Teacher's Guide to

Understanding Mathematics

Textbook for Class VIII

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Department of School Education
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MINISTRY OF EDUCATION
THIMPHU :BHUTAN
Cultivating the Grace of Our Mind

MINISTER

December 15, 2008

Foreword

I am at once awed and fascinated by the magic and potency of numbers. I am amazed at the marvel of the human mind that conceived of fantastic ways of visualizing quantities and investing them with enormous powers of representation and symbolism. As my simple mind struggles to make sense of the complexities that the play of numbers and formulae presents, I begin to realize, albeit ever so slowly, that, after all, all mathematics, as indeed all music, is a function of forming and following patterns and processes. It is a supreme achievement of the human mind as it seeks to reduce apparent anomalies and to discover underlying unity and coherence.

Abstraction and generalization are, therefore, at the heart of meaning-making in Mathematics. We agreed, propped up as by convention, that a certain figure, a sign, or a symbol, would carry the same meaning and value for us in our attempt to make intelligible a certain mass or weight or measure. We decided that for all our calculations, we would allow the signifier and the signified to yield whatever value would result from the tension between the quantities brought together by the nature of their interaction.

One can often imagine a mathematical way of ordering our surrounding and our circumstance that is actually finding a pattern that replicates the pattern of the universe – of its solid and its liquid and its gas. The ability to engage in this pattern-discovering and pattern-making and the inventiveness of the human mind to anticipate the consequence of marshalling the power of numbers give individuals and systems tremendous privilege to the same degree which the lack of this facility deprives them of.

Small systems such as ours cannot afford to miss and squander the immense power and privilege the ability to exploit and engage the resources of Mathematics have to present. From the simplest act of adding two quantities to the most complex churning of data, the facility of calculation can equip our people with special advantage and power. How intelligent a use we make of the power of numbers and the precision of our calculations will determine, to a large extent, our standing as a nation.

I commend the good work done by our colleagues and consultants on our new Mathematics curriculum. It looks current in content and learner-friendly in presentation. It is my hope that this initiative will give the young men and women of our country the much-needed intellectual challenge and prepares them for life beyond school. The integrity of the curriculum, the power of its delivery, and the absorptive inclination of the learner are the eternal triangle of any curriculum. Welcome to Mathematics.

Imagine the world without numbers! Without the facility of calculation!

Tashi Delek.

Thakur S Powdyel.

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INTRODUCTION

HOW MATHEMATICS HAS CHANGED

Mathematics is a subject with a long history. Although newer mathematical ideas are always being created, much of what your students will be learning is mathematics that has been known for hundreds of years, if not longer.

The learning of mathematics helps a person to solve problems. While solving problems, skills related to, representation of mathematical ideas, making connections with other topics in mathematics and connections with the real world, providing reasoning and proof, and communicating mathematically would be required. The textbook is designed to promote the development of these process skills.

Worldwide, there is now a greater emphasis on the need for students to understand the mathematics they learn rather than to memorize rote procedures.

Worldwide, there is now a greater emphasis on the need for students to understand the mathematics they learn rather than to memorize rote procedures. There are many reasons for this.

- In the long run, it is very unlikely that students will remember the mathematics they learn unless it is meaningful. It is much harder to memorize “nonsense” than to learn something that relates to what they already know.
- Some approaches to mathematics have not been successful; there are many adults who are not comfortable with mathematics even though they were successful in school.

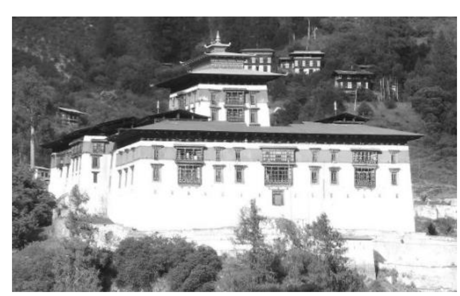
It is important always to talk about why something is true, not simply that it is true.

In this new program, you will find many ways to make mathematics more meaningful.

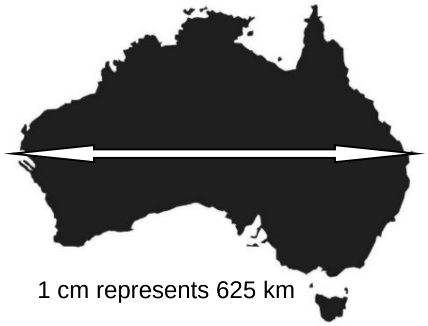
- We will always talk about why something is true, not simply that it is true.
  For example, the reason why it is useful to write numbers in scientific notation is explained.
- Mathematics should be taught using contexts that are meaningful to students. They can be mathematical contexts or real-world contexts. These contexts will help students see and appreciate the value of mathematics.

  For example, in Unit 1 (Number), a task with a real-world context involves square roots and the area inside a dzong. Unit 5 (Measurement) has a task involving both area and ratio. Another task in Unit 5 compares the size of Bhutan to the size of Australia.

a) The area inside the square wall around a dzong is about 3500 m^2. Estimate the length of one of the side walls.
b) What is the side length to one decimal place?



A 5 kg bag of grass seed covers about 650 m^2. How many bags should Tshering buy to seed a 60 m-by-80 m football field?



1 cm represents 625 km

a) What is the width of Australia? (Measure the map to the nearest tenth of a centimetre.)
b) Bhutan is about 300 km wide. About how many times would Bhutan fit across Australia?

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- To discuss mathematical ideas, we expect students to use the processes of problem solving, communication, reasoning, making connections (connecting mathematics to the everyday world and connecting mathematical topics to each other) and representation (representing mathematical ideas in different ways, such as manipulatives, graphs, and tables). For example, students represent integers using counters and number lines to help them see how the rules for multiplying and dividing make sense.
- There is an increased emphasis on problem solving because the reason we learn mathematics is to solve problems. When students become adults, they will not be told when to factor or when to multiply; they need to know how to apply those skills to solve certain problems. Students will be given opportunities to make decisions about which concepts and skills they need in certain situations.

We expect students to use the processes of problem solving, communication, reasoning, making connections, and representation.

A significant amount of research evidence has shown that these more meaningful approaches work. Scores on international tests are higher when the emphasis is on higher level thinking and not only on the application of skills.

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THE DESIGN OF THE STUDENT TEXTBOOK

Each unit of the textbook has the following features:

- a *Getting Started* to review prerequisite content
- two or three chapters, which cluster the content of the unit into sections that contain related content
- regular lessons and at least one *Explore* lesson
- a *Game*
- at least one *Connections* feature
- a *Unit Revision*

Getting Started

There are two parts to the *Getting Started*. They are designed to help you know whether students are missing critical prerequisites and to remind students of knowledge and terminology they have already learned that will be useful in the unit.

- The *Use What You Know* section is an activity that takes 20 to 30 minutes. Students are expected to work in pairs or in small groups. Its purpose is consistent with the rest of the text's approach, that is, to engage students in learning by working through a problem or task rather than being told what to do and then just carrying it through.
- The *Skills You Will Need* section is a more straightforward review of required prerequisite skills for the unit. This should usually take about 20 to 30 minutes.

The Getting Started is designed to help you know whether students are missing critical prerequisites and to remind students of knowledge and terminology they will need for the unit.

Regular Lessons

- Each lesson might be completed in one or two hours (i.e., one or two class periods), although some are shorter. The time is suggested in this *Teacher's Guide*, but it is ultimately at your discretion.
- Lessons are numbered #.#.#. The first number tells the unit, the second number the chapter, and the third number the lesson within the chapter. For example, Lesson 4.2.1 is Unit 4, Chapter 2, Lesson 1.

Lessons are numbered #.#.#. The first number tells the unit, the second number the chapter, and the third number the lesson within the chapter.

• Each lesson is divided into five parts:

- A *Try This* task or problem
- The exposition (the main points of the lesson)
- A question that revisits the *Try This* task, called *Revisiting the Try This* in this guide
- one or more *Examples*
- *Practising and Applying* questions

Try This

• The *Try This* task is in a shaded box, like the one below from **lesson 2.3.2** on **page 45**.

Try This

A. Padam works in a motorcycle store. He sold a motorcycle for Nu 45,000 and earned Nu 2700 for making the sale. What percent of the selling price of the motorcycle was Padam's earnings?

• The *Try This* is a brief task or problem that students complete in pairs or small groups. It serves to motivate new learning. Students can do the *Try This* without the new concepts or skills that are the focus of the lesson, but the problem is related to the new learning. It should be completed in 5 to 10 minutes. The reason why a lesson starts with a *Try This* is so that students do some mathematics independently before you intervene.

The Try This is a brief task or problem that students complete in pairs or small groups to motivate new learning.

• The answer to the *Try This* is not found in the back of the student textbook (but it is in this *Teacher's Guide*).

The Exposition

• The exposition presents the main concepts and skills of the lesson. Examples are often included to clarify the points being made.

The exposition presents the main concepts and skills of the lesson and is taught in different ways, as suggested in this guide.

• You will help students through the exposition in different ways (as suggested in this *Teacher's Guide*). Sometimes you will present the ideas first, using the exposition as a reference. Other times students will work through the exposition independently, in pairs, or in small groups.

• Key mathematical terms are introduced and described in the exposition. When
a key term first appears in a unit of the textbook, it is highlighted in **bold** type to indicate that it is found in the glossary (at the back of the student textbook).

• Students are not expected to copy the exposition into their notebooks either directly from the student text or from your recitation.

Revisiting the Try This

• The *Revisiting the Try This* question follows the exposition and appears in a shaded lozenge, like this example from **lesson 2.3.2** on **page 46**.

The Revisiting the Try This question links the Try This task to the new ideas presented in the exposition.

B. Use the percent you calculated in **part A** as the commission percent.

i) How much commission would Dorji earn for selling a motorcycle for Nu 30,000?

ii) Dorji earned a commission of Nu 3000 for selling a motorcycle. What was the selling price of the motorcycle?

• The *Revisiting the Try This* question links the *Try This* task or problem to the new ideas presented in the exposition. This is designed to build a stronger connection between the new learning and what students already understand.

Examples

- The *Examples* are designed to provide additional instruction by modelling how to approach some of the questions students will meet in *Practising and Applying*. Each example is a bit different from the others so that students have multiple models from which to work.
- The *Examples* show not only the formal mathematical work (in the left hand *Solution* column), but also student reasoning (in the right hand *Thinking* column). This model should help students learn to think and communicate mathematically. Photographs of students are used to reinforce this notion.
- Some of the *Examples* present two different solutions. The example below, from **lesson 6.1.1** on **page 154**, shows two possible ways to approach the task,
Solution 1 and *Solution 2*.

The Examples model how to approach some of the questions students will meet in Practising and Applying

Example 1 Solving a Probability Problem

Choki and Sithar are playing a game where they flip two Nu 1 coins.

- Choki wins when the coins both show Khorlo.
- Sithar wins when both coins show Tashi Ta-gye.
- If the two coins show different faces, no one wins.

a) What is the theoretical probability that Choki will win?

b) Which is greater, P(Choki does not win) or P(Sithar wins)?

Solution 1

a) First coin Second coin Outcomes

K — K **KK**
K — T KT
T — K TK
T — T TT

P(Choki wins) = $\frac{1}{4}$

b) P(Sithar wins) = $\frac{1}{4}$

P(Not Choki wins) $= 1 - \text{P(Choki wins)}$
$= 1 - \frac{1}{4}$
$= \frac{3}{4}$

P(Not Choki wins) > P(Sithar wins)

Thinking

a) I made a tree diagram to list all the possible outcomes.

- Each outcome has a probability of $\frac{1}{4}$ because there are 4 equally likely outcomes.
- The event that Choki wins is represented by the outcome KK.

b) The TT outcome represents a win for Sithar. It is 1 out of 4 possible outcomes.

- I knew that Choki not winning is the complement of Choki winning, so I used the formula:

P(Not Choki wins) = 1 – P(Choki wins)

Solution 2

a)

Second coin		
T	KT	TT
K	**KK**	TK
	K	T

First coin

P(Choki wins) = $\frac{1}{4}$

Thinking

a) I used an area model to represent the possible outcomes.

- The event that Choki wins is represented by the outcome KK. Its area represents $\frac{1}{4}$ of the area of the whole square.

The Examples show the formal mathematical work in the Solution column and model student reasoning in the Thinking column. This model should help students learn to think and communicate mathematically.

b) P(Sithar wins) = $\frac{1}{4}$ P(Not Choki wins) = $\frac{1+1+1}{4} = \frac{3}{4}$ P(Not Choki wins) > P(Sithar wins)	**b)** The TT outcome represents a win for Sithar. It is 1 out of 4 possible outcomes. • The event of Choki not winning consists of the 3 outcomes KT, TT, and TK. That's 3 out of 4 possible outcomes.

• The treatment of *Examples* varies and is discussed in the *Teacher's Guide*. Sometimes students work through these independently, sometimes they work in pairs or small groups, and sometimes you are asked to lead them through some or all of the examples.

• A number of the questions in the *Practising and Applying* section are modelled in the *Examples* to make it more likely that students will be successful.

Practising and Applying

• Students work on the *Practising and Applying* questions independently, with a partner, or in a group, using the exposition and *Examples* as references.

Students work on the Practising and Applying questions independently, with a partner, or in a group, using the exposition and Examples as references.

• The questions usually start like the work in the *Examples* and get progressively more conceptual, requiring more explanations and problem solving later in the exercise set.

• The last question in the section always brings closure to the lesson by asking students to summarize the main learning points. This question could be done as a whole class.

Explore Lessons

• *Explore* lessons provide an opportunity for students to work in pairs or small groups to investigate some mathematics in a less directed way. Often, but not always, the content is revisited more formally in a regular lesson immediately before or after the *Explore* lesson. The *Teacher's Guide* indicates whether the *Explore* lesson is optional or essential.

Explore lessons provide an opportunity for students to work with a partner or in small groups to investigate some mathematics in a less directed way.

• There is no exposition or teacher lecture in an *Explore* lesson, so the parts of the regular lesson are not there. Instead, a problem or task is posed and students work through it by following a sequence of questions or instructions that direct their investigation.

• The answers for these lessons are not found in the back of the textbook, but are found in this *Teacher's Guide*.

Connections

• The *Connections* is an optional feature that relates the content to something else.

The Connections is sometimes a relevant and interesting historical note. Sometimes it relates the content of a unit to the content of a different unit. Other times it relates the content to the real world.

• There are always one or more *Connections* features in a unit. The placement of a *Connections* feature in a unit is not fixed; it depends on the content knowledge required. Sometimes it will be early in the unit and sometimes later.

• The *Connections* feature always gives students something to do beyond simply reading it.

• Students usually work in pairs or small groups to complete these activities.

Game

• There is at least one *Game* per unit.

Games provide an enjoyable way to apply and practise skills and concepts introduced in the unit.

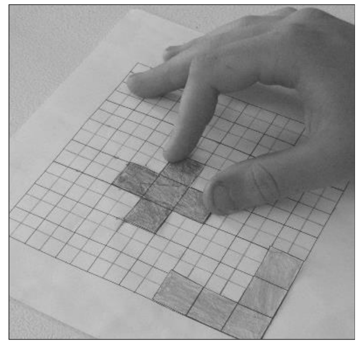
• The *Game* provides an enjoyable way to practise skills and concepts introduced in the unit.

• Its placement in the unit is based on where it makes most sense in terms of the content required to play the *Game*.

• In most *Games* students work in pairs or small groups, as indicated in the instructions.

• The materials and rules are listed in the text. Usually a sample is shown to make sure that students understand the rules.

• Most *Games* require 15 to 20 minutes, but students can often benefit from playing them more than once.



The Pentomino game in UNIT 5

Unit Revision

The Unit Revision provides an opportunity for review for students and allows you to gather informal assessment data.

• The *Unit Revision* provides an opportunity for review for students and for you to gather informal assessment data. *Unit Revisions* review all lesson content except the *Getting Started* feature, which is based on previous class content. There is always a mixture of skill, concept, and problem solving questions.

• The order of the questions in the *Unit Revision* generally follows the order of the lessons in the unit. Sometimes, if a question reflects more than one lesson, it is placed where questions from the later lesson would appear.

• Students can work in pairs or on their own, as you prefer.

• The *Unit Revision*, if done in one sitting, requires one or more hours. If you wish, you might break it up and assign some questions earlier in the unit and some questions later in the unit.

Glossary

The glossary contains key mathematical terminology introduced in the units, important terms from previous classes, and a set of instructional terms.

• At the end of the student textbook, there is a glossary of key mathematical terminology introduced in the units. When new terms are introduced in the units, they are in **bold** type. All of these terms are found in the glossary.

• The glossary also contains important mathematical terms from previous classes that students might need to refer to.

• In addition, there is a set of instructional terms commonly used in the *Practising and Applying* questions (for example, explain, predict, …) along with descriptions of what those terms require the student to do.

Answers

The answers to most of the numbered questions are found in the back of the student text. This Teacher's Guide contains a full set of answers.

• Answers to most numbered questions are provided in the back of the student text. In most cases, only the final answers are shown, not full solutions and explanations. For example, if students are asked to solve a problem and then "Show your work" or "Explain your thinking", only the final answer to the problem will be included, not the work or the reasoning.

• There is often more than one possible answer. This is indicated by the phrase *Sample Response*.

• Full solutions to the questions and explanations that show reasoning are provided in this *Teacher's Guide*, as are the answers to the lettered questions (such as A or B) in the *Try This* and the *Explore* lessons. Note that when an answer or any part of an answer is enclosed in square brackets, this indicates that it has been omitted from the answers at the back of the student text.

THE DESIGN OF THE TEACHER'S GUIDE

The Teacher's Guide is designed to complement and support the use of the student textbook.

The *Teacher's Guide* is designed to complement and support the use of the student text.

• The sequencing of material in the guide is identical to the sequencing in the student text.

• The elements in the *Teacher's Guide* for each unit include:
- a *Unit Planning Chart*
- *Math Background* for the unit
- a *Rationale for Teaching Approach*
- support for each lesson
- a *Unit Test*
- a *Performance Task*
- an *Assessment Interview* (Units 4 and 8)

The support for each lesson includes:
- *Curriculum outcomes* covered in that lesson
- *Outcome relevance* (*Lesson relevance* in the case of optional *Explore* lessons)
- *Pacing* in terms of minutes and hours
- *Materials* required to teach the lesson
- *Prerequisites* that the lesson assumes students possess
- *Main Points to be Raised* explicitly in the lesson
- suggestions for working through the parts of the lesson
- *Suggested assessment* for the lesson
- *Common errors* to be alert for
- *Answers*, often with more complete solutions than in the student text
- suggestions for *Supporting Students* who are struggling and/or for enrichment

Unit Planning Chart

This chart provides an overview of the unit and indicates, for each lesson, which curriculum outcomes are being covered, the pacing, the materials required, and suggestions for which questions to use for formative assessment.

Math Background and Rationale for Teaching Approach

This explains the organization of the unit or provides some background for you that you might not have, particularly in the case of less familiar content.
In addition, there is an indication of why the material is approached the way it is.

Regular Lesson Support

• Suggestions for grouping and instructional strategies are offered under the headings *Try This, Revisiting the Try This, The Exposition — Presenting the Main Ideas, Using the Examples,* and *Practising and Applying — Teaching Tips.*

• *Common errors* are sometimes included. If you are alert for these and apply some of the suggested remediation, it is less likely that students will leave the lessons with mathematical misunderstandings.

• A number of *Suggested assessment questions* are listed for each lesson. This is to emphasize the need to collect data about different aspects of students' performance — sometimes the ability to apply skills, sometimes the ability to solve problems or communicate mathematical understanding, and sometimes the ability to show conceptual understanding.

• It is not necessary to assign every *Practising and Applying* question to each student, but they are all useful. You should go through the questions and decide where your emphasis will be. It is important to include a balance of skills, concepts, and problem solving. You might use the *Suggested assessment questions* as a guide for choosing questions to assign.

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• You may decide to use the last *Practising and Applying* question to focus a class discussion that revisits the main ideas of the lesson and to bring closure to the lesson.

Explore Lesson Support

• As with regular lessons, for each *Explore* lesson there is an indication of the curriculum outcomes being covered, the relevance of the lesson, and whether the lesson is optional or essential.

• Because the style of the lesson is different, the support provided is different than for the regular lessons. There are suggestions for grouping students for the exploration, a list of *Observe and Assess* questions to guide your informal formative assessment, and *Share and Reflect* ideas on how to consolidate and bring closure to the exploration.

Because the style of an Explore lesson is different than a regular lesson, the support provided in the Teacher's Guide is also different.

Unit Test

A pencil-and-paper unit test is provided for each unit. It is similar to the unit revision, but not as long. If it seems that the test might be too long (for example, if students would require more than one class period to complete it) some questions may be omitted. It is important to balance the items selected for the test to include questions involving skills, concepts, and problem solving, and to include at least one question requiring mathematical communication.

If the test seems too long, some questions may be omitted but it is important to ensure a balance of questions involving skills, concepts, problem solving, and communication.

Performance Task

• The *Performance Task* is designed as a summative assessment task. Performance on the task can be combined with performance on a *Unit Test* to give a mark for a student on a particular unit.

• The task requires students to use both problem solving and communication skills to complete it. Students have to make mathematical decisions to complete the task.

• It is not appropriate to mark a performance task using percentages or numerical grades. For that reason, a rubric is provided to guide assessment. There are four levels of performance that can be used to describe each student’s work on the task. A level is assigned for each different aspect of the task and, if desired, an overall profile can be assigned. For example, if a student’s performance is level 2 on most of the aspects of the task, but level 3 on one aspect, an overall profile of level 2 might be assigned.

• A sample solution is provided for each task.

The Performance Task is designed as a summative assessment task that can be combined with a student’s performance on the Unit Test to give an overall mark.

Unit Assessment Interviews

• Selected units (4 and 8) provide a structure for an interview that can be used with one or several students to determine their understanding of the outcomes.

• Interviews are a good way to collect information about students because they allow you to interact with students and to follow up if necessary. Interviews are particularly appropriate for students whose performance is inconsistent or who do not perform well on written tests, but who you feel really do understand the content.

• You may use the data you collect in combination with class work or even a unit test mark.

Interviews are a good way to collect information about students, since interviews allow you to interact with students and to follow up if necessary.

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ASSESSING MATHEMATICAL PERFORMANCE

Forms of Assessment

It is important to consider both formative (continuous) and summative assessment.

Formative

- Formative assessment is observation to guide further instruction. For example, if you observe that a student does not understand an idea, you may choose to re-teach that idea to that student using a different approach.

Formative assessment is observation to guide further instruction.

- Formative assessment opportunities are provided through
  - prerequisite or diagnostic assessment in the *Getting Started*
  - suggestions for assessment questions in each regular lesson
  - questions that might be asked while students work on the *Try This* or during an *Explore* lesson
  - the *Unit Revision*
  - the unit *Assessment Interview* (for the units with interviews)
- Formative assessment can be supplemented by
  - everyday observation of students' mathematical performance
  - formal or informal interviews to reveal students' understanding
  - journals in which students comment on their mathematical learning
  - short quizzes
  - projects
  - a portfolio of work so students can see their progress over time, for example, in problem solving or mathematical communication (see *Portfolios* below)

Summative

- Summative assessment is used to see what students have learned and is often used to determine a mark or grade.

Summative assessment is used to see what students have learned and is often used to determine a mark.

- Summative assessment opportunities are provided through
  - the *Unit Test*
  - the *Performance Task*
  - the *Assessment Interview*
- Summative assessment can be supplemented with
  - short quizzes
  - projects
  - a portfolio that is assessed with respect to progress in, for example, problem solving or communication

Portfolios

One of the advantages of using portfolios for assessment is that students can observe their own growth over time, leading to greater confidence. In each unit, the section on math background identifies items that pertain to the various mathematical processes. The portfolio could be made up of student work items related to one of the mathematical processes: problem solving, communication, reasoning, or representation.

One of the advantages of using portfolios for assessment is that students can observe their own growth over time, leading to greater confidence.

Assessment Criteria

- It is right and fair to inform students about what will be assessed and how it will be assessed. For example, students should know whether the intent of a particular assessment task is to focus on application or on problem solving.

It is right and fair to inform students about what will be assessed and how it will be assessed.

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• A student's mark and all assessments should reflect the curriculum outcomes for Class VIII. The proportions of the mark assigned for each unit should reflect both the time spent on the unit and the importance of the unit. The modes of assessment used for a particular unit should be appropriate for the content.

For example, if the unit focuses mostly on skills, the main assessment might be a paper-and-pencil test or quizzes. If the unit focuses on concepts and application, more of the student's mark should come from activities like performance tasks.

The modes of assessment used for a particular unit should be appropriate for the content.

• The focus of this curriculum is not on procedures for their own sake but on a conceptual understanding of mathematics so that it can be applied to solve problems. Procedures are important too, but only in the context of solving problems. All assessment should balance procedural, conceptual, and problem solving items, although the proportions will vary in different situations.

All assessment should balance procedural, conceptual, and problem solving items, although the proportions may vary in different situations.

• Students should be informed whether a test is being marked numerically, using letter grades, or with a rubric. If a rubric is being used, then it should be shared with students before they begin the task to which it is being applied.

It is important to remember that the mark should reflect student competence with the mathematical outcomes of the course and not behaviour, neatness, participation, and so on.

Determining a Mark

• In determining a student's mark, you can use the tools described above along with other information such as work on a project or poster. It is important to remember that the mark should, as closely as possible, reflect student competence with the mathematical outcomes of the course. The mark should not reflect behaviour, neatness, participation, and other non-mathematical aspects of the student's learning. These are important to assess as well, but not as part of the mathematics mark.

• In looking at a student's mathematics performance, the most recent data might be weighted more heavily. For example, suppose a student does poorly on some of the quizzes given early in Unit 1, but later you observe that he or she has a better understanding of the material in the unit. You might choose to give the early quizzes less weight in determining the student's mark for the unit.

• At present, you are required to produce a numerical mark for a student, but that should not preclude your use of rubrics to assess some mathematical performance. One of the values of rubrics is their reliability. For example, if a student performs at level 2 on a particular task one day, he or she is very likely to perform at the same level on that task another day. On the other hand, a student who receives a test mark of 45 one day might have received a mark of 60 on a different day if one question had changed on the test or if he or she had read an item more carefully.

• You can combine numerical and rubric data using your own judgment. For example, if a student's marks on tests average 50%, but the rubric performance is higher, for example, level 3, it is fair and appropriate to use a higher average for that student's class mark.

This new curriculum requires a change in the classroom environment to include more pair and group work and an increased emphasis on communication.

THE CLASSROOM ENVIRONMENT

This new curriculum requires a change in the classroom environment to include more pair and group work and an increased emphasis on communication.

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It is only in this way that students will really become engaged in mathematical thinking instead of being spectators.

• In every lesson, students should be engaged in some pair or small group work (for the *Try This*, selected *Practising and Applying* questions, or during an *Explore* lesson).

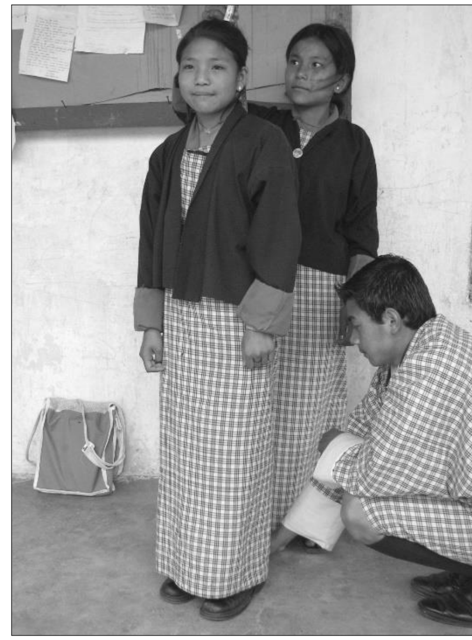
• Students should be encouraged to communicate with each other to share responses that are different from those offered by other students or from the responses you might expect. Communication involves not only talking and writing but also listening and reading.

Pair and Group Work

• There are many reasons why students should be working in pairs or groups, including

- to ensure that students have more opportunities to communicate mathematically (instead of competing with the whole class for a turn to talk)
- to make it easier for them to take the risk of giving an answer they are not sure of (rather than being embarrassed in front of so many other people if they are incorrect)
- to see the different mathematical viewpoints of other students
- to share materials more easily
- to share responsibilities for a task

• Sometimes students can work with students who sit near them, but other times you may wish to form the groups so that students who are struggling are working together. Then you can help them while the other students move forward. Students who need enrichment can also work together so that you can provide an extra challenge for them all at once.



Working as a group to collect height data

• For students who are not used to working in pairs or groups, you should set down rules of behaviour that require them to attend to the task and to participate fully. You need to avoid a situation where four students are working together, but only one of them is really doing the work. You might post Rules for Group Work, as shown here to the right.

• Once students are used to working in groups, you might sometimes be able to base assessment on group performance rather than on individual performance.

Communication

Students should be communicating regularly about their mathematical thinking. It is through communication that they clarify their own thinking as well as show you and their classmates what they do or do not understand. When they give an answer to a question, you can always be asking questions like, *How did you get that? How do you know? Why did you do that next?*

It is through communication that students clarify their own thinking as well as show you and their classmates what they do or do not understand.

• Communication is practised in small group settings, but is also appropriate when the whole class is working together.

• Students will be reluctant to communicate unless the environment is risk-free. In other words, if students believe that they will be reprimanded or made to feel bad if they say the wrong thing, they will be reluctant to communicate. Instead, show your students that good thinking grows out of clarifying muddled thinking. It is reasonable for students to have some errors in their thinking and it is your job to help shape that thinking. If a student answers incorrectly, it is your job to ask follow-up questions that will help the student clarify his or her own thinking.

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• Many of the questions in the textbook require students to explain their thinking. The sample *Thinking* in the *Examples* is designed to provide a model for mathematical communication.

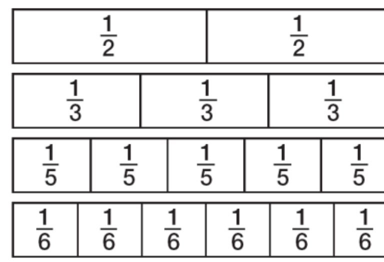
The sample Thinking in the Examples is designed to provide a model for mathematical communication.

• One of the ways students communicate mathematically is by describing how they know an answer is right. Even when a question does not ask students to check their work, you should encourage them to think about whether their answer makes sense. When they check their work, they should usually check using a different way than the way they used to find their answer so that they do not make the same reasoning error twice. This will enhance their mathematical flexibility.

MATHEMATICAL TOOLS

Manipulatives

There is great value in using manipulative materials in mathematics instruction; sometimes, it is essential. For example, integer work in Unit 3 will be better understood if it is modelled with black and white counters. The work in Unit 4 will be enhanced if students have access to fraction strips. Unit 8 cannot be completed without using linking cubes. Other times, for example, in Unit 3, some students can be successful without manipulative materials, but all students will benefit from using them. Students will start to see not only how to perform arithmetic calculations, but why they are done the way they are.



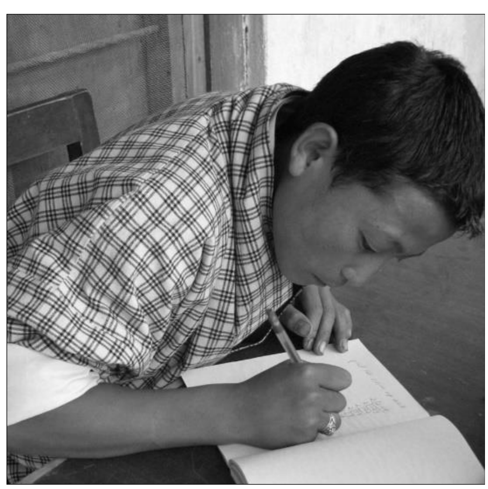
Fraction strips

+3 –3

White and black counter models for integers

Linking cubes

THE STUDENT NOTEBOOK



It is valuable for students to have a well-organized, neat notebook to look back at to review the main mathematical ideas they have learned. However, it is also important for students to feel comfortable doing rough work in that notebook or doing rough work elsewhere without having to waste time copying it neatly into their notebooks. In addition to the things you tell students to include in their notebooks, they should be allowed to make some of their own decisions about what to include in their notebooks.

Students should be allowed to make some of their own decisions about what to include in their notebooks.

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STRAND A: NUMBER

KSO Numeration: *by the end of* ***Class 8*** *students should:*

• *understand meanings and appropriate applications (number sense) with respect to integers, rational numbers, and common irrational numbers, and be able to draw on a wide variety of relationships and strategies to solve problems in relevant situations*

• *move flexibly from one form of representation of numbers to another, as might be appropriate in a given situation to understand or solve a particular problem*

• *interpret numbers in many ways, through reading, writing, illustrating, modelling and talking about numbers*

• *apply reasoning to order integers, rational numbers, and common irrational numbers to aid in estimation and developing strong number sense*

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-A1 Negative Exponents: develop meaning concretely and symbolically
• encountered in the context of place value charts (tenths, hundredths, thousandths as 10^{-1}, 10^{-2}, 10^{-3})
• introduce using a pattern ($100 = 10^2$, $10 = 10^1$, $1 = 10^0$, $0.1 = 10^{-1}$, $0.01 = 10^{-2}$)
• work with base ten models

8-A2 Large and Small Numbers: scientific notation to standard form and vice versa
• establish connection with multiplying by 0.1, 0.01, and 0.001
• make real-life connections (e.g., diameter of cell, diameter of electrons, mass of a bug)
• relate small numbers to large numbers to show the difference between the two in scientific notation form
• translate numbers from one form to another

8-A3 Square Roots: modelling and representing
• model perfect squares and square roots using blocks or grid paper
• establish link between concrete and numerical representations
• on grids, view the area as the perfect square and the side length of the square as the square root

8-A4 Perfect Squares: patterns between 1 and 144
• recognize each of the perfect squares from 1 through 144
• expose to perfect squares up to 400
• relate patterns to perfect squares
• understand that the differences in perfect squares increase in a constant way
• work with patterns related to perfect squares of any size

8-A5 Square Roots: find using an appropriate number
• estimate where the square root will fall
• apply factorization in a variety of ways (e.g., $576 = 4 \times 144 = 2 \times 2 \times 144 = 2 \times 2 \times 12 \times 12 = 24 \times 24$, then $\sqrt{576} = 24$)
• approximate to the point where students can identify which whole number is closer to the square root (e.g., square root of 22 is between 4 and 5 and is closer to 5 than 4)
• use patterns and/or reasoning to determine whether the square root of a number is another number (e.g., 16 is 4 therefore 1600 is 40 and 2200 is between 40 and 50)

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8-A6 Square Roots: exact square root and its decimal approximation
• emphasize the difference between exact square root and the decimal approximation
• model square roots for non-perfect squares

8-A7 Percent: greater than 100
• recognize that values greater than a whole are described by percents greater than 100%
• relate percent greater than 100 to other subjects and topics (e.g., social studies: inflation rates and population growth)

8-A8 Integers and Rational Numbers: comparing and ordering (fractional and decimal form)
• understand that placement of the negative sign does not affect the value (e.g., $\frac{-2}{3}$, $\frac{2}{-3}$, and $-\frac{2}{3}$ are equivalent)
• understand that a negative is always less than a positive
• understand that positive fractions with common denominators can be compared by examining numerators (e.g., $\frac{3}{8}$ is less than $\frac{5}{8}$ because 3 is less than 5)
• understand that positive fractions with common numerators can be compared by examining denominators (e.g., $\frac{3}{5}$ is greater than $\frac{3}{6}$ because 5 is less than 6)
• use reference points (1, $\frac{1}{2}$, −1, etc.)
• change numbers to a common form

STRAND B: OPERATIONS

KSO Operations: *by the end of **Class 8** students should:*
• *establish the relationship between algebraic and arithmetic operations and use this relationship in solving computational problems with algebraic expressions*
• *model, explain and use rational numbers and integers to solve problems*
• *model and solve computational problems involving fractions, ratios, percent, proportion, integers, exponents by selecting appropriate operations and procedures for computation, estimation, and mental math*
• *efficiently select and apply appropriate estimation strategies to problems involving rational numbers and integers, to answer real life questions, make predictions and check for reasonableness of answer in calculation*

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-B1 Proportion: solve problems
• use a variety of strategies to solve problems of proportionality:
- find relationships between the various terms of proportion and use these relations to solve for missing values (e.g., use equivalent fractions to solve $\frac{2.2}{5} = \frac{x}{5}$)
- apply the unit ratio/rate method
• recognize uses for and importance of proportion
• investigate problem solving opportunities (e.g., study of scale, transformational geometry, i.e., dilatations)

8-B2 Percent: solving and creating real problems in context (including estimation)
• estimate and calculate a percent of a given number (a% of $b = c$, e.g., 25% of 1500)
• find the percent one number is of another number (e.g., what percent of 20 is 15?)
• find the whole when a specified percent is given (e.g., 28% of what number is 42?)
• use mental strategies when an exact answer is required (e.g., 28% of 1200 = 20% of 1200 + 8% of 1200)
• use percents that are not whole numbers

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8-B3 Percent: increase and decrease
• investigate mark-ups and mark-downs of retail items (e.g., a dress cost Nu 7 to make and is being sold for Nu 15. What is the percent of mark-up?)
• develop formula (% increase = $\frac{\text{increase}}{\text{original amount}} \times 100\%$)
• develop formula (% decrease = $\frac{\text{decrease}}{\text{original amount}} \times 100\%$)
• investigate commissions and simple interest; develop formula $I = Prt$

8-B4 Multiply and Divide Integers: solve problems
• connect visual models, such as counters and number lines, to symbols
• interpret multiplication as repeated addition and vice versa
• relate multiplication and division

8-B5 Properties of Operations for Integers: commutative, associative, and distributive
• apply properties and understand their usefulness: commutative property (order, e.g., $(-5) \times 4 = 4 \times (-5)$), associative property (grouping, e.g., $((-2) \times 4) \times (-3) = (-2) \times (4 \times (-3))$, and distributive property (e.g., $(-2)\,(3 + (-2)) = (-2)(3) + (-2)(3)$)
• recognize the property of closure (e.g., $2 - 5$ is not defined within the set of whole numbers, therefore needing the introduction to integers)

8-B6 Multiply and Divide Integers: mentally
• develop and use mental strategies such as the following:
- front-end
- compatible numbers/factors
- working by parts
- double and halves

8-B7 Order of Operations for Integers: solve problems
• apply the proper conventions for order of operations

8-B8 Add and Subtract: fractions — develop algorithm (pictorially and symbolically)
• apply prior understanding of equivalent fractions, lowest terms, and LCM
• use manipulatives to develop operations with fractions concretely (e.g., fraction strips, grids, fraction circles, number lines)
• record equivalent fractions when moving from the concrete to symbolic
• represent both fractions using the same subdivision of the whole

8-B9 Add and Subtract: fractions mentally
• attempt mental calculation first when denominators are the same or easily determined (e.g., $\frac{1}{2} + \frac{1}{4}$)
• when addition or subtraction can not be done mentally, determine if estimation is sufficient or an exact answer is required

8-B10 Multiply: fractions — develop algorithm (pictorially and symbolically)
• construct concrete and pictorial models to develop meaning
• understand that "of" means multiplication and can be shown by comparing results in questions such as $\frac{1}{4}$ of 8 and $\frac{1}{4} \times 8$
• multiply a whole number by a fraction less than 1 (e.g., $4 \times \frac{1}{3}$ uses repeated addition)
• multiply a fraction less than 1 by another fraction especially when the numerator is 1 (e.g., $\frac{1}{4}$ of $\frac{2}{3}$)

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8-B11 Divide: fractions— develop algorithm (pictorially and symbolically)
• derive a personal algorithm from carefully chosen examples, e.g.:
- a simple fraction divided by a whole number (e.g., for $\frac{1}{2} \div 3$, divide $\frac{1}{2}$ into 3 equal parts. What does each part represent?)
- a whole number divided by a simple fraction (e.g., $4 \div \frac{1}{2}$, asks, "How many $\frac{1}{2}$s there are in 4?")
- a simple fraction divided by simple fraction where the numerator of the divisor is 1 and both denominators are the same (e.g., $\frac{5}{6} \div \frac{1}{6}$ asks, "How many $\frac{1}{6}$s are there in $\frac{5}{6}$?")
- a simple fraction divided by a simple fraction where the numerator of the divisor is 1 and the fractions are compatible (e.g., $\frac{1}{2} \div \frac{1}{4}$)
• use a number line to model division
• apply prior knowledge of reciprocal

8-B12 Fractions: estimate and mentally compute products and quotients
• appropriate number situations to use for multiplication include:
- a fraction by a whole number when the numbers are compatible
- any two proper fractions when the numerators and denominators are relatively simple to work with
- a whole number by a mixed number (distributive property should be used)
• appropriate number situations to use for division include:
- a simple fraction divided by a whole number
- a whole number divided by a fraction
- a simple fraction divided by a simple fraction when the denominators are the same
- a simple fraction divided by a simple fraction
- a mixed number divided by a whole number
- a whole number divided by a mixed number
- a mixed number divided by a mixed number
• use estimation to check reasonableness of results
• round to nearest whole and sometimes to nearest half to reach rough estimates

8-B13 Operations: positive and negative decimal numbers
• use prior experience to construct concrete and pictorial representations
• connect visual representations to symbols
• use a variety of models to illustrate the operations (e.g., coloured counters, number lines)
• develop computational algorithms with decimals, using estimation, mental computation, pencil and paper
• apply prior knowledge of order of operations in the context of positive and negative decimals
• continue to estimate to check reasonableness of answers

8-B14 Order of Operations: fractions
• understand that the order is the same as for whole numbers, and why that makes sense
• understand how improper order impacts results

8-B15 Add and Subtract Simple Algebraic Terms: solve problems
• establish a parallel between a measurement situation and a variable situation (e.g., for 3 m + 0.2 m, 3 m and 20 cm need to be “like terms” before you can add or subtract)
• add and subtract simple expressions with concrete materials such as algebra tiles (know which like terms can and cannot be combined)

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8-B16 Polynomial Expressions: add and subtract visually
• use concrete materials such as algebra tiles for conceptual development
• for subtraction, consider different representations of subtraction, including the following:
- comparison (which refers to comparing and finding the difference between two quantities)
- taking away (which refers to starting with a quantity and removing a specific amount
- adding the opposites (which refers to subtracting by first changing the question to an addition and then adding the opposite of a quantity (e.g., subtracting x instead of $-x$)
- missing addend (What would be added to the number being subtracted to get the starting amount? (e.g., for $(3x-2)-(2x+1)$, what is added to $2x+1$ to get $3x-2$?)

8-B17 Multiplying by a Scalar (Polynomials): visually and symbolically
• develop with concrete materials and diagrams using repeated addition (e.g., for $3(2x+1) = 2x+1+2x+1+$
$2x+1$, model the binomial three times and combine the like terms using algebra tiles)
• explore the area model to associate repeated multiplication

STRAND C: PATTERNS AND RELATIONSHIPS

KSO Pattern: *by the end of* ***Class 8*** *students should:*

• *represent patterns as algebraic expressions, equations, inequalities, and exponents*

• *interpret patterns through algebraic description and apply generalizations to make predictions of unknown values and solve real world and mathematical problems*

• *explore and generalize how a change in one quantity in a function affects another, in order to efficiently solve similar problems*

• *solve linear equations and inequalities through algebraic methods*

• *understand the meaning of non-linear equations*

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-C1 Patterns and Relations: represent in a variety of formats
• move interchangeably among a variety of formats which describe relationships
• describe in words, and use expressions and equations to represent patterns given in tables, graphs, charts, pictures and/or problems situations
• use information presented in a variety of formats to derive mathematical expressions and predict unknown values
• investigate linear situations and those which create a regular pattern (broken line or curved graph)
• predict unknown values once algebraic description of a pattern is established
• interpolate and extrapolate to predict unknown values when patterns are not regular

8-C2 Graphs (Linear and Non-linear): interpret
• understand, when looking at tabular data, that when an equal spacing between the values of one variable produces an equal spacing between values of another variable, the relationship is linear
• interpret graphs of non-smooth situations
• use information from tables, graphs, or algebraic expressions to describe change
• match situations to corresponding graphs
• sketch graphs for a variety of situations, leading to linear and broken-line graphs
• understand that a variety of representations may be used to show relationships and that choices are available
• construct graphs to determine if a relationship is linear (e.g., graph the sum of the interior angles of a polygon against the number of side)

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8-C3 Graphs and Tables (linear and non-linear): how changing one quantity affects the other
- use information from tables, graphs, or equations to investigate the impact of changing related quantities
- explore patterns associated with parameter changes in a linear equation (e.g., understand how changes in the equation affect the slant of the graph)

8-C4 Slope: link visual characteristics with numerical values
- understand that, for linear relationships, the ratio of vertical change to horizontal change is constant anywhere along the line
- use the terms rise and run to describe vertical and horizontal change in a line graph
- investigate practical situations: slope of a staircase, slope of a roof, and the steepness of roads
- determine the slope of a line
- understand that ratios for a graph that rises to the right are positive
- understand that ratios for a graph that rises to the left are negative

8-C5 Single Variable Equations: solve algebraically
- use prior knowledge developed through concrete experiences to transfer to symbolic representation of single variable equations
- solve one- and two-step equations symbolically using integer and simple fraction and decimal coefficients
- use the "balance method" to solve problems

8-C6 Linear Equations: create and solve problems
- create and solve relevant problems for which algebraic solutions are required
- justify strategies used to create and solve problems
- appreciate the use of an algebraic equation in problems involving large numbers (as opposed to a guess and check approach)

8-C7 Intersection of Two Lines: solve problems
- compare tables of values, equations, or verbal descriptions of two linear situations to identify where lines will intersect
- use tables of values to generate ordered pairs for each equation and identify coordinates for points of intersection

STRAND D: MEASUREMENT

KSO Measurement: *by the end of **Class 8** students should:*
- *use concepts of rate to solve real-life and mathematical problems*
- *use direct and indirect measurement to make comparisons and interpret scales*
- *understand how a change in one measurement affects another in problems of rate*
- *understand relationships and move freely among all SI units, and choose appropriate units to solve measurement problems in given situations*
- *estimate effectively using a variety of strategies to solve measurement problems and understand when estimation is appropriate*
- *use relationships and reasoning to develop and apply procedures for measuring in a wide variety of measurement problems*

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Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-D1 Pythagorean Relationship: understanding
• investigate the side relationships of a variety of right triangles
• explore the 3-4-5 rule for establishing a right angle
• understand, through investigation, that if a square is made on each side of a right triangle, the sum of the two smaller squares will equal the area of the longer side ($c^2 = a^2 + b^2$ where a, b, and c are sides of a right triangle)
• explore triangles of a variety of orientations to discover that the hypotenuse (the longest side) is the side opposite the right angle regardless of orientation

8-D2 Pythagorean Relationship: application
• use the Pythagorean relationship to determine if three given side lengths are, or are not, the sides of a right triangle
• understand usefulness of Pythagorean relationships to solve problems in real life (whenever a triangle has a right angle and two known sides)
• investigate real world problems to determine the length of the hypotenuse, as well as the length of the other side when the hypotenuse and one side are given
• understand that the Pythagorean relationship can be used if only one side is given when the right triangle is isosceles
• find distance between two points using Pythagorean relationship (e.g., determine the reach of a ladder)

8-D3 Area and Perimeter: patterns and relationships of quadrilaterals and circles
• understand, through investigation, that area can vary when perimeter is fixed (e.g., for a perimeter of rectangle of 16 cm, determine all possible whole-number dimensions)
• understand, through investigation, that perimeter can vary when the area of a rectangle is fixed
• determine what happens to the area of a regular polygon as the number of sides increases (e.g., if perimeter is 24 cm, what is the area when the figure has 4 sides? 6 sides?)

8-D4 SI Units: solve measurement problems
• integrate measurement problems as other mathematical ideas are explored
• use the prefixes kilo, hecto, deka, deci, centi, and milli in problem solving contexts
• investigate and create problems involving length and perimeter (mm, cm, m, km), area (mm^2, cm^2, m^2, km^2, and hectare (1 hm^2))
• continue to make decisions in real world situations about when estimating is close enough
• develop sense of relative size of units (e.g., compare 1 cm^3 to 1 m^3; 1 kL to 1 L)
• apply relationships between capacity and volume for water (1 mL = 1 cm^3; 1 L = 1000 cm^3) to solve problems
• when choosing between capacity and volume, understand which is more appropriate in a given situation
• establish link between capacity and mass of pure water (1 mL of water has a mass of 1 g)
• investigate and create problems involving volume (cm^3 and m^3), mass (mg, g, kg) and capacity (mL, L, kL)

8-D5 Proportion: solve indirect measurement problems
• link proportion to ideas of ratio and rate
• read, interpret, and discuss scale drawings
• understand usefulness of proportion ideas in relevant real-world problems

8-D6 Area of Circles: estimate
• understand why it makes sense to estimate by squaring the diameter
• understand why a closer estimate is $3 \times r^2$

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8-D7 Area of Circles: develop formula
• apply prior knowledge of area for a parallelogram to develop a formula for the area of a circle
• investigate to determine the radius when the area of a circle is given
• apply prior knowledge of square root
• understand that $\frac{22}{7}$ and 3.14 are approximations and that a calculator must be used for more precision

8-D8 Volume and Total Surface Area: estimate and calculate right prisms
• estimate volume in a variety of situations
• find total surface area of rectangular prisms
• investigate changes in total surface area based on changes in dimensions

STRAND E: GEOMETRY

KSO Geometry: *by the end of **Class 8** students should:*

• *build and analyse physical and pictorial models of 2-D and 3-D shapes to understand relationships and properties, and enhance spatial sense in mathematical and real world situations*

• *analyse the results of transforming shapes to understand and apply transformation properties to mathematical and real world situations and to explain geometrical ideas*

• *compare, classify, and apply geometric properties to figures*

• *appreciate the importance of geometry in understanding mathematical ideas in art and the world around them*

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-E1 Interpret and Make Orthographic Drawings and Isometric Drawings: to represent 3-D shapes
• use interlocking cubes to explore attributes of 3-D shapes
• compare constructions to determine how they are different and the same
• use cubes to build structures from isometric drawings
• make isometric drawings of cube structures
• continue to develop visualization skills by physically exploring the results of moving objects and structures in a variety of ways
• use cubes to build structures from a set of orthographic drawings
• make orthographic drawings of cube structures

8-E2 Dilatations: represent, analyse, and apply
• understand that the dilatation centre and scale factor must be identified in order to locate the position of the dilatation image
• explore combinations of transformations that include dilatations, such as an enlargement followed by a reflection

8-E3 Polygons: properties and interrelationships
• develop a chart to observe and extend patterns and generalize about the sum of the measures of the interior angles and exterior angles of various polygons, and the measure of each interior angle of a regular polygon
• understand that the dilatation centre and scale factor must be identified in order to locate the position of the
• understand, through investigating, that the sum of the measures of the interior angles of a polygon is found by dividing the polygon into triangles

8-E4 Angle Pair Relationships: parallel and non-parallel lines
• understand that corresponding angles and alternate angles are only equal when a transversal intersects two parallel lines
• understand that interior angles are supplementary when a transversal intersects two parallel lines
• apply transformational geometry to discover why the various angle pairs are equal

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STRAND F: DATA MANAGEMENT

KSO Data Management: *by the end of* ***Class 8*** *students should:*

- *understand issues in data collection, including bias, repeated sample variability, randomness, collect, record, organize and describe data in multiple ways to draw conclusions about everyday issues*
- *understand, choose, and apply appropriate data collection methods (real or simulated data) to answer questions on meaningful issues*
- *predict, read, and draw inferences for a variety of data displays, including interpolation and extrapolation (draw conclusions about things not specifically represented by the data)*
- *construct and analyse a variety of data displays, including circle graphs, box and whisker plots, and scatter plots, and choose the most appropriate display for a given situation*
- *analyse measures of central tendency in terms of the effect on mean, median, and mode when changes in data occur, in order to draw conclusions and make decisions*
- *examine interpretations of data displays for validity, considering data collection issues to form opinions and make predictions*

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-F1 Repeated Sampling (of Same Population): variability
- understand that survey results of two different samples of the same population will not exactly be the same
- recognize the variability among repeated samples and provide a basic and informal introduction to the notion of sampling distribution
- conduct probability experiments to demonstrate variability of repeated sampling
- use real and simulated data in interesting investigations

8-F2 Randomness: concepts
- understand that a random sample is a sample collected from a population so that every member of the population has an equal chance of being selected
- understand that members are chosen independently of each other
- understand that common devices and methods used in selecting random sample are coins, dice, sampling boxes, a table of random numbers

8-F3 Circle Graphs: construct and interpret
- understand usefulness of circle graphs in situations where a comparison of the part to the whole is needed (e.g., budgets)
- apply prior knowledge about percent and using a protractor in construction of circle graphs
- focus on when a circle graph is the most appropriate data display

8-F4 Box and Whisker Plots: construct and interpret
- understand that this is an easy method for visually displaying the median, the range, and the distribution
- construct plots
- identify the median and the median of the upper half of the data (upper quartile)
- identify the median of the lower half of the data (lower quartile)
- identify the extremes, that is, the lower value and the higher value

8-F5 Scatter Plots: construct and interpret
- use data collected by students to construct scatter plots

8-F6 Variations: on mean, median, and mode
- consider and compare, through investigation, the impact of alterations to data sets to each of mean, median and mode

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STRAND G: PROBABILITY

KSO Probability: *by the end of **Class 8** students should:*
• explore, interpret, and make predictions for everyday events by estimating and conducting experiments
• determine theoretical and experimental probability, understand the difference between the two and determine when each is relevant to a particular situation
• express probabilities as ratios, fractions, decimals, percents and choose appropriate expressions given a particular situation
• conduct simulations and experiments to determine the probability of single and complementary events in real life situations of planning and making decisions (e.g., patterns in population growth, traffic)

Toward this, students in **Class 8** will be expected to master the following **SO** (Specific Outcomes):

8-G1 Theoretical Probability: single and complementary events
• apply formula from Class 7: $P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$
• understand that this formula can only be used when dealing with equally likely outcomes or events
• find the probability of a complementary event using the formula $1 - P(E)$
• understand that, if the probability of an event occurring is, e.g., $\frac{1}{4}$, then the probability of it not occurring is $1 - \frac{1}{4} = \frac{3}{4}$)

8-G2 Simulations and Experiments: single and complimentary events
• understand that, in situations for which the probability of various events occurring is not equally likely, experimentation is often the only method of determining probability

8-G3 Compare Results: theoretical and experimental
• compare theoretical and experimental probability for a given situation and discuss results
• investigate strategies to allow greater accuracy in experimental results (e.g., larger sample size)

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UNIT 1 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 1** **TG p. 4**	Review prerequisite concepts, skills, and terminology, and pre-assessment	1 h	• Rulers	All questions
Chapter 1 Powers				
1.1.1 Negative Exponents **SB p. 3** **TG p. 7**	**8-A1 Negative Exponents: develop meaning concretely and symbolically** • encountered in the context of place value charts (tenths, hundredths, thousandths as 10^{-1}, 10^{-2}, 10^{-3}) • introduce using a pattern ($100 = 10^2$, $10 = 10^1$, $1 = 10^0$, $0.1 = 10^{-1}$, $0.01 = 10^{-2}$) • work with base ten models	1 h	None	Q1, 2, 3, 4
GAME: Getting to a Half (Optional) **SB p. 6** **TG p. 9**	Practise working with powers, specifically powers with negative exponents, in a game situation.	30 min	• Dice	N/A
1.1.2 Scientific Notation **SB p. 7** **TG p. 10**	**8-A2 Large and Small Numbers: scientific notation to standard form and vice versa** • establish connection with multiplying by 0.1, 0.01, and 0.001 • make real-life connections (e.g., diameter of cell, diameter of electrons, mass of a bug) • relate small numbers to large numbers to show the difference between the two in scientific notation form • translate numbers from one form to another	1 h	None	Q3, 6, 7
Chapter 2 Square Roots				
1.2.1 Perfect Squares **SB p. 10** **TG p. 13**	**8-A3 Square Roots: modelling and representing** • model perfect squares and square roots using blocks or grid paper • establish link between concrete and numerical representations • on grids, view the area as the perfect square and the side length of the square as the square root **8-A4 Perfect Squares: patterns between 1 and 144** • recognize each of the perfect squares from 1 through 144 • expose to perfect squares up to 400 • relate patterns to perfect squares • understand that the differences in perfect squares increase in a constant way • work with patterns related to perfect squares of any size	1 h	None	Q1, 5, 8

UNIT 1 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
1.2.2 EXPLORE: Squaring Numbers Ending in 5 (Optional) **SB p. 12** **TG p. 16**	**8-A4 Perfect Squares: patterns between 1 and 144** • expose to perfect squares up to 400 • relate patterns to perfect squares • work with patterns related to perfect squares of any size	1 h	None	Observe and Assess questions
1.2.3 Interpreting Square Roots **SB p. 13** **TG p. 18**	**8-A3 Square Roots: modelling and representing** • model perfect squares and square roots using blocks or grid paper • on grids, view the area as the perfect square and the side length of the square as the square root **8-A5 Square Roots: find using an appropriate number** • estimate where the square root will fall • apply factorization in a variety of ways (e.g., 576 = 4 × 144 = 2 × 2 × 144 = 2 × 2 × 12 × 12 = 24 × 24, then $\sqrt{576}$ = 24)	1 h	• Grid paper, or Small Grid Paper (BLM)	Q1, 3, 4
1.2.4 Estimating and Calculating Square Roots **SB p. 16** **TG p. 21**	**8-A6 Square Roots: exact square root and its decimal approximation** • emphasize the difference between exact square root and the decimal approximation • model square roots for non-perfect squares **8-A5 Square Roots: find using an appropriate number** • estimate where the square root will fall • approximate to the point where students can identify which whole number is closer to the square root (e.g., square root of 22 is between 4 and 5 and is closer to 5 than 4) • use patterns and/or reasoning to determine whether the square root of a number is another number • apply factorization in a variety of ways	1 h	None	Q4, 7, 8
CONNECTIONS: The Square Root Algorithm (Optional) **SB p. 19** **TG p. 24**	Make a connection between the history of mathematical calculation and techniques of computation	30 min	None	N/A
UNIT 1 Revision **SB p. 20** **TG p. 25**	Review the concepts and skills in the unit	2 h	• Grid paper, or Small Grid Paper (BLM) (optional)	All questions
UNIT 1 Test **TG p. 27**	Assess the concepts and skills in the unit	1 h	• Grid paper, or Small Grid Paper (BLM) (optional)	All questions
UNIT 1 Performance Task **TG p. 29**	Assess concepts and skills in the unit	40 min	• Grid on **page 29** • Rulers	Rubric provided
UNIT 1 Blackline Masters **TG p. 32**	BLM 1 Small Grid Paper			

Math Background

• This unit extends students' understanding of exponents to negative exponents. In particular, it applies exponents to work with scientific notation. This is important because students meet scientific notation in science courses. The unit also extends earlier work with perfect squares to understanding the concepts of squaring and taking the square root of all whole numbers, including numbers that are not perfect squares.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 7** in **lesson 1.2.1**, where they count the number of perfect squares within a particular range, in **lesson 1.2.2**, where they explore a pattern to develop with a rule for squaring numbers that end in 5, and in **question 7** in **lesson 1.2.4**, where they calculate a square root to solve a problem.

• Students use communication in **question 6** in **lesson 1.1.1**, where they talk about the concept of place value to develop meaning for negative powers, in **question 8** in **lesson 1.1.2**, where they discuss how to compare numbers written in scientific notation, and in **question 9** in **lesson 1.2.3**, where they explain the term square root in a simple way.

• Students use reasoning in **question 5** in **lesson 1.1.1**, where they extend their knowledge of 10^{-1} to conjecture about what 2^{-1} means, in **question 3** in **lesson 1.2.1**, where they use reasoning to continue a pattern, in **question 5** in **lesson 1.2.1**, where they relate the factored form of a number to its potential to be a perfect square, and in **question 5** in **lesson 1.2.4**, where they use a pattern to develop a relationship involving the products of square roots.

• Students consider representation in the **Try This** in **lesson 1.2.1**, where they look at the pattern for perfect squares in terms of adding consecutive odd numbers, in **lesson 1.2.1**, where they recognize that a perfect square can be represented as the area of a square with whole number side lengths, and in **question 1** in **lesson 1.2.3**, where they represent a square root using a diagram.

• Students use visualization skills in **question 4** in **lesson 1.2.1**, where they develop a strategy to count the number of triangles in a complex diagram, and in **lesson 1.2.3,** where they estimate a square root using a geometric representation of squares with given areas.

• Students make connections in **question 3** in **lesson 1.1.2**, where they relate their knowledge of the meaning of negative exponents to the height of a pass, and in **question 7**, where they relate place value concepts to square root concepts.

Rationale for Teaching Approach

• This unit is divided into two chapters:

Chapter 1 focuses on powers and exponents. It extends student work with exponents to negative exponents and applies the use of exponents to scientific notation.

Chapter 2 focuses on the concept of the square root, looking at it from geometric and numerical perspectives.

• The two **Explore** lessons allow students to see an interesting pattern related to finding the squares of whole numbers with a 5 in the ones place.

• The **Connections** introduces students to a traditional algorithm for calculating square roots. Although this algorithm is not used often (since calculators are generally available to do this work), it provides some historical interest and some insight into properties of square roots.

• The unit's **Game** provides an opportunity to practise working with negative exponents.

Getting Started

Curriculum Outcomes	Outcome relevance
6 Prime Numbers: distinguish from composites **6** Factors: of whole numbers **6** Large Numbers: reading and writing **6** Whole Numbers and Decimals: Single-digit Division **6** Divide Mentally: whole numbers by 0.1, 0.01, 0.001 **7** Add, Subtract, Multiply, Divide: whole numbers and decimals **7** Large Numbers: model	Students will find the work in the unit easier after they review prime factors, standard and expanded forms of numbers, place value, and calculations with decimals.

Pacing	Materials	Prerequisites
1 h	• Rulers	• calculating areas of squares and volumes of cubes • comparing two numbers using fractions • familiarity with the terms *prime factor*, *standard form,* and *expanded form* • place value from billions through thousandths • multiplying and dividing by powers of 10 • multiplying and dividing by simple decimals

Main Points to be Raised

Use What You Know

• The number $n \times n$ represents the area of a square with side length n.

• The number $n \times n \times n$ represents the volume of a cube with side length n.

• If the side length of a square is multiplied by n, its area is multiplied by $n \times n$.

• If the side length of a cube is multiplied by n, its volume is multiplied by $n \times n \times n$.

• As the value of n increases, the value of $n \times n$ or $n \times n \times n$ increases even more if $n > 1$.

Skills You Will Need

• You can write every whole number as the product of primes.

• We write a number in expanded form as the sum of the values of the digits of the number.

• You can use exponential notation to describe the place value columns.

• To multiply two decimals, you can multiply the related whole numbers and then place the decimal points appropriately.

• To multiply or divide by a power of 10, you can calculate mentally by moving the digits to the appropriate place value positions.

Use What You Know — Introducing the Unit

• Before assigning the activity, you may wish to review how to calculate the area of a square and the volume of a cube.

For example, tell students that a square has a side length of 4 cm and ask them to tell you the area of the square. Similarly, tell them a cube has a side length of 4 cm and ask for its volume.

• Students can work alone or in pairs to complete the activity.

While you observe students at work, you might ask questions such as the following:

• *How do you know the area of the large square is a number multiplied by itself?* (I know that the length and width are the same, and the area is found by multiplying the length by the width.)

• *What should the denominator of your fraction be? Why?* (It should be 100 since I am taking a fraction of the area of the larger square and 100 is the area of the larger square.)

• *How do you know that the fractions will decrease if more 5s and 10s are multiplied?* (I am really multiplying by one half each time because 5 is half of 10.)

Skills You Will Need

- To ensure students have the required skills for this unit, assign all questions.
- First review the terms *prime factor, standard form,* and *expanded form* to make sure students can interpret the questions successfully.
- You should review how to represent large numbers by asking students to write the standard form of some examples that you say aloud.

  For example:

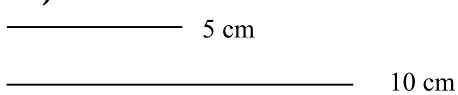
  thirteen million (13,000,000)

  two hundred and sixty-five thousand (265,000)

  five billion (5,000,000,000)
- Parts of **questions 5** may be done as a whole class using mental math. The remaining parts can be done individually using the chart.
- Students can work individually on other questions.

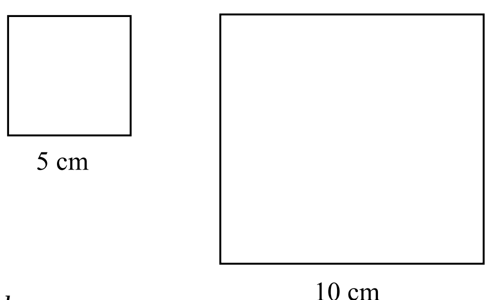
Answers NOTE: *Read about* ***Answers*** *on* ***page xvi*** *in the* ***Introduction*** *to this Teacher's Guide.*

A. i)



ii) $\frac{1}{2}$

B. i)



ii) *Sample response*:
The formula for the area of a square says to multiply the length by the width. Both are 5 cm.

iii) 10 × 10 **iv)** $\frac{1}{4}$

C. i) 5 × 5 × 5 and 10 × 10 × 10

ii) $\frac{1}{8}$

D. *Sample response*:
$\frac{1}{16}$ since the pattern of the denominators was to multiply by 2 each time.
5 × 5 × 5 × 5 = 25 × 25 = 625
10 × 10 × 10 × 10 = 100 × 100 = 10,000
625 × 16 = 10,000, so I was right.

E. Each time 5 is half of 10, so if I have more factors, I am multiplying by more halves. Whenever I multiply by one half, I am taking one half of something, which is only part of it.

1. a) 2 × 2 × 2 × 2 × 3 × 5 **b)** 3 × 3 × 5
c) 2 × 2 × 2 × 2 × 3 × 3 **d)** 31 (prime number)

2. a) 4 hundred thousands + 1 ten thousand + 2 thousands + 1 hundred + 5 tens;
4 × 100,000 + 1 × 10,000 + 2 × 1000 + 1 × 100 + 5 × 10
b) 3 hundred thousands + 6 ten thousands + 5 thousands + 1 hundred + 2 tens + 4 ones;
3 × 100,000 + 6 × 10,000 + 5 × 1000 + 1 × 100 + 2 × 10 + 4
c) 1 million + 3 thousands + 1 ten;
3 × 1,000,000 + 3 × 1000 + 1 × 10
d) 1 billion + 9 hundred thousands + 1 thousand + 1 hundred + 4 tens + 2 ones;
1 × 1,000,000,000 + 9 × 100,000 + 1 × 1000 + 1 × 100 + 4 × 10 + 2

3. a) 8,052,000; 8 millions + 5 ten thousands + 2 thousands
b) 40,070,000,637; 4 ten billions + 7 ten millions + 6 hundreds + 3 tens + 7 ones

4. a) 1.07 **b)** 1.98
c) 0.21 **d)** 0.0096

5. a) 3740 **b)** 230 **c)** 0.03028
d) 0.6234 **e)** 4000 **f)** 821.13
g) 0.00312 **h)** 0.234

6. a) Ten thousandths
b) Hundredths
c) Tens
d) Hundreds

Supporting Students

Struggling students

• Review prime factorisation. Check that students know the prime numbers less than 20 and, if necessary, replace 31 with 17 in **part d)** of **question 1** to illustrate the concept in this question.

• For **questions 2 and 3**, you may ask students to write each number directly in the place value chart. Then they can write that number in standard and both expanded forms.

• Remind students that the products in **question 4** are exactly the same as the products of whole numbers except for the position of the decimal point.

Enrichment

• Provide a value, such as 30.245. Ask students to write three decimal calculations, including at least one product and one quotient, that would give that value as a result, for example, 0.030245 ÷ 0.001 and 3024.5 × 0.01.

Chapter 1 Powers

1.1.1 Negative Exponents

Curriculum Outcomes	Outcome relevance
8-A1 Negative Exponents: develop meaning concretely and symbolically • encountered in the context of place value charts (tenths, hundredths, thousandths as 10^{-1}, 10^{-2}, 10^{-3}) • introduce using a pattern ($100 = 10^2$, $10 = 10^1$, $1 = 10^0$, $0.1 = 10^{-1}$, $0.01 = 10^{-2}$) • work with base ten models	Students will be able to work more effectively with decimals when they understand the connection between negative exponents and place values less than 1 (tenths, hundredths, and so on). This prepares them for further work with exponents, including scientific notation.

Pacing	Materials	Prerequisites
1 h	None	• familiarity with standard and expanded forms of numbers • connecting place value and powers of 10 for positive exponents • multiplying decimals

Main Points to be Raised

• We need negative exponents to represent powers of 10 (place values) less than 1 in exponential form. Each time you divide by 10, the exponent decreases by 1.

• In a power of 10 greater than 1 in standard form, the number of zeros after the 1 tells the exponent. In a power of 10 less than 1 in standard form, the number of zeros after the decimal point is one less than the opposite of the exponent.

• Place value charts can be extended to values of 0.1, 0.01, 0.001, and so on to represent place values to the right of the decimal point. The names of the place values to the right of the decimal point match the names to the left, symmetrically.

For example, ten thousandths match ten thousands.

• You can express a number less than 1 in exponential form using negative exponents.

For example: $20.35 = 2 \times 10^1 + 3 \times 10^{-1} + 5 \times 10^{-2}$

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Why is the number of digits not always the same as the number of parts?* (The digit 0 does not require a part.)

• *Does a greater number always have more parts? Explain your thinking.* (No; One million has only one part.)

• *How can you quickly count the number of parts without writing the number in expanded form?* (I can count the number of digits that are not zero.)

The Exposition — Presenting the Main Ideas

• Referring back to the **Try This**, write the number 810,053.1146 on the board. Have the students prepare a 10-column place value chart that will accommodate the number. Ask students to write the number in the chart and then write the expanded form of the number using words and using numbers.

• Using the same place value chart, have students add an additional row. In this row, ask them to write the powers of 10 for each place value. For example, 100,000 is 10^5, 10,000 is 10^4, and so on. They will find it challenging to write the ones, tenths, hundredths, and thousandths.

• Work through the pattern, showing how the exponents decrease by 1 as the place values move to the right in the number. This will lead to the connection that you must use an exponent of 0 for ones. Further, you need negative exponents for place values to the right of the decimal point.

• Return to the number used above to show how to write a number in exponential form:

$810{,}053.1146 = 8 \times 10^5 + 1 \times 10^4 + 5 \times 10^1 + 3 \times 10^0 + 1 \times 10^{-1} + 4 \times 10^{-3} + 6 \times 10^{-4}$

• Point out that the number of parts that you need to add in the expanded form of a number is the same as the number of products that appear in the exponential form. There are eight parts in the expanded form of 810,053.1146.

• Encourage students to read through the exposition.

Revisiting the Try This

B. This question allows students to make a formal connection between the decimal portions expressed as numbers in **part A** and their corresponding exponential forms that use negative exponents.

Using the Examples

• Have students work in pairs on **example 1**. Each student should write both numbers in standard form. One student can write the number in expanded form using words in **part a)** and numbers in **part b)**. The other student can do the opposite. They can then compare answers. If the students do not use place value charts in their solutions, they can see how the charts are used in the book.

• **Example 2** has three parts. You can model the first part in class and students can work alone on the remaining parts. They can then compare their answers to the solutions in the text.

Practising and Applying

Teaching points and tips

Q 1: Students can refer to **example 1** and use place value charts if necessary.

Q 2: Students can refer to **example 2** for a model.

Q 3: Encourage students to check their answers by doing some parts in two ways: comparing exponents only and calculating values before comparing quantities.

Q 4: Although **part d)** is the final part, it is likely to be the easiest part of the question.

Q 5: Some students will find this idea to be abstract. This question is less important than other questions at this stage. You can remind students to consider the values of other powers of 2 and look for a pattern. Or, they might use the powers of 10 to find a pattern to help them.

Q 6: This question is suitable for whole class discussion.

Common errors

• Many students mishandle negative exponents in calculations. Remind them that a negative exponent does not result in a negative number but rather that it represents a power of 10 that has a value less than 1.

• Students may want to write 10^{-2} as 0.001 because they will use the exponent to indicate the number of zeros rather than the number of places after the decimal point. Remind them that $10^{-1} = 0.1$.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can express numbers in standard form and in both expanded forms
Question 2	to see if students can perform calculations involving negative exponents
Question 3	to see if students can compare numbers written with negative exponents
Question 4	to see if students recognize that different forms of expanded notation can be used to write decimal equivalents

Answers

A. i) 5 **ii)** 4 **iii)** 8 Each nonzero digit represents a part of the number in expanded form.	**B. i)** $6 \times 10^{-1} + 1 \times 10^{-2}$ **ii)** 3×10^{-3} **iii)** $1 \times 10^{-1} + 1 \times 10^{-2} + 4 \times 10^{-3} + 6 \times 10^{-4}$

NOTE: Answers or parts of answers to numbered questions that are in square brackets throughout the Teacher's Guide are NOT found in the answers at the back of the student text. (See ***Answers*** *on* ***page xvi*** *in the* ***Introduction*** *to this Teacher's Guide.)*

1. a) 0.1407; 1 tenth + 4 hundredths + 7 ten thousandths;
$1 \times 0.1 + 4 \times 0.01 + 7 \times 0.0001$
b) 306.057008; 3 hundreds + 6 ones + 5 hundredths + 7 thousandths + 8 millionths;
$3 \times 100 + 6 \times 1 + 5 \times 0.01 + 7 \times 0.001 + 8 \times 0.000001$
c) 0.00075; 7 ten thousandths + 5 hundred thousandths;
$7 \times 0.0001 + 5 \times 0.00001$
d) 5,060,030.047003; 5 millions + 6 ten thousands + 3 tens + 4 hundredths + 7 thousandths + 3 millionths;
$5 \times 1{,}000{,}000 + 6 \times 10{,}000 + 3 \times 10 + 4 \times 0.01 + 7 \times 0.001 + 3 \times 0.000001$

2. a) 0.011 **b)** 2.58 **c)** 0.000001

3. a) 3×10^{-2} **b)** 2×10^{-3} **c)** $10^2 \times 10^{-2}$

4. a) 13; [The smallest part of the number in exponential form is 4×10^{-13} which means that the standard form of the number will have a nonzero digit 13 places to the right of the decimal.]
b) Yes. [There could be seven zeros after the last nonzero digit. The number would be written as 0.00050060509040000000.]

4. c) Greater ($5 \times 10^{-4} + 6 \times 10^{-7} + 5 \times 10^{-9} + 9 \times 10^{-11} + 4 \times 10^{-13} > 5 \times 10^{-4}$);
[*Sample response*:
$5 \times 10^{-4} + 6 \times 10^{-7} + 5 \times 10^{-9} + 9 \times 10^{-11} + 4 \times 10^{-13}$ is 5×10^{-4} with more added.]

5. $\frac{1}{2}$; [*Sample response*:
$2^3 = 2 \times 2 \times 2 = 8$
$2^2 = 2 \times 2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$

The pattern is to divide by 2 each time the exponent decreases by 1.

[**6.** *Sample response*:
In the place value system, the exponent of 10 is 1 less each time you go from left to right. That means that after 10^2, 10^1, and 10^0, you have to have negative exponents.]

Supporting Students

Struggling students

• **Question 5** may not be suitable for students who struggle with more abstract mathematical thinking.

• If students have difficulty working with negative exponents, you may wish to encourage them to create their own place value chart for reference.

• Some students may need extra time if they need to do the calculations because they do not recognize the relationships between the number of parts and the digits, or the importance of the exponents in comparing quantities. Help these students by requiring them to complete fewer parts of some questions.

Enrichment

• You might challenge students to write numbers that meet given requirements.

For example, write a number that has all of these features:

- It has five parts when it is written in expanded form
- It has eight digits and ends with a 4.
- The largest place value is millions and the smallest place value is represented by 10^{-3}.

GAME: Getting to a Half

• This game provides practice with negative exponents, standard form, and multiplication and addition of decimals.

• Students need to recognize that rolling two dice gives two possible numbers, except when they roll doubles. Their choices about which number to use for the whole number and which number to use for the exponent will depend upon the numbers and the situation in the game.

• For a variation to the game, students can change the sum or the number of rolls permitted in a round. Or, they can use three dice, where they add two dice together to be either the powers of 10 or the whole number.

For example:

A roll of 2, 3, and 4 could result in 5×10^{-4} (5 is 2 + 3), 6×10^{-3} (6 is 2 + 4), 7×10^{-2} (7 is 3 + 4),
2×10^{-7} (7 is 3 + 4), 3×10^{-6} (6 is 2 + 4), or 4×10^{-5} (5 is 2 + 3).

1.1.2 Scientific Notation

Curriculum Outcomes	Outcome relevance
8-A2 Large and Small Numbers: scientific notation to standard form and vice versa • establish connection with multiplying by 0.1, 0.01, and 0.001 • make real-life connections (e.g., diameter of cell, diameter of electrons, mass of a bug) • relate small numbers to large numbers to show the difference between the two in scientific notation form • translate numbers from one form to another	Scientific notation is a practical application of negative exponents. This work will strengthen students' skills with powers and exponents to prepare them for future work with applied problems and contexts, particularly in the sciences.

Pacing	Materials	Prerequisites
1 h	None	• familiarity with negative exponents • writing expanded and exponential forms of numbers

Main Points to be Raised

- Scientific notation has a particular form that helps us compare numbers.
- A number in scientific notation is written as a whole number or decimal multiplied by a power of 10.
  The whole number or decimal is called the multiplier. The multiplier is always greater than or equal to 1 and less than 10.
- The exponent in the power of 10 is positive if the original number is greater than the multiplier. The exponent is negative if the original number is less than the multiplier.
- An exponent of 0 is used with the power of 10 when the original number is the multiplier.
  For example, 3.42 is written 3.42×10^0.
- It is easy to compare numbers written in scientific notation. The number with the greater power of 10 is greater. If the powers of 10 are the same, the number with the greater multiplier is greater.

Try This — Introducing the Lesson

A. Allow students to try this alone. While you observe students at work, you might ask questions such as the following:

- *How do you know that* $0.3456 \times 10^3 = 345.6$*?* (I moved the digits three places to the left to multiply by 1000.)
- *Which of the expressions can you quickly tell is equal to 345.6?* (3456×10^{-1}, as it is 3456 with the digits moved one place to the right.)
- *Have you found an expression that does not equal 345.6?* (Not yet.)

The Exposition — Presenting the Main Ideas

- Students should have noticed that the examples in the **Try This** are all equivalent expressions. Talk with students about why they could not see immediately that the numbers were equivalent. Point out that it would be useful to use a consistent form so that you can readily compare numbers. Scientific notation offers such a form.
- Explain how to write a number in scientific notation: you express the number as a product of a multiplier and a power of 10. The multiplier is always greater than or equal to 1 and less than 10.

For example, to write 56738.209 in scientific notation, the digits are shifted to obtain a multiplier of 5.6738209 (a number in the permissible range). The power of 10 is 10^4 because that is what 5.6738209 needs to be multiplied by to get to the original number.

$$56738.209 = \underbrace{5.6738209}_{\text{Multiplier}} \times \underbrace{10^4}_{\text{Power of 10}}$$

• Show students why the comparison of numbers is straightforward when the numbers are in scientific notation.

- *How do you know that* 2.31×10^5 *greater than* 4.6701×10^4*?* (2.31×10^5 is more than 20,000, but 4.6701×10^4 is only a bit more than 46,000.)

- *Which is greater:* 4.713×10^{-3} *or* 6.03×10^{-3}*? How do you know?* (6.03×10^{-3}; Both numbers are in scientific notation. The powers of 10 are the same so I compared the multipliers. 6.03 is greater than 4.713 because about 6 groups of something is more than about 4.7 groups of the same thing.)

Revisiting the Try This

B. Students should recognise that none of the expressions in **part A** are written in scientific notation. You may wish to ask them to write 345.6 in scientific notation before starting work with **example 1**.

Using the Examples

• List the five numbers in **example 1** on the board. Ask students to work in pairs to write each of the numbers in scientific notation and then compare their answers to the solutions in the text. Students can then read **example 2** and ask any questions they may have about the solutions. **Example 3** can be done as a class.

Practising and Applying

Teaching points and tips

Q 1: You can direct students to review **example 1** to help them see how to deal with any concerns. This is a basic question that should be done first by all students.

Q 3: Students must use reasoning to understand how the exponent affects the value of the number.

Q 5: Refer students to the exposition if they need information to answer the question.

Q 6: Students should be directed back to **question 2** if they are unsure about how to make comparisons.

Q 8: Encourage students to incorporate examples into the explanation of their thinking.

Common errors

• Students sometimes record the exponents in the powers of 10 as the opposite of what they should be. This happens particularly when there is no context for the number. If there is a context, for example, **question 3** (about the height of the pass), help them see why a negative exponent would result in a very small number, which would not make sense.

• Students may struggle to express numbers properly in scientific notation. It is helpful to keep in mind that if the multiplier is less than the original number, then it needs to be multiplied by a positive power of 10.

• Errors caused by using incorrect powers of 10 can result in serious problems in real-world contexts. It is worthwhile for students to check to see if their results are reasonable. They may find their own errors through checking.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can demonstrate number sense with scientific notation in a real-world context
Question 6	to see if students can interpret and compare numbers expressed in scientific notation
Question 7	to see if students can write numbers using scientific notation

Answers

A. All of them	**B.** None; None of the multipliers is greater than or equal to 1 and less than 10.
1. a) 2.3196×10^2 **b)** 4.356×10^6 **c)** 2.1×10^{-4} **d)** 1.367×10^{-1} **2. a)** 2×10^{-2}; [Its power of 10 is greater and both numbers are in scientific notation.] **b)** 2×10^6; [The powers of 10 are the same and $2 > 1.5996$.] **c)** 1.99×10^5; [Its power of 10 is greater and both numbers are in scientific notation.] **3.** 3.8×10^3 m is a reasonable height; [It is 3800 metres. Since 10^{-3} is less than 1, the other number is much too small.] **4. a)** 4.7×10^4 **b)** 7.982×10^8	**5.** Yes; [The multiplier in a number in scientific form must be greater than or equal to 1 and less than 10, and $1.0 = 1$.] **6.** Dorji; [$9.31 \times 10^4 > 9.86 \times 10^3$ because $10^4 > 10^3$; Dorji has Nu 83,240 more.] **7. a)** 1.496×10^8 **b)** 5.1×10^8 **c)** 7.4×10^{-4} **8.** Powers of 10; [*Sample response*: I would compare the powers of 10 first because if they are different, the number with the greater power of 10 is greater. I would not have to look at the multipliers. If the powers of 10 are the same, then I would compare the multipliers.]

Supporting Students

Struggling students

• Struggling students might have difficulty with **question 6**. You may wish to have them work with a non-struggling partner on this question. It is likely the language rather than the mathematical content that makes this question appear difficult.

• You might encourage students who struggle with **question 1 or 2** to revisit the examples before proceeding further in the lesson.

Enrichment

• You might ask students to research examples where scientific notation is used or to find real-world situations where numbers would be more meaningful if they were expressed in scientific notation.

Chapter 2 Square Roots

1.2.1 Perfect Squares

Curriculum Outcomes	Outcome relevance
8-A3 Square Roots: modelling and representing • model perfect squares and square roots using blocks or grid paper • establish link between concrete and numerical representations • on grids, view the area as the perfect square and the side length of the square as the square root **8-A4 Perfect Squares: patterns between 1 and 144** • recognize each of the perfect squares from 1 through 144 • expose to perfect squares up to 400 • relate patterns to perfect squares • understand that the differences in perfect squares increase in a constant way • work with patterns related to perfect squares of any size	Recognizing and modelling perfect squares are important skills for developing number sense. These concepts are the foundation for work with square roots, and will help students understand more advanced number concepts in secondary school.

Pacing	Materials	Prerequisites
1 h	None	• finding the prime factors of a number

Main Points to be Raised

• A perfect square is the product of a whole number multiplied by itself.

For example, 49 is a perfect square since 7 × 7 = 49.

• A square with whole number side lengths has an area that is a perfect square.

For example, an area of 49 square units can be represented by a square with sides of length 7 units.

• You can look at the prime factors of a number to determine whether it is a perfect square. If you can pair each of the prime factors with an identical prime factor, then the product is a perfect square.

For example:

$36 = \underline{2 \times 2} \times \underline{3 \times 3}$; 36 is a perfect square.

$12 = \underline{2 \times 2} \times 3$; 12 is not a perfect square.

• The differences between consecutive perfect squares form the pattern 3, 5, 7, ... (4 – 1 = 3; 9 – 4 = 5; 16 – 9 = 7; ...).

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *What sum comes next?* (1 + 3 + 5 + 7 + 9 = 25 = 5 × 5.)

• *Do you see the connection between the picture and the pattern?* (Yes. The numbers 1, 3, 5, and so on are the number of squares in the L-shape each time. The total number of squares in each big square is the sum.)

• *If you keep the pattern going, what is a greater number that will be a sum? How do you know?* (81; 9 × 9 = 81 and any number times itself will appear as a sum eventually.)

The Exposition — Presenting the Main Ideas

• Read through the exposition with students. Draw their attention to the connection between a model of a square with whole number side lengths and the concept of a perfect square.

• Do some mental math with the class to make sure students are familiar with perfect squares up to and including 144.

• Have students write out the prime factors of 24, 49, 80, 81, and 196. Ask them to see if they can pair the prime factors in each case. Students can then determine which of these numbers are perfect squares.

Revisiting the Try This

B. This question allows students to revisit **part A** with increased attention to the visual model. It is important to recognise the connections between visual models and corresponding numerical representations.

Using the Examples

• **Example 1** can be done as a class. Note that the numbers do not have to be broken down into prime factors. Instead, students can see that factors pair off as with 400 = 20 × 20. This makes 400 a perfect square. You can point out that a number like 200 is not a perfect square because although 100 is a perfect square, 2 is not, and 200 = 100 × 2. The factor of 2 will be left on its own when 100 is factored into a pair of 10s.

Practising and Applying

Teaching points and tips

Q 2: Remind students that the ones digit in a product is the ones digit of the product of the ones digits of the factors.

For example, 397 × 397 ends in 9 because 7 × 7 = 49, which ends in 9.

Q 3: This simple patterning question prepares students for the more complex patterning in **question 4**.

Q 4: This question could be reviewed by pairs of students or by the entire class.

Q 5: This question reinforces the importance of pairing all factors to produce a perfect square.

Q 6: This question could be done with the entire class or in groups. You may wish to have students share the work and summarize the results in a chart.

Q 7: You may wish to come back to this question as you are teaching **lesson 1.2.3**, as it connects nicely to square roots.

Q 8: It may be helpful for some students first to list perfect squares that are divisible by 3. This will help suggest the claim is true. An explanation, rather than examples, is required to verify the claim.

Common errors

• Some students think that a number with perfect squares inside of it must be a perfect square.

For example, 2 × 2 × 3 × 3 × 3 may be mistaken for a perfect square because its prime factors include a pair of 2s and a pair of 3s.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can identify whether a number is a perfect square
Question 5	to see if students recognize the connection between factoring and determining whether a number is a perfect square
Question 8	to see if students can reason about divisibility and perfect squares

Answers

A. 1 + 3 + 5 + 7 + 9 = 25 = 5 × 5 1 + 3 + 5 + 7 + 9 + 11 = 36 = 6 × 6 1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 = 7 × 7	**B.** *Sample responses*: **i)** Starting at the bottom left corner there is 1 grey square. The next white L-shape has 3 squares. Added together (1 + 3), these make a 2 × 2 = 4 square. • The next grey L-shape has 5 squares. Added together (1 + 3 + 5), this makes a 3 × 3 = 9 square. • The next white L-shape has 7 squares. Added together (1 + 3 + 5 + 7), this makes a 4 × 4 = 16 square. • The next grey L-shape has 9 squares. Added together (1+ 3 + 5 + 7 + 9), this makes a 5 × 5 = 25 square. **ii)** Each square in the large square is made up of an L shape with an odd number of squares and a smaller square. For example, the 5-by-5 square is made up of an L-shape with 9 squares and a smaller square of 16 squares. The L-shapes, which are each made up of an odd number of squares, represent the (odd) difference between the squares.

1. 144

2. a) 0, 1, 4, 5, 6, and 9
b) No; [*Sample response*:
The digit 7 does not appear in the ones place when you multiply 1 × 1, 2 × 2, 3 × 3, …, 10 × 10. All greater perfect squares end in the same digits (0, 1, 4, 5, 6, and 9). For example, 73 × 73 ends in the same digit as 3 × 3.]

3. a) 4, 9, 16, 25
b) The sums are perfect squares.
c) 36

4. a) 4
b) 9
c) 16
d) 36; [The total number of small triangles = the square of the number of rows.]

5. a) No; [One of the 2s and the 7 cannot be paired.]
b) 14; [The extra 2 and the 7 each need factors to pair with, so you would multiply both by 2 and by 7. 2 × 7 = 14.]

6.

a)	**b)**
1: 1	1 factor
2: 1, 2	2 factors
3: 1, 3	2 factors
4: 1, 2, 4	3 factors
5: 1, 5	2 factors
6: 1, 2, 3, 6	4 factors
7: 1, 7	2 factors
8: 1, 2, 4, 8	4 factors
9: 1, 3, 9	3 factors
10: 1, 2, 5, 10	4 factors
11: 1, 11	2 factors
12: 1, 2, 3, 4, 6, 12	6 factors
13: 1, 13	2 factors
14: 1, 2, 7, 14	4 factors
15: 1, 3, 5, 15	4 factors
16: 1, 2, 4, 8, 16	5 factors

c) The perfect squares each have an odd number of factors and the other numbers each have an even number of factors.

7. a) 20
[**b)** The least perfect square is 100 = 10 × 10 and the greatest is 841 = 29 × 29. There are 20 perfect squares altogether since there are 20 numbers from 10 up to and including 29.]

8. Yes; [*Sample response*:
Each factor in a perfect square must have a matching factor to pair with, so the 3 must have another 3. That means there is a factor of 3 × 3, or 9.]

9. *Sample response*:
A perfect square
• has an odd number of factors
• can form a square with whole number length sides
• has factors that can be paired when factored into prime numbers

Supporting Students

Struggling students

• Make sure students can express a number as a product of prime factors. If necessary, you may provide the prime factorisation of numbers so that students can focus on determining whether a number is a perfect square. Students who take a long time to do the work will likely gain more from the lesson if less prime factorisation is required of them.

Enrichment

• Pose claims like the one made in **question 8**. Have students discuss their validity. Here are some examples:

- *A perfect square ending in 25 is divisible by 5.* (True. The number is divisible by 25 and 25 = 5 × 5.)
- *A perfect square ending in 9 is divisible by 3.* (False. 49 is not divisible by 3.)
- *A perfect square that is divisible by 4 is divisible by 16.* (False. 4 is not.)
- *A perfect square that is divisible by 2 is divisible by 4.* (True. Each 2 must have another 2 to pair with, making 2 × 2 a factor.)

Reprint 2024

1.2.2 EXPLORE: Squaring Numbers Ending in 5

Curriculum Outcomes	Lesson Relevance
8-A4 Perfect Squares: patterns between 1 and 144 • expose to perfect squares up to 400 • relate patterns to perfect squares • work with patterns related to perfect squares of any size	Mental math is an important tool for success in everyday mathematics. If students think about when they could use mental math, they will be more likely to use it.

Pacing	Materials	Prerequisites
1 h	None	• multiplying whole numbers

Main Points to be Raised

• There is a pattern for perfect squares of numbers with a 5 in the ones place.

• To square the number *a*5 (i.e., *a* tens + 5 ones), multiply $a \times (a + 1)$ and put a 25 at the end.
For example, $35^2 = 1225$, where $12 = 3 \times (3 + 1)$.

Exploration

• Explain the meaning of *square root* to the class. Select suitable examples to review the perfect squares up to 144, using mental math.

• Write the chart for **part A** on the board. Fill in the first row or two with the entire class. Then have students work in pairs to complete the chart. Try to make sure all students have time to complete the chart. Any students who have completed the chart can complete **part A**.

• Discuss the answers to **part A ii)**, **iii)**, **and iv)** with the class as a whole.

• Have students proceed with **parts B**, **C**, **and D**. These questions can be done alone, in pairs, or as a class. For **part B**, make sure that when students think about how many tens are in 115, they do not focus on the tens digit of 1, but rather on the fact that 115 is 11 tens and 5 more.

Observe and Assess

As students work, notice the following:

• Do they need to write down products or can they calculate the perfect squares mentally?

• Do they complete the final rows of the chart quickly? Have they observed the pattern?

• Do they calculate correctly using mental math?

Share and Reflect

After students have had sufficient time to work through the exploration, discuss the generalisations and how they apply to square roots with more than two digits, as with 115 or 195.

• *How do you know how many tens there are in 115? 195?*

• *Why is it useful to think of 5 as 05 when you are trying to explain why the method works?*

• *What is 105 × 105? How did you get that answer?*

• *Can you tell me another perfect square ending in 5 that has a square root greater than 100?*

Answers

A. i)

Product	**Perfect square**	**Number of tens in square root**	**Number of hundreds in perfect square**
15 × 15	225	1	2
25 × 25	625	2	6
35 × 35	1225	3	12
45 × 45	2025	4	20
55 × 55	3025	5	30
65 × 65	4225	6	42
75 × 75	5625	7	56

ii) They all end in 25.
iii) The number of hundreds in the perfect square is equal to the product of the number of tens in the square root and the number of tens in the square root plus 1.
For example, for the square root 15 and perfect square 225, 1 × (1 + 1) = 1 × 2 = 2, so there are 2 hundreds.
iv) 7225 and 9025;
85 × 85 = [8 × (8 + 1)] hundreds + 25 =
(8 × 9) hundreds + 25 = 7225
95 × 95 = [9 × (9 + 1)] hundreds + 25 =
(9 × 10) hundreds + 25 = 9025

B. i) 11 tens
ii) 115 × 115 = [11 × (11 + 1)] hundreds + 25 =
(11 × 12) hundreds + 25 = 13,225
.
C. You find the number of tens in the whole number and multiply it by a number that is 1 greater. This is the number of hundreds in the square. Then you add 25.
195 × 195 = [19 × (19 + 1)] hundreds + 25 =
(19 × 20) hundreds + 25 = 38,025

D. Yes; The perfect square has (0 × 1) hundreds + 25 = 25.
5 × 5 = 25, so it works.

Supporting Students

Struggling students

• Some students prefer to write out calculations rather than performing them mentally because this gives them more confidence in the answer. You may wish to encourage these students by showing them how to perform calculations mentally using the shortcut.

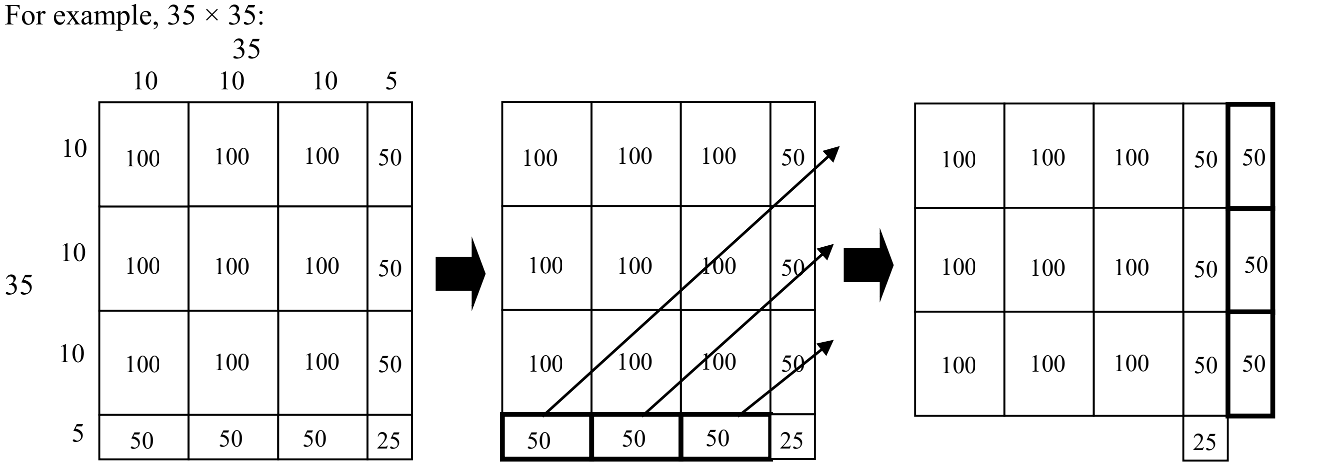
• After discussing **part A** with the class, you may not wish to assign more individual work to struggling students.

Enrichment

• Pair students and have them individually square numbers of their choice ending in 5. The partners can trade the perfect squares and try to determine each other's square roots using mental math.

• Students might be interested in seeing how an area model can be used to explain visually how this mental math strategy works.

For example, 35 × 35:



Multiply 35 × 35 in parts.

Rearrange the six 50s into three 100s.

You end up with 3 rows of 4 hundreds and an extra 25.

1.2.3 Interpreting Square Roots

Curriculum Outcomes	Outcome relevance
8-A3 Square Roots: modelling and representing • model perfect squares and square roots using blocks or grid paper • on grids, view the area as the perfect square and the side length of the square as the square root **8-A5 Square Roots: find using an appropriate number** • estimate where the square root will fall • apply factorization in a variety of ways (e.g., 576 = 4 × 144 = 2 × 2 × 144 = 2 × 2 × 12 × 12 = 24 × 24, then $\sqrt{576} = 24$)	Students will form a better connection between perfect squares and square roots as they work with models. Understanding the difference between exactness and estimation is important as the basis for more advanced secondary level mathematics. The focus on interpretation is intended to support the subsequent lesson on estimating and calculating square roots.

Pacing	Materials	Prerequisites
1 h	• Grid paper, or Small Grid Paper (BLM)	• familiarity with perfect squares up to and including 144 • familiarity with rectangular area models

Main Points to be Raised

• The *square root* of a given number is the number that can be multiplied by itself to get the given number.

• The symbol $\sqrt{\ }$ is used to show the square root.

For example, $\sqrt{81} = 9$.

• The side length of a square is the square root of the area of the square.

• The square root of a number can be exact or it can be an estimate.

• One method for estimating a square root is to average the length and width of a rectangle. The more square-like the rectangle, the better the estimate will be.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Why is there only one rectangle with an area of 13 square units?* (13 is a prime number. 13 × 1 is the only possibility.)

• *Are any of your rectangles square?* (Yes. I have a square with an area of 16 square units.)

Make sure students understand that there would be many more possible rectangles if there were no restriction to use sides with whole-number lengths.

The Exposition — Presenting the Main Ideas

• Ask students to determine the square roots of 100 and 90. Use these examples to explain the difference between an exact square root and an estimate of a square root. (Note that the **Explore lesson 1.2.2** can be connected to estimates by squaring numbers ending in .5; for example, 9.5 × 9.5 = 90.25.)

• Have students draw a picture on grid paper to show that the square root of 9 is 3. Then ask them how the picture can tell them something about the square root of 8.

• Continue with the example to show how the average of the length and width of the most square-like rectangle can be used as an estimate of the square root of the rectangle’s area. As an example, contrast the average of 1 and 8 (the side lengths of a 1-by-8 rectangle) and the average of 2 and 4 (the side lengths of a 2-by-4 rectangle) as estimates for the side length of a square with area 8.

• Make sure students are familiar with the approximately equal sign, ≈. Note that it is sometimes shown as an equal sign with a dot above it.

• Encourage students to read through the exposition and ask any questions they have.

Revisiting the Try This

B. Students estimate the side lengths of squares. Comparing the squares of the estimates with actual areas provides examples of an exact square root, a good estimate, and a poor estimate.

Using the Examples

• **Example 1 parts a) and b)** can be done as a class to reinforce work in the exposition. Ask students to do **part c)** in pairs. Check that the students realise that doubling the area of a square does not mean the same as doubling the side length.

• Ask students to read **example 2** to see how prime factorisation may help in estimating side lengths of squares.

Practising and Applying

Teaching points and tips

Q 1: Students can refer to **example 1** for a model, if needed.

Q 2 and 3: These questions work together to illustrate the difference between an exact square root and an estimate. Mental math can be used because students only need to know perfect squares up to and including 144.

Q 4 and 5: Students can initially try examples with numbers to help them better understand the questions if they are confused.

Q 6 and 7: These questions provide one example of a good estimate and one example of a poor estimate. Students could be assigned one or the other question and then you could compare the two situations in a class discussion.

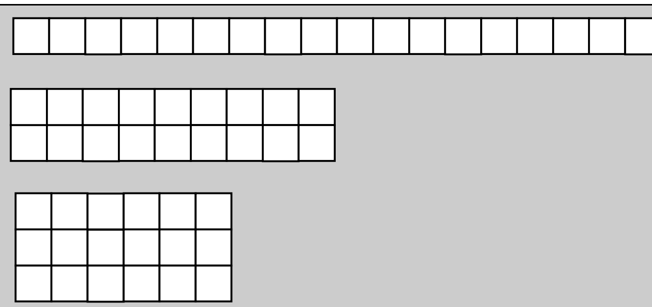
Q 9: Encourage students to use an example to support their explanations.

Suggested assessment questions from Practising and Applying

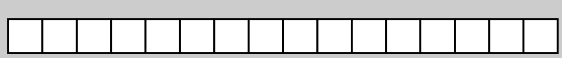
Question 1	to see if students can draw a diagram to illustrate the concept of the square root
Question 3	to see if students can apply knowledge of perfect squares up to and including 144 in determining the values of square roots
Question 4	to see if students can reason about the relationship between side lengths and the area of a square

Answers

A. i) 1 × 18, 2 × 9, and 3 × 6



ii) 1 × 16, 2 × 8, and 4 × 4

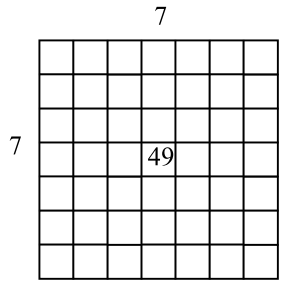


iii) 1 × 13

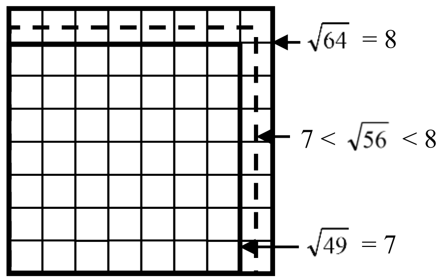
B. 18:
i) 6 and 3 **ii)** 4.5
iii) 20.25, which is greater than the original area.
16:
i) 4 and 4 **ii)** 4
iii) 16, which is the exact area.
13:
i) 13 and 1 **ii)** 7
iii) 49, which is much greater than the original area.

Answers [Continued]

1. a)



b)



2. a) 9 **b)** 10
c) 8 **d)** 11

3. a) 5 **b)** 6
c) 10 **d)** 12

4. No; [*Sample response*:
If I take a square of side length 3 and triple the length to 9, the area becomes 81 square units, which is actually 9 times greater than the original area of 9 square units.]

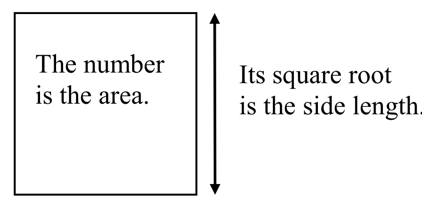
5. *Sample response*: 625, 169, and 8100.

6. a) 1 × 72, 2 × 36, 3 × 24, 4 × 18, 6 × 12, and 8 × 9
b) About 8.5; [(8 + 9) ÷ 2 = 8.5]
c) Yes; [*Sample response*:
8.5 × 8.5 = 72.25, which is very close to 72.]

7. a) 1 × 95 and 5 × 19
b) About 12; [(5 + 19) ÷ 2 = 12]
c) No; [*Sample response*:
12 × 12 = 144, which is much greater than 95.
The square root of 95 must be less than the square root of 100, which is 10.]

8. *Sample response*:
About 5.4 cm; No; [5.4 × 5.4 = 29.16, which is not between 30 and 31.]

[**9.** *Sample response*:
The square root of a given number is a number that, when multiplied by itself, results in the given number. For example, the square root of 25 is 5 because 5 × 5 = 25.
The square root of a number can be represented by the side length of a square with an area equal to the given number.



Supporting Students

Struggling students

• Some students may find **questions 2 and 3** difficult because they do not know the perfect squares up to and including 144. If so, reduce the number of parts of these questions or allow them to write out a reference list containing the perfect squares.

Enrichment

• Students should do all questions in this lesson. They can discuss the explanations in **questions 8 and 9** in pairs, and possibly share them with the class.

1.2.4 Estimating and Calculating Square Roots

Curriculum Outcomes	Outcome relevance
8-A6 Square Roots: exact square root and its decimal approximation • emphasize the difference between exact square root and the decimal approximation • model square roots for non-perfect squares **8-A5 Square Roots: find using an appropriate number** • estimate where the square root will fall • approximate to the point where students can identify which whole number is closer to the square root (e.g., square root of 22 is between 4 and 5 and is closer to 5 than 4) • use patterns and/or reasoning to determine whether the square root of a number is another number • apply factorization in a variety of ways (e.g., 576 = 4 × 144 = 2 × 2 × 144 = 2 × 2 × 12 × 12 = 24 × 24, then $\sqrt{576}$ = 24)	Students gain skills with estimating and calculating square roots, which enhances their number sense and opens up another connection to applied mathematical situations.

Pacing	Materials	Pre-requisites
1 h	None	• multiplying decimals

Main Points to be Raised

• A square root can be a whole number or an exact decimal, but a square root is often only an approximation.

- The square root of 64 is 8.
- The square root of 1.44 is 1.2.
- The square root of 3 is 1.7 to one decimal place.

• Square roots of even powers of 10 such as 100, 10,000, and 1,000,000 can be helpful in estimating or calculating square roots. This is because the even powers of 10 have 10s that can be paired up, so they are perfect squares.

For example: 10,000 = $\underline{10 \times 10} \times \underline{10 \times 10}$

The square root of 10,000 is 10 × 10 = 100.

• You can find the square root of a number by breaking the number into the product of numbers for which you can more easily find square roots.

For example, to calculate $\sqrt{14,400}$:

14,400 = 144 × 100 [$\sqrt{144} = 12$ and $\sqrt{100} = 10$]

$\sqrt{14,400}$ = 12 × 10 = 120

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Observe students while they work. You might ask questions such as the following:

• *Is the side length less than 100 m? How do you know?* (Yes. 100 × 100 is too large.)

• *What number near 6510 could be a perfect square? Why?* (6400; 80 × 80 = 6400)

• *Is the side length more or less than 80?* (It is more than 80 because 6510 > 6400)

The Exposition — Presenting the Main Ideas

• Write three numbers on the board: 64, 1.44, and 3. Ask students to work out the square root of each of these numbers.

• As they work, observe whether students recognize that 1.44 has an exact square root. When students seem to know that they cannot find an exact square root for 3, tell them that they should estimate the square root to one decimal place.

• Ask students to find the square root of 12,100. As a class, discuss how 12,100 = 121 × 100 and this leads to finding $\sqrt{12{,}100} = 11 \times 10 = 110$. You may need to explain to students why they can find the square roots of 121 and 100 separately and then multiply them. Because 11 is the square root of 121, then 11 × 10 × 11 × 10 = 11 × 11 × 100 = 12,100.

• With the class try another example that will work exactly, such as $\sqrt{4900}$. Extend the idea to greater powers of 10 using 490,000 or 49,000,000.

• Then write $\sqrt{487200}$ and guide the class to find an estimate of its value. Note that even powers of 10 and values near perfect squares are most helpful. Observe that 487,200 = 48.72 × 10,000. You might ask:

- *What perfect square is near 48.72?* (49)
- *What is the square root of 49?* (7)
- *What is the square root of 10,000?* (100)
- *What is a good estimate of the square root of 487,200?* (700 = 7 × 100)

• Have students read the exposition for a review of the main ideas.

• Depending on how comfortable students are with the ideas presented, you might show them how they can use $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ to simplify calculations if *a* and *b* both have the same extra factor (see *Enrichment* on **page 23**).

Revisiting the Try This

B. Students have the opportunity to apply a method of estimation learned in the lesson.

Using the Examples

• Write the question from **example 1** on the board. Ask students to work alone or in pairs to complete the question and then compare their work with the solution in the text. Students can read **example 2** and work through it in pairs.

Practising and Applying

Teaching points and tips

Q 1: Students could do this question in pairs. One person could do **part a)** and the other **part b)**. They could explain their answers to one another.

Q 2 and 3: Encourage students to use mental math in answering these questions.

Q 4: **Example 1** is a helpful reference.

Q 5: This question is not required. It is intended for enrichment and/or for advanced understanding of the connection between products and square roots.

Q 6: Encourage students to use an example in their explanations.

Q 7: Students need to realize that they must calculate the square root of each height before multiplying by 0.45.

Q 8: This question is excellent for reviewing factorisation skills and the square roots of perfect squares. (9216 is a perfect square.)

Common errors

• Students may identify the powers of 10 incorrectly or get mixed up by thinking of 9000 as a perfect square. They may place decimal points incorrectly if they make such errors in estimating square roots.

Suggested assessment questions from Practising and Applying

Question 4	to see if students can estimate a square root
Question 7	to see if students can apply square roots in a formula
Question 8	to see if students can use factoring to calculate a square root

Answers

A. About 80 m

B. *Sample response*:
$6510 = 65.1 \times 100$
$65.1 \approx 64$, $\sqrt{64} = 8$, and $\sqrt{100} = 10$.
$\sqrt{6510} \approx 8 \times 10 = 80$.

1. a) 6.2 **b)** 9.8

2. B

3. a) 82 **b)** 820

4. a) $70 \times 70 = 4900$ and $4900 \approx 4823$.
b) Less than
c) 69.4 m

5. a) i) $\sqrt{64} = \sqrt{4} \times \sqrt{16}$ and $64 = 4 \times 16$
ii) $\sqrt{225} = \sqrt{9} \times \sqrt{25}$ and $225 = 9 \times 25$
iii) $\sqrt{324} = \sqrt{36} \times \sqrt{9}$ and $324 = 36 \times 9$
b) i) 25 **ii)** 81 **iii)** 576

6. Yes; [*Sample response*:
Any number greater than 1 becomes greater when it is squared. So, the square root of any number greater than 1 must be less than the number.]

7. a) 3 or 4 s **b)** 14 s **c)** 45 s

8. **a)** Yes; [When I factor 142,884, there is a pair of 9s, a pair of 7s, and a pair of 6s. So, I can write 142,884 as $(9 \times 7 \times 6) \times (9 \times 7 \times 6)$.
Since $9 \times 7 \times 6$ is multiplied by itself to get 142,884, $\sqrt{142{,}884} = 9 \times 7 \times 6$.]
b) $9216 = 3 \times 3 \times 2 \times 2 \times 16 \times 16 = (3 \times 2 \times 16) \times (3 \times 2 \times 16)$, so $\sqrt{9216} = 3 \times 2 \times 16 = 96$.

9. a) Yes; [*Sample response*:
The square root of 100 is 10 and the square root of 1600 is 40.
b) No; [*Sample response*:
The square root of the least 5-digit number, 10,000, is 100 because $100 \times 100 = 10{,}000$. The square root of the greatest 4-digit number, 9999, has only two digits; it must be less than 100 because $100 \times 100 = 10{,}000$.]
c) The area of the square must have 7 or 8 digits; [*Sample response*:
It could have at least seven digits because $1000 \times 1000 = 1{,}000{,}000$.
It could have eight digits because $5000 \times 5000 = 25{,}000{,}000$.
It cannot have nine digits because it must be less than $10{,}000 \times 10{,}000 = 100{,}000{,}000$.]

[**10.** *Sample response*:
One way:
$50 \times 50 = 2500$ (79 too low)
$51 \times 51 = 2601$ (22 too high)
So $\sqrt{2579}$ is between 50 and 60 but closer to 60.
$\sqrt{2579}$ is a bit less than 60.
Another way:
2579 is a bit more than 2500.
$2500 = 25 \times 100$
$\sqrt{2500} = \sqrt{25} \times \sqrt{100} = 5 \times 10 = 50$
$\sqrt{2579}$ is a bit more than 50.]

Supporting Students

Struggling students

• Mental math is important in this lesson for doing some questions and for checking the reasonableness of answers. Students with weak mental math skills are likely to find this lesson more difficult.

• Students who are struggling can omit **questions 5 and 9**.

• Although **question 6** may appear abstract, students can try some examples of numbers to help them understand the question and prepare an explanation.

Enrichment

• Encourage students to extend the ideas in **question 5** to examples where the numbers that make up the products are not exact, though the product itself is exact.

For example, $\sqrt{18} \times \sqrt{2} = \sqrt{36}$. 18 and 2 are not perfect squares but $18 = 3 \times 3 \times 2$ and 2 both have a 2 that needs a partner. Multiplying them together gives 3×2 as the square root, which is the value of $\sqrt{36}$.
Students can make up other examples.

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CONNECTIONS: The Square Root Algorithm

• Students who enjoy mathematics will find this method interesting. The square root algorithm is designed for enrichment of mathematical scope as well as connecting mathematical history to the topic at hand.

• The square root algorithm is a historical method used to find the square root of a number.

• With the class you can work through the detailed example that shows how to find the square root of 19,044.

• Students can extend the idea to calculation involving decimals by placing pairs of zeros after the decimal point to increase accuracy.

For example, the square root of 238 can be found using 238.00 or 238.0000.

Answers

1. a) 27

$$\left(\begin{array}{r|r} & \mathbf{2\ \ 7} \\ \hline 2 & 7\ \ 29 \\ & -\underline{4} \\ 47 & 329 \\ & \underline{-329} \\ & 0 \end{array}\right)$$

b) 51

$$\left(\begin{array}{r|r} & \mathbf{5\ \ 1} \\ \hline 5 & 26\ \ 01 \\ & -\underline{25} \\ 101 & 101 \\ & \underline{-101} \\ & 0 \end{array}\right)$$

UNIT 1 Revision

Pacing	Materials
2 h	• Grid paper, or Small Grid Paper (BLM) (optional)

Question(s)	Related Lesson(s)
1 – 3	Lesson 1.1.1
4 – 7	Lesson 1.1.2
8 – 11	Lesson 1.2.1
12 and 13	Lesson 1.2.3
14	Lesson 1.2.4
15	Lessons 1.2.3 and 1.2.4
16 and 17	Lesson 1.2.4

Revision Tips

Q 4: It is possible for students to answer **question 3** without writing the number in standard form. It will be helpful if they write the standard form in **part a)** before using scientific notation.

Q 6: Students must pay attention to the units themselves as well as to the number of units.

Q 7: In **part a)**, you may need to point out that 34×10^6 represents 34 million, but not in scientific notation. In **part b)**, some students may write only 3.1 in scientific notation rather than 3.1% or 0.031.

Q 10: Encourage students to use examples to facilitate their explanation.

Q 12: The diagram can be a sketch and need not be drawn to scale.

Q 15: Students may not notice at first that **part a)** requires them to begin by identifying dimensions of rectangles with an area of 70 square units.

Q 16: Remind students that they do not necessarily need to break the number down into prime factors. They should be looking to factor out perfect squares.

Answers

1. a) 90,040.057008;
9 ten thousands + 4 tens + 5 hundredths + 7 thousandths + 8 millionths;
$9 \times 10{,}000 + 4 \times 10 + 5 \times 0.01 + 7 \times 0.001 + 8 \times 0.000001$
b) 4.5007;
4 ones + 5 tenths + 7 ten thousandths;
$4 \times 1 + 5 \times 0.1 + 7 \times 0.0001$

2. a) 1.2×10^{-3} **b)** 6×10^{-2}

3. a) 3; [The most negative exponent is –3, which means 0.001, so three digits are needed after the decimal point.]
b) 8; [The exponents of the nonzero digits range from 4 to –3. One digit is needed for each place value, from ten thousands to thousandths. That is 8 places.]
[**c)** Zeros can be placed after the last nonzero digit in a decimal without changing the value. There is no limit to the number of digits that can be used to represent the number.]

4. a) 2.0030905×10^4 **b)** 2.395×10^3
c) 2.0030905×10^4 in **question 3** is greater;
[Its power of 10 is 4, which is greater than the power of 10 for 2395 (2.395×10^3).]

5. a) 5.198723×10^4 **b)** 1.93567×10^{-1}
c) 7.4×10^{-3} **d)** 1.017×10^1

6. A. 1.39×10^2 cm; [This is a reasonable height because it represents a height of 139 cm or 1.39 m.
A height of 830,000 mm (8.3×10^5 mm) is 830 m, which is very tall.
A height of 14.8 m (1.48×10^1 m) is also too tall.]

7. a) 3.4×10^7 **b)** 3.1×10^{-2}

8. B. 121

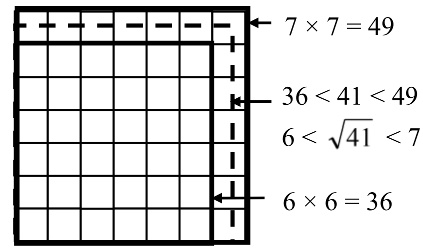
Answers [Continued]

9. a) No; [The factor 11 does not have a partner. A perfect square requires all prime factors to be paired.]
b) 11; [Since only the factor 11 does not have a partner, multiplying by 11 will produce a perfect square.]
c) $308 = 2 \times 2 \times 7 \times 11$

[**10.** *Sample response*:
The factors of any number can be paired. For example, 24 has four factor pairs: 1 and 24; 2 and 12; 3 and 8; and 4 and 6. So there is an even number of factors. However, a perfect square always includes a factor pair that has the same two factors, which only counts as one factor, making the total number of factors odd. For example, the pairs of factors that make 81 are
1 and 81, 3 and 27, and 9 and 9. The factor pair 9 and 9 counts as only one factor, so 81 has 5 factors.]

[**11.** *Sample response*:
The powers of 10 are made up of tens multiplied together. The number is a perfect square if there is an even number of tens, as in $100 = 10 \times 10$. It is impossible to pair up all of the tens in a number like $1000 = 10 \times 10 \times 10$, so it cannot be a perfect square.]

12.



13. a) 12 or 13 **b)** 7 **c)** 9

14. a) 12.4 **b)** 6.9 **c)** 9.1

15. a) $8.5 = \frac{7+10}{2}$
b) Estimate to one decimal place: 8.4;
Both estimates are about the same.
c) $7.5 = \frac{5+10}{2}$; Estimate to one decimal place: 7.1
The estimate of 7.1 suggests that the estimate of 7.5 is too high.

16. 270;
$72{,}900 = 729 \times 100 = 9 \times 81 \times 100$
$= 3 \times 3 \times 9 \times 9 \times 10 \times 10$
$\sqrt{72{,}900} = 3 \times 9 \times 10 = 270$

17. 3 or 4 digits; [The square root of the area is at least 10 and at most 99. Therefore the area is at least 100 and at most 9801, so the area of the square could have 3 or 4 digits.]

UNIT 1 Number Test

1. Which number in each pair is greater? How do you know?

a) 5.23×10^3 or 2×10^4

b) $7 \times 10^{-3} + 7 \times 10^{-4}$ or 7×10^{-3}

2. a) When the number below is in standard form, how many digits are to the right of the decimal point? How do you know?

$3 \times 10^4 + 5 \times 10^3 + 9 \times 10^{-1} + 6 \times 10^{-2} + 5 \times 10^{-4} + 7 \times 10^{-7}$

b) What is the least number of digits required to write the number in **part a)**? Explain your thinking.

c) Write the number using 15 digits.

3. Write each in scientific notation.

a) 1,674,378.029 **b)** 0.37491

4. A report states that a town in Bhutan has a population of 9.834×10^2 people. Could this report be accurate? Explain your thinking.

5. a) Which of the choices below is a perfect square?

A. $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5$

B. $3 \times 3 \times 3 \times 3 \times 5 \times 5$

C. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

b) For each number not chosen in **part a)**, what is the least whole number by which you can multiply it to get a perfect square?

6. a) Draw a diagram to show that $\sqrt{17}$ is between 4 and 5.

b) Estimate $\sqrt{17}$ to one decimal place.

c) Suppose you estimated $\sqrt{17}$ by averaging the length and width of the most square-like rectangle. Why would it be a poor estimate?

7. Factor 1,440,000 to find its square root.

8. a) Write a number between 100 and 125 that is a perfect square.

b) Write a number between 1 and 2 that has an exact square root.

c) Write a number between 1 and 10 that does not have an exact square root.

9. The side length of a square is a 3-digit whole number. What do you know about the number of digits in the area of the square? Explain your thinking.

10. Which powers of 10 are perfect squares? Explain your thinking.

11. How are negative exponents useful?

12. How do you know that 1,440,000 has an odd number of factors without listing all the factors?

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UNIT 1 Test

Pacing	Materials
1 h	• Grid paper, or Small Grid Paper (BLM) (optional)

Question(s)	Related Lesson(s)
1 and 2	Lesson 1.1.1
3 and 4	Lesson 1.1.2
5	Lesson 1.2.1
6	Lesson 1.2.3
7 – 9	Lesson 1.2.4
10	Lesson 1.2.1
11	Lesson 1.1.1
12	Lesson 1.2.1

Select questions to assign according to the time available.

Answers

1. a) 2×10^4; Both numbers are expressed in scientific notation and 2×10^4 has a greater power of 10.

b) $7 \times 10^{-3} + 7 \times 10^{-4}$; 7×10^{-3} is common to both numbers. Since 7×10^{-4} is positive and added to 7×10^{-3}, $7 \times 10^{-3} + 7 \times 10^{-4}$ is greater than 7×10^{-3}.

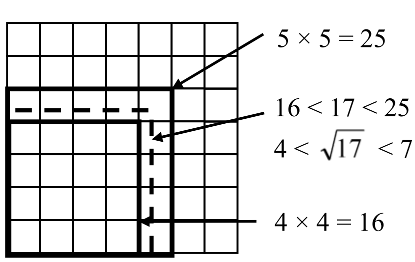
2. a) 7; There is one term involving 10^{-7}.
b) 12; The powers of 10 go from 4 to –7. The number must have one digit for each of these place values.
c) 35,000.9605007000

3. a) 1.674378029×10^6 **b)** 3.79491×10^{-1}

4. No; $9.834 \times 10^2 = 983.4$ and it is impossible to have a population of 983.4.

5. a) B
b) A needs to be multiplied by 15 (3×5).
C needs to be multiplied by 2.

6. a)



b) 4.1

c) The only rectangle has dimensions of 1 and 17, which is not like a square. The average is 9, or $(1 + 17) \div 2$, which is a poor estimate of $\sqrt{17}$.

7. $1,440,000 = 144 \times 10,000 = 12 \times 12 \times 100 \times 100$
$\sqrt{1,440,000} = 12 \times 100 = 1200$

8. a) 121
b) *Sample response*: 1.21
(Among the other correct responses are 1.44, 1.69, and 1.96. Numbers with more digits are also possible.)
c) *Sample response*: 2

9. The area could have five or six digits; The smallest possible side length is 100, and $100 \times 100 = 10,000$. The largest possible side length is 999, and 999×999 has a product with six digits since it is a little less than $1000 \times 1000 = 1,000,000$.

10. All of the even powers of 10 are perfect squares. $10 \times 10 \times 10 \times ...$ is a perfect square if all the factors of 10 can be paired. This happens if there is an even number of tens like 10^2 or 10^{14}, for example.

11. *Sample response*:
Negative exponents are used to represent small numbers.

12. *Sample response*:
I know 1,440,000 is a perfect square because $1,440,000 = 144 \times 10,000$, and 144 is a perfect square (the square of 12) and 10,000 is also a perfect square (the square of 100). Every perfect square has an odd number of factors since one factor pair repeats a factor.

UNIT 1 Performance Task – Estimating Side Lengths

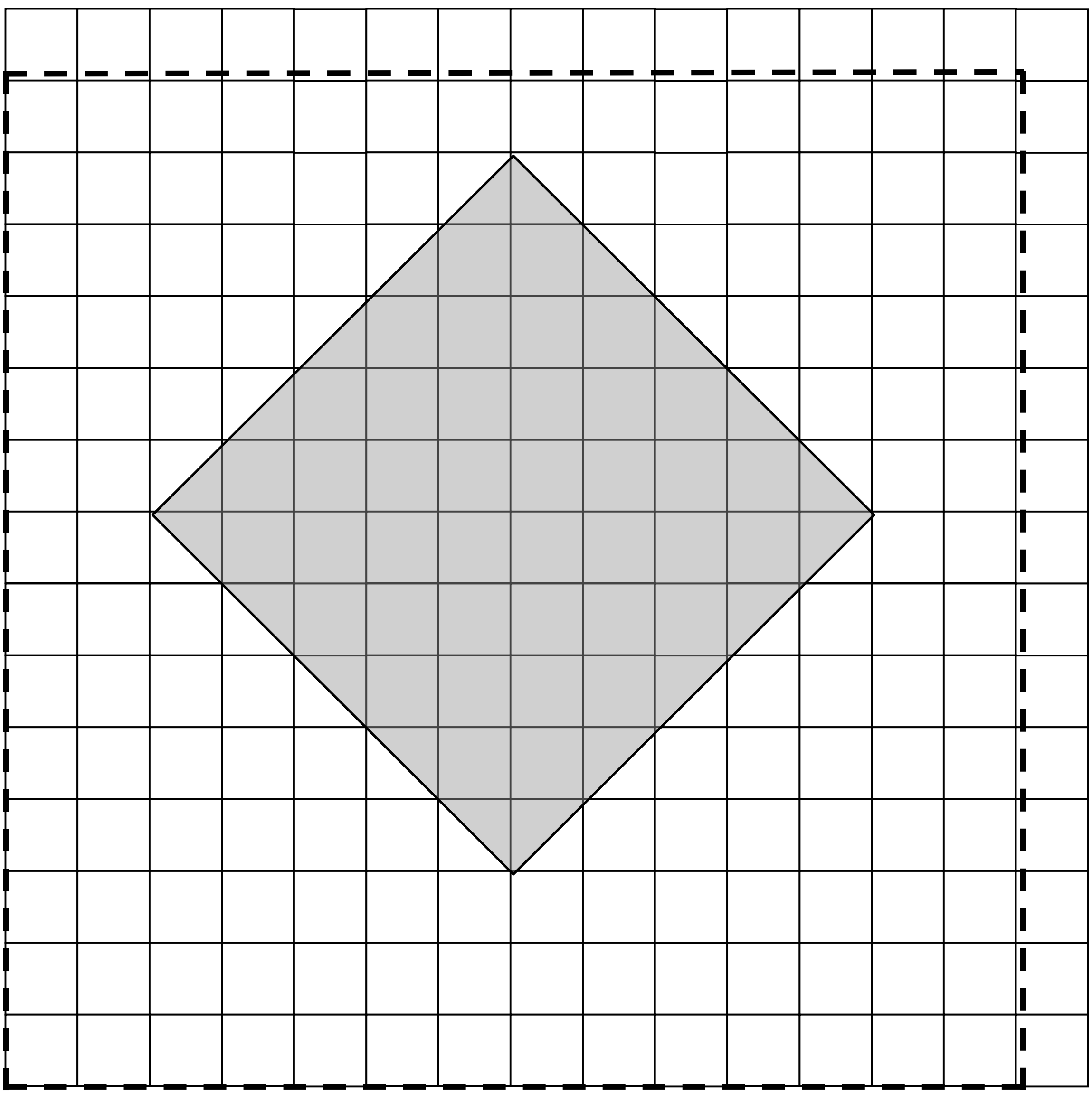
Work with the grey square.

A. i) Find the area of the grey square by counting grid squares.

ii) Write the side length of the grey square as a square root. How do you know you are right?

iii) Measure the side length to the nearest tenth of a centimetre.

iv) Check your measurement in **part iii)** by squaring it and comparing the result with the area of the square from **part i)**.



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Work with the large dashed square and centimetres.

B. The area of the large square is 200 cm^2.

i) Write the side length as a square root.

ii) Measure the side length to the nearest tenth of a centimetre. (Note that the side length is not exactly 14 cm.)

iii) Check your measurement in **part ii)** by squaring it and comparing the result with the area of the square (200 cm^2).

iv) Write the side length in scientific notation.

Work with the large dashed square and millimetres.

C. The area of the dashed square is 20,000 mm^2.

i) Write the side length as a square root.

ii) Measure the side length to the nearest millimetre.

iii) Check your measurement in **part ii)** by squaring it and comparing the result with the area of the square.

iv) Write the side length in scientific notation.

Work with the large dashed square and decimeters.

D. The area of the dashed square is 2 dm^2.

i) Write the side length as a square root.

ii) Measure the side length to the nearest hundredth of a decimetre.

iii) Check your measurement in **part ii)** by squaring it and comparing the result with the area of the square.

iv) Write the side length in scientific notation.

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UNIT 1 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
8-A4 Large and Small Numbers: scientific notation to standard form and vice-versa **8-A5** Square Root: exact square and its decimal approximation **8-A6** Square Root: find using an appropriate number	40 min	• Grid on **page 29** • Rulers

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could supplement the unit test. It could also be used as enrichment material for some students.

• You can assess performance on the task using the rubric on the next page.

Sample Solution

A. i) 50 cm^2 **ii)** $\sqrt{50}$ cm; The area of the square is the square of its side length. So the side length is the square root of the area. **iii)** 7.1 cm **iv)** $7.1 \times 7.1 = 50.41$ **B. i)** $\sqrt{200}$ cm **ii)** 14.1 cm **iii)** $14.1 \times 14.1 = 198.81$, which is almost 200. **iv)** 1.41×10^1 cm	**C. i)** $\sqrt{20{,}000}$ mm **ii)** 141 mm **iii)** $141 \times 141 = 19{,}881$, which is almost 20,000. **iv)** 1.41×10^2 mm **D. i)** $\sqrt{2}$ dm **ii)** 1.41 dm **iii)** $1.41 \times 1.41 = 1.9881$, which is almost 2. **iv)** 1.41×10^0 dm

UNIT 1 Performance Task Assessment Rubric

The student	Level 4	Level 3	Level 2	Level 1
Calculates square roots	Effectively identifies and works with squares and square roots	Shows familiarity with squares and square roots	Identifies some values correctly but not others	Identifies few correct values
Writes scientific notation and powers of 10	Effectively uses scientific notation and works accurately with powers of 10	Demonstrates knowledge of scientific notation	Shows minor errors in the use of scientific notation	Shows significant errors in the use of scientific notation
Calculates and measures side lengths	Performs accurate calculations and measurements	Performs mostly accurate calculations and measurements	Shows minor errors in calculations and measurements	Shows significant errors in calculations and measurements

BLM 1 Small Grid Paper

UNIT 2 PROPORTION AND PERCENT

UNIT 2 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 21** **TG p. 36**	Review prerequisite concepts, skills, and terminology, and pre-assessment	1 h	• 100 charts (BLM) (optional)	All questions
Chapter 1 Proportions				
2.1.1 Solving Proportions **SB p. 22** **TG p. 38**	**8-B1 Proportion: solve problems** • use a variety of strategies to solve problems of proportionality - find relationships between the various terms of proportions and use these relations to solve for missing values (e.g., use equivalent fractions to solve $\frac{2.2}{5} = \frac{x}{5}$) - apply the unit ratio/rate method • recognize uses for and importance of proportion	1 h	• Weigh scale (optional)	Q1, 4, 9
2.1.2 EXPLORE: Scale Drawings and Similar Figures (Essential) **SB p. 26** **TG p. 41**	**8-B1 Proportion: solve problems** • recognize uses for and importance of proportions • investigate problem solving opportunities (e.g., study of scale, transformational geometry, i.e., dilatations)	40 min	• Blank paper • Rulers	Observe and Assess questions
Chapter 2 Percent				
2.2.1 Percents Greater Than 100% **SB p. 27** **TG p. 43**	**8-A7 Percent: greater than 100** • recognize that values greater than a whole are described by percents greater than 100% • relate percent greater than 100 to other subjects and topics (e.g., social studies and inflation rates, population growth)	1 h	None	Q1, 4, 6, 8
2.2.2 Solving Percent Problems **SB p. 31** **TG p. 46**	**8-B2 Percent: solving and creating real problems in context (including estimation)** • estimate and calculate a percent of a given number (a% of $b = c$, e.g., 25% of 1500) • find the percent one number is of another number (e.g., what percent of 20 is 15?) • find the whole when a specified percent is given (e.g., 28% of what number is 42?) • use mental strategies when an exact answer is required (e.g., 28% of 1200 = 20% of 1200 + 8% of 1200)	1 h	None	Q1, 2, 3, 5
GAME: Equivalent Concentration (Optional) **SB p. 34** **TG p. 48**	Practise relating percent to fraction and decimal equivalents in a game situation	30 min	• Student- or teacher-made game cards	N/A

UNIT 2 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
2.2.3 Fractional Percents **SB p. 35** **TG p. 49**	**8-B2 Percent: solving and creating real problems in context (including estimation)** • use percents that are not whole numbers	1 h	• Thousandths Grids (Rectangular) (BLM)	Q1, 3, 5, 7
2.2.4 Solving Percent Problems Using Familiar Percents **SB p. 38** **TG p. 52**	**8-B2 Percent: solving and creating real problems in context (including estimation)** • estimate and calculate a percent of a given number (a% of b = c, e.g., 25% of 1500) • find the percent one number is of another number (e.g., what percent of 20 is 15?) • find the whole when a specified percent is given (e.g., 28% of what number is 42?) • use mental strategies when an exact answer is required (e.g., 28% of 1200 = 20% of 1200 + 8% of 1200)	1 h	None	Q1, 2, 5
Chapter 3 Consumer Problems				
2.3.1 Markup and Discount Consumer Problems **SB p. 41** **TG p. 55**	**8-B3 Percent: increase and decrease** • investigate markups and mark-downs of retail items (e.g., a dress cost Nu 7 to make and is being sold for Nu 15. What is the percent of markup?) • develop formula (% increase = increase/original amount × 100%) • develop formula (% decrease = decrease/original amount × 100%)	1 h	None	Q1, 2, 4, 8
2.3.2 Simple Interest and Commission **SB p. 45** **TG p. 58**	**8-B3 Percent: increase and decrease** • investigate commissions and simple interest; develop formula $I = Prt$	1 h	None	Q3, 5, 6, 7
CONNECTIONS: Currency Conversion (Optional) **SB p. 49** **TG p. 60**	Make a connection between proportions and calculating rates of exchange for currencies.	30 min	None	N/A
UNIT 2 Revision **SB p. 50** **TG p. 61**	Review the concepts and skills in the unit	2 h	None	All questions
UNIT 2 Test **TG p. 62**	Assess the concepts and skills in the unit	1 h	None	All questions
UNIT 2 Performance Task **TG p. 64**	Assess concepts and skills in the unit	1 h	None	Rubric provided
UNIT 2 Blackline Masters **TG p. 66**	BLM 1 100 Charts BLM 2 Thousandths Grids (Rectangular)			

Math Background

• This unit develops strategies for solving proportions and extends the percent work from Class VII to include percents greater than 100%, fractional percents, and percents in consumer applications such as discount, markup, interest, and commission problems.

• The focus of the unit is on developing a variety of strategies for solving proportion problems and percent problems.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in every lesson in the unit. In **question 8** in **lesson 2.1.1**, they solve for various missing terms in proportions. In **question 8** in **lesson 2.2.1**, they solve a population growth problem using their understanding of percents greater than 100%. In **question 7** in **lesson 2.2.2**, they infer from given information to solve a problem. They also use problem solving in **question 5** in **lesson 2.2.3**, where they use their knowledge of both measurement equivalents and fractional percents to solve a problem, in **question 12** in **lesson 2.2.4**, where they explain why they chose to use certain benchmark fractions, and in all of **Chapter 3**, where they solve many consumer problems.

• Students use communication frequently as they explain their thinking. In **question 10 a)** in **lesson 2.1.1**, they explain what a ratio tells them, in **question 9 b)** in **lesson 2.2.1**, they explain the effects of increasing and decreasing by a given percent, in **question 7** in **lesson 2.2.3**, they explain why different strategies can be used to find a fractional percent of a number, and in **question 12** in **lesson 2.3.2**, they communicate about the formula for calculating interest.

• They use reasoning in answering questions such as **question 7** in **lesson 2.1.1**, where they use a given ratio to create and solve a proportion involving themselves, in **question 2** in **lesson 2.2.1**, where they determine how to combine various percents of a number to find a different percent of that number, in **question 11** in **lesson 2.2.2**, where from given information they extrapolate new information related to themselves, in **question 3** in **lesson 2.2.4**, where they select benchmarks that make sense in particular situations, and in **question 8** in **lesson 2.3.1**, where they compare two ways of giving discounts.

• They consider representation in **question 1** in **lesson 2.2.1**, where they use figures to represent various percents greater than 100%, in **lesson 2.1.2**, where they decide what scale diagram best represents an actual object, and in **question 1** in **lesson 2.2.3**, where they represent various fractional percents on a thousandths grid.

• Students use visualization skills in **question 10** in **lesson 2.1.1**, where they recognize that the length of AD must be greater than the length of AB in the given rectangle, in **part D** in **lesson 2.1.2**, where they use the picture of a window to estimate its actual measurements, and in **question 2** in **lesson 2.2.3**, where they look at a thousandths grid and determine the percent that is shaded.

• They make real-world connections throughout the unit, including in **question 7** in **lesson 2.1.1**, **question 8** in **lesson 2.2.1**, **question 6** in **lesson 2.2.2**, in **question 4** in **lesson 2.2.4**, and in the consumer problems throughout **Chapter 3**. They connect percent to measurement ideas in **question 8** in **lesson 2.2.3**.

Rationale for Teaching Approach

• This unit is divided into three chapters:

Chapter 1 is about proportions.

Chapter 2 focuses on percent.

Chapter 3 involves using percent to solve consumer problems.

• The **Explore** lesson allows students to apply their understanding of proportions to scale drawings and similar figures.

• The **Connections** helps students connect the concept of percent to currency conversions and currency rates of exchange.

• The **Game** provides an opportunity to recall the relationships among fractions, decimals, and percents and to practise working with equivalences in a pleasant way.

• Throughout the unit, it is important to encourage development of a variety of strategies for solving proportions and percent problems, and to accept different approaches from students.

Getting Started

Curriculum Outcomes	Outcome relevance
7 Equivalent Ratios and Rates: solve problems **7** Percent: as a special ratio **7** Percent: number sense **7** Percent: develop algorithms	Students will find the work in the unit easier after they review the concepts and skills related to ratio and percent they encountered in Class VII.

Pacing	Materials	Prerequisites
1 h	• 100 charts (BLM) (optional)	• familiarity with the terms *ratio*, *equivalent ratio*, *percent,* and *lowest terms* • finding equivalent ratios for a given ratio • finding the percent of a number • finding the whole when a percent is known

Main Points to be Raised

Use What You Know

• You can use equivalent ratios to solve problems.

• To construct an equivalent ratio, you multiply or divide the two terms of the given ratio by the same non-zero amount.

• Finding the percent of a number is a special case of finding equivalent ratios. The second term of the percent ratio is always 100.

Skills You Will Need

• There are many equivalent ratios for any given ratio.

• You can use various strategies to find the percent of a number. It is often useful to relate to 10% of a number by taking one tenth of it. You can also multiply by the decimal that is equivalent to the percent.

• The same number can represent different percents, depending on what the whole is.

• When you know the percent of a whole, you can use equivalent ratios to find the whole.

Use What You Know — Introducing the Unit

• Students can work in pairs or small groups.

Observe students as they work. While they work, you might ask questions such as the following:

• *What equivalent ratio did you solve in* ***part A****?* (4 : 5 = ? : 10)

• *Why did you use that ratio?* (The ratio was given as 4 : 5, which would be 5 for archery, but my chart had 10 for archery, not 5.)

• *What equivalent ratio could you write for* ***part B****?* (10 : 25)

• *Why was the second term 25?* (It was a part-to-whole ratio. The whole was 25 students.)

• *How might finding 10% of 40 and then 5% of 40 help you with* ***part D****?* (It is easy to find 10% of 40 and divide that by 2 to find 5% of 40 in my head. Once I know 5% of 40, I can multiply that by 9 to get 45% of 40.)

Skills You Will Need

• Students can work individually.

• To ensure students have the required skills for this unit, assign all of these questions.

• Review the term *equivalent ratio* to make sure students can successfully complete **question 1**.

• Remind students what the whole numbers are for **question 2** (1 to 50).

• Encourage students to use a variety of strategies to answer **question 4**.

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Answers

A. 8 **B.** 2 : 5 **C.** 5 **D.** 18; 45% of 40 is 0.45 × 40 = 18	**E.** *Sample Response*: I surveyed 40 students. 18 chose archery, 16 chose football, 4 chose track, and 2 chose other. So, 45% chose archery, 40% chose football, 10% chose track, and 5% chose other.
1. 36 : 45 and 4 : 5 **2. a)** 32% **b)** 20% **3.** 10 apples for Nu 100 [*Sample response*: 10 apples for Nu 100 is Nu 10 for 1 apple; 6 apples for Nu 72 is Nu 12 for 1 apple.]	**4. a)** 0.6 **b)** 2.5 **c)** 9.6 **d)** 25.5 **5. a)** 2000 **b)** 80 **c)** 50 **d)** 40

Supporting Students

Struggling students

• If students are struggling with finding the percents for their class in **part E**, you might have them survey a "friendly number" of students (e.g., 10, 20, or 25 students).

• Some students might benefit from the use of a 100 chart (see BLM on **page 66** in this guide) for **question 2**. They can shade and count the multiples of 3 or the numbers greater than 40.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Enrichment

• For **question 5**, you might challenge students to find other statements where 20 is a given percent of a whole number (e.g., it is 2 % of 1000, 4% of 500, 5% of 400, and so on).

Chapter 1 Proportions

2.1.1 Solving Proportions

Curriculum Outcomes	Outcome relevance
8-B1 Proportion: solve problems • use a variety of strategies to solve problems of proportionality - find relationships between the various terms of a proportion and use these relations to solve for missing values (e.g., $\frac{2.2}{5} = \frac{x}{5}$, students in this situation should consider equivalent fractions) - apply the unit ratio/rate method • recognize uses for and importance of proportion	Many situations in everyday life require us to solve proportions, so it is important for students to become familiar with a variety of strategies to solve them. Proportions are also used a great deal in higher classes in mathematics and science.

Pacing	Materials	Prerequisites
1 h	• Weigh scale (optional)	• writing equivalent fractions • familiarity with rates, and part-to-part and part-to-whole ratios

Main Points to be Raised

• A proportion is a statement or equation that shows two equivalent ratios or rates.

• A unit ratio (or rate) is an equivalent ratio (or rate) where the second term is one unit.

• You can use a scale factor to find a missing term in a proportion. You multiply each term by the scale factor.

• A ratio or rate table is a useful tool for solving a series of related proportion problems.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *What is the ratio of the amount of pork in the original recipe to the amount of pork in the adjusted recipe?* (500 : 300 or 5 : 3)

• *How can you find the amount of tomatoes Yeshi needs?* (I need to find what ? is in 150 : ? = 5 : 3.)

• *Does it make sense that the adjusted recipe serves fewer people than the original recipe?* (Yes. There will be less of every ingredient, so there will be less Pork Fing altogether.)

The Exposition — Presenting the Main Ideas

• Ask 3 girls and 4 boys to come to the front of the classroom. Tell the class you want to make another group with the same ratio of girls to boys, but with 9 girls in the group. Ask how many boys need to be in the group. When they answer, write the following on the board: 3 : 4 = 9 : 12 (or $\frac{3}{4} = \frac{9}{12}$). Explain that, when we write an equation with two equivalent ratios, it is called a *proportion.*

• Remind students why you multiplied (to make sure there were 3 girls every time there were 4 boys) rather than adding 6 girls and 6 boys (because you would have 9 girls and 10 boys and there would not be 3 girls for every 4 boys).

• Present the paint example in the exposition on **page 22** of the student text. Write the proportion to be solved on the board. Explain what a *scale factor* is. Have students note that since 2 was multiplied by the scale factor 25 to get 50, 3 has to be multiplied by the same scale factor for the equation to stay balanced. If students ask what the scale factor is when you divide, you might point out, for example, that since dividing by 4 is the same as multiplying by $\frac{1}{4}$, the scale factor is $\frac{1}{4}$.

• Make sure students understand what a *unit ratio* and a *unit rate* are. Go through the bread example in the exposition with the students to show how to use a unit rate to solve a proportion.

• Draw the ratio table for the uncooked/cooked rice example in the exposition on **page 23** of the student text. Complete the table with students. Make sure students understand that you divide or multiply all the numbers in a column of a ratio table by the same number. You may wish to add another column to the ratio table and ask students to describe two different ways to complete that column.

For example, if you had 50 mL of uncooked rice, how much cooked rice would it make? Students might multiply the column with 10 mL of uncooked rice by 5, or divide the column with 100 mL of uncooked rice by 2.

Some students might wonder about the use of millilitres for flour. Explain to students that we often use millilitres to measure solids like flour because it pours like a liquid.

Revisiting the Try This

B. Students apply the strategies presented in the exposition (scale factor, unit ratio/rate, and ratio/table table) to adjust the recipe from **part A** for one ingredient (tomatoes).

Using the Examples

• Ask pairs of students to read through **solutions 1 and 2** of the example. Ask them to choose the solution that most closely matches what they would have done and to say why they would have done it that way.

Practising and Applying

Teaching points and tips

Q 1: Remind students that sometimes they need to divide by the scale factor rather than multiplying (**parts a), e), and f)**).

Q 2, 3, and 4: Students need to write the ratios in the proportion in the same order (e.g., in **question 2**, if the first ratio is paint concentrate to water, then the second ratio has to be written in the same way).

Q 5: Students will likely benefit by first finding a unit ratio in this problem.

Q 7 b): If possible, have available a weigh scale for students to find their own mass. Or, they can estimate their masses. Note that weight is often used informally to mean mass, but we try to model the correct use in the classroom — do not penalize students if they refer to weight rather than mass.

Q 8 b) and c): Some students may choose to answer these questions using fractions of an hour rather than minutes. Note that this question involves both ratios (5 : 3) and rates (90 seedlings/h and 150 seedlings/h).

Q 9: Some students may not recognize that they need to either determine the ratio of boys to total students, or solve the question in two parts, first finding the number of girls and then adding to find the total number of students.

Q 10: It is important for students to realize that knowing the ratio of side lengths is not enough information to determine the length of a side.

Q 12: Use this last question as a closure question. to highlight the types of situations that are most effectively solved by first finding a unit ratio or rate.

Common errors

• In **question 9**, many students will solve the proportion for the number of girls and not notice that they are asked to find the total number of students. You might encourage them first to write the ratio of boys to total students and then to solve a proportion using that ratio and the fact that there are 196 boys at the school.

• Students may be confused with the use of the terms *ratio* versus *rate* in this lesson. You might review the distinction between the terms but do not put too much emphasis on the difference, as some mathematicians define a rate as a special type of ratio. In Class VII, these terms were defined:

- A ratio is a comparison of items with the same unit (e.g., 3 boys to 2 girls (the unit being people), 50 mL of yellow paint to 75 mL of red paint, and 30 g of uncooked rice to 105 g of cooked rice). That is why the units are dropped for the symbolic form of the ratio (3 : 2, 50 : 75, and 30 : 105). Ratios can have two or more terms.

- A rate is a comparison of two items with different units, so the units are part of the rate (e.g., 750 mL per 2 loaves, or 30 calories/30 min). Rates have two terms.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can apply strategies such as scale factors in solving proportions
Question 4	to see if students can set up and solve a proportion to solve a contextual problem
Question 9	to see if students can recognize difference between a part-to-part ratio and a part-to-whole ratio when solving a proportion

Answers

A. i) 90 g tomatoes, 60 g butter, 72 mL water, 27 g green chillis, 48 g onions
ii) 3 people

B. *Sample response*:

$$\frac{300}{500} = \frac{3}{5} \text{ and } \frac{3}{5} = \frac{\blacksquare}{150} \rightarrow \frac{3}{5} = \frac{90}{150}$$

× 30

÷ 10 × 3

Original recipe	500	50	150
Adjusted recipe	300	30	**90**

÷ 10 × 3

1. a) 2 **b)** 9 **c)** 25
d) 30 **e)** 10 **f)** 6

2. 72 L

3. 39 people are sitting.

4. 32 kg

5. 6 times

6.

Concentrate (mL)	Water (mL)
1	3
350	**1050**
475	**1425**
175	525
400	1200

7. a) 20 g
b) *Sample response*:
I have a mass of 45 kg. If I were as strong as an ant, I could carry 2250 kg.

8. a) 150 seedlings
b) 20 min ($\frac{1}{3}$ h)
c) 12 min ($\frac{1}{5}$ h)

9. 343 students [196 boys, 147 girls]

10. a) No; [*Sample response*:
It just tells you how one side compares to the other side, not the length of any of the sides.]
b) 25 cm

11. *Sample response*:
If 4 apples cost Nu 80, how much do 7 apples cost?
(Answer: Nu 140)
[The unit rate method is best for this problem because if I try to solve it using a proportion, it is difficult to know what to multiply 4 by to get 7. If I find the cost of 1 apple, I can multiply that cost by 7 to find the cost of 7 apples. I can use the unit rate to find the cost of any number of apples.]

Supporting Students

Struggling students

• Some students may have trouble with finding and solving a proportion for **question 5**. You might ask if they can find a simpler ratio for 9 : 3, and use that ratio to set up their proportion.

• If students have difficulty creating their own problems for **question 11**, you might have them look back at the problems in the exercises to decide which problem they think is easiest to solve using a unit ratio and why.

Enrichment

• Students might create additional proportion problems that are of interest to them for partners to solve.

For example, if a student is interested in football, he might use information he knows about football to create his problems. Similarly, if a student is interested in clothing, she might create proportion problems related to the amount of fabric needed to make different sizes of an item of clothing.

2.1.2 EXPLORE: Scale Drawings and Similar Figures

Curriculum Outcomes	Outcome Relevance
8-B1 Proportion: solve problems • recognize uses for and importance of proportions • investigate problem solving opportunities (e.g., study of scale, transformational geometry, i.e., dilatations)	This essential exploration shows how to use proportional reasoning for drawing and interpreting scale drawings.

Pacing	Materials	Prerequisites
40 min	• Blank paper • Rulers	• finding the missing term in a proportion

Main Points to be Raised

• A scale drawing is a proportional representation of a shape or a figure.

• The scale tells how a linear distance on the drawing relates to the corresponding linear dimension of the shape or figure being represented.

For example, a scale of "1 cm represents 10 m" means that 1 cm is drawn for each 10 m of length on the shape or figure.

• The scale relates to all linear dimensions of the shape or figure.

• If the same units are used for the scale drawing and for the actual shape or figure, you can use a ratio to describe the scale.

For example, a scale of 1 : 1000 means 1 cm represents 1000 cm (or 10 m).

• The chosen scale should be reasonable so that the drawing is not too small or too large.

• You can use relationships in a scale drawing to deduce information about the actual shape or figure.

Exploration

• Work through the introduction (in white) with the students. Make sure they understand that a *scale* on a scale drawing is like a ratio between a measurement on the drawing and a measurement on the object you are drawing. If the units are the same, you can write the scale as a ratio, as shown in the example.

• Have students work, alone, in pairs, or in small groups on **parts A to D**. You may wish to give them an example of how to select a scale.

For example, if the longer side of their paper is about 35 cm, they need to consider what the maximum length of an object they are drawing can be if they use scales of 1 cm represents 2 cm, 1 cm represents 3 cm, 1 cm represents 4 cm, and so on.

• They also need to consider how large they want their drawing to be.

For example, if they use a scale of 1 cm represents 20 cm, and the object they were drawing is only 40 cm long, their drawing will be only 2 cm long and might be hard to see.

While you observe students at work, you might ask questions such as the following:

• *What is the greatest length you can show on your paper if you use a scale of 1 cm represents 4 cm? How do you know?* (The greatest length I can show is 160 cm because the longest side of my paper is 35 cm and 4 × 35 = 160)

• *Why would you not choose a scale of 1 cm represents 50 cm for your scale drawing of the person?* (If I used a scale of 1 cm represents 50 cm, my drawing would be less than 4 cm high, which would be hard to draw and to see.)

• Distribute rulers and blank paper for students to complete **part B**. Make sure students notice the measurement unit for the football field. Since many scales are possible, you might have different students or groups discuss the reasons for their choice of scale.

For example, some might choose 1 cm represents 4 m because it fits on the paper and because both 100 m and 64 m are divisible by 4, so the centimetre measurements are in whole numbers of centimetres.

Observe and Assess

As students work, notice the following:

- Do they successfully determine an appropriate and practical scale for **part A**?
- Do they use proportions to find the lengths they need for the scale drawing of the football field in **part B**? Do they measure correctly?
- Do they recognize the proportions they need to set up and solve for **parts B iii), C, and D**?
- Do they solve the proportions correctly?

Share and Reflect

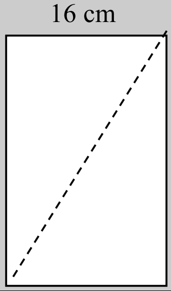
After students have had sufficient time to work through the exploration, they could discuss their observations and answer these questions.

- *How do you know what scale(s) will work best in **part A**?*
- *What is the advantage of choosing a scale for **part B** that uses most of your paper for the scale drawing? (Hint: Think of what you need to do to answer **part B iii)**.)*
- *Is there more than one way to set up the proportions for **parts C and D**? Explain your thinking.*

Answers

A. *Sample responses*:
i) If I make the drawing vertical on the paper, the tallest height possible is only 4 × 35 cm = 140 cm, not 170 cm. (I do not want the figure to be diagonal on the paper.)
ii) 1 cm represents 5 cm; 5 × 35 cm = 175 cm, which is tall enough to show the full height (170 cm) of the person

B. *Sample responses*:
i) 1 cm represents 4 m; In 35 cm, I can draw 4 × 35 m = 140 m, which is long enough, and in 25 cm, I can draw 4 × 25 m = 100 m, which is wide enough.
ii)



iii) 29.7 cm → 118.8 m; $\frac{1\text{ cm}}{4\text{ m}} = \frac{29.7\text{ cm}}{?\text{ m}}$ (× 29.7, × 29.7)

$4 \times 29.7 = 118.8$

C. *Sample responses*:
i) Measure the height of the window and the width of the pane in the photo. Then set up a proportion to compare the actual height and width of the window to the height and width in the photo. Then solve the proportion.
ii) $\frac{\text{Photo width of pane}}{\text{Photo height of window}} = \frac{\text{Actual width of pane}}{\text{Actual height of window}}$

$\frac{1.5}{6} = \frac{\text{Actual width of pane}}{150} \rightarrow \frac{1.5}{6} = \frac{37.5}{150}$ (× 25, × 25)

The pane is about 37.5 cm wide.

D. i) About 62.5 cm **ii)** About 150 cm

Supporting Students

Struggling students

- If students are struggling with choosing a scale in **part B i)**, you might have them organize a list showing the maximum length and width that will fit on their paper for a chosen scale, such as:

Scale	**Maximum length**	**Maximum width**
1 cm represents 1 m	35 m	25 m
1 cm represents 2 m	70 m	50 m
1 cm represents 3 m	105 m	75 m
1 cm represents 4 m	140 m	100 m

Enrichment

- For **part B**, you might challenge students to determine what scale they would use if they were drawing the football field on a 40 cm-by-60 cm sheet of poster board or on a 10 cm-by-15 cm notepad.

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Chapter 2 Percent

2.2.1 Percents Greater Than 100%

Curriculum Outcomes	Outcome relevance
8-A7 Percent: greater than 100 • recognize that values more than a whole are described by percents greater than 100% • relate percent greater than 100 to other subjects (e.g., social studies and inflation rates, population growth, etc.)	Students have previously considered percent as "part of 100". However, there are many real-world applications of percent that use percents greater than 100%. It is important for students to extend their understanding of percent to these situations.

Pacing	Materials	Prerequisites
1 h	None	• writing percents as decimals • using percent (ratio) tables

Main Points to be Raised

• When one value is more than another, the first value can be written as a percent greater than 100% of the second value.

• Percents greater than 100% can be calculated in a similar way to percents less than 100%, e.g., rewriting a percent as a decimal and multiplying.

• A percent increase is the sum of 100% and the percent increase.

• Percent tables can be used just like other ratio tables to solve problems involving percents greater than 100%

• You can use a double number line to solve percent problems involving percents greater than 100%. Write the percent values on one line and write the matching actual values on the matching line.

• To solve a percent problem, it is critical first to identify what value actually represents 100%.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know what percent of the original selling price the reduced price is?* (Since the original selling price is 100% of the price, reducing it by 20% means the price is now 80% of the original selling price.)

• *How can you find the reduced price given the original price?* (I can find how much the 20% reduction is and subtract it from the original selling price, or I can calculate 80% of the original price in one step.)

The Exposition — Presenting the Main Ideas

• With students, go through the first illustration in the exposition on **page 27** of the student text. Be sure students see that if a figure represents "one whole", then it represents 100%, so two of that same figure are 200%, three of that same figure are 300%, and so on. In particular, make sure they understand why the figure together with half of the figure represents 150%. To be sure they understand, you might ask them to show what 400% looks like, and what 125% might look like.

• Ask students to write the following percents as decimals: 25%, 50%, 75%, 100%. Have them continue the pattern so they see that 125% is 1.25. Remind them that they have often found percents of a number by multiplying the number by the decimal equivalent of that percent, and that they can do the same thing when the percent they are multiplying is greater than 100%. In this case, the decimal will always be greater than 1.

• Go through the percent increase example in the exposition on **page 27**, reminding students of how to use a percent table to solve a problem.

• Draw the double number line from the exposition on **page 28** on the board. Discuss with the students the reasoning that if 360 radios is a 150% increase, it represents 250% of the radios sold last year. Make sure that students understand that the top number line shows five equal intervals, each representing 50% (250% ÷ 5 = 50%), so the bottom number line also needs to show five equal intervals, ending at 360.

[Continued]

Ask students how to determine the number of radios in each interval (360 radios divided into 5 equal intervals means each interval is 360 ÷ 5, or 72).

To solve the problem, find the number of radios that corresponds with the 100% point on the number line (since 100% represents the number of radios sold last year).

• You might have students compare a double number line and a percent table (a number line has the values in order from least to greatest in equal intervals, whereas the percent table does not have to have values in order and the size of the intervals is not important).

Revisiting the Try This

B. Students apply what they learned in the exposition about solving problems involving a percent increase to a problem like the problem posed in the **Try This**.

Using the Examples

• Work through **example 1** with the whole class. Make sure students can explain why two hexagons represent 200% and $1\frac{1}{2}$ hexagons represent 150%. They should understand that the two hexagons for 200% can be joined in different ways or not joined at all; the same is true of the $1\frac{1}{2}$ hexagons for 150%.

• Present the problems in **examples 2, 3, and 4** to the students. You may choose to write them on the board. Ask each student to choose two of the problems to solve. Then each student can compare his or her work to what is shown in the matching example. Suggest that students may wish to read through the other example. Some students might realize that the problem in **example 4** can be solved by dividing 180 by 2.25.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to use shapes that are easy to divide into halves and fourths to make their illustrations easier to create and interpret.

Q 2: This is an important generalization. It not only illustrates a pattern for finding percents that are multiples of 10 (such as 2%, 20%, and 200%), but it also reminds students that percents of a number are additive (e.g., if they know 2% , 20%, and 200% of a number, they can add those results to determine 222% (200% + 20% + 2%) of the number).

Q 5: Some students may choose to find 25% of 13,600 and add it to 13,600, while others will find 125% of 13,600 directly. Encourage students to find ways to solve problems like this in one step.

Q 6: Some students may not recognize that 800,000 represents 127%, not 100%. Encourage students to read the question carefully to determine what they are being asked to find. You may wish to explain that the National Exchequer is the government's treasury.

Q 7: Make sure students notice that the increase in attendance for Day 3 is based on the attendance for Day 2, not Day 1.

Q 8: Some students may not recognize that each year's population increase is based on the population of the year before, not on the current population.

Q 9: Encourage students to consider what price the 25% increase was based on and what price the 25% reduction was based on.

Q 10: This question focuses on the difference between a given percent of a number and a number increased by that given percent. This is a critical concept in this lesson.

Common errors

• Many students will assume incorrectly in **question 9** that a price increased by 25% and then reduced by 25% will return to the original price. To address this, start with a simpler number like Nu 100. Students should be able to calculate quickly that a 25% increase is Nu 25, making the new price Nu 125. Then ask them to consider how 25% of 125 compares with 25% of 100. This should help them see that the 25% reduction, based on the increased price, is more than the original price increase, so the final price will be lower than the original price.

Suggested assessment questions from Practising and Applying

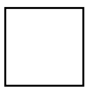
Question 1	to see if students can draw representations of percents greater than 100%
Question 4	to see if students can apply various strategies to solve questions involving percents greater than 100%
Question 6	to see if students can to solve a real-world problem involving a given percent greater than 100%
Question 8	to see if students can explain the effect of repeated percent increases (i.e., that each percent increase changes the base number for the next percent increase)

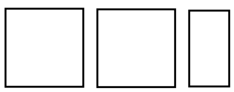
Answers

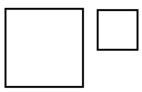
A. i) 80%
ii) *Sample response*:
I can multiply the original price by 0.8:
0.8 × Nu 80 = Nu 64

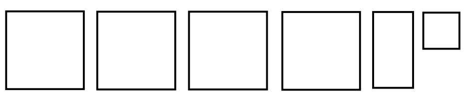
B. i) 120%
ii) *Sample response*:
I can multiply the original price by 1.2:
1.2 × Nu 80 = Nu 96

1. *Sample response*:

 = 100%

a) 250% = 

b) 125% = 

c) 475% = 

2. a) i) 8.4 **ii)** 84 **iii)** 840
b) 932.4 (840 + 84 + 8.4)

3. a) 286 **b)** 1375 **c)** 1329.8 **d)** 300

4 a) 500 **b)** 350 **c)** 150

5. 17,000 tourists

6. *Sample response*: About 630,000 U.S. dollars

7. 1500 on Day 2 and 3375 on Day 3

8. a) 816,000, **b)** 979,200 c) 998,784
[**d)** *Sample response*:
In 1 year, the population will increase 2% over the current population. In 2 years, the population will increase 2% increase over the population after 1 year, which is a greater number than the current population. In 3 years, the population will increase 2% over the population after 2 years, which is an even greater number than the current population. So the total increase is more than 2% + 2% + 2% = 6% of the current population.]

9. a) Nu 2500
b) Less than the original price.
[*Sample response*:
The 25% reduction is 25% of 2500. The 25% increase was 25% of 2000.]

10. No; [125% of a number is the number increased by 25%, which is different from the number increased by 125%; *Sample response*:
125% of 200 is 250.
200 increased by 125% is 450.]

Supporting Students

Struggling students

• Some students may have trouble knowing which familiar percents to use if they are making percent tables for **question 4**. You might have some students share their percent table steps with the class to help others see the variety of possibilities.

• Many students will need support in identifying what value should represent 100% in each calculation.

Enrichment

• For **question 9**, you might challenge students to determine whether the final price would be any different if the shopkeeper had first decreased the price by 25% and then increased it by 25% (No; the resulting price is the same as the answer to **question 9**.)

2.2.2 Solving Percent Problems

Curriculum Outcomes	Outcome relevance
8-B2 Percent: solving and creating real problems in context (including estimation) • estimate and calculate a percent of a given number (a% of $b = c$, e.g., 25% of 1500) • find the percent one number is of another number (e.g., what percent of 20 is 15?) • find the whole when a specified percent is given (e.g., 28% of what number is 42?) • use mental strategies when an exact answer is required (e.g., 28% of 1200 = 20% of 1200 + 8% of 1200)	The ability to calculate with percents is an everyday life skill. It is important to be able to recognize what you are being asked to find, whether the percent of a number, the total when a percent is known, or what percent one number is of another.

Pacing	Materials	Prerequisites
1 h	None	• writing percents as decimals • relating fractions and percents • setting up and solving proportions

Main Points to be Raised

- You can sometimes solve a percent problem by rewriting the percent as a decimal or a fraction and then multiplying.
- You can use a proportion to model and solve a percent problem. A proportion is a statement that two fractions are equivalent.
- To determine what percent one number is of another, you might form a fraction using the numbers as numerator and denominator and then calculate an equivalent fraction with a denominator of 100.

Try This — Introducing the Lesson

A. and B. Allow students to try these alone or with a partner. While you observe students at work, you might ask questions such as the following:

- *What familiar percents might help you with your estimate?* (I could use 10%, since it is easy to calculate, and then take half of the result. Or, I could use 50% and then divide the result by 10.)
- *How do you know what the denominator of the fraction will be in* ***part B*** *before you write it in lowest terms?* (The school has 420 students in all, so that will be the denominator of the fraction.)
- *How can you tell that the fraction is not in its lowest terms?* (140 and 420 can both be divided by 10. I also know that both 14 and 42 can be divided by 2 and by 7.)

The Exposition — Presenting the Main Ideas

- With students, go through the first example in the exposition on **page 31** of the student text. Note that to find the percent of a number, it works well to use the strategy of rewriting the percent as a decimal and then multiplying.
- Review with students how a percent can be rewritten as a fraction with a denominator of 100.
- Write the proportion on the board for the second question in the exposition (finding the number when you know that 4% of the number is 28). To begin, ask students how they know that the number must be much greater than 28 (4% is a very small part of the number). Be sure students understand why 28 is the numerator of the second fraction (28 represents 4% of the total, so it will be in the same position as the 4.) Discuss how to solve the proportion.
- Go through the last example in the exposition about finding what percent 45 is of 75. Have students note that, when possible, it is helpful to write fractions in lowest terms, especially if the denominator is not a factor of 100, to make calculations easier. Reinforce the relationship between a percent and a fraction with a denominator of 100.

Revisiting the Try This

C. Students apply what they learned in the exposition about using proportions to the first problem posed in the **Try This**.

D. Students use what they learned in the exposition about using fractions to find what percent one number is of another.

Using the Examples

• Ask pairs of students to read through **solutions 1 and 2** of **example 1**. Ask them to choose which solution most closely matches what they would have done and to say why. Repeat the process for **example 2** and **example 3**. It is important for students to recognize the three different types of percent problems.

• The use of division shown in **solution 2** of **example 2** is a new approach. You may wish to discuss it further using additional examples such as: *If 30% of a number is 51, what is the number?* (divide 51 ÷ 0.30).

Practising and Applying

Teaching points and tips

Q1, 2, and 3: Encourage students to estimate so they can see if their answers are reasonable.

For example, in **question 1 b)**, 120% of 256 should be somewhat greater, or about 300. In **question 2 c)**, if 240 is 150% of a number, the number must be less than 240.

Q 4: Although **parts b) and c)** can be solved by subtracting the answer to **part a)** from 120 and subtracting 55% from 100%, respectively, you might encourage students to check their answers by solving a one-step percent problem (i.e., find 45% of 120).

Q 5 c): Some students may not recognize that multiple steps are needed to answer this question. You may wish first to ask t students to find how many pieces of fruit were neither apples nor plums, and then ask how they would use that information to find the percent of pieces of fruit that were neither apples nor plums.

Q 7: This question might be assigned only to selected students.

Q 8: Some students may need to be reminded of how to find the area of a rectangle.

Q 11: Some students may wonder how to estimate if they are not one of the given ages. If they are unsure of their height at a target age, allow them to use their best guess of their height at that age for their estimate.

Common errors

• Some students will have difficulty with **question 7**. You might ask students what percent of last year's total the 40 children represent, and how they know this (since this year's league is 110% of last year's, the league increased by 10%, so the 40 students represents 10% of last year's total.)

Suggested assessment questions from Practising and Applying

Question 1	to see if students can apply different strategies for finding the percent of a number
Question 2	to see if students can apply different strategies for finding the total when a percent is known
Question 3	to see if students can apply different strategies for finding what percent one number is of another
Question 5	to see if students can solve a real-world problem that involves finding the percent of a number and finding what percent one number is of another

Answers

A. 21 students; *Sample response*:

I know 10% is $\frac{1}{10}$.

$\frac{1}{10}$ of 420 is 42 and 5% is half of that.

B. $\frac{1}{3}$

C. *Sample responses*:

i) $\frac{5}{100} = \frac{1}{20} \rightarrow \frac{1}{20} = \frac{?}{420} \rightarrow \frac{1}{20} = \frac{21}{420}$

ii) 0.05 × 420 = 21

D. 33%

Answers [Continued]

1. a) 26.88 **b)** 307.2 **c)** 189 **d)** 121.52 **2. a)** 30 **b)** 320 **c)** 160 **d)** 600 **3. a)** 60% **b)** 115% **c)** 30% **4. a)** 66 girls **b)** 54 boys **c)** 45% **5. a)** 16 **b)** 24 **c)** 50% **6. a)** 12,750 **b)** 30,000 **7.** 400 children **8.** 20%	**9.** *Sample responses*: **a)** About 167 cm (166.6...) **b)** About 163 cm (163.3...) **10. a)** About 160 cm **b)** *Sample response*: About 157 cm (156.8) **11.** *Sample response*: I was 153 cm tall when I was 11 years old. Because I am a girl, that is 90% of my adult height. I will be about 153 ÷ 9 × 10 = 170 cm tall. **12.** *Sample response*: 80% of the boys in my class play football. There are 20 boys in my class. How many of them play football? (16 boys)

Supporting Students

Struggling students

• If students are struggling with **question 5 c)**, you might advise them to find, in order, the total number of plums and apples, the number of pieces of fruit that are not plums or apples (i.e., 80, which is the number of apples and plums), and what percent of the fruit that is.

Enrichment

• For **question 12**, you might challenge students to create a problem that involves all three types of percent problems (i.e., finding the percent of a number, finding the total when a percent is known, and finding what percent one number is of another.)

GAME: Equivalent Concentration

• This optional game is designed to allow students to practise identifying equivalent fractions, decimals, and percents. Students can make their cards using small pieces of paper.

• You may wish to create another nine cards with the "third form" of each value (e.g., for $\frac{9}{4}$ and 225%, the third card would be 2.25) and play a Rummy-type game where players (in pairs) get five cards each and the remainder of the cards are placed face down. Players take turns drawing one card from the pile and discarding one card. When they have collected three cards that match, they lay those cards down on the table. Play continues until one player has laid down of all his or her cards.

2.2.3 Fractional Percents

Curriculum Outcomes	Outcome relevance
8-B2 Percent: solving and creating real problems in context (including estimation) • use percents that are not whole numbers	Many real-world problems require calculation with fractional percents.

Pacing	Materials	Prerequisites
1 h	• Thousandths Grids (Rectangular) (BLM)	• writing percents as decimals • multiplying and dividing by powers of 10 mentally • using percent tables to solve problems

Main Points to be Raised

- Percents are not always whole numbers. They can also be fractions and decimals, such as $33\frac{1}{3}\%$.
- A fractional percent can be written as a fraction, a mixed number, or a decimal.
- You can use a thousandths grid to represent a fractional percent. 10 small parts of the grid represent 1%, so 1 small part of the grid represents $0.1\% = \frac{1}{10}\%$.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

- *How many 100 mL containers fill 1 L? How do you know?* (Ten; 1 L is 1000 mL, so it takes ten 100 mL containers to fill 1 L.)
- *Does it make sense that if each 100 mL container has enough sugar to taste sweet, the whole 1 L tastes sweet?* (Yes. To make the 1 L of water taste sweet, you can think of it as putting 0.5 g of sugar into each of ten 100 mL containers.)
- *How can you find out if 7 g of sugar is enough to make 1.5 L of water taste sweet?* (I can think of 1.5 L as fifteen 100 mL containers and find out whether 15×0.5 g is more or less than 7 g.)

The Exposition — Presenting the Main Ideas

- Review with students how to write a decimal and a percent for the fraction $\frac{3}{4}$ ($\frac{3}{4} = 0.75 = 75\%$).
- Have the class find a decimal representation for the fraction $\frac{5}{8}$ ($\frac{5}{8} = 0.625$). Then, using what they have learned about decimal/percent relationships, discuss why it makes sense to write this as 62.5% (between 62% and 63%).
- With students, go through the first part of the exposition on **page 35** of the student text, making sure students understand why 52.5% and $52\frac{1}{2}\%$ are the same.
- You may wish to review how to use a hundredths grid (a 10 by 10 square grid) to represent a “whole” percent (e.g., for 15%, 15 of the 100 squares are shaded, and so on). Then show students a thousandths grid. Discuss what percent one 10-by-10 grid represents if the whole grid represents one whole (since there are ten 10-by-10 grids in the whole, each 10-by-10 grid is $\frac{1}{10}$ of the whole, or 10%).
- Ask students to determine what percent 10 small squares represent on the grid (10 small squares represent $\frac{1}{100}$ of the whole, or 1%).
- Go through the last example in the exposition.

Revisiting the Try This

B. This question allows students to apply what they learned about fractional percents in the exposition to the **Try This** problem.

Using the Examples

• Provide students with thousandths grids and have them pair up to work through **example 1**. Have one student represent **part a)** on a grid, and the other student represent **part b)** on another grid. Have them check each other's results.

• Ask pairs of students to read through **solutions 1 and 2** of **example 2**. Ask them to choose which solution more closely matches what they would have done and to explain why.

• Work through **example 3** with students. Make sure they understand why finding 5.5% of 5000 is the same as finding 12.5% of 5000 and then subtracting 7% of 5000.

Practising and Applying

Teaching points and tips

Q 1 and 2: Remind students that the whole grid is 100%, so each 10-by-10 square represents 10%, and each strip of 10 squares represents 1%.

Q 3: Different students may choose different strategies

For example, in **part a)**, some students may divide by 5, while others might first multiply by 2 and then divide by 10.

Q 4: Remind students to think of benchmark percents such as 1% in determining their estimation strategies.

Q 5 and 6: Students need to recall that there are 1000 mL in 1 L.

Q 7: Some students might determine that the first way is correct and then stop. Encourage students to try all the suggested ways. This question is intended to show students that there are many ways of approaching a percent problem.

Q 8: Encourage students first to determine what fraction of a metre 1 mm is, and then to use that information to verify the percent.

Q 9: If students are unsure, have them refer back to **question 8**.

Q 10: This question is intended to ensure that students understand the difference between $\frac{1}{2}$ (and its equivalences) and $\frac{1}{2}$% (and its equivalences). You might remind students of the meaning of the symbol $\neq$.

Common errors

• Some students will wonder how the very different methods in **question 7** can all be correct. You might have them try each of the methods to find 2.5% of a number that is easy to calculate with, e.g., 2.5% of 100.

• Many students make the kind of error highlighted in **question 10**. This question is provided to help minimize the likelihood of that error.

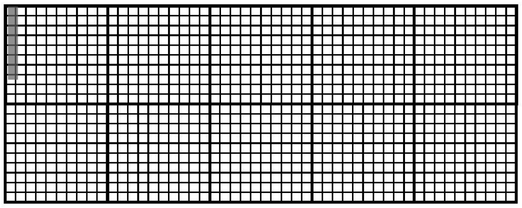
Suggested assessment questions from Practising and Applying

Question 1	to see if students can represent fractional percents
Question 3	to see if students can use efficient strategies to solve a mathematical problem involving fractional percents
Question 5	to see if students can use fractional percents to solve a real-world problem
Question 7	to see if students can explain various strategies for finding a given fractional percent of a number

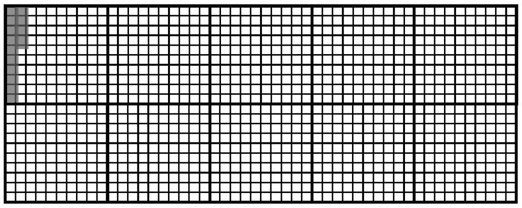
Answers

A. i) 5 g **ii)** No; *Sample response*: 1.5 L would need 1.5 × 5 = 7.5 g of sugar.	**B.** 0.5%

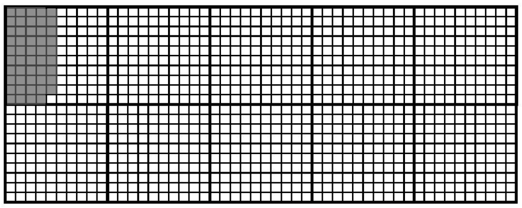
1. a)



b)



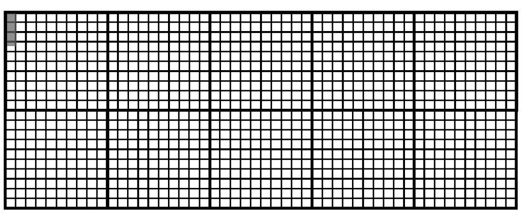
c)



2. 7.6%

[**3.** *Sample responses*:
a) Divide 25 by 5 to get 5 g.
b) Divide 25 by 5 and then by 10 to get 0.5 g.
c) Divide 25 by 2 to get 12.5 g.]

4. [**a)** *Sample response*:
Divide by 100 to get 1% or 6.3. Divide by 3 to get about one third of 6.3 or about 2.]
b)



c) 1.08; [*Sample response*:
Divide by 1000 to get 0.1% or 0.36. Multiply 0.36 by 3 to get 1.08.]

5. 2.5 mL

6. a) 9.3 g
b) 0.3 g

7. All the ways are correct.
[*Sample response*:
A. 5% ÷ 2 = 2.5%
B. 25% ÷ 10 = 2.5%
C. Dividing by 4 and then 10 is the same as dividing by 40, and 1 ÷ 40 = 0.025 = 2.5%.
D. 1% + 1% + 0.5% = 2.5%]

8. [**a)** *Sample response*:
1 m = 1000 mm so 1 mm = (1 ÷ 1000) m = 0.001 m.
0.001 = 0.1%.]
b) 0.32%

9. Yes; [This happens when the number is a multiple of 1000; *Sample response*: 0.1% of 5000 = 5.]

[**10.** *Sample response*:
a) $50\% = \frac{50}{100} = \frac{1}{2}$ but $\frac{1}{2}\% = 0.5\% = \frac{0.5}{100} = \frac{1}{200}$.
b) 0.5% = 0.5 ÷ 100 = 0.005, but $\frac{1}{2} = 0.5$.]

11. No, because it depends on the size of the number; [A small number has a small difference and a big number has a big difference; *Sample response*:
5% of 100 = 5 and 5.1% of 100 = 5.1. 5 and 5.1 are only 0.1 apart.
5% of 1,000,000 = 50,000 and 5.1% of 1,000,000 = 51,000. 50,000 and 51,000 are 1000 apart, which is much more than 0.1.]

Supporting Students

Struggling students

• Many students will have difficulty making the transition from 1% to 0.1%. Encourage them first to calculate 10%, 1%, and 0.1% of a number and then to think about their calculations in terms of these values.

Enrichment

• For **question 9**, you might challenge students to generalize:
For what numbers will 0.1% of the number be a whole number? (For any multiple of 1000)

2.2.4 Solving Percent Problems Using Familiar Percents

Curriculum Outcomes	Outcome relevance
8-B2 Percent: solving and creating real problems in context (including estimation) • estimate and calculate a percent of a given number (a% of $b = c$, e.g., 25% of 1500) • find the percent one number is of another number (e.g., what percent of 20 is 15?) • find the whole when a specified percent is given (e.g., 28% of what number is 42?) • use mental strategies when an exact answer is required (e.g., 28% of 1200 = 20% of 1200 + 8% of 1200)	The ability to estimate and calculate with percents is an everyday life skill. In many situations, estimation is as important a skill as calculation. Knowing how to work with benchmark or familiar percents is very useful in estimating and mentally calculating percent problems.

Pacing	Materials	Prerequisites
1 h	None	• familiarity with percent/fraction equivalents for benchmark fractions • using a percent table to solve a problems

Main Points to be Raised

• Percents such as 10% and 1% are considered *benchmark percents* because they are easy to calculate with mentally. Sometimes other percents like 25% and 50% are used as benchmarks.

• Benchmark percents are useful for finding other percents of a number.

• You can use a percent table, together with percents such as 1% and 10%, to solve a percent problem.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How can you find 10% of a number in your head?* (10% is the same as $\frac{1}{10}$, so finding 10% of a number is the same as dividing the number by 10. I can do that by moving the digits one place to the right.)

• *Does it make sense that your answer to **part ii)** is twice as much as your answer to **part i)**?* (Since 20% is twice as much as 10%, it makes sense that 20% of a number is twice as much as 10% of that number.)

The Exposition — Presenting the Main Ideas

• Have students discuss any strategies they have developed for mentally calculating the following:

50% of a number (e.g., 50% is the same as $\frac{1}{2}$. To find 50% of a number, divide the number by 2.)

25% of a number (e.g., 25% is the same as $\frac{1}{4}$. To find 25% of a number, divide the number by 4.)

10% of a number (e.g., 10% is the same as $\frac{1}{10}$. To find 10% of a number, divide the number by 10.)

1% of a number (e.g., 1% is the same as $\frac{1}{100}$. To find 1% of a number, divide the number by 100.)

• With students, go through the exposition on **page 38** of the student text. Have them discuss why percents such as 10% and 1% are considered to be familiar or benchmark percents. Students might observe that these percents are the same as dividing by 10 and by 100, which could help with estimation and calculation.

• Remind students of how to set up and use a percent table. Discuss why the useful first step in setting up a percent table is to find a benchmark percent like 10% or 1%.

Revisiting the Try This

B. Students apply what they learned in the exposition about benchmark percents to the problem in the **Try This**.

Using the Examples

• Have students work in pairs. One of the pair should become an expert on **example 1** and the other should become an expert on **example 2**. Each should then explain his or her example to the other student.

Practising and Applying

Teaching points and tips

Q 1: Some students may choose to find the answers by previous methods such as rewriting the percent as a decimal and multiplying. There is nothing wrong with solving the problems that way, but you should encourage students to devise strategies for calculating the percents at least partially in their heads.

Q 2: You might encourage students to use a percent table together with benchmark percents to solve the exercises. For **part b)**, students might use as 100% as a benchmark. For **part e)**, they might use 25%.

Q 3 d): Many students will not see a way to use benchmark percents for this problem. You might ask them to think about how it might be helpful to find 5%.

Q 4 a): Students may choose a variety of paths in a percent table to answer this question.

For example, they might divide by 4 to find 4% and then multiply by 25 to get 100%. Or, they might divide by 16 to find 1% and then multiply by 100 to find 100%.

Q 6 b): Have students recall how many grams are in a kilogram. Be sure that students notice the question asks for how much is NOT water.

Q 12: This is a closure question that highlights the benchmark percents strategy emphasized in this lesson.

Common errors

• Some students will think incorrectly that **question 2** requires them to find the percent of a number rather than to find the total when a percent is known. You might have them rewrite the question in a way that makes it more clear. For example, for 9% is 36 m, they might write 9% of ■ m is 36 m.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can apply familiar/benchmark percents to finding a percent of a number
Question 2	to see if students can apply familiar/benchmark percents to find the total when a percent is known
Question 5	to see if students can solve a real-world problem that involves finding a percent of a number

Answers

<table>
<tr>
<td>A. i) 30 m^2
ii) Sample response:
Since 10% of the garden is 30 m^2, 20% of the garden is twice as much, or 60 m^2.</td>
<td>B. Sample response:
1 % of the garden is 3 m^2, so
15% is 15 × 3 = 45 m^2.
÷ 10 ÷ 2 × 3
<table>
<tr><td>Percent</td><td>100</td><td>10</td><td>5</td><td>15</td></tr>
<tr><td>Area (m^2)</td><td>300</td><td>30</td><td>15</td><td>45</td></tr>
</table>
</td>
</tr>
<tr>
<td>1. a) 88; [10% is 80 and 1% is 8. 80 + 8 = 88]
b) 648; [100% is 540, 10% is 54, and 20% is 2 × 54 = 108. 540 + 54 + 108 = 648]
c) 344; [10% is 430, 1% is 43, and 2% is 2 × 43 = 86. 8% = 10% – 2%. 430 – 86 = 344]
d) 1058; [100% is 920, 10% is 92, and 5% is 92 ÷ 2 = 46. 920 + 92 + 46 = 1058]

2. a) 400 m; [1% is 36 ÷ 9 = 4, 100% = 100 × 4 = 400]
b) 80 g; [100% = 240 ÷ 3 = 80]
c) Nu 175; [20% is 140 ÷ 4 = 35, 100% = 5 × 35 = 175]
d) 30 kg [50% is 135 ÷ 9 = 15, 100% = 2 × 15 = 30]
e) 60 L [25% is 105 ÷ 7 = 15, 100% = 4 × 15 = 60]</td>
<td>3. a) Nu 25; [10% of 250 = 250 ÷ 10 = 25]
b) Nu 75; [10% of 1500 = 150 so 5% is 150 ÷ 2 = 75]
c) Nu 35; [25% of 140 is 140 ÷ 4 = 35]
d) Nu 14; [10% is 8, 5% is 4, 2.5% = 8 ÷ 4 = 2, 8 + 4 + 2 = 14]
e) Nu 2700; [10% is 1800, 5% is 900, 1800 + 900 = 2700]

4. a) About 766,667 b) About 2080 males
5. 14 games</td>
</tr>
</table>

Answers [Continued]

6. a) 168 g **b)** About 0.27 kg (270 g) **7.** 25 questions **8.** 35 students **9.** 400 mL **10. a)** 5,000,000 **b)** 30,000,000 **c)** 33,000,000	**11.** Archery, 180; Soccer, 120; Track & Field, 60; Other, 40. [**12** *Sample response*: I would start with 10% and use that to find 5%. I would use 5% to find 15% and then use 15% to find both 30% and 45%. I would use these because I can calculate them mentally.]

Supporting Students

Struggling students

• If students are struggling with **question 3 d)**, you might scaffold the question for them as follows:

To find 17.5% of Nu 80:

find 10% of 80;

find half of that result (5%);

find half of that result (2.5%);

put all the results together (10% + 5% + 2.5% = 17.5%).

Enrichment

• For **question 11**, you might challenge students to find each percent in at least two different ways.

2.3.1 Markup and Discount Consumer Problems

Curriculum Outcomes	Outcome relevance
8-B3 Percent: increase and decrease • investigate markups and mark-downs of retail items (e.g., a dress cost Nu 7 to make and is being sold for Nu 15. What is the percent of markup?) • develop formula (% increase = increase/original amount × 100%) • develop formula (% decrease = decrease/original amount × 100%)	Most people use percents every day in consumer applications. The ability to estimate and calculate with percents in these settings is an important life skill.

Pacing	Materials	Prerequisites
1 h	None	• writing a percent as a decimal • calculating benchmark percents such as 10% and 1% mentally • multiplying a whole number by a decimal

Main Points to be Raised

• The price a shopkeeper pays for an item is the *cost price*. The price the shopkeeper sells it for is the *selling price*.

• The increase from cost price to selling price is the *markup*, which can be an amount or a percent.

• A *discount* or *markdown* is an amount or a percent by which a shopkeeper reduces a regular selling price to encourage customers to buy the item. The reduced price is called the *sale price*.

• If you know the percent markup, you can find the selling price by finding that percent of the cost price and adding it to the cost price. Or, you can find (100% + percent markup) of the cost price.

• If you know the percent discount or markdown, you can find the sale price by finding the discount and subtracting it from the regular price. Or, you can find (100%-percent discount) of the regular selling price.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Which CD player has the highest percent discount?* (Player Y, 30%).

• *Why can you not tell immediately whether Player X or Player Z costs more?* (The price of Player Z is greater, but it also has a greater percent off.)

• *How do you know that you save more by buying Player Z than by buying Player X?* (The discount on Player Z is a greater percent of a greater amount.)

• *How can you figure out which CD player gives you the greatest savings?* (Since 30% is the greatest percent and 2000 is the greatest number, Player Y has the greatest savings.)

The Exposition — Presenting the Main Ideas

• Ask students to discuss how stores make money (They buy things for one price and sell the things to others at a higher price. The difference is their profit less expenses.)

• Ask students to tell you the meaning of an advertisement of “20% off”, such as the advertisement in the **Try This** (the price you pay is 20% less than the listed price, or the price is discounted by 20%).

• Go through the exposition on **pages 41 and 42** of the student text with the students. Make sure they understand all the terms listed below and how to calculate each:

Cost price
Selling price, or regular price
Markup
Discount or mark-down
Sale price

Revisiting the Try This

B. and C. Students apply what they learned in the exposition about calculating discounts to the problem posed in the **Try This**.

Using the Examples

• Work through **example 1** with the students to make sure they understand it. In particular, note the use of benchmark percents (10% and 5%) to find the discount.

• Ask pairs of students to read through **solutions 1 and 2** of **example 2**. Ask them to choose which solution most closely matches what they would have done and to say why. Mention to the students that in the percent table for **solution 1** they could have added the first two columns to get the third column.

• Have students work in pairs. Have one student in each pair become the expert on **example 3 a)** and the other become the expert on **example 3 b)**. Each student should then explain his or her example to the other student in the pair. Note that if students use the fraction $\frac{1600}{2000}$, calculate the equivalent percent of 80%, and then subtract from 100%, their answer will be incorrect. Reinforce that they must consider the percent of the original price and not the increased price.

Practising and Applying

Teaching points and tips

Q 1: If students choose to calculate each sale price directly, remind them that the question also asks for the discount amount (which they can find by subtracting the sale price from the regular selling price).

Q 2: If students choose to calculate each selling price directly, remind them that the question also asks for the markup amount.

Q 4 and 5: Remind students to make sure they are using the correct price for the denominator when they calculate the percent, i.e., the regular selling price in the case of a discount, and the cost price in the case of a markup.

Q 7: Some students may not notice that the price given in the question is the sale price. They need to work backwards to find the regular selling price.

Q 8: This is an important generalization: successive discounts of 10%, then 10%, and then 10% give
a smaller discount than a single discount of 30%.

Q 10: This question highlights the method of calculating a selling price directly.

Common errors

• In **question 6**, many students will assume incorrectly that the second 20% discount is the same as the first discount. Make sure students recognize that the sale price after 3 weeks becomes the regular selling price for calculating the 20% discount after 6 weeks.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can solve simple discount consumer problems
Question 2	to see if students can solve simple markup consumer problems
Question 4	to see if students can select and use the relevant information from a real-world problem that involves finding a percent discount
Question 8	to see if students can recognize that the selling price changes each time in successive discounts

Answers

A. Player Y; *Sample response*: 30% is the greatest percent and Nu 2000 is the highest price, so I would save the most money by if I bought Player Y. **B.** Player X: Nu 1200 Player Y: Nu 1400 Player Z: Nu 1350	**C.** No; *Sample response*: I might save more money on one CD player, but if it is a lot more expensive than another player to start with, it is not the least expensive, even after the savings. Player Y has the greatest savings but it is still the most expensive.

1. a) Discount amount: Nu 21; Sale price: Nu 119
b) Discount amount: Nu 18; Sale price: Nu 54
c) Discount amount: Nu 52; Sale price: Nu 598
d) Discount amount: Nu 5400; Sale price: Nu 12,600

2. a) Mark-up amount: Nu 6;
Regular selling price: Nu 36
b) Markup amount: Nu 70;
Regular selling price: Nu 350
c) Markup amount: Nu 75;
Regular selling price: Nu 825
d) Markup amount: Nu 750;
Regular selling price: Nu 3250

3. Nu 210 per kg

4. 10%

5. 150%

6. a) Nu 240 **b)** Nu 192

7. Nu 1500

8. Shop 2; [*Sample response*:
At the end of 3 weeks, if the item had a regular selling price of Nu 1000, it would sell for Nu 729 in Shop 1 (week 1: Nu 900; week 2: Nu 810; week 3: Nu 729) and for Nu 700 in Shop 2.]

9. Yes; [*Sample response*:
The percent discount is $\frac{160}{800}$, or 20%, and the percent markup is also $\frac{160}{800}$ or 20%.]

10. *Sample response*:
The cost price of an item is Nu 300. The markup is 25%. What is the selling price?
(125% of Nu 300 = Nu 375)

Supporting Students

Struggling students

• If students are struggling with **question 7**, encourage them to use a percent table. Ask them what percent of the regular selling price Nu 1200 represents (80%, since it is the price after a 20% discount). Have them use that information to get to a benchmark percent (such as 10%), which will allow them to find the regular selling price more easily.

Nu 1200	Nu 150	Nu 1500
80%	10%	100%

Enrichment

• For **question 8**, you might challenge students to calculate the total discount from the regular selling price when three successive 10% discounts are given (27.1%).

2.3.2 Simple Interest and Commission

Curriculum Outcomes	Outcome relevance
8-B3 Percent: increase and decrease • investigate commissions and simple interest; develop formula $I = Prt$.	Most people use percents every day in consumer applications. The ability to estimate and calculate with percents in these settings is an important life skill.

Pacing	Materials	Prerequisites
1 h	None	• writing a percent as a decimal • calculating mentally benchmark percents such as 10% and 1% • multiplying a whole number by a decimal

Main Points to be Raised

• When you borrow money from a bank or when you buy something on credit and pay for it later, you are charged interest.

• Interest is calculated as a percent of the amount borrowed or owed — the greater the amount borrowed or owed, the greater the amount of interest you pay.

• Banks also pay interest to you when you put or invest money in the bank, since the bank is borrowing your money to lend to others.

• When the interest charged (or earned) is based only on the amount of money that you originally borrowed (or invested), the interest is called *simple interest*.

• The formula for finding the amount of simple interest on a loan or an investment is $I = Prt$, where

I is the amount of simple interest,

P is the principal, the amount borrowed or invested,

r is the annual interest rate (rate per year), and

t is the time period in years.

• Interest can be charged or earned for a time period less than one year.

• Shopkeepers sometimes pay salespeople a commission to sell items for them. They calculate commission as a percent of the money received from the sale of an item or from their total sales. The greater the amount sold, the greater the commission.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know the equivalent percent will be less than 10?* (Because 10% of 45,000 is 4500.)

• *What estimate would you use for the percent?* (6% since 5% is half of 4500 and that is just a bit too little.)

• *What fraction of the selling price were Padam's earnings?* ($\frac{2700}{45{,}000}$)

• *How can you write that as a percent?* (Write the fraction in its lowest terms, then find an equivalent fraction with a denominator of 100.)

The Exposition — Presenting the Main Ideas

• Ask if any students are familiar with the term *interest* with regard to a loan or a savings account.

• With students, go through the exposition on **page 45** of the student text. Make sure they understand what simple interest is (based only on the original amount borrowed or invested), and that it can be an amount or a percent. Point out that an interest rate is usually quoted as a percent per year even if the term of the loan or saving is less than a year.

• Write the formula $I = Prt$ on the board, write what each letter in the formula means, and then ask the students to put the information from the example (Ugyen borrowing money) into the formula: I = principal (5000) × rate (12% written as a decimal: 0.12) × time (1 year). If students ask why the percent has to be written as a decimal, explain that otherwise the answer will be too high.

For example, if you borrow \$100 at 10% interest for one year, you should pay \$10 in interest. If you multiply 100 × 10 × 1, you get 1000 (which is 100 times too much).

• In the second example, make sure students can explain why the time is written as 0.5 (6 months is $\frac{1}{2}$ or 0.5 a year).

• Discuss with the students that most banks charge (and pay) *compound interest*, not simple interest. Explain what the difference is, i.e., the interest for the second or later period of time is based on the amount of money that is still owing or on the amount of money that has accumulated, not on the original amount.

• Have students look at the commission example in the exposition on **page 46** of the student text. Discuss how it is similar and how it is different from calculating interest (similar: both are a percent of an amount; different: there is no time component to commission).

Revisiting the Try This

B. Students apply what they learned in the exposition about calculating commissions to the problem posed in the **Try This** and to similar problems.

Using the Examples

• Present the problems in the three examples to the students. Ask each student to choose two of the problems to solve. Then each student should compare his or her work to what is shown in the matching example. Students should then read through the other example, as it is important for them to be familiar with all the problem types.

Practising and Applying

Teaching points and tips

Q 1 c): Students may first write $6\frac{1}{4}\%$ using the decimal equivalent for $\frac{1}{4}$ (6.25%) and then write that percent as a decimal (0.0625).

Q 5: Remind students that in paying back a loan, you have to pay back the principal as well as the interest owing.

Q 7: This is an important generalization: if you double the interest rate and halve the term, the interest will remain the same.

Q 8: You may wish to discuss with students the effect of paying only the interest on a loan (you can end up paying more than the amount of the original loan in interest, and still owe the principal).

Q 12: This question highlights the different ways the interest formula can be used, a critical notion brought out in this lesson.

Common errors

• Some students will substitute 6 and 3, the number of months, for *t* in the interest formula for **question 5 parts a) and b)**, respectively. Remind them that they must rewrite a time given in months as fraction of a year.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can apply the simple interest formula
Question 5	to see if students can use the interest formula to solve a real-world problem
Question 6	to see if students can solve a problem that involves calculating commission
Question 7	to see if students can explain the effects of changes in the rate and time on interest earned

Answers

A. 6%	**B. i)** Nu 1800 **ii)** Nu 50 000
1. a) 0.07 **b)** 0.045 **c)** 0.0625 **2. a)** Nu 1558 **b)** Nu 8100 **c)** Nu 2550 **3. a)** Nu 648 **b)** Nu 1820 **c)** Nu 2340 **4.** Nu 640	**5. a)** Interest: Nu 450; Total amount paid: Nu 6450 **b)** Interest: Nu 250; Total amount paid: Nu 12,750 **c)** Interest: Nu 2880; Total amount paid: Nu 10,880 **6.** Nu 3250 [Nu 2250 + Nu 1000]

Answers [Continued]

7. They both earn the same amount of interest.
[*Sample response*:
The first investment earns half the interest rate but for double the time. Since you multiply by the amount of time to find the amount of interest, they both work out to the same amount.]

8. a) Nu 150 **b)** Nu 300
c)

Number of years	Interest paid (Nu)
3	450
4	600
5	750
6	900
7	1050
8	1200
9	1350
10	1500

9. a) Nu 7200 **b)** Nu 36,000 **c)** Nu 1500

10. Dawa earned more;
[Dawa: Nu 4000 + Nu 2400 (6% × 40,000) = Nu 6400
Nima: Nu 4500 + Nu 1600 (4% × 40,000) = Nu 6100]

11. 12%

[**12.** *Sample response*:
If I know that the deposit earned Nu 100 in interest at 10% for 2 years, I could multiply the interest rate by the time and then divide the interest earned by that result:
10% = 0.1
0.1 × 2 = 0.2
100 ÷ 0.2 = 500
The amount deposited was Nu 500.]

Supporting Students

Enrichment

• For **question 10**, you might challenge students to find the amount of sales for which Dawa and Nima would both earn the same commission.

CONNECTIONS: Currency Conversion

• This optional Connection can be used with all students.

• For people who travel between countries, currency conversion is a very common and important application of percent.

• The rate of exchange between currencies depends on many factors, and can change very quickly. It is affected not only by the country's economy, but things such as political forces and natural disasters.

• The currency board pictured in the student text indicates two rates — the rate at which the bank will sell you that currency, and the rate at which the bank will buy that currency from you. You might point out to students that the bank always sells the currency at a higher rate than it buys it. The difference is the profit the bank makes for the service of exchanging currency.

• For **question 4**, you may wish to write on the board the following list, which shows in order the countries that are on the currency table:

United States Dollar
British Pound
Euro
Japanese Yen
Swiss Franc
Hong Kong Dollar
Canadian Dollar
Australian Dollar
Singapore Dollar
Danish Kroner
Norwegian Kroner
Swedish Kroner

Answers

1. a) 5.6 baht **b)** 56 baht **c)** 560 baht

2. a) Nu 19 **b)** Nu 190 **c)** Nu 1900

3. Nu 1 = 0.02 Canadian dollars

4. *Sample response*:
1 US dollar = Nu 67.05
Nu 1 ≈ 0.015 US dollars

1 £ = Nu 83.80
Nu 1 = 0.012 £

UNIT 2 Revision

Pacing	Materials
2 h	None

Question	Related Lesson(s)
1 – 3	Lesson 2.1.1
4 – 6	Lesson 2.2.1
7	Lesson 2.2.2
8	Lesson 2.2.3
9	Lesson 2.2.4
10 – 12	Lesson 2.3.1
13 – 15	Lesson 2.3.2

Revision Tips

Q 1: Remind students that sometimes they need to divide by the scale factor and sometimes they need to multiply.

Q 2: Remind students to write both ratios of the proportion in the same order:

3 boys : 4 girls → 150 boys : ? girls

Q 7: Encourage students to use benchmark percents where appropriate.

Q 10: Some students may choose to calculate the sale price directly and then subtract to find the discount amount.

Q 12: Students might benefit from using an example with specific values to solve this problem.

Answers

1. a) 5 **b)** 27 **c)** 28

2. 200 girls

3. 15 min

4. 350%

5. a) 1300 **b)** 1428 **c)** 1210

6. 50,400

7. a) 105 **b)** 180 **c)** 640
d) Nu 240 **e)** 20%

8. 40

9. 615 g

10. a) Nu 540 **b)** Nu 1260

11. 37.5%

12. No; It's less expensive at the end of June;
[15% of the starting price is not as much as 15% of the increased price. For example:
15% of Nu 100 is Nu 15, so the increased price is Nu 115.
15% of Nu 115 is Nu 17.25, so the decreased price is Nu 97.75.]

13. a) Nu 630 **b)** Nu 4130

14. Nu 4200

15. 8%

UNIT 2 Proportion and Percent Test

1. The ratio of girls to boys in a school is 2 : 3. If there are 150 girls in the school, how many students are in the school?

2. Draw a picture for each.

a) show 250% if ☐ is 100%

b) show 125% if ☐ is 100%

c) show 400% if ☐ is 200%

3. Complete each.

a) 175% of 400 is ___

b) 210% of ___ is 315

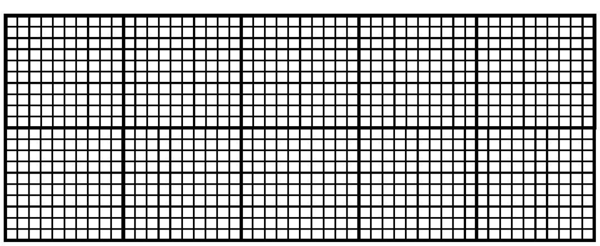
c) 125% of ___ is 280

4. a) In the 2005 Census of Bhutan, the population was about 670,000, with about 30% of the population living in urban areas. About how many people in Bhutan lived in urban areas in 2005?

b) In Lhuentse Dzongkhag, 1350 people live in rural areas. This is about 90% of the population of Lhuentse. If 1350 people live in rural areas, estimate the population of Lhuentse.

5. How many small sections of a thousandths grid would you colour to show each percent?

a) 0.1% **b)** 8% **c)** 40.5%



6. Seawater is 1.6% argon and 1.4% carbon dioxide. In 1 L of seawater, how many grams are there of each?

a) argon

b) carbon dioxide

7. Find each.

a) 11% of 700 is ▲

b) 15% of 820 is ▲

c) 80% of ▲ is 320

d) 125% of ▲ is 480

8. 40% of the number ▲ is 46.

a) How do you know ▲ is greater than 92?

b) How do you know 10% of ▲ is more than 10?

c) What is 90% of ▲?

9. A bed is on sale for a discount of 20%. The regular selling price is Nu 1400.

a) What is the amount of the discount?

b) What is the sale price of the bed?

10. A gho with a cost price of Nu 750 was sold for Nu 900. What was the percent markup?

11. Deki borrowed Nu 8000 for 18 months at an annual simple interest rate of 7%.

a) How much interest did Deki pay after 18 months?

b) How much did Deki have to pay back altogether after 18 months?

12. A salesperson earns a rate of commission of 5% on the first Nu 70,000 of sales and a rate of 7% on sales over Nu 70,000. What is the total amount of commission on sales of Nu 85,000?

UNIT 2 Test

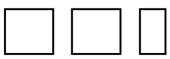
Pacing	Materials
1 h	None

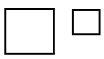
Question	Related Lesson(s)
1	Lesson 2.1.1
2 and 3	Lesson 2.2.1
4	Lesson 2.2.2
5 and 6	Lesson 2.2.3
7 and 8	Lesson 2.2.4
9 and 10	Lesson 2.3.1
11 and 12	Lesson 2.3.2

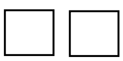
Select questions to assign according to the time available.

Answers

1. 375 students (150 girls and 225 boys)

2. a) 

b) 

c) 

3. a) 700 **b)** 150 **c)** 224

4. a) 201,000 **b)** 1500

5. a) 1 **b)** 80 **c)** 405

6. a) 16 g **b)** 14 g

7. a) 77 **b)** 123 **c)** 400 **d)** 384

8. a) *Sample response*:
If the number were 92, then 50% would be 46. Since a smaller percent was 46, the number must be greater.
b) If 40% is 46, then 10% = 40% ÷ 4 = 11.5, which is more than 10.
c) 103.5 (46 + 46 + 11.5)

9. a) Nu 280 **b)** Nu 1120

10. 20%

11. a) Nu 840 **b)** Nu 8840

12. Nu 4550

UNIT 2 Performance Task — Vegetable Market

Dorji and Tandin sell vegetables at the market.

This chart shows how much they paid to buy the vegetables and how much they sold for one day:

Vegetable	Cost price (per kg)	Selling price (per kg)	Amount sold (kg)
Potatoes	Nu 12	Nu 15	200
Onions	Nu 10	Nu 14	50
Tomatoes	Nu 30	Nu 40	80

A. For each vegetable, what percent is the selling price of the cost price?

B. How much is the total profit they earned? (Profit is the difference between the amount of money they received for the vegetables and the amount of money they paid to buy the vegetables.)

C. i) The two sellers worked together at the market for 6 h. Dorji worked alone for 8 h and Tandin worked alone for 5 h. Describe a fair way for them to share the profits.

ii) What percent of the total profit should each seller get?

iii) How much profit did each seller earn?

D. Suppose that one day, instead of using their money to buy vegetables, they invest the money. Estimate the interest rate needed to earn the same amount of money as their profit from a day selling vegetables, if the money is invested for one year.

UNIT 2 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
8-B1 Proportion: solve problems **8-B2** Percent: solving and creating real problems in context **8-B3** Percent: increase and decrease	1 h	None

How to Use This Performance Task

- You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could supplement the unit test. It could also be used as enrichment material for some students.
- You can assess performance on the task using the rubric provided below.

Sample Solution

A. Potatoes: 125%; onions 140%; tomatoes; $133\frac{1}{3}$%

B. Nu 1600

C. i) Each gets a share according to the total hours worked at the market.
Total hours worked: 6 + 6 + 8 + 5 = 25
Dorji worked 14 h and Tandin worked 11 h.

Dorji should get $\frac{14}{25}$ of Nu 1600.

Tandin should get $\frac{11}{25}$ of Nu 1600.

ii) Dorji: $\frac{14}{25}$ = 56%

Tandin: $\frac{11}{25}$ = 44%

iii) Dorji earned Nu 896 and Tandin earned Nu 704.

D. About 33%; They spent Nu 5300 to buy the vegetables and the profit was Nu 1600.

1600 ÷ 5300 = 16 ÷ 53, which is about 17 ÷ 51 = $\frac{1}{3}$, which is about 33%.

UNIT 2 Performance Task Assessment Rubric

The student	Level 4	Level 3	Level 2	Level 1
Calculates	Makes completely accurate calculations	Makes mostly accurate calculations	Makes a few errors in calculations	Makes many errors in calculations
Communicates	Expresses answers in correct form with units or percent signs; explains clearly why the shares are fair	Expresses answers mostly in correct form with units or percent signs; shows a proportional basis for distributing fair shares	Expresses some answers in correct form with units or percent signs; shows a reasonable basis for distributing fair shares but does not use proportional reasoning	Does not express answers in correct form with units or percent signs; shows an arbitrary distribution of shares

UNIT 2 Blackline Masters

BLM 1 100 Charts

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

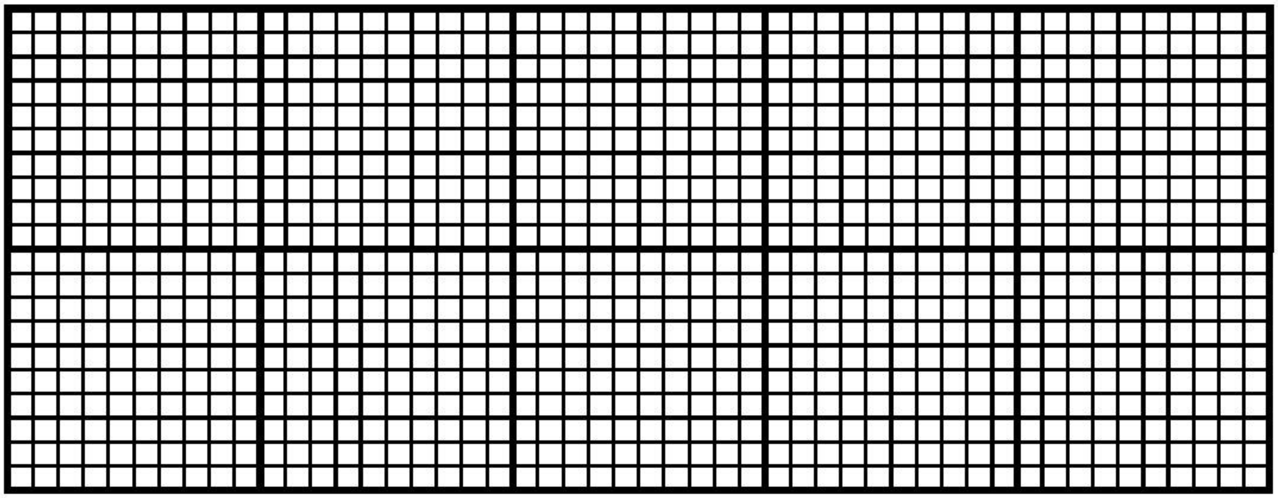
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

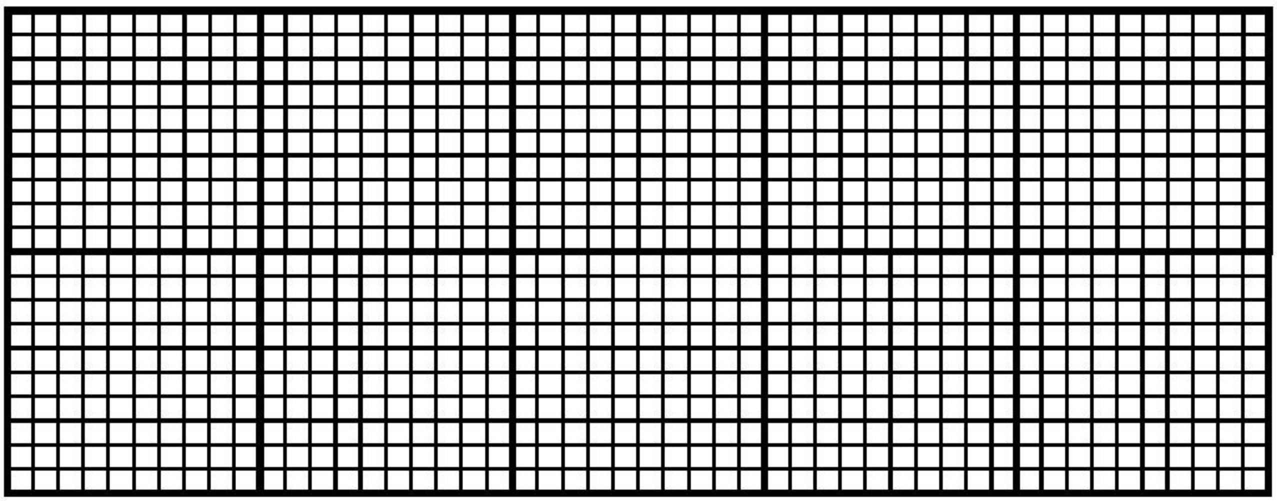
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

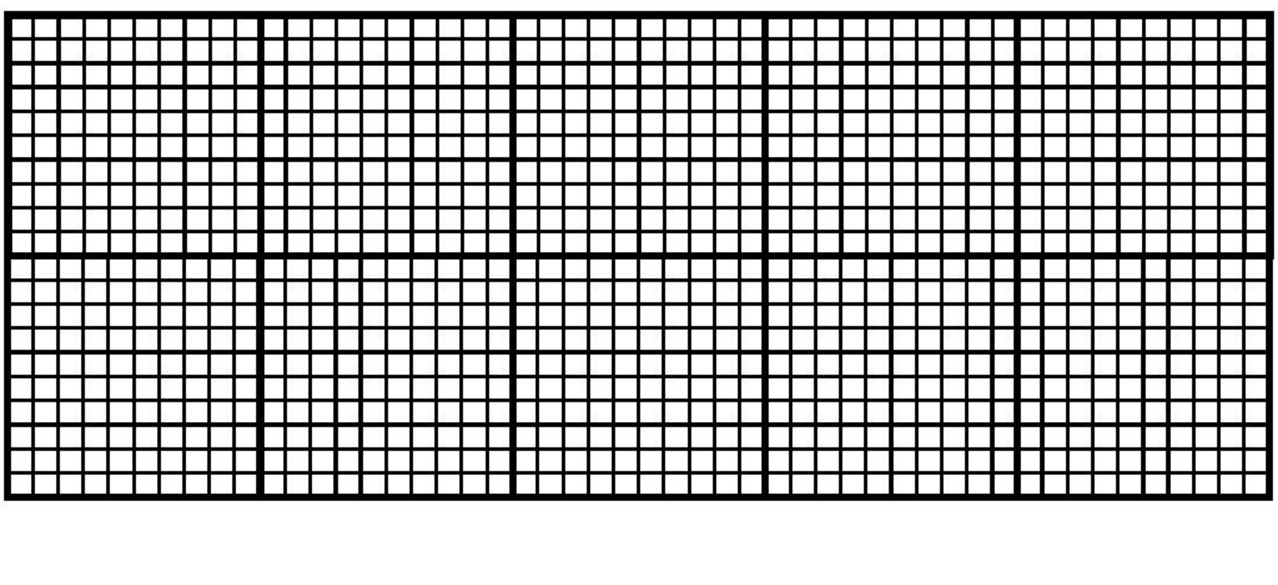
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Reprint 2024

BLM 2 Thousandths Grids (Rectangular)







UNIT 3 INTEGERS

UNIT 3 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 51** **TG p. 72**	Review prerequisite concepts, skills, and terminology, and pre-assessment	1 h	• Black and white counters	All questions
Chapter 1 Multiplying Integers				
3.1.1 Multiplying Integers Using Counters and Patterns **SB p. 53** **TG p. 75**	**8-B4 Multiply Integers: solve problems** • connect visual models, such as counters and number lines, to symbols • interpret multiplication as repeated addition and vice versa **8-B5 Properties of Operations for Integers: commutative and associative** • apply properties and understand their usefulness: commutative property (order, e.g., $(-5) \times 4 = 4 \times (-5)$) and associative property (grouping, e.g., $((-2) \times 4) \times (-3) = (-2) \times (4 \times (-3))$	1 h	• Black and white counters	Q2, 5, 6, 9
3.1.2 Multiplying Integers Using a Number Line **SB p. 56** **TG p. 78**	**8-B4 Multiply Integers: solve problems** • connect visual models, such as counters and number lines, to symbols • interpret multiplication as repeated addition and vice versa **8-B5 Properties of Operations for Integers: commutative and associative** • apply properties and understand their usefulness: commutative property and associative property • recognize the property of closure (e.g., $2 - 5$ is not defined within the set of whole numbers, therefore needing the introduction to integers)	1 h	None	Q2, 6, 7
3.1.3 EXPLORE: Pattern Grids (Optional) **SB p. 58** **TG p. 81**	**8-B4 Multiply Integers: solve problems** • connect visual models, such as counters and number lines, to symbols • interpret multiplication as repeated addition and vice versa **8-B5 Properties of Operations for Integers: commutative and associative** • apply properties and understand their usefulness: commutative property and associative property	40 min	None	Observe and Assess questions
3.1.4 Renaming Factors to Multiply Mentally **SB p. 59** **TG p. 83**	**8-B6 Multiply Integers: mentally** • develop and use mental strategies such as the following: - front-end - compatible numbers/factors - working by parts - double and halves **8-B4 Multiply Integers: solve problems** • connect visual models, such as counters and number lines, to symbols	1 h	• Black and white counters (optional)	Q1, 4

UNIT 3 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
3.1.4 Renaming Factors to Multiply Mentally [Continued]	**8-B5 Properties of Operations for integers: distributive** • apply the distributive property (e.g., (−2)(3 + (−2)) = (−2)(3) + (−2)(3)) and understand its usefulness			
GAME: Order the Integers (Optional) **SB p. 61** **TG p. 85**	Practise multiplying integers in a game situation	30 min	• 42 integer game cards (student or teacher-made), −10 to 10 (two of each)	N/A
Chapter 2 Dividing Integers				
3.2.1 Dividing Integers Using Models and Patterns **SB p. 62** **TG p. 86**	**8-B4 Divide Integers: solve problems** • connect visual models, such as counters and number lines, to symbols	1 h	• Black and white counters	Q1, 2, 4, 6
3.2.2 Relating Division of Integers to Multiplication **SB p. 66** **TG p. 89**	**8-B4 Divide Integers: solve problems** • connect visual models, such as counters and number lines, to symbols • relate multiplication and division **8-B6 Divide Integers: mentally** • develop and use mental strategies	1 h	• Black and white counters (optional)	Q2, 4, 5
CONNECTIONS: Mean Temperatures (Optional) **SB p. 68** **TG p. 91**	Make a connection between the mathematics of integers and a practical use of them	30 min	None	N/A
3.2.3 Order of Operations with Integers **SB p. 69** **TG p. 92**	**8-B7 Order of Operations for Integers: solve problems** • apply the proper conventions for order of operations	1 h	None	Q2, 4
GAME: Target (Optional) **SB p. 71** **TG p. 94**	Practise applying properties and order of operations of integers in a game situation	30 min	• 42 integer game cards (student- or teacher-made), −10 to 10 (two of each)	N/A
UNIT 3 Revision **SB p. 72** **TG p. 95**	Review the concepts and skills in the unit	1.5 h	• Black and white counters	All questions
UNIT 3 Test **TG p. 97**	Assess the concepts and skills in the unit	1 h	• Black and white counters	All questions
UNIT 3 Performance Task **TG p. 100**	Assess concepts and skills in the unit	1.5 h	None	Rubric provided

Math Background

• This unit extends students' understanding of integer principles and calculations. It builds on the models for comparing, adding, and subtracting integers presented in Class VII.

• The focus of the unit is on using models to represent integers. This promotes a deep understanding of integers. Students use these models to they learn to multiply and divide integers. Patterns are used for concepts where the use of models does not foster understanding.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 4** in **lesson 3.1.2**, where they find many ways to express a negative integer as a product, in **lesson 3.1.3**, where they determine the missing numbers or rules using multiplication grids, in **question 8** in **lesson 3.2.2**, where they look for two integers to meet a set of conditions, and in **question 3** in **lesson 3.2.3**, where they find the least possible answer by applying rules for the order of operations.

• Students use communication frequently as they explain their thinking in **question 8** in **lesson 3.1.1**, where they explain their understanding of the sign of the product of two consecutive integers, in **question 10** in **lesson 3.1.2**, where they explain the sign of a product in relation to the signs of the numbers being multiplied, and in **question 8** in **lesson 3.2.2**, where they explain how to solve a problem involving multiple clues for dividing integers.

• Students use reasoning in **question 7** in **lesson 3.1.2**, where they make and test a conjecture, in **part C** in **lesson 3.1.3**, where they use reasoning to decide whether there is more than one possible solution**,** throughout **lesson 3.1.4**, when they decide how to rearrange products to make multiplication easier, in **question 6** in **lesson 3.2.2**, where they consider the possible results in a game, and in **question 2** in **lesson 3.2.3**, where they reason about the order in which to use cards to obtain as great a result as possible in a game.

• Students consider representation in **question 1** in **lesson 3.1.1**, where they use counters to represent integer multiplication, and in **question 2** in **lesson 3.1.2**, where they use number lines to model multiplication. Models of counters and number lines are used frequently throughout the unit. In **question 3** in **lesson 3.2.1**, students are asked to choose the model that best supports their way of thinking.

• Students use visualization skills in **question 9** in **lesson 3.1.2** where they multiply two negative integers by visualizing a number line, and in **question 2** in **lesson 3.2.1**, where they model integer quotients.

• Students make connections to real-world contexts in **question 3** in **lesson 3.1.2**, about spending money, and in **question 6** in **lesson 3.2.1**, about measuring temperature. They connect division to multiplication in **lesson 3.2.2**.

Rationale for Teaching Approach

• This unit is divided into two chapters.

Chapter 1 is about multiplying integers mentally and with counters, number lines, and patterns.

Chapter 2 is about dividing integers with models and patterning.

• In the **Explore** lesson students investigate patterns generated by rules. They use reasoning skills to determine the number of possible sets of rules.

• The **Connections** has students use mean temperatures in a practical application of using integers to determine the best date for sowing seeds.

• Two **Games** are included in the unit. In the first game students apply and practise integer multiplication. In the second game students use the order of operations rules to create integer expressions that are closest to a given integer.

• Throughout the unit, it is important to encourage students to use models to multiply and divide integers. When they use integers that are far from zero, students should try to visualize counters or number lines. It is important to accept a variety of approaches from students.

Getting Started

Curriculum Outcomes	Outcome relevance
7 Add and Subtract Integers: to solve problems	Students will find the work in the unit easier after they review the concepts and skills related to counter and number line models for integer addition and subtraction that were introduced in Class VII.

Pacing	Materials	Prerequisites
1 h	• Black and white counters	• representing integers with counter and number line models • using integers to describe real-world situations • interpreting recorded temperatures • comparing integers • using models to add and subtract integers

Main Points to be Raised

Use What You Know

• You can use a number line to compare numbers.

• Integers have many real-world applications, such as denoting temperature.

• One way to calculate $a - b$ is to figure out what to add to b to get to a ($a - b \rightarrow b + ? = a$). You can show this on a number line. The solution is a distance and direction on the number line.

Skills You Will Need

• You can use the zero property to add or subtract a positive integer and a negative integer.

• You can use counters or a number line to model integer addition and integer subtraction.

• Changing the order of an integer addition does not change the result.

• To add three integers, you can first combine any pair and then add the third integer.

• When you add or subtract integers, the result is always another integer.

Use What You Know — Introducing the Unit

• Before assigning the activity, you may wish to remind students what integers are and ask students to brainstorm different ways integers are used in daily contexts. This could be done as a whole class.

• You may also wish to review with students how 0°C is used as a benchmark to which other temperatures are compared. You can ask students how a number line is similar to, or different from, a thermometer.

Students can work in pairs to complete the activity. While you observe students at work, you might ask questions such as the following:

• *How did you find the greatest high temperature?* (Since all the high temperatures were positive integers, I knew that +18 was greater than the other temperatures. If I had used a number line, I would have seen the order from lowest to highest going from left to right.)

• *How did you figure out the difference between the low and high temperatures for any one place?* (I put the high and low temperatures for the place on a number line and then find the distance between them by counting.
For example, for Thimphu, there are 22 spaces between +15 and –7 so the difference is 22°C.)

• *What strategies did you use to find a high temperature 23° greater than a low temperature?* (I added the positive amounts to the opposite of the negative amounts and looked for a sum of 23.)

• *How did you know that the difference between high and low temperatures in Trongsa was less than the difference for Thimphu?* (The area from –1 to 13 is completely inside the part of the line that goes from –7 to 15.)

Skills You Will Need

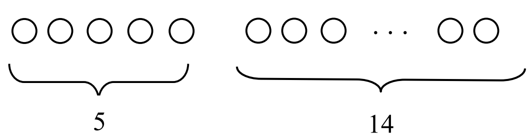
- Students can work individually.
- To ensure students have the required skills for this unit, assign all questions.
- Before students begin, you may wish to talk about how to use counters to model integer addition. Make sure students take the time to read the reminder about how to add integers using a counter model, shown at the top of **page 52** of the student text. You may also wish to ask students how to apply the zero principle when subtracting a positive integer from a negative integer.
- Encourage students to sketch the counter or number line model they use to answer these questions.

Answers

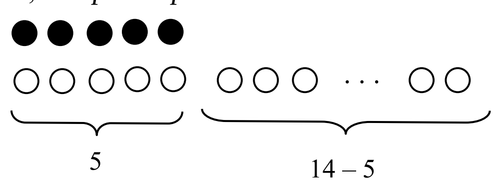
A. i) Punahka
ii) Thimphu
iii) Paro
iv) Punahka

B. *Sample response*:
- a low temperature that is 14ºC lower than its daily high temperature; Trongsa.
- a high temperature that is 8°C higher than the highest low temperature; Punakha.
- a low temperature that is 20°C lower than the high temperature in Trongsa; Thimphu.
- a high temperature that is 5°C higher than the highest low temperature; Thimphu.

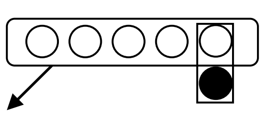
1. a) +19, *Sample response*:



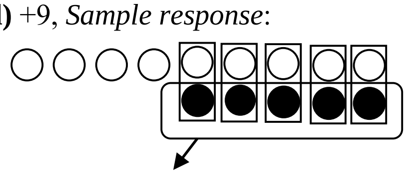
b) +9, *Sample response*:



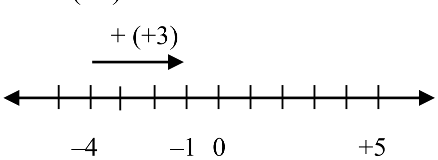
c) –1, *Sample response*:



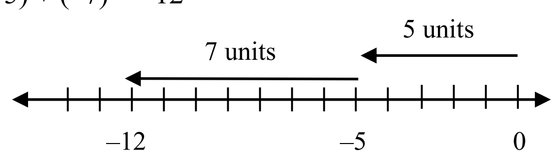
d) +9, *Sample response*:



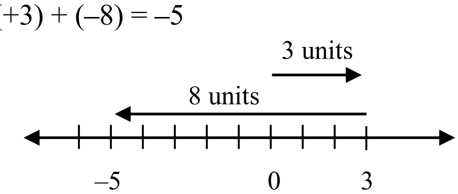
2. *Sample response*:
–4 + (+3) = –1



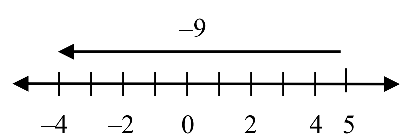
3. a) –12; *Sample response*:
(–5) + (–7) = –12



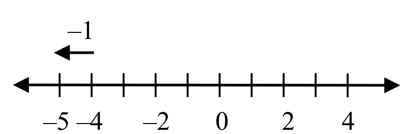
b) –9; *Sample response*:
(+3) + (–8) = –5



c) –9; *Sample response*:
(–4) – (+5) = –9



d) –1; *Sample response*:
(–5) – (–4) = –1



4. a) –14 **b)** 0 **c)** –35
d) +50 **e)** –70

Supporting Students

Struggling students

• For **question 1**, some students may need to manipulate some counters and then sketch a corresponding picture of the model. This will help them link the concrete manipulation to the pictorial image.

• For the subtraction parts of **question 3**, you may wish to remind students that one way to calculate $a - b$ is to figure out what to add to b to get to a ($a - b = ? \rightarrow b + ? = a$). You can show this easily on a number line as
the distance (and direction) to get from b to a.

• If students are struggling with subtracting more than two integers in **question 4**, you might have them break each question into two stages by sketching a number line and plotting the first pair of integers on it. After finding the difference between these two integers, they can then locate the third integer on the number line and find
the difference between this integer and the previous difference.

Enrichment

• For **question 4**, you might challenge students to show why subtracting an integer is like adding its opposite and to explain how this can help them figure out the answers mentally.

For example, you can show (+30) – (–11) by first modelling +30 with 30 white counters along with 11 more white counters and 11 black counters. When you remove the 11 black counters model –11, there are 41 white counters left. That is the same as starting with 30 white counters and adding 11 white counters (+30) + (+11),
so (+30) – (–11) = (+30) + (+11).

Chapter 1 Multiplying Integers

3.1.1 Multiplying Integers Using Counters and Patterns

Curriculum Outcomes	Outcome relevance
8-B4 Multiply Integers: solve problems • connect visual models, such as counters and number lines, to symbols • interpret multiplication as repeated addition and vice versa **8-B5 Properties of Operations for Integers: commutative and associative** • apply properties and understand their usefulness: commutative property (order, e.g., $(-5) \times 4 = 4 \times (-5)$) and associative property (grouping, e.g., $((-2) \times 4) \times (-3) = (-2) \times (4 \times (-3))$	• Students extend ideas from Class VII by using visual models to show multiplication as repeated addition of integers. • Experience with visualizing and drawing pictures of counter and number line models will help students connect these models to symbols. They will understand why procedures work and not just apply rules for addition without understanding.

Pacing	Materials	Prerequisites
1 h	• Black and white counters	• using counters to add integers

Main Points to be Raised

• Multiplying a negative integer by a positive integer can be interpreted as repeated addition.

• You can use counters to model integer multiplication when one factor is positive.

• Changing the order of the multiplication does not change the result because the commutative property of multiplication works with integers.

• To multiply three integers, you can multiply them in any order. This is based on combining the associative and commutative properties.

• When you multiply integers, the result is always another integer.

• To understand how to multiply two negative integers, you can use a pattern.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *What information does the first part give you?* (Drakpa has +50 from correctly answering some questions.)

• *How can you use counters to model the loss of marks for incorrect answers?* (5 groups of 2 black counters)

• *How can you write the loss of marks as an integer expression?* (–2 + (–2) + (–2) + (–2) + (–2) or –2 – 2 – 2 – 2 – 2)

• *How can you model the final mark using counters?* (I can start with 50 white counters and add 5 groups of 2 black counters to it. Using the zero property, I can pair 10 black and 10 white counters to make zero. I am left with 40 white counters.)

If students represent the loss incorrectly by repeatedly subtracting –2, encourage them to model the situation using counters. Emphasize that they need to take away 2 white counters 5 times or add 2 black counters 5 times.

The Exposition — Presenting the Main Ideas

• With students, read through the beginning parts of the exposition (about counter models) on **page 53** of the student text. Demonstrate how to use counters to show that multiplication is repeated addition. Invite students to use counters to show other examples of multiplication as repeated addition.

• Point out that most of the examples in the exposition involve multiplying a positive integer and a negative integer. Ask students for ideas about how to think about multiplying two negative integers. Ask students to make conjectures about the sign of the answer based on the sign of the factors. Read through the part of the exposition on **page 54** about using a pattern to multiply two negative integers.

Revisiting the Try This

B. Students connect multiplication to the concept of repeated addition. They might do this by sketching a counter model for the problem in the **Try This** and then representing the repeated addition as multiplication. Students may need help to remember how to use brackets to show the correct order of operations.

Using the Examples

• Present the problems in the example and have students try to solve them using counter models. They can then check their work and thinking against the solution and thinking in the text.

• Remind students that they are no longer required to use the + sign to indicate a positive integer. If an integer is written without a sign, they should assume it is positive. This is a mathematical convention.

Practising and Applying

Teaching points and tips

Q 1: Students may write $2 \times (-4)$ or $4 \times (-2)$ for **part b)**. Ask them how the model supports their conclusion. Make sure students understand how multiplication is shown as an array of 2 rows of -4 or an array of 4 columns of -2.

Q 2: Give students counters to use for modelling.

Q 3: Encourage students to sketch counters if they need to visualize the answer. Make sure students realize that they can sketch and that they need not draw to scale.

Q 4: Encourage students to use a counter model to explain their thinking.

Q 5: Students may need some prompting to set up patterns that will help them with this question. They should start the pattern with a related fact they know.
For example, to calculate $(-4) \times (-2)$, students can start a pattern with $(-4) \times 2 = (-8)$ and decrease the second factor by 1:

$(-4) \times 2 = (-8)$
$(-4) \times 1 = (-4)$
$(-4) \times 0 = (-8)$
$(-4) \times (-1) = 4$
$(-4) \times (-2) = 8$

Q 7: Encourage students to sketch a counter model to show each of the parts of the expression. They may need to be reminded that a negative is like subtracting its opposite. This was introduced in Class VII and can be demonstrated using a number line model.

Q 8: Students may need to be reminded of what *consecutive* means, i.e., two numbers in a row.

Q 9: You might have students use counters to explain how this can be modelled.

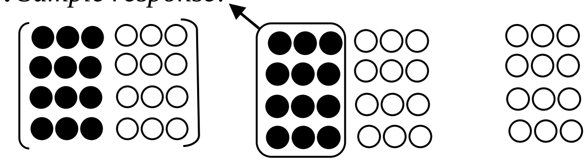
Common errors

• Some students may assume that multiplying two negative integers like those in **question 5** gives a negative product. This might seem reasonable to them because of their work with adding two negative integers. Remind them to use patterns to determine whether they are looking for a positive product or a negative product.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can use counters to model multiplication of a positive and negative integer
Question 5	to see if students can multiply two negative integers
Question 6	to see if students can represent a situation using the product of integers
Question 9	to see if students can describe the value of the commutative property

Answers

A. i) *Sample response*: (+50) + (–2) + (–2) + (–2) + (–2) + (–2) **ii)** 40	**B.** *Sample response*: 50 + [5 × (–2)] = 40
1. a) 6 × (–4) **b)** 2 × (–4) or 4 × (–2) **2. a)** –6; *Sample response*: (–2) × 3 = 3 × (–2) (●●) + (●●) + (●●) **b)** –25; *Sample response*: 5 × (–5) (●●●●●) + (●●●●●) + (●●●●●) + (●●●●●) + (●●●●●) **3. a)** –4 **b)** –6 **c)** –6 **4.** No; [*Sample response*: A negative integer multiplied by a positive integer is the same = as a positive integer multiplied by a negative integer, e.g., –4 × 5 = 5 × (–4). If you use counters to model multiplying a positive integer by a negative integer, e.g., 5 × (–4), there are only groups of black counters, e.g., 5 groups of 4 black counters, so the product is always negative.]	**5. a)** 8 **b)** 0 **c)** 12 **d)** 20 **6. a)** 3 × (–5) **b)** –15°C **7.** *Sample response*:  [I knew I had to subtract 4 groups of 3 black counters from zero. I showed zero as 4 groups of 3 white counters along with 4 groups of 3 black counters. When I took away the 4 groups of 3 black counters, there were 4 groups of 3 white counters left, and that is +12.] **8.** No; [*Sample response*: Any two consecutive integers are either both positive or both negative. They could also be –1 and 0, or 0 and 1. In any case, the product is either positive (negative × negative = positive and positive × positive = positive) or 0 (–1 × 0 = 0 and 0 × 1 = 0).] [**9.** *Sample response*: I cannot model –4 × 5 as –4 groups of 5. I can use the commutative property to change the order so I can instead model 5 × (–4) as 5 groups of –4.]

Supporting Students

Struggling students

• If students are struggling with visualizing a counter model in **question 3**, you might have them sketch a model so that they can figure out the number of counters.

For example, for **part b)**, 3 × (–2), students can sketch a counter model of 3 groups of 2 black counters. Then they can count the number of black counters to get –6 as the product.

• You may choose to not assign **question 7** to struggling students; it requires a level of abstraction.

Enrichment

• For **question 5**, you might challenge students to generate patterns for multiplying three factors.

For example, for **part a)** (–4) × (–2), you could include a third factor such as –5 in each expression. Students could generate a pattern to calculate (–4) × (–2) × (–5):

(–4) × 2 × (–5) = 40

(–4) × 1 × (–5) = 20

(–4) × 0 × (–5) = 0

(–4) × (–1) × (–5) = –20

(–4) × (–2) × (–5) = –40

Reprint 2024

3.1.2 Multiplying Integers Using a Number Line

Curriculum Outcomes	Outcome relevance
8-B4 Multiply Integers: solve problems • connect visual models, such as counters and number lines, to symbols • interpret multiplication as repeated addition and vice versa **8-B5 Properties of Operations for Integers: commutative and associative** • apply properties and understand their usefulness: commutative property and associative property • recognize the property of closure (e.g., 2 − 5 is not defined within the set of whole numbers, therefore needing the introduction to integers)	A number line provides students with a way to visualize the multiplication of integers. This helps to develop their sense of numbers.

Pacing	Materials	Prerequisites
1 h	None	• using a number line to add integers

Main Points to be Raised

• You can use a number line to model the product of a positive integer and a negative integer. The positive integer tells how many jumps to take and the negative integer tells the size of each jump.

• With the idea that a negative integer is like subtracting its opposite from zero, you can use a number line to model the product of two negative integers. This model is more abstract and difficult for some students than the model of the product of a positive integer and a negative integer.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know that Sonam will have less money left after 3 days than after 1 day?* (She is spending money each day so the amount of money left is decreasing.)

• *How can you find the day when Sonam had exactly Nu 0 left?* (I know that it was sometime during the 13th day because after the 12th day she still had Nu 20 and after the 13th day she owed Nu 20.)

• *How else can you describe the amount she had left after the 13th day?* (She owed Nu 20 or she had negative Nu 20.)

The Exposition — Presenting the Main Ideas

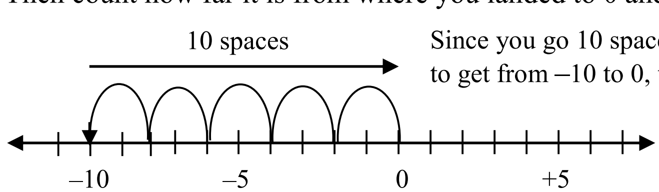
• With students, read through the exposition on **page 56** of the student text.

• Ask students how they could use a number line to model the multiplication of two negative integers. You may need to remind them that when they multiply two negative integers, they can think of one of the integers as the difference between zero and its opposite.

For example, to multiply (–5) × (–2):

(–5) = 0 – (+5) = **0 – 5**, so **(–5)** × (–2) = **0 – 5** × (–2) = 0 – [5 × (–2)]

To solve 0 – [5 × (–2)] on a number line, find 5 × (–2) by making 5 jumps of –2.
Then count how far it is from where you landed to 0 and in what direction.



(–5) × (–2) = 10

Since you go 10 spaces to the right (in a positive direction) to get from –10 to 0, the answer is +10.

This model is addressed in **question 9**. You may wish to wait until students try it in that question before showing them this model.

Revisiting the Try This

B. Make sure students connect the number line model to the expression by writing 13 jumps of 40 to the left as
13 × (–40). Remind students that they need to show this amount in relation to the amount Sonam started with (Nu 500).

Using the Examples

• Present **parts a) and b)** of the example to the whole group. Let each student answer on his or her own and then compare his or her solution to the solution and thinking in the text.

Practising and Applying

Teaching points and tips

Q 2: For **part a)**, remind students that they can use the commutative property to show 2 jumps of –4.

Q 3: Make sure students understand that they should use the answer from **part a)** to answer **part c)**.

Q 4: Ensure students understand that a multiplication such as (–1) × (–2) × (–6) = –12 is the same as
(–2) × (–1) × (–6) = –12 so they are not different multiplications and they count as only one way.

Q 5: Encourage students to look for patterns in the work they did in the example.

Q 6: Encourage students to sketch a vertical number line if they have difficulty visualizing the changing temperatures.

Q 7: You may wish to have students try a few examples to help them recognize a pattern.

Q 8: You may wish to review the distributive property so students understand why [5 + (–5)] × (–2) = [5 × (–2)] + [(–5) × (–2)].

Q 9: Encourage students to sketch a number line if they have difficulty visualizing –6 as 0 – 6 (the distance from +6 to 0 on a number line).

Common errors

• Some students may have difficulty with **question 3 part a)** because they will think that spending money involves multiplying by a negative number.

For example, students might write 55 + (–10 × 13).

Encourage students to sketch a number line so that they can see that they need to add 13 jumps of 10 to the right to find the amount of money Karma started with.

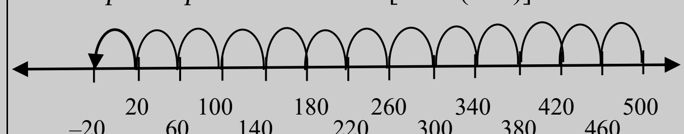
Suggested assessment questions from Practising and Applying

Question 2	to see if students can model integer multiplication on a number line
Question 6	to see if students can solve a simple problem involving multiplication by a negative integer
Question 7	to see if student can make and test a conjecture about multiplying by –1

Answers

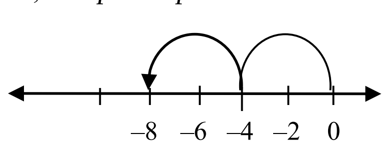
A. i) Nu 460
ii) Nu 420
iii) Nu 380
iv) Nu 20
v) *Sample response*: She owed Nu 20.

B. *Sample response*: Nu 500 + [13 × (–40)] = –20

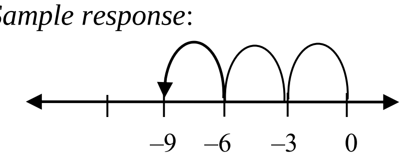


1. 5 × (–2) = –10

2. a) –8; *Sample response*:



b) –9; *Sample response*:



Answers [Continued]

3. a) *Sample response*: 55 + (10 × 13)
b) Nu 185
c) –75, which means he will owe Nu 75;
[*Sample response*:
He starts with Nu 185, or +185. He spends 13 × 20 = Nu 260, which is –260. –260 + 185 = –75.]

4. 14 ways:
[(–1) × (–2) × (–6) = –12
(–1) × 2 × 6 = –12
1 × (–2) × 6 = –12
1 × 2 × (–6) = –12
(–1) × (–3) × (–4) = –12
(–1) × 3 × 4 = –12
1 × (–3) × 4 = –12
1 × 3 × (–4) = –12
(–2) × (–2) × (–3) = –12
2 × (–2) × 3 = –12
2 × 2 × (–3) = –12
(–1) × (–1) × (–12) = –12
(–1) × 1 × 12 = –12
1 × 1 × (–12) = –12]

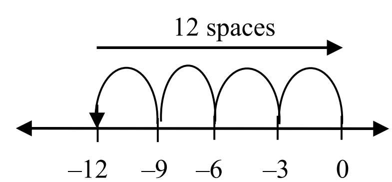
5. a) Positive; [*Sample response*:
Negative × negative = positive.]
b) Negative; [*Sample response*:
Negative × negative = positive,
then positive × negative = negative.]
c) Positive; [*Sample response*:
Negative × a negative = positive,
then positive × positive = positive.]
d) Positive; [*Sample response*:
The first pair is negative × negative = positive,
the last pair is negative × negative = positive,
then positive × positive = positive.]

6. –12°C

7. The product is the opposite of the original integer;
Sample response: (–3) × (–1) = +3

[**8.** *Sample response*:
5 + (–5) = 0, so
[5 + (–5)] × (–2) = 0 × (–2) = 0.
But [5 + (–5)] × (–2) = [5 × (–2)] + [(–5) × (–2)].
So 0 = –10 + [(–5) × (–2)].
That means (–5) × (–2) = +10.]

[**9.** *Sample response*:
–6 × (–2) = +12
If –6 = 0 – 2, then –6 × (–2) = 0 – 6 × (–2)
0 – 6 × (–2) = 0 – [6 × (–2)]
= 0 – (–12)
0 – (–12) is the distance from –12 to 0 on a number line



12 spaces to the right is +12, so 0 – (–12) = +12
That means –6 × (–2) = +12.

10. Yes; [*Sample response*:
5 × 3 = 15, (–5) × (–3) = 15, 5 × (–3) = –15]

11. a) Positive
b) Negative

Supporting Students

Struggling students

• For **question 5**, you might provide students with numbers close to zero so that they can make a prediction based on visualizing these numbers.

For example, instead of presenting (–1245) × (–2678), you could ask students to predict the sign of the product of (–12) × (–2).

• You might not assign **questions 8 and 9**, which are more abstract.

3.1.3 EXPLORE: Pattern Grids

Curriculum Outcomes	Lesson Relevance
8-B4 Multiply Integers: solve problems • connect visual models, such as counters and number lines, to symbols • interpret multiplication as repeated addition and vice versa **8-B5 Properties of Operations for Integers: commutative and associative** • apply properties and understand their usefulness: commutative property and associative property	This optional exploration provides a problem-solving opportunity for students to use what they have learned about multiplication of integers. Students apply the commutative property when they decide that it does not matter whether they go right first or up first. Students are also encouraged to recognize and apply patterns when multiplying integers.

Pacing	Materials	Prerequisites
40 min	None	• familiarity with multiplication facts • multiplying integers

Main Points to be Raised

• If you know the result of multiplying by a given number one or more times, you can figure out the number that was multiplied.

For example, if you know that a number was multiplied by –2 twice and the result was –16, you know the original number was –4.

• When a number is multiplied by two different integers, the order of multiplying does not matter.

• When you multiply repeatedly by a negative value, the signs of the resulting products alternate.

Exploration

• Read through the introduction (in white) with students to help them understand how the grids work. They should begin in the bottom left box, multiply this integer following the rules, and multiply the resulting numbers to complete the pattern.

For example, to fill in the box above –12, multiply –12 × 3. To fill in the box to the right of 8, multiply 8 × –2.

Have students work in pairs for **parts A and B**. While you observe students at work, you might ask questions such as the following:

• *For **part B**, can you complete the grid by moving to the right and multiplying by –2 and moving up and multiplying by 3?* (No, I can only complete part of the first two rows that way.)

• *Does it make any difference whether you move right before moving up?* (No, I can multiply in any order.)

• *What patterns do you notice when you move diagonally up and to the right?* (If I multiply both by –2 and by 3, it is the same as multiplying by –6.)

• *How does your answer for **part A** help you with **part B**?* (I can use the pattern of integer signs to know that I am looking for a negative number.)

Have students continue to work in pairs for **part C**.

Observe and Assess

As students work, notice the following:

• Do they multiply correctly?

• Do they apply sign patterns when they multiply?

• Do they observe diagonal patterns as well as vertical and horizontal patterns?

• Do they work backwards when required?

Share and Reflect

After students have had sufficient time to work through the exploration, they could form small groups to discuss their observations and answer these questions.

- *How did you decide how to start each time?*
- *Was there more than one order that worked to complete the grids?*
- *How do you know that you found all the ways to complete the grid in* **part C***?*
- *How can you know which integers are necessary* to complete the grid in **part C***?*

Answers

A. i)

–36	72	–144	288
–12	24	–48	96
–4	8	–16	32

ii) *Sample response*:
- All integers in a column have the same sign.
- The integers in each row have alternating signs.
- The integers on the diagonal going up and to the right are –6 times as much each time.

iii) *Sample response*:
- I multiply each column by a positive number, so the signs do not change.
- A negative × a positive = a negative. Since I multiply by a negative across each row, the signs alternate.
- If I go up and to the right, then I am multiplying both by 3 and by –2. That is the same as multiplying by –6.

B.

–9	18	–36	72
–3	6	–12	24
–1	2	–4	8

C. i)
- Multiply by –4 when you move to the right.
- Multiply by –2 when you move up.

ii)

4	–16	64	–256
2	–8	32	–128
1	–4	16	–64

iii) No; *Sample response*:
I know that I have to multiply by a negative number to move across each row so that the right hand box can be –64. Since I multiply 1 by the same number three times to get –64, the number I multiply by has to be –4. That means the number to the right of 1 is –4.
To get to –8 above –4, I have to multiply by 2, so I must multiply by 2 each time I move up.

iv) Only one of –8 or 64 is necessary, not both. Once I knew that I had to multiply by –4 to move to the right, I could use either the –8 or the 64 to figure out what to multiply by when I move up.

Supporting Students

Struggling students

- If students are struggling to find missing numbers in **parts B and C**, you might provide them with one or two more integers. If you give students integers in the far left columns and bottom rows, they will be able to calculate the missing integers more easily than by working backwards from the given integers.

Enrichment

- For **part C**, you might provide a grid with integers far from zero. You can easily construct this by multiplying each integer in the existing grid by a constant.

For example, they could solve a grid like the grid shown to the right.

		1536	
	192		
24			–1536

- You might challenge students to create their own grid and rules. They could exchange their grids with classmates to solve.

3.1.4 Renaming Factors to Multiply Mentally

Curriculum Outcomes	Outcome relevance
8-B6 Multiply Integers: mentally • develop and use mental strategies such as the following: - front-end - compatible numbers/factors - working by parts - double and halves **8-B4 Multiply Integers: solve problems** • connect visual models, such as counters and number lines, to symbols **8-B5 Properties of Operations for integers: distributive** • apply the distributive property (e.g., $(-2)(3 + (-2)) = (-2)(3) + (-2)(3)$) and understand its usefulness	• Work with mental strategies will help support students' number sense. • It is important that students use the distributive property to help them understand why procedures work so they do not just apply rules without understanding.

Pacing	Materials	Prerequisites
1 h	• Black and white counters (optional)	• multiplying integers using counters and number lines

Main Points to be Raised

You can use a variety of mental math strategies to multiply integers:

• Rearrange factors so that they are easier to multiply.

• Break up factors to create compatible factors.

• Rearrange factors in order to double one factor and take half of the other factor.

• Work by parts using the distributive property.

Try This — Introducing the Lesson

A. Allow students to work in pairs. While you observe students at work, you might ask questions such as the following:

• *How can you solve* 35 × (–42) *mentally?* (I know that 35 is 30 + 5, so I can multiply −42 by 30, then multiply
–42 by 5, and finally add the results.)

• *How can you solve* 70 × (–21) *mentally?* (70 = 7 × 10, so I can multiply −21 first by 7 and then by 10.)

• *Why do these strategies work?* (For the first strategy, I know that 35 groups of something is 30 groups and then
5 more groups. For the second strategy, I use the idea that 70 groups of something is 10 sets of 7 groups of it.)

The Exposition — Presenting the Main Ideas

• Read through the exposition with students.

• When you get to the counter model for working by parts on **page 60** of the student text, you may wish to use counters to demonstrate. Make sure students understand how the distributive property helps them multiply mentally — it allows you to multiply by a multiple of 10 and by a single digit number, both of which are easier to do than to multiply by a two-digit number that is not a multiple of 10.

Revisiting the Try This

B. Students should conclude that they can choose different strategies for mental multiplication based on what makes the calculation easier for them. Not all students may choose to use the same strategy.

Using the Examples

• Present the questions in the example to students. Have students work in pairs to solve them. Then ask students to compare their work to the solution in the student text.

Practising and Applying

Teaching points and tips

Q 1: Some students may not recognize which number to double or halve. Encourage them to try different combinations and think about which one is easier.

Q 2: Remind students that they can ignore the signs, work with the numbers, and then write each answer with the appropriate sign.

Q 3: Students may use diagrams to help them work by parts. Encourage them to think about how to rename one or more of the numbers to simplify the calculations.

For example:

Part e), they could rename –11 as (–10) + (–1).

Part d), they could rename –27 as (–25) + (–2).

Q 4: Encourage students to think about the most helpful strategy for each calculation. Students may use a variety of strategies; be sure to accept all reasonable explanations.

Common errors

• In **question 2**, some students might not realize that they can rearrange the order of the factors when they multiply. You can explore with them whether it makes any difference if the numbers are multiplied in the order given or in a different order.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can apply the doubling and halving strategy
Question 4	to see if students can use and explain a variety of mental math strategies

Answers

A. *Sample response*: 70 × (–21); I can multiply by 10 by putting a 0 on the end of the number, so I really only have to do one multiplication instead of two multiplications.	**B.** 35 × (–42) = [35 × 2] × [(–42) × $\frac{1}{2}$] = 70 × (–21) = –210; It has the same result as the other calculation.
1. a) –450; [9 × (–50) = –450] **b)** 960; [(–30) × (–32) = 960] **c)** 140; [(–14) × (–10) = 140] **d)** –140; [(–2) × 70 = –140] **e)** –480; [(–10) × 48 = –480] **2. a)** 300; [25 × (–4) × (–3) = (–100) × (–3) = 300] **b)** 240; [*Sample response*: (–5) × (–8) × 6 = 40 × 6 = 240] **c)** 60; [(–2) × (–5) × 6 = 10 × 6 = 60] **d)** –80; [20 × (–4) = –80] **e)** 60; [*Sample response*: (–6) × (–10) = 60] **3. a)** –115; [[5 × (–20)] + [5 × (–3)] = –115] **b)** 192; [[(–20) × (–8)] + [(–4) × (–8)] = 192] **c)** –372; [*Sample response*: [(–12) × 30] + [(–12) × 1] = –372] **d)** 108; [*Sample response*: [(–4) × (–25)] + [(–4) × (–2)] = 108] **e)** –572; [*Sample response*: [(–10) × 52] + [(–1) × 52] = –572]	**4.** *Any two sample responses*: **a)** –33; [*Sample response*: Work by parts: 10 × (–3) and 1 × (–3)] **b)** 80; [*Sample response*: Double –5 and halve –16.] **c)** 300; [*Sample response*: Use compatible factors: first multiply 6 × (–5) and then multiply the product (–30) by 10.] **d)** 300; [*Sample response*: Use compatible factors: first multiply –2 by –50 and then multiply 100 by 3.] **e)** –3100; [*Sample response*: Use compatible factors: first multiply –25 by –4 first and then multiply 100 by –31.]

Supporting Students

Struggling students

• In **question 3**, students may struggle with choosing which part to work with, especially with 2-digit by 2-digit multiplication. They may not understand how the distributive property works. They might benefit from the use of pictures of counters.

For example, for **part c)**, (–12) × 31, they might use pictures of counters to show that 31 groups of 12 black counters is the same as 31 groups of 10 black counters and 31 groups of 2 black counters. Then they can rearrange the factors to show that 12 groups of 31 black counters is the same as 12 groups of 30 black counters and 12 groups of 1 counter.

By using the pictures, they can see that it is easier to model the first expression than to model the second expression, and so the first expression might be easier to calculate mentally.

Enrichment

Students might create computations that involve multiplying integers that they think might look very difficult initially, but turn out to be easy if you rearrange factors.

For example, 25 × (–68) × (–50) × 4 × (–2).

GAME: Order the Integers

• This optional game is designed to provide practice with multiplication of integers.

• In rounds 1 and 2, students must make a quick decision about which integer to select so that they get the greatest product possible. They apply mental math strategies and what they know about the sign of the product when multiplying three factors.

• Encourage students to visualize or to use counter or number line models when multiplying.

• Students can adapt the game by using two-digit cards to create greater products.

3.2.1 Dividing Integers Using Models and Patterns

Curriculum Outcomes	Outcome relevance
8-B4 Divide Integers: solve problems • connect visual models, such as counters and number lines, to symbols	Experience with visualizing and drawing counter and number line models will help students connect these models to symbols. Students will understand why procedures work and not just apply rules for addition without understanding.

Pacing	Materials	Prerequisites
1 h	• Black and white counters	• multiplying integers using counters and number line models.

Main Points to be Raised

• You can use counters to model dividing a negative integer by a negative integer and dividing a negative integer by a positive integer. To divide a negative by a negative, think of how many groups of the divisor are in the dividend. To divide a negative by a positive, think of how big each group would be if the dividend were divided into the number of groups that the divisor represents.

• To divide a positive integer by a negative integer, you can use a pattern.

Try This — Introducing the Lesson

A. Allow students to try this alone. Provide students with black and white counters. While you observe students at work, you might ask questions such as the following:

• *What operation are you performing when you arrange the counters into groups of 2? What equation can you write to represent what you are doing?* (Division; 30 ÷ 2 = 15.)

• *How does the counter model show multiplication?* (There are 15 groups of 2 black counters, so 15 × 2 = 30.)

• *What operation are you performing when you arrange the counters into 6 groups? What equation can you write to represent what you are doing?* (Division; 30 ÷ 6 = 5.)

• *How does the counter model show multiplication?* (There are 6 groups of 5 black counters, so 6 × 5 = 30.)

The Exposition — Presenting the Main Ideas

• Give a mini-lesson on two possible meanings of division: grouping and sharing. Use whole numbers to remind students of these meanings.

For example, ask students to think of 8 ÷ 4 = ? as dividing 8 items into groups of 4 to find the number of groups. Then ask students to think of 8 ÷ 4 = ? as 8 items shared by 4 people to find the number of items in each share.

You may wish to model each of these meanings using counters and have students note the actions you are taking when you move the counters.

• Divide students into small groups. Have students read through the first part of the exposition on **page 62** of the student text. For each of the questions in the exposition, have students model the grouping or sharing. Ask them to pay attention to the actions they are taking and to talk about whether these actions involve grouping or sharing.

• Remind students that when they use a number line model, –6 is the same as 0 – 6, or the distance from 0 to –6.

• Have students continue to read through the exposition on **page 63** in their groups. Ask them to demonstrate how to use counters and a number line to show division of integers with different signs.

• Point out to students that we use a pattern for dividing a positive integer by a negative integer because it does not make sense to model this using counters or a number line. For example, for 8 ÷ (–4) we cannot divide 8 positive counters into groups of negative 4 or into negative 4 groups. Ask students to show how they might think of 8 ÷ (–4) = ? by starting with what they know and building a pattern as shown to the right. This is shown in **example 3** on **page 64** of the student text.

$$\begin{aligned}(-8) \div (-4) &= \blacksquare\\ (-4) \div (-4) &= \blacksquare\\ 0 \div (-4) &= \blacksquare\\ 4 \div (-4) &= \blacksquare\\ 8 \div (-4) &= \blacksquare\end{aligned}$$

Revisiting the Try This

B. Students connect the models they used in **part A** to symbolic expressions of integer division.

Using the Examples

• Have students work in pairs. One student should become an expert on **example 1** and the other should become an expert on **example 2**. They should each model and explain their example to the other student. Have students work through **example 3** together.

• Make sure students understand that **example 1** uses the grouping meaning of division in **part a)** and the sharing meaning in **part b)**. **Example 2 part a)** uses the sharing meaning and **example 2 part b)** uses the grouping meaning, but in the context of jumps instead of groups.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to identify whether they are writing an expression for division using sharing or grouping. Either is possible for both situations.

For example, **part a)** could be interpreted as 12 negative counters divided into 4 groups or as 12 negative counters divided into group of negative 3.

Q 2: Give students counters to use to model the question.

Q 3: Make sure students realize that they can sketch and that they need not draw to scale.

Q 4: Encourage students to visualize or to use a counter or number line model when dividing.

Q 5: You may wish to review the terms as a class before assigning the exercises. You may choose to assign this question only to select students.

Q 6: Encourage students to sketch a vertical number line model to help them understand the question.

Q 7: Students may need some prompting to set up patterns to help them with this question. They should start the pattern with a related calculation they know.

For example, they could start with (–21) ÷ (–7) = 3.

Q 8: You might have students use counters or number lines to explain what they know. This question is linked to work in the next lesson.

Common errors

• For **question 1 a)**, students may think that because all the counters are black, both the dividend and divisor are negative, i.e., 12 negative counters divided into groups of 4 negative counters (–12) ÷ (–4).

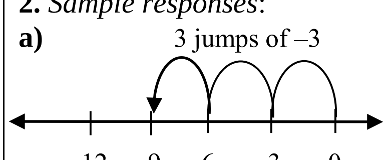
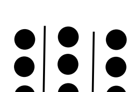
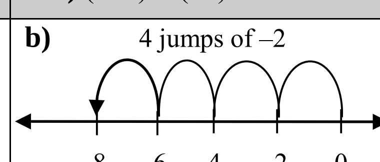
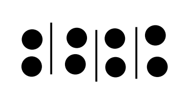
• Students have trouble distinguishing between the sharing and grouping models of division. Use simple examples and real objects to help them.

For example, in **question 1 a)**, 12 items can be shared equally among 3 students so there are 4 for each student (12 ÷ 3 = 4). Or, if 12 items are divided into groups of 3, there are enough for 4 students (12 ÷ 3 = 4).

Suggested assessment questions from Practising and Applying

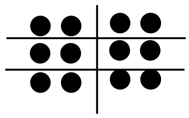
Question 1	to see if students can connect models to symbols in division situations
Question 2	to see if students can connect symbols to models in division situations
Question 4	to see if students can divide integers
Question 6	to see if students can use reasoning to solve a real-world problem involving division of integers

Answers

A. i) 15 **ii)** 6	**B. i)** (–30) ÷ (–2) = 15 **ii)** (–30) ÷ 5 = –6
1. *Sample responses*: **a)** (–12) ÷ (–3) = 4 **b)** (–6) ÷ (–2) = 3 **2.** *Sample responses*: **a)**   3 groups of 3 black counters	**b)**   4 groups of 2 black counters

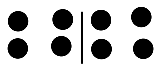
Answers [Continued]

3. a) –2; *Sample response*:



6 groups of 2 black counters

b) 2; *Sample response*:



2 groups of 4 black counters

4. a) –6 **b)** 8 **c)** –2 **d)** 2 **e)** –4 **f)** –8

[**5. a) i)** The quotient decreases;
Sample response: 12 ÷ 4 = 3, 12 ÷ 6 = 2
ii) The quotient could decrease or increase;
Sample response: 12 ÷ (–4) = –3, 12 ÷ (–2) = –6
12 ÷ (–4) = –3, 12 ÷ 4 = 3
b) i) The quotient increases;
Sample response: –12 ÷ 4 = –3, –12 ÷ 6 = –2
ii) The quotient could decrease or increase;
Sample response: –12 ÷ (–4) = 3, –12 ÷ 4 = –3
–12 ÷ (–4) = 3, –12 ÷ (–2) = 6]

6. 9 h

7. *Sample response*:
(–21) ÷ (–7) = 3
(–14) ÷ (–7) = 2
(–7) ÷ (–7) = 1
0 ÷ (–7) = 0
7 ÷ (–7) = –1
14 ÷ (–7) = –2
21 ÷ (–7) = –3

8. When you divide a negative by a negative, or a positive by a positive, the quotient is positive. When the signs are different, the quotient is negative.

Supporting Students

Struggling students

• Students and teachers often use the terms “large” or “small” to describe integers without their signs.

For example, they might describe –45 as large and –2 as small. There is a hidden danger in this. The language may be natural but it is not mathematically correct. One problem is that –45 is actually “small”. Another problem is that we often think of large and small in terms of the physical size of the numerals on a page.

Make sure students talk about numbers that are nearer to or farther from zero.

• If students are struggling with visualizing negative numbers far from zero in **question 4**, you might have them work with simpler numbers. Once students become better at mentally dividing integers close to zero, have them visualize those farther from zero using a pile of counters or a sketched number line.

For example, for **part b)**, you might ask students to calculate (–16) ÷ (–8) by visualizing 16 black counters divided into groups of 8 black counters. Then ask them to consider (–32) ÷ (–8), (–40) ÷ (–8), and so on.

• You may choose not to assign **question 5** to struggling students because of its abstract nature.

Enrichment

• For **question 5**, you might challenge students to extend their understanding of integer division by asking:

Suppose you divided two integers and then increased both integers and divided again. Is each possible?

a) for the quotient to increase

b) for the quotient to decrease

c) for the quotient to be 0

Use examples to support each answer above.
Make sure you consider:

• positive ÷ positive
• negative ÷ negative
• negative ÷ positive
• positive ÷ negative

Answer for **part a)**:
Yes; *Sample response*:
For two negative integers:
(–12) ÷ (–4) = 3 → (–10) ÷ (–2) = 5 and 5 > 3.
For two positive integers:
12 ÷ 4 = 3 → 20 ÷ 5 = 4 and 4 > 3
For a negative ÷ a positive:
(–12) ÷ 4 = –3 → 12 ÷ 6 = 2 and 2 > –3
For a positive ÷ a negative:
12 ÷ (–4) = –3 → 14 ÷ 2 = 7 and 7 > –3

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3.2.2 Relating Division of Integers to Multiplication

Curriculum Outcomes	Outcome relevance
8-B4 Divide Integers: solve problems • connect visual models, such as counters and number lines, to symbols • relate multiplication and division **8-B6 Divide Integers: mentally** • develop and use mental strategies	• Students extend previous ideas of multiplication of integers by using visual models to relate multiplication and division. • Work with mental strategies supports students' number sense. • In everyday life, mental math and estimation are very important skills, often more important than paper and pencil calculation.

Pacing	Materials	Prerequisites
1 h	• Black and white counters (optional)	• multiplying and dividing integers using counter and number line models

Main Points to be Raised

• You can write an integer division as a multiplication with a missing factor.

• You can divide integers by ignoring the signs, dividing them like whole numbers, and then applying rules for the sign of the quotient.

Try This — Introducing the Lesson

A. Allow students to try this with a partner. While you observe students at work, you might ask questions such as the following:

• *What did you notice about the signs of the quotients when you divided?* (Integers with like signs have a positive quotient, integers with different signs have a negative quotient.)

• *How might you figure out the quotients without using a counter or number line model?* (I could ignore the signs and divide the numbers like I would divide whole numbers. Then I could apply what I know about the sign of the quotient from the previous lesson.)

• *How do you know which integers will have the greatest quotient?* (I know that I do not want a negative answer, so I want to divide integers with like signs. I also know that if the signs are the same, I can divide the integer farthest from zero by the integer closest to zero to get the greatest quotient.)

• *What multiplication is related to* (–15) ÷ (–3)*?* (__ × –3 = –15 or "What integer multiplied by –3 makes –15?")

The Exposition — Presenting the Main Ideas

• Ask students to share what they have learned about the sign of the product in an integer multiplication. Write their ideas on the board.

• Present problems that involve dividing negative integers close to zero. Ask students to solve these and explain how they solved each. Ask students to look for a general way to divide negative numbers.
For example, you might give students the following calculations:
(–4) ÷ (–2); (–15) ÷ (–3); (–21) ÷ (–7)

Students might notice that you can ignore the signs, divide the integers, and then make each quotient positive. Ask them to justify this with counter or number line models.

• Present problems that involve dividing negative and positive integers close to zero. Ask students to solve these and explain how they solved each. Ask students to look for a general way to divide numbers with different signs.

For example, you might give students the following calculations: (–4) ÷ 2; (–15) ÷ 3; (–21) ÷ 7

Students might notice that you can ignore the signs, divide the integers, and then make each quotient negative. Ask them to justify this with counter or number line models. This will help students understand why procedures work so they do not just apply rules for division without understanding.

• Read through the exposition on **page 66** of the student text with students.

Revisiting the Try This

B. Students need to think about whether to divide integers with like signs or with different signs to make the greatest quotient.

Using the Examples

• Have students work in pairs. Present the three questions in **example 1** to students. Ask each pair to choose two questions to solve. Then the pair can compare their work to what is shown in the matching example. Suggest that they may then wish to read through the other solution and **example 2**.

Practising and Applying

Teaching points and tips

Q 1: Encourage students to visualize counter or number line models. Refer students to **example 1** to see how to do this.

Q 2: A flexible understanding of the relationship between multiplication and division will help students develop number sense.

Q 4: Remind students to visualize a number line if they are having difficulty doing this mentally.

Q 5: You might encourage students to talk in small groups about how they are making sense of division. Encourage them to communicate their thinking using models like those in **example 1**.

Q 6: Some students may not recognize that the least integer is the negative integer that is farthest from zero.

For example, some students may think that –3 is less than –60.

Q 7: Remind students that a divisor is the number you are dividing by (the second number in a division).

Q 8: This question might be assigned only to selected students.

Q 9: You might have students share their understandings in a group discussion.

Common errors

• Many students will attempt to memorize a set of rules generated from statements such as those given in the exposition. Encourage students to explain their thinking in **questions 5 and 8**. This could be done in ways other than writing.

For example, students could justify their thinking by showing the class a counter model example.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can relate multiplication and division
Question 4	to see if students can divide integers to solve a real-world problem
Question 5	to see if students can communicate about choosing the appropriate sign for an integer division

Answers

A. –120 and –3; *Sample response*: • She needs a positive quotient and there is only one positive integer so the dividend and the divisor must both be negative. • She needs the dividend farthest from zero and the divisor closest to zero to get the greatest quotient.	**B.** There is only one positive number so I knew that she had to pick two negative integers to get a positive quotient.
1. a) –2 **b)** –17 **c)** –5 **d)** 25 **2. a)** 4 × ■ = –64 **b)** (–7) × ■ = –84 **3. a)** –16 **b)** 12 **4.** 16 h	[**5.** *Sample response*: 24 ÷ 6 = 4, and a negative divided by a positive is a negative, so (–24) ÷ 6 = –4.] **6. a)** (–60) ÷ (–3) = 20 **b)** (–60) ÷ 4 = –15 **7.** *Sample responses*: **a)** –2, –3 **b)** 2, 6 **c)** –2, –9 **d)** 2, 5

8. a) The first integer is 1 and the second integer is –1. [**b)** *Sample response*: For the first clue, I knew that if the quotient was –1, the integers could be any negative integer and its opposite, such as –3 and 3, –2 and 2, or –1 and 1. For the second clue, I tried each number pair, using each number as the first number. Only 1 and –1 worked: $-3 \div (3 + 2) = -3 \div 5 \neq 1$ $3 \div (-3 + 2) = 3 \div (-1) \neq 1$ $-2 \div (2 + 2) = -2 \div 4 \neq 1$ $2 \div (-2 + 2) = 2 \div 0 \neq 1$ $-1 \div (1 + 2) = -1 \div 3 \neq 1$ $1 \div ((-1) + 2) = 1 - 1 = 1$]	[**9.** *Sample response*: When you divide integers with opposite signs, the quotient is negative. It does not matter whether it is the dividend or the divisor that is negative. That means $(-32) - 8 = 32 \div (-8)$.]

Supporting Students

Struggling students

• If students are struggling with the idea that dividing integers with like signs makes a positive quotient or that dividing integers with different signs makes a negative quotient in **question 1**, you may wish to have students use counters. The counter model might make more intuitive sense for some students. It is very important that
the students have some way of visualizing this process of division. Otherwise they will rely on memorizing this rule. It is easy to mix up memorized rules.

Enrichment

• For **question 6**, you might challenge students to play a game similar to the game Lobzang is playing.

Arrange students in groups of 2 or 3. Have students make a set of 20 integer cards.

Remind them that there must be positive and negative integers and that they need to consider numbers that are factors of each other.

Have one person deal five cards to each player.

Each player finds the greatest integer quotient possible with his or her 5 cards.

The player with the greatest integer quotient is awarded 1 point, while the other players each get –1 point.

At the end of 5 rounds, the player with the highest score wins.

• For **question 7**, you might challenge students to find a pattern that describes the different division expressions that can be made with two integers.

CONNECTIONS: Mean Temperatures

• This optional connection is intended for all students. It makes a link to the use of integers in daily life. By connecting integers to growing seasons in Bhutan, students can begin to understand the usefulness of this part of the number system.

• This connection can be extended by asking students to find mean temperatures for two or three weeks of data.

• Students might research alternative methods for determining optimum sowing and harvest dates for crops in their region. These methods might involve using environmental clues, for example, the amount of moisture in the soil or the amount of daylight.

Answers

1. a) 11°C	**b)** –3°C
2. a) 12°C	**b)** 24 weeks

3.2.3 Order of Operations with Integers

Curriculum Outcomes	Outcome relevance
8-B7 Order of Operations for Integers: solve problems • apply the proper conventions for order of operations	Students need to be aware of the order of operations for integers in order to deal with computations involving more than two numbers.

Pacing	Materials	Prerequisites
1 h	None	• adding, subtracting, multiplying, and dividing integers

Main Points to be Raised

- An expression that has many calculations and different operations can be interpreted in different ways and lead to different results unless there are rules for the order of operations.
- The rules state that you do calculations inside brackets first, then you calculate division and multiplication in order from left to right, and finally you calculate addition and subtraction in order from left to right.

Try This — Introducing the Lesson

A. Allow students to try this with a partner. While you observe students at work, you might ask questions such as the following:

- *Which operation might it be helpful to start with?* (I want the greatest score so I probably want to multiply first).
- *What strategies did you try?* (I tried different combinations until I found the greatest score.)
- *Why is it important to start with a score of 3?* (I need a number that can divide evenly by 3 to use the second card.)
- *How does thinking about subtracting –3 as adding its opposite help you?* (Usually subtracting a number makes the result less, but when I subtract a negative I actually get a greater result.)
- *Does it make sense that dividing first makes the greatest score?* (Yes. If I divide at the end, my result is a lot less than if I divide at the beginning.)

The Exposition — Presenting the Main Ideas

- With students, read through the exposition on **page 69** of the student text. Point out to students that brackets are used to help people interpret the expressions.
- Divide the class into small groups. Write an expression on the board and have each group calculate it.

For example, you might have the groups calculate [(–4) – 2] × (7 + 1) + 2.

- Circulate among the groups and check to see if students are using the order of operation rules correctly. Encourage them to talk about their strategies. Have students create and calculate two or three other similar expressions.

Revisiting the Try This

B. Students now have a method to show the steps in **part A** using the rules for order of operations.

Using the Examples

- Assign students to pairs. Have one student in each pair become an expert on **part a)** of **example 1** and the other become an expert on **part b)**. Each student should then explain his or her example to the other student.
- Work through **example 2** with students to make sure they understand it. Students may need help in understanding the reasoning of working backwards through the calculations. Provide them with several more examples and have a class discussion about students' reasoning.

For example, students might complete the following expressions:

3 ■ 6 ■ (5 ■ 4) ÷■ 3 ■ 7 = 14 (answer: 3 + 6 × (5 + 4) ÷ 3 – 7)

9 ■ 5 ■ (8 ■ 3) ×■ 2 ■ 6 = 13 (answer: 9 – 5 ÷ (8 – 3) × 2 + 6)

Practising and Applying

Teaching points and tips

Q 1: You may need to explain the difference between brackets indicating the order of operations and brackets used to show negative integers.

Q 2: This is an important question because it emphasizes that the order in which you calculate integers makes a difference in the result.

Q 3: This question provides students with practice in applying the bracket notation.

Q 4: Encourage students to guess and check if they have difficulty working backwards through the order of operations.

Q 5: You might have students share their understandings in a group discussion.

Common errors

• In **question 1,** some students may do the calculations in the order they appear. You might have a class discussion about all the possible results and then talk about the need to make rules to avoid confusion. Some students might find it helpful to break the expression into steps that show the order of operations.

For example, (–6) + (–4) × (–3) ÷ 2 could be written like this:

Step 1 There are no brackets.

Step 2 Multiply and divide, left to right: (–4) × (–3) ÷ 2 = 12 ÷ 2 = 6

Step 3 Add or subtract, left to right: (–6) + 6 = 0

Suggested assessment questions from Practising and Applying

Question 2	to see if students can apply order of operations rules
Question 4	to see if students can solve calculation problems using order of operations rules

Answers

<table>
<tr>
<td>A. Sample response:
First divide by 3, then subtract –3, and then multiply by 3. The final score is 12.</td>
<td>B. i) No; Sample response:
Applying the order of operations gives an answer of 10, not 12:
3 ÷ 3 – (–3) × 3
= 1 – (–3) × 3 [Divide: 3 ÷ 3 = 1]
= 1 – (–9) [Multiply: (–3) × 3 = –9]
= 10 [Subtract 1: – (–9) = 10]
ii) [3 ÷ 3 – (–3)] × 3</td>
</tr>
<tr>
<td>1. a) 0 b) –65 c) 10 d) –2
e) 8 f) 12

2. First divide by –3, then subtract –9, and then multiply by 3;
[0 ÷ (–3) – (–9)] × 3 = 27</td>
<td>3. [(40 × 6) – 3] × (4 – 5) = –237

4. a) 36 – [4 – 1] × 2 = 30
b) (–12) + 4 × (–3) = –24

5. Different people may get different answers to the same question if the rules are not followed.</td>
</tr>
</table>

Supporting Students

Struggling students

• If students are struggling with multiple operations in **question 1**, provide them with expressions using only one or two operations to familiarize them with the rules.

For example, you could change **part a)** to (–6) + (–4) × (–3) + 2.

• You might focus on applying the rules for order of operations rather than on asking students which operation signs are missing to result in a certain value (for example, **question 4**).

Enrichment

• For **question 2,** you might have students create a game like Devika's. This will challenge them as they create cards that result in integer answers.

• For **question 4**, you might change the values on the right and challenge students to find other possible solutions.

GAME: Target

• This optional game is designed to provide practice with integer operations. In addition, students create expressions that apply the rules for the order of operations.

• Students use the same cards as were used in the previous **Game** called **Order the Integers**.

• Since students are trying to figure out the best order for calculating integers, they are likely to estimate sums, differences, products, and quotients.

• Students can adapt the game by using two-digit cards to create larger sums, differences, products, and quotients.

• An alternative rule is to have students try to get a value as far as possible from the value on the target card. This encourages them to apply their understanding of negative and positive integers.

UNIT 3 Revision

Pacing	Materials
1.5 h	• Black and white counters

Question	Related Lesson(s)
1	Lessons 3.1.1 and 3.1.2
2 – 4	Lesson 3.1.2
5 and 6	Lesson 3.1.4
7	Lesson 3.2.1
8 – 12	Lesson 3.2.2
13 and 14	Lesson 3.2.3

Revision Tips

Q 1 and 7: Provide students with counters to help them solve these.

Q 1: Students might use a counter model for **part b)**
if they interpret the question as subtracting 3 sets of negative 3 from 0.

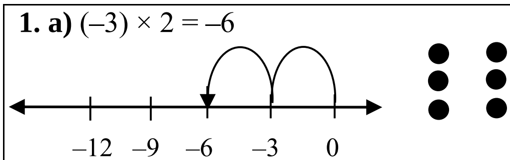
Q 2 and 8: Some students may choose to use a counter or number line model to help them find the products.

Q 9: Students might use a vertical number line to help them solve this.

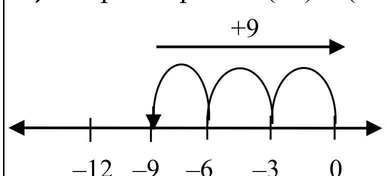
Q 10: If students did not complete the **Connections** section, this will be new information. Thus, instructions for calculating the mean temperature for the year are included in the question.

Answers

1. a) (–3) × 2 = –6



b) *Sample response*: (–3) × (–3) = +9



2. a) 30 **b)** –48 **c)** –28

3. No; [*Sample response*:
If you model multiplying a positive integer by a negative integer with counters, there are only groups of black counters, so the product is always negative.]

4. 14 ways:
[(–1) × (–2) × (–9) = –18
(–1) × 2 × 9 = –18
1 × (–2) × 9 = –18
1 × 2 × (–9) = –18
(–1) × (–3) × (–6) = –18
(–1) × 3 × 6 = –18
1 × (–3) × 6 = –18
1 × 3 × (–6) = –18
(–2) × (–3) × (–3) = –18
2 × (–3) × 3 = –18
–2 × 3 × 3 = –18
(–1) × (–1) × (–18) = –18
(–1) × 1 × 18 = –18
1 × 1 × (–18) = –18]

[**5.** To change the order of factors to model 5 × (–3) on a number line with 5 jumps of –3 from 0.]

Answers [Continued]

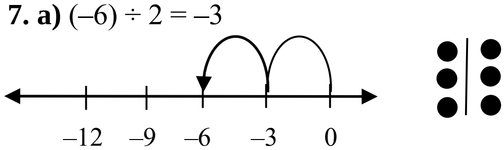
6. a) 2300; [*Sample response*:
I doubled –50 and halved –46.
(–50) × (–46) = (–100) × (–23) = 2300]
b) –2860; [*Sample response*:
I wrote 110 as 100 + 10 and worked by parts to multiply using the distributive property.

$$\begin{aligned}(-26) \times 110 &= (-26) \times (100 + 10)\\ &= (-26) \times 100 + (-26) \times 10\\ &= (-2600) + (-260)\\ &= -2860]\end{aligned}$$

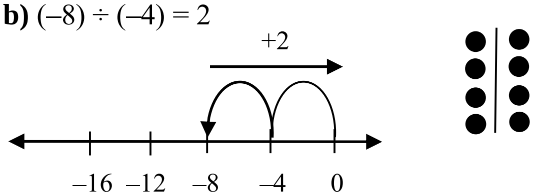
c) 670; [*Sample response*:
I looked for compatible factors and multiplied the first and third integers to get –10.

$$\begin{aligned}(-5) \times (-67) \times 2 &= (-5) \times 2 \times (-67)\\ &= (-10) \times (-67)\\ &= 670]\end{aligned}$$

7. a) (–6) ÷ 2 = –3



b) (–8) ÷ (–4) = 2



8. a) (–92) ÷ 4 → ♦ × (4) = –92; ♦ = –23
b) (–91) ÷ (–7) → ♦ × (–7) = –91; ♦ = 13

9. 4 h

10. 6°C

[**11.** *Sample response*:
I know that 4 ÷ 2 = 2. I also know that a negative divided by a positive is negative, so (–4) ÷ 2 = –2.]

12. a) Positive; [*Sample response*:
Two negatives multiplied together make a positive.]
b) Negative; [*Sample response*:
Two negatives multiplied together make a positive, and then a positive multiplied by a negative makes a negative.]
c) Positive; [*Sample response*:
A negative divided by a negative gives a positive.]
d) Negative; [*Sample response*:
A negative divided by a positive gives a negative.]

13. a) 112 **b)** –29 **c)** 153
d) 2 **e)** –1

14. 40 × **(**6 – 3**)** × 4 – 5 = 475

UNIT 3 Integers Test

1. Multiply each expression using counters or a number line. Sketch your solution.

a) (–4) × 2 **b)** (–6) × (–3)

2. Calculate.

a) (–5) × (–7)

b) (–3) × 8

c) 4 × (–9)

3. Multiply mentally. Explain your strategy for one question.

a) (–15) × (–64)

b) (–79) × 101

c) (–25) × (–32) × 4

4. How does knowing (–4) × 3 = 3 × (–4) help you use a model to calculate (–4) × 3?

5. Divide one expression below using counters. Divide the other expression using a number line. Sketch your solutions.

a) (–4) ÷ 2 **b)** (–6) ÷ (–3)

6. Divide.

a) (–42) ÷ (–7)

b) (–36) ÷ 6

c) 81 ÷ (–9)

7. The temperature in Bumthang was 3°C. In one evening, the temperature fell 2°C every hour.
What was Bumthang's temperature after 3 h?

8. The daily low temperatures were recorded for Paro during a week in December. Find the mean low temperature for the week.

Sunday	–3°C	**Thursday**	–3°C
Monday	–1°C	**Friday**	2°C
Tuesday	3°C	**Saturday**	–1°C
Wednesday	–4°C		

9. How do you know that (–8) ÷ (–4) = 2?

10. Without calculating, predict whether each answer is negative or positive. Explain each prediction.

a) (–44) × (–908)

b) (–353) × (–7927) × (–815)

c) (–1098) ÷ (–183)

d) (–3003) ÷ 429

11. Calculate.

a) –3 + (–4) × [2 × (–6)]

b) –15 ÷ (–3) + 2 × (–8)

c) [(–6) + (–10)] ÷ [(–4) × 2]

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UNIT 3 Test

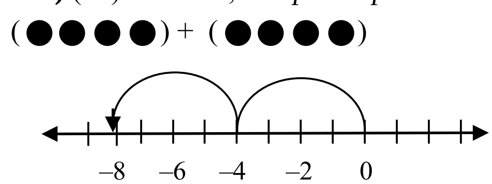
Pacing	Materials
1 h	• Black and white counters

Question	Related Lesson(s)
1	Lessons 3.1.1 and 3.1.2
2 and 10	Lesson 3.1.2
3 and 7	Lesson 3.1.4
4, 5, and 9	Lesson 3.2.1
6, 10	Lesson 3.2.2
11	Lesson 3.2.3

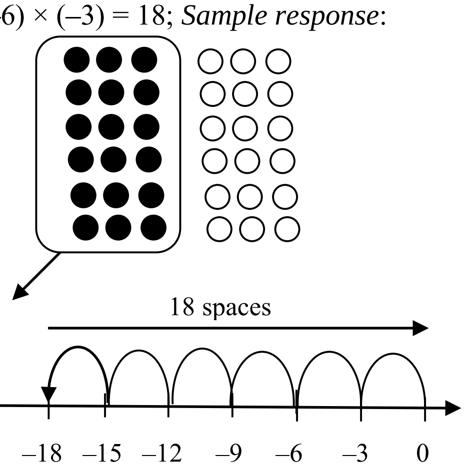
Select questions to assign according to the time available.

Answers

1. a) (–4) × 2 = –8; *Sample response*:



b) (–6) × (–3) = 18; *Sample response*:



2. a) 35 **b)** –24 **c)** –36

3. a) 960; *Sample response*:

(–15) × (–64)
= (–30) × (–32) [Doubled and halved]
= (–3) × 10 × (–32) [Broke up a factor to get compatible factors]
= (–3) × (–320)
= 960

b) –7979; *Sample response*:
I wrote 101 as 100 + 1 and worked by parts to multiply using the distributive property.

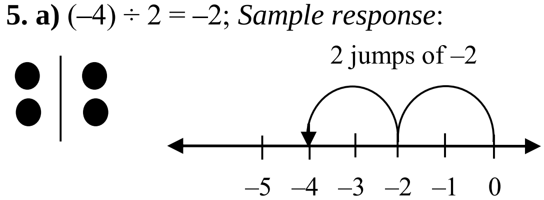
(–79) × 101 = (–79) × (100 + 1)
= (–79) × 100 + (–79) × 1
= (–790) + (–79)
= –7979

c) 3200; *Sample response*:
I found compatible factors and multiplied the first and third integers to get –100:

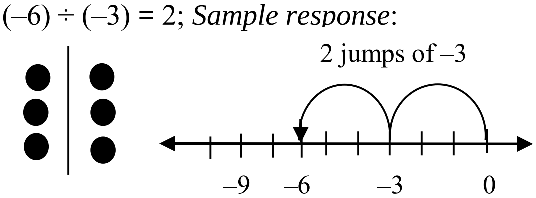
(–25) × (–32) × 4 = (–25) × 4 × (–32)
= (–100) × (–32)
= 3200

4. It is easy to model 3 jumps of –4 but it is not easy to model –4 jumps of 3.

5. a) (–4) ÷ 2 = –2; *Sample response*:



b) (–6) ÷ (–3) = 2; *Sample response*:



6. a) 6 **b)** –6 **c)** –9

7. –3°C

8. –1°C **9.** *Sample response*: I know that (–8) ÷ (–4) could mean the number of groups of –4 that are in –8. I can model this using counters by showing 8 black counters arranged into groups of 4. There are 2 groups.	**10. a)** Positive; *Sample response*: Two negatives multiplied together make a positive. **b)** Negative; *Sample response*: Two negatives multiplied together make a positive, then a positive multiplied by a negative makes a negative. **c)** Positive; *Sample response*: A negative divided by another negative gives a positive. **d)** Negative; *Sample response*: A negative divided by a positive gives a negative. **11. a)** 45 **b)** –11 **c)** 2

UNIT 3 Performance Task — Mystery Integers

Part 1

A. The following clues describe four mystery integers:

A	C	B	D

Clue 1: The sum of the four integers is –5.

$$A + C + B + D = -5$$

Clue 2: If you order the integers from least to greatest,
the product of the middle two integers is –6.

$$A, B, C, D \qquad B \times C = -6$$

Clue 3: If you subtract the least integer from the greatest integer
and then divide the difference by 2, the quotient is 7.

$$A, B, C, D \qquad (D - A) \div 2 = 7$$

i) Find three possible sets of four mystery integers. Note that the same number can appear in more than one set.

ii) Choose one set of mystery integers. Create a *Clue 4* that works for this set but not for the other two sets.

iii) Show how the set of mystery integers you chose in **part ii)** satisfies each of the four clues.

Part 2

B. i) Select any four mystery integers and make up four clues.

- Use all four operations in the clues.
- All clues must be necessary to solve the problem.
- Use clues that are different from the clues in **part A**.

ii) Is only one set of numbers possible for your clues in **part i)**? How do you know? If not, what clue could you add to make your set the only possible set?

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UNIT 3 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
8-B4 Multiply and Divide Integers: solve problems **8-B7** Order of Operations for Integers: solve problems	1.5 h	None

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could replace or supplement the unit test. It could also be used as enrichment material for some students.

• Depending on the time available, you may choose to assign only **part 1**.

• You can assess performance on the task using the rubric provided on **page 102**.

Sample Solution

A. i) −9, −3, 2, 5; −7, −6, 1, 7; −10, −2, 3, 4

ii) I chose −9, −3, 2, and 5.
Clue 4: The product of the least integer and the greatest integer is −45.

iii) *Clue 1*: The sum of the four integers is –5: −9 + −3 + 2 + 5 = –5

Clue 2: If you order the integers from least to greatest, the product of the middle two integers is –6: −9, −3, 2, 5 → –3 × 2 = –6.

Clue 3: If you subtract the least integer from the greatest integer and then divide
the difference by 2, the quotient is 7: −9, −3, 2, 5 → 5 – (–9) = 14 → 14 ÷ 2 = 7.

Clue 4: The product of the least and the greatest integers is −45: −9, −3, 2, 5 → –9 × 5 = –45.

B. i) −7, 2, 4, 9

Clue 1: The product of the least integer and the greatest integer is −63.

Clue 2: If you order the integers from least to greatest, the sum of the two least integers is −5.

Clue 3: If you subtract the least integer from the greatest integer, you get 16.

Clue 4: If you order the integers from least to greatest and then divide the greater of the two middle integers by the other middle integer, you get 2.

ii) Yes; I used *Clue 1* to start listing possibilities: –63 and 1, –21 and 3, –9 and 7, –7 and 9, –3 and 21, –1 and 63.

Using *Clue 2*, I could figure out possibilities for the two least integers. Since I knew the possibilities for the two least integers and the greatest integer, this resulted in the following possibilities for the solution:

–63, 58, ■, 1: does not work because 58 > 1.

–21, 16, ■, 3: does not work because 16 > 3.

–9, 4, ■, 7

–7, 2, ■, 9

–3, –2, ■, 21

–1, –4, ■, 63: does not work because –1 > –4, and –1 must be least.

Using *Clue 3*, I eliminated –3, –2, ■, 21.

I was left with –9, 4, ■, 7 and –7, 2, ■, 9.

Using *Clue 4*, I got –9, 4, 8, 7 and –7, 2, 4, 9. The first possibility does not work because 8 > 7.

UNIT 3 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Solves problems	• Completes all three solutions correctly using the given information • Provides a clue that leads to a unique solution that matches one possibility	• Completes one or two solutions correctly using the given information • Provides a clue that works but does not lead to a unique solution	• Makes minor mathematical errors leading to wrong answers or incomplete solutions using the given information • Makes minor errors in providing a clue that leads to a wrong answer	• Incorrectly completes the solutions; makes major mathematical errors using the given information • Provides an incorrect clue
Creates a problem	Provides clues that lead to a unique solution and uses all four operations	Provides clues that lead to a unique solution but does not use all four operations	Makes minor errors in creating clues that lead to a wrong answer	Provides incorrect clues
Communicates the solutions	Presents clear, coherent, and insightful explanations	Presents organized explanations that include justifications	Presents acceptable but brief explanations	Presents explanations that are disorganized and difficult to follow

UNIT 4 FRACTIONS AND RATIONAL NUMBERS

UNIT 4 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 73** **TG p. 109**	Review prerequisite concepts, skills, and terminology, and pre-assessment	1 h	• Pattern blocks or Pattern Block Fraction Pieces (BLM) • Counters in two colours (optional) • Fraction Circles (BLM) (optional)	All questions
Chapter 1 Adding and Subtracting Fractions				
4.1.1 Adding and Subtracting Fractions Mentally **SB p. 75** **TG p. 111**	**8-B9 Add and Subtract: fractions mentally** • attempt mental calculation first when denominators are the same or easily determined (e.g., $\frac{1}{2} + \frac{1}{4}$) • when addition or subtraction can not be done mentally, determine if estimation is sufficient or an exact answer is required	1 h	• Measuring cups (optional) • Pattern blocks or Pattern Block Fraction Pieces (BLM) • Fraction Circles (BLM) (optional)	Q1, 2, 4, 10
4.1.2 Adding and Subtracting Fractions Symbolically **SB p. 78** **TG p. 114**	**8-B8 Add and Subtract: fractions — develop algorithm (pictorially and symbolically)** • apply prior understanding of equivalent fractions, lowest terms, and LCM • use manipulatives to develop operations with fractions concretely (e.g., fraction strips, grids, fraction circles, number lines) • record equivalent fractions when moving from the concrete to symbolic • represent both fractions using the same subdivision of the whole	1 h	None	Q1, 2, 4
Chapter 2 Multiplying and Dividing Fractions				
4.2.1 EXPLORE: Multiplying Fractions (Optional) **SB p. 82** **TG p. 117**	**8-B10: Multiply: fractions — develop algorithm (pictorially and symbolically)** • construct concrete and pictorial models to develop meaning	30 min	• Grid paper or Small Grid Paper (BLM) (optional) • Coloured pencils (optional)	Observe and Assess questions
4.2.2 Multiplying Fractions **SB p. 83** **TG p. 119**	**8-B10 Multiply: fractions — develop algorithm (pictorially and symbolically)** • construct concrete and pictorial models to develop meaning • understand that "of" means multiplication and can be shown by comparing results in questions such as $\frac{1}{4}$ of 8 and $\frac{1}{4} \times 8$ • multiply a whole number by a fraction less than 1 (e.g., $4 \times \frac{1}{3}$ uses repeated addition) • multiply a fraction less than 1 by another fraction especially when the numerator is 1 (e.g., $\frac{1}{4}$ of $\frac{2}{3}$)	1 h	• Grid paper or Small Grid Paper (BLM) (optional) • Coloured pencils (optional)	Q1, 2, 3, 10

UNIT 4 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
4.2.2 Multiplying Fractions [Cont'd]	**8-B12 Fractions: estimate and mentally compute products and quotients** • appropriate number situations to use for multiplication include: - a fraction by a whole number when the numbers are compatible - any two proper fractions when the numerators and denominators are relatively simple to work with - a whole number by a mixed number (distributive property should be used) • use estimation to check reasonableness of results			
CONNECTIONS: The Sierpinski Triangle (Optional) **SB p. 87** **TG p. 122**	Make a connection between the multiplication of fractions and the geometry of fractals.	30 min	• Rulers • Coloured pencils (optional)	N/A
4.2.3 Multiplying Mixed Numbers **SB p. 88** **TG p. 123**	**8-B10 Multiply: fractions — develop algorithm (pictorially and symbolically)** • construct concrete and pictorial models to develop meaning **8-B12 Fractions: estimate and mentally compute products and quotients** • appropriate number situations to use for multiplication include: - a whole number by a mixed number (distributive property should be used) • use estimation to check reasonableness of results • round to nearest whole and sometimes to nearest half to reach rough estimates	1 h	• Coloured pencils (optional)• Grid paper or Small Grid Paper (BLM) (optional)	Q1, 2, 3, 10
4.2.4 Dividing Fractions With a Common Denominator **SB p. 92** **TG p. 126**	**8-B11 Divide: fractions — develop algorithm (pictorially and symbolically)** • derive a personal algorithm from carefully chosen examples, e.g.: - a simple fraction divided by simple fraction where the numerator of the divisor is 1 and both denominators are the same (e.g., $\frac{5}{6} \div \frac{1}{6}$ asks, "How many $\frac{1}{6}$ s are in $\frac{5}{6}$?") - a simple fraction divided by a simple fraction where the numerator of the divisor is one and the fractions are compatible (e.g., $\frac{1}{2} \div \frac{1}{4}$) • use a number line to model division **8-B12 Fractions: estimate and mentally compute products and quotients** • appropriate number situations to use for division include: - a simple fraction divided by a simple fraction when the denominators are the same - a simple fraction divided by a simple fraction	1 h	• Pattern blocks or Pattern Block Fraction Pieces (BLM) (optional)	Q 1, 4, 5

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
4.2.5 Dividing Fractions in Other Ways **SB p. 95** **TG p. 129**	**8-B11 Divide: fractions — develop algorithm (pictorially and symbolically)** • derive a personal algorithm from carefully chosen examples, e.g.: - a simple fraction divided by a whole number (e.g., for $\frac{1}{2} \div 3$, divide $\frac{1}{2}$ into 3 equal parts. What does each part represent?) - a whole number divided by a simple fraction (e.g., $4 \div \frac{1}{2}$, asks, "How many $\frac{1}{2}$ s there are in 4?") - a simple fraction divided by simple fraction where the numerator of the divisor is 1 and both denominators are the same (e.g., $\frac{5}{6} \div \frac{1}{6}$ asks, "How many $\frac{1}{6}$ s are in $\frac{5}{6}$?") - a simple fraction divided by a simple fraction where the numerator of the divisor is 1 and the fractions are compatible (e.g., $\frac{1}{2} \div \frac{1}{4}$) • use a number line to model division • apply prior knowledge of reciprocal **8-B12 Fractions: estimate and mentally compute products and quotients** • appropriate number situations to use for division include: - a simple fraction divided by a whole number - a whole number divided by a fraction - a simple fraction divided by a simple fraction when the denominators are the same - a simple fraction divided by a simple fraction	1 h	• Coloured pencils (optional)	Q1, 2, 3, 5
4.2.6 Dividing Mixed Numbers **SB p. 99** **TG p. 132**	**8-B11 Divide: fractions — develop algorithm (pictorially and symbolically)** • use a number line to model division • apply prior knowledge of reciprocal **8-B12 Fractions: estimate and mentally compute products and quotients** • appropriate number situations to use for division include: - a mixed number divided by a whole number - a whole number divided by a mixed number - a mixed number divided by a mixed number • use estimation to check reasonableness of results • round to nearest whole and sometimes to nearest half to reach rough estimates	1 h	None	Q2, 3, 6, 9

UNIT 4 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Chapter 3 Rational Numbers				
4.3.1 Introducing Rational Numbers **SB p. 102** **TG p. 135**	**8-A8 Integers and Rational Numbers: comparing and ordering (fractional and decimal form)** • understand that placement of the negative sign does not affect the value (e.g., $\frac{-2}{3}$, $\frac{2}{-3}$, and $-\frac{2}{3}$ are equivalent) • understand that a negative is always less than a positive • understand that positive fractions with common denominators can be compared by examining numerators (e.g., $\frac{3}{8}$ is less than $\frac{5}{8}$ because 3 is less than 5) • understand that positive fractions with common numerators can be compared by examining denominators (e.g., $\frac{3}{5}$ is greater than $\frac{3}{6}$ because 5 is less than 6) • use reference points (1, $\frac{1}{2}$, −1, etc.) • change numbers to a common form	1 h	None	Q3, 4, 6
4.3.2 Operations with Rational Numbers **SB p. 106** **TG p. 138**	**8-B13 Operations: positive and negative decimal numbers** • use prior experience to construct concrete and pictorial representations • connect visual representations to symbols • use a variety of models to illustrate the operations (e.g., coloured counters, number lines) • develop computational algorithms with decimals, using estimation, mental computation, pencil and paper • apply prior knowledge of order of operations in the context of positive and negative decimals • continue to estimate to check reasonableness of calculations	1 h	None	Q2, 3, 4, 5
4.3.3 Order of Operations **SB p. 110** **TG p. 141**	**8-B14 Order of Operations: fractions** • understand that the order is the same as for whole numbers, and why that makes sense • understand how improper order impacts results **8-B13 Operations: positive and negative decimal numbers** • use prior experience to construct concrete and pictorial representations • connect visual representations to symbols • use a variety of models to illustrate the operations (e.g., coloured counters, number lines)	1 h	None	Q1, 2, 8

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
4.3.3 Order of Operations [Cont'd]	• develop computational algorithms with decimals, using estimation, mental computation, pencil and paper • apply prior knowledge of order of operations in the context of positive and negative decimals • continue to estimate to check reasonableness of calculations			
GAME: Target One (Optional) **SB p. 114** **TG p. 143**	Practise operations with fractions in a game situation	30 min	• Teacher- or student-made game cards	N/A
UNIT 4 Revision **SB p. 115** **TG p. 144**	Review the concepts and skills in the unit	2 h	• Grid paper or Small Grid Paper (BLM) (optional) • Coloured pencils (optional)	All questions
UNIT 4 Test **TG p. 146**	Assess the concepts and skills in the unit	1 h	None	All questions
UNIT 4 Performance Task **TG p. 148**	Assess concepts and skills in the unit	1 h	None	Rubric provided
UNIT 4 Assessment Interview **TG p. 150**	Assess concepts and skills in the unit	15 min	None	All questions
UNIT 4 Blackline Masters **TG p. 151**	BLM 1 Pattern Block Fraction Pieces BLM 2 Fraction Circles Small Grid Paper on **page 32** in UNIT 1			

Math Background

• This unit continues the work on addition and subtraction of fractions and mixed numbers from Class VII. It also extends multiplication and division to include multiplying and dividing fractions and mixed numbers. Operations with positive and negative fractions, mixed numbers, and decimals are among the new ideas introduced in this unit.

• The focus of the unit is on computation with fractions, mixed numbers, and decimals.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 7** in **lesson 4.1.2**, where they determine the missing numbers in a Magic Square, in **question 9** in **lesson 4.2.2**, where they look for a pattern in the multiplication of particular fractions, in **question 4** in **lesson 4.2.5**, where they solve a real-world problem using division of fractions, and in **question 2** in **lesson 4.3.3**, where they determine the effects of the order of operations on a sequence of calculations and insert brackets to make an equation true.

• Students use communication in **question 4** in **lesson 4.1.1**, where they explain why an estimate is high or low, in **question 5** in **lesson 4.1.2**, where they explain why the associative property works for addition but not for subtraction, in **question 9** in **lesson 4.2.6**, where they explain an estimation strategy, and in **question 5** in **lesson 4.3.3**, where they explain a prediction about the sign of the result of a series of calculations.

• Students use reasoning in **question 9** in **lesson 4.1.1**, where they estimate a reasonable sum, in **question 8** in **lesson 4.2.2**, where they compare/relate multiplication of decimals with multiplication of fractions, in **question 9** in **lesson 4.2.3**, where they determine possibilities that satisfy a particular condition for a computation, and in **question 4** in **lesson 4.3.2**, where they predict whether a result will be positive or negative and explain their prediction.

• Students consider representation in **question 1** in **lesson 4.2.2** and **question 1** in **lesson 4.2.3**, where they use models to represent multiplication of fractions and mixed numbers, and in **question 2** in **lesson 4.2.4** and **question 1** in **lesson 4.2.6**, where they draw number lines to represent division of fractions and mixed numbers.

• Students use visualization skills in **question 5** in **lesson 4.2.4**, where they visualize a division situation using pattern blocks, and in **question 1** in **lesson 4.2.5**, where they interpret area and number line models of division.

• Students make connections in **question 8** in **lesson 4.2.2** and **question 8** in **lesson 4.2.3**, where they link multiplication of fractions and multiplication of decimals, in **lesson 4.3.2**, where they link sign rules for calculating with integers to rules for calculating with rational numbers, and in **lesson 4.3.3**, where they link rules for order of operations with fractions to rules for rational numbers. There are also many real-world connections, such as **question 10** in **lesson 4.2.2**, **question 3** in **lesson 4.2.3**, and **question 6** in **lesson 4.3.1**.

Rationale for Teaching Approach

• This unit is divided into three chapters:

Chapter 1 is about adding and subtracting fractions and mixed numbers.

Chapter 2 focuses on multiplying and dividing fractions.

Chapter 3 examines operations with positive and negative rational numbers.

• The **Explore** lesson allows students to explore the use of pictorial models to develop meaning for multiplication of fractions.

• The **Connections** allows students to examine the fraction patterns in the Sierpinski Triangle.

• The **Game** provides an opportunity to apply and practise work with fraction operations in an enjoyable way.

• Throughout the unit, it is important to encourage students to estimate as a way of determining whether the results of their computations make sense and to demonstrate flexibility in computational strategies. You should accept a variety of approaches from students.

Getting Started

Curriculum Outcomes	Outcome relevance
7 Add and Subtract Integers Mentally: develop and use strategies **7** Add and Subtract: simple fractions and mixed numbers of various denominators **7** Multiply: fraction by a whole number **8** Multiply and Divide Integers: to solve problems **8** Multiply and Divide Integers: mentally **8** Order of Operations for Integers: to solve problems	Students will find the work in the unit easier after they review the concepts and skills related to fractions, operations on fractions, and integers from Class VII.

Pacing	Materials	Prerequisites
1 h	• Pattern blocks or Pattern Block Fraction Pieces (BLM) (optional) • Counters in two colours (optional) • Fraction Circles (BLM) (optional)	• familiarity with the terms *mixed number* and *equivalent fraction* • representing fractions • representing multiplication as repeated addition • multiplying a fraction by a whole number • performing operations with integers

Main Points to be Raised

Use What You Know

• You can represent a fraction as equal parts of a whole.

• You can use the same pattern block many times to model a fraction multiplied by a whole number.

• To write an improper fraction as a mixed number, divide the numerator by the denominator to get the whole number part.

Skills You Will Need

• You can represent multiplication of a whole number by a fraction as repeated addition of the fraction.

• You can find equivalent fractions by multiplying the numerator and denominator by the same non-zero number. Or, you can divide the numerator and denominator by a common factor.

• There are a variety of ways to add and subtract fractions and mixed numbers.

• You can use the zero property to add and subtract integers.

• The order of operations is: operations inside brackets, then multiplication and division in order from left to right, and finally addition and subtraction from left to right.

• The product or quotient of integers with the same sign is positive. The product or quotient of integers with different signs is negative.

Use What You Know — Introducing the Unit

• Students can work in pairs or small groups.

While you observe students at work, you might ask questions such as the following:

• *How do the answers to* ***part A*** *help you to answer the other question?* (When I know what part of the whole the trapezoid, rhombus, and triangle are, it is easier to write the equations for **parts B and C**. I also know which shapes I need for **part D**.)

• *How did you know what fraction the rhombus represents?* (Since it takes 3 rhombuses to cover the hexagon, which is the whole, each rhombus must be $\frac{1}{3}$.) *What about the trapezoid? the triangle?*

• *How did you know what mixed number to write?* (I divided the numerator by the denominator to figure out the number of wholes. There was a fraction left over.)

Skills You Will Need

- Students can work individually.
- To ensure students have the required skills for this unit, assign all of these questions.
- Before students begin work, review the terms *equivalent fraction* and *mixed number* to make sure students can interpret **questions 1 and 3**. Refer students to the glossary at the back of the student text.
- Encourage students to use different computational strategies for answering **question 4**.

Answers

Note: *Students may not use lowest terms for fractions. This should not be considered incorrect.*

A. i) One half, or $\frac{1}{2}$ **ii)** One third, or $\frac{1}{3}$

iii) One sixth, or $\frac{1}{6}$

B. i) $3 \times \frac{1}{6} = \frac{1}{2}$ **ii)** $2 \times \frac{1}{6} = \frac{1}{3}$ **iii)** $6 \times \frac{1}{6} = 1$

C. i) $3 \times \frac{1}{2} = 1\frac{1}{2}$ **ii)** $5 \times \frac{1}{3} = 1\frac{2}{3}$ **iii)** $7 \times \frac{1}{6} = 1\frac{1}{6}$

iv) $4 \times \frac{1}{2} = 2$ **v)** $10 \times \frac{1}{3} = 3\frac{1}{3}$ **vi)** $9 \times \frac{1}{6} = 1\frac{1}{2}$

D. i) Five trapezoids; $2\frac{1}{2}$ **ii)** Seven rhombuses; $2\frac{1}{3}$

iii) Eight triangles; $1\frac{1}{3}$ **iv)** Seven trapezoids; $3\frac{1}{2}$

v) 11 rhombuses; $3\frac{2}{3}$

1. *Sample responses*:

a) $\frac{4}{6} = \frac{6}{9} = \frac{8}{12}$ **b)** $\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$

c) $\frac{10}{16} = \frac{15}{24} = \frac{20}{32}$ **d)** $\frac{3}{5} = \frac{6}{10} = \frac{15}{25}$

2. a) Yes; [You add the same amount, $\frac{3}{5}$, 6 times, $6 \times \frac{3}{5}$.]

b) No; [You do not add the same number many times.]

3. a) $3\frac{3}{4}$ **b)** $2\frac{1}{2}$ **c)** $3\frac{1}{9}$

4. a) $\frac{1}{12}$ **b)** $\frac{13}{24}$ **c)** $4\frac{1}{8}$

d) $1\frac{1}{5}$ **e)** $2\frac{11}{12}$ **f)** $1\frac{7}{12}$

5. $2\frac{1}{4}$ cups

6. a) +10 **b)** +300 **c)** –79 **d)** –15

7. a) –268 **b)** +136 **c)** –110

8. a) 96 **b)** –126 **c)** –90

[**9.** *Sample response*:
I can divide –12 into 6 groups of –2; (–12) ÷ 6 = –2.]

10. a) 180 **b)** –53 **c)** 3 **d)** –1

Supporting Students

Struggling students

- For **question 1**, some students might find it helpful to use fraction circles to review simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$ and $\frac{1}{3} = \frac{2}{6}$. You could use BLM 2 Fraction Circles to review. You can create fraction fourths can easily from the fraction circle in halves, create fraction sixths from the fraction circle in thirds, and so on.
- If students are struggling with **question 1 b) and d)**, remind them that they can divide as well as multiply to find equivalent fractions.
- Some students will benefit from a review of subtraction of mixed numbers for **question 4**, the use of the zero property to add and subtract integers for **questions 6 and 7**, and the order of operations rules for **question 10**.
- You may choose to start with simpler calculations and provide counters in two colours for the integer questions, e.g., for **question 6 a)**, start with –5 + (+7) using counters and then do –25 + (+35) without counters.

Enrichment

- For **question 10**, you might have students explore what happens to the answer when the brackets are removed for **part a)** or when brackets are placed in different places for **part b)**.

Reprint 2024

Chapter 1 Adding and Subtracting Fractions

4.1.1 Adding and Subtracting Fractions Mentally

Curriculum Outcomes	Outcome relevance
8-B9 Add and Subtract: fractions mentally • attempt mental calculation first when denominators are the same or easily determined (e.g., $\frac{1}{2}+\frac{1}{4}$) • when addition or subtraction can not be done mentally, determine if estimation is sufficient or an exact answer is required	There are many everyday situations where we need to be able to add simple fractions mentally. In many situations an estimate is sufficient, so estimation strategies for fractions and mixed numbers are very important.

Pacing	Materials	Prerequisites
1 h	• Measuring cups (optional) • Pattern blocks or Pattern Block Fraction Pieces (BLM) (optional) • Fraction Circles (BLM) (optional)	• finding common denominators • using benchmarks (e.g., 0, $\frac{1}{2}$, 1) to estimate fraction sums and differences • adding and subtracting fractions with a common denominator • expressing fractions in words, e.g., $\frac{1}{4}$ is 1 fourth

Main Points to be Raised

• You can often find the answer to a fraction addition or subtraction mentally if the denominators are the same or if one denominator is a multiple of the other.

• If the denominators are the same, you add or subtract the numerators. If they are different, use an equivalent fraction for at least one of the denominators.

• You can often estimate fraction sums and differences mentally. Sometimes an estimate is all that is needed.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Does it make sense that one piece of the first bar is the same amount as two pieces of the second bar?* (Yes. The first bar has four pieces. If you cut each of those pieces into two equal parts, you have eight pieces, which is the number of pieces in the second bar.)

• *How do you know how much of the bar Pema ate altogether?* (Since one piece of the first bar is the same as two pieces of the second bar, the total amount eaten is equivalent to two pieces of the second bar and three more pieces of the second bar, which is five pieces, or five eighths.)

The Exposition — Presenting the Main Ideas

• Ask students a sequence of questions similar to the following:

- *If you eat* $\frac{1}{2}$ *of a cake and then eat another* $\frac{1}{2}$*, how much of the cake have you eaten?* (The whole cake)

- *If you eat* $\frac{1}{4}$ *of a cake and then eat another* $\frac{1}{4}$*, how much of the cake have you eaten?* ($\frac{1}{2}$ of the cake)

- *If you eat* $\frac{1}{2}$ *of a cake and then eat another* $\frac{1}{4}$*, how much of the cake have you eaten?* ($\frac{3}{4}$ of the cake)

• Discuss why it is easy to answer the above questions mentally. (They have the same denominator. Or, it is easy to give them the same denominator. Or, it is easy to work with halves and fourths.)

• Go through the exposition with students.

• Make sure students understand the difference between calculating mentally (finding the exact answer in your head) and estimating mentally (finding what a calculated value might be close to in your head).

Revisiting the Try This

B. Students apply what they learned in the exposition about adding fractions mentally to the problem posed in the **Try This** where one denominator was a multiple of the other denominator.

Using the Examples

• Pose the question in **example 1** to the students. Let them try it and then compare their work to the solution in the student text. Then have students work in pairs on **example 2**.

Practising and Applying

Teaching points and tips

Q1 and 2: Remind students that to add mixed numbers mentally, they can first add the fraction parts and the whole number parts separately, and then combine them for their answer. To subtract mixed numbers, the fraction part of the first number must be greater than the fraction part of the second number for you to be able to subtract the whole numbers and fractions in parts. In **question 2 c) and d)**, the first fraction is greater, so this method will work.

Q3: You might ask for volunteers to share their strategy for each part of **questions 1 and 2** so that all of the questions are discussed.

Q4: Remind students that there are different ways to estimate (e.g., some students may choose to round mixed numbers to whole numbers, while others might use some benchmark fractions). In **question 4 b)**, students need to explain how they estimated in order for their rationale about whether the estimate is high or low to make sense.

Q5, 6, and 7: Remind students that they need to explain their answers to each question, and not just answer yes or no. For **questions 5 and 6**, students might benefit from referring to an actual measuring cup.

Q 10: This question provides another opportunity to discuss the difference between mental computation and mental estimation. In **part a)**, it is the context that determines whether an estimate will do. In **part b),** the numbers used in the calculation determine when mental math can be used.

Common errors

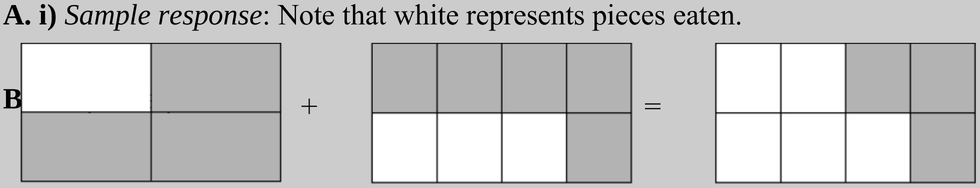
• Many students will use paper and pencil to answer **questions 1 and 2**. Encourage them to think of ways to find the answers mentally and to share their strategies with classmates.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can apply mental computation strategies to add fractions and mixed numbers
Question 2	to see if students can apply mental computation strategies to subtract fractions and mixed numbers
Question 4	to see if students can use and explain estimation strategies for adding and subtracting fractions and mixed numbers
Question 10	to see if students understand that context determines whether an estimate will do and that the numbers involved in a calculation determine whether mental math can be used easily

Answers

A. i) *Sample response*: Note that white represents pieces eaten. **ii)** $\frac{5}{8}$



B. I know 1 fourth is 2 eighths, so 1 fourth + 3 eighths = 2 eighths + 3 eighths = 5 eighths.

Note: All answers are written in lowest terms, but you should accept correct answers that are not in lowest terms.

1. a) $\frac{7}{9}$ **b)** $\frac{7}{8}$ **c)** $8\frac{1}{2}$ **d)** $16\frac{5}{8}$

2. a) $\frac{1}{6}$ **b)** $\frac{3}{8}$ **c)** $2\frac{4}{9}$ **d)** $3\frac{1}{16}$

[**3.** *Sample responses*:

1 b) I know $\frac{1}{2}$ is $\frac{4}{8}$, so $\frac{1}{2} + \frac{3}{8} = \frac{4}{8} + \frac{3}{8} = \frac{7}{8}$.

2 b) I know $\frac{3}{4}$ is $\frac{6}{8}$, so $\frac{3}{4} - \frac{3}{8} = \frac{6}{8} - \frac{3}{8} = \frac{3}{8}$.]

4. *Sample responses*:

a) and b)

i) 7; Higher [because $\frac{1}{8}$ is less than $\frac{1}{5}$ (assuming you round $2\frac{1}{8}$ to 2 and $4\frac{4}{5}$ to 5).]

ii) 6; Lower [because $\frac{1}{8}$ is greater than $\frac{1}{10}$ (assuming you round $4\frac{7}{8}$ to 5 and $1\frac{1}{10}$ to 1).]

iii) $8\frac{1}{2}$; Not sure [It is very close.]

iv) 5; Higher [because $\frac{1}{3}$ is greater than $\frac{1}{4}$ (assuming you round $7\frac{1}{4}$ to 7 and $2\frac{1}{3}$ to 2).]

v) $2\frac{1}{2}$; Higher [because $\frac{9}{10}$ is less than 1 (assuming you round $1\frac{9}{10}$ to 2 and subtract from $4\frac{1}{2}$).]

vi) $\frac{1}{4}$; Lower [because $\frac{11}{12}$ is less than 1 (assuming you round $\frac{11}{12}$ to 1 and subtract $\frac{3}{4}$).]

5. Yes; [*Sample response*:
$\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$ and $\frac{7}{8} > \frac{3}{4}$, so $\frac{3}{4} + \frac{7}{8} > 1\frac{1}{2}$.]

6. No; [*Sample response*:
$1\frac{3}{4} + 1\frac{3}{4} = 3\frac{1}{2}$, so $3\frac{1}{3}$ cups is not enough.]

7. Yes; [*Sample response*: $\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$]

8. $\frac{1}{6}$; [*Sample response*:
$\frac{1}{3} = \frac{2}{6}$ and $\frac{1}{2} = \frac{3}{6}$, so the difference is $\frac{1}{6}$.]

9. $18\frac{1}{2}$; [*Sample response*:
$6 + 4 + 7 = 17$, $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, and $\frac{3}{4} + \frac{2}{3}$ is almost $1\frac{1}{2}$.]

10. *Sample responses*:

a) Wondering whether a 5-cup bowl will be big enough to hold $1\frac{1}{2}$ cups flour, $2\frac{1}{4}$ sugar, and $1\frac{2}{3}$ cups butter.

b) $1\frac{1}{4} + 2\frac{3}{4}$

Supporting Students

Struggling students

• Some students may try to subtract $3\frac{1}{3} - 1\frac{3}{4}$ to answer **question 6**. Suggest that it might be easier to find out the total amount Chandra needs to use and compare that result with $3\frac{1}{3}$.

• Some students will have difficulty explaining how they calculated in **questions 3, 5, 6, 7, and 9**. Tell them that they can explain what they did by showing their calculations instead of using words.

• If some students would benefit from the continued use of materials, such as fraction circles, pattern blocks, or measuring cups, make these available.

4.1.2 Adding and Subtracting Fractions Symbolically

Curriculum Outcomes	Outcome relevance
8-B8 Add and Subtract: fractions — develop algorithm (pictorially and symbolically) • apply prior understanding of equivalent fractions, lowest terms, and LCM • use manipulatives to develop operations with fractions concretely (e.g., fraction strips, grids, fraction circles, number lines) • record equivalent fractions when moving from the concrete to symbolic • represent both fractions using the same subdivision of the whole	The ability to add and subtract fractions and mixed numbers is a life skill. Students need to understand why the algorithms work and not just apply rules without understanding.

Pacing	Materials	Prerequisites
1 h	None	• familiarity with equivalent fractions • familiarity with the terms *lowest common multiple* and *greatest common factor* • writing fractions in lowest terms • rewriting improper fractions as mixed numbers and vice versa

Main Points to be Raised

• When fractions have the same denominator, you can find the sum or difference by adding or subtracting the numerators.

• You can find a common denominator for fractions by finding equivalent fractions with the same denominator. One way to do this is to find a common multiple of the denominators and use it as the common denominator.

• The least common multiple of the denominators is usually the easiest common denominator to use, but any common multiple will work.

• When a fraction sum or difference is an improper fraction, it is helpful to write it as a mixed number and/or in lowest terms.

• A fraction is in lowest terms when the numerator and denominator have no common factor other than 1.

• You can write a fraction in lowest terms by dividing the numerator and denominator by their greatest common factor.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Which benchmark fractions might help you with your estimates?* ($\frac{3}{4}$, because $\frac{2}{3}$ is close to $\frac{3}{4}$, and 0, because $\frac{1}{8}$ is close to 0.)

• *What do you need to find in order to calculate the exact answers?* (I need equivalent fractions for $\frac{2}{3}$ and $\frac{1}{8}$ with the same denominator.)

The Exposition — Presenting the Main Ideas

• Write the example $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$ from the exposition on the board, along with the drawing to show combining 1 fifth and 3 fifths to make 4 fifths. Also write the example to show $\frac{3}{5} - \frac{1}{5}$. Remind students that the denominator tells them the size of the piece and the numerator tells them how many pieces they have. If the denominators are the same, they only need to add or subtract the numerators to find the sum or difference.

• Review the term *multiple* with students. Write the number 8 on the board. Ask students to tell you the first several multiples of 8. Write them on the board.

• With students, go through the exposition on **page 78** of the student text.

• Some students may notice that you can always find a common denominator by multiplying the denominators together. Remind them that it is usually best to use the lowest common denominator because there will be less or no simplifying at the end.

• Review the terms *factor*, *common factor*, and *greatest common factor* with students.

• Finish going through the exposition on **page 79**. Be sure students understand that answers that are not in lowest terms, or that are left as improper fractions rather than as mixed numbers, are mathematically correct. However, for the sake of clarity, it is usually best to put answers in their lowest terms and to write answers that are improper fractions as mixed numbers. You might write a few improper fractions and their mixed number equivalents on the board. Ask students which form gives a better sense of each number.

Revisiting the Try This

C. Students apply what they learned in the exposition, about finding a common denominator and that any common multiple of the denominators can be used as a common denominator, to the problem posed in the **Try This**.

Using the Examples

• Have students work in pairs. Have one student in each pair do **example 1 a)** and the other do **example 1 b)**. Each student should then explain his or her work to his or her partner.

• Work through **example 2 a)** with students. In particular, discuss that when you add mixed numbers, it is usually easier to do the whole number and fraction parts separately. If an answer turns out to be a whole number together with an improper fraction, you must rewrite it as a mixed number. Make sure students are comfortable with renaming a whole number as a fraction when necessary for subtracting.

• Work through **example 2 b)** with students. For mixed number subtraction situations, make sure students understand when you can subtract the whole number parts and fraction parts separately and when you cannot (when the first fraction is less than the second fraction, or if you are subtracting a mixed number from a whole number). Make sure students are aware that, in this example, the whole number 10 is renamed as a mixed number with a fraction that is greater than the second fraction, so that the whole number and fraction parts can be subtracted separately.

Practising and Applying

Teaching points and tips

Q1: You might encourage students to discuss the strategies they used to create equivalent fractions.

Q3 a): Some students may not recognize that they need to add down the columns (not across the rows) to find the time spent exercising each day.

Q 4: Encourage students to see that they could add Bijoy's and Arun's values and then subtract from $33\frac{1}{2}$, or subtract each value, one at a time.

Q7: Make sure students are aware of how to solve a Magic Square: First, determine the Magic Sum (the sum of any row, column or diagonal). Next, find each row, column, and diagonal that has two numbers filled in. Find each missing number by finding the sum of the given numbers and subtracting the result from the Magic Sum.

Q8: This question highlights one of the reasons why it is better to write a fraction as a mixed number rather than as an improper fraction.

Common errors

• Some students will just add the three given numbers in **question 4**. Have students note that they are looking for the number they can add to $8\frac{1}{2}$ and $13\frac{1}{4}$ to get a total of $33\frac{1}{2}$.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can add fractions and mixed numbers symbolically
Question 2	to see if students can subtract fractions and mixed numbers symbolically
Question 4	to see if students can solve a problem using addition and subtraction of mixed numbers

Answers

A. i) *Sample response*:
About $\frac{3}{4}$ cup, because $\frac{2}{3}$ is a little less than $\frac{3}{4}$,
and $\frac{1}{8}$ is a little more to add.
ii) $\frac{19}{24}$ cup

B. i) *Sample response*:
About $\frac{1}{2}$ cup, because $\frac{2}{3}$ is a little more than $\frac{1}{2}$,
and $\frac{1}{8}$ is a small part of $\frac{1}{2}$.
ii) $\frac{13}{24}$ cup

C. i) 24, 48, 72, ... (any multiple of 24 except 0)
ii) 24; It is the lowest and that means I will not have to simplify at the end.

1. a) $\frac{17}{24}$ **b)** $1\frac{5}{18}$ **c)** $10\frac{1}{20}$ **d)** $7\frac{3}{10}$

2. a) $\frac{5}{24}$ **b)** $\frac{11}{30}$ **c)** $1\frac{17}{24}$ **d)** $4\frac{5}{8}$

3. a) Day 1: $1\frac{1}{12}$ h, Day 2: $1\frac{7}{12}$ h **b)** $\frac{1}{2}$ h

4. $11\frac{3}{4}$ laps

5. a) Yes;
[*Sample response*:
$(\frac{3}{4} + \frac{1}{2}) + \frac{3}{8} = 1\frac{5}{8}$ and $\frac{3}{4} + (\frac{1}{2} + \frac{3}{8}) = 1\frac{5}{8}$
$(2\frac{1}{2} + 1\frac{2}{3}) + 3\frac{3}{4} = 7\frac{11}{12}$ and $2\frac{1}{2} + (1\frac{2}{3} + 3\frac{3}{4}) = 7\frac{11}{12}$]

b) No;
[*Sample response*:
$(\frac{2}{3} - \frac{1}{3}) - \frac{1}{5} = \frac{2}{15}$ and $\frac{2}{3} - (\frac{1}{3} - \frac{1}{5}) = \frac{8}{15}$
$(2\frac{7}{8} - 1\frac{2}{3}) - \frac{5}{6} = \frac{9}{24}$ and $2\frac{7}{8} - (1\frac{2}{3} - \frac{5}{6}) = 2\frac{1}{24}$]

7.

$1\frac{2}{3}$	$7\frac{1}{2}$	$3\frac{1}{3}$
$5\frac{5}{6}$	$4\frac{1}{6}$	$2\frac{1}{2}$
5	$\frac{5}{6}$	$6\frac{2}{3}$

[**8.** *Sample response*:
With mixed numbers, you can round up or down to the nearest whole numbers and then use those whole numbers to estimate.
For example:
$7\frac{5}{24} + 4\frac{5}{8} \approx 7 + 5 = 12$
$10\frac{11}{30} - 1\frac{17}{24} \approx 10 - 2 = 8$]

Supporting Students

Struggling students

• If students are struggling with **question 7**, scaffold the question by first calculating the Magic Sum ($12\frac{1}{2}$), then asking students to find what needs to be added to ($1\frac{2}{3} + 5$) to get $12\frac{1}{2}$, and so on. Or, provide them with the centre number ($4\frac{1}{6}$) and the number in the bottom right corner ($6\frac{2}{3}$).

Enrichment

• For **question 7**, you might challenge students to make up their own Magic Square for others to solve. Remind them that they must provide enough numbers for others to be able to determine the Magic Sum, as well as a starting place for filling in the other missing numbers.

4.2.1 EXPLORE: Multiplying Fractions

Curriculum Outcomes	Outcome Relevance
8-B10: Multiply: fractions — develop algorithm (pictorially and symbolically) • construct concrete and pictorial models to develop meaning	This optional exploration provides students with a pictorial representation of multiplication of fractions to prepare them for the next lesson.

Pacing	Materials	Prerequisites
30 min	• Grid paper or Small Grid Paper (BLM) (Optional) • Coloured pencils (optional)	• multiplying whole numbers • multiplying a fraction by a whole number

Main Points to be Raised

- You can model the product of two fractions by creating a rectangle whose dimensions are those fractions.
- The product of two fractions is a fraction whose numerator is the product of the two numerators and whose denominator is the product of the two denominators.

Exploration

• Work through the introduction (in white) with students. Make sure they understand that they are dividing a unit square (i.e., a 1 × 1 square, which has an area of 1). You may wish to begin by asking what would be the area of a square that is $1 \times \frac{3}{4}$, and then ask if a square that is $\frac{2}{3} \times \frac{3}{4}$ would have more or less area.

• Have students work, alone, in pairs, or in small groups for **parts A and B**. You may wish to give them an example of how to identify the area that is coloured with both colours.

For example, if students colour the top 2 out of 3 rows (for $\frac{2}{3}$) and the first 3 out of 4 columns (for $\frac{3}{4}$), then the rectangle that is 2 small squares by 3 small squares in the upper left corner of the unit square is coloured with both colours. If students colour the bottom 2 rows and the last 3 columns, then the rectangle that is 2 small squares by 3 small squares in the lower right hand corner is coloured with both colours. No matter how they colour the $\frac{2}{3}$ (rows) and $\frac{3}{4}$ (columns), 6 small squares will be coloured with both colours.

You may wish to provide the students with grid paper to make it easier to draw the diagrams. While you observe students at work, you might ask questions such as the following:

• *In **part B i)**, into how many parts is the grid divided? How do you know?* (The grid is divided into 10 equal parts — there are 2 rows and 5 columns.)

Note: There might also be 5 rows and 2 columns, depending on how students draw the grid. This will not affect the results.

• Discuss **parts A and B** with students to make sure they are proceeding successfully.

• Ask students to complete **part C**. Discuss their answers with the class.

Observe and Assess

As students work, notice the following:

• Do they successfully draw and colour grids to model and find the products for **part B**?

• Do they recognize that the denominator of one fraction tells the number of rows and the denominator of the other fraction tells the number of columns?

• Do they recognize that the product of the numerators gives them the number of parts that are coloured with both colours and the product of the denominators gives them the number of parts into which the grid is divided?

• Do they understand how the grid model explains the multiplication of fractions?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and answer these questions.

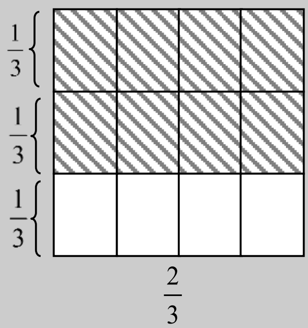
• *How do you know the number squares that will be in your grid after you divide it?*

• *What does the part of the grid that is coloured with both colours represent?*

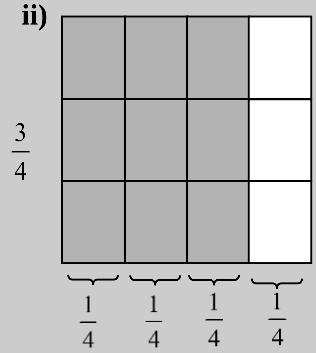
• *How can you find the product of two fractions without drawing a grid?*

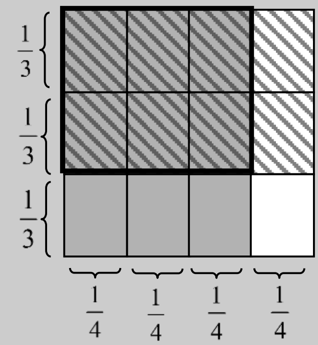
Answers

A. i)



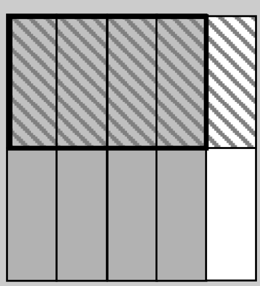
ii)





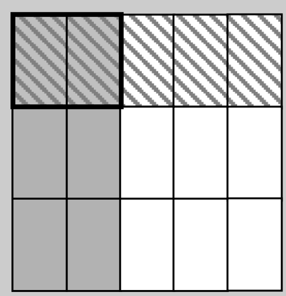
iii) 12 **iv)** 6 **iv)** $\frac{6}{12}$ **v)** $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$

B. i)



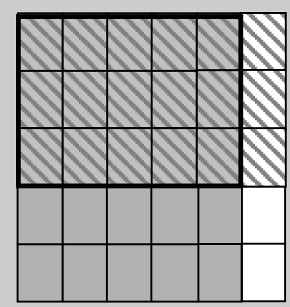
$\frac{4}{10}$

ii)



$\frac{2}{15}$

iii)



$\frac{15}{30}$

C. i) $\frac{A}{B} \times \frac{C}{D} = \frac{A \times C}{B \times D}$

ii) The denominators of the fractions I multiply are the dimensions of the grid. The numerators are the dimensions of the area that has been coloured twice.

The denominator of the product is the product of the dimensions of the grid. The numerator of the product is the product of the dimensions of the area that has been coloured twice.

Supporting Students

Struggling Students

• If students are struggling in **part A** to see the relation between the part of the grid that is coloured with both colours and the product of $\frac{2}{3} \times \frac{3}{4}$:

- First ask them to show the part of the grid that is $1 \times \frac{3}{4}$.

- Next, ask them to show the part of the grid that is $\frac{1}{3} \times \frac{3}{4}$.

- Finally, ask them to show the part of the grid that is $\frac{2}{3} \times \frac{3}{4}$.

4.2.2 Multiplying Fractions

Curriculum Outcomes	Outcome relevance
8-B10 Multiply: fractions — develop algorithm (pictorially and symbolically) • construct concrete and pictorial models to develop meaning • understand that "of" means multiplication and can be shown by comparing results in questions such as $\frac{1}{4}$ of 8 and $\frac{1}{4} \times 8$ • multiply a whole number by a fraction less than 1 (e.g., $4 \times \frac{1}{3}$ uses repeated addition) • multiply a fraction less than 1 by another fraction especially when the numerator is 1 (e.g., $\frac{1}{4}$ of $\frac{2}{3}$) **8-B12 Fractions: estimate and mentally compute products and quotients** • appropriate number situations to use for multiplication include: - a fraction by a whole number when the numbers are compatible - any two proper fractions when the numerators and denominators are relatively simple to work with - a whole number by a mixed number (distributive property should be used) • use estimation to check reasonableness of results	The ability to multiply fractions is a life skill. Since multiplying by a whole number makes a number greater, it is especially important for students to understand why the answer is less than either of the fractions being multiplied when they multiply two proper fractions. Students should not just apply the algorithm without understanding.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM) (optional) • Coloured pencils (optional)	• multiplying whole numbers • factoring a whole number • writing a fraction in lowest terms

Main Points to be Raised

• You can use an area grid to model the multiplication of fractions. The model shows why $\frac{A}{B} \times \frac{C}{D} = \frac{A \times C}{B \times D}$.

• You can think of multiplying a whole number and a fraction as repeated addition of the fraction. Or, you can write the whole number as a fraction and then multiply the fractions.

• When you multiply fractions, it is usually helpful to write the final answer in lowest terms. If the answer is an improper fraction, write it as a mixed number.

• You can simplify as you go by dividing any numerator and any denominator by a common factor. The product will be in lower or lowest terms.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know what denominator to use in* ***part A ii)****?* (I am finding a fraction of the Asian population. 4 billion is the whole, which makes it the denominator.)

• *Why were the denominators different for* ***part ii) and part iii)****?* (The wholes were different; one was the population of Asia and the other was the population of the world.)

Source: www.world-gazetteer.com

The Exposition — Presenting the Main Ideas

• With students, go through the exposition on **page 83** of the student text. Discuss how the grid model explains why $\frac{A}{B} \times \frac{C}{D} = \frac{A \times C}{B \times D}$.

• You may wish to do a simple example with the students, e.g., $\frac{1}{2} \times \frac{4}{5}$, to show that, whether they first do the multiplication and then write the answer in lowest terms, or whether they simplify as they go, they will still get the same final answer. Continue to go through the exposition on **page 84**.
• Remind students that they learned in Class VII to think of multiplication of a fraction by a whole number as repeated addition. Show an example. Point out that this new multiplication algorithm gives the same product as solving the problem by repeated addition.

Revisiting the Try This

B. Students apply what they learned in the exposition about multiplying fractions to the problem posed in the **Try This**.

Using the Examples

• Present the problems in **examples 1 and 2** to students. Let them try the problems alone or in pairs and compare their work to the solutions in the student text.

Practising and Applying

Teaching points and tips

Q1: You may wish to provide grid paper for students to use for drawing their grids.

Q2: Some students may choose to simplify the multiplication before multiplying, while others will first multiply and then write the product in lowest terms.

Q3: Allow some time for students to share their estimation strategies with others.

Q4: Some students may wish to draw a grid diagram for this problem.

Q5: Encourage students to think of the factors of 6 and the factors of 40 to get ideas for fractions they might use.

Q6: Help students to see that this problem asks them to find $\frac{3}{4}$ of $\frac{1}{7}$.

Q7: This question might be assigned only to selected students. It requires a subtraction, a multiplication, and an addition of fractions.

Q8: This is an important generalization: whether you calculate the product using decimals or using fractions, the product is the same.

Q9: You might discuss with the class why this happens (if you "simplify as you go", all numerators and denominators except the first numerator and the last denominator will simplify to 1).

Q10: Karatage, or Karats, is a measure of the purity of gold, measured in 24ths. 24 K gold is $\frac{24}{24}$ gold, which is pure gold.

Q11: This question highlights the concept that when you multiply proper fractions, the product is less than either factor. This is a critical notion in this lesson.

Common errors

• Some students will leave their answers to **question 3** as improper fractions. Remind them that it is better to write their answers as mixed numbers.

• When they multiply a whole number and a fraction, some students will multiply the whole number by the denominator of the fraction instead of by the numerator. Encourage them to estimate to see whether they answer makes sense.

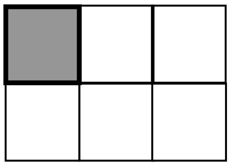
Suggested assessment questions from Practising and Applying

Question 1	to see if students can model the multiplication of fractions with a grid model
Question 2	to see if students can multiply two fractions
Question 3	to see if students can multiply a whole number by a fraction and apply estimation strategies
Question 10	to see if students can solve a problem that involves multiplying fractions

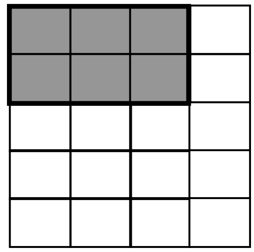
Answers

A. i) $\frac{4}{7}$ **ii)** $\frac{1}{4}$ **iii)** $\frac{1}{7}$	**B.** $\frac{4}{7} \times \frac{1}{4} = \frac{4}{28} = \frac{1}{7}$

1. a)



b)



2. a) $\frac{4}{15}$ **b)** $\frac{5}{16}$ **c)** $\frac{1}{2}$

d) $\frac{9}{35}$ **e)** $\frac{1}{5}$ **f)** $\frac{8}{15}$

3. a) About 12; $12\frac{4}{5}$ **b)** About 8; $8\frac{1}{4}$

c) About 20; $21\frac{7}{8}$ **d)** About 8; $8\frac{1}{3}$

4. $\frac{1}{5}$

5. *Sample response*: $\frac{2}{5} \times \frac{3}{8} = \frac{1}{40} \times \frac{6}{1} = \frac{3}{4} \times \frac{2}{10} = \frac{6}{40}$

6. $\frac{3}{28}$

7. $\frac{3}{4}$ day

8. a) 0.12

b) $\frac{3}{10} \times \frac{4}{10} = \frac{12}{100}$

c) They are equal.

9. a) $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{1}{5}$

The numerator of the first fraction is the numerator of each product. The denominator of the last fraction is the denominator of each product.

b) $\frac{1}{100}$

10. a) i) 5000 g **ii)** 7500 g **iii)** 9170 g

b) $450 U.S.

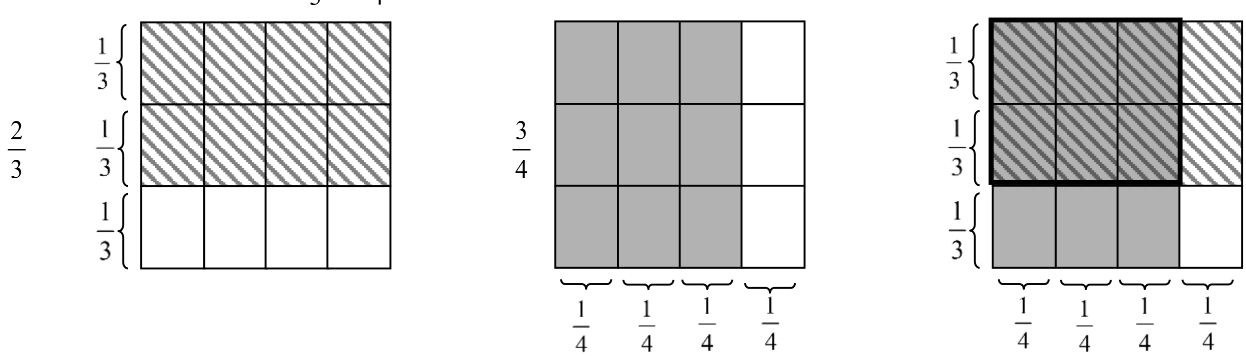
11. The product is less than each fraction; [Multiplying a fraction by a fraction means finding a fraction of a fraction, or part of a fraction.]

Supporting Students

Struggling students

• If you did not do **Explore lesson 4.2.1**, you might show students the more step-by-step approach to using a grid model to multiply fractions that was the focus of that lesson.

For example, to multiply $\frac{2}{3} \times \frac{3}{4}$



$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$

• If students are struggling with **question 7**, begin by asking them what fraction of the day Chandra was awake ($\frac{2}{3}$), then what fraction of that time he was at home ($\frac{5}{8} \times \frac{2}{3}$, or $\frac{10}{24}$). Then ask what they need to do to find the fraction of the whole day that Chandra spent at home (add $\frac{10}{24}$ and $\frac{1}{3}$).

Enrichment

• For **question 5**, challenge students to find as many different fractions pairs as they can find with the product $\frac{6}{40}$.

CONNECTIONS: The Sierpinksi Triangle

• This optional connection can be used as enrichment for some students. It provides an interesting geometric application of the multiplication of fractions.

• The word fractal was first used by Benoit Mandelbrot in 1975. The word was derived from the Latin *fractus*, which means broken or fractured.

Answers

1. a) $\frac{3}{4}$

b) i) $\frac{3}{4}$ **ii)** $\frac{9}{16}$

c) $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$; [$\frac{3}{4}$ (step 3) of $\frac{3}{4}$ (step 2) of the large triangle is not coloured.]

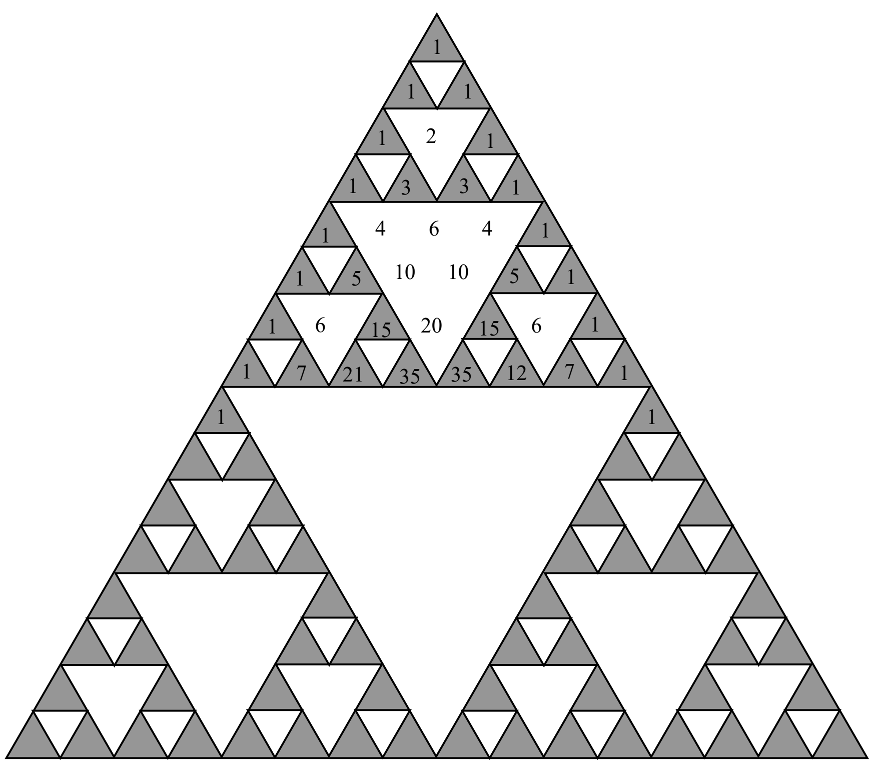
2. a) $\frac{27}{64}$; [$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$]

b) I counted the total number of small white triangles (64) and the number that were not coloured (27).

$\frac{27}{64}$ of the large triangle was not coloured.

Enrichment

• If you colour the odd numbers in *Pascal's triangle*, you will see the *Sierpinski triangle*.



4.2.3 Multiplying Mixed Numbers

Curriculum Outcomes	Outcome relevance
8-B10 Multiply: fractions — develop algorithm (pictorially and symbolically) • construct concrete and pictorial models to develop meaning **8-B12 Fractions: estimate and mentally compute products and quotients** • appropriate number situations to use for multiplication include: - a whole number by a mixed number (distributive property should be used) • use estimation to check reasonableness of results • round to nearest whole and sometimes to nearest half to reach rough estimates	The ability to multiply with mixed numbers is a life skill. In many situations an estimate is sufficient, so students need to develop good estimation skills. Students need to understand why the procedures work and not just apply rules without understanding.

Pacing	Materials	Prerequisites
1 h	• Coloured pencils (optional)	• multiplying whole numbers • multiplying a whole number by a fraction • multiplying fractions • using the distributive property of multiplication over addition

Main Points to be Raised

- Multiplying with mixed numbers involves the same strategies as multiplying fractions and multiplying whole numbers.
- The distributive property of multiplication over addition is useful for multiplying with mixed numbers.

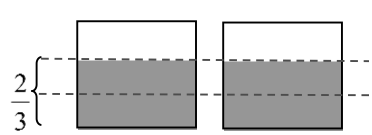
Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

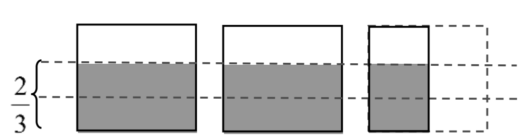
- *How do you know if Indra Maya earned more or less than Choki earned?* (Since $1\frac{1}{2}$ is more than 1, Indra Maya earned more than Choki.)
- *How might you estimate your answer?* (1 × Nu 8000 is Nu 8000, and 2 times Nu 8000 is Nu 16,000, so $1\frac{1}{2}$ times Nu 8000 will be between Nu 8000 and Nu 16,000.)

The Exposition — Presenting the Main Ideas

- Ask students how they could draw a model to show $\frac{2}{3} \times 2$. Draw the model on the board: show two square regions, each divided into thirds with two of the thirds shaded.



- Ask how the model will change if you show $\frac{2}{3} \times 2\frac{1}{2}$. Students should see that you need to show another $\frac{1}{2}$ region, divided into thirds with two thirds shaded. Discuss with the students that the model shows $\frac{2}{3}$ of 2 together with $\frac{2}{3}$ of $\frac{1}{2}$.



• With students, go through the exposition on **page 88** of the student text. Draw attention to the relationship between what the model shows and the distributive property, i.e., to find $\frac{2}{3} \times 2\frac{1}{2}$, you can find $\frac{2}{3} \times 2$ and $\frac{2}{3} \times \frac{1}{2}$, and then combine them.

Revisiting the Try This

B. Students apply what they learned in the exposition about multiplying a mixed number and a fraction to the problem posed in the **Try This.**

Using the Examples

• Assign students to pairs. Have one student in each pair become the expert on **example 1** and have the other student become an expert on **example 2**. Each student should then explain his or her example to his or her partner.

• Ask pairs of students to read through **solutions 1 and 2** of **example 3**. Ask them to choose the solution that most closely matches what they would have done and say why they would have done it that way.

• Present the problem in **example 4**. Let students work through it and then check their work against the solution in the student text. Some students might realize that 750 g is $\frac{3}{4}$ of 1 kg and multiply $\frac{8}{3}$ by $\frac{3}{4}$.

Practising and Applying

Teaching points and tips

Q2: Remind students that they should write any answers that are improper fractions as mixed numbers.

Q6: Remind students that multiplying a number by $\frac{1}{6}$ is the same as dividing the number by 6.

Q7: You might encourage students to share their estimation strategies with others.

Q8: This is an important generalization: whether it is calculated using decimals or using fractions, the product is the same.

Q9: This question might be assigned only to selected students. They need to realize that if the denominator of the new fraction is 5, the result is a whole number.

Q11: This question highlights the concept that a multiplication of mixed numbers can be broken down into multiplying whole numbers, multiplying fractions and whole numbers, and multiplying fractions. This is an important notion from this lesson.

Common errors

• If students multiply by changing the mixed numbers to improper fractions in **question 2**, they might be more likely to leave their answers as improper fractions. Encourage them to write each answer as a mixed number.

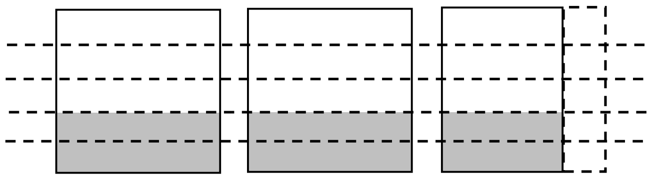
Suggested assessment questions from Practising and Applying

Question 1	to see if students can model the multiplication of a fraction and a mixed number.
Question 2	to see if students can calculate products involving fractions, whole numbers, and mixed numbers
Question 3	to see if students can solve a problem that involves multiplying mixed numbers
Question 10	to see if students can think of a situation that calls for the multiplication of mixed numbers

Answers

A. Nu 12,000	**B.** $1\frac{1}{2} \times 8000 = 12,000$

1. *Sample response*:



2. a) $\frac{7}{12}$ **b)** 72 **c)** $2\frac{2}{7}$

d) 8 **e)** $1\frac{1}{3}$ **f)** 355

g) 16 **h)** $4\frac{7}{32}$

3. a) $\frac{7}{8}$ cups **b)** $4\frac{3}{8}$ cups

4. 33 eggs

5. 5 h

6. *Sample response*: About 8800 m

7. *Sample responses*:

a) About 333 **b)** About $20\frac{1}{2}$

8. a) $\frac{81}{100}$
b) $0.3 \times 2.7 = 0.81$
c) They are the same.

9. *Sample response*: $1\frac{1}{5}$

10. *Sample response*:
To find the area of a wall or a floor with dimensions $3\frac{1}{2}$ m × $2\frac{1}{3}$ m.

[**11.** *Sample response*:
Multiplying mixed numbers involves multiplying whole numbers and multiplying fractions. If you always change the mixed numbers to improper fractions before multiplying, you only have to know how to multiply fractions.]

Supporting Students

Struggling students

- If students are struggling to estimate their answer in **question 6**, you might suggest that they consider $1\frac{1}{6} \times 6000$ and then $1\frac{1}{6} \times 9000$. Since 7554 is about halfway between 6000 and 9000, a good estimate would be about half way between those answers.

Enrichment

- For **question 9**, you might challenge students to find several possibilities.
- You might ask students to invent a game that requires the multiplication of mixed numbers.

4.2.4 Dividing Fractions With a Common Denominator

Curriculum Outcomes	Outcome relevance
8-B11 Divide: fractions — develop algorithm (pictorially and symbolically) • derive a personal algorithm from carefully chosen examples, e.g.: - a simple fraction divided by simple fraction where the numerator of the divisor is 1 and both denominators are the same (e.g., $\frac{5}{6} \div \frac{1}{6}$ asks, "How many $\frac{1}{6}$ s are in $\frac{5}{6}$?") - a simple fraction divided by a simple fraction where the numerator of the divisor is one and the fractions are compatible (e.g., $\frac{1}{2} \div \frac{1}{4}$) • use a number line to model division **8-B12 Fractions: estimate and mentally compute products and quotients** • appropriate number situations to use for division include: - a simple fraction divided by a simple fraction when the denominators are the same - a simple fraction divided by a simple fraction	Students who understand how to divide fractions using common denominators can visualize how the division of fractions works. It also provides them with an alternative to the traditional fraction division algorithm, the invert and multiply algorithm, which they will learn in the next lesson.

Pacing	Materials	Prerequisites
1 h	• Coloured pencils (optional) • Pattern blocks or Pattern Block Fraction Pieces (BLM) (optional)	• dividing whole numbers • finding common denominators • using an area region model for fractions • dividing a fraction by a whole number

Main Points to be Raised

• Dividing fractions with the same denominator is similar to dividing whole numbers.

• You can use area models and number lines to model fraction division.

• When you divide fractions with the same denominator, you only have to divide the numerators.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How can you figure out how long it takes Choki to walk to school?* (If it takes $\frac{3}{4}$ h to walk halfway to school,
it would take twice that long, $\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$ h, to walk all the way to school.)

• *How can you figure out how long it will take Choki to go both ways?* (To find out how long it will take her to go both ways, I have to double the amount of time it takes for her to walk to school.)

• *How can you figure out how many half hours it will take?* (Count the number of half hours in the total time.)

The Exposition — Presenting the Main Ideas

• Begin by asking ten students to come forward. Have students pair up, and ask the class how many pairs of students there are. Write on the board an equation that illustrates the problem, $10 \div 2 = ?$. Say "This division is asking us to find how many 2s there are in 10".

• With students, go through the area model in the exposition on **page 92** of the student text. Make sure students can make sense of the model. Now go through the number line model. Make sure students fully understand that the last jump of $\frac{1}{6}$ leads to a quotient of $2\frac{1}{2}$, not $2\frac{1}{6}$, since the $\frac{1}{6}$ is $\frac{1}{2}$ of $\frac{2}{6}$ (the size of the unit jump).

• Finish going through the exposition, paying particular attention to last part about how to deal with fractions that have different denominators by finding equivalent fractions with the same denominator and then dividing the numerators.

Revisiting the Try This

B. Students apply what they learned in the exposition about dividing fractions with common denominators to the problem posed in the **Try This**.

Using the Examples

• Present the three questions in the example to the students. Ask each student to choose two of the questions to solve. Then each student can compare his or her work to what is shown in the matching solution. Suggest that they may wish to solve and read through the other problem as well.

Practising and Applying

Teaching points and tips

Q1 b): Some students may not recognize the arrow from $\frac{1}{6}$ to 0 as being "a half of $\frac{1}{3}$". They may need help to write the answer to the division question.

Q2: Students might draw an area model or they might use a number line.

Q4 c) to f): Some students may choose to draw a model to help them. Others may write the fractions as equivalent fractions with a common denominator and then divide the numerators.

Q5: This question provides students with another way to model the division of fractions. You might provide pattern blocks to students to work with.

Q6: This is an important generalization about dividing fractions with a common denominator.

Common errors

• Some students will write the division correctly in **question 1 b)** but will write the answer as $2\frac{1}{6}$. Remind them that they are determining what part of $\frac{1}{3}$ that last arrow represents, not what part of the whole it represents.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can recognize a model for dividing fractions
Question 4	to see if students can divide one fraction by another fraction
Question 5	to see if students can solve a problem involving the division of fractions

Answers

A. 6 half hours; *Sample response*:

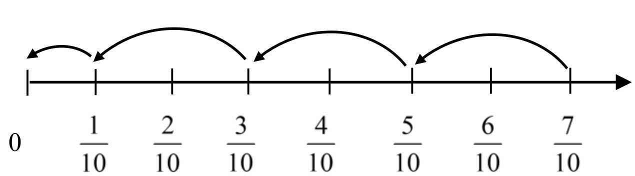
It will take another $\frac{3}{4}$ of an hour to walk to school.

$\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$ and $1\frac{1}{2} = \frac{3}{2}$, which is 3 half hours to walk one way and 6 half hours to go both ways.

B. $2(\frac{3}{4} + \frac{3}{4}) \div \frac{1}{2} = 2(1\frac{1}{2}) \div \frac{1}{2} = 3 \div \frac{1}{2} = 6$

1. a) $\frac{2}{4} \div \frac{1}{8} = 4$ **b)** $\frac{5}{6} \div \frac{1}{3} = 2\frac{1}{2}$

2. *Sample response*:



3. $1\frac{1}{5}$; [$\frac{3}{5} \div \frac{1}{2} = \frac{6}{10} \div \frac{5}{10} = 6 \div 5 = \frac{6}{5} = 1\frac{1}{5}$]

4. a) 3 **b)** 2 **c)** $1\frac{3}{4}$

d) 4 **e)** $2\frac{7}{10}$ **f)** $10\frac{1}{2}$

5. $1\frac{1}{2}$; $\frac{1}{2} \div \frac{1}{3} = 1\frac{1}{2}$

[**6.** Dividing 6 parts of a whole by 2 equal-sized parts of a whole will always give a quotient of 3.]

Supporting Students

Struggling students

• If students are struggling with any parts of **question 4**, you might suggest that they draw a number line model or an area model.

Enrichment

• For **question 5**, you might challenge students to write and solve other division questions using the pattern blocks.

For example, write a fraction division equation to model the solution to how many triangles are in a trapezoid.

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4.2.5 Dividing Fractions in Other Ways

Curriculum Outcomes	Outcome relevance
8-B11 Divide: fractions — develop algorithm (pictorially and symbolically) • derive a personal algorithm from carefully chosen examples, e.g.: - a simple fraction divided by a whole number (e.g., for $\frac{1}{2} \div 3$, divide $\frac{1}{2}$ into 3 equal parts. What does each part represent?) - a whole number divided by a simple fraction (e.g., $4 \div \frac{1}{2}$, asks, "How many $\frac{1}{2}$s there are in 4?") - a simple fraction divided by simple fraction where the numerator of the divisor is 1 and both denominators are the same (e.g., $\frac{5}{6} \div \frac{1}{6}$ asks, "How many $\frac{1}{6}$s are in $\frac{5}{6}$?") - a simple fraction divided by a simple fraction where the numerator of the divisor is 1 and the fractions are compatible (e.g., $\frac{1}{2} \div \frac{1}{4}$) • use a number line to model division • apply prior knowledge of reciprocal **8-B12 Fractions: estimate and mentally compute products and quotients** • appropriate number situations to use for division include: - a simple fraction divided by a whole number - a whole number divided by a fraction - a simple fraction divided by a simple fraction when the denominators are the same - a simple fraction divided by a simple fraction	It is important for students to understand why the procedures for the division of fractions work, and not just to apply rules with no understanding. The invert and multiply rule for dividing fractions is the rule that is most useful in higher class mathematics when algebraic expressions are involved.

Pacing	Materials	Prerequisites
1 h	• Coloured pencils (optional)	• dividing whole numbers • finding a common denominator • using an area model for fractions • dividing a fraction by a whole number • dividing fractions with a common denominator

Main Points to be Raised

• To divide a fraction by a whole number, you can figure out the size of each share if the fraction is equally divided into that whole number of pieces.

• To divide a whole number by a fraction, you can think about how many of that fraction are in the whole number. If you divide by a unit fraction, you can multiply by the denominator of that fraction. If you divide by a non-unit fraction, you can multiply by the denominator of the fraction and then divide by its numerator.

• Whenever you divide by a fraction, the result is the same as multiplying by the reciprocal of that fraction.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How many hours would Tshering practise in 4 days?* (He would practise 3 h in 4 days. $4 \times \frac{3}{4} = 3$.)

• *How can you use that information to help you find how many days it would take to practise 9 h?* (If he practises 3 h in 4 days, he would practise 6 h in 8 days and 9 h in 12 days.)

The Exposition — Presenting the Main Ideas

• The exposition is constructed to deal with different types of divisions involving fractions. All of the divisions could be described with one rule: multiply by the reciprocal. As students see each different situation, they will develop a better understanding of why the rule makes sense.

• You may need to go through the parts slowly and provide additional examples of each type of situation to make sure that students understand the processes that are modelled.

• Draw a rectangle area model on the board. Tell students that it represents 1 whole. Ask how we could show $1 \div 4$ (divide the rectangle into 4 equal parts).

• With student, go through the part of the exposition about dividing a fraction by a whole number. Discuss with students how the diagrams illustrate the given division. Make sure students can see why, for any fraction divided by a whole number, the following is true: $\frac{A}{B} \div C = \frac{A}{B \times C}$. You might show another example, such as

$\frac{5}{6} \div 3$,

by modelling $\frac{5}{6}$ and then splitting it into 3 equal parts.

• With students, go through the part of the exposition about dividing a whole number by a fraction. Discuss how the diagrams illustrate the given divisions. Make sure students can see that for a whole number divided by a fraction, $A \div \frac{B}{C} = A \times \frac{C}{B}$ (i.e., dividing a whole number by a fraction gives the same result as multiplying the whole number by the *reciprocal* of that fraction). Again, you may wish to follow up with another example. You might model $5 \div \frac{3}{4}$ by modelling $5 \div \frac{1}{4}$ on a number line and seeing that there are 20 groups of $\frac{1}{4}$ in 5. Show that there are only one third as many groups of $\frac{3}{4}$ in 5.

• Go through the last part of the exposition on **page 97** of the student text. Show at least one more example, e.g., $\frac{5}{8} \div \frac{2}{9}$. Note that $\frac{5}{8} \div \frac{1}{9} = \frac{5}{8} \times 9$ and show that $\frac{5}{8} \div \frac{2}{9}$ is half as much, or $\frac{5}{8} \times \frac{9}{2}$. Write the generalization that for dividing any fraction by a fraction, the following is true: $\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C}$ (i.e., dividing by a fraction gives the same result as multiplying by the reciprocal of that fraction).

Revisiting the Try This

B. Students apply what they learned in the exposition about dividing a whole number by a fraction to the problem posed in the **Try This**.

Using the Examples

• Present to students the problems in the three examples. Ask each student to choose two of the problems to solve. Then each student can compare his or her work to what is shown in the matching example. Suggest that students may wish to read through the other example as well.

Practising and Applying

Teaching points and tips

Q2: Students might use an area model or they might draw a number line.

Q3: You might encourage students to draw an illustration of one or two of their solutions to show that their answers make sense.

Q7: This question highlights the relationship between dividing by a fraction and multiplying by its reciprocal. This is a critical notion brought out in this lesson.

Common errors

• Some students will have difficulty determining the division equation in **question 1 a)**. You might first draw the picture showing just the $\frac{3}{4}$. Next, show how to divide the $\frac{3}{4}$ by 3. Then have students explain what fraction of the whole the shaded portion represents.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can relate a model to a fraction division
Question 2	to see if students can draw a model for a fraction division situation and solve it
Question 3	to see if students can calculate fraction divisions
Question 5	to see if students can solve a problem involving division by a fraction

Answers

A. i) 45 min **ii)** 12 days

B. $9 \div \frac{3}{4} = 12$

1. a) $\frac{3}{4} \div 3 = \frac{3}{12}$ or $\frac{1}{4}$ **b)** $3 \div \frac{3}{8} = 8$

2. a) 4

b) *Sample response*:

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

3. a) 8 **b)** $\frac{1}{8}$ **c)** $\frac{3}{4}$

d) $\frac{4}{5}$ **e)** $\frac{9}{10}$ **f)** $2\frac{1}{10}$

4. 3 subjects

5. $\frac{7}{40}$ m

6. 4 parts

[**7.** *Sample response*:

$A \div \frac{B}{C} = A \times \frac{C}{B} \rightarrow 7 \div \frac{1}{5} = 7 \times \frac{5}{1} = 7 \times 5 = 35$

You can first think about how many $\frac{1}{5}$ s are in 1.

There are five $\frac{1}{5}$ s in 1.

Then, you can multiply 5 by 7 to find the number of $\frac{1}{5}$ s in 7. There are $5 \times 7 = 35$ $\frac{1}{5}$ s in 1.

So, dividing by $\frac{1}{5}$ is the same as multiplying by 5, which is the reciprocal of $\frac{1}{5}$.]

Supporting Students

Struggling students

• If students are having difficulty drawing a picture for **question 2 b)**, you might suggest that they rewrite $\frac{1}{5}$ as an equivalent fraction with a denominator of 10. The question becomes "How many $\frac{2}{10}$ s are there in $\frac{8}{10}$?"

Enrichment

• You might ask students to create problems to match various fraction division computations.

For example, ask them to create a problem that could be solved by dividing, such as $\frac{2}{3}$ by $\frac{3}{4}$.

4.2.6 Dividing Mixed Numbers

Curriculum Outcomes	Outcome relevance
8-B11 Divide: fractions — develop algorithm (pictorially and symbolically) • use a number line to model division • apply prior knowledge of reciprocal **8-B12 Fractions: estimate and mentally compute products and quotients** • appropriate number situations to use for division include: - a mixed number divided by a whole number - a whole number divided by a mixed number - a mixed number divided by a mixed number • use estimation to check reasonableness of results • round to nearest whole and sometimes to nearest half to reach rough estimates	It is important that students understand why the procedures for dividing mixed numbers work, and not just apply rules. In many real-world situations involving mixed numbers, estimation is as important a skill as calculation.

Pacing	Materials	Prerequisites
1 h	None	• dividing whole numbers • dividing fractions

Main Points to be Raised

• Dividing a mixed number by a mixed number is just like dividing fractions.

• Writing mixed numbers as improper fractions allows you to divide them using the same methods as for dividing fractions. The methods include using a common denominator and dividing the numerators, and multiplying by the reciprocal of the divisor.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How could you have predicted that the answer would be less than 5 weeks?* (They did not even have 5 dozen eggs. If they only used 1 dozen each week, the eggs would be gone after 5 weeks. They used more than 1 dozen each week, so the eggs would be gone before 5 weeks passed.)

• *How can you figure out how many dozen eggs the family uses in 2 weeks?* (If they use $1\frac{1}{2}$ dozen eggs in 1 week, in 2 weeks they use $2 \times 1\frac{1}{2} = 3$ dozen eggs.)

The Exposition — Presenting the Main Ideas

• Remind students that they can interpret a division like 8 ÷ 2 as "How many 2s are in 8?" Then ask how they might interpret $3 \div 1\frac{1}{2}$ (How many $1\frac{1}{2}$s are in 3?). Find the answer as a class.

• With students, go through the number line model in the exposition on **page 100** of the student text. Make sure students follow the number line modelling of the division.

• Go through the remaining part of the exposition with students. Have them note that any division calculation involving mixed numbers can be rewritten as a division with improper fractions. Tell them that the methods that worked for dividing proper fractions will also work for improper fractions.

• Have students read through the discussion about multiplying by the reciprocal of the divisor. Allow time for them to ask any questions to clarify their understanding. Another approach you can use is to write the division as a fraction. The fraction will still be the same if it is multiplied by a form of 1.

For example: $\dfrac{8\frac{3}{4}}{1\frac{1}{4}} = \dfrac{\frac{35}{4}}{\frac{5}{4}} \rightarrow \dfrac{\frac{35}{4}}{\frac{5}{4}} \times \dfrac{\frac{4}{5}}{\frac{4}{5}} = \dfrac{\frac{35}{4} \times \frac{4}{5}}{1} = \dfrac{35}{4} \times \dfrac{4}{5}$. So $\dfrac{35}{4} \div \dfrac{5}{4} = \dfrac{35}{4} \times \dfrac{4}{5}$.

Revisiting the Try This

B. Students apply what they learned in the exposition about modelling the division of mixed numbers and strategies for calculating the quotient of mixed numbers to the problem posed in the **Try This**.

Using the Examples

• Assign students to pairs. One student in each pair should become the expert on **example 1** and the other should become the expert on **example 2**. Each should explain his or her example to his or her partner. Be sure to have each student work through both solutions for the example he or she is doing.

Practising and Applying

Teaching points and tips

Q3: Encourage students to use a number line model if it helps them.

Q4: This is an important generalization to discuss with the class.

Q5: This question might be assigned only to selected students. Remind students that 1 ha = 10,000 m^2.

Q9: Encourage students to share their estimation strategies with the class.

Q10: This question highlights an estimation strategy that can be applied to any mixed number division to determine a rough estimate.

Common errors

• When they use a "multiply by the reciprocal of the divisor" strategy for the divisions in **question 3**, some students will use the reciprocal of the dividend instead the reciprocal of the divisor. Encourage students first to estimate the answer (the error will result in the answer being the reciprocal of the correct answer, so even a rough estimate should show the student that their answer does not make sense.)

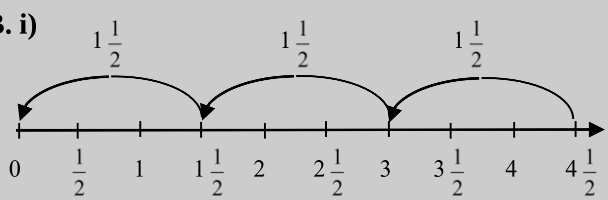
Suggested assessment questions from Practising and Applying

Question 2	to see if students can apply a number line model to a division involving mixed numbers
Question 3	to see if students can calculate quotients in mixed number division
Question 6	to see if students can solve a problem involving the division of mixed numbers
Question 9	to see if students can estimate a quotient for a division of mixed numbers

Answers

A. 3 weeks

B. i)

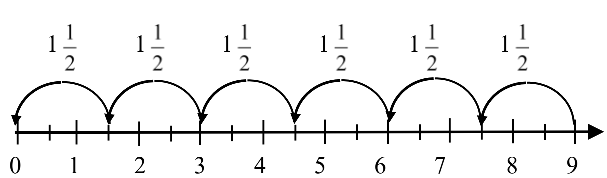


ii) $4\frac{1}{2} \div 1\frac{1}{2} = \frac{9}{2} \div \frac{3}{2} = 9 \div 3 = 3$

$4\frac{1}{2} \div 1\frac{1}{2} = \frac{9}{2} \div \frac{3}{2} = \frac{9}{2} \times \frac{2}{3} = \frac{18}{6} = 3$

1. $4 \div 1\frac{1}{3} = 3$

2.



3. a) 2 **b)** $1\frac{5}{6}$

c) $2\frac{1}{18}$ **d)** $1\frac{1}{3}$

Answers [Continued]

4. a)

A: $\frac{3}{2} = 1\frac{1}{2}$ and $\frac{2}{3}$

B: $\frac{5}{3} = 1\frac{2}{3}$ and $\frac{3}{5}$

b) The quotients are reciprocals.

c) *Sample response*:

$2\frac{1}{4} \div 4\frac{1}{2} = \frac{9}{4} \div \frac{9}{2} = \frac{9}{4} \times \frac{2}{9} = \frac{2}{4}$

$4\frac{1}{2} \div 2\frac{1}{4} = \frac{9}{2} \div \frac{9}{4} = \frac{9}{2} \times \frac{4}{9} = \frac{4}{2}$

5. a) 7 crops

b) No; [$7 \times 1\frac{1}{4} = 8\frac{3}{4}$, so he will not be using $9\frac{1}{2} - 8\frac{3}{4} = \frac{3}{4}$ ha.]

6. 8 instruments

7. $1\frac{1}{4}$ h

[**8.** *Sample response*:

$9 \div 2$ is $4\frac{1}{2}$. Dividing a number greater than 9 by a number less than 2 has a quotient greater than $4\frac{1}{2}$ since you have more altogether, and it is divided into smaller sections.]

9. a)

i) *Sample response*:

about 9; [$6\frac{5}{6}$ is close to 7, and $7 \div \frac{3}{4}$ is $7 \times \frac{4}{3}$, which is $\frac{28}{3}$, or $9\frac{1}{3}$.]

ii) $9\frac{1}{9}$; [*Sample response*: My estimate, 9, was a bit low but it was close.]

b)

i) *Sample response*:

about $5\frac{1}{2}$; [$11\frac{2}{5}$ is close to 11, and $2\frac{1}{9}$ is close to 2, so $11 \div 2$ is $5\frac{1}{2}$.]

ii) $5\frac{2}{5}$; [*Sample response*: My estimate, $5\frac{1}{2}$, was a bit low but it was close.]

[**10. a)** If the dividend is greater than the divisor, the quotient will be greater than 1;

Sample response: $5\frac{1}{2} \div 3 > 1$

b) If the dividend is less than the divisor, the quotient will be less than 1;

Sample response: $3 \div 5\frac{1}{2} < 1$

c) If the dividend and divisor are equal, the quotient will be 1;

Sample response: $6 \div \frac{12}{2} = 1$]

Supporting Students

Struggling students

- Some students may have trouble with **question 5**. Point out that the number of crops the farmer can plant has to be a whole number. Although $9\frac{1}{2} \div 1\frac{1}{4} = 7\frac{3}{5}$, he cannot plant $7\frac{3}{5}$ different crops. In **part b)**, they need to determine how much land the farmer will use if he plants 7 crops that each use $1\frac{1}{4}$ ha.

Enrichment

- Students can investigate patterns that involve the division of fractions.

For example, they might use the pattern below to try to predict the answer to $\frac{99}{100} \div \frac{100}{101}$:

$\frac{2}{3} \div \frac{3}{4} =$

$\frac{3}{4} \div \frac{4}{5} =$

$\frac{4}{5} \div \frac{5}{6} =$

4.3.1 Introducing Rational Numbers

Curriculum Outcomes	Outcome relevance
8-A8 Integers and Rational Numbers: comparing and ordering (fractional and decimal form) • understand that placement of the negative sign does not affect the value (e.g., $\frac{-2}{3}$, $\frac{2}{-3}$, and $-\frac{2}{3}$ are equivalent) • understand that a negative is always less than a positive • understand that positive fractions with common denominators can be compared by examining numerators (e.g., $\frac{3}{8}$ is less than $\frac{5}{8}$ because 3 is less than 5) • understand that positive fractions with common numerators can be compared by examining denominators (e.g., $\frac{3}{5}$ is greater than $\frac{3}{6}$ because 5 is less than 6) • use reference points (1, $\frac{1}{2}$, −1, etc.) • change numbers to a common form	Although negative rational numbers are mostly used in higher math, all students can learn about them to consolidate their understanding of both fractions and integers.

Pacing	Materials	Prerequisites
1 h	None	• comparing and ordering fractions • comparing and ordering decimals • comparing and ordering integers

Main Points to be Raised

• A rational number is a number that can be written as the quotient of two integers (the divisor ≠ 0).

• All integers and all fractions are rational numbers.

• To express a negative rational number, the negative sign can be part of the numerator, part of the denominator, or in front of the fraction.

• Rational numbers can be compared and ordered like integers.

• Any positive rational number is greater than any negative rational number.

• You can write a rational number in fraction, mixed number, and decimal form.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. Some students will need to draw a number line to help them. While you observe students at work, you might ask questions such as the following:

• *What is the opposite of 10? How many units apart are 10 and its opposite on a number line?* (The opposite of 10 is –10. There are 10 units from 10 to 0, and another 10 units from 0 to –10, so 10 and –10 are 20 units apart.)

• *Does it make sense that the opposite of a positive number is twice that number of units apart from the number?* (Yes. It would be that number of units to 0, and then that number of units again to the number's opposite.)

The Exposition — Presenting the Main Ideas

• Draw a number line on the board. Label a point in the middle to be 0. Ask students where 2 is located on the number line, and label the point. Ask students where –2 is located on the number line, and label the point. Ask students to locate and label the point for $\frac{1}{2}$. Mark a point that same distance from 0 on the negative side of the number line. Ask students how they might write the number for that point.

• With students, go through the first part of the exposition on **page 102** of the student text. Make sure students recognize that $-\frac{6}{2}$, $\frac{-6}{2}$, and $\frac{6}{-2}$ are all names for the same number (all are equivalent to –3).

• Go through the second part of the exposition. Have students pay particular attention to the statements that the rational numbers increase as you move to the right on a horizontal number line, and that any positive rational number is greater than any negative rational number.

• Go through the final part of the exposition on **page 103**. Give students one rational number, such as $-\frac{13}{10}$, and have them write it as a mixed number and as a decimal.

Revisiting the Try This

B. Students can apply what they learned in the exposition about positive and negative rational numbers to the problem posed in the **Try This**.

Using the Examples

• Ask pairs of students to read through **solutions 1 and 2** of **example 1**. Ask them to choose which solution most closely matches what they would have done and to say why they would have done it that way.

• Present **example 2** to the students. Ask each student to choose three of the parts of the example to solve. Then each student can compare his or her work to what is shown in the matching solution. Suggest that students may wish to read through the parts of the example they did not select.

Practising and Applying

Teaching points and tips

Q1: Remind students to place the point for 0 such that there is room to place both positive and negative numbers on the number line.

Q3: Some students may choose to place the numbers on a number line to compare them.

Q4: To get students started ordering the numbers, remind them that any positive rational number is greater than any negative rational number. They can then order by whole number part, and finally by fraction or decimal part.

Q6: If students are not familiar with stock quotations, you may wish to bring in a newspaper or another written example of stock market quotes.

Q9: This question reinforces an important generalization: if two fractions (or positive rational numbers) have the same numerator, then the fraction with the greater denominator is less.

Q10: This question highlights that numbers farther left on a horizontal number line are less than numbers to the right, a critical notion brought out in this lesson.

Common errors

• In **question 3 a)**, some students will write that $-5\frac{1}{3} < -7\frac{1}{8}$ (because 5 is less than 7). You might have students place the numbers on a number line. Remind them that any number to the right of another number is greater.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can compare rational numbers
Question 4	to see if students can order a set of rational numbers
Question 6	to see if students can solve a problem involving rational numbers

Answers

A. i) +12 and – 12
ii) 6.5 and –6.5

B. If the opposites were integers, they would be an even number of units apart.

1. a)

b) $-\frac{7}{4}$; [It is the number that is farthest left.]

[**2.** *Sample response*:
$\frac{-7}{3} = -7 \div 3$. If you divide –7 by 3, you get the negative of 7 ÷ 3, which is $-\frac{7}{3}$, so $\frac{-7}{3} = -\frac{7}{3}$.

$\frac{7}{-3} = 7 \div -3$. If you divide 7 by –3, you get the negative of 7 ÷ 3, which is $-\frac{7}{3}$, so $\frac{7}{-3} = -\frac{7}{3}$.

That means $\frac{-7}{3} = -\frac{7}{3} = \frac{7}{-3}$.]

3. a) >; [*Sample response*:
$-5\frac{1}{3}$ is right of –6 and $-7\frac{7}{8}$ is left of –6,
so $-5\frac{1}{3} > -7\frac{7}{8}$.]
b) <; [*Sample response*:
Any negative number is less than any positive number.]
c) >; [*Sample response*:
The whole number parts are equal, but $\frac{5}{9} > \frac{2}{9}$.]
d) <; [*Sample response*:
$\frac{7}{-4}$ is the opposite of $\frac{7}{4}$ and $\frac{4}{-7}$ is the opposite of $\frac{4}{7}$. Since $\frac{7}{4} > \frac{4}{7}$, the opposite of $\frac{7}{4}$ ($\frac{7}{-4}$) is less than the opposite of $\frac{4}{7}$ ($\frac{4}{-7}$).]
e) <; [*Sample response*:
Since $\frac{2}{3} > \frac{2}{5}$, then $-1\frac{2}{3} < -1\frac{2}{5}$.]
f) <; [*Sample response*:
$3\frac{1}{3} = 3.333 \ldots > 3.3$, so $-3\frac{1}{3} = -3.333 \ldots < -3.3$]

4. a) $-6\frac{1}{2}$, -3, $-\frac{9}{4}$, $-\frac{4}{5}$
b) $-\frac{5}{4}$, $-\frac{1}{2}$, 0, $\frac{7}{12}$, $\frac{11}{12}$
c) -5, -3, $\frac{15}{8}$, $\frac{11}{4}$, $\frac{15}{4}$
d) $-5\frac{2}{5}$, -5.2, 0, 4.7, $4\frac{3}{4}$

5. Paro; [–1.7 < –1.2 so –1.7ºC is colder than –1.2ºC.]

6. Stock B; [*Sample response*:
I compared the three negative numbers, since they were the only losses. The least value was the greatest loss.]

7. a) $-\frac{11}{2}$
b) "It is less than –3" is not needed; [–5 is less than –3.]

8. *Sample response*: $-\frac{11}{4}$, $-\frac{5}{2}$, $-2\frac{1}{3}$, $-2\frac{1}{6}$

[**9.** The number with the least denominator is greater.
Sample response:
$\frac{10}{2} > \frac{10}{5}$ because $\frac{1}{2} > \frac{1}{5}$.]

[**10.** As you move farther to the right on a number line, the value of the numbers increases, so $15 > \frac{1}{2}$. As you move farther to the left on a number line, the value of the numbers decreases, so $-15 < -\frac{1}{2}$.]

Supporting Students

Struggling students

• If students are struggling with **question 8**, you might have them draw a number line and note that numbers from –3 to –2 are the opposites of numbers from 2 to 3. They might find it easier to locate four fractions between 2 and 3, and then write their opposites, which are in reverse order from least to greatest.

For example, if students wrote $2\frac{1}{5}$, $2\frac{2}{5}$, $2\frac{3}{5}$, and $2\frac{4}{5}$, then their answer, in order from least to greatest, would be $-2\frac{4}{5}$, $-2\frac{3}{5}$, $-2\frac{2}{5}$, and $-2\frac{1}{5}$.

Enrichment

• For **question 7**, you might challenge students to write their own "mystery number" questions for other students to solve. They need to make sure that their clues identify a unique number.

4.3.2 Operations with Rational Numbers

Curriculum Outcomes	Outcome relevance
8-B13 Operations: positive and negative decimal numbers • use prior experience to construct concrete and pictorial representations • connect visual representations to symbols • use a variety of models to illustrate the operations (e.g., coloured counters, number lines) • develop computational algorithms with decimals, using estimation, mental computation, pencil and paper • apply prior knowledge of order of operations in the context of positive and negative decimals • use estimation to check reasonableness of answers	The ability to relate operations with rational numbers to operations with integers and decimals is important for making sense of the concepts students will need for higher level mathematics.

Pacing	Materials	Prerequisites
1 h	None	• adding, subtracting, multiplying, and dividing integers • adding, subtracting, multiplying, and dividing decimals • adding, subtracting, multiplying, and dividing fractions and mixed numbers

Main Points to be Raised

• The number line model used for adding and subtracting integers can also be a model for adding and subtracting rational numbers.

• When you add rational numbers, the sum has the sign of the number with the greater value without its sign.

For example, the sum of –65.2 + 47.5 is negative because 65.2 > 47.5.

• Subtracting a number is the same as adding its opposite.

• To multiply and divide rational numbers, you use the same methods as you do for multiplying and dividing fractions and decimals.

• The sign of the answer in multiplying and dividing rational numbers is determined in the same way as it is for integers.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How can you figure out how much the stock lost in 3 days?* (If it lost 1.05 Hong Kong dollars each day for 3 days, it lost 3 × 1.05 = 3.15 in 3 days.)

• *How can you figure out the value of the stock after 3 days?* (Subtract 3.15 from 39.45.)

The Exposition — Presenting the Main Ideas

• Begin by drawing an integer number line on the board. Use the number line to solve a few simple integer addition and subtraction examples.

• With students, go through the exposition on **page 106** of the student text. Make sure students see the similarities in modelling the addition and subtraction of integers, the addition and subtraction of rational numbers expressed as decimals, and the addition and subtraction of rational numbers expressed as fractions. Give them an opportunity to ask questions.

• Even if students do not use a number line to determine their answers, it is helpful for them to visualize the numbers they are working with on a number line so that they can determine ahead of time what the sign will be for each answer.

• Go through the exposition on **page 107**. Discuss how to determine the sign of each of these operations with rational numbers: the sum when adding; the difference when subtracting; the product when multiplying; and the quotient when dividing.

Revisiting the Try This

B. Students apply what they learned about calculations with positive and negative rational numbers to the problem posed in the **Try This**.

Using the Examples

• Ask pairs of students to read through **solutions 1 and 2** of **example 1**. Ask them to choose the solution that most closely matches what they would have done and to say why they would have done it that way.

• In their pairs, have one student in the pair become the expert on **example 2** and the other become the expert on **example 3**. Each should then explain his or her example to the other student in the pair.

Practising and Applying

Teaching points and tips

Q1: This question reinforces the concept that subtracting a rational number gives the same result as adding the opposite of the rational number. At the same time, it shows that the order in which two numbers are added does not affect the sum.

Q2 and 3: Encourage students to determine the sign of each answer before performing the calculation.

Q4: Allow time for students to discuss the estimation strategies they used and the method they used to predict the sign of the answer.

Q6: This question highlights the strategies used to determine the sign of an answer for calculations with rational numbers, a critical notion brought out in this lesson.

Common errors

• Some students will subtract the two numbers in **question 2 b)**. Have them visualize the numbers on a number line. To get from 24.25 to –110.3, they have to move 24.25 to the left (a negative direction) to get to 0, and then the move another 110.3 to the left to get to –110.3. So, they will have moved in a negative direction a total of 24.25 + 110.3 = 134.55, or –134.55.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can add and subtract positive and negative rational numbers
Question 3	to see if students can multiply and divide positive and negative rational numbers
Question 4	to see if students can explain the estimation strategies and strategies for determining the sign of the answer in a calculation with rational numbers
Question 5	to see if students can solve a problem that involves calculation with rational numbers

Answers

A. i) 36.30 Hong Kong dollars **ii)** 31.05 Hong Kong dollars	**B.** *Sample responses*: **i)** 3 × 1.05 = 3.15 39.45 – 3.15 = 36.30 **ii)** 5 × 1.05 = 5.25 36.30 – 5.25 = 31.05
1. a) i) $-\frac{17}{24}$ **ii)** $-\frac{17}{24}$ **iii)** $-\frac{17}{24}$ **iv)** $-\frac{17}{24}$ **b)** Same: Each had the same answer. Different: Two involved adding two rational numbers but in different orders. The other two involved subtracting a rational number from a rational number but in different orders. **2. a)** 36.18 **b)** –134.55 **c)** –22.1 **d)** 0.95 **3. a)** $-\frac{3}{4}$ **b)** 83.13 **c)** $-\frac{7}{8}$ **d)** 9.5	**4. a) i)** Negative; [I am multiplying a negative by a positive.] **ii)** Positive; [$12\frac{1}{3} > 5\frac{7}{8}$, and $12\frac{1}{3}$ is positive.] **iii)** Negative; [I am dividing a positive by a negative.] **iv)** Positive; [To get from –7.15 to –5.86, I move in a positive direction (to the right).] **b)** *Sample responses*: **i)** About –900 [–30 × 30] **ii)** About 6 [–6 + 12] **iii)** About –9 [36 ÷ –4] **iv)** About 1 [7 – 6]

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Answers [Continued]

5. a) i) 4.61 **ii)** 36.80 **iii)** 198.75 **iv)** 6.50 **b)** +5.36 **c)** –10.70 [**6. a)** The answer has the sign of the greater number, if you ignore the signs. For example, –6.1 + (–5.2) will have a negative sum because 6.1 > 5.2 and 6.1 is negative. **b)** I think about the direction I move on a number line from the number I am subtracting to the number I am subtracting from. For example, 7 – (–3.4) will be positive because I move in a positive direction (to the right) to get from –3.4 to 7.	**c)** If the signs are the same, the answer will be positive. If they are different, the answer will be negative. For example: 5.2 × 3.4 and –1.2 × (–7.9) have positive products. 5.2 ÷ 3.4 and –1.2 ÷ (–7.9) have positive quotients. –5.2 × 3.4 and 1.2 × (–7.9) have negative products. –5.2 ÷ 3.4 and 1.2 ÷ (–7.9) have negative quotients.]

Supporting Students

Enrichment

• For **question 5**, you might challenge students to find current quotes for the Hong Kong Stock Exchange and to make up questions based on those quotes for other students to solve.

4.3.3 Order of Operations

Curriculum Outcomes	Outcome relevance
8-B14 Order of Operations: fractions • understand that the order is the same as for whole numbers, and why that makes sense • understand how improper order impacts results **8-B13 Operations: positive and negative decimal numbers** • use prior experience to construct concrete and pictorial representations • connect visual representations to symbols • use a variety of models to illustrate the operations (e.g., coloured counters, number lines) • develop computational algorithms with decimals, using estimation, mental computation, pencil and paper • apply prior knowledge of order of operations in the context of positive and negative decimals (e.g., $-0.2 + 4.5\ (-5 + 2.24) - (-6) \div 0.2$ • continue to estimate before calculating	Using and understanding the convention of the order of operations is essential for communicating mathematically when a sequence of operations is involved.

Pacing	Materials	Prerequisites
1 h	None	• adding, subtracting, multiplying, and dividing rational numbers • familiarity with the order of operations for whole numbers and integers

Main Points to be Raised

• When you add, subtract, multiply, and divide rational numbers, the order in which you do the calculations can affect the answer.

• The order of operations rules for rational numbers are the same as for integers, decimals, and fractions. First, do any calculations inside brackets. Next, do all multiplications and divisions from left to right. Finally, do all additions and subtractions from left to right.

• When an expression has brackets inside brackets, you can use both round and square brackets to make clear which numbers are being combined.

• Brackets are also used around negative numbers that follow an operation sign to make the calculation easier to interpret.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How might you begin with 10 and end up with an answer of 8?* (Subtract 2 or multiply by $\frac{8}{10}$.)

• *How do you know which to try?* (There does not seem to be any way to get 2 using $\frac{1}{2}$ and $\frac{5}{8}$, so I need to see if I can find a way to use $\frac{1}{2}$ and $\frac{5}{8}$ to get $\frac{8}{10}$.)

The Exposition — Presenting the Main Ideas

• On the board, write a sequence of calculations, such as $3 + 6 \times (5 + 4) \div 3 - 7$. Have students recall how the order of operations for whole numbers determines the order in which the calculations are done, and what the final answer is. The given example is done like this:

First, do the addition in brackets:	$5 + 4 = 9$	$3 + 6 \times (5 + 4) \div 3 - 7$
Then, do the multiplication:	$6 \times 9 = 54$	$3 + 6 \times 9 \div 3 - 7$
Then, do the division:	$54 \div 3 = 18$	$3 + 54 \div 3 - 7$
Then, do the addition:	$3 + 18 = 21$	$3 + 18 - 7$
Finally, do the subtraction:	$21 - 7 = 14$	$21 - 7$
		14

Make sure students understand that doing the operations in a different order from the order of operations would result in a different answer.

• With students, go through the first part of the exposition on **page 109** of the student text. Point out that the order of operations for rational numbers is the same as for whole numbers and integers.

• Go through the rest of the exposition on **pages 110 and 111**. Discuss when and why both square and round brackets might be used in an expression (to make clear which numbers are being combined), e.g., 50 ÷ [3 × (6 + 2)].

• Talk about why brackets are sometimes used around individual numbers (to separate their signs from the preceding operation signs in order to make it easier to interpret the expression), e.g., 5 × (–2) vs. 5 × –2.

• Have students note that when an expression is written as a fraction involving operations in the numerator and/or denominator, you do the calculations as though everything in the numerator was inside brackets divided by everything in the denominator inside another set of brackets, e.g., $\frac{6 \times 3 + 2}{4 + 10 \div 2} = (6 \times 3 + 2) \div (4 + 10 \div 2)$.

Revisiting the Try This

B. Students apply what they learned in the exposition about order of operations solve a calculation using the numbers from **part A** of the **Try This**.

Using the Examples

• Present the problems in the three examples to students. Ask each student to choose two of the problems to solve. Then each student can compare his or her work to what is shown in the matching example. Suggest that they may wish to read through the other example as well.

Practising and Applying

Teaching points and tips

Q1: It might be helpful to choose one or two of the examples and discuss with the class which operations would be done first, second, third, and so on.

Q2: Students can experiment by placing brackets in different places and estimating to see if they are close to the desired answer.

Q3: Some students may choose to try all the possible combinations.

Q4: This question may be assigned to selected students.

Q5: A few students might discover or suggest this generalization: If only multiplications and divisions are involved in a sequence of calculations, and there is an even number of negative signs, the result will be positive. If there are is odd number of negative signs, the result will be negative.

Q6: Encourage students to share their estimation strategies with others.

Q7: Remind students first to do all the calculations in the numerator, and then to divide the result by the result of all the calculations in the denominator.

Q8: This question highlights how brackets can be used to communicate the intent to have one operation performed before another, an important concept in this lesson.

Common errors

• Some students will just do the calculations from left to right in **questions 1 e) and f)**. Remind them that when there are no brackets, they should begin by doing multiplications and divisions, and then do the additions and subtractions. You might mention that sometimes brackets are used even when they are not necessary, to reinforce that certain calculations should be done first, e.g., 19.5 – 15.8 × 3 → 19.5 – (15.8 × 3).

Suggested assessment questions from Practising and Applying

Question 1	to see if students can apply the order of operations rules to a sequence of calculations
Question 2	to see if students can solve a mathematical problem using order of operations rules
Question 8	to see if students can create an expression with given criteria using order of operations rules

Answers

A. *Sample responses*: **i)** $\frac{1}{2} \div \frac{5}{8} \times 10$ **ii)** $\frac{1}{2} \div 10 \times \frac{5}{8}$	**B.** Using no brackets: $\frac{1}{2} + \frac{5}{8} \div 10 = \frac{45}{80} = \frac{9}{16}$. Using brackets around the addition: $(\frac{1}{2} + \frac{5}{8}) \div 10 = \frac{9}{80}$.

1. a) $\frac{5}{7}$ **b)** $\frac{33}{40}$ **c)** $\frac{13}{24}$ **d)** 5.9 **e)** –27.9 **f)** –11.5 **2. a)** $(3.6 + 6) \div (3.5 - 1.1) + 3 = 7$ **b)** $\frac{5}{8} \div \left(\frac{1}{2} + \frac{1}{3}\right) \times \frac{3}{5} = \frac{9}{20}$ **3. a)** Multiply by $\frac{3}{4}$, add $\frac{2}{3}$, and then divide by $\frac{1}{2}$. ($2\frac{5}{6}$) **b)** Divide by $\frac{1}{2}$, add $\frac{2}{3}$, and then multiply by $\frac{3}{4}$. (2) **4.** $11.2 - (-5.4) \div 2.7 + (-9) = 4.2$ **5. a)** Positive; [*Sample response*: $-3.2 \times (-5.5) \div (-1.6) \times 3.2 \div (-0.5)$ Because there are no brackets and only multiplication and division, you calculate from left to right. $-3.2 \times (-5.5)$ is positive. The positive from $-3.2 \times (-5.5)$ divided by (-1.6) is negative. The negative result so far multiplied by 3.2 is negative. The negative result so far divided by (-0.5) gives a positive result.] **b)** Negative; [*Sample response*: $-1\frac{3}{4} \times 4\frac{2}{3} \div (-1\frac{1}{2}) \times 1\frac{3}{8} \times (-3\frac{1}{6})$ Because there are no brackets and only multiplication and division, you calculate from left to right. $-1\frac{3}{4} \times 4\frac{2}{3}$ is negative. The negative result from $-1\frac{3}{4} \times 4\frac{2}{3}$ divided by $-1\frac{1}{2}$ is positive. [Continued]	**5. b)** [Continued] The positive result so far multiplied by $1\frac{3}{8}$ is positive. The positive result so far multiplied by $(-3\frac{1}{6})$ gives a negative result.] **6.** *Sample responses*: **a)** About 54; [$-3.2 \times (-5.5) \div (-1.6) \times 3.2 \div (-0.5)$ $\approx (-3) \times (-6) \div (-2) \times 3 \div (-\frac{1}{2})$ $= (-3) \times (-6) \div (-2) \times 3 \times (-2)$ $= 54$] **b)** About –30; [$-1\frac{3}{4} \times 4\frac{2}{3} \div (-1\frac{1}{2}) \times 1\frac{3}{8} \times (-3\frac{1}{6})$ $\approx (-2) \times 5 \div (-2) \times 1 \times (-3)$ $= -15$] **7. a)** $\frac{1}{10}$ **b)** $-\frac{1}{10}$ **8.** *Sample response*: $-2.5 \times [4 + (-5.3)] \div \frac{3}{4}$; [• It has three operations (×, +, ÷). • Since $4 + (-5.3)$ is in brackets, you add first ($4 + (-5.3) = -1.3$). • Since $-2.5 \times (-1.3)$ is to the left of $(-1.3) \div \frac{3}{4}$, you multiply, then divide.]

Supporting Students

Struggling students

• If students are struggling with **question 2**, you might have them first try some simpler expressions with only whole numbers. In the given expressions, encourage them to estimate as they place brackets to see the effect of inserting brackets in different places.

Enrichment

• For **question 5**, you might create other similar expressions for the students to solve.
For example: 9 ■ 5 ■ (8 ■ 3) ■ 2 ■ 6 = 13.

GAME: Target One

• This optional game allows students to practise adding, subtracting, multiplying, and dividing fractions.

• Students might realize that if they have a whole number and a fraction among their cards, they can use them to form a mixed number (for example, $2 + \frac{1}{3}$ is the same as $2\frac{1}{3}$).

• If desired, the target number can be changed.

$\frac{1}{5}$

UNIT 4 Revision

Pacing	Materials
2 h	• Grid paper or Small Grid Paper (BLM) (optional) • Coloured pencils (optional)

Question	Related Lesson(s)
1	Lessons 4.1.1 and 4.1.2
2 and 3	Lesson 4.1.2
4 and 5	Lesson 4.2.2
6 and 7	Lesson 4.2.3
8	Lessons 4.2.4 and 4.2.5
9 – 11	Lesson 4.2.5
12 and 13	Lesson 4.2.6
14	Lesson 4.3.1
15	Lesson 4.3.2
16 and 17	Lesson 4.3.3

Revision Tips

Q1 b): You may wish to have students share their mental math strategies with the class.

Q3: You might encourage students to do these calculations using a common denominator.

Q4: You may wish to provide students with grid paper if they want to use area models.

Q10: Point out that the question calls for an estimate. Students do not need to solve the division question; they should estimate and then represent it with a model.

Q12: Encourage students to use a variety of strategies.

Q14: Remind students that a negative change means a loss in the value of the stock for the day.

Q15: Encourage students to determine the sign of each answer before doing the calculation.

Answers

1. a) i) $7\frac{5}{8}$ **ii)** $10\frac{7}{9}$ **iii)** $5\frac{1}{4}$ **iv)** $5\frac{1}{6}$

[**b)** *Sample response*:

part ii), $3\frac{5}{9} + 7\frac{2}{9}$: The denominators are equal, so I could add the fraction parts and the whole number parts mentally: $3\frac{5}{9} + 7\frac{2}{9} = 3 + \frac{5}{9} + 7 + \frac{2}{9} = 3 + 7 + \frac{5}{9} + \frac{2}{9} = 10 + \frac{7}{9} = 10\frac{7}{9}$.]

2. $9\frac{1}{2}$; [*Sample response*:

$5\frac{7}{8} + 7\frac{1}{4}$ is a bit more than 13 and $3\frac{5}{9}$ is a bit more than $3\frac{1}{2}$. Subtracting a bit more than $3\frac{1}{2}$ from a bit more than 13 will give an answer close to $9\frac{1}{2}$.]

3. a) $15\frac{7}{12}$ **b)** $2\frac{1}{12}$

4. a) $\frac{21}{32}$ **b)** $\frac{7}{18}$ **c)** $\frac{1}{3}$ **d)** $\frac{3}{5}$

5. $\frac{3}{28}$

6. a) $5\frac{1}{7}$ **b)** $8\frac{2}{3}$

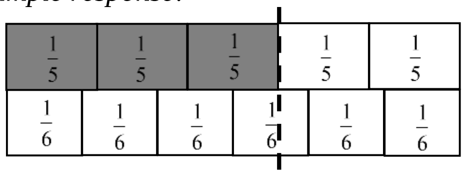
7. a) $\frac{5}{8}$ cups **b)** $4\frac{3}{8}$ cups

8. a) $\frac{2}{3}$ **b)** $\frac{3}{4}$ **c)** $\frac{4}{5}$ **d)** 16

9. 6 times

10. a) About $3\frac{1}{2}$ times

b) *Sample response*:



11. a) $1\frac{13}{14}$ **b)** $1\frac{5}{16}$

12. a) $1\frac{5}{9}$ **b)** $1\frac{4}{5}$ **c)** $\frac{5}{6}$ **d)** $\frac{2}{3}$

d) $\frac{2}{3}$

13. a) 5 **b)** Yes; $\frac{5}{8}$ m

14. a) Aluminum Corp.

b) Associated Int. Hotel

c) –0.50, –0.07, –0.03, +0.06, +1.18

15. a) $-11\frac{1}{6}$ **b)** 15.13 **c)** –15.95 **d)** $1\frac{1}{2}$

16. a) $-\frac{1}{2}$ **b)** –6

17. a) $\left(1\frac{3}{4} - \frac{5}{8}\right) \times \frac{2}{3} + \frac{4}{9} \div 1\frac{1}{3}$

b) $1\frac{3}{4} - \frac{5}{8} \times \left(\frac{2}{3} + \frac{4}{9} \div 1\frac{1}{3}\right)$

UNIT 4 Fractions and Rational Numbers Test

Express all fractions in lowest terms. Write all improper fractions as mixed numbers.

1. Calculate.

a) $\frac{2}{3} + \frac{3}{5}$

b) $\frac{8}{10} - \frac{1}{4}$

c) $2\frac{1}{4} + 3\frac{2}{5}$

d) $6\frac{3}{4} - 2\frac{5}{6}$

e) $5\frac{1}{2} - 3\frac{7}{10}$

f) $2\frac{3}{4} + 1\frac{3}{8} + 3\frac{1}{5}$

2. Calculate each product.

a) $\frac{3}{4} \times \frac{4}{9}$

b) $\frac{5}{9} \times \frac{6}{10}$

c) $\frac{3}{5} \times 11$

d) $\frac{5}{8} \times 30$

3. Kamala can read a chapter of a novel in $1\frac{1}{4}$ h. How long will it take her to read 8 chapters?

4. A glass holds $\frac{3}{5}$ cup. How many glasses can be filled from a 10-cup pitcher of water?

5. When you divide a proper fraction by an improper fraction, is the quotient less than 1, greater than 1, or equal to 1? Use an example to explain your answer.

6. a) Estimate each quotient. Explain how you estimated.

i) $2\frac{1}{4} \div \frac{3}{8}$

ii) $3\frac{3}{8} \div \frac{3}{4}$

iii) $3\frac{1}{3} \div \frac{2}{5}$

iv) $6\frac{2}{5} \div 9\frac{3}{5}$

v) $8\frac{8}{9} \div 3\frac{1}{5}$

b) Divide each and compare the exact answer to your estimate from **part a)**.

7. Order from least to greatest.

a) $-6\frac{1}{3}, -\frac{5}{6}, -\frac{9}{4}, -2$

b) $-3\frac{1}{5}, 0, 3.4, -4.2, 4\frac{1}{4}$

8. Write three different rational numbers that are greater than –5 but less than –4. Order them from least to greatest.

9. Calculate.

a) $27.25 + (-15.3)$

b) $-1\frac{7}{8} - (-\frac{3}{4})$

c) $-7.22 + (-12.77)$

d) $4\frac{1}{2} - (-2\frac{1}{3})$

e) $-4.1 \times (-15.2)$

f) $2\frac{3}{4} \div (-3\frac{1}{7})$

g) $-38.25 \div 4.5$

10. On a recent day on the Hong Kong Stock Exchange, a stock had an opening price of 44.75 Hong Kong Dollars. It had a change of –2.35 Hong Kong Dollars for the day.

a) What was the closing price for the stock?

b) The stock changed by the same amount for 4 days in a row.

i) What was the total change in price for the stock?

ii) What was the closing price?

11. Calculate.

a) $12.4 - (8.6 \div 2 + 1.1) + (-1.3)$

b) $\frac{3}{4} - \frac{1}{3} \times (\frac{1}{6} + \frac{3}{4})$

c) $\frac{-3.5 - (-20.5 \div 2)}{3.2 \times 5 - 2.5}$

UNIT 4 Test

Pacing	Materials
1 h	None

Question	Related Lesson(s)
1	Lessons 4.1.1 and 4.1.2
2	Lesson 4.2.2
3	Lesson 4.2.3
4	Lessons 4.2.4 and 4.2.5
5	Lesson 4.2.5
6	Lesson 4.2.6
7 and 8	Lesson 4.3.1
9 and 10	Lesson 4.3.2
11	Lesson 4.3.3

Select questions to assign according to the time available.

Answers

1. a) $1\frac{4}{15}$ **b)** $\frac{11}{20}$ **c)** $5\frac{13}{20}$

d) $3\frac{11}{12}$ **e)** $1\frac{4}{5}$ **f)** $7\frac{13}{40}$

2. a) $\frac{1}{3}$ **b)** $\frac{1}{3}$ **c)** $6\frac{3}{5}$ **d)** $18\frac{3}{4}$

3. 10 h **4.** $16\frac{2}{3}$ glasses

5. Less than 1; *Sample response*:
A proper fraction is less than 1 and an improper fraction is greater than 1. A number less than 1 divided by a number greater than 1 will always be less than 1 because there is not even one group of the improper fraction in the proper fraction,
e.g., $\frac{2}{3} \div \frac{4}{3} = \frac{1}{2}$.

6. a) *Sample responses*:

i) About 5; $\frac{3}{8}$ is close to $\frac{1}{2}$ and $2\frac{1}{4}$ is close to $2\frac{1}{2}$.
There are about five $\frac{1}{2}$s in $2\frac{1}{2}$.

ii) About 4; I rounded $\frac{3}{4}$ up to 1 and I rounded $3\frac{3}{8}$ up to 4. $4 \div 1 = 4$.

iii) A bit more than 7; $3\frac{1}{3}$ is close to $3\frac{1}{2}$ and $\frac{2}{5}$ is close to (but less than) $\frac{1}{2}$. There are seven $\frac{1}{2}$s in $3\frac{1}{2}$

iv) About $\frac{2}{3}$; $6\frac{2}{5}$ is close to 6 and $9\frac{3}{5}$ is close to 9.
$6 \div 9$ is $\frac{6}{9}$, or $\frac{2}{3}$.

v) About 3; $8\frac{8}{9}$ is close to 9 and $3\frac{1}{3}$ is close to 3.
$9 \div 3 = 3$.

b) i) 6; *Sample response*:
The exact answer is a bit higher than my estimate.

ii) $4\frac{1}{2}$; *Sample response*:
The exact answer is a bit higher than my estimate.

iii) 8; *Sample response*:
The exact answer is higher than my estimate.

iv) $\frac{2}{3}$; *Sample response*:
The exact answer is the same as my estimate.

v) $2\frac{7}{9}$; *Sample response*: The exact answer is a bit lower than my estimate.

7. a) $-6\frac{1}{3}, -\frac{9}{4}, -2, -\frac{5}{6}$ **b)** $-4.2, -3\frac{1}{5}, 0, 3.4, 4\frac{1}{4}$

8. *Sample response*: $-4.8, -4\frac{1}{2}, -4.25$

9. a) 11.95 **b)** $-1\frac{1}{8}$ **c)** –19.99 **d)** $6\frac{5}{6}$

e) 62.32 **f)** $-\frac{7}{8}$ **g)** –8.5

10. a) 42.40 Hong Kong dollars
b) i) –9.40 Hong Kong dollars
ii) 35.35 Hong Kong dollars

11. a) 5.7 **b)** $\frac{4}{9}$ **c)** $\frac{1}{2}$

UNIT 4 Performance Task — Ordering Operation Results

Suppose you were to add, subtract, multiply, and divide the two rational numbers $\frac{1}{2}$ and $\frac{1}{4}$.

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4} \qquad \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8} \qquad \frac{1}{2} \div \frac{1}{4} = 2$$

The order of the sum, difference, product, and quotient, from least to greatest, is

$$\underset{\frac{1}{8}}{\text{Product}} < \underset{\frac{1}{4}}{\text{Difference}} < \underset{\frac{3}{4}}{\text{Sum}} < \underset{2}{\text{Quotient}}$$

A. i) Find a pair of rational numbers that has a quotient that is less than the sum.
ii) Find a pair of rational numbers that has a difference that is greater than the quotient.

B. Find six different pairs of rational numbers that have a sum, difference, product, and quotient that are in a different order for each pair.
For example, only one pair should have this order: sum < difference < product < quotient
Explain why you chose each pair of numbers.
You could use a chart like this to organize your solution:

Pair of numbers	Why I chose this pair of numbers	Sum	Difference	Product	Quotient	Order (least to greatest)
$\frac{1}{2}$ and $\frac{1}{4}$	I wanted one where the greatest value was the quotient and I thought 2 positive unit fractions would have a sum, difference, and product smaller than the quotient.	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	2	Product Difference Sum Quotient

UNIT 4 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
8-A8 Integers and Rational Numbers: comparing and ordering **8-B8** Add and Subtract: fractions— develop algorithm **8-B10** Multiply and Divide: fractions — develop algorithm **8-B12** Fractions: estimate and mentally compute products and quotients **8-B13** Operations: positive and negative decimal numbers	1 h	None

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could supplement the unit test. It could also be used as enrichment material for some students.

• You can assess performance on the task using the rubric provided below.

Sample Solution

A. i) $\frac{1}{4}$ and $\frac{1}{2}$. The sum is $\frac{3}{4}$ and the quotient is $\frac{1}{2}$; $\frac{3}{4} > \frac{1}{2}$

ii) $-\frac{1}{2}$ and $\frac{1}{4}$. The difference is $-\frac{3}{4}$ and the quotient is -2; $-\frac{3}{4} > -2$

B.

Pair of numbers	Why I chose this pair of numbers	Sum	Difference	Product	Quotient	Order (least to greatest)
$2\frac{1}{2}$ and $1\frac{1}{4}$	I wanted the sum and product to be greatest. I knew that for positive mixed numbers, the sum and product is usually greater than the difference.	$3\frac{3}{4}$	$1\frac{1}{4}$	$3\frac{1}{8}$	2	Difference Quotient Product Sum
$-2\frac{1}{2}$ and $1\frac{1}{4}$	I switched $2\frac{1}{2}$ to $-2\frac{1}{2}$ so all the results are negative.	$-1\frac{1}{4}$	$-3\frac{3}{4}$	$-3\frac{1}{8}$	–2	Difference Product Quotient Sum
$2\frac{1}{2}$ and $-1\frac{1}{4}$	I switched $1\frac{1}{4}$ to $-1\frac{1}{4}$ so product and quotient are negative but the sum and difference are positive.	$1\frac{1}{4}$	$3\frac{3}{4}$	$-3\frac{1}{8}$	–2	Product Quotient Sum Difference
$-2\frac{1}{2}$ and $-1\frac{1}{4}$	I switched $2\frac{1}{2}$ to $-2\frac{1}{2}$ and $1\frac{1}{4}$ to $-1\frac{1}{4}$ so product and quotient are positive but the sum and difference are negative.	$-3\frac{3}{4}$	$-1\frac{1}{4}$	$3\frac{1}{8}$	2	Sum Difference Quotient Product
$\frac{5}{6}$ and $\frac{1}{6}$	I knew that, if I used two proper fractions, the product would not be very big. I picked one fraction close to 1 and one fraction close to 0.	1	$\frac{4}{6}$	$\frac{5}{36}$	5	Product Difference Sum Quotient
$-\frac{5}{6}$ and $-\frac{1}{6}$	I switched $\frac{5}{6}$ to $-\frac{5}{6}$ and $\frac{1}{6}$ to $-\frac{1}{6}$ so the product and quotient are the same and positive but the sum and difference are negative.	–1	$-\frac{4}{6}$	$\frac{5}{36}$	5	Sum Difference Product Quotient

UNIT 4 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Calculates with and compares rational numbers	Correctly, consistently, and efficiently uses all four operations with rational numbers; efficiently compares rational numbers	Is mostly correct in using all four operations with rational numbers and in comparing rational numbers	Is sometimes correct in using all four operations with rational numbers and in comparing rational numbers	Shows errors in most of the calculations with or comparisons of rational numbers
Find many different orderings	Insightfully uses information from one solution to suggest ideas for another solution; uses a strategy to find many (at least 5 or 6) solutions; makes reasonable predictions about what might occur	Correctly analyses the problem and creates many different possible solutions	Creates at least a few correct solutions to the problem by using a reasonable strategy	Has difficulty organizing to create more than a couple of solutions; work appears random

UNIT 4 Assessment Interview

• You may wish to take the opportunity to interview selected students to assess their understanding of the work of this unit.

• Interviews are most effective when done with individual students, although pair and small group interviews are sometimes appropriate.

• The results can be used as formative assessment or as summative assessment data.

Ask students the following questions. As students work, ask them to explain their thinking:

• *When do you find it easiest to add fractions? Why?*

• *Name two fractions that are easy for you to subtract. Show me how you subtract them.*

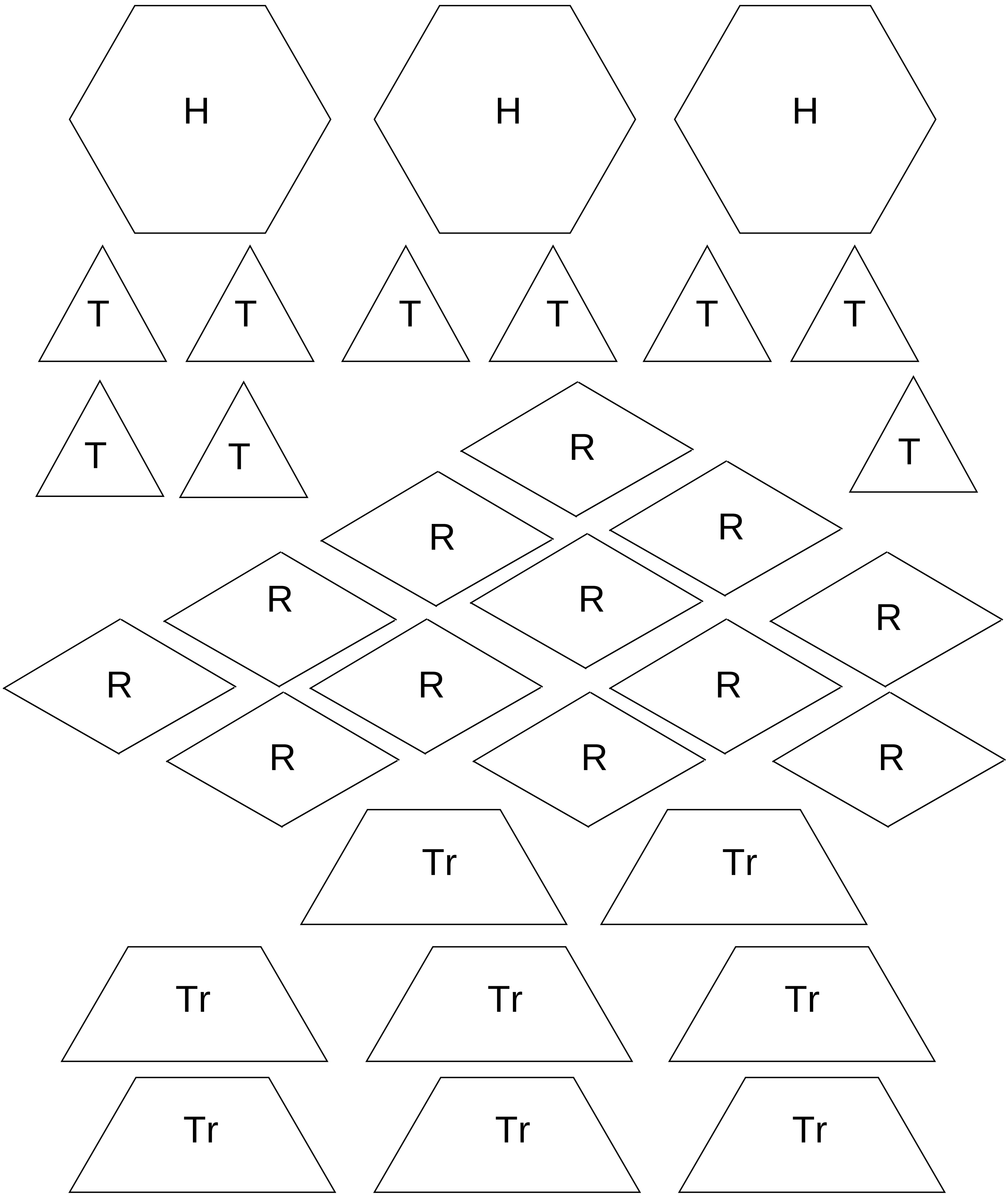
• *Karma says that when you multiply two fractions, the answer is always less than the fractions you multiplied. Do you agree? Show me why or why not.*

• *You can use a common denominator to add, subtract, and divide fractions. Can you use a common denominator to multiply fractions? Does it make the multiplication easier?*

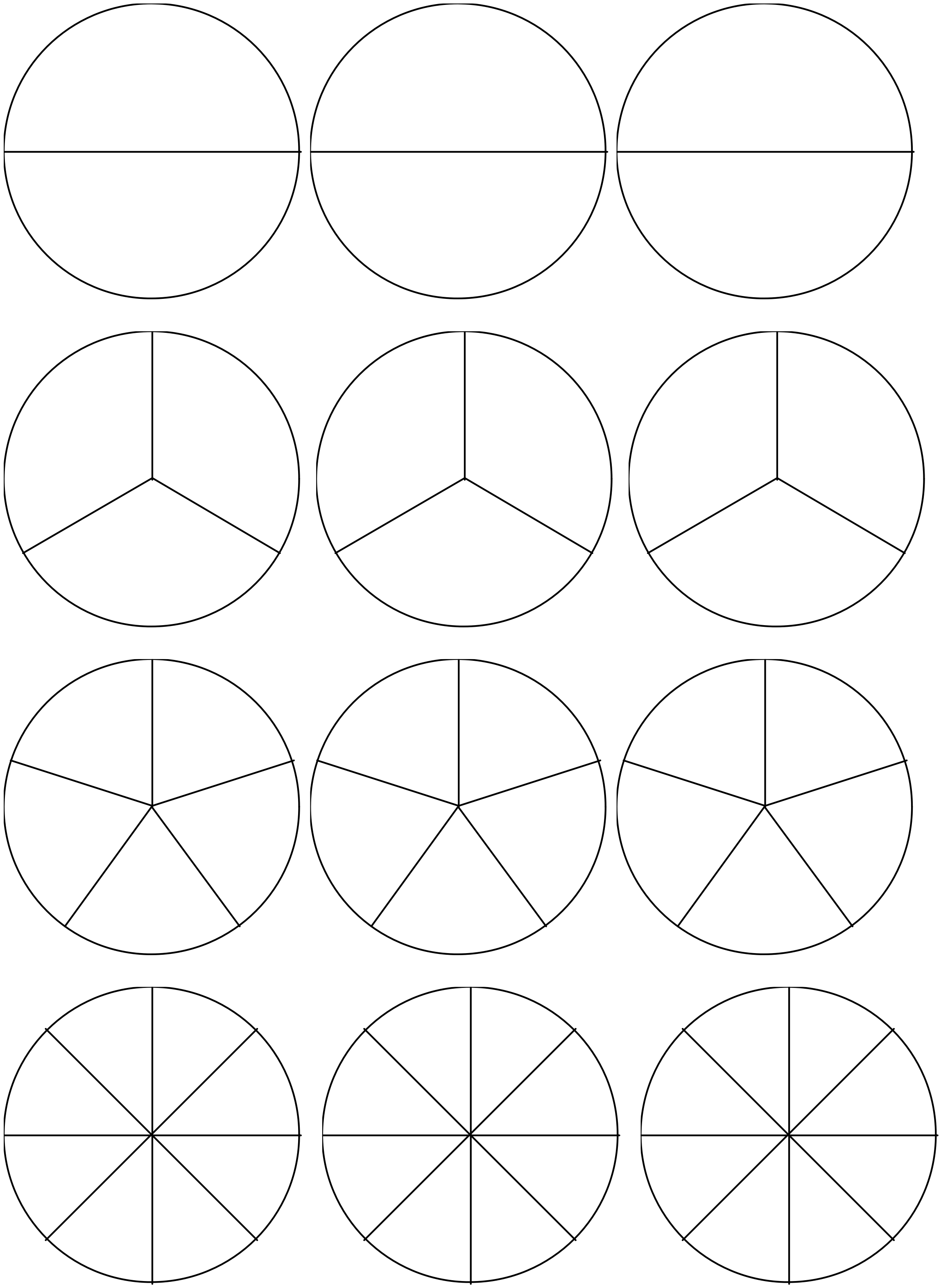
• *What is the product of* $-\frac{2}{3} \times (-\frac{1}{2})$*? Why did you give that answer?*

• *Which is less,* $-\frac{2}{3}$ or $\frac{1}{2}$*? How do you know?*

BLM 1 Pattern Block Fraction Pieces



BLM 2 Fraction Circles



UNIT 5 MEASUREMENT

UNIT 5 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 117** **TG p. 158**	Review prerequisite concepts, skills, and terminology, and pre-assessment	1 h	None	All questions
Chapter 1 The Pythagorean Theorem				
5.1.1 The Pythagorean Theorem **SB p. 119** **TG p. 160**	**8-D1 Pythagorean Relationship: understanding** • investigate the side relationships of a variety of right triangles • explore the 3-4-5 rule for establishing a right angle • understand, through investigation, that if a square is made on each side of a right triangle, the sum of the two smaller squares will equal the area of the longer side ($c^2 = a^2 + b^2$ where a, b, and c are sides of a right triangle) • explore triangles of a variety of orientations to discover that the hypotenuse (the longest side) is the side opposite the right angle regardless of orientation **8-D2 Pythagorean Relationship: application** • use the Pythagorean relationship to determine if three given side lengths are, or are not, the sides of a right triangle	1 h	• Grid paper or Small Grid Paper (BLM) (Optional)	Q2, 4
5.1.2 Applying the Pythagorean Theorem **SB p. 122** **TG p. 163**	**8-D2 Pythagorean Relationship: application** • understand usefulness of Pythagorean relationships to solve problems in real life (whenever a triangle has a right angle and two known sides) • investigate real world problems to determine the length of the hypotenuse, as well as the length of the other side when the hypotenuse and one side are given • understand that the Pythagorean relationship can be used if only one side is given when the right triangle is isosceles • find distance between two points using Pythagorean relationship (e.g., determine the reach of a ladder)	1 h	• Rulers (optional)	Q2, 4, 5
Chapter 2 Linear and Area Relationships				
5.2.1 Area and Perimeter Relationships **SB p. 125** **TG p. 166**	**8-D3 Area and Perimeter: patterns and relationships of quadrilaterals and circles** • understand, through investigation, that area can vary when perimeter is fixed (e.g., for a perimeter of rectangle of 16 cm, determine all possible whole-number dimensions) • understand, through investigation, that perimeter can vary when the area of a rectangle is fixed **8-D4 SI Units: solve measurement problems** • integrate measurement problems as other mathematical ideas are explored • investigate and create problems involving length and perimeter (mm, cm, m, km), area (mm^2, cm^2, m^2, km^2, and hectare ($1\ hm^2$))	1 h	• Grid paper or Small Grid Paper (BLM) (optional)	Q3, 4, 11

UNIT 5 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
CONNECTIONS: Pentominos (Optional) **SB p. 128** **TG p. 169**	Make a connection between geometry and area in a problem solving situation	1 h	• Grid paper or Small Grid Paper (BLM) or Grid Paper (1 cm by 1 cm) (BLM)	N/A
GAME: Pentominos (Optional) **SB p. 129** **TG p. 170**	Practise visualization skills in a strategy game using shapes of a given area	30 min	• Grid paper or Small Grid Paper (BLM) or Grid Paper (1 cm by 1 cm) (BLM) • Pentominos from the previous **Connections**	N/A
5.2.2 Scale Drawings **SB p. 130** **TG p. 171**	**8-D5 Proportion: solve indirect measurement problems** • link proportion to ideas of ratio and rate • read, interpret, and discuss scale drawings • understand usefulness of proportion ideas in relevant real-world problems **8-B1 Proportion: solve problems** • use a variety of strategies to solve problems of proportionality: - find relationships between the various terms of proportion and use these relations to solve for missing values (e.g., use equivalent fractions to solve $\frac{2.2}{5} = \frac{x}{5}$) • recognize uses for and importance of proportion	1.5 h	• Rulers	Q1, 4, 6, 8
5.2.3 EXPLORE Estimating the Area of a Circle (Optional) **SB p. 134** **TG p. 174**	**8-D3 Area and Perimeter: patterns and relationships of quadrilaterals and circles** • understand, through investigation, that area can vary when perimeter is fixed (e.g., for a perimeter of rectangle of 16 cm, determine all possible whole-number dimensions) • determine what happens to the area of a regular polygon as the number of sides increases (e.g., if perimeter is 24 cm, what is the area when the figure has 4 sides? 6 sides?)	40 min	• Rulers • Grid Paper (1 cm by 1 cm) (BLM) (Optional)	Observe and Assess questions
5.2.4 The Formula for the Area of a Circle **SB p. 136** **TG p. 176**	**8-D6 Area of Circles: estimate** • understand why it makes sense to estimate by squaring the diameter • understand why a closer estimate is $3 \times r^2$ **8-D7 Area of Circles: develop formula** • apply prior knowledge of area for a parallelogram to develop a formula for the area of a circle • investigate to determine the radius when the area of a circle is given • apply prior knowledge of square root • understand that $\frac{22}{7}$ and 3.14 are approximations and that a calculator must be used for more precision	1 h	• Large paper circle divided into 12 equal sectors (optional)	Q1, 3

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
CONNECTIONS: The History of Pi (Optional) **SB p. 138** **TG p. 178**	Make a connection between various approximations for π	10 min	None	N/A
5.2.5 Applying Area Formulas **SB p. 139** **TG p. 179**	**8-D4 SI Units: solve measurement problems** • integrate measurement problems as other mathematical ideas are explored • use the prefixes kilo, hecto, deka, deci, centi, and milli in problem solving contexts • investigate and create problems involving length and perimeter (mm, cm, m, km), area (mm^2, cm^2, m^2, km^2, and hectare (1 hm^2)) • continue to make decisions in real world situations about when estimating is close enough	1 h	None	Q1, 2, 5, 12
CONNECTIONS: Tangrams (Optional) **SB p. 142** **TG p. 181**	Make a connection between areas of parts of squares	30 min	• Tangrams (BLM)	N/A
Chapter 3 Volume and Surface Area				
5.3.1 Volume of a Rectangular Prism **SB p. 143** **TG p. 182**	**8-D8 Volume and total surface area: estimate and calculate right prisms** • estimate volume in a variety of situations **8-D4 SI Units: solve measurement problems** • integrate measurement problems as other mathematical ideas are explored • develop sense of relative size of units (e.g., compare 1 cm^3 to 1 m^3; 1 kL to 1 L) • apply relationships between capacity and volume for water (1 mL = 1 cm^3; 1 L = 1000 cm^3) to solve problems • when choosing between capacity and volume, understand which is more appropriate in a given situation • establish link between capacity and mass of pure water (1 mL of water has a mass of 1 g) • use the prefixes kilo, hecto, deka, deci, centi, and milli in problem solving contexts • investigate and create problems involving volume (cm^3 and m^3), mass (mg, g, kg) and capacity (mL, L, kL) • continue to make decisions in real world situations about when estimating is close enough	1 h	• Linking cubes (optional) • Grid paper or Small Grid Paper (BLM) (optional)	Q1, 6, 9

UNIT 5 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
5.3.2 Surface Area of a Rectangular Prism **SB p. 147** **TG p. 185**	**8-D8 Volume and Total Surface Area: estimate and calculate right prisms** • find total surface area of rectangular prisms • investigate changes in total surface area based on changes in dimensions **8-D4 SI Units: solve measurement problems** • integrate measurement problems as other mathematical ideas are explored • use the prefixes kilo, hecto, deka, deci, centi, and milli in problem solving contexts • investigate and create problems involving length and perimeter (mm, cm, m, km), area (mm^2, cm^2, m^2, km^2, and hectare (1 hm^2)) • continue to make decisions in real world situations about when estimating is close enough	1 h	• Linking cubes (optional) • Rectang-ular prism items such as boxes (optional)	Q1, 6, 7
UNIT 5 Revision **SB p. 150** **TG p. 188**	Review the concepts and skills in the unit	2 h	None	All questions
UNIT 5 Test **TG p. 189**	Assess the concepts and skills in the unit	1 h	None	All questions
UNIT 5 Performance Task **TG p. 191**	Assess concepts and skills in the unit	1 h	• Rulers	Rubric provided
UNIT 5 Blackline Masters **TG p. 194**	BLM 1 Sample Net of Rectangular Prism (for **lesson 5.3.2**) BLM 2 Tangrams BLM 3 Grid Paper (1 cm by 1 cm) Small Grid Paper (BLM) on **page 32** in UNIT 1			

Math Background

• This measurement unit develops concepts of linear, area, surface area, and volume. It also provides a basis for understanding and using the Pythagorean theorem.

• The lessons on area emphasize the area of the circle. The lessons on volume and surface area deal with the volume and surface area of rectangular prisms.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 5** in **lesson 5.1.2**, where they use the Pythagorean theorem to calculate a distance, in **question 6** in **lesson 5.2.1**, where they find a rectangle with a given perimeter and area, in **question 5** in **lesson 5.2.2**, where they solve problems related to a scale drawing, in **question 4** in **lesson 5.2.4**, where they calculate the areas of sections of a circle, in **question 1** in **lesson 5.2.5**, where they solve a problem involving areas of circles inside rectangles, in **question 7** in **lesson 5.3.1**, where they calculate the best price, and in **question 7** in **lesson 5.3.2**, where they calculate a surface area in a real-world situation.

• Students use communication in **question 4** in **lesson 5.1.1**, where they explain how they know whether a triangle has a right angle, in **part C** in **lesson 5.2.3**, where they explain their conjecture about the area of a dodecagon, and in **question 3** in **lesson 5.3.1**, where they describe the difference in growth of volume as compared to area.

• Students use reasoning in **question 3** in **lesson 5.1.1**, where they infer a pattern from given information and use it to solve a problem, in **question 8** in **lesson 5.1.2**, where they recognize that knowing two side lengths of a right triangle may not define a triangle if they do not know to which sides the values refer, in **question 8** in **lesson 5.2.1**, where they use relationships between perimeter and side lengths to draw a conclusion about the possible values of the perimeter, in **question 7** in **lesson 5.2.4,** where they recognize the relationship between various formulas, in **question 13** in **lesson 5.2.5**, where they reason about changes in area as a result of changes in length measures, and in **question 4** in **lesson 5.3.2**, where they make and verify a conjecture about how surface area changes when dimensions change.

• Students consider representation in **lesson 5.1.1**, where they represent the Pythagorean theorem both numerically and geometrically, in **lesson 5.2.2**, where they consider how to represent a large object in a small diagram, and in the **Try This** in **lesson 5.3.2**, where they use a net to represent an object in order to calculate its surface area.

• Students use visualization skills in **question 7** in **lesson 5.1.2**, where they recognize whether a triangle is acute or obtuse by looking at it, in **question 7** in **lesson 5.2.1,** where they use visual cues to see why an irregular boundary has a greater perimeter than a shape of similar size with straight sides, in **question 8** in **lesson 5.2.2**, where they visualize how to orient the paper to make a scale drawing, in **question 2** in **lesson 5.2.4**, where they look at a whole shape as a composite of parts, and in **question 8** in **lesson 5.3.2**, where they visualize halves of a cube to discuss surface area.

• Students make connections in **lesson 5.2.1**, where they connect different metric units of measure, in **lesson 5.2.2**, where they relate the measurement concepts involved in scale drawings to the number concepts involved in solving proportions, in **lesson 5.2.4**, where the connect the area of a circle to the area of a related parallelogram, and in **question 6** in **lesson 5.3.1**, where they relate metric measures.

Rationale for Teaching Approach

• This unit is divided into three chapters.

Chapter 1 focuses on developing and applying the Pythagorean theorem.

Chapter 2 focuses on length and area relationships, with a special focus on the circle for area and on similarity and scale diagrams for length.

Chapter 3 focuses on working with the volume and surface area of rectangular prisms.

• The **Explore** lesson allows students to estimate the area of a circle before they learn the formula. In this way, they can relate what they learn to what they have already predicted.

• There are three **Connections**. The first focuses on perimeter and area relationships, using interesting shapes called pentominos rather than ordinary rectangles. The second provides a history of the value of π. The third allows students to explore areas of the pieces of geometric puzzles called tangrams.

• The unit's **Game** is a strategy game which follow ups and uses materials from one of the **Connections**.

• Throughout the unit, it is important to encourage flexibility and to accept a variety of approaches from students.

Getting Started

Curriculum Outcomes	Outcome relevance
7 Volume: rectangular prisms **7** SI Units: identify, use and convert **7** Area: composite shapes **7** Circles: solve problems with diameter, radius, circumference	Students will find the work in the unit easier after they review measurement formulas and metric conversions.

Pacing	Materials	Prerequisites
1 h	None	• familiarity with formulas for areas of rectangles and triangles • calculating the area of composite shapes • converting metric units • familiarity with the formula for the circumference of circle • familiarity with the formula for the volume of rectangular prism

Main Points to be Raised

Use What You Know

• It can be easier to calculate the area of a composite shape, if you first divide it into rectangles and triangles.

Skills You Will Need

• To convert metric lengths, you multiply and divide by appropriate powers of 10.

• The formula for the area of a rectangle is base × height. The formula for the area of a triangle is base × height ÷ 2.

• π (pi) is the ratio of the circumference of a circle to its diameter.

• You can calculate the volume of a rectangular prism by multiplying the area of a base by the corresponding height.

• 1 cm^3 of water is equivalent to 1 mL of water.

Use What You Know — Introducing the Unit

• Before assigning the activity, you may wish to review the formula for the area of a rectangle.

• Students can work alone or in pairs.

While you observe students at work, you might ask questions such as the following:

• *How did you estimate the area of the whole shape?* (It looked like the rectangle was about $\frac{1}{3}$ of the whole shape, so I estimated 3 × 24 = 72 cm^2.)

• *How did you divide the shape?* (I cut the top part into a rectangle and triangle so there were two rectangles and one triangle altogether.)

• *Why did you pick those shapes?* (I knew the formulas for the areas of those shapes.)

Skills You Will Need

• To ensure students have the required skills for this unit, assign all of these questions.

• You may wish to review the formulas for the area of a triangle, the circumference of a circle, and the volume of a rectangular prism. You might also review the meaning of the metric prefixes *milli, kilo,* and *centi*. You can refer students to the **Measurement Reference** at the back of the student text.

• Students can work individually.

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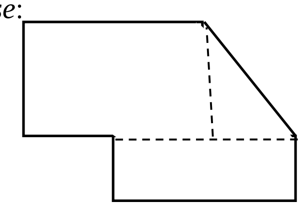
Answers

A. 24 cm^2

B. 82 cm^2; *Sample response*:
About two and a bit more of the rectangle can fit in the upper part of the shape, so the whole area is about 82 cm^2.
[(24 + 48 + 10) cm^2 = 82 cm^2]

C. i) *Sample response*:



ii) 84 cm^2

1. a) 1200 **b)** 45 **c)** 1200

2. a) $A = l \times w$
b) *Sample response*: $A = b \times h \div 2$

3. a) 24 cm^2 **b)** $12\frac{1}{2}$ cm^2 **c)** 6 m^2

4. a) *Sample response*:
The outside outline of the circle.
b) π : 1 or about $\frac{22}{7}$
c) $C = 2 \times \pi \times r$ or $C = 2r\pi$

5. a) 22 cm **b)** 88 mm

6. a) 63 cm^3 **b)** 48 m^3

7. a) 1 mL **b) i)** 1000 mL **ii)** 1 L

8. 2 cm

9. No; [*Sample response*:
The box has a volume of 960 cm^3, which is less than 1000 cm^3, so it cannot hold 1000 mL, or 1 L.]

Supporting Students

Struggling students

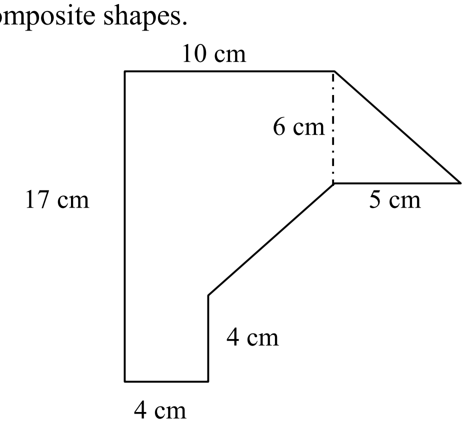
• If students are struggling with the formulas for various measurements, review them as meaningfully as possible. Rather than just telling students the formulas, try to redevelop them using the concepts in the Class VII student text.

For example, to show why the area of a triangle is the product of the base and height divided by 2, show how any triangle is half of a parallelogram, whose area is the product of the length of the base and the height of the triangle.

Enrichment

• Some students might enjoy calculating the areas of more challenging composite shapes.

For example, you could provide shapes made up of several triangles and rectangles, such as this shape:



Or, students may wish to create their own shapes made up of triangles, rectangles, and trapezoids to trade with
a partner.

Chapter 1 The Pythagorean Theorem

5.1.1 The Pythagorean Theorem

Curriculum Outcomes	Outcome relevance
8-D1 Pythagorean Relationship: understanding • investigate the side relationships of a variety of right triangles • explore the 3-4-5 rule for establishing a right angle • understand, through investigation, that if a square is made on each side of a right triangle, the sum of the two smaller squares will equal the area of the longer side ($c^2 = a^2 + b^2$ where a, b, and c are sides of a right triangle) • explore triangles of a variety of orientations to discover that the hypotenuse (the longest side) is the side opposite the right angle regardless of orientation **8-D2 Pythagorean Relationship: application** • use the Pythagorean relationship to determine if three given side lengths are, or are not, the sides of a right triangle	The Pythagorean theorem is fundamental to the development of many concepts in higher mathematics, including trigonometry. It is also useful in everyday life to calculate measurements that cannot be determined directly.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM) (Optional)	• familiarity with the term *right triangle* • familiarity with the formula for the area of a square • raising a number to the second power (or squaring it)

Main Points to be Raised

• The hypotenuse is the name for the longest side of a right triangle.

• The Pythagorean theorem states that the area of the square on the hypotenuse is the sum of the areas of the squares on the other two sides of every right triangle.

• You can use the Pythagorean theorem to calculate the length of one side of a right triangle if you know the lengths of the other two sides.

• If a set of three whole numbers can be the side lengths of a right triangle, it is called a Pythagorean triple.

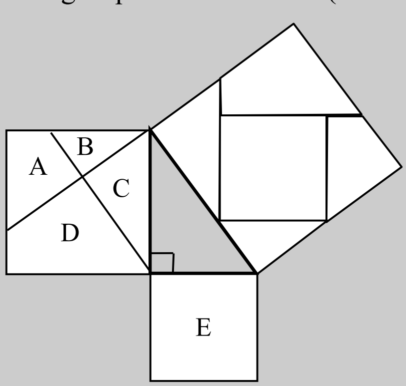
Try This — Introducing the Lesson

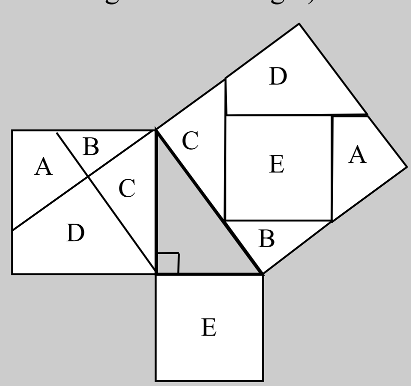
A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *What did you notice about the five pieces?* (They covered the large square exactly.)

• *How did the pieces go together?* (E in the centre, B and D on opposite sides of E, A and C on the other opposite sides of E.)

As an alternative, you could show students the diagram below on the left and ask them to identify where each part of the large square comes from (as shown in the diagram on the right).





The Exposition — Presenting the Main Ideas

• Have students look at the 3-4-5 right triangle shown in the middle of the exposition on **page 119** of the student text. They should count the grid squares to see the areas of the three squares on the three sides of the right triangle; they can notice that 9 + 16 = 25. Have them look at the shading of the grid squares on the hypotenuse to show where the squares come from: the 9 shaded squares come from the square on the short side of the triangle and the 16 white squares come from the square on the other side of the triangle.

• Explain the meaning of $c^2 = a^2 + b^2$, i.e., if the short sides of the right triangle are a and b and the hypotenuse is c, then the area of the square on the hypotenuse is $c \times c$, or c^2 and the other two areas are $a \times a$, or a^2, and $b \times b$, or b^2. This explains why the formula describes a relationship among areas.

• Point out that the formula also describes a relationship among numbers. Draw a right triangle and mark the two short sides as 6 cm and 8 cm. Ask students to use the Pythagorean theorem to see why the hypotenuse must be 10 cm.

• Then draw a right triangle with a hypotenuse of 13 cm and a small side labelled 5 cm. Ask students to use the Pythagorean theorem to calculate the length of the other side ($13^2 – 5^2 – 169 – 25 = 144$; $12 \times 12 = 144$, so 12 is the length of the other side.)

• Remind students that if the side lengths of a triangle do not satisfy the Pythagorean theorem, the triangle cannot be a right triangle.

• Indicate that 5-12-13 and 6-8-10 are called Pythagorean triples because they are whole number side lengths of right triangles.

Revisiting the Try This

B. This question allows students to make a connection between the geometry problem they solved in **part A** and the Pythagorean theorem.

Using the Examples

• Have students work through the examples in pairs. You may wish to remind students that dm is the abbreviation for 1 decimetre, which is a length of 10 cm.

• Provide time for students to ask any questions they might have.

Practising and Applying

Teaching points and tips

Q 1: This question is designed to get students accustomed to the idea that the hypotenuse is always opposite the right angle and is always the longest side of a right triangle.

Q 2: Students can use the examples as a model for this question. They should realize that the Pythagorean theorem is not true for non-right triangles.

Q 3: This question is designed to help students see that once a Pythagorean triple is found, any multiple of that triple is another Pythagrean triple. For **part d)**, students might note that a third triangle is possible (22.5, 30, 37.5) but that it does not continue the pattern.

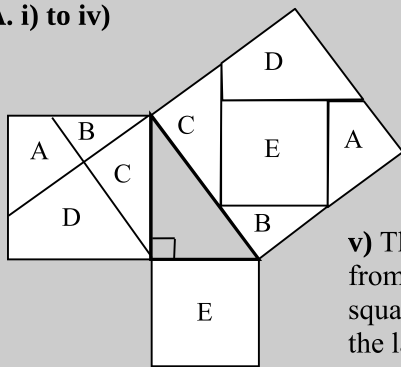
Common errors

• To find a missing side length, many students square and add the values of whichever two sides of the right triangle are given, even if one side is the hypotenuse. Remind students to check their work to make sure that the hypotenuse is the longest side of the triangle.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can apply the Pythagorean theorem to see whether a triangle is a right triangle
Question 4	to see if students can communicate the usefulness of the Pythagorean theorem

Answers

A. i) to iv)  **v)** The five pieces from the two smaller squares exactly cover the large square without overlapping.	**B.** *Sample response*: The sum of the areas of the smaller squares is equal to the area of the large square. If the side lengths of the squares on the shorter sides are *a* and *b* and the side length of the square on the hypotenuse is *c*, then $a^2 + b^2 = c^2$.
1. a) AC **b)** EF **c)** GH **2. a)** Yes; [$13^2 = 169$ and $5^2 + 12^2 = 25 + 144 = 169$.] **b)** No; [$25^2 = 625$ and $8^2 + 23^2 = 64 + 529 = 593$, not 625.] **c)** Yes; [$17^2 = 289$ and $8^2 + 15^2 = 64 + 225 = 289$.]	**3. a)** Yes **b)** *Sample response*: Multiply the values of Row 1 by 2 to get Row 2, multiply the values of Row 1 by 3 to get Row 3, multiply the values of Row 1 by 4 to get Row 4, and so on. **c)** 18, 24, 30; 21, 28, 35 **d)** 2 triangles: 18, 24, 30 and 30, 40, 50 [**e)** *Sample response*: Each side is half of a 3-4-5 triangle, so it is part of the pattern if you work backwards, dividing the values in Row 1 in half.] [**4.** *Sample response*: Square the short sides (7^2 and 24^2) to see if the sum of the squares (625) is equal to 25^2 (625).]

Supporting Students

Struggling students

• Some students might benefit from working on grid paper so that they can count the number of square units in each square on the sides of the right triangle. This might help them better understand the theorem.

Enrichment

• Students might solve problems involving Pythagorean triples.

For example, they might look for all the triples where at least two side lengths are 10 or less. Or, they might look for all the triples that include a certain value, such as 24.

5.1.2 Applying the Pythagorean Theorem

Curriculum Outcomes	Outcome relevance
8-D2 Pythagorean Relationship: application • understand the usefulness of Pythagorean relationships to solve problems in real life (whenever a triangle has a right angle and two known sides) • investigate real world problems to determine the length of the hypotenuse, as well as the length of the other side when the hypotenuse and one side are given • understand that the Pythagorean relationship can be used if only one side is given when the right triangle is isosceles • find distance between two points using Pythagorean relationship (e.g., determine the reach of a ladder)	The Pythagorean theorem is fundamental to the development of many concepts in higher mathematics including trigonometry. It is also useful in everyday life to calculate measurements that cannot be determined directly.

Pacing	Materials	Prerequisites
1 h	• Rulers (optional)	• squaring numbers • familiarity with the terms *isosceles triangle* and *right triangle*

Main Points to be Raised

• The shorter sides of a right triangle are called *legs*.

• If you know the lengths of any two sides of a right triangle, you can find the length of the third side using the Pythagorean theorem.

• If you know that a right triangle is isosceles, you only need to know the length of one side and whether it is a leg or the hypotenuse to find the lengths of the other two sides.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Which part of the triangle does the ladder represent?* (The hypotenuse)

• *What are the lengths of the two shorter sides of the triangle?* (4 m and 11 m)

• *How do you know whether the ladder is long enough?* (The hypotenuse of the right triangle with sides 4 m and 11 m has to be less than 12 m. If the hypotenuse is longer than 12 m, the ladder will not be able to extend the extra 1 m above the wall.)

The Exposition — Presenting the Main Ideas

• Remind students of the Pythagorean theorem, $c^2 = a^2 + b^2$.

• On the board, draw a right triangle with leg lengths of 4 units and 8 units. Identify the shorter sides of the triangle as the *legs* of the triangle. Ask students to tell you where the hypotenuse is.

• Ask students how to use the Pythagorean theorem to calculate the length of the hypotenuse.

• Now draw a triangle with a hypotenuse of 13 units and one leg length of 5 units. Ask students to figure out how to use the Pythagorean theorem to calculate the missing leg length. If necessary, help students see why they must subtract 5^2 from 13^2.

• Have students read through the exposition on **page 122** of the student text.

• Ask students why it must be the leg lengths that are equal for an isosceles right triangle and not a leg and the hypotenuse.

• Make sure that students understand the last part of the exposition by asking them to calculate the length of the hypotenuse of an isosceles right triangle with a leg length of 5 cm. Then ask them to calculate the leg length of another isosceles right triangle with an hypotenuse of 10 cm.

Revisiting the Try This

B. This question allows students to make a formal connection between what was done in **part A** and the application of the Pythagorean theorem.

Using the Examples

• Present the two questions in the two examples on the board. Ask students to try the questions alone or in pairs and then to compare their results with the solutions provided in the student text.

Practising and Applying

Teaching points and tips

Q 3: Make sure students understand that the ramp is the hypotenuse.

Q 5: Students need to calculate two distances:

- the distance from A to the vertex of the right angle
- the distance from that vertex to B

They must then add the parts.

Q 6: Students need to use two triangles. The height divides the base into two equal lengths, so the base of each right triangle is 1.5 m.

Q 7: This question is designed to highlight the fact that the Pythagorean theorem is true *only* for right triangles:

- For acute triangles, the sum of the squares of the two legs is always greater than the square of the hypotenuse.
- For obtuse triangles, the sum of the squares of the two legs is always less than the square of the hypotenuse.

Q 8: Students have to realize that the 30 cm could be the hypotenuse or the length of the second leg. The
16 cm length cannot be the hypotenuse since it is less than 30 cm.

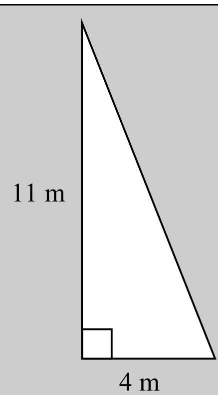
Common errors

• Many students will use the 3 cm base length in **question 6** rather than dividing it in half. If a student does this, you may need to ask him or her to identify the two right triangles and ask for the dimensions of each. Then you need to ask for the total length (of the base) he or she used.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can apply the Pythagorean theorem directly
Question 4	to see if students can solve a simple real-world problem using the Pythagorean theorem
Question 5	to see if students can solve a more complex real-world problem using the Pythagorean theorem

Answers

A. i) 	**ii)** Yes; *Sample response*: The triangle formed by the 4 m and 11 m legs has an hypotenuse equal to the square root of 137. ($4^2 + 11^2 = 16 + 121 = 137$) The square root of 137 is less than 12 because $137 < 144$. A 13 m ladder is longer than necessary, so it is safe. **B.** *Sample response*: I used it to figure out the hypotenuse, which is the minimum length of the ladder. I knew the lengths of the two legs.

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1. a) 15 cm **b)** 26 m

2. a) 8.9 km **b)** 11.3 cm

3. 10.2 m

4. 9 km

5. About 2500 m (or 2482 m)

6. 1.3 m

7. a)

	a^2	b^2	c^2
i)	36	49	64
ii)	64	64	81
iii)	225	400	576

b) $a^2 + b^2 > c^2$

c)

	a^2	b^2	c^2
i)	100	144	400
ii)	36	36	100
iii)	225	400	676

d) $a^2 + b^2 < c^2$

[**e)** I can measure the three sides, with c being the longest. If $c^2 = a^2 + b^2$, I know it is a right triangle.
If $c^2 > a^2 + b^2$, I know it is an obtuse triangle.
If $c^2 < a^2 + b^2$, I know it is an acute triangle.]

8. No; [*Sample response*:
The third side could be the hypotenuse or one of the legs. If the third side is the hypotenuse, it is the square root of $16^2 + 30^2$, or 34 cm.
If the third side is one of the legs, it is the square root of $30^2 - 16^2$, or about 25 cm.]

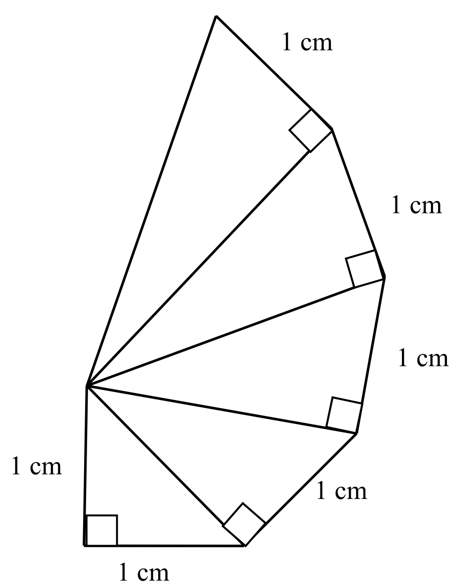
Supporting Students

Struggling students

• **Questions 5 and 6** may be the most challenging for struggling students since two triangles are involved. You might pair them up with other students for these questions.

Enrichment

• You might have students explore the Archimedes spiral. This spiral is created by repeatedly drawing new triangles on a given triangle as shown below:



Students can extend the shape by a few more triangles to see what happens. They can then calculate each hypotenuse length as a square root to view the pattern:

$\sqrt{2}$ cm, $\sqrt{3}$ cm, $\sqrt{4}$ cm, $\sqrt{5}$ cm, $\sqrt{6}$ cm, $\sqrt{7}$ cm, $\sqrt{8}$ cm, $\sqrt{9}$ cm, $\sqrt{10}$ cm, $\sqrt{11}$ cm, $\sqrt{12}$ cm

As you move to the next triangle, the number you take the square root of is 1 greater.

5.2.1 Area and Perimeter Relationships

Curriculum Outcomes	Outcome relevance
8-D3 Area and Perimeter: patterns and relationships of quadrilaterals and circles • understand, through investigation, that area can vary when perimeter is fixed (e.g., for a perimeter of rectangle of 16 cm, determine all possible whole-number dimensions) • understand, through investigation, that perimeter can vary when the area of a rectangle is fixed **8-D4 SI Units: solve measurement problems** • integrate measurement problems as other mathematical ideas are explored • investigate and create problems involving length and perimeter (mm, cm, m, km), area (mm^2, cm^2, m^2, km^2, and hectare (1 hm^2))	• As students explore the relationship between the perimeter and the area of a rectangle, they will be prepared to solve many optimization problems (for example, the most area for the least perimeter). • Because the world is increasingly metric, it is important for students to be able to move easily from one metric unit to another to solve real-world problems.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM) (optional)	• familiarity with the formulas for the perimeter and area of a rectangle • familiarity with some metric relationships (e.g., 100 cm = 1 m; 1000 m = 1 km)

Main Points to be Raised

• Metric length units include millimetres, centimetres, metres, and kilometres.

• The area of shapes is measured in square units, which are the squares of the length units.

• To relate different square units, you must square the corresponding length units.

For example, since 1 m = 100 cm, then
$1\ m^2 = 100 \times 100\ cm^2$.

• There are several ways to write the formula for the perimeter of a rectangle:

- You can double the length, double the width, and then add them.
- You can add the length and width, and then double the sum.

• Two rectangles can have the same perimeter but different areas. Or, they can have the same area but different perimeters.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. You may choose to have them use grid paper. While you observe students at work, you might ask questions such as the following:

• *What do you know about the length and width if the perimeter is 16?* (The total of length and width is 8.)

• *What might be the dimensions of a long and skinny rectangle?* (It could be 1 cm by 7 cm.)

• *How did you get a new rectangle once you had a rectangle that worked?* (I increased the width by 1 cm and decreased the length by 1 cm.)

• *How do you know you have all the possibilities?* (Once I got to the square, when I increased the width by 1 cm, I was back to a rectangle I had already used. It was turned 90°.)

The Exposition — Presenting the Main Ideas

• With students, read through the exposition on **page 125** of the student text. Make sure they understand that although no equivalence is shown for 1 mm^2, we could create equivalence using either smaller units or decimals.

For example, since 1 mm = 0.1 cm, then $1\ mm^2 = 1\ mm \times 1\ mm = 0.1\ cm \times 0.1\ cm = 0.01\ cm^2$.

• The hectare (ha) may be less familiar to students. Be sure to define it as the area of a 100 m square. It is for this reason that 1 ha = 10,000 m^2.

• Students have already met the idea that rectangles with different areas can have the same perimeter and that rectangles with different perimeters can have the same area, so this part of the exposition should be familiar.

Revisiting the Try This

B. This question is designed to make students informally aware that for rectangles with a fixed perimeter, the rectangle with the greatest area is the square.

Using the Examples

• Assign students to pairs and have them read through the two examples. One student in each pair should be responsible for **example 1** and the other for **example 2**. They should then teach each other what they have learned.

Practising and Applying

Teaching points and tips

Q 1 b) and c): You may need to remind students to change all measures to the same unit

Q 2: You may have to reassure students that perimeter and circumference are synonymous for **part b)**. Circumference is just a special name for the perimeter of a circle. Refer students to **example 2** for help with this question.

Q 5: This question is designed to remind students that the unit square centimetres is not reserved for measuring squares, i.e., 1 cm^2 is the area equivalent to the area of a square that is 1 cm by 1 cm.

Q 6: Students need to experiment. They could start with combinations of numbers with a product of 14 or with combinations of numbers with a sum of 7.5 to represent the length and width.

Q 7: This question alerts students to the notion that when a shape has very irregular sides, its perimeter is usually greater than the perimeter of a simpler polygon of a similar size.

Q 8: This question requires reasoning. The fact that the side lengths are whole numbers of centimetres is
an essential part of the question.

Q 9: Students begin by calculating the side length.

Common errors

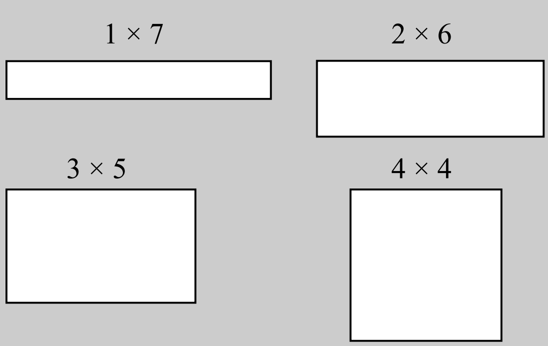
• Some students continue to confuse the terms perimeter and area. You may wish to draw a shape on the board and mark the two terms on the shape.

Suggested assessment questions from Practising and Applying

Question 3	to see if students understand that area can vary when perimeter is fixed
Question 4	to see if students understand that perimeter can vary when area is fixed
Question 11	to see if students can solve problems involving area and perimeter

Answers

A. 4 rectangles;



B. 7 cm^2, 12 cm^2, 15 cm^2, 16 cm^2

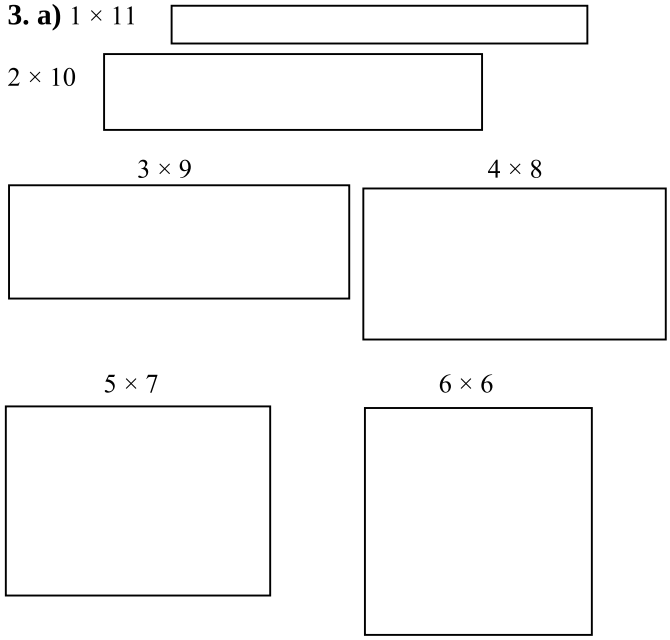
C. *Sample response*:
The smallest area is long and thin and the greatest area is square.

1. a) P = 2.6 km or 2600 m
A = 0.3 km^2, or 30 ha, or 300,000 m^2
b) P = 18 cm, A = 12 cm^2
c) P = 16 m, A = 12.5 m^2
d) P = 80 cm, A = 300 cm^2

2. *Sample responses*:
a) Metres and squares metres or hectares
b) Millimetres and square millimetres (or centimetres and square centimetres)
c) Centimetres and square centimetres
d) Kilometres and square kilometres or hectares

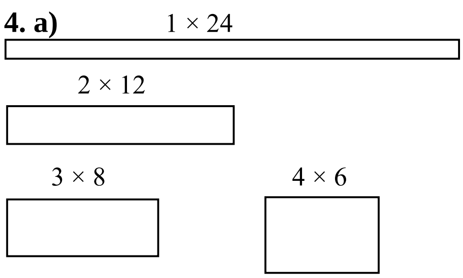
Answers [Continued]

3. a) 1 × 11



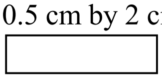
b) The smallest area (11 cm^2) is long and thin (1 cm by 11 cm) and the greatest area (36 cm^2) is a square (6 cm by 6 cm).

4. a)



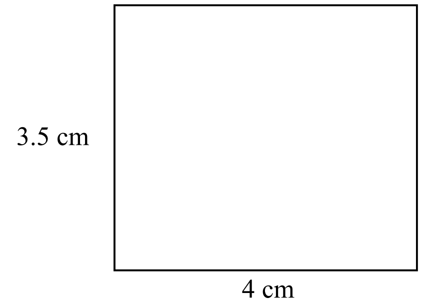
b) The greatest perimeter (50 cm) is long and thin (1 cm by 24 cm) and the least perimeter (20 cm) is closest to the shape of a square (4 cm by 6 cm).

5. *Sample responses*:
a) 0.5 cm by 2 cm



b) 5 cm

6. 4 cm by 3.5 cm

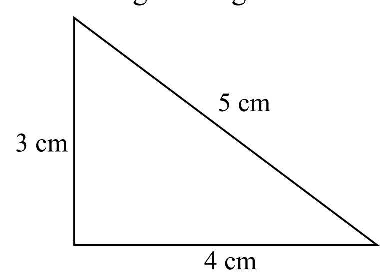


7. a) 45,000 km^2
b) It is close to the real area of 47,000 km^2
c) 900 km
[**d)** The boundaries of Bhutan follow the contours of the land, making the perimeter longer.]

8. a) The perimeter is twice the sum of length and the width, so it will be a multiple of 2.
b) The area will be an even number when one of the sides is an even number [because the product of two evens or an even and an odd is even.]

9. a) 6.25 cm^2 **b)** 2 km

10. A 3-4-5 right triangle:



11. a) 6 cm by 8 cm
b) *Sample response*:
A 1-by-48 rectangle is long and thin and has a much greater perimeter than the rectangle in **part a)**.

12. No; [*Sample response*:
Rectangles with the same perimeter can have different areas. You can only figure out the area from the perimeter of a square.]

Supporting Students

Struggling students

• **Questions 1 d), 6, 7 d), 8, 9 b), and 11** will be most difficult for struggling students. You might choose to assign them only some of these questions. You might also have them work with other students for these more difficult problems.

Enrichment

• Students might create problems like **question 6** to trade with classmates.

For example, a student might draw a rectangle, calculate its perimeter and area, provide the perimeter and area without the dimensions, and ask another student to figure out the dimensions.

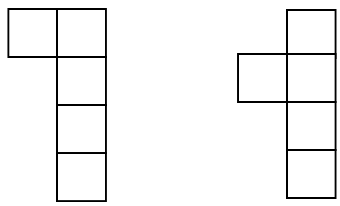
CONNECTIONS: Pentominos

• Provide grid paper students can use to cut out their pentominos.

- If students use regular grid paper, each 2 cm^2 square will be a 3-by-3 grid.
- If they use Small Grid Paper (BLM), each 2 cm^2 square will be a 4-by-4 grid.
- If they use Grid Paper (1 cm by 1 cm), each 2 cm^2 square will be a 2-by-2 grid.

• Many students will need support for drawing all the pentominos. You might suggest an organizational strategy like this:

- Begin with a line that is 5 squares long (there is only one such shape).
- Make a line of 4 squares and place your fifth square in the next row. Use all possible positions for that fifth square.

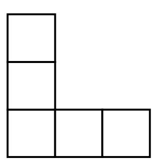
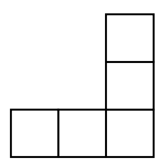


(Students might notice that there are only two possibilities, since attaching a square to the top of the row is no different than attaching it to the bottom; attaching it to the left side is no different from attaching it to the right side.)

- Next, make a line of 3 squares, placing the 4th and 5th squares in other rows and in all possible arrangements.

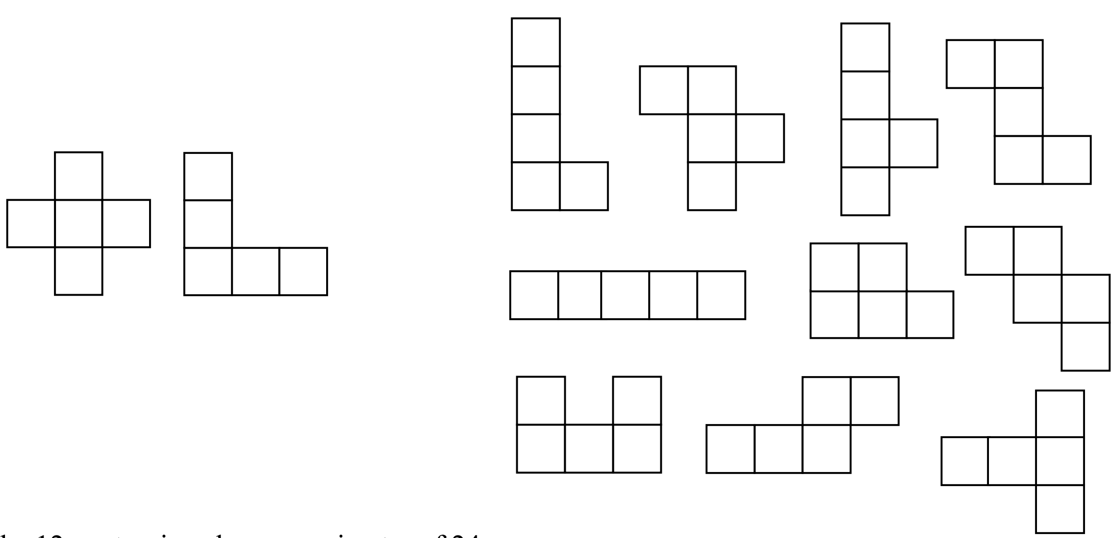
• If students have trouble manipulating the small 2 cm-by-2 cm square shapes, they might cut out larger versions. They will need to take the larger size into account when they describe the perimeters they find.

• Talk about how rotations and reflections do not make different pentominos.

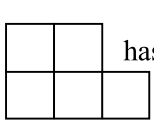
For example:  and  are the same pentomino.

Answers

1. There are 12 pentominos altogether — the two from the student book, on the left, plus these 10:



2. 11 of the 12 pentominos have a perimeter of 24 cm.

Only  has a perimeter of 20 cm.

GAME: Pentominos

• Provide grid paper students can use as their game board:

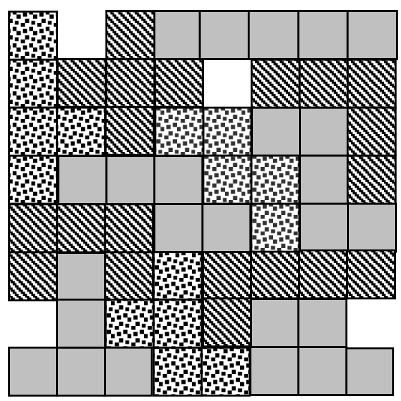
- If students use regular grid paper, they will need a 24-by-24 grid.
- If they use Small Grid Paper (BLM), they will need a 32-by-32 grid.
- If they use Grid Paper (1 cm by 1 cm) (BLM), a 16-by-16 grid will work.

• If students use Grid Paper (1 cm by 1 cm) (BLM), each square of each pentomino will cover four grid squares,
as shown in the photo and visual in the student text.

• If students use Small Grid Paper (BLM), each square will cover 16 grid squares.

• If students use regular grid paper, each square will cover 9 grid squares.

• As an individual challenge, students can investigate whether it is possible to place all 12 pentominos on the board without overlapping. First ask them to compare the total area of all the pieces (240 cm^2) and the area of the board (16 by 16 cm^2). There are many ways to fit all the pieces on the board. Here is one example:



5.2.2 Scale Drawings

Curriculum Outcomes	Outcome relevance
8-D5 Proportion: solve indirect measurement problems • link proportion to ideas of ratio and rate • read, interpret, and discuss scale drawings • understand usefulness of proportion ideas in relevant real-world problems **8-B1 Proportion: solve problems** • use a variety of strategies to solve problems of proportionality: - find relationships between the various terms of proportion and use these relations to solve for missing values (e.g., use equivalent fractions to solve $\frac{2.2}{5} = \frac{x}{5}$) • recognize uses for and importance of proportion	Scale drawings and maps are an important part of everyday life. Students need to learn to interpret and create these drawings, which are based on proportion and length concepts.

Pacing	Materials	Prerequisites
1.5 h	• Rulers	• familiarity with metric length units • solving simple proportions

Main Points to be Raised

• A scale drawing represents a real object in a reduced or enlarged size. Every scale drawing has information, called a key or scale ratio, to tell how the drawing's measurements compare to the measurements of the real object.

• The key can be in the form of a ratio if the same units are used for the drawing and for the real measurements.

• A map is a special type of scale drawing that represents a geographical area.

• You can use the key on a map to calculate real distances.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How many 20 m widths are in the field? How wide is the field?* (3 sets of 20 m. It is 60 m wide.)

• *How many 20 m lengths are in the field? How long is the field?* (4 sets of 20 m. It is 80 m long.)

• *How do you calculate the area?* (Multiply the total length by the total width.)

• *Is there another way to calculate the area?* (Yes. Get the area of one 20 m square and multiply it by 12.)

The Exposition — Presenting the Main Ideas

• Ask students if they already know what a scale drawing is. If not, introduce them to the idea by taking a piece of paper and showing how you could make a scale drawing of the paper on the board. Let 1 cm on the drawing represent 2 cm on the perimeter of the paper. Next to the drawing, write a key or scale ratio 1 : 2 and describe it as a *key*. Show that another way to write the key is, "1 cm represents 2 cm". You might relate this to the term *representative fraction* used in geography.

• Show how you can measure the drawing to figure out the length and width of the paper if you did not already know these measurements.

• Have pairs of students read through the exposition on **pages 130 and 131** of the student text. Let them ask any questions they might have.

Make sure that they understand why 50 km was converted to 5,000,000 cm (50 km = 50,000 m = 5,000,000 cm).

Revisiting the Try This

B. Students need to measure the drawing in **part A** with their rulers to determine the key for the drawing. To write it as a ratio, they need to use the fact that 20 m = 2000 cm.

Using the Examples

- Write the problems from **examples 1 and 2** on the board. Ask students to solve them and then compare their solutions with the two solutions in the text.
- Work through **examples 3 and 4** together as a class.

Practising and Applying

Teaching points and tips

Q 3: Students should measure the arrow lines to be 2.5 cm for height and 3 cm for width. Students need not worry about the roof in their drawings for **part b)**.

Q 5: This question lets students consider the use of the scale diagram in both directions. If you know real distances, you must decide how to represent them on the map. If you know distances on the scale diagram, you can calculate real distances.

Q 6: Students should measure the arrow lines to be 6 cm for **part a)**, 3 cm for **part b)**, and 4 cm for **part c)**.

Q 7: Students should measure the arrow line to be 6.5 cm.

Q 8: Students need to decide how the drawing could be oriented on the page.

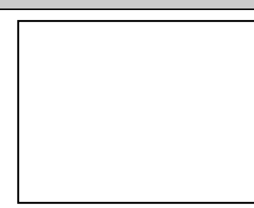
Common errors

- Students may have difficulty writing the scale as a ratio. For a scale like 1 cm represents 3 km, students often incorrectly use the ratio 1 : 3. Remind students that this means that the second value is 3 times the first and that 3 km is 3000 times as long as 1 cm, not 3 times as long. Have them check to see if that makes sense for the situation they are diagramming.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use provided information to describe a scale
Question 4	to see if students can use a scale to determine the size of a scale drawing
Question 6	to see if students can use a scale to determine real measurements
Question 8	to see if students can decide on a scale for a drawing

Answers

A. i) 60 m and 80 m **ii)** 4800 m^2	**B. i)** 1 cm represents 10 m, or 0 10 m **ii)** 1 : 1000
1. a) 1 cm represents 16 cm **b)** 1 cm represents 12.5 cm **c)** 1 cm represents 2 m **d)** 1 cm represents 6 m **e)** 1 cm represents 3 m **f)** 1 cm represents 0.3 mm **2. a)** The key is for a real object that is smaller than the scale drawing. **b)** *Sample response*: To show details of very small things, like insects. **3.** *Sample responses*: **a)** Height: 5.5 m; width: 6.6 m	**b)**  1 cm represents 2 m **4.** 4.5 cm by 6 cm **5. a)** 160 km **b)** 2.5 cm **6. a)** 1.8 m **b)** 3 m **c)** 2 cm

7. a) About 4000 km **b)** About 13 times	**8.** 22 cm represents 12 m or 1 cm represents about 0.6 m. [*Sample response*: The drawing area of the paper is 22 cm (26 cm – 4 cm) by 30 cm (34 cm – 4 cm). The key that compares the length of the drawing area (in cm) to the width of the building (in m) is 30 cm represents 15 m, which needs a scale of 1 cm represents 0.5 m. Using this scale for the height, I would need a drawing area that is 24 cm wide, but my drawing area is only 22 cm wide. So I need to create a key that works for both the height of the building and the width of the drawing area. The key that compares the width of the drawing area (in cm) to the height of the building (in m) is 22 cm represents 12 m, which needs a scale of 1 cm represents about 0.6 m. Then I could represent 15 m using 25 cm, which fits.]

Supporting Students

Struggling students

• Students who have difficulty with certain computations may still understand the underlying concepts.

For example, you might choose not to assign **question 6 a)** or **question 8**, which involve some spatial reasoning.

• You might allow struggling students to simplify some of the numbers they work with.

For example, in **question 5 b)**, they could use 120 km instead of 100 km.

Enrichment

• Students might create scale drawings of 2-D figures that are of interest to them.

5.2.3 EXPLORE: Estimating the Area of a Circle

Curriculum Outcomes	Lesson Relevance
8-D3 Area and Perimeter: patterns and relationships of quadrilaterals and circles • understand, through investigation, that area can vary when perimeter is fixed (e.g., for a perimeter of rectangle of 16 cm, determine all possible whole-number dimensions) • determine what happens to the area of a regular polygon as the number of sides increases (e.g., if perimeter is 24 cm, what is the area when the figure has 4 sides? 6 sides?)	This optional exploration will make the more formal work with the area of a circle more meaningful. The lesson prepares students for the next lesson, which compares the area of a square with the area of a circle using the estimate $3 \times r^2$.

Pacing	Materials	Prerequisites
40 min	• Rulers • Grid Paper (BLM) (optional)	• measuring perimeters of polygons • using a grid to estimate area • familiarity with the formula for the circumference of a circle • multiplying by a fraction

Main Points to be Raised

- If different regular polygons have the same perimeter, the polygon with the most sides has the greater area.
- There are a number of ways to estimate the area of a given polygon.
- The perimeter of a circle is referred to as the circumference, although it is not incorrect to refer to it as the perimeter.
- You can estimate the area of a circle by examining regular polygons with the same perimeter. As the number of sides of a polygon increases, the area of the polygon gets closer and closer to the area of the circle with the same perimeter (or circumference).

Exploration

- Make sure students understand that they are to measure perimeter and area in grid units, which are centimetres and square centimetres.
- Assign students to work in pairs on **parts A to C**.

While you observe students at work, you might ask questions such as the following:

- *Why do you only have to measure one side length to calculate the perimeters of the square, the hexagon and the octagon?* (They are all regular polygons, which means they have equal side lengths.)
- *How did you determine the area of the hexagon?* (I divided it into a rectangle and two triangles. The rectangle was 4 by 7, for a total of 28 squares. The triangles each had a height of 2 units and a base of 7 units, so the total area for the two triangles was 14 squares. That is a total of 42 squares.)
- *How did you determine the area of the octagon?* (I estimated. I noticed that there are 37 whole squares, another 4 squares that are almost whole, another 4 half-squares, and a bit more, for a total of about 43 squares.)
- *How did you determine the area of the circle?* (I imagined the circle inside a square that was about 7.6 units on each side. Then I had to subtract the 4 corner areas. I thought each corner area was about 3 squares. That is 57.8 – 12, or about 45.8 squares.)

Observe and Assess

As students work, notice the following:

- Do they calculate successfully the areas of rectangles and triangles?
- Do they estimate reasonably the areas of the octagon and circle?
- Do they realize that the perimeter is the same, but the area increases as the number of sides increases?
- Do they recognize that the circle has the greatest area?

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and these questions.

• *How do you know that the shapes have the same perimeter?*

• *Which areas were easiest to figure out? Why?*

• *There is more than one way to calculate the area of the hexagon. What are some of the ways?*

Answers

A. i) 24 cm; They are all the same. **ii)** Square: 36 cm^2 Hexagon: *Sample response*: 42 cm^2 (There is a rectangle of 28 squares and two triangles each with an area of 7 squares, so the total area is 42 cm^2.) Octagon: *Sample response*: About 43 cm^2 (There are 37 whole square centimetres inside the octagon, another 4 squares that are almost whole, another 4 half squares, and a bit more. That is a total area of about 43 cm^2.) **iii)** Square, hexagon, octagon **iv)** The area increases as the number of sides increases.	**B. i)** 76 or 77 mm (7.6 or 7.7 cm); 24 cm; It is the same as the perimeter of the three polygons. **ii)** More than 43 cm^2; About 45 cm^2 (There are 32 whole square centimetres inside the circle and 26 part squares. Counting each part square as 0.5 cm^2 gives a total area of 45 cm^2.) **C. i)** 44 cm^2; It is between the area of the regular octagon at 43 cm^2 and the area of the circle at 45 cm^2.

Supporting Students

Struggling students

• If students are struggling with counting partial squares to estimate area, you might suggest that they count any square that is more than halfway in the shape as a whole square and that they not count any square that is less than halfway in the shape.

Enrichment

• You might challenge students to attempt to draw the dodecagon referred to in **part C** on a centimetre grid to test their conjecture. Each side should be 2 cm long.

5.2.4 The Formula for the Area of a Circle

Curriculum Outcomes	Outcome relevance
8-D6 Area of Circles: estimate • understand why it makes sense to estimate by squaring the diameter • understand why a closer estimate is $3 \times r^2$ **8-D7 Area of Circles: develop formula** • apply prior knowledge of area for a parallelogram to develop a formula for the area of a circle • investigate to determine the radius when the area of a circle is given • apply prior knowledge of square root • understand that $\frac{22}{7}$ and 3.14 are approximations and that a calculator must be used for more precision	Circles are important shapes in our world. Students need to be able to calculate their area to solve everyday measurement problems.

Pacing	Materials	Prerequisites
1 h	• Larger paper circle divided into 12 equal sectors (optional)	• familiarity with the formulas for the circumference of a circle and the area of a parallelogram

Main Points to be Raised

• The formula for the area of a circle is $A = \pi r^2$, where r represents the radius of the circle.

• To develop the formula, you can cut up the circle into a number of identical almost-triangles that you can rearrange to form a parallelogram. The area of the circle is about the same as the area of the parallelogram. The parallelogram has a height equal to the radius of the circle and a base equal to half the circumference of the circle.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know the circle has less area than the square?* (The circle is completely inside the square.)

• *Based on the picture, what fraction of the area of the square do you estimate the area of the circle to be?* (About $\frac{3}{4}$ of the square)

• *How does the length "s" relate to the diameter of the circle?* (It is equal to the diameter.)

The Exposition — Presenting the Main Ideas

• With students, read through the exposition on **page 136** of the student text.

• If possible, demonstrate how to make the parallelogram using a large circle cut into 12 triangles as shown on the page. Colour the circumference of the circle before cutting it out.

• Help students understand why the base of the parallelogram is half the circumference of the circle by showing how half the coloured circumference of the circle is on the top of the parallelogram and the other half of the coloured circumference is on the bottom of the parallelogram.

Revisiting the Try This

B. Students need to realize that π is a little greater than 3 to see why they should use a little more than 75% of the area of the square to calculate the area of the circle.

Using the Examples

• Present the problems from the two examples on the board. Ask students to try the problems and then check their work against the solutions in the student text.

• Highlight the use of the approximately equals sign, ≈, and how it is used in the calculations. It is used to show that a calculation using 3.14 or $\frac{22}{7}$ for π is only approximate, but subsequent calculations use the equals sign if they are exact. It is also used to show a rounded answer.

For example:

In **example 2**, $A_{\text{circle}} = \pi r^2 \approx 3.14 \times 1^2 = 3.14\ \text{m}^2$, 3.14 is an approximation of π so $\pi r^2 \approx 3.14 \times 1^2$, however, $3.14\ \text{m}^2$ is the exact product of 3.14×1^2, so the equals sign is used, $3.14 \times 1^2 = 3.14\ \text{m}^2$.

Practising and Applying

Teaching points and tips

Q 2: For **parts b) and d)**, students need to subtract one area from another.

For **parts c) and e)**, they need to add areas.

For **part a)**, they need to divide the area of the full circle by 4.

For **part e)**, they need to see that the two half circles make a full circle and that there are two triangles with base 3 and height 4.

Q 3: The diameter of the largest circle is 50 cm. The diameter of the second largest circle is 30 cm.

Q 4: The area of the outer ring is $\frac{8}{9}$ of the area of the whole circle. So students need to multiply the area of a circle with radius 0.5 m by $\frac{8}{9}$ and divide 6 to get the area of each section.

Q 5: Students use the area formula in reverse.

Q 7: This question is designed to help students see that because both the circumference formula and area formula relate to the radius, and the radius is half
the diameter, if you know one value you can calculate all the other values.

Common errors

• Students may use the diameter instead of the radius when they calculate areas if it is the diameter that is given. Encourage students always to mark the length of the radius before calculating an area.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use the formula to find the area of a circle
Question 3	to see if students can solve a problem using the area formula

Answers

A. i) $s = 2r$ **ii)** $A = 2r \times 2r = 4r^2$ **iii)** High, because the circle is actually a bit smaller than the square.	**B.** Yes; Multiplying r^2 by 4 gave a high estimate. It makes sense to multiply by 3.14 instead.
1. a) 154 cm^2 **b)** 39 m^2 **c)** 50 km^2 **d)** 79 mm^2 **2. a)** 3 m^2 [exact answer is π m^2] **b)** 11 m^2 **c)** 89 cm^2 **d)** 16 m^2 **e)** 19 cm^2 **3.** Centre: 314 cm^2; Inner band: 2512 cm^2; Outer band: 5024 cm^2. **4.** Centre: 900 cm^2; Each other section: 1200 cm^2.	**5. a)** 5 m **b)** 314 m^2 **6.** 2π cm [**7.** If I know the radius, I can substitute into the formula to find the area. • If I know the diameter, I can divide by 2 to get the radius and then substitute it into the formula to find the area. • If I know the circumference, I can divide it by π to get the diameter and then divide the diameter by 2 to get the radius. Finally, I substitute the radius into the formula to find the area.]

Supporting Students

Struggling students

• Some students may need help to figure out how to approach **questions 2 b), d, and e)**. They may also struggle with **question 4**. You may wish to provide some hints for these questions or you may substitute simpler shapes for given shapes.

Enrichment

• Students might create interesting shapes (like those in **question 2**) by combining parts of circles, rectangles, and triangles and then using formulas to calculate their areas.

CONNECTIONS: The History of Pi

Answers

1. a) 0.0013 **b)** $\frac{22}{7}$ (3.142...) is closer to 3.1415... than 3.14 is.	**2. a)** 3.3397 (this may vary with different calculation methods) **b)** No; [$\frac{22}{7}$ (3.142...) and 3.14 are both closer to π = 3.1415... than 3.3397 is.]

5.2.5 Applying Area Formulas

Curriculum Outcomes	Outcome relevance
8-D4 SI Units: solve measurement problems • integrate measurement problems as other mathematical ideas are explored • use the prefixes kilo, hecto, deka, deci, centi, and milli in problem solving contexts • investigate and create problems involving length and perimeter (mm, cm, m, km), area (mm^2, cm^2, m^2, km^2, and hectare (1 hm^2)) • continue to make decisions in real world situations about when estimating is close enough	Many real-world problems require the calculation of areas of various basic shapes, including rectangles, triangles, parallelograms, trapezoids, and circles.

Pacing	Materials	Prerequisites
1 h	None	• familiarity with the formulas for the area of a rectangle, a triangle, a trapezoid, a parallelogram, and a circle

Main Points to be Raised

• Area formulas for 2-D shapes involve multiplying two linear measures. The formulas sometimes involve a constant like π or $\frac{1}{2}$.

• Multiplying two linear units results in square units.

• When you multiply linear measures to calculate area, the units must be the same.

• When you use π or numbers with several decimal places, it is appropriate to estimate the value for an area.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How did you figure out the length of wall?* (I divided 25 by 2.)

• *How much paint is needed for 1 coat over 90 m^2?* (More than 3 cans, but less than 4 cans since 90 is between 75 and 100.)

• *Would you need 8 cans of paint for 2 coats?* (Yes, since 2 × 90 = 180 and 7 cans of paint only cover 7 × 25 = 175 m^2.)

The Exposition — Presenting the Main Ideas

• Have students read the exposition on **page 139** of the student text.

• Assign students to pairs and have them take turns telling each other one important idea in the exposition until they feel they have covered all the important concepts.

Revisiting the Try This

B. When students estimate, they must consider the context. In this case, they must purchase one more whole can of paint even if only part of a can is needed, because otherwise there will not be enough paint.

Using the Examples

• Assign students to pairs to work through **examples 1 and 3**. Each student should becomes an expert on one example and teach it to his or her partner.

• Go through **example 2** together as a class. Help students understand that the value of 90 could just as easily have been, for example, 89, 91, or 92.

Practising and Applying

Teaching points and tips

Q 1: Students first have to recognize that the radius of a tin must be 4 cm and so its diameter is 8 cm. Then they have to figure out how many rows and how many tins in a row would fit in the box. Since 100 ÷ 8 =
12 R 4 and 60 ÷ 8 = 7 R 4, 7 rows of 12 tins will fit.

Q 2: The small circle (5 cm in diameter) in **part a)** has exactly the same area as the shaded area. The difference in the areas will be due to rounding.

Part b) requires students to use the Pythagorean theorem.

For **part c)**, students need to recognize that each of the four grey parts is one quarter of a circle with a radius of 6 cm.

Q 3: To determine the base and height of each triangle, students must use the fact that the hypotenuse of each right triangle is 10 cm.

Q 8: Students must realize that they have to overestimate in order to have enough seed.

Q 9: Remind students that a hectare is 10,000 m^2.

Q 10: Students must first calculate the area of the circle and then take the square root.

Q 13: Students should try a variety of shapes before drawing a conclusion.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can solve a two-step area problem with a real-world context
Question 2	to see if students can calculate the areas of parts of shapes or composite shapes
Question 5	to see if students can solve a problem using formulas for the areas of trapezoids and circles
Question 12	to see if students can create and solve an area problem

Answers

A. 12.5 m **B.** 8 cans of paint are needed; (90 × 2) ÷ 25 = 7.2	**C.** The answer rounds to 7 cans but 7 cans are not enough.
1. 84 tins **2. a)** 19.6 cm^2 **b)** 204 cm^2 **c)** 30.9 cm^2 **3.** B is greater; [A_A = 39.3 – 25 = 14.3 cm^2 A_B = 39.3 – 24 = 15.3 cm^2] **4.** 8 cm by 32 cm; [*Sample response*: I needed a rectangle with a perimeter of 80, so the length and width had to add to 40. I tried different combinations for the length and width until the area came out to be 256.] **5.** 35.7 m^2; [A = 75 – 39.3 = 35.7 m^2.] **6.** A, the square; *Sample response*: For both shapes, the area is the product of the height or width and the base or length. The square is bigger because the slanted sides of the rhombus make its height less than the square's height.	**7.** 2 cm **8.** 8 bags **9.** about 30 tonnes **10.** about 12.4 cm **11.** No; [*Sample response*: She needs almost 4 m for the four 95 cm sides. She needs a lot more than 2 m (4 times a half metre) for the four 65 cm sides. So she needs more than 6 m in total.] **12.** *Sample response*: A circle of diameter 4 cm is cut out of a triangle with a base of 10 cm and a height of 8 cm. What is the area of the resulting shape? (40 – 12.56 = 27.4 cm^2) **13.** The area is multiplied by 9; [*Sample response*: Area is a product of two dimensions, so tripling each dimension means multiplying the area by 9. For example: If $A = l \times w$, then tripling the l and the w makes $A = 3 \times l \times 3 \times w = 9 \times l \times w$.]

Supporting Students

Struggling students

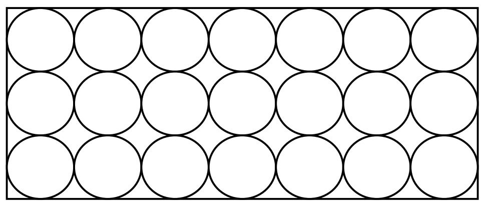
• Questions like **questions 6 and 7** require reasoning, which may be difficult for some students. You may choose not to assign these questions to struggling students.

• Some students may have difficulty with the number of problems on the page. You may wish to assign fewer problems to struggling students.

Enrichment

• Students might create and solve additional problems like **question 12**.

• They might also figure out the percent of the space that is left over when circles are packed into a rectangle.



CONNECTIONS: Tangrams

• Tangram pieces are available on BLM 2 at the end of this teacher's guide.

• There are many web sites about using tangrams to teach mathematical concepts. A simple web search using the word "tangrams" will help you find them.

Answers

1. 25 cm^2 **2.** 12.5 cm^2 **3.** 6.25 cm^2 **4.** 12.5 cm^2 **5.** 12.5 cm^2

Reprint 2024

Chapter 3 Volume and Surface Area

5.3.1 Volume of a Rectangular Prism

Curriculum Outcomes	Outcome relevance
8-D8 Volume and total surface area: estimate and calculate right prisms • estimate volume in a variety of situations **8-D4 SI Units: solve measurement problems** • integrate measurement problems as other mathematical ideas are explored • develop sense of relative size of units (e.g., compare 1 cm^3 to 1 m^3; 1 kL to 1 L) • apply relationships between capacity and volume for water (1 mL = 1 cm^3; 1 L = 1000 cm^3) to solve problems • when choosing between capacity and volume, understand which is more appropriate in a given situation • establish link between capacity and mass of pure water (1 mL of water has a mass of 1 g) • use the prefixes kilo, hecto, deka, deci, centi, and milli in problem solving contexts • investigate and create problems involving volume (cm^3 and m^3), mass (mg, g, kg) and capacity (mL, L, kL) • continue to make decisions in real world situations about when estimating is close enough	Many real-world objects are shaped like rectangular prisms. It is important to be able to calculate their volumes to solve everyday problems.

Pacing	Materials	Prerequisites
1 h	• Linking cubes (optional) • Grid paper or Small Grid Paper (BLM) (Optional)	• familiarity with the formula for area of a rectangle • familiarity with common metric capacity and volume units • familiarity with metric prefix relationships

Main Points to be Raised

• The volume of an object is the amount of space it takes up.

• The most common metric volume units are the cubic centimetre, the cubic decimetre, and the cubic metre.

• A cubic decimetre cube has an edge length of 10 cm.

• You can understand the formula for the volume of a rectangular prism by thinking of the prism in layers.

• The volume of a rectangular prism is the area of its base multiplied by its height. You can write it as $V = l \times w \times h$.

• Capacity tells how much something holds. The most common metric capacity units are the millilitre, the litre, and the kilolitre. The centilitre and decilitre are also sometimes used.

• There is a relationship between volume and capacity units. A 1 cm^3 cube has a capacity of 1 mL. A 1 dm^3 cube has a capacity of 1 L.

• 1 mL of water has a mass of 1 g.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How do you know you need more than 2 cubes?* (If there were 2 cubes, the shape would be only 4 cm on one dimension and so it would not be a cube.)

• *How many layers are there?* (2 layers, so the height is 4 cm.)

• *How many cubes are in each layer?* (4 cubes, so it is 2 cubes wide and 2 cubes deep.)

The Exposition — Presenting the Main Ideas

• If possible, use linking cubes to create a 4-by-2 rectangle. Have students tell you the area of its base (8 square units). Discuss why the volume is 8 cubic units since 8 small cubes make up the object.

• Add another layer of linking cubes to the 4-by-2 rectangle. Talk about the volume of the new object and why it is twice as much as the original volume. Relate the fact that there are twice as many cubes to the fact that the height is 2.

• Add one more layer of linking cubes. Ask for the new volume. Bring attention the fact that the height is now 3 and the volume is now 3 times the area of the base.

• Ask students to read through the exposition on **pages 143 and 144** of the student text. Discuss with them the capacity/volume/mass relationships that are described. Make sure they have a sense of the size of 1 cm^3, 1 dm^3, and 1 m^3.

Revisiting the Try This

B. Students use their knowledge of the volume formula to make sense of the solution to **part A**.

Using the Examples

• Write the questions from **examples 1 and 2** on the board. Ask students to work alone or in pairs to complete the questions and then check their work against the solutions in the student text.

• Then have students read through **examples 3 and 4.** Provide time for them to ask any questions they might have.

Practising and Applying

Teaching points and tips

Q 1: Students might refer to **examples 3 and 4** to help them with this question. Remind them that they need to make sure that the units for the linear dimensions are the same before they compute the volume.

Q 5: You might point out that there is only one solution to each problem because a cube is required. If a rectangular prism were required, there would be many solutions.

Q 7: Students might compare the different volumes for a fixed amount of money or the different amounts of money for a fixed volume. They should assume the three choices are the same product in different forms.

Q 8 and 9: Students should first calculate the equivalent volume using 1 mL = 1 cm^3.

Q 10: Students must consider that it is not possible to break up the cubes, so dividing the volume of the box by the volume of the cube will not give the correct result.

Q 12: Encourage students to require only estimation, rather than calculation.

Common errors

• For **question 10**, many students will divide the volume of the box by the volume of a cube. This will not give the correct answer since partial dice cannot be used. Have students draw diagrams to help them see why.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can apply the volume formula
Question 6	to see if students can relate volume, capacity, and mass
Question 9	to see if students can solve a volume and capacity problem

Answers

A. 8	**B.** The volume increases by a factor of 8; $V = l \times w \times h \rightarrow 2 \times l \times 2 \times w \times 2 \times h$ $= 2 \times 2 \times 2 \times l \times w \times h$ $= 8 \times l \times w \times h$

Answers [Continued]

1. a) 12 cm^3; 12 mL; 12 g
b) 30 dm^3; 30 L; 30 kg
c) 25 cm^3; 25 mL; 25 g
d) 1000 dm^3 or 1 m^3; 1000 L; 1000 kg or 1 t

2.

V **(cm^3)**	***l*** **(cm)**	***w*** **(cm)**	***h*** **(cm)**	**Capacity (mL)**
48	4	3	**4**	**48**
105	10	**3.5**	3	**105**
720	**12**	12	5	720

3. A has the greatest volume;
A. 729 cm^3
B. 720 cm^3
C. 704 cm^3

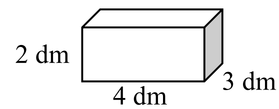
4. a) 512 cm^3
b) 2 cm on each edge

5. a) 4 cm **b)** 5 cm **c)** 100 cm

6. a) 1,000,000 cm^3
b) 1000 L
c) 1,000,000 g; 1,000 kg; 1 t

7. B; A is 1000 cm^3, B is 512 cm^3, C is 700 cm^3
[*Sample response*:
For A, you get 250 cm^3 for Nu 10
For B, you get 256 cm^3 for Nu 10.
For C, you get about 233 cm^3 for Nu 10.]

8. *Sample response*:



9. 20 cm

10. a) 12 dice **b)** 44 cm^3

11. No; [*Sample response*:
A 20 cm or 2 dm cube holds 8 dm^3 or 8 L, so 8 L is not enough to fill a 22.5 cm cube.]

12. *Sample response*:
A cube has a volume of 12 m^3. What are its dimensions to the nearest tenth of a metre? (2.3 m)

13. a) It is multiplied by 9. **b)** It is multiplied by 27.
[**c)** *Sample response*:
• Area is a product of two dimensions, so tripling each means the area increases by a factor of 3 × 3 = 9.
• Volume is a product of three dimensions, so tripling each means the volume increases by a factor of
3 × 3 × 3 = 27.
So, volume increases by an extra factor each time.]

Supporting Students

Struggling students

• Some students may have difficulty keeping the equivalent capacity/mass/volume relationships in mind. Have them record these on a chart and refer to it.

• For **question 10**, encourage struggling students to draw a diagram to help them see what is required.

• If students have trouble creating a problem, have them solve problems created by other students. They might then use a similar problem with different values.

Enrichment

• Students might draw a graph that relates the volume of a cube to its side length. On the same grid, they could draw a graph that relates the area of a square to its side length. They could then use that graph to help explain **question 13 c)** in a different way.

Reprint 2024

5.3.2 Surface Area of a Rectangular Prism

Curriculum Outcomes	Outcome relevance
8-D8 Volume and Total Surface Area: estimate and calculate right prisms • find total surface area of rectangular prisms • investigate changes in total surface area based on changes in dimensions **8-D4 SI Units: solve measurement problems** • integrate measurement problems as other mathematical ideas are explored • use the prefixes kilo, hecto, deka, deci, centi, and milli in problem solving contexts • investigate and create problems involving length and perimeter (mm, cm, m, km), area (mm^2, cm^2, m^2, km^2, and hectare (1 hm^2)) • continue to make decisions in real world situations about when estimating is close enough	Many real-world objects are shaped like rectangular prisms. It is important to be able to calculate their surface areas to solve everyday problems.

Pacing	Materials	Prerequisites
1 h	• Linking cubes (optional) Rectangular prism items such as boxes (optional)	• familiarity with the area formula for a rectangle • familiarity with the concept of net of a 3-D shape • solving simple algebraic equations

Main Points to be Raised

• The total surface area of a 3-D shape is a measure of the total amount of area that covers all of its surfaces.

• Common metric units for surface area are square millimetres, square centimetres, and square metres.

• The total surface area of a rectangular prism is the sum of the areas of its six faces. Since opposite faces are identical, you only need to add the areas of
the three faces that could be different (top, front, and one side) and then double the sum.

• The formula for the surface area if the dimensions are l, w, and h is $SA = 2 \times (l \times w + w \times h + l \times h)$.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *Which part of the net shows you the base?* (The big rectangle in the middle)

• *What are the dimensions of the base?* (12 cm by 8 cm)

• *Which part of the net shows the height of the box?* (The width of the narrow rectangle next to the big rectangle in the middle.)

• *Why do six rectangles making up the net?* (A rectangular prism has six faces.)

• *How many separate areas do you have to calculate to determine the area of the net?* (Only three, since there are duplicates of each area in the net.)

The Exposition — Presenting the Main Ideas

• If possible, show students a rectangular prism and point to the six surfaces whose areas are added together to calculate the total surface area. Point out how the top and bottom, the two sides, and the front and back pairs are congruent shapes. Discuss how this simplifies the calculation of the total surface area; only three areas need to be calculated. The values are then doubled and added.

• Have students read through the exposition on **page 147** of the student text. Make sure they have a sense of the size of 1 m^2, 1 cm^2, and 1 mm^2.

Revisiting the Try This

B. This question reveals to students why it is often easier to visualize the surface area by sketching a net for a rectangular prism.

C. This question allows students to notice that even though it is a 3-D shape that is being examined, the surface area is still measured in square units.

Using the Examples

• Write the questions from **examples 1 and 3** on the board and allow students to try them. They can then compare their answers with those in the student text.

• Read through **example 2** with students to make sure they understand that the formula for the surface area of a cube is based on the surface area formula for a rectangular prism, but that it is simpler. Discuss why it makes sense that the formula involves multiplying by 6 since there are 6 identical faces. After you calculate the area of one face, the total surface area is 6 times as much.

Practising and Applying

Teaching points and tips

Q 1: Observe whether students remember to make sure all units are the same before a surface area is calculated.

Q 3: Note whether students realize they can use the simpler formula for surface area of a cube or whether they use the more complex general rectangular prism formula.

Q 4: Encourage students to experiment by using dimensions of their choice for the smaller prism.

Q 6: Make sure students notice that it is the volume, not the surface area, that must be 12 cm^3. The purpose of the question is to preview the concept that the cube has the least surface area among rectangular prisms with a given volume.

Common errors

• Many students forget to double after they calculate the surface area of three faces. Remind them to make sure they have accounted for all faces.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can apply the surface area formula
Question 6	to see if students understand the difference between total surface area and volume
Question 7	to see if students can solve a problem involving total surface area

Answers

A. 352 cm^2

B. *Sample response*:
The net represents the six faces of the prism so its area is the total area of the six faces, which is the total surface area.

C. *Sample response*:
The total surface area is the sum of the areas of six rectangle faces. Area is measured in square units because it is the product of two dimensions.

1. a) 32 cm^2 **b)** 26 cm^2
c) 59 cm^2 **d)** 600 dm^2

2.

SA **(cm^2)**	***l*** **(cm)**	***w*** **(cm)**	***h*** **(cm)**
33	6	1.5	1
10	2	**1**	1
76	**5**	4	2
150	5	5	**5**

3. a) 216 cm^2
b) 216 cm^3
c) The numbers are the same; It is usually not true; [*Sample response*:
A 1 cm cube has a total surface area of 6 cm^2 but a volume of 1 cm^3. It only happens in this case because the edge length is 6.]

4. a) The area is multiplied by 4; [Each dimension is multiplied by 2 and so the area of each rectangular face is multiplied by 4. So the total surface area is also multiplied by 4.] **b)** The total surface area is multiplied by 9. **5. a)** 96 cm^2 **b)** 96 cm^2 **c)** The cube has the greater volume; A_{cube} = 64 cm^3 is greater than A_{prism} = 36 cm^3. **6. a)** 1 by 1 by 12, 1 by 3 by 4, 1 by 2 by 6, 2 by 2 by 3 **b)** 1 by 1 by 12; It is flat, long, and wide. **c)** 2 by 2 by 3; It is closest to cube-shaped.	**7.** 38,100 cm^2 or 3.81 m^2 **8. a)** The total volume does not change; [The two resulting shapes occupy the same amount of space as the cube.] **b)** The total surface area increases; [I create two additional faces when I cut the cube.]

Supporting Students

Struggling students

• Some students have trouble visualizing all the faces of a rectangular prism. Have available some linking cubes so students can make their own prisms.

• Some students continue to mix up the volume and surface area of a shape. Remind them to read the question carefully and think about which measure is being described.

Enrichment

• Students could calculate the surface area of rectangular prism-shaped items in the classroom to see which is greatest. Or, they might investigate the maximum dimensions a box could have if they have a certain amount of paper with which to wrap it.

UNIT 5 Revision

Pacing	Materials
2 h	None

Question(s)	Related Lesson(s)
1 and 2	Lesson 5.1.1
3	Lesson 5.1.2
4 and 5	Lesson 5.2.1
6 – 8	Lesson 5.2.2
9	Lesson 5.2.3
10	Lesson 5.2.4
11	Lesson 5.2.5
12 – 14	Lesson 5.3.1
15 and 16	Lesson 5.3.2

Revision Tips

Q 3: Students need to recognize that the two triangles are both isosceles.

Q 5: Encourage students to work in an organized way. For example, they might start with 20 by 1, then 10 by 2, then 5 by 4. Each time they decrease one dimension and increase the other.

Q 7: Students can write the scale as a ratio or in words.

Q 9 b): Observe whether students realize they do not have to recalculate once they have completed **part a)**.

Q 10: Students must assume that each end and each white section is a semi-circle.

Q 11: Students need to recognize that the base of the right triangle with the hypotenuse of 5 is 3 cm (15 cm – 12 cm) and therefore its height is 4 cm, using the Pythagorean theorem.

Answers

1. a) Yes **b)** No

2. 11.4 cm

3. 14.1 cm

4. 3 rectangles;
1 by 20: P = 42 cm
2 by 10: P = 24 cm
4 by 5: P = 18 cm

5. 5 rectangles;
1 by 9: $A = 9$ cm^2
2 by 8: $A = 16$ cm^2
3 by 7: $A = 21$ cm^2
4 by 6: $A = 24$ cm^2
5 by 5: $A = 25$ cm^2

6. 125 m^2

7. 1 : 13,000,000

8. 1500 km

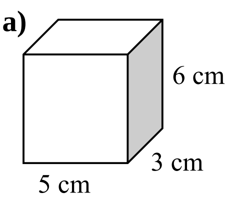
9. a) 38.5 cm^2 **b)** 38.5 km^2

10. 49 cm^2 **11.** 84 cm^2

12. 343 m^3 **13.** 7 cm

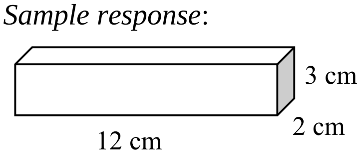
14. a) 19.3 L **b)** 17.9 L **c)** 17.9 kg

15. a)

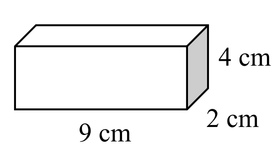


b) 126 cm^2

16. *Sample response*:



SA = 132 cm^2

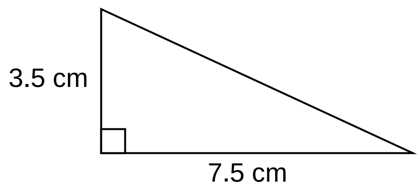


SA = 124 cm^2

UNIT 5 Measurement Test

Round to one decimal place, when necessary.

1. What is the length of the hypotenuse of this right triangle?

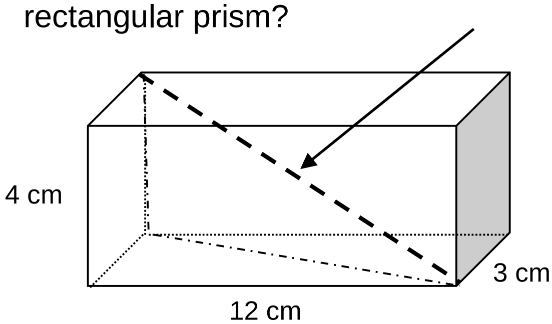


2. Could each set of numbers be the side lengths of a right triangle?

a) 0.15, 0.20, 0.25 **b)** 5, 15, 17

c) 8, 30, 31 **d)** 11, 60, 61

3. What is the length of the diagonal of this rectangular prism?



4. A rectangle has an area of 12 cm^2. How many different rectangles are possible, if each side length is a whole number of centimetres? What is the perimeter of each?

5. A rectangle has a perimeter of 12 cm. How many different rectangles are possible, if each side length is a whole number of centimetres? What is the area of each?

6. What is the area of a circle with a diameter of 10 m?

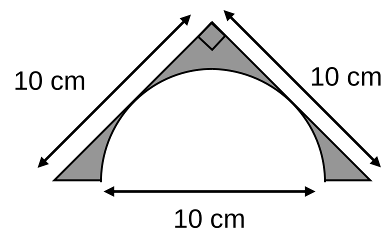
7. The perimeter of a rectangular field is 216 m. A map of the field is 12 cm by 15 cm.

a) What is the scale ratio of the map?

b) Draw the rectangle to scale. Include a key.

c) What are the real dimensions of the field?

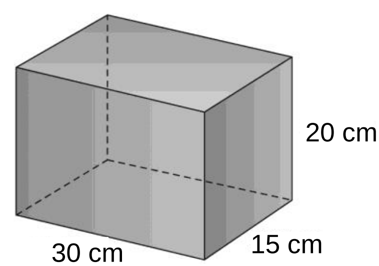
8. What is the area of the shaded region?



9. What is the volume of a 6 m cube?

10. A box has a volume of 1 m^3. Its base has an area of 5000 cm^2. What is its height?

11. a) What is the volume of this container?



b) How much water will it hold (in L) if it is filled until the water is 2 cm from the top?

c) What is the mass of the water in **part b)**?

12. a) Sketch a rectangular prism that is 2 cm by 3 cm by 4 cm.

b) What is its total surface area?

13. Sketch two different boxes, each with a capacity of 6 L. What is the total surface area of each?

UNIT 5 Test

Pacing	Materials
1 h	• Rulers

Question(s)	Related Lesson(s)
1 and 2	Lesson 5.1.1
3	Lesson 5.1.2
4 and 5	Lesson 5.2.1
6	Lesson 5.2.4
7	Lesson 5.2.2
8	Lesson 5.2.5
9 – 11	Lesson 5.3.1
12 and 13	Lesson 5.3.2

Select questions to assign according to the time available.

Answers

1. 8.3 cm

2. a) Yes **b)** No
c) No **d)** Yes

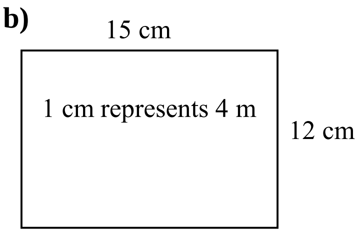
3. 13 cm

4. 3 rectangles;
1 by 12: perimeter 26 cm,
2 by 6: perimeter 16 cm,
3 by 4: perimeter 14 cm.

5. 3 rectangles;
1 by 5: area 5 cm^2,
2 by 4: area 8 cm^2,
3 by 3: area 9 cm^2.

6. 78.5 cm^2 or 78.6 cm^2

7. a) 1 : 400
b)



c) 60 m by 48 m

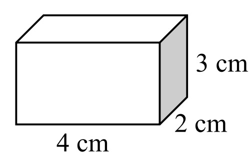
8. 10.7 cm^2 or 10.8 cm^2

9. 216 m^3

10. 200 cm or 2 m

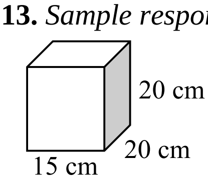
11. a) 9000 cm^3 **b)** 8.1 L **c)** 8.1 kg

12. a)

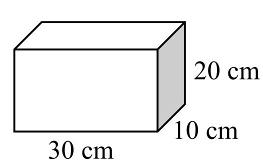


b) 52 cm^2

13. *Sample response*:



SA = 2000 cm^2

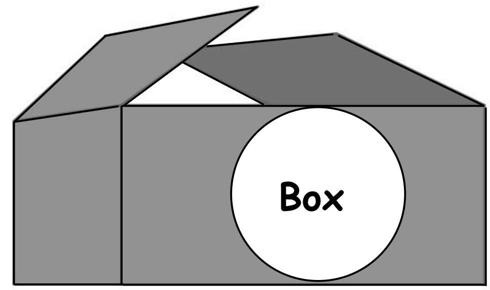


SA = 2200 cm^2

UNIT 5 Performance Task — Designing a Box

Your task is to design a box to meet these criteria:

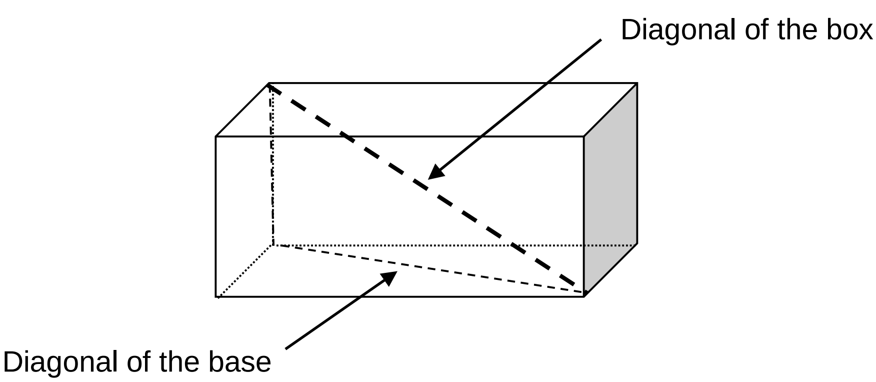
- The volume must be 6000 cm^3.
- The total surface area must be less than 2500 cm^2.
- The height must be less than 20 cm.
- The dimensions must be whole numbers.
- It must be easy to handle and carry.



A. Sketch the box and label its dimensions.
Explain how and why you chose those dimensions.

B. Draw a scale diagram of the net of your box. Include a key or scale ratio.

C. i) Calculate the length of the diagonal of the base of your box.
ii) Could the diagonal of your box be less than 20 cm? Explain your thinking.



D. The label on your box is a circle. It is centered halfway between the edges of the front and it is as big as possible. What is the area of the label?

UNIT 5 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
8-D2 Pythagorean Relationship: application **8-D4** SI Units: solve measurement problems **8-D5** Proportion: solve indirect measurement problems **8-D7** Area of Circles: develop formula **8-D8** Volume and Total Surface Area: estimate and calculate right prisms	1 h	• Rulers

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could supplement the unit test. It could also be used as enrichment material for some students.

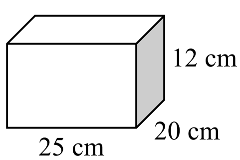
• You can assess performance on the task using the rubric provided on the next page.

Sample Solution

Here are some values that will work:

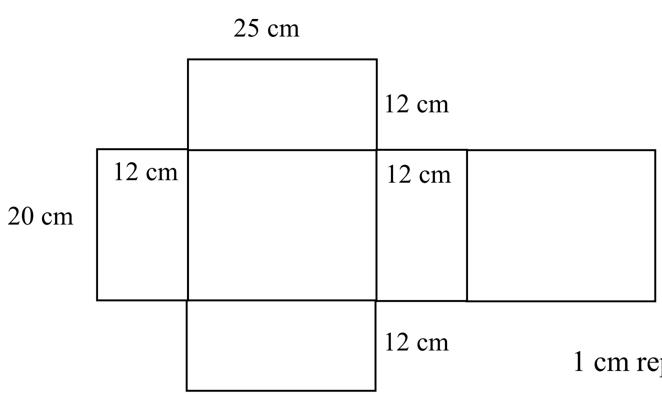
l	*w*	*h*	*SA*
20	25	12	2080
30	20	10	2200
24	25	10	2180
12	50	10	2440
25	30	8	2380

A.



The volume is 6000 cm^3.
$6000 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5$
I needed three factors of 6000 that give a surface area less than 2500 cm^3.
One of the factors must be less than 20, so I could choose from 2, 3, 4, 6, 8, 10, 12 and 15.
I tried 2, 30, and 100: *SA* = 6520 cm^3. The box is much too large.
Small heights will make the other dimensions large, so I had to increase the height to reduce the total surface area.
I tried 8, 15, and 50: *SA* = 2540 cm^3. It is still a bit too large.
I tried 12, 20, and 25. *SA* = 2080 cm^3. It works.
I checked the volume: $15 \times 20 \times 25 = 6000$.
The box is easy to carry since none of the dimensions is too large.

B.



1 cm represents 10 cm

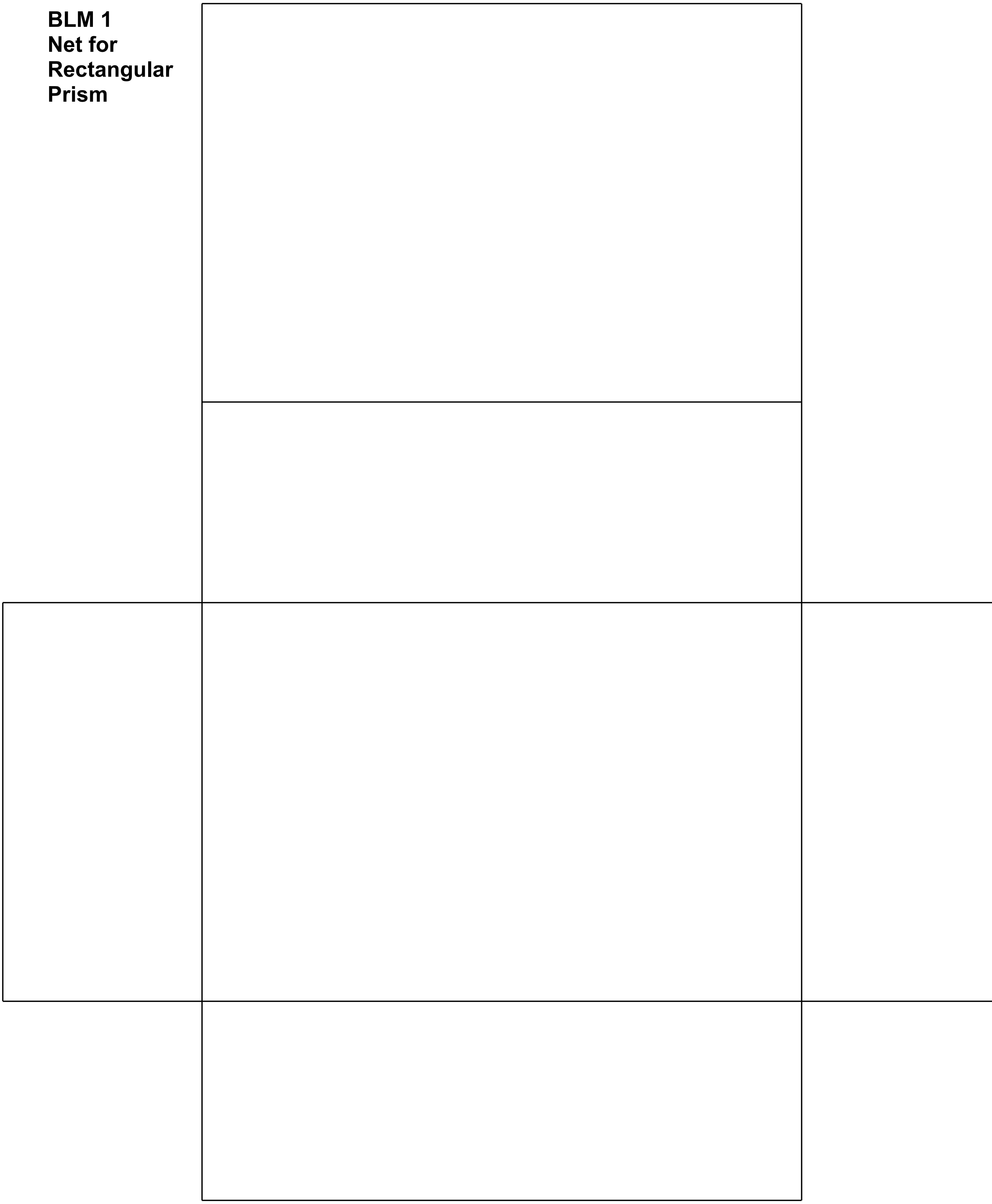
C. i) About 32 cm
ii) No; The diagonal of the base is the hypotenuse of a triangle with legs equal to *l* and *w*. The diagonal of the box is the hypotenuse of a right triangle with legs equal to *h* and the diagonal of the base. Since $20 \times 20 \times 20 = 8000$, I figure that even if the box were a cube, its edge length would be almost 18 and so the diagonal would be greater than 20. If one of the dimensions were longer than that, the diagonal would be even longer.

D. My circle has a diameter of 20 cm, so the radius is 10 cm and the area is about 314 cm^2.

UNIT 5 Performance Task Assessment Rubric

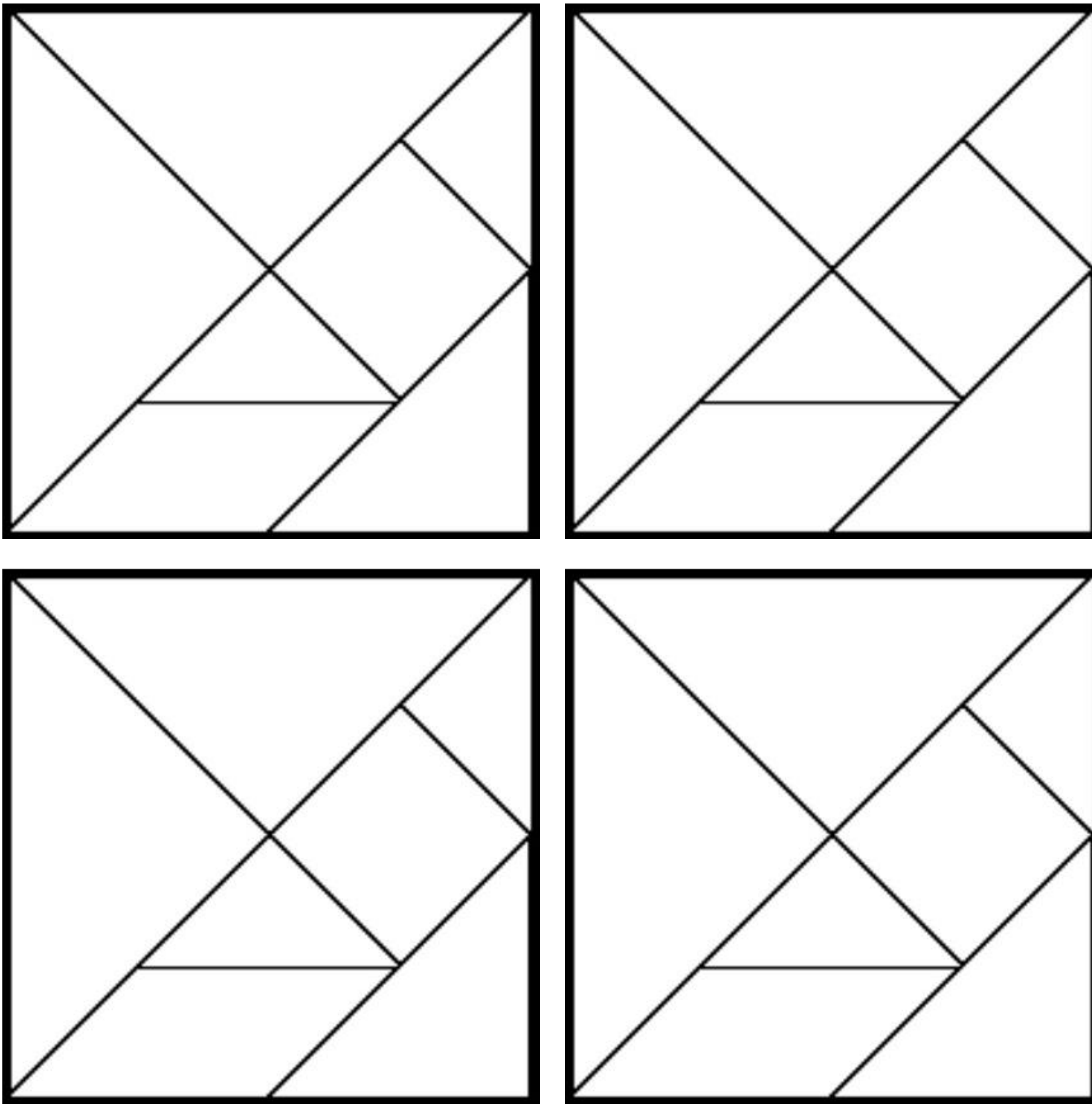
The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Finds dimensions	Uses creative strategies to find dimensions that work; shows understanding of volume and surface area	Uses appropriate strategies to find dimensions that work, with minor errors in calculations or results that do not meet the exact criteria	Uses strategies to find dimensions that work; shows some understanding of the concepts, but not consistently	Does not employ clear or logical strategies for approaching the problem
Calculates	Efficiently and correctly calculates volumes, surface areas, diagonals, and the area of the circle	Completely correctly calculates volumes, surface areas, diagonals, and the area of the circle	Mostly correctly calculates volumes, surface areas, diagonals, and the area of the circle	Does not correctly calculate even two of volumes, surface areas, diagonals, or the area of the circle
Creates a scale diagram	Draws an appropriate scale diagram for the net with a correct and appropriate scale and a labelled key	Draws an appropriate scale diagram for the net with a correct scale and a labelled key	Draws a scale diagram for the net with few errors; indicates a key	Does not draw a reasonable scale diagram for the net
Reasons about the diagonal	Insightfully explains why the diagonal cannot be less than 20 cm	Explains why the diagonal cannot be less than 20 cm	Partially explains why the diagonal cannot be less than 20 cm	Cannot explain why the diagonal cannot be less than 20 cm

BLM 1
Net for Rectangular Prism



Reprint 2024

BLM 2 Tangrams



Reprint 2024

BLM 3 Grid Paper (1 cm by 1 cm)

UNIT 6 PROBABILITY AND DATA

UNIT 6 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 151** **TG p. 201**	Review prerequisite concepts, skills, and terminology, and pre-assessment	1 h	• Grid paper or Small Grid Paper (BLM)	All questions
Chapter 1 Probability				
6.1.1 Complementary Events **SB p. 152** **TG p. 204**	**8-G1 Theoretical Probability: single and complementary events** • apply formula from Class 7: P(E) = Number of favourable outcomes / Number of possible outcomes) • understand that this formula can only be used when dealing with equally likely outcomes or events • find the probability of a complementary event using the formula $1 - P(E)$ • understand that, if the probability of an event occurring is, e.g., $\frac{1}{4}$, then the probability of it not occurring is $1 - \frac{1}{4} = \frac{3}{4}$	1 h	• Counters or marbles in three colours (optional) • Ch 20, Ch 25, Ch 50, and Nu 1 coins (optional) • Deck of playing cards (optional)	Q1, 4, 7, 9
GAME: Unlucky Ones (Optional) **SB p. 155** **TG p. 207**	Practise computing and applying probability when making a decision in a game situation.	20 to 30 min	• Dice	N/A
CONNECTIONS: Simpson's Paradox (Optional) **SB p. 156** **TG p. 207**	Make a connection between the probabilities of related events.	15 to 20 min	• 2 paper bags and counters in two colours (optional)	N/A
6.1.2 Simulations **SB p. 157** **TG p. 208**	**8-G2 Simulations and Experiments: single and complementary events** • understand that, in situations for which the probability of various events occurring is not equally likely, experimentation is often the only method of determining probability **8-G3 Compare Results: theoretical and experimental** • compare theoretical and experimental probability for a given situation and discuss results	1.5 h	• Fraction Circles (for spinners) (BLM) • Nu 1 coins • Opaque containers such as bags or bangchung • Dice • Coloured counters or marbles • Identical slips of paper • Playing cards	Q2, 3 (or 4 or 5), 8

UNIT 6 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Chapter 2 One-Variable Data				
6.2.1 EXPLORE: Sample Size (Optional) **SB p. 161** **TG p. 212**	**8-F1 Repeated Sampling (of Same Population): variability** • understand that survey results of two different samples of the same population will not exactly be the same • recognize the variability among repeated samples and provide a basic and informal introduction to the notion of sampling distribution • conduct probability experiments to demonstrate variability of repeated sampling • use real and simulated data in interesting investigations	1.5 h	• Opaque containers such as bags or bangchung • 100 identical slips of paper	Observe and Assess questions
6.2.2 Selecting a Random Sample **SB p. 162** **TG p. 215**	**8-F2 Randomness: concepts** • understand that a random sample is a sample collected from a population so that every member of the population has an equal chance of being selected • understand that members are chosen independently of each other • understand that common devices and methods used in selecting random sample are coins, dice, sampling boxes, a table of random numbers	1 h	Optional: • Random Number Table (BLM) • A variety of probability devices such as coins, dice, spinners, and identical slips of paper • An opaque container	Q1, 4, 5
6.2.3 Circle Graphs **SB p. 165** **TG p. 218**	**8-F3 Circle Graphs: construct and interpret** • understand usefulness of circle graphs in situations where a comparison of the part to the whole is needed (e.g., budgets) • apply prior knowledge about percent and using a protractor in construction of circle graphs • focus on when a circle graph is the most appropriate data display	1.5 h	• Protractors • Compasses • Coloured pencils (optional)	Q1, 3, 5
6.2.4 Box and Whisker Plots **SB p. 168** **TG p. 222**	**8-F4 Box and Whisker Plots: construct and interpret** • understand that this is an easy method for visually displaying the median, the range, and the distribution or spread • construct plots • identify the median and the median of the upper half of the data (upper quartile) • identify the median of the lower half of the data (lower quartile) • identify the extremes, that is, the lower value and the higher value	1.5 h	• Rulers • Grid paper or Small Grid Paper (BLM) (optional)	Q2, 3, 4 (or 5)
6.2.5 EXPLORE: The Impact of Altering a Data Set (Essential) **SB p. 175** **TG p. 226**	**8-F6 Variations: on mean, median, and mode** • consider and compare, through investigation, the impact of alterations to data sets to each of mean, median and mode	40 min	• Rulers • Grid paper or Small Grid Paper (BLM) (optional)	Observe and Assess questions

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Chapter 3 Two-Variable Data				
6.3.1 EXPLORE: The Relationship Between Two Variables (Essential) **SB p. 176** **TG p. 229**	**8-C1 Patterns and Relations: represent in a variety of formats** • describe in words, and use expressions and equations to represent patterns given in tables, graphs, charts, pictures and/or problems situations **8-C2 Graphs (linear and non-linear): interpret** • use information from tables, diagrams, pictures, graphs or equations to describe change	40 min	• Rulers	Observe and Assess questions
6.3.2 Using a Scatter Plot to Represent a Relationship **SB p. 177** **TG p. 231**	**8-C1 Patterns and Relations: represent in a variety of formats** • move interchangeably among a variety of formats which describe relationships • describe in words, and use expressions and equations, to represent patterns given in tables, graphs, charts, pictures and/or problems situations • use information presented in a variety of formats to derive mathematical expressions and predict unknown values • investigate linear situations and those which create a regular pattern (broken line or curved graph) • predict unknown values once algebraic description of a pattern is established • interpolate and extrapolate to predict unknown values when patterns are not regular **8-F5 Scatter Plots: construct and interpret** • use data collected by students to construct scatter plots	1.5 h	• Rulers • Grid paper or Small Grid Paper (BLM)	Q1, 2, 7
UNIT 6 Revision **SB p. 181** **TG p. 237**	Review the concepts and skills in the unit	2 h	• Rulers • Grid paper or Small Grid Paper (BLM) • Protractors and compasses Optional: • Probability devices such as spinners, coins, containers, dice, and playing cards	All questions
UNIT 6 Test **TG p. 240**	Assess the concepts and skills in the unit	1 h	• Rulers • Grid paper or Small Grid Paper (BLM)	All questions
UNIT 6 Performance Task **TG p. 243**	Assess concepts and skills in the unit	1 h	• Rulers • Grid paper or Small Grid Paper (BLM) (optional) • Probability devices such as slips of paper and a bag	Rubric provided
UNIT 6 Blackline Masters **TG p. 246**	BLM 1 Random Number Table Small Grid Paper on **page 32** in UNIT 1 Fraction Circles (for spinners) on **page 152** in UNIT 4			

Math Background

• The probability portion of this unit extends students' previous work on the probability of independent events to include complementary events. It also introduces simulation experiments to generate the experimental probability of an event, which is used to estimate the theoretical probability. In this unit, simulations are used for events that have outcomes that are not equally likely and for events for which the theoretical probability is difficult or impossible to determine.

• The data portion of the unit introduces strategies for selecting a random sample from a population. It also extends the work with circle graphs and scatter plots that was begun in Class VII. Students also meet a new graph — the box and whisker plot. Box and whisker plots enable students to visualize the spread and distribution of the values in a set of numeric data.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 2** in **lesson 6.1.1**, where they figure out all the possible combinations, in **questions 3 to 7** in **lesson 6.1.2**, where they design simulations for estimating the probability of events for which the theoretical probability is difficult to determine, and in **question 1** in **lesson 6.2.2**, where they develop a sampling method.

• Students use communication when they are asked for explanations and descriptions. They use communication in **question 7** in **lesson 6.1.1**, where they describe the difference between an event and an outcome, in **part D** in **lesson 6.2.1**, where they explain the sample size they would choose and say why they would choose it, **questions 1 to 4** in **lesson 6.2.2**, where they describe a specific procedure for selecting a random sample, and in **question 4** in **lesson 6.2.3**, where they describe a situation that might be represented by a circle graph.

• Students use reasoning in **question 1** in **lesson 6.1.2**, where they select a device that is appropriate for a particular simulation situation, in the **Revisiting the Try This** in **lesson 6.2.2,** where they reflect on the quality of their samples from **parts A and B**, and in **lesson 6.2.5**, where they reason about the effect on a box plot of removing data values.

• Students use representation in **lesson 6.1.1**, where they use tree diagrams and area models to determine the sample space for an event, in **lesson 6.1.2**, where they use a simulation model to represent a real event, and in **lessons 6.2.3 and 6.2.4**, where they use graphs to represent numerical data. These graphic representations make it easier to interpret data and to compare sets of data.

• Students use visualization skills throughout **Chapters 2 and 3**, where they use box and whisker plots and scatter plots to make inferences about data.

• Students make connections in **lesson 6.2.3**, where they link operations (percent and proportions) and geometry (circles and sector angles) to the creation of circle graphs. They also connect statistical measures such as greatest and least values and the median to graphing when they interpret and create box and whisker plots in **lesson 6.2.4**. The focus in **Chapter 3** on relating the values of two variables also involves making connections.

Rationale for Teaching Approach

• This unit is divided into three chapters:

Chapter 1 focuses on probability.

Chapter 2 focuses on one-variable data.

Chapter 3 focuses on two-variable data.

• There are three **Explore** lessons. The first shows students how different samples can produce variable results, but that as sample size increases, the range of the results tends to decrease. The second has students use box and whisker plots to display the effects of removing selected values from a data set. The third involves using actual measurements to discern a relationship between sizes of humans.

• The **Connections** allows students to explore a surprising situation involving probability.

• The **Game** provides an informal opportunity to determine and apply the probability of two independent events.

• The data portion of the unit requires students to compute statistical measures for relatively large sets of numeric data. You might consider allowing students to use calculators, if they are available, so that the effort associated with a tedious computation does not distract from the development of the statistical concepts.

Getting Started

Curriculum Outcomes	Outcome relevance
7 Circle Graphs: construct and interpret **7** Histograms: construct and interpret **7** Central Tendency: examine the effect of changing data **7** Compare Results: theoretical versus experimental **7** Independent Events: identify all possible outcomes	Students will find the work in the unit easier after they review the concepts and skills they encountered in Class VII related to histograms and circle graphs, measures of central tendency, and probability.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM)	• identifying outliers in a set of data • computing the mean, median, mode, and range for a set of data • constructing histograms • interpreting circle graphs • constructing tree diagrams • calculating theoretical probability

Main Points to be Raised

Use What You Know

• Outliers are data values that are much lower or much higher than most of the other data values in a set. When you remove an outlier from a data set, the effect on the mean is usually greater than the effect on the median or the mode.

• The intervals in a histogram appear to overlap, but the value at the end of each interval actually belongs in the following interval.

• The height of each bar in a histogram is the frequency of the data that the bar represents.

• You can choose the number of intervals to use for a histogram, but the size must be appropriate for the set of data being displayed.

Skills You Will Need

• In a circle graph, the whole circle represents the whole set of data, or 100%.

• You can calculate the mean by adding all the data values and dividing the sum by the number of data values. The mean represents a "fair share" of the total data.

• The mode in a set of data is the data value that occurs most often in the set of data.

• The median is the middle data value when the data values are in order.

• The range is the difference between the least and greatest data values.

• A tree diagram is a useful tool for organizing the possible outcomes in a probability situation.

• The theoretical probability of an event is the fraction that compares the number of favourable outcomes to the number of possible outcomes.

Use What You Know — Introducing the Unit

• Before assigning the activity, you might discuss the factors students need to consider in choosing the number
of intervals and the end points of the intervals when they construct a histogram.

For example:

- If the data values range from 0 to 50 and there are many pieces of data, you might decide on five intervals, from 0 to 10, 10 to 20, and so on.

- If the data values range from 15 to 45, you might choose three intervals, from 15 to 25, 25 to 35, and 35 to 45, or you might choose four intervals, from 10 to 20, 20 to 30, 30 to 40, and 40 to 50. With four intervals, it is easier to identify the interval for each number.

Discuss why each interval in a set of data that has a small range would also have a small range.

• Students can work in pairs or small groups. As you observe students at work, you might ask questions such as the following:

- *How did you know that Throw 7 is an outlier?* (It is much shorter than all the others.)

- *Is Throw 17 an outlier?* (It might not be. It could be his best throw and evidence that he is improving. If it had happened earlier, like Throw 2, it might be considered an outlier.)

- *Why would it make sense to use an interval size of 0.5 for this set of data?* (The data values go from 5.5 to 9.1, so an interval of 0.5 would result in 8 bars. That is a few more than I would prefer, but if I use a greater interval size, like 1, there will be too many pieces of data in the 8.0 – 8.5 and 8.5 – 9.0 intervals. I would not see the difference between the higher and lower values in that range.)

Skills You Will Need

• Students can work individually.

• To ensure students have the required skills for this unit, assign all these questions.

• You may have to review the terms *histogram*, *outlier*, *tree diagram, mean, median, mode,* and *range* for some students. You might suggest that students refer to the glossary at the back of the student text.

Answers

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A. i) *Sample response*:
5.5 is the most obvious outlier. It is the only number less than 7 and it is a lot less than 7. It might be a mistake. 9.1 is a possible outlier, but it might represent a very good throw as he improved.
ii) If I remove 5.5, the mean goes up, but the median and mode do not change.
If I remove 5.5 and 9.1, the same thing happens.

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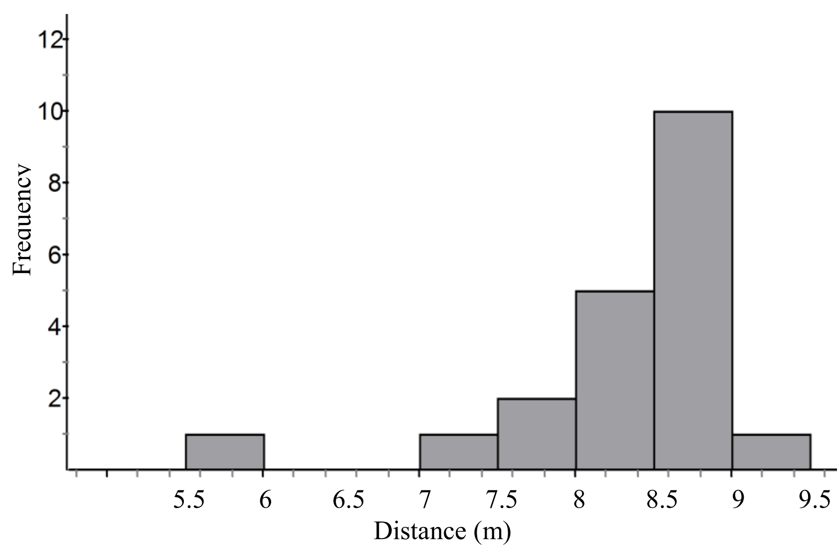
B. i) *Sample response*:

Bhagi's Shot Put Throws



ii) It seemed reasonable to use an interval of 0.5. That gave 9 intervals, which is a reasonable number for the graph.

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1. a) Tea; *Sample response*:
Fruit juice, but there might have been another less popular drink in "Other".
b) Tea: 40%, Other: 25%,
Water: 25%, Juice: 10%

2. a) 161.27
b) 161.7
c) 162.7
d) 47.9

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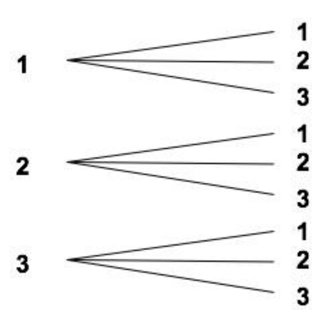
3. a)



b) $\frac{4}{9}$

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Supporting Students

Struggling students

• Some students may have trouble making the tree diagram in **question 3**. You might draw the first set of branches on the board to help them get started.

Enrichment

• For **question 3**, encourage students to increase the number of equal sectors on the spinner or to experiment with sectors of different sizes. You might also ask them to consider the probability of getting two odd numbers if you spin the spinner three times.

Chapter 1 Probability

6.1.1 Complementary Events

Curriculum Outcomes	Outcome relevance
8-G1 Theoretical Probability: single and complementary events • apply formula from Class 7: P(E) = Number of favourable outcomes / Number of possible outcomes • understand that this formula can only be used when dealing with equally likely outcomes or events • find the probability of a complementary event using the formula $1 - P(E)$ • understand that, if the probability of an event occurring is, e.g., $\frac{1}{4}$, then the probability of it not occurring is $1 - \frac{1}{4} = \frac{3}{4}$	Students need to learn about calculating the probability of complementary events because sometimes it is easier to determine the probability that an event will occur by calculating the probability that it will not occur.

Pacing	Materials	Prerequisites
1 h	• Counters or marbles in three colours (optional) • Ch 20, Ch 25, Ch 50, and Nu 1 coins (optional) • Deck of playing cards (optional)	• constructing an area model or tree diagram • calculating the theoretical probability of an event

Main Points to be Raised

• Outcomes that have the same probability are equally likely.

• You use the formula P(Event) = $\frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$ to calculate the theoretical probability of an event E if all possible outcomes are equally likely.

• You can use a tree diagram or an area model to determine the theoretical probability of an event.

• The probabilities of an event and its complement add to 1.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *What is the probability that the spinner will land on a particular sector?* ($\frac{1}{8}$)

• *Why are the probabilities of landing on each shape different even though the probabilities for each sector are the same?* (Each shape is represented by a different number of sectors of the circle. For example, three sectors are triangles, but only two are circles. So it is more likely that the spinner will land on a triangle than a circle.)

• *What is the probability that the spinner will land on a square?* (0)

The Exposition — Presenting the Main Ideas

• Read the first part of the exposition with the students. Copy the spinner from **page 152** of the student text onto the board. Beside it, draw another spinner of the same size with the same numbers, but make each sector a different size. Ask why you cannot use the formula for theoretical probability to find the probability of spinning any one number on the second spinner. (The sectors are not the same size so the outcomes of spinning each sector are not equally likely).

• Before students read the second part of the exposition, draw a circle on the board and shade part of it. Ask students to explain why you can add the shaded and unshaded parts to form the whole circle. Ask students why this is true regardless of the size of the shaded region.

• Tell students that the whole circle you drew represents 1. Ask them to write an equation that shows the relationship between the shaded and unshaded parts (Shaded + Unshaded = 1).

• Read the second part of the exposition with students.

Revisiting the Try This

B. Students should notice that the event of landing on a polygon (a triangle or a hexagon) is the complement of landing on a circle. Discuss why you can write this as P(polygon) + P(circle) = 1, P(polygon) = 1 – P(circle) or P(circle) = 1 – P(polygon).

Using the Examples

• Ask pairs of students to read through **Solutions 1 and 2** of the example. Ask them to choose which solution most closely matches what they would have done and to say why they would have done it that way.

Practising and Applying

Teaching points and tips

Q 2: Students could use a tree diagram or an area model to list all possible outcomes (as sum of the two values). Some students might benefit from working with real coins.

Q 4 to 6: For each part, students might trace the spinner and write the different number descriptions in place of the numbers.

For example, for **question 4 a)**, they could draw a spinner divided into twelfths with an E (for even) where the 0, 2, 4, 6, 8, 10, and 12 would be.

You could suggest students list the outcomes that constitute an event and its complement before they answer the questions.

For example, for **question 4 c)**, (not prime), they could list: 1, 4, 6, 8, 9, 10, 12.

Q 8 to 11: Some students will benefit from working with a deck of cards, specifically the suit of clubs. For example, for **question 9 b)**, they could sort the cards into face cards and not face cards.

Q 11: This question highlights the important idea in this lesson about the relationship between complementary events. You might have a volunteer suggest one possible answer and then ask students to find another answer.

Common errors

• Students may create an incomplete list of the outcomes that make up all possible outcomes. Encourage students to list the outcomes for the event itself and the outcome for its complement so that they can see whether they have formed all possible outcomes.

• Students confuse the term *event* with *outcome*. **Question 7** is designed to address this, but it is important to model the proper use of the terms. An event consists of one or more outcomes. The confusion may arise because students often work with events consisting of a single outcome so the outcome and event are the same.

For example, for the event of spinning the spinner on **page 154** and getting a 1, the outcome describing the event is spinning 1. For the event of spinning an even number, the outcomes 0, 2, 4, 6, 8, 10, and 12 are involved.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can determine the theoretical probability of an event
Question 4	to see if students can describe the complement of an event
Question 7	to see if students understand the difference between an outcome and an event
Question 9	to see if students can determine the probability of the complement of an event

Answers

NOTE: *Probability fractions do not need to be in lowest terms, although the answers show lowest terms.*

A. i) $\frac{3}{8}$ **ii)** $\frac{1}{4}$ **iii)** $\frac{3}{4}$	**B.** P(circle) = 1 – P(polygon); They are complementary probabilities.

Answers [Continued]

1. a) $\frac{1}{2}$ **b)** $\frac{1}{3}$ **c)** $\frac{1}{6}$

d) $\frac{1}{2}$ **e)** $\frac{2}{3}$ **f)** $\frac{5}{6}$

2. a) $\frac{1}{16}$ **b)** $\frac{15}{16}$ **c)** $\frac{1}{2}$

3. a) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
[**b)** Yes; Each number is on one sector of the circle and the sectors are all the same size.]

4. *Sample responses*:
a) spinning an odd number
b) spinning a number that is not a multiple of 4
c) spinning a number that is 10 or less
d) spinning a prime number or 1

5. a) 2, 4, 6, 8, 10, 12 **b)** 1, 3, 5, 7, 9, 11
c) 2, 3, 5, 7, 11 **d)** 1, 4, 6, 8, 9, 10,
12
e) 6, 12 **f)** 1, 2,
3

f) 1, 2, 3

6. a) $\frac{1}{2}$ **b)** $\frac{1}{2}$ **c)** $\frac{5}{12}$

d) $\frac{1}{3}$ **e)** $\frac{2}{3}$ **f)** $\frac{5}{6}$

[**7.** *Sample response*:
The probability of each outcome (choosing each marble) is $\frac{1}{6}$, the probability of each event (choosing a marble of a certain colour), depends on the number of marbles of that colour in the bag.]

8. a) Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King
[**b)** The cards were shuffled and a card was selected randomly, without looking.]

9. a) $\frac{12}{13}$ **b)** $\frac{10}{13}$ **c)** $\frac{4}{13}$

10. a) $\frac{1}{3}$ **b)** $\frac{5}{9}$
[**c)** *Sample response*:
The event (< 5) does not include 5, and neither does the event (> 5), so 5 is not included in either event.]
d) Card value ≥ 5

11. *Sample response*:
P(not 6 of clubs) = $\frac{12}{13}$ and P(6 of clubs) = $\frac{1}{13}$

$1 - \frac{12}{13} = \frac{1}{13}$

Supporting Students

Struggling students

• Some students may have difficulty visualizing the outcomes that make up the complement of an event. It might be helpful to work with the actual materials.

For example:

For **question 1**, you could supply students with coloured counters or marbles. Draw a rectangle that has been divided into two parts with one part labelled “event” and the other part labelled “complement”. Have students sort the counters into the event sector and the complement sector. They can then count to determine the number of outcomes in each sector.

For **questions 8 to 11**, you could supply students with cards so they can place the cards that correspond to an event and its complement on their desks.

Enrichment

• You could assign probabilities in advance for events such as the events described in **question 4 or 5**. Have students design a spinner for which the probabilities are correct.

GAME: Unlucky Ones

• Students can follow either a conservative approach or a risk-taking approach when they play this game:

- Some students may say to themselves that they will stop on each round after rolling three times (unless they roll a 1).
- Some students may decide to try to roll to the next decade (or some other benchmark) on a turn and then stop.
- Other students will simply keep rolling until they roll a 1 (or get to 100) in a turn.

• After the game, you might ask students the following questions:

- *What is the probability of rolling a 1? Two 1s?*
- *If you have not rolled a 1 in a few rolls, do you think it is time for a 1 to come up? Why or why not?*
- *Do you think it is possible to roll all the way to 100 in one turn?*
- *If you did not roll a 1, what is the least number of rolls you would have to roll with two dice to reach 100?*
- *What strategy did you use to win the game?*

Variations

• For practice with mental subtraction, the game can be played in reverse, starting at 100 with a goal of reaching zero. The numbers on the two rolled dice are mentally added together but then subtracted, first from 100, and then, on each turn, from a player's subtotal.

• For practice in multiplication, the numbers rolled on the two dice can be multiplied together mentally. Players can continue rolling as many times as they wish, adding together the products of the rolls. The same rules about rolling a 1 apply, but the target number can be greater than 100.

CONNECTIONS: Simpson's Paradox

• This optional Connection demonstrates that sometimes the probability of a combination of events is not what they would expect based on the probability of each individual event.

• Simpson's Paradox says that sometimes conclusions from combining small data sets into a large data set contradict the probabilities obtained from the smaller sets.

• Students are often surprised to discover that even though it is more likely that a white ball will be chosen from Bag 1 in each small group, this is not the case when you combine the groups.
Some students might benefit by working with real bags and coloured marbles or counters.

Answers

1. Situation 1 Bag 1: P(white) = $\frac{5}{11}$ or about 45% Bag 2: P(white) = $\frac{3}{7}$ or about 43% Situation 2 Bag 1: P(white) = $\frac{6}{9}$ or about 67% Bag 2: P(white) = $\frac{9}{14}$ or about 64%	**2. a)** The combined Bag 1 has 11 white marbles and 9 black marbles. P(white) = $\frac{11}{20}$ or 55% **b)** The combined Bag 2 has 12 white marbles and 9 black marbles. P(white) = $\frac{12}{21}$ or about 57% **c)** I have a higher probability of winning with Bag 1 in each single situation. When I combine the bags, the probability of winning is higher with Bag 2.

6.1.2 Simulations

Curriculum Outcomes	Outcome relevance
8-G2 Simulations and Experiments: single and complimentary events • understand that, in situations for which the probability of various events occurring is not equally likely, experimentation is often the only method of determining probability **8-G3 Compare Results: theoretical and experimental** • compare theoretical and experimental probability for a given situation and discuss results	Students need to learn about simulations because sometimes it is impossible or extremely complicated to determine the theoretical probability of an event. In these cases, you can conduct a simulation experiment to approximate the probability.

Pacing	Materials	Prerequisites
1.5 h	• Fraction circles (for spinners) (BLM) • Nu 1 coins • Opaque containers such as bags or bangchung • Dice • Coloured counters or marbles • Identical slips of paper • Playing cards	• using a tally chart to collect data • calculating experimental probability

Main Points to be Raised

• Sometimes it is not practical or possible to determine the theoretical probability of an event. You can carry out an experiment to estimate the probability.

• In a simulation, you let one situation represent another comparable situation. You can use a simulation when it is not possible or practical to perform an experiment directly.

• The probability of the simulated outcome should match the probability of the outcome it represents.

• You can use a simulation experiment to estimate the theoretical probability of an event. The more trials you do, the more the estimated probability is like
a theoretical probability.

Try This — Introducing the Lesson

A. Allow students to discuss their response to the question in small groups. Have each group share their responses with the class.

• *Why is difficult to find a theoretical probability to predict how Tshering will do on his next two shots?* (The wind might blow harder or change direction. The sun might be lower in the sky and shine in his eyes.
His arrows might be damaged from previous shots.)

• *Is it reasonable to use past performance to estimate the probability for the next two shots?* (Yes, unless the conditions change or the data set is from a long time ago.)

The Exposition — Presenting the Main Ideas

• Explain to students that it is not always possible to determine the theoretical probability of an event. There are a lot of things you might not know about that could influence what happens, for example, who might win a race. However, you can use what has happened in the past to make a reasonable estimate. In these situations you can use an experiment with many trials, using an appropriate model to estimate a theoretical probability.
The experiment is called a simulation when you are using a model rather than the actual objects or events.

• Emphasize to students that a simulation models the actual situation as closely as possible using probability devices such as dice, coins, and spinners.

• Ask students to answer these questions in their groups after they have read through the exposition on **page 157** of the student text:

- *How does each model described in the chart on* ***page 157*** *match the situation it represents?* (The number of possible outcomes in the model is the same as the number of possible outcomes in the situation. The probability of each outcome in the model is the same as the probability presented in the situation.)

- *Why should you repeat a simulation many times?* (The experimental probability of an event becomes closer to the theoretical probability as the number of trials increases.)

Revisiting the Try This

B. This question allows students to apply the principles developed in the exposition to design a simulation based on Tshering's past archery performance.

Using the Examples

• You may wish to have students work in small groups to complete each experiment described in the examples. As students complete each experiment, they could record the experimental probability on the board. When all the groups have completed the experiments, you could discuss the range of values recorded for each simulation.

• You could also combine the class data for each experiment to determine a single experimental probability for each situation. You could compare the single probability to the answer provided and talk about the effect of a greater number of trials.

• Make sure students understand why the model selected in each of the examples was appropriate.

Practising and Applying

Teaching points and tips

Q 1: Remind students that the number of outcomes that make up the event has to match the number of outcomes in the simulation.

Q 2: Remind students that the denominator of the probabilities in a situation corresponds to the total number of equally likely possible outcomes.

Q 8: This question highlights the important ideas in the lesson about the results of a simulation and the relationship between experimental and theoretical probability.

Common errors

• Students may have difficulty identifying the total number of outcomes required for the model they must use for the simulation because they are not able to determine the number of possible outcomes in the situation. Or, they may not be able to determine the number of favourable outcomes they must build into their simulation. Students might benefit by doing the reverse, which is looking at a probability device and coming up with a possible situation for a given context.

For example, describe a probability situation about children in a family that you could model by rolling a die.

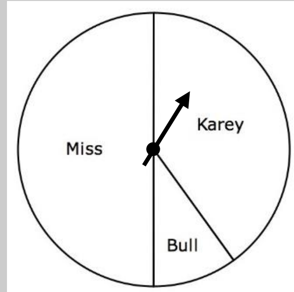
Suggested assessment questions from Practising and Applying

Question 2	to see if students can design a simulation for a given situation
Question 3, 4, or 5	to see if students can design and conduct a simulation to determine probability
Question 8	to see if students understand the use and the limitations of simulations and the relationship between experimental and theoretical probability

Answers

A. *Sample response*: $\frac{1}{100}$

B. i)



ii) *Sample response*:
I could do this:
- Spin 2 times and record whether I got 2 bullseyes.
- Repeat the experiment 99 more times.
- Record what percent of the time I got 2 bullseyes and see if it was 1 out of 100.

1. a) A, B (odd vs. even), C (odd vs. even), and D (odd vs. even)
b) C
c) B and D

2. *Sample responses*:
a) Flip a Nu 1 coin 3 times. Khorlo represents a girl being born. Repeat the experiment 20 times and count the percent of the time you get 3 Khorlos in 3 flips.
b) Cut out 3 pieces of paper and label one with RAIN. Put them in a bag. Draw one piece of paper from the bag, put it back, and then draw another. Repeat the drawing two more times. Repeat the entire experiment 25 times and count the percent of the time you choose the paper that says RAIN on each of the three draws.
c) Label 28 papers with BOY and 12 papers with GIRL. Put them into a bag. Draw two papers. Repeat the experiment 25 times and count the percent of the time you choose two papers that say GIRL.
d) Make a spinner with 10 equal sectors. Label 3 sectors as kareys. This is because 15 out of 50 is the same ratio as 3 out of 10. Spin the spinner two times. Repeat the experiment 20 times and count the percent of the time you spin karey both times.

3. *Sample response*:
The simulation could involve a spinner with four equal sectors. One sector is labelled CORRECT. This is because 1 out of 4 choices is correct. Spin the spinner five times and count how many times you get CORRECT. Repeat the experiment 20 times.
Count the percent of the time that you got correct 3, 4, or 5 times. The probability is likely to be close to 0.1 (actual answer ≈ 0.12).

4. *Sample response*:
The simulation could involve a spinner with 8 equal sectors. One sector is labelled HIT. This is because 25 out of 200 is the same as 1 out of 8.
Spin the spinner five times and record whether all five spins are HITs. Repeat the experiment 20 times.
Count the percent of the time that you got all HITs.
The probability is likely to be close to 0
(actual answer ≈ 0.00003).

5. *Sample response*:
5% is 1 out of 20. The simulation could involve 20 slips of paper, with one of the slips labelled BAD. Choose a slip of paper from a bag. Record whether it says BAD and return the slip to the bag. Repeat twice more. Record whether all of the slips chosen said BAD. Repeat the experiment 20 times. Count the percent of the time that you got all BADs.
The probability is likely to be close to 0
(actual answer = 0.000125).

6. *Sample response*:
Print the numbers on cards and draw two numbers 20 times. Record the numbers chosen. Repeat the experiment 20 times. Count the percent of the time that each outcome occurs.
Actual probabilities should be close to:

a) $\frac{5}{6}$ **b)** $\frac{5}{6}$ **c)** $\frac{1}{2}$

7. *Sample response*:
Toss a Nu 1 coin until you get 4 outcomes the same, with Khorlo representing a win for Maaros. Do this 20 times. Use the fraction that describes each probability.
The probabilities might be:

a) $\frac{1}{10}$ **b)** $\frac{1}{5}$ **c)** $\frac{1}{2}$

[**8.** *Sample response*:
A simulation determines an experimental probability, and not the theoretical probability. Improbable events sometimes occur, and so it is not likely that a simulation will exactly reflect the theoretical probability.]

Supporting Students

Struggling students

• If students are struggling with determining an appropriate model for a simulation, suggest one model and explain why it is appropriate. Then ask them to create another.

For example, in **question 4**, you could explain how 200 slips of paper with 25 labelled HIT could represent the 200 previous attempts and 25 previous hits. Then you could suggest that since 1 in every 8 shots was a hit, they could use those numbers to design a simpler model based on a spinner or drawing a HIT card from a deck of 8 cards with 7 blank and 1 marked HIT. Or, you could suggest a model for **question 2 a)** using a die, where even rolls are girls and odd rolls are boys, and then ask students how they might instead use a coin, spinner, or deck of cards.

Enrichment

• For **question 7**, you might challenge students to conduct the experiment and then compare their experimental results to the results they would obtain using a tree diagram of the possible tournament outcomes.

6.2.1 EXPLORE: Sample Size

Curriculum Outcomes	Outcome Relevance
8-F1 Repeated Sampling (of Same Population): variability • understand that survey results of two different samples of the same population will not exactly be the same • recognize the variability among repeated samples and provide a basic and informal introduction to the notion of sampling distribution • conduct probability experiments to demonstrate variability of repeated sampling • use real and simulated data in interesting investigations	Students encounter statistical information all the time. It is important for them to recognize that the samples upon which conclusions are based must be selected thoughtfully.

Pacing	Materials	Prerequisites
1.5 h	• Opaque containers such as bags or bangchung • 100 identical slips of paper	• understanding simulation models • recording data in a frequency table • determining what percent a number is of 10, 20, or 30 (rounding to the nearest whole number)

Main Points to be Raised

• You can create an experimental situation to match, or simulate a known set of data. You can then investigate how well different sample sizes estimate the known values.

• In an experimental situation, results for different trials can vary.

• Variation is usually greatest from one trial to the next if the sample size is smaller.

Exploration

• Read through the introduction (in white) with students. You might sketch a circle graph that displays the 53% male and 47% female.

• Have students work, alone, in pairs, or in small groups for **parts A to C**. Emphasize that all 100 slips of paper must look the same so there is no bias when they select a sample from the container.

While you observe students at work, you might ask questions such as the following:

• *Why is it important to mix up the pieces of paper before each sampling*? (To make sure I do not reselect the slips that were placed on top from the previous sample.)

• *What do you notice about the results from trial to trial for samples of the same size?* (The results are not always the same.)

• When all groups have completed **parts A and B**, ask them to report on their mean, median and range.

• Have students report again after they have completed **part C**.

• You might have students discuss **part D** in their groups or with another group and then discuss it as a class.

Observe and Assess

As students work, notice the following:

• Do they understand why the population must be remixed after each sample so that every slip of paper has the same chance of being chosen?

• Do they recognize that samples of the same size will show variable results?

• Do they observe that the trial statistics tend to be more similar when the sample size is greater?

Share and Reflect

After students have worked through the exploration, they can discuss their observations and answer these questions.

- *How did you make sure that every piece of data (paper) had the same chance of being chosen?*
- *Why would you expect more similarity among trials with a sample size of 30 than with sample size of 10?*
- *Which sample size resulted in statistics that seem most like the known percent of males in the population?*
- *Suppose you did not know that the percent of males in Bhutan was 53%. How could you use an experiment like this one to help you predict that percent?*

Answers

A. and B. i) to iv) *Sample responses*:

Sample Size = 10		
Trial #	**Number of males**	**% Male**
1	6	60
2	4	40
3	5	50
4	7	70
5	2	20
6	8	80
7	8	80
8	4	40
9	4	40
10	7	70
11	5	50
12	4	40
13	5	50
14	4	40
15	4	40
16	6	60
17	6	60
18	5	50
19	2	20
20	7	70

B. v) Mean = 51.5%, median = 50%, range = 60%; On average, the mean and median are fairly close to 53%, but the range of 60% shows that the trials were very different. Sometimes the percent was as high as 80% and sometimes it was as low as 20%. Although I would expect that to happen once in a while, I was surprised to see several 80% and 70% values.

C. i) and ii) *Sample responses*:

Sample Size = 20		
Trial #	**Number of males**	**% Male**
1	9	45
2	5	25
3	7	35
4	9	45
5	9	45
6	12	60
7	8	40
8	11	55
9	16	80
10	12	60
11	14	70
12	11	55
13	8	40
14	15	75
15	11	55
16	10	50
17	7	35
18	12	60
19	10	50
20	9	45

Mean = 49.3%, median = 50%, range = 55%; On average, the mean and median are fairly close to 53% and the range is fairly high. There are fewer values that are very high or very low than when the sample size was 10.

Answers [Continued]

Sample Size = 30		
Trial #	**Number of males**	**Male percent**
1	17	57
2	15	50
3	15	50
4	18	60
5	14	47
6	16	53
7	17	57
8	19	63
9	15	50
10	17	57
11	13	43
12	15	50
13	16	53
14	21	70
15	18	60
16	18	60
17	19	63
18	11	37
19	18	60
20	13	43

Mean = 54.2%, median = 55%, range = 33%;
On average, the mean and median are fairly close to 53% and the range is of a medium size. There are not many values that are not between 45% and 60%.

D. *Sample response*:
I would use the largest sample size since more of the values were close to the values they should be, which is 53%.

Supporting Students

Struggling students

• If you think a student might have difficulty with the procedures in **parts A to C**, you could place him or her in a group with students whose language and organizational skills will allow them to provide support.

• You could prepare a recording sheet on which the students can enter their experimental data.

6.2.2 Selecting a Random Sample

Curriculum Outcomes	Outcome relevance
8-F2 Randomness: concepts • understand that a random sample is a sample collected from a population so that every member of the population has an equal chance of being selected • understand that members are chosen independently of each other • understand that common devices and methods used in selecting random sample are coins, dice, sampling boxes, a table of random numbers	It is important for students to recognize that the samples upon which conclusions are based must be selected thoughtfully. This lesson will help them understand why a random sample is more likely to represent a population.

Pacing	Materials	Prerequisites
1 h	Optional: • Random Number Table (BLM) • A variety of probability devices such as coins, dice, spinners, and identical slips of paper • An opaque container	• calculating a mean

Main Points to be Raised

• A representative sample includes members from throughout the population. It must be as likely for any one member to be selected as for any other member to be selected.

• Using a strategy that involves randomly selecting a sample from all the members of a population helps increases the likelihood that the sample is representative of the population. There are many possible strategies.

Try This — Introducing the Lesson

A. and B. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *In which unit do you think the word angle will occur most frequently?* (Geometry) *least frequently?* (Number)

• *Do you predict that the difference in the means will be a high amount or a low amount? Why?* (A high amount; I would not expect to see the word *angle* at all in a number chapter, but I would expect it a lot in a geometry chapter.)

The Exposition — Presenting the Main Ideas

Have students read the exposition on **page 162** of the student text. Then ask the following question:

• *Why is it important for every member of the population to be as likely to be selected as any other member?*
(This will make sure no subgroup within the population gets unfairly overrepresented or underrepresented.
That way the sample represents the population.)

• Ask students why they think devices like coins, dice, spinners, and slips of paper might be useful for random selection.

• Indicate that **example 1** will provide more detail on how to use a random number table. **Example 2** will clarify how other devices can be used for random selection.

Revisiting the Try This

C. This question has students reflect on the samples selected in **parts A and B** with the new knowledge about the importance of random selection. Students might also discuss sample size, as this will extend their learning from the previous lesson.

Using the Examples

• You might choose to do **example 1** as a class since it involves the use of the random number table, which is a new concept for students. Have students use the table on **page 162** or the Random Number Table (BLM) to select their own samples of three numbers, using their pencils while closing their eyes. You might discuss whether a sample of three pages is large enough. If students are interested, you might share this information from the 2005 Bhutan Census:

- About 57% of households have electric lighting.
- About 37% use kerosene lighting.
- About 6% use some other method of lighting.

• Assign students to read the **examples 2 and 3** in pairs. For **example 2**, you might take a vote about which solution students prefer. For **example 3**, you might ask for other examples of bad samples and/or ask students how they might select an appropriate sample.

Practising and Applying

Teaching points and tips

Q 1 to 5: Although students are not being asked to actually use the probability devices to select samples, many students will benefit from working with the actual devices. You might have students choose one question and demonstrate the sampling method they have described.

Q 1: Emphasize that the strategy students design must select a random sample with exactly 100 volunteers. Point out that selecting the 50 who receive the sugar pills is not part of the problem.

Q 2 and 4: Suggest that students consider the size of the sample as well as the strategy for selecting the sample.

Q 4 b): This question is not about omitting people from the population as much as it is about knowing what the population is. In this case, even though we are looking at people in a particular dzongkhag, the population in question includes only those who can buy a cell phone (i.e., not children). Therefore, no children should be selected as part of the sample.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can design a suitable sampling strategy to select randomly a required number of members of the population
Question 4	to see if students can design a suitable sampling strategy and recognize that they must understand what the population is before they select a sample
Question 5	to see if students understand that, due to the element of chance, it is possible but not likely that a random sample will not represent the population

Answers

A. and B. The word angle does not appear in Unit 1 Chapter 1. The word appears frequently in Unit 8 Chapter 3, but answers will depend on the actual pages chosen.	**C.** *Sample responses*: **i)** Unit 1 is about number and the word "angle" is not likely to come up, while Unit 8 is about geometry and the word "angle" is mentioned a lot. Also, a sample size of two pages for a book of about 300 pages is too small a sample. **ii)** I would use a random number table to select pages randomly from throughout the book. I would also select a larger number of pages so that my sample would better represent the population.
1. *Sample response*: Assign each volunteer a number from 001 to 500. Use a random number table and select 100 three-digit numbers. You can start anywhere and keep moving three digits ahead. Ignore any numbers that are greater than 500 or that are repeats.	**2.** *Sample response*: Decide on how big a sample you want, such as 10. Have the yaks pass in single file through a narrow opening or a gate. Select every tenth yak that passes through.

3. *Sample response*: Put each renter's name on a piece of paper in a bag. Select 5 pieces of paper from the bag. **4.** *Sample responses*: **a)** Obtain a list of people old enough to buy a cell phone plan (e.g., no children) who live in the dzongkhag. Assign each person a number in sequence from 0000 to 9999. Choose 4-digit random numbers from a table. **b)** Children should not be included because they are too young to own cell phones.	**5. a)** About 4 [**b)** *Sample responses*: The element of chance means that you can never predict with certainty. So although it is likely that you will get 4 slips with dots, anything is possible. There are at least 10 slips with dots so it is possible to get all dots.]

Supporting Students

Struggling students

• Many of the questions ask students to analyse a situation and then discuss or explain their reasoning. Pair students who may have difficulty reading or explaining their reasoning with students who have strong reading and speaking skills.

Enrichment

• For **question 5**, you could ask students to carry out an experiment in which they determine the experimental probability that all 10 slips have black dots. Another variation is to determine the experimental probability that you draw 2, 3, 4, or 5 slips in a row that have black dots.

6.2.3 Circle Graphs

Curriculum Outcomes	Outcome relevance
8-F3 Circle Graphs: construct and interpret • understand usefulness of circle graphs in situations where a comparison of the part to the whole is needed (e.g., budgets) • apply prior knowledge about percent and using a protractor in construction of circle graphs • focus on when a circle graph is the most appropriate data display	Students learn that organizing data in a different way can allow for more insight into the data. A circle graph allows the reader to see how the categories of a set of data compare to each other and how each category compares to the whole set of data. Circle graphs are commonly used, so it is important to know how to create and interpret them.

Pacing	Materials	Prerequisites
1.5 h	• Protractors • Compasses • Coloured pencils (optional)	• calculating what percent a given part is of a whole • calculating a given percent of a number • using a protractor to draw a given sector angle in a circle

Main Points to be Raised

• To find the sector that represents a category in a circle graph, you find the percent of 360° that is equal to the percent that category is of the whole set of data.

• A circle graph is useful for comparing the parts (categories) of a set of data to the whole set of data and for comparing the categories to each other.

• A circle graph is only appropriate if the set of data represents a whole.

For example:

- You can use a circle graph to represent data about how a family spends its money each month.
- You cannot use a circle graph for data about the number of ghos sold each week in a shop.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask:

• *How might you have known which sector represented the male population even if it was not labelled?* (It has to be white since 53% is greater than half and only the white sector is greater than half.)

• *How could you use the graph to show which sector represents 3% of the population?* (It is the amount between the straight line that comes down from the vertical line at the top half of the circle and the radius between the white and dark sectors at the bottom of the circle.)

• *How can you estimate the size of the angles for the sectors representing Male and Female?*
(The angle for Male must be slightly larger than the angle for Female. There are only two categories, so that means the angle for Male must be more than 180° and the angle for Female must be less than 180º.)

The Exposition — Presenting the Main Ideas

• You may wish to review how circle graphs were created in Class VII (using a percent circle) by sketching a percent circle on the board and talking about how you could graph the data in the **Try This**.

• Ask students whether the order in which you construct the sectors matters (it does not matter except that each new sector you create must be adjacent to one of the sectors already drawn). Ask how you might choose which sector to create first (it might be an easier calculation or you may wish certain categories next to each other).

• You might remind students how to calculate the answers to questions like "What is ___% of ___?"

Revisiting the Try This

B. Students use what they learned in the exposition about calculating sector angles for the circle graph in **part A**.

Using the Examples

• Have students through the example in pairs. Have them discuss how the way this circle graph is made compares to how the circle graph in the exposition was made.

• Discuss why it might be reasonable to use 70% and 30% to estimate the two sectors.

• Make sure students know that labelling the circle graph with its percent is optional. Also mention that circle graph sectors are often coloured or shaded differently to highlight the different categories.

Practising and Applying

Teaching points and tips

Q 1: Suggest that students follow the procedure shown in the exposition. Discuss why it is necessary to calculate the angle for only two of the sectors since the third sector must represent the rest of the whole. Ask students which sector they created first and why. Students can colour or shade their graphs if they wish.

Q 3: You might have students who created circle graphs for the same data sets compare their graphs and talk about why they might be different (sectors are in a different order).

Q 4: This could be any data set that assigns students into non-overlapping categories and for which the sum of the categories is the total number of students.

Q 5: Use this last question as a closure question to highlight the important idea in the lesson about the value of circle graphs. You might have students think about their personal answers to the question and then discuss them with two or three other students until they agree on a "best" answer.

Common errors

• Any errors students make in this lesson will probably be calculation errors. You might have students work in pairs and check each other's calculations.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can calculate the percent a category represents of a whole, use the information to calculate the sector angle needed to represent that category on a circle graph, and construct the circle graph
Question 3	to see if students can create a circle graph from a set of raw data
Question 5	to see if students can explain why a circle graph might be more appropriate than another type of graph for a given context and data set

Answers

A. *Sample responses*:
i) It is like a picture that shows how the two categories of the population compare.
ii) Use a circle that has percent markers around the circumference. Use the percent markers to draw radii for one of the category sectors.

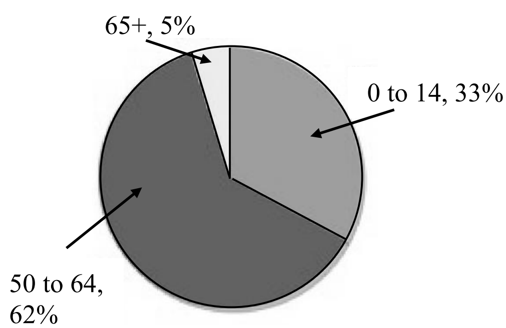
B.
Male: 191°
Female: 169°

1. a) and b)

Age group	Persons	%	Angle
0–14	210,000	33	119°
15–64	395,000	62	223°
65+	30,000	5	18°
Total	635,000	100	360°

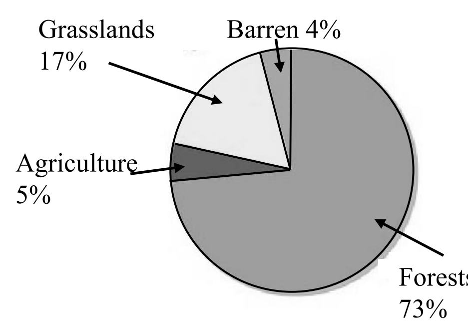
c)

Age Distribution



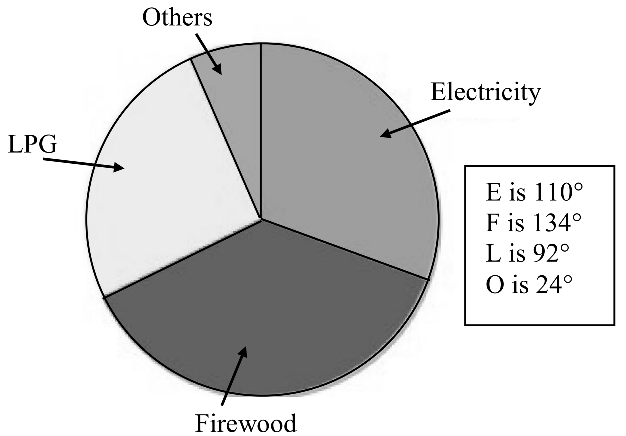
2.

Ecosystems in Bhutan



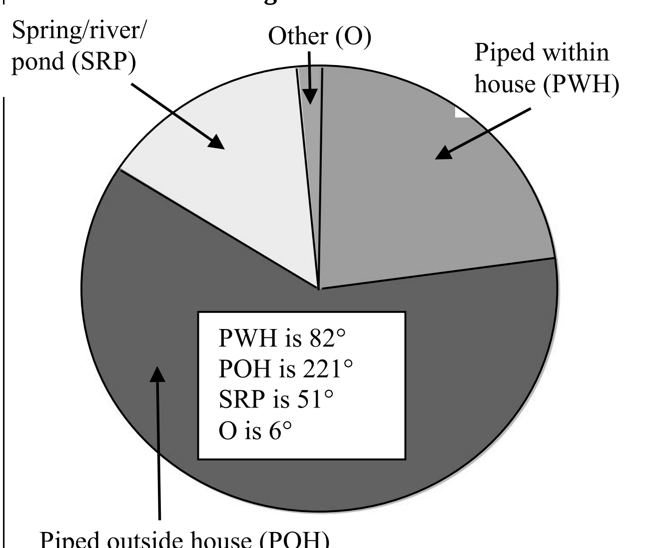
3. a)

Cooking Fuel



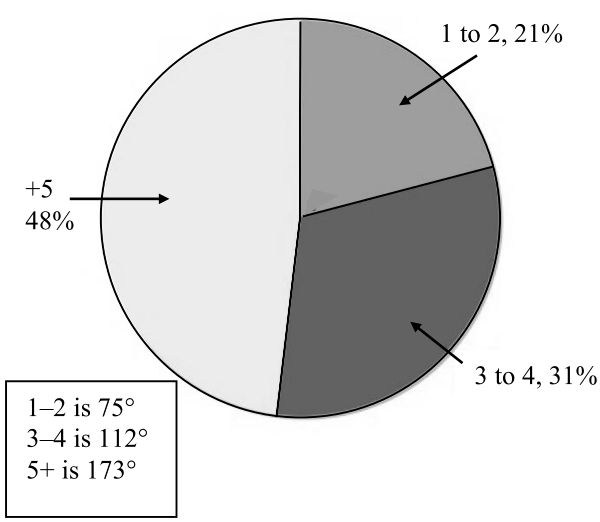
b)

Drinking Water Resources



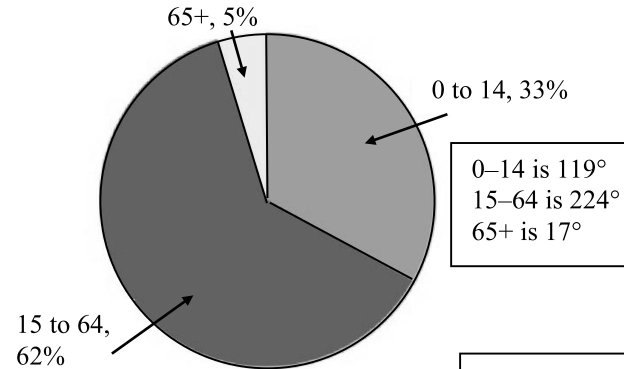
c)

People in Households

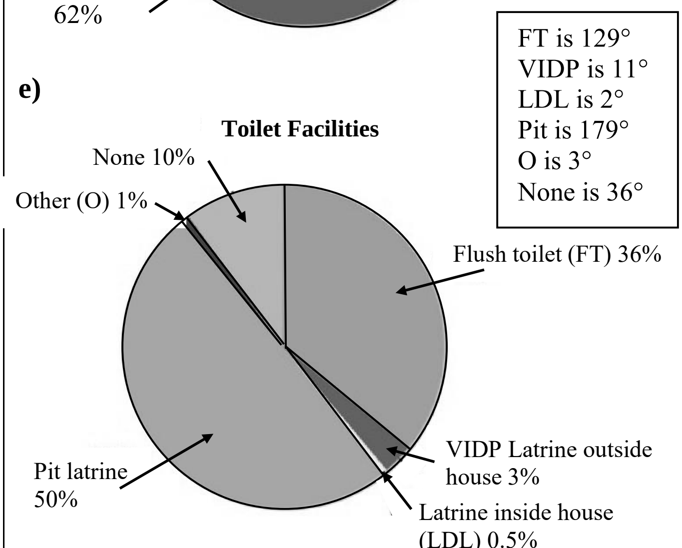


d)

Population by Age Group

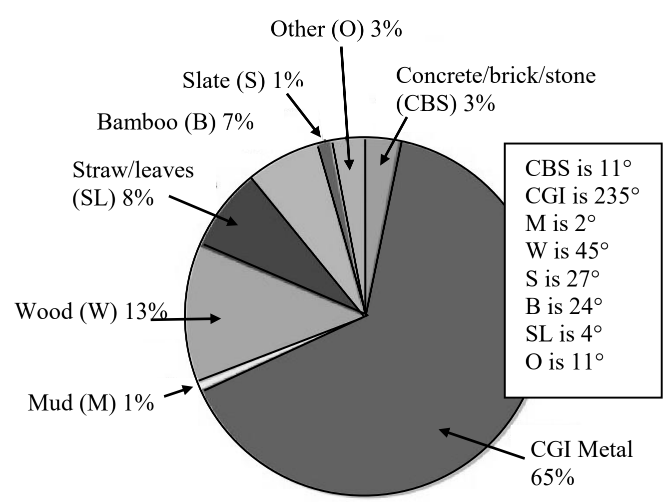


e)



f)

Households by Roofing Material



4. *Sample response*:
The percent of students in different classes of a school. [It makes sense to use a circle graph because the circle represents the whole school and the classes make up all the parts of the school.]

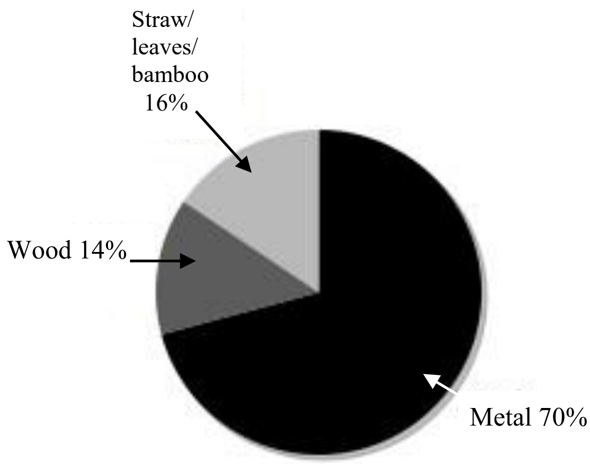
[**5.** The circle graph allows you to see easily how each category compares to the whole set of data; Bar graphs and histograms are better at showing how the parts compare to each other.]

Supporting Students

Struggling students

• If students are struggling with the number of calculations required for **Question 3 f)**, suggest that they simplify the data categories to Metal, Wood, and Straw/leaves/bamboo.

Roofing Materials



6.2.4 Box and Whisker Plots

Curriculum Outcomes	Outcome relevance
8-F4 Box and Whisker Plots: construct and interpret • understand that this is an easy method for visually displaying the median, the range, and the distribution or spread • construct plots • identify the median and the median of the upper half of the data (upper quartile) • identify the median of the lower half of the data (lower quartile) • identify the extremes, that is, the lower value and the higher value	Students learn that a different way of organizing data can allow for more insight into the data A box and whisker plot allows the reader to see how the data values are distributed. A box plot displays only particular exact data values — the greatest and least values.

Pacing	Materials	Prerequisites
1.5 h	• Rulers • Grid paper or Small Grid Paper (BLM) (optional)	• calculating the mean and median of a set of data

Main Points to be Raised

• A box and whisker plot allows you to display the median, the spread, the range, and the outliers in a set of data.

• A box and whisker plot allows you to see how the values in a set of data are distributed. You can use box plots to compare the distributions of several related sets of data.

Try This — Introducing the Lesson

A. Spend a few moments reviewing what students learned in **lesson 6.2.1** about the effect of sample size on the results of a simulation experiment.

You might handle this question as a class after allowing students to discuss their thoughts in pairs or small groups. Then ask students what they would do with the three sets of data in order to compare them. Students might suggest calculating the mean, median, mode, and range. Some might suggest graphing. If students suggest graphing, ask them what type of graph they would use and why they would use it.

The Exposition — Presenting the Main Ideas

• On the board, demonstrate all the steps used in the example in the exposition. Have the students perform the steps with you and suggest that they check one another's work at the end of each step. Some students find that using grid paper makes the task easier.

• Explain that you can think of the upper and lower quartiles as medians of the upper and lower halves of the data set. The quartiles including the median if there is an odd number of pieces of data.

• Make sure students understand the difference between extreme data values and outliers. Extreme values are the least and greatest values. An outlier is a value that is so distant from the other data values that you wonder if it is an error.

• It is important to make sure students realize that when they create a box and whisker plot, they are creating a picture of the set of data so it is easier to analyse.

Revisiting the Try This

B. Discuss with the class how the box and whisker plots show all the information necessary to compare visually the distributions of the three data sets: the medians, the extremes (and ranges), and the quartiles.

• You might ask students why it is important to use the same scale/number line for all three plots.

• Mention that **example 2** goes into detail about how to use the three box plots to compare the distribution of the three data sets.

Using the Examples

• Ask students to work in pairs. One student in each pair should become the expert on **example 1** and the other should become the expert on **example 2**. They should then teach each other about their examples.

Practising and Applying

Teaching points and tips

Q 1, 4, and 5: Remind students that they must order the data values to determine the median, quartiles, and extreme values.

Q 2: Before students start working on this question, you might discuss whether the last four values should be included. (If you are only interested in student ages, they should be omitted before plotting the data since they are probably the ages of teachers and are therefore outliers. If you are interested in everyone in the school, they should be included.)

Common errors

• Some students may have difficulty determining the first and third quartiles when the upper and lower halves have an even number of data values. You might review how to find the two middle values and their mean.

• Students might inadvertently omit a data value when they copy and order the data. Have students count the data values to make sure they have the same number of values as in the question.

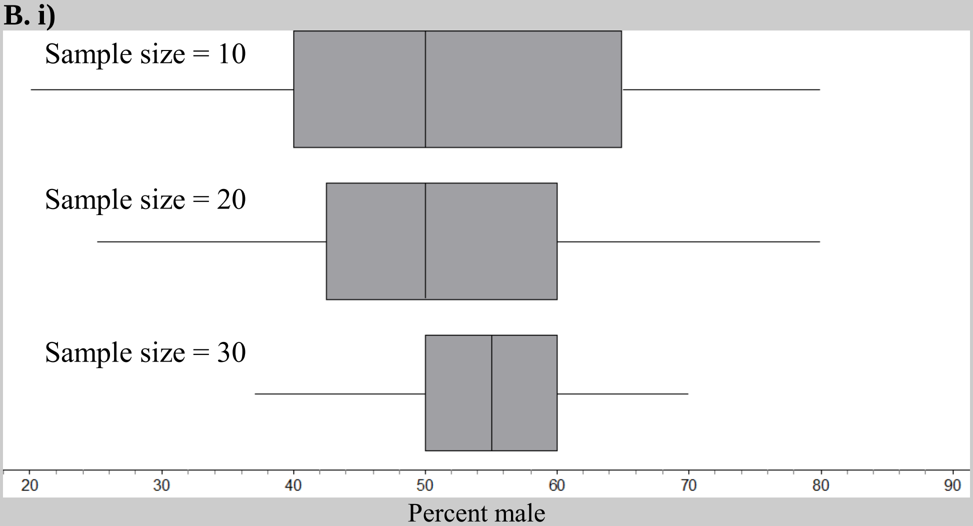
Suggested assessment questions from Practising and Applying

Question 1	to see if students can construct a box and whisker plot from raw data
Question 3	to see if students can use box and whisker plots to compare the distributions of two sets of related data
Question 4 or 5	to see if students can construct box and whisker plots from raw data and use the plots to compare the distributions of related sets of data

Answers

A. *Sample response*:
To compare the data sets, you need to compare the extremes, the ranges, and the medians and/or means. These are not easily gleaned from the data in the charts.

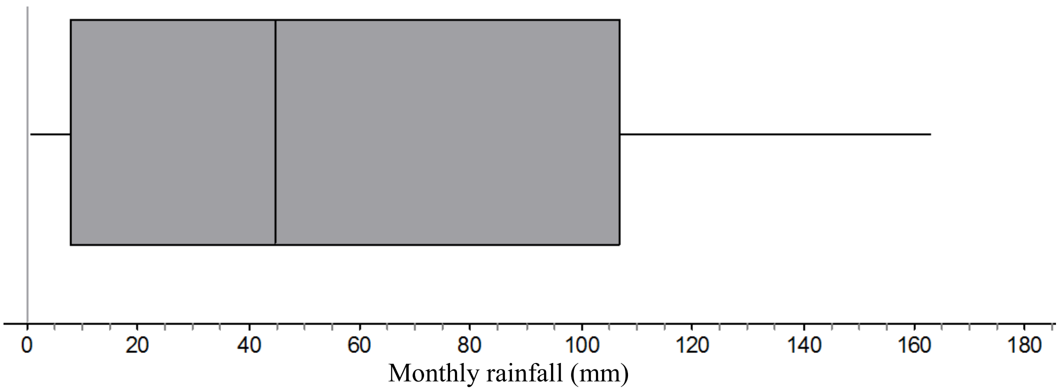
B. i)



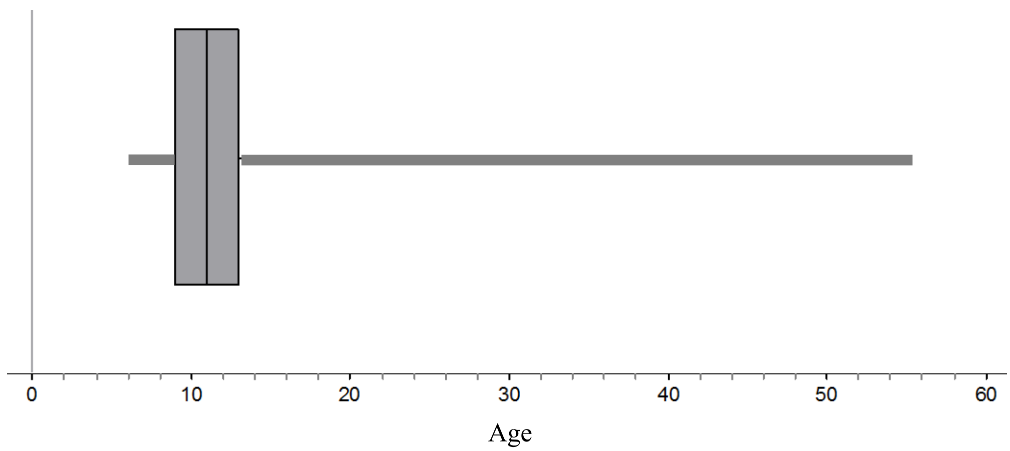
ii) The plots show all the information necessary to compare the distributions of the three data sets: the medians, the extremes and ranges, and the quartiles. Since the plots are all on the same scale/number line, you can compare them visually.

Answers [Continued]

1.



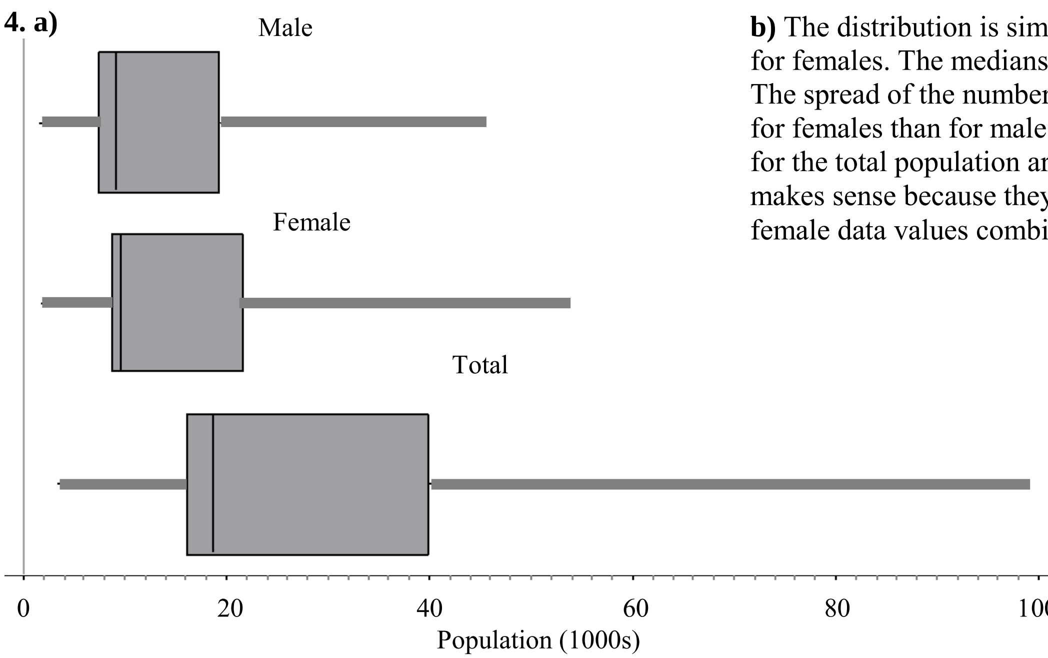
2. a)



b) *Sample response*:
Most of the data values are clustered close to the median but the very long right whisker shows that there are some very high values, which represent the teachers' ages. These values make the range of the data very large.

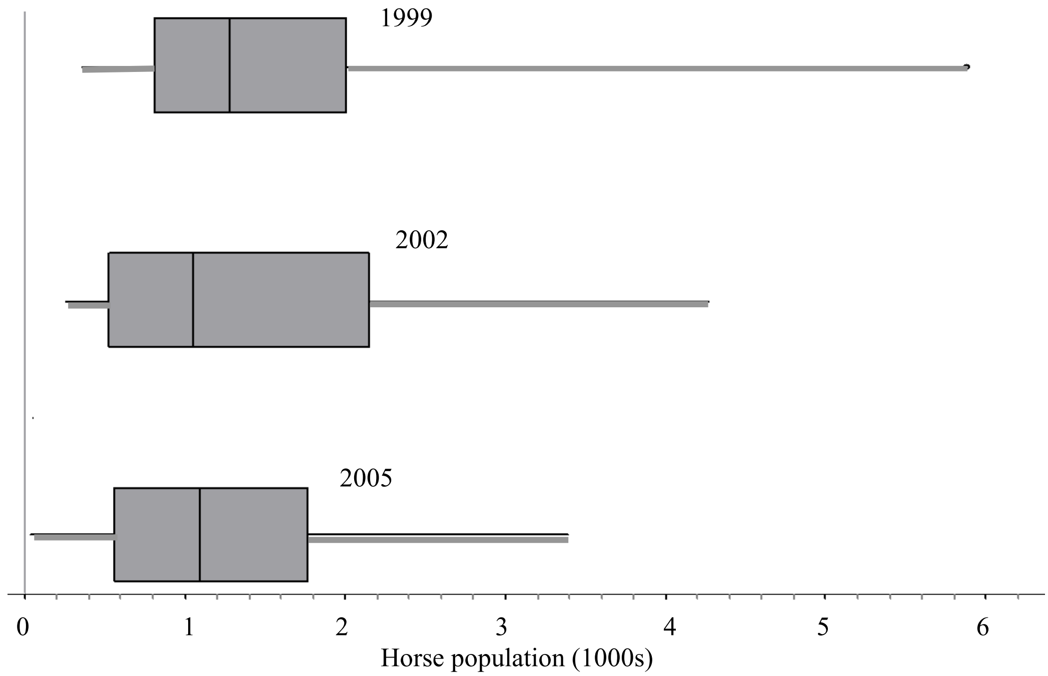
3. *Sample response*:
It looks like Econo Bulb has the best quality. They have fewer problem bulbs overall and the extremes are low values rather than high values, which means that fewer bulbs do not work.

4. a)



b) The distribution is similar for males and for females. The medians are close.
The spread of the numbers is a bit greater for females than for males. The data values for the total population are greater, which makes sense because they are the male and female data values combined.

5. a)



b) *Sample response*:
The spread of the data was greatest in 1999. That year there was an unusually large number of horses in Trashigang that should likely be considered an outlier.
The distributions for 2002 and 2005 had almost identical medians, but there was a greater range of values in 2002.
Trashigang provided the extreme value each year, but it decreased from 1999 to 2005.

6. The right whisker will be a lot longer than the left whisker [because there is a very high extreme of 200 and a low extreme of 1.]
The box will be very narrow with no median line [because the median will be 10, which is also the first quartile, and the upper quartile will be 10.5.]

Supporting Students

Struggling students

• Students may make mistakes based on not copying or sorting data correctly. Suggest that they work in pairs to avoid copying data incorrectly from the textbook. Have them check one another's sorted data sets before they calculate the median and quartiles.

• Some students would benefit from creating their box plots on grid paper.

Enrichment

• You could ask students to create a set of data directly from a box plot.

6.2.5 EXPLORE: The Impact of Altering a Data Set

Curriculum Outcomes	Outcome Relevance
8-F6 Variations: on mean, median, and mode • consider and compare, through investigation, the impact of alterations to data sets to each of mean, median and mode	Valid alterations to a data set occur from time to time. It is important for students to understand the implications of these alterations. For example, if you choose to remove values that you consider to be outliers, how will this affect the results? In this essential exploration, students explore what happens to a set of data when certain values are removed. They use box and whisker plots to display the impact of omitting values from a data set.

Pacing	Materials	Prerequisites
40 min	• Rulers • Grid paper or Small Grid Paper (BLM) (optional)	• constructing a box and whisker plot • calculating the mean

Main Points to be Raised

- You may sometimes choose to remove the least value, the greatest value, or both of these from a set of data. You should only do this if you believe that the extreme values distort the impression the data gives.
- If you remove the extreme data values and plot a set of data using a box and whisker plot, it is more likely that the whiskers will change than that the box will change.

Exploration

- Work through the introduction (in white) with the students.
- Have students work in pairs or in small groups for **parts A to C**. Suggest that they refer to **pages 169 and 170** in the student text if they have trouble constructing the required box plots. Provide grid paper to students, if you think it will help make the task easier for them.
- Encourage students to use the box plot on **page 175** to help them visualize the impact of removing the extreme data values.

While you observe students at work, you might ask questions such as the following:

- *Should removing one value have a major impact on the value of the median or on a quartile?* (Probably not, but it depends. It would change the length of the whisker.)
- *What effect should you see after removing both extreme values?* (Other than changing the lengths of the whiskers, removing the greatest and least values should leave the median the same and the quartiles mostly unchanged.)

Observe and Assess

As students work, notice the following:

- Do they determine the mean correctly?
- Can they determine the median and other quartiles?
- Can they construct neat and readable box plots?

Share and Reflect

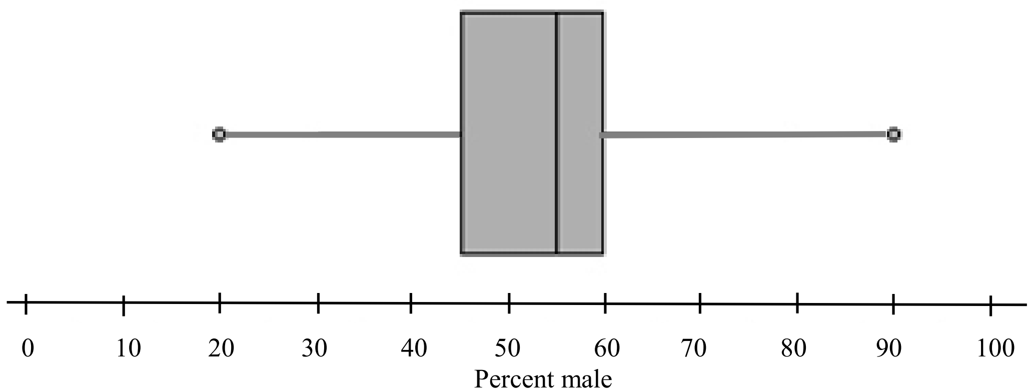
After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and answer these questions.

- *Does the box plot show the distribution of the data more accurately if you leave in both extreme value, remove only one extreme value, or remove both extreme values?*
- *How did the box plots show the impact of removing the data values?*

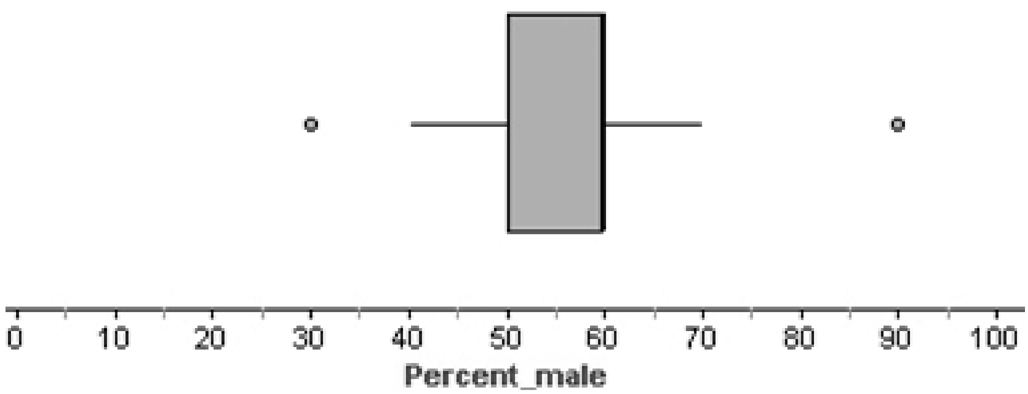
Answers

A. i), ii), and iii) *Sample response*:
I think the lengths of the whiskers will change, but not much else will change.

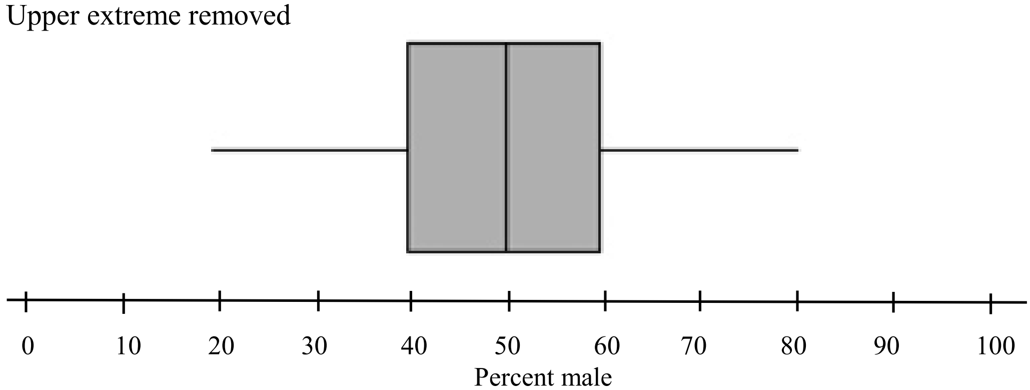
B. Original data



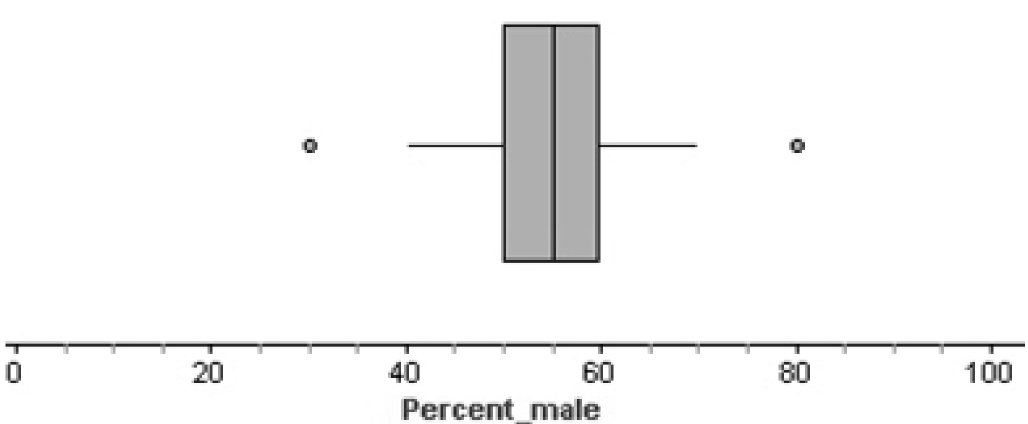
Lower extreme removed



Upper extreme removed



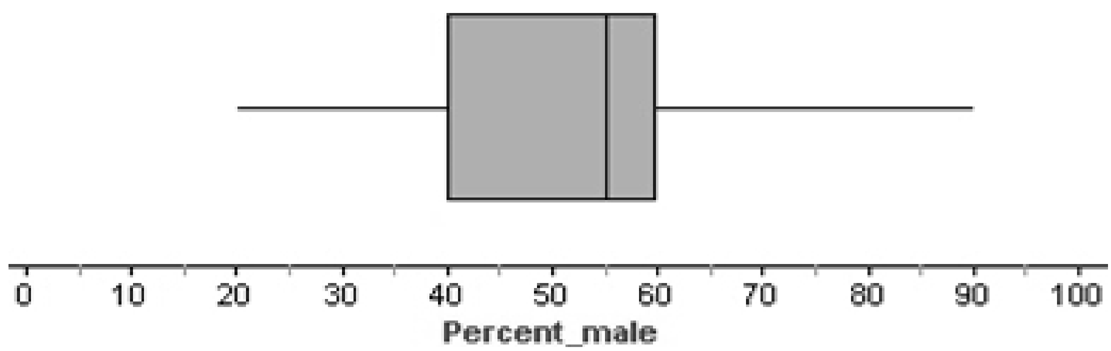
Both extremes removed



Answers [Continued]

What was changed	Mean	Median	Effect on box plot
Lower extreme removed	54.7 (increased)	60 (increased)	• median shifts to the right • the median is the upper quartile • box moves to the right and is narrower
Upper extreme removed	51.1 (decreased)	50 (decreased)	• median shifts to the left • box becomes wider to the left of the median
Both extremes removed	52.8 (about the same)	55 (same)	• extremes move closer to box • median is unchanged • box becomes narrower

C. i) *Sample response*: There will minor changes, if any, to the mean, median, and the shape of the box plot.
ii) Two middle values removed:



Mean	Median	Effect on box plot
52.8 (about the same)	55 (no change)	• median is unchanged • left side of the box is wider

My predictions were almost right.

Supporting Students

Enrichment

• For **Part C**, suggest that students investigate to determine whether it is possible to remove two or more values from the data set and not change the box plot at all.

6.3.1 EXPLORE: The Relationship Between Two Variables

Curriculum Outcomes	Outcome Relevance
8-C1 Patterns and Relations: represent in a variety of formats • describe in words, and use expressions and equations to represent patterns given in tables, graphs, charts, pictures and/or problems situations **8-C2 Graphs (linear and non-linear): interpret** • use information from tables, diagrams, pictures, graphs or equations to describe change	This essential exploration re-introduces students to the concept of a relation between two variables. It is important for students to know about strategies for determining whether a relationship exists between two sets of variables.

Pacing	Materials	Prerequisites
40 min	• Rulers	• measuring length to the nearest centimetre • ordering decimals from least to greatest

Exploration

• Work through the introduction (in white) with the students. Make sure they understand how they are to measure the three lengths. You may wish to demonstrate using your own hands and feet.

• While students are carrying out the measurements, you can prepare the class summary chart on the board. Have students enter their own values on the chart and then make sure that each student makes a copy of the class summary.

Main Points to be Raised

• Two different variables can be related. You can sometimes observe the relationship using a table of values, especially if the values for one variable are ordered.

• Sometimes you can see relationships within a table of values. Often, it is easier to see the relationships when you use a graph (which students will see when they complete **question 6** on **page 180** in the next lesson).

Observe and Assess

As students work, notice the following:

• *Do they look at the overall relationship between the measurements or are they focusing on their own measurements or only a few measurements?*

• *Do they look for a relationship between hand span and foot length, even though they are not asked to?*

• *Do they look for linear relationships or do they look for any trend?*

Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and answer these questions.

• *What seems to happen to foot length as hand length increases?* (Foot length also increases.)

• *Do you think you could accurately predict a person's hand span or foot length if you knew his or her hand length?* (Not exactly, but if the hand span was large, I would predict that the hand length would also be large.)

• *How did ordering the data according to hand length help you see any possible relationships?* (When the data was all mixed up, it was hard to see a pattern. When I sorted the hand length so it was in increasing order, I saw that the data values in the other two columns were also in increasing order.)

Answers

A. *Sample responses*:
i) 19.0 cm **ii)** 21.0 cm **iii)** 21.2 cm

B. *Sample responses:*
i)

ii)

Hand length (cm)	Hand span (cm)	Foot length (cm)
17.0	15.4	20.1
17.3	15.7	21.6
17.8	17.0	23.6
18.1	17.8	23.9
18.2	17.8	22.0
18.3	18.3	22.2
18.7	19.5	22.8
18.7	19.1	21.9
18.8	19.2	23.7
19.0	21.0	22.7
19.0	20.3	23.7
19.3	20.6	24.6
19.3	20.7	23.3
19.8	22.6	23.5
20.0	22.2	25.9
20.1	22.5	24.8
20.1	22.9	25.5
20.2	22.7	24.2
20.5	23.3	24.2
20.7	24.5	25.4
20.8	24.5	25.8
21.0	24.6	27.3
21.3	25.4	25.1
21.6	26.1	28.3
22.0	27.4	28.6
22.4	28.4	26.7
22.8	29.8	27.4
22.9	30.2	29.5
22.9	29.5	28.7

C. i) As hand length increases, hand span also increases.
ii) As hand length increases, foot length also increases.

Supporting Students

Struggling students

• Some students may have measuring their own hands and feet. You might ask students to find a partner to help them complete the measurements.

• You may need to suggest that students reorder the values in the chart so that one measure, e.g. hand length, goes from least to greatest or greatest to least to make it easier to see the relationships.

Enrichment

• You could ask students to see if similar relationships exist between facial characteristics like nose length and the distance between the eyes.

6.3.2 Using a Scatter Plot to Represent a Relationship

Curriculum Outcomes	Outcome relevance
8-C1 Patterns and Relations: represent in a variety of formats • move interchangeably among a variety of formats which describe relationships • describe in words, and use expressions and equations, to represent patterns given in tables, graphs, charts, pictures and/or problems situations • use information presented in a variety of formats to derive mathematical expressions and predict unknown values • investigate linear situations and those which create a regular pattern (broken line or curved graph) • predict unknown values once the algebraic description of a pattern is established • interpolate and extrapolate to predict unknown values when patterns are not regular **8-F5 Scatter Plots: construct and interpret** • use data collected by students to construct scatter plots	• The idea that a relationship between two numeric variables may be represented graphically is an essential concept. The development of algebra and analytic geometry in higher classes is based in this concept. • This lesson follows from the previous explore lesson, where students tried to look for a relationship between variables by examining numbers in a table of values.

Pacing	Materials	Prerequisites
1.5 h	• Rulers • Grid paper or Small Grid Paper (BLM)	• writing a pattern rule (as an algebraic expression or using words) • organizing data in a table of values • plotting points on a coordinate grid (Quadrant I)

Main Points to be Raised

• You can represent the relationship between two numerical variables visually using a scatter plot.

• When the horizontal axis represents time, the scatter plot may help you identify a trend over time.

• You can use the shape formed by the plotted points
to predict the values of points that are not actually recorded in the data set.

Try This — Introducing the Lesson

A. and B. Allow students to try this alone or with a partner. You may wish to review what a pattern rule is (an expression using a variable or words that you can use to predict the number of tiles if you know the figure number).

While you observe students at work, you might ask questions such as the following:

• *How can you predict the number of tiles in the 6th, 7th, or 8th figure?* (I can sketch the figures and then count the tiles in each figure. Or, I can extend the pattern. I see that I need two more shaded tiles each time, so I could add on two tiles for each figure.)

• *Why do you think your pattern rule is correct?* (I tried it with all five figures in the pattern that I had drawn and it worked each time.)

• *What led you to your rule?* (I noticed there was one more column of dark tiles each time, so I knew the rule had to show that the number of tiles increased by two when the figure number increased by one.)

The Exposition — Presenting the Main Ideas

• On the board, show how to construct the graph and how to plot the points. Keep this graph on the board while you work through **part C** of the **Try This,** so students can refer to it throughout the lesson.

• As you plot each point, write the coordinates of the point beside the point. Make sure students realize that you would not normally do this when you create a scatter plot.

• Explain that the variable we usually choose to go on the vertical axis is the variable that depends on the value of the other variable. This is not required, but it is a convention.

Revisiting the Try This

C. Students apply what they learned about creating scatter plots in the exposition to the problem presented in the **Try This**.

• On the board, demonstrate all the steps required to solve this part. Then ask the students to repeat the same steps in their notes.

• Discuss why it is reasonable to assume that the number of tiles depends on the figure number. That is why the values for number of tiles used are on the vertical axis. Do not introduce the formal terms *dependent* or *independent variables*. These will be dealt with in Class IX.

Using the Examples

• Read through the example with the class. Ask whether students believe that the number of telephones should be thought of as depending on the year or whether the year depends on the number of telephones.

• Explain that even though it is not possible to plot each data point with absolute precision, you can approximate the vertical value well enough to see whether there is a trend in the data values. Suggest that if students were to use a very large piece of graph paper, they could graph the data points with greater accuracy.

• Ask students whether or not they agree with conclusions reached in the solution to the example.

Practising and Applying

Teaching points and tips

Q 1: You might assign different parts to different groups of students and then have each group present the solution for its assigned part. Students might notice that if they write the pattern rule algebraically, the coefficient of the variable that represents the figure number describes the increase from one figure to the next figure.

Q 2: Explain that the dots are not connected because there is no such thing as, for example, Figure 2.5. If you connect the dots with an imaginary line you can predict values that are not plotted.

Q 3: Make sure students first think about what scale to use. They need to look at the ranges for both variables. Point out how this time the graph goes down rather than up.

Q 4: The source for this data is the International Telecommunications Union (ITU). Note that the axis label "Year" is optional.

Q 5: Remind students of the discussion about how to decide which axis to use for which variable.

Q 6: This question uses the data values collected in **explore lesson 6.3.1**.

Common errors

• Many students will not use the conventional approach for selecting the axis for a variable. Remind them that the vertical axis is generally used to represent the variable that depends on, or is affected by the other variable. This is a difficult concept for many students and is more difficult in some contexts than in others. Note that, in some contexts, either variable could be considered dependent, depending on the interpretation. Students will learn to identify which variable is dependent through exposure.

• Some students will plot the points in the order they appear and not set up the axes first. This will result in points seeming as if they are randomly distributed on the graph. Remind them that each axis is a number line and the spacing and intervals have to be established before the coordinates can be plotted.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can set up a table of values for a pattern and create a scatter plot
Question 2	to see if students can use a scatter plot to determine values not recorded in the data
Question 7	to see if students can identify the essential features of a scatter plot

Answers

A. i) 9 and 11

ii)

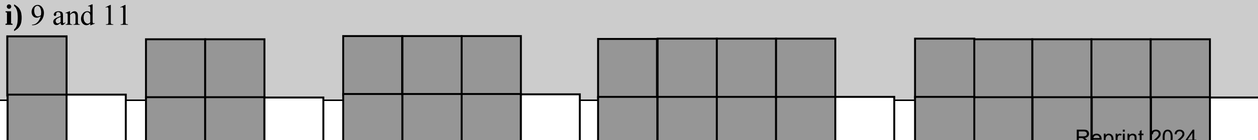


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

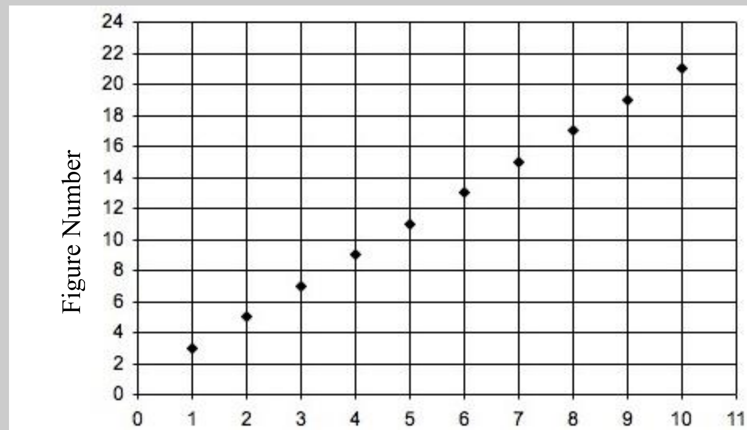
B. $t = 2f + 1$, or the number of tiles is one more than twice the figure number. t is the number of tiles and f is the figure number.

C. i)

Figure Number	Number of tiles
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	21

ii)

Tile Pattern



Number of tiles

iii) They form a straight diagonal line that goes up from left to right.

1. a) i)

Figure number	Number of squares
1	3
2	4
3	5
4	6
5	7
6	8

ii)

Geometric Pattern

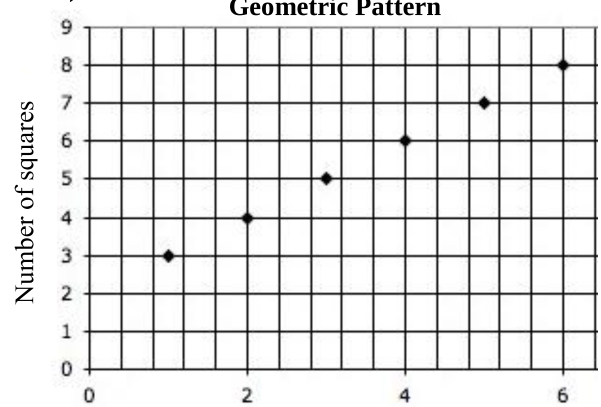


Figure number

b) i)

Figure number	Number of circles
1	5
2	8
3	11
4	14
5	17
6	20
7	23
8	26

ii) **Geometric Pattern**

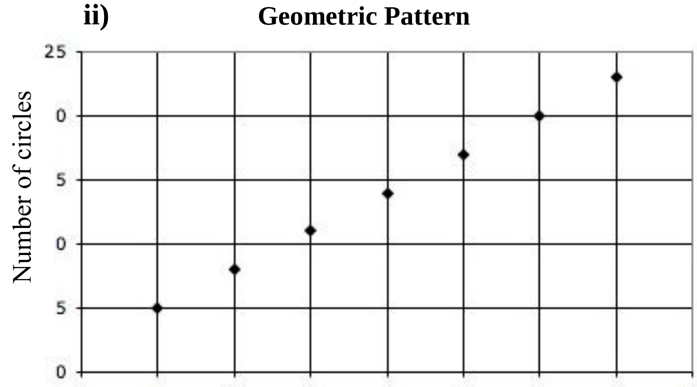


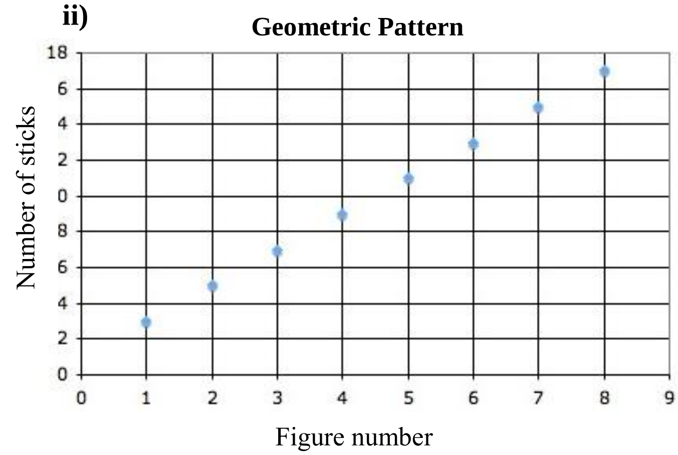
Figure number

Answers [Continued]

c) i)

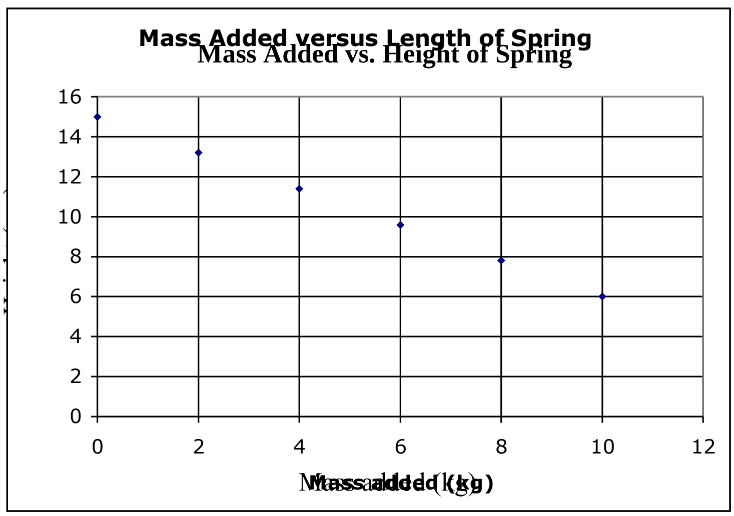
Figure number	Number of sticks
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17

ii)

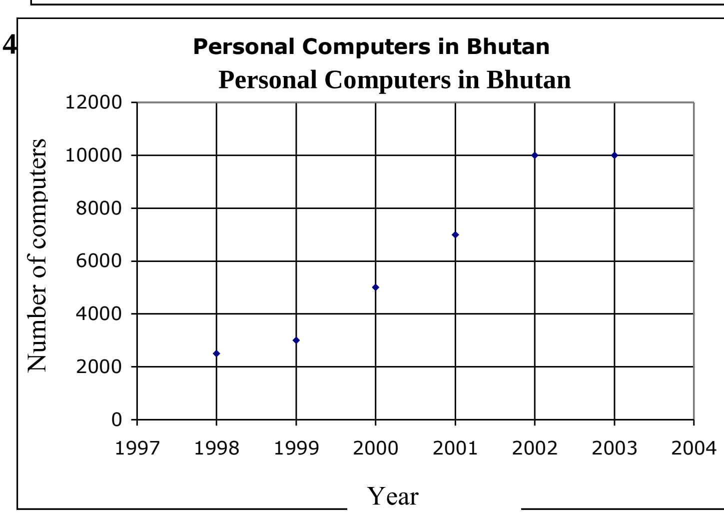


2. 5, 7, 11, 14; [I imagined a straight line passing through the plotted points and then used that line to find each missing point.]

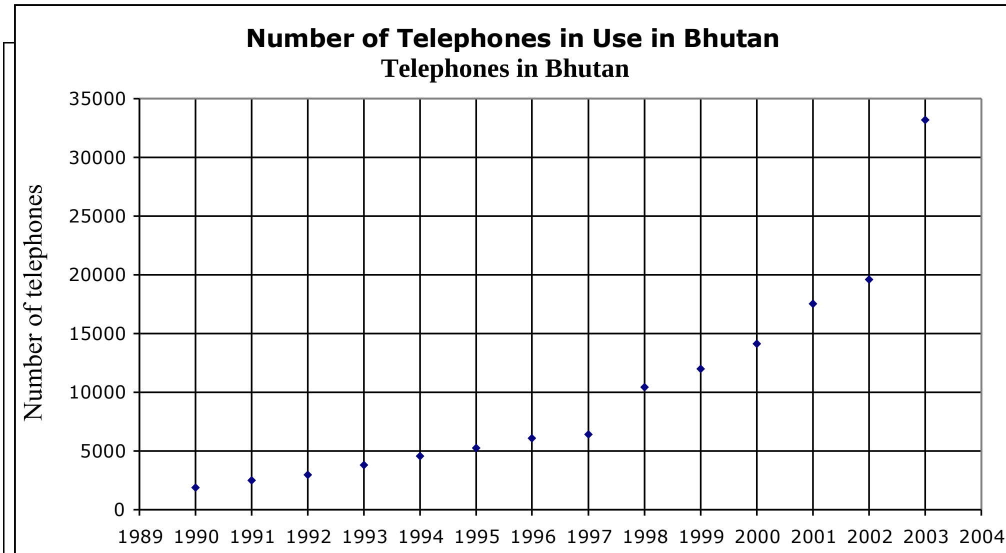
3.



4



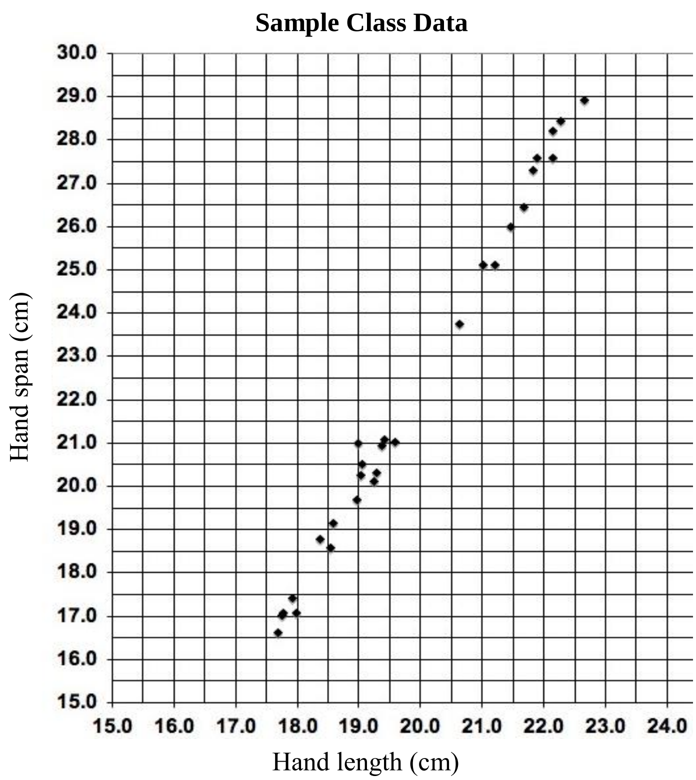
ii) *Sample response*:
The number of computers grew quickly for a few years but then stopped growing.



ii) *Sample response*: The number of telephones grew by an increasing amount each year. In the last few years, the change from one year to the next also increased.

5. A; [Because time is plotted along the horizontal axis, it shows the population as time goes by from left to right, so it is easier to see the trend.]

6. a) *Sample response*:



b) If hand length increases, so does hand span. The plotted points seem to fall closely along a straight line.

[**c)** I could first find the hand length on the *x*-axis. I would go up from there until I hit the “line”. Then I could look over to the *y*-axis from that point to see the hand span for that point.]

7. *Sample responses*:

• The data values in one row go together. Selecting data values from two different rows will give a false piece of information, for example, a year with the incorrect population.

• Each column in a table of values usually represents a specific type of data, for example, year (time) in one column and population in the other column. I must use the same coordinate for the same type data so that the data will be displayed accurately and the trend will be displayed correctly.

Supporting Students

Struggling students

• If students are struggling with setting up the axis scales for a scatter plot, you might provide them with a graph on which you have already drawn the axes and labelled the scales.

• Some students may continue to confuse the order of the coordinates of a point. Leave the explanation you provided for the exposition on the board and encourage them to refer to it as they work through the questions.

Enrichment

• You might encourage students to use the graph in **example 1** to predict the number of telephones per 100 people for the current year. They could then do some research to find the most recent data and compare it with their prediction.

UNIT 6 Revision

Pacing	Materials
2 h	• Probability devices such as spinners, coins, containers, dice, and playing cards (optional) • Rulers • Grid paper or Small Grid Paper (BLM) • Protractors and compasses

Question(s)	Related Lesson(s)
1 and 2	Lesson 6.1.1
3	Lesson 6.1.2
4	Lesson 6.2.2
5 and 6	Lesson 6.2.3
7 – 9	Lesson 6.2.4
10 and 11	Lesson 6.3.2

Revision Tips

Q 1 to 4: Some students might benefit from working with actual probability devices.

Q 1: Fraction answers do not have to be in lowest terms.

Q 3: Students can use any device where the probability of one outcome is $\frac{1}{2}$ to represent the first day.

Q 4: Encourage students to share answers so they will see that there are many possibilities.

Q 6: Remind students to label their graphs with the percents and to indicate in their work the angle sizes they used.

Q 7: Provide grid paper to help students construct the box plots.

Answers

1. a) $\frac{4}{52} = \frac{1}{13}$ **b)** $1 - \frac{1}{13} = \frac{12}{13}$ **c)** $\frac{36}{52} = \frac{9}{13}$

2. a) Drawing the white ball or one of three grey balls.

b) i) $\frac{4}{10}$ **ii)** $\frac{9}{10}$

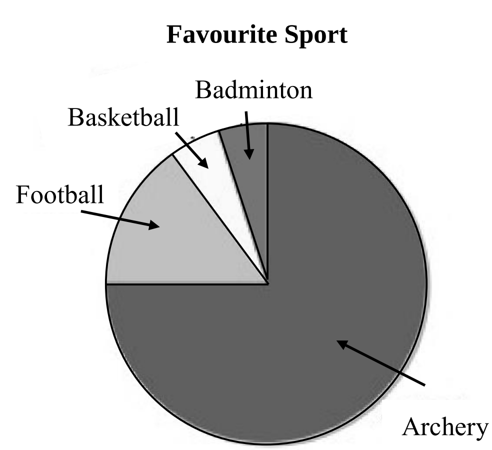
3. *Sample response*:
Use a coin, where one side represents "rain" and the other side represents "no rain". Also use a 3-sector spinner with one sector marked "rain". Flip the coin and spin the spinner and record whether both come out as "no rain". Repeat 20 times.
Record the fraction or percent of the 20 times that you got "no rain" for both the flip and the spin.

4. *Sample responses*:
a) Write the 100 student names on identical slips of paper. Put them in a hat, mix them up, and draw out
10 names at random.
b) A sample that consists of only girls or only boys.

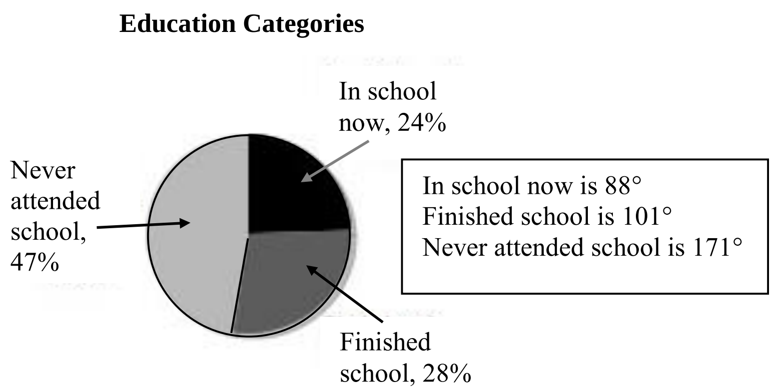
5. a)

Favourite Sport	Percent of students	Angle
Archery	75%	270°
Football	15%	54°
Basketball	5%	18°
Badminton	5%	18°

b)

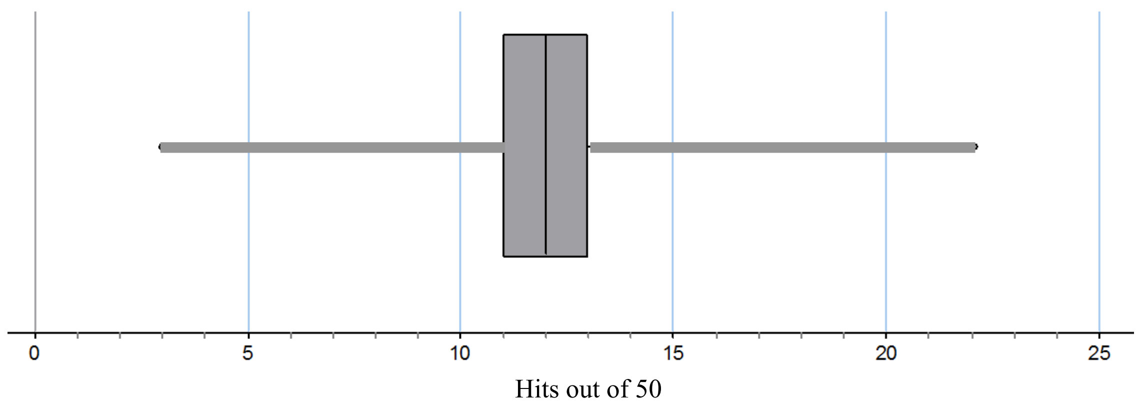


6.



7. a) Mean = 11.95, median = 12, range = 19

b)

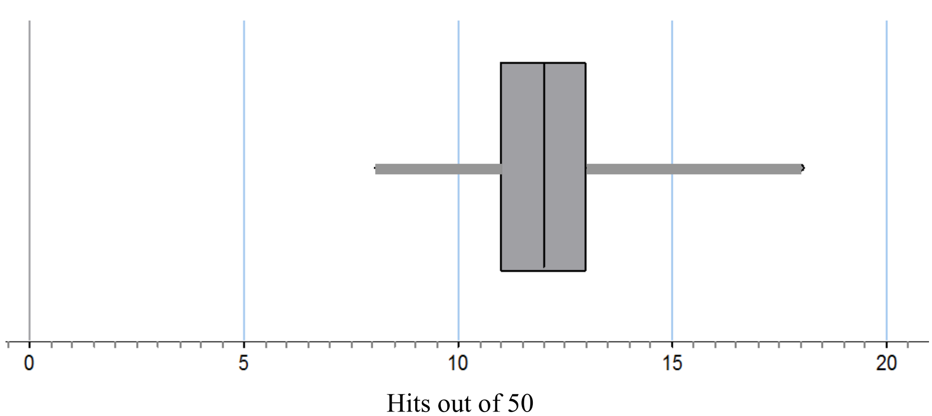


c) 3 and 22

d) The mean increases to about 12.4; the median does not change; the range decreases to 14.

e) The mean decreases to about 11.4; the median does not change; the range decreases to 15.

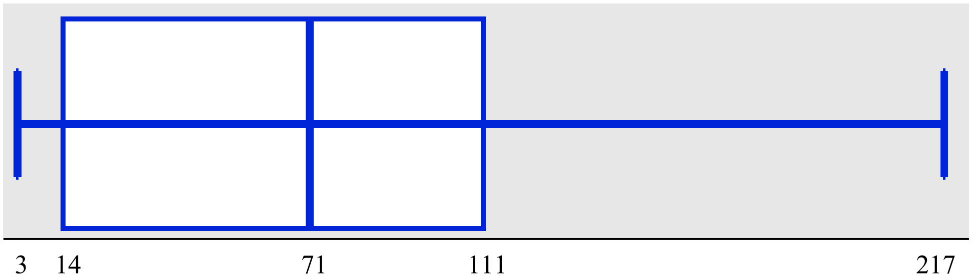
f) The whiskers would be shorter.



My prediction was correct.

8. Plot B; [Even though both box plots have the same range and same extreme values, I think a larger sample of 1000 would shows a median closer to 50 and quartiles close to 25 and 75, which are shown in plot B. This is what I would expect to find for a sample of random numbers between 0 and 100.]

9. a)

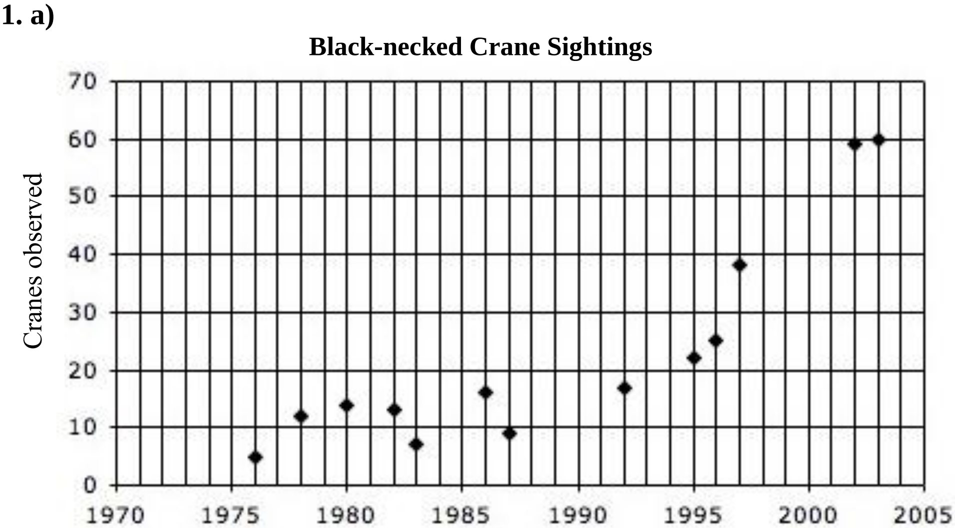


b) *Sample response*: The box plot shows that there is at least one month with very, very high rainfall compared to the median and that the high rainfall is much farther away from the median than the least rainfall amount is from the median.

10. a) The number was fairly constant until about 1987. Then it began to increase, with a large increase in 1996.

b) *Sample response*: 2

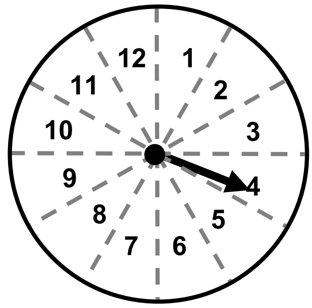
11. a)



b) The trends seem almost identical.

UNIT 6 Probability and Data Test

1. Determine the probability of each event for this spinner.



a) P(not a prime number)

b) P(not even)

2. a) For each event below, tell which of these two devices would you use to design a simulation model for estimating the theoretical probability. Explain your thinking.

Coin Die

i) The next two students to enter a classroom are boys. There are equal numbers of boys and girls in the school.

ii) It will rain for the next two days.

The probability of rain each day is $\frac{2}{3}$.

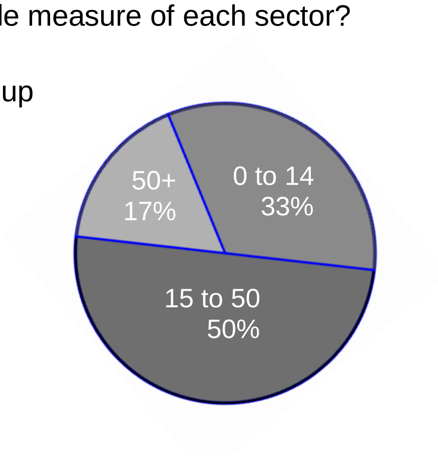
b) Choose **i) or ii)** from **part a)**. Design a simulation to estimate the probability of the event.

3. Mrs. Dorji wants to determine whether her 40 students have completed the work she assigned. She does not want to check the work of student. Describe one or more methods she can use to select a random sample of 5 students.

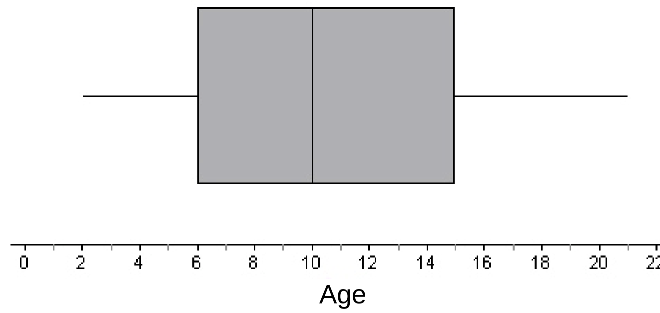
4. What is the angle measure of each sector? Show your work.

a) the 50+ age group sector

b) the 0 to 14 age group sector



5. This box and whisker plot shows the ages of children in a village.



Determine each.

a) the median age of the children

b) the greatest age

c) the range of all the ages

d) the range of the middle 50% of the ages

6. This chart shows the number of points a basketball team scored in 10 games.

a) Construct a box plot to show the distribution of the scores.

b) Describe how the data values are distributed.

Game	1	2	3	4	5
Points	40	45	38	35	50

Game	6	7	8	9	10
Points	60	53	49	50	52

7. Mr. Yeshi recorded how well each of his students did on the math examination and how long each said he or she had studied. Is there a relationship between the amount of time a student studied and his or her examination score? Use a scatter plot to help you explain.

Hours of study	Examination score
3	80
5	90
2	75
6	80
7	90
1	50
2	65
7	85
1	40
7	100

Reprint 2024

UNIT 6 Test

Pacing	Materials
1 h	• Grid paper or Small Grid Paper (BLM) • Rulers

Question(s)	Related Lesson(s)
1	Lesson 6.1.1
2	Lesson 6.1.2
3	Lesson 6.2.2
4	Lesson 6.2.3
5 and 6	Lesson 6.2.4
7 and 8	Lesson 1.2.1

Select questions to assign according to the time available.

Answers

1. a) $\frac{7}{12}$ **b)** $\frac{6}{12}$ or $\frac{1}{2}$

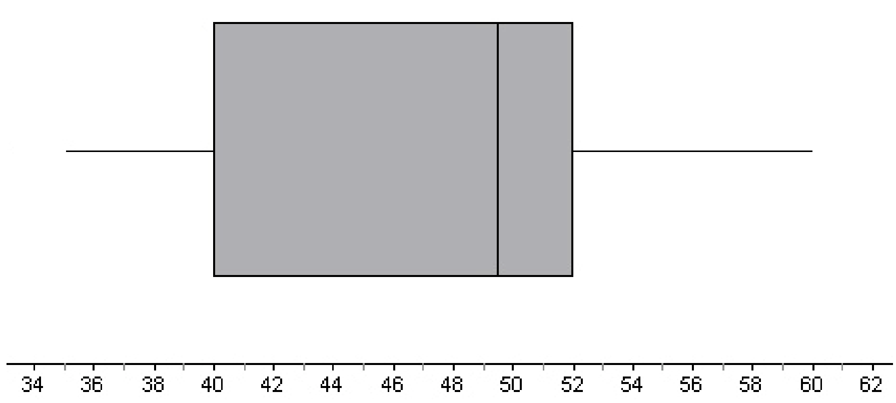
2. a) i) Coin; When I flip a coin, there are two equally likely outcomes. I need a device with two equally likely outcomes to model the probability that each student who enters is a boy. The die would also work if I used odds and evens or the numbers 1, 2, 3 and 4, 5, 6.
ii) Die; The die lets me model an event that happens 2 out of 3 times. I would let a roll of under 5 represent
the probability of rain each day and a roll of 5 or 6 represent no rain.
b) *Sample response*:
I chose **part i)**. I would flip a coin twice to represent two students entering the room. Flipping Khorlo would mean a boy entered and Tashi-gye would mean a girl entered. I would repeat this 100 times. I would keep track of the number of times I flipped Khorlo on both flips. That would be the percent probability of the next two students being boys.

3. *Sample responses*:
• She could put the names of all the students into a bag and randomly select 5 to check.
• She could put 5 black marbles and 35 white marbles into a bag and ask each student to select a marble. Once
a student has taken a marble, it is not returned to the bag. She would check the work of the students who drew the black marbles.

4. a) 61.2°; 17% of 360º = 0.17 × 360 = 61.2.
b) 118.8º; *Sample response*: 360º – 180º = 180º and 180º – 61.2º = 118.8º.

5. a) 10 years **b)** 21 years **c)** 19 years (from 2 to 21) **d)** 9 years (from 6 to 15)

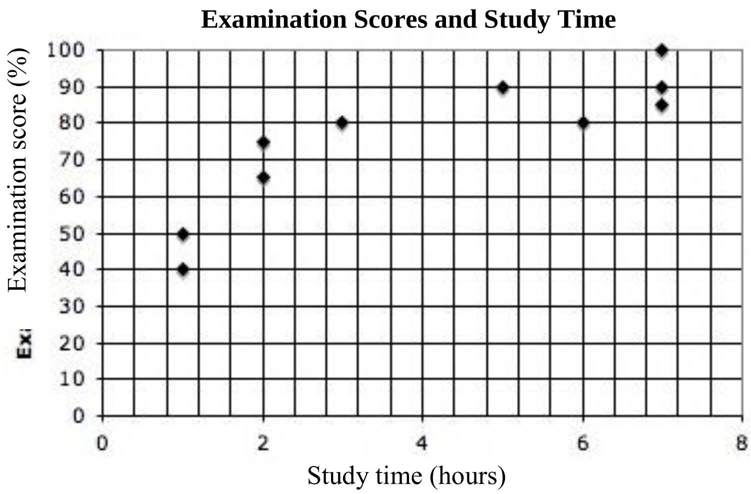
6. a)



Answers [Continued]

6. b) *Sample response*:
The scores range from 35 to 60, with 50% of the scores between 40 and 52. The median score is 49.5.
I can tell there are more high scores than low scores because the right whisker is longer and the median is in
the right side of the box.

7. Yes; *Sample response*:
The graph shows a trend in which the longer a student studied, the higher the score he or she got on the examination.



UNIT 6 Performance Task — Detecting Bird Flu

Duptho has a flock of 100 chickens. If 10% or more of the chickens have bird flu, the entire flock has to be destroyed.

A. Decide on a simulation model to represent a population of chickens where 11% are infected, so the flock should be destroyed. You need to choose a device to use and a sample size.

i) Describe your simulation.

ii) Conduct your simulation. Repeat the simulation 10 times to have 10 trials.

Sample Size = ?

Trial number	Number infected	Percent infected
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

B. Examine your data.

i) What is the probability that the sample you used was correct in predicting that the flock should be destroyed? Explain your thinking.

ii) Calculate the mean, median, and range of the percent data.

iii) Do your results match what you expected to find? Explain your thinking.

iv) Create a box plot for the percent data. What does the box plot show about the set of data?

C. Suppose you repeated the simulation using a larger sample size than you used in **part A**. Predict how these results would change:

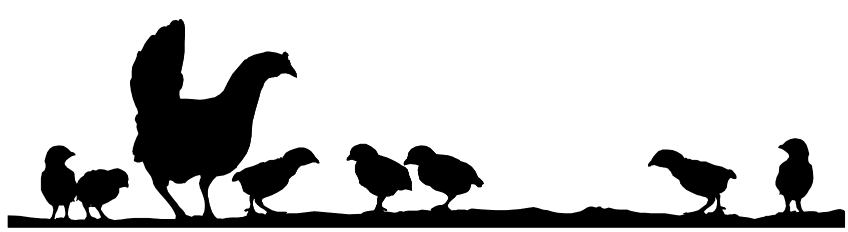
• the probability that the larger sample would be correct in predicting that the flock should be destroyed

• the mean, median, and range of the percent of chickens infected

• the shape of a box plot of the percent data

Explain your predictions.

D. Do you think the sample size you used in **part A** would be good enough to determine whether a flock should be destroyed? Explain your thinking.



UNIT 6 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
8-F1 Repeated Sampling (of Same Population): variability **8-F2** Randomness: concepts **8-F4** Box and Whisker Plots: construct and interpret **8-G2** Simulations and Experiments: single and complimentary events	1 h	• Rulers • Grid paper or Small Grid Paper (BLM) • Probability devices such as slips of paper and a bag

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could supplement the unit test. It could also be used as enrichment material for some students.

• You can assess performance on the task using the rubric provided on **page 245**.

Sample Solution

A. i) I will use 100 slips of paper. I will write the word INFECTED on 11 slips. I will put those 11 slips and the 89 blank slips into a bag. I will draw out 20 slips and note how many say INFECTED. I will record that number in the second column of my chart. I will repeat this 10 times altogether for 10 trials. I will then calculate the percent infected for each trial to complete the third column.

ii)

Sample size = 20		
Trial number	**Number infected**	**Percent infected**
1	3	15%
2	2	10%
3	2	10%
4	0	0%
5	2	10%
6	0	0%
7	1	5%
8	2	10%
9	4	20%
10	2	10%

B. i) $\frac{7}{10}$ or 70%; In 7 out of the 10 trials, 10% or more of the chickens were infected.

ii) Mean = 9%, median = 10%, range = 20% (0% to 20%)

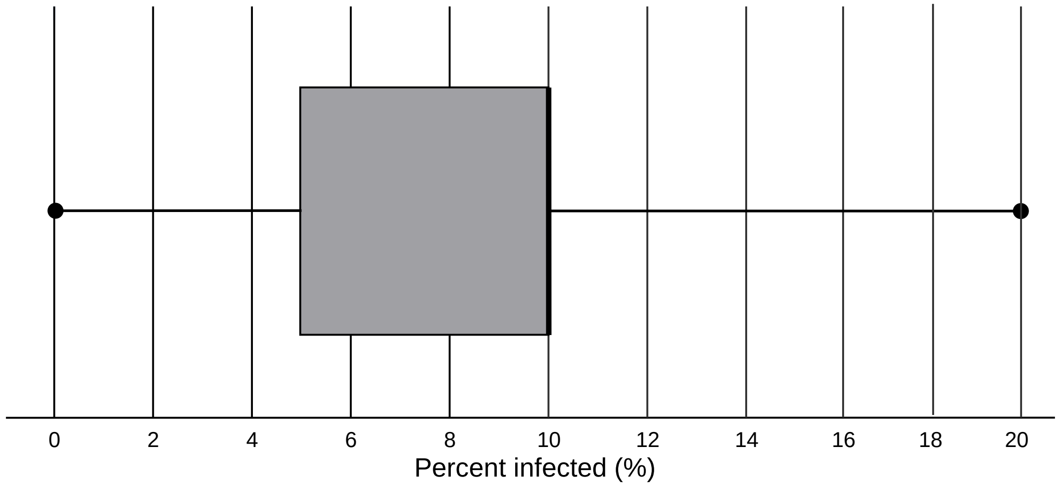
iii) No and yes;

No because I expected the probability to be higher than 70%. Being correct 70% of the time is not very accurate when it comes to making such an important decision.

Yes because I expected the mean and median to be close to 11% and they were. But if I used the mean (9%) to make a decision about destroying the flock, it would say not to destroy the flock when the flock should be destroyed because 11% were actually affected. If I used the median (10%) to make the decision, then the flock would be destroyed, as it should be.

iv)

Lower extreme	Lower quartile	Median	Upper quartile	Upper extreme
0	5	10	10	20



The box plot shows that the middle 50% of the data values are between 5% and 10%, with the median at 10%. That means there were a lot of data values that were either less than 10% or just 10%. Maybe that is why
the median and mode showed 10%, which indicate correctly that the flock should be destroyed, but the probability of the sample being correct was only 70%.

C. For a sample size of 30, I predict the following:
• The probability will be greater than 70%.
• The mean and median will still be close to 11%.
• The range will be less than 20%.
• The middle 50% of the values in the box plot will be on both sides of 11%.
Because the sample size is bigger, the data values in the results should be more accurate. The sample will be more like the actual population.

D. No; Even though the mean and median at 10% would be good predictors of whether the flock should be destroyed, a 70% probability of being correct of is not good enough to make such an important decision. Also, the range of 20%, from 0% to 20%, shows that sometimes the sample showed 0%, which means the flock should not be destroyed and sometimes it showed 20%, which means the flock should be destroyed. Those are highly unpredictable results.

UNIT 6 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Constructs a box and whisker plot	• Calculates median, quartiles, and extremes correctly • Places box ends, median indicator, and whiskers correctly; uses a scale that makes the plot easy to interpret	• Calculates median, quartiles, and extremes correctly • Places box ends, median indicator, and whiskers correctly; uses a reasonable scale	• Makes minor errors in the calculation of median, quartiles, and extremes • Makes minor errors in placement of box ends, median indicator; makes a poor choice for the scale	• Makes major errors in the calculation of median, quartiles, and extremes • Makes major errors in the placement of box ends, median indicator, and whiskers; makes major errors in the scale
Makes conclusions	• States a valid conclusion • Provides thorough, clear, and insightful justification for the conclusion • Makes appropriate reference to probability and statistics measures to support the conclusion	• States a valid conclusion • Provides reasonably complete, clear, and logical justification for the conclusion • Makes appropriate reference to probability and statistics measures to support the conclusion	• States a valid conclusion • Provides partial justification for the conclusion • Makes limited reference to probability and statistics measures to support the conclusion	• States an invalid conclusion • Provides incomplete or inappropriate justification for the conclusion • Makes limited and incorrect reference to probability and statistics measures to support the conclusion

Random Number Table

07804	14559	14133	13724	08887	05636
09450	07281	07926	11449	09748	13576
09379	13150	08987	13329	07215	09402
12315	10468	07256	13799	10638	09470
11408	14297	05945	11980	08055	12770
13854	10725	06720	14484	12212	08309
09777	12420	10909	14565	10032	07085
10088	13340	11700	06459	05581	10524
12739	14822	12680	08374	05454	06634
05040	13460	07218	14095	14231	10777
14294	09112	09015	13588	11556	14700
05617	14562	10277	08670	08221	12273
06550	14753	14075	13286	14686	12816
10385	13995	08207	11425	07941	05138
08314	05036	10712	10151	14621	12776
11882	08549	07131	13408	14225	08997
06754	11078	07011	06931	13373	07652
11138	11449	07792	06540	14107	07568
05151	12187	06779	06588	05502	10449
12725	12502	14903	05357	08517	09531
10336	11289	07164	08258	05328	14568
13540	13930	11323	07868	08026	11835
09693	09024	08256	08667	10533	06665
10381	14839	11960	06790	07076	14208
08042	05107	05640	05963	14095	09730
05082	08436	11326	11626	08661	08331
08283	11243	12789	11544	10599	06539
13079	08918	07976	10731	09266	07815
08099	13842	10050	10819	07604	11419
14611	07254	12393	05114	06203	05963
07030	08238	13117	07398	05000	09377
05729	07045	05531	09439	14755	09546
10337	06658	09799	07341	14267	06608
12921	11094	14339	08221	11999	10091
13814	13177	06345	08233	09082	07341

07030	08238	13117	07398	05000	09377
05729	07045	05531	09439	14755	09546
10337	06658	09799	07341	14267	06608
12921	11094	14339	08221	11999	10091
13814	13177	06345	08233	09082	07341

UNIT 7 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 183** **TG p. 252**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• Grid paper or Small Grid Paper (BLM)	All questions
Chapter 1 Describing Relationships				
7.1.1 EXPLORE: Representing Relationships (Optional) **SB p. 185** **TG p. 255**	**8-C1 Patterns and Relations: represent in a variety of formats** • move interchangeably among a variety of formats which describe relationships • describe in words, and use expressions and equations to represent patterns given in tables, graphs, charts, pictures and/or problems situations • investigate linear situations and those which create a regular pattern • predict unknown values once an algebraic description of a pattern is established	1 h	• Grid paper or Small Grid Paper (BLM)	Observe and Assess questions
7.1.2 Describing Relationships and Patterns **SB p. 186** **TG p. 257**	**8-C1 Patterns and Relations: represent in a variety of formats** • move interchangeably among a variety of formats which describe relationships • describe in words, and use expressions and equations to represent patterns given in tables, graphs, charts, pictures and/or problems situations • use information presented in a variety of formats to derive mathematical expressions and predict unknown values • investigate linear situations and those which create a regular pattern • predict unknown values once an algebraic description of a pattern is established	1 h	• Grid paper or Small Grid Paper (BLM)	Q3, 5, 7
7.1.3 Recognizing Linear Relationships **SB p. 191** **TG p. 262**	**8-C1 Patterns and Relations: represent in a variety of formats** • investigate linear situations and those which create a regular pattern **8-C2 Graphs (Linear and Non-linear): interpret** • understand, when looking at tabular data, that when an equal spacing between the values of one variable produces an equal spacing between values of another variable, the relationship is linear • use information from tables, graphs, or algebraic expressions to describe change • match situations to corresponding graphs • sketch graphs for a variety of situations, leading to linear and broken-line graphs • understand that a variety of representations may be used to show relationships and that choices are available	1 h	• Grid paper or Small Grid Paper (BLM) (optional)	Q1, 4, 7

UNIT 7 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
CONNECTIONS: Adding Values in a Linear Relationship (Optional) **SB p. 196** **TG p. 265**	An opportunity to use patterns in a linear relationship to solve number problems	15 min	None	N/A
7.1.4 Slope **SB p. 197** **TG p. 266**	**8-C3 Graphs and Tables (linear and non-linear): how changing one quantity affects the other** • use information from tables, graphs, or equations to investigate the impact of changing related quantities • explore patterns associated with parameter changes in a linear equation (e.g., understand how changes in the equation affect the slant of the graph) **8-C4 Slope: link visual characteristics with numerical values** • understand that, for linear relationships, the ratio of vertical change to horizontal change is constant anywhere along the line • use the terms rise and run to describe vertical and horizontal change in a line graph • investigate practical situations: slope of a staircase, slope of a roof, and the steepness of roads • determine the slope of a line • understand that ratios for a graph that rises to the right are positive • understand that ratios for a graph that rises to the left are negative	1 h	• Grid paper or Small Grid Paper (BLM)	Q1, 2, 5, 9
Chapter 2 Solving Linear Equations				
7.2.1 Solving an Equation Using Inverse Operations **SB p. 203** **TG p. 270**	**8-C5 Single Variable Equations: solve algebraically** • use prior knowledge developed through concrete experiences to transfer to symbolic representation of single variable equations • solve one- and two-step equations symbolically using integer and simple fraction coefficients • use the "balance method" to solve problems	1 h	None	Q2, 3, 5, 7
7.2.2 Using an Equation to Solve a Problem **SB p. 207** **TG p. 273**	**8-C6 Linear Equations: create and solve problems** • create and solve relevant problems for which algebraic solutions are required • justify strategies used to create and solve problems • appreciate the use of an algebraic equation in problems involving large numbers (as opposed to a guess and check approach)	1 h	None	Q1, 4, 7, 8

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
7.2.3 Solving a Problem Involving Two Relationships **SB p. 210** **TG p. 276**	**8-C7 Intersection of Two Lines: solve problems** • compare tables of values, equations, or verbal descriptions of two linear situations to identify where lines will intersect • use tables of values to generate ordered pairs for each equation and identify coordinates for points of intersection	40 min	• Grid paper or Small Grid Paper (BLM)	Q1, 4, 6
GAME: Alge-Scrabble (Optional) **SB p. 213** **TG p. 278**	Practise solving equations in a game setting	30 min	• Alge-Scrabble Game Board (BLM 2A) • Alge-Scrabble Game Tiles (BLM 2B) • Scissors	N/A
Chapter 3 Linear Polynomials				
7.3.1 Adding Polynomials **SB p. 214** **TG p. 279**	**8-B15 Add and Subtract simple algebraic terms: to solve problems** • establish a parallel between a measurement situation and a variable situation (e.g., for 3 m + 0.2 m, 3 m and 20 cm need to be "like terms" before you can add or subtract) • add simple expressions with concrete materials such as algebra tiles (know which like terms can and cannot be combined) **8-B16 Polynomial Expressions: Add and subtract visually** • use concrete materials such as algebra tiles for conceptual development	1 h	• Algebra tiles or Algebra Tiles (BLM)	Q2, 6, 7, 9
7.3.2 Subtracting Polynomials **SB p. 217** **TG p. 282**	**8-B15 Add and Subtract Simple Algebraic Terms: solve problems** • subtract simple expressions with concrete materials such as algebra tiles (know which like terms can and cannot be combined) **8-B16 Polynomial Expressions: add and subtract visually** • use concrete materials such as algebra tiles for conceptual development • for subtraction, consider different representations of subtraction, including the following: - comparison (which refers to comparing and finding the difference between two quantities) - taking away (which refers to starting with a quantity and removing a specific amount - adding the opposites (which refers to subtracting by first changing the question to an addition and then adding the opposite of a quantity (e.g., subtracting x instead of $-x$) - missing addend (What would be added to the number being subtracted to get the starting amount? (e.g., for $(3x - 2) - (2x + 1)$, what is added to $2x + 1$ to get $3x - 2$?)	1 h	• Algebra tiles or Algebra Tiles (BLM)	Q4, 5, 6

UNIT 7 PLANNING CHART [Continued]

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
7.3.3 EXPLORE: Multiplying a Polynomial by an Integer (Essential) **SB p. 220** **TG p. 285**	**8-B17 Multiplication by a Scalar (Polynomials): visually and symbolically** • develop with concrete materials and diagrams using repeated addition (e.g., for $3(2x + 1) = 2x + 1 + 2x + 1 + 2x + 1$, model the binomial three times and combine the like terms using algebra tiles) • explore the area model to associate repeated multiplication	40 min	• Algebra tiles or Algebra Tiles (BLM)	Observe and Assess questions
UNIT 7 Revision **SB p. 221** **TG p. 287**	Review the concepts and skills in the unit	2 h	• Grid paper or Small Grid Paper (BLM) • Algebra tiles or Algebra Tiles (BLM)	All questions
UNIT 7 Test **TG p. 292**	Assess the concepts and skills in the unit	1 h	• Grid paper or Small Grid Paper (BLM) • Algebra tiles or Algebra Tiles (BLM)	All questions
UNIT 7 Performance Task **TG p. 295**	Assess concepts and skills in the unit	1 h	• Grid paper or Small Grid Paper (BLM) • Algebra tiles or Algebra Tiles (BLM) (optional)	Rubric provided
UNIT 7 Blackline Masters **TG p. 298**	BLM 1 Algebra Tiles (paper models) BLM 2A Alge-Scrabble Game Board BLM 2B Alge-Scrabble Game Tiles Small Grid Paper on **page 32** in UNIT 1			

Math Background

• This algebra unit focuses on linear relationships, whether algebraic, graphical, or using polynomial models, as well as on modelling and solving problems based on linear relationships. It is a bridge between the pattern work students did in Class VII and the more formal algebra that they will meet in Class IX.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Students use problem solving in **question 2** in **lesson 7.2.2**, where they solve problems they have modelled algebraically, in the **Try This** in **lesson 7.2.3**, where they solve a problem about ages, and in **question 5** in **lesson 7.3.1**, where they create a polynomial addition that meets specific conditions.

• Students use communication in **question 11** in **lesson 7.1.4**, where they explain slope, in **question 8** in **lesson 7.2.2**, where they talk about why the same equation can model different problems or the same problem can be modelled by different equations, and in **question 9** in **lesson 7.3.1**, where they describe how to use the zero property in adding polynomials.

• Students use reasoning in **question 5** in **lesson 7.1.2**, where they decide which algebraic expression describes a pattern, in **lesson 7.1.3**, where they decide whether or not relationships are linear, in **question 5** in **lesson 7.1.4**, when they reason about which line will be steeper, in **question 8** in **lesson 7.1.4**, where they reason about what points can be on a line with a given slope, and in **question 8** in **lesson 7.2.1**, where they reason about why a particular solution method does not work.

• Students consider representation in **part C** in **lesson 7.1.1**, where they use a graph to decide why a certain situation is not possible, in **question 8** in **lesson 7.1.2**, where they decide which representation of a situation is easiest for them, and throughout **Chapter 3**, where they use algebra tiles to represent algebraic expressions.

• Students use visualization in **question 2** of **lesson 7.1.2**, where they use a graph to predict a value in a pattern, in **question 8** in **lesson 7.1.3**, where they see the difference between linear and non-linear relationships that have some features in common, and in **question 3** in **lesson 7.1.4**, where they draw lines with particular slopes.

• Students make connections between graphical and algebraic representations of information in **lesson 7.1.2**, between measurement concepts and algebraic concepts in **question 4** in **lesson 7.1.3**, and between verbal descriptions of situations and their graph in **question 7** in **lesson 7.1.4**. They also make connections in **question 3** in **lesson 7.1.1**, where they recognize how different equations can have the same solution, in **lesson 7.2.3**, where they connect the common solution of two equations to the single equation that represents both situations, and in **question 8** in **lesson 7.3.1**, where they connect measurement with using like terms of polynomials.

Rationale for Teaching Approach

• This unit is divided into three chapters.

Chapter 1 focuses on describing relationships between quantities using both graphs and algebraic expressions. There is a focus on linear relationships and an explanation of how the slope of a line relates to a description of the relationship.

Chapter 2 focuses on methods for solving one or more linear equations, both connected to and not connected to solving real-world problems.

Chapter 3 introduces students to adding and subtracting linear polynomials and multiplying linear polynomials by integers.

• The first of two **Explore** lessons provides students with an informal opportunity to see how a linear relationship can be described algebraically or with a graph. The second **Explore** lesson introduces students to multiplication of a polynomial by an integer. This topic is handled as an exploration because of the nature of the content.

• The **Connections** has historical information about how a famous German mathematician solved a problem at a very young age. Students get a chance to use the insightful approach Gauss developed.

• The unit's **Game** allows students to practise solving equations.

Getting Started

Curriculum Outcomes	Outcome relevance
7 Summarize Patterns: make predictions **7** Single Variable Linear Equations: represent solutions **7** Single Variable Linear Equations: one and two step **7** Linear Equations: graph using table of values **7** Graphs: linear and non-linear	Students will find the work in the unit easier after they review what they already know about describing patterns using algebraic expressions, graphing algebraic relationships, and solving linear equations in a variety of ways.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM)	• interpreting algebraic expressions • graphing algebraic relationships on a coordinate grid • completing tables of values that show relationships • adding and subtracting integers and simple polynomials • writing an equation to describe a simple linear relationship • solving a simple linear equation • familiarity with the terms *coefficient* and *constant*

Main Points to be Raised

Use What You Know

- You can use a table of values to look for a pattern.
- You can use an algebraic expression to describe a pattern rule.
- By graphing an algebraic relationship, you can extend a pattern.

Skills You Will Need

- A table of values is a good way to see a pattern.
- You can use a table of values or a graph to help solve an equation.
- You can use integer operations to help simplify a linear algebraic expression.
- You can use a pattern rule by substituting values to find unknown entries in the table.
- You can solve a simple linear equation using a variety of methods.

Use What You Know — Introducing the Unit

Suggest that students work alone or in pairs. While you observe students at work, you might ask questions such as the following:

- *What did you notice about the numbers in the second column?* (The number of squares increased by 2 each time.)
- *How could you use that to figure out the number of white squares for 15 grey squares?* (I could extend the table to 15 rows and add 2 to the number in the second column each time.)
- *How did you decide which expression to use?* (I substituted the number 1 for g and tested all of the expressions. The only expressions that worked were $w = 8g$ and $w = 6 + 2g$. Then I substituted $g = 2$ to see which of those expressions worked.)
- *How did you decide on a scale to use on the y-axis*? (I estimated that if it was going up by 2 each time, I would have to get past 30 for the 15 grey squares, so I used a scale of 5 for one square.)

Skills You Will Need

- To ensure students have the required skills for this unit, assign these questions.
- You may wish to review the idea of simplifying in preparation for **question 3**.

For example, you might ask students why $3n$ is a simplified version of $n + 2n$.

- You may also wish to review the methods for solving equations students learned in Class VII. The methods include guess and test, using a model where rectangular tiles represent the variable and small square tiles represent 1 or –1, using a diagram where small rectangles represent the variable, and jumps on a number line.
- Students can work individually.

Reprint 2024

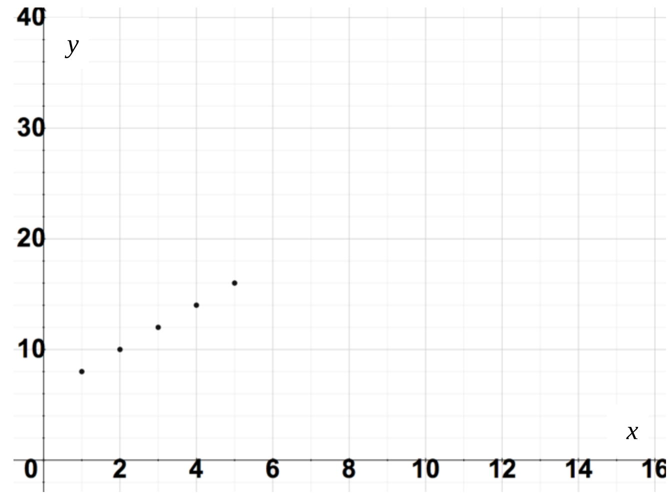
Answers

A.

Number of grey squares	Number of white squares
1	8
2	10
3	**12**
4	**14**
5	**16**

B. i) 36; *Sample response*:
I noticed that I added 2 white squares each time there was 1 more grey square. So when there are 15 grey squares, I have added 2 fourteen times starting at 8, which is 28 + 8 = 36.
ii) $w = 6 + 2g$; *Sample response*:
I substituted different values for g from the table. The results all matched the corresponding values for w.
iii) $6 + 2 \times 15 = 6 + 30 = 36$

C. i)



ii) The points form a line
iii) If I extend the line, the y-coordinate is 36 when the
x-coordinate is 15.

1. a) i)

x	y
1	8
2	13
3	**18**
4	**23**
5	28

ii)

x	y
1	18
2	**16**
3	**14**
4	12
5	**10**

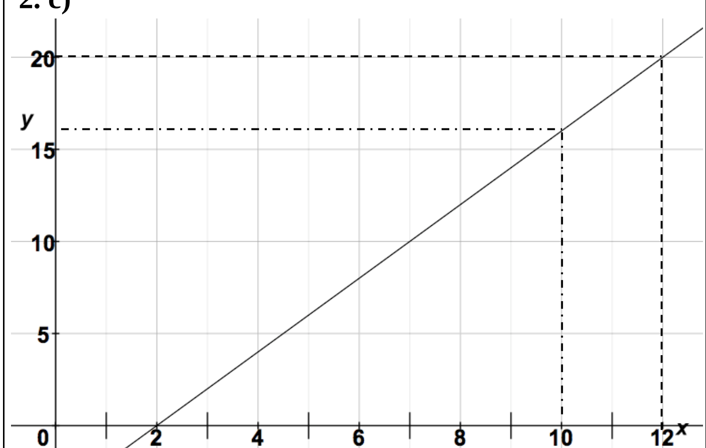
b) i) $y = 5x + 3$ **ii)** $y = 20 - 2x$

2. a)

x	y
1	– 2
2	0
3	2
4	4
5	6

b) i) $y = 2x - 4$ so $y = 2 \times 10 - 4 = 16$
ii) $20 = 2x - 4$, so $x = 12$

2. c)



[**i)** Find the x-value of 10 on the graph and then look across to see what the value of y is at that point.
ii) Find the y-value of 20 on the graph and then look down to see what the value of x is at that point.]

3. a) i) $7n - 3$ **ii)** $-n - 10$
b) i) $7 \times 8 - 3 = 56 - 3 = 53$ **ii)** $-8 - 10 = -18$

4. a) $90 = 4(f + 1) - 2$
b) $f = 22$; [*Sample response*:
Guess and test:
If $f = 10$, then $4(f + 1) - 2 = 42$. Too low
If $f = 20$, then $4(f + 1) - 2 = 82$. Close, but low
If $f = 24$, then $4(f + 1) - 2 = 98$. Too high
If $f = 22$, then $4(f + 1) - 2 = 90$. The solution is Figure 22.

Answers [Continued]

Rectangle model:

<table>
<tr><td>f</td><td>1</td><td>f</td><td>1</td><td>f</td><td>1</td><td>f</td><td>1</td></tr>
<tr><td colspan="7">90</td><td>2</td></tr>
</table>

Rearrange:

<table>
<tr><td>f</td><td>f</td><td>f</td><td>f</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td colspan="6">90</td><td colspan="2">2</td></tr>
</table>

$4f = 90 - 2$
$4f = 88$
$f = 22$]

5. a) Coefficient = 2, constant = –3
b) Coefficient = 3, constant = 4
c) Coefficient = –3, constant = 6

Supporting Students

Struggling students

• If students are struggling to solve equations, go through some of the procedures that they learned in Class VII. Emphasize pictorial and concrete models first.

• If students are struggling with creating an equation to describe a pattern rule, start with some simpler examples and work toward more difficult examples.

For example, consider these patterns first. In each case, it is easy to see how the *y*-value relates to the *x*-value.

x	*y*
1	2
2	3
3	4
4	5
5	6

x	*y*
1	8
2	16
3	24
4	32
5	40

Then extend the tables to tables that are similar, but slightly more complicated, such as those below. Relate them back to the simpler examples.

x	*y*
1	3
2	4
3	5
4	6
5	7

x	*y*
1	10
2	18
3	26
4	34
5	42

Enrichment

• Some students might enjoy creating patterns like those in the **Use What You Know** activity or in **question 4** and challenging other students to figure out the pattern rules.

Chapter 1 Describing Relationships

7.1.1 EXPLORE: Representing Relationships

Curriculum Outcomes	Lesson Relevance
8-C1 Patterns and Relations: represent in a variety of formats • move interchangeably among a variety of formats which describe relationships • describe in words, and use expressions and equations to represent patterns given in tables, graphs, charts, pictures and/or problems situations • investigate linear situations and those which create a regular pattern • predict unknown values once an algebraic description of a pattern is established	This optional exploration sets the stage for additional work in **lesson 7.1.2** on relating two variables. It allows students to discover relationships in their own way before being shown more formal methods.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM)	• using a table of values to represent data • graphing on a coordinate grid • using algebraic equations and expressions to describe and relate amounts • solving simple equations

Main Points to be Raised

- You can use a graph, a table of values, or an equation to represent the relationship between two related variables.
- You can use your graph, table of values, or equation to deduce information about one variable by using information about the other variable.

Exploration

• Read through the introduction (in white) with the students. Make sure they understand the situation by asking, for example, why you might write 50 and 0 in the first column.

Assign students to work in pairs on **parts A to F**. While you observe students at work, you might ask questions such as the following:

• *Why must the number of Nu 2 coins be 50 or fewer? How do you know?* (50 coins is already Nu 100. If there were any more Nu 2 coins, there would be too much money.)

• *Could there be exactly one Nu 5 note? How do you know?* (No. That would leave Nu 95. I cannot show that amount using only Nu 2 coins because 95 is odd and any value using only Nu 2 coins is even.)

• *Could there be exactly two Nu 5 notes? How do you know?* (Yes. There could be 45 Nu 2 coins to make Nu 90 and two Nu 5 notes to make a total of Nu 100.)

• *How does the graph show that there can be two Nu 5 notes, but not one Nu 5 note?* (The y-coordinate is 45 for an x-coordinate of 2, but the y-coordinate is 45.5 for an x-coordinate of 1. There cannot be half a coin.)

• *Why does 2t tell the value of t Nu 2 coins*? (Each coin counts for Nu 2, so the value is twice as much as the number of coins, which is represented by t.)

• *Why does the equation* $2t + 5f = 100$ *describe the problem?* (The equation says there are so many Nu 2 amounts and so many Nu 5 amounts to add to Nu 100. That is what the problem asks for.)

Observe and Assess

As students work, notice the following:

• Do they use the table of values correctly?

• Do they graph the information correctly? Do they use reasonable scales on their graphs?

• Can they interpret the graph to answer **part C**?

• Do they use either the table or graph to figure out **part D**?

• Do they use algebraic expressions and equations correctly to describe the problem?

• Do they solve their equation correctly and interpret the solution by using the graph?

Share and Reflect

After students have had time to work through the exploration, discuss their observations and these questions.

- *How did the number of Nu 2 coins change when the number of Nu 5 notes increased by 2? Why?*
- *How did the number of Nu 5 notes change when the number of Nu 2 coins increased by 5? Why?*
- *Why was 20 the maximum number of Nu 5 notes? What was the maximum number of Nu 2 coins? Why were the two maximum numbers different?*
- *Was it easier for you to see the relationships and all possible solutions with the graph, the table, or the equation? Why?*

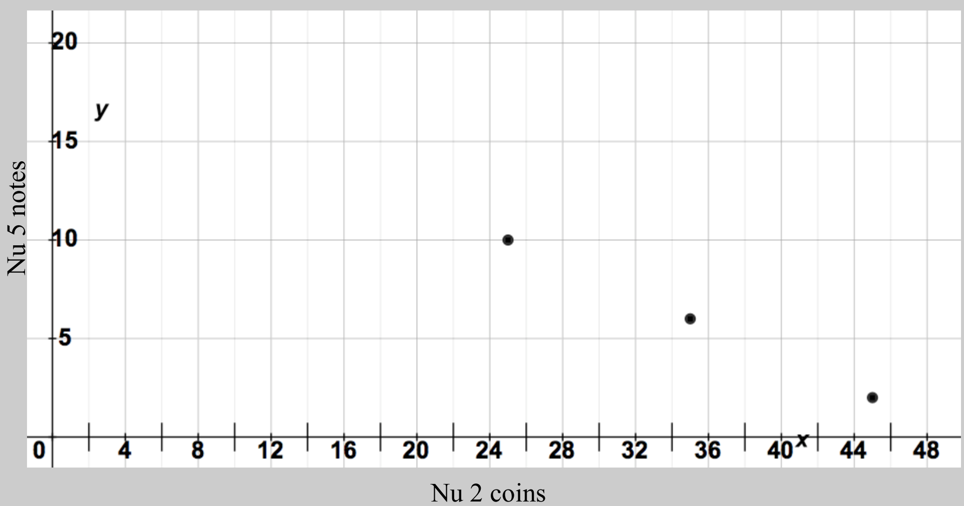
Answers

A. *Sample response*:

Combinations With a Total Value of Nu 100

Number of Nu 2 coins	45	35	25
Number of Nu 5 notes	2	6	10

B. *Sample response*:



C. *Sample responses*:
i) When the y-coordinate is 5 (meaning five Nu 5 notes), the x-coordinate (the number of Nu 2 coins) is between 37 and 38, so it is not a whole number. It is not possible to have part of a coin.
ii) Five Nu 5 notes would be worth Nu 25, which leaves Nu 75 to be in Nu 2 coins, which is impossible since 75 is an odd number.

D. 5 fewer coins;
Sample response:
Two more Nu 5 notes means a value of Nu 10 more, so there would have to be Nu 10 less in coins. Since each coin is Nu 2, that would be 5 fewer coins.

E. i) $2t$ **ii)** $5f$ **iii)** $2t + 5f = 100$
iv) If $t = 25$, then $50 + 5f = 100$, so $5f = 50$ and $f = 10$; The solution is at (25, 10).

F. *Sample responses*:
i)

Number of Nu 2 coins	50	45	40	35	30	25	20	15	10	5	0
Number of Nu 5 notes	0	2	4	6	8	10	12	14	16	18	20

ii) I used a table because it was easier for me to see the pattern that way.

Supporting Students

Struggling students

- Some students may be able to use the table and graph, but they may have trouble with the reasoning required in **part D**. These issues should become clearer to these students as other students discuss their solutions with the class.
- Some students may have difficulty writing the equation in **part E**. Remind them that the goal is to translate the words into algebra. The equation involves adding two values to get 100. The two values are the values of the coins and notes. They should represent each of those amounts algebraically and then add them.

Enrichment

- You might challenge students to consider how the solutions to the problem would change if Nu 2 coins and Nu 10 notes were used instead and/or the total value was Nu 200.

7.1.2 Describing Relationships and Patterns

Curriculum Outcomes	Outcome relevance
8-C1 Patterns and Relations: represent in a variety of formats • move interchangeably among a variety of formats which describe relationships • describe in words, and use expressions and equations to represent patterns given in tables, graphs, charts, pictures and/or problems situations • use information presented in a variety of formats to derive mathematical expressions and predict unknown values • investigate linear situations and those which create a regular pattern • predict unknown values once an algebraic description of a pattern is established	The ability to describe patterns and relationships algebraically is an important foundation for work in higher classes of mathematics. Algebraic descriptions are used because of their efficiency in capturing a lot of information in very few symbols.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM)	• using a table of values to describe data • graphing on a coordinate grid • using algebraic equations and expressions to relate amounts

Main Points to be Raised

- You can represent a pattern or relationship using a table of values, a graph, or an algebraic equation in order to solve a problem related to the pattern or relationship.
- To create the graph of a pattern or relationship, you can use the figure number as the x-coordinate and the term value (e.g., number of dots) as the y-coordinate.
- In a number pattern, there is a relationship between the term or figure number and the value of the term.
- When a pattern increases by a given amount, it makes sense to compare the values in the pattern to the values in the multiplication table for the increase amount. This is helpful for writing an algebraic equation to describe the pattern.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

- *How are the first two figures different?* (The second figure has 11 dots. The first figure has only 8 dots.)
- *What is the pattern?* (You add 3 more dots each time.)
- *How can you predict the number of dots in the 20th figure?* (I could add 3 nineteen times, but it is easier to think 3 × 19 = 57 and add 57 to the 8 dots in Figure 1.)

The Exposition — Presenting the Main Ideas

- Draw this pattern on the board.

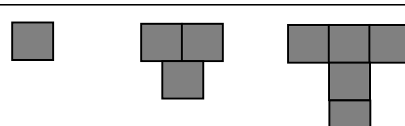


Figure number	Number of squares
1	1
2	3
3	
4	

Then create a related table of values like the table to the right.

- Ask students to tell you what values should be written in the empty cells and why they chose those values.
- Show students how to graph the relationship.
- Discuss why the equation $y = 2x - 1$ describes the relationship between x and y.
- Have students read the exposition on **pages 186 and 187** of the student text.
- Point out how you could have compared the values in the table (the Number of squares) to multiples of 2 to see why the equation is $y = 2x - 1$, since the number of squares goes up by 2 but each value is 1 less than the corresponding multiple of 2,.

Revisiting the Try This

B. and C. These questions allow students to re-examine their work in **part A** in a more thorough way.

Using the Examples

• Assign students to pairs to work through the exercises. One in the pair should become an expert on **example 1**, while the other should become an expert on **example 2**. Once each has become comfortable with his or her own example, they can teach their partner what they learned.

Practising and Applying

Teaching points and tips

Q 1: Remind students to compare the term values to the multiples of the increase or decrease in the patterns.

Q 2: Students do not have to use both the equation and the graph; either method is acceptable.

Q 6: Encourage students to consider how many dots are added to go from one figure to the next.

Q 8: This question has no single correct answer. Students can choose what they prefer, as long as they can support their choices with a suitable explanation.

Common errors

• Many students have difficulty with relationships for decreasing patterns. Even if they realize that there should be a negative coefficient for *x*, they often do not know what to do to get the rest of the expression. Provide a variety of examples so that students can refer to them.

Suggested assessment questions from Practising and Applying

Question 3	to see if students can observe a relationship from a table of values, describe it algebraically, and graph it
Question 5	to see if students can determine whether a given algebraic relationship describes a situation
Question 7	to see if students can create a table of values to describe a pattern, describe the pattern algebraically, and use the algebraic description to predict other values

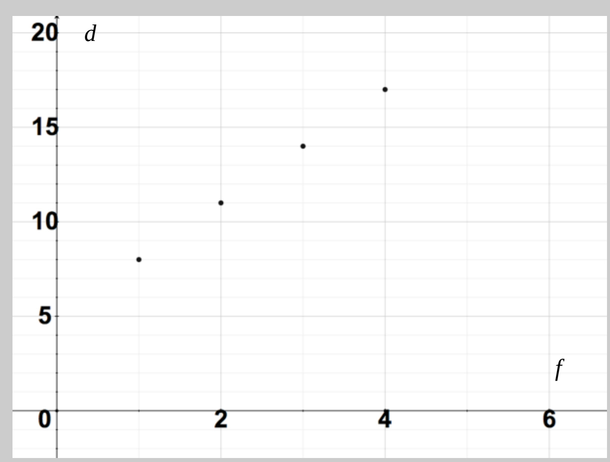
Answers

A. i) Each figure has 3 more dots: 8, 11, 14, …
ii) Figure 4 has 17 dots.
Figure 12 has 41 dots.
Figure 20 has 65 dots.

B. i)

Figure number	Number of dots
1	8
2	11
3	14
4	17

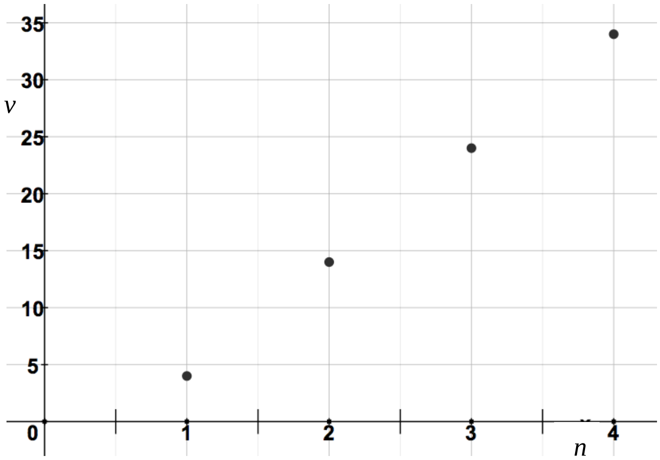
ii)



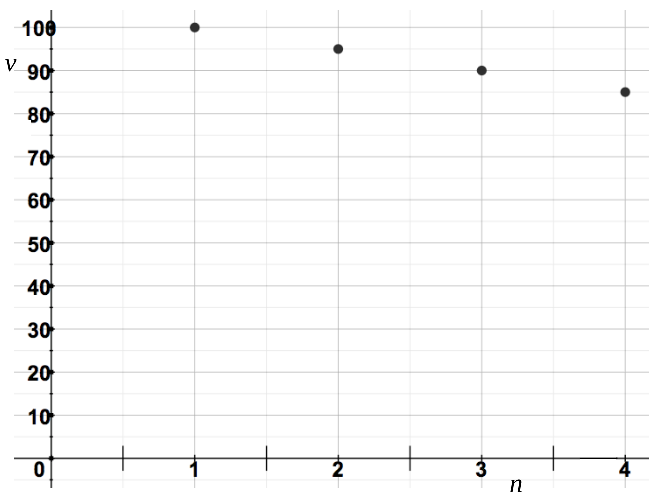
iii) If you multiply the figure number by 3 and add 5, you get the number of dots.
iv) $d = 3f + 5$, where d is the number of dots and f is the figure number.

C. *Sample response*:
$d = 3f + 5$, so for $f = 4$, d = 3 × 4 + 4 = 16.
For $f = 12$, d = 3 × 12 + 4 = 41.
For $f = 20$, d = 3 × 20 + 4 = 65.

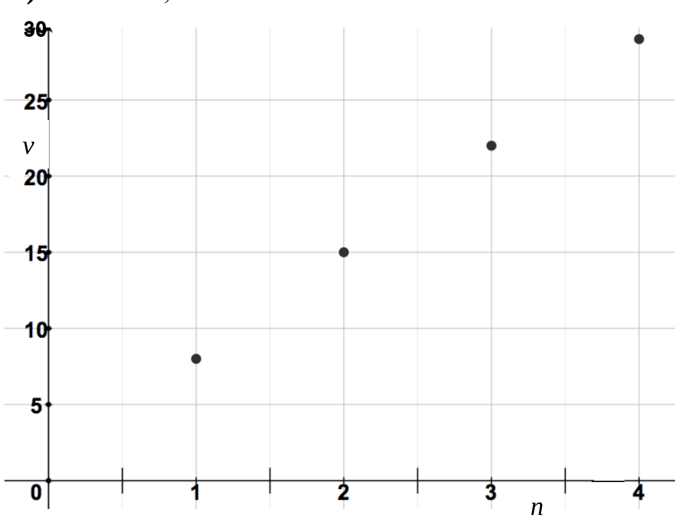
1. a) $v = 10n - 6$, where n is the term number and v is the term value.



b) $v = 105 - 5n$, where n is the term number and v is the term value.

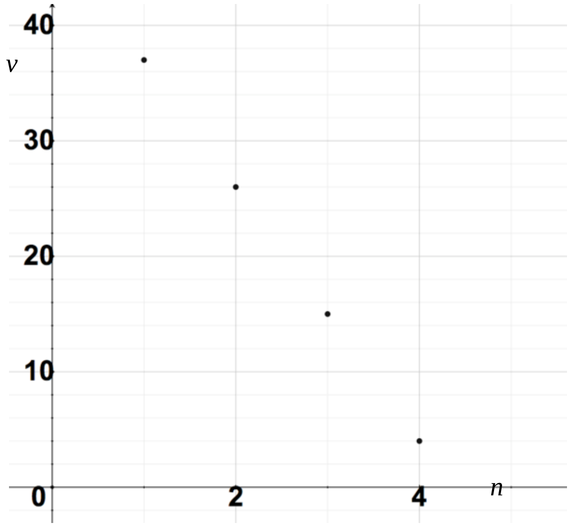


c) $v = 7n + 1$, where n is the term number and v is the term value.



Answers [Continued]

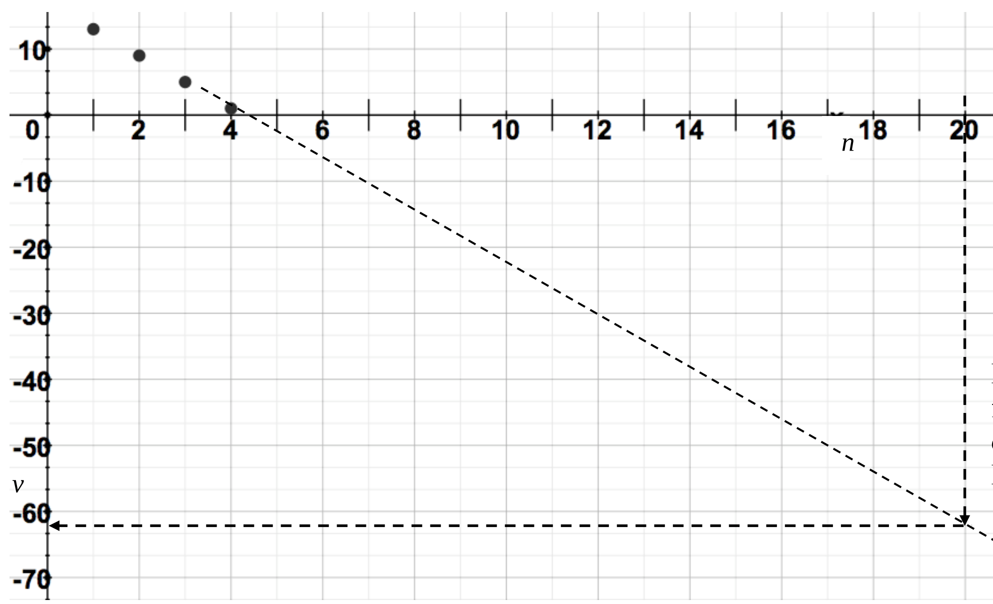
d) $v = 48 - 11n$ or $v = -11n + 48$, where n is the term number and v is the term value.



2. a) $v = 7n - 4$; The 20th term is $140 - 4 = 136$.

b) $v = 22 - 7n$ (or $v = -7n + 22$); The 20th term is $22 - 140 = -118$.

c)



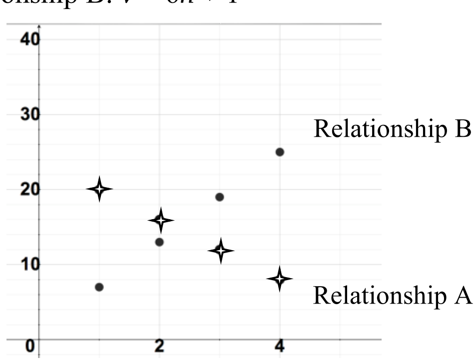
I extended the line formed by the pattern of dots until n was 20. Then I read the v-value of –63.

d) $v = \dfrac{-4n + 13}{2}$; The 20th term is $\dfrac{-67}{2}$.

3. *Sample response*:

a) Relationship A: $v = -4n + 24$ or $v = 24 - 4n$

Relationship B: $v = 6n + 1$

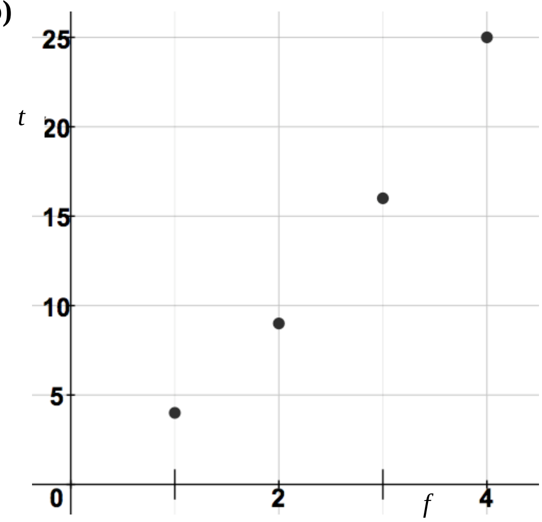


b) The equation for Relationship A involves multiplying by a negative number and then adding, while the equation for Relationship B involves multiplying by a positive number and then adding.

The graph for Relationship A slopes down, while the graph for Relationship B slopes up.

4. a) No; [*Sample response*:
I made a table that went up to Figure 20. Figure 19 has 400 small triangles, so Figure 20 must have more small triangles than that.]

b)

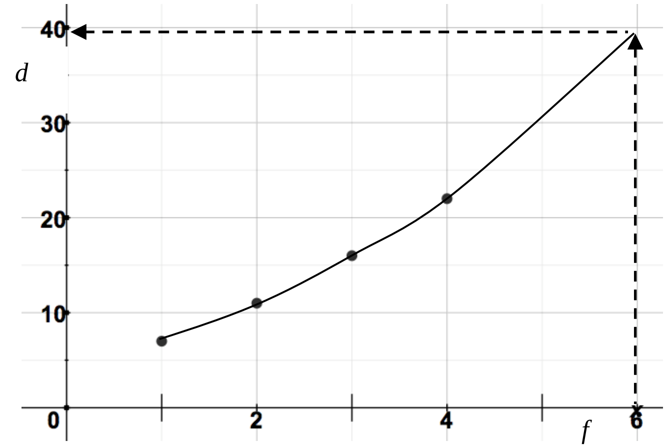


(f is the figure number and t is the number of triangles.)

c) The points form a curve that slopes upward; [*Sample response*:
The increase is not at a steady rate since it goes up 5, then 7, then 9, and so on.]

5. Rinzin is right; [*Sample response*:
I substituted different values for n. If $n = 1$, they both could be right, but when $n = 2$, Rinzin is correct and Kinley is not.]

6. *Sample responses*: **a) and b)**



(f is the figure number and d is the number of dots.)

I connected the points with a curve and then extended the curve to Figure 6. It looks like Figure 6 has 40 dots.
c) I noticed that the curve went up by 4, then by 5, then by 6. I started with the 7 dots in Figure 1 and added $4 + 5 + 6 + 7 + 8$. I got 37, so I was close.

7. $d = 3f + 3$, where d is the number of dots and f is the figure number.

Figure number	Number of dots
1	6
2	9
3	12
4	15

8. *Sample responses*:
A. An algebraic equation; [All I have to do is substitute a value and then calculate.]
B. A graph; [I can easily compare how two graphs slope up or down, how quickly they slope up and down, or if they slope up and down in a straight line or in a curve.]

Supporting Students

Struggling students

• **Questions 4 and 6** may be the most challenging for struggling students because the relationships are not linear. You need not assign these two questions to struggling students.

Enrichment

• Students might use graphs or tables of values to create patterns where a specific term, for example, the 10th term, is a fixed amount such as 80. They can then create various graphs, patterns, and algebraic equations to fit the required condition.

For example, some possible patterns for a 10th term of 80 might be:

8, 16, 24, … algebraic equation: $v = 8n$
62, 64, 66, … algebraic equation: $v = 60 + 2n$
35, 40, 45, ... algebraic equation: $v = 30 + 5n$

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7.1.3 Recognizing Linear Relationships

Curriculum Outcomes	Outcome relevance
8-C1 Patterns and Relations: represent in a variety of formats • investigate linear situations and those which create a regular pattern **8-C2 Graphs (Linear and Non-linear): interpret** • understand, when looking at tabular data, that when an equal spacing between the values of one variable produces an equal spacing between values of another variable, the relationship is linear • use information from tables, graphs, or algebraic expressions to describe change • match situations to corresponding graphs • sketch graphs for a variety of situations, leading to linear and broken-line graphs • understand that a variety of representations may be used to show relationships and that choices are available	Graphing is an important tool for understanding the relationship between two quantities. Students need to learn to interpret graphs so they know what kind of relationship is being described. Linear relationships are particularly important since they are so prevalent.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM) (optional)	• using a table of values to describe a data set • graphing on a coordinate grid • using equations and expressions to relate amounts

Main Points to be Raised

• A linear relationship is a relationship whose graph is a straight line.

• The consecutive values for a linear relationship increase or decrease by a constant amount.

• If a table of values represents a linear relationship, then each *y*-value is the same amount more (or less) than the corresponding multiple of the increase (or decrease) in each consecutive *x*-value.

• Some common linear relationships are distance travelled compared to time when the speed is constant, earnings compared to time when the rate of pay is constant, or the perimeter of a square compared to its side length.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *What does the x-coordinate represent?* (The day number.)

• *What does the y-coordinate represent?* (The number of emails for that day.)

• *Who sent more emails on Day 1?* (Mindu. He sent 20 and Choki only sent 5.)

• *Who sent more emails on Day 20?* (Choki. He would be send 19 more emails than on Day 1. That is 24 emails, and Mindu sent only 20.)

The Exposition — Presenting the Main Ideas

• With students, read through the exposition on **pages 191 and 192** of the student text. Help them contrast a linear relationship to a non-linear relationship by using the pattern 1, 4, 9, 16, 25, …, where the increases are not constant. Note that students might better see this pattern as a relationship (between term number and term value), if it is presented in table form:

Term number	Term value
1	1
2	4
...	...

Graph this relationship to see that the points (1, 1), (2, 4), (3, 9), (4, 16), and (5, 25) do not form a line.

• Make sure students understand that either graphing and seeing the line, or observing the constant increase in consecutive terms in the table of values are equally appropriate ways to see whether a relationship is linear.

Revisiting the Try This

B. Students can use what they have learned about linear relationships being associated with constant increases to revisit their answer to **part A.**

Using the Examples

• Ask students to work through the two examples with a partner. Provide time for them to ask questions.

• It might be helpful to discuss the use of the words *line* versus *graph*. Some mathematicians use the term graph to describe only the coordinate grid and the line together. Others use the word graph to describe the line by itself as well as to describe the coordinate grid and the line together. You will notice the latter use of the word graph in the student text and this teacher guide.

Practising and Applying

Teaching points and tips

Q 1: Remind students that linear relationships are associated with constant decreases as well as constant increases.

Q 2 b): To find the missing values, students need to realize that the same number was added three times to get from 20 to 27.5.

Q 4: Students can use either graphs or tables of values to make their decisions.

Q 6: Students need to create their own table of values to calculate the volumes for different side lengths.

Q 7: Some students will recognize that the form $8 - 2x$ has to describe a linear relationship. Encourage them to use a graph or table of values to support their thinking.

Common errors

• For **question 5**, many students will use the form $1000 + 400c$ rather than $400 + 1000c$. They will still deduce that the relationship is linear. Encourage them to figure out the values using reasoning rather than substitution to see why their algebraic expression is not correct.

Suggested assessment questions from Practising and Applying

Question 1	to see if students recognize a linear relationship from a table of values
Question 4	to see if students can create a table of values or graph to see whether a relationship is linear
Question 7	to see if students can recognize whether an algebraic equation describes a linear relationship

Answers

A. i) Mindu's graph;
Sample response: I sketched it to see.

B. *Sample response*:
While the day number increased by 1 each time for both people, the increase in the number of e-mails each day was constant for Mindu but not for Choki, so only the relationship for Mindu was linear.

1. a) Yes; [*Sample response*:
The y-values go down by a constant amount while the x-values go up by a constant amount.]
b) No; [*Sample response*:
The y-values go down by a different amount each time.]
c) Yes; [*Sample response*:
The y-values go up a constant amount while the x-values go up by a constant amount.]

2. a)

x	y
1	20
2	23
3	**26**
4	**29**

b)

x	y
1	20
2	**22.5**
3	**25**
4	27.5

Answers [Continued]

[**3.** *Sample responses*:
A. The distance increases by 550 km for every 1 hour increase.
B. The amount earned increases by Nu 920 for every 1 month increase.]

4. a) Yes; [*Sample response*:
$d = 2r$ is the algebraic equation for the relationship. When r increases by 1, d always increases by 2, so the relationship is linear.]
b) No; [*Sample response*:

r	$A = \pi r^2$
1	π
2	4π
3	9π
4	16π

The increase in A is not constant (even though the increase in r is constant).]
c) Yes; [*Sample response*:
$C = 2r\pi$ is the algebraic equation for the relationship. When r increases by 1, C always increases by 2π, so the relationship is linear.]

5. a)

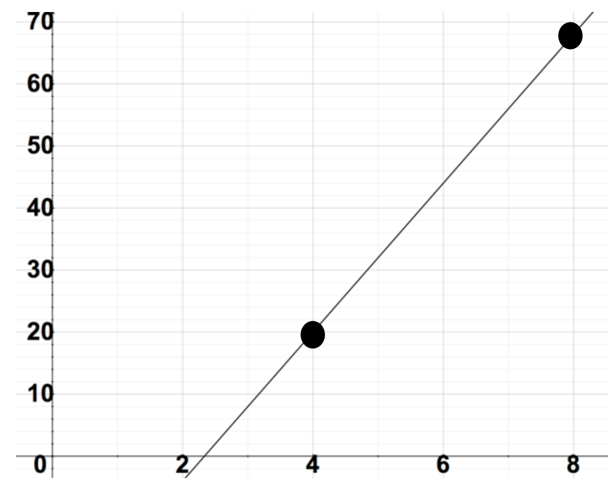
Cars	Nu
0	400
1	1400
2	2400
3	3400
4	4400

b) Yes; [The increase is Nu 1000 for every increase of 1 in the number of cars.]

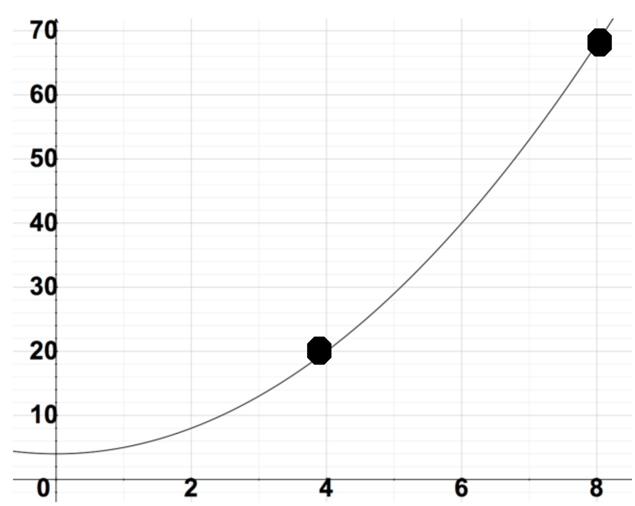
6. a) Graph 2; [*Sample response*:
I substituted different values for the edge length into $V = e^3$ and it seemed to match Graph 2.]
b) No; [The graph is not a straight line.]

7. a) Yes; [*Sample response*:
I tried some values for x and found that y always decreases by 2 when x increases by 1.]
b) No; [*Sample response*:
For $x = 0$, 1, and 2, y decreases by different amounts.]

8. *Sample responses*:
a) A straight line joining the two points shows a linear relationship.



b) A curve joining the two points does not show a linear relationship.



Supporting Students

Struggling students

• Some students may have difficulty with **question 4** until you suggest that they need to create a table of values, a graph, or an equation to make their decision. Encourage them to think about the formulas they already know as algebraic equations.

Enrichment

• Students might determine other measurement relationships that are linear.

For example, it could be the perimeter of an equilateral triangle compared to the side length or the areas of rectangles that are 5 cm in width compared to their lengths.

CONNECTIONS: Adding Values in a Linear Relationship

• Students might be interested to know a bit more about Gauss. Gauss discovered the relationship shown in the connection when he was just seven years old. Gauss did a variety of work in mathematics, on the subjects of geometry, number theory, and differential equations used in physics applications.

Answers

1. 50 × 101 = 5050 **2. a)** 420; [10 × 42 = 420] **b)** 590; [10 × 59 = 590] **c)** 14,340; [20 × 717 = 14,340]	**3.** No; [*Sample response*: 1 + 36 is not equal to 4 + 25.]

7.1.4 Slope

Curriculum Outcomes	Outcome relevance
8-C3 Graphs and Tables (Linear and Non-linear): how changing one quantity affects the other • use information from tables, graphs, or equations to investigate the impact of changing related quantities • explore patterns associated with parameter changes in a linear equation (e.g., understand how changes in the equation affect the slant of the graph) **8-C4 Slope: link visual characteristics with numerical values** • understand that, for linear relationships, the ratio of vertical change to horizontal change is constant anywhere along the line • use the terms rise and run to describe vertical and horizontal change in a line graph • investigate practical situations: slope of a staircase, slope of a roof, and the steepness of roads • determine the slope of a line • understand that ratios for a graph that rises to the right are positive • understand that ratios for a graph that rises to the left are negative	The slope of a line is often the most important piece of information about a linear relationship. It is important that students understand how to calculate the slope and how to use the slope to draw conclusions about a linear relationship.

Pacing	Materials	Prerequisites
1 h	• Grid paper or Small Grid Paper (BLM)	• graphing on a coordinate system • simplifying a fraction

Main Points to be Raised

• In a linear relationship, the *y*-value changes by a constant amount when the *x*-value changes by a constant amount. The ratio of those amounts is called the slope.

• The slope is defined as $\frac{\text{rise}}{\text{run}}$, where the *rise* is the associated change in *y* for a particular *run* that is the change in *x*. The slope is the same anywhere on the line.

• If a line rises as the values of *x* increase, the slope is positive. If a line falls as the values of *x* increase, the slope is negative.

Try This — Introducing the Lesson

A. and B. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How did you draw the graph?* (I substituted three different values for *x*, plotted the points, and saw that they formed a line.)

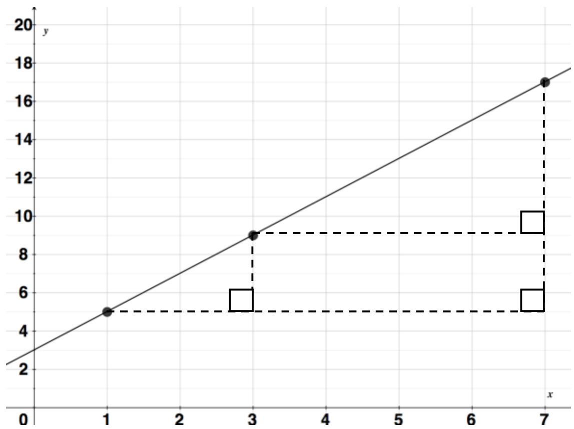
• *Did the y-values change by the same amount or by different amounts for the different increases in x that were described?* (They changed by different amounts for **part B i), ii), and iii).** When *x* increased more, so did *y*.)

• *What did you notice about **part B iii) and iv)**?* (The amount of the *y*-increase was the same.)

The Exposition — Presenting the Main Ideas

• Draw the line $y = 2x + 3$ on the board. Ask students to name the coordinates at $x = 1$, $x = 3$, and $x = 7$.

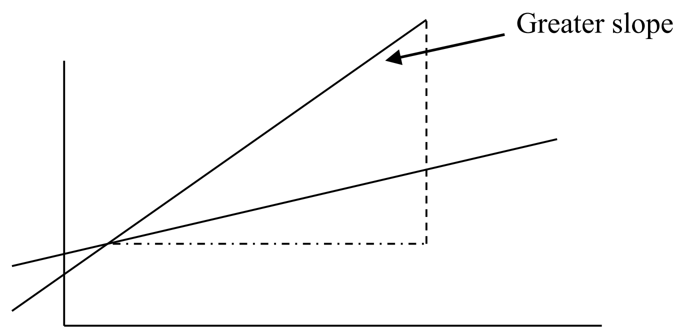
• Have them calculate the change in *y* going from the first point to the second, the second to the third, and the first to the third.



• Draw horizontal and vertical lines to show the changes in y and x. Point out how three right triangles are formed. Have students observe that the triangles are similar. For each, point out the *rise* and the *run*. Have students calculate the ratio $\frac{\text{rise}}{\text{run}}$. They should observe that the three ratios, $\frac{4}{2}$, $\frac{8}{4}$, and $\frac{12}{6}$, are all equal to 2. Tell them that the slope of this line is 2.

• With students, read through the exposition on **pages 197 and 198** of the student text to reinforce what you have shown on the board.

• Make sure students understand that if a line rises more quickly, then the ratio that describes the slope is greater because the change in y is greater for the same change in x.



Revisiting the Try This

C. and D. These questions allow students to practise what they learned about slope using the line from **part A**.

Using the Examples

• Present the problems from **examples 1 and 2** on the board. Ask students to try them and then compare their solutions to the solutions in the student text.

• Work through **example 3** with the class. Make sure students understand why the side lengths in each case can be represented as x and $\frac{x}{2}$, respectively. The number of times each is added into the perimeter is shown in the example.

Practising and Applying

Teaching points and tips

Q 1: Students can use any two points on the graph to make the calculation. Encourage them to use points that make the mental work simpler.

For example, they might use (3, 10) and (2, 6) for **line c)**.

Q 3: There is an infinite number of correct answers, but all the possible lines that could be drawn are parallel.

Q 4: Encourage students to try a variety of negative values for x.

Q 6: Some students will be able to analyse the situation theoretically, while others will benefit from using particular numbers of candy bars to figure out possible points on the graph.

Q 8: For **part a)**, students might reason that going from an x-value of 5 to an x-value of 3 means a run of –2, so the rise must be –12.

Q 9: Students need to recognize that the unit rate is the slope.

Q 11: You may wish to handle this as a whole class discussion.

Common errors

• Many students calculate the change in *y*-values without remembering to divide by the appropriate change in *x*. Remind students that the slope is a ratio.

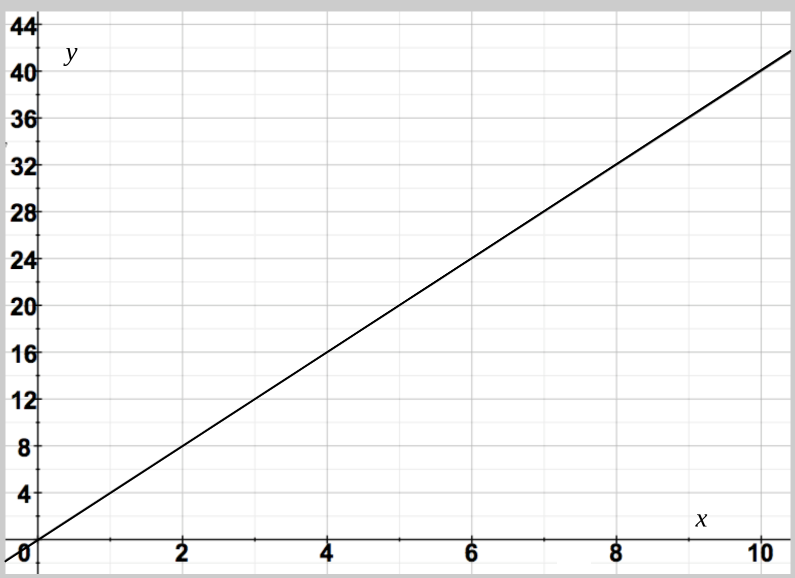
• Some students get confused about whether it is the constant or the coefficient of *x* in the expression $y = mx + b$ that tells the slope. Remind them that they can always calculate the value using two points on the line.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can calculate a slope from a drawn graph
Question 2	to see if students can calculate a slope given coordinates for points on the line
Question 5	to see if students relate the value of a slope to the steepness of a line
Question 9	to see if students can solve a problem involving slope

Answers

A.



B. i) Increases by 4; When *x* is 1, *y* is 4. When *x* is 2, *y* is 8. 8 – 4 = 4.
ii) Increases by 8; When *x* is 1, *y* is 4. When *x* is 3, *y* is 12.12 – 4 = 8.
iii) Increases by 12; When *x* is 1, *y* is 4. When *x* is 4, *y* is 16. 16 – 4 = 12.

iv) Increases by 12; When *x* is 2, *y* is 8. When *x* is 5, *y* is 20. 20 – 8 = 12.

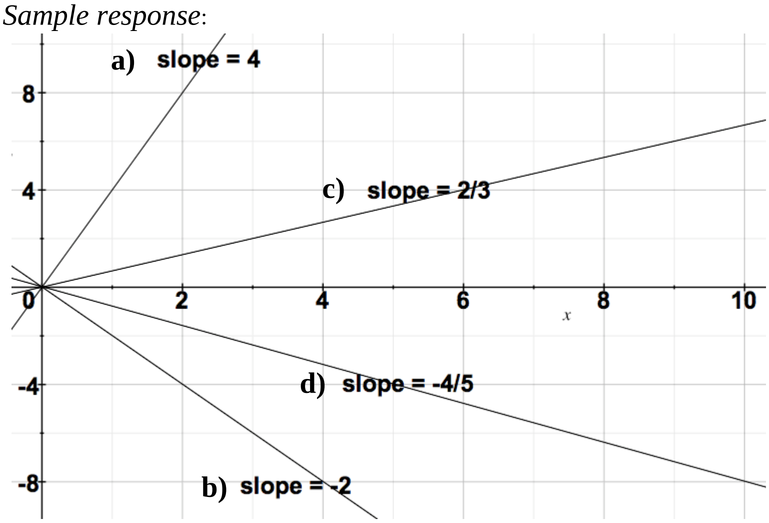
C. The slope is 4; For each run of 1, there is a rise of 4, and $\frac{4}{1} = 4$.

D. Since the slope is 4, I multiply the run by 4 to get the rise each time.
i) A change in the *x*-value from 1 to 2 (the run) is +1, so the rise is 1 × 4 = 4. The rise is the change in the *y*-value: 4.
ii) A change in the *x*-value from 1 to 3 (the run) is +2, so the rise is 2 × 4 = 8. The rise is the change in the *y*-value: 8.
iii) A change in the *x*-value from 1 to 4 (the run) is +3, so the rise is 3 × 4 = 12. The rise is the change in the *y*-value: 12.
iv) A change in the *x*-value from 2 to 5 (the run) is +3, so the rise is 3 × 4 = 12. The rise is the change in the *y*-value: 12.

1. a) –1 **b)** 3 **c)** 4

2. a) 3 **b)** 3 **c)** 4

3. *Sample response*:



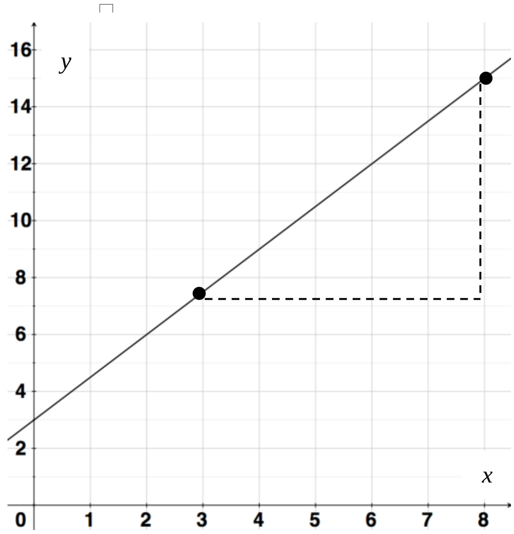
4. Positive; [*Sample response*: Going from 2 to 3 horizontally is an increase, so the rise is a positive value. Going from a negative number to 5 vertically is also an increase, so the rise is also a positive value.]

5. $y = 3x – 10$; [*Sample response*: I think the slope is the coefficient of *x* in the equation, and 3 > 2. I checked by plotting two points on each line.
For the first line, I used (10, 20) and (20, 50). The slope is $\frac{30}{10} = 3$.
For the second line, I used (0, 8) and (1, 10). The slope is $\frac{2}{1} = 2$.]

6. 60; [*Sample response*:
An increase of 1 bar (run) results in a cost increase of Nu 60 (rise), so rise ÷ run = 60 ÷ 1 = 60.]

7. a) The change is $\frac{7.5}{5}$ cm per minute since it changes 7.5 cm in 5 minutes. That is 1.5 cm per minute.

b)



The slope of the graph is 1.5.

8. a) (5, 4), (3, **–8**), (**7**, 16)
b) (3, 10), (8, **12**), (**18**, 16)

9. 15 km/h; [The slope from (2, 55) and (4.5, 17.5) is $-\frac{37.5}{2.5} = -15$.]

10. $\frac{2}{3}$ m

[**11.** *Sample responses*:
a) The slope tells how steep the line is and whether it goes up or down to the right.
b) The slope does not tell the actual points on the line, and exactly where the line is located on the grid.]

Supporting Students

Struggling students

• **Questions 6, 7, and 9** require students to make the connection between slope and rate of change in real-world situations. This may be difficult for some students. You may need to reinforce this relationship as you go over the questions with them.

• Some students may have difficulty with **question 4**, which requires reasoning. Encourage students to use specific values to give them a starting point, but then encourage them to see how they might have predicted the result without using the specific values.

Enrichment

• You might have students create other measurement situations like those in **example 3** to explore and graph.

For example, they might consider the perimeter of a shape made up of a square and an attached square with a side length that is less than half the side length of the original square.

Chapter 2 Solving Linear Equations

7.2.1 Solving an Equation Using Inverse Operations

Curriculum Outcomes	Outcome relevance
8-C5 Single Variable Equations: solve algebraically • use prior knowledge developed through concrete experiences to transfer to symbolic representation of single variable equations • solve one- and two-step equations symbolically using integer and simple fraction coefficients • use the “balance method” to solve problems	Solving equations is a critical skill for higher mathematics. This outcome takes students beyond intuitive models to more formal approaches.

Pacing	Materials	Prerequisites
1 h	None	• using a balance model for an equation • recognizing that the inverse of multiplication is division and the inverse of addition is subtraction

Main Points to be Raised

- A linear equation is an equation that involves variables only to the first power.
- Solving an equation means finding a value for the variable that makes the equation true.
- You can think of an equation as a situation where you input a value for the variable, apply the rules of the algebraic expression, and get the output.
- To solve an equation, you work backwards from the output. Working backwards means using the inverse (or opposite) operations.
- If the variable appears on both sides of the equation, it is a good idea to get the variable on only one side of the equation by using inverse operations.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

- *What answer would you give if you chose 2?* (8)
- *What answer would you give if you chose 6?* (16)
- *How do you know that the answer you give is usually more than the number you started with?* (When I double the number, it gets bigger if it is positive. Also, I add more to it than I subtract from it.)
- *How could you use a diagram to model what happened to the number you chose?* (If I start with $\square$, I end up with $\square\square - 4 + 8$.)

The Exposition — Presenting the Main Ideas

- Draw a graph of the line $y = 2x + 7$ on the board. Ask students to name some of the points on the line, e.g., (0, 7), (1, 9), and (2, 11).
- Ask students how they could use the graph to solve the equation $2x + 7 = 9$. Remind them that they have learned to look for the point on the line where the y-value is 9 and determine the corresponding x-value. Show how this is done on the graph. Note that the x-value is 1 because (1, 9) is on the line.
- Show how you can use the same line to solve $2x + 7 = 7$ or $2x + 7 = 11$. In fact, you can use it to solve any equation of the form $2x + 7 = \text{k}$. Students should see that the equation $y = 2x + 7$ has many solutions. You can substitute different values for x and find the corresponding values for y. But an equation like $2x + 7 = \text{k}$, where k is a particular number, has only one solution.

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• Show students that another way to solve $2x + 7 = 9$ is to think about what happened to the number x as a result of applying the expression $2x + 7$. You doubled the value of x and then added 7. If the result is 9 after that happens, the number x has to have been 2 before the 7 was added $(9 - 7)$ and it has to have been 1 before the doubling $(2 \times 1 = 1)$.

• Tell students that 1 was the input (in this case the x-value) and the 9 was the output, or result (in this case, the y-value).

• With that as background, have students read through the exposition in the student text on **page 203** and the first part of **page 204**.

• With students, discuss the section at the end of the exposition, where it shows to maintain a balance by adding or subtracting numbers or variables to or from both sides of the equation.

Revisiting the Try This

C. This question is designed to make a more formal connection between the number trick described in **parts A and B** and the formal way of solving an equation that was discussed in the exposition.

Using the Examples

• Present the questions from both examples on the board. Ask students to try them. They can then compare their work with the solutions in the student text.

Practising and Applying

Teaching points and tips

Q 2: Students can use any method they wish to solve the equation. They do not have to use inverse operations.

Q 3: Encourage students to refer back to **example 2**.

Q 4: You may need to remind some students that $\frac{x}{4} = \frac{1}{4}x$.

Q 5: Some students may use informal methods to solve this problem. Others may recognize that they are actually solving the equation $2x + 8 = 11 - x$.

Q 6 and 7: These questions require students to estimate a solution. This was not explicitly taught. Students should try a value for x as a possible solution and then decide if, and by about how much, they should go up or down to be close to the actual value.

For example, for $80 = 6 + 8x$, they might decide to ignore the 6 and estimate $8x = 80$ with $x = 10$. Or, they might try a value like 2, realize that $6 + 8 \times 2 = 22$ is way too low, and try a higher value for x. There is no single correct answer.

Q 8: This question highlights a common error people make in solving equations. It is useful to bring this to students' attention so they can avoid the error. You might point out what Gembo did that is incorrect; he divided only some of the terms in the equation by 3, rather than dividing all the terms by 3.

Q 9: This question is designed to review the critical ideas in the lesson.

Common errors

• Many students will make the error highlighted in **question 8**, where they apply multiplication or division to some, but not all, of the terms of an equation. Encourage students to keep the balance metaphor in mind. They might visualize a balance like the one shown in the exposition.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can solve a simple linear equation
Question 3	to see if students can create equations with a given solution
Question 5	to see if students can create an equation to match a situation and then solve it
Question 7	to see if students can estimate the solution to an equation

Answers

A. *Sample response*: If you think of 8, you would say 20 and your partner would have to say 8. **B.** *Sample response*: If your partner says 24, you would say the secret number was 10.	**C.** *Sample response*: $2s - 4 + 8 = 20$, $s = 8$ $2s - 4 + 8 = 24$, $s = 10$
1. a) $3m - 4 = 12$ **b)** *Sample response*: $(x + 3) \times 2 = -6$ **c)** *Sample response*: $(n - 4) \times 3 = 1.5$ **2. a)** $x = -1$ **b)** $x = 7$ **c)** $x = \frac{5}{6}$ **d)** $x = 7$ **3.** *Sample responses*: **a)** $3m = 33$; $2 - m = -9$; $m + 1 = 12$ **b)** $5n = 4$; $10n = 8$; $80 - 10n = 72$ **c)** $8x = 1$; $16x = 2$; $-16x = -2$ **d)** $2k = -16$; $-k = 8$; $30 - k = 38$ [**4.** *Sample response*: If you substitute the same value for x into both equations, both equations are true. It works for any value of x.] **5.** 1	[**6.** *Sample response*: 19 is about 20, so $5x$ is about 48, which is about 50. That means x is about 10.] **7.** *Sample responses*: **a)** x is about 9 **b)** x is about 10 **c)** x is about 1 **8.** Pema; [*Sample response*: I tried it both ways and got the right number using Pema's method, but not Gembo's.] [**9.** *Sample response*: You can get the answer right away without having to try a lot of possibilities. The numbers are not easy to work with mentally if you have to use them many times.] [**10.** *Sample response*: I know 2 was added last, and before that x was divided by 3, so I would undo them in reverse order. I would subtract 2 and then multiply by 3.]

Supporting Students

Struggling students

• **Questions 8, 9, and 10** may be difficult for struggling students since these questions require them to reflect on how equations are solved rather than on just solving them. You may choose not to assign these questions to struggling students.

• Some students may find it easier to solve equations than to create equations with a given solution. You may wish to refer them back to **example 2** to see that this is not difficult to do if you start with the very simplest possible equation and then modify it.

• Some struggling students may not recognize the equation implicit in **question 5**. You may need to let them use other means to solve the equation and then later bring the relevant equation to their attention.

7.2.2 Using an Equation to Solve a Problem

Curriculum Outcomes	Outcome relevance
8-C6 Linear Equations: create and solve problems • create and solve relevant problems for which algebraic solutions are required • justify strategies used to create and solve problems • appreciate the use of an algebraic equation in problems involving large numbers (as opposed to a guess and check approach)	One of the main reasons we study mathematics is so that we can use mathematical symbols to solve real-world problems. It is important for students to understand this.

Pacing	Materials	Prerequisites
1.5 h	None	• solving linear equations • representing situations algebraically

Main Points to be Raised

• You can use linear equations to model many real-world situations. Solving an equation can help solve a real-world problem.

• You can create a problem to match a linear equation by considering when the various operations on the input (x) might actually be performed.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How much money will Eden have after two months?* (Nu 200)

• *How much more does Eden need in order to have Nu 300?* (Nu 100)

• *How long will it take to save that much extra? How do you know?* (Five more months, since there are five 20s in 100.)

The Exposition — Presenting the Main Ideas

• Present this problem to students: *An equilateral triangle has a perimeter of 42 cm. What is its side length?*

Ask students how they could express that information in an equation. Lead them to see why $3s = 42$ (or $s + s + s = 42$) describes the situation. Ask them how they would solve the equation.

• Then present the equation $3s = 48$. Ask students why this problem matches the equation: *An equilateral triangle has a perimeter of 48 cm. What is its length?*

• Ask students if they can create another problem that matches the same equation.

For example, they might say this:

I have some stools with 3 legs. Altogether there are 48 legs. How many stools do I have?

• Have students read through the exposition on **page 207** of the student text.

Revisiting the Try This

B. Students did not need to use an equation to solve **part A**, but this shows that they could have done so. It also allows students to think about the approach they might take to solve the equation.

Using the Examples

• Ask pairs of students to read through the two examples to make sure they understand them.

• Provide time for students to ask questions about the examples.

Practising and Applying

Teaching points and tips

Q 1: You might help students get started by having them think about what sorts of situations are described by each operation: addition, subtraction, multiplication, and division.

Q 2: Allow students to use any method they wish for solving the equations.

Q 4: Students need to realize that the variable in this case refers to the number of minutes.

Q 5: Encourage students to draw a diagram to model this problem.

Q 6: Alert students to the fact that since the problem is asking how many hours, the variable should represent the number of hours.

Q 7: Refer students to **example 2**.

Q 8: You may choose to handle this question in a whole class discussion.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can use a linear equation to model a situation
Question 4	to see if students can solve a real-world problem involving a linear relationship
Question 7	to see if students can create a problem to match an equation
Question 8	to see if students can communicate about the link between linear equations and real-world situations they might model

Answers

A. 7 months

B. *Sample responses*:
i) $160 + 20m = 300$

ii) Inverse operations;

$$\begin{aligned} 160 + 20m &= 300 \\ 160 - 160 + 20m &= 300 - 160 \\ 20m \div 20 &= 140 \div 20 \\ m &= 7 \end{aligned}$$

1. *Sample responses*:
a) $24b = 744$
b) $k + k + 2 = 82$ or $2k + 2 = 82$
c) $4s + 4 = 28$
d) $4b + 7 = 103$

2. a) $b = 31$ **b)** $k = 40$
c) $s = 6$ **d)** $b = 24$

3. $100 = 2.2k$; $k \approx 45$

4. $400 - 112 = 32m$; $m = 9$ min

5. *Sample response*:
a) $l - 4 + l - 4 + l + l = l + 64$; $l = 24$ cm
b) $w = l - 4$; $w = 20$ cm

6. *Sample response*:
$150 = 80 + 28h$; $h = \frac{70}{28} = 2\frac{1}{2}$ h

7. *Sample responses*:
a) Four equal groups of students and another group of 97 students attended an event. Altogether, there were 489 students. How many students were in each of the four equal groups?
b) Four pieces of string of equal length were cut from a string of length 100 cm, or 1 m. There were 44 cm of string left. How long was each of the equal pieces?
c) Dechen followed a recipe that called for two containers of flour plus another six cups. The total amount of flour was equal to four full containers of flour. How many cups of flour does a full container hold?

[**8.** *Sample responses*:
a) The equation $4x = 28$ could represent these two problems:
• How many circles can you make out of 28 quarter circles?
• How many groups of four students could be made out of a class with 28 students?
b) Both equations $2(w + 3 + w) = 80$ and $2(l + l - 3) = 80$ could represent this problem:
The perimeter of a rectangle is 80 cm. Its length is 3 more than its width. What is its width?

Supporting Students

Struggling students

• Many students who can solve a given equation by using procedures they have learned will have difficulty modeling a real-world situation with an equation. This takes time and a great deal of practice. For some students, you may have to provide additional situations similar to the situations in the text to help them make the necessary connections.

• Some students will have difficulty creating problems to match equations. Suggest that these students use the various situations found in the **Practicing and Applying** questions as models they could copy.

Enrichment

• Students might think of other situations that can be modelled by linear relationships. They can then create problems to solve. Students can exchange problems and solve each other's problems. Encourage students to choose problems that are relevant to their own lives and/or that use contexts that are of interest to them.

7.2.3 Solving a Problem Involving Two Relationships

Curriculum Outcomes	Outcome relevance
8-C7 Intersection of Two Lines: solve problems • compare tables of values, equations, or verbal descriptions of two linear situations to identify where lines will intersect • use tables of values to generate ordered pairs for each equation and identify coordinates for points of intersection	In a number of real-world situations, two relationships must hold simultaneously. Students must be able to consider both relationships at the same time in order to solve those real-world problems.

Pacing	Materials	Prerequisites
40 min	• Grid paper or Small Grid Paper (BLM)	• graphing linear equations • solving linear equations

Main Points to be Raised

• If the graphs of two lines intersect, the intersection point is called a solution to the two equations. The point shows the one set of values for which both equations are true.

• You can also find the common solution to two linear equations by using a single equation that includes the information from both equations. This is usually done by writing both equations in the form $v = \ldots$ and then setting the two right sides of the equation equal.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How could you use an equation to say that Karma's mother is $6\frac{1}{2}$ times as old as he is?* ($m = 6\frac{1}{2}k$)

• *How do you know that Karma could not be 10 years old?* (If he were 10, his mother would be 65. Two years ago, when he was 8, his mother would have been 63. 63 is not 12×8.)

• *Could Karma be 6 years old now?* (No. If he were 6 now, his mother would be 39. Two years ago, he would have been 4 and his mother would have been 37. 37 is not 12×4.)

• *Could Karma be 4 years old?* (Yes. If he were 4, his mother would be 26 years old. Two years ago, he would have been 2 and she would have been 24. $24 = 12 \times 2$.)

The Exposition — Presenting the Main Ideas

• Ask students to keep their books closed. Present Dorji's situation, found near the bottom of the exposition on **page 210** of the student text. Have students work with you to see why $y = 3x + 8$ describes the first situation and $y = 4x - 1$ describes the second situation.

• Graph both lines and have students notice there is only one point in common. Read the coordinates of the intersection point, (9, 35). Point out that this means that if the x-value is 9, both y-values are 35. Explain that $x = 9$ is a solution to the equation $3x + 8 = 4x - 1$, which is a single equation that says that the two y-values are equal.

• Have students read through the exposition on **pages 210 and 211**.

Revisiting the Try This

B. Students can use both methods of looking at the two relationships holding simultaneously, using one equation or looking for the intersection of two graphs, as they re-examine the problem from **part A**.

Using the Examples

• Have students read through the example. Provide time for them to ask questions.

Practising and Applying

Teaching points and tips

Q 1: Allow students to draw the graphs if they wish. Some students will not need to do so because they will write a single equation to represent both of the given equations.

Q 3: Students need to understand that the figure number is based on the patterns continuing in the same way. The solution is not among the displayed figures.

Q 4: Students should first model each situation with an equation.

Q 5: Students need to use reasoning to solve this problem since they cannot try every possible value.

Common errors

• For **question 5,** some students may simply try one value that does not work and conclude that the prices could not be the same. They need to understand that they have to be able to show that there is no possible value that works and not simply to show that one particular value does not work.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can find the common solution to two linear equations
Question 4	to see if students can model two linear situations with equations and find a common solution
Question 6	to see if students can recognize when a situation requires that two relationships have a common solution

Answers

A. 4 years old **B.** *Sample responses*: **i)** If Karma's age is k, $12(k - 2) + 2 = 6.5k$.	**ii)** I would graph $m - 2 = 12(k - 2)$ and $m = 6.5k$ and see where they intersect. The k-coordinate of the intersection point is Karma's age now.
1. a) $(-2, 5)$ **b)** $(8, 0)$ **c)** $(15, 124)$ **2.** *Sample response*: Since the point is on both lines, the coordinates of that point make both equations true. **3.** $4f - 3 = 3f + 14$; $f = 17$, so Figure 17 has the same number of dots for both patterns. **4.** 8 h; $60h + 220 = 50h + 300$ [**5.** *Sample response*: Sonam's four tins cost more than Rinzin's five tins. If each tin cost the same amount, this would be impossible.]	**6.** *Sample response*: An isosceles triangle has two equal long sides. Its third side is 5 cm shorter than each long side. The perimeter is the same as the perimeter of a square whose side length is the same as the short side of the triangle. What are the dimensions of each shape? (The triangle has sides of 10 cm, 15 cm, and 15 cm. The square has four sides of 10 cm.) [Perimeter of triangle = $s + 5 + s + 5 + s = 3s + 10$ Perimeter of square = $4s$ $4s = 3s + 10$, so $s = 10$ cm. The triangle has sides of 10 cm, 15 cm, and 15 cm. The square has four sides of 10 cm.]

Supporting Students

Struggling students

• Struggling students might be more successful drawing both graphs rather than trying to come up with the single equation that can be solved to find the common solution. Allow them to do that.

• For **question 3**, make sure students understand that they must first use an equation to describe each pattern rule. They must look for how the figures within each pattern are related.

Enrichment

• Students might begin with a common solution and come up with equations that have that common solution.

• They might also create other number tricks like Dorji's in the exposition, where two rules are applied and the resulting outputs are the same for the same input. They can then trade their problems with other students.

GAME: Alge-Scrabble

- Provide the alge-scrabble game board and game tiles to pairs of students.
- They will need to cut out the game tiles if they have not been pre-cut.
- Encourage students to set the first equation in the centre of the board to allow for more flexibility of play.
- As students play the game, observe to see whether students realize that they should create equations with high solutions and whether they try different equations to maximize their scores.

7.3.1 Adding Polynomials

Curriculum Outcomes	Outcome relevance
8-B15 Add and Subtract simple algebraic terms: to solve problems • establish a parallel between a measurement situation and a variable situation (e.g., for 3 m + 0.2 m, 3 m and 20 cm need to be "like terms" before you can add or subtract) • add simple expressions with concrete materials such as algebra tiles (know which like terms can and cannot be combined) **8-B16 Polynomial Expressions: Add and subtract visually** • use concrete materials such as algebra tiles for conceptual development	The ability to simplify algebraic expressions is an important first step to being able to solve equations and problems using the equations model. It is important for students to understand how and why simplification works.

Pacing	Materials	Prerequisites
1 h	• Algebra tiles or Algebra Tiles (BLM)	• adding integers

Main Points to be Raised

- A linear polynomial is an algebraic expression that includes a variable with an exponent of 1 and no other powers. It usually involves more than one term.
- You can represent polynomials using algebra tiles. Rectangles represent the variable and small squares represent the constant. One colour is used for negative values and a different colour for positive values. In this book, black is used for negative and white for positive, but this colour choice is arbitrary. The actual colours are also arbitrary — some commercial algebra tiles are green and white or yellow and red.
- The zero property for integers also applies to $+x$ and $-x$ tiles.
- When you combine like terms and eliminate unnecessary terms using the zero property, it is called simplifying.
- Adding like terms is similar to adding numbers, with tens added to tens, hundreds to hundreds, and so on.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

- *Why does* $6 \times 423 + 4 \times 423$ *mean 6 groups of 423 and 4 more groups of 423?* (Multiplication means finding that many groups of something and addition means putting them together.)
- *How many groups of 423 are there in total?* (10 groups)
- *How can you calculate that amount using mental math?* (Write 423 with a 0 on the end to be 4230.)
- *Are there only groups of 423 in the second expression?* (No)

The Exposition — Presenting the Main Ideas

- If possible, distribute algebra tiles to students. In this teacher's guide and student text, the tiles are assumed to be white and black. If your tiles use a different colour combination, revise the instructions accordingly. It is advised that you use the light colour for positive and the darker colour for negative to parallel the use of white and black in the textbook.
- Explain that the white rectangles are called x-tiles, the black rectangles are called $-x$-tiles, the small white squares are worth 1, and the small black squares are worth –1. (See the note above in **Main Points to be Raised** about the arbitrary colours for the tiles.) Remind students that using black and white for negative and positive is much like what was done with integers.
- Ask students to represent various amounts, e.g., $x + 4$ (as one x-tile and four 1-tiles), $2x - 3$ (as two x-tiles and three –1-tiles), and so on. Write the expressions on the board. Mention that each expression is called a polynomial. Tell students that a polynomial as an expression that involves variables and constants.
- Have one student choose a polynomial for the rest of the class to model. Ask another student to suggest another polynomial. Then ask pairs or small groups of students to figure out what the sum of those two polynomials might be. Most students will naturally combine the polynomials and get the correct result.

• Make sure students have a chance to use the zero property by asking them to add the polynomials $4 - 2x$ and $3x - 8$. Ask them why the xs combine to form only $1x$ and why the result of adding the constants is -4.

• Then have students read through the exposition on **pages 214 and 215** of the student text.

• Discuss the last statement in the exposition. Line up two numbers to add them.

For example:

$$\begin{array}{r} 345 \\ +\ \underline{287} \end{array}$$

• Point out that when you add ones to ones, tens to tens, and hundreds to hundreds, you are combining like terms.

• You might mention that this idea also comes into play in work with measurements.

For example, if the dimensions of a rectangle are 1.2 m and 57 cm, the reason that you change either 1.2 m to 120 cm or 57 cm to 0.57 m is so that you can combine like terms. Using the same units is like using like terms.

Revisiting the Try This

B. Students should see that in **part i)** the like terms are the groups of 423.

Using the Examples

• Present the problem from the example on the board. Have students try to solve it using algebra tiles. They can then compare their solutions with the solution in the student text.

Practising and Applying

Teaching points and tips

Q 1: Students should understand that the order in which they place the tiles does not matter.

Q 4: Make sure students understand that they can use x-tiles, 1-tiles, or any combination of these.

Q 5: Students might observe that the sum could use anywhere from 1 tile to 9 tiles.

For example, for $5x + (-4x)$, you start with 9 tiles and end with 1 tile, but for $5x + 4x$, you start with 9 tiles and end with 9 tiles.

Q 7: You might help students get started by showing an example where the sum is x. Show how you could have used $2x$ and $-x$ or maybe $(2x + 1)$ and $(-x - 1)$.

Q 9: Students can refer to the example.

Q 10: You might handle this question as a class discussion.

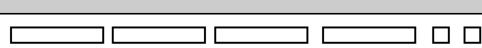
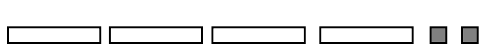
Common errors

• Students sometimes combine terms that are not like terms if they are not careful. These students might benefit from using a small chart where they list the x-terms in one column and the constants in another column.

Suggested assessment questions from Practising and Applying

Question 2	to see if students can identify like terms in a polynomial
Question 6	to see if students can add polynomials by simplifying them
Question 7	to see if students can work backwards from a polynomial sum to the addends
Question 9	to see if students can communicate about the process of adding polynomials

Answers

A. $6 \times 423 + 4 \times 423$; *Sample response*: I only have to multiply 10×423.	**B.** The 423s are like terms. There are six 423s and another four 423s, so there are ten 423s altogether.
1. a)  **b)**  **c)**  **d)** 	**2. a)** $5x$ and $2x$; 7 and – 2 **b)** $-5x$, $2x$, and x; 3 and – 2 **c)** $-2x$ and $-4x$; 6 and -1 **3. a)** $7x + 5$ **b)** $-2x + 1$ **c)** $-6x + 5$

4. *Sample responses*: **a)** $-6x$, $-5x-1$, $-4x-2$, $-3x-3$, $-2x-4$ **b)** $8x$, $7x+1$, $6x+2$, $5x+3$, $4x+4$ [**5. a)** *Sample response*: I can use the zero property to get rid of some of the tiles.] **b)** *Sample response*: $(-3x+1)+(4x+1)=x+2$ **6. a)** $6-2x$ **b)** $7x$ **c)** $2x-2$ **d)** 3 **7.** *Sample responses*: **a)** $(4x+2)+(-6x)$ $(2x+1)+(-4x+1)$ $(-2x+1)+1$ **b)** $(2x+1)+x$ $4x+2+(-x+-1)$ $(6x+-2)+(-3x+3)$ **c)** $x+(x-2)$ $2x+(-2)$ $(7x-9)+(-5x+7)$	[**8.** Metres are like terms and centimetres are like terms. You can add the metres and then add the centimetres to find the total length.] [**9.** *Sample response*: You can use the zero property to get rid of either x-tiles or 1-tiles that you do not need. For example, to add $3x-4$ to $-2x+7$, you can combine $2x$ from the $3x$ with the $-2x$ and be left with one x. You can also combine 4 from the 7 with the -4 and be left with three 1s.] [**10.** *Sample response*: You can use the zero property as many times as you want to create other possibilities. For example, for $2x+1$, you can use $2x$ added to 1 but you could also use $2x+1$ added to $1-1$ or $2x+7$ added to $1-7$.]

Supporting Students

Struggling students

• Some students may have difficulty with **questions 7 and 10**, where they must work backwards from the sum to the addends. Have them look at some of the questions where they have added two polynomials to get a sum and suggest they look at their steps backwards.

Enrichment

• Students could create polynomials to fit various conditions.

For example, you added a polynomial you can model with 4 tiles to a polynomial you can model with 6 tiles. The sum requires 2 tiles. What could the polynomials have been? [*Sample response*: $(2x+2)+(-4x-2)$]

7.3.2 Subtracting Polynomials

Curriculum Outcomes	Outcome relevance
8-B15 Add and Subtract Simple Algebraic Terms: solve problems • subtract simple expressions with concrete materials such as algebra tiles (know which like terms can and cannot be combined) **8-B16 Polynomial Expressions: add and subtract visually** • use concrete materials such as algebra tiles for conceptual development • for subtraction, consider different representations of subtraction, including the following: - comparison (which refers to comparing and finding the difference between two quantities) - taking away (which refers to starting with a quantity and removing a specific amount - adding the opposites (which refers to subtracting by first changing the question to an addition and then adding the opposite of a quantity (e.g., subtracting x instead of $-x$) - missing addend (What would be added to the number being subtracted to get the starting amount? (e.g., for $(3x - 2) - (2x + 1)$, what is added to $2x + 1$ to get $3x - 2$?)	The ability to simplify algebraic expressions is an important first step in solving equations. It is important for students to understand how and why simplification works.

Pacing	Materials	Prerequisites
1 h	• Algebra tiles or Algebra Tiles (BLM)	• adding and subtracting integers

Main Points to be Raised

• Subtraction can mean take away, compare, find the missing addend, or add the opposite. Each of these meanings can be applied to subtracting polynomials.

• If you treat subtraction as take away or comparing and the starting amount does not include the tiles needed for taking away or comparing, you can use the zero property to add tiles to make take away or comparing possible.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How would you model* $4x - 2$*?* (I would use four x-tiles and two (-1)-tiles.)

• *What would you do to subtract?* (I figure it is just like integers, so I would add the opposite.)

• *What would the difference be?* ($(4x - 3x) + (5 - 2) = x + 3$)

The Exposition — Presenting the Main Ideas

• Display the polynomial $7x + 5$. Tell students you want to take away $2x + 3$. Ask what would be left. Ask why you might write the situation as $(7x + 5) - (2x + 3)$.

• Then have students model both polynomials: $7x + 5$ and $2x + 3$. Ask them to arrange the tiles for the first polynomial in a line. Have them place the tiles for the second polynomial in a line under the tiles for the first polynomial. Then ask how much more $7x + 5$ is than $2x + 3$.

• Next, ask students to think about how they would solve $7x - (-2x)$. They might suggest adding the opposite or using the zero property. They could add $2x + (-2x)$ and then subtract $(-2x)$. They would be left with $9x$, the result as adding the opposite. You can note that if they think of $7x - (-2x) = ?$ as the related addition equation $(-2x) + ? = 7x$, the result for the missing amount is still $9x$.

• With students, read through the exposition on **pages 217 and 218** of the student text. Make sure they relate what they read to the calculations you have performed with them. They should observe that they can subtract like terms separately to find the difference.

Revisiting the Try This

B. This question allows students to review the various meanings of subtraction to see what the possibilities are.

Using the Examples

• Let students read through **examples 1 and 2** so they can see some of the different ways they can approach polynomial subtraction.

Practising and Applying

Teaching points and tips

Q 1: Even though for most questions students can subtract using whichever method they choose, this question is designed to make sure they are aware of how each method might be used.

Q 3: Students should realize that there are many possible solutions for **parts a) and b)**. They need to find only one solution for each part.

Q 5: This question might be easiest for those students who think about adding the opposite.

Q 6: There are many possible solutions. Students should realize that all they need to do is add whatever they want. Then they use the sum as the minuend and what they added as the subtrahend.

Q 7: Some students will benefit by using particular values for a, b, c, and d.

Q 9: This question provides students with a model for making and testing a conjecture.

Common errors

• Many students forget to apply the subtraction to both parts of the polynomial.

For example, for $(8 + 4x) – (2x + 5)$, they might subtract the $2x$ but then add the 5.

Suggested assessment questions from Practising and Applying

Question 4	to see if students can subtract polynomials
Question 5	to see if students can reason about the difference of positive and negative polynomials
Question 6	to see if students can figure out two polynomials to subtract to get a particular difference

Answers

A. $x + 3$; *Sample response*: I subtracted the x-terms and then subtracted the integers just like I would subtract integers.	**B.** *Sample response*: I would add the opposite.
[**1. a) i)** Take away two x-tiles from five x-tiles, leaving three x-tiles. Take away one 1-tile from two 1-tiles, leaving one 1-tile. $(5x + 2) – (2x + 1) = 3x + 1$ **ii)** Compare the model for $5x + 2$ with the model for $2x + 1$. $5x$ is 3 more x-tiles than $2x$. Two 1-tiles is one more 1-tile than one 1-tile. $(5x + 2) – (2x + 1) = 3x + 1$ **iii)** Add three x-tiles and one 1-tile to $2x + 1$ to get $5x + 2$. $(5x + 2) – (2x + 1) = 3x + 1$ **iv)** Add two $–x$-tiles and one (-1)-tile to five x-tiles and two 1-tiles to get three x-tiles and one 1-tile. $(5x + 2) – (2x + 1) = (5x + 2) + (–2x – 1) = 3x + 1$ **b)** *Sample response*: Take away was easiest because I did not have to add any tiles to be able to subtract.	**c)** *Sample response*: No; Take away would not be as easy since I do not have any negative x-tiles to take away. I would first have to add tiles.] **2. a)** $2x + 3$ **b)** $2x + 13$ **c)** $6x + 13$ **d)** $6x + 3$ **3. a)** *Sample response*: $(10 + \mathbf{10}x) – (\mathbf{8}x + 5) = (2x + 5)$ **b)** *Sample response*: $(4 – \mathbf{2}x) – (–\mathbf{4}x – 5) = (2x + \mathbf{9})$ **c)** $(6 + \mathbf{2}x) – (3x + 5) = (–x + 1)$ **4. a)** $–1 + 5x$ **b)** $–12 + x$ **c)** $5x + 6$

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Answers [Continued]

5. All white tiles; [*Sample response*: For $2x - (-4x)$, the result is $6x$. When I subtract, I can think of it as adding the opposite, so if I am subtracting black tiles (negatives) from white tiles (positives), I am adding positives to positives.] **6.** *Sample responses*: **a)** $4x + 2 - (6x)$ $x + 2 - (3x)$ $8x + 5 - (10x + 3)$ **b)** $9x + 3 - (6x + 2)$ $9x + 5 - (6x + 4)$ $10x + 3 - (7x + 2)$ **7.** The differences are opposites: $5x - (3x + 2) = 2x - 2$ and $- 5x - (-3x - 2) = -2x + 2$	**8.** *Sample response*: More tiles in the difference: For $x - (-2x) = 3x$, A uses 1 tile, but C uses 3 tiles. Fewer tiles in the difference: For $3x - 2x = x$, A uses 3 tiles, but C uses 1 tile. The same number of tiles in the difference: For $(2x + 4) - (x - 1) = x + 3$, A uses 6 tiles and C uses 6 tiles. **9.** No; [*Sample response*: $(3x + 3) + (2x + 4)$ is represented with 12 tiles, but $(3x + 3) - (2x + 4)$ is represented with only two tiles.] **10.** Yes; [*Sample response*: You always subtract x-tiles from x-tiles and 1-tiles from 1-tiles.]

Supporting Students

Struggling students

• Some students may not recognize that you can test examples to answer **questions 5, 7**, **and 9**. Tell them that using examples to test ideas is an appropriate part of mathematics learning.

• For **question 8**, encourage students to try a variety of examples. After they have tried enough examples, they will be able to answer the question.

Enrichment

• Students could create polynomials to fit various conditions.

For example, you subtract a polynomial you can model with 4 tiles from a polynomial you can model with 10 tiles. The difference requires 14 tiles. What could the polynomials be? (*Sample response*: $(8x - 2) - (-x + 3)$)

7.3.3 EXPLORE: Multiplying a Polynomial by an Integer

Curriculum Outcomes	Outcome Relevance
8-B17 Multiplication by a Scalar (Polynomials): visually and symbolically • develop with concrete materials and diagrams using repeated addition (e.g., for $3(2x + 1) = 2x + 1 + 2x + 1 + 2x + 1$, model the binomial three times and combine the like terms using algebra tiles) • explore the area model to associate repeated multiplication	This essential exploration sets the stage for work in higher classes on multiplying polynomials. This is the first step in developing that skill.

Pacing	Materials	Prerequisites
40 min	• Algebra tiles or Algebra Tiles (BLM)	• representing a product as the area of a rectangle • familiarity with multiplication as repeated addition

Main Points to be Raised

• You can think of the product of a constant and an algebraic expression in terms of repeated addition.

For example, $2 \times (2x + 5) = (2x + 5) + (2x + 5)$.

• The variable x can be represented as a rectangle that is 1 unit by x units.

• You can think of the product of a constant and an algebraic expression as the area of a rectangle whose dimensions are the constant and the algebraic expression.

• To model negative integers, you can use black tiles instead of white tiles. The use of coloured algebra tiles is explained on **page 279**.

• To multiply an algebraic sum or difference by an integer, you can multiply each part of the sum or difference by that integer and then combine the parts.

Exploration

• With students, read through the introduction (in white) on **page 220** of the student text. Make sure they understand that the area of the rectangle is the product of the length and width because it shows another form of repeated addition, i.e., $3 \times 4 = 4 + 4 + 4$. Tell students that repeated addition means that the same value is added over and over.

• Use another example or two to remind students that to show any product, you can always draw a rectangle with the given dimensions and then calculate its area, e.g., for 5×6 you calculate the area of a rectangle with dimensions 5 and 6.

• Provide algebra tiles to pairs of students.

Assign students to work in pairs through **parts A to F**. While you observe students at work, you might ask questions such as the following:

• *How do you know that the width of the x tile is 1?* (It matches the width of the 1 tile, which is 1 by 1.)

• *How do you know the length of the tile must be x?* (The area is x, so if the width is 1, the length must be x. That is because you multiply the two numbers to get x.)

• *Why would you use both white and black tiles to model* $4 \times (3x - 2)$*?* (I have 4 copies of $3x - 2$. The xs are all white and the -2s are all black.)

• *Why do you think* $-2(2x + 5)$ *should be the opposite of* $2(2x + 5)$*?* (If I add -2 groups of $2x + 5$ to 2 groups of $2x + 5$, I have 0 groups, or zero. Opposites are numbers you add to get zero.)

Observe and Assess

As students work, notice the following:

• Can they represent a product as repeated addition?

• Do they relate the factors of a product to the dimensions of a rectangle with a given area?

• Do they show good judgment in predicting products, even when negative values are involved?

• Do they communicate well about the connection between multiplying integers and multiplying a polynomial by an integer?

Share and Reflect

After students have had sufficient time to work through the exploration, discuss these questions.

- *How are the models for* $3 \times (2 + 5)$, $4 \times (2x + 5)$, *and* $6 \times (2x + 5)$ *alike? How are they different?*
- *How would you model* $-2 \times (2x + 5)$*?*
- *How did using algebra tiles help you understand how to multiply a polynomial by an integer?*

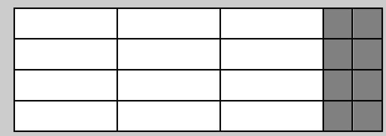
Answers

A. $(2x + 5) + (2x + 5) + (2x + 5) = 6x + 15$

B. i) The length of the x-tile is x and the width is 1;
The length and width of the 1-tile are both 1.
ii) *Sample response*:
The rectangle has dimensions 3 by $2x + 5$ so its area is the product of $3 \times (2x + 5)$.
iii) The area of the rectangle is made up of 6 x-tiles and 15 one-tiles so the product is $6x + 15$.

C. *Sample response*:
I could have multiplied 3 by 2 and multiplied 3 by 5, and just included x when I multiplied the x-term, $2x$.

D. *Predictions will vary*
i) $12x - 8$



ii) $12 - 4x$

E. $-4x - 10$; *Sample response*:
It would be the opposite of $2 \times (2x + 5)$. Since that product is $4x + 10$, this product must be $-4x - 10$.

F. *Sample responses*:
You multiply the integer by the number that is the coefficient. Then you multiply the integer by the constant. Last, you add the two terms together.

Supporting Students

Struggling students

- Most students will have little difficulty with the idea of multiplying a polynomial by an integer. Some students may understand the underlying concepts better if they use repeated addition rather than using the area of rectangles. Allow them to do that.

Enrichment

- Students might explore the product of a polynomial multiplied by another polynomial.

For example, $2x(3x + 5)$ or $(2x + 1)(3x + 5)$.

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UNIT 7 Revision

Pacing	Materials
2 h	• Grid paper or Small Grid Paper (BLM) • Algebra tiles or Algebra Tiles (BLM)

Question(s)	Related Lesson(s)
1 – 4	Lesson 7.1.2
5 – 7	Lesson 7.1.3
8 – 11	Lesson 7.1.4
12 – 14	Lesson 7.2.1
15	Lesson 7.2.2
16	Lesson 7.2.3
17 – 19	Lesson 7.3.1
20 and 21	Lesson 7.3.2
22	Lesson 7.3.3

Revision Tips

Q 1: Students should focus on what happens when you double or triple odd or even numbers and what happens when you add odd or even numbers.

Q 3: If students use a graph rather than an equation, they might need to estimate rather than calculating an exact value for the answer.

Q 6: Students could choose to draw graphs, consider whether each equation contains an exponent no greater than 1, or look for constant increases to decide whether each relationship is linear.

Q 8: Because of the scale, it will be hard for students to be precise. You should accept estimates.

Q 9 b): Students need to use only two of the points, but they may wish to check with the third point.

Q 10: There are many correct answers. All of the possible lines are parallel.

Q 11: Students should think about whether the y-value increases or decreases with an increase in the x-value.

Q 13 c): The simplest equation for a fractional solution like this is to multiply the fraction by the denominator to get the numerator.

Q 16: Students can solve the equations using a graph or a single equation that includes both pieces of information.

Answers

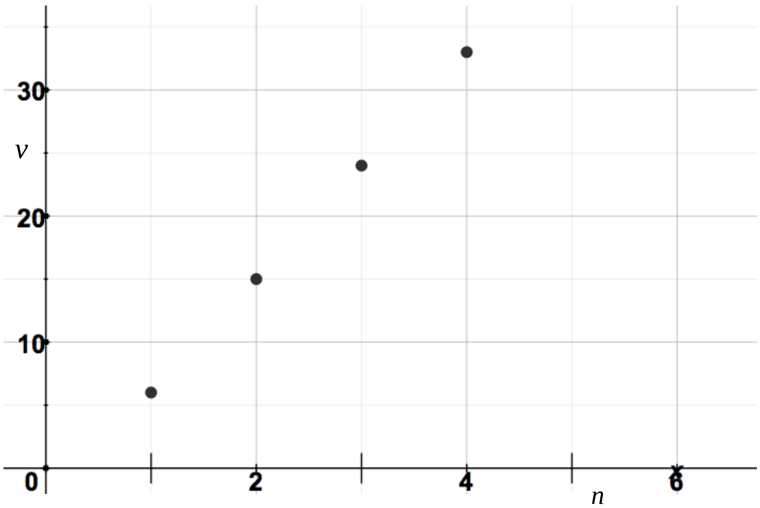
1. *Sample responses*:
a) 20 and 0; 2 and 27; 10 and 15
[**b)** Any whole number that is doubled is even, so the double of B is even. Any odd number that is tripled is odd. If you multiply an odd number by 3 and then add the resulting odd number to double B (an even number), you get an odd number, not 60.
c) I could use the three pairs from **part a)** as ordered pairs. Plot the points and draw a line through all three. Then I could look for other whole number pairs along the line.]

[Continued]

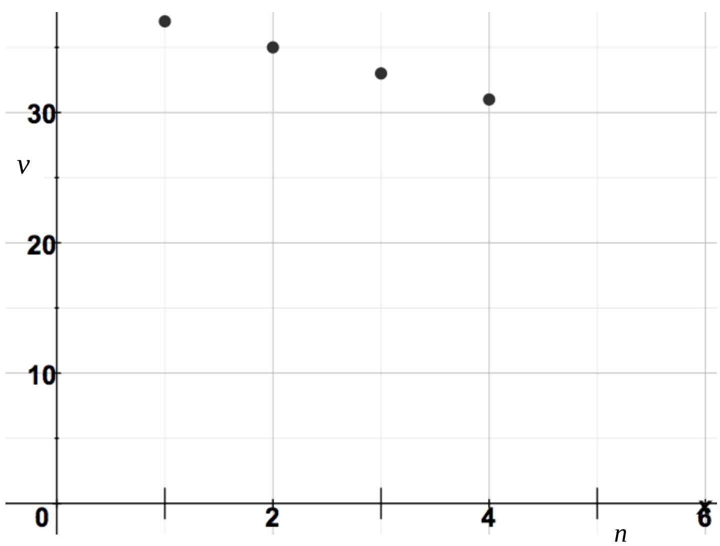
Answers [Continued]

2. If n is the term number and v is the term value:

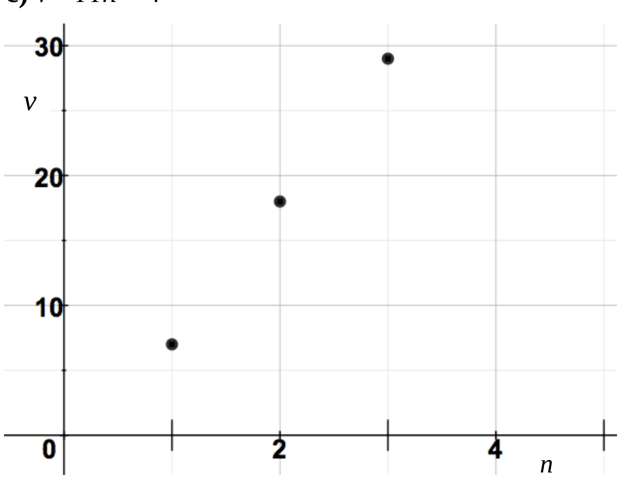
a) $v = 9n - 3$



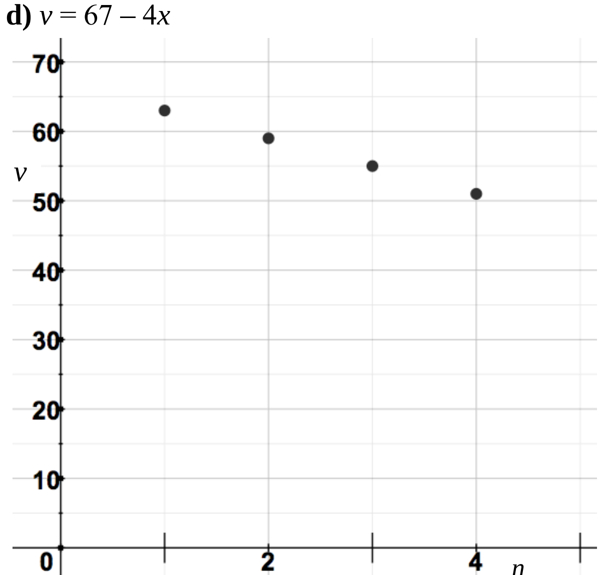
b) $v = 39 - 2n$



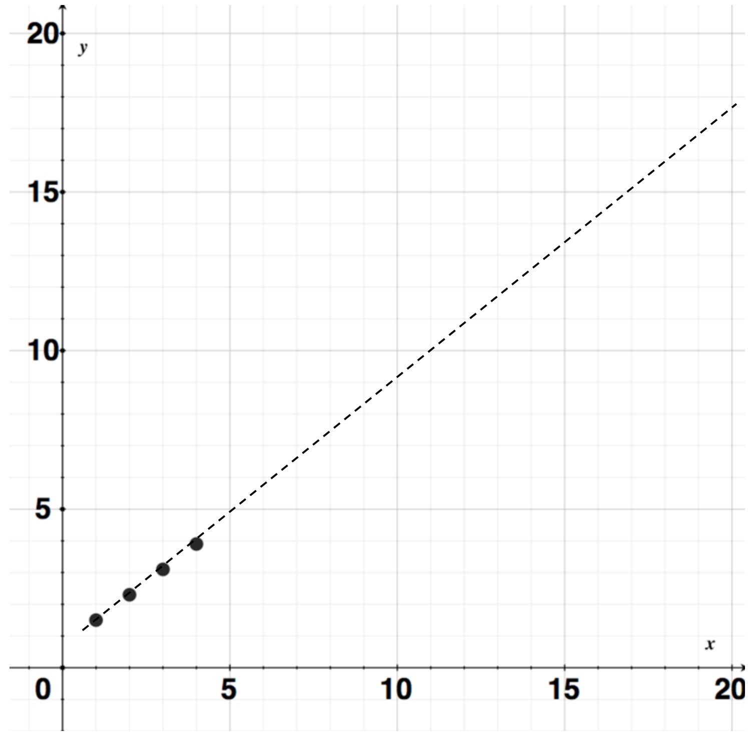
c) $v = 11n - 4$



d) $v = 67 - 4x$

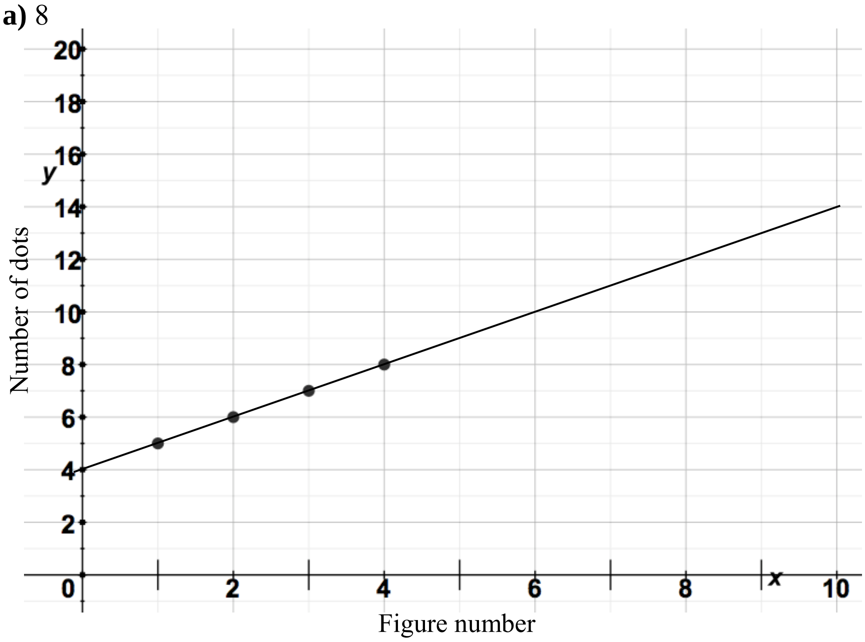


3. a) $v = 7n + 1$, so the 20th term is $7 \times 20 + 1 = 141$.
b) $v = 48 - 6n$, so the 20th term is $48 - 6 \times 20 = 48 - 120 = -72$.
c) $v = 5n + 12$, so the 20th term is $5 \times 20 + 12 = 112$.
d)



The y-value for $x = 20$ is about 16.5. (If you use an equation, it is actually 16.7.)

4. a) 8
b)



c) 14
d) *Sample response*:
I can test with the algebraic expression $y = x + 4$ (x is the figure number and y is the number of dots).
$y = 10 + 4 = 14$.

5. B is linear; [The increases in x and in y are always the same; A is not linear because y increases by a different amount each time even though x increases by the same amount each time.]

6. a) Linear; [*Sample response*:
The perimeter is 4 times the side length, so the perimeter increases by a constant amount of 4 for each increase of 1 in the side length.]
b) Linear; [*Sample response*:
I measured the side length and diagonal of several squares. Each time the side length increased by 1, the diagonal length increased by about 1.4.]
c) Not linear; [*Sample response*:
When the side length increases by 1, from 1 to 2, the area increases by 3.
When the side length increases by 1, from 5 to 6, the area increases by 11.
The side length increases by a constant amount but the area does not.]

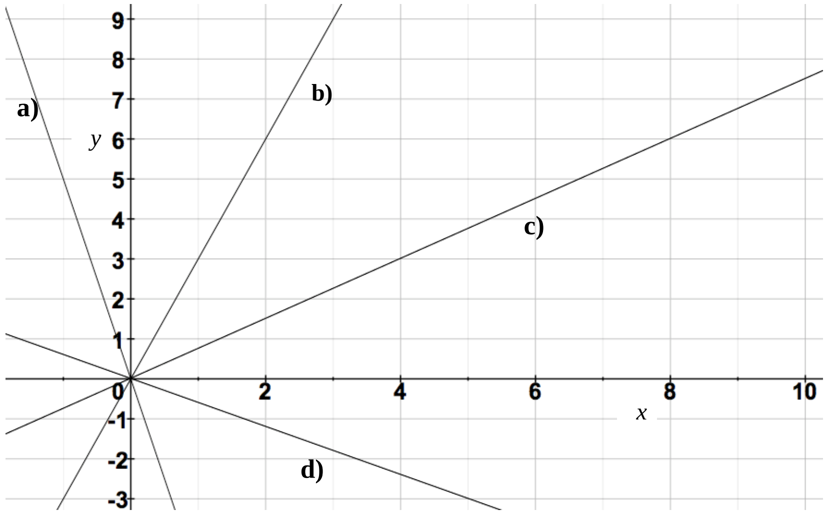
7. No; [*Sample response*:
One of variables has an exponent of 2. A linear equation only has variables with an exponent of 1.]

8. *Sample responses*:
a) –2 **b)** 4 **c)** 3

9. a) 2 **b)** –1 **c)** 3

Answers [Continued]

10.



11. a) (5, 4), (3, **8**), (**11**, –8)
b) (3, 10), (8**, 0**), (**–6**, 28)

12. a) $x = -5$ **b)** $x = 10$

c) $x = \frac{5}{8}$ or 0.375 **d)** $x = \frac{39}{5}$ or $7\frac{4}{5}$ or 7.8

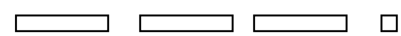
13. *Sample responses*:
a) $2m = 14$ $8 - m = 1$ $3m + 7 = 28$
b) $5n = 1$ $10n = 2$ $11 - 5n = 10$
c) $5x = 3$ $10x = 6$ $-15x = -9$
d) $-k = 3$ $2k + 11 = 5$ $30 + 5k = 15$

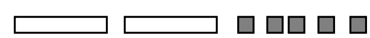
14. a)
• First, add 2 to both sides to get rid of –2 on the left side.
• Then multiply both sides by 5 to get rid of the denominator 5 on the left side.
• Then divide both sides by 2 to get rid of the coefficient of 2 on the left side.
• *x* is now alone on the left side so I know its value, which is on the right side.
b) $x = 47.5$

15. a) $30 + 24n = 95$; *n* is number of hours

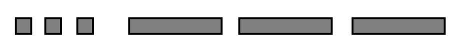
b) $n = \frac{65}{24} = 2\frac{17}{24}$ h

16. a) (–1, –1)
b) (10, 73)
c) $\left(\frac{50}{7}, \frac{40}{7}\right)$ or $\left(7\frac{1}{7}, 5\frac{5}{7}\right)$

17. a) 

b) 

c) 

d) 

18. a) $6x$, x, and $2x$
b) $5x$, $9x$, and $-3x$; 13 and –2
c) $-4x$ and $-7x$; 8 and –5

19. a) $9x - 8$ **b)** $11x + 11$ **c)** $-11x + 3$

20. *Sample response*:
$(4x + 8)$ and $(3x + 6)$; [$(4x + 8) - (3x + 6) = x + 2$; $x + 2$ can be modelled with 3 tiles: one *x*-tile and two 1-tiles.]

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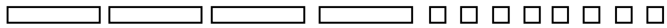
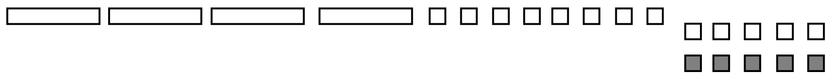
21. *Sample responses*:

a) $(4x + 8)$ and $(2x + 13)$; $[(4x + 8) - (2x + 13) = 2x - 5]$

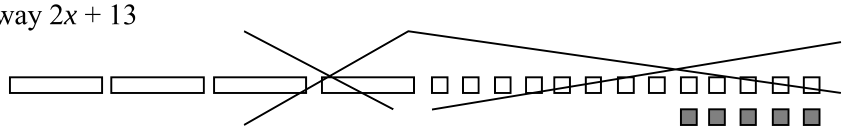
b)

$4x + 8$ 

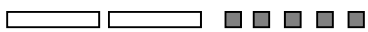
Add five (–1)-tiles and five 1-tiles



Take away $2x + 13$



$2x - 5$ is left



22. $28 - 8x$; [*Sample response*: $(7 - 2x) + (7 - 2x) + (7 - 2x) + (7 - 2x) = 28 - 8x$]

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UNIT 7 Algebra Test

1. Represent the relationship between the term number and the term value for each pattern. Use both an algebraic equation and a graph for each.

a) 67, 64, 61, 58, …

b) 21, 25, 29, 33, …

2. Use your equation or graph from **question 1** to predict the 20th term in each pattern.

3. Complete the table so the relationship is linear.

x	***y***
1	13
2	
3	
4	34

4. Which of these relationships is linear? How do you know?

A. the distance you travel in *h* hours if you go at a speed of 28 km/h

B. the area of a circle compared to its radius

C. the volume of a cube compared to its edge length

D. the money you would earn in *d* days, if your salary starts at Nu 50 per day and increases after you have worked for one month

5. A line passes through these points. Calculate each slope.

a) (4, 8) and (7, 15)

b) (–2, 4) and (1, –9)

6. Draw a line that goes through (2, 1) and has a slope of –4.

7. Solve each equation.

a) $5 - 3x = -16$

b) $5 - 3x = 8x - 39$

8. Yangka is 150 km from home when he starts a trip toward home. He travels at a speed of 29 km/h.

a) Write an equation that you could solve to find how many hours he needs to travel before he is 20 km from home.

b) Solve your equation.

9. At what point does each pair of lines intersect?

a) $y = -x + 6$ and $y = -4x + 15$

b) $y = 2x - 9$ and $y = -3x + 15$

10. Dechen multiplies a number by 4 and then subtracts 8. The result is the same as if she subtracted 8 times the number from –32. What number did Dechen start with?

11. Show the steps to calculate each sum or difference using algebra tiles. Write each result as a polynomial.

a) $(2x - 8) + (-x + 5)$

b) $(8x - 5) - (3x - 2)$

c) $(x + 8) - (-x - 4)$

12. List two pairs of possible polynomials for each.

a) the sum of the pair is $-4x - 7$

b) the difference of the pair is $-x - 5$

13. a) Draw a picture to show what $4 \times (3x - 4)$ means.

b) Calculate the product.

UNIT 7 Test

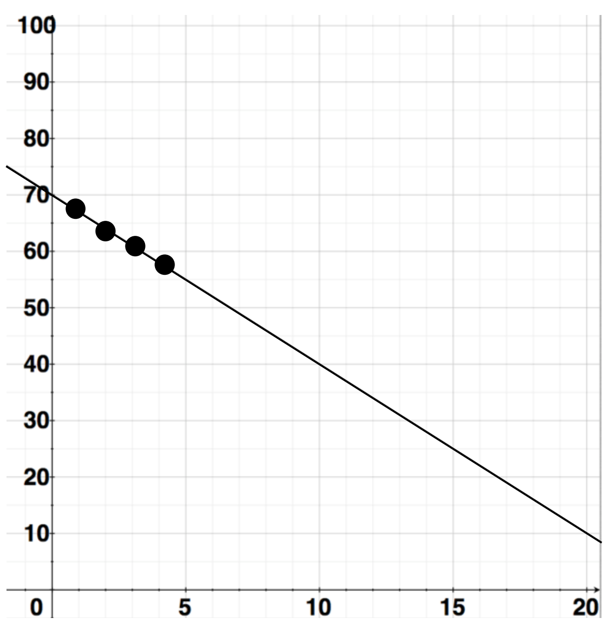
Pacing	Materials
1 h	• Grid paper or Small Grid Paper (BLM) • Algebra tiles or Algebra Tiles (BLM)

Question(s)	Related Lesson(s)
1 and 2	Lesson 7.1.2
3 and 4	Lesson 7.1.3
5 and 6	Lesson 7.1.4
7	Lesson 7.2.1
8	Lesson 7.2.2
9 and 10	Lesson 7.2.3
11 and 12	Lessons 7.3.1 and 7.3.2
13	Lesson 7.3.3

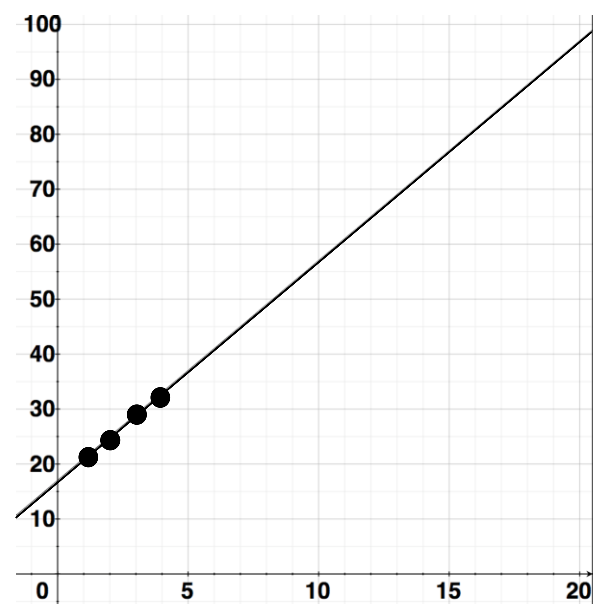
Select questions to assign according to the time available.

Answers

1. a) $v = 70 - 3n$



b) $v = 17 + 4n$



2. a) 10 **b)** 97

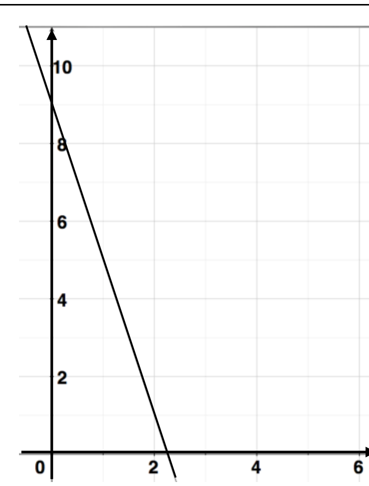
3.

x	y
1	13
2	**20**
3	**27**
4	34

4. A; It is the only relationship where the increase in y is a constant amount for each constant increase in x.

5. a) $\frac{7}{3}$ **b)** $-\frac{13}{3}$

6.



Answers [Continued]

7. a) $x = 7$ **b)** $x = 4$

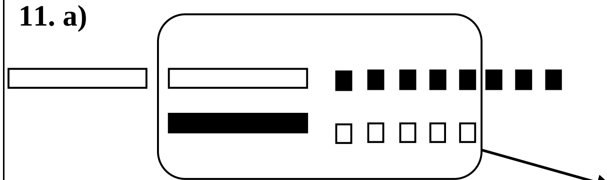
8. a) *Sample response*:
$20 = 150 - 29n$, n is number of hours

b) $n = \frac{130}{29}$, or $4\frac{14}{29}$ h (or about $4\frac{1}{2}$ h)

9. a) $x = 3$, $y = 2$ **b)** $x = 4\frac{4}{5}$, $y = \frac{3}{5}$

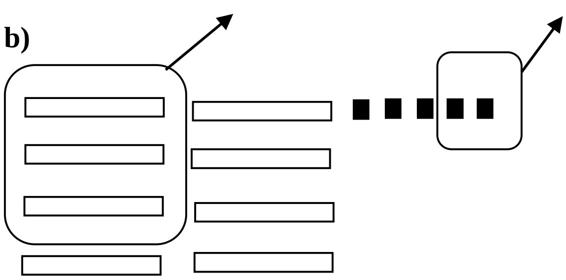
10. –2

11. a)



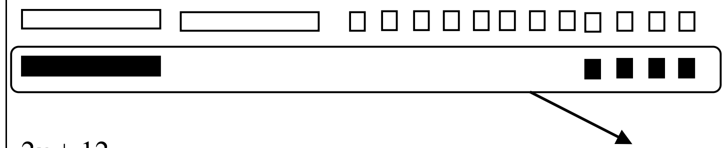
$x - 3$

b)



$5x - 3$

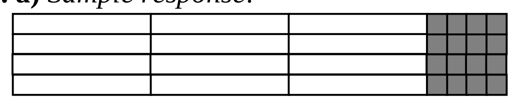
11. c)



$2x + 12$

12. *Sample responses*:
a) $(-2x - 4) + (-2x - 3)$ or $(5x + 4) + (-9x - 11)$
b) $(2x + 3\ ) - (3x\ 8)$ or $(-2x - 4) - (-x + 1)$

13. a) *Sample response*:



b) $12x - 16$

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UNIT 7 Performance Task — Relating Ages

A. i) Create a pattern that begins with the number 35.
Subtract 3 to get the next term. List the first five terms.

ii) Write an algebraic equation for in the pattern in **part i)** that relates the term value (*v*) to the term number (*n*).

iii) Graph the relationship in **part ii)**.

iv) What is the slope of the line in **part iii)**? How can you predict the slope from the information in **part i)**?

B. Rinzin says that if you take his sister's age, triple it, and then subtract the result from 38, you get his age.

i) Suppose Rinzin is 14 years old. Write an equation that you could solve to figure out his sister's age.

ii) Solve the equation.

iii) How does the equation you wrote relate to the graph from **part A**?

C. How can you tell from the graph in **part A** that Rinzin's sister must be less than 13 years old?

D. Recall the algebraic equation you wrote in **part A ii)**. Write the algebraic expression from that equation.

i) Find two polynomial expressions that you can add together to give you the algebraic expression from **part A ii)** as the sum. Describe how you would use algebra tiles to add the expressions.

ii) Find two polynomial expressions that you can subtract to give you the algebraic expression from **part A ii)** as the difference. Describe how you would use algebra tiles to subtract the expressions.

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UNIT 7 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
8-C1 Patterns and Relations: represent in a variety of formats **8-C2** Graphs (Linear and Non-linear): interpret **8-C3** Graphs and Tables (Linear and Non-linear): how changing one quantity affects the other **8-C5** Single Variable Equations: solve algebraically **8-C6** Linear Equations: create and solve problems **8-B16** Polynomial Expressions: add and subtract visually	1 h	• Grid paper or Small Grid Paper (BLM) • Algebra tiles or Algebra Tiles (BLM) (optional)

How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could supplement the unit test. It could also be used as enrichment material for some students.

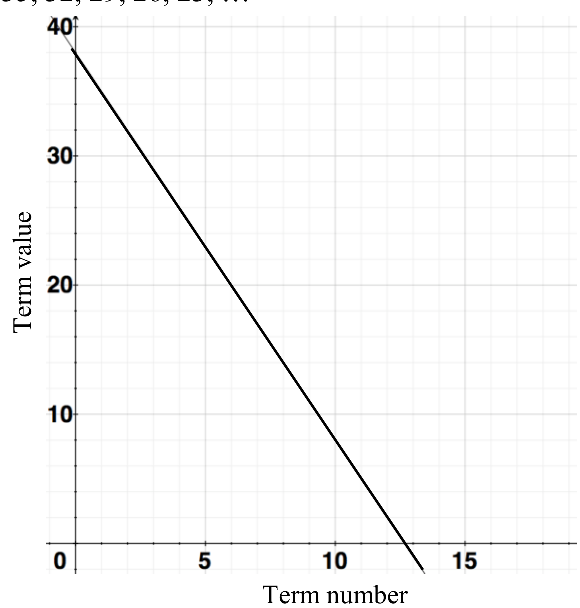
• You can assess performance on the task using the rubric provided on the next page.

Sample Solution

A. i) 35, 32, 29, 26, 23, …

ii) $v = 38 - 3n$

iii)



iv) –3; I know from the pattern that when the term number increases by 1, the value goes down by 3.

B. i) $14 = 38 - 3n$, where n is Rinzin's sister's age.

ii) $n = 8$

iii) The graph shows $v = 38 - 3n$. The equation $14 = 38 - 3n$ is one point along that graph, where $v = 14$. If I find the point on the line with a v-coordinate of 14, the n-coordinate is Rinzin's sister's age.

C. The value of v is negative when n is 13 or more, so if Rinzin's sister's age (the n-coordinate) is 13 or more, Rinzin's age (the v-coordinate) is negative, and that is not possible.

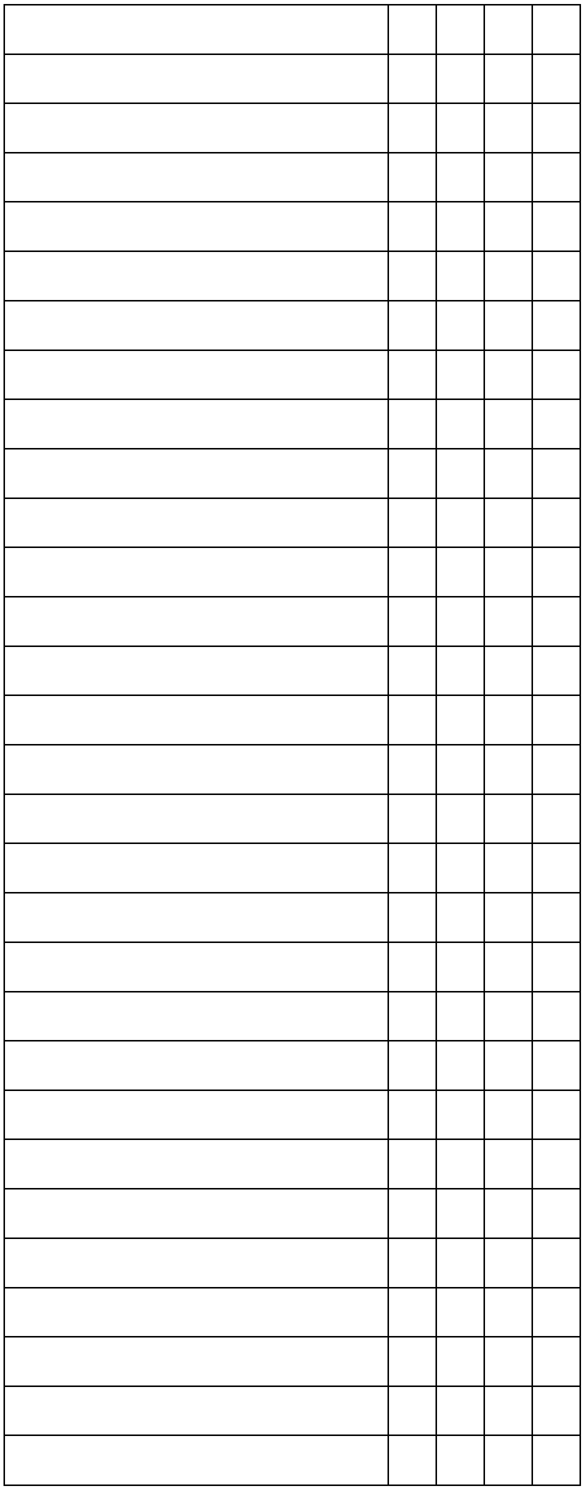
D. i) $(24 + 4x) + (14 - 7x) = 38 - 3x$ or $(24 + 4n) + (14 - 7n) = 38 - 3n$;
• Add 24 white 1-tiles and 4 white x-tiles to 14 white 1-tiles and 7 black x-tiles.
• Combine the 24 white x-tiles and 14 white x-tiles to get 38 white x-tiles altogether.
• Combine the 1-tiles and end up with 3 black 1-tiles because there are 4 pairs of white and black 1-tiles that have a value of zero.

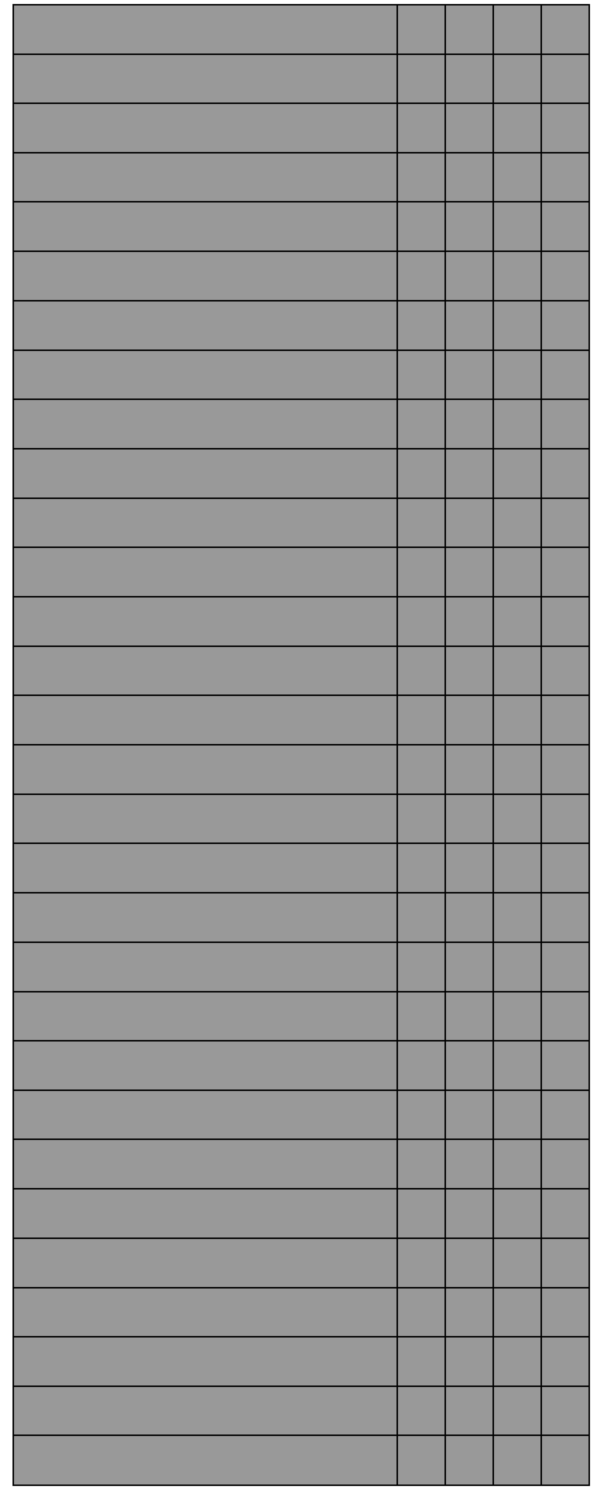
ii) $(5x + 40) - (8x + 2) = 38 - 3x$ or $(5n + 40) - (8n + 2) = 38 - 3n$
• Start with 5 white x-tiles and 40 white 1-tiles.
• Add 3 white tiles x-tiles and 3 black x-tiles so I can take away 8 white x-tiles.
• Take away 8 white x-tiles and end up with 3 black x-tiles.
• Take away 2 white 1-tiles and end up with 38 white 1-tiles.

Unit 7 Performance Task Assessment Rubric

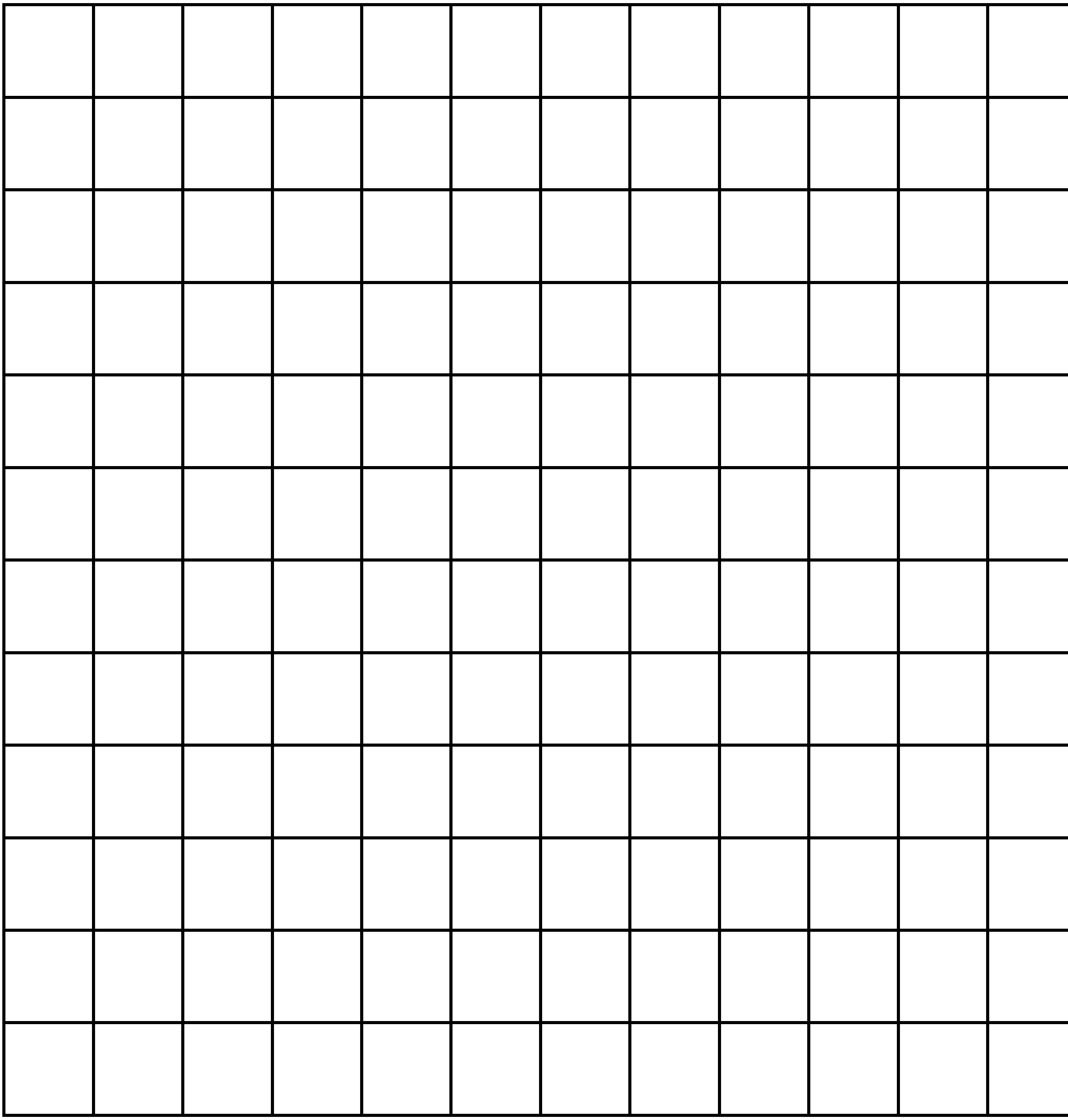
The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Uses algebraic expressions to describe situations	Insightfully and efficiently uses algebraic expressions to describe patterns and solve equations	Correctly uses algebraic expressions to describe patterns and solve equations	Mostly correctly uses algebraic expressions to describe patterns and solve equations	Has difficulty using algebraic expressions to describe patterns and solve equations
Interprets and creates graphs	Efficiently and accurately graphs algebraic relationships and insightfully uses the graph to interpret information about a situation and to solve an equation	Accurately graphs algebraic relationships and uses the graph to interpret information about a situation and to solve an equation	Correctly graphs algebraic relationships and uses the graph either to interpret information about a situation or to solve an equation, but not both	Has difficulty graphing and interpreting algebraic relationships
Uses models to add and subtract polynomials	Correctly uses and clearly describes how to use models to describe polynomial addition and subtraction	Correctly uses and somewhat clearly describes how to use models to describe polynomial addition and subtraction	Correctly provides polynomials that add or subtract to a given polynomial but has difficulty describing the process	Has difficulty using or describing models to show polynomial addition and/or subtraction

BLM 1 Algebra Tiles





BLM 2A Alge-Scrabble Game Board



BLM 2B Alge-Scrabble Game Tiles

+	+	+	–	–	–	÷	÷	÷	×	×	×
a	*a*	*a*	*b*	*b*	*b*	*c*	*c*	*c*	*n*	*n*	*n*
=	=	=	=	=	=	=	=	=	=	=	=
1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	25										

UNIT 8 GEOMETRY

UNIT 8 PLANNING CHART

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Getting Started **SB p. 223** **TG p. 305**	Review prerequisite concepts, skills, and terminology and pre-assessment	1 h	• Linking cubes • Rulers and protractors • Isometric Dot Paper (BLM)	All questions
Chapter 1 Representing Objects				
8.1.1 Isometric Drawings **SB p. 225** **TG p. 307**	**8-E1 Interpret and Make Orthographic Drawings and Isometric Drawings: to represent 3-D shapes** • use interlocking cubes to explore attributes of 3-D shapes • compare constructions to determine how they are different and the same • use cubes to build structures from isometric drawings • make isometric drawings of cube structures • continue to develop visualization skills by physically exploring the results of moving objects and structures in a variety of ways	1 h	• Linking cubes • Isometric Dot Paper (BLM)	Q4, 5, 7
8.1.2 Orthographic Drawings **SB p. 229** **TG p. 311**	**8-E1 Interpret and Make Orthographic Drawings and Isometric Drawings: to represent 3-D shapes** • use interlocking cubes to explore attributes of 3-D shapes • use cubes to build structures from a set of orthographic drawings • make orthographic drawings of cube structures • compare constructions to determine how they are different and the same • continue to develop visualization skills by physically exploring the results of moving objects and structures in a variety of ways	1 h	• Linking cubes • Isometric Dot Paper (BLM) • Grid paper or Small Grid Paper (BLM) (optional)	Q2, 6, 10
Chapter 2 Transformations				
8.2.1 Dilatations **SB p. 233** **TG p. 315**	**8-E2 Dilatations: represent, analyse, and apply** • understand that the dilatation centre and scale factor must be identified in order to locate the position of the dilatation image	1 h	• Rulers • Torch (flashlight) • Cut-out cardboard triangle	Q2, 3, 7
8.2.2 Combining Transformations **SB p. 239** **TG p. 319**	**8-E2 Dilatations: represent, analyse, and apply** • explore combinations of transformations that include dilatations, such as an enlargement followed by a reflection	2 h	• Protractors, rulers, and compasses • Tracing paper	Q1, 5, 7
GAME: Isometry (Optional) **SB p. 244** **TG p. 322**	To help students practise identifying rotations, translations, and reflections in a game situation	30 min	• Isometric Dot Paper (BLM)	N/A

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
Chapter 3 Angle Relationships				
8.3.1 EXPLORE: Measuring Angles in Polygons (Essential) **SB p. 246** **TG p. 323**	**8-E3 Polygons: properties and interrelationships** • develop a chart to observe and extend patterns and generalize about the sum of the measures of the interior angles and exterior angles of various polygons, and the measure of each interior angle of a regular polygon **8-C2 Graphs (Linear and Non-linear): interpret** • construct graphs to determine if a relationship is linear (e.g., graph the sum of the interior angles of a polygon against the number of sides)	1 h	• Rulers and protractors • Grid paper or Small Grid Paper (BLM) • Regular Polygons (BLM)	Observe and Assess questions
8.3.2 Angles in Polygons **SB p. 247** **TG p. 325**	**8-E3 Polygons: properties and interrelationships** • understand, through investigating, that the sum of the measures of the interior angles of a polygon is found by dividing the polygon into triangles	1 h	• Rulers • Regular Polygons (BLM)	Q1, 5, 7
8.3.3 Angles With Parallel and Intersecting Lines **SB p. 251** **TG p. 328**	**8-E4 Angle Pair Relationships: parallel and non-parallel lines** • understand that corresponding angles and alternate angles are only equal when a transversal intersects two parallel lines • understand that interior angles are supplementary when a transversal intersects two parallel lines • apply transformational geometry to discover why the various angle pairs are equal	1 h	• Rulers, protractors, and compasses • Tracing paper	Q1, 2, 7, 10
CONNECTIONS: Tools for Geometry (Optional) **SB p. 256** **TG p. 333**	Make a connection between geometric work and experiences outside the classroom	20 min	None	N/A
UNIT 8 Revision **SB p. 257** **TG p. 334**	Review the concepts and skills in the unit	2 h	• Rulers, protractors, and compasses • Tracing paper • Isometric Dot Paper (BLM) • Linking cubes	All questions
UNIT 8 Test **TG p. 337**	Assess the concepts and skills in the unit	1 h	• Rulers, protractors, and compasses • Tracing paper • Isometric Dot Paper (BLM) • Linking cubes	All questions

	Outcomes or Purpose	Suggested Pacing	Materials	Suggested Assessment
UNIT 8 Performance Task **TG p. 340**	Assess concepts and skills in the unit	1 h	• Rulers, protractors, and compasses • Tracing paper • Isometric Dot Paper (BLM) • Linking cubes • Grid paper or Small Grid Paper (BLM)	Rubric provided
UNIT 8 Assessment Interview **TG p. 342**	Assess concepts and skills in the unit	15 min	• Linking cubes • Protractors • Isometric Dot Paper (BLM) • Grid paper or Small Grid Paper (BLM)	All questions
UNIT 8 Blackline Masters **TG p. 343**	BLM 1 Isometric Dot Paper BLM 2 Regular Polygons Small Grid Paper on **page 32** in UNIT 1			

Math Background

• This unit extends students' understanding of geometry.

• The focus of the unit is on representing 3-D objects, transformations that include dilatations, and angle properties with polygons and parallel lines.

• As students proceed through this unit they will use a variety of mathematical processes, including problem solving, communication, reasoning, representation, visualization, and making connections.

For example:

• Most lessons involve problem solving. Students use problem solving in **question 4** in **lesson 8.1.1**, where they identify possible hidden cubes, in **question 6** in **lesson 8.2.1**, where they find dilatation centres to meet certain conditions, in **question 6** in **lesson 8.2.2**, where they find combinations of transformations that fit challenging criteria, and in **question 7** in **lesson 8.3.2**, where they draw on knowledge from previous years to calculate angle measurements.

• Students use communication frequently as they explain their thinking in **question 3** in **lesson 8.1.1**, where they explain why isometric drawings of the same structure could be different, in **question 2** in **lesson 8.2.1**, where they explain why a pair of rectangles cannot represent a dilatation, and in **questions 2, 4, and 6** in **lesson 8.2.2,** where they explain why the order of the transformations matters or does not matter in each situation.

• Students use reasoning in **question 4** in **lesson 8.1.1**, where they explain why there can only be a limited number of hidden cubes in a drawing, in **question 4** in **lesson 8.1.2**, where they recognize that the same view can apply to several structures, in **question 7** in **lesson 8.2.1**, where they explain why the image of a dilatation enlargement cannot be inside the original shape, in **question 3** in **lesson 8.3.2**, where they explain the limitations of a formula, and in **question 5** in **lesson 8.3.2**, where they explain how to use a formula backwards, finding the input value given a result.

• Students consider representation throughout the unit, as all their geometric work involves paper and pencil representations of physical phenomena. This is especially powerful in **chapter 1**, where they draw 3-D shapes and make 3-D shapes given 2-D drawings. Another form of representation, which is not usually considered to be geometry, appears in **part B** of **lesson 8.3.1**, where students use a graph to help them identify numeric patterns.

• Students use visualization skills throughout the unit, as all of geometry involves visualization. In **question 6** in **lesson 8.1.1**, students identify drawings that represent the same structure, in **question 2** in **lesson 8.1.2**, they look at a structure and figure out how to build it, in **question 6** in **lesson 8.2.1**, they locate centres of dilatations, in **questions 5 and 8** in **lesson 8.2.2**, they visualize intermediate images in combinations of transformations, and in **question 6** in **lesson 8.3.2**, they imagine themselves walking a path in the snow.

• Students make connections between areas of mathematics in **lesson 8.2.1**, where they connect the concepts of dilatation and similarity, in **questions 6 and 10** in **lesson 8.3.3**, where they bring together knowledge from various previous experiences to solve problems, in **questions 7, 8, and 9** in **lesson 8.3.3**, where they use transformations to support their understanding of angle properties in parallel lines, and in **lesson 8. 3.1**, where they use graphing skills to solve a geometry problem. They also make connections between their school mathematics and everyday experiences throughout **chapter 1,** where they connect 2-D representations with simple real-world physical objects, and in the **Connections**, where they think about improvised geometric tools.

Rationale for Teaching Approach

• This unit is divided into three chapters:

Chapter 1 focuses on representing 3-D objects in 2-D drawings.

Chapter 2 involves transformations and introduces dilatations.

Chapter 3 examines angle properties in polygons and in situations involving parallel lines.

• The **Explore** lesson allows students to discover patterns involving the angles in polygons. It gives students experience in developing formulas that reflect the patterns they uncover.

• The **Connections** helps students discuss ways of applying their geometry skills in real-world situations in which tools are limited.

• The **Game** provides an opportunity to practise identifying transformations and to visualize shapes on isometric grid paper in a pleasant way.

• Throughout the unit, it is important to encourage and accept a variety of approaches from students.

Getting Started

Curriculum Outcomes	Outcome relevance
5 Isometric drawings: make and interpret **6** Orthographic drawings: make and interpret **7** Angles: estimate and measure angles using a protractor **7** Triangles: classify **7** Transformations: properties of translations, reflections, and rotations	Students will find the work in the unit easier after they review related geometry skills and concepts from earlier classes.

Pacing	Materials	Prerequisites
1 h	• Linking cubes • Rulers and protractors • Isometric Dot Paper (BLM)	• familiarity with the terms *translation, rotation, reflection, transformation, regular hexagon, equilateral triangle, isosceles triangle, scalene triangle, acute triangle, right triangle,* and *obtuse triangle* • performing and recognizing reflections, rotations, and translations • making isometric or orthographic drawings of simple cube structures

Main Points to be Raised

Use What You Know

• 3-D structures with the same volume (made of the same number of cubes) can have different shapes.

• You can build a cube structure according to specific guidelines by visualizing different arrangements of the cubes.

• There are different ways of drawing 3-D shapes.

Skills You Will Need

• 2-D shapes are classified by angles, lengths of sides, and number of sides.

• A translation is described by its direction and distance.

• A rotation is described by the turn centre and the angle of rotation.

• A reflection is described by its line of reflection.

• Different transformations may result in the same image.

Use What You Know — Introducing the Unit

• Show students a few simple cube structures, including the structures shown in **part A**. Ask them how many cubes are in each structure. Ask them which of the structures you built are the same as the structures shown in **part A**, and how they know that they are the same.

• Let students work in pairs to do **parts A and B**. Give them plenty of cubes to use (40 cubes will be enough to make all the cube structures. It would be good to give each group a few more than 40).

While you observe students at work, you might ask questions such as the following:

• *How do you know that there are no more structures with three cubes?* (No matter where I put this second cube, the result is the same. When I add a third cube, there are only two possible results.)

• *Why do you predict there will be more structures if there are more cubes?* (If I have an extra cube, I can add it to different places on the structure with fewer cubes, so that means one structure could become more than one other structure when I add more cubes.)

• After students have answered these questions, ask groups to report their results for **part A**. Where there are discrepancies in their answers, ask the groups that found more structures to show the class their extra shapes.

• Ask students to describe the methods they used for drawing their structures. Invite individuals to show examples of their methods of drawing on the board. Show both orthographic and isometric drawings.

Skills You Will Need

• Students can work individually.

• To ensure students have the required skills for this unit, assign all of the questions.

• Encourage students to think about side length, angle measure, and number of sides to answer **question 1**.

• Review the terms *translation, rotation, reflection,* and *transformation* to make sure students can interpret **questions 2 and 3**. Refer students to the glossary at the back of the student text.

Answers

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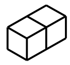
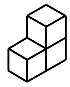
A. i) 1 **ii)** 2 **iii)** 8

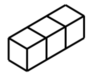
B. *Sample responses*:

i)



ii)





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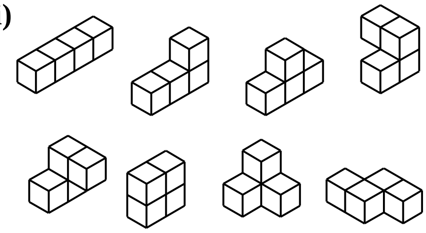
B. iii)



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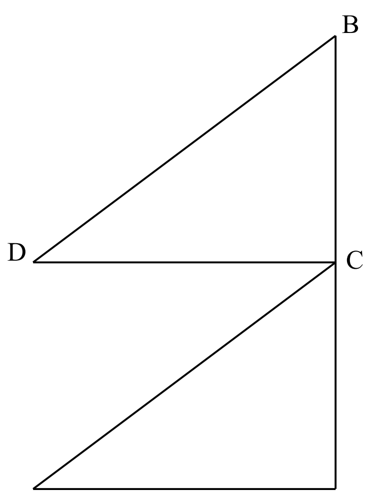
1. a) Angles are all 60º, side lengths are all 3.4 cm; Acute equilateral triangle.
b) Angles are 30º, 60º, and 90º, side lengths are 3.4 cm, 5.9 cm, and 6.9 cm; Scalene right triangle.
c) Angles are 35º, 35º, and 110º, side lengths are 3.4 cm, 3.4 cm, and 5.5 cm; Isosceles obtuse triangle.
d) Angles are all 120º, side lengths are all 3.4 cm; Regular hexagon.

2. a) 3 cm down along BC.
b) Turn centre is the midpoint of CD, angle of rotation is 180º.
c) Reflection line is CD.

3. a)



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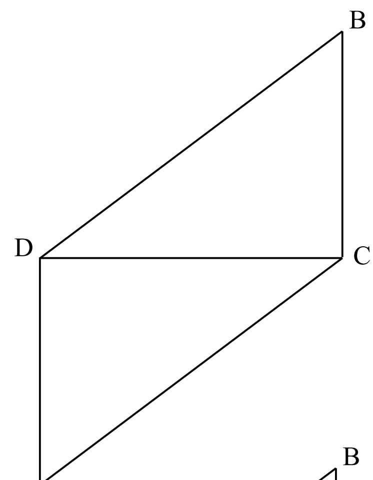
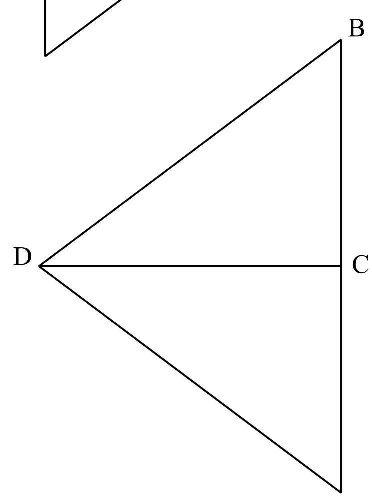
b)



c)



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Supporting Students

Struggling students

• If students are trying to decide whether they have all the possibilities in **part A**, ask them to build the structures systematically.

For example, for a four-cube structure, start with a three-cube stick and think about the different places you can put the fourth cube. Then start with a two-cube stick, think about the different places you could put the third cube, and then for each of these possibilities, think about the different places you could put the fourth cube.

• Some students may have trouble drawing their structures for **part B**. You might give them an example by taking one of their structures and explaining what you are doing as you draw it.

• For **question 1**, you might list some of the terms students might use to classify the triangles.

Enrichment

• For **part A**, you might challenge students to predict a pattern that tells them how many structures can be built with five cubes, six cubes, and so on. They could then test their prediction by finding all the possible cube structures possible. This gets much harder each time you add another cube.

8.1.1 Isometric Drawings

Curriculum Outcomes	Outcome relevance
8-E1 Interpret and Make Orthographic Drawings and Isometric Drawings: to represent 3-D shapes • use interlocking cubes to explore attributes of 3-D shapes • compare constructions to determine how they are different and the same • use cubes to build structures from isometric drawings • make isometric drawings of cube structures • continue to develop visualization skills by physically exploring the results of moving objects and structures in a variety of ways	It is becoming increasingly commonplace for people to represent 3-D objects in two dimensions and to visualize 3-D objects given 2-D representations. Whenever students encounter printed diagrams or pictures of 3-D objects, they use these skills. Students have done some of this kind of work in Classes V and VI, with simpler structures.

Pacing	Materials	Prerequisites
1 h	• Linking cubes • Isometric Dot Paper (BLM)	• building and drawing simple cube structures

Main Points to be Raised

• An isometric drawing is a 2-D representation of a 3-D object.

• There can be ambiguity in an isometric drawing — some parts of the shape may not be visible from certain viewpoints.

• It is helpful to label the front (or back or one side) of an isometric drawing, and/or to shade or colour parts of the drawing to make the drawing easier to interpret. This helps to minimize some of the ambiguity.

Try This — Introducing the Lesson

A. and B. Allow students to try this alone or with a partner.

While you observe students at work, you might ask questions such as the following:

• *Could there be parts of the building you cannot see?* (Yes. There could be a short section behind the tall section.)

• *The front face looks like a parallelogram in the diagram. How did you decide that it was a rectangle?* (When I look at a rectangle from the side, it does not look like a rectangle.)

• *How did you know that the tall section was not twice as tall as the short section?* (When I looked at the diagram, the tall section looked only a little taller, not a lot taller.)

• If students make structures with incorrect shapes, suggest that they hold their structures in the air and look at them with one eye to see what they look like.

• If students make structures that resemble the structure but with different dimensions, you can encourage them to measure the lengths of the edges in the diagram of the structure and look for relationships (for example, the short section is almost twice as long as the tall section).

The Exposition — Presenting the Main Ideas

• Choose one of the students' structures from the **Try This**. Ask the other students how their structures differ from it. In this discussion, try to make as many connections as possible between the diagram in the text and the structures students have built.

• Demonstrate how to make an isometric drawing by using one of the students' structures from the **Try This**. Compare this drawing with the diagram in the **Try This**. (Remember that students have made isometric drawings in previous classes.)

• Identify one place on the structure where there could be a hidden cube that does not appear in your drawing. Ask a student to put that cube on the structure. Ask students to identify other places there could be hidden cubes.

• Then have students read through the exposition on **page 225** of the student text.

Revisiting the Try This

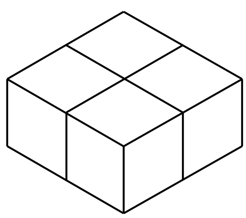
C. Students apply the method demonstrated in the exposition for creating their isometric drawings. Hide the example you did in the exposition, because it will be the solution for at least one student in the class.

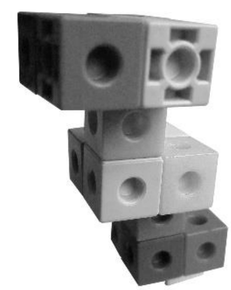
Using the Examples

• Have students work in pairs to read **example 1.** Ask them to make an isometric drawing of the same structure from a perspective different than the two perspectives in the example solutions. Ask individuals to draw the structure on the board. For each drawing, ask the class to decide whether or not it is correct.

• In pairs, have students make the structure in **example 2**, first with no hidden cubes and then with all the hidden cubes.

• Point out that in **example 2**, and for many other drawings, there is really no maximum number of cubes if you can only see a structure from one perspective and if you do not assume that some of the visible cubes are sitting on a flat surface. However, we generally assume that part of the visible structure is sitting on a flat surface, so
a limited number of hidden cubes are possible, as in the answer to **example 2**. This set of pictures shows three views of the same structure, revealing many hidden cubes for the given isometric diagram. The pattern, with four cubes in every other row, and one cube in the rows between, could continue indefinitely. All the rows would be hidden by the first row in an isometric drawing or a photograph taken from the right perspective.





Practising and Applying

Teaching points and tips

Q 1: It is possible to make an isometric drawing without building the cube structure. Encourage students to build the structure even if they think they can bypass this step. Building the structures will help them visualize the more difficult structures that follow.

Q 2: Students could use many different heights, widths, and thicknesses for their letter A.

Q 4: As explained above in the note on **example 2**, there is really no maximum number of cubes unless you assume that the structure is sitting on a flat surface with the bottom of the front face touching the surface.

Q 4 and 5: As with **question 1**, it is possible to do these questions without building the structure, but it will be easier to think about hidden cubes if students work with a physical structure.

Q 6: This question does not ask students to make the structures, but you could encourage them to do so. This will help them visualize the rotations.

Q 7: Encourage students to use examples in their explanation.

Common errors

• Many students will worry about their answers to **questions 2 and 5** because they differ from classmates' answers. Remind students that this is to be expected because there are many correct answers.

For example, in **question 5**, different students will turn their structures in different directions and thus have different isometric drawings.

Suggested assessment questions from Practising and Applying

Question 4	to see if students can identify ambiguities in isometric drawings, locating possible hidden cubes
Question 5	to see if students can make a structure given an isometric drawing and if students can make an isometric drawing given a structure
Question 7	to see if students know why a single isometric drawing might not represent a structure exactly

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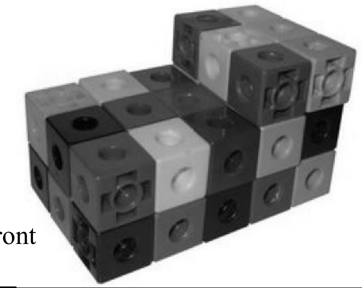
Answers

A. *Sample response*:



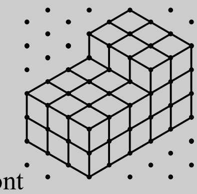
Front

B. *Sample responses*:

i) The tall section is about one and a half times the height of the short section, so I made the short section two cubes high and the tall section three cubes high.

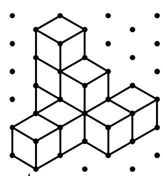
B. ii) The short section is almost twice as long as the tall section so I made the short section three cubes long and the tall section two cubes long. That made the whole structure five cubes long at the bottom. The short and tall sections are both the same width. The width seems to be a bit more than half the length of the structure at the bottom so I made it three cubes wide.

C. *Sample response*:



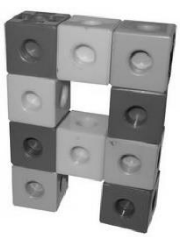
Front

1. *Sample response*:



Front

2. *Sample response*:



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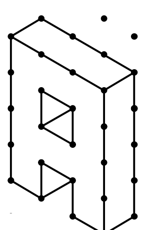
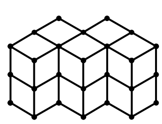


Front

3. *Sample responses*:

a) and b)

Front



Front

[**c) and d)** Drawings could be different if the structure is drawn from different viewpoints.]

4. a)

Front

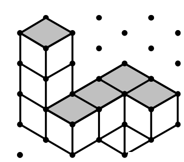
b) 14; [The 14 cubes that are visible in the drawing.]

c) 25; [14 + 11 = 25; the 14 visible in the drawing plus 11 more: 4 more cubes in the middle layer at the back, 4 more cubes in the bottom layer underneath the four in the middle layer, and 3 more cubes on the bottom layer attached to the back corner.]

5. *Sample responses*:

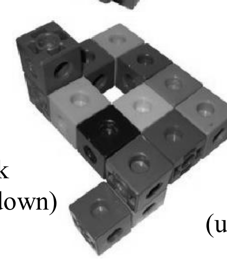
a)

Back

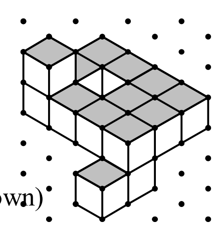


Back

b)



Back (upside down)



Back (upside down)

6. A and D; [*Sample response*:
I made all four structures and then turned them to view them from different viewpoints. I was able to position A and D so they matched.]

[**7.** *Sample response*:
If you only use one drawing, some cubes in the structure could be hidden from view so the drawing does not really represent the structure. Another drawing from a different viewpoint would show those cubes. Together, both drawings would represent the structure.]

Supporting Students

Struggling students

• If students are struggling with their isometric drawings, for example, in **question 1**, encourage them to start by building part of the structure. Have them draw that part. Then they can add another cube to the structure and add that one cube to their drawing. They can continue in this way until they have finished.

• Some students may have trouble visualizing structures. Encourage them to make the structures they are imagining, and to turn them and manipulate them as much as possible. Show students how to look at a structure from one corner to see the isometric outline. This works especially well if you look with only one eye.

Enrichment

• For **question 4**, you might challenge students to compare the possible number of hidden cubes using this perspective to the possible number using a different perspective.

• Students could make their own puzzle like **question 6**, in which others have to identify which two drawings could represent the same structure. Encourage them to make the question as difficult as possible by drawing structures that have only minor differences.

• Ask students to explain why there is only one possible isometric drawing for a $2 \times 2 \times 2$ cube.

8.1.2 Orthographic Drawings

Curriculum Outcomes	Outcome relevance
8-E1 Interpret and Make Orthographic Drawings and Isometric Drawings: to represent 3-D shapes • use interlocking cubes to explore attributes of 3-D shapes • use cubes to build structures from a set of orthographic drawings • make orthographic drawings of cube structures • compare constructions to determine how they are different and the same • continue to develop visualization skills by physically exploring the results of moving objects and structures in a variety of ways	It is important for people to be fluent in connecting 3-D structures with 2-D representations of the structures. There are multiple ways of representing a 3-D structure. This lesson describes another commonly-used approach for representing a 3-D object.

Pacing	Materials	Prerequisites
1 h	• Linking cubes • Isometric Dot Paper (BLM) • Grid paper or Small Grid Paper (BLM) (optional)	• making isometric drawings of cube structures • building cube structures from isometric drawings

Main Points to be Raised

• An orthographic drawing of a 3-D structure includes direct views of three faces.

• Using opposite views in a set of orthographic drawings often provides less information than using non-opposite views.

• It is possible for a set of orthographic drawings to describe more than one structure.

Try This — Introducing the Lesson

A. Suggest that students stand up and move around until each student`s view of his or her desktop makes the desktop look like a parallelogram, as in the given diagram.

While you observe students at work, you might ask questions such as the following:

• *What happens when you move your point of view higher?* (The desktop looks more like a rectangle.)

• *Where should you stand to make the desktop look the least like a rectangle? most like a rectangle?* (With my eye just above the top of the desk, at the corner of the desk, as close as possible to the desk; Straight on from above.)

• It may take time for some students to see their desktops as non-rectangles because they know the desktops are rectangles.

The Exposition — Presenting the Main Ideas

• Build a simple cube structure that is different from the structures shown in the exposition. Show the different views of the structure to the class by turning it to face them for each view. Draw each face view on the board.

• Build several prisms and one more irregular shape. Draw every face view for each structure. Show students that using two opposite views in a set of three or four orthographic drawings often does not provide as much information as using only non-opposite views, except in the case of very irregular shapes.

• Point out that when students draw the top view of a structure, they should always position it as if they are looking at the structure from the front. All other views should also be consistent with this.

Revisiting the Try This

B. Students apply the instructions from the exposition for making orthographic drawings to draw face views of a cube structure that resembles a desk or table.

Using the Examples

• Have students work in pairs. Ask each pair to read **example 1** to decide which set of views gives the most information about the structure, and which gives the least information — the front-right-top views as in **solution 1**, the top-front-left views as in **solution 2**, or the back-left-bottom views (not shown). (**Solution 1** gives the most information and the back-left-bottom views give the least information because there are no thicker lines to show the change in depth.) Have some students share their findings with the whole class. Ask other students whether they agree. Have them explain why or why not.

• Work through **example 2** with the class. As you build the structure in **part a)**, ask the class if you could put a cube in various places. If students say no, ask them why not. They will have to justify their reasoning by referring to the orthographic drawings. As you build, you can also draw attention to the choices you make, which addresses **part b)**.

Practising and Applying

Teaching points and tips

Q 1 to 5 and 8: It is possible to do the non-building parts of these questions without building the structures, but students will develop much better visualization skills if they build the structures and manipulate them.

Q 2: The back, left, and bottom views are the easiest
to draw because there are no depth changes. Encourage students to choose the views that show the most complexity (the most depth changes) because they give the most information about a shape.

Q 4: There are an infinite number of possible structures that have this side view.

Q 6: This question is challenging. Encourage students to make a structure that is as simple as possible. You might ask students why the face views in this question are shaded instead of white. They should notice that the structure that has a cube missing from the centre needs another colour to show which cubes are used.

Q 8: Some students will want to make very challenging structures, while others will want to make simple structures. Keep this in mind when you arrange students in pairs.

Q 10: It would be good to have students read their answers aloud so they can hear each other's ways of reasoning. Encourage students to use examples in their answers.

Common errors

• Many students will have difficulty making decisions about where to draw thicker lines to indicate change in depth, especially in **question 2**. Remind them to turn their structure to look at each view.

• If students build their structures incorrectly in **questions 1 and 2**, their drawings will not be correct. Encourage students to compare their structures with classmates' structures before making their orthographic drawings.

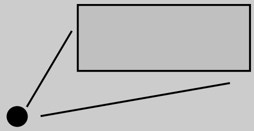
• Watch out for students that confuse the word "view" when used with isometric drawings and "view" in a set
of orthographic drawings. You might model and encourage them to always refer to the views in orthographic drawings as "face views."

Suggested assessment questions from Practising and Applying

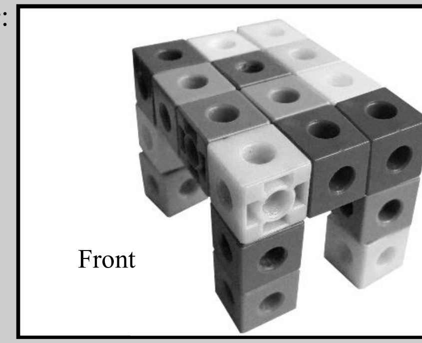
Question 2	to see if students can make orthographic drawings of a cube structure
Question 6	to see if students build cube structures given orthographic drawings
Question 10	to see if students understand and can explain why a set of orthographic drawings does not always give complete information about a structure

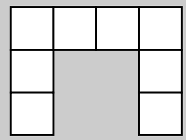
Answers

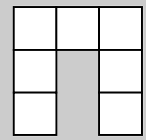
A. My eyes could be above the level of the desktop, but not directly over the top, in front, behind, or in line with the side. For example, I could be standing near a corner.

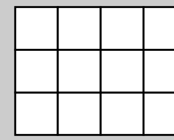


B. i) *Sample response*:



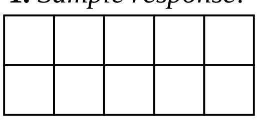

Front view

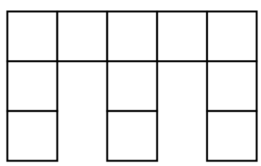

Side view

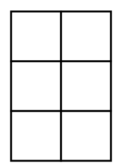

Top view

ii) No. The view is not from directly above the desk. An orthographic view of a rectangular desktop would be a rectangle.

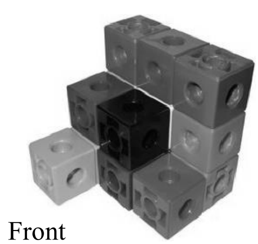
1. *Sample response*:

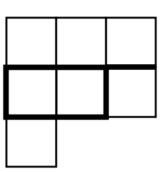

Top view

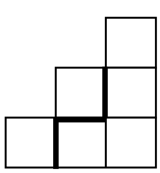

Front view

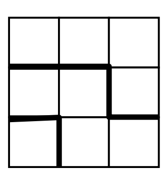

Right view

2. *Sample response*:



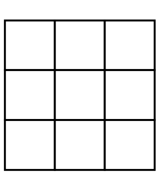

Top view


Right view

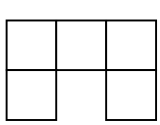

Front view

3. a)

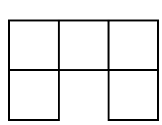
Top view



Right and left view



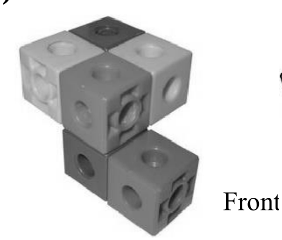
Front view



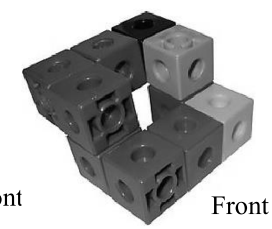
b) *Sample response*: See **part B i)** above

4. *Sample responses*:

a) Structure 1



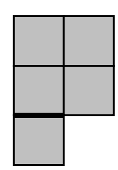
Structure 2



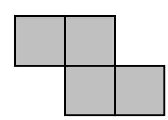
b)

Structure 1

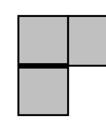
Top view



Left view

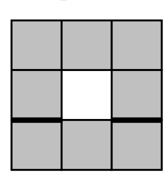


Front view

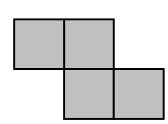


Structure 2

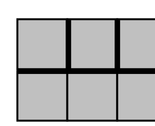
Top view



Left view



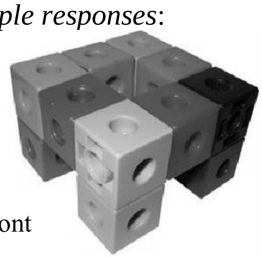
Front view



Answers [Continued]

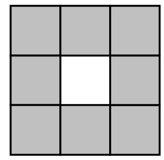
5. *Sample responses*:

a)



Front

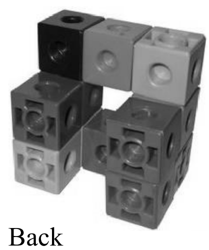
b)



6. *Sample responses*:

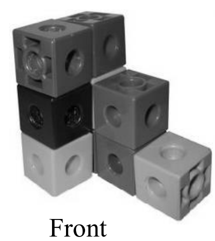
a) Two different views of the structure:

Front



Back

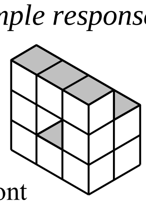
b)



Front

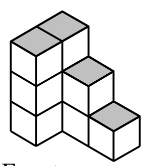
7. *Sample responses*:

a)



Front

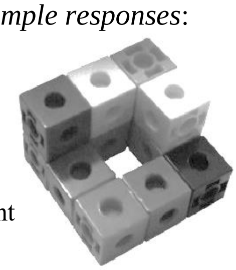
b)



Front

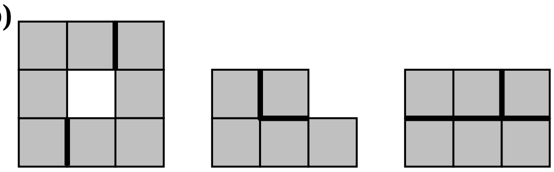
8. *Sample responses*:

a)



Front

b)



Top view Front view Right view

9. a) A cube

b) 27 cubes; [It could be a 3-by-3-by-3 cube.]
26 cubes; [It could be a 3-by-3-by-3 cube with one cube missing from the centre. All cubes except the centre cube would be visible in at least one of the views.]

10. No; [*Sample response*:
In **example 2**, the three views do not give enough information. In **question 9**, all six views of the cube do not give enough information.]

Supporting Students

Struggling students

• If students are struggling with building cube structures from the face views, you could build the structures for them to copy.

• Some students may have trouble drawing squares for their orthographic diagrams. You could supply them with grid paper.

Enrichment

• Challenge students to draw orthographic views of other objects in class, objects that are not made from linking cubes, such as a book, a desk, or a chair.

Chapter 2 Transformations

8.2.1 Dilatations

Curriculum Outcomes	Outcome relevance
8-E2 Dilatations: represent, analyse, and apply • understand that the dilatation centre and scale factor must be identified in order to locate the position of the dilatation image	Students will often encounter images that are projected onto other surfaces. Simple projections like this are dilatations. Students have been working with other transformations that have congruent images. This is their first experience with similar but non-congruent images.

Pacing	Materials	Prerequisites
1 h	• Rulers • Torch (flashlight) • Cut-out cardboard triangle	• familiarity with the term *transformation* • familiarity with basic 2-D shape names and properties • familiarity with congruence and similarity (including scale factor)

Main Points to be Raised

• A dilatation is a transformation that changes a shape to a larger size or a smaller size without changing anything else about the shape.

• A dilatation is described by its dilatation centre and scale factor. If the scale factor is 1, the original shape and image are congruent.

• These things are true for all dilatations:

- Any line that passes through a point on the original shape and its image point passes through the dilatation centre.
- A scale factor greater than 1 results in an enlargement. A scale factor between 0 and 1 results in a reduction.
- The original shape and image are similar. If the scale factor is 1, they are congruent. The dilatation scale factor is also the similarity scale factor (the ratio of corresponding side lengths).
- The original shape and the image have the same orientation (unless the scale factor is negative, but all scale factors are positive in Class VIII).

Try This — Introducing the Lesson

A. Allow students to make a prediction alone or with a partner. Ask some students to explain their predictions to the class. Test their predictions using a torch and a cut-out triangle.

B. Allow students to work alone or with a partner. Then invite some students to report their findings to the class.

While you observe students at work, you might ask questions such as the following:

• *Where do you think the third line will cross the other two lines?* (At the same place the other two crossed.)

• *Are the two triangles similar or congruent? How do you know?* (They are similar because they seem to be the same shape. I know they are not congruent because one triangle is larger than the other.)

If students' lines do not pass through the vertices exactly, they may not meet at one point. Encourage students to be careful, but assure them that it is impossible to be completely accurate with tracing and pencils. They should not be discouraged, but they should know that it is important to be as accurate as possible.

The Exposition — Presenting the Main Ideas

• Use the triangles on the board from the **Try This** to show that the lines that pass through corresponding points meet at one point.

• Ask students if they think the triangles are similar. Tell students that the larger triangle is a dilatation of the smaller triangle, and vice-versa. Write the word *dilatation* on the board, along with a brief summary of the first two key points from exposition.

• Demonstrate another dilatation of one of the triangles. Use a dilatation centre outside the original shape and a scale factor of 2. Students will see how it is different when the centre is outside the shape, rather than inside.

• Show how the lines through corresponding points always go through the centre, and that the original shape and image are similar (the ratio of corresponding side lengths is the same for all side lengths).

• Have students look at the exposition on **pages 233 and 234** of the student text. Talk about the difference between a scale factor greater than 1 and a scale factor less than 1 (but greater than 0).

Revisiting the Try This

C. Students apply their new understanding of dilatations and scale factors to the triangles in **parts A and B**.

Using the Examples

• For **example 1**, allow students to work with a partner to decide which solution they prefer. Ask students to share their preferences and justifications with the class. Ask if they have any questions about the two solutions.

• Draw the triangles on the board and work through **example 2** with the class.

Practising and Applying

Teaching points and tips

Q 2: Some students may wonder how to describe where the centre is for the dilatation because it is outside the shapes. They can show the centre visually without explaining its location.

Q 3: As students work on this question, you might ask them how the result would be different if C were in a different location.

Q 5: You could ask some students to do the same question using a scale factor of 3. Students can compare their answers to see a greater range in possibilities using not only different centre locations but also two different scale factors.

Q 6: You might ask students why you can locate the centre of dilatation using only two lines when you know that an image is a dilatation image.

Common errors

• Because of inevitable imperfections in students' instruments, they may find that their lines through corresponding points do not actually meet in one place when they try to locate the centre of a dilatation. Instead, the lines will probably meet in a vicinity. Encourage students to be as careful as possible, but allow for some margin of error.

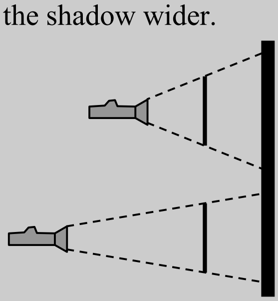
• Many students will have difficulty visualizing the result of a dilatation. **Question 5** is designed to help them get a feel for the range of possibilities. You might ask them first to do **question 5** and then to go back to the question that was difficult for them.

Suggested assessment questions from Practising and Applying

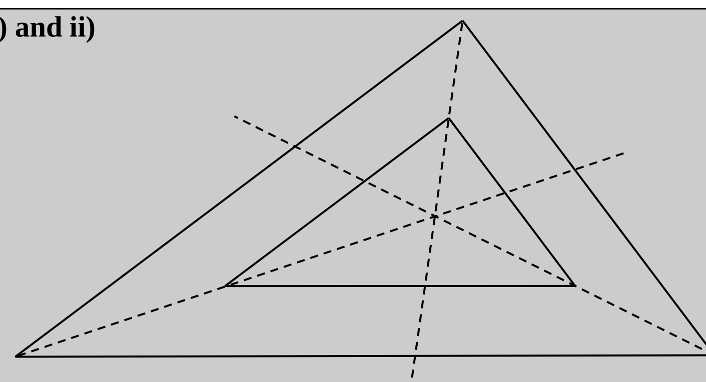
Question 2	to see if students can identify a dilatation
Question 3	to see if students can perform a dilatation
Question 7	to see if students can reason about dilatations

Answers

A. The larger image is from when the torch was closer; *Sample response*: The angle of the light is greater, so it spreads the shadow wider.



B. i) and ii)



iii) All three lines intersect at one place.

C. i) Scale factor 2, dilatation centre O.

ii) Scale factor $\frac{1}{2}$, dilatation centre O.

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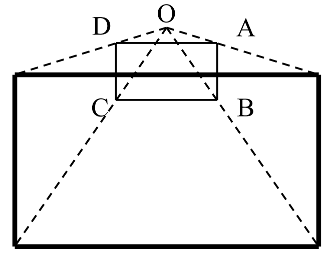
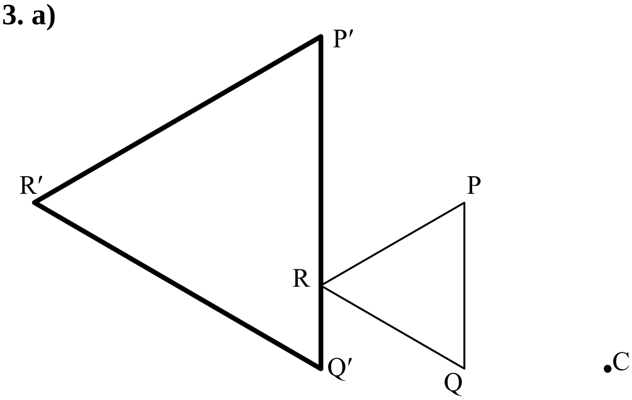
1. [**a)** All the lines through pairs of corresponding points intersect at A.]

b) Dilatation centre A, scale factor $1\frac{1}{2}$

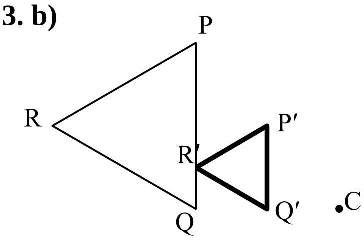
2. a) B; [A cannot be a dilatation because the lines that connect pairs of corresponding points are parallel so they will never meet.
B is a dilatation because all the lines that connect pairs of corresponding points intersect at one point, O.]
b) The dilatation centre is O and the scale factor is 3.



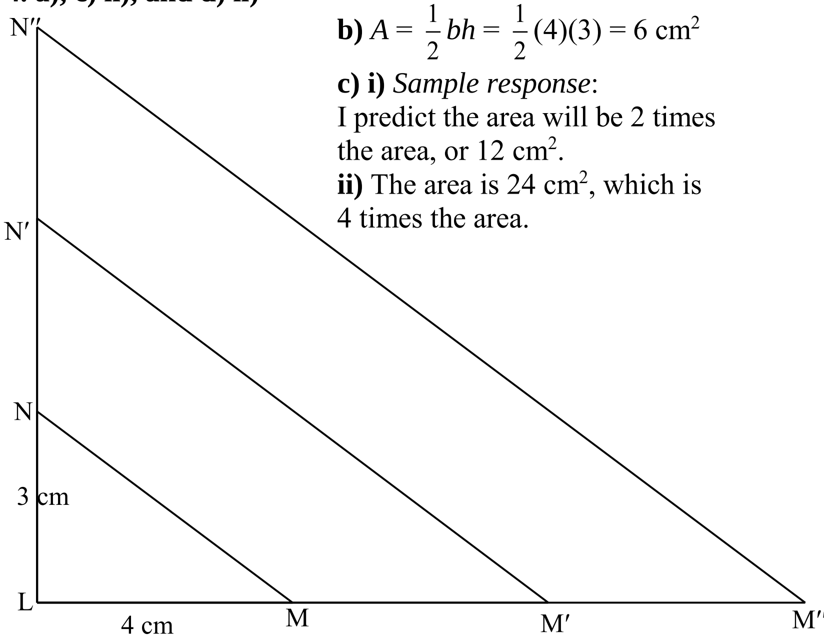
3. a)



3. b)



4. a), c) ii), and d) ii)

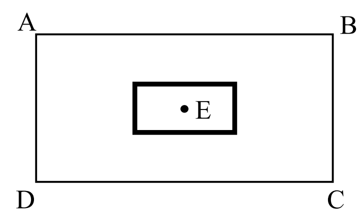


b) $A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6 \text{ cm}^2$

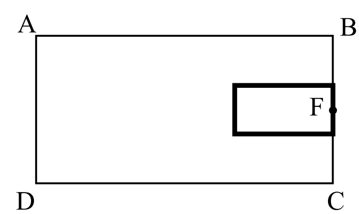
c) i) *Sample response*:
I predict the area will be 2 times the area, or 12 cm^2.
ii) The area is 24 cm^2, which is 4 times the area.

d) i) *Sample response*:
I predict the area will be 3 times the area, or 18 cm^2.
ii) The area is 54 cm^2, which is 9 times the area.
e) Multiply the area by the square of the scale factor.

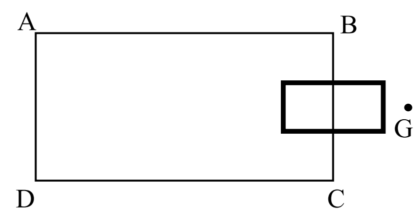
5. a)



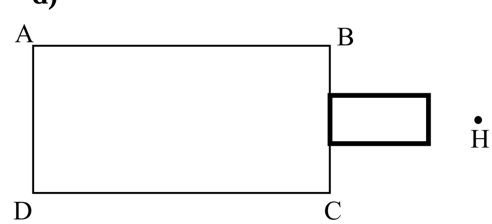
b)



c)



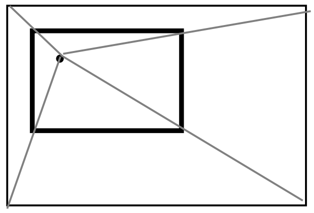
d)



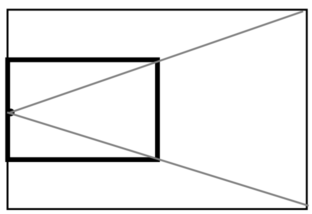
Answers [Continued]

6. *Sample response*:

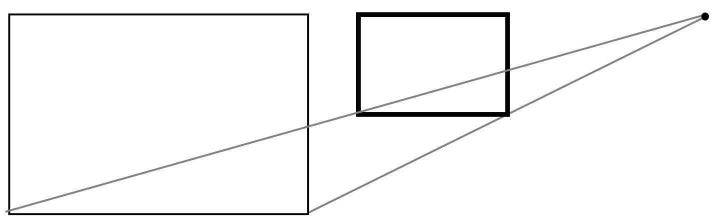
a) The dilatation centre is somewhere inside the rectangle.



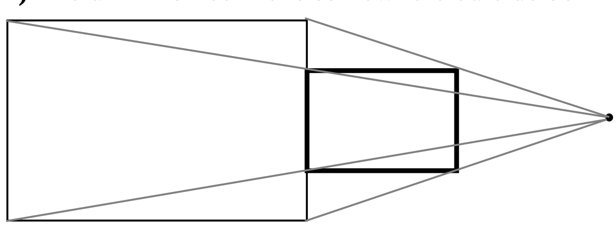
b) The dilatation centre is somewhere on the side of the rectangle.



c) The dilatation centre is somewhere outside both the rectangle and its image.



d) The dilatation centre is somewhere outside both the rectangle and its image.



7. No; [The image will be larger than the original shape because a scale factor of 2 means an enlargement. That means there is no way the image can be completely inside the original shape.]

Supporting Students

Struggling students

• If students are unsure of their understanding of dilatations, you might remind them that they can check their answers using their knowledge that the image is similar to the original shape. If the result does not look similar, then they should double-check their work by locating each dilatated vertex again.

• Students may have difficulty with **question 6** because it asks them to predict the location of each centre of dilatation. Encourage them to try different centres even if they are not sure what the result will be. As they try different possibilities, they will find centres that bring them close to answering the question.

Enrichment

• **Question 6** asks for one dilatation centre location that will meet each condition. Challenge students to find several centres to meet each condition and to look for what those centres have in common.

• Extend **question 6** to enlargements, asking students to use a scale factor of 2. If they cannot answer one part, they should explain why it is not possible.

8.2.2 Combining Transformations

Curriculum Outcomes	Outcome relevance
8-E2 Dilatations: represent, analyse, and apply • explore combinations of transformations that include dilatations, such as an enlargement followed by a reflection	Outside the mathematics classroom, people often manipulate objects in a number of ways to get a desired result. These are combinations of transformations. Students have already worked with combinations of transformations in previous classes. Combinations that include dilatations are new in this class.

Pacing	Materials	Prerequisites
2 h	• Protractors, rulers, and compasses • Tracing paper	• familiarity with translations, reflections, rotations, and dilatations • familiarity with combinations of transformations that include translations, reflections, and rotations

Main Points to be Raised

• A series of transformations can be performed in a sequence.

• Sometimes the order of the sequence makes a difference and sometimes it does not.

For example, two translations can be performed in either order, but for a reflection and a translation the order usually matters.

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *How did you find the centre of the dilatation?* (I visualized lines through corresponding vertices in the image and the original shape.)

• *How did you find the scale factor?* (Since the smaller triangles have sides that are half the length of sides of the larger triangle, I knew the scale factor was $\frac{1}{2}$.)

• *How did you locate the reflection line?* (I visualized a line of symmetry between the original shape and the image.)

• *How did you find the turn centre?* (One vertex does not move in the rotation, so it has to be the turn centre.)

If students identify the properties of the required transformations incorrectly, you might demonstrate the transformation they are having trouble with. Assure them that you will soon review these transformations.

The Exposition — Presenting the Main Ideas

• On the board, copy the chart from the exposition, filling in only the top row and the left column. Ask students to share their answers to the **Try This**. Fill in the corresponding parts of the chart with students' answers.

• Ask students to read the two examples in the exposition. Point out these key features:

- Showing the intermediate images can help you do the combination of transformations.
- There is usually more than one combination that will achieve the final image.
- The order of the sequence of transformations sometimes makes a difference and sometimes does not.

For example, two translations can be performed in either order, but for a reflection and a translation the order usually makes a difference.

Revisiting the Try This

B. Students apply their knowledge of combinations of transformations to the tiling on Purna Bahadur's wall from **part A**.

Using the Examples

• Work through **examples 1 and 3** with students to make sure they understand them. Ask students why they think the order of the transformations makes a difference in **example 3** but not in **example 1**.

• Have students work in pairs to discuss **example 2**. Ask them to try to find a different combination of transformations that has the same result.

Practising and Applying

Teaching points and tips

Q 2, 4, and 7: Students are not expected to know in general when the sequence of transformations matters. It is sufficient for them to tell why they think the result was or was not different in each case.

Q 3: Remind students that they can use tracing paper to help them with rotations.

Q 6: Encourage students to try transformations even if they cannot visualize the final result.

Q 8: This question might take a lot of time. It is a challenging problem.

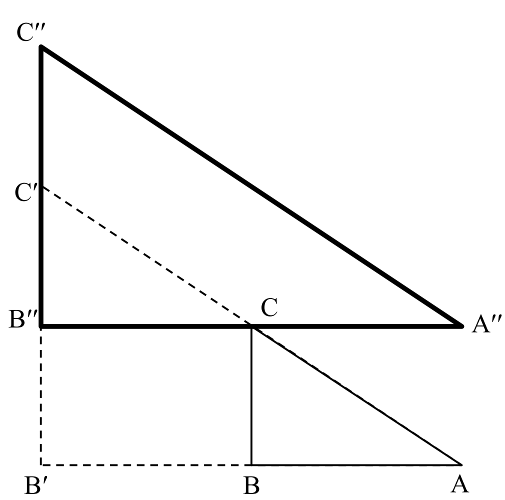
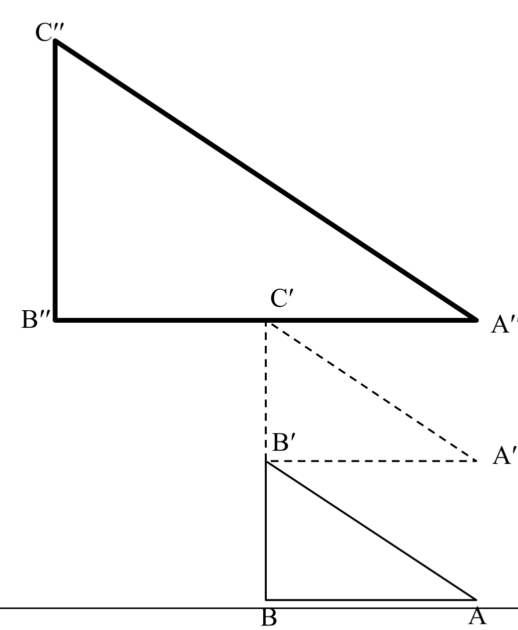
Common errors

• There are many possibilities for errors in a combination of transformations because there are multiple steps. Encourage students to compare their work with their classmates' work. Remind them that various answers are possible for some of the questions.

Suggested assessment questions from Practising and Applying

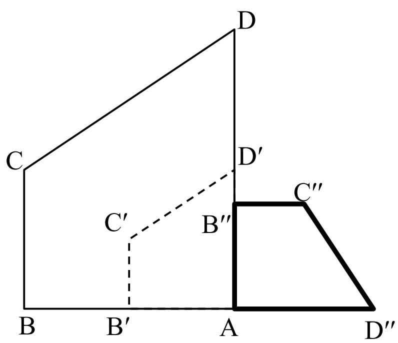
Question 1	to see if students can follow instructions to perform a combination of transformations
Question 5	to see if students can identify a combination of transformations from an original shape and final image
Question 7	to see if students can solve a problem involving combinations of transformations

Answers

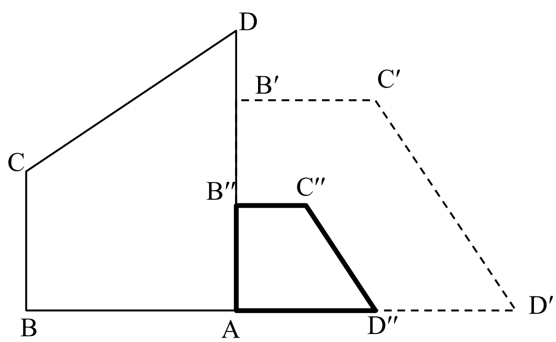
<table>
<tr>
<td>A. i) The scale factor is 2 and the turn centre is at the bottom left vertex of the small triangle to the left of the grey triangle.
ii) The translation the length of a side of the striped triangle, to the left.
iii) The reflection line is the vertical line through the point that is a vertex of both triangles.
iv) The turn centre is the point that is a vertex of both triangles and the angle of rotation is 120º clockwise.</td>
<td>B. i) Do a dilatation as in part A i), followed by a translation to the right the length of a side of the black triangle.
ii) Do a translation as in part A ii), followed by a dilatation with centre at the bottom left vertex of the small triangle to the left of the grey triangle, with a scale factor of $\frac{1}{2}$.</td>
</tr>
<tr>
<td>1. a)

</td>
<td>b)

</td>
</tr>
</table>

[**2.** *Sample response*:
The image is higher in **part b)** because doing the dilatation first stretches the 2 cm distance that it travels upwards in the translation.]

3. a)

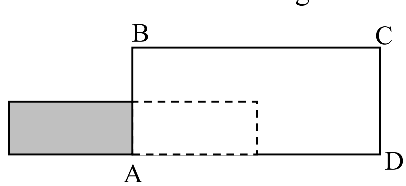


b)

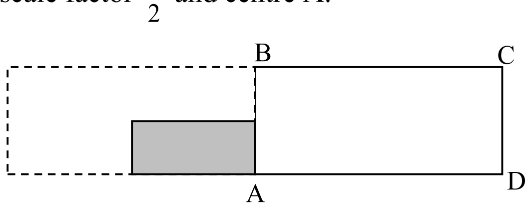


[**4.** *Sample response*:
It is the same because the centre is the same for both transformations and I did the same transformations.]

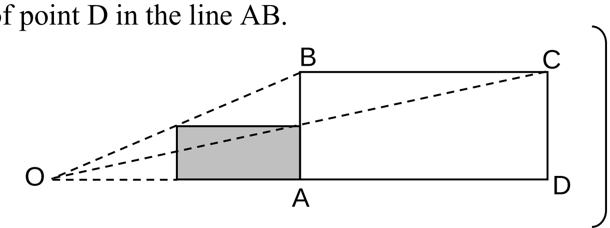
5. a) A dilatation with scale factor $\frac{1}{2}$ and centre A results in the dashed rectangle. This is followed by a translation to the left half the length of DA.



b) A translation to the left along DA results in the dashed rectangle. This is followed by a dilatation with scale factor $\frac{1}{2}$ and centre A.

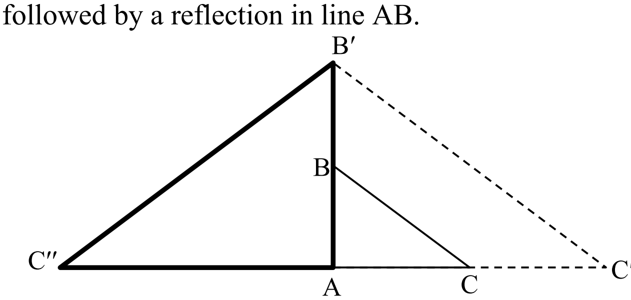


c) Yes; [A dilatation with centre O and scale factor $\frac{1}{2}$ has the grey rectangle as its image. O is the reflection of point D in the line AB.

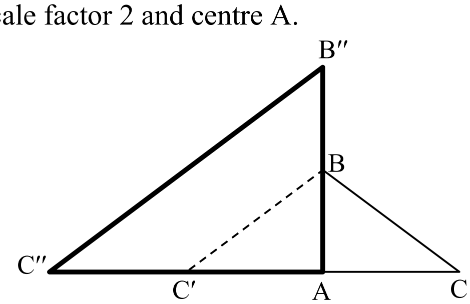


]

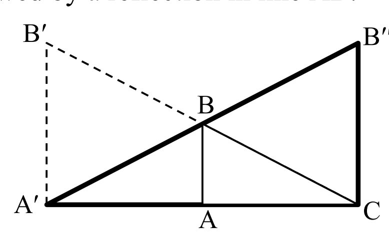
6. *Sample responses*:
a) A dilatation with scale factor 2 and centre A followed by a reflection in line AB.



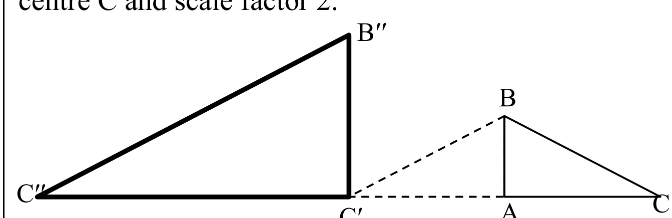
A reflection in line AB followed by a dilatation with scale factor 2 and centre A.



b) A dilatation with centre C and scale factor 2 followed by a reflection in line AB.



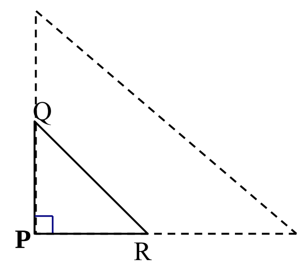
The final image of this combination in the opposite order is congruent, but in a different position.
A reflection in line AB followed by a dilatation with centre C and scale factor 2.



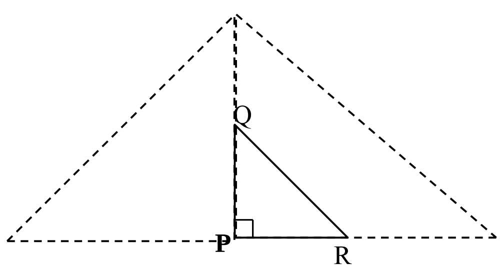
Answers [Continued]

[**7.** *Sample response*:
In a reflection, the only points that do not move are the points on the refection line. In a dilatation, the only point that does not move is the dilatation centre.
So, if the dilatation centre is not on the refection line, the order matters, but if the dilatation centre is on the reflection line, then the order does not matter.]

8. a) The triangle on the left [because it is similar. The triangle on the right cannot be an image because it is not similar.]
b) *Sample response*:
- Enlarge by a scale factor of 2 with centre P.

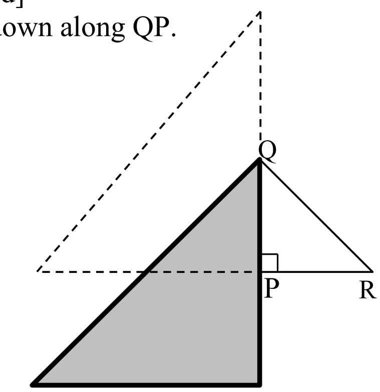


- Rotate 90º counter-clockwise around centre P.



[Cont'd]

8. [Continued]
- Translate down along QP.



[**9.** *Sample responses*:
a) One transformation must be a dilatation if the image and original shape are similar but not congruent.
b) One transformation must be a rotation if the image is turned from the direction of the original shape, but the orientation is the same.
c) One transformation must be a reflection if the orientation is opposite (the direction of the vertices is cw in the image and ccw in the original shape or vice-versa).]

Supporting Students

Struggling students

• Many students will struggle to explain why the order of the transformations matters or does not matter in **questions 2, 4, and 7**. Suggest that they pay attention to which points are fixed (i.e., do not move) in each transformation. If the fixed points are different for two transformations, it is likely that the order of the transformations matters.

Enrichment

• Challenge students to determine whether any combination of a dilatation and a translation could instead be done with a single dilatation.

GAME: Isometry

• This optional game has students work with isometric grid paper, which can help them develop a feel for possibilities in isometric drawing. It also helps them visualize transformations.

• Students have to find a way to keep track of the last triangle drawn, since each transformation is based on the just-completed triangle and the triangle completed before that.

Observe students as they play. You might ask questions such as the following:

• *How can you tell when a completed triangle is a translation image of another triangle?*

• *How can you tell when a completed triangle is a reflection image of another triangle?*

• *How can you tell when a completed triangle is a rotation image of another triangle?*

• *Is it possible for a completed triangle to be a dilatation image of another triangle?*

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Chapter 3 Angle Relationships

8.3.1 EXPLORE: Measuring Angles in Polygons

Curriculum Outcomes	Outcome Relevance
8-E3 Polygons: properties and interrelationships • develop a chart to observe and extend patterns and generalize about the sum of the measures of the interior angles and exterior angles of various polygons, and the measure of each interior angle of a regular polygon **8-C2 Graphs (Linear and Non-linear): interpret** • construct graphs to determine if a relationship is linear (e.g., graph the sum of the interior angles of a polygon against the number of sides)	This essential exploration helps students see how total angle measure formulas are developed. One of the most powerful applications of mathematics outside of school is to recognize patterns, develop formulas that describe the patterns, and use the formulas for making predictions to inform our decisions.

Pacing	Materials	Prerequisites
1 h	• Rulers and protractors • Grid paper or Small Grid Paper (BLM) • Regular Polygons (BLM)	• measuring angles • familiarity with the names of the various regular polygons • drawing a line graph from a tables of values • interpreting line graphs

Main Points to be Raised

• The sum of the angles of a triangle is 180°.

• The sum of the interior angles of a polygon with *n* sides is the same, no matter what the shape looks like.

• The sum of the angles of a polygon is 180° greater for each extra side, beyond three side lengths. You can see this using a graph or a table of values.

• The sum of the exterior angles of a polygon is always 360°. An exterior angle is formed by extending a side length.

Exploration

• Work through the introduction (in white) with the students. Make sure they understand how to measure an exterior angle. Walk along the path of a pentagon on the floor to demonstrate how an exterior angle is the degree to which you turn your body at each vertex.

• Have students work, alone, in pairs, or in small groups for **parts A to D**. If students work in pairs or groups, tell them to write their results on one sheet of paper so they can share it with other groups at the end of the class. Remind students to measure carefully so that their results will be close to the true results.

While you observe students at work, you might ask questions such as the following:

• *Why do you think your results are the same as Kamala's results for the pentagon?* (It is like a triangle, which always has an interior angle sum of 180º.)

• *Are all your results for the exterior angle sum close to a certain number?* (Yes, they are all close to 360º, so I think they are all supposed to be 360º. I am probably making some measuring errors.)

• *Why do you think the exterior angle sum is always the same and always 360º?* (If I walk around the outside of the shape, these are the angles at which I turn so I end up doing a full rotation, which is 360º.)

Observe and Assess

As students work, notice the following:

• Do they recognize the patterns in their charts?
• Do they describe their patterns well?
• Do they use the patterns to predict results for other polygons?
• Do they recognize that both the graph and the numerical pattern are helpful for predicting?

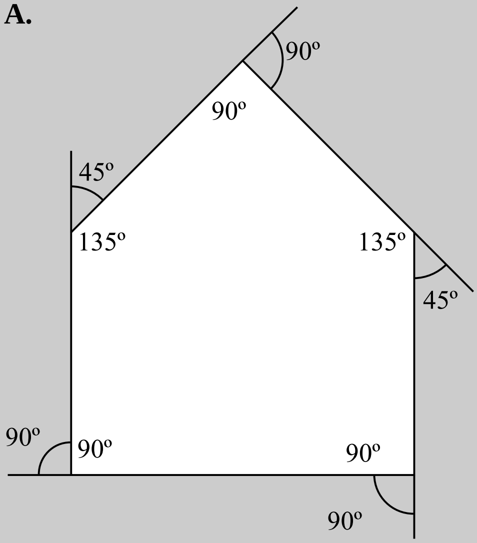
Share and Reflect

After students have had sufficient time to work through the exploration, they should form small groups to discuss their observations and answer these questions. You can follow up with a class discussion:

- *What patterns did you find for predicting the angle sums in polygons?*
- *What patterns did you find for predicting the angle sums in regular polygons?*
- *Why do you think these patterns work the way they do?*

Answers

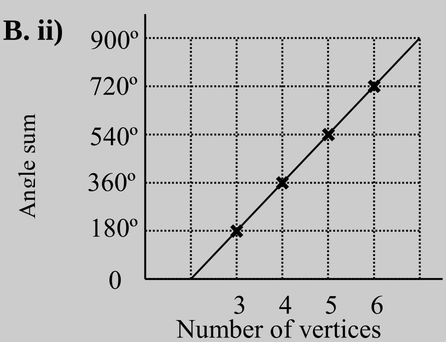
A.



i) 540º
ii) 360º
iii) The sum of the interior angles in any pentagon is 540º. The sum of the exterior angles in any pentagon is 360º.

B. i)	**Triangle**	**Quadrilateral**	**Pentagon**	**Hexagon**
Number of vertices	3	4	5	6
Interior angle sum	180º	360º	540º	720º
Exterior angle sum	360º	360º	360º	360º

B. ii)



Yes; The points form a straight line.
iii) For any number of vertices, the exterior angle sum is always the same, 360º; Yes; if it was graphed, it would be a (horizontal) straight line.

C. i) Visualize a vertical line from the number of vertices up to the line and read across to the vertical axis to find the angle sum.
ii) Moving to the right in the table, there is an increase of 180º each time the number of vertices increases. This helps me see that the interior angle sum of any polygon is 180º × (the number of vertices – 2), e.g., the interior angle sum of a hexagon is 180º × (6 – 2) = 180º × 4 = 720º.
iii) The sum of the exterior angles in any polygon is 360º.

D. To find each interior angle, divide the sum of the interior angles by the number of vertices. To find each exterior angle, divide the sum of the exterior angles by the number of vertices. This works because all the angles in a regular polygon are equal and the number of vertices is the same as the number of angles.

Supporting Students

Struggling students

- If students are struggling with measuring angles in **part A**, you might suggest that they extend the lengths of the sides to make it easier to read the protractor accurately. (This will only help if their pentagon is small enough that the sides are shorter than the radius of the protractor.)
- If students do not recognize the patterns in **part C**, suggest that they round their results to the nearest 10º. They will likely round to the correct results. Then help them see the patterns by asking guiding questions such as: *What operation do you do to 180º to get 360º? What operation do you do to 360º to get 540º?* and so on.

Enrichment

- Challenge students to predict the sum of the interior angles in a 100-gon (a polygon with 100 sides), and to predict the interior angle in a regular 100-gon. Then ask them to do the same for a 1000-gon. Last, ask them to generalize for an *n*-gon (a polygon with *n* sides).

8.3.2 Angles in Polygons

Curriculum Outcomes	Outcome relevance
8-E3 Polygons: properties and interrelationships • understand, through investigating, that the sum of the measures of the interior angles of a polygon is found by dividing the polygon into triangles	It is useful to know the formula that relates the number of sides of a polygon to the total angle sum in order to deduce other geometric information.

Pacing	Materials	Prerequisites
1 h	• Rulers • Regular Polygons (BLM)	• substituting values into a formula and evaluating • familiarity with angle relationships in isosceles and equilateral triangles

Main Points to be Raised

- The sum of the exterior angles in any polygon is 360º.
- The sum of the interior angles in any polygon is 180º × (number of vertices – 2).
- The interior angle of a regular polygon is $\frac{180°(n-2)}{n}$

Try This — Introducing the Lesson

A. Allow students to try this alone or with a partner. While you observe students at work, you might ask questions such as the following:

- *How can you divide the hexagon into triangles without placing a vertex inside the hexagon?* (By placing the vertex at one of the angles of the hexagon.)
- *How can you divide the hexagon into more triangles?* (By placing a vertex inside the hexagon.)
- *How can you use the triangles to find the measures of the interior angles?* (I can add them up: 180º for each triangle.)
- *Do your results fit with what you did in the last lesson?* (Yes. I got the same answer when I measured the angles in a hexagon.)

If students divide the hexagon incorrectly so that it does not use the least number of triangles, you might ask them how they could make more triangles. Then ask how they could make fewer triangles.

The Exposition — Presenting the Main Ideas

- Remind students of their results from the previous lesson. Ask them to repeat their explanations of how to find the sum of the exterior angles and the sum of the interior angles in a polygon. Use their results to develop the formulas for these properties in this lesson.
- Show why these formulas are true by explaining that the exterior angles should add to a complete rotation. Explain that when you walk around the perimeter of a polygon, all your turns together make a complete rotation. Also explain how a polygon can be divided into triangles to find the sum of the interior angles. (The number of triangles is two less than the number of vertices because it takes three vertices to make the first triangle and then each new vertex adds another triangle.)
- With students, read through the exposition on **pages 247 and 248** of the student text. This section explores different ways to come to the formula for the interior angle of a regular polygon. Point out that the two formulas look different but give the same result. Use an example to show this is true. Let students choose a value for *n* to try.

Revisiting the Try This

B. Students apply the formulas discussed in the exposition to the hexagon from **part A**.

Using the Examples

- Present the problems in **examples 1 and 2** for students to try. Students can compare their solutions to the solutions in the student text.

Practising and Applying

Teaching points and tips

Q 1: Some students may choose to divide the shapes into triangles to calculate each sum. Others may use
the formula. Either way is acceptable.

Q 4: There are a number of acceptable methods for this. There are two formulas. Or, a student might find the angle sum by dividing the polygon into triangles and then dividing by the number of vertices.

Q 7: Students have to draw on what they know about angles in equilateral and isosceles triangles.

Q 8: This question helps students verify that both formulas do indeed get the same result. Many students find it too difficult to manipulate the expressions algebraically to show that they are the same.

Q 11: Students do not need to calculate the interior or exterior angle sums. They can use reasoning, along with an understanding of the formulas.

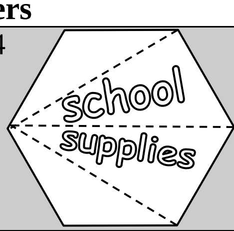
Common errors

• Many students confuse the formulas; they are unsure when to use each formula. Encourage them to think about how they would find the answers without the formulas (e.g., by dividing the polygon into triangles). This approach may make it unnecessary for them to use the formula, but it will give them a clue as to which formula applies because the formulas relate to the students' understanding of the shapes.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can apply a formula or knowledge about polygons to find the sum of interior angles
Question 5	to see if students can use reasoning to make conclusions about a shape, given information about its angles
Question 7	to see if students can solve problems using their knowledge of angles in polygons

Answers

A. i) 4  **ii)** 180º × 4 = 720º	**B. i)** 720º **ii)** 60º **iii)** 120º
1. *Choice of formula will vary*: **a)** 1080º; [180º(n – 2) = 180º(8 – 2) = 180º × 6 = 1080º] **b)** 720º; [180º(n – 2) = 180º(6 – 2) = 180º × 4 = 720º] **c)** 720º; [180º(n – 2) = 180º(6 – 2) = 180º × 4 = 720]º **d)** 360º; [180º(n – 2) = 180º(4 – 2) = 180º × 2 = 360º] **2. a)** 120º; [360º ÷ n = 360º ÷ 3 = 120º] **b)** 90º; [360º ÷ n = 360º ÷ 4 = 90º] **c)** 36º; [360º ÷ n = 360º ÷ 10 = 36º] **d)** 30º; [360º ÷ n = 360º ÷ 12 = 30º] [**3.** Not all the interior angles of a regular polygon are equal, so not all the exterior angles are the same. That formula assumes that all the angles are equal.]	**4.** *Choice of formula will vary*: **a)** 60º; [180º ÷ 3 = 60º] **b)** 140º; [180º – 360º ÷ n = 180º – 360º ÷ 9 = 180º – 40º = 140º] **c)** 135º; [180º – 360º ÷ n = 180º – 360º ÷ 8 = 180º – 45º = 135º] **5. a)** Yes; [It is a nonagon (9 sides). 360º ÷ n = 40º n = 360º ÷ 40º = 9] **b)** No; [No regular polygon has an exterior angle of 55º. 360º ÷ n = 55º n = 360º ÷ 55º = 6.546] **6.** An octagon; [360º ÷ n = 45º n = 360º ÷ 45º= 8]

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7. a) 22.5º; [The interior angles in the octagon: 180º – 360º ÷ n = 180º – 360º ÷ 8 = 180º – 45º = 135º. The isosceles triangle has angles x, x, and 135º, which add up to 180º. 180º – 135º = 45º 45º ÷ 2 = 22.5º 22.5º + 22.5º + 135º = 180º So x is 22.5º.] **b)** 72º; [Interior angles in the regular pentagon: 180º – 360º ÷ n = 180º – 360º ÷ 5 = 180º – 72º = 108º. Interior angles in the regular hexagon: 180º – 360º ÷ n = 180º – 360º ÷ 6 = 180º – 60º = 120º. Interior angles in the bottom equilateral triangle: 180º ÷ 3 = 60º. The four angles in the middle add up to 360º: 108º + 120º + 60º + x = 360º 288º + x = 360º x = 360º – 288º x = 72º] **c)** 135º; [Sum of the interior angles in the pentagon: 180º(n – 2) = 180º(5 – 2) = 180º × 3 = 540º. Three interior angles are 90º and two interior angles are x: 90º + 90º + 90º + x + x = 540º 270º + 2x = 540º 2x = 540º – 270º 2x = 270º x = 135º] **8. a)** 140º; [180 × (n – 2) ÷ n = 180 × (9 – 2) ÷ 9 = 180 × 7 ÷ 9 = 1260 ÷ 9 = 140º] **b)** 140º; [180º – 360º ÷ n = 180 – 360 ÷ 9 = 180 – 40 = 140º] **c)** 177º; [180 × (n – 2) ÷ n = 180 × (120 – 2) ÷ 120 = 180 × 118 ÷ 120 = 21240 ÷ 120 = 177º 180º – 360º ÷ n = 180 – 360 ÷ 120 = 180 – 3 = 177º]	[**9.** *Sample response*: I prefer the second formula; It lets me work with smaller numbers.] **10. a)** Regular polygons with these numbers of sides: 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360. [If the number of sides is a factor of 360, the result is a whole number when you divide.] **b)** Regular polygons with these numbers of sides: 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360 (same as **part a)**). [If the exterior angle is a whole number, then the interior angle will also be a whole number because they add to 180º.] **11. a)** A regular polygon with 100 sides; [*Sample response*: The greater the number of sides or vertices, the greater the interior angle sum. If you keep increasing the value of n in the formula 180 × (n – 2), you will get increasing results.] **b)** An equilateral triangle; [*Sample response*: The greater the number of vertices or sides, the smaller the exterior angle sum. If you keep increasing the value of n in the formula 180 ÷ 2, you will get decreasing results.]

Supporting Students

Struggling students

• Students will have difficulty with **question 7**. You might ask them if they see any isosceles or equilateral triangles in each figure. Ask them which angles they are able to calculate. Tell them that these other angles might help them find the required angle.

• Some students might benefit from doing **question 6** before **question 5**. Students who like to think spatially could benefit from this. Students who think more in terms of the formulas may find **question 5** easier.

Enrichment

• For **question 7**, you might challenge students to invent their own question that draws on knowledge of angles in polygons. Encourage them to make their questions such that as few angle measurements as possible are given.

8.3.3 Angles With Parallel and Intersecting Lines

Curriculum Outcomes	Outcome relevance
8-E4 Angle Pair Relationships: parallel and non-parallel lines • understand that corresponding angles and alternate angles are only equal when a transversal intersects two parallel lines • understand that interior angles are supplementary when a transversal intersects two parallel lines • apply transformational geometry to discover why the various angle pairs are equal	Parallel lines are very common inside and outside of school. Students who know about angle relationships related to parallel lines can deduce geometric properties about shapes. The basis for the proof of why the sum of the angles in a triangle is 180° is based on these relationships.

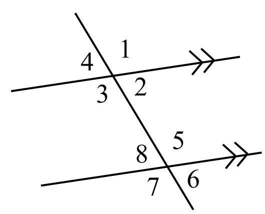
Pacing	Materials	Prerequisites
1 h	• Rulers, protractors, and compasses • Tracing paper	• naming angles and identifying an angle given its angle name, e.g., ∠ ACD or ∠ C • determining the angles in a regular polygon • familiarity with translations and rotations

Main Points to be Raised

• When two lines or line segments cross, these angle relationships are formed:

- Opposite angles are equal (1 = 3, 2 = 4, 5 = 7, and 6 = 8).

- Adjacent angles are supplementary (supplementary angles add up to 180º) (1 + 2 = 180º, 4 + 3 = 180º, 1 + 4 = 180º, 2 + 3 = 180º, 5 + 6 = 180º, 7 + 8 = 180º, 5 + 8 = 190º, and 6 + 7 = 180º).



• A transversal is a line that crosses two lines. If the two lines are parallel, these angle relationships are formed:

- Corresponding angles are equal (1 = 5, 2 = 6, 4 = 8, and 3 = 7).

- Alternate angles are equal (2 = 8 and 3 = 5).

- Interior angles are supplementary (2 + 5 = 180º and 3 + 8 = 180º).

Try This — Introducing the Lesson

Make sure students know how to locate ∠ GFL and ∠ EFL in the diagram in the **Try This**. Then allow students to try **parts A and B** alone or with a partner. While you observe students at work, you might ask questions such as the following:

• *What does a translation along FA mean?* (It means vertex F moves to where vertex A is. The rest of the points in ∠ FGL move accordingly.)

• *Which angle corresponds to ∠ GFL in the image of ΔFGL after a translation along FA?* (∠ FAB or ∠ BAF)

• *How do you decide in what order to list the vertices when you describe the image triangle?* (I make sure they are in the same order as in the original shape's corresponding vertices.)

If students do not identify which angles correspond to each other, show them and explain how you know. This is not an outcome of this lesson, but it is integral to students' understanding of the parallel line properties.

The Exposition — Presenting the Main Ideas

• On the board, draw the diagram with two triangles from the bottom of **page 251** in the student text.

• Show which angles are *adjacent*. Explain why these adjacent angles add to 180º (they form a straight line, which is a 180º angle).

• Show which angles are *opposite*. Point out that opposite angles are always equal. Tell students that this is proven in **example 2**.

• As you refer to the diagram of the two triangles, show how to identify and record angles. Emphasize that the vertex of an angle must be the middle letter when you write the angle. Talk about how you could name ∠ A, ∠ B, ∠ D, and ∠ E using one letter. The angles at C need more information because there are four possible angles: ∠ ACD, ∠ ACB, ∠ BCE, and ∠ DCE.

• Tell students that the word for angles that have a sum of 180º is *supplementary*. Use this word as you talk about the pairs of adjacent angles.

• Review with students the meaning of the word *parallel*. Give examples of parallel lines in your surroundings (e.g., the top and bottom of the board are parallel; the left and right sides of a road are roughly parallel).

• Draw the diagram from the centre of **page 252** showing two parallel lines cut by a transversal. Label the *transversal* and explain what it means.

• Demonstrate how to show that lines are parallel using matching arrows, and how to write that two lines are parallel (e.g., AB $\parallel$ CD).

• In the diagram of the lines and the transversal, identify pairs of *corresponding angles*. Tell students that these are always equal and that they will be asked to show why this is true in **question 7**.

• Identify pairs of *alternate angles*. Tell students that these are always equal and that they will be asked show why this is true in **question 8**.

• Identify pairs of *interior angles*. Tell students that these are always supplementary and that they will show why this is true in **question 9**.

Revisiting the Try This

B. Students identify angle pairs in the diagram from **part A** using their new knowledge from the exposition.
You can do this question as a class, allowing students to identify various pairs of each kind of angle pair.

Using the Examples

• Ask a few pairs of students to do **example 1** using $\angle$ DEH instead of $\angle$ BED. They can follow the reasoning of the student in the example. Have students compare their findings with their classmates.

• Explain to students that **example 2** demonstrates how you can use a transformation of a triangle to show why
a certain angle property is true. This case demonstrates why opposite angles are equal. Tell students that they will do similar work in **questions 7 and 8** to show why corresponding angles and alternate angles are equal. Work through **example 2** with the students to make sure they understand it.

Practising and Applying

Teaching points and tips

Note: *Students may use various prepositions to describe the properties of angles. For example, they might say "$\angle$ ABE is alternate to $\angle$ BED", but it is also okay to say "$\angle$ ABE is alternate with $\angle$ BED". It is the same for the other kinds of angles.*

Q 1 and 2: Some students may wish to measure the angles to verify their conjectures.

Q 3: Students may find it difficult to identify the transversals in each case. Have them focus on the letters that identify each, e.g., LD.

Q 6: This question draws on the previous lesson.

Q 7: This question asks students to demonstrate why corresponding angles are equal.

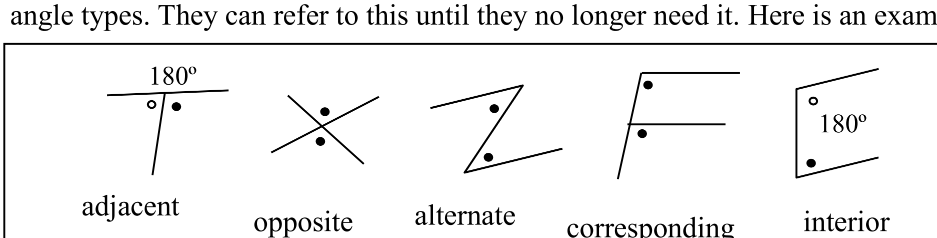
Q 8: This question asks students to demonstrate why alternate angles are equal.

Q 9: This question asks students to demonstrate why interior angles are supplementary. Encourage students to use the word *supplementary* instead of saying that the angles add to 180º.

Q 10: This question is like **question 6**, but it is more challenging.

Common errors

• Many students will confuse the angle terminology because the words carry different meanings outside of mathematics. You might encourage students to make a reminder sheet with a set of sketches showing the angle types. They can refer to this until they no longer need it. Here is an example:



Some students associate these angle pairings with letters of the alphabet. The sketches resemble the letters T, X, Z, F, and C, respectively.

• Students may have difficulty with **questions 6 and 10** because it is hard for them to identify the parallel lines and transversals. You might outline for them the parallel lines and the transversals. In **question 10**, you could use one colour to outline one set of parallel lines and another colour to outline the other set.

Suggested assessment questions from Practising and Applying

Question 1	to see if students can identify angle relationships given lines that intersect
Question 2	to see if students can identify angle relationships given parallel lines and a transversal
Question 7	to see if students can use transformational geometry to show why corresponding angles are equal
Question 10	to see if students can solve a problem using their knowledge of angle properties

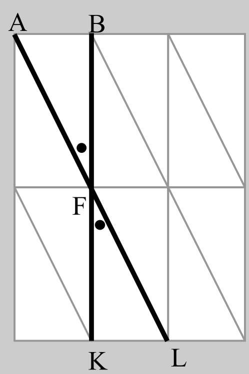
Answers

A. i) EK, AL, BM, and CH are parallel. AD, EH, and JM are parallel.
ii) Together they make a straight angle.

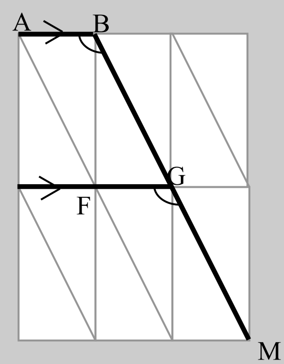
B. i) ΔABF
ii) ΔFEA
iii) ΔLKF

C. *Sample responses*:

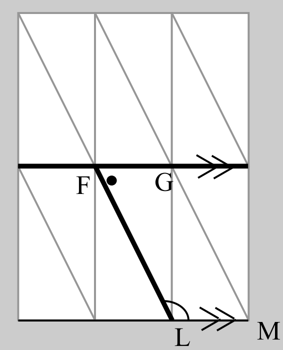
i) ∠ AFB and ∠ LFK



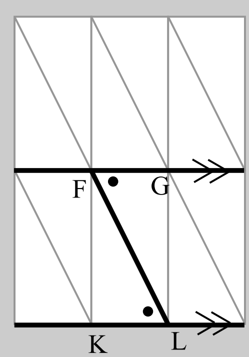
ii) ∠ ABG and ∠ FGM



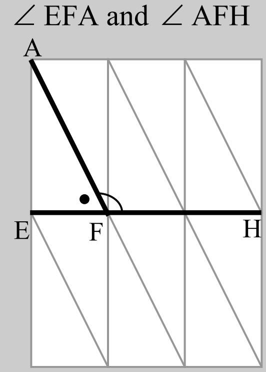
iii) ∠ GFL and ∠ MLF



iv) ∠ GFL and ∠ KLF



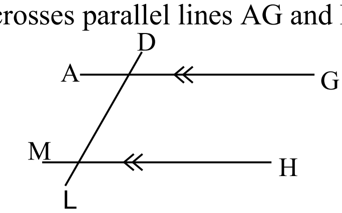
v) ∠ EFA and ∠ AFH



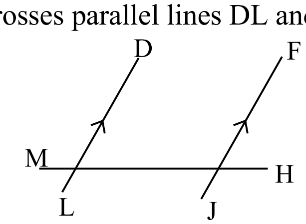
1. a) 135º; [∠ ABC and the given 45º angle are adjacent, so together they make a straight angle.]
b) 45º; [∠ ABE is opposite the given 45º angle.]

2. a) 120º; [It is an interior angle with ∠ UYX.]
b) 60º; [It is an alternate angle to ∠ UYX.]
c) 60º; [It is a corresponding angle to ∠ UYX.]
d) 120º; [It is opposite the angle that is interior with ∠ WUY.]

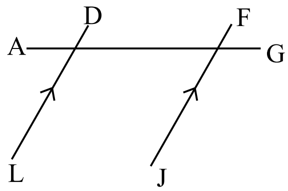
3. a) LD crosses parallel lines AG and MH.



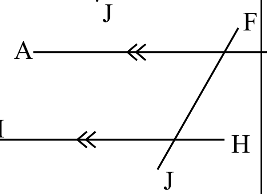
b) HM crosses parallel lines DL and FJ.



3. c) AG crosses parallel lines DL and FJ.



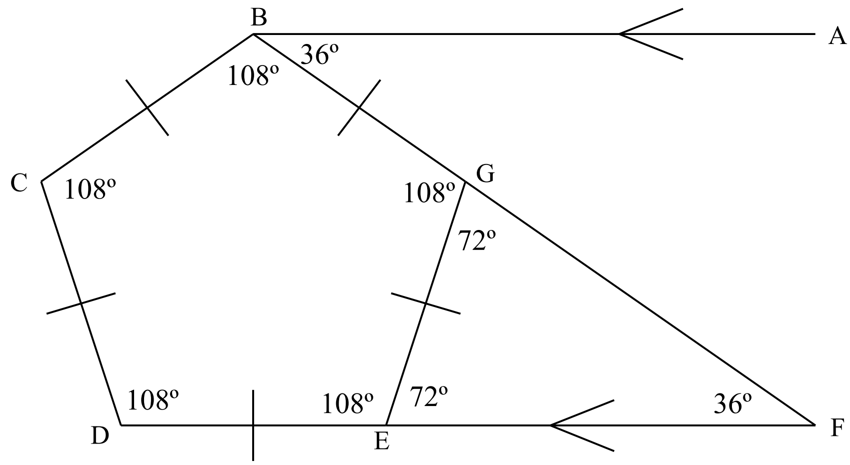
d) FJ crosses parallel lines AG and MH.



4. a) ∠ BKI, ∠ BEI, ∠ ABM, and ∠ LKM
b) ∠ BKI, ∠ BEI, ∠ ABM, and ∠ LKM
c) *Sample response*:
The angles across from each other are equal.

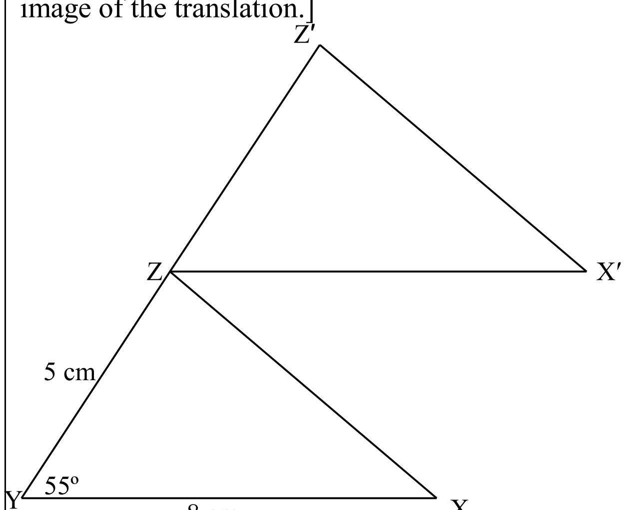
5. a) ∠ LKI and ∠ GEI
b) ∠ MKB and ∠ BEF

6.

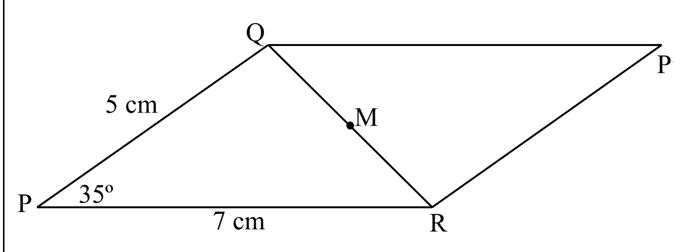


[Each interior angle in the regular pentagon is 108º:
180º – (360º ÷ *n*) = 180º – (360º ÷ 5) = 180º – 72º = 108º.
∠ FGE and ∠ FEG are each 72º because each is adjacent to a 108º angle.
∠ GFE is 36º because the three angles in the triangle should add up to 180º: 72º + 72º + 36º = 180º.
∠ ABF is 36º because it is alternate to ∠ GFE.]

7. a) and b) See diagram below.
c) *Sample response*:
ZX′ ∥ YX; [With a translation, the image stays facing the same way so that image lines are parallel to the original lines.]
d) ∠ ZYX and ∠ Z′ZX; [*Sample response*:
The angles are equal because they are corresponding angles in the original shape and the image of the translation.]



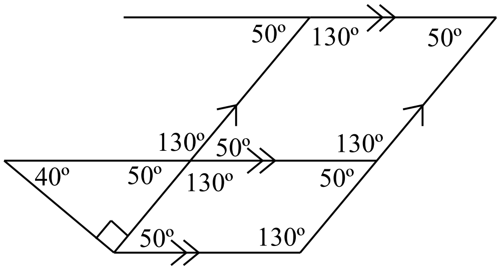
8. a), b), and c) See diagram below.
d) *Sample response*:
PR ∥ QP′; [The rotation turns the original line PR 180º, which means it is parallel to QP′.]
e) ∠ QRP and ∠ RQP′; [*Sample response*:
The angles are equal because they are corresponding angles in the original shape and image of a rotation.]



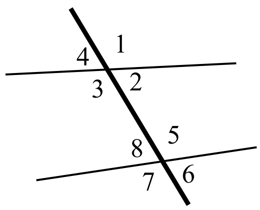
9. a) 133º; [This angle and the 47º angle are adjacent. Together they make a straight angle, and 180º – 47º = 133º.]
b) 47º; [It is a corresponding angle with ∠ DCE.]
[**c)** *Sample response*:
The interior angle forms a straight angle with the corresponding angle, so they must add to 180º.]

Answers [Continued]

10. *This diagram shows all possible angle measures*:



11. a) Yes; [*Sample response*:
I drew two non-parallel lines and a transversal to find out:

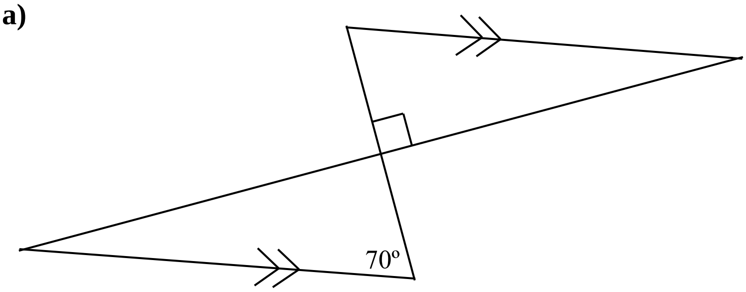


Corresponding angles are not congruent:
1 ≠ 5, 2 ≠ 6, 4 ≠ 8, and 3 ≠ 7.
Alternate angles are not congruent:
2 ≠ 8 and 3 ≠ 5.]

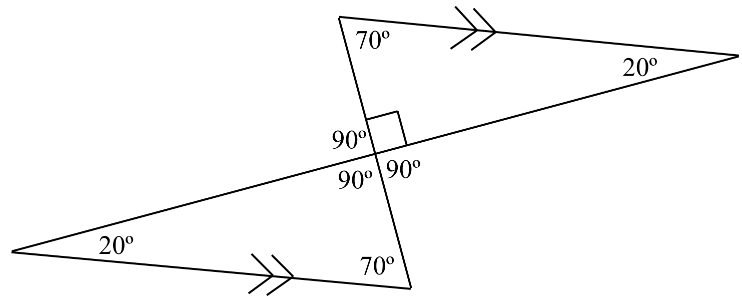
b) Yes; [in the diagram that I drew above, opposite angles are still congruent:
1 = 3, 2 = 4, 5 = 7, and 6 = 8.]

12. *Sample responses*:

a)



b)



Supporting Students

Struggling students

• If students struggle with the transformations in **questions 7 and 8**, you might help them perform the transformations and ask them to focus their attention on the questions about the parallel lines and angle properties.

• Some students might have trouble identifying angle pairs, but will not have trouble deciding which angles are equal and which angles are supplementary. Encourage these students to focus their attention on identifying these angle measurements and writing down the measurement of every possible angle in a diagram. Then they can use a reminder sheet (like the sheet described in **Common errors**) to help them with identification.

Enrichment

• You might challenge students to make a diagram that uses as few lines as possible and that includes at least 15 pairs of corresponding angles. They could do the same for opposite angles.

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CONNECTIONS: Tools for Geometry

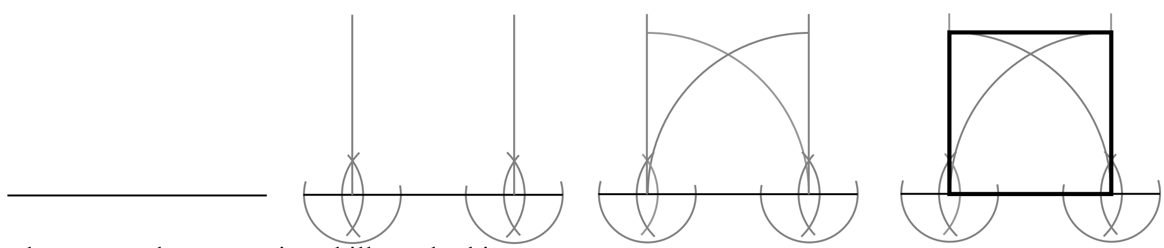
• This optional connection asks students to think about how they can apply the geometrical thinking they have learned in school to situations outside of school.

• The focus of this section is on tools and how they can be improvised. In discussion with students you could think about situations in which you might need to construct a large square or rectangle, or in which you would want to make parallel lines. Talk with them about how they could do these things with the materials that would be available in each situation. Let them explain their own ideas. Do not only share your ideas.

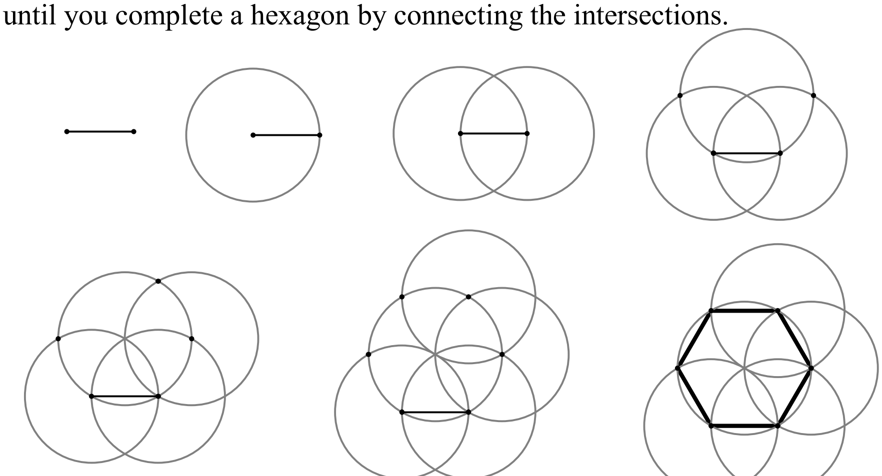
Answers

[**1.** *Sample response*:
a) Make one fold. The folded edge is straight.
b) Make one fold. Fold again so that the folded edge goes over onto itself. Fold the edge onto itself one more time. Unfold the paper. You should have two fold lines that are perpendicular to your first fold line. You have two parallel lines intersected by a perpendicular transversal.]

[**2.** *Sample responses*:
a) Hold two sticks in one hand so their end points stay an equal distance apart. Walk in a straight line, dragging the sticks in the dirt. Their paths will be parallel.
b) Use the rope and construction skills to do this:
- Make a straight line.
- Make two right angles on the line.
- From each right angle, make an arc with radius equal to the distance between the two right angles. The two arcs intersect the perpendiculars at the two other points of the square.



c) Use the rope and construction skills to do this:
- Make a straight line as long as the rope.
- Hold the rope at one endpoint and mark a circle using the length of the rope.
- Mark another circle from the other endpoint.
- Mark another circle from the place where the two circles intersect.
- Mark another circle from the intersection of the last two circles.
- Make a straight line to connect the points where the circles intersect.
- Continue making circles until you complete a hexagon by connecting the intersections.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UNIT 8 Revision

Pacing	Materials
2 h	• Rulers, protractors, and compasses • Tracing paper • Isometric Dot Paper (BLM) • Linking cubes

Question(s)	Related Lesson(s)
1 and 2	Lesson 8.1.1
3 – 5	Lesson 8.1.2
6 and 7	Lesson 8.2.1
8 and 9	Lesson 8.2.2
10 and 11	Lesson 8.3.2
12 and 13	Lesson 8.3.3

Revision Tips

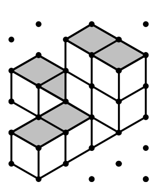
Q 1 to 5: Some students may choose to do these questions by visualizing and not by using linking cubes. Others will choose to use linking cubes.

Q 2: Students need to consider alternate viewpoints to make the other isometric drawings.

Q 8 c): This question relates to work students have done but it is approached somewhat differently. They need to use their problem solving skills.

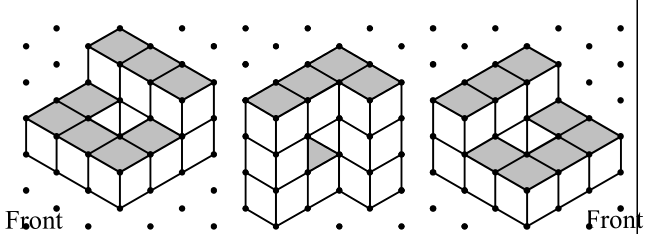
Answers

1. *Sample response*:



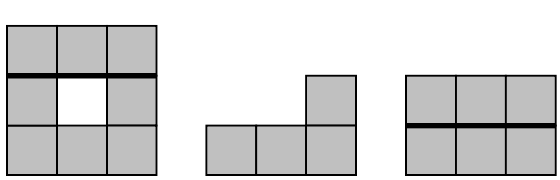
Back (but turned $\frac{1}{4}$ turn)

2. *Sample response*:



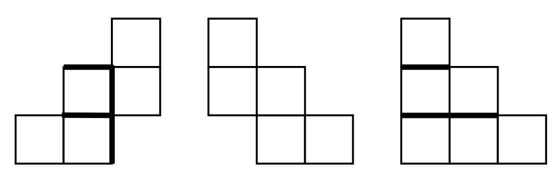
3. *Sample response*:

Top face view Right face view Back face view



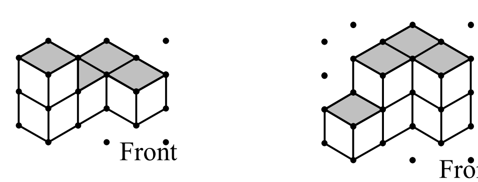
4. *Sample response*:

Right face view Left face view Front face view



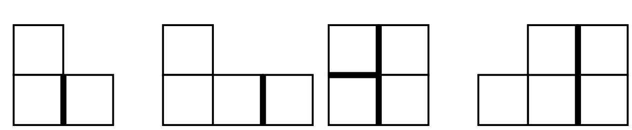
5. *Sample responses*:

a) Structure A Structure B



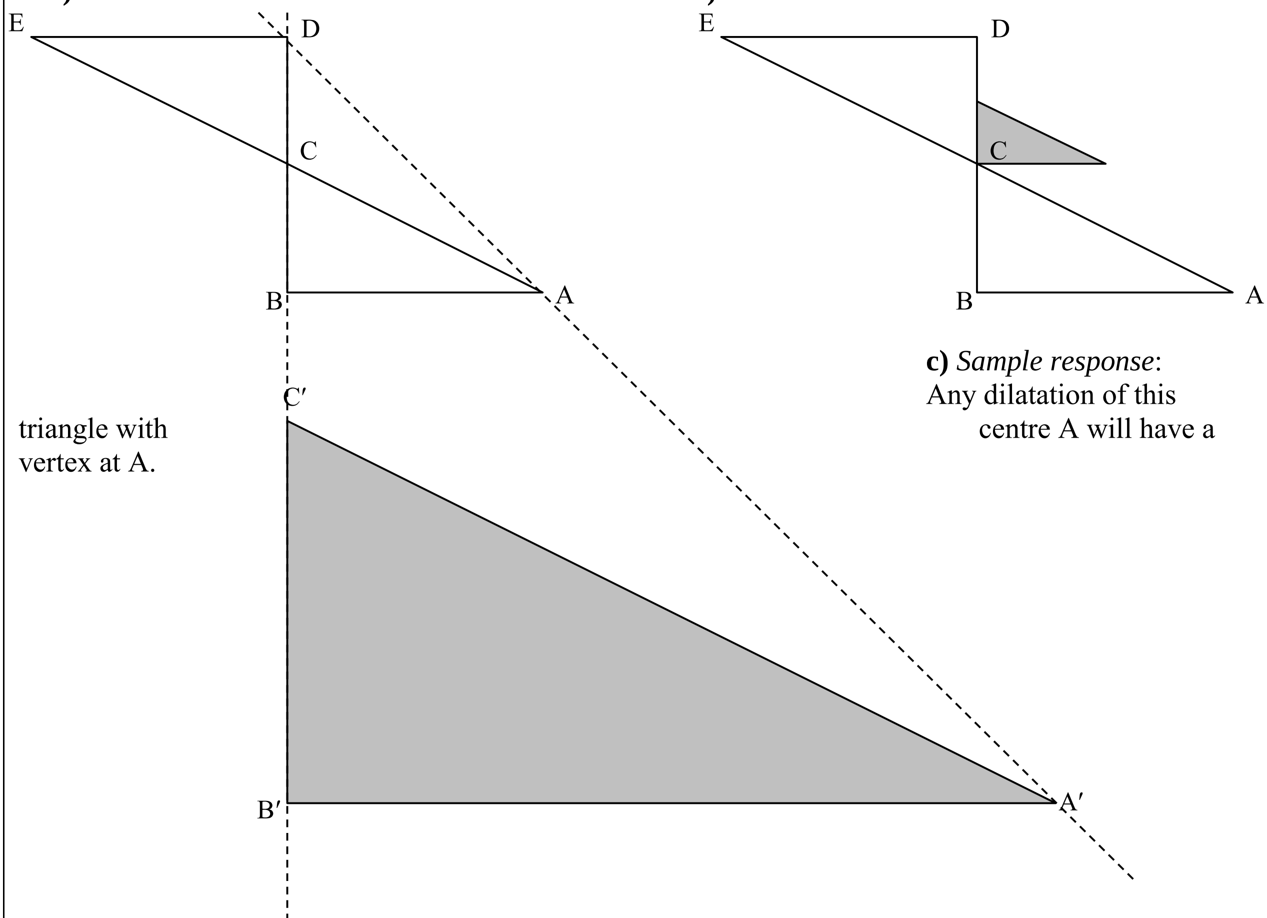
b) Structure A Structure B

Left face view Front face view Left face view Front face view



6. A; [*Sample response*: A is similar to B, and if you draw lines through corresponding vertices they all meet at one point, the centre of the dilatation.
C could not be a dilatation image because it is turned. Also, if you draw lines through corresponding vertices, they do not meet at one point.]

7. a)

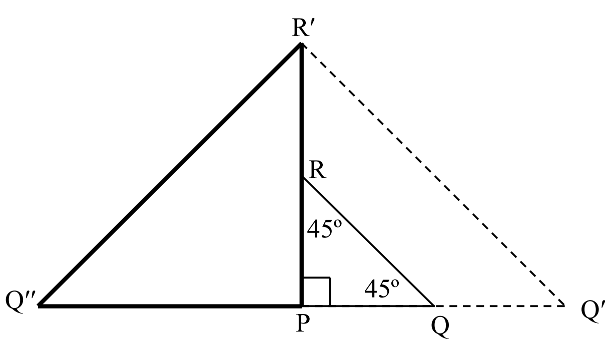


b)

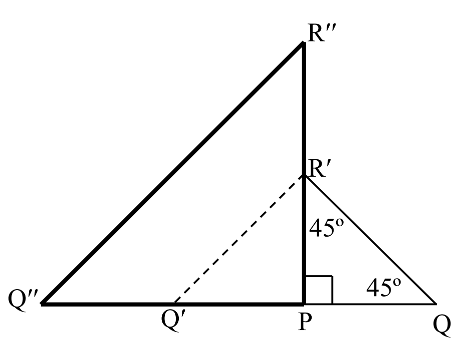
c) *Sample response*:
Any dilatation of this
centre A will have a

triangle with
vertex at A.

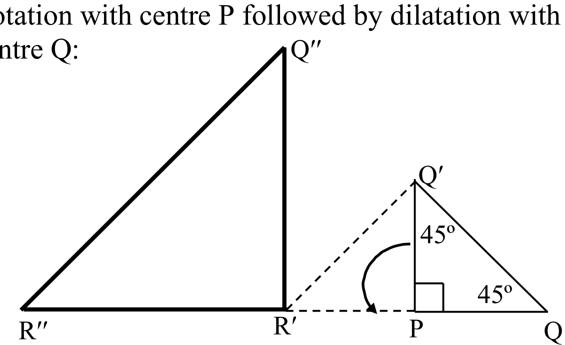
8. a)



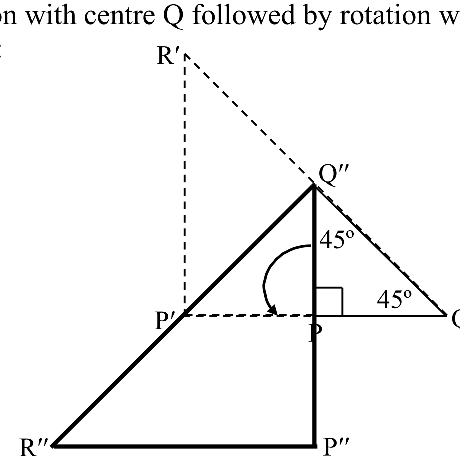
b) Yes.



c) If the centre of the dilatation is Q, then the order matters:
Rotation with centre P followed by dilatation with centre Q:



Dilatation with centre Q followed by rotation with centre P:



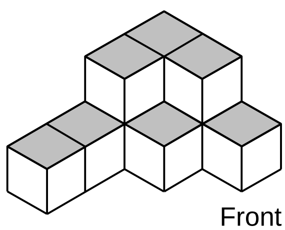
Answers [Continued]

9. *Sample response*: A rotation of 180º (cw or ccw) around C followed by a dilatation with centre D′ and scale factor 3. **10. a)** 540º; [$180º(n-2) = 180º(5-2) = 180º \times 3 = 540º$] **b)** 1260º; [$180º(n-2) = 180º(9-2) = 180º \times 7 = 1260º$] **c)** 1800º; [$180º(n-2) = 180º(12-2) = 180º \times 10 = 1800º$] **11. a)** 135º; [$180º - 360º \div n = 180º - 360º \div 8 = 180º - 45º = 135º$] **b)** 40º; [$360º \div n = 360º \div 9 = 40º$] **c)** 36 sides; [$360º \div n = 10º$ $n = 360º \div 10º$ $n = 36$] **12. a)** $\angle 4$ **b)** $\angle 7$ **c)** $\angle 3$	**13. a)** 63º; [The three angles in the triangle add to 180º.] **b)** 105º; [$\angle$ CEF and $\angle$ FEG (75º) are supplementary angles, and 180º – 105º = 75º.] **c)** 75º; [It is opposite the 75º angle, $\angle$ FEG.] **d)** 42º; [$\angle$ AEG and $\angle$ CGF (42º) are alternate angles.] **e)** 42º; [$\angle$ CED and $\angle$ CGF (42º) are corresponding angles.] **f)** 138º; [$\angle$ DEG and $\angle$ CGF (42º) are interior angles, and 180º – 42º = 138º.]

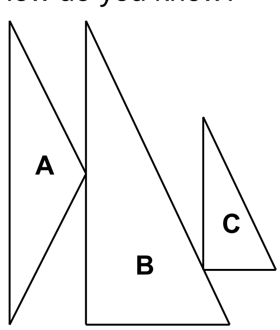
UNIT 8 Geometry Test

1. Make an isometric drawing of this cube structure.

2. Draw three orthographic face views of this structure made from 10 cubes.



3. Triangle B is the original shape. Which of the other triangles could be a dilatation image of Triangle B? How do you know?



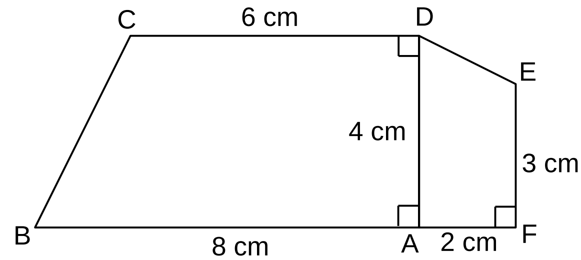
4. a) Draw $\triangle ABC$ with $\angle B = 90^\circ$, AB = 6 cm, and BC = 3 cm.

b) Dilatate $\triangle ABC$ by a scale factor of 2 with centre A.

c) Dilatate $\triangle ABC$ by a scale factor of $\frac{2}{3}$ with centre B.

d) Describe a dilatation of $\triangle ABC$ in which the image of vertex C is on one of the vertices of $\triangle ABC$.

5. This diagram shows two similar trapezoids.



a) Describe a combination of transformations that has trapezoid ABCD as the original shape and trapezoid AFED as its final image.

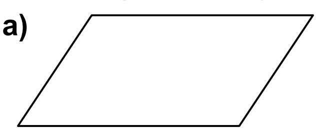
b) Does the order of the transformations in **part a)** matter?

c) Copy trapezoid AFED. Find its image after this combination of transformations:

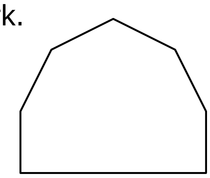
- Dilatate with centre A and scale factor 3.
- Then reflect across line AD.

6. What is the sum of the interior angles of each shape? Show your work.

a)



b)



7. a) Find the measure of an interior angle in a regular pentagon. Show your work.

b) Find the measure of an exterior angle in a regular decagon (ten sides). Show your work.

c) How many sides does a regular polygon have if one of its exterior angles is 20°? How do you know?

8. a) Draw two parallel lines with a transversal.

b) Identify a pair of corresponding angles.

c) Identify a pair of alternate angles.

d) Identify a pair of interior angles.

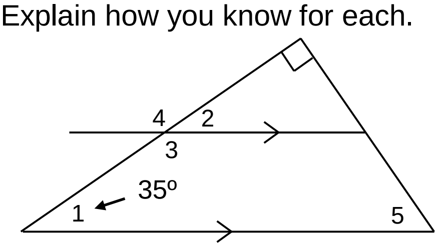
9. Determine the measure of each angle. Explain how you know for each.

a) $\angle 2$

b) $\angle 3$

c) $\angle 4$

d) $\angle 5$



UNIT 8 Test

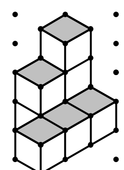
Pacing	Materials
1 h	• Rulers, protractors, and compasses • Tracing paper • Isometric Dot Paper (BLM) • Linking cubes

Question(s)	Related Lesson(s)
1	Lesson 8.1.1
2	Lesson 8.1.2
3 and 4	Lesson 8.2.1
5	Lesson 8.2.2
6 and 7	Lesson 8.3.2
8 and 9	Lesson 8.3.3

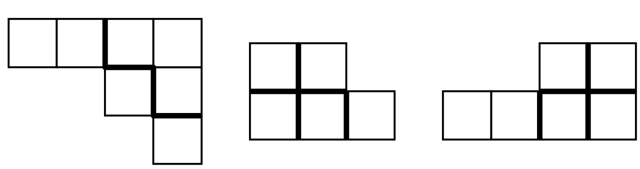
Select questions to assign according to the time available.

Answers

1. *Sample response*:

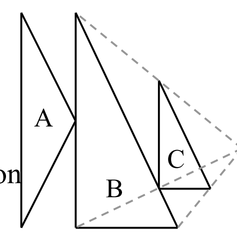


2. *Sample response*:

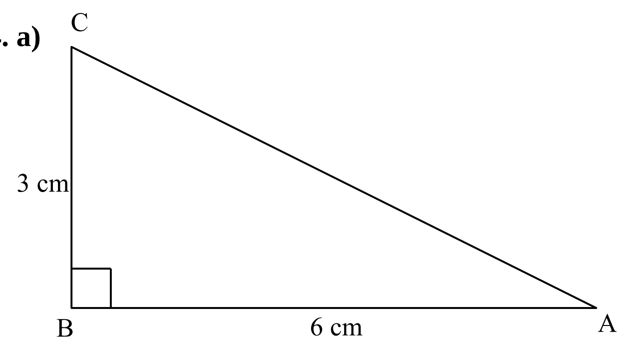


Top face view Left face view Front face view

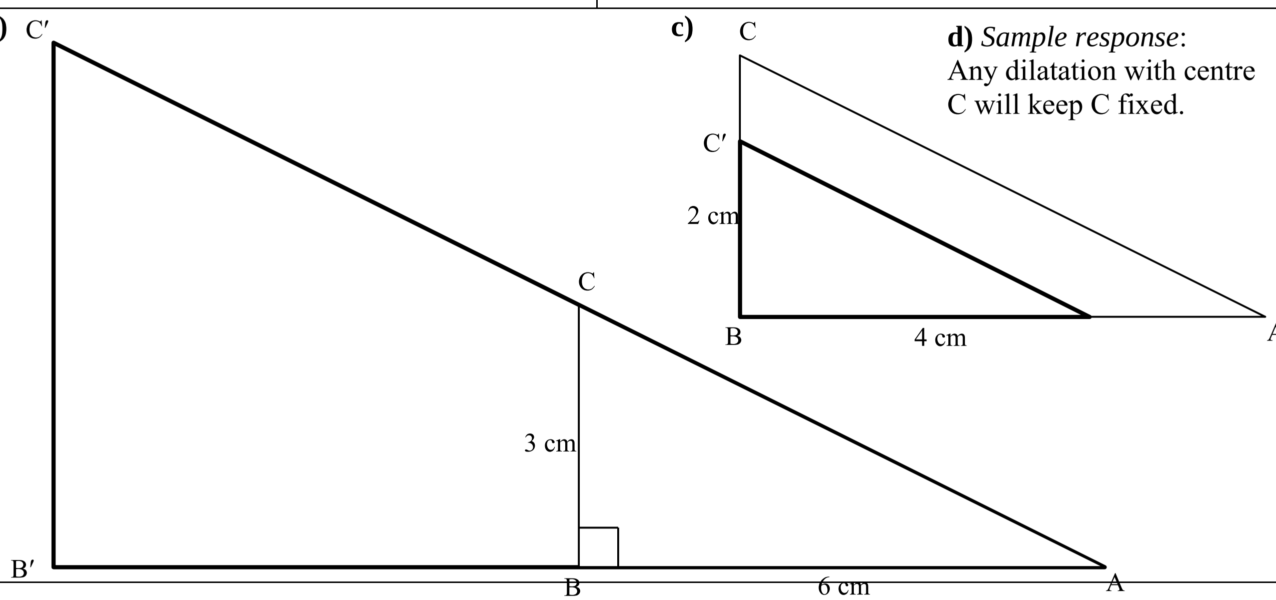
3. Triangle C could be a dilatation because it is similar to Triangle B, and if you draw lines through corresponding points they meet at one point, the dilatation centre. (Triangle A could not be a dilatation image because it is not similar to Triangle B.)



4. a)



4. b)



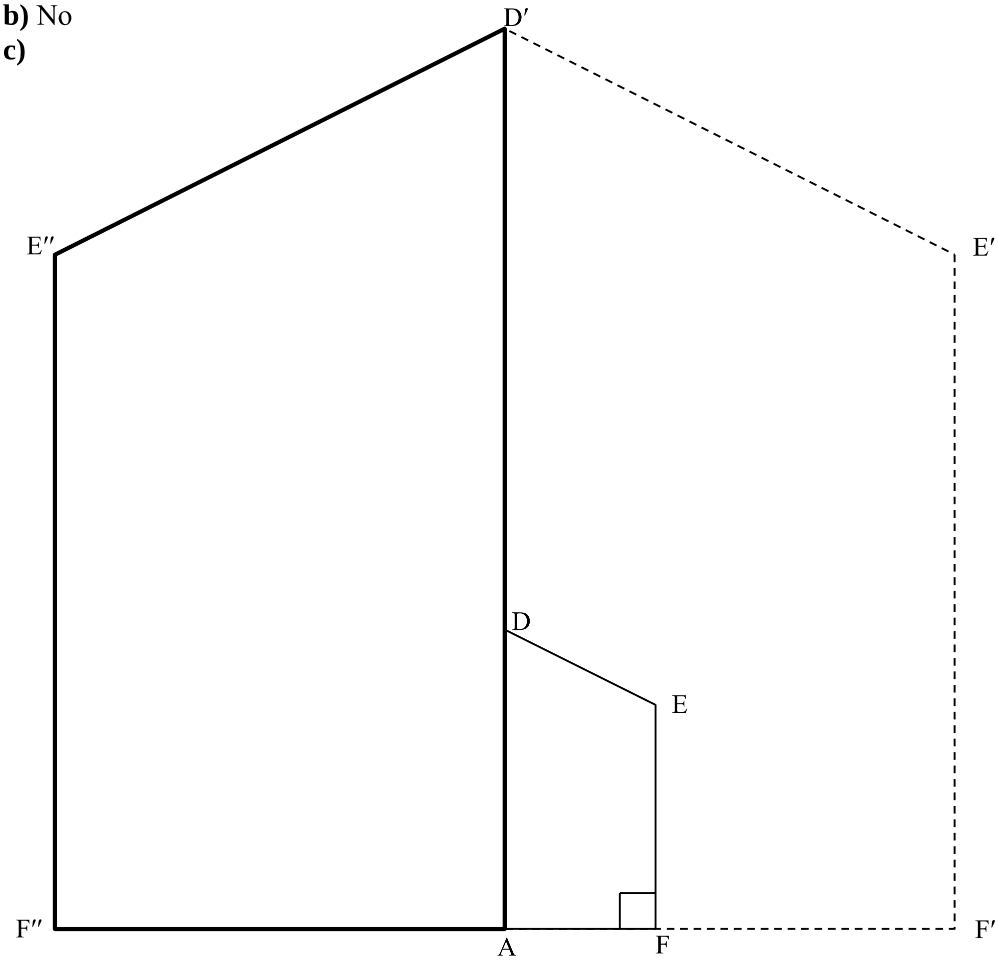
d) *Sample response*:
Any dilatation with centre C will keep C fixed.

5. a) *Sample response*:

Dilatate ABCD with centre A and scale factor $\frac{1}{2}$. Then rotate 90º cw around point A.

b) No

c)



6. a) 360º; 180º(n – 2) = 180º(4 – 2) = 180º × 2 = 360º

b) 900º; 180º(n – 2) = 180º(7 – 2) = 180º × 5 = 900º

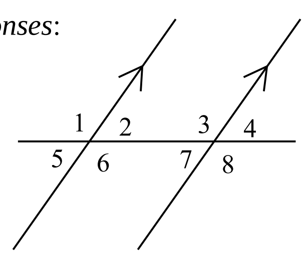
7. a) 108º; 180º – 360º ÷ n = 180º – 360º ÷ 5 = 180º – 72º = 108º

b) 36º; 360º ÷ n = 360º ÷ 10 = 36º

c) 18 sides; 360º ÷ n = 20º, n = 360º ÷ 20º, n = 18

8. *Sample responses*:

a)



8. b) ∠ 1 and ∠ 3 are corresponding angles.

c) ∠ 7 and ∠ 2 are alternate angles.

d) ∠ 3 and ∠ 2 are interior angles.

9. *Sample responses*:

a) 35º; ∠ 1 and ∠ 2 are corresponding angles so they are equal.

b) 145º; ∠ 1 and ∠ 3 are interior angles so they are supplementary. ∠ 3 is 180º – 35º = 145º.

c) 145º; ∠ 4 is opposite to ∠ 3.

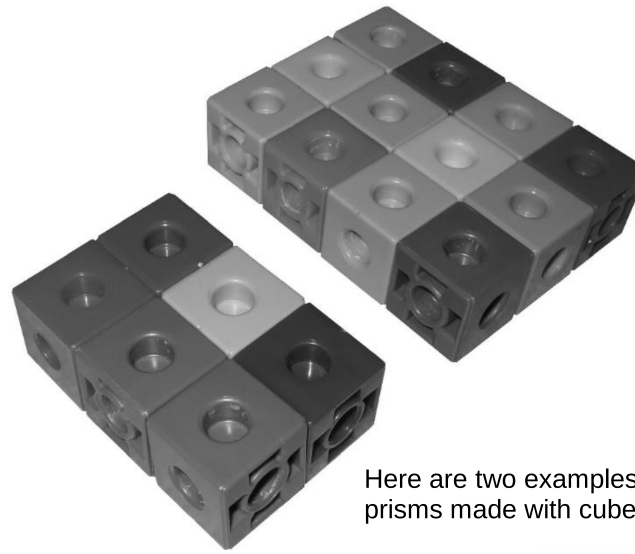
d) 55º; ∠ 1, ∠ 5, and the right angle are the angles in a triangle, so they add to 180º.
∠ 5 is 180º – 35º – 90º = 55º.

UNIT 8 Performance Task — Rectangles

A. i) Build three rectangular prisms using cubes. Build the prisms so that the three orthographic views from the top show these three shapes:

- a rectangle, labelled as the original
- a rectangle that could be the image of the original rectangle after a dilatation
- a rectangle that could not be an image of the original rectangle

ii) Draw the top face view of each prism. Label them “Original”, “Image”, and “No image”. Draw all three views on the same piece of grid paper.



Here are two examples of prisms made with cubes

B. i) Explain how you know that the “No image” top view cannot be an image of the “Original” top view.

ii) Describe the transformations needed to transform the “Original” top view to the “Image” top view.

C. i) Add six more linking cubes to your smallest prism without changing the orthographic view from the top.

ii) Draw three orthographic views of this structure.

iii) Draw two different isometric views of this structure.

D. Examine the lines in one of your isometric drawings.

i) Identify two parallel lines and a transversal.

ii) Use the parallel lines and the transversal from **part i)** to identify each:

- a pair of corresponding angles
- a pair of interior angles
- a pair of alternate angles
- a pair of opposite angles
- a pair of adjacent angles

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UNIT 8 Performance Task

Curriculum Outcomes Assessed	Pacing	Materials
8-E1 Interpret and Make Orthographic Drawings and Isometric Drawings: to represent 3-D shapes **8-E2** Dilatations: represent, analyse, and apply **8-E4** Angle Pair Relationships: parallel and non-parallel lines	1 h	• Rulers, protractors, and compasses • Tracing paper • Isometric Dot Paper (BLM) • Linking cubes • Grid paper or Small Grid Paper (BLM)

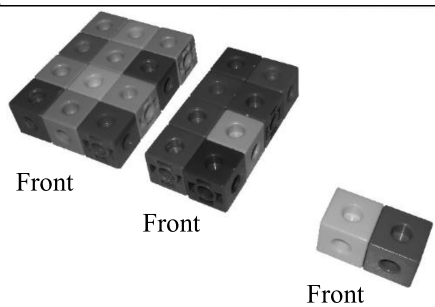
How to Use This Performance Task

• You might use this task as a rich problem to assess student understanding of a number of outcomes in this unit. It could supplement the unit test. It could also be used as enrichment material for some students.

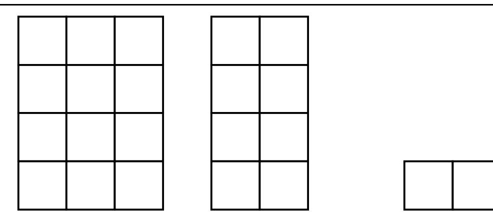
• You can assess performance on the task using the rubric on the next page.

Sample Solution

A. i)



ii)

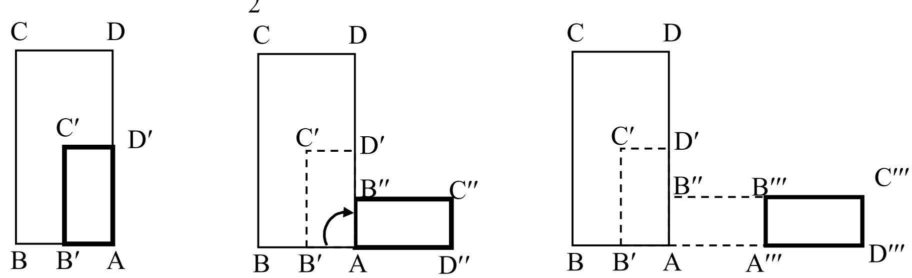


No image Original Image

Top views

B. i) It cannot be an image because it is not similar or congruent.

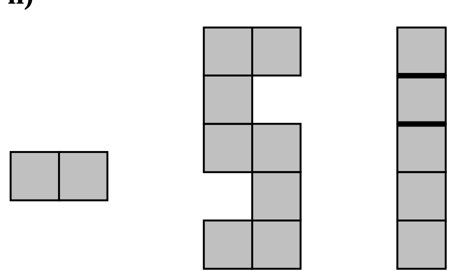
ii) First, dilatate with centre A and scale factor $\frac{1}{2}$. Then rotate 90º cw around centre A. Finally, translate along BA.



C. i)

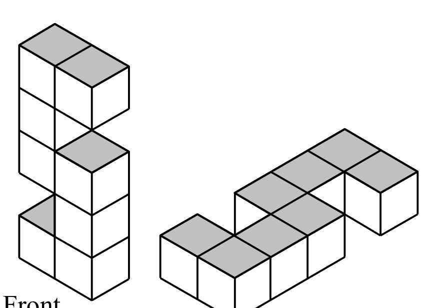
Front

ii)



Top view Front view Right view

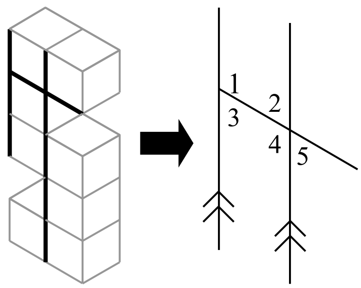
iii)



Front Bottom

Sample Solution [Continued]

D. i)



ii) ∠ 3 and ∠ 5 are corresponding angles.
∠ 3 and ∠ 4 are interior angles.
∠ 3 and ∠ 2 are alternate angles.
∠ 2 and ∠ 5 are opposite angles.
∠ 3 and ∠ 1 are adjacent angles.

UNIT 8 Performance Task Assessment Rubric

The student	**Level 4**	**Level 3**	**Level 2**	**Level 1**
Builds cube structures	Builds all cube structures according to specifications	Builds most cube structures according to specifications, with minor errors	Builds some of the cube structures according to specifications, with a few minor errors	Makes major errors in the cube structures
Explains transformations	Explains transformations completely and accurately	Explains transformations well but not completely clearly	Explains some transformations correctly	Makes major errors in explaining transformations
Represents shapes	Draws clearly and accurately all required shapes	Draws accurately all required shapes	Draws accurately some of the required shapes	Shows major flaws in the drawings
Identifies angle relationships	Clearly marks and correctly identifies angles	Correctly identifies most angles with adequate markings to show the angle relationships	Correctly identifies some angle relationships	Makes major errors in identifying angle relationships

UNIT 8 Assessment Interview

- You may wish to interview selected students to assess their understanding of the work of this unit.
- Interviews are most effective when done with individual students, although it is sometimes appropriate to interview students in pairs or small groups.
- The results can be used as formative assessment or as a piece of summative assessment data.
- As students work, ask them to explain their thinking.

Have available linking cubes, protractors, Isometric Dot Paper (BLM) and grid paper or Small Grid Paper (BLM).

Part 1

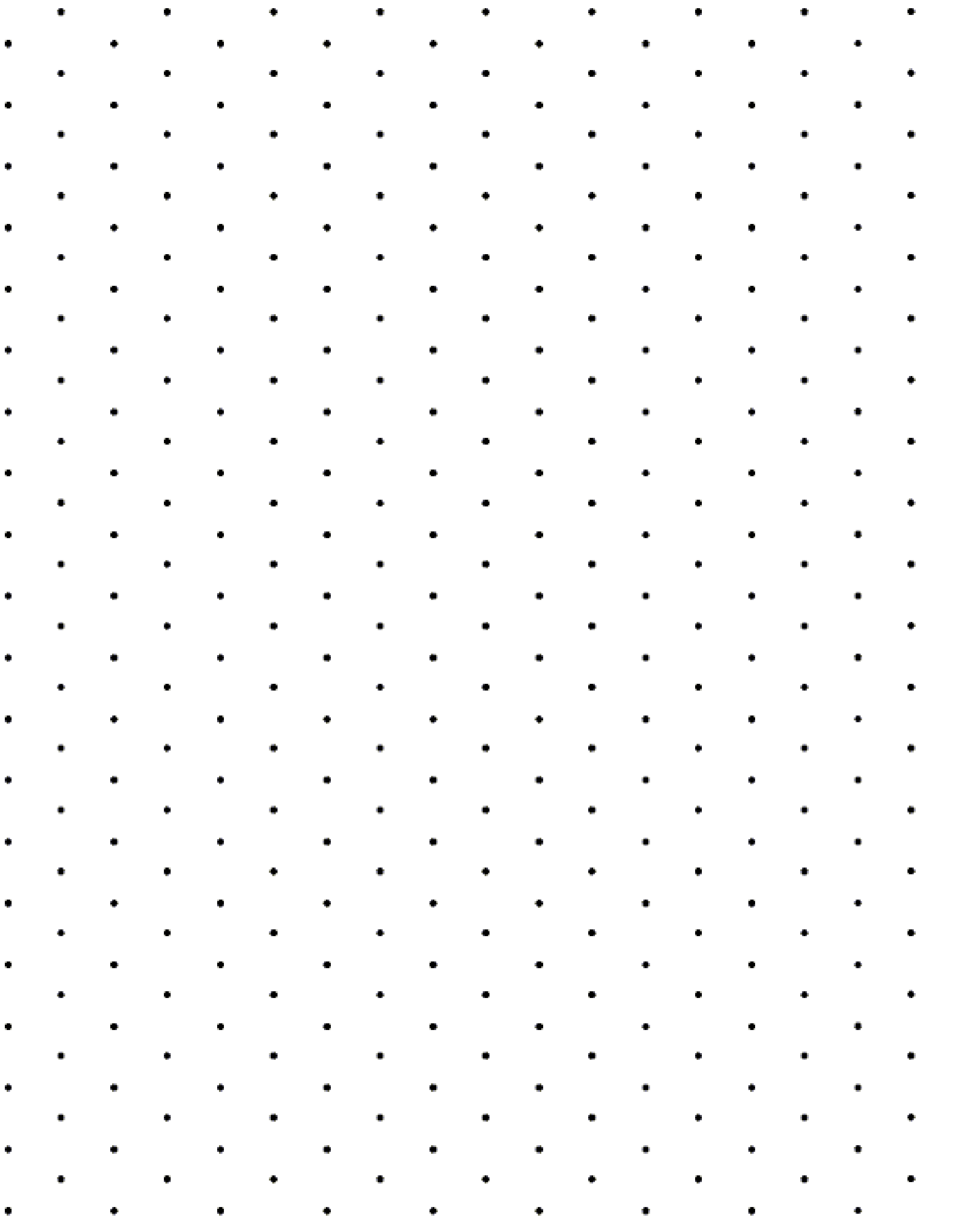
Ask students to do each of these tasks:
- Build a structure using nine linking cubes.
- Draw the front face view of the structure.
- Change the structure so that only two of its face views change.
- Explain why isometric drawings of the two structures would be different.

Part 2

Ask students to do each of these tasks:
- Draw a shape with five or more sides and with one pair of parallel sides.
- Tell what is the sum of the interior angles. How can you prove it?
- Draw a transversal across the two parallel sides. Measure one angle that is formed.
- Describe the measures of as many other angles in the shape as you can, using the known angle measure. Explain your reasoning.

UNIT 8 Blackline Masters

BLM 1 Isometric Dot Paper



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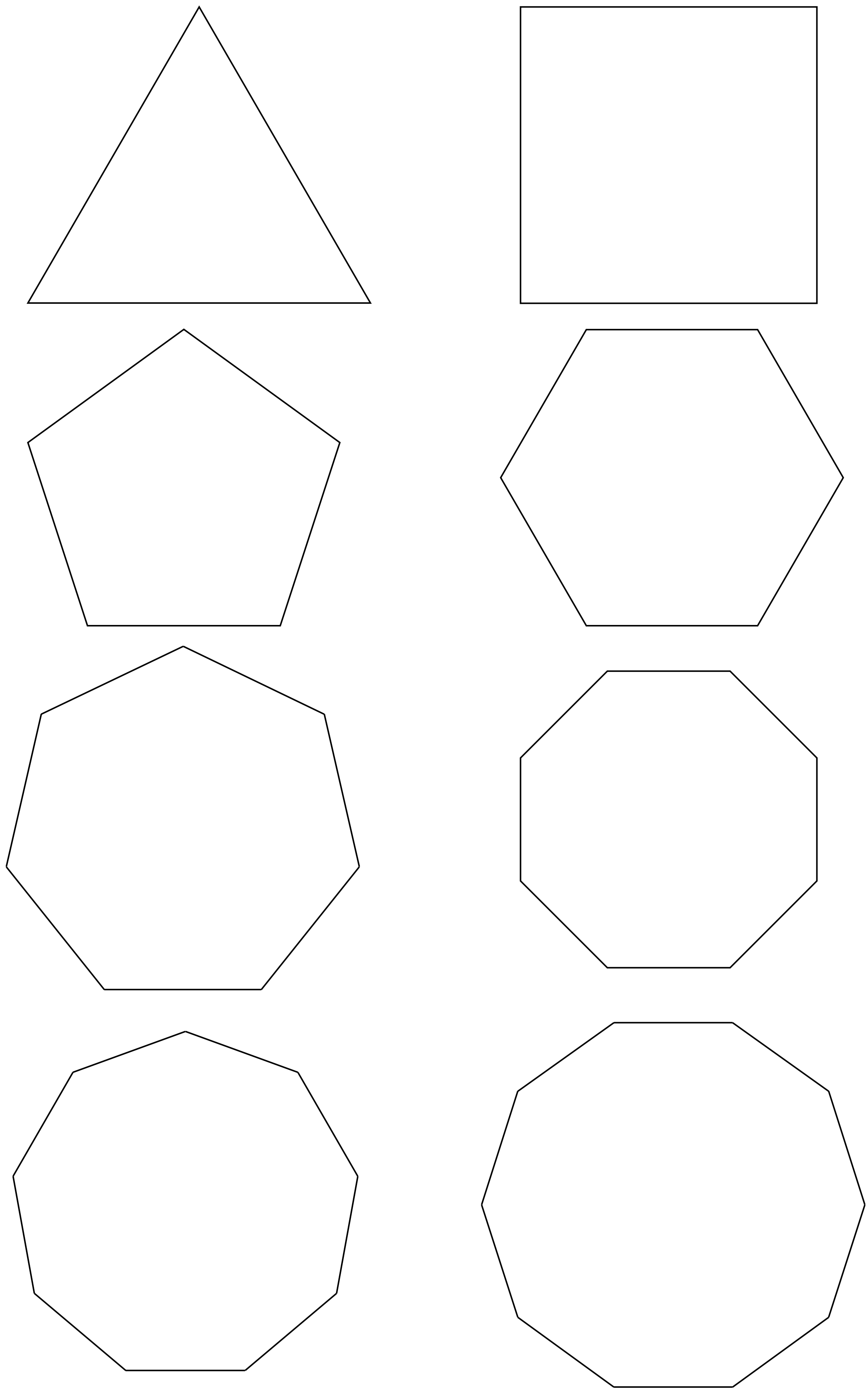
BLM 2 Regular Polygons



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