UNIT 3 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 77** **TG p. 100**	Review prerequisite concepts, skills, and terminology, and pre-assessment	1 h	• Black and white slips of paper	All questions
Chapter 1 Ratio and Rate				
3.1.1 Solving Ratio Problems **SB p. 79** **TG p. 102**	**7-A9 Equivalent Ratios and Rates: solve problems** • solve problems involving equivalent ratios	1 h	None	Q2, 3, 7
3.1.2 Solving Rate Problems **SB p. 83** **TG p.105**	**7-A9 Equivalent Ratios and Rates: solve problems** • solve problems involving equivalent rates **7-D6 Rate: Compare two quantities** • understand rate as the comparison between two quantities with different units • write as a rate (e.g., m/s, km/h, beats per minutes)	1 h	None	Q1, 7, 8
Chapter 2 Percent				
3.2.1 Percent as a Special Ratio **SB p. 87** **TG p. 108**	**7-A10 Percent: as a special ratio** • understand percent as a special ratio • understand that parts should always add up to 100% • relate visual and symbolic representations of percent	1 h	• Percent Grids (BLM) or grid paper	Q3, 5, 7, 9
3.2.2 Relating Percents, Fractions, and Decimals **SB p. 90** **TG p. 111**	**7-A10 Percent: as a special ratio** • relate percent to fraction and decimal equivalents • use benchmark percents **7-A11 Percent: number sense** • estimate and calculate percents for familiar fractions concretely and symbolically	1.5 h	• Percent Grids (BLM) or grid paper	Q2, 3, 10, 12
CONNECTIONS: The Golden Ratio (Optional) **SB p. 95** **TG p. 114**	Make a connection to the use of ratios in architecture and art	20 min	• Rulers	N/A
GAME: Ratio Concentration (Optional) **SB p. 96** **TG p. 114**	Practise equivalent ratios in a game situation	25 min	• Ratio Concentration Game Cards (BLMs 2A and 2B)	N/A
3.2.3 Estimating and Calculating Percents **SB p. 97** **TG p. 115**	**7-A10 Percent: as a special ratio** • relate visual and symbolic representations of percent **7-B9 Percent: develop algorithms** • use a variety of strategies in calculating percent of a number (including invented strategies): - change percent to a decimal and multiply - compute 1% and then multiply - change to a fraction and divide • calculate percents symbolically	1 h	None	Q1, 3, 4, 7

UNIT 3 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
3.2.4 EXPLORE: Representing Numbers Using Percents (Optional) **SB p. 102** **TG p. 118**	**7-B9 Percent: develop algorithms** • calculate percents symbolically	40 min	None	Observe and Assess questions
UNIT 3 Revision **SB p. 103** **TG p. 120**	Review the concepts and skills in the unit	2 h	• Percent Grids (BLM) or grid paper	All questions
UNIT 3 Test **TG p. 122**	Assess the concepts and skills in the unit	1 h	• Percent Grids (BLM) or grid paper	All questions
UNIT 3 Performance Task **TG p. 123**	Assess concepts and skills in the unit	1 h	None	Rubric provided
UNIT 3 Blackline Masters **TG p. 126**	BLM 1 Percent Grids BLM 2A Ratio Concentration Game Cards BLM 2B Ratio Concentration Game Cards			

Math Background

• This unit extends the work students did with fractions in **Unit 2** to the concepts of ratio, rate, and percent, which were introduced in Class VI.

• The focus of the unit is on recognizing and creating equivalent ratios and rates, recognizing and using fraction/decimal/percent equivalents, and using equivalents to solve problems involving rates, ratios, and percents.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in every lesson of the unit, including in **question 3** in **lesson 3.1.1**, where they determine various possible equivalent ratios for a given ratio, in **question 8** in **lesson 3.1.2**, where they calculate and compare several unit rates, in **question 7** in **lesson 3.2.1**, where they apply their knowledge of number concepts to a percent problem, and in **question 6** in **lesson 3.2.3**, where they calculate a whole amount given a percent of it.

• Students use communication frequently as they explain their thinking in answering questions, such as in **question 5 b)** in **lesson 3.1.1**, where they explain their strategies for solving a ratio problem, in **question 7** in **lesson 3.1.2**, where they describe a strategy for comparing rates, and in **question 11** in **lesson 3.2.2**, where they explain how they estimated.

• Students use reasoning in answering questions such as **question 1** in **lesson 3.1.1**, where they use different strategies to find equivalent ratios, **question 2** in **lesson 3.1.2**, where they find various rates and use the information to answer a rate question related to their class, **question 8** in **lesson 3.2.1**, where they determine what percent remains after some percents are given, and **question 2** in **lesson 3.2.3**, where they calculate a whole when they know 50% of it.

• Students consider representation in **question 4** in **lesson 3.1.1**, where they group students in different ways to maintain a given ratio, in **question 9** in **lesson 3.2.1**, where they represent different percents on a grid, and in **lesson 3.2.4**, where they consider different ways to represent a quantity as a percent.

• Students use visualization skills in **question 7** in **lesson 3.1.1**, where they picture the placement of a Bhutanese flag on a piece of paper, **question 4** in **lesson 3.2.1**, where they figure out the result of doubling a particular portion of a grid, and in **question 5** in **lesson 3.2.2**, where they estimate the percent of a grid shown by different regions of the grid.

• Students make real world connections throughout the unit, such as in **question 5** in **lesson 3.1.1**, **question 7** in **lesson 3.1.2**, and **question 12** in **lesson 3.2.2**. They make connections between fractions, decimals, and percents through questions like **question 4** in **lesson 3.2.2**.

Rationale for Teaching Approach

• This unit is divided into two chapters.

Chapter 1 is about ratio and rate.

Chapter 2 focuses on percent.

• The **Explore lesson** allows students to find out how to express a number as a percent of many other numbers. This supports development of number sense.

• The **Connections** section helps them see how the concept of ratio is used in architecture and art.

• The **Game** provides an opportunity to apply and practise work with equivalent ratios in an enjoyable way.

• Throughout the unit it is important to encourage different strategies for solving rate, ratio, and percent problems, and to accept a variety of approaches from students.

Getting Started

Curriculum Outcomes	Outcome relevance
6 Renaming: simple fractions to decimals **6** Comparing Fractions: develop procedures **6** Ratio: part-to-part, part-to-whole **6** Equivalent Ratios: using models and symbols **6** Percent: developing benchmarks (number sense)	Students will find the work in the unit easier after they review the concepts and skills related to fractions, ratio, and percent they encountered in Class VI.

Pacing	Materials	Prerequisites
1 h	• Black and white strips of paper	• identifying and representing fractions and decimals • creating equivalent fractions and ratios • identifying percents of a whole on a 10-by-10 grid • familiarity with the metric prefixes

Main Points to be Raised

Use What You Know

• The same representation can show different ratios: part-to-part or part-to-whole.

For example, 2 : 4 can mean 2 white slips to 4 black slips or it can mean 2 white slips to a total of 4 slips (2 white and 2 black).

• Any given ratio has many equivalent ratios.

Skills You Will Need

• You can represent percents on a 10-by-10 grid.

• You can write a decimal as a fraction with a denominator of 10, 100, 1000, and so on.

• You can find an equivalent ratio for a given ratio by multiplying both terms by the same number.

Use What You Know — Introducing the Unit

Students can work in pairs or small groups. While you observe students at work, you might ask questions such as the following:

• *How do you know your slips of paper show the ratio 7 : 3?* (I used 7 black strips and 3 white strips, so the ratio of black strips to white strips is 7 : 3)

• *Why can you not use all the strips to represent the ratio 7 : 2?* (I have to use 10 strips, not just 9, so the terms have to add to 10.)

• *How could you represent the fraction $\frac{1}{10}$ with 11 strips of paper?* (I could think of $\frac{1}{10}$ as the ratio 1 : 10 and use a part-to-part ratio. But I notice that there are $\frac{1}{10}$ as many black strips as white strips, so it also makes sense to think of it as a fraction.)

• *How many black strips do you need to use to show the ratio 4 : 6 using 20 slips of paper? How do you know?* (I need 8 black strips. Because I am using twice as many strips in all, I need to use twice as many of each colour.)

Skills You Will Need

• Students can work individually.

• To ensure students have the required skills for this unit, assign these questions.

• You may have to review equivalent ratio and percent to make sure students can successfully interpret **questions 2, 3, and 4**. Refer students to the glossary at the back of the student text.

Answers

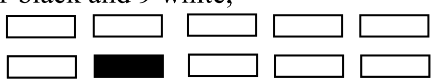
A. *Sample responses*:
i) 4 black strips and 6 white strips
ii) 7 white strips and 3 black strips
iii) 1 black strip and 9 white strips

B. *Sample responses (based on answers to part A)*:
i) 4 black to 10 altogether
ii) 7 white to 10 altogether
iii) 1 black to 10 altogether

C. *Sample response*:
You could switch the black and white strips, e.g., 4 : 6 could be 4 black to 6 white or 4 white to 6 black.

D. *Sample responses*:
i) 2 : 8; 2 black strips and 8 white strips
ii) 5 : 5; 5 black strips and 5 white strips

E. *Sample responses*:
i) 1 black and 9 white;



E. ii) 1 : 9 (1 black : 9 white)
$\frac{9}{10}$ or 9 : 10 (9 white : 10 in all)

F. *Sample responses*:
i) 4 : 6; 7 : 3; 2 : 8 **ii)** 3 : 10; 5 : 10, 4 : 10

G. *Sample responses*:
i) 4 black : 6 white = 8 black : 12 white
ii) 7 white : 3 black = 14 white : 6 black
iii) 3 black: 10 in all = 6 black : 20 in all

H. i) 30 : 100
ii) 30%; 30 : 100 means 30 white out of 100 in total and 30% means 30 out of 100

NOTE: Answers and parts of answers that are in square brackets throughout the Teacher's Guide are NOT found in the answers in the student textbook.

1. a) 8 **b)** 40 **c)** 5 **d)** 14

2. *Sample response*: 6 : 14; 9 : 21; 12 : 28; 15 : 35

3. a) 14 : 7 or 2 : 1
b) 28 : 42 (or 14 : 21 or 2 : 3)

4. a) B **b)** A
[**c)** 25%; *Sample response*:
A and B add to 75%. The whole grid is 100%, so C and D together are 25%.]

5. a) 13 **b)** 6 **c)** 90 **d)** 4

6.

Measurement unit	gram	**kilogram**	**millilitre**	**litre**	metre	**kilometre**	**hour**	second	**minute**
Symbol	**g**	kg	mL	L	**m**	km	h	**s**	min

Supporting Students

Struggling students

• If students are struggling with **parts c) and d)** of **question 1**, you might provide some very simple examples, such as $\frac{1}{?} = \frac{2}{4}$ and $\frac{?}{8} = \frac{3}{4}$. This will remind them that equivalent fractions and ratios can be found by dividing or multiplying both the numerator and the denominator by the same number.

Enrichment

• For **part G**, you might challenge students to represent an equivalent ratio for 4 black to 6 white using 25 slips of paper.

3.1.1 Solving Ratio Problems

Curriculum Outcomes	Outcome relevance
7-A9 Equivalent Ratios and Rates: solve problems • solve problems involving equivalent ratios	Many situations in everyday life require the ability to solve problems involving ratios. Using ratio tables builds number sense and helps students see how multiplication and division connect to work with fractions and ratios.

Pacing	Materials	Prerequisites
1 h	None	• interpreting fractions as part of a group • creating equivalent fractions • understanding that a ratio can be part-to-part or part-to-whole

Main Points to be Raised

• A ratio describes a relationship, or comparison, between amounts or items.

• You can find an equivalent ratio by multiplying or dividing the terms of a ratio by the same number. An equivalent ratio describes the same comparison in a different way.

• A ratio is in lowest terms if the term values are whole numbers that are as low as possible to represent the same comparison.

• A ratio table is a way of recording equivalent ratios in an organized chart. You can use a ratio table to solve problems involving equivalent ratios.

Try This — Introducing the Lesson

A. and B. Allow students to try this alone or with a partner. Make sure students understand that in this context changing a ratio means preserving the same relationship among ingredients so that the taste is the same. While you observe students at work, you might ask questions such as the following:

• *How do you know how many more tomatoes you will need in order to serve 8 people?* (Since 2 tomatoes are needed for 4 people, I would need 2 more tomatoes to serve another 4 people)

• *How does the recipe have to change to serve only 2 people?* (The original recipe serves 4 people, so you would need only half as much of each ingredient to serve 2 people.)

The Exposition — Presenting the Main Ideas

• Ask three boys and two girls to come to the front of the classroom. Ask the students to tell you the ratio of boys to girls in that group. Tell the class you would like to have ten students in the group, but that you want to keep the ratio the same. Have them determine how many more boys and girls need to come forward.

• Present the example in the exposition on **page 79**. You may wish to extend the example by asking how many squares would be needed to make an equivalent ratio if there were 12 triangles. Make sure students understand how to determine whether a ratio is in lowest terms.

• Draw the ratio table for the footballs and basketballs example on **page 80**. Complete the ratio table with the students. You may wish to add another column to the ratio table and ask the students to describe two different ways they could find out how many footballs there would be if there were 30 basketballs (They could multiply 6 × the number of footballs for 5 basketballs, or 2 × the number of footballs for 15 basketballs, or 3 × the number of footballs for 10 basketballs).

Revisiting the Try This

C. Students use a ratio table to solve the problems posed in **part A** and extend it to solve a problem about their own class.

Using the Examples

• Have students work in pairs. One of the pair should become an expert on **example 1** and the other should become an expert on **example 2**. Each student should then explain his or her example to the other student.

Practising and Applying

Teaching points and tips

Q 2: Remind students that they can divide as well as multiply to find equivalent ratios.

Q 3: Some students may not realize that Thinley could have fewer total stamps than Deki.

Q 4 b): Many students will not know that this question requires them first to put the given ratio into lowest terms, and then to find how many groups of that size can be made from the 18 boys and 24 girls in the class.

Q 5: You might encourage students to use a ratio table to solve this problem.

Q 6: Students might find this question easier to explain if they write both ratios in lowest terms.

Q 7: If students choose first to find an equivalent ratio using the length of the paper, they will discover that the width of a flag drawn in that ratio will not fit on the given piece of paper.

Common errors

• In **question 4 a)**, some students will find equivalent ratios for 18 boys to 24 girls that have more than 18 boys and 14 girls, such as 36 boys and 48 girls. You should make sure students realize that they are forming groups of boys and girls from the 18 boys and 24 girls that are in the class, so that all the groups need to contain fewer than 18 boys and fewer than 24 girls. They cannot add more students to the class.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can find equivalent ratios and recognize when a ratio is in lowest terms
Question 3	to see if students can use equivalent ratios to solve a problem
Question 7	to see if students can use reasoning to solve a problem involving equivalent ratios

Answers

A. i) 6 tomatoes
ii) 4 tomatoes
iii) 1 tomato

B. i) 15 cloves of garlic
ii) 10 cloves of garlic
iii) $2\frac{1}{2}$ cloves of garlic

C. i)

Number of people	4	12	8	2	1
Number of tomatoes	2	**6**	**4**	**1**	$\mathbf{\frac{1}{2}}$

ii) *Sample response*:
I need $\frac{1}{2}$ tomato for each person. There are 37 people in our class, so I need $18\frac{1}{2}$ tomatoes (but I would buy 19 tomatoes).

1. a) 5 : 8 = 15 : 24
2 : 1 = 10 : 5
2 : 3 = 8 : 12
3 : 4 = 12 : 16
[**b)** *Sample response*:
Each time, I multiplied both terms by the same amount.
5 × **3** = 15 and 8 × **3 =** 24, so 5 : 8 = 15 : 24.
2 × **5** = 10 and 1 × **5** = 5, so 2 : 1 = 10 : 5.
2 × **4** = 8 and 3 × **4** = 12, so 2 : 3 = 8 : 12.
3 × **4** = 12 and 4 × **4** = 16, so 3 : 4 = 12 : 16.]

2. *Sample responses*:
a) 5 : 2, 20 : 8, 30 : 12
b) Divide each term of the original ratio by 2.
Multiply each term of the original ratio by 2.
Multiply each term of the original ratio by 3.
c) Yes; [5 : 2 is in lowest terms because 1 is the only factor that will divide into both terms.]

3. *Sample responses*:
a) Thinley might have 10 Bhutanese and 6 Ugandan stamps.
b) Yes; [Many answers are possible because many ratios are equivalent to 20 : 12.]

Answers [Continued]

4. a) Groups of 7 (3 boys, 4 girls), groups of 14 (6 boys, 8 girls), and groups of 21 (9 boys, 12 girls). **b)** 6 groups **c)** 3 boys and 4 girls **5. a) i)** 720 g **ii)** 120 g **b)** 480 g; [*Sample response*: Find the amount needed to serve 1 person and multiply by 4: 120 g × 4 = 480 g.] **c) i)** 12 **ii)** 4 **6.** Yes; [*Sample response*: I know 14 : 4 = 7 : 2 because I can divide each term by 2, and I know 7 : 2 = 21 : 6 because I can multiply each term by 3, so 14 : 4 = 21 : 6.]	**7.** 18 cm by 27 cm [2 : 3 = 18 : 27 and 20 : 30 but if you draw the flag 30 cm long, the width of 20 cm will not fit on the paper so it has to be 18 cm by 27 cm.] **8.** *Sample response*: There are 39 students in my class. The ratio of girls to boys is 6 : 7. How many girls are there? (Answer: 18)

Supporting Students

Struggling students

• If students are struggling to find the possible groups in **question 4**, you might have them begin by listing all the factors of 18 and all the factors of 24. They should then find all the common factors for 18 and 14 (2, 3, and 6). This will determine the number of groups (there could be 2 groups, 3 groups, or 6 groups). By dividing the original ratio by 2, 3, and 6, they will find the number of boys and girls in 2 groups (9 boys, 6 girls), 3 groups (6 boys, 8 girls), and 6 groups (3 boys, 4 girls).

Enrichment

• For **question 7**, you might challenge students to determine what size flag could be drawn on a variety of different paper sizes.

3.1.2 Solving Rate Problems

Curriculum Outcomes	Outcome relevance
7-A9 Equivalent Ratios and Rates: solve problems • solve problems involving equivalent rates **7-D6 Rate: Compare two quantities** • understand rate as the comparison between two quantities with different units • write as a rate (e.g., m/s, km/h, beats per minutes)	Many situations in everyday life require the ability to solve problems involving rates. It is useful to understanding rate tables, in particular the concept of a unit rate, to solve a variety of problems.

Pacing	Materials	Prerequisites
1 h	None	• equivalent fractions • understanding rate as a relationship involving two different units of measure • knowing 60 s = 1 min and 1 dozen is 12 items

Main Points to be Raised

• A rate describes a relationship, or comparison, between two numbers with different units of measure. Speed is one of the most common types of rate.

• A unit rate is a rate that has 1 as its second term.

• An average rate is usually written as a unit rate.

• Rate tables are a useful way to solve rate problems and to find unit rates. You can use them to display many equivalent rates at the same time.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How can you find how many ngultrums you would get for 2 Canadian dollars?* (If 1 Canadian dollar can be exchanged for Nu 40, then I would get twice as many ngultrums, or Nu 80, for 2 Canadian dollars.)

• *Does it make sense that you would get twice as many ngultrums for 20 Canadian dollars as you did for 10 Canadian dollars?* (Yes. Since 20 Canadian dollars are worth twice as much as 10 Canadian dollars, the equivalent number of ngultrums should also be twice as much.)

• *Can you describe two different ways to find out how many ngultrums you would get for 100 Canadian dollars?* (I could take the number of ngultrums received for 1 Canadian dollar and multiply that amount by 100. Or, I could take the number of ngultrums received for 50 Canadian dollars and double it.)

The Exposition — Presenting the Main Ideas

• Have a finger-snapping or hand-clapping contest. Have students count how many times they can snap their fingers in 10 seconds (you should tell them when to start and stop). Ask students to share how many finger snaps they were able to do in 10 s. You might have the person who did the most finger snaps in 10 seconds demonstrate his or her technique to the class. Tell students that, for example, "30 finger snaps in 10 s" is a *rate*, because it describes a relationship between two numbers with different units: number of snaps/time (seconds).

• Work through the exposition with students.

• You might have the class calculate a unit rate for the person who did the most finger snaps in 10 s (either in finger snaps per minute or in finger snaps per second).

• Make sure students realize that you can add or subtract equivalent rates to find other equivalent rates.

For example:

If 8 pineapples cost Nu 160 and 1 pineapple costs Nu 20, you can subtract the first terms and then subtract the second terms to find the cost of 7 pineapples:

8 – 1 = 7 and 160 – 20 = 140, so 7 pineapples cost Nu 140.

[Continued]

If the ratios are in a rate table, you add or subtract terms in the same row:

Cost (Nu)	80	**160**	**20**	**140**
Pineapples	4	**8**	**1**	**7**

160 – 20 = 140

8 – 1 = 7

Revisiting the Try This

B. and C. Students use a rate table to solve the problem in **part A**. In **part C**, they use the rate table to solve more rate problems.

Using the Examples

• Ask students to read through **solutions 1 and 2** of **example 1 a)**. Ask them to choose the solution that most closely matches what they would have done and to explain why. Then work through **example 1 b)** with the students. Ask how it is similar to **example 1 a)**, and how it is different from it.

• Present the problem in **example 2**. Let students work through it and then have them check their work against the work in the student text.

Practising and Applying

Teaching points and tips

Q 1 b): You might mention that unit rates are often speeds, but not always.

Q 2: Since this question involves finding several rates related to a given rate, you might encourage students to use a rate table. Remind them that 1 dozen is 12 items.

Q 3: Remind students to observe the units of measure in the question. They will need to recall the number of seconds in a minute.

Q 5: You might encourage students to find the answer in more than one way, such as finding a unit rate and multiplying, or using other equivalent rates.

Q 6: Make sure students are aware that the rate has been reversed in this question (time/distance).

Q 7: This question highlights the usefulness of considering equivalent rates for comparing prices. Some students will use unit rates. Others will figure out the cost of 4 oranges at each rate or the cost of 48 oranges at each rate. You might ask what the better price is for the seller.

Q 8 b): Remind students to observe the units of measure in the question (minutes instead of seconds).

Common errors

• Some students might double both terms in the rate in **question 4**. Have them observe that 140 beats in 2 min is the same rate as 70 beats in 1 min, not double the rate.

Suggested assessment questions from Practising and Applying

Question 1	to see if students understand unit rates
Question 7	to see if students can solve a problem using rates
Question 8	to see if students can apply equivalent rates to solve a variety of rate problems

Answers

A. i) Nu 400 **ii)** Nu 800
iii) Nu 2000 **iv)** Nu 4000

B.

Canadian dollars	1	10	20	50	100
Ngultrums	40	**400**	**800**	**2000**	**4000**

C. i) Nu 480; *Sample response*:
If 1 Canadian dollar is about Nu 40,
12 Canadian dollars is about 12 × Nu 40 = Nu 480
ii) 4 Canadian dollars; *Sample response*:
If Nu 40 is about 1 Canadian dollar,
Nu 160 is about 4 × 1 Canadian dollars = 4 Canadian dollars.

Answers [Continued]

1. a) 25 km/h **b)** 180 kg/day **c)** 25 m/min **2. a)** Nu 72 **b)** Nu 12 **c)** Nu 2 **d)** *Sample response*: Nu 78 for 39 students; [It costs Nu 2 for 1 banana and there are 39 in the class, so it would cost 39 × Nu 2 = Nu 78 to buy a banana for each person.] **3.** 66 beats/min **4.** 140 beats/min **5.** 25 L **6.** 3 min/km	**7.** Nu 60 for 12 oranges; [*Sample response*: Nu 60 for 12 oranges is Nu 5 per orange and Nu 48 for 8 oranges is Nu 6 per orange.] **8. a) i)** Elephant **ii)** Lion **b) i)** Rabbit **ii)** Tortoise **c)** Cheetah; [*Sample response*: The Cheetah has the fastest unit rate ($31\frac{1}{4}$ m/s).] **d)** Tortoise; [*Sample response*: The unit rate of the tortoise is the slowest (much less than 1 m/s).] [**9.** *Sample response*: Multiply how far it travels in 10 min by 6 to find how far it travels in 60 min, or 1 h. That is the rate in kilometres per hour.]

Supporting Students

Struggling students

• If students are struggling with **question 8 a)**, you might discuss some estimation strategies with the class.

For example, students can look for “friendly numbers” for dividing, such as about 180 m in about 6 s for the cheetah, and about 20 m in about 2 s for the elephant.

For some problems, a rate table is ideal.

For example, for the lion, 400 m in 16 s is the same as 200 m in 8 s, 100 m in 4 s, 25 m in 1 s, and so on.

Encourage students to examine the numbers to decide which strategy might be best for finding or estimating a unit rate for each animal.

Enrichment

• For **question 7,** you might challenge students to create other price comparison questions for others to solve. They might even be able to find examples from a local market.

3.2.1 Percent as a Special Ratio

Curriculum Outcomes	Outcome relevance
7-A10 Percent: as a special ratio • understand percent as a special ratio • understand that parts should always add up to 100% • relate visual and symbolic representations of percent	It is much easier for students to make connections among fractions, decimals, and percents when they can visualize a percent as an amount "out of 100".

Pacing	Materials	Prerequisites
1 h	• Percent Grids (BLM) or grid paper	• interpreting fractions as part of a whole • representing hundredths as shaded regions of a grid.

Main Points to be Raised

- A percent is a special ratio that always has a second term of 100. Percent means "out of 100".
- 100% is another way to say one whole, so the total of the parts of a whole is always 100%.
- There are many ways to show the same percent on a grid.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

- *How do you read 0.45*? (0.45 is 45 hundredths.)
- *How do you know how many squares to use for your picture or shape?* (There are 100 squares, so I need to use 45 squares to show 0.45.)
- *Does it make sense that your shape/picture might look different from your classmate's?* (Yes. I could have drawn any shape that used 45 of the small squares.)

The Exposition — Presenting the Main Ideas

- Draw a percent sign on the board. Ask students if they recall how to read the % sign, and what it means.
- Students can use the Percent Grid BLM or create their own on grid paper. Make sure students know that there are 100 squares in the percent grid.
- Have the students shade one row of the grid. Ask how many squares have been shaded. Then ask students how they think they would write the shaded amount as a percent. Turn the grid so that the shaded row is now a shaded column. Ask students if the percent of the grid that is shaded has changed.
- Work through the exposition with the students, drawing particular attention to the fact that the parts of the whole always add up to 100%, as long as all parts are considered.

Revisiting the Try This

B. and C. Students connect what they know about part-to-whole ratios with a second term of 100 to percent.

Using the Examples

• Work through the example with the students to make sure they understand it. For **part b)**, make sure students recognize that they can find the percent of the grid that is not shaded either by counting the non-shaded squares or by adding the percents of the shaded parts and subtracting that amount from 100%, the whole.

Practising and Applying

Teaching points and tips

Q 1: This question is designed to make sure students recognize the various ways of representing a ratio.

Q 3 d): You might encourage students to show two different ways to find this answer (counting the white squares, or adding the percents for the other questions and subtracting from 100%).

Q 4: Some students may need to draw and shade another copy of the black region to answer the question.

Q 7: You might encourage students to look for patterns to help answer the question.

For example, they may notice that 5 of 10 columns are even, so 5 out of 10, or 50% are even.

Q 8: Remind students to think of what percent represents the whole, and use that information to help answer the questions.

Q 9 c): Many students may not know how many Xs and Os to use, and will just start filling in the remainder of the grid with two Xs and one O until the grid is filled. You might discuss how they could have used the information in **parts a) and b)** to determine ahead of time how many Xs and Os they would need to draw (60% of the grid, or 60 squares with an X or O in a ratio of 2 : 1, means 40 Xs and 20 Os).

Common errors

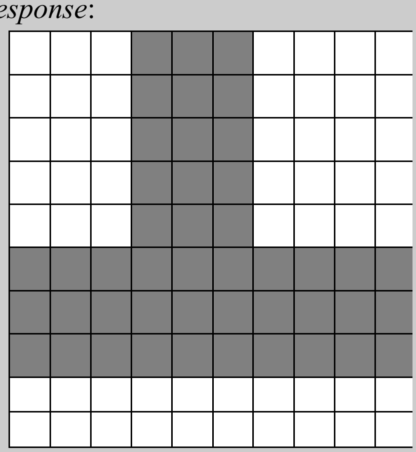
• Some students will fail to consider what the whole is in **question 8 c)** and will answer 20% (thinking only of those who chose bananas). Ask, “Did all the students who did not choose apples choose bananas?”

<u>Suggested assessment questions from Practising and Applying</u>

Question 3	to see if students can identify visual representations of percent, and if they realize that 100% is another name for one whole
Question 5	to see if students can use their understanding of percent to solve a problem
Question 7	to see if students can solve a problem using percent
Question 9	to see if students can produce a visual representation of various percents, and if they recognize that 100% is equivalent to one whole

Answers

A. i) 100
ii) *Sample response*:



B. i) 45 : 100
ii) 55 : 100

C. 45%; 55%

Answers [Continued]

1. a) 9% **b)** 19% **c)** 87%
d) 43% **e)** 100%

2. a) 71% **b)** 29%

3. a) 18% **b)** 25% **c)** 10% **d)** 47%

4. 50%

5. a) 68% **b)** 32%

6. 28%

7. a) 50% **b)** 50% **c)** 20%

8. a) 70% **b)** 20% **c)** 30% **d)** 10%

9. a), b), and c) *Sample response*:

x	x	x	x	x					
x	x	x	x	x					
o	o	o	o	o					
x	x	x	x	x	x	x	x	x	x
o	o	o	o	o	x	x	x	x	x
					x	x	x	x	x
					o	o	o	o	o
					x	x	x	x	x
					x	x	x	x	x
					o	o	o	o	o

d) 40% **e)** 20%
f) 100%; [The total of all the percents of any whole region is one whole, or 100%.]

Supporting Students

Struggling students

• If students are struggling with **question 7 c)**, remind them of how to determine whether a number is a multiple of 5 (if it ends in a 5 or a 0).

• Some students will have difficulty with **question 9 c)**. Help them see that whenever they fill one row with Os, they must fill two rows with Xs.

Enrichment

• For **question 9**, you might challenge students to make up their own grid colouring problem for other students to solve. Remind them that they have to make sure to give enough information so that students can fill in the grid completely.

3.2.2 Relating Percents, Fractions, and Decimals

Curriculum Outcomes	Outcome relevance
7-A10 Percent: as a special ratio • relate percent to fraction and decimal equivalents • use benchmark percents **7-A11 Percent: number sense** • estimate and calculate percents for familiar fractions concretely and symbolically	For students to have maximum flexibility in solving problems involving fractions, decimals, and percents, it is important for them to be able to identify fraction, decimal, and percent equivalents.

Pacing	Materials	Prerequisites
1.5 h	• Percent Grids (BLM) or grid paper	• connecting fractions and division • renaming fractions and mixed numbers as decimals, and vice versa

Main Points to be Raised

• Because percent means "out of 100", it is easy to write both a fraction with a denominator of 100 as a percent and a decimal hundredths as a percent.

• To write a fraction as a percent, first determine whether you can write the fraction as an equivalent fraction with a denominator of 100.

• You can write a percent as a ratio, a fraction, and a decimal.

• For some fractions, it is not possible to write an exact whole number percent; you have to estimate.

• A number line is a useful model for showing how fractions, decimals, and percents relate.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How many chhertums are equal to one ngultrum*? (100 chhertums)

• *Why is the denominator always the same?* (1 Nu is always divided into hundredths.)

• *Does it make sense that Nu 1 could be written as a fraction of Nu 1? Explain your thinking*. (Yes. When the numerator and denominator are the same, the fraction is equal to 1.)

The Exposition — Presenting the Main Ideas

• Students can use the Percent Grid BLM or draw their own percent grids on grid paper. Have the students shade in 11 small squares on the grid. Ask them to write what fraction of the grid is shaded. Then ask them what percent of the grid is shaded. They should see that $\frac{11}{100}$ is the same as 11%.

• Repeat the above procedure, but this time have them shade in 50 small squares. Ask them to write what fraction of the grid is shaded. Many students are likely to say $\frac{1}{2}$. They should notice that $\frac{1}{2}$ and $\frac{50}{100}$ are equivalent.

• Work through the exposition with the students, in particular noting that when you are given any one form of a number (fraction, decimal, or percent), you can write the other two forms.

• Students might recall from **Unit 2** that some fractions result in repeating decimals. Direct students' attention to the discussion in the exposition regarding estimating percents for such fractions.

• You might ask students to locate other fraction/decimal/percent equivalents on the number line, such as $\frac{1}{2}$, 0.5, 50% and $\frac{3}{4}$, 0.75, and 75%.

Revisiting the Try This

B. Students write their fractions from **part A** as percents.

Using the Examples

• Assign students to pairs. Have one student in the pair become the expert on **examples 1 and 2**, and the other student become the expert on **examples 3 and 4**. Alternately, have students work in groups of four, making each student an expert on one example. Each student should then explain his or her example(s) to the other student(s) in the pair or group.

Practising and Applying

Teaching points and tips

Q 2: Encourage students to write the fractions in lowest terms.

Q 3: Remind students to find equivalent fractions with a denominator of 100 to make it easier to write each fraction as a decimal and a percent.

Q 4: Suggest to students that they make their number lines long enough to make it easy to locate the various fractions, decimals, and percents.

Q 5: You might ask students to think first about what fraction of the grid each region represents, and then about how to estimate a percent for that fraction.

Q 7 and 13: You may choose not to assign these to struggling students.

Q 9: Some students may not recognize that the given ratio is part-to-part, and that they need to find the ratio of boys to students before comparing the two classes.

Q 12: Encourage students to decide whether it is easier to compare the quantities using fractions, decimals, or percents.

Q 14: This question highlights the effective use of percent equivalents, a critical idea brought out in this lesson.

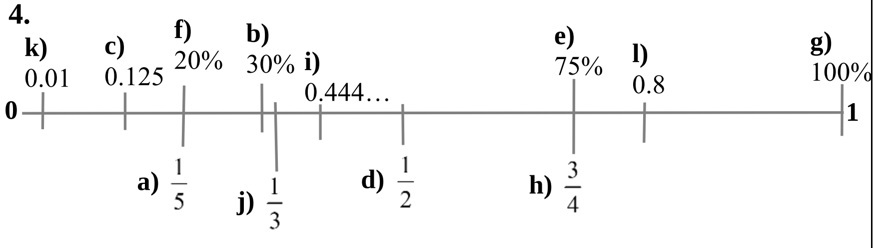
Common errors

• Many students will answer 8% for **question 1 d)**. Have students compare 0.8, 0.80, and 0.08.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can identify fraction and decimal equivalents for given percents
Question 3	to see if students can rewrite fractions as decimals and percents
Question 10	to see if students can solve a problem using what they know about percents
Question 12	to see if students can use equivalent fractions/decimals/percents to solve a problem

Answers

A) i) $\frac{5}{100}$ or $\frac{1}{20}$ **ii)** $\frac{10}{100}$ or $\frac{1}{10}$ **iii)** $\frac{25}{100}$ or $\frac{1}{4}$ **iv)** $\frac{50}{100}$ or $\frac{1}{2}$ **v)** $\frac{100}{100}$ or 1	**B) i)** 5% **ii)** 10% **iii)** 25% **iv)** 50% **v)** 100%
1. a) 47% **b)** 63% **c)** 5% **d)** 80% **2. a)** $\frac{75}{100}$ or $\frac{3}{4}$; 0.75 **b)** $\frac{24}{100}$ or $\frac{6}{25}$; 0.24 **c)** $\frac{90}{100}$ or $\frac{9}{10}$; 0.9 **d)** $\frac{1}{100}$; 0.01 **e)** $\frac{2}{100}$ or $\frac{1}{50}$; 0.02 **f)** $\frac{35}{100}$ or $\frac{7}{20}$; 0.35 **3. a)** 0.25; 25% **b)** 0.6; 60% **c)** 0.7; 70% **d)** 0.04; 4% **e)** 0.16; 16% **f)** 0.14; 14% **g)** 0.05; 5% **h)** 0.55; 55% **4.** 	**5.** About 33%; [*Sample response*: Each shaded region is $\frac{1}{3}$ of the grid, and $\frac{1}{3}$ is about 33%.] **6.** *Sample responses*: **a)** About 83% **b)** About 11% **c)** About 44% **d)** About 9% **7.** *Sample responses*: **a)** About $\frac{6}{10}$ **b)** About $\frac{3}{4}$ **c)** About $\frac{1}{6}$ **d)** About $\frac{5}{6}$

8. a) $\frac{7}{20}$ **b)** $\frac{9}{20}$

c) $\frac{13}{20}$ **d)** $\frac{19}{20}$

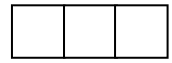
9. Chabilal's class

10. a) 36% **b)** 64%

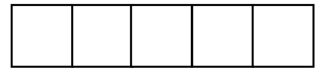
11. About 30%; [*Sample response*:
$\frac{10}{30}$ is $\frac{1}{3}$**,** which is about 33%, so $\frac{8}{30}$ is a bit less than that, or about 30%.]

12. a) 0.27 (Services),
$\frac{1}{3}$ (Agriculture),
40% (Industry).
[**b)** *Sample response*:
I changed each to a percent:
0.27 is 27% , $\frac{1}{3}$ is about 33%.]

13. *Sample responses*:
a)



b)



c) Yes; [The squares can be in any arrangement as long as there are 3 (for 60%) and 5 (for the whole figure).]

[**14.** *Sample response*:
$\frac{17}{20}$, $\frac{2}{3}$, and $\frac{6}{10}$ can easily be calculated or estimated as percents, but $\frac{1}{15}$, $\frac{1}{16}$, and $\frac{1}{17}$ are more easily ordered by comparing the denominators.]

Supporting Students

Struggling students

• If students are struggling with **question 4 i)**, you might suggest that they recall from **Unit 2** how to write 0.444… as a fraction. The repeating decimal is at the same mark on the number line as the fraction equivalent.

• You may choose not to assign **question 7** or **question 13** to struggling students.

Enrichment

• For **question 13**, you might challenge students to draw all the possible rectangles that could be made if the given figure were 5% of a larger rectangle.

For example, it could be a rectangle that is 1 "figure" wide and 20 "figures" long; or 2 by 10, or 4 by 5.

CONNECTIONS: The Golden Ratio

• You can use this optional connection with all students.

• Two approximations for the golden ratio are given: 8 : 5 and 1.61803. Students can use these to create their own figures with golden rectangles.

• If a calculator is available, you could ask students to find the golden ratio as a decimal to the limit of the calculator display.

• Some students might be curious about what the exact golden ratio is, although most students may not be ready to use the square root sign. The exact ratio is $\frac{2}{\sqrt{5}-1}$ or $\frac{\sqrt{5}+1}{2}$.

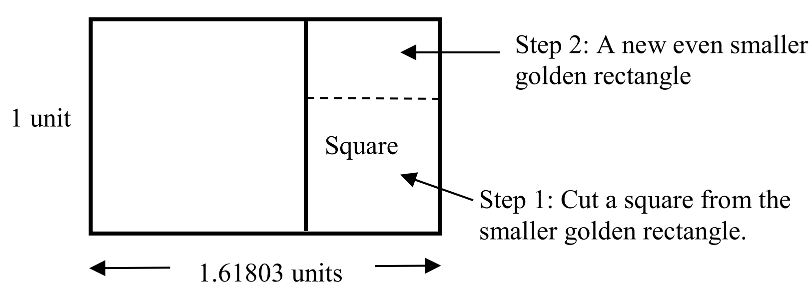
• For **question 3**, you may have to provide some examples if the students do not have access to resources that would illustrate examples.

• The golden ratio can be seen in the construction of a Bhutanese Spirit Trap.

Answers

1. Yes; *Sample response*:
3.2 cm and 2 cm; 3.2 cm ÷ 2 cm = 1.6 is close to the given value.

2. *Sample response*:



3. *Sample response*:
The base of the Parthenon in Athens; the Pantheon in Rome; the face of Leonardo da Vinci's Mona Lisa fills a golden rectangle.

GAME: Ratio Concentration

• This optional game is designed to allow students to practise identifying equivalent fractions, decimals, and percents.

• Copy and cut out the BLM Ratio Concentration Game Cards found on **pages 127 and 128**.

• If 30 cards seem too many for some students, they can use fewer cards as long as you make sure that there is a match for every card used.

3.2.3 Estimating and Calculating Percents

Curriculum Outcomes	Outcome relevance
7-A10 Percent: as a special ratio • relate visual and symbolic representations of percent **7-B9 Percent: develop algorithms** • use a variety of strategies in calculating percent of a number (including invented strategies): - change percent to a decimal and multiply - compute 1% and then multiply - change to a fraction and divide	Being able to estimate and calculate with percents is an everyday life skill. In many situations, estimation is as important a skill as calculation.

Pacing	Materials	Prerequisites
1 h	None	• percent/fraction equivalents for common fractions (benchmarks) • writing percents as decimals • multiplying a fraction by a whole number

Main Points to be Raised

• Familiar percents that are easy to work with mentally can be used to estimate and calculate percent problems.

• Rewriting a percent as a decimal and multiplying is one strategy for finding the percent of a number.

• Finding the unit percent (1%) can be useful for finding other percents of a number.

• A percent table can be a useful strategy for solving percent problems, whether calculating a percent when you know the whole, or calculating the whole when you know the value of a percent.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe while students at work, you might ask questions such as the following:

• *What will be the length of the strip if you fold it in half? What percent of the strip is that?* (Half of the strip is 10 cm. That is 50% of the strip.)

• *How can you use this information to find 25% of the strip?* (If I fold the strip again, it will be 5 cm long, which is $\frac{1}{4}$ of the strip. $\frac{1}{4}$ is the same as 25%.)

The Exposition — Presenting the Main Ideas

• Begin by asking students questions such as the following:

- *If you receive a mark of 50% on a test with 100 questions (of the same value), how many questions did you answer correctly?*

- *If the test had 50 questions (of the same value), how many questions did you answer correctly? How do you know?*

• Work through the first part of the exposition with the students. Have them discuss why percents such as 10%, 25%, 50%, and 75% are considered "familiar percents". Explain that the term *benchmark* in the student text means that these are values to which you can relate other values. Students might observe that these percents have easy fraction equivalents that can help with estimation and calculation.

• To help students understand why you can multiply, for example, 0.7 by 40 to calculate 70% of 40, remind students of what they learned about multiplying by decimals in Class VI.

• Go through the discussion in the exposition about how to use a percent table. Students should recognize that using a percent table is like using a ratio table or a rate table. Make sure they understand that the percent table allows them to find the whole when they know the percent or to find the percent when they know the whole.

Revisiting the Try This

B. Students calculate lengths for given percents in **part A**. Here they calculate percents for given lengths.

Using the Examples

• Present the problems in the first three examples to the students. Ask each student to choose two of the problems to solve. Then they can compare their work to what is shown in the matching examples. Suggest that they may wish to read through the other example.

• Ask pairs of students to read through **solutions 1 and 2** of **example 4**. Ask them to choose the solution that most closely matches what they would have done and to explain why they would have done it that way.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to estimate first, but remind them that they also need to calculate an exact answer.

Q 3: Some students may choose to answer **part b)** first to find 10% and then multiply by 4 to get the 40% for **part a)**.

Q 4: Remind students that a percent table is particularly useful for finding various percents of the same number.

Q 6: Make sure students understand that they are being asked to estimate, and that they should be prepared to explain their strategies for estimating.

Q 7: This is an important generalization.

Q 8: This question emphasizes that there are different ways to solve percent problems, an important notion in this lesson.

Common errors

• Some students will have difficulty with **questions 2 and 6 b)**, treating them as “percent of a number” questions rather than “finding the whole when a percent is known” questions. Review with those students the difference between the two types of question.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can apply various strategies for calculating the percent of a number
Question 3	to see if students can to use the strategies for finding the percent of a number to solve a problem
Question 4	to see if students recognize problems where using a percent table is an effective strategy
Question 7	to see if students can apply the generalization that ■% of ▲ has the same result as ▲% of ■

Answers

A i) 5 cm **ii)** 2 cm **iii)** 8 cm **iv)** 16 cm	**B. i)** 10 cm **ii)** 4 cm **iii)** 1 cm **iv)** 15 cm
1. a) 30 **b)** 10 **c)** 4 **d)** 27 **e)** 77 **f)** 285 **2.** 14 cm **3. a)** 36 kareys **b)** 9 dobjeys **4.** Reading, 4 h; TV, 12 h; Games, 56 h; Song and dance, 4 h; Other, 4 h. **5. a) i)** 1000 **ii)** 5000 **iii)** 7500 **iv)** 9000 **b) i)** 3000 **ii)** 15,000 **iii)** 22,500 **iv)** 27,000	**6.** *Sample responses*: **a)** About 20 mm in March; [If 100% is 650, then 10% is 650 ÷ 10 = 65 and 3% is about 65 ÷ 3, which is about 20 mm. About 215 mm in August; [If 100% is 650, then 33% is about 650 ÷ 3, which is about 215 mm.] **b)** About 6500 mm; [11% is about 10%. If 10% is 650, then 100% is 650 × 10 = 6500.] **7. a)** 39; 39; Answer is the same. **b)** 22.5; 22.5; Answer is the same. **c) [***Sample response*: 40% of 50 and 50% of 40 (answer for both is 20) 12% of 25 and 25% of 12 (answer for both is 3)] Yes the results were the same.

[**8.** *Sample response*:
You could divide 25% of the number by 5 and then multiply your answer by 3.
You could multiply the 25% by 4 to find the number and then find 15% of your answer.]

Supporting Students

Struggling students

• If students are struggling with the large numbers in **question 5**, you might have them find the percents for 100, then ask how they would use that to find the percents for 1000 and 10,000 (and 30,000).

Enrichment

• For **question 7**, you might ask students:

Why does this happen? Use an example to help you explain.

Sample response:
50% of 78 = 50 × 0.01 × 78
78% of 50 = 78 × 0.01 × 50
The commutative property of multiplication says that 50 × 0.01 × 78 = 78 × 0.01 × 50.

3.2.4 EXPLORE: Representing Numbers Using Percents

Curriculum Outcomes	Outcome Relevance
7-B9 Percent: develop algorithms • calculate percents symbolically	This optional exploration applies what students learned about percents in the previous lesson.

Pacing	Materials	Prerequisites
40 min	None	• calculating the whole when a percent is known

Exploration

• Work through the introduction (in white) with the students. Make sure they understand that the same number can represent different percents, depending on what the whole is.

For example, 10 could be 100% (of 10), 50% (of 20), 10% (of 100), and so on.

• Have students work alone, in pairs, or in small groups for **part A i) and ii)**. You may want to give them an example of a pattern in the chart that will help them find other possibilities.

For example, $30 = 0.01 \times 3000$ and $30 = 0.02 \times 1500$. When the first factor is doubled, the second factor is halved, so $30 = 0.04 \times 750$, ….

While you observe students at work, you might ask questions such as the following:

• *How did you find the number such that 5% of the number is 30?* (Since I knew $30 = 0.01 \times 3000$, I divided 3000 by 5 to find $30 = 0.05 \times 600$, so 30 is 5% of 600.)

• *Why could you not put 7% in your chart?* (When I tried to divide 3000 by 7, I did not get a whole number answer. 30 is not 7% of a whole number, and I only wanted to use whole numbers.)

Discuss **parts B to D** with the students to make sure they are proceeding successfully. Then ask students to complete those parts of the exploration.

Observe and Assess

As students work, notice the following:

• Do they find all the possible whole number answers for **part A**?

• Do they recognize the inverse relationship in **part B**? (e.g., if the percent doubles, the number will be divided by two, if the percent is halved, the number will be multiplied by two, and so on.)

• Are they successful in creating their own examples in **part D**?

Note: Some students may think of using fractional percents or percents over 100 even though these have not been introduced. Do not stop them from considering these, but do not expect this of most students.

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and answer these questions.

• *How do you know what numbers will work in your charts in **part A**?*

• *What is the relationship between 30% and 90% in **part C i)**? What is the relationship between Number C and Number D?*

• *How did you choose the percents to use in creating your own questions for **part D**?*

Answers

A. i)

	Percent	Number
30	1	3000
30	2	1500
30	3	1000
30	4	750
30	5	600
30	6	500
30	8	375
30	10	300
30	12	250
30	15	200
30	20	150
30	25	120
30	30	100
30	40	75
30	50	60
30	60	50
30	75	40
30	100	30

ii)

	Percent	Number
25	1	2500
25	2	1250
25	4	625
25	5	500
25	10	250
25	20	125
25	25	100
25	50	50
25	100	25

B. i) The second percent is $\frac{1}{2}$ of the first percent.

The second number is 2 times the first number.

ii) Number A; *Sample response*:
40 is 1 of 5 parts of Number A but 2 of 5 parts of Number B.
Because it is a bigger part of Number B, Number B must be less than Number A.

iii) Number B is $\frac{1}{2}$ of Number A

C. i) Number D is $\frac{1}{3}$ of Number C.

ii) Number F is 2 times Number E

iii) Number G is 3 times Number H.

D. *Sample response*:
15 is 5% of Number J but 50% of Number K.
How are Numbers J and K related?
(Number J is 10 times Number K.)

Supporting Students

Struggling students

• If students are struggling with **part A**, you might start with an easier number, such as 5.

Enrichment

• For **part A**, you might challenge students to determine which number from 1 to 40 has the greatest number of possible answers.

UNIT 3 Revision

Pacing	Materials
2 h	• Percent Grids (BLM) or grid paper

Question	Related Lesson(s)
1 – 5	Lesson 3.1.1
6 – 9	Lesson 3.1.2
10 – 12	Lesson 3.2.1
13	Lesson 3.2.2
14 – 17	Lesson 3.2.3

Revision Tips

Q 1: You might encourage students to use a ratio table to answer this question.

Q 3: Some students may not recognize the ratio that is described. Make sure students have found the correct boy to girl ratio for the question before proceeding. In **part b)**, students need to consider a part–to-whole ratio to answer the question.

Q 4: Some students may choose first to write the ratio in lowest terms, although it is not necessary to do so.

Q 6: You might encourage students to answer this question in more than one way (e.g., by finding unit rate or by using other equivalent rates).

Q 7: Because this question involves finding several rates related to a given rate, you might encourage students to use a rate table.

Q 9: This question generalizes the convenience of finding a unit rate.

Q 10 b): You might encourage students to show two different ways to find this answer (counting the remaining squares, or adding the percents for **part a)** and subtracting the total from 100%).

Q 12 a): Some students may choose to shade in the multiples of 3 on the grid (and observe the pattern) to help them.

Q 13: Encourage students to decide whether it is easier to compare the quantities using fractions, decimals, or percents.

Q 14: Encourage students to estimate first, but remind them that they also need to calculate an exact answer.

Q 15 a) and b): Some students may not realize that first writing the fractions ($\frac{28}{35}$ and $\frac{72}{600}$) in lowest terms will make it much easier to find the percent.

Answers

1. a) i) 20 mL **ii)** 5 mL
b) 10 servings

2. a) 18 girls
b) 20 boys and 15 girls

3. a) 24
[**b)** *Sample response*:
The ratio of boys to girls is 8 : 3, so the whole is a multiple of 8 + 3.
8 + 3 = 11 and you cannot evenly divide 35 by 11.]

4. a) 45 : 75, 60 : 100
[**b)** *Sample response*:
45 was the first term because 45 is a multiple of 15 and it is not a multiple of 25.
100 is the second term because it is a multiple of 25 but not of 15.]

5. No; [You cannot write an equivalent ratio for 5 : 2 with 3 as the second term and a whole number as the first term.]

6. 1 dozen apples for Nu 60; [*Sample response*:
9 apples for Nu 60 is 3 apples for Nu 20, or 12 apples for Nu 80.]

7. a) i) Nu 30 **ii)** Nu 60
b) Nu 5 per orange
[**c)** If the rate is Nu 5 per orange, then 8 oranges cost 8 × Nu 5 = Nu 40.]
[**d)** *Sample response*: Using a ratio table:

× 4 × 2 ÷ 3

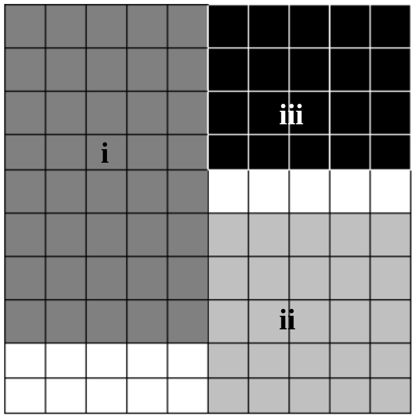
Oranges	3	12	24	8
Cost (Nu)	15	60	120	**40**

× 4 × 2 ÷ 3

8. 4 h

[**9.** Divide by 3 and then multiply by 2.]

10. a) *Sample response*:



b) 15%

11. a) 35% **b)** 65%
[**c)** Since 35% of the grid is shaded and the whole grid is 100%, the part that is not shaded is 100% – 35% = 65%.]

12. a) 33% **b)** 25%

13. 20%, $\frac{1}{4}$, 0.55, $\frac{3}{5}$, 0.75, 90%

14. a) 3 **b)** 13
c) 2 **d)** 5.4

15. a) 80% **b)** 12%
c) 300 **d)** 500

16. 300 students

17. a) 5% **b)** 135 people

UNIT 3 Ratio, Rate, and Percent Test

1. a) The ratio of boys to girls in a class is 5 : 3. If there are 20 boys, how many girls are there?

b) A class of 40 students has a ratio of boys to girls of 5 : 3. How many boys and girls are there? Show your work.

2. Write two equivalent ratios for 16 : 30.

- One ratio should include 48 as a term.
- The other ratio should include 15 as a term.

3. a) A recipe for Pork Fing calls for 3 green chilli peppers. It serves 6 people. How many chilli peppers are needed for each?

i) 12 servings **ii)** 2 servings

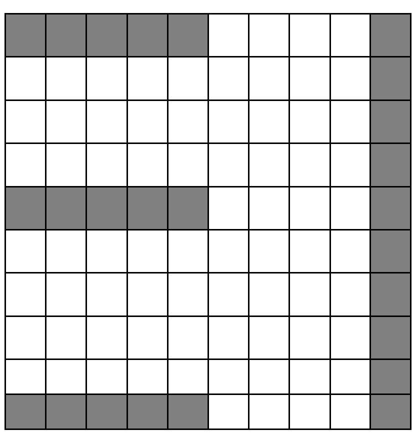
b) How many servings can be made using 5 chilli peppers?

4. Which is a better price for the buyer? How do you know?

3 tomatoes for Nu 60
or
5 tomatoes for Nu 110

5. a) What percent of the grid below is shaded?

b) How can you use the answer to **part a)** to figure out the percent of the grid that is not shaded?



6. a) Draw these two shapes on a 10 × 10 grid:

i) The first shape covers 30% of the grid.

ii) The second shape covers another 50% of the grid.

b) How much of the grid is not covered?

7. What percent of the numbers in this 100 chart are each?

a) multiples of 5 **b)** less than 30

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

8. Order from least to greatest:

$\frac{1}{3}$, 10%, 0.35, $\frac{5}{9}$, 0.85, 45%

9. Calculate.

a) 15% of 40 **b)** 75% of 48

c) 2% of 70 **d)** 31% of 50

10. In the Population and Housing Census of Bhutan for 2005, data about drinking water showed the following:

- about 23% of the homes had piped water within the house,
- about 62% of the homes had piped water outside the house, and
- the remaining homes got drinking water from other sources such as a spring, river, or pond.

a) About what percent got their drinking water from other sources?

b) An area has 300 homes. Use the percents above to calculate the number of homes that get their drinking water from other sources.

UNIT 3 Test

Pacing	Materials
1 h	• Percent Grids (BLM) or grid paper

Question	Related Lesson(s)
1 – 3	Lesson 3.1.1
4	Lesson 3.1.2
5 – 7	Lesson 3.2.1
8	Lesson 3.2.2
9 and 10	Lesson 3.2.3

Select questions to assign according to the time available.

Answers

1. a) 12
b) *Sample response*:
25 boys and 15 girls

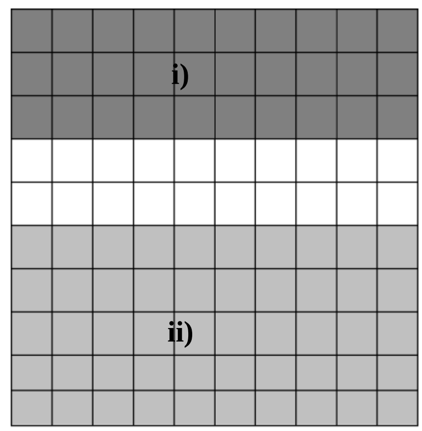
2. 48 : 90, 8 : 15

3. a) i) 6 **ii)** 1
b) 10 servings

4.) 3 tomatoes for Nu 60
Sample response:
3 tomatoes for Nu 60 is Nu 20 per tomato.
5 tomatoes for Nu 110 is Nu 22 per tomato.

5. a) 25%
b) Since 25% of the grid is shaded and the whole grid is 100%, the part that is not shaded is 75%.

6. a) *Sample responses*:



b) 20%

7. a) 20% **b)** 29%

8. 10%, $\frac{1}{3}$, 0.35, 45%, $\frac{5}{9}$, 0.85

9. a) 6 **b)** 36 **c)** 1.4 **d)** 15.5

10. a) 15% **b)** 45

UNIT 3 Performance Task — Bhutan Population

A recent report estimates the population of Bhutan to be about 750,000.

• About 40% of the population is under age 15 and about 4% of the population is over age 65.

• About 70% of the population lives on farms and about 20% of the population lives in urban areas.

A. About what percent of people in Bhutan are each?

i) ages 15 to 65 **ii)** do not live on farms or in urban areas

B. About how many people in Bhutan are each?

i) under the age of 15 **ii)** over the age of 65

C. Write each ratio in lowest terms.

i) Population under age 15 : Total population

ii) Population over age 15 : Total population

D. Write each ratio in **part C** as a fraction.

E. Using the above information to write your own report that makes comparisons using fractions, ratios, and percents in different ways. Use different comparisons than those in **parts A to D.**

For example, you might report on how the population over age 65 compares with the population under age 15.

UNIT 3 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
7-A9 Solve problems involving equivalent ratios **7-A10** Understand percent as a special ratio **7-B9** Use a variety of strategies in calculating percent of a number	1 h	None

How to Use This Performance Task

You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. It could also be used as enrichment material for some students. You can assess performance on the task using the rubric provided below.

Sample Solution

A. i) 56% **ii)** 10%

B. i) About 300,000 **ii)** About 30,000

C. i) 2 : 5 **ii)** 3 : 5

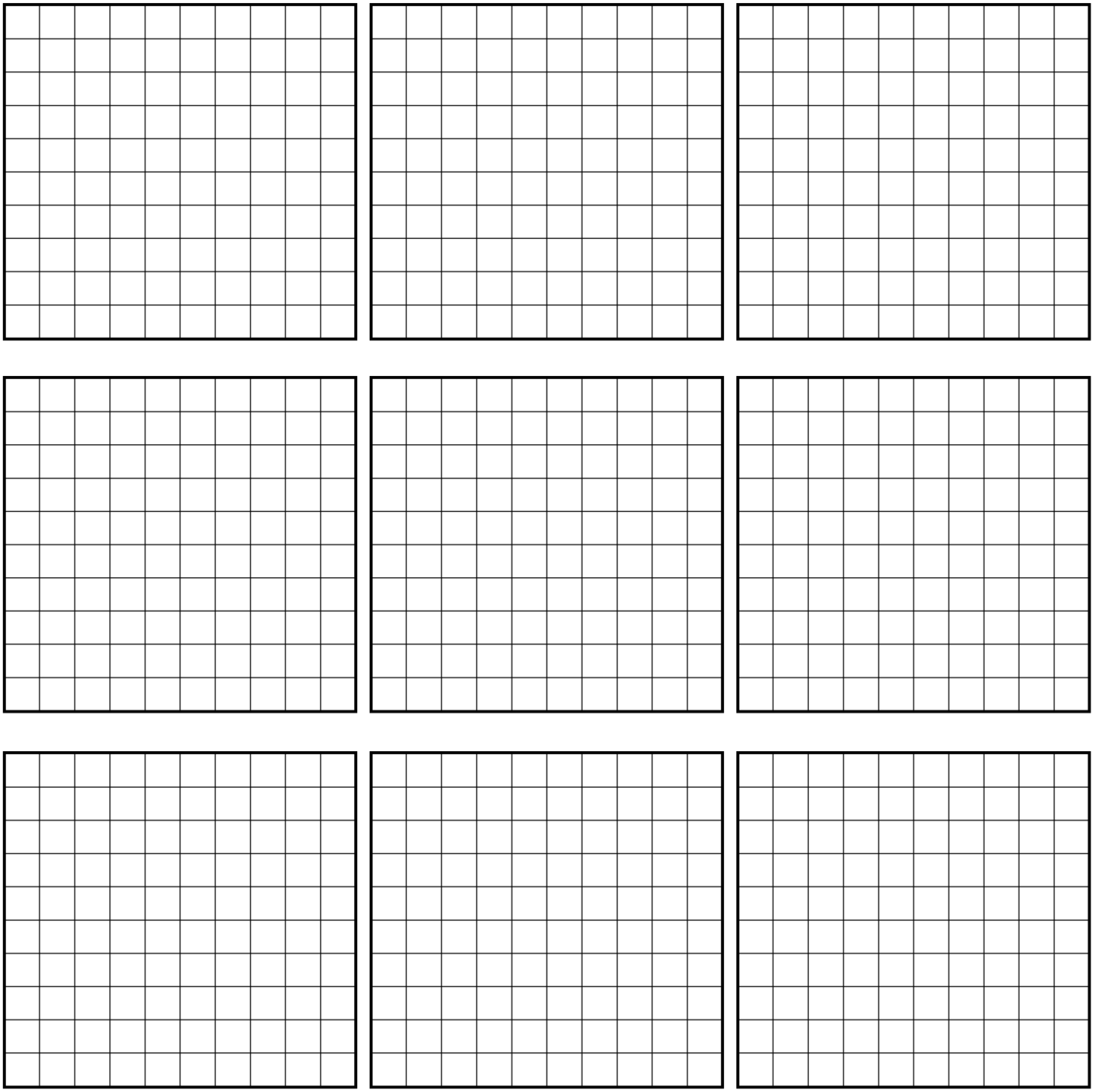
D. $\frac{2}{5}$ and $\frac{3}{5}$

E. In Bhutan, about 525,000 people live on farms.
The ratio of those living on farms to those living in urban areas is 7 : 2.
About 420,000 people are between ages 15 and 65.
The ratio of the population under 15 to the population over 15 is 2 : 3.
The number of people over age 65 is $\frac{1}{10}$ or 10% of the number of people under age 15.

UNIT 3 Performance Task Assessment Rubric

The student	Level 4	Level 3	Level 2	Level 1
Calculates percents	Calculates the number of people correctly in each case	Makes at least one calculation error but shows overall understanding of percents	Makes multiple calculation errors but shows some understanding of percents	Makes many calculation errors and does not show understanding of percents
Expresses ratios	Expresses all ratios in lowest terms in the correct order	Expresses most ratios in the correct order	Makes errors in expressing several ratios, but shows an overall understanding of ratio	Makes errors in expressing most ratios
Presents information	Shows correct calculations and expresses information comprehensively in a variety of ways	Shows mostly correct calculations and expresses information in a variety of ways	Makes errors in calculations or shows information in a limited number of ways	Makes errors in calculations and shows information in a limited number of ways

BLM 1 Percent Grids



BLM 2A Ratio Concentration Game Cards

$\frac{1}{5}$	$\frac{2}{5}$	100%	$\frac{4}{5}$	2 : 5
$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	1 : 5	4 : 5
0.60	1 : 1	1 : 2	0.25	0.80

1 : 4	2 : 4	3 : 4	3 : 5	75%
2 : 8	30%	4 : 8	40%	8 : 10
50%	$\frac{50}{100}$	6 : 8	3 : 10	4 : 10

UNIT 4 GEOMETRY AND MEASUREMENT

UNIT 4 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 105** **TG p. 133**	Review prerequisite concepts, skills, and terminology, and pre-assessment	1 h	• Rulers • Square Dot Grid Paper (BLM)	All questions
Chapter 1 Angle Relationships				
4.1.1 EXPLORE: Angles in a Triangle (Essential) **SB p. 107** **TG p. 136**	**7-E1 Angles: sum** • understand through investigation that the sum of angles of any triangle is 180º **7-E2 Relationships: triangles** • make associations between side length and opposite angle size • draw conclusions about angle measures within an isosceles triangle **7-D1 Angles: estimate and measure using a protractor** • use the appropriate scale on a double scale protractor • estimate angles as a way of checking that the appropriate scale was used	60 min	• Paper for cutting • Rulers • Protractors • Scissors • Compasses	Observe and Assess questions
CONNECTIONS: Angle Measurement Units (Optional) **SB p. 109** **TG p. 138**	Make a connection between different units for measuring angles	15 min	None	N/A
4.1.2 Drawing and Classifying Triangles **SB p. 110** **TG p. 139**	**7-E1 Angles: sum** • understand through investigation that the sum of angles of any triangle is 180º **7-E3 Triangles: classify** • classify triangles as scalene, isosceles, equilateral, acute, obtuse, and right • determine if certain combinations of classifications can exist at the same time (e.g., is a right isosceles triangle possible?) **7-E2 Relationships: triangles** • make associations between side length and opposite angle size • draw conclusions about angle measures within an isosceles triangle	1.25 h	• Rulers • Protractors • Compasses	Q2, 4, 7
4.1.3 Constructing and Bisecting Angles **SB p. 114** **TG p. 144**	**7-E4 Bisectors: construct** • construct angle bisectors • explore the basic use of a compass and straightedge **7-D1: Angles: estimate and measure using a protractor** • use the appropriate scale on a double scale protractor • estimate angles as a way of checking that the appropriate scale was used	2 h	• Rulers • Protractors • Compasses	Q1, 7, 9

UNIT 4 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Chapter 2 Transformations				
4.2.1 Translations **SB p. 119** **TG p. 149**	**7-E5 Transformations: properties of translations, reflections, and rotations** • use formal language: translations for slides • emphasize what changes and what stays the same as a result of a transformation • investigate congruency and orientation in transformations • use tessellations as a context for transformations	1.25 h	• Rulers • Protractors • Compasses • Scissors	Q1, 2, 7
4.2.2 Reflections **SB p. 123** **TG p. 152**	**7-E5 Transformations: properties of translations, reflections, and rotations** • use formal language: reflections for flips • emphasize what changes and what stays the same as a result of a transformation • investigate congruency and orientation in transformations • use tessellations as a context for transformations	1.25 h	• Rulers • Protractors • Compasses	Q 1, 3, 10, 11
GAME: Reflection Archery (Optional) **SB p. 128** **TG p. 157**	Practise reflections in a game situation	20 min	• Rulers • Compasses	N/A
4.2.3 Rotations **SB p. 129** **TG p. 158**	**7-E5 Transformations: properties of translations, reflections, and rotations** • use formal language: rotations for turns • emphasize what changes and what stays the same as a result of a transformation • investigate congruency and orientation in transformations • use tessellations as a context for transformations	1.5 h	• Rulers • Protractors • Compasses • Tracing paper or transparencies (optional)	Q1, 4, 5
Chapter 3 3-D and 2-D Measurement				
4.3.1 Volume of a Rectangular Prism **SB p. 133** **TG p. 162**	**7-D2 Volume**: **rectangular prisms** • relate volume to dimensions • understand that each of the three dimensions of a prism affects the volume	1 h	• Linking cubes	Q1, 3, 10
4.3.2 Measurement Units **SB p. 137** **TG p. 166**	**7-D3 SI Units**: **identify, use, and convert** • identify, use, and convert SI units to measure, estimate, and solve problems • understand the approximate nature of measurement • examine milli, centi, deci, deca, hecto, and kilo as prefixes for measures of length, mass, and capacity • apply principles of conversion using common units (relate the size of a number to the size of the unit) • establish the link between volume, capacity, and mass	1 h	• Grid paper or Small Grid Paper (BLM)	Q1, 5, 8

4.3.3 Area of a Composite Shape **SB p. 141** **TG p. 169**	**7-D4 Area**: **composite shapes** • estimate and calculate the area of shapes on grids • understand that composite shapes can be broken down into familiar shapes for which there are area formulas available	1.25 h	• Square Dot Grid Paper (BLM) • Rulers	Q2, 4, 6
4.3.4 Area of a Trapezoid **SB p. 144** **TG p. 173**	**7-D4 Area**: **composite shapes** • develop and apply the formula for the area of a trapezoid	1 h	• Square Dot Grid Paper (BLM) • Rulers	Q3, 6
4.3.5 Circumference of a Circle **SB p. 147** **TG p. 176**	**7-D5 Circles: solve problems with diameter, radius, circumference** • relate diameter, radius, and circumference to solve problems • investigate π as $C \div d$ for a number of circles and cylinders • develop the formulas $C = \pi d$ and $C = 2\pi r$ **7-D4 Area: composite shapes** • understand that composite shapes can be broken down into familiar shapes for which there are area formulas available	1 h	• Circular objects (tins, etc.) • Compasses • Rulers • String.	Q1, 3, 7
UNIT 4 Revision **SB p. 150** **TG p. 179**	Review the concepts and skills in the unit	2 h	• Rulers • Compasses • Protractors • Square Dot Grid Paper (BLM)	All questions
UNIT 4 Test **TG p. 182**	Assess the concepts and skills in the unit	1 h	• Rulers • Compasses • Protractors • Square Dot Grid Paper (BLM)	All questions
UNIT 4 Performance Task **TG p. 185**	Assess concepts and skills in the unit	1 h	• Rulers • Compasses • Protractors	Rubric provided
UNIT 4 Blackline Masters **TG p. 189**	BLM 1 Square Dot Grid Paper BLM 2 Tangrams Small Grid Paper on **page 53** in UNIT 1			

Math Background

• This unit extends student understanding of both geometry and measurement.

• The unit focuses on angle constructions, transformations done without the help of dot paper, and measurement concepts involving metric units, length, area, and volume.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in most lessons, for example, in **question 9** in **lesson 4.1.2**, where they combine pieces of one shape to create triangles, in **question 9** in **lesson 4.2.2**, where they create a word that has certain reflection properties, in **question 5** in **lesson 4.2.3**, where they must determine a turn centre and the amount of turn in a specific situation, in **question 6** in **lesson 4.3.1**, where they design a container that meets a given set of requirements, and in **questions 3 and 8** in **lesson 4.3.5**, where they use what they know about the circumference of circles to calculate the perimeters of unusual shapes.

• They use communication as they explain their thinking in **question 12** in **lesson 4.1.2**, where they explain how they classify a triangle, in **question 2** in **lesson 4.1.3**, where they describe how they created certain angles, in **question 3** in **lesson 4.2.1**, where they describe how to use transformations to create particular shapes, and in **question 9** in **lesson 4.3.1**, where they explain why an estimate rather than a calculation is appropriate.

• Students use reasoning in answering questions in the **Explore lesson 4.1.1**, where they explain their understanding of the sum relationship between angles in a triangle, in **questions 6, 7, and 8** in **lesson 4.1.2**, where they consider why certain descriptions of triangles are possible and others are not, in **question 5** in **lesson 4.2.1**, where they explain why certain actions create a shape that tiles, and in **question 11** in **lesson 4.2.2**, where they explain how they know what transformation was performed.

• They consider representation as they connect written descriptions of triangles with sketches and accurate diagrams throughout **chapter 1**, convert measurements from one unit to another in **lesson 4.3.2**, and represent formulas for the area of a trapezoid in two different ways in **lesson 4.3.4**.

• Students use visualization skills in **questions 6 and 7** in **lesson 4.1.3**, where they sketch angle estimates using benchmarks, throughout **chapter 2**, where they transform shapes, in **question 3** in **lesson 4.3.2**, where they visualize objects with particular measurements, in **question 6** in **lesson 4.3.3**, where they visualize a parallelogram cut up into a square and two triangles, and in **question 4** in **lesson 4.3.3**, where they imagine a way to make a shape of a given area.

• They make connections to mathematics done outside school in **question 4** in **lesson 4.2.3**, where they connect the concept of rotations to clocks, in **question 8** in **lesson 4.3.2**, where they represent the area of a field, in **question 9** in **lesson 4.3.2**, where they connect standard measurements to a traditional Bhutanese measurement, in **question 7** in **lesson 4.3.4**, where they find the area of a section of roof, and in **question 5** in **lesson 4.3.5**, where they find the length of materials required to build a cylindrical water tank.

Rationale for Teaching Approach

• This unit is divided into three chapters.

Chapter 1 is about angle constructions and reasoning about angle relationships in triangles.

Chapter 2 focuses on transformations done without the help of dot paper.

Chapter 3 extends various measurement skills, including converting units, calculating the area of a polygon, extending area understanding into three dimensions and volume, and calculating the circumference of a circle.

• The **Explore lesson** allows students to experience concretely some relationships between angles in triangles.

• The **Connections** section helps students see that the angle measurement system most familiar to them is arbitrary, and that other systems are used for various purposes.

• The **Game** provides an opportunity to apply and practise work with reflections.

• Throughout the unit, it is important to encourage students to explain their reasoning and to accept a variety of approaches from them.

Getting Started

Curriculum Outcomes	Outcome relevance
4 Congruence: polygons **4** Reflective Symmetry: generalize for properties of various quadrilaterals **5** Area: irregular shapes- estimate and measure **5** Perimeter and area: rectangles and squares **5** Divide Mentally **5** Multiply Mentally: whole numbers by 0.1, 0.01, 0.001 **5** Translations & Reflections: generalize & apply **5** SI Units: reinforce relationships among various SI units **5** Similarity: name, describe & represent **6** Area of a Triangle: relate to area of a parallelogram **6** Divide Mentally: whole numbers by 0.1, 0.01, 0.001 **6** Angles: estimate, measure, and draw **6** Parallelograms: relate bases, heights, and area **6** Rotations: $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ turns	Students will find the work in the unit easier after they review related geometry and measurement skills and concepts from earlier classes.

Pacing	Materials	Prerequisites
1 h	• Rulers • Square Dot Grid Paper (BLM)	• familiarity with the terms *translation, rotation, reflection, transformation, symmetry, similar, congruent,* and *dimension* • area formulas for triangles and rectangles • mirror (reflective) symmetry • converting metric units • multiplying and dividing by powers of ten

Main Points to be Raised

Use What You Know

• You can find the area of a polygon by counting squares on dot paper or by using formulas.

• There are many triangles that have the same area.

• Translations, rotations and reflections do not change the size of shape of a shape; the new shape is congruent to the original shape.

Skills You Will Need

• You can use a formula to find the area of a simple shape or to find a missing dimension if you are given the area.

• The formula for the area of a rectangle is $A = lw$.

• The formula for the area of a triangle is $A = bh \div 2$.

• Multiplication and division by powers of ten does not change the digits, it only changes their place value.

• Metric conversions involve multiplication and division by powers of ten.

Use What You Know — Introducing the Unit

• Before assigning the activity, you may wish to review with students how to find the area of polygons on dot paper. You could draw a rectangle on a dot grid and then ask the students to count the squares to find the area. Ask them to suggest other strategies for finding the area more efficiently. Repeat the activity using a triangle.

• Model a transformation of your example triangle so that the image overlaps. Ask students to find the area of the combined shape.

• Review the terms *translation, rotation, reflection,* and *transformation* to make sure students can interpret **part C**. Refer students to the glossary at the back of the student text.

• Students can work in pairs or small groups on the activity.

While you observe students at work, you might ask questions such as the following:

• *How did you find the dimensions of your three triangles?* (I listed pairs of factors that have a product of 12. Then I doubled one of the factors.)

• *Did you count squares to find the area or did you use a different strategy?* (For the triangles I used the formula, but for the more complicated shapes I counted squares.)

• *Why can you always create another triangle with an area of 12 cm*^{2}*?* (With a base of 6 cm, I could put the third vertex anywhere on the line that is 4 cm from the base.)

• *Why did you choose a translation (or reflection, or rotation)?* (I find it easiest to visualize a translation.)

Skills You Will Need

• Students can work individually.

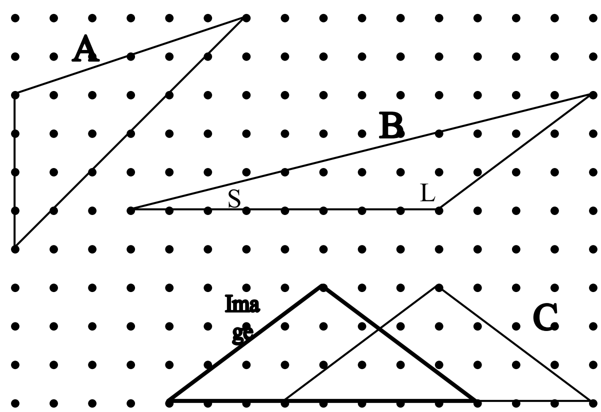
• To ensure students have the required skills for this unit, assign these questions.

• First review the terms *symmetry, similar, congruent,* and *dimension* to make sure students can interpret **questions 2, 3, and 4**. Refer students to the glossary at the back of the student text.

Answers

A. i) 8 cm^2 **ii)** 14 cm^2

B. *Sample responses*:
i) Triangles A, B, and C each have an area of 12 cm^2.
ii) B has the largest angle.
iii) B has the smallest angle.

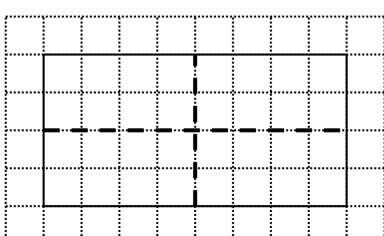


C. *Sample responses*:
i) I translated triangle C to the left; it is a pentagon.
ii) Translate the triangle 1 to 4 spaces farther to the left.

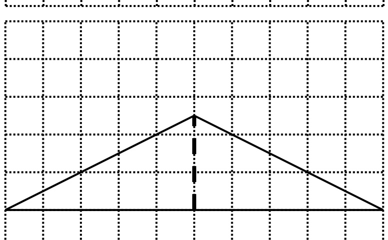
NOTE: Answers or parts of answers that are in square brackets throughout the Teacher's Guide are NOT found in the answers in the student textbook.

1. a) 32 cm^2; [$A = b \times h = 8 \times 4 = 32$ cm^2]
b) 32 cm^2; [$A = b \times h = 8 \times 4 = 32$ cm^2]
c) 12.5 cm^2; [$A = b \times h \div 2 = 10 \times 2.5 \div 2 = 12.5$ cm^2]
d) 6.75 cm^2; [$A = b \times h \div 2 = 4.5 \times 3 \div 2 = 6.75$ cm^2]

2. a)



c)

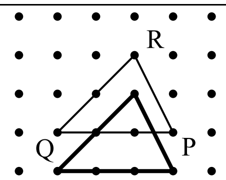


3. A is neither congruent nor similar, [because it is a right isosceles triangle and ΔDEF is a right scalene triangle.]
B is similar but not congruent [because the sides are in the same proportion, but shorter: $3 \times 2 = 6$, $4 \times 2 = 8$, $5 \times 2 = 10$, but it is smaller.]
C is congruent and similar [because the sides are the same length and the angles are the same.]

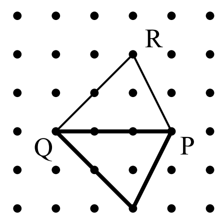
4. a) 2.5 cm; [$A = b \times h$, $35 = b \times 14$, $b = 35 \div 14 = 2.5$ cm]
b) 6 cm; [$A = b \times h \div 2$, $30 = 10 \times h \div 2$, $30 = h \times 10 \div 2$, $30 = h \times 5$, $h = 30 \div 5 = 6$ cm]

5. 12.25 cm^2; [3.5 cm × 3.5 cm = 12.25 cm^2]

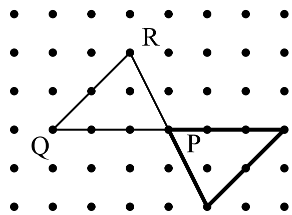
6. a)



b)



c)



7. a) 52 mm
b) 0.052 m
c) 50 g

8. a) 4030
b) 7.2
c) 600,000
d) 5.3

Supporting Students

Struggling students

• If students are struggling with creating triangles with area 12 cm^2 in **part B**, you might model for them how you would create a triangle with an area they suggest (not 12).

For example, to make a triangle with area 20 cm^2, you could think of the numbers 5 × 4 = 20. Because the formula is (base × height) ÷ 2, you have to double one of the numbers, for example, (10 × 4) ÷ 2 = 20. Use these numbers to make the triangle with a base of 10 cm and a height of 4 cm.

• Some students may have trouble writing an explanation in **part C ii)**. You might ask them to tell another student their reasoning before writing it down.

Enrichment

• For **question 1**, you might challenge students to write an explanation for the area of the triangle formula.

4.1.1 EXPLORE: Angles in a Triangle

Curriculum Outcomes	Outcome Relevance
7-E1 Angles: sum • understand through investigation that the sum of angles of any triangle is 180º **7-E2 Relationships: triangles** • make associations between side length and opposite angle size • draw conclusions about angle measures within an isosceles triangle **7-D1 Angles: estimate and measure using a protractor** • use the appropriate scale on a double scale protractor • estimate angles as a way of checking that the appropriate scale was used	• This essential exploration of the angles in a triangle gives students a concrete experience that shows them why the sum of the angles is 180º. • Students also experience a relationship between side lengths and angle measurements. Because this relationship cannot be proven, it is called an axiom; it is an assumption that mathematicians agree to. It is how a degree is defined.

Pacing	Materials	Prerequisites
40 min	• Paper for cutting • Rulers • Protractors • Scissors • Compasses	• familiarity with the terms *scalene, isosceles,* and *equilateral*

Exploration

• Work through the introduction (in white) with the students. Model the folding to help them understand the steps.

• Demonstrate the use of a protractor. Ask students to place the protractor in one of the correct positions for measuring and then ask them to read the measurement.

• Have a few students measure the angles in a triangle you give them and record their results so the other students do not see. Then compare the results. It is likely that their results will not be exactly the same because of measuring error and inaccuracy. Use this experience to explain that there are always measuring errors and inaccuracy, and that these issues will be a factor in their exploration. They will do more careful work with rounding in future years.

Have students work alone, in pairs, or in small groups for **parts A to D**. While you observe students at work, you might ask questions such as the following:

• *How is your scalene (or isosceles, or equilateral) triangle different from Dorji's (or any other classmate's)?* (The largest angle in mine is smaller than Dorji's, so they are different shapes.)

• *Why is the longest (or shortest) side of a triangle always across from the largest (or smallest) angle?* (The largest angle opens wider than the others so the triangle is bigger across from it.)

• *How are you dealing with the different angle sums (when they are not all 180 º)?* (Most are 180º, and the others are very close, so I am quite sure that the differences are due to measurement error.)

Observe and Assess

As students work, notice the following:

• Do they successfully create triangles that are scalene, isosceles, and equilateral?

• Do they place their compasses correctly for angle measurement?

• Do they understand how the folding helps them see that the sum of the angles in a triangle is 180º?

• Do they recognize relationships for the questions that ask "What do you notice ..."?

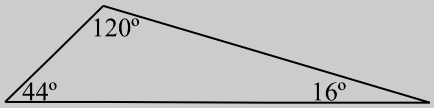
Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and answer these questions.

- *How can you show that the sum of the angles in a triangle is 180º?*
- *How do you know that the sum of the angles in any triangle is 180º?*
- *What relationships did you find between the sides and vertices you marked S and L?*
- *How can you know these relationships are always true?*

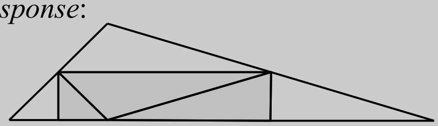
Answers

A. i) and ii) *Sample response*:



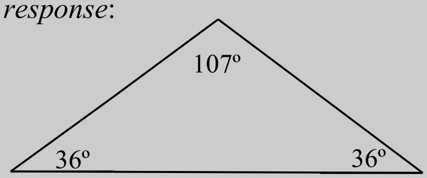
ii) About 180º (results will vary due to measurement error)

iii) *Sample response*:



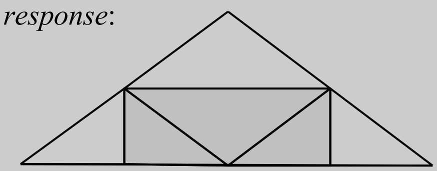
The three angles together match up with the side of the triangle, which is a straight line, so the sum of the angles is a straight angle, 180º.

B. i) *Sample response*:

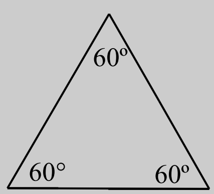


ii) About 180º (results will vary due to measurement error)

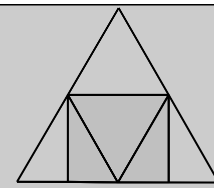
iii) *Sample response*:



The three angles together match up with the side of the triangle, which is a straight line, so the sum of the angles is a straight angle, 180º.



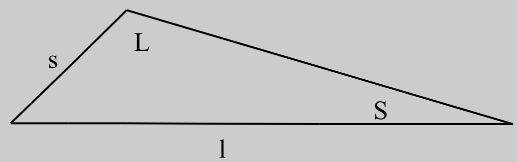
C. i) *Sample response*:



ii) About 180º (results will vary due to measurement error)

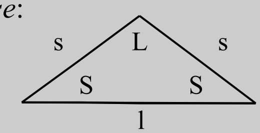
iii) *Sample response*:
The three angles together match up with the side of the triangle, which is a straight line, so the sum of the angles is a straight angle, 180º.

D. i) *Sample response*:



The longest side is across from the largest angle; the shortest side is across from the smallest angle.

ii) *Sample response*:



The longest side is across from the largest angle; my isosceles triangle has two shortest sides and the two smallest angles are across from these sides.

iii) There are no longest or shortest sides or largest or smallest angles in an equilateral triangle because they are all the same.

Supporting Students

Struggling students

- If students are struggling with following the instructions for folding in **parts A, B, and C**, you can allow them to tear off the corners of the triangle and join the three vertices together to see that they form a straight angle.

[Continued]

• Students are likely to be unsure how to make the triangles they need to make for **parts A, B, and C**. Encourage them to use their common sense. It should be possible to make a scalene or isosceles triangle using only a ruler. An equilateral triangle is more challenging. You could encourage them to estimate and adjust or you might provide a template they could copy.

• Some students may have trouble with writing explanations in **part A iv)**. Remind them that a 180° angle forms a straight line and ask them to find a vertex with such an angle in their folded rectangle.

Enrichment

• Students might investigate the angle sums in other polygons, for example, quadrilaterals.

CONNECTIONS: Angle Measurement Units

• This optional connection helps students understand that measurement units are arbitrary.

• You can compare the different ways of measuring angles to the different ways of measuring length (metric vs. Imperial system), capacity (metric millilitres and litres vs. cups), temperature (Celsius vs. Fahrenheit) or any other attribute.

• You may choose simply to tell students that units are arbitrary and invite them to read the Connections and do the questions on their own if they are interested. Or, you could explain each system of measuring angles, using diagrams on the board, and then ask the class **questions 1 to 3**, discussing and answering the questions as a large group.

• The historical references are not certain. We are not sure why Babylonians made 360 degrees represent a full rotation, but we can think about why it is a good idea.

Answers

1. a) 200 gradients
b) 400 gradients
c) 67.7 gradients; [200 ÷ 3 ≈ 67.7 gradients]

2. a) 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360
b) 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 200, 400

3. a) 3.14 radians; [6.28 ÷ 2 = 3.14 radians]
b) 1.57 radians; [3.14 ÷ 2 = 1.57 radians]

4.1.2 Drawing and Classifying Triangles

Curriculum Outcomes	Outcome relevance
7-E1 Angles: sum • understand through investigation that the sum of angles of any triangle is 180º **7-E3 Triangles: classify** • classify triangles as scalene, isosceles, equilateral, acute, obtuse, and right • determine if certain combinations of classifications can exist at the same time (e.g., is a right isosceles triangle possible?) **7-E2 Relationships: triangles** • make associations between side length and opposite angle size • draw conclusions about angle measures within an isosceles triangle	• It is important for students to be able to draw triangles with particular properties because triangles are one of the most basic shapes in our lives and in higher mathematics. • Students need to be able to explain their understanding of mathematical relationships, including reasoning about triangles.

Pacing	Materials	Prerequisites
1 h	• Rulers • Protractors • Compasses	• measuring angles and lengths • determining lines of symmetry in a triangle

Main Points to be Raised

- The sum of the angles in a triangle is 180°.
- The longest side of a triangle is opposite the largest angle and the shortest side is opposite the smallest angle.
- You can describe a triangle based on the relationship between its side lengths or based on the sizes of its angles.
- An equilateral triangle has three equal sides, an isosceles triangle has two equal sides, and a scalene triangle has no equal sides.
- A right triangle has one 90° angle, an obtuse triangle has one angle greater than 90°, and an acute triangle has three angles less than 90°.
- We can classify a triangle in more than one way, for example, a right triangle can also be scalene or isosceles. However, certain combinations of triangle classifications cannot be represented by the same triangle.

  For example, it is not possible to have an equilateral scalene triangle.
- An angle is labelled with one or three letters that name the vertex or vertices of the triangle. A side is labelled with the names of its two end points.
- You can use a protractor to draw an angle with a given measure.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Encourage them to use visualization as much as possible and to measure side lengths only if they are unsure whether two sides are equal. While you observe students at work, you might ask questions such as the following:

- *How did you visualize triangle 1 (or 4, or 7) to see that two sides have the same length?* (I imagined myself standing at this vertex and two friends standing at the other vertices. My friends both seemed to be about the same distance away.)
- *How do you know this angle is greater than 90º?* (I imagined a corner of a piece of paper in the angle. I could see that the angle is larger than the page corner.)
- *What would happen to the other angles in this triangle if you made this angle larger?* (They would have to become smaller because the sum of the angles does not change.)
- If students visualize side lengths or angle measures incorrectly, encourage them by saying that it is good to try visualizing even though it does not always produce accurate results. Then ask them to explain their visualization so you can talk about any misconceptions they might have had.

The Exposition — Presenting the Main Ideas

- After students have completed the **Try This**, ask them what types of triangles they created (e.g., isosceles, right, …). List the types on the board. For each type, ask a student to describe the characteristics of that kind of triangle.
- Ask students if they know other types of triangles and ask them to describe the characteristics of the other types.
- Add any missing triangle types and descriptions to the list compiled by the students. Make sure that the six types of triangles referred to in the exposition are included. Refer students to **page 110** to see all six types.
- Explain while demonstrating how to use a protractor to draw an angle. You could ask a student to suggest the size for an angle. Repeat the process with a different angle, but this time, have a student both suggest and draw the angle.
- Tell students that if they need further help on drawing angles they can refer to the description in the exposition.

Revisiting the Try This

B. Students should look back at their classifications for the triangles in **part A**.

Using the Examples

- Explain that **example 1** asks a student to draw and classify a triangle with sides 5 cm, 6 cm, and 7 cm. The largest angle turns out to be 78º so the triangle is acute scalene. Lead students through the example. You may need to explain the term *draw an arc* by demonstrating. Point out that the small curved part is called the *arc*.
- Demonstrate how to draw a triangle with different side lengths: 6 cm, 7 cm, and 8 cm (each side is 1 cm larger than in the example). Ask students to predict whether the largest angle will be larger than, smaller than, or the same as 78º. (It is still an acute triangle, with the largest angle 76º.)
- Point out that there are issues with accuracy in this question and in some of the **Practising and Applying** questions. When you measure a shape you have drawn, there is always inaccuracy. Results will vary but they should be close.
- Ask students to read **example 2** and try to find a way of drawing a right isosceles triangle without using a compass. (Instead of making two equal length sides using the compass, we could use a ruler to measure 6 cm along each angle arm.)

Practising and Applying

Teaching points and tips

Q 3: Make sure students realize they could start with any of the three sides. They might then use a compass at one end to mark all the possible positions of one of the other sides, and at the other end to mark the possible positions of the third side.

Q 4: If students are having trouble knowing how to start drawing a triangle, suggest that they first draw the longest side and work from there.

Q 6, 7, and 8: You might encourage students to do these questions with a classmate. It is often difficult for students to write good explanations without first trying out their reasoning on a peer.

Q 9: Make sure students understand that the diagonal line in the top half of the triangle joins the midpoints of two sides of the large square. Many students will not know that a tangram is a puzzle used to make many different shapes. Here is an explanation of the answer to **part c)**: There are only three angles in the tangram: 45°, 90° and 135°. A triangle can have a 135° angle but its other two angles must be smaller than 45° because 135° + 45° = 180°. If a triangle made from these pieces has a 90° angle, then the other two angles must be 45° for the three angles to add to 180°. The only other possibility is that all the angles are 45°, which is not enough to make 180°. Thus every triangle made from tangram pieces has one 90° angle and two 45° angles.

Q 10: Students might first draw the 5 cm side and then put a 90° angle at one end and a 45° angle at the other.

Q 11: Some students might benefit from cutting strips of paper of the given lengths and putting the strips together to make the triangles.

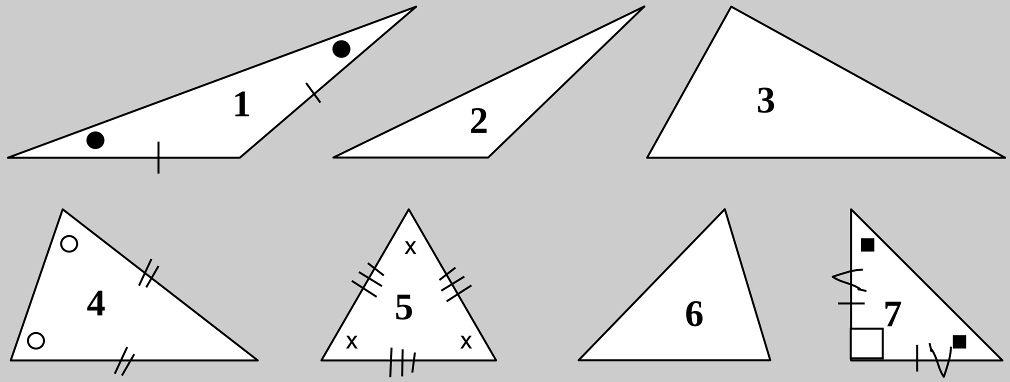
Q 13: You might encourage students to make a chart to help them consider the different types of triangles with various angles.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can apply their knowledge of the sum of the angles in a triangle
Question 4	to see if students can classify a triangle using their experience of drawing triangles and their understanding of the relationships between side lengths and angles
Question 7	to see if students can determine which combinations of triangle classifications are possible

Answers

A. i)



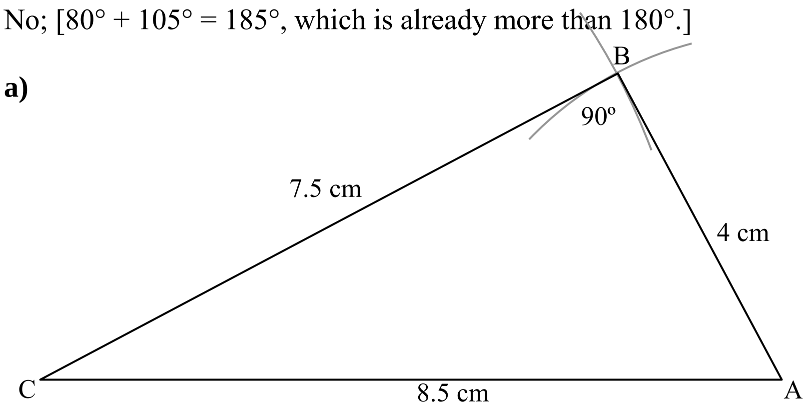
ii) *Sample response:*
Triangles 1, 4, and 7 each have two equal sides and two equal angles.
Triangles 2, 3, and 6 have no equal sides or angles.
Triangle 5 has all equal sides and angles.

B. i) Triangle 4 and Triangle 5 **ii)** No

1. 95°; [The sum of the angles is 180° and 180° – 30° – 55° = 95°.]

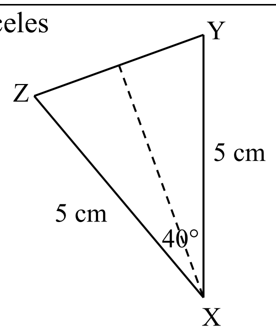
2. No; [80° + 105° = 185°, which is already more than 180°.]

3. a)

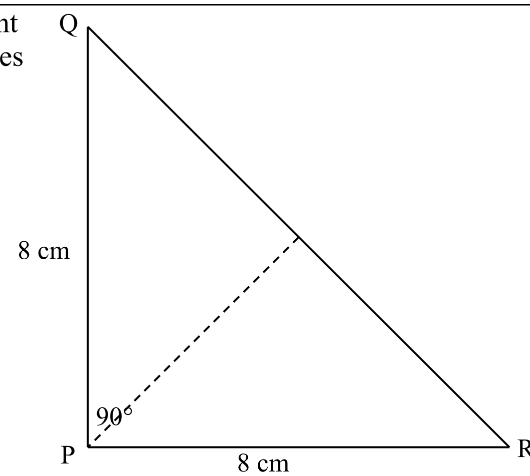


b) Right scalene

4. a) Acute isosceles

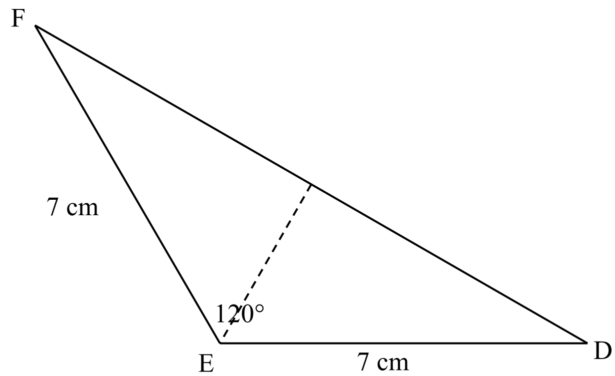


b) Right isosceles



Answers [Continued]

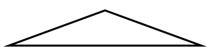
4. c) Obtuse isosceles



5. Any isosceles triangle has one line of symmetry.

6. *Sample responses*:

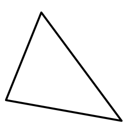
a)



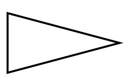
b) Not possible; [all angles in an equilateral triangle are 60° so none are obtuse (between 90° and 180º).]
c) Not possible; [if there is a right angle in a triangle, it is the largest angle because the three angles add to 180° and 90° is already half of this.]

7. *Sample responses*:

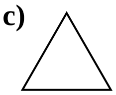
a)



b)

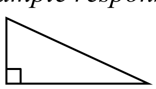


c)

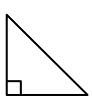


8. *Sample responses*:

a)

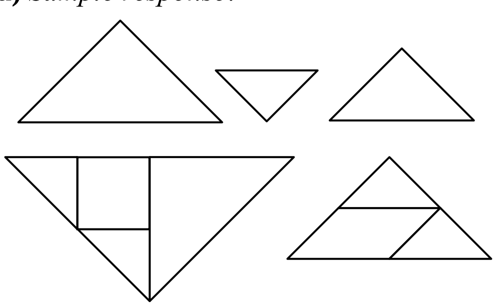


b)



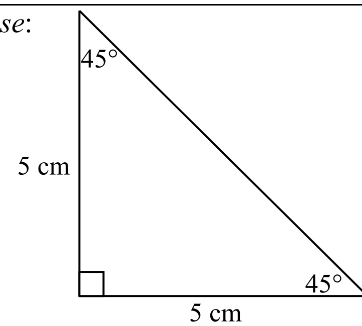
c) Not possible, [since all the angles in an equilateral triangle must be 60°; none can be 90°.]

9. a) *Sample response*:



b) They are all right isosceles.
c) They are all right triangles.
They are all different sizes.

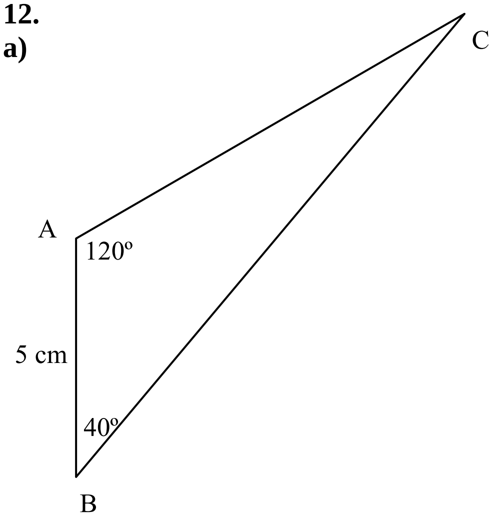
10. a) *Sample response*:



b) Right isosceles
c) A right triangle with two 45º angles and a long side that is 5 cm long.

11. a) Acute scalene
b) Obtuse isosceles
c) Not possible [because 4 + 4 < 9 so they will not form a closed shape.]
d) Acute equilateral
e) Not possible [because 5 + 6 = 11 so they will not form a triangle.]

12.
a)



b) Obtuse scalene;
[Its largest angle is obtuse (120°) so it is an obtuse triangle.
∠C = 20° (because the angles have to add to 180°) so all the angles are different, making it scalene.]

13. Yes; [You can find the third angle because the sum of the angles is 180°. If two angles are equal, it is isosceles. If three angles are equal, it is equilateral.
If no angles are equal, it is scalene. You can tell if it is right, obtuse, or acute by looking at the greatest angle.]

Supporting Students

Struggling students

• If students are having trouble knowing how to start drawing a triangle in **questions 4, 10, and 12**, suggest that they first draw the longest side. If they still struggle, you might demonstrate how to draw a different triangle with a given angle and two given side lengths, or with a given side length and two given angles.

• Some students may have trouble writing explanations in **questions 6, 7, 8, 11, 12, and 13**. If they cannot find a triangle that is not possible, encourage them to figure out which of the classifications is most limiting and consider that one first.

For example, for an obtuse equilateral triangle, the equilateral designation is the more limiting part, so they might start there.

Enrichment

• For **question 11**, you might challenge students to find sets of side lengths that make right triangles. These are called Pythagorean triples (e.g., 3-4-5, 5-12-13, and 8-15-17)

• For **question 11**, you might also ask students to find a rule for deciding whether or not a triangle is possible when three side lengths are given.

4.1.3 Constructing and Bisecting Angles

Curriculum Outcomes	Outcome relevance
7-E4 Bisectors: **construct** • construct angle bisectors • explore the basic use of a compass and straightedge **7-D1 Angles: estimate and measure using a protractor** • use the appropriate scale on a double scale protractor • estimate angles as a way of checking that the appropriate scale was used	Constructions are an historical part of the mathematics curriculum. Although constructions are less important today because we have many other technological tools, they are still of historical interest.

Pacing	Materials	Prerequisites
2 h	• Rulers • Protractors • Compasses	• familiarity with the characteristics of similar triangles

Main Points to be Raised

• To bisect something means to cut it in half.

• You can bisect an angle using a protractor or using a construction.

• To construct means to use a straight edge and compass as the only tools. To draw means to create carefully using other tools like a protractor as well as a straight edge and compass. To sketch means that an estimate is sufficient.

• All points on an angle bisector are equidistant from the endpoints of angle arms that are of equal length. The construction of the angle bisector is based on that principle.

• You can construct a 90° angle by constructing a straight angle and bisecting it.

• You can construct a 60° angle by creating a line segment and using a compass at each end point to mark other segments of the same length.

• You can use constructions of 90° and 60° angles to construct other angles.

Try This — Introducing the Lesson

A. Set up the game described in the **Try This**. Use chalk to draw the angles on the floor and have students stand with equal spacing. They do not actually have to run to the ball, but they can see which student in any pair is closer. It is more important to talk about positioning the ball to make the races fair than it is to actually race.
You might have a couple of demonstration races, but most of the time should be spent discussing the game.

Ask the questions given in **part A**. You could also ask the following questions:

• *How could we check that both Number 3 students are the same distance from the vertex?* (Measure the distance for one of the students by putting a mark on a rope and use the same rope to measure the distance to the other student.)

• *How could we check that the distance to the ball is equal for both Number 3s?* (Measure the distance for one of the students by putting a mark on a rope, and then use that rope for the other student.)

The Exposition — Presenting the Main Ideas

• First, distinguish between the verbs *construct, draw*, and *sketch.* It is important that students understand that the word *construct* has a special meaning in mathematics.

• Demonstrate how to construct an angle bisector using a compass. Start with an acute angle.

• Ask students to explain how this construction relates to the game in the **Try This**. Your questions about checking the distances for the two students labelled Number 3 will help students recognize comparisons.

• Repeat the construction with an obtuse angle close to 90º. You might ask a student to do this construction.

• Repeat with an angle close to 180º, perhaps asking a student to do the construction this time.

• Repeat with a 180º angle to show that this construction makes a perpendicular.

- Tell students that 90º angles are very common, so it is good to be able to construct them.
- Demonstrate how to construct a 60º angle. Then ask a student to repeat the procedure.
- Explain how you can make other angles using 90º and 60º angles and bisections.
- Show one way of making a 120º angle (choose a relatively difficult way, for example, making a 90º angle, adding a 60º angle, and bisecting the 60º angle), and then ask students to suggest other ways of constructing a 120º angle.
- Tell students that if they need further help on constructing angles or bisectors, they can refer to the description in the exposition.

Revisiting the Try This

B. This question allows students to make a formal connection between what was done in **part A** and the more formal approach students have learned for constructing angle bisectors.

Using the Examples

- Before asking students to open their books, work through **example 1** by demonstrating the constructions on the board. Ask students to suggest what to do at each step.
- While you work through the examples, give students tips about how to use a compass well.

For example, to make a smooth arc, hold the compass on the top or on the arm that has the point on it, not on the arm with the pencil.

- After working through **example 1**, ask students if it is possible to use a 90º angle to construct a 15º angle. (It is possible but not practical. You could bisect 90º to make 45º and then construct a 30º angle on one arm by making a 60º and bisecting it. 45º – 30º = 15º.)
- Have students work through **example 2** and **example 3** with a partner.
- Ask students to suggest different ways of visualizing at 20º angle (e.g., visualize 60º, visualize its bisection, which is 30º, and then visualize about two thirds of that).

Practising and Applying

Teaching points and tips

Q 1: Some students may find it helpful to write out an expression that represents their strategy before they make the constructions (e.g., 60º ÷ 2 = 30º).

Q 2: Encourage students to use various number operations, such as adding, dividing by 2, dividing by 4, and so on.

Q 4: This question might be assigned only to selected students. If necessary, remind students of the characteristics of similar triangles.

Q 6 and 7: Some students may worry about accuracy in their sketches. Remind them that they will not be evaluated on the accuracy of the drawing. Rather, they will be evaluated on the strategy they describe as long as the sketch is close to accurate.

Q 9: This question is a way for students to apply what they have learned throughout the lesson. You might begin the next class by asking students to explain their answers to this question.

Common errors

- Many students will find that their angle bisectors do not meet in a single point in **question 3**. This is likely to happen even if there are no errors because there are inaccuracies involved in constructions. You might remind students that these errors are likely to occur, and that they will be evaluated on their method, which is shown by their compass markings, more than on their accuracy.

Suggested assessment questions from Practising and Applying

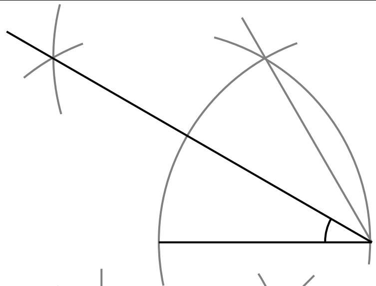
Question 1	to see if students can use what they know about bisection and number sense to construct angles
Question 7	to see if students can use estimation to sketch angles
Question 9	to see if students can construct other angles using a given angle

Answers

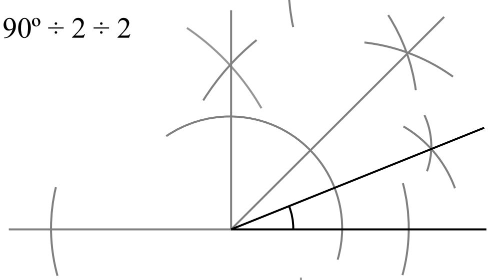
A. i) *Sample response*:
Yes; they both have the same distance to run to the ball.
ii) Anywhere on the shared arm.

B. She can construct the angle bisector of the angle formed by the two outer arms.

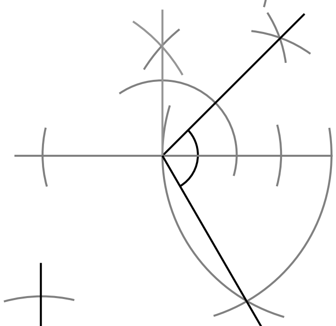
1. a) 30° = 60º ÷ 2



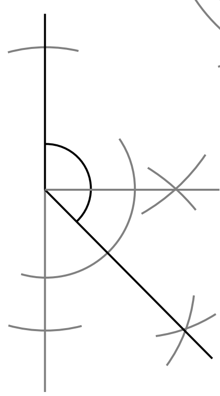
b) 22.5° = 90º ÷ 2 ÷ 2



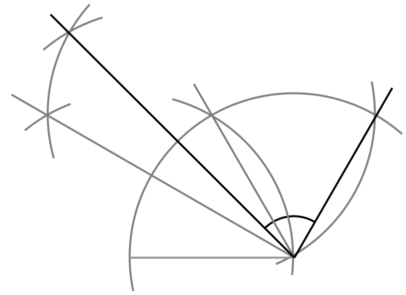
c) 105° = 60º + 45º



d) 135° = 180º – 45º

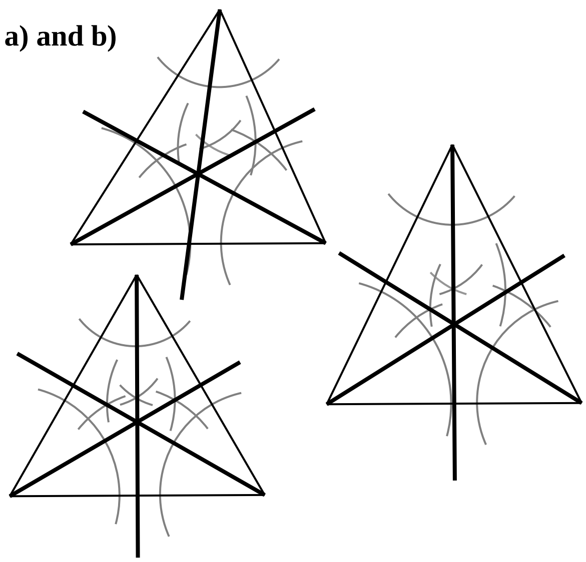


e) 75° = 60º + (60º ÷ 2 ÷ 2)



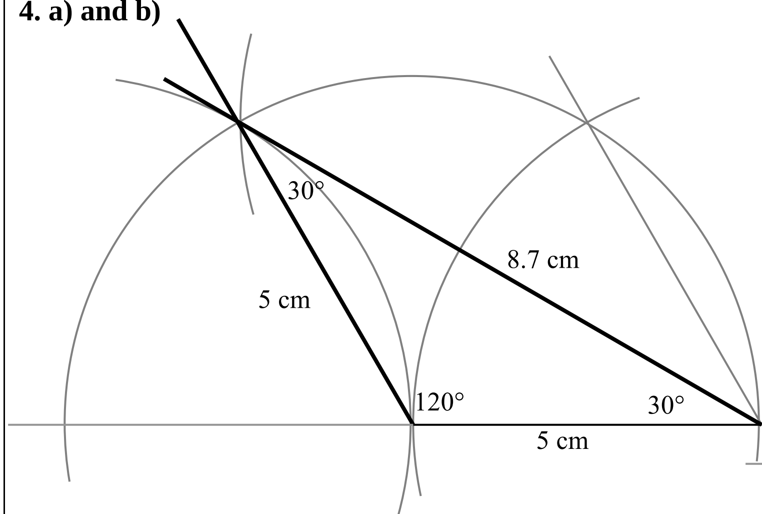
2. *Sample response*:
15°; 7.5°; 37.5°; 82.5°º; 97.5°
[15° = 60º ÷ 2 ÷ 2; 7.5° = 60º ÷ 2 ÷ 2 ÷ 2;
37.5° = 30º + 7.5º; 82.5° = 90º – 7.5º;
97.5° = 90º + 7.5º]

3. a) and b)

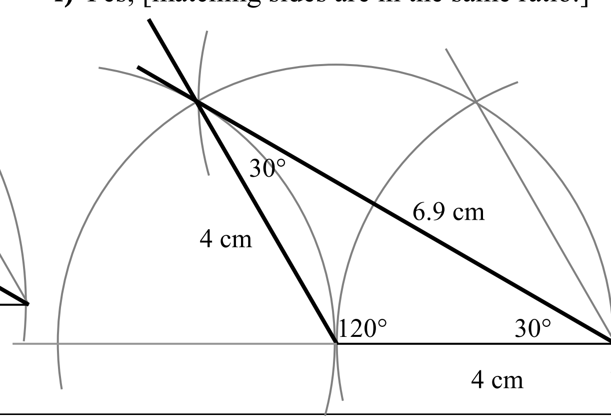


c) In each triangle, all three bisectors go through the same point in the middle of the triangle.

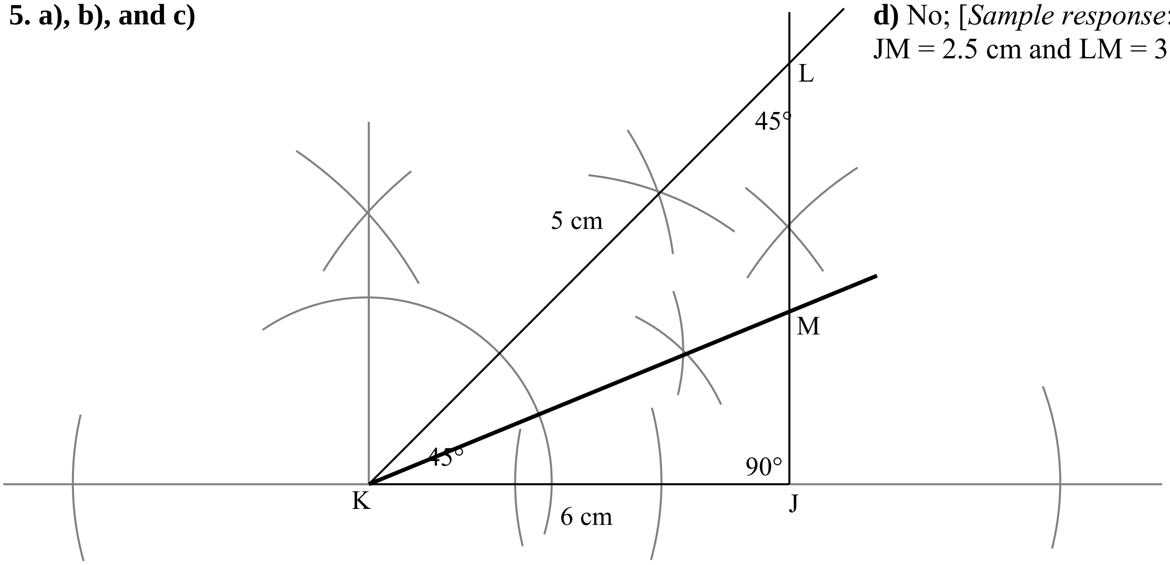
4. a) and b)



c) 5 ÷ 4 = 1.25 **d)** 8.7 ÷ 6.9 = 1.26
e) The result are the same (answers may vary due to constructing and measuring errors).
f) Yes; [matching sides are in the same ratio.]

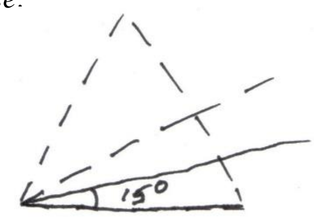


5. a), b), and c)



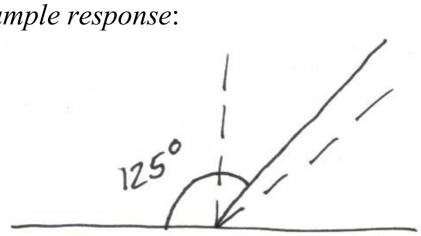
d) No; [*Sample response*:
JM = 2.5 cm and LM = 3.5 cm.]

6. *Sample response*:
a)



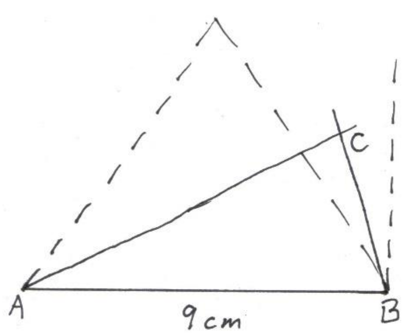
[**b)** I sketched an equilateral triangle, then bisected one of its 60º angles. Next, I bisected one of those 30º angles
(60° ÷ 2 ÷ 2 = 15°).]

7. *Sample response*:
a)

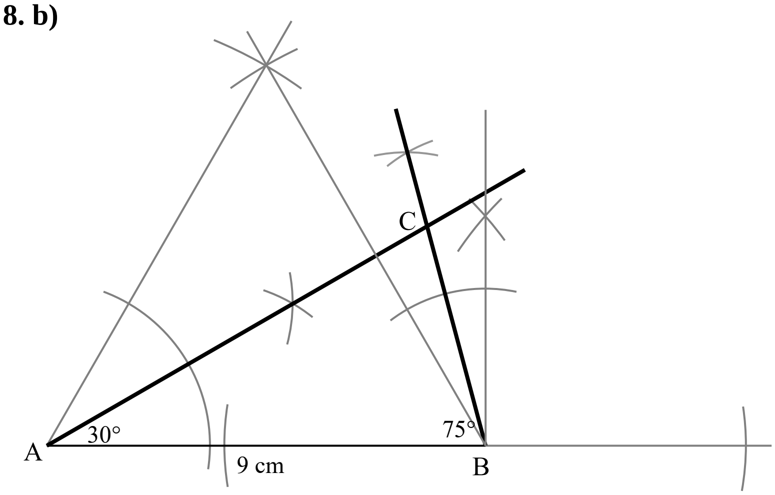


[**b)** I sketched a 90° angle on a line segment. Then I bisected the right side of the angle. I used the 90° angle on the left and half the 90° on the right to make a 135º angle (90° + (90° ÷ 2) = 135°). Then I drew the angle a little less than 135° to make 125°.]

8. a)



8. b)



c) *Sample response*:
In my sketch BC is 4.1 cm, and in my construction BC is 4.7 cm.
In my sketch AC is 8.7 cm, and in my construction AC is 9 cm.
My sketch is a good estimate.

[**9.** *Sample responses*:
a) Bisect the 18º angle.
b) Construct a 60º angle on one arm of the 18º angle to make a 78º angle, then bisect the 78º angle to make the 39º angle.
c) 108º: Construct a 90º on one arm of the 18º angle.
4.5º: Bisect the 18º angle to make two 9º angles, then bisect one of those to make two 4.5º angles.]

Supporting Students

Struggling students

• Some students may have trouble finding a method for constructing some of the angles in **question 1**. You might invite them to ask a classmate to describe a method. Even doing a construction explained by someone else will help students understand how these angles can be constructed.

Enrichment

• Related to **questions 6 and 7**, you could ask students what angles would be relatively easy to estimate with sketches. Also ask them to explain their choices.

• You could do the constructions from **question 1** outside with students or invite students to do them in groups. Use a rope for a compass: one person stands still holding the rope and another rotates around this person while holding the rope taught. Use another rope for a straight edge: pull the rope tight and it will be straight.

• Students who have access to a computer might do some research on the kinds of constructions that are traditionally used in mathematics.

4.2.1 Translations

Curriculum Outcomes	Outcome relevance
7-E5 Transformations: properties of translations, reflections, and rotations • use formal language: translations for slides • emphasize what changes and what stays the same as a result of a transformation • investigate congruency and orientation in transformations • use tessellations as a context for transformations	By exploring transformations without the benefit of a grid, students will gain a deeper understanding of the properties of the transformations.

Pacing	Materials	Prerequisites
1 h	• Rulers • Protractors • Compasses • Scissors	• familiarity with basic 2-D shapes • drawing and constructing 2-D shapes

Main Points to be Raised

• A translation is a slide. All points on a shape move the same distance and in the same direction. You can describe the slide by stating a distance right or left and a distance up or down. Or, you can use a slide arrow that shows the distance and direction. The slide arrow can be named by its end points.

• We name the image of a translated shape by using the same vertex letters as the original shape, each with a mark called a prime next to it.

For example, A moves to A'.

• Some of the properties of translations are:

- corresponding sides in the original shape and the image are the same length
- corresponding angles in the original shape and the image have the same size
- the image is congruent to the original shape
- the orientation of the shape remains the same

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How did you calculate the distance the brick would have to be moved?* (It would be a slide two bricks to the right. Each brick is 20 cm long, so the distance is 40 cm.)

• *What is the least information you would have to write to describe each slide?* (Direction and distance, for example, “right 40 cm”.)

• If students count the space between the original shape and image bricks instead of counting the distance of the slide (getting answers of 20 cm for **part i)** and 10 cm for **part ii)**), you might direct their attention to a point on the original shape brick.

For example, ask them to focus on the top right corner of the black brick while they imagine the slide.

The Exposition — Presenting the Main Ideas

• On the board, draw a diagram of a triangle and its image after a translation.

• Use the diagram to talk about the different ways you can describe the translation, including using an arrow and referring to the endpoints of a line segment.

• Ask students what measurements are the same in the image as in the original shape. Mark these equivalencies on your diagram.

• Draw students' attention to the translation shown on **page 119**. Make sure they understand what the markings on the triangles near the bottom of the page show, i.e., if the marks are identical, the side lengths or angle measures are identical.

• Lead students through the description of orientation on **page 120**.

• Show that the orientation is the same in the translation. To help students understand, draw an example of two shapes with opposite orientation.

Revisiting the Try This

B. Students probably described the translations in **part A** without slide arrows. This provides an opportunity for students to see the connection between the different ways of describing a translation.

Using the Examples

• Draw the diagrams from **example 1** on the board, and ask the students to think about which pairs are translations. Then ask them to explain their reasoning. Different students will describe their reasoning in different ways, so be sure to hear from more than one student for each part of the example. They can check their thinking against what is shown on **page 120**.

• Assign students to pairs to work on **examples 2 and 3**.

• For **example 2**, you might encourage students to cut some paper to make a shape that resembles shape A. They could then use this shape to physically model each possible slide and to help them explain why some pairs of shapes could not be translations. If students do not cut the shape out, you might encourage them to imagine a cut-out shape.

• Ask the student pairs to read **example 3** and then to translate the shape along arrow CB.

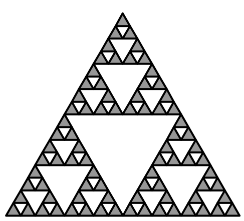
• After students have worked on **example 3**, test their understanding. Draw the shape on the board and ask a student to draw the image after a translation along CB. Ask students what is the same and different about the results with BC and with CB.

Practising and Applying

Teaching points and tips

Q 1: Some students may choose to cut out shape A to help them visualize and explain.

Q 3: The shape in this question is the basic shape of this famous fractal image called Sierpinski's Gasket. Imagine starting with a gray equilateral triangle. Cut a triangular hole in it with vertices at the midpoint of each side. Repeat this procedure on the remaining equilateral triangles. Repeat again, and again, and again.



Q 4: Remind students they need to use a straight edge and compass to perform the construction.

Q 5: You might encourage students to do this question with a classmate.

Q 7: This is an important generalization: the area does not change with a translation.

Common errors

• Many students will think of some, but not all, of the translations that fit the description in **questions 2 and 3**.

• For transformations, it is more likely that students will fail to recognize a possibility than make an error in their work. Encourage them to move their hands and fingers to model the translations or to hold and move physical objects to model the slides. This will help them visualize. Encourage them to use such visualization aids at first and then to try moving away from using them to strengthen their visual imaginations.

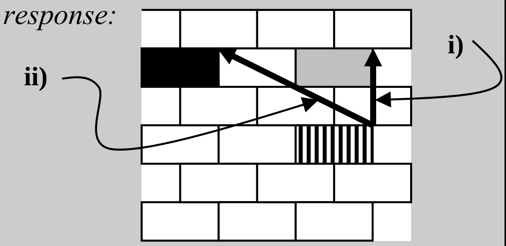
Suggested assessment questions from Practising and Applying

Question 1	to see if students can identify a translation image
Question 2	to see if students can identify and describe a translation
Question 7	to see if students understand the properties of translations

Answers

A. i) 40 cm to the right **ii)** 20 cm up

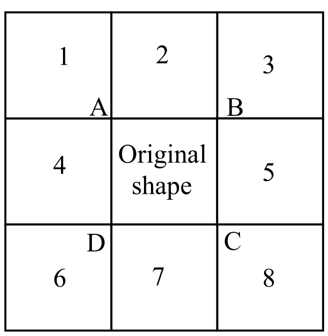
B. *Sample response:*



1. B is not a translation [because it doesn't have the same orientation (it is flipped so it faces the opposite way).]
C is not a translation [because it doesn't have the same orientation (it is flipped so it faces the opposite way).]
D is a translation [because it has the same orientation and it is congruent.]

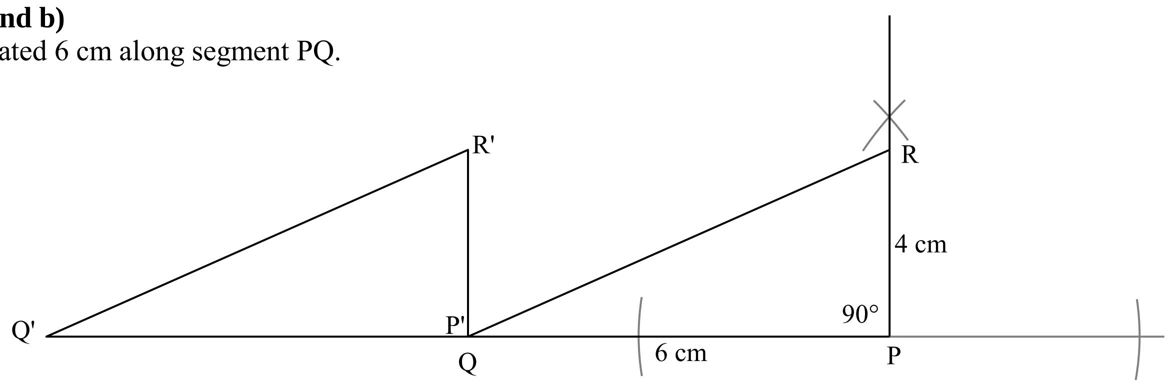
2. a) and b)

- Square 1 is the result of a translation along CA.
- Square 2 is the result of a translation along DA or CB.
- Square 3 is the result of a translation along DB.
- Square 4 is the result of a translation along BA or CD.
- Square 5 is the result of a translation along AB or DC.
- Square 6 is the result of a translation along BD.
- Square 7 is the result of a translation along AD or BC.
- Square 8 is the result of a translation along AC.



3. a) and b)
ΔABF can be translated along arrow AF to create ΔFED. F is the image of A. E is the image of F. D is the image of B.
ΔABF can be translated along arrow AB to create ΔBDC. B is the image of A. D is the image of F. C is the image of B.
c) No. [It has a different orientation (it would have to be flipped).]
d) No, [because it is not congruent.]

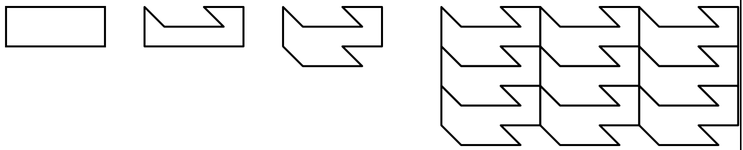
4. a) and b)
Translated 6 cm along segment PQ.



c) Area of ΔPQR: 12 cm^2; [$A = b \times h \div 2 = 6 \times 4 \div 2 = 12$ cm^2]
Area of ΔQQ'R' (or P'Q'R'): 12 cm^2; [$A = b \times h \div 2 = 6 \times 4 \div 2 = 12$ cm^2]

Answers [Continued]

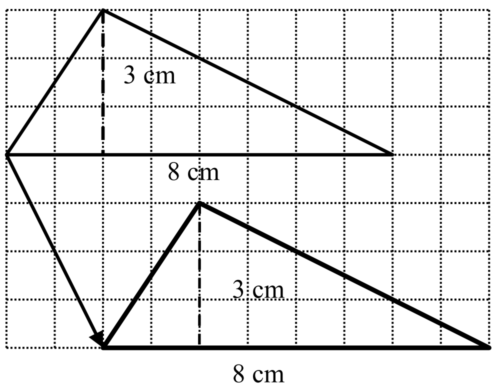
5. a) to d) *Sample response*:



[**e)** The cut-out pieces that are translated from one side to the other side are congruent so the extra part on the bottom of one tile fits into the missing top part on another one.]

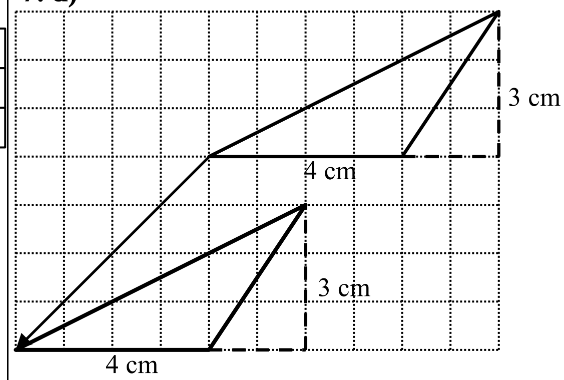
6. a) Along PR **b)** PQ, QR, RP, QP, or RQ

7. a), b), and c)



The areas of the original shape and the image are the same, 12 cm^2. [$A = b \times h \div 2 = 8 \times 3 \div 2 = 12$ cm^2]

7. d)



The areas of the original shape and the image are the same, 6 cm^2. [$A = b \times h \div 2 = 4 \times 3 \div 2 = 6$ cm^2]

[**8.** Any translation image and its original shape are congruent.]

Supporting Students

Struggling students

• If students are struggling with visualizing a translation in **question 1** or any other question, you might ask them to cut out the original shape and physically slide it to see the translation. If they do this for a few translations it will help them visualize others without having to cut out the shapes.

• Some students may have trouble understanding the instructions in **question 5**. You might show them an example by cutting out the shape shown in the question and tracing it to make the tiling.

Enrichment

• For **question 5**, you might challenge students to use the same approach for both pairs of opposite sides in the original rectangle. This would result in more complex tilings. In the example, the left and right side are modified. But the top and bottom can be modified in a similar way. The shape will still tile.

• As a further extension to **question 5**, you could challenge students to think of some other starting shapes (instead of a rectangle) that could be used as a base for tilings. They could then use one or some of these starting shapes to make some tilings as they did in **question 5**.

4.2.2 Reflections

Curriculum Outcomes	Outcome relevance
7-E5 Transformations: properties of translations, reflections, and rotations • use formal language: reflections for flips • emphasize what changes and what stays the same as a result of a transformation • investigate congruency and orientation in transformations • use tessellations as a context for transformations	By exploring transformations without the benefit of a grid, students will gain a deeper understanding of the properties of the reflections, specifically reflections involving curves, reflection lines passing through the original shape, and diagonal reflection lines.

Pacing	Materials	Prerequisites
1 h	• Rulers • Protractors • Compasses	• familiarity with basic 2D shapes • drawing and constructing 2-D shapes • constructing a perpendicular

Main Points to be Raised

• A reflection is a flip. It is described by indicating the reflection line, or mirror line.

• The line segment that joins any point to its image after reflection is bisected by and at right angles to the reflection line. To draw a reflection image, you can draw a perpendicular line segment from a point to the mirror line and extend it an equal distance from the mirror line to locate the image point.

• To reflect a shape, you can reflect its vertices (if it is a polygon) and then join the images.

• A reflection line can be located outside a shape, along an edge of a shape, or inside a shape.

• When you reflect a shape, the only points that do not move are those on the reflection line.

• To locate a reflection line, you can connect corresponding vertices on the original and image shapes. The perpendicular bisector of that segment is the reflection line. The same reflection line must work no matter which pair of corresponding vertices is used.

• A reflection image is congruent to its original shape but it has the opposite orientation.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Is it possible to slide the black brick into the grey hole? Why not?* (No, it is not possible because the brick is angled the other way.)

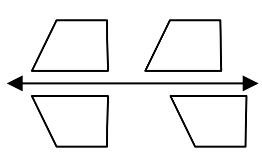
• *Is it the same first to flip and then to slide the striped brick as it is first to slide and then to flip?* (Yes, the result is the same either way.)

• If students say that the striped brick can be reflected in the line halfway between it and the grey brick, they are correct. However, with real bricks such a flip might not really work because of the space between the brick and the reflection line. You would have to lift the brick, flip it, and place it on the other side of the reflection line, the same distance away from the line. In this lesson, it is assumed that shapes are flat (with no depth).

The Exposition — Presenting the Main Ideas

• On the board, draw two diagrams, one that shows a reflection and one that does not (because the image is shifted).

For example:



• Ask students which pair looks like a reflection and which does not.

• Use the diagrams to show that if you connect corresponding points in the original and image shapes of a reflection, the connecting line is perpendicular to the reflection line and its midpoint is on the reflection line. Show that this is not the case when the situation is not a reflection.

• Show students that to reflect on paper, you can fold along the reflection line. The original shape points match up with their image points.

• Ask students to look at the diagram of the hexagon tiling on **page 123**.

• Have the students describe the reflection they see if the reflection line is the one marked.

• Ask a student to read the vertices for the original shape starting at A and going clockwise. Then do the same for the image.

• Students should notice that the letters are read in the opposite direction, and so the image has the opposite orientation.

• On the board, sketch the example on **page 124** to show how to find the image of each point.

• Ask students if the orientation of the image is the same or opposite this time. Point out that it is always opposite for reflections.

• Ask students why point B did not move when it was reflected.

• Have students read through the exposition on **pages 123 and 124** to make sure they are comfortable with the concepts discussed.

Revisiting the Try This

B. This question allows students to apply the properties of reflections. If the issue about the impracticality of physically flipping a brick in a reflection line that is not along its edge did not come up for discussion in **part A**, talk about it with students now.

Using the Examples

• Ask the students to close their books. Copy the diagrams from **examples 1, 2, and 3** on the board.

• Discuss **example 1** first. Ask students to say which triangles could be reflections of A. Start with triangle B. Ask students to raise their hands if they think it is a reflection. Then ask a student who has not raised his or her hand to explain why it is not a reflection. Ask if other students have another way of explaining this (or if they have more to say).

• Repeat this procedure with triangles C and D. For C, choose a student who has raised his or her hand to explain how he or she knows it is a reflection.

• Now have pairs of students read through **examples 1 and 2**. Inform them that you will choose a pair to come to the board to do each reflection.

• After they have had enough time to read the examples carefully, ask a student pair to do the reflection of the shape in **example 1**. Then have a pair do the reflection from **example 2**.

Practising and Applying

Teaching points and tips

Q 1, 2, and 3: If you have a semi-transparent piece of glass or plastic, you can use it for a mirror that allows reflection in both directions. Hold the glass/plastic with its edge along the reflection line. This tool can help students visualize reflections. This tool is commercially available and is called a Mira.

Q 2: Make sure students understand that their reflection lines can go in any two directions they wish.

Q 4: Remind students to use construction techniques for the 90º angle. They should be aware of the word *construct* in this question.

Q 5: Make sure students reflect both parts of the shape, i.e., the part below the reflection line and the part above the line.

Q 7: When a shape reflects onto itself, the shape has reflective symmetry.

Q 8: Remind students to use construction techniques for the angle bisectors.

Q 10: Remind students to use construction techniques for the 90º and 60º angles.

Q 11: This question is designed to help students consider a variety of possibilities when they see a transformation.

Common errors

• Many students will think of some, but not all, the reflection lines in **questions 6 and 7**. When you talk about their results, be sure to focus on the students' success in finding reflection lines rather on any omissions.

• For transformations, it is more likely that students will fail to recognize a possibility than make an error in their work. Encourage them to move their hands and fingers to model the translations or to hold and flip physical objects to model the reflections. This will help them visualize. Encourage them to use such visualization aids at first.

Suggested assessment questions from Practising and Applying

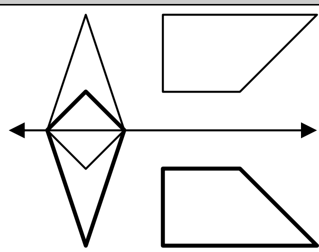
Question 1	to see if students can perform a reflection whether the shape crosses the reflection line or not
Question 3	to see if students can identify a reflection image
Question 10	to see if students can describe and perform a reflection
Question 11	to see if students understand the properties of reflections and translations

Answers

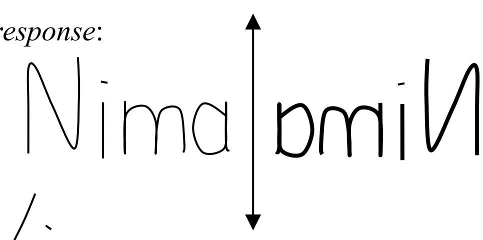
A. *Sample responses*:
i) Flip it along the top edge.
ii) Slide it down until its edge touches the top of the grey hole, then flip it. Or, flip it across a horizontal line halfway between the brick and the hole.

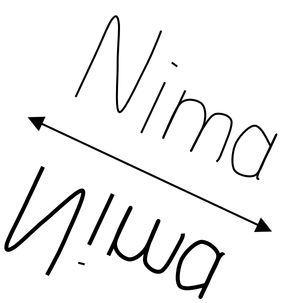
B. i) *Sample response*:
For **part i)**, the reflection line is the top edge of the black brick
For **part ii)**, the reflection line is the line between the two white parallelograms that are between the hole and striped brick.
ii) It is not possible; both parallelograms face the same way, i.e., they have the same orientation. A reflection always has an opposite orientation.

1.



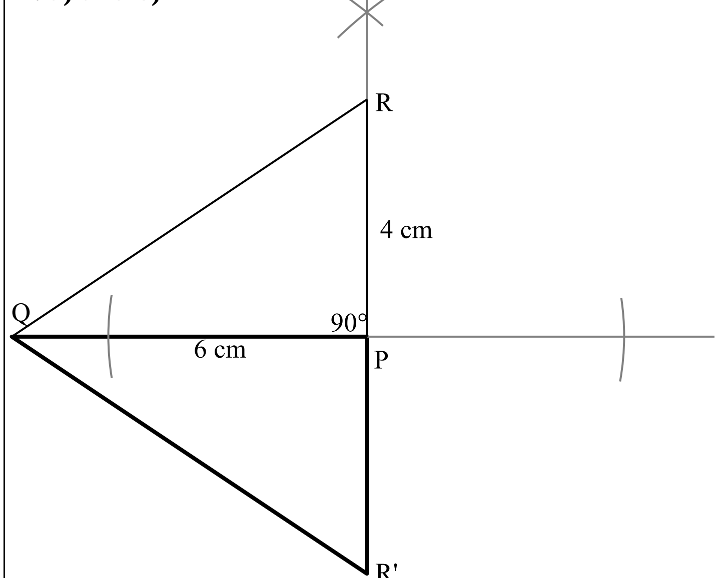
2. *Sample response*:





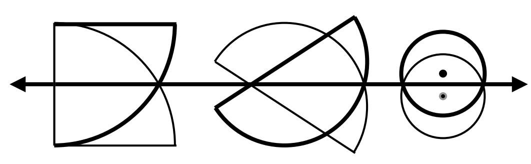
3. B is a reflection; [it is congruent and the orientation is opposite.]
C is not a reflection; [the orientation is the same and it should be opposite.]
D is not a reflection; [even though it is congruent and the orientation is opposite, I could not draw one reflection line though the midpoints of the line segments connecting corresponding vertices.]

4. a) and b)

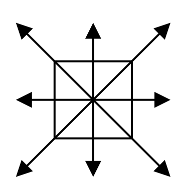


c) $\angle RQP = \angle R'Q'P' = 34°$
d) Acute isosceles

5.

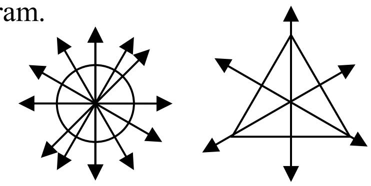


6. 3; *Sample response*:



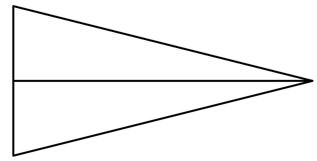
Answers [Continued]

7. The circle and the triangle, but not the parallelogram.

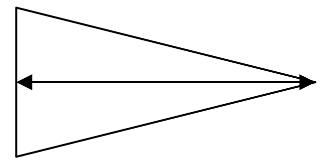


Any line that connects two points on the circumference of a circle through the centre is a reflection line.

8. a) and b)



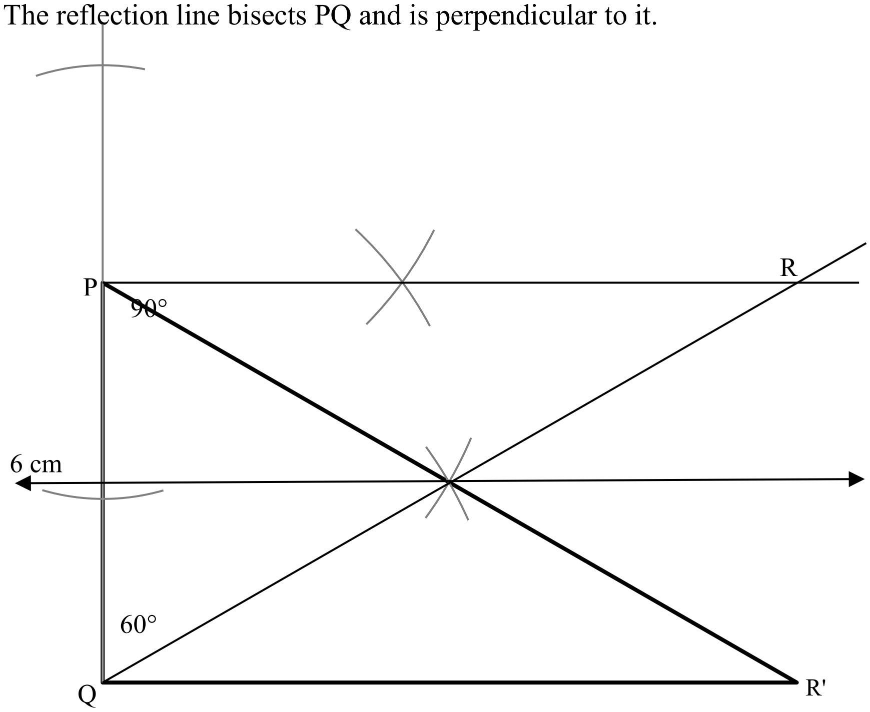
c)



The angle bisector is the reflection line.

9. *Sample responses*:
a) TOT **b)** BOB **c)** BED or ICE

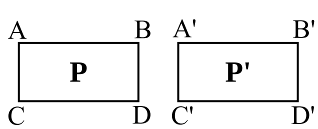
10. a) and b)
The reflection line bisects PQ and is perpendicular to it.



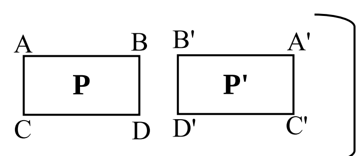
c) The areas of the original and image triangles are the same, 31.2 cm^2.
[$A = b \times h \div 2 = 6 \times 10.4 \div 2 = 31.2$ cm^2]

11. P' could be a reflection or a translation of P. [If the points in P and its image were labelled, it would be clear which transformation it is.

Translation:



Reflection:



Supporting Students

Struggling students

• If students are struggling with reflections that are neither horizontal nor vertical, for example, in **questions 4, 6, 7, 8, and 10**, you might encourage them to turn the page so that the reflection line looks horizontal or vertical. Doing this will help them visualize these reflections and will likely become unnecessary soon enough.

• Some students may have trouble visualizing reflections in any of the questions. You might encourage them to fold their page along the reflection line to see how the original shape points match up with the image points.

Enrichment

• For **question 9**, you might challenge students to extend by considering numbers that are palindromes. You might ask them to write properties of numbers that are palindromes (e.g., every digit must be 0, 1, or 8). Or, you might ask them if there are more palindromes that are even numbers or more that are odd, and ask them to explain their reasoning. (The answer to this is tricky because there are an infinite number of even-number palindromes and an infinite number of odd-number palindromes, but there are twice as many even-numbered palindromes as odd-numbered within a given range.)

• You might ask students which shapes can be tiled by doing reflections only, and which shapes cannot be tiled in this way (e.g. a rectangle can be tiled this way and a parallelogram cannot).

GAME: Reflection Archery

• This optional game helps students to better visualize reflections.

• They may choose to use informal tools (for example, their hands) to mark equal lengths. Using informal tools will help them to better use the formal tools (rulers, compasses, and so on) because they are developing their understanding about how each tool works and what it is used for.

• Encourage students to watch how their classmates find their reflection images. Ask them what they are thinking about. It is good for them to explain their reasoning in this way, and it is especially good for them to see how valuable their invented strategies can be. This will increase their confidence in their ability to do mathematics.

4.2.3 Rotations

Curriculum Outcomes	Outcome relevance
7-E5 Transformations: properties of translations, reflections, and rotations • use formal language: rotations for turns • emphasize what changes and what stays the same as a result of a transformation • investigate congruency and orientation in transformations • use tessellations as a context for transformations	By exploring transformations without the benefit of a grid, students will gain a deeper understanding of the properties of rotations other than quarter turns.

Pacing	Materials	Prerequisites
1.5 h	• Rulers • Protractors • Compasses • Tracing paper or transparencies (optional)	• familiarity with basic 2-D shapes • drawing and constructing 2-D shapes • using a protractor

Main Points to be Raised

• A rotation can be any angle.

• A rotation is described by its turn centre, the angle of rotation, and a direction, either clockwise (cw) or counterclockwise (ccw).

• To rotate a point, draw a line segment from the point to the centre of rotation, measure an angle of the stipulated size at the centre of rotation where one arm is the segment you drew, and use a compass to mark a point on the new arm the same distance as the original point from the centre of rotation. This is the image of the point.

• To rotate a polygon, you can rotate each of its vertices and then connect the images of the vertices.

• You can also rotate using tracing paper. You rotate the tracing to form the appropriate angle with the original side of the shape and mark the appropriate image point.

• The image of a rotated shape has the same orientation as and is congruent to the original shape.

• The only point that does not move as a result of a rotation is the centre of rotation.

• There is always more than one rotation that would result in the same image. You can add either 360° or a multiple of 360° and rotate in the same direction, or you can subtract from 360° and rotate in the opposite direction.

Try This — Introducing the Lesson

A. Place a table along a wall. Inform the students that you would like to move the table to the left (or to the right) but that it needs to stay against the wall. Ask how one person could move the table if it were very heavy. Demonstrate how to move the table by pivoting it on one of its legs.

Then ask students to work alone or with a partner on the **Try This**. While you observe students at work, you might ask questions such as the following:

• *Is there more than one path that works for moving the cupboard in this way?* (There are many ways, but because the cupboard is heavy we would want the shortest path.)

• *How do you know you have found the shortest path?* (The cupboard is always touching the wall, so I did not add any extra distance to the path.)

The Exposition — Presenting the Main Ideas

• Draw a triangle on the board and demonstrate how you would use a compass and protractor to rotate it 57º clockwise around one of the vertices.

• Have students turn to **page 129** to see how a similar procedure moved point R to R'.

• Use your rotation to show how you could measure the angle of rotation, with and without tracing paper.

• Ask a student to explain why one of the vertices did not move in the rotation.

• Ask students if the image is congruent to the original shape, and to explain how they know. Be sure that you use the formal terminology (e.g. *image*, *original shape*), but do not demand that students use it. You may point out the words that are considered to be more formal.

• Ask students if the image has the same orientation as the original shape, and to explain how they know.

• Have students read the bottom of **page 129** and observe how they could have used tracing paper to do the same rotation. If tracing paper is available, you may wish to have students try this.

• On the board, write the properties of rotations: the image is congruent and has the same orientation.

• Ask students if you could have used a different angle to get the same result. Show how 417º clockwise and 303º counterclockwise would give the same result.

• Inform students that some people use the word *anticlockwise* instead of *counterclockwise*. Both are correct, but *counterclockwise* is more common internationally (*counter* means *against*).

• When you write the degree of a rotation on the board, use the short forms cw and ccw, explaining what they stand for.

Revisiting the Try This

B. This question allows students to make a formal connection between what was done in **part A** and the main ideas presented in the exposition. In this case, they apply the terminology of rotations to their visualization from **part A**. For **part ii)**, there may be some argument about whether or not the counterclockwise turn is permissible because you cannot turn the cupboard through the wall. Make sure students understand that the rotation would give the same result even though it is not physically possible in this situation.

Using the Examples

• Have pairs of students read through **examples 1 and 2**. Inform them that you will choose a pair to come to the board to demonstrate each reflection. While they are reading, you could draw the triangle from **example 1** and the parallelogram from **example 2** on the board.

• After they have had enough time to read the examples carefully, ask a student pair to demonstrate the rotation of the shape in **example 1**. While one student does the rotation, have the other student explain what he or she is doing. Repeat this process for **example 2**.

Practising and Applying

Teaching points and tips

Q 1: Some students may choose to use tracing paper or transparencies, if they are available.

Q 3 c): This is an important generalization.

Q 5: Students should find the turn centre by inspection. One method to locate the turn centre is described below, but it requires the construction of a perpendicular bisector, which is learned next year. However, students may choose to draw a perpendicular bisector by measuring. Here is how:

1) Construct the perpendicular bisector of the segment that joins an image point to its original.
2) Repeat the above step for more than one set of corresponding points.
3) These perpendicular bisectors will intersect at the turn centre.

Q 6: Students should find the turn centre by inspection. (See the tip for **Q 5**.)

Q 8: You might point out that the bases and heights of the triangles did not change as a result of the rotations.

Q 9: When a shape rotates onto itself, it has turn symmetry.

Q 10: You might start the next day's class by drawing a triangle with different angles than this triangle and asking students to describe rotations that would have an image side coincide with a side of the original shape. Each time a student answers, you might ask him or her to explain how he or she determined the result.

Common errors

• Students will sometimes not be careful about whether their turn is clockwise or counterclockwise. Alert them to pay attention to this.

• Students will sometimes use the vertex being moved, rather than the turn centre, as the point at which the angle of rotation is drawn. Again, remind them that this angle is always at the turn centre.

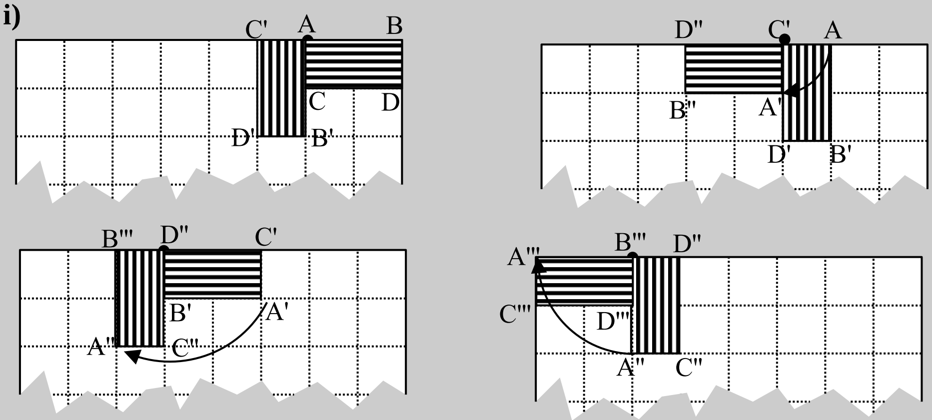
Suggested assessment questions from Practising and Applying

Question 1	to see if students can perform a rotation
Question 4	to see if students can apply rotations to a real-world situation
Question 5	to see if students can describe a rotation

Answers

A. *Sample response*:

i)



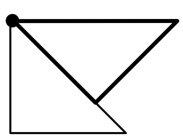
ii) A, C', D", and B'''

iii) The path of A is shown above using curved arrows. On the first turn, A does not move.

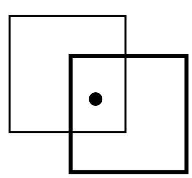
B. i) 90º cw around turn centre A.

ii) *Sample response*: 270º ccw around turn centre A.

1. a)



b)



2. a) 315° cw

b) 180° ccw

3. a) 290° cw

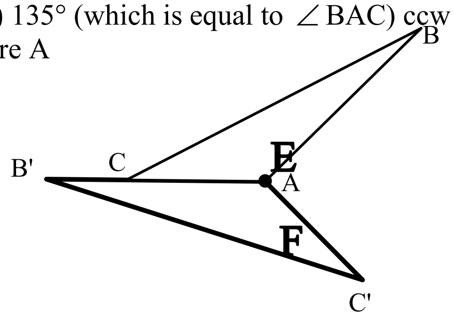
b) 245° cw

c) Subtract the angle from 360°

4. a) 90° cw

b) 180° cw

5. a) 135° (which is equal to $\angle$BAC) ccw around turn centre A

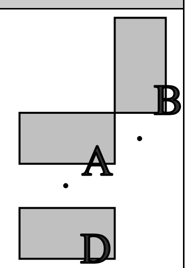


b) 225º cw around turn centre A

6. B is a rotation [of 90° cw (or 270° ccw) around the turn centre shown.]

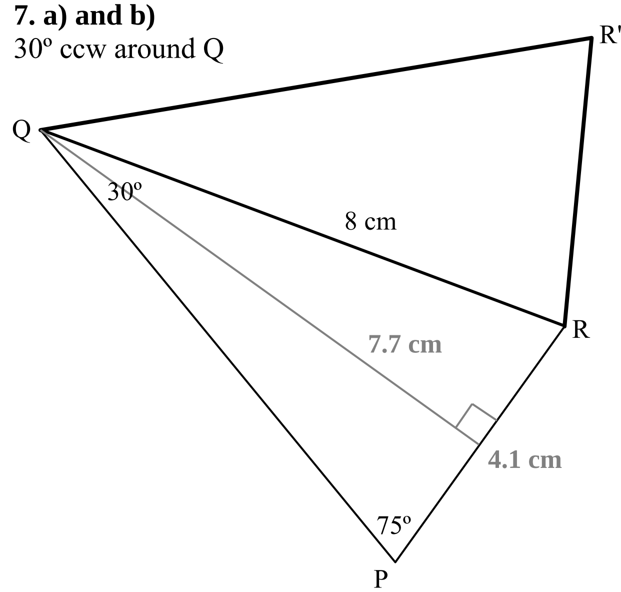
D is a rotation [of 180° cw (or ccw) around the turn centre shown.]

C cannot be a rotation [because it is not congruent to A.]



7. a) and b)

30º ccw around Q



c) The areas of the triangles are the same, 15.8 cm^2.

[$A = b \times h \div 2 = 4.1 \times 7.7 \div 2 = 15.8$ cm^2

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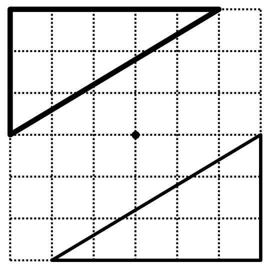
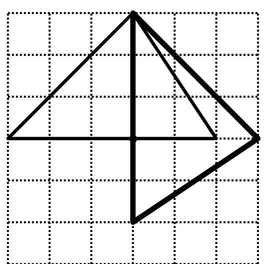
8. a), b), and c) *Sample response*:
7.5 square units; [$A = b \times h \div 2 = 5 \times 3 \div 2 = 7.5$ square units]
The triangle is rotated 180º cw around the turn centre shown.
7.5 square units; [$A = b \times h \div 2 = 5 \times 3 \div 2 = 7.5$ square units]



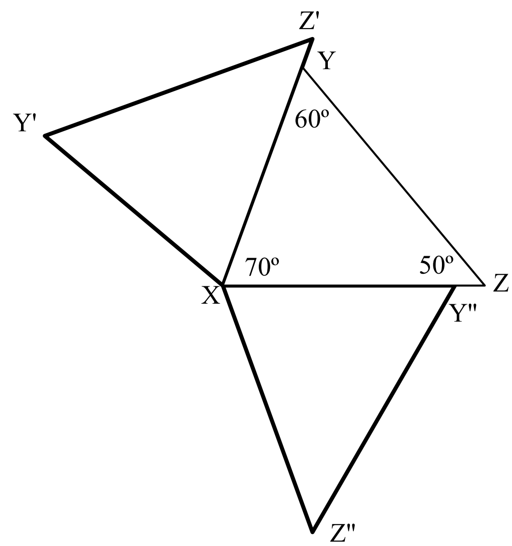
d) 7.5 square units; [$A = b \times h \div 2 = 5 \times 3 \div 2 = 7.5$ square units]
The triangle is rotated 90º cw around the turn centre shown.
7.5 square units; [$A = b \times h \div 2 = 5 \times 3 \div 2 = 7.5$ square units]



e) The area of the image is the same as the area of the original triangle [because rotation images are congruent.]

9. a) 180° or 270° cw, and 90°, 180°, or 270° ccw around the centre of the square
b) *Sample response*: circle, regular hexagon

10. If the image is ΔXY'Z', the rotation could have been:
• turn centre X and angle 70° ccw, or
• turn centre X and angle 290° cw.
If the image is ΔXY"Z", the rotation could have been
• turn centre X and angle 70° cw, or
• turn centre X and angle 290° ccw.



Supporting Students

Struggling students

• Of all the transformations, rotations seem to be the most difficult for students to visualize. It is important to spend lots of time on simple shapes so that students fully understand the process. If they continue to struggle with visualizing a rotation, they might trace and cut out the original shape and physically turn it.

• Some students might benefit from the use of tracing paper or transparencies for **questions 6 and 9**.

Enrichment

• You might ask students to make a tiling using rotations.

For example, if they take any quadrilateral, rotate it 180º around the midpoint of a side, and repeat the rotation in all directions, they will develop a tiling.

• They could also explore different starting shapes.

For example, they could try different quadrilaterals (it will work for all quadrilaterals).

• You might extend the **Try This** and ask students to explore challenges for furniture rotation.

For example, if the cupboard is supposed to move a little farther than it had to be moved in the given situation, it would not be possible for the cupboard to keep touching the wall. Students might invent different situations (with different furniture shapes and different lengths of required moves) and find ways of making the move with only pivots on corners.

4.3.1 Volume of a Rectangular Prism

Curriculum Outcomes	Outcome relevance
7-D2 Volume: **rectangular prisms** • relate volume to dimensions • understand that each of the three dimensions of a prism affects the volume	Rectangular prisms are common in our everyday lives. It is important that students be able to calculate their volumes to solve real-world problems.

Pacing	Materials	Prerequisites
1 h	• Linking cubes	• multiplying and factoring • formula for the area of a rectangle • familiarity with square units for area measurement

Main Points to be Raised

• The volume of an object describes how much space it takes up.

• You can find the volume of a rectangular prism by multiplying the area of the base by the height.

• It is arbitrary which face you consider to be the base when you calculate the volume in this way.

• You can find the volume of a rectangular prism by multiplying the length, width, and height.

• You can use cubes to demonstrate these volume calculations.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Make sure they realize that the diagram shown does not fully describe the situation in the problem; there would need to be more cubes in all three dimensions. While you observe students at work, you might ask questions such as the following:

• *Why does 7 × 10 tell you the number of cubes on the bottom of the box?* (There are 7 rows of cubes, with 10 cubes in each row.)

• *How did you find the total number of cubes that would fit in the box?* (70 × 4 because there are 70 cubes in each layer and there are 4 layers.)

The Exposition — Presenting the Main Ideas

• Ask students to tell about when they have measured things outside of school. Use their replies to demonstrate that people usually measure a thing so they can compare it to something else.

• Write on the board, "Volume describes how much space an object takes up".

• Remind students that volume is measured in cubic units because it has three dimensions. Ask them to recall a cubic centimetre and cubic metre. Point out that these are examples of cubic units that are commonly used.

• Use cubes to make a rectangle with dimensions 5 units by 4 units. Ask students how many cubes are in the rectangle, and how they know (i.e., Did they count all of them or did they multiply 4 × 5?).

• Ask two students to make two more rectangles like the rectangle described above. Have students watch as you place these other rectangles on top of the original rectangle. Ask students how many cubes there are in this shape, and how they know.

• On the board, write the formula for volume: V = Area of base × height. Indicate that this formula is for a rectangular prism.

• Ask a student to use your example with the stacked cubes to explain why the formula works.

• Point out to students that a cube is an example of a rectangular prism, so the formula also works for a cube.

• Note that the volumes you calculated were based on using a single cube as a unit. If that single cube were 1 cm^3, then the volume could be reported in cubic centimetres. Make sure they understand that the area of the base is $l \times w$, so the "area of base" part of the original formula can be replaced by $l \times w$.

• Direct students to **page 133** of the student text to see representations of cubic centimetres and cubic metres, as well as the process you just modelled for calculating volume in terms of layers.

• Have students turn to **page 134**. Ask how the two prisms shown are the same and how they are different.

• Have students calculate the volume of each prism. When they report their answers to the class, ask them to explain why the results are the same. Ask which face they used for the base of each rectangular prism. Make sure they understand that it is arbitrary (unimportant) which face is used as the base of a rectangular prism for calculating the volume.

• Write another formula for volume on the board underneath the first formula: $V = l \times w \times h$.

• Ask a student to use your example with the stacked cubes to explain why the formula works.

Revisiting the Try This

B. Students should apply one of the volume formulas they have learned.

Using the Examples

• Have students work in pairs. One student should become the expert on **example 1** and the other should become the expert on **example 2**. The pairs of students should teach each other about their examples.

• Follow up by asking how else they could have calculated the volume for **example 1**. Also ask why a factor pair was determined for **example 2** and what other prisms might have had the same volume.

Practising and Applying

Teaching points and tips

Q 1: Remind students that they can choose which face they want to think of as the base (many will choose to use one of the shaded faces). Thus their work will look different but they should get the same answer.

Q 2: You may need to show students how to sketch a rectangular prism.

Q 5: This is an important alternative formula. If you teach the lesson in the way described above, students will already have discussed this question, so you could omit this question. If they are working through the book without classroom interaction, this will be the first time they see this formula. **Question 3** is designed to guide students to discover this formula.

Q 6: Encourage students to use diagrams in their explanations.

Q 8: Remind students to estimate without calculating.

Q 9: Remind students to estimate without calculating, and to explain their thinking.

Q 11: You might encourage students to use diagrams in their answers.

Common errors

• Some students may have difficulty explaining their estimation methods as required in **questions 8 and 9**. Encourage them by saying that most explanations do not need specialized vocabulary. They can explain in the way they would tell a friend what they have done.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can calculate the volume of a prism
Question 3	to see if students can calculate volume and find missing dimensions
Question 10	to see if students understand how to use the volume formula in a variety of ways

Answers

Note: Answers in the teacher guide do not follow the rules for significant digits because students have not yet learned these. For this reason, it is likely that their answers will be slightly different from the answers in the teacher guide. If the method is correct and the answer is close, it should be considered correct.

A. i) 70 **ii)** 4 layers **iii)** 280

B. The area of the base is $7 \times 10 = 70$ cm^2.
Volume = Area of base × height = $70 \times 4 = 280$ cm^3

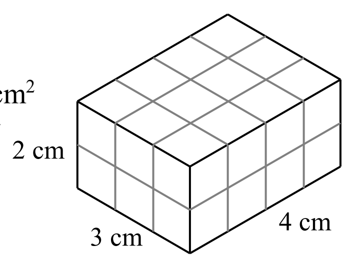
1. a) 24 m^3; [*Sample response*:
Base area: $A = 2 \times 4 = 8$ m^2
V = *base area* × *height* = $8 \times 3 = 24$ m^3]
b) 1260 cm^3; [*Sample response*:
Base area: $A = 10 \times 14 = 140$ cm^2
V = *base area* × *height* = $140 \times 9 = 1260$ cm^3]
c) 1200 mm^3; [*Sample response*:
Base area: $A = 6 \times 10 = 60$ mm^2
V = *base area* × *height* = $60 \times 20 = 1200$ mm^3]

2. a) 125 mm^3; [Base area: $A = 5 \times 5 = 25$ mm^2
V = *base area* × *height* = $25 \times 5 = 125$ mm^3]
b) 9 m^3; [Base area: $A = 3 \times 1 = 3$ m^2
V = *base area* × *height* = $3 \times 3 = 9$ m^3]
c) 9 m^3; [Base area: $A = 3 \times 3 = 9$ m^2
V = *base area* × *height* = $9 \times 1 = 9$ m^3]
d) 48 cm^3; [Base area: $A = 4 \times 2 = 8$ cm^2
V = *base area* × *height* = $8 \times 6 = 48$ cm^3]

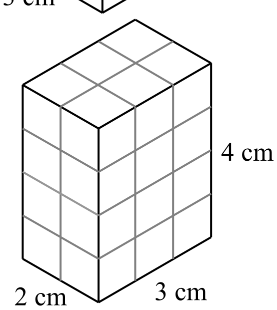
3.

	Length (cm)	**Width (cm)**	**Height (cm)**	**Volume (cm^3)**
a)	5	3	2	**30**
b)	10	6	4	**240**
c)	**4**	4	4	64
d)	5	**8**	6	240
e)	4	5	**3**	60

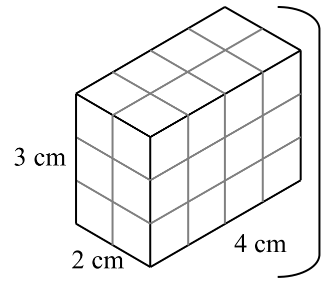
[**4.** *Sample response*:
Height = 2 cm
Base area = $3 \times 4 = 12$ cm^2
V = *base area* × *height*
= $12 \times 2 = 24$ cm^3



Height = 3 cm
Base area = $2 \times 4 = 8$ cm^2
V = *base area* × *height*
= $8 \times 3 = 24$ cm^3



4. [Cont'd]
Height = 4 cm
Base area = $2 \times 3 = 6$ cm^2
V = *base area* × *height*
= $6 \times 4 = 24$ cm^3



5. Yes; [The area of the base is the length of the base times the width of the base so all you do is replace *base area* in the formula with $l \times w$:
V = *base area* × *height* = $l \times w \times h$]

6. *Sample responses*:
a) 3 cm × 2 cm × 10 cm
[**b)** There are many sets of three whole number factors that multiply to 60; 3 cm × 4 cm × 5 cm, 2 cm × 6 cm × 5 cm, 15 cm × 1 cm × 4 cm.]

7. *Sample responses*:
a) First way: 4 boxes wide (8 cm each) × 5 boxes deep (12 cm each) × 6 boxes high (25 cm each)
Second way: 4 boxes wide (8 cm each) × 6 boxes deep (12 cm each) × 5 boxes high (25 cm each)
Third way: 5 boxes wide (8 cm each) × 8 boxes deep (12 cm each) × 3 boxes high (25 cm each)
b) First way: 32 cm × 60 cm × 150 cm, V = 288,000 cm^3
Second way: 32 cm × 72 cm × 125 cm, V = 288,000 cm^3
Third way C: 40 cm × 96 cm × 75 cm, V = 288,000 cm^3

8. a) *Sample response*:
Estimate: Box A: about 100,000 cm^3; [$50 \times 20 \times 100$ = 100,000 cm^3]
Estimate: Box B: about 125,000 cm^3; [$50 \times 50 \times 50$ = 125,000 cm^3]
Box B probably has the greater volume.
b) Box A: 87,768 cm^3; [$53 \times 18 \times 92$ = 87,768 cm^3]
Box B: 132,651 cm^3; [$51 \times 51 \times 51$ = 132,651 cm^3]
Box B has the greater volume.

[**9.** *Sample response*:
Even if you round each dimension down to estimate the volume, the volume is still more than 23,000 cm^3.
The base is about 30 cm × 20 cm = 600 cm^2.
Estimate: V = *base area* × *height* = 600×40 = 24,000 cm^3]

10. Yes; [because you can multiply the two dimensions to get the area of a base and then divide the volume by this area to find the height.]	[**11.** *Sample response*: Volume is the area of the base × height, so there are two things that affect volume, the area of the base and the height. The area of the base of the tall prism could be much less than the area of the base of the shorter prism and the height of the shorter prism might be just a bit shorter.]

Supporting Students

Struggling students

• If students are struggling with visualizing prism dimensions, you might encourage them to manipulate linking cubes or other small cubes to help them with their visualization.

• For **question 3 parts c), d), and e)**, students could use small cubes to help them understand their calculations.

For example, in **part c)**, they could make 4 × 4 rectangles until they have used up 64 cubes. They will see that they can make four of these 4-by-4 rectangles, which they can then stack on top of each other to make a 4-by-4-by-4 cube.

Enrichment

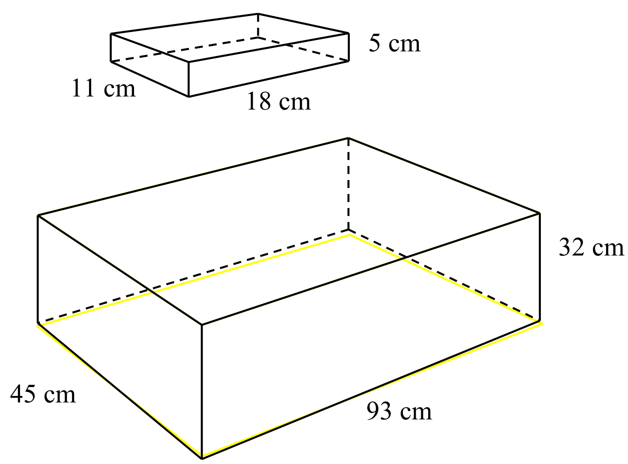
• For **question 7**, ask students how Chhimi could stack the boxes so that the overall shape would be the most like a cube? (The result will depend on the criteria students use to decide what shape looks enough like a cube.)

• Here is an additional problem you could use to challenge students:

Rinzin is storing boxes of tea in a cupboard.

• Each box is 5 cm tall with an 18 cm-by-11 cm base.

• The cupboard is 93 cm wide, 45 cm deep, and 32 cm tall.



a) Devi says that you can figure out how many boxes will fit into the cupboard by dividing the volume of the cupboard by the volume of a tea box. What is wrong with his thinking?

b) For each possible arrangement of tea boxes, calculate the number that will fit in the cupboard. Show your work.

i) with the 18 cm-by-5 cm face facing the front

ii) with the 11 cm-by-5 cm face facing the front

c) Calculate using Devi's method from **part a)**.
How does his answer compare with what you found out in **part b)**?

Sample answer:
a) Devi's idea only works if you are storing a commodity like rice or flour, which takes the shape of the container. Because the tea boxes have a certain set of dimensions, there may be spaces left over after you pack them into the cupboard, so you are not using the whole volume of the cupboard.
b) i) Along the front: 93 ÷ 18 = 5 R 3, so 5 boxes will fit, with a 3 cm space on the side.
Depth: 45 ÷ 11 = 4 R 1, so 4 boxes will fit, with a 1 cm space in front.
Number of boxes = 5 × 4 = 20
ii) Along the front: 93 ÷ 11 = 8 R 5, so 8 boxes will fit, with a 5 cm space on the side.
Depth: 45 ÷ 18 = 2 R 5, 2 boxes will fit, with a 5 cm space in front.
Number of boxes = 5 × 2 = 10
c) Volume of a tea box = 20 × 10 × 5 = 1000 cm^2
Estimated volume of the cupboard = 50 × 30 × 90 = 135,000 cm^3
135,000 ÷ 1000 = 135 boxes
The most that will actually fit is 20 boxes, which is a lot fewer than 135.

4.3.2 Measurement Units

Curriculum Outcomes	Outcome relevance
7-D3 SI Units: **identify, use, and convert** • identify, use, and convert SI units to measure, estimate, and solve problems • understand the approximate nature of measurement • examine milli, centi, deci, deca, hecto, and kilo as prefixes for measures of length, mass, and capacity • apply principles of conversion using common units (relate the size of a number to the size of the unit) • establish the link between volume, capacity, and mass	Measurement is an important skill for everyday life and for further work in mathematics. Students need to know how to convert between units in many everyday situations.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM)	• familiarity with units of measure including metres, litres, and grams • familiarity with simple conversions using prefixes. • multiplying and dividing by powers of ten

Main Points to be Raised

• We use different measurement units so that the numbers that describe the units are reasonable in size.

• To convert units in the metric system, you multiply or divide the measurement by a power of ten.

• To convert to a larger unit, you divide. To convert to a smaller unit, you multiply.

• The factor by which you multiply or divide is based on the two prefixes you use.

• For square units, two linear dimensions are involved, so if the prefixes indicate a conversion factor of 100, the factor will actually be 100 × 100 (10,000).

• For cubic units, three linear dimensions are involved, so if the prefixes indicate a conversion factor of 10,
the factor will actually be 10 × 10 × 10 (1000).

• Volume and capacity units are related: 1 cm^3 = 1 mL.

• 1 mL of water has a mass of 1 g.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Encourage them to use grid paper to help with their reasoning. They could outline the rectangle that contains the 200 squares (10 tiles by 20 tiles). While you observe students at work, you might ask questions such as the following:

• *How much area does a 10 cm × 10 cm tile take up?* (100 cm^2)

• *How much area does a 20 cm × 20 cm tile take up?* (400 cm^2)

• *How do you know when you have explained* ***part iv)*** *sufficiently?* (When my partner understands what I am saying, I have given a good explanation.)

• If students try to use unit conversions incorrectly (e.g., 2 m^2 = 2 × 100 cm^2 = 200 cm^2), ask them to do their unit conversions on the lengths, not on the area.

The Exposition — Presenting the Main Ideas

• Give a local example of directions like those given at the beginning of the exposition on **page 137**. This will show students why it is important to have different units for the same attribute.

• On the board, copy the first chart on **page 137**, starting with only the headings (Prefix, Symbol, …).

Fill in the base unit on the chart, show an example, and then write two rows above it under Meaning ($\frac{1}{100}$ of the base unit). Ask students if they know the other parts of the row. Continue this with the other rows until you have exhausted what students recall. Then fill in the rest of the chart, explaining what you are writing.

• Write the meanings of the special units, hectare and tonne, and tell students some examples of how they are used.

• Use drawings like those on **page 138** of the student text to explain why area and volume conversions have different factors than their linear equivalents.

• On the board, write the relationship between capacity and volume and the relationship for the mass of water. Explain that these relationships are what give the base units their meaning. These units are not like the body-based units that are common in most cultures (e.g., the *tho* in Bhutan and the *span* in the United Kingdom).

• Ask students to open their texts and look at the step chart on **page 138**. Invite them to use this chart whenever they need to. Explain why the chart makes sense. If you are using a smaller unit, then it takes more of them to make a given amount, so you multiply. If you are using a larger unit, then it takes fewer of them to make a given amount, so you divide. Demonstrate how to use the step chart by doing the example under it (or a similar example) on the board.

Revisiting the Try This

B. Students apply their understanding of unit conversions for area to the problem they solved in **part A**.

Using the Examples

• Work through **example 1** with the students to make sure they understand it. Sketch the unit conversion steps and use them to work on the example.

• Assign students to pairs and have them look at **example 2**. Ask them to decide what would be different in the calculations if the container were 1 m wide instead of 1.5 m wide. The purpose of this task is to get them to read the example with understanding and to report back in a way that helps you know whether they understand.

• After they have had time to consider the example in their pairs, ask students to explain what would be different.

Practising and Applying

Teaching points and tips

Q 1 and 2: You might encourage students to sketch unit conversion step charts to do help them think about these questions.

Q 3 and 4: Because the answers to these questions will vary according to students' experiences, it would be worthwhile to have students share their answers with their classmates. After students have finished their work, and while they are doing other work, you might write each question on a separate piece of paper and have students pass them around. Ask the students to write their answers on this paper. When they have all finished, you could post the papers on the wall.

Q 5: Remind students to be careful about units. One dimension is in mm and the others are in cm.

Q 6: Students need to use some algebra skills to find the missing linear dimension in the formula.

Q 7: Some students may struggle with this question because they will not be able to plan their entire work before starting. You might ask struggling students first to make any rectangular container that holds 6 kg and then to modify the dimensions until it fits all the criteria.

Common errors

• Students often do the opposite conversion in questions like **questions 1 and 2**.

For example, they might multiply by 10 when they should divide by 10.

You might encourage them to think about the reasonableness of their answers.

For example, in **question 1 a)**, 0.3 cm is less than one centimetre. *Centi* means 100, so this will involve multiplying by 100 or dividing by 100. If you multiply by 100, the result is 300 cm, which is more than a metre. The length does not change, so it cannot be both less than a metre and more than a metre.

• Many students will forget to convert 5 mm to 0.5 cm in **question 5**. Remind them to think about reasonableness. How could the height be 5 cm if the length is also 5 cm? Look at how different they are.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can convert metric units
Question 5	to see if students understand the link between cubic units and capacity
Question 8	to see if students can use SI units to solve problems

Answers

A. i) 20,000 cm^2 **ii)** 20,000 cm^2 **iii)** 40,000 cm^2
iv) Doubling the dimensions of a tile multiplies the area by four, not by two, because it doubles both dimensions. That means he needs only fifty 20 cm tiles.

B. 20,000 cm^2 = 2 m^2 (10,000 cm^2 in 1 m^2);
1 m × 2 m = 2 m^2 and 100 cm × 200 cm = 20,000 cm^2.

1. a) 0.003 m **b)** 520 L
c) 3000 mg **d)** 42,000 dm
e) 40.7 mm^2 **f)** 0.0054 m^3
g) 1 L; [1000 cm^3 = 1000 mL = 1 L]
h) 40 ha; [40 hm^2 = 40 ha]
i) 1.5 kg
j) 1 m^3; [1000 kg = 1,000,000 g which is 1,000,000 cm^3 = 1 m^3]

2. a) Divide by 10,000 **b)** Multiply by 10,000
c) Multiply by 10 **d)** Multiply by 1,000,000
e) Divide by 100 **f)** Multiply by 10,000

3. *Sample responses*:
a) mL
b) ha
c) kg

4. a) 1.4 g; *Sample response*: A pencil
b) 5.4 km; *Sample response*: A distance along a road
c) 5400 cm^3; *Sample response*: A small sack of rice
d) 5 mm^2; *Sample response*: The area of the top of a push pin
e) 2 L; *Sample response*: The capacity of a jug
f) 35 g; *Sample response*: The mass of a roll of tape

5. a) 7.5 mL; [V = 0.5 cm × 3 cm × 5 cm = 7.5 cm^3 and 7.5 cm^3 = 7.5 mL]
b) 2.5 g; [7.5 mL of water = 7.5 g and 10 – 7.5 = 2.5 g]

6. 7 mm; [1 kg = 1000 g and 1000g of water = 1000 cm^3
$V = 36 \times 40 \times h = 1000\ \text{cm}^3$
$36 \times 40 \times h = 1000\ \text{cm}^{3;}$
$1440 \times h = 1000\ \text{cm}^3$
$h = 1000 \div 1440 = 0.7$ cm = 7 mm]

7. *Sample response*: 20 × 15 × 20 cm
[6 kg = 6000 g and 6000 g of water = 6000 mL = 6000 cm^3
20 × 15 × 20 cm = 6000 cm^3]

8. More; [Area of field: 1 ha = 1 hm^2 =
1 hm × 1 hm = 100 m × 100 m
Depth: 1 cm = 0.01 m
Volume of water: V = 0.01 m × 100 m × 100 m = 100 m^3
The mass of 1 m^3 of water is 1 t, so 100 m^3 has a mass of 100 t.]

9. a) 900 mm; [6 th = 6 × 15 cm = 90 cm = 900 mm]
b) 9 m; [60 th = 60 × 15 cm = 900 cm = 9 m]
c) 225 cm^2; [1 th^2 = 1 th × 1 th = 15 cm × 15 cm = 225 cm^2]
d) 400 th^2; [Because 1 th = 15 cm, 20 th = 3 m;
9 m^2 = 3 m × 3 m = 20 th × 20 th = 400 th^2]

10. a) The number of units used for the measurement becomes smaller [because you need fewer of them.]
b) The number of units used for the measurement becomes greater [because you need more of them.]

Supporting Students

Struggling students

• If students are struggling to connect the measurements with everyday items in **questions 3 and 4**, you might suggest that they model the lengths, areas, and volumes with classroom objects and convert capacity and mass to their volume equivalents for water. Then they can think about objects that are similar in size.

• Some students might benefit from using the step chart for **questions 1 to 4**. Allow them to use it, but remind them that they will want to work toward doing conversions without the chart. They will not have the chart with them when they are not in school.

Enrichment

• Relating to **question 9**, you might ask students to measure in centimetres other traditional Bhutanese measures based on body parts, then to measure various things with the traditional measure, and finally to convert those measurements to centimetres.

• Also relating to **question 9**, you might give students other conversion factors for comparing measurements in feet and inches to metric measures, and for comparing pounds to kilograms (1 inch is about 2.54 cm, 1 foot is about 30.48 cm, 1 yard is about 91.44 cm, and 1 kg is about 2.205 pounds).

4.3.3 Area of a Composite Shape

Curriculum Outcomes	Outcome relevance
7-D4 Area: **composite shapes** • estimate and calculate the area of shapes on grids • understand that composite shapes can be broken down into familiar shapes for which there are area formulas available	Students have found the areas of irregular shapes on dot paper and grid paper by counting. The ability to calculate the areas of composite shapes will help them not only with irregular shapes, but also in understanding the formula for the area of a trapezoid.

Pacing	Materials	Prerequisites
1 h	• Square Dot Grid Paper (BLM) • Rulers	• applying area formulas for rectangles and triangles.

Main Points to be Raised

• You can think of a composite shape as a combination of simpler shapes.

• You can find the area of a composite shape by adding up the areas of the simpler shapes that make up the larger shape.

• Sometimes you can find the area of a composite shape by embedding it in a larger, simpler shape and subtracting the excess area.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Are you using any short cuts to speed up your counting?* (Yes, I skip-counted by counting pairs of squares. I also multiplied to find the number of squares in this rectangle, which is part of the larger shape.)

• *How are you counting the squares that are not completely enclosed by the shape?* (These two squares are each divided in half by this diagonal line, so together they make one square inside the shape here.)

• If students count incorrectly, you might explain that this is not surprising. Miscounts are expected when there are so many things to count. In fact, this is why it is important to develop strategies that make it possible to avoid too much counting.

The Exposition — Presenting the Main Ideas

• Ask students how people in your dzongkhag usually describe the area of a rice paddy.

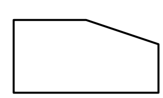
For example, if someone divides up an area amongst his or her children or sells an area, how do they talk about the size of the rice paddy?

Ask what are the advantages of the ways they mention and of methods that give a measurement in square units. Students may also describe practices from other dzongkhags.

• On the board, write “composite shapes”. Explain that the word *composite* is the root word of *composition*, which means putting things together. In the same way, the word *composite* is used to describe something that is made by putting together parts.

• Explain that to find the area of an irregular shape, you can think of it as a composite shape. You can divide a shape into parts for which you know formulas — rectangles and triangles work very well when the composite shape is a polygon.

• Draw a simple irregular shape like the one shown to the right. Show students how you can think of it as two rectangles and a triangle. Then show how you can also think of it as a large rectangle with a triangle cut out of it.



• Make sure students understand that there are multiple ways of dividing up or embedding any composite shape.

Revisiting the Try This

B. Observe whether students divide the shape into a rectangle and two triangles or whether they use a larger rectangle and cut out the area of two triangles. Either way is correct.

Using the Examples

• Work through **solution 1** of **example 1** with the students to make sure they understand it.

• Assign students to pairs. Ask them to decide which **solution**, **1 or 2**, they find easier for the question. After they have had time to decide, ask pairs of students to present their arguments to the class.

• Then ask them how they could decide in advance which approach to take for any given composite shape – adding together shapes inside the composite shape or subtracting shapes around the outside of the composite shape. Make it clear that if they struggle with one approach they can try the other.

Practising and Applying

Teaching points and tips

Q 1 c): This part is particularly challenging because it is not easy to visualize the shape as a composition of triangles and rectangles. It is easier to see it as a large rectangle with triangles cut out of three of its corners.

Q 2 b): If students visualize this as a rectangle with a triangle on top of it, they will have to calculate the length of the triangle's base. It is 7 – 4, using dimensions from the rectangle beneath it. They also need to see that the height of the triangle is 8 – 4. The 5 cm edge is unnecessary information to answer the question.

Q 4 and 5: There are many solutions to each of these questions. You may think that a simple solution suggests that a student lacks ingenuity, but for students to be able to visualize the simplest approach they need to understand areas and compositions very well.

Q 5: You may have to help students understand that they need to mark off a shape inside the grid that leaves 25 squares as part of the park.

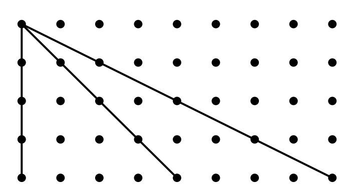
Q 6: This is a very important way of understanding the formula for the area of a parallelogram. It relates closely to work on trapezoids in the next lesson.

Common errors

• Many students will forget to divide by two when they calculate the areas of some triangles. This is especially likely if they are not showing their work. Remind them that this is the reason that you expect them to show their work.

• Some students will sometimes count lengths incorrectly on dot paper. They often count the dots instead of counting the spaces between the dots. You might help them visualize the spaces by moving a pencil tip from one dot to the next and counting the number of jumps for each successive dot.

• Another way that students count incorrectly on dot paper sometimes occurs when they want to find the length of a diagonal line segment. If they count the spaces between the dots, the length will not be correct because they are counting diagonals. You might show that this cannot be correct by making a sketch like the one at right. Ask which line is the longest and which is the shortest. Then show that they each have four spaces between dots when you follow along each line.



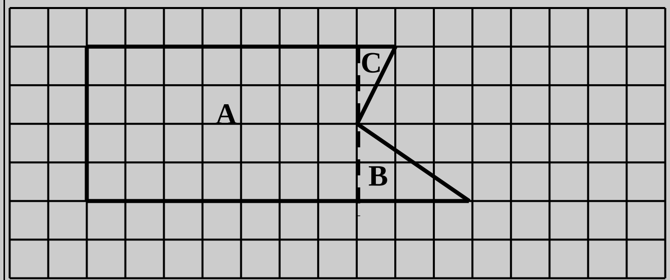
Suggested assessment questions from Practising and Applying

Question 2	to see if students can find the area of a composite shape
Question 4	to see if students can solve problems involving composite shapes
Question 6	to see if students can visualize the components of a figure

Answers

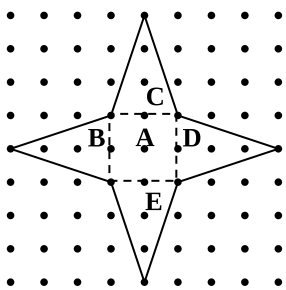
A. 32 m^2;
Sample response:
A rectangle of 40 squares – 4 whole squares –
$\frac{1}{2}$ of a 2-by-1 rectangle – $\frac{1}{2}$ of a 2-by-3 rectangle =
40 – 4 – 1 – 3 = 40 – 8 = 32

B.

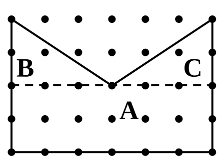


A: $A = l \times w = 7 \times 4 = 28$ m^2
B: $A = b \times h \div 2 = 3 \times 2 \div 2 = 3$ m^2
C: $A = b \times h \div 2 = 1 \times 2 \div 2 = 1$ m^2
Total Area = 28 + 3 + 1 = 32 m^2

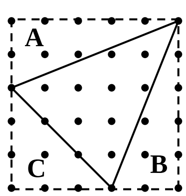
1. a) 16 cm^2; [Four congruent triangles (B, C, D, E):
$A = b \times h \div 2 = 2 \times 3 \div 2 = 3$
Square in middle (A): $A = b \times h = 2 \times 2 = 4$
Total area = 4 × 3 + 4 = 16 cm^2]



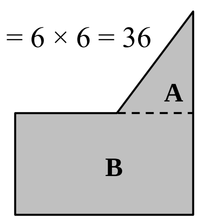
b) 18 cm^2; [Each congruent triangle (B, C):
$A = b \times h \div 2 = 2 \times 3 \div 2 = 3$
Rectangle (A): $A = b \times h = 2 \times 6 = 12$
Total area = 2 × 3 + 12 = 18 cm^2]



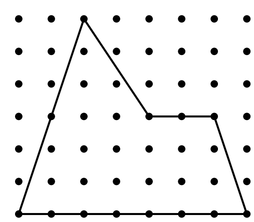
c) 10.5 cm^2; Outside square: $A = b \times h = 5 \times 5 = 25$
Top left triangle (A): $A = b \times h \div 2 = 2 \times 5 \div 2 = 5$
Bottom left triangle (C): $A = b \times h \div 2 = 3 \times 3 \div 2 = 4.5$
Bottom right triangle (B): $A = b \times h \div 2 = 2 \times 5 \div 2 = 5$
Total area = 25 – 5 – 5 – 4.5 = 10.5 cm^2]



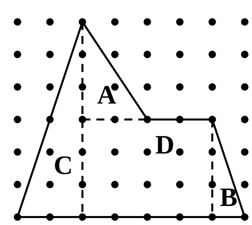
2. a) 24 m^2; [Outside rectangle: $A = b \times h = 5 \times 9 = 45$
Inside rectangle: $A = b \times h = 7 \times 3 = 21$
Total Area = 45 – 21 = 24 m^2]
b) 34 cm^2; [Rectangle (B): $A = b \times h = 7 \times 4 = 28$
Triangle (A): $A = b \times h \div 2 = 3 \times 4 \div 2 = 6$
Total Area = 28 + 6 = 34 cm^2]
c) 27 cm^2; [Large square: $A = b \times h = 6 \times 6 = 36$
Small square: $A = b \times h = 3 \times 3 = 9$
Total Area = 36 – 9 = 27 cm^2]



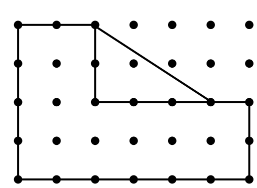
3. a) *Sample response*:



Area: 22.5 cm^2
[Triangle A: $A = b \times h \div 2 = 2 \times 3 \div 2 = 3$
Triangle B: $A = b \times h \div 2 = 1 \times 3 \div 2 = 1.5$
Triangle C: $A = b \times h \div 2 = 2 \times 6 \div 2 = 6$
Rectangle D: $A = b \times h = 4 \times 3 = 12$
Total area: $A = 3 + 1.5 + 6 + 12 = 22.5$ cm^2]

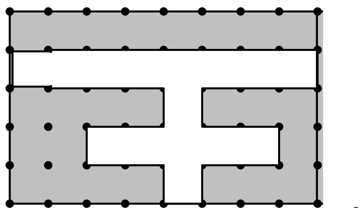


4. *Sample response*:

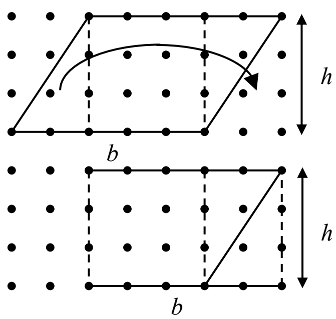


Answers [Continued]

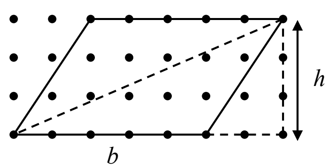
5. *Sample response*:



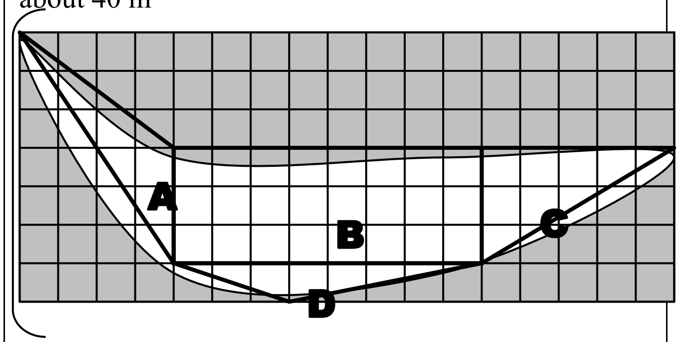
[**6. a)** Translate one of the triangles to the opposite side to make a rectangle with the same base and height as the parallelogram. The area of the rectangle is $A = b \times h$. The parallelogram covers the same area.



b) The two triangles are congruent. Each triangle has the same base and height as the parallelogram. The area of one triangle is $A = b \times h \div 2$ so the area of both is double that: $A = b \times h$. The parallelogram covers the same area.



7. *Sample response*:
about 40 m^2



A: $A = b \times h \div 2 = 3 \times 4 \div 2 = 6$
B: $A = b \times h = 3 \times 8 = 24$
C: $A = b \times h \div 2 = 3 \times 5 \div 2 = 7.5$
D: $A = b \times h \div 2 = 1 \times 8 \div 2 = 4$
Estimate: Total Area = 6 + 24 + 7.5 + 4 = 41.5 m^2
The area is about 40 m^2.]

Supporting Students

Struggling students

• If students are struggling with visualizing the composition of shapes involved in a situation, remind them of the importance of sketching the shape and drawing lines to show the outlines of the shapes they visualize.

• Some students may have trouble organizing their work. You might encourage them to label each region with letters or numbers. They can refer to these labels in their calculations.

• Some students may struggle with finding the area of triangles that are not right triangles, for example, in **question 1 c)**. You might suggest that they imagine sliding the top vertex along a line parallel to the base. This kind of sliding does not change the area because neither the height nor the base changes in length.

Enrichment

• If you have a tile floor, you might extend **question 7** by drawing two curvy shapes on the floor with chalk (or spilling some liquid in two places) and asking the students to figure out which area covers more of the floor.

4.3.4 Area of a Trapezoid

Curriculum Outcomes	Outcome relevance
7-D4 Area: **composite shapes** • develop and apply the formula for the area of a trapezoid	When they see the trapezoid as a special case of a composite shape, students will be able to develop formulas for any other less common shape they encounter outside of school.

Pacing	Materials	Prerequisites
1 h	• Square Dot Grid Paper (BLM) • Rulers	• applying the area formulas for a rectangle, a parallelogram, and a triangle

Main Points to be Raised

• You can find the area of a trapezoid by multiplying the height by the sum of the base lengths and then dividing by two.

• A trapezoid can be seen as half a parallelogram with the same height, with bases that are equal to the sum of the two bases of the trapezoid.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How many ways could you divide the polygon into two triangles?* (Two)

• *Are there other ways you could divide up the polygon?* (I can think of it as four right triangles by dividing the rectangle in the middle into two right triangles.)

• For **part i)**, do not discourage students from dividing the trapezoid into parts in a way that is different from how you would divide it. This is fine as long as students answer the questions correctly.

The Exposition — Presenting the Main Ideas

• Introduce the definition of a trapezoid by drawing an example on the board and pointing out the key characteristics.

• Sketch an isosceles trapezoid and explain its special characteristic.

• Make a trapezoid out of a piece of paper by cutting off two corners. Trace it on the board and then trace its image after a rotation of 180º around the midpoint of one of the non-parallel sides (like the example at the end of the exposition). Use this diagram to explain to students why the formula makes sense.

• Draw students' attention to **page 144** to see another example of how a rotated trapezoid combines with the original to create a parallelogram. Make sure students note the formula for the area of a trapezoid that is recorded on the page.

Revisiting the Try This

B. Students can now calculate the area in **part A** more directly using the formula they have learned.

Using the Examples

• Have students work in pairs. Assign half the pairs to become experts on **example 1** and the other half to become experts on **example 2**.

• Have each student discuss his or her example with his or her partner so that they both understand. Next, have each pair separate and ask each student to form a pair with someone who was an expert on the other example.

• In their new pairings, each student should explain to the other student the example he or she is expert on. Encourage them to explain without looking in the student text.

• When they have done their expert sharing, ask the whole class which example shows more challenging work. Ask the students to justify their choices by explaining why they think one is more difficult than the other.

Practising and Applying

Teaching points and tips

Q 2: Shape D is unlike the other shapes because you cannot easily use the trapezoid formula. The bases and height are diagonals, so their lengths are not integers and cannot easily be read on the diagram. Students can find the area by counting squares.

Q 4: This way of dividing up a trapezoid connects to another good way to think about the area of a trapezoid formula. This will be developed in **question 8**.

Q 5: This is a composite shape that is divided into two trapezoids instead of into triangles like in **question 4**.

Q 7: It is hard to see the trapezoid in the picture. It is outlined in white. When people experience a trapezoid outside of mathematics class, they often will not recognize it as a trapezoid unless they are looking for the trapezoid, so this picture is like a real-world situation.

Q 8: You might work through this question with the whole class as a follow-up to **question 4**.

Common errors

• Some students will count lengths incorrectly on dot paper. They may count the dots instead of counting the spaces between the dots. You might help them visualize the spaces by moving a pencil tip from one dot to the next, counting the number of jumps for each successive dot.

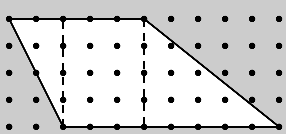
• Some students will have difficulty identifying the bases of the trapezoids, especially in **questions 1 d) and 3 b)**. This is a new way of thinking about what a base is. You might ask them to turn the book or paper until the parallel sides face them.

Suggested assessment questions from Practising and Applying

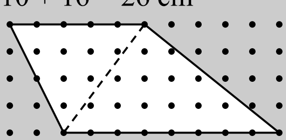
Question 3	to see if students can find the area of a trapezoid
Question 6	to see if students can find a dimension of a trapezoid, given the area

Answers

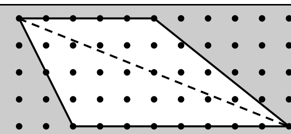
A. i) Triangle on the left: $A = b \times h \div 2 = 2 \times 4 \div 2 = 4$
Rectangle: $A = b \times h = 3 \times 4 = 12$
Triangle on the right: $A = b \times h \div 2 = 5 \times 4 \div 2 = 10$
Total Area = 4 + 12 + 10 = 26 cm^2



ii) Triangle on the left: $A = b \times h \div 2 = 5 \times 4 \div 2 = 10$
Triangle on the right: $A = b \times h \div 2 = 8 \times 4 \div 2 = 16$
Total Area = 10 + 16 = 26 cm^2



A. ii)



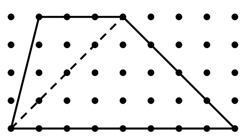
B. i) There are exactly two parallel sides.
ii) $A = (a + b) \times h \div 2 = (8 + 5) \times 4 \div 2 = 13 \times 4 \div 2 = 52 \div 2 = 26$ cm^2

1. They are all trapezoids; A and D are isosceles trapezoids.

2. A: 8 square units; [$A = (a + b) \times h \div 2 = (6 + 2) \times 2 \div 2 = 8 \times 2 \div 2 = 16 \div 2 = 8$ square units]
B: 7 square units; [$A = (a + b) \times h \div 2 = (4 + 3) \times 2 \div 2 = 7 \times 2 \div 2 = 14 \div 2 = 7$ square units]
C: 7.5 square units; [$A = (a + b) \times h \div 2 = (4 + 1) \times 3 \div 2 = 5 \times 3 \div 2 = 15 \div 2 = 7.5$ square units]
D: 4 square units; [I divided it into a right triangle with area 2 square units and a parallelogram with area 2 square units, which is 4 square units.]

3. a) 18 cm^2; $A = (a + b) \times h \div 2 = (3 + 6) \times 4 \div 2 = 9 \times 4 \div 2 = 36 \div 2 = 18$ cm^2.]
b) 36 cm^2; $A = (a + b) \times h \div 2 = (9 + 3) \times 6 \div 2 = 12 \times 6 \div 2 = 72 \div 2 = 36$ cm^2.]

4. a) 22 cm^2; [$A = (a + b) \times h \div 2 = (3 + 8) \times 4 \div 2 = 11 \times 4 \div 2 = 44 \div 2 = 22$ cm^2]

b) 

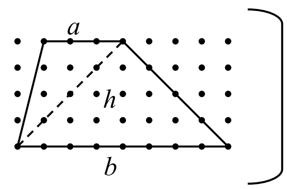
c) Triangle on the left: 6 cm^2; [$A = b \times h \div 2 = 3 \times 4 \div 2 = 12 \div 2 = 6$ cm^2]
Triangle on the right: 16 cm^2; [$A = b \times h \div 2 = 8 \times 4 \div 2 = 32 \div 2 = 16$ cm^2]
d) The triangles have the same height (which is the same as the height of the trapezoid). They have different bases.

5. 42 m^2; [The hexagon is made up of two congruent trapezoids. The height of each trapezoid is half of 7 m = 3.5 m.
Trapezoid: $A = (a + b) \times h \div 2 = (4 + 8) \times 3.5 \div 2 = 12 \times 3.5 \div 2 = 42 \div 2 = 21$ m^2
Hexagon: $A = 2 \times 21 = 42$ m^2]

6. a) 3 m
b) 4 m

7. 228 m^2; [One trapezoid: $A = (a + b) \times h \div 2 = (14 + 5) \times 6 \div 2 = 19 \times 6 \div 2 = 114 \div 2 = 57$ m^2
Total area = $4 \times 57 = 228$ m^2]

[**8.** The area of one triangle is $A = a \times h \div 2$.
The area of the other triangle is $A = b \times h \div 2$.
Putting the two together, $A = (a \times h \div 2) + (b \times h \div 2)$.



Supporting Students

Struggling students

• If students are struggling with identifying the bases and heights of trapezoids, suggest that they first label the parts they know. Have them label the parallel sides first. This will help them identify the other parts.

• Some students may have trouble with **question 6** because they do not yet use algebra. If they can neither follow the reasoning of **example 2** nor develop their own reasoning, you might suggest that they rotate the trapezoid as shown in the exposition. This will turn the combined shape into a parallelogram.

For example, for **question 6 a)**, the parallelogram would have an area of 24 m^2 (i.e., double 12), and the bases would be 8 m (i.e., 2 + 6). The calculations are simpler using the formula for the area of a parallelogram.

$A = b \times h$, so $24 = 8 \times x$.

$24 = 8 \times 3$, so $x = 3$ cm.

Enrichment

• Extending **question 5**, you might challenge students to create other composite shapes by putting together a number of trapezoids. A student could choose a shape to make into a problem and then trade with another student who also made a problem. They could then solve each other's problem.

4.3.5 Circumference of a Circle

Curriculum Outcomes	Outcome relevance
7-D5 Circles: solve problems with diameter, radius, circumference • relate diameter, radius, and circumference to solve problems • investigate π as $C \div d$ for a number of circles and cylinders • develop the formulas $C = \pi d$ and $C = 2\pi r$ **7-D4 Area**: **composite shapes** • understand that composite shapes can be broken down into familiar shapes for which there are area formulas available	The formula for the circumference of a circle is widely used outside of school. Because it is often much easier to measure diameter than circumference, the formula is valuable.

Pacing	Materials	Prerequisites
1 h	• Circular objects (tins, etc.) • Compasses • Rulers • String	• multiplying and dividing fractions and decimals

Main Points to be Raised

• The circumference of a circle is the distance around the outside.

• The diameter of a circle is the distance across at the widest place.

• The radius of a circle is the distance from any point to the centre. It is half the diameter.

• The ratio between the circumference and the diameter of any circle is always the same. The factor is just over 3. It is called pi and is written π.

• You can use the formula for the circumference of a circle, $C = \pi d$, to find the circumference if you know the diameter, or vice-versa.

• The formula can also be connected to the radius because the diameter is twice as long as the radius, i.e., $C = \pi(2r)$ or $2\ \pi r$.

Try This — Introducing the Lesson

A. Using chalk, sketch on the floor (inside the classroom or outside) the 1 m square, the 1 m circle, and the two points that are 1 metre apart. Ask a student to use string to outline the square. Then ask another student to outline the circle. Ask another student to use string to make a loop around the two points, as shown in the **Try This**.

• Ask the **Try This** questions of the whole class, making sure that the students explain their answers. Their answers for **part v)** will vary, so you should allow many students to speculate on the length.

The Exposition — Presenting the Main Ideas

• Sketch a circle on the board and label the parts. Mention the term *radius* and its plural, *radii*.

• Make sure that students understand, from their work on the **Try This**, that the circumference must be greater than two times the diameter and that it must be less than four times the diameter.

• Write the formula for the circumference of the circle, pointing out that the special symbol pi (π) represents the ratio between circumference and diameter. Students will be interested to know that people around the world have been trying to calculate this ratio for thousands of years, and that no one knows its value with complete accuracy. However, we do know the value with more much more accuracy than is necessary for virtually all real situations.

• Assure students that they may use either 3.14 or $\frac{22}{7}$ as an approximation of pi. Sometimes it will be easier to use one than the other.

• Use the formula to calculate the circumference of a round object in the classroom (e.g., a tin of fish). Check the result by measuring the circumference with string.

Revisiting the Try This

B. This question allows for students to revisit why it makes sense that π is about 3 in light of the problem they solved in the **Try This**.

Using the Examples

• Present the question in **example 1** and have students try it. They can then compare their solutions to the solution on **page 148**.

• Ask students to read through **example 2**. Allow students to ask any questions they might have about either example.

• Note that the answers to these example questions are given as "close to" in the solutions. This is because you are using an approximation of pi. Also, no measurement is ever completely accurate. Inform students that they will learn more about accuracy in future years, but that for this lesson they may just write "close to". If the method they show in their work makes sense, and their answer is reasonably close to the right value, you should consider their work to be correct.

Practising and Applying

Teaching points and tips

Q 1: It is preferable to have students measure physical objects that are circular instead of drawing different circles. It would be helpful for you to have such objects available, such as a clock, a tin, and so on.

Q 3: As with composite shapes, it will help students to label the parts and to use these labels to refer to the parts as they work.

Q 9: This question is hard to visualize without looking at a cylindrical object. Encourage students to handle some of the cylinders they used in **question 1** to help them visualize the shape of the label. Students may or may not allow for overlapping of the label, where the two ends are glued together. Either way is reasonable.

Q 10: You might discuss this question with the whole class about five minutes before the class period ends.

Common errors

• Because students are working with both radius and diameter, they may get mixed up and use the wrong one in a situation. You might encourage students always to look at the original question with their answer in mind to see if it makes sense.

For example, in **question 2 b)**, if a student mistakenly uses 21 as the diameter instead of as the radius, the result will be about 60, which is clearly incorrect when you look at the diagram.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can investigate the value of π through measurement
Question 3	to see if students can solve problems involving circumference
Question 7	to see if students can develop a formula for radius, given circumference

Answers

A. i) There are four sides, 1 m each, so the string is 4 m long.
ii) There are two lengths, 1 m long each, so it is about 2 m long.
iii) Less; *Sample response*:
Because the corners are pulled in.
iv) More; *Sample response*:
Because you have to pull the two strings apart in the middle to make a circle.
v) A bit longer than 3 m

B. $C = \pi \times d = 3.14 \times 1 = 3.14$ m; 3.14 m is very close to my estimate of a bit longer than 3 m.

Answers [Continued]

1. a), b), and c) *Sample response*:

Diameter (d)	**Circumference (C)**	$C \div d$
8 cm	25 cm	3.125
5 cm	16 cm	3.2
14 cm	44 cm	3.14

d) Almost; [they should be the same because for any circle, the circumference divided by the diameter has the same value so any differences are due to measurement error.]

2. a) 13 mm;
[$C = \pi \times d \approx 3.14 \times 4 = 12.56$ mm ≈ 13 mm]
b) 132 cm ;
[$C = 2 \times \pi \times r \approx 2 \times \frac{22}{7} \times 21 = 132$ cm ≈ 132 cm]
c) 66 cm; [$C = \pi \times d \approx \frac{22}{7} \times 21 = 66$ cm ≈ 66 cm]

3. a) 63 cm;
[One semicircle: $\pi \times d \div 2 \approx 3.14 \times 10 \div 2 = 15.7$
Total perimeter $= 4 \times 15.7 = 62.8 \approx 63$ cm]
b) 114 cm;
[The radius of the quarter circle is 15 cm (30 – 15).
The perimeter of the quarter circle is $2 \times \pi \times r \div 4 \approx 2 \times 3.14 \times 15 \div 4 = 23.55$.
Total perimeter $= 30 + 30 + 15 + 15 + 23.55 = 113.55 \approx 114$ cm]

4. 75 cm;
[$C = \pi \times d \approx \frac{22}{7} \times 24 = \frac{528}{7} = 75\frac{3}{7} \approx 75$ cm]

5. 28 m long;
[$C = \pi \times d \approx \frac{22}{7} \times 9 = \frac{198}{7} = 28\frac{2}{7} \approx 28$ m long]

6. a) 10 cm; [$31.4 \div 3.14 = 10$ cm]
b) 31.8 cm; [$100 \div 3.14 = 31.8$ cm]
c) $d = C \div \pi$

7. a) 5 cm; [$31.4 \div 3.14 \div 2 = 5$ cm]
b) 15.9 cm; [$100 \div 3.14 \div 2 = 15.9$ cm]
c) $r = C \div \pi \div 2$

8. a) They are the same.
[Semicircle: $\pi \times d \div 2 \approx 3.14 \times 10 \div 2 = 15.7$ cm
Circle: $C = \pi \times d \approx 3.14 \times 5 = 15.7$ cm]
b) They would be equal [7. 9 cm each].

9. a) The label is a rectangle.
b) *Sample response*: about 22.5 cm by 11.5 cm
[It is about 11.5 cm high (just less than the height of the tin) and about 22.5 cm wide (a bit more than the circumference of the tin); $C = \pi \times d \approx 3 \times 7.5 = 22.5$ cm.]

[**10.** *Sample response*:
You can use a ruler to measure diameter directly and get an accurate measure and then use the formula to find the circumference.
If you measure the circumference directly, you have to use either string or a measuring tape, and that is not very accurate.]

Supporting Students

Struggling students

• Many students will have challenging calculations if they use the less convenient estimation of π in a question. You might encourage students to think about changing the estimation they are using if the calculation seems difficult.

For example, in **question 2 b)**, it is convenient to use the fraction $\frac{22}{7}$ for π because 21 is divisible by 7.

Enrichment

• Challenge students to think about how they use the formula for the circumference of a circle by writing a rule for deciding when it is easier to use the $\frac{22}{7}$ approximation and when it is easier to use the 3.14 approximation.

• To connect this question to other parts of the unit, challenge students to return to the **Connections** to explain why the number of radians in a full rotation is 6.28.

UNIT 4 Revision

Pacing	Materials
2 h	• Rulers • Compasses • Protractors • Square Dot Grid Paper (BLM)

Question	Related Lesson(s)
1 – 4	Lesson 4.1.2
5 and 6	Lesson 4.1.3
7	Lesson 4.2.1
8	Lesson 4.2.2
9 and 10	Lesson 4.2.3
11 – 13	Lesson 4.3.1
14 and 15	Lesson 4.3.2
16	Lessons 4.3.3 and 4.3.4
17	Lesson 4.3.4
18 and 19	Lesson 4.3.5

Revision Tips

Q 3: Encourage students to try saying their explanation to a classmate before writing it down. The writing will be clearer this way.

Q 5: Make sure students realize they cannot use a protractor to construct.

Q 7: Students can create the right angle using a protractor if they wish.

Q 8: You may have to remind students that the only points in a reflection that do not move are the points on the reflection line.

Q 13: Remind students to pay attention to units. The height is in cm and the other dimensions are in m.

Q 16: Some students may use composite shapes to find the areas of the two trapezoids. The results should be the same, but remind students that they should be able to use the trapezoid formula. For **parts b) and d)**, remind students to show clearly in their work which calculations go with which parts of the shape.

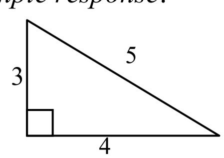
Q 17: This question ought to test students' understanding of the trapezoid formula even if they did not use the formula as expected in **question 16**.

Q 19: For **part b)**, remind students to show clearly in their work which calculations go with which parts of the shape.

Answers

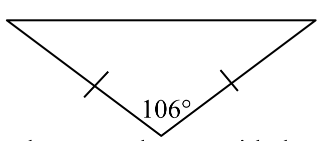
1. *Sample response*:

a)



[It has a right angle; three side lengths are different.]

b)



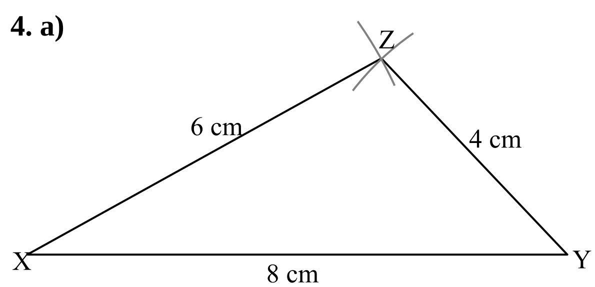
[It has an obtuse angle; two side lengths are equal.]

2. a) 92° [180° – 51° – 37° = 92º]

b) Obtuse scalene [Because the largest angle is obtuse, it is an obtuse triangle and because the angles are all different, it is a scalene triangle.]

3. No; [*Sample response*:
Both classifications refer to the largest angle, which cannot be both acute and obtuse.]

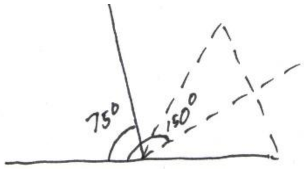
4. a)



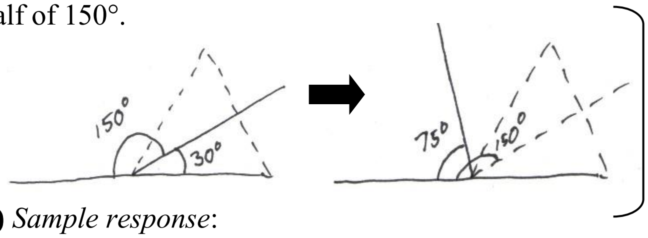
b) Obtuse scalene [$\angle Z = 104°$ so this is an obtuse triangle. The sides are different lengths, so it is scalene.]

Answers [Continued]

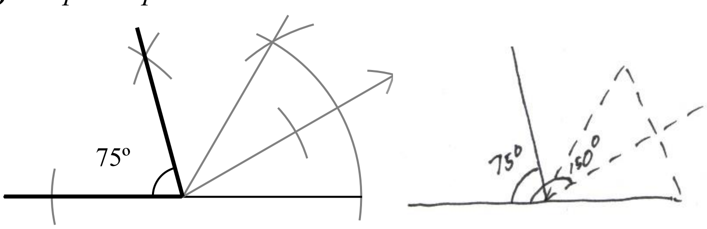
5. a) *Sample response*:



[• I started with a straight angle.
• I imagined an equilateral triangle on it to visualize a 60° angle.
• I visualized the bisection of this angle to get 30°.
• I "subtracted" the 30° angle from the straight angle (180º) and was left with 150°.
• I visualized bisecting this 150° angle to get 75°, which is half of 150°.

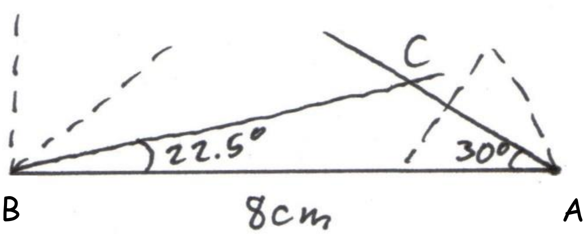


b) *Sample response*:



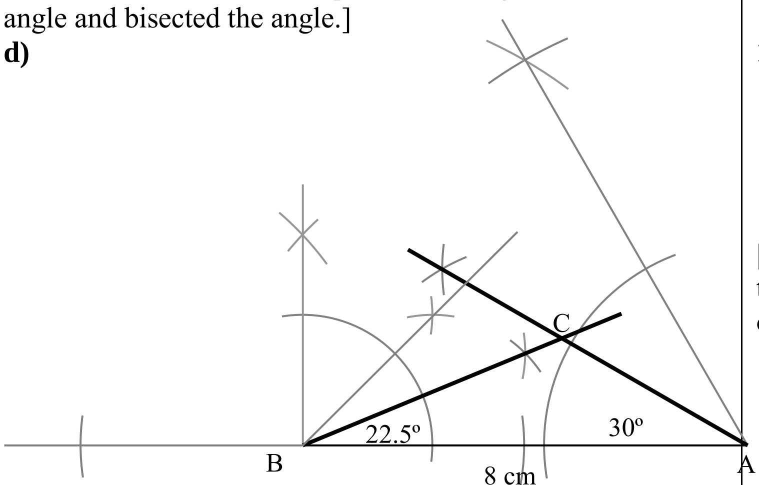
It is close to my sketch.

6. a) and b) *Sample response*:

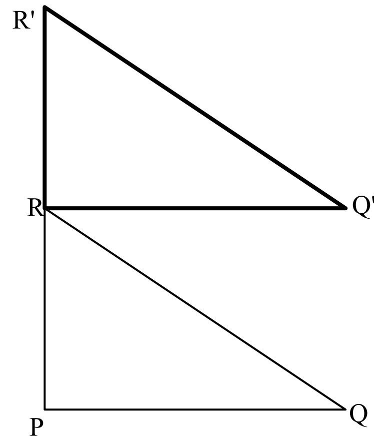


[**c)** *Sample response*: For the 22.5° angle I imagined the corner of a piece of paper, then I bisected twice. 90 ÷ 2 ÷ 2 = 22.5°.
For the 30°, I sketched an equilateral triangle with a 60º angle and bisected the angle.]

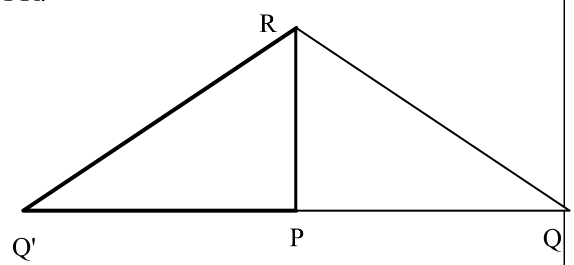
d)



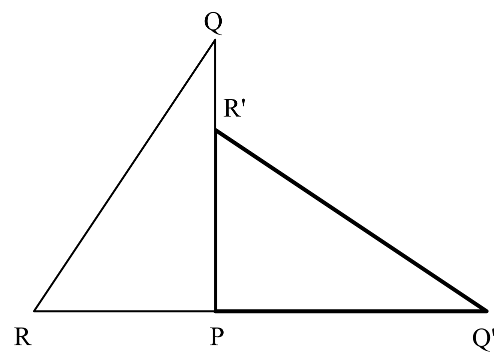
7. a), b), and c) The triangle is translated up 4 cm along PR.



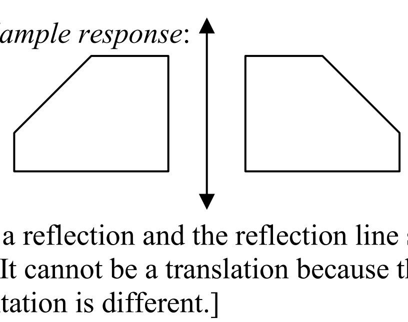
8. a), b), and c) The triangle is reflected in PR.



9. a), b), and c) *Sample response*:
The triangle is rotated 90° cw around turn centre P.

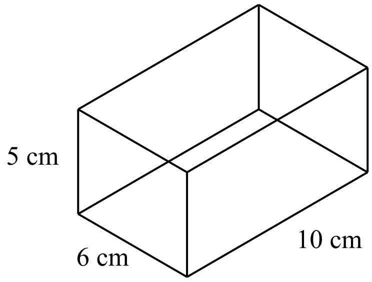


10. *Sample response*:



[It is a reflection and the reflection line shows that. It cannot be a translation because the orientation is different.]

11.



12.

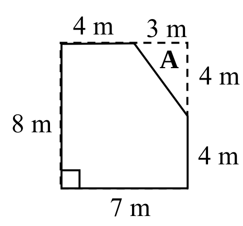
	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
a)	**5**	5	4	100
b)	2.0	**2.5**	6.0	30

13. 2400 L [Dimensions are 40 cm × 200 cm × 300 cm. $V = l \times w \times h = 40 \times 200 \times 300 = 2{,}400{,}000\ cm^3$ = 2,400,000 mL = 2400 L.]

14. a) 36 mm^2
b) 5.4 hg
c) 210 daL [because 2.1 t of water = 2100 kg = 2,100,000 g, and this mass of water fills 2,100,000 mL = 2100 L = 210 daL.]

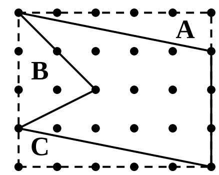
15. a) Divide by 100,000
b) Multiply by 1000

16. a) 222 m^2 [$A = (a + b) \times h \div 2 = (14 + 23) \times 12 \div 2 = 37 \times 12 \div 2 = 444 \div 2 = 222\ m^2$]
b) 50 m^2
[The base of the triangle A is 3 m because 7 – 4 = 3.
The height of the triangle is 4 m because 8 – 4 = 4.
Rectangle outside: $A = b \times h = 8 \times 7 = 56\ m^2$
Triangle A: $A = b \times h \div 2 = 3 \times 4 \div 2 = 12 \div 2 = 6$
Total area: $A = 56 - 6 = 50\ m^2$]

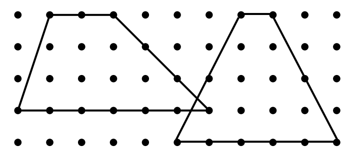


c) 16 cm^2 [$A = (a + b) \times h \div 2 = (5 + 3) \times 4 \div 2 = 8 \times 4 \div 2 = 32 \div 2 = 16\ cm^2$]

d) 12 cm^2
[Area of rectangle around outside:
$A = b \times h = 4 \times 5 = 20$
Triangle A: $A = b \times h \div 2 = 5 \times 1 \div 2 = 2.5$
Triangle B: $A = b \times h \div 2 = 3 \times 2 \div 2 = 3$
Triangle C: $A = b \times h \div 2 = 5 \times 1 \div 2 = 2.5$
Total Area: $A = 20 - 2.5 - 3 - 2.5 = 12\ cm^2$]



17. *Sample response*:



18. About 88 cm [$C = \pi \times d \approx \frac{22}{7} \times 28 = 88$ cm]

19. a) About 46 mm
[$C = 2 \times \pi \times r = 2 \times \pi \times 7.3 \approx 45.844$ mm ≈ 46 mm]
b) About 398 cm
[Half circle = $\pi \times d \div 2 \approx 3.14 \times 87 \div 2 = 136.59$ cm
Total perimeter = $136.59 + 3 \times 87 = 136.59 + 261 = 397.59$ cm ≈ 398 cm]

UNIT 4 Geometry and Measurement Test

1. Sketch an example of a right isosceles triangle. Explain how you know it is correct.

2. $\angle X = 52°$ and $\angle Z = 38°$ in $\triangle XYZ$.

a) What is $\angle Y$?

b) Classify the triangle by angle and side length.

3. Draw $\triangle PQR$ with PQ = 8 cm, QR = 6 cm, and PR = 6 cm.

4. a) Estimate to sketch a 105º angle. Explain how you did it.

b) Construct a 105º angle. Explain how you did it.

5. a) Draw $\triangle ABC$ with $\angle C = 105°$, AC = 4 cm, and BC = 3 cm.

b) Translate $\triangle ABC$ so A is translated to point B.

c) Describe the translation.

d) Draw $\triangle ABC$ again and then reflect it so that vertex C does not move.

e) Describe the reflection.

f) Draw $\triangle ABC$ again and then rotate it so that the image of vertex C is on AB.

g) Describe two rotations that will result in the same image.

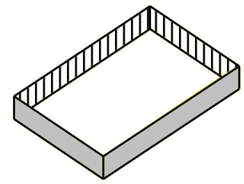
6. Sketch a pair of shapes that show a rotation but not a translation. Explain how you know you are right.

7. Sketch a rectangular prism with a volume of 240 cm^3 and edges that are a whole number of centimetres.

8. Copy and complete the chart for each rectangular prism.

	Length **(cm)**	Width **(cm)**	Height **(cm)**	Volume **(cm^3)**
a)	6		7	210
b)	6	5		105

9. What is the capacity of this container? It is 8 mm deep and its base is 4 cm × 6.5 cm.



10. Complete.

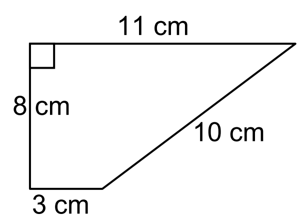
a) 0.760 km^2 = ☐ ha

b) ☐ dL = 3.2 daL

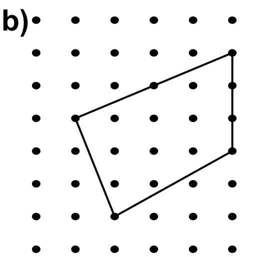
c) ☐ kL of water = 4.3 t

11. Find the area of each shape.

a)



b)

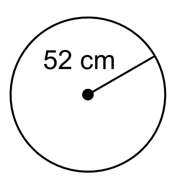


12. Draw two different trapezoids with area of 20 cm^2 on dot paper.

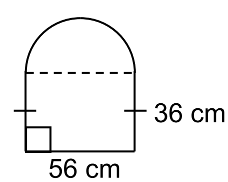
13. Determine the circumference in centimetres of a circle with diameter 84 mm.

14. Determine each perimeter to the nearest whole unit.

a)



b)



UNIT 4 Test

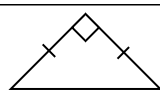
Pacing	Materials
1 h	• Rulers • Compasses • Protractors • Square Dot Grid Paper (BLM)

Question	Related Lesson(s)
1 – 3	Lesson 4.1.2
4 and 5	Lesson 4.1.3
5 and 6	Lessons 4.2.1 – 4.2.3
7 – 9	Lesson 4.3.1
10	Lesson 4.3.2
11	Lessons 4.3.3 and 4.3.4
12	Lesson 4.3.4
13 and 14	Lesson 4.3.5

Select questions to assign according to the time available.

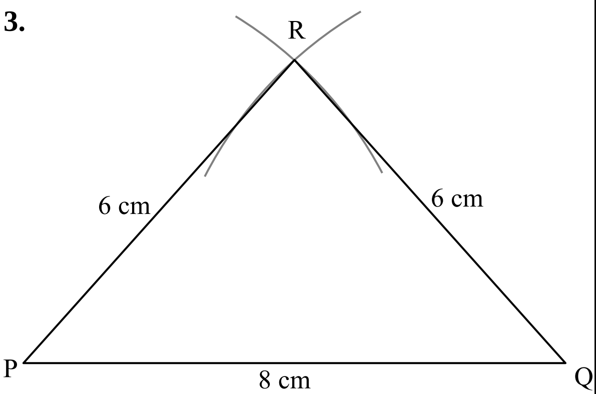
Answers

1. It has a right angle and two sides are the same length.

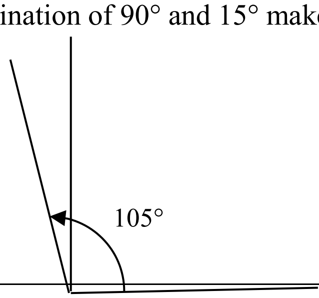


2. a) 90°; 180° – 52° – 38° = 90°
b) Right scalene; Because the side lengths are all different, it is a scalene triangle and because the largest angle is a right angle, it is a right triangle.

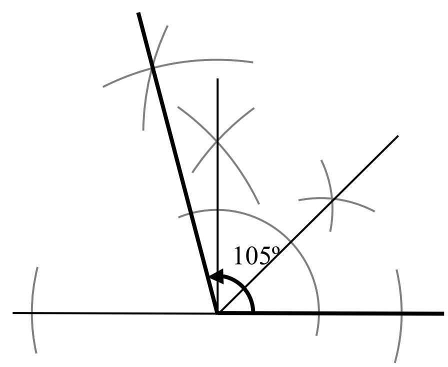
3.



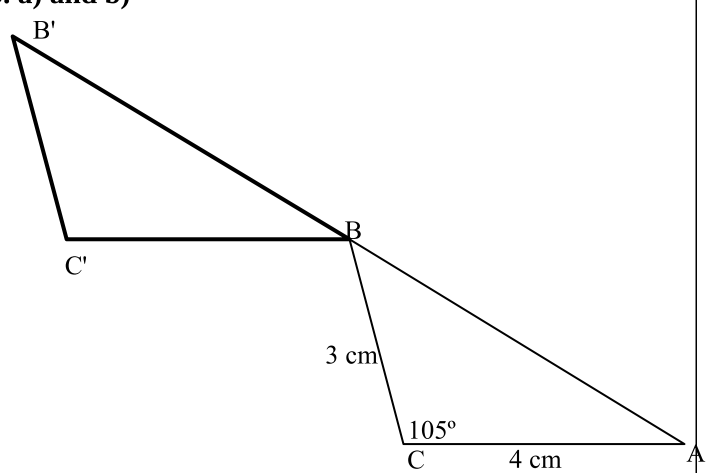
4. *Sample responses*:
a) • I first drew a right angle by imagining the corner of a page.
• Then I drew a bit more that looked like 15°.
• The combination of 90° and 15° makes 105°.



b) I constructed a right angle by bisecting a straight angle.
• Then I bisected it to get 45°.
• Then I constructed a 60° angle on its arm.
• The combination of 45° and 60° makes 105°.



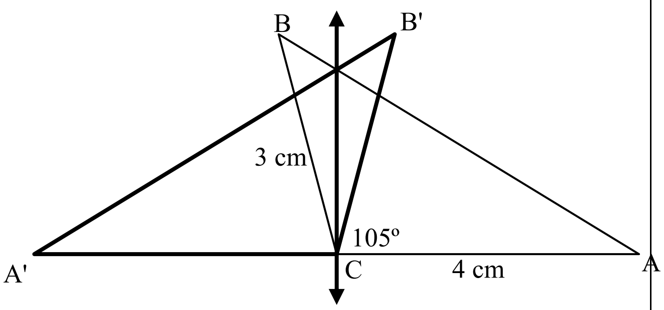
5. a) and b)



c) The triangle was translated along AB.

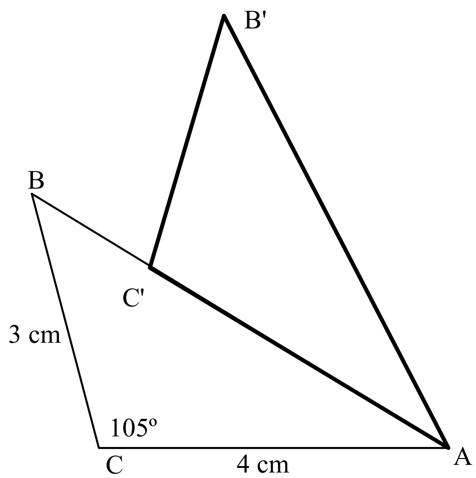
Answer [Continued]

d) *Sample response*:



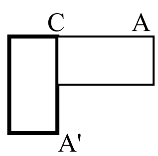
e) *Sample response*:
The triangle was reflected in a line through C that is perpendicular to AC

f)



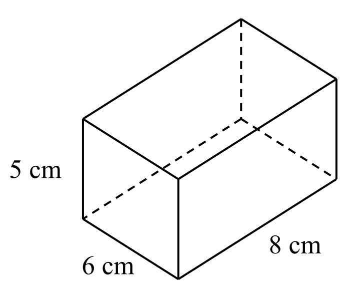
g) Rotate 31° cw around turn centre A or rotate 329° ccw around turn centre A.

6. *Sample response*:



This shows a rectangle rotated around vertex C. It cannot be a translation because side AC was horizontal in the original and A'C is vertical in the image.

7.



8.

	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
a)	6	**5**	7	210
b)	6	5	**3.5**	105

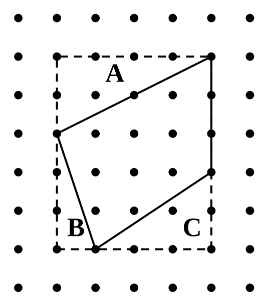
9. 20.8 mL; Dimensions are 8 mm × 40 mm × 65 mm.
$V = l \times w \times h = 8 \times 40 \times 65 = 20{,}800\ \text{mm}^3 =$
$20.8\ \text{cm}^3 = 20.8$ mL.

10. a) 76 ha

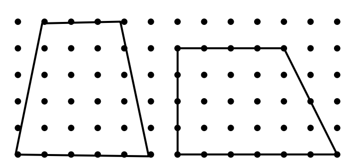
b) 320 dL

c) 4.3 kL

11. a) 56 cm^2;
$A = (a + b) \times h \div 2 = (11 + 3) \times 8 \div 2 = 14 \times 8 \div 2 =$
$56\ \text{cm}^2$
b) 11.5 square units
Area of rectangle around outside: $A = b \times h = 4 \times 5 =$
20
Triangle A: $A = b \times h \div 2 = 4 \times 2 \div 2 = 4$
Triangle B: $A = b \times h \div 2 = 3 \times 1 \div 2 = 1.5$
Triangle C: $A = b \times h \div 2 = 3 \times 2 \div 2 = 3$
Total Area: $A = 20 - 4 - 1.5 - 3 = 11.5$ square units



12. *Sample response*:



13. About 26.4 cm;

$C = \pi \times d \approx \frac{22}{7} \times 84 = 264\ \text{mm} = 26.4\ \text{cm}.$

14. a) About 327 cm;

$C = 2 \times \pi \times r \approx 2 \times 3.14 \times 52 = 326.56\ \text{cm} \approx 327\ \text{cm}$

b) About 216 cm;

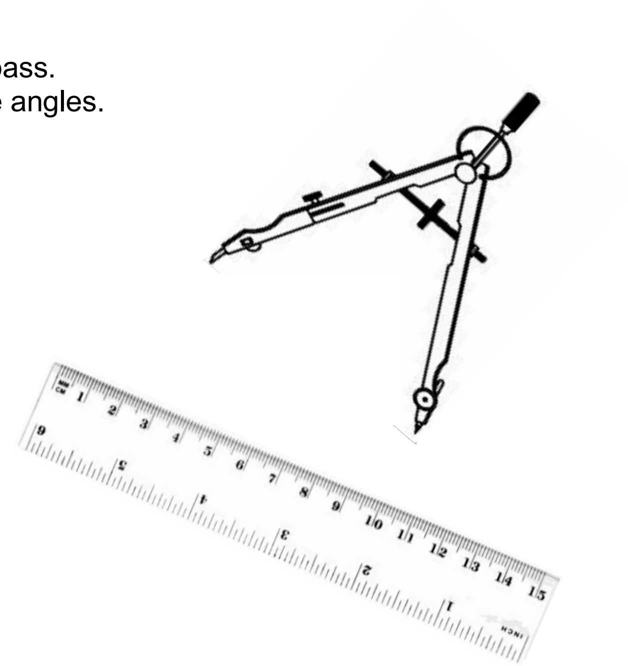
Half circle $= \pi \times d \div 2 \approx \frac{22}{7} \times 56 \div 2 = 88$ cm

Total perimeter $= 88 + 56 + 2 \times 36 = 88 + 56 + 72 =$
216 cm

UNIT 4 Performance Task — Trapezoid Transformations

Show all your work.

In **part A**, you will draw a trapezoid using a ruler and a compass.
Do not use your protractor, as you will be constructing all the angles.



A. i) $\triangle ABD$ has $\angle B = 45^\circ$, $\angle D = 30^\circ$, and BD = 6 cm.
What is $\angle A$?

ii) Draw $\triangle ABD$. Classify it by angle and side length.

iii) Draw $\triangle BCD$ on side length BD of $\triangle ABD$.
$\triangle BCD$ has DB = 6 cm, $\angle BDC = 45^\circ$, and DC = 7 cm.

iv) Classify $\triangle BCD$ by angle and side length.

B. i) What shape did you create in **part A**?
How do you know?

ii) Which parts can you measure in order to calculate the area of the shape using a single formula?

iii) Calculate the area in square centimetres and in square millimetres.

C. Describe a translation that would slide the image of vertex B onto one of the sides or vertices of shape ABCD. Make a sketch to show this translation.

D. Describe two different reflections of ABCD for which the image of vertex B remains at vertex B. Make a sketch to show these reflections.

E. Describe two different rotations of ABCD that rotate vertex A onto side CD.

F. Suppose shape ABCD is rotated 90º cw around vertex D.
What is the length of the path travelled by vertex C?

UNIT 4 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
7-E3 Triangles: classify **7-E4** Bisectors: construct **7-E5** Transformations: properties of translations, reflections, and rotations **7-D4** Area: composite shapes (trapezoid)	1 h	• Rulers • Compasses • Protractors

How to Use This Performance Task

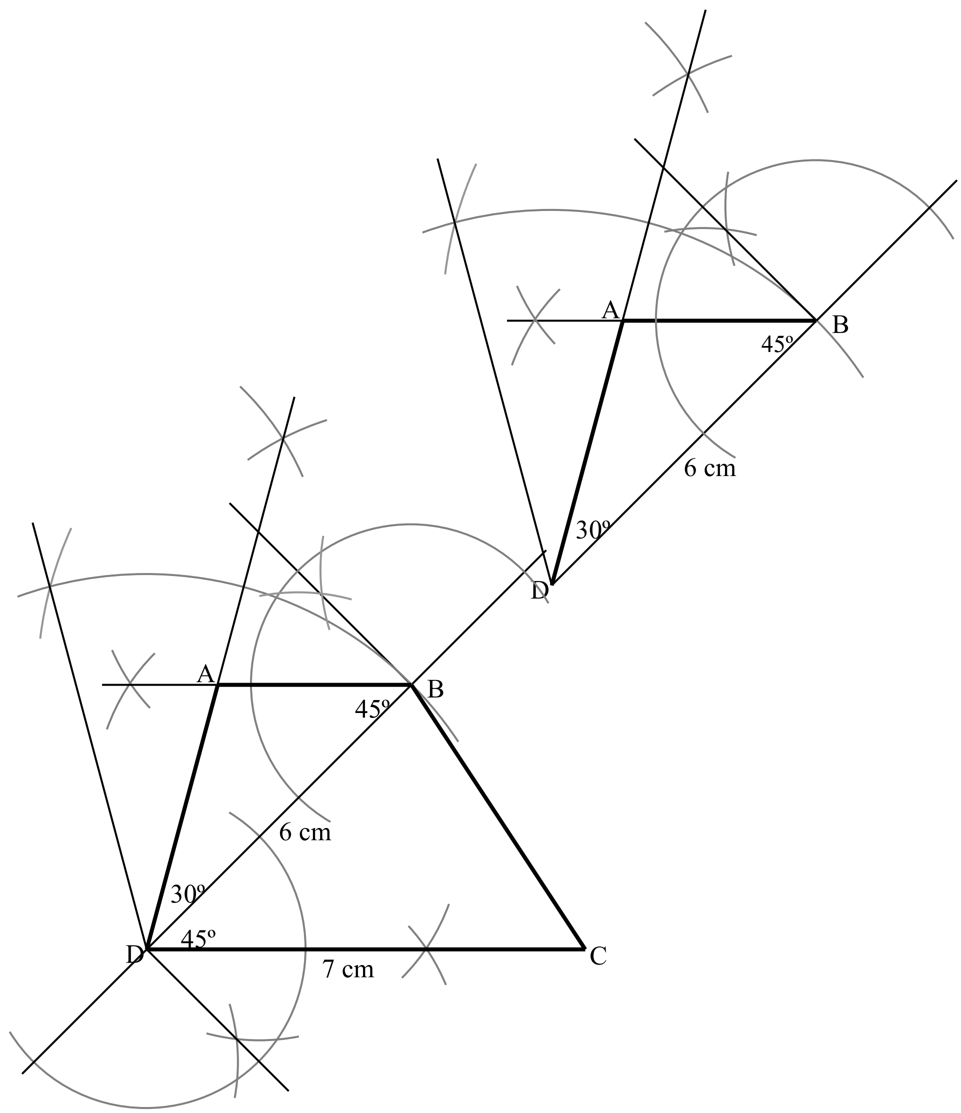
You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. It could also be used as enrichment material for some students. You can assess performance on the task using the rubric provided on the next page.

Sample Solution

A. i) 105º [180 – 45 – 30 = 105]

ii) Obtuse scalene [All the sides are different lengths and the largest angle is between 90º and 180º.]

iii)



iv) Acute scalene [All the sides are different lengths and the largest angle, ∠DBC, is less than 90 º.]

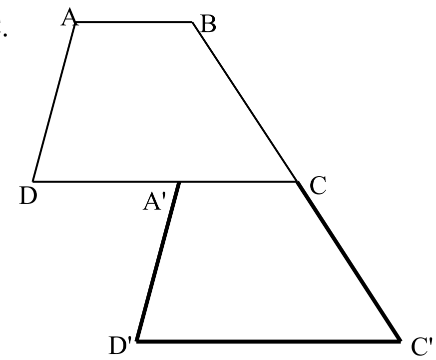
B. i) Trapezoid; there are exactly two parallel sides, AB and DC.

ii) The height, which is the distance from AB to CD, and the two bases, AB and CD.

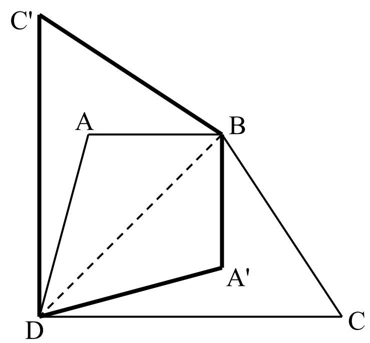
iii) Height = 4.2 cm, AB = 3.1 cm, CD = 7 cm

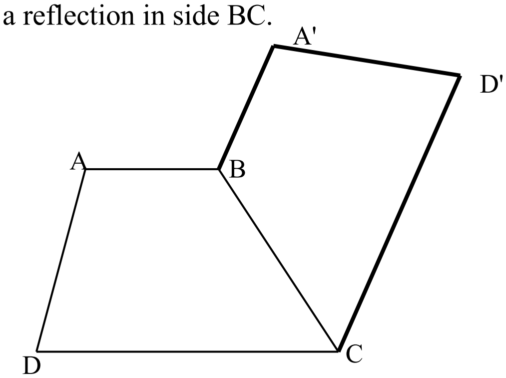
$A = (a + b) \times h \div 2 = (3.1 + 7) \times 4.2 \div 2 = 10.1 \times 4.2 \div 2 = 21.21 \text{ cm}^2 = 2121 \text{ mm}^2$

C. A translation along side BC.



D. A reflection in diagonal DB or a reflection in side BC.





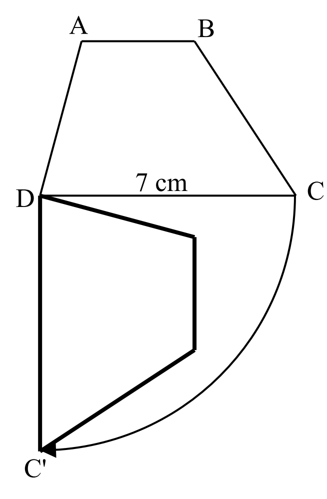
E. A rotation 75º cw around D or a rotation 285º ccw around D.

F. About 11 cm

The path is a quarter circle.

The radius of the circle is the length of DC.

Path length = $2 \times \pi \times r \div 4 \approx 2 \times \frac{22}{7} \times 7 \div 4 = 11$ cm

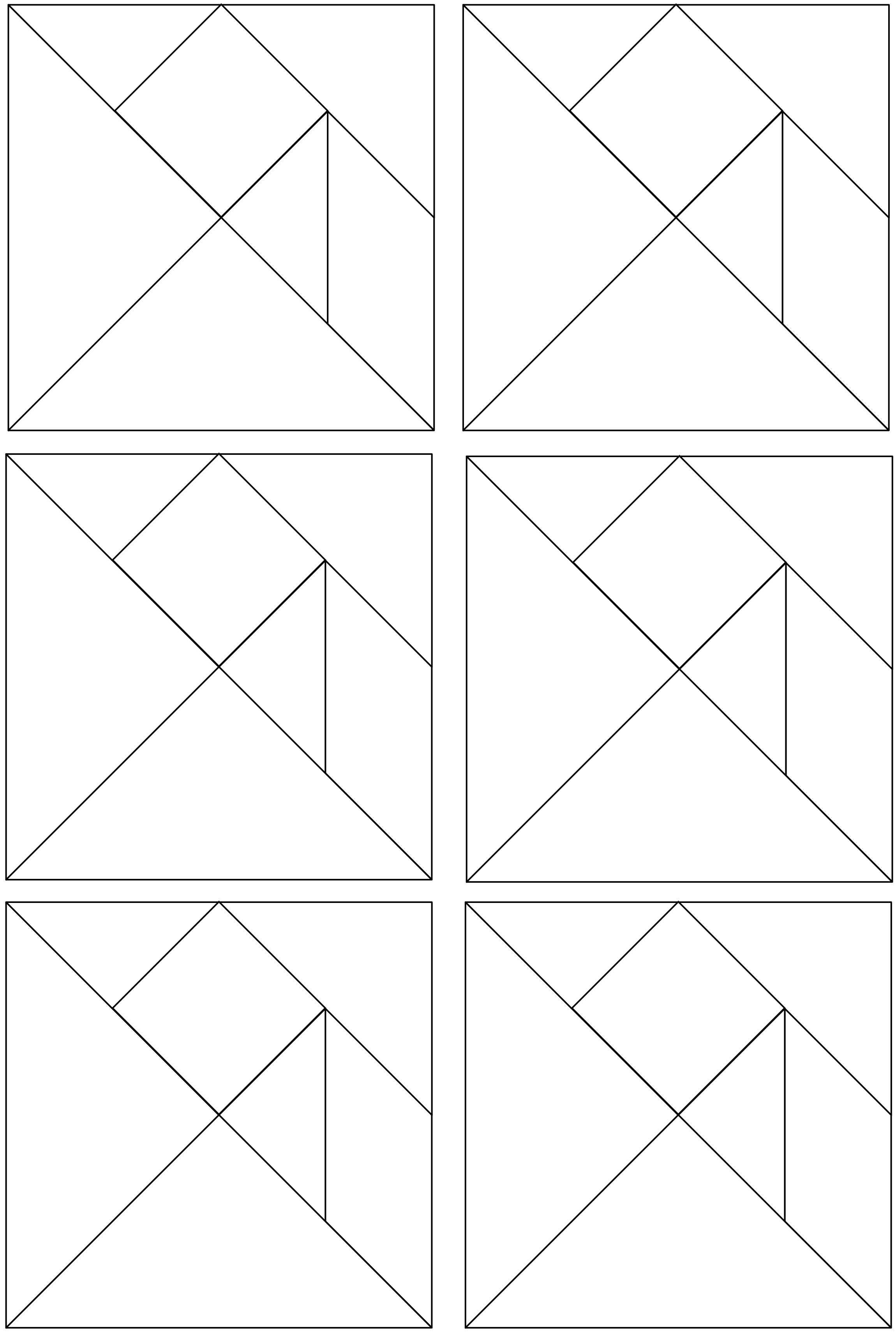


UNIT 4 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Constructs	Shows constructions done carefully with clear markings	Shows reasonably accurate constructions with markings and sufficient description to indicate reasoning	Shows sufficient understanding in most of the constructions and explanations	Makes major errors in constructions and explanations
Calculates area	Calculates area completely accurately with good measurement and sufficient explanation	Measures reasonably accurately, calculates area correctly using these measurements, and provides enough explanation to show understanding	Measures reasonably accurately and calculates area correctly using these measurements	Makes major errors in measurement or in calculation of area
Chooses and describes transformations	Chooses correctly and describes properly transformations that fit the criteria, with clear sketches	Chooses most transformations correctly and describes them properly, with clear sketches	Indicates some understanding of transformations and describes them	Shows major flaws in choosing and describing transformations
Finds path length	Applies the circumference formula correctly and shows work	Uses the circumference formula correctly with minor errors in applying it to the situation	Recognizes that the circumference formula may apply	Uses inappropriate methods for finding the length of the quarter circle

BLM 1 Square Dot Grid Paper

BLM 2 Tangrams



UNIT 5 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 153** **TG p. 194**	Review prerequisite concepts, skills, and terminology, and pre-assessment	1 h	• Thermometer (optional) • Number lines (BLM) (optional)	All questions
Chapter 1 Representing Integers				
5.1.1 Integer Models **SB p. 155** **TG p. 196**	**7-A12 Integers: compare and order** • represent integers in a variety of ways	1 h	• Counters in two colours, e.g., black and white counters • Number lines (BLM) (optional)	Q1, 4, 7
5.1.2 Comparing and Ordering Integers **SB p. 158** **TG p. 199**	**7-A12 Integers: compare and order** • compare and order integers with number lines and using real life situations	1 h	• Number lines (BLM) (optional)	Q3, 8, 9
CONNECTIONS: Time Zones (Optional) **SB p. 161** **TG p. 202**	Make a connection between the mathematics of integers and a practical use of them	20 min	• Time Zone Map (BLM)	N/A
5.1.3 The Zero Property **SB p. 162** **TG p. 203**	**7-A12 Integers: compare and order** • understand the zero principle: balance of positive and negative values	1 h	• Nu 1 coins • Counters in two colours, e.g., black and white counters	Q1, 3
Chapter 2 Adding and Subtracting Integers				
5.2.1 Adding Integers using the Zero Property **SB p. 164** **TG p. 206**	**7-B7 Add Integers: to solve problems** • connect visual models to symbols • use counters, number lines, and real-life contexts • understand that, when adding two integers, it is necessary to first model each integer, then match positive and negative values to make zeros **7-B2 Properties of Operations: integers** • apply commutative and associative properties • explore the concept of "closure"	1 h	• Nu 1 coins • Counters in two colours, e.g., black and white counters	Q2, 4, 10
5.2.2 Adding Integers that are Far from Zero **SB p. 168** **TG p. 210**	**7-B7 Add Integers: to solve problems** • connect visual models to symbols **7-B8 Add Integers Mentally: develop and use strategies** • develop and use mental strategies: - front-end - compatible numbers	1 h	• Counters in two colours, e.g., black and white counters	Q3, 5, 10, 11

UNIT 5 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
GAME: Target Sum –50 (Optional) **SB p. 171** **TG p. 213**	Practice adding integers in a game situation	30 min	• Target Sum –50 Game Cards (BLM)	N/A
5.2.3 Subtracting Integers using Counters **SB p. 172** **TG p. 214**	**7-B7 Subtract Integers: to solve problems** • connect visual models to symbols • use counters and real-life contexts **7-B8 Subtract Integers Mentally: develop and use strategies** • develop and use mental strategies: - compatible numbers	1 h	• Counters in two colours, e.g., black and white counters	Q2, 3, 11
5.2.4 Subtracting Integers using a Number line **SB p. 175** **TG p. 217**	**7-B7 Subtract Integers: to solve problems** • connect visual models to symbols • use number lines and real-life contexts **7-B2 Properties of Operations: integers** • apply commutative and associative properties • explore the concept of "closure"	1 h	• Number Lines (BLM) (optional)	Q1, 2, 6, 9
5.2.5 EXPLORE: Integer Representations (Optional) **SB p. 179** **TG p. 220**	**7-A12 Integers: compare and order** • represent integers in a variety of ways **7-B7 Add Integers: to solve problems** • connect visual models to symbols • use counters, number lines, and real-life contexts **7-B2 Properties of Operations: integers** • apply commutative and associative properties	1 h	• Counters in two colours, e.g., black and white counters	Observe and Assess questions
UNIT 5 Revision **SB p. 180** **TG p. 222**	Review the concepts and skills in the unit	1 h	• Number Lines (BLM) (optional) • Counters in two colours, e.g., black and white counters	All questions
UNIT 5 Test **TG p. 224**	Assess the concepts and skills in the unit	1 h	• Number Lines (BLM) (optional) • Counters in two colours, e.g., black and white counters	All questions
UNIT 5 Performance Task **TG p. 227**	Assess concepts and skills in the unit	1 h	None	Rubric provided
UNIT 5 Assessment Interview **TG p. 229**	Assess concepts and skills in the unit	10 to 15 min	See **p. 229**	All questions
UNIT 5 Blackline Masters **TG p. 230**	BLM 1 Number Lines (blank) BLM 2 Time Zone Map (for **Connections**) BLM 3 Target Sum –50 Game Cards			

Math Background

• This unit extends student understanding of number principles and calculations from whole numbers to negative integers. It builds on introductory content presented in Class VI.

• The focus of the unit is on using models to represent integers in order to promote a deep understanding. Students will use the models to compare, add, and subtract integers.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 7** in **lesson 5.1.1**, where they solve a problem involving multiple clues comparing integers, in **question 9** in **lesson 5.1.2**, where they find an integer that satisfies conditions, in **question 3** in **lesson 5.1.3**, where they find missing integers by applying the zero property, and in **question 6** in **lesson 5.2.3**, where they create two subtractions for a given model.

• They use communication frequently as they explain their thinking in **question 4** in **lesson 5.1.2**, where they explain how to compare two integers, in **question 10** in **lesson 5.2.1**, where they explain the signs of a sum in relation to the signs of the numbers being added, and in **question 7** in **lesson 5.2.4**, where they use a number line to consider why the order of subtracting makes a difference.

• They use reasoning in answering questions such as **question 9** in **lesson 5.1.2**, where they are asked to figure out which clues in a question are not needed, in **question 4** in **lesson 5.1.3**, where they consider the possibilities for scores in a game, in **question 4** in **lesson 5.2.2**, where they compare sums of two related sets of integers, and in **question 9** in **lesson 5.2.4**, where they make conjectures from patterns evident when subtracting.

• They consider representation in **question 1** in **lesson 5.1.1**, where they use number lines to represent integers, and in **question 2** in **lesson 5.2.3**, where they use counters to model subtraction. Models of counters and number lines are used frequently throughout the unit and students are often asked to choose the model that best supports their way of thinking, for example, in **question 3** in **lesson 5.2.2**.

• Students use visualization skills in **question 6** in **lesson 5.1.2**, where they compare integers, in **question 8** in **lesson 5.2.2**, where they add numbers far from zero, and in **question 3** in **lesson 5.2.4**, where they mentally subtract integers.

• They make connections in situations like **question 9** in **lesson 5.2.1**, where they make connections between models and mental math, and **question 6** in **lesson 5.2.2**, which makes a connection to a real-world sports context.

Rationale for Teaching Approach

• This unit is divided into two chapters.

Chapter 1 is about representing integers using two models: counters and number lines.

Chapter 2 examines adding and subtracting integers with models and mentally.

• The **Explore lesson** allows students to find ways to combine counters to make many representations of any given integer. Students investigate patterns within these representations and use reasoning skills to explore relationships between the integers and the number of possible representations.

• The **Connections** section helps students see some connections to time zones. It offers a practical application of how integers are used to determine the local time of places around the world.

• The **Game** provides an opportunity to apply and practise integer addition in a problem-solving context. Students use estimation skills to get a sum closest to –50.

• Throughout the unit, it is important to encourage students to use models to add and subtract integers. When they use integers that are far from zero, encourage students to visualize counters or a number line. It is important to accept a variety of approaches from students.

Getting Started

Curriculum Outcomes	Outcome relevance
6 Integers: negative and positive • develop meaning with models and symbols • explore negative integers in context • understand that zero is neither positive nor negative • compare integers	Students will find the work in the unit easier after they review the concepts and skills related to number line models introduced in Class VI.

Pacing	Materials	Prerequisites
1 h	• Thermometer (optional) • Number Lines (BLM) (optional)	• representing whole numbers with a number line model • using integers to describe some situations • interpreting a recorded temperature

Main Points to be Raised

Use What You Know

• Integers are positive or negative relative to zero. Zero is neither positive nor negative.

• Integers greater than zero are positive; they are above zero on a number line. Integers less than zero are negative; they are below zero on a number line.

• If an integer is below another integer, it is less than that integer.

Skills You Will Need

• You can use a number line to compare numbers.

• Integers are used in many real-world situations, such as temperature and altitude relative to sea level.

Use What You Know — Introducing the Unit

• Before assigning the activity, you may wish to remind students what integers are and ask students to brainstorm different ways integers are used in daily contexts. This could be done as a whole class. You may also wish to review the terms *elevation* and *sea level* and remind students of the location of the various continents. Explain to students that sea level is used as a benchmark to which other heights or depths are compared.

• Students can work in pairs to complete the activity.

While you observe students at work, you might ask questions such as the following:

• *How did you know that Cerro Aconcagua was between Mount Everest and Mount McKinley?* (The elevations are all positive, and Cerro Aconcagua has an elevation greater than Mount McKinley and less than Mount Everest.)

• *How did you find the elevation closest to sea level?* (I looked at places whose elevations were two-digit numbers to see whether they were above or below sea level. Then I thought about how these elevations would look on a number line and I figured out which elevation was the closest to zero.)

Skills You Will Need

• Students can work individually.

• To ensure students have the required skills for this unit, assign these questions.

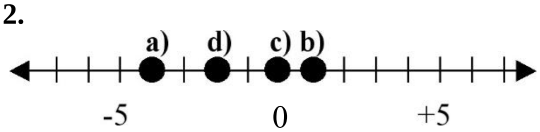
• Before they begin, you could build on the brainstormed ideas from the beginning of the lesson by showing students a thermometer and talking about how integers are used to express temperature. You may wish to ask students how a number line is similar to, or different from, a thermometer.

• Encourage students to use a number line to answer **question 4**.

Answers

A. i) Mount Everest, +8,850 **ii)** Bentley Subglacial Trench, –2555 **iii)** Lake Eyre, –16 **iv)** Cerro Aconcagua**,** +6959 **v)** Dead Sea, –408	**B. i)** Mount Everest; *Sample response*: Because +8850 is greater than +7553. **ii)** Death Valley; *Sample response*: Using sea level as 0, Drangme Chhu is 97 m away from 0 and Death Valley is 86 m away from 0. –86 is closer to 0 than +97 is.

NOTE: Answers or parts of answers that are in square brackets throughout the Teacher's Guide are NOT found in the answers in the student textbook.

1. $a < b$; [*Sample response*: I know this because *a* is left of *b* on the number line.] **2.**  **3.** Yes; [*Sample response*: –4 < – 2 because it is farther away from 0 in a negative direction.]	**4. a)** > **b)** < **c)** < **d)** > **5. a)** +3747 **b)** –86 **c)** –14 **d)** +3 **6. a)** +10°C **b)** 7 degrees colder

Supporting Students

Struggling students

• If students are struggling with comparing integers in **question 4**, you might have them draw a number line and plot pairs of integers on it. This will allow them to see which integer is to the right of the other and is therefore greater.

• For **question 6**, some students may not be able to visualize the increments between degrees using the diagram in the textbook. Encourage them to draw a vertical number line from –5°C to +10°C and plot the temperatures. This will allow them to count degrees by moving up or down.

Enrichment

• For **part A**, you might challenge students to write comparisons of the elevations using integers.

For example, they could write various elevation comparisons between the Dead Sea and Mount Vinson, such as –408 < +4897, +4897 > –408, and +4897 is 5305 m higher than –408.

• For **part A**, you could ask students to choose a place from the chart and write several clues using integers to describe it.

For example, Mount Elbrus could be described as having an elevation between Mount Vinson and Mount Kilimanjaro, or as having an elevation that is 1326 m less than Ceero Aconcagua.

5.1.1 Integer Models

Curriculum Outcomes	Outcome relevance
7-A12 Integers: compare and order • represent integers in a variety of ways	Work with integer models will help support students' later work with comparing, ordering, adding, and subtracting integers.

Pacing	Materials	Prerequisites
1 h	• Counters in two colours, e.g., black and white counters • Number lines (BLM) (optional)	• arranging integers on a number line

Main Points to be Raised

• Every integer has an opposite. The opposite of a positive integer is a negative integer and the opposite of a negative integer is a positive integer. Zero is its own opposite.

• You can represent or model integers using number lines or counters.

• You can use a number line to locate opposite integers.

For example, you can find the opposite of +3 by finding a negative integer that is the same distance away from 0 as +3.

• You can use counters with two different colours to represent opposite integers.

For example, three white counters can represent +3 and three black counters can represent –3.

• You can use a model to find an integer that is greater or less than another integer by a given amount.

For example, to find two integers that are three units away from +1, draw a number line and move three units to the right of +1 and three units to the left of +1.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know the temperature is negative?* (It is between two negative numbers so it cannot be right of 0.)

• *What information does the first clue give you?* (I know that possibilities are –4°C, –5°C, –6°C, –7°C, –8°C, and –9°C. I also know that the first three are closer to –3°C than the last three.)

• *Why can the temperature not be –7°C?* (–7 is closer to –10 than to –3.)

• *Does it make sense that there is only one answer?* (Yes; –6°C is the only integer that is colder than –5°C but closer to –3°C than –10°C.)

If students incorrectly identify –4°C as the answer, ask them to show you how they compare two negative integers. They may not understand that integers are arranged symmetrically around zero on a number line. They may also think that because 4 is less than 6, –4 must be less than –6.

The Exposition — Presenting the Main Ideas

• Write all the integers from –7 to +7 on separate pieces of paper. Distribute these papers to 15 students. Ask the 15 students to stand in a line along the front of the class in order from least to greatest. Have the rest of the students make up clues similar to those in the **Try This**.

For example, a student might say that the integer she is thinking of is between –4 and –1 and is closer to –4 than to –1. Have classmates show how they would find the answer using the number line of students.

• Tell students that –5 and +5 are called opposites. Ask students for reasons why they might be called opposites. Use the student number line to show other pairs of opposites. Draw attention to the relationship between zero and each of the integers. Ensure students see that opposite integers are the same distance away from zero.

• Read through the exposition with the students. Demonstrate how to use counters to show that –3 and +3 are opposites. Invite students to use counters to show other examples of opposite integers.

Revisiting the Try This

B. This question allows students to make a formal connection between what was done in **part A** and the main ideas presented in the exposition. In this case, students use the model of a number line to show how the clues are satisfied. Students might draw a number line from –3 to –10 and label each integer between these points. Then they could use different-coloured crayons to mark the possible integers that satisfy each individual condition. The place where the colours overlap would indicate the answer.

Using the Examples

• Present the problems in **examples 1 and 2** and have students try to solve them. They can then check their work and thinking against the solution and thinking in the student text.

Practising and Applying

Teaching points and tips

Q 1: Some students may choose to fold their paper to find opposites. Some students will simply know that they should use the same number with the opposite sign. Ask them how the number line supports their conclusion.

Q 2: Give students counters to model the question.

Q 3: Encourage students to draw a number line to show how they know the answer. Make sure they realize that they can sketch; they need not draw to scale.

Q 4: Encourage students to sketch a number line to help them with these questions.

Q 8: Use this last question to highlight the important ideas in the lesson. You might have students use counters to model the answer.

Common errors

• Some students may list two answers for each part of **question 6**. Remind them to read the entire statement to determine whether they are looking for a positive integer or a negative integer.

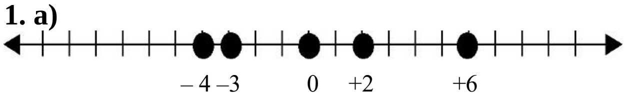
For example, students might answer +4 and –8 for **part a)**. They do not need to find both integers, only the integer that is positive.

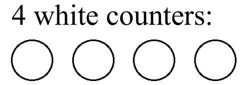
• Remind students that a number line has negative integers to the left of zero and positive integers to the right of zero. Some students may reverse this and will be confused when faced with this convention in **lesson 5.1.2**.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can locate integers and opposites on a number line
Question 4	to see if students can locate an integer that is a given distance from another integer
Question 7	to see if students can solve a problem involving integer comparisons using a number line

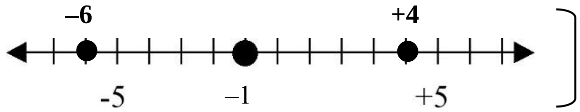
Answers

<table>
<tr><td>A. –6°C</td><td>B. Sample response:
I would find +9 and then count 15 spaces to the left to get to –5. It was colder than that so it had to be –6, –7, –8, or colder. Then I looked at the numbers between –3 and –10. I saw that –4, –5, and –6 were all closer to –3. That meant the answer had to be –6 because it was the only number that fit both clues.</td></tr>
<tr><td>1. a)


[b) Sample response:
I can fold the number line at zero and see that the opposite of +6 is –6, the opposite of –4 is +4, the opposite of –3 is +3, and the opposite of +2 is –2.]</td><td>2. a) –4
b) Sample response:
4 white counters:

c) +4

3. –6, +6</td></tr>
</table>

Answers [Continued]

4. a) –6 and +4; [*Sample response*: I used a number line to count 5 spaces to the left of –1 and 5 spaces to the right of –1.  **b)** +10 and –10 **c)** –2 and +6 **5.** Part b); [*Sample response*: These integers are the same distance from zero.] **6. a)** +4 **b)** –14 **c)** –8 **d)** +8	**7. a)** +11°C **b)** The last clue; [*Sample response*: I need only the last clue because once I know what 7ºC less than –4ºC is (–11ºC), I can find its opposite.] [**8.** *Sample response*: An integer and its opposite are always the same distance from zero on a number line. One integer is on one side of zero and the other integer is on the other side, so one is always negative and the other is always positive.]

Supporting Students

Struggling students

• If students are struggling with visualizing a number line in **question 6**, you might have them sketch a number line so that they can count the given number of units in the appropriate direction.

For example, for **part a)** students can sketch a number line from –10 to +10 and find –2. Then they can count 6 units to the right.

Enrichment

• For **question 7**, you might challenge students to create their own clues for the usual high temperatures for other places. These could be shared with classmates to solve.

5.1.2 Comparing and Ordering Integers

Curriculum Outcomes	Outcome relevance
7-A12 Integers: compare and order • compare and order integers with number lines and using real life situations	Using a number line provides students with a way to visualize integers in relation to each other and helps develop number sense. A real-world context makes the meaning of the numbers easier to grasp.

Pacing	Materials	Prerequisites
1 h	• Number lines (BLM) (optional)	• addition facts • representing integers using a number line

Main Points to be Raised

• You can use a number line to compare and order integers. By marking numbers on a number line, you can see when an integer is to the left of (or right of) or below (or above) another integer and is therefore less (or greater).

For example:

- If you were comparing –4 and –7, you could mark these on a number line. You would notice that –7 is to the left of, or below –4 and must be less.

- To order –4, +2, +4, and –1 from least to greatest, you could mark each integer on a number line and then read the numbers from left to right or bottom to top.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know that Trongsa is colder than Punakha?* (It is below Punakha on the thermometer.)

• *How do you know that a place colder than Punakha by 10° has to have a negative temperature?* (Punakha is only 4° above zero, so any temperature more than 4° colder has to be negative.)

• *How else could you describe the place that is warmer than Paro by 6°?* (I could say that it is warmer than Thimphu by 3°.)

The Exposition — Presenting the Main Ideas

• Use the integer cards you made in **lesson 5.1.1**. Tape these to the wall at the front of the classroom to make a number line. Choose two integers. Have students compare these two integers in as many ways as possible.

For example, if +5 and –2 are selected, students could make the following comparisons: +5 > –2, –2 < +5, +5 is 7 more than –2, and –2 is 7 less than +5.

For each comparison offered, point out the integers on the number line and ask students to tell how they know they are right.

• Read through the exposition with the students.

Revisiting the Try This

B. Some students might consider the thermometer to be a number line. They are right. Others may assume that a thermometer is not a number line because they think a number line must be horizontal. Make sure students understand that number lines (including thermometers) can be horizontal or vertical.

Using the Examples

• Have students work in pairs. One of the pair should become an expert on **example 1** and the other should become an expert on **example 2**. Each student should then explain his or her example to the other student. Present the question in **example 3** to the whole group. Let each student answer individually and then check his or her thinking against the solution in the text.

Practising and Applying

Teaching points and tips

Q 2: Remind students that there may be more than one possible pair of answers for each part.

Q 3: Make sure students realize that they can sketch and that they need not draw to scale. The same number line can be used for all parts.

Q 6: Encourage students to sketch a number line if they have difficulty visualizing integers like –100 or +210.

Q 8: You may wish to explain to students that a low score in golf wins because it means the player took fewer shots to get the ball into each hole.

Common errors

• Some students may have difficulty with **questions 4 and 6** because they will think that a “larger” (meaning farthest from zero) number is greater.

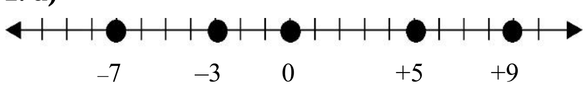
For example, some students may think that –47 is greater than –30 because the number without its sign is greater.

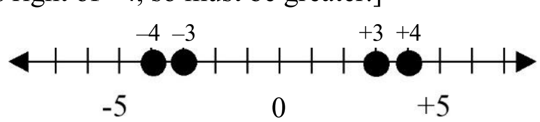
You may wish to encourage students to draw a number line so that they can see that numbers decrease as they move to the left. This can be linked to what they know about whole numbers.

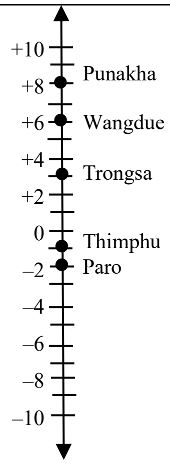
Suggested assessment questions from Practising and Applying

Question 3	to see if students can order integers
Question 8	to see if students can compare integers in a real-world situation
Question 9	to see if students can use reasoning to solve a problem involving integer comparisons

Answers

<table>
<tr>
<td>A. i) Trongsa
ii) Thimphu
iii) Paro</td>
<td>B. Sample response:
You think about going up and down the thermometer one space at a time just like you go right and left on a number line. Both have a zero place and both have positive and negative numbers spaced the same way.</td>
</tr>
<tr>
<td>1. a)

b) –7, –3, 0, +5, +9

2. a) Any two of –4, –5, –6, –7, –8, –9
b) Any two of –1, 0, +1, +2, +3
c) –2, –1
d) Any two of 0 or any positive integer
e) Any two of –15, –16, –17, …
f) Any pair of –4 and +4, or –3 and +3, or –2 and +2, or –1 and +1
g) One pair in this pattern:
<table>
<tr><td>+1</td><td>+3</td></tr>
<tr><td>0</td><td>+4</td></tr>
<tr><td>–1</td><td>+5</td></tr>
<tr><td>–2</td><td>+6</td></tr>
<tr><td>–3</td><td>+7</td></tr>
<tr><td></td><td></td></tr>
</table>
</td>
<td>3. a) +4; [Sample response:
+4 is right of +3, so it must be greater.]
b) +4; [Sample response:
+4 is right of –3, so it must be greater.]
c) –3; [Sample response:
–3 is right of –4, so it must be greater.]
d) +3; [Sample response:
+3 is right of –4, so must be greater.]

4. a) <; [Sample response:
0 < +4 because 4 is left of 0 on a number line.]
b) >; [Sample response:
+3 > –5 because +3 is right of –5 on a number line.]
c) <; [Sample response:
–23 < +18 because –23 is left of +18 on a number line.]
d) <; [Sample response:
–65 < +65 because –65 is left of +65 on a number line.]</td>
</tr>
</table>

5. a)  **b)** Paro, Thimphu, Trongsa, Wangdue, Punakha	**6. a)** –25, –12, +8, +16, +25 **b)** –140, –120, –100, –10 **c)** –48, –6, 0, +4, +210 **7. a)** Nov. 17 **b)** Nov. 18 **c)** Nov. 16 **d)** Nov. 17 **8. a)** –18, –11, –7, +3, +4, +8 **b)** Tiger Woods **9. a)** –5 **b)** I am a negative number. I am less than –2. [**10.** *Sample response*: All negative integers are less than zero and all positive integers are greater than zero, so –3 is less than any positive number because it is a negative integer.]

Supporting Students

Struggling students

• If students are struggling to keep track of all the clues in **question 9**, you might have them list the possibilities for each clue separately. They can then compare these, looking for integers that appear in all five lists. They can use a number line to help them list the possibilities.

For example:

For the first clue, the integers are –5, –6, –7, ….

For the second clue, answers are –6, –5, –4, …. Students can see that there are only two numbers in both lists: –5 and –6.

For the third clue, both sets of listed numbers still work.

For the fourth clue only –5 works.

–5 also satisfies the fifth clue, so –5 must be the answer.

Enrichment

• For **question 9**, you might challenge students to choose a mystery integer and make up some clues that describe it uniquely.

CONNECTIONS: Time Zones

• This optional connection is intended for all students. It makes a link to a way integers are used in daily life. By connecting integers to local times around the world, students can begin to understand the usefulness of this part of the number system.

• Time zones within geographical areas have been used since 1675, but they became more widespread with the advent of railroads because of the need to coordinate transportation schedules. Today, most countries use a system of standard time zones. Some countries change their time seasonally to daylight saving time. Parts of some countries do not advance their time by a full hour increment.

• The values in the answers may change if some areas are not on standard time. The blackline master time zone map assumes all local zones are on standard time.

• You might discuss how the International Date Line works and challenge some students to travel over the IDL to find each time in **question 2**.

For example, students will need to know that if it is 3 p.m. on August 12 immediately west of the International Date Line, then it is 4 p.m. on August 11 immediately to the east of the IDL.

Answers

1. a) –11 **b)** +1 **c)** –5 **d)** –14 **e)** +2	**2.** 24 h clock time (12 h clock time) **a)** 1:00 (1:00 am) **b)** 13:00 (1:00 pm) **c)** 7:00 (7:00 am) **d)** 22:00 the previous day (10:00 pm the previous day) **e)** 14:00 (2:00 pm)

5.1.3 The Zero Property

Curriculum Outcomes	Outcome relevance
7-A12 Integers: compare and order • understand the zero principle: balance of positive and negative values	• Work with the zero property will support students' later work with adding and subtracting integers. • It is important for students to use the zero property so they understand why procedures work and do not just apply rules without understanding.

Pacing	Materials	Prerequisites
1 h	• Nu 1 coins • Counters in two colours, e.g., black and white counters	• representing integers using counters and number lines

Main Points to be Raised

- When you add an integer to its opposite, the result is zero.
- You can represent the zero property using counters and number lines as models.

Try This — Introducing the Lesson

A. Have students play the game individually. While you observe students at work, you might ask questions such as the following:

- *What prediction did you make? Why?* (I predicted my pencil would be at zero because I thought the chance of flipping a Tashi Ta-gye was equal to the chance of flipping a Khorlo.)
- *How close was your prediction?* (My pencil was at +1, so my prediction was close.)
- *What would you expect to see if you flipped the coin 20 times?* (My pencil might be at zero.)

The Exposition — Presenting the Main Ideas

- Play the number line and coin game one more time as a class. Have student volunteers take turns flipping the coin. Instead of recording movements on a number line, have students record them in a chart like this:

Flip number	1	2	3	4	5	6	7	8	9	10
Result										
Move										
Integer										
Location (start at 0)										

- Ask students if they see any relationship between the move and the location.
- Read through the exposition with the students.

Revisiting the Try This

B. and C. Make sure students understand that when they combine equal flips of K and T, they will get zero no matter what the order of the flips. Students should conclude that to determine the final result they can combine equal numbers of K and T flips to get zero and then see what flips are left over.

Using the Examples

• Work through the example with the students to make sure they understand it. Demonstrate using counter and number line models to show the solutions.

Practising and Applying

Teaching points and tips

Q 1: Be sure students have counters to use for this question.

Q 3: Some students may not recognize that the numbers can be put into the boxes in many different orders for most parts of the question. This concept will be important when students begin adding and subtracting integers.

Q 4: Some students may think that getting 30 identical flips is impossible. Remind them that it is highly unlikely but still possible.

Q 5: Encourage students to use models to help them answer this question.

Common errors

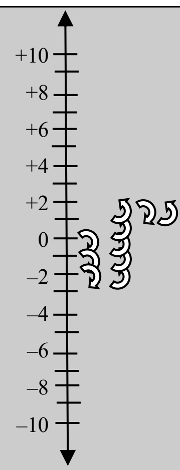
• In **question 2**, some students might not realize that they can combine all the Ts and then combine an equal number of Ks (or vice versa if Ts outnumber Ks) before finding a total. You can explore with them whether it makes any difference if you move all the Ts before moving any Ks.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use models to demonstrate the zero property
Question 3	to see if students can apply the zero property

Answers

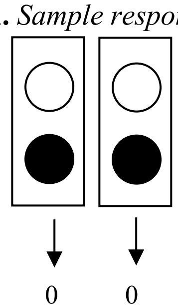
A. *Sample responses*:
i) I predict I will be at +4.
ii) T, T, T, K, K, K, K, K, T, K
I ended at +2.



B. *Sample responses*:
i) Maya ended at 0 and I ended at +2.
ii) If I end below 0, my location is negative. If I end above 0, my location is positive.

C. If I pair each K with a T, they will make zero. There are four pairs of KTs, so these make zero. There are two Ks left over. They each represent +1, so the final location is +2.

1. *Sample response*:



+2 + (–2) = 0

2. a) 0
[**b)** *Sample response*:
Find pairs of +1 and –1 to make zero. Flips 1 and 2 make zero, flips 3 and 4 make zero, and flips 5 and 6 make zero.]
c) +1

3. a) –1
b) +1
c) +1, –1 *in any order*
d) +1, –1, –1, –1 *in any order*
e) +1, +1, –1, –1 *in any order*
f) –1, –1, –1

4. a) +30 [because if you flipped all Ks then you would have +1 thirty times.] **b)** –30 [because if you flipped all Ts then you would have –1 thirty times.] [**5.** *Sample response*: **a)** To add +1 to a negative number: I can represent the negative number with black counters and +1 with a white counter. I pair one black counter with the white counter and remove them. The answer is the remaining counters. It will be one fewer than the number of black counters at the start.	**b)** To add –1 to a positive number: I can represent the positive number with white counters and –1 with a black counter. I pair one white counter with the black counter and remove them. The answer is the remaining counters. It will be one fewer than the number of white counters at the start.]

Supporting Students

Enrichment

• Students can play the number line and coin game with a partner with the following changes:

Player 1 begins. This person flips the coin and keeps track mentally of his or her score. Player 1 may continue flipping the coin as long as the flip does not result in a negative accumulated score. When Player 1 decides to stop flipping the coin, he or she keeps the points accumulated for that round. If the flip results in a negative score, the total for that round is 0 and play moves to Player 2. The game continues until 10 rounds have been played. The total for each round is added and the player with the higher total score wins.

For example:

- If Player 1 flips a Khorlo (K), the accumulated point is +1.

- If his or her next flip is a Tashi Ta-gye (T), the accumulated points are still not negative, so he or she can continue the turn (this is advised because the score is 0 at this point and Player 1 has nothing to lose).

- If Player 1 now flips 3 Ks in a row, the accumulated score is +3. Player 1 may decide to continue to flip the coin. As long as the accumulated score is not negative, his or her turn continues as he or she attempts to maximize the points. To be safe, Player 1 might decide to stop flipping when he or she has a score of +3. These points are recorded and play moves to Player 2.

5.2.1 Adding Integers Using the Zero Property

Curriculum Outcomes	Outcome relevance
7-B10 Add Integers: to solve problems • connect visual models to symbols • use counters, number lines, and real-life contexts • understand that, when adding two integers, it is necessary to first model each integer, then match positive and negative values to make zeros **7-B2 Properties of Operations: integers** • review use of commutative and associative properties • explore the concept of "closure"	• Students will apply what they learned in the previous lesson about the zero property to add integers. • Experience with counter and number line models will allow students connect these models to symbols. This will help students understand why procedures work so they do not just apply rules for addition without understanding.

Pacing	Materials	Prerequisites
1 h	• Nu 1 coins • Counters in two colours, e.g., black and white counters	• addition facts • understanding of the closure property of whole numbers

Main Points to be Raised

• You can use the zero property to add a positive integer and a negative integer.

• You can use counters and number lines to model integer addition.

• Changing the order of the addition does not change the result because the commutative property of addition works with integers.

• To add three integers, you can first combine any pair and then add the third integer.

• When you add integers, the result is always another integer.

Try This — Introducing the Lesson

A. Provide students with counters. Allow students to try this with a partner. While you observe students at work, you might ask questions such as the following:

• *What numbers can you model with 3 counters? with 5 counters?* (I can model +3 or –3 with 3 counters and +5 or –5 with 5 counters.)

• *How do you know the two counters in the sum must both be black or both be white?* (If they were all white or all black, there would be 8 white or 8 black counters and you would need all 8 of them to show the number.)

• *How do you know the answer you have is correct?* (I used 3 counters for my first number and 5 counters for my second number. After I removed the 3 pairs of black and white counters, I had 2 counters left.)

• *Can you find other solutions?* (I can use opposite integers because it does not say whether the 2 counters in the sum are both white or both black.)

The Exposition — Presenting the Main Ideas

• Divide students into small groups. Have them play the number line and coin game from **lesson 5.1.3**, this time without a number line. Have them complete the chart below by taking turns to flip the coin, record the result, and assign a coloured counter and the corresponding integer value of the flip. When they have finished, have them figure out the total score at the end of the game.

Flip number	1	2	3	4	5	6	7	8	9	10
Result										
Counter										
Value										

• Discuss with the students how they can use the zero property to help figure out the total score. Ask student volunteers to demonstrate how to use of the zero property.

For example, have students show how to model 3 flips of K and 7 flips of T with counters. Ask them to group the counters to demonstrate the zero property.

• Read through the exposition with the students. Be sure to demonstrate the use of the number line model. Make sure they understand that they move right to add a positive value and move left to add a negative value, and that the solution is found by naming the location at the end of the last movement.

• Discuss with students the meaning of *associative*, *commutative*, and *closure*. While it is not necessary for students to memorize these terms, it is important that they be familiar with the concepts.

Revisiting the Try This

B. This question allows students to make a formal connection between what was done in **part A** and the main ideas presented in the exposition. In this case, students apply the zero property to adding integers.

Using the Examples

• Present the problem in the **example** and ask students to try it alone or with a partner. Then ask students to read through **solutions 1, 2, and 3**. Ask them to choose which solution most closely matches what they did. Discuss the third solution and link this to the other two solutions.

Practising and Applying

Teaching points and tips

Q 1: Many students may choose to use models to find the sums.

Q 4: Encourage students to use the associative principle and mental math.

Q 6: Remind students to visualize counters or number lines if they are having difficulty doing this mentally.

Q 7: Some students may not recognize that there is more than one answer for each of these.

Q 10: Use this last question to highlight the important ideas students have learned in the lesson. You might have students debate each statement in a class discussion. Encourage them to justify their thinking and to find counter examples.

Common errors

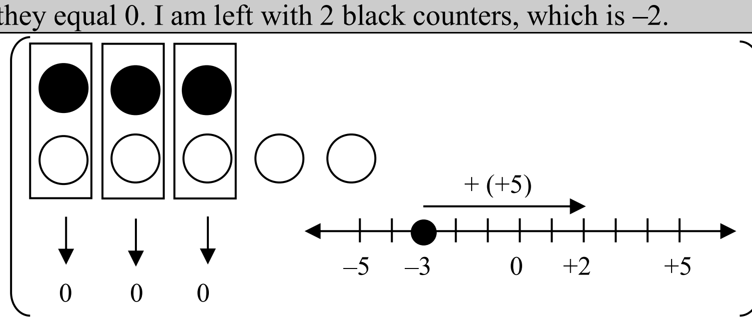
• Many students will attempt to memorize a set of rules generated from statements like those given in **question 10**. You should emphasize the second part of the question, where students are encouraged to explain their thinking. They can do this in a variety of ways, not only by writing.

For example, students could justify their thinking by showing the class an example using counters.

Suggested assessment questions from Practising and Applying

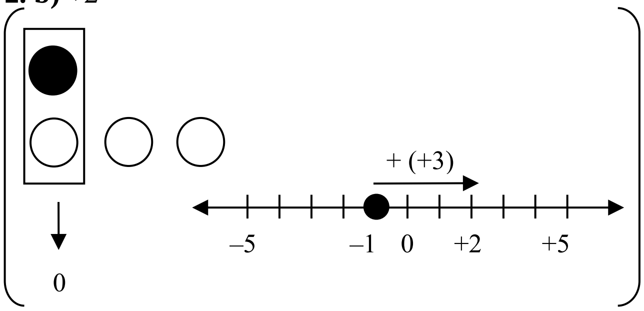
Question 2	to see if students can use models to add integers
Question 4	to see if students can add more than two integers
Question 10	to see if students can form generalizations about adding integers and communicate their thinking

Answers

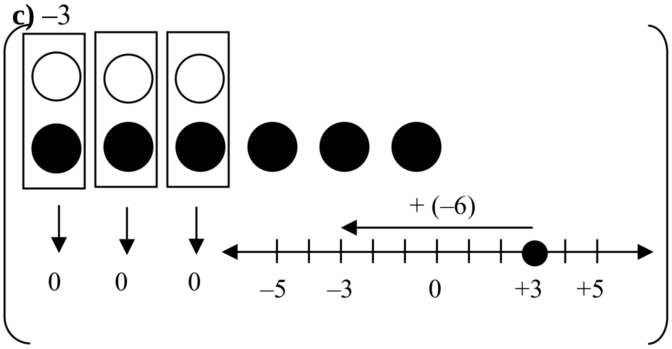
A. +3 and –5; *Sample response*: (+3) + (–5) = –2	**B.** *Sample response*: I can pair up 3 white counters with 3 black counters and remove them because they equal 0. I am left with 2 black counters, which is –2.
1. a) +1 **b)** +1 **c)** –4 **d)** +9 **e)** –4 **f)** –8	**2. a)** +2 

Answers [Continued]

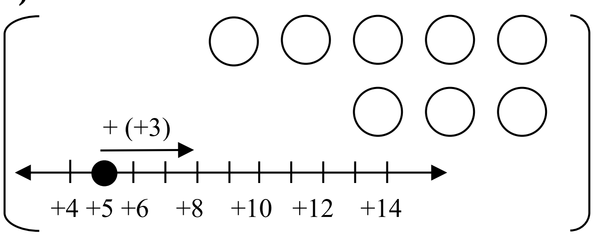
2. b) +2



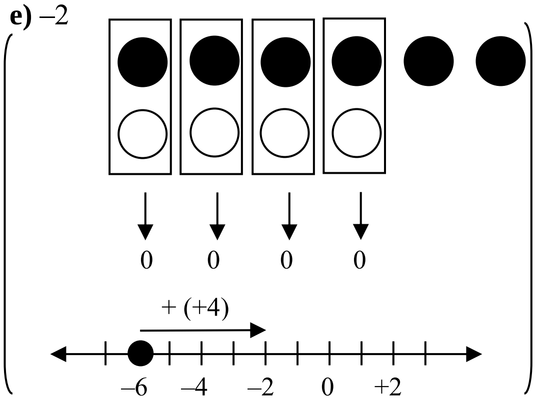
c) –3



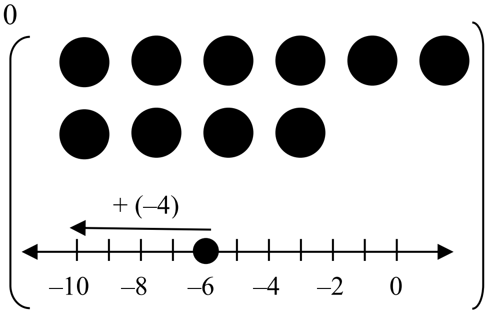
d) +8



e) –2



f) –10



3. a) +15°C **b)** +16°C **c)** +14°C

4. a) –6 **b)** –2 **c)** –3
d) –8 **e)** –1 **f)** –7

[**5.** *Sample response*:
You can first add the opposite integers to get zero. What you have left is the answer. Sometimes you have to rearrange them first, e.g.,
(–6) + (–7) + (+6) = (–6) + (+6) + (–7) = 0 + (–7)]

6. a) > **b)** < **c)** = **d)** > **e)** <

7. *Sample responses*:
a) (+1) + (–1) + (+1) + (–1) = 0
b) (+1) + (+1) + (+1) + (–1) = (+2)
c) (–1) + (+1) + (–1) + (+1) + (–1) = –1
d) (+1) + (+1) + (+1) + (+1) = (+4)

[**8.** *Sample response*:
To get an answer of –1, you need three –1s or +1s that add to 0. This is impossible because the +1s and –1s have to be in pairs. Adding an odd number of integers will not work.]

9. 0; [*Sample response*:
I imagine all the integers from –20 to +20 on a number line and then I pair up the opposites to make 0:
(–20) + (+20) + (–19) + (+19) + (–18) + (+18) +
… + (–1) + (+1) + 0
= 0 + 0 + 0 + ... + 0 + 0
= 0]

10. a) True; [*Sample response*:
Imagining counters, if I am adding white counters to white counters, I am going to end up with white counters.]
b) True; [*Sample response*:
Imagining counters, if I am adding black counters to black counters, I am going to end up with black counters.]
c) False; [*Sample response*:
This is only sometimes true because it depends on which integer is farthest from zero. If the positive integer is farthest from zero, the sum will be positive, but if the negative integer is farthest from zero, the sum will be negative.]
d) False; [*Sample response*:
This is only true if the numbers are opposites.]

Supporting Students

Struggling students

• If students are struggling with using a number line model in **question 2**, you might have them physically act it out by standing on a number line and moving. For this, you will have to create a number line on the floor. This is easily done using tape and small cards.

For example, if students are struggling to represent (–3) + (+5), have them stand at –3 and move 5 spaces to the right.

Enrichment

• For **part A** of the **Try This**, you could encourage students to use a combination of counters that are not the same colour for each part of the problem.

For example, the first number modelled with three counters could be +1 (one black counter and two white counters) and the second number modelled with five counters could be +1 (three white counter and two black counters). The solution of +2 could be modelled with two counters.

Students could be asked to find as many combinations as they can that satisfy the conditions. This extends the idea that any integer can be represented as a sum of two or more integers.

• For **question 7**, you might challenge students to find multiple answers and then to generalize about the pattern of their responses.

5.2.2 Adding Integers that are Far from Zero

Curriculum Outcomes	Outcome relevance
7-B10 Add Integers: to solve problems • connect visual models to symbols **7-B8 Add Integers Mentally: develop and use strategies** • develop and use mental strategies - front-end - compatible numbers	• This lesson builds on the previous lesson and prompts students to visualize models in order to connect them to the symbols. This will help students understand why procedures work and not just apply rules for addition without understanding. • In everyday life, mental math and estimation are very important skills, often more important than paper and pencil calculation.

Pacing	Materials	Prerequisites
1 h	• Counters in two colours, e.g., black and white counters	• addition facts • representing integers using counters and number lines

Main Points to be Raised

• You can sketch number lines or use counters to model and determine the sum of numbers that are far from zero.

• The sum of two negative integers is always negative.

• The sum of two positive integers is always positive.

• The sum of a positive integer and negative integer can be either positive or negative, depending on which addend is farther from zero.

• If the sum of two integers is zero, the integers must be opposites.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How might you visualize +28?* (A pile of 28 white counters or 28 units to the right of zero on a number line.)

• *How might you visualize –12?* (A pile of 12 black counters or 12 units to the left of zero on a number line.)

• *How can you use the zero property to combine +28 and –12?* (Combine –12 and +12 to make 0. There will be 28 – 12 = 16 white counters left.)

The Exposition — Presenting the Main Ideas

• Make a pile of black counters. Tell students to imagine that there are 46 counters in the pile. Make another pile of white counters. Tell students there are 79 counters in the pile. Ask them to figure out mentally how to combine these piles using the zero property. Ask, "Why can you think of 79 white counters as 46 + 33 white counters?" and, "How does this help you add the two integers?"

• Ask students to explain how to model the same addition using a number line. If they have difficulty with this, have them sketch a number line and place –46 on it. They can make a rough sketch, not listing every value. Have them use arrows to show a movement of 79 spaces to the right. Ask them where they would end.

• Read through the exposition with the students. Emphasize that students should visualize the models and sketch them but that they need not draw them to scale. For example, they might show –34 black counters as a big black circle with –34 written in it, and then show the circle separated into two black circles, one labelled –16 and the other labelled –18.

Revisiting the Try This

B. Students could compare their method for solving addition of integers far from zero with the counter and number line models presented in the exposition. (Some students may have used a number line to answer **part A**.) Allow time for students to share their responses with each other.

Using the Examples

• Present several problems that involve adding negative integers close to zero. Ask students to solve these and explain how they solved each. Ask students to look for a general way to add negative numbers.

For example, you might give students the following calculations:

(–4) + (–2) (–5) + (–3) (–2) + (–7)

Students might notice that you can ignore the signs, add the integers, and then make the sum negative. Ask them to justify this procedure with counter or number line models.

• Present several problems that involve adding negative and positive integers close to zero. Ask students to solve these and explain how they solved each. Ask students to look for a general way to add numbers with different signs.

For example, you might give students the following calculations:

(–4) + (+2) (–5) + +3) (–2) + (+7)

Students might notice that you can ignore the signs, subtract the integers, and then make the sum the same sign as the integer farthest from zero. (Mathematicians call this the absolute value of the number. Students do not yet need to use this language, but you can introduce it if you wish.) Ask them to justify this rule with counter or number line models. This will help students understand why procedures work so they do not just apply rules for addition without understanding.

• Work through each example with the students to make sure they understand it. Draw students' attention to the techniques used to show greater numbers on a number line (using a scale of 5 or 10, for example) and with counters (using the bags, which could also be shown as circles).

Practising and Applying

Teaching points and tips

Q 3: Some students may wish to use the same model for each part. Encourage them to try using both number lines and counters.

Q 4: Some students may not be able to solve these questions without calculating each sum. Encourage them to compare the first part to the second part and to look for patterns. Remind them that they can use reasoning to help them answer these questions.

Q 5 and 6: These questions apply integer addition to real-world situations.

Q 7: Many students will not realize that they can estimate rather than finding the actual sum.

Q 9: Encourage students to break up numbers in a way that will allow them to use the zero property to simplify the calculation.

Q 11: Use this last question to highlight the important ideas students have learned in the lesson. You might encourage students to justify their ideas using models.

Common errors

• Students and teachers will often use the term "large" or "small" to describe integers without their signs.

For example, they might describe –45 as large and –2 as small. There is a hidden danger in this, as the language may be natural but it is not mathematically correct. One problem is that –45 is actually "small"; another is that we often think of large and small in terms of the physical size of the numerals on a page. Make sure students talk about numbers that are nearer to or farther from zero.

• Some students will have difficulty calculating the missing addends in **question 5**. You might have them write an addition sentence for the numbers in the chart.

For example, in **part b)**, students could write (+9) + ■ = +16. This might help them think of what number must be added to +9 to get +16.

• Some students will reverse the inequality signs in **question 7** and may need to be reminded that > means "greater than" and < means "less than".

• In **question 8**, many students will assume incorrectly that a number like –50 is greater than +10 because 50 > 10. Have students sketch a number line to compare these integers.

For example, in **part b)** students might assume that –560 is greater than –40.

Suggested assessment questions from Practising and Applying

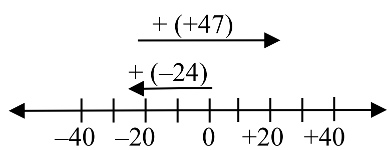
Question 3	to see if students can add numbers far from zero using a model
Question 5	to see if students can add integers in a real-world situation
Question 10	to see if students can generalize about the sign of the sum
Question 11	to see if students can justify a conjecture using a model

Answers

A. *Sample response*:
$(-12) + (+28) = (-12) + (+12) + (+16)$
$= 0 + (+16)$
$= +16$

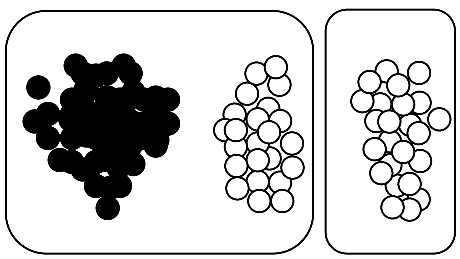
B. *Sample response*:
My method was similar to using counters because I used the zero property to pair (+12) with (–12) to get zero.

1.



I end at +23 because 47 – 24 = 23.

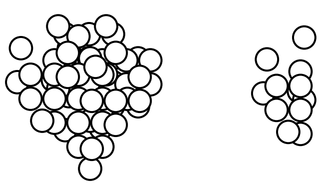
2.



24 pairs = 0 47 – 24 = 23 counters

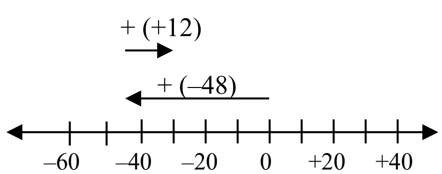
I match 24 of my 47 white counters with the 24 black counters to make 0 and I am left with 23 white counters, or +23.

3. a) +60; [*Sample response*:



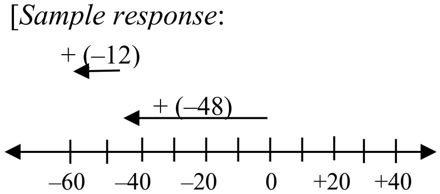
48 counters + 12 counters]

b) –36; [*Sample response*:



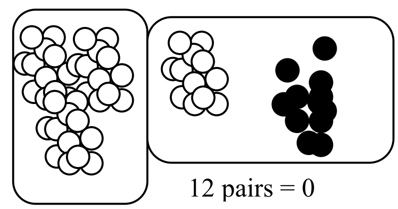
]

c) –60; [*Sample response*:



]

d) +36; [*Sample response*:



12 pairs = 0
48 – 12 = 36 counters]

4. a) 10; [*Sample response*:
I add the same amount each time, so I can just compare –24 and –34.]
b) 20; [*Sample response*:
I start with the same amount each time, so I can just compare –26 and –46.]

5.

	Start (°C)	Change (°C)	Final (°C)
a)	–12	+15	**+3**
b)	+9	**+7**	+16
c)	**–22**	+12	–10
d)	–15	**+3**	–12

6. *Sample response*: (+159) + (–3) = 156

7. a) > **b)** > **c)** = **d)** > **e)** <

8. a) i) –40 **ii)** –40 **iii)** –560 **iv)** +10
[**b)** *Sample response*:
Part iii) has two negative numbers very far to the left of zero. If I think about a number line, when I add two negative numbers, I move farther away from zero — the farther away from zero, the smaller the number.]

9. a) i) +12 **ii)** +80 **iii)** –27 **iv)** +25 [**b)** *Sample responses*: **i)** I imagined 25 on a number line and then moved 13 to the left, so I subtracted 23 – 13 = 12. **ii)** I thought about adding 17 white counters to 63 white counters, so I added 17 + 63 = 80. **iii)** I added the first two integers to get +10. Then imagined 37 black and 10 white counters. I subtracted 37 – 10 = 27 to find how many more white counters I had. **iv)** I added the first and last integers to get +30. Then I imagined 30 white and 5 black counters. I subtracted 30 – 5 = 25 to find how many more white counters I had.]	**10.** Yes; [*Sample response*: If I think of negatives as black counters and positives as white counters, all I have to do is decide whether there are more white counters or more black counters.] [**11.** *Sample responses*: **a)** If I start at a place left of zero on the number line and move farther to the left, the answer will always be negative. **b)** If two numbers add to zero, that means that they are the same distance from zero, which makes them opposite integers.]

Supporting Students

Struggling students

• If students are struggling with visualizing negative numbers far from zero in **question 8**, you might have them work with simpler numbers. Once students become better at mentally adding integers close to zero, have them visualize integers farther from zero using a pile of counters or a sketched number line.

For example:

For **part a) i)**, you might first ask students to calculate (–5) + (+1) and then ask them to consider (–50) + (+10).

For **part a) iv)**, you might have them find the sum for (+11) + (–10).

Enrichment

• For **question 11**, you might challenge students to make alternative conjectures that they feel are always true and ask them to justify their reasoning.

GAME: Target Sum –50

• This optional game is designed to allow students to practice adding 2-digit integers.

• As students try to figure out the best order and combination of digits, they are likely to estimate sums.

• Students should be encouraged to visualize counter or number line models when adding.

• Students can adapt the game by choosing to use two extra digit cards to create 3-digit numbers and targeting –500, or by targeting a different sum.

• A BLM of the game cards is provided but you can easily make your own cards using paper and markers.

5.2.3 Subtracting Integers Using Counters

Curriculum Outcomes	Outcome relevance
7-B10 Subtract Integers: to solve problems • connect visual models to symbols • use counters and real-life contexts **7-B8 Subtract Integers Mentally: develop and use strategies** • develop and use mental strategies - compatible numbers	Having experience with counter models will help students understand why subtraction of integers works so they will not just apply rules for subtraction without understanding.

Pacing	Materials	Prerequisites
1 h	• Counters in two colours, e.g., black and white counters	• subtraction facts • representing integers using counters

Main Points to be Raised

- You can use counters to model integer subtraction using a take-away meaning.
- If you do not have enough of one type of counter to take away, you can use the zero property to add more counters that you can take away.
- The zero property allows you to add pairs of opposite integers without affecting the sum.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

- *What addition is related to (–4) – (+2)?* (What integer added to +2 would make –4?)
- *How can you represent –4 as an integer addition?* ((–8) + (+4), (–7) + (+3), (–6) + (+2), and so on.)
- *Which addition can you use if you want to subtract +2?* ((–6) + (+2))
- *How can you represent this with counters?* (6 black counters and 2 white counters.)
- *Does it make sense that 6 counters and 2 counters can represent –4?* (Yes. I can pair up 2 white counters with 2 black counters and make 0. The 4 remaining black counters are –4.)

The Exposition — Presenting the Main Ideas

- Read through the exposition with the students. Use counters to demonstrate. Point out to students that you use the zero property when you do not have enough of one type of counter to take away. Make sure they realize that you cannot just add any counters you want, because that changes the problem; you have to add a form of zero because only zero does not change the value.
- Divide students into small groups. Provide each group with some counters. Present a subtraction calculation and have each group demonstrate it using the counters.

For example, you might have the groups model (–4) – (+2).

Circulate among the groups and check to see whether students are using the zero property. Encourage them to talk about their strategies. Have students demonstrate four or five similar problems.

Revisiting the Try This

B. Students now have a method to model the question in **part A** using a take-away meaning for subtraction.

Using the Examples

- Have each student work through **example 1** individually. This will allow you to see whether each student understands how to use the counter model. Ask students to explain why only 1 black and 1 white counter were added for **part a)** of **example 1**, but 4 white and 4 black counters were added for **part b)**.

• Work through **example 2** with the students, making sure that understand how to show integer subtraction by adding the opposite. Reinforce the concept with one or two more examples.

For example, you can show (+20) – (–4) by putting down 20 white counters and then placing another 4 white and 4 black counters. When you remove the 4 black counters, there are 24 white counters left. That is the same as starting with 20 white counters and adding the 4 white counters (the counters that you added with the 4 black counters using the first strategy), so (+20) – (–4) = +20 + (+4).

Practising and Applying

Teaching points and tips

Q 2: Remind students to draw diagrams similar to the diagrams in the examples.

Q 3: This is an important generalization that students should understand rather than just apply in a rote fashion. Counter models will provide this understanding

Q 4: Encourage students to do the calculations in the order they appear, but ask them afterwards if they could have subtracted the last number before the middle number.

Q 6: Many students will not know how to find two subtractions. This question might be assigned only to selected students.

Q 8: Encourage students to connect the procedure to a visualization of the counter model. Refer students to **example 2** to see how it is done.

Q 9: Some students may not recognize that they have to subtract Day 1 from Day 2, instead of the other way around.

Q 10: You might encourage students to talk in small groups about how they found each difference. Encourage them to communicate their thinking in a similar manner as shown in the Thinking part of the examples.

Q 11: You might have students share their understandings in a group discussion.

Common errors

• Many students will have difficulty finding the difference when taking away more than one integer in **questions 4 and 5**. You might allow some students to simply subtract the middle number from the first number.

• When adding the opposite, some students take the opposite of the minuend (the number they are subtracting from) instead of the subtrahend (the number they are subtracting). Ask students to use a model to check.

• Although it is not an error, some students choose to focus only on procedure even if it is not always efficient.

For example, for (–20) – (–4), it is more efficient to think of taking 4 black counters from 20 black counters, leaving 16 black counters, than to add (–20) + 4. It is for this reason that it is not a good idea to overemphasize the rule.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can subtract integers using counters
Question 3	to see if students can explain how subtracting an integer is the same as adding its opposite
Question 11	to see if students can generalize about the sign of the difference and explain their thinking

Answers

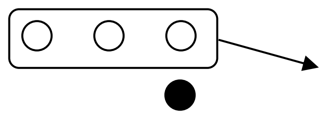
A. *Sample response*: She can rewrite it as an addition expression. If (–4) – (+2) = ?, then ? + (+2) = –4. She can think of what to add to +2 to get – 4: (–6) + (+2) = –4, so (–4) – (+2) = –6.	**B.** *Sample response*: I would add 2 white and 2 black counters, which would not add any value. Then I could take away 2 white counters. I would be left with 6 black counters, which is –6.

Answers [Continued]

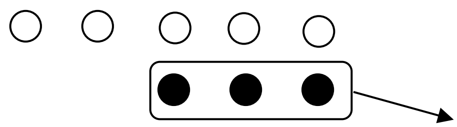
1. a) –3

[**b)** *Sample response*:
I started with 5 black counters and was able to take away 2 black counters without adding any other counters.]

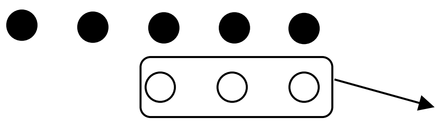
2. a) –1; *Sample response*:



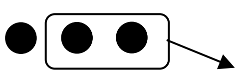
b) +5; *Sample response*:



c) –5; *Sample response*:

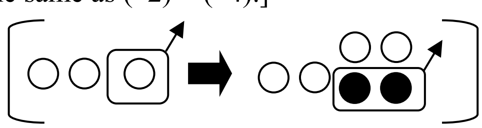


d) –1; *Sample response*:

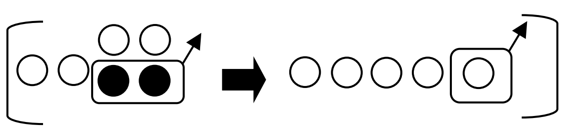


3. Yes; [*Sample response*:
To subtract (–2) – (+4), I would start with 2 black counters, add 4 white and 4 black counters, and then take away 4 white counters. That leaves me with 2 original black counters and 4 new black counters, which is the same as (–2) + (–4).]

4. a) +4;



b) +4;



5. a) –6 **b)** +17 **c)** –75

6. a) (–5) – (+2); (–5) – (–7)
b) (+4) – (–3); (+4) – (+7)

7. a) –7; +2 **b)** +7; –3

8. a) +27 **b)** –3
c) –40 **d)** –16

9.

	Golfer	Day 1	Day 2	Change (Day 2 – Day 1)
a)	Dechen	–4	–1	**+3**
b)	Dawa	+2	+6	**+4**
c)	Novin	–2	+4	**+6**
d)	Meto	**–7**	+3	+10
e)	Karma	–7	**–8**	–1

10. a) +21; [*Sample response*:
I subtracted just like with whole numbers, 44 – 23.]
b) –17; [*Sample response*:
I thought about subtracting 45 black counters from 62 black counters, which is 62 – 45.]
c) –136; [*Sample response*:
I imagined adding 33 pairs of black and white counters. Then I took away 33 white counters, leaving me with 33 + 103 black counters.]
d) –245; [*Sample response*:
I imagined adding 30 pairs of black and white counters. Then I took away 30 white counters, leaving me with 215 + 30 black counters.]

11. Yes; [*Sample response*:
Think of negatives as black counters and of positives as white counters. You cannot take away white counters from black counters. You always have to add enough pairs of black and white counters so you can take away the white counters. When you take away the white counters, you are left with only black counters, so the answer must be negative.]

Supporting Students

Struggling students

• If students are struggling with the idea of adding the opposite in **question 3**, you might have them use counters to show the first part of (–2) – (+4). It might be helpful to review **example 2** with them and provide them with additional questions.

Enrichment

• For **question 11**, you might challenge students to create additional conjectures about integer subtraction that are always, sometimes, and never true.

5.2.4 Subtracting Integers Using a Number Line

Curriculum Outcomes	Outcome relevance
7-B10 Subtract Integers: to solve problems • connect visual models to symbols • use number lines and real-life contexts **7-B2 Properties of Operations: integers** • review use of commutative and associative properties • explore the concept of "closure"	Students who have used number line models have an additional strategy to help them understand why procedures work so they do not just apply rules for subtraction without understanding.

Pacing	Materials	Prerequisites
1 h	• Number Lines (BLM) (optional)	• subtraction facts • representing integers using a number line

Main Points to be Raised

• You can use a number line to model integer subtraction.

• One way to calculate $a - b$ is to figure out what to add to b to get to a ($a - b \rightarrow b + ? = a$). This can easily be shown on a number line. The solution to the problem is a distance and direction on the number line.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How can you figure out the differences without counting on a number line (or thermometer)?* (I can subtract the usual low temperature from the usual high temperature.)

• *How do you know which places have the greatest differences?* (They are places that have both a high temperature that is very high and a low temperature that is very low.)

The Exposition — Presenting the Main Ideas

• Draw a thermometer on the board. Have students calculate the difference between the daily low and high temperatures of the place where they live. Encourage students to link a thermometer to a vertical number line.

• Write a subtraction sentence for finding this difference, e.g., (+13) – (–2). Ask students to give you a corresponding addition sentence, e.g., (–2) + ? = (+13). Encourage students to see that one way to calculate the difference is to find out what to add to the second number to get the first number.

• Read through the exposition with the students. Make sure they understand that when they add integers, the solution is the position where the arrow ends up, but when they subtract, the solution is a distance moved, either to the right or to the left.

Revisiting the Try This

B. Students need to think about whether to subtract the low temperature from the high temperature or the high temperature from the low temperature. They can then use a number line to calculate the difference.

Using the Examples

• Present the problems in the three examples to the students. Ask each student to choose two of the problems to solve. Then each student can compare his or her work with what is shown in the matching example. Suggest that students then read through the other example.

Practising and Applying

Teaching points and tips

Q 1: Remind students to look at the direction of the arrows to help them.

Q 2: This is an important question because it emphasizes that the order in which you subtract integers makes a difference in the result.

Q 3: Encourage students to think about how to rename one or more of the numbers to simplify the calculations.

For example, for **part d)**, they could rename +40 as +21 + (+9) + (+10).

Q 4: A flexible understanding of the relationship between subtraction and addition will help students develop number sense.

Q 6: Some students may notice that you can start with any correct number sentence and then increase or decrease both values by the same amount.

For example, since (–4) – 0 = (–4), then (–5) – (–1) = (–4) [subtract 1 from –4 and from 0].

Q 8: The direction of travel is important when finding each answer. This will affect the sign of the answer.

Q 9: You might have students look for counterexamples.

Common errors

• In **question 1**, some students may not be able to create a subtraction sentence from the model. You may wish to have them first write an addition sentence and then change it to a subtraction sentence.

For example, students can see that the arrow begins at +5 and ends at +25. They can write (+5) + ■ = +25. Then they can change this into a subtraction sentence: (+25) – (+5) = +20.

• Many students will subtract the elevations in the wrong order in **question 8**. You might have them think about whether the change in elevation is negative or positive and then check their answers using this context.

For example, traveling from Trongsa to Thimphu involves going from 2120 m to 2320 m. This is an increase in altitude so the change in elevation will be positive: (+2320) – (+2120) = +200.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can interpret a number line model for subtraction
Question 2	to see if students can subtract integers using a number line
Question 6	to see if students can represent a difference in many ways
Question 9	to see if students can communicate and justify conjectures about subtracting a positive and a negative integer

Answers

A. i)

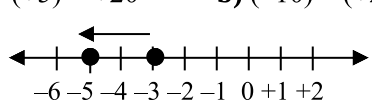
	Difference in temperature
Punakha	7°C colder
Paro	13°C colder
Thimphu	16°C colder
Wangdue	13°C colder
Bumthang	14°C colder

ii) Thimphu; *Sample response*:
16°C is the greatest difference.

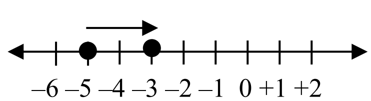
B. *Sample response*:
For each place, I can find the usual high temperature and the usual low temperature on the number line and then count the spaces between them to find the greatest distance.

1. a) (+25) – (+5) = +20 **b)** (–10) – (+20) = –30

2. a) i) –2;



ii) +2;

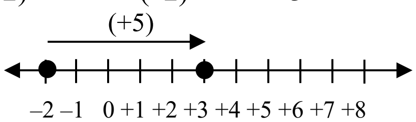


2. b) *Sample response*:
There are the same number of spaces between –3 and –5, but the arrows go in opposite directions.

3. a) –10 **b)** 0 **c)** –35 **d)** +70 **e)** –90

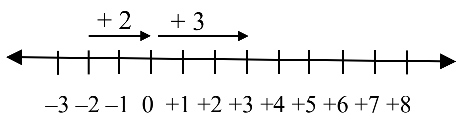
4. a) –2°C
b) (+10) – (+12) = –2
c) (+12) + (–2) = +10

5. [**a)** *Sample response*: (+3) – (–2)
(+3) – (–2) = ■ → (–2) + ■ = +3



The arrow from –2 to +3 is 5 spaces long.

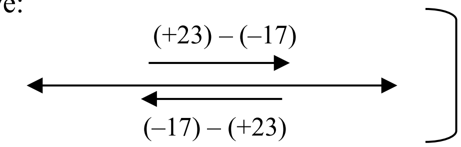
(+3) + (+2)   (+2) + (+3) = +5



The +5 arrow is broken into 2 parts, one from –2 to 0 and another from 0 to +3, a total of +2 + (+3) = +5.]
b) Yes; [*Sample response*:
If I want to subtract a number *n,* I can add *n* black and *n* white counters and it does not change the result (because I am adding zero). So, when I want to subtract *n* black counters (a negative integer), I add *n* white counters (a positive integer).]

6. *Sample response*:
(+4) – (+8) = –4, (+5) – (+9) = –4, (–3) – (+1) = –4

[**7.** *Sample response*:
When you model them on a number line, the arrows go in opposite directions, so one is positive and the other is negative:



8. a) (+2320) – (+2120) = +200
b) (+2120) – (+2235) = –115
c) (+1250) – (+2120) = –870
d) (+2120) – (+2320) = –200
e) (+2320) – (+2235) = +85
f) (+2235) – (+1250) = +985

9. a) The second expression, █ – (–1), is greater.
b) Yes; [*Sample response*:
When you subtract a positive integer from a number, it is like you are adding a negative. The answer will be less than the number, e.g., (+2) – (+1) = (+2) + (–1) = +1.
When you subtract a negative integer from a number, it is like you are adding a positive. The answer will be greater than the number, e.g., (+2) – (–1) = +1 and (+2) + (+1) = +3.]

Supporting Students

Struggling students

• If students are struggling with the idea that subtracting an integer is the same as adding the opposite in **question 5**, you may wish to have them use counters. The counter model might be more intuitive for some students. It is very important that they have a way of visualizing this process. Otherwise they will rely on memorizing this rule, and memorized rules are easily mixed up.

Enrichment

• For **question 6**, you might challenge students to find a pattern that describes the different subtraction expressions that can be made with two integers. Students could also generate several subtraction expressions using three integers and four integers.

5.2.5 EXPLORE: Integer Representations

Curriculum Outcomes	Lesson Relevance
7-A12 Integers: compare and order • represent integers in a variety of ways **7-B10 Add Integers: to solve problems** • connect visual models to symbols • use counters, number lines, and real-life contexts **7-B2 Properties of Operations: integers** • review use of commutative and associative properties	This optional exploration provides a problem-solving opportunity for students to use what they have learned about addition and subtraction of integers. It emphasizes that an integer can be represented in many ways. The ability to represent an integer many ways will help students simplify calculations.

Pacing	Materials	Prerequisites
1 h	• Counters in two colours, e.g., black and white counters	• addition facts • representing integers using counters

Exploration

• Work through the first part of the introduction (in white) with the students. Make sure they understand that the zero property can be used to represent an integer in many different ways. Use the example given and ask students to provide alternative representations of +6 using 8 counters, 12 counters, and 14 counters.

For example, + 6 could look like:

●○○○○○○○

or ●●●○○○○○○○○○

or ●●●●○○○○○○○○○○

• Work through the second part of the introduction with the students. Use the example given and ask students to show the addition using counters. Have them provide examples of how +6 can be represented as a sum of 3 integers, 5 integers, and 6 integers.

For example,

+6 = (–8) + (–4) + (+18)

+6 = (–8) + (–4) + (+18) + (+3) + (–3) OR (+2) + (–5) + (–3) + (+4) + (+8)

+6 = (–8) + (–4) + (+18) + (+3) + (–3) + (+2) + (–2) OR (–3) + (–7) + (+6) + (–1) + (+4) + (+7)

Encourage the students to use both positive and negative integers. Make sure students understand that there is more than one answer.

• Have students work in pairs for **part A**. Distribute counters for them to use. While you observe students at work, you might ask questions such as the following:

• *Can you represent –10 using 11 counters?* (No, I need 10 black counters to represent it, so if I add an eleventh counter, it will not be –10 anymore.)

• *Why can you use an even number of counters to represent –10 but not an odd number of counters?* (I can represent –10 with 10 black counters and then I can add pairs of black and white counters without changing the value. If I add pairs to an even number like 10, I always end up with an even number.)

• *How did you decide what integers to use in **part A ii)**?* (I just kept trying until something worked.)

• *How did your answer for **part A ii)** help you with the rest of the sums?* (I used my work for **part A ii)** and just added opposites using the zero property.)

• Discuss **parts A and B** with the students to make sure they are proceeding successfully.

• Have students continue to work in pairs for **parts C and D** to complete the exploration.

Observe and Assess

As students work, notice the following:

- Do they successfully apply the zero property?
- Do they understand how to use counters to represent an integer in many ways?
- Do they recognize patterns when they describe numbers using a sum of many integers?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and to answer these questions:

- *How do you know that you have found all the ways to represent each integer in* ***part D****?*
- *Why is the zero property helpful for finding more than one way to represent an integer?*
- *How can you predict the number of ways you can write +150 as a sum of two integers between 1 and 149?*
- *How can you organize your work to show patterns when you list the sums?*

Answers

A. *Sample responses*:

i) 10 black counters and 7 pairs of black and white counters
ii) (–2) + (–9) + (+1)
iii) (–2) + (–9) + (–1) + (+2)
iv) (–2) + (–9) + (+1) + (+1) + (–1)
v) (–2) + (–9) + (–1) + (+2) + (+1) + (–1)
vi) (–2) + (–9) + (+1) + (+1) + (–1) + (+1) + (–1)
vii) (–2) + (–9) + (–1) + (+2) + (+1) + (–1) + (+1) + (–1)
viii) (–2) + (–9) + (+1) + (+1) + (–1) + (+1) + (–1) + (+1) + (–1)
ix) (–2) + (–9) + (–1) + (+2) + (+1) + (–1) + (+1) + (–1) + (+1) + (–1)

D. i) 6 ways
ii) 4 ways
iii) You can divide the opposite of the integer by 2 to get the number of ways.
iv) 50 ways

B. *Sample response*:
Once I have described a number as a sum of a certain number of integers, I can always describe it as the sum of two more, four more, or six more integers just by adding pairs of + 1 and –1. For example, –10 as a sum of three integers could be (–2) + (–9) + (+1) and as a sum of five integers could be (–2) + (–9) + (+1) **+ (+1) + (–1)**.

C. *Sample responses*: I chose +4.

i) 4 white counters and 10 pairs of black and white counters
ii) (–2) + (+5) + (+1)
iii) (–2) + (+5) + (–1) + (+2)
iv) (–2) + (+5) + (+1) + (+1) + (–1)
v) (–2) + (+5) + (–1) + (+2) + (+1) + (–1)
vi) (–2) + (+5) + (+1) + (+1) + (–1) + (+1) + (–1)
vii) (–2) + (+5) + (–1) + (+2) + (+1) + (–1) + (+1) + (–1)
viii) (–2) + (+5) + (+1) + (+1) + (–1) + (+1) + (–1) + (+1) + (–1)
ix) (–2) + (+5) + (–1) + (+2) + (+1) + (–1) + (+1) + (–1) + (+1) + (–1)

D. i) Six ways **ii)** Four ways **iii)** Divide the number by 2 **iv)** 50 ways

Supporting Students

Struggling students

- If students are struggling with finding a sum of –10 using three integers in **part A**, have them choose any two integers and then find the third integer to add to make the sum work. Breaking the question into smaller parts might be helpful. For example, if students choose –5 and +3, this makes a sum of –2. Then they need to add –8 as the third number to make –10.

Enrichment

- For **parts A and C**, you might challenge students to use integers far from zero.

For example, they could use integers greater than 100 and less than –100. Have students look for patterns and organize their work to show patterns.

UNIT 5 Revision

Pacing	Materials
2 h	• Number lines (BLM) (optional) • Counters in two colours, e.g., black and white counters

Question	Related Lesson(s)
1 and 2	Lesson 5.1.1
3, 5, and 6	Lesson 5.1.2
4	Lesson 5.1.3
7	Lesson 5.2.1
8 and 9	Lesson 5.2.2
10	Lesson 5.2.3
11 and 12	Lesson 5.2.4

Revision Tips

Q 2: Some students may choose to use a number line to eliminate integers that do not meet the conditions.

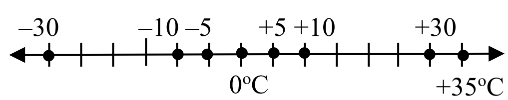
Q 4, 7, and 10: Students should be encouraged to use counters or a number line to help them solve these.

Q 5: Students might use a vertical number line to help them solve this.

Q 12: These are important generalizations.

Answers

1. a)

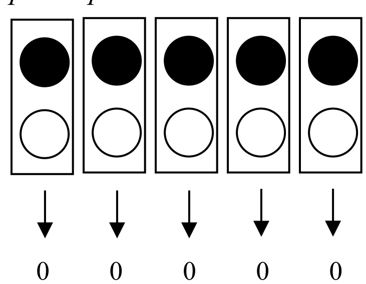


b) –30 and +30, –10 and +10, –5 and +5;
[*Sample response*:
These integers are the same distance away from zero.]

2. +3

3. a) False; [*Sample response*:
If an integer is farther from zero in a negative direction, then it is less. For example, –2 is farther from zero than –1 but it is less than –1.]
b) True; [*Sample response*:
All integers less than –5 are to the left of –5 on a number line, which means they are to the left of zero and must be negative.]
c) True; [*Sample response*:
0 is the next integer less than +1, and it is neither positive nor negative. All integers less than zero are negative.]

4. *Sample response*:



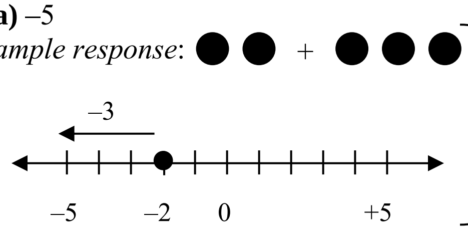
(–5) + (+5) = 0

5. a) +2ºC; [*Sample response*:
I added +4 to –2.]
b) +4ºC; [*Sample response*:
I added +2 to +2.]
c) +1ºC; [*Sample response*:
I subtracted +3 from +4.]

[**6.** *Sample response*:
–5 is left of +5 on a number line, so –5 is less than +5. The distance from zero is not important unless you are comparing two positives or two negatives.]

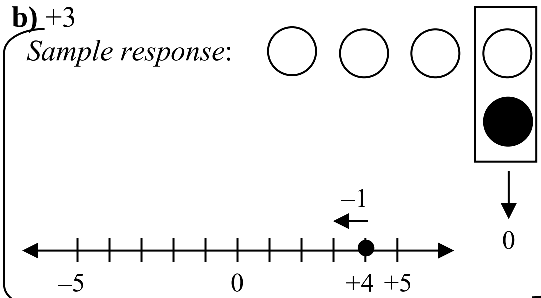
7. a) –5

Sample response:



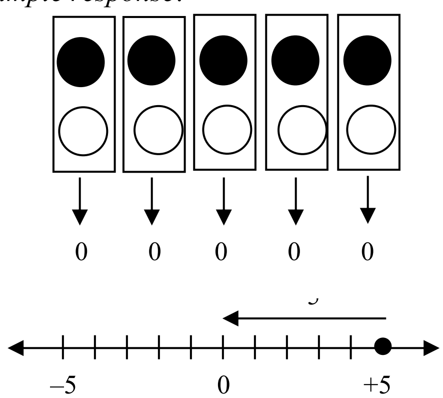
b) +3

Sample response:



c) 0

Sample response:

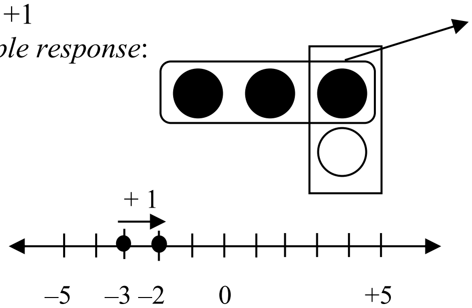


8. a) +10 **b)** +70 **c)** –70 **d)** –10

9. a) <; [*Sample response*:
100 – (+4) = 96 and +96 is left of +98 on a number line.]
b) <; [*Sample response*:
(–31) – (–3) = –28 and –37 is left of –28 on a number line.]

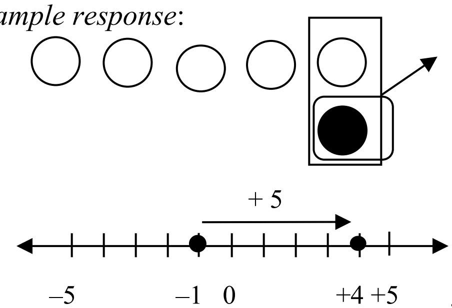
10. a) +1

Sample response:



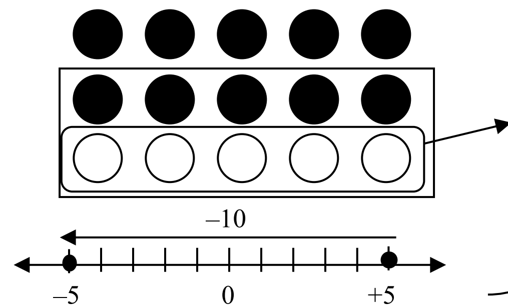
10. b) +5

Sample response:



c) –10

Sample response:



11. a) –340
b) +81
c) –60

[**12.** *Sample responses*:
a) If the distance between the positive integer and zero is greater than the distance between the negative integer and zero, the answer is positive.
b) Subtracting a negative integer is the same as adding its opposite, so if you are subtracting a negative integer from a positive integer, your answer will always be positive and it will be greater than the number you started with.]

UNIT 5 Integers Test

1. Draw a vertical number line from
–10 to +10.

a) Mark each integer on the number line.

0, +9, +4, –6, +7, –9, –4

b) Which pairs of integers are opposites?
Explain how you know.

2. Explain why +6 is greater than –6 even though both integers are the same distance from zero.

3. An integer is between –6 and +15.
It is half as far from +15 as it is from –6.
What is the integer?

4. Is each statement below true or false?
Explain your thinking.

a) A positive integer and a negative integer can be equally distant from a positive integer.

b) It is not possible for there to be 15 integers between a pair of integers.

c) There is no least negative integer.

5. Use a model to show that an integer added to its opposite makes zero. Sketch your model.

6. Suppose the low temperature for Thimphu on February 10 was +12ºC. What was the low temperature on each successive day?
Explain how you know.

a) February 11, down 4 degrees

b) February 12, up 2 degrees from February 11

c) February 13, down 5 degrees from February 12

7. Can two integers have a sum that is less than their difference? Explain your thinking using an example.

8. Add each using a number line.
Draw each solution.

a) (–3) + (–1)

b) (+2) + (–1)

c) (–4) + (+4)

d) (+1) + (+3)

e) (–2) + (–2)

9. Calculate each without a model.

a) (–20) + (+30) **b**) (+20) + (+30)

c) (–20) + (–30) **d)** (+20) + (–30)

10. Replace ■ with <, >, or =.
Explain how you know you are right.

a) –246 ■ –243 + (–42)

b) +27 ■ +39 – (–12)

11. Subtract using counters. Sketch each solution.

a) (–1) – (–2)

b) (+4) – (–6)

c) (–3) – (+3)

d) (+2) – (+1)

e) (–5) – (–1)

12. Subtract.

a) (–120) – (+330)

b) (+48) – (–23)

c) (–168) – (–38)

13. Suppose you know that 43 – x is positive. What do you know about x? Explain how you know.

UNIT 5 Test

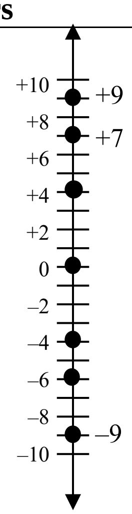
Pacing	Materials
1 h	• Number lines (BLM) (optional) • Counters in two colours, e.g., black and white counters

Question	Related Lesson(s)
1 and 3	Lesson 5.1.1
2 and 4	Lesson 5.1.2
5	Lesson 5.1.3
6 and 8	Lesson 5.2.1
9	Lesson 5.2.2
7 and 11	Lesson 5.2.3
10, 12, and 13	Lesson 5.2.4

Select questions to assign according to the time available.

Answers

1. a)



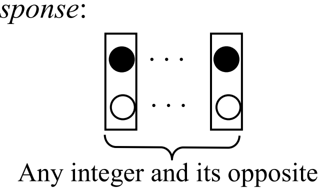
b) –9 and +9, –4 and +4; *Sample response*:
If I fold my number line at zero, these numbers are the same distance away from zero.

2. *Sample response*:
+6 is right of –6 on a number line. This means that +6 is greater than –6.

3. +8

4. a) True; *Sample response*:
–1 and +3 are equally far from +1.
b) False; *Sample response*:
Use two integers with a difference of 16, like 0 and 16 or –1 and 15.
c) True; *Sample response*:
There is always an integer to the left. For example, –1002 is left of –1001.

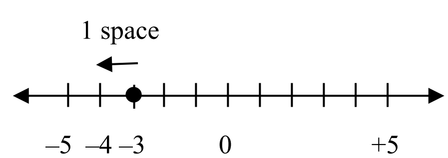
5. *Sample response*:



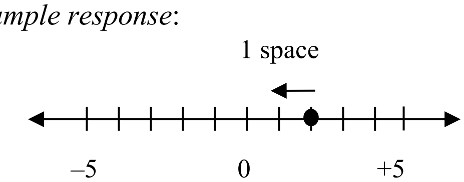
6. a) +8 ºC; *Sample response*:
I subtracted +4 from +12.
b) +10ºC; *Sample response*:
I added +2 to +8.
c) +5ºC; *Sample response*:
I subtracted +5 from +10.

7. Yes; *Sample response*:
(–3) + (–5) = (–8) but (–3) – (–5) = (+2)

8. a) –4
Sample response:

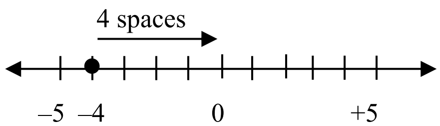


b) +1
Sample response:

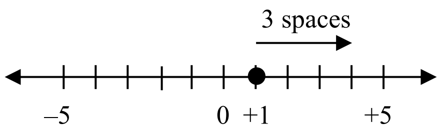


Answers [Continued]

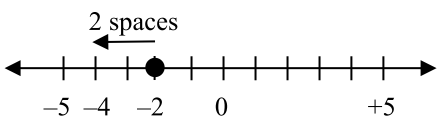
8. c) 0
Sample response:



d) +4
Sample response:



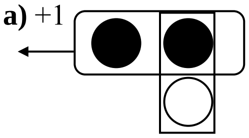
e) –4
Sample response:



9. a) +10
b) +50
c) –50
d) –10

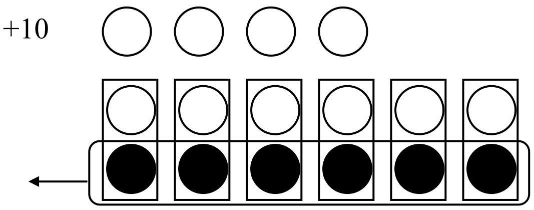
10. a) >; *Sample response*:
–246 is right of –286 on a number line;
(–243) + (–42) = –286.
b) <; *Sample response*:
+39 is right of +27 on a number line. If you add the opposite of –12, the result moves even farther to the right, so +27 must be less.

11. a) +1



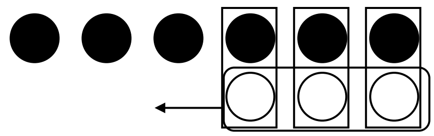
Add 1 pair and then take away 2 black counters.

b) +10



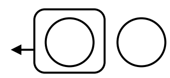
Add 6 pairs and then take away 6 black counters.

c) –6



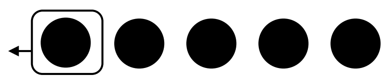
Add 3 pairs and then take away 3 white counters.

d) +1



Take away 1 white counter.

e) –4



Take away 1 black counter.

12. a) –450
b) +71
c) –130

13. x is any integer less than 43.
Sample response:
On a number line, the distance from x to 43 has to be a positive arrow, so x can be any integer left of 43, whether positive, zero, or negative.

UNIT 5 Performance Task — Magic Square

In a magic square, all rows, columns, and diagonals have the same sum, called the magic sum, and no number appears more than once.

A. In this magic square, every row, column, and diagonal has a magic sum of zero.

i) Copy and complete the square.

ii) Order the nine integers in the square from least to greatest.

-1	?	+3
?		-4
?	?	?

B. i) Add –5 to each value in the magic square from **part A** to make a new magic square.

-1 + (-5)	? + (-5)	+3 + (-5)
? + (-5)	? + (-5)	-4 + (-5)
? + (-5)	? + (-5)	? + (-5)

ii) Is it still a magic square? How do you know?

iii) What is the magic sum? How could you have predicted this?

C. i) Create a magic square that uses integers from –6 to +2.

ii) What is the magic sum? How could you have predicted the magic sum?

UNIT 5 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
7-B10 Add and Subtract Integers: to solve problems **7-B2** Properties of Operations: integers	1 h	None

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. It could also be used as enrichment material for some students. You can assess performance using the rubric provided below.

• If you find that the task takes too long, you can skip **part C**. You can also give students one more number in the original square in **part A** to simplify the task.

For example, you could tell them that –3 goes in the bottom left corner.

Sample Solution

A. i)

–1	–2	+3
+4	0	–4
–3	+2	+1

ii) –4, –3, –2, –1, 0, 1, 2, 3, 4

B. i)

–6	–7	–2
–1	–5	–9
–8	–3	–4

ii) Yes; The rows, columns, and diagonals all have the same sum.
iii)–15; I added –5 to each of three integers in each row, column, and diagonal.

C. i)

–3	+2	–5
–4	–2	0
+1	–6	–1

ii) –6; The new square uses –6, –5, –4, –3, –2, –1, 0, +1, +2. The square with the magic sum of 0 used –4, –3, –2, –1, 0, 1, 2, 3, 4. Each integer in the new square is 2 less than the corresponding integer in the square with a magic sum of 0. Because there are three integers in each row and the previous sum was 0, the change in the magic sum is (–2) + (–2) + (–2) = –6.

UNIT 5 Performance Task Assessment Rubric

The student	Level 4	Level 3	Level 2	Level 1
Completes the magic square	Correctly and independently completes the magic square; correctly orders the integers	Correctly completes the magic square with guidance; correctly orders the integers	Makes a minor mathematical error in completing the magic square; orders the integers incorrectly because of the minor error or requires an extra clue	Incorrectly completes the magic square even with direction or an additional clue and makes major mathematical errors
Transforms the magic square	Correctly and independently transforms the magic square; provides an insightful explanation for why it is a magic square; clearly describes a method for predicting the sum	Correctly transforms the magic square with guidance; provides a well-organized explanation for why it is a magic square; describes a method for predicting the sum	Makes a minor mathematical error in transforming the magic square; provides an explanation with teacher assistance; describes a method for predicting the sum with teacher assistance	Incorrectly transforms the magic square even with direction and makes major mathematical errors; explanation is disorganized; reasons for the prediction are difficult to follow
Creates a magic square	Correctly creates the magic square independently; provides the sum; clearly describes a method for predicting the sum	Correctly creates the magic square with guidance; provides the sum; describes a method for predicting the sum	Makes a minor mathematical error in a creating the magic square; provides an incorrect sum as a result; describes a method for predicting the sum with teacher assistance	Incorrectly creates the magic square even with direction and makes major mathematical errors; sum is incorrect; reasons for the prediction are difficult to follow

UNIT 5 Assessment Interview

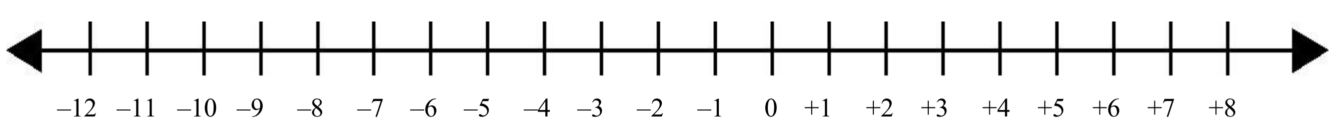
You may wish to take the opportunity to interview selected students to assess their understanding of the work of this unit. Interviews are most effective when done with individual students, although pair and small group interviews are sometimes appropriate. The results can be used as formative assessment or as a piece of summative assessment data. As the students work, ask them to explain their thinking.

Have available counters in two colours for the student to model integers.

• Ask the student to use the counters to do the following:

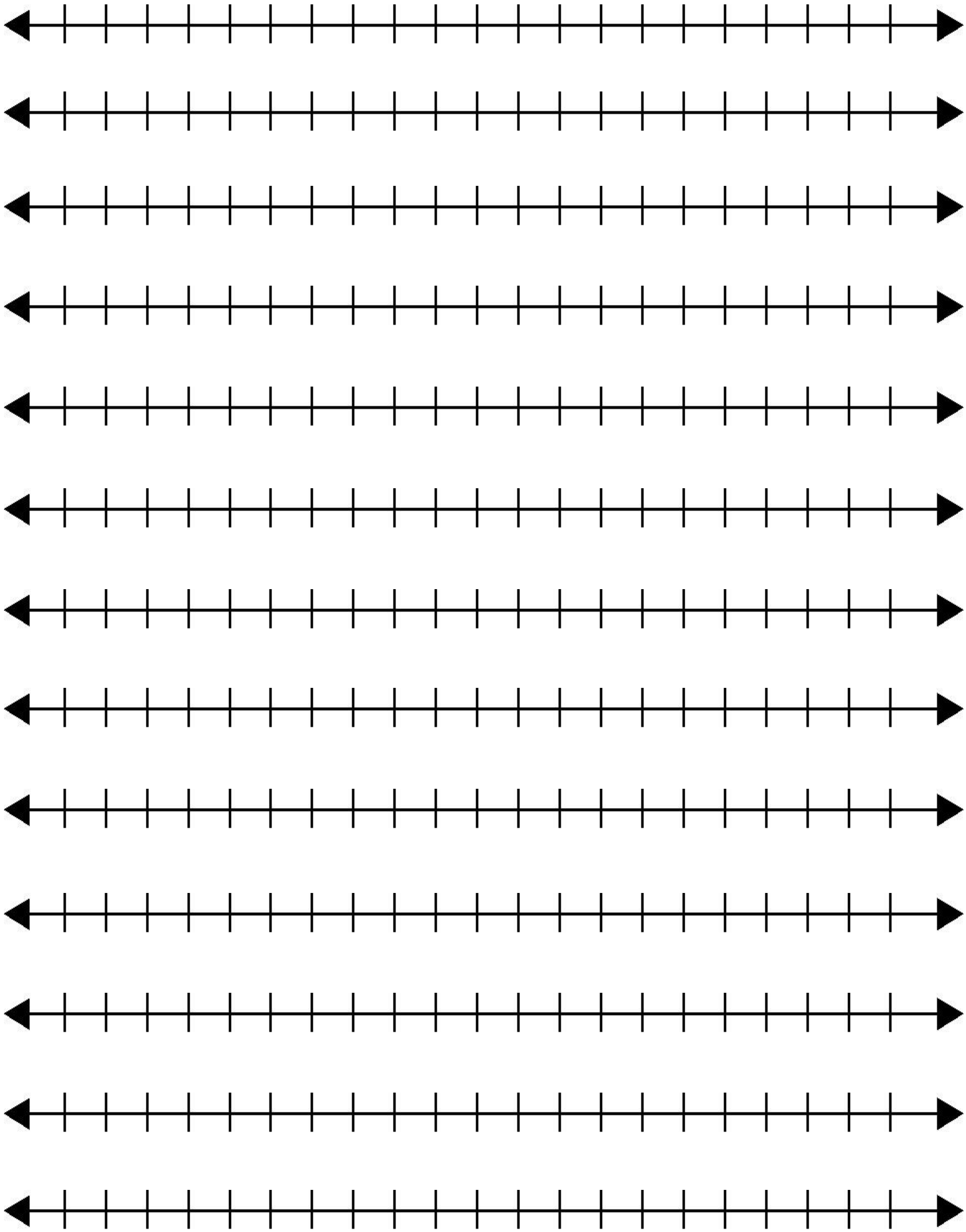
- *Show me four different integers that you can represent with 5 counters each. Tell the value of each integer.*
- *Put your four integers in order from least to greatest.*
- *Show how to add –4 and –7* [(–4) + (–7)].
- *Show how to add +4 and –7* [(+4) + (–7)].
- *Show how to subtract –7 from –4* [(–4) – (–7)].
- *Show how to subtract –4 from +2* [(+2) – (–4)].

• Then ask the student to use a number line to show all of the same calculations listed above.



• Finally, ask the student to pick a number greater than 38 and a number less than –49. Ask him or her to show how to add the two numbers and how to subtract the two numbers (in either order).

BLM 1 Number Lines



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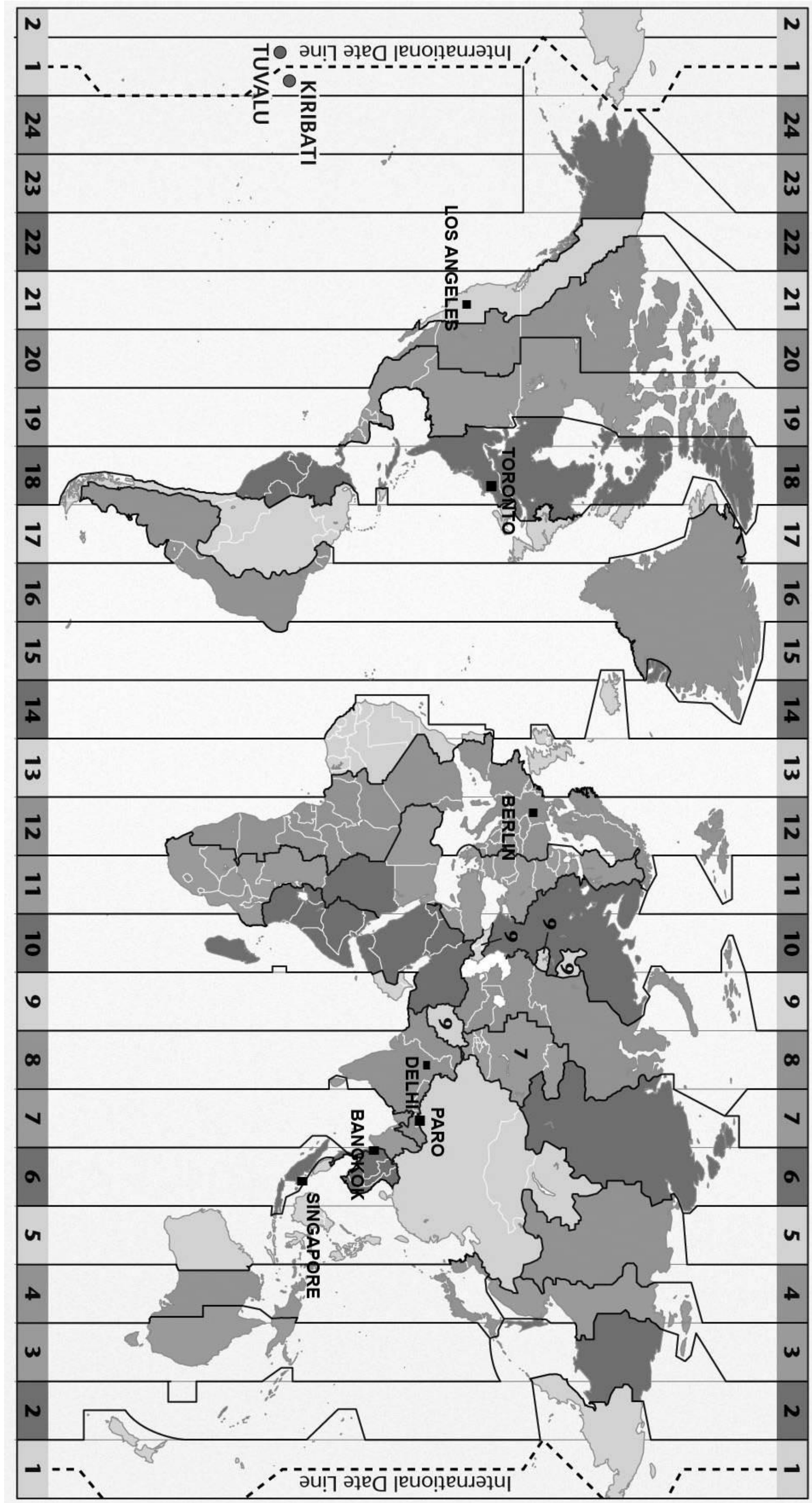
BLM 2

Time Zone Map



BLM 3 Target Sum –50 Game Cards

0	1	2	3	4
5	6	7	8	9
0	1	2	3	4
5	6	7	8	9
0	1	2	3	4
5	6	7	8	9
+	+	+	+	+
–	–	–	–	–

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UNIT 6 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 181** **TG p. 237**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• Grid paper or Small Grid Paper (BLM)	• All questions
Chapter 1 Patterns and Relationships				
6.1.1 Using Variables to Describe Pattern Rules **SB p. 182** **TG p. 240**	**7-B10 Simple Variable Expressions: relate to numerical expressions** • understand that quantities that change are called variables • develop a sense of why we need variables • use simple patterning **7-C1 Summarize Patterns: make predictions** • use constants, variables, and algebraic expressions to make predictions • recognize that variables can represent a changing quantity (e.g., $x = 4y$) • use tables to organize the information that a pattern provides	1 h	• Grid paper or Small Grid Paper (BLM) (optional) • Matchsticks (optional) • Linking cubes (optional)	Q1, 3, 6
6.1.2 Creating and Evaluating Expressions **SB p. 183** **TG p. 244**	**7-B10 Simple Variable Expressions: relate to numerical expressions** • recognize that the four operations apply in the same way as they do for numerical expressions • understand that quantities that change are called variables • develop a sense of why we need variables • use simple patterning • evaluate simple variable expressions by substituting for a variable in the expression • understand that what was true in evaluating numerical expressions applies to variable expressions, once the variable has been given a numerical value	1 h	None	Q1, 2, 5
6.1.3 Simplifying Expressions **SB p. 184** **TG p. 247**	**7-B10 Simple Variable Expressions: relate to numerical expressions** • recognize that the four operations apply in the same way as they do for numerical expressions • evaluate simple variable expressions by substituting for a variable in the expression • understand that what was true in evaluating numerical expressions applies to variable expressions, once the variable has been given a numerical value **7-B11 Like and Unlike Terms: develop meaning** • develop meaning visually • distinguish between like and unlike terms • add and subtract like terms by recognizing the parallel with numerical situations, using concrete and pictorial models	1 h	None	Q2, 4, 5

UNIT 6 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
CONNECTIONS: Using Variables to Solve Number Tricks (Optional) **SB p. 193** **TG p. 250**	Make a connection between computational strategies and algebra	20 min	None	N/A
Chapter 2 Solving Equations				
6.2.1 Solving Equations Using Models **SB p. 194** **TG p. 251**	**7-C1 Summarize Patterns: make predictions** • recognize that variables can represent a changing quantity (e.g., $x = 4y$) or a single value (e.g., $x + 3 = 9$) **7-C2 Single Variable Linear Equations: represent solutions** • show solution concretely and pictorially (one step, two step) using a variety of methods including a balance • use concrete models to show a solution to a simple equation (e.g., $e + 3 = 7$; how many are in the envelope?)	1 h	None	Q1, 4, 8
6.2.2 Solving Equations Using Guess and Test **SB p. 198** **TG p. 255**	**7-C3 Single Variable Linear Equations: one and two step** • solve equations using systematic trials	1 h	None	Q3, 6, 9
6.2.3 Solving Equations Using Inverse Operations **SB p. 201** **TG p. 258**	**7-C1 Summarize Patterns: make predictions** • use the term algebraic equation to describe a number sentence with a variable • distinguish between equations and expressions **7-C2 Single Variable Linear Equations: represent solutions** • show a solution pictorially (one step, two step) using a variety of methods including a balance • recognize that adding/subtracting the same value to/from both sides of an equation maintains balance **7-C3 Single Variable Linear Equations: one and two step** • solve equations using reasoning	1 h	None	Q3, 5, 6
GAME: Equations, Equations (Optional) **SB p. 203** **TG p. 260**	Practise solving simple equations in a game setting	20 min	• Game cards: digit, variable, and operation cards	N/A
6.2.4 EXPLORE: Solving Equations Using Reasoning (Optional) **SB p. 204** **TG p. 261**	**7-C3 Single Variable Linear Equations: one and two step** • solve equations using reasoning	40 min	None	Observe and Assess questions

Chapter 3 Graphical Representations				
6.3.1 Graphing a Relationship **SB p. 205** **TG p. 263**	**7-C4 Linear Equations: graph using a table of values** • use the x-axis and y-axis for the horizontal and vertical axes • use a table of values for graphing • interpolate (find a point between two known points) • extrapolate (find a point that lies beyond the existing data)	1.5 h	• Grid paper or Small Grid Paper (BLM)	Q1, 3, 6
6.3.2 Examining a Straight Line Graph **SB p. 209** **TG p. 267**	**7-C1 Summarize Patterns: make predictions** • use constants, variables, algebraic expressions and equations to make predictions • recognize that variables can represent a changing quantity (e.g., $x = 4y$) or a single value (e.g., $x + 3 = 9$) **7-C4 Linear Equations: graph using table of values** • use the x-axis and y-axis for the horizontal and vertical axes • determine if an ordered pair satisfies a given equation: - by plotting the points to see if they are in keeping with the rest of the points in the pattern - by substituting them into the equation to see if they make the equation true or false • equate an ordered pair that makes an equation true with the fact that it is a solution to the equation	1.5 h	• Grid paper or Small Grid Paper (BLM)	Q2, 3, 5
6.3.3 Describing Change on a Graph **SB p. 212** **TG p. 271**	**7-D6 Rate: compare two quantities** • construct and analyse graphs to show change • understand rate as the comparison of two quantities • write as a rate with different units (e.g., m/s, km/h, beats per min) • solve indirect problems	1.5 h	• Grid paper or Small Grid Paper (BLM)	Q1, 4, 5
6.3.4 EXPLORE: Are all Relationship Graphs Straight Lines? (Essential) **SB p. 216** **TG p. 276**	**7-C5 Graphs**: **linear and non-linear** • understand how changing one quantity affects the other • develop a sense of how the value of an expression changes with the value of the variable	60 min	• Grid paper or Small Grid Paper (BLM)	Observe and Assess questions
UNIT 6 Revision **SB p. 217** **TG p. 280**	Review the concepts and skills in the unit	2 h	• Grid paper or Small Grid Paper (BLM)	All questions
UNIT 6 Test **TG p. 283**	Assess the concepts and skills in the unit	1 h	• Grid paper or Small Grid Paper (BLM)	All questions
UNIT 6 Performance Task **TG p. 287**	Assess concepts and skills in the unit	1 h	• Grid paper or Small Grid Paper (BLM)	Rubric provided
UNIT 6 Blackline Masters	100 Charts on **page 49** in UNIT 1 Small Grid Paper on **page 53** in UNIT 1			

Math Background

• This algebra unit moves students from earlier work with patterns, relationships, and equation-solving into more formal work with algebra. Algebra is the study of relationships that form the basis of patterns.

• Students will explore the use of variables to express and generalize mathematical concepts, learn a variety of methods for solving equations, and see how graphing provides a great deal of information about relationships.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 4** in **lesson 6.1.2**, where they represent a problem algebraically to solve it, in **question 5** in **lesson 6.2.3**, where they solve a real-world problem using algebra, in **question 3** in **lesson 6.3.3**, where they solve rate problems using graphs, and in **lesson 6.3.4**, where they solve measurement problems using graphs.

• They use communication in **question 8** in **lesson 6.1.1**, where they explain the value of using variables to describe situations, in **question 7** in **lesson 6.1.3**, where they explain the value of algebraic simplification, in **question 10b** in **lesson 6.2.1**, where they describe the advantages and disadvantages of alternate models for solving equations, in **question 8** in **lesson 6.2.2**, where they describe why one approach is better than another for solving an equation, in **question 6** in **lesson 6.3.1**, where they talk about the usefulness of a graph for making predictions, and in **part D** of **lesson 6.3.4**, where they explain a prediction.

• They use reasoning in answering questions such as **question 5** in **lesson 6.1.3**, where they come to generalizations by using algebraic thinking, **question 9** in **lesson 6.2.1**, where they compare two models for the same equation, **question 6** in **lesson 6.3.2**, where they think about which graph would help them solve a particular equation and how two graphs are alike, and **question 5** in **lesson 6.3.3**, where they reason about where to look on a graph to compare different situations.

• They consider representation in **question 7** in **lesson 6.1.1**, where they notice how a different model for the same situation can lead someone to use different algebraic expressions, in **question 1** in **lesson 6.1.2**, where they represent situations algebraically, in **question 6** in **lesson 6.2.1**, where they represent a pattern problem algebraically, in **question 4** in **lesson 6.2.2**, where they represent a word sentence as an equation before solving it, and in **question 2** in **lesson 6.3.1**, where they represent a visual pattern in a table of values and graph.

• Students use visualization skills in **lesson 6.1.3**, where they represent variables and constants using geometric models to help clarify the concept of like terms, in **question 2** in **lesson 6.2.1**, where they use a model for an equation to help solve it, in **lesson 6.2.3**, where they use a balance model to visualize an equation, in **question 5** in **lesson 6.3.1**, where they use a visual display to solve a problem, and in **question 2** in **lesson 6.3.3**, where they use a graph to visualize a rate.

• They make connections in situations like those in **lesson 6.1.2**, where they relate word phrases to algebraic expressions, in **question 4** in **lesson 6.1.3**, where they connect measurement formulas to algebraic thinking, in **lesson 6.3.3**, where they relate the concepts of rate to the concepts of graphing, and in **lesson 6.3.4**, where they connect measurement situations to types of graphs.

Rationale for Teaching Approach

• This unit is divided into three chapters.

Chapter 1 focuses on using variables to describe mathematical concepts and relationships.

Chapter 2 focuses on multiple ways to solve a simple equation.

Chapter 3 explores the use of graphical representations to describe relationships, learn more about relationships, and solve equations.

• The first **Explore lesson** allows students to develop more intuitive strategies for solving equations. The second **Explore lesson** helps students see that not all relationships are linear.

• The **Connections** is an engaging way for students to learn the power of variables in explaining computational situations.

• The **Game** in the unit practises equation-solving skills.

Getting Started

Curriculum Outcomes	Outcome relevance
6 Equivalent Ratios: change in one term affects the other term **6** Equivalent Ratios: represent in tables and graphs **6** Area Patterns: explore **6** Square and Triangular Numbers: represent pictorially and symbolically **6** Linear Equations: using open frames **6** Coordinates: plotting	Students will find the work in the unit easier after they review the concepts of patterns, equivalent ratios, using variables in equations and formulas, coordinates, and square and triangular numbers.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM)	• familiarity with the terms *ray, vertex, triangular number, square number, solution, ordered pair,* and *ratio* • factoring simple whole numbers • plotting on a coordinate grid • writing ratios and equivalent ratios • using measurement formulas

Main Points to be Raised

Use What You Know

• Triangular numbers are the numbers 1, 3, 6, 10, 15, …. The distance between the values increases by one more each time.

• Triangular and square numbers are useful and common patterns.

Skills You Will Need

• Square numbers are the numbers 1, 4, 9, 16, …. Each is the square of a counting number. You can represent each square number as an array with an equal number of rows and columns.

• Solving an equation means determining the values of a variable that make the two sides of the equation equal. Different equations can have the same solution.

• You can use ordered pairs to locate points on a plane.

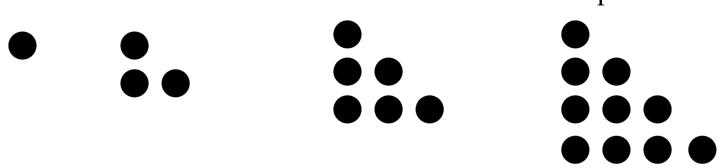
• Whenever a ratio describes a situation, there are always other ratios that can describe the same situation.

• Equivalent ratios form a line when they are graphed.

• When you describe measurements using a formula, you can predict how the changes in one measurement will affect some of the other measurements in the formula.

Use What You Know — Introducing the Unit

• Before assigning the activity, you may wish to review the meaning of some of the terms that will come up in the activity, particularly the terms *ray, vertex*, and *triangular number*. You might review the first two terms by drawing an angle and pointing to each of the two rays and to the vertex. To review the concept of triangular numbers, you might ask students if they recall what these are. If they do not remember, draw a picture of the triangular numbers and ask students to describe the number pattern that goes with the picture (1, 3, 6, 10, …).



• Students can work in pairs or individually to complete the activity.

• Although this activity may not seem to relate to algebra, it does. Triangular numbers are one example of relationships that are generally described algebraically. By completing the activity, students see that the same mathematical idea can be embodied in a number of different situations; this is at the heart of algebraic thinking.

While you observe students at work, you might ask questions such as the following:

• *Where are the three angles?* (The two little angles and the big angle made of both angles together.)

• *When you added a new ray, how many angles did you add?* (Three angles — the new little angle, a new big angle that includes the new angle, and a new middle-sized angle when I combined the new small angle with the angle next to it.)

• *How many dots do you predict for the 10th diagram (after you have added 9 rays to the vertex)?* (I predict 55 because I would be adding 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10.)

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions.

• First, review the terms *square number*, *solve, ordered pair, ratio, equivalent ratio,* and *expression* to make sure students can interpret the questions. Refer students to the glossary at the back of the student text.

• Provide grid paper or Small Grid Paper (BLM) for students to complete **question 5**.

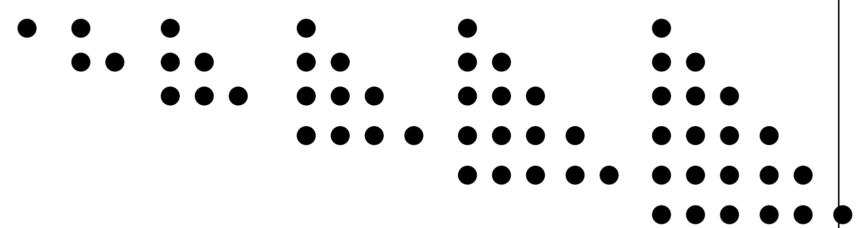
• Assure students that the purpose of this activity is to find out what they remember. If they have difficulty remembering an idea, they should feel free to ask.

• Students can work individually.

Answers

A. ii) There are the two small angles and one large angle made up of the two small angles.
iii) 6 angles
iv) 10, 15, and 21 angles

B. i)

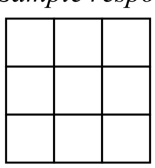


1 3 6 10 15 21

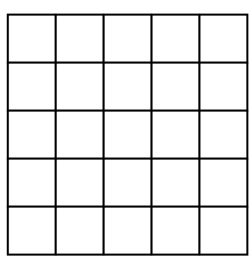
ii) For each number, you can take that number of items and form a triangle.

1. *Sample responses*:

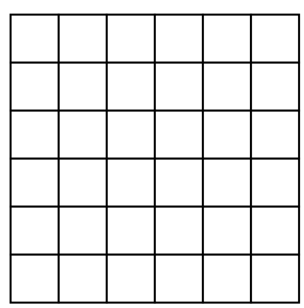
a)



b)



c)

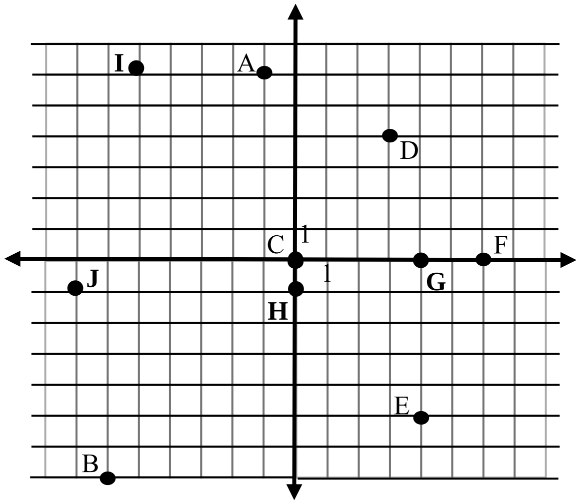


2. a) $n = 3$ **b)** $h = 47$
c) $n = 9$ **d)** $a = 3$

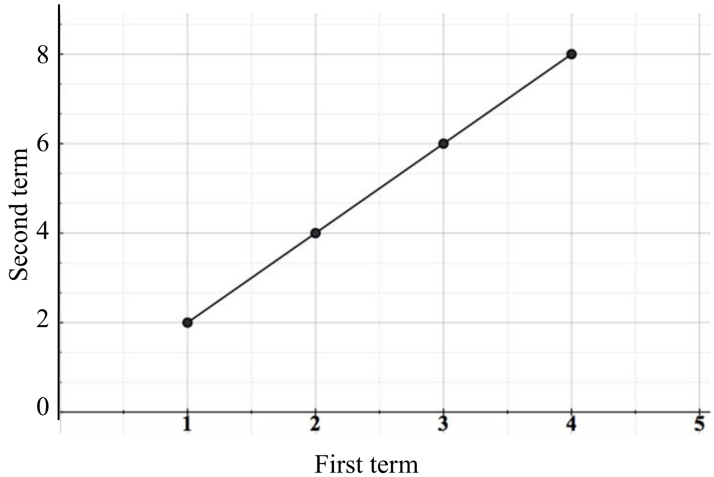
3. a) and d)

4. A(–1, 6); B(–6, –7); C(0, 0); D(3, 4); E(4, –5); F(6, 0)

5.



6. a) 2:4
b) *Sample response*: 1:2, 4:8, 3:6
c) and d) (1, 2), (2, 4), 3, 6), (4, 8)



e) They form a straight line.

7. a) i) $bh \div 2$ **ii)** bh
b) i) The area would double for both shapes.
ii) The area would be multiplied by 4 for both shapes.
iii) The area would not change for either shape.

Supporting Students

Struggling students

• If students are struggling with the activity, you may wish to model one or two additional steps in each of **parts A and B**.

• If students struggle with **question 2 or 4**, you may wish to review these topics. They will be critical for the unit.

Enrichment

• You might challenge students to draw other pictures similar to the dot picture in **question 6**, create equivalent ratios, plot the points, and notice the shapes of the new graphs.

Chapter 1 Patterns and Relationships

6.1.1 Using Variables to Describe Pattern Rules

Curriculum Outcomes	Outcome Relevance
7-B10 Simple Variable Expressions: relate to numerical expressions • understand that quantities that change are called variables • develop a sense of why we need variables • use simple patterning **7-C1 Summarize Patterns: make predictions** • use constants, variables, and algebraic expressions to make predictions • recognize that variables can represent a changing quantity (e.g., $x = 4y$) • use tables to organize the information that a pattern provides	Variables are an important tool for solving problems and describing mathematical situations. Pattern rules are a natural starting place for the use of variables.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM) (optional) • Matchsticks (optional) • Linking cubes (optional)	• formula for the area of a parallelogram

Main Points to be Raised

- A table of values is a way to describe a formula, or relationship.
- A pattern rule can be thought of as a sort of formula.
- You can write a pattern rule using variables, coefficients, and constants.
- A coefficient is always multiplied by a variable. A constant is always the same value added or subtracted no matter what the value of the variable is. Sometimes the constant is 0 (it does not appear).

Try This — Introducing the Lesson

A. and B. Allow students to try these alone or with a partner. While you observe students at work, you might ask questions such as the following:

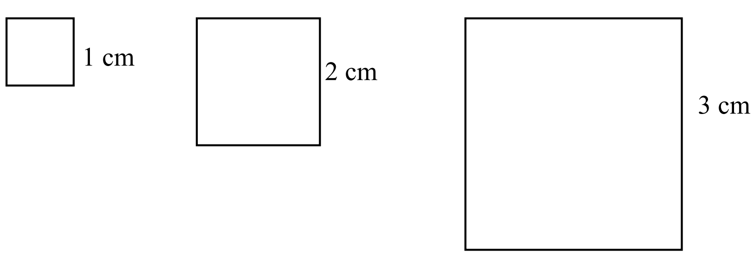
• *Why was it easy to complete* ***part A iii)*** *after the first two parts were completed?* (You add the number of white squares and black squares to get the total number of squares.)

• *How is each pattern changing?* (The number of black squares never changes. The number of white squares grows by two each time. So does the total number of squares.)

• *How did you make the prediction in* ***part B****?* (I added 11 twos to the number of white squares and to the total number of squares from Figure 1 The number of black squares does not change, so I still used three.)

The Exposition — Presenting the Main Ideas

• Draw several squares on the board. Write the dimensions on each. Ask students how to calculate the perimeter.

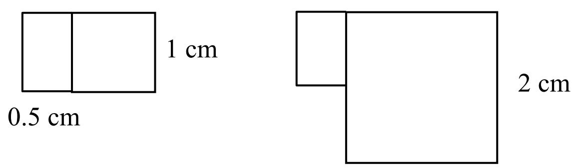


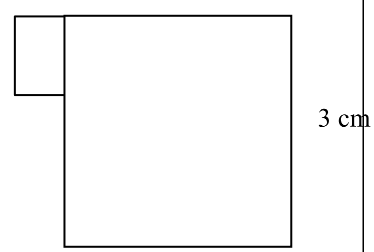
- Show students how to create a table of values to relate the area of a square to the side length.

Side length (s)	1	2	3	4	5	6
Area ($A = 4 \times s$)	4	8	12	16	20	24

- Point out how this could be written with variables using a formula: $A = 4 \times s$.
- Mention that s is the *variable* because the values you can use to replace it *vary*.
- Explain that 4 is the *coefficient*. It is the value the variable is multiplied by.
- Show how you can use both the formula and the table of values to predict the perimeter for a square with side length 20 cm. You use the formula by substituting the number 20 for s. You use the table by observing the pattern and extending it.
- The pattern for the table is: starts at 4 and goes up by 4, so the *pattern rule* can be applied. You add 4 nineteen times to the original value of 4 (to get the perimeter for a square of side 20).

• Draw shapes like these:





- Help students see that the perimeter of the square increases by 4 cm each time minus the 1 cm hidden by the rectangle.
- Ask students why the expression $4s - 1$ can be used the find the perimeter of the square in each figure if s is the side length in centimetres. (You multiply the side length by 4 but subtract 1 for the hidden part.)
- For the total perimeter, you need to add on the 2 cm on the outside of the rectangle that is attached to the square's left side.
- Ask students why the expression $4s + 1$ can be used the find the total perimeter when the square has a side length of s. (You multiply the side length of the square by 4, subtract 1 for the hidden part, and then add 2 for the rectangle, which means you end up adding 1).
- Tell students that the 1 is called a *constant* because you add the same value of 1 to the perimeter no matter what the value of s is.

• Mention that for an expression such as $4f$, you can say either that the constant is 0 ($4f + 0$) or you can say that there is no constant.

• You may wish to quickly go through the exposition with the students to solidify the ideas presented.

Revisiting the Try This

C. Students can create the table of values vertically (as suggested in the question) or horizontally as shown in the exposition.

Using the Examples

• Pair up students to work through the examples. One student should focus on **example 1** and the other student should focus on **example 2**. Each student should then explain his or her example to the other.

• Here is an alternative solution for **example 1**:

The pattern started at 4 and added 3 each time:

The **1**st number was 4 + 3(**0**).

The **2**nd number, 7, was 4 + 3(**1**).

The **3**rd number, 10, was 4 + 3(**2**).

The **4**th number, 13, was 4 + 3(**3**).

The **5**th number, 16, was 4 + 3(**4**)

The number of matchsticks is equal to 4 plus 3 times 1 less than the figure number.

• Have the students discuss how colouring (or shading) the constant in **example 2** helps them identify the constant in order to write the pattern rule.

Practising and Applying

Teaching points and tips

Q 2: Some students may not list the variable on the left hand side because it is not part of the pattern rule. Talk about why this decision is reasonable, but that it is also reasonable to list the variable because it does vary.

Q 3: Encourage students to explain how they extended the table each time.

Q 4: Some students will notice, without counting the total number of cubes each time, that there are 4 more cubes each time the figure number increases, one on each of the arms. Some may even notice that you can multiply the figure number by 4 (one for each arm) and then add in the centre cube.

Q 5: If necessary, draw students' attention to the increase of 5 each time.

Q 7: There are always many ways to write a pattern rule. The shading is there simply to encourage specific ways.

Q 8: You may wish to handle this question in a class discussion rather than assigning it to individual students.

Common errors

• Many students incorrectly predict values that are not in the table by multiplying the figure number by the increase.

For example, for the sequence 4, 6, 8, 10, …, a student who notices that the sequence goes up by 2 may say that the 20th term is 20 × 2, or maybe 4 + 20 × 2, when in fact it is 4 + 19 × 2.

Encourage those students to extend the table to check their predictions.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use the terms *variable, coefficient,* and *constant*
Question 3	to see if students can extend a table of values
Question 6	to see if students can create and use a table of values to describe a geometric situation

Answers

A. i) 4, 8, 12, 16
ii) 3, 3, 3, 3
iii) 7, 11, 15, 19

B. i) 48 **ii)** 3 **iii)** 51

C. i)

Figure number	Total number of squares
1	7
2	11
3	15
4	19

ii) $4f + 3$

1. a) Variable is h; coefficient is 3; constant is 5.
b) Variable is m; coefficient is –2; constant is –4.
c) Variable is q; coefficient is 1; constant is 6.
d) Variable is n; coefficient is 5; constant is 3.

2. a) Variables are s and P; coefficient is 4 (or 4 for s and 1 for P); (constant is 0).
b) Variables are A and r; coefficient is π (or π for r^2 and 1 for A); (constant is 0).

3. a)

Figure number	Number of shapes
1	6
2	8
3	10
4	12
5	**14**
6	**16**
7	**18**

b)

Figure number	Number of shapes
1	35
2	30
3	25
4	20
5	**15**
6	**10**
7	**5**

4. a)

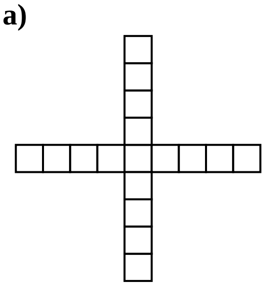


Figure 4

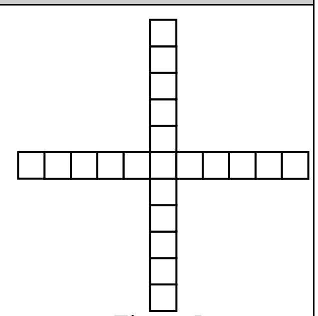


Figure 5

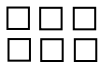
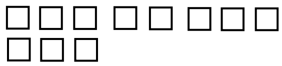
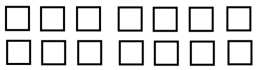
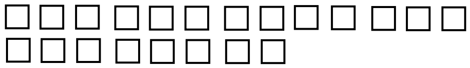
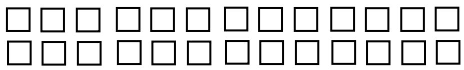
b)

Figure number	Number of squares
1	5
2	9
3	13
4	17
5	21
6	25
7	29
8	33

NOTE: Answers or parts of answers that are in square brackets throughout the Teacher's Guide are NOT found in the answers in the student textbook.

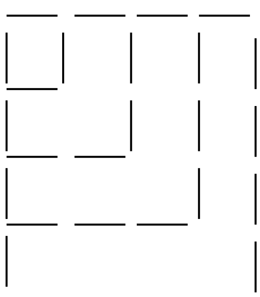
4. c) $4f + 1$ or $5 + 4(f - 1)$
d) 81

5. *Sample responses*:
a)

Figure number	Figure	Number of squares
1		6
2		11
3		16
4		21
5		26

b) $6 + 5(f - 1)$ or $5f + 1$

6. a)



b)

Figure number	Number of matchsticks
1	4
2	10
3	18
4	28
5	40
6	54
7	70
8	**88**

7. $2(n + 1) + 2$ and $2n + 4$

[**8.** *Sample response*:
You can use it for any figure number instead of for just one figure.]

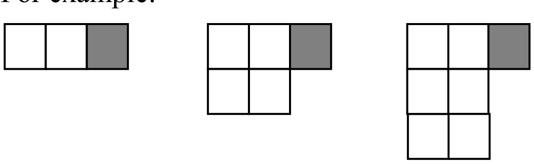
Supporting Students

Struggling students

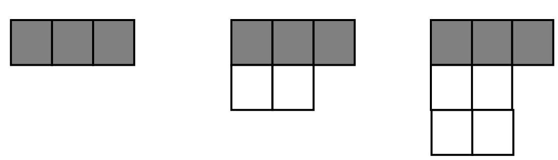
• Some students may need your help in starting the tables for **questions 4 and 6**. For **question 7**, you may allow struggling students to write only one pattern rule rather than two equivalent rules.

Enrichment

• Ask students to create other patterns like those in **question 7** with shading to suggest alternative pattern rules.
For example:



OR



6.1.2 Creating and Evaluating Expressions

Curriculum Outcomes	Outcome relevance
7-B10 Simple Variable Expressions: relate to numerical expressions • recognize that the four operations apply in the same way as they do for numerical expressions • understand that quantities that change are called variables • develop a sense of why we need variables • use simple patterning • evaluate simple variable expressions by substituting a variable in the expression • understand that what was true in evaluating numerical expressions applies to variable expressions, once the variable has been given a numerical value	Students need to become comfortable with writing algebraic expressions to describe situations so they will have success with algebra in higher classes.

Pacing	Materials	Prerequisites
1 h	None	• recognizing situations that can be described using multiplication

Main Points to be Raised

• You can think of an expression as one side of an equation.

• Expressions can involve numbers, variables, and operations.

• You can translate a mathematical expression into a word phrase, or vice versa.

For example, 2 more than a number can be written as $n + 2$.

• When an expression involves a variable, it is called an algebraic expression. When you replace the variable by a particular number, it is called substitution.

• You can use algebraic expressions and equations to represent situations. Solving an equation can give you the answer to a problem.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know the total sales were more than Nu 1250?* (That would only be multiplying by 10, so it would mean selling only 10 bangchung.)

• *How much would 100 bangchung cost? How do you know?* (Nu 12,500; multiplied by 100 by adding two zeros.)

• *How does knowing how much 100 banchung cost help you figure out the price for 50?* (Take half of the total.)

The Exposition — Presenting the Main Ideas

• Write 3 + 5, 3 + ■, $n + 3$, and $4n - 4$ on the board. Explain to students that each of these is called a mathematical expression. Help them notice that some have no variables at all (3 + 5), some have a variable represented by an open frame (3 + ■), and some have a variable that is a letter.

• Ask students to tell what $n + 3$ and $4n - 4$ mean ("three more than a number" and "four less than four times a number"). Talk about how these algebraic expressions are a way to translate the word expressions. Ask them why you could write "four more than twice a number" as either $2n + 4$ or as $4 + 2n$, but not as $4n + 2$.

• Ask them what the value of $2n + 4$ would be for different values of n, e.g., $n = 0$, 2, or 3 (4, 8, 10). Tell them that this is called substituting for the variable.

• Then ask them how they might write an algebraic translation for the phrase "a number added to its double" ($n + 2n$, or $3n$). Point out that any letter could have been used to represent "a number".

• Show students how you can use an algebraic expression to solve a problem.

For example, state that a number and its double add up to 387. Ask students to write an algebraic expression to represent a number and its double ($n + 2n$). Then show how you can substitute values for n to figure out what the value of n must be.

If $n = 100$, then $n + 2n = 100 + 200 = 300$, which is too low.

If $n = 150$, then $n + 2n = 150 + 300 = 450$, which is too high.

If $n = 120$, then $n + 2n = 120 + 240 = 360$, which is too low, but close.

Eventually students will find that $n = 129$ works.

• Have students turn to **page 187** in the student text and examine the chart in the middle of the page. Make sure they understand why each word phrase has been translated into an algebraic expression.

Revisiting the Try This

B. This question allows students to make a formal connection between what was done in **part A** and the main ideas presented in the exposition.

Using the Examples

• Read aloud the expressions to be translated in **example 1**. Ask students to write down the algebraic expressions they would use. Then have them check their answers against the solution in the student text.

• Work through **example 2** with the students. Ask why $m(m + 2)$ represents multiplying a number by another number that is 2 greater than itself.

Practising and Applying

Teaching points and tips

Q 1: Observe how students make the matches.

For example:

- When a phrase says “the total cost”, do they realize there has to be a + sign in the expression?

- When the phrase says “average”, do they realize there must be a sum and a division?

Q 2 d): Make sure students realize that they must substitute the value for n each time the variable appears.

Q 3: Students will be more successful if they use the definition of even number as a number that is a multiple of 2 and not a definition that looks at the last digit of the number.

Q 5: Note that students might write $7000m + 500m$ or they might write $7500m$.

Q 6: Students should use a single expression that combines the two costs.

Common errors

• Many students have more difficulty translating from words into algebra than vice versa. Help students deal with each part of the phrase separately. They can ask themselves what operation signs they would expect to see based on the words used.

For example, for a phrase like "the sum of a multiple of three and a number one greater than a multiple of three":

- "multiple of three" suggests a multiplication
- "sum" suggests an addition
- "one greater" suggests an addition

Suggested assessment questions from Practising and Applying

Question 1	to see if students can translate a word phrase into algebra
Question 2	to see if students can substitute into an algebraic expression
Question 5	to see if students can describe a real-world problem situation algebraically and then solve it

Answers

A. Nu 8500	**B. i)** 125p **ii)** p + 50 **iii)** 18
1. a) vi) **b)** v) **c)** ii) **d)** vii) **e)** iv) **f)** i) **g)** iii) **2. a)** 23 **b)** –23 **c)** 44 **d)** 1.7 **e)** 452 **f)** 29 [**3.** *Sample responses*: **a)** An even number is a multiple of 2 and 2n is also a multiple of 2. **b)** 3n]	**4. a)** 72r **b)** You would subtract 38 from 72r, 72r – 38 **5. a)** 7500m **b)** 7500 × 12 = Nu 90,000 **6. a)** 20 + 15n **b)** 20 + 15 × 25 = Nu 395 **7.** *Sample response*: Lemo bought 2 kg of meat. The price per kilogram is Nu x. She also bought some oranges worth Nu 60. How much did she spend altogether?

Supporting Students

Struggling students

• Struggling students will benefit from additional modelling of translation from words to algebra and vice-versa. You might provide a number of extra examples before asking them to complete the exercises.

• For **question 7**, you might provide some word problems and first ask for the related algebraic expression before asking students to complete the question.

Enrichment

• Students who find the exercises simple may enjoy making up their own situations like those in **questions 4 and 5** for other students to try.

6.1.3 Simplifying Expressions

Curriculum Outcomes	Outcome relevance
7-B10 Simple Variable Expressions: relate to numerical expressions • recognize that the four operations apply in the same way as they do for numerical expressions • evaluate simple variable expressions by substituting a variable in the expression • understand that what was true in evaluating numerical expressions applies to variable expressions, once the variable has been given a numerical value **7-B11 Like and Unlike Terms: develop meaning** • develop meaning visually • distinguish between like and unlike terms • add and subtract like terms by recognizing the parallel with numerical situations, using concrete and pictorial models	It is important for students to be able to simplify expressions in order to take advantage of the algebraic techniques they will learn for solving equations.

Pacing	Materials	Prerequisites
1 h	None	• the zero principle for integers • adding and subtracting integers

Main Points to be Raised

• We can use different shapes to represent variables. If you use more than one variable, you should use more than one shape. (In this lesson, only one variable is used within each question.)

• The variable letter does not affect the way you represent it, nor what values you can substitute for it.

• You should use different coloured shapes to represent positive and negative copies of a variable or constant.

• Like terms are terms that represent different multiples of the same variable. You can collect them to simplify expressions.

• We model collecting like terms by collecting shapes that are the same.

• You can apply the zero principle to help simplify expressions.

Try This — Introducing the Lesson

A. and B. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Why do you think the addition will take more time?* (I have to add six numbers in part i), but in part ii) there are only three operations: two multiplications and one addition.)

• *Do you think the amount of time would change if you changed the order of the numbers you are adding?* (I think it would be easier to add all the 59s first. I would keep adding 60 and then taking away 1.)

The Exposition — Presenting the Main Ideas

• Work through the exposition with the students. Make sure they understand that the choice of a rectangle to represent the variable and a square to represent one was arbitrary. The shape choices could have been reversed or a completely different shape, like a larger square, could have been used for the variable. **Example 1** uses a different shape to make this point.

• Make sure students understand why white and dark shapes need to be used to represent positive and negative copies of the variable. Again, tell students that the choice of white for positive and grey for negative was arbitrary.

• You may need to remind students of the zero principle for adding integers before they apply it to combining like terms.

• To check student understanding, you might ask them to model and simplify $(3n + 8) + (–2n – 4)$.

Revisiting the Try This

B. This question allows students to see how the concept of like terms underlies our use of multiplication. It helps make the connection between work with variables and familiar work with numbers.

Using the Examples

• You may wish to review the formulas for the area of a triangles and a rectangle before students examine **example 1**. Ask students to work with a partner as they read through the two examples. For each example, ask them to identify which step in the solution they thought was most important and why.

For example, they might decide, in **example 1**, that the most important step is writing the areas using the same variable. In **example 2**, they might decide that the most important step is writing the other numbers in the square in terms of the top left corner number. Discuss their choices and any questions they might have about the examples.

Practising and Applying

Teaching points and tips

Q 1: Most students will use a rectangle for the variable and small squares for the ones. If they choose to use
a square for the variable, make sure they understand that it is important that the shape for the constant be either a different shape or a different size. Otherwise, there will be errors in collecting like terms.

Q 2: Students can simplify either using models or not.

Q 3: Students should realize that there are as many possibilities for the two expressions as they might want. They could start with any two expressions and add the same amount to one expression as they subtract from the other.

Q 4: You may wish to suggest that students use the same variable for the base in each case.

Q 5 a): Students could choose to use a variable to represent any of the four numbers in the square. As long as they represent the other three numbers correctly in terms of that choice, students should be able to solve the problem.

For example, if they call the top right corner n, the four numbers would be $n - 1$, n, $n + 9$, and $n + 10$.

Q 7: You may wish to have students address this question in a class discussion rather than individually.

Common errors

• Many students add variable terms and constants incorrectly when simplifying.

For example, some students simplify $3n + 2 + 4n$ as $9n$, treating the 2 as if it were $2n$.

Emphasize the importance of the distinction by showing how the situation is parallel to a computation.

For example, $30 + 30 + 50 + 50 + 50 + 50$ is $2 \times 30 + 4 \times 50$ and not 6×30 or 6×50.

Suggested assessment questions from Practising and Applying

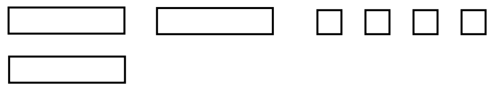
Question 2	to see if students can simplify algebraic expressions
Question 4	to see if students can represent a situation using algebraic expressions and then simplify those expressions
Question 5	to see if students can use reasoning to solve problems that can best be approached using algebraic representations

Answers

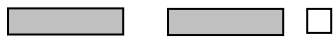
A. Calculation **i)** will likely take longer. **B.** Calculation **i)** will likely take longer.

C. The second calculations collect like terms — they collect the 23s and 47s for **part A** and the 59s for **part B**.

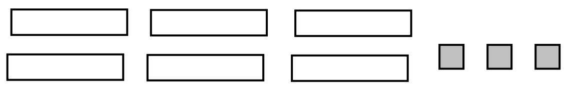
1. a)



b)



c)



d)

2. a) $8n + 8$ **b)** $2m - 4$
c) $2m - 8$ **d)** $5k - 5$

3. *Sample responses*:
a) $n + (n + 4)$ and $(n + 2) + (n + 2)$
b) $n + (2n - 6)$ and $(n - 6) + n + n$
c) $(3 + 2x) + (2 - 3x)$ and $(6 - x) - 1$
d) $4k - 2 + (9 - 7k)$ and $2 - (3k - 5)$

4. a) $7b \div 2$; $8b \div 2$; $9b \div 2$
b) $7b \div 2 + 8b \div 2 + 9b \div 2$; $12b$

5. a) *Sample response:*

N	***N* + 1**
***N* + 10**	***N* + 11**

The sum of one diagonal is $N + (N + 11) = 2N + 11$.
The sum of the other diagonal is $(N + 1) + (N + 10) = 2N + 11$.
b) The five numbers are n, $n + 10$, $n + 20$, $n + 30$, and $n + 40$. If you add them you get $5n + 100$.
If you multiply $(n + 20)$ by 5, you get $5n + 100$.

6. a) $9x + 7$
b) *Sample response*: $(3x + 5) + (6x + 2)$

[**7.** *Sample response*:
It takes less time to substitute and evaluate if you first simplify.]

Supporting Students

Struggling students

• Struggling students might have difficulty working with negative coefficients for modelling or simplifying. You might use only examples with all positive coefficients until students become more comfortable with the process.

• Struggling students might need help with the type of reasoning required in **question 5**. You might help them get started by suggesting a way to represent one of the numbers in the square or in the column.

Enrichment

• Students might find other patterns in the 100 chart (like in **question 5)** and then use algebraic reasoning to try to show that the patterns are always true.

For example, they might notice that if you add the four numbers in the corner of a 3-by-3 square in the table, the sum is four times the value of the middle number of the square.

CONNECTIONS: Using Variables to Solve Number Tricks

• Before students open their books, write the steps of this number trick on the board:

Number Trick

A. Think of a number.

B. Double it.

C. Add 8.

D. Take half.

- Call on a student to follow the steps of the trick without telling you the number he or she has selected, but only telling you the result. You can subtract 4 and tell him what number he or she chose.
- Repeat the trick using another student's calculations.
- Tell students that they will have the chance to figure out why a trick like this works.

• Have them open their student texts to **page 193** and work on the **Connections** questions.

Answers

1. *Sample response*: **A.** 20 **B.** 40 **C.** 36 **D.** 18 **E.** 20; I got same number I started with. **2.** *Sample response*: **A.** 10 **B.** 20 **C.** 16 **D.** 8 **E.** 10; I got same number I started with.	**3. a) A.** n **B.** $2n$ **C.** $2n - 4$ **D.** $n + 2$ **E.** n [**b)** *Sample response*: You start with n and end with n, no matter what n is.] **4.** *Sample response*: **A.** Think of a number. **B.** Add 8. **C.** Double it. **D.** Subtract 14. **E.** Take half. **F.** Subtract 1.

6.2.1 Solving Equations Using Models

å

Curriculum Outcomes	Outcome relevance
7-C1 Summarize Patterns: make predictions • recognize that variables can represent a changing quantity (e.g., $x = 4y$) or a single value (e.g., $x + 3 = 9$) **7-C2 Single Variable Linear Equations: represent solutions** • show solution concretely and pictorially (one step, two step) using a variety of methods including a balance and the "cover-up" method • use concrete models to show a solution to a simple equation (e.g., $e + 3 = 7$; how many are in the envelope)	We can use linear equations to solve many real-world problems. It is important to expose students to a variety of strategies from which to choose to solve them.

Pacing	Materials	Prerequisites
1 h	None	• the zero principle for integers • adding and subtracting integers • using an algebraic expression for a pattern rule

Main Points to be Raised

• You can use the term *unknown* to describe a variable.

• To solve an equation means to find the value(s) you can substitute for the variable that makes both sides equal.

• You can sometimes use addition or multiplication facts to help you solve an equation.

• One way to solve an equation is to use tiles to represent the side with a variable and then match the tiles with the side that is a constant. You can solve by adding, subtracting, multiplying, or dividing both sides of the equation by the same amount to preserve the balance. Your goal is to get the variable alone on one side of the equation. The number on the other side is the solution.

• Another way to solve an equation is a variation of the tile strategy where we use rectangles to represent each copy of the variable and the constant. You represent the two sides that are equal by combining rectangles into large rectangles of the same size.

• You can jump on a number line to model an equation. You solve it by calculating the number of jumps.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *What does it mean when a number is a solution to the equation?* (When you substitute the number into the equation, the two sides of the equation have the same value.)

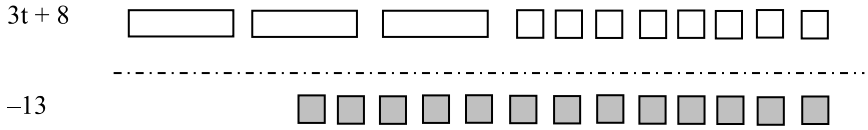
• *How did you calculate the value of the left hand side?* (I multiplied 8 by 6 and then added 2.)

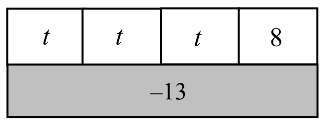
• *How did you come up with that equation?* (I wrote $x = 8$ and then I added 1 to both sides to get $x + 1 = 9$.)

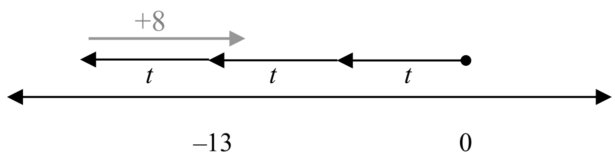
The Exposition — Presenting the Main Ideas

• Write on the board the equations $n - 2 = 8$ and $3n = 12$. Ask students to tell you the solution for each. Make sure they understand the meaning of the terms *solution* and *solve the equation*.

• Next, model the solution of the equation $3t + 8 = -13$ using the three methods shown in the exposition. Explain the steps as you model.







• Suggest that students use the equation solved in the student text as another example for reference.

Revisiting the Try This

B. This question allows students to try out different equation solving strategies with a familiar equation.

Using the Examples

• Write the three equations from **examples 1, 2, and 3** on the board. Ask students to solve each equation using one of the new strategies they have learned. Then have them check their work against the worked solutions in the student text.

Practising and Applying

Teaching points and tips

Q 1: Observe whether students immediately recognize which model goes with which equation by using the coefficient of the variable as a starting point.

Q 3: This question requires students to translate one side of each equation into a word phrase.

Q 5: There are many possible equations a student could use.

Q 7: Students have an opportunity to use whichever strategy they prefer.

Q 8: Some students will refer to the idea of keeping the balance while adding the same amount to both sides, but others may refer to the word phrase equivalent.

For example, if you subtract 2 from something to get 10, then the something must be 12.

Q 9: This question highlights the fact that you can model addition on a number line by starting with zero or by starting with the first addend.

Common errors

• Some students will use the operation sign shown in the equation rather than the reverse operation to solve an equation.

For example, to solve $4n + 8 = 28$, they might solve $4n = 28 + 8 = 36$. Encourage students to substitute their solution into the equation to check their work.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use a model to represent an equation
Question 4	to see if students can solve equations
Question 8	to see if students can reason about equivalent equations

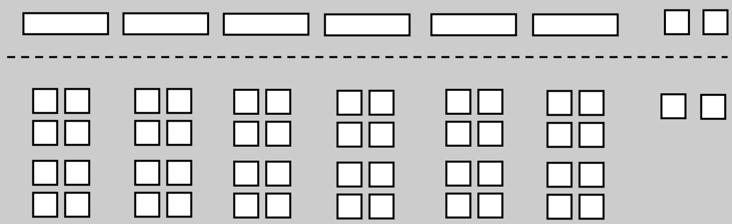
Answers

A. i) Because $6 \times \mathbf{8} + 2 = 48 + 2 = 50$. **ii)** *Sample response*: $2x + 7 = 23$, $40 - 5x = 0$

B. *Tile model*

6*x* + 2

50



Each *x* is worth 8.

Rectangle model

6*x* + 2
50

x	*x*	*x*	*x*	*x*	*x*	2
50						

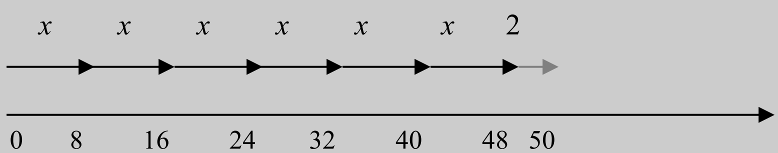
x	*x*	*x*	*x*	*x*	*x*	2
48						2

x	*x*	*x*	*x*	*x*	*x*	2
8	8	8	8	8	8	2

Each *x* is worth 8.

Number line model

$6x + 2 = 50$



8 jumps of 6 take you to 48 and 2 more take you to 50.
$x = 8$

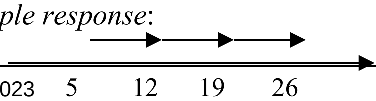
1. a) ii) **b)** iii) or v) **c)** iv)
d) i) **e)** iii)

2. a) $m = 2$ **b)** $m = 3$ **c)** $m = 4$
d) $m = 27$ **e)** $m = 3$

3. a) Add a number to 10. The result is 28.
b) A number is multiplied by 7. The result is 35.
c) Multiply a number by 4 and then add 8. The result is 28.
d) The difference between triple a number and 5 is 16.
e) Multiply a number by 5 and add 10. The result is 55.
f) The difference between 30 and triple a number is 27.

4. a) $p = 18$ **b)** $k = 5$ **c)** $n = 5$
d) $p = 7$ **e)** $m = 9$ **f)** $k = 1$

5. a) *Sample response*:



b) $5 + 7k = 26$
[**c)** There were 3 jumps of equal length from 5 to 26.]

6. a) $4f + 3 = 71$ **b)** $f = 17$

7. *Sample responses*:
Number line; I can just add to get the answer.

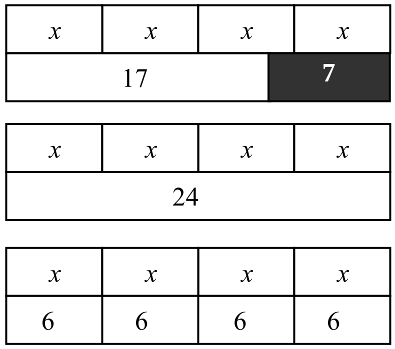
[**8.** If in the first equation you subtract 2 from 5*x* to get to 10, then the 5*x* must be 12, which is the same as
the second equation.]

[**9.** *Sample response*:
The first number line shows counting by 5s to 15 and then adding 12 to get to 27, which is $5k + 12 = 17$.
The second number line shows starting at 12 and then jumping by 5s to get to 27, which is $12 + 5k = 17$.
$5k + 12 = 17$ is the same as $12 + 5k = 17$.]

Answers [Continued]

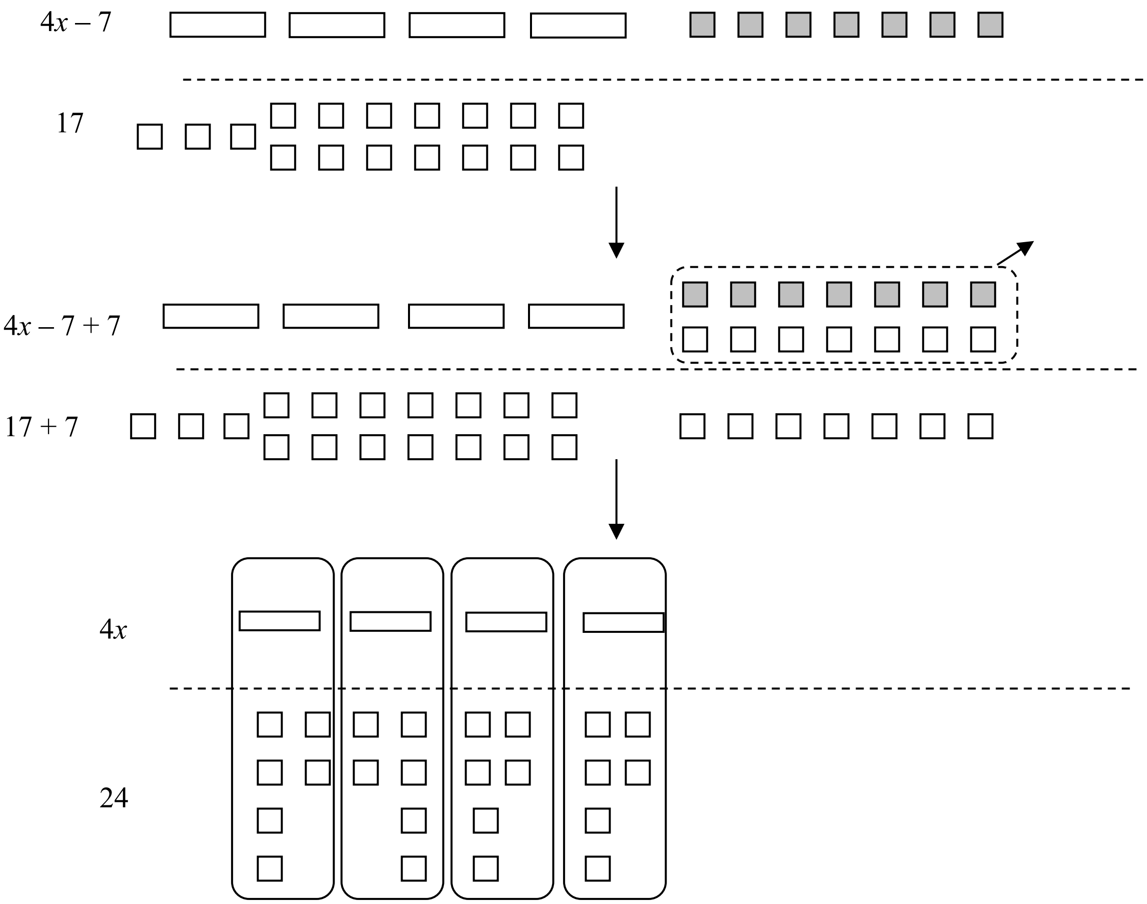
10. $x = 6$; [*Sample responses*:

a) Rectangle model



$x = 6$

Tile Model



Each $x = 6$.

b) The tile model lets you count the number of squares for each copy of the variable, but you have to have tiles or draw a lot of squares.

• The number line is easiest because you can draw it for any equation and just use arrows to show the jumps.

• The rectangle model is quicker to draw than tiles, but it takes longer to draw than the number line because you usually have to draw several pictures.]

Supporting Students

Struggling students

• Some students will find the number of strategies presented overwhelming. Let them focus on one strategy of their choice. Do not require them to use the other models or to answer **questions 7 and 10**.

Enrichment

• Encourage students who find these questions simple to try to create alternate models for solving equations.

6.2.2 Solving Equations Using Guess and Test

Curriculum Outcomes	Outcome relevance
7-C3 Single Variable Linear Equations: one and two step • solve equations using systematic trials	We can use linear equations to solve many real-world problems. It is important to expose students to a variety of strategies from which to choose to solve them.

Pacing	Materials	Prerequisites
1 h	None	• operations with whole numbers and decimals

Main Points to be Raised

• Guessing and testing is a good way to solve equations. Your first guess could be anything, although it makes sense to estimate to get a good first guess.

• You base your further guesses on the result of substituting the previous guess. You increase or decrease your guess, depending on the results of the substitution.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know that the numbers cannot all be less than 50?* (The sum would only be 150 if they were all as high as 50.)

• *Why might you try numbers near 100?* (100 + 100 + 100 = 300 and 297 is close to 300.)

• *How does knowing that 100 + 100 + 100 = 300 help you?* (If I take away 1 from the first 100 and add it to the last 100, I have three consecutive numbers that add to 300. Then I could take away 1 from each of those numbers and the sum would be 3 less. Because 297 is the sum I want, these numbers are the answer to the question.)

The Exposition — Presenting the Main Ideas

• Write the equation $2x - 18 = 94$ on the board.

- Ask students why 50 might be a reasonable first guess for a solution (because $2x = 94$ is close to 100 if x = 50 and 50 is an easy number to substitute). Have them substitute 50 for x to see why 50 is too low.

- Encourage students to continue to make better guesses until they find that 56 works. Make sure they understand that if the substitution results in $2x - 18$ being lower than 94, they need a higher guess. If the result is higher, they need a lower guess.

• Repeat the above process using the equation $5m = 24$. Students will realize they need to use a decimal between 4 and 5 to solve the problem. They might try 4.5 and see that they must go up. They should continue guessing and testing until they reach the solution of $m = 4.8$.

• Reinforce that it does not matter what letter you use for the variable; you solve the equation in the same way.

• Tell students that they can see how these same two equations are solved in the exposition on **page 198**.

Revisiting the Try This

B. Students are asked to look at the problem they solved in **part A** using the guess and test strategy featured in this lesson.

Using the Examples

• Write the two equations from the example on the board. Ask students to work in pairs, using guess and test to solve the equations. They should then check their solutions against the solutions in the student text.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to estimate to come up with a first guess.

For example, for **part d)**, they might realize they need to subtract about 250 from 617 to get to 382, so they might use 50 as a first guess.

Q 2: It is important to make sure students understand how to use the information from one guess to improve the next guess.

Q 4: Some students may need help translating the word expression in **part a)** into an equation involving subtraction.

Q 5: Most students will recognize that the total number of counters is 4 more than the number of black counters. They might still struggle with the equation. The equation might be written as $t = b + 4$ or as
$b = t - 4$. They can then substitute 83 for t and solve their equation for b.

Q 7: Students will need to recognize that the way to write the sum of two consecutive integers could be $n + (n + 1)$ or $n + (n - 1)$, depending on whether they use the variable to represent the lesser or the greater number.

Common errors

• Some students will not use the information from one guess appropriately to get the next guess, especially in a subtraction situation.

For example, if they substitute $n = 30$ into $600 - 2n = 500$ and find that $600 - 2n$ is 540, they do not realize that the next guess should go up rather than down in order to subtract more.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can use the guess and test strategy to solve linear equations
Question 6	to see if students can translate a real-world situation into an algebraic equation and then solve it
Question 9	to see if students can communicate about the guess and test strategy as a way to solve equations

Answers

A. 98, 99, 100	**B. i)** $m + (m + 1) + (m + 2) = 297$ **ii)** *Sample response*: 100
1. *Sample responses*: **a)** 130 [because 100 more than 378 is 478 and I need more than that.] **b)** 5 [because 350 ÷ 70 = 5] **c)** 80 [because 5m is about 400] **d)** 50 [because 5m is between 200 and 300] **e)** 50 [because 6t is about 300] **f)** 90 [because 6t is about 540] **2.** *Sample responses*: **a)** 120; [600 was too low so I would guess higher.] **b)** 65; [297 was too high so I would guess lower.] **c)** 60; [300 was not low enough, so I would guess higher to subtract more.] **3. a)** $k = 21$ **b)** $k = 11$ **c)** $m = 52$ **d)** $t = 0.2$ **4. a)** $n + (n + 10) = 124$; $n = 57$; the numbers are 57 and 67. **b)** $8k = 344$; $k = 43$ **c)** $2m - 35 = 79$; $m = 57$ **5. a)** $t = b + 4$ **b)** 79	**6. a)** C = K – 273 **b)** 150°C **7. a)** $n + (n + 1) = 284$ **[b)** *Sample response*: I tried $n = 141$ and it was too low. I tried $n = 142$ and it was too high. There cannot be any integer solution because the solution is between 141 and 142.] **8. a)** 20; [*Sample response*: 6 × 50 is 300 and if you subtract about 50, you would be way too high.] **b)** $x = 24$ [**9.** *Sample response*: To figure out the value of the variable in an equation, try different values that make sense. Each number you try is based on the number you tried before. If you are way off, change your number a lot and if you are close, change the number a bit.] [**10.** *Sample response*: If the numbers are big or if they are decimals, I would not bother with a model. I would guess and test.]

Supporting Students

Struggling students

• Some students may have difficulty with **questions 5 and 6**. These questions require students to represent a situation with an equation. You might provide these students with the equations for these situations and let them then solve the equations.

• For **question 7**, you may wish to suggest which of the numbers to represent with the variable.

• For **question 10**, you may wish to provide some equations and ask students which equations they would solve with guess and test rather by using a model.

6.2.3 Solving Equations Using Inverse Operations

Curriculum Outcomes	Outcome relevance
7-C1 Summarize Patterns: make predictions • use the term algebraic equation to describe a number sentence with a variable • distinguish between equations and expressions **7-C2 Single Variable Linear Equations: represent solutions** • show a solution pictorially (one step, two step) using a variety of methods including a balance and the "cover-up" method • recognize that adding and subtracting the same values from both sides of an equation maintains balance **7-C3 Single Variable Linear Equations: one and two step** • solve equations using reasoning	We can use linear equations to solve many real-world problems. It is important to expose students to a variety of strategies from which to choose to solve them. The formal strategy of inverse operations will be especially useful in subsequent mathematics classes.

Pacing	Materials	Prerequisites
1 h	None	• operations with integers

Main Points to be Raised

- Addition and subtraction are inverse operations; so are multiplication and division.
- When you subtract a number that has been added or add a number that has been subtracted, the result is zero and does not affect a computation.
- When you multiply a number that has been divided or divide a number that has been multiplied, the result is one and does not affect a computation.
- You can create an equivalent equation with the same solutions if you add, subtract, multiply, or divide both sides of an equation. This is because you are maintaining the balance the equation represents.
- To solve an equation, it is helpful to perform inverse operations until you get the variable alone on one side of the equation, with a coefficient of 1.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

- *Why might you add and subtract in a different order than the order in which the numbers are given?* (I do not have to add in the 358 if I subtract the other 358 at the same time. The same is true for the 269.)
- *Would it have been as easy if you had been asked to divide by a different number? What number?* (Yes; dividing by 35,215 instead of by 487 would have been just as easy.)

The Exposition — Presenting the Main Ideas

- Ask students to turn to **page 201** in the student text. Talk about how an equation represents a balance; the value on the left balances the value on the right. Then discuss how solving an equation that has a variable means finding a value to substitute for the variable that makes the equation balance.
- Lead the students through the solution of the equation $3x + 8 = 32$ in the student text, using the terminology *inverse operations* as you work through it.
- Make sure they understand why you subtract the 8 from both sides before dividing the two sides by 3.
- Use another equation like $5m - 12 = 48$ and go through a similar process with the students.

Revisiting the Try This

B. Even though the inverse operations used in **part A** did not involve equations, students can see the effect and value of using inverse operations in those situations.

Using the Examples

• Ask students to read through the example. Ask why addition was the first inverse operation performed.

Practising and Applying

Teaching points and tips

Q 1 b): Although it is not required, encourage students to state the inverse operations in the order of their use.

Q 2: Students can use a simple rectangle or square to represent the variable rather than the "bag" shown in the art in the lesson.

Q 3: Ask students to list the steps in order. Note that for the last question the variable can remain on the right.

Q 5: Watch to make sure that students realize that only the food amount is multiplied by the number of days, not the amount for books.

Q 6: You may have to remind students that an equivalent equation is an equation with exactly the same solutions.

Common errors

• Some students use inverse operations incorrectly and in the wrong order.

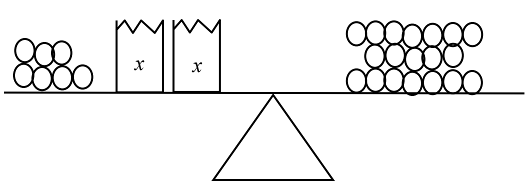
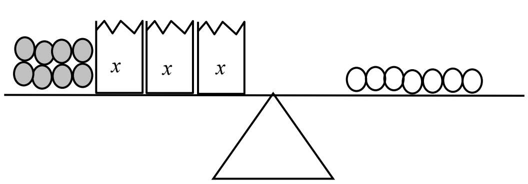
For example, to solve $3x + 5 = 30$, they might first divide by 3 rather than subtracting 5, but they forget to also divide the 5 by 3. They would get $x + 5 = 10$, which leads to a wrong answer.

Encourage students always to check their answers. Remind them that when you divide or multiply one side of an equation, the operation must be applied to all the terms on that side of the equation.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can show how to solve a linear equation using inverse operations
Question 5	to see if students can solve a word problem using a linear equation
Question 6	to see if students can explain why the process for using inverse operations is valid

Answers

A. *Sample responses*: **i)** You subtract each number you add, so there is no calculating to do — the answer is 0. **ii)** If you multiply and then divide by the same number, it is the same as multiplying or dividing by 1.	**B.** *Sample response*: In the first calculation, three times I subtracted the same number I added. In the second calculation, I divided by the same number after multiplying.
1. a) $2x - 1 = 11$ **b)** Add 1 and then divide by 2. **c)** $x = 6$ **2. a)**  **b)** 	**3. a)** Subtract 18, divide by 12. **b)** Add 19, divide by 7. **c)** Subtract 200, divide by 9. **d)** Subtract 16, divide by 6. **4. a)** $k = 87$ **b)** $m = 7$ **c)** $t = 32$ **d)** $k = 8$ **5. a)** $2700 = 120 + 200d$ **b)** $d = 12.9$; the money will last for 12 days. [**6.** When you subtract or add to both sides of an equation, you get a new equation but it has the same solution.]

Supporting Students

Struggling students

• Some students may find **question 5** difficult to interpret. You may choose not to assign this question to struggling students.

Enrichment

• You might ask students to create equations that meet particular conditions. They can give their equations to a partner to solve.

For example, they might create an equation where the coefficient of the variable is 20 and the solution is –8, or an equation where the constant in the expression on one side of the equation is 6 more than the solution (e.g., $4x + 13 = 41$).

GAME: Equations, Equations

This game provides practice with creating and solving equations. Because they have to create the equations, students are likely to solve many more equations mentally than they would solve if the equations were given.

6.2.4 EXPLORE: Solving Equations Using Reasoning

Curriculum Outcomes	Lesson Relevance
7-C3 Single Variable Linear Equations: one and two step • solve equations using reasoning	It is often more efficient to use reasoning to solve equations than to use a formal process. This lesson encourages that strategy.

Pacing	Materials	Prerequisites
40 min	None	• adding and multiplying whole numbers

Exploration

• Ask students to turn to **page 204** in the student text. Make sure they understand that:

- Each number on the right represents the sum of all numbers in that row.
- Each number at the bottom represents the sum of all numbers in that column.
- The value for each shape is the same throughout the puzzle.

Ask students to work in pairs. While you observe students at work, you might ask questions such as the following:

• *How would combining the information from Rows 1 and 3 help you?* (I could tell that the triangle is 2 less than the square.)

• *How would combining the information from Row 1 and Column 1 help you?* (I could find the value of the square because I know the value of 2 circles + a triangle from row 1. I can add it to the value of the square in column 1.)

• *Which shape's value did you figure out first?* (I first figured out the value of the circle. I subtracted the 47 for two triangles and a square from row 2 from the 57 for two triangles, a square, and a circle from column 3.)

• *How did you make up your puzzle?* (I first decided on values for the shapes. Then I put shapes in different places and added the values in the rows and the columns.)

Observe and Assess

As students work, notice:

• Do they choose to put together useful combinations of information to solve the problem?

• Do they use equations to represent the information?

• Do they use reasonable strategies to solve the problem?

• Do they calculate correctly?

• When they create a puzzle, do they test that someone who does not know the values can actually solve it?

Share and Reflect

After students have had sufficient time to work through the exploration, discuss both how they solved the given puzzle and how they created their new puzzles.

• *How could combining the information from Row 2 and Column 3 help you?*

• *Why could you not figure out any of the values using information from just one row or just one column?*

• *Why was it important first to test whether the puzzle you gave your partner could be solved by someone who did not know the answers?*

Answers

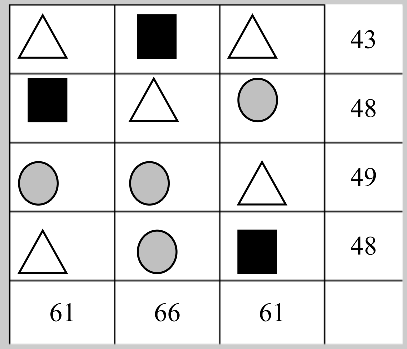
A. *Sample response*:
The numbers in the first row add up to an odd number (35). Since there are two circles, their sum must be even, so the triangle must be odd.

B. *Sample response*:
The numbers in the second row add up to an odd number (47). But since there are two triangles, their sum must be even, so the square must be odd.

C. *Sample response*:
The second column includes one of each shape and an extra circle, but the sum (52) is less than the sum in the third column (57) that includes one of each shape and an extra triangle. That means the circle is 5 less than the triangle.

D. Circle = 10; square = 17; triangle = 15

E. *Sample response*:



Triangle = 13; square = 17; circle = 18

Supporting Students

Struggling students

• You may have to help students who struggle by giving them a value for one of the variables in the given puzzle. You may also have to give them a starting point for creating their own puzzles.

6.3.1 Graphing a Relationship

Curriculum Outcomes	Outcome relevance
7-C4 Linear Equations: graph using table of values • use the x-axis and y-axis for the horizontal and vertical axes • use a table of values for graphing • interpolate (find a point between two known points) • extrapolate (find a point that lies beyond the existing data)	Graphs are a useful tool for solving both real-world problems and mathematical problems. Students need to learn how to use graphs to describe problem situations.

Pacing	Materials	Prerequisites
1.5 h	• Grid paper or Small Grid Paper (BLM)	• plotting ordered pairs

Main Points to be Raised

• You can show a relationship visually by plotting the ordered pairs from a table of values.

• If you extend the graph of the points in a table of values, you can estimate or calculate information about other values by extrapolating or interpolating.

• Whether a graph can be used to estimate or calculate may depend on the scale of the graph.

• To use a graph to estimate or calculate, you might use an x-value to determine a y-value, or vice versa.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know he bought fewer than 10 bars?* (10 bars would cost Nu 500 even if he had not spent the other Nu 120. That is too much, because he paid only Nu 470.)

• *Why did you subtract 120 from 470?* (I wanted to find out how much he spent just on chocolate.)

• *How could you use guess and test to solve the problem?* (I could guess the number of bars and see if I guessed too high or too low.)

• *What equation are you solving?* (The equation is $50c + 120 = 470$.)

The Exposition — Presenting the Main Ideas

•Ask students to open their student texts to **page 205**. Discuss the situation presented: 12 oranges are needed to create 1 L of juice. Discuss with students why this seems reasonable (there is only a small amount of juice inside each orange).

• Make sure students recall how to plot an ordered pair and talk about how the numbers in the table of values describe an ordered pair. Discuss why this is reasonable because a graph shows a relationship and there is a relationship between the number of oranges and the amount of juice.

• Make sure students realize that the 1.5 L based on 18 oranges or the 3.3 L based on 40 oranges are estimates and that the only way to be more precise is to use a graph that has a more precise scale near the point being examined. Even then, the result is only an estimate.

Revisiting the Try This

B. The graph cannot be precise in showing that an answer is correct, but it is useful to see whether or not an answer is reasonable.

Using the Examples

• Assign students to work in pairs on the two examples. One student should study **example 1** and the other student should study **example 2**. Each student should then teach the other student about the example he or she studied.

Practising and Applying

Teaching points and tips

Q 1: Students can stop the graph at $x = 5$ and $y = 32$ or they can extend it.

Q 2: Encourage students to think about how far to extend the two axes in order to solve the problem. Ask them why it is easier to use the graph for the final prediction than to extend the table.

Q 4: This question requires students to work backwards. For **part a)**, they must read off the ordered pairs. They can choose which values to read off the graph. For **part b)**, they must be creative in thinking of a situation that would lead to those ordered pairs.

Common errors

• Some students get confused about which axis to start from when they look for a solution on a graph. Encourage them to pay attention to the labels on the axes to know where to start.

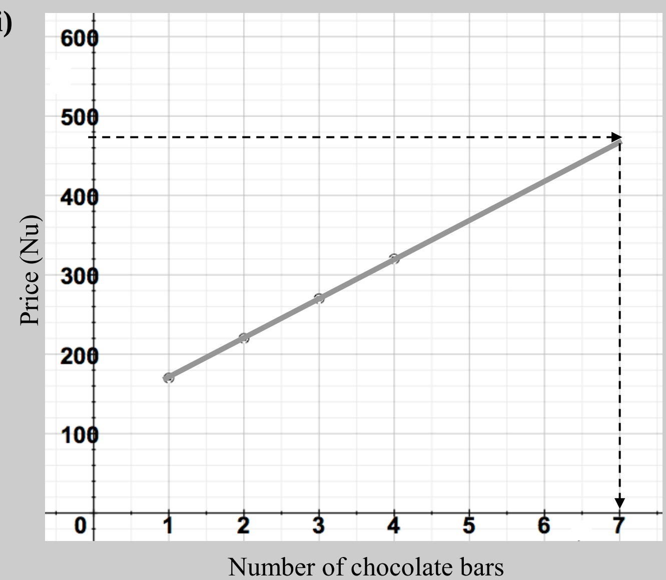
Suggested assessment questions from Practising and Applying

Question 1	to see if students can use a table of values to create a graph
Question 3	to see if students can create a table of values and the related graph, and use the graph to extrapolate to solve a problem
Question 6	to see if students can communicate about the value of a graph to describe a relationship

Answers

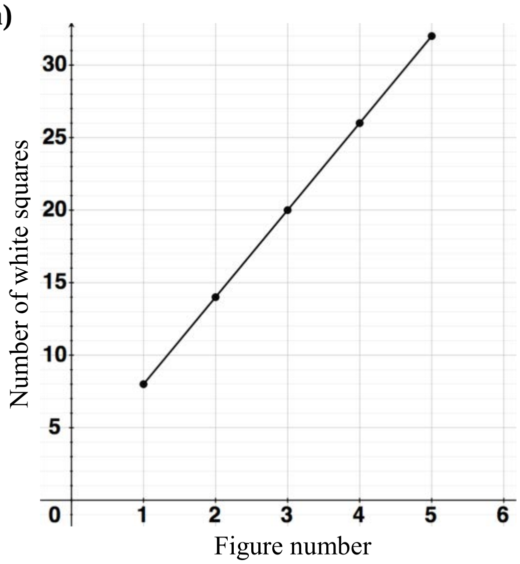
A. 7 bars

B. i)

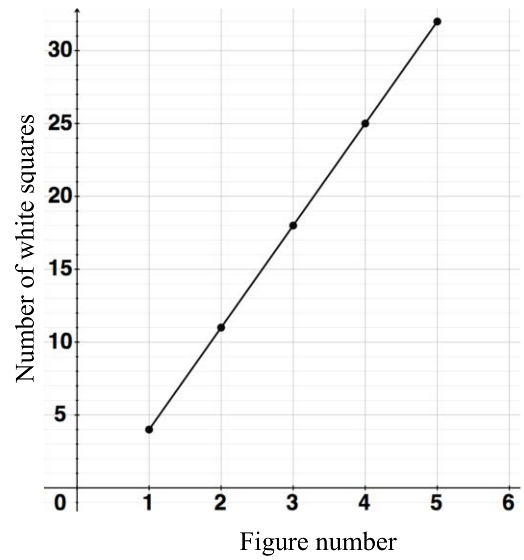


ii) I drew a horizontal line over from Nu 470 and then looked down to the x-axis. It showed 7 bars. That is the same as my answer to **part A.**

1. a)



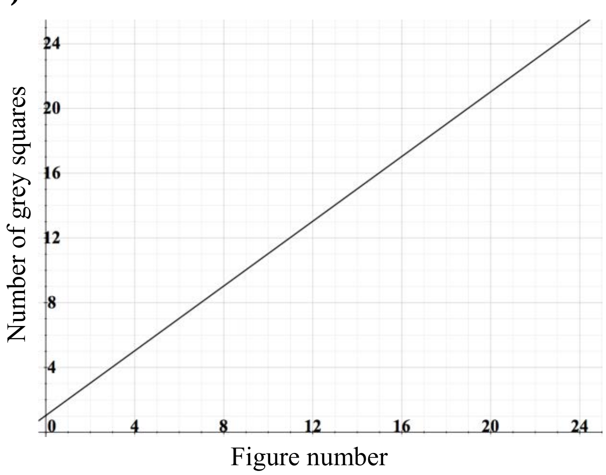
b)



2. a)

Figure number	Number of grey squares
1	2
2	3
3	4
4	5

b)

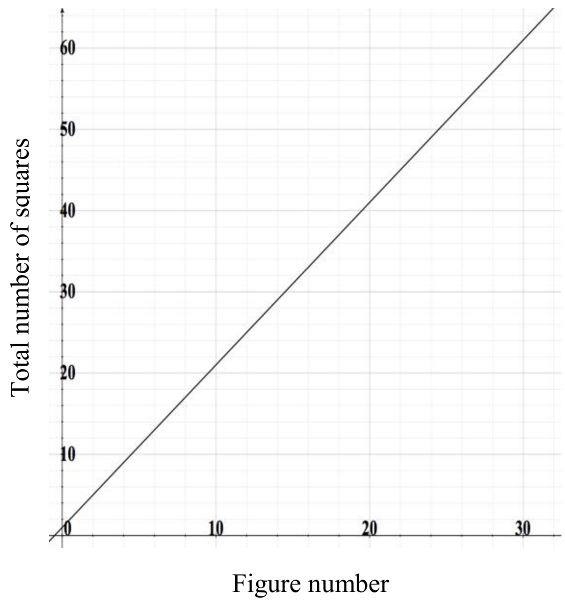


c) Figure 16 **d)** 22

e)

Figure number	Total number of squares
1	3
2	5
3	7
4	9

f)

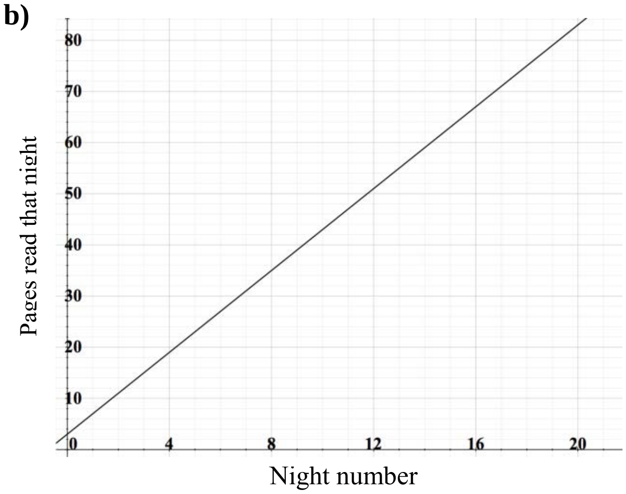


g) Figure 30

3. a)

Night number	Pages read that night
1	10
2	14
3	18
4	22

b)

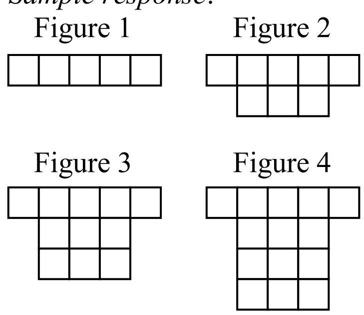


c) Night 18

4. a)

x	*y*
1	5
2	8
3	11
4	14

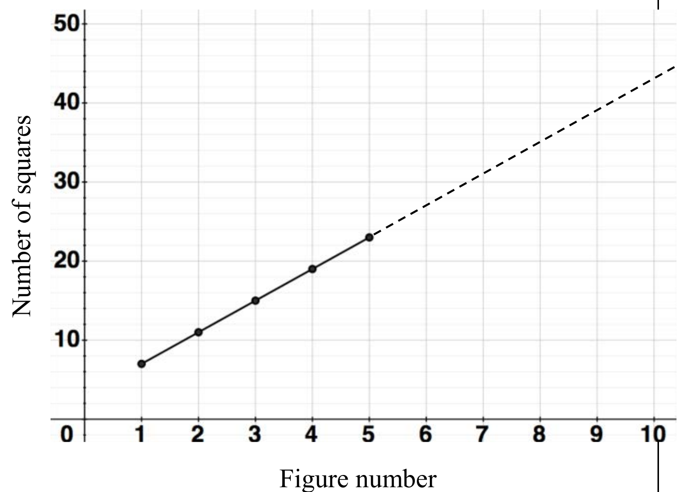
b) *Sample response*:



5. a)

Figure number	Total number of squares
1	7
2	11
3	15
4	19
5	23

b) and c)
The 10th figure has 43 squares.



d) *Sample response*:
I used the pattern in the right column, which was adding 4 for each new figure number; 19 + (6 × 4) = 43.

[**6.** *Sample responses*:
a) If you have a graph, you can easily answer questions about data that you have not observed.
b) If you use a big scale on the axes, you might have trouble being exact for values that fall in between the values marked on the axes.]

Supporting Students

Struggling students

• Some students may have difficulty when they must first create the tables of values to match a problem and then use the graph. You may at first provide the tables of values for these students.

• Other students may have more difficulty extrapolating (going beyond the plotted points) than interpolating (reading between the plotted points). Help them by showing how to label the axes beyond the plotted points in both directions and then extending the line of the graph.

Enrichment

• Students might create other patterns of figures like those in **questions 2 and 5** and create problems involving graphs of those patterns for other students to solve.

6.3.2 Examining a Straight Line Graph

Curriculum Outcomes	Outcome relevance
7-C1 Summarize Patterns: make predictions • use constants, variables, algebraic expressions and equations to make predictions • recognize that variables can represent a changing quantity (e.g., $x = 4y$) or a single value (e.g., $x + 3 = 9$) **7-C4 Linear Equations: graph using table of values** • use the x-axis and y-axis for the horizontal and vertical axes • determine if an ordered pair satisfies a given equation; - by plotting the points to see if they are in keeping with the rest of the points in the pattern - by substituting them into the equation to see if they make the equation true or false • equate an ordered pair that makes an equation true with the fact that it is a solution to the equation	A graphs is a useful tool for solving a linear equation. Students need to make the link between drawing a line graph and solving the associated linear equation.

Pacing	Materials	Prerequisites
1.5 h	• Grid paper or Small Grid Paper (BLM)	• division

Main Points to be Raised

• Once you have plotted two points on a line, you can extend the line without plotting other points.

• If you have an equation of the form ax + b = c, you can solve it or check a solution by graphing y = ax + b, locating the point on the line with a y-coordinate of c, and finding the x-coordinate for that point.

• Unless the scale allows for it, you often have to estimate a solution when you use a graph.

• You can solve many equations using the same graph because you can locate points with various y-coordinates.

For example, you can solve $3x + 7 = 12$, $3x + 7 = 18$, and $3x + 7 = 32$ using the graph of $y = 3x + 7$.

Try This — Introducing the Lesson

A. Allow students to try this with a partner. While you observe students at work, you might ask questions such as the following:

• *What number would Buthri say if he thought of the number 5?* (11)

• *Where would Lobzang look for 11 on the graph?* (On the vertical axis)

• *What would Lobzang do after he found 11 on the vertical axis?* (He would find the place on the graph that is at the same height as 11 and then look down at the x-coordinate for that point.)

• *Why would it be harder to use the graph if Buthri thought of a number like 20?* (The graph only goes to x = 6.)

The Exposition — Presenting the Main Ideas

• Ask students to open their student texts to **page 209**. Point out the table of values and the associated graph in the exposition. Discuss how the graph is based on the table.

For example, show that the points (1, 5), (2, 7), (3, 9), and (4,11) appear on the line. On the board, demonstrate that even if you plotted only (1, 5) and (2, 7), you would end up with the same line. Talk about why you can graph the table of values using the line $y = 2x + 3$; in each case, the y-value is 3 more than double the x-value.

• Next, have students look at the second graph where the equation $2x + 3 = 6$ (which is the same as $6 = 2x + 3$) is solved. Make sure students understand how to use the dotted lines: You first follow the horizontal line to locate a point on the line $y = 2x + 3$ where the y-value is 6. Then you follow the vertical line figure out the corresponding x-value.

• Help students see that although it is hard to be precise on the graph, the value $x = 1\frac{1}{2}$ is a reasonable estimate.

• Then look at the last graph with the students, showing how you could use the graph as a check for a solution that you found using a different method, for example, guess and test or inverse operations.

• Finally, point out how you can use the same graph, in this case $y = 2x + 15$, to solve any equation of the form $2x + 15 = k$, no matter what the value of k is, by locating the point on the line with that y-coordinate, k. They should notice that there is never more than one point on the line with a particular y-coordinate (unless the line is horizontal, so the equation would be $y = k$, which is already solved).

Revisiting the Try This

B. This question helps students see how they can represent the number trick by an equation that they can solve in different ways, including using a graph.

Using the Examples

• Ask students to work through the two examples in pairs. Answer any questions they might have.

Practising and Applying

Teaching points and tips

Q 1: Remind students to choose a scale that allows them to see the y-value when the x-value is 5.

Q 2: Students may need to extend the graphs they created in **question 1** or change the scales to accommodate solving these equations.

For example, the graph for **part a)** must allow for a y-value of 33.

Q 3: Students should realize that they only need to change the value on the side of the equation with a single number to answer this question.

Q 5: The graph students create must allow for y-values as high as 58 and x-values as high as 13.

Q 6: Students need not use high values of x to answer this question.

Q 7: You might ask students to discuss this question with a partner before recording a response.

Common errors

• Sometimes students do not use a scale that will allow them to answer a question. Make sure they realize that they must choose an appropriate scale, but also make sure they realize that if they use a big scale, they may have to estimate a solution less precisely.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can use a graph to solve a linear equation
Question 3	to see if students recognize the range of equations that can be solved using a graph
Question 5	to see if students can use a graph to check a solution to a linear equation

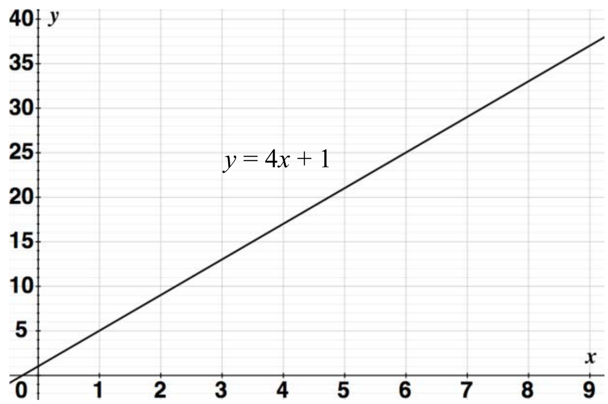
Answers

A. ii) Find the answer on the *y*-axis, draw a horizontal line over to the graph, and then go down to see the original number on the *x*-axis.
iii) No; You can use any point on the graph and it works.

B. i) The *x*-axis is the secret number and the *y*-axis is the answer.
ii) $y = 3x - 4$; *Sample response*:
I know by checking a few ordered pairs: (2, 2), (3, 5), and (4, 8).

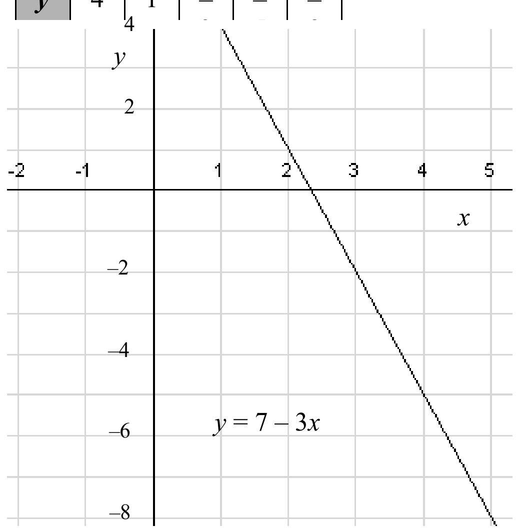
1. a)

x	1	2	3	4	5
y	5	9	13	17	21



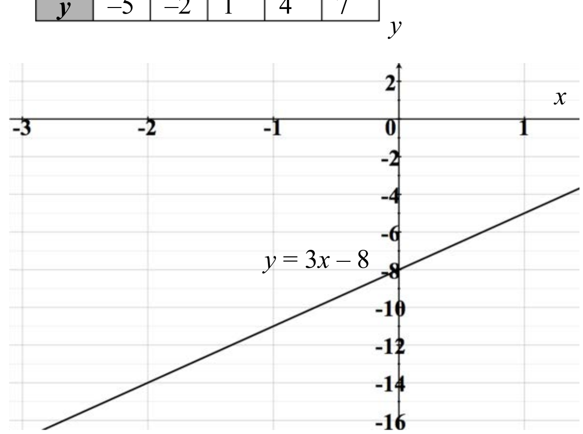
b)

x	1	2	3	4	5
y	4	1	–	–	–



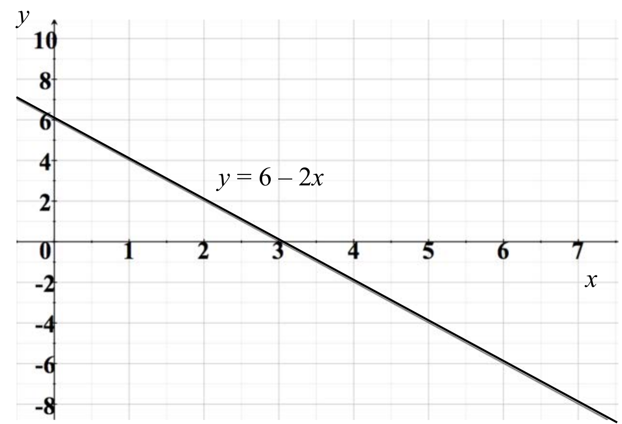
c)

x	1	2	3	4	5
y	–5	–2	1	4	7



d)

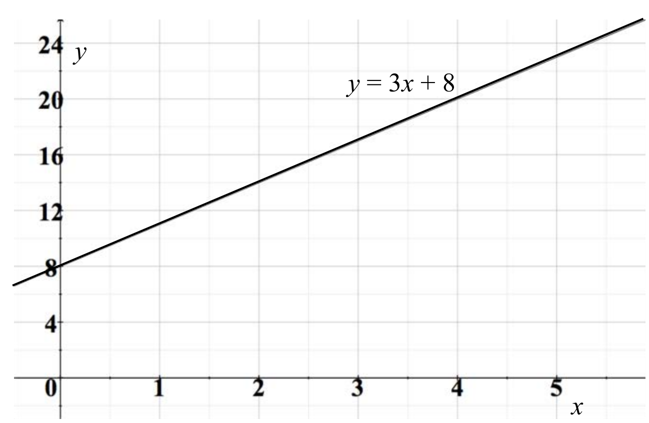
x	1	2	3	4	5
y	4	2	0	–2	–4



2. a) $x = 8$ **b)** $x = -2$
c) $x = -2$ **d)** $x = 2$

3. *Sample responses*:
a) $4x + 1 = 37$; $x = 9$
b) $7 - 3x = 1$; $x = 2$
c) $3x - 8 = -2$; $x = 2$
d) $6 = 6 - 2x$; $x = 0$

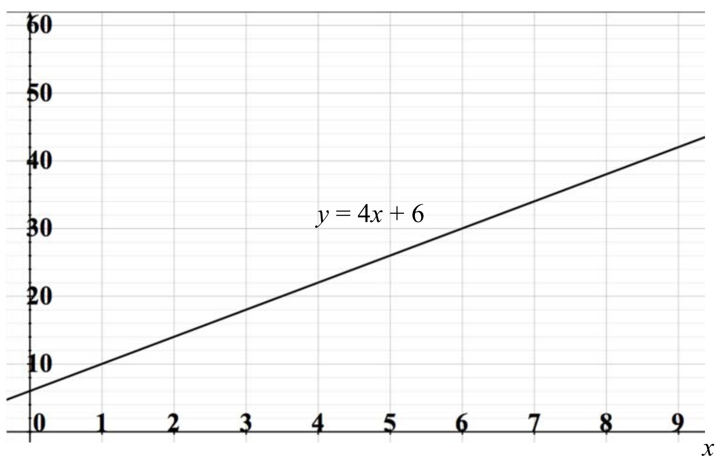
4.



[*Sample response*:
When I looked up from 3 on the *x*-axis, the *y*-value that met the graph in that place was not 10.]

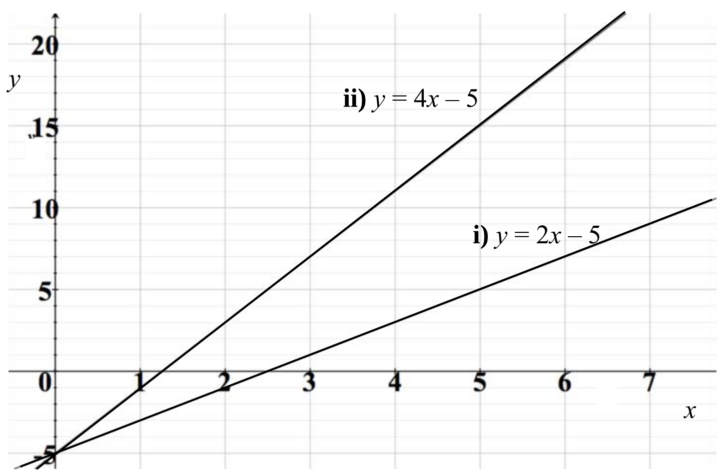
Answers [Continued]

5. B is correct;



For **A**, I looked up from 7 on the *x*-axis to see if the *y*-coordinate was 29. It was not.
For **B**, I looked over from 42 on the *y*-axis across to the line and then looked down. It was at $x = 9$, so 9 is a solution.

6. a)



b) They both go through (0, –5). They have different slopes.
c) $y = 4x - 5$
d) $x = 4.5$

[**7.** By drawing one graph, you can solve many equations with different values for *y.* A disadvantage is that you may only be able to estimate a solution because the scale might be big and not easy to read for values in between the values that are marked on the axes.]

Supporting Students

Struggling students

• Some students may have trouble recognizing that the number 33 in an equation like $33 = 4x + 1$ does not help you know what relationship to graph; it only tells you how to use the graph afterwards. Help students see that they would graph exactly the same relationship to solve $33 = 4x + 1$, or $23 = 4x + 1$, or $-4 = 4x + 1$.

6.3.3 Describing Change on a Graph

Curriculum Outcomes	Outcome relevance
7-D6 Rate: compare two quantities • construct and analyse graphs to show change • understand rate as the comparison of two quantities • write as a ratio (e.g., m/s, km/h, beats per minute) • solve indirect problems	Proportional thinking is important for solving real-world problems. Students need to realize that graphing is one way to solve proportions.

Pacing	Materials	Prerequisites
1.5 h	• Grid paper or Small Grid Paper (BLM)	• operations with decimals and whole numbers

Main Points to be Raised

- A rate is a relationship that compares quantities.
- You can use a graph to describe a rate; the graph is a line.
- You can use graphs to solve problems involving rates.
- A greater rate is described by a steeper line.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know that the amount of cheese will be less than 0.5 kg?* (It would take 5 kg of milk to make 0.5 kg of cheese, and 2.4 < 5.)

• *Why might you estimate a number that is about one fourth of 10?* (2.5 is $\frac{1}{4}$ of 10 and 2.4 is close to 2.5.)

• *How did you solve the problem?* (I figured out that it would take 1 kg of milk to make 0.1 kg of cheese. Then, I multiplied 0.1 by 2.4 because 2.4 is 2.4 times as much as 1.)

The Exposition — Presenting the Main Ideas

• Ask students if they recall what a rate is. See if they can think of different sorts of rates such as speeds in kilometres per hour, prices in Nu per item, and heart rates in beats per minute. Discuss how knowing a rate gives you lots of information.

For example, if you know that a car is going 31 km/h, you know how far it will go in 1 h, 2 h, 3 h, and so on.

- Discuss how you can use rate information to create a table of values and a graph. Demonstrate with one of the rates students suggest.

For example, you might draw a graph through the points (1, 31), (2, 62), and (3, 93) to describe the car speed above (31 km/h).

- Show students how you can use the graph to find out even more information.

For example, by extending the graph for car speed and reading the y-coordinate for the point on the line where the x-coordinate is 10, you can see the total distance travelled in 10 hours. Similarly, you can tell how long it would take to travel a distance of, for example, 200 km, by reading the x-coordinate for the point on the line where the y-coordinate is 200.

• Have students turn to **page 212** in the student text. Ask them to look at the graph of distance against time for a speed of 25 km/h. Discuss with them the parts of the graph. Talk about how to use the graph to solve the problems at the bottom of the page.

• Ask students how the graph would look different if you were going faster, say 30 km/h. They can check their predictions by looking at the graph on **page 213**.

• Students might also look at the beginning of the exposition to see other examples of rates that might be graphed. You may need to inform students that a pon (mentioned in the picture of oranges) is 80 items and is used only for oranges or betel nuts.

Revisiting the Try This

B. Students can look back at their solution to **part A** and compare it to a solution that uses a graph.

Using the Examples

• Lead students through the two examples. Make sure they understand why the particular graphs that are shown were drawn and how to use them to solve each problem.

Practising and Applying

Teaching points and tips

Q 1: Students must realize that they need to compare the coordinates of two different points in order to figure out the speed.

Q 2: Encourage students first to create a table of values for each rate. They can choose which variable to use for the x-axis and which to use for the y-axis.

Q 3: Whether students start at the x-axis or y-axis will depend on what choices they made for the axis labels for **question 2**.

Q 4 c): Students need to compare several pairs of coordinates to answer this question.

Q 5: Students need to look at several pairs of points to answer this question.

Common errors

• Students might not label their axes in a way that allows them to solve the required problems. You may have to remind them that they should consider the size of the values involved in the rate when they determine the scale for the axes.

Suggested assessment questions from Practising and Applying

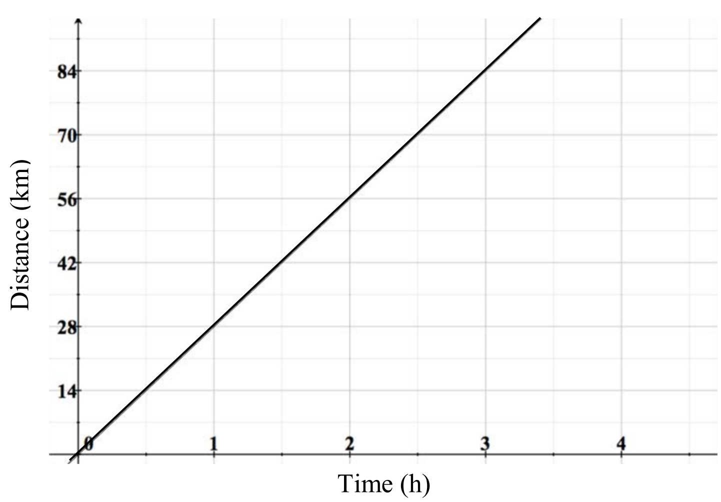
Question 1	to see if students can interpret and use a rate graph
Question 4	to see if students can create and use a graph that describes a rate
Question 5	to see if students can relate a graph of a rate to its underlying meaning

Answers

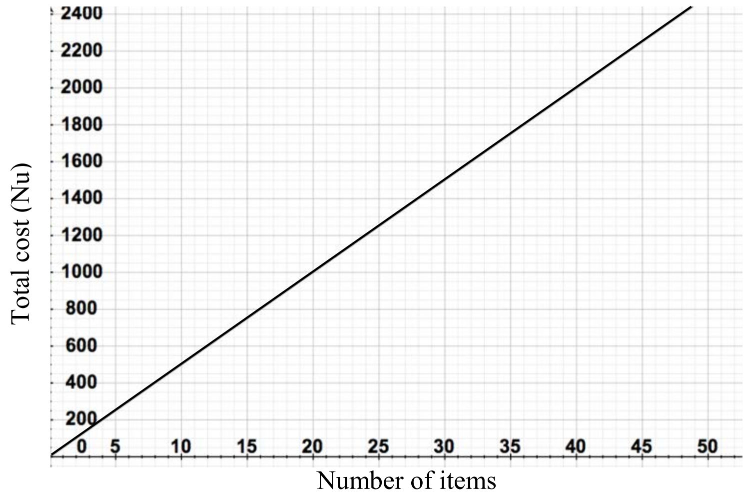
A. 0.24 kg	**B. i)** The rate is 10 kg milk per 1 kg cheese. **ii)** If you graph $y = 10x$, you look for the x-value that meets the graph at the same place as a y-value of 2.4.

1. a) About 67.5 km
[b) *Sample response*:
Look at the graph between (2, 45) and (4, 90). Because there is a 45 km change in 2 h, the rate must be 22.5 km/h. I know it is a constant rate because the graph is a straight line.]

2. a)



b)



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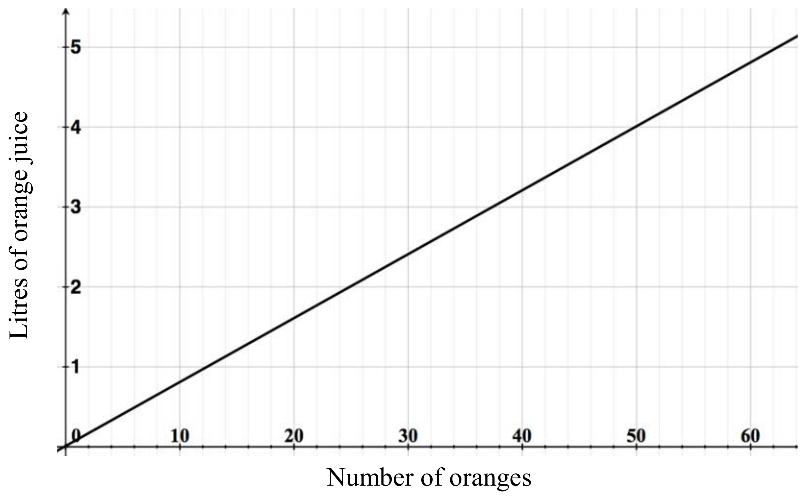
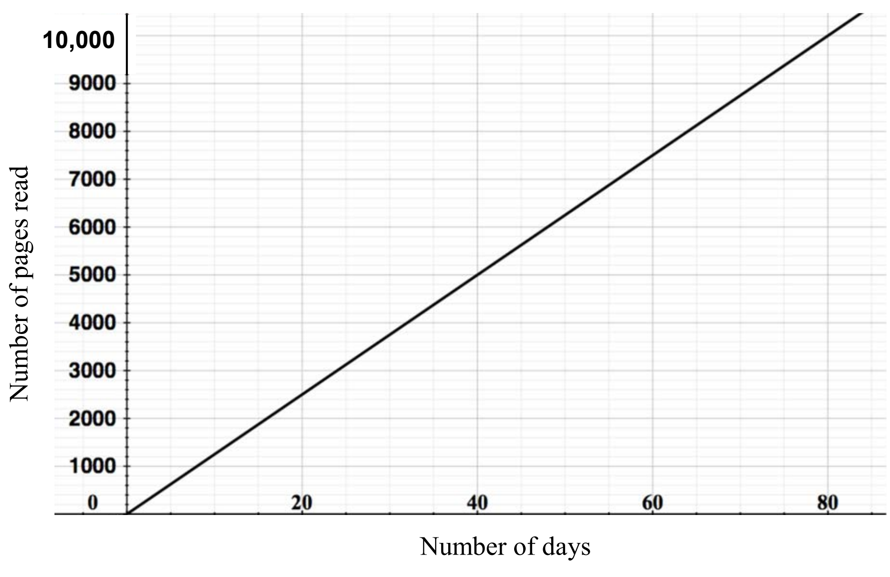


d)



3. a) i) 70 km **ii)** About 3.1 h

b) 46 items

c) i) 4.8 L **ii)** 25 oranges

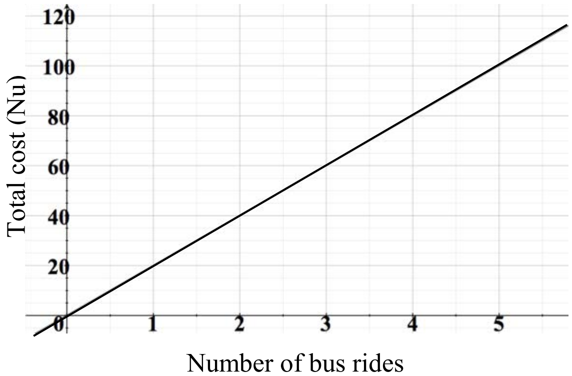
d) 80 days

Answers [Continued]

4. a)

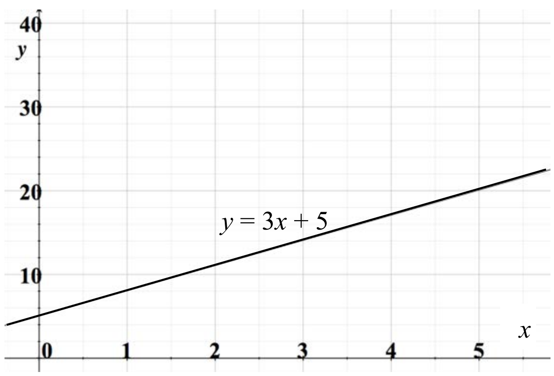
Number of bus rides	1	2	3	4	5
Total Cost (Nu)	20	40	60	80	100

b)



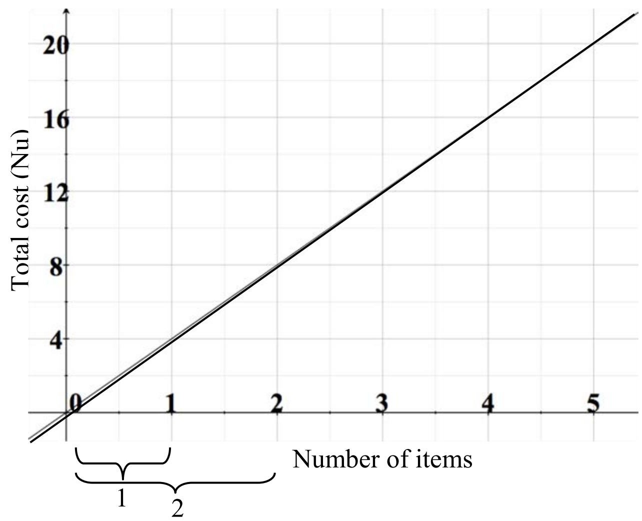
[**c)** *Sample response*:
If you go from 1 to 4 bus rides on the graph, the cost increases by Nu 60 (from Nu 20 to Nu 80). The same thing happens if you go from 2 to 5 bus rides or from 3 to 6 bus rides. The same thing happens anywhere on the graph.]

5. a)



[**b)** *Sample response*:
I located the points (2, 11), (3, 14), (4, 17), and (5, 20) on the graph and saw that an increase of 1 in x matched with an increase of 3 in y each time.]
[**c)** *Sample response*:
I used the same points as for **part b)**, but I checked the values backwards.]
d) No; [*Sample response*:
It seems to be true for whichever two points I picked that had x-coordinates that were one unit apart.]

6. It will increase by 8.
[*Sample response*:



7. [**a)** Each graph shows the cost per item for any number of items.]
b) $y = 120x$ is steeper [because if you substitute the same values for x into both equations, the total cost, y, increases faster for the same change in the number of items, x.]

Supporting Students

Struggling students

• Struggling students might have difficulty changing the information about the rate into an equation to graph. You may need to provide additional models beyond the examples in the student text.

For example, you might show how rates like 5 items for Nu 300, 60 min/h, and 1 birth/1000 can be described by the relationships $y = 300x \div 5$, $y = 60x$, and $y = 0.001x$.

• You may assign struggling students to work with a partner for **questions 5 and 6**, which are more abstract.

Enrichment

• Some students might enjoy creating problems for their peers that involve unusual rates.

For example, they might use the information in the table below as a source for such problems.

Animal Speeds

Cheetah	112 km/h
Lion	80 km/h
Elephant	40 km/h
Chicken	14.4 km/h
Giant tortoise	0.27 km/h
Snail	0.05 km/h

6.3.4 EXPLORE: Are All Relationship Graphs Straight Lines?

Curriculum Outcomes	Lesson Relevance
7-C5 Graphs: **linear and non-linear** • understand how changing one quantity affects the other • develop a sense of how the value of an expression changes with the value of the variable	As students move up to higher classes, they will learn that many relationships are not described by lines. This essential exploration introduces this idea because most of the relationships they have seen so far have been linear.

Pacing	Materials	Prerequisites
1.5 h	• Grid paper or Small Grid Paper (BLM)	• formulas for the perimeter and area of a rectangle and the volume of a cube

Exploration

• Draw a rectangle on the board and indicate its dimensions. Ask students how to calculate its perimeter and area. Ask how they would write these as formulas, for example, $A = lw$ and $P = 2l + 2w$. Ask them what formulas they would use if they know that the length is 10 units ($A = 10w$ and $P = 20 + 2w$).

• Draw a cube on the board and ask students how they would calculate its volume.

• Ask students to work with a partner to read through the box at the top of **page 216** and work through the parts. While you observe students at work, you might ask questions such as the following:

• *Why did the perimeter increase by 4?* (The length increased by 2.)

• *Why did a change in length of 1 affect the perimeter less than a change in length of 4?* (I add only 2 to the perimeter if the length increases by 1, but I add 8 if the length increases by 4.)

• *I notice you are working on **part C**. If one dimension is 4 cm, what could the other dimension be?* (Either 3 or 5; it is 3 if the length is 4, but it is 5 if the width is 4.)

• *Why might someone say that the volume grows very quickly?* (When the side length grows from 4 cm to 5 cm, the volume grows by over 60 units.)

Observe and Assess

As students work, notice:

• Do they correctly calculate the required measurements?

• Are their tables of values clear and organized?

• Do they graph correctly based on the tables of values?

• Do they compare the graphs in suitable ways?

• Is their prediction about volume reasonable?

Share and Reflect

After students have had sufficient time to work through the exploration, ask them to share what they observed using questions such as these:

• *Why do you think the perimeter graph looks different from the area graph?*

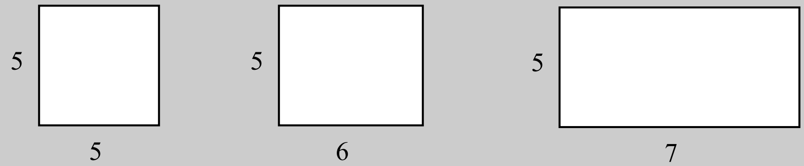
• *Which graphs formed lines?*

• *For* ***parts A and B****, do you think the shape of the graphs would be different if the width of the rectangles were a number other than 5?*

• *Why might someone predict that the volume graph would not be a straight line?*

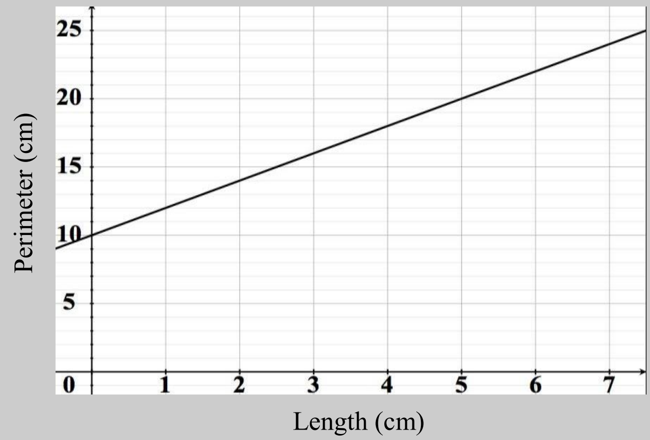
Answers

A. i)



ii) and iii)

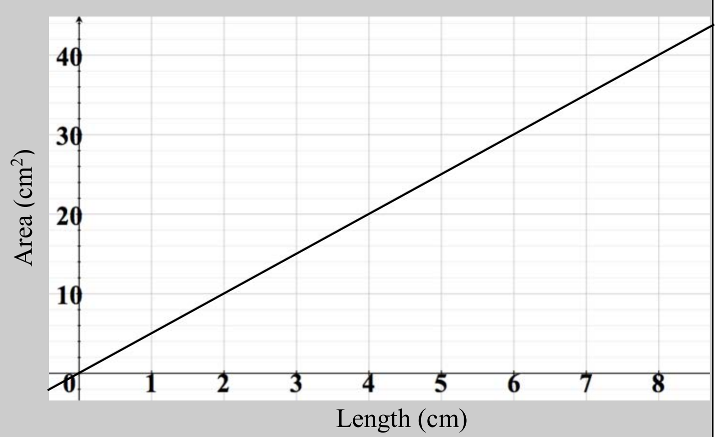
Length (cm)	5	6	7
Perimeter	20	22	24



iv) The graph is a straight line.
v) It always increases the perimeter by 2.
vi) It always increases the perimeter by 8.

B.

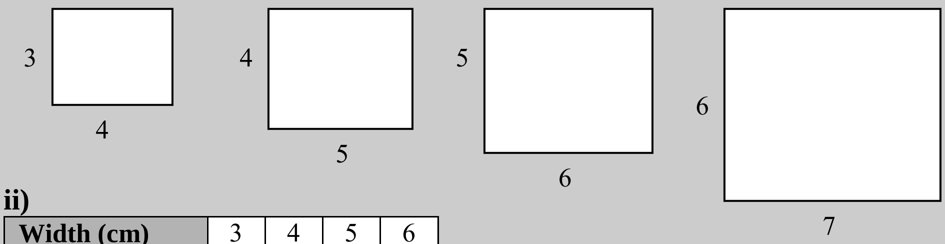
Length (cm)	5	6	7
Area (cm^2)	25	30	35



The graph is a straight line.
A change of 1 in the width always increases the area by 5.
A change of 4 in the width always increases the area by 20.

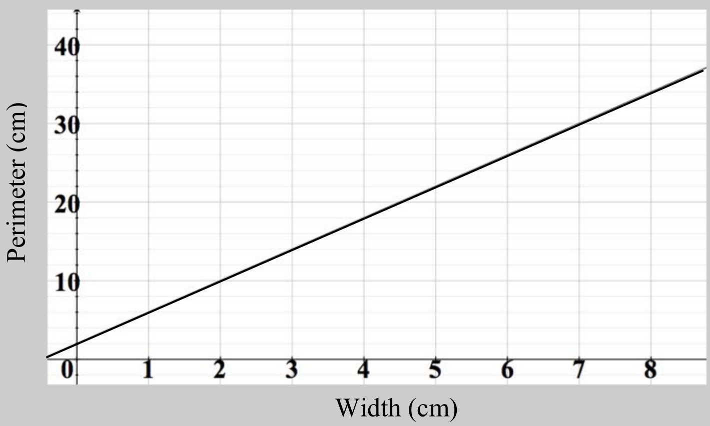
Answers [Continued]

C. i) *Sample response*:



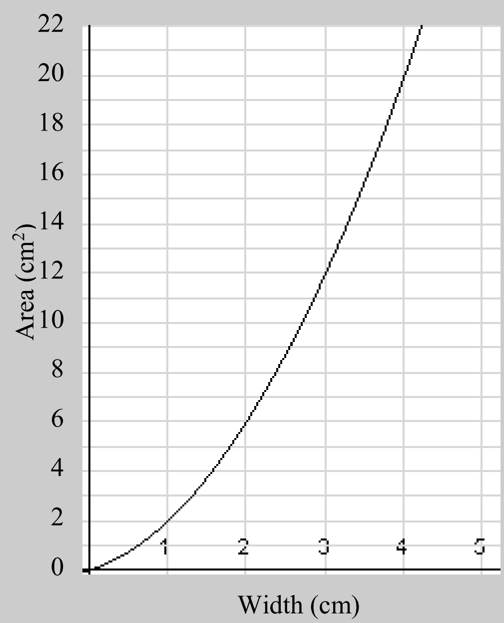
ii)

Width (cm)	3	4	5	6
Perimeter (cm)	14	18	22	26



The graph is a straight line.
An increase of 1 in the width always results in an increase of 4 in the perimeter.
An increase of 4 in the width always results in an increase of 16 in the perimeter.

Width (cm)	3	4	5	6
Area (cm^2)	12	20	30	42



The graph is a curve.
An increase of 1 in the width results in different increases in the area. It depends on the starting width.
An increase of 4 in the width results in different increases in the area. It depends on the starting width.

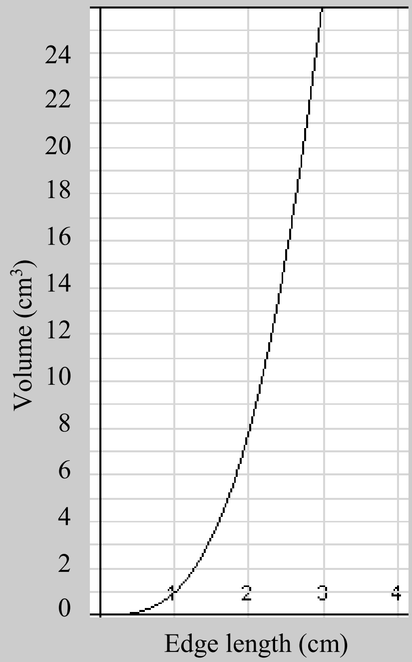
For the three rectangles, the graphs of perimeter vs. length and area vs. length were both straight lines.
For the four rectangles, the graph of width vs. perimeter was a straight line but the graph of width vs. area was curved.

D. i)

Edge length (cm)	1	2	3	4
Volume (cm^3)	1	8	27	64

ii) *Sample response*: It will not be straight.
iii) *Sample response*:
Volume is more like area than like perimeter.
The area graph in **part C ii)** was a curve.

iv) Yes, the graph is not straight.



Supporting Students

Struggling students

• Some students may not be good at using the measurement formulas. Because the focus in this lesson is on the graphs and not on the formulas, you may wish to provide these students with the tables of values and concentrate on discussing the shapes of the graphs.

Enrichment

• Some students may predict what the graphs for other types of measurements would look like and then test their predictions.

For example, they might predict the graphs for perimeters of equilateral triangles of different side lengths, for perimeters of regular hexagons of different side lengths, for areas of rectangles where the length is 4 greater than the width, or for volumes of rectangular prisms where the three side lengths are in the ratio 1 : 2 : 4, for example, 1 cm by 2 cm by 4 cm, or 3 cm by 6 cm by 12 cm, etc.

UNIT 6 Revision

Pacing	Materials
2 h	• Grid paper or Small Grid Paper (BLM)

Question(s)	Related Lesson(s)
1 – 3	Lesson 6.1.1
4 and 5	Lesson 6.1.2
6 – 8	Lesson 6.1.3
9 – 11	Lesson 6.2.1
12 and 13	Lesson 6.2.2
14 and 15	Lesson 6.2.3
16 and 17	Lesson 6.3.1
18	Lesson 6.3.2
19 and 20	Lesson 6.3.3
21	Lesson 6.3.4

Revision Tips

Q 3: Students might notice that a is constant shown by the shading in the second picture, but not in the first picture.

Q 4: Students can use either 200 or 4 × 50 as the constant in their expression.

Q 5: The problem should be a word problem.

Q 7: To solve this, students might use a variable such as *n* to represent any of the four numbers in the T. If students are struggling, it might be easier to have the variable represent the middle top number.

Q 14 b): The operations should be written in sequence.

Q 17: Students might look ahead to **question 18** to decide on the scales to use on their graphs.

Answers

1. a) Variable is *k;* coefficient is –1; constant is 5.

b) Variable is *m;* coefficient is 3; constant is $\frac{1}{2}$

2. a)

x	*y*
1	10
2	13
3	16
4	**19**
5	**22**

b)

x	*y*
1	28
2	26
3	24
4	**22**
5	**20**

3. $3 + 2(f - 1) + (f - 1)$ and $2f + f$

4. a) $200 + 20n$ **b)** Nu 360

5. *Sample response*:
How far would you have travelled if you drove 15 km and then drove for *x* hours at 30 km/h?

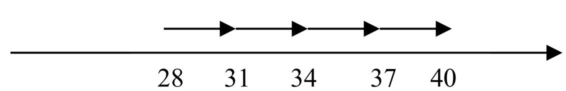
6. a) $12n + 5$ **b)** $-m - 12$ **c)** $-8n + 11$ **d)** $5m - 6$

7. a) $4n + 10$;
[The four numbers are $(n - 1)$, n, $(n + 1)$, and $n + 10$, if *n* is the middle number, and $(n - 1) + n + (n + 1) + n + 10 = 4n + 10$.]
[**b)** $4n + 10$ means 10 more than 4 times the middle number.]

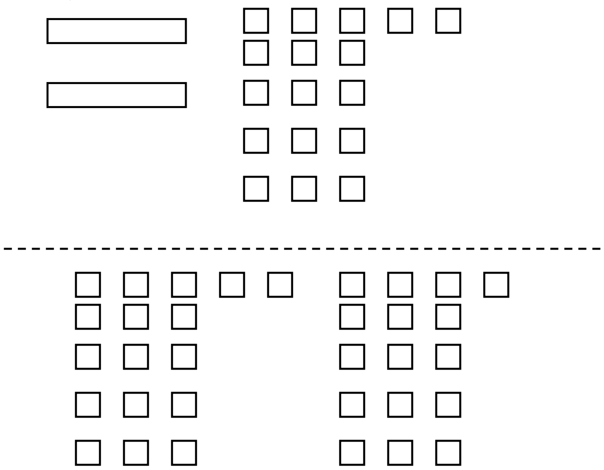
8. *Sample responses*:
a) $(2n + 6) + 2n$
b) $8n - 3n + 10 - 20$

9. a) The difference between 4 times a number and 5 is 23.
b) 8 more than 6 times a number is 50.

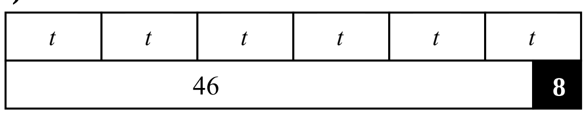
10. *Sample responses*:
a)



10. b)



c)



11. a) $5f + 1 = 101$ **b)** $f = 20$

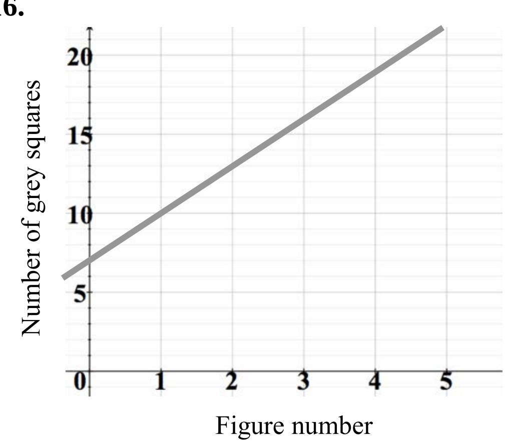
12. *Sample responses*:
a) $k = 100$ [because $500 - 400 = 100$.]
b) $k = 11$ [because $220 \div 20 = 11$.]

13. a) $n + (n + 8) = 164$; $n = 78$
b) $3 + 3n = 255$; $n = 84$

14. a) $2x + 4 = 12$
b) Subtract 4 and divide by 2.
c) $x = 4$

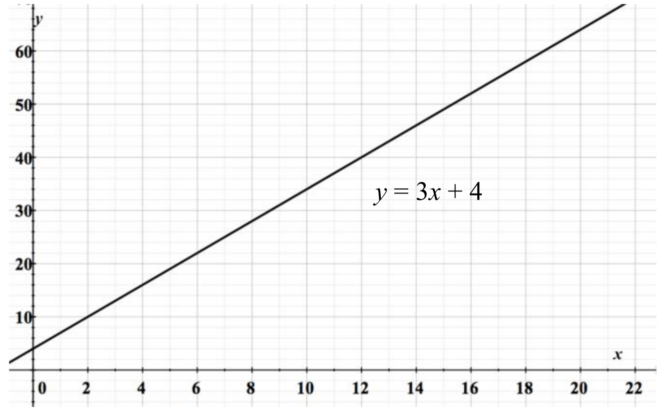
15. a) $k = 117$ **b)** $t = 87$

16.



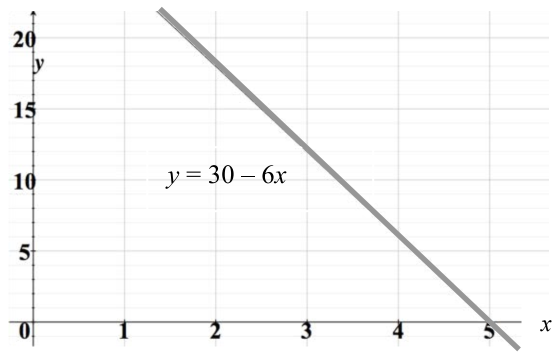
17. a)

x	1	2	3	4	5
y	7	10	13	16	19



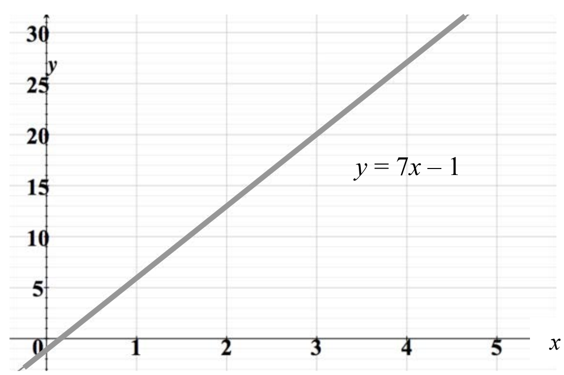
b)

x	1	2	3	4	5
y	24	18	12	6	0



c)

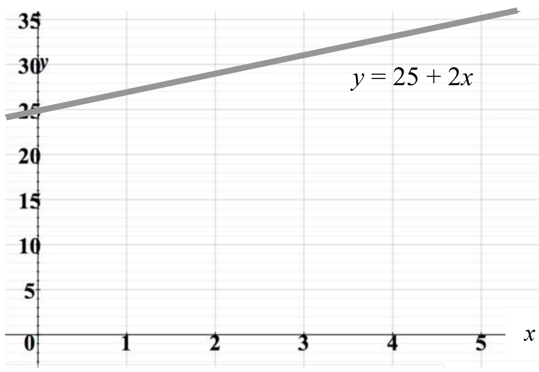
x	1	2	3	4	5
y	6	13	20	27	34



Answers [Continued}

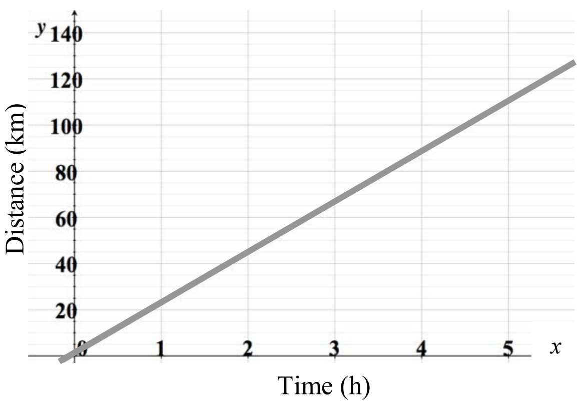
d)

x	1	2	3	4	5
y	27	29	31	33	35

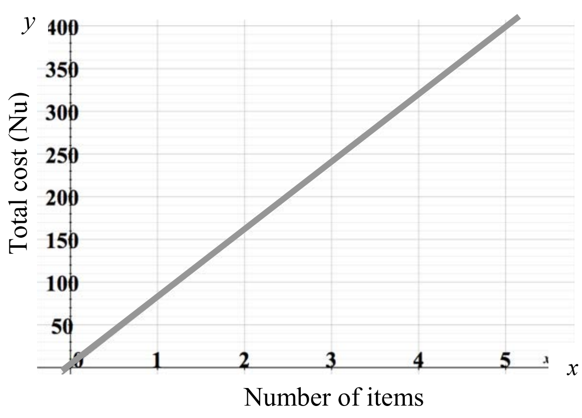


18. a) $x = 21$ **b)** $x = 3$
c) $x = 4$ **d)** $x = 3$

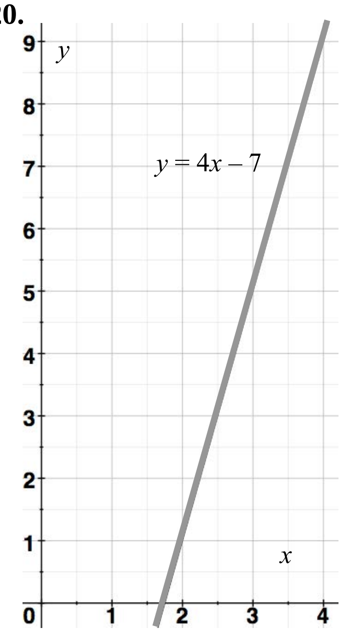
19. a)



b)

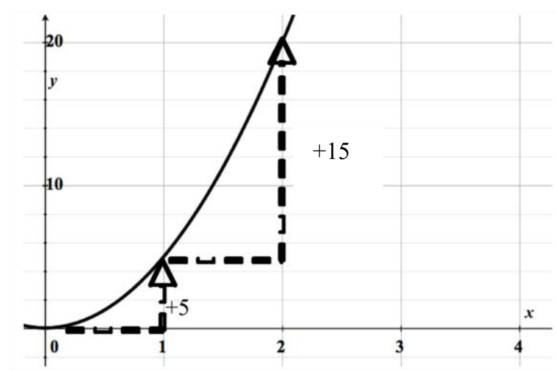


20.



[*Sample response*:
From (2, 1) to (3, 5) to (4, 9), the y increased by 4 each time when x increased by 1.]

[**21.**



An increase of 1 from $x = 0$ to $x = 1$ results in an increase of 5 for y.
An increase of 1 from $x = 1$ to $x = 2$ results in an increase of 15 for y.]

UNIT 6 Algebra Test

1. a) Copy and complete the table.

x	1	2	3	4	5	6	7
y	3	7	11	15			

b) Write a pattern rule you can use to find the value of y if you know x.

c) What is the coefficient in your pattern rule?

2. a) Explain how the pattern rule $f + (f - 1) + 2$ describes this pattern.

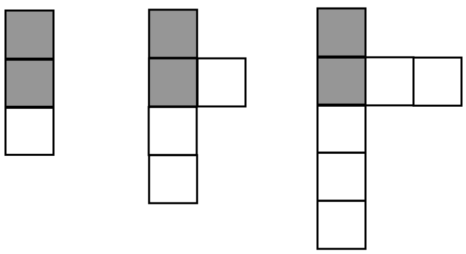


Figure 1 Figure 2 Figure 3

b) Simplify the pattern rule.

3. a) Write an algebraic expression to describe this situation:

Sonam bought some Nu 8 stamps and two fewer Nu 15 stamps.

b) Create a problem you could solve using your expression.

4. a) Simplify $(3x - 4) - (-2x - 5)$.

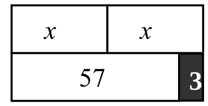
b) Evaluate the expression for $x = 4$.

5. Write an expression that simplifies to $-3x + 2$.

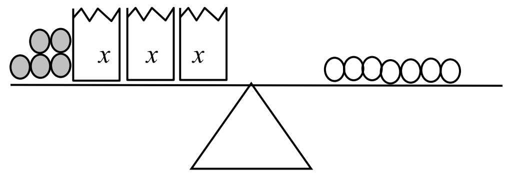
6. You double a number, add 4, and then divide by 3. The result is 4. Write an equation to represent this.

7. What equation does each represent?

a)



b)



8. Represent and solve this equation using each strategy below. Show your work.

$$4n - 2 = 30$$

a) a model

b) a graph

c) inverse operations

d) guess and test

9. Create an equation you could use to find the number of the figure that has 25 squares in the pattern from **question 2**. Solve the equation.

10. a) For the relationship $y = 3x + 8$, create a table of values up to $x = 5$ and graph it.

b) Use your graph to solve $3x + 8 = 35$.

11. Graph this relationship:

How the total price of a number of items is related to the number of items purchased, if two items cost Nu 60

12. Graph $y = 2x - 16$. How does the graph show that for every increase of 1 in x, there is an increase of 2 in y?

UNIT 6 Test

Pacing	**Materials**
1 h	• Grid paper or Small Grid Paper (BLM)

Question(s)	**Related Lesson(s)**
1 and 2	Lesson 6.1.1
3	Lesson 6.1.2
4 – 6	Lesson 6.1.3
7	Lesson 6.2.1
8 and 9	Lessons 6.2.1 – 6.2.3
10	Lessons 6.3.1 and 6.3.2
11 and 12	Lesson 6.3.3

Select questions to assign according to the time available.

Answers

1. a)

x	1	2	3	4	5	6	7
y	3	7	11	15	**19**	**23**	**27**

b) $4x - 1$ or $y = 4x - 1$
c) 4

2. a) *Sample response*:
There are f white squares at the bottom, $(f - 1)$ white squares on the side, and 2 grey squares in each figure, if f is the figure number.
b) $2f + 1$

3. a) $8x + 15(x - 2)$
b) *Sample response*:
How much did Sonam spend if she bought ten Nu 8 stamps?

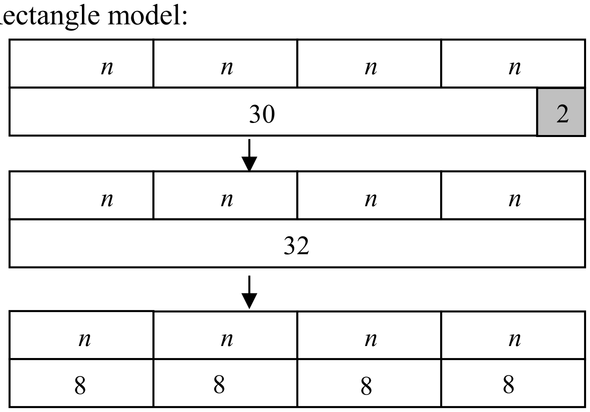
4. a) $5x + 1$ **b)** 21

5. *Sample response*: $(4x + 1) - (7x - 1)$

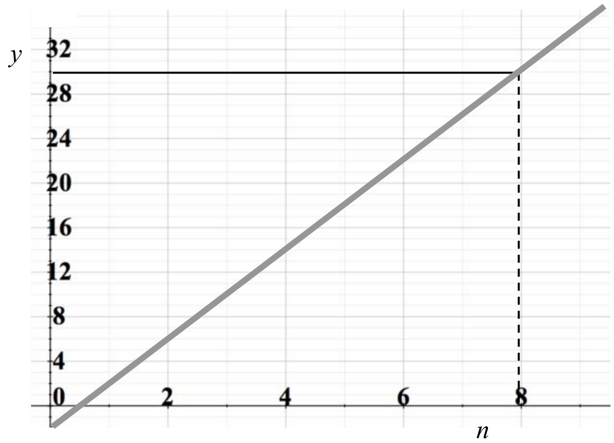
6. $(2x + 4) \div 3 = 4$

7. a) $2x - 3 = 57$ **b)** $3x - 5 = 7$

8. *Sample responses*:
a) Rectangle model:



b) Graph:



c) Inverse operations:

Add 2	$4n - 2 = 30$
	$4n = 30 + 2$
Divide by 4	$4n = 32$
	$n = 8$

d) Guess and test:

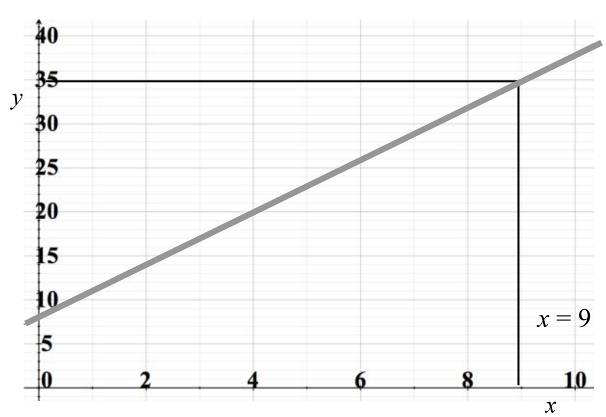
Guess 10	$4 \times \mathbf{10} - 2 = 38$	Too high. Try a lower number.
Guess 9	$4 \times \mathbf{9} - 2 = 34$	Still too high. Try a slightly lower number.
Guess 8	$4 \times \mathbf{8} - 2 = 30$	Got it!

9. *Sample response*:
$2f + 1 = 25$; $f = 12$; Figure 12 has 25 squares.

10. a)

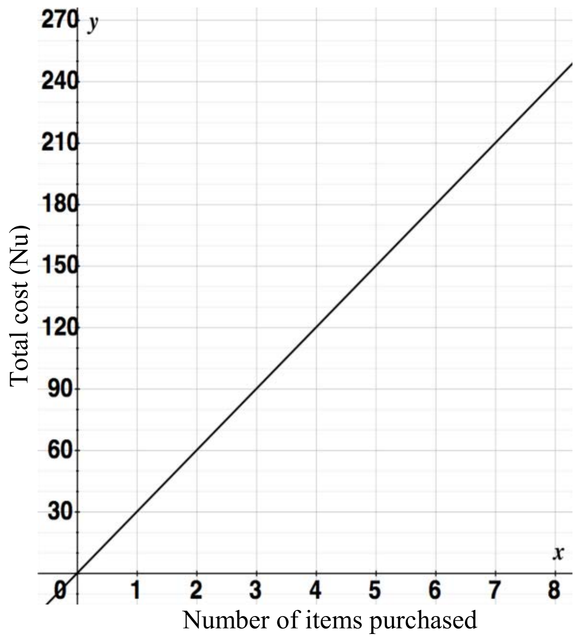
x	1	2	3	4	5
y	11	14	17	20	23

a) and b)

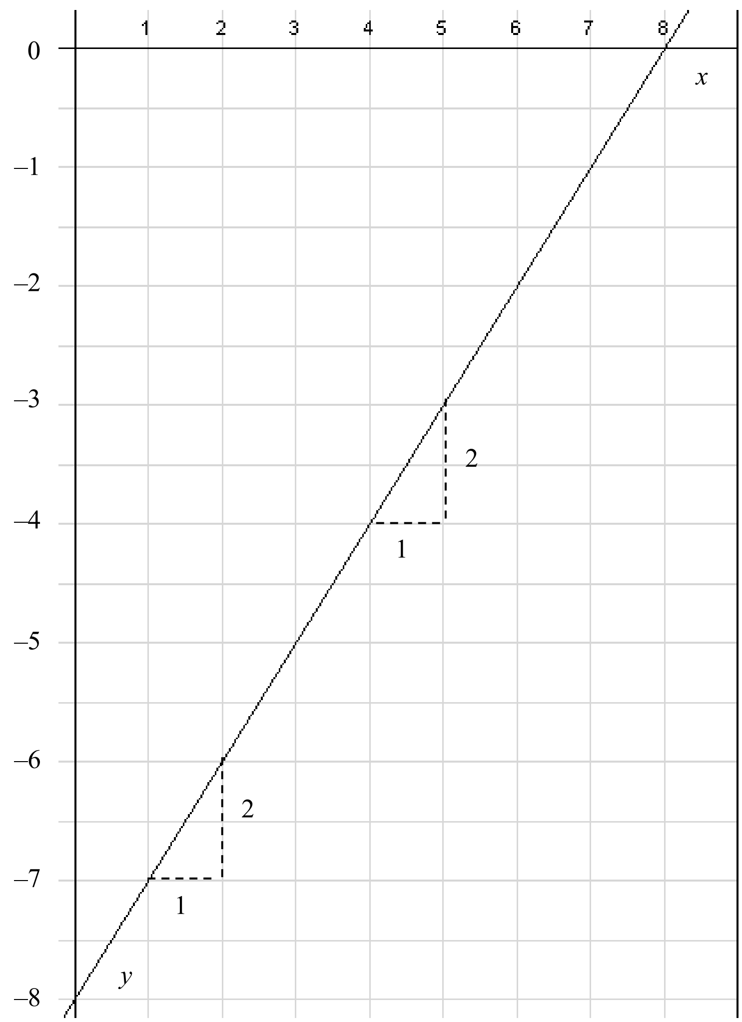


Answers [Continued]

11.



12. The dashed lines show that for every increase in 1 for x, there is an increase of 2 for y. It is the same anywhere along the graph.



UNIT 6 Performance Task — Names, Patterns, and Equations

A. i) Choose a letter from your name.
Shade in whole squares on grid paper to create the letter.
An example of the letter A is shown on the right.
Call the letter Figure 1.

ii) Build a pattern using bigger versions of the letter.

Arjun's letter A

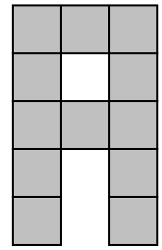


Figure 1

B. i) Write a pattern rule that relates the figure number to the total number of squares.
ii) Explain how you determined the rule.
iii) What is the constant in your expression? What is the coefficient?

C. i) Create an equation from your pattern rule and graph it.
ii) Create a problem about your pattern that could be solved using the graph.
iii) Write the answer to your problem.

D. Create a different problem about your pattern that could be solved using the equation. Solve it two of these three ways:
• using a model
• using inverse operations
• using guess and test

UNIT 6 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
7-B10 Simple Variable Expressions: relate to numerical expressions **7-C1** Summarize Patterns: make predictions **7-C2** Single Variable Linear Equations: represent solutions **7-C3** Single Variable Linear Equations: one and two step	1 h	• Grid paper or Small Grid Paper (BLM)

How to Use This Performance Task

You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. It could also be used as enrichment material for some students. You can assess performance on the task using the rubric on the next page.

Sample Solution

A. i) and ii)

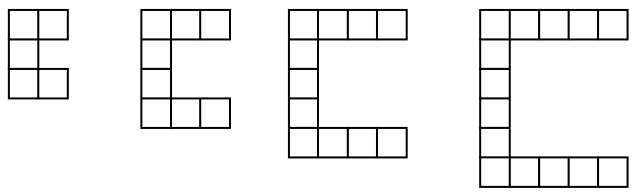
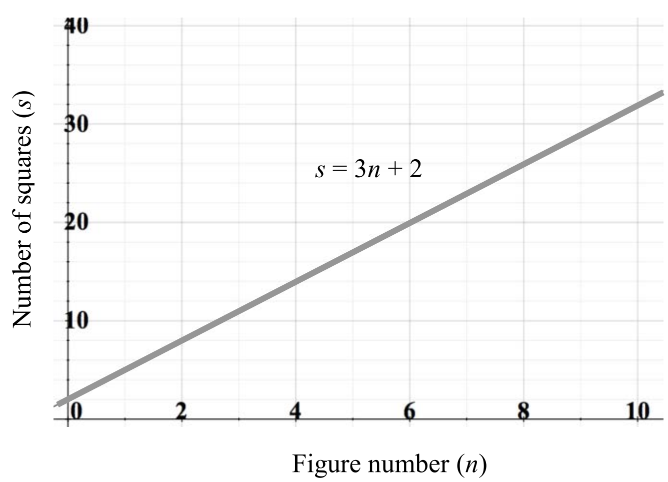


Figure 1 Figure 2 Figure 3 Figure 4

B. i) $3n + 2$
ii) I made a table of values and realized that I was going up by 3 with each new letter. Instead of the 3 times table: 3, 6, 9, 12, …, my numbers were always two greater: 5, 8, 11, 14, …, so I added 2 to the multiple of 3.
iii) Constant is 2; coefficient is 3.

C. i)



ii) How many squares will there be in Figure 10?
iii) 32

D. Which figure number has 50 squares in it?

Model

$3n + 2 = 50$

n	n	n	2
50			

n	n	n	2
48			2

n	n	n	2
16	16	16	2

$n = 16$

Inverse Operations

Subtract 2: $3n + 2 = 50$

Divide by 3: $3n = 48$

$n = 16$

Guess and Test $3n + 2 = 50$

Guess: 10	$3 \times \mathbf{10} + 2 = 32$	Too low. Try a higher number.
Guess: 20	$3 \times \mathbf{20} + 2 = 62$	Too high, but closer than 10. Try a number between 10 and 20.
Guess: 16	$3 \times \mathbf{16} + 2 = 50$	Got it!

Figure 16 has 50 squares.

UNIT 6 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Recognizes and describes a pattern	Builds a pattern that follows the instructions, identifies an appropriate pattern rule and explains the rule's development insightfully, and correctly identifies the components of the expression of the rule	Builds a pattern that follows the instructions, identifies an appropriate pattern rule and explains the rule's development, and correctly identifies the components of the expression of the rule	Builds a pattern, identifies an appropriate pattern rule but has difficulty explaining the rule's development, and correctly identifies at least one of the components of the expression of the rule	Builds a pattern, cannot identify an appropriate pattern rule, and correctly identifies at least one of the components of the expression of the rule
Graphs a relationship	Creates a completely correct graph of the pattern rule and uses it to create an interesting problem the graph could help solve	Creates a correct graph of the pattern rule and uses it to create a problem the graph could help solve	Creates a graph of the pattern rule with only minor errors, but has difficulty describing a problem to match the graph	Cannot create a correct graph of the pattern rule and has difficulty creating an appropriate equation for that graph
Solves an equation	Correctly and insightfully solves an equation using two different strategies	Correctly solves an equation using two different strategies	Correctly solves an equation one way	Cannot solve an equation

UNIT 7 PROBABILITY AND DATA

UNIT 7 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 219** **TG p. 295**	Review prerequisite concepts, skills, and terminology, and pre-assessment	1 h	• Grid paper or Small Grid Paper (BLM) or lined paper	All questions
Chapter 1 Probability				
7.1.1 Determining Theoretical Probability **SB p. 221** **TG p. 298**	**7-G2 Compare Results: theoretical versus experimental** • understand theoretical probability as: P(event) = number of favorable outcomes divided by total number of outcomes • understand that the theoretical probability formula can be used only when dealing with equally likely events (e.g., the probability of rolling 1, 2, 3, 4, 5, or 6 on a die) **7-G3 Independent Events: identify all possible outcomes** • construct tree diagrams to identify possible outcomes of independent events • use the area model to identify possible outcomes of independent events where one event is represented by one dimension, the other event by the other dimension of a rectangle	1 h	None	Q2, 3, 5
7.1.2 EXPLORE: Experimental Probability (Essential) **SB p. 224** **TG p. 302**	**7-G1 Describe Theoretical Probability: identify probability situations near 0, 1, $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{3}{4}$** • understand that performing more trials usually results in an experimental probability that approaches the theoretical probability **7-G2 Compare Results: theoretical versus experimental** • understand theoretical probability as: P(event) = number of favorable outcomes divided by total number of outcomes • understand experimental probability as the result of actual trials, where P (event) = number of times favoured outcome occurs divided by the total number of trials	1 h	• Dice	Observe and Assess questions
7.1.3 Matching Events and Probabilities **SB p. 226** **TG p. 305** [Cont'd]	**7-G1 Describe Theoretical Probability: identify probability situations near 0, 1, $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{3}{4}$** • understand that impossible events have a probability of 0 • understand that events that are certain have a probability of 1 • understand that uncertain events have a probability between 0 and 1	1 h	None	Q1, 3, 4

UNIT 7 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
[Cont'd] **7.1.3** Matching Events and Probabilities	**7-G2 Compare Results: theoretical versus experimental** • understand theoretical probability as: P(event) = number of favorable outcomes divided by total number of outcomes • understand experimental probability as the result of actual trials, where P (event) = number of times favoured outcome occurs divided by the total number of trials			
GAME: No Tashi Ta-gye! (Optional) **SB p. 229** **TG p. 308**	Practise probability concepts in a game situation	20 min	• Nu 1 coins	N/A
Chapter 2 Collecting Data				
7.2.1 Formulating Questions to Collect Data **SB p. 230** **TG p. 309**	**7-F1 Data Collection Methods: select and defend** • select, defend, and use appropriate data collection methods in real-world applications: - interview - observation - questionnaire • consider advantages disadvantages of different data collection methods • consider sensitivities such as privacy, cost, and political agenda **7-F2 Formulate Questions for Data Collection: real world application** • consider the following when formulating questions: - whether the question as asked will collect the data that is desired - simplicity and clarity of question - how data will be displayed	1 h	None	Q1, 2, 4
7.2.2 Sampling and Bias **SB p. 233** **TG p. 312**	**7-F2 Formulate Questions for Data Collection: for real world application** • explore issue of bias **7-F3 Bias: determine in questions and samples** • understand the distinction between first- and second-hand data • evaluate the reliability of second-hand data • understand bias in samples	40 min	None	Q2, 3
CONNECTIONS: Estimating a Fish Population (Optional) **SB p. 235** **TG p. 314**	Make a connection between the concept of sampling and probability	20 min	None	N/A
7.2.3 EXPLORE: Conducting a Survey (Essential) **SB p. 236** **TG p. 315**	**7-F2 Formulate Questions for Data Collection: for real world application** • explore issue of bias **7-F3 Bias: determine in questions and samples** • understand bias in samples	2 h (over several days)	• Paper for recording	Observe and Assess questions

	7-F1 Data Collection Methods: select and defend • select, defend, and use appropriate data collection methods in real-world applications: - interview - questionnaire			
Chapter 3 Graphing Data				
7.3.1 Circle Graphs **SB p. 237** **TG p. 317**	**7-F4 Circle Graphs: construct and interpret** • create a circle graph using a fraction circle in hundredths • represent proportions as percent of total circle • identify appropriate applications for circle graphs	1.5 h	• Percent Circles (BLM)	Q1, 3, 4
7.3.2 Histograms **SB p. 241** **TG p. 320**	**7-F5 Histograms: construct and interpret** • construct histograms to show the frequency distribution of data grouped in intervals • identify appropriate applications for histograms	1.5 h	• Grid paper or Small Grid Paper (BLM) or lined paper	Q1, 2, 3
Chapter 4 Describing and Analysing Data				
7.4.1 Mean, Median, Mode, and Range **SB p. 246** **TG p. 323**	**7-F6 Central Tendency: examine the effect of changing data** • understand that, if values are added to a set of data, any of the measures of central tendency can be affected • understand that adding, subtracting, multiplying, or dividing every value in a data set by the same value has the same effect on its mean, median, and mode	1 h	None	Q1, 3, 6
7.4.2 Outliers and Measures of Central Tendency **SB p. 250** **TG p. 326**	**7-F6 Central Tendency: examine the effect of changing data** • discuss the effect on mean, median, and mode if outliers are removed • understand that the measure of central tendency best suited to a particular situation is dependent on the situation (e.g., the median or mode is not affected by outliers as much as the mean) **7-F7 Variability: make inferences and predictions** • understand that range is the difference between the two extreme data values • find gaps and clusters in a set of data by observing and analysing the data • use range, outliers, gaps, and clusters to make inferences and predictions	40 min	None	Q1, 3, 5
UNIT 7 Revision **SB p. 254** **TG p. 329**	Review the concepts and skills in the unit	2 h	• Percent Circles (BLM)	All questions
UNIT 7 Test **TG p. 331**	Review the concepts and skills in the unit	1 h	None	All questions
UNIT 7 Performance Task **TG p. 334**	Review concepts and skills in the unit	1 h	• Fraction Circle Spinners (BLM) • Percent Circles (BLM)	Rubric provided
UNIT 7 Blackline Masters **TG p. 337**	BLM 1 Percent Circle BLM 2 Fraction Circle Spinners (in fifths for the **Performance Task**) Small Grid Paper in UNIT 1 on **p. 53**			

Math Background

• This data unit deals with many different data and probability topics including theoretical and experimental probability, data collection, data display, and data analysis.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 5** in **lesson 7.1.1**, where they use a tree diagram to find a probability, and in **questions 4 and 5** in **lesson 7.4.1**, where they create data sets to match conditions.

• Students use communication in **part E** in **lesson 7.1.2**, where they explain why they might be more confident with one prediction than with another, in **question 2** in **lesson 7.1.3**, where they use probability language to describe a situation, in **question 2** in **lesson 7.2.1**, where they explain their thinking about why one question is better others for collecting data, in **question 2** in **lesson 7.2.2**, where they explain why a sample might be biased, in **question 6** in **lesson 7.4.1**, where they describe an efficient process for calculating a mean, and in **question 3** in **lesson 7.4.2**, where they describe why one measure of central tendency is more appropriate than another in a particular case.

• Students use reasoning in **question 3** in **lesson 7.1.3**, where they use past experience to predict future events, in **lesson 7.2.1**, where they reason about why one question is more appropriate than another to collect data, in **question 4** in **lesson 7.2.2**, where they consider the effect of sample size on bias, and in **question 5** in **lesson 7.3.1**, where they use reasoning to match a set of data with the correct circle graph.

• Students consider representation in **question 7** in **lesson 7.1.1**, where they decide how two different representations of a situation are alike and different, and in **lesson 7.3.2**, where they consider how the choice of intervals for a histogram affects the representation of the data.

• Students use visualization skills in **lesson 7.3.1** and in **lesson 7.3.2**, where they use circle graphs and histograms to make sense of data, and in **question 5** in **lesson 7.4.2**, where they use a histogram to interpret a data set.

• Students make connections in **question 3** in **lesson 7.1.3**, where they relate probability concepts to a real-world situation, in **lesson 7.2.3** where they conduct a survey to gain information about a real-world issue, and in **lesson 7.3.2**, where they connect histograms to bar graphs.

Rationale for Teaching Approach

• This unit is divided into four chapters.

In **Chapter 1**, students explore both experimental and theoretical probability because it is important that students make the distinction between the two. This chapter also extends students' ability to work backwards, creating situations to match numerical probabilities.

Chapter 2 focuses on issues related to collecting data. Students need to consider not only what questions to ask to collect good data, but whom to ask. They apply the skills they have learned by conducting a survey.

Chapter 3 develops students' skills in creating two types of graphs: circle graphs and histograms. This initial work with circle graphs separates the skill of understanding what a circle graph is about from the ability to measure angles properly to create a circle graph; the latter skill is developed in Class VIII.

Chapter 4 extends students' understanding of various statistics, specifically mean, median, mode, and range, and shows students the effect of outliers (extreme pieces of data) on some of those statistics.

• Both **Explore lessons** are essential to accomplishing the outcomes because a more exploratory approach is required. One exploration lets students gather data to compare experimental and theoretical probability, and the other exploration has students apply acquired skills in surveying to conduct a survey.

• The **Connections** section shows students how scientists can use statistical concepts to study the world.

• The **Game** provides an opportunity to use probability concepts.

Getting Started

Curriculum Outcomes	Outcome relevance
6 Collect, Organize and Describe Data: real world issues **6** Line Graphs: construct and interpret **6** Bar and Double Bar Graphs: construct and interpret **6** Stem and Leaf Plots: grouping data **6** Mean, Median, and Mode: concepts **6** Inference: interpret data **6** Theoretical Probability: determine	Students will find the work in the unit easier after they review the concepts of factors and multiples, prime numbers, place value, and calculations with decimals.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM) or lined paper	• familiarity with the terms *factor*, *multiple*, *common factor*, and *prime number* • place value from billions through thousandths • multiplying and dividing by powers of 10 • multiplying and dividing by simple decimals

Main Points to be Raised

Use What You Know

• A stem and leaf plot is a useful way to organize data into categories based on place value.

• The mean of a set of data is the result of sharing the data equally among all the data values.

• The median of a set of data is the middle number if the data values are displayed in order.

• The mode of a set of data is the piece of data that appears most frequently.

• The mean, median, and mode are all measures of central tendency. Depending on the situation, one measure may represent a data set better than the others.

• A double bar graph is a way to display two sets of data with similar ranges organized into the same categories at the same time.

• One type of graph might display a particular set of data more effectively than another type of graph.

Skills You Will Need

• Two fractions are equivalent if the numerator and denominator of one can be multiplied or divided by the same amount to create the other fraction.

• A percent is a fraction with a denominator of 100. Percents can be expressed as equivalent fractions with other denominators.

• A line graph is useful to show a trend. It makes sense to interpolate (read between plotted values) and sometimes to extrapolate (extend beyond plotted values) to describe data not explicitly collected.

• A stem and leaf plot is a useful way to organize data into categories based on place value.

Use What You Know — Introducing the Unit

• Before beginning this unit, ask each student to find out the ages of his or her mother and father and bring that information to school. If some students cannot get the information, suggest reasonable values they could use.

• You may wish to review the meaning of the terms *mean, median, mode, stem and leaf plot,* and *double bar graph*. You might use the data set 3, 4, 5, 5, 13 to review the first three terms (the mean is 6 because the sum of the data is 30 and there are 5 pieces of data; the median is 5 because 5 is the middle number; the mode is also 5 because 5 is the most frequent number). You can use the graphs on **page 219** of the student text to remind students about stem and leaf plots and double bar graphs.

• Before beginning the activity, write the ages of mothers and fathers for the whole class on the board. You may choose to put them in numerical order, but that is not required. Make sure that students understand that the stem and leaf plots in the book are just samples; they should use the stem and leaf plot with the class data to answer **parts B to D**. If it is not possible to collect data from students, they can use the plots in the student text.

Ask students to work alone or in pairs on the activity. While you observe students at work, you might ask the following questions:

• *How did you decide what row to put 33 in?* (It has to be in the row with a 3 as the stem.)

• *How did you know there would be [7] numbers in that row?* (There were 7 numbers in the 30s.)

• *How can you use the stem and leaf plot to determine the mode? the median?* (For the mode, I look for the leaf that is repeated the most. For the median, I count how many numbers there are. Since there are [41], I know there are [20] numbers before the median, so I start from the top and find the [21st] number.)

• *How did you decide that the mode was not the best number to represent the data?* (I thought it was too low because the ages go from 29 to 51 and a mode of 35 would not represent that range.)

• *Why did it make sense to use a double bar graph for this information?* (The ages of mothers and fathers could all be put in the same categories.)

• *Why was it easier to use the stem and leaf plots than the double bar graph to tell the age of the oldest parent?* (With the double bar graph, I cannot see the individual ages, but only the number of ages in a category.)

Skills You Will Need

• To ensure students have the required skills for this unit, assign these questions.

• Although your goal is to see what students recall, you may wish to review the content in some questions if many students seem to be unsure of how to proceed for that question.

• Students can work individually.

Answers

A. ii) *Sample response*: See the sample on **page 219** of the student text. **B.** *Sample responses*: **i)** Mothers' ages: mean is 39.2; median is 38.5; mode is 35. Fathers' ages: mean is 41; median is 41; mode is 41. **ii)** For mothers: The mean or median best represents the mothers' ages. The mode is too low. For fathers: It does not matter which measure you use because they are all the same.	**C.** *Sample response*: See the sample on **page 219** of the student text. **D.** *Sample responses*: **i)** 55; I used the stem and leaf plot because it shows each data value. The bar graph only shows the number of data values in each interval. **ii)** 15; I could use either graph but the bar graph might be easier because I only have to add 11 + 4 for the two bars, but in the stem and leaf plot I have to count all the values that are 40 or more.

NOTE: Answers or parts of answers that are in square brackets throughout the Teacher's Guide are NOT found in the answers in the student textbook.

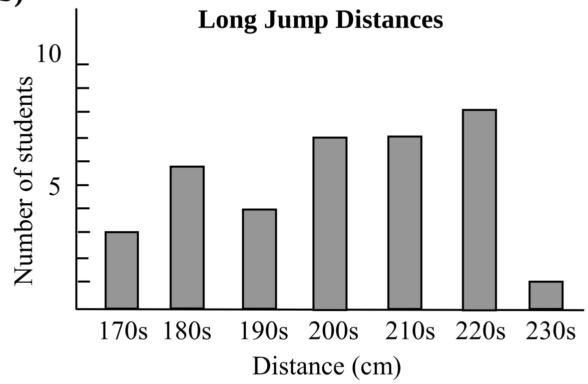
1. a) 15 **b)** 40 **c)** 11 **d)** *Sample response*: $\frac{9}{10}$ **e)** 56 **f)** 85 **g)** 15 **h)** *Sample response*: About 53 **2.** $\frac{2}{5}$, 0.4, 40% **3.** *Sample responses*: **a) i)** About 55% **ii)** About 72% **iii)** About 83% **b)** 62% **c)** Yes; [the graph extends to reach 100% before 2015.]	**4. a)** *Sample response*: About 54 kg per year **b)** 30 kg per year **c)** No; [The bar is probably too small to show on the scale of this graph.] **5. a)** Nu 11 **b)** Nu 5 **c)** Nu 5

6. *Sample responses*:

a) Long Jump Distances (cm)

17	2 3 5
18	5 6 6 7 8 9
19	3 3 6 8
20	0 2 5 5 6 7 8
21	2 2 2 4 5 5 8
22	0 1 1 2 3 7 8 9
23	0

b)



Supporting Students

Struggling students

• Some students might need brief reviews of any of the following:

- creating stem and leaf plots
- creating double bar graphs
- finding mean, median, and/or mode
- creating equivalent fractions or percents
- finding a theoretical probability in a simple situation

• Some students might need to be reminded of the importance of organizing data in order from least to greatest before creating a stem and leaf plot.

Enrichment

• You might ask students to create stem and leaf plots to meet various criteria.

For example, you could ask for a plot where there are 15 pieces of data, the median is 19, and there are more pieces of data in each category than in the category above it.

Chapter 1 Probability

7.1.1 Determining Theoretical Probability

Curriculum Outcomes	Outcome relevance
7-G2 Compare Results: theoretical versus experimental • understand theoretical probability as: P(event) = number of favorable outcomes divided by total number of outcomes • understand that the theoretical probability formula can be used only when dealing with equally likely events (e.g., the probability of rolling 1, 2, 3, 4, 5, or 6 on a die) **7-G3 Independent Events: identify all possible outcomes** • construct tree diagrams to identify possible outcomes of independent events • use the area model to identify possible outcomes of independent events where one event is represented by one dimension, the other event by the other dimension of a rectangle	Most probability theory, especially in upper classes, is based on theoretical probability, not on experimental probability. Students need to understand the difference between the two and be able to determine all possible outcomes to find theoretical probability.

Pacing	Materials	Prerequisites
1 h	None	• understanding of fractions of a whole

Main Points to be Raised

• Theoretical probability is a fraction that compares the number of favourable outcomes to the number of possible outcomes.

• Theoretical probability is useful for predicting what will happen in the future.

• If more than one event is involved in a probability situation, you can use a tree diagram that shows the combinations of what can happen in the first event and what can happen in the second event.

• If two events are involved in a probability situation, you can draw a rectangle model that shows the combinations of what can happen in the first event and what can happen in the second event. The dimensions of the rectangle should be proportional to the likelihood of each event.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Why is the sum of 7 not possible?* (The most I could spin is 3 + 3 = 6.)

• *What is the least sum possible? How do you know?* (2, because it is 1 + 1 and 1 is the least I can spin.)

• *Why do you think that a sum of 4 is more likely than a sum of 2?* (I can only get 2 if I spin 1 and then another 1, but I can get 4 in three different ways: 1 and then 3, 2 and then 2, or 3 and then 1.)

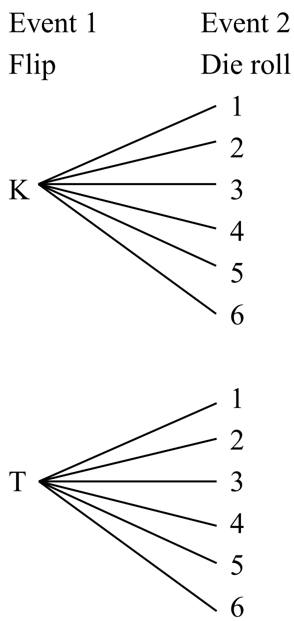
The Exposition — Presenting the Main Ideas

• Remind students about the meaning of the term *theoretical probability* by discussing what can happen when you toss a coin. Ask what the probability of Khorlo is ($\frac{1}{2}$). Point out how the 2 tells that there are two possible outcomes (and they are equally likely) and the 1 tells that you are only interested in one of those outcomes.

• Have students open their texts to **page 221**. Have them look first at the tree diagram to see how it shows the two possibilities for the first flip (in the first column) and the two possibilities for the second flip, no matter what happened on the first flip (in the second column). Point out that the probability of two Khorlos is $\frac{1}{4}$ since only 1 of the 4 branches at the far right shows KK. The probability of one Khorlo and one Tashi Ta-gye is $\frac{2}{4}$ because 2 of the 4 branches describe that situation.

• Next, point out the rectangular diagram on **page 221** that shows the same situation. Make sure students understand that the length and width are each split in half because K and T are equally likely. Again, only 1 of the 4 sections relates to KK but 2 of the 4 sections relate to flipping one Khorlo and one Tashi Ta-gye.

• Make sure students understand that if, for example, the first event had been the flip of a coin and the second event had been the roll of a die, there would have been 12 possible outcomes and so there would be 12 branches on a tree diagram and 12 sections in a rectangle diagram.



	1	**2**	**3**	**4**	**5**	**6**
K	K1	K2	K3	K4	K5	K5
T	T1	T2	T3	T4	T5	T6

Revisiting the Try This

B. This question allows students to make a formal connection between what was done in **part A** and the main ideas presented in the exposition.

Using the Examples

• Describe the situation presented in the example. Lead students through the two solutions. For **solution 1**, point out that a tree diagram that just shows B and W in the left column would not represent the situation because it would make it seem that black and white are equally likely when they are not. Point out that the two black counters are described as B_1 and B_2 to show that black is twice as likely as white. Ask students what the left column would have looked like if there had been 3 black counters and 2 white counters (e.g., B_1, B_2, B_3, W_1, W_2).

• Similarly, point out why the two rows in the rectangle diagram in **solution 2** have different depths. The B row occupies twice as much space as the W row because B is twice as likely as W. The dotted line is there just to help show that it is twice as deep.

Practising and Applying

Teaching points and tips

Q 1: Make sure students realize they are to multiply and not add the two numbers spun. Check that their diagrams and rectangles have 16 branches or sections.

Q 2: Make sure students notice that for **part a)** no tree diagram or rectangle is needed because there is only one event, but either a tree diagram or a rectangle is needed for **part b)** because there are two events.

Q 3: Students need to understand that even though the values of the coins are different, each is equally likely to be drawn and is associated with the same number of branches on the tree diagram or the same number of sections in a rectangle diagram.

Q 5: Students might realize that they do not need to use a tree diagram or a rectangle model for **part a)**, but they do need a model for **part b)**.

Q 6: This question extends the learning as students must create a tree diagram with three columns rather than only two. To find the answer, they must realize that the Khorlo could happen on the first, second, or third flip.

Q 7: You may wish to have a class discussion to deal with this question or you might have students discuss it in pairs. Some students might realize that tree diagrams can be used for any number of events but rectangle models are suitable for two events (because rectangles are two-dimensional).

Common errors

• Many students will have difficulty recognizing when a tree diagram or rectangle diagram is required and when it is not. They should focus on whether or not more than one outcome must be considered at the same time.

Suggested assessment questions from Practising and Applying

Question 2	to see if students recognize whether a tree diagram or rectangle diagram is required and, if it is, whether they can use it to calculate a probability
Question 3	to see if students can solve a problem that requires the calculation of a probability involving two outcomes
Question 5	to see if students can set up a tree diagram for a given situation

Answers

A. *Sample responses*:

i) 2, 3, 4, 5, and 6

ii) I think they are the same because there is one of each number on the spinner — 1, 2, and 3 — and it is equally likely that I will spin each of them.

B. *Sample responses*:

i)

	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

P(2) = $\frac{1}{9}$, P(3) = $\frac{2}{9}$, P(4) = $\frac{3}{9}$, P(5) = $\frac{2}{9}$, and P(6) = $\frac{1}{9}$.

ii) No; A sum of 4 is more likely than sums of 2, 3, 5, or 6, and sums or 3 and 5 are more likely than sums of 1 and 6. I did not realize that the pairs 1 + 2 and 2 + 1, 1 + 3 and 3 + 1, and 2 + 3 and 3 + 2 would each count as two possible outcomes.

1. a) $\frac{1}{16}$ **b)** $\frac{1}{8}$ **c)** $\frac{3}{16}$ **d)** $\frac{1}{2}$ **e)** $\frac{3}{4}$

2. a) i) $\frac{1}{13}$ **ii)** $\frac{1}{4}$ **iii)** $\frac{3}{13}$ **iv)** $\frac{5}{13}$ **v)** $\frac{1}{26}$

b) $\frac{1}{16}$

3. $\frac{7}{16}$

4. a) $\frac{4}{25}$ **b)** $\frac{8}{25}$ **c)** $\frac{8}{25}$ **d)** $\frac{4}{25}$ **e)** $\frac{1}{25}$

5. a) $\frac{1}{3}$ **b)** $\frac{5}{9}$

6. $\frac{3}{8}$

[**7.** *Sample response*:
Similar:
Both show ways of combining outcomes of events in an organized way to show all possible outcomes.
Different:
The tree diagram shows outcomes as if they happened one after the other, but the rectangle shows them as if they were happening at the same time.]

Supporting Students

Struggling students

• If some students seem to be struggling with the lesson, you may wish to have them focus only on the tree diagram or only on the rectangle diagram, rather than on both. In such a case, you would not ask them to complete **question 7**. You might also not assign **question 6** to struggling students because it extends the learning in the lesson.

Enrichment

• You might challenge students to create other probability problems involving two events for classmates to solve.

7.1.2 EXPLORE: Experimental Probability

Curriculum Outcomes	Outcome Relevance
7-G1 Describe Theoretical Probability: identify probability situations near 0, 1, $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{3}{4}$ • understand that performing more trials usually results in an experimental probability that approaches the theoretical probability **7-G2 Compare Results: theoretical versus experimental** • understand theoretical probability as: P(event) = number of favorable outcomes divided by total number of outcomes • understand experimental probability as the result of actual trials, where P = number of times favoured outcome occurs divided by the total number of trials	This essential exploration allows students to see how experimental probability is different from theoretical probability, but how the value of an experimental probability usually approaches the value of the theoretical probability with more trials.

Pacing	Materials	Prerequisites
1 h	• Dice	• calculating theoretical probability for two simple outcomes • using a tally chart

Exploration

• Show a die to the students. Ask them to tell you the probability of rolling a 1 ($\frac{1}{6}$). Roll the die 6 times and see if you actually do roll only one 1. That might happen, but it might not. If it does occur, continue to do sets of 6 rolls until one set of 6 does not include a 1. Point out the difference between experimental and theoretical probability.

• Provide a pair of dice to a pair or small group of students. Let them go through the exploration. Make sure they understand that they are looking at the total of the two values rolled.

• Make sure the students use either the tree diagram or the rectangle model (not both) to find the theoretical probabilities. They should use the data from 36 rolls of the dice to complete the column for the experimental probability.

• The students do not need to roll again for **part B**, but each group should combine its data with another group's data.

• For **part C**, each group should combine data with yet another group. The purpose of the activity is to help them see that with more trials, the experimental values approach theoretical values, although this does not always happen in a particular circumstance.

• For each **part iii)**, students might add a column to their charts to record the actual differences between the experimental and theoretical probabilities and note whether each difference is $\frac{1}{36}$ ($\frac{2}{72}$, $\frac{3}{108}$) or less apart (as is shown in the answers on **pages 303 and 304**). Or they might circle the sums that are $\frac{1}{36}$ ($\frac{2}{72}$, $\frac{3}{108}$) or less apart.

While you observe students at work, you might ask questions such as the following:

• *For **part A**, do all of your experimental probabilities match your theoretical probabilities?* (No. Some do, but not all of them.)

• *With more data, were the experimental probabilities and theoretical probabilities closer?* (Yes. More of them were very close.)

• *Why can you be more confident in estimating theoretical probability using 100 flips than using 10 flips?* (It was just like the dice rolls. With more times, fewer surprises seem to happen.)

Students should keep this data for later use in **lesson 7.3.1**.

Observe and Assess

As students work, notice:

- Do they calculate the theoretical probabilities correctly?
- Do they calculate the experimental probabilities correctly?
- Do they recognize that, with more trials, experimental probability tends to be closer to theoretical probability?
- Do they realize that $\frac{1}{36} = \frac{2}{72} = \frac{3}{108}$?

Share and Reflect

After students have had sufficient time to work through the exploration, discuss their observations with them using questions such as these.

- *How often were your experimental and theoretical values* $\frac{1}{36}$ *or less apart for* ***part A****?*
- *How often were your experimental and theoretical values* $\frac{1}{36}$ *or* $\frac{2}{72}$ *or less apart for* ***part B****?*
- *How often were your experimental and theoretical values* $\frac{1}{36}$ *or* $\frac{3}{108}$ *or less apart for* ***part C****?*
- *Why do you think your answers to these three questions might be different?*

Answers

A. i), ii), and iii)

Sample response:

Rolling two dice 36 times

Sum	Theoretical probability	Experimental probability		iii)	
2	$\frac{1}{36}$	//	$\frac{2}{36}$	$\frac{1}{36}$	Y
3	$\frac{2}{36}$	/	$\frac{1}{36}$	$\frac{1}{36}$	Y
4	$\frac{3}{36}$	//	$\frac{2}{36}$	$\frac{1}{36}$	Y
5	$\frac{4}{36}$	~~////~~	$\frac{5}{36}$	$\frac{1}{36}$	Y
6	$\frac{5}{36}$	~~////~~ //	$\frac{7}{36}$	$\frac{2}{36}$	N
7	$\frac{6}{36}$	////	$\frac{4}{36}$	$\frac{2}{36}$	N
8	$\frac{5}{36}$	////	$\frac{4}{36}$	$\frac{1}{36}$	Y
9	$\frac{4}{36}$	//	$\frac{2}{36}$	$\frac{2}{36}$	N
10	$\frac{3}{36}$	///	$\frac{3}{36}$	0	Y
11	$\frac{2}{36}$	///	$\frac{3}{36}$	$\frac{1}{36}$	Y
12	$\frac{1}{36}$	///	$\frac{3}{36}$	$\frac{2}{36}$	N

iii) The probabilities were $\frac{1}{36}$ or less apart a bit more than half the time.

B. i), ii), and iii)

Sample response:

Rolling two dice 72 times

Sum	Theoretical probability	Experimental probability	iii)	
2	$\frac{2}{72}$	$\frac{2}{72}$	0	Y
3	$\frac{4}{72}$	$\frac{6}{72}$	$\frac{2}{72}$	Y
4	$\frac{6}{72}$	$\frac{7}{72}$	$\frac{1}{72}$	Y
5	$\frac{8}{72}$	$\frac{7}{72}$	$\frac{1}{72}$	Y
6	$\frac{10}{72}$	$\frac{10}{72}$	0	Y
7	$\frac{12}{72}$	$\frac{9}{72}$	$\frac{3}{72}$	N
8	$\frac{10}{72}$	$\frac{9}{72}$	$\frac{1}{72}$	Y
9	$\frac{8}{72}$	$\frac{7}{72}$	$\frac{1}{72}$	Y
10	$\frac{6}{72}$	$\frac{6}{72}$	0	Y
11	$\frac{4}{72}$	$\frac{5}{72}$	$\frac{1}{72}$	Y
12	$\frac{2}{36}$	$\frac{3}{72}$	$\frac{1}{72}$	Y

iii) The probabilities were $\frac{2}{72}$ or less apart almost every time.

Answers [Continued]

C. i), ii), and iii)
Sample response:

Rolling two dice 108 times

Sum	Theoretical probability	Experimental probability	iii)
2	$\frac{3}{108}$	$\frac{2}{108}$	$\frac{1}{108}$ Y
3	$\frac{6}{108}$	$\frac{6}{108}$	0 Y
4	$\frac{9}{108}$	$\frac{9}{108}$	0 Y
5	$\frac{12}{108}$	$\frac{9}{108}$	$\frac{3}{108}$ N
6	$\frac{15}{108}$	$\frac{14}{108}$	$\frac{1}{108}$ Y
7	$\frac{18}{108}$	$\frac{18}{108}$	0 Y
8	$\frac{15}{108}$	$\frac{18}{108}$	$\frac{3}{108}$ Y
9	$\frac{12}{108}$	$\frac{12}{108}$	0 Y
10	$\frac{9}{108}$	$\frac{9}{108}$	0 Y
11	$\frac{6}{108}$	$\frac{9}{108}$	$\frac{3}{108}$ Y
12	$\frac{3}{108}$	$\frac{3}{108}$	0 Y

iii) The probabilities were $\frac{3}{108}$ or less apart every time.

D. The number of times the theoretical probabilities were close to the experimental probabilities was higher with more rolls. It probably happened because unexpected high results were balanced by unexpected low results when there were more rolls.

E. Flipping the coins 100 times would give a more reasonable prediction; *Sample response*:
With more flips, it will be more like what happened in my experiment and more results will be close to what they should be.

Supporting Students

Struggling students

• If students are struggling with calculating theoretical probabilities, suggest that they roll the dice separately. Help them figure out that the denominators have to be 36 because there are six possible second values for each of the six possible rolls on the first die.

Enrichment

• Some students might enjoy carrying out a similar investigation where, instead of the sums of two dice, they consider the theoretical and experimental probabilities based on the difference of the two numbers rolled.

7.1.3 Matching Events and Probabilities

Curriculum Outcomes	Outcome relevance
7-G1 Describe Theoretical Probability: identify probability situations near 0, 1, $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{3}{4}$ • understand that impossible events have a probability of 0 • understand that events that are certain have a probability of 1 • understand that uncertain events have a probability between 0 and 1 **7-G2 Compare Results: theoretical versus experimental** • understand theoretical probability as: P(event) = number of favorable outcomes divided by total number of outcomes • understand experimental probability as the result of actual trials, where P = number of times favoured outcome occurs divided by the total number of trials	By working backwards to describe events with particular probabilities, students gain a deeper understanding of the use of fractions to describe probabilities.

Pacing	Materials	Prerequisites
1 h	None	• meaning of fractions • equivalent fractions

Main Points to be Raised

• Theoretical probability is based on analysing all possible results and considering the likelihood that each of those possibilities will occur. Experimental probability is based on what actually happens in a particular set of trials.

• Words that describe probabilities include *impossible, very unlikely, unlikely, even chance, likely, very likely,* and *certain*. By knowing which of those words best describes an event, you can have a good idea of how to predict the likelihood that the event will occur in the future.

• An event with a probability of 1 is certain to happen.

• An event with a probability of 0 will never happen.

• As the fraction describing a probability gets closer to 1, the event is more and more likely to happen. If the fraction gets farther from 1 and closer to 0, the event is less and less likely to happen.

• An event with a probability of $\frac{1}{2}$ is as likely to happen as not to happen.

• You can create an event with a particular probability by setting up a situation where the number of possible equally likely outcomes is the denominator of the fraction for the probability and the number of favourable events is the numerator of the fraction.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Suggest that they first create a chart to show all possible outcomes. While you observe students at work, you might ask questions such as the following:

• *What are the possible outcomes when you roll a die?* (The numbers 1, 2, 3, 4, 5, or 6.)

• *How will you decide whether an event is likely?* (I will call it likely if it happens more than it does not happen.)

• *Why are those two events equally likely?* (There are three ways to roll an even number and three ways to roll an odd number.)

The Exposition — Presenting the Main Ideas

• Ask students to look at the probability line on **page 226** and help them see the following:

- The words show that an event that is more likely to happen should be placed farther to the right than one that is less likely to happen.

- The numbers increase from left to right, so the least possible probability of 0 matches an impossible event, the greatest possible probability of 1 matches a certain event, and the fraction that describes the probability of a possible event is greater if the event is more likely to happen.

• Ask students why the probability of flipping a coin and getting Khorlo is $\frac{1}{2}$. Point out the place on the probability line that describes this probability. Also point out why the probability is $\frac{1}{2}$ (because there are 2 possible outcomes (the denominator) and only 1 is favourable (the numerator)).

• Draw a spinner with 4 equal sections marked 1, 1, 1, and 2. Ask why the probability of spinning 1 is $\frac{3}{4}$. Point out that the denominator is 4 is because there are 4 equal sections and the numerator is 3 because there are 3 sections that are favourable (where you spin a 1). Ask how they would set up the spinner to make the probability of spinning a 1 to be $\frac{2}{5}$ instead (a spinner with 5 equal sections, where two of them are marked 1).

• Discuss another way to set up a situation where you might have a $\frac{2}{5}$ probability of getting a 1: Take 5 slips of paper, write a 1 on two of those slips, write other numbers on the other slips, and place the all strips in a bag. Ask the students why the probability of selecting a slip with a 1 on it is $\frac{2}{5}$.

• Ask students why you might use a die to create a situation where the probability is given in sixths, but not for
a situation where the probability is given in fifths.

• Point out how using slips of paper in a bag or using spinners allows you to set up a situation to match any given probability. You first adjust the number of slips in the bag or the number of equal spinner sections to make the denominator correct, and then you adjust the number of slips or the number of sections marked with the value of interest to make the numerator correct.

Revisiting the Try This

B. This question allows students to connect probabilities described as fractions with probabilities described using words in **part A**.

Using the Examples

• Present the question in **example 1** to the students and ask them to respond. They can then check their responses against those provided in the student text.

• Work through **example 2** with the students. Make sure they understand why equivalent fractions with a denominator of 100 were used in each case.

Practising and Applying

Teaching points and tips

Q 1: Refer students back to the probability line in the exposition or draw a copy of it on the board. Be sure to include both the fractions and the probability words.

Q 2: Make sure students realize that two different students could describe the same event to match very different probability words.

For example, if one student never has left his village and another has left it frequently, each would use different probability words to describe the likelihood of leaving his or her own village.

Q 3: Students will need to think about equivalent fractions with denominators of 100 to address this question.

Q 4: Make sure students understand that they should look at all four choices before answering this question.

Q 5: This question is designed to help students focus on the distinction between experimental and theoretical probability.

Q 6: Students may have varying opinions on what is easy or difficult.

Common errors

• Some students will have difficulty attaching the word likely (or unlikely) to events with probabilities close to $\frac{1}{2}$. Help them understand that as soon as a probability is greater than $\frac{1}{2}$, it should be called likely. (It could be called very likely if it is considerably greater than $\frac{1}{2}$, or certain if it is 1.)

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use probability words appropriately
Question 3	to see if students can create events to match probabilities
Question 4	to see if students can compare probabilities

Answers

A. *Sample responses*: **i)** Unlikely **ii)** Very likely **iii)** No. They are equally likely. **B**. **i)** P(less than 3) = $\frac{2}{6}$ or $\frac{1}{3}$ P(greater than 1) = $\frac{5}{6}$ P(even) = $\frac{3}{6}$ or $\frac{1}{2}$ and P(odd) = $\frac{3}{6}$ or $\frac{1}{2}$	**ii)** Yes; • On the probability line, $\frac{1}{3}$ is unlikely, so P(less than 3) is unlikely. • On the probability line, $\frac{5}{6}$ is very likely, so P(greater than 1) is very likely. • $\frac{1}{2} = \frac{1}{2}$ so P(even) is not greater than P(odd).
1. a) Very unlikely **b)** Certain **c)** Very unlikely **d)** Likely (or very likely) **e)** Impossible **f)** Likely (or very likely) **g)** Even chance **2.** *Sample responses*: Impossible: I will grow two heads. Certain: My next birthday will be on <insert date>. Very likely: I will go straight home after school. Likely: We will have rice for dinner. Even chance: My mother's new baby will be a girl. Unlikely: I will go to the next Tshechu in Thimphu. Very unlikely: I will get a perfect mark on my next math test. **3. a)** P(Gorthibu-karey) **b)** P(other karey) **c)** P(karey) **d)** P(miss) or P(no karey)	**4.** B and D; [The chances of drawing a spade are 13 in 52, or $\frac{13}{52}$ or 1 chance in 4, or $\frac{1}{4}$. There are four seasons so the chance of a summer birthday is also 1 in 4, or $\frac{1}{4}$.] **5. a)** 1 or $\frac{100}{100}$; certain **b)** No; [*Sample response*: Not hitting the target is always a possible outcome, even if it did not happen for the experimental results.] [**6.** *Sample response*: If the denominator of the fraction is equal to the number of possible outcomes, it is easy to match the number of favourable outcomes with the numerator. It is harder when the denominator is different.]

Supporting Students

Struggling students

• Struggling students might have trouble coming up with situations to match probabilities when they must first create equivalent fractions. For **question 3**, you might have them use fractions with denominators of 100 instead of the given fractions.

• You might need to lead struggling students through **question 5** because they may have trouble seeing the main point — experimental probability and theoretical probability are different.

Enrichment

• You might ask students to create situations to match other fractional probabilities.

GAME: No Tashi Ta-gye!

• This game provides a hands-on opportunity for students to see that prior results do not affect the probability of any one individual event. Many students find it difficult to believe that the probability of flipping Khorlo is the same whether you have already flipped KKKK or whether you have flipped only one K.

• Students' personalities will influence how they play this game. Some students are naturally cautious, but others are more likely to take risks.

• Once the game has been played, be sure to discuss with students why you are not more likely to flip a Tashi Ta-gye on the next turn, even if you have not flipped a Tashi Ta-gye for many turns before that.

Chapter 2 Collecting Data

7.2.1 Formulating Questions to Collect Data

Curriculum Outcomes	Outcome relevance
7-F1 Data Collection Methods: select and defend • select, defend, and use appropriate data collection methods in real-world applications: - interview - observation - questionnaire • consider advantages disadvantages of different data collection methods • consider sensitivities such as privacy, cost, and political agenda **7-F2 Formulate Questions for Data Collection: real world application** • consider the following when formulating questions: - whether the question as asked will collect the data that is desired - simplicity and clarity of question - how data will be displayed	An important aspect of using and interpreting data is understanding that the way data is collected can influence the results. Students need to recognize the importance of good, clear questions for collecting data. They also need to see that different collection strategies are appropriate in different situations.

Pacing	Materials	Prerequisites
1 h	None	None

Main Points to be Raised

• You need to be thoughtful about how to collect data in order to gather good information.

• Three ways to collect data personally are interviews, questionnaires, and observation. In some situations, one method is better, or more practical, than another.

• An interview is where a person directly asks other people a question and records their answers.

• A questionnaire involves the preparation of questions to be asked of others, without personal interaction.

• Observation involves watching to collect data.

• When you ask a question to collect information, it is important to ask the question simply and clearly so that everyone understands it and interprets it in the same way.

• It is good to avoid questions with more than one part, questions with too many negatives, and questions that people might not be willing to answer honestly.

• Sometimes it is good to have choices for the answer to a question. This makes it easier to create bar graphs to display the data in different categories.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Make sure they understand the task and what is meant by a "survey company". While you observe students at work, you might ask questions such as the following:

• *Why might you get a different response if you asked "Are you happy?" than if you asked "Do you think most of the people in Bhutan are happy?"?* (The person who answers might be happier than a lot of other people, or she might be more unhappy than most people.)

• *Why would you not ask, "Are people in your village happy?"?* (The survey is supposed to be about the whole country, not about just one village. One village may be quite different from others.)

• *Why might it be helpful to give a choice of responses, such as "very happy", "happy", and so on?* (Without a choice, different people might think you mean different things. For example, some people might think they should only say yes if they think people are very happy but other people might think they should say yes as long as people are not unhappy.)

The Exposition — Presenting the Main Ideas

• Ask students if they know the meaning of the terms *interview, questionnaire*, and *observation*. If so, ask them how they think these terms might be related to the way we collect data. If not, introduce and explain the meaning of the terms.

- Make sure students understand that an interview involves a personal interaction, but a questionnaire can be on paper or via other media, such as the Internet.

- Ask for an example of when they might use observation to find out something.

For example, they might observe students playing to find out the students' favourite game.

• Ask students why they might interview to find out the favourite foods of their friends, but use a questionnaire to find out the favourite foods of all the students in the school. Ask why they would not use observation to find this out (for example, they would not be able see most of the students eating).

• Ask students to open their texts to **page 230**. Read through the boxes where good questions are compared to poor questions. In each case, discuss why the good question is better than the poor question. For example, in the last box, it is better not to ask a person if he has lied because he might not wish to admit this, but he might be willing to talk about how people in general might pretend in order to avoid a situation.

• Read with students the last box in the exposition on **page 231**. Point out how using these choices makes it possible to create a bar graph to show how a group of people responded. Discuss why it would otherwise be difficult, since there might be too many categories to make a useful bar graph.

• Finally, you might point out that often people collect information that is not numerical. For example, to find out how satisfied people are with Bhutan's bus service, someone might ask many people to "Describe a bad experience with Bhutan's bus service". This information can still be analysed and interpreted, although it may be harder to graph.

Revisiting the Try This

B. This question allows students to use the criteria for better questions to assess their own original question.

Using the Examples

• Read the question in **example 1** to the class. Ask each student to respond. Then have them compare their responses to the solutions and thinking in the student text.

• Write the four possible questions for **example 2** on the board. Ask pairs or small groups of students to discuss which question they would choose and why. Have them compare their opinions to those in the student text.

Practising and Applying

Teaching points and tips

Q 2: Students may differ in their responses to this question. They should realize that choice A might be a poor choice because a student may think that a particular answer is expected. Students may have different ideas about what it means to read more. They should also realize that choice D leads students to a particular response. Choice B is unclear in its intent. Choice C or E might both seem reasonable.

Q 5: You might have to remind students what a scale is so that they can answer this question. Link the diagram showing the scale to the concept of a number line or a probability line, i.e., there is a least and greatest value and a number more to the right is meant to represent more (in this case, a stronger agreement).

Q 6: You might encourage students to work in pairs to answer this question.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can select an appropriate strategy for collecting data
Question 2	to see if students can compare the quality of possible survey questions
Question 4	to see if students can improve a survey question

Answers

A. *Sample response*: How would you describe how happy you are?	**B.** *Sample response*: How happy are you, on a scale of 1 to 5? Not happy Very happy 1 2 3 4 5 This question is better because it gives clear choices for an answer.
1. *Sample responses*: **a)** Questionnaire; [It is not practical to interview everyone and you only need to know about most people, not about everyone.] **b)** Interviews; [It would be easy to ask everyone in a class.] **c)** Observation; [I would have to take their temperatures because they would not necessarily know if they had a high temperature if I asked them.] **2.** *Sample response*: C will give the best information about reading habits (if "reading habits" is about reading frequency); [The wording of A and D might influence answers. B and E will not collect information about reading habits.] **3.** *Sample response*: Add choices like this: *How many books have you read in the past month?* None 1 or 2 3 or 4 5 or more **4.** *Sample responses*: **a)** *It is a good idea to have a math test every week. Do you agree or disagree?*	**4. b)** *On a scale of 1 to 5, how would you rate your enjoyment of studying each subject?* **Math** Do not enjoy Enjoy a lot 1 2 3 4 5 **Science** Do not enjoy Enjoy a lot 1 2 3 4 5 **English** Do not enjoy Enjoy a lot 1 2 3 4 5 **c)** *Has doing homework improved your success at school?* Not helped Helped a lot 1 2 3 4 5 **5.** *Sample response*: *On a scale of 1 to 5, how do you feel about the following statement? There should be more holidays during the school year.* **6.** *Sample responses*: **a)** *On a scale of 1 to 5, how would you rate the need for new road construction in your area?* Not needed Urgently needed 1 2 3 4 5 **b)** *Does your area needs more and better roads?* [The question might influence the answer.]

Supporting Students

Struggling students

• Some students will find it easier to identify what is wrong with a question than to create a question of their own. For these students, you may wish to continue giving them choices of possible questions rather than having them create good survey questions.

Enrichment

• Some students will enjoy looking at and evaluating survey questions found elsewhere.

For example, the following questions were used by *Kuensel* online in 2007:

· *To what extent do you think rural and urban Bhutan would vote differently?*

· *Do we need a third party for a "free and fair" election in 2008?*

· *Will financial incentives to teachers improve the quality of education?*

· *Did the Yellow party win the mock elections mainly because of its colour and significance?*

· *Will the Civil Service (RCSC) remain scrupulously neutral in the new political scenario?*

· *How do you rate the recently held mock primary elections?*

· *Will you participate in the upcoming mock election?*

7.2.2 Sampling and Bias

Curriculum Outcomes	Outcome relevance
7-F2 Formulate Questions for Data Collection: for real world application • explore issue of bias **7-F3 Bias: determine in questions and samples** • understand the distinction between first- and second-hand data • evaluate the reliability of second-hand data • understand bias in samples	An important aspect of using and interpreting data is understanding that who you ask can influence the results when collecting data. Students need to recognize the importance of asking enough people and ensuring that those people are representative of the population.

Pacing	Materials	Prerequisites
40 min	None	• theoretical probability

Main Points to be Raised

• A small sample might give a misleading result, whether in a survey or in a probability experiment.

• A sample is a small set of all possible results.

• A sample that is too small can affect results.

• *Bias* means that the result from the sample is likely to be different than the result would be from the whole population.

• A census is a survey where every single member of the population answers the question of interest.

• A large enough sample that is representative of the whole population often gives a good indication of the result that would have been obtained using a census.

• First-hand data are collected personally, whether through interviews, questionnaires, or observation.

• Second hand data are collected from references like books or the Internet. The same information is often reported somewhat differently in different second hand resources.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Why might it not be good to ask only people who live in cities?* (Maybe people who live in rural areas have very different lives than those in the cities. They may be either more or less happy.)

• *Why might you get different responses from teenagers than from adults?* (Maybe teenagers are happier because they have fewer responsibilities.)

• *Why might you get different answers if you asked women as well as men?* (Men and women sometimes have different responsibilities in life and maybe that makes one group happier or less happy than the other group.)

• *Why is it important to ask a large number of people?* (If you do ask only a few people, you might happen to ask only people who are especially happy or especially unhappy.)

The Exposition — Presenting the Main Ideas

• Ask students the theoretical probability of flipping Khorlo on a coin. Once they indicate that the probability is $\frac{1}{2}$, you should flip a coin, perhaps 10 times. Talk about how this is a sample of all possible flips and that the fraction of Khorlos you flip may or may not be close to the theoretical probability of $\frac{1}{2}$.

• Discuss how this is like a situation where you want to find out how many hours of homework students in your school do each night and then you ask only five students because it is easier than asking everyone. Indicate that when you ask all students, it is called a census, but when you ask on a portion of the population, you are only taking a sample. Write the words *census* and *sample* on the board.

• Discuss why it is possible that the responses from five students might give you a good idea about how much homework all students do, but that it might not.

For example, if you only ask students in PP, it is probably not a good sample. Mention that a sample from PP would be called a biased sample because these students are not enough like the full population in terms of the amount of homework they are likely to do. Point out that if the five students were five different ages, it might still not be a good sample because these five students might do a lot more or a lot less homework than most students of similar ages. • Mention that data that are collected directly are is called first-hand data, but data that are found using references like books or the Internet are called second-hand. You might write the two phrases *first-hand data* and *second-hand data* on the board. Students may find it strange to think of the word "data" as plural. Explain that the word comes from the Latin word "datum", which means "something given". Its plural is "data". • Have students open their texts to **page 234** and look at the reports of the population of Bhutan from different second-hand data sources. Talk about how whenever we use second-hand data, we must always question its accuracy. The example shows that the same data are often reported quite differently. • Let students know they can use the exposition for later reference.

Revisiting the Try This

B. Students re-examine their answers to **part A** with the idea of biased samples in mind.

Using the Examples

• Read out the situations described in the example. Ask students to talk to a partner about their thoughts on whether the samples are biased. Then they can compare their thinking to the thinking in the student text.

Practising and Applying

Teaching points and tips

Q 1: Students may not be sure of what some of the sources are and so may have difficulty answering this question. However, most will still believe that the government census is the most reliable information.

Q 2: Emphasize that the focus is on collecting information that describes *all* of the people in Bhutan.

Q 3: Remind students first to think of what the entire population is in order to decide whether a census or a sample is to be used. Remind them to think about whether they could interview, use a questionnaire, or observe to decide whether the data can be collected first-hand.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can recognize why a sample might be biased
Question 3	to see if students can select an appropriate data collection strategy

Answers

A. *Sample response*: **i)** Yes; I think people that live in cities are not as happy because it is more expensive to live there. **ii)** Yes; I think teenagers are not as happy because they are worried about their school marks. **iii)** Yes; I think men might be happier.	**A. iv)** No; I think they would be the same because you are still asking a variety of people. **B.** *Sample response*: I would change part iv) to say that the results would probably be very different because the sample size is so small.
1. *Sample response*: Census data; [it is probably most accurate because it was carried out by the government.] The Millennium Report may also be accurate; [it may have been estimated at a later time based on the census.]	**2.** Yes; [*Sample responses*: **a)** The sample is too small. **b)** The sample will give information only about teenagers, not about the whole population. **c)** The sample will give information only about people in the east, not about the whole population.]

Answers [Continued]

3. *Sample responses*:
a) Sample; [There are too many people to do a census, and second-hand information would not be available.]
b) Sample; [You would need to observe and measure speeds of many vehicles at different locations. This likely would not be available as second-hand data.]
c) Second-hand data; [If the government census has this information, you could use it. If not, you could sample the population.]
d) Census; [to find the total population in all classes, you need to count them all, not just a sample.]

4. No; [*Sample response*:
If the survey question is not a good one, the results will not be accurate no matter how large the sample.]

Supporting Students

Struggling students

• Most students will not struggle with this topic. You may wish to help some students with **question 2** by making sure they think about why the results might be biased.

• For **question 3 d)**, you may have to tell students that government ministries keep records of these types of data.

Enrichment

• Students might imagine a certain sample and describe two situations: one for which the sample is likely to be biased and one for which the same sample is likely not to be biased.

CONNECTIONS: Estimating a Fish Population

• To set up this situation, you might do this small experiment first:

- Have the students watch you place 2 coloured slips of paper and 8 white slips of paper in a bangchung. Ask them what percent of the slips are coloured (20%).
- Then, without showing the students, tell them you are adding some coloured and some white slips of paper using the same ratio. (Add 2 more coloured slips and 8 more white slips.)
- Draw a slip from the bangchung and record its colour. Return the slip to the bangchung. Repeat this 10 times.
- Count the number of coloured slips that were drawn in ten draws. Call the number c.
- Remind the students that 20% of the slips were coloured, so c represents 20%. Ask how many slips they think are in the bangchung. Call the total number of slips x.

For example if 3 coloured slips were drawn in ten draws, then 3 is 20% of x, so you might multiply 3 by 5 to get a value of 15 slips altogether because 20% × 5 = 100%.

• Mention that this experiment is based on the belief that the correct percent will normally be chosen, although you know it will not always be exact. The more times you draw a slip, the greater the chance of drawing a coloured slip 20% of the time.

• Read through the connection with the students and help them see that the coloured slips were like the tagged fish.

Answers

1. *Sample response*:
You cannot see them all underwater and they move too quickly to count. You would need to drain the lake to count them all, but then they would all die.

2. 500

7.2.3 EXPLORE: Conducting a Survey

Curriculum Outcomes	Outcome/Lesson Relevance
7-F2 Formulate Questions for Data Collection: for real world application • explore issue of bias **7-F3 Bias: determine in questions and samples** • understand bias in samples **7-F1 Data Collection Methods: select and defend** • select, defend, and use appropriate data collection methods in real-world applications: - interview - questionnaire	This essential exploration allows students to apply what they have learned about good questions and unbiased samples.

Pacing	Materials	Prerequisites
2 h (over several days)	• Paper for recording	• creating clear and simple questions • using an unbiased sample

Exploration

• Inform students that they will be using what they have learned about formulating good survey questions and using an unbiased sample to conduct an actual survey.

• Ask them to work through **parts A to E** with a partner.

Observe and Assess

As students work, notice:

• Do they formulate good questions?

• Do they choose an appropriate method to collect their data?

• Do they describe an appropriate sample?

• Are they able to organize the data they collect?

• Do their clearly and accurately report the data they collect?

Share and Reflect

After students have had sufficient time to work through the exploration, encourage some of them to share their work by reading all or part of their reports aloud to the class. You might discuss the following questions:

• *What issue did you investigate?*

• *Why do you think that was a good question?*

• *How did you collect the data?*

• *What difficulties did you have when you collected the data?*

Answers

A. *Sample response*:
I chose to find out whether students in Classes VII and VIII think we have too much homework.

B. *Sample responses*:
i) Survey questions:

1. On an average school night, how many hours do you spend on homework? Pick one of these choices.
0 to 30 min
30 min to 1 h
1 h to 1.5 h
1.5 h to 2 h

2. Which do you think describes you best?
• I spend more time on homework than most of my classmates.
• I spend less time on homework than most of my classmates.
• I spend about the same amount of time on homework as most of my classmates.

3. With which of these statements do you agree?
• We should have about the same amount of homework as we get now.
• We should have less homework.
• We should have more homework.

ii)
• I have decided to give choices that cover all the possibilities for the first question.
• I want to know how much time the person who answers actually spent so I can interpret the reasons for his or her answer to the third question.
• I realize that each person's answers might depend on how fast or slow he or she is, so I will ask the second question.
• I will ask the main question (question 3) last so that it does not make it seem that one thing is better than another.

C. *Sample response*:
I will use a questionnaire because it will be quicker and the choices will be easy to understand.
I have decided to ask 10 students from each class in case different classes have different amounts of homework. I think 10 out of 40 is a pretty good sample. I will be sure not to ask only good or poor students or only girls or boys.

D and E. *Sample response*:
I surveyed a total of 50 students, 30 in Class VII and 20 in Class VIII.

My survey results:

Question 1

Time	0 to 30 min	30 min to 1 h	1 h to 1.5 h	1.5 h to 2 h	More than 2 h
Number	2	5	15	18	10

Question 2

Compared to others	More time	Less time	About the same
Number	11	7	32

Question 3

Same amount	Less	More
29	16	5

• I noticed that it was the people who spent the most time on homework and the people who took longer than their classmates who wanted less time. That makes sense.
• Most people who spend the same amount of time as other people are happy with the amount of homework they have now.

Supporting Students

Struggling students

• Encourage struggling students to work with stronger students to organize the planning of the survey and the writing of the report. Struggling students might play a bigger role in the actual data collection.

7.3.1 Circle Graphs

Curriculum Outcomes	Outcome relevance
7-F4 Circle Graphs: construct and interpret • create a circle graph using a fraction circle in hundredths • represent proportions as percentage of total circle • identify appropriate applications for circle graphs	Circle graphs are widely used in the media. When students know how to create circle graphs, they will better understand how these graphs work.

Pacing	Materials	Prerequisites
1.5 h	• Percent Circles (BLM)	• calculating and interpreting percents • interpreting bar graphs

Main Points to be Raised

• A circle graph, also called a pie chart, shows how data values are distributed in different categories.

• You can use percents to describe the fraction of the data that is in each category.

• Although a bar graph and a circle graph can show the same information, a bar graph might show the actual number in each category, while a circle graph might not.

• Whether you should use a bar graph or circle graph depends on the information you need. With a circle graph it is easier to compare each category to the whole, but with a bar graph it is easier to compare different categories.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know that red is about 25% of the circle?* (The whole circle is 100% and red is $\frac{1}{4}$ of the circle; $\frac{1}{4}$ of 100 is 25.)

• *How did you decide the percent for blue?* (I knew it was a bit less than halfway between 25% and 50%, so I used 35%.)

• *How do you know that the percent for yellow is less than the percent for green?* (The yellow part of the circle is smaller than the green part.)

• *How did you choose the percent for the section marked other?* (It looks like about 10 of those sections would fit in the whole circle, so I said 10%.)

The Exposition — Presenting the Main Ideas

• Ask students to open their texts to **page 237**. Point out the percent circle that could be used to make a circle graph. Draw a sample circle graph. For example, show how to draw a line from the centre to the top mark and then from the centre to the 30% mark to create a circle graph that shows 30% of the data in one category and 70% in another category.

• After you have drawn the circle graph, ask students how many people are in each category. They should realize that they cannot answer this question.

• Have them look at the bar graph and the corresponding circle graph about forest fires on **page 237**. Talk about how both graphs show that most of the forest fires occur in the winter and that the fewest fires occur in the summer, but that only the bar graph tells you how many fires there were in each season.

• Have them notice that it is easier to see from the circle graph that the percent of fires in winter is between 50% and 75%. The choice of an appropriate graph depends on what information you want to display.

• Rather than asking students to measure angles to create the percent graphs, which is left to Class VIII, the focus in this class should be on using the simple template that is provided as a blackline master to create the circle graphs. This allows the students to focus on the concept, rather than on the mechanics, in their first formal work with these types of graphs.

Revisiting the Try This

B. Students who may not have recognized earlier that the percents in **part A** need to add to 100 will see how this might affect their estimates.

Using the Examples

• Assign pairs of students to work on the examples. One student in each pair should study **example 1** and the other should study **example 2**. Each student should then explain his or her example to the other student.

Practising and Applying

Teaching points and tips

Q 1: Make sure students understand why the "other" section percent was not required, but ask them what it is (18%).

Q 2: Some students may not have kept the data for this question. If they did not, do not assign the question.

Q 4: Make sure students understand that they should use the given graph to answer the question.

Q 5: Students will probably not have difficulty deciding which ecosystem each section represents, but they may have trouble calculating the percents. They should use the graph to estimate, but they should calculate using the actual values.

Q 6: Students need to realize that the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ is too much because it is greater than 1. Some students will sum the fractions and others the percents.

Common errors

• Many students have difficulty knowing what to do after they create the first section of a circle graph.

For example, after making a section for 28%, they draw a line at the 42% mark to make the next section for 42%. Instead, they need to draw a line at the 70% mark (adding the 28% to the 42%).

You may need to show students more models of circle graphs than just **examples 1 and 2**.

Suggested assessment questions from Practising and Applying

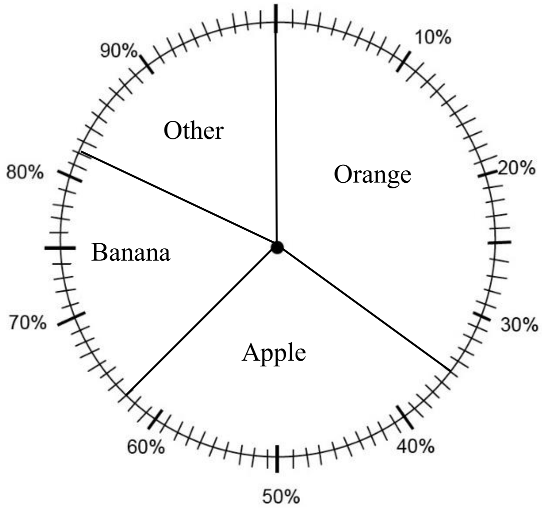
Question 1	to see if students can create a circle graph given the percents for each category
Question 3	to see if students recognize when a circle graph is appropriate
Question 4	to see if students can interpret a circle graph

Answers

A. *Sample response*: Red: 25%: blue: 30%; yellow: 15%; green: 20%; other: 10%.	**B. i)** 100%; *Sample response*: 100% is the whole sample so all the percentages for all possible responses must add up to 100%. **ii)** 25 + 30 + 15 + 20 + 10 = 100. My estimates add to 100%.

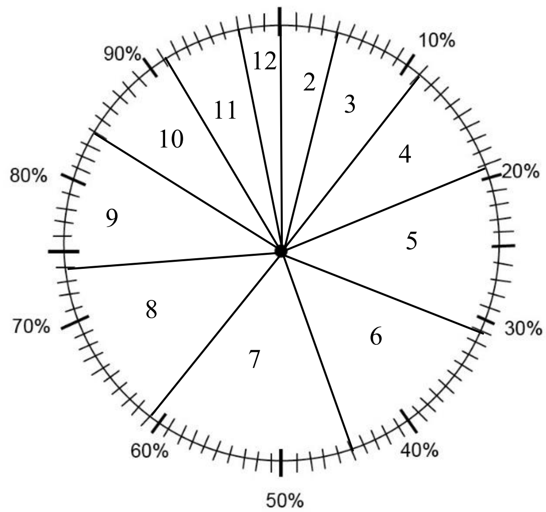
1.

Favourite Fruit of a Group of College Students



2. *Sample response*:

Experimental Results of Rolling and Adding Two Dice



3. *Sample responses*:
a) Line graph; [It will show change over time.]
b) Bar graph; [I can compare actual amounts.]
c) Circle graph; [I can compare each part with the whole.]

4. a) 34%
b) 37%
c) 29%

5. a)
A is forests
B is grasslands
C is agriculture
D is barren
b) *Sample response*:
A = 73%; **B** = 18%; **C** = 5%; **D** = 4%

6. No; [*Sample response*:
The fraction parts have to add up to 1 (which is 100%). The sum of Nima's parts is greater than 1.]

Supporting Students

Struggling students

• Some students might find it much more difficult to create circle graphs than to interpret them. For these students, you might start the graphs in **questions 1 and 2** for them and have them complete the graphs.

Enrichment

• Some students might try to create circle graphs without the circle graph template. This will preview work they will do in Class VIII where they learn to use angles to construct circle graphs.

7.3.2 Histograms

Curriculum Outcomes	Outcome relevance
7-F5 Histograms: construct and interpret • construct histograms to show the frequency distribution of data grouped in intervals • identify appropriate applications for histograms	Histograms are used to describe the information in many official documents, like the census. It is important that students be able to interpret these graphs. By constructing them, they will better understand how they work.

Pacing	Materials	Prerequisites
1.5 h	• Grid paper or Small Grid Paper (BLM) or lined paper	• knowledge of bar graphs

Main Points to be Raised

• A histogram is like a bar graph, but there are no gaps between the bars because the categories are based on numbers that could be whole numbers of fractions.

• The sections of the graph are called intervals.

• One interval ends where the next interval begins. A piece of data that is the same as the end value of an interval goes in the higher interval.

• The height of each bar is called the frequency for that interval.

• You can choose the number of intervals and then use the range of the data and the number of intervals you have chosen to decide on the size of the intervals. Normally, each interval is the same size.

• You might create a frequency table before you draw the histogram to make it easier to draw.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Why did you count the number of pieces of data?* (The median is the middle number, so I have to know how many numbers there are to find the middle.)

• *How did you find the median?* (There are 35 numbers so I used the 18th number. There are 17 numbers before it and 17 numbers after it.)

• *Why might you have guessed that the middle number was in the third row?* (It looks like there are about the same number of values in rows 4 and 5 as in rows 1 and 2.)

• *How did you find the mode?* (I looked for a leaf that appeared a lot of times.)

The Exposition — Presenting the Main Ideas

• Ask students to tell what time they usually go to sleep. Ask students how you might show that information. If they do not know, suggest using a bar graph with different categories.

For example, you might count how many go to sleep between 6:30 p.m. and 7:00 p.m., between 7:00 p.m. and 7:30 p.m., between 7:30 p.m. and 8:00 p.m., between 8:00 p.m. and 8:30 p.m., or after 8:30 p.m..

• Create a frequency table to show the number of students in each category. Tell students that it is called a frequency table.

• Create the bar graph to match the data, but do not leave spaces between the bars. If any students go to sleep at exactly 7:00, 7:30, 8:00, or 8:30, inform them that you included those values in the next higher category.

• Point out to students that you did not leave spaces between the bars because there are no times between the categories. (Note that if there are no data values in a category, for example, if no one went to bed between 8:00 and 8:30, there is no bar. This will appear to be a space but it is really just an empty category.) Tell them that this sort of graph is called a *histogram*. Tell students that each section of time that is graphed separately is called an interval. The height of each bar, which tells how many people go to bed in each section of time, is called a frequency.

• Point out that you could have used different categories to show the same data, such as 6:15 to 7:15, 7:15 to 8:15, 8:15 to 9:15, and after 9:15. The graph would look different but it would still represent the same data.

• Tell students that when they create a histogram they are free to choose the intervals first or to choose the number of bars first, creating intervals that will give that number of bars. Let them know that the intervals should normally be the same size.

• Tell students they can refer to the exposition in the text, which summarizes the information you have presented.

Revisiting the Try This

B. This question provides a context for students to create a histogram from a stem and leaf plot. If they use the same intervals as the stem and leaf plot (i.e., 10 to 19, 20 to 29, etc.), the shape of the two graphs will be the same. If they choose to use different intervals, it might look different.

Using the Examples

• Work through **example 1** with the students to make sure they follow the thinking. Emphasize that in this case, unlike the case in the introductory activity, the values for the intervals were determined by looking at the range of the data and deciding on the number of intervals. Make sure that they see that the shape of the graph changes when different numbers of intervals are used, but emphasize that no one graph is more correct than another.

• Present the situations in **example 2** orally and ask students to tell what sorts of graphs they would use. They can check their responses against those in the student text.

Practising and Applying

Teaching points and tips

Q 1: In this case, the intervals are already provided and students do not have to make any decisions about them. Source: www.bhutan studies.org.bt/admin/pubFiles/YouthBhutan.pdf

Q 2: Students need to consider the range of data and the fact that seven intervals are suggested to decide what the intervals should be. There could still be some variation.

For example, one student might choose intervals of 0 to 10, 10 to 20, ..., 60 to 70, whereas another student might choose 5 to 15, 15 to 25, ..., 65 to 75. It is essential that the final interval include the highest value, 69, and that each data value have a place on the graph.

Q 4: Students need to realize that neither the length of the marathon nor the number of participants is relevant to choosing the intervals. What is relevant is the range of hours it takes runners to finish.

Q 5: This question is designed to make a connection, but also to show the distinction, between bar graphs and histograms.

Common errors

• Many students will misplace a data value that is on the edge of an interval. Remind them that a data value at the edge of an interval always goes in the higher interval.

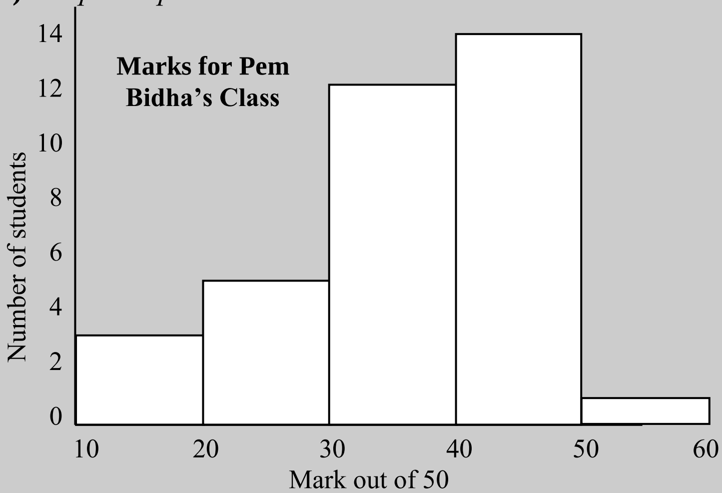
Suggested assessment questions from Practising and Applying

Question 1	to see if students can create a histogram given a frequency table
Question 2	to see if students can decide on intervals to create an appropriate frequency table and associated histogram
Question 3	to see if students can interpret a histogram

Answers

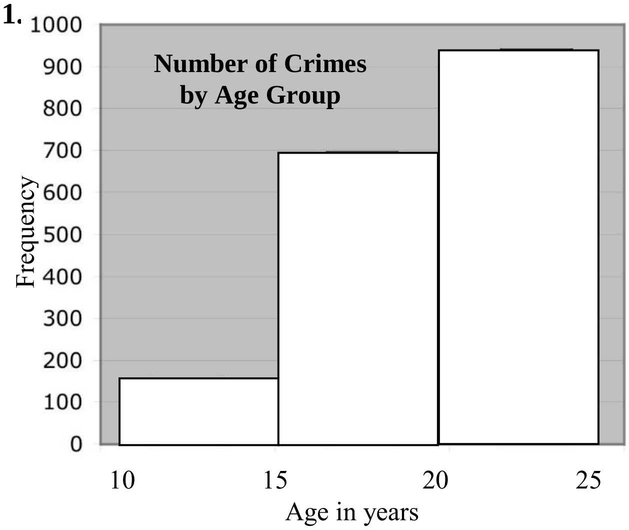
A. i) 38 **ii)** 15 **iii)** There are two: 38 and 42.

B. i) *Sample response*:



B. ii)
It has the same shape — the number of students becomes greater and greater as the mark increases and then it drops to 1 because only 1 student got a mark of 50. It is the same because the plot and the histogram use the same intervals and they have the same number of data values in each interval.

1.

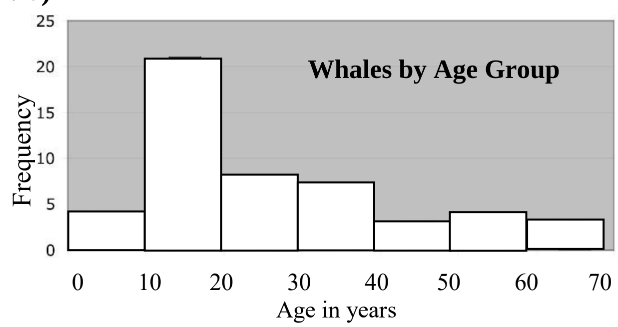


2. *Sample responses*:

a)

Age group	Tally	Frequency
0 – 9	||||	4
10 – 19	~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ |	21
20 – 29	~~||||~~ |||	8
30 – 39	~~||||~~ ||	7
40 – 49	|||	3
50 – 59	||||	4
60 – 69	|||	3

2. b)



3. a) i) 9 **ii)** 27 **iii)** 45 **iv)** 19
b) 36
c) 64

4. *Sample response*:
2 h – 3 h, 3 h – 4 h, 4 h – 5 h, 5 h – 6 h, 6 h – 7 h;
[The times go from just over 2 h to 7 h, and 1 h intervals are easy to work with.]

[**5.** *Sample response*:
Using a bar graph might make it look as if there were gaps in the times, but there are no gaps.]

Supporting Students

Struggling students

• You might suggest to struggling students what intervals to use rather than having them decide on the intervals.

Enrichment

• Encourage students to look at the Government of Bhutan census information to see how histograms are used to report information about the country.

7.4.1 Mean, Median, Mode, and Range

Curriculum Outcomes	Outcome relevance
7-F6 Central Tendency: examine the effect of changing data • understand that, if values are added to a set of data, any of the measures of central tendency can be affected • understand that adding, subtracting, multiplying, or dividing every value in a data set by the same value has the same effect on its mean, median, and mode	Although students already know how to calculate a mean, understanding the effect on the mean of various calculations with the data help them calculate means more easily. The same is true, to a lesser extent, for the calculation of the median, mode, and range.

Pacing	Materials	Prerequisites
1 h	None	• familiarity with mode, mean, range, and median

Main Points to be Raised

• The mode of a set of data is the data value that occurs most often.

• To find the mean of a set of data, you add the data values and divide the sum by the number of pieces of data. The mean represents an equal sharing of the data.

• The median is the middle value when data values are put in order.

• The range is the difference between the greatest value and the least value in a data set.

• When a constant is added to or subtracted from each value in a data set, the mean, median, and mode each increase or decrease by that constant, but the range does not change.

• When all the values in a data set are multiplied or divided by a constant, the mean, median, mode, and range are all multiplied or divided by the constant.

• You can sometimes predict how the mean, median, mode, and range of a set of data will be affected if a piece of data is removed, but not always.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How did you calculate the mean?* (I added all the numbers and divided by 10.)

• *How did you calculate the median?* (I put the numbers in order from least to greatest and then took the mean of the fifth and sixth numbers.)

• *Why was it easier to see the mode after you put the values in order?* (Because then it was easy to see whether any number occurred more than once or twice.)

• *What other values might students pick for the typical score?* (*S*ome people might pick the mean, but others might pick the median, the mode, or maybe even some other value.)

The Exposition — Presenting the Main Ideas

• Record these data values on the board: 8, 8, 9, 10, 15, 22. Ask students to calculate the mean, median, mode, and range to see if they recall these statistics. If they have difficulty with any of them, remind them of the necessary calculations (see **page 246** in the student text).

- Now tell the students you are going to add 5 to each value. Have them calculate the new data values and then calculate the new mean, median, mode, and range. Ask what they notice. They should observe that the range did not change, but the other statistics all increased by 5.

- Ask what they think would happen if 5 were subtracted from each data value and why. Have them check.

- Next, multiply each of the original data values by 2. Have students re-calculate the new mean, median, mode, and range. They should notice that all of these values are doubled.

[Continued]

- Ask the students to predict what will happen to the mean, median, mode, and range if you divide each of the original values by 2. Let them investigate to check. (They are all divided by 2.)
- Summarize the results for the students.
- Point out the usefulness of this information.

For example, to calculate the mean of test scores of, for example, 60, 65, and 73, a teacher could subtract 60 from each value, calculate the mean of 0, 5, and 13 (which is much easier to do mentally), and then add the 60 back.

• Return to the original values of 8, 8, 9, 10, 15, 22. Ask students to work in pairs to find out what would happen to each statistic in each of these situations:
- One of the 8s is removed from the data
- The value 22 is removed
- The value 9 is removed

Help them understand the results they obtain.

For example, if you remove one of the 8s, the mean increases because you have removed a value below the mean, the median increases because you have removed a value below the median, the range is not affected because there are two equal low values, and the mode is affected only because 8 was the mode.

• You might have students read through the exposition to reinforce their learning before starting the exercises.

Revisiting the Try This

B. Students apply the concepts they have learned to a problem they have already solved.

Using the Examples

• Write the problems from **examples 1 and 2** on the board. Ask students to work in pairs to try to solve the problems. They should check their work against the student thinking and solutions in the student text.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to predict before they actually do the calculations.

Q 2: Remind students that measures of central tendency are the mean, median, and mode.

Q 3: Some students will suggest that it is efficient to add the numbers and divide by 6. Others will choose to remove the 100 from each number, calculate the mean of 10, 17, 18, 20, 13, and 15, and then add 100. Still others might choose a value like 115 to subtract from each data value, so that some positive and negative results cancel each other out.

Q 4: Students might begin with a simple data set like 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 or perhaps 10, 10, 10, 10, 10, 10, 10, 10, 10, 10 and then adjust it.

For **part a)**, students need to realize that the original mean should be less than 100.

For **part b)**, they need to realize that the original mean should be greater than 100.

For **part c),** they should realize that the original mean is 50.

For **part d)**, they should realize that the original mode is 200.

Q 5: Students might begin with a data set with an odd number of values where the median is not repeated and then add a high value.

Common errors

• Students sometimes forget to put data values in order before calculating the median. Remind them that this is essential.

Suggested assessment questions from Practising and Applying

Question 1	to see if students recognize the effect on statistics of various changes in the data
Question 3	to see if students recognize the usefulness of knowing how statistics change when the data values change in a consistent way
Question 6	to see if students can calculate a statistic efficiently using what they know about the effect on statistics of constant changes to the data

Answers

A. i) Mean: 115.2; median: 115.5; mode: 108. **ii)** *Sample response*: I would use the median of 115.5 because it is easier to calculate than the mean. I would not use the mode because 108 seems a bit low and it would not show that there were many scores in the 120s and 130s.	**B.** The median would change to 114 because there would be another low value. The mode would not change because the new value would not repeat an existing value. The mean would change to 112. It would go down because a low value was added.
1. a) Mean, median, and mode increase (by 20), range does not change. **b)** Mean, median, and mode decrease (by 100), range does not change **c)** Mean, median, mode, and range increase (multiplied by 3). **d)** Mean, median, mode, and range decrease (divided by 5). **e)** Mean and median increase, mode does not change, range decreases. **f)** Mean, median, and range increase, mode does not change. **2. a)** Mean: 11.8; median: 12; mode: 12; range: 2. **b)** The mean and range will decrease. The mode and median will stay the same. **3.** *Sample response*: Subtract 110 from each data value, calculate the mean of the new data, and then add 110 to the result.	**4.** *Sample responses*: **a)** 5, 6, 7, 7, 8, 9, 10, 10, 10, 10 **b)** 305, 306, 307, 307, 308, 309, 310, 310, 310, 310 **c)** 50, 50, 50, 50, 50, 50, 50, 50, 50, 50 **d)** 110, 200, 200, 200, 200, 300, 400, 500, 600, 700 **5.** *Sample response*: The data values are 4, 5, 6, 7, 8, 9, 10. The median increases when 11 is added to the data. **6.** *Sample response*: • Multiply each value by 10 to get the mean using whole numbers and then divide the value by 10. [• Think of the values in terms of 40 (subtracting 4 and multiplying by 10): –2, –1, 0, 5, 7, 9. Find the mean of these whole numbers, add 40, and then divide by 10.] **7.** *Sample response*: A set of data that has decimal values; [I would change the data by multiplying by 100 to get rid of the decimals in order to calculate the mean with whole numbers. Then I would divide by 100 to get the correct values.]

Supporting Students

Struggling students

• Struggling students may need to re-calculate many statistics when there are data changes before they become comfortable predicting what will happen. They may also have difficulty with **questions 3 and 7**, where they have to think about why this strategy is useful rather than simply performing it. You may choose not to assign those two questions to these students.

Enrichment

• Students might investigate what happens to the mean, median, mode, and range if other types of data changes are made, for example, if half the data values are increased and the other half are decreased by the same amount. They will learn that they can only predict the results when the changes are consistent.

7.4.2 Outliers and Measures of Central Tendency

Curriculum Outcomes	Outcome relevance
7-F6 Central Tendency: examine the effect of changing data • discuss the effect on mean, median, and mode if outliers are removed • understand that the measure of central tendency best suited to a particular situation is dependent on the situation (e.g., the median or mode is not affected by outliers as much as the mean) **7-F7 Variability: make inferences and predictions** • understand that range is the difference between the two extreme data values • find gaps and clusters in a set of data by observing and analysing the data • use range, outliers, gaps, and clusters to make inferences and predictions	In order to interpret data effectively, students must think critically about each data set and about the likelihood that other data values might belong to the set.

Pacing	Materials	Prerequisites
40 min	None	• familiarity with mean, median, and mode

Main Points to be Raised

• The three measures of central tendency are the mean, median, and mode. Sometimes, if only one measure can be chosen to represent the data, one is more appropriate or meaningful than the others.

• An outlier is a data value that is much lower or much higher than most of the other values in a set of data. For this reason, an outlier is sometimes, but not always, an error that does not belong in the data set.

• If you ignore outliers in calculating statistics, there is usually a greater effect on the mean than on the median.

• Sometimes data values is described in terms of how they cluster. This happens when a group of data values are very close, with a gap between them and the next group of data values.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Why do you think the median will be in the 20s?* (There are only four values above the 20s and so the middle looks like it will be in the 20s.)

• *Why is the mode the easiest statistic to find?* (You do not have to do any calculations. You just look for repeated numbers.)

• *Why do you think the mean is greater than the median?* (There are more high numbers than low numbers.)

The Exposition — Presenting the Main Ideas

• From the last lesson, students should recall the definitions of mean, median, and mode. Check to make sure this information is clear to them.

• Work through the exposition with the students. Make sure they understand what outliers are, that they can sometimes but not always be ignored, and that the mean is affected more than the median when outliers are ignored.

- Demonstrate this last point using the data 2, 2, 2, 2, 22. The median remains 2 when the 22 is removed, but the mean changes dramatically (from 6 to 2).
- Show how in the data set 2, 2, 2, 2, 22, there is a cluster at 2 and a gap between the 2s and the 22.

Revisiting the Try This

B. This question allows students to apply a number of the concepts taught in the exposition to the data set from **part A**.

Using the Examples

• Have pairs of students work through the first two examples. Ask students if they agree more with **solution 1** or with **solution 2** in **example 1**. Ask them why someone might disagree with the conclusion in **example 2**. On the board, write the problem in **example 3**. Ask students to solve it and then compare their answers with the student text.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to look for both very high and very low values as outliers. Observe whether they actually calculate to answer **part b)** or whether they use the shape of the data to help them decide on the answer.

Q 2: This question requires students to have a good understanding of both mode and outlier.

Q 3: Some students may realize that without knowing what the data represents, it is hard to know whether or not the outliers belong.

Q 4: The intent of **part a)** is that the median, mode, and range remain the same.

Q 5: Students will normally be able to find the gap easily. For the clusters, some may suggest that the first interval represents a cluster, whereas others may think that two intervals are needed to define a cluster. Either response is reasonable. Some students may realize that the actual cluster may be a small part of each interval, but that this cannot be determined with a histogram.

Q 6: Students need to think about particular contexts in which it is reasonable to remove outliers and not focus just on the numbers.

Common errors

• Some students will automatically remove outliers even when it is not appropriate to do so. Remind them that they must consider the context before they decide whether it is appropriate to remove outliers.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can describe the shape of a set of data
Question 3	to see if students can communicate about which measure of central tendency is most appropriate in a situation
Question 5	to see if students can make the connection between a histogram and the notion of clusters and gaps

Answers

A. Mean: 28.7 Median: 27 Modes: 20 and 41 **B.** There are two clusters: 20, 20, 22, 26, 28 and 41, 41, 42, 44, with a gap in between 28 and 41. There is a low outlier, 3.	**C. i)** *Sample response*: The mean will go up if the low outlier is dropped. The median may increase a bit. The modes will probably stay the same. **ii)** Mean: 31.6; median: 28; modes: 20 and 41. The mean increased from 28.7 to 31.6, the median increased from 27 to 28, and the modes stayed the same. **D.** *Sample response*: The mean of 31.6 (without the outlier)

1. Data in order: 9, 26, 31, 35, 35, 35, 52, 71, 77, 96, 97, 104, 107 **a)** *Sample response*: There are gaps between 9 and 26, 35 and 52, 52 and 71, and between 77 and 96. There are clusters from 26 to 35 and from 96 to 107. There is one outlier of 9. **b)** *Sample response*: The mean is 63.8 and the median is 61.5 without the outlier 9. [The mean is 59.6. The median is 52. The mode is 35. Because of the low outlier, the mean is a bit low but the median is even lower, so I would drop the outlier and re-calculate. Without the outlier: The mean is 63.8. The median is 61.5. The mode is 35. Without the outlier, the mean and mean are closer. The mean is 63.8 and the median is 61.5. Both represent the set of data equally well because they are so close together.] [**2.** *Sample response*: The mode only changes when you remove the outlier(s) if the mode is an outlier. The mode is a value that is repeated two or more times, so it is unlikely that a mode would be an outlier because outliers are unusually small or large data values.]	**3.** *Sample responses*: **a)** The mean of 7 [because it shows that there is a data value that is greater than the rest.] **b)** The mean of 40 [because it's a bit low so it better represents/includes the outlier of 5.] **4.** *Sample responses*: **a)** 5, 10, 10, 10, 15, 20, 25, 30, 45, **55** **b)** **0**, 10, 10, 10, 15, 20, 25, 30, 45, **55** **5.** There are clusters between 100 and 200 and between 300 and 500. There is a gap between 200 and 300. [**6.** *Sample response*: If the outlier is pulling the mean way up or way down so it is very different from the median, I would remove the outlier. I would leave an outlier in if it's also the mode.]

Supporting Students

Struggling students

• The concept of outliers is difficult because there are no rules about what makes a number an outlier. Some students find this difficult. It is better to acknowledge the difficulty rather than to make up rules that students can use to decide whether a number is an outlier.

Enrichment

• Students might try to create or describe situations where outliers are likely to occur. They can discuss why the outliers appear and how they would handle those values if they were to calculate a measure of central tendency.

UNIT 7 Revision

Pacing	Materials
2 h	• Percent Circles (BLM)

Question(s)	Related Lesson(s)
1 and 2	Lesson 7.1.1
3 – 5	Lesson 7.1.3
6 and 7	Lesson 7.2.1
8 and 9	Lesson 7.2.2
10 and 11	Lesson 7.3.1
12 and 13	Lesson 7.3.2
14 and 15	Lesson 7.4.1
16 and 17	Lesson 7.4.2

Revision Tips

Q 1: Students should first focus on the fact that the denominator for the probability has to be 15 because there are 15 balls.

Q 3: Students need to sort and classify numbers in a variety of ways to answer this question.

Q 4 and 5: Students should use theoretical probability, not experimental probability, to answer these questions.

Q 8: You may need to remind students of the definitions of first-hand and second-hand data.

Q 11: You may need to remind some students first to calculate a total in order to estimate the percents. [Source: LUPP Dzongkhag Data Sheets, 1995; Roder *et al*. 2001. 11,995 excluding pigs and poultry.]

Q 12: Some students may focus on the frequencies of the different intervals, but you should encourage them to make comparisons and to generalize.

Q 15: For **part a)**, make sure students understand that 12 must be one of the data values. For **part c)**, 10 must be one of the data values.

Answers

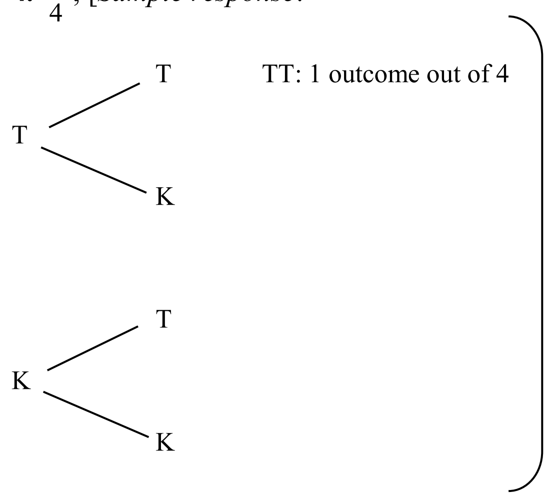
1. a) $\frac{1}{15}$ **b)** $\frac{8}{15}$ **c)** $\frac{7}{15}$

d) $\frac{7}{15}$ **e)** $\frac{6}{15}$ **f)** 1

2. Choosing an even number and a striped ball

3. *Sample responses*:
a) Choosing a number greater than 1
b) Choosing a number less than 12
c) Choosing a white ball
d) Choosing the 3 ball (or any single given number)
e) Choosing a number greater than 15

4. $\frac{1}{4}$; [*Sample response*:



Answers [Continued]

5. *Sample responses*:
a) Spinning a sum of 2 (a 1 and a 1)
b) Spinning a sum of 4 (a 1 and a 3, a 2 and a 2, or a 3 and a 1)

6. [**a)** *Sample responses*:
i) It might influence the person to say yes.
ii) It assumes the person eats meat.
iii) It is too personal and may embarrass the person.]
b) *Sample response*:
What is your favourite meat?
Seven choices are: Pork, Beef, Chicken, Yak, Goat, Other, and I do not eat meat.

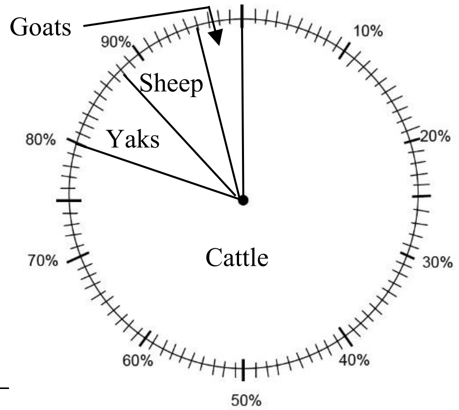
7. *Sample responses*:
a) Distribute a questionnaire to a sample of people of all ages throughout the country; [too many people to interview and it is not something I can observe.]
b) Observe chickens in an experimental situation; [I could interview farmers or send them a questionnaire to complete but I might not get information about all breeds of chickens.]
c) Interview all the students in my class; [It is easy to get answers from a small group in one place.]

8. a) First-hand; [*Sample response*:
The restaurant is getting data directly from customers.]
b) Second-hand; [*Sample response*:
The tourist relies on others to predict the weather.]
c) Second-hand; [*Sample response*:
The researcher relies on data in the book rather than actually studying takins.]

[**9.** *Sample responses*:
a) She did not ask enough people to represent a large place like Thimphu.
b) People without telephones are not represented. Their answers might be different.]

10. *Sample response*:
A small percent were not happy. More than 95% are happy or very happy.

11. Cattle: 80%; yaks: 8%; sheep: 8%; goats: 4%



12. a) *Sample response*:
The data values range between 100 cm and 170 cm. The greatest number of students is in the middle (120 cm to 140 cm) and the number of students decreases in both directions.
b) 26
c) 30

13. 12 –14, 14 – 16, 16 – 18, 18 – 20, 20 – 22, 22 – 24, 24 – 26; [The data values start at 13 min and end at 25 min. Intervals of 2 min are easy to work with.]

14. a) Mean, median, mode, and range decrease (divided by 10).
b) Mean, median, and mode decrease (by 10). Range does not change.
c) Now there are 4 modes. Mean and median decrease. Range does not change.
d) Mean and median decrease. Range increases. Mode does not change.

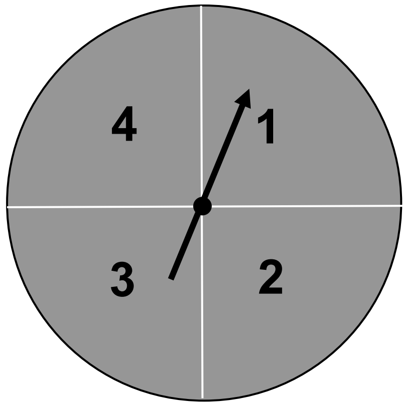
15. *Sample responses*:
a) 10, 10, 10, 10, 10, 12
b) 17, 18, 19, 17, 18, 19
c) 8, 9, 10, 10, 11, 12

16. a) *Sample response*:
37 is an outlier.
There are two clusters, from 58 to 69 and from 80 to 91, with a gap between the clusters.
b) *Sample response*:
The mean with or without the outlier;
[Mean: 70; median: 66.5; mode: 60.
Without the outlier:
Mean: 71.7; median: 67; mode: 60.
The mean with or without the outlier best represents the data because the outlier of 37 does not affect the mean or median much. There is a cluster of higher data, so the mean represents that data better than the median because the mean is higher.]

17. The mean changes from 50 to 40.
The median changes from 45 to 37.5
The mode of 55 does not change.
The range changes from 85 to 30.

UNIT 7 Probability and Data Test

Use this spinner to answer questions 1 to 4.



1. What is the probability of getting each in one spin?

a) a 4

b) an odd number

c) a number greater than 1

2. Suppose you spin the spinner twice and add the numbers. What is the probability of getting each?

a) a sum of 2 **b)** a sum of 7

3. a) Name two sums that are equally likely to occur for two spins.

b) What sum is most likely for two spins?

c) Name a sum that is impossible.

4. Describe an event that has each probability of occurring for two spins.

a) $\frac{1}{4}$ **b)** $\frac{3}{8}$

5. Describe what is wrong with each survey question. Change each question to make it better.

a) Do you like apples and oranges?

b) Is happiness important to you?

c) Don't you agree that criminals should be punished harshly?

6. Which method would you use to collect the following data: observation, interviews, or a questionnaire? Explain your choice.

a) the effects of long-term smoking

b) the average family size in Bhutan

c) the average height of students in your class

7. Is each an example of first-hand data or second-hand data? Explain your thinking.

a) a forester tests trees for diseases

b) a student finds information about tree populations in an encyclopedia

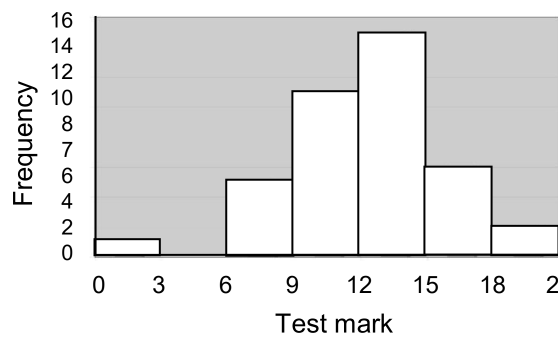
8. Why might each survey be biased?

a) A questionnaire about favourite sports is handed out at an archery contest.

b) An e-mail survey attempts to find out how many people in Bhutan plan to get a computer.

9. The 2005 census showed that Bhutan had about 360,000 males and about 300,000 females. Estimate a percent for each. Create a circle graph of the data.

10. This histogram shows the results of a quiz marked out of 20 for a class.



a) What is the size of each interval?

b) How many students got a mark between 18 and 20? How do you know?

c) Identify gaps in the data.

d) Is there an outlier? How can you tell?

11. How will the mean, median, mode, and range of this set of data change in each situation?

5, 20, 25, 30, 35, 55, 55, 65, 65, 85

a) Each value is multiplied by 10

b) 5 is subtracted from each value

c) One 65 value is removed

d) Another value 35 is added

12. a) Describe the data in **question 11** in terms of gaps, clusters, and outliers.

b) How does removing one outlier affect each measure of central tendency?

UNIT 7 Test

Pacing	Materials
1 h	None

Question(s)	Related Lesson(s)
1 and 2	Lesson 7.1.1
3 and 4	Lesson 7.1.3
5	Lesson 7.2.1
6	Lesson 7.2.3
7 and 8	Lesson 7.2.2
10	Lesson 7.3.2
11	Lesson 7.4.1
12	Lesson 7.4.2

Select questions to assign according to the time available.

Answers

1. a) $\frac{1}{4}$ **b)** $\frac{1}{2}$ **c)** $\frac{3}{4}$

2. a) $\frac{1}{16}$ **b)** $\frac{1}{8}$

3. a) *Sample response*: 3 and 7
b) 5
c) *Sample response*: 0, 9, or greater

4. *Sample responses*:
a) A sum of 5
b) A sum less than 5 (or greater than 5)

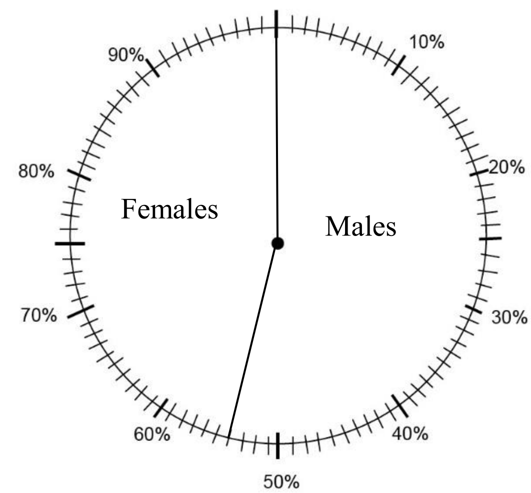
5. *Sample responses*:
a) You should ask one question at a time; Do you like apples? Do you like oranges?
b) The question is too vague; How important is happiness to you on a scale of 1 to 5? 1 means not at all and 5 means very important.
c) The question influences the answer; Do you think harsh penalties for criminals are appropriate?

6. *Sample responses*:
a) Observation; Interviews and questionnaires rely on people's opinions rather than on observed facts.
b) Questionnaire; There are too many people to interview over too large an area, and observation does not make sense.
c) Observation; Actual measurement would be most accurate, but you could also ask everyone and record the results (interview) or have them write it down (questionnaire).

7. a) First-hand data; *Sample response*:
The forester actually tests the trees rather than getting information from another source.
b) Second-hand data; *Sample response*:
The encyclopedia is a second-hand source of someone else's data.

8. *Sample responses*:
a) Only people who like archery are surveyed. The results might be inaccurate for other sports.
b) The survey only includes people who already have to have access to a computer to get e-mail. Other people would not get counted.

9. *Sample response*:
$\frac{360}{660} = \frac{36}{66} = \frac{6}{11} = \frac{54}{99}$
That is about 54%. Females would be about 46%.



<table>
<tr>
<td>10. a) 3
b) 2; The bar for 18 to 20 shows a frequency of 2.
c) There is a gap from 3 to 6.
d) Sample response:
It is hard to be sure whether the single value in the first interval is an outlier without knowing the actual values in the 0 to 3 and 6 to 9 intervals. If there is a 2 in the first interval, and the five values in the third interval are 6, 6, 6, 7, 8, then the 2 may not be an outlier. If there is a 1 in the first interval and the five values in the third interval are 8, 8, 8, 9, 9, then is the 1 is probably an outlier.</td>
<td>11. a) Mean and median increase.
Modes change from 55 and 65 to 550 and 650.
Range does not change.
b) Mean and median decrease.
Modes change from 55 and 65 to 50 and 60.
Range does not change.
c) Mean and median decrease.
Modes change from 55 and 65 to just 55.
Range does not change.
d) Mean and median decrease.
Modes change from 55 and 65 to 35, 55, and 65.
Range does not change.

12. a) Sample response:
There is a cluster of data in the 55 – 65 range.
There are gaps from 5 to 20, from 35 to 55, and from 65 to 85.
5 and 85 are outliers.
b) Removing the 5 changes the mean from 44 to 48 and changes the median from 40 to 55. The mode stays the same.</td>
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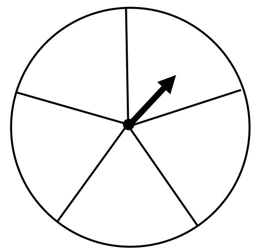
UNIT 7 Performance Task — Making a Game Spinner

You will create a spinner for a game. Players will take turns spinning the spinner and will score the same number of points as the number they spin on each turn.

A. Design and make a spinner with these theoretical probabilities using a fraction circle divided into fifths.



- Spinning a 10 is possible.
- Spinning a 2 has a probability of $\frac{2}{5}$
- Spinning a 1 is impossible.
- Spinning a 3 or a 5 are equally likely events.

B. Spin your spinner 50 times and record the results as a list of 50 data values.

C. i) Organize your data from **part B** to see how often you spun each score.

ii) Create a circle graph of your results.

iii) How does the circle graph of the experimental results compare with the spinner that shows the theoretical probabilities?

D. i) Find the mean, median, mode, and range of your data from **part B**. Which measure of central tendency would you use to describe the average score?

ii) How would these values change if you had spun three more 10s and three fewer 2s?

E. Suppose you were to spin the spinner twice and find the sum of the numbers. What sum or sums would you expect to get most often? Explain your answer.

UNIT 7 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
7-G1 Describe Theoretical Probability: identify probability situations near 0, 1, $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{3}{4}$ **7-G2** Compare Results: theoretical versus experimental **7-F4** Circle Graphs: construct and interpret **7-F6** Central Tendency: examine the effect of changing data	1 h	• Fraction Circle Spinners (BLM) • Percent Circles (BLM)

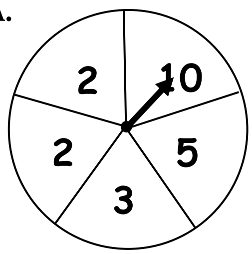
How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. It could also be used as enrichment material for some students.

• You can assess performance on the task using the rubric on the next page.

Sample Solution

A.

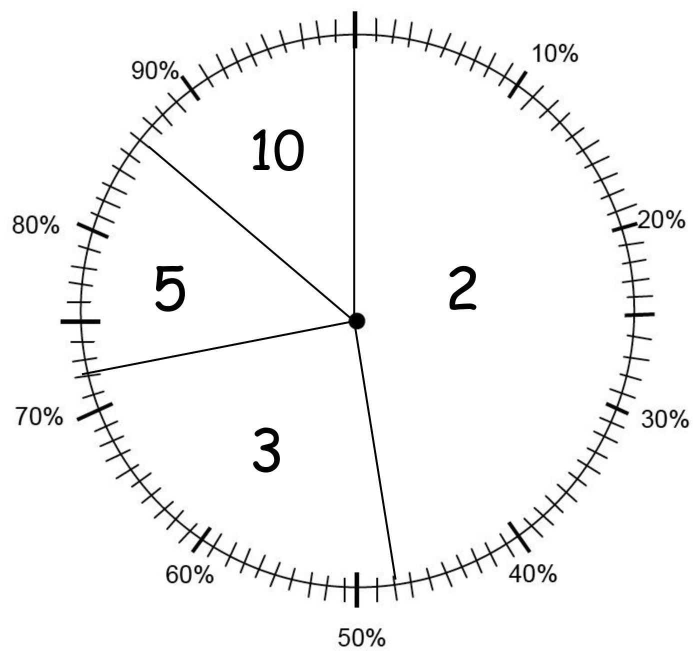


B. 2, 3, 2, 2, 10, 3, 2, 2, 5, 2, 2, 2, 10, 2, 3, 5, 3, 2, 2, 2, 3, 2, 2, 2, 10, 10, 3, 2, 2, 2, 2, 3, 3, 5, 3, 10, 5, 5, 2, 2, 2, 2, 2, 3, 3, 5, 5, 3, 10, 10

C. i)

Number spun	2	3	5	10
How many times	24	12	7	7
Percent	48%	24%	14%	14%

ii)



iii) The circle graph and the spinner are similar, but the circle graph of the experimental results does not show 3, 5, and 10 in equal sections. The section for 2 is larger than the two 2 sections combined on the spinner.

Sample Solution [Continued]

D. i) 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, 10, 10, 10, 10, 10, 10, 10

Mean: 3.78; median: 3; mode: 2; range: 8.

I would use the mean to describe the data because there are several high values, so the median of 3 might be a little misleading.

ii) The mean would go up but none of the other statistics would change.

E. Because 2 is the biggest section of the spinner, and 3, 5, and 10 are all equal sections, I would expect to spin sums of 2 + 2 = 4, 3 + 2 = 5, 5 + 2 = 7, and 10 + 2 = 12.

I created a rectangle model:

	2	2	3	5	10
2	4	4	5	7	12
2	4	4	5	7	12
3	5	5	6	8	13
5	7	7	8	10	15
10	12	12	13	15	20

P(sum of 4) = $\frac{4}{25}$ P(sum of 5) = $\frac{4}{25}$ P(sum of 7) = $\frac{4}{25}$ P(sum of 12) = $\frac{4}{25}$

P(sum of 8) = $\frac{2}{25}$ P(sum of 13) = $\frac{2}{25}$ P(sum of 15) = $\frac{2}{25}$

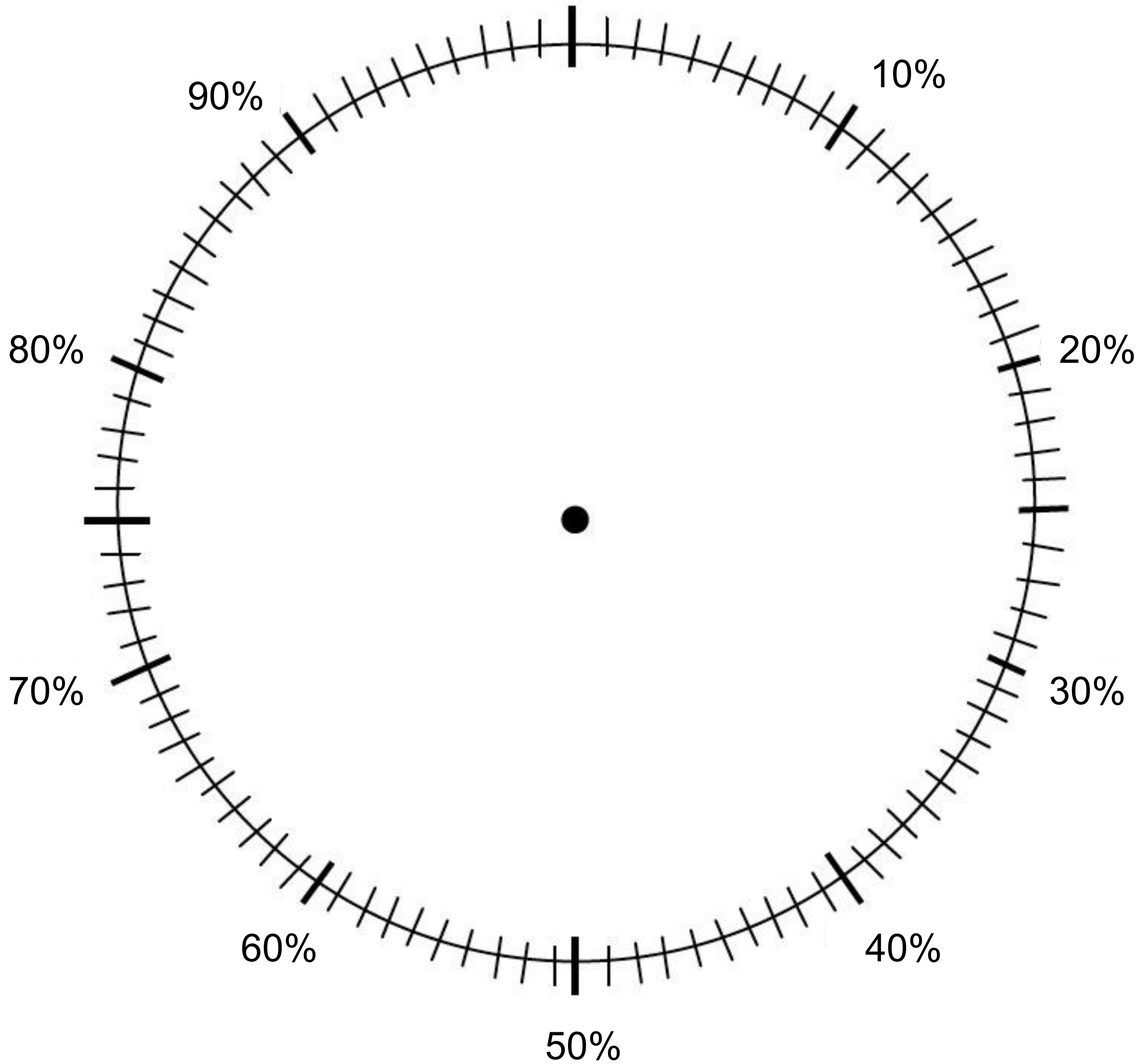
P(sum of 6) = $\frac{1}{25}$ P(sum of 10) = $\frac{1}{25}$ P(sum of 20) = $\frac{1}{25}$

The sums of 4, 5, 7, and 12 are most likely to occur.

UNIT 7 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Designs the spinner	Demonstrates sophisticated ability to apply mathematical knowledge to design a spinner with given probabilities	Demonstrates considerable ability to apply mathematical knowledge to design a spinner with given probabilities	Demonstrates some ability to apply mathematical knowledge to design a spinner with given probabilities	Demonstrates limited ability to apply mathematical knowledge to design a spinner with given probabilities
Records the data	Is organized, careful, and accurate in recording the results of the experiment for the correct number of trials	Is careful and accurate in recording the results of the experiment for the correct number of trials	Conducts the experiment as described but does not organize data and may conduct too few or too many trials	Makes errors in recording data and conducts an incorrect number of trials
Compares probabilities	Demonstrates a sophisticated understanding of the concepts of theoretical and experimental probability	Demonstrates a considerable understanding of the concepts of theoretical and experimental probability	Demonstrates some understanding of the concepts of theoretical and experimental probability	Expects theoretical and experimental results to be exactly equal
Analyses the data	Makes accurate calculations and predictions of the measures of central tendency and range; accurately presents information in a histogram; gives clear and insightful justification for predicting results of two spins	Makes accurate calculations and predictions of the measures of central tendency; accurately presents information in a histogram; gives reasonable justification for predicting results of two spins	Demonstrates understanding but makes errors in calculations; presents information in a histogram in correct proportions to data; makes a reasonable prediction for two spins but gives a poor explanation	Demonstrates limited understanding of the measures of central tendency; makes major errors in graphical representation; does not predict reasonable sums for two spins

BLM 1 Percent Circle



BLM 2 Fraction Circle Spinners