Understanding Mathematics

Textbook for Class VII

ཤེས་རིག

Department of School Education
Ministry of Education and SKills Development
Royal Government of Bhutan

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ROYAL GOVERNMENT OF BHUTAN
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MINISTER

FOREWORD

Provision of quality education for our children is a cornerstone policy of the Royal Government of Bhutan. Quality education in mathematics includes attention to many aspects of educating our children. One is providing opportunities and believing in our children's ability to understand and contribute to the advancement of science and technology within our culture, history and tradition. To accomplish this, we need to cater to children's mental, emotional and psychological phases of development, enabling, encouraging and supporting them in exploring, discovering and realizing their own potential. We also must promote and further our values of compassion, hard work, honesty, helpfulness, perseverance, responsibility, *thadamtsi* (for instance being grateful to what I would like to call '*Pham Kha Nga',* consisting of parents, teachers, His Majesty the King, the country and the Bhutanese people, for all the goodness received from them and the wish to reciprocate these in equal measure) and *ley-ju-drey* — the understanding and appreciation of the natural law of cause and effect. At the same time, we wish to develop positive attitudes, skills, competencies, and values to support our children as they mature and engage in the professions they will ultimately pursue in life, either by choice or necessity.

While education recognizes that certain values for our children as individuals and as citizens of the country and of the world at large, do not change, requirements in the work place advance as a result of scientific, technological, and even political advancement in the world. These include expectations for more advanced interpersonal skills and skills in communications, reasoning, problem solving, and decision-making. Therefore, the type of education we provide to our children must reflect the current trends and requirements, and be relevant and appropriate. Its quality and standard should stem out of collective wisdom, experience, research, and thoughtful deliberations.

Mathematics, without dispute, is a beautiful and profound subject, but it also has immense utility to offer in our lives. The school mathematics curriculum is being changed to reflect research from around the world that shows how to help students better understand the beauty of mathematics as well as its utility.

The development of this textbook series for our schools, *Understanding Mathematics,* is based on and organized as per the new School Mathematics Curriculum Framework that the Ministry of Education has developed recently, taking into consideration the changing needs of our country and international trends. We are also incorporating within the textbooks appropriate teaching methodologies including assessment practices which are reflective of international best practices. The *Teacher's Guides* provided with the textbooks are a resource for teachers to support them, and will definitely go a long way in assisting our teachers in improving their efficacy, especially during the initial years of teaching the

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new curriculum, which demands a shift in the approach to teaching and learning of Mathematics. However, the teachers are strongly encouraged to go beyond the initial ideas presented in the Guides to access other relevant resources and, more importantly to try out their own innovations, creativity and resourcefulness based on their experiences, reflections, insights and professional discussions.

The Ministry of Education is committed to providing quality education to our children, which is relevant and adaptive to the changing times and needs as per the policy of the Royal Government of Bhutan and the wish of our beloved King.

I would like to commend and congratulate all those involved in the School Mathematics Reform Project and in the development of these textbooks.

I would like to wish our teachers and students a very enjoyable and worthwhile experience in teaching, learning and understanding mathematics with the support of these books. As the ones actually using these books over a sustained period of time in a systematic manner, we would like to strongly encourage you to scrutinize the contents of these books and send feedback and comments to the Curriculum and Professional Support Division (CAPSD) for improvement with the future editions. On the part of the students, they can and should be enthusiastic, critical, venturesome, and communicative of their views on the contents discussed in the books with their teachers and friends rather than being passive recipients of knowledge.

Trashi Delek!

Thinley Gyamtsho

MINISTER Ministry of Education

October of 2007

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INTRODUCTION

HOW MATHEMATICS HAS CHANGED

Mathematics is a study of quantity, space, structure, patterns and change. This study at the school level is divided into 5 strands of content, namely, numbers and operations, algebra, geometry, measurement, and data and probability.

The learning of mathematics helps a person to solve problems. While solving problems, skills related to, representation of mathematical ideas, making connections with other topics in mathematics and connections with the real world, providing reasoning and proof, and communicating mathematically would be required. The textbook is designed to promote the development of these process skills.

Nowadays, greater emphasis is given to conceptual understanding rather than on memorizing and applying rote procedures. There are many reasons for this.

• In the real world, you are not told when to factor or when to multiply but rather you need to figure out when to do so. You need to know and how to apply the concepts and skills you are learning in order to solve problems.

• Over time, it is very unlikely that you will remember the mathematics you learn unless it is meaningful. It is much harder to memorize something that does not make sense than something that relates to what you already know.

In this textbook, mathematics is made meaningful in many ways:

Using problems about Bhutan and around the world. These problems will help you see the value of math. For example:

• One problem will ask you to calculate with fractions.

Kamala has $\frac{7}{2}$ cups of rice. Does she have enough to make a meal that needs $3\frac{2}{3}$ cups of rice? Show your work.

• In another lesson you will answer questions about animal speeds.

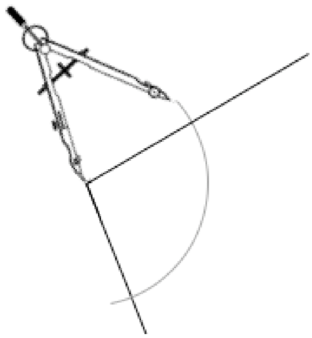
Animal	Distance (m)	Time (s)
Cheetah	200	6.4
Bear	500	36.0
Zebra	250	14.0
Elephant	20	1.8
Tortoise	10	120.0
Rabbit	300	20.0
Lion	400	16.0

a) Which animal runs at each speed?
i) about 11 m/s **ii)** about 25 m/s
b) Which animal could travel each?
i) 900 m in 1 min **ii)** about 5 m in 1 min
c) Which animal is fastest? How do you know?
d) Which is slowest? How do you know?

In the textbook, you will be often required to use objects and tools to learn the math. For example:

- You will build with cubes to learn about geometry.

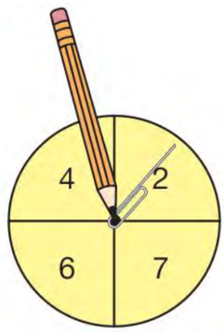
- You will use a compass to draw angles.



- You will use white and black counters to represent positive and negative integers.

+3 –3

- You will use spinners and dice in probability experiments.



This textbook will also ask to explain *why* things are true. It will not be enough to just say something is either true or false. For example, you might be asked to show why a number is divisible by 9 if the sum of the digits is 18.

You will solve many types of problems and you will be encouraged to use your own way of thinking to solve them.

USING YOUR TEXTBOOK

Each unit has

- a *Getting Started* section
- two or three chapters
- regular lessons and at least one *Explore* lesson
- a *Game*
- a *Connections* activity
- a *Unit Revision*

Getting Started

There are two parts to the *Getting Started*. You will complete a *Use What You Know* activity and then you will answer *Skills You Will Need* questions. Both remind you of things you already know that will help you in the unit.

- The *Use What You Know* activity is done with a partner or in a group.
- The *Skills You Will Need* questions help you review skills you will use in the unit. You will usually do these by yourself.

Regular Lessons

- Lessons are numbered #.#.# — the first number tells the unit, the second number is the chapter, and the third number is the lesson in the chapter.

For example, Lesson 4.2.1 is Unit 4, Chapter 2, Lesson 1.

- Each regular lesson is divided into five parts:

- A *Try This* problem or task
- A box that explains the main ideas of the lesson; it is called the exposition
- A question that asks you to think about the *Try This* problem again, using what you have learned in the exposition
- one or more *Examples*
- *Practising and Applying* questions

Try This

- The *Try This* is in a grey box, like this one from lesson 1.1.2 on page 5.

Try This

Yuden bought 9 kg of chicken. Each kilogram cost Nu 85.
The shopkeeper said that Yuden owed a total of Nu 755.

A. Describe two or more ways Yuden could have known the total was incorrect.

You will solve the *Try This* problem with a partner or in a small group. The math you learn later in the lesson will relate back to this problem.

The Exposition

• The exposition comes after the *Try This*.

• It presents and explains the main ideas of the lesson.

• Important math words are in **bold** text. You will find the definitions of these words in the glossary at the back of the textbook.

• You are not expected to copy the exposition into your notebook.

Going Back to the Try This

• There is always a question after the exposition that asks you to think about the *Try This* problem again. You can use the new ideas presented in the exposition. In the example below from lesson 1.1.2 on page 6, the exposition shows to use rectangle models to multiply. You can use these rectangles to solve the *Try This* problem again but in a different way.

> **B.** How could Yuden have used a divisibility test to know the total was incorrect?

Examples

• The *Examples* prepare you for the *Practising and Applying* questions. Each example is a bit different from the others so that you can refer to many models.

• You will work through the examples sometimes on your own, sometimes with another student, and sometimes with your teacher.

• What is special about the examples is that the *Solutions* column shows you what you should write when you solve a problem, and the *Thinking* column shows you what you might be thinking as you solve the problem.

• Some examples show you two different solutions to the same problem. The example below from lesson 1.1.4 on page 14 shows two possible ways to answer the question, *Solution 1* and *Solution 2*.

Examples

Example 1 Calculating the GCF of Three Numbers

Calculate GCF (50, 70, 140).

Solution 1

50 → **5** × 5 × **2**

70 → **5** × 7 × **2**

140 → 7 × 10 × 2 → 7 × **2** × **5** × 2

The common factors are 2 and 5.

GCF (50, 70, 140) = 2 × 5 = 10

Thinking

• I used a factor tree to find the factors of each.

• I looked for the prime factors that were common to all three.

• I multiplied the common factors to get the GCF.

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Solution 2

Factors of 50

1 2 5 **<u>10</u>** 25 50

Factors of 70

1 2 5 7 **<u>10</u>** 14 35 70

Factors of 140

1 2 4 5 7 **<u>10</u>** 14 20 28 35 70 140

The greatest number in all three lists is 10.

GCF (50, 70, 140) = 10

Thinking

• I used a factor rainbow to find factor pairs for each number.

• I divided each number by 1, then by 2, then by 3, and so on until I had a list of all possible different factors.

Practising and Applying

• You might work on the *Practising and Applying* questions by yourself, with a partner, or in a group. You can use the exposition and examples to help you.

• The first few questions are similar to the questions in the *Examples* and the exposition.

• The last question helps you think about the most important ideas you have learned in the lesson.

Explore Lessons

• An *Explore* lesson gives you a chance to work with a partner or in a small group to investigate some math.

• Your teacher does not tell you about the math in an *Explore* lesson. Instead, you work through the questions and learn your own way.

Connections Activity

• The *Connections* activity is usually something interesting that relates to the math you are learning. For example, in Unit 7, the *Connections* on page 235 explains how scientists use probability to estimate fish populations.

• There is always a *Connections* in a unit.

• You usually work in pairs or small groups to complete the task or answer the question(s).

Game

• Each unit usually has at least one *Game*.

• The *Game* is a way to practise skills and concepts from the unit with a partner or in small group.

• The materials you need and the rules are listed in the textbook. Usually the textbook shows a sample game to help you understand the rules.

Unit Revision

• The *Unit Revision* is a chance to review the lessons in the unit.

• The order of the questions in the *Unit Revision* is usually the same as the order of the lessons in the unit.

• You can work with a partner or on by yourself, as your teacher suggests.

Glossary

• At the end of the textbook, you will find a glossary of new math words and their definitions. The glossary also contains other important math words from Class VI that you need to remember.

• The glossary also has definitions of instructional words such as "explain", "predict", and "estimate". These will help you understand what you are expected to do.

Answers

• You will find answers to most of the numbered questions at the back of the textbook. Answers to questions that ask for explanations (Explain your thinking or How do you know?) are not included in your textbook. Your teacher has those answers.

• Questions with capital letters, such as A or B, do not have answers in the back of the textbook. Your teacher has the answers to these questions.

• If there could be more than one correct answer to a question, the answer will start with *Sample Response*. Even if your answer is different than the answer at the back of the textbook, it may still be correct.

ASSESSING YOUR MATHEMATICAL PERFORMANCE

Your teacher will be checking to see how you are doing in your math learning. Sometimes your teacher will collect information about what you understand or do not understand in order to change the way you are taught. Other times your teacher will collect information in order to give you a mark.

Assessment Criteria

• Your teacher should tell you about what she or he will be checking and how it will be checked.

• The amount of the mark assigned for each unit should relate to the time the class spent on the unit and the importance of the unit.

• Your mark should consider how you are doing on skills, applications, concepts, and problem solving.

• Your teacher should tell you whether the mark for a test will be a number such as a percent, a letter grade such as A, B, or C, or a level on a rubric (level 1, 2, 3, or 4). A rubric is a chart that describes criteria for your work, usually in four levels of performance. If a rubric is used, your teacher should let you see the rubric before you start to work on the task.

Determining a Mark or Grade

In determining your overall mark in mathematics, your teacher might use a combination of tests, assignments, projects, performance tasks, exams, interviews, observations, and homework.

THE CLASSROOM ENVIRONMENT

In almost every lesson you will have a chance to work with other students. You should always share your responses, even if they are different from the answers offered by other students. It is only in this way that you will really be engaged in the mathematical thinking.

YOUR NOTEBOOK

• It is valuable for you to have a well-organized, neat notebook to look back at to review the main ideas you have learned. You should do your rough work in this same notebook. Do not do your rough work elsewhere and then waste valuable time copying it neatly into your notebook.

• Your teacher will sometimes show you important points to write down in your notebook. You should also make your own decisions about which ideas to include in your notebook.

UNIT 1 NUMBER

Getting Started

Use What You Know

Bijoy is thinking of a secret whole number less than 100.

Here are her clues for guessing the number
- It has factors of 2, 3, and 5.
- It is a multiple of 4.

A. How do you know Bijoy's secret number is even?

B. i) Use a 100 chart.
- Find all the numbers that have 2 as a factor.
- Find all the numbers that have 5 as a factor.
- Shade all the numbers that have both 2 and 5 as factors.
- Circle the shaded numbers that have 3 as a factor.
- Put an X on the circled numbers that are multiples of 4.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	747	5	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

ii) What is Bijoy's secret number? How do you know?

C. i) Think of your own secret whole number less than 100.
Make up clues that involve factors and multiples.

ii) Test your clues to see if they result in only one possible number.

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Skills You Will Need

1. List the common factors of each pair of numbers.

a) 12 and 16 **b)** 15 and 21 **c)** 24 and 50

2. Which are prime numbers?

23 18 17 35 91

3. Write each number below in expanded form in two ways.

For example, 312,056 is

$3 \times 100{,}000 + 1 \times 10{,}000 + 2 \times 1000 + 5 \times 10 + 6 \times 1$

3 hundred thousands + 1 ten thousand + 2 thousands + 5 tens + 6 ones

a) 412,150 **b)** 365,124 **c)** 1,003,010 **d)** 1,000,901,142

4. What is the product of 0.4×0.6? Draw a picture to show 0.4×0.6.

5. Calculate.

a) 5×31.4 **b)** 6×89.04 **c)** 3×0.28 **d)** 9×11.5

e) 0.3×0.7 **f)** 0.8×1.2 **g)** 19×0.001 **h)** 23×0.01

6. What is the quotient of $0.8 \div 0.2$? Draw a picture to show $0.8 \div 0.2$.

7. Calculate.

a) $315.6 \div 3$ **b)** $89.2 \div 4$ **c)** $124.8 \div 6$ **d)** $1003.5 \div 9$

e) $0.18 \div 0.09$ **f)** $0.3 \div 0.04$ **g)** $2.7 \div 0.4$ **h)** $15.3 \div 0.6$

You might find this place value chart helpful for questions 8, 9, and 10.

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
		1	4	5	2

8. Calculate.

a) 3.4×100 **b)** 0.245×100 **c)** $3.28 \div 10$ **d)** $23.4 \div 1000$

9. In which place value column is the digit 3 after each calculation?

a) $42\underline{3}.6 \times 0.01$ **b)** $42\underline{3}.6 \times 0.001$

c) $42\underline{3}.6 \div 0.1$ **d)** $42\underline{3}.6 \div 0.01$

10. Round each to the nearest tenth and to the nearest hundredth.

a) 0.235 **b)** 14.913 **c)** 1.998

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Chapter 1 Whole Numbers and Decimals

1.1.1 EXPLORE: Divisibility by 3 and 9

• A number is **divisible** by another number if the **quotient** is a whole number.
For example:
24 is divisible by 3 since 24 ÷ 3 = 8 and there is no remainder.
29 is not divisible by 3 since 29 ÷ 3 = 9 R 2.

• If a number is divisible by another number, it is a **multiple** of the other number.
For example, 24 is divisible by 3, so 24 is a multiple of 3.

• One way to test if a number is divisible by 3 or 9 is to divide the number by 3 or 9 to see if the quotient is a whole number. But there are easier ways to test. You will learn about them in this lesson.

A. i) Choose five multiples of 3 between 100 and 1000.

ii) Copy the chart below. Record the five numbers in the first column.

Number	Sum of digits	Sum of digits ÷ 3

iii) Find the sum of the digits in each chosen number. Record those sums in the second column of the chart.

iv) Divide each digit sum by 3. Record the quotients in the third column.

v) Is each digit sum a multiple of 3?

B. i) The number 605 is not a multiple of 3 since 605 ÷ 3 = 201 R 2.
Is the sum of its digits a multiple of 3? How do you know?

ii) List four other numbers between 100 and 1000 that are not multiples of 3. Show that the sums of their digits are not multiples of 3.

C. Repeat **part A i) to iv)** using multiples of 9, but divide the sum of the digits by 9 instead of by 3. Is each digit sum a multiple of 9?

D. i) The number 901 is not a multiple of 9 since 901 ÷ 9 = 100 R 1.
Show that the sum of its digits is not a multiple of 9.

ii) List four other numbers between 100 and 1000 that are not multiples of 9. Show that the sums of their digits are not multiples of 9.

[Continued]

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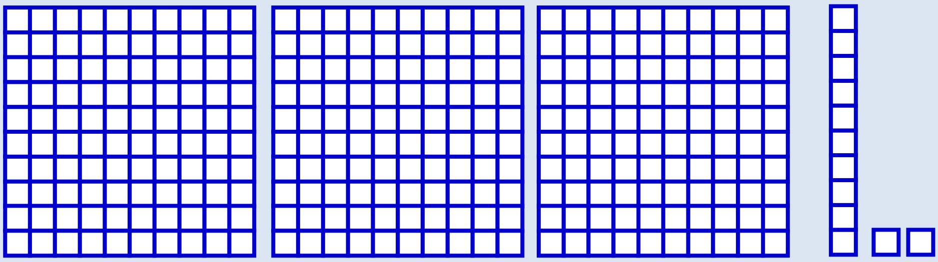
E. Write a rule that uses the digit sum of a number that you think you could use to test if a number is each.

i) divisible by 3 **ii)** divisible by 9

In parts F to H, you will use models to see why your rules work.

F. i) Use models or pictures of hundreds, tens, and ones to represent 312.

The model will look like this:



ii) If you divide each model for 100 and 10 into groups of 3, how many ones would be left over from each?

iii) If you were to remove the groups of 3, how many ones would remain? How does this number relate to the sum of the digits?

iv) How do you know the entire number can be divided into groups of 3 with none left over?

v) Model another number that is divisible by 3 and repeat **parts ii) to iv)**.

vi) Use models to explain why your rule from **part E i)** about divisibility by 3 makes sense.

G. i) Use models to represent 279.

ii) If you divide each model for 100 and 10 into groups of 9, how many ones would be left over from each?

iii) If you were to remove the groups of 9, how many ones would remain? How does this number relate to the sum of the digits?

iv) How do you know the entire number can be divided into groups of 9 with none left over?

v) Model another number that is divisible by 9 and repeat **parts ii) to iv)**.

vi) Use models to explain why your rule from **part E ii)** about divisibility by 9 makes sense.

H. Show that your rules for divisibility by 3 and divisibility by 9 also work for testing larger numbers like 4005 and 59,130.

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1.1.2 Divisibility Tests

Try This

Yuden bought 9 kg of chicken. Each kilogram cost Nu 85.
The shopkeeper said that Yuden has to pay a total of Nu 755.

A. Describe two or more ways Yuden could have known the total was incorrect.

• Sometimes you might want to know whether a number is divisible by another number without figuring out the **quotient**.

For example:

If you want to know whether 404 kg of rice can be divided into 4 kg packages with no rice left over, all you need to know is whether 404 is divisible by 4.

• If a number is **divisible** by another number, it is a **multiple** of the other number.

For example, 404 is divisible by 4, which means 404 is a multiple of 4.

• For some numbers you can use **divisibility tests** to tell if one number is divisible by (or a multiple of) another number without actually dividing.

Divisibility Tests for 2, 3, 4, 5, 9, and 10

A whole number is divisible by	**If …**
2	the ones digit is 0, 2, 4, 6, or 8.
3	the sum of the digits is a multiple of 3.
4	2 × tens digit + the ones digit is a multiple of 4.
5	the ones digits is 0 or 5.
9	the sum of the digits is a multiple of 9.
10	the ones digit is 0.

For example:

You can use the tests to see if 360 is a multiple of 2, 5, 10, 3, 9, and 4:

- The ones digit is 0, so 360 is divisible by 2, 5, and 10.
- The sum of the digits, 9, is a multiple of 3 and 9, so 360 is divisible by 3 and 9.
- 2 × 6 + 0 = 12, which is divisible by 4, so 360 is divisible by 4.

• The divisibility tests for 2, 5, and 10 make sense because of the pattern in the ones digits of the multiples of 2, 5, and 10:

- The multiples of 2 are **0**, **2**, **4**, **6**, **8**, 1**0**, 1**2**, 1**4**, 1**6**, 1**8**, 2**0**, …, so the ones digit is 0, 2, 4, 6, or 8.
- The multiples of 5 are **0**, **5**, 1**0**, 1**5**, 2**0**, 2**5**, 3**0**, …, so the ones digit is 0 or 5.
- The multiples of 10 are **0**, 1**0**, 2**0**, 3**0**, 4**0**, …, so the ones digit is always 0.
- To understand the other divisibility tests, you can use models.

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Divisibility tests for 3 and 9

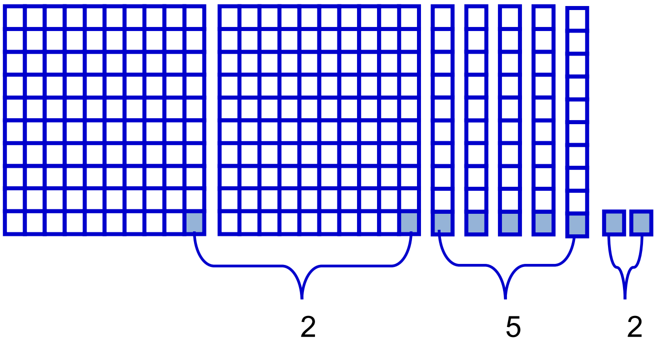
A number is divisible by 3 if the sum of the digits is a multiple of 3.

A number is divisible by 9 if the sum of the digits is a multiple of 9.

For example:

The model for 252 has 2 hundreds, 5 tens, and 2 ones.

- Each hundred is 1 more than a multiple of 3 or 9 (99 + 1 = 100).
- Each ten is 1 more than a multiple of 3 or 9 (9 + 1 = 10).
- If you divide each hundred and each ten into groups of 3 or 9, then 2 ones are left over from the 2 hundreds and 5 ones are left over from the 5 tens.
- If you add the 2 + 5 leftover ones to the 2 ones, there are 2 + 5 + 2 = 9 ones.



2 + 5 + 2 = 9 is also the sum of the digits in 252.

- Since you can divide the 9 ones into groups of 3 (or a group of 9), 252 must be divisible by 3 (or 9).

Divisibility test for 4

A number is divisible by 4 if 2 × tens digit + ones digit is a multiple of 4.

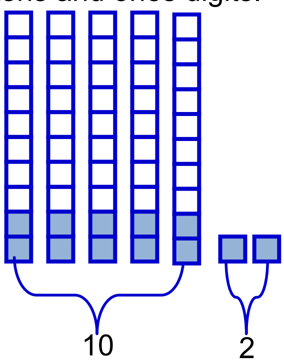
In any number with three or more digits, each hundreds digit, thousands digit, ten thousands digit, and so on has a value that is a multiple of 4:

1 hundred = 4 × 25 1 thousand = 4 × 250 1 ten thousand = 4 × 2500

So to test for divisibility by 4, you need to look only at the tens and ones digits.

For example, the model for 252 has 5 tens and 2 ones:

- Each ten is 2 more than a multiple of 4 (8 + 2 = 10).
- If you divide each ten into groups of 4, then 2 ones are left over from each, or 5 × 2 = 10 ones are left over from all 5 tens.
- If you add the 10 leftover ones to the 2 ones, there are 10 + 2 = 12 ones altogether.
- Since you can divide 12 into groups of 4, 252 must be divisible by 4.



10 + 2 is 2 × the tens digit plus the ones digit of 252.

B. How could Yuden have used a divisibility test to know the total was incorrect?

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Examples

Example 1 Using Divisibility Rules to Find a Remainder

Test 1025 for divisibility by 4 and by 3. What is the remainder if it is not divisible?

Solution

1025 → 2 × 2 + 5 = 9

9 is 1 more than a multiple of 4, so 1025 is not divisible by 4.

If the number were 1 less, it would be a multiple of 4, so the remainder is 1.

1025 → 1 + 0 + 2 + 5 = 8

8 is 2 more than a multiple of 3, so 1025 is not divisible by 3.

If the number were 2 less, it would be a multiple of 3, so the remainder is 2.

Thinking

• I used the divisibility test for 4 but I realized afterwards that I didn't need to because an odd number cannot be divisible by 4.

• 9 is not divisible by 4, but 8 is, so I knew 1025 is not divisible by 4 but 1024 is divisible by 4.

• I used the divisibility test for 3.

• 8 is not a multiple of 3, but 6 is, so I knew 1025 is not divisible by 3, but 1023 is divisible by 3.

Example 2 Creating New Divisibility Tests from Known Ones

Three students made up new divisibility tests. Are they right?

a) Duptho says that since 6 = 2 × 3, a number is divisible by 6 if it is divisible both by 2 and by 3.

b) Kinley says that since 8 = 2 × 4, a number is divisible by 8 if it is divisible both by 2 and by 4.

c) Dechen says that since 8 = 2 × 4, a number is divisible by 8 if half its value is divisible by 4.

Solution

a) Multiple of 2 and 3 = 2 × 3 × ■
= 6 × ■

Duptho's test is right.

b) 12 is divisible by 2 and by 4, but not by 8.

Kinley's test is not right.

c) $\frac{8 \times [\,]}{2} = 4 \times [\,]$

Dechen's test is right.

Thinking

a) If a number is a multiple of 2 and of 3, it would have both 2 and 3 as factors, so it would be a multiple of 2 × 3, or 6.

b) As soon as I found one number that was divisible by 2 and by 4, but not by 8, I knew Kinley's test was not right.

c) Any multiple of 8 can be written as 8 × ■.

• I knew a multiple of 8 could be divided into groups of 8. If I took half of each group of 8, the new number would be in groups of 4. So half the number is a multiple of 4.

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Practising and Applying

1. Test each number for divisibility by 3.

a) 254 **b)** 387 **c)** 1395

2. Test each number for divisibility by 9.

a) 789 **b)** 602 **c)** 2052

3. Test each number for divisibility by 4.

a) 812 **b)** 70,102 **c)** 5896

4. Use divisibility tests to find the remainder for each.

a) 518 ÷ 9 **b)** 3057 ÷ 4

c) 2419 ÷ 3 **d)** 18,205 ÷ 3

5. Some items were sold for Nu 200, some for Nu 500, and some for Nu 300. The total income from the sale of the items is shown below.

Item	Total income from sales
A	Nu 18,500
B	Nu 3,400
C	Nu 6,900

Use divisibility tests to decide which item cost each price.

a) Nu 200 **b)** Nu 300 **c)** Nu 500

6. List all possible values for each missing digit.

a) 82■3 is divisible by 3.

b) 32,67■ is divisible by 3.

c) 65■8 is divisible by 4.

d) 2■,352 is divisible by 4.

e) ■78 is divisible by 9.

f) 6■49 is divisible by 9.

7. A farmer plants 987 apple trees in 9 rows. Can each row have the same number of trees? How do you know?

8. 1219 kg of Swiss cheese is packaged in 3 kg bags. Will there be any cheese left over? How do you know?

9. What is the least number that is divisible by 3 and greater than 32,152?

10. Find three 4-digit numbers less than 5000 that end in 6 and are divisible by 6.

11. Tell whether each statement is true or false. Explain your thinking.

A. A number that is divisible by 3 and 4 is divisible by 12.

B. A number that is divisible by 3 and 9 is divisible by 27.

12. Use the digits 4, 6, 7, 8, and 9 each once to create the greatest 5-digit number that is divisible by 8.

■ ■ ■ ■ ■ ÷ 8

13. Find the smallest 3-digit number that is not divisible by 5 but is divisible both by 4 and by 9.

14. Explain why you do not have to think about the digits to the left of the tens place to decide whether a number is divisible by 4. Use an example to help you explain.

15. Why is it quicker to use a divisibility test to see if a number is divisible by 9 than to actually divide it?

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CONNECTIONS: Casting Out Nines

You can check calculations using the divisibility test for 9. The method is called casting out nines.

For example, suppose you calculate 1458 + 814 = 10,362.

You can check to see if 10,362 is incorrect by doing the following:

Is 1458 + 814 = 10,362?	**1458**	**814**	**10,362**
Add the digits	1 + 4 + 5 + 8 = 18	8 + 1 + 4 = 13	1 + 0 + 3 + 6 + 2 = 12
Cast out the nines (subtract the greatest multiple of 9)	18 – 18 = **0**	13 – 9 = **4**	12 – 9 = **3**
Add the leftovers	0 + 4 = 4		3
Compare	4 ≠ 3 so 1458 + 814 = 10,362 is incorrect.		

• To use casting out nines to check a subtraction, you subtract the leftovers after you cast out the nines.
• To use casting out nines to check a multiplication, you multiply the leftovers after you cast out the nines.

Perform each calculation. Use casting out nines to check your work.

1. 3489 + 2379 **2.** 1425 – 387 **3.** 25 × 38

GAME: Divisibility Spin

You can play this game in groups of 2 to 4 players.

You can use a set of 40 digit cards (0 to 9) or a deck of playing cards. (If you use playing cards, remove the face cards. The Aces represent ones, the tens represent zeros, and the remaining cards represent the numbers on the cards.)

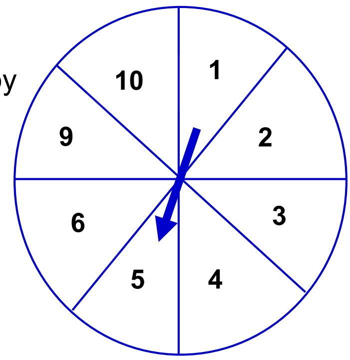
The goal of the game is to create numbers that are divisible by another number.

• The dealer shuffles the deck and deals three cards to each player.

• Each player arranges the cards to form as many 3-digit numbers as possible and records the numbers.

• Each player then spins a number on the spinner.

• Players score 1 point for each number that is divisible by the number spun.



Players take turns being the dealer.

The first player to get 30 points wins.

For example:

You are dealt 4, 5, and 2 and then you spin a 5.

Possible numbers are: 452, 425, 542, 524, 254, 245

Only 245 and 425 are divisible by 5, so you get 2 points.

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1.1.3 Lowest Common Multiple

Try This

A teacher grouped the students in her class into groups of 12 one day and into groups of 9 another day. Each time there were no students left over.

A. How many students do you think she has in her class?

• A **common multiple** (other than 0) is a number that is a multiple of two or more different numbers.

For example, 18 is a common multiple of 2 and 3:

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, <u>18</u>, ...

Multiples of 3: 3, 6, 9, 12, 15, <u>18</u>, ...

• Numbers can have many common multiples, but only one **Lowest Common Multiple (LCM)**.
For example, the lowest common multiple of 2 and 3 is 6:

Multiples of 2: 2, 4, <u>**6**</u>, 8, 10, <u>12</u>, 14, 16, <u>18</u>, 20, 22, <u>24</u>, 26, 28, <u>30</u>, ...

Multiples of 3: 3, <u>**6**</u>, 9, <u>12</u>, 15, <u>18</u>, 21, <u>24</u>, 27, <u>30</u>, ...

Common multiples of 2 and 3: 6, 12, 18, 24, 30, ….

Notice that every common multiple of 2 and 3 is a multiple of the LCM, 6.

• There are different strategies for calculating the lowest common multiple.

Strategy 1

List the multiples of each number until they have a number in common. The common number is the LCM.

For example, for the LCM of 14 and 35:

Multiples of 14: 14, 28, 42, 56, <u>**70**</u>, 84, 98, …

Multiples of 35: 35, <u>**70**</u>, 105, …

} 70 is the lowest number that is common to both lists, so LCM (14, 35) = 70

Strategy 2

- Write each number as a product of **prime factors**.
- Use all of the factors of one number and include any additional prime factors that are in the other number.
- Multiply these prime factors to get the LCM.

For example, for the LCM of 14 and 35:

14 = <u>2</u> × <u>7</u>

35 = <u>5</u> × 7

} Use all the prime factors of 14 but only the prime factor 5 from 35. The prime factor 5 is not already included in the factors of 14. So LCM (14, 35) = 2 × 7 × 5 = 70

If the extra prime factor 7 is also included, it would result in a common multiple and not the LCM. 2 × 7 × 5 × 7 = 490 which is a multiple of both 14 and 35.

- Knowing the LCM of two or more numbers can be useful for finding **equivalent fractions** and for solving problems like the one in **Example 3**.

Reprint 2023

B. How does the problem in **part A** relate to finding the lowest common multiple?

Examples

Example 1 Finding the LCM of Three Numbers

What is the LCM of 12, 50, and 8?

Solution

$12 = \underline{3} \times \underline{2} \times \underline{2}$

$50 = \underline{5} \times \underline{5} \times 2$

$8 = \underline{2} \times 2 \times 2$

$\text{LCM} = 3 \times 2 \times 2 \times 5 \times 5 \times 2$

$= 600$

Thinking

- I factored each into prime numbers.
- For the LCM, I used all the factors without repeating factors I had:

- 3 × 2 × 2 from 12
- 5 × 5 from 50, but not 2 since I already had 2
- only one 2 from 8, since I already had two 2s

Example 2 Using Divisibility Tests to Find Prime Factors and the LCM

Use divisibility tests to find the LCM of 96 and 312.

Solution

Find the prime factors of 96

$9 + 6 = 15 \rightarrow 15 \div 3 = 5$

So 96 is divisible by 3.

$96 \div 3 = 32$ and $32 = 4 \times 8$

$4 = 2 \times 2$ and $8 = 2 \times 2 \times 2$

$96 = 3 \times 2 \times 2 \times 2 \times 2 \times 2$

Find the prime factors of 312

$3 + 1 + 2 = 6 \rightarrow 6 \div 3 = 2$

So 312 is divisible by 3.

312 is even, so it is also divisible by 2.

$312 \div 6 = 52$

$2 \times 5 + 2 = 12$, so 52 is divisible by 4.

$52 \div 4 = 13$

$312 = 2 \times 3 \times 2 \times 2 \times 13$

Find the LCM of 96 and 312

$96 = \underline{3} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{2}$

$312 = 2 \times 3 \times 2 \times 2 \times \underline{13}$

$\text{LCM}(96, 312) = 96 \times 13$

$= 1248$

Thinking

- I used divisibility tests for 3 and 4 to find some of the prime factors of each number.
- Then I divided to get the other prime factors.

- Since I knew I was going to use all the prime factors of 96, I used 96 instead of writing 3 × 2 × 2 × 2 × 2 × 2.
- I included with 96 the only prime factor of 312 that didn't repeat a factor from 96.

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Example 3 Solving a Problem Using the LCM

Every day, Karma walks a 9 km path and Dorji walks a 6 km path. Each boy walks the same total distance. What is the least number of times each boy could be walking his path?

Solution

Prime factors of 9 and 6

$9 = \underline{3} \times \underline{3}$

$6 = 3 \times \underline{2}$

LCM (9, 6) = $3 \times 3 \times 2 = 18$

$18 \div 9 = 2$

$18 \div 6 = 3$

Karma walks his path 2 times.

Dorji walks his path 3 times.

Thinking

- I knew Karma's total distance was a multiple of 9 and Dorji's total distance was a multiple of 6.
- Since they walked the same distance and I wanted to know the least number of times, I knew I wanted the LCM of 9 and 6.
- I divided the LCM by the length of each path to find out how many times each boy walks it.

Practising and Applying

1. Find the lowest common multiple of each group. Show the factors you used.

a) 5, 20, and 28

b) 4, 8, and 32

c) 38 and 57

d) 5, 22, and 121

2. Tell whether each statement is true or false. Explain how you know.

a) LCM (7, 18) = LCM (14, 18)

b) LCM (5, 8) = LCM (10, 8)

c) LCM (6, 11) = LCM (12, 11)

3. The LCM of two numbers is 45. What could the two numbers be? List three or more possible pairs of values.

4. The lowest common multiple of three numbers is one of the three numbers. What do you know about the three numbers? Use an example to help you explain.

5. Can the lowest common multiple of two numbers ever be less than both numbers? How do you know?

6. a) What is the lowest common multiple of 6 and 10?

b) Explain why every common multiple of 6 and 10 is a multiple of the LCM.

7. Ugyen helps to cook every second day and helps to wash dishes every third day. How many times will she do both on the same day in September? How do you know?

8. Sonam says that you can always calculate the LCM of two numbers by multiplying them. For example:

LCM (8, 15) = $8 \times 15 = 120$

Is he right? How do you know?

Reprint 2023

1.1.4 Greatest Common Factor

Try This

Students drew pictures on cardboard squares of the same size. The squares were put together to make a rectangle 135 cm by 120 cm.

A. What is the largest square the students could have used?

• A **common factor** is a number that is a factor of two or more different numbers.

For example, 1 and 2 are the common factors of 6 and 8.

Factors of 6: **1**, **2**, 3, 6

Factors of 8: **1**, **2**, 4, 8

• A set of numbers can have many common factors but the **greatest common factor (GCF)** is the greatest number that is a common factor.

For example:

GCF (6, 8) = 2

• There are different strategies for calculating the greatest common factor.

Strategy 1

List the factors of both numbers. Look for common factors and choose the greatest common factor.

For example, for GCF (12, 20):

Factors of 12 : 1, 2, 3, **4**, 6, 12
Factors of 20: 1, 2, **4**, 5, 10, 20

4 is the greatest number that is common to both lists, so GCF (12, 20) = 4.

Strategy 2

- Write each number as the product of prime factors.
- Look for all prime factors that are common. Their product is the greatest common factor.

For example, for GCF (12, 20):

$12 = 3 \times \underline{2} \times \underline{2}$
$20 = 5 \times \underline{2} \times \underline{2}$

2 and 2 are common prime factors, so GCF (12, 20) = 2 × 2 = 4.

• Knowing the GCF of two or more numbers can be useful for writing fractions in **lowest terms** and for solving problems like the one in **question 7** on **page 15**.

Reprint 2024

B. How does the problem in **part A** relate to finding the greatest common factor?

Examples

Example 1 Calculating the GCF of Three Numbers

Calculate GCF (50, 70, 140).

Solution 1

50 → **5** × 5 × **2**

70 → **5** × 7 × **2**

140 → 7 × 10 × 2 → 7 × **2** × **5** × 2

The common factors are 2 and 5.

GCF (50, 70, 140) = 2 × 5 = 10

Thinking

- I used a factor tree to find the factors of each.
- I looked for the prime factors that were common to all three.
- I multiplied the common factors to get the GCF.

Solution 2

Factors of 50

1 2 5 **10** 25 50

Factors of 70

1 2 5 7 **10** 14 35 70

Factors of 140

1 2 4 5 7 **10** 14 20 28 35 70 140

The greatest number in all three lists is 10.

GCF (50, 70, 140) = 10

Thinking

- I used a factor rainbow to find factor pairs for each number.
- I divided each number by 1, then by 2, then by 3, and so on until I had a list of all possible different factors.

Example 2 Multiplying the LCM and GCF of Two Numbers

a) Calculate GCF (30, 8) × LCM (30, 8) and 30 × 8.

b) What do you notice?

Solution

a) 30 = 3 × 2 × 5 8 = 2 × 2 × 2

GCF (30, 8) = 2

LCM (30, 8) = 3 × 2 × 5 × 2 × 2 = 120

GCF (30, 8) × LCM (30, 8) = 2 × 120 = 240

30 × 8 = 240

b) GCF (30, 8) × LCM (30, 8) = 30 × 8

Thinking

- I found the prime factors of each number because I knew I could use them to find both the GCF and the LCM.

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Practising and Applying

1. Calculate.

a) GCF (40, 42)

b) GCF (56, 49)

c) GCF (96, 120)

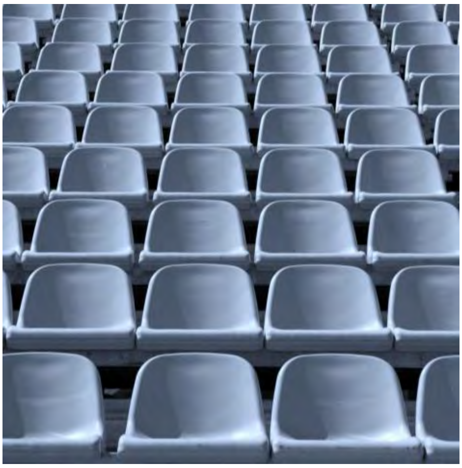
d) GCF (14, 18, 20)

2. How much more is the first value than the second?

a) GCF (40, 28), GCF (39, 27)

b) GCF (100, 105), GCF (99, 104)

3. 64 chairs were arranged in equal rows. 56 students came to sit in the chairs. If the same number of students sat in each row, how many chairs could have been in each row? Explain your thinking.



4. Is it possible for two numbers to have a GCF of 6 and a LCM of 50? How do you know?

5. How does knowing the GCF of 30 and 40 help you to write $\frac{30}{40}$ in lowest terms?

6. Two numbers have a GCF of 10 and a LCM of 300. What could be the numbers? List two possible pairs.

7. Store A and Store B sell the same cakes for different prices.

Store A
6 cakes for Nu 540

Store B
4 cakes for Nu 340

a) Use the GCF of 4 and 6 to figure out which store has the lower price. Show your work.

b) Use the LCM of 4 and 6 to figure out which store has the lower price. Show your work.

8. a) What is the GCF of any two different prime numbers? How do you know?

b) What is the GCF of any two consecutive numbers, such as 15 and 16? Explain your answer.

9. a) Why does every pair of whole numbers have a GCF of 1 or greater?

b) When is the GCF of two numbers the same as one of the numbers?

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CONNECTIONS: Carrom Math

In the game of Carrom, you shoot a striker to hit other discs ("gotte") into a hole. The game board is a square with a hole in each corner.

Mathematicians study a game like Carrom.

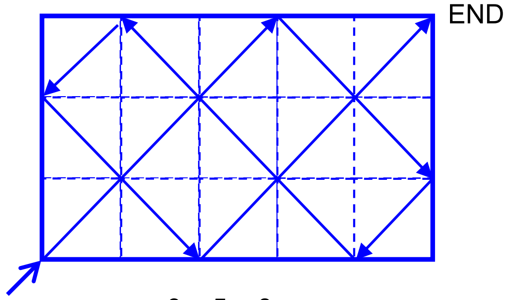
• The board is a rectangle that has a length and width that are a whole number of units.

• Only the striker is used and it always travels at a 45° angle to the sides of the board.

• The striker always rebounds off the side of the board at the same angle at which it hits the side.

• When the striker reaches a corner, it drops into the hole.

Here is the path of the striker on a 3-by-5 board.

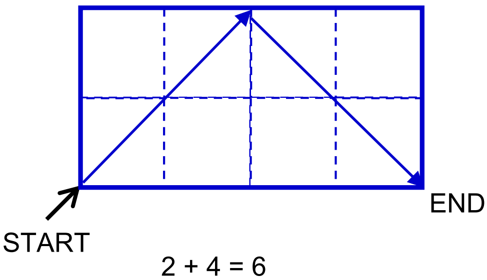


3 + 5 = 8

GCF (3, 5) = 1

The striker touches the sides of the board at 8 ÷ 1 = 8 points, including the start and end points.

Here is the path on a 2-by-4 board.



2 + 4 = 6

GCF (2, 4) = 2

The striker touches the sides of the board at 6 ÷ 2 = 3 points, including the start and end points.

1. Draw game boards with these dimensions.

a) 12 by 6 **b)** 5 by 8 **c)** 6 by 8 **d)** 10 by 15

For each board, how many times does the striker touch the sides of the board in its path from start to end?

2. a) Create a rule that you could use to predict the number of times the striker will touch the sides. The rule should involve the dimensions of the board and their greatest common factor.

b) Use your rule to make a prediction about a 9-by-15 board. Test your prediction by drawing a 9-by-15 board and drawing the path of the striker. Did your rule work?

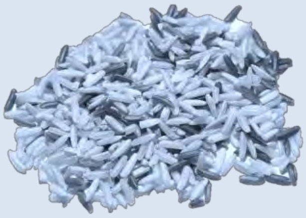
Reprint 2023

Chapter 2 Powers

1.2.1 Introducing Powers

Try This

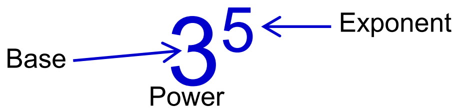
There is a folk tale about a man who asked a king for a reward. He wanted
1 grain of rice on day 1,
2 grains of rice on day 2,
4 grains of rice on day 3,
8 grains of rice on day 4,
and so on, for 64 days.



A. i) How many grains of rice would he get on day 8?
ii) How many grains would he get on day 10?

• Multiplication is a short way to show repeated addition.
For example:
It is quicker to write 5 × 10 than to write 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5.

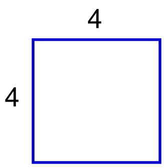
• There is also a short way to show repeated multiplication — using a **power**.
For example:
Instead of writing 3 × 3 × 3 × 3 × 3, you can write it as the power 3^5.



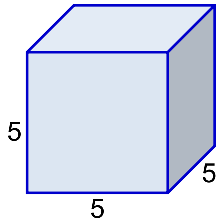
The **exponent** 5 tells you that the **base** 3 is multiplied by itself 5 times.
The expression 3^5 is a power of 3. It is called "the fifth power of 3" and is read aloud as "three to the fifth power" or "three to the fifth".

• There are special exponents, 2 and 3, that relate to squares and cubes.
For example:

The power 4^2, which is read as "four squared", means 4 × 4.
It describes the area of a square with a side length of 4 units.



The power 5^3, which is read as "five cubed", means 5 × 5 × 5.
It describes the volume of a cube with an edge length of 5 units.



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B. Write the values you calculated in **part A** as powers.

Examples

Example 1 Describing Area Using a Power

Each dimension is doubled when a photograph is enlarged. How does the area change?

Solution

Original area is $l \times w$.

Enlarged area is $2l \times 2w$.

$$2l \times 2w = 2 \times l \times 2 \times w$$
$$= 2 \times 2 \times l \times w$$
$$= 2^2 \times l \times w$$
$$= 4 \times l \times w$$

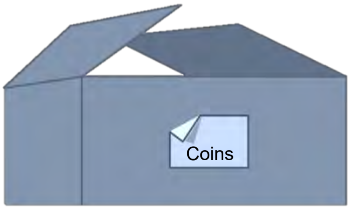
The area is multiplied by 4.

Thinking

- I used $l \times w$ to represent the original area, so the new area was $2l \times 2w$.
- I used a power of 2 to represent 2×2.

Example 2 Using Powers to Solve a Problem

A large box holds 5 containers.
Each container holds 5 small boxes.
Each small box holds 5 cans.
Each can holds 5 coins.



a) Use powers to represent the number of coins in each small box and the large box.

b) How many coins are in the large box?

Solution

a) There are 5 coins in a can, so there are

$5 \times 5 = 5^2$ coins in a small box.

$5 \times 5 \times 5 = 5^3$ coins in a container.

$5 \times 5 \times 5 \times 5 = 5^4$ coins in the large box.

b) $5^4 = 5 \times 5 \times 5 \times 5 = 25 \times 25 = 625$

There are 625 coins in the large box.

Thinking

a) Since there are 5 coins in a can and 5 cans in a small box, there are 5×5 coins in each small box.

- Since there are 5×5 coins in a small box and 5 small boxes in a container, there are $5 \times 5 \times 5$ coins in a container.
- Since there are $5 \times 5 \times 5$ coins in a container and 5 containers in the large box, there are $5 \times 5 \times 5 \times 5$ coins in the large box.

Reprint 2023

Practising and Applying

1. What is the base and exponent of each power?

a) 3^6 **b)** 4^{10} **c)** 1^2 **d)** 0^4

2. Write each power as a repeated multiplication.

a) 7^8 **b)** 9^4

3. Write each as a power.

a) 6 × 6 × 6 × 6 × 6 × 6 × 6

b) 8 × 8 × 8 × 8 × 8 × 8

c) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

4. Draw a picture to show what each power means.

a) 7^2 **b)** 9^3

5. How would you read aloud the two powers in **question 4**?

6. Which is greater in each pair? How much greater?

a) 2^3 or 3^2 **b)** 5^3 or 10^2

c) 2^4 or 4^2 **d)** 4^3 or 3^5

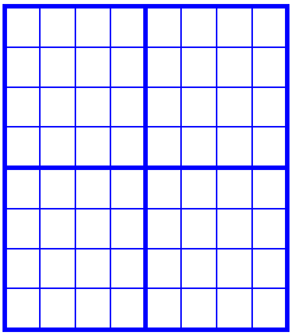
7. How many powers with an exponent of 2 are there between 1 and 100? How do you know?

8. A quilt is made of 4 large squares:

• Each large square has 4 medium sized squares in it.

• Each medium sized square has 4 small squares in it.

How many small squares would it take to make the quilt? Write it as a power.



9. Is the square of a number always less than the cube of the number? Use an example to show how you know.

10. Can 3 × 2 × 2 × 3 be written as a power with an exponent of 4? Explain your thinking.

11. How are exponents related to multiplication?

GAME: Rolling Powers

In this game, you create a power and score points by predicting its value.

You need a die.

Take turns with a partner.

• Roll the die to get the base for your power.

• Predict whether the value of your power will be greater or less than 28.

• Roll the die to get the exponent.

• Calculate the value of your power.

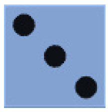
• Score 1 point if you predicted correctly.

The first player to score 15 points wins.

For example:

Roll a 3

$3^?$

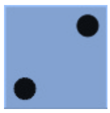


Predict

$3^? < 28$

Roll a 2

3^2



Calculate

$3^2 = 9$

This player gets 1 point.

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1.2.2 Expanded, Standard, and Exponential Forms

Try This

A. When you write 4,680,002 in expanded form, it has four parts:

4 one millions + 6 hundred thousands + 8 ten thousands + 2 ones

Without writing each number in expanded form, tell how many parts there will be. How do you know?

i) 10,002,003 **ii)** 1,300,020,000 **iii)** 200,000,032

You can write a large whole number in different ways other than **standard form**.

• You can use **expanded form** using place value words or numbers.

For example, 1,002,300,040 is 1 one billion + 2 one millions + 3 hundred thousands + 4 tens, or

$1 \times 1{,}000{,}000{,}000 + 2 \times 1{,}000{,}000 + 3 \times 100{,}000 + 4 \times 10$

• You can also use **exponential form**, which is a special type of expanded form that uses powers of 10 instead of words like hundreds, thousands, ten thousands, and so on.

10^1 = ten because 10 = 10

10^2 = hundred because $100 = 10 \times 10$

10^3 = thousand because $1000 = 10 \times 10 \times 10$

10^4 = ten thousand because $10{,}000 = 10 \times 10 \times 10 \times 10$

and so on.

Each **place value** has a value equivalent to a power of 10. As you move left, the exponent increases by 1 since you are multiplying by another 10.

Billions	**Millions**			**Thousands**			**Ones**		
Ones	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**	**Hundreds**	**Tens**	**Ones**
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	1
1	0	0	2	3	0	0	0	4	0

A place value chart can help you write numbers in expanded and exponential forms.

For example, using the place value chart above, you can see that:

1,002,300,040 = 1 one billion + 2 one millions + 3 hundred thousands + 4 tens

$= 1 \times 10^9 + 2 \times 10^6 + 3 \times 10^5 + 4 \times 10^1$

• If a power of 10 is written in standard form, the exponent tells you the number of zeros after the 1.

For example:

$10^9 = 1{,}000{,}000{,}000$ There are 9 zeros after the 1.

$10^5 = 100{,}000$ There are 5 zeros after the 1.

$10^4 = 10{,}000$ There are 4 zeros after the 1.

$10^1 = 10$ There is 1 zero after the 1.

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• Recall that the digits of numbers in standard form are in groups of three called **periods**, separated by commas.

- They are grouped in threes because each period has three place value columns: hundreds (H), tens (T), and ones (O).

- The place value chart below has the full billions period in order to show the number 120,004,000,005:

Billions period			**Millions period**			**Thousands period**			**Ones period**		
H	T	O	H	T	O	H	T	O	H	T	O
10^{11}	10^{10}	10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	1
1	2	0	0	0	4	0	0	0	0	0	5

120,004,000,005 = 1 hundred billion + 2 ten billions + 4 one millions + 5 ones

$= 1 \times 10^{11} + 2 \times 10^{10} + 4 \times 10^6 + 5$

B. i) Write each number in **part A** in exponential form.

ii) What do the exponential forms have in common? How could you have predicted this?

Examples

Example 1 Exponential Form to Standard and Expanded Forms

Write each in expanded form (using place value words), and in standard form.

a) $3 \times 10^7 + 4 \times 10^4 + 3 \times 10^2 + 1 \times 10^1 + 1$

b) $6 \times 10^{10} + 4 \times 10^8 + 5 \times 10^6 + 5 \times 10^4 + 2 \times 10^2 + 4$

Solution 1

Thinking

• I wrote each number in a place value chart to match the powers of 10 with the digits in standard form.

Billions			**Millions**			**Thousands**			**Ones**		
H	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**
10^{11}	10^{10}	10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	1
				3	0	0	4	0	3	1	1
	6	0	4	0	5	0	5	0	2	0	4

a) 30,040,311

3 ten millions + 4 ten thousands + 3 hundreds + 1 ten + 1 one

b) 60,405,050,204

6 ten billions + 4 hundred millions + 5 one millions + 5 ten thousands + 2 hundreds + 4 ones

• The chart reminded me which place value word to use for each non-zero digit when I wrote the expanded form.

b) I could have written 5 millions instead of 5 one millions.

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Example 1 Exponential Form to Standard and Expanded Forms [Continued]	
Solution 2	**Thinking**

Solution 2

a) $3 \times 10^7 + 4 \times 10^4 + 3 \times 10^2 + 1 \times 10^1 + 1$

10^7 = 10,000,000, or ten million
10^4 = 10,000, or ten thousand
10^2 = 100, or hundred
10^1 = 10, or ten

Expanded form

3 ten millions + 4 ten thousands + 3 hundreds + 1 ten + 1 one

Standard form 30,040,311

b) $6 \times 10^{10} + 4 \times 10^8 + 5 \times 10^6 + 5 \times 10^4 + 2 \times 10^2 + 4$

10^{10} = 10,000,000,000, or ten billion
10^8 = 100,000,000, or one hundred million
10^6 = 1,000,000, or one million
10^4 = 10,000, or ten thousand
10^2 = 100, or hundred

Expanded form

6 ten billions + 4 hundred millions + 5 one millions + 5 ten thousands + 2 hundreds + 4 ones

Standard form 60,405,050,204

Thinking

- I wrote each power of 10 in standard form. I knew that when the base was 10, the exponent told how many zeros were after the 1.
- I used the standard form of each power of 10 to write it in place value language. Then I wrote the expanded form.
- I used the expanded form to help me write the standard form.

Practising and Applying

1. Write each in exponential form.

a) 34,000,200

b) 3,004,502

c) 620,350,000

d) 118,000,342

e) 22,304,205,032

2. Write each in standard form and in expanded form.

a) $4 \times 10^9 + 5 \times 10^7 + 6 \times 10^3$

b) $3 \times 10^{10} + 5 \times 10^6 + 6 \times 10^2 + 3 \times 10^1 + 6$

c) $7 \times 10^8 + 4 \times 10^5 + 4 \times 10^3 + 2 \times 10^2 + 9$

d) $5 \times 10^{11} + 6 \times 10^9 + 8 \times 10^8 + 8 \times 10^5 + 2 \times 10^3 + 3 \times 10^2 + 6$

3. How are 3×10^5 and 3×10^8 alike? How are they different?

4. The exponential form of a number has 4×10^8 as the first part.

a) How many digits does the standard form of the number have? How do you know?

b) Tell two other things you know about the number.

5. When a number is in exponential form, which power of 10 tells you the most about the size of the number? Explain your thinking.

6. When 10^5 is written in standard form, how many zeros does it have after the digit 1? Why does that make sense?

7. Why does each column in a place value chart represent a power of 10?

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Chapter 3 Decimal Operations

1.3.1 Multiplying Decimals

Try This

A rectangular plot of land measures 42.3 m by 26.2 m.

A. i) How do you know the plot's area is about 1200 m^2?

ii) Draw a picture to show that the area is made up of these four parts:

- 42 m by 26 m
- 42 m by 0.2 m
- 0.3 m by 26 m
- 0.3 m by 0.2 m

iii) What is the exact area of the plot of land?

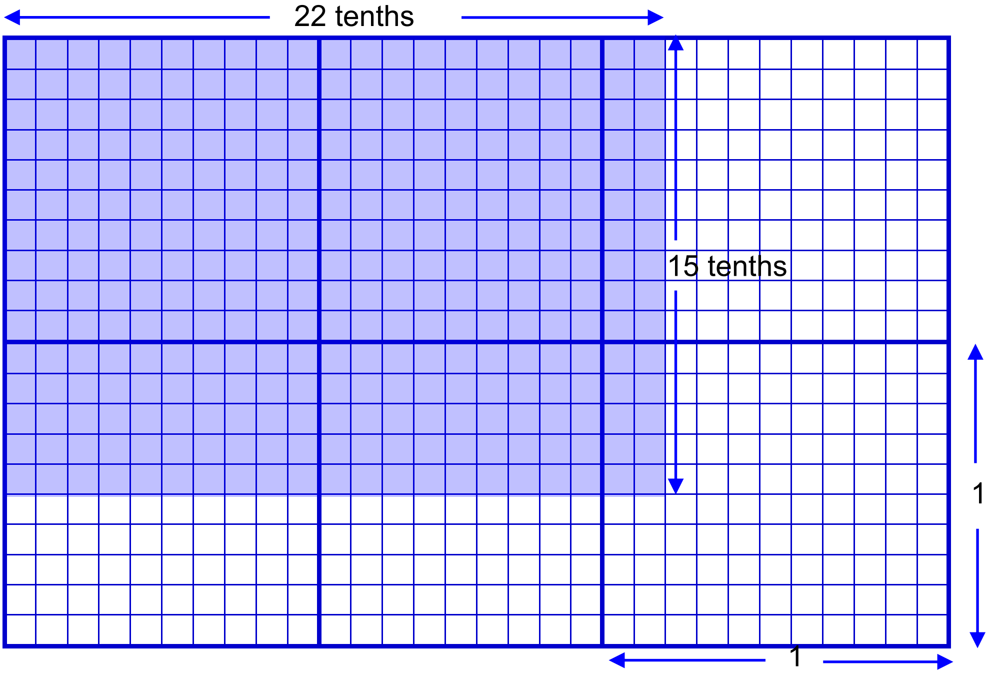
• To multiply two decimal tenths, you can **rename** each decimal using place value and then multiply whole numbers.

For example, 2.2 × 1.5 = 22 tenths × 15 tenths = 330 hundredths, which is 3.30.

The grid model below explains why tenths × tenths = hundredths.

- Each large square represents 1 and has 10 × 10 = 100 small squares. That means each small square is 1 hundredth or 0.01.

- Multiplying 2.2 × 1.5 is like finding the area of a rectangle that is 22 tenths by 15 tenths.



- There are 22 × 15 = 330 small squares in the shaded rectangle.

- Each small square is 1 hundredth, or 0.01, so 330 small squares are 330 hundredths, or 3.30.

- So, 2.2 × 1.5 = 3.30. This makes sense since 2.2 × 1.5 is about 2 × 1.5 = 3.

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• When you multiply decimals, you can figure out the number of decimal places in the **product** by counting the total number of decimal places in the factors.

For example:

For 2.2 × 1.5, since there is a total of 2 decimal places in the two factors, there must be 2 decimal places in the product, so a product of 3.30 makes sense.

• To multiply decimal hundredths, you can rename each decimal using place value and then multiply whole numbers.

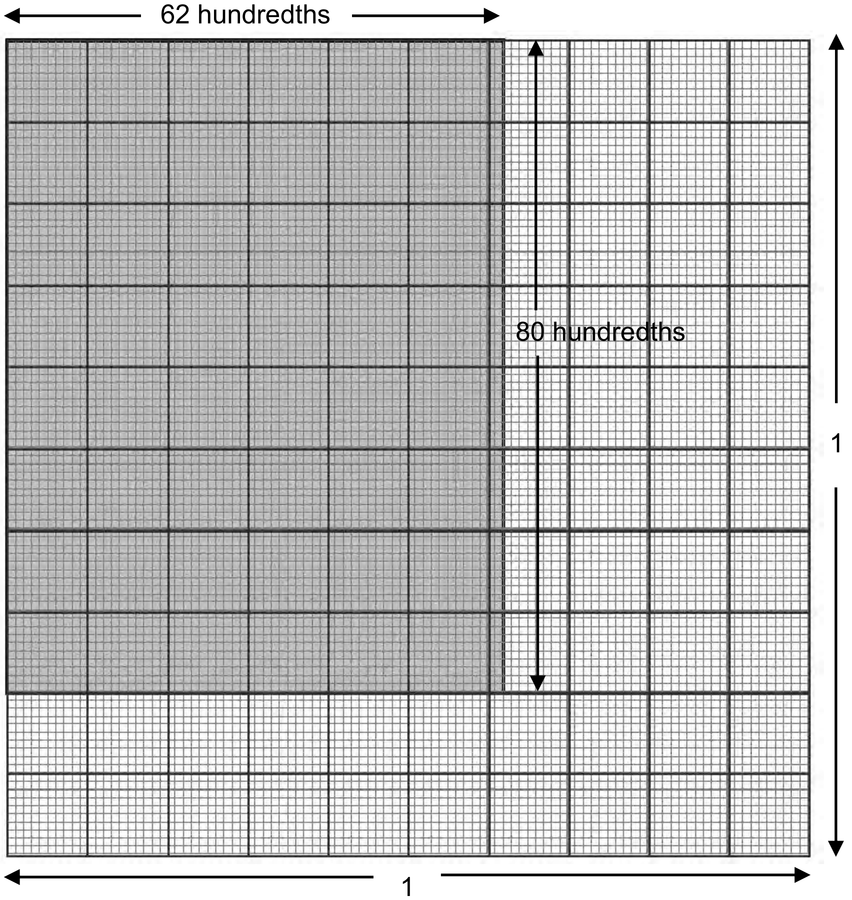
For example:

0.80 × 0.62 = 80 hundredths × 62 hundredths = 4960 ten thousandths, or 0.4960

The grid model below explains why hundredths × hundredths = ten thousandths.

- The whole grid represents 1 and has 100 × 100 = 10,000 tiny squares. That means each tiny square is 1 ten thousandth or 0.0001.

- Multiplying 0.80 × 0.62 is like finding the area of a rectangle that is 80 hundredths by 62 hundredths.



- There are 80 × 62 = 4960 tiny squares in the shaded rectangle.

- Each tiny square is 1 ten thousandth, or 0.0001 so 4960 tiny squares are 4960 ten thousandths, or 0.4960.

- So 0.80 × 0.62 = 0.4960. This makes sense since there are 4 decimal places altogether in the two factors and the product also has 4 decimal places.

B. How could you have predicted that the area in **part A iii)** would have 2 decimal places?

Examples

Example 1 Multiplying Decimals Using Place Value

Use 13 × 200 = 2600 to find each product.

a) 1.3 × 200 **b)** 1.3 × 20.0 **c)** 0.13 × 200 **d)** 0.13 × 0.200

Solution

13 × 200 = 2600

a) 1.3 × 200 = 13 tenths × 200
= 2600 tenths
= 260.0

b) 1.3 × 20.0 = 13 tenths × 200 tenths
= 2600 hundredths
= 26.00

c) 0.13 × 200 = 13 hundredths × 200
= 2600 hundredths
= 26.00

d) 0.13 × 0.200
= 13 hundredths × 200 hundredths
= 2600 ten thousandths
= 0.0260

Thinking

a) When you multiply tenths by a whole number, the product has 1 decimal place.

b) When you multiply tenths by tenths, the product has 2 decimal places.

c) When you multiply hundredths by a whole number, the product has 2 decimal places.

d) When you multiply hundredths by hundredths, the product has 4 decimal places.

Example 2 Counting Decimal Places in Products

Tell the number of decimal places there will be in each product.
Use 14 × 512 = 7168 to calculate each.

a) 1.4 × 5.12 **b)** 0.14 × 5.12 **c)** 0.14 × 51.2 **d)** 0.14 × 0.512

Solution

a) There will be 3 decimal places, so 1.4 × 5.12 = 7.168.
1 × 5 = 5, so 7.168 is right.

b) There will be 4 decimal places, so 0.14 × 5.12 = 0.7168.
0.1 × 5 = 0.5, so 0.7168 is right.

c) There will be 3 decimal places, so 0.14 × 51.2 = 7.168.
0.1 × 50 = 5.0, so 7.168 is right.

d) There will be 5 decimal places, so 0.14 × 0.512 = 0.07168.
0.1 × 0.5 = 0.05, so 0.07168 is right.

Thinking

- I knew the number of decimal places in the product was equal to the total number of decimal places in the factors.
- I knew each product had the digits 7168, so all I did was count the correct number of decimal places starting at the right of 7168.
- I checked each answer by estimating.

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Practising and Applying

1. Use 78 × 4156 = 324,168 to calculate each product.

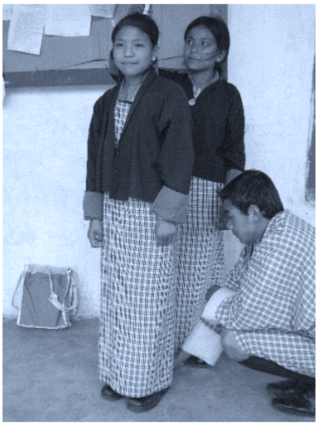
a) 78 × 41.56 **b)** 7.8 × 41.56

c) 0.78 × 4.156 **d)** 0.078 × 41.56

2. Use estimation to show that any two answers in **question 1** make sense.

3. A straight road is 1.2 km long and 0.004 km wide. What area does it cover?

4. A 12-year-old girl is about 1.5 m tall. When she becomes an adult, she will be 1.07 times as tall as she is now. Estimate her adult height.



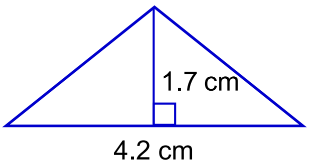
5. Tshering says 4.5 × 8.12 has three decimal places because

4.5 × 8.12 = 45 × 0.1 × 812 × 0.01
= 45 × 812 × 0.1 × 0.01
= 45 × 812 × 0.001

Do you agree? Explain his thinking.

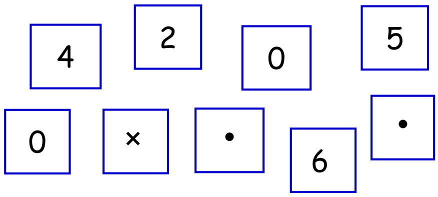
6. Use two different methods to show that 1.5 × 4.048 = 6.072.

7. What is the area of this triangle? ($A = bh \div 2$)



8. Give two examples of pairs of decimals you would multiply using mental math. Describe what strategy you would use for each.

9. You can rearrange the cards below to create a multiplication.



How would you arrange these cards to create each?

a) a product with 3 decimal places

b) a product with 6 decimal places

c) a product of about 200

d) a product of about 0.3

10. Tashi rode a bicycle at an average speed of 14.8 km/h for 2.25 h. About how far did he go? Explain how you estimated.

11. Tenzin says that the rule for the number of decimal places in the product does not always work because 0.5 × 2.0 = 1. Do you agree with Tenzin? Explain your thinking.

12. Why might someone say that you only need to know how to multiply whole numbers to be able to multiply decimals?

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1.3.2 Dividing Decimals

Try This

A takin's body length including the tail is about 1.7 m. The tail is about 0.15 m long.

A. Estimate how many times longer the body is than the tail. Explain how you estimated.

• To divide decimals, you can rename each decimal using the same place value and then divide whole numbers.

For example:

  0.4 ÷ 0.2
= 4 tenths ÷ 2 tenths
= 4 ÷ 2
= 2

0.4 ÷ 0.2 means,
"How many 2 tenths are in 4 tenths?"
This is the same as,
"How many 2s are in 4?"

• Sometimes you have to find an **equivalent decimal** before you can rename the decimals using the same place value.

For example:

  0.4 ÷ 0.02
= 0.40 ÷ 0.02
= 40 hundredths ÷ 2 hundredths
= 40 ÷ 2
= 20

0.40 ÷ 0.02 means,
"How many 2 hundredths are in 40 hundredths?"
This is the same as,
"How many 2s are in 40?"

• Using the long or short division format, the divisions above would look like this:

$0.2\overline{)0.4} \rightarrow 2\overline{)4}$ $0.02\overline{)0.4} \rightarrow 0.02\overline{)0.40} \rightarrow 2\overline{)40}$

You move the digits of both the **dividend** and the **divisor** the same number of places. It appears as if the decimal point is moving to the right the same number of places.

For $0.2\overline{)0.4}$, the digits move one place: $0.2\overline{)0.4} \rightarrow 2.\overline{)4.} = 2\overline{)4}$

For $0.02\overline{)0.4}$, the digits move two places, which requires adding an extra 0 in the hundredths place.

$$0.02\overline{)0.4} \rightarrow 0.02\overline{)0.40} \rightarrow 2.\overline{)40.} = 2\overline{)40}$$

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• For $0.02\overline{)0.4}$, the quotient is a whole number, 20. But the quotient of a decimal division is often not a whole number. If you rename the dividend by adding zeros, you can continue dividing to get a decimal quotient.

For example:

0.8 ÷ 0.3 means, "How many 3 tenths are in 8 tenths?" The answer will be a decimal between 2 (6 ÷ 3 = 2) and 3 (9 ÷ 3 = 3).

- Rename the divisor and the dividend so you are dividing whole numbers.
- Write an equivalent decimal for the dividend by adding zeros.
Add 2 zeros if you want to round the quotient to the nearest tenth.
- Use your usual division **algorithm** and then round to the nearest tenth.

$0.3\overline{)0.8} \rightarrow 3\overline{)8} \rightarrow 3\overline{)8.00} \rightarrow$

$$\begin{array}{r} 2.66 \\ 3\overline{)8.00} \\ -6 \\ 2\,0 \\ -1\,8 \\ 20 \\ -18 \\ 2 \end{array}$$

2.66 rounded to the nearest tenth (1 decimal place) is 2.7, so

0.8 ÷ 0.3 = 2.7

B. Calculate an answer for **part A** to the nearest thousandth.

Examples

Example 1 Comparing Decimal Quotients

Order these quotients from least to greatest.

A. 0.15 ÷ 0.2 **B.** 15 ÷ 0.02 **C.** 1.5 ÷ 2 **D.** 0.015 ÷ 0.2

Solution

Rename each with the same divisor

A. 0.15 ÷ 0.2 = 1.5 ÷ 2

B. 15 ÷ 0.02 = 1500 ÷ 2

C. 1.5 ÷ 2 = 1.5 ÷ 2

D. 0.015 ÷ 0.2 = 0.15 ÷ 2

Compare the dividends

• B is greatest because it has the greatest dividend.

• D is least because it has the least dividend.

• A and C have the same value and are between B and D.

In order from least to greatest:
D, A and C, then B

Thinking

• I wrote an equivalent division for each with 2 as the divisor. I did this by moving the digits to the left the same number of places for both the dividend and divisor.

• Once the divisions all had the same divisor, I just compared the dividends.

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Example 2 Dividing to Get a more Exact Answer

Calculate 6.2 ÷ 0.07 to the nearest hundredth.

Solution

$0.07\overline{)6.2} \rightarrow 7\overline{)620} \rightarrow$

```
      88.571
  7 ) 620.000
    - 56
       60
     - 56
        40
      - 35
         50
       - 49
          10
        -  7
           3
```

6.2 ÷ 0.07 = 88.57

Thinking

- I renamed the dividend and divisor so I could divide whole numbers.
- I knew 620 was not divisible by 7, so I added zeros to the dividend to get a decimal quotient.
- I added 3 zeros because I wanted to round the quotient to 2 decimal places (the nearest hundredth).

Practising and Applying

1. Use 500 ÷ 0.16 = 3125 to calculate each quotient.

a) 500 ÷ 1.6 **b)** 50 ÷ 0.16

c) 50 ÷ 0.016 **d)** 5 ÷ 0.16

2. Show that any two of the answers in **question 1** make sense by estimating.

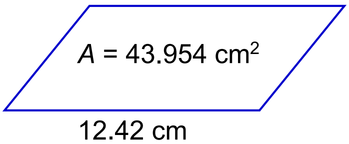
3. Calculate to the nearest hundredth.

a) 7.82 ÷ 0.8

b) 312.4 ÷ 0.05

c) 135.07 ÷ 0.9

4. What is the parallelogram's height, to the nearest tenth of a centimetre? (*A* = *bh*)



5. Is each statement below possible when you divide tenths by tenths, for example, 0.3 ÷ 0.6? For each, use an example to help you explain.

a) The quotient is a whole number.

b) The quotient is a tenth.

c) The quotient is a thousandth.

6. How fast is each train traveling (km/h)? Round to the nearest whole number.

a) Train A travels 520.4 km in 4.8 h

b) Train B travels 72.1 km in 0.8 h

7. A snail moves about 0.013 m each second. About how many hours would it take the snail to travel 174 km? Show your work.

8. You are dividing 4.2 by 0.■, where ■ is a digit from 1 to 9.

a) What is the least possible quotient?

b) What is the greatest quotient?

9. Lhakpa made a division error:

4.12 ÷ 0.03 = 137 R 1

a) What is his error?

b) Show how to calculate it correctly.

10. Without dividing, explain why these divisions have the same quotient.

0.8 ÷ 0.4 = 0.08 ÷ 0.04

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1.3.3 EXPLORE: Mental Math with Decimals

Some calculations with decimals are easy to do mentally.

• For example, 4.02 ÷ 0.1:

Since there are 10 tenths in each 1, finding the number of 0.1s in 4.02 is the same as multiplying 4.02 by 10.

4.02 ÷ 0.1 = 4.02 × 10 = 40.2

So 4.02 ÷ 0.1 = 4.02 × 10 = 40.2.

• Another example is 4.67 + 1.11:

You can add 1.11 in parts to 4.67 using mental math.

1.11 = 1 + 0.1 + 0.01, so 4.67 + 1.11 = 4.67 + 1 + 0.1 + 0.01:

4.67 + 1 = 5.67 → 5.67 + 0.1 = 5.77 → 5.77 + 0.01 = 5.78

So 4.67 + 1.11 = 5.78.

A. Suppose you were to mentally add a decimal thousandth to 3.099.
List three values that would be easy to add. Explain how you would add each.

3.099 + 0.■ ■ ■

B. Suppose you were to mentally subtract a decimal thousandth from 4.1.
List three values that would be easy to subtract. Explain how you would subtract each.

4.1 – 0.■ ■ ■

C. Suppose you were to mentally multiply 2.48 by a decimal.
List three numbers that would be easy to multiply, other than 0.1, 0.01, or 0.001. Explain how you would multiply each.

2.48 × 0.■ or 2.48 × 0.■ ■

D. Suppose you were to mentally divide 4.2 by a decimal.
List three numbers that would be easy to divide, other than 0.1, 0.01, or 0.001. Explain how you would divide each.

4.2 ÷ 0.■ or 4.2 ÷ 0.■ ■

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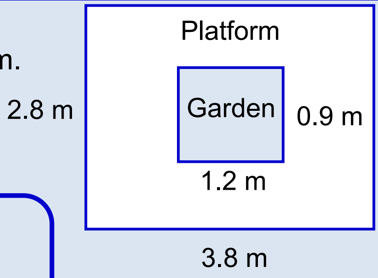
1.3.4 Order of Operations

Try This

Yeshi is building a wooden platform to go around a garden. He has a piece of wood that is 3.8 m by 2.8 m. He plans to cut a hole that is 0.9 m by 1.2 m for the garden. He wants to paint the platform so he needs to know its area.



A. Calculate the area of the wooden platform.

B. Yeshi said, "I need to find 3.8 × 2.8 – 1.2 × 0.9, so I will calculate 3.8 × 1.6 × 0.9." Is he right? Explain your thinking.

• It is sometimes hard to understand expressions that have many calculations and different operations.

For example: 3.5 + 6.8 + 2 × 6.4

- One person might choose to add 3.5 + 6.8 + 2 and then multiply the sum by 6.4. The answer would be 78.72.
- Someone else might first add 3.5 + 6.8 and then add the sum to the product of 2 × 6.4. The answer would be 23.1.

It is confusing to have more than one answer for the same calculation.

• To get rid of the confusion, people have agreed on rules for calculating. They are called the **order of operations** rules:

Step 1 Calculate anything inside **B**rackets first.

Step 2 Apply the **E**xponents next.

Step 3 **D**ivide and **M**ultiply numbers next to each other, in order from left to right.

Step 4 **A**dd and **S**ubtract numbers next to each other, in order from left to right.

For example: $(5 + 3.1)^2 + 7 \div 0.1 \times 1.5^2 - 3.2$

Step 1	$(\mathbf{8.1})^2 + 7 \div 0.1 \times 1.5^2 - 3.2$	[**B**rackets]
Step 2	$\mathbf{65.61} + 7 \div 0.1 \times \mathbf{2.25} - 3.2$	[**E**xponents]
Step 3	$65.61 + \mathbf{70} \times 2.25 - 3.2$	[**D**ivide and **M**ultiply left to right]
	$65.61 + \mathbf{157.5} - 3.2$	
Step 4	$\mathbf{223.11} - 3.2$	[**A**dd and **S**ubtract left to right]
	219.91	

C. i) Describe the error in Yeshi's thinking in **part B**.

ii) Yeshi said that, if the expression included brackets, **(**3.8 × 2.8**)** – **(**1.2 × 0.9**)**, he would have calculated correctly. What would you say to him?

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Examples

Example Writing an Expression Using Order of Operations

You want someone to multiply 4.1 by 5, cube it, subtract 2, and then divide by 4. How should you write the expression?

Solution	Thinking
$(4.1 \times 5)^3$	• I put 4.1 × 5 in brackets so it would be multiplied before it was cubed. (The brackets tell that the whole expression should be cubed, and not just the 5.) 
$[(4.1 \times 5)^3 - 2] \div 4$	• I put the $(4.1 \times 5)^3 - 2$ inside different square brackets so it would be calculated before being divided by 4. (Without the brackets, you would divide 2 by 4 to get 0.5 and then subtract 0.5 from $(4.1 \times 5)^3$.) • Because I had brackets inside brackets, I used square brackets for the outside and round brackets for the inside. (If I had used round brackets for both, $((4.1 \times 5)^3 - 2) \div 4$, it would have been hard to tell which pairs of brackets belonged together.)

Practising and Applying

1. Calculate using the order of operations rules.

a) $(2.5 \times 2.0) - 4.3$

b) $27.12 - (4.8^2 - 2 \times 2)$

c) $5.9 + 4.1 \times 3 - 2.5 \times 4$

2. Sometimes brackets are not necessary. For which questions below are the brackets unnecessary?

A. $(2.5 \times 2.0) - 4.3$

B. $(7.12 - 4.8^2) - 2 \times 2$

C. $5.9 + 4.1 \times (3 - 2.5) \times 4$

3. Tell whether each calculation is correct. If it is not, then correct the answer.

a) $12 + 0.8 \times 3 = 38.4$

b) $5.6^2 + 5 - 4.2 \div 2 = 16.08$

c) $4.2 \times 7 - 10.4 \div 2 + 2.9 \times 3 = 32.9$

d) $4 \times 1.5^2 \div 8 + 9 \times 2.1 = 20.025$

4. Write each as a calculation.

a) Divide 8 by 0.1, add 12, multiply by 3, and then subtract 2.

b) Add 4.2 and 3.5, multiply the sum by 3, square the product, and subtract 4.

c) Multiply 6.2 by 2, add 5.6 to the product, square it, add 3, and then divide by 2.

5. Describe the error in each calculation. Then calculate correctly.

a) $1.8 - 0.2^3 + 6.4 = 10.496$

b) $8.1 + 9.3 \times 4 - 5.6 \times 2 = 128$

c) $30 - 4.2 \div 0.2 + 8 \div 0.4 = 42.5$

6. Explain why it is important to have rules for the order of operations. Provide an example to support your explanation.

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UNIT 1 Revision

1. a) Use base ten models to model 522 as 5 hundreds + 2 tens + 2 ones.

b) Use the model to show why 522 is divisible by 9.

2. Use divisibility tests to see if each number is divisible by 2, 3, 4, 5, 9, or 10.

a) 40,240 **b)** 32,157

c) 8325 **d)** 8236

3. What are the remainders when you divide each by 3? divide each by 4?

a) 794 **b)** 8021

c) 1738 **d)** 20,047

4. Use the divisibility tests to see if 1485 is divisible by 15. Show your work.

5. List all the possible missing digits that will make each true.

a) 41■2 is divisible by 3.

b) 20,■03 is divisible by 9.

c) 89,■12 is divisible by 4.

6. Calculate.

a) LCM (28, 48)

b) LCM (30, 45)

c) LCM (3, 7, 10)

7. Which three values, other than 15, will make this true?

LCM (15, 35) = LCM (■, 35)

8. The LCM of two numbers is 90.

a) Could one of the numbers be 4? How do you know?

b) Could one number be a multiple of the other? How do you know?

c) List two possible pairs of numbers.

9. Calculate.

a) GCF (20, 30) **b)** GCF (15, 35)

c) GCF (25, 35, 40)

10. Two numbers have a GCF of 30 and an LCM of 600. List two possible pairs of numbers.

11. a) How many ways can you arrange 64 squares in equal rows? Show your work.

b) How many ways can you arrange 36 squares in equal rows? Show your work.

c) Which arrangements in **part a)** and **part b)** have the same number of rows? How could you have predicted that?

12. Write each power as a repeated multiplication.

a) 4^8 **b)** 11^3

13. Write each as a power.

a) 9 × 9 × 9 × 9 × 9 × 9

b) 3 × 3 × 3 × 3 × 3 ×3 × 3 × 3

14. Tell why 2^{10} is twice as much as 2^9.

15. Suppose 3 people each called 3 friends on the phone.

Then, each of the friends called 3 other friends.

Then, each of those friends called 3 other friends.

Then, each of those friends called 3 other friends.

Write a calculation using powers that you could use to figure out to how many people were called altogether.

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16. How could you predict that 5^{30}, when written in standard form, will have the digit 5 in the ones place?

17. Write each in exponential form.

a) 3,120,003

b) 3,120,003,400

18. Write each in standard form and in expanded form.

a) $3 \times 10^8 + 2 \times 10^4 + 3 \times 10^2 + 8$

b) $6 \times 10^{11} + 7 \times 10^7 + 3 \times 10^3 + 2 \times 10^2 + 5$

19. Use 352 × 108 = 38,016 to calculate each product.

a) 35.2 × 10.8

b) 3.52 × 1.08

c) 3.52 × 0.108

d) 0.352 × 10.8

20. The product of two decimal numbers has 5 decimal places. What number of decimal places could have been in the two factors? How do you know?

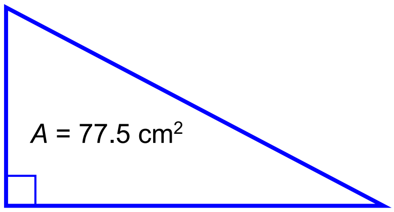
21. Use all of the digits 1, 2, 3, 4, and 5 to make this true.

```
   ■.■
 × ■.■
 ■.1 6
```

22. The area of a triangle is 77.5 cm^2. Both the base and height are each greater than 6 cm.

a) List a possible pair of dimensions for the base and height.

b) Find another possible pair of dimensions.



23. Explain how you know that 32.5 ÷ 0.5 = 325 ÷ 5.

24. Calculate.

a) 42.5 ÷ 0.05 **b)** 12.3 ÷ 0.2

25. Calculate to the nearest hundredth.

a) 5.1 ÷ 0.7 **b)** 0.80 ÷ 0.6

26. a) List three decimal multiplications that are easy to do mentally.

b) Explain how you would do each.

27. Calculate each.

a) $4.2 - 1.5^2 + 0.3 \times 0.7$

b) $(3.2 - 1.4) \times 7 + 6 \div 0.1$

c) $5.2 \times 1.5 + 3 \times (2.5 - 1)$

28. Tell whether the brackets are necessary for each. How do you know?

a) $(5.2 + 3.6) - 4.1 \times 3.4$

b) $4.5 \times (3.6 \times 0.1) \div 2$

c) $(3.2 + 5)^3 \times 4.9$

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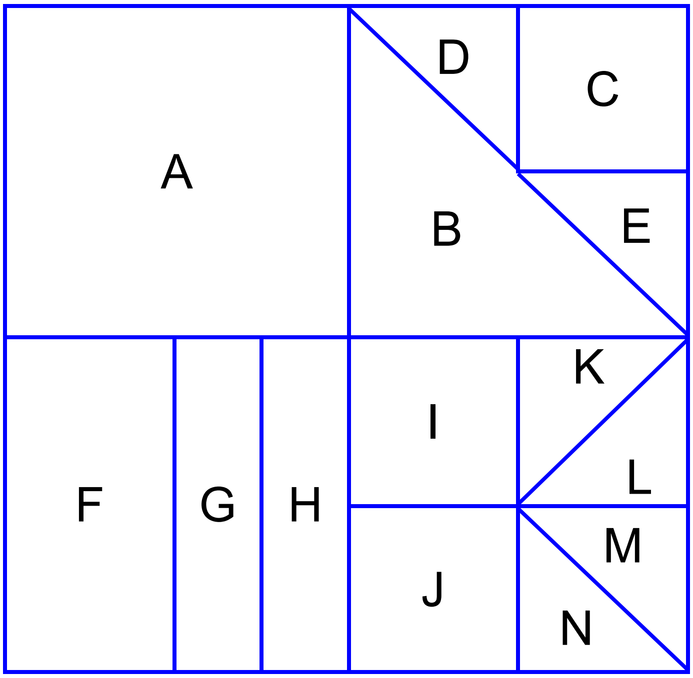
UNIT 2 FRACTIONS

Getting Started

Use What You Know

In this fraction puzzle, you can compare pieces of the puzzle to the whole puzzle and you can compare the puzzle pieces to each other.



A. i) What fraction of the whole puzzle is piece A?

ii) What fraction of piece A is piece B?

iii) What fraction of the whole puzzle is piece B?

iv) What fraction of piece A is piece F?

v) How can pieces F and B be the same fraction of the puzzle even though they are different shapes?

B. i) What fraction of the puzzle does each piece, A to N, represent?

ii) Which pieces are the same fraction of the puzzle?

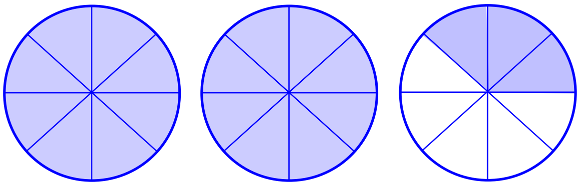
C. Order the fractions in **part B i)** from least to greatest.

D. Create your own fraction puzzle for a partner to solve. Include four or more puzzle pieces. Be sure there is a way to determine what fraction of the puzzle each piece is.

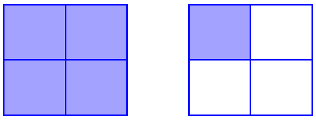
Skills You Will Need

1. Write an improper fraction and a mixed number for each picture.

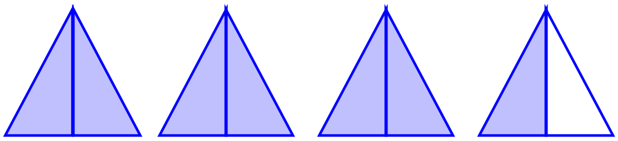
a)



b)



c)



2. Draw a picture to represent each.

a) $1\frac{3}{4}$ **b)** $\frac{7}{2}$ **c)** $1\frac{5}{8}$ **d)** $\frac{8}{3}$

3. Write each as an improper fraction.

a) $3\frac{2}{5}$ **b)** $6\frac{1}{2}$ **c)** $4\frac{1}{3}$ **d)** $2\frac{3}{8}$

4. Which of these numbers are between 3 and 4?

A. $\frac{11}{3}$ **B.** $\frac{17}{4}$ **C.** $\frac{7}{2}$ **D.** $2\frac{1}{8}$

5. Complete each with >, <, or =.

a) $\frac{3}{8}$ ■ $\frac{6}{11}$ **b)** $\frac{3}{4}$ ■ $\frac{9}{12}$ **c)** $\frac{12}{5}$ ■ $\frac{12}{7}$

6. Each day Bhagi runs for $\frac{1}{2}$ h before school and $\frac{3}{4}$ h after school. How many hours does Bhagi run each day?

7. Samten is studying for a math test. He studied $\frac{1}{2}$ of his notes on Friday and $\frac{1}{4}$ of his notes on Saturday. How much of his notes does he have left to study?

8. Write a decimal for each.

a) $\frac{3}{10}$ **b)** $\frac{27}{100}$ **c)** $\frac{1}{2}$ **d)** $\frac{3}{5}$

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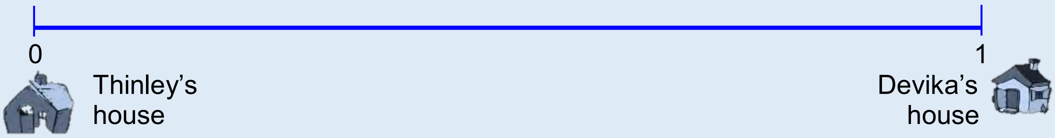
Chapter 1 Fraction Addition and Subtraction

2.1.1 Comparing and Ordering Fractions

Try This

Thinley's house is located at 0 on the number line. Devika's house is at 1.



A. Copy the number line. Mark the location of each house below.

i) Jigme's house is halfway between Thinley's house and Devika's house.

ii) Sithar's house is halfway between Thinley's house and Jigme's house.

iii) Kachap's house is halfway between Jigme's house and Devika's house.

B. i) Label each location in **part A** with a fraction.

ii) What fraction would describe the location of a house halfway between Kachap's house and Devika's house?

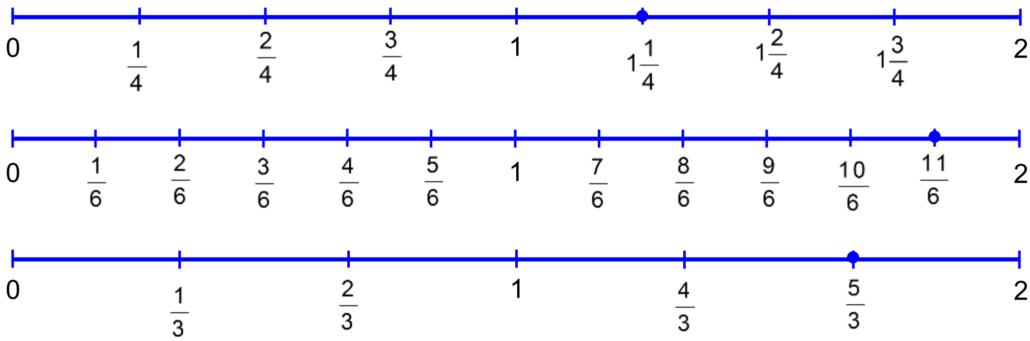
iii) What fraction would describe the location of a house halfway between Thinley's house and Sithar's house?

There are different ways to compare and order **fractions** and **mixed numbers**.

• You can use number lines.

For example, to order $1\frac{1}{4}$, $\frac{11}{6}$, and $\frac{5}{3}$:

Use number lines of the same length marked in fourths, sixths, and thirds.



The order from least to greatest is $1\frac{1}{4}$, $\frac{5}{3}$, $\frac{11}{6}$, since $1\frac{1}{4}$ is farthest left and $\frac{11}{6}$ is farthest right.

• You can also compare and order fractions using equivalent fractions with the same denominator.

For example, to order $1\frac{1}{4}$, $\frac{11}{6}$, and $\frac{5}{3}$:

- First write $1\frac{1}{4}$ as an **improper fraction**: $1\frac{1}{4} = \frac{5}{4}$

- Find a **common denominator** for fourths, sixths, and thirds by looking for a common multiple (other than zero) of 4, 6, and 3:

Multiples of 4: 4, 8, **12**

Multiples of 6: 6, **12**

Multiples of 3: 3, 6, 9, **12**

12 is a common multiple of 3, 4, and 6, so 12 is a common denominator.

- Find equivalent fraction twelfths: $\frac{5}{4} = \frac{15}{12}$ (× 3), $\frac{11}{6} = \frac{22}{12}$ (× 2), $\frac{5}{3} = \frac{20}{12}$ (× 4)

- Order from least to greatest: $\frac{15}{12} < \frac{20}{12} < \frac{22}{12}$

So $1\frac{1}{4} < \frac{5}{3} < \frac{11}{6}$

When denominators are equal, a fraction with a greater numerator is greater than a fraction with a lower numerator.

• Sometimes you can compare and order fractions using a **common numerator**.

For example, to order $1\frac{1}{4}$, $\frac{5}{4}$, $\frac{11}{6}$, and $\frac{5}{3}$:

- Find a common multiple of 5 and 11:

Multiples of 5: 5, 10, 15, 20, 25, ..., 50, **55**

Multiples of 11: 11, 22, 33, 44, **55**

55 is a common multiple of 5 and 11, so 55 can be a common numerator.

- Find equivalent fractions with 55 as a numerator:

$\frac{5}{4} = \frac{55}{44}$ $\frac{11}{6} = \frac{55}{30}$ $\frac{5}{3} = \frac{55}{33}$

- Order from least to greatest: $\frac{55}{44} < \frac{55}{33} < \frac{55}{30}$

So $1\frac{1}{4} < \frac{5}{3} < \frac{11}{6}$

When numerators are equal, a fraction with a greater denominator is less than a fraction with a lower denominator.

C. i) Use a common denominator to show that $\frac{3}{4}$ (Kachap's house) is between $\frac{1}{2}$ (Jigme's house) and 1 (Devika's house).

ii) Repeat **part i)** using a common numerator.

Examples

Example 1 Using a Common Denominator to Compare Fractions

Kamala has $\frac{7}{2}$ cups of rice. Does she have enough to make a meal that needs $3\frac{2}{3}$ cups of rice? Show your work.

Solution

Comparing $\frac{7}{2}$ and $3\frac{2}{3}$:

$3\frac{2}{3} = \frac{11}{3}$

$\frac{7}{2} = \frac{21}{6}$ $\qquad$ $\frac{11}{3} = \frac{22}{6}$

$\frac{21}{6} < \frac{22}{6}$, so $\frac{7}{2} < 3\frac{2}{3}$

Kamala does not have enough rice.

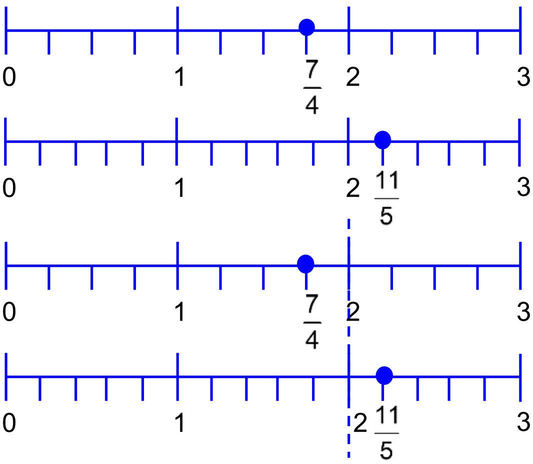
Thinking

- I needed to figure out whether $\frac{7}{2}$ was equal to or more than $3\frac{2}{3}$.
- I wrote $3\frac{2}{3}$ as an improper fraction.
- I found a common denominator for halves and thirds by finding a common multiple of 2 and 3, which was 6.
- Then, I wrote an equivalent fraction for each using a common denominator, which was sixths.

Example 2 Comparing Fractions Using Number Lines

Compare $\frac{11}{5}$ and $\frac{7}{4}$.

Solution



$\frac{7}{4} < \frac{11}{5}$

Thinking

- I drew a number line for fourths and marked $\frac{7}{4}$.
- I drew a number line of the same length but for fifths and marked $\frac{11}{5}$.
- I used the number 2 on both number lines to help me compare.
- I could see $\frac{7}{4}$ was less than 2, and $\frac{11}{5}$ was greater than 2, so I knew $\frac{7}{4}$ was less than $\frac{11}{5}$.

Example 3 Using a Common Numerator to Compare Fractions

In Bal's class, $\frac{8}{15}$ of the students are boys.

In Kuenga's class, $\frac{16}{25}$ of the students are boys.

Which class has the greater fraction of boys?

Solution

Compare $\frac{8}{15}$ and $\frac{16}{25}$:

$$\frac{8}{15} = \frac{16}{30}$$

$$\frac{16}{30} < \frac{16}{25}$$

Kuenga's class has a greater fraction of boys.

Thinking

- I used a common numerator to compare the fractions because the numerator of $\frac{16}{25}$ was 2 times the numerator of $\frac{8}{15}$.
- I renamed $\frac{8}{15}$ as an equivalent fraction with a numerator of 16 by multiplying the numerator and denominator by 2.
- I knew $\frac{1}{30} < \frac{1}{25}$, so $\frac{16}{30} < \frac{16}{25}$.

Practising and Applying

1. Complete each with >, <, or =.

a) $\frac{29}{5}$ ■ $6\frac{3}{10}$

b) $4\frac{1}{2}$ ■ $\frac{18}{4}$

c) $1\frac{5}{6}$ ■ $\frac{11}{5}$

d) $3\frac{2}{3}$ ■ $3\frac{6}{9}$

2. Order from least to greatest. For each, describe the method you used to order the fractions.

a) $1\frac{3}{4}$, $\frac{7}{3}$, $\frac{7}{6}$

b) $\frac{11}{4}$, $2\frac{1}{3}$, $\frac{9}{2}$

c) $\frac{11}{6}$, $\frac{21}{12}$, $1\frac{5}{9}$

3. Pelden studied $1\frac{3}{4}$ h altogether for a test. Padam studied for $\frac{1}{2}$ hour before school, $\frac{1}{2}$ hour after school, and another $\frac{1}{2}$ hour before bed. Who studied more?

4. Rupak, Arjun, and Yuden sold cakes by the piece at the market.

Rupak sold $\frac{11}{6}$ cakes.

Arjun sold $\frac{17}{8}$ cakes.

Yuden sold $\frac{10}{4}$ cakes.

a) Who sold the most cakes?
Who sold the least cakes?

b) What method did you use to compare the fractions in **part a)**?

5. a) What values could ■ be?

$$\frac{■}{5} < \frac{22}{3} ▲$$

b) List three possible pairs of values for ? and #.

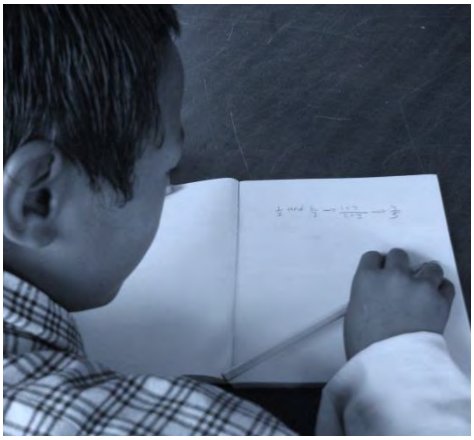
$$\frac{?}{3} < \frac{▲}{5}$$

c) What values could ■ be? List two or more.

$$\frac{5}{■} < \frac{10}{9}$$

6. Namgyel created a new fraction by adding the numerators and denominators of two fractions.

$$\frac{1}{2} \text{ and } \frac{2}{3} \rightarrow \frac{1+2}{2+3} \rightarrow \frac{3}{5}$$



a) Order Namgyel's three fractions $\left(\frac{1}{2}, \frac{2}{3}, \text{ and } \frac{3}{5}\right)$ from least to greatest.
What do you notice about the position of the new fraction?

b) i) Create a new fraction using the fractions $\frac{5}{4}$ and $\frac{7}{5}$.

ii) Order the three fractions.
What do you notice about the position of the new fraction?

c) i) Create any two non-equivalent fractions and use them to create a new fraction.

ii) Order the three fractions. What do you notice about the position of the new fraction?

d) Do you think the new fraction will always be in this position? Use examples to show this.

7. Write a word problem that could be solved by comparing $\frac{2}{3}$ and $\frac{3}{5}$.

8. a) Find all the fractions with a denominator of 3 that are between 2 and 3.

b) Find all the fractions with a denominator of 4 that are between 2 and 3.

c) Find all the fractions with a denominator of 5 that are between 2 and 3.

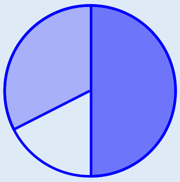
d) Do you think it is possible to find all the fractions that are between 2 and 3? Explain your thinking.

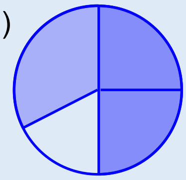
9. Why might you use a different method to compare $\frac{2}{3}$ and $\frac{11}{12}$ than to compare $\frac{3}{10}$ and $\frac{3}{17}$?

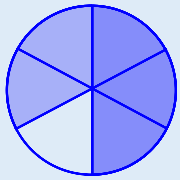
2.1.2 Adding Fractions Using Models

Try This

A. These fraction circles chow different models for 1. In each circle the white part, the light blue part and the slightly dark blue part represent different fractions. What three-fraction combination does each model show?

i) 

ii) 

iii) 

Different models can help you add fractions.

- You can use fraction strips.

For example, to add $\frac{1}{2} + \frac{2}{3}$:

- Estimate first:

$\frac{1}{2} + \frac{1}{2} = 1$ and $\frac{2}{3}$ is more than $\frac{1}{2}$, so $\frac{1}{2} + \frac{2}{3}$ is a bit more than 1.

- Combine fraction strips for $\frac{1}{2}$ and $\frac{2}{3}$ end to end in a line:

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$

- Since the sum is estimated to be a bit more than 1, line up a 1 strip and then find another strip to match the length of $\frac{1}{2} + \frac{2}{3}$:

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$
1		$\frac{1}{6}$

$\frac{1}{2} + \frac{2}{3} = 1\frac{1}{6}$

- You can also use fraction strips to find equivalent fractions with a common denominator.

For example, to add $\frac{1}{2} + \frac{2}{3}$:

$\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1\frac{1}{6}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

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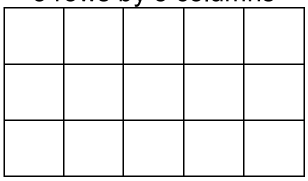
• You can use a grid model to add fractions.

For example, to add $\frac{2}{3} + \frac{1}{5}$:

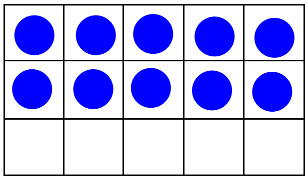
- Create a 3 row-by-5 column grid:

The denominator of the first fraction, $\frac{2}{3}$, tells the number of rows.

The denominator of the second fraction, $\frac{1}{5}$, tells the number of columns.



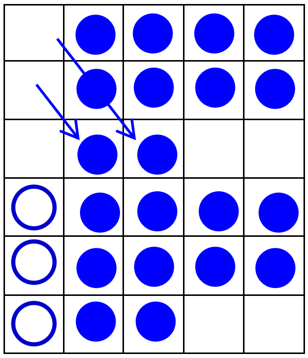
- Model $\frac{2}{3}$ using the rows — fill 2 of the 3 rows with counters. Notice that $\frac{2}{3} = \frac{10}{15}$.



- Model $\frac{1}{5}$ using the columns — fill 1 of the 5 columns with counters but first move counters to clear a column.

Notice that $\frac{1}{5} = \frac{3}{15}$.

$$\frac{2}{3} + \frac{1}{5} = \frac{10}{15} + \frac{3}{15} = \frac{13}{15}$$



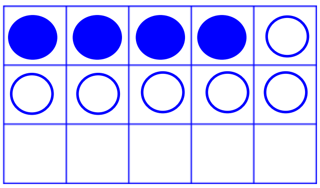
• Sometimes a sum is not in lowest terms. That means the **numerator** and **denominator** have a common factor other than 1.

For example:

$\frac{4}{15} + \frac{6}{15} = \frac{10}{15}$ but $\frac{10}{15}$ is not in lowest terms because 5 is a factor of 10 and 15.

- You can use a grid to help you write $\frac{10}{15}$ in lowest terms.

Since $\frac{10}{15}$ is 2 out of 3 rows, you can rewrite $\frac{10}{15}$ as $\frac{2}{3}$.



- You can also write $\frac{10}{15}$ in lowest terms by dividing the numerator and the denominator by their greatest common factor, which is 5.

$$\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$$

C. i) Write each fraction combination in **parts A and B** as an addition sentence.

$$\frac{?}{?} + \frac{?}{?} + \frac{?}{?} = 1$$

ii) How would you use a model to show that each sentence is true?

Examples

Example 1 Renaming Fractions in Lowest Terms

Tandin added two fractions and the sum was $\frac{12}{15}$.

How could he write the sum in lowest terms?

Solution

$$\frac{12}{15} = \frac{?}{?}$$

$12 = 2 \times 2 \times \underline{3}$

$15 = \underline{3} \times 5$

$$\frac{12}{15} = \frac{12 \div 3}{15 \div 3} = \frac{4}{5}$$

Thinking

- I needed an equivalent fraction using the lowest terms possible.
- I knew the greatest common factor of 12 and 15 was 3.
- I divided the numerator and denominator by 3 to get an equivalent fraction.

Example 2 Adding Fractions

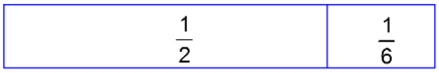
Pema and Mindu are keeping track of how much milk they drink for a week.

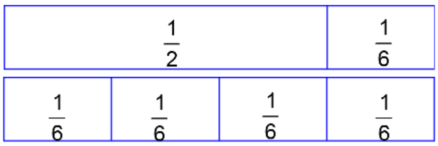
On Monday, Pema drank $\frac{1}{2}$ L and Mindu drank $\frac{1}{6}$ L.

How much milk did they drink in total on Monday?

Solution 1

$$\frac{1}{2} + \frac{1}{6} = ?$$





$$\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$$

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

Pema and Mindu drank $\frac{2}{3}$ L of milk.

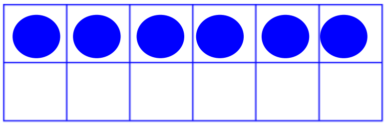
Thinking

- I used a $\frac{1}{2}$ strip and a $\frac{1}{6}$ strip to show $\frac{1}{2} + \frac{1}{6}$.
- I used four $\frac{1}{6}$ strips to match the length.
- I wrote the sum in lowest terms by dividing the numerator and the denominator by their greatest common factor 2.

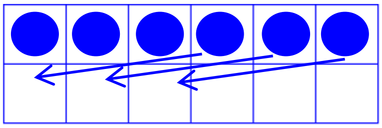
Solution 2

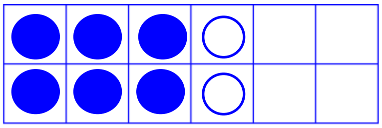
$$\frac{1}{2} + \frac{1}{6} = ?$$

Model $\frac{1}{2}$ using counters.



Model $\frac{1}{6}$ using counters.





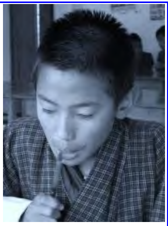
$\frac{8}{12}$ of the grid is filled.

$$\frac{1}{2} + \frac{1}{6} = \frac{6}{12} + \frac{2}{12} = \frac{8}{12}$$

$$\frac{8}{12} = \frac{2}{3}$$

Pema and Dawa drank $\frac{2}{3}$ L of milk.

Thinking



- I used a 2-by-6 grid since the denominators were 2 and 6.
- 1 out of 2 rows is $\frac{1}{2}$ of the grid. I noticed that it's also $\frac{6}{12}$.
- I moved some counters so I could model $\frac{1}{6}$ using 1 of the 6 columns.
- 1 out of 6 columns is $\frac{1}{6}$ of the grid. I noticed that it's also $\frac{2}{12}$.
- I wrote the sum in lowest terms. I divided the numerator and denominator by 4.

Example 3 Using a Model to Find Fractions for a Given Sum

R. K. needs to measure $1\frac{1}{2}$ cups of flour. He has two different measuring cups, a $\frac{1}{6}$-cup measuring cup and a $\frac{1}{3}$-cup measuring cup.

How could he use the two measuring cups to measure $1\frac{1}{2}$ cups? Write an addition sentence to show your answer.

[Continued]

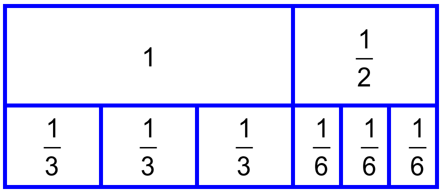
$\frac{1}{3}$-cup and $\frac{1}{6}$-cup measuring cups

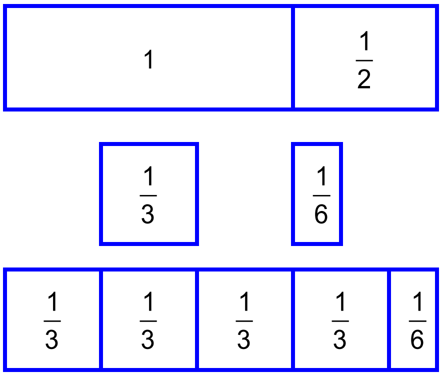
Example 3 Using a Model to Find Fractions for a Given Sum [Continued]

Solution

$$\frac{4}{3} + \frac{1}{6} = 1\frac{1}{2}$$

R. K. could use four $\frac{1}{3}$ - cup measures and one $\frac{1}{6}$ - cup measure.





$$\frac{3}{3} + \frac{3}{6} = 1\frac{1}{2}$$

R.K. could use three $\frac{1}{3}$ - cup measures and three $\frac{1}{6}$ - cup measures.

Thinking

- I used a 1 strip and a $\frac{1}{2}$ strip to represent $1\frac{1}{2}$ cups.
- I used a $\frac{1}{3}$ strip and a $\frac{1}{6}$ strip to represent the two measuring cups.
- I tried different combinations of $\frac{1}{3}$ and $\frac{1}{6}$ to make $1\frac{1}{2}$.
- I could see that there were other possible answers — I could replace any of the $\frac{1}{3}$ strips with two $\frac{1}{6}$ strips.

Practising and Applying

1. Write each in lowest terms.

a) $\frac{6}{8}$ **b)** $\frac{10}{16}$ **c)** $\frac{12}{20}$ **d)** $\frac{9}{15}$

2. What common denominator could you use to add each pair of fractions?

a) $\frac{3}{4}, \frac{2}{3}$ **b)** $\frac{3}{5}, \frac{2}{10}$

c) $\frac{1}{6}, \frac{2}{9}$ **d)** $\frac{1}{3}, \frac{3}{5}$

3. Estimate each sum. Then add each using any model.

a) $\frac{5}{6} + \frac{2}{3}$ **b)** $\frac{7}{8} + \frac{1}{2}$

c) $\frac{7}{10} + \frac{2}{5}$ **d)** $\frac{1}{9} + \frac{1}{3}$

e) $\frac{1}{3} + \frac{2}{5}$ **f)** $\frac{3}{4} + \frac{2}{3}$

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4. a) Create two fractions that have a sum of $\frac{3}{4}$. One should have 9 in the denominator.

b) Create two fractions that have a sum of $\frac{3}{2}$. Find five different pairs of fractions.

5. Lobzang ate $\frac{3}{8}$ of a chocolate bar.

Tashi ate $\frac{1}{4}$ of the same bar.

What fraction of the bar was eaten? Draw a diagram to show the addition.

6. Three students were given a whole cake. Which combination below is possible? Explain your thinking.

A. Ngawang ate $\frac{1}{2}$.

Jamyang ate $\frac{1}{10}$.

Radhika ate $\frac{3}{5}$.

B. Ngawang ate $\frac{1}{2}$.

Jamyang ate $\frac{1}{5}$.

Radhika ate $\frac{1}{10}$.

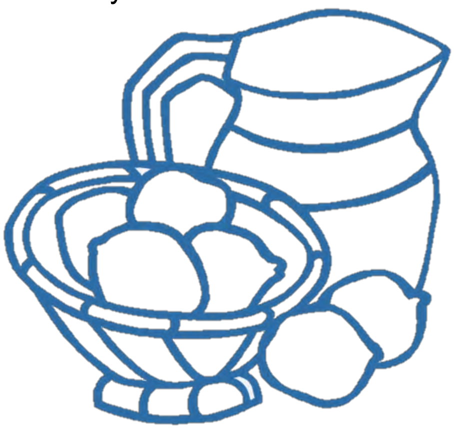
7. a) Create a fraction addition with a sum between 1 and 2. Use each of the digits 3, 4, 5, and 6. Then, find the sum.

$$1 < \frac{\blacksquare}{\blacksquare} + \frac{\blacksquare}{\blacksquare} < 2$$

b) Repeat **part a)**. Find four or more different sums.

8. A jug is half full of lemonade.

If you were to add $\frac{1}{2}$ cup of lemonade, it would be $\frac{3}{4}$ full. How many cups of lemonade can the jug hold? How do you know?



9. Why is it easier to add fractions when they have a common denominator? Use an example to help you explain.

10. How is finding a common denominator using the grid and counters model different from finding a common denominator using the fraction strip model? Use an example to help you explain.

2.1.3 Adding Fractions and Mixed Numbers Symbolically

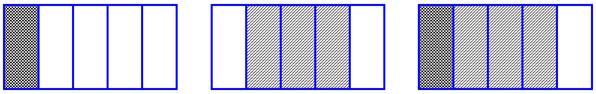
Try This

A runner follows the same training schedule each day. He runs for $\frac{1}{2}$ of his training time, walks for $\frac{1}{3}$ of his training time, then runs for the last $\frac{1}{6}$ of his training time.

A. What fraction of his training time does he run?

• When fractions have the same denominator, it is easy to find the sum. You add the numerators.

For example: $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$



• To add fractions with denominators that are not the same, find equivalent fractions that have the same denominator and then add.

For example, to add $\frac{1}{4} + \frac{2}{3}$:

- List fractions equivalent to each fraction and then look for fractions with a common denominator:

$\frac{1}{4} = \frac{2}{8} = \boxed{\frac{3}{12}} = \frac{4}{16}$ $\qquad$ $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \boxed{\frac{8}{12}}$

- Add the equivalent fractions:

$\frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$

• You can also find a common denominator by finding a common multiple of the denominators.

For example, to add $\frac{1}{4} + \frac{2}{3}$:

Multiples of 4: 4, 8, <u>12</u>

Multiples of 3: 3, 6, 9, <u>12</u>

A common multiple of 4 and 3 is 12, so 12 is a common denominator for $\frac{1}{4}$ and $\frac{2}{3}$.

$$\frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$

Any common multiple of 4 and 3 (12, 24, 36, …) can be used as a common denominator, but it is usually a good idea to use the lowest common multiple.

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• You can **simplify** sums by writing them as mixed numbers or in lowest terms.

For example: As a mixed number In lowest terms

$\frac{3}{4}+\frac{5}{6}=\frac{8}{12}+\frac{10}{12}=\frac{18}{12} \rightarrow \frac{18}{12}=1\frac{6}{12} \rightarrow 1\frac{6}{12}=1\frac{6\div 6}{12\div 6}=1\frac{1}{2}$

Note that you could have found $\frac{18}{12}$ in lowest terms first and then written the mixed number:

$$\frac{18}{12}=\frac{18\div 6}{12\div 6}=\frac{3}{2}=1\frac{1}{2}$$

B. Use a common denominator to answer **part A**. Show your work.

Examples

Example 1 Adding More Than Two Fractions

Kamal recorded his exercise one day. He stretched for $\frac{1}{2}$ h, he jogged for $\frac{3}{4}$ h, and then he walked for $\frac{2}{3}$ h. How many hours did he spend exercising that day?

Solution

$$\frac{1}{2}+\frac{3}{4}+\frac{2}{3}=?$$

Estimate

$\frac{3}{4}<1$ and $\frac{2}{3}<1$, so $\frac{3}{4}+\frac{2}{3}<2$.

$\frac{1}{2}+\frac{3}{4}+\frac{2}{3}$ will be close to 2.

Add using common denominators

Multiples of 3: 3, 6, 9, <u>12</u>

Multiples of 4: 4, 8, <u>12</u>

12 is a common multiple of 2, 3, and 4.

$\frac{1}{2}=\frac{6}{12}$ $\frac{3}{4}=\frac{9}{12}$ $\frac{2}{3}=\frac{8}{12}$

$\frac{6}{12}+\frac{9}{12}+\frac{8}{12}=\frac{23}{12}=1\frac{11}{12}$

Kamal exercised for $1\frac{11}{12}$ h.

Thinking

• I estimated a sum close to 2.

• Since any multiple of 4 is also a multiple of 2, I only had to look for a common multiple of 3 and 4 to find a common denominator for halves, thirds, and fourths.

• I checked the answer against my estimate: $1\frac{11}{12}$ is close to 2.

Example 2 Adding Mixed Numbers

A recipe calls for $1\frac{3}{4}$ cups of red rice and $2\frac{2}{3}$ cups of white rice.

How much rice is needed altogether?

Solution

$$1\frac{3}{4} + 2\frac{2}{3} = ?$$

Estimate

$$1\frac{3}{4} + 2\frac{2}{3} = 1 + \frac{3}{4} + 2 + \frac{2}{3}$$

$$= 1 + 2 + \frac{3}{4} + \frac{2}{3}$$

$1 + 2 = 3$ $\frac{3}{4} + \frac{2}{3}$ is between 1 and 2.

The sum should be between 4 (3 + 1) and 5 (3 + 2).

Add

$$1\frac{3}{4} + 2\frac{2}{3} = 1 + \frac{3}{4} + 2 + \frac{2}{3}$$

$$= 3 + \frac{3}{4} + \frac{2}{3}$$

$$= 3 + \frac{9}{12} + \frac{8}{12}$$

$$= 3 + \frac{17}{12}$$

$$= 3 + 1\frac{5}{12}$$

$$= 4\frac{5}{12}$$

$4\frac{5}{12}$ cups of rice are needed.

Thinking

- To estimate, I knew there were 3 whole cups, and $\frac{3}{4} + \frac{2}{3}$ more cups.
- I found a common denominator for $\frac{3}{4} + \frac{2}{3}$ using common multiples:

4: 4, 8, <u>12</u> 3: 3, 6, 9, <u>12</u>

$\frac{3}{4} = \frac{9}{12}$ and $\frac{2}{3} = \frac{8}{12}$

- I checked the answer against my estimate: $4\frac{5}{12}$ is between 4 and 5.

Practising and Applying

1. a) Add. Write the answer in lowest terms and as a mixed number, if necessary.

i) $\frac{1}{3} + \frac{4}{9}$ **ii)** $\frac{3}{8} + \frac{1}{2}$

iii) $\frac{2}{3} + \frac{4}{5}$ **iv)** $\frac{3}{4} + \frac{7}{10}$

b) Choose one question from **part a)**. Estimate to show why your answer is reasonable.

2. Add.

a) $\frac{5}{8} + \frac{1}{2} + \frac{3}{4}$

b) $\frac{2}{3} + \frac{1}{6} + \frac{5}{9}$

3. a) Add. Write the sum in lowest terms, if necessary.

i) $1\frac{1}{6} + 2\frac{1}{2}$ **ii)** $1\frac{7}{10} + 1\frac{1}{5}$

iii) $7\frac{1}{3} + 2\frac{2}{5}$ **iv)** $4\frac{2}{3} + 3\frac{1}{4}$

b) Choose one question from **part a)** and estimate to show why your answer is reasonable.

4. a) Find three fractions with different denominators that add to 1.

For example: $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$

b) Explain how you got your answer to **part a)**.

5. A punch recipe uses $1\frac{1}{3}$ cups of fruit concentrate and $3\frac{1}{4}$ cups of water. How many cups of punch will the recipe make?

6. Dawa and Nima were painting a wall white. Dawa painted $\frac{1}{3}$ of the wall. Nima painted the same amount as Dawa, then painted another $\frac{1}{5}$ of the wall.

a) What fraction of the wall did Nima paint?

b) What fraction of the wall did both Dawa and Nima paint?

7. A biscuit recipe calls for these ingredients:

- $\frac{3}{8}$ cup of brown sugar
- $2\frac{1}{2}$ cups of flour
- $\frac{1}{3}$ cup of white sugar

a) How much sugar is needed altogether?

b) Would a bowl that holds $3\frac{1}{2}$ cups be big enough to hold all three ingredients? Explain how you know.

8. a) Replace each ■ with any of the digits 1 to 5. Do not use any digit more than once. Find the least possible sum. Use each digit only once.

$\frac{■}{■} + \frac{■}{■}$

b) Repeat **part a)** to find the greatest possible sum.

c) Repeat **part a)** to find the sum that is closest to 1.

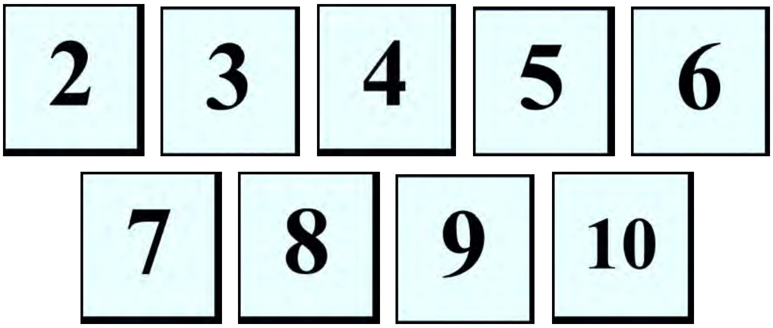
9. How is it helpful to use equivalent fractions with a common denominator to add fractions?

GAME: A "Whole" in One

Play with a partner.

Here is how to play:

- Write the numbers 2 to 10 on slips of paper.

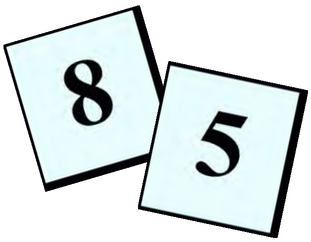


- Put the slips into a bag and mix them up.
- Pull out two numbers and use them as the denominators of two fractions. You can use any numbers you want for the numerators. The goal is to make the sum of your two fractions as close as possible to 1.
- When you have both made your two fractions, compare the results. The player with the sum closer to 1 scores 1 point. If that player gets a sum of exactly 1, the player scores 2 points.
- Return the slips to the bag and pull out numbers for the next round.

The winner is the first player to get 5 points.

For example:

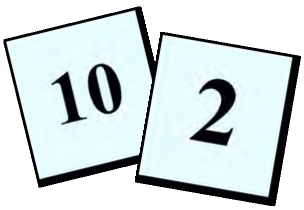
Player A draws 8 and 5.



$\frac{?}{8} + \frac{?}{5}$ must be as close as possible to 1.

$$\frac{5}{8} + \frac{2}{5} = \frac{25}{40} + \frac{16}{40} = \frac{41}{40} = 1\frac{1}{40}$$

Player B draws 10 and 2.



$\frac{?}{10} + \frac{?}{2}$ must be as close as possible to 1.

$$\frac{5}{10} + \frac{1}{2} = \frac{5}{10} + \frac{5}{10} = 1$$

Player B got exactly 1 and scores 2 points for the round.

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2.1.4 Subtracting Fractions and Mixed Numbers

Try This

Tshering spent $\frac{1}{4}$ h longer than Meghraj working on a project.

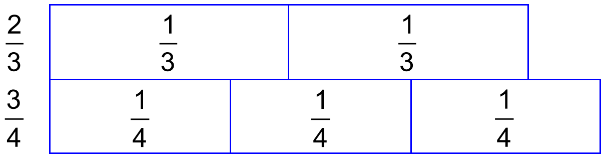
A. How much time might each student have spent on the project?
Find five or more different answers. You can use fraction strips to help you.

Different models can help you subtract fractions.

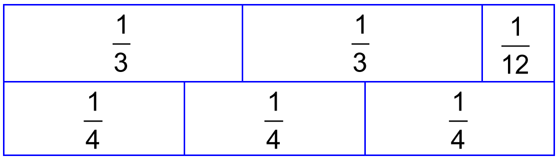
• You can use a fraction strip model.

For example, to subtract $\frac{3}{4} - \frac{2}{3}$:

- compare the length of two $\frac{1}{3}$ strips $\left(\frac{2}{3}\right)$ with the length of three $\frac{1}{4}$ strips $\left(\frac{3}{4}\right)$.



- find a fraction strip that makes up the difference in lengths:



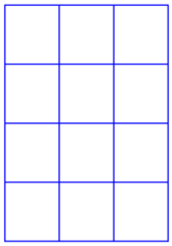
$\frac{3}{4} - \frac{1}{3} = \frac{1}{12}$

• You can use a grid model.

For example, to subtract $\frac{3}{4} - \frac{2}{3}$:

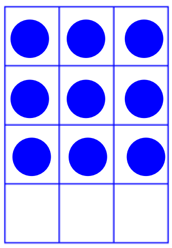
- Create a 4-by-3 grid:
The denominator of the first fraction tells the number of rows.
The denominator of the second fraction tells the number of columns.

4 rows by 3 columns



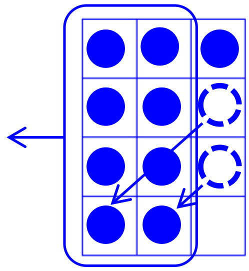
- Represent $\frac{3}{4}$ by filling 3 of the 4 rows with counters.

Notice that $\frac{9}{12}$ of the grid is filled.



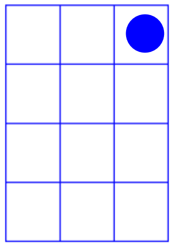
[Continued]

- To subtract $\frac{2}{3}$, you need to take away counters from 2 of the 3 columns. Move counters to fill 2 columns and then take away 2 of the columns.



Notice that 2 out of 3 columns is $\frac{8}{12}$ of the grid.

- 1 of the 12 squares of the grid has a counter left:



$$\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$$

If you think of the subtraction as squares in the grid instead of as rows and columns:

$$\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$$

• To subtract symbolically, you can use equivalent fractions with a common denominator.

For example, to subtract $\frac{3}{4} - \frac{2}{3}$:

$$\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$$

B. In **part A**, you found subtraction sentences with a difference of $\frac{1}{4}$.

Pem Bidha says, "You can make as many subtraction sentences as you want with a difference of $\frac{1}{4}$". Is she right? Explain your thinking.

Examples

Example 1 Subtracting Fractions with Unlike Denominators

Subtract $\frac{5}{8} - \frac{1}{6}$.

Solution

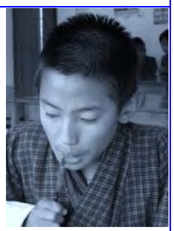
Multiples of 8: 8, 16, <u>24</u>

Multiples of 6: 6, 12, 18, <u>24</u>

$$\frac{5}{8} = \frac{15}{24} \qquad \frac{1}{6} = \frac{4}{24}$$

$$\frac{5}{8} - \frac{1}{6} = \frac{15}{24} - \frac{4}{24} = \frac{11}{24}$$

Thinking



• I found a common denominator for eighths and sixths by finding a common multiple of 8 and 6.

• Then I found equivalent fractions for $\frac{5}{8}$ and $\frac{1}{6}$ with a denominator of 24.

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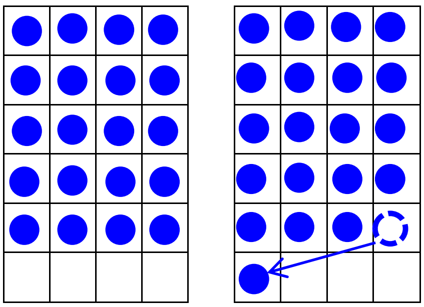
Example 2 Solving a Problem with Unlike Denominators

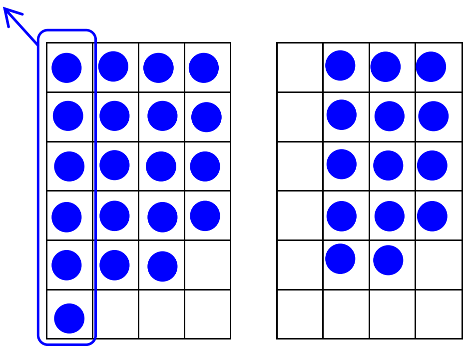
A truck was carrying $\frac{5}{6}$ full of manure. It delivered $\frac{1}{4}$ of the load to a farm. What fraction of a full truckload does the truck have now?

Solution 1

$$\frac{5}{6} - \frac{1}{4} = ?$$

$\frac{5}{6}$





$\frac{5}{6} - \frac{1}{4}$ $\qquad$ $\frac{5}{6} - \frac{1}{4} = \frac{14}{24}$

$$\frac{14}{24} = \frac{7}{12}$$

The truck is now $\frac{7}{12}$ full

Thinking

- I drew a 6-by-4 grid because the fractions were $\frac{5}{6}$ and $\frac{1}{4}$.
- I put counters in 5 of the 6 rows to represent $\frac{5}{6}$.
- To subtract $\frac{1}{4}$, I had to take away counters from 1 of the 4 columns, so I moved a counter to fill in 1 column.
- I took away 1 of the 4 columns.
- $\frac{14}{24}$ of the grid still had counters.
- Subtracting $\frac{5}{6} - \frac{1}{4} = \frac{14}{24}$ on the grid was like subtracting $\frac{20}{24} - \frac{6}{24} = \frac{14}{24}$.
- I wrote the answer in lowest terms.

Example 2 Solving a Problem with Unlike Denominators [Continued]

Solution 2

$$\frac{5}{6} - \frac{1}{4} = ?$$

Multiples of 6: 6, <u>12</u>

Multiples of 4: 4, 8, <u>12</u>

$$\frac{5}{6} = \frac{10}{12} \qquad \frac{1}{4} = \frac{3}{12}$$

$$\frac{5}{6} - \frac{1}{4} = \frac{10}{12} - \frac{3}{12} = \frac{7}{12}$$

The truck is now $\frac{7}{12}$ full.

Thinking

- I looked for a common denominator for sixths and fourths.
- I wrote equivalent fractions with the common denominator of 12.
- Since the denominators were the same, I just subtracted the numerators.

Example 3 Subtracting Mixed Numbers to Solve a Problem

One weekend, Karma worked $3\frac{3}{4}$ h on Saturday and $2\frac{1}{2}$ h on Sunday.
How many hours more did she work on Saturday than on Sunday?

Solution

$$3\frac{3}{4} - 2\frac{1}{2} = ?$$

4 is a multiple of 2, so 4 is a common denominator for fourths and halves.

$$3\frac{3}{4} - 2\frac{1}{2} = 3\frac{3}{4} - 2\frac{2}{4} = 1\frac{1}{4}$$

Karma worked $1\frac{1}{4}$ h more on Saturday.

Thinking

- I found a common denominator for fourths and halves.
- I wrote equivalent fractions with the common denominator of 4.
- I subtracted the whole numbers and then subtracted the fractions.

Practising and Applying

1. a) Subtract.

i) $\frac{5}{8} - \frac{1}{2}$ **ii)** $\frac{5}{9} - \frac{1}{3}$

iii) $\frac{7}{10} - \frac{1}{4}$ **iv)** $\frac{4}{5} - \frac{2}{3}$

b) Choose one question from **part a)**. Explain why your answer is reasonable.

2. Subtract.

a) $1\frac{4}{5} - \frac{3}{5}$ **b)** $2\frac{5}{6} - 1\frac{1}{3}$

c) $5\frac{4}{7} - 2\frac{2}{7}$ **d)** $4\frac{3}{10} - 2\frac{1}{5}$

3. A recipe calls for $\frac{3}{4}$ cup of red rice and $\frac{2}{3}$ cup white rice. Does the recipe call for more red rice or more white rice? How much more?

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4. The fuel tank of a car was $\frac{3}{4}$ full at the beginning of the week. At the end of the week, there was $\frac{1}{8}$ of a tank left.

a) Did the car use more or less than $\frac{1}{2}$ of a tank of fuel? How do you know?

b) How much more or less than $\frac{1}{2}$ of a tank did it use?

5. Without calculating, how do you know that both subtractions below have a difference that is less than $\frac{1}{2}$?

$\frac{5}{6} - \frac{1}{2} < \frac{1}{2}$ $\frac{5}{6} - \frac{2}{3} < \frac{1}{2}$

6. Write three different subtractions with a difference of $\frac{3}{4}$.

$$\frac{■}{■} - \frac{■}{■} = \frac{3}{4}$$

7. A class voted on whether to use red or yellow on a school banner:

- $\frac{2}{5}$ of the class voted for red.
- $\frac{1}{3}$ of the class voted for yellow.

a) Which colour got more votes?

b) What is the difference between the fractions?

c) What fraction of the class did not vote? How do you know?

8. The difference between two fractions is $\frac{1}{2}$. The smaller fraction is between 0 and $\frac{1}{4}$. What do you know about the greater fraction? Explain your thinking.

9. In a community land-use survey, the following results were found:

- $\frac{5}{8}$ of the land was used for farming.
- $\frac{1}{5}$ was used for housing.
- $\frac{1}{20}$ was used for roads.
- $\frac{1}{8}$ was used for other purposes.

a) How much more land is used for farming than for housing?

b) What fraction is used for roads and housing?

c) Is the fraction in **part b)** more or less than the land used for "other"? By how much?

10. a) Replace each ■ with one of the digits 2, 3, 4, and 5 to find the greatest possible difference.

$$\frac{■}{■} - \frac{■}{■}$$

b) Repeat **part a)** to find the least possible difference.

c) Repeat **part a)** to find the difference that is closest to $\frac{1}{2}$.

11. Which method do you prefer for subtracting fractions with different denominators? Use two examples to show how you would subtract.

2.1.5 Subtracting Mixed Numbers in Different Ways

Try This

A. In the long jump event at a school track meet,

- Ngawang jumped 5 m.
- Jamyang jumped $3\frac{3}{4}$ m.

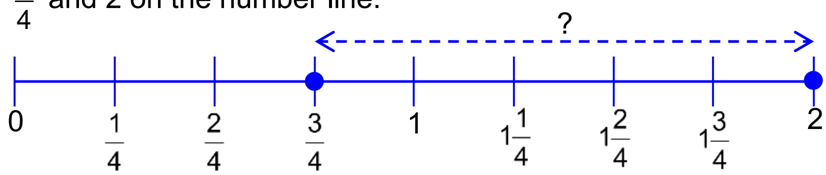
How many metres farther did Ngawang jump?

- To subtract a fraction or mixed number from a whole number, you can use a number line.

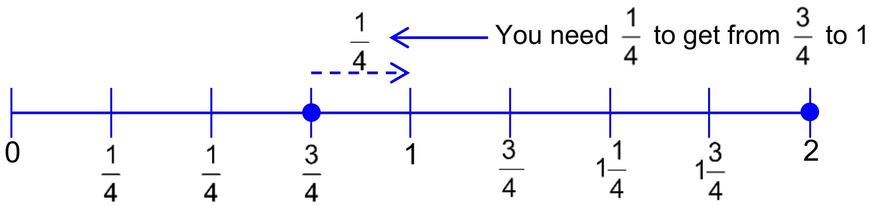
For example:

To subtract $2 - \frac{3}{4}$ on a number line, you find the distance between the numbers.

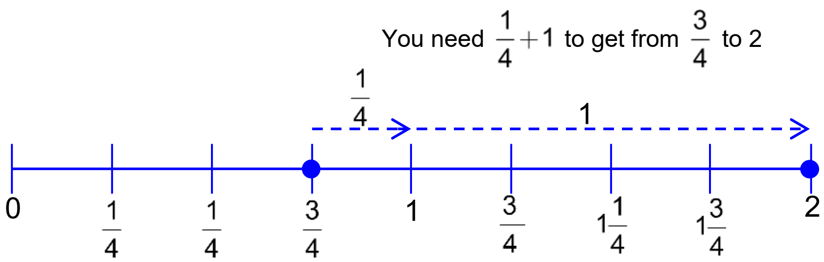
- Find $\frac{3}{4}$ and 2 on the number line.



- Find the fraction you need to get from $\frac{3}{4}$ to the next whole number.



- Now find the number you need to get to 2.



The distance between 2 and $\frac{3}{4}$ is $1\frac{1}{4}$, so $2 - \frac{3}{4} = 1\frac{1}{4}$.

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• You learned in the last lesson that you can sometimes subtract mixed numbers by subtracting the whole-number parts and then subtracting the fraction parts.

For example: $2\frac{2}{3} - 1\frac{1}{2} = 2\frac{4}{6} - 1\frac{3}{6} = 1\frac{1}{6}$

• There are other ways to subtract, as well. These may be easier to use when the fraction in the greater mixed number is less than the fraction in the mixed number being subtracted.

For example:

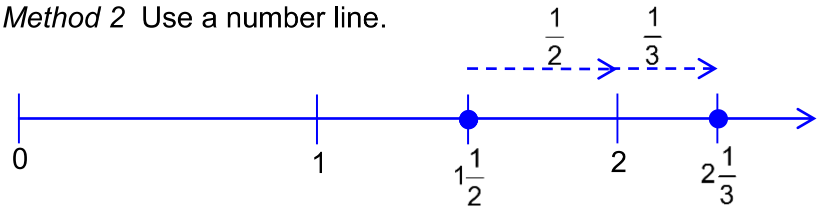
To subtract $2\frac{1}{3} - 1\frac{1}{2}$, you can subtract 1 from 2 but not $\frac{1}{2}$ from $\frac{1}{3}$.

Here are some different methods for subtracting $2\frac{1}{3} - 1\frac{1}{2}$:

Method 1 Write both mixed numbers as improper fractions and then subtract.

$2\frac{1}{3} - 1\frac{1}{2} = \frac{7}{3} - \frac{3}{2} = \frac{14}{6} - \frac{9}{6} = \frac{5}{6}$

Method 2 Use a number line.



The distance between $1\frac{1}{2}$ and $2\frac{1}{3}$ on a number line is $\frac{1}{2} + \frac{1}{3}$.

$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$, so $2\frac{1}{3} - 1\frac{1}{2} = \frac{5}{6}$.

Method 3 Rename the first mixed number and then subtract.

$2\frac{1}{3} = 1 + 1\frac{1}{3} = 1\frac{4}{3}$

$$\begin{array}{r} 2\frac{1}{3} \\ -\,1\frac{1}{2} \\ \hline \end{array} \Rightarrow \begin{array}{r} 1\frac{4}{3} \\ -\,1\frac{1}{2} \\ \hline \end{array} \Rightarrow \begin{array}{r} 1\frac{8}{6} \\ -\,1\frac{3}{6} \\ \hline \frac{5}{6} \end{array}$$

$2\frac{1}{3} - 1\frac{1}{2} = \frac{5}{6}$

B. The youth long jump record in Asia is $8\frac{1}{2}$ m. How much farther would Jamyang have to jump to reach the world record? Show your work.

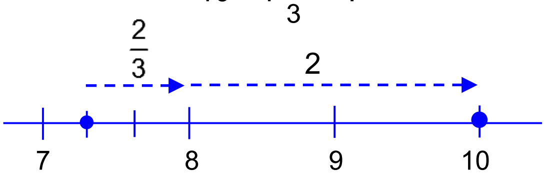
Examples

Example 1 Subtracting from a Whole Number

Nima ran 10 laps of the track. Rinzin ran $7\frac{1}{3}$ laps. How many more laps did Nima run than Rinzin?

Solution 1

$$10 - 7\frac{1}{3} = ?$$



$\frac{2}{3} + 2 = 2\frac{2}{3}$, so $10 - 7\frac{1}{3} = 2\frac{2}{3}$

Thinking

- I found $7\frac{1}{3}$ and 10 on a number line.
- The distance from $7\frac{1}{3}$ to 8 was $\frac{2}{3}$, and from 8 to 10 was 2 more.

Solution 2

$$10 - 7\frac{1}{3} = 9\frac{3}{3} - 7\frac{1}{3} = 2\frac{2}{3}$$

Thinking

- I regrouped 10 as $9\frac{3}{3}$ so I could subtract the fraction parts and the whole-number parts.

Example 2 Subtracting Mixed Numbers

A recipe for Hapai Hoenteh calls for $2\frac{1}{4}$ cups of white flour and $1\frac{1}{2}$ cups of buckwheat flour. How much more white flour is needed than buckwheat flour?

Solution 1

$$2\frac{1}{4} - 1\frac{1}{2} = ?$$

$2\frac{1}{4} = 1\frac{5}{4}$

$$\begin{array}{r} 1\frac{5}{4} \\ -\,1\frac{1}{2} \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 1\frac{5}{4} \\ -\,1\frac{2}{4} \\ \hline \frac{3}{4} \end{array}$$

$\frac{3}{4}$ cup more white flour is needed.

Thinking

- I renamed $2\frac{1}{4}$ as $1 + \frac{5}{4}$ because I couldn't subtract $\frac{1}{2}$ from $\frac{1}{4}$.

- I found a common denominator for halves and fourths and then subtracted.

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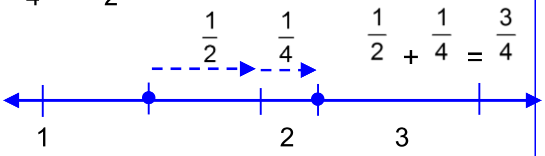
Solution 2

$2\frac{1}{4} - 1\frac{1}{2} = ?$



$2\frac{1}{4} - 1\frac{1}{2} = \frac{3}{4}$

$\frac{3}{4}$ cup more white flour is used.

Thinking

- From $1\frac{1}{2}$ to 2 is $\frac{1}{2}$ and from 2 to $2\frac{1}{4}$ is $\frac{1}{4}$, so the distance between $1\frac{1}{2}$ and $2\frac{1}{4}$ is $\frac{1}{2} + \frac{1}{4}$.

Practising and Applying

1. Subtract.

a) $4 - \frac{3}{5}$ **b)** $6 - \frac{3}{7}$

c) $3 - \frac{5}{6}$ **d)** $5 - \frac{4}{9}$

2. Subtract.

a) $3\frac{2}{5} - 1\frac{5}{8}$ **b)** $6\frac{1}{3} - 2\frac{4}{9}$

c) $5\frac{1}{4} - 3\frac{1}{2}$ **d)** $4\frac{1}{6} - 1\frac{2}{9}$

3. Eden has completed $7\frac{5}{8}$ laps of a 10-lap race. How many more laps does she still need to run?

4. It took Tandin 4 h to paint a fence. He painted for $2\frac{3}{4}$ h on Friday and then finished painting on Saturday. How many hours did he spend painting on Saturday?

5. Solve each.

a) $3\frac{5}{6} = 2\frac{■}{6}$ **b)** $7\frac{3}{10} = 6\frac{■}{10}$

6. Hari Maya worked $7\frac{1}{4}$ h on Saturday and $4\frac{1}{2}$ h on Sunday. How much longer did she work on Saturday than on Sunday?

7. Nima ran $10\frac{1}{2}$ laps on a track. Bina Gurung ran $12\frac{1}{4}$ laps. How many fewer laps did Nima run?

8. A curtain pattern calls for $3\frac{1}{2}$ m of material. Choki had two pieces of cloth, one $5\frac{1}{8}$ m and another $4\frac{1}{4}$ m.

a) Choki made one curtain from each piece. How much material did she have left over from each piece?

b) If she sews the remaining pieces together, will she have enough to make a third curtain? How do you know?

9. a) Copy and complete the magic square.

	$3\frac{9}{10}$	$2\frac{2}{5}$
	$2\frac{7}{10}$	
	$1\frac{1}{2}$	

b) What is the magic sum?

10. How does renaming a mixed number help you subtract? Use an example to explain.

Chapter 2 Fraction Multiplication and Division

2.2.1 Multiplying a Fraction by a Whole Number

Try This

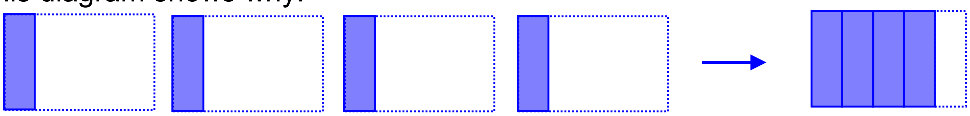
Kuenga spends $\frac{3}{4}$ h each day walking to and from school.

A. How much time does he spend walking to and from school in 6 days?

• Multiplying a fraction by a whole number is like multiplying two whole numbers.
A sum of identical whole numbers can be written as a product.
For example: 3 + 3 + 3 + 3 = 4 × 3 = 12
A sum of identical fractions can also be written as a product.

For example: $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 4 \times \frac{1}{5} = \frac{4}{5}$

This diagram shows why:



$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{4}{5}$$

• To multiply a fraction such as $5 \times \frac{2}{3}$, think, "What is 5 groups of 2 thirds?"

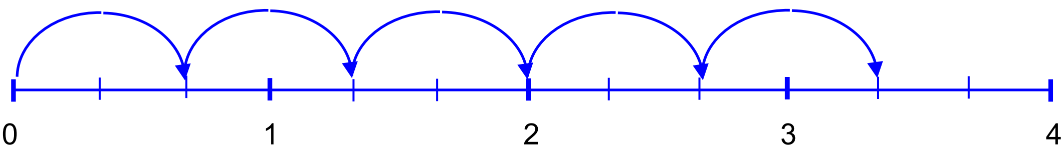
$$5 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{10}{3}$$

5 groups of 2 thirds is 10 thirds.

It is usually a good idea to rename improper fractions as mixed numbers.

$$5 \times \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$$

This number line model shows that $5 \times \frac{2}{3} = 3\frac{1}{3}$ makes sense:



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B. Write the time you spend getting to and from school each day as a fraction of an hour.

i) How much time do you spend getting to and from school in 6 days?

ii) Use a diagram to show that your answer makes sense.

iii) Could you multiply to find how much everyone in the class spends getting to school in 1 day? Why or why not?

Examples

Example Multiplication as Repeated Addition

Chandra reads $\frac{5}{6}$ h every day. How long does he read in a week?

Solution

$\frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6} + \frac{5}{6}$

$= 7 \times \frac{5}{6}$

$7 \times \frac{5}{6}$ = 7 × 5 sixths = 35 sixths

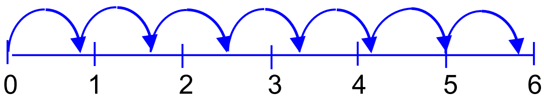
$= \frac{35}{6}$

$\frac{35}{6} = 5\frac{5}{6}$

Chandra spends $5\frac{5}{6}$ h reading.

Thinking

- Adding seven $\frac{5}{6}$s is like multiplying $\frac{5}{6}$ by 7.
- $7 \times \frac{5}{6}$ is 7 groups of 5 sixths.
- I wrote $\frac{35}{6}$ as a mixed number.
- I pictured 7 jumps of $\frac{5}{6}$ on a number line:



$7 \times \frac{5}{6} = 5\frac{5}{6}$ made sense.

Practising and Applying

1. Write each as a multiplication and then calculate the product.

a) $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

b) $\frac{7}{10} + \frac{7}{10} + \frac{7}{10}$

c) $\frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9}$

2. Which of these additions can be written as a multiplication? Explain your thinking.

A. $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$

B. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

C. $\frac{5}{3} + \frac{5}{3} + \frac{5}{3} + \frac{5}{3} + \frac{5}{3} + \frac{5}{3} + \frac{5}{3}$

3. Calculate and then write each product as a mixed number.

a) $4 \times \frac{3}{5}$ **b)** $11 \times \frac{1}{2}$

c) $6 \times \frac{5}{8}$ **d)** $5 \times \frac{3}{10}$

4. a) Calculate and then write each product in lowest terms.

i) $6 \times \frac{5}{6}$ **ii)** $12 \times \frac{7}{12}$ **iii)** $8 \times \frac{3}{8}$

b) What do you notice about the product of each multiplication in **part a)**?

5. A parrot eats $\frac{2}{3}$ of an apple each day.

How many apples does it eat in a week?

6. It takes $\frac{3}{4}$ h to pick the peaches from one tree. How many hours will it take to pick the peaches from 28 trees?

7. a) Write each product as an improper fraction.

$7 \times \frac{3}{10}$ $3 \times \frac{7}{10}$

b) What do you notice?

c) Write another pair of multiplications like this.

8. A biscuit recipe calls for $\frac{2}{3}$ cup of walnuts.

How many cups of walnuts are needed to make 4 batches of biscuits?

9. a) Use a model to show why

$3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$. Sketch your model.

b) Explain how your model shows that

$3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$.

c) Sketch a different model that shows the same thing.

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2.2.2 Dividing a Fraction by a Whole Number

Try This

A recipe that serves 4 people calls for $\frac{2}{3}$ cup of rice.

A. How much rice is needed for 1 serving?

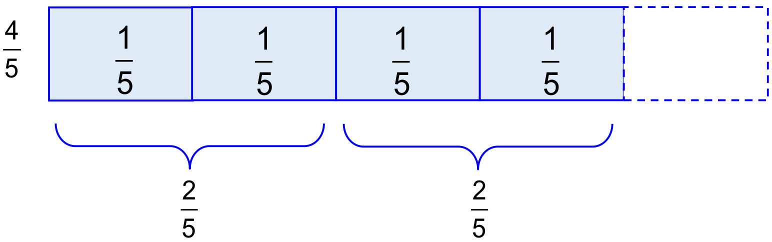
• Dividing a fraction by a whole number is like dividing a whole number by another whole number.

- To divide 8 by 2, you can think of 2 people sharing 8 cakes.

Each person gets 8 ÷ 2 = 4.

- To divide $\frac{4}{5}$ by 2, you can think of 2 people sharing $\frac{4}{5}$ of a cake.

Each person gets $\frac{4}{5} \div 2 = ?$

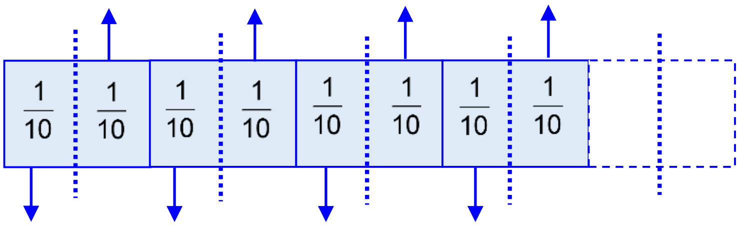


$$\frac{4}{5} \div 2 = \frac{2}{5}$$

If 4 fifths are shared between 2 people, each person gets 2 fifths.

• Another way to divide $\frac{4}{5}$ by 2 is to divide each fifth by 2. Half of $\frac{1}{5}$ is $\frac{1}{10}$.

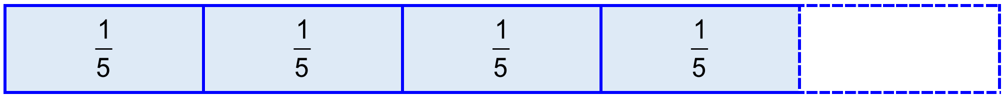
One person gets $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10}$, or $\frac{2}{5}$.



The other person gets $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10}$, or $\frac{2}{5}$.

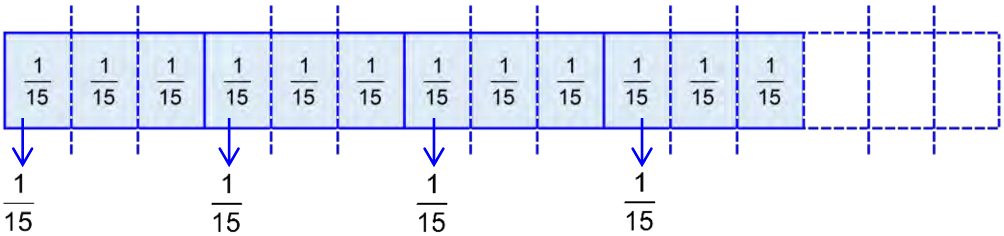
$$\text{So, } \frac{4}{5} \div 2 = \frac{4}{10} = \frac{2}{5}$$

• If $\frac{4}{5}$ of a cake is shared by 3 people, each share is not an exact number of fifths.



To divide $\frac{4}{5}$ by 3, you can divide each fifth by 3.

A third of $\frac{1}{5}$ is $\frac{1}{15}$:



Each person gets $\frac{1}{15} + \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{4}{15}$.   So $\frac{4}{5} \div 3 = \frac{4}{15}$

B. Draw a picture to model sharing the rice in **part A** among 4 people.

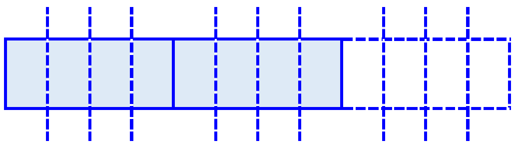
Examples

Example 1 Dividing a Fraction by a Whole Number

Jamyang wants to share $\frac{2}{3}$ of a large chocolate bar equally among herself and three friends in her family. What fraction of the bar will each person get?

Solution 1

$$\frac{2}{3} \div 4 = ?$$

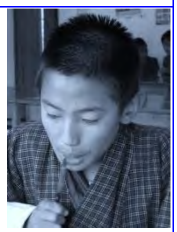


$$\frac{2}{3} \div 4 = \frac{8}{12} \div 4 = \frac{2}{12}$$

$$\frac{2}{12} = \frac{1}{6}$$

Each person will get $\frac{1}{6}$ of the bar.

Thinking



• There were 4 people sharing, so I divided each third into 4 equal parts.

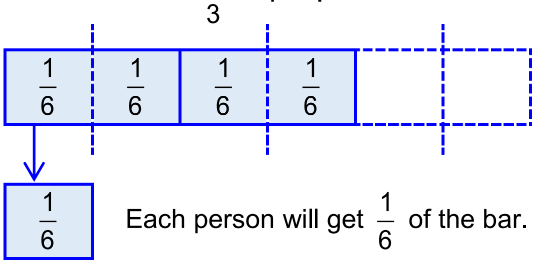
• Each part was 1 twelfth, so there were 8 twelfths to share among 4 people.

• 8 twelfths ÷ 4 = 2 twelfths

• $\frac{2}{12}$ in lowest terms is $\frac{1}{6}$.

Solution 2

$\frac{2}{3} \div 4 = ?$



Each person will get $\frac{1}{6}$ of the bar.

Thinking

- I cut each third in half so there will be 4 sixths to share among 4 people.

Practising and Applying

1. Find each quotient. Draw a picture of what you did.

a) $\frac{1}{2} \div 2$ **b)** $\frac{1}{4} \div 4$ **c)** $\frac{2}{3} \div 3$

2. Find each quotient.

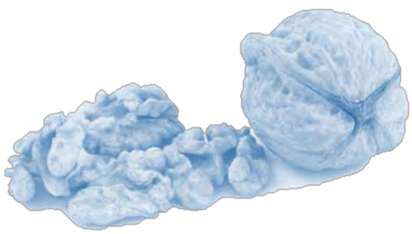
a) $\frac{1}{3} \div 5$ **b)** $\frac{5}{8} \div 2$ **c)** $\frac{4}{5} \div 3$

d) $\frac{7}{2} \div 4$ **e)** $\frac{10}{3} \div 5$ **f)** $\frac{21}{4} \div 7$

g) $\frac{9}{5} \div 6$ **h)** $\frac{6}{7} \div 8$ **i)** $\frac{33}{10} \div 3$

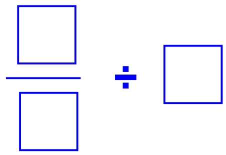
3. Choki has $\frac{5}{6}$ h to study math and English. She wants to spend the same amount of time studying each. What fraction of an hour should she spend on each subject?

4. Nima has $\frac{9}{4}$ cups of walnuts to use in 3 equal batches of biscuits. How many cups of walnuts will be in each batch?



5. Write a word problem that could be solved by dividing a fraction by 5. Solve your problem.

6. Draw boxes like these:



a) Write the digits 2, 4, and 6 in the boxes. Write as many different division expressions as possible.

b) Which expressions have the greatest quotient?

c) Which expressions have the least quotient?

d) Which expressions have equal quotients?

7. If you divide $\frac{2}{5}$ by 2, you get a quotient with a denominator of 5.

$$\frac{2}{5} \div 2 = \frac{\#}{5}$$

If you divide $\frac{3}{5}$ by 2, you get a quotient with a denominator of 10.

$$\frac{3}{5} \div 2 = \frac{\#}{10}$$

Explain why this happens.

Chapter 3 Relating Fractions and Decimals

2.3.1 Naming Fractions and Mixed Numbers as Decimals

Try This

There are 4 jugs of water to share among 5 people.

A. What fraction of a jug does each person get? Write it as a decimal.

You can find a decimal equivalent to any fraction. How you do this will depend on the fraction.

• For fractions that have a denominator that is a factor of 10, 100, 1000, or another place value amount, you can create an equivalent fraction with a denominator of 10, 100, 1000, and so on, and then write the decimal.

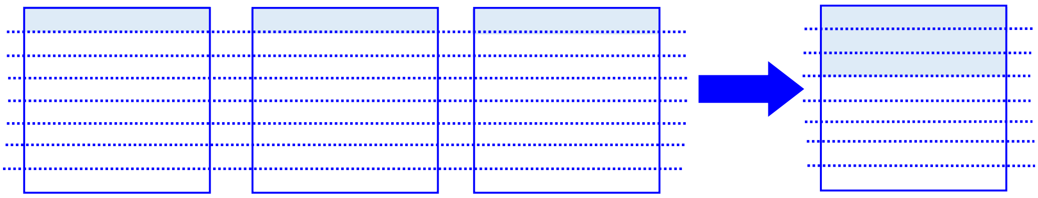
For example:

$\frac{2}{5} = \frac{4}{10} \rightarrow \frac{2}{5} = 0.4$ $\qquad$ $\frac{3}{4} = \frac{75}{100} \rightarrow \frac{3}{4} = 0.75$

• Some fractions have a denominator that is a factor of 10, 100, 1000, and so on, but it is not obvious. This makes it difficult to find an equivalent fraction tenth, hundredth, thousandth, and so on. In these cases, you can use the division meaning of a fraction to find the **equivalent decimal**.

For example, to find an equivalent decimal for $\frac{3}{8}$, you can divide 3 ÷ 8.

$\frac{3}{8}$ = 3 ÷ 8 because you can think of $\frac{3}{8}$ as dividing 3 wholes into 8 equal parts.



If 3 wholes are divided into 8 equal parts, each part is $\frac{3}{8}$ of one whole.

To write $\frac{3}{8}$ as a decimal, you divide 3 by 8.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -\underline{24} \\ 60 \\ -\underline{56} \\ 40 \\ -\underline{40} \\ 0 \end{array}$$

• A fraction that can be written as an equivalent fraction with a denominator of 10, 100, 1000, and so on, results in a decimal called a **terminating decimal**. Terminating means that the decimal terminates or ends.

For example:

$\frac{3}{8}$ is a terminating decimal because it can be written as a fraction with a denominator of 1000: $\frac{3}{8} = \frac{375}{1000} = 0.375$

• Some fractions cannot be written as an equivalent fraction with a denominator of 10, 100, 1000, or so on. For these fractions, you can use the division meaning of a fraction to find the equivalent non-terminating decimal.

For example:

To find an equivalent decimal for $\frac{1}{3}$, divide 1 by 3:

$$
\begin{array}{r}
0.333\ldots \\
3\overline{)1.000} \\
\underline{-9} \\
10 \\
\underline{-9} \\
10 \\
\underline{-9} \\
1
\end{array}
$$

Look at what happens when you divide 1 by 3. You never get a remainder of 0, so the dividing goes on forever.

$\frac{1}{3} = 0.333333333\ldots$ The three dots mean the 3s go on forever.

Showing enough digits to see the pattern followed by three dots is enough to represent the decimal: $\frac{1}{3} = 0.333\ldots$

• A fraction that cannot be written as an equivalent fraction with a denominator of 10, 100, 1000, and so on, results in a decimal called a **repeating decimal**.

• Repeating decimals can have more than one repeating digit.

For example: $\frac{3}{11} = 0.272727\ldots$

$$
\begin{array}{r}
0.272727\ldots \\
11\overline{)3.000000} \\
\underline{-22} \\
80 \\
\underline{-77} \\
30 \\
\underline{-22} \\
80 \\
\underline{-77} \\
30 \\
\underline{-22} \\
80 \\
\underline{-77} \\
3
\end{array}
$$

• The length of the repeating part of a decimal is called its **period**.

For example: 0.333… has a period of 1 because one digit repeats.
0.272727… has a period of 2 because two digits repeat.

B. Suppose you had 4 jugs of water to share among each number of people. Write, as a decimal, how much each person would get in each case.

i) 2 people **ii)** 3 people **iii)** 4 people **iv)** 5 people
v) 6 people **vi)** 7 people **vii)** 8 people

C. Which decimals in **part B** terminate? How could you have predicted this?

Examples

Example 1 Renaming Fractions as Decimals

Write $\frac{11}{5}$ as a decimal.

Solution 1

$$\begin{array}{r} 2.2 \\ 5\overline{)11.0} \\ -\underline{10} \\ 10 \\ -\underline{10} \\ 0 \end{array}$$

$\frac{11}{5} = 2.2$

Thinking

• I knew $\frac{11}{5}$ was 11 ÷ 5.

• I divided 11 by 5 — it stopped, or terminated, after one decimal place.

Solution 2

$\frac{11}{5} = 2\frac{1}{5}$

$2\frac{1}{5} = 2\frac{2}{10}$

$2\frac{2}{10} = 2.2$

Thinking

• I wrote the improper fraction as a mixed number.

• I found an equivalent mixed number with a denominator of 10.

• I wrote the mixed number as a decimal.

Example 2 Using Decimal Equivalents to Compare Fractions

a) Use $\frac{1}{3}$ = 0.333… to write $\frac{2}{3}$ as a decimal. **b)** Compare $\frac{2}{3}$ and $\frac{3}{5}$.

Solution

a) $\frac{1}{3}$ = 0.333..., so $\frac{2}{3} = 2 \times \frac{1}{3}$

$2 \times \frac{1}{3} = 2 \times 0.333\ldots = 0.666\ldots$

b) $\frac{2}{3}$ = 0.666...

$\frac{3}{5} = \frac{6}{10} = 0.6$

0.666… > 0.6

So $\frac{2}{3} > \frac{3}{5}$

Thinking

• I knew that $\frac{1}{3}$ was 0.333... and that $\frac{2}{3}$ was 2 times $\frac{1}{3}$.

• I found an equivalent fraction for $\frac{3}{5}$ with a denominator of 10 so I could write it as a decimal.

• I knew that 0.6 was equal to 0.600.

Practising and Applying

1. Write an equivalent fraction with a denominator of 10, 100, or 1000 and a decimal for each.

a) $\frac{1}{2}$ **b)** $\frac{3}{5}$ **c)** $\frac{3}{4}$

d) $\frac{7}{20}$ **e)** $\frac{11}{25}$ **f)** $\frac{7}{8}$

2. Write $\frac{1}{5}$ as a decimal and then use it to write each as a decimal.

a) $\frac{3}{5}$ **b)** $\frac{4}{5}$ **c)** $\frac{7}{5}$

d) $\frac{8}{5}$ **e)** $\frac{11}{5}$ **f)** $2\frac{2}{5}$

3. Write each as a decimal.

a) $\frac{3}{11}$ **b)** $\frac{5}{8}$ **c)** $\frac{2}{9}$

d) $\frac{12}{25}$ **e)** $\frac{5}{6}$ **f)** $\frac{7}{12}$

4. a) For this set of fractions:

$\frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \frac{4}{9}, \frac{5}{9}, \frac{6}{9}, \frac{7}{9}, \frac{8}{9}$

• write the first three fractions as decimals

• look for and describe the pattern

• use the pattern to write the other fractions in the set as decimals

b) Repeat **part a)** for this set of fractions.

$\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11}, \frac{7}{11}, \frac{8}{11}, \frac{9}{11}, \frac{10}{11}$

c) Repeat **part a)** for this set of fractions.

$\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$

5. a) Which number in each pair is greater? How do you know?

i) 0.3 or $\frac{1}{3}$

ii) 0.33 or $\frac{1}{3}$

iii) 0.333 or $\frac{1}{3}$

b) Rename $\frac{1}{3}$ as an equivalent fraction to make it easy to see how it compares to each decimal in **part a)**.

6. Write each pair of fractions as decimals and then compare them.

a) $\frac{12}{25}$ and $\frac{23}{50}$ **b)** $\frac{7}{8}$ and $\frac{22}{25}$

7. Use equivalent decimals to compare.

a) $\frac{7}{8}$ and $\frac{5}{6}$ **b)** $\frac{1}{6}$ and $\frac{4}{25}$

c) $\frac{11}{50}$ and $\frac{2}{9}$ **d)** $\frac{4}{11}$ and $\frac{18}{50}$

8. The number π is a non-repeating, non-terminating decimal.

$\pi = 3.141592653589\ldots$

$\frac{22}{7}$ is often used to approximate π.

For how many digits is the decimal for $\frac{22}{7}$ the same as π?

9. a) What are the possible remainders when you divide a number by 7?

b) The period for a fraction with a denominator of 7 is 6 digits. Use the answer to **part a)** to explain why a decimal equivalent for a fraction seventh must repeat after 6 digits or fewer.

10. When might someone find it easier to compare fractions using their decimal equivalents? Use an example to explain.

2.3.2 EXPLORE: Relating Repeating Decimals and Fractions

You can use fractions and their repeating decimal equivalents to find other repeating decimals.

For example:

Since $\frac{1}{9}$ = 0.111... and $\frac{1}{9} = \frac{11}{99}$, then $\frac{11}{99}$ = 0.111111….

If $\frac{11}{99}$ = 0.111111..., then $\frac{1}{99}$ = 0.111111… ÷ 11.

So $\frac{1}{99}$ = 0.010101…

$$
\begin{array}{r}
0.010101...\\
11\,\overline{)\,0.111111...}\\
-\underline{0}\\
11\\
-\underline{11}\\
01\\
-\underline{0}\\
11\\
-\underline{11}\\
01
\end{array}
$$

A. Use $\frac{1}{9}$ = 0.111… to write an equivalent fraction for each.

i) 0.222222… **ii)** 0.555555… **iii)** 0.777777…

B. Why does it make sense that 0.999999… = 1?

C. Use $\frac{1}{99}$ = 0.010101… to write a repeating decimal for each.

i) $\frac{13}{99}$ **ii)** $\frac{37}{99}$ **iii)** $\frac{51}{99}$ **iv)** $\frac{74}{99}$

D. Use the pattern from **part C** to write an equivalent fraction for each.

i) 0.414141… **ii)** 0.424242…

iii) 0.434343… **iv)** 0.444444…

E. Use the patterns from **parts A and C** to write each as a fraction.
Express in lowest terms, if necessary.

i) 0.666666… **ii)** 0.444444... **iii)** 0.272727

iv) 0.818181… **v)** 0.151515… **vi)** 0.060606…

F. How would you find an equivalent fraction for each?

i) a repeating decimal with a period of 1

ii) a repeating decimal with a period of 2

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CONNECTIONS: Repeating-Decimal Graphs

Devi Charan created graphs of the fraction thirteenth family. This is how he did it:

x	y
0	7
7	6
6	9
9	2
2	3
3	0

• He wrote $\frac{1}{13}$ as 0.076923076923…

• He put pairs of digits from the repeating part of the decimal into a table and then plotted the points on a graph.

• He connected the points in the order they were plotted, joining the last point with the first.

Devi Charan did this for all the other fractions in the family:

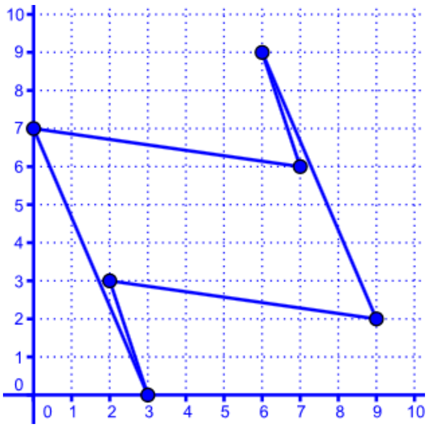
$\frac{2}{13}$ = 0.153846153846…, $\frac{3}{13}$ = 0.230769230769…,

and so on, all the way to $\frac{12}{13}$ = 0.923076923076….

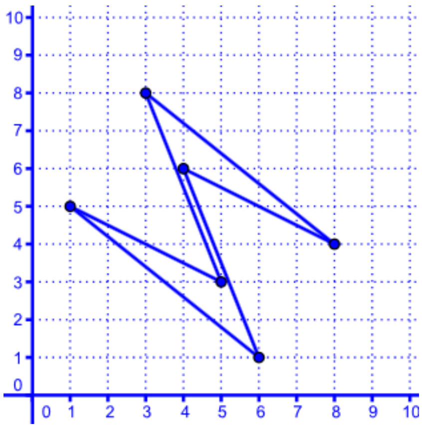
He found that all the graphs were of one of two shapes. Here are his graphs:

This is the graph he got for each of these fractions: $\frac{1}{13}, \frac{3}{13}, \frac{4}{13}, \frac{9}{13}, \frac{10}{13}$ and $\frac{12}{13}$.

.



This is the graph he got for each of these fractions: $\frac{2}{13}, \frac{5}{13}, \frac{6}{13}, \frac{7}{13}, \frac{8}{13}$ and $\frac{11}{13}$



1. a) Write each fraction in the fraction seventh family as a repeating decimal:

$\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$.

b) Create a graph for each decimal.

c) How many different shapes does this fraction family have?

2. a) Predict the graph shapes for the fraction eleventh family: $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \ldots, \frac{10}{11}$

b) Graph two or more of these to check your prediction.

UNIT 2 Revision

1. Which symbol is missing, >, <, or = ?

a) $\frac{8}{6}$ ■ $\frac{16}{3}$

b) $\frac{24}{5}$ ■ $\frac{67}{15}$

c) $2\frac{1}{4}$ ■ $\frac{19}{8}$

d) $\frac{6}{7}$ ■ $\frac{12}{14}$

2. Order from least to greatest. For each, describe how you ordered the fractions.

a) $\frac{13}{8}$, $1\frac{7}{8}$, $\frac{7}{4}$

b) $3\frac{1}{5}$, $\frac{21}{6}$, $3\frac{3}{10}$

c) $\frac{9}{6}$, $1\frac{3}{9}$, $\frac{27}{15}$

3. Pema is 85 months old and Rinzin is $7\frac{2}{3}$ years old. Who is older? How do you know?

4. Penjor ate $\frac{1}{3}$ of a cake.

Namgay ate $\frac{3}{8}$ of the same cake.

a) How much of the cake was eaten?

b) Draw a diagram to show how much was eaten.

5. Add.

a) $\frac{1}{4} + \frac{3}{10}$

b) $\frac{5}{6} + \frac{7}{8}$

c) $2\frac{1}{2} + 4\frac{3}{10}$

6. Subtract.

a) $\frac{5}{6} - \frac{3}{8}$

b) $\frac{7}{8} - \frac{2}{3}$

c) $9\frac{1}{2} - 2\frac{3}{8}$

d) $4 - 1\frac{3}{5}$

e) $7\frac{1}{3} - 2\frac{3}{4}$

f) $5\frac{1}{6} - 3\frac{2}{3}$

7. a) Dorji was able to subtract $6\frac{■}{5} - 3\frac{1}{2}$ without renaming $6\frac{■}{5}$ first. What are the possible values for ■? Explain your thinking.

b) To subtract $6\frac{■}{5} - 3\frac{1}{2}$, Dorji found it easier to rename $6\frac{■}{5}$ first. What are the possible values for ■? Explain your thinking.

c) Choose one value for ■ from **part b)** and complete the subtraction.

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8. In a class survey, students voted for their favourite sport:

- $\frac{3}{5}$ of the students chose archery
- $\frac{1}{4}$ chose football
- the rest chose other sports

a) Which sport got the most votes?

b) What is the difference between the fractions for archery and for football?

c) What fraction of the class did not vote for archery or for football? How do you know?

9. Calculate and then write each product in lowest terms.

a) $9 \times \frac{4}{9}$

b) $7 \times \frac{3}{5}$

c) $10 \times \frac{5}{6}$

10. Divide.

a) $\frac{8}{9} \div 4$

b) $\frac{11}{5} \div 2$

c) $\frac{5}{6} \div 3$

11. a) Lobzang runs $\frac{3}{4}$ h each day.

How many hours does he run in a week?

b) He wants to break his training session into two equal times for running and stretching. What fraction of an hour should he spend on each in a day?

12. Write $\frac{1}{8}$ as a decimal and then use it to help write each as a decimal.

a) $\frac{2}{8}$ **b)** $\frac{3}{8}$ **c)** $\frac{5}{8}$

d) $\frac{7}{8}$ **e)** $1\frac{3}{8}$

13. a) Is $\frac{4}{9}$ greater or less than each decimal below? How do you know?

i) 0.4

ii) 0.44

iii) 0.444

b) Rename $\frac{4}{9}$ as an equivalent fraction to make it easy to see how it compares to each decimal in **part a)**.

14. Write each as a fraction in lowest terms.

a) 0.545454…

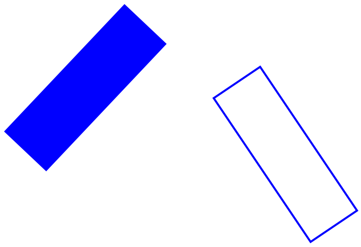
b) 0.424242…

c) 0.777…

UNIT 3 RATIO, RATE, AND PERCENT

Getting Started

Use What You Know



A. Use 10 black and white slips of paper in total to represent each ratio. Sketch a picture of each.

i) 4 to 6 **ii)** 7 : 3 **iii)** 1 : 9

B. How does each of your pictures from **part A** show one of the ratios below?

i) 4 : 10 **ii)** 7 to 10 **iii)** $\frac{1}{10}$

C. How can you make a different picture for each ratio in **part A**?

D. i) Write a ratio with each term 10 or less. Represent it with 10 black and white slips. Sketch a picture of what you made.

ii) Repeat **part i)** for another ratio.

E. i) Represent the ratio $\frac{1}{10}$ with 10 slips of paper. Sketch a picture.

ii) Write two other ratios to describe your picture. Tell how each ratio describes your picture.

F. i) Which ratios from **parts A and D** compare a part to a part?

ii) Which ratios compare a part to a whole?

G. Represent each ratio in **part A** using 20 slips of paper. For each, write equivalent ratios.

For example, for **part i)**:

4 black : 6 white = ___ black : ___ white

H. Suppose the ratio of white slips to total slips is 3 : 10.

i) Write an equivalent ratio for 100 slips in total.

ii) What percent of the slips are white? How do you know?

Skills You Will Need

1. Complete.

a) $\frac{2}{5} = \frac{\square}{20}$ **b)** $\frac{5}{8} = \frac{25}{\square}$ **c)** $\frac{\square}{9} = \frac{20}{36}$ **d)** $\frac{3}{\square} = \frac{21}{98}$

2. List four ratios equivalent to 3 : 7.

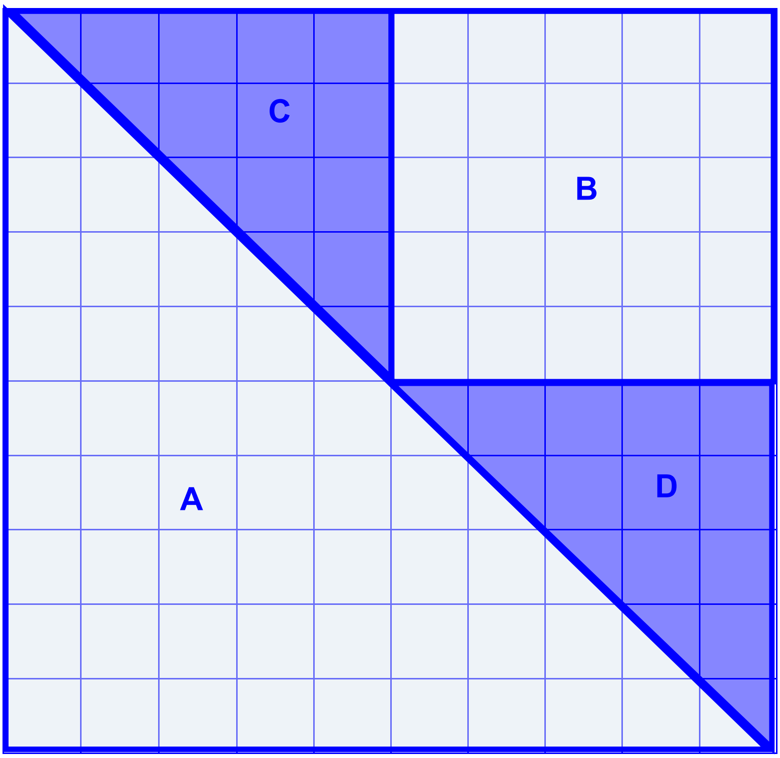
3. a) The ratio of boys to girls in Dorji's class is 28 : 14. Write an equivalent ratio with lower values.

b) What is the ratio of boys to the total number of students in Dorji's class?

4. a) Which figure below covers 25% of the large square?

b) Which figure covers 50% of the large square?

c) What percent of the large square is covered by the remaining two figures? How do you know?



5. Complete.

a) $0.13 = \frac{?}{100}$ **b)** $0.6 = \frac{?}{10}$ **c)** $0.9 = \frac{?}{100}$ **d)** $0.40 = \frac{?}{10}$

6. Copy and complete.

Measurement unit	gram				metre			second	
Symbol		kg	mL	L		km	h		min

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3.1.1 Solving Ratio Problems

Try This

A recipe for Ema Datshi serves 4 people.
It calls for 2 tomatoes and 5 cloves of garlic.
You can change the recipe to serve a different number of people.

A. How many tomatoes would you need to serve each?

i) 12 people **ii)** 8 people **iii)** 2 people

B. How many cloves of garlic would you need to serve each?

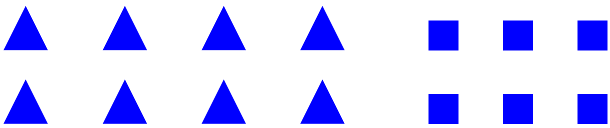
i) 12 people **ii)** 8 people **iii)** 2 people

A bulb of garlic and a clove of garlic

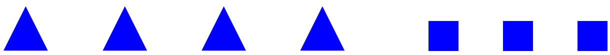
• Recall that a **ratio** is about a relationship between amounts or items.

For example, for the group of shapes below:

The ratio 8 : 6 ("8 to 6") means that, for every **8** triangles, there are **6** squares.

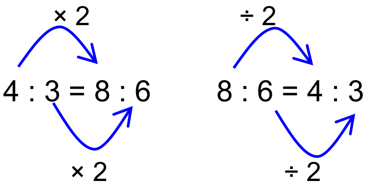


The ratio 4 : 3 ("4 to 3") means that, for every **4** triangles, there are **3** squares.



Notice that you can see the ratio 4 : 3 in the ratio 8 : 6 above.

• The ratios 8 : 6 and 4 : 3 are **equivalent ratios**. You can find an equivalent ratio by multiplying or dividing the **terms** of a ratio by the same number.



• The ratio 4 : 3 is in **lowest terms**. A ratio is in lowest terms when the only common factor for the terms is 1. It is often easier to imagine a ratio when it is in lowest terms.

For example:

32 : 24 in lowest terms is 4 : 3. It is often easier to picture 4 : 3 than 32 : 24.

• Ratio tables can be used to solve problems involving equivalent ratios.
For example:
The ratio of footballs to basketballs in a set of balls is 2 : 5.

That means for every 2 footballs, there are 5 basketballs.
You can use the ratio table below to solve problems like these:
- *How many footballs are there, if there are 15 basketballs?*
- *How many basketballs are there, if there are 4 footballs?*

Footballs	2	**?**	4
Basketballs	5	15	**?**

- To find how many footballs there are if there are 15 basketballs, notice that 5 **× 3** = 15 basketballs. That means there are 2 **× 3** = **6** footballs.

Footballs	2	**?**	4
Basketballs	**5**	**15**	**?**

× 3

× 3

Footballs	**2**	**6**	4
Basketballs	5	15	?

- To find how many basketballs there are, if there are 4 footballs, notice that 2 **× 2** = 4. That means there are 5 **× 2** = **10** basketballs.

× 2

Footballs	**2**	6	**4**
Basketballs	5	15	?

Footballs	2	6	4
Basketballs	**5**	15	**10**

× 2

C. i) Complete this ratio table for the Ema Datshi recipe in **part A**.

Number of people	4	12	8	**2**	**1**
Number of tomatoes	2	?	?	**?**	**?**

ii) How many tomatoes would you need to make Ema Datshi for your class?

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Examples

Example 1 Using a Ratio Table to Solve a Ratio Problem

In a class, the ratio of boys to girls is 5 : 4. There are 15 boys in the class.
How many girls are in the class?

Solution

Boys	5	15
Girls	4	?

× 3

Boys	**5**	**15**
Girls	4	?

Boys	5	15
Girls	**4**	**12**

× 3

There are 12 girls in the class.

Thinking

- I put the information I knew into a ratio table.
- I noticed 5 was multiplied by 3 to get 15.
- That's why I multiplied 4 by 3 to get 12.

Example 2 Using a Ratio Table to Write a Ratio in Lowest Terms

Choki has 24 apples and 20 peaches.
She wants to make as many fruit baskets
as she can with the same ratio of apples
to peaches (24 : 20) in each basket.
How many apples and peaches will be in each basket?

Solution

Ratio of apples to peaches is 24 : 20

Apples	24	12		
Peaches	20	10		

Apples	24	12	6	
Peaches	20	10	5	

There will be 6 apples and 5 peaches in each basket.

Thinking

- She can make the most baskets by putting the fewest apples and peaches in each basket.
- I knew that 24 : 20 in lowest terms would give me the fewest apples and oranges in the same ratio.
- I used a ratio table to find equivalent ratios for 24 : 20.
- I stopped when I had a ratio with terms that had no common factor other than 1.

Practising and Applying

1. a) Match each ratio in the first row with an equivalent ratio in the second row.

5 : 8	2 : 1	2 : 3	3 : 4
8 : 12	15 : 24	10 : 5	12 : 16

b) Tell how you know each pair is equivalent.

2. a) Write three ratios equivalent to 10 : 4.

b) How did you find each ratio in **part a)**?

c) Is one of the ratios in lowest terms? How do you know?

3. Deki has a stamp collection with 20 Bhutanese and 12 Ugandan stamps. Thinley has a stamp collection with a different total number of stamps but the same ratio of Bhutanese to Ugandan stamps as Deki's.

a) How many of each kind of stamp might be in Thinley's collection?

b) Is more than one answer possible? Explain your thinking.

4. There are 18 boys and 24 girls in Class VII. They want to form groups.

a) List the ways that the students could be grouped in the same ratio.

b) What is the greatest number of groups that can be formed with boys and girls in the same ratio?

c) How many boys and girls will be in each group?

5. A recipe calls for 240 g of pork and 360 g of potatoes. It serves 3 people.

a) How many grams of potatoes would you need to serve each?

i) 6 people **ii)** 1 person

b) How many grams of potatoes are needed to serve 4 people? Explain how you got your answer.

c) i) If you have 960 g of pork, how many people would the recipe serve?

ii) How many people would 320 g of pork serve?

6. Is 14 : 4 equivalent to 21 : 6? How do you know?

7. The Bhutanese flag has a width-to-length ratio of 2 : 3. If you draw the largest possible Bhutanese flag on an 18 cm by 30 cm piece of paper, what are the width and length of the flag?

8. Write your own word problem that can be solved using equivalent ratios. Solve your problem.

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3.1.2 Solving Rate Problems

Try This

A. Suppose one Canadian dollar can be exchanged for about Nu 50. About how many ngultrums would you get if you exchanged each?

i) 10 Canadian dollars **ii)** 20 Canadian dollars
iii) 50 Canadian dollars **iv)** 100 Canadian dollars

Nu 50 = $1 Canadian

• Recall that a **rate** is like a ratio, but it describes a relationship between two numbers with different units of measure.

For example, an arrow might travel at a rate of 150 m in 2 seconds.

• A **unit rate** is an **equivalent rate** that has 1 as its second term.

For example, if a person walks 15 km in 3 hours, the equivalent unit rate would be 5 km in 1 hour.

A unit rate like 5 km in 1 hour is usually written as 5 km/h even though it means 5 km/1 h. It is read as "5 kilometres per hour."

• An **average rate** is usually a unit rate.

For example, if a person walks 15 km in 3 h, we say the average rate is 5 km/h.

• **Speed** is a special kind of rate. It is a rate of distance per unit of time.

For example, 5 km/h and 45 m/s are both speeds.

• Rate tables can be used to solve rate problems and to find unit rates.

For example:

Pineapples are sold at a rate of 4 pineapples per Nu 80. You can use a rate table to find the cost of other numbers of pineapples.

How much would 8 pineapples cost?

8 pineapples cost Nu 160.

× 2

Cost (Nu)	**80**	**160**		
Pineapples	**4**	**8**		

× 2

How much would 1 pineapple cost?

1 pineapple costs Nu 20.

÷ 4

Cost (Nu)	**80**	160	**20**	
Pineapples	**4**	8	**1**	

÷ 4

[Continued]

If you know the cost per pineapple (the unit rate), you can easily find the cost of any number of pineapples.

How much would 7 pineapples cost?

7 pineapples cost Nu 140.

Cost (Nu)	80	160	**20**	**140**
Pineapples	4	8	**1**	**7**

× 7 (from 20 to 140, and from 1 to 7)

You can use the rate table to find how many pineapples you can buy for a certain amount of money.

How many pineapples could you buy for Nu 280?

14 pineapples cost Nu 280.

Cost (Nu)	80	160	20	**140**	**280**
Pineapples	4	8	1	**7**	**14**

× 2 (from 140 to 280, and from 7 to 14)

• In a rate table, it does not matter which unit is in which row. For example, the table above could be set up like this:

Pineapples	4	8	1	7
Cost (Nu)	80	160	20	140

B. Complete this rate table for exchanging Canadian dollars and ngultrums.

Canadian dollars ($)	1	10	20	50	100
Ngultrums (Nu)	50	?	?	?	?

C. i) About how many ngultrums would you get for 12 Canadian dollars? Explain how you got your answer.

ii) About how many Canadian dollars would you get for Nu 200? Explain how you got your answer.

Examples

Example 1 Solving Rate Problems

Meghraj can walk 12 km in 2 h.

a) How far can he walk in 3 h, if he walks the same speed?

b) How long will it take him to walk 9 km?

a) Solution 1

Time (h)	2	3
Distance (km)	12	

× 1.5

Time (h)	**2**	**3**
Distance (km)	12	

× 1.5

Time (h)	2	3
Distance (km)	**12**	**18**

× 1.5

Meghraj can walk 18 km in 3 h.

Thinking

• I put the information that I knew into a rate table.

• I noticed that 2 could be multiplied by 1.5 to get 3.

• That's why I multiplied 12 by 1.5 to get 18.

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a) Solution 2

÷ 2

Distance (km)	12	6	
Time (h)	2	1	

÷ 2

× 3

Distance (km)	12	6	18
Time (h)	2	1	3

× 3

Meghraj can walk 18 km in 3 h.

Thinking

- I used a rate table to find the unit rate of speed — how many kilometres in 1 h.
- I knew that if he walked 6 km in 1 h, he would walk 3 times that distance in 3 h.

b) Solution

÷ 2

Distance (km)	12	6	18	9
Time (h)	2	1	3	

Distance (km)	12	6	18	9
Time (h)	2	1	3	1.5

÷ 2

It will take Meghraj 1.5 h to walk 9 km.

Thinking

- I put 9 km into my rate table from **part a)** and noticed that 9 was 18 divided by 2.
- That's why I divided 3 km by 2 to get 1.5 km.

Example 2 Finding Unit Rates and Solving Rate Problems

A motorbike travelled 154 km in 7 h.

a) What is the average speed of the motorbike?

b) How far would it travel in 30 min at that speed?

[Continued]

Solution

a) 154 km in 7 h = ____ km in 1 h

↑

154 ÷ 7 = 22

The average speed is 22 km/h.

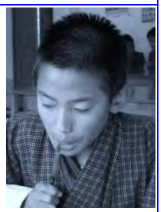
b) 22 km in 1 h = ____ km in $\frac{1}{2}$ h

↑

22 ÷ 2 = 11

In 30 min, the bike would travel 11 km.

Thinking



- The average speed is the unit rate.
- Since the bike went 154 km in 7 h, it could go 154 ÷ 7 in 1 h.
- If the bike went 22 km in 1 h, it would go half that distance in half the time.

Practising and Applying

1. Express each as a unit rate.

a) An elephant travelled 50 km in 2 h.

b) An elephant ate 540 kg of food in 3 days.

c) A person swam 125 m in 5 min.

2. A dozen bananas cost Nu 24.

a) How much do 3 dozen bananas cost?

b) What is the cost of 6 bananas?

c) What is the cost of 1 banana?

d) How much would it cost to buy a banana for each person in your class? Show your work.

3. Lhamo's heart beats 22 times in 20 s. What is her heart rate in beats/min?

4. Before running a race, Tandin's heart rate was 70 beats/min. After the race his heart rate had doubled. What was his heart rate after the race?

5. A car uses 10 L of fuel to go 100 km. How much fuel will it use to go 250 km?

6. An Olympic runner travels 10 km in about 30 min. What is the runner's rate in minutes per kilometre?

7. Which is the better price per orange for the buyer?

- 12 oranges for Nu 60
- 8 oranges for Nu 48

How do you know?

8. This table shows the distance various animals can run in different times.

Animal	Distance (m)	Time (s)
Cheetah	200	6.4
Bear	500	36.0
Zebra	250	14.0
Elephant	20	1.8
Tortoise	10	120.0
Rabbit	300	20.0
Lion	400	16.0

a) Which animal runs at each speed?

i) about 11 m/s

ii) about 25 m/s

b) Which animal could travel each?

i) 900 m in 1 min

ii) about 5 m in 1 min

c) Which animal is fastest? How do you know?

d) Which is slowest? How do you know?

9. Suppose you know how far a car travels in 10 min. How would you find its speed in kilometres per hour?

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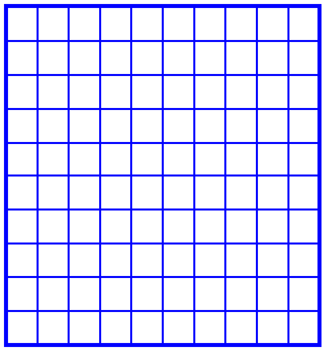
Chapter 2 Percent

3.2.1 Percent as a Special Ratio

Try This

A. Outline a 10 by 10 grid on a piece of grid paper.
i) How many small squares are in a 10 by 10 grid?
ii) Draw a picture or shape that covers 0.45 of the area of the 10 by 10 grid.



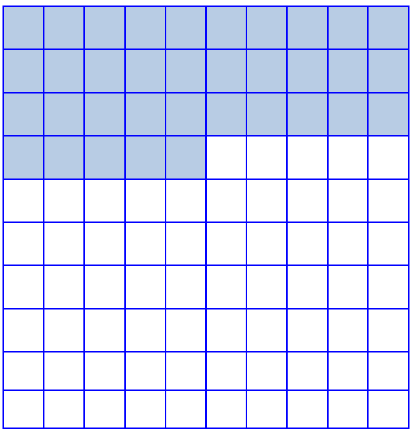
- Recall that **percent** (%) means "out of 100". Percent is a special ratio that compares one amount to another and always has a second term of 100.

For example: 35% means 35 out of 100, 35 : 100, or $\frac{35}{100}$.

- 100% means 100 out of 100, or $\frac{100}{100}$. It is another way to say one whole.

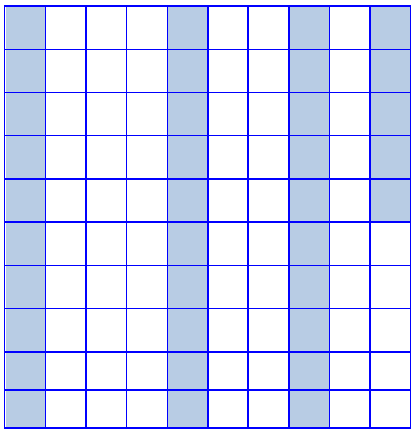
- For the 10 by 10 grid below:
  The whole grid is 100%.
  35% is shaded.
  65% is not shaded.

The parts of the whole always add up to 100%.



35% of the grid is shaded.

There are many ways to represent the same percent.



35% of the grid is shaded.

B. Look at your picture from **part A**. What is each ratio described below?

i) the amount of grid covered by the picture compared to the whole grid

ii) the amount of grid not covered by the picture compared to the whole grid

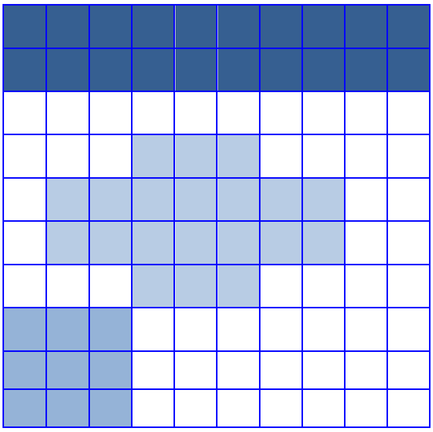
C. Write each ratio in **part B** as percent.

Examples

Example Using a Percent to Describe a Picture

a) Express each shape as a percent of the total grid.

b) What percent of the grid is not shaded?



Solution

a)

The rectangle covers 20 out of 100 squares, so it is 20%.

The square covers 9 out of 100 squares, so it is 9%.

The cross shape covers 20 out of 100 squares, so it is 20%.

b)

20% + 9% + 20% + ___ = 100%

49% + ___ = 100%

49% + **51%** = 100%

51% of the grid is not shaded

Thinking

a)

- 20 out of 100 = $\frac{20}{100}$ = 20%.
- 9 out of 100 = $\frac{9}{100}$ = 9%
- 20 out of 100 = $\frac{20}{100}$ = 20%.

b) The parts of a whole add to 100%.

- I could have just counted the number of white squares.

Practising and Applying

1. Express each ratio as a percent.

a) 9 out of 100

b) 19 : 100

c) $\frac{87}{100}$

d) 43 : 100

e) 100 out of 100

2. On a math test, Eden answered 71 of the 100 questions.

a) What percent of the questions did she answer?

b) What percent did she not answer?

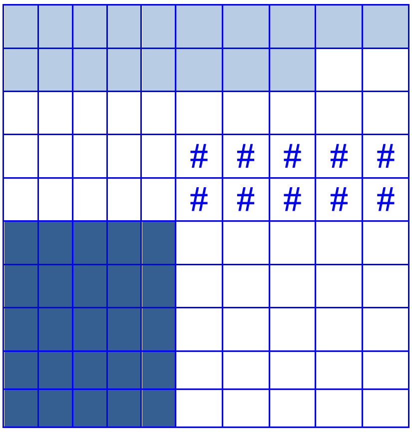
3. Express each as a percent of the total grid below.

a) Grey squares

b) Black squares

c) Squares containing a #

d) White squares



4. If you double the black region on the grid in **question 3**, what percent will be covered by black squares?

5. In an archery contest, the ratio of kareys to total arrows shot was 68 : 100.

a) Write this ratio as a percent.

b) What percent were not kareys?

6. Water covers about 72% of the Earth's surface. About what percent of the Earth's surface is dry land?

7. a) In the chart below, what percent of the numbers are even?

b) What percent are odd?

c) What percent are multiples of 5?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	6	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

8. 100 students were asked about their favourite fruit.

- 70 students chose apples
- 20 chose bananas
- the rest chose another fruit

a) What percent of the students chose apples?

b) What percent chose bananas?

c) What percent did not choose apples?

d) What percent chose something other than apples or bananas?

9. Use a 10 by 10 grid.

a) Colour 15% of the grid grey.

b) Colour another 25% of the grid black.

c) Fill the rest of the grid with Xs and Os in the ratio of 2 : 1.

d) What percent of the squares have X?

e) What percent of the squares have O?

f) What is the total of the percent in **parts a) to c)**? How do you know without adding?

3.2.2 Relating Percent, Fractions, and Decimals

Try This

1 chhertum (1 Ch) has a value that is $\frac{1}{100}$ of a ngultrum (Nu 1).

A. What fraction of Nu 1 is each coin?

i) Ch 5

ii) Ch 10

iii) Ch 25

• If a fraction has a denominator of 100, it is easy to write as a percent.
A part-whole ratio with a second term of 100 is also easy to write as a percent.

For example: $\frac{41}{100} = 41\%$ $\quad$ 35 : 100 = 35%

• If the denominator of a fraction is not 100, you can write the fraction as a percent after you find an equivalent fraction with a denominator of 100.

For example:

If $\frac{3}{4}$ of a set of beads are black, 75% are black.

$$\frac{3}{4} = \frac{75}{100} \quad (\times 25 \text{ numerator}, \times 25 \text{ denominator})$$

• If the second term of a ratio is not 100, you can write the ratio as a percent after you find an equivalent ratio with a second term of 100.

For example:

If the ratio of girls to the whole class is 20 : 25, 80% are girls.

× 4

20 : 25 = 80 : 100

× 4

• You can easily write a decimal as a percent after you write it as hundredths.

For example:

$0.15 = \frac{15}{100} = 15\%$ $\quad$ $0.2 = 0.20 = \frac{20}{100} = 20\%$

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• You can also write percent as ratio, fraction, and decimal.

For example:

$40\% = 40 : 100 = 2 : 5$ — percent as ratio

$40\% = \frac{40}{100} = \frac{4}{10} = \frac{2}{5}$ — percent as fraction

$40\% = 0.40 = 0.4$ — percent as decimal

• For some fractions it may not be possible to write an exact whole number percent, but you can estimate.

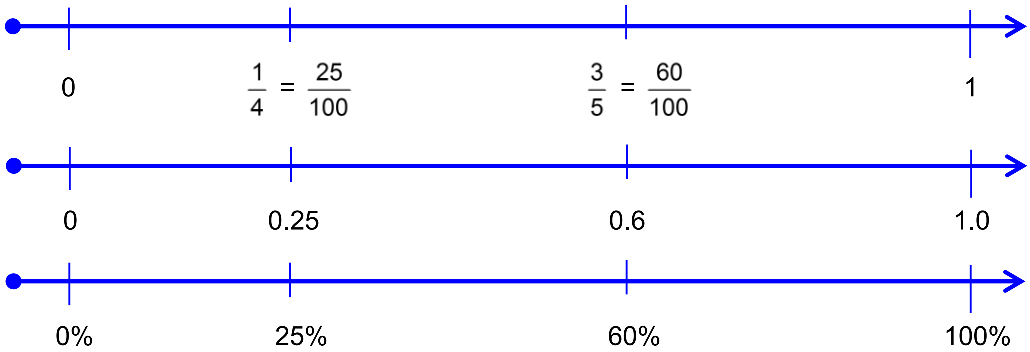
For example:

You cannot write $\frac{5}{6}$ as an equivalent fraction with a denominator of 100, but you can estimate by finding a denominator that is close to 100.

$$\frac{5}{6} = \frac{85}{102}\text{, so } \frac{5}{6} \text{ is about } 85\%$$

• Number lines are useful for showing how fraction, decimal, and percent relate.

For example:



From these number lines, you can see these relationships:

$\frac{1}{4} = \frac{25}{100} = 0.25 = 25\%$ $\quad \frac{3}{5} = \frac{60}{100} = 0.60 = 60\%$ $\quad 1 = \frac{100}{100} = 1.0 = 100\%$

B. What percent of a ngultrum is each coin in **part A**?

Examples

Example 1 Writing a Ratio as a Percent to Compare

In survey A, 8 out of 10 people said that they were happy.

In survey B, 82% of the people said that they were happy.

Which survey indicates greater happiness?

[Continued]

Example 1 Writing a Fraction and Ratio as a Percent to Compare [Cont'd]

Solution

Survey A: 8 out of 10 Survey B: 82%

8 out of 10 = $\frac{8}{10}$

$$\frac{8}{10} \overset{\times 10}{=} \frac{80}{100}$$

$\frac{80}{100}$ = 80%

Survey A: 80% Survey B: 82%

80% is less than 82%, so

Survey B indicates greater happiness.

Thinking

- I changed the ratio for Survey A to a percent.
- It was easy to compare the survey results when they were both in percent.

Example 2 Writing Common Fractions as Percent

Order these fractions from greatest to least: $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{10}$

Solution

$\frac{1}{2} = \frac{50}{100}$ = 50%

$\frac{1}{4}$ = 25%

$\frac{3}{4} = 3 \times \frac{1}{4} = 3 \times 25\% = 75\%$

$\frac{1}{5} = \frac{20}{100}$ = 20%

$\frac{2}{5} = 2 \times \frac{1}{5} = 2 \times 20\% = 40\%$

$\frac{3}{10} = \frac{30}{100}$ = 30%

The percent from greatest to least:

75% , 50% , 40% , 30% , 25%, 20%

The fractions from greatest to least:

$\frac{3}{4}, \frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{1}{4}, \frac{1}{5}$

Thinking

- I changed all of them to percent because I find it easier to order percent. It's hard to order fractions when the denominators are not all the same.
- I was able to use my results for one fraction to figure out the percent for other fractions:

  - I used $\frac{1}{4}$ = 25% to figure out $\frac{3}{4}$.

  - I used $\frac{1}{5}$ = 20% to figure out $\frac{2}{5}$.

- I wrote the percent in order and then wrote the matching fractions in order.

Example 3 Estimating Fractions as Percent

Estimate each fraction as a percent. **a)** $\frac{1}{3}$ **b)** $\frac{2}{3}$ **c)** $\frac{1}{6}$

Solution 1

a) $\frac{1}{3} = \frac{33}{99}$, so $\frac{1}{3}$ is about 33%.

b) $\frac{2}{3} = 2 \times \frac{1}{3}$, so $\frac{2}{3}$ is about 66%.

c) $\frac{1}{6} = \frac{1}{3} \div 2$, so $\frac{1}{6}$ is about 17%.

Thinking

a) I wrote an equivalent fraction with a denominator as close as possible to 100.

b) I used my estimate for $\frac{1}{3}$ to estimate a percent for $\frac{2}{3}$.

c) I used my estimate for $\frac{1}{3}$ to estimate $\frac{1}{6}$.

Solution 2

a) $\frac{1}{3}$ = 0.333... or about 0.33 = $\frac{33}{100}$ = 33%

b) $\frac{2}{3}$ = 0.666... or about 0.67 = $\frac{67}{100}$ = 67%

c) $\frac{1}{6}$ = 0.166... or about 0.17 = $\frac{17}{100}$ = 17%

Thinking

- I changed each fraction to an equivalent decimal.
- I rounded each repeating decimal to the nearest hundredth.

Example 4 Estimating Percent as Fractions

Estimate a fraction for each percent. Use a denominator of 12 or less.

a) 31% **b)** 42% **c)** 65%

Solution

a) 33% is about $\frac{1}{3}$, so 31% is close to $\frac{1}{3}$.

b) 20% = $\frac{1}{5}$, so 40% = $\frac{2}{5}$, so 42% is close to $\frac{2}{5}$.

c) 67% is about $\frac{2}{3}$, so 65% is close to $\frac{2}{3}$.

Thinking

- For each percent, I found a percent that was close and had a fraction that I knew.
- I know the fractions for 33%, 20%, and 66%.

Practising and Applying

1. Write each decimal as a percent.

a) 0.47 **b)** 0.63 **c)** 0.05 **d)** 0.8

2. Write each percent as a fraction and as a decimal.

a) 75% **b)** 24% **c)** 90%

d) 1% **e)** 2% **f)** 35%

3. Write each fraction as a decimal and as a percent.

a) $\frac{1}{4}$ **b)** $\frac{3}{5}$ **c)** $\frac{7}{10}$ **d)** $\frac{1}{25}$

e) $\frac{4}{25}$ **f)** $\frac{7}{50}$ **g)** $\frac{1}{20}$ **h)** $\frac{11}{20}$

4. Draw a number line from 0 to 1. Mark on it each number below. (Two or more numbers may be at the same mark.)

a) $\frac{1}{5}$ **b)** 30% **c)** 0.125

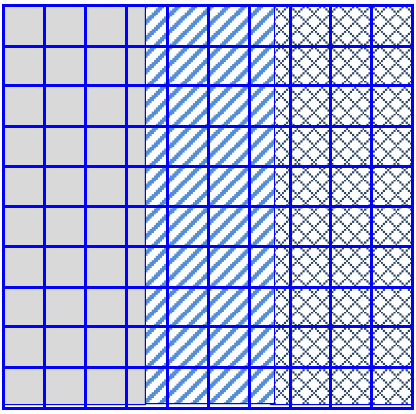
d) $\frac{1}{2}$ **e)** 75% **f)** 20%

g) 100% **h)** $\frac{3}{4}$ **i)** 0.444…

j) $\frac{1}{3}$ **k)** 0.01 **l)** 0.8

5. About what percent of the grid does each region represent? How do you know?

Region 1 Region 2 Region 3



6. Estimate a percent for each.

a) $\frac{5}{6}$ **b)** $\frac{1}{9}$ **c)** $\frac{4}{9}$ **d)** $\frac{1}{11}$

7. Estimate a fraction for each. Use a denominator of 12 or less.

a) 59% **b)** 76% **c)** 16% **d)** 83%

8. Use 5% = $\frac{1}{20}$ to help you write a fraction for each percent.

a) 35% **b)** 45% **c)** 65% **d)** 95%

9. In Chabilal's class, there are 3 boys for every 2 girls. In Rinzin's class, 55% of the students are boys. Whose class has a higher percent of boys?

10. A class has 25 students. 9 are playing football and the rest are practising archery.

a) What percent of the class is playing football?

b) What percent is practising archery?

11. There are 30 players in a table tennis tournament. Only 8 will make it to the final round. Estimate the percent of players that will make the finals. Explain how you estimated.

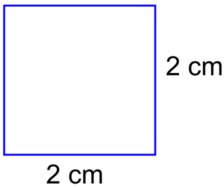
12. Of Bhutan's Gross Domestic Product (GDP), it is estimated that

- about $\frac{1}{3}$ comes from agriculture
- about 40% comes from industry
- about 0.27 comes from services

a) Order the sources of Bhutan's GDP from least to greatest.

b) Explain how you ordered.

13. This square is 20% of a larger shape.



a) Draw 60% of the larger shape. How do you know it is 60%?

b) Draw the larger shape.

c) Is there more than one way to draw these figures? Explain your thinking.

14. Why might you use equivalent percent to order $\frac{17}{20}$, $\frac{2}{3}$, and $\frac{6}{10}$, but not to order $\frac{1}{15}$, $\frac{1}{16}$ and $\frac{1}{17}$?

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CONNECTIONS: The Golden Ratio

The golden ratio is a special ratio that is used in art, architecture, and in mathematics. A useful approximation for the golden ratio is 8 to 5.

• Artists and architects have created many works using the golden ratio. They use the ratio to design golden rectangles that have a ratio of length to width that is about 8 units to 5 units. People seem to find rectangles with these dimensions pleasing to the eye.

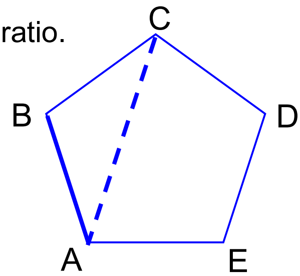
For example:

The United Nations building in New York has an approximate height to width ratio equal to the golden ratio.

• There are many mathematical ideas that have the golden ratio.

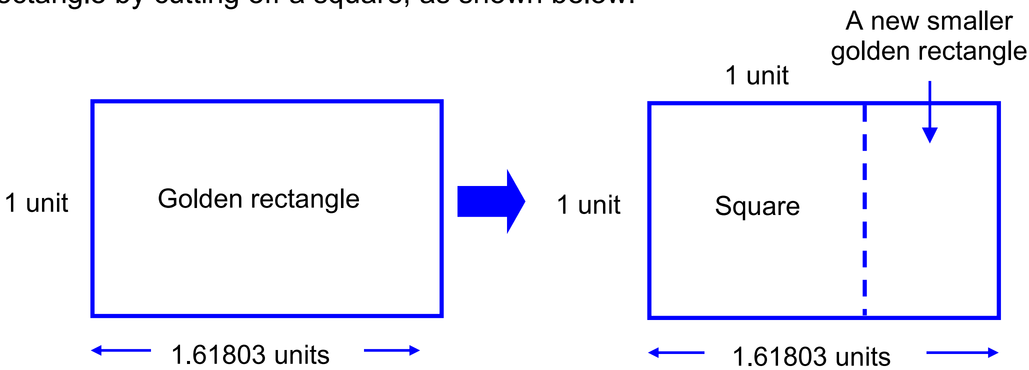
One is the ratio of the length of a diagonal of a regular pentagon to the length of a side of that pentagon.

In this pentagon, the ratio of the length of AC to the length of AB is the golden ratio, AC : AB.



• The golden ratio can be written as a decimal, but it has to be rounded because it does not end. Rounded to five decimal places, the golden ratio is 1.61803. (The ratio 8 : 5 = 1.6, so it is close to the golden ratio.)

• A unique feature of the golden rectangle is that you can make a smaller golden rectangle by cutting off a square, as shown below:



1. Measure AB and AC in the pentagon at the top of the page. Divide AC by AB. Do you get a value that close to 1.61803? Show your work.

2. How would you make the next smaller golden rectangle in the diagram above?

3. Find some examples of the golden ratio in architecture and art.

GAME: Ratio Concentration

This game is for 2 or 3 players.

You will need a set of Ratio Concentration Cards.

Here is how to play:

- Shuffle the cards and place them face down in a 6 by 5 array.
- On your turn, flip over any two cards.

\- If the ratios, percent, fractions, or decimals shown are equivalent, you keep those cards and take another turn.
\- If the cards are not equivalent, flip them face down. Your turn is over.

- Play continues until all the cards have been turned over and matched.

The player with the most cards at the end wins.

For example:

1 : 1 and 100% are equivalent. The player keeps these cards and

3.2.3 Estimating and Calculating Percent

Try This

A strip of paper is 20 cm long.

20 cm

A. What would be the length of each?

i) 25% of the strip

ii) 10% of the strip

iii) 40% of the strip

iv) 80% of the strip

• You can use familiar **benchmark percent** to estimate and calculate. Benchmark percent are percent that are easy to work with mentally, such as 10%, 25%, 50%, and 75%.

For example:

Passang received a mark of 70% on a test with 40 questions. Each question had the same value. How many questions did he answer correctly?

Estimate first

70% is about 75%. A mark of 75% would mean $\frac{3}{4}$ of the answers were correct:

25% of 40 = $\frac{1}{4}$ of 40 = 40 ÷ 4 = 10

75% of 40 = 3 × 25% = 3 × 10 = 30

Passang answered about 30 questions correctly.

Calculate the exact answer

Change 70% to a decimal and multiply.

70% = 0.70

70% of 40 = 0.70 × 40 = 28

Passang answered 28 questions correctly. This is close to the estimate of 30.

Calculate the exact answer a different way

Another way to find 70% of 40 is to find 10% of 40 and then multiply by 7.

10% of 40 = 40 ÷ 10 = 4 → 7 × 4 = 28

• Sometimes it is useful to first find the **unit percent** (1%) and then use it to find other percent. A percent table can help.

For example:

In 200 mL of milk, 87% is water. The rest is milk solids: 7% sugar, 4% protein, and 2% fat. How many millilitres are there of each milk solid?

Find the unit percent
How much is 1%?

÷ 100

100%	**1%**	7%	4%	2%
200 mL	**2 mL**			

÷ 100

[Continued]

If 1% is 2 mL, then
7% is 7 × 2 mL = 14 mL

× 7

100%	**1%**	**7%**	4%	2%
200 mL	**2 mL**	**14 mL**		

× 7

If 1% is 2 mL, then
4% is 4 × 2 mL = 8 mL

× 4

100%	**1%**	7%	**4%**	2%
200 mL	**2 mL**	14 mL	**8 mL**	

× 4

If 4% is 8 mL, then
2% is 8 ÷ 2 mL = 4 mL

÷ 2

100%	1%	7%	**4%**	**2%**
200 mL	2 mL	14 mL	**8 mL**	**4 mL**

÷ 2

• Sometimes, you know the percent and you need to find the whole amount (100%). There are different ways to do this.

For example:
Nine students in Yangchen's class play football. That is 25% of the class. How many students are in Yangchen's class?

You could use a percent table

If 25% is 9 students, then
100% is 9 × 4 = 36 students.

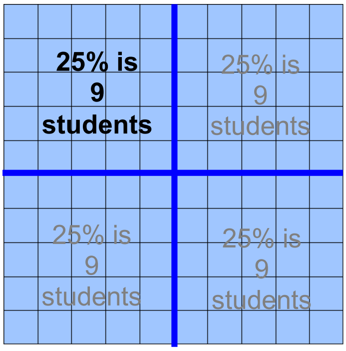
× 4

Percent	**25**	**100**
Number of students	**9**	**36**

× 4

Another way is to use a percent grid

If you divide a 100-grid representing the whole class (100%) into fourths, each is 25%. If 25% is 9 students, then there must be 36 students in the whole class.



B. Use the percents you calculated in **part A** to figure out the length of each percent of the 20 cm strip.

i) 50% **ii)** 20% **iii)** 5% **iv)** 75%

Examples

Example 1 Multiplying to Find Percent

A class has 60 students. 55% of the students are girls. How many girls are there?

Solution

Estimate

50% of 60 would be $\frac{1}{2}$ of 60 = 30

Calculate

55% of 60 = 0.55 × 60 = 33

33 students are girls.

Thinking

- 50% is close to 55%.
- 55% is 0.55 in decimal form, so 55% of 60 = 0.55 × 60.

Example 2 Using a Known Percent to Calculate Other Percent

120 people were surveyed about their favourite sport. Here are the results:

Archery	Football	Running	Cricket	No favourite
35%	30%	20%	10%	5%

How many people chose each sport?

Solution

If 100% is 120, then 10% is 12.

(÷ 10)

Percent (%)	**100**	35	30	20	**10**	5
Number	**120**				**12**	

(÷ 10)

If 10% is 12, then 20% is 24, 30% is 36, and 5% is 6.

(× 3)

Percent (%)	100	35	**30**	**20**	**10**	**5**
Number	120		**36**	**24**	**12**	**6**

(× 3)

If 5% is 6, then 35% is 42.

(× 7)

Percent (%)	100	35	**30**	**20**	**10**	**5**
Number	120	**42**	**36**	**24**	**12**	**6**

(× 7)

42 people chose archery, 36 chose football, 24 chose running, 12 chose cricket, and 6 have no favourite sport.

Thinking

- I began by setting up my percent table with all the percent in the question.
- I found 10% first because it was easy to calculate and I could use it to find other percents.
- I used 10% to find 20%, 30%, and 5%.
- I used 5% to find 35%.

Example 3 Calculating Percent in Different Ways

Show three ways to calculate 95% of 60.

Solution

i) 95% of 60 = 0.95 × 60 = 57

ii) 10% of 60 = 6, so 5% of 60 = 3

95% = 9 × 10% + 5% = 9 × 6 + 3 = 57

iii) 10% of 60 = 6, so 5% of 60 = 3

100% of 60 = 60 and 95% = 100% – 5%

So 95% of 60 = 60 – 3 = 57.

Thinking

- For the first way, I multiplied 60 by 0.95 on paper.
- I did the other two ways mentally using familiar percent.

Example 4 Finding the Whole When a Percent is Known

In a survey, 30 students chose momos as their favourite snack.
This was 15% of the group surveyed. How many students were in the group?

Solution 1

If 15% of the group is 30 students, then
1% of the group is 2 students.

÷ 15

Percent (%)	15	1	
Number of students	30	2	

÷ 15

If 1% of the group is 2 students, then
100% of the group is 200 students.

× 100

Percent (%)	15	1	100
Number of students	30	2	200

× 100

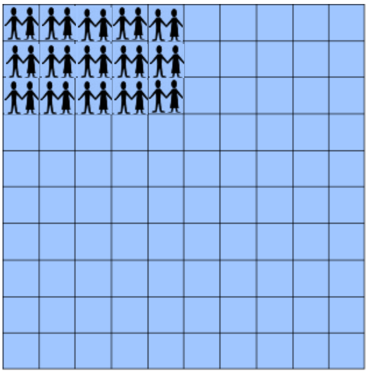
200 students were in the group.

Thinking

- I knew the whole group was 100%.
- I found the unit percent (1%) first and then used it to find 100%.

Solution 2

If 15 squares represent 30 students, then
1 square represents 2 students.



If 1 square represents 2 students, then
100 squares represent 200 students.

Thinking

- I used a 10 by 10 grid to represent the whole group.
- I figured out how many students each square represented and used that to figure out how many students the whole grid represented.

Practising and Applying

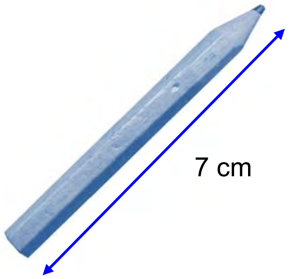
1. Calculate.

a) 10% of 300 **b)** 20% of 50

c) 1% of 400 **d)** 18% of 150

e) 22% of 350 **f)** 95% of 300

2. A used pencil is 7 cm long. This is 50% of the length of a new pencil. What is the length of a new pencil?



3. a) Tshering Dhendup played in an archery tournament. He shot 90 arrows and 40% were kareys. How many kareys did he hit?

b) 10% of his arrows were dobjeys. How many dobjeys did he hit?

4. This chart shows how some teens in Thimphu spend their leisure time.

Activity	Time spent (%)
Reading	5
TV	15
Games	70
Song and dance	5
Other	5

If teens have about 80 h a month of leisure time, how much time would they spend on each activity in a month?

5. a) Changlimithang National Stadium in Thimphu currently seats about 15,000 people. How many people would be in the stadium at each capacity?

i) 10% full **ii)** 50% full

iii) 75% full **iv)** 90% full

b) The stadium is being renovated to seat 30,000 people. How many people would be in the stadium at each capacity in **part a)**?

6. Thimphu gets an average annual precipitation of about 650 mm.

a) About 3% is in March and about 33% is in August. Estimate how much precipitation falls each month. Explain how you estimated.

b) The average annual precipitation in Thimphu is about 11% of the average in Southern Bhutan. Estimate how much precipitation Southern Bhutan gets annually. Explain how you estimated.

7. a) Find 50% of 78. Then find 78% of 50. What do you notice?

b) Find 25% of 90. Then find 90% of 25. What do you notice?

c) Create two examples like **parts a) and b)**. Were the results the same?

8. Suppose you know 25% of a number. Describe two ways you could find 15% of the same number.

3.2.4 EXPLORE: Representing Numbers Using Percent

The number 30 can be written as different percent of different numbers.

For example: 30 is 1% of 3000.
30 is 2% of 1500.
30 is 3% of 1000.

A. i) Find as many percents of different numbers as you can for 30.
Use a chart like this to organize your answers. Use only whole numbers.

	Percent	Number
30	1	3000
30	2	1500
30	3	1000
30	4	

ii) Now find as many percent of different numbers as you can for 25.

	Percent	Number
25	1	
25	2	
25		

B. 50 is 50% of 100 but it is 25% of 200.

i) How are the two percent, 50% and 25%, related?
How are the two numbers, 100 and 200, related?

ii) Suppose 40 is 20% of Number A but 40% of Number B.
Which number is greater, Number A or Number B? How do you know?

iii) How are Numbers A and B related?

C. Use the idea in **part B** to tell how these numbers are related.

i) 33 is 30% of Number C but 90% of Number D.

ii) 48 is 50% of Number E but 25% of Number F.

iii) 4 is 8% of Number G but 24% of Number H.

D. Create your own question like **part C**. Share it with another student to solve.

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UNIT 3 Revision

1. a) An Ema Datshi recipe calls for 10 mL of vegetable oil for 4 servings. How much vegetable oil is needed for each?

i) 8 servings **ii)** 2 servings

b) How many servings can be made with 25 mL of vegetable oil?

2. The ratio of boys to girls in a class is 4 : 3.

a) If there are 24 boys in the class, how many girls are there?

b) How many boys and girls are in the class if there are 35 students?

3. Another class has twice as many boys for every 3 girls as the class described in **question 2**.

a) If there are 9 girls in the class, how many boys are there?

b) How do you know there are not 35 students in the class?

4. a) Write two equivalent ratios for 15 : 25. One ratio should include 45 as a term. Another ratio should include 100 as a term.

b) How did you decide which term in the ratio each would be?

5. The ratio of flour to sugar in a recipe is 5 : 2. If you use a whole number of cups of flour, can you use 3 cups of sugar? Explain your thinking.

6. Which is a better price per apple for the buyer?

9 apples for Nu 60

or

1 dozen apples for Nu 60

How do you know?

7. a) If 3 oranges cost Nu 15, what is the cost of each?

i) 6 oranges

ii) 1 dozen oranges

b) Express the cost as a unit rate.

c) How could you use the unit rate to find the cost of 8 oranges?

d) How else could you find the cost of 8 oranges?

8. It takes the staff 7.2 h to clean all 27 rooms in a hotel. How many hours would it take them to clean 15 rooms?

9. You know how much three chocolate bars cost. How could you figure out the cost of two chocolate bars?

10. a) Draw three shapes on a 10 by 10 grid to match these descriptions.

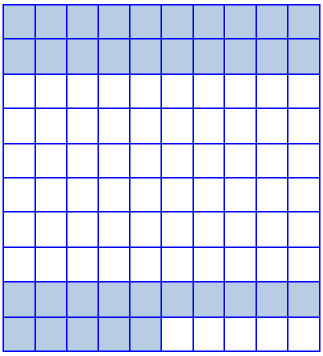
- The first shape covers 40% of the grid.
- The second shape covers 25%.
- The third shape covers 20%.

b) How much of the grid is not covered if the three shapes do not overlap?

11. a) What percent of the grid below is shaded?

b) What percent of the grid is not shaded?

c) Explain how you could use your answer to **part a)** to answer **part b)**.



12. What percent of the numbers in the 100 chart below are each?

a) multiples of 3 **b)** greater than 75

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	6	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

13. Order from least to greatest.

$\frac{1}{4}$, 90%, 0.55, $\frac{3}{5}$, 20%, 0.75

14. Calculate.

a) 5% of 60 **b)** 25% of 52

c) 1% of 200 **d)** 27% of 20

15. Copy and complete.

a) 28 is ___% of 35.

b) 72 is ___% of 600.

c) 24 is 8% of _____.

d) 30 is 6% of _____.

16. 186 boarders represent 62% of the students who attend a school. How many students attend the school?

17. In the Population and Housing Census of Bhutan for 2005, the following data was collected about the state of happiness for people living in rural areas:

- about 45% were "Very Happy".
- about 50% were "Happy".
- the remainder were "Not Very Happy".

a) About what percent were "Not Very Happy"?

b) If 300 people live in a rural area that has these same percent, how many people in that area are "Very Happy"?

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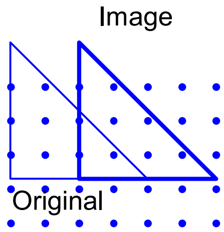
UNIT 4 GEOMETRY AND MEASUREMENT

Getting Started

Use What You Know



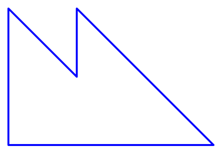
A triangle was drawn on 1 cm square dot grid paper.

It was translated 2 cm to the right.

When the original and image triangles were combined, they made a pentagon.

A. i) What is the area of the original triangle?

ii) What is the area of the pentagon?



Combined shape

B. i) Draw three different triangles on 1 cm dot paper, each with an area of 12 cm^2.

ii) Which triangle has the largest angle? Mark this angle with the letter L.

iii) Which has the smallest angle? Mark this angle with the letter S.

C. i) Translate, rotate, or reflect one of your triangles so that the original shape and its image overlap.

• Describe the transformation.

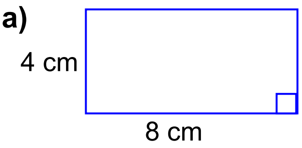
• Identify the new combined shape.

ii) How could you change your transformation so that the combined shape would have a greater area?

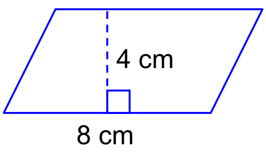
Skills You Will Need

1. Determine the area of each. Show your work.

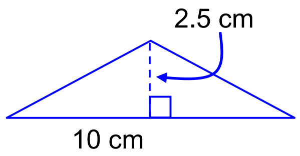
a)



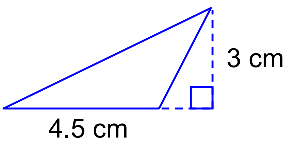
b)



c)

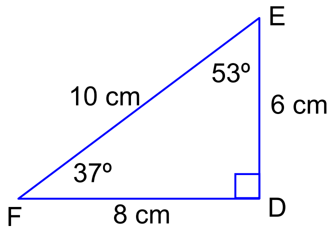


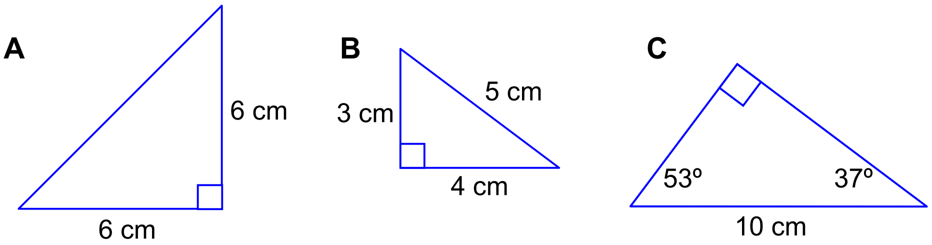
d)



2. Which shapes in **question 1** have one or more lines of symmetry? Draw them on centimetre grid paper and sketch all the lines of symmetry.

3. For each triangle below, tell whether it is similar or congruent to ΔDEF or neither. Explain your thinking.



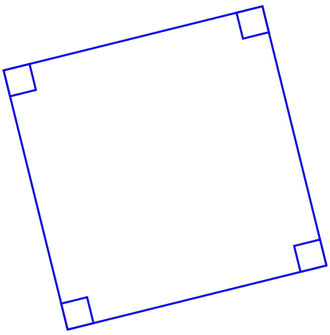


4. Calculate each dimension. Show your work.

a) the length, or base, of a rectangle with area 35 cm^2 and height 14 cm

b) the height of a triangle with area 30 cm^2 and base 10 cm

5. Measure this square and calculate its area. Show your work.

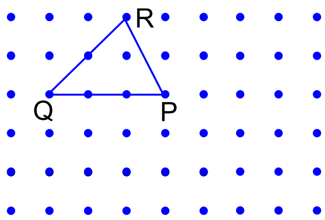


6. Copy ΔPQR onto grid paper or square dot paper.

a) Translate ΔPQR 1 unit down.

b) Reflect ΔPQR using PQ as the reflection line.

c) Rotate ΔPQR 180° clockwise around vertex P.



7. Complete each statement.

a) 5.2 cm = ■ mm **b)** 5.2 cm = ■ m **c)** 0.05 kg = ■ g

8. Calculate.

a) 0.403 × 10,000 **b)** 7200 × 0.001

c) 6000 ÷ 0.01 **d)** 5300 ÷ 1000

4.1.1 EXPLORE: Angles in a Triangle

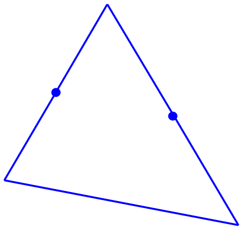
What is the sum of angles in a triangle?

- Take a piece of paper (about half the size of this page). Cut out a scalene triangle from the paper.
- Measure the angles.
- Add the 3 angles. What is the sum?

You can test the relation between the three angles by folding your triangle into a rectangle in three folds.

Step 1

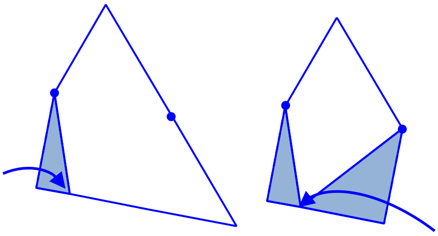
Mark the middle of the two shortest sides of your triangle. (You can find the middle of a side by folding it in half or by measuring.)



Step 1

Step 2

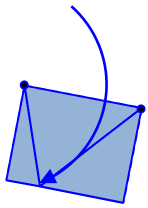
Fold the two bottom **vertices** inwards, so that each crease starts at the middle of the side and the vertex lies along the base of the triangle.



Step 2

Step 3

Fold the third vertex down to meet the other two vertices.



Step 3

The three angles form a straight angle.
What is the angle measure in a straight angle?

From the two processes above you see that the sum of angles in a triangle is 180°.

A. Repeat the process explained above for an

i) **isosceles triangle**.

ii) **equilateral triangle**

iii) What do you notice about the sum of angles in both the triangles?

B. Use the scalene triangle that you used before.

i) On your scalene triangle, label the largest angle L and the longest side l. Label the smallest angle S and the smallest side s.

- What do you notice about the longest side and largest angle?
- What do you notice about the shortest side and smallest angle?

ii) Repeat **part B i)** for your isosceles triangle.

iii) Can you not repeat **part B i)** for your equilateral triangle? Why?

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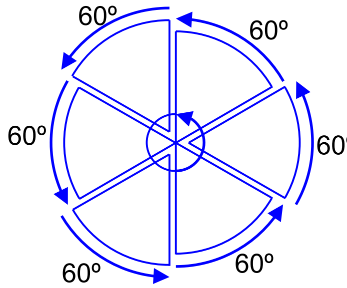
CONNECTIONS: Angle Measurement Units

There are different ways of measuring angles. The degree is just one unit of measurement.

Degrees

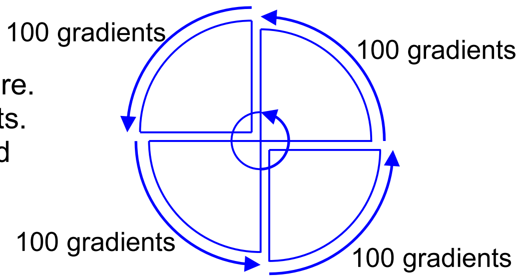
The ancient Babylonians decided that a full rotation should have 360 degrees (360º), perhaps because 360 is close to the number of days in a year.

An advantage of the number 360 is that it has lots of factors. It is divisible by 2, 3, 4, 5, 6, 8, 9, 10, and so on.



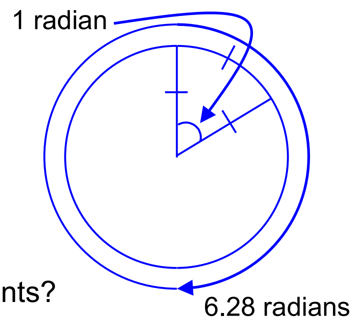
Gradients

The British military used a different measure. It is based on a right angle having 100 units. The steepness of a road is often measured in gradients.



Radians

A radian is another unit for measuring angles. It is the angle formed by two arms of equal length (radii of the circle) and an arc of the same length on the circumference. A full rotation is about 6.28 radians.



1. A right angle is 100 gradients.

a) What is the sum of the angles in a triangle in gradients?

b) What is a full rotation in gradients?

c) What is the size of each angle in an equilateral triangle in gradients? Show your work.

2. a) List all the ways you can divide a circle into congruent angles of a whole number of degrees.

b) List all the ways you can divide a circle into congruent angles of a whole number of gradients.

3. A full rotation is about 6.28 radians.

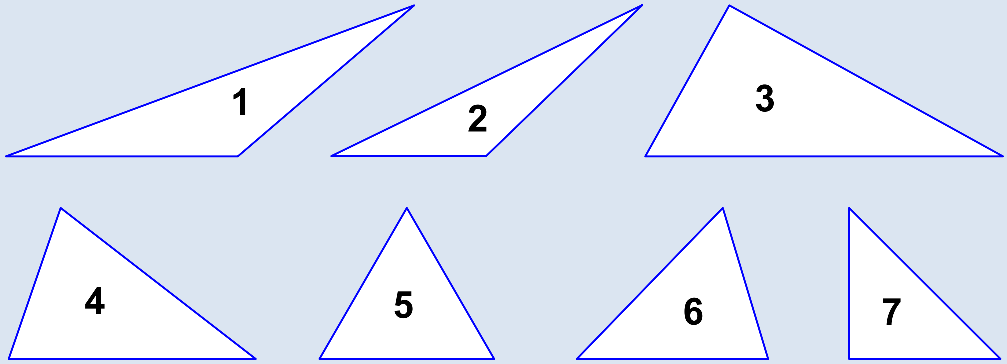
a) What is the sum of the angles in a triangle in radians? Show your work.

b) What is the size of a right angle in radians? Show your work.

4.1.2 Drawing and Classifying Triangles

Try This

Penjor Wangdi cut seven triangles out of paper.



A. Trace the triangles.

i) On each triangle, mark all the equal sides and angles.

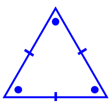
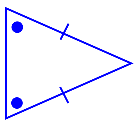
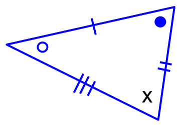
ii) Use the markings to help sort the triangles. Describe how you sorted.

• In the last lesson, you saw that some things are always the same in triangles:

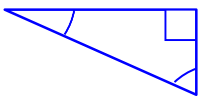
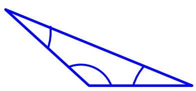
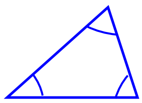
- In *all* triangles, the sum of the angles is 180°.
- The largest angle in a triangle is *always* across from the longest side.
- The smallest angle is *always* across from the shortest side.

• There are also ways that triangles can be different. These differences are used to classify triangles.

Classifying by Side Length

Equilateral triangle	Isosceles triangle	Scalene triangle
Three **congruent** sides  (There are also three congruent angles.)	Two congruent sides  (There are also two congruent angles.)	No congruent sides  (There are no congruent angles.)

Classifying by Angle

Right triangle	Obtuse triangle	Acute triangle
Largest angle is a **right angle** 	Largest angle is an **obtuse angle** 	Largest angle is an **acute angle** 

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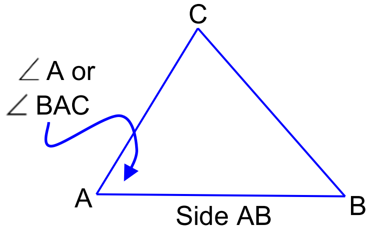
Labelling diagrams and identifying triangle parts



- A corner of a triangle is called a vertex (plural is vertices) and is usually labelled with a capital letter.
- A side is described by the vertices it joins, such as side AB.
- An angle is described by its vertex using the symbol ∠, such as ∠A. The three vertices that form an angle can also be used, so ∠A could also be described as ∠BAC. (Notice that A is the middle letter.)

Drawing angles

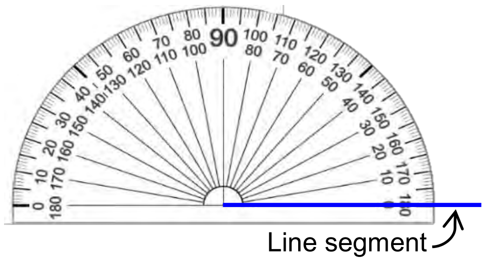
Sometimes you need to draw a triangle with a certain angle measure.

For example, to draw a triangle with an angle of 65°:

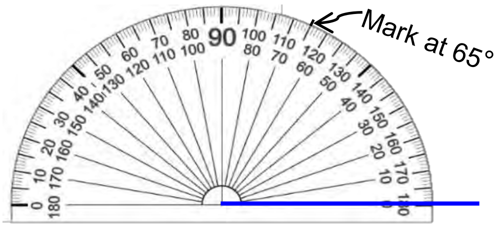
- Draw a **line segment** (one of the sides).

- Place the protractor on the line segment.

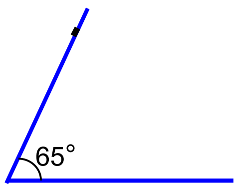
  - One of the zero lines should be on top of the line segment.
  - The centre of the protractor should be at an endpoint.



- Mark the angle using the appropriate scale.

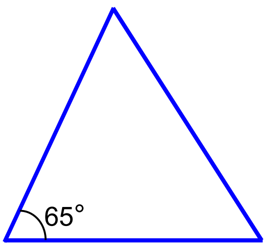


- Take out the protractor and draw a line from the end point through your mark.



- Complete the triangle.

If the other angles need to have certain angle
measures, construct those angles following
the same steps.



B. Are there any triangles in **part A** that fit each description below?

i) a triangle that is both isosceles and acute

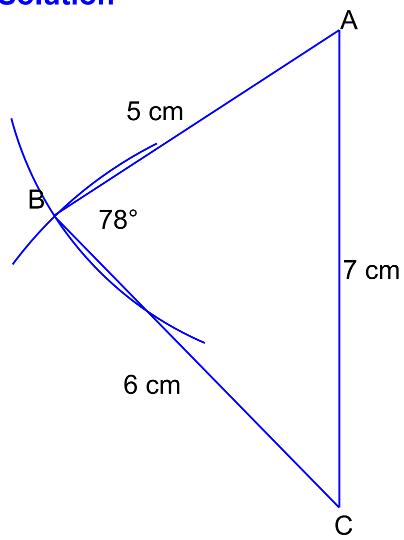
ii) a triangle that is both equilateral and obtuse

Examples

Example 1 Drawing and Classifying a Triangle

Draw and classify ∆ABC with AB = 5 cm, BC = 6 cm, and AC = 7 cm.

Solution



∆ABC is an acute scalene triangle.

Thinking

- I could tell it was scalene by the different side lengths.
- To draw the triangle:
  - I used a ruler to draw a 7 cm line segment for the longest side, AC.
  - I set my compass to 6 cm and made an arc from C because CB = 6 cm.
  - I set my compass to 5 cm and made an arc from A because AB = 5 cm.
  - I labelled the point where the arcs intersected B.
  - I connected A to B and C to B.
- I measured the largest angle, ∠B, which was across from the longest side, AC.

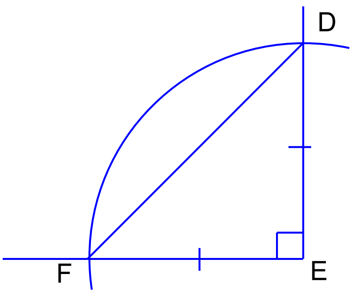
Example 2 Reasoning about Triangle Classification

Is it possible for a right triangle to also be isosceles? How do you know?

Solution

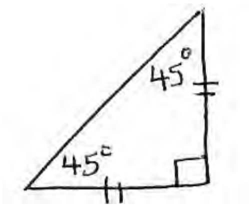
Yes, because I was able to draw one, ∆DEF.

- I drew a right angle using my protractor and a ruler. I labelled the vertex E.
- I used a compass to mark two points, D and F, on the arms of the angle. Each was an equal distance from the vertex.
- I connected D and F.



Thinking

- Before I tried to draw it, I thought about whether it was possible.
- I knew an isosceles triangle had two equal angles and a right triangle had a 90° angle.
- I also knew the sum of the angles of a triangle was 180°.
- So, in a right isosceles triangle, there had to be a 90° angle and two 45° angles.
- I sketched the triangle and it looked possible. My sketch also helped me draw the triangle.



Practising and Applying

1. A triangle has a 30° angle and a 55° angle. What is the other angle? How do you know?

2. Is it possible for a triangle to have an 80° angle and a 105° angle? How do you know?

3. a) Draw ΔABC with AB = 4 cm, BC = 7.5 cm, and AC = 8.5 cm.

b) Classify the triangle by angle and side length.

4. Draw each triangle below. Classify each by angle and side length. Show the lines of symmetry.

a) ΔXYZ with ∠X = 40°, XY = 5 cm, and XZ = 5 cm

b) ΔPQR with ∠P = 90°, PQ = 8 cm, and PR = 8 cm

c) ΔDEF with ∠E = 120°, DE = 7 cm, and EF = 7 cm

5. Use the results from **question 4** to predict the number of lines of symmetry in any isosceles triangle.

6. Sketch an obtuse triangle that fits each classification. If it is not possible, explain why.

a) isosceles triangle

b) equilateral triangle

c) right triangle

7. Sketch an acute triangle that fits each classification. If it is not possible, explain why.

a) scalene triangle

b) isosceles triangle

c) equilateral triangle

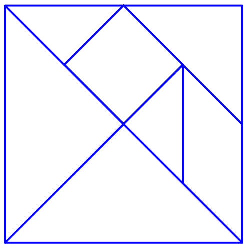
8. Sketch a right triangle that fits each classification. If it is not possible, explain why.

a) scalene triangle

b) isosceles triangle

c) equilateral triangle

9. A tangram has seven pieces: five triangles, one square, and one parallelogram, that fit together to make a square. Make the tangram as shown.



a) Use one or more of the pieces to make five or more different triangles. Sketch each triangle.

b) Classify each triangle from **part a)** by angle and by side length.

c) How are all the triangles the same? How are they different?

10. a) Draw a triangle with a 90° angle, a 45° angle, and a 5 cm side.

b) Classify it by angle and side length.

c) Describe a different triangle that fits the description in **part a)**.

11. Tandin is making triangles using drinking straws for the sides. For each set of straw lengths below, classify the triangle they will form by angle and side length. If they cannot form a triangle, explain why.

a) 7 cm, 8 cm, 9 cm

b) 5 cm, 5 cm, 8 cm

c) 4 cm, 4 cm, 9 cm

d) 7 cm, 7 cm, 7 cm

e) 5 cm, 6 cm, 11 cm

12. a) Draw ΔABC with AB = 5 cm, ∠B = 40°, and ∠A = 120°.

b) Classify ΔABC by angle and by side length. Explain how you know you are right.

13. Suppose you are given two angles of a triangle. Can you classify it without first drawing it? Explain your thinking.

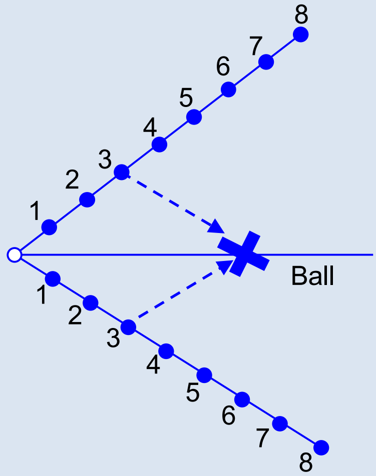
4.1.3 Constructing and Bisecting Angles

Try This

A group of 16 students are playing a ball game. On the floor their teacher draws two equal angles that share a vertex and an arm.

The students are numbered. They stand on the outer arms of the two angles. The teacher places a ball on the shared arm.

When the teacher says a number, the two students with that number race to get the ball.



A. i) If the two Number 3 students are called, will the race be fair? Why?

ii) Where else could the teacher put the ball for the race to be fair for the Number 3 students?

• If you **bisect** something you divide it exactly in half. A line segment or point that bisects something is called the **bisector**.

• You can bisect an angle using a protractor or you can **construct** an **angle bisector**.

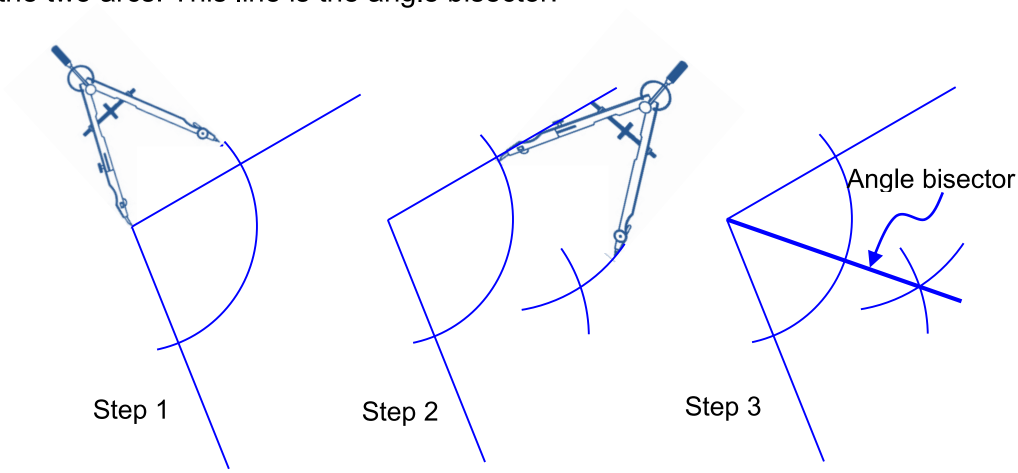
• In mathematics, to construct means to draw using only a straight edge (like a ruler with no markings) and a compass — no other tools. If the instructions have a different word, like **draw**, you can use other tools, like a ruler and protractor. To **sketch** usually means to draw free-hand using estimation.

To construct an angle bisector

Step 1 Put the compass point on the angle vertex and make an arc that intersects both arms of the angle.

Step 2 Keep the compass set at the same distance, and make arcs from both of the intersection points.

Step 3 Draw a line from the angle vertex through the intersection point of the two arcs. This line is the angle bisector.



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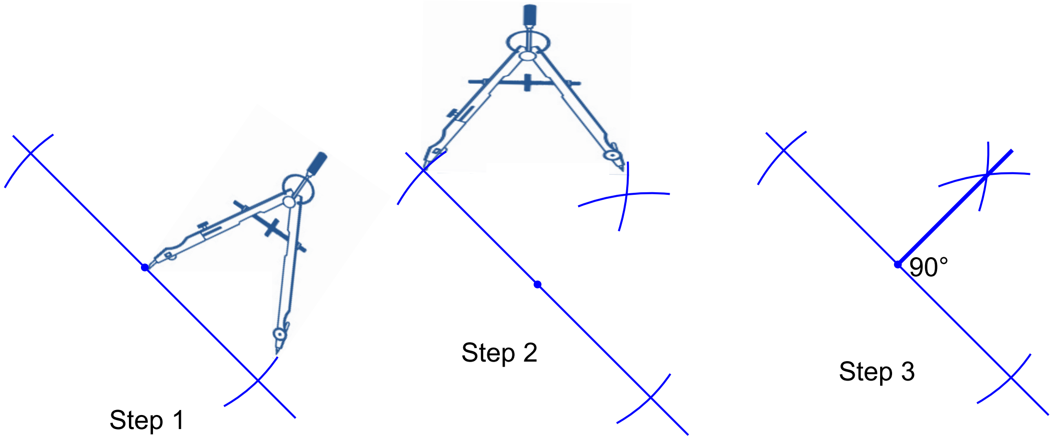
To construct a 90º angle

You can think of this as a special angle bisector because it bisects a **straight angle**.

Step 1 Draw a line segment. Put the compass point on a given point on the line segment. Make two arcs that intersect the line segment. Keep the compass set at the same distance for both arcs.

Step 2 Open the compass up a little and make an arc from each intersection. Keep the compass set at the same distance for both.

Step 3 Draw a line segment through the original given point and the point where the arcs intersect. This new line segment makes a 90° angle with the original line segment and is called a **perpendicular**.



To construct a 60° angle

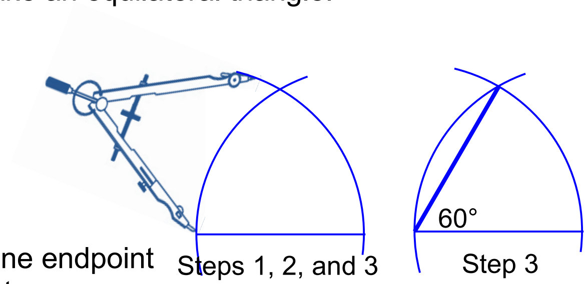
You can use this construction to make an equilateral triangle.

Step 1 Draw a line segment.

Step 2 Set your compass to the same distance as the length of the line segment.

Step 3 Make an arc from each endpoint of the line segment.

Step 4 Draw a line segment from one endpoint to the place where the arcs intersect.

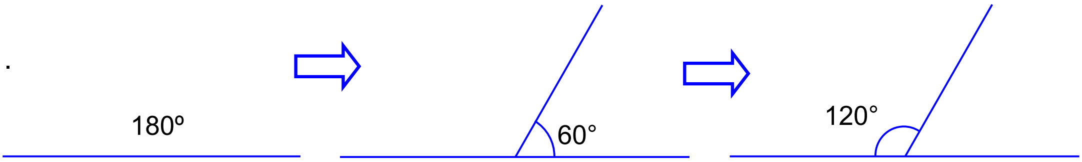


Now that you have a 60º angle and two arms of equal length, you can create an equilateral triangle by drawing a line segment from the other endpoint.

To construct other angles

You can use a combination of constructions to construct other angles.

- To make a 120° angle you can construct two 60° angles (60º + 60º = 120º).
- Or, you can draw a 180º straight angle and then construct a 60° angle in order to "subtract" it from the 180º angle (180º – 60º = 120º).



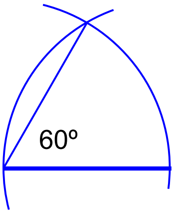
B. If the teacher in **part A** draws the outer arms first, how can she construct the shared arm?

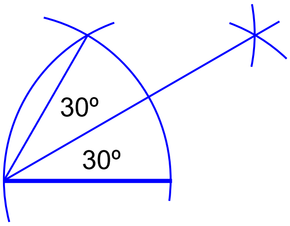
Examples

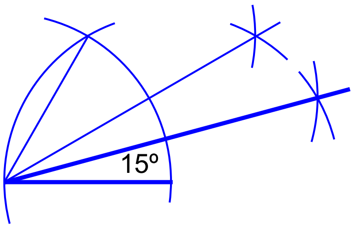
Example 1 Constructing a 15° Angle

Construct a 15° angle.

Solution





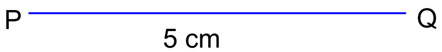


Thinking

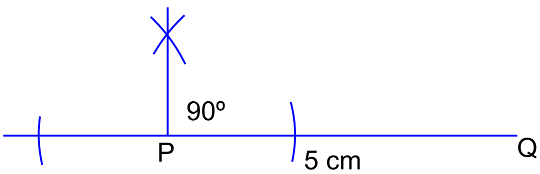
- I constructed a 60° angle.
- I bisected the 60° angle to make two 30° angles.
- I bisected one of the 30° angles to make two 15° angles.

Example 2 Constructing a Triangle Given Two Angles and a Side

Given line segment PQ, construct ΔPQR with ∠P = 45° and ∠R = 75°.



Solution



Thinking

- I extended PQ past P so I could construct a perpendicular at point P.

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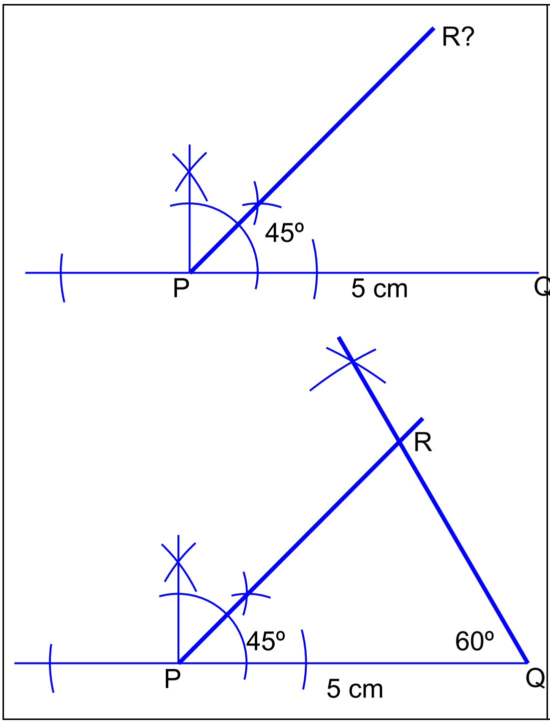
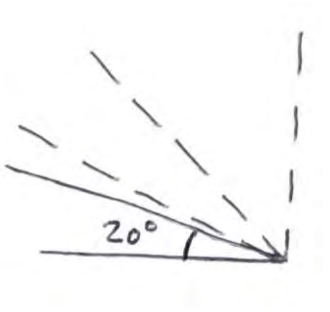
	• I bisected the 90° angle to construct a 45° angle. • I knew ∠R was supposed to be 75° but I didn't know its distance from P so I first constructed ∠Q. • I knew ∠Q was 60° because 180° - 45° - 75° = 60°. • I constructed a 60° angle at Q and extended one of its arms to intersect with the arm from ∠P. • ∠R was at the intersection of the arms.

Example 3 Estimating a 20° Angle

Sketch a 20° angle using estimation.

Solution	**Thinking**
	 • I knew that if I bisected a 90° angle twice I'd have a 22.5° angle (because 90° ÷ 2 ÷ 2 = 22.5). • I sketched a 90° angle by visualizing the corner of a square. • I estimated where the bisector of the 90° angle would be and then sketched it. • Then I estimated where the bisector of one of the 45° angles would be and then sketched it. • Since 20° is a bit smaller than 22.5°, I marked an angle slightly smaller.

Practising and Applying

1. Construct each angle. Show your work.

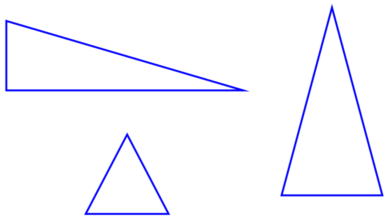
a) 30° **b)** 22.5°

c) 105° **d)** 135°

e) 75°

2. List five other angles you could construct using the 60°, 90°, and angle bisector constructions. Explain how you would make each angle.

3. a) Draw three different triangles — scalene, isosceles, and equilateral.



b) Construct all the angle bisectors in each triangle.

c) What do you notice about the bisectors? Compare your results with several classmates.

4. a) Construct two different (not congruent) isosceles triangles, each with angles 30°, 30°, and 120°.

b) Measure the lengths of the sides in each triangle.

c) Divide the length of the shortest side in the larger triangle by the length of the shortest side in the smaller triangle.

d) Divide the length of the longest side in the larger triangle by the length of the longest side in the smaller triangle.

e) What do you notice? Compare your results with several classmates.

f) Are the triangles similar? How do you know?

5. a) Draw line segment JK = 6 cm. Construct isosceles triangle ΔJKL with ∠J = 90°, ∠K = 45°, and ∠L = 45°.

b) Construct the bisector of ∠K.

c) Label the intersection of the bisector and side JL as point M.

d) Meto says that because KM is a bisector, point M bisects JL. Do you agree? How do you know?

6. a) Estimate to sketch a 15° angle.

b) Describe what you did.

7. a) Estimate to sketch a 125° angle.

b) Describe what you did.

8. a) Draw line segment AB = 9 cm. Estimate to sketch ΔABC with ∠A = 30° and ∠B = 75°.

b) Draw line segment AB = 9 cm. Construct ΔABC with ∠A = 30° and ∠B = 75°.

c) Compare the sketch of ΔABC with the constructed ΔABC. Would you say your sketch is a good estimate?

9. Suppose you have an 18° angle.

a) How could you construct a 9° angle?

b) How could you construct a 39° angle?

c) List two other angles you could construct. Explain how to construct each.

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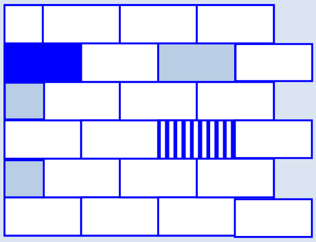
Chapter 2 Transformations

4.2.1 Translations

Try This

Meghraj was tiling a sidewalk with 10 cm by 20 cm rectangular bricks, as shown.



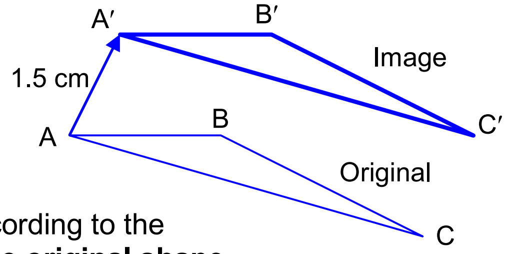
A. How far, and in what direction, would he have to slide
i) the black brick to fill the grey hole?
ii) the striped brick to fill the grey hole?

Moving a shape is called a **transformation**.

• To **translate** a shape, you slide it a certain distance in a certain direction without changing the shape.

For example:

∆ABC has been translated to the right and up 1.5 cm. If it was on grid paper, you would say how many units right and how many units up.



• The vertices on the **image** are named according to the names of their corresponding vertices on the **original shape** using the symbol **'** (read aloud as "prime").

For example:

The original shape is ∆ABC and its image is ∆A′B′C′, read as “A prime, B prime, C prime”.

∠A and ∠A′ are **corresponding angles**. AB and A′B′ are **corresponding sides**.

• A translation can also be described by an arrow that connects any point on the original shape to its image.

For example:

You could describe the translation from ∆ABC to ∆A′B′C′ by saying that it was translated 1.5 cm along arrow AA′ (meaning from A to A′).

Properties of Translations

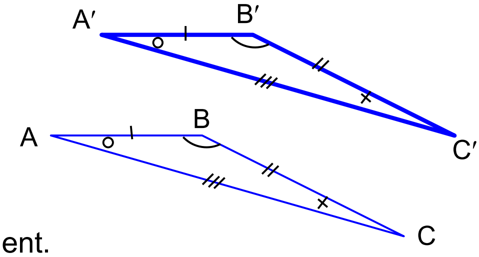
You have already learned some things that are always true about translations:

• Corresponding sides in the original shape and the image are the same length.

AB = A′B′ BC = B′C′ CA = C′A′

• Corresponding angles in the original shape and the image are the same size.

∠A = ∠A′ ∠B = ∠B′ ∠C = ∠C′



• The original shape and the image are congruent.

Here is a new property of translations:

• The **orientation** of the original shape and its image is the same. This means that the vertices in the original shape and the corresponding vertices in the image go in the same direction, **clockwise** or **counter clockwise**.

For example, the vertices in both ∆ABC and its image ∆A′B′C′ go clockwise. That means they have the same orientation.

B. Sketch the rectangle bricks in **part A**. Draw an arrow to represent the translation of the striped rectangle to each.

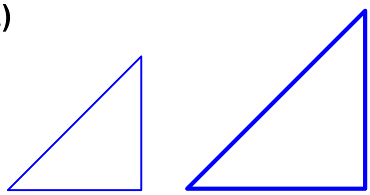
i) the grey rectangle **ii)** the black rectangle

Examples

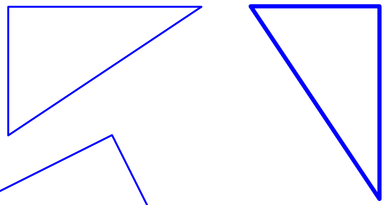
Example 1 Identifying a Translation

For each pair of images, describe how you know whether or not it is a translation.

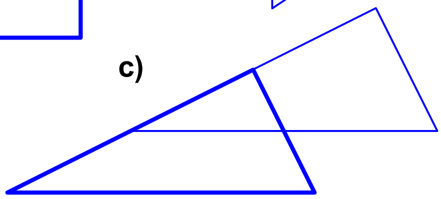
a)



b)



c)



Solution

a) This is not a translation because the triangles are not congruent.

b) This is not a translation because they face different ways.

c) This is a translation because they are congruent and you can slide one to get to the other.

Thinking

a) I noticed one triangle was larger than the other.

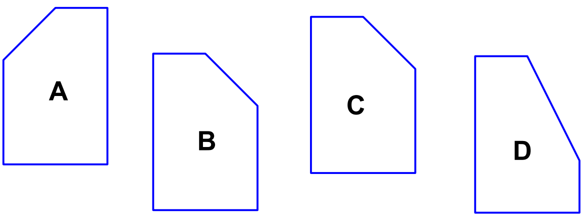
b) I could not slide one triangle onto the other without turning and flipping it.

c) The triangles looked congruent but I measured the angles and sides to be sure.

• The angles went in the same direction so it faced the same way.

Example 2 Describing Transformations

Describe each translation in this set of pentagons.



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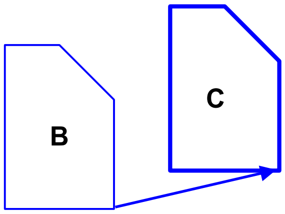
Solution

If B is the original shape and C is the image, the translation is to the right and up, along the arrow shown.



If C is the original shape and B is the image, the translation is to the left and down.

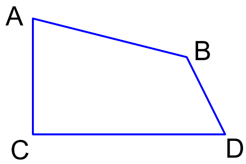
This translation would use the same arrow but in the opposite direction.

Thinking

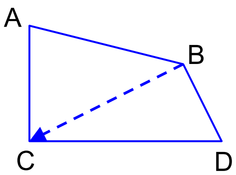
- First I looked for congruent shapes.
- A, B, and C looked congruent but I measured to be sure.
- I knew A was not a translation image of B or C (or vice versa) because it faces the opposite way. You would have to flip it and then slide it.
- I knew B could be translation of C (and vice versa), because they were congruent and had the same orientation.

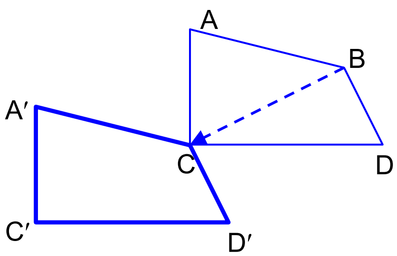
Example 3 Translating a Quadrilateral

Translate the quadrilateral along line segment BC.

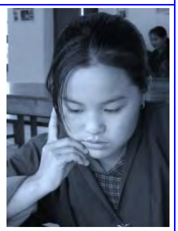


Solution





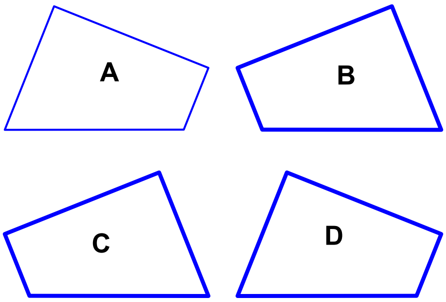
Thinking



- I traced the quadrilateral.
- I drew a line segment from B to C. Since the translation was along BC, I added an arrow to show that it went from B to C.
- I could see that the image of vertex B would end up in the same place as vertex C.
- I traced the quadrilateral again in the new position, with the image of B at C.
- I labelled the image vertices using prime markings. I didn't label the image of B because it is the same as C, but I could have named it B′.

Practising and Applying

1. Tell whether or not B, C, and D are translation images of A.
Explain your thinking.

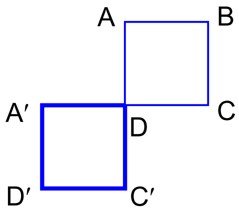


2. a) Draw any square ABCD.

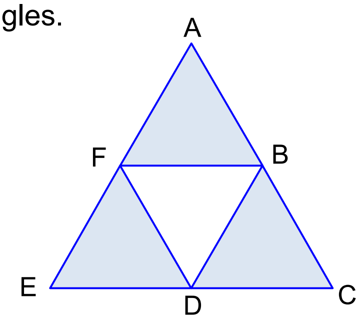
b) Describe all the ways you could translate the square so that one of the image vertices is in the same place as one of the original vertices

For example:

Square ABCD has been translated along DB to A′DC′D′.



3. a) Describe how ΔABF could be translated to create the other two grey triangles.



b) For each translation, which points are the images of vertices A, F, and B?

c) Could ΔFBD be a translation of ΔABF? Explain your thinking.

d) Could ΔACE be a translation of ΔABF? Explain your thinking.

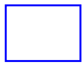
4. a) Draw ΔPQR with base PQ = 6 cm, ∠P = 90°, and PR = 4 cm.
Use constructions to create ∠P.

b) Translate ΔPQR so that Q is the image of P. Describe the translation.

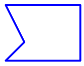
c) Find the area of ΔPQR and its image.

5. Follow these instructions to make a tiling design.

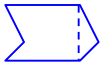
a) Cut out a rectangle.



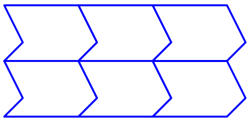
b) Cut out of one side to make it more interesting.



c) Translate the cut-out piece to the opposite side to create a new shape.

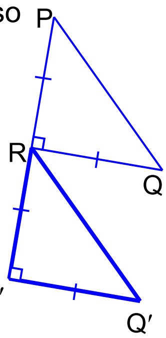


d) Trace your new shape, translate it, trace it again, translate it again, and so on.



e) Why do the shapes fit together?

6. ΔPQR has been translated so that its image has a vertex at one of the original vertices.



a) What arrow describes the translation?

b) Which other translations would move ΔPQR so that its image has a vertex at P, Q, or R?

7. a) Draw a triangle on grid paper and calculate its area.

b) Translate the triangle.

c) Calculate the area of the image. What do you notice?

d) Repeat **parts a) to c)** for another triangle.

8. Why does it make sense that the area of a translation image is the same as the area of the original shape?

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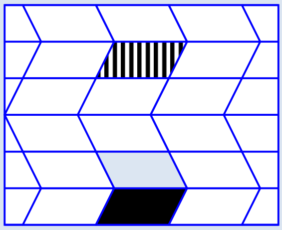
4.2.2 Reflections

Try This

Meghraj made a tiling design with parallelogram bricks.

A. i) How could he move the black brick so that it would fill the grey hole?

ii) How could he move the striped brick so that it would fill the grey hole?

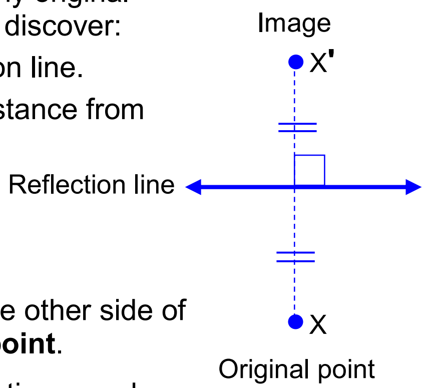


- **Reflections** are transformations that create an image like the one you see in a mirror or on water.

- A reflection is described by showing or locating the **reflection line**, sometimes called a mirror line.
- You have worked with **horizontal** and **vertical** reflection lines on grid paper. Reflection lines can also be diagonal, without using grid paper.
- If you were to draw a line segment connecting any original point in a shape to its reflection image, you would discover:

- The line segment is perpendicular to the reflection line.

- The original point and its image are the same distance from the reflection line.



- To reflect a point:

- Draw a perpendicular line segment from the original point to the reflection line.

- Extend the line segment an equal distance on the other side of the reflection line — the image point is at its **endpoint**.

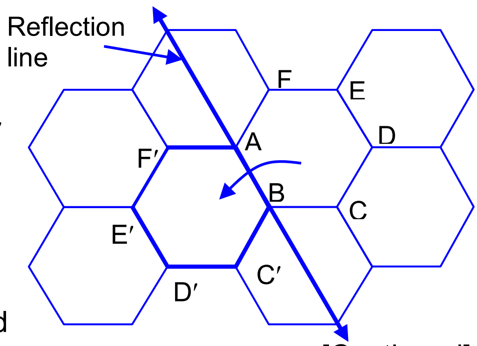
- To reflect a shape, you can reflect each of its vertices as above and then connect them with line segments.
- A reflection line can be outside of a shape, along one of the sides of the shape, or inside the shape.

For example, in this **tiling pattern** of **regular hexagons**, each hexagon can be reflected onto the hexagons around it across a reflection line along one of its sides.

- Hexagon ABCDEF reflects onto ABC′D′E′F′ across the reflection line alongside AB.

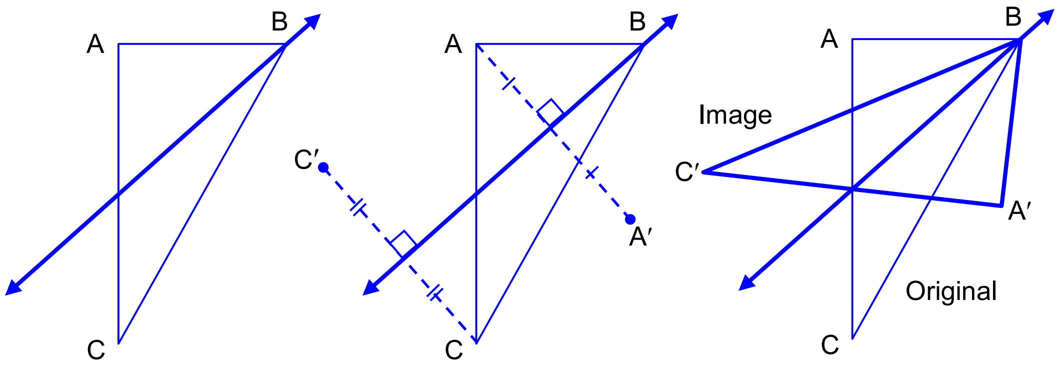
- The points alongside AB do not move because they are on the reflection line.

- Vertex C and its image C' are the same distance from the reflection line, as measured by the perpendicular line segment CC'.



[Continued]

In this example, the reflection line is inside the shape.
- Vertex B does not move because it is on the reflection line.



To locate a reflection line for a shape

• Connect each original vertex and its reflection image with a line segment.

• Mark the **midpoint** of each line segment.

• Draw a line through each midpoint perpendicular to its line segment. This is the reflection line.

If you cannot draw a single line through all three midpoints, the image is not a reflection.

Properties of Reflections

Some things are always true about reflections:

• The original shape and its image are congruent.

• The original shape and its image have opposite orientations. In the example above, the vertices ABC go clockwise, but their image points, A′BC′, go counter-clockwise.

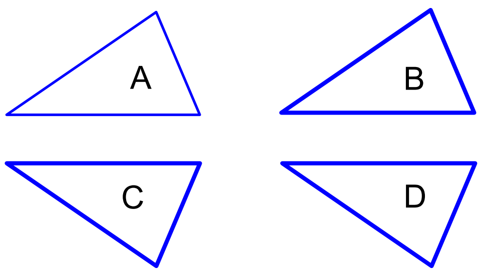
• The reflection line goes through the midpoint of the line segment that connects any point and its image at a 90º angle.

B. i) Describe the reflection lines for the reflections in **part A**.
ii) Is it possible to reflect the black brick onto the striped brick? Explain your thinking.

Examples

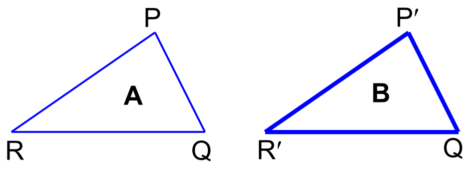
Example 1 Determining Reflections

Which of triangles B, C, and D could be reflections of A?
Explain your thinking.

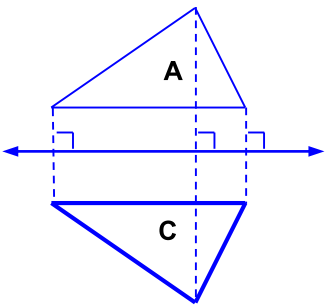


Solution

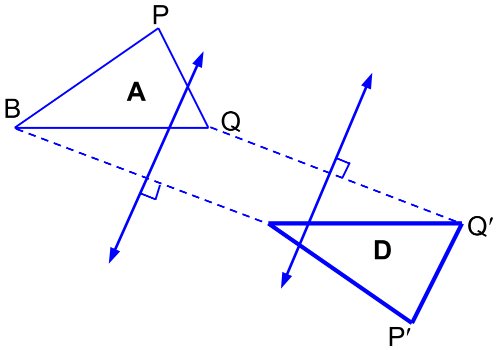
B is not a reflection of A because A has the same orientation as B.



C is a reflection, with the reflection line as shown.



D is not a reflection. The reflection lines between corresponding vertices are not the same.



Thinking

- PQR in A and P′Q′R′ in B both go clockwise, so I knew B could not be a reflection (reflections have an opposite orientation).

- C and D were congruent and had the opposite orientation to A so I knew they could be reflections.
- To check C:

- I drew line segments between corresponding vertices.

- I measured to find their midpoints.

- I drew one perpendicular line through all three midpoints (the reflection line).

- Since the reflection line was the same for all three vertices, I knew C was a reflection of A.

- To check D:

- I drew line segments between two pairs of corresponding vertices.

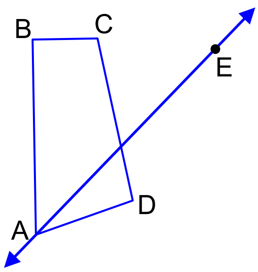
- I measured to find their midpoints.

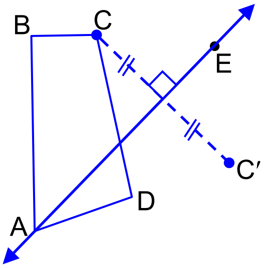
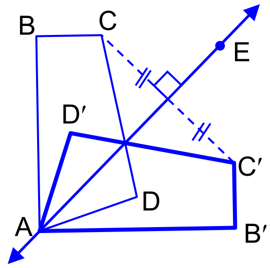
- I drew a perpendicular line (reflection line) through each midpoint.

- Since the reflection lines for Q to Q′ and for R to R′ were different, I knew D was not a reflection of A.

Example 2 Reflecting a Quadrilateral

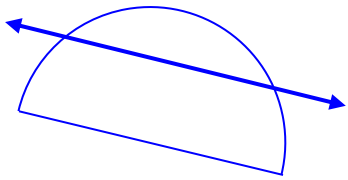
Reflect quadrilateral ABCD in reflection line AE.

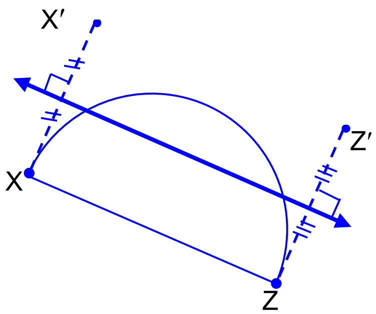
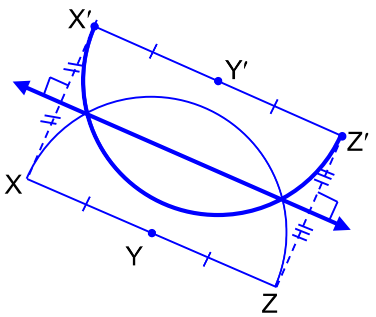


Solution	Thinking
 	 • To locate C′: - I drew a perpendicular line segment from C to AE and measured it. - I continued the line segment beyond AE the same distance and labelled the end point C′. • I located B′ and D′ the same way as C′. • Because A was on the reflection line, I knew that it didn't move. • I connected the image points to create AB′C′D′.

Example 3 Reflecting a Shape with a Curve

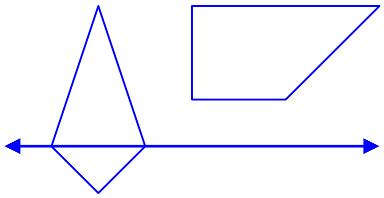
Reflect this half circle in the reflection line.



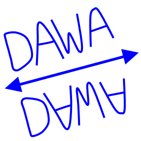
Solution	Thinking
 	 • To locate X′: - I drew a perpendicular line segment from X to the reflection line and measured it. - I continued the line segment beyond the reflection line the same distance and labelled the end point X′. • I located Z′ the same way as X′. • I connected X′ and Z′. • I knew the centre of the original half circle was Y, the midpoint of XZ. So the centre of its image was the midpoint of X′Z′. I measured X′Z′ and marked the midpoint Y'. • I put my compass point on Y′ and the pencil tip on X′ and made an arc from X′ to Z′.

Practising and Applying

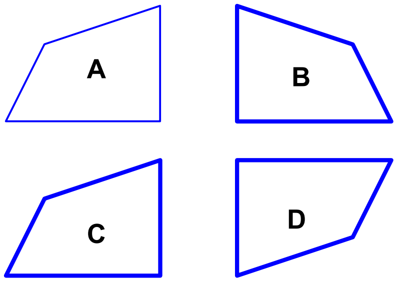
1. Copy and reflect these shapes in the reflection line.



2. Print your name. Reflect it in two different reflection lines.



3. Which of shapes B, C, and D are reflections of A? Explain your thinking.



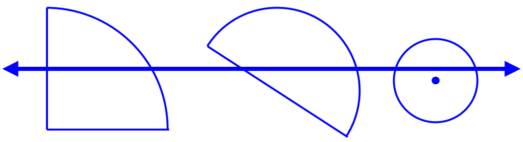
4. a) Draw ΔPQR with base PQ = 6 cm and PR = 4 cm. Construct $\angle P = 90°$.

b) Reflect ΔPQR in reflection line PQ.

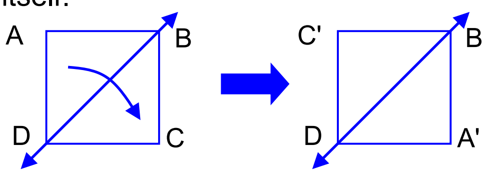
c) Measure $\angle RQP$ and $\angle R'Q'P'$.

d) Classify $\Delta R'QR$ by angle and side length.

5. Copy and reflect each shape in the reflection line.

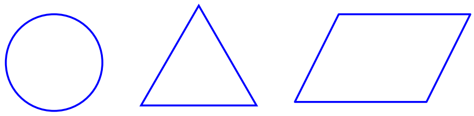


6. When a square is reflected across one of its diagonals, it reflects onto itself.



How many other reflection lines for the square will have the same result?

7. Which shapes could reflect onto themselves? Copy each shape and sketch the reflection line.

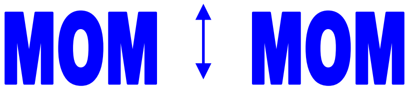


8. a) Draw an isosceles triangle.

b) Construct the angle bisector for the angle that is different from the other angles.

c) Draw the reflection line that will reflect the triangle onto itself. What do you notice?

9. The word MOM is a palindrome because it is spelled the same forwards and backwards. MOM is a special palindrome because it also spells MOM when it is reflected in a vertical line.



a) Find another special palindrome that reflects in a vertical reflection line.

b) Find a palindrome that does not reflect in a vertical line.

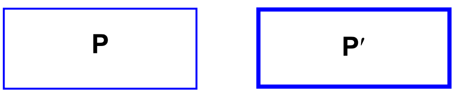
c) Find two words that reflect in a horizontal line to spell the same thing. They do not have to be palindromes.

10. a) Draw line segment PQ = 6 cm. Construct ΔPQR with $\angle P = 90°$ and $\angle Q = 60°$.

b) Reflect ΔPQR so that Q is the image of P. Describe the reflection line.

c) Find the areas of ΔPQR and $\Delta P'Q'R'$.

11. Was rectangle P reflected or translated to make P′? Explain your thinking.



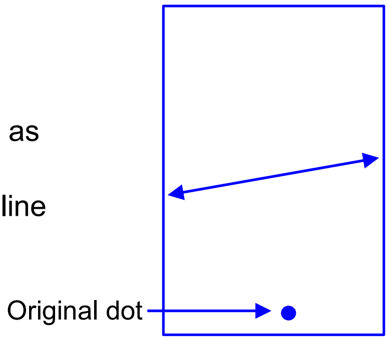
GAME: Reflection Archery

Play with a partner.

You need eight small pieces of paper, a pencil, and a ruler.

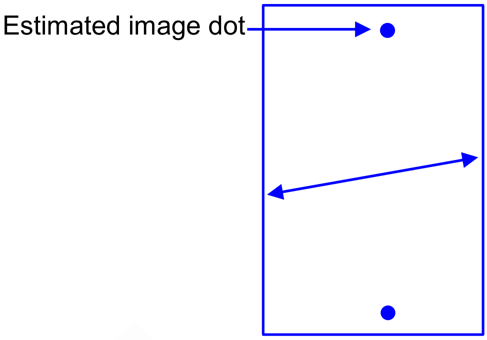
The goal is to estimate the position of a reflection as closely as possible.

• Each player marks a dot and draws a reflection line anywhere on four pieces of paper.

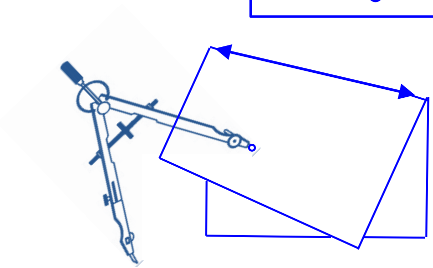


• Take turns. On your turn:

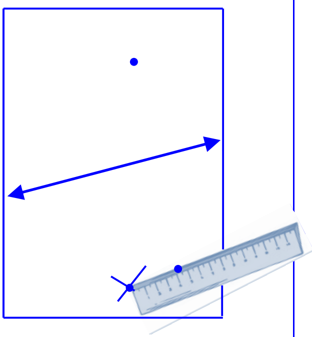
- Choose one of the eight papers.
- Place a dot where you estimate the reflection image of the dot will be.



- Fold the paper along the reflection line so the line is on the outside.
- Poke a hole through the dot image and the other side of the paper (you could use a compass point).



- Measure the distance from the original dot to the hole (to the nearest millimetre), which represents the image of the dot — this measure is your score for the turn.



• After each player has taken four turns, add each player's distances.

The player with the least total is the winner.

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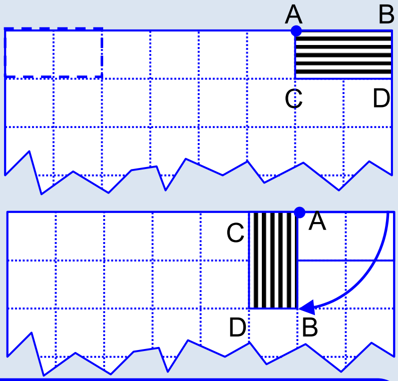
4.2.3 Rotations

Try This

Pem Bidha moved a heavy cupboard from one corner of a room to another.

• She let it rest on one leg (A) while she turned it out of the corner, as shown to the right.

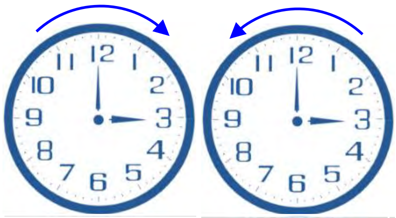
• Next she let it rest on a different leg to turn the cupboard again. She continued like this until the cupboard was in the other corner.



A. i) Describe how Pem Bidha moved the cupboard to the other corner.
ii) Write, in order, the names of the legs on which she turned the cupboard.
iii) Sketch the path of leg A from its original position to its final position.

You have rotated shapes with quarter turns (90°) and half turns (180°), but a rotation can be any angle.

• A **rotation** is described by its **turn centre**, **angle of rotation**, and direction of rotation, which can be clockwise (cw) or counter-clockwise (ccw).

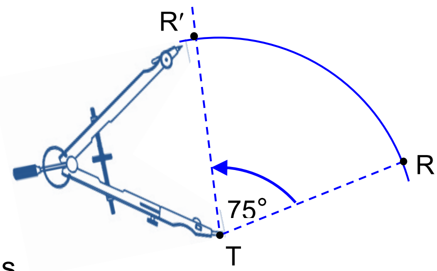


Clockwise Counter-clockwise

• To rotate point R 75° ccw around turn centre T:

- Draw a line segment from R to T.
- Measure or construct the angle of rotation at T.
- Use a compass to measure RT and mark point R′ on the angle arm so that R′T = RT.

R′ is the rotation image of R.

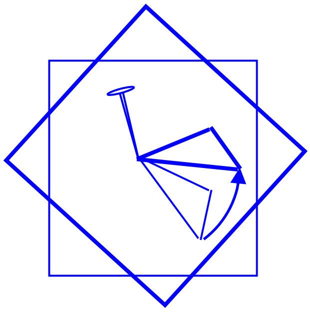


Rotating point R 75º ccw around T

• To rotate a shape, you can rotate each vertex as above and then connect the vertices.

• You can also use tracing paper to do rotations.

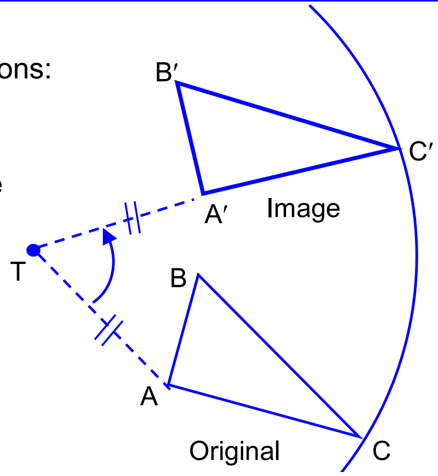
- Trace the shape onto the tracing paper.
- Pin the tracing paper to the paper underneath at the turn centre.
- Rotate the tracing paper around the angle of rotation.
- Copy the image. You can use a compass to poke holes through the tracing paper to mark the vertices.



Properties of Rotations

These are things that are always true about rotations:

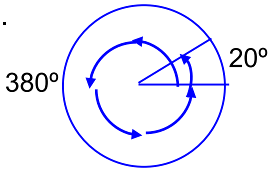
- The original shape and its image are congruent.
- The original shape and its image have the same orientation. The image may be turned but the direction of the vertices is the same.
- If the turn centre is on the shape, it is the only point that does not move. Otherwise all points move in a rotation.
- Each point and its image are the same distance from the turn centre.



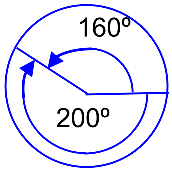
- You can get the same image with more than one rotation.

For example:

- Rotating 20° ccw is the same as rotating 380° ccw because a full rotation of 360° returns the shape to its original position and 360° + 20° = 380°.



- Rotating 200° cw is the same as rotating 160° ccw because 360º – 160º = 200º. These two rotations go in opposite directions to get the same image.

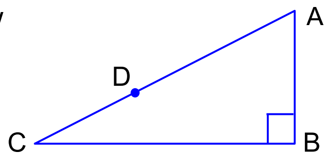


B. i) Describe the angle and turn centre of the first rotation in **part A**.
ii) What other rotation would have the same result?

Examples

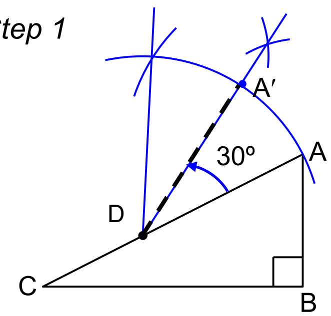
Example 1 Rotating a Triangle

Use constructions to rotate the ΔABC 30° ccw around point D.



Solution

Step 1



Thinking

- To construct a 30° ccw angle at D, I used DA as one arm and constructed a 60° ccw angle. Then I bisected it.
- I used my compass first to measure DA and then to mark A′ on the angle arm so that DA′ = DA.

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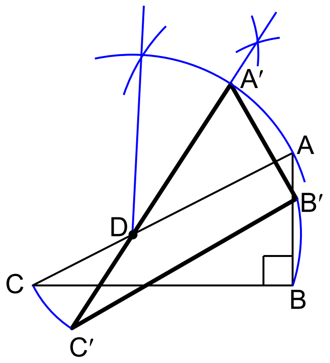
Step 2

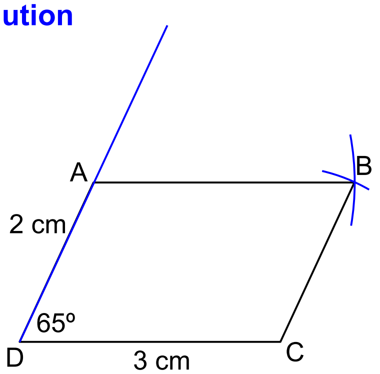


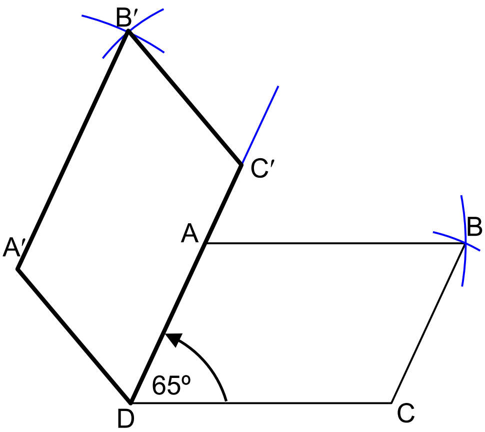
- I rotated B the same way as I did A, by constructing a 30° angle on BD.
- I rotated C the same way as I did A, by constructing a 30° angle on CD.
- D, the turn centre, was on the shape, so it was the only point that didn't move.

Example 2 Determining a Rotation

Draw a parallelogram that has a 65° angle. Rotate the parallelogram so that one side of the image lies on top of one side of the original shape. Describe the rotation.

Solution



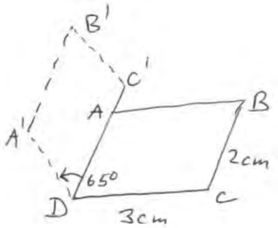


A 65° ccw rotation around vertex D

Thinking

- To draw the parallelogram:
- I drew a 3 cm line segment and labelled it DC.
- I drew a 65° angle at D and marked vertex A, 2 cm from D along the arm.
- To locate B, I used my compass to make a 3 cm arc from A and a 2 cm arc from C (because opposite sides of a parallelogram are equal). B was at their intersection.

- To visualize how to rotate the shape, I drew a sketch:

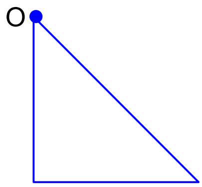


- I knew if I used any of the vertices as the turn centre, one side would rotate onto another.
- I used D as the turn centre so I could use 65° as the angle of rotation.
- I could have rotated each vertex 65°, but it was easier to just draw the parallelogram again along DA.

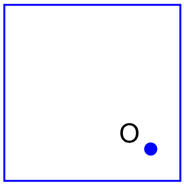
Practising and Applying

1. Copy and rotate each shape

a) 45° ccw around O



b) 180° cw around O



2. For each rotation in **question 1**, describe another rotation with the same image.

3. a) What cw rotation is the same as a 70° ccw rotation?

b) What cw rotation is the same as a 115° ccw rotation?

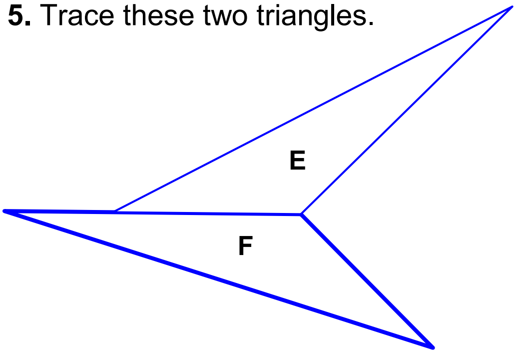
c) Describe a rule for finding the cw rotation that is the same as a ccw rotation.

4. What is the angle of rotation made by the minute hand of a clock for each?

a) from 3:30 to 3:45

b) from 1:20 to 1:50

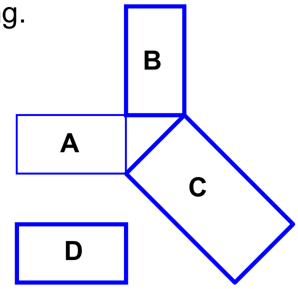
5. Trace these two triangles.



a) What turn centre and angle of rotation would rotate E onto F?

b) What other rotation has the same result?

6. Which of the shapes B, C, and D could be rotations of A? Explain your thinking.



7. a) Draw isosceles ΔPQR, with PQ = 8 cm, ∠P = 75°, and ∠Q = 30°.

b) Rotate ΔPQR so that R is the image of P. Describe the rotation.

c) Find the areas of ΔPQR and ΔP′Q′R′.

8. a) Draw a triangle on grid paper and find its area.

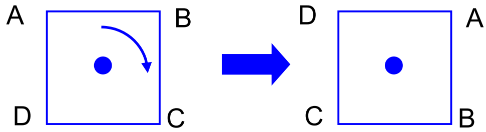
b) Rotate the triangle using any turn centre and any angle of rotation. Describe the rotation.

c) Find the area of the image.

d) Repeat **parts a) to c)** using a different triangle.

e) What do you notice? Explain why this happens.

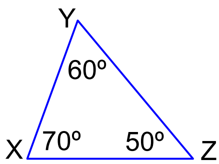
9. When a square is rotated 90° cw around its centre, it rotates onto itself.



a) What other angles of rotation less than 360° have the same result?

b) Name two other shapes that can rotate onto themselves.

10. ΔXYZ is rotated around X so that one side of the image lies on top of one side of the original shape.



Describe four different rotations that give this result.

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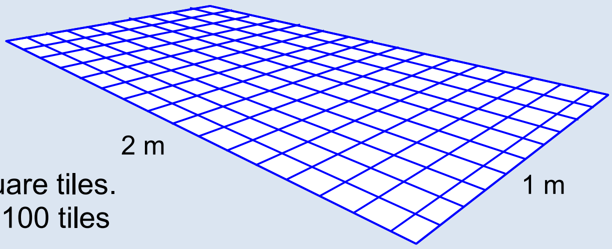
Chapter 3 3-D and 2-D Measurement

4.3.1 Measurement Units

Try This

Meghraj wants to buy square tiles to pave a rectangular patio that measures 1 m by 2 m. He says that he needs 200 tiles that are each 10 cm square.



The shopkeeper has only 20 cm square tiles. He tells Meghraj that he needs only 100 tiles because the tiles are twice as big.

A. i) What area (in cm^2) does Meghraj need to tile?

ii) How much area will 200 square tiles cover if they are 10 cm by 10 cm each?

iii) How much area will 100 square tiles cover if they are 20 cm by 20 cm each?

iv) Explain the error in the shopkeeper’s thinking.

• There are different units for measurements of each type so that the numbers used will be reasonable in size.

For example, there are both metre and kilometre units for measuring length:

You might tell someone, "Drive 23 km and then walk 40 m up a hill." This is clearer than saying, "Drive 23,000 m and walk 40 m," or "Drive 23 km and walk 0.04 km."

• The different units for each measurement type are related to place value. They use different **metric prefixes**.

Prefix	Symbol	Meaning
milli	m	$\frac{1}{1000}$ of the base unit
centi	c	$\frac{1}{100}$ of the base unit
deci	d	$\frac{1}{10}$ of the base unit
The base unit		
deca	da	10 times the base unit
hecto	h	100 times the base unit
kilo	k	1000 times the base unit

Example	As a metre
5 mm	$5 \times \frac{1}{1000} = 0.005$ m
7 cm	$7 \times \frac{1}{100} = 0.07$ m
6 dm	$6 \times \frac{1}{10} = 0.6$ m
3 m	3 m
5 dam	5 × 10 = 50 m
7 hm	7 × 100 = 700 m
2 km	2 × 1000 = 2000 m

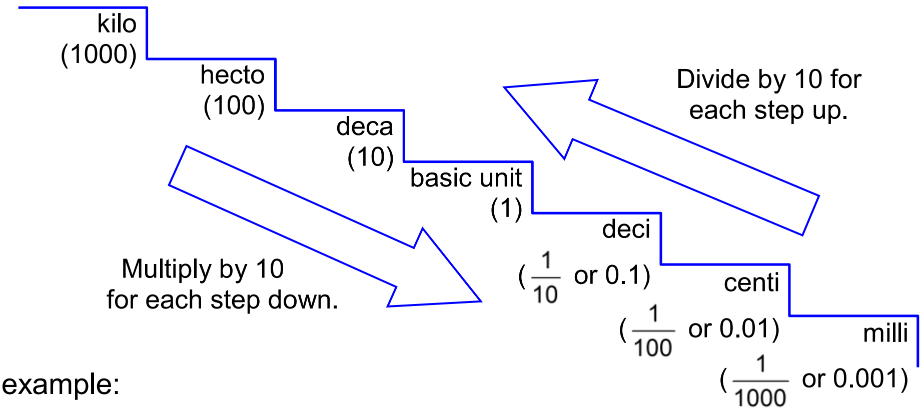
• Some special units for measuring large areas and large masses do not use prefixes: the **hectare** and the **tonne**.

1 ha (hectare) = 1 hm^2 1 t (tonne) = 1000 kg

• Metric units are sometimes called **SI units** — from the French words *Système International* (International System of Units).

Converting units using prefixes

You can visualize this step chart to convert one measurement to another.



For example:

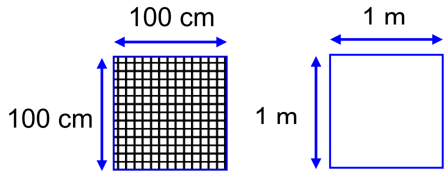
To convert millilitres to decilitres, you divide by 100.
So 43 mL = 43 ÷ 100 = 0.43 dL.

This makes sense since 100 mL = 1 dL, so 1 mL = 0.01 dL.

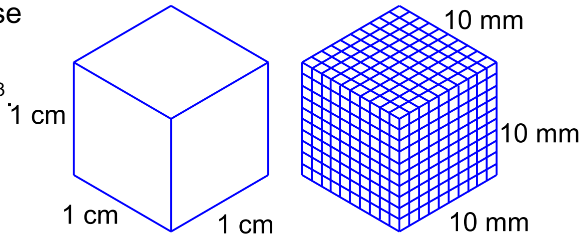
Area and Volume

When you convert units of area and volume, you need to consider all the dimensions involved.

• There are 10,000 cm^2 in 1 m^2 because
1 m × 1 m = 1 m^2, and
100 cm × 100 cm = 10,000 cm^2.



• There are 1000 mm^3 in 1 cm^3 because
1 cm × 1 cm × 1 cm = 1 cm^3, and
10 mm × 10 mm × 10 mm = 1000 mm^3.



Capacity

• Litres and millilitres measure **capacity.** You can think of capacity as the volume of liquid that a container can hold.

Special relationships: 1 cm^3 = 1 mL = 1 g

• 1 mL of water has a volume equivalent to 1 cm^3 and a mass of 1 g.

• You can use these relationships for estimating with liquids that are similar to water.

For example:

What volume does a 2 L juice jug hold? What is its mass when it is full of juice?

2 L = 2000 mL → 2000 cm^3 → 2000 g

It has a volume of about 2000 cm^3 and a mass of about 2000 g or 2 kg when full.

B. Convert the area in **part A** to square metres. Show how the result relates to the dimensions of the rectangle.

Examples

Example 1 Thinking about Reasonable Measurements

Convert each measurement to a more reasonable unit.
Give an example of something that might have this measurement.

a) 15,400 kg **b)** 0.75 dm^3

Solution

a) 1 kg = $\frac{1}{1000}$ t, so

15,400 ÷ 1000 = 15.4 t

A bus could have this mass.

b) 1 dm^3 = 1000 cm^3

0.75 × 1000 = 750 cm^3

This is the amount of water in a 750 mL bottle.

Thinking

a) Large masses can be reported in kilograms, but I think the tonne is a better unit for this situation.

b) Cubic decimetres aren't used very often, so I converted it to cubic centimetres.

- 1 dm^3 = 1000 cm^3 because

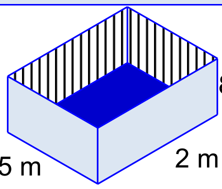
1 dm × 1 dm × 1 dm = 10 cm × 10 cm × 10 cm = 1000 cm^3.

Example 2 Determining the Capacity of a Box

The mass of this container when empty is 150 kg.

a) How many litres of liquid could this container hold?

b) What would the mass be if it were full of water?



Solution

a) 2 m × 1.5 m × 80 cm
= 200 cm × 150 cm × 80 cm

Volume

$V = l \times w \times h$ = 200 × 150 × 80
= 2,400,000 cm^3

Capacity

Since 1 cm^3 = 1 mL,

2,400,000 cm^3 = 2,400,000 mL,

Since 1 mL = $\frac{1}{1000}$ L,

2,400,000 ÷ 1000 = 2400 L.

The container could hold 2400 L.

b) 1 mL has a mass of 1 g, so

2,400,000 mL = 2,400,000 g

Since 1 g = $\frac{1}{1000}$ kg,

2,400,000 ÷ 1000 = 2400 kg.

Total mass

2400 + 150 = 2550 kg

Thinking

a) I converted the dimensions to centimetres so I could convert from cubic centimetres to millilitres.

b) I found the mass of the water using the special relationship between 1 cm^3, 1 mL, and 1 g of water:

1 cm^3 = 1 mL = 1 g

Practising and Applying

1. Complete each.

a) 0.3 cm = ■ m

b) 5.2 hL = ■ L

c) 30 dg = ■ mg

d) 420 dam = ■ dm

e) 0.407 cm^2 = ■ mm^2

f) 5400 cm^3 = ■ m^3

g) 1 dm^3 = ■ L

h) 0.4 km^2 = ■ ha

i) 1500 mL of water = ■ kg

j) 1 t of water = ■ m^3

2. Complete each sentence.

For example:

"To change from metres to centimetres, *multiply by 100*."

a) To change from millimetres to decametres, …

b) To change from hectolitres to centilitres, …

c) To change from centigrams to milligrams, …

d) To change from cubic metres to cubic centimetres, …

e) To change from square millimetres to square centimetres, …

f) To change from hectares to square metres, …

3. Which unit would you use for each?

a) the capacity of a serving spoon

b) the area covered by a rice paddy

c) the mass of a 1 L bottle of water

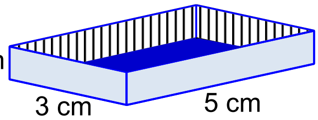
4. Convert each measurement to a more reasonable unit. Give an example of something that might have each measurement.

a) 0.0014 kg **b)** 5,400,000 mm

c) 5,400,000 mm^3 **d)** 0.05 cm^2

e) 0.2 daL **f)** 0.000035 t

5. a) How much water can this container hold? Show your work.



b) The container has a mass of 10 g when it is full of water. What is its mass when it is empty? Show your work.

6. The base of a rectangular box is 36 cm by 40 cm. How many millimetres deep is the box, if it holds 1 kg of water? Show your work.

7. What are the dimensions of a container that has these properties?

• It is a rectangular prism.

• It has edge lengths that are whole centimetres.

• It holds about 6 kg of water.

• It would fit in your lap.

Show your work.

8. Would it take more or less than 1 t of water to flood a level 1 ha field 1 cm deep? Show your work.

9. Some Bhutanese use the tho (hand span) to estimate length. One tho is about 15 cm (1 th = 15 cm). Complete each. Show your work.

a) 6 th = ■ mm **b)** 60 th = ■ m

c) 1 th^2 = ■ cm^2 **d)** 9 m^2 = ■ th^2

10. a) What happens to a measurement when you convert it to a larger metric unit? Explain why this happens.

b) What happens when you convert to a smaller unit? Explain why this happens.

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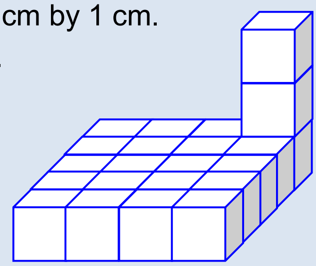
4.3.2 Volume of a Rectangular Prism

Try This

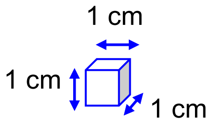
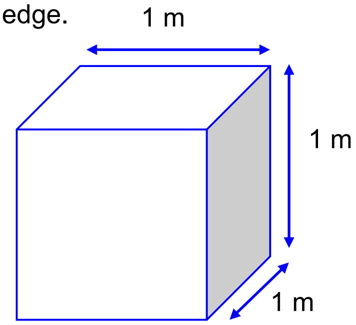
Kamala is filling a box with sugar cubes that are 1 cm by 1 cm by 1 cm.
The base of the box is 7 cm by 10 cm. The box is 4 cm tall.



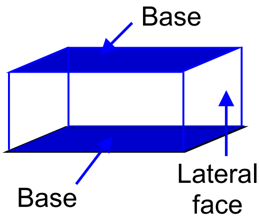
A. i) How many sugar cubes will cover the bottom of the box?
ii) How many layers of cubes can Kamala pack into the box?
iii) How many sugar cubes will fit in the box?

Usually we measure objects when we need to make comparisons. One way to compare the size of objects is to compare their **volume**.

- The volume of an object describes how much space it takes up.
- Volume is usually measured in these cubic units:

Cubic centimetre (cm^3)	**Cubic metre** (m^3)
A cubic centimetre is the amount of space taken up by a cube that measures 1 cm on each edge.  1 cubic centimetre, or 1 cm^3	A cubic metre is the amount of space taken up by a cube that measures 1 m on each edge.  1 cubic metre, or 1 m^3

- A **rectangular prism** has two opposite **bases**, which are congruent rectangles. The bases are connected by four **lateral faces**, which are also rectangles.



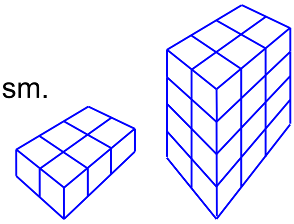
- To find the volume of a rectangular prism, you multiply the area of one of its bases by its height.

Volume = Area of base × height

- This formula makes sense if you think of a prism as layers of rectangles.

- Imagine a rectangle made with 6 cubes as the base of a prism.
- If there are 4 layers of these rectangles in the prism, there will be 6 × 4 cubes altogether.

So the volume of the prism is 6 × 4 = 24 cubes.

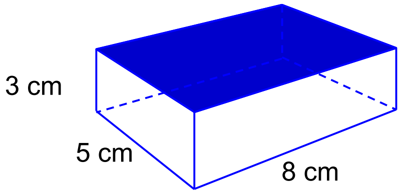


• With a rectangular prism, you can choose which rectangle to use as the base when you calculate the volume.

For example:

In this 5 cm-by-8 cm-by-3 cm prism,

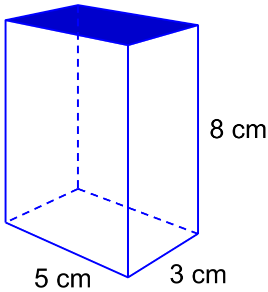
- you can use the 5 cm-by-8 cm face as the base:



The area of the base is $5 \times 8 = 40$ cm^2.

Volume = *Area of base* × *height*
= 40 × 3
= 120 cm^3

- or, you can use the 5 cm-by-3 cm face as the base:



The area of the base is $3 \times 5 = 15$ cm^2.

Volume = *Area of base* × *height*
= 15 × 8
= 120 cm^3

The volume is the same when you calculate each way.

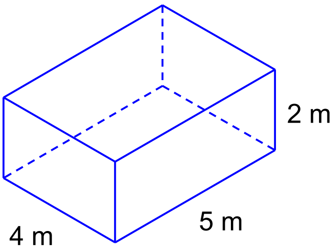
B. Use the formula to calculate the volume of sugar Kamala's box can hold.

Examples

Example 1 Calculating the Volume of a Rectangular Prism

Calculate the volume of a box that is 5 m × 4 m × 2 m.

Solution



Area of base = $l \times w$
= 5 × 4
= 20 m^2

Volume = *Area of base* × *height*
= 20 × 2
= 40 m^3

The volume is 40 m^3.

Thinking

• I sketched and labelled the box to help me visualize the situation.

• I used the 5 m-by-4 m faces as the bases.

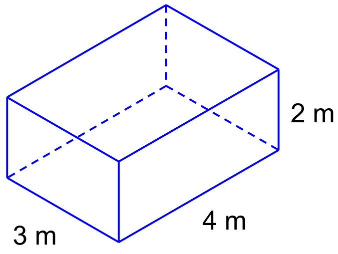
• I used the formula for the area of a rectangle to calculate the area of a base.

• I used the formula for the volume of a rectangular prism.

Example 2 Finding the Dimensions of a Box

What might be the dimensions of a rectangular prism with a volume of 24 cm^3?

Solution



Check

Volume = *Area of base* × *height*

= 3 cm × 4 cm × 2 cm

= 24 cm^3

Thinking

- I found a factor pair for 24:

  12 × 2 = 24

I used the first factor for the area of the base and the second factor for the prism height.

- Then I found a factor pair for 12:

  3 × 4 = 12

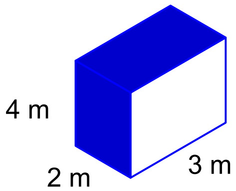
I used the first factor for the width of the base and the second factor for its length.

- I checked to make sure the dimensions resulted in a volume of 24 cm^3.

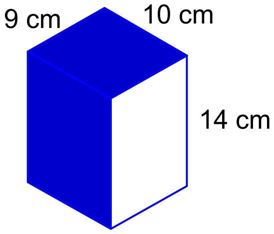
Practising and Applying

1. Calculate the volume of each. Show your work.

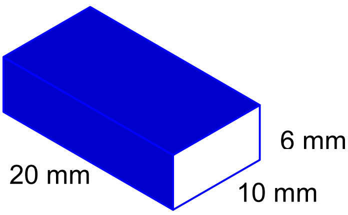
a)



b)



c)



2. Calculate the volume of each rectangular prism. You can sketch the prism first to help you.

a) l = 5 mm, w = 5 mm, h = 5 mm

b) l = 3 m, w = 1 m, h = 3 m

c) l = 3 m, w = 3 m, h = 1 m

d) l = 4 cm, w = 2 cm, h = 6 cm

3. Copy and complete the chart for each rectangular prism.

	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
a)	5	3	2	
b)	10	6	4	
c)		4	4	64
d)	5		6	240
e)	4	5		60

4. When you are deciding which base dimensions to use to calculate the volume of a rectangular prism, you have three choices. Use an example to show why this is true.

5. Anjali says that you can calculate the volume of a rectangular prism using this formula:

$$V = l \times w \times h$$

Do you agree? Explain your thinking.

6. a) Give a possible set of dimensions for a rectangular prism with edges that are whole numbers of centimetres and a volume of 60 cm^3.

b) Explain how you know there is more than one answer to **part a)**. Give three or more possible sets of dimensions.

7. Chhimi was given 120 boxes to stack to form a rectangular prism. Each box was 8 cm wide by 12 cm deep by 25 cm tall.
a) Describe three different ways to stack the boxes.
b) For each way to stack the boxes in **part a)**, give the overall dimensions and volume of the rectangular prism formed.

8. a) Estimate to determine which box has the greater volume. Show how you estimated.

Box A: l = 53 cm, w = 18 cm, h = 92 cm

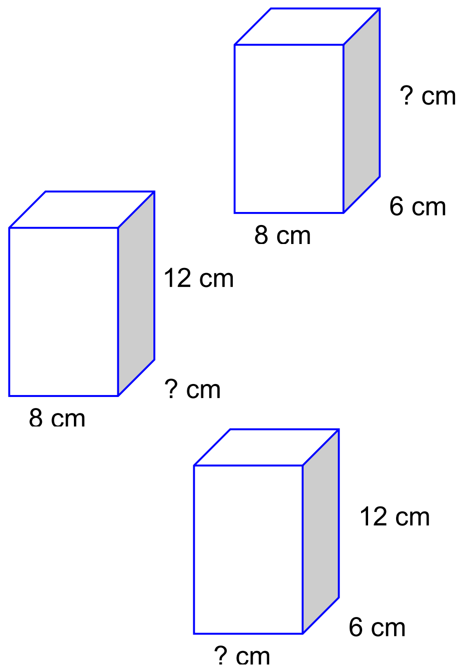
Box B: l = 51 cm, w = 51 cm, h = 51 cm

b) Calculate each volume to see if you estimated correctly.

9. The following problem can be solved by estimating. An exact answer is not necessary. Explain why.

Will 23,000 cm^3 of rice fit into a box with a base that is 31 cm × 22 cm and a height of 43 cm?

10. Dechen says that if she knows the volume of a rectangular prism and two dimensions, she can calculate the third dimension. Is she correct? Explain your thinking.



11. How can a tall prism have less volume than a shorter prism?

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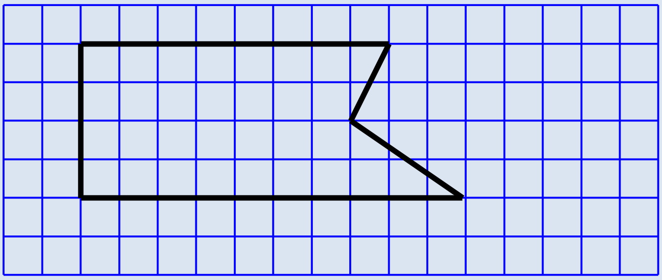
4.3.3 Area of a Composite Shape

Try This

Hari Maya wanted to find the area of a paddy field because her family was thinking of selling it. She drew a scale map of the field on grid paper.

1 square = 1 m^2

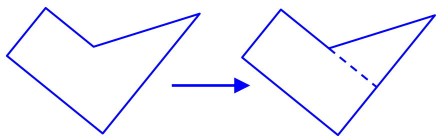


A. Count squares to find the area of the field. Explain how you counted.

Sometimes you need to find the area of a shape and there is no formula for it.

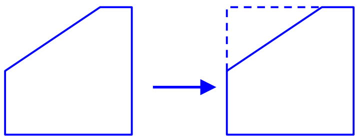
- **Composite shapes** are shapes that you can divide into familiar shapes.
- To find the area of a composite shape, divide it into shapes that you know formulas for.
- You can add shapes together to find the total area or you can subtract them, whichever makes more sense.
- Some possible strategies for composite **polygons** include looking for rectangles and right triangles.

Adding shapes to find total area



Area = Area of rectangle + Area of triangle

Subtracting shapes to find total area



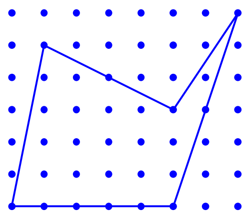
Area = Area of square – Area of triangle

B. Find the total area of the paddy field by dividing it into familiar shapes and then using area formulas. Show your work.

Examples

Example 1 Determining the Area of a Composite Shape

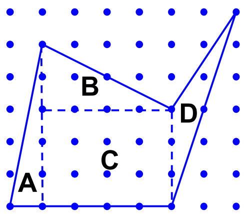
Find the area of this shape, which is on 1 cm square dot paper.



[Continued]

Example 1 Determining the Area of a Composite Shape [Continued]

Solution 1



Region A

$A = b \times h \div 2 = 1 \times 5 \div 2 = 2.5 \text{ cm}^2$

Region B

$A = b \times h \div 2 = 4 \times 2 \div 2 = 4 \text{ cm}^2$

Region C

$A = b \times h = 4 \times 3 = 12 \text{ cm}^2$

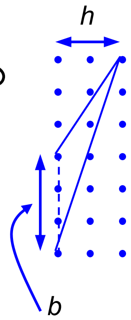
Region D

$A = b \times h \div 2 = 3 \times 2 \div 2 = 3 \text{ cm}^2$

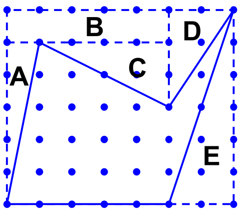
Total Area = 2.5 + 4 + 3 + 12 = 21.5 cm^2

Thinking

- I divided the shape into rectangles and triangles.
- I used the formulas for the area of a triangle and a rectangle.
- I had to visualize region D differently because the base was vertical.
- I found the sum of the four areas to get the total area.



Solution 2



Region A

$A = b \times h \div 2 = 1 \times 5 \div 2 = 2.5 \text{ cm}^2$

Region B

$A = b \times h = 5 \times 1 = 5 \text{ cm}^2$

Region C

$A = b \times h \div 2 = 2 \times 4 \div 2 = 4 \text{ cm}^2$

Region D

$A = b \times h \div 2 = 2 \times 3 \div 2 = 3 \text{ cm}^2$

Region E

$A = b \times h \div 2 = 2 \times 6 \div 2 = 6 \text{ cm}^2$

Area of outside rectangle

$A = b \times h = 7 \times 6 = 42 \text{ cm}^2$

Total Area = 42 – (2.5 + 5 + 4 + 3 + 6)
= 42 – 20.5
= 21.5 cm^2

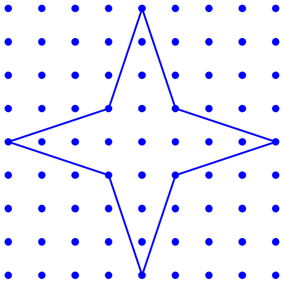
Thinking

- I made a rectangle around the shape.
- I divided the area outside the shape into triangles and rectangles.
- I found the area of each outside region.
- I found the area of the large rectangle.
- I subtracted the sum of the areas of the regions outside the shape from the area of the large rectangle.

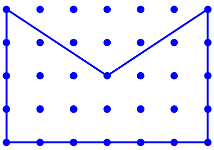
Practising and Applying

1. Find the area of each shape drawn on 1 cm dot paper. Show your work.

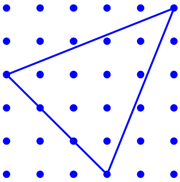
a)



b)

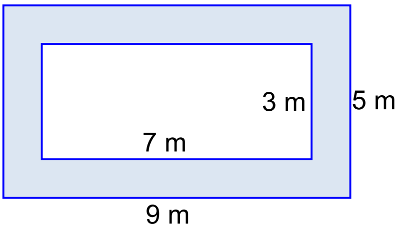


c)

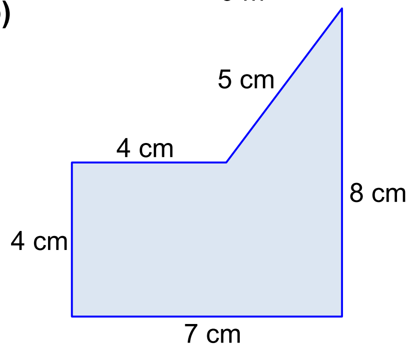


2. Determine the area of the grey region in each diagram. Show your work.

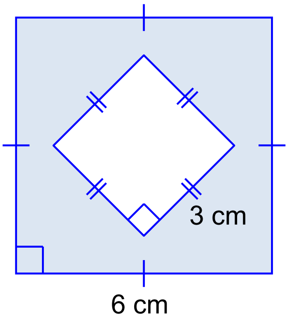
a)



b)



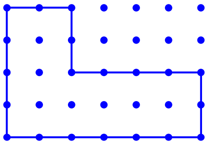
c)



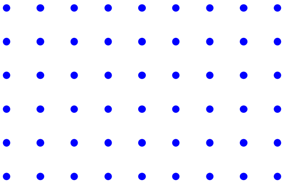
3. a) Draw a composite shape on grid paper. Calculate its area.

b) Exchange shapes with a classmate and calculate the area. Compare results.

4. Add a triangle to this shape so that the total area is 19 square units.



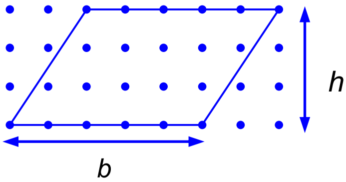
5. This grid represents a 40 m^2 park.



Design a park with gardens and paths. The paths should be 1 m wide and should allow walkers to visit the different parts of the park. There should be 25 m^2 of garden area.

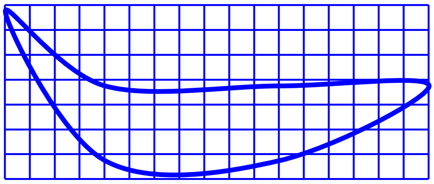
6. Show that the area of this parallelogram can be found using the formula $A = b \times h$ these two ways:

a) By dividing the parallelogram into a square and two triangles and arranging the three pieces to form a rectangle.



a) By dividing the parallelogram into two congruent triangles.

7. How can you use what you have learned in this lesson to estimate the area of this shape?



4.3.4 Area of a Trapezoid

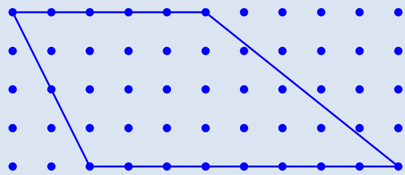
Try This

This polygon is drawn on 1 cm dot paper.

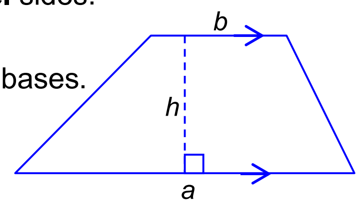
A. i) Find its area by dividing it into a rectangle and two triangles.

ii) Find its area by dividing it into two triangles.

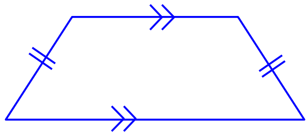
iii) Show another way you can divide the polygon into two triangles.



- A **trapezoid** is a quadrilateral with exactly two **parallel** sides.
  - *a* and *b* represent the two bases — the parallel sides.
  - *h* represents the height, which is perpendicular to the bases.



- In an **isosceles trapezoid**, the sides that are not parallel are equal in length.



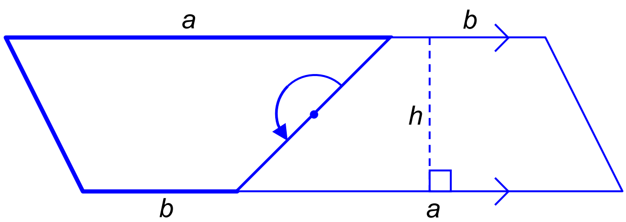
Isosceles trapezoid

- The formula for the area of a trapezoid is

$$\text{Area of a trapezoid} = (a + b) \times h \div 2$$

The formula makes sense if you think of a parallelogram as two congruent trapezoids:

- Rotate a trapezoid 180° around the midpoint of one of its non-parallel sides.
- The two congruent trapezoids together make a parallelogram.
- The height of the parallelogram is *h* and its base is *a* + *b*.
- The area of a parallelogram is the length of the base times the height. For this parallelogram, it is $A = (a + b) \times h$.
- Since the parallelogram is made up of two congruent trapezoids, one of the trapezoids is half the parallelogram. Its area is $A = (a + b) \times h \div 2$.



$A_{\text{parallelogram}} = \text{base} \times \text{height}$

$A_{\text{parallelogram}} = (a + b) \times h$

$A_{\text{trapezoid}} = (a + b) \times h \div 2$

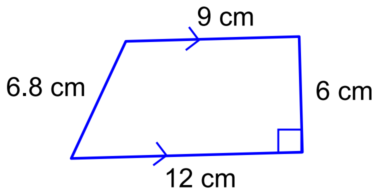
B. i) How do you know the polygon in **part A** is a trapezoid?

ii) Use the formula to find its area.

Examples

Example 1 Finding the Area of a Trapezoid

Determine the area of this trapezoid.



Solution

$A = (a + b) \times h \div 2$

$a = 12$ cm

$b = 9$ cm

$h = 6$ cm

$A = (a + b) \times h \div 2$

$= (12 + 9) \times 6 \div 2$

$= 21 \times 6 \div 2$

$= 21 \times 3$

$= 63 \text{ cm}^2$

The area is 63 cm^2.

Thinking

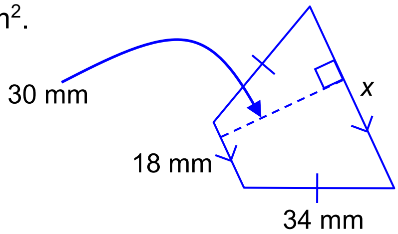
• I knew it was a trapezoid because the arrow marks showed that it had exactly two parallel sides.

• I identified the bases and the height. I noticed that the 6.8 cm side length was extra information that I didn't need.

• I used the formula.

Example 2 Finding a Missing Dimension in a Trapezoid

The area of this isosceles trapezoid is 1020 mm^2.
What is the length of the side marked x?



Solution

$A = (a + b) \times h \div 2$

$A = 1020 \text{ mm}^2$

$a = 18$ mm

$h = 30$ mm

$A = (a + b) \times h \div 2$

$1020 = (a + b) \times 30 \div 2$

$1020 = (a + b) \times 15$

$1020 \div 15 = a + b$

$68 = a + b$

Since $a = 18$, then $b = 50$ because $68 - 18 = 50$.

The side marked x is 50 mm.

Thinking

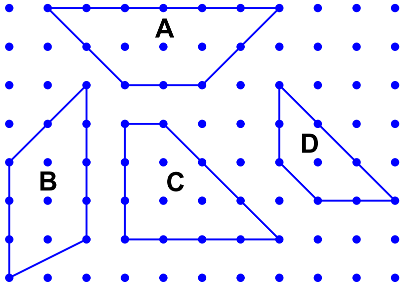
• I wrote down what I knew about the parts of the area formula.

• I put these measurements into the formula.

• I knew the sum of $a + b$ and I knew what a was. So all I had to do was subtract to find b.

Practising and Applying

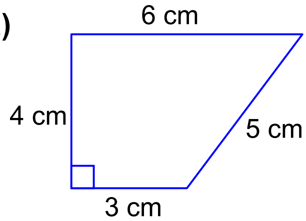
1. Which polygons are trapezoids? Which are isosceles trapezoids?



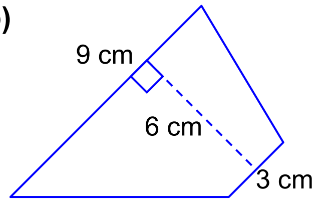
2. Determine the area of each shape in **question 1**. Show your work.

3. Find the area of each trapezoid. Show your work.

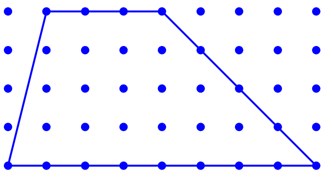
a)



b)



4. Copy this trapezoid on dot paper or grid paper.



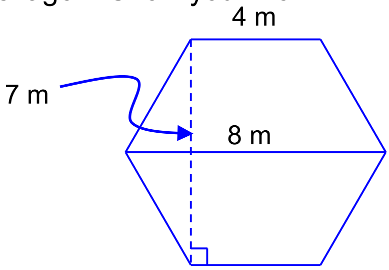
a) Use the formula to find the area of the trapezoid. Show your work.

b) Divide the trapezoid into two triangles.

c) Determine the area of each triangle. Show your work.

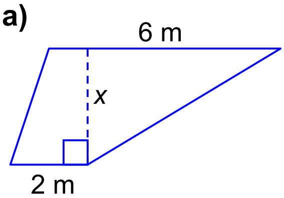
d) How are the triangles the same and how are they different?

5. Find the area of this regular hexagon. Show your work.

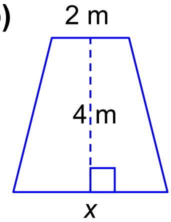


6. Find the length of each x.
The area of each trapezoid is 12 m^2.

a)



b)



7. The lower roof of this pavilion in Samste is covered by four congruent trapezoids. One of the trapezoids is outlined in white.

The bases of each trapezoid are 14 m and 5 m long and the height is 6 m. How much roof material is needed to cover the lower roof? Show your work.

8. Draw any trapezoid on grid paper. Divide it into two triangles. Create a formula for the area of a trapezoid using these two triangles. Show your work.

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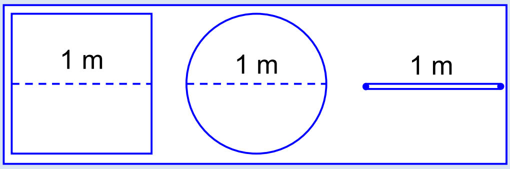
4.3.5 Circumference of a Circle

Try This

Jamyang used string to make a square, a circle, and a loop.



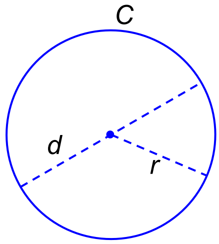
A. i) What length of string did Jamyang use for the square? How do you know?

ii) What length did she use for the loop? How do you know?

iii) Did the circle require more or less string than the square? How do you know?

iv) Did the circle require more or less string than the loop? How do you know?

v) Estimate the length of string Jamyang used for the circle.



• A circle has several measurements. They are all related:

- The **circumference**, C, is the perimeter of a circle.
- The **diameter**, d, is the distance across the circle through the centre.
- The **radius**, r, is the distance from the centre to the circumference.

• Relating radius and diameter

The radius is half the diameter, so a formula for finding the radius or diameter is

$$r = d \div 2 \qquad d = 2 \times r$$

• Relating diameter and circumference

- The ratio of the circumference to the diameter, $C \div d$, is the same for any circle.
- Mathematicians have been trying to find the exact ratio of $C \div d$ for thousands of years. The ratio is called **pi** and is written as the Greek letter $\boldsymbol{\pi}$.
- The formula for the circumference of a circle is

$$C = \pi \times d \qquad C = 2 \times \pi \times r$$

• Approximate values for π

- You can use approximate values for π to calculate circumference.

You can use the fraction $\frac{22}{7} = 3\frac{1}{7}$ or the decimal value of 3.14.

- A calculator will give you a more accurate value. Look for the π key next time you use a calculator.
- The value of π is known to many thousands of decimal places, but the approximations above are reasonable enough for most situations.

B. Calculate the circumference of the circle in **part A**. How does this compare with your estimate in **part A v)**?

Examples

Example 1 Finding the Circumference Given the Radius

Is 40 cm of string enough to wrap once around a circular pole with radius 7 cm?

Solution

$C = 2 \times \pi \times r$ and $r = 7$ cm

$C = 2 \times \pi \times r$

$\approx 2 \times \frac{22}{7} \times 7$

$= 2 \times 22$

$= 44$ cm

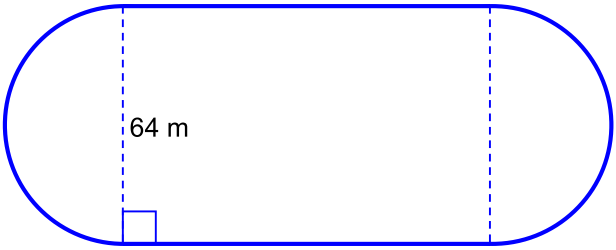
The circumference is about 44 cm, so there is not enough string.

Thinking

- I knew I needed to find the circumference of a pole with a radius of 7 cm.
- I used the fraction approximation for π because I was multiplying by 7.
- I used the approximately equals sign, ≈, to show that the next step was an approximation.
- I used the words "about" because I knew the value of π was only approximate, so the circumference was an approximation.

Example 2 Finding the Perimeter of a Composite Shape

What is the distance around this running track?



Solution

Circumference of the circle

$C = \pi \times d$ and $d = 64$

$C \approx 3.14 \times 64$

$= 200.96$ m

Total perimeter

Circle + 2 straight stretches

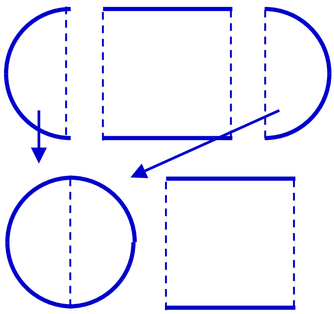
$P = 200.96 + 100 + 100$

$= 400.96$

The distance around the track is about 400 m.

Thinking

- I imagined dividing the shape into two semicircles and a rectangle.
- I combined the semicircles to make a full circle.



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Practising and Applying

Show your work for each question.

1. a) Draw three different circles. Measure each diameter.

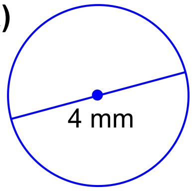
b) Use string to measure the circumference of each circle.

c) For each circle, calculate $C \div d$.

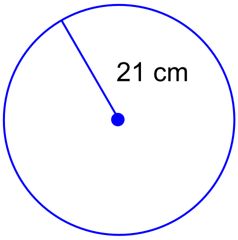
d) Did you get the same value in **part c)** for each circle? Why or why not?

2. Find the circumference of each. Round to the nearest whole unit.

a)



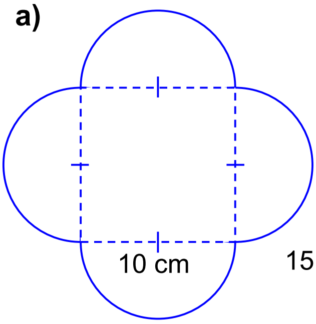
b)



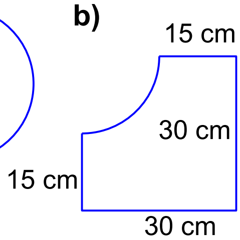
c) A circle with diameter 21 cm.

3. Find the perimeter of each. Round to the nearest whole unit.

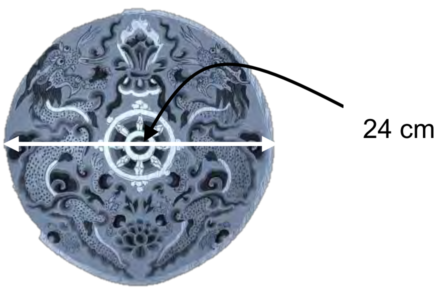
a)



b)



4. An artist is estimating how much silver paint she will need to paint an outline around the circumference of this design. What is its circumference? Round to the nearest whole unit.



5. A community is building a cylindrical water tank with a diameter of 9 m. There are reinforcing horizontal steel bars around its circumference. How long is each horizontal bar, to the nearest whole unit?

6. a) What is the diameter of a circle with circumference 31.4 cm?

b) What is the diameter of a circle with circumference 100 cm?

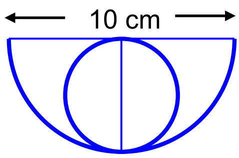
c) Determine a formula for the diameter of a circle, given the circumference.

7. a) What is the radius of a circle with circumference 31.4 cm?

b) What is the radius of a circle with circumference 100 cm?

c) Determine a formula for the radius of a circle, given the circumference.

8. a) Compare the circumference of the round part of the semicircle to the circumference of the circle.



b) How would they compare if the diameter was 5 cm instead?

9. a) What is the shape of the label on this tin?

b) The tin is 11.5 cm tall and its diameter is 7.5 cm. Estimate the dimensions of the label. Explain your thinking.

10. Why might someone measure the diameter of a circle in order to find its circumference instead of measuring the circumference directly?

Unit 4 Revision

1. Sketch an example of each triangle.
Explain how you know you are right.

a) a right scalene triangle

b) an obtuse isosceles triangle

2. ∠A = 51° and ∠B = 37° in ΔABC.

a) What is ∠C?

b) Classify the triangle by angle and side length. Explain how you know you are right.

3. Is it possible to have an acute obtuse triangle? Explain your thinking.

4. a) Draw ΔXYZ with XY = 8 cm, YZ = 4 cm, and XZ = 6 cm.

b) Classify the triangle by angle and side length. Explain how you know you are right.

5. a) Estimate to sketch a 75° angle. Explain how you did it.

b) Construct a 75° angle and compare it to your sketch. How close is your sketch to your constructed angle?

6. a) Draw line segment AB = 8 cm.

b) Estimate to sketch ΔABC with ∠A = 30° and ∠B = 22.5°.

c) Explain how you did **part b)**.

d) Draw line segment AB = 8 cm. Construct ΔABC with ∠A = 30° and ∠B = 22.5°.

7. a) Draw ΔPQR with ∠P = 90°, PQ = 6 cm, and PR = 4 cm.

b) Translate ΔPQR so that the image of vertex P is on vertex R.

c) Describe the translation.

8. a) Draw ΔPQR with ∠P = 90°, PQ = 6 cm, and PR = 4 cm.

b) Reflect ΔPQR so that vertices P and R do not move.

c) Describe the reflection.

9. a) Draw ΔPQR with ∠P = 90°, PQ = 6 cm, and PR = 4 cm.

b) Rotate ΔPQR so that the image of vertex R is on side PQ.

c) Describe the rotation.

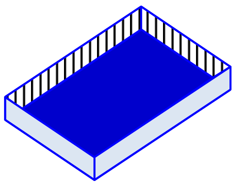
10. Sketch a pair of shapes that show a reflection but not a translation. Explain why they fit the description.

11. Sketch a rectangular prism with a volume of 300 cm^3. Its edges should be a whole number of centimetres long.

12. Copy and complete the chart for each rectangular prism.

	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
a)		5	4	100
b)	2		6	30

13. What is the capacity of this container? It is 40 cm deep and it covers an area that is 2 m by 3 m.



14. Complete.

a) 0.360 cm^2 = ■ mm^2

b) 5400 dg = ■ hg

c) ■ daL of water = 2.1 t

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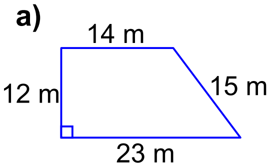
15. Complete each sentence.

a) To change from centimetres to kilometres, …

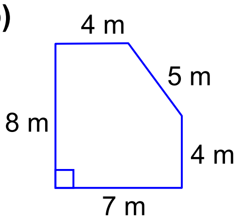
b) To change from cubic centimetres to cubic millimetres, …

16. Find the area of each shape. (Assume **parts c) and d)** are on centimetre dot paper.) Show your work.

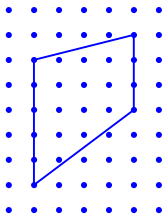
a)



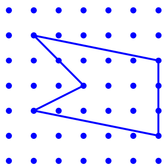
b)



c)



d)

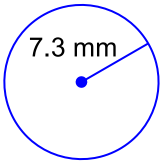


17. On grid paper, sketch two different trapezoids, each with area 12 cm^2.

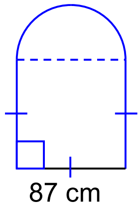
18. Determine the circumference of a circle with diameter 28 cm.

19. Determine the perimeter of each. Round to the nearest whole unit.

a)



b)



UNIT 5 INTEGERS

Getting Started

Use What You Know

This chart shows the highest and lowest elevations on each continent.

Continent	Highest location	Elevation above sea level	Lowest location	Elevation below sea level
Africa	Mount Kilimanjaro	5895 m	Lac Assal	153 m
Antarctica	Mount Vinson	4897 m	Bentley Subglacial Trench	2555 m
Asia	Mount Everest	8850 m	Dead Sea	408 m
Australia/ Oceania	Puncak Jaya	4884 m	Lake Eyre	16 m
Europe	Mount Elbrus	5633 m	Caspian Sea	28 m
North America	Mount McKinley	6194 m	Death Valley	86 m
South America	Cerro Aconcagua	6959 m	Valdés Peninsula	40 m

If sea level is 0, elevations above and below sea level can be written as positive and negative integers.

A. Name each location and describe its elevation using an integer.

i) the highest elevation

ii) the lowest elevation

iii) the elevation closest to sea level

iv) the elevation between Mount Everest and Mount McKinley

v) the elevation between the Bentley Subglacial Trench and Lac Assal

Mount Jomolhari — the highest elevation in Bhutan

B. The highest elevation in Bhutan is Mount Jomolhari, at 7314 m above sea level. The lowest elevation is Drangme Chhu, at 97 m above sea level.

i) Which location in the chart is higher than Mount Jomolhari? How do you know?

ii) Which is closer to sea level, Drangme Chhu or Death Valley? How do you know?

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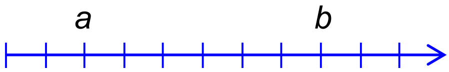
Skills You Will Need

1. Which is true, $a < b$ or $a > b$? Use the number line to explain how you know.



2. Draw a number line from –5 to +5. Place these integers on your number line.

a) –4 **b)** +1 **c)** 0 **d)** –2

3. Is –4 < –2? How do you know?

4. Replace each ■ with < or > to make the statement true.

a) +4 ■ 0 **b)** +11 ■ +20 **c)** –3 ■ +2 **d)** –4 ■ –10

5. Use + or – to write each as an integer.

a) 3747 m above sea level

b) 86 m below sea level

c) 14°C below zero

d) 3°C above zero

6. Use the thermometer to help you answer each question.

a) Today's temperature is +7°C.
Tomorrow's temperature is predicted to be 3°C higher than today's temperature.
What is tomorrow's predicted temperature?

b) How many degrees colder is a temperature of ⁻3°C than a temperature of +4°C?

Reprint 2023

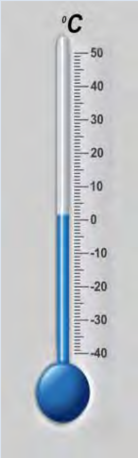
Chapter 1 Representing Integers

5.1.1 Integer Models

Try This

This thermometer uses a Celsius temperature scale.
On the Celsius scale, 0ºC is the freezing point of water.

- Temperatures warmer than, or above the freezing point of water are positive (+).
- Temperatures colder than, or below the freezing point of water are negative (–).



A. The usual high January temperature in Paro is +9°C.
Use the clues below to figure out the temperature for the usual low January temperature:

- It is between –3ºC and –10ºC but closer to –3ºC.
- It is colder than –5ºC.

- A positive sign (+) indicates that a number is greater than zero.
  A negative sign (–) indicates that a number is less than zero.
  Zero is neither positive nor negative.

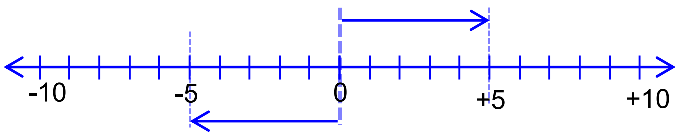
- Every positive number, such as +5, has an **opposite** negative number.
For example, the opposite of +5 is –5.
- Every negative number has an opposite positive number.
For example, the opposite of –5 is +5.
- One number is the opposite of another if the numbers are the same except for their sign. Zero is its own opposite.

- The set of numbers that contains zero and all the whole numbers and their opposites is called the set of **integers**.

Integers can be represented or modelled using a number line.

Opposite integers are the same distance from zero but on opposite sides of zero.

For example, –5 is 5 spaces left of 0 and +5 is 5 spaces right of 0.



Integers can also be represented or modelled with counters

A white counter could represent +1 and a black counter could represent –1.

For example, you can represent both +3 and –3 with three counters:
three white counters for +3 and three black counters for –3.

+3 –3

+3 and –3 are opposite integers

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B. Explain how could you use a number line to find the usual low January temperature in Paro from **part A**.

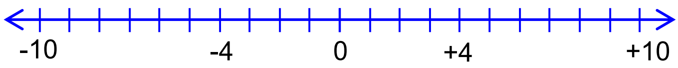
Examples

Example 1 Locating Opposite Integers on a Number Line

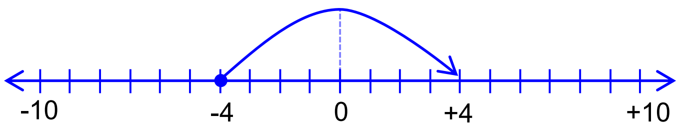
a) What number is the opposite of –4?

b) What number is the opposite of +10?

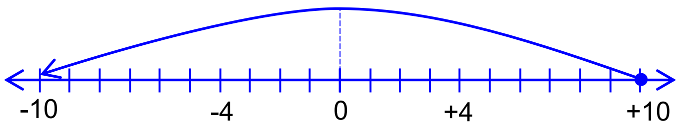
Solution



a) The opposite of –4 is +4.



b) The opposite of +10 is –10.



Thinking

I drew a number line from -10 to +10.

a) I knew -4 was 4 spaces left of 0 because I counted. So I counted 4 spaces right of 0 to find +4.

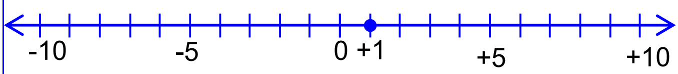
b) I knew +10 was 10 spaces right of 0 because I counted. So I counted 10 spaces left of 0 to find -10.

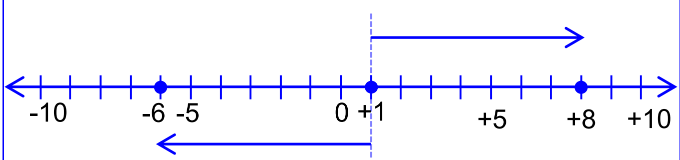
Example 2 Identifying Integers a Given Distance Away

a) What two integers are 7 units away from +1?

b) Why are they not opposites?

Solution





a) +8 and –6 are 7 units away from +1.

b) They are not opposites because they are not the same distance from zero.

Thinking

a) I drew a number line and marked +1.

- I counted 7 spaces to the right of +1 and then counted 7 spaces to the left of +1.

b) +8 is 8 units right of zero and -6 is 6 units left of zero.

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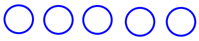
Example 3 Representing Opposite Integers Using Counters

Which numbers can you represent using either five black or five white counters?

Solution	Thinking
+5 	• I represented +5 using 5 white counters.
–5 	• I represented –5 using 5 black counters.

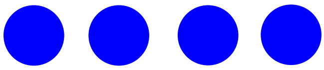
Practising and Applying

1. a) Draw a number line from –10 to +10. Mark these integers on your number line.

+6, –4, –3, +2

b) Explain how to find the opposite of each integer.

2. a) What integer do these counters represent?



b) How would you represent the opposite integer using counters?

c) What is the opposite integer?

3. Two opposite integers are 12 units apart on a number line. What are the integers?

4. a) Find two integers that are 5 units from –1. How did you find the integers?

b) Find two integers that are 10 units from 0.

c) Find two integers that are 4 units from +2.

5. Which part of **question 4** resulted in opposite integers? Explain how you know.

6. a) Which positive integer is 6 units from –2?

b) Which negative integer is 9 units from –5?

c) Which negative integer is 4 units from –4?

d) Which positive integer is 5 units from the opposite of –3?

7. The usual high temperature in January in Trashigang is 20ºC.

Which one of the following hint can be used to find the usual low temperature?

- The usual low temperature is colder than 6ºC below the usual high temperature.
- The usual low temperature is between +10ºC and +15ºC but closer to +10ºC than to +15ºC.

a) What is the usual low temperature?

b) Which of the three clues is necessary? Explain your thinking.

8. Why does the opposite of an integer always have a different sign unless the integer is zero?

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5.1.2 Comparing and Ordering Integers

Try This

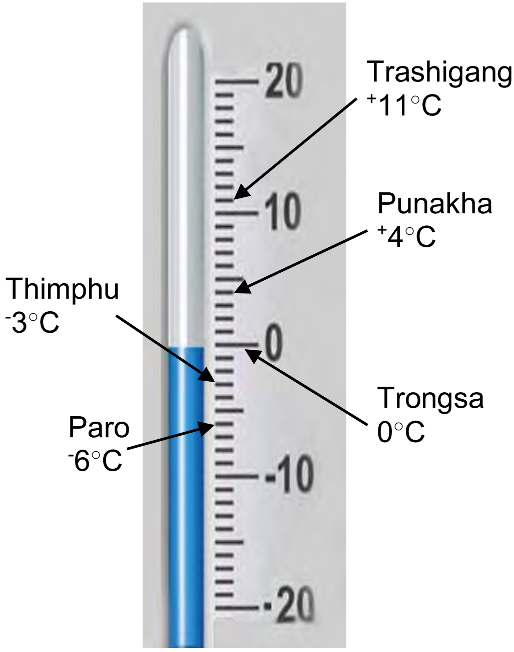
The usual low temperatures in January for several places are shown on the thermometer.

A. Identify each place.

i) It is warmer than Paro by 6°C.

ii) It is colder than Trashigang by 14°C.

iii) It is colder than Punakha by 10°C.



Temperature increases as you move up a thermometer and decreases as you move down. You can think of a thermometer as a vertical number line for modelling integers.

- On a horizontal number line, the numbers increase as you move to the right.
- On a vertical number line, the numbers increase as you move up.

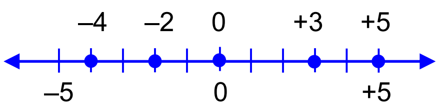
B. How is using a thermometer like using a number line?

Examples

Example 1 Ordering Integers

Order –2, +5, +3, 0, and –4 from least to greatest.

Solution



From least to greatest:

–4, –2, 0, +3, +5

Thinking

- I marked each number on a number line by
  - deciding if it was right (+) or left (–) of 0
  - counting spaces right or left, for example, –2 was 2 spaces left of zero.
- When you move from left to right on a horizontal number line, you go from least to greatest.

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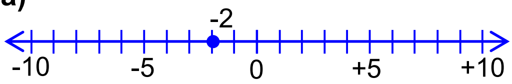
Example 2 Comparing Integers Using Symbols

Replace each ■ with < or > to compare each pair of integers.

a) –2 ■ –5 **b)** –3 ■ +1

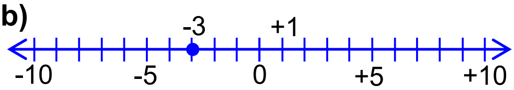
Solution

a)



–2 > –5

b)



–3 < +1

Thinking

I marked each integer on a number line.

a) -2 is right of -5, so -2 is greater than -5.

b) -3 is left of +1, so -3 is less than +1.

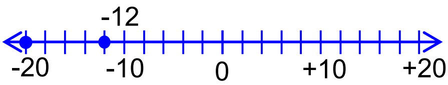
Example 3 Comparing Integers Far From Zero

Which integer in each pair is greater?

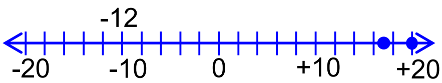
a) –12, –20 **b)** +17, +20 **c)** –47, +35

Solution

a) –12 is greater than –20.



b) +20 is greater than +17.



c) +35 is greater than –47.

Thinking

a) I sketched a number line from -20 to 20.

- I saw that -20 was 20 to the left of 0 and -12 was only 12 to the left of 0.
- Since -12 was farther right, I knew it was greater.

b) I used a number line again.

- I saw that +20 was 20 to the right of 0 and +17 was only 17 to the right of 0.
- Since 20 was farther right, I knew +20 > +17.

c) Since any negative integer is less than any positive integer, I knew that +35 > -47.

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Practising and Applying

1. a) Draw a number line from –10 to +10. Mark these integers:
+9, –7, –3, 0, +5

b) Order the integers from least to greatest.

2. Write two integers for each.

a) between –3 and –10
b) less than +4 and greater than –2
c) between 0 and –5 and greater than –3
d) greater than –1
e) less than –14
f) opposites between –5 and +5
g) the same distance from +2

3. Which integer is greater? Use a number line to show how you know.

a) +3, +4 **b)** –3, +4
c) –3, –4 **d)** +3, –4

4. Replace each ■ with < or > to compare each pair of integers. Explain your thinking.

a) 0 ■ +4 **b)** +3 ■ –5
c) –23 ■ +18 **d)** –65 ■ +65

5. Here are some usual low temperatures for December.

Paro: –2°C
Thimphu: –1°C
Punakha: +8°C
Wangdue: +6°C
Trongsa: +3°C

a) Draw a vertical number line from –10 to +10 and mark each temperature.

b) Order the places from coldest to warmest.

6. Order each set of integers from least to greatest.

a) +8, –25, +16, + 25, –12
b) –140, –100, –120, –10
c) –6, –48, +4, +210, 0

7. Here is a weather forecast for Thimphu.

Date	Daily high	Daily low
Nov. 15 	+13°C	–1°C
Nov. 16 	+14°C	0°C
Nov. 17 	+15°C	–3°C
Nov. 18 	+9°C	–2°C

Write the date for each.

a) the highest daily high temperature
b) the lowest daily high temperature
c) the highest daily low temperature
d) the lowest daily low temperature

8. In golf, the player with the lowest score wins. The final scores for six players in the 2006 PGA Championship are listed below.

PGA Championship 2006

Player	Country	Score
Hideto Tanihara	Japan	+4
Tiger Woods	U.S.A.	–18
Henrik Stenson	Sweden	–7
Miguel Angel Jimenez	Spain	+8
Nathan Green	Australia	+3
Mike Weir	Canada	–11

a) List the scores from lowest to highest.
b) Who won the tournament?

9. a) Find the mystery number using these clues.

- I am less than –4.
- I am greater than –7.
- I am a negative number.
- I am in this counting sequence: +7, +5, +3, +1, –1, ...
- I am less than –2.

b) Which clues are not needed?

10. Why is –3 less than any positive number?

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CONNECTIONS: Time Zones

The earth rotates on its axis once each day. Since there are 24 hours in a day, the earth is divided into 24 time zones.

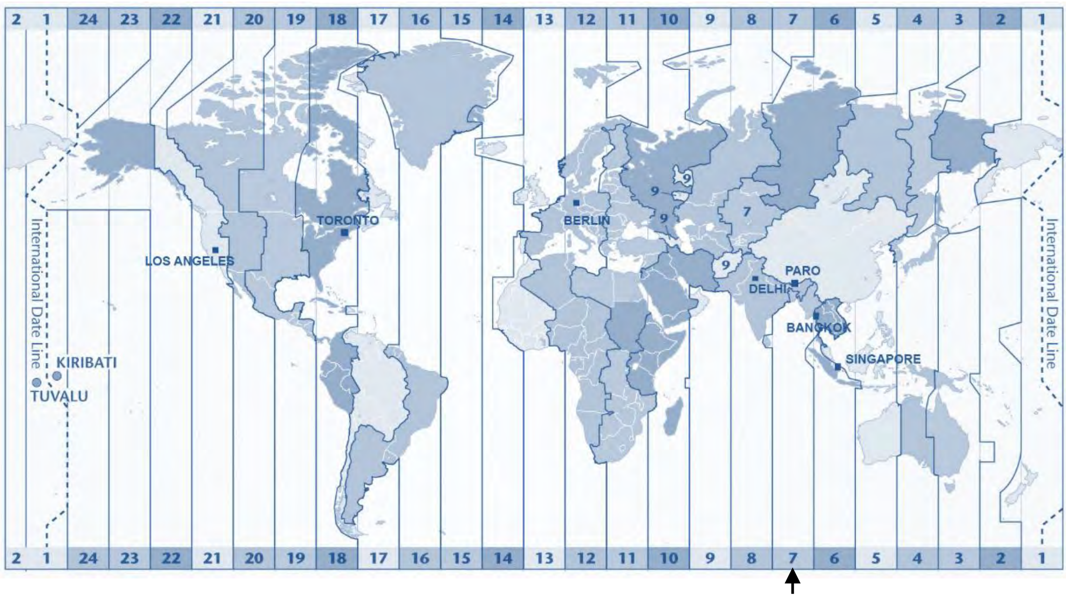
• Places that are in the same time zone have the same time. That means, when you travel north and south within a time zone, the time remains the same.

• When you travel from east to west, you subtract one hour each time you cross into a new time zone.

For example, if it is 1:30 p.m. in Zone 9, it is 12:30 pm in Zone 10.

• When you travel from west to east, you add one hour each time you cross into a new time zone.

The 24 time zones are shown on the map below.



1. Bhutan is in time zone 7. How far behind or ahead of Bhutan time is each city? Write your answer as an integer.

a) Toronto, Canada (Zone 18)

b) Bangkok, Thailand (Zone 6)

c) Berlin, Germany (Zone 12)

d) Los Angeles, United States of America (Zone 21)

e) Singapore (Zone 5)

2. If it is noon (12:00) in Bhutan, what time is it in each city listed in **question 1**?

5.1.3 The Zero Property

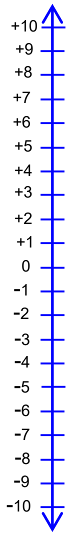
Try This

A. Draw a vertical number line from –10 to +10.

- Place your pencil on 0. Then flip a Nu 1 coin.
- Move up 1 if you get a Khorlo (K).
- Move down 1 if you get a Tashi Ta-gye (T).

i) Predict where your pencil will be after 10 flips.

ii) Test your prediction. Flip a coin 10 times. Record your progress by drawing arrows on the number line.



- On a vertical number line, movements down are negative and movements up are positive.

- Maya played the number line and coin game in the **Try This**.
Here are her results:

Flip number	1	2	3	4	5	6	7	8	9	10
Result	K	T	T	K	K	T	K	K	T	T
Move	up	down	down	up	up	down	up	up	down	down
Integer	+1	-1	-1	+1	+1	-1	+1	+1	-1	-1
Location (Start at 0)	+1	0	-1	0	+1	0	+1	+2	+1	0

- A combination of K and then T gets her back to 0. So does a combination of T and then K.

- Maya flipped the same number of Khorlos as Tashi Ta-gyes, so she ended up at 0. This happened because any integer added to its opposite has a value of 0. This is called the **zero property**.

For example:

- Each flip of K followed by a flip of T is (+1) + (–1) = 0.

- Five flips of K and five flips of T is (+5) + (–5) = 0.

B. i) How do your results from **part A ii)** compare with Maya's results?

ii) Explain how your number line shows positive and negative values.

C. Use the zero property to figure out the result of these 10 flips:

K K T T K K K T T K

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Examples

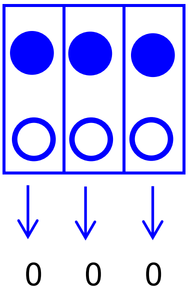
Example Representing the Zero Property

Show that (–3) + (+3) = 0.

Solution 1

−3

+3



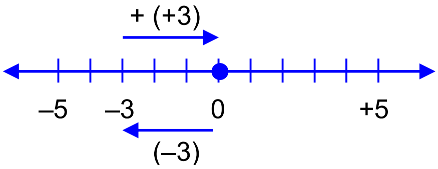
$-3 + +3 = 0$

Thinking

I used 3 black counters to represent -3 and 3 white counters to represent +3.

- I made 3 pairs of black and white counters.
- Each pair was worth 0 because (+1) + (-1) = 0.

Solution 2



(–3) + (+3) = 0

Thinking

I drew a number line and started at 0:

- -3 means 3 to the left.
- +3 means 3 to the right.
- I went 3 to the left and then 3 to the right and ended up back at 0.

Practising and Applying

1. Use counters to model this addition.

+2 + (–2) = 0

2. Gembo played the number line and coin game from **page 162**. He flipped a Nu 1 coin six times and got these results:

Flip number	1	2	3	4	5	6
Result	K	T	T	K	K	T
Move	+1	-1	-1	+1	+1	-1

He started at 0 on a vertical number line. He moved his pencil tip up 1 when he got a Khorlo (K) and moved it down 1 when he got a Tashi Ta-gye (T).

a) Where was Gembo's pencil tip after six flips?

b) Explain how you can use the zero property to calculate his final location.

c) Where will his pencil tip be, if his seventh flip is a Khorlo (K)?

3. Replace each ■ with +1 or –1.

a) (+1) + ■ = 0

b) (–1) + (–1) + ■ = –1

c) (+1) + ■ + ■ = +1

d) (+1) + ■ + ■ + ■ + ■ = –1

e) (–1) + ■ + ■ + ■ + ■ = –1

f) (+1) + ■ + ■ + ■ + (+1) = –1

4. Suppose you played the number line game and flipped a coin 30 times. Your final score is the same as your final location.

a) What would be the greatest possible score? Explain your thinking.

b) What would be the least possible score? Explain your thinking.

5. How can you use the zero property to do each?

a) add +1 to any negative integer

b) add –1 to any positive integer

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Chapter 2 Adding and Subtracting Integers

5.2.1 Adding Integers Using the Zero Property

Try This

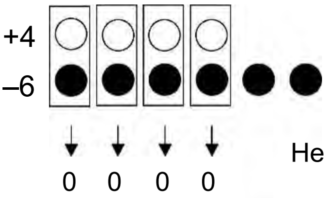
Therchung modelled one number with 3 counters and another number with 5 counters. When he added the numbers, the sum was modelled with 2 counters.

A. What could the numbers be? Explain your thinking.

- Recall the number line and coin game from **lesson 5.1.3** on **page 162**. The game described below is similar but, instead of moving up and down a number line, you earn white and black counters. Your final score is the total value of your counters at the end of the game.
- Maya played the game. Here are her results:

Flip number	1	2	3	4	5	6	7	8	9	10
Result	K	T	T	K	K	T	K	T	T	T
Counter	W	B	B	W	W	B	W	B	B	B
Value	+1	-1	-1	+1	+1	-1	+1	-1	-1	-1

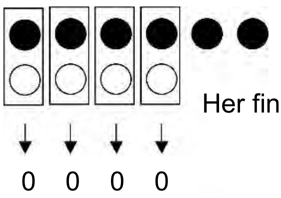
- Maya has 4 white counters worth +4 and 6 black counters worth –6.
- To find her final score, she uses the zero property to add the integers.



(+4) + (–6) = –2

Her final score is –2.

- Adding (–6) + (+4) has the same result as adding (+4) + (–6).



(–6) + (+4) = –2

Her final score is still –2.

This is an example of the **commutative** property of addition with integers:

(+4) + (–6) = (–6) + (+4)

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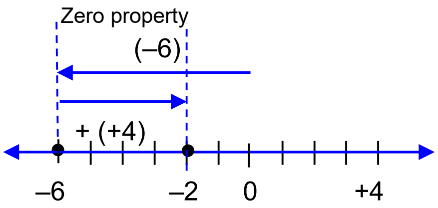
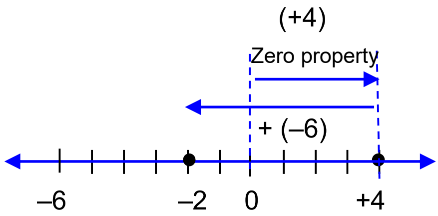
• You can also show integer addition on a number line. Again, changing the order of the addition does not change the result because of the commutative property.

To add (–6) + (+4):	To add (+4) + (–6):
Start at 0 and move left 6 to –6.	Start at 0 and move right 4 to +4.
Add +4 by moving right 4 to –2.	Add –6 by moving left 6 to –2.



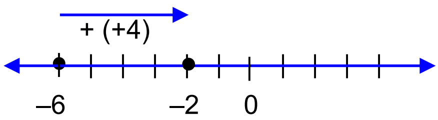
(–6) + (+4) = –2



(+4) + (–6) = –2

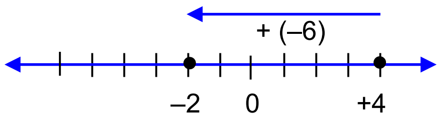
• When you add two numbers on a number line you can use a single arrow instead of two arrows. You can start at either number.

To add (–6) + (+4), start at –6 and add +4 by moving right 4 to –2.



(–6) + (+4) = –2

To add (+4) + (–6), start at +4 and add –6 by moving left 6 to –2.

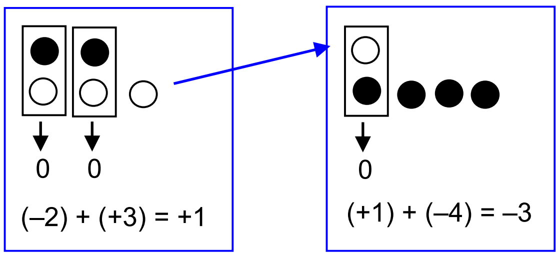


(+4) + (–6) = –2

• To add three integers, you can first combine any pair and then add the third integer.

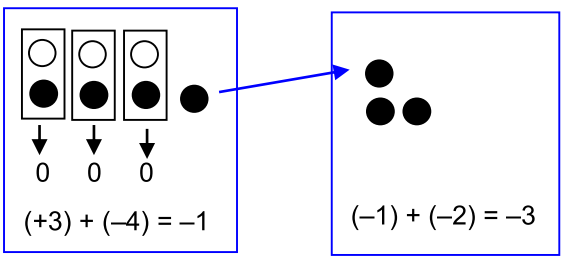
For example, to add (–2) + (+3) + (–4):

First add (–2) + (+3) to get +1, then add (+1) + (–4):



So (–2) + (+3) + (–4) = –3

Or, add (+3) + (–4) to get –1. Then add (–1) and (–2):



So (–2) + (+3) + (–4) = –3

This is an example of the **associative** property of addition of integers:

[(–2) + (+3)] + (–4) = (–2) + [(+3) + (–4)]

• When you add integers, the result is always another integer. This is called the **closure property** of integers under addition.

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B. How can you use the zero property to help you find the numbers modelled in **part A**?

Examples

Example Adding Positive and Negative Integers

Dorji flipped a coin 10 times. Here are his results.

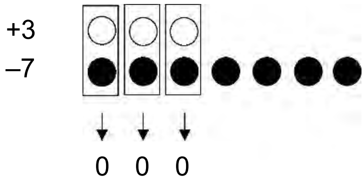
Flip number	1	2	3	4	5	6	7	8	9	10
Result	+1	-1	-1	-1	-1	-1	+1	+1	-1	-1

He earned 3 white counters and 7 black counters. What is his final score?

Solution 1

Using counters

3 white counters is +3.
7 black counters is –7.



(+3) + (–7) = –4, so his score is –4.

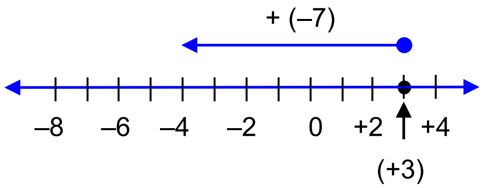
Thinking

- I showed his results with white and black counters.
- I used the zero property to make pairs of white and black counters each worth 0.
- I was left with 4 black counters, or -4.

Solution 2

Using a number line

(+3) + (–7)



(+3) + (–7) = –4, so his score is –4.

Thinking

I modelled his results with a number line.

- 3 white counters are +3 and 7 black counters are -7, so I added (+3) + (-7) to get his score.
- I started at +3 and then moved 7 left to add -7.

Solution 3

Using numbers

(+3) + (–7)

= (+3) + (–3) + (–4)

= 0 + (–4)

= –4

(+3) + (–7) = –4, so his score is –4.

Thinking

I added (+3) + (-7) to get his score:

- Since 7 black counters was 3 black counters plus 4 black counters, I knew that -7 = (-3) + (-4).
- I used the zero property to match +3 with the -3 part of - 7 to get 0.

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Practising and Applying

1. Replace each ■ with an integer.

a) $(-3) + (+4) = \blacksquare$

b) $(-1) + (+2) = \blacksquare$

c) $(+3) + (-7) = \blacksquare$

d) $(+5) + (+4) = \blacksquare$

e) $(-6) + (+2) = \blacksquare$

f) $(-6) + (-2) = \blacksquare$

2. Add each two ways: using counters and then using a number line. Sketch your work.

a) $(-3) + (+5)$

b) $(-1) + (+3)$

c) $(+3) + (-6)$

d) $(+5) + (+3)$

e) $(-6) + (+4)$

f) $(-6) + (-4)$

3. Here is a weather forecast for Paro.

Date	Daily high temperature
Nov. 25 sunny	+12°C

What is the forecasted daily high temperature for these days?

a) November 26, up 3 degrees

b) November 27, up 1 more degree

c) November 28, down 2 degrees from November 27

4. Replace each ■ with an integer.

a) $(-6) + (-2) + (+2) = \blacksquare$

b) $(+5) + (-2) + (-5) = \blacksquare$

c) $(-3) + (-1) + (+1) = \blacksquare$

d) $(+4) + (-4) + (-8) = \blacksquare$

e) $(-1) + (-1) + (+1) = \blacksquare$

f) $(-6) + (-7) + (+6) = \blacksquare$

5. Explain how to use the zero property to solve the equations in **question 4**.

6. Replace each ■ with =, <, or >.

a) $(-2) + (+4) \blacksquare -5$

b) $(+1) + (-1) \blacksquare +2$

c) $0 + (-3) \blacksquare -3$

d) $(-6) + (+6) \blacksquare -1$

e) $(-5) + (+4) \blacksquare +1$

7. Use only +1s and –1s to create an addition sentence with each sum. Use four or more numbers for each. Check your answer with counters.

For example, for a sum of -3:

$(-1) + (-1) + (-1) + (+1) + (-1) = -3$

a) 0 **b)** +2

c) –1 **d)** +4

8. Explain why you cannot complete this using only +1s and –1s.

$$(-1) + \blacksquare + \blacksquare + \blacksquare = -1$$

9. Use mental math to add all the integers from –20 to +20. Explain how you calculated.

10. State whether each statement is true or false. Explain your thinking.

a) The sum of two positive integers is always positive.

b) The sum of two negative integers is always negative.

c) The sum of a positive integer and a negative integer is always positive.

d) The sum of a positive integer and a negative integer is always zero.

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5.2.2 Adding Integers that are Far from Zero

Try This

Lemo is trying to calculate (+28) + (–12) but she does not have enough counters.

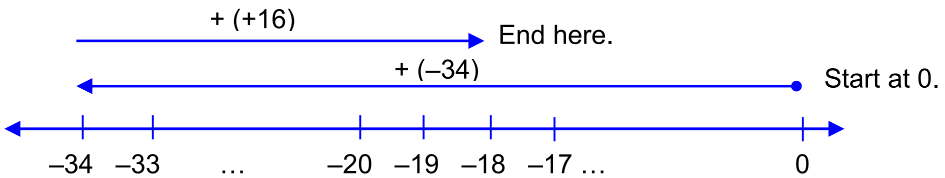
A. How could Lemo calculate (+28) + (–12)?

When you add numbers that are far from zero, you can still use number lines or counters to help you determine the sum.

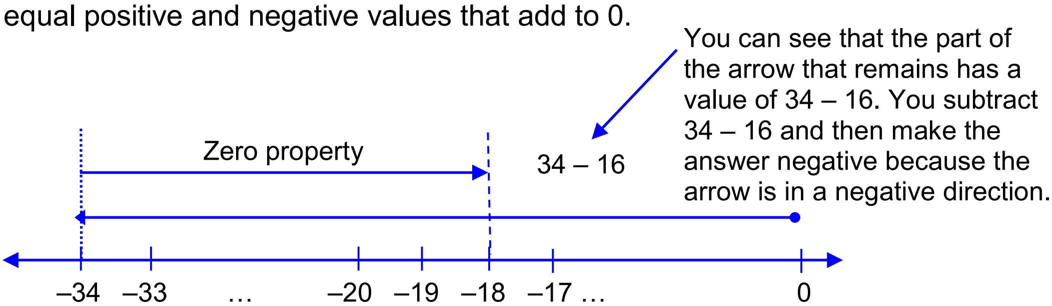
For example:

Using a number line

To add –34 + (+16), you can sketch a number line without all the numbers.



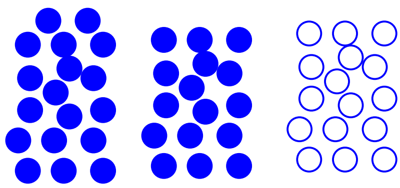
Where the two arrows overlap, you can use the zero property, which represents equal positive and negative values that add to 0.



(–34) + (+16) = –1

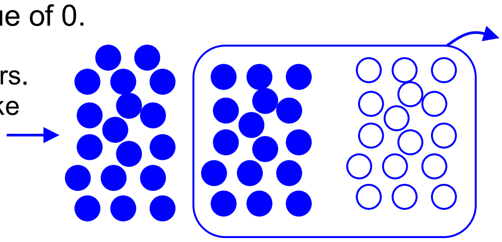
Using counters

To add (–34) + (+16), use a pile of 34 black counters and a pile of 16 white counters.



Match 16 of the black counters with 16 white counters.
The zero property tells you they have a value of 0.

You are left with 34 – 16 black counters. So you subtract 34 – 16 and then make the answer negative because the counters are black.



(–16) + (+16) = 0

(–34) + (+16) = –18

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B. How does your method for calculating (+28) + (–12) from **part A** compare to using a number line or counters?

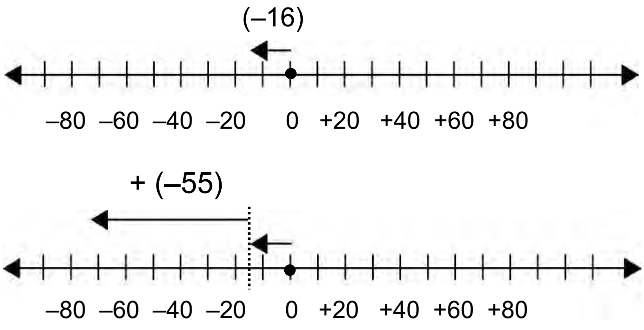
Examples

Example 1 Adding Two Negative Integers

Add –16 and –55.

Solution 1

(–16) + (–55)



(–16) + (–55) = –71

Thinking

- I started at 0 and represented -16 by moving left 16 to -16.
- I added -55 by moving left another 55 from -16. I ended up at -71.
- It's like adding 16 + 55 but making the sum negative because I was always moving in a negative (left) direction.

Solution 2

(–16) + (–55)

(–16) + (–55) = –71

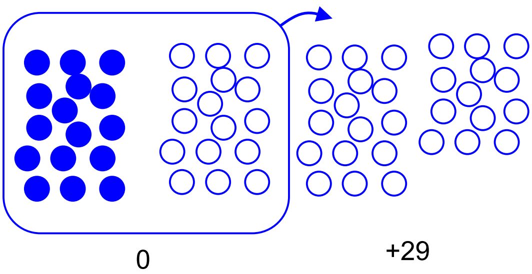
Thinking

- I visualized a bag of 16 black counters and a bag of 55 black counters.
- Since they were all black counters, I just combined them. It was like adding 16 + 55 but making the sum negative because all the counters were black.

Example 2 Adding a Positive and a Negative Integer

Add –16 and +45.

Solution 1



(–16) + (+45) = +29

Thinking

- I used a pile of 16 black counters and another pile of 45 white counters.
- I matched 16 white counters with 16 black counters.
- 16 pairs of black and white counters have a total value of 0 and 29 white counters were left.

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Example 2 Adding a Positive and a Negative Integer [Continued]	
Solution 2 (–16) + (+45) = +29	**Thinking** • There were more positives than negatives so I knew the answer would be positive. • There were 45 – 16 = 29 more positives than negatives.

Practising and Applying

1. Use a number line to model the sum of (–24) + (+47). Sketch your model.

2. Use counters to model the sum of (–24) + (+47). Sketch your model.

3. Add. Sketch a model for each.

a) (+48) + (+12)

b) (–48) + (+12)

c) (–48) + (–12)

d) (+48) + (–12)

4. How much greater is the first sum than the second sum? How do you know?

a) (–24) + (+27) = ?
(–34) + (+27) = ?

b) (+251) + (–26) = ?
(+251) + (–46) = ?

5. Copy and complete this chart.

	Start (°C)	Change (°C)	Final (°C)
a)	–12	+15	
b)	+9		+16
c)		+12	–10
d)	–15		–12

6. In the 2004 Olympic Women's Archery Competition, Bhutan's Tshering Chhoden scored 159 points to beat Lin Sang by 3 points. Write an addition sentence using positive and negative integers to show Lin Sang's score.

7. Replace each ■ with =, <, or >.

a) (–22) + (+44) ■ –55

b) (+61) + (–16) ■ +42

c) (+30) + (–33) ■ –3

d) (–60) + (+60) ■ –1

e) (–25) + (+14) ■ +39

8. a) Find each sum.

i) (–50) + (+10)

ii) (+30) + (–70)

iii) (–220) + (–340)

iv) (+115) + (–105)

b) How could you have predicted which answer in **part a)** would be least?

9. a) Add mentally.

i) (+25) + (–13)

ii) (+63) + (+17)

iii) (+12) + (–2) + (–37)

iv) (+21) + (–5) + (+9)

b) Explain how you calculated any two of the additions in **part a)**.

10. Can you predict whether the sum of two integers will be positive or negative without adding? Explain your thinking.

11. How do you know each statement is *always* true?

a) The sum of two negative integers is always negative.

b) If the sum of two integers is zero, the integers must be opposites.

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GAME: Target Sum – 50

This is a game for two or three players.
You need a set of Target Sum –50 Game Cards (digit cards and sign cards).

How to play:

• In each round, deal four digit cards and two sign cards to each player.

• Each player arranges the six cards as two 2-digit integers that have a sum as close as possible to –50.

• The player with the sum closest to –50 gets 1 point. A sum that is exactly –50 gets 2 points. The player with the sum farthest from –50 gets –1 point.

Play continues for ten rounds. The player with the most points wins.

Sample round:

Player 1 and Player 2 are dealt these cards:

Player 1

–	+	3	6	5	5

Player 2

–	–	1	8	2	3

They make these addition expressions:

–	6	5	plus	+	3	5

(–65) + (+35) = –30

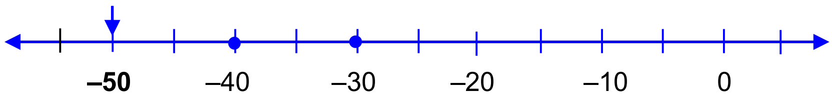
Player 1 has a sum of –30.

–	2	8	plus	–	1	3

(–28) + (–13) = –41

Player 2 has a sum of –41.

Player 2 gets 1 point because –41 is closer than –30 to –50.



Player 1 gets –1 point.

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5.2.3 Subtracting Integers Using Counters

Try This

Yuden wanted to subtract (–4) – (+2) by taking away +2 from –4.

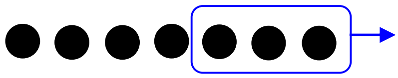
She used 4 black counters to represent –4.
She did not know what to do next because she did not have any white counters to take away.

A. What should she do? Explain your thinking.

• You can use counters to subtract integers.

For example, (–7) – (–3):

If you take away –3 from –7,

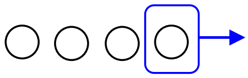


you are left with –4:

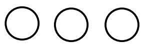
(–7) – (–3) = –4

For example, (+4) – (+1):

If you take away +1 from +4,



you are left with +3:



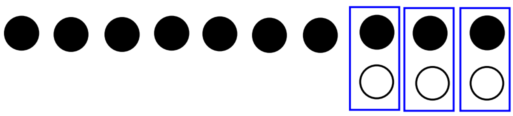
(+4) – (+1) = +3

• If you do not have enough of one type of counter to take away, you can use the zero property to add more counters that you can take away without changing the value.

For example, (–7) – (+3):

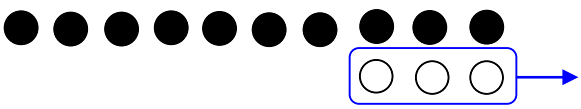
To subtract (–7) – (+3), start with 7 black counters to represent –7.

To take away 3 white counters (+3), you can add 3 white counters (+3) and 3 black counters (–3):

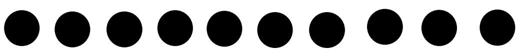


The zero property allows you to add pairs of opposite integers without affecting the sum. Since (–1) + (+1) = 0, then (–3) + (+3) = 0.

Now you can take away 3 white counters,



and you are left with 10 black counters:



(–7) – (+3) = –10

B. How can you use the zero property to subtract (–4) – (+2)?

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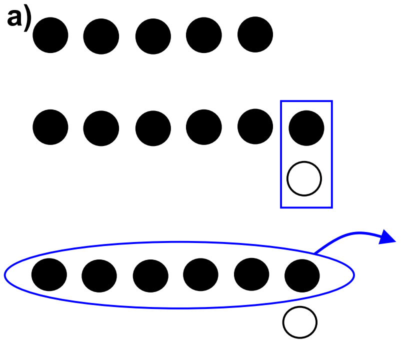
Examples

Example 1 Subtracting Integers with Same and Different Signs

Subtract. **a)** (–5) – (–6) **b)** (+3) – (–4)

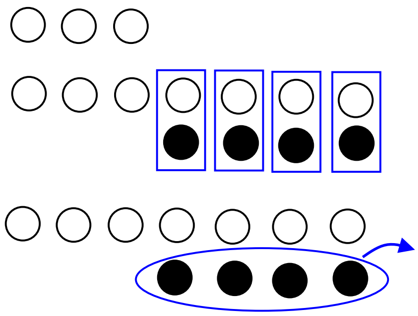
Solution

a)



(–5) – (–6) = +1

b)



(+3) – (–4) = +7

Thinking

a) I used 5 black counters to represent –5.

• I needed to take away 6 black counters but I had only 5, so I added 1 black and 1 white counter. (The new counters didn't change the value because it's like adding 0.)

• When I took away 6 black counters, 1 white counter was left.

b) I used 3 white counters to represent +3.

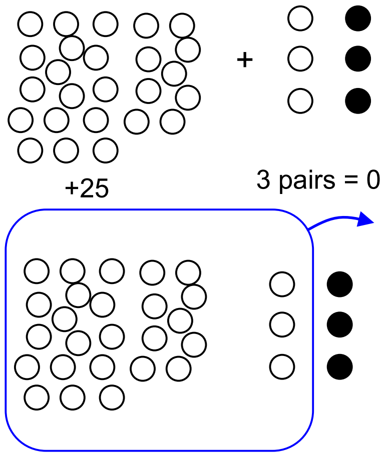
• I needed to take away 4 black counters and I had none, so I added 4 black counters and 4 white counters using the zero property.

• When I took away 4 black counters, 7 white counters were left.

Example 2 Subtracting Integers by Adding the Opposite

Find the difference: (+25) – (+28)

Solution



(+25) – (+28) = (–3)

[Continued]

Thinking

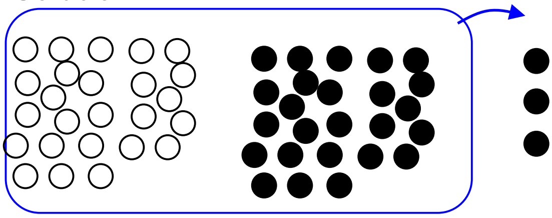
• I began with 25 white counters for +25. I needed to take away 28 white counters, so I needed 3 more white counters to take away.

• I added 3 white and 3 black counters using the zero property.

• Then I took away 28 white counters, leaving 3 black ones.

Example 2 Subtracting Integers by Adding the Opposite [Continued]

Solution



$(+25) - (+28) = (+25) + (-28)$
$= (-3)$

Thinking

• I realized I would have had the same result if I had added 28 black counters to the 25 white ones instead of trying to subtract 28 white counters. I would have taken away 25 white counters and there would have been 3 black counters left.

Practising and Applying

1. a) Use counters to subtract
(–5) – (–2).

b) Why did you not have to use the zero property?

2. Draw a counter model to show how you would subtract each.

a) (+2) – (+3) **b)** (+2) – (–3)
c) (–2) – (+3) **d)** (–3) – (–2)

3. Does (–2) – (+4) = (–2) + (–4)? How do you know?

4. Subtract using counters.

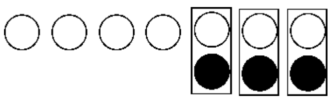
a) (+3) – (+1) – (–2)
b) (+3) – (–2) – (+1)

5. Subtract.

a) (–4) – (–2) – (+4)
b) (+13) – (–7) – (+3)
c) (–45) – (+25) – (+5)

6. Write two subtractions that can be calculated using each model.

a) 

b) 

7. Calculate each difference in **question 6**.

8. Subtract by adding the opposite.

a) (+15) – (–12) **b)** (–15) – (–12)
c) (–23) – (+17) **d)** (+37) – (+53)

9. A golf tournament lasted two days. The last column of the chart shows how well each player played on Day 2 compared to Day 1. Copy and complete the chart.

	Golfer	Day 1	Day 2	Change (Day 2 – Day 1)
	Sithar	–4	+2	+6
a)	Dechen	–4	–1	
b)	Dawa	+2	+6	
c)	Novin	–2	+4	
d)	Meto		+3	+10
e)	Karma	–7		–1

10. Find each difference. Explain how you calculated.

a) (+44) – (+23)
b) (–62) – (–45)
c) (–103) – (+33)
d) (–215) – (+30)

11. When you subtract a positive integer from a negative integer, is the answer always negative? Use counters to explain.

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5.2.4 Subtracting Integers Using a Number Line

Try This

Usual high and low temperatures in December are listed in the chart below.

	Usual high temperature	Usual low temperature
Punakha	+15°C	+8°C
Paro	+11°C	–2°C
Thimphu	+15°C	–1°C
Wangdue	+19°C	+6°C
Bumthang	+12°C	–2°C

A. i) How much colder is the usual low temperature than the usual high temperature for each place?

ii) Which place has the greatest difference between the usual high and low temperatures? How do you know?

• One way to calculate *a* – *b* is to figure out what to add to *b* to get to *a*.

For example:

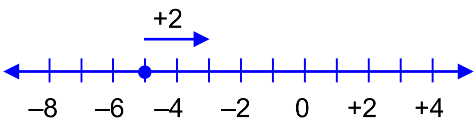
To calculate 10 – 3 = ■, you can add 3 + ■ = 7. Since 3 + **7** = 10, then 10 – 3 = **7**.

• This method also works for integers.

For example:

To calculate (–3) – (–5) = ■, you can add (–5) + ■ = –3.

Since you move 2 to the right (add +2) to go from –5 to –3, ■ is +2.



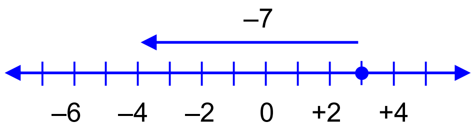
Since (–5) + (**+2**) = –3, then (–3) – (–5) = **+2.**

• If you move to the right to get from *b* to *a*, the difference is positive (as shown above). If you move to the left, the difference is negative, as shown below.

For example:

To subtract (–4) – (+3), you can add (+3) + ■ = –4.

Since you move 7 spaces to the left to go from +3 to –4, then ■ is –7.



Since (+3) + (–7) = –4, then (–4) – (+3) = –7.

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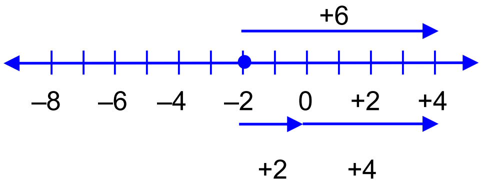
• When you subtract a negative integer from a positive integer, it is like adding the opposite.

For example, to calculate (+4) – (–2):

You can subtract –2 by figuring out what to add to –2 to get to –4.
Since (–2) + (**+6**) = +4, then (+4) – (–2) = **+6**.



You can also subtract (+4) – (–2) by adding the opposite:

(+4) **–** (**–2**) = (+4) **+** (**+2**) = +6

(+4) – (–2) is the same as (+4) + (+2), since you move 2 spaces to the right to get from –2 to 0 (that is +2) and then you move 4 more spaces to the right to get from 0 to +4 (that is another +4).

B. How could you have used a number line to find the differences in temperature in **part A**?

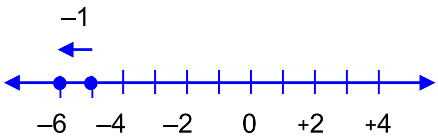
Examples

Example 1 Subtracting Integers with the Same Sign

Calculate (–6) – (–5).

Solution 1

(–6) – (–5) = ■, so (–5) + ■ = –6



Since (–5) + (**–1**) = –6, then
(–6) – (–5) = **–1**.

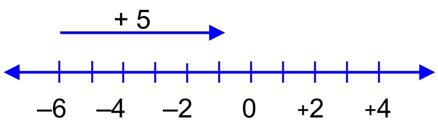
Thinking

• Instead of subtracting -5 from -6, I figured out what to add to -5 to get -6.

• I had to move 1 space to the left to get from -5 to -6, so the answer was -1.

Solution 2

(–6) – (–5) = (–6) + (+5)



(–6) – (–5) = –1

Thinking

• Instead of subtracting -5 from -6, I added the opposite of -5 to -6.

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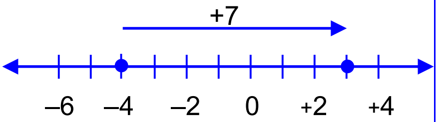
Example 2 Subtracting Integers with Different Signs

Calculate (+3) – (–4).

Solution 1

(+3) – (–4) = ■, so (–4) + ■ = +3



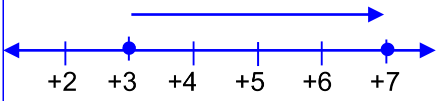
Since (–4) + (**+7**) = +3,
then (+3) – (–4) = **+7**.

Thinking

- Instead of subtracting –4 from +3, I figured out what to add to –4 to get +3.
- To get from –4 to +3, I moved 7 spaces to the right, so the answer was +7.

Solution 2

(+3) – (–4) = (+3) + (+4)



(+3) + (+4) = +7

Thinking

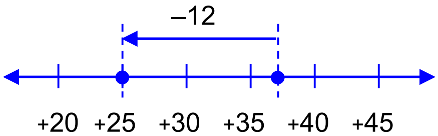
- I knew (+3) – (–4) = (+3) + (+4) because you can subtract an integer by adding its opposite.
- I started at +3 and then added +4 by moving 4 to the right.

Example 3 Subtracting Integers Far from Zero

Subtract. **a)** (+25) – (+37) **b)** (–61) – (+48)

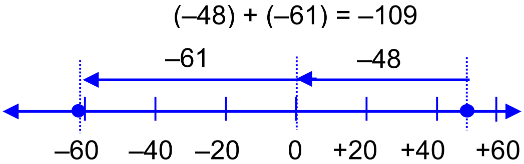
Solution 1

a) (+25) – (+37) = ■, so (+37) + ■ = +25



Since (+37) + (**–12**) = +25,
then (+25) – (+37) = **–12**

b) (–61) – (+48) = ■, so (+48) + ■ = –61



Since (+48) + (**–109**) = –61,
then (–61) – (+48) = **–109**.

Thinking

a) Instead of subtracting, I figured out what to add to +37 to get +25.

- I sketched a number line that included +25 and +37.
- To get from +37 to +25, I moved 12 to the left, or –12.

b) Instead of subtracting, I figured out what to add to +48 to get –61.

- I sketched a number line that included –61 and +48.
- To get from +48 to –61, I moved 48 to the left (–48) to get to 0. Then I moved another 61 to the left (–61) to –61.
- Altogether I moved 48 + 61 = 109 to the left, which is –109.

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Example 3 Subtracting Integers Far from Zero [Continued]

Solution 2

a) (+25) – (+37)
= (+25) + (–37)
= [(+25) + (–25)] + (–12) = –12

b) (–61) – (+48) = (–61) + (–48)
= –109

Thinking

I added the opposite.

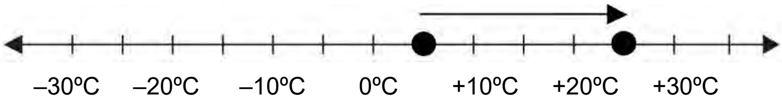
a) *I used the zero property.*

b) *Adding two negatives is like adding two whole numbers, but you make the sum negative.*

Practising and Applying

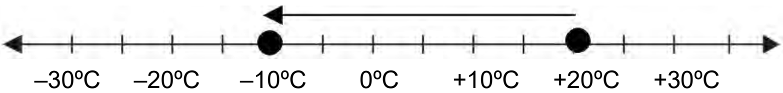
1. Write a subtraction sentence for each.

a)



(+25) – (■) = ■

b)



(■) – (■) = –30

2. a) Subtract each on a number line.

i) (–5) – (–3) **ii)** (–3) – (–5)

b) How do the answers compare?

3. Calculate mentally.

a) (–5) – (+5)

b) (+13) – (+3) – (+10)

c) (–35) – (–15) – (+15)

d) (+40) – (–21) – (–9)

e) (–33) – (+60) – (–3)

4. In Trongsa, the temperature fell by 12ºC in one day from a high of +10ºC.

a) What was the low temperature?

b) Write a subtraction expression that describes this situation.

c) Write an addition expression that describes this situation.

5. a) Use a number line to show why this is true: (+3) – (–2) = (+3) + (+2)

b) Is subtracting an integer always the same as adding its opposite? Explain your thinking.

6. Write three different subtractions that have a result of –4.

(■) – (■) = –4

7. Use a number line to explain why (+23) – (–17) and (–17) – (+23) have opposite values.

8. Use the information in the chart to find the change in elevation for each trip below. Express your answer as an integer. Show your work.

Place	Metres above sea level
Paro	2235
Punakha	1250
Thimphu	2320
Trongsa	2120

a) Trongsa to Thimphu

b) Paro to Trongsa

c) Trongsa to Punakha

d) Thimphu to Trongsa

e) Paro to Thimphu

f) Punakha to Paro

9. a) ■ represents the same integer in each calculation. Which calculation is greater?

■ – (+1) or ■ – (–1)

b) If you subtract a positive integer from a number, is the answer *always* less than if you subtract its opposite from that same number? Explain your thinking. Use an example.

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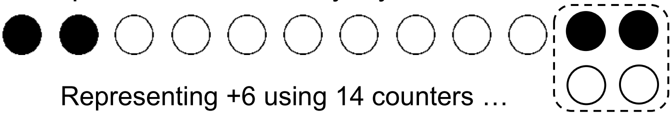
5.2.5 EXPLORE: Integer Representations

There are many ways to represent an integer.
For example:
You can represent +6 using 10 counters. You can represent this with two integers:

$+6 = (-2) + (+8)$

To represent +6 other ways, you could add an even number of counters:



$+6 = (-2) + (+8)$ $\mathbf{+ (+2) + (-2)}$

Representing +6 using 14 counters … and four integers

You could also represent +6 as the sum of five integers, instead of four integers, by breaking up one of the integers you already have:

$+6 = (-2) + \mathbf{(+8)} + (+2) + (-2)$
$\quad = (+2) + \mathbf{(+5) + (+3)} + (-2) + (-2)$

A. Describe –10 using each.

i) 24 counters

ii) a sum of 3 integers

iii) a sum of 4 integers

iv) a sum of 5 integers

v) a sum of 6 integers

vi) a sum of 7 integers

vii) a sum of 8 integers

viii) a sum of 9 integers

ix) a sum of 10 integers

B. What patterns do you see when you describe –10 using a sum of integers?

C. Choose another integer and describe it using the conditions in **part A**.

D. You can express –10 as a sum of two integers between –1 and –9 in the five ways shown here.

$-10 = (-9) + (-1)$
$-10 = (-8) + (-2)$
$-10 = (-7) + (-3)$
$-10 = (-6) + (-4)$
$-10 = (-5) + (-5)$

i) How many ways can you express –12 as a sum of two integers between –1 and –11?

ii) How many ways can you express –8 as a sum of two integers between –1 and –7?

iii) What do you notice about the relationship between an even integer and the number of ways you can express it as a sum of two integers?

iv) Use what you learned in **parts i) and ii)** to predict how many ways you can write –100 as a sum of two integers between –1 and –99.

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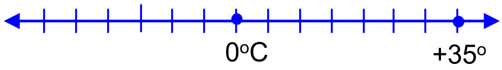
UNIT 5 Revision

1. Draw a number line like this.



a) Mark each temperature on your number line.

–10ºC +5ºC +30ºC

+10ºC –30ºC –5ºC

b) Which pairs of temperatures are opposites? Explain how you know.

2. An integer is between –2 and +12.

• It is less than 6 units away from –2.

• It is less than 10 units away from +12.

What is the integer?

3. Tell whether each statement is true or false. Explain your thinking.

a) The farther an integer is from zero on a number line, the greater it is.

b) All integers less than –5 are negative.

c) No positive integer is less than +1.

4. Use a model to show (–5) + (+5) = 0.

5. Suppose the low temperature for Bumthang on Nov. 1 was –2ºC.
What was the low temperature on each day? Explain how you got your answer.

a) Nov. 2, up 4 degrees

b) Nov. 3, up 2 degrees

c) Nov. 4, down 3 degrees

6. Explain why –5 is less than +5, even though both numbers are the same distance from 0.

7. Add each expression using counters or a number line. Sketch your work.

a) (–2) + (–3)

b) (+4) + (–1)

c) (+5) + (–5)

8. Add.

a) (–30) + (+40)

b) (+30) + (+40)

c) (–30) + (–40)

d) (+30) + (–40)

9. Replace each ■ with < or >.
How do you know you are right?

a) +100 – (+4) ■ +98

b) –37 ■ –31 – (–3)

10. Subtract each using counters or a number line. Sketch your work.

a) (–2) – (–3)

b) (+4) – (–1)

c) (–5) – (+5)

11. Subtract.

a) (–230) – (+110)

b) (+68) – (–13)

c) (–134) – (–74)

12. a) If you add a negative integer to a positive integer, when is the sum positive?

b) If you subtract a negative integer from a positive integer, when is the difference negative?

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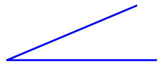
UNIT 6 ALGEBRA

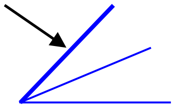
Getting Started

Use What You Know

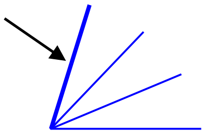
A. i) Draw an angle like this.



ii) Add a ray at the vertex. Explain why there are now three angles in the diagram.



iii) Add another ray at the vertex.
How many angles are there now?



iv) Repeat **part iii)** three more times.

B. i) The drawings below represent the number of angles in **part A i) and ii)**.
Continue the pattern for **parts iii) and iv)**.

1 3

ii) Why do you think the numbers in **part i)** are called triangular numbers?

Skills You Will Need

1. Draw a diagram to show why each number is called a square number.

a) 9 **b)** 25 **c)** 36

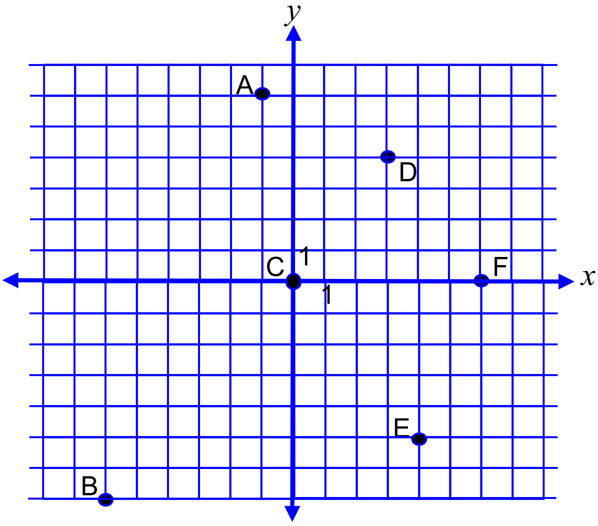
2. Solve each equation.

a) $12 + n = 15$ **b)** $h - 15 = 32$

c) $15 + 2n = 33$ **d)** $15 - a = 12$

3. Which equations in **question 2** have the same solution?

4. Name the ordered pair for each point on the graph.

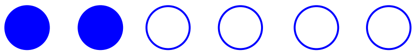


5. Copy the graph in **question 4** and then plot these points on it.

G (4, 0) **H** (0, –1)

I (–7, 6) **J** (–7, –1)

6. a) Write a ratio that compares the number of blue circles to the number of white circles.



b) Write three ratios that are equivalent to the ratio in **part a)**.

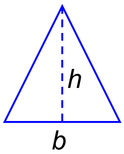
c) For each of the four ratios from **parts a) and b)**, create an ordered pair. For example, 2 : 4 would be (2, 4).

d) Plot the points from **part c)** on a grid.

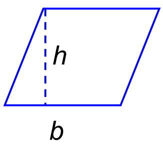
e) Describe the pattern in the plotted points.

7. a) Write an expression (formula) for the area of each shape using *b* and *h*.

i)



ii)



b) How would the area of each shape in **part a)** change in each case?

i) if *h* is doubled

ii) if both *b* and *h* are doubled

iii) if *b* is doubled and *h* is divided by 2

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Chapter 1 Patterns and Relationships

6.1.1 Using Variables to Describe Pattern Rules

Try This

In this pattern, the number of squares changes in each new figure.

A. Write the first four terms of each number pattern below.

i) the number of white squares

ii) the number of black squares

iii) the total number of squares

B. Use the patterns in **part A** to predict each number of squares in Figure 12.

i) the number of white squares

ii) the number of black squares

iii) the total number of squares

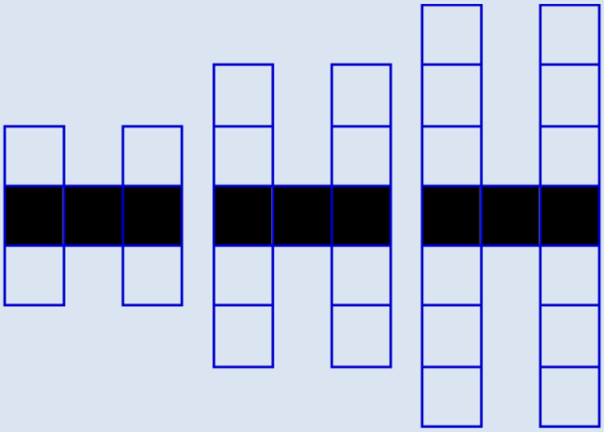
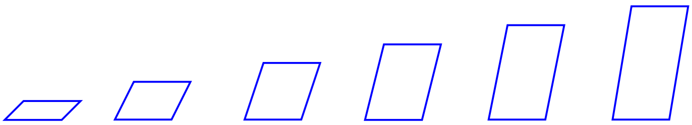


Figure 1 Figure 2 Figure 3

• You can use a **table of values** to describe or represent an **equation** or **formula**. For example, the table of values below compares the areas of parallelograms with the same base but different heights.

Parallelograms with a base of 4 units

Height (h)	1	2	3	4	5	6
Area ($A = bh$)	4	8	12	16	20	24



- Notice that the area goes up by 4 each time, so the next area will be 28.

- The table is based on the formula for the area of a parallelogram ($A = bh$) using a base of 4 units ($A = 4h$). The formula $A = 4h$ represents the **relationship** between area and height.

• Formulas are made up of **expressions** that include **variables**.

- The formula $A = 4h$ from above means $A = 4 \times h$.

- A and h are variables because their values can change. The example above uses whole numbers, but the values can also be decimals.

- The multiplier of a variable is called a **coefficient**. In the expression $4h$, 4 is the coefficient of h. A coefficient can be an integer, a fraction, or a decimal.

- If you think of an expression like $4 - 3x$ as $4 + (-3)x$, you can see that -3 is the coefficient of x.

• A table of values is useful for making predictions about a relationship.

For example, the table of values to the right shows the relationship between the number of squares and the figure number for the pattern below.

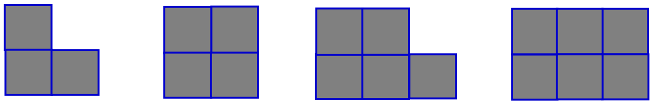


Figure 1 Figure 2 Figure 3 Figure 4

Figure number	Number of squares
1	3
2	4
3	5
4	6

Since the number of squares is always 2 greater than the figure number, you can predict that the number of squares for Figure 15 will be 15 + 2 = 17 squares.

• You can also use an expression to make a prediction. An expression for a relationship that is based on a pattern is sometimes called a **pattern rule**.

- For the pattern above, the pattern rule $f + 2$ tells the number of squares if you know the figure number (the variable f represents the figure number).

- In the expression $f + 2$, the variable f has a coefficient of 1 (since $f = 1 \times f$).

- The expression $f + 2$ also has a **constant** of 2. A constant is a number that is not a coefficient. It is called a constant because it remains constant.
This means it does not change when the variable changes.

- You can use the pattern rule to predict the number of squares for Figure 15:
If $f = 15$, then $f + 2 = 15 + 2 = 17$ squares.

C. i) Create a table of values for the pattern in **part A**.

Figure number	Total number of squares

ii) Write a pattern rule that you could use to predict the total number of squares if you know the figure number.

Examples

Example 1 Using a Table of Values to Create a Pattern Rule

Imagine using matchsticks to make each figure in the pattern below.

a) Create a table of values to show the number of matchsticks in each figure.

b) Use the table to create a pattern rule.

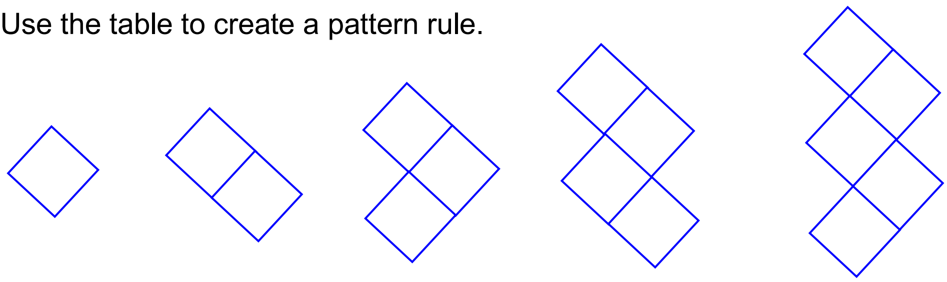


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

Solution

a)

Figure number	Number of matchsticks
1	4
2	7
3	10
4	13
5	16

b) $3f + 1$, where f is the figure number.

Thinking

a) Each side of each small square is 1 matchstick.
I counted the number of sides in each small square.
If two squares shared one side, I counted the side only once.

b) I looked for a relationship between the number of matchsticks and the figure number.

- I noticed that the number of matchsticks was 3 times the figure number plus 1:

4 = 3 × **1** + 1, **7** = 3 × **2** + 1, **10** = 3 × **3** + 1, **13** = 3 × **4** + 1, and **16** = 3 × **5** + 1.

Example 2 Creating Equivalent Pattern Rules

Create two different but equivalent pattern rules to describe the number of squares in each figure.

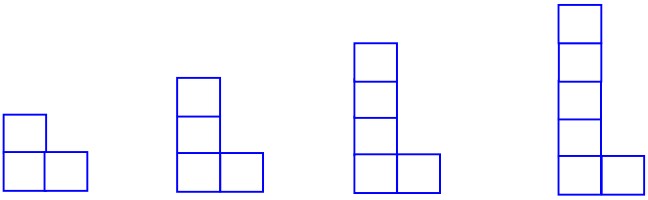


Figure 1 Figure 2 Figure 3 Figure 4

Solution

If the 2 bottom squares are constant

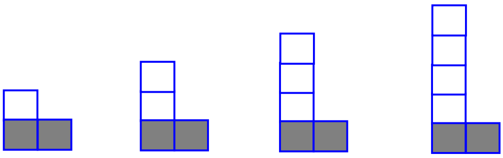


Fig 1	Fig 2	Fig 3	Fig 4
1 + 2	2 + 2	3 + 2	4 + 2

Pattern rule: $f + 2$

If the 1 square on the right is constant

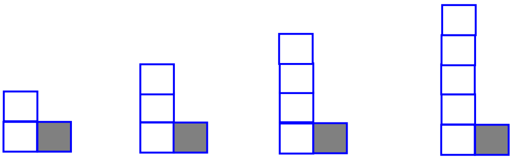


Fig 1	Fig 2	Fig 3	Fig 4
2 + 1	3 + 1	4 + 1	5 + 1

Pattern rule: $(f + 1) + 1$

Thinking

- First pattern rule:

I decided to make the 2 squares at the bottom constant so the number of remaining squares would match the figure number.
So, Figure f has f white squares plus the 2 constant grey squares at the bottom.

- Second pattern rule:

I let the single square on the right be the constant. That made the number of remaining squares 1 more than the figure number.
So, Figure f has $f + 1$ vertical squares plus the 1 constant grey square on the right.

Practising and Applying

1. For each pattern rule, list the variable, the coefficient, and the constant.

a) $3h + 5$ **b)** $-2m - 4$
c) $6 + q$ **d)** $5n + 3$

2. For each formula, list the variables, the coefficients, and the constants.

a) $P = 4s$ **b)** $A = \pi r^2$

3. Copy and complete each table.

a)

Figure number	Number of shapes
1	6
2	8
3	10
4	12
5	
6	
7	

b)

Figure number	Number of shapes
1	35
2	30
3	25
4	20
5	
6	
7	

4. Look at this pattern made of cubes.

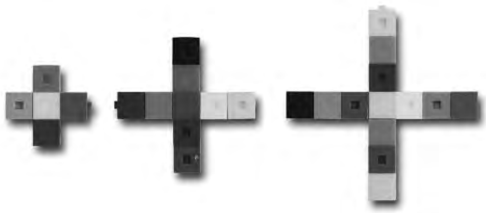


Figure 1 Figure 2 Figure 3

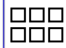
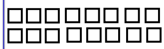
a) Sketch the next two figures.

b) Create a table of values to show the number of cubes for each figure number. Extend the table to include Figure 8.

c) Write a pattern rule that you could use to find the number of cubes if you know the figure number.

d) Predict the number of cubes in Figure 20.

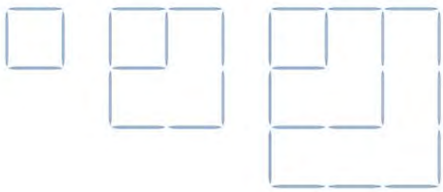
5. a) Copy and complete the table.

Figure number	Figure	Number of squares
1		6
2		11
3		16
4		
5		

b) Write a pattern rule that could be used to find the number of squares if you know the figure number.

6. a) Sketch Figure 4 in this pattern.

Figure 1 Figure 2 Figure 3



b) Use a table of values to find the number of sticks in Figure 8.

7. Write two equivalent pattern rules for the number of squares in this pattern.

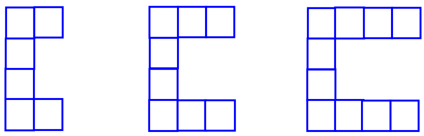
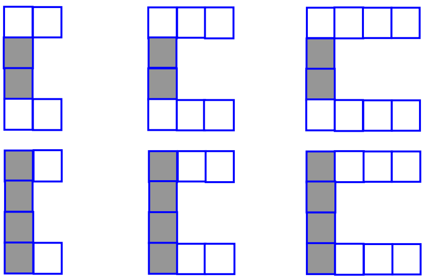


Figure 1 Figure 2 Figure 3

Use these two shadings to help you.



8. Why does it make sense to use a variable in a pattern rule?

6.1.2 Creating and Evaluating Expressions

Try This

Yuden sells bangchung for Nu 125 each. She sold Nu 2250 worth of bangchung one morning.

A. If she sold 50 bangchung in the afternoon, what were her total sales that day?

- You have used mathematical equations for many years. An equation is a sentence that relates two expressions.

For example:

$3 + ■ = 10$ is an equation. It means that the expression $3 + ■$ is equal to the expression 10.

- Mathematical expressions can involve numbers, numbers with operations, variables, or variables with operations. All of these are expressions:

$12 \qquad 3 \times 6 + 12 \qquad \frac{3}{2} + 8 \div 4 \qquad n \qquad n + 3 \qquad 2x - 4$

- To represent a mathematical situation, you sometimes have to translate from words to mathematics in order to write an expression using a variable. A variable represents something that can change. You can use any letter or symbol for a variable. Here are some examples.

Words	4 more than a number	1 less than double a number	Sum of a number and its double	Sum of two numbers in a row
Mathematics	$n + 4$	$2n - 1$	$x + 2x$	$m + (m + 1)$

- When an expression involves a variable, it is called an **algebraic expression**. You can **evaluate** it by **substituting** a value for the variable.

For example:

The expression $m + (m + 1)$ represents the sum of two numbers in a row, where m is the first number. You can substitute values for m to determine the sum.

If $m = 10$, then $m + (m + 1) = 10 + (10 + 1) = 10 + 11 = 21$.

- You can also use algebraic expressions to represent and solve problems.

For example:

Karma sells items at Nu 50 each. How much can she earn if she sells 70 items?

If n is the number of items sold, you can use $50n$ to represent the situation.

If Karma sells 70 items, substitute $n = 70$.

She earns $50n = 50 \times 70 =$ Nu 3500.

B. i) Write an expression you could use to determine how much Yuden will earn if she sells *p* bangchung.

ii) Write an expression you could use to determine how many bangchung she has sold if she sells 50 more than *p* bangchung.

iii) What value of *p* (number of bangchung) results in sales of Nu 2250?

Examples

Example 1 Translating From Words to Algebraic Expressions

Create an algebraic expression to represent each situation.

a) the total price of a number of items that cost Nu 120 each

b) the cost per person of a Nu 250 item shared by a number of people, plus Nu 200

c) the number of weeks in a certain number of years

Solution	Thinking
a) $120k$	**a)** I let *k* be the number of items. I multiplied the price per item by *k* to represent the total price.
b) $\frac{250}{s} + 200$	**b)** If people are sharing the cost of something, you divide by the number of people sharing. I let *s* be the number of people sharing. I added on the "plus Nu 200."
c) $\frac{365y}{7}$	**c)** I let *y* represent the number of years. There are 365*y* days in *y* years. I divided by the number of days in a week to find the number of weeks.

Example 2 Creating and Substituting into an Expression

Buthri says that when you multiply a number by a number that is 2 greater and then add 1, the answer is always a perfect square. Do you agree? Show your work.

Solution

Write an expression

$m(m + 2) + 1$, where *m* is the lower number

Test Buthri's rule using the expression and different values for m

If $m = 2$, then $2(2 + 2) + 1 = 8 + 1 = 9 = 3^2$

If $m = 4$, then $4(4 + 2) + 1 = 24 + 1 = 25 = 5^2$

If $m = 5$, then $5(5 + 2) + 1 = 35 + 1 = 36 = 6^2$

I agree. I got a perfect square each time.

Thinking

- I used *m* for the lower number, so *m* + 2 was the number that was 2 greater.
- To create the expression, I multiplied *m* and *m* + 2 and added 1.
- It didn't matter what value I used for *m*. I got a perfect square each time.

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Practising and Applying

1. Match each algebraic expression with a word phrase below.

a) $n + 2$

b) $300 - k^2$

c) $3x + 3x + 1$

d) $\frac{300}{m}$

e) $\frac{n+(n+1)}{2}$

f) $30k$

g) $30 - d$

Word phrases:

i) The total cost of a number of items that cost Nu 30 each

ii) The sum of a multiple of 3 and a number 1 greater than the multiple of 3

iii) The difference between 30 and a number

iv) The average of two consecutive numbers

v) How much less a perfect square is than 300

vi) A number increased by 2

vii) How much each person pays if they share the cost of an item that is Nu 300

2. Evaluate each expression.

a) $3 + 2m$, when $m = 10$

b) $2 - x^2$, when $x = 5$

c) $3x + 2x^2$, when $x = 4$

d) $\frac{(3n+2)(n-4)}{2n}$, when $n = 5$

e) $50p + 2$, when $p = 9$

f) $(3n + 4) + 2(n - 5)$, when $n = 7$

3. a) How do you know the expression $2n$ represents an even number?

b) What algebraic expression could represent a multiple of 3?

4. Eden's family makes many trips to her grandmother's home. A return trip (to and from) is 72 km.

a) Use an expression to represent the number of kilometres they travel in r return trips.

b) How would the expression in **part a)** change if they stayed at Eden's grandmother's home on the last trip?

5. Kinley earns Nu 7000 each month. He also receives an allowance of Nu 500 each month.

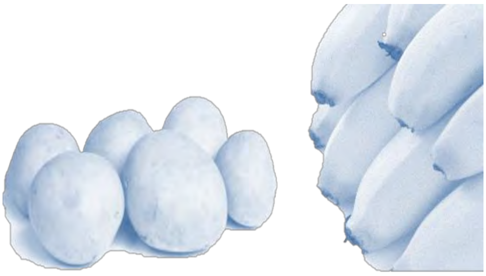
a) Write an algebraic expression that represents how much Kinley receives in m months.

b) Evaluate the expression to determine his annual income.

6. A kilogram of potatoes costs Nu 35. A dozen bananas cost Nu 80.

a) Write an algebraic expression for the total cost of n kilograms of potatoes and one dozen bananas.

b) Evaluate your expression to determine the cost of 25 kg of potatoes and one dozen bananas.



7. Write a word problem that could be solved using the expression $2x + 60$.

6.1.3 Simplifying Expressions

Try This

A. Work with a partner. One person should perform both calculations below while the other person times how long each calculation takes. First predict which you think will take less time and why.

i) 23 + 47 + 23 + 23 + 47 + 23 + 23 **ii)** 5 × 23 + 2 × 47

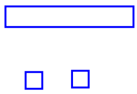
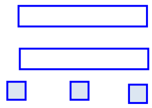
B. Switch roles and repeat with this pair of calculations.

i) 59 + 59 + 59 + 36 + 59 + 59 + 59 + 59 + 59 + 59 + 59 **ii)** 10 × 59 + 36

• You can use a model to represent an algebraic expression.

For example:

You can use or draw a rectangle-shaped tile to represent a variable and a small square tile to represent a constant value. White can represent positive values and grey can represent negative values. For example:

Expression	***n***	***n* + 2**	**2*n* – 3**	**–3*n* – 1**
Model				

• You can **simplify** expressions by collecting **like terms**. Like terms are terms that use the same size tile in your model.

For example:

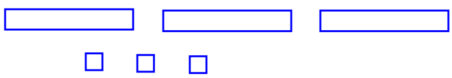
3*n*, 2*n*, and –2*n* are like terms since they are all modelled with rectangles.

3, 2, and –2 are like terms since they are all modelled with squares.

• The sum of three numbers in a row can be described as $n + (n + 1) + (n + 2)$. If you create a tile model for the expression, you can combine like shapes to simplify the expression:

Model the expression: n + $(n + 1)$ + $(n + 2)$

Combine like tiles:



$3n + 3$

Combining like tiles is the same as combining like terms.

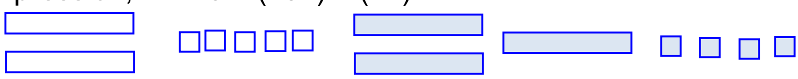
So, $n + (n + 1) + (n + 2) = 3n + 3$.

• When subtraction is involved, you can subtract by adding the opposite.

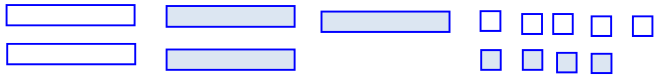
For example, to simplify $2n + 5 – (3n + 4)$:

Rewrite the expression: $2n + 5 – (3n + 4) = 2n + 5 \mathbf{+} (–3n) \mathbf{+} (–4)$

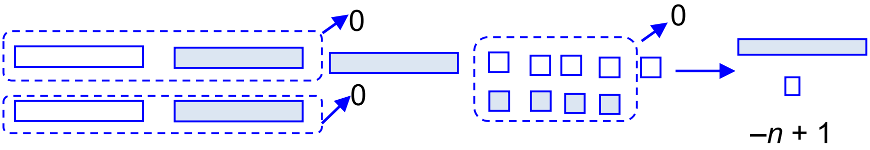
Model the new expression, $2n + 5 \mathbf{+} (–3n) \mathbf{+} (–4)$:



Combine like tiles, $(2n - 3n) + (5 - 4)$:



Pair up opposite values that add to 0 and remove them:



You are left with a simplified expression.

This is what it looks like algebraically:

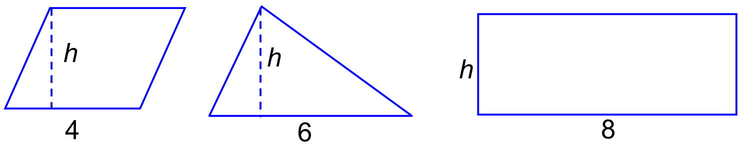
$$2n + 5 - (3n + 4) = 2n + 5 \mathbf{+} (\mathbf{-}3n) \mathbf{+} (\mathbf{-}4)$$
$$= (2n - 3n) + (5 - 4)$$
$$= -n + 1$$

C. How do the two expressions in **part A** relate to the idea of collecting like terms to simplify an expression?

Examples

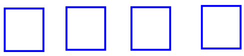
Example 1 Collecting Like Terms to Simplify

Pema wants to calculate the combined area of the three shapes below. He knows the height, *h*, is the same for all three shapes. How can he use an algebraic expression to make the calculation easier?

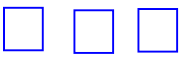


Solution 1

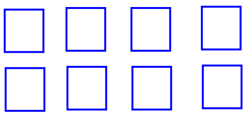
Area of parallelogram, 4h



Area of triangle, 6h ÷ 2, or 3h



Area of rectangle, 8h



There are 15*h* squares altogether, so the combined area is 15*h*.

Once Pema knows the height, he can multiply it by 15 to get the combined area.

Thinking

- I decided to draw a square to represent the area of each shape.
- Each square represents one *h*.
- I counted all 15 squares to write an expression for the combined area.

Example 1 Collecting Like Terms to Simplify [Continued]

Solution 2

$A = 4h + (6h \div 2) + 8h$
$= 4h + 3h + 8h$
$= 15h$

Once Pema knows the height, he can multiply it by 15 to get the combined area.

Thinking

All the shapes had the same height, so I combined these three formulas:

$A_{parallelogram} = bh \rightarrow 4h$

$A_{triangle} = bh \div 2 \rightarrow 6h \div 2$

$A_{rectangle} = bh \rightarrow 8h$

Example 2 Describing a Pattern by Simplifying Expressions

Choki says that if you add the nine numbers in any 3-by-3 square of a calendar, the sum is 9 times the middle number.

Use algebra to show why this is always true.

		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

Solution

Test Choki's rule on two 3-by-3 squares of numbers

3 + 4 + 5 + 10 + **11** + 12 + 17 + 18 + 19 = 99
= 9 × **11**

13 + 14 + 15 + 20 + **21** + 22 + 27 + 28 + 29 = 189
= 9 × **21**

Write an expression for the sum of the nine numbers

N	$N + 1$	$N + 2$
$N + 7$	$N + 8$	$N + 9$
$N + 14$	$N + 15$	$N + 16$

$N + (N + 1) + (N + 2) + (N + 7) + (N + 8) +$
$(N + 9) + (N + 14) + (N + 15) + (N + 16)$
$= 9N + 1 + 2 + 7 + 8 + 9 + 14 + 15 + 16$
$= 9N + 72$

Write an expression for 9 times the middle number

$9 \times (N + 8)$

Compare the expressions

$9N + 72 = 9 \times (N + 8)$ because
$9 \times (N + 8) = 9 \times N + 9 \times 8 = 9N + 72$

The sum of the nine numbers will always be equal to 9 times the middle number.

Thinking

- I tried it with the two shaded squares. It looked like Choki was right.
- I drew a 3-by-3 square and used N for the lowest number. I knew that when you go to the right, the numbers increase by 1 and when you go down, they increase by 7.
- I represented the middle number with $N + 8$.

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Practising and Applying

1. Model each algebraic expression by sketching a tile model.

a) $3s + 4$ **b)** $-2m + 1$

c) $6t - 3$ **d)** $-2n - 2$

2. Simplify each.

a) $(3n + 2) + (5n + 6)$

b) $(4m - 1) + (-2m - 3)$

c) $(6m - 3) - (4m + 5)$

d) $(2k - 1) - (-3k + 4)$

3. The expressions below have been simplified. Write two possible unsimplified expressions for each.

a) $2n + 4$ **b)** $3n - 6$

c) $5 - x$ **d)** $7 - 3k$

4. Three triangles have equal bases. The heights are 7, 8, and 9 units.

a) Write an algebraic expression to describe the area of each triangle.

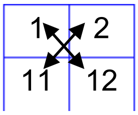
b) Combine the three expressions from **part a)** to create an expression for the total area of the three triangles. Simplify the expression.

5. The first three rows of a 100 chart are shown here.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

Use algebraic expressions to show that each statement below is true.

a) The sum of the two numbers in a diagonal of any 2-by-2 square is equal to the sum of the numbers in the other diagonal. For example:



$1 + 12 = 2 + 11$

b) If you add five consecutive numbers in a column, the sum is 5 times the middle number. For example:

$7 + 17 + 27 + 37 + 47 = 5 \times 27$

6. a) Simplify.

$(4x - 3) + (2x + 6) - (-3x - 4)$

b) The expression $(12x + 2) + (-3x + 5)$ simplifies to the same expression as the one in **part a)**. Write another expression that would simplify to the same expression.

7. Why is simplifying an expression a good thing to do?

CONNECTIONS: Using Variables to Solve Number Tricks

1. Try the number trick. What do you notice after **Step E**?

2. Repeat the trick starting with a different number.

3. a) Write an algebraic expression for each step of the trick. Use n to represent the starting number. Simplify each expression as you go.

b) Explain how the trick works.

4. Make up your own number trick.

Number Trick

A. Think of a number.
B. Double it.
C. Subtract 4.
D. Take half.
E. Add 2.

Chapter 2 Solving Equations

6.2.1 Solving Equations Using Models

Try This

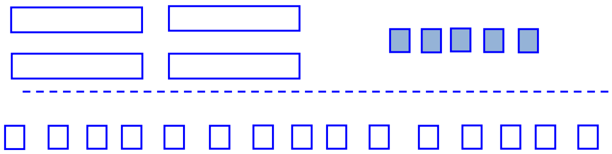
A. i) How do you know that 8 is the solution to the equation $6 \times \blacksquare + 2 = 50$?
ii) Write two other equations that have 8 as a solution.

• A variable in an equation can represent a value that is unknown.
For example:
Only one value of n (6) makes the equation $n + 6 = 12$ true because **6** + 6 = 12.

• Some equations are easy to **solve** using addition or multiplication facts.
For example:
$n + 6 = 12$ is easy to solve if you know 6 + 6 = 12.

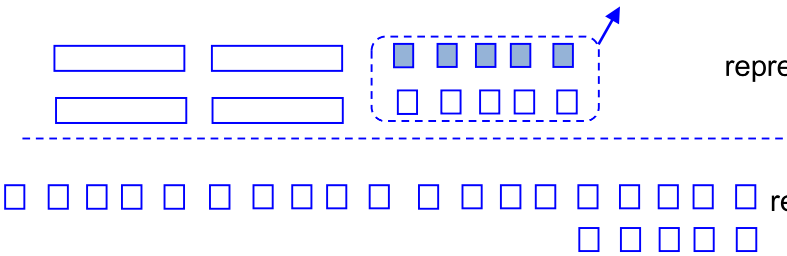
• There are times when you need a model to solve an equation.
The following three models show how to solve $4k - 5 = 19$.

Tile Model for $4k - 5 = 19$
- Use rectangular and square tiles to represent $4k - 5 = 19$:



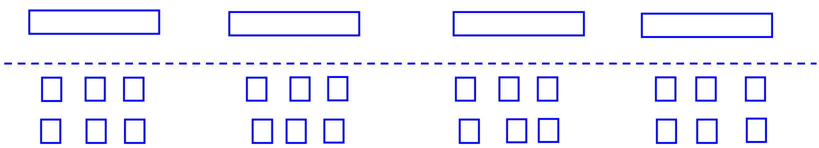
represents $4k - 5$
represents the equals sign
represents 19

- Add 5 white squares to each side so that you end up with only *k*s on one side:
(An equation is like a balance. Since $4k - 5$ and 19 are equal, if you add 5 to one expression, you must also add 5 to the other expression to keep the equation balanced.)



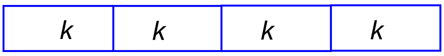
represents $4k - 5 \mathbf{+ 5} = 4k$
represents $19 \mathbf{+ 5} = 24$

- Since there are 4 *k*s, arrange the square tiles on the other side into 4 groups:
One group of 6 square tiles matches each *k* tile, so $k = 6$.

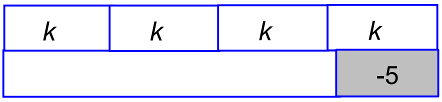


Rectangle Model for $4k - 5 = 19$

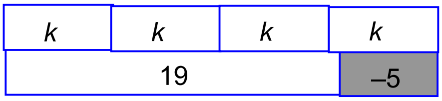
Step 1 Model $4k$ as four copies of k.



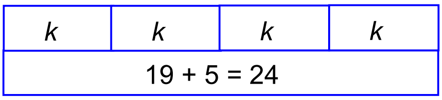
Step 2 Model –5 as 5 units going back from the end (to subtract 5).



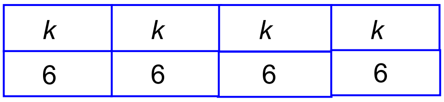
Step 3 Model 19 as the difference between $4k$ and –5 because $4k - 5 = 19$.



Step 4 Find a value equivalent to $4k$. $4k$ is equivalent to $19 + 5 = 24$



Step 5 Since $4k = 24$, then $k = 6$.

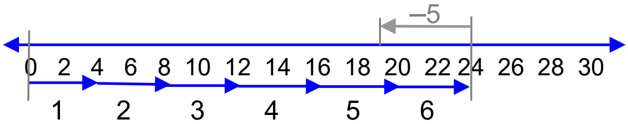


Number Line Model for $4k - 5 = 19$

For $4k - 5 = 19$, start at 0, make k jumps of 4, go back 5, and end at 19.

If you ended at 19 after going back 5, that means you must have jumped to 24.

If you were at 24 after k jumps of 4 from 0, you must have taken 6 jumps, as shown on the number line below.



There are 6 jumps of 4 from 0 to 24, so $k = 6$.

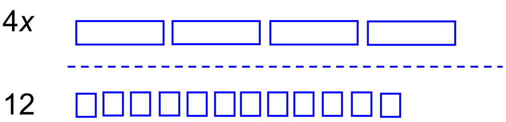
B. Apply two of the models shown to solve the equation in **part A.**

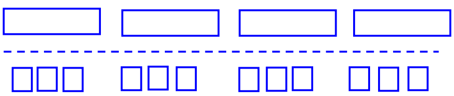
Examples

Example 1 Solving an Equation Involving One Operation

Solve $4x = 12$.

Solution 1





$x = 3$

Thinking

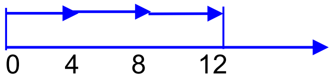
- I used a tile model.
- Each rectangle represented x.
- Each square represented 1.

- Since I had 4 rectangles, I grouped the squares into 4 groups.
- Each rectangle matched 3 small squares.

Example 1 Solving an Equation Involving One Operation [Continued]

Solution 2

$4x = 12$



It takes 3 jumps of 4 get to 12, so $x = 3$.

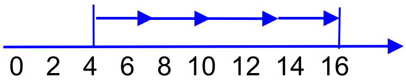
Thinking

- I used a number line model. I knew $4x = 12$ meant x jumps of 4 to 12.
- I counted the number of jumps of 4 from 0 to 12 on a number line.

Example 2 Solving an Equation Involving Two Operations

Solve the equation $3t + 4 = 16$.

Solution 1



There are 4 jumps of 3 from 4 to 16.

$t = 4$

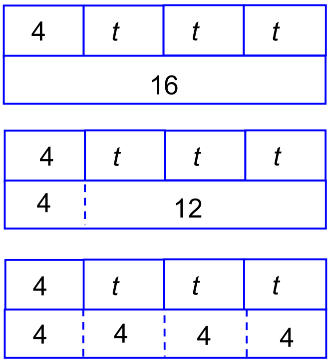
Thinking

- I knew $4 + 3t = 16$ meant starting at 4 on a number line and taking t jumps of 3 to get to 16.
- I knew I needed to find the number of jumps of 3 from 4 to 12.

Solution 2

$4 + 3t$

16



Each t box is worth 4, so $t = 4$.

Thinking

- I drew a rectangle that was made up of a box of 4 and three t boxes.
- To match the first rectangle, I drew another rectangle below it that was 16 units long.
- Since 3 t boxes were 12 units long ($16 - 4 = 12$), each t box was 4 units long.

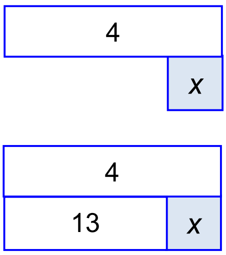
Example 3 Solving an Equation Involving a Negative Variable

Solve $4 - x = 13$.

Solution

4

$-x$



x is $4 - 13 = -9$.

Thinking

- I drew a rectangle that was 4 units long and then showed a box that was x units long subtracted from it.
- Since $4 - x = 13$, I knew the difference between 4 and $-x$ was 13, so I completed the model with a box for 13.
- I subtracted 13 from 4 to find x.

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Practising and Applying

1. Match each equation to one or more models below.

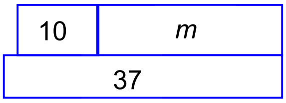
a) $3m + 4 = 10$

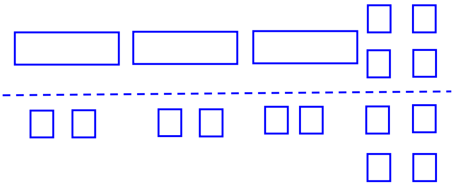
b) $30 - 3m = 21$

c) $25 + 3m = 37$

d) $10 + m = 37$

e) $21 + 3m = 30$

i)


ii)


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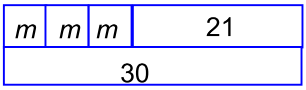

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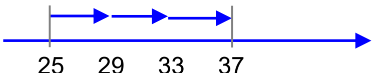

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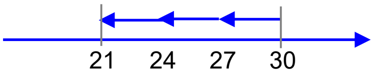


2. Solve each equation in **question 1**. Use models if you wish.

3. Describe what each equation below means.

For example:

The equation $2m + 3 = 12$ means that if you add 3 to a number (m) that is multiplied by 2, the result is 12.

a) $p + 10 = 28$

b) $7k = 35$

c) $4n + 8 = 28$

d) $3p - 5 = 16$

e) $10 + 5m = 55$

f) $27 = 30 - 3k$

4. Solve each equation in **question 3**. Use models if you wish.

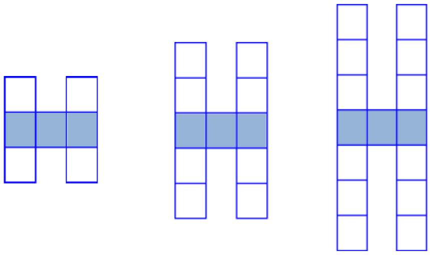
5. The solution to an equation is 3.

a) Use a model to represent the equation.

b) Write the equation.

c) How can you tell from the model that the solution is 3?

6. The pattern rule for the number of squares in this pattern is $4f + 3$.



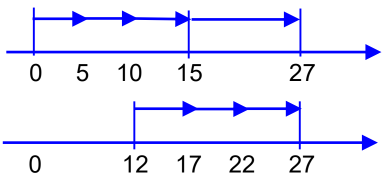
a) Write an equation you could use to determine the number of the figure that has exactly 71 tiles.

b) Solve the equation.

7. Which model (tile, number line. or rectangle) would you use to solve $5x = 20$? Why would you choose that model?

8. Without solving, how do you know that $5x - 2 = 10$ and $5x = 12$ have the same solution?

9. You can use either number line model below to represent $3k + 12 = 27$. Explain why.



10. a) Solve $17 = 4x - 7$ using two different models (number line, tile, or rectangle).

b) Describe the advantages and disadvantages of each model.

6.2.2 Solving Equations Using Guess and Test

Try This

A. Three consecutive numbers have a sum of 297. What are the numbers?

$$? + ? + ? = 297$$

• You can solve an equation by guessing and testing. Try different values for the variable, solve the equation, and then make changes to your guess if you were too high or too low.

For example, to solve $2x - 18 = 94$:

Start by guessing x = 50

Substitute $x = 50$ into the equation $2x - 18 = 94$.

$2 \times \mathbf{50} - 18 = 82$ 82 is too low. Try a higher number.

Try 60

$2 \times \mathbf{60} - 18 = 102$ 102 is too high. Try a lower number.

Try 55, which is between 50 and 60

$2 \times \mathbf{55} - 18 = 92$ 92 is almost there. Try a slightly higher number.

Try 56

$2 \times \mathbf{56} - 18 = 94$ That works, so the solution is $x = 56$.

• This method works even if the solution is a decimal.

For example, to solve $5m = 24$:

Start by guessing x = 5

Substitute $x = 5$ into the equation $5m = 24$.

$5 \times \mathbf{5} = 25$ 25 is a bit too high. Try a lower number.

Try 4

$5 \times \mathbf{4} = 20$ 20 is too low. Try a higher number.

Try 4.5, which is between 4 and 5

$5 \times \mathbf{4.5} = 22.5$ 22 is almost there. Try a slightly higher number.

Try 4.8

$5 \times \mathbf{4.8} = 24$ That works, so the solution is $m = 4.8$.

B. i) What equation did you solve in **part A**?

ii) What would be a good first guess to solve this equation?

Example 1 Using Guess and Test

Solve each equation.

a) $50 + 4k = 338$ **b)** $100 - 8s = 56$

Solution

a) $50 + 4k = 338$

Starting guess of 50

$50 + 4k = 50 + 4 \times \mathbf{50}$

$= 250$ is too low. Try a higher number.

Try 75

$50 + 4k = 50 + 4 \times \mathbf{75}$

$= 350$ is a bit too high. Try a bit lower.

Try 70

$50 + 4k = 50 + 4 \times \mathbf{70}$

$= 330$ is a bit too low. Try a bit higher.

Try 72

$50 + 4k = 50 + 4 \times \mathbf{72}$

$= 338$ It works. The solution is $k = 72$.

b) $260 - 8s = 56$

Starting guess of 20

$260 - 8s = 260 - 8 \times \mathbf{20}$

$= 100$ is not low enough. Try higher.

Try 25

$260 - 8s = 260 - 8 \times \mathbf{25}$

$= 60$ is almost low enough. Try higher.

Try 26

$260 - 8s = 260 - 8 \times \mathbf{26}$

$= 52$ is a bit too low. Try a lower number.

Try 25.5

$260 - 8s = 260 - 8 \times \mathbf{25.5}$

$= 56$ It works. The solution is $s = 25.5$.

Thinking

a) My first guess was $k = 50$ because it was easy to calculate and I knew 100 was way too much.

- When the result was too low, I tried a higher number next.
- When the result was too high, I tried a lower number next.

b) I guessed 20 first because it seemed reasonable and was easy to calculate with.

- Because I was subtracting $8s$ from 260, I had to try a higher number when the result was too high. If the result was too low, I had to try a lower number next.

Practising and Applying

1. What would be a good starting guess to solve each equation? Tell why it would be a good guess.

a) $378 + k = 512$ **b)** $365 = 72k$

c) $212 + 5m = 627$ **d)** $382 = 617 - 5m$

e) $6t + 8 = 314$ **f)** $6t - 54 = 474$

2. You are solving equations using guess and test. The results of the first guess are shown. What would be a good second guess for each? Why?

a) $6x = 752$ *First guess*: 100
$6 \times \mathbf{100} = 600$

b) $3k + 87 = 272$ *First guess*: 70
$3 \times \mathbf{70} + 87 = 297$

c) $500 - 4x = 252$ *First guess*: 50
$500 - 4 \times \mathbf{50} = 300$

3. Use guess and test to solve each.

a) $5k + 62 = 167$

b) $8k - 62 = 26$

c) $84 - 3m = -72$

d) $7t + 0.5 = 1.9$

4. Write each sentence as an equation and then solve it.

a) Two numbers that are 10 apart have a sum of 124.

b) Eight times a number is 344.

c) If you reduce the double of a number by 35, the result is 79.

5. a) Write an equation that relates the number of black counters to the total number of counters in each figure.

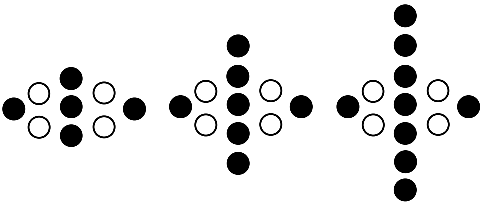


Figure 1   Figure 2   Figure 3

b) How many black counters are needed to make a figure with exactly 83 counters?

6. A scientist measured the temperature of a liquid with a thermometer marked in kelvins (K). The temperature was 423 K.

Celsius temperature is 273 degrees less than kelvin temperature.

a) Write an equation that relates degrees Celsius to kelvins.

b) What is the temperature of the liquid in degrees Celsius?

7. a) Write an equation for this sentence.

The sum of two consecutive integers is 284.

b) Explain why there is no solution to your equation in **part a)**.

8. a) Would 20 or 50 be a better first guess to solve $6x - 52 = 92$? Explain your thinking.

b) Solve the equation.

9. If you were asked to explain the guess and test method for solving equations, what would you say?

10. Sometimes it makes more sense to use guess and test rather than a model to solve an equation. Describe an equation that it makes sense to solve using guess and test.

6.2.3 Solving Equations Using Inverse Operations

Try This

A. i) Why is it easy to calculate 358 + 269 – 85 – 269 + 85 – 358, even though there are many numbers?

ii) Why is it easy to calculate 487 × 35,215 ÷ 487, even though the numbers are large?

• You can use **inverse operations** to solve equations. You "undo" an operation by using its inverse operation. Subtraction is the inverse of addition and division is the inverse of multiplication.

For example:

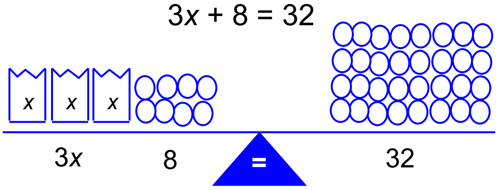
3 **+ 4 – 4** = 3 means you start with 3 and add 4. Then you subtract 4 to undo the addition of 4. You end up with 3, which is where you started.

• If you think of an equation as a balance, this will help you understand how inverse operations work in solving equations.

For example, to solve $3x + 8 = 32$:

Imagine the equation as a balance with the equals sign as the balance point.

$3x + 8 = 32$

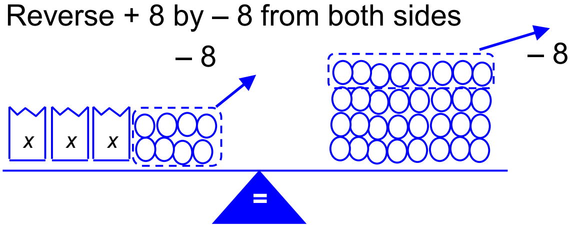


• One side has 3 bags of x balls, plus 8 more balls. The other side has 32 balls.

• Since each bag holds x balls, you can solve the equation by finding out how many balls are in one bag. To do that you need to get one bag alone on one side of the balance.

Think about the operations in $3x + 8 = 32$. x is multiplied by 3 and then 8 is added.

Reverse + 8 by – 8 from both sides

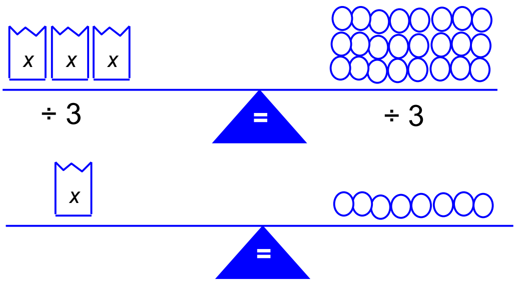


Reverse the addition

$3x + 8 \mathbf{- 8} = 32 \mathbf{- 8}$

$3x = 24$

Reverse × 3 by ÷ 3 on both sides



Reverse the multiplication

$3x = 24$

$3x \mathbf{\div 3} = 24 \mathbf{\div 3}$

$x = 8$

There are 8 balls in one bag, so $x = 8$.

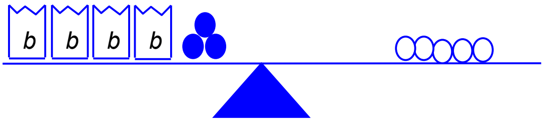
B. How did using inverse operations simplify the calculations in **part A**?

Examples

Example 1 Writing an Equation to Represent a Balance

a) What equation does this balance represent? How do you know?

b) Solve the equation using inverse operations.



Solution

a) $4b - 3 = 5$

Each bag represents b, so 4 bags represent $4b$.

3 grey balls represent –3.

5 white balls represent 5.

b)
$$4b - 3 = 5$$
$$4b - 3 + 3 = 5 + 3$$
$$4b = 8$$
$$4b \div 4 = 8 \div 4$$
$$b = 2$$

Thinking

a) I figured the grey balls were negative just like with the tiles.

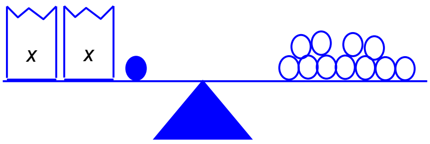
- Adding a negative is like subtracting a positive, so I knew $4b + (-3) = 4b - 3$.

b) I used inverse operations to get b alone on one side of the equation:

- Since 3 was being subtracted on the left, I reversed it by adding 3 to both sides.
- Since b was being multiplied by 4 on the left, I reversed it by dividing both sides by 4.

Practising and Applying

1. a) What equation does this balance represent?



b) What operations do you have to perform to solve the equation?

c) Solve the equation.

2. Sketch a balance to represent each.

a) $2x + 7 = 19$

b) $3x - 8 = 7$

3. List the steps you would follow to solve each using inverse operations.

a) $12f + 18 = 114$

b) $7t - 19 = 100$

c) $200 + 9l = 119$

d) $106 = 16 + 6m$

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4. Solve each using inverse operations.

a) $7k = 609$

b) $35 + 18m = 161$

c) $24 + 5t = 184$

d) $41 = 16k - 87$

5. Karma has Nu 2700 to spend. He is going to spend Nu 120 on a book and he needs Nu 200 each day for food.

a) Write an equation you could solve to see how many days his money will last.

b) Solve the equation to solve the problem.

6. When you solve an equation using inverse operations, the result is a simpler equivalent equation. Explain what this means.

GAME: Equations, Equations

Play with a partner. You need a set of 14 cards like this:

0	1	2	3	4	5	6	7	8	9

+	–	=	*m*

For each round of play:

- Spread the cards out facing up between the players.
- Take turns choosing cards to make an equation with an integer solution:
  - The numbers in the equation can only be 1-digit numbers.
  - You can use + or –.
  - After you make each equation, return the variable and operation cards to the middle to be used again. Do not use the number cards again.
  - You lose your turn if the solution is not an integer.
- The round is over when no more equations can be made.
- The winner of the round is the person who creates the last equation.

Take turns going first each round. You should start each round with an equation that was not used before.

Here is a sample round:

Player 1:	$4m + 1 = 9$	Numbers remaining: 0, 2, 3, 5, 6, 7, 8
Player 2:	$3m = 6$	Numbers remaining: 0, 2, 5, 7, 8
Player 1:	$2m - 5 = 7$	Numbers remaining: 0, 8
Player 2:	$m - 8 = 0$	

Player 2 wins the round.

Play as many rounds as you have time for, but always play an even number of rounds.

6.2.4 EXPLORE: Solving Equations Using Reasoning

The sum of each row and each column is shown in the chart.

For example:

Row 1

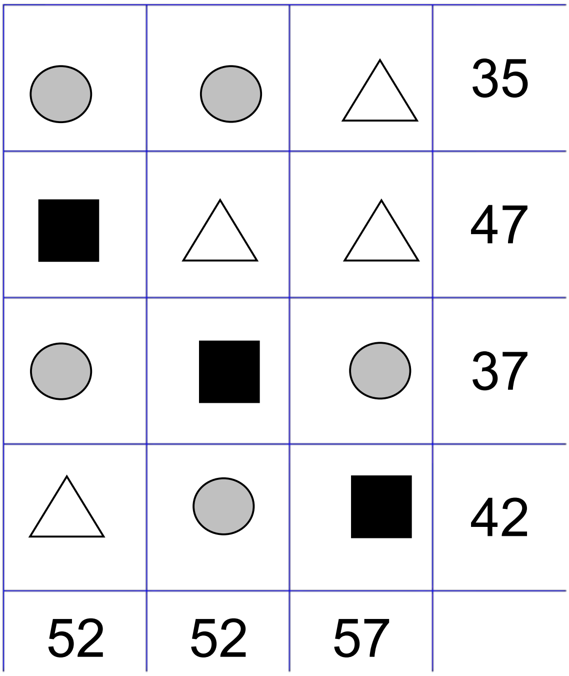
2 circles and 1 triangle = 35

Column 1

2 circles + 1 square + 1 triangle = 52

The circle is worth the same amount no matter where it is. The same is true for the triangle and the square.

Each shape has a different integer value.



Answer the questions below to figure out the value of each shape.

A. How do you know the value of the triangle is an odd number?

B. How do you know that the value of the square is an odd number?

C. How do you know the value of the circle is less than the value of the triangle?

D. What is the value of each shape?

E. Create a similar puzzle for a partner to solve.

6.3.1 Graphing a Relationship

Try This

A. Rinchen bought some chocolate bars at a shop. Each bar cost Nu 50. He also bought some other items that cost Nu 120 altogether. How many chocolate bars did he buy if he paid a total of Nu 470?

A table of values represents a relationship. A **graph** of the values in the table gives you a picture of the relationship. The graph, if extended, also gives you more information about the relationship.

For example:

Suppose it takes 12 oranges to make 1 L of orange juice.

• Create a table of values to show the relationship between the number of oranges and the amount of juice that can be made:

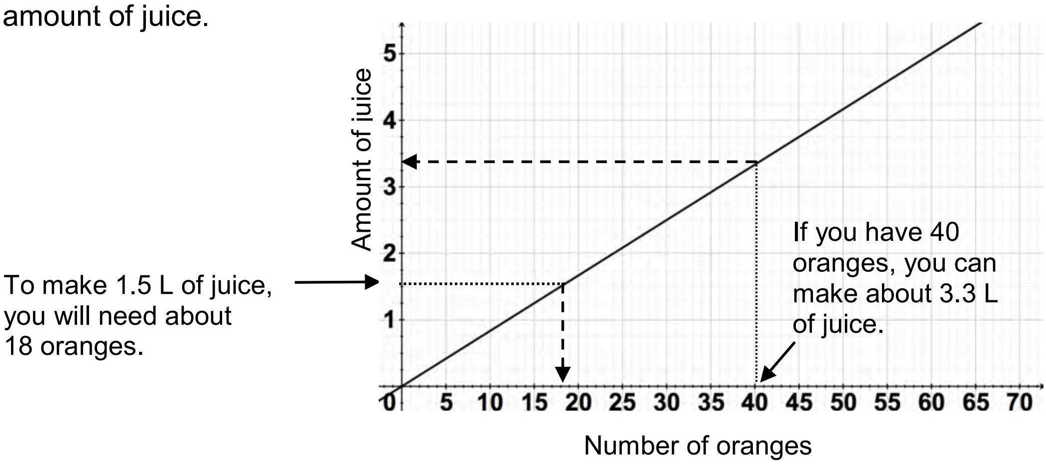
Number of oranges	12	24	36	48	60	72
Litres of orange juice	1	2	3	4	5	6

• Use the table to write **ordered pairs**:

(12, 1), (24, 2), (36, 3), (48, 4), (60, 5), (72, 6)

• Create a graph by plotting the ordered pairs. (Recall that the first number, the ***x*-coordinate**, tells how far right to move from the **origin** (0, 0) along the ***x*-axis**. The second number, the ***y*-coordinate**, tells how far to go up from the origin along the ***y*-axis**.)

• You can use the graph to estimate how much juice you can make with any number of oranges. Or, you can tell how many oranges you need to make any amount of juice.



• Depending on the **scale** used along the axes of your graph, you may be able to use the graph to find an exact answer or just an estimate. Generally, the larger the scale, the more difficult it is to read exact values on the graph.

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B. i) Sketch a graph of the relationship between the number of chocolate bars purchased and the total cost in **part A**.

ii) How does the graph show that your answer to **part A** was correct?

Examples

Example 1 Using a Graph to Describe a Pattern

A window with three panes uses four rectangular vertical pieces of wood.

a) Create a table of values to show the relationship between the number of three-pane windows and the number of vertical pieces of wood needed to build them.

b) Graph the relationship.

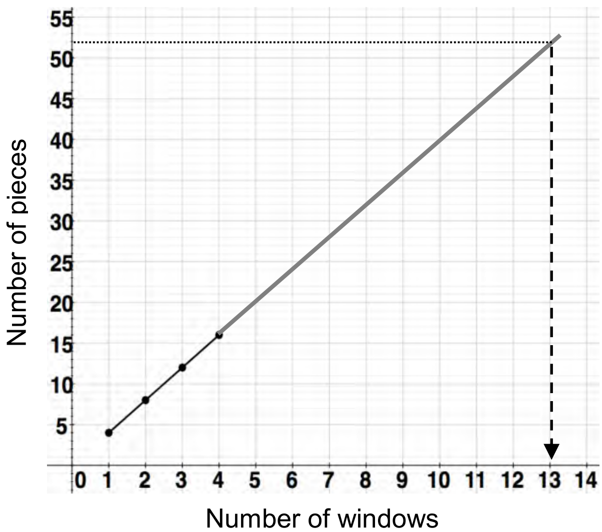
c) About how many three-pane windows could be made using 52 vertical pieces?

Solution

a)

Windows	Number of pieces
1	4
2	8
3	12
4	16

b)



c) 13 windows could be made with 52 pieces.

Thinking

- My table only went to 4 windows and 16 pieces. So, to answer **part c)**, I knew I had to extend the axes to have more windows so I could have 52 pieces.
- I found 52 first on the *y*-axis and then on the graph. Then I looked down to see how many windows 52 pieces would make.

Example 2 Using a Graph to Analyse a Pattern

a) Create a table to relate the number of grey squares to the figure number.

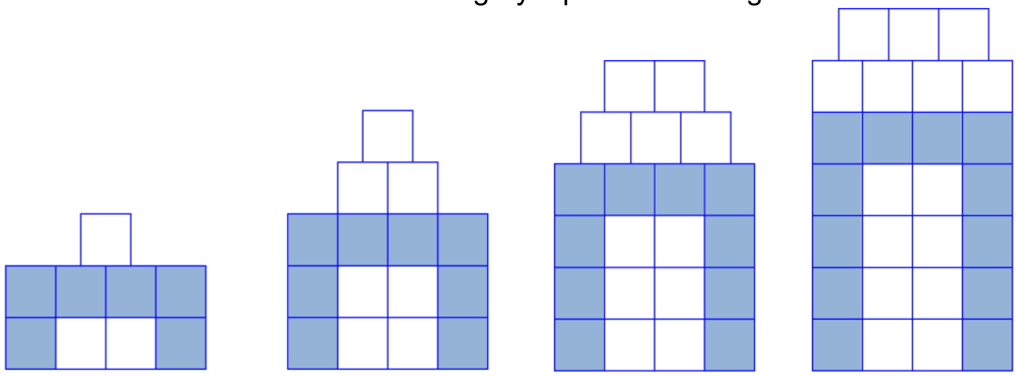


Figure 1 Figure 2 Figure 3 Figure 4

b) Graph the relationship.

c) Use the graph to estimate which figure has 32 grey squares.

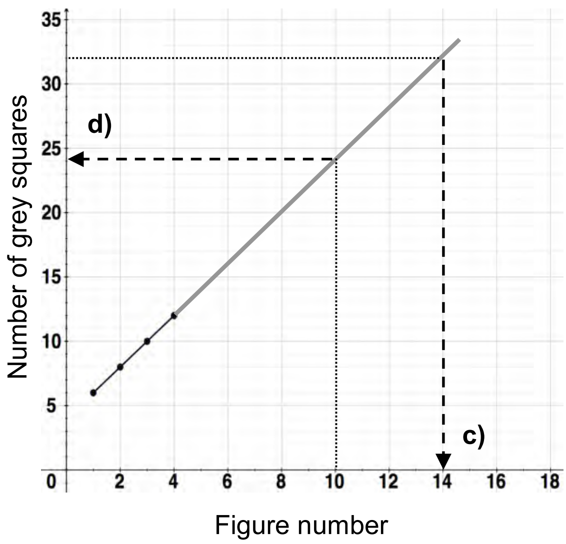
d) Use the graph to estimate the number of grey squares in Figure 10.

Solution

a)

Figure number	Number of grey squares
1	6
2	8
3	10
4	12

b)



c) Figure 14 has about 32 grey squares.

d) There are about 24 grey squares in Figure 10.

Thinking

a) I noticed there were two more grey squares in each new figure.

b) I knew that I would need to extend both axes. The vertical axis had to go to at least 32 and I extended the horizontal axis enough to make that work.

c) I looked for which figure goes with 32 squares by looking for the x-coordinate that went with a y-coordinate of 32.

d) I looked for the y-coordinate that went with an x-coordinate of 10.

Practising and Applying

Use your graphs to estimate or find an exact answer, depending on the scale used. Be prepared to extend the axes far enough to solve the problem.

1. Graph each relationship.

a)

Figure number	Number of white squares
1	8
2	14
3	20
4	26
5	32

b)

Figure number	Number of white squares
1	4
2	11
3	18
4	25
5	32

2. Examine this pattern.

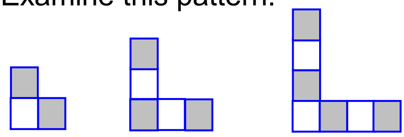


Figure 1 Figure 2 Figure 3

a) Create a table of values that relates the number of grey squares and the figure number up to Figure 5.

b) Graph the relationship in **part a)**.

c) Predict which figure will have 17 grey squares.

d) Predict the number of grey squares in Figure 21.

e) Create a table that relates the total number of squares and the figure number up to Figure 4.

f) Graph the relationship in **part e)**.

g) Predict which figure will have a total of 61 squares.

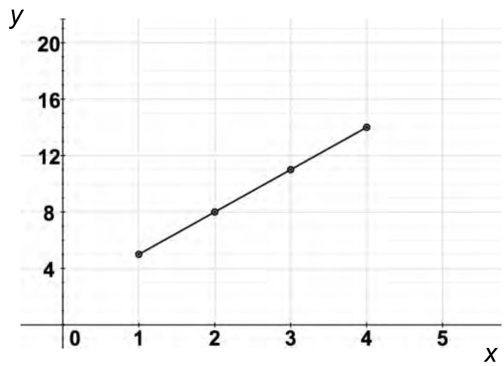
3. Sonam reads 10 pages of a book on night 1. Each night after that, she reads 4 pages more than the night before.

a) Create a table that relates the number of pages read each night to the night number up to night 4.

b) Graph the relationship in **part a)**.

c) On what night will she read 78 pages?

4. a) Create a table of values for this graph.



b) Sketch a pattern or describe a situation that the graph and table in **part a)** might describe.

5. Examine this pattern.

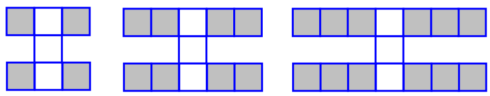


Figure 1 Figure 2 Figure 3

a) Create a table to show the total number of squares for the first six figures in the pattern up to Figure 5.

b) Graph the relationship.

c) Use your graph to predict the total number of squares in the 10th figure.

d) Check your prediction another way.

6. a) Explain why a graph is useful for making predictions.

b) Explain why sometimes only an estimate is possible when you use a graph to make predictions.

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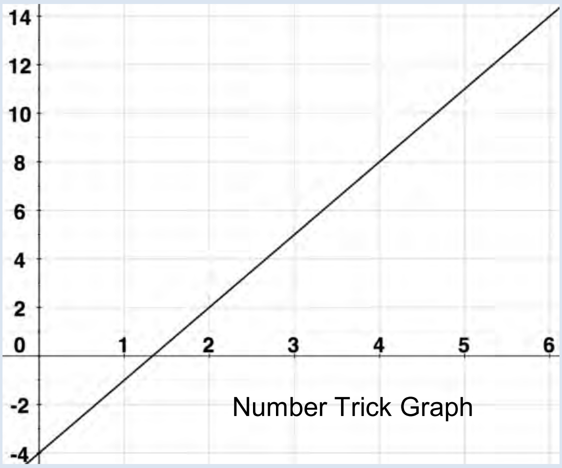
6.3.2 Examining a Straight Line Graph

Try This

Lobzang asked Buthri to think of a secret number from 0 and 6, multiply it by 3, subtract 4, and then tell Lobzang the answer. Lobzang was then able to use the graph to the right to figure out Buthri's secret number.



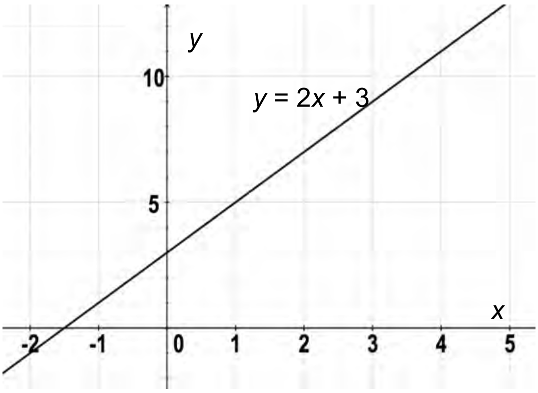
A. i) Try the trick with a partner several times.

ii) How did you use the graph to figure out the secret number?

iii) Did the success of the trick depend on what number your partner started with? Explain your thinking.

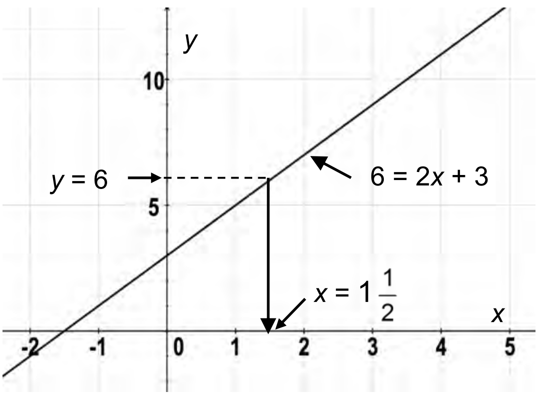
• This table of values and graph both display the relationship represented by the equation $y = 2x + 3$.

x	$y = 2x + 3$
1	5
2	7
3	9
4	11



When you create the graph from the table, you need to plot only a few points. Once you see that the graph is a straight line, you can join the points and extend the line.

Solving $6 = 2x + 3$



• You can use the graph of the equation $y = 2x + 3$ to solve equations that have values for y.

For example:

To solve $6 = 2x + 3$, draw a horizontal line at $y = 6$. Where it hits the graph, draw a vertical line down. The x-value on the x-axis at that point is the solution to the equation, $x = 1\frac{1}{2}$. You may sometimes have to estimate, depending on the scale.

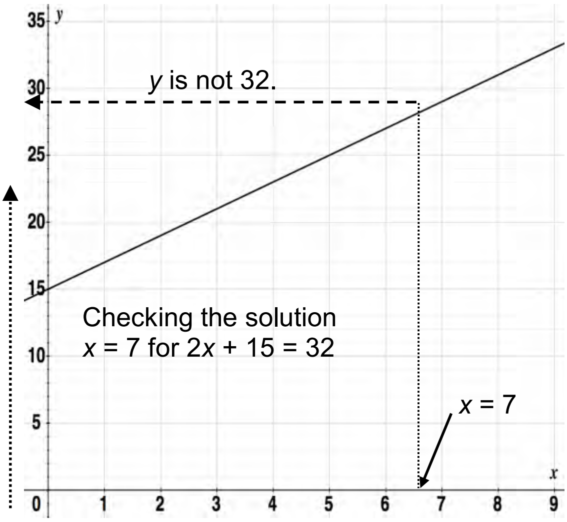
• You can also use a graph to check a solution to an equation.

For example:

Suppose you get a solution of $x = 7$ for the equation $2x + 15 = 32$. This is how to check it:

- Graph $y = 2x + 15$.
- Find 7 on the x-axis and draw a vertical line up to meet the graph.
- Draw a horizontal line from the graph to the y-axis to see if the y-value is 32.

The solution $x = 7$ is too low.



• You can solve many equations using the same graph.

For example:

You can also solve $2x + 15 = 10$ and $2x + 15 = 40$ using the graph above.

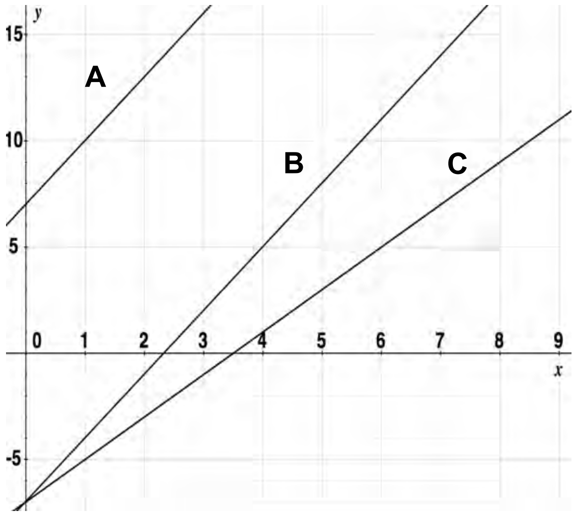
B. i) What labels would you give each axis for the graph in **part A**?

ii) What equation does the graph represent? How do you know?

Examples

Example 1 Matching a Graph to its Equation

Which graph(s) can be used to solve the equation $3x – 7 = 2$? How do you know?



Solution

B is the graph of $y = 3x – 7$, so I can use it to solve the equation $3x – 7 = 2$.

Thinking

- I thought it was line B, so I tested it using $x = 3$ to see if y was 2.
- The value of y was $3(3) – 7 = 2$.
- This worked for graph B, but not for graphs A or C.

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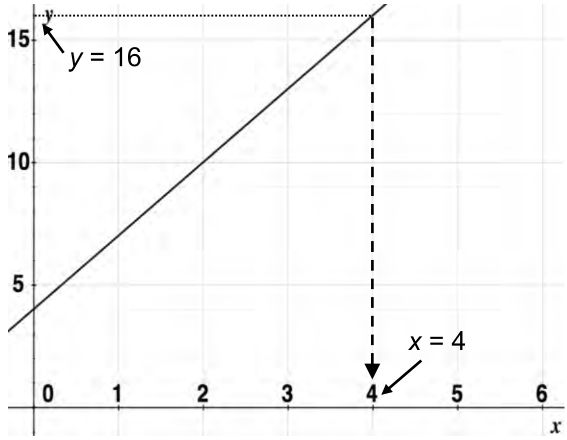
Example 2 Solving an Equation using a Graph
Solve the equation 16 = 3*x* + 4 using its graph.

Solution

x	$y = 3x + 4$
0	4
1	7
2	10
3	13



The solution to 16 = 3*x* + 4 is *x* = 4.

Thinking

• I made a table of values and then used the ordered pairs from it to graph the equation.

• I extended the axes to include higher values of *x* and *y*. I knew that *y* had to go up to 16 to be able to solve the equation.

• I drew a horizontal line at *y* = 16 and looked for where it touched the graph. Then I drew a vertical line down to see what the *x*-value was at that point on the graph.

Practising and Applying

Use your graphs to estimate or find an exact answer, depending on the scale used. Be prepared to extend the axes far enough to solve the problem.

1. For each equation, create a table of values up to $x = 5$ and then graph the relationship.

a) $y = 4x + 1$ **b)** $y = 7 - 3x$

c) $y = 3x - 8$ **d)** $y = 6 - 2x$

2. Use your graphs from **question 1** to solve these equations.

a) $33 = 4x + 1$ **b)** $13 = 7 - 3x$

c) $3x - 8 = -14$ **d)** $2 = 6 - 2x$

3. List one more equation you could solve using each graph from **question 1**. Solve each equation.

4. Graph the relationship $y = 3x + 8$. How does the graph show you that $x = 3$ is not a solution to $3x + 8 = 10$?

5. Graph $y = 4x + 6$ to test which solution is correct. Explain your thinking.

A. $x = 7$, if $4x + 6 = 29$

B. $x = 9$, if $4x + 6 = 42$

6. a) Graph each equation.

i) $y = 2x - 5$

ii) $y = 4x - 5$

b) How are the graphs the same? How are they different?

c) Which graph could you use to solve the equation $18 = 4x$?

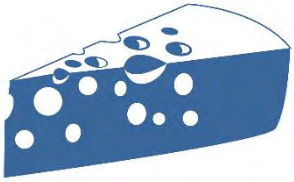
d) What is the solution to $18 = 4x$?

7. Why is a graph a good way to solve an equation? What might be some disadvantages of using a graph?

6.3.3 Describing Change on a Graph

Try This

A. It takes 10 kg of milk to make 1 kg of cheese.
How much cheese can you make with 2.4 kg of milk?

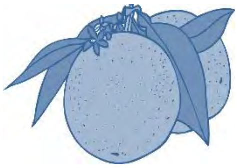


People often use rates to compare quantities.

For example:

- Speed, such as 3 km/h, is a rate that compares distance to time.
- A price of Nu 200/*pon* (*1 pon* = *80*) is a rate that compares price to amount of mandarin.

Mandarin
Nu 200/pon
at Dorokha
Orchard



• A rate provides a lot of information in one simple statement and you can use one rate to solve many problems.

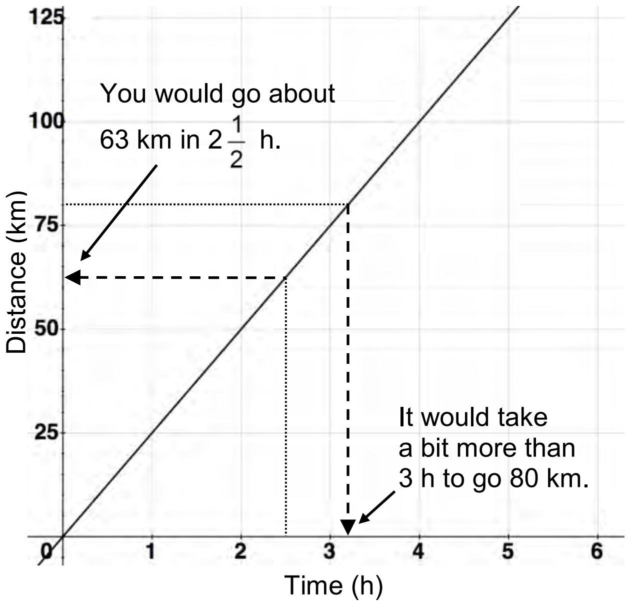
For example:

A price of Nu 200/pon (meaning Nu 200 for 1 *pon*) allows you to calculate prices for 2 *pons* (Nu 200 × 2 = Nu 400), 3 pons (Nu 200 × 3 = Nu 300), or for any number of mandarin.

• You can also create a graph of a rate and use it to estimate to solve problems.

For example:

This graph shows the distance
you could travel in kilometres at
a speed of 25 km/h for different
numbers of hours.



Here are some problems you
can solve with this graph:

- To estimate how far you go in $2\frac{1}{2}$ h, draw a vertical line up from the *x*-axis at $2\frac{1}{2}$ and read the *y*-coordinate. It is about 63 km.

- To estimate how long it would take to go 80 km, draw a horizontal line across from the *y*-axis at 80 and read the *x*-coordinate, which is a bit more than 3 h.

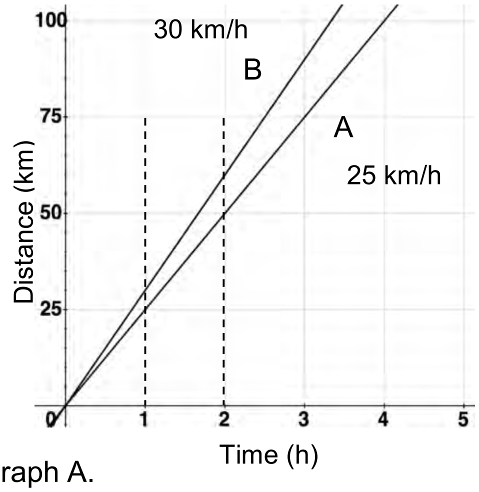
• The rate of change in a graph is shown by its steepness.
The steeper the graph, the faster something is changing.

For example:

These two graphs show the change in distance over time for two speeds:
- Graph A is 25 km/h
- Graph B is 30 km/h

The change in distance over 1 h is marked with dashed lines. (The change could be shown anywhere on the graph because it is a straight line.)
- In 1 h, graph A goes from 25 km to 50 km, an increase of 25 km.
- In 1 h, graph B goes from 30 km to 60 km, an increase of 30 km.

Graph B has a greater rate of change than Graph A.



B. i) What rate is described in **part A**?

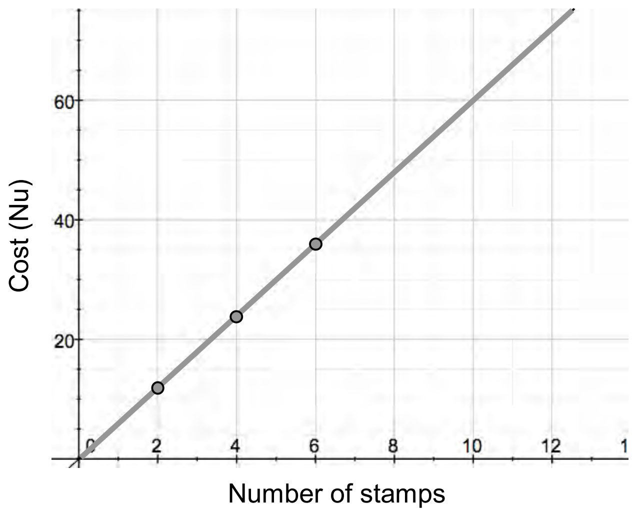
ii) How could a graph help you estimate to solve the problem in **part A?**

Examples

Example 1 Constructing a Graph to Describe a Rate

Draw a graph that could be used to determine the cost of any number of stamps, if each stamp costs Nu 6

Solution



Thinking

• I plotted three points.

• To get each point, I multiplied the number of stamps by the cost per stamp. For example, 2 stamps at Nu 6 each was Nu 12 so I plotted (2, 12).

• I joined my points with a straight line.

• I can use my graph for up to 12 stamps.

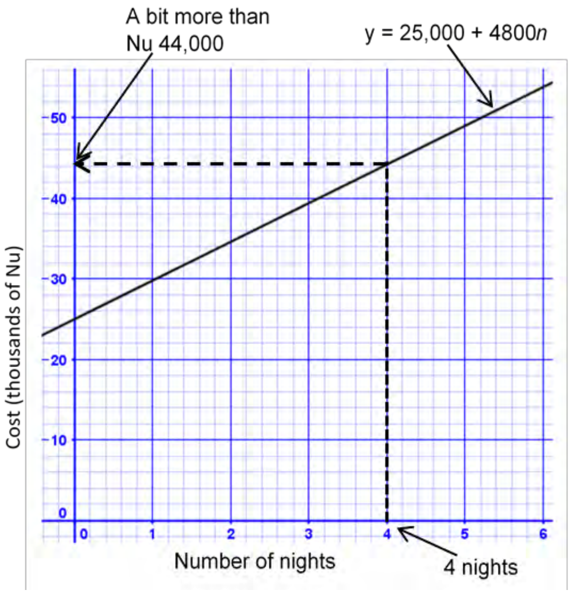
Example 2 Solving a Rate Problem using a Graph

The air fare from Paro to Bangkok and back to Paro costs about Nu 25,000. A hotel room in Bangkok costs about Nu 4800 per night.

Estimate the total cost to fly from Paro to Bangkok, staying for four nights and returning to Paro.

A statue at the Grand Palace, Bangkok

Solution



It would cost a bit more than Nu 44,000 for the trip.

Thinking

- I estimated using a graph. That way, if the number of nights changed, it would be easy to estimate again.
- I graphed the equation $y = 25{,}000 + 4800n$, where n is the number of nights.
- Each tick mark on the y-axis is worth Nu 2000, so 42 means Nu 44,000.
- I looked at $x = 4$ and read the corresponding y-value.

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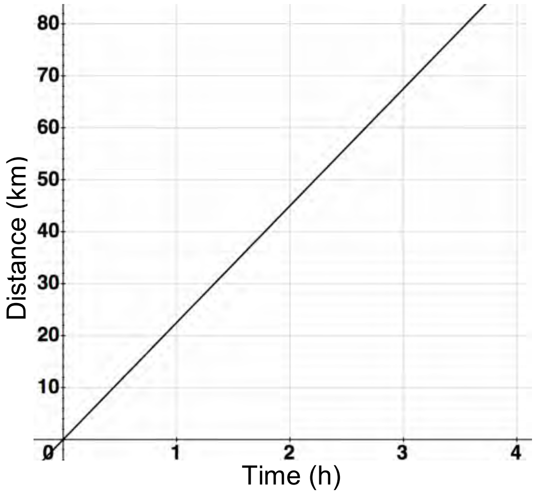
Practising and Applying

Use your graphs to estimate or find an exact answer, depending on the scale used.

1. This graph shows the distance driven over different numbers of hours.



a) How far could the driver go in 3 h?

b) How can you figure out the speed the driver is going from the graph?

2. Draw a graph to represent each rate.

a) distance compared to time at a speed of 28 km/h

b) total cost compared to number of items at a cost of Nu 50 per item

c) litres of orange juice compared to numbers of oranges at a rate of 0.08 L/orange

d) pages read compared to number of days at a rate of 125 pages/day

3. Use your graphs from **question 2** to answer these questions.

a) i) How far can you go in 2.5 h at 28 km/h?

ii) How long would it take to go 88 km at 28 km/h?

b) How many Nu 50 items can you purchase for Nu 2300?

c) i) How many litres of orange juice can you make with 60 oranges?

ii) How many oranges do you need to make 2 L of orange juice?

d) How long would it take you to read 10,000 pages if you read 125 pages/day?

4. The bus ride from Paro to Thimphu takes about 1 h and 30 min and costs Nu 60.

a) Create a table to show the total cost for different numbers of bus rides.

b) Graph the table of values.

c) If you take 3 more bus rides, it will always cost an extra Nu 180. How does the graph show this?

5. a) Graph $y = 3x + 5$.

b) How does the graph show that y increases by 3 whenever x increases by 1?

c) How does the graph show that y decreases by 3 whenever x decreases by 1?

d) Does the amount of increase or decrease change if you use different points on the graph? Explain your thinking.

6. The y-value on a graph increases by 4 whenever x increases by 1. How will the y-value change if you increase x by 2? Use a graph to explain.

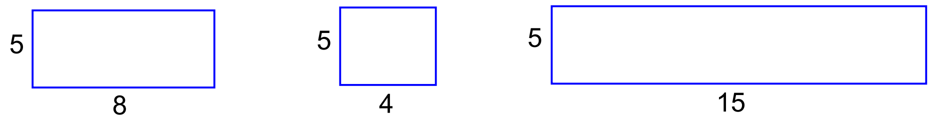
7. The graphs for $y = 100x$ and $y = 120x$ describe the total cost of different numbers of items that cost Nu 100 and Nu 120 each.

a) Why do both graphs describe rates?

b) Which graph is steeper? How could you have predicted this without actually graphing?

6.3.4 EXPLORE: Are All Relationship Graphs Straight Lines?

These rectangles all have a width of 5 units, but different lengths.



You can graph different relationships for these rectangles to see if each graph is a straight line.

A. i) Draw three rectangles, each with a width of 5 cm but with different lengths.

ii) Calculate the perimeter of each.

iii) Create a table of values that relates the perimeter to the length.

iv) Graph the relationship. What shape is the graph?

v) How does a change of 1 cm in length affect the perimeter?

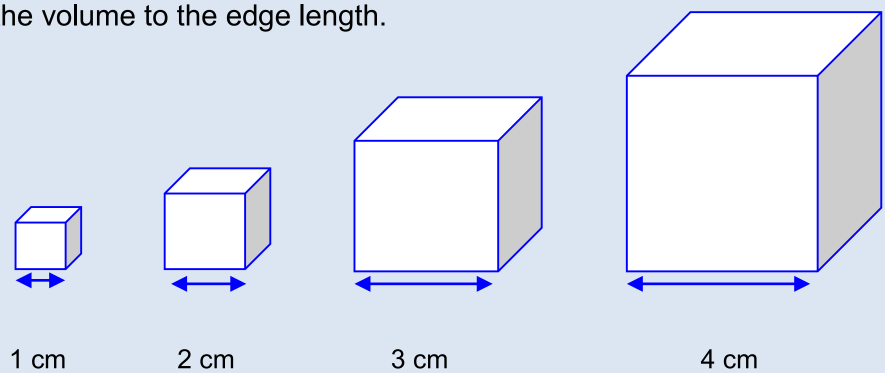
vi) How does a change of 4 cm in length affect the perimeter?

B. Repeat **part A ii) to vi)** for the same three rectangles, but use the area instead of the perimeter.

C. i) Draw four rectangles, each with a length that is 1 cm longer than its width.

ii) Repeat **part A ii) to vi)** and **part B** for the four rectangles. Compare your results to what happened with the three rectangles.

D. i) Now imagine cubes with edge lengths 1 cm, 2 cm, 3 cm, and 4 cm. Determine the volume of each. Then create a table of values that relates the volume to the edge length.



ii) Predict whether the graph will be a straight line.

iii) Explain your prediction.

iv) Test your prediction by creating a graph. Was your prediction correct?

1. Name the variable, the coefficient, and the constant in each pattern rule.

a) $5 - k$ **b)** $3m + \frac{1}{2}$

2. Copy and complete each table.

a)

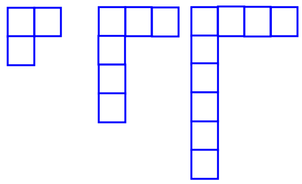
x	*y*
1	10
2	13
3	16
4	
5	

b)

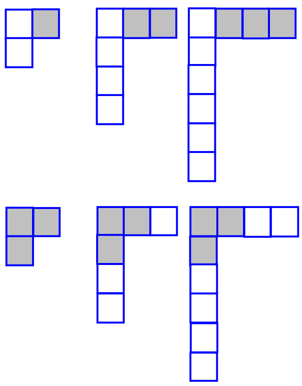
x	*y*
1	28
2	26
3	24
4	
5	

3. Write two equivalent pattern rules for the number of squares in the pattern below.

Fig 1 Fig 2 Fig 3



You can use these two shadings to help you.



4. A man has four Nu 50 notes and some Nu 20 notes.

a) Write an algebraic expression to represent how much money he has altogether if he has *n* Nu 20 notes.

b) Use your expression to determine how much money he has if $n = 8$.

5. Write a word problem that could be solved using $30x + 15$.

6. Simplify each.

a) $(2n + 8) + (10n - 3)$

b) $(2m - 9) + (-3m - 3)$

c) $(2n + 8) - (10n - 3)$

d) $(2m - 9) - (-3m - 3)$

7. If you add the numbers in any T-shape of four numbers in a 100 chart, the sum is always 10 greater than 4 times the number in the middle of the top row.

For example:

1	2	**3**	4	5	6	7	8	9	10
11	12	13	14	15	16	17	**18**	19	20
21	22	23	24	25	26	27	28	29	30

$2 + 3 + 4 + 13 = 4 \times 3 + 10$

a) Write an algebraic expression to represent this relationship. Show your work.

b) How do you know you are right?

8. Write an expression that would simplify to each.

a) $4n + 6$ **b)** $5n - 10$

9. Write a description using words for each equation below.

For example:

The equation $3m - 2 = 13$ in words means "the difference between 3 times a number and 2 is 13".

a) $4k - 5 = 23$ **b)** $6t + 8 = 50$

10. Represent each equation using a square tile model, a number line model, or a rectangle model.

a) $40 - 3m = 28$

b) $2k + 17 = 33$

c) $6t - 8 = 46$

11. The pattern rule for the number of squares in this pattern is $5f + 1$.

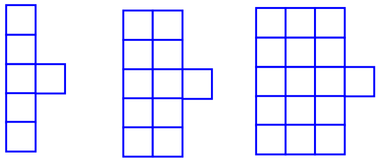


Figure 1 Figure 2 Figure 3

a) Write an equation you can use to find the number of the figure that has exactly 101 squares.

b) Solve the equation.

12. What would be a good first guess to solve each equation? Explain your choice.

a) $563 - k = 412$

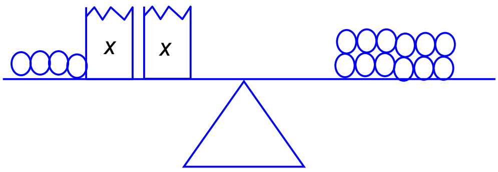
b) $221 = 17k$

13. Write each sentence below as an equation and then solve it.

a) The sum of two numbers that are 8 apart is 164.

b) If you add 3 to 3 times a number, the result is 255.

14. a) What equation does this represent?



b) What operations do you have to perform to solve the equation using inverse operations?

c) Solve the equation.

15. Solve each using inverse operations.

a) $4k + 29 = 497$

b) $624 - 4t = 276$

16. Create a graph for this table.

Figure number	Number of grey squares
1	10
2	13
3	16
4	19
5	22

17. For each, create a table of values up to $x = 5$ and then graph.

a) $y = 3x + 4$ **b)** $y = 30 - 6x$

c) $y = 7x - 1$ **d)** $y = 25 + 2x$

18. Use your graphs in **question 17**. Estimate the solution to each equation.

a) $67 = 3x + 4$ **b)** $12 = 30 - 6x$

c) $7x - 1 = 27$ **d)** $31 = 25 + 2x$

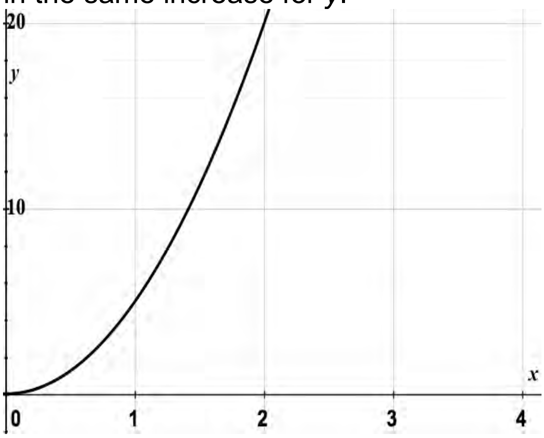
19. Draw a graph to represent each rate.

a) distance compared to time at a speed of 22 km/h

b) total cost compared to number of items purchased at Nu 80 per item

20. Graph $y = 4x - 7$. Explain how the graph shows that, whenever x increases by 1, the value of y always increases by 4?

21. Explain how this graph shows that an increase of 1 for x does not always result in the same increase for y.



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UNIT 7 PROBABILITY AND DATA

Getting Started

Use What You Know

A. i) If you know the ages of your mother and father, write them on a piece of paper. Your teacher will collect the data from everyone in the class for you to use.

ii) Graph the data in two stem and leaf plots. Use stems of 20s, 30s, 40s, and 50s. Samples are shown to the right.

Sample Stem and Leaf Plots

Mothers' Ages

2	9
3	0 2 2 3 4 5 5 5 6 7 8 9
4	0 0 1 2 3 5 6 8 9
5	0 1

Fathers' Ages

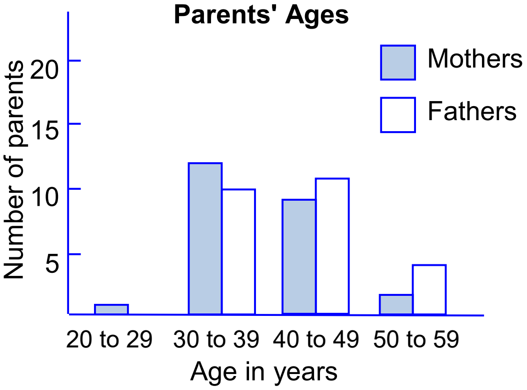
2	
3	1 2 3 4 5 5 6 7 8 9
4	0 0 1 1 1 2 2 3 4 5 8
5	0 1 2 5

B. i) Find the mean, median, and mode for the mothers' ages and for the fathers' ages.

ii) Which measure in **part i)** best represents each set of data? Explain your answer.

C. Use the stems of your plots to display the data in a double bar graph. A sample is shown to the right.

Sample Double Bar Graph



D. Use the stem and leaf plots or the double bar graph to answer each question below. For each question, tell which graph you used and why.

i) How old is the oldest parent?

ii) How many fathers are 40 years of age or older?

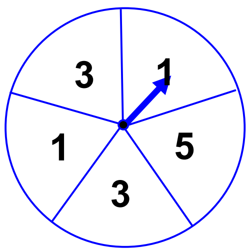
Skills You Will Need

1. Complete.

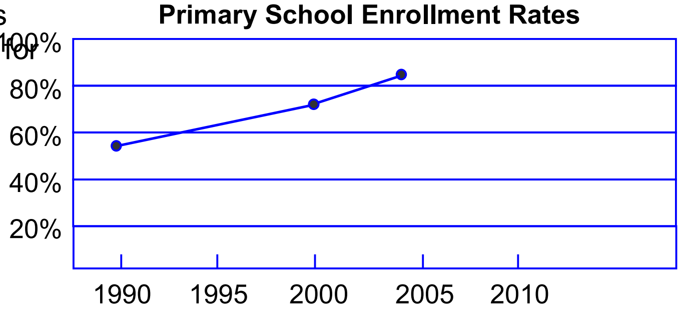
a) $\frac{3}{4} = \frac{[]}{20}$ **b)** $\frac{15}{[]} = \frac{3}{8}$ **c)** $\frac{49}{77} = \frac{7}{[]}$ **d)** $\frac{18}{20} = \frac{[]}{[]}$

e) $\frac{14}{25}$ = ___% **f)** $\frac{17}{20}$ = ___% **g)** $\frac{6}{40}$ = ___% **h)** $\frac{16}{30}$ = ___%

2. What is the probability of spinning 3 on this spinner? Express the probability as a fraction, as a decimal, and as a percent.



3. This broken line graph shows primary school enrolment rates for Bhutan.



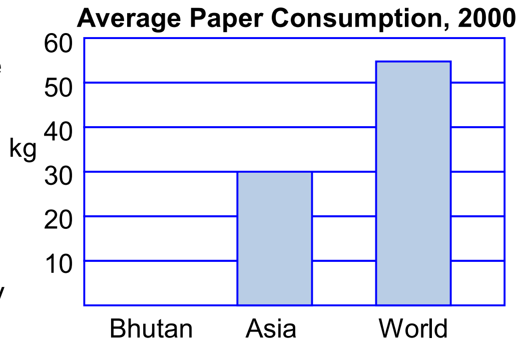
a) What was the enrolment rate for each year?

i) 1990 **ii)** 2000 **iii)** 2004

b) Estimate the enrolment rate for 1995.

c) Bhutan's Millennium Development Goals call for 100% enrolment by 2015. Based on the data in the graph, do you think this goal is likely to be achieved? Explain your thinking.

4. The bar graph on the right shows the average amount of paper (in kilograms) one person uses in one year.



a) How much paper does the average person in the world use in one year?

b) How much does the average Asian use?

c) It appears as though no paper is used in Bhutan. Do you think this is true? If not, why is there no bar for Bhutan?

5. A group of students saved money for a trip to the Samtse Tshechu. The chart on the right shows the amount each student saved. Calculate each measure for the set of data.

a) mean **b)** median **c)** mode

Student	Savings (Nu)
Tshering	50
Rinzin	10
Nima	5
Chandra	5
Passang	10
Chhimi	2
Dawa	5
Pema	5
Karma	5
Tandin	4
Tshewang	20

6. The following set of data is the result of a long jump contest for the students in Eden's class.

Long Jump Distances (cm)

187	205	221	186	185	212	222	215	198
200	205	207	193	186	172	208	223	175
206	215	227	228	230	218	188	173	196
202	221	214	220	229	189	193	212	212

a) Create a stem and leaf plot of the data.

b) Use the stems to make a bar graph of the data.

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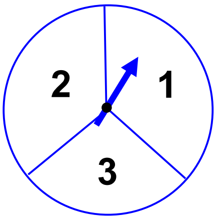
7.1.1 Determining Theoretical Probability

Try This

A. Suppose you spin this spinner twice and add the numbers.
i) What different sums are possible?
ii) Is one sum more likely than another sum? Explain your



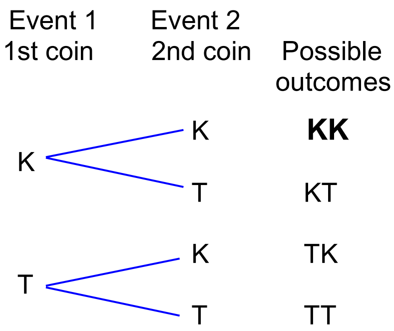
• The **theoretical probability** of an **event** is the fraction that compares the number of **favourable outcomes** to the number of **possible outcomes**.

The theoretical probability of an event is $P(\text{event}) = \dfrac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$

.
Theoretical probability is useful for making predictions about future events.

• To determine theoretical probability, you need to know the number of possible outcomes. You can do this by making a **tree diagram** when more than one event is involved.

For example, you want to find the probability of two Khorlos when you flip two Nu 1 coins. This tree diagram shows all the possible outcomes.



There is 1 favourable outcome (**KK**) out of 4 possible (and equally likely) outcomes, so the theoretical probability of getting two Khorlos with two coins is 1 out of 4, or $P(KK) = \frac{1}{4}$.

• When there are two events, another way to determine theoretical probability is with a rectangle model. You label each side of the rectangle with the possible outcomes of each event.

For example:

To find the probability of two Khorlos when flipping two Nu 1 coins, you divide each side of the rectangle in half because the probability of getting a K or a T with each coin is $\frac{1}{2}$.

1st coin \ 2nd coin	K	T
T	KT	TT
K	**KK**	TK

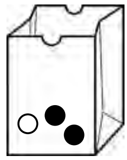
Since Khorlo-Khorlo (KK) is $\frac{1}{4}$ of the area, $P(KK) = \frac{1}{4}$.

B. i) Create a tree diagram or a rectangle model to find the theoretical probability of each sum in **part A i)**.
ii) Were your answers to **part A** correct? Explain your thinking.

Examples

Example Determining Theoretical Probability to Solve a Probability Problem

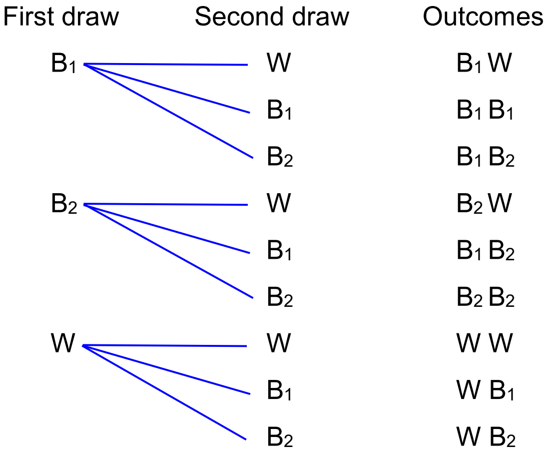
There are two black counters and one white counter in a bag. Chabilal draws a counter from the bag without looking. He then replaces it and draws a second counter without looking.



a) What is the probability that he draws two black counters?

b) What is the probability that he draws at least one black counter?

Solution 1



a) P(two black counters) = $\frac{4}{9}$

b) P(at least one black counter) = $\frac{8}{9}$

Thinking

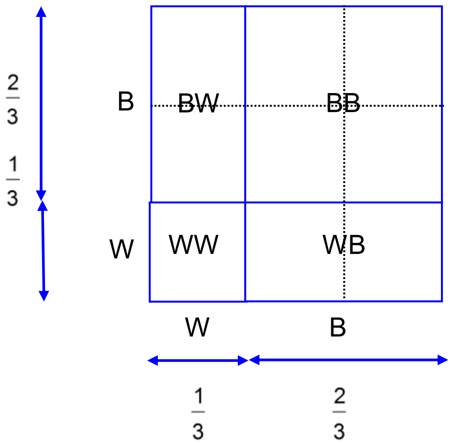
• My tree diagram showed every possible outcome of drawing two counters.

• I called the two black counters B_1 and B_2.

• There were 9 equally likely possible outcomes:

- 4 of the 9 outcomes had two black counters.
- 8 of the 9 outcomes had at least one black counter (BB, WB, or BW).

Solution 2



a) P(two blacks) = $\frac{4}{9}$

b) P(at least one black) = $\frac{8}{9}$

Thinking

• The probability of drawing black on each draw was $\frac{2}{3}$ and the probability of drawing white was $\frac{1}{3}$, so I divided the sides of my rectangle to show that.

• Then I divided the rectangle into 9 equal area units so I could determine the area of each part.

• I counted area units for each favourable outcome:

- Drawing two blacks (BB) covered 4 of the 9 area units.
- Drawing at least one black (BB, WB, or BW) covered 8 of the 9 area units.

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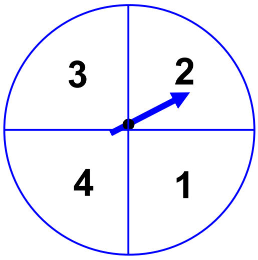
Practising and Applying

1. Choki spun this spinner twice and then multiplied the two numbers.

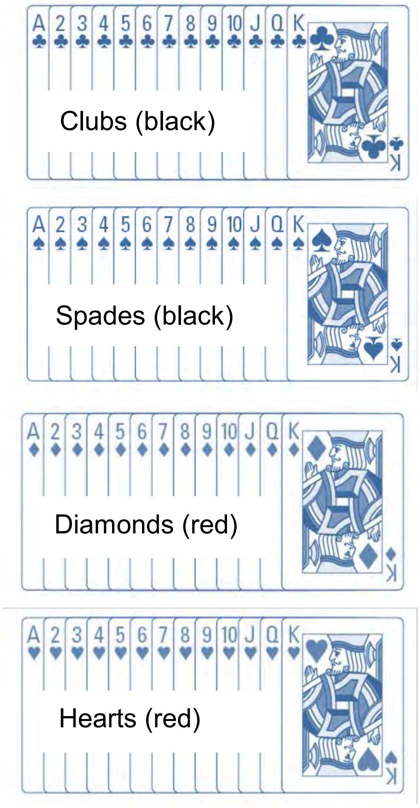


What is the probability of each product?

a) 1 **b)** 2 **c)** 4

d) less than 5 **e)** an even number

2. In a deck of 52 playing cards there are 4 suits, with 13 cards in each suit. In each suit, there are three face cards: a Jack (J), a Queen (Q), and a King (K).



a) If you draw one card from the deck, what is the probability of drawing each?

i) an ace (A) **ii)** a club (♣)

iii) a face card **iv)** an even number

v) a red queen (Q)

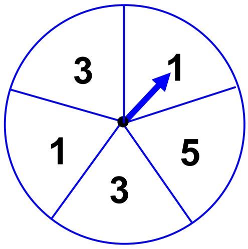
b) Suppose you draw a card, replace it and then draw a card again. What is the probability that you will draw two clubs?

3. A bag contains one of each coin:

Ch 20 Ch 25 Ch 50 Nu 1

Lhamo draws a coin from the bag and replaces it. Then she draws another coin. What is the probability that she draws two coins with a total value of more than Nu 1?

4. Dorji spun this spinner twice and then added the numbers.



What is the probability of each sum?

a) 2 **b)** 4 **c)** 6 **d)** 8 **e)** 10

5. A bag contains three counters: one red, one green, and one blue.

What is the probability of?

a) drawing a blue counter in the first draw

b) drawing one or more blue counters in two draws

6. What is the probability of getting one Khorlo and two Tashi Ta-gyes when you flip three Nu 1 coins?

7. How are these two methods for determining probability similar? How are they different?

- a tree diagram
- a rectangle model

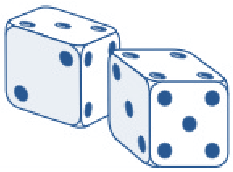
7.1.2 EXPLORE: Experimental Probability

Theoretical probability is about analysing a situation before it happens in order to make a reasonable prediction about what is likely to happen in the future.

Experimental probability is about the results of what actually happened.

Experimental probability is P(event) = $\dfrac{\textit{number of favourable results}}{\textit{number of trials}}$.

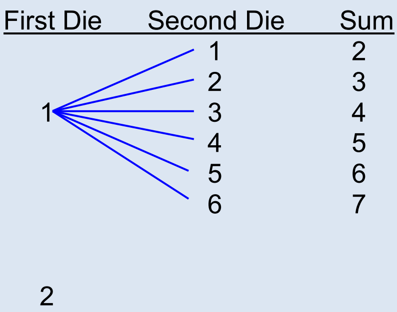
In this lesson, you will use theoretical probability to predict the results of rolling two dice and finding the sum. You will then conduct an experiment with the dice to see if your predictions match the results.



Save the experimental data you get from this lesson. You will need it to answer questions in other parts of the unit.

A. i) Work in a group of two or three students.

Copy the chart to the right. Complete the Theoretical probability column. Express each probability as a fraction with a denominator of 36.
You can use a tree diagram to find the theoretical probability.



Or, you can use a rectangle model.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2						
3						
4						
5						
6						

Rolling two dice 36 times

Sum	Theoretical probability	Experimental probability	
		Tally	Fraction
2	$\frac{1}{36}$	//	$\frac{2}{36}$
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

ii) Conduct an experiment. Roll two dice 36 times and find the sum each time. Tally the results in the chart. Then express each experimental probability as a fraction with a denominator of 36.

iii) For each sum in the chart, subtract to find the difference between the theoretical and experimental probabilities. Are they $\frac{1}{36}$ or less apart?

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B. i) In your group, copy the chart to the right. Complete the middle column. Express each theoretical probability as a fraction with a denominator of 72.

ii) Combine the data from your experiment in **part A** with the data from another group. This will be just like rolling 72 times. Complete the last column of the chart.

iii) For each sum in the chart, subtract to find the difference between the theoretical probability and the experimental probability.

Are they $\frac{2}{72}$ or less apart?

Rolling two dice 72 times

Sum	Theoretical probability	Experimental probability
2	$\frac{2}{72}$	$\frac{?}{72}$
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

C. i) In your group, copy the chart to the right. Complete the middle column. Express each theoretical probability as a fraction with a denominator of 108.

ii) Add the data from a third group in order to complete the last column of the chart. This will be just like rolling 108 times. Express each as a fraction with a denominator of 108.

iii) For each sum in the chart, subtract to find the difference between the theoretical probability and the experimental probability.

Are they $\frac{3}{108}$ or less apart?

Rolling two dice 108 times		
Sum	**Theoretical probability**	**Experimental probability**
2	$\frac{3}{108}$	$\frac{?}{108}$
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

D. What do you notice about your answers to **part A iii)**, **part B iii)**, and **part C iii)**? Why do you think this happened?

E. For which experiment below would you be able to make a more reasonable prediction about the results using theoretical probability? Explain your thinking.

- Flipping two coins 10 times to see how often you get both coins the same.
- Flipping two coins 100 times to see how often you get both coins the same.

7.1.3 Matching Events and Probabilities

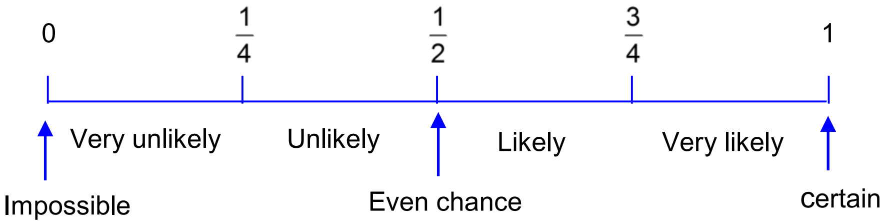
Try This

A. Answer each question about rolling a die. Explain your thinking.
i) Is rolling a number less than 3 likely or unlikely?
ii) Is rolling a number greater than 1 likely or very likely?
iii) Is rolling an even number more likely than rolling an odd number?

• You have already learned that theoretical probability is about analysing a situation in order to make a reasonable prediction about what will happen, and that experimental probability is about the results of what actually happened.

• For some events, it is impossible or difficult to determine theoretical probability. In these cases, you can use make reasonable predictions using an experimental probability that is based on a lot of data.

• Whether probability is theoretical or experimental, it can be described with words or numbers. Words that are often used are shown on the **probability line** below.



• Probability words can help you match an event to a particular probability.
For example:

What event has a probability of 1?
Think of things that are certain to happen, such as the event that a student in your class is either a boy or a girl.

What event has a probability of 0?
Think of things that are impossible, such as the event of rolling a die and getting a 7.

What event has a probability of $\frac{1}{2}$ *?*

Think of things that have an even chance of happening or not happening, such as the event of flipping a Nu 1 coin and getting a Khorlo.

P(Khorlo) = $\frac{1}{2}$

What event has a probability of $\frac{3}{4}$ *?*

Think of things that are likely but not certain, such as the event of someone picking a number less than 4 (1, 2, or 3) if asked to pick a number from 1 to 4.

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• Sometimes the denominator of the probability fraction can help you decide on an event that has that probability.

For example:

For a probability of $\frac{1}{6}$, think of something that has 6 possible outcomes, like rolling a die. The probability of rolling a 2 on a die is $\frac{1}{6}$.

For a probability of $\frac{3}{5}$, think of something that has 5 possible outcomes, like drawing a particular cube from a bag of 5 cubes. The probability of drawing a red cube from a bag with 2 blue cubes and 3 red cubes is $\frac{3}{5}$.

B. i) What is the theoretical probability of each event in **part A**?

ii) Were your answers to **part A** correct? How do you know?

Examples

Example 1 Describing Events to Match Probabilities

Describe an event to match each probability.

a) almost 0 **b)** almost 1 **c)** about $\frac{1}{4}$ **d)** about $\frac{1}{2}$ **e)** about $\frac{2}{3}$

Solution	Thinking
a) Winning first prize in a lottery	**a)** Many lottery tickets are sold but only one is the winning ticket.
b) Rain during the monsoon season	**b)** Almost every day has rain during monsoon season, so it's almost certain.
c) Being born during a leap year	**c)** Leap years happen once every four years, so about 1 in 4 people are born in a leap year.
d) A baby girl being born	**d)** About half of all people are female, so there is 1 chance in 2 that a new born will be a girl.
e) Being born in a month with an "r" in its name	**e)** 8 out of 12 months have an "r", so the chances are 8 in 12, or 2 in 3.

Example 2 Matching Events With Probabilities

In 100 shots, a basketball player had these results:

- He missed the basket 29 times.
- He scored two points 63 times.
- He scored three points the rest of the time.

Based on these past results, describe an event with each probability.

a) about $\frac{3}{5}$ **b)** about $\frac{3}{10}$

c) about $\frac{1}{10}$ **d)** about $\frac{7}{10}$

Solution

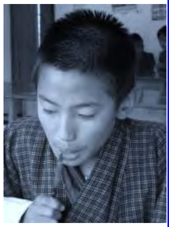
a) P(two points) is about $\frac{3}{5}$.

b) P(miss) is about $\frac{3}{10}$.

c) P(three points) is about $\frac{1}{10}$.

d) P(two or three points) is about $\frac{7}{10}$.

Thinking



a) $\frac{63}{100}$ is about $\frac{60}{100} = \frac{6}{10} = \frac{3}{5}$.

b) $\frac{29}{100}$ is about $\frac{30}{100} = \frac{3}{10}$.

c) He scored three points 8 times (100 – 63 + 29 = 8). $\frac{8}{100}$ is about $\frac{1}{10}$.

d) He scored two points 63 times and three points 8 times. So he scored two or three points 71 times (63 + 8 = 71). $\frac{71}{100}$ is about $\frac{7}{10}$.

Practising and Applying

1. Use probability words from the list to make predictions about these events. You may not need to use each word, and you may use some words more than once.

Impossible, certain, very unlikely, very likely, unlikely, likely, even chance

a) If you flip a Nu 1 coin 100 times you will get 100 Khorlos

b) The sun will set tonight

c) No dogs will bark tonight

d) A person is right-handed

e) A person is more than 4 m tall

f) An average student will pass a test

g) If you roll a die, you will get 1, 2, or 3

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2. Describe an event in your own life that matches each probability word in **question 1**.

3. In 100 shots, an archer hit a Gorthibu-Karey 8 times. He hit 25 other kareys. His other shots missed the target. Based on these past results, describe an event with each probability.

a) about $\frac{1}{12}$

b) $\frac{1}{4}$

c) about $\frac{1}{3}$

d) about $\frac{2}{3}$

4. Which two events below are equally likely? Explain your answer.

A. Rolling a die and getting 1 or 2

B. Drawing a spade from a deck of cards

C. Tossing a Nu 1 coin and getting a Tashi Ta-gye

D. Having a birthday during the summer

5. An archer hit the target 100 times in 100 shots.

a) What number and word describes the experimental probability of hitting the target?

b) Is the archer certain to hit the target with the next shot? Explain your answer.

6. When is it easy to match a probability expressed as a fraction with an event? When is it more difficult?

GAME: No Tashi Ta-gye!

Play with a partner. You need a Nu 1 coin.

Take turns.

- On your turn, flip a coin and score 1 point for each Khorlo. You can stop flipping the coin at any time.
- If you flip a Tashi Ta-gye, your turn ends and you lose all your points.
- If you choose to stop before flipping a Tashi Ta-gye, you keep all your points.

When you stop, it is your partner's turn to begin flipping the coin.

The game ends when a player has 10 points.

As you play, think about these questions:

- *Is there an advantage to being the first player to flip?*
- *Should you always stop after the first Khorlo you flip?*

Sample Record for One Player

Turn	Flips	Turn points	Total points
1	T	0	0
2	KKK	3	3
3	KT	0	3
4	KK	2	5
5			

Chapter 2 Collecting Data

7.2.1 Formulating Questions to Collect Data

Try This

A. A survey company wants to find out how many people in Bhutan are happy. Design a survey question to find out what the survey company wants to know.

There are many ways to collect data.

• Sometimes **interviews** are conducted. An interviewer asks questions and records the answers. Interviews are often used when you can talk to the people you want to **survey**.

• When it is not easy to interview, people may be asked to answer a list of questions called a **questionnaire**.

• **Observation** is another way to collect data. Instead of asking questions directly, an observer watches, measures, and records the data.

For example, if a store wants to know what colour kira to stock, they may keep a record of what people buy instead of asking them what they want.

For some kinds of data, observation is the only way to collect information.

For example, weather data can be collected only by observing what happens.

• Whether the data is being collected by interview, by questionnaire, or by observation, the survey question must be created thoughtfully.

- The question should be simple and clear:

Good question: *About how many times have you taken the bus in the last 30 days?*

Poor question: *What is the frequency of your travel by public transit in the last 30 days?*

- The question should be asked so everyone will understand the question in the same way:

Good question: *Bhutan Post is now using six more buses to deliver mail. Are you getting mail more quickly now?*

Poor question: *How effective is Bhutan Post's new bus service?*

• There should be one topic per question:

Good question: (People can answer each question separately.)

1. *Are you satisfied with Bhutan Post's service?*

2. *Is the service good value for the cost?*

Poor question: *Are you satisfied with the service and cost of Bhutan Post buses?*

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- The question should avoid too many negatives, such as “do not", as they can confuse people. The question should not influence the answer.

Good question: *Do you think it is good for people to exercise daily?*

Poor question: *Most doctors do not believe that daily exercise is not good for you. Don’t you agree?*

• The question should not ask for information people may not want to give:

Good question: *Do you think it is all right for a student to pretend to be sick to avoid going to school once in a while?*

Poor question: *Do you ever lie about being sick so you can stay home from school?*

• A survey question should also consider how the data will be displayed. If you provide answer choices, you can graph the numbers of people who chose each answer. It is important to include all the possible answers in the choices.

For example, the first question on **page 230** about bus service could be improved by giving answer choices as shown below. Note that without **choice d)**, some people would not be able to answer the question.

About how many times have you taken the bus in the last 30 days?

a) 0 times **b)** 1 to 10 times **c)** 11 to 20 times **d)** more than 20 times

B. Rewrite your survey question in **part A** to make it a better question. Tell why it is better.

Examples

Example 1 Choosing and Justifying Data Collection Methods

Choose a method for collecting each type of data. Explain your choice.

a) to find out the favourite colour of every student in your class

b) to find out the favourite colour of most people in Bhutan

c) to find out if animals have colour preferences

Solution	Thinking
a) Interview — to make sure everybody in the class answers	**a)** I could also use a questionnaire but I might not get everyone.
b) Questionnaire — you cannot observe or interview everyone in Bhutan because there are too many people	**b)** I only need to know about most people, not everyone.
c) Observation — you cannot ask questions to animals	**c)** I would design an experiment.

Example 2 Choosing a Good Survey Question

Suppose you want to find out about what people think about the state of happiness in Bhutan. Which survey question would you use? Explain why.

A. Do you not agree that most people in Bhutan are happy?

B. Are most people in Bhutan happy or unhappy?

C. Most people in Bhutan should be happy. Do you think they are?

D. What does it mean to be happy or not happy in Bhutan?

Solution

I would use question **B**.

- It is simple and clear and offers two answer choices.
- It will not influence people's answers.
- It will collect the data needed to find out what people think about the state of happiness.

Thinking

- The wording of questions *A* and *C* might influence people's answers.
- Question D isn't going to collect the data that I need. It asks a different question.

Practising and Applying

1. Which method would you use to collect health data to answer each of the following questions? Explain your choices.

a) How many times did most people in your town visit a doctor in the past year?

b) How many students in your class missed school due to illness this year?

c) How many patients in a hospital have an above-normal temperature?

2. Which is the best survey question to ask to find out about reading habits? Explain why you think it is best.

A. *Do you think you should read more?*

B. *What do you think about reading?*

C. *How many books have you read in the past month?*

D. *Reading can strain your eyes. Do you not think people should read less?*

E. *What books have you read recently?*

3. What could you do to make the question you chose in **question 2** an even better question?

4. Rewrite each survey question to make it better.

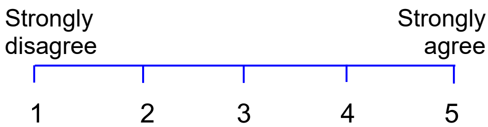
a) *Should the teacher not schedule a math test every week?*

b) *How would you rate your enjoyment of studying math, science, and English?*

c) *How effective is homework in improving your learning?*

5. Scales like the one below are often used in survey questions. Write a survey question that could use this scale.

Choose 1, 2, 3, 4, or 5:



6. The Department of Roads (DoR) wants to determine the part of the country that needs new road construction the most. They decide to survey all drivers in the country.

a) Design a good survey question for the DoR.

b) Write a poor survey question and tell why it is poor.

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7.2.2 Sampling and Bias

Try This

In a survey on happiness, 500 people were chosen to represent all age groups and regions of the country. They were asked,

How do you feel about your life as a whole?

A scale of 1 to 5 was used for their answers. The results are shown to the right.

5	Happy	25%
4	Mostly happy	40%
3	Neither happy nor unhappy	10%
2	Mostly unhappy	15%
1	Unhappy	10%

A. Do you think the results would be different in each case? Explain your thinking.

i) if the 500 people were from urban areas only

ii) if the 500 people were all teenagers

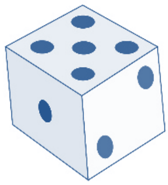
iii) if the 500 people were all men

iv) if only 100 people were asked

• When you do an experiment, you get better results if you have a lot of data.

For example:

Suppose you conduct a probability experiment to see how many times each number is rolled on a die. Based on theoretical probability, you predict that each number has an equal chance of being rolled.



- If you roll 6 times, you likely will not get each number once. You might roll a five several times and never roll a three. The result of this small **sample** might mislead you to think that fives are more likely than threes.

- If you increase your sample and roll, for example, 600 times, you are more likely to roll each number about the same number of times.

• A sample that is too small can affect the results and be misleading.

• A survey is like an experiment — if you ask more people, you will get a more accurate result.

• A survey of an entire country or **population** is called a **census**. You get good results from a census because you are surveying everyone.

• If you cannot survey everyone, you can survey a sample of people has been carefully selected to represent the population.

- If the sample is large enough, the results of a survey will usually be close to the results you would get if you did a census.

Population and Housing Census 2005

• The sample you choose can influence, or **bias** the results. This is because the wrong sample will not represent the population.

For example:

Suppose you want to know what most people in Bhutan think about tourism. If you ask only your friends, the results will be biased towards your age group and neighbourhood. The sample does not represent all the people in Bhutan.

• The source of data is also important:

- **First-hand data** come directly from interviews, questionnaires, or observation.
- **Second-hand data** come from sources such as books, radio and TV, or the Internet. You need to be very careful when you use second-hand data.

For example:

If you research Bhutan's population on the Internet, you will find many different figures. This shows why you must be careful when you use second-hand data.

Population of Bhutan	Internet source
2,237,849	www.cia.gov/cia/publications/factbook
2,279,723	http://www.countryreports.org
1,745,700	http://www.populationworld.com
752,700	Millennium Development Goals Progress Report (http://www.undp.org.bt/mdg/MDG_PR05.pdf)
672,425	Bhutan Census 2005 (www.bhutancensus.gov.bt)

B. Would you change your answers to **part A** now that you know more about samples? Why or why not?

Examples

Example Identifying Bias in Survey Samples

A researcher wants to find out about the diets of Bhutanese people.
Why might each of these samples be biased?

a) interviewing a sample of 50 people by telephoning each person

b) sending questionnaires to all addresses in cities of more than 10,000 people

c) observing people in restaurants

Solution

a) It is a small sample and might not give accurate results. It only includes people who have telephones.

b) It will not give any information about people in rural areas or in smaller cities and towns.

c) It tells you only what people eat in restaurants, not at home. Many people do not eat in restaurants.

Thinking

a) 50 people is only a small fraction of the population of Bhutan.

b) Most people in Bhutan don't live in large cities.

c) People who eat in restaurants wouldn't represent the population of Bhutan.

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Practising and Applying

1. Look at the Bhutan population data on **page 234**. Which population figures do you trust most? Tell why.

2. Suppose you want to find out the favourite foods of people in Bhutan. You send out a questionnaire to 1000 people. Do you think the results for each situation below might be biased? Explain your thinking.

a) Only 45 people completed and returned the questionnaire.

b) Most of the responses came from teenagers.

c) All the questionnaires were sent to the eastern Dzongkhags.

3. Which method of data collection: census, sample, or second-hand data, would you use for each situation below? Explain your choice.

a) to find out how the people in a city are likely to vote in the next election

b) to find out how fast people drive on Bhutan's roads

c) to find out how many children are in an average Bhutanese family

d) to find out how many children attend Class VII in Bhutan

4. Recall what you learned about survey questions in **lesson 7.2.1**.
Will increasing the sample size *always* give you better survey results? Why?

CONNECTIONS: Estimating a Fish Population

In the 1930s, the Government of Bhutan stocked the rivers and lakes of Bhutan with brown trout. Since then, the fish population has increased.

How do you estimate the fish population of a lake?

Scientists do this using sampling and probability.

For example:

- They catch a sample of 100 fish from the lake, tag them with labels, and then release them.
- They allow time for the tagged fish to mix in with the rest of the fish population.
- They catch a sample of 100 fish again and count the tagged fish.
- They use the ratio of tagged fish to the 100 fish in the sample to estimate the population (they have to assume that the probability of catching each fish, whether tagged or not, is the same).

For example:

If 4% of the fish in the sample are tagged, then about 4% of the population is tagged. That means the 100 tagged fish represent about 4% of the population.

If, $\frac{4}{100} = \frac{100}{P}$, where P is the population, then $\frac{4}{100} = \frac{100}{2500}$ (× 25) and P = 2500.

There are about 2500 fish in the lake.

1. Why is it impossible to do a census of the fish population in the lake?

2. Suppose 50 tagged fish are put into the lake. Scientists then catch 200 fish and 20 of these fish are tagged. Estimate the fish population.

7.2.3 EXPLORE: Conducting a Survey

Now that you know about survey questions, sampling, and bias, you can apply these ideas by conducting a survey.

A. Select an issue that is of interest to you.

Some examples might be:

- whether the school should provide a shorter or longer lunch time
- whether students think they have too much or not enough homework
- whether students would like a change in school dress

B. i) Create a series of one to three questions you could ask to collect the information you want.

ii) How did you make sure your survey questions were good questions?

C. Describe how you will collect the data, including whether you will interview or hand out a questionnaire. Describe a sample you can use if you do not want to ask every student.

D. Carry out your survey over the next few days.

E. Write a brief report about what you found out.

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Chapter 3 Graphing Data

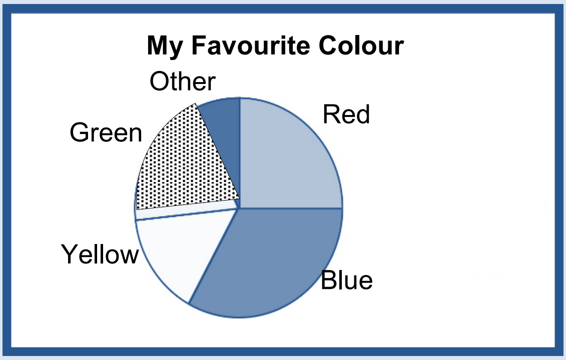
7.3.1 Circle Graphs

Try This

A group of students were surveyed about their favourite colours. The results are displayed in this circle graph.

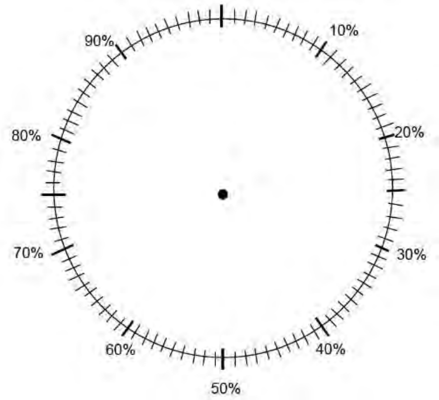
Each part of the circle represents the percent of students who chose that colour as their favourite.

A. Estimate the percent of students who chose each colour.



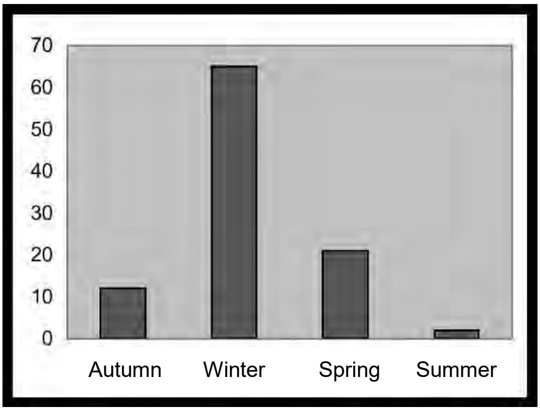
• A **circle graph** is a way to display data. Each part of the circle shows a percent of the whole set of data.

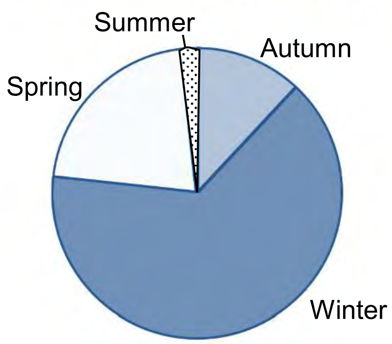
Circle graphs are sometimes called pie charts.



• The **bar graph** and circle graph below display the same data about forest fires.

Percent of Forest Fires by Season





- The bar graph tells the actual percent of forest fires in each season and makes it easy to see how the seasons compare.

- The circle graph shows the fraction of fires that occur in each season of the year. It also shows how the seasons compare, but not as well as the bar graph.

You can choose to display data in a bar graph or in a circle graph, depending on the data and what you want to show about the data.

B. i) What is the sum of all the percents in **part A**? How do you know?
ii) Use your answer to **part B i)** to check your estimates from **part A**.

Examples

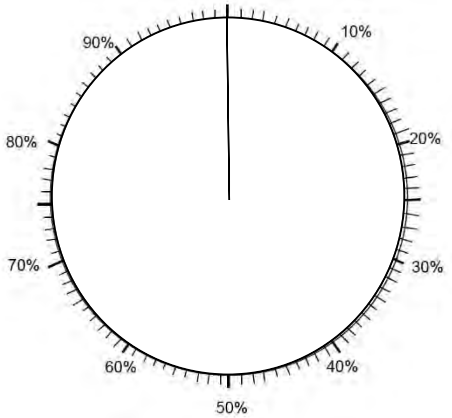
Example 1 When to Use a Circle Graph

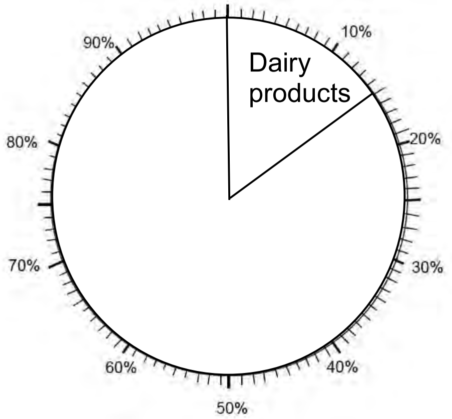
Chandra was doing a report on the type of food he ate during one week.

Dairy products	Meat and fish	Vegetables and fruit	Rice and grain
15%	10%	34%	41%

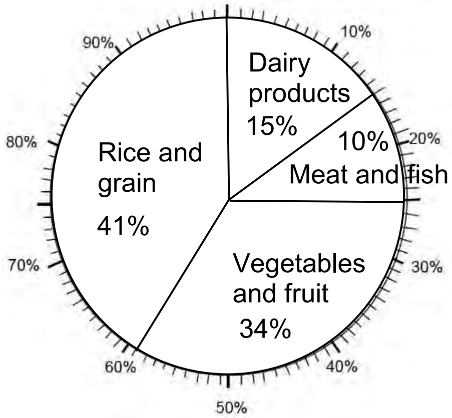
Display the information in a graph.

Solution





Chandra's Food Groups for a Week



Thinking

- I chose a circle graph because each data value is a part of a whole. The whole is everything he ate in one week.
- I used a percent circle.
- I started by drawing a radius from the centre to the top, which is 0%.
- To show Dairy Products, I drew another radius from the centre to the 15% mark on the circumference and then labelled the section.
- For the 10% for Meat and fish, I counted 10 tick marks clockwise from 15% and added a radius.
- For the 34% for Vegetables and fruit, I counted 34 tick marks from the last radius and added a radius.
- Rice and grain took up the space that was left.
- I added a title to describe the data.
- I labelled each part with its percent so I could compare the parts more exactly.

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Example 2 Estimating Percents to Make a Circle Graph

The chart shows the population by age group in Bhutan according to the 2005 census.

a) Show this information in a circle graph.

b) Would a bar graph be suitable to display this set of data? What about a line graph?

Population by Age Group

Age group	Persons
0 – 14	210,000
15 – 64	395,000
65+	30,000
Total	635,000

Solution

a) *Estimate the 0 – 14 age group*

$\frac{210{,}000}{635{,}000} = \frac{210}{635}$ is about $\frac{200}{600} = \frac{1}{3}$.

$\frac{1}{3}$ is about 33%.

Estimate the 65+ age group

$\frac{30{,}000}{635{,}000} = \frac{30}{635}$ is about $\frac{30}{600} = \frac{5}{100}$.

$\frac{5}{100}$ is 5%.

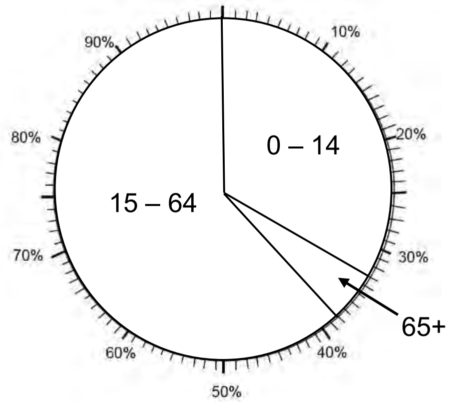
Estimate the 15 – 64 age group

33% + 5% = 38%

100% – 38% = 62%

Draw the circle graph

Population by Age Group
Bhutan 2005 Census



b) A bar graph would be suitable but a line graph would not.

Thinking

a) To make a circle graph, I needed the percent each age group was of the whole population.

- I estimated percents for the two parts of the population that were easiest to estimate.

- Since the whole population was 100%, I subtracted the estimates for the two parts from 100 to get the third percent.

- I used the percents to draw the circle graph.

Age group	Percent
0 - 14	33
15 - 64	62
65+	5

b) A bar graph might show how the age groups compare better than the circle graph does. However, it wouldn't show how each age group compared to the whole population as well as the circle graph does.

- A line graph is used to show how things change over time, so it wouldn't be suitable.

Practising and Applying

1. Students at a college were asked to choose one fruit as their favourite. Display the data in a circle graph.

Fruit	Percent that chose each
Orange	35%
Apple	28%
Banana	19%
Other	?

2. Create a circle graph to display the experimental results of rolling two dice 108 times in **lesson 7.1.2**. You can estimate the percents.

3. What type of graph would you use to display the data for each: bar graph, line graph, or circle graph? Explain your choice.

a) The growth of Bhutan's population from 1990 to today.

b) The amounts (in tonnes) of four different types of farm crops

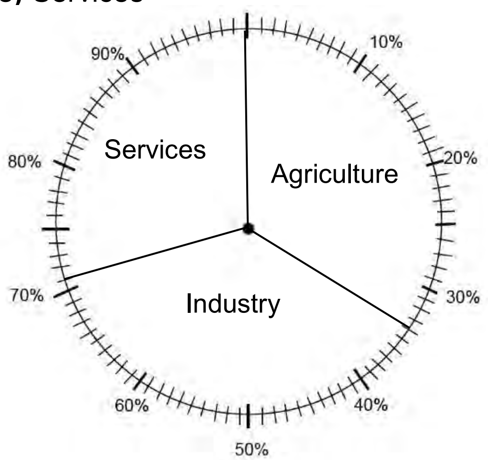
c) How you spend your time over a 24-hour period

4. Bhutan's economy is made up of the three main sources listed below. Use the circle graph to tell the percent for each.

a) Agriculture

b) Industry

c) Services

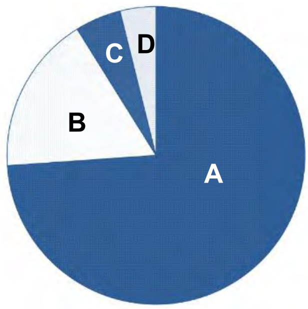


5. Bhutan is made up of four different ecosystems.

Ecosystem	Area (millions of hectares)
Forests	3400 ha
Agriculture	240 ha
Grasslands	800 ha
Barren (snow and ice)	190 ha

a) Match each ecosystem in the chart above to a part of the circle graph below.

Bhutan's Ecosystems



b) Estimate the percent of Bhutan's area that is each ecosystem.

6. Nima is looking at a circle graph that shows the favourite subjects of her classmates. She says that

- half the class chose math,
- one fourth chose English, and
- one third chose science.

Can she be reading the graph correctly? Explain your answer.

7.3.2 Histograms

Try This

Pem Bidha's class wrote a math test that was marked out of 50. Her teacher displayed the marks in a stem and leaf plot. Pem Bidha's mark was the median.

A. i) What was Pem Bidha's mark?

ii) How many students had higher marks than Pem Bidha?

iii) What is the mode for the data?

Stem	Leaves
1	5 8 9
2	3 5 8 9 9
3	0 1 4 4 4 6 6 7 8 8 8 8
4	0 0 1 2 2 2 2 3 4 6 7 8 9 9
5	0

• A **histogram** looks like a bar graph but it displays a different type of data.

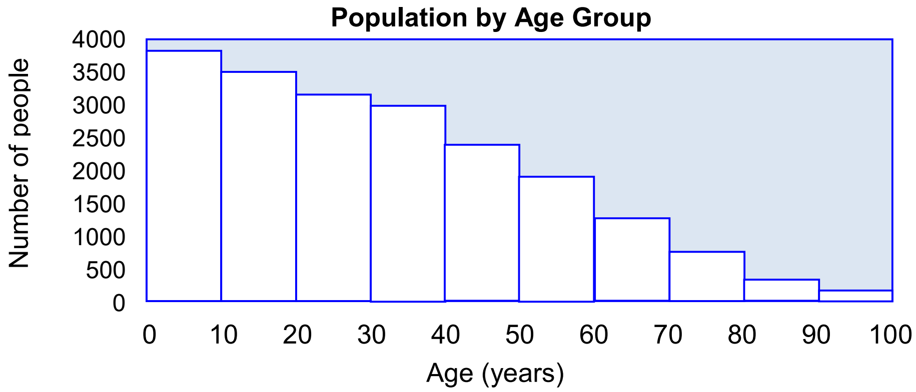
The data in a histogram is grouped into equal **intervals**. Each interval continues on from the previous interval, with no gaps. That is why the bars in a histogram have no spaces between them.

For example:

Suppose you collected data about a population by age.

- The first interval might be ages 0 to 10, but not including 10.
- The next interval could be ages 10 to 20, but not including 20.
- The remaining intervals could be 20 – 30, 30 – 40, and so on.

This histogram shows the population by age for a town of about 20,000 people.



• The intervals in a histogram seem to overlap but the value at the end of each interval actually belongs in the following interval.

For example:

Age 10 appears to be between the first and second intervals but we include it in the second interval. Age 20 is in the third interval, age 30 in the fourth, and so on.

• The height of each bar is the **frequency** of the data that the bar represents.

For example:

The height of the first bar in the histogram above shows there are about 3750 people between the ages of 0 and 10.

• To create a histogram, you need to decide on the size of the intervals you want. Once you know the interval size, you count the number or frequency of data for each interval.

• You can tally and record the data in a **frequency table** before displaying it in a histogram.

For example:

Set of Data

2	9	14	11	15	19	1	12	13	16
5	4	2	7	8	9	19	4	3	12

Frequency Table

Interval	Tally	Frequency
0 – 5	~~IIII~~ I	6
5 – 10	~~IIII~~	5
10 – 15	~~IIII~~	5
15 – 20	IIII	4

Note that the data value 5 is included in 5 – 10, and 15 is included in 15 – 20.

B. i) Sketch a histogram for the data in the stem-and-leaf plot in **part A**. Use the stems of the stem and leaf plot as the intervals of the histogram.

ii) How does the shape of the data in the histogram compare to the shape of the data in the stem and leaf plot? Why does this happen?

Examples

Example 1 Creating a Histogram

The following times, in seconds, were recorded for a 100 m race.

13.9	14.3	13.7	14.4	15.2	15.4	13.9	13.9	14.5	14.7	13.8
13.1	13.8	14.4	12.4	13.8	12.7	13.4	13.9	14.0	14.0	14.3
14.5	11.8	12.9	12.3	12.8	13.7	13.1	15.0	14.8	14.2	14.4
14.8										

a) i) Create a frequency table for the data in three intervals.

ii) Create a frequency table for the data in five intervals.

b) Which number of intervals would you use to create a histogram? Explain your choice.

c) Create the histogram.

d) What does your histogram tell you about the data?

Solution

a) i) *Three intervals*

Interval (s)	Tally	Frequency
11 – 12.5	III	3
12.5 – 14	~~IIII~~ ~~IIII~~ ~~IIII~~	15
14 – 15.5	~~IIII~~ ~~IIII~~ ~~IIII~~ I	16

Thinking

a) i) The data went from 11.8 s to 15.4 s, which is a range of 3.6 s, so I used three intervals of 1.5 s each from 11 s to 15.5 s in order to include all the data.

a) ii) *Five intervals*

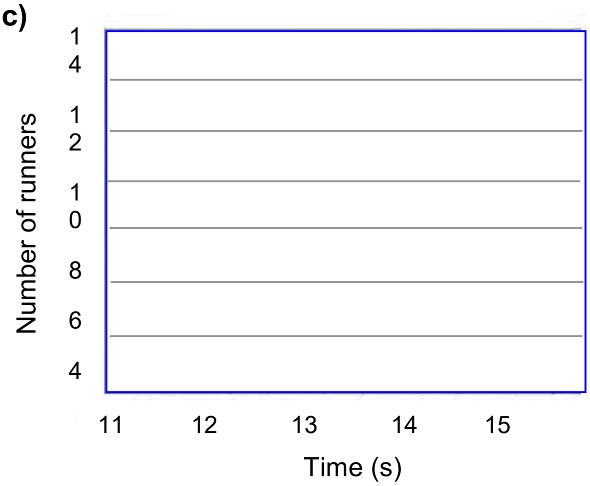
Interval (s)	Tally	Frequency
11 – 12	I	1
12 – 13	~~IIII~~	5
13 – 14	~~IIII~~ ~~IIII~~ II	12
14 – 15	~~IIII~~ ~~IIII~~ III	13
15 – 16	III	3

a) ii) From 11.8 s to 15.4 s, there were five whole-number parts from 11 to 15, so I used them as intervals.

b) I would choose five intervals because the intervals are simpler (they use whole numbers and they are 1 second each). Also, five bars will show how the numbers are greater in the middle while three intervals will not show this.

b) Three intervals made it look like the number of runners increased as times got longer.

c)

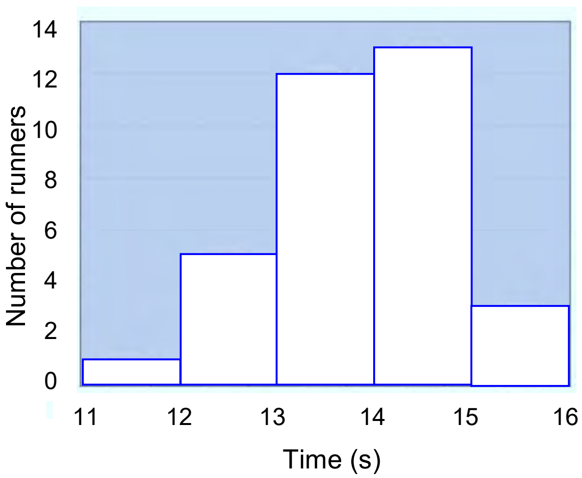


c) I labelled the vertical axis from 0 to 14 because the number of runners in each interval went from 1 to 13.

• I labelled the horizontal axis with the five intervals.

• I gave each axis a heading — I could have named the vertical axis "Frequency" instead of "Number of runners".

Running times for 100 m



• For each interval, I drew a bar that showed its frequency.

• I remembered to give my graph a title.

[Continued]

Example 1 Creating a Histogram [Continued]	
d) The histogram tells me that most of the runners ran 100 m in a time between 13 s and 15 s.	**d)** I could figure out how many ran the race in a time between 13 s and 15 s by adding up the frequencies of the two bars: 12 + 13 = 25.

Example 2 Choosing a Suitable Graph

What type of graph would you use to display each set of data?

a) the amount of rain each month (in millimetres) for 12 months

b) the change in Bhutan's population from 1960 to today

c) the percent of Bhutan's population in different age categories

d) the heights of everyone in your school

Solution	Thinking
a) Bar graph	**a)** I would use a bar graph so it would be easy to see which months had more or less rain. • I could use a circle graph but I would have to figure out the percent for each month. 
b) Line graph	**b)** A line graph will show how the population increased or decreased over time.
d) Circle graph	**d)** I would use a circle graph because they're good for data that is about how a whole (the population) is divided into parts (the age categories). • I could also use a bar graph, but each bar will show percent instead of the actual number.
c) Histogram	**c)** A histogram would be good because there will be a lot of data and I could group the data in intervals. Measurements like heights are continuous, so I need a graph that has no gaps between the intervals.

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Practising and Applying

1. Data values were collected about the number of crimes committed in Bhutan by people between 10 and 25 years of age. Create a histogram of the data.

Age group	Frequency, or Number of crimes
10 – 15	155
15 – 20	695
20 – 25	940

2. Data values were collected about the ages of whales in a pod of 50 whales.

Ages of Whales (years)

17	57	12	35	18
13	10	69	8	42
11	20	14	58	13
62	13	11	62	19
15	35	14	27	22
36	16	36	55	28
32	46	7	8	9
10	12	11	24	19
29	28	52	46	14
31	17	25	32	13

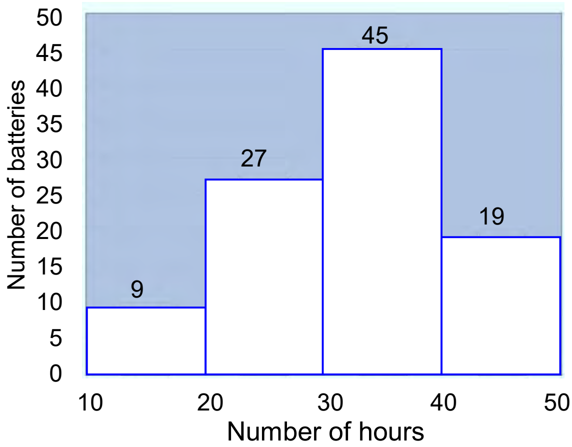
Beluga whale

a) Create a frequency table of the data. Use seven intervals. Include a tally column in your chart.

Age group	Tally	Frequency

b) Create a histogram of the data.

3. One hundred batteries were tested to see how long they lasted. The data values were graphed in this histogram.



a) How many batteries lasted each amount of time?

i) 10 h to 20 h **ii)** 20 h to 30 h

iii) 30 h to 40 h **iv)** 40 h to 50 h

b) How many lasted less than 30 h?

c) How many lasted 30 h or more?

4. A marathon is a foot race over 42 km. The world record time is about 2 h. Runners are sometimes given up to 7 h to finish. In one marathon, about 25,000 runners participated.

Suppose you want to create a histogram to show the number of runners that finished the race in different intervals of time. If you choose to use five intervals, what intervals would you use? Explain your thinking.

5. Why is a histogram better than a bar graph to show the data in **question 4**?

7.4.1 Mean, Median, Mode, and Range

Try This

The game scores in a regional basketball tournament are shown in the chart at right.

A. i) Calculate the mean, median, and mode for this set of data.

ii) What value would you call a typical score for this set of data? Explain your thinking.

Basketball Scores

132	108	117	129
99	108	114	94
127	124		

Recall the definitions for the three **measures of central tendency**: **mean**, **median**, and **mode**.

These values are used to describe a set of data.

• *The Mean*

- The mean is a number that tells what would happen if all the data values were spread evenly across the set of data.

- You can find the mean by adding all the data values and then dividing the total by the number of data values.

For example, for the set of data 1, 2, 2, 2, 3, 5, 6, 6, 6, 7, 8, 8, 9:

$\frac{1+2+2+2+3+5+6+6+6+7+8+8+9}{13} = \frac{65}{13}$ = 65 ÷ 13 = 5. The mean is 5.

• *The Median*

- The median is the middle data value when the data values are in order.

For example, for the set of data 1, 2, 2, 2, 3, 5, 6, 6, 6, 7, 8, 8, 9:

The median is 6.

1, 2, 2, 2, 3, 5, **6**, 6, 6, 7, 8, 8, 9

5 values below 6 5 values above 6

• *The Mode*

The mode is the data value that occurs most often in the set of data.

For example:

The set of data 1, 2, 2, 2, 3, 5, 6, 6, 6, 7, 8, 8, 9 has two modes, 2 and 6.

• The **range** is another value that can be used to describe a set of data.

The range is the difference between the greatest and least data values.

For example:

← Range is 9 – 1 = 8 →

For the set of data **1**, 2, 2, 2, 3, 5, 6, 6, 6, 7, 8, 8, **9**, the range is 8.

• Sometimes a change in a set of data affects all data values in the same way.

- When the same amount is subtracted or added to each data value, the mean, median, and mode each decrease or increase by that amount. The range stays the same.

For example:

A teacher calculates the mean, median, mode, and range of a set of test scores and then decides to add 5 points to everyone's score.

For the set of test scores 42, 50, 50, 56, 65, 70, and 80:

Mean = 59 Median = 56 Mode = 50 Range = 38

If 5 is added to each score, the scores become 47, 55, 55, 61, 70, 75, and 85:

Mean = 64 (59 **+ 5**) Median = 61 (56 **+ 5**) Mode = 55 (50 **+ 5**) Range = 38

- When each data value is multiplied or divided by the same amount, the mean, median, mode, and range are all multiplied or divided by that amount.

For example:

The usual number of calories that five people eat on a particular day is 1800, 1900, 2000, 2000, and 2300.

Mean = 2000 Median = 2000 Mode = 2000 Range = 500

The number they eat in two days is double those values: 3600, 3800, 4000, 4000, and 4600.

Mean = 4000 Median = 4000 Mode = 4000 Range = 1000

• If a data value is removed from or added to a set of data, you can sometimes predict whether the mean, median, mode, and range will increase or decrease.

For example, for the set of data 20, 25, 30, 30, 30, 42, 68:

Mean = 35 Median = 30 Mode = 30 Range = 48

If the data value 68 is removed: 20, 25, 30, 30, 30, 42, ~~68~~

- The mean decreases because a high value is removed (from 35 to 29.5).
- The median is not affected, but it could have changed if there had not been so many repeated values in the middle of the set of data (remains 30).
- The mode is not affected because 68 was not the mode (remains 30).
- The range decreases because the highest value is removed (from 48 to 22).

If a data value of 5 is added: **5**, 20, 25, 30, 30, 30, 42, 68

- The mean decreases because another low value is added (from 35 to 31.25).
- The median and mode are not affected this time (remain 30).
- The range increases because a value lower than all the others is added (from 48 to 63).

• If another data value is included in the middle of a set of data:

- The mean, median, and mode might be affected, but the effect is less than if an extra high or low data value were included.
- The range is not affected.

B. How would the mean, median, and mode in **part A** change if another game with a score of 80 were included in the set of data? Explain your thinking.

Examples

Example 1 Predicting Changes in the Mean, Median, Mode, and Range

The high temperatures each day for one week in Thimphu are shown below.

–3°C, –1°C, –2°C, –2°C, 0°C, +5°C, +3°C

a) Find the measures of central tendency and the range for the set of data.

b) Suppose the highest value were +7°C instead of +5°C. Predict how the mean, median, mode, and range would change. Explain your thinking and then check your predictions.

Solution

a) –3°C, –2°C, –2°C, –1°C, 0°C, +3°C, +5°C

- The mean is 0°C:

  (–3 + (–2) + (–2) + (–1) + 0 + 3 + 5) ÷ 7

  = 0 ÷ 7 = 0

- The median is –1°C:

  –3°C, –2°C, –2°C, <u>–1°C</u>, 0°C, +3°C, +5°C

- The mode is –2°C:

  –3°C, <u>–2°C, –2°C</u>, –1°C, 0°C, +3°C, +5°C

- The range is 8°C:

  (+5) – (–3) = 8

b) –3°C, –2°C, –2°C, –1°C, 0°C, +3°C, **+7°C**

Predict: The mean will increase because the sum of the data values will increase but the number of data values will not change.

Check:

The mean increases from 0 to $\frac{2}{7}$: 0 + 2) ÷ 7 = $\frac{2}{7}$.

Predict: The median and mode will not change because the middle value is the same and +5°C is not the mode.

Check:

The median does not change.

The mode does not change.

Predict: The range will increase by 2 because the maximum value increased by 2.

Check:

The range increases from 8 to 10: 8 + 2 = 10.

Thinking

- When the values are in order from least to greatest, it's easier to find the median, mode, and range.
- I recalculated the mean by adding 2 to the original total of 0.
- I didn't recalculate the median or mode, as I could see that changing the highest value didn't affect them.
- I recalculated the range by adding 2 to the original range of 8.

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Example 2 Calculating Data Measures Efficiently

A teacher wants to know how a group of 10 students are doing in math.
Their marks on a recent quiz are 53, 54, 55, 55, 55, 57, 58, 59, 59, 60.
What are the mean, median, mode, and range?

Solution

53, 54, 55, 55, 55, 57, 58, 59, 59, 60

Subtract 50 from each value

3, 4, 5, 5, 5, 7, 8, 9, 9, 10

The mean is 6.5. (65 ÷ 10 = 6.5)

The median is 6. (between 5 and 7)

The mode is 5.

The range is 7. (10 – 3 = 7)

Add 50 back to the mean, median, and mode

The mean is 50 + 6.5 = 56.5.

The median is 50 + 6 = 56.

The mode is 50 + 5 = 55.

The range stays the same, 7.

Thinking

- I subtracted 50 from each data value to make the calculations easier.
- I knew the mean, median, and mode would also decrease by 50, so I planned to add the 50 back at the end.
- I knew the range wouldn't be affected by subtracting 50.

Practising and Applying

1. Will the mean, median, mode, and range in each situation below increase, decrease, or stay the same?

Use this set of data:

189, 112, 321, 207, 308, 189

a) 20 is added to each value.

b) 100 is subtracted from each value.

c) Each value is multiplied by 3.

d) Each value is divided by 5.

e) The value 112 is removed.

f) The value 602 is added.

2. These are the ages of the members of a football team:

11, 11, 12, 13, 12, 12, 11, 12, 12, 12

a) Find the measures of central tendency and the range.

b) Will each measure increase, decrease, or stay the same if the 13-year-old is replaced by an 11-year-old?

3. What would be an efficient way to calculate the mean of this set of data?

110, 117, 118, 120, 113, 115

4. Create a set of 10 data values for each.

a) The mean increases when 100 is added to the set.

b) The mean decreases when 100 is added to the set.

c) The mean is 100 when each data value is doubled.

d) The mode is 100 when each data value is divided in half.

5. Create a set of data where the median changes when an additional value is included.

6. The long jump distances, in metres, for a group of athletes are shown here:

3.8, 3.9, 4.0, 4.5, 4.7, 4.9

Describe one or more ways to calculate the mean distance efficiently.

7. Describe a set of data for which you would change the data in order to calculate the mean. Explain your thinking.

7.4.2 Outliers and Measures of Central Tendency

Try This

A. For this set of data, determine the mean, median, and mode.

3, 20, 20, 22, 26, 28, 41, 41, 42, 44

The three measures of central tendency, mean, median, and mode, are often used to make predictions and decisions. It is important to use the measure that best describes the data and suits the situation.

For example:

- If the mean running time for a race is 14 s, you might predict that most runners will finish the next race in about 14 s.

- If the mode shoe size sold in a store is size 7, the store owner would predict that this size will continue to be popular and would decide to buy more of that size.

• When you decide which measure of central tendency best describes a set of data, it is important to consider **outliers**. An outlier is a data value that is much lower or much higher than most of the other data values in the set. Outliers can be valid data values, but they can also be caused by errors in data collection.

For example, this set of data shows the ages of 50 people in a school:

6	6	6	6	7	7	7	8	8	8	9	9	9	
9	9	9	9	9	9	9	9	9	10	10	10	10	
10	10	10	11	11	11	11	11	11	11	11	11	11	
11	12	12	12	13	13	13	13	**39**	**50**	**55**			← Outliers

The last three data values are outliers because they are much greater than the other values. They are probably the ages of three teachers.

• Sometimes you can ignore outliers when you calculate measures of central tendency. This usually has a greater effect on the mean than on the median. It likely will not affect the mode.

For example:

In the set of data above, the median age is 10, the mode is 9, and the mean is 12. If you remove the outliers, the median is 10, the mode is 9, but the mean is 9.7.

- If you want to describe the set of data to include teachers' ages, the mean of 12 is probably the best measure because it shows there are some high values.

- If you think it is wrong to include the teachers' ages because the data set is supposed to be about students, the mean of 9.7 might best describe the data because it is both close to and between the median and the mode.

• You should consider each outlier carefully before removing it because it might be important.

• Some other terms that are used to describe sets of data are **clusters**, which are large amounts of data that fall within a narrow range, and **gaps**.

For example, in the set of data below, there are two clusters with a gap in between them. There is another gap between the value 17 and the outlier 42.

Cluster (1 1 3 3 4 5 6) — Gap — Cluster (13 13 13 15 15 17) — Gap — Outlier (42)

1 1 3 3 4 5 6 13 13 13 15 15 17 42

B. Identify gaps, clusters, and outliers in the set of data in **part A**.

C. i) Predict the mean, median, and mode if the outlier is removed.

ii) Determine each measure of central tendency to check your predictions in **part i)**. What was the effect on each when you removed the outlier?

D. Which measure of central tendency best describes the set of data? Consider the values with and without the outlier.

Examples

Example 1 Describing Data to Make Predictions

The goals scored by one team in ten football matches were 0, 2, 3, 2, 3, 3, 2, 15, 1, and 3. Which measure of central tendency would you use to predict the team's goals in the next game?

Solution 1	Thinking
0, 1, 2, 2, 2, 3, 3, 3, 3, 15 The mean is 3.4. The median is 2.5. The mode is 3. I would choose the mode to predict the goals in the next game.	 • I first ordered the data. • The mean was high because of the outlier, so I wouldn't use the mean. • The median was reasonable but it was a decimal and scores are whole numbers. • The mode was close to the median and it was a whole number so I chose the mode.

Solution 2	Thinking
0, 1, 2, 2, 2, 3, 3, 3, 3, 15 The mean is 3.4. The median is 2.5. The mode is 3. 0, 1, 2, 2, 2, 3, 3, 3, 3 The mean is 2.1. The median is 2. The mode is 3. I would choose the median (without the outlier) to predict the goals in the next game.	 • I first ordered the data. • I decided to remove the outlier (15) because I know that it is very unusual to score 15 goals in a game. • Without the outlier, all three measures of central tendency described the set of data reasonably well. • I chose the median because it's close to the mean. It's also a whole number, and the number of goals is always a whole number.

Example 2 Analysing a Set of Data to Describe It

This chart shows the average monthly rainfall in millimetres in Trashigang.

a) Describe the set of data in terms of clusters, gaps, and outliers.

b) Would you use the mean or the median to describe this set of data?

Jan	Feb	Mar	Apr	May	Jun
0.6	8.0	28.0	53.7	62.3	135.0
Jul	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
163.2	120.2	94.0	36.0	8.0	0.3

a) Solution

0.3	0.6	8.0	8.0
28.0	36.0	53.7	62.3
94.0	120.2	135.0	163.2

The data set is in three clusters — the months with light rainfall, the months with moderate rainfall, and the months with heavy rainfall.

There are two gaps between the clusters and another gap between 135.0 and 163.2, which is an outlier.

Thinking

• I ordered the data values so I could describe them and find the measures of central tendency.

b) Mean: 59.1 mm

Median: 44.9 mm

Without the outlier

Mean: 49.6 mm

Median: 36.0 mm

0.3	0.6	8.0	8.0
28.0	36.0	53.7	62.3
94.0	120.2	135.0	163.2

Median: 44.9 mm Mean: 59.1 mm

I would use the median of 44.9 mm to describe the set of data.

b) The mean was much higher than the median, so I decided to figure out what they would be without the outlier.

• Even without the outlier, the mean is still much higher than the median.

• I looked at the data again, along with the mean and median that were calculated with the outliers included.

• I decided that the reason the mean was so high compared to the median was that there are many high values.

• I figured the median described the data better than the mean because it shows that there are low values too.

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Example 3 The Effect of Outliers on Measures of Central Tendency

What happens to the mean, median, and mode if you remove the outliers?

a) 3, 19, 20, 21, 22, 24, 25, 25, 26, 27, 30

b) 7, 8, 8, 8, 10, 11, 15, 16, 16, 41

Solution

a) *Removing 3*:

The mode is still 25.

The median increases from 24 to 24.5.

The mean increases from 22 to 23.9.

b) *Removing 41*:

The mode is still 8.

The median decreases from 10.5 to 10.

The mean decreases from 14 to 11.

Thinking

- Removing a low outlier increases the mean and the median.
- Removing a high outlier decreases the mean and, this time, decreases the median.
- Removing an outlier affects the mean more than it affects the median.

Practising and Applying

1. The film called *The Cup* was shown in the United States for 13 weeks in 2000. This chart shows the ticket sales each weekend in thousands of US dollars.

Date	Sales $US (000s)
Jan 30	35
Feb 6	9
Feb 13	77
Feb 20	104
Feb 27	107
Mar 5	97
Mar 12	96
Mar 19	71
Mar 26	52
Apr 2	35
Apr 9	35
Apr 16	31
Apr 23	26

a) Describe the data in terms of gaps, clusters, and outliers.

b) Which measure of central tendency best describes the data? Consider including and not including outliers. Explain your thinking.

2. Why does the mode rarely change when outliers are removed from a set of data?

3. Which measure of central tendency best describes each set of data without removing the outliers? Explain your thinking.

a) 0, 1, 2, 3, 4, 6, 6, 6, 6, 36

b) 5, 35, 35, 40, 45, 45, 45, 55, 55

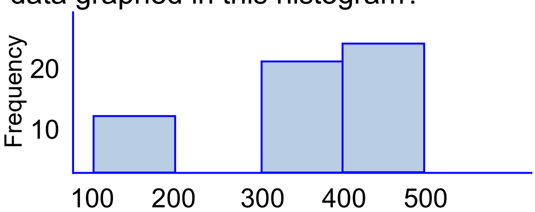
4. a) Change one data value so that only the mean will change.

5, 10, 10, 10, 15, 20, 25, 30, 45, 50

b) Change two data values so that no measures of central tendency change.

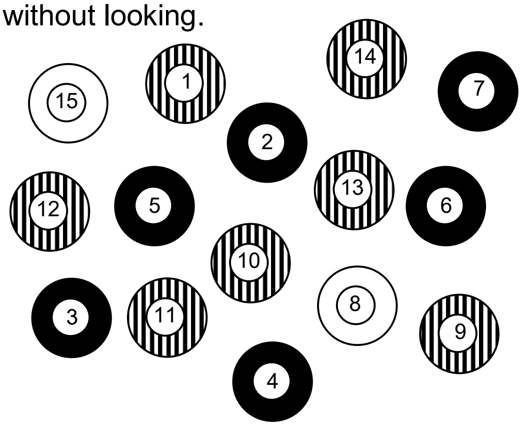
5, 10, 10, 10, 15, 20, 25, 30, 45, 50

5. Where are the clusters and gaps in the data graphed in this histogram?



6. When might it be a good idea to remove an outlier from a set of data? When might it be better to leave it in?

1. Lhakpa chose one of these balls without looking.



What is the probability of choosing each?

a) 8

b) an odd number

c) an even number

d) a striped ball

e) a black ball

f) a number less than 20

2. Name two events in **question 1** that are equally likely.

3. Describe an event that fits each probability of choosing a ball from **question 1**.

a) very likely

b) likely

c) unlikely

d) very unlikely

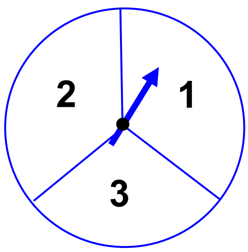
e) impossible

4. You toss two Nu 1 coins. What is the probability that you will get two Tashi Ta-gyes? Show your work.

5. You spin this spinner twice. Describe an event with each probability.

a) $\frac{1}{9}$

b) $\frac{1}{3}$



6. a) Explain why each survey question below is a poor question.

i) Do you agree that archery is the world's greatest sport?

ii) What is your favourite meat?

iii) Would you lie about an answer to a survey question?

b) Choose one of the questions and make it a better question.

7. Choose a method: observation, interviews, or questionnaire, to collect data about each. Explain your choice.

a) the most popular musician in Bhutan

b) the breed of chicken that produces the largest eggs

c) the favourite song of the students in your class

8. Are the data sets described below first-hand or second-hand data? Explain your thinking.

a) A restaurant tests a new recipe on customers and asks for their opinion on a scale of 1 to 5.

b) A tourist looks on the Internet for weather predictions before taking a trip to Bhutan.

c) A researcher reads in a science book to find the average mass of a takin.

9. Why might each survey be biased?

a) Dechen asked 20 people in Thimphu about their favourite sport to find out about popular sports in Thimphu.

b) Eden called everyone in Zhemgang with a telephone to determine the average income in the district.

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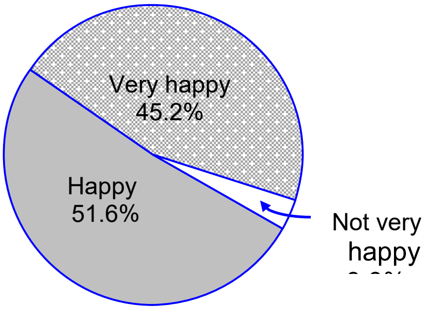
10. What does the graph below tell you about happiness in Bhutan?

Happiness in Bhutan



11. This chart shows data about the number of each type of ruminant in Bhutan. (A ruminant is an animal with a special stomach for digesting its food.)

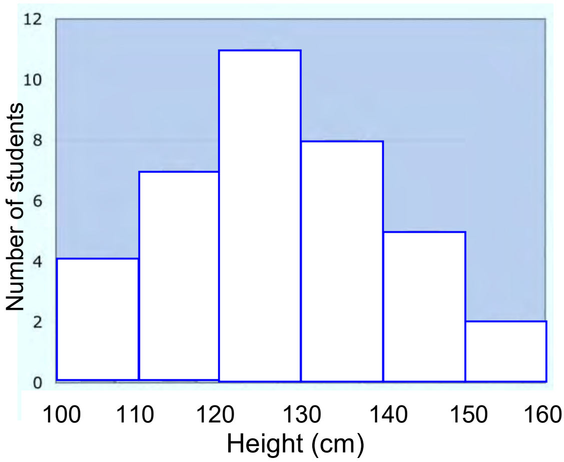
Ruminant Livestock Population

Animal	Number (thousands)
Cattle	305
Yaks	30
Sheep	31
Goats	16

Estimate a percent for each type of ruminant and then draw a circle graph.

12. A class made a histogram of the heights of all its students.

Student Heights



a) What does the histogram tell you about the data?

b) How many students are 120 cm or taller?

c) How many students are shorter than 140 cm?

13. The times for 24 swimmers in a 1000 m event ranged from 13 min to 25 min. What intervals would you use to create a histogram, if you wanted seven intervals? Explain your answer.

14. Will the mean, median, mode, and range increase, decrease, or stay the same in each situation below?

Use this set of data:

18, 19, 19, 12, 17, 16, 18, 18, 12, 16

a) Each value is divided by 10.

b) 10 is subtracted from each value.

c) The value 18 is removed.

d) The value 7 is added.

15. Create a set of data for each.

a) The mean decreases when the data value 12 is removed.

b) The mean is 9 when each data value is divided by 2.

c) The mode changes when the data value 10 is removed.

16. These are math test scores.

37, 60, 66, 59, 81, 83, 90, 63, 64, 69,
67, 58, 80, 88, 60, 58, 91, 82, 60, 84

a) Describe the data in terms of gaps, clusters, and outliers.

b) Which measure of central tendency best represents the data? Explain your choice.

17. How does removing the outlier from this set of data affect each measure of central tendency and the range?

25, 30, 30, 45, 55, 55, 110

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Instructional Terms

calculate: Figure out the number that answers a question; compute

classify: Put things into groups according to a rule and name the groups; e.g., classify triangles as right, acute, or obtuse by the size of their angles

compare: Look at two or more objects or numbers and identify how they are the same and how they are different; e.g., compare the numbers 6.5 and 5.6; compare the size of the students' feet; compare two shapes

conclude: Judge or make a decision after looking at all the data

construct: Draw using only a compass and straight edge

create: Make your own example or problem

describe: Tell, draw, or write about what something is or what something looks like; tell about a process in a step-by-step way

determine: Decide what the answer or result is for a calculation, a problem, or an experiment

draw: **1.** Show something using a picture **2.** Take out an object without looking; e.g., draw a card from a deck

estimate: Use what you know to make a sensible decision about an amount; e.g., estimate how long it takes to walk from your home to school; estimate how many leaves are on a tree; estimate the sum of 3210 + 789

evaluate: **1.** Determine whether something makes sense; judge **2.** Calculate the value as a number; e.g., evaluate the expression $m + 3$ for $m = 5$

explain (your thinking): Tell what you did and why you did it; write about what you were thinking; show how you know you are right

explore: Investigate a problem by questioning and trying new ideas

justify: Give convincing reasons for a prediction, an estimate, or a solution; tell why you think your answer is correct

measure: Use a tool to tell how much; e.g., use a ruler to measure a height or distance; use a protractor to measure an angle; use balance scales to measure mass; use a measuring cup to measure capacity; use a stopwatch to measure elapsed time

model: Show an idea using objects, pictures, words, and/or numbers; e.g., model integers using black and white counters:

Modelling integers with counters

predict: Use what you know to figure out what is likely to happen; e.g., predict the number of times you will roll a number greater than 2 when you roll a die 30 times

relate: Describe how two or more objects, drawings, ideas, or numbers are similar

represent: Show information or an idea in a different way; e.g., draw a graph of an equation; make a model from a word description; create an expression to model a situation

show (your work): Record all the calculations, drawings, numbers, words, or symbols that you used to calculate an answer or to solve a problem

simplify: Write a number or expression in a simpler form; e.g., combine like terms of a polynomial, write an equivalent fraction with a lower numerator and denominator

sketch: Make a quick drawing to show your work; e.g., sketch a picture of a field with given dimensions

solve: **1.** Find an answer to a problem **2.** Find the value of a variable in an equation; e.g., solve $3 + x = 7$ by finding the value of x that makes it true, which is 4

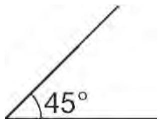
sort: Separate a set of objects, drawings, ideas, or numbers into groups according to an attribute; e.g., sort 2-D shapes by the number of sides

visualize: Form a picture in your head of what something is like; e.g., visualize the number 6 as 2 rows of 3 dots like you would see on a die

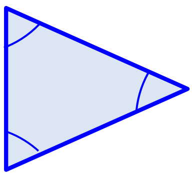
Definitions of Mathematical Terms

A

acute angle: An angle less than 90º; e.g.,



acute triangle: A triangle in which all angles are acute angles; e.g.,



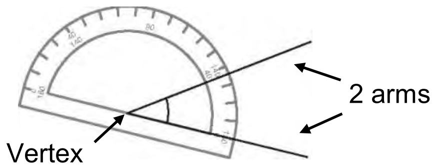
algebraic equation: An equation that includes algebraic expressions and an equals sign; e.g., $3x + 5 = 8$

algebraic expression: A combination of one or more terms with at least one variable; it may include numbers and operation signs; e.g., $8x + 9$

algorithm: A specific set of instructions or a procedure for finding the solution to a problem or the answer to a calculation; e.g., here is one of several possible algorithms for dividing 620 by 7:

```
     88.571
 7 ) 620.000
   – 56
      60
    – 56
       40
     – 35
        50
      – 49
         10
       –  7
          3
```

angle: A figure formed by two arms with a shared endpoint, or vertex; the measure of an angle is the amount of turn between the two arms; angles are often measured in degrees (º)



angle bisector: A line through the vertex of an angle that separates the angle into two equal parts

angle of rotation: The angle through which a shape has moved after a rotation See *rotation*

anticlockwise: See *counter clockwise*

area: The number of square units needed to cover a shape; common units are square centimetres or square metres

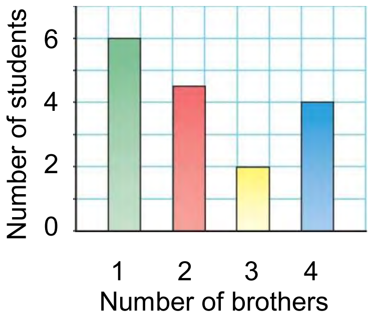
average: Average is a term we can use instead of the term mean See *mean*

average rate: A rate expressed as a unit rate; an average rate assumes that the rate stayed the same over the entire time period; e.g., if someone walks 15 km in
5 h, the average rate would be 3 km in 1 h or 3 km/h

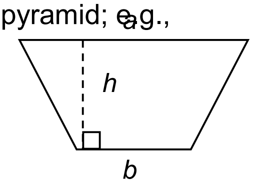
B

bar graph: A graph that compares the sizes of bars that each represent the number associated with a category in a set of data; e.g.,

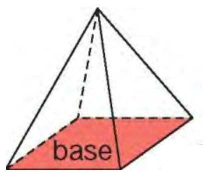
Number of Brothers



base: 1. The number that is repeatedly multiplied in a power; e.g., in the power 5^3, 5 is the base **2.** In a 2-D shape, the line segment(s) that is perpendicular to the height **3.** In a 3-D shape, the face(s) that determines the name of a prism or pyramid; e.g.,



A trapezoid has two bases, a and b



A square-based pyramid

bias: When the results of data collection are affected or influenced, often as
a result of a poorly-chosen sample; e.g., if a survey about colour preferences of school-age children involves only girls, the results will be biased

bisect: Divide in half; e.g., an angle bisector divides an angle in half; if line segment AB passes through the midpoint of line segment CD, AB bisects CD

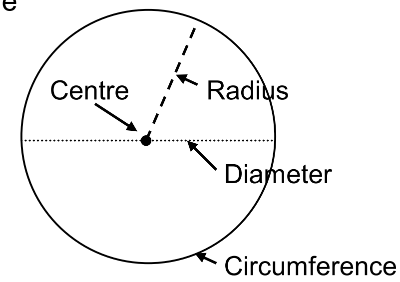
bisector: See *bisect*

C

capacity: The amount that a container can hold, often measured in millilitres (mL) or litres (L)

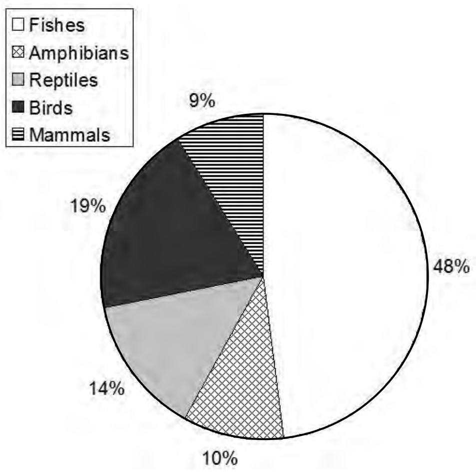
census: A survey of an entire population; e.g., a government conducts a census of its people to collect information for making decisions about creating laws and spending tax money

circle: A 2-D shape made up of a set of points that are the same distance, called the radius (r), from a point called the centre



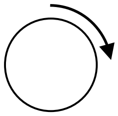
circle graph: A graph that shows how a complete set of data is broken into categories, each represented by a section of a circle; e.g.,

Known Vertebrate Species

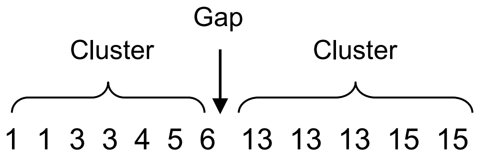


circumference: 1. The boundary of a circle **2.** the length of the boundary of a circle calculated using the formula $C = 2 \times \pi \times r$, where r is the radius, or $C = \pi \times d$, where d is the diameter
See *circle*

clockwise (cw): The direction that the hands of a clock move; describes the direction of a rotation



cluster: A large amount of data that falls within a small range; e.g., in the set of data below, there are two clusters with a gap in between them



coefficient: The number by which a variable is multiplied; e.g., in the term 3*z* the coefficient is 3

common denominator: A common multiple of the denominators of two or more fractions; e.g., you can use the common multiple of 6 for 2 and 3 to create fractions with a common denominator and then add them:

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

common factor: A number that divides into two or more other numbers with no remainder; e.g., 4 is a common factor of 8 and 12 because 8 ÷ 4 = 2 and 12 ÷ 4 = 3.

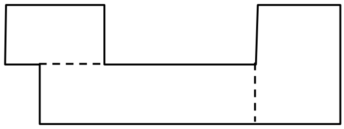
common multiple: A number that is a multiple of two or more given numbers; e.g., 12, 24, and 36 are common multiples of 4 and 6

common numerator: A common multiple of the numerators of two or more fractions; e.g., you can use the common multiple of 12 for 3 and 4 to create fractions with a common numerator and then compare them:

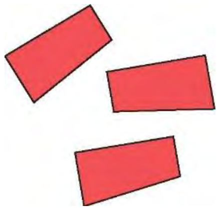
$$\frac{3}{4} = \frac{12}{16} \text{ and } \frac{4}{7} = \frac{12}{21}$$

$$\text{so } \frac{12}{16} > \frac{12}{21} \text{ and } \frac{3}{4} > \frac{4}{7}$$

composite shape: A shape that is made up of several simple shapes; e.g., this composite shape is a polygon that can
be divided into three rectangles

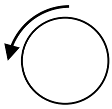


congruent: Identical in size and shape; shapes, side lengths, and angles can be congruent; e.g., these three shapes are congruent

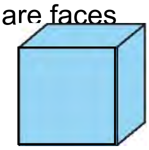


constant: In an algebraic expression or equation, a value that does not change; e.g., in the equation $y = 3x + 7$, the term 7 is the constant

counter clockwise (ccw): The direction opposite to the direction the hands of a clock move; sometimes called anticlockwise; describes the direction of a rotation



cube: A 3-D shape that has six congruent square faces



cubic centimetre (cm^3): A standard unit of measure for volume that is equivalent to the amount of space taken up by a cube that is 1 cm along each edge

cubic metre (m^3): A standard unit of measure for volume that is equivalent to the amount of space taken up by a cube that is 1 m along each edge

cuboid: Another name for a rectangular prism See *rectangular prism*

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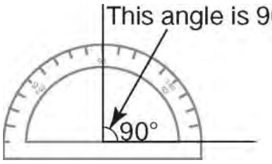
D

data: Information collected in a survey, in an experiment, or by observing; the word data is plural, not singular; e.g., a set of data can be a list of students' names or it can be the numerical scores of a set of quiz marks

degree: A unit of measure for angle size; e.g.,



denominator: The number in a fraction that represents the total number of parts in a whole set or the number of parts the whole has been divided into; e.g., in $\frac{4}{5}$, the denominator is 5

diameter: **1.** A line segment that joins two points on a circle and passes through the centre **2.** The length of the line segment described in **1** See *circle*

difference: The result of a subtraction; e.g., in 45 – 5 = 40, the difference is 40

dimension: The size or measure of an object, usually length; e.g., the width and length of a rectangle are its dimensions

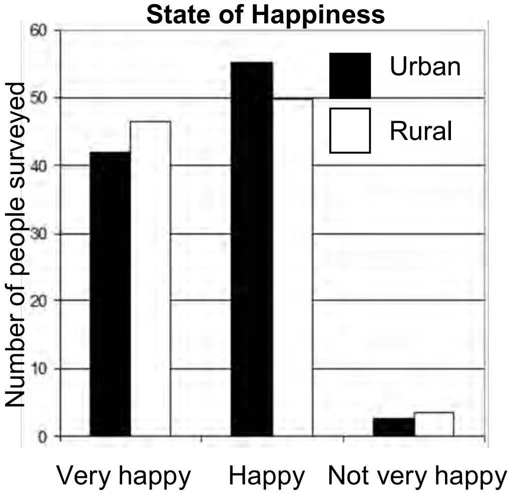
dividend: A number that is being divided; e.g., in 45 ÷ 5 = 9, the dividend is 45

divisible: When a number can be divided by second number without a remainder, the first number is divisible by the second number; e.g., 24 is divisible by 8 because 24 ÷ 8 = 3 and the remainder is 0

divisibility rule or test: A way to determine whether one number is a factor or multiple of another number without actually dividing; e.g., a number is divisible by 3 if the sum of the digits is divisible by 3, so 81 is divisible by 3 because 8 + 1 = 9 and 9 ÷ 3 = 3

divisor: The number by which another number is divided; e.g., in 45 ÷ 5 = 9, the divisor is 5

double bar graph: A special bar graph that shows two sets of data using the same categories; e.g.,

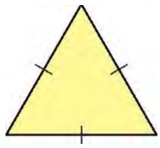


E

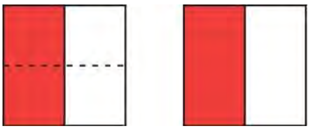
endpoint: The point where a line segment begins or ends

equation: A mathematical statement in which the value on the left side of the equals sign is the same as the value on the right side of the equals sign; e.g., the equation $5n + 4 = 39$ means that 4 more than the product of 5 and a number equals 39

equilateral triangle: A triangle with three sides of equal length (and with all angles equal and 60º)



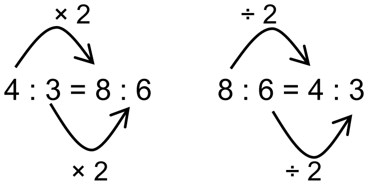
equivalent fractions: Fractions that represent the same part of a whole or set; e.g., $\frac{2}{4}$ is equivalent to $\frac{1}{2}$



equivalent decimal: A decimal that represents the same part of a whole or set; e.g., 0.5 is equivalent to 0.50; 0.5 is also equivalent to $\frac{1}{2}$

equivalent rates: Rates that describe the same relationship; you can find an equivalent rate by multiplying or dividing each term by the same number; e.g., a rate of 26 km in 2 days is equivalent to a rate of 52 km in 4 days or 13 km in 1 day

equivalent ratios: Ratios that makes the same comparison; you can find an equivalent ratio by multiplying or dividing each term by the same number; e.g., 4 : 3 and 8 : 6 are equivalent ratios



event: A set of outcomes for a probability experiment; e.g., if you roll a die with
the numbers 1 to 6, the event of rolling an even number has the outcomes 2, 4, or 6

expanded form: A way of writing a number that shows the place value of each digit; e.g., 1209 in expanded form is 1 × 1000 + 2 × 100 + 9 × 1 or 1 thousand + 2 hundreds + 9 ones

experimental probability: The probability of an event based on the results of an experiment with many trials; it is calculated using this expression:

$$\frac{\text{Number of favourable results}}{\text{Number of trials}}$$

exponent: A superscript in mathematics that denotes repeated multiplication; sometimes referred to as a power or an index; e.g., 4^3 means 4 × 4 × 4 because the exponent is 3

exponential form: A way of writing a number that shows the value of each digit as a power of 10; e.g., in exponential form 1209 is $1 \times 10^3 + 2 \times 10^2 + 9 \times 1$

expression: See *algebraic expression*

F

face: A 2-D shape that forms a flat surface of a 3-D shape; e.g.,

factor: **1.** One of the numbers you multiply in a multiplication operation; e.g., 3 and 4 are the factors in 3 × 4 = 12 **2.** A number that divides into another number with no remainder; e.g., the factors of 24 are 1, 2, 3, 4, 6, 8, and 12

favourable outcome: The desired outcome when you calculate a theoretical probability; e.g., when you find the theoretical probability of rolling a number less than 3 on a die, rolls of 1 and 2 are the favourable outcomes

favourable result: The desired result when you calculate an experimental probability; e.g., when you find the experimental probability of rolling an even number on a die, rolls of 2, 4, and 6 are the favourable results

first-hand data: Data values that are collected directly using surveys, interviews, or observations

formula: A general rule stated in mathematical language; e.g., the formula for the area of a rectangle is Area = length × width, or $A = l \times w$

fraction: A number that is written in the form of a numerator and a denominator; e.g., $\frac{4}{5}$ and $\frac{13}{5}$ are fractions

frequency: The number of times a data value or range of data values occurs in a data set; e.g., in the frequency table below, ages between 0 and 11 happened 50 times so the frequency is 50

frequency table: A table that organizes a set of data into intervals and indicates the number of times data values occur in each interval

Age	Frequency
0 – 11	50
11 – 22	300
22 – 33	250
33 – 44	400
44 – 55	550

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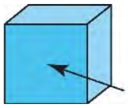


A square face of a cube

G

gap: See *cluster*

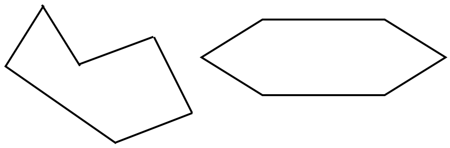
graph: A picture of a set of data or a mathematical relationship between two sets of data; e.g., when you plot the ordered pairs in a table of values, you create a graph of the relationship between the two sets of values in the table

greatest common factor (GCF): The greatest whole number that divides into two or more other whole numbers with no remainder; e.g., 4 is the greatest common factor of 8 and 12

H

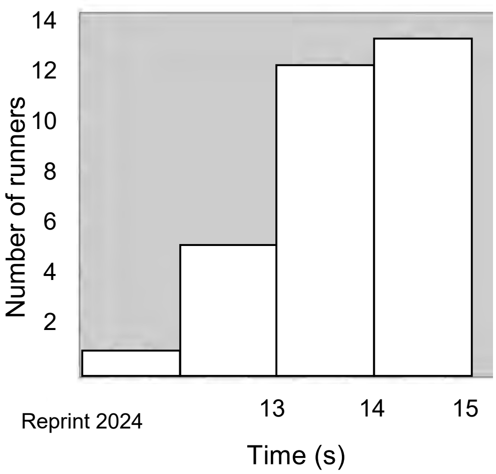
hectare (ha): A standard unit of measure for area; 1 ha is equivalent to the area of a square with a side length of 100 m

hexagon: A six-sided polygon; e.g.,



histogram: A graph with vertical or horizontal bars that show frequencies of data organized into intervals. The bars line up side by side without gaps on the scale because there are no gaps between the intervals of data; e.g., the histogram below shows the number of runners that ran 100 m in different amounts of time

Running times for 100 m



horizontal: A left-right or across direction as opposed to a vertical (up-down) or diagonal direction; e.g., a horizontal line segment

I

image: The new shape that you create when you apply a transformation to a shape; e.g., after a reflection, the new shape is called the refection image

improper fraction: A fraction in which the numerator is greater than or equal to the denominator; e.g., $\frac{5}{4}$ and $\frac{6}{6}$

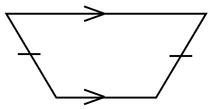
integers: The set of whole numbers and their opposites (zero is its own opposite): …, –2, –1, 0, 1, 2, …

interval: A range of values, often used in creating a histogram; e.g., 0–10 is the interval from 0 to 10 See *frequency table* and *histogram*

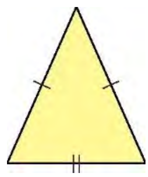
interview: A method for collected first-hand data by asking people questions e.g., Class III students interviewed Class VIII students about their favourite sports

inverse operation: An operation you use to undo another operation, often used in solving an equation; e.g., addition is the inverse of subtraction

isosceles trapezoid: A trapezoid with exactly two congruent sides; e.g.,



isosceles triangle: A triangle with two congruent sides



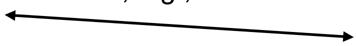
L

lateral face: The surface of a prism or pyramid that is not a base

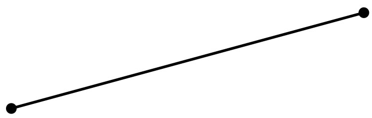
least common multiple (LCM): The lowest whole number that has two or more given numbers as factors; e.g., 12 is the LCM of 4 and 6

like terms: Terms of an expression that have the same variables raised to the same power but may have different coefficients; e.g., in $2x + 6x + 5$, the like terms are $2x$ and $6x$

line: A set of points that form a straight path that goes on forever in both directions; e.g.,



line segment: A part of a line; it consists of two end points and all the points in between; e.g.,



lowest common multiple: The least multiple that is common to two or more numbers; e.g., the lowest common multiple of 2 and 3 is 6:

Multiples of 2: 2, 4, **6**, 8, 10, 12, ...

Multiples of 3: 3, **6**, 9, 12, 15, ...

lowest terms: 1. When a fraction is in lowest terms, the only common factor of the numerator and the denominator is 1; e.g., $\frac{5}{10}$ is $\frac{1}{2}$ in lowest terms

2. When a ratio is in lowest terms, the only common factor of the terms is 1;
e.g., 12 : 9 in lowest terms is 4 : 3

M

mean: A single number that represents all the values in a data set; to calculate the mean, you add the values together and then divide the total by the number of values in the set; it is often called the average; e.g., the mean of 3, 4, 5, 6 is $(3 + 4 + 5 + 6) \div 2 = 4.5$

measure of central tendency: A value (usually a single value) that can be used to represent all of the values in a set of data See *mean*, *median*, and *mode*

median: The middle value of a set of data arranged in order. If there is an even number of values in the set, the median is the mean of the two middle values; e.g., in the data set below, the median is 10:

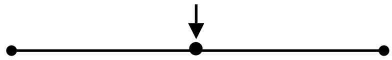
1 7 9 11 11 13

The median is the mean of 9 and 11.

metre (m): A unit of measurement for length; e.g., 1 m is about the distance from a doorknob to the floor; 1000 mm = 1 m; 100 cm = 1 m; 1000 m = 1 km

metric system/prefixes: A standard system of units and prefixes for measuring and reporting length, area, mass, volume, capacity, and so on, where each unit is made up of ten of the next smallest unit See *Measurement Reference* on **page 271**

midpoint: The point that divides a line segment into two equal parts; e.g.,



mixed number: A number made up of a whole number and a proper fraction; e.g., $5\frac{1}{7}$

mode: The piece(s) of data that occurs most often in a set of data; there can be more than one mode or there might be no mode; e.g., in the data set below, the modes are 12 and 14:

4 7 9 12 12 13 14 14 16

multiple: The product of a whole number and any other whole number; e.g., when you multiply 10 by the whole numbers
0, 1, 2, 3, and 4, you get the multiples 0, 10, 20, 30, and 40

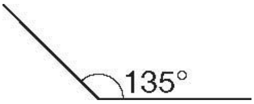
N

numerator: The number in a fraction that shows the number of parts of a given size the fraction represents; e.g., in $\frac{4}{5}$, the numerator is 4

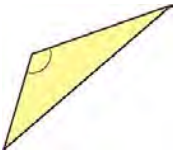
O

observation: A method for collecting data directly by watching and recording

obtuse angle: An angle greater than 90º and less than 180º; e.g.,



obtuse triangle: A triangle in which one of the angles is an obtuse angle; e.g.,



ones period: The cluster of three digits in a whole number that contains the hundreds digit, the tens digit, and the ones digit; e.g., in the number 123,456, the digits 456 make up the ones period

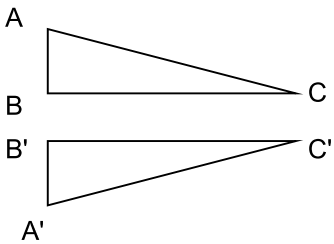
opposite integers: Two integers that are the same distance away from zero in opposite directions; e.g., 6 and –6 are opposite integers

order of operations (rules): Rules that describe the sequence to use to evaluate an expression:

1 Evaluate within brackets
2 Calculate exponents
3 Divide and multiply from left to right
4 Add and subtract from left to right

ordered pair: A pair of numbers in a particular order that describe the location of a point in a coordinate grid; e.g., the ordered pairs (3, 5) and (5, 3) describe the locations of two different points on a grid See *x-axis*

orientation: The direction around a shape when you name the vertices in order, clockwise or counterclockwise; e.g., the orientation of the vertices of ΔABC is counterclockwise, but the orientation of the vertices of its reflection image, ΔA'B'C', is clockwise



origin: The intersection of the axes in a coordinate grid, represented by the ordered pair (0, 0)

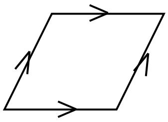
outliers: Data values that are much lower or much higher than the other data values in a set; e.g., the values 3, 23, and 24 appear to be outliers in this set of data:

3 11 11 13 13 13 15 15 23 24

P

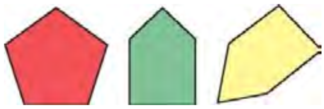
parallel lines or line segments: Lines or line segments that never meet, so they are always the same distance apart; e.g.,

parallelogram: A quadrilateral with pairs of opposite sides that are parallel; e.g.,



pattern rule: An algebraic expression that you can use to figure out the value of a term if you know the term number, or vice versa; e.g., for the pattern 4, 7, 10, 13, 16, .., the pattern rule is $3f + 1$, where f is the term number and the value of the expression is the term value

pentagon: A polygon with five sides; a regular pentagon has five congruent sides and five congruent angles; e.g.,



These are all pentagons. The first shape is a regular pentagon

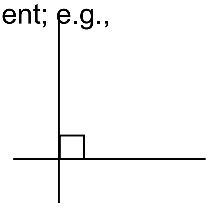
percent: A special ratio that compares a number to 100 using the symbol %; e.g., if 3 out of 4 students are girls, then 75% are girls because $\frac{3}{4} = \frac{75}{100} = 75\%$

perimeter: 1. The boundary or outline of a 2-D shape **2.** The length of the boundary

period: A group of three digits in a number, often separated by a comma or a space; e.g., in the number 458,675, the thousands period is 458 and the ones period is 675

perpendicular: At a right angle

perpendicular (line segment): A line segment that is at a right angle to another line segment; e.g.,



***π* (pi):** The result of dividing the circumference of any circle by its diameter; it has a value of 3.141592654 …, or about 3.14 or $\frac{22}{7}$

place value: The value of a digit depends on its place in the number; e.g., in the number 123.4, the digit 3 has a value of 3 because it is in the ones place, the digit 2 has a value of 20 because it is in the tens place, and so on

plot (a point): Locate a point on a coordinate grid using its coordinates

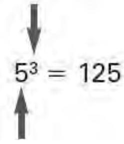
polygon: A closed 2-D shape with three or more sides; e.g., triangle, quadrilateral, pentagon, and so on

population: The entire group of subjects that you are interested in collecting data about; e.g., for collecting data about the favourite type of memo of students at a school, the population is all of the students in the school

possible outcome: A thing that could happen in a probability situation; e.g., when you roll a die, there are six possible outcomes: 1, 2, 3, 4, 5, and 6

power: A numerical expression that shows repeated multiplication; a power has a base and an exponent; the exponent tells how many equal factors there are in a power; sometimes the exponent is also called the power; e.g., the power 5^3 is a shorter way of writing 5 × 5 × 5:

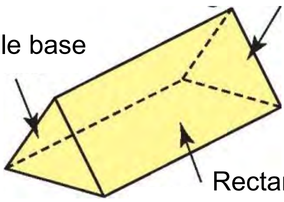
3 is the exponent of the power



5 is the base of the power

prime factors: The factors of a number that are prime numbers; usually written as a product; e.g., the prime factors of 24 are 2 × 2 × 2 × 3

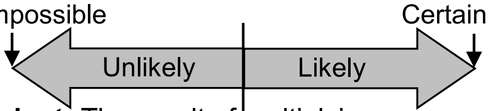
prism: A 3-D shape with two parallel and opposite congruent bases; the other faces are parallelograms (usually rectangles); the shape of the bases determines the name of the prism; e.g.,



A triangle-based prism

probability: A number from 0 (will never happen) to 1 (certain to happen) that tells how likely it is that an event will happen; it can be a decimal or fraction; sometimes it is called chance

probability line: A number line from 0 to 1 used to compare probabilities



product: The result of multiplying numbers; e.g., in 5 × 6 = 30, the product is 30

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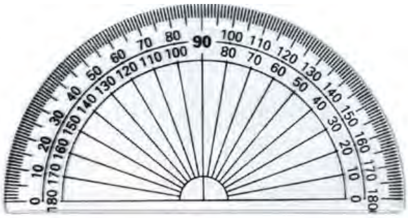
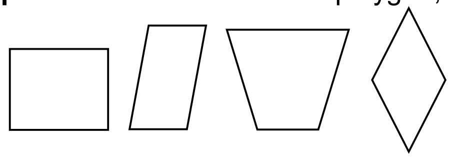
proper fraction: A fraction in which the denominator is greater than the numerator; e.g., $\frac{1}{7}$, $\frac{4}{5}$, $\frac{29}{40}$

protractor: A tool used to measure the size of an angle



Q

quadrilateral: A four-sided polygon; e.g.,



questionnaire: A set of survey questions, often on a related topic, that is used to collect information from people

quotient: The result of dividing one number by another number; e.g., in 45 ÷ 5 = 9, the quotient is 9

R

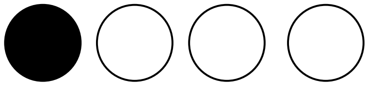
radius (plural is radii)**:** The name of the line segment that joins the centre of a circle to any point on its circumference; it is also the length of this line segment See *circle*

range: The difference between the extremes (minimum and maximum) of a set of data

rate: A comparison of two quantities measured in different units; unlike ratios, rates include units; e.g., 45 km/h

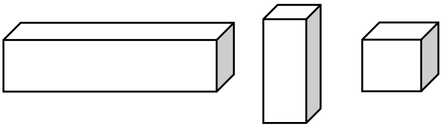
rate of change (on a graph): In a graph of a relationship between two variables, the rate of change describes how one variable changes when the other variable changes; in a straight line graph, the steeper the graph, the faster the rate of change

ratio: A number or quantity compared with another, expressed in symbols as $a : b$ or $\frac{a}{b}$; it can be a part-to-part comparison or a part-to-whole comparison; e.g., all three ratios below describe this set of counters:

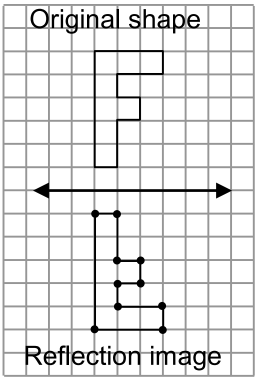


1 : 3, 1 : 4, and 3 : 4

rectangular prism: A prism with rectangle bases; e.g.,

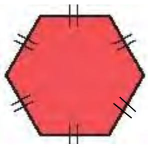


reflection: A transformation that produces a mirror image of a shape across a reflection line; also called a flip; e.g., this is a reflection of the F-shape across a horizontal refection line:



reflection line: See *reflection*

regular hexagon: A six-sided polygon with all sides and angles congruent; e.g.,



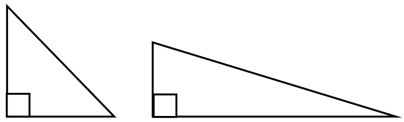
relation/relationship: A property that connects two sets of numbers or two variables; a relation can be expressed mathematically as a table of values, a graph, or an equation

rename (a number): Change a number to another form to make it easier to calculate or compare, but without changing its value; e.g., you can rename 0.4 as 0.40 or as 0.400

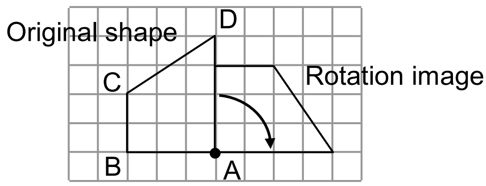
repeating decimal: A decimal in which a block of one or more digits eventually repeats in a pattern; e.g., 0.124444..., 0.252525252…, or 990.142857142857…

right angle: An angle that measures 90º; sometimes called a square corner See the right angles in the *right triangle*s below

right triangle: A triangle with one right angle; e.g.,



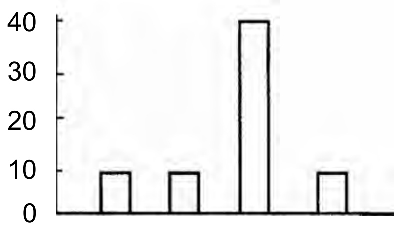
rotation: A transformation in which each point in a shape moves around a point (the turn centre) through the angle of rotation; e.g., this is a 90º cw rotation of trapezoid ABCD around vertex A:



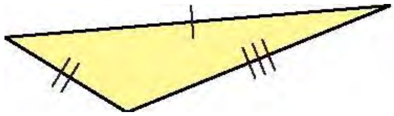
S

sample: If you cannot collect data from the entire population you are interested in, you can collect data from a carefully chosen sample; e.g., to collect data about the favourite type of memo of all the students at a school, a good sample might be five students chosen randomly from each classroom

scale (on a graph): The numbers and marks at regular intervals on the axes of a graph; it is also the value of each interval on an axis; the scale tells how to interpret a graph; e.g., the scale on the vertical axis of the graph below is 10



scalene triangle: A triangle with no congruent sides; e.g.,



second-hand data: Information collected from sources such as books, radio, TV, and the Internet

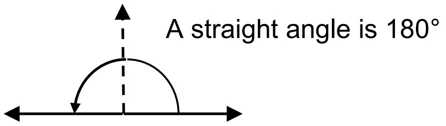
simplify: 1. To simplify a fraction means to write it in lowest terms or as a mixed fraction; e.g., you can simplify $\frac{18}{10}$ as $\frac{9}{5}$ and then as $1\frac{4}{5}$ **2.** To simply an expression means to collect and combine like terms; e.g., to simplify $2x + 3x + 4 + 7$, combine the x terms and then the constants to get $5x + 11$

solution: 1. The complete answer to a problem **2.** The value that makes an equation true; e.g., in $x + 4 = 39$, the solution is $x = 35$ because $35 + 4 = 39$

speed: The rate at which a moving object changes position with time, often given as a unit or average rate; e.g., a sprinter who runs 100 m in 10 s has an average speed of 10 m/s

standard form (of a number): The usual way to write a number; e.g., 23,650 is in standard form

straight angle: An angle that measures 180º; e.g.,



substitute: Replace a variable in an expression or equation with a value, usually to evaluate the expression; e.g., if $y = 3x + 5$ and $x = 2$, you can substitute 2 for x to evaluate the expression and find the value of y as $y = 3(2) + 5 = 11$

sum: The result of adding numbers; e.g., in 5 + 4 + 7 = 16, the sum is 16

survey: A method of collecting data using observation, questionnaires, and so on

symmetry: A property of a shape; line or reflectional symmetry means that when a 2-D shape is folded or reflected across a line (the reflection line), the two sides of the shape match

T

table of values: An arrangement of numerical values, usually arranged in rows and columns, that represents a relationship between two variables

term: 1. Part of an algebraic expression that is separated from the rest of the expression by addition or subtraction signs; e.g., the expression 3x + 3 has two terms **2.** Each number or item in a sequence; e.g., in the sequence 1, 3, 5, 7, …, the third term is 5 **3.** The numbers in a ratio or rate; e.g., the ratio 2 : 3 has two terms

terminating decimal: A decimal that is complete after a certain number of digits; e.g., $\frac{29}{40} = 0.725$

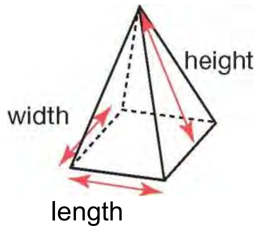
theoretical probability: A number from 0 to 1 that tells how likely an event is to occur, calculated using the expression:

$$\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}};$$

e.g., the theoretical probability of rolling a 4 on a six-sided die is $\frac{1}{6}$

thousands period: The group of three digits in a whole number that contains the hundreds thousands digit, the ten thousands digit, and the one thousands digit; e.g., in the number 123,456, the digits 123 make up the thousands period

three-dimensional (3-D): A shape with three dimensions: length, width (or breadth or depth), and height; e.g.,

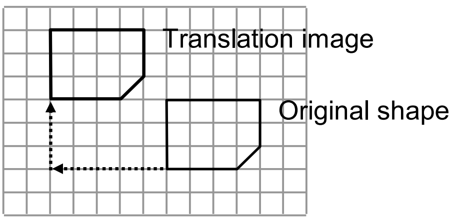


tiling (pattern): An arrangement of 2-D shapes that covers a plane (in all directions) without gaps or overlapping

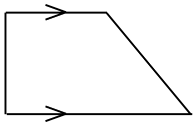
tonne (t): A standard unit of measure for mass; 1 t is equivalent to 1000 kg

transformation: Changing a shape according to a rule; transformations include translations, rotations, and reflections See *translation*, *reflection*, and *rotation*

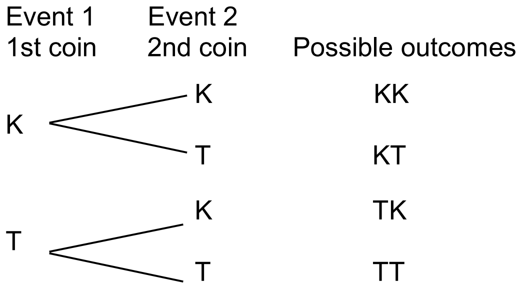
translation: A transformation in which each point of a shape moves the same distance and in the same direction; also called a slide; e.g., the pentagon below has been translated 5 units left and 3 units up



trapezoid: A quadrilateral in which one pair of opposite sides are parallel; e.g.,

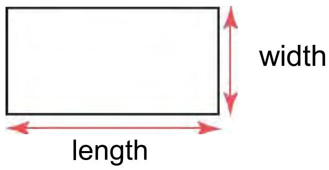


tree diagram: A way to determine all possible outcomes in a probability situation; e.g., this tree diagram shows all possible outcomes if you flip two coins



turn centre: The point around which all the points in a shape turn or rotate in a clockwise (cw) or counter-clockwise (ccw) direction See *rotation*

two-dimensional (2-D): A shape with two dimensions: length and width (or breadth); e.g.,



U

unit percent: The value that is equivalent to 1%; e.g., if 100% is 200 mL, the unit percent or 1% is 2 mL

unit rate: A rate with a second term of 1; e.g., 4 km/h is a unit rate because it means 4 km in 1 h

V

variable: A letter or symbol, such as *a*, *b*, *x*, or *n*, that represents a number; e.g.,in the formula for the area of a rectangle, A = *l* × *w*, the variables *A*, *l*, and *w* represent the area, length, and width of the rectangle

vertex (plural is vertices)**:** The point at the corner of an angle or shape where two or more sides or edges meet; e.g., a cube has eight vertices, a triangle has three vertices, and an angle has one vertex

vertical: An up-down direction as opposed to a horizontal (left-right) or diagonal direction; e.g., a vertical line:

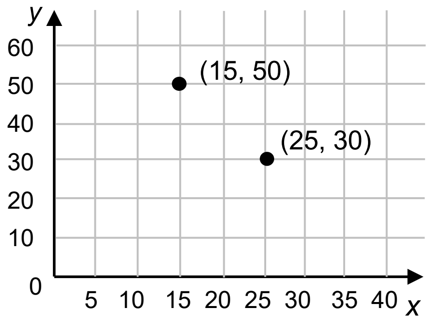
volume: The amount of space occupied by an object; often measured in cubic centimetres or cubic metres

W

whole numbers: The set of numbers that begins at 0 and continues forever in this pattern: 0, 1, 2, 3, …

X

***x*-axis:** One of the two axes in a coordinate grid; sometimes called the horizontal axis; e.g., the *x*-axis below goes from 0 to 40



***x*-coordinate:** The first value in an ordered pair that represents the distance along the *x*-axis from (0, 0); e.g., in (15, 50), the *x*-coordinate is 15 See *x-axis*

Y

***y*-axis:** One of the two axes in a coordinate grid; sometimes called the vertical axis; e.g., the *y*-axis of the grid shown above goes from 0 to 60
See *x-axis*

***y*-coordinate:** The second value in an ordered pair; it represents the distance along the *y*-axis from (0, 0); e.g., in (15, 50), the *y*-coordinate is 50
See *y-axis*

Z

zero property or principle: When you add two opposite integers, they have a sum of zero so they do not change the value of anything they are added to; often used when doing operations with integers; e.g., if you add (–2) and (+2) to an integer, the value of the integer stays the same because (–2) + (+2) = 0

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MEASUREMENT REFERENCE

Measurement Abbreviations and Symbols

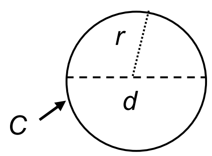
Time second minute hour	 s min h	**Capacity** millilitre litre	 mL L
Length millimetre centimetre metre kilometre	 mm cm m km	**Volume** cubic centimetre cubic metre cubic millimetres	 cm^3 m^3 mm^3
Mass milligram gram kilogram tonne	 mg g kg t	**Area** square centimetre square metre hectare (10,000 m^2) square kilometre	 cm^2 m^2 ha km^2

Metric Prefixes

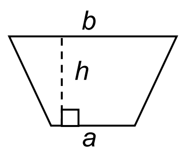
Prefix	kilo × 1000	hecto × 100	deka × 10	unit 1	deci × 0.1 or $\frac{1}{10}$	centi × 0.01 or $\frac{1}{100}$	milli × 0.001 or $\frac{1}{1000}$
Example	*kilo*metre km	*hecto*metre hm	*deka*metre dam	metre m	*deci*metre dm	*centi*metre cm	*milli*metre mm
	1000 m	100 m	10 m	1 m	0.1 m	0.01 m	0.001 m

Measurement Formulas and Relationships

Perimeter
rectangle $P = 2(l + w)$
square $P = 4 \times s$
circle (circumference) $C = \pi \times d$ or $C = 2 \times \pi \times r$



Area
rectangle $A = l \times w$
square $A = s \times s$
parallelogram $A = b \times h$
triangle $A = b \times h \div 2$
trapezoid $A = (a + b) \times h \div 2$



Volume
rectangular prism $V = Area\ of\ base \times height$ or $V = l \times w \times h$

Volume, Capacity, and Mass of Water
1 cm^3 = 1 mL = 1 g

ANSWERS

NOTE: Read the information about **Answers** on **page xiv** at the front of your textbook.

UNIT 1 NUMBER pp. 1–34

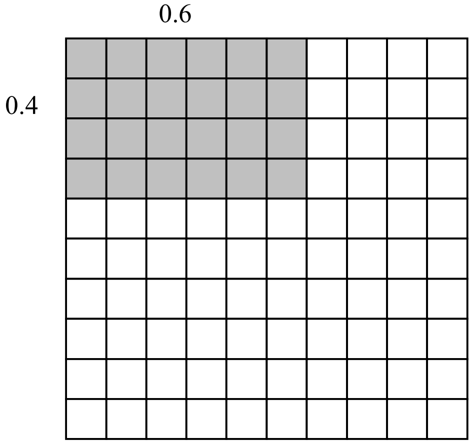
Getting Started — Skills You Will Need p. 2

1. a) 1, 2, 4 **b)** 1, 3 **c)** 1, 2

2. 23, 17

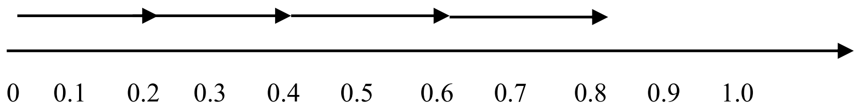
3. a) 4 × 100,000 + 1 × 10,000 + 2 × 1000 + 1 × 100 + 5 × 10
4 hundred thousands + 1 ten thousand + 2 thousands + 1 hundred + 5 tens
b) 3 × 100,000 + 6 × 10,000 + 5 × 1000 + 1 × 100 + 2 × 10 + 4 × 1
3 hundred thousands + 6 ten thousands + 5 thousands + 1 hundred + 2 tens + 4 ones
c) 1 × 1,000,000 + 3 × 1000 + 1 × 10
1 million + 3 thousands + 1 ten
d) 1 × 1,000,000,000 + 9 × 100,000 + 1 × 1000 + 1 × 100 + 4 × 10 + 2 × 1
1 billion + 9 hundred thousands + 1 thousand + 1 hundred + 4 tens + 2 ones

4. 0.24; *Sample response*:



5. a) 157.0 **b)** 534.24 **c)** 0.84
d) 103.5 **e)** 0.21 **f)** 0.96
g) 0.019 **h)** 0.23

6. 4; *Sample response*:



7. a) 105.2 **b)** 22.3 **c)** 20.8
d) 111.5 **e)** 2 **f)** 7.5
g) 6.75 **h)** 25.5

8. a) 340 **b)** 24.5
c) 0.328 **d)** 0.0234

9. a) Hundredths place **b)** Thousandths place
c) Tens place **d)** Hundreds place

10. a) 0.2, and 0.24 or 0.23
b) 14.9 and 14.92
c) 2.0 and 2.00

1.1.2 Divisibility Tests p. 8

1. a) No **b)** Yes **c)** Yes

2. a) No **b)** No **c)** Yes

3. a) Yes **b)** No **c)** Yes

4. a) 5 **b)** 1 **c)** 1 **d)** 1

5. a) Item B **b)** Item C **c)** Item A

6. a) 2, 5, or 8 **b)** 0, 3, 6, or 9
c) 0, 2, 4, 6, or 8 **d)** 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9
e) 3 **f)** 8

7. No

8. Yes

9. 32,154

10. *Sample response*: 1206, 2106, 4266

11. A is true; **B** is false

12. 97,864

13. 108

CONNECTIONS: Casting Out Nines p. 9

1. 3489 + 2379 = 5868
Check:
3 + 4 + 8 + 9 = 24; 24 – 18 = 6
2 + 3 + 7 + 9 = 21; 21 – 18 = 3
6 + 3 = 9; 9 – 9 = 0
5 + 8 + 6 + 8 = 27; 27 – 27 = 0
It works.

2. 1425 – 387 = 1047
Check:
1 + 4 + 2 + 5 = 12; 12 – 9 = 3
3 + 8 + 7 = 18; 18 – 18 = 0
3 – 0 = 3
1 + 0 + 4 + 7 = 12; 12 – 9 = 3
It works.

3. 25 × 38 = 950
Check:
2 + 5 = 7
3 + 8 = 11; 11 – 9 = 2
7 × 2 = 14; 14 – 9 = 5
9 + 5 + 0 = 14; 14 – 9 = 5
It works.

1.1.3 Lowest Common Multiple p. 12

1. a) 140 **b)** 32 **c)** 114 **d)** 1210

2. a) True **b)** True **c)** False

3. *Sample response*:
1 and 45, 3 and 45, 15 and 9

5. No

6. a) 30

7. 5 times

8. No

1.1.4 Greatest Common Factor p. 15

1. a) 2 **b)** 7 **c)** 24 **d)** 2

2. a) 1 **b)** 4

3. *Sample response*: 8

4. Yes

6. *Sample response*:
10 and 300; 20 and 150

7. a) The price is less in Store B
b) The price is less in Store B.

8. a) 1 **b)** 1

CONNECTIONS: Carrom Math p. 16

1 a) 3 times **b)** 13 times
c) 7 times **d)** 5 times

2. b) 8

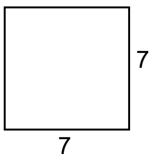
1.2.1 Introducing Powers

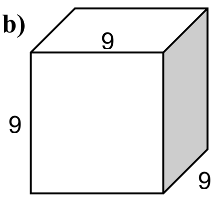
1. a) Base = 3, Exponent = 6
b) Base = 4, Exponent = 10
c) Base = 1, Exponent = 2
d) Base = 0, Exponent = 4

2. a) 7 × 7 × 7 × 7 × 7 × 7 × 7 × 7

b) 9 × 9 × 9 × 9

3. a) 6^7 **b)** 8^6 **c)** 2^8

4. *Sample responses*:
a)


b)


5. *Sample responses:*
a) Seven squared
b) Nine cubed

6. a) 3^2; by 1
b) 5^3; by 25
c) Same value
d) 3^5; by 179

7. 6

8. 4^3

9. No

10. No

1.2.2 Expanded, Standard, and Exponential Forms p. 22

1. a) $3 \times 10^7 + 4 \times 10^6 + 2 \times 10^2$
b) $3 \times 10^6 + 4 \times 10^3 + 5 \times 10^2 + 2$
c) $6 \times 10^8 + 2 \times 10^7 + 3 \times 10^5 + 5 \times 10^4$
d) $1 \times 10^8 + 1 \times 10^7 + 8 \times 10^6 + 3 \times 10^2 + 4 \times 10^1 + 2$
e) $2 \times 10^{10} + 2 \times 10^9 + 3 \times 10^8 + 4 \times 10^6 + 2 \times 10^5 + 5 \times 10^3 + 3 \times 10^1 + 2$

2. a) 4,050,006,000;
4 (one) billions + 5 ten millions + 6 thousands
b) 30,005,000,636;
3 ten billions + 5 one millions + 6 hundreds + 3 tens + 6 ones
c) 700,404,209;
7 hundred millions + 4 hundred thousands + 4 one thousands + 2 hundreds + 9 ones
d) 506,800,802,306;
5 hundred billions + 6 (one) billions + 8 hundred millions + 8 hundred thousands + 2 (one) thousands + 3 hundreds + 6 ones

3. *Sample response*:
Alike: In standard form, they both have one digit that is 3 and the other digits are all 0.
Different: One number is greater than the other because 30 million is more than 30 thousand.

4. a) 9
b) *Sample responses*:
- It is less than 5 hundred million.
- It is more than 4 thousand.

5. The power with the greatest exponent tells the most.

6. 5

1.3.1 Multiplying Decimals p. 26

1. a) 3241.68 **b)** 324.168
c) 3.24168 **d)** 3.24168

3. 0.0048 km^2

4. *Sample response:* 1.6 m

5. Yes

6. *Sample response*:
1.5 is one and a half, so add 4.048 to half of 4.048: 4.048 + 2.024 = 6.072

7. 3.57 cm^2

8. *Sample responses*:
- 3.45 × 0.01
- 0.5 × 40.444

9. *Sample responses*:
a) 20.4 ×
5.06

b) .204 × .065 (or 0.204 × 0.065)
c) 40.6 ×
5.20

d) .502 × .604 (or 0.502 × 0.604)

10. *Sample response*:
About 34 km

11. No

1.3.2 Dividing Decimals p. 29

1. a) 312.5 **b)** 312.5
c) 3125 **d)** 31.25

3. a) 9.78 **b)** 6248.00 **c)** 150.08

4. 3.5 cm

5. *Sample responses*:
a) Yes **b)** Yes **c)** Yes

6. a) 108 km/h **b)** 90 km/h

7. *Sample response*: About 4000 h

8. a) 4.667 (rounded to nearest thousandth)
b) 42

9. a) The remainder is $\frac{1}{3}$ or 0.333, not 1

1.3.4 Order of Operations — p. 32

1. a) 0.7 **b)** 8.08 **c)** 8.2

2. **A** and **B**

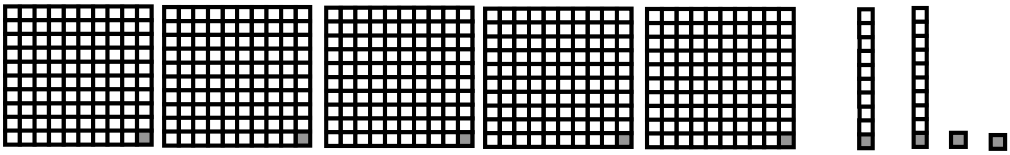
3. a) Not correct; 14.4 **b)** Not correct; 34.26
c) Correct **d)** Correct

4. a) $(8 \div 0.1 + 12) \times 3 - 2$
b) $[(4.2 + 3.5) \times 3]^2 - 4$
c) $[(6.2 \times 2 + 5.6)^2 + 3] \div 2$

5. a) 8.192 **b)** 34.1 **c)** 29

UNIT 1 Revision — pp. 33–34

1. a) 522



2. a) Divisible by 2, 4, 5, and 10
b) Divisible by 3 and 9
c) Divisible by 3, 5, and 9
d) Divisible by 2 and 4

3. a) By 3: remainder is 2; by 4: remainder is 2
b) By 3: remainder is 2; by 4: remainder is 1
c) By 3: remainder is 1; by 4: remainder is 2
d) By 3: remainder is 1; by 4: remainder is 3

4. 1485 is divisible by 15

5. a) 2, 5, or 8 **b)** 4 **c)** Any digit 0 to 9

6. a) 336 **b)** 90 **c)** 210

7. 3, 21, 105

8. a) No
b) Yes
c) *Sample responses*: 1 and 90 or 30 and 45

9. a) 10 **b)** 5 **c)** 5

10. *Sample response*:
30 and 600 or 150 and 120

11. a) 4 ways
b) 5 ways:
c) 1 row, 2 rows, or 4 rows

12. a) $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
b) $11 \times 11 \times 11$

13. a) 9^6 **b)** 3^8

15. $3^2 + 3^3 + 3^4 + 3^5 = 360$

17. a) $3 \times 10^6 + 1 \times 10^5 + 2 \times 10^4 + 3$
b) $3 \times 10^9 + 1 \times 10^8 + 2 \times 10^7 + 3 \times 10^3 + 4 \times 10^2$

18. a) 300,020,308;
3 hundred millions + 2 ten thousands +
3 hundreds + 8 ones
Or, $3 \times 100{,}000{,}000 + 2 \times 10{,}000 + 3 \times 100 + 8$
b) 600,070,003,205;
6 hundred billions + 7 ten millions +
3 thousands + 2 hundreds + 5 ones
Or, $6 \times 100{,}000{,}000{,}000 + 7 \times 10{,}000{,}000 +$
$3 \times 1000 + 2 \times 100 + 5$

19. a) 380.16 **b)** 3.8016
c) 0.38016 **d)** 3.8016

20. *Sample response*:
1 decimal place and 4 decimal places

21. 4.3 × **1.2** = **5**.16

22. *Sample responses*:
a) 10 cm base and 15.5 cm height
b) 20 cm base and 7.75 cm height

24. a) 850 **b)** 61.5

25. a) 7.29 **b)** 1.33

26. *Sample responses*:
a) 43.5×0.1, 8.4×0.5, 3.0×3.1

27. a) 2.16 **b)** 72.6 **c)** 12.3

28. a) Not necessary
b) Not necessary
c) Necessary

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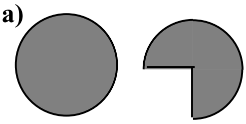
Getting Started — Skills You Will Need p. 36

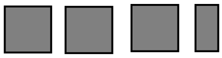
1. a) $\frac{19}{8}, 2\frac{3}{8}$ **b)** $\frac{5}{4}, 1\frac{1}{4}$ **c)** $\frac{7}{2}, 3\frac{1}{2}$

2. *Sample responses:*

a)

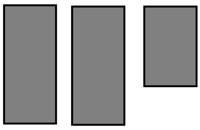


b)



c)

d)



3. a) $\frac{17}{5}$ **b)** $\frac{13}{2}$ **c)** $\frac{13}{3}$ **d)** $\frac{19}{8}$

4. A and C

5. a) < **b)** = **c)** >

6. $1\frac{1}{4}$ h

7. $\frac{1}{4}$

8. a) 0.3 **b)** 0.27 **c)** 0.5 **d)** 0.6

2.1.1 Comparing and Ordering Fractions pp. 40–41

1. a) < **b)** = **c)** < **d)** =

2. a) $\frac{7}{6}, 1\frac{3}{4}, \frac{7}{3}$ **b)** $2\frac{1}{3}, \frac{11}{4}, \frac{9}{2}$

c) $1\frac{5}{9}, \frac{21}{12}, \frac{11}{6}$

3. Pelden

4. Yuden; Rupak

5. a) Any value from 1 to 36.

b) *Sample response*:

?	#
1	3
2	6
3	9

c) Any number greater than 4.

6. a) $\frac{1}{2}, \frac{3}{5}, \frac{2}{3}$;
The new fraction is in the middle.

b) i) $\frac{12}{9}$

ii) $\frac{5}{4}, \frac{12}{9}, \frac{7}{5}$;
The new fraction is in the middle.

c) *Sample responses*:

i) $\frac{1}{2}$ and $\frac{3}{4}$ to create $\frac{4}{6}$

ii) $\frac{1}{2}, \frac{4}{6}, \frac{3}{4}$; The new fraction is in the middle.

7. *Sample response*:
Dechen served two cakes of the same size to her guests. $\frac{2}{3}$ of the first cake and $\frac{3}{5}$ of the second cake were left over. Which cake had more left over?

8. a) $\frac{7}{3}, \frac{8}{3}$ **b)** $\frac{9}{4}, \frac{10}{4}, \frac{11}{4}$

c) $\frac{11}{5}, \frac{12}{5}, \frac{13}{5}, \frac{14}{5}$ **d)** No

2.1.2 Adding Fractions Using Models pp. 46–47

1. a) $\frac{3}{4}$ **b)** $\frac{5}{8}$ **c)** $\frac{3}{5}$ **d)** $\frac{3}{5}$

2. *Sample responses*:
a) 12 **b)** 10 **c)** 18 **d)** 15

3. *Sample responses for estimates*:

a) Estimate: about 2; $\frac{9}{6}$ or $1\frac{1}{2}$

b) Estimate: about $1\frac{1}{2}$; $\frac{11}{8}$ or $1\frac{3}{8}$

c) Estimate: about $1\frac{1}{2}$; $\frac{11}{10}$ or $1\frac{1}{10}$

d) Estimate: about $\frac{1}{2}$; $\frac{4}{9}$

e) Estimate: about $\frac{3}{4}$; $\frac{11}{15}$

f) Estimate: about 2; $\frac{17}{12}$ or $1\frac{5}{12}$

2.1.2 Adding Fractions Using Models [Continued] pp. 46–47

4. *Sample responses*:

a) $\frac{3}{9} + \frac{5}{12}$

b) $\frac{2}{2} + \frac{1}{2}$; $\frac{3}{4} + \frac{3}{4}$; $\frac{1}{4} + \frac{5}{4}$; $\frac{1}{8} + \frac{11}{8}$; $\frac{5}{8} + \frac{7}{8}$, and so on.

5. $\frac{5}{8}$

6. B

7. a) and b) *Sample responses*:

$\frac{3}{4} + \frac{5}{6} = \frac{19}{12}$ or $1\frac{7}{12}$;

$\frac{3}{4} + \frac{6}{5} = \frac{39}{20}$ or $1\frac{19}{20}$;

$\frac{3}{5} + \frac{4}{6} = \frac{19}{15}$ or $1\frac{4}{15}$;

$\frac{3}{6} + \frac{4}{5} = \frac{13}{10}$ or $1\frac{3}{10}$;

$\frac{3}{6} + \frac{5}{4} = \frac{7}{4}$ or $1\frac{3}{4}$.

8. 2 cups

2.1.3 Adding Fractions and Mixed Numbers Symbolically pp. 50–51

1. a) i) $\frac{7}{9}$ **ii)** $\frac{7}{8}$

iii) $\frac{22}{15} = 1\frac{7}{15}$ **iv)** $\frac{29}{20} = 1\frac{9}{20}$

2. a) $\frac{15}{8} = 1\frac{7}{8}$ **b)** $\frac{25}{18} = 1\frac{7}{18}$

3. a) i) $3\frac{4}{6} = 3\frac{2}{3}$ **ii)** $2\frac{9}{10}$

iii) $9\frac{11}{15}$ **iv)** $7\frac{11}{12}$

4. *Sample response*: **a)** $\frac{1}{2} + \frac{2}{5} + \frac{1}{10}$

5. $4\frac{7}{12}$ cups

6. a) $\frac{8}{15}$ **b)** $\frac{13}{15}$

7. a) $\frac{17}{24}$ cup **b)** Yes

8. a) $\frac{2}{5} + \frac{1}{4} = \frac{13}{20}$

b) $\frac{5}{1} + \frac{4}{2} = 7$

c) $\frac{1}{2} + \frac{3}{5} = \frac{11}{10}$ or $\frac{2}{4} + \frac{3}{5} = \frac{11}{10}$

2.1.4 Subtracting Fractions and Mixed Numbers pp. 56–57

1. a) i) $\frac{1}{8}$ **ii)** $\frac{2}{9}$ **iii)** $\frac{9}{20}$ **iv)** $\frac{2}{15}$

2. a) $1\frac{1}{5}$ **b)** $1\frac{3}{6}$ or $1\frac{1}{2}$ **c)** $3\frac{2}{7}$ **d)** $2\frac{1}{10}$

3. Red rice; $\frac{1}{12}$ cup more

4. a) More **b)** $\frac{1}{8}$ more than $\frac{1}{2}$ of a tank

6. *Sample response*: $\frac{1}{1} - \frac{1}{4}$; $\frac{5}{4} - \frac{1}{2}$; $\frac{3}{2} - \frac{3}{4}$

7. a) Red **b)** $\frac{1}{15}$ **c)** $\frac{4}{15}$

8. The other fraction is between $\frac{1}{2}$ and $\frac{3}{4}$;

9. a) $\frac{17}{40}$ **b)** $\frac{5}{20}$ or $\frac{1}{4}$ **c)** More; $\frac{1}{8}$

10. a) $\frac{5}{2} - \frac{3}{4} = \frac{7}{4}$

b) $\frac{3}{5} - \frac{2}{4} = \frac{2}{20}$ or $\frac{1}{10}$

c) $\frac{5}{4} \square \frac{2}{3} = \frac{7}{12}$

2.1.5 Subtracting Mixed Numbers in Different Ways p. 61

1. a) $3\frac{2}{5}$ **b)** $5\frac{4}{7}$ **c)** $2\frac{1}{6}$ **d)** $4\frac{5}{9}$

2. a) $1\frac{31}{40}$ **b)** $3\frac{8}{9}$ **c)** $1\frac{3}{4}$ **d)** $2\frac{17}{18}$

3. $2\frac{3}{8}$ laps

4. $1\frac{1}{4}$ h

5. a) $3\frac{5}{6} = 2\frac{11}{6}$ **b)** $7\frac{3}{10} = 6\frac{13}{10}$

6. $2\frac{3}{4}$ h longer

7. $1\frac{3}{4}$ fewer laps

8. a) $1\frac{5}{8}$ m from one; $\frac{3}{4}$ m from the other

b) No

9. a)

$1\frac{4}{5}$	$3\frac{9}{10}$	$2\frac{2}{5}$
$3\frac{3}{10}$	$2\frac{7}{10}$	$2\frac{1}{10}$
3	$1\frac{1}{2}$	$3\frac{3}{5}$

b) The magic sum is $8\frac{1}{10}$.

2.2.1 Multiplying a Fraction by a Whole Number pp. 63–64

1. a) $5 \times \frac{1}{3} = \frac{5}{3}$ or $1\frac{2}{3}$

b) $3 \times \frac{7}{10} = \frac{21}{10}$ or $2\frac{1}{10}$

c) $6 \times \frac{2}{9} = \frac{12}{9}$ or $\frac{4}{3}$ or $1\frac{3}{9}$ or $1\frac{1}{3}$

2. A $(5 \times \frac{3}{8})$ and C $(7 \times \frac{5}{3})$

3. a) $2\frac{2}{5}$ **b)** $5\frac{1}{2}$

c) $3\frac{6}{8}$ or $3\frac{3}{4}$ **d)** $1\frac{5}{10}$ or $1\frac{1}{2}$

4. a) i) 5 **ii)** 7 **iii)** 3

5. $\frac{14}{3}$ or $4\frac{2}{3}$ apples

6. 21 h

7. a) $\frac{21}{10}$ and $\frac{21}{10}$

c) *Sample response*:

$3 \times \frac{2}{5} = \frac{6}{5}$ and $2 \times \frac{3}{5} = \frac{6}{5}$

8. $\frac{8}{3}$ or $2\frac{2}{3}$ cups of walnuts

2.2.2 Dividing a Fraction by a Whole Number p. 67

1. a) $\frac{1}{4}$ **b)** $\frac{1}{16}$ **c)** $\frac{2}{9}$

2. a) $\frac{1}{15}$ **b)** $\frac{5}{16}$ **c)** $\frac{4}{15}$

d) $\frac{7}{8}$ **e)** $\frac{2}{3}$ **f)** $\frac{3}{4}$

g) $\frac{3}{10}$ **h)** $\frac{3}{28}$ **i)** $1\frac{1}{10}$

3. $\frac{5}{12}$ h **4.** $\frac{3}{4}$ cup

5. *Sample response*:

Five students share $\frac{1}{2}$ of a cake equally.

What fraction of the whole cake does each student get? $\left(\frac{1}{10}\right)$

2.2.2 Dividing a Fraction by a Whole Number [Continued] p. 67

6. a) $\frac{2}{4} \div 6 = \frac{1}{12}$; $\frac{4}{2} \div 6 = \frac{1}{3}$; $\frac{2}{6} \div 4 = \frac{1}{12}$;
$\frac{6}{2} \div 4 = \frac{3}{4}$; $\frac{4}{6} \div 2 = \frac{1}{3}$; $\frac{6}{4} \div 2 = \frac{3}{4}$.

b) $\frac{6}{2} \div 4$ and $\frac{6}{4} \div 2$

c) $\frac{2}{4} \div 6$ and $\frac{2}{6} \div 4$

d) $\frac{6}{2} \div 4$ and $\frac{6}{4} \div 2$; $\frac{2}{4} \div 6$ and $\frac{2}{6} \div 4$;
$\frac{4}{2} \div 6$ and $\frac{4}{6} \div 2$

2.3.1 Naming Fractions and Mixed Numbers as Decimals p. 71

1. a) $\frac{5}{10} = 0.5$ **b)** $\frac{6}{10} = 0.6$
c) $\frac{75}{100} = 0.75$ **d)** $\frac{35}{100} = 0.35$
e) $\frac{44}{100} = 0.44$ **f)** $\frac{875}{1000} = 0.875$

2. 0.2
a) 0.6 **b)** 0.8 **c)** 1.4
d) 1.6 **e)** 2.2 **f)** 2.4

3. a) 0.272727… **b)** 0.625 **c)** 0.222…
d) 0.48 **e)** 0.8333… **f)** 0.58333…

4. a) 0.11…, 0.22…, 0.33…, 0.44…, 0.55…, 0.66…, 0.77…, 0.88…
b) 0.0909…, 0.1818…, 0.2727…, 0.3636…, 0.4545…, 0.5454…, 0.6363…, 0.7272…, 0.8181…, 0.9090…
c) 0.142857142857…, 0.285714285714…, 0.428571428571…, 0.571428571428…, 0.714285714285…, 0.857142857142….

5. a) i) $\frac{1}{3}$ **ii)** $\frac{1}{3}$ **iii)** $\frac{1}{3}$
b) i) $\frac{3}{10} < \frac{3}{9}$ **ii)** $\frac{33}{100} < \frac{33}{99}$
iii) $\frac{333}{1000} < \frac{333}{999}$

6. a) 0.48 > 0.46 **b)** 0.875 < 0.88

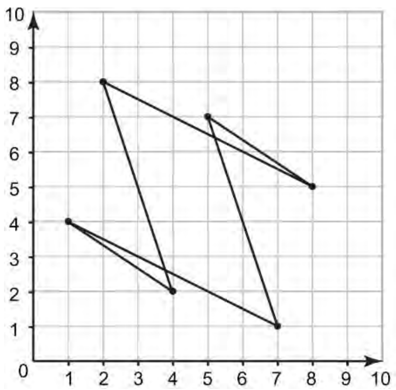
7. a) 0.875 > 0.833…, so $\frac{7}{8} > \frac{5}{6}$
b) 0.166… > 0.16, so $\frac{1}{6} > \frac{4}{25}$
c) 0.22 < 0.222…, so $\frac{11}{50} < \frac{2}{9}$
d) 0.3636… 0. , so $\frac{11}{4} > \frac{18}{50}$

8. 3 digits; 3.14

9. a) 0, 1, 2, 3, 4, 5, and 6

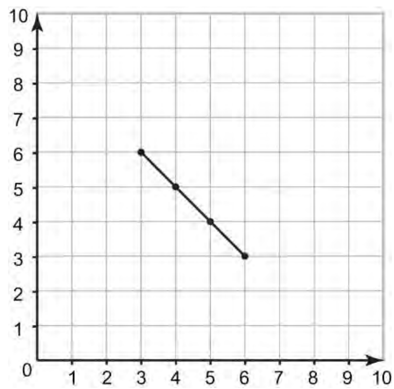
CONNECTIONS: Repeating Decimal Graphs p. 73

1. a) 0.142857, 0.285714, 0.428571, 0.571428, 0.714285, 0.857142
b)



b) One shape

2. a) A straight line
b)



UNIT 2 Revision pp. 74–75

4. a) $\frac{17}{24}$ of the cake was eaten.

5. a) $\frac{11}{20}$ **b)** $\frac{41}{24}$ or $1\frac{17}{24}$ **c)** $6\frac{8}{10}$ or $6\frac{4}{5}$

6. a) $\frac{11}{24}$ **b)** $\frac{5}{24}$ **c)** $7\frac{1}{8}$

d) $2\frac{2}{5}$ **e)** $4\frac{7}{12}$ **f)** $1\frac{3}{6}$ or $1\frac{1}{2}$

7. a) 3 or 4 **b)** 1 or 2

c) *Sample response*: $2\frac{7}{10}$

8. a) Archery **b)** $\frac{7}{20}$ **c)** $\frac{3}{20}$

9. a) 4 **b)** $\frac{21}{5}$ or $4\frac{1}{5}$ **c)** $\frac{25}{3}$ or $8\frac{1}{3}$

10. a) $\frac{2}{9}$ **b)** $\frac{11}{10}$ or $1\frac{1}{10}$ **c)** $\frac{5}{18}$

11. a) $\frac{21}{4}$ or $5\frac{1}{4}$ h **b)** $\frac{3}{8}$ h

12. 0.125
a) 0.25 **b)** 0.375 **c)** 0.625
d) 0.875 **e)** 1.375

13. a) i) $\frac{4}{9}$ is greater

ii) $\frac{4}{9}$ is greater

iii) $\frac{4}{9}$ is greater

b) i) $\frac{4}{9} > \frac{4}{10}$

ii) $\frac{4}{9} = \frac{44}{99}$ and $\frac{44}{99} > \frac{44}{100}$

iii) $\frac{4}{9} = \frac{444}{999}$ and $\frac{444}{999} > \frac{444}{1000}$

14. a) $\frac{6}{11}$ **b)** $\frac{14}{33}$ **c)** $\frac{7}{9}$

UNIT 3 RATE RATIO, AND PERCENT pp. 77–104

Getting Started — Skills You Will Need p. 78

1. a) 8 **b)** 40 **c)** 5 **d)** 14

2. *Sample response*:
6 : 14; 9 : 21; 12 : 28; 15 : 35

3. a) 14 : 7 or 2 : 1
b) 28 : 42 (or 14 : 21 or 2 : 3)

4. a) B **b)** A

5. a) 13
b) 6
c) 90
d) 4

6.

Measurement unit	gram	**kilogram**	**millilitre**	**litre**	metre	**kilometre**	**hour**	second	**minute**
Symbol	**g**	kg	mL	L	**m**	km	h	**s**	min

3.1.1 Solving Ratio Problems p. 82

1. a) 5 : 8 = 15 : 24 2 : 1 = 10 : 5
2 : 3 = 8 : 12 3 : 4 = 12 : 16

2. *Sample responses*:
a) 5 : 2, 20 : 8, 30 : 12
b) Divide each term of the original ratio by 2.
Multiply each term of the original ratio by 2.
Multiply each term of the original ratio by 3.
c) Yes

3. *Sample responses*:
a) Thinley might have 10 Bhutanese and 6 Ugandan stamps.
b) Yes

4. a) Groups of 7 (3 boys, 4 girls), groups of 14 (6 boys, 8 girls), and groups of 21 (9 boys, 12 girls)
b) 6 groups
c) 3 boys and 4 girls

3.1.1 Solving Ratio Problems [Continued] p. 82

5. a) i) 720 g **ii)** 120 g
b) 480 g
c) i) 12 **ii)** 4

6. Yes

7. 18 cm by 27 cm

8. *Sample response*: There are 39 students in my class. The ratio of girls to boys is 6 : 7. How many girls are there? (Answer: 18)

3.1.2 Solving Rate Problems p. 86

1. a) 25 km/h **b)** 180 kg/day **c)** 25 m/min

2. a) Nu 72 **b)** Nu 12 **c)** Nu 2
d) *Sample response*: Nu 78 for 39 students

3. 66 beats/min

4. 140 beats/min

5. 25 L **6.** 3 min/km

7. Nu 60 for 12 oranges

8. a) i) Elephant **ii)** Lion
b) i) Rabbit **ii)** Tortoise
c) Cheetah
d) Tortoise

3.2.1 Percent as a Special Ratio p. 89

1. a) 9% **b)** 19% **c)** 87%
d) 43% **e)** 100%

2. a) 71% **b)** 29%

3. a) 18% **b)** 25% **c)** 10% **d)** 47%

4. 50%

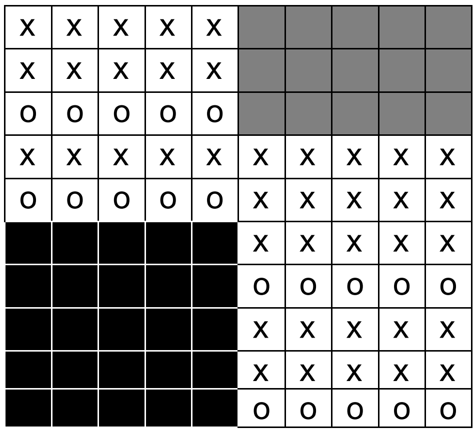
5. a) 68% **b)** 32%

6. 28%

7. a) 50% **b)** 50% **c)** 20%

8. a) 70% **b)** 20%
c) 30% **d)** 10%

9. a), b), and c) *Sample response*:



d) 40% **e)** 20% **f)** 100%

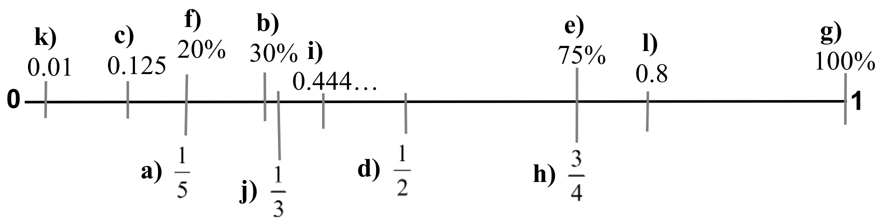
3.2.2 Relating Percents, Fractions, and Decimals pp. 93–94

1. a) 47% **b)** 63% **c)** 5% **d)** 80%

2. a) $\frac{75}{100}$ or $\frac{3}{4}$; 0.75 **b)** $\frac{24}{100}$ or $\frac{6}{25}$; 0.24
c) $\frac{90}{100}$ or $\frac{9}{10}$; 0.9 **d)** $\frac{1}{100}$; 0.01
e) $\frac{2}{100}$ or $\frac{1}{50}$; 0.02 **f)** $\frac{35}{100}$ or $\frac{7}{20}$; 0.35

3. a) 0.25; 25% **b)** 0.6; 60%
c) 0.7; 70% **d)** 0.04; 4%
e) 0.16; 16% **f)** 0.14; 14%
g) 0.05; 5% **h)** 0.55; 55%

4.



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5. About 33%

6. *Sample response*:
a) About 83% **b)** About 11%
c) About 44% **d)** About 9%

7. *Sample response*:
a) About $\frac{6}{10}$ **b)** About $\frac{3}{4}$
c) About $\frac{1}{6}$ **d)** About $\frac{5}{6}$

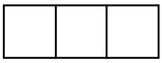
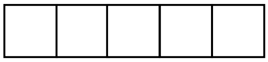
8. a) $\frac{7}{20}$ **b)** $\frac{9}{20}$ **c)** $\frac{13}{20}$ **d)** $\frac{19}{20}$

9. Chabil's class

10. a) 36% **b)** 64%

11. About 30%

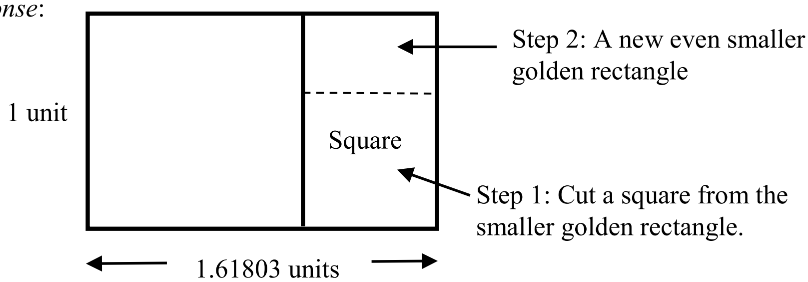
12. a) 0.27 (Services), $\frac{1}{3}$ (Agriculture) , 40% (Industry)

13. *Sample responses*:
a)

b)

c) Yes

CONNECTIONS: The Golden Ratio p. 95

1. Yes

2. *Sample response*:



3. *Sample response*: The base of the Parthenon in Athens; the Pantheon in Rome; the face of Leonardo da Vinci's Mona Lisa fills a golden rectangle.

3.2.3 Estimating and Calculating Percents p. 101

1. a) 30 **b)** 10 **c)** 4
d) 27 **e)** 77 **f)** 285

2. 14 cm

3. a) 36 kareys **b)** 9 dobjeys

4. Reading, 4 h; TV, 12 h; Games, 56 h; Song and dance, 4 h; Other, 4 h.

5. a) i) 1000 **ii)** 5000
iii) 7500 **iv)** 9000
b) i) 3000 **ii)** 15,000
iii) 22,500 **iv)** 27,000

6. *Sample responses*:
a) About 20 mm in March
About 215 mm in August
b) About 6500 mm

7. a) 39; 39; Answer is the same.
b) 22.5; 22.5; Answer is the same.
c) Yes

UNIT 3 Revision pp. 103–104

1. a) i) 20 mL **ii)** 5 mL
b) 10 servings

2. a) 18 girls **b)** 20 boys and 15 girls

3. a) 24

4. a) 45 : 75, 60 : 100

5. No

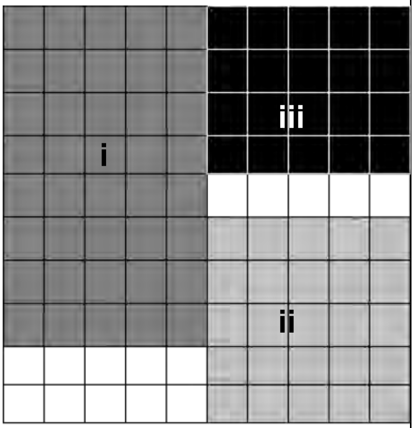
6. 1 dozen apples for Nu 60

UNIT 3 Revision [Continued] pp. 103–104

7. a) i) Nu 30 **ii)** Nu 60
b) Nu 5 per orange

8. 4 h

10. a) *Sample response*:



d) 15%

11. a) 35% **b)** 65%

12. a) 33% **b)** 25%

13. 20%, $\frac{1}{4}$, 0.55, $\frac{3}{5}$, 0.75, 90%

14. a) 3 **b)** 13
c) 2 **d)** 5.4

15. a) 80% **b)** 12%
c) 300 **d)** 500

16. 300 students

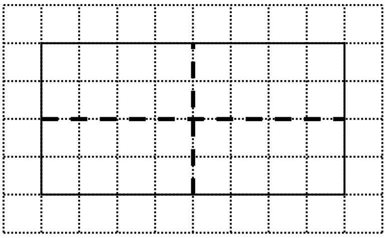
17. a) 5% **b)** 135 people

UNIT 4 GEOMETRY AND MEASUREMENT pp. 105–151

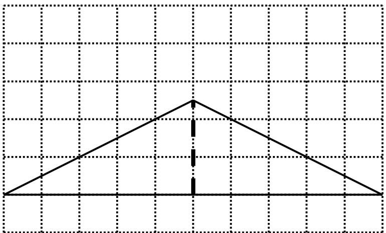
Getting Started — Skills You Will Need pp. 105–106

1. a) 32 cm^2 **b)** 32 cm^2
c) 12.5 cm^2 **d)** 6.75 cm^2

2. a)



c)

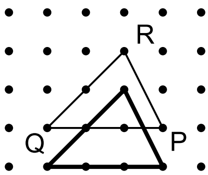


3. A is neither congruent nor similar.
B is similar but not congruent
C is congruent and similar.

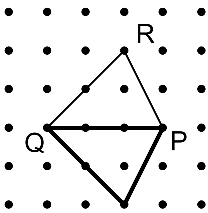
4. a) 2.5 cm **b)** 6 cm

5. 12.25 cm^2

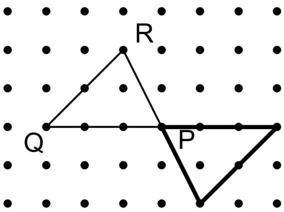
6. a)



b)



c)



7. a) 52 mm **b)** 0.052 m **c)** 50 g

8. a) 4030 **b)** 7.2
c) 600,000 **d)** 5.3

CONNECTIONS: Angle Measurement Units p. 109

1. a) 200 gradients **b)** 400 gradients **c)** 67.7 gradients

2. a) 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360
b) 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 200, 400

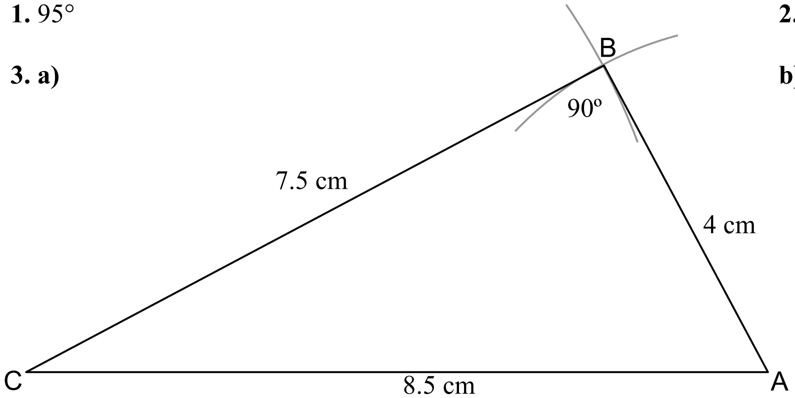
3. a) 3.14 radians **b)** 1.57 radians

4.1.2 Drawing and Classifying Triangles p. 113

1. 95°

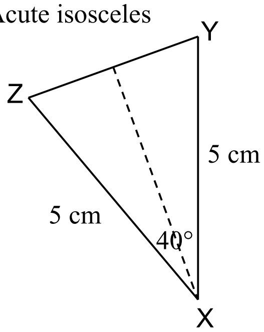
2. No

3. a)

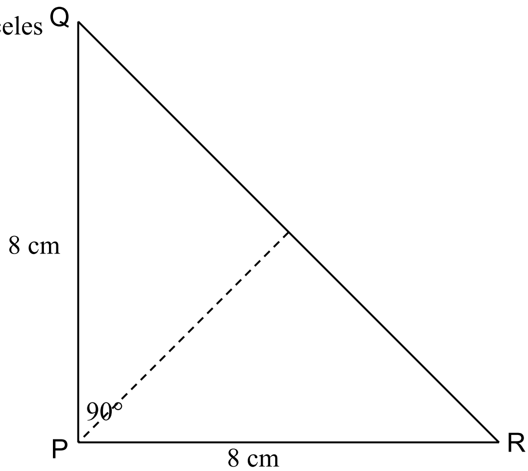


b) Right scalene

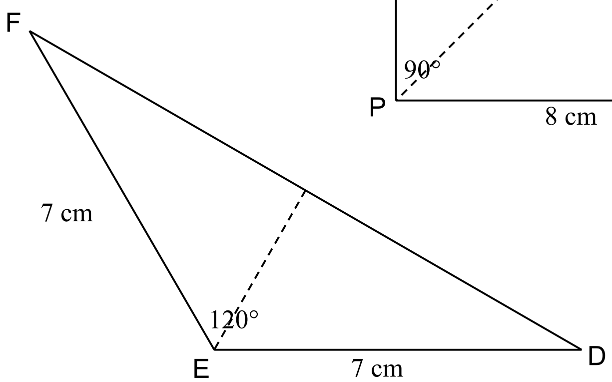
4. a) Acute isosceles



b) Right isosceles



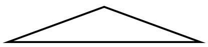
c) Obtuse isosceles



5. Any isosceles triangle has one line of symmetry.

6. *Sample responses*:

a)



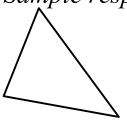
b) Not possible

c) Not possible

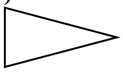
4.1.2 Drawing and Classifying Triangles [Continued] p. 113

7. *Sample responses*:

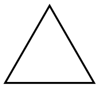
a)



b)

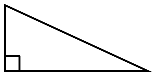


c)

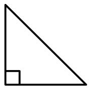


8. *Sample responses*:

a)

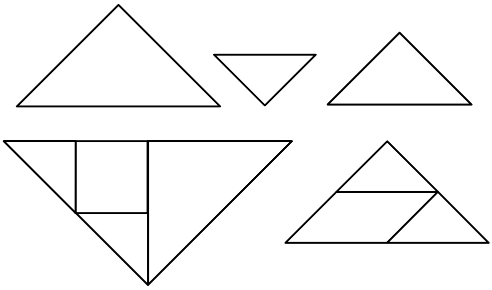


b)



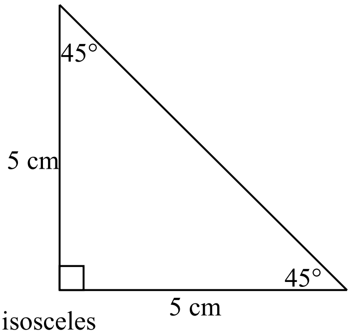
c) Not possible

9. a) *Sample response*:



b) They are all right isosceles.
c) They are all right triangles.
They are all different sizes.

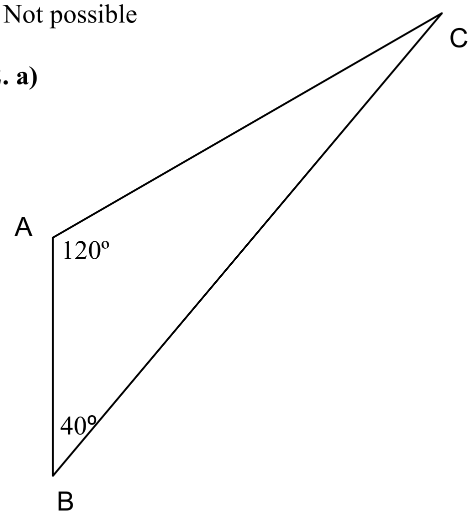
10. a) *Sample response*:



b) Right isosceles
c) A right triangle with two 45º angles and a long side that is 5 cm long.

11. a) Acute scalene
b) Obtuse isosceles
c) Not possible
d) Acute equilateral
e) Not possible

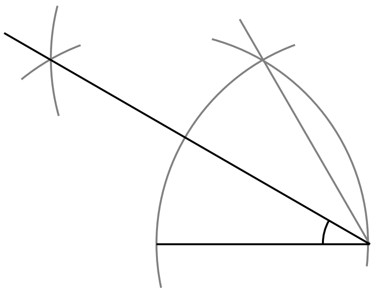
12. a)



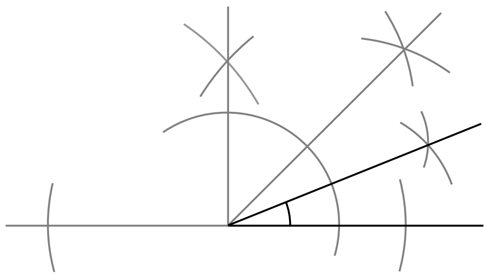
13. Yes

4.1.3 Constructing and Bisecting Angles pp. 117–118

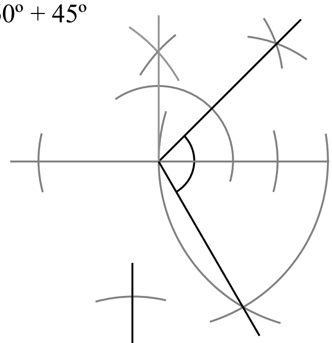
1. a) 30° = 60º ÷ 2



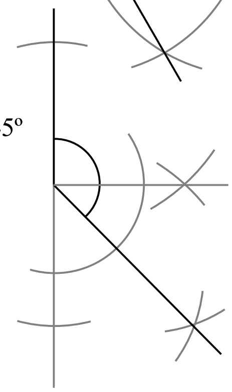
b) 22.5° = 90º ÷ 2 ÷ 2



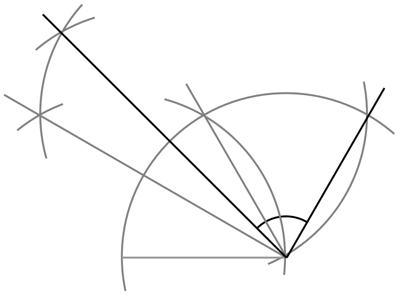
c) 105° = 60º + 45º



d) 135° = 180º – 45º

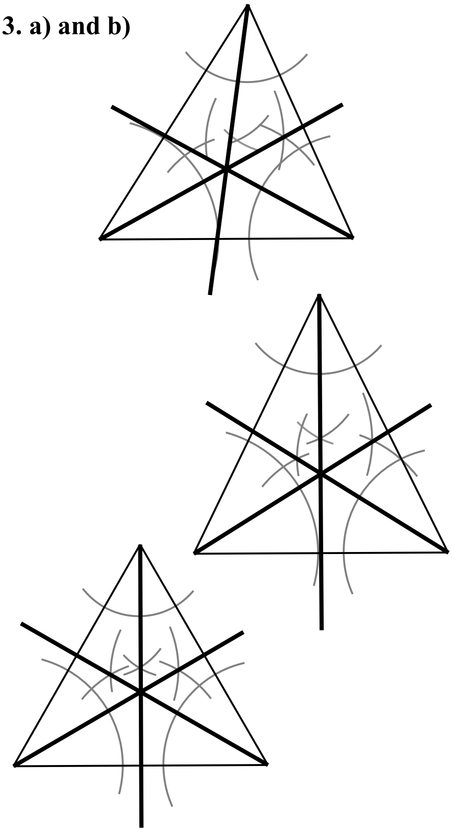


e) 75° = 60º + (60º ÷ 2 ÷ 2)



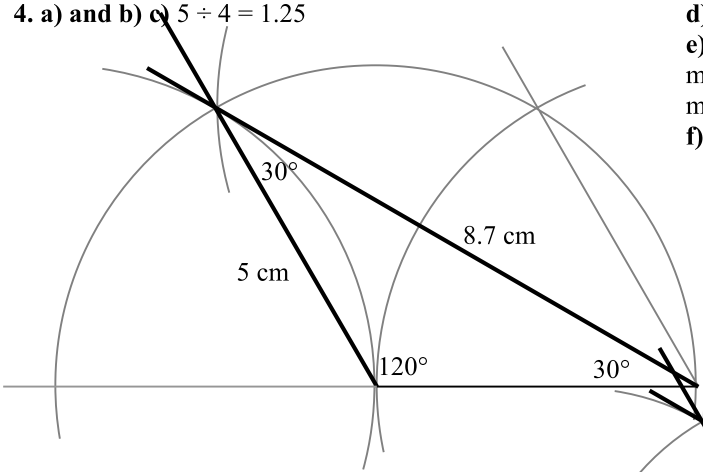
2. *Sample response*:
15°; 7.5°; 37.5°; 82.5ºº; 97.5°

3. a) and b)

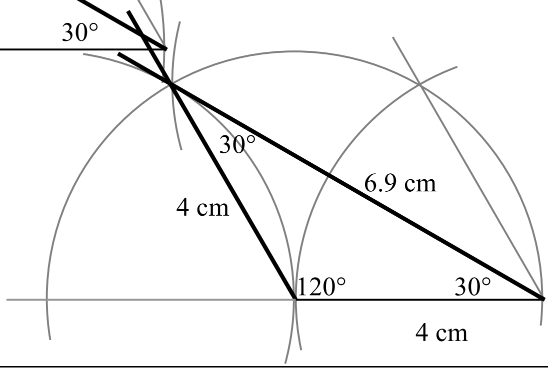


c) In each triangle, all three bisectors go through the same point in the middle of the triangle.

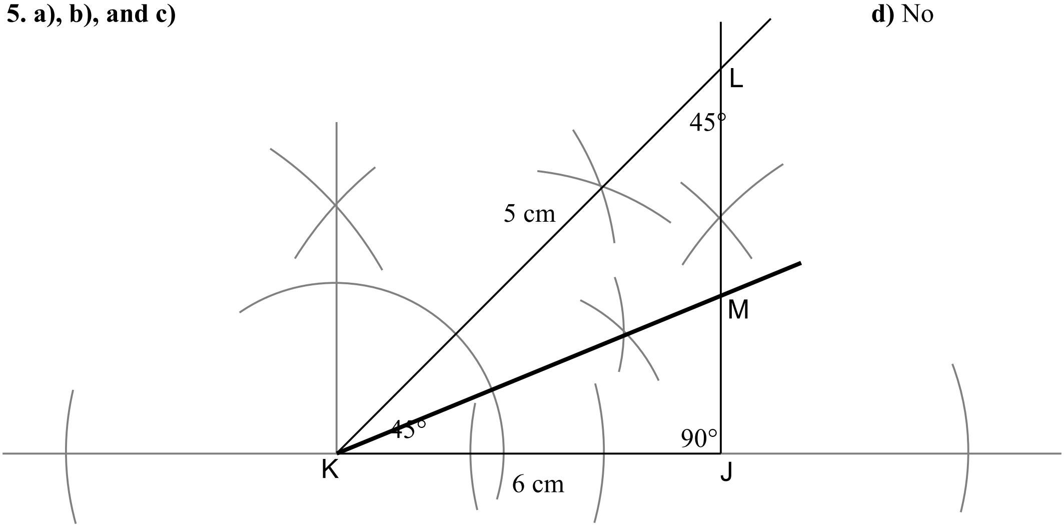
4. a) and b) c) 5 ÷ 4 = 1.25



d) 8.7 ÷ 6.9 = 1.26
e) The result are the same (answers may vary due to constructing and measuring errors).
f) Yes



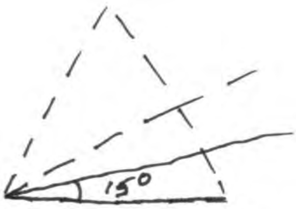
5. a), b), and c)



d) No

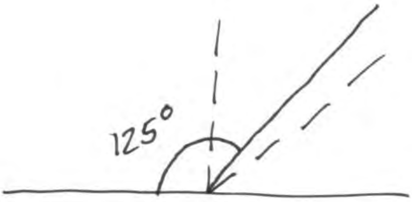
6. *Sample response*:

a)

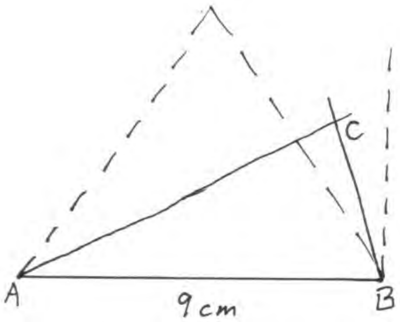


7. *Sample response*:

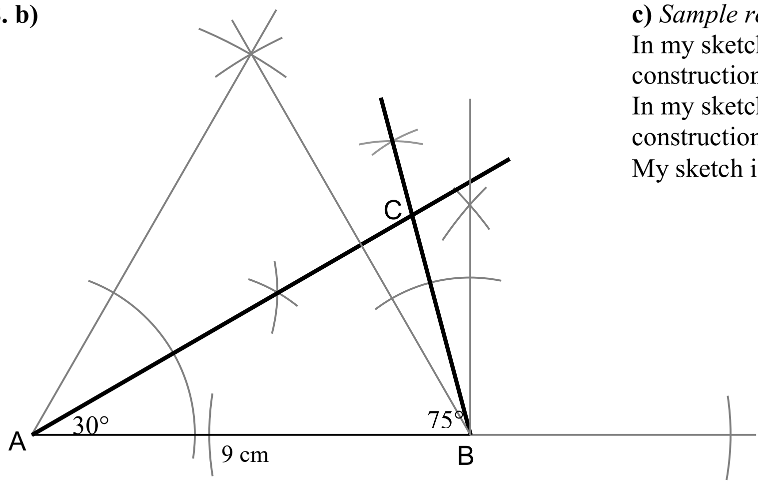
a)



8. a)



8. b)



c) *Sample response*:

In my sketch BC is 4.1 cm, and in my construction BC is 4.7 cm.

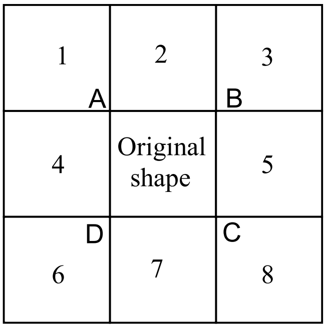
In my sketch AC is 8.7 cm, and in my construction AC is 9 cm.

My sketch is a good estimate.

4.2.1 Translations p. 122

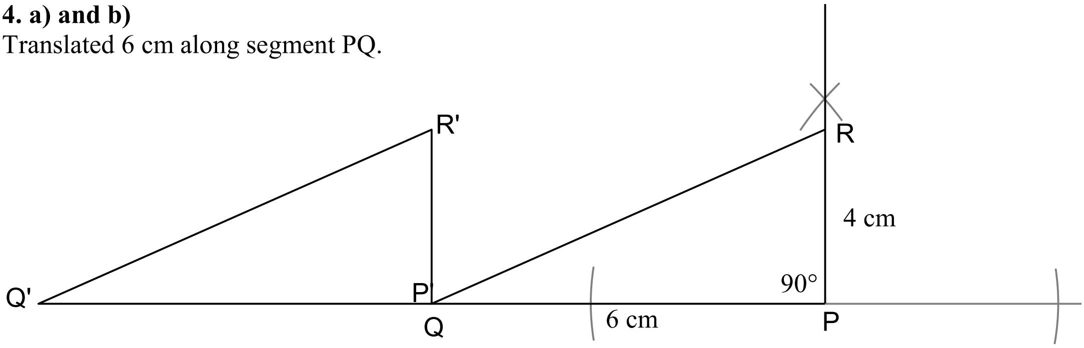
1. B is not a translation.
C is not a translation.
D is a translation.

2. a) and b)
- Square 1 is the result of a translation along CA.
- Square 2 is the result of a translation along DA or CB.
- Square 3 is the result of a translation along DB.
- Square 4 is the result of a translation along BA or CD.
- Square 5 is the result of a translation along AB or DC.
- Square 6 is the result of a translation along BD.
- Square 7 is the result of a translation along AD or BC.
- Square 8 is the result of a translation along AC.



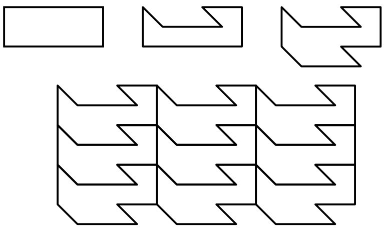
3. a) and b)
ΔABF can be translated along arrow AF to create ΔFED. F is the image of A. E is the image of F. D is the image of B.
ΔABF can be translated along arrow AB to create ΔBDC. B is the image of A. D is the image of F.
C is the image of B.
c) No.
d) No.

4. a) and b)
Translated 6 cm along segment PQ.



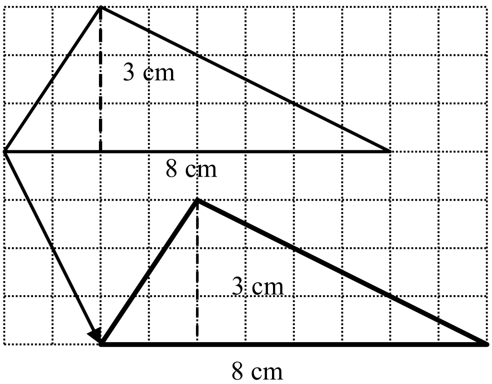
c) Area of ΔPQR: 12 cm^2; Area of ΔQQ'R' (or P'Q'R'): 12 cm^2

5. a) to d) *Sample response*:



6. a) Along PR
b) PQ, QR, RP, QP, or RQ

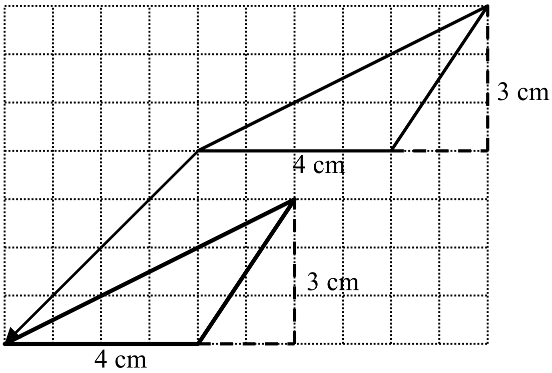
7. a), b), and c)



The areas of the original shape and the image are the same, 12 cm^2.

4.2.1 Translations p. 122

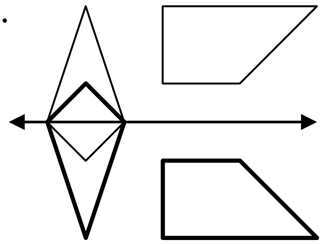
7. d)



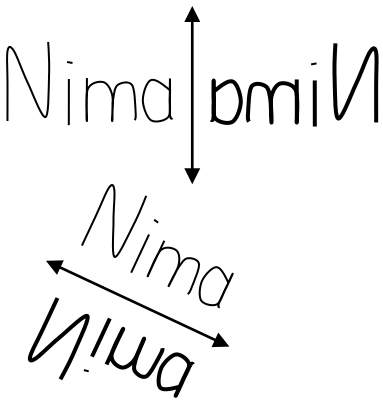
The areas of the original shape and the image are the same, 6 cm^2.

4.2.2 Reflections p. 127

1.

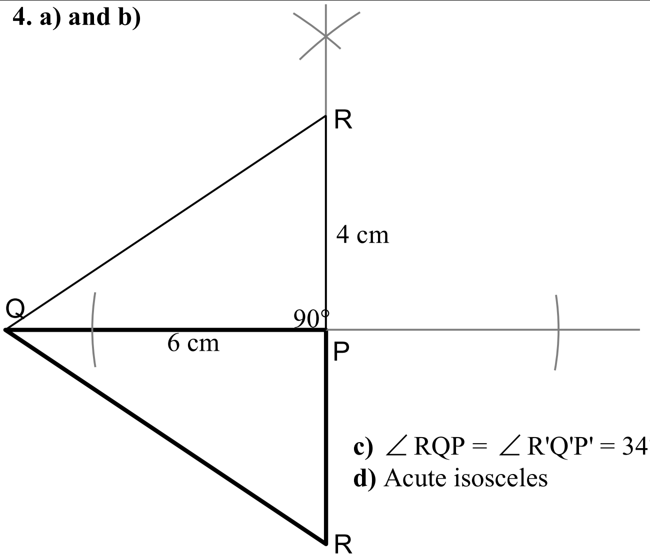


2. *Sample response*:



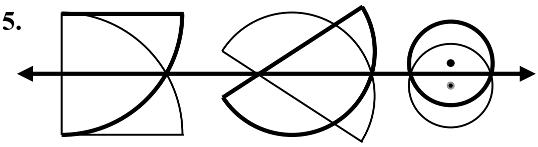
3. B is a reflection
C is not a reflection
D is not a reflection

4. a) and b)

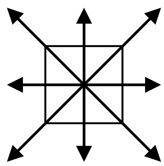


c) ∠RQP = ∠R'Q'P' = 34°
d) Acute isosceles

5.

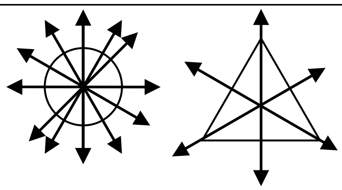


6. 3; *Sample response*:

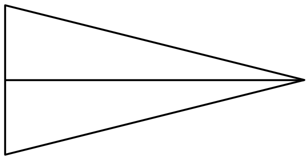


7. The circle and the triangle, but not the parallelogram.

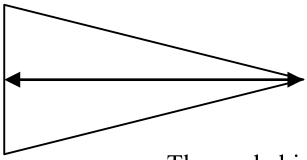
Any line that connects two points on the circumference of a circle through the centre is a reflection line.



8. a) and b)



c)

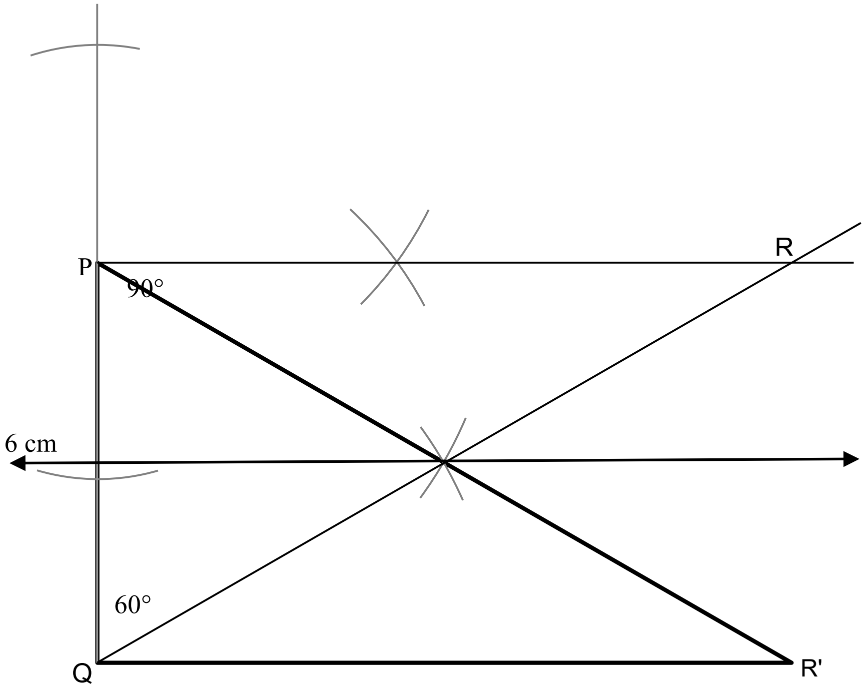


The angle bisector is the reflection line.

9. *Sample responses*:

a) TOT **b)** BOB **c)** BED or ICE

10. a) and b) The reflection line bisects PQ and is perpendicular to it.

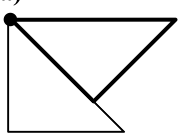


c) The areas of the original and image triangles are the same, 31.2 cm^2.

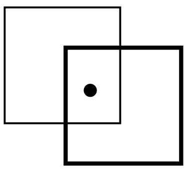
11. P' could be a reflection or a translation of P.

4.2.3 Rotations p. 132

1. a)



b)



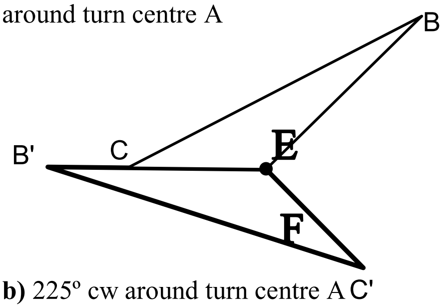
2. a) 315° cw **b)** 180° ccw

3. a) 290° cw **b)** 245° cw

c) Subtract the angle from 360°

4. a) 90° cw **b)** 180° cw

5. a) 135° (which is equal to ∠BAC) ccw around turn centre A

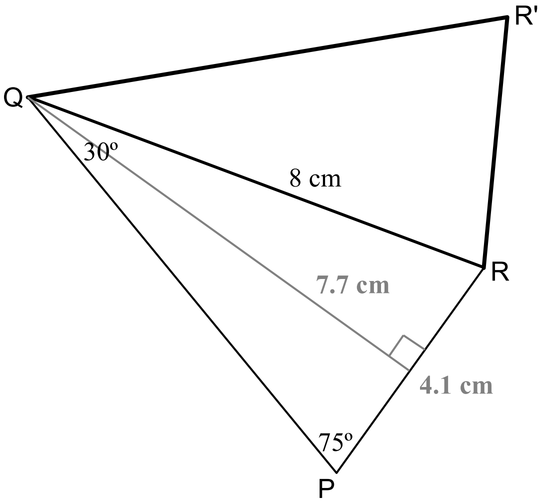


b) 225° cw around turn centre A

4.2.3 Rotations p. 132

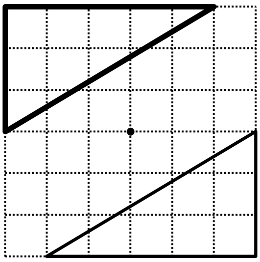
6. B is a rotation.
D is a rotation.
C cannot be a rotation.

7. a) and **b)** 30º ccw around Q

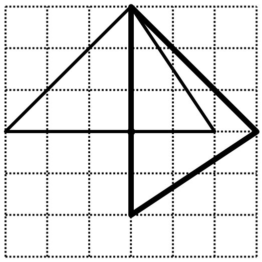


c) The areas are the same, 15.8 cm^2.

8. a), b), and c) *Sample response*:
7.5 square units
The triangle is rotated 180º cw around the turn centre shown.
7.5 square units



d) 7.5 square units
The triangle is rotated 90º cw around the turn centre shown.
7.5 square units

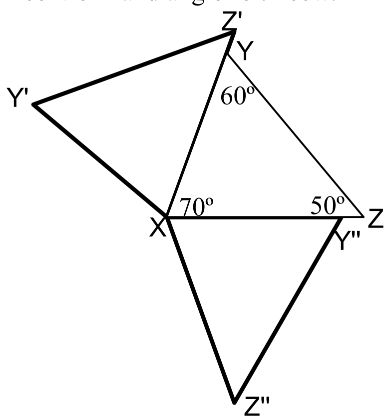


e) The area of the image is the same as the area of the original triangle.

9. a) 180° or 270° cw, and 90°, 180°, or 270° ccw around the centre of the square
b) *Sample response*: circle, regular hexagon

10. If the image is ΔXY'Z', the rotation could have been:
- turn centre X and angle 70° ccw, or
- turn centre X and angle 290° cw.

If the image is ΔXY"Z", the rotation could have been
- turn centre X and angle 70° cw, or
- turn centre X and angle 290° ccw.



4.3.1 Volume of a Rectangular Prism pp. 135–136

1. a) 24 m^3
b) 1260 cm^3
c) 1200 mm^3

2. a) 125 mm^3
b) 9 m^3
c) 9 m^3
d) 48 cm^3

3.

	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
a)	5	3	2	**30**
b)	10	6	4	**240**
c)	**4**	4	4	64
d)	5	**8**	6	240
e)	4	5	**3**	60

5. Yes

6. *Sample response*:
a) 3 cm × 2 cm × 10 cm

7. *Sample responses*:
a) First way: 4 boxes wide (8 cm each) × 5 boxes deep (12 cm each) × 6 boxes high (25 cm each)
Second way: 4 boxes wide (8 cm each) × 6 boxes deep (12 cm each) × 5 boxes high (25 cm each)
Third way: 5 boxes wide (8 cm each) × 8 boxes deep (12 cm each) × 3 boxes high (25 cm each)

b) First way: 32 cm × 60 cm × 150 cm, $V = 288{,}000$ cm³
Second way: 32 cm × 72 cm × 125 cm, $V = 288{,}000$ cm³
Third way: 40 cm × 96 cm × 75 cm, $V = 288{,}000$ cm³

8. a) *Sample response*:
Estimate: Box A: about 100,000 cm^3
Estimate: Box B: about 125,000 cm^3
Box B probably has the greater volume.
b) Box A: 87,768 cm^3
Box B: 132,651 cm^3
Box B has the greater volume.

10. Yes

4.3.2 Measurement Units p. 140

1. a) 0.003 m **b)** 520 L
c) 3000 mg **d)** 42,000 dm
e) 40.7 mm^2 **f)** 0.0054 m^3
g) 1 L **h)** 40 ha
i) 1.5 kg **j)** 1 m^3

2. a) Divide by 10,000
b) Multiply by 10,000
c) Multiply by 10
d) Multiply by 1,000,000
e) Divide by 100
f) Multiply by 10,000

3. *Sample responses*:
a) mL **b)** ha **c)** g

4. a) 1.4 g; *Sample response*: A pencil
b) 5.4 km; *Sample response*:
A distance along a road
c) 5400 cm^3; *Sample response*:
A small sack of rice
d) 5 mm^2; *Sample response*:
The area of the top of a push pin
e) 2 L; *Sample response*: The capacity of a jug
f) 35 g; *Sample response*:
The mass of a roll of tape

5. a) 7.5 mL **b)** 2.5 g

6. 7 mm]

7. *Sample response*: 20 × 15 × 20 cm

8. More

9. a) 900 mm **b)** 9 m
c) 225 cm^2 **d)** 400 th^2

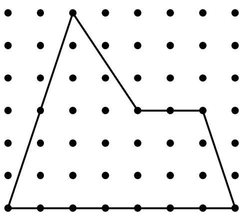
10. a) The number of units used for the measurement becomes smaller
b) The number of units used for the measurement becomes greater

4.3.3 Area of a Composite Shape p. 143

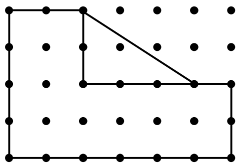
1. a) 16 cm^2 **b)** 18 cm^2 **c)** 10.5 cm^2

2. a) 24 m^2 **b)** 34 cm^2 **c)** 27 cm^2

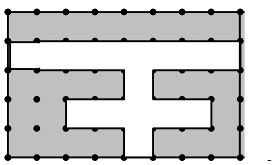
3. a) *Sample response*:
Area: 22.5 cm^2



4. *Sample response*:



5. *Sample response*:



7. *Sample response*: about 40 m^2

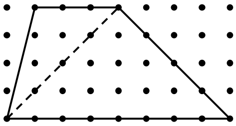
4.3.4 Area of a Trapezoid — p. 146

1. They are all trapezoids;
A and D are isosceles trapezoids.

2. A: 8 square units **B**: 7 square units
C: 7.5 square units **D**: 4 square units

3. a) 18 cm^2 **b)** 36 cm^2

4. a) 22 cm^2 **b)**



c) Triangle on the left: 6 cm^2
Triangle on the right: 16 cm^2
d) The triangles have the same height.
They have different bases.

5. 42 m^2

6. a) 3 m
b) 4 m

7. 228 m^2

4.3.5 Circumference of a Circle — p. 149

1. a), b), and c) *Sample responses*:

Diameter (*d*)	Circumference (*C*)	$C \div d$
8 cm	25 cm	3.125
5 cm	16 cm	3.2
14 cm	44 cm	3.14

d) Almost

2. a) 13 mm **b)** 132 cm **c)** 66 cm

3. a) 63 cm **b)** 114 cm

4. 75 cm

5. 28 m long

6. a) 10 cm **b)** 31.8 cm

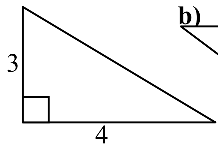
7. a) 5 cm **b)** 15.9 cm

8. a) They are the same.
b) They would be equal.

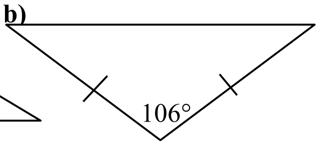
9. a) The label is a rectangle.
b) *Sample response*: about 22.5 cm by 11.5 cm

UNIT 4 Revision — pp. 150–151

1. *Sample response*:
a)



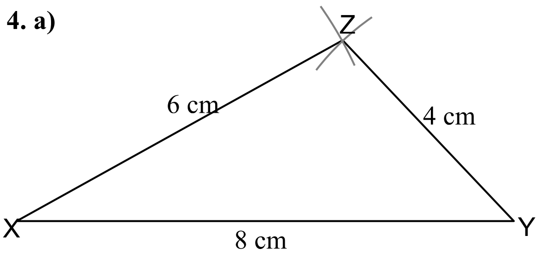
b)



2. a) 92° **b)** Obtuse scalene

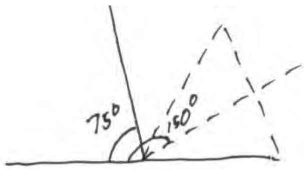
3. No

4. a)

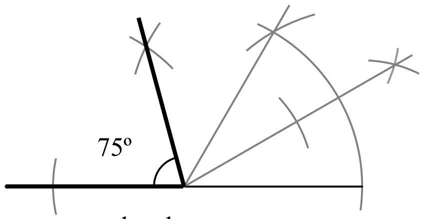


b) Obtuse scalene

5. a) *Sample response*:

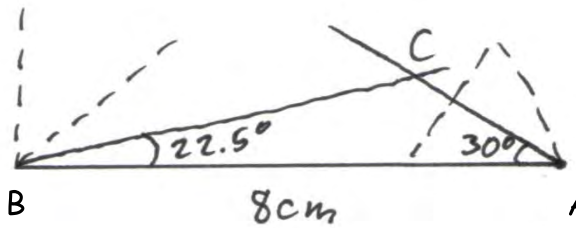


b) *Sample response*:

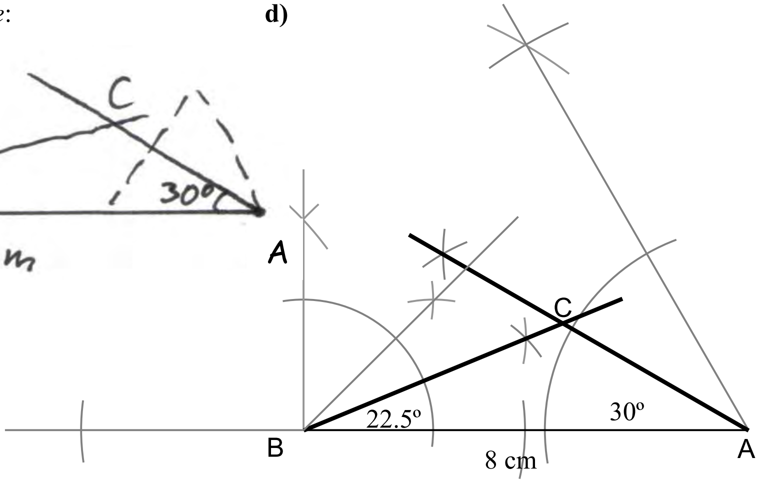


It is close to my sketch.

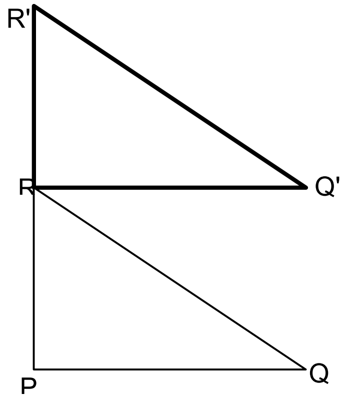
6. a) and b) *Sample response*:



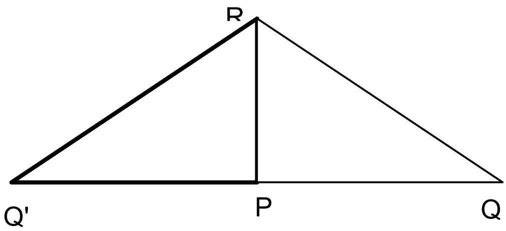
d)



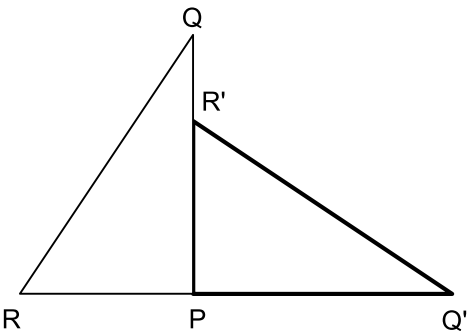
7. a), b), and c) The triangle is translated up 4 cm along PR.



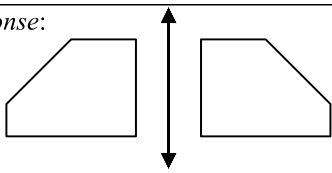
8. a), b), and c) The triangle is reflected in PR.



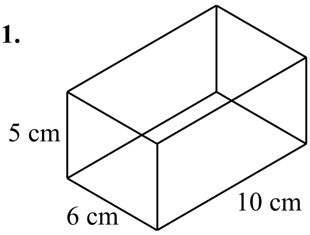
9. a), b), and c) *Sample response*: The triangle is rotated 90° cw around turn centre P.



10. *Sample response*:



11.



12.

	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
a)	**5**	5	4	100
b)	2.0	**2.5**	6.0	30

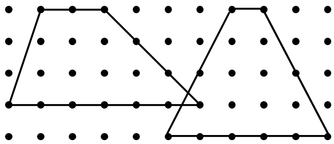
13. 2400 L

14. a) 36 mm^2 **b)** 5.4 hg **c)** 210 daL

15. a) Divide by 100,000 **b)** Multiply by 1000

16. a) 222 m^2 **b)** 50 m^2
c) 16 cm^2 **d)** 12 cm^2

17. *Sample response*:



18. About 88 cm

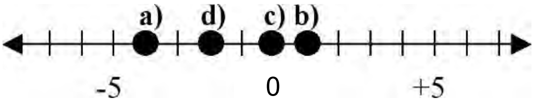
19. a) About 46 mm **b)** About 398 cm

UNIT 5 INTEGERS pp. 153–180

Getting Started — Skills You Will Need p. 154

1. $a < b$

2.



3. Yes

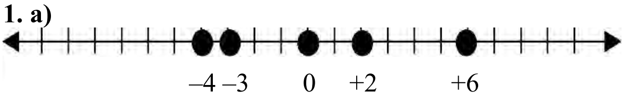
4. a) > **b)** < **c)** < **d)** >

5. a) +3747 **b)** –86
c) –14 **d)** +3

6. a) +10°C
b) 7 degrees colder

5.1.1 Integer Models p. 157

1. a)



2. a) –4
b) *Sample response*: 4 white counters: 
c) +4

3. –6, +6

7. a) +11°C **b)** The last clue

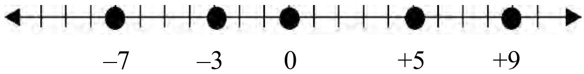
4. a) –6 and +4
b) +10 and –10
c) –2 and +6

5. Part b)

6. a) +4 **b)** –14
c) –8 **d)** +8

5.1.2 Comparing and Ordering Integers p. 160

1. a)



b) –7, –3, 0, +5, +9

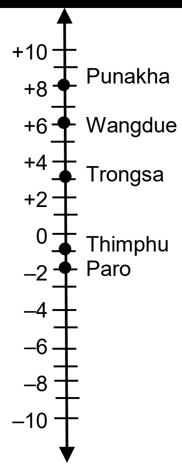
2. a) *Any two of* –4, –5, –6, –7, –8, –9
b) *Any two of* –1, 0, +1, +2, +3
c) –2, –1
d) *Any two of* 0 or any positive integer
e) *Any two of* –15, –16, –17, …
f) *Any pair of* –4 and +4, or –3 and +3, or –2 and +2, or –1 and +1
g) *One pair in this pattern*:

+1	+3
0	+4
–1	+5
–2	+6
–3	+7

3. a) +4 **b)** +4
c) –3 **d)** +3

4. a) < **b)** >
c) < **d)** <

5. a)



b) Paro, Thimphu, Trongsa, Wangdue, Punakha

6. a) –25, –12, +8, +16, +25
b) –140, –120, –100, –10
c) –48, –6, 0, +4, +210

7. a) Nov. 17 **b)** Nov. 18
c) Nov. 16 **d)** Nov. 17

8. a) –18, –11, –7, +3, +4, +8
b) Tiger Woods

9. a) –5 **b)** I am a negative number. I am less than –2.

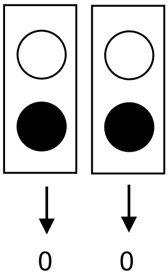
CONNECTIONS: Time Zones p.161

1. a) –11
b) +1
c) –5
d) –14
e) +2

2. 24 h clock time (12 h clock time)
a) 1:00 (1:00 am)
b) 13:00 (1:00 pm)
c) 7:00 (7:00 am)
d) 22:00 the previous day (10:00 pm the previous day)
e) 14:00 (2:00 pm)

5.1.3 The Zero Property p. 163

1. *Sample response*:



+2 + (–2) = 0

2. a) 0 **c)** +1

3. a) –1 **b)** +1
c) +1, –1 *in any order*
d) +1, –1, –1, –1 *in any order*
e) +1, +1, –1, –1 *in any order*
f) –1, –1, –1

4. a) +30 **b)** –30

5.2.1 Adding Integers Using the Zero Property p. 167

1. a) +1 **b)** +1 **c)** –4
d) +9 **e)** –4 **f)** –8

2. a) +2 **b)** +2 **c)** –3
d) +8 **e)** –2 **f)** –10

3. a) +15°C **b)** +16°C **c)** +14°C

4. a) –6 **b)** –2 **c)** –3
d) –8 **e)** –1 **f)** –7

6. a) > **b)** < **c)** = **d)** > **e)** <

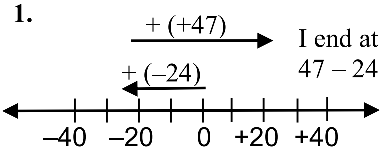
7. *Sample responses*:
a) (+1) + (–1) + (+1) + (–1) = 0
b) (+1) + (+1) + (+1) + (–1) = (+2)
c) (–1) + (+1) + (–1) + (+1) + (–1) = –1
d) (+1) + (+1) + (+1) + (+1) = (+4)

9. 0

10. a) True **b)** True **c)** False **d)** False

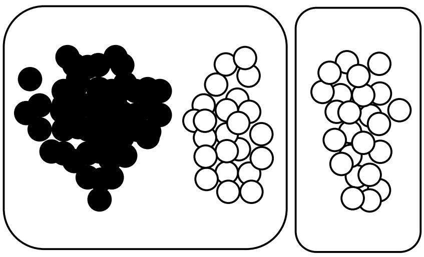
5.2.2 Adding Integers that are Far from Zero p. 170

1.



I end at +23 because 47 – 24 = 23.

2.



24 pairs = 0 47 – 24 = 23 counters

I match 24 of my 47 white counters with the 24 black counters to make 0 and I am left with 23 white counters, or +23.

3. a) +60 **b)** –36
c) –60 **d)** +36

4. a) 10
b) 20

5.

	Start (°C)	Change (°C)	Final (°C)
a)	–12	+15	**+3**
b)	+9	**+7**	+16
c)	**–22**	+12	–10
d)	–15	**+3**	–12

6. *Sample response*: (+159) + (–3) = 156

5.2.2 Adding Integers that are Far from Zero [Continued] p. 170

7. a) > **b)** > **c)** = **d)** > **e)** <

8. a) i) –40 **ii)** –40 **iii)** –560 **iv)** +10

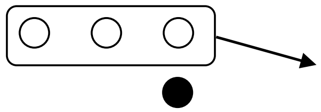
9. a) i) +12 **ii)** +80 **iii)** –27 **iv)** +25

10. Yes

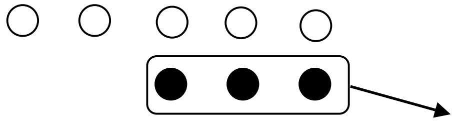
5.2.3 Subtracting Integers Using Counters p. 174

1. a) –3

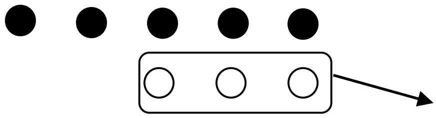
2. a) –1; *Sample response*:



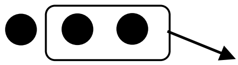
b) +5; *Sample response*:



c) –5; *Sample response*:



d) –1; *Sample response*:



3. Yes

4. a) +4 **b)** +4

5. a) –6 **b)** +17 **c)** –75

6. a) (–5) – (+2); (–5) – (–7)
b) (+4) – (–3); (+4) – (+7)

7. a) –7; +2 **b)** +7; –3

8. a) +27 **b)** –3
c) –40 **d)** –16

9.

	Golfer	Day 1	Day 2	Change (Day 2 – Day 1)
a)	Dechen	–4	–1	**+3**
b)	Dawa	+2	+6	**+4**
c)	Novin	–2	+4	**+6**
d)	Meto	**–7**	+3	+10
e)	Karma	–7	**–8**	–1

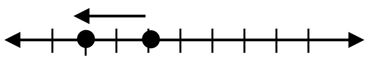
10. a) +21 **b)** –17
c) –136 **d)** –245

11. Yes

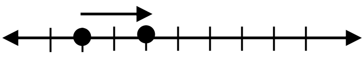
5.2.4 Subtracting Integers Using a Number Line p. 178

1. a) (+25) – (+5) = +20 **b)** (–10) – (+20) = –30

2. a) i) –2;



ii) +2;



b) *Sample response*:
There are the same number of spaces between –3 and –5, but the arrows go in opposite directions.

3. a) –10 **b)** 0 **c)** –35 **d)** +70 **e)** –90

4. a) –2°C
b) (+10) – (+12) = –2
c) (+12) + (–2) = +10

5. b) Yes

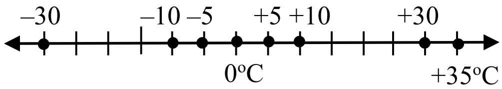
6. *Sample response*:
(+4) – (+8) = –4
(+5) – (+9) = –4
(–3) – (+1) = –4

8. a) (+2320) – (+2120) = +200
b) (+2120) – (+2235) = –115
c) (+1250) – (+2120) = –870
d) (+2120) – (+2320) = –200
e) (+2320) – (+2235) = +85
f) (+2235) – (+1250) = +985

9. a) The second expression, ■ – (–1), is greater.
b) Yes

UNIT 5 Revision — p. 180

1. a)

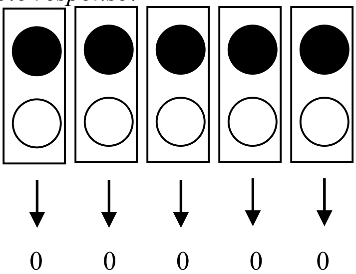


b) –30 and +30, –10 and +10, –5 and +5;

2. +3

3. a) False **b)** True **c)** True

4. *Sample response*:



(–5) + (+5) = 0

5. a) +2ºC **b)** +4ºC **c)** +1ºC

7. a) –5 **b)** +3 **c)** 0

8. a) +10 **b)** +70
c) –70 **d)** –10

9. a) < **b)** <

10. a) +1 **b)** +5 **c)** –10

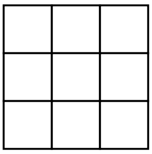
11. a) –340 **b)** +81 **c)** –60

UNIT 6 ALGEBRA pp. 181–218

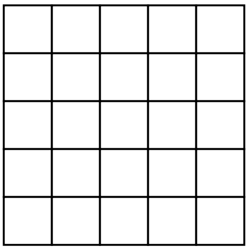
Getting Started — Skills You Will Need pp. 181–182

1. *Sample responses*:

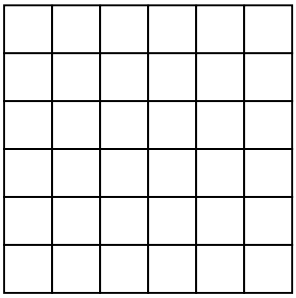
a)



b)



c)

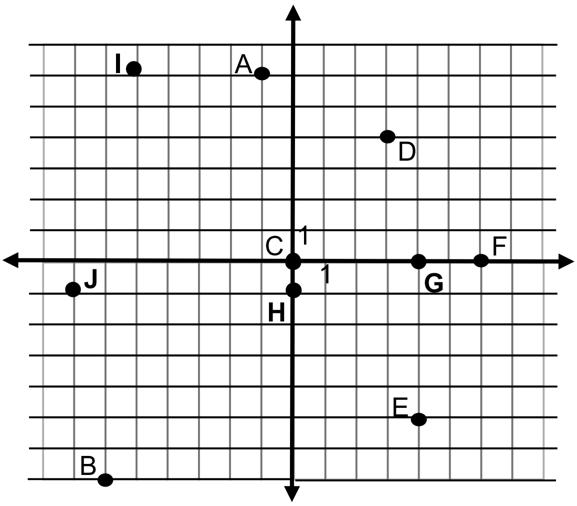


2. a) $n = 3$ **b)** $h =$ 47
c) $n = 9$ **d)** $a = 3$

3. a) and d)

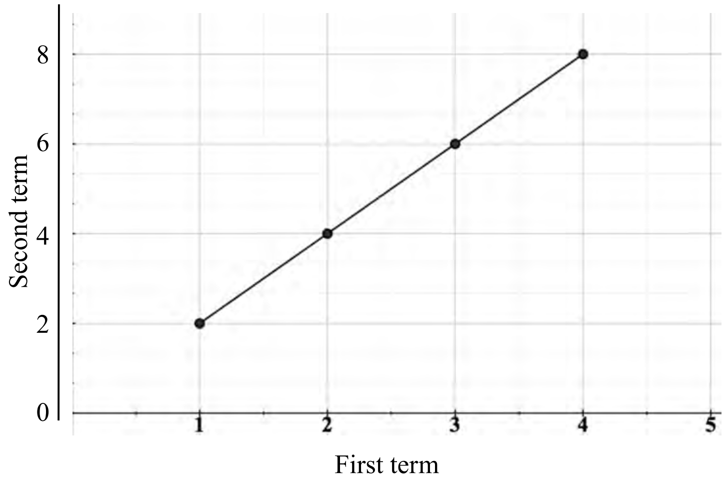
4. A(–1, 6); B(–6, –7); C(0, 0); D(3, 4); E(4, –5); F(6, 0)

5.



Getting Started — Skills You Will Need [Continued] pp. 181–182

6. a) 2:4
b) *Sample response*: 1:2, 4:8, 3:6
c) and d) (1, 2), (2, 4), 3, 6), (4, 8)



e) They form a straight line.

7. a) i) *bh* ÷ 2
ii) *bh*
b) i) The area would double for both shapes.
ii) The area would be multiplied by 4 for both shapes.
iii) The area would not change for either shape.

6.1.1 Using Variables to Describe Pattern Rules p. 186

1. a) Variable is *h*; coefficient is 3; constant is 5.
b) Variable is *m*; coefficient is –2; constant is –4.
c) Variable is *q*; coefficient is 1; constant is 6.
d) Variable is *n*; coefficient is 5; constant is 3.

2. a) Variables are *s* and *P*; coefficient is 4 (or 4 for *s* and 1 for *P*); (constant is 0).
b) Variables are *A* and *r*; coefficient is π (or π for r^2 and 1 for *A*); (constant is 0).

3. a)

Figure number	Number of shapes
1	6
2	8
3	10
4	12
5	**14**
6	**16**
7	**18**

b)

Figure number	Number of shapes
1	35
2	30
3	25
4	20
5	**15**
6	**10**
7	**5**

4. a)

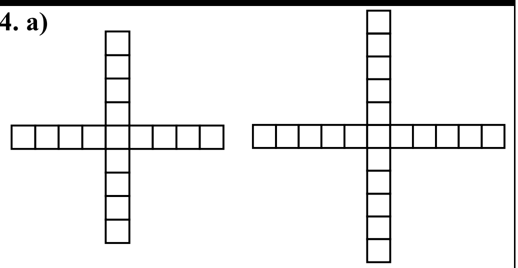


Figure 4 Figure 5

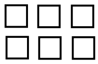
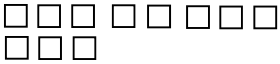
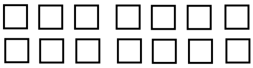
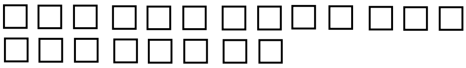
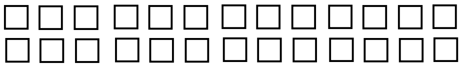
b)

Figure number	Number of squares
1	5
2	9
3	13
4	17
5	21
6	25
7	29
8	33

4. c) $4f + 1$ or $5 + 4(f - 1)$
d) 81

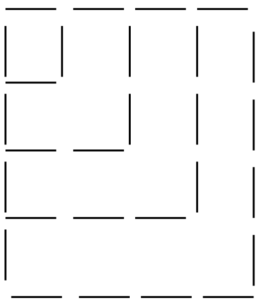
5. *Sample responses*:

a)

Figure number	Figure	Number of squares
1		6
2		11
3		16
4		21
5		26

b) $6 + 5(f - 1)$ or $5f + 1$

6. a)



b)

Figure number	Number of matchsticks
1	4
2	10
3	18
4	28
5	40
6	54
7	70
8	**88**

7. $2(n + 1) + 2$ and $2n + 4$

6.1.2 Creating and Evaluating Expressions p. 189

1. a) vi) **b)** v) **c)** ii)
d) vii) **e)** iv) **f)** i)
g) iii)

2. a) 23 **b)** –23 **c)** 44
d) 1.7 **e)** 452 **f)** 29

4. a) $72r$
b) You would subtract 38 from $72r$, $72r - 38$

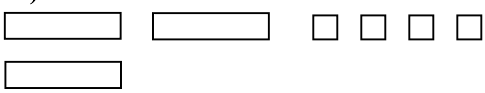
5. a) $7500m$ **b)** $7500 \times 12 =$ Nu 90,000

6. a) $80 + 30n$ **b)** $80 + 30 \times 25 =$ Nu 830

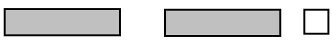
7. *Sample response*:
Lemo bought 2 kg of meat. The price per kilogram is Nu x. She also bought some oranges worth Nu 60. How much did she spend altogether?

6.1.3 Simplifying Expressions p. 193

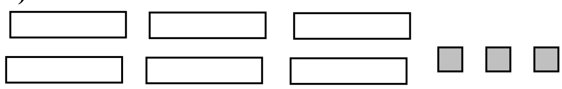
1. a)



b)



c)



d)

6.1.3 Simplifying Expressions [Continued] p. 193

2. a) $8n + 8$ **b)** $2m - 4$
c) $2m - 8$ **d)** $5k - 5$

3. *Sample responses*:
a) $n + (n + 4)$ and $(n + 2) + (n + 2)$
b) $n + (2n - 6)$ and $(n - 6) + n + n$
c) $(3 + 2x) + (2 - 3x)$ and $(6 - x) - 1$
d) $4k - 2 + (9 - 7k)$ and $2 - (3k - 5)$

4. a) $7b \div 2$; $8b \div 2$; $9b \div 2$
b) $7b \div 2 + 8b \div 2 + 9b \div 2$; $12b$

5. a) *Sample response:*
The sum of one diagonal:
$N + (N + 11) = 2N + 11$
The sum of the other diagonal:
$(N + 1) + (N + 10) = 2N + 11$

N	***N* + 1**
***N* + 10**	***N* + 11**

b) The five numbers are:
n, $n + 10$, $n + 20$, $n + 30$, and $n + 40$
If you add them you get $5n + 100$.
If you multiply $(n + 20)$ by 5, you get $5n + 100$.

6. a) $9x + 7$
b) *Sample response*: $(3x + 5) + (6x + 2)$

CONNECTIONS: Using Variables to Solve Number Tricks p. 193

1. *Sample response*:
A. 20 **B.** 40 **C.** 36 **D.** 18 **E.** 20
I got same number I started with.

2. *Sample response*:
A. 10 **B.** 20 **C.** 16 **D.** 8 **E.** 10
I got same number I started with.

3. a) A. n **B.** $2n$ **C.** $2n - 4$ **D.** $n + 2$ **E.** n

4. *Sample response*:
A. Think of a number.
B. Add 8.
C. Double it.
D. Subtract 14.
E. Take half.
F. Subtract 1.

6.2.1 Solving Equations Using Models p. 197

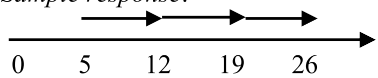
1. a) ii) **b)** iii) or v) **c)** iv)
d) i) **e)** iii)

2. a) $m = 2$ **b)** $m = 3$ **c)** $m = 4$
d) $m = 27$ **e)** $m = 3$

3. a) Add a number to 10. The result is 28.
b) A number is multiplied by 7. The result is 35.
c) Multiply a number by 4 and then add 8. The result is 28.
d) The difference between triple a number and 5 is 16.
e) Multiply a number by 5 and add 10. The result is 55.
f) The difference between 30 and triple a number is 27.

4. a) $p = 18$ **b)** $k = 5$ **c)** $n = 5$
d) $p = 7$ **e)** $m = 9$ **f)** $k = 1$

5. a) *Sample response*:



b) $5 + 7k = 26$

6. a) $4f + 3 = 71$ **b)** $f = 17$

7. *Sample responses*:
Number line; I can just add to get the answer.

10. $x = 6$

6.2.2 Solving Equations Using Guess and Test pp. 199–200

1. *Sample responses*:
a) 130 **b)** 5 **c)** 80
d) 50 **e)** 50 **f)**

2. *Sample responses*:
a) 120 **b)** 65 **c)** 60

3. a) $k = 21$ **b)** $k = 11$
c) $m = 52$ **d)** $t = 0.2$

4. a) $n + (n + 10) = 124$; $n = 57$; the numbers are 57 and 67.
b) $8k = 344$; $k = 43$ **c)** $2m - 35 = 79$; $m = 57$

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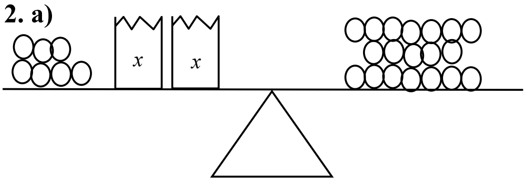
5. a) $t = b + 4$ **b)** 79	**7. a)** $n + (n + 1) = 284$
6. a) C = K – 273 **b)** 150°C	**8. a)** 20 **b)** $x = 24$

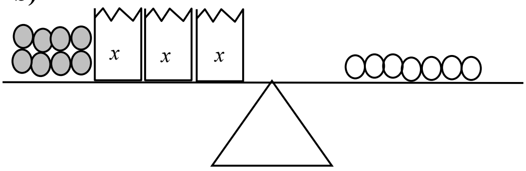
6.2.3 Solving Equations Using Inverse Operations pp. 202–203

1. a) $2x - 1 = 11$
b) Add 1 and then divide by 2.
c) $x = 6$

2. a)



b)



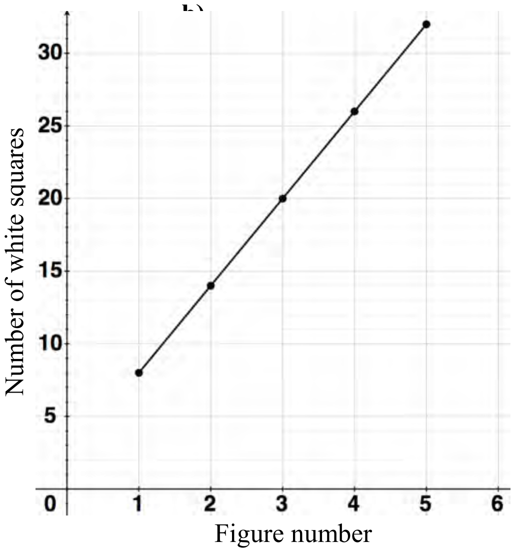
3. a) Subtract 18, divide by 12.
b) Add 19, divide by 7.
c) Subtract 200, divide by 9.
d) Subtract 16, divide by 6.

4. a) $k = 87$ **b)** $m = 7$
c) $t = 32$ **d)** $k = 8$

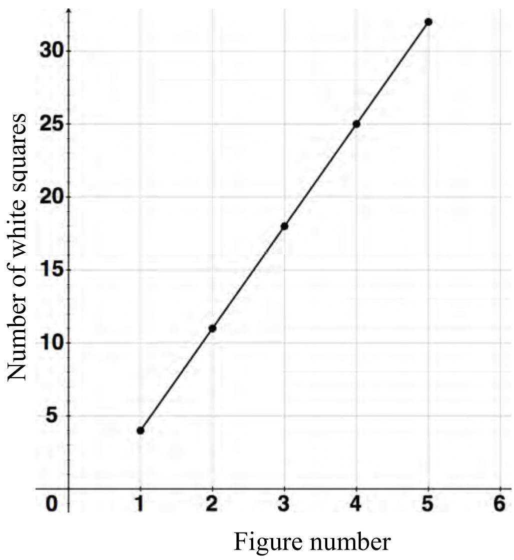
5. a) $2700 = 120 + 200d$
b) $d = 12.9$; the money will last for 12 days.

6.3.1 Graphing a Relationship p. 208

1. a)



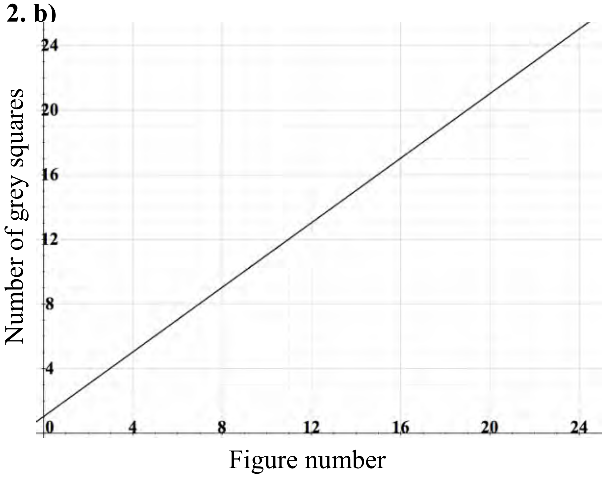
b)



2. a)

Figure number	Number of grey squares
1	2
2	3
3	4
4	5
5	6

2. b)



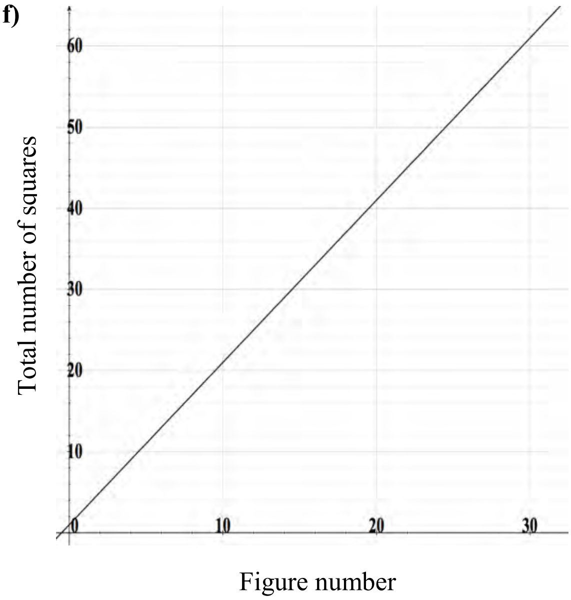
c) Figure 16

d) 22

e)

Figure number	Total number of squares
1	3
2	5
3	7
4	9

f)

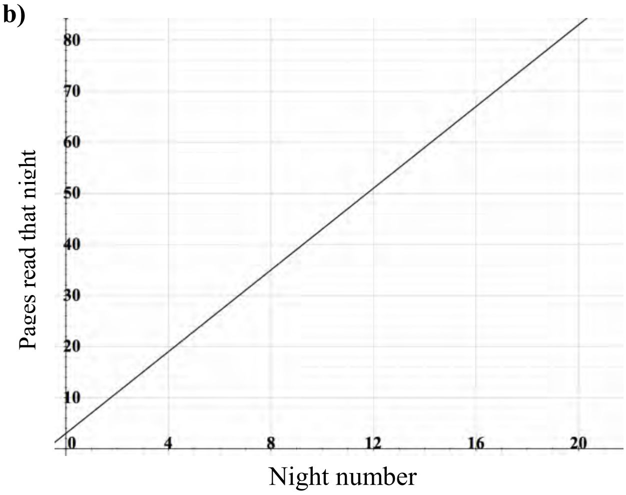


g) Figure 30

3. a)

Night number	Pages read that night
1	10
2	14
3	18
4	22

b)

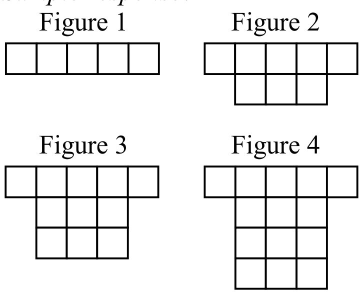


c) Night 18

4. a)

x	y
1	5
2	8
3	11
4	14

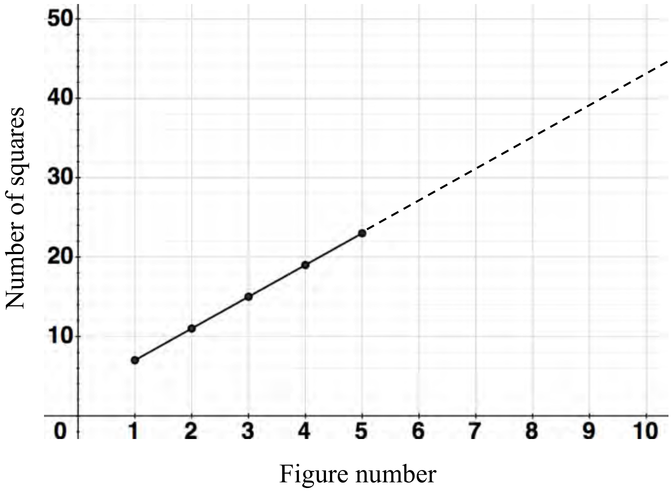
b) *Sample response*:



5. a)

Figure number	Total number of squares
1	7
2	11
3	15
4	19
5	23

b) and c)
The 10th figure has 43 squares.

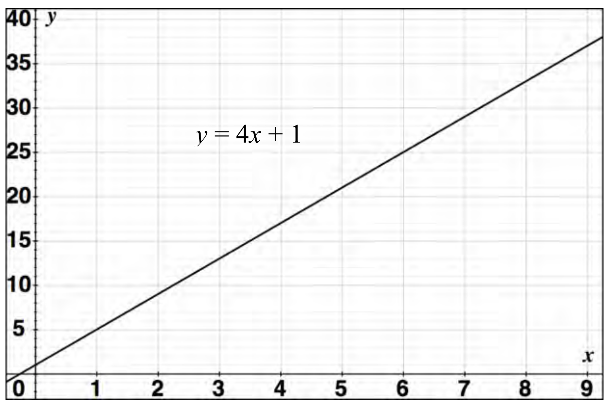


d) *Sample response*:
I used the pattern in the right column, which was adding 4 for each new figure number; $19 + (6 \times 4) = 43$.

6.3.2 Examining a Straight Line Graph p. 211

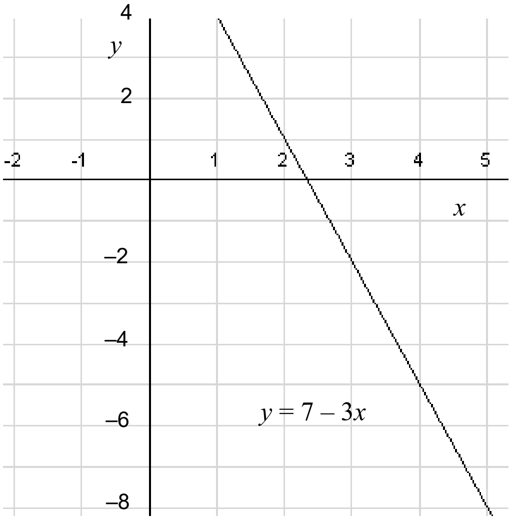
1. a)

x	1	2	3	4	5
y	5	9	13	17	21



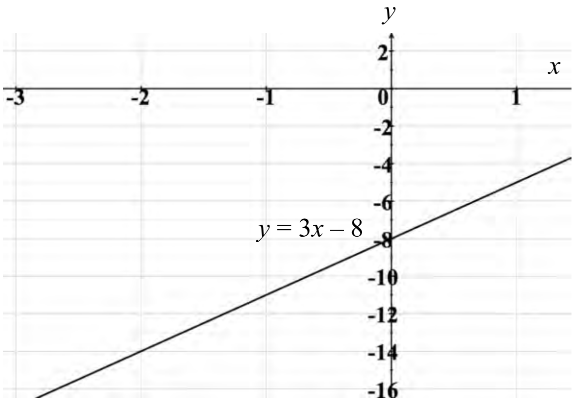
b)

x	1	2	3	4	5
y	4	1	– 2	– 5	– 8



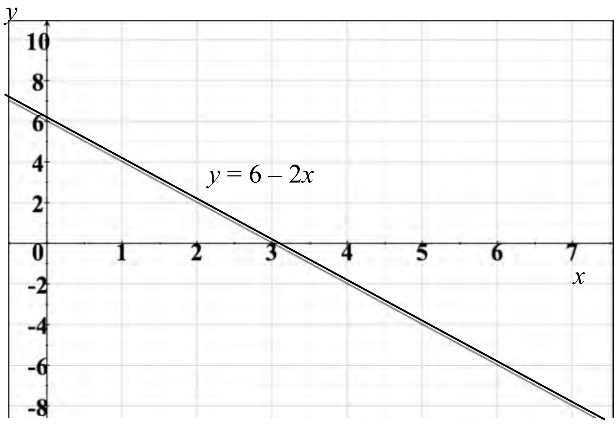
c)

x	1	2	3	4	5
y	–5	–2	1	4	7



d)

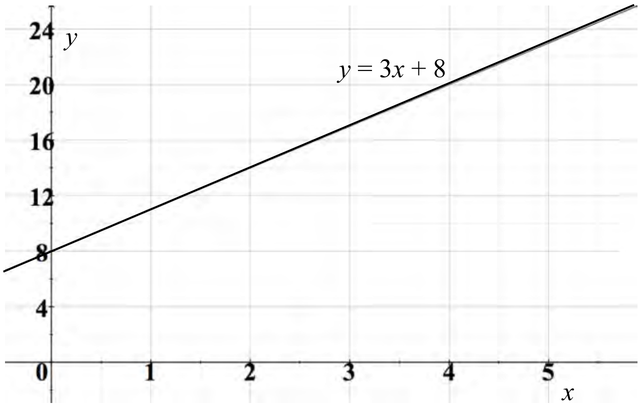
x	1	2	3	4	5
y	4	2	0	–2	–4



2. a) $x = 8$ **b)** $x = -2$ **c)** $x = -2$ **d)** $x = 2$

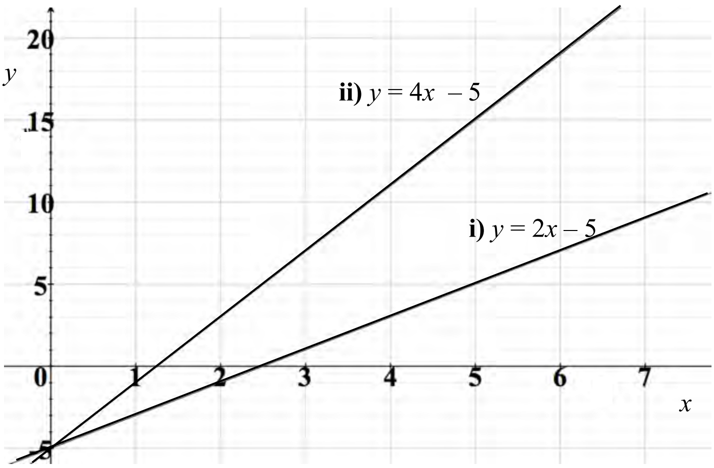
3. *Sample responses*:
a) $4x + 1 = 37; x = 9$ **b)** $7 - 3x = 1; x = 2$ **c)** $3x - 8 = -2; x = 2$ **d)** $6 = 6 - 2x; x = 0$

4.



5. B is correct

6. a)



b) They both go through (0, –5). They have different slopes.

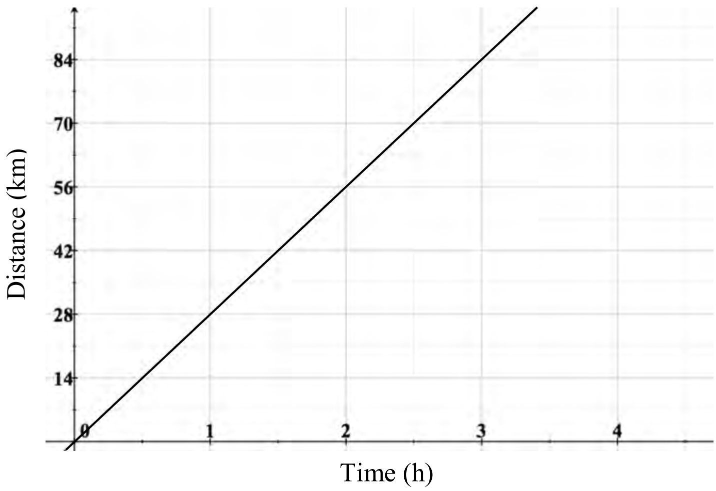
c) $y = 4x - 5$

d) $x = 4.5$

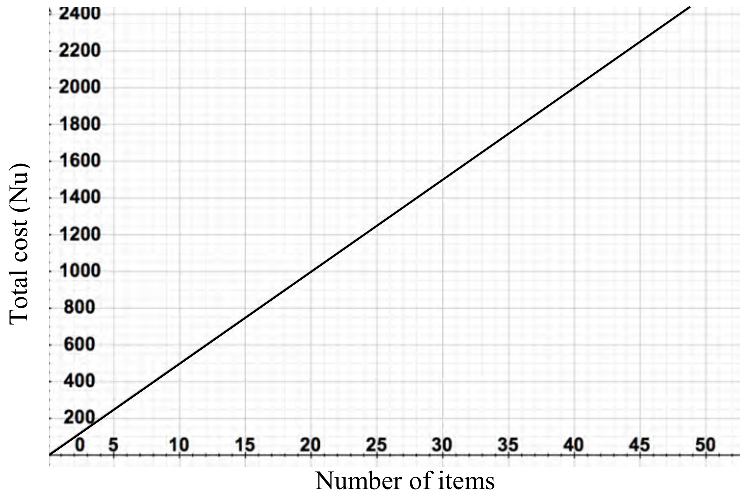
6.3.3 Describing Change on a Graph p. 215

1. a) About 67.5 km

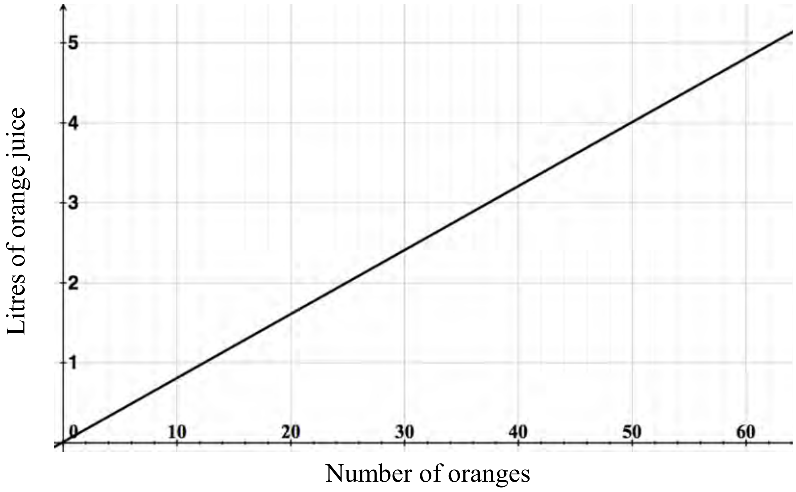
2. a)



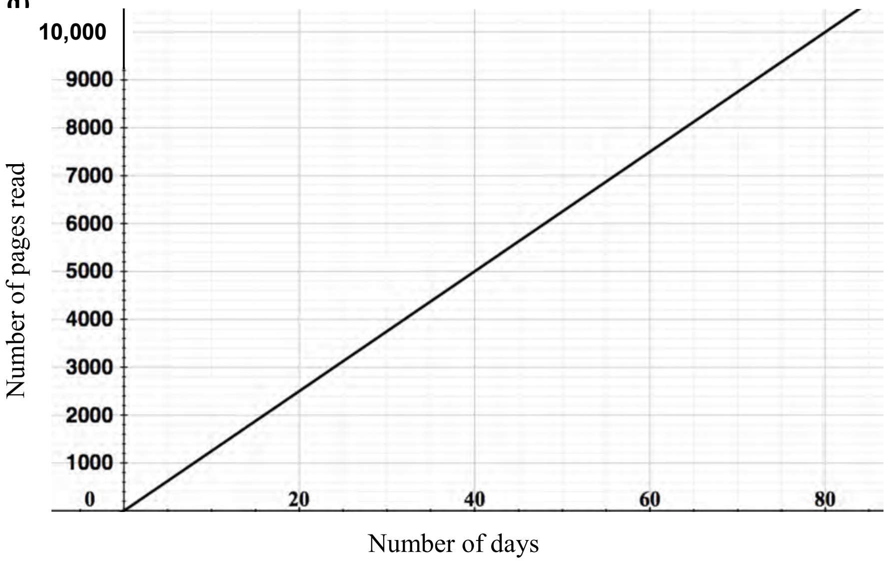
b)



c)



d)



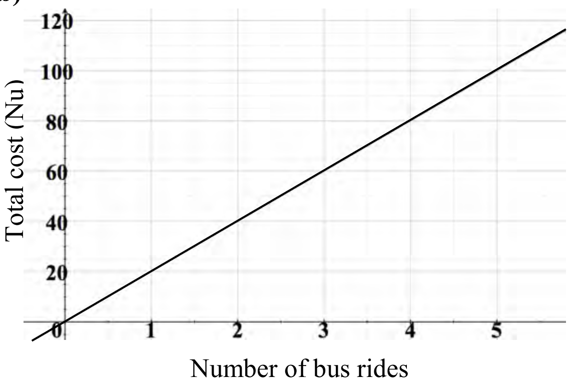
3. a) i) 70 km **ii)** About 3.1 h **b)** 46 items

c) i) 4.8 L **ii)** 25 oranges **d)** 80 days

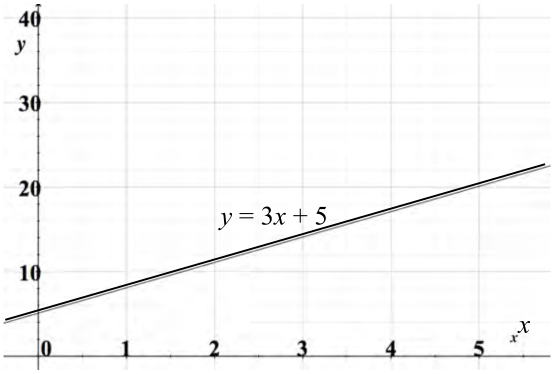
4. a)

Number of bus rides	1	2	3	4	5
Total Cost (Nu)	20	40	60	80	100

b)



5. a)



d) No

6. It will increase by 8.

UNIT 6 Revision pp. 217–218

1. a) Variable is k; coefficient is –1; constant is 5.

b) Variable is m; coefficient is 3; constant is $\frac{1}{2}$.

2. a)

x	y
1	10
2	13
3	16
4	**<u>19</u>**
5	**<u>22</u>**

b)

x	y
1	28
2	26
3	24
4	**<u>22</u>**
5	**<u>20</u>**

3. $3 + 2(f - 1) + (f - 1)$ and $2f + f$

4. a) $200 + 20n$ **b)** Nu 360

5. *Sample response*:
How far would you have travelled if you drove 15 km and then drove for x hours at 30 km/h?

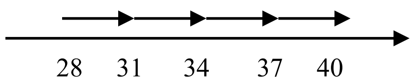
6. a) $12n + 5$ **b)** $-m - 12$
c) $-8n + 11$ **d)** $5m - 6$

7. a) $4n + 10$

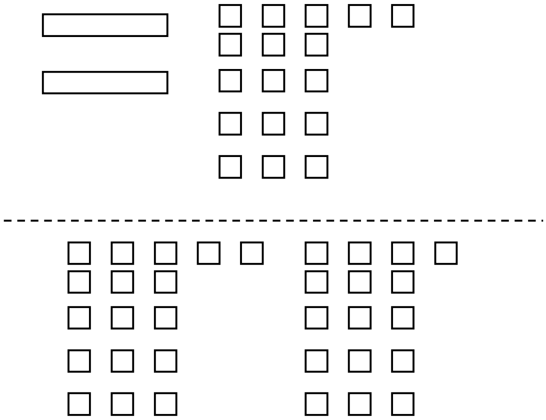
8. *Sample responses*:
a) $(2n + 6) + 2n$
b) $8n - 3n + 10 - 20$

9. a) The difference between 4 times a number and 5 is 23.
b) 8 more than 6 times a number is 50.

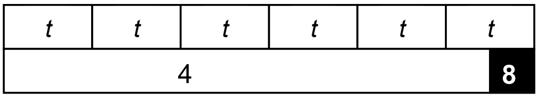
10. *Sample responses*:
a)



10. b)



c)



11. a) $5f + 1 = 101$ **b)** $f = 20$

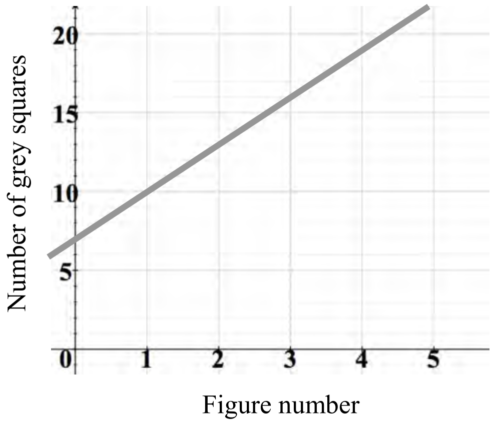
12. *Sample responses*:
a) $k = 100$ **b)** $k = 11$

13. a) $n + (n + 8) = 164$; $n = 78$
b) $3 + 3n = 255$; $n = 84$

14. a) $2x + 4 = 12$
b) Subtract 4 and divide by 2.
c) $x = 4$

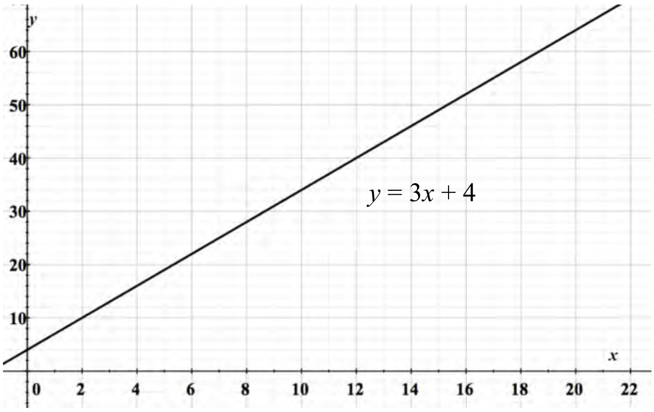
15. a) $k = 117$ **b)** $t = 87$

16.



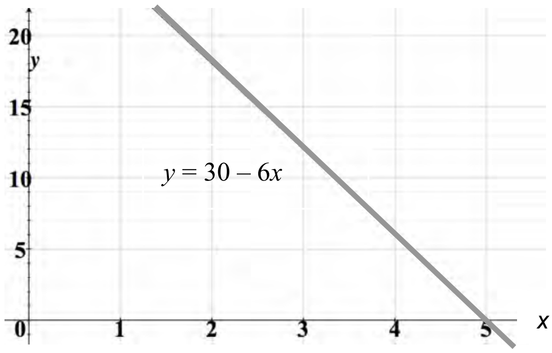
17. a)

x	1	2	3	4	5
y	7	10	13	16	19



b)

x	1	2	3	4	5
y	24	18	12	6	0



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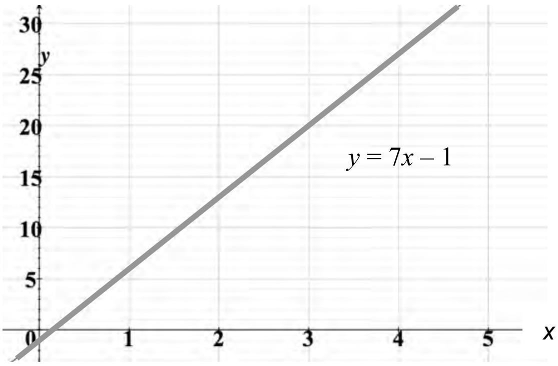
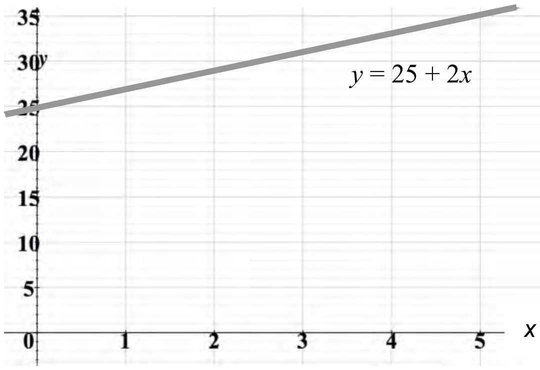
c)

x	1	2	3	4	5
y	6	13	20	27	34



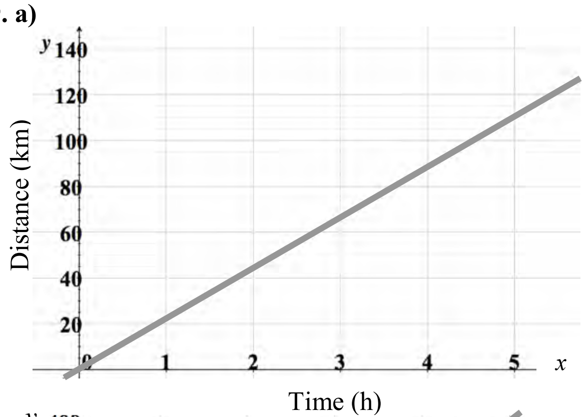
d)

x	1	2	3	4	5
y	27	29	31	33	35

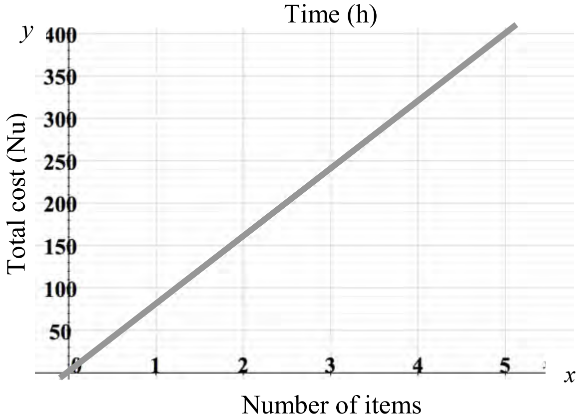


18. a) $x = 21$ **b)** $x = 3$ **c)** $x = 4$ **d)** $x = 3$

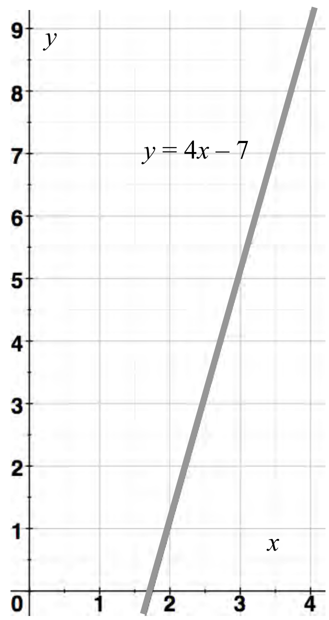
19. a)



b)



20.



Getting Started — Skills Your Will Need pp. 219–220

1. a) 15 **b)** 40
c) 11 **d)** *Sample response*: $\frac{9}{10}$
e) 56 **f)** 85
g) 15 **h)** *Sample response*: About 53

2. $\frac{2}{5}$, 0.4, 40%

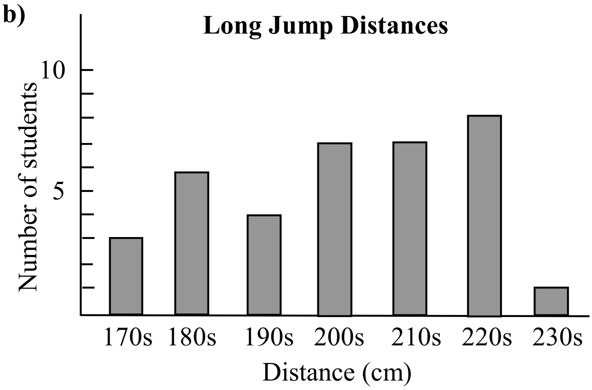
3. *Sample responses*:
a) i) About 55%
ii) About 72%
iii) About 83%
b) 62%
c) Yes.

4. a) *Sample response*:
About 54 kg per year
b) 30 kg per year
c) No.

5. a) Nu 11 **b)** Nu 5 **c)** Nu 5

6. *Sample responses*:
a) Long Jump Distances (cm)

17	2 3 5
18	5 6 6 7 8 9
19	3 3 6 8
20	0 2 5 5 6 7 8
21	2 2 2 4 5 5 8
22	0 1 1 2 3 7 8 9
23	0

b)


7.1.1 Describing Theoretical Probability p. 223

1. a) $\frac{1}{16}$ **b)** $\frac{1}{8}$ **c)** $\frac{3}{16}$ **d)** $\frac{1}{2}$ **e)** $\frac{3}{4}$

2. a) i) $\frac{1}{13}$ **ii)** $\frac{1}{4}$ **iii)** $\frac{3}{13}$ **iv)** $\frac{5}{13}$ **v)** $\frac{1}{26}$

b) $\frac{1}{16}$

3. $\frac{7}{16}$

4. a) $\frac{4}{25}$ **b)** $\frac{8}{25}$ **c)** $\frac{8}{25}$ **d)** $\frac{4}{25}$ **e)** $\frac{1}{25}$

5. a) $\frac{1}{3}$ **b)** $\frac{5}{9}$

6. $\frac{3}{8}$

7.1.3 Matching Events and Probabilities pp. 228–229

1. a) Very unlikely **b)** Certain
c) Very unlikely **d)** Likely (or very likely)
e) Impossible **f)** Likely (or very likely)
g) Even chance

2. *Sample responses*:
Impossible: I will grow two heads.
Certain: My next birthday will be on <birth date>.
Very likely: I will go straight home after school.
Likely: We will have rice for dinner.
Even chance: My mother's new baby will be a girl.
Unlikely: I will go to the next Tshechu in Thimphu.
Very unlikely: I will get a perfect mark on my next math test.

3. a) P(Gorthibu-karey)
b) P(other karey)
c) P(karey)
d) P(miss) or P(no karey)

4. B and D

5. a) 1 or $\frac{100}{100}$; certain
b) No.

7.2.1 Formulating Questions to Collect Data p.232

1. *Sample responses*:
a) Questionnaire **b)** Interviews **c)** Observation

2. *Sample response*:
C will give the best information about reading habits (if "reading habits" is about reading frequency).

3. *Sample response*:
Add choices like this:
How many books have you read in the past month?
None 1 or 2 3 or 4 5 or more

4. *Sample responses*:
a) *It is a good idea to have a math test every week. Do you agree or disagree?*
b) *On a scale of 1 to 5, how would you rate your enjoyment of studying each subject?*

Math
Do not enjoy Enjoy a lot
1 2 3 4 5

Science
Do not enjoy Enjoy a lot
1 2 3 4 5

English
Do not enjoy Enjoy a lot
1 2 3 4 5

c) *Has doing homework improved your success at school?*
Not helped Helped a lot
1 2 3 4 5

5. *Sample response*:
On a scale of 1 to 5, how do you feel about the following statement?
There should be more holidays during the school year.

6. *Sample responses*:
a) *On a scale of 1 to 5, how would you rate the need for new road construction in your area?*
Not needed Urgently needed
1 2 3 4 5
b) *Does your area needs more and better roads?*

7.2.2 Sampling and Bias p. 235

1. *Sample response*:
Census data; The Millennium Report may also be accurate.

2. Yes.

3. *Sample responses*:
a) Sample **b)** Sample
c) Second-hand data **d)** Census

4. No.

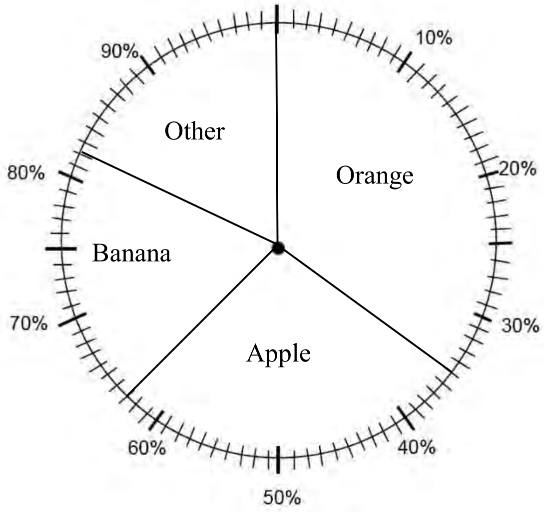
CONNECTIONS: Estimating Fish Populations p. 235

1. *Sample response*:
You cannot see them all underwater and they move too quickly to count. You would need to drain the lake to count them all, but then they would all die.

2. 500

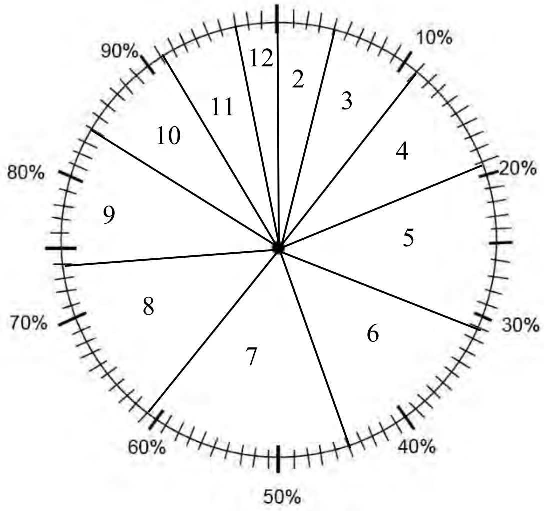
7.3.1 Circle Graphs p. 240

1. Favourite Fruit of a Group of College Students



2. *Sample response*:

Experimental Results of Rolling and Adding Two Dice



3. *Sample responses*:
a) Line graph
b) Bar graph
c) Circle graph

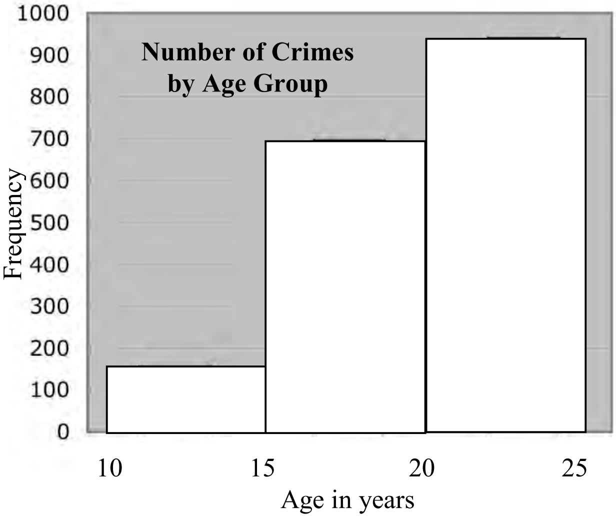
4. a) 34%
b) 37%
c) 29%

5. a)
A is forests
B is grasslands
C is agriculture
D is barren
b) *Sample response*:
A = 73%; **B** = 18%; **C** = 5%; **D** = 4%

6. No.

7.3.2 Histograms p. 245

1.

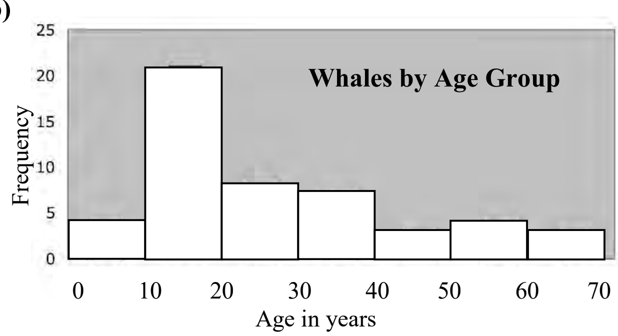


2. *Sample responses*:

a)

Age group	Tally	Frequency
0 – 9	||||	4
10 – 19	𝍸 𝍸 𝍸 𝍸 |	21
20 – 29	𝍸 |||	8
30 – 39	𝍸 ||	7
40 – 49	|||	3
50 – 59	||||	4
60 – 69	|||	3

b)



3. a) i) 9
ii) 27
iii) 45
iv) 19
b) 36
c) 64

4. *Sample response*:
2 h – 3 h,
3 h – 4 h,
4 h – 5 h,
5 h – 6 h, and
6 h – 7 h

7.4.1 Mean, Median, Mode, and Range p. 249

1. a) Mean, median, and mode increase (by 20), range does not change.
b) Mean, median, and mode decrease (by 100), range does not change
c) Mean, median, mode, and range increase (multiplied by 3).
d) Mean, median, mode, and range decrease (divided by 5).
e) Mean and median increase, mode does not change, range decreases.
f) Mean, median, and range increase, mode does not change.

7.4.1 Mean, Median, Mode, and Range [Continued] p. 249

2. a) Mean: 11.8; median: 12; mode: 12; range: 2
b) The mean and range will decrease.
The mode and median will stay the same.

3. *Sample response*:
Subtract 110 from each data value, calculate the mean of the new data, and then add 110 to the result.

4. *Sample responses*:
a) 5, 6, 7, 7, 8, 9, 10, 10, 10, 10
b) 305, 306, 307, 307, 308, 309, 310, 310, 310, 310
c) 50, 50, 50, 50, 50, 50, 50, 50, 50, 50
d) 110, 200, 200, 200, 200, 300, 400, 500, 600, 700

5. *Sample response*:
The data values are 4, 5, 6, 7, 8, 9, 10.
The median increases when 11 is added to the data.

6. *Sample response*:
Multiply each value by 10 to get the mean using whole numbers and then divide the value by 10.

7. *Sample response*:
A set of data that has decimal values

7.4.2 Outliers and Measures of Central Tendency p. 253

1. Data in order:
9, 26, 31, 35, 35, 35, 52, 71, 77, 96, 97, 104, 107
a) *Sample response*:
There are gaps between 9 and 26, 35 and 52, 52 and 71, and between 77 and 96.
There are clusters from 26 to 35 and from 96 to 107.
There is one outlier of 9.
b) *Sample response*:
The mean is 63.8 and the median is 61.5 without the outlier 9.

3. *Sample responses*:
a) The mean of 7
b) The mean of 40

4. *Sample responses*:
a) 5, 10, 10, 10, 15, 20, 25, 30, 45, **<u>55</u>**
b) **<u>0</u>**, 10, 10, 10, 15, 20, 25, 30, 45, **<u>55</u>**

5. There are clusters between 100 and 200 and between 300 and 500. There is a gap between 200 and 300.

UNIT 7 Revision pp. 254–255

1. a) $\frac{1}{15}$ **b)** $\frac{8}{15}$ **c)** $\frac{7}{15}$
d) $\frac{7}{15}$ **e)** $\frac{6}{15}$ **f)** 1

2. Choosing an even number and a striped ball

3. *Sample responses*:
a) Choosing a number greater than 1
b) Choosing a number less than 12
c) Choosing a white ball
d) Choosing the 3 ball
e) Choosing a number greater than 15

4. $\frac{1}{4}$

5. *Sample responses*:
a) Spinning a sum of 2 (a 1 and a 1)
b) Spinning a sum of 4 (a 1 and a 3, a 2 and a 2, or a 3 and a 1)

6. b) *Sample response*:
What is your favourite meat?
Seven choices are:
Pork, Beef, Chicken, Yak, Goat, Other, and I do not eat meat.

7. *Sample responses*:
a) Distribute a questionnaire to a sample of people of all ages throughout the country
b) Observe chickens in an experimental situation
c) Interview each student in my class

8. a) First-hand
b) Second-hand
c) Second-hand

10. *Sample response*:
A small percent were not happy. More than 95% are happy or very happy.

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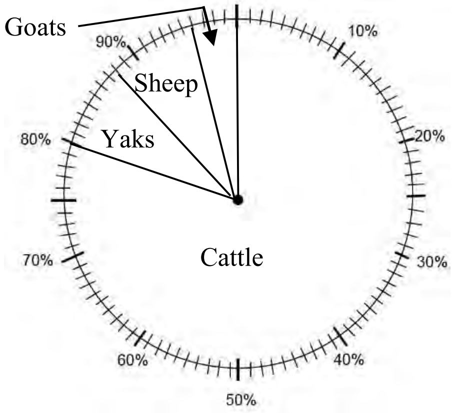
11. Cattle: 80%; yaks: 8%; sheep: 8%; goats: 4%



12. a) *Sample response*:
The data values range between 100 cm and 170 cm. The greatest number of students is in the middle (120 cm to 140 cm) and the number of students decreases in both directions.
b) 26
c) 30

13. 12 –14, 14 – 16, 16 – 18, 18 – 20, 20 – 22, 22 – 24, 24 – 26

14. a) Mean, median, mode, and range decrease (divided by 10).
b) Mean, median, and mode decrease (by 10). Range does not change.
c) Now there are 4 modes. Mean and median decrease. Range does not change.
d) Mean and median decrease. Range increases. Mode does not change.

15. *Sample responses*:
a) 10, 10, 10, 10, 10, 12
b) 17, 18, 19, 17, 18, 19
c) 8, 9, 10, 10, 11, 12

16. a) *Sample response*:
37 is an outlier.
There are two clusters, from 58 to 69 and from 80 to 91, with a gap between the clusters.
b) *Sample response*:
The mean with or without the outlier;

17. The mean changes from 50 to 40.
The median changes from 45 to 37.5
The mode of 55 does not change.
The range changes from 85 to 30.

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