Understanding Mathematics

Textbook for Class VIII

ཤེས་རིག

Department of School Education
Ministry of Education and Skills Development
Royal Government of Bhutan

Published by:
Department of School Education (DSE)
Ministry of Education and Skills Development (MoESD)
Royal Government of Bhutan
Tel: +975-8-271226 Fax: +975-8-271991

ACKNOWLEDGEMENTS

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The Ministry of Education wishes to thank
- all teachers in the field who have given support and feedback on this project
- the World Bank, for ongoing support for School Mathematics Reform in Bhutan
- the students at Drugyal LSS and Lango LSS for their photos
- Nelson Publishing Canada, for its publishing expertise and assistance

1st edition 2008
Reprint 2024

ISBN: 99936-0-323-6

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ROYAL GOVERNMENT OF BHUTAN
ཤེས་རིག་ལྷན་ཁག།
MINISTRY OF EDUCATION
THIMPHU :BHUTAN
Cultivating the Grace of Our Mind

MINISTER

December 15, 2008

Foreword

I am at once awed and fascinated by the magic and potency of numbers. I am amazed at the marvel of the human mind that conceived of fantastic ways of visualizing quantities and investing them with enormous powers of representation and symbolism. As my simple mind struggles to make sense of the complexities that the play of numbers and formulae presents, I begin to realize, albeit ever so slowly, that, after all, all mathematics, as indeed all music, is a function of forming and following patterns and processes. It is a supreme achievement of the human mind as it seeks to reduce apparent anomalies and to discover underlying unity and coherence.

Abstraction and generalization are, therefore, at the heart of meaning-making in Mathematics. We agreed, propped up as by convention, that a certain figure, a sign, or a symbol, would carry the same meaning and value for us in our attempt to make intelligible a certain mass or weight or measure. We decided that for all our calculations, we would allow the signifier and the signified to yield whatever value would result from the tension between the quantities brought together by the nature of their interaction.

One can often imagine a mathematical way of ordering our surrounding and our circumstance that is actually finding a pattern that replicates the pattern of the universe – of its solid and its liquid and its gas. The ability to engage in this pattern-discovering and pattern-making and the inventiveness of the human mind to anticipate the consequence of marshalling the power of numbers give individuals and systems tremendous privilege to the same degree which the lack of this facility deprives them of.

Small systems such as ours cannot afford to miss and squander the immense power and privilege the ability to exploit and engage the resources of Mathematics have to present. From the simplest act of adding two quantities to the most complex churning of data, the facility of calculation can equip our people with special advantage and power. How intelligent a use we make of the power of numbers and the precision of our calculations will determine, to a large extent, our standing as a nation.

I commend the good work done by our colleagues and consultants on our new Mathematics curriculum. It looks current in content and learner-friendly in presentation. It is my hope that this initiative will give the young men and women of our country the much-needed intellectual challenge and prepares them for life beyond school. The integrity of the curriculum, the power of its delivery, and the absorptive inclination of the learner are the eternal triangle of any curriculum. Welcome to Mathematics.

Imagine the world without numbers! Without the facility of calculation!

Tashi Delek.

Thakur S Powdyel.

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INTRODUCTION

HOW MATHEMATICS HAS CHANGED

Mathematics is a study of quantity, space, structure, patterns and change. This study at the school level is divided into 5 strands of content, namely, numbers and operations, algebra, geometry, measurement, and data and probability.

The learning of mathematics helps a person to solve problems. While solving problems, skills related to, representation of mathematical ideas, making connections with other topics in mathematics and connections with the real world, providing reasoning and proof, and communicating mathematically would be required. The textbook is designed to promote the development of these process skills.

Nowadays, greater emphasis is given to conceptual understanding rather than on memorizing and applying rote procedures. There are many reasons for this.

- In the real world, you are not told when to factor or when to multiply but rather you need to figure out when to do so. You need to know and how to apply the concepts and skills you are learning in order to solve problems.
- Over time, it is very unlikely that you will remember the mathematics you learn unless it is meaningful. It is much harder to memorize something that does not make sense than something that relates to what you already know.

In this textbook, mathematics is made meaningful in many ways:

Using problems about Bhutan and around the world. These problems will help you see the value of math. For example:

For example:

- One problem will ask you to estimate and calculate a square root.

a) The area inside the square wall around a dzong is about 3500 m^2. Estimate the length of one of the side walls.

b) What is the side length to one decimal place?

- In another lesson, you will answer a question about planting grass.

A 5 kg bag of grass seed covers about 650 m^2. How many bags should Tshering buy to seed a 60 m-by-80 m football field?

- Other problems are related to your country and the world.

According to a May 2007 Kuensel report, 8×10^6 litres of water are used each day in Thimphu. If about 100,000 people live in Thimphu, estimate how many litres of water are used daily per person.

a) What is the width of Australia? (Measure the map to the nearest tenth of a centimetre.)

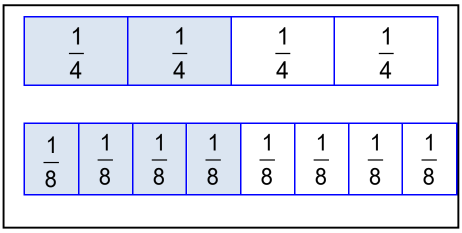
b) Bhutan is about 300 km wide. About how many times would Bhutan fit across Australia?

Your textbook will often ask you to use objects and tools to learn the math.

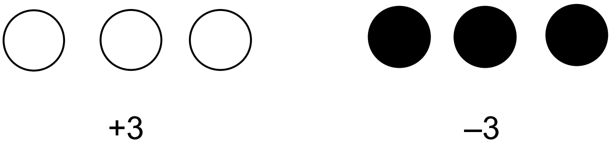
For example:

- You will build with cubes to learn about geometry.

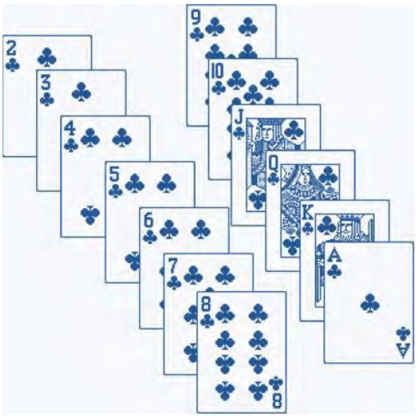
- You will use fraction strips to work with fractions.

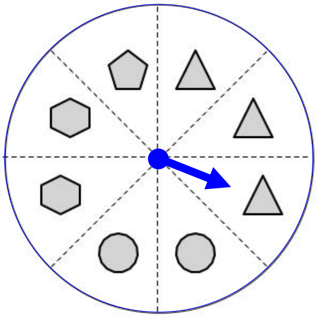


- You will use white and black counters to represent positive and negative integers.



- You will use spinners and cards in probability experiments.





This textbook will also ask to explain *why* things are true. It will not be enough to just say something is either true or false. For example, you might be asked to calculate the quotient of a negative number divided by a negative number and then explain why the quotient is positive.

You will solve many types of problems and you will be encouraged to use your own way of thinking to solve and explain them.

USING YOUR TEXTBOOK

Each unit has

- a *Getting Started* section
- two or three chapters
- regular lessons and at least one *Explore* lesson
- a *Game*
- a *Connections* activity
- a *Unit Revision*

Getting Started

There are two parts to the *Getting Started*. You will complete a *Use What You Know* activity and then you will answer *Skills You Will Need* questions.
Both remind you of things you already know that will help you in the unit.

- The *Use What You Know* activity is done with a partner or in a group.
- The *Skills You Will Need* questions help you review skills you will use in the unit. You will usually do these by yourself.

Regular Lessons

- Lessons are numbered #.#.# — the first number tells the unit, the second number is the chapter, and the third number is the lesson in the chapter.

For example, Lesson 4.2.1 is Unit 4, Chapter 2, Lesson 1.

- Each regular lesson is divided into five parts:

- A *Try This* problem or task

- A box that explains the main ideas of the lesson; it is called the exposition

- A question that asks you to think about the Try This problem again, using what you have learned in the exposition

- one or more *Examples*

- *Practising and Applying* questions

Try This

- The *Try This* is in a grey box, like this one from **lesson 2.3.2** on **page 45**.

Try This

A. Padam works in a motorcycle store. He sold a motorcycle for Nu 45,000 and earned Nu 2700 for making the sale. What percent of the selling price of the motorcycle was Padam's earnings?

You will solve the *Try This* problem with a partner or in a small group.
The math you learn later in the lesson will relate back to this problem.

The Exposition

- The exposition comes after the *Try This*.
- It presents and explains the main ideas of the lesson.
- Important math words are in **bold** text. You will find the definitions of these words in the glossary at the back of the textbook.
- You are not expected to copy the exposition into your notebook.

Going Back to the Try This

- There is always a question after the exposition that asks you to think about the *Try This* problem again. You can use the new ideas presented in the exposition. In the example below from **lesson 2.3.2** on **page 46**, the exposition shows how to use a formula to calculate a commission.

B. Use the percent you calculated in **part A** as the commission percent.
i) How much commission would Dorji earn for selling a motorcycle for Nu 30,000?
ii) Dorji earned a commission of Nu 3000 for selling a motorcycle.
What was the selling price of the motorcycle?

Examples

- The *Examples* prepare you for the *Practising and Applying* questions. Each example is a bit different from the others so that you can refer to many models.
- You will work through the examples sometimes on your own, sometimes with another student, and sometimes with your teacher.
- What is special about the examples is that the *Solutions* column shows you what you should write when you solve a problem, and the *Thinking* column shows you what you might be thinking as you solve the problem.
- Some examples show you two different solutions to the same problem.

The example below from **lesson 6.1.1** on **page 153** shows two possible ways to answer the question, *Solution 1* and *Solution 2*.

Example 1 Solving a Probability Problem

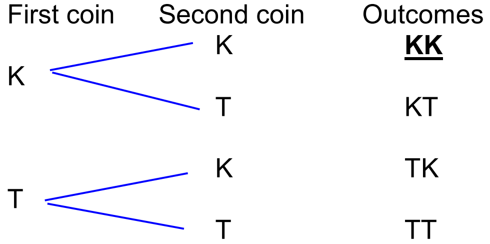
Choki and Sithar are playing a game where they flip two Nu 1 coins.

- Choki wins when the coins both show Khorlo.
- Sithar wins when both coins show Tashi Ta-gye.
- If the two coins show different faces, no one wins.

a) What is the theoretical probability that Choki will win?
b) Which is greater, P(Choki does not win) or P(Sithar wins)?

Solution 1

a)



P(Choki wins) = $\frac{1}{4}$

Thinking

a) I made a tree diagram to list all the possible outcomes.

- Each outcome has a probability of $\frac{1}{4}$ because there are 4 equally likely outcomes.
- The event that Choki wins is represented by the outcome KK.

b) P(Sithar wins) = $\frac{1}{4}$

P(Not Choki wins) $= 1 - \text{P(Choki wins)}$

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

P(Not Choki wins) > P(Sithar wins)

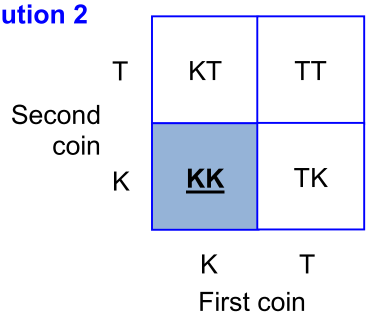
b) The TT outcome represents a win for Sithar. It is 1 out of 4 possible outcomes.

- I knew that Choki not winning is the complement of Choki winning, so I used the formula:

P(Not Choki wins) = 1 – P(Choki wins)

Solution 2

a)



P(Choki wins) = $\frac{1}{4}$

Thinking

a) I used an area model to represent the possible outcomes.

- The event that Choki wins is represented by the outcome KK. Its area represents $\frac{1}{4}$ of the area of the whole square.

b) P(Sithar wins) = $\frac{1}{4}$

P(Not Choki wins) = $\frac{1+1+1}{4} = \frac{3}{4}$

P(Not Choki wins) > P(Sithar wins)

b) The TT outcome represents a win for Sithar. It is 1 out of 4 possible outcomes.

- The event of Choki not winning consists of the 3 outcomes KT, TT, and TK. That's 3 out of 4 possible outcomes.

Practising and Applying

- You might work on the *Practising and Applying* questions by yourself, with a partner, or in a group. You can use the exposition and examples to help you.
- The first few questions are similar to the questions in the *Examples* and the exposition.
- The last question helps you think about the most important ideas you have learned in the lesson.

Explore Lessons

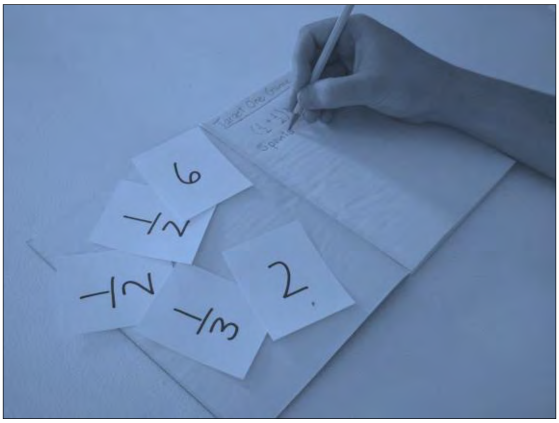
- An *Explore* lesson gives you a chance to work with a partner or in a small group to investigate some math.
- Your teacher does not tell you about the math in an *Explore* lesson. Instead, you work through the questions and learn in your own way.

Connections Activity

- The *Connections* activity is usually something interesting that relates to the math you are learning. For example, in Unit 2, the *Connections* on page 49 relates to changing money from one currency to another currency.
- There is always a *Connections* in a unit.
- You usually work in pairs or small groups to complete the task or answer the question(s).

Game

- Each unit usually has at least one *Game*.
- The *Game* is a way to practise skills and concepts from the unit with a partner or in small group.
- The materials you need and the rules are listed in the textbook. Usually the textbook shows a sample game to help you understand the rules.



Target One game from UNIT 4

Unit Revision

- The *Unit Revision* is a chance to review the lessons in the unit.
- The order of the questions in the *Unit Revision* is usually the same as the order of the lessons in the unit.
- You can work with a partner or by yourself, as your teacher suggests.

Glossary

- At the end of the textbook, you will find a glossary of new math words and their definitions. The glossary also contains other important math words from previous classes that you need to remember.
- The glossary also has definitions of instructional words such as “explain”, “predict”, and “estimate”. These will help you understand what you are expected to do.

Answers

- You will find answers to most of the numbered questions at the back of the textbook. Answers to questions that ask for explanations, such as “Explain your thinking” or “How do you know?” are not included in your textbook. Your teacher has those answers.
- Questions with capital letters, such as A or B, do not have answers at the back of the textbook. Your teacher has the answers to these questions.
- If there could be more than one correct answer to a question, the answer will start with the words *Sample response*. Even if your answer is different than the answer at the back of the textbook, it may still be correct.

ASSESSING YOUR MATHEMATICAL PERFORMANCE

Forms of Assessment

Your teacher will be checking to see how you are doing in your math learning. Sometimes your teacher will collect information about what you understand or do not understand in order to change the way you are taught. Other times your teacher will collect information in order to give you a mark.

Assessment Criteria

- Your teacher should tell you about what she or he will be checking and how it will be checked.
- The amount of the mark assigned for each unit should relate to the time the class spent on the unit and the importance of the unit.
- Your mark should consider how you are doing on skills, applications, concepts, and problem solving.
- Your teacher should tell you whether the mark for a test will be a number such as a percent, a letter grade such as A, B, or C, or a level on a rubric (level 1, 2, 3, or 4). A rubric is a chart that describes criteria for your work, usually in four levels of performance. If a rubric is used, your teacher should let you see the rubric before you start to work on the task.

Determining a Mark or Grade

In determining your overall mark in mathematics, your teacher might use a combination of tests, assignments, projects, performance tasks, exams, interviews, observations, and homework.

THE CLASSROOM ENVIRONMENT

In almost every lesson you will have a chance to work with other students. You should always share your responses, even if they are different from the answers offered by other students. It is only in this way that you will really be engaged in the mathematical thinking.

YOUR NOTEBOOK

- It is valuable for you to have a well-organized, neat notebook to look back at to review the main ideas you have learned. You should do your rough work in this same notebook. Do not do your rough work elsewhere and then waste valuable time copying it neatly into your notebook.

- Your teacher will sometimes show you important points to write down in your notebook. You should also make your own decisions about which ideas to include in your notebook.

Reprint 2024

Getting Started

Use What You Know

A. i) Draw two lines: one 5 cm long and another 10 cm long.

ii) What fraction of the 10 cm line is the 5 cm line?

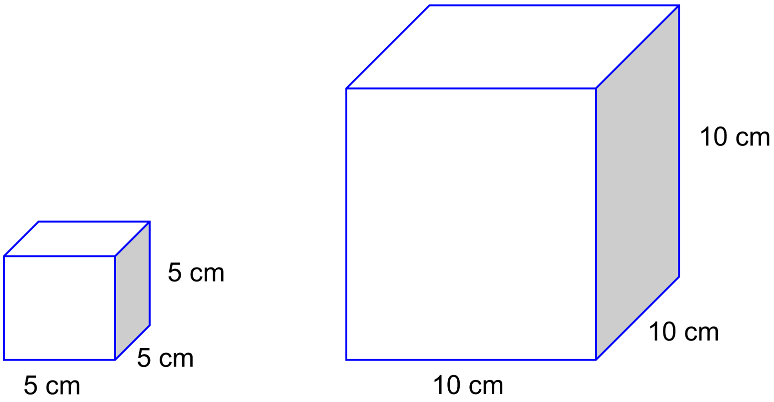
B. i) Draw two squares: a 5 cm square and a 10 cm square.

ii) Why can you write the product 5 × 5 to describe the area of the small square?

iii) Describe the area of the large square as a product.

iv) What fraction of the area of the 10 cm square is the area of the 5 cm square?

C. Think about two cubes: a cube with an edge length of 5 cm and another cube with an edge length of 10 cm.



i) Write the volume of each cube as the product of three numbers.

ii) What fraction of the volume of the 10 cm cube is the volume of the 5 cm cube?

D. Estimate what fraction 5 × 5 × 5 × 5 is of 10 × 10 × 10 × 10. Explain your estimate. Then test it by multiplying.

E. Why do you think the value of the fractions in **parts A ii)**, **B iv)**, **C ii)**, **and D** decreased as you multiplied more 5s and more 10s together?

Skills You Will Need

1. Express each as a product of prime factors.

a) 240 **b)** 45 **c)** 144 **d)** 31

You might find this place value chart helpful for questions 2 and 3.

Billions			Millions			Thousands			Ones		
H	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**
10^{11}	10^{10}	10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	1

2. Write each number below in expanded form in two ways.

For example, the number 312,056 in expanded form is

3 hundred thousands + 1 ten thousand + 2 thousands + 5 tens + 6 ones

$3 \times 100{,}000 + 1 \times 10{,}000 + 2 \times 1000 + 5 \times 10 + 6$

a) 412,150 **b)** 365,124 **c)** 1,003,010 **d)** 1,000,901,142

3. Write each number in standard form and in expanded form.

a) $8 \times 10^6 + 5 \times 10^4 + 2 \times 10^3$

b) $4 \times 10^{10} + 7 \times 10^7 + 6 \times 10^2 + 3 \times 10^1 + 7$

4. Calculate each.

a) 0.5 × 2.14 **b)** 9 × 0.22 **c)** 0.3 × 0.7 **d)** 0.08 × 0.12

You might find this place value chart helpful for questions 5 and 6.

1000	100	10	1	0.1	0.01	0.001	0.0001

5. Calculate each.

a) 37.4 × 100
b) 0.23 × 1000
c) 3.028 ÷ 100
d) 623.4 ÷ 1000
e) 0.4 × 10,000
f) 8.2113 × 100
g) 3.12 ÷ 1000
h) 23.4 ÷ 100

6. The digit 3 is in the ones column in the number $42\underline{3}.6$. In which place value column is the digit 3 in 423.6 after each calculation?

a) 423.6 × 0.0001
b) 423.6 × 0.01
c) 423.6 ÷ 0.1
d) 423.6 ÷ 0.01

Chapter 1 Powers

1.1.1 Negative Exponents

Try This

A. How many parts are added together, when you write each number below in expanded form? How do you know?

For example:

1245 = 1 thousand + 2 hundreds + 4 tens + 5 ones, so there are four parts.

i) 2059.61 **ii)** 725.003 **iii)** 810,053.1146

- You can describe a decimal using **expanded form**.

For example:

$$\underbrace{10{,}060.407}_{\textbf{Standard form}}\ \underbrace{\begin{aligned}&= 1 \text{ ten thousand} + 6 \text{ tens} + 4 \text{ tenths} + 7 \text{ thousandths}\\ &= 1 \times 10{,}000 + 6 \times 10 + 4 \times 0.1 + 7 \times 0.001\end{aligned}}_{\textbf{Expanded forms}}$$

- You can also describe a decimal using **exponential form**. A place value chart that includes **powers of 10** is a helpful tool for understanding exponential form.

Each time you move one column to the right in a place value chart, you divide by 10. That means there will be one fewer 10 in the product of the power of 10, so the **exponent** of 10 decreases by 1.

For example: $10^4 \div 10 = (10 \times 10 \times 10 \times 10) \div 10 = 10^3$

$10^3 \div 10 = (10 \times 10 \times 10) \div 10 = 10^2$

$10^2 \div 10 = (10 \times 10) \div 10 = 10^1$

10,000	1000	100	10	1	0.1
10^4	10^3	10^2	10^1	**?**	**?**

The pattern continues: $10^1 \div 10 = 10 \div 10 = 10^0 = 1$

$10^0 \div 10 = 1 \div 10 = 10^{-1} = 0.1$

- Notice that powers of 10 less than 1, such as tenths (0.1), hundredths (0.01), and so on, have negative exponents.

For example: $0.1 = 10^{-1}$ $0.01 = 10^{-2}$ $0.001 = 10^{-3}$

Ones	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths
1	0.1	0.01	0.001	0.0001	0.00001
10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}

- If you continue the pattern, you get $10^{-6} = 0.000001$ in the millionths place.
- When you write a power of 10 greater than 1 in standard form, the number of zeros after the 1 equals the exponent.

For example: $10^4 = 10{,}000$ (there are four zeros after the 1).

- When you write a power of 10 less than 1 in standard form, the number of zeros after the decimal point is one less than the opposite of the exponent. For example:

Place Value	Power of 10	Standard form	Exponent	Zeros after the decimal point
Thousandths	10^{-3}	0.001	–3	2
Ten thousandths	10^{-4}	0.0001	–4	3
Hundred thousandths	10^{-5}	0.00001	–5	4
Millionths	10^{-6}	0.000001	–6	5

- Just as you can write numbers greater than 1 using exponential form, you can write numbers less than 1 using exponential form.

For example: $245 = 2 \times 10^2 + 4 \times 10^1 + 5$

$0.245 = 2 \times 10^{-1} + 4 \times 10^{-2} + 5 \times 10^{-3}$

B. Here is the decimal portion of each number from **part A**. Write each in exponential form. **i)** 0.61 **ii)** 0.003 **iii)** 0.1146

Examples

Example 1 Changing Exponential Form to Standard and Expanded Forms

Write each number in standard form and in both expanded forms.

a) $7 \times 10^{-1} + 3 \times 10^{-2} + 1 \times 10^{-6}$

b) $6 \times 10^{0} + 4 \times 10^{-1} + 3 \times 10^{-3} + 7 \times 10^{-4} + 2 \times 10^{-5}$

Solution

a) and b)

Ones	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths	Millionths
1	**0.1**	**0.01**	**0.001**	**0.0001**	**0.00001**	**0.000001**
10^{0}	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}
0	**7**	**3**	0	0	0	**1**
6	**4**	0	**3**	**7**	**2**	

a) The standard form is 0.730001.

The expanded forms are:

7 tenths + 3 hundredths + 1 millionth

$7 \times 0.1 + 3 \times 0.01 + 1 \times 0.000001$

b) The standard form is 6.40372.

The expanded forms are:

6 ones + 4 tenths + 3 thousandths
+ 7 ten thousandths
+ 2 hundred thousandths

$6 \times 1 + 4 \times 0.1 + 3 \times 0.001 + 7 \times 0.0001 + 2 \times 0.00001$

Thinking

a) I put each of the digits, 7, 3, and 1, in the correct place in a place value chart. Then I put zeros in the places that were blank.

b) Since there were no millionths (10^{-6}) in the number in exponential form, I didn't need a zero in that place because no digits came after it.

Example 2 Calculating With Negative Exponents

Calculate.

a) $75 \times 10^{-2} + 4 \times 10^{-3}$ **b)** $10^{-1} + 23 \times 10^{-3} + 6 \times 10^{-5}$ **c)** $10^{-2} \times 10^{-2}$

Solution	**Thinking**
a) $75 \times 10^{-2} + 4 \times 10^{-3}$ $= 0.75 + 0.004$ $= 0.754$	**a)** $75 \times 10^{-2} = 0.75$ because the exponent −2 means hundredths. • $4 \times 10^{-3} = 0.004$ because the exponent −3 means thousandths.
b) $10^{-1} + 23 \times 10^{-3} + 6 \times 10^{-5}$ $= 0.1 + 0.023 + 0.00006$ $= 0.12306$	**b)** The three parts of the number are 0.1, 0.023, and 0.00006. I added them together.
c) $10^{-2} \times 10^{-2}$ $= 0.01 \times 0.01$ $= 0.0001$	**c)** I knew that $10^{-2} = 0.01$, so I multiplied the decimals.

Practising and Applying

You might find the place value chart on page 4 helpful.

1. Write each number in standard form and in both expanded forms.

a) $1 \times 10^{-1} + 4 \times 10^{-2} + 7 \times 10^{-4}$

b) $3 \times 10^{2} + 6 \times 10^{0} + 5 \times 10^{-2} + 7 \times 10^{-3} + 8 \times 10^{-6}$

c) $7 \times 10^{-4} + 5 \times 10^{-5}$

d) $5 \times 10^{6} + 6 \times 10^{4} + 3 \times 10^{1} + 4 \times 10^{-2} + 7 \times 10^{-3} + 3 \times 10^{-6}$

2. Calculate.

a) $10^{-2} + 10^{-3}$

b) $25 \times 10^{-1} + 8 \times 10^{-2}$

c) $10^{-3} \times 10^{-3}$

3. Which number in each pair is greater?

a) 3×10^{-2} or 6×10^{-3}

b) $5 \times 10^{-4} + 7 \times 10^{-5}$ or 2×10^{-3}

c) $10^{2} \times 10^{-2}$ or 9×10^{-1}

4. a) How many digits are there to the right of the decimal point if you write this number in standard form? How do you know?

$5 \times 10^{-4} + 6 \times 10^{-7} + 5 \times 10^{-9} + 9 \times 10^{-11} + 4 \times 10^{-13}$

b) Could you write the number using 20 digits after the decimal point? Explain your thinking.

c) Is the number greater than or less than 5×10^{-4}? How do you know?

5. What do you think 2^{-1} means? Explain your thinking.

6. How is understanding the place value system useful for understanding negative powers of 10?

GAME: Getting to a Half

The goal of the game is to make numbers with a sum as close as possible to 0.5 without going over 0.5. Each number consists of a whole number multiplied by a negative power of 10.

$$\blacksquare \times 10^{-\blacksquare}$$

Any number of people can play. Take turns.

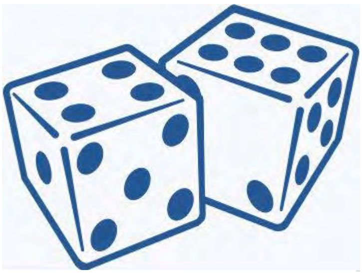
Do this on your turn:

- Roll a pair of dice. The numbers on the two dice represent the whole number and the negative exponent of the power of 10.

You can choose which number represents the whole number and which represents the exponent.

For example:

A roll of 4 and 6 could make 4×10^{-6} or 6×10^{-4}.



This roll could be 4×10^{-6} or 6×10^{-4}.

- Write the number in standard form.

For example, $6 \times 10^{-4} = 0.0006$.

- Continue rolling up to three more times. You can stop at any time.
Add your new number to your total from the numbers before.
If your sum goes above 0.5, you lose.

- Compare the sums of all the players at the end of each round.
The player with the score closest to 0.5 (but not more than 0.5) receives 1 point.

- The first person to get 3 points wins the game.

Players take turns going first for each round.

Sample round for one player

Dice throw		**Number**	**Standard form**	**Sum**
4	6	6×10^{-4}	0.0006	0.0006
1	3	3×10^{-1}	0.3	0.3006
5	2	5×10^{-2}	0.05	0.3506
1	1	1×10^{-1}	0.1	0.4506

1.1.2 Scientific Notation

Try This

A. Which expressions below are equal to 345.6?

i) 0.3456×10^3 **ii)** $34,560 \times 10^{-2}$ **iii)** 3456×10^{-1}

iv) $3,456,000 \times 10^{-4}$ **v)** 0.03456×10^4 **vi)** 0.003456×10^5

- To compare numbers like 0.12467×10^3 and $12,467 \times 10^{-1}$, it is helpful to write them in a similar form. One way is to use **scientific notation**.
- A number in scientific notation is written as a whole number or decimal multiplied by a power of 10. The whole number or decimal part is called the **multiplier**. The multiplier is always greater than or equal to 1 and less than 10.

For example:

$235 = 2.35 \times 10^2$

$0.1345 = 1.345 \times 10^{-1}$

$0.000467 = 4.67 \times 10^{-4}$

$$\underbrace{2.35}_{\text{Multiplier}} \times \underbrace{10^2}_{\text{Power of 10}}$$

- The power of 10 is positive if the original number is greater than the multiplier. The power of 10 is negative if the original number is less than the multiplier.
- The exponent is equal to the number of places that each digit moves when you change a number from standard form to scientific notation.
  - \- A positive exponent means the digits moved to the right.
  - \- A negative exponent means the digits moved to the left.

$$235 = 2.35 \times 10^2$$

The digits move 2 places to the right.

$$0.1345 = 1.345 \times 10^{-1}$$

The digits move 1 place to the left.

- If a number is already greater than or equal to 1 and less than 10, write the number in scientific notation using 10^0 as the power of 10 because $10^0 = 1$.

For example, 5.301 in scientific notation is 5.301×10^0.

- It is easy to compare numbers when they are in scientific notation:
  - \- If the powers of 10 are the same, compare the multipliers. The greater number is the number with the greater multiplier.

    For example, $3.15 \times 10^4 > 1.56 \times 10^4$ since $3.15 > 1.56$.
  - \- If the powers of 10 are not the same, compare the powers of 10. The greater number is the number with the greater power of 10.

    For example, $2.68 \times 10^{-2} > 9.51 \times 10^{-3}$ since $-2 > -3$.

B. Are any of the expressions in **part A** in scientific notation? How do you know?

Examples

Example 1 Expressing Numbers in Scientific Notation

Write each number in scientific notation.

a) 3406 **b)** 0.021 **c)** 0.10832 **d)** 1,200,000 **e)** 5.67

Solution	Thinking
a) $3406 = 3.406 \times 10^3$	 **a)** I knew that the decimal point had to go between the digits 3 and 4 so that the multiplier would be greater than or equal to 1 and less than 10. • I multiplied by a positive power of 10 so the digits would move to the right.
b) $0.021 = 2.1 \times 10^{-2}$	**b)** I knew that the decimal point had to go between the digits 2 and 1 so that the multiplier would be 2.1. • I multiplied by a negative power of 10 so the digits would move to the left.
c) $0.10832 = 1.0832 \times 10^{-1}$	**c)** I knew the decimal point had to go between the digits 1 and 0 so that the multiplier would be 1.0832. • I multiplied by 10^{-1} so the digits would move one place to the left.
d) $1,200,000 = 1.2 \times 10^6$	**d)** I knew the decimal point had to go between the digits 1 and 2. • I multiplied by 10^6 so the digits would move six places to the right.
e) $5.67 = 5.67 \times 10^0$	**e)** 5.67 was already between 1 and 10 so the digits did not need to move. Multiplying by 10^0 is the same as multiplying by 1.

Example 2 Comparing Quantities using Scientific Notation

Which value is greater in each?

a) 4.6×10^6 or 2×10^7 **b)** 2.3×10^4 or 1.99×10^4 **c)** 1.83×10^{-2} or 2.053×10^{-1}

Solution	Thinking
a) $2 \times 10^7 > 4.6 \times 10^6$	 **a)** The powers of 10 were different, so all I had to do was compare the powers. I knew the number with the greater power of 10 would be greater.
b) $2.3 \times 10^4 > 1.99 \times 10^4$	**b)** The powers of 10 were the same, so I compared the multipliers. $2.3 > 1.99$, so $2.3 \times 10^4 > 1.99 \times 10^4$.
c) $2.053 \times 10^{-1} > 1.83 \times 10^{-2}$	**c)** The powers of 10 were different, so I compared the exponents. $-1 > -2$, so-$2.053 \times 10^{-1} > 1.83 \times 10^{-2}$.

Example 3 Interpreting Scientific Notation

According to a May 2007 Kuensel report, 8 × 10^6 litres of water are used each day in Thimphu. If about 100,000 people live in Thimphu, estimate how many litres of water are used daily per person.

Solution

8 × 10^6 = 8,000,000

8,000,000 ÷ 100,000

80 hundred thousands ÷ 1 hundred thousand = 80

About 80 L of water are used daily per person.

Thinking

- I worked out the total number of litres of water that was represented by 8 × 10^6.
- I knew that 8,000,000 was 80 hundred thousands and 100,000 was 1 hundred thousand, so I divided 80 by 1.

Practising and Applying

1. Write each in scientific notation.

a) 231.96 **b)** 4,356,000

c) 0.00021 **d)** 0.1367

2. Which is greater in each pair? Explain how you know.

a) 3.45 × 10^{-3} or 2 × 10^{-2}

b) 1.5996 × 10^6 or 2 × 10^6

c) 2 × 10^4 or 1.99 × 10^5

3. One of the highest passes in Bhutan is Thrumshing La. Which value below is a reasonable estimate of its height? How do you know?

3.8 × 10^3 m or 3.8 × 10^{-3} m

4. Write each underlined number in scientific notation.

a) Bhutan's area is about 47,000 km².

b) Bhutan's Gross National Income in 2005 was 798,200,000 United States dollars (according to the World Bank).

5. Can the multiplier in a number in scientific notation be 1.0? How do you know?

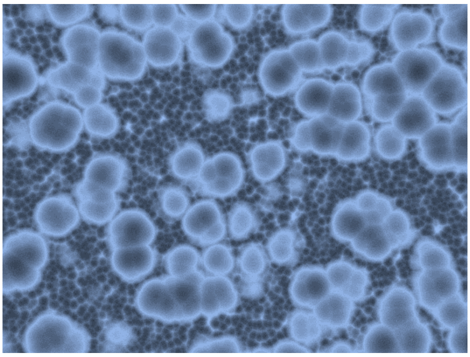
6. Dorji's savings account has a value of Nu 9.31 × 10^4. Kinley's account has Nu 9.86 × 10^3. Who has more money? How do you know? How much more?

7. Write each underlined number in scientific notation.

a) The average distance from Earth to the sun is 149,600,000 km.

b) The area of Earth's surface is about 510,000,000 km².

c) The diameter of a red blood cell is 0.00074 cm.



Red blood cells magnified many times.

8. If you were comparing two numbers in scientific notation, which would you look at first, the multipliers or the powers of 10? Explain your thinking.

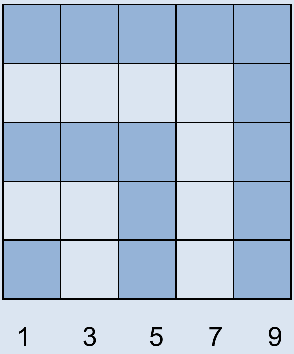
Chapter 2 Square Roots

1.2.1 Perfect Squares

Try This

Tshering noticed a pattern when adding odd numbers.

1 = 1 = 1 × 1
1 + 3 = 4 = 2 × 2
1 + 3 + 5 = 9 = 3 × 3
1 + 3 + 5 + 7 = 16 = 4 × 4

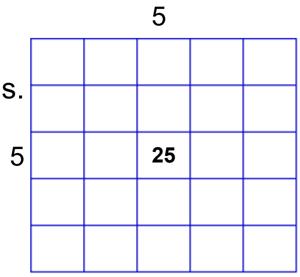


A. Continue the pattern for three more rows.

- A **perfect square** is the **product** of a whole number multiplied by itself.

For example, 25 is a perfect square since 5 × 5 = 25.
You can say “5 squared equals 25”.

- A number such as 25 is called a perfect square because it can be formed into a square with whole number side lengths.



- Here are the first twelve perfect squares.

1 × 1 = **1**
2 × 2 = **4**
3 × 3 = **9**
4 × 4 = **16**
5 × 5 = **25**
6 × 6 = **36**
7 × 7 = **49**
8 × 8 = **64**
9 × 9 = **81**
10 × 10 = **100**
11 × 11 = **121**
12 × 12 = **144**

- You can look at the **prime factors** of a number to see if it is a perfect square. If you can pair each factor with another factor and no factors are left over, the number is a perfect square.

$9 = \underline{3 \times 3}$ The prime factors of 3 can be paired, so 9 is a perfect square.

$20 = \underline{2 \times 2} \times 5$ The prime factor of 5 does not have a partner, so 20 is not a perfect square.

$8 = \underline{2 \times 2} \times 2$ Only two of the three 2s can be paired, so 8 is not a perfect square.

$225 = \underline{3 \times 3} \times \underline{5 \times 5}$ Each of the prime factors can be paired, so 225 is a perfect square.

- The **differences** between **consecutive** perfect squares form this pattern:

$$\left.\begin{array}{r}4-1=3\\9-4=5\\16-9=7\end{array}\right\} \quad 3, 5, 7, \ldots$$

B. How does the diagram in **part A** show each?
i) that the sums of consecutive odd numbers are perfect squares
ii) the pattern of differences between consecutive perfect squares

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Examples

Example 1 Identifying Perfect Squares

Which of these numbers are perfect squares?

A. 400 **B.** 80 **C.** 10,000 **D.** 31 **E.** 111

Solution	**Thinking**
A. 400	A. 400 = 20 × 20 
	B. 80 = $\underline{2 \times 2} \times \underline{2 \times 2} \times 5$. There is no matching factor for 5.
C. 10,000	C. 10,000 = 100 × 100
	D. 31 is a prime number so it cannot be a perfect square because its only factors are 1 and 31.
	E. $10^2 = 100$ and $11^2 = 121$, so 111 cannot be a perfect square because it is between two consecutive perfect squares.

Practising and Applying

1. Which number is a perfect square?

48 103 4000 144

2. Look at the list of perfect squares on **page 10**.

a) Which digits appear in the ones place of these perfect squares?

b) Can a perfect square have the digit 7 in the ones place? Why?

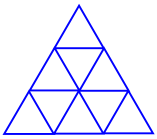
3. a) What are the missing sums?

1 + (1 + 2) = ■
1 + 2 + (1 + 2 + 3) = ■
1 + 2 + 3 + (1 + 2 + 3 + 4) = ■
1 + 2 + 3 + 4 + (1 + 2 + 3 + 4 + 5) = ■

b) What do you notice about the sums?

c) Predict the sum of 1 + 2 + 3 + 4 + 5 + (1 + 2 + 3 + 4 + 5 + 6).

4. a) How many small triangles are in the top two rows of the larger triangle?



b) How many small triangles are in all three rows?

c) Copy the triangle. Draw a fourth row at the bottom. How many small triangles are in all four rows?

d) How many small triangles will be in six rows? How do you know?

5. The prime factorisation of a number is 2 × 2 × 2 × 7 × 13 × 13.

a) Is the number a perfect square? How do you know?

b) What is the least number that you could multiply this number by to get a perfect square? Explain your thinking.

6. a) List the numbers from 1 to 16 and identify all the factors of each.

b) How many factors does each number have?

c) How is the number of factors different for the perfect squares than for the other numbers?

7. a) How many perfect squares are there between 90 and 890?

b) How can you answer **part a)** without listing all the perfect squares?

8. Duptho says that a perfect square that is divisible by 3 is also divisible by 9. Do you agree? Explain your thinking.

9. Describe two or more things that make a perfect square a special number.

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1.2.2 EXPLORE: Squaring Numbers Ending in 5

- The **square root** of a perfect square is the whole number that is multiplied by itself to get the perfect square.

For example, the square root of 225 is 15 because 15 × 15 = 225.

- In this lesson, you will learn about the relationship between any square root ending in 5 and its perfect square.

A. The chart below will help you relate the number of tens in each square root to the number of hundreds in its perfect square.

For example, 15 × 15 = 225 → <u>1</u>5 has 1 ten and <u>2</u>25 has 2 hundreds.

i) Copy and complete this chart.

Square root		**Perfect square**	**Number of tens in square root**	**Number of hundreds in perfect square**
15	15 × 15 =	225	1	2
25	25 × 25 =			
35	35 × 35 =			
45	45 × 45 =			
55	55 × 55 =			
65	65 × 65 =			
75	75 × 75 =			

ii) What do you notice about the last two digits of each perfect square?

iii) Describe the relationship between the number of tens in the square root and the number of hundreds in the perfect square.

iv) Use what you noticed in **parts ii) and iii)** to find 85 × 85 and 95 × 95. Explain how you got each product. Then multiply to see if you were correct.

B. i) How many tens are in 115?

ii) Use the pattern you noticed in **part A** to find 115 × 115. Multiply to check.

C. Explain how to find the perfect square of any whole number that ends in 5. Use 195 × 195 as an example.

D. Does the method you described in **part C** work for 5 × 5? Explain your thinking.

1.2.3 Interpreting Square Roots

Try This

A. On grid paper, sketch all possible rectangles with whole-number dimensions that have each area.

i) 18 square units **ii)** 16 square units **iii)** 13 square units

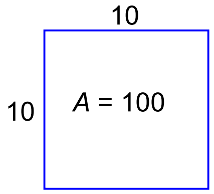
- The **square root** of a given number is the number that you can multiply by itself to get the given number.
- A square root can be a whole number, a fraction, or a decimal.
- The symbol $\sqrt{}$ is used to show a square root.

For example, 10 × 10 = 100, so $\sqrt{100} = 10$.

- The side length of a square is the square root of its area.

For example:

A square with an area of 100 square units has a side length of 10 units, so 10 is the square root of 100.



- The square root of a number can be exact or it can be an estimate.

For example:

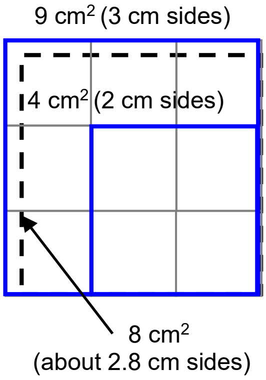
- Since 3 × 3 = 9, 3 is the exact square root of 9 ($\sqrt{9} = 3$). A square with an area of 9 cm^2 has a side length of exactly 3 cm.

- You can estimate the square root of 8 ($\sqrt{8}$) by visualizing a square with an area of 4 cm^2 inside a square with an area of 9 cm^2. A square with an area of 8 cm^2 is much larger than the 4 cm^2 square and just a bit smaller than the 9 cm^2 square.

That means the side length of the 8 cm^2 square must be between 2 cm and 3 cm, but closer to 3 cm.

So the square root of 8 is a bit less than 3, or about 2.8.

$$2 < \sqrt{8} < 3 \rightarrow \sqrt{8} \approx 2.8$$

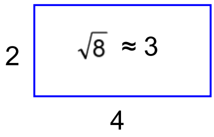


- You can sometimes estimate a square root by finding the average of the length and width of a rectangle with the same area as a square. The closer the rectangle is to the shape of a square, the better the estimate.

For example:

To estimate the square root of 8, you can visualize a 2-by-4 rectangle.

The average of 2 and 4 is 3, so you can estimate that 3 is close to the square root of 8. This is a reasonable estimate, although it is a bit high when you remember that the square root of 9 is 3.



B. For each area in **part A**, answer these questions.

i) What is the length and width of the rectangle that is closest to a square?

ii) Find the average of the length and the width to estimate the side length of a square with the same area.

iii) Square your estimate from **part ii)**. Compare your answer to the area of the original rectangle.

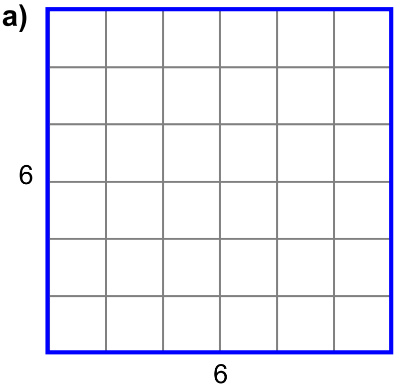
Examples

Example 1 Interpreting the Square Root

a) The square root of 36 is 6. Draw a diagram to show what this means.

b) A square has an area of 37 m^2. How do you know that its side length is between 6 m and 6.1 m?

c) When you double the area of a square, does the side length double? Explain your thinking.

Solution	**Thinking**
a) 	 **a)** I knew that $\sqrt{36}$ = 6, so a square that has an area of 36 square units has a side length of 6 unit.
b) $6 \times 6 = 36$ $6.1 \times 6.1 = 37.21$ $36 < 37 < 37.21$, so $6 < \sqrt{37} < 6.1$.	**b)** I found the square of 6 and the square of 6.1. Then I compared the result to 37.
c) No, the side length is not doubled. For a square with an area of 25: An area of 25 means a side length of 5. An area of 2 × 25 = 50 does not have a side length 2 × 5 = 10 because the square root of 50 is between 7 and 8: $7 \times 7 = 49 \rightarrow \sqrt{49} = 7$ $8 \times 8 = 64 \rightarrow \sqrt{64} = 8$ This example shows that, when you double the area of a square, the side length does not double.	**c)** I used an example to explain. • I found the square root of 50 using the perfect square just less than 50 (49) and the perfect square just greater than 50 (64).

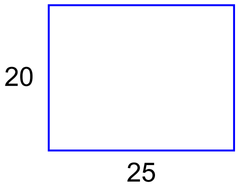
Example 2 Using a Rectangle to Estimate a Square Root

The area of a square is 500 cm^2. Estimate its side length. Check your estimate.

Solution

$500 = \underline{5 \times 5} \times \underline{5 \times 2 \times 2}$

$= 25 \times 20$



$\frac{20+25}{2} = 22.5$

A side length of about 22.5 cm.

Check

22.5 × 22.5 = 506.25

506.25 is close to 500.

Thinking

- I factored 500 into primes to find the dimensions of the rectangle with an area of 500 that is closest to a square.
- I tried combining the factors in different ways:

$\underline{5 \times 5 \times 5} \times \underline{2 \times 2} \rightarrow 125 \times 4$

$\underline{5} \times \underline{5 \times 5 \times 2 \times 2} \rightarrow 5 \times 100$

$\underline{5 \times 5 \times 5 \times 2} \times \underline{2} \rightarrow 250 \times 2$

$\underline{5 \times 5} \times \underline{5 \times 2 \times 2} \rightarrow 25 \times 20$

I chose to use 25 and 20.

- The average of 25 and 20 is 22.5. When I squared 22.5, the result was close to 500 so I knew the estimate was reasonable.

Practising and Applying

1. Draw a diagram to show each.

a) The square root of 49 is 7.

b) The square root of 56 is between 7 and 8.

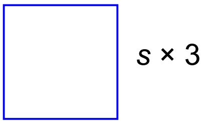
2. What is each square root?

a) $\sqrt{81}$ **b)** $\sqrt{100}$ **c)** $\sqrt{64}$ **d)** $\sqrt{121}$

3. What whole number is nearest to each square root?

a) $\sqrt{22}$ **b)** $\sqrt{40}$ **c)** $\sqrt{105}$ **d)** $\sqrt{139}$

4. Tshering claims that if she triples the side length of a square (*s*), the area of the square will also be tripled.
Do you agree? Explain your thinking.



5. List three numbers that have square roots that are whole numbers greater than 12.

6. a) List all possible whole number dimensions of a rectangle with an area of 72 cm^2.

b) Estimate the square root of 72 by finding the average of the length and width of the most square-like rectangle. Show your work.

c) Is your estimate in **part b)** reasonable? Explain your thinking.

7. Repeat **question 6** for a rectangle with an area of 95 cm^2.

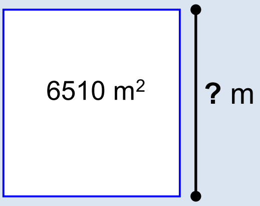
8. The area of a square is between 30 cm^2 and 31 cm^2. Estimate the side length to the nearest tenth of a centimetre. Is your estimate reasonable? How do you know?

9. Explain the meaning of *square root* for someone who does not know the term. Use words and pictures.

1.2.4 Estimating and Calculating Square Roots

Try This

A park is in the shape of a square. The area is 6510 m^2.



A. Estimate the side length of the park.

- A square root can be a whole number or an exact decimal, but more often it is only an approximation.

For example:

- The square root of the perfect square 100 is an exact whole number: $\sqrt{100} = 10$

- The square root of 17.64 is an exact decimal: $\sqrt{17.64} = 4.2$ [$4.2 \times 4.2 = 17.64$]

- The square root of 8 is approximately 2.8, rounded to one decimal place:

$\sqrt{8} \approx 2.8$ [$2.8 \times 2.8 = 7.84$ and $2.9 \times 2.9 = 8.41$, so $\sqrt{8}$ is closer to 2.8 than to 2.9)]

- To calculate or estimate the square roots of larger numbers, you can use the square roots of the perfect squares from 1 to 144 and the square roots of even powers of 10 such as 100, 10,000, and 1,000,000.

For example, to calculate $\sqrt{8100}$:

$8100 = 81 \times 100$, $\sqrt{81} = 9$, and $\sqrt{100} = 10$

So $\sqrt{8100} = 9 \times 10 = 90$.

For example, to estimate $\sqrt{1{,}438{,}200}$:

$1{,}438{,}200 = 143.82 \times 10{,}000$

$143.82 \approx 144$, $\sqrt{144} = 12$, and $\sqrt{10{,}000} = 100$

So $\sqrt{14{,}382} \approx 12 \times 100 = 1200$.

- The perfect squares of numbers ending in 5 can be helpful for finding square roots.

For example, to find $\sqrt{56}$:

$\sqrt{5625} = 75$ because $75 \times 75 = 5625$.

$5625 = 56.25 \times 100$

So $\sqrt{56} \approx 7.5$, rounded to one decimal place.

It is helpful to be familiar with these square roots.

Number	Square root
1	1
4	2
9	3
16	4
25	5
36	6
49	7
64	8
81	9
100	10
121	11
144	12
10,000	100
1,000,000	1000

B. Use one of the methods described above to estimate the side length of the park in **part A**.

Examples

Example 1 Estimating and Calculating a Square Root

a) The area inside the square wall around a dzong is about 3500 m^2. Estimate the length of one of the side walls.

b) What is the side length to one decimal place?

Solution

a) $50 \times 50 = 2500$
$60 \times 60 = 3600$

$$\begin{array}{r} 59 \\ \times\ \underline{59} \\ 531 \\ \underline{295} \\ 3481 \end{array}$$

$\sqrt{3500} \approx 59$

Each side wall is about 59 m.

b)

$$\begin{array}{r} 59.1 \\ \times\ \underline{59.1} \\ 591 \\ 5319 \\ \underline{2955} \\ 3492.81 \end{array} \qquad \begin{array}{r} 59.2 \\ \times\ \underline{59.2} \\ 1184 \\ 5328 \\ \underline{2960} \\ 3504.64 \end{array}$$

Each side wall is approximately 59.2 m.

Thinking

a) To estimate $\sqrt{3500}$, I began with whole number square roots.

- I tried 50, since 5 × 5 = 25, and then 60, since 6 × 6 = 36.
- It looked like $\sqrt{3500}$ was between 50 and 60 but much closer to 60. So I tried 59 next.
- When I squared 59, it was almost 3500, so I knew 59 was a good estimate for $\sqrt{3500}$.

b) From part **a)**, I knew $\sqrt{3500}$ was much closer to $\sqrt{3481}$ (59) than to $\sqrt{3600}$ (60), so I tried values close to 59.

- 59.1 × 59.1 = 3492.81 was about 7 too low.
- 59.2 × 59.2 = 3504.64 was only about 5 too high.
- So $\sqrt{3500}$ is closer to 59.2 than to 59.1.

Example 2 Calculating a Square Root to One Decimal Place

The area of Bhutan is about 47,000 km^2. If Bhutan were shaped like a square, what would its side length be? Round to the nearest ten kilometres.

Solution

$47{,}000 = 4.7 \times 10{,}000$

$\sqrt{47{,}000} = \sqrt{4.7} \times \sqrt{10{,}000}$

$\sqrt{4} = 2$, so $\sqrt{4.7}$ is a bit more than 2:

$2.1 \times 2.1 = 4.41$ (about 0.3 too low)

$2.2 \times 2.2 = 4.84$ (about 0.1 too high)

$\sqrt{4.7} \approx 2.2$ and $\sqrt{10{,}000} = 100$

$\sqrt{47{,}000} \approx 2.2 \times 100 = 220$

The side length would be about 220 km.

Thinking

- I knew I needed to find $\sqrt{47{,}000}$.
- I factored 47,000 into the product of a number and an even power of 10.
- I estimated the square root of 4.7 by trying numbers just greater than 2.
- It looked like $\sqrt{4.7}$ was between 2.1 and 2.2, but closer to 2.2.

Practising and Applying

Estimate to one decimal place when necessary.

1. Estimate.

a) $\sqrt{39}$ **b)** $\sqrt{97}$

2. Estimate to decide which one of these answers is incorrect.

A. $\sqrt{5612} \approx 74.9$

B. $\sqrt{91{,}230} \approx 30.2$

C. $\sqrt{517{,}432} \approx 719.3$

3. Use $\sqrt{68} \approx 8.2$ to estimate each.

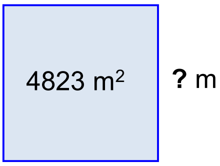
a) $\sqrt{6800}$ **b)** $\sqrt{680{,}000}$

4. A square field has an area of 4823 m^2.

a) How do you know the side length is about 70 m?

b) Is the side length less than or more than 70 m?

c) Calculate the side length.



5. a) Calculate each and then tell what you notice.

i) $\sqrt{64} = ?$ $\sqrt{4} \times \sqrt{16} = ?$

ii) $\sqrt{225} = ?$ $\sqrt{9} \times \sqrt{25} = ?$

iii) $\sqrt{324} = ?$ $\sqrt{36} \times \sqrt{9} = ?$

b) Complete the following, using what you noticed in **part a)**.

i) $\sqrt{100} = \sqrt{?} \times \sqrt{4}$

ii) $\sqrt{8100} = \sqrt{100} \times \sqrt{?}$

iii) $\sqrt{?} = \sqrt{36} \times \sqrt{16}$

6. Tashi says that, for any whole number greater than 1, the square root is always less than the whole number. Do you agree? Explain your thinking.

7. This is the formula for estimating how long it takes for an object to fall:

$$t \approx 0.45 \times \sqrt{h}$$

t is the time in seconds.

h is the height in metres.

Estimate the amount of time (to the nearest second) it will take for an object that is at each height to fall.

a) 60 m

b) 915 m

c) 10,000 m

8. **a)** Tenzin factored 142,884 to find its square root:

$142{,}884 = 9 \times 7 \times 9 \times 7 \times 6 \times 6$

He says that $\sqrt{142{,}884} = 9 \times 7 \times 6$.

Is he right? How do you know?

b) Factor 9216 to find $\sqrt{9216}$.

9. Use examples to explain your thinking for each.

a) Can the square root of a 3-digit perfect square and the square root of a 4-digit perfect square have the same number of digits?

$\sqrt{***}$ $\sqrt{****}$

b) Can the square root of a 4-digit perfect square and the square root of a 5-digit perfect square have the same number of digits?

$\sqrt{****}$ $\sqrt{*****}$

c) The side length of a square is a 4-digit whole number. What do you know about the number of digits in the area of the square?

10. Describe two ways to estimate $\sqrt{2579}$.

CONNECTIONS: The Square Root Algorithm

Over the centuries, people have developed different ways of finding the square root of a number. One way is called the square root **algorithm**.

For example, this is how to find the square root of 19,044: $\sqrt{19{,}044}$

Step 1: Write the number in groups of two digits starting from the right. If necessary, use a single digit on the left.

$$\mathbf{1\ \ 90\ \ 44}$$

Step 2: Write the greatest whole number that, when multiplied by itself, is no greater than the number in the first group (working from left to right).

In this example, the first group is “1”.
The greatest whole number needed is also 1.

$$1 \times 1 = 1$$

Put this number above in the left column in a long division format as shown. Subtract to get any remainder.
Bring down the next pair of digits.

$$\begin{array}{r|l} \multicolumn{1}{r}{} & \ \ \mathbf{1} \\ \cline{2-2} \mathbf{1} & \ \ 1\ \ 90\ \ 44 \\ & -\underline{1} \\ & \ \ 0\ \ 90 \end{array}$$

Step 3: Double the number in the left column.
Then add a space to insert a digit in the ones place.

$$2 \times 1 = 2, \text{ so the number is } 2\ \_\ .$$

$$\begin{array}{r|l} \multicolumn{1}{r}{} & \ \ 1 \\ \cline{2-2} 1 & \ \ 1\ \ 90\ \ 44 \\ & -\underline{1} \\ \mathbf{2\ \_} & \ \ 0\ \ 90 \end{array}$$

Find the greatest digit that can go in the space so that, when the number formed is multiplied by that digit, the product is not greater than the amount in the dividend.

$$23 \times 3 = 69 < 90$$

Subtract to get any remainder. Bring down the next pair of digits.

$$\begin{array}{r|l} \multicolumn{1}{r}{} & \ \ 1\ \ \ \mathbf{3} \\ \cline{2-2} 1 & \ \ 1\ \ 90\ \ 44 \\ & -\underline{1} \\ \mathbf{2\underline{3}} & \ \ 0\ \ 90 \\ & \ \ \underline{\mathbf{-69}} \\ & \ \ \ \ \mathbf{21\ \ 44} \end{array}$$

Step 4: Double the number on top. Then add a space to insert a digit in the ones place.

$$2 \times 13 = 26, \text{ so the number is } 26\_\ .$$

$$\begin{array}{r|l} \multicolumn{1}{r}{} & \ \ \mathbf{1\ \ \ 3} \\ \cline{2-2} 1 & \ \ 1\ \ 90\ \ 44 \\ & -\underline{1} \\ 23 & \ \ 0\ \ 90 \\ & \ \ \underline{-69} \\ \mathbf{26\ \_} & \ \ \ \ 21\ \ 44 \end{array}$$

Find the greatest digit that can go in the space:

$$8 \times 268 = 2144$$

If necessary, continue the method until no more pairs of digits remain to be brought down.

The square root of 19,044 is 138. (138 × 138 = 19,044)

$$\begin{array}{r|l} \multicolumn{1}{r}{} & \ \ 1\ \ \ 3\ \ \ \mathbf{8} \\ \cline{2-2} 1 & \ \ 1\ \ 90\ \ 44 \\ & -\underline{1} \\ 23 & \ \ 0\ \ 90 \\ & \ \ \underline{-69} \\ \mathbf{26\underline{8}} & \ \ \ \ 21\ \ 44 \\ & \ \ \underline{\mathbf{-21\ \ 44}} \\ & \ \ \ \ \ \ \ \ \ \ \ \mathbf{0} \end{array}$$

1. Use the algorithm to find the square root of each. Show your work.

a) 729 **b)** 2601

UNIT 1 Revision

1. Write each number in standard form and in both expanded forms.

a) $9 \times 10^4 + 4 \times 10^1 + 5 \times 10^{-2} + 7 \times 10^{-3} + 8 \times 10^{-6}$

b) $4 \times 10^0 + 5 \times 10^{-1} + 7 \times 10^{-4}$

2. Which number in each pair is greater?

a) 2.3×10^{-4} or 1.2×10^{-3}

b) $5 \times 10^{-2} + 7 \times 10^{-4}$ or 6×10^{-2}

3. a) Without writing the number below in scientific form, predict how many digits there will be to the right of the decimal point? How do you know?

$2 \times 10^4 + 3 \times 10^1 + 9 \times 10^{-1} + 5 \times 10^{-3}$

b) Predict the least number of digits required to write this number. Explain your prediction.

c) Why is it impossible to predict the greatest number of digits required?

4. a) Write the number in **question 3** in scientific notation.

b) Write 2395 in scientific notation.

c) Which is greater, the number in **part a)** or the number in **part b)**? How do you know?

5. Write each in scientific notation.

a) 51,987.23 **b)** 0.193567

c) 0.0074 **d)** 10.17

6. Which measurement is a reasonable height for a child? Explain your choice.

A. 1.39×10^2 cm

B. 8.3×10^5 mm

C. 1.48×10^1 m

7. Express each underlined number in scientific notation.

a) The population of Canada is about 34 million.

b) About 3.1% of the people in Bhutanese schools are teachers.

8. Which number is a perfect square?

A. 60 **B.** 121 **C.** 9000

D. 164 **E.** 745 **F.** 47

9. a) Is the number below a perfect square? How do you know?

$2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 11$

b) If the number in **part a)** is not a perfect square, what is the least whole number you could multiply it by to get a perfect square? Explain your thinking.

c) What is the square root of the perfect square from **part b)**?

10. Why does each perfect square have an odd number of factors?

11. Explain why some powers of 10 are perfect squares and others are not. Include examples in your answer.

12. Draw a diagram to show that the square root of 41 is between 6 and 7.

13. What is the nearest whole number to each square root?

a) $\sqrt{155}$ **b)** $\sqrt{48}$ **c)** $\sqrt{83}$

14. Estimate each square root from **question 13** to one decimal place.

15. a) Estimate $\sqrt{70}$ by averaging the length and width of the most square-like rectangle.

b) Estimate the square root of 70 to one decimal place. Compare this estimate to your result from **part a)**.

c) Repeat **parts a) and b)** for $\sqrt{50}$.

16. Factor 72,900 to find its square root. Show your work.

17. The side length of a square is a 2-digit whole number. What do you know about the number of digits in the square's area? Explain your thinking.

UNIT 2 PROPORTION AND PERCENT

Getting Started

Use What You Know

This chart shows some of the results from a survey of 25 students.

What is your favourite sport?	
Sport	**Number of students**
Archery	10
Football	
Track	
Other	

A. The ratio of those who chose football to those who chose archery is 4 : 5. How many chose football?

B. What is the ratio of those who chose archery to the total group? Write the ratio in lowest terms.

C. The percent of students who chose track was 20%. How many chose track?

D. In another group, 45% of 40 students chose archery. How many students chose archery in that group? Show your work.

E. Survey the students in your own class about their favourite sport. Record the results. Find the percent of students who chose each sport.

Skills You Will Need

1. Write two equivalent ratios for 12 : 15. One ratio should include the term 45 and the other ratio should include the term 4.

2. What percent of the whole numbers from 1 to 50 are each?

a) multiples of 3 **b)** greater than 40

3. Which is a better price for the buyer? Explain your thinking.

6 apples for Nu 72 or 10 apples for Nu 100

4. Find each amount.

a) 1% of 60 **b)** 10% of 25

c) 20% of 48 **d)** 85% of 30

5. Complete each statement.

a) 20 is 1% of ■ **b)** 20 is 25% of ■

c) 20 is 40% of ■ **d)** 20 is 50% of ■

Chapter 1 Proportions

2.1.1 Solving Proportions

Try This

Yeshi is making Pork Fing. He has only 300 g of pork, so he has to adjust the recipe.

A. i) How much of each of the other ingredients should he use to make the Pork Fing?

ii) How many people will the adjusted recipe serve?

Pork Fing

500 g pork
150 g tomatoes
100 g butter
120 mL water
45 g green chillis
80 g onions
Serves 5 people

- A statement that shows two **equivalent ratios** or **equivalent rates** is called a **proportion**.

For example: 3 : 4 = 18 : 24 or $\frac{3}{4} = \frac{18}{24}$

- You can use a **scale factor** to find a missing **term** in a proportion.

For example:

To make orange paint, an artist mixes 2 parts of yellow paint with 3 parts of red paint. The artist has 50 mL of yellow paint. How much red paint does she need to make orange paint?

To solve this problem, you can write and **solve** a proportion:

$$\frac{\textit{yellow}}{\textit{red}} \text{ is } \frac{2}{3} = \frac{50}{\blacksquare}$$

$$\overset{\times 25}{\curvearrowright} \qquad \frac{2}{3} = \frac{50}{\mathbf{75}} \qquad \underset{\times 25}{\curvearrowright}$$

2 × **<u>25</u>** = 50

You also multiply 3 by 25 to find the missing term in the proportion.

3 × **<u>25</u>** = 75

The scale factor is 25.

The artist needs 75 mL of red paint.

- You can also solve a proportion by finding the **unit ratio** or **unit rate**. A unit ratio or unit rate is an **equivalent ratio or equivalent rate** with a second term of 1 unit.

For example:

A recipe calls for 750 mL of flour to make 2 loaves of bread. How much flour is needed to make 5 loaves?

To solve this problem you can find the equivalent unit rate (which is the amount of flour needed to make 1 loaf) and then use it to solve the problem:

750 mL of flour for 2 loaves = 375 mL for 1 loaf because 750 mL ÷ 2 = 375 mL.

375 mL per 1 loaf × 5 loaves = 1875 mL of flour

For 5 loaves, 1875 mL of flour is needed.

- A ratio or rate table can be useful for solving a series of proportion problems.

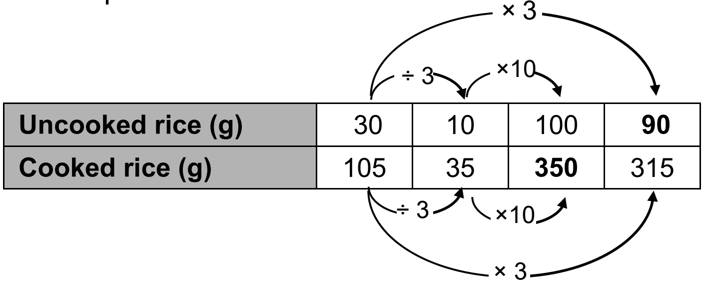
For example:

30 g of uncooked rice makes 105 g of cooked rice.

- How much cooked rice will you get from 100 g of uncooked rice?

- How much uncooked rice does it take to make 315 g of cooked rice?

You begin the ratio table with the known ratio, 30 : 105, and then use it to find other equivalent ratios.



Uncooked rice (g)	30	10	100	**90**
Cooked rice (g)	105	35	**350**	315

- 100 g of uncooked rice will make 350 g of cooked rice.
- 90 g of uncooked rice will make 315 g of cooked rice.

Notice that the equivalent ratio 10 : 35 was found first because that made it easy to find the equivalent ratio 100 : 350.

B. Use two of the following methods to find the amount of tomatoes in the adjusted recipe from **part A**.

- solving a proportion
- using a unit ratio or rate
- using a ratio or rate table

Examples

Example Solving a Proportion Problem

A cyclist burns about 300 calories of energy cycling for 30 min.

a) How many calories does the cyclist burn in 50 min?

b) How long does the cyclist have to cycle to burn 1000 calories?

Solution 1

a) 300 calories in 30 min
 =   10 calories in   1 min

50 min × 10 calories/min = 500 calories

The cyclist burns 500 calories in 50 min.

b)

$$\overset{\times 100}{\frac{10\text{ calories}}{1\text{ min}} = \frac{1000\text{ calories}}{\blacksquare\text{ min}}}$$

$$\underset{\times 100}{\frac{10\text{ calories}}{1\text{ min}} = \frac{1000\text{ calories}}{100\text{ min}}}$$

The cyclist has to cycle for 100 min to burn 1000 calories.

Thinking

a) I divided by 30 to find the number of calories burned in 1 min.

- I multiplied the number of calories burned in 1 min by 50 to find how many calories are burned in 50 min.

b) I wrote a proportion using what I knew about the number of calories burned in 1 min.

- I found the number I needed to multiply by to get 1000 calories. I multiplied the number of minutes by the same number.

Example Solving a Proportion Problem [Continued]

Solution 2

÷ 3 × 5 × 2

Calories	300	100	**500**	1000
Minutes	30	10	50	**100**

÷ 3 × 5 × 2

a) **b)**

a) The cyclist burns 500 calories in 50 min.

b) It takes the cyclist 100 min to burn 1000 calories.

Thinking

- I wrote the known rate, 300 cal/30 min, in a rate table. Then I divided both terms by 3 to get an equivalent rate that was easy to work with, 100 cal/10 min.

a) I multiplied both terms in the second rate by 5 to find the number of calories the cyclist burns in 50 min.

b) I multiplied both terms in the third rate by 2 to find how many minutes it takes to burn 1000 calories.

Practising and Applying

1. Solve each proportion by finding the missing term.

a) $\frac{\blacksquare}{7} = \frac{8}{28}$ **b)** $\frac{3}{8} = \frac{\blacksquare}{24}$

c) $\frac{4}{5} = \frac{20}{\blacksquare}$ **d)** $\frac{3}{5} = \frac{18}{\blacksquare}$

e) $\frac{4}{\blacksquare} = \frac{20}{50}$ **f)** $\frac{3}{15} = \frac{\blacksquare}{30}$

2. To make tempera paint, you mix 3 parts of paint concentrate with 18 parts of water. If you have 12 L of paint concentrate, how many litres of water do you need?

3. The ratio of the number of people standing to the number of people sitting in a bus is 3 : 13. If 9 people are standing, how many are sitting?

4. The ratio of Bhagi's mass to Samten's mass is 5 : 4. If Bhagi's mass is 40 kg, what is Samten's mass?

5. The rear wheel of Thinley's bicycle turns 9 times for every 3 times the pedals turn. How many times does the rear wheel turn when the pedals turn twice?

6. Jigme is mixing concentrated orange juice with water in the ratio 1 : 3. Complete the table.

Concentrate (mL)	Water (mL)
1	3
350	
475	
	525
	1200

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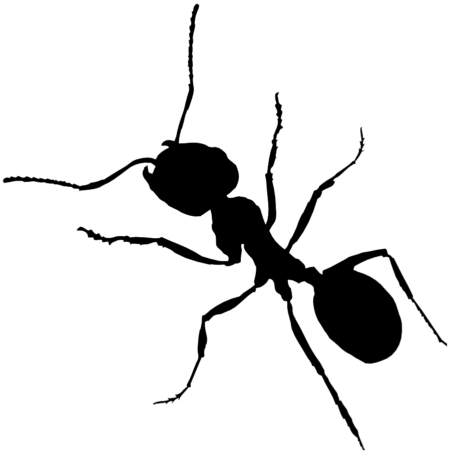
7. The ratio of the mass of an ant to the mass it can carry is 1 : 50.

a) If an ant has a mass of 0.4 g, how much mass can it carry?

b) If you were as strong as an ant, how much mass could you carry?



8. Kachap planted 5 seedlings for every 3 seedlings Sithar planted. Sithar planted 90 seedlings in 1 h.

a) How many seedlings did Kachap plant in 1 h?

b) How long did it take Sithar to plant 30 seedlings?

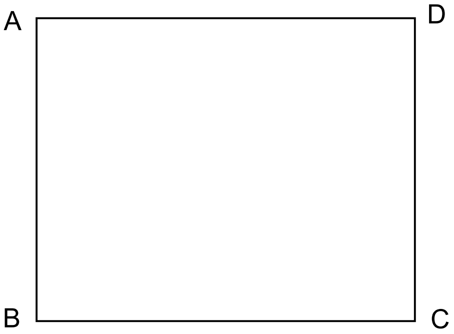
c) How long did it take Kachap to plant 30 seedlings?

9. In a school there are 3 girls for every 4 boys. If there are 196 boys, how many students are there altogether?

10. In rectangle ABCD, the ratio of side length AB to side length AD is 4 : 5.

a) Does this information tell you how long AB is? Explain your thinking.

b) If AB is 20 cm long, how long is AD?



11. Write and solve a proportion problem that you can solve using a unit ratio or rate. Explain why you think it is the best method for solving the problem.

2.1.2 EXPLORE: Scale Drawings and Similar Figures

- Suppose you want to draw a picture of a person who is 170 cm tall and your piece of paper is only 25 cm by 35 cm. That means you will have to create a **scale drawing**.
- In a scale drawing, each centimetre on the paper represents a certain length of the object you are drawing.

For example:

A **scale** of “1 cm represents 10 m” means 1 cm on the scale drawing represents 10 m on the actual item. If you use the same unit for both numbers and write it as "1 cm represents 1000 cm" instead (since 10 m = 1000 cm), it can be written as the ratio 1 : 1000 and is called a **scale ratio**.

- The scale that you use depends on the size you want the scale drawing to be.

A. i) Why would you not choose a scale of "1 cm represents 4 cm" for a scale drawing of a person who is 170 cm tall?

ii) What scale might you choose instead? Why?

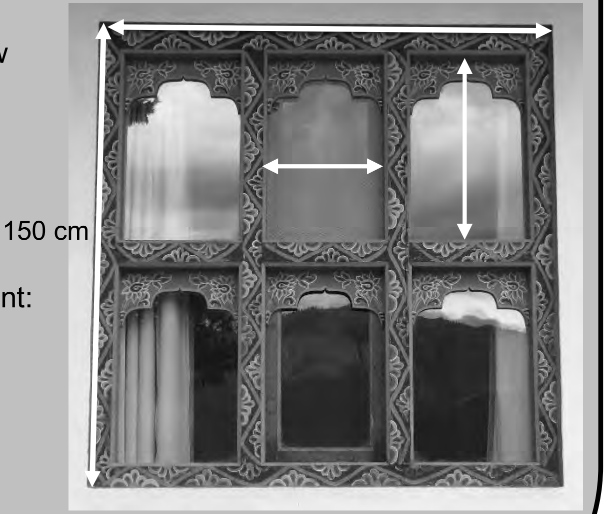
B. i) Suppose you want to make a scale drawing of a rectangular football field that is 100 m long by 64 m wide on a piece of paper that is only 35 cm by 25 cm. What scale would you use? Explain your thinking.

ii) Create a scale drawing of the football field. Measure the diagonal of the drawing.

iii) Use your answer to **part ii)** to estimate the length of the diagonal of the actual field.

C. i) The actual height of this window is about 150 cm. Explain how you could estimate the actual width of one of the panes in the window.

ii) Estimate the actual width of the pane.



D. Estimate each actual measurement:

i) the height of one of the panes

ii) the width of the whole window

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Chapter 2 Percent

2.2.1 Percents Greater Than 100%

Try This

A. A store reduced the original selling price of an item by 20%.

i) What percent of the original selling price is the reduced price?

ii) How can you find the reduced price if you know the original price was Nu 80?

- 100% (100 **percent**) of a quantity means the "whole" quantity.

If ☐ represents 100%, then ☐☐ is 200%,

☐☐☐ is 300%, and ☐▯ is 150%.

- You can calculate a percent greater than 100 much like you calculate a percent less than 100.

For example, to find 125% of 600, you can use the fact that 125% = 100% + 25%.

You know that 100% of 600 is the whole 600, but you also need to calculate 25%.

$$25\% \times 600 = 600 \div 4 = 150$$

Add 100% of 600 to 25% of 600.

$$600 + 150 = 750$$

So 125% of 600 is 750.

- You can also calculate a percent by rewriting the percent as a decimal and then multiplying:

$$\begin{aligned} 125\% &= 1.25 \\ 125\% \text{ of } 600 &= 1.25 \times 600 \\ &= 750 \end{aligned}$$

```
    600
 × 1.25
   3000
  1200
+ 600
 750.00
```

- When you know the percent by which something increased, you can use a percent table to solve a problem.

For example:

A store sold 360 radios this year, which was a 50% increase over last year. How many radios did the store sell last year?

- If 360 radios is a 50% increase over last year, then 360 is 150% of last year's sales.
- Last year's sales is 100%, so you need to find how many radios that is. To find 100%, you can first find 50% by dividing 150% by 3.

Radios	**360**		
Percent	**150**		

÷ 3

Radios	360	**120**	
Percent	150	**50**	

÷ 3

[Continued]

- Then you can double 50% to get 100% and double 120 to get 240.

Last year the store sold 240 radios.

× 2

Radios	360	120	**240**
Percent	150	50	**100**

× 2

• Another tool for solving a percent problem is a double number line. A double number line organizes the same values you might put in a percent table, but in order from least to greatest.

For example:

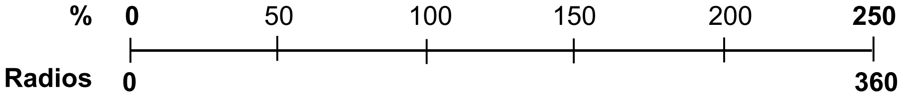
Suppose the 360 radios sold this year was a 150% increase over the number sold last year. How many radios were sold last year?

- A 150% increase means that the number of radio sold is 250% of the number sold last year. It is the 100% sold last year plus another 150%.

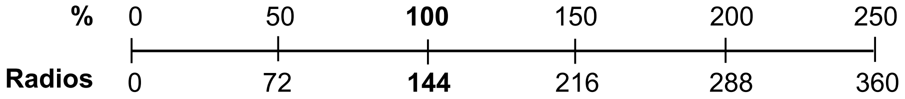
- On one side of the number line, write the percents from 0 to as high as you want. To solve this radio problem, you need percents from 0% to 250%.

- On the other side of the number line, write the corresponding values. In this case, you write the number of radios.

- You know 0% is 0 radios and 250% is 360 radios.



- Since there are 5 equal jumps of 50% to get from 0% to 250%, there must be 5 equal jumps of 72 radios from 0 to 360 (since 360 ÷ 5 = 72).



- Since the number of radios sold last year is 100%, 144 radios must have been sold last year.

B. A second store increased the selling price of the item in **part A** by 20%.

i) What percent of the original price is the increased price?

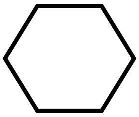
ii) How can you find the increased price of the Nu 80 item?

Examples

Example 1 Modelling Percents Greater than 100%

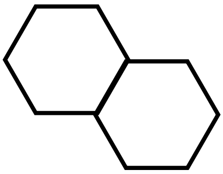
If this hexagon represents 100%, make a shape that represents each.

a) 200% **b)** 150%



Solution

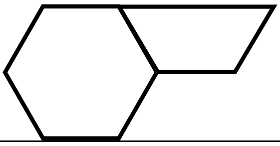
a) 200%



Thinking

• 200% is twice as much as 100%, so I need 2 hexagons to show 200%.

b) 150%



- 150% is $1\frac{1}{2}$ times as much as 100%, so I need a hexagon and half of another hexagon to show 150%.

Example 2 Using 100% to Find a Percent Greater than 100%

The current population of Gasa is about 3000 people. Over the next five years, the population is expected to increase by about 11%. Estimate the population of Gasa five years from now.

Solution 1

Find 111% of 3000

111% = 1.11

3000 × 1.11 = 3330

The population of Gasa will be about 3330 in five years.

Thinking

- I knew an 11% increase meant that I needed to find 100% + 11% = 111% of the current population.
- I rewrote 111% as a decimal and then multiplied the current population by that decimal.

Solution 2

÷10 ÷10

Population	3000	300	30	**3330**
Percent	100	10	1	111

100 + 10 + 1 = 111

3000 + 300 + 30 = 3330

The population of Gasa will be about 3330 in five years.

Thinking

- I used a percent table to find 10% and 1% of 3000.
- Because 111% is 100% + 10% + 1%, I knew that 111% of 3000 was 3000 + 300 + 30.

Example 3 Using a Percent Greater than 100% to Find 100%

The current population of Paro is about 33,000. This is about a 20% increase over its population in 2000. Estimate the population of Paro in 2000.

Solution

÷ 6 × 5

Population	33,000	5,500	**27,500**
Percent	120	20	100

Thinking

- I knew that the current population is 120% of the population in 2000 and that the population in 2000 was 100%.
- I used a percent table to find 100%.
- I could not divide 120% easily to get 100%, so I divided it by 6 to get 20% and then multiplied by 5.

Example 4 Using a Percent Greater than 100% to Find the Original Amount

225% of ■ is 180. What is ■?

Solution

÷ 9 × 4

Amount	180	20	**80**
Percent	225	25	100

225% of 80 is 180.

Thinking

I divided by 225 by 9 to get 25 because 25 was a percent I could multiply by easily to get 100%.

Practising and Applying

1. Select a figure to represent the whole, or 100%. Use it to show each percent.

a) 250% **b)** 125% **c)** 475%

2. a) Find each.
i) 2% of 420
ii) 20% of 420
iii) 200% of 420

b) Use **part a)** to find 222% of 420.

3. Calculate.

a) 130% of 220 **b)** 275% of 500
c) 109% of 1220 **d)** 400% of 75

4. Find ■ for each.

a) 125% of ■ is 625
b) 220% of ■ is 770
c) 350% of ■ is 525

5. In the year 2005, 13,600 tourists visited Bhutan. In 2006, the number of tourists had increased by 25%. How many tourists visited Bhutan in 2006?

6. The National Exchequer received about 800,000 U.S. dollars from tourism in the year 2006. This is an increase of 27% compared to 2005. About how much did the National Exchequer receive in 2005?

7. On Day 1 of an archery tournament, 1200 people attended. On Day 2, the attendance increased by 25%. On Day 3, the attendance increased by 125% over Day 2's attendance. What was the attendance on Days 2 and 3?

8. The population of Bhutan is about 670,000. It is predicted that the population will increase by about 2% a year for each of the next three years.

a) Estimate the population in one year.

b) Use your result from **part a)** to estimate the population in two years.

c) Use your result from **part b)** to estimate the population in three years.

d) Explain why the population in **part c)** is not a 6% increase over the current population, even though there was a 2% increase each year for three years.

9. A shopkeeper bought a desk for Nu 2000.

a) He increased the price of the desk by 25% to sell it. What is the selling price of the desk?

b) He was unable to sell the desk at this price, so he lowered the price by 25%. How does the lower price compare to the original price of Nu 2000? Explain your thinking.

10. Bijoy found 125% of a number. Tashi said that it was the same as finding the result of the number increased by 125%. Do you agree? Use examples to help you explain.

2.2.2 Solving Percent Problems

Try This

A school has 420 students.

5% of the students ride bikes to school.

140 students walk 1 km or more.

A. How many students ride their bikes to school? Explain how you calculated.

B. What fraction of all the students walk 1 km or more? Write the fraction in lowest terms.

There are many strategies for solving a percent problem.

- As you saw in the last lesson, a percent problem can sometimes be solved by rewriting the percent as a decimal or as a fraction and then multiplying.

For example:

To find 45% of 80, rewrite 45% as 0.45 and then multiply:

0.45 × 80 = 36, so 45% of 80 is 36.

- You can also use a proportion to solve a percent problem.

For example:

If you know that 4% of a number is 28, you can use a proportion to find the number. Percent means "out of 100," so 4%, or 4 out of 100, must be equivalent to 28 out of the number you want: $\frac{4}{100} = \frac{28}{\blacksquare}$

$$\frac{4}{100} = \frac{28}{\blacksquare} \rightarrow \overset{\times 7}{\underset{\times 7}{\frac{4}{100} = \frac{28}{700}}}$$

4 × 7 = 28

So you multiply 100 by 7 to solve the proportion:
100 × 7 = 700

If 4% of a number is 28, the number is 700.

- You can use what you know about fractions to solve some percent problems.

For example, to find what percent 45 is of 75:

You want to find what percent is equal to $\frac{45}{75}$. Since $\frac{45}{75} = \frac{3}{5}$ in **lowest terms**, you can find an **equivalent fraction** for $\frac{3}{5}$ with a denominator of 100.

$$\overset{\times 20}{\underset{\times 20}{\frac{3}{5} = \frac{60}{100}}}$$

Since $\frac{45}{75} = \frac{60}{100}$ = 60%, 45 is 60% of 75.

C. i) Show how to use a proportion to calculate the number of students who ride their bikes to school in **part A**.

ii) Use another way to calculate the number of students who ride their bikes.

D. Write the fraction in **part B** as the nearest whole percent.

Examples

Example 1 Finding the Percent of a Number

In a school of 450 students, 60% chose archery as their favourite sport. How many students chose archery as their favourite sport?

Solution 1

60% = 0.6 → 0.6 × 450 = 270

270 students chose archery.

Thinking

- I wrote 60% as a decimal and then multiplied it by 450.

Solution 2

$$\frac{60}{100} = \frac{?}{450} \rightarrow \overset{\times 45}{\underset{\times 45}{\frac{6}{10} = \frac{?}{450}}} \rightarrow \frac{6}{10} = \frac{270}{450}$$

270 students chose archery.

Thinking

- I solved a proportion. First, I changed $\frac{60}{100}$ to $\frac{6}{10}$ so the scale factor would be a whole number.

Example 2 Finding the Total When a Percent is Known

Karma has read 30% of a book. She has read 72 pages. How many pages are there in the book?

Solution 1

$$30\% = \frac{30}{100} = \frac{3}{10}$$

$$\overset{\times 24}{\frac{3}{10} = \frac{72}{\blacksquare}} \rightarrow \underset{\times 24}{\frac{3}{10} = \frac{72}{\mathbf{240}}}$$

There are 240 pages in the book.

Thinking

- I wrote 30% as a fraction and then wrote it in lowest terms.
- I set up a proportion, then I found the scale factor and solved the proportion.

Solution 2

30% of ■ is 72 → 0.3 × ■ = 72

$$0.3 \times \blacksquare = 72$$
$$\blacksquare = 72 \div 0.3$$
$$\blacksquare = 240$$

There are 240 pages in the book.

Thinking

- I set up an equation with the information I knew: 30% of the book is 72 pages.
- I divided 72 by 0.3 to solve the equation.

Example 3 Finding What Percent One Number is of Another

Last year, Kuenga's class had 40 students. This year, there are 48 students. What percent of last year's class size is this year's class?

Solution 1

$$\underset{\times 2.5}{\frac{48}{40} = \frac{\blacksquare}{100}} \rightarrow \overset{\times 2.5}{\frac{48}{40} = \frac{\mathbf{120}}{100}}$$

This year's class size is 120% of last year's.

Thinking

- I knew that it would be more than 100%.
- I set up a proportion, found the scale factor, and then solved the proportion.

Solution 2

$$\frac{48}{40} = \frac{6}{5} \rightarrow \underset{\times 20}{\frac{6}{5} = \frac{\blacksquare}{100}} \rightarrow \overset{\times 20}{\frac{6}{5} = \frac{\mathbf{120}}{100}}$$

This year's class size is 120% of last year's.

Thinking

- I wrote $\frac{48}{40}$ in lowest terms. Then I found an equivalent fraction for $\frac{6}{5}$ with a denominator of 100.

Practising and Applying

1. Find each amount. Use more than one strategy.

a) 24% of 112

b) 120% of 256

c) 54% of 350

d) 98% of 124

2. Solve. Use more than one strategy.

a) 12 is 40% of what number?

b) 128 is 40% of what number?

c) 240 is 150% of what number?

d) 588 is 98% of what number?

3. Find each percent. Use more than one strategy.

a) 48 is ■% of 80

b) 230 is ■% of 200

c) 270 is ■% of 900

4. A school has 120 students. 55% of the students are girls.

a) How many girls are there?

b) How many boys are there?

c) What percent are boys?

5. Devika bought 80 pieces of fruit at the market. 20% were apples.

a) How many apples did she buy?

b) The number of plums she bought was 150% of the number of apples. How many plums did she buy?

c) What percent of the fruit was neither plums nor apples?

6. Changlingmithang stadium in Thimphu holds 15,000 people.

a) At a recent tournament, 85% of the seats were occupied. How many people were at the tournament?

b) When renovations are completed, the stadium will hold 100% more people. What will be the new capacity of the stadium?

7. This year 40 more children joined a football league than joined last year. The number of children in this year's league is 110% of the number in last year's league. How many children played in the league last year?

8. A rectangular garden plot measures 9 m by 5 m. Chillies are planted in a rectangular area of the garden that measures 4.5 m by 2 m. What percent of the garden is planted in chillies?

9. The average boy is 90% of his adult height by age 13. He is 98% of his adult height by age 18.

a) Tandin is 13 years old and 150 cm tall. Estimate his adult height.

b) Estimate his height at age 18.

10. The average girl is 90% of her adult height by age 11. She is 98% of her adult height by age 17.

a) Pem Bidha is 11 years old and 144 cm tall. Estimate her adult height.

b) Estimate her height at age 17.

11. Use the information in **question 9 or 10** to estimate your adult height.

12. Write your own problem that involves percents. Solve your problem.

GAME: Equivalent Concentration

The goal of this game is to collect pairs of cards with equivalent values.

Make 18 cards like the cards shown here.

Play with a partner.

This is how to play:

- Shuffle the cards and place them face down on a desk in a 3-by-6 array (as shown).
- On your turn, turn over two cards.

- If the cards match (the values are equivalent), take the cards and play again.

- If the cards do not match, turn both cards face down again. It is the other player’s turn.

- Take turns until all the cards have been matched.

The winner is the player with more cards at the end.

0.57	0.75	6
$\frac{1}{4}$	$\frac{9}{4}$	2.3
$\frac{6}{8}$	65%	$\frac{26}{25}$
$\frac{13}{20}$	0.06	104%
57%	225%	25%
230%	600%	6%

2.2.3 Fractional Percents

Try This

If just 0.5 g of sugar is dissolved in 100 mL of water, the water will taste sweet.

A. i) How much sugar do you need to add to 1 L of water to make it taste sweet?
ii) If you add 7 g of sugar to 1.5 L of water, will it taste sweet? How do you know?

- A percent is not always a whole number. It can be a fraction or a decimal.

For example:

There are 63 girls in a school of 120 students. What percent are girls?

$\xrightarrow{\div 6}$ $\xrightarrow{\times 5}$

Girls	63	10.5	52.5
Students	120	20	100

$\xrightarrow[\div 6]{}$ $\xrightarrow[\times 5]{}$

This means that 52.5%
("fifty-two and one half percent")
of the students are girls.

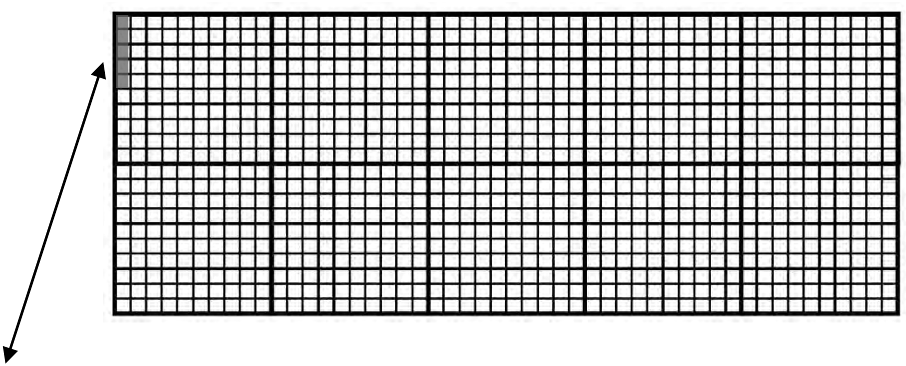
- You can also write a percent as a fraction or as a **mixed number**.

For example, $52\frac{1}{2}$ % of the students in the school are girls.

- A half percent (0.5%), or half of one percent, can be represented on a rectangular grid of 1000 squares, called a thousandths grid.

10 small squares make up 1% of the grid since $1\% = \frac{1}{100} = \frac{10}{1000}$.

So 5 small squares represent half of one percent.



$0.5\% = \frac{5}{1000} = \frac{1}{200}$

B. What percent of a solution needs to be sugar in order for it to taste sweet? (Hint: 1 mL of water is 1 g.)

Examples

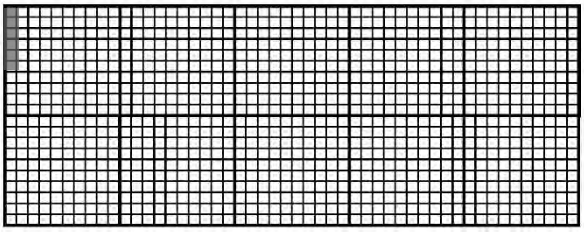
Example 1 Representing Percents Less Than 1%

Use a thousandths grid to show each percent.

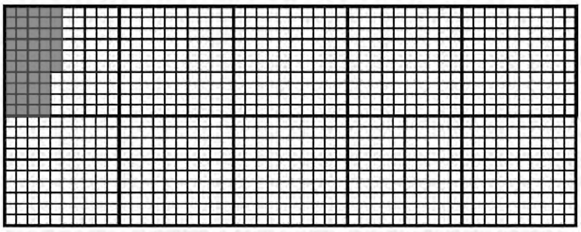
a) 0.6% **b)** 4.6%

Solution

a)



b)



Thinking

- I knew $\frac{1}{100}$ = 1%, so $\frac{1}{1000}$ = 1% ÷ 10 = 0.1%.
- That meant each small square was $\frac{1}{1000}$, or 0.1%.

a) 0.6% = 6 x 0.1%, which is 6 small squares.

b) 4.6% = 46 × 0.1%, which is 46 small squares.

Example 2 Solving a Problem Involving Percents Less Than 1%

The population of Bhutan was about 670,000 in 2005. About 0.7% of the population was over age 75. About how many people were over age 75?

Solution 1

1% of 670,000 = 6700

0.1% of 670,000 = 6700 ÷ 10 = 670

0.7% of 670,000 = 7 × 670 = 4690

About 4690 people were over age 75.

Thinking

This is how I found 0.7%:

- I first found 1% by dividing 670,000 by 100.
- Then I found 0.1% by dividing 6700 by 10.
- I multiplied 670 by 7 to get 0.7%.

Solution 2

7% of 670,000 = 0.07 × 670,000
= 46,900

46,900 ÷ 10 = 4690

About 4690 people were over age 75.

Thinking

- I knew 0.7 = 7 ÷ 10, so I first found 7% and then divided by 10 to get 0.7%.

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Example 3 Comparing Percents

The sales tax rate in New Zealand is 12.5%. In Thailand it is 7%. How much more tax would you pay in New Zealand than in Thailand for a Nu 5000 item?

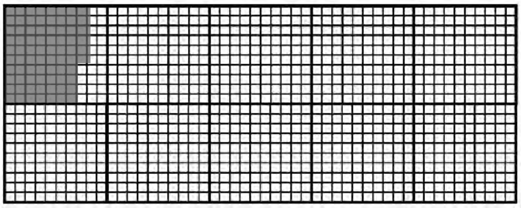
Solution	**Thinking**
12.5% – 7% = 5.5% 5.5% = 5% + 0.5% 10% of 5000 = 500 5% of 5000 = 500 ÷ 2 = 250 0.5% of 5000 = 250 ÷ 10 = 25 250 + 25 = 275 I would pay Nu 275 more tax in New Zealand.	 • Instead of calculating 12.5% of Nu 5000 and 7% of Nu 5000 and subtracting, I subtracted to find the difference between the percents and then found that percent of Nu 5000. • I thought of 5.5% as 5% + 0.5%. I calculated each separately and then added them: - For 5%, I took half of 10%. - For 0.5%, I divided the 5% amount by 10.

Practising and Applying

1. Use a thousandths grid to represent each percent.

a) 0.75% **b)** 1.4% **c)** 4.9%

2. What percent of this grid is shaded? The whole grid is 100%.



3. 5% of a mass is 25 g. How can you calculate each percent of the same mass?

a) 1% **b)** 0.1% **c)** 2.5%

4. a) How would you estimate 0.3% of 630?

b) Represent 0.3% on a thousandths grid.

c) Calculate 0.3% of 360. Explain your strategy.

5. You can taste saltiness if 0.25% of a mixture is salt. How many millilitres of salt would there have to be in 1 L of water for it to taste salty?

6. Air is 0.93% argon and 0.03% carbon dioxide. In 1 L of air, how many grams are there of each?

a) argon **b)** carbon dioxide

7. Which of these ways of calculating 2.5% of a number are correct? Explain your thinking.

A. Calculate 5%, then divide by 2.

B. Calculate 25%, then divide by 10.

C. Divide by 4, then divide by 10.

D. Calculate 1%, double it, and then add another half of 1%.

8. a) How do you know that 1 mm is 0.1% of a metre?

b) What percent of 1 m is 3.2 mm?

9. Can 0.1% of a number be a whole number? Use an example to help you explain your thinking.

10. a) Explain why 50% ≠ $\frac{1}{2}$%.

b) Explain why 0.5% ≠ $\frac{1}{2}$.

11. Is 5.1% of a number always close to 5% of the number? Explain your thinking using examples.

2.2.4 Solving Percent Problems Using Familiar Percents

Try This

The area of a garden is 300 m^2.

A. i) Onions are planted in 10% of the garden. What is the area planted in onions?

ii) Peppers are planted in 20% of the garden. How can you use your answer from **part i)** to find the area of the garden planted in peppers?

- 10% and 1% are familiar percents that you can relate to other percents.

For example:

- To find 30% of 950, first calculate 10% and then multiply by 3:

  10% of 950 = 950 ÷ 10 = 95

  3 × 95 = 285

  So 30% of 950 is 285.

- To find 7% of 1600, first calculate 1% and then multiply by 7:

  1% of 1600 = 1600 ÷ 100 = 16

  7 × 16 = 112

  So 7% of 1600 is 112.

- You can use a percent table and familiar percents such as 1% and 10% to solve percent problems.

For example:

In a school of 360 students, students were surveyed about their favourite meal. 25% chose Pork Fing, 15% chose Kewa Phagsha, and 40% chose Ema Datshi. *How many students chose each meal as their favourite?*

- First calculate 10% and use the result to calculate 5% and 40%.
- You can use 5% to calculate 25% and 15%.

× 4 × 3

÷ 10 ÷ 2 × 5

Percent	100	10	5	25	40	15
Number of students	360	36	18	**90**	**144**	**54**

÷ 10 ÷ 2 × 5

× 4 × 3

For their favourite meal, 90 students chose Pork Fing, 144 chose Ema Datshi, and 54 chose Kewa Phagsha.

You can also find 15% by adding the numbers for 10% (36 students) and 5% (18 students).

10% + 5% = 15% → 36 + 18 = 54 students

B. Suppose chillies are planted in 15% of the garden in **part A**. Show two different ways to find the area of the garden that is planted in chillies.

Examples

Example 1 Finding a Percent of a Number

A school has 620 students. A survey found that 15% of the students own bicycles. How many students own bicycles?

Solution	**Thinking**
10% of 620 = 62 5% of 620 = 62 ÷ 2 = 31 15% = 10% + 5%, so 15% of 620 = 62 + 31 = 93. 93 students own bicycles.	• I found 10% of 620 by dividing by 10. Then I divided the result by 2 to find 5%. 

Example 2 Finding the Total When a Percent is Known

A store sold 40% more radios this year than last year. If the store sold 112 radios this year, how many did it sell last year?

Solution	**Thinking**
If 140% of ■ is 112, then 10% of ■ is 112 ÷ 14 = 8, and 100% of ■ is 8 × 10 = 80. The store sold 80 radios last year.	• I knew that "40% more" meant that if last year was 100%, this year is 140%. 

Practising and Applying

Show your work for questions 1 to 3.

1. Find each. Use familiar percents.

a) 11% of 800 **b)** 120% of 540

c) 8% of 4300 **d)** 115% of 920

2. Find the value of each. Use familiar percents.

a) 9% of a length is 36 m

b) 300% of a mass is 240 g

c) 80% of the money is Nu 140

d) 450% of the mass is 135 kg

e) 175% of the capacity is 105 L

3. Find the amount of sales tax that is charged for each. Use familiar percents.

	Country	Price (Nu)	Sales tax (%)	Sales tax (Nu)
a)	Paraguay	250	10%	
b)	Singapore	1500	5%	
c)	Sweden	140	25%	
d)	United Kingdom	80	17.5%	
e)	Mexico	18,000	15%	

4. a) The population of Thimphu is about 102,000. This is about 16% of the population of Bhutan. Estimate the population of Bhutan.

b) Gasa has a population of about 3000. 52% of the population are males. About how many males live in Gasa?

5. A football team won 40% of its games. If the team played 35 games altogether, how many games did it win?

6. a) An apple is about 80% water. About how many grams of water are in a 210 g apple?

b) A watermelon is about 97% water. About how many grams of a 9 kg watermelon are <u>not</u> water?

7. Sonam got 19 questions right on a math test. These questions made up 76% of the test. How many questions were on the test?

8. Last year, a class had 28 students. This year, the number of students is 125% of last year's number. How many students are in the class this year?

9. A 250 mL glass of juice contains 100% of the daily amount of vitamin C that a person needs. Pelden drank enough juice to get 160% of his daily vitamin C needs. How much juice did Pelden drink?

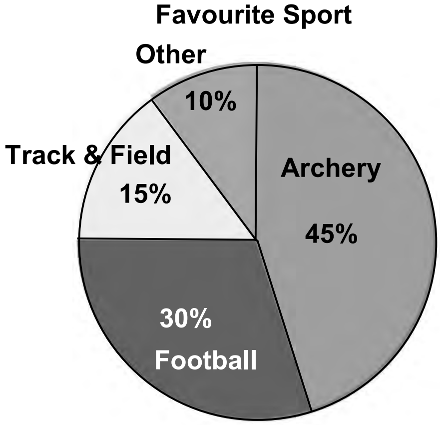
10. Canada's population in the year 1984 was about 25 million people. From 1984 to 2001, the population grew by about 20%.

a) By how many people did Canada's population increase between 1984 and 2001?

b) About how many people lived in Canada in 2001?

c) From 2001 to 2006, the population grew by about 10%. Estimate Canada's population in 2006.

11. The circle graph below shows the results of a survey about the favourite sport of 400 Class VIII students. Find the number of students who chose each sport.



12. What familiar percents would you use to calculate the number of students who chose each sport in **question 11**? Why did you choose those percents?

Chapter 3 Consumer Problems

2.3.1 Markup and Discount Consumer Problems

Try This

CD Player **X**
Regular price Nu 1500
20% OFF

CD Player **Y**
Regular price Nu 2000
30% OFF

CD Player **Z**
Regular price Nu 1800
25% OFF

A. Suppose you want to buy a CD player. With which CD player do you think you would save the most money? Explain your thinking.

- The price that shopkeepers pay for items they sell is called the **cost price**.
- To pay for **expenses** and to make a **profit**, shopkeepers sell items for a price that is higher than their cost price. This price is called the **regular** or **selling price**.
- The increase in price from cost price to selling price is called a **markup**.
The markup can be an amount or a percent.
- When a shopkeeper places a 30% markup on an item, the selling price of the item is 130% of the cost price.

100% (cost price) + 30% (markup) = 130% (selling price)

For example:

Suppose a gho with a cost price of Nu 1000 is sold at a markup of 30%.

Markup = 30% of Nu 1000
= Nu 300

Selling price = Nu 1000 + Nu 300
= Nu 1300

The selling price can also be calculated by finding
130% of Nu 1000 = 1.3 × Nu 1000
= Nu 1300

- To encourage customers to buy, shopkeepers often reduce the price and put items on sale. The decrease in price is called a **discount** or **markdown**.
The discount can be an amount or a percent.
- When an item is discounted or marked down by 20%, the **sale price** of the item is 80% of the regular selling price.

100% (regular selling price) – 20% (discount) = 80% (sale price)

[Continued]

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For example:

Suppose a mobile with a regular selling price of Nu 15000 is on sale for a discount of 20%.

Discount = 20% of Nu 15000
= Nu 3000

Sale price = Nu 15000 – Nu 3000
= Nu 12000

The sale price can also be calculated by finding 80% of Nu 15000 = 0.8 × 15000
= Nu 12000

- You can find the percent discount if you know the sale price and the regular selling price.

For example:

A table that is usually priced at Nu 2000 is on sale for Nu 1700. The discount amount is Nu 2000 – Nu 1700 = Nu 300.

This is how to calculate the percent discount:

- Divide the discount amount by the regular selling price: $\frac{300}{2000} = 0.15$

- Multiply by 100% to change the decimal to a percent: 0.15 × 100% = 15%

- Similarly, you can find the percent markup if you know the markup amount and the cost price. You divide the markup amount by the cost price and then multiply by 100% (as shown in **Example 3 part b)** on **page 43)**.

B. Find the sale price of each CD player described in **part A**.

C. When you say that you save the most money on a certain CD player, is it the same as saying that the CD player is the least expensive? Explain your thinking.

Examples

Example 1 Finding a Discount Amount and Sale Price

An item with a regular selling price of Nu 720 is discounted by 15%.

a) What is the discount amount?

b) What is the sale price of the item?

Solution	**Thinking**
a) 10% of Nu 720 = 720 ÷ 10 = Nu 72 5% of Nu 720 = Nu 72 ÷ 2 = Nu 36 15% of Nu 720 = Nu 72 + Nu 36 = Nu 108 The discount amount is Nu 108. **b)** Sale price = Nu 720 – Nu 108 = Nu 612 The sale price is Nu 612.	 **a)** To find 15% of the selling price, I found 10% and 5% and added them. **b)** I subtracted the discount amount from the regular selling price to find the sale price.

Example 2 Finding a Markup and Selling Price

A shopkeeper buys radios for Nu 1200 each and marks them up 25% to sell them.

a) What is the markup amount?

b) What is the selling price of each radio?

Solution 1

÷ 4 × 5

Amount (Nu)	1200	**300**	**1500**
Percent	100	25	125

÷ 4 × 5

a) The markup amount is Nu 300.

b) The selling price is Nu 1500.

Thinking

- I used a percent table.

a) Since 100 ÷ 4 = 25,
I divided 1200 by 4 to find
25% of Nu 1200.

b) Since the markup was 25%, the selling price was 125% of the cost price.

Solution 2

a) 25% of Nu 1200 = $\frac{1}{4}$ of 1200
= 300

The markup amount is Nu 300.

b) Nu 1200 + Nu 300 = Nu 1500

The selling price is Nu 1500.

Thinking

a) I calculated 25% of Nu 1200 by finding $\frac{1}{4}$ of Nu 1200, which is 1200 ÷ 4.

b) I added the markup amount to the cost price to find the selling price.

Example 3 Finding the Percent of Discount and Markup

a) A table that usually sells for Nu 2000 is on sale for Nu 1600. What is the percent discount?

b) A shopkeeper buys a table for Nu 1600 and sells it for Nu 2000. What is the percent markup?

Solution

a) $\frac{1600}{2000} = \frac{800}{1000} = 0.8 = 80\%$

The sale price is 80% of the regular selling price.

Percent discount = 100% – 80%
= 20%

The percent discount is 20%.

b) $\frac{2000}{1600} = \frac{125}{100} = 1.25 = 125\%$

The selling price is 125% of the cost price.

Percent markup = 125% – 100%
= 25%

The percent markup is 25%.

Thinking

a) I divided the sale price by the regular selling price and wrote it as a percent.

- I subtracted the percent from 100% to find the percent discount.

b) I divided the selling price by the cost price and wrote it as a percent.

- I subtracted 100% to find the percent markup.

Practising and Applying

1. Find the discount amount and sale price for each.

	Regular selling price (Nu)	Discount percent
a)	140	15%
b)	72	25%
c)	650	8%
d)	18,000	30%

2. Find the markup amount and regular selling price for each.

	Cost price (Nu)	Markup percent
a)	30	20%
b)	280	25%
c)	750	10%
d)	2500	30%

3. Laxmi was selling Avocados at Nu 200 per kilogram. She raised the price by 5%. What is the new price per kilogram?

4. A toy that regularly sells for Nu 250 is on sale for Nu 225. What is the percent discount?

5. Radhika paid Nu 2000 for a dress with a cost price of Nu 800. What was the percent markup?

6. A shopkeeper reduces the price of an item by 20% if the item is not sold after 3 weeks. She reduces the price a further 20% if it is not sold after 6 weeks.

a) After 3 weeks, what is the sale price of a shirt that regularly sells for Nu 300?

b) After 6 weeks, what is the sale price of the same shirt?

7. The sale price of a bed is Nu 1200. This is a savings of 20% off the regular selling price. What is the regular selling price of the bed?

8. Two shops sell the same item for the same price.

Shop 1 offers a discount of 10% one week, 10% the next week, and 10% in the third week.

Shop 2 offers a discount of 30% in the third week.

Which shop offers the greater discount in the third week? Use an example to explain your answer.

9. The selling price of an item is Nu 800. It is marked down by Nu 160. Another item that sells for Nu 800 is marked up by Nu 160. Is the percent markdown (discount) equal to the percent markup? Explain your thinking.

10. Create and solve a consumer problem that involves calculating 125% of a number.

2.3.2 Simple Interest and Commission

Try This

A. Padam works in a motorcycle store. He sold a motorcycle for Nu 45,000 and earned Nu 2700 for making the sale. What percent of the selling price of the motorcycle was Padam's earnings?

- A bank makes money by charging you **interest** when you borrow money. You are also charged interest when you buy something on **credit** and pay for it later.
- If you deposit or **invest** money in the bank, you earn interest. The bank pays you interest, since the bank is borrowing your money to lend to others.
- When the interest charged or interest earned is based only on the money that was originally borrowed or invested, the interest is called **simple interest**. Interest can be an amount or a percent.

For example:

Ugyen borrowed Nu 5000 from the bank to buy a computer. She paid the money back at the end of 1 year. She was charged 12% simple interest for the year. *What amount did Ugyen pay in interest? How much did she have to pay back?*

12% of 5000 = $0.12 \times 5000 = 600$	Ugyen paid Nu 600 in interest.
Nu 5000 + Nu 600 = Nu 5600	Ugyen paid back a total of Nu 5600.

- The **formula** for finding the amount of simple interest on a loan or investment is

$$I = Prt$$

I is the amount of simple interest.

P is the **principal**, the amount of money borrowed or invested.

r is the **annual interest rate** (rate per year), usually written as a decimal.

t is the time period in years.

- Simple interest may be charged or earned for a time period less than one year.

For example:

Tshering borrowed Nu 3000 for 6 months. The annual interest rate was 7%. *What amount did Tshering pay in interest? How much did he have to pay back?*

The principal, P, is Nu 3000.

The interest rate, r, is 7% or 0.07 per year.

The time, t, is $\frac{6}{12} = \frac{1}{2} = 0.5$.

Substitute $P = 3000$, $r = 0.07$, and $t = 0.5$ into $I = Prt$:

$$\begin{aligned} I &= 3000 \times 0.07 \times 0.5 \\ &= 105 \end{aligned}$$

Tshering paid Nu 105 in interest.

He paid back the principal and the interest: Nu 3000 + Nu 105 = Nu 3105

- Many salespeople earn a **commission.** A commission is money received by a salesperson for the sale of an item or from total sales. Commission can be an amount or a percent. Percent commission is often called **rate of commission**.

For example:

Rupak is a salesperson who earns a rate of commission of 5% on sales. He sold a TV for Nu 7500. *How much commission did he earn?*

The rate of commission is 5% of Nu 7500: $0.05 \times 7500 = 375$

Rupak earned Nu 375 in commission.

B. Use the percent you calculated in **part A** as the commission percent.

i) How much commission would Dorji earn for selling a motorcycle for Nu 30,000?

ii) Dorji earned a commission of Nu 3000 for selling a motorcycle. What was the selling price of the motorcycle?

Examples

Example 1 Finding Interest and Total Amount Received

A bank offers an interest rate of 6% per year. Arjun deposits Nu 2500.

a) How much interest will he earn at the end of one year?

b) How much money will be in the account at the end of the year if he does not withdraw any money?

Solution	**Thinking**
a) $I = Prt = 2500 \times 0.06 \times 1 = 150$ He will receive Nu 150 in interest.	**a)** I substituted the information I knew into the simple interest formula. 
b) Nu 2500 + Nu 150 = Nu 2650 He will have Nu 2650 in the account at the end of the year.	**b)** I added the interest earned to the principal to find the total amount in the account at the end of the year.

Example 2 Finding Interest Rate

Namgyel borrowed Nu 1500 for a year and a half. He was charged Nu 270 in interest. What was the annual interest rate?

Solution	**Thinking**
$I = Prt$ $270 = 1500 \times r \times 1.5$ $r = 270 \div 1500 \div 1.5$ $= 0.12$ $= 12\%$ The interest rate was 12% per year.	• I substituted what I knew into the simple interest formula. 

Example 3 Finding Commission and Rate of Commission

a) Karchung earns a rate of commission of 6%. What is his commission for selling an item priced at Nu 1400?

b) Karchung's commission was Nu 168 for the sale of a table priced at Nu 2100. What was the rate of commission for that sale?

Solution

a) 6% × 1400 = 1400 × 0.06 = 84

His commission is Nu 84.

b) $\frac{168}{2100} = \frac{84}{1050} = \frac{42}{525} = \frac{14}{175} = \frac{2}{25}$

$\frac{2}{25} = \frac{8}{100}$ = 8%

The rate of commission was 8%.

Thinking

a) I wrote the rate as a decimal and then multiplied it by the amount of the sale to find the amount of commission.

b) I wrote the commission as a fraction of the selling price in lowest terms. Then I wrote the fraction as a percent.

Practising and Applying

1. Write each percent as a decimal.

a) 7% **b)** 4.5% **c)** $6\frac{1}{4}\%$

2. Calculate the simple interest charged for each loan.

	Loan (Nu)	Annual interest rate (%)	Time (years)
a)	4900	15.9	2
b)	18,000	9.0	5
c)	10,000	8.5	3

3. Calculate the simple interest earned on each deposit.

	Deposit (Nu)	Annual interest rate (%)	Time (years)
a)	5400	6.0	2
b)	6500	7.0	4
c)	12,000	6.5	3

4. Lobzang earns a rate of commission of 4%. What is his commission on a sale of Nu 16,000?

5. Calculate the simple interest amount and the total amount that has to be paid back on each loan.

a) Nu 6000 at an annual interest rate of 15% for 6 months

b) Nu 12,500 at an annual interest rate of 8% for 3 months

c) Nu 8000 at an annual interest rate of 12% for 3 years

6. Each month, Mindu earns a 3% commission on sales up to Nu 75,000 and 5% on sales over Nu 75,000. Last month he had sales of Nu 95,000. How much commission did Mindu earn?

7. Which investment earns more total interest? Explain your thinking.

A. Nu 1000 for 2 years at 5% per year

B. Nu 1000 for 1 year at 10% per year

8. At one time, the Bhutan National Bank charged 15% interest per year for a personal loan. Suppose you had borrowed money and paid the interest at the end of each year.

a) If you had borrowed Nu 1000, how much would you have paid in interest at the end of one year?

b) How much interest would you have paid at the end of two years?

c) Make a chart to show how much interest you would have paid after 3 years, 4 years, and so on, up to 10 years.

9. Pema borrows Nu 28,800 for 2 years at an annual interest rate of 12.5%.

a) How much interest will Pema pay?

b) How much will Pema have to pay back altogether?

c) Pema decides to pay back the money in equal monthly payments over the two years. What is the amount of each monthly payment?

10. Dawa and Nima both work in stores that sell radios and TVs.

- Dawa earns a salary of Nu 4000 a month and a commission of 6% on sales.
- Nima earns a salary of Nu 4500 a month and a commission of 4% on sales.

One month they both sold Nu 40,000 worth of goods. Who earned more money that month? Show your work.

11. What is the annual interest rate when you borrow Nu 6000 for one year and pay Nu 720 in interest?

12. Suppose you know these three things about a bank deposit:

- the annual interest rate
- how much simple interest was earned
- the length of time the money was in the account

How can you find the principal amount that was deposited? Use an example to explain your thinking.

CONNECTIONS: Currency Conversion

- The foreign exchange rate, or **rate of exchange**, is used to convert money of one country into the money of another country. Travellers change money from their own currency into the currency of the country they are travelling to.
- A rate of exchange is usually given as one unit of one currency to the equivalent number of units of another.

For example:

Recently the exchange rate between the ngultrum and Thai baht was:

1 Thai baht = Nu 1.91

Sometimes this rate is given as a percent:

1 Thai baht = 191% of a ngultrum

Exchange Rates

CURRENCY	BUY	SELL
USD	63.45	65.55
GBP	82.25	84.55
EUR	72.45	74.45
JPY(100)	56.00	57.60
CHF	65.40	67.25
HKD	8.10	8.35
CAD	49.80	51.20
AUD	49.25	50.60
SGD	46.15	47.45
DKK	9.75	10.00
SEK	7.60	7.80
NOK	7.70	7.95

Foreign Currency Exchange Rate with Bhutanese Currency (*https://www.rma.org.bt/*)

- You can solve a proportion to calculate the value of a ngultrum in bahts:

$$\text{Nu } 1.91 = \Delta \text{ baht}$$

$$\frac{1.91}{1.91} = \frac{\Delta}{1.91}$$

$$1 = \Delta \times 1.91$$

$$\Delta = \frac{1}{1.91}$$

$$\text{Nu } 1 = 0.52 \text{ baht}$$

1. Use the rate of exchange given above. How much Thai money would you get for each amount?

a) Nu 10 **b)** Nu 100 **c)** Nu 1000

2. How many ngultrums would you get for each amount of Thai money?

a) 10 baht **b)** 100 baht **c)** 1000 baht

3. If one Canadian dollar is worth Nu 40, what is the value of Nu 1 in Canadian dollars?

Different currency notes from around the world

4. Choose two different currencies and find the current rate of exchange.

Find the rate both ways:

- what one unit of the first currency is in relation to the second currency
- what one unit of the second currency is in relation to the first currency

UNIT 2 Revision

1. Solve each proportion.

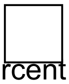
a) $\frac{\blacksquare}{15} = \frac{20}{60}$

b) $\frac{9}{7} = \frac{\blacksquare}{21}$

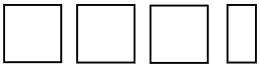
c) $\frac{3}{4} = \frac{21}{\blacksquare}$

2. The ratio of boys to girls in a school is 3 : 4. If there are 150 boys, how many girls are there?

3. Cycling burns about 4 calories each minute. An apple has about 60 calories. About how many minutes would you have to cycle to burn off the calories in an apple?

4.  represents 100%. What percent

is represented by these shapes?



5. Calculate each.

a) 325% of 400

b) 119% of 1200

c) 275% of 440

6. Trashiyangtse has a population of about 36,000. Sarpang's population is about 140% of Trashiyangtse's population. Estimate the population of Sarpang.

7. a) What is 42% of 250?

b) What is 15% of 1200?

c) 256 is 40% of what number?

d) 75% of a number is Nu 180. What is the number?

e) What percent of 2150 is 430?

8. Tshering got 62.5% on a test. Each question on the test was worth 1 mark and he got 25 questions correct.
How many questions were on the test?

9. Bananas are about 75% water. About how much water is in 820 g of bananas?

10. A portable CD player is on sale for a discount of 30%. The regular selling price is Nu 1800.

a) What is the discount amount?

b) What is the sale price?

11. A kilogram of beans has a cost price of Nu 40 and a selling price of Nu 55. What is the percent markup?

12. The price of a house increased by 15% in January. The price then decreased by 15% in June.

Is the price of the house at the end June equal to the price of the house at the beginning of January? Use an example to help explain your thinking.

13. Rinzin borrows Nu 3500 for 18 months at an annual simple interest rate of 12%.

a) How much interest will Rinzin pay after 18 months?

b) How much will Rinzin have to pay back altogether after 18 months?

14. A salesperson earns a 6% commission. What is the commission on sales of Nu 70,000?

15. Dorji invested Nu 5000 for two and half years. He earned Nu 1000 in interest. What was the simple interest rate?

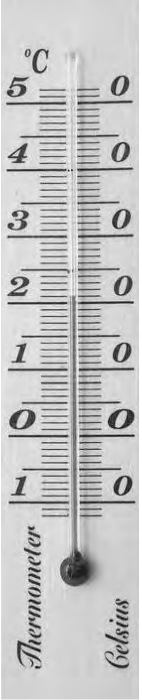
UNIT 3 INTEGERS

Getting Started

Use What You Know

This chart shows the temperature for three places on January 10.

Place	High temperature	Low temperature
Paro	+17ºC	–6ºC
Thimphu	+15ºC	–7ºC
Punakha	+18ºC	+2ºC



A. Use the chart to identify the place that has each.

i) the greatest high temperature

ii) a low temperature 22ºC lower than its high temperature

iii) a high temperature 23ºC higher than its low temperature

iv) a low temperature 9ºC higher than the lowest low temperature

B. This chart shows the temperature for two more places on January 10.

Place	High temperature	Low temperature
Trongsa	+13ºC	–1ºC
Trashigang	+20ºC	+10ºC

Create four or more clues, like the clues in **part A**, to describe temperatures in four or more of the places in the two charts. Include the answer to each clue.

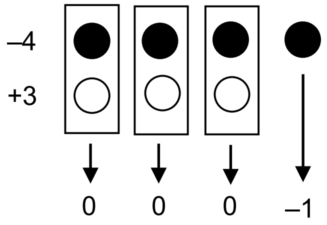
Reprint 2024

Skills You Will Need

This is how how to calculate (–4) + (+3) using a counter model:

- Use 4 black counters to represent –4.
- Use 3 white counters to represent +3.
- Pair each black counter with a white counter to represent 0.

Each pair makes 0 and there is 1 black counter left:



That means (–4) + (+3) = –1.

Use a counter model as shown above to answer question 1.

1. Use a counter model to add or subtract each.

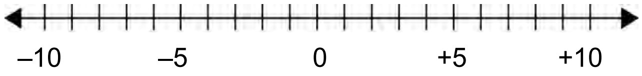
a) (+5) + (+14)

b) (–5) + (+14)

c) (+4) – (+5)

d) (+4) – (–5)

2. Use a number line model to show that (–4) + (+3) = –1.



3. Use a number line model to add or subtract each.

a) (–5) + (–7)

b) (+3) + (–8)

c) (–4) – (+5)

d) (–5) – (–4)

4. Calculate. Use models if you wish.

a) (–7) – (+7)

b) (+14) – (+4) – (+10)

c) (–35) – (–25) – (+25)

d) (+30) – (–11) – (–9)

e) (–33) – (+40) – (–3)

Reprint 2024

Chapter 1 Multiplying Integers

3.1.1 Multiplying Integers Using Counters and Patterns

Try This

Drakpa wrote a quiz.

- He got 50 marks for the questions he answered correctly.
- He lost 2 marks for each of the 5 questions he answered incorrectly.

A. i) Use integers to write an expression that could be used to calculate Drakpa's final mark on the quiz.
ii) What was his final mark?

- Multiplying whole numbers is the same as **repeated addition**. You can also use repeated addition to multiply **integers**.

For example:

(+4) × (–2) is the same as adding 4 groups of (–2). You can use counters to represent the multiplication, with each black counter representing (–1).

(+4) × (–2) = (–2) + (–2) + (–2) + (–2) = –8

(●●) + (●●) + (●●) + (●●) = –8

(+4) × (–2)

- Since positive integers are like whole numbers, you do not have to show the + sign and the brackets.

For example: (+4) × (–2) = –8 → 4 × (–2) = –8

- If you want to multiply (–2) × 4, it does not make sense to use (–2) groups of 4. You can use the **commutative property** of multiplication to change the order to 4 × (–2). Then you can multiply 4 groups of (–2).

For example: (–2) × 4 → 4 × (–2) = –8

- The **associative property** of multiplication allows you to multiply three integers. First you multiply the first pair and then you multiply by the third integer. Or, you can first multiply the last pair and then multiply by the first integer.

For example, to multiply 3 × (–2) × 4:

First multiply 3 × (–2): 3 × (–2) = –6

(●●) + (●●) + (●●) = –6

Then multiply the product by 4: (–6) × 4 = 4 × (–6) = –24

(●●●●●●) + (●●●●●●)
+ (●●●●●●) + (●●●●●●) = –24

[Continued]

Or, to multiply 3 × (–2) × 4, you can first multiply (–2) × 4 and then multiply by 3.

(–2) × 4 = 4 × (–2) = (–8)

3 × (–8) = –24

[(●●) + (●●) + (●●) + (●●)] + [(●●) + (●●) +
(●●) + (●●)] + [(●●) + (●●) + (●●) + (●●)] = –24

- To multiply two negative integers, you can use a pattern.

Start the pattern with products you know.

(+3) × (–2) = –6

(+2) × (–2) = –4

(+1) × (–2) = –2

0 × (–2) = 0

(–1) × (–2) = ? →

(–2) × (–2) = ? →

Notice that each time the first **factor** decreases by 1, the product increases by 2. This makes sense since you add one fewer group of –2 each time, so the answer should be 2 greater each time.

That means

(–1) × (–2) = +2

(–2) × (–2) = +4

The pattern shows that, when you multiply two negatives, the result is positive.

That means, to multiply two negative numbers, you can first multiply them without the signs and then make the product positive.

For example: (–4) × (–5) = +20, which can be written as just 20.

B. Write an expression using integers and multiplication to calculate Drakpa's final score in **part A**.

Examples

Example Multiplying Integers with Different Signs

Use counters to model and calculate each.

a) 4 × (–3) **b)** 2 × (–4) × 3 **c)** (–5) × 3

Solution	**Thinking**
a) 4 × (–3) = –12 (●●●) + (●●●) + (●●●) + (●●●)	**a)** I used 4 groups of 3 black counters to represent 4 × (-3). 
b) 2 × (–4) × 3 → 2 × 3 × (–4) 2 × 3 = 6 6 × (–4) = –24 (●●●●) + (●●●●) + (●●●●) + (●●●●) + (●●●●) + (●●●●)	**b)** I changed the order of the integers so I could first multiply 2 × 3. I did this because I wanted a positive number of groups. • I used 6 groups of 4 black counters to show 6 × (-4).

c) (–5) × 3 = 3 × (–5) = –15 (●●●●●) + (●●●●●) + (●●●●●)	**c)** I changed the order so I would have a positive number of groups. • I used 3 groups of 5 black counters to show 3 × (-5).

Practising and Applying

1. Write a multiplication expression for each model.

a) (●●●●) + (●●●●)
\+ (●●●●) + (●●●●)
\+ (●●●●) + (●●●●)

b) ●●●●
●●●●

2. Sketch a picture to show how you would model each multiplication using counters.

a) (–2) × 3

b) 5 × (–5)

3. Calculate.

a) 4 × (–1)

b) 3 × (–2)

c) 2 × (–3)

4. Is it possible to multiply a negative integer by a positive integer and get a positive product? Explain your thinking.

5. Calculate.

a) (–4) × (–2)

b) (–3) × 0

c) (–4) × (–3)

d) (–5) × (–4)

6. The temperature fell 5°C each hour for 3 h.

a) Write an integer multiplication to represent the temperature change.

b) What was the temperature change?

7. Use counters, and the idea that a negative is like subtracting its opposite from 0 (–4 = 0 – 4) to show why this is true:

–4 × (–3) = 0 – 4 × (–3) = +12

Explain what you did.

8. Is it possible for the product of two consecutive integers to be negative? Explain your thinking.

9. Why might you use the commutative property to multiply (–4) × 5?

3.1.2 Multiplying Integers Using a Number Line

Try This

Sonam had Nu 500. She spent Nu 40 each day.

A. How much money did she have left

i) after 1 day? **ii)** after 2 days?

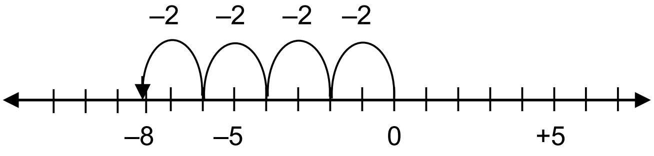
iii) after 3 days? **iv)** after 12 days?

v) after 13 days?

- You can use a number line to model the product of a positive integer and a negative integer.

For example, to multiply 4 × (–2):

4 × (–2) is 4 jumps of –2, starting at 0 and ending at –8.



4 × (–2) = –8

Jumps to the left are negative jumps.

B. Write an expression and use a number line to show how much money Sonam has left after 13 days.

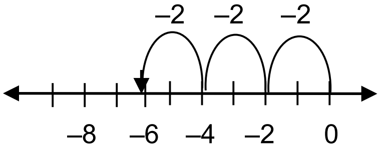
Examples

Example Multiplying Integers with Different Signs

Use a number line to model and calculate each. **a)** (–2) × 3 **b)** 3 × 4 × (–2)

Solution

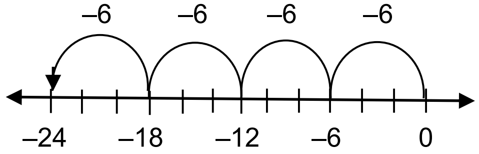
a) (–2) × 3 → 3 × (–2)



(–2) × 3 = –6

b) 3 × 4 × (–2) → 3 × (–2) × 4

3 × (–2) = –6 and (–6) × 4 → 4 × (–6)



4 × (–6) = –24

3 × (–2) × 4 = –24

Thinking

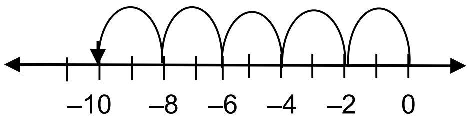
a) I changed the order of (-2) × 3 to 3 × (-2) so I could make 3 jumps of -2 from 0.

b) I first multiplied the first and third integers because I knew the answer was -6 from **part a)**.

- I changed (–6) × 4 to 4 × (-6) so I could make 4 jumps of -6 from 0.

Practising and Applying

1. Write the multiplication expression represented by this model.



2. Multiply. Use a number line to show each solution. Sketch your work.

a) (–4) × 2

b) 3 × (–3)

3. Karma spends Nu 10 each day.
At the end of 13 days, he has Nu 55.

a) Write an integer expression to show how much money Karma started with.

b) Calculate the value of the expression.

c) If Karma spends Nu 20 each day, how much money will he have at the end of 13 days? Use positive and negative integers to explain your answer.

4. How many different ways are there to express –12 as a product of three integers? Show your work.

(Hint: (–1) × (–2) × (–6) = –12 and (–2) × (–1) × (–6) = –12 are the same multiplication and only count as one way.)

5. Without multiplying, predict whether each product will be negative or positive. Explain your prediction.

a) (–1245) × (–2678)

b) (–837) × (–672) × (–7782)

c) (–733) × (–1355) × 267

d) (–64) × (–467) × (–222) × (–535)

6. The temperature in Paro fell 2°C each hour for 6 h. What was the total change in temperature?

7. What happens when you multiply any integer except 0 by –1?

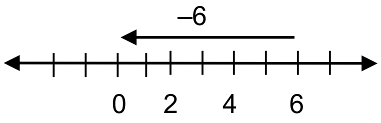
8. Pema is trying to figure out why
(–5) × (–2) = +10.

He knows these three things:

- 5 + (–5) = 0
- [5 + (–5)] × (–2) =
  [5 × (–2)] + [(–5) × (–2)]
- 5 × (–2) = –10

How can Pema use what he knows to figure out that (–5) × (–2) = +10?

9. You can think of –6 as 0 – 6, because –6 is the distance from 6 to 0 on a number line.



If –6 = 0 – 6, then

(–6) × (–2) = 0 – 6 × (–2).

Use the idea above and a number line to show why it makes sense that (–6) × (–2) has a positive product:

(–6) × (–2) = +12

10. Prem says that he ignores the signs when he multiplies integers. He multiplies them as if they are whole numbers and then makes sure the product follows these rules:

- positive × positive = positive
- negative × negative = positive
- positive × negative = negative

Do you agree? Explain using examples.

11. Predict the sign of each.

a) the product of an even number of negative integers

b) the product of an odd number of negative integers

3.1.3 EXPLORE: Pattern Grids

You can make a pattern grid by following a set of rules.
For example:

- Start the pattern with the bottom left number (–4).
- Multiply by –2 when you move to the right.
- Multiply by 3 when you move up.

× 3 ↑

-12			
-4	8		

→ × (–2)

A. i) Copy and complete the grid above.
ii) What patterns do you notice?
iii) Explain why the patterns occur.

B. Using the same rules as above, copy and complete this grid.

	6		

C. The rules and many of the integers for this grid are missing.

		64	
	-8		
1			-64

i) Create a set of pattern rules that use integer multiplication.
ii) Copy and complete the grid using your rules.
iii) Is there more than one possible set of rules? Explain your thinking.
iv) Which original integers in this grid are not needed to figure out the pattern? Explain your thinking.

3.1.4 Renaming Factors to Multiply Mentally

Try This

Devika is finding two products: 35 × (–42) and 70 × (–21).
She says that one product is easier to calculate mentally than the other.

A. Which calculation do you think she feels is easier? Explain your thinking.

You can use different strategies for multiplying integers mentally.

- One strategy is to rearrange the factors so they are easy to multiply mentally.

For example:

20 × 9 × (–5) = [20 × (–5)] × 9 [rearrange the factors to multiply 20 × (–5)]
= –100 × 9 [negative × positive = negative]
= –900

Numbers that are easy to multiply, like 20 × (–5), are called *compatible factors*.

- Sometimes you can break up factors to create compatible factors.

For example:

250 × (–40) = [25 × 10] × [4 × (–10)] [250 = 25 × 10 and (–40) = 4 × (–10)]
= [25 × 4] × [10 × (–10)] [rearrange the factors]
= 100 × (–100) [positive × negative = negative]
= –10,000

- You can break up factors and rearrange them in order to double one factor and take half of the other. This is called *doubling and halving*.

For example:

(–35) × 82 = (–35) × 2 × 41 [82 = 2 × 41]
= [(–35) × 2] × 41 [rearrange the factors]
= (–70) × 41
= 7 × (–10) × 41 [(–70) = 7 × (–10)]
= [7 × 41] × (–10) [rearrange the factors]
= 287 × (–10) [positive × negative = negative]
= –2870

- You can also *work by parts* to multiply mentally. Write one factor as a sum of two parts, multiply each part, and then add the products.

For example:

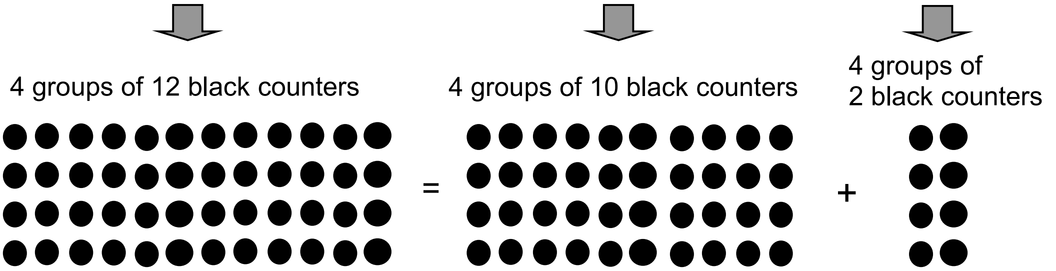
4 × –36 = 4 × [(–30) + (–6)] = [4 × (–30)] + [4 × (–6)]
= (–120) + (–24)
= –144

[Continued]

The model below shows how to work by parts:

4 × (–12) = 4 × [(–10) + (–2)] = [4 × (–10)] + [4 × (–2)]

4 × (–12) = [4 × (–10)] + [4 × (–2)]



4 × (–12) = (–40) + (–8) = –48

Working by parts uses the **distributive property**, which allows you to multiply an integer by either the **sum** of or the **difference** between two other integers.

B. Use doubling and halving to calculate 35 × (–42) from **part A**. What do you notice?

Examples

Example Multiplying Integers Using Mental Math Strategies

Multiply mentally. **a)** (–5) × (–36) **b)** (–42) × 11 **c)** 25 × (–68) × (–4)

Solution	**Thinking**
a) (–5) × (–36) = (–10) × (–18) = 180	 **a)** I doubled and halved. I know a negative times a negative is positive.
b) (–42) × 11 = (–42) × 10 + (–42) × 1 = (–420) + (–42) = –462	**b)** I wrote 11 as (10 + 1) and then worked in parts. I know a negative times a positive is negative.
c) 25 × (–68) × (–4) = [25 × (–4)] × (–68) = (–100) × (–68) = 6800	**c)** I knew 25 × (-4) was -100, so I first multiplied those two factors.

Practising and Applying

Multiply mentally.

1. Use doubling and halving.

a) 18 × (–25) **b)** (–15) × (–64)

c) (–28) × (–5) **d)** (–4) × 35

e) (–20) × 24

2. Use compatible factors.

a) 25 × (–3) × (–4) **b)** (–5) × (–8) × 6

c) (–2) × 6 × (–5) **d)** (–4) × (–5) × (–4)

e) (–2) × 3 × (–10)

3. Work by parts.

a) 5 × (–23) **b)** (–24) × (–8)

c) (–12) × 31 **d)** (–4) × (–27)

e) (–11) × 52

4. Multiply. Explain your strategy for two of the calculations.

a) 11 × (–3) **b)** (–5) × (–16)

c) (–10) × 6 × (–5) **d)** (–2) × 3 × (–50)

e) (–25) × (–31) × (–4)

GAME: Order the Integers

Play in a group of 2 to 4. You need to make 42 integer cards from –10 to 10 (two of each). The goal of the game is to get the greatest product after three rounds.

Choose a player to shuffle the cards and deal five cards face up in a row to each player. You cannot rearrange your cards once they have been dealt.

Each player plays all three rounds with his or her own cards.

Round 1

- Select an integer that is not the first or last card. Multiply the integer by the integers to its left and to its right.
- Record the product as your score. Remove the card.

Round 2

- Select an integer from the remaining cards that is not the first or last card. Multiply the integer by the integers to its left and to its right.
- Record the product as the score. Remove the card.

Round 3

- Multiply the three integers on the last three cards.
- Record the product as the score.

Final score

- Find the total of your three scores. The player with the greatest score wins.

For example:

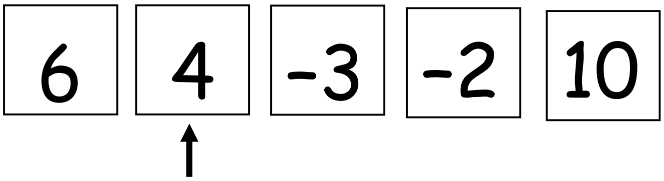
Player A gets these cards.

Round 1

Player A selects 4.

Score: 6 × 4 × (–3) = –72

Player A removes the 4 card.

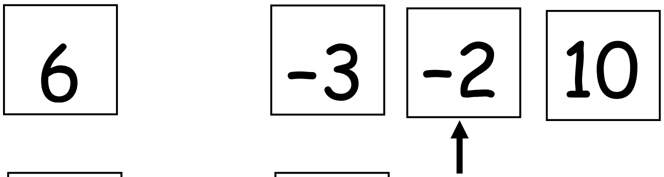


Round 2

Player A selects –2.

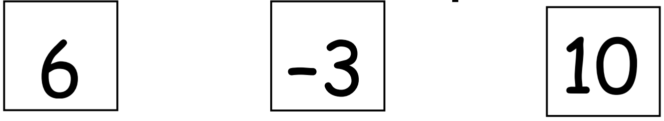
Score: (–3) × (–2) × 10 = 60

Player A removes the –2 card.



Round 3

Score: 6 × (–3) × 10 = –180



Final score for Player A: (–72) + 60 + (–180) = –192

Note that Player A could have selected the cards in a different order.
If he had selected –3, –2, and then 4, his score would have been much higher:

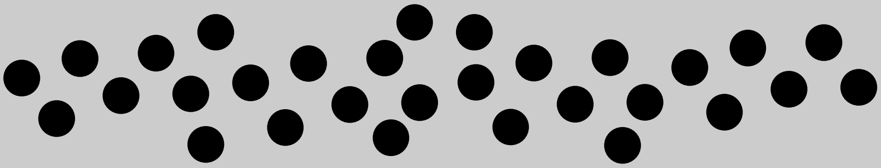
$$\left.\begin{array}{l} 4 \times (-3) \times (-2) = 24 \\ 4 \times (-2) \times 10 = -80 \\ 6 \times 4 \times 10 = 240 \end{array}\right\} \quad \text{Final score: } 24 + (-80) + 240 = 184$$

Chapter 2 Dividing Integers

3.2.1 Dividing Integers Using Models and Patterns

Try This

Seldon wants to arrange these 30 black counters into equal groups.



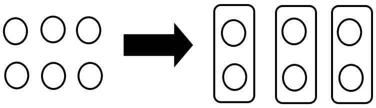
A. i) How many groups will there be if she puts 2 counters in each group?
ii) How many counters will be in each group if she shares them with 5 classmates?

- Dividing a negative integer by a negative integer is like dividing whole numbers.

For example:

You can think of 6 ÷ 2 = ? in this way:

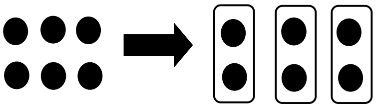
"If 6 items are divided into groups of 2, how many groups are there?"



There are 3 groups of 2 in 6, so 6 ÷ 2 = 3.

You can think of (–6) ÷ (–2) = ? in the same way:

"If –6 items are divided into groups of –2, how many groups are there?"



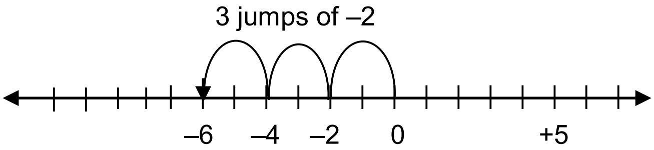
There are 3 groups of –2 in –6, so (–6) ÷ (–2) = 3.

- You can use a number line to model dividing a negative integer by a negative integer.

For example:

Think of (–6) ÷ (–2) = ? in this way:

"To go from 0 to –6 in jumps of –2, how many jumps do I need?"



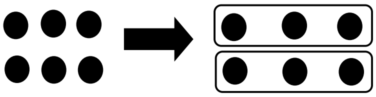
(–6) ÷ (–2) = 3

- To divide a negative integer by a positive integer, you can also use counters or a number line.

For example, using counters:

Think of (–6) ÷ 2 = ? in this way:

"If –6 is shared by 2 people, how many are in each share?"

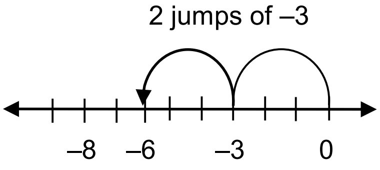


There are 3 black counters in each share, so (–6) ÷ 2 = –3.

For example, using a number line:

Think of (–6) ÷ 2 = ? in this way:

"To go from 0 to –6 in 2 equal jumps, what size jumps do I need?"



(–6) ÷ 2 = –3

- To divide a positive integer by a negative integer, you can use a pattern. (See **Example 3** on **page 64**.)

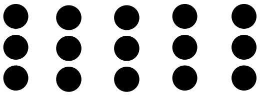
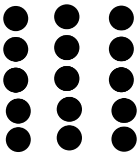
B. Use integers to represent each calculation in **part A**.

Examples

Example 1 Dividing Integers Using Counters

Use counters to model and calculate each.

a) (–15) ÷ (–3) **b)** (–15) ÷ 3

Solution	**Thinking**
a)  (–15) ÷ (–3) = 5	 **a)** I thought of (-15) ÷ (-3) as "If -15 is divided into groups of -3, how many groups are there?" • I arranged 15 black counters into groups of -3. There were 5 groups.
b)  (–15) ÷ 3 = –5	**b)** I thought of (-15) ÷ -3 as "If -15 is shared among 3 people, how many are in each person's share?" • I arranged 15 black counters into 3 groups. There were -5 in each group.

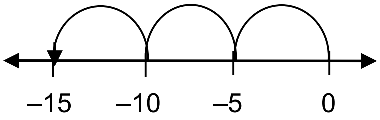
Example 2 Dividing Integers Using a Number Line

Use a number line to model and calculate each.

a) (–15) ÷ 3 **b)** (–15) ÷ (–3)

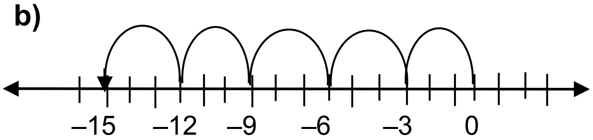
Solution

a)



(–15) ÷ 3 = –5

b)



(–15) ÷ (–3) = 5

Thinking

a) To get from 0 to -15 in 3 equal jumps, each jump had to be -5.

b) To get from 0 to -15 in jumps of -3, I had to make 5 jumps.

Example 3 Dividing a Positive Integer by a Negative Integer Using a Pattern

a) Complete the pattern.

(–12) ÷ (–4) = ■
(–8) ÷ (–4) = ■
(–4) ÷ (–4) = ■
0 ÷ (–4) = ■
4 ÷ (–4) = ■
8 ÷ (–4) = ■
12 ÷ (–4) = ■

b) Use what you learned in **part a)** to calculate 15 ÷ (–3). Explain your thinking.

Solution

a)

Dividend increases by 4 ↓

(–12) ÷ (–4) = 3
(–8) ÷ (–4) = 2
(–4) ÷ (–4) = 1
0 ÷ (–4) = 0
4 ÷ (–4) = –1
8 ÷ (–4) = –2
12 ÷ (–4) = –3

Quotient decreases by 1 ↓

b) 15 ÷ (–3) = –5

Since 15 ÷ 3 is 5 and a positive integer divided by a negative integer is negative, then 15 ÷ (–3) = –5.

Thinking

a) For the first three calculations, I thought of dividing sets of black counters into groups of 4 black counters. The quotient was the number of groups.

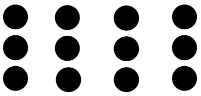
- It made sense that 0 ÷ (-4) = 0 because 0 counters in groups of -4 is 0 groups.
- Once I had the first four calculations, I saw a pattern. I used it to finish the calculations.

b) I noticed, in the last three calculations of **part a)**, that a positive integer divided by a negative integer had a negative quotient.

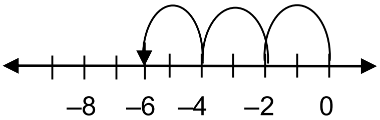
Practising and Applying

1. Write a division expression for each.

a)



b)



2. Model each division two ways:

- using counters
- using a number line

a) (–9) ÷ (–3)
b) (–8) ÷ 4

3. Divide each using a model. Then, sketch each model.

a) (–12) ÷ 6
b) (–8) ÷ (–4)

4. Divide.

a) 18 ÷ (–3)
b) (–64) ÷ (–8)
c) (–12) ÷ 6
d) (–4) ÷ (–2)
e) (–12) ÷ 3
f) 72 ÷ (–9)

5. In division, the first number is called the dividend, the second number is the divisor, and the result is the quotient.

A ÷ B = C

Dividend Divisor Quotient

Suppose the dividend in an integer division always stays the same. What happens to the quotient when you increase the divisor in each situation below?

a) The dividend is positive and
  i) the divisor is positive
  ii) the divisor begins as a negative integer

b) The dividend is negative and
  i) the divisor is positive
  ii) the divisor begins as a negative integer

6. The temperature in Paro was 4°C. It fell 2°C each hour until it was –14°C. Over how many hours did the temperature fall?

The Rinpung Dzong in Paro

7. Use a number pattern to show why 21 ÷ (–7) has a negative quotient.

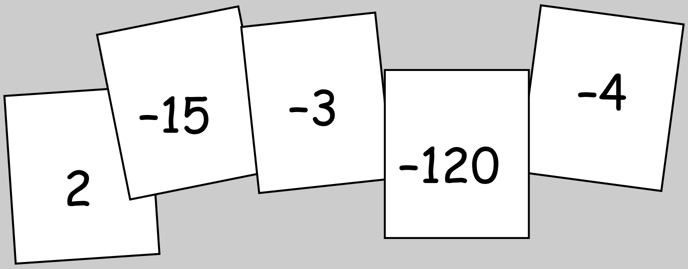
8. What do you notice about the sign of the quotient when you divide integers in each case below?

a) positive ÷ positive
b) negative ÷ negative
c) positive ÷ negative
d) negative ÷ positive

3.2.2 Relating Division of Integers to Multiplication

Try This

Lhamo is playing a game. She has been dealt five integer cards. She must divide one of the integers by another integer to get the greatest quotient possible.



A. Which two integers should she choose? Why?

- You can write an integer division as a multiplication with a missing factor because of the relationship between the two operations.

A ÷ B = ? → B × ? = A

That means you can divide integers using the related multiplication.

For example:

12 ÷ (–3) = ■ → ■ × (–3) = 12

Since **(–4)** × (–3) = 12, then 12 ÷ (–3) = **–4**.

- Negative × Negative = Positive
- Positive × Positive = Positive
- Negative × Positive = Negative
- Positive × Negative = Negative

- You can use what you have learned about the sign of the product of an integer multiplication to predict the sign of the **quotient** of an integer division.

For example:

12 ÷ (–3) = ■ → (–3) × ■ = 12

Since a negative times a negative is positive, the missing factor must be positive.

(–15) ÷ (–3) = ■ → ■ × (–3) = –15

Since a positive times a negative is negative, the missing factor must be positive.

- To divide integers, you can ignore the signs and divide them like whole numbers. Then, you follow these rules to determine the sign of the quotient:

- Negative ÷ Negative = Positive
- Positive ÷ Positive = Positive
- Negative ÷ Positive = Negative
- Positive ÷ Negative = Negative

B. How did you use the rules for the sign of the quotient to help you decide which pair of integers to divide in **part A**?

Examples

Example 1 Dividing Integers Using Missing Factors

Write a multiplication for each division. Use it to calculate the quotient.

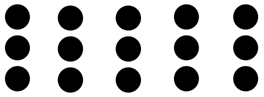
a) 15 ÷ 3 **b)** (–15) ÷ (–3) **c)** (–15) ÷ 3

Solution

a) 15 ÷ 3 = ■ → ■ × 3 = 15

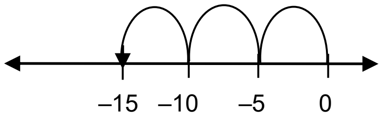
Since **5** × 3 = 15, then 15 ÷ 3 = **5**.

b) (–15) ÷ (–3) = ■ → ■ × (–3) = –15



Since **5** × (–3) = –15,
then (–15) ÷ (–3) = **5**.

c) (–15) ÷ 3 = ■ → 3 × ■ = –15



Since 3 × (**–5**) = –15,
then (–15) ÷ 3 = **–5**.

Thinking

a) I knew that the missing factor was the quotient.

b) I modelled and solved the multiplication by using 5 groups of 3 black counters to make 15 black counters.

c) I modelled and solved the multiplication by making 3 jumps of -5 on a number line to get from 0 to -15.

Example 2 Dividing Integers Using Rules for the Product

Calculate (–16) ÷ 8.

Solution

(–16) ÷ 8 = ■
Since 16 ÷ 8 = 2 and
negative ÷ positive = negative,
(–16) ÷ 8 = –2.

Thinking

- I ignored the signs and divided 16 by 8. I remembered to make sure the quotient had the correct sign.

Practising and Applying

1. Calculate.

a) (–10) ÷ 5 **b)** (–34) ÷ (+2)

c) 35 ÷ (–7) **d)** (–100) ÷ (–4)

2. Write a multiplication for each.

a) (–64) ÷ 4 **b)** (–84) ÷ (–7)

3. Calculate each quotient in **question 2**.

4. The temperature in Bumthang was 0°C. It fell 1°C every 2 h until it was –8°C. Over how many hours did the temperature fall?

5. How do you know (–24) ÷ 6 = –4?

6. Lobzang is playing a game.

- He has been dealt these five integers:

–3, 5, –12, –60, 4

- And, he must divide one integer by another integer.

a) What is the greatest quotient possible?

b) What is the least integer quotient?

7. Divide each integer below by another integer so that the quotient is negative. Write two possible divisors for each.

a) 6 **b)** –12 **c)** 18 **d)** –10

8. a) Use both clues to find two integers:

- If you divide the first integer by the second integer, the quotient is –1.
- If you divide the first integer by an integer that is 2 greater than the second integer, the quotient is +1.

b) Explain how you found the answer.

9. How does knowing (–32) ÷ 8 = –4 help you find 32 ÷ (–8)?

CONNECTIONS: Mean Temperatures

Growing seasons in Bhutan are determined by rainfall, altitude, and temperature.

- To predict the best dates for sowing seeds, growers use the mean low temperature.
- Here is how to calculate the mean low temperature for a week:
  - Find the sum of the low temperatures for each day the week.
  - Divide the sum by 7 (the number of days).

You calculate the mean high temperature in the same way, using high temperatures instead of low temperatures.

Daily Temperatures (January)

Day	High	Low
Monday	13°C	–5°C
Tuesday	12°C	–3°C
Wednesday	12°C	–3°C
Thursday	10°C	–4°C
Friday	13°C	–4°C
Saturday	9°C	–1°C
Sunday	8°C	–1°C

Use the information in the chart above to answer these questions.

1. a) Calculate the mean high temperature for the week.

b) Calculate the mean low temperature for the week.

2. It is safe to plant maize when the mean low temperature is greater than 9°C.

a) By how many degrees does the mean low temperature need to increase before maize can be planted?

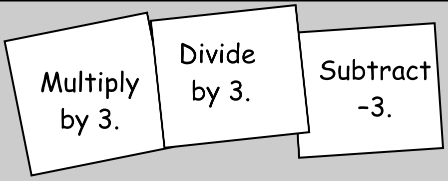
b) Suppose the mean low temperature increases by 1°C every 2 weeks. After how many weeks will it be safe to plant maize?

Reprint 2024

3.2.3 Order of Operations with Integers

Try This

Devika is playing a game. She starts with a score of 3 and three cards. She can use the cards in any order to increase her score as much as possible.



A. In what order should Devika use her three cards?

- It can be hard to interpret an **expression** that has many different calculations.

For example, Maya and Namgay interpreted this expression in different ways:

(–3) – (–5) + (–2) × (–4)

- Maya subtracted (–3) – (–5) = 2, then added the 2 to (–2), 2 + (–2) = 0, and then multiplied the 0 by (–4), 0 × (–4) = 0. Her answer was 0.
- Namgay multiplied (–2) × (–4) = 8, then added the 8 to (–5), (–5) + 8 = 3, and then subtracted the 3 from (–3), (–3) – 3 = –6. His answer was –6.

It is confusing to have more than one answer for the same calculation.

- To get rid of the confusion, people have agreed upon rules for calculating called the **order of operations** rules:

Step 1 If there are *Brackets*, first calculate anything inside them.

Step 2 *Divide* and *Multiply* numbers next to each other, in order *from left to right*.

Step 3 *Add* and *Subtract* numbers next to each other, in order *from left to right*.

For example: (–3) – [(–5) + (–2)] × (–4)
Step 1 (–3) – (**–7**) × (–4) [Add inside brackets: (–5) + (–2) = **–7**]
Step 2 = (–3) – **28** [Multiply: (–7) × (–4) = **28**]
Step 3 = **–31** [Subtract: (–3) – 28 = **–31**]

- The two different types of brackets in the expression above, square brackets and round brackets, make it easier to interpret the expression.

If you write (–3) – **[**(–5) + (–2)**]** × (–4) as (–3) – **(**(–5) + (–2)**)** × (–4), it is difficult to tell which brackets belong together.

- If you calculate the expression at the top, (–3) – (–5) + (–2) × (–4), using the order of operations rules, the answer is 10.

(–3) – (–5) + (–2) × (–4)
= (–3) – (–5) + **8** [Multiply: (–2) × (–4) = **8**]
= **2** + 8 [Subtract: (–3) – (–5) = **2**]
= **10** [Add: 2 + 8 = **10**]

B. Devika wrote the expression 3 ÷ 3 – (–3) × 3 to show the order in which she used the three cards in **part A**.

i) If she uses the order of operations rules to calculate her expression, will she still get the greatest possible score? Why?

ii) How should she write the expression?

Examples

Example 1 Applying the Order of Operations Rules	
Calculate. **a)** (–5) + (–4) × (–3) ÷ 2	**b)** –4 + (–2) × [(–5) + 3]
Solution	**Thinking**
a) (–5) + (–4) × (–3) ÷ 2 = (–5) + **12** ÷ 2 = (–5) + **6** = **1**	**a)** I first multiplied and divided from left to right. • Then I added. 
b) –4 + (–2) × [(–5) + 3] = –4 + (–2) × (**–2**) = –4 + (**+4**) = **0**	**b)** I first did the addition in the brackets. • Then I multiplied. • Then I added.

Example 2 Using the Order of Operations Rules to Solve a Problem	
Complete the expression with operation signs: 43 ■ [3 ■ 12] ■ (–7) = 14	
Solution	**Thinking**
$43 \blacksquare \underbrace{[3 \blacksquare 12] \blacksquare (-7)} = 14$ 43 **–** 29 = 14 $\underbrace{[3 \blacksquare 12]} \blacksquare (-7) = 29$ 36 + (–7) = 29 3 ■ 12 = 36 → 3 **×** 12 = 36 43 **–** [3 **×** 12] ■ (–7) = 14 43 **–** [3 **×** 12] **–** (–7) = 14	 • I knew that 43 - 29 = 14, so that meant the first operation was subtraction and [3 ■ 12] ■ (-7) = 29. • I knew that 36 + (-7) = 29, so that meant 3 ■ 12 was 3 × 12 and, because the whole expression, [3 × 12] + (-7), had to be subtracted from 43, each part needed a negative sign, -[3 x 12] and -(-7). • I realize that the square brackets in 43 - [3 × 12] - (-7) = 14 aren't really necessary, since you would do the multiplication first anyway.

Practising and Applying

1. Calculate.

a) (–6) + (–4) × (–3) ÷ 2

b) 6 ÷ (–3) + [(4 – (–5)) × (–7)]

c) (–2) + (–3) × [(–8) + 4]

d) (–6) ÷ (–3) – [(–8) ÷ (–2)]

e) 2 × (–3) – (–7) × 2 + (–5) × 0

f) (–3) + (–6) × (–5) ÷ 2

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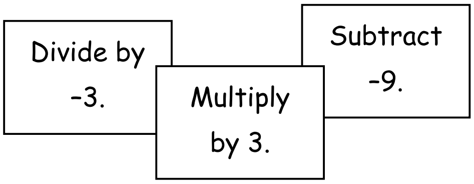
2. Therchung is playing Devika's game. He starts with a score of 0 and can use these three cards in any order.



In what order should he play his cards to get the greatest possible score? Write an expression to show this.

3. Add brackets to 40 × 6 – 3 × 4 – 5 to get the least possible answer.

4. Copy and complete each equation with operation signs to make each true.

a) 36 ■ [4 ■ 1] ■ 2 = 30

b) (–12) ■ 4 ■ (–3) = –24

5. Why is it important to follow the order of operations rules?

GAME: Target

Play in a group of 2 to 4.

You need to make 42 integer cards, –10 to 10 (two of each).

The goal is to create an expression that is as close as possible to a given integer.

- For each round, deal five integer cards to each player. Place the remaining cards in a pile with the top card facing up. This is the target card.

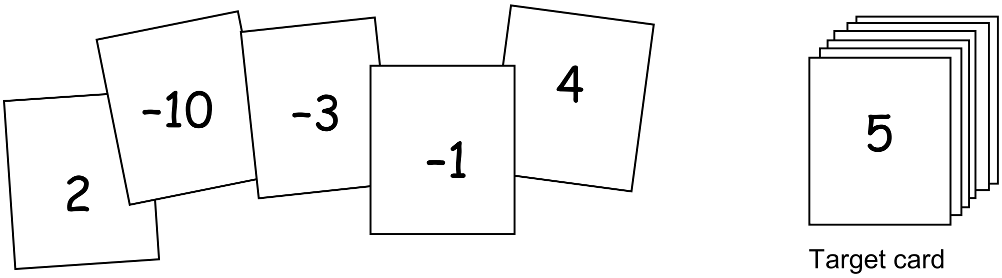
Each player should do this:

- Create an expression using all five integers that has a value as close as possible to the value on the target card.
- Calculate your expression.

- You receive a score equal to the positive difference between the value of your expression and the value of the target card. An exact match gets 0 points.
- The player with the lowest score after 10 rounds wins.

For example:

Player A has been dealt these cards. The target integer is 5.



Target card

Player A creates the following expression:

(–10) ÷ 2 × (–1) + (–3) + 4 = 6

Player A's score for this round is 1 because 6 – 5 = 1.

UNIT 3 Revision

1. Multiply each expression by modelling with counters or a number line. Then, sketch your model.

a) (–3) × 2 **b)** (–3) × (–3)

2. Multiply.

a) (–5) × (–6)

b) (–6) × 8

c) 4 × (–7)

3. Can the product of a negative integer and a positive integer be positive? Explain your thinking.

4. How many different ways are there to express –18 as a product of three integers? Show your work.

■ × ■ × ■ = –18

5. Why might you use the commutative property first before modelling (–3) × 5?

6. Multiply mentally. Explain your strategy for each.

a) (–50) × (–46)

b) (–26) × 110

c) (–5) × (–67) × 2

7. Divide each expression by modelling in two ways:

- using counters
- using a number line

Then, sketch your models.

a) (–6) ÷ 2 **b)** (–8) ÷ (–4)

8. Write a related multiplication for each division. Use it to calculate the quotient.

a) (–92) ÷ 4 **b)** (–91) ÷ (–7)

9. The temperature in Paro started at 3°C. It fell 2°C every hour until it was –5°C. Over how many hours did the temperature fall?

10. The monthly mean low temperatures were recorded for Bumthang.

January	–5°C	**July**	12°C
February	–1°C	**August**	15°C
March	4°C	**September**	13°C
April	5°C	**October**	6°C
May	10°C	**November**	1°C
June	14°C	**December**	–2°C

This is how to calculate the mean low temperature for the year:

- Find the sum of the 12 monthly low temperatures.
- Divide the sum by 12 (the number of months).

Find the mean low temperature for the year.

11. How do you know (–4) ÷ 2 = –2?

12. Without calculating, predict whether each answer is negative or positive. Explain your prediction.

a) (–446) × (–9087)

b) (–935) × (–279) × (–5481)

c) (–8528) ÷ (–164)

d) (–5022) ÷ 279

13. Calculate.

a) 7 × [(–3) – (–5)] × 8

b) 10 + (–4) – 7 × 5

c) [(–14) + (–23)] – [((–17) – 2) × 10]

d) [(–6) + (–10)] ÷ [(–4) × 2]

e) [49 ÷ (–7)] ÷ [1 + (2 × 3)]

14. Add brackets to the expression below to get the greatest possible answer.

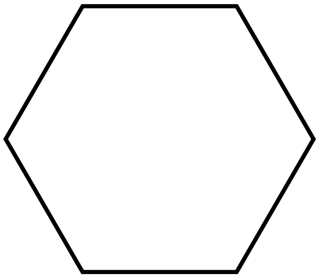
40 × 6 – 3 × 4 – 5

UNIT 4 FRACTIONS AND RATIONAL NUMBERS

Getting Started

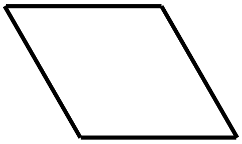
Use What You Know

Examine these four shapes:

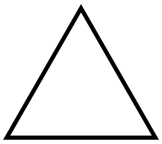


Hexagon

Trapezoid



Rhombus



Triangle

A. The hexagon represents one whole, or 1. What fraction does each shape represent?

i) trapezoid **ii)** rhombus **iii)** triangle

B. Write a multiplication equation to match each sentence below.

For example, if two trapezoids equal one hexagon, then write $2 \times \frac{1}{2} = 1$.

i) Three triangles equal one trapezoid.

ii) Two triangles equal one rhombus.

iii) Six triangles equal one hexagon.

C. Write a multiplication equation to represent each set of shapes below. Express the product as a whole number or as a mixed number.

For example:

seven trapezoids → $7 \times \frac{1}{2} = 3\frac{1}{2}$

i) three trapezoids **ii)** five rhombuses **iii)** seven triangles

iv) four trapezoids **v)** ten rhombuses **vi)** nine triangles

D. Describe what set of shapes each multiplication represents. Find each product.

i) $5 \times \frac{1}{2}$ **ii)** $7 \times \frac{1}{3}$ **iii)** $8 \times \frac{1}{6}$ **iv)** $7 \times \frac{1}{2}$ **v)** $11 \times \frac{1}{3}$

Skills You Will Need

1. Write three equivalent fractions for each.

a) $\frac{2}{3}$ **b)** $\frac{18}{24}$ **c)** $\frac{5}{8}$ **d)** $\frac{30}{50}$

2. Can each sum be written as a multiplication? How do you know?

a) $\frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5}$ **b)** $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10}$

3. Multiply. Express each product as a mixed number.

a) $5 \times \frac{3}{4}$ **b)** $4 \times \frac{5}{8}$ **c)** $7 \times \frac{4}{9}$

4. Subtract.

a) $\frac{5}{6} - \frac{3}{4}$ **b)** $\frac{7}{8} - \frac{1}{3}$ **c)** $5\frac{1}{2} - 1\frac{3}{8}$

d) $3 - 1\frac{4}{5}$ **e)** $6\frac{2}{3} - 3\frac{3}{4}$ **f)** $4\frac{1}{4} - 2\frac{2}{3}$

5. A recipe calls for $\frac{3}{4}$ cup of rice. How much rice is needed to triple the recipe?

6. Add.

a) –25 + (+35) **b)** 250 + 50

c) –32 + (–47) **d)** 32 + (–47)

7. Subtract.

a) –123 – (+145) **b)** 89 – (–47) **c)** –185 – (–75)

8. Multiply.

a) –8 × (–12) **b)** –14 × 9 **c)** 6 × (–15)

9. How do you know that –12 ÷ 6 = –2?

10. Calculate.

a) 9 × [(–5) – (–7)] × 10 **b)** 20 + (–7) – 11 × 6

c) [–17 + (–19)] ÷ [(–3) × 4] **d)** [72 ÷ (–8)] ÷ [1 + (4 × 2)]

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Chapter 1 Adding and Subtracting Fractions

4.1.1 Adding and Subtracting Fractions Mentally

Try This

Pema cuts two chocolate bars of the same size in different ways:

- He cuts the first bar into four equal pieces.
- He cuts the second bar into eight equal pieces.

Then he eats one piece of the first bar and three pieces of the second bar.

A. i) Draw a diagram to show what fraction of a whole bar Pema ate altogether.

ii) What fraction of a whole bar did Pema eat?

- You can use mental math to find the answer to a fraction addition or subtraction, if the **denominators** are the same.

For example, this is how to add $\frac{1}{8} + \frac{3}{8}$:

Think about adding 1 item to 3 of the same item: 1 eighth + 3 eighths = 4 eighths

$$\frac{1}{8} + \frac{3}{8} = \frac{4}{8}$$

- You can often add or subtract fractions mentally when one denominator is a **multiple** of the other because it is easy to find an equivalent fraction with the same denominator.

For example, this is how to subtract $\frac{5}{8} - \frac{1}{2}$ (notice that 8 is a multiple of 2):

Since $\frac{1}{2} = \frac{4}{8}$, then $\frac{5}{8} - \frac{1}{2} = \frac{5}{8} - \frac{4}{8} = \frac{1}{8}$ [5 eighths – 4 eighths = 1 eighth]

$$\frac{5}{8} - \frac{1}{2} = \frac{1}{8}$$

- Even when it is not easy to calculate mentally, you can often estimate mentally. Sometimes an estimate is all you need.

For example, this is how to estimate $\frac{12}{13} + \frac{7}{8}$:

$\frac{12}{13}$ and $\frac{7}{8}$ are both close to 1, so $\frac{12}{13} + \frac{7}{8} \approx 1 + 1 = 2$.

$$\frac{12}{13} + \frac{7}{8} \approx 2$$

B. Explain how you could find the answer to **part A ii)** mentally.

Examples

Example 1 Adding and Subtracting Fractions Mentally

Lhamo spent $\frac{3}{4}$ h on homework before dinner and $\frac{1}{2}$ h after dinner.

a) How much time did she spend on homework altogether?

b) How much more time did she spend on homework before dinner than after dinner?

Solution	**Thinking**
a) $\frac{3}{4} + \frac{1}{2} = \frac{5}{4} = 1\frac{1}{4}$ She spent $1\frac{1}{4}$ h on homework.	 **a)** I knew $\frac{1}{2} = \frac{2}{4}$. • I added 3 fourths + 2 fourths in my head.
b) $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$ She spent $\frac{1}{4}$ h more on homework before dinner.	**b)** I used $\frac{1}{2} = \frac{2}{4}$ again. • I subtracted 3 fourths – 2 fourths in my head.

Example 2 Estimating Sums and Differences of Mixed Numbers

Recipe for Fruit Punch

$2\frac{2}{3}$ cups orange juice

3 cups pineapple juice

$2\frac{1}{4}$ cups water

$4\frac{3}{4}$ cups ginger ale

a) i) About how much more ginger ale than orange juice is in the recipe?

ii) Without calculating, explain how your estimate compares with the exact answer.

b) Is a 15-cup punch bowl big enough to hold all the punch? Explain your thinking.

Solution	**Thinking**
a) i) There are about 2 cups more ginger ale than orange juice.	 **a) i)** I knew $4\frac{3}{4}$ cups was about 5 cups, and $2\frac{2}{3}$ cups was about 3 cups.
ii) The estimate is a bit low since $4\frac{3}{4}$ is closer to 5 cups than $2\frac{2}{3}$ is to 3 cups.	**ii)** I subtracted 5 – 3 = 2 to estimate.
b) A 15-cup punch bowl is big enough. There are 11 whole cups plus 3 fraction parts, each less than 1.	**b)** I added the whole number parts of cups: 2 + 3 + 2 + 4 = 11 • I realized I didn't need an exact answer to decide whether the bowl was big enough because each fraction part was less than 1 cup, and 11 + 3 < 15.

Practising and Applying

1. Add using mental math.

a) $\frac{4}{9} + \frac{3}{9}$

b) $\frac{1}{2} + \frac{3}{8}$

c) $5\frac{1}{12} + 3\frac{5}{12}$

d) $7\frac{1}{4} + 9\frac{3}{8}$

2. Subtract using mental math.

a) $\frac{7}{12} - \frac{5}{12}$

b) $\frac{3}{4} - \frac{3}{8}$

c) $8\frac{5}{9} - 6\frac{1}{9}$

d) $7\frac{13}{16} - 4\frac{3}{4}$

3. Choose one part from **question 1** and one part from **question 2**. Explain how you would find each answer mentally.

4. a) Estimate.

i) $2\frac{1}{8} + 4\frac{4}{5}$

ii) $4\frac{7}{8} + 1\frac{1}{10}$

iii) $3\frac{3}{4} + 2\frac{1}{8} + 2\frac{3}{5}$

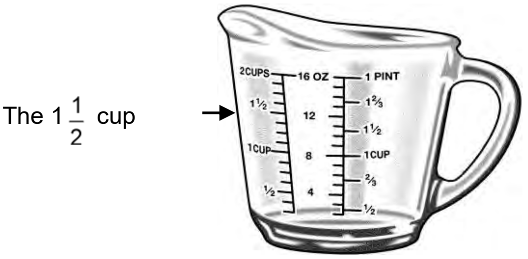
iv) $7\frac{1}{4} - 2\frac{1}{3}$

v) $4\frac{1}{2} - 1\frac{9}{10}$

vi) $\frac{11}{12} - \frac{3}{4}$

b) Choose three calculations from **part a)**. Is each estimate higher or lower than the exact answer? How do you know?

5. Chandra and his brother need $1\frac{1}{2}$ cups of rice. Chandra measured $\frac{3}{4}$ cup and his brother measured another $\frac{7}{8}$ cup. Do they need to measure more rice? How do you know?



A 2-cup measure

6. Chandra had with $3\frac{1}{3}$ cups of flour. He used $1\frac{3}{4}$ cups yesterday and needs the same amount today. Does he have enough flour? How do you know?

7. Choki says,
"The difference between a third and a fourth is a twelfth."
Is Choki right? Show how you know.

8. How much do you need to add to $\frac{1}{3}$ to make $\frac{1}{2}$? How do you know?

9. Without calculating an exact sum, decide whether $6\frac{1}{2} + 4\frac{2}{3} + 7\frac{1}{4}$ is closer to $17\frac{1}{2}$, to 18, to $18\frac{1}{2}$, or to 19. Explain how you know.

10. Give an example of each.

a) a fraction situation where you can use mental math to estimate the answer

b) a fraction calculation where you can use mental math to find an exact answer

4.1.2 Adding and Subtracting Fractions Symbolically

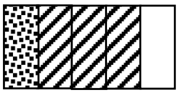
Try This

A cake recipe calls for $\frac{1}{8}$ cup of brown sugar and $\frac{2}{3}$ cup of white sugar.

A. i) Estimate how much sugar is needed altogether. Explain your estimate.
ii) Exactly how much sugar is needed altogether?

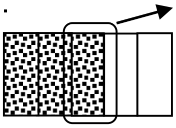
B. i) Estimate how much more white sugar than brown sugar is needed. Explain your estimate.
ii) Exactly how much more white sugar than brown sugar is needed?

- When fractions have the same denominator, you can find the sum or difference by adding or subtracting the **numerators**.

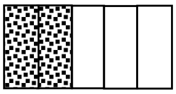


$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$

1 fifth + 3 fifths = 4 fifths



$\frac{3}{5} - \frac{1}{5}$



$\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

3 fifths – 1 fifth = 2 fifths

- To add or subtract fractions with unlike denominators, you can first find a **common denominator**.

- You can do this by finding equivalent fractions with the same denominator.

For example, this is how to add $\frac{3}{4}$ and $\frac{1}{5}$:

$\frac{3}{4} = \frac{15}{\mathbf{20}}$ and $\frac{1}{5} = \frac{4}{\mathbf{20}}$, so $\frac{3}{4} + \frac{1}{5} = \frac{15}{20} + \frac{4}{20} = \frac{19}{20}$.

- You can also find a common denominator by finding a **common multiple** of the denominators.

For example, this is how to subtract $\frac{7}{8} - \frac{5}{12}$:

Multiples of 12: 12, **24**, 36, …
Multiples of 8: 8, 16, **24**, 32, …

24 is a common multiple of 8 and 12, so 24 is a common denominator for $\frac{5}{12}$ and $\frac{7}{8}$.

Once you have a common denominator, you can create an equivalent fraction for each fraction and then subtract:

$$\overset{\times 3}{\underset{\times 3}{\frac{7}{8} = \frac{21}{24}}} \qquad \overset{\times 2}{\underset{\times 2}{\frac{5}{12} = \frac{10}{24}}} \qquad \frac{7}{8} - \frac{5}{12} = \frac{21}{24} - \frac{10}{24} = \frac{11}{24}$$

You could use any common multiple of 8 and 12 (24, 48, 72, …) as a common denominator, but the **lowest common multiple** is usually the easiest one to use.

- You should express a fraction sum or difference as an equivalent fraction in lowest terms. A fraction is in lowest terms when the numerator and denominator have no **common factor** other than 1.

For example:

$\frac{11}{24}$ is in lowest terms because 1 is the only common factor of 11 and 24.

$\frac{18}{24}$ is not in lowest terms because 18 and 24 have the common factors 1, 2, 3, 6.

- You can write a fraction in lowest terms by dividing the numerator and denominator by the **greatest common factor** of the numerator and denominator.

For example: $\underset{\div 6}{\overset{\div 6}{\frac{18}{24} = \frac{3}{4}}}$

Factors of 18: 1, 2, 3, **6**, 9, 18
Factors of 24: 1, 2, 3, 4, **6**, 8, 12, 24
The GCF of 18 and 24 is **6**.

- It is a good idea to write a fraction sum or difference that is an **improper fraction** as a mixed number because it usually makes the fraction easier to understand.

C. i) What common denominators could you have used to find the exact answers to **parts A and B**?

ii) Which common denominator do you think is best to use? Why?

Examples

Example 1 Using Common Denominators to Add/Subtract Fractions

Tandin, Pema, and Dawa worked together on a school project.

- Tandin spent $\frac{3}{4}$ h on the project and Pema spent $\frac{7}{12}$ h.
- Dawa spent $\frac{1}{4}$ h less than Pema spent.

a) How much time did Dawa spend on the project?

b) How much time did the three students spend on the project altogether?

Solution	**Thinking**
a) $\frac{7}{12} - \frac{1}{4} = ?$ $\frac{1}{4} = \frac{3}{12}$ $\frac{7}{12} - \frac{1}{4} = \frac{7}{12} - \frac{3}{12} = \frac{4}{12}$ $\frac{4}{12} = \frac{1}{3}$ Dawa spent $\frac{1}{3}$ h on the project.	 **a)** I needed to subtract $\frac{1}{4}$ from $\frac{7}{12}$ but they had different denominators. • I found an equivalent fraction for $\frac{1}{4}$ with a denominator of 12. • I subtracted the two fractions. Then I wrote the answer in lowest terms.

Example 1 Using Common Denominators to Add/Subtract Fractions [Cont'd]

Solution

b) $\frac{3}{4} + \frac{7}{12} + \frac{1}{3} = ?$

Multiples of 3: 3, 6, 9, **<u>12</u>**, 15, …

Multiples of 4: 4, 8, **<u>12</u>**, 16, ...

Multiples of 12: **<u>12</u>**, 24, 36, ...

$$\frac{3}{4} = \frac{9}{12} \qquad \frac{1}{3} = \frac{4}{12}$$

$$\frac{3}{4} + \frac{7}{12} + \frac{1}{3} = \frac{9}{12} + \frac{7}{12} + \frac{4}{12} = \frac{20}{12}$$

$$\frac{20}{12} = \frac{5}{3} = 1\frac{2}{3}$$

Altogether, the three students spent $1\frac{2}{3}$ h on the project.

Thinking

- I found the lowest common multiple of 3, 4, and 12, which was 12.
- I wrote the fractions as equivalent fractions with the common denominator. Then I added them.
- I wrote the answer in lowest terms, and then as a mixed number.

Example 2 Adding and Subtracting Mixed Numbers

A jug filled with juice holds 10 cups. Yuden poured $3\frac{1}{3}$ cups of juice from the jug.

Yangchen then poured another $2\frac{3}{4}$ cups of juice from the jug.

a) How much juice was poured from the jug altogether?

b) How much juice is left in the jug?

Solution

a) $3\frac{1}{3} + 2\frac{3}{4} = ?$

$3 \times 4 = 12$

$$\frac{1}{3} = \frac{4}{12} \qquad \frac{3}{4} = \frac{9}{12}$$

$$\begin{aligned} 3\frac{1}{3} + 2\frac{3}{4} &= 3\frac{4}{12} + 2\frac{9}{12} \\ &= 5 + \frac{13}{12} \\ &= 5 + 1\frac{1}{12} \\ &= 6\frac{1}{12} \end{aligned}$$

$6\frac{1}{12}$ cups were poured from the jug.

Thinking

a) I knew I needed to add $3\frac{1}{3}$ and $2\frac{3}{4}$.

- I multiplied the denominators to find a common multiple to use as a common denominator.
- I wrote $3\frac{1}{3}$ and $2\frac{3}{4}$ with a common denominator.
- I added the whole number parts, 3 + 2.
  Then I added the fraction parts, $\frac{4}{12} + \frac{9}{12}$.
- I changed the improper fraction to a mixed number.
- I added the whole number parts, 5 + 1, to get the final answer.

b) $10 - 6\frac{1}{12} = ?$ $10 = 9 + 1 = 9 + \frac{12}{12} = 9\frac{12}{12}$ $10 - 6\frac{1}{12} = 9\frac{12}{12} - 6\frac{1}{12} = 3\frac{11}{12}$ $3\frac{11}{12}$ cups are left in the jug.	**b)** I knew I needed to subtract $6\frac{1}{12}$ from 10. • There were no twelfths in 10 to subtract $\frac{1}{12}$ from so I renamed the 10 as $9\frac{12}{12}$. • I subtracted the fraction parts, $\frac{12}{12} - \frac{1}{12}$, and then the whole number parts, 9 – 6.

Practising and Applying

1. Add.

a) $\frac{3}{8} + \frac{1}{3}$

b) $\frac{4}{9} + \frac{5}{6}$

c) $2\frac{3}{4} + 7\frac{3}{10}$

d) $4\frac{5}{6} + 1\frac{2}{3} + \frac{4}{5}$

2. Subtract.

a) $\frac{5}{6} - \frac{5}{8}$

b) $\frac{2}{3} - \frac{3}{10}$

c) $5\frac{1}{3} - 3\frac{5}{8}$

d) $7 - 2\frac{3}{8}$

3.

Type of exercise	**Number of hours on**	
	Day 1	**Day 2**
Stretching	$\frac{1}{4}$ h	$\frac{1}{3}$ h
Walking	$\frac{1}{3}$ h	$\frac{1}{2}$ h
Jogging	$\frac{1}{2}$ h	$\frac{3}{4}$ h

a) How much time was spent exercising each day?

b) How much more time was spent exercising on Day 2 than on Day 1?

4. Bijoy Kumar, Arun Kumar, and Binod Chhetri ran laps around a track.

- Altogether they ran $33\frac{1}{2}$ laps.
- Bijoy Kumar ran $8\frac{1}{2}$ laps.
- Arun Kumar ran $13\frac{1}{4}$ laps.

How many laps did Binod Chhetri run?

5. a) Sithar says that you can add three fractions in any order.

For example:

$$\left(\frac{3}{4} + \frac{1}{2}\right) + \frac{3}{8} = \frac{3}{4} + \left(\frac{1}{2} + \frac{3}{8}\right)$$

Do you agree? Explain your thinking using examples.

b) Sithar wonders whether you can also subtract three fractions in any order.

For example: Is it true that

$$\left(2\frac{7}{8} - 1\frac{2}{3}\right) - \frac{5}{6} = 2\frac{7}{8} - \left(1\frac{2}{3} - \frac{5}{6}\right)?$$

What would you tell him? Explain your thinking using examples.

7. Copy and complete the Magic Square. (Hint: The sums of the rows, columns, and diagonals must be equal.)

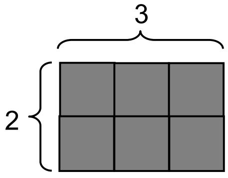
$1\frac{2}{3}$	$7\frac{1}{2}$	$3\frac{1}{3}$
5		

8. Why is it usually easier to estimate the sum or difference of mixed numbers than the sum or difference of improper fractions? Use examples to help you explain.

Chapter 2 Multiplying and Dividing Fractions

4.2.1 EXPLORE: Multiplying Fractions

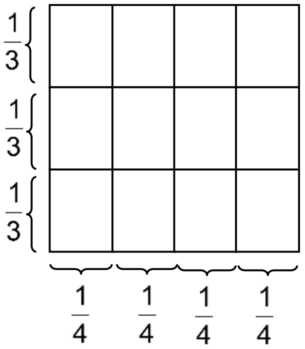
- To multiply 2 × 3, you can think of the **area** of a rectangle with **dimensions** 2 and 3.



2 × 3 = 6

- To multiply $\frac{2}{3} \times \frac{3}{4}$, you can think of the area of a rectangle with dimensions $\frac{2}{3}$ and $\frac{3}{4}$:

- Draw a large square to represent 1.
- Divide the square into thirds along one side.
- Divide it into fourths along the other side.
- Remember that the whole square has an area of 1.



A. To use the grid to model to find $\frac{2}{3} \times \frac{3}{4}$, follow these steps:

i) Copy the 3-by-4 grid above. Colour two rows to represent $\frac{2}{3}$.

ii) Use a different colour to colour three columns to represent $\frac{3}{4}$.

iii) Into how many parts is the whole grid divided?

iv) How many parts are coloured with both colours?

v) Use your answers to **parts iii) and iv)** to find $\frac{2}{3} \times \frac{3}{4}$.

B. Draw and colour a grid to model and find each product.
Do <u>not</u> write the products in lowest terms.

i) $\frac{1}{2} \times \frac{4}{5}$ **ii)** $\frac{1}{3} \times \frac{2}{5}$ **iii)** $\frac{3}{5} \times \frac{5}{6}$

C. i) How do the numerator and denominator of the product (E and F) relate to the numerator and denominator of the fractions you multiply ($\frac{A}{B}$ and $\frac{C}{D}$)?

$$\frac{A}{B} \times \frac{C}{D} = \frac{E}{F}$$

ii) How do the models that you created in **part B** show the relationship you described in **part C i)**?

4.2.2 Multiplying Fractions

Try This

In 2007, estimated populations (to the nearest billion) were as follows:

- The world was 7 billion.
- Asia was 4 billion.
- India was 1 billion.

A. i) What fraction of the world population lived in Asia?

ii) What fraction of the Asian population lived in India?

iii) What fraction of the world population lived in India?

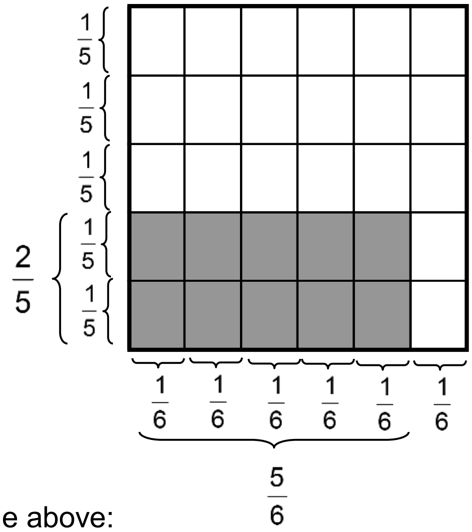
- To multiply two fractions, you can think of the area of a rectangular grid.

For example:

The grid to the right is a model for $\frac{2}{5} \times \frac{5}{6}$.

- The grid is divided into fifths along one dimension and into sixths along the other dimension.
- The shaded part of the grid is $\frac{2}{5}$ of $\frac{5}{6}$.
- Since 10 out of 30 grid squares are shaded,

$$\frac{2}{5} \times \frac{5}{6} = \frac{10}{30}$$



- Here are some things to notice in the example above:
- The numerator of the product is the product of the two numerators.
- The denominator of the product is the product of the two denominators.

$$\frac{2}{5} \times \frac{5}{6} = \frac{2 \times 5}{5 \times 6} = \frac{10}{30}$$

This happens with all fraction multiplications. Here is the reason why:

- The denominators of the fractions you are multiplying represent the number of grid units along each dimension of the *whole* grid, so the product of the denominators is the *total* number of grid squares.
- The numerators of the fractions you are multiplying represent the number of grid units along each dimension of the *shaded part* of the grid, so the product of the numerators is the number of *shaded* grid squares.

So, for any two fractions, $\frac{A}{B} \times \frac{C}{D} = \frac{A \times C}{B \times D}$.

- As with addition and subtraction of fractions, it is usually a good idea to **simplify** the final answer by writing it in lowest terms or as a mixed number, if necessary.

$$\frac{2}{5} \times \frac{5}{6} = \frac{10}{30} \underset{\div 10}{\overset{\div 10}{=}} \frac{1}{3}$$

- Sometimes you can get the product in lowest terms by simplifying as you go. You do this by dividing any numerator and denominator by a common factor.

For example:

$$\frac{2}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{6} = \frac{2}{1} \times \frac{1}{6}$$ Divide a numerator and a denominator by 5.

$$= \frac{\overset{1}{\cancel{2}}}{1} \times \frac{1}{\underset{3}{\cancel{6}}}$$ Divide a numerator and a denominator by 2.

$$= \frac{1}{1} \times \frac{1}{3}$$ Note that dividing by 5 and then by 2 is the same as dividing the numerator and denominator of the final product by 10 to write it in lowest terms, as shown above.

$$= \frac{1}{3}$$

- There are different ways to think about multiplying a whole number and a fraction.

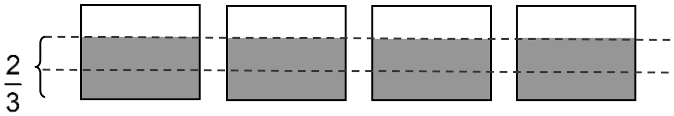
For example, here are some ways to find $4 \times \frac{2}{3}$:

- Repeated addition: $4 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3}$ or $2\frac{2}{3}$

- Write the whole number as a fraction: $4 \times \frac{2}{3} = \frac{4}{1} \times \frac{2}{3} = \frac{4 \times 2}{1 \times 3} = \frac{8}{3}$ or $2\frac{2}{3}$

- Use the commutative property and a model: $4 \times \frac{2}{3} = \frac{2}{3} \times 4$, which is $\frac{2}{3}$ of 4

$\frac{2}{3}$ of 4 can be modelled as $\frac{2}{3}$ of each of 4 wholes.



$$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3} \text{ or } 2\frac{2}{3}$$

B. Write a multiplication equation to represent the solution to **part A iii)**.

Examples

Example 1 Multiplying Fractions

Multiply. Write each product in lowest terms.

a) $\frac{1}{2} \times \frac{4}{9}$ **b)** $\frac{3}{4} \times \frac{5}{12}$ **c)** $\frac{3}{8} \times 10$

Solution	**Thinking**
a) $\frac{1}{2} \times \frac{4}{9} = \frac{1\times4}{2\times9} = \frac{4}{18} = \frac{2}{9}$	 **a)** I divided the numerator and denominator of the product by 2 to write it in lowest terms.
b) $\frac{3}{4} \times \frac{5}{12} = \frac{\overset{1}{\cancel{3}}}{4} \times \frac{5}{\underset{4}{\cancel{12}}}$ $= \frac{1}{4} \times \frac{5}{4}$ $= \frac{5}{16}$	**b)** I divided the numerator of the first fraction and the denominator of the second fraction by the common factor 3 to simplify the multiplication. • Then I multiplied the fractions.
c) $\frac{3}{8} \times 10 = \frac{3}{8} \times \frac{10}{1}$ $= \frac{30}{8}$ $= \frac{15}{4}$ $= 3\frac{3}{4}$	**c)** I knew that $10 = \frac{10}{1}$. • I wrote the product as a mixed number in lowest terms. • $3\frac{3}{4}$ makes sense because $\frac{3}{8} \times 10$ is a bit less than $\frac{1}{2} \times 10 = 5$ (since $\frac{3}{8}$ is a bit less than $\frac{1}{2}$).

Example 2 Solving a Fraction Problem using Multiplication

- About $\frac{2}{7}$ of the population of Bhutan lives in urban areas.
- About $\frac{1}{4}$ of the urban population lives in Thimphu.

What fraction of the population of Bhutan lives in Thimphu?

Solution	**Thinking**
$\frac{1}{4} \times \frac{2}{7} = \frac{1\times2}{4\times7} = \frac{1}{14}$ About $\frac{1}{14}$ of the population of Bhutan lives in Thimphu.	 • The population of Thimphu is $\frac{1}{4}$ of the urban population so I knew I had to find $\frac{1}{4}$ of $\frac{2}{7}$. • To find a fraction of another fraction, you can multiply the fractions.

Practising and Applying

1. Draw a grid model for each multiplication.

a) $\frac{1}{2}$ of $\frac{1}{3}$ **b)** $\frac{2}{5} \times \frac{3}{4}$

2. Find each product in lowest terms.

a) $\frac{3}{5} \times \frac{4}{9}$ **b)** $\frac{3}{8} \times \frac{5}{6}$

c) $\frac{5}{8} \times \frac{4}{5}$ **d)** $\frac{3}{7} \times \frac{3}{5}$

e) $\frac{3}{10} \times \frac{2}{3}$ **f)** $\frac{8}{9} \times \frac{6}{10}$

3. Estimate each product. Then find each product in lowest terms.

a) $\frac{4}{5} \times 16$ **b)** $\frac{3}{4} \times 11$

c) $\frac{7}{8} \times 25$ **d)** $\frac{5}{9} \times 15$

4. Lhakpa's bed takes up $\frac{1}{3}$ of the width of her room and $\frac{3}{5}$ of its length. What fraction of the floor area is covered by the bed?

5. Write three different pairs of fractions that have a product of $\frac{6}{40}$.

6. About $\frac{1}{7}$ of Bhutan's population lives in Thimphu Dzongkhag. Chhukha Dzongkhag has about $\frac{3}{4}$ of the population of Thimphu Dzongkhag. What fraction of Bhutan's population lives in Chhukha Dzongkhag?

7. Chandra slept at home for $\frac{1}{3}$ of the day and spent $\frac{5}{8}$ of his waking hours at home on the same day. What fraction of the whole day did he spend at home?

8. a) Calculate 0.3 × 0.4.

b) Use fractions to rewrite the multiplication in **part a)**. Find the product as a fraction.

c) What do you notice about the products in **parts a) and b)**?

9. a) Find each product in lowest terms and then describe what you notice.

$\frac{1}{2} \times \frac{2}{3} = ?$

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = ?$

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = ?$

b) Use what you noticed in **part a)** to predict the product of this multiplication:

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \ldots \times \frac{99}{100} = ?$

10. Pure gold is 24 K (karats).

Two 10 kg bars of 24 K gold

a) When we say that gold is 18 K, it means that pure gold is mixed with other metals so that it is $\frac{18}{24}$ or $\frac{3}{4}$ gold.

Estimate how much gold is in 10 kg of each. Give each answer in grams.

i) 12 K **ii)** 18 K **iii)** 22 K

b) Suppose the value of 30 g of pure gold is $600 U.S. Estimate the value of 30 g of 18 K gold.

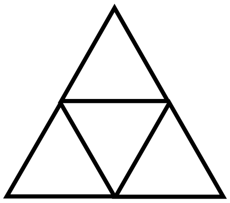
11. You are multiplying two fractions, each less than 1. How does each fraction compare to the product? Why does this happen?

CONNECTIONS: The Sierpinski Triangle

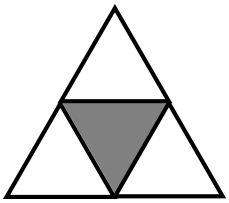
A **fractal** is a geometric shape that can be subdivided in parts. Each part is a reduced copy of the original shape.

- The word “fractal” is related to the word “fracture”, which means to break.
- Many modern artists create fractal art.
- The Sierpinski triangle is a fractal named after Wacław Sierpiński, who described it in 1915. You can make it by following these steps:

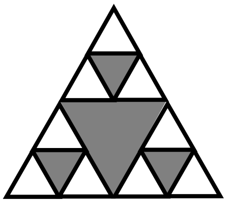
Step 1
Draw an equilateral triangle and connect the midpoints of each side.



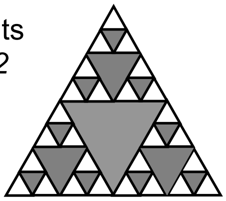
Step 2
Colour or shade the triangle in the centre. Think of this as cutting a hole in the triangle.



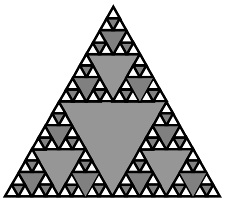
Step 3
Connect the midpoints and repeat *Step 2* with each of the uncoloured small triangles from *Step 2*. You now have one large coloured triangle and three smaller coloured triangles.



You can continue to connect the midpoints of the smaller triangles and repeat *Step 2* as many times as you wish.



Step 4



Step 5

1. a) In *Step 2*, what fraction of the large triangle is not coloured?

b) i) In *Step 3*, what fraction of each white triangle from *Step 2* is not coloured?

ii) In *Step 3*, what fraction of the large triangle is not coloured?

c) What do you notice about the fractions in **parts a) and b)**? Why do you think this happened?

2. a) Use what you noticed in **question 1 c)** to predict the fraction of the large triangle in *Step 4* that is not coloured. Explain your prediction.

b) Check your prediction to see if you were right.

4.2.3 Multiplying Mixed Numbers

Try This

A. Choki earned Nu 8000 working at a call centre. Indra Maya earned $1\frac{1}{2}$ times as much as Choki earned. How much did Indra Maya earn?

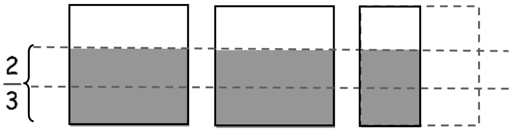
To multiply a mixed number by a fraction, you use the same strategies you have already learned for multiplying fractions and for multiplying whole numbers.

For example:

- To find $\frac{2}{3} \times 2\frac{1}{2}$, you can use a model to find $\frac{2}{3}$ of $2\frac{1}{2}$:

- Draw two and a half squares to represent $2\frac{1}{2}$.

- Divide each square or half square into thirds and shade two thirds of each.



$$\frac{2}{3} \quad + \quad \frac{2}{3} \quad + \quad \frac{2}{6} \quad \rightarrow \quad \frac{2}{3} + \frac{2}{3} + \frac{1}{3} = \frac{5}{3} = 1\frac{2}{3}$$

- To find $\frac{2}{3} \times 2\frac{1}{2}$, you can use also use the distributive property.

Since $2\frac{1}{2} = 2 + \frac{1}{2}$, you can multiply each part of the mixed number by $\frac{2}{3}$.

$$\begin{aligned} \frac{2}{3} \times 2\frac{1}{2} &= \frac{2}{3} \times (2 + \frac{1}{2}) \\ &= \frac{2}{3} \times 2 + \frac{2}{3} \times \frac{1}{2} \\ &= \frac{4}{3} + \frac{1}{3} \\ &= \frac{5}{3} \\ &= 1\frac{2}{3} \end{aligned}$$

Recall the order of operations:

You do any multiplications in an expression before you do any additions.

$$\frac{2}{3} \times 2 + \frac{2}{3} \times \frac{1}{2} = (\frac{2}{3} \times 2) + (\frac{2}{3} \times \frac{1}{2})$$

B. Write a multiplication equation to represent the solution to the problem in **part A**.

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Examples

Example 1 Multiplying a Mixed Number by a Whole Number

Estimate each product. Then calculate each product in lowest terms.

a) $8 \times 2\frac{3}{4}$ **b)** $1\frac{3}{8} \times 10$ **c)** $\frac{2}{3} \times 4\frac{1}{2}$

Solution	**Thinking**
a) $8 \times 2\frac{3}{4}$ is a bit less than 24. $8 \times 2\frac{3}{4} = 8 \times (2 + \frac{3}{4})$ $= 8 \times 2 + 8 \times \frac{3}{4}$ $= 16 + \frac{24}{4}$ $= 16 + 6$ $= 22$	 **a)** I knew that the product was a bit less than 8 × 3 = 24 because $2\frac{3}{4}$ is a bit less than 3. • I used the distributive property to multiply. • The product was close to my estimate.
b) $1\frac{3}{8} \times 10$ is a bit less than 15. $1\frac{3}{8} \times 10 = 1 \times 10 + \frac{3}{8} \times 10$ $= 10 + \frac{30}{8}$ $= 10 + \frac{15}{4}$ $= 10 + 3\frac{3}{4}$ $= 13\frac{3}{4}$	**b)** I knew the product was a bit less than $1\frac{1}{2} \times 10 = 15$ because $1\frac{3}{8}$ is a bit less than $1\frac{1}{2}$. • I used the distributive property to multiply. • I wrote the answer in lowest terms as a mixed number. • The product was close to my estimate.
c) $\frac{2}{3} \times 4\frac{1}{2}$ is about $2\frac{1}{2}$. $\frac{2}{3} \times 4\frac{1}{2} = \frac{2}{3} \times \frac{9}{2}$ $= \frac{18}{6}$ $= 3$	**c)** I knew the product was about $\frac{1}{2} \times 5 = 2\frac{1}{2}$. To estimate, I rounded $\frac{2}{3}$ down to $\frac{1}{2}$ and I rounded $4\frac{1}{2}$ up to 5. • I wrote $4\frac{1}{2}$ as an improper fraction. Then I multiplied. • The product was close to my estimate.

Example 2 Estimating a Product

Estimate each.

a) $2\frac{5}{8} \times 50$ (to the nearest whole number) **b)** $1\frac{7}{10} \times 12$ (to the nearest half)

Solution

a)
$$\begin{aligned} 2\frac{5}{8} \times 50 &= 2 \times 50 + \frac{5}{8} \times 50 \\ &= 100 + \frac{250}{8} \\ &= 100 + \frac{125}{4} \approx 100 + \frac{124}{4} \\ &= 100 + 31 \\ &= 131 \end{aligned}$$

b)
$$\begin{aligned} 1\frac{7}{10} \times 12 = \frac{17}{10} \times 12 &= \frac{204}{10} \\ &= \frac{200+4}{10} \\ &= \frac{200}{10} + \frac{4}{10} \\ &= 20 + \frac{4}{10} \approx 20\frac{1}{2} \end{aligned}$$

Thinking

a) I used the distributive property to multiply.

- To estimate, I changed 125 to 124 so that the numerator was divisible by the denominator.

b) I changed the mixed number to an improper fraction to multiply.

- I knew that $\frac{4}{10}$ was close to $\frac{1}{2}$.

Example 3 Multiplying Two Mixed Numbers

Multiply $2\frac{5}{8} \times 3\frac{3}{7}$.

Solution 1

$$\begin{aligned} 2\frac{5}{8} \times 3\frac{3}{7} &= (2 \times 3\frac{3}{7}) + (\frac{5}{8} \times 3\frac{3}{7}) \\ &= (2 \times 3 + 2 \times \frac{3}{7}) + (\frac{5}{8} \times 3 + \frac{5}{8} \times \frac{3}{7}) \\ &= 6 + \frac{6}{7} + \frac{15}{8} + \frac{15}{56} \\ &= 6 + \frac{48}{56} + \frac{105}{56} + \frac{15}{56} \\ &= 6 + \frac{168}{56} = 6 + 3 = 9 \end{aligned}$$

Thinking

- I multiplied using the distributive property twice.

Solution 2

$$\begin{aligned} 2\frac{5}{8} \times 3\frac{3}{7} = \frac{21}{8} \times \frac{24}{7} = \frac{21}{\underset{1}{\cancel{8}}} \times \frac{\overset{3}{\cancel{24}}}{7} &= \frac{\overset{3}{\cancel{21}}}{1} \times \frac{3}{\underset{1}{\cancel{7}}} \\ &= \frac{3 \times 3}{1 \times 1} = \frac{9}{1} = 9 \end{aligned}$$

Thinking

- I changed each mixed number to an improper fraction to multiply.

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Example 4 Solving a Fraction Multiplication Problem

A large bag of rice holds $2\frac{2}{3}$ times as much as a small bag holds. If a small bag holds 750 g of rice, how much does the large bag hold, in kilograms?

Solution

$$\begin{aligned} 2\frac{2}{3} \times 750 &= 2 \times 750 + \frac{2}{3} \times 750 \\ &= 1500 + \frac{1500}{3} \\ &= 1500 + 500 \\ &= 2000 \quad \rightarrow \quad 2000\text{ g} = 2\text{ kg} \end{aligned}$$

The large bag holds 2 kg.

Thinking

- I used the distributive property to multiply.

Practising and Applying

1. Draw a model to show $\frac{2}{5} \times 2\frac{3}{4}$.

2. Find each product in lowest terms.

a) $\frac{1}{4} \times 2\frac{1}{3}$ **b)** $42 \times 1\frac{5}{7}$

c) $\frac{4}{5} \times 2\frac{6}{7}$ **d)** $\frac{3}{4} \times 10\frac{2}{3}$

e) $\frac{3}{4} \times 1\frac{7}{9}$ **f)** $8\frac{7}{8} \times 40$

g) $4\frac{4}{7} \times 3\frac{1}{2}$ **h)** $1\frac{7}{8} \times 2\frac{1}{4}$

3. A recipe calls for $1\frac{3}{4}$ cups of rice.

a) How much rice is needed to make half the recipe?

b) How much rice is needed to make two and one half times the recipe?

4. How many eggs are in $2\frac{3}{4}$ dozen?

5. Kamala worked for $7\frac{1}{2}$ h. She spent $\frac{2}{3}$ of the time on her computer. How long was she on her computer?

6. Kula Kangri is 7554 m high. Mount Everest is about $1\frac{1}{6}$ the height of Kula Kangri. About how high is Mount Everest?

7. Estimate each.

a) $3\frac{1}{3} \times 100$ (nearest whole number)

b) $2\frac{5}{9} \times 8$ (nearest half)

8. a) Calculate $\frac{3}{10} \times 2\frac{7}{10}$.

b) Use decimals to rewrite the multiplication in **part a)**. Find the product as a decimal.

c) What do you notice about the products in **parts a) and b)**?

9. Dorji multiplied $2\frac{1}{2}$ by another mixed number. The product was a whole number. What could the other mixed number have been? Give one answer.

10. Describe a situation for which you might have to multiply $3\frac{1}{2}$ by $2\frac{1}{3}$.

11. Why can you always multiply mixed numbers if you know how to multiply fractions?

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4.2.4 Dividing Fractions With a Common Denominator

Try This

A. Choki walked halfway to school in $\frac{3}{4}$ h. If she continues at the same rate, how many half hours will it take her to walk to school and back? Show your work.

- Dividing fractions with the same denominator is like dividing whole numbers.

For example:

Just like 10 ÷ 2 means "How many 2s are there in 10?",

$\frac{5}{6} \div \frac{1}{6}$ means "How many 1 sixths are in 5 sixths?"

- You can use a model to understand fraction division.

For example:

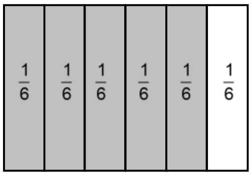
\- For $\frac{5}{6} \div \frac{1}{6}$, you can use a model like this for $\frac{5}{6}$.

You can see that $\frac{5}{6}$ has 5 shaded parts. Each part is $\frac{1}{6}$.

Since 5 ÷ 1 = 5, there are five 1 sixths in 5 sixths.

So, $\frac{5}{6} \div \frac{1}{6} = 5$.

A model for $\frac{5}{6}$

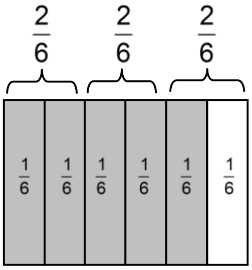


There are five $\frac{1}{6}$ s in $\frac{5}{6}$.

\- For $\frac{5}{6} \div \frac{2}{6}$, you need to find how many 2 sixths there are in 5 sixths.

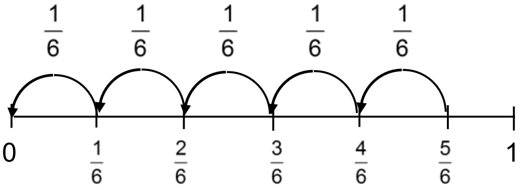
Since $5 \div 2 = 2\frac{1}{2}$, there are $2\frac{1}{2}$ sets of $\frac{2}{6}$ in $\frac{5}{6}$.

So, $\frac{5}{6} \div \frac{2}{6} = 2\frac{1}{2}$.



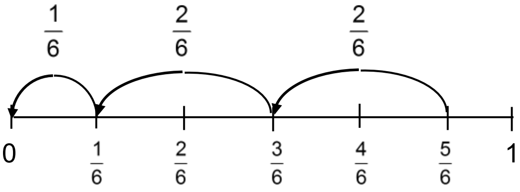
- You can also model fraction division on a number line.

It takes 5 jumps of $\frac{1}{6}$ to get from $\frac{5}{6}$ to 0.



So, $\frac{5}{6} \div \frac{1}{6} = 5$.

It takes $2\frac{1}{2}$ jumps of $\frac{2}{6}$ to get from $\frac{5}{6}$ to 0.



So, $\frac{5}{6} \div \frac{2}{6} = 2\frac{1}{2}$.

- When you divide two fractions with the same denominator, you only need to divide the numerators.

$$\frac{5}{6} \div \frac{1}{6} = 5 \div 1 = 5 \qquad \frac{5}{6} \div \frac{2}{6} = 5 \div 2 = 2\frac{1}{2}$$

- If two fractions in a division have different denominators, you can rename the fractions as equivalent fractions with the same denominator. Then you can divide the numerators.

For example:

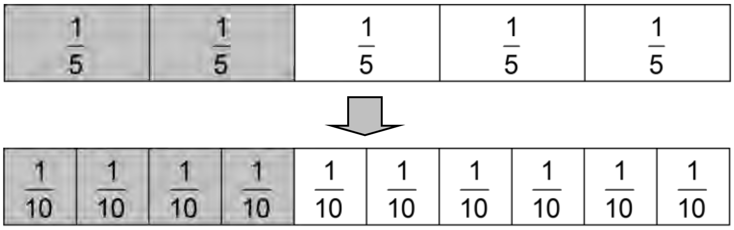
$$\frac{2}{5} \div \frac{1}{10}$$

$$= \frac{4}{10} \div \frac{1}{10}$$

$$= 4 \div 1$$

$$= 4$$

This model shows why a quotient of 4 makes sense.



There are four 1 tenths in 2 fifths.

B. Write a division equation using fractions that you could use to solve the problem in **part A**.

Examples

Example 1 Dividing Fractions

Solution

a) $\frac{5}{8} \div \frac{1}{8} = 5 \div 1$

$= 5$

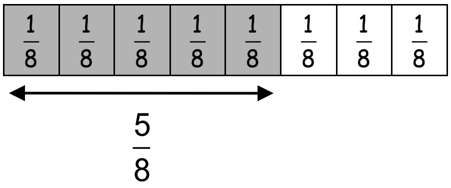
b) $\frac{5}{8} \div \frac{3}{8} = 5 \div 3$

$= \frac{5}{3}$

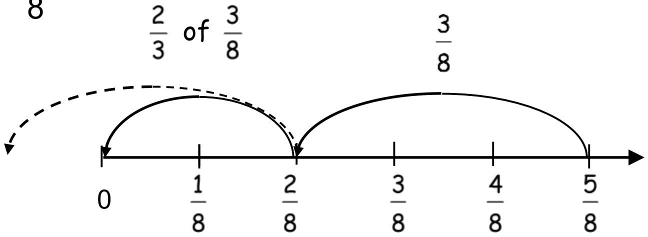
$= 1\frac{2}{3}$

Thinking

a) I knew there were five $\frac{1}{8}$s in $\frac{5}{8}$.



b) It takes 1 jump of $\frac{3}{8}$ and another $\frac{2}{3}$ of a jump of $\frac{3}{8}$ to get from $\frac{5}{8}$ to 0.



Example 2 Dividing Fractions [Continued]

Solution

c) $\frac{5}{8} \div \frac{1}{3} = ?$

$\frac{5}{8} = \frac{15}{24} \quad \frac{1}{3} = \frac{8}{24}$

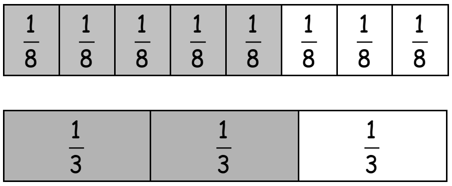
$$\frac{5}{8} \div \frac{1}{3} = \frac{15}{24} \div \frac{8}{24}$$
$$= \frac{15}{8}$$
$$= 1\frac{7}{8}$$

Thinking

c) I wrote $\frac{5}{8}$ and $\frac{1}{3}$ as equivalent fractions with a common denominator.

- I multiplied the denominators to find a common denominator: 8 × 3 = 24

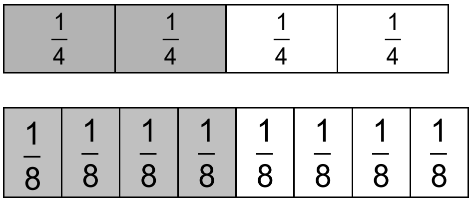
- A quotient of $1\frac{7}{8}$ makes sense since there are about 2 sets of $\frac{1}{3}$ in $\frac{5}{8}$.



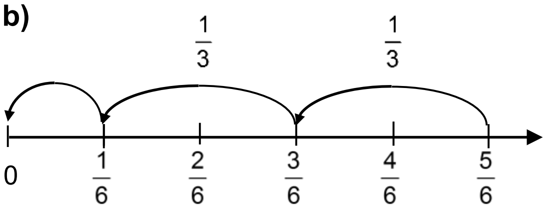
Practising and Applying

1. What division equation does each model show?

a)



b)



2. Draw a picture to show how many $\frac{1}{5}$s there are in $\frac{7}{10}$.

3. Calculate $\frac{3}{5} \div \frac{1}{2}$ using equivalent fractions with a common denominator. Show your work.

4. Find each quotient.

a) $\frac{3}{4} \div \frac{1}{4}$

b) $\frac{4}{5} \div \frac{2}{5}$

c) $\frac{7}{8} \div \frac{1}{2}$

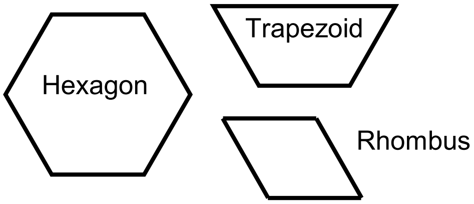
d) $\frac{2}{3} \div \frac{1}{6}$

e) $\frac{9}{10} \div \frac{1}{3}$

f) $\frac{7}{8} \div \frac{1}{12}$

5. The trapezoid is $\frac{1}{2}$ of the hexagon.

The rhombus is $\frac{1}{3}$ of the hexagon.



How many rhombuses are there in one trapezoid? Write a division equation to represent the solution.

6. Explain why $\frac{6}{\blacksquare} \div \frac{2}{\blacksquare} = 3$ is always true, as long as the denominators are the same.

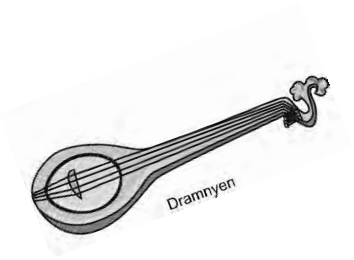
4.2.5 Dividing Fractions in Other Ways

Try This

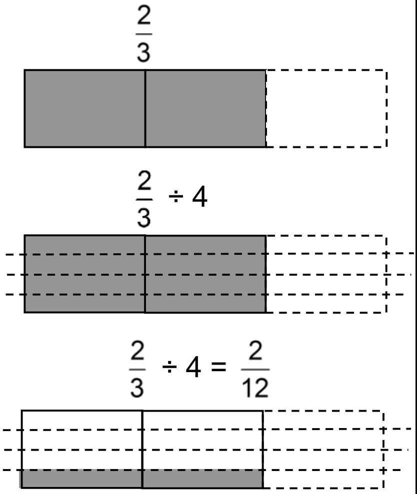
Tshering practised playing his dramnyen for $\frac{3}{4}$ h each day before a festival. He practised for a total of 9 h.

A. i) How many minutes are in $\frac{3}{4}$ h?

ii) How many days did he practise?



- To divide a fraction by a whole number, you can think about dividing the fraction into equal parts.



For example, this is how to divide $\frac{2}{3} \div 4$:

- Draw a rectangle to represent 1, and then shade it to model $\frac{2}{3}$.
- Divide the rectangle into 4 equal parts so that the shaded $\frac{2}{3}$ is also divided into 4 equal parts.
- The rectangle is now divided into 12 equal parts and each part is $\frac{1}{12}$.
- You can see that, when $\frac{2}{3}$ is divided by 4,

you get 2 of those parts, or $\frac{2}{12}$.

So, $\frac{2}{3} \div 4 = \frac{2}{12}$. Notice that the denominator of the product is the product of the denominator of the original fraction (3) and the divisor (4): 3 × 4 = 12

This is true when you divide any fraction by a whole number because the whole number divides the fraction into more parts:

$$\frac{A}{B} \div C = \frac{A}{B \times C}$$

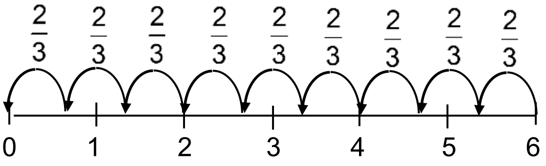
This makes sense because, when a fraction with B parts is divided by C, the quotient has B × C parts.

- To divide a whole number by a fraction, you can think about how many of the fractions are in the whole number.

For example, to find $6 \div \frac{2}{3}$:

You can use a number line to count the number of $\frac{2}{3}$s in 6.

There are nine $\frac{2}{3}$s in 6, so $6 \div \frac{2}{3} = 9$.



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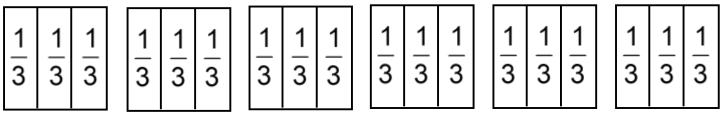
- To divide a whole number by a **unit fraction**, you can think about the number of unit fractions there are in the whole number.

For example, this is how to divide $6 \div \frac{1}{3}$:

There are three $\frac{1}{3}$s in 1, so there must be six times that many, or eighteen $\frac{1}{3}$s in 6.



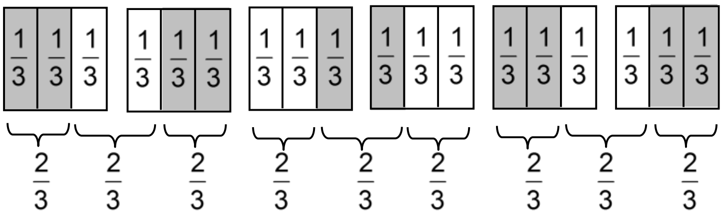
$$1 \div \frac{1}{3} = 3\text{, so } 6 \div \frac{1}{3} = 6 \times 3 = 18$$

This is true when you divide any whole number by a unit fraction: $A \div \frac{1}{C} = A \times C$

- To divide a whole number by a fraction that is not a unit fraction, you can first divide by the unit fraction and then divide by the number of parts in the fraction (the numerator).

For example, this is how to divide $6 \div \frac{2}{3}$:

There are eighteen $\frac{1}{3}$s in 6, so there must be half that many $\frac{2}{3}$s, or nine $\frac{2}{3}$s, in 6.



$$6 \div \frac{1}{3} = 6 \times 3 = 18,$$

$$\text{so } 6 \div \frac{2}{3} = 6 \times 3 \div 2 = 9.$$

- Multiplying by 3 and then dividing by 2 is the same as multiplying by $\frac{3}{2}$.

You can use this relationship to help you divide by $\frac{2}{3}$.

For example:

$$6 \times \frac{3}{2} = \frac{6 \times 3}{2} = 6 \times 3 \div 2$$

Since $6 \div \frac{2}{3}$ is also equal to $6 \times 3 \div 2$, then $6 \div \frac{2}{3} = 6 \times \frac{3}{2}$.

This is true for dividing any whole number by a fraction: $A \div \frac{B}{C} = A \times \frac{C}{B}$

$\frac{C}{B}$ is called the **reciprocal** of $\frac{B}{C}$

- To divide a fraction by a fraction, you can find a common denominator and then divide the numerators.

For example:

$\frac{3}{4} \div \frac{1}{5} = \frac{15}{20} \div \frac{4}{20} = 15 \div 4$ or $\frac{15}{4}$

To divide a fraction by a fraction, you can also multiply the first fraction by the reciprocal of the divisor.

If you think of 5 as $\frac{5}{1}$, then $\frac{3}{4} \div \frac{1}{5} = \frac{3}{4} \times \frac{5}{1}$.

This is true when you divide any number by a fraction: $\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C}$

B. Write a division equation using a fraction that could be used to represent and solve the problem in **part A ii)**.

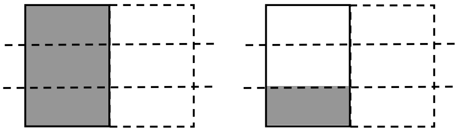
Examples

Example 1 Dividing Whole Numbers and Fractions

Divide. **a)** $\frac{1}{2} \div 3$ **b)** $4 \div \frac{1}{2}$

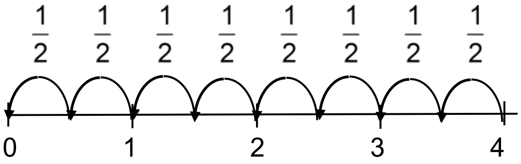
Solution

a)



$\frac{1}{2} \div 3 = \frac{1}{6}$

b)



$4 \div \frac{1}{2} = 8$

Thinking

a) I drew a model for $\frac{1}{2}$ and divided it into three parts.

Each part was $\frac{1}{6}$.

b) I counted back from 4 to 0 by $\frac{1}{2}$s on a number line to see how many $\frac{1}{2}$s were in 4.

Example 2 Dividing Fractions Using Reciprocals

Divide $\frac{5}{12} \div \frac{3}{8}$. Write the quotient as a mixed number in lowest terms.

Solution

$\frac{5}{12} \div \frac{3}{8} = \frac{5}{12} \times \frac{8}{3}$

[Continued]

Thinking

- To find how many $\frac{3}{8}$s there were in $\frac{5}{12}$, I knew I could multiply by 8 to find the number of $\frac{1}{8}$s and then divide by 3 to find the number of $\frac{3}{8}$s. That's the same as multiplying by $\frac{8}{3}$.

Example 2 Dividing Fractions Using Reciprocals [Continued]

Solution

$$\frac{5}{12} \div \frac{3}{8} = \frac{5}{\cancel{12}} \times \frac{\cancel{8}^{2}}{3}$$
$$= \frac{5^{3}}{3} \times \frac{2}{3}$$
$$= \frac{10}{9} = 1\frac{1}{9}$$

Thinking

- I found a common factor of 4 in the numerators and denominators and used it to simplify the multiplication.

Example 3 Solving a Fraction Problem

Dawa has a glass that holds $\frac{2}{3}$ of a cup. How many times does he need to fill the glass to measure out 6 cups for a recipe?

Solution

a) $6 \div \frac{2}{3} = 6 \times \frac{3}{2} = \frac{18}{2} = 9$

Dawa needs to fill the glass 9 times.

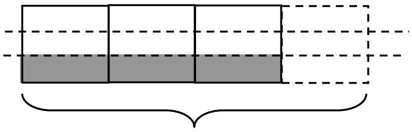
Thinking

- I knew I needed to figure out how many $\frac{2}{3}$ cups were in 6 cups, so I divided $6 \div \frac{2}{3}$.

Practising and Applying

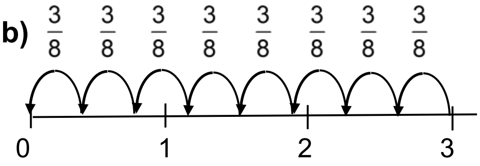
1. What division equation does each picture represent?

a)



The large rectangle is the whole.

b)



2. a) How many $\frac{1}{5}$ s are there in $\frac{8}{10}$?

b) Draw a picture to show why your answer makes sense.

3. Find each quotient.

a) $2 \div \frac{1}{4}$ **b)** $\frac{1}{4} \div 2$ **c)** $\frac{3}{8} \div \frac{1}{2}$

d) $\frac{2}{3} \div \frac{5}{6}$ **e)** $\frac{3}{5} \div \frac{2}{3}$ **f)** $\frac{7}{8} \div \frac{5}{12}$

4. Yangchen spent 2 h studying for exams. For each subject, she studied $\frac{2}{3}$ h. How many subjects did she study?

5. To frame a square picture, Rupak cut a $\frac{7}{10}$ m board into four equal pieces. How long was each piece?

6. $\frac{3}{4}$ of a circle is divided into equal parts. Each part is $\frac{3}{16}$ of the whole circle. How many parts are there?

7. Choose one of the rules below. Use an example to help you explain why the rule makes sense.

- $A \div \frac{B}{C} = A \times \frac{C}{B}$
- $\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C}$

4.2.6 Dividing Mixed Numbers

Try This

A. A family uses $1\frac{1}{2}$ dozen eggs each week. How long does it take for them to use $4\frac{1}{2}$ dozen eggs?

- Dividing a mixed number by a mixed number is just like dividing whole numbers and other fractions. You are finding the number of times the **divisor** fits into the **dividend**. A number line model can help show this.

For example:

To divide $8\frac{3}{4} \div 1\frac{1}{4}$, you find how many $1\frac{1}{4}$s are in $8\frac{3}{4}$.

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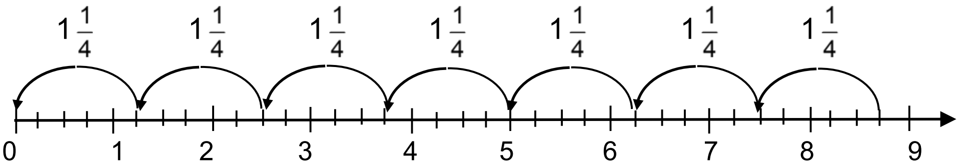


$$8\frac{3}{4} \div 1\frac{1}{4} = 7$$

- If you write mixed numbers as improper fractions, you can divide them using the same methods you learned for dividing fractions.

For example, here are two ways to divide $8\frac{3}{4} \div 1\frac{1}{4}$:

- Use a common denominator: $8\frac{3}{4} \div 1\frac{1}{4} = \frac{35}{4} \div \frac{5}{4} = 35 \div 5 = 7$

- Or, multiply by the reciprocal of the divisor:

$$8\frac{3}{4} \div 1\frac{1}{4} = \frac{35}{4} \div \frac{5}{4} = \frac{\overset{7}{\cancel{35}}}{4} \times \frac{4}{\underset{1}{\cancel{5}}} = \frac{28}{4} = 7$$

Multiplying by the reciprocal of the divisor makes sense because, when you divide $8\frac{3}{4} \div 1\frac{1}{4}$, you find the number of $1\frac{1}{4}$s or $\frac{5}{4}$s in $8\frac{3}{4}$:

- The number of $\frac{1}{4}$s in $8\frac{3}{4}$ is $8\frac{3}{4} \times 4$.

- So, the number of $\frac{5}{4}$s in $8\frac{3}{4}$ is $8\frac{3}{4} \times 4 \div 5 = 8\frac{3}{4} \times \frac{4}{5}$.

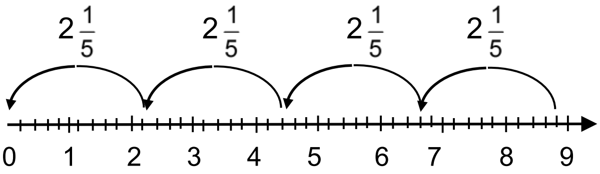
B. i) Draw a number line model to show how you can find the answer for **part A**.
ii) Show how you can calculate the answer for **part A** with both methods — using a common denominator and multiplying by the reciprocal.

Examples

Example 1 Dividing Mixed Numbers

Divide $8\frac{4}{5} \div 2\frac{1}{5}$.

Solution 1



$$8\frac{4}{5} \div 2\frac{1}{5} = 4$$

Thinking

- I drew a number line from 0 to 9 in fifths.
- I counted how many jumps of $2\frac{1}{5}$ it would take to get from $8\frac{4}{5}$ to 0.
- I had estimated that it would be about 4 because 8 ÷ 2 = 4.

Solution 2

$$8\frac{4}{5} \div 2\frac{1}{5} = \frac{44}{5} \div \frac{11}{5} = 44 \div 11 = 4$$

Thinking

- I wrote the mixed numbers as improper fractions.
- Since the denominators were the same, I divided the numerators.

Example 2 Solving a Problem with Mixed Numbers

Choki is making scarves. She needs $1\frac{1}{4}$ m of fabric for each scarf and has 6.3 m of fabric. Does she have enough fabric to make five scarves? Show your work.

Solution 1

Yes, she has enough fabric to make five scarves.

$$6.3 \div 1\frac{1}{4} = 6\frac{3}{10} \div 1\frac{1}{4}$$
$$= \frac{63}{10} \div \frac{5}{4}$$
$$= \frac{63}{\cancel{10}_{5}} \times \frac{\cancel{4}^{2}}{5}$$
$$= \frac{126}{25}$$

$\frac{126}{25} > \frac{125}{25}$, which is 5.

Thinking

- I knew that I had to divide 6.3 m by $1\frac{1}{4}$ m to see if $1\frac{1}{4}$ fit into 6.3 at least 5 times.
- I wrote the decimal 6.3 and the mixed number $1\frac{1}{4}$ as improper fractions. Then I multiplied by the reciprocal of the divisor.
- Since the quotient was greater than 5, I knew she had enough fabric.

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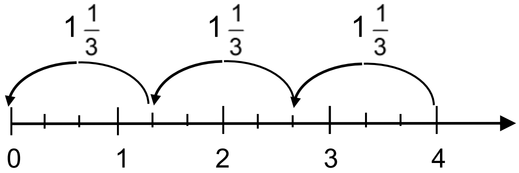
Solution 2	**Thinking**
Yes, she has enough fabric to make five scarves. $5 \times 1\frac{1}{4} = 5 \times \frac{5}{4} = 6\frac{1}{4} = 6.25$ $6.25 < 6.3$	• I calculated the amount she would need for five scarves and then compared that with the amount of fabric she has. 

Practising and Applying

1. What division equation does this number line model represent?



2. Draw a number line to show $9 \div 1\frac{1}{2}$.

3. Calculate.

a) $4\frac{2}{3} \div 2\frac{1}{3}$

b) $5\frac{1}{2} \div 3$

c) $4\frac{5}{8} \div 2\frac{1}{4}$

d) $2\frac{8}{9} \div 2\frac{1}{6}$

4. a) Calculate each pair of quotients.

Pair A $2\frac{1}{2} \div 1\frac{2}{3}$ and $1\frac{2}{3} \div 2\frac{1}{2}$

Pair B $5\frac{1}{3} \div 3\frac{1}{5}$ and $3\frac{1}{5} \div 5\frac{1}{3}$

b) What do you notice about the quotients in each pair?

c) Write another pair of division calculations with the same pattern.

5. a) A farmer has $9\frac{1}{2}$ ha (hectares) of land. He needs $1\frac{1}{4}$ ha for each crop. How many different crops can he plant?

b) Will all the land be used for planting? Explain your thinking.

6. A musical instrument is made from a $1\frac{1}{4}$ m length of wood. How many instruments can be made from 10 m of wood?

7. Chabilal practises archery the same amount of time each day of the week. He practises a total of $8\frac{3}{4}$ h each week. How much does he practise each day?

8. Without dividing, how can you tell that $9\frac{1}{3} \div 1\frac{8}{9}$ is greater than $4\frac{1}{2}$?

9. For each division below:

i) Estimate each quotient and then explain how you estimated.

ii) Find the exact quotient. Compare each exact quotient to your estimate.

a) $6\frac{5}{6} \div \frac{3}{4}$

b) $11\frac{2}{5} \div 2\frac{1}{9}$

10. a) How can you tell, without actually dividing, that the quotient of two mixed numbers will be greater than 1? Use an example to illustrate.

b) Repeat **part a)** for a quotient that is less than 1.

c) Repeat **part a)** for a quotient that is equal to 1.

Chapter 3 Rational Numbers

4.3.1 Introducing Rational Numbers

Try This

A. The opposite of +2 is –2. They are 4 units apart on a number line.

i) Which two opposite numbers are 24 units apart on a number line?

ii) Which two opposite numbers are 13 units apart on a number line?

- A fraction is a number that can be written as the quotient of two whole numbers. A **rational number** is a number that can be written as a quotient of two integers. In both cases, the divisor cannot be zero.

For example: $\frac{3}{4}$ and $\frac{-2}{3}$ are rational numbers because –2, 3, and 4 are integers.

- Every fraction is a rational number. Every integer is also a rational number.

For example:

–2 is a rational number because it can be written as $\frac{-4}{2}$ (–4 and 2 are integers).

- You can use what you know about integers to understand why a rational number can be written in different ways.

For example:

3 is a rational number because it can be written as $\frac{6}{2}$.

The opposite of 3 or $\frac{6}{2}$ is –3 or $-\frac{6}{2}$, so $-3 = -\frac{6}{2}$.

–3 can also be written other ways:

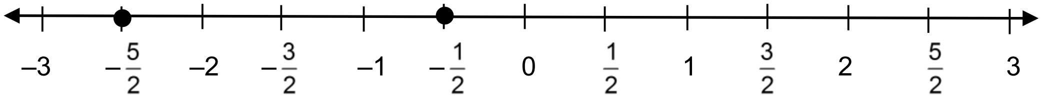
Since $\frac{-6}{2} = (-6) \div 2 = -3$ and $\frac{6}{-2} = 6 \div (-2) = -3$, then $-3 = \frac{-6}{2} = \frac{6}{-2}$.

That means $-\frac{6}{2}$, $\frac{-6}{2}$, and $\frac{6}{-2}$ are all equivalent to –3.

- You can compare and order rational numbers just like you do integers.

- On a horizontal number line, the numbers increase as you move to the right.

For example:



$-\frac{1}{2}$ is right of $-\frac{5}{2}$, so $-\frac{1}{2} > -\frac{5}{2}$.

- Any positive rational number is greater than any negative rational number.

For example: $\frac{1}{2} > -\frac{3}{2}$

- You can write a rational number in fraction, mixed number, or decimal form.

For example: $-\frac{5}{2} = -2\frac{1}{2} = -2.5$

- To compare rational numbers in mixed number form, you need to understand the meaning of the whole number part and the fraction part of the mixed number.

For example: $-2\frac{1}{2} = -2 - \frac{1}{2}$ or $-2 + (-\frac{1}{2})$. This shows why $-2\frac{1}{2} < -2$.

B. Why are the opposites for **part A ii)** called rational numbers and not integers?

Examples

Example 1 Ordering Rational Numbers

Order from least to greatest. $-\frac{2}{3}$ $2\frac{1}{2}$ $-1\frac{1}{3}$ $\frac{3}{2}$ $\frac{4}{3}$

Solution 1

Write as improper fractions

$-\frac{2}{3}$ $2\frac{1}{2}$ $-1\frac{1}{3}$ $\frac{3}{2}$ $\frac{4}{3}$

$-\frac{2}{3}$ $\frac{5}{2}$ $-\frac{4}{3}$ $\frac{3}{2}$ $\frac{4}{3}$

Order the negative numbers

$-\frac{4}{3} < -\frac{2}{3}$

Order the positive numbers

$\frac{4}{3} < \frac{3}{2}$ and $\frac{3}{2} < \frac{5}{2}$

Order from least to greatest

$-1\frac{1}{3}, -\frac{2}{3}, \frac{4}{3}, \frac{3}{2}, 2\frac{1}{2}$

Thinking

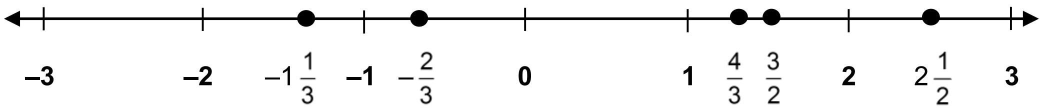
- I wrote them all as improper fractions to make it easier to compare them.
- I knew that $-\frac{4}{3} < -\frac{2}{3}$ since $-\frac{4}{3}$ is 4 thirds below 0 and $-\frac{2}{3}$ is only 2 thirds below 0.
- $\frac{4}{3} < \frac{3}{2}$ because $\frac{4}{3} = 1\frac{1}{3}$ and $\frac{3}{2} = 1\frac{1}{2}$ and $\frac{1}{3} < \frac{1}{2}$.
- $\frac{3}{2} < \frac{5}{2}$ because 3 halves < 5 halves.

Solution 2

$-1\frac{1}{3}, -\frac{2}{3}, \frac{4}{3}, \frac{3}{2}, 2\frac{1}{2}$

Thinking

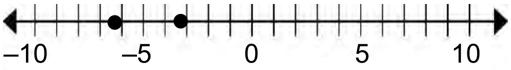
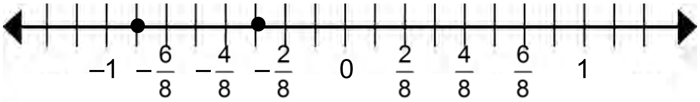
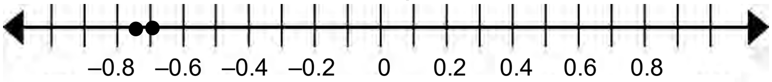
- I estimated where each number was on a number line. I put positive numbers right of 0 and negative numbers left of 0.
- When I placed $\frac{4}{3}$ and $\frac{3}{2}$ on the number line, I thought of them as $1\frac{1}{3}$ and $1\frac{1}{2}$.



Example 2 Comparing Rational Numbers

Which number in each pair is greater?

a) $-\frac{3}{4}$, 0.1 **b)** $-3\frac{2}{5}$, $-6\frac{1}{4}$ **c)** $-23\frac{3}{8}$, $-23\frac{7}{8}$ **d)** $\frac{9}{7}$, $\frac{9}{4}$ **e)** $-\frac{3}{4}$, -0.7

Solution	**Thinking**
a) $0.1 > -\frac{3}{4}$	**a)** Any positive number is greater than any negative number, so $0.1 > -\frac{3}{4}$.
b) $-3\frac{2}{5} > -6\frac{1}{4}$	**b)** I imagined a number line. I knew that $-3\frac{2}{5}$ was right of -5 and $-6\frac{1}{4}$ was left of -5. So, $-3\frac{2}{5} > -6\frac{1}{4}$. 
c) $-23\frac{3}{8} > -23\frac{7}{8}$	**c)** The integer parts were the same, so I compared the fraction parts: $-23\frac{3}{8} = (-23) + (-\frac{3}{8})$ and $-23\frac{7}{8} = (-23) + (-\frac{7}{8})$ • Since $-\frac{3}{8}$ was right of $-\frac{7}{8}$ on a number line, $-\frac{3}{8} > -\frac{7}{8}$, which means $-23\frac{3}{8} > -23\frac{7}{8}$. 
d) $\frac{9}{4} > \frac{9}{7}$	**d)** Since both numbers were positive and the numerators were the same, I compared the denominators. • Since 1 fourth > 1 seventh, 9 fourths > 9 sevenths.
e) $-0.7 > -\frac{3}{4}$	**e)** I changed $-\frac{3}{4}$ to -0.75 so the numbers would be in the same form and easier to compare. • Since -0.7 was right of -0.75 on a number line, I knew that $-0.7 > -0.75$ or $-\frac{3}{4}$.  • I also knew that, if $0.7 < \frac{3}{4}$, then $-0.7 > -\frac{3}{4}$.

Practising and Applying

1. a) Draw a number line. Mark these rational numbers on the line.

$-\frac{1}{2}$ $1\frac{1}{4}$ $-\frac{7}{4}$ $\frac{7}{2}$

b) Which rational number in **part a)** is the least? How do you know?

2. Explain how you know that $-\frac{7}{3}$, $\frac{-7}{3}$, and $\frac{7}{-3}$ are equivalent.

3. Replace each ■ with < or > to compare each pair of rational numbers. Explain how you compared each pair.

a) $-5\frac{1}{3}$ ■ $-7\frac{7}{8}$

b) $-\frac{4}{5}$ ■ $\frac{1}{2}$

c) $23\frac{5}{9}$ ■ $23\frac{2}{9}$

d) $\frac{7}{-4}$ ■ $\frac{4}{-7}$

e) $-1\frac{2}{3}$ ■ $-1\frac{2}{5}$

f) $-3\frac{1}{3}$ ■ -3.3

4. Order from least to greatest.

a) $-\frac{9}{4}$, -3, $-\frac{4}{5}$, $-6\frac{1}{2}$

b) $-\frac{1}{2}$, 0, $\frac{7}{12}$, $-\frac{5}{4}$, $\frac{11}{12}$

c) $\frac{15}{8}$, -3, $\frac{15}{4}$, -5, $\frac{11}{4}$

d) $-5\frac{2}{5}$, 0, 4.7, -5.2, $4\frac{3}{4}$

5. The temperature in Thimphu one day was –1.2°C. On the same day, it was –1.7°C in Paro. Which city was colder? How do you know?

6. The change in value of four stocks on the Hong Kong Stock Exchange was given as follows:

Stock A –2.09	Stock B –2.50
Stock C +0.23	Stock D –1.98

(Hint: A negative change means a loss in value. A positive change means a gain in value.)

Which stock had the greatest loss in value? How do you know?

7. a) Find the mystery rational number that matches all four clues:

- It is less than –5.
- It has a denominator of 2.
- It is less than –3.
- It is greater than –6.

b) One of the clues is not necessary. Which clue is it? How do you know?

8. Write four different rational numbers that are greater than –3 but less than –2. Order them from least to greatest.

9. Two rational numbers in fraction form are positive. Their numerators are the same. How can you compare them using the denominators? Use an example to help you explain.

10. How can $-15 < -\frac{1}{2}$ when $15 > \frac{1}{2}$?

4.3.2 Operations with Rational Numbers

Try This

The price of a stock on the Hong Kong Stock Exchange started at a value of 39.45 Hong Kong dollars. The stock lost 1.05 Hong Kong dollars in value each day for three days.

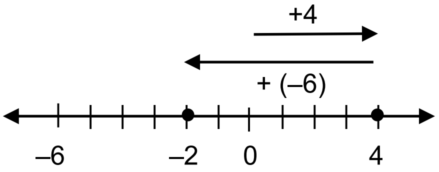
A. i) What was the price of the stock after the three days?

ii) If it continued to lose value at the same rate, what would be the price of the stock after 5 more days?

- The number line model you used to add integers can also be used to add rational numbers.

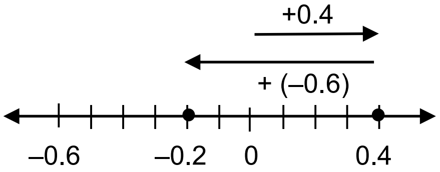
For example:

Adding integers



$4 + (-6) = -2$

Adding rational numbers



$0.4 + (-0.6) = -0.2$

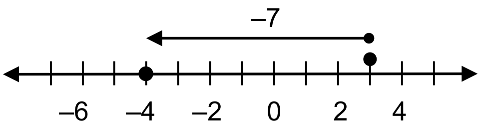
You can also express this calculation in fraction form:

$$\frac{4}{10} + \left(-\frac{6}{10}\right) = -\frac{2}{10}$$

- The number line model you used to subtract integers can also be used to subtract rational numbers.

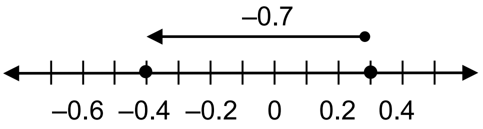
For example:

To subtract (–4) – 3, find 3 and –4 on the number line. Figure out how far and in what direction you need to go to get from 3 to –4.



$-4 - 3 = -7$

To subtract (–0.4) – 0.3, find 0.3 and –0.4 on the number line. Figure out how far and in what direction you need to go to get from 0.3 to –0.4.



$-0.4 - 0.3 = -0.7$

You can also express this calculation in fraction form:

$$-\frac{4}{10} - \frac{3}{10} = -\frac{7}{10}$$

- When you add rational numbers, it helps to predict the sign of the sum. To do this, ignore the signs of the numbers and look for the greater number. The sum will have the sign of that number. This is because the greater number moves you farther to the left than to the right of zero if it is negative, or farther to the right than to the left of zero if it is positive.

For example:

0.4 + (–0.6) is negative because 0.6 > 0.4 and –0.6 is negative.

–0.4 + 0.6 is positive because 0.6 > 0.4 and 0.6 is positive.

- When you subtract rational numbers, you can determine the sign of the difference by thinking of which direction you move on a number line.

\- If you are going to a number farther left, the difference will be negative.

For example: –0.4 – (+0.6) = –1 because from +0.6 to –0.4 is 1 unit to the left.

\- If you are going to a number farther right, the difference will be positive.

For example: +0.4 – (–0.6) = +1 because from –0.6 to +0.4 is 1 to the right.

- You can also think of subtraction as adding the opposite.

For example:

0.4 – 0.6 is negative because 0.4 – 0.6 = 0.4 + (–0.6), and 0.6 > 0.4.

–0.4 – (–0.6) is positive because –0.4 – (–0.6) = –0.4 + 0.6, and 0.6 > 0.4.

- You can multiply and divide rational numbers using the same methods you used for fractions and decimals. To determine the sign of the answer, use what you know about multiplying and dividing integers.

negative × negative = positive positive × positive = positive negative × positive = negative positive × negative = negative	negative ÷ negative = positive positive ÷ positive = positive negative ÷ positive = negative positive ÷ negative = negative

B. Write equations to represent the solutions to **part A i) and ii)**.

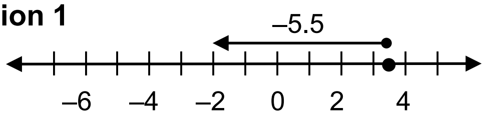
Examples

Example 1 Adding and Subtracting Rational Numbers in Decimal Form

Calculate. **a)** 3.5 + (–5.5) **b)** 2.5 – (–3.1)

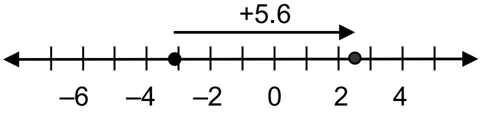
Solution 1

a)



3.5 + (–5.5) = –2

b)



2.5 – (–3.1) = 5.6

Thinking

a) I started at 3.5 and moved to the left 5.5 in order to add -5.5

b) To get from -3.1 to 2.5, I had to go 5.6 to the right.

Example 1 Adding and Subtracting Rational Numbers in Decimal Form [Cont'd]

Solution 2

a) $3.5 + (-5.2) = -1.7$

b) $2.5 - (-3.1) = 2.5 + (+3.1) = +5.6$

Thinking

a) Since 5.2 > 3.5, I knew the difference would be negative.

b) I knew that subtracting is the same as adding the opposite.

Example 2 Adding and Subtracting Rational Numbers in Fraction Form

Calculate. **a)** $-\frac{3}{4} + 1\frac{1}{3}$ **b)** $1\frac{1}{6} - 3\frac{1}{2}$

Solution

a) $-\frac{3}{4} + 1\frac{1}{3} = 1\frac{1}{3} - \frac{3}{4}$

$= \frac{4}{3} - \frac{3}{4}$

$= \frac{16}{12} - \frac{9}{12} = \frac{7}{12}$

b) $1\frac{1}{6} - 3\frac{1}{2} \rightarrow 3\frac{1}{2} - 1\frac{1}{6}$

$= \frac{7}{2} - \frac{7}{6} \quad = \frac{21}{6} - \frac{7}{6}$

$= \frac{14}{6} = \frac{7}{3} = 2\frac{1}{3}$

$3\frac{1}{2} - 1\frac{1}{6} = 2\frac{1}{3}$,

so $1\frac{1}{6} - 3\frac{1}{2} = -2\frac{1}{3}$.

Thinking

a) $-\frac{3}{4} + 1\frac{1}{3} = 1\frac{1}{3} + (-\frac{3}{4})$ and $1\frac{1}{3} + (-\frac{3}{4}) = 1\frac{1}{3} - \frac{3}{4}$ (because adding a negative is the same as subtracting its positive opposite).

- A positive sum makes sense since $1\frac{1}{3} > \frac{3}{4}$, and $1\frac{1}{3}$ is positive.

b) Instead of subtracting $1\frac{1}{6} - 3\frac{1}{2}$, I subtracted $3\frac{1}{2} - 1\frac{1}{6}$ and then gave the answer the opposite sign.

- A negative difference makes sense since $3\frac{1}{2} > 1\frac{1}{6}$, and $3\frac{1}{2}$ is negative.

Example 3 Multiplying and Dividing Rational Numbers

Calculate. **a)** $-\frac{3}{8} \div (-1\frac{1}{4})$ **b)** -5.2×6.4

Solution

a) $-\frac{3}{8} \div (-1\frac{1}{4}) \rightarrow \frac{3}{8} \div 1\frac{1}{4}$

$= \frac{3}{8} \div \frac{5}{4} \quad = \frac{3}{8} \div \frac{10}{8}$

$= \frac{3}{10}$

Thinking

a) I knew the quotient of two negative numbers is positive, so I ignored the signs and divided.

- I divided by using common denominators.

b) -5.2×6.4 If $52 \times 64 = 3328$, then $5.2 \times 6.4 = 33.28$. If $5.2 \times 6.4 = 33.28$, then $-5.2 \times 6.4 = -33.28$ $-5 \times 6 = -30$, so -33.28 makes sense.	**b)** I knew the product would have two decimal places because there was a total of two decimal places in the two factors. • I multiplied 52 × 64 to find the digits for the answer. Then I inserted the decimal point where it belonged. • I knew that the product of a negative and positive number is negative. • I checked my answer by estimating.

Practising and Applying

1. a) Calculate.

i) $-\frac{7}{8} + \frac{1}{6}$ **ii)** $\frac{1}{6} - \frac{7}{8}$

iii) $-\frac{7}{8} - (-\frac{1}{6})$ **iv)** $\frac{1}{6} + (-\frac{7}{8})$

b) What was the same about each calculation in **part a)**? What was different?

2. Calculate.

a) $73.48 + (-37.3)$

b) $-110.3 - 24.25$

c) $-5.33 + (-16.77)$

d) $-0.75 - (-1.7)$

3. Calculate.

a) $\frac{2}{3} \times (-1\frac{1}{8})$

b) $-5.1 \times (-16.3)$

c) $3\frac{3}{4} \div (-4\frac{2}{7})$

d) $-52.25 \div (-5.5)$

4. a) Tell whether each answer is negative or positive. Explain how you know.

i) -27.4×32.1 **ii)** $-5\frac{7}{8} + 12\frac{1}{3}$

iii) $34\frac{2}{3} \div (-4\frac{1}{5})$ **iv)** $-5.86 - (-7.15)$

b) Estimate the answer to each calculation in **part a)**.

5. a) The chart below shows the results of one day's trading for some stocks on the Hong Kong Stock Exchange. Complete the last column of the table:

Company	Opening price	Change	Closing price
i) China Foods	4.66	–0.05	
ii) BYD Co.	38.35	–1.55	
iii) Cisco Systems	204.00	–5.25	
iv) Cosco	6.14	+0.36	

b) The year's high for China Foods was 10.02. What change would be needed from its opening price to match the year's high?

c) The year's low for BYD Co. was 27.65. What change would be needed from its opening price to match the year's low?

6. How do you know the sign of the answer in each case? Use examples to help you explain.

a) when you add two rational numbers

b) when you subtract two rational numbers

c) when you multiply or divide two rational numbers

4.3.3 Order of Operations

Try This

A. Use all of the numbers $\frac{1}{2}$, $\frac{5}{8}$, and 10, and any operations (+, –, ×, ÷).

Write an expression that has each value. **i)** 8 **ii)** $\frac{1}{32}$

- When you add, subtract, multiply, or divide rational numbers, the order in which you do the calculations affects the answer. It is the same as with whole numbers and integers.

For example:

If you calculate the expression (–2.5) + 4.5 × 6 ÷ 0.2 from left to right, you get an answer of 60.

If you calculate the expression (–2.5) + 4.5 × 6 ÷ 0.2 from right to left, you get an answer of 132.5.

(–2.5) + 4.5 = 2
2 × 6 = 12
12 ÷ 0.2 = **60**

6 ÷ 0.2 = 30
4.5 × 30 = 135
(–2.5) + 135 = **132.5**

- To make sure everyone gets the same answer for the same rational number calculation, use the same order of operations rules that you used for integers, decimals, and fractions.

Order of Operations Rules

- First, do any calculations inside brackets.
- Next, divide and multiply, from left to right.
- Finally, add and subtract, from left to right.

For example:

$$
\begin{aligned}
(-2.5) + 4.5 \times 6 \div 0.2 &= (-2.5) + \mathbf{27} \div 0.2 && \text{First, multiply } 4.5 \times 6. \\
&= (-2.5) + \mathbf{135} && \text{Next, divide } 27 \div 0.2. \\
&= \mathbf{132.5} && \text{Finally, add } (-2.5) + 135.
\end{aligned}
$$

- When an expression has brackets inside brackets, you can use both round and square brackets to show which numbers are being combined.

For example:

$$
\begin{aligned}
[-\tfrac{1}{2} + \tfrac{3}{4} \times (\tfrac{2}{3} - \tfrac{1}{5})] \div 1\tfrac{1}{4} &= [-\tfrac{1}{2} + \tfrac{3}{4} \times \tfrac{7}{15}] \div 1\tfrac{1}{4} && \text{First, subtract } (\tfrac{2}{3} - \tfrac{1}{5}). \\
&= [-\tfrac{1}{2} + \tfrac{7}{20}] \div 1\tfrac{1}{4} && \text{Then multiply } \tfrac{3}{4} \times \tfrac{7}{15}. \\
&= -\tfrac{3}{20} \div \tfrac{5}{4} && \text{Then add } [-\tfrac{1}{2} + \tfrac{7}{20}]. \\
&= -\tfrac{3}{25} && \text{Finally, divide } -\tfrac{3}{20} \div \tfrac{5}{4}.
\end{aligned}
$$

- The brackets used around individual numbers serve a special purpose. When they are used around a negative number that follows an operation sign, it is to make the calculation easier to interpret. Sometimes brackets are used around every negative number.

For example: You might write a calculation as –3.5 × **(**–5.4**)** or as **(**–3.5**)** × **(**–5.4**)**.

- Sometimes a numerator or denominator contains operations to be performed. First calculate the expression in the numerator as though all of it is in brackets. Then calculate the expression in the denominator as though all of it is in brackets. Last, divide the numerator by the denominator.

For example:

$$\frac{-3+\frac{1}{2}}{-\frac{2}{3}-\frac{4}{5}} = (-3 + \frac{1}{2}) \div (-\frac{2}{3} - \frac{4}{5}) = -2\frac{1}{2} \div (-\frac{10}{15} - \frac{12}{15}) = -2\frac{1}{2} \div (-\frac{22}{15})$$

$$= -\frac{5}{2} \div (-\frac{22}{15}) = \frac{5}{2} \times \frac{15}{22} = \frac{75}{44}$$

B. Use what you have learned about the order of operations to show how to use brackets to get two different answers for $\frac{1}{2} + \frac{5}{8} \div 10$.

Examples

Example 1 Applying the Order of Operations Rules

Calculate. **a)** 10.7 – (4.3 + 5.7 × 5.1) + (–7.1) **b)** $\frac{-\frac{1}{2}-(-\frac{3}{4})}{\frac{2}{3}-\frac{1}{6}}$

Solution

a)

$$\begin{aligned} & 10.7 - (4.3 + 5.7 \times 5.1) + (-7.1) \\ &= 10.7 - (4.3 + 29.07) + (-7.1) \\ &= 10.7 - 33.37 + (-7.1) \\ &= -22.67 + (-7.1) \\ &= -29.77 \end{aligned}$$

b) $-\frac{1}{2} - (-\frac{3}{4}) = -\frac{1}{2} + \frac{3}{4} = \frac{1}{4}$

$\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

$$\frac{-\frac{1}{2}-(-\frac{3}{4})}{\frac{2}{3}-\frac{1}{6}} = \frac{1}{4} \div \frac{1}{2} = \frac{1}{4} \times 2 = \frac{1}{2}$$

Thinking

a) I first did the calculations inside the brackets, starting with the multiplication.

- Then I did the subtraction and addition from left to right.

b) I first calculated the expressions in the numerator and the denominator.

- Then I divided the numerator by the denominator.

Example 2 Using the Order of Operations Rules to Solve Problems

Place brackets in $\frac{3}{4} \times \frac{1}{2} + \frac{1}{6} \div \frac{1}{3} + \frac{1}{2}$ so that it has each value. **a)** 2 **b)** $\frac{3}{5}$

Solution

a) $\frac{3}{4} \times \frac{1}{2} + \frac{1}{6} \div \frac{1}{3} + \frac{1}{2}$

$= \frac{3}{8} + \frac{1}{2} + \frac{1}{2}$

$= 1\frac{3}{8}$

$\frac{3}{4} \times \left(\frac{1}{2} + \frac{1}{6}\right) \div \frac{1}{3} + \frac{1}{2}$

$= \frac{3}{4} \times \frac{2}{3} \div \frac{1}{3} + \frac{1}{2}$

$= \frac{1}{2} \div \frac{1}{3} + \frac{1}{2}$

$= \frac{3}{2} + \frac{1}{2}$

$= 2$

b) $\frac{3}{4} \times \left(\frac{1}{2} + \frac{1}{6}\right) \div \left(\frac{1}{3} + \frac{1}{2}\right)$

$= \frac{3}{4} \times \frac{4}{6} \div \left(\frac{2}{6} + \frac{3}{6}\right)$

$= \frac{3}{4} \times \frac{4}{6} \div \frac{5}{6}$

$= \frac{3}{6} \div \frac{5}{6}$

$= \frac{3}{5}$

Thinking

a) I first calculated the expression without any brackets, but that didn't give me an answer of 2.

- Then I tried it with brackets around the first addition so I would have to do it first. That gave me an answer of 2.

b) I had to use two sets of brackets, each around an addition, to get an answer of $\frac{3}{5}$.

Example 3 Estimating

Estimate $5\frac{3}{4} - (8\frac{1}{3} + 6\frac{7}{8} \times 5\frac{1}{9}) \div (-7\frac{3}{8})$.

Solution

$5\frac{3}{4} - (8\frac{1}{3} + 6\frac{7}{8} \times 5\frac{1}{9}) \div (-7\frac{3}{8})$

$\approx 6 - (8 + 7 \times 5) \div (-7)$

$= 6 - 43 \div (-7)$

$\approx 6 - 42 \div (-7)$

$= 6 - (-6)$

$= 12$

Thinking

- I rounded each mixed number to the nearest integer.
- I used 42 ÷ (–7) as an estimate for 43 ÷ (–7) because 42 was divisible by 7.

Practising and Applying

1. Calculate.

a) $(\frac{1}{2} + \frac{1}{3}) \times \frac{6}{7}$

b) $\frac{2}{5} \times \frac{1}{2} + \frac{3}{4} \div 1\frac{1}{5}$

c) $\frac{2}{3} - \frac{1}{7} \times (\frac{1}{2} + \frac{3}{8})$

d) $11.5 - (3.4 \div 2 + 3) + (-0.9)$

e) $19.5 - 15.8 \times 3$

f) $-4.5 - 4.4 + (-5.2) \div 2$

2. Use brackets to make each true.

a) $3.6 + 6 \div 3.5 - 1.1 + 3 = 7$

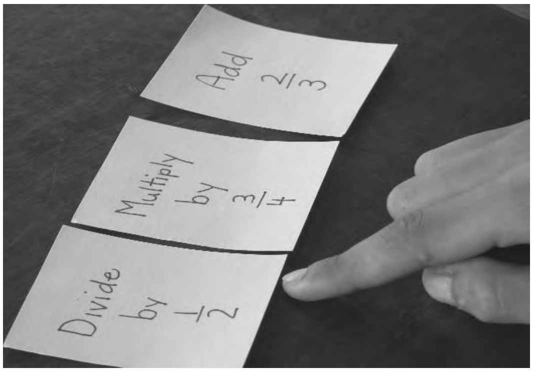
b) $\frac{5}{8} \div \frac{1}{2} + \frac{1}{3} \times \frac{3}{5} = \frac{9}{20}$

3. Eden is playing a card game.
She gets three cards at a time.
She starts with a score of 1 and can use the cards in any order to change her score. Her three cards say these things:

- Divide by $\frac{1}{2}$.
- Add $\frac{2}{3}$.
- Multiply by $\frac{3}{4}$.

a) In what order should Eden play her cards to get the greatest score?

b) In what order would she play her cards to get the least score?



4. Fill in the missing operation signs to make this equation true.

11.2 ■ (–5.4) ■ 2.7 ■ (–9) = 4.2

5. Predict whether the result of each expression is positive or negative. Explain how you know, without calculating.

a) $-3.2 \times (-5.5) \div (-1.6) \times 3.2 \div (-0.5)$

b) $-1\frac{3}{4} \times 4\frac{2}{3} \div (-1\frac{1}{2}) \times 1\frac{3}{8} \times (-3\frac{1}{6})$

6. Estimate the result of each expression in **question 5**. Show how you estimated.

7. Calculate.

a) $\frac{-1.7-(-5.4\div 2)}{2.3\times 5-1.5}$

b) $\frac{\frac{1}{5}+\frac{3}{4}\times\frac{2}{3}}{(-\frac{5}{2})\div\frac{1}{6}+8}$

8. Karchung calculated an expression.

- It had three different operations.
- He added before he divided.
- He multiplied before he divided.

Write a possible expression. Explain how you know it could be Karchung's expression.

GAME: Target One

This game can be played by two or three players.

Make three of each of the following cards:

$\boxed{\frac{1}{2}}$ $\boxed{\frac{1}{3}}$ $\boxed{\frac{2}{3}}$ $\boxed{\frac{1}{4}}$ $\boxed{\frac{3}{4}}$ $\boxed{\frac{1}{6}}$ $\boxed{\frac{5}{6}}$ $\boxed{\frac{1}{8}}$ $\boxed{\frac{3}{8}}$ $\boxed{\frac{5}{8}}$ $\boxed{\frac{7}{8}}$

Make one of each of the following cards:

$\boxed{\frac{3}{2}}$ $\boxed{\frac{4}{3}}$ $\boxed{\frac{8}{3}}$ $\boxed{\frac{6}{5}}$ $\boxed{\frac{8}{5}}$ $\boxed{\frac{8}{7}}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$ $\boxed{6}$ $\boxed{8}$

This is how to play the game:

- Shuffle the cards and deal five cards face up to each player.
- The object of the game is to add, subtract, multiply, and divide the numbers on the cards to get an answer of 1, using as many cards as possible. You get one point for each card you use.

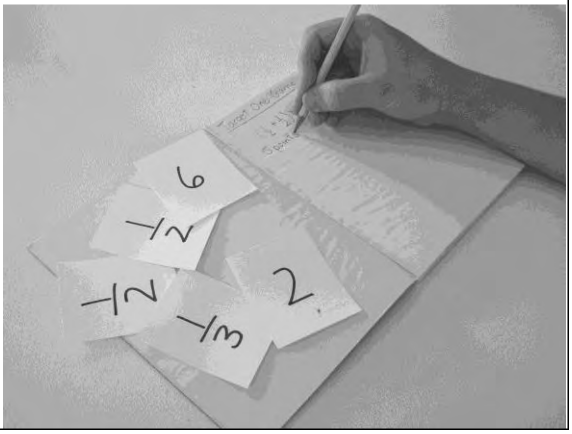
For example:

A player with $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{3}$, 6, and 2 might add two cards to get 1: $\frac{1}{2} + \frac{1}{2} = 1$

The player earns two points because he or she used two cards.

If he or she used all five cards like this: $(\frac{1}{2} + \frac{1}{2}) \times 6 \times \frac{1}{3} \div 2 = 1$, he or she would get five points.

- At the end of each round, each player draws as many additional cards as necessary to have five cards to play the next round.
- If you cannot get an answer of 1 on your turn, choose two of your cards to discard at the bottom of the deck. Draw two new cards from the top of the deck.
- The player with the most points after three rounds wins.



UNIT 4 Revision

1. a) Calculate.

i) $4\frac{1}{4} + 3\frac{3}{8}$

ii) $3\frac{5}{9} + 7\frac{2}{9}$

iii) $11\frac{3}{4} - 6\frac{1}{2}$

iv) $8\frac{5}{6} - 3\frac{2}{3}$

b) Choose one calculation from **part a)**. Explain how you can find the answer using mental math.

2. Which of these three numbers is closest to $5\frac{7}{8} + 7\frac{1}{4} - 3\frac{5}{9}$?

9, $9\frac{1}{2}$, 10, or $10\frac{1}{2}$

Explain how you know.

3. Chali Maya ran $8\frac{5}{6}$ laps on a running track. Then Jamyang ran $6\frac{3}{4}$ laps.

a) Altogether, how many laps did they run?

b) How many more laps did Chali Maya run than Jamyang?

4. Find each product.

a) $\frac{3}{4} \times \frac{7}{8}$

b) $\frac{2}{3} \times \frac{7}{12}$

c) $\frac{3}{7} \times \frac{7}{9}$

d) $\frac{2}{3} \times \frac{9}{10}$

5. $\frac{3}{4}$ of Chandra's garden is planted in vegetables. About $\frac{1}{7}$ of the vegetable area contains turnips. What fraction of the garden area is planted in turnips?

6. Multiply.

a) $3\frac{3}{7} \times 1\frac{1}{2}$

b) $3\frac{1}{3} \times 2\frac{3}{5}$

7. A recipe calls for $1\frac{1}{4}$ cups flour.

a) How much flour would you need if you were making half the recipe?

b) How much flour would you need if you were making $3\frac{1}{2}$ times the recipe?

8. Divide.

a) $\frac{2}{7} \div \frac{3}{7}$

b) $\frac{3}{8} \div \frac{1}{2}$

c) $\frac{2}{3} \div \frac{5}{6}$

d) $6 \div \frac{3}{8}$

9. Tandin has a $\frac{1}{8}$ cup measuring cup. How many times does he have to fill his cup to measure out $\frac{3}{4}$ cup of rice?

10. a) About how many times does $\frac{1}{6}$ fit into $\frac{3}{5}$?

b) Draw a picture to show this.

11. Find each quotient.

a) $\frac{6}{7} \div \frac{4}{9}$

b) $\frac{3}{4} \div \frac{4}{7}$

12. Divide.

a) $2\frac{1}{3} \div 1\frac{1}{2}$

b) $3\frac{3}{8} \div 1\frac{7}{8}$

c) $5\frac{1}{3} \div 6\frac{2}{5}$

d) $7 \div 10\frac{1}{2}$

13. a) How many scarves can be made from $7\frac{1}{2}$ m of fabric, if each scarf requires $1\frac{3}{8}$ m of fabric?

b) Will any fabric be left over? If so, how much?

14. The chart shows the changes in the prices of stocks on the Hong Kong Stock Exchange in one day.

Stock	Change in value (Hong Kong Dollars)
AKM Industrial	–0.07
Aluminum Corp.	+1.18
Asian Union	–0.03
Associated Int. Hotel	–0.50
China Water	+0.06

a) Which stock performed the best on that day?

b) Which stock performed the worst?

c) Put the losses and gains in order from greatest loss to greatest gain.

15. Calculate.

a) $-7\frac{1}{3} + (-3\frac{5}{6})$

b) $5.43 - (-9.7)$

c) $2.75 \times (-5.8)$

d) $-6\frac{3}{4} \div (-4\frac{1}{2})$

16. Calculate.

a) $-2\frac{3}{4} \div 1\frac{3}{8} + (\frac{2}{5} + \frac{3}{10}) \times 2\frac{1}{7}$

b) $\frac{-2.5 \div 5 + 6.25 \times 2}{2.4 \div (1.2 - 2.4)}$

17. Insert one pair of brackets in this expression to give each answer.

$$1\frac{3}{4} - \frac{5}{8} \times \frac{2}{3} + \frac{4}{9} \div 1\frac{1}{3}$$

a) $1\frac{1}{12}$

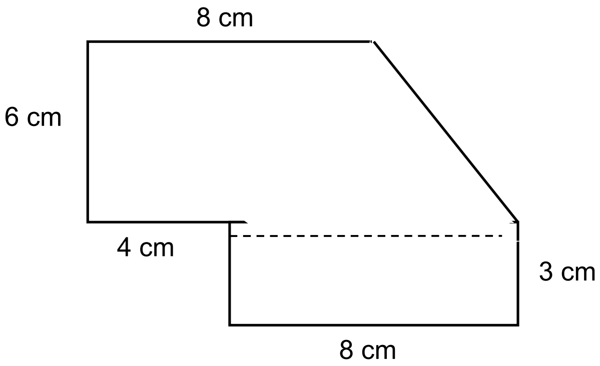
b) $1\frac{1}{8}$

UNIT 5 MEASUREMENT

Getting Started

Use What You Know

A. What is the area of the rectangle part of this composite shape?



B. Use the rectangle's area in **part A** to estimate the area of the whole shape. Explain how you estimated.

C. i) Copy the whole shape. Divide it into parts so you can calculate the area of each part.

ii) What is the actual area of the whole shape?

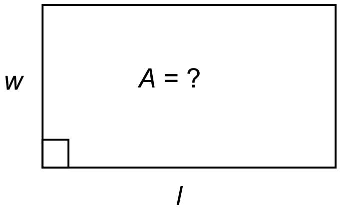
Skills You Will Need

1. Complete.

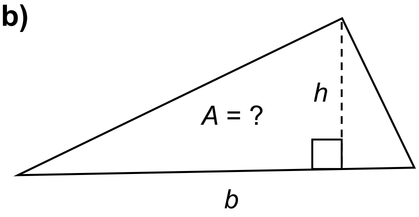
a) 12 m = ___ cm
b) 45,000 m = ___ km
c) 120 cm = ___ mm

2. What is the formula for the area of each shape?

a)

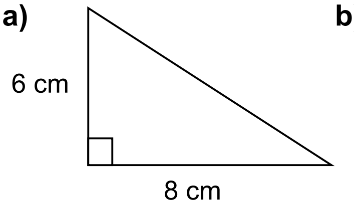


b)

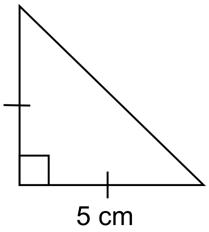


3. Calculate the area of each.

a)



b)



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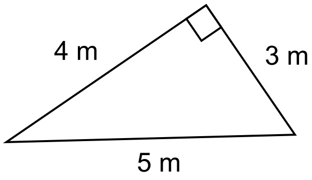
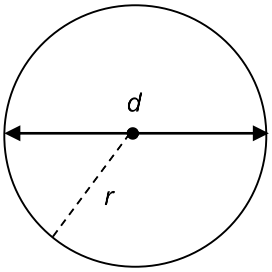


4. a) Where is the circumference of a circle?

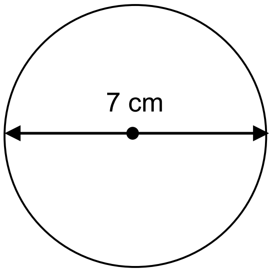
b) What is the ratio of the circumference of a circle to its diameter?

c) Write the formula for the circumference, C, of a circle with radius r.

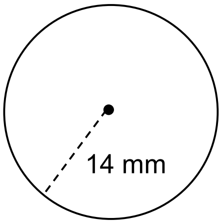


5. Use $\pi = \frac{22}{7}$ to calculate the circumference of each circle.

a)

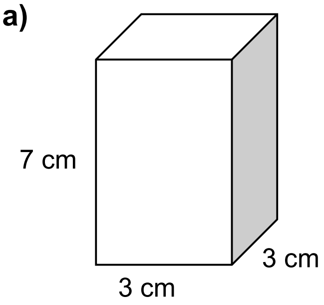


b)

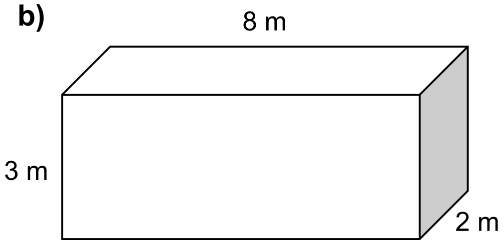


6. Calculate the volume of each rectangular prism.

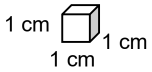
a)



b)

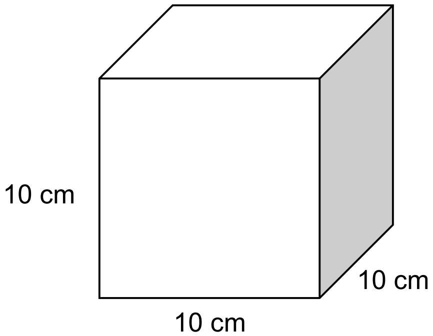


7. a) What is the capacity of a 1 cm cube in millilitres?



b) i) What is the capacity of a 10 cm cube in millitres?

ii) What is the capacity of a 10 cm cube in litres?



8. The volume of a rectangular prism box is 50 cm^3. The base is a 5 cm square. What is the height of the box?

9. The base of a rectangular prism juice carton measures 8 cm by 10 cm. Its height is 12 cm. Can the carton hold 1 L of juice? Explain your answer.

Chapter 1 The Pythagorean Theorem

5.1.1 The Pythagorean Theorem

Try This

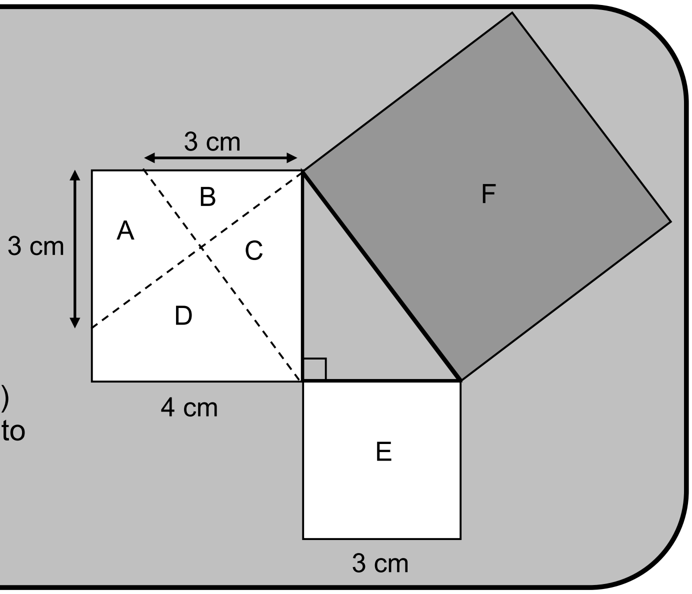
A. i) Copy the diagram using the measurements shown.

ii) Cut out the 3 cm square and the 4 cm square.

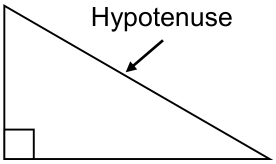
iii) Cut the 4 cm square into four pieces along the dashed lines.

iv) Place the five pieces you have made (A, B, C, D, and E) on top of the large square (F) to cover it without overlapping.

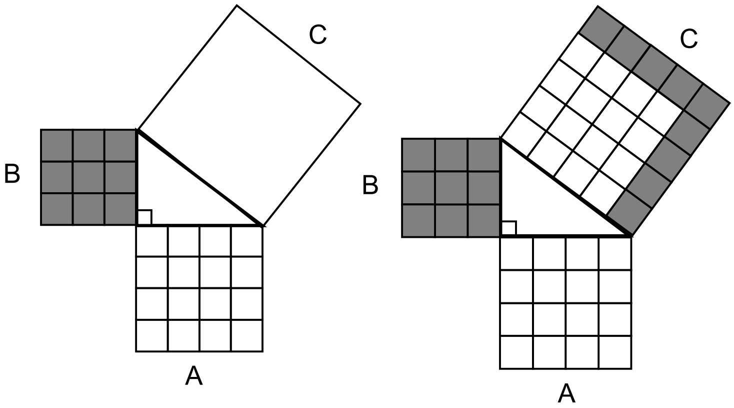
v) What do you notice?



- The longest side of a **right triangle** is called the **hypotenuse**. It has a special relationship with the other two sides, as described below.

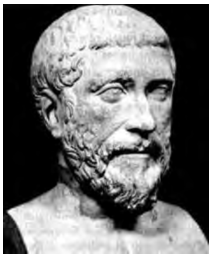


If you build a square on each of the other two sides, the total area of the two smaller squares is the same as the area of the square on the hypotenuse.



Area of A + Area of B = Area of C

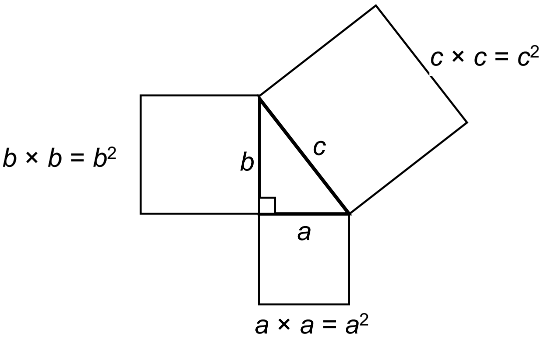
This relationship is named after the Greek mathematician Pythagoras, who lived in the 6th century BCE (Before Common Era).



Pythagoras

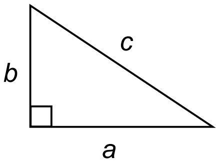
- The **Pythagorean theorem** states that the sum of the **squares** of the lengths of the two shorter sides of a right triangle is equal to the square of the length of the hypotenuse.

This relationship is called a **theorem** because it is always true.



If the hypotenuse of a right triangle is c and the other two sides are a and b, then the following equations show the Pythagorean relationship.

$$\boxed{c^2 = a^2 + b^2 \quad \text{or} \quad a^2 + b^2 = c^2}$$



- If the relationship is true for a triangle, it means the triangle is a right triangle. If the relationship is not true for a triangle, it means it is not a right triangle.
- A set of whole numbers that makes the relationship true is called a **Pythagorean triple**.

For example, {3, 4, and 5} is a Pythagorean triple because $3^2 + 4^2 = 5^2$.

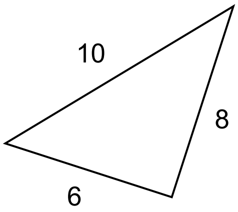
B. How does what you did in **part A** prove the Pythagorean relationship?

Examples

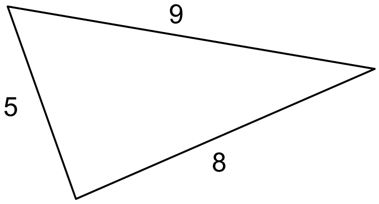
Example 1 Checking for Right Triangles

Is each triangle a right triangle? Show your work.

a)



b)



Solution

a) $6^2 + 8^2 = 36 + 64 = 100$

$10^2 = 100$

$100 = 100$

The triangle is a right triangle.

b) $5^2 + 8^2 = 25 + 64 = 89$

$9^2 = 81$

$81 \neq 89$

The triangle is not a right triangle.

Thinking

For **parts a) and b)**,

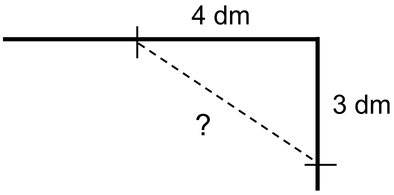
- I squared the length of each of the shorter sides and found the sum of the values.
- Then I squared the length of the longest side to see if it was equal to the sum.

Example 2 Applying the 3-4-5 Rule

The corner of a room usually forms a right angle. How can you tell whether a corner is a right angle, using only a measuring tape or a ruler?

Solution

Put a mark 3 dm from the corner on one wall and another mark 4 dm from the corner on the other wall, both at the same height.



If the direct distance between the two marks is 5 dm, then the corner is a right angle.

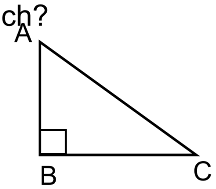
Thinking

- I knew that $3^2 + 4^2 = 5^2$ because $9 + 16 = 25$.
- So the dashed line had to be 5 dm for the corner to be a right angle.

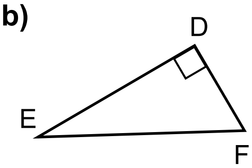
Practising and Applying

1. Which side is the hypotenuse in each?

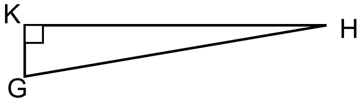
a)



b)

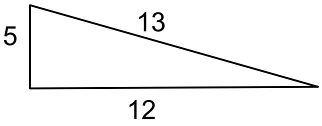


c)

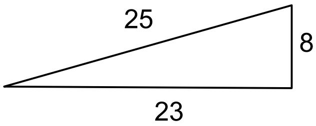


2. Tell whether each triangle is a right triangle. Explain how you know.

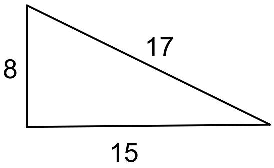
a)



b)



c)



3. a) Is the set of numbers in each row of this chart a Pythagorean triple?

3	4	5
6	8	10
9	12	15
12	16	20
15	20	25

b) What pattern do you notice in the chart above?

c) Continue the pattern to make two more rows of Pythagorean triples.

d) Suppose that each row in the chart represents the sides of a right triangle.

How many right triangles in this pattern have either a 30 cm side or a 30 cm hypotenuse? List all three side lengths for each triangle.

e) A triangle has sides of 1.5 cm, 2 cm, and 2.5 cm. Use the pattern in the chart to explain how you know it is a right triangle.

4. A triangle has sides of 7 cm, 24 cm, and 25 cm. How can you tell whether it is a right triangle?

5.1.2 Applying the Pythagorean Theorem

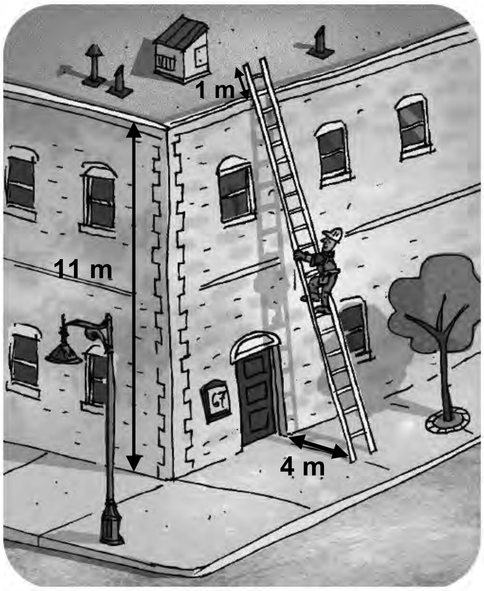
Try This

A wall is 11 m high. For safety reasons

- the base of a ladder must be 4 m away from the bottom of the wall, and
- the ladder must extend 1 m or more above the top of the wall.

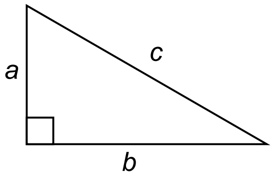
A. i) Sketch and label the triangle formed by the ladder, the wall, and the ground. Mark the right angle.

ii) Is it safe to use a ladder that is 13 m long? How do you know?



- The two shorter sides of a right triangle are called **legs**.
- If you know the lengths of any two sides of a right triangle (two legs, or one leg and the hypotenuse), you can use the Pythagorean theorem to find the length of the third side.

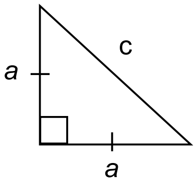
$$c^2 = a^2 + b^2$$
$$a^2 = c^2 - b^2$$
$$b^2 = c^2 - a^2$$



- In an **isosceles right triangle**, the legs are the same length, so *a* = *b*.

\- If you know the length of one leg (*a*), you can find the hypotenuse using this formula:

$$c^2 = 2a^2$$



\- If you know the length of the hypotenuse, you can find the length of the legs using this formula:

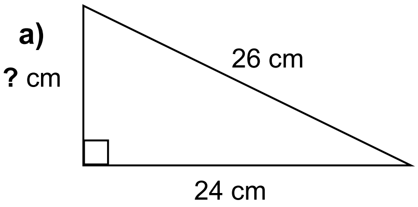
$$a^2 = c^2 \div 2 \text{ or } a^2 = \frac{c^2}{2}$$

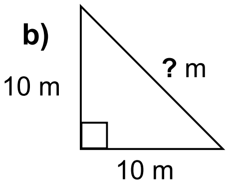
B. How did you use the Pythagorean theorem to solve **part A ii)**?

Examples

Example 1 Finding the Length of a Missing Side

What is the missing side length of each triangle? Round to one decimal place, if necessary.

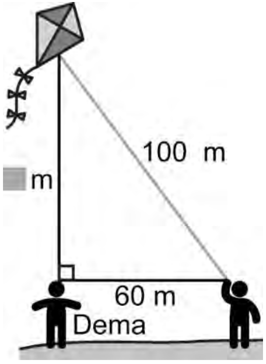




Solution	Thinking
a) $a^2 = c^2 - b^2$ $= 26^2 - 24^2$ $= 676 - 576$ $a^2 = 100$ $a = \sqrt{100} = 10$ The other leg is 10 cm.	**a)** I knew the hypotenuse (c) and one leg (b), so I used $a^2 = c^2 - b^2$ to find the other leg (a). 
b) $c^2 = 2a^2$ $= 2 \times 10^2$ $c^2 = 200$, so $c = \sqrt{200}$ $\sqrt{200} \approx 14.1$ The hypotenuse is about 14.1 m.	**b)** The right triangle was isosceles and I knew the lengths of the two legs (a), so I used $c^2 = 2a^2$ to find the hypotenuse (c). • The square root of 200 is a bit more than 14 because 14 × 14 = 196, 14.1 × 14.1 = 198.81, and 14.2 × 14.2 = 201.64, so $\sqrt{200}$ is closer to 14.1.

Example 2 Using the Pythagorean Theorem to Solve a Problem

Karma is flying a kite with 100 m of string. Dema stands directly under the kite, 60 m from Karma. How high is the kite above Dema?



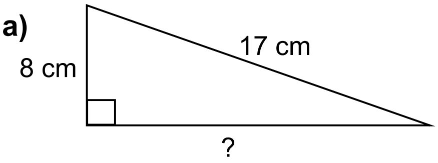
Solution	Thinking
$a^2 = c^2 - b^2$ $= 100^2 - 60^2$ $= 10{,}000 - 3600$ $a^2 = 6400$, so $a = \sqrt{6400}$ $\sqrt{6400} = 80$ The kite is 80 m above Dema.	• I knew what the measure of the hypotenuse and one leg was, so I used $a^2 = c^2 - b^2$. 

Practising and Applying

***Round your answers to one decimal place, if necessary*.**

1. What is each missing side length?

a)



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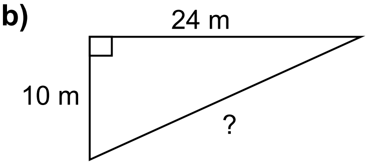
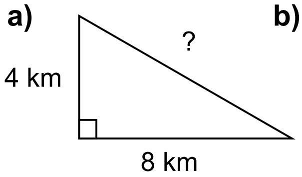


2. What is each missing side length?

a)



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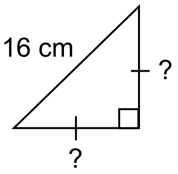
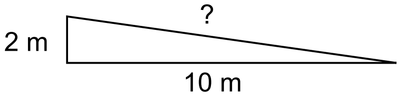
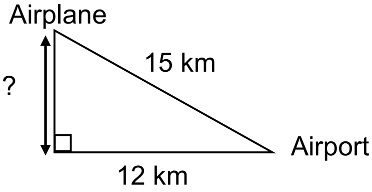


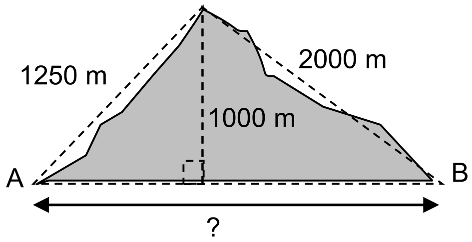
3. A ramp descends 2 m from a point 10 m away. How long is the ramp?



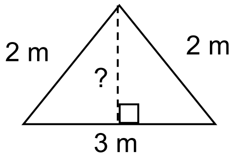
4. An airplane travels 15 km while it climbs after take-off. At that point it is 12 km from the airport. How many kilometres high is the airplane?



5. The diagram shows the distances up each side of a 1000 m peak. Estimate the distance from A to B.



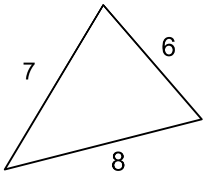
6. How tall is this tent?



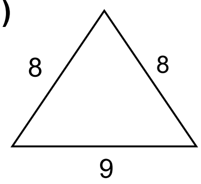
7. a) Copy and complete this chart for the acute triangles below (c is the length of the longest side; a and b are the lengths of the other two sides).

	a^2	b^2	c^2
i)			
ii)			
iii)			

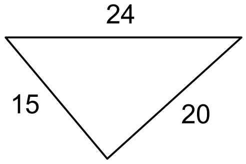
i)



ii)



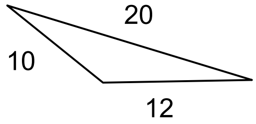
iii)



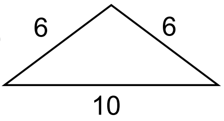
b) Which is greater: c^2 or $a^2 + b^2$?

c) Copy and complete the same chart for these obtuse triangles.

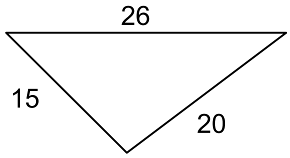
i)



ii)



iii)



d) Which is greater: c^2 or $a^2 + b^2$?

e) How can you use what you learned in **parts a) to d)** to decide whether a triangle is right, acute, or obtuse?

8. Two sides of a right triangle are 16 cm and 30 cm. Can you be certain about the length of the third side? Explain your answer.

Chapter 2 Linear and Area Relationships

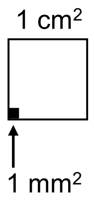
5.2.1 Area and Perimeter Relationships

Try This

A. How many different rectangles can you draw, each with a perimeter of 16 cm? Draw them. (The length of each side must be a whole number of centimetres.)

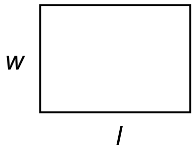
- The **perimeter** of a shape is measured in **linear units** such as millimetres (mm), centimetres (cm), metres (m), or kilometres (km).
- The **area** of a shape is measured in **square units**. The chart below shows the units that are commonly used to measure area.

Area unit	Symbol	Equivalence
Square millimetre	mm^2	
Square centimetre	cm^2	$1\ cm^2 = 100\ mm^2$
Square metre	m^2	$1\ m^2 = 10{,}000\ cm^2$
Square hectometer, or hectare (ha)	ha	$1\ ha = 10{,}000\ m^2$
Square kilometre	km^2	$1\ km^2 = 1{,}000{,}000\ m^2$



- Here are formulas for the area (*A*) and perimeter (*P*) of a rectangle with sides *l* and *w*:

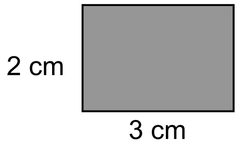
$$A = l \times w \qquad P = 2 \times (l + w) \text{ or } P = 2 \times l + 2 \times w$$



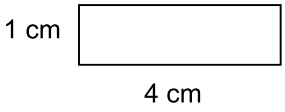
- Two different rectangles can have the same perimeter or the same area.

For example:

- These two rectangles have the same perimeter, but different areas.

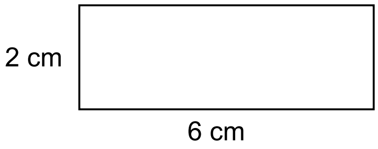


$P = 10$ cm; $A = 6\ cm^2$

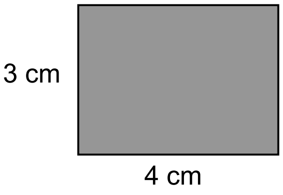


$P = 10$ cm; $A = 4\ cm^2$

- These two rectangles have the same area, but different perimeters.



$P = 16$ cm; $A = 12\ cm^2$



$P = 14$ cm; $A = 12\ cm^2$

B. What is the area of each rectangle you found in **part A**?

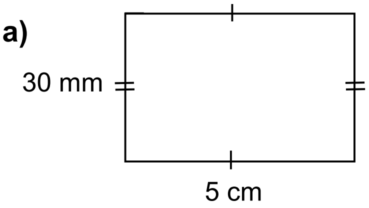
C. Compare the shape of the rectangle with the smallest area to the shape of the rectangle with the greatest area. What do you notice?

Examples

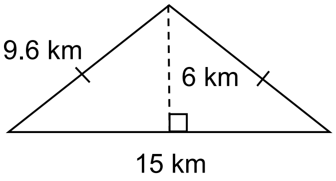
Example 1 Calculating Area and Perimeter

Calculate the area and perimeter of each.

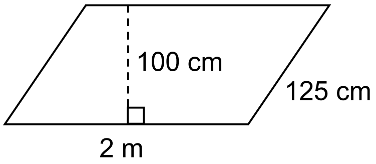
a)



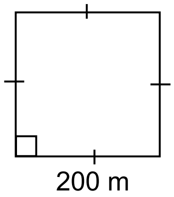
b)



c)



d)



Solution	**Thinking**
a) $P = 2 \times (l + w) = 2 \times (5 + 3)$ $= 2 \times 8$ $= 16$ cm $A = l \times w = 5 \times 3 = 15 \text{ cm}^2$	 **a)** I knew the sides had to be in the same units, so I changed 30 mm to 3 cm (rather than change 5 cm to 500 mm), so I could work with lower numbers.
b) $P = 9.6 + 9.6 + 15 = 34.2$ km $A = b \times h \div 2 = 6 \times 15 \div 2$ $= 90 \div 2$ $= 45 \text{ km}^2$	**b)** The perimeter of a triangle is the sum of the sides.
c) $P = 2 \times (2 + 1.25) = 2 \times 3.25 = 6.5$ m $A = b \times h = 2 \times 1 = 2 \text{ m}^2$	**c)** The perimeter of a parallelogram is twice the sum of any two adjacent sides. • The sides must be in the same units, so I changed 125 cm to 1.25 m and 100 cm to 1 m to work with lower numbers.
d) $P = 4 \times s = 4 \times 200 = 800$ m $A = s^2 = 200 \times 200 = 40{,}000 \text{ m}^2$ $40{,}000 \text{ m}^2 = 4$ ha	**d)** I changed the area to hectares to get a lower number. • I knew that 1 ha = $10{,}000 \text{ m}^2$.

Example 2 Using Appropriate Units

Which units would you use to measure the perimeter and area of each?

a) a postage stamp

b) a small town

c) a large room

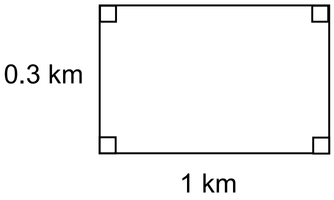
d) a sheet of paper

Solution	**Thinking**
a) Millimetres and square millimetres	**a)** Stamps are small and need small units. 
b) Metres and hectares	**b)** Metres and hectares are appropriate for a small town. A large town would likely be measured in kilometres and square kilometres.
c) Metres and square metres	**c)** Metres, with decimals, are accurate enough for a large room.
d) Centimetres and square centimetres	**d)** I could use millimetres, but the area would be a very large number, so centimetres are better.

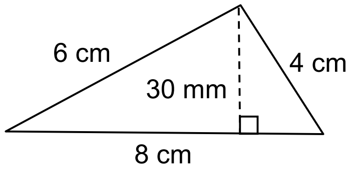
Practising and Applying

1. Find the area and perimeter of each.

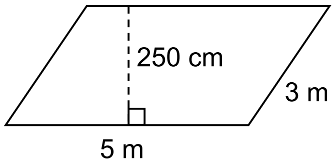
a)



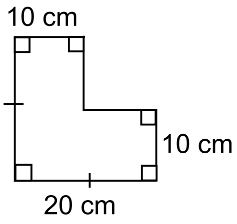
b)



c)



d)



2. What unit would you use to measure the perimeter and area of each?

a) a football field

b) a Nu 1 coin

c) a photograph

d) a dzongkhag

3. a) Draw and label all the rectangles with a perimeter of 24 cm. Each side must be a whole number of centimetres.

b) Describe the rectangles with the least area and the greatest area.

4. a) Draw and label all the rectangles with an area of 24 cm^2. Each side must be a whole number of centimetres.

b) Describe the rectangles with the least perimeter and the greatest perimeter.

5. a) Draw and label a rectangle that is not a square and has an area of 1 cm^2.

b) What is the perimeter?

6. A rectangle has a perimeter of 15 cm and an area of 14 cm^2. Draw the rectangle and label its dimensions.

7. The rectangle on the map below has about the same area as Bhutan.

a) Calculate the area of the rectangle in square kilometres.

b) How does it compare with the actual area of Bhutan (about 47,000 km^2)?

c) What is the perimeter of the rectangle?

d) Why do you think the perimeter of the rectangle is less than the perimeter of Bhutan (about 1075 km)?

8. A rectangle has whole number dimensions.

a) Why will its perimeter always be an even number?

b) When will its area be an even number? Explain your thinking.

9. a) A square has a perimeter of 10 cm. What is its area?

b) A square has an area of 0.25 km^2. What is its perimeter?

10. Draw and label a right triangle with a perimeter of 12 cm and an area of 6 cm^2.

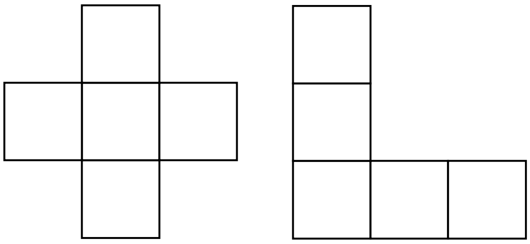
11. a) What are the dimensions of a rectangle with an area of 48 cm^2 and a perimeter of 28 cm?

b) A 1 cm-by-48 cm rectangle has the same area as the rectangle in **part a)**. Compare the shapes and perimeters of the two rectangles.

12. Does knowing the perimeter of a rectangle give you enough information to calculate its area? Explain your thinking.

CONNECTIONS: Pentominos

These two shapes are called pentominos. Each shape has an area of 20 cm^2 because it is made up of five 4 cm^2 squares. The squares are attached to each other along one or more sides.



1. a) Copy the two pentominos onto grid paper. Draw all the other possible pentominos. Cut out all the pentominos.

b) How many different pentominos are there?

2. What do you notice about the perimeters of the pentominos?

GAME: Pentominos

Pentominos is a game of logic and strategy.

Play the game with a partner.

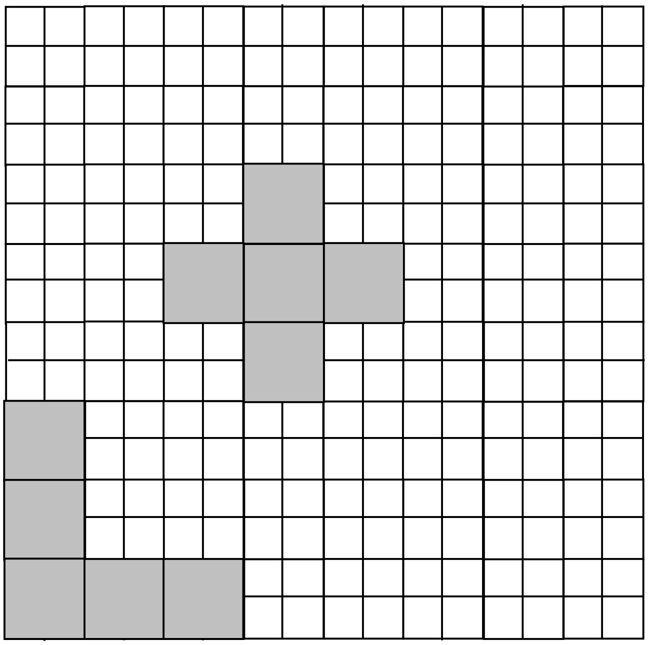
You need the pentominos from the **Connections** on **page 128** and a 16 cm-by-16 cm grid for a game board.

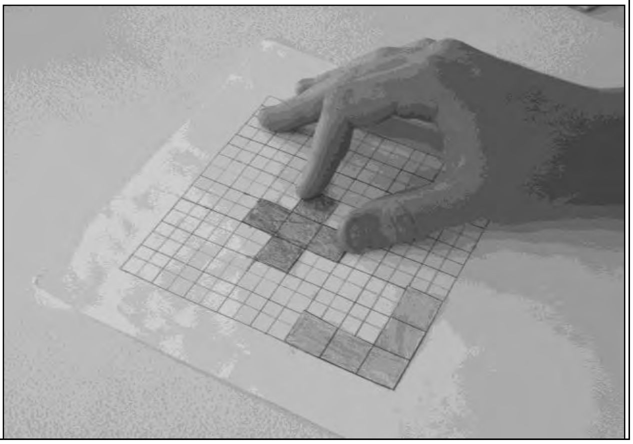
Here are the rules:

- Take turns placing pentominos on the grid.
  - On your turn, choose any unused pentomino and place it on the grid.
  - The pentominos must not overlap or extend beyond the grid.
- The last player who is able to place a pentomino on the board is the winner.

For example:

Here is the game board after each player has placed one pentomino.





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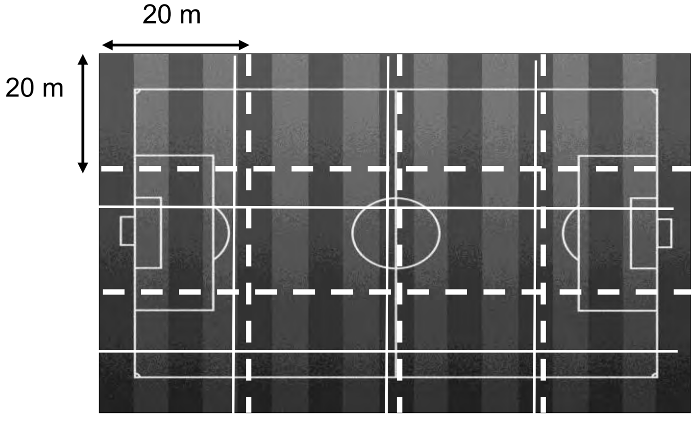
5.2.2 Scale Drawings

Try This

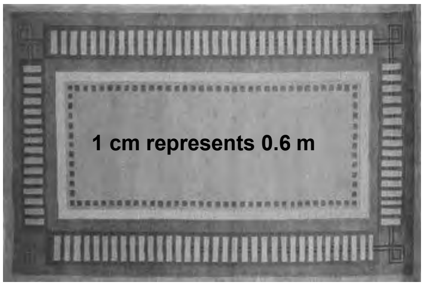
Each of the 12 squares on this football field has 20 m sides.

A. i) What is the length and width of the field?

ii) What is the total area of the field?



- A scale drawing represents a real object or figure in two dimensions, usually at a reduced size but sometimes at an enlarged size.
- A scale drawing has a **key** to show the scale of the drawing, that is, how the drawing's measurements compare to the real measurements.



For example:

The key "1 cm represents 0.6 m" on this scale drawing of a rug means that 1 cm on the drawing represents 0.6 m on the real rug.

- To express the scale as a ratio, the measurements must have the same units:

1 cm represents 0.6 m → 0.6 m = 60 cm → 1 cm : 60 cm → 1 : 60

So the scale ratio of measurements in the scale drawing to the real rug is 1 : 60.

- You can use the scale ratio of a scale drawing to figure out the real dimensions of the object.

- A map is a form of scale drawing with a key or a scale ratio to show the real distances. The key is sometimes a diagram that looks like part of a ruler.

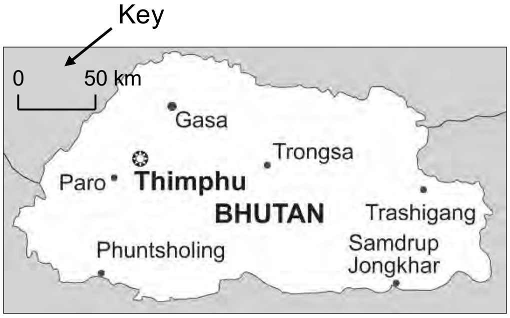
For example:

On the map of Bhutan at the right, the key means "1 cm represents 50 km",
so 1 cm on the map represents a real distance of 50 km.

1 cm = 50 km → 1 cm = 5,000,000 cm

1 : 5,000,000

So the scale ratio of a distance on the map to a real distance is 1 : 5,000,000.



- You can also use the key on a map to figure out the real distances.

For example:

The distance from Thimphu to Phuntsholing on the map is about 1.5 cm.

If 1 cm represents 50 km, then 1.5 cm represents 1.5 × 50 km = 75 km.
The real distance from Thimphu to Phuntsholing is 75 km.

(Note that this is a straight-line distance, not the distance by road, which is 172 km.)

B. i) Make a key for the scale drawing of the football field in **part A**.

ii) What is the scale ratio?

Examples

Example 1 Using the Scale Ratio of a Scale Drawing

If 1 cm represents 10 m, what are the real dimensions of the football ground?

Solution

The width and length of the scale drawing are 7.5 cm by 9 cm.

1 cm represents 10 m

The real width is 7.5 × 10 = 75 m.

The real length is 9 × 10 = 90 m.

The real dimensions of the football ground are 75 m by 90 m.

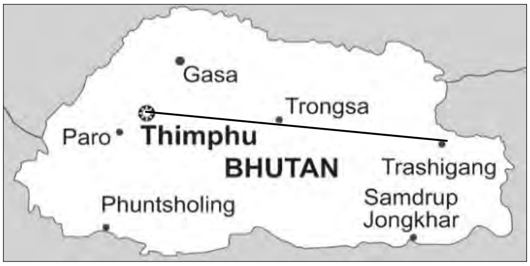
Thinking

- I multiplied the measurements of the scale drawing given in the diagram by the second term of the scale ratio to get the real dimensions.

Example 2 Using the Key From a Map

Use the map to find the straight-line distance from Thimphu to Trashigang.

The scale ratio is 1 : 5,000,000



Solution

The distance is 4 cm on the map.

So the real distance is 4 × 5,000,000 = 20,000,000 cm.

$= \frac{20,000,000}{100,000} = 200 \text{ km}$

The real straight-line distance from Thimphu to Trashigang is 200 km.

Thinking

- I knew that 1 cm on the map represented 5,000,000 cm
- So 4 cm was 4 × 5,000,000
- I divided the answer by 100,000 to change cm to km.

Example 3 Making a Key for a Map

Suppose 2.2 cm on a map represents a real distance of 33 km.

a) What does 1 cm on the map represent?

b) What is the scale ratio of distances on the map to real distances?

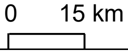
c) Make a key for the map.

Solution

a) If 2.2 cm represents 33 km, then
1 cm represents (33 ÷ 2.2) km = 15 km
1 cm on the map represents 15 km.

b) 15 km = 15 × 1000 m
= 15 × 1000 × 100 cm
= 1,500,000 cm
The ratio is 1 : 1,500,000

c) The key is
1 cm represents 15 km or



Thinking

a) I divided 33 ÷ 2.2 to figure out what 1 cm represents.

b) To make a ratio, both distances need to be in the same units, so I changed kilometres to centimetres.

c) I created a key that showed the number of kilometres each 1 cm represented as an equation and a drawing.

Example 4 Planning a Scale Drawing

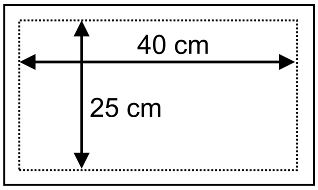
An architect is making plans for a house. The front of the house will have a height of 5 m and a width of 8 m. The drawings for the house will be printed on paper that measures 29 cm by 44 cm. There must be at least a 2 cm margin.

a) What is the height and width of a scale drawing that will fill the drawing area of the paper?

b) What is the key for the scale drawing?

Solution

a) The 2 cm margins leave a drawing area of 25 cm by 40 cm.



The ratio of the sides of the drawing area is 25 : 40 = 5 : 8.

The ratio of the height to the width of the house is 5 : 8.

So the drawing could be 25 cm by 40 cm.

b) If 25 cm represents 5 m, then
1 cm represents 0.2 m (5 ÷ 25 = 0.2).
The key is 1 cm represents 0.2 m.

Thinking

a) I made a sketch to find the actual drawing area.

- I figured that both dimensions would fill the paper if 25 cm represented 5 m.

b) I divided 5 by 25 to find what 1 cm would represent.

Practising and Applying

1. Make a key for each scale drawing in the form “1 cm represents ___”.

a) 5 cm represents 80 cm

b) 5 cm represents 10 m

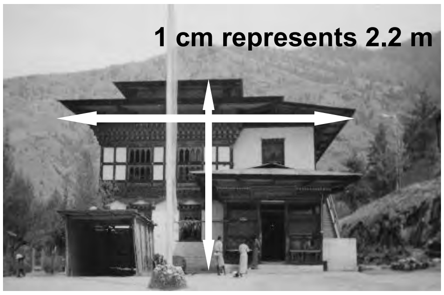
c) 0.5 cm represents 1.5 m

d) 5 cm represents 1.5 mm

2. a) How is **part d)** of **question 1** different from the other parts?

b) When might you use a scale like the scale in **part d)**?

3. a) What is the real height and width of the building in this photo? (Measure the scale photo to the nearest tenth of a centimetre.)



b) Draw a rectangle to represent a scale drawing of the front of the building. Use a key that is different from the key shown.

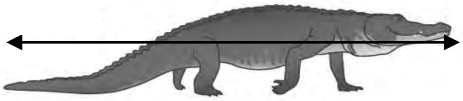
4. An architect makes a drawing of a room that is 9 m by 12 m, using a key of 1 cm represents 2 m. What are the dimensions of the room in the drawing?

5. On a map, 1 cm represents 40 km.

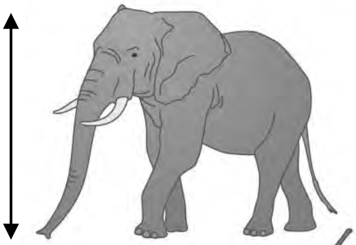
a) What is the real distance between two towns that are 4 cm apart on the map?

b) The real distance between two other towns is 100 km. How far apart are they on the map?

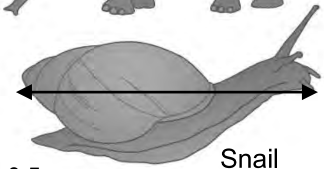
a) 1 cm represents 0.3 m



b) 1 : 100

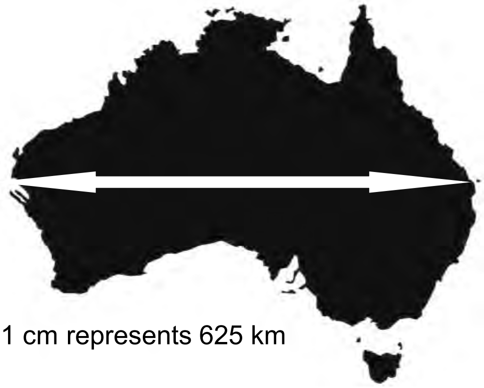


c)



1 cm represents 0.5 cm

7. a) What is the width of Australia? (Measure the map to the nearest tenth of a centimetre.)



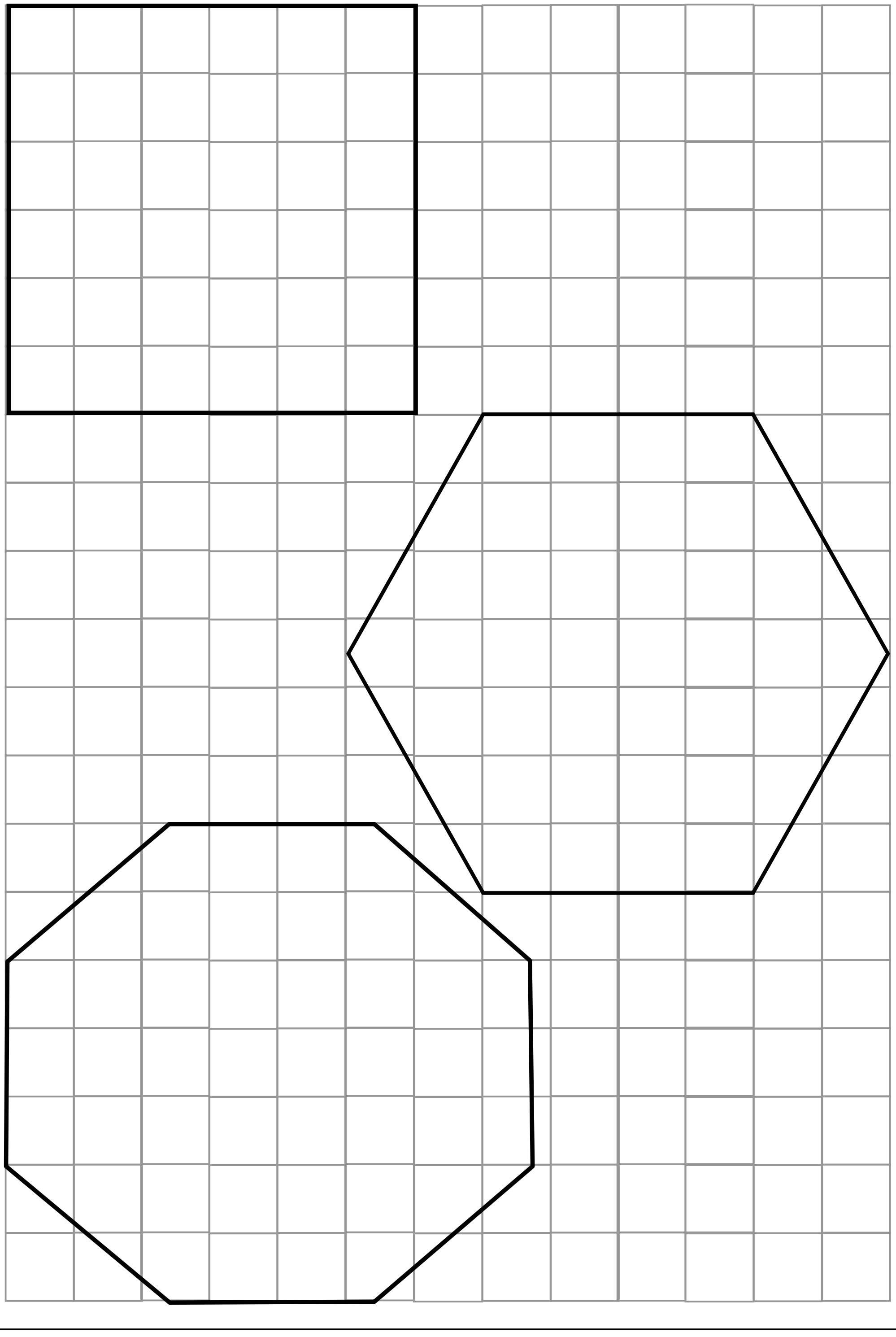
1 cm represents 625 km

b) Bhutan is about 300 km wide. About how many times would Bhutan fit across Australia?

8. You have to make a scale drawing of a building on a sheet of paper that is 26 cm by 34 cm. You need a 2 cm margin all the way around. The real building is 15 m wide and 12 m high. What key would you use? Show your work.

5.2.3 EXPLORE: Estimating the Area of a Circle

Here are three polygons — a square, a **regular hexagon**, and a **regular octagon** — on a centimetre grid.



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A. i) What is the perimeter of each polygon (to the nearest centimetre)? What do you notice?

ii) Find the area of the square. Estimate the area of the hexagon and octagon (to the nearest square centimetre) by counting whole and part grid squares.

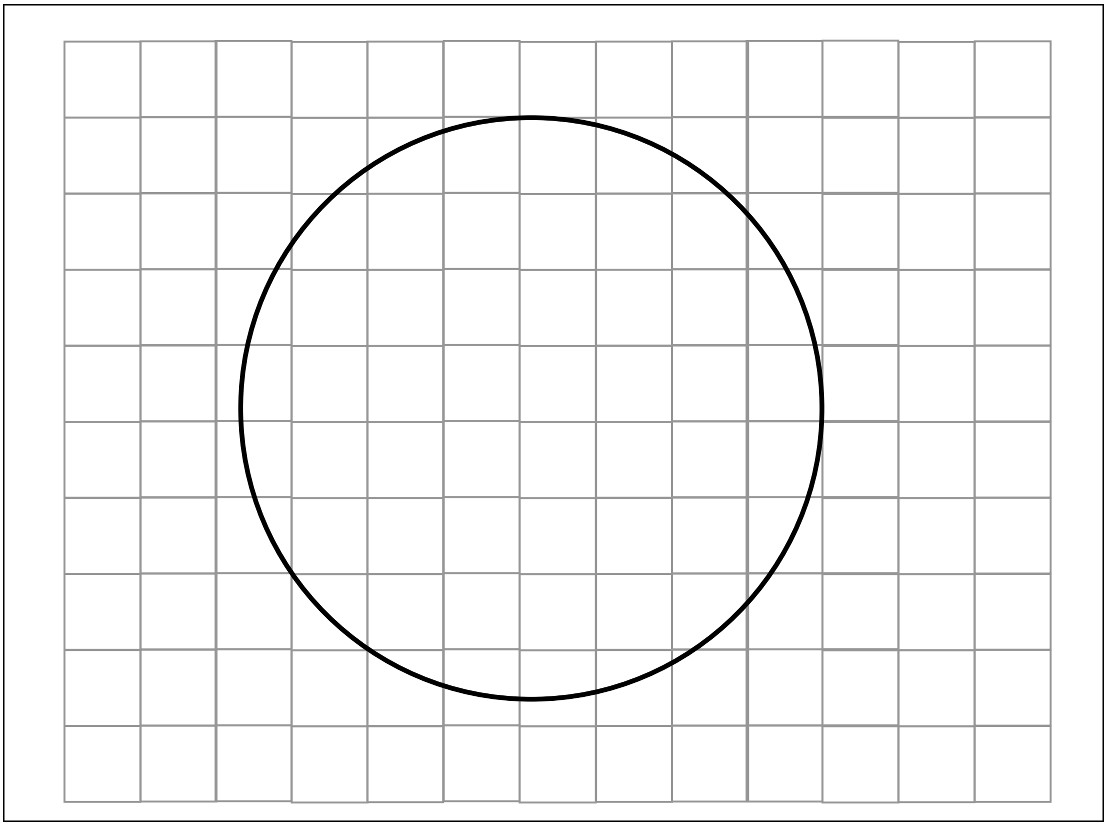
iii) Order the polygons from least to greatest area.

iv) What happened to the area of the three shapes as the number of sides increased? Why do you think that happened, even though the perimeter stayed the same?

B. i) Measure the diameter of the circle below (to the nearest millimetre). Use the diameter and $\pi = \frac{22}{7}$ to calculate the circumference of the circle (to the nearest centimetre). What do you notice?

ii) Use what you noticed in **part A iv)** to predict the area of the circle. Then estimate the area of the circle (to the nearest whole centimetre) by counting whole and part grid squares.

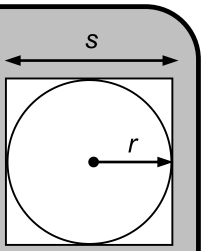
C. Predict the area of a 12-sided regular polygon (a dodecagon) with a perimeter of 24 cm. Explain your prediction.



5.2.4 The Formula for the Area of a Circle

Try This

You can use the area of a square to estimate the area of a circle.



A. i) What is the side length, *s*, of the square shown to the right in terms of the radius, *r*, of the circle?

ii) Write a formula for the area of the square, $A = s^2$, in terms of the radius of the circle, *r*.

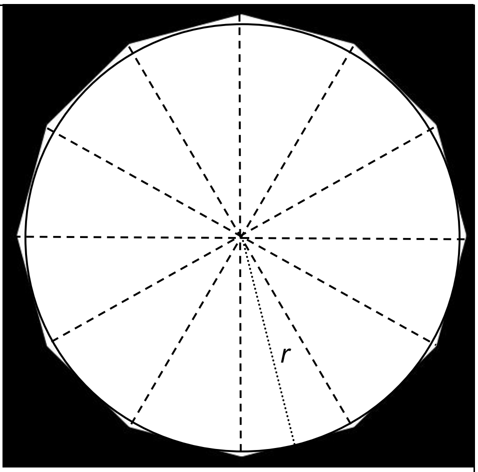
iii) If you use the formula from **part ii)** to estimate the area of the circle, will your estimate be high or low? How do you know?

The formula for the area of a circle is

$$\boxed{A_{\text{circle}} = \pi r^2}$$

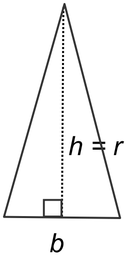
Here is the reason this formula makes sense:

If you draw a circle inside a regular **dodecagon** (a polygon with 12 sides), as shown on the right, you can use the area of a parallelogram to understand the formula for the area of a circle.

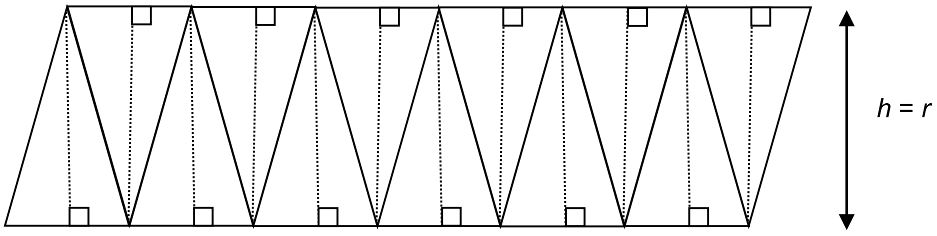


A circle inside a dodecagon

- The area of the dodecagon is almost equal to the area of the circle.
- The perimeter of the dodecagon is almost equal to the **circumference** of the circle.
- The dodecagon is made up of 12 **congruent** triangles like this:
  - The height of each triangle is equal to the **radius** of the circle.
  - The **base** of each triangle is about $\frac{1}{12}$ the circumference of the circle.



- If you arrange the 12 triangles as shown below, they make a **parallelogram**. Its height is equal to the circle's radius and its base is about half of the circle's circumference.



b is about $\frac{1}{2} \times 2\pi r = \pi r$

So, the area of both parallelogram and the circle is $A = b \times h \approx \pi r \times r = \pi r^2$.

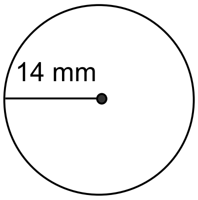
B. Compare the formula from **part A ii)** to the formula for calculating area of the circle. Does the formula for the area of a circle make sense? Why?

Examples

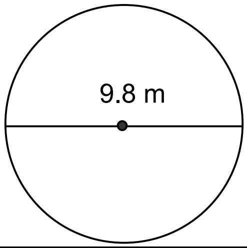
Example 1 Calculating the Area of a Circle

Find the area of each circle. Calculate to one decimal place.

a)



b)



Solution

a) $A = \pi r^2 \approx 3.14 \times 14^2$

$= 615.4 \text{ mm}^2$

b) $A = \pi r^2 = 3.14 \times 4.9^2$

$\approx 75.4 \text{ m}^2$

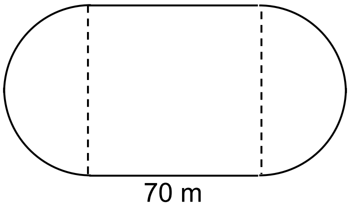
Thinking

a) Because I used 3.14 or $\frac{22}{7}$ for π, I used the approximately equals sign, ≈.

b) The radius is half the diameter, so I used 4.9 instead of 9.8.

Example 2 Solving an Area Problem

A track is made of two semi-circles attached to opposite sides of a 70 m square.
What is the total area inside the track?



Solution

$$A = \pi r^2 + s^2 \approx \frac{22}{7}(35)^2 + 70^2$$

$$\approx 3850 + 4900$$

$$= 8750 \text{ m}^2$$

The area is 8750 m^2.

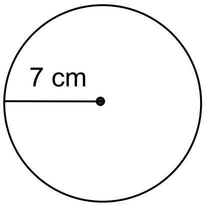
Thinking

- The two semi-circles combine to make a whole circle with a radius that is half the side of the square.
- I combined the formulas for the area of a circle and of a square.

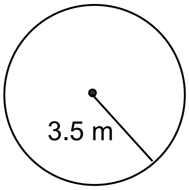
Practising and Applying

1. Calculate the area of each circle to the nearest whole unit. (Use $\pi = \frac{22}{7}$ or 3.14.)

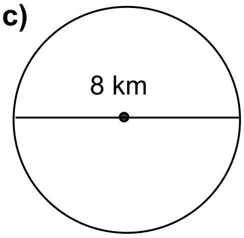
a)



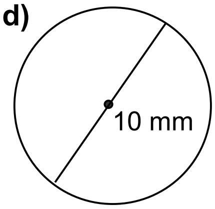
b)



c)



d)



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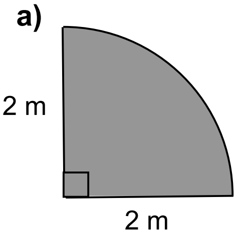
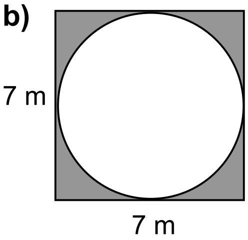
2. Calculate the area of the shaded part to the nearest whole unit.

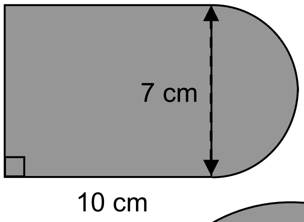
a)



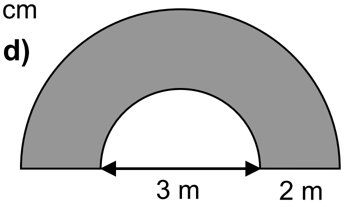
b)



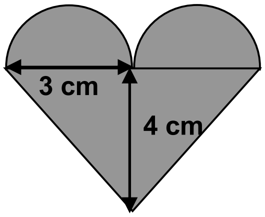
c)



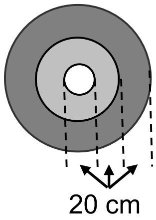
d)



e)

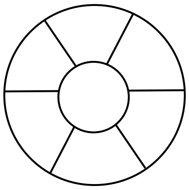


3. The diameter of the inner circle of a target is 20 cm. Each ring is 20 cm wide. What is the area of each section? (Use $\pi = 3.14$.)



4. The diameter of the outer circle of this Wheel of Life is 1 m. The diameter of the inner circle is $\frac{1}{3}$ of the diameter of the outer circle.

What is the area of each section, to the nearest hundred square centimetres?



5. a) A circle has an area of 78.5 m^2. What is its radius? (Use $\pi = 3.14$.)

b) A circle has a circumference of 62.8 cm. What is its area?

6. A circle has an area of π cm^2. What is its circumference in terms of π?

7. Why do you need only one value (the radius, diameter, or circumference) to calculate the area of a circle?

CONNECTIONS: The History of Pi

The Greek mathematician Archimedes used $\frac{22}{7}$ as an approximation for π (pi) in the third century BCE. However, he knew the real value was a little less than $\frac{22}{7}$.

The real value is a decimal that continues forever without a pattern:

$\pi = 3.141592653589793238462643383279502884197169399375105...$

To this day, mathematicians are still trying to figure out more decimal places for pi.

1. a) What is the difference between $\frac{22}{7}$ and π? Write your answer to four decimal places?

b) Which value is closer to π: 3.14 or $\frac{22}{7}$?

2. In the 17th century, this formula for π was used: $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - ...$

a) Use the five terms shown to estimate π to four decimal places.

b) Is the answer to **part a)** as close an estimate as 3.14 or $\frac{22}{7}$? Explain.

5.2.5 Applying Area Formulas

Try This

A 4 L can of paint covers a 25 m^2 area.

A. The walls of a room are 2 m high. What length of wall can be covered by one can of paint?

B. A room has a total area of 90 m^2 to paint. How many cans of paint are needed to give it two coats of paint? Show your work.

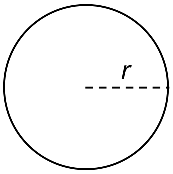
Measurement formulas can help you solve everyday problems.

- The area formulas you have worked with so far all have one thing in common: each involves the product of two appropriate linear measurements (width, length, base, height, and radius). In the case of a **trapezoid**, the area is the product of the trapezoid's height and the **mean** length of its **parallel** sides.
- Some formulas involve multiplying by a **constant**.

For example:

- The area of a triangle is the product of its base and its height multiplied by the constant $\frac{1}{2}$.
- The area of a circle is the product of its radius and its radius multiplied by the constant π.

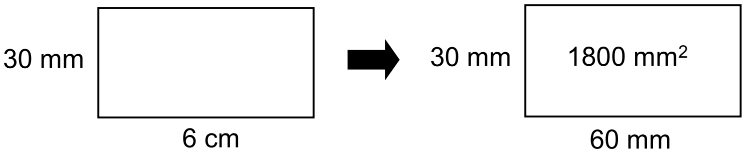
$$A_{\text{circle}} = \pi \times r \times r$$



- When you use any area formula, you need to make sure the dimensions are in the same units.

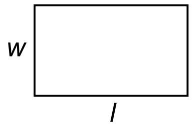
For example:

To express the area of the rectangle below in square millimetres, both measurements have to be in millimetres.



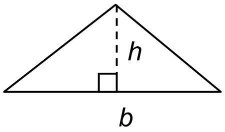
- When you calculate with decimals and π, you often have to round your answers to reasonable approximations.

Rectangle



$A = l \times w$

Triangle

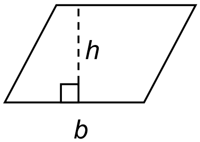


$A = b \times h \div 2$

or

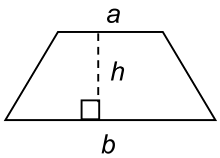
$A = \frac{1}{2} \times b \times h$

Parallelogram



$A = b \times h$

Trapezoid



$A = \frac{a+b}{2} \times h$

C. Why is it not appropriate to round the number of cans in **part B** to the nearest whole number?

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Examples

Example 1 Solving an Area Problem

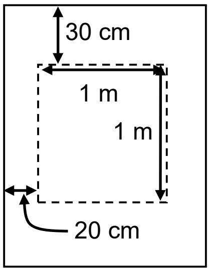
A cloth on a 1 m square table hangs down 30 cm over the edge along two edges and 20 cm along the other two edges. What is the area of the cloth?

Solution

The length is
(0.3 + 1 + 0.3) m = 1.6 m.

The width is
(0.2 + 1 + 0.2) m = 1.4 m.

The area is
1.4 × 1.6 = 2.24 m^2.



Thinking

- I drew a diagram to show what the flat cloth looked like and labelled it with the information I knew.
- I changed centimetres to metres so all measurements were the same.

Example 2 Estimating Area

Tandin wants to tile a circular area that is 2 m in diameter using 20 cm square tiles. Estimate how many tiles he will need.

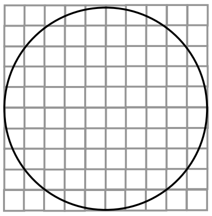
Solution

$A_{\text{circle}} = \pi r^2 \approx 3.14 \times 1^2 = 3.14 \text{ m}^2$

Each tile is 0.2 × 0.2 = 0.04 m^2

3.14 m^2 ÷ 0.04 m^2 = 78.5 tiles

100 tiles will cover the square.



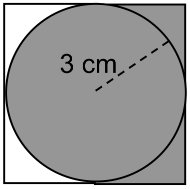
Tandin will need about 90 tiles.

Thinking

- I divided the area of the circle by the area of a tile to find the minimum number of tiles needed.
- Some of the tiles will need to be broken to create the circle, so I knew Tandin would need more than 78.5 tiles.
- The circle just fits inside a 20 cm square, which would need 100 tiles to cover it.
- I estimated the circle would need between 78.5 and 100 tiles, or about 90 tiles.

Example 3 Finding the Area of a Composite Shape

What is the area of the shaded region?
Round to one decimal place.



Solution

$A_{\text{rectangle}} = 3 \times 6 = 18 \text{ cm}^2$

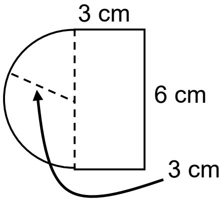
$$A_{\text{semi-circle}} = \frac{1}{2} \times \pi \times 3^2$$

$$\approx \frac{1}{2} \times \frac{22}{7} \times 9$$

$$\approx 14.1 \text{ cm}^2$$

$A \approx 18 \text{ cm}^2 + 14.1 \text{ cm}^2 = 32.1 \text{ cm}^2$

The total area is about 32.1 cm^2.



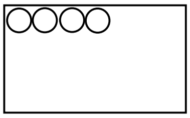
Thinking

- I divided the region into a rectangle and a semi-circle.
- The width of the rectangle is the circle's radius and the length of the rectangle is its diameter.

Practising and Applying

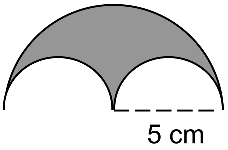
Round all answers to one decimal place, when appropriate.

1. Round tins are packed in a box with a 60 cm-by-100 cm bottom. The area of the base of a tin is 50 cm^2. How many tins will fit in the box in a single layer?

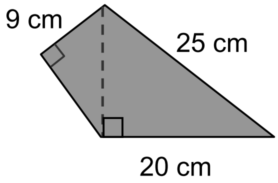


2. What is the area of each shaded region?

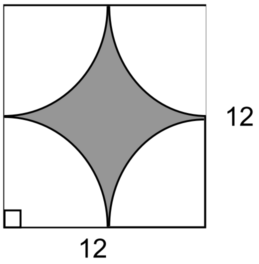
a)



b)

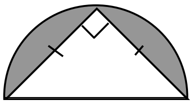


c)

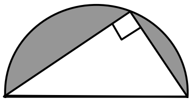


3. Two right triangles are drawn inside a semi-circle of radius 5 cm as shown below. One triangle is isosceles and the other triangle has an 8 cm leg. Which has the greater shaded region, A or B? Make a prediction and then check your answer. Show your work.

A.

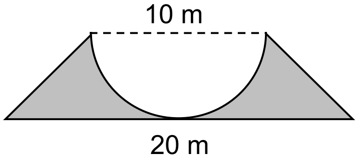


B.



4. Eden made a rectangular mosaic design using exactly 256 one-centimetre square tiles. She used exactly 80 cm of wood to make the frame. What are the dimensions of her design? Explain your thinking.

5. A semi-circle is cut out of a trapezoid. What is the area of the shaded region? Show your work.



6. Which shape has the greater area?

A. a square with 10 cm sides

B. a non-square rhombus with 10 cm sides

7. The circumference of a circle in centimetres is the same number of units as its area in square centimetres. What is the radius of the circle?

8. A 5 kg bag of grass seed covers about 650 m^2. How many bags should Tshering buy to seed a 60 m-by-80 m football field?

9. A grower produces about 3 tonnes of rice per hectare of land. He has a paddy field that is 0.5 km long and 200 m wide. About how much rice can he produce?

10. A square has about the same area as a circle with radius 7 cm. About what is the side length of the square?

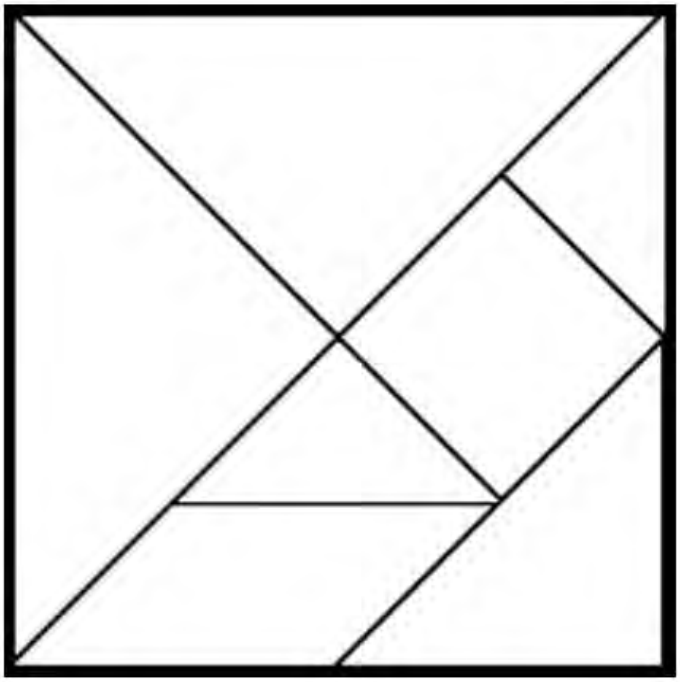
11. Choki has 6 m of wood to make frames for posters. She wants to frame two posters that are each 65 cm by 95 cm. Does she have enough wood? How can you tell without calculating exactly?

12. Create and solve a problem that involves finding the area of a composite shape that includes a circle or part of a circle.

13. When you triple each dimension in any area formula, what happens to the area? Explain your answer.

CONNECTIONS: Tangrams

The tangram is an ancient Chinese puzzle consisting of seven pieces (called tans) that form a square.



There are two large right triangles, two small right triangles, and one medium-sized right triangle. There are also a parallelogram and a square.

The tans can be combined to make pictures and designs. The challenge of the puzzle is to put a given number of tans together to match a given design.

If the tangram is a 10 cm square, what is the area of each tan?

1. Large triangle

2. Medium-sized triangle

3. Small triangle

4. Square

5. Parallelogram

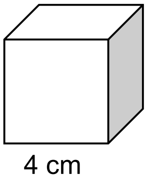
See if you can figure out how the seven tans are arranged to make each picture above.

Chapter 3 Volume and Surface Area

5.3.1 Volume of a Rectangular Prism

Try This

A. Nidup is using 2 cm cubes to build larger cubes. How many 2 cm cubes does he need to build a cube with an edge length of 4 cm?



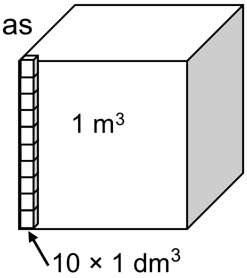
- The **volume** of a 3-D object is a measure of the amount of space it takes up. Here are the most common units of volume:

Cubic metre (m^3)	Cubic decimetre (dm^3)	Cubic centimetre (cm^3)
1,000,000 cm^3	1,000 cm^3	1 cm^3
1 m^3	0.001 m^3	0.000001 m^3

- A cubic centimetre, 1 cm^3, takes up the same amount of space as a 1 cm-by-1 cm-by-1 cm cube.

- A cubic decimetre, 1 dm^3, takes up the same amount of space as
a 1 dm-by-1 dm-by-1 dm cube or
a 10 cm-by-10 cm-by-10 cm cube.

- A cubic metre, 1 m^3, takes up the same amount of space as
a 1 m-by-1 m-by-1 m cube,
a 10 dm-by-10 dm-by-10 dm cube, or
a 100 cm-by-100 cm-by-100 cm cube.

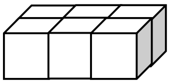


- To understand the formula for the volume of a **rectangular prism**, you can find a pattern by using 1 cm^3 cubes to create prisms with the same base but different heights:

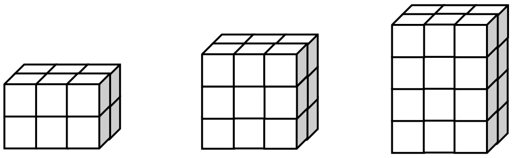
- The area of the base of this rectangular prism is 2 cm × 3 cm = 6 cm^2.

The prism's height is 1 cm.

The prism's volume is 6 cm^3. Notice that 6 cm^2 × 1 cm = 6 cm^3.



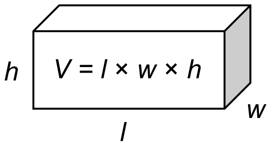
- If you build prisms with heights of 2 cm, 3 cm, and 4 cm, the volumes are 6 cm^2 × 2 cm = 12 cm^3, 6 cm^2 × 3 cm = 18 cm^3, and 6 cm^2 × 4 cm = 24 cm^3.



$V = (2 \times 3) \times 2$ $V = (2 \times 3) \times 3$ $V = (2 \times 3) \times 4$

- So, the formula for the volume of a rectangular prism is

V = Area of base × height or $V = l \times w \times h$



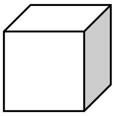
- The amount that a container can hold is a measure of its **capacity**.

Here are the most common units of capacity:

kilolitre (kL)	litre (L)	decilitre (dL)	centilitre (cL)	millilitre (mL)
1000 L	1 L	0.1 L	0.01 L	0.001 L
1,000,000 mL	1000 mL	100 mL	10 mL	1 mL

- In the metric system, there is a special relationship among units of volume, capacity, and mass for water.

- A 1 cm^3 cube has a capacity of 1 mL.
- 1 mL of water has a mass of 1 g.



1 cm^3 of water = 1 mL and has a mass of 1 g.

You can use this relationship to estimate the mass of liquids that are mostly water.

B. When you double each dimension of a cube, what happens to its volume? Use the formula for the volume of a rectangular prism to explain why this happens.

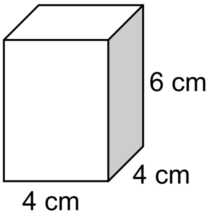
Examples

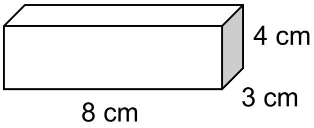
Example 1 Relating Dimensions and Volume

Draw and label two different rectangular prisms, each with a volume of 96 cm^3.

Solution

4 cm × 4 cm × 6 cm = 96 cm^3





8 cm × 4 cm × 3 cm = 96 cm^3

Thinking

- I needed to find three dimensions with a product of 96.
- I factored 96 and then chose two different sets of factors:

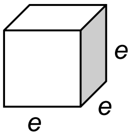
$96 = (2 \times 2) \times (2 \times 2) \times (2 \times 3)$
$= 4 \times 4 \times 6$

$96 = (2 \times 2 \times 2) \times (2 \times 2) \times 3$
$= 8 \times 4 \times 3$

Example 2 Developing the Formula for the Volume of a Cube

Write a formula for the volume of a cube with edge length *e*.

Solution



$V = l \times w \times h = e \times e \times e = e^3$

The formula for the volume of a cube is $V = e^3$.

Thinking

- I used the volume formula for a rectangular prism.
- For each of the three dimensions, *l*, *w*, and *h*, I used *e* instead because the length, width, and height of a cube are equal so *1 = e*, *w = e*, and *h = e*.

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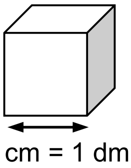
Example 3 Relating Volume, Capacity, and Mass

a) What single unit of volume is equivalent to 1 L?

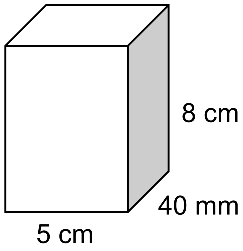
b) What is the mass of 1 L of water in kilograms?

Solution	**Thinking**
a) 1 L = 1000 mL = 1000 cm^3 1000 cm^3 = 10 cm × 10 cm × 10 cm = 1 dm^3  1 L = 1 dm^3	 **a)** I knew that 1 L = 1000 mL, 1 mL = 1 cm^3, and 1 dm = 10 cm.
b) 1 L of water has a mass of 1 kg.	**b)** I knew that 1 mL of water has a mass of 1 g, so 1000 mL has a mass of 1000 g.

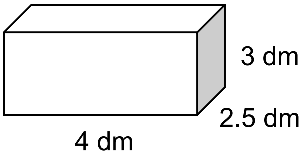
Example 4 Calculating Volume, Capacity, and Mass

Find the volume and capacity of each rectangular prism. Then find the mass of water each prism could hold if it were a container.

a)



b)

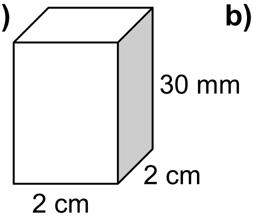


Solution	**Thinking**
a) 40 mm = 4 cm $V = l \times w \times h = 5 \times 4 \times 8 = 160$ The volume is 160 cm^3. The capacity is 160 mL. It holds a mass of 160 g of water.	 • For **part a),** I changed 40 mm to 4 cm so all the dimensions would be in the same units.
b) $V = l \times w \times h = 4 \times 2.5 \times 3 = 30$ The volume is 30 dm^3. The capacity is 30 L. It holds a mass of 30 kg of water.	• I used these volume-capacity-mass relationships for water to find the capacity and mass for both prisms: 1 cm^3 = 1 mL 1 dm^3 = 1 L 1 mL = 1 g 1 L = 1 kg

Practising and Applying

1. If these containers are filled with water, what is the volume, capacity, and mass of the water in each?

a)



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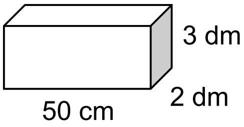
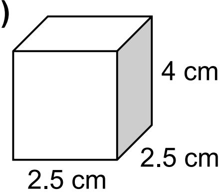
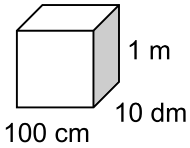


c)



d)



2. Copy and complete the chart.

V (cm^3)	*l* (cm)	*w* (cm)	*h* (cm)	Capacity (mL)
48	4	3		
105	10		3	
		12	5	720

3. Which rectangular prism has the greatest volume? Estimate and then check your answer.

A. 9 cm by 9 cm by 9 cm

B. 10 cm by 12 cm by 6 cm

C. 8 cm by 8 cm by 11 cm

4. a) What is the volume of an 8 cm cube?

b) What are the dimensions of a cube with a volume of 8 cm^3?

5. What is the edge length of a cube with each volume?

a) 64 cm^3

b) 125 cm^3

c) 1,000,000 cm^3

6. a) How many cubic centimetres are there in 1 m^3?

b) How many litres are in 1 m^3?

c) What is the mass of 1 m^3 of water in grams? in kilograms? in tonnes?

7. Which is the best price for the buyer? Explain your choice.

A. A 1 L carton for Nu 40

B. An 8 cm cube for Nu 20

C. 700 cm^3 for Nu 30

8. A rectangular prism container holds 24 L of water. Sketch and label the container, showing one possible set of dimensions.

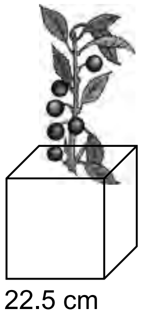
9. The base of a 1 L rectangular prism carton has an area of 50 cm^2. How tall is the carton?

10. a) How many of these dice will fit into a box that measures 5 cm by 4 cm by 7 cm? Estimate and then check.

b) How much extra space is there?

11. Gom Raj has a cube-shaped planter that is 22.5 cm on each edge. He has 8 L of soil.

Does he have enough soil to fill the planter? How can you tell without calculating the exact volume?



12. Create and solve a problem that involves estimating the dimensions of a cube when you know its volume.

13. a) What happens to the area of a rectangle when you triple each dimension?

b) What happens to the volume of a rectangular prism when you triple each dimension?

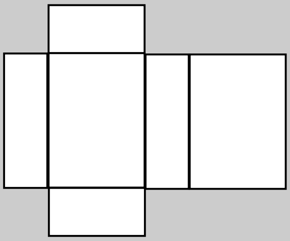
c) Why is the volume increase in **part a)** greater than the area increase in **part b)**, even though each dimension was multiplied by the same amount?

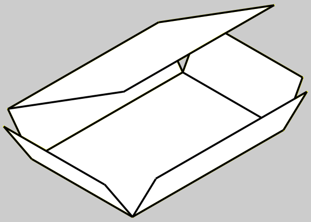
5.3.2 Surface Area of a Rectangular Prism

Try This

This net folds to make a box that is 12 cm by 8 cm by 4 cm.

A. What is the area of the net?





- The total **surface area** of a 3-D object is a measure of the total amount of area that covers all of its surfaces. Surface area units are the same as area units:

Square metre (m^2)	Square centimetre (cm^2)	Square millimetre (mm^2)
1 m^2	0.0001 m^2	0.01 cm^2
10,000 cm^2	100 mm^2	1 mm^2

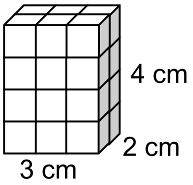
For example:

- This rectangular prism has dimensions 2 cm by 3 cm by 4 cm.
- Its total surface area is the sum of the areas of all six **faces**:

  Two faces are 2 cm by 3 cm, each with an area of 6 cm^2.

  Two faces are 3 cm by 4 cm, each with an area of 12 cm^2.

  Two faces are 2 cm by 4 cm, each with an area of 8 cm^2.



- The total area of all six faces is 2 × (6 + 12 + 8) cm^2 = 2 × 26 cm^2 = 52 cm^2

The total surface area of the prism is 52 cm^2.

- To develop a formula for the total surface area of a rectangular prism, you can find the total surface area of a prism with dimensions l units, w units, and h units.
- Total surface area of a rectangular prism is the sum of the areas of all six faces:

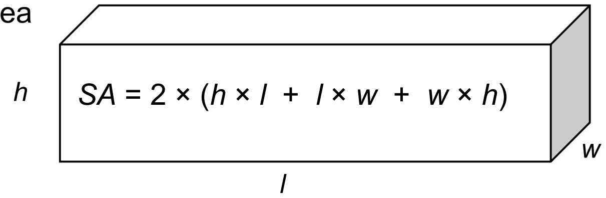
  Two faces (front and back) are h units by l units, each with an area of $h \times l$ square units.

  Two faces (top and bottom) are l units by w units, each with an area of $l \times w$ square units.

  Two faces (at the ends) are w units by h units, each with an area of $w \times h$ square units.

- The total area of all six faces is $2 \times (h \times l + l \times w + w \times h)$ square units.

The formula for the total surface area of a rectangular prism is



$$SA = 2 \times (h \times l + l \times w + w \times h)$$

B. Why is the area of the net in **part A** equal to the total surface area of the box?

C. Why is surface area measured in square units?

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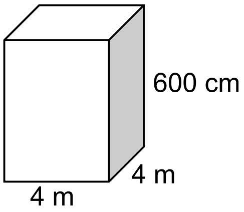
Examples

Example 1 Calculating Surface Area

What is the total surface area of this box?



Solution

$SA = 2 \times (h \times l + l \times w + w \times h)$

$SA = 2 \times (4 \times 6 + 4 \times 4 + 6 \times 4)$

$= 2 \times (24 + 16 + 24)$

$= 2 \times 64$

$= 128$

The total surface area is 128 m^2.

Thinking

- I changed 600 cm to 6 m so all the units would be the same.
- I found the sum of the areas of the front, top, and side rectangular faces. Then I doubled the sum.

Example 2 The Formula for the Total Surface Area of a Cube

Write a formula for the total surface area of a cube with sides *e.*

Solution

$SA = 2 \times (h \times l + l \times w + w \times h)$

$SA = 2 \times (e \times e + e \times e + e \times e)$

$= 2 \times (e^2 + e^2 + e^2)$

$= 6e^2$

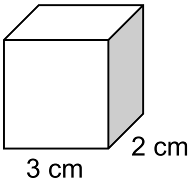
The formula for the total surface area of a cube is $SA = 6e^2$.

Thinking

- I used the surface area formula for a rectangular prism.
- For each of the three dimensions, *l*, *w*, and *h*, I used *e* instead because the length, width, and height of a cube are equal, so $1 = e$, $w = e$, and $h = e$.

Example 3 Solving a Surface Area Problem

The total surface area of this box is 62 cm^2.
What is the height of the box?



Solution

$SA = 2 \times (h \times l + l \times w + w \times h)$

$62 = 2 \times (h \times 3 + 3 x 2 + 2 x h)$

$62 = 2 \times (3h + 6 + 2h)$

$62 = 12 + 10h$

$50 = 10h$

$5 = h$

The height of the box is 5 cm.

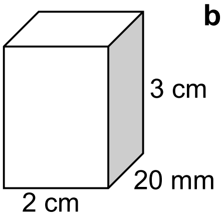
Thinking

- I used the formula for the total surface area to create an equation.
- I solved the equation for *h* (the height of the box).

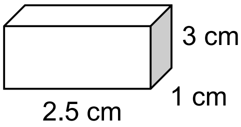
Practising and Applying

1. Find the total surface area of each.

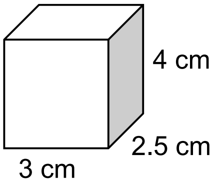
a)



b)



c)



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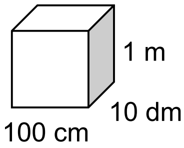
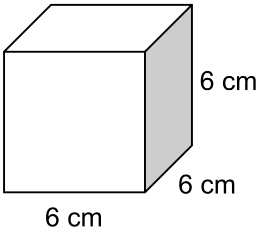


2. Copy and complete the chart.

SA **(cm^2)**	***l*** **(cm)**	***w*** **(cm)**	***h*** **(cm)**
	6	1.5	1
10	2		1
76		4	2
150	5	5	

3. a) Find the total surface area of this cube.



b) What is the volume of the cube?

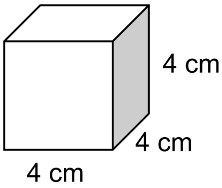
c) What do you notice about the two measurements? Do you think this is true for all cubes? Explain your answer.

4. a) What happens to the total surface area of a rectangular prism when you double each dimension? Explain your answer.

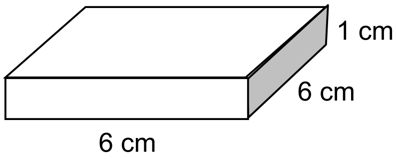
b) What happens to the total surface area of a rectangular prism when you triple each dimension?

5. Find the total surface area of each.

a)



b)



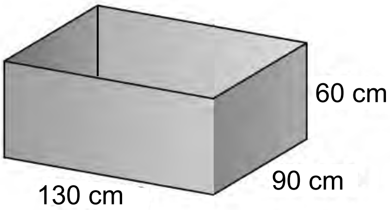
c) Predict which prism has the greater volume. Check your prediction.

6. a) List the dimensions of all the rectangular prisms with a volume of 12 cm^3. Each dimension should be a whole number of centimetres.

b) Which prism has the greatest total surface area? Describe its shape.

c) Which prism has the least total surface area? Describe its shape.

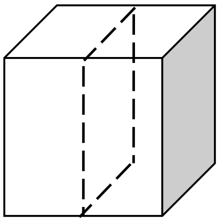
7. How much wood (in cm^2) does Nima need to make this box with no top?



8. Suppose you cut a cube in half.

a) How does the combined volume of the two halves compare to the original volume? Why does this happen?

b) How does the combined surface of the two halves compare to the original surface area? Why does this happen?



UNIT 5 Revision

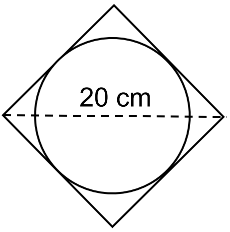
Round to one decimal place, when necessary.

1. Could each set of numbers be the side lengths of a right triangle?

a) 10, 24, 26 **b)** 9, 11, 14

2. Find the length of the hypotenuse of a right triangle with legs 7 cm and 9 cm.

3. A round post is cut from a square piece of wood with a diagonal of 20 cm. What is the greatest possible diameter of the post?



4. How many different rectangles with an area of 20 cm^2 are possible, if each side length is a whole number of centimetres? What is the perimeter of each rectangle?

5. How many different rectangles with a perimeter of 20 cm are possible, if each side length is a whole number of centimetres? What is the area of each rectangle?

6. On a scale drawing, 1 cm represents 2.5 m. A rectangle in the drawing is 4 cm by 5 cm. What is the area of the real rectangle?

7. The distance from Hong Kong to Kolkata is about 2600 km. The distance on a map is 20 cm. What is the scale ratio of the map?

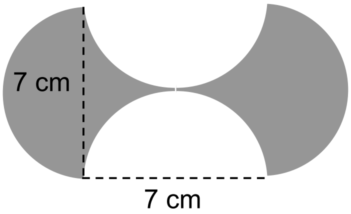
8. On a map, the distance between two cities is 7.5 cm. The key on the map reads “1 cm represents 200 km”. What is the real distance between the cities?

9. Estimate the area of each.

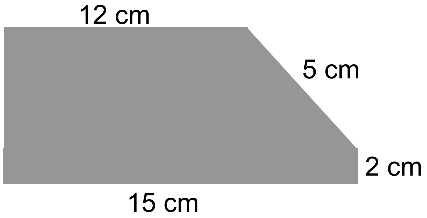
a) A circle with radius 3.5 cm

b) A circle with diameter 7 km

10. What is the area of the shaded region?



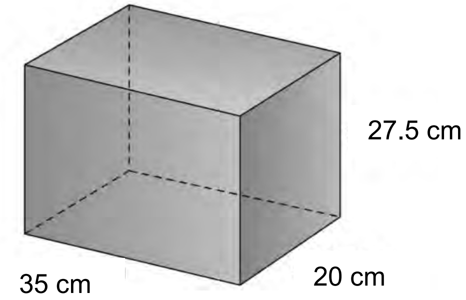
11. What is the area of this shape?



12. What is the volume of a 7 m cube?

13. A box holds 336 mL. Its base has an area of 48 cm^2. What is its height?

14. a) What is the capacity of this aquarium in litres?



b) If the aquarium is filled until the water is 2 cm from the top, what is the volume of the water in litres?

c) What is the mass of the water in **part b)**?

15. a) Sketch a rectangular prism that is 3 cm by 5 cm by 6 cm.

b) What is its total surface area?

16. Sketch two different boxes, each with a volume of 72 cm^3. What is the total surface area of each?

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UNIT 6 PROBABILITY AND DATA

Getting Started

Use What You Know

Bhagi is practising the shot put. This chart shows the results of 20 throws.

Throw number	1	2	3	4	5	6	7	8	9	10
Distance (m)	8.5	8.2	8.5	8.5	8.4	8.5	5.5	8.6	8.3	8.6

Throw number	11	12	13	14	15	16	17	18	19	20
Distance (m)	7.5	7.2	8.5	8.5	8.4	8.5	9.1	7.9	8.3	8.6

A. i) Identify possible outliers in the set of data. Why do you think they are outliers?

ii) How will the mean, median, and mode be affected, if you remove the outliers from the set of data?

B. i) Create a histogram to display the data for the 20 throws.

ii) How did you decide on the number of intervals to use?

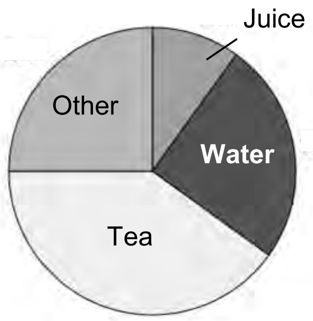
Skills You Will Need

1. This circle graph shows the favourite lunch-time drinks for a class of students.

a) What is the favourite? the least favourite?

b) Estimate the percent of students who chose each drink.

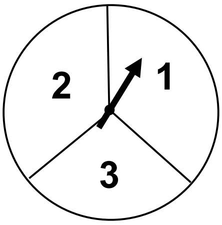
Lunch Time Drink Choices



2. The heights (in cm) of 20 students are shown in the chart to the right. Determine each.

a) mean
b) median
c) mode
d) range

181.6	169.3	141.5	144.8	169.7
152.7	160.7	142.8	176.6	189.4
149.8	157.2	162.7	164.6	159.2
162.7	149.2	170.2	143.8	176.9



3. Suppose you spin this spinner twice.

a) Draw a tree diagram to represent all possible outcomes.

b) Use your tree diagram to determine the theoretical probability of spinning two odd numbers in a row.

Chapter 1 Probability

6.1.1 Complementary Events

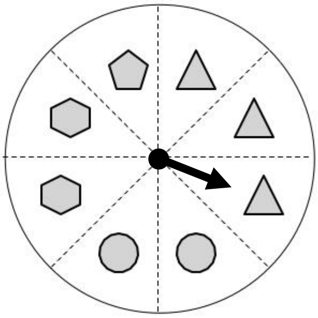
Try This

A. What is the theoretical probability of spinning each shape on this spinner?

i) a triangle

ii) a circle

iii) a polygon



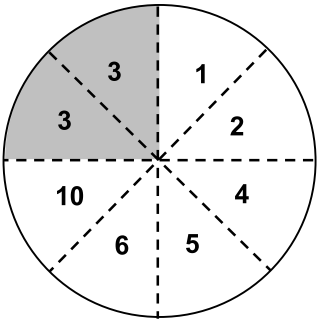
- You can use this formula to find the **theoretical probability** that an **event** (E) will occur:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

The formula is used when each of the **possible outcomes** of a probability experiment has an equal chance of happening, or is **equally likely**.

For example:

The eight **sectors** of this spinner are identical, so the probability of landing on a particular sector is the same as the probability of landing on any other sector. Since landing on one sector is an outcome, all possible outcomes are equally likely. That means you can use the formula above for finding the theoretical probability of an event.



circle divided into 8 sectors

Suppose you want to find the probability that the event of spinning the number 3 will occur.

The probability of landing on 3 is

$$P(3) = \frac{\text{number of sections labelled "3"}}{\text{total number of sections}} = \frac{2}{8} = \frac{1}{4}$$

- The **complement** of an event (E) consists of all the possible outcomes that are <u>not</u> part of the event. The complement of E is called “Not E”.

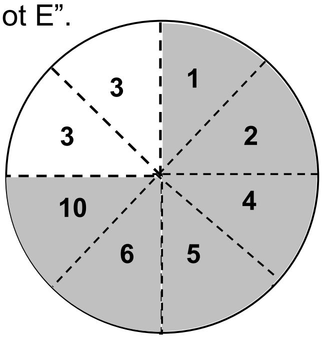
“E” and “Not E” together represent all possible outcomes:

P(E) + P(Not E) = 1 so P(Not E) = 1 – P(E)

For example:

The probability of <u>not</u> landing on 3 is P(Not 3).

$$P(\text{Not } 3) = 1 - P(3) = 1 - \frac{1}{4} = \frac{3}{4}$$



circle divided into 8 sectors

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B. What is the relationship between P(circle) and P(polygon) in **parts A ii) and iii)**? Explain your thinking.

Examples

Example 1 Solving a Probability Problem

Choki and Sithar are playing a game where they flip two Nu 1 coins.

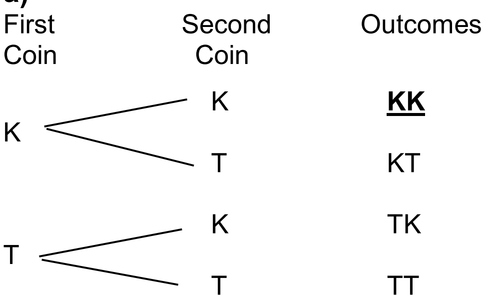
- Choki wins when the coins both show Khorlo.
- Sithar wins when both coins show Tashi Ta-gye.
- If the two coins show different faces, no one wins.

a) What is the theoretical probability that Choki will win?

b) Which is greater, P(Choki does not win) or P(Sithar wins)?

Solution 1

a)



$$P(\text{Choki wins}) = \frac{1}{4}$$

b) $P(\text{Sithar wins}) = \frac{1}{4}$

$$\begin{aligned} P(\text{Not Choki wins}) &= 1 - P(\text{Choki wins}) \\ &= 1 - \frac{1}{4} \\ &= \frac{3}{4} \end{aligned}$$

P(Not Choki wins) > P(Sithar wins)

Thinking

a) I made a tree diagram to list all the possible outcomes.

- Each outcome has a probability of $\frac{1}{4}$ because there are 4 equally likely outcomes.
- The event that Choki wins is represented by the outcome KK.

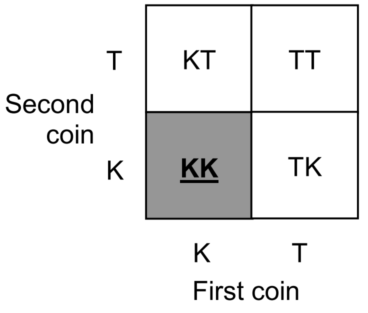
b) The TT outcome represents a win for Sithar. It is 1 out of 4 possible outcomes.

- I knew that Choki not winning is the complement of Choki winning, so I used the formula:

P(Not Choki wins) = 1 – P(Choki wins)

Solution 2

a)



$$P(\text{Choki wins}) = \frac{1}{4}$$

Thinking

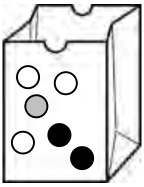
a) I used an area model to represent the possible outcomes.

- The event that Choki wins is represented by the outcome KK. Its area represents $\frac{1}{4}$ of the area of the whole square.

Example 1 Solving a Probability Problem [Continued]

b) P(Sithar wins) = $\frac{1}{4}$ P(Not Choki wins) = $\frac{1+1+1}{4}$ = $\frac{3}{4}$ P(Not Choki wins) > P(Sithar wins)	**b)** The TT outcome represents a win for Sithar. It is 1 out of 4 possible outcomes. • The event of Choki not winning consists of the 3 outcomes KT, TT, and TK. That's 3 out of 4 possible outcomes.

Practising and Applying



1. A bag contains 2 black, 1 grey, and 3 white marbles. Dorji took a marble from the bag. Determine the theoretical probability of each event.

a) P(white) **b)** P(black)
c) P(grey) **d)** P(not white)
e) P(not black) **f)** P(not grey)

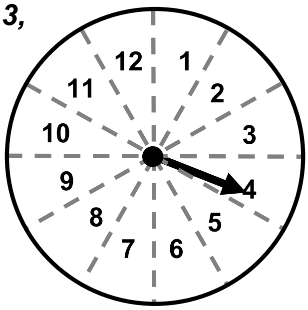
2. A bag contains one of each coin:

Ch 20 Ch 25 Ch 50 Nu 1

Lhamo took a coin from the bag and then replaced it. Then she took another coin and found the sum of the two coins she had taken. Determine each theoretical probability.

a) P(sum = Ch 40)
b) P(sum ≠ Ch 40)
c) P(sum < Nu 1)

Use this spinner for questions 3, 4, 5, and 6.



3. a) What are the possible outcomes of spinning the spinner?

b) Are the outcomes equally likely? How do you know?

4. What is the complement of each event?

a) spinning an even number
b) spinning a multiple of 4
c) spinning a number greater than 10
d) spinning a composite number

5. What are the favourable outcomes for each event?

a) spinning an even number
b) spinning a number that is not even
c) spinning a prime number
d) spinning a number that is not prime
e) spinning a multiple of both 2 and 3
f) spinning a number less than 4

6. What is each probability?

a) P(an even number)
b) P(an odd number)
c) P(a prime number)
d) P(a multiple of 3)
e) P(not a multiple of 3)
f) P(not a multiple of 5)

7. In **question 1**, each possible outcome is drawing a marble from the bag. Each event is drawing or not drawing a particular colour. Why are all the possible outcomes equally likely but the events not equally likely?

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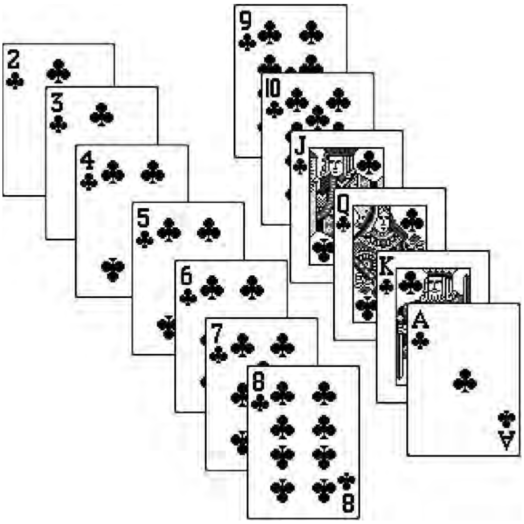
Use the following information for questions 8, 9, 10, and 11.

A suit in a standard deck of playing cards contains number cards (2 to 10), face cards (Jack, Queen, and King), and an Ace.

Thinley took the suit of clubs, shuffled the cards, and then randomly selected one card.



8. a) List the possible outcomes of selecting a card.

b) How do you know they are equally likely?

9. Determine each probability.

a) P(not the Ace of Clubs)

b) P(not a face card)

c) P(not a number card)

10. Thinley removed the face cards and the Ace and then randomly selected one card from the remaining number cards. Determine the probability of each.

a) P(card value < 5)

b) P(card value > 5)

c) Explain why the events in **parts a) and b)** are not complementary.

d) Describe the event that is the complement of the event in **part a)**.

11. Create an example that shows two complementary events, using the 13 cards in the suit of clubs.

GAME: Unlucky Ones

Any number of players can play. You need two dice.

The goal of this game is to score 100 points.

Players take turns.

- On your turn, roll two dice and add the numbers. The sum is your score for that roll.
  - If you roll a 1 on either die, you lose all the points you scored *on this turn*. Your turn ends.
  - If you roll two 1s, you lose *all* the points you have scored so far. Your turn ends.
- As long as you do not roll a 1, you may continue rolling. Add the sum of each roll to your point total. Or, you may stop at any time and pass the dice to the next player.
- When you stop, add all your points from this turn to your points from all previous turns.
- The first player to reach or pass 100 points wins.

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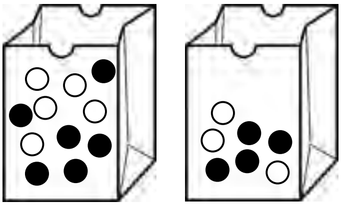
CONNECTIONS: Simpson’s Paradox

Suppose that you have two bags, each with some white balls and some black balls. If you draw a white ball from a bag, you win.

Here are two situations to investigate.

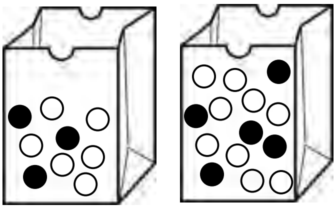
Situation 1

Balls	Bag 1	Bag 2
White	5	3
Black	6	4



Situation 2

Balls	Bag 1	Bag 2
White	6	9
Black	3	5



1. In both situations, you have a higher probability of winning if you draw a ball from Bag 1. Show how you know this is true.

2. a) Suppose you combine the balls from both Bag 1s. What is the theoretical probability of winning with this bag?

b) Suppose you combine the balls from both Bag 2s. What is the theoretical probability of winning with this bag?

c) What do you notice about the probability of winning with Bag 1 compared to the probability of winning with Bag 2?

6.1.2 Simulations

Try This

Tshering is a good archer. In a recent competition, he shot 100 arrows altogether. He got 50 kareys, 10 of which were bullseyes.

A. Based on his performance in the recent competition, estimate the probability that Tshering will score two bullseyes in a row on his next two shots.

- The theoretical probability of an event is sometimes difficult or impossible to determine. To estimate a theoretical probability, you can use an experiment that models the event to find the **experimental probability**. This experiment is called a **simulation**.
- For each simulation, you must use a model that matches the event.

For example:

Event	What you need to model	Simulation model
6 people in a race are evenly matched. What is the probability of a runner winning 2 races in a row?	6 equally likely outcomes, repeated twice	Roll a 6-sided die 2 times
On a true-false test of 3 questions, what is the probability of guessing all 3 answers correctly?	2 equally likely outcomes, repeated 3 times	Flip a coin 3 times
The chance of rain on any day is 20%. What is the probability that it will rain 2 days in a row?	1 outcome with probability 20% and 1 outcome with probability 80%, repeated twice	Spin a spinner with 1 section of 20% and 1 section of 80%, 2 times
A box contains 2 white, 3 red, and 5 blue cubes. What is the probability of choosing 2 cubes of the same colour?	1 outcome with probability 20%, 1 with probability 30%, and 1 with probability 50%, repeated twice	Randomly draw an object from a bag with 2 items of one kind, 3 of another kind, and 5 of yet another kind, 2 times

- When you are designing an experiment to simulate an event, remember this:

- The model must have the same number of possible outcomes as the situation you are modelling.
- The probability of each outcome in the simulation should match the probability it is meant to represent as closely as possible.
- The number of **trials**, or repetitions of the experiment, must be large enough that you can use the experimental probability to estimate a theoretical probability.

B. i) Sketch a spinner you could use to model Tshering's performance in **part A**.

ii) How could you use the spinner to estimate the probability in **part A**?

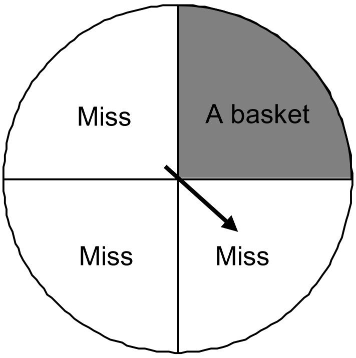
Examples

Example 1 Simulating With a Spinner

Yeshi plays basketball. His past performance suggests that his average score was one basket in every four shots.

What is the probability that he will score three or more baskets in his next five shots?

Solution



Results of 5 Spins	
3 or more baskets	Fewer than 3 baskets
||||	𝍸 𝍸 𝍸 |

P(3 or more baskets in 5 shots) = $\frac{4}{20}$ or about 20%

Thinking

- I made a spinner to simulate Yeshi shooting baskets.
- The spinner had four equal sections since the probability of scoring is 1 basket in 4 shots.
- I labelled one section to represent scoring a basket and three sections to represent missing the shot, so the probability of spinning "A basket" is $\frac{1}{4}$.
- I spun the spinner five times and recorded whether I spun three or more baskets.
- I did 20 trials: 4 out of 20 trials had three or more baskets.
- I could repeat the simulation more times to improve the estimate.

Example 2 Comparing Experimental and Theoretical Probabilities

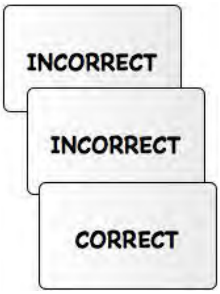
On a multiple choice test, each question has three answer choices: A, B, and C. Only one choice is correct for each.

a) Simulate guessing the correct answers to two questions on the test.

b) Compare the experimental probability of answering both questions correctly to the theoretical probability of answering both questions correctly.

Solution

a) *Simulation (Experimental probability)*



Thinking

a) I wrote CORRECT on one slip of paper and INCORRECT on two slips. I put the slips into a bag.

- I drew one slip of paper from the bag and noted whether it said CORRECT or INCORRECT. Then I put the slip back into the bag and mixed them up.

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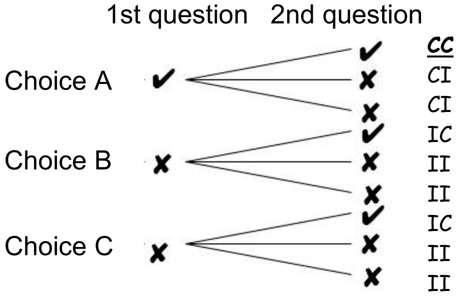
Results of Drawing Slips	
Both correct	Not both correct
𝍸	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸
5	35

$$P(2\text{ correct}) = \frac{5}{40} \approx 0.13$$

b) *Theoretical probability*



$$P(2\text{ correct}) = \frac{1}{9} \approx 0.11$$

The experimental probability is close to the theoretical probability.

- I drew a second slip and recorded whether both slips said CORRECT (both correct) or whether at least one was INCORRECT (not both correct).
- I repeated this for 40 trials.
- I wrote the probability as a decimal so I could compare it to the theoretical probability in **part b)**.

b) For the theoretical probability, I drew a tree diagram to list all possible outcomes. I used C for correct and I for incorrect.

- There were nine equally likely possible outcomes and only one outcome was both guesses correct (CC).
- I wrote the probability as a decimal so I could compare it to the experimental probability from **part a)**.
- I think the probabilities were close because I did a large number of trials.

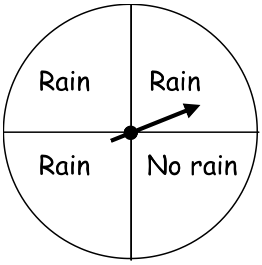
Example 3 Designing a Simulation

A forecast predicts the chance of rain to be 50% for Thursday and 75% for Friday. Design a simulation to find the probability that it will not rain either day.

Solution

Coin model for probability of rain on Thursday

Spinner model for probability of rain on Friday



Flip the coin and spin the spinner 20 times. Record how often Tashi Ta-gye-No rain occurs.

Thinking

- It was equally likely that there would be rain or no rain on Thursday, so I thought flipping a Nu 1 coin would be a good model — Khorlo would represent rain and Tashi Ta-gye would represent no rain.
- I knew P(rain Friday) = 75% $= \frac{75}{100} = \frac{3}{4}$, so I thought a spinner in fourths would be a good model for Friday's weather.
- I decided to do many trials so the probability would be a good estimate of the theoretical probability.

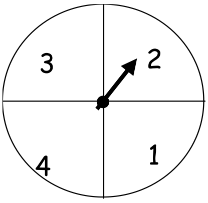
Practising and Applying

1. Here are four devices for conducting different probability experiments.

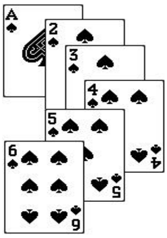
A.

B.

C.



D.



Which device above could you use to simulate each event? List more than one, if possible.

a) an event with two equally likely outcomes

b) an event with four equally likely outcomes

c) an event with six equally likely outcomes

2. Describe a simulation to estimate each probability.

a) What is the probability that a family with three children has three girls?

b) The probability of rain each day is $\frac{1}{3}$. What is the probability that it will rain on the next three days?

c) A class has 28 boys and 12 girls. What is the probability of randomly choosing two girls from the class?

d) An archer has previously scored 15 kareys in 50 shots. What is the probability that the archer will score a karey on each of his next two shots?

***Choose two of questions 3 to 5 To complete*.**

3. Each of the five questions in a multiple-choice test has four choices. Design and conduct a simulation to determine this probability:

What is the probability of answering three or more questions correctly by guessing the answer to each question?

4. In his last 200 attempts, an archer hit the target 25 times. Design and conduct a simulation to determine this probability:

What is the probability that the archer will hit the target in each of his next five attempts?

5. A company makes light bulbs. 5% of the bulbs it made last year did not work. Design and conduct a simulation to determine this probability:

What is the probability that three bulbs selected at random do not work?

***Choose one of questions 6 and 7 to complete*.**

6. Two numbers are randomly chosen from the numbers 3, 5, 7, and 12. Design and conduct a simulation to determine each probability.

a) P(product > 20) **b)** P(sum > 9)

c) P(both numbers are prime)

7. Maaros and Yarab are equally skilled archery teams competing against each other. The first team to win four out of seven games wins. Design and conduct a simulation to determine each.

a) P(Maaros wins the tournament in 7 games)

b) P(either team wins in 7 games)

c) P(Maaros wins the tournament)

8. A probability determined using a simulation is only an estimate of the theoretical probability. Why is this true?

6.2.1 EXPLORE: Sample Size

- According to the 2005 Bhutan **Census**, about 53% of the **population** is male. That means the theoretical probability that a person in Bhutan is male is $\frac{53}{100}$ or 53%.
- If you create a model of the population and conduct a simulation experiment, you would expect to find an experimental probability that was close to 53% of the population being male.
- In this lesson, you will explore how the size of the **sample** used in a probability experiment affects the results.

(Keep the data you collect in this lesson to use later in **Lesson 6.2.5**.)

A. Create a model to represent the population of Bhutan:

- Cut out 100 identical slips of paper.
- Label 53 of the slips MALE and the rest of the slips FEMALE.

Place all the slips in a container (a bag, a box, or a bangchung).

B. i) Create a chart like this to record the results of your simulation.

Sample Size = 10		
Trial #	**Number of Males**	**% Male**
1		
2		
3		
4		
5		
6		
...		
...		
19		
20		

ii) Mix up the slips and then draw a random sample of 10 slips from the container without looking.

iii) In your chart, for Trial 1 record the number and percent of the slips that say MALE. Return the slips to the container.

iv) Repeat **steps ii) and iii)** until you have done 20 trials.

v) Examine the data in your chart. Were the results similar from trial to trial? Consider the mean, median, and range to make your decision.

C. i) Repeat **part B** for a sample size of 20 and then for a sample size of 30.

ii) Were the results similar from trial to trial? Consider the mean, median, and range to make your decision.

D. Which sample size would you use to predict the percent of males? Why?

6.2.2 Selecting a Random Sample

Try This

Suppose you want to estimate the average number of times the word “angle” appears on a page in this textbook. You could count the number of times “angle” appears on each page in the book (the population) and then find the mean. Or, you could use a sample of pages instead.

A. Select two pages from Unit 1 Chapter 1 (pages 3 to 9). Count the number of times the word “angle” appears on each page. Find the mean.

B. Repeat **part A** using two pages from Unit 8 Chapter 3 (pages 246 to 255).

- When you use a **sample** to obtain data about a **population**, it is important to use a sample size that is large enough. It is also important that every member of the population has an equal chance of being included in the sample. This way, the results will closely represent the results of a **census**. In a census, you collect data from every member of the population.

For example:

If you want information about the height of the students in your school, it would not be a good idea to survey a sample of only the oldest students, or only boys. Your sample should include students from all age groups and both genders.

- In a **random sample**, each member of the population is as likely to be selected as any other member.

- Methods for randomly selecting members of a population for a sample can involve probability devices such as dice, coins, or drawing slips from a container.

- Computers and scientific calculators can be used to generate **random numbers** that can be used to select random samples.

- These random numbers are often listed in a **random number table**, like the table on the right. A random number table is another device that can be used to select a random sample.

Random Number Table

75995	76569	78527	56724	92100
88907	78395	36288	38738	93994
45372	16824	31669	35608	78167
82116	72889	69830	78171	20394
63131	17041	34191	56945	64061
01011	93856	15424	09156	27120
79048	35784	58118	28217	07125
33434	75190	37791	28295	74988
90665	99792	41264	75904	26250
78160	88194	43984	03233	79918
30039	40898	85062	68256	22415
19205	08500	52732	26334	85971
62672	71771	53822	00551	94523
31593	69255	78559	21081	54107
09474	44973	65937	12715	00469
36230	80790	27290	39075	99185
46257	46357	51897	63106	14429
59734	74549	68776	32350	21146

C. i) Why might the pages you selected in **parts A and B** be poor samples for estimating the average number of times the word “angle” appears on a page in this textbook?

ii) How would you select a better sample? Why would you use that method?

Examples

Example 1 Selecting a Random Sample Using a Random Number Table

Use a random number table to select a random sample of three pages in this book.

Solution

93789	31360	71077
$\boxed{279}68$	51207	85236
42444	02599	$\boxed{111}45$
48850	14565	17240
74823	34092	$5\boxed{088}0$
33769	41363	38017
31509	43782	57290
78144	98926	32250
78287	03656	41734
41866	65965	18042
81216	16207	67008

I will use pages 279, 111, and 88.

Thinking

- I randomly picked a 3-digit number from the random number table by closing my eyes and putting my pencil tip on the table. I used the digit that the tip of my pencil was on as the first digit of a 3-digit number. I did this three times to get three random numbers.
- If a number was too high, I shifted to the right one digit until I found a page number that was not too high.
- I knew that 1- and 2-digit page numbers were possible because you can have 3-digit numbers like 008 and 077.

Example 2 Selecting a Random Sample in Different Ways

A class has 36 students. The teacher wants to know how well the students understand what they are learning. He wants to select randomly a sample of six students to be interviewed. Describe a way the teacher could select a sample.

Solution 1

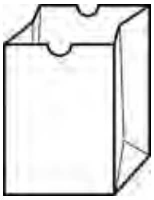
Have each student choose a 2-digit number (from 10 to 99). Ask students to say their numbers aloud.

Use the six students who chose the six highest numbers in the sample.

Thinking

- If there's a tie between two students, the teacher can flip a coin to determine which student to interview.

Solution 2



Write all the students' names on identical pieces of paper and put them into a bag.

Draw six names out of the bag without looking and without replacing each one.

Thinking

- I knew if I put all the names into a bag and took out six without looking, each student would have the same chance of being selected and the sample would be random.

Solution 3

Have students mix themselves up and then walk through the classroom door in a single line. The teacher could select every sixth student.

Thinking

- This method is good because it doesn't require any materials.
- It's important that the students are mixed up before walking through the door, so the sample is random.

Example 3 Thinking about Representative Samples

People in Bhutan light their homes using a variety of sources, including electricity, wood, and kerosene.

Suppose you wanted to find out what percent of homes use each different lighting source.

Would a random sample of households in Thimphu be a good sample to use? Explain your thinking.

Solution	**Thinking**
No. Homes in Thimphu are more likely to have electric lighting because they are in a big city, so they would not represent the whole population of Bhutan.	- Even if the households in Thimphu were selected randomly, it would not be a good sample from which to make a prediction for the whole country. 

Practising and Applying

1. A drug company has developed a new antimalarial drug. To test it, the company needs 100 subjects. Half will be given the drug and the other half will receive sugar pills. Describe a method for selecting 100 subjects from the 500 people who have volunteered.

2. A yak herder has 100 yaks. A disease that affects yaks is present in the area. The herder wants to know whether his herd is infected. He cannot afford to test every animal. Describe a method the herder can use to select a sample of animals to test.

3. A man rents out 30 homes. He wants to find out whether his tenants are happy. Describe how he might select a sample of five tenants to survey.

4. A company wants to estimate the percent of people in the dzhongkhag who plan to buy cell phones next year.

a) Describe how the company might choose a sample of people to survey.

b) Are there any groups or individuals who should not be included? Why?

5. Consider the following experiment.

- Cut out 50 slips of paper.
- Mark a black dot on 20 of the slips.
- Put all the slips into a container and mix them up.
- Draw a sample of 10 slips.

a) How many of the 10 slips would you expect to have black dots? Explain your answer.

b) Why is it possible that all 10 slips could all have black dots?

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6.2.3 Circle Graphs

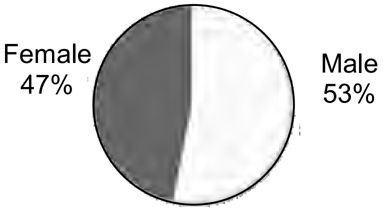
Try This

According to the 2005 census, 53% of Bhutan's population is male and 47% is female.

A. i) Why is a circle graph a good way to display this information?

ii) How would you create the circle graph?

Gender Distribution in Bhutan's Population



- A **circle graph** shows the parts that make up a set of data. You can use a circle graph to do these things:

- Compare the parts (each part is a sector of the circle).
- Compare each part to the whole set of data (represented by the whole circle).

- Follow these steps to create a circle graph:

Step 1 Determine what percent of the whole set of data each part, or **category** is.

Step 2 Calculate the angle for the sector that represents each category by calculating the appropriate percent of 360 (the whole circle is 360°).

Step 3 Draw a circle and locate its centre.

Step 4 Use a protractor to draw each sector and label it with the category name. You can also label each category with its percent, if you wish.

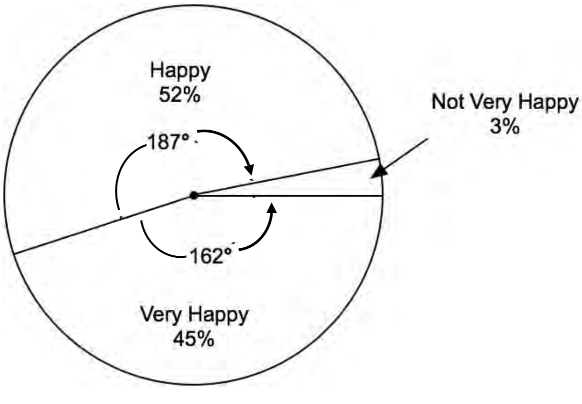
For example:

Here are some results from the 2005 Bhutan Census:

45% of the population said they were *Very Happy*

52% said they were *Happy*

3% said they were *Not Very Happy*



- A sector with an angle of 162° represents the category *Very Happy* since

45% of 360° = 0.45 × 360° = 162°

Use a protractor to mark a sector with a 162° angle at the centre of a circle. Label the sector *Very Happy*.

- A sector with an angle of 187° represents the category *Happy* since

52% of 360° = 0.52 × 360° = 187°

Use a protractor to mark a 187° angle starting with one of the radii of the previous sector. Label the sector *Happy*.

- The remaining sector represents the category *Not Very Happy* because there are three categories altogether.

B. Calculate the angles of both sectors of the circle graph in **part A**.

Examples

Example Calculating Percents and Angles for a Circle Graph

According to the 2005 Bhutan Census, 196,111 people live in urban areas and 438,871 people live in rural areas. Create a circle graph of this information.

Solution

Rural percent

$$\frac{438,871}{438,871 + 196,111} \times 100\%$$

$$= \frac{438,871}{634,982} \times 100\%$$

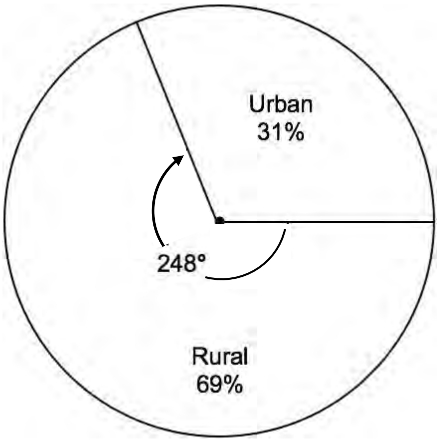
$$\approx 69\%$$

Rural sector angle

69% of 360° = 0.69 × 360° ≈ 248°

Urban percent

100% – 69% = 31%



Thinking

- I calculated the percent the rural population was of the total population.
- I did rural first because it was less than half the total so its sector was going to be less than half the circle. I find it easier to draw angles less than 180°.
- I used the percent that was rural to calculate the angle for the rural sector. Then I drew the rural sector and labelled it with its percent.
- Since there were only two categories, the urban sector was what was left of the circle.
- To find the urban percent, I subtracted the rural percent from 100%.

Practising and Applying

Round to the nearest whole percent, if necessary

1. This chart shows the population by age group in Bhutan, according to the 2005 census.

Population by Age Group

Age group	Persons
0–14	210,000
15–64	395,000
65+	30,000
Total	635,000

a) Calculate the percent of the population each group represents.

b) Determine the sector angle for each group in a circle graph.

c) Create a circle graph of the data.

2. Bhutan has four ecosystems. Create a circle graph to represent the information below.

Type of ecosystem	Area (millions of hectares)
Forests	3400
Agriculture	240
Grasslands	800
Barren (snow and ice)	190

3. The following data sets are from the 2005 Bhutan Census. Create a circle graph to represent each set of data.

Choose two sets of data from parts a) to f) to graph.

a) Major sources of cooking fuel

Source of cooking fuel	Percent
Electricity	30.6%
Firewood	37.2%
LPG	25.5%
Others	6.7%

b) Main sources of drinking water

Source of drinking water	Percent of households
Piped within house	22.7%
Piped outside house	61.5%
Spring/river/pond	14.3%
Other	1.5%

c) Households by number of members

Number in household	Number of households
1–2	26,139
3–4	39,381
5+	60,595
Total	**126,115**

d) Population by age group

Age group	Number of persons
0–14	209,959
15–64	395,278
65+	29,745
Total	**634,982**

e) Households by type of toilet

Toilet facilities	Number of households
Flush toilets	45,074
VIDP latrine outside house	4006
Long drop latrine inside house	579
Pit latrine	62,806
Others	903
No toilet facility	12,747
Total	**126,115**

f) Households by roofing material

Roof material	Number of households
Concrete/brick/stone	3875
CGI/Metal	82,432
Mud	636
Wood	15,852
Straw/leaves	9580
Bamboo	8539
Slate	1498
Others	3703

4. Describe some data about the students in your school that you could display on a circle graph. Tell why it makes sense to use a circle graph.

5. You could have used a bar graph to display the data in **question 2** or a histogram for the data in **question 1**. What does a circle graph show about the data that the other graphs might not show as well?

6.2.4 Box and Whisker Plots

Try This

Recall the exploration on **page 161**. In that lesson, you created a simulation model of the population of males and females in Bhutan (53% male and 47% female). Then you used the model to investigate the effect of different sample sizes on the results. These three charts show one group's results.

Sample Size = 10			Sample Size = 20		Sample Size = 30	
Trial #	**Number of males**	**Percent male**	**Number of males**	**Percent male**	**Number of males**	**Percent male**
1	6	60	9	45	17	57
2	4	40	5	25	15	50
3	5	50	7	35	15	50
4	7	70	9	45	18	60
5	2	20	9	45	14	47
6	8	80	12	60	16	53
7	8	80	8	40	17	57
8	4	40	11	55	19	63
9	4	40	16	80	15	50
10	7	70	12	60	17	57
11	5	50	14	70	13	43
12	4	40	11	55	15	50
13	5	50	8	40	16	53
14	4	40	15	75	21	70
15	4	40	11	55	18	60
16	6	60	10	50	18	60
17	6	60	7	35	19	63
18	5	50	12	60	11	37
19	2	20	10	50	18	60
20	7	70	9	45	13	43

A. Why is it difficult to use the charts to compare the three sets of data?

- A **box and whisker plot**, or **box plot** divides a set of data into fourths, or **quartiles**, in order from least to greatest. Each quartile contains 25% of the data values.
- The dividing lines between the quartiles are called the **lower quartile** (Q_1), the **median** (Q_2), and the **upper quartile** (Q_3).
  - The lower quartile is the median of the lower half of the data. It includes the median if there are an odd number of pieces of data.
  - The upper quartile is the median of the upper half of the data. It also includes the median, if there are an odd number of pieces of data.

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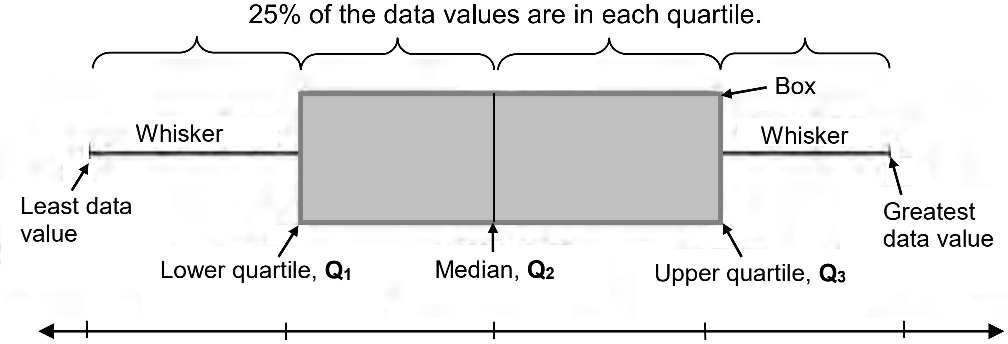


- You can follow these steps to create a box and whisker plot.

Step 1 Put the data in order from least to greatest.

For example:

The price of the same item (in Nu) in 15 different stores is shown below:

124	135	158	110	128	131	158	95
137	165	152	144	129	143	162	

In order: 95, 110, 124, 128, 129, 131, 135, 137, 143, 144, 152, 158, 158, 162, 165

Step 2 Find the median (Q_2), the lower quartile (Q_1), and the upper quartile (Q_3).

95, 110, 124, **128, 129**, 131, 135, **137**, 143, 144, **152, 158**, 158, 162, 165

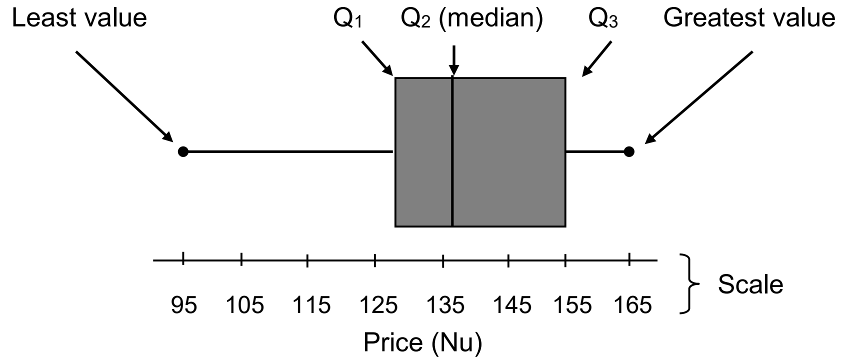
$Q_1 = 128.5$ Q_2 or Median $Q_3 = 155$

Step 3 Use Q_1, Q_2, and Q_3 to draw the box and locate the median.

- Draw a number line or **scale** that includes the least and greatest values.
- Find the lower quartile, the median, and the upper quartile on the scale.
- Draw a rectangular box above the scale. The left side should be above the lower quartile and the right side should be above the upper quartile. You can make the box any height.
- Draw a vertical line inside the box above the median.

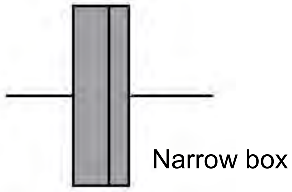
Step 4 Use the least and greatest values to draw the whiskers:

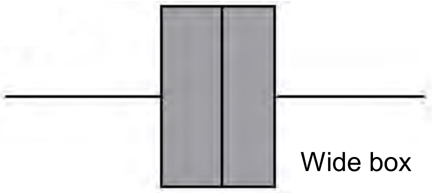
Make a dot outside the box at the least and greatest data values. Join each side of the box to one of the dots.



- A box and whisker plot shows the median, the **spread** of the data, and the **range**. It can also help to identify **outliers**.
- The shape of the box and whisker plot tells you a lot about how the data values in the set are **distributed**.

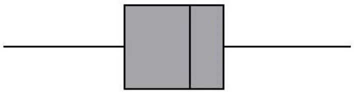
- A narrow box means the middle part of the data is clustered around the median. A wide box means that the middle part of the data is more spread out.



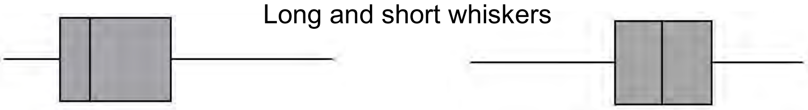


[Continued]

- Long whiskers on both sides mean there are some very high and very low values compared to the median.


Long equal whiskers

- A long whisker on only one side means that there are extreme values either lower or higher than the median, but not both.


Long and short whiskers

B. i) Create a box plot for each sample size in **part A**.

ii) Why does having the data graphed in three box plots make it easier to compare the distributions of the data sets?

Examples

Example 1 Creating a Box and Whisker Plot

Here are the heights (in cm) of 20 students. Create a box and whisker plot to display the heights.

181.6	169.3	141.5	144.8	169.7
152.7	160.7	142.8	176.6	189.4
149.8	157.2	162.7	164.6	159.2
162.7	149.2	170.2	143.8	176.9

Solution

141.5
142.8
143.8
144.8
149.2

$$Q_1 = \frac{149.2 + 149.8}{2} = 149.5$$

149.8
152.7
157.2
159.2
160.7

$$Q_2 \text{ or Median} = \frac{160.7 + 162.7}{2} = 161.7$$

162.7
162.7
164.6
169.3
169.7

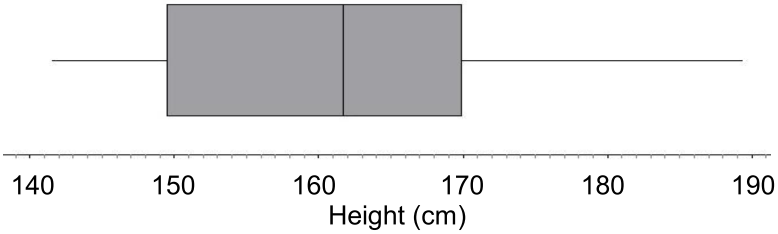
$$Q_3 = \frac{170.2 + 169.7}{2} \approx 170.0$$

170.2
176.6
176.9
181.6
189.4

Thinking

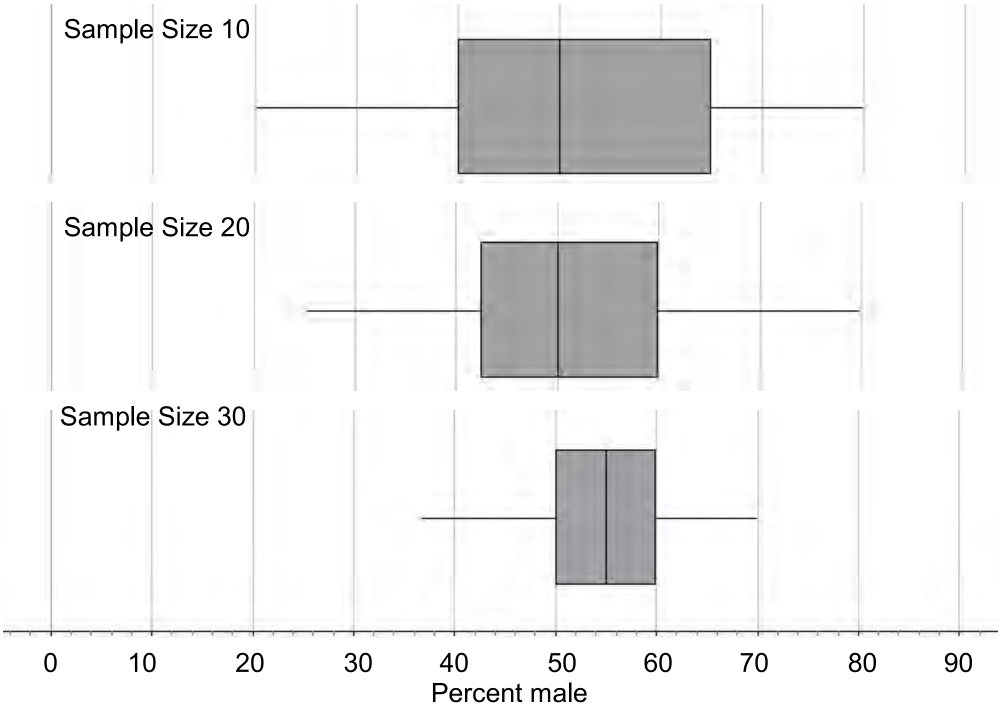
- I ordered the data.
- The median was the mean of the 10th and 11th numbers.
- There are 10 numbers below the median and 10 numbers above the median:
  - The lower quartile is the mean of the 5th and 6th numbers.
  - The upper quartile is the mean of the 15th and 16th numbers.
- I drew a scale from 140 to 192 to include the extreme values of 141.5 cm and 189.4 cm.

Student Heights



Example 2 Using Box and Whisker Plots to Compare Distributions

Here are the box and whisker plots for the data in the **Try This** on **page 168**.
How do the three sets of data compare?



Solution

Compare the medians

The median for sample sizes 10 and 20 is 50% and the median for sample size 30 is 53%.

The larger sample size best represents the population, which is 53% male.

Compare the quartiles

The lower and upper quartiles are farthest apart for sample size 10 and closest together for sample size 30.

That means, as the sample size increases, the percents are less spread out.

Compare whiskers

For sample size 30, the ends of the whiskers are closer to the lower and upper quartiles than the extreme values of the other data sets.

This means, as the sample size increases, the data values are less spread out and there are fewer or no outliers.

Thinking

- I knew the population was 53% male, so I compared each median to that percent.

- I looked at the lower and upper quartiles in each data set to compare the spread of the data values.

- I checked the extreme values to see whether possible outliers existed in the data sets.

Practising and Applying

1. The chart on the right shows the average monthly rainfall (in millimetres) in Trashigang. Create a box plot to display the data.

Average Monthly Rainfall in Trashigang

J	F	M	A	M	J
0.6	8.0	28.0	53.7	62.3	135.0
J	**A**	**S**	**O**	**N**	**D**
163.2	120.2	94.0	36.0	8.0	0.3

2. The chart on the right shows the ages of 60 people in a school.

a) Create a box plot to display the data.

b) Describe the distribution of the data.

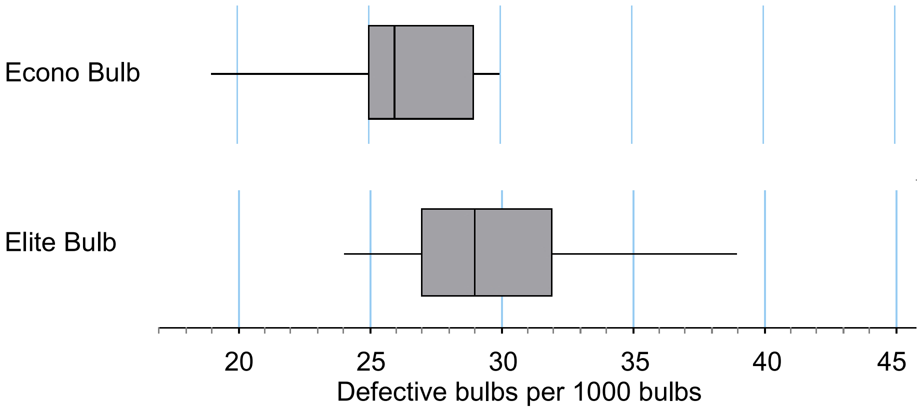
Ages of 60 People in a School

6	6	6	6	7	7	7
8	8	8	9	9	9	9
9	9	9	9	9	9	9
9	10	10	10	10	10	10
10	11	11	11	11	11	11
11	11	11	11	11	12	12
12	13	13	13	13	13	13
14	14	14	14	14	14	27
30	39	50	55			

3. Each week, workers at the Econo Bulb Company randomly choose 1000 light bulbs and test them to see how many work. They record the number of defective bulbs. They collected data values for 25 weeks and displayed them a box plot.

The Elite Bulb Company does the same thing with their bulbs.

Describe what the plots below tell you about how the quality of the bulbs compare.



Choose either question 4 or question 5 to complete.

4. a) Create box and whisker plots to show the distribution of the data in each column of this population table from the 2005 Census.

b) Use the plots to describe and compare the distribution of males, females, and total population.

Dzongkhag	Male	Female	Total
Bumthang	8,751	7,365	16,116
Chhukha	42,298	32,089	74,387
Dagana	9,168	9,054	18,222
Gasa	1,635	1,481	3,116
Ha	6,284	5,364	11,648
Lhuentse	7,727	7,668	15,395
Monggar	18,694	18,375	37,069
Paro	19,294	17,139	36,433
Pemagatshel	6,856	7,008	13,864
Punakha	8,989	8,726	17,715
Samdrupjongkhar	20,555	19,406	39,961
Samtse	31,306	28,794	60,100
Sarpang	21,664	19,885	41,549
Thimphu	53,496	45,180	98,676
Trashigang	26,056	25,078	51,134
Trashiyangtse	8,861	8,879	17,740
Trongsa	6,869	6,550	13,419
Tsirang	9,517	9,150	18,667
Wangdue	16,083	15,052	31,135
Zhemgang	9,492	9,144	18,636

5. a) Create box and whisker plots to show the distribution of the horse population in Bhutan for each year shown in the chart on the right.

b) Use the plots to describe and compare how the distribution changed from year to year.

Dzongkhag	1999	2002	2005
Bumthang	1,978	1,193	1,398
Chhukha	865	655	912
Dagana	822	412	325
Gasa	793	906	971
Ha	1,832	1,661	604
Lhuentse	1,810	2,759	1,968
Mongar	2,558	3,037	2,642
Paro	1,348	1,780	1,334
Pemagatshel	915	728	516
Punakha	1,056	761	789
Samdrup Jongkhar	2,728	2,457	2,148
Samtse	466	246	27
Sarpang	694	455	846
Thimphu	1,300	482	1,217
Trashigang	5,873	4,277	3,399
Trongsa	360	575	391
Tsirang	349	363	199
Wangdue	2,050	1,502	1,393
Yangtse	2,195	2,449	1,837
Zhemgang	1,263	1,868	1,692

6. Without drawing a box and whisker plot, predict what the plot would look like for this data set. Explain your prediction.

1, 10, 10, 10, 10, 10, 10, 11, 200

6.2.5 EXPLORE: The Impact of Altering a Data Set

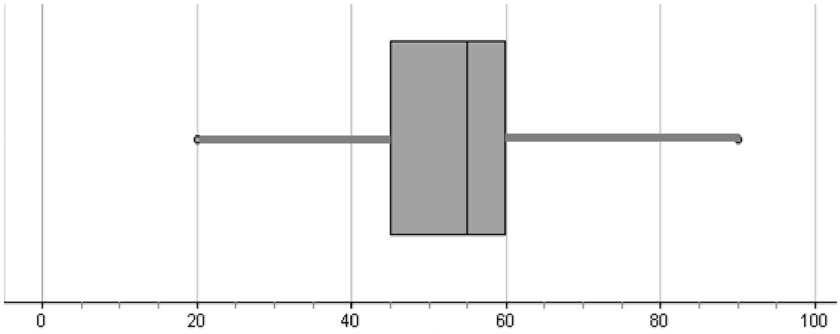
Samten graphed the following data in a box and whisker plot.

Trial #	Percent male
1	60
2	80
3	60
4	30
5	40
6	60
7	90
8	30
9	50
10	50

Trial #	Percent male
11	60
12	70
13	20
14	50
15	60
16	50
17	60
18	30
19	60
20	50

Minimum = 20
Q_1 = 45
Median = 55
Q_3 = 60
Maximum = 90

Mean = 53



Percent male

After analysing the data and the box plot, Samten wondered what would happen if he excluded the two extreme values, 20 and 90.

A. Predict what will happen to the mean, the median, and the shape of the box plot in each case:

i) if the least value is removed

ii) if the greatest value is removed

iii) if both extremes are removed

B. Check your predictions in **part A** by creating a box plot for each case. Were your predictions correct? Explain your thinking.

C. i) Suppose two data values are removed from the middle of the data set. Predict what will happen to the mean, median, and shape of the box plot.

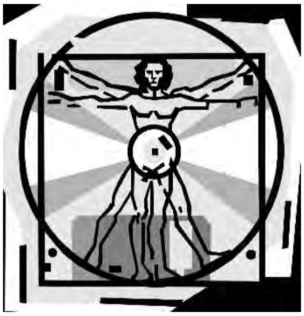
ii) Check your predictions by creating a box plot. Were your predictions correct? Explain you thinking.

6.3.1 EXPLORE: The Relationship Between Two Variables

Artists and scientists who study the structure of human bodies have discovered **relationships** between the measures of various parts of our bodies.

In this lesson, you will try to determine whether there is
- a relationship between hand span and hand length, and
- a relationship between foot length and hand length.

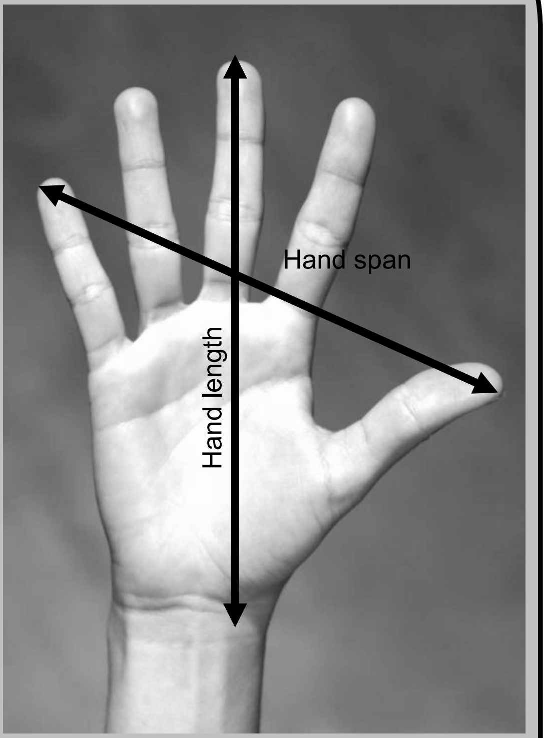
(Keep your data to use for **question 6** on **page 180**.)



A. i) Measure your hand length — the distance from your wrist to the tip of your middle finger — to the nearest tenth of a centimetre.

ii) Spread your fingers as wide as you can. Measure your hand span — the distance from the tip of your little finger to the tip of your thumb.

iii) Measure your foot length.



B. i) Record the data values for all the students in your class in a chart like this.

Student	Hand length (cm)	Hand span (cm)	Foot length (cm)

ii) Reorganize the data so the values in the hand length column go from least to greatest. (Remember to keep each student's data together.)

C. Look for relationships in the data in the chart.

i) What happens to the hand span measurements as the hand length measurements increase?

ii) What happens to the foot length measurements as the hand length measurements increase?

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6.3.2 Using a Scatter Plot to Represent a Relationship

Try This

Examine this pattern.

Figure 1

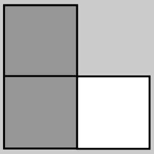


Figure 2

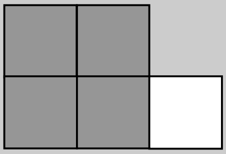
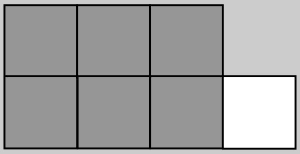


Figure 3



A. i) Predict the number of square tiles in each of the next two figures.

ii) Sketch all five figures in the pattern to check your predictions.

B. What pattern rule describes the relationship between the figure number and the number of tiles in each figure?

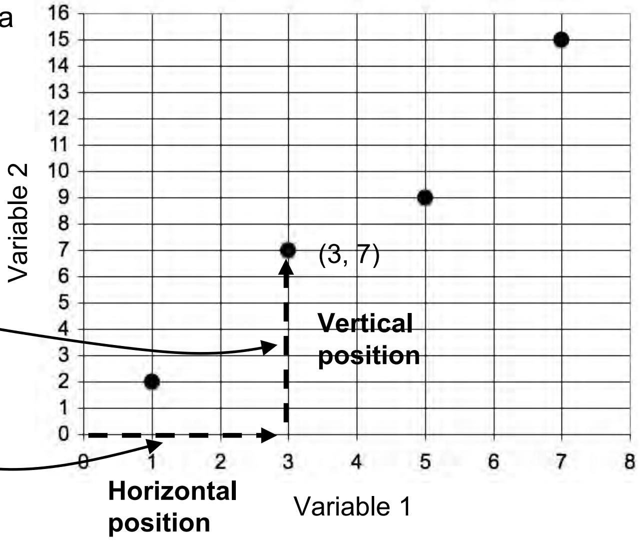
- A **scatter plot** is a graph you can use to see if there is a relationship between two **variables**.
- To create a scatter plot, follow these steps:
  - Create a **table of values** in which the related values of each variable appear in each row.
  - Create the **horizontal axis** and **vertical axis** of the graph. The horizontal axis is often called the ***x*-axis**. The vertical axis is called the ***y*-axis**.
  - Plot each pair of values in the table as a point. The value in the left column is the ***x*-coordinate** and the value in the second column is the ***y*-coordinate**.

For example:

This scatter plot displays the data in this table of values.

x-coordinate *y*-coordinate

Variable 1	Variable 2
1	2
3	7
5	9
7	15



- If you see a pattern formed by the plotted points, you can use it to predict the values for other points.

For example, it looks like (2, 4.5) and (9, 20) would also be on this graph, so they could be part the same relationship as the other pairs of values.

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C. i) Create a table of values that relates the number of tiles in each figure of the pattern in **part A** to the figure number for the first 10 figures.

ii) Create a scatter plot using the data in the table of values.

iii) Describe the shape formed by the plotted points.

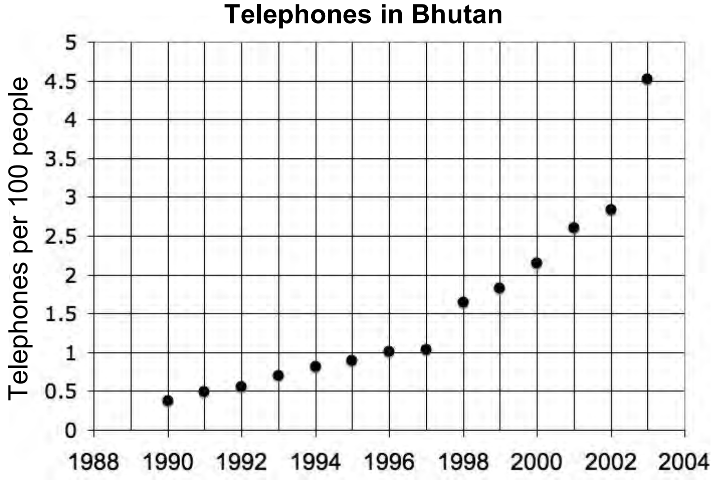
Examples

Example Using a Scatter Plot to Show a Trend

The table shows the number of telephones per 100 people in Bhutan from 1990 to 2003. Describe the trend in the number of phones from 1990 to 2003.

Year	Telephones per 100 people
1990	0.37
1991	0.49
1992	0.56
1993	0.70
1994	0.81
1995	0.90
1996	1.01
1997	1.04
1998	1.64
1999	1.82
2000	2.15
2001	2.60
2002	2.84
2003	4.52

Solution



The number of phones per 100 people grew each year. From 1998 to 2003, the yearly growth increased more in the later years than in earlier years.

Thinking

- I drew a scatter plot to show the trend in the number of phones.
- The shape that the points formed showed a relationship between time and the number of phones — over time, the number of phones increased.

Practising and Applying

1. ***Choose two of a), b), and c) below.*** For each pattern, do this:

i) Create a table of values that relates the figure number to the number of items in the figure.

ii) Create a scatter plot to represent the relationship.

a)

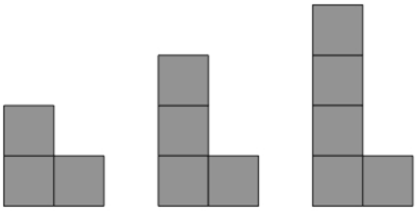


Figure 1 Figure 2 Figure 3

Figure number	Number of squares
1	3

b)

Figure 1 Figure 2 Figure 3

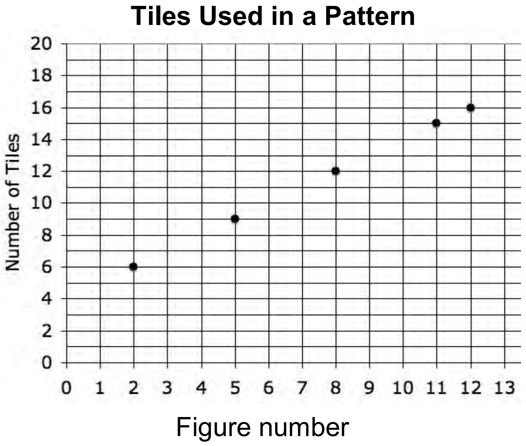
Figure number	Number of circles
1	5

c)

Figure 1 Figure 2 Figure 3

Figure number	Number of sticks
1	3

2. Karma made the following graph.



Karma's graph shows the relationship between the number of tiles in each figure of a pattern and the figure number. Use the graph to determine the number of tiles needed to make Figures 1, 3, 7, and 10. Explain what you did.

3. The students in a science class conducted an experiment in which they added different masses to the top of a spring. Draw a scatter plot to display their data.

Mass added (kg)	Length of spring (cm)
0	15.0
2	13.2
4	11.4
6	9.6
8	7.8
10	6.0

4. ***Choose one of the following sets of data, a) or b).***

i) Create a scatter plot.

ii) Describe the trend the graph shows.

a) This table shows the number of personal computers in Bhutan.

Year	Number of computers
1998	2500
1999	3000
2000	5000
2001	7000
2002	10,000
2003	10,000

b) This table shows the number of telephones in Bhutan.

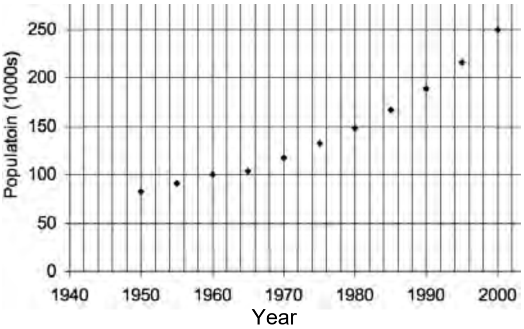
Year	Number of telephones
1990	1871
1991	2500
1992	2959
1993	3809
1994	4572
1995	5243
1996	6074
1997	6430
1998	10,437
1999	11,990
2000	14,145
2001	17,553
2002	19,615
2003	33,190

5. This table shows the population in Bhutan of people aged 10 to 14.

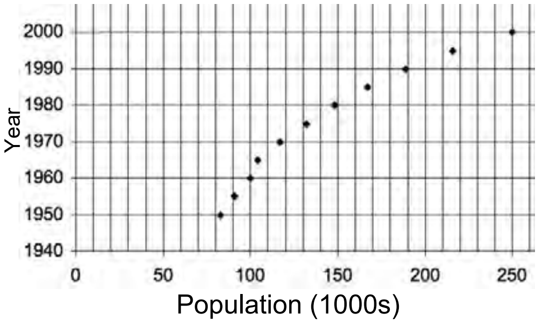
Year	Population age 10 to 14 (1000s)
1950	83
1955	91
1960	100
1965	104
1970	117
1975	132
1980	148
1985	167
1990	189
1995	216
2000	250

5. [Cont'd] Which scatter plot below better represents the data? Why?

A. Population of Bhutan, Ages 10 to 14



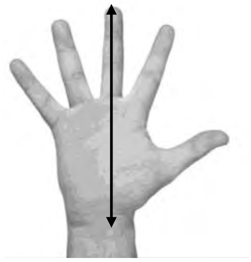
B. Population of Bhutan, Ages 10 to 14



6. a) Create a scatter plot to display the data you collected about hand length and hand span in **lesson 6.3.1** on **page 176**.

b) Examine your graph. Describe the relationship between hand span and hand length.

c) How could you use the graph to predict someone's hand span if you knew his or her hand length?



7. Name two or more things that you need to keep in mind when you create a scatter plot.

UNIT 6 Revision

1. Determine the theoretical probability of drawing each from a deck of 52 cards.

a) P(any Queen)

b) P(not a Queen)

c) P(a number card from 2 to 10)

2. A bag contains one white ball, three grey balls, and six black balls. You draw one ball from the bag.

a) What outcomes make up the event "not black"?

b) Determine each theoretical probability.

i) P(not black) **ii)** P(not white)

3. The weather forecast says that the probability it will rain today is $\frac{1}{2}$ and the probability it will rain tomorrow is $\frac{1}{3}$.

Describe a simulation you could do to estimate the probability that it will not rain over the two days.

4. Suppose you wanted to find out the favourite type of music of 100 students in Class VIII.

a) Describe how you might choose a random sample of 10 students.

b) Describe a sample that would not represent the population very well.

5. The chart shows the results of a survey of students' favourite sports at a school.

Favourite sport	Percent of students
Archery	75%
Football	15%
Basketball	5%
Badminton	5%

5. (Cont'd) **a)** Calculate the angle needed to display each category as a sector in a circle graph.

b) Create the circle graph.

6. Create a circle graph to display the following data about education levels in Bhutan from the 2005 census.

Education Levels in Bhutan (2005)

Education	Number of people (1000s)
In school now	136,368
Finished school	157,227
Never attended school	264,927

7. Arun Kumar shoots 50 arrows every time he practises. The following scores represent the number of times he hit the target each time he practised.

8	13	12	11	12
11	10	3	12	13
18	12	12	13	15
11	11	22	12	8

a) Calculate the mean, median, and range for the data.

b) Create a box and whisker plot to display the data.

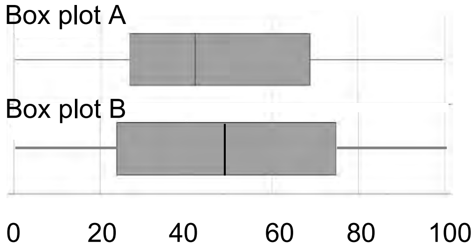
c) Identify the extreme values in the data.

d) Describe the effect on the measures calculated in **part a)** of removing only the least value.

e) Describe the effect on the measures calculated in **part a)** of removing only the greatest value.

f) Predict how the box and whisker plot would change if you removed both extreme values. Create a box and whisker plot to check your prediction.

8. Chandra used a calculator to generate random numbers between 0 and 100. He generated a set of 100 random numbers and then another set of 1000 random numbers.
He created a box plot for each set.
Which box plot below do you think represents the sample of 1000 random numbers? How do you know?



9. a) Create a box plot to display this set of data about the number of millimetres of rain each month in Mongar.

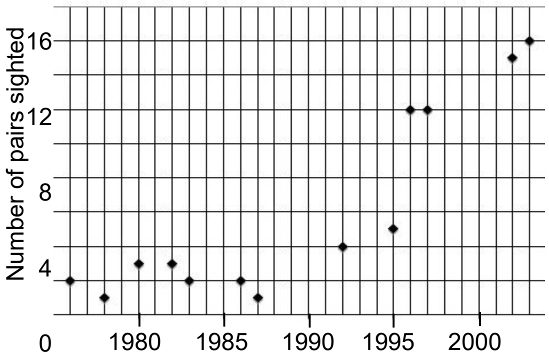
Monthly Rainfall in Mongar

J	F	M	A	M	J
7	12	36	71	89	133
J	**A**	**S**	**O**	**N**	**D**
217	178	81	71	16	3

b) What does the box plot tell you about the data?

10. The black-necked crane was in danger of extinction for many years.
This graph shows the number of breeding pairs sighted in Ladakh, India over several years.

Black-necked Crane Breeding Pairs



10. [Cont'd] **a)** Use the scatter plot to describe the trend in the number of breeding pairs sighted.

b) There is no data value for 1985.
If there were, what number do you think it would be?

11. a) Create a scatter plot to display this data set about the number of black-necked crane sightings each year in Ladakh.

Black-necked Crane Sightings

Year	Number of cranes sighted
1976	5
1978	12
1980	14
1982	13
1983	7
1986	16
1987	9
1992	17
1995	22
1996	25
1997	38
2002	59
2003	60

b) Compare the trend in crane sightings in **part a)** with the trend in breeding pair sightings in **question 10**.

Black-necked crane

UNIT 7 ALGEBRA

Getting Started

Use What You Know

Kinley made a pattern using grey and white squares.

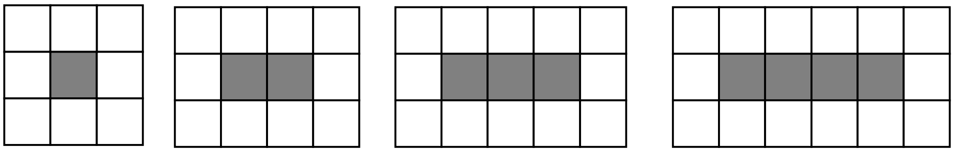


Figure 1 Figure 2 Figure 3 Figure 4

A. Copy and complete this table to show the pattern.

Number of grey squares	Number of white squares
1	8
2	10
3	
4	
5	

B. i) Predict the number of white squares when there are 15 grey squares. Explain your prediction.

ii) Which expression below could you use to find the number of white squares (*w*), if you know the number of grey squares (*g*)? How do you know?

$w = 8g$

$w = 8g + g + 2$

$w = 8 + 2g$

$w = 6 + 2g$

iii) Use the expression you chose in **part ii)** to check your prediction from **part i)**.

C. i) Graph the information in the table. Use the number of grey squares as the *x*-coordinate of each point and the number of white squares as the *y*-coordinate.

ii) What pattern do the points form?

iii) How can you use the graph to check your prediction from **part B i)**?

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Skills You Will Need

1. a) Each table represents a pattern. Copy and complete each table.

i)

x	*y*
1	8
2	13
3	
4	
5	28

ii)

x	*y*
1	18
2	
3	
4	12
5	

b) For each table in **part a)**, write an equation that you can use to find y if you know x.

2. a) Create a table like those in **question 1** to represent the equation $y = 2x - 4$.

b) i) Use the equation to find the value of y that relates to an x-value of 10.

ii) Use the equation to find the value of x that relates to a y-value of 20.

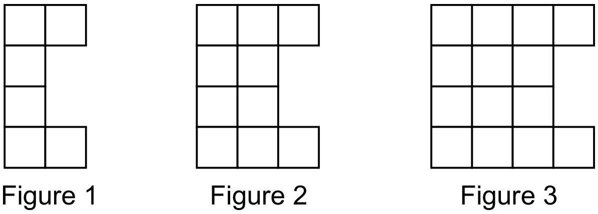
c) Graph the points in the table from **part a)**. Show how you can use the graph to answer **part b) i) and ii)**.

3. a) Simplify each.

i) $(3n + 2) + (4n - 5)$ **ii)** $(2n - 4) - (3n + 6)$

b) Evaluate each expression in **part a)** for $n = 8$.

4. A pattern rule for the number of squares in the pattern below is $4(f + 1) - 2$. You can use the rule to find the number of squares in any figure if you know the figure number (f).



a) Write an equation you can use to find the number of the figure that has 90 squares.

b) Solve the equation in two different ways. Show your work.

5. What is the coefficient of x in each expression? What is the constant?

a) $2x - 3$ **b)** $4 + 3x$ **c)** $-3x + 6$

7.1.1 EXPLORE: Representing Relationships

- Nima has some Nu 2 coins and some Nu 5 notes. The total value of the coins is Nu 100. There are many different combinations of Nu 2 coins and Nu 5 notes that he could have.
- You can use a table of values, a **graph**, or an **algebraic equation** to find all the different possible combinations that have a total value of Nu 100.

A. Copy and complete the table of values below. List three combinations of Nu 2 coins and Nu 5 notes that have a total value of Nu 100.

Combinations With a Total Value of Nu 100

Number of Nu 2 coins			
Number of Nu 5 notes			

B. Graph the information in your table. Use the number of Nu 2 coins as the *x*-coordinate and the number of Nu 5 notes as the *y*-coordinate.

C. i) Use the graph to show why Nima cannot have exactly five Nu 5 notes.

ii) Use reasoning to explain why Nima cannot have exactly five Nu 5 notes.

D. One combination has two more Nu 5 notes than another combination. How many fewer Nu 2 coins does it have? How do you know?

E. i) Write an algebraic expression to represent the value of t Nu 2 coins.

ii) Write an algebraic expression to represent the value of f Nu 5 notes.

iii) Use your expressions for t and f to write an equation to represent the combination of Nu 2 coins and Nu 5 notes that have a total value of Nu 100.

iv) Solve your equation for $t = 25$. Where is the solution located on your graph?

F. i) Use your table, graph, or equation to find all possible combinations of Nu 2 coins and Nu 5 notes that have a total value of Nu 100.

ii) Why did you make the choice you made (table, graph, or equation) in **part i)**?

7.1.2 Describing Relationships and Patterns

Try This

A. i) Describe the pattern in the number of dots.

ii) Use the pattern to predict the number of dots in each figure:

- Figure 4
- Figure 12
- Figure 20

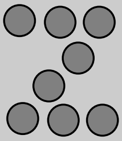


Figure 1

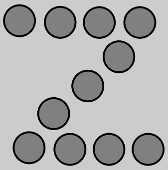


Figure 2

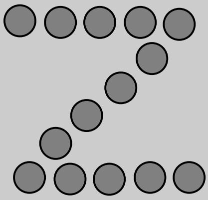


Figure 3

You can represent or describe a **pattern** or relationship in different ways:

- using a table of values

- using a graph

- using an algebraic equation

You can use your table, graph, or equation to solve problems related to the pattern or relationship.

For example:

For the pattern below, you can describe the relationship between the number of dots in each figure and the figure number.

Figure 1

Figure 2

Figure 3

• You can create a table of values.

Figure number	Number of dots
1	10
2	14
3	18
4	

- The table makes it easy to see that the number of dots in the pattern is 10, 14, 18, …, while the figure number is 1, 2, 3, ... This means that the number of dots increases by 4 as the figure number increases by 1.

- You can use the relationship between the number of dots and the figure number to determine the number of dots in any figure:

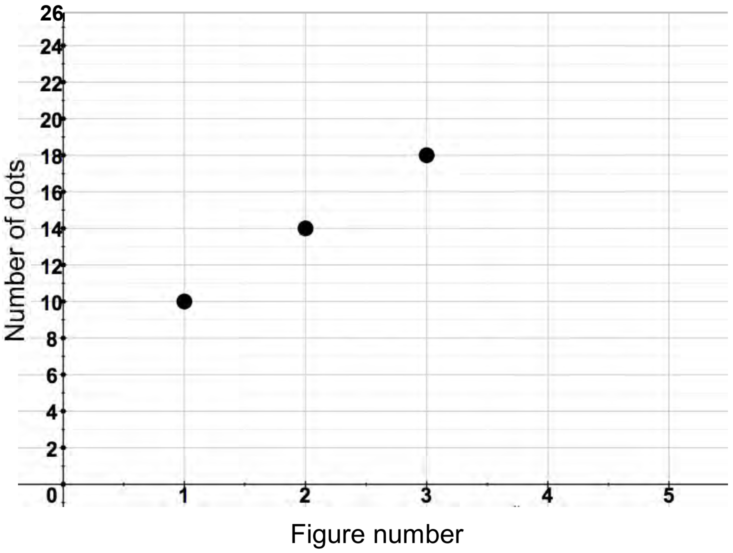
Since there are 18 dots in Figure 3, there are

18 + 4 = 22 dots in Figure 4

18 + 2 × 4 = 26 dots in Figure 5

- You can create a graph of the information in the table.

Plot each row in the table as a point. The figure number is the *x*-coordinate and the number of dots is the *y*-coordinate. The points are (1, 10), (2, 14), and (3, 18).



You can use the pattern in the graph to see that the number of dots for Figure 4 is 22 and the number of dots for Figure 5 is 26.

- You can also describe the relationship using an algebraic equation.

Notice that the number of dots increases by 4 just like the multiples of 4. If you compare the pattern in the number of dots to the multiples of 4, you will see that each is 6 more than the corresponding multiple of 4.

Figure number (*f*)	1	2	3
Number of dots (*d*)	10	14	18
Multiples of 4 (4 ×)	4	8	12
	4 + 6	8 + 6	12 + 6

The algebraic equation for the relationship is $d = 4f + 6$, where d is the number of dots and f is the figure number.

You can use the equation to determine the number of dots in any figure:

For Figure 4, substitute $f = 4$: $d = 4 \times 4 + 6 = 22$

For Figure 5, substitute $f = 5$: $d = 4 \times 5 + 6 = 26$

B. Describe or represent the relationship between the figure number and the number of dots for the pattern in **part A** using each form.

i) a table of values

ii) a graph

iii) in words

iii) an algebraic equation

C. Use the table, graph, or equation from **part B** to determine the number of dots in Figures 4, 12, and 20. Were your predictions in **part A** correct?

Examples

Example 1 Comparing Relationships Using Tables, Equations, and Graphs

Each table of values represents the number pattern listed above it. Each table shows the relationship between the position of the term (the term number) and the value of the term (the term value).

Relationship A

20, 23, 26, 29, ...

Term number	Term value
1	20
2	23
3	26
4	29

Relationship B

10, 6, 2, –2, ...

Term number	Term value
1	10
2	6
3	2
4	–2

a) How are the algebraic equations for the two relationships alike? different?

b) How are the graphs for the two relationships alike? different?

Solution

a) **Relationship A**

Term number	Term value	3 ×
1	20	3
2	23	6
3	26	9
4	29	12

Each term value is 17 more than the product of 3 and the term number.

Term value = 3 × *Term number* + 17

Relationship B

Term number	Term value	–4 ×
1	10	–4
2	6	–8
3	2	–12
4	–2	–16

Each term value is 14 more than the product of –4 and the term number.

Term value = –4 × *Term number* + 14

Alike: The equations both involve multiplying the term number by a number and adding another number to get the term value.

Different: The multiplier and constant values are different.

Thinking

a) Since the term values in the first table go up by 3, I compared them to the multiples of 3, which is the same as the 3 times table (3 ×).

- Since the term values in the second table go down by 4, I compared them to the opposite values of the multiples of 4.

- I could have written the equation as

Term value
= 14 – *Term number* × 4

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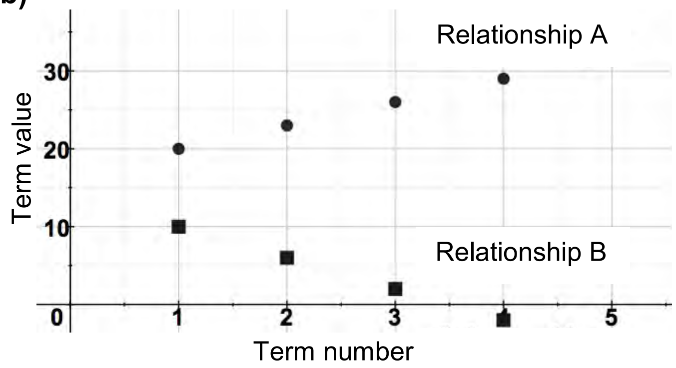
b)



Alike: The points form a straight line.

Different: One line slants upward and the other line slants downward.

b) I drew both graphs on the same grid to make it easier to compare them.

Example 2 Predicting an Unknown Value in a Pattern Using an Equation

A sequence of numbers is 3, 12, 21, 30, ….

a) Create an algebraic equation to describe the relationship between any term value and its position in the pattern.

b) Use your equation to predict the value of the 20th term.

Solution

a)

Term number	Term value	9 ×
1	3	9
2	12	18
3	21	27
4	30	36

Each term value is 6 less than the product of 9 and the term number.

If v represents the term value and n represents the term number, then:

$$v = 9n - 6$$

b) For $n = 20$, $v = 9(20) - 6$

$$= 180 - 6$$

$$= 174$$

The 20th term in the pattern is 174.

Thinking

a) I created a table so I could look for patterns.

- I noticed that the term values in the sequence increased by 9 each time, so I compared the term values to the multiples of 9.

b) I substituted $n = 20$ into the equation to find the value of the 20th term.

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Practising and Applying

1. Use an equation and a graph to describe the relationship between the term number and term value in each pattern.

a) 4, 14, 24, 34, …

b) 100, 95, 90, 85, …

c) 8, 15, 22, 29, …

d) 37, 26, 15, 4, …

2. Use an equation or a graph to predict the value of the 20th term in each pattern.

a) 3, 10, 17, 24, …

b) 15, 8, 1, –6, …

c) 13, 9, 5, 1, …

d) $\frac{9}{2}, \frac{5}{2}, \frac{1}{2}, -\frac{3}{2}, \ldots$

3. a) Use an equation and a graph to describe the relationship in each table.

Relationship A

Term number	Term value
1	20
2	16
3	12
4	8

Relationship B

Term number	Term value
1	7
2	13
3	19
4	25

b) Compare the relationships by comparing the equations and graphs.

4. a) Sonam says that the 20th figure in this pattern has 400 small triangles. Do you agree? Explain your thinking.

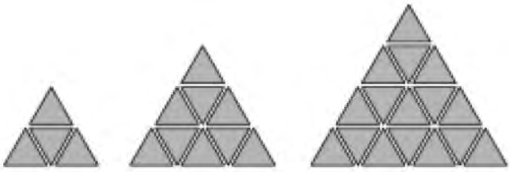

Figure 1 Figure 2 Figure 3

b) Create a graph to show the relationship between the figure number and the number of small triangles.

c) Describe the shape of the graph. Why do you think it has that shape?

5. Kinley says that he can use the equation $v = 2n + 3$ to describe the relationship in the table below.

Term number (n)	1	2	3	4
Term value (v)	5	8	11	14

Rinzin says the equation is $v = 3n + 2$.

Who is right, Kinley or Rinzin? How do you know?

6. a) Draw a graph to show the relationship between the figure number and the number of dots in this pattern.

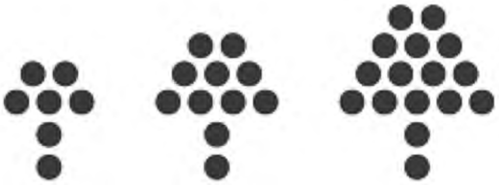

Figure 1 Figure 2 Figure 3

b) Use your graph to predict the number of dots in Figure 6.

c) Find a way to check your prediction.

7. Use a table and an equation to describe this pattern.

Figure 1 Figure 2 Figure 3

8. Which form of describing a relationship would you use for each situation below?

- a table of values or
- a graph or
- an algebraic equation

Explain your thinking.

Situation A
to predict the value of an unknown term in a sequence

Situation B
to compare two relationships

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7.1.3 Recognizing Linear Relationships

Try This

Mindu sent 20 e-mails every day for seven days. Choki sent five e-mails one day. Then he sent one more e-mail than the previous day every day for seven days.

Suppose you were asked to graph the relationship between the day number (Day 1 to Day 7) and the total number of e-mails sent both for Mindu and for Choki.

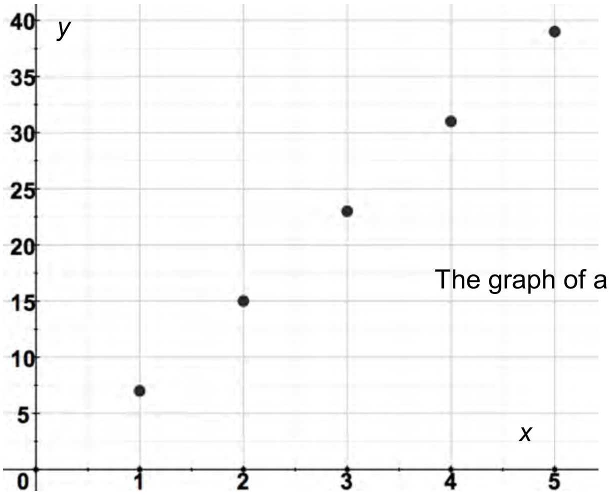
A. Whose graph will be a straight line, Mindu's or Choki's? Why do you think that?

- When you plot points to graph a relationship, the points sometimes form a straight line. When this happens, it means the relationship is **linear**.

For example:

When you graph the linear relationship in this table of values, the points form a straight line.

***x*-value**	1	2	3	4	5
***y*-value**	7	15	23	31	39



The graph of a linear relationship

- You can always graph a relationship to tell whether it is linear. You can also use its table of values to predict whether a relationship is linear.

- You can see if the *y*-values change by a **constant** amount as the *x*-values also change by a constant amount.

For example:

In this table, the *y*-values increase by 8 each time, as the *x*-values increase by 1.

A constant increase in both values means the relationship is linear.

+ 1 + 1 + 1 +1

***x*-value**	1	2	3	4	5
***y*-value**	7	15	23	31	39

+ 8 + 8 + 8 + 8

- You can also see if the *y*-values are the same amount greater or less than the multiples of a number.

For example:

In this table, each *y*-value is 1 less than the corresponding multiple of 8.

This means the relationship is linear.

***x*-value**	1	2	3	4	5
***y*-value**	7	15	23	31	39
Multiples of 8	8	16	24	32	40

8 – 1   16 – 1   and so on

Comparing the *y*-values to multiples is also helpful for writing equations. You can use the equation $y = 8x - 1$ represent the relationship in the table above.

These two ways to tell whether a relationship is linear are related, since multiples always increase by a constant amount.

- Here are some common linear relationships:

The relationship between the distance travelled and the time spent travelling, when the speed is constant

For example, when the speed is 30 kilometres per hour:

Time (h)	1	2	3	4	5
Distance (km)	30	60	90	120	150

This relationship can be represented by the equation $y = 30x$.

The relationship between the amount earned and the amount of time worked, when the rate of pay is constant

For example, when the rate of pay is Nu 20 per hour:

Time (h)	1	2	3	4	5
Earnings (Nu)	20	40	60	80	100

This relationship can be represented by the equation $y = 20x$.

Some measurement situations, such as the relationship between perimeter and side length

For example, the perimeter of a square compared to its side length:

Side length	1	2	3	4	5
Perimeter	4	8	12	16	20

This relationship can be represented by the equation $y = 4x$.

There are also non-linear relationships. You will study them in higher classes.

B. How could you have predicted which relationship in **part A** was linear?

Examples

Example 1 Describing Change With a Table, a Graph, and an Expression

Dawa Dem has saved Nu 150. Each month, she will add Nu 25 to her savings.

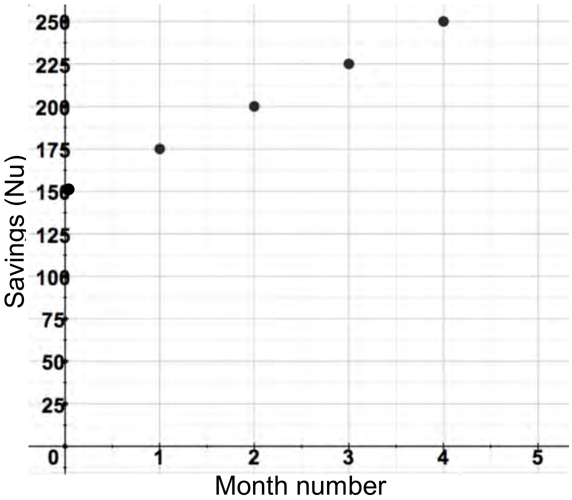
a) Describe the relationship that compares the month number and the total amount of money she has saved. Use a table of values, a graph, and an algebraic equation.

b) Explain how you know the relationship is linear.

Solution

a)

Month	Total saved at end of month
Start (0)	150
1	175
2	200
3	225
4	250



Month	Total saved at end of month	Multiples of 25
0	150	0
1	175	25
2	200	50
3	225	75
4	250	100

The total saved at the end of each month is Nu 150 more than 25 times the month number. If *m* is the month number and *s* is the amount of savings at the end of the month, the equation is

$$s = 25m + 150.$$

b) The graph is a straight line, so the relationship is linear.

The table shows that the values for the savings are all the same amount (150) more than a multiple of 25, so the relationship is linear.

Thinking

a) I first created the table of values because it would help me draw the graph.

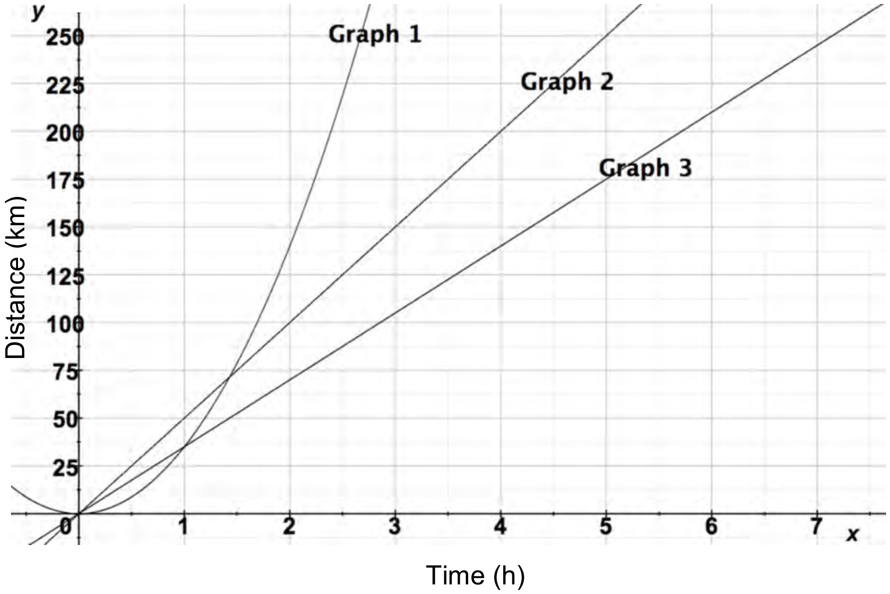
- I started with the Nu 150 Dawa Dem had already saved and then added Nu 25 for each month.

- By comparing the savings to the multiples of 25, I was able to figure out the algebraic equation. I used multiples of 25 because the increase was Nu 25 each month.

b) I used the graph and the table of values to tell that the relationship was linear.

Example 2 Matching a Situation to a Graph

Which graph below describes the distance travelled each hour by a car that is travelling at a constant speed of 35 km/h? How do you know?

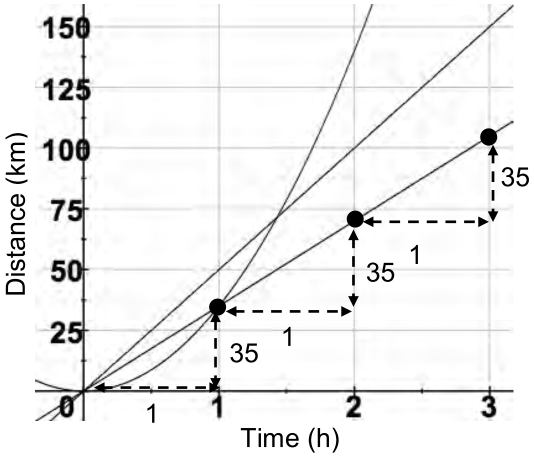


Solution

Graph 3 shows the distance travelled at a speed of 35 km/h.

The *y*-value increases by 35 for each *x*-value increase of 1.

I used the points (1, 35), (2, 70), and (3, 105) to tell.



Thinking

- I knew the graph had to be a straight line because the speed was constant. That meant it was a linear relationship, so it was either Graph 2 or Graph 3.

- To decide between Graph 2 or 3, I looked to see which graph showed a *y*-value increase of 35 km for each *x*-value increase of 1 h. (I had to estimate because not all the values were marked on the *y*-axis.)

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Practising and Applying

1. Tell whether the relationship in each table is linear. How do you know?

a)

x	y
1	30
2	28
3	26
4	24

b)

x	y
1	30
2	29
3	27
4	24

c)

x	y
1	15
2	17
3	19
4	21

2. Copy and complete each table so that it represents a linear relationship.

a)

x	y
1	20
2	23
3	
4	

b)

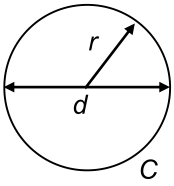
x	y
1	20
2	
3	
4	27.5

3. How do you know that each situation described below is a linear relationship?

Situation A the distance travelled by a plane flying at 550 km/h compared to the numbers of hours flying

Situation B a person's total earnings compared to the number of months worked, at a pay rate of Nu 920 a month

4. Tell whether each relationship is linear. Explain your thinking.



a) the radius of a circle compared to its diameter

b) the radius of a circle compared to its area

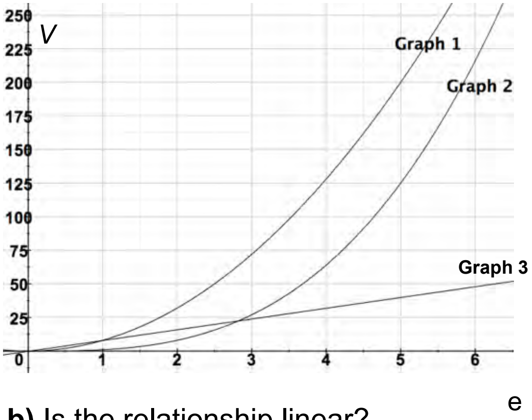
c) the radius of a circle compared to its circumference

5. Chabilal sells cars. He is paid Nu 400 each month plus Nu 1000 for each car he sells.

a) Create a table of values to describe Chabilal's monthly earnings based on the number of cars he sells.

b) Is the relationship linear? How do you know?

6. a) Which graph might describe the relationship between the volume (V) and edge length (e) of a cube? Explain how you know.



b) Is the relationship linear? Explain your thinking.

7. a) Is the expression $y = 8 – 2x$ linear? How do you know?

b) Repeat **part a)** for $y = 8 – 2x^2$.

8. A relationship between x and y is graphed. Two points on the graph are (4, 20) and (8, 68).

a) Sketch a graph to show that the relationship could be linear.

b) Sketch a graph to show that the relationship might not be linear.

CONNECTIONS: Adding Values in a Linear Relationship

- One of the simplest linear relationships is based on the pattern 1, 2, 3, 4, 5, …. It is a linear relationship because the term values change by a constant amount each time (+1) while the term number changes by a constant amount (+1).
- A famous German mathematician named Gauss (1777-1855) discovered how to add the values in a linear relationship quickly and easily. He made this discovery when he was very young.

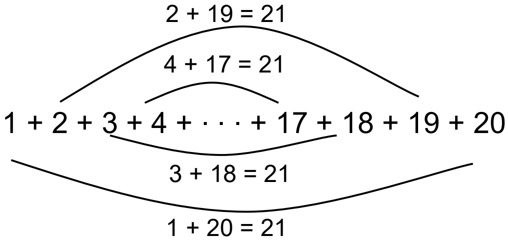
For example:

Consider adding the first 20 numbers in the sequence:

$$1 + 2 + 3 + 4 + \cdots + 17 + 18 + 19 + 20$$

Gauss noticed that you could pair the terms. Each pair adds to the same amount.

Gauss (1777–1855)



Since each pair adds to 21, and there are 10 pairs, the sum is 10 × 21 = 210.

1. Use Gauss's method to find the sum of all the whole numbers from 1 to 100.

2. Add the numbers in these linear patterns. Show your work.

a) The first 20 numbers in 2, 4, 6, 8, . . .

b) The first 20 numbers in 1, 4, 7, 10, . . .

c) The first 40 numbers in 300, 303, 306, 309, . . .

3. Does Gauss's method work for adding numbers in a non-linear pattern, such as 1 + 4 + 9 + 16 + 25 + 36? How do you know?

7.1.4 Slope

Try This

A. Graph the relationship represented by the equation $y = 4x$.

B. How do the *y*-values in your graph change in each instance?
Use the graph to explain how you know you are right.

i) when *x* increases from 1 to 2 **ii)** when *x* increases from 1 to 3

iii) when *x* increases from 1 to 4 **iv)** when *x* increases from 2 to 5

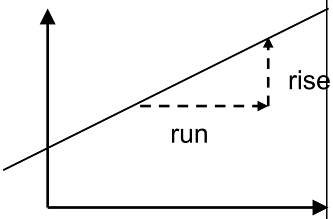
- Recall that the graph of a linear relationship is a straight line. It is straight because the *y*-value changes by a constant amount each time the *x*-value changes by a constant amount.

For example:

If *y* increases by 4 for each increase of 1 in *x*, then
y increases by 8 for an increase of 2 in *x*,
y increases by 12 for an increase of 3 in *x*, and so on.

- The change in the *x*-value is called the **run**.
The change in the *y*-value is called the **rise**.

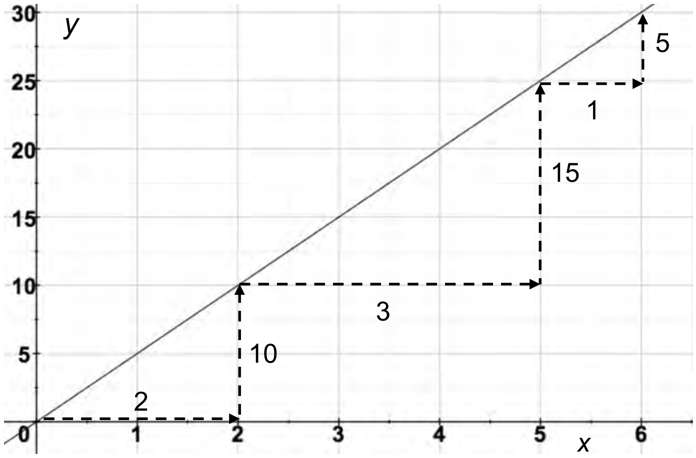
The ratio $\frac{rise}{run}$ (or $\frac{change\ in\ y}{change\ in\ x}$) is called the **slope**.



The slope of a straight line graph is constant.
The slope is equal to the change in *y* for a change of 1 in *x* anywhere along the graph.

For example:

The graph below of $y = 5x$ was created by plotting and connecting the points



Notice the following:

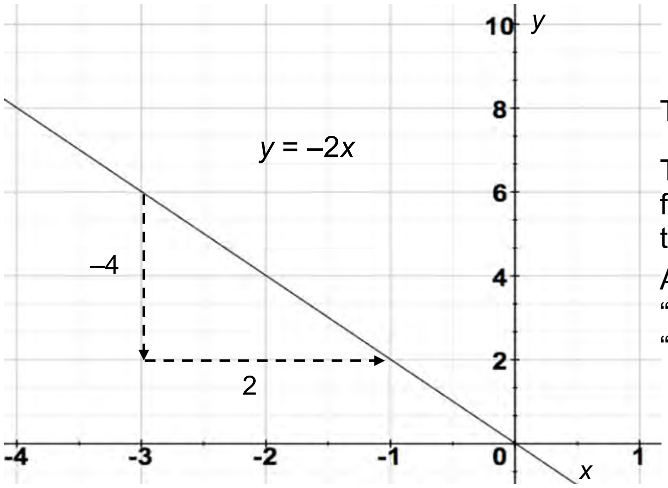
- The slope $\frac{10}{2}$ (bottom left), $\frac{15}{3}$ (centre of graph), and $\frac{5}{1}$ (top right) are all equal to 5, which is the change in *y* for a change of 1 in *x*.
- The three right triangles

The slope of the graph of $y = 5x$ is $\frac{5}{1} = 5$.

- Sometimes a graph falls to the right instead of rising to the right. When this happens, the rise is described using a negative value but the run remains positive. This means the slope is negative.

For example:

The graph below of $y = -2x$ was created by plotting and connecting the points (–3, 6), (–2, 4), and (–1, 2).



The slope is $\frac{-4}{2} = -2$.

This makes sense because for every increase of 1 in x, the y-value decreases by 2.

Another way of saying "a decrease of 2" is to say "an increase of –2".

The slope of the graph of $y = -2x$ is $\frac{-4}{2} = -2$.

C. What is the slope of the graph of $y = 4x$ in **part A**? How do you know?

D. Use what you know about the slope of the graph to describe how the y-value changes in each situation.

i) when x increases from 1 to 2
ii) when x increases from 1 to 3
iii) when x increases from 1 to 4
iv) when x increases from 2 to 5

Examples

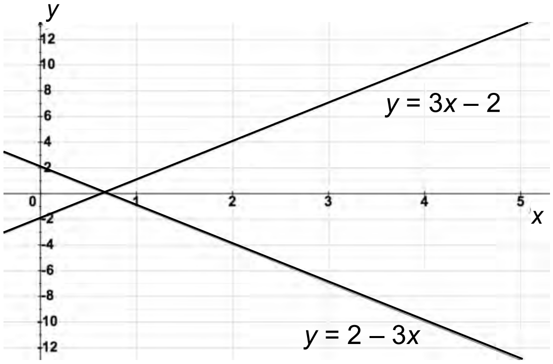
Example 1 Using a Graph to Determine Slope

a) Graph each relationship.

$y = 3x - 2$
$y = 2 - 3x$

b) Which graph has a negative slope? How do you know?

c) Consider the graph of $y = 3x - 2$. If the rise is 12, what is the run? Show your work.

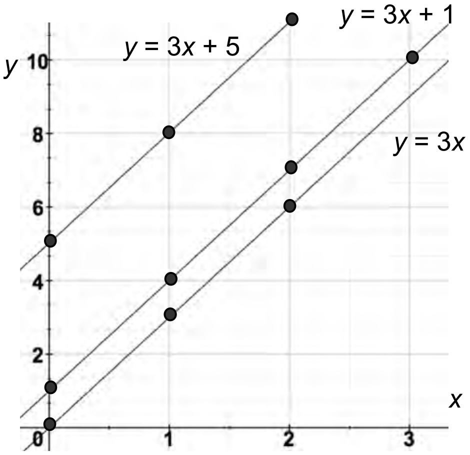
Solution	**Thinking**
a) 	 **a)** For each line, I plotted points for $x = 1$, $x = 2$, and $x = 3$. I connected the points with straight lines to create the two graphs.
b) The graph of $y = 2 – 3x$ has a negative slope because it falls to the right.	**b)** A negative slope means that the y-value decreases while the x-value increases.
c) The points (1, 1) and (2, 4) are on the graph of $y = 3x – 2$, so the slope is $\frac{rise}{run} = \frac{4-1}{2-1} = 3$. If $\frac{rise}{run} = 3$ and the rise is 12, then $\frac{12}{run} = 3$. Since $\frac{12}{4} = 3$, the run must be 4.	**c)** I could have used any two points on the line to calculate the slope. • Because I subtracted the y-value in (1, **1**) from the y-value in (2, **4**) for the rise, I knew I had to subtract the x-value in (**1**, 1) from the x-value in (**2**, 4).

Example 2 Relating the Slope of a Line to the Equation of the Line

a) Compare the slopes of the graphs of $y = 3x$, $y = 3x + 1$, and $y = 3x + 5$.

b) Compare the slopes of the graphs of $y = 2x$, $y = 3x$, and $y = 4x$.

c) Predict the slope of the graph of $y = 4x – 5$. Explain and check your prediction.

Solution	**Thinking**
a)  [Continued]	 **a)** I drew each graph by substituting 0, 1, and 2 for x in the equation to find y. Then I plotted the three points.

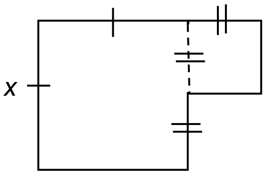
Example 2 Relating the Slope of a Line to the Equation of the Line [Cont'd]

Two points on $y = 3x$ are (0, 0) and (1, 3). Slope = 3 ÷ 1 = 3 Two points on $y = 3x + 1$ are (0, 1) and (1, 4). Slope = 3 ÷ 1 = 3 Two points on $y = 3x + 5$ are (0, 5) and (1, 8). Slope = 3 ÷ 1 = 3 The slope of each of the three graphs is 3.	• For all three graphs, I decided to find the rise when the run was 1. This was an easy way to calculate the slope. I used the points where $x = 0$ and $x = 1$.
b) For $y = 2x$, the points are (0, 0) and (1, 2). The slope is $\frac{rise}{run} = \frac{2}{1} = 2$. For $y = 3x$, the points are (0, 0) and (1, 3). The slope is $\frac{rise}{run} = \frac{3}{1} = 3$. For $y = 4x$, the points are (0, 0) and (1, 4). The slope is $\frac{rise}{run} = \frac{4}{1} = 4$.	**b)** I didn't have to graph the equations to figure out each slope. • I substituted $x = 0$ and $x = 1$ into each equation to get two points that I knew would be on the graph. Then I used the two points to calculate each slope.
c) Prediction: I think the slope of $y = 4x – 5$ is 4 because the slope of each graph I have seen so far seems to be the coefficient of x in its equation. Check: Substitute $x = 0$ and $x = 5$ into $y = 4x – 5$ to get two points that would be on the graph: (0, – 5) and (5, 15). The slope is $\frac{15-(-5)}{5} = \frac{20}{5} = 4$.	**c)** I thought that the slope might be the coefficient of x in the equation. I checked to be sure.

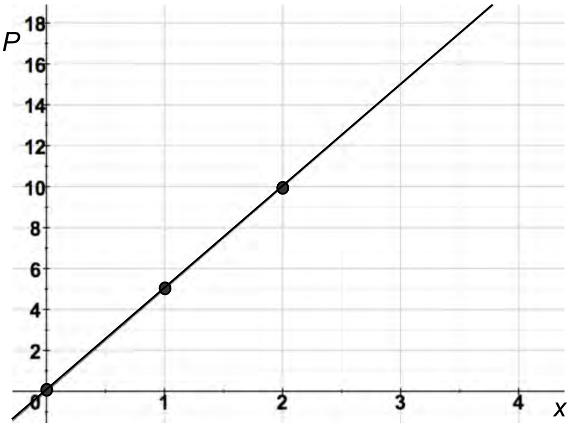
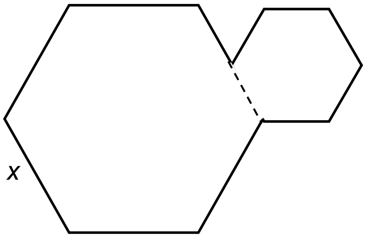
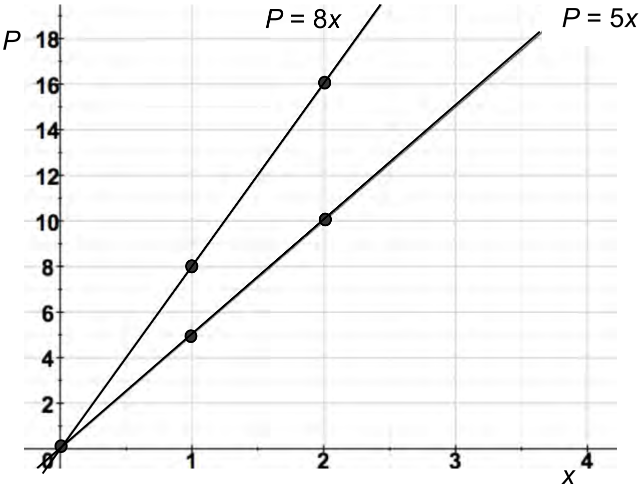
Example 3 Exploring Changes in a Linear Relationship

a) Write an algebraic equation and draw a graph to show the relationship between the value of x and the perimeter of a shape like the shape on the right.

b) Suppose the two shapes are regular hexagons instead of squares. The side length of the small hexagon is half the side length of the larger hexagon. How do the equation and the graph change?

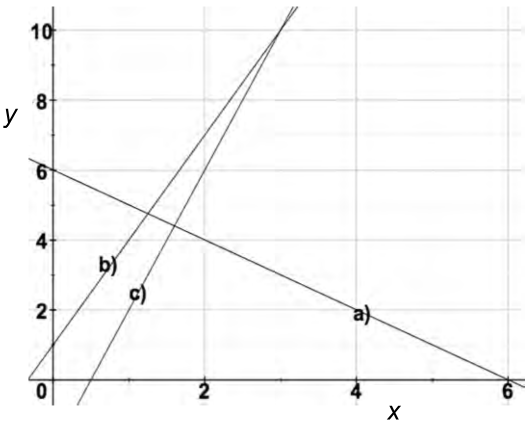


Solution	Thinking
a) $P = 3\frac{1}{2}x + 3(\frac{x}{2})$ $= \frac{7x}{2} + \frac{3x}{2}$ $= \frac{10x}{2}$ $= 5x$	 **a)** I knew that to find the perimeter of the whole shape I would add these parts: • $3\frac{1}{2}$ sides of the large square and • 3 sides of the small square, which were each half the size of the large square.

Substitute *x* = 0, 1, and 2 into $P = 5x$: Three points on the graph are (0, 0), (1, 5), and (2, 10). 	• Since I wasn't sure if it was a linear relationship, I graphed three points instead of just two. • It looks like the relationship between side length *x* and the perimeter *P* is linear.
b)  $P = 5\frac{1}{2}x + 5(\frac{x}{2}) = \frac{11x}{2} + \frac{5x}{2} = \frac{16x}{2} = 8x$ The equation for the squares used 5*x* and the equation for the hexagons used 8*x*.	**b)** I knew the perimeter of the whole shape includes: - $5\frac{1}{2}$ sides of the large hexagon and - 5 sides of the small hexagon, which were each half the size of the large hexagon.
Substitute *x* = 0, 1, and 2 into $P = 8x$: Three points on the graph are (0, 0), (1, 8), and (2, 16).  Both graphs are straight lines but the graph for the hexagons ($P = 8x$) is steeper than graph for the squares ($P = 5x$).	• Since I wasn't sure if this was a linear relationship, I graphed three points. • It looks like the relationship between side length *x* and the perimeter *P* is linear both for the squares and for the hexagons. • The graph for $P = 8x$ was steeper than the graph of $P = 5x$ because a slope of 8 is greater than a slope of 5.

Practising and Applying

1. Calculate the slope of each graph.



2. What is the slope of a line that passes through each set of points?

a) (2, 5) and (3, 8)

b) (1, 4), (3, 10), and (5, 16)

c) (9, 6), (8, 2), and (6, –6)

3. Draw a line with each slope.

a) 4 **b)** –2 **c)** $\frac{2}{3}$ **d)** $-\frac{4}{5}$

4. Two points on a line are (3, 5) and (2, *x*), where *x* is a negative number. Is the slope of the line positive or negative? How do you know?

5. If you graph these two equations on the same grid, which line will be steeper? How do you know?

$y = 3x - 10$ $y = 2x + 8$

6. A chocolate bar costs Nu 60. Suppose you were to graph how the cost of different numbers of bars compared to the number of bars purchased. What will be the slope of the line? Explain your thinking.

7. a) A water trough is being filled with water at a constant rate. After 3 min, the water is 7.5 cm deep. After 8 min, the water is 15 cm deep. Calculate the change in depth (in centimetres) for 1 minute.

b) Draw a graph of the information. What does your answer to **part a)** tell about the graph?

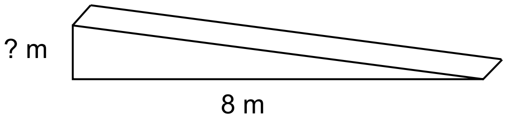
8. Complete the blanks to name two other points on the line.

a) Slope is 6: (5, 4), (3, ☐), (☐, 16)

b) Slope is $\frac{2}{5}$: (3, 10), (8, ☐), (☐, 16)

9. Dorji was cycling toward his home. After 2 h of cycling, he was 55 km from home. After 4.5 h of cycling he was 17.5 km from home. He was cycling at a constant rate. How fast was he cycling? How do you know?

10. It is easy to move materials up a ramp (inclined plane) that has a slope no greater than $\frac{1}{12}$. How high should this ramp be, if it must cover a horizontal distance of 8 m?



11. Your friend does not yet understand slope. How would you explain each?

a) what the slope of a line tells you about the line

b) what the slope does not tell you about the line

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Chapter 2 Solving Linear Equations

7.2.1 Solving an Equation Using Inverse Operations

Try This

A. Try this number trick with a partner.

- Think of a secret number. Do not tell your partner the number.
- Double the number, subtract 4, and then add 8. Tell your partner the answer.
- Ask your partner to figure out your secret number using your rules.

B. Change roles and let your partner try the trick on you.

- An equation that describes a linear relationship is called a **linear equation**.
- Each **variable** in a linear equation has an exponent of 1. The exponent does not need to be written because $x^1 = x$.

For example:

$3x + 2 = 8$ is a linear equation because the exponent of the variable x is 1.

$x^2 = 9$ is not a linear equation because the exponent of the variable x is 2.

- To **solve** an equation means to find the value of the variable that makes the equation true.

For example:

Since 3(**2**) + 2 = 8, the value 2 is a **solution** to the equation $3\boldsymbol{x} + 2 = 8$.

- You can solve an equation by using a model or a graph, as you did in Class VII. Another way to solve an equation is to think of the equation as describing the output when the variable is the input.

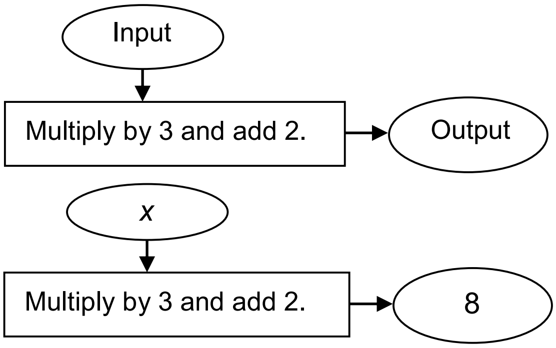
For example:

Suppose you use this rule to get an output for a given input:

Take a number (the input), multiply it by 3, and then add 2.

The output is 3 times the input plus 2.

If the input is x, the output is $3x + 2$.



- To solve an equation is to find the input (the value of the variable) when you know the output. You can do this by undoing each operation in the equation.

For example:

For $3x + 2 = 8$, you triple the input value, add 2, and the output is 8.
To solve for x, which is the input value, you can reverse the process:

- The last thing you did was add 2. To undo adding 2, you subtract 2: $8 – 2 = 6$
- To undo multiplying by 3, you divide by 3. $6 \div 3 = 2$

So the value of the input is 2. $x = 2$

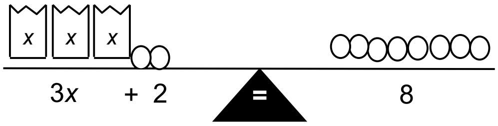
[Continued]

The thinking goes like this:

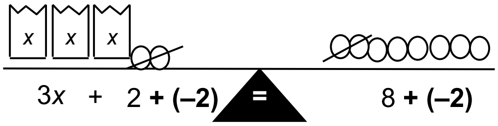
$3x + 2 = 8$	**The output is 8 when the input is x. What is x?**
$3x + 2 = 8$	Subtract 2 from both sides of the equation to keep it balanced: $2 – 2 = 0$ and $3x + 0 = 3x$
$3x + 2 \mathbf{- 2} = 8 \mathbf{- 2}$	
$3x = 6$	Divide both sides by 3 to find the value of one x: $3 \div 3 = 1$ and $6 \div 3 = 2$
$3x \mathbf{\div 3} = 6 \mathbf{\div 3}$	
$\mathbf{x = 2}$	**The input is 2.**

- Your goal is to get the variable with a **coefficient** of 1 on one side of the equation so you can see what its value is. The process is called using **inverse operations**, or maintaining a balance (as you learned in Class VII).

For example:

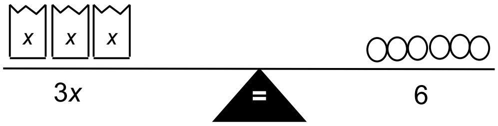


$3x + 2 = 8$

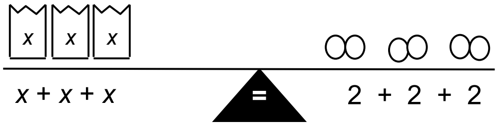


Undo adding 2 by subtracting 2:

$3x + 2 \mathbf{- 2} = 8 \mathbf{- 2}$

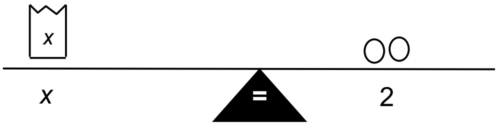


$3x = 6$



Undo multiplying by 3 by dividing by 3:

$3x \mathbf{\div 3} = 6 \mathbf{\div 3}$



x is now alone on one side of the equation:

$x = 2$

- Sometimes at the start the variable appears on both sides of the equation. In this case, you should first get the **term** with the variable on one side of the equation by adding or subtracting the same amount to both sides.

For example:

$$2x + 7 = 4x + 3$$

$$2x + 7 \mathbf{- 2x} = 4x + 3 \mathbf{- 2x}$$

Subtract $2x$ from both sides.

$$7 = 2x + 3$$

Then you can continue to solve the equation as before.

C. What equation did each partner solve in **part A?**

Examples

Example 1 Solving Equations with Negative, Fractional, or Decimal Values

Solve each equation.

a) $5 - 4x = 17$ **b)** $\frac{2}{3}x - \frac{3}{4} = \frac{5}{4}$ **c)** $3 + 2.5x = 9.25$

Solution	**Thinking**
a) $5 - 4x = 17$ $5 - 5 - 4x = 17 - 5$ $-4x = 12$ $-4x \div (-4) = 12 \div (-4)$ $x = -3$	 **a)** This is how I got the x term alone on one side of the equation: • I subtracted 5 from both sides of the equation to undo adding 5. • Then I divided both sides by –4 to undo multiplying by –4.
b) $\frac{2}{3}x - \frac{3}{4} = \frac{5}{4}$ $\frac{2}{3}x - \frac{3}{4} + \frac{3}{4} = \frac{5}{4} + \frac{3}{4}$ $\frac{2}{3}x = \frac{8}{4}$ $\frac{2}{3}x \div \frac{2}{3} = 2 \div \frac{2}{3}$ $x = 2 \times \frac{3}{2} = 3$	**b)** This is how I got the x term by itself on one side of the equation: • I added $\frac{3}{4}$ to both sides of the equation. • Then I divided both sides by $\frac{2}{3}$. To divide $2 \div \frac{2}{3}$, I multiplied by the reciprocal.
c) $3 + 2.5x = 9.25$ $3 - 3 + 2.5x = 9.25 - 3$ $2.5x = 6.25$ $2.5x \div 2.5 = 6.25 \div 2.5$ $x = 2.5$	**c)** I estimated the solution would be more than 2 because $3 + 2.5 \times 2 = 8$ and 8 is less than 9.25. • I got x alone on one side of the equation by - subtracting 3 from both sides and then - dividing by 2.5, the coefficient of x.

Example 2 Creating an Equation For a Given Solution

Create three possible equations for each solution. **a)** –5 **b)** 0.5

Solution	**Thinking**
a) $x = -5$ $2x = -10$ $2x + 6 = -4$	 **a)** I started with the easiest equation I could write, which was $x = -5$. • I multiplied both sides by 2 to make another equation. • Then I added 6 to both sides to make a third equation.
b) $x = \frac{1}{2}$ $2x = 1$ $2x - 4 = -3$	**b)** I started with the easiest equation, which was $x = \frac{1}{2}$. • I multiplied both sides by 2 to make another equation. • Then I subtracted 4 from both sides for a third equation.

Practising and Applying

1. Write an equation for each.

a) When you input *m*, triple it, and subtract 4, the output is 12.

b) When you input *x*, add 3, and multiply the sum by 2, the output is – 6.

c) When you input *n*, subtract 4, and triple the difference, the output is 1.5.

2. Solve each equation.

a) $7 = 4 - 3x$

b) $5x - 4 = 31$

c) $5 = 6x$

d) $5 - 7x = -44$

3. Create three equations with each solution. For each solution, one or more of the equations should have a negative coefficient for the variable.

a) $m = 11$

b) $n = 0.8$

c) $x = \frac{1}{8}$

d) $k = -8$

4. How do you know these equations have the same solution?

$$\frac{x}{4} - 8 = 11 \text{ and } \frac{1}{4}x = 19$$

5. Mindu got the same output using these two rules:

Rule A *Double the input number and then add 8.*

Rule B *Start with 11 and then subtract the input number.*

What was his input number?

6. Without solving the equation, how do you know the solution to $5x - 19 = 28$ is about 10?

7. Estimate each solution without solving the equation.

a) $80 = 6 + 8x$

b) $9 - 3x = -20$

c) $8x = 7.4$

8. Gembo and Pema started to solve the equation $3x + 4 = 40$.

Gembo wrote this:

$$3x + 4 = 40$$
$$x + 4 = 40 \div 3$$
$$x = 40 \div 3 - 4$$

Pema wrote this:

$$3x + 4 = 40$$
$$3x = 40 - 4$$
$$x = (40 - 4) \div 3$$

Who will get the correct solution? How do you know?

9. Why might it be better to use inverse operations than to guess and test to solve this equation?

$$4x + 17.9 = 382.7$$

10. Which inverse operations would you use, and in which order would you use them, to solve this equation?

$$\frac{x}{3} + 2 = 17$$

7.2.2 Using an Equation to Solve a Problem

Try This

A. Eden has Nu 160 in the bank. She has decided to start saving an additional Nu 20 each month. How long will it take for her savings to total Nu 300?

- In **Chapter 1**, you learned that an algebraic equation can describe a linear relationship.

For example:

The relationship between a square's perimeter and its side length is a linear relationship. It can be described by the linear equation $P = 4s$.

- You can model a problem with a linear equation. You solve the equation to solve the problem.

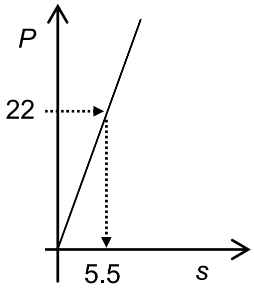
For example:

Suppose the perimeter of a square is 22 cm. Since you know that the perimeter is 4 times the side length (s), you can model the situation with the equation $4s = 22$. The solution to this equation is the side length of the square.

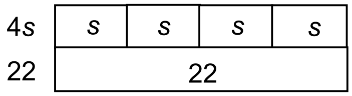
- You can use any method you know for solving equations to solve a problem described by a linear equation.

For example, to solve $4s = 22$, you can choose one of these methods:

- Guess and test:
  Guess $s = 5$ (too low), then $s = 6$ (too high), and eventually $s = 5.5$ cm.
- Draw a graph:
  Graph the linear relationship $P = 4s$. Find the s-value that corresponds to the P-value of 22.



- Draw a diagram:



$s = 5.5$ cm

- Use inverse operations:

$$\begin{aligned} 4s &= 22 \\ 4s \div 4 &= 22 \div 4 \\ s &= 5.5 \text{ cm} \end{aligned}$$

- You can create a problem to match a given equation.

For example:

Suppose you were given the equation $2s - 8 = 12$. This could be the problem:

Meghraj started with **2** pieces of wood that were the same length (***s***). He cut **8** cm off one piece of wood. He had a total of **12** cm of wood left. How long was each piece of wood that he started with (***s***)?

B. i) What equation could you use to solve the problem in **part A**?
ii) Which method did you use to solve the equation? Show your work.

Examples

Example 1 Representing a Problem with a Linear Equation

Which equations below represent this problem?

Chimi Dorji walked for four days. He walked the same distance on the first day and the second day. On the third day, he walked $\frac{3}{4}$ of the distance he walked on the second day. On the fourth day, he walked 7 km. He walked 32 km altogether. How far did he walk on the first day?

A. $2x + \frac{3}{4}x + 7 = 32$

B. $2\frac{3}{4}x = 32 - 7$

C. $(2\frac{3}{4} + 7)x = 32$

D. $32 = x + x + \frac{3}{4}x - 7$

Solution

Distances walked each day

Day 1: x Day 2: x Day 3: $\frac{3}{4}x$ Day 4: 7

Total distance walked

$x + x + \frac{3}{4}x + 7 = 32 \rightarrow 2\frac{3}{4}x + 7 = 32$

Equations A and B both represent the problem.

Thinking

- I knew that the equation had to add the distances for the four days, with a sum of 32.
- Equation C used $7x$ instead of 7 because $(2\frac{3}{4} + 7)x = 2\frac{3}{4}x + 7x$.
- In Equation D, 7 was subtracted instead of added.

Example 2 Creating a Problem to Match an Equation and Solving It

a) Create a problem that can be modelled by the equation $300 - 3x = 27$.

b) Solve the problem by solving the equation using a method of your choice.

Solution

a) 300 people were at an archery tournament. After the tournament, three equal groups of people left the range. 27 people were still there. How many people were in each group that left?

b)

$$\begin{aligned} 300 - 3x &= 27 \\ 300 - 3x + 3x &= 27 + 3x \\ 300 &= 27 + 3x \\ 300 - 27 &= 27 - 27 + 3x \\ 273 &= 3x \\ 273 \div 3 &= 3x \div 3 \\ 91 &= x \end{aligned}$$

Three groups of 91 people left the range.

Thinking

a) I thought of a situation where I could subtract 3 equal groups of something ($3x$) from 300 and there would be 27 left over.

b) I used inverse operations to solve the equation.

Practising and Applying

1. Write an equation to model each problem.

a) A box holds 24 tins. How many boxes are needed to hold 744 tins?

b) In an archery competition, Passang hit 2 more kareys than Tandin. Altogether they hit 82 kareys. How many kareys did Tandin hit?

c) There are four bags. Each bag holds the same number of stones. Four extra stones are not in a bag. In total, there are 28 stones. How many stones are in each bag?

d) Four identical bags of cement and a cinder block have a combined mass of 103 kg. The cinder block has a mass of 7 kg. What is the mass of each bag of cement?

2. Solve each equation in **question 1**.

3. To convert kilograms to pounds, you can multiply by 2.2. Write an equation you could solve to find the number of kilograms that are equivalent to
100 pounds. Solve the equation.

4. A tank contains 400 L of water. It is emptied at a rate of 32 L/min. Write an equation you could solve to determine to how long it will take until the tank contains only 112 L of water. Solve the equation. Round to the nearest whole number.

5. The width of a rectangle is 4 cm less than the length. The perimeter is 64 cm greater than the length.

a) Write and solve an equation to determine the length of the rectangle.

b) Write and solve an equation to determine the width of the rectangle.

6. Nima was 80 km from home when he started a trip that took him even farther from home. He travelled at a speed of 28 km/h. Write an equation you could solve to determine how many hours he travelled to get 150 km from home. Solve the equation.

7. Create a problem that could be modelled by each equation.

a) $4x + 97 = 489$

b) $100 - 4x = 44$

c) $2x + 6 = 4x$

8. Explain each statement using examples.

a) You can use the same linear equation to model and solve more than one problem.

b) You can model a problem using more than one linear equation.

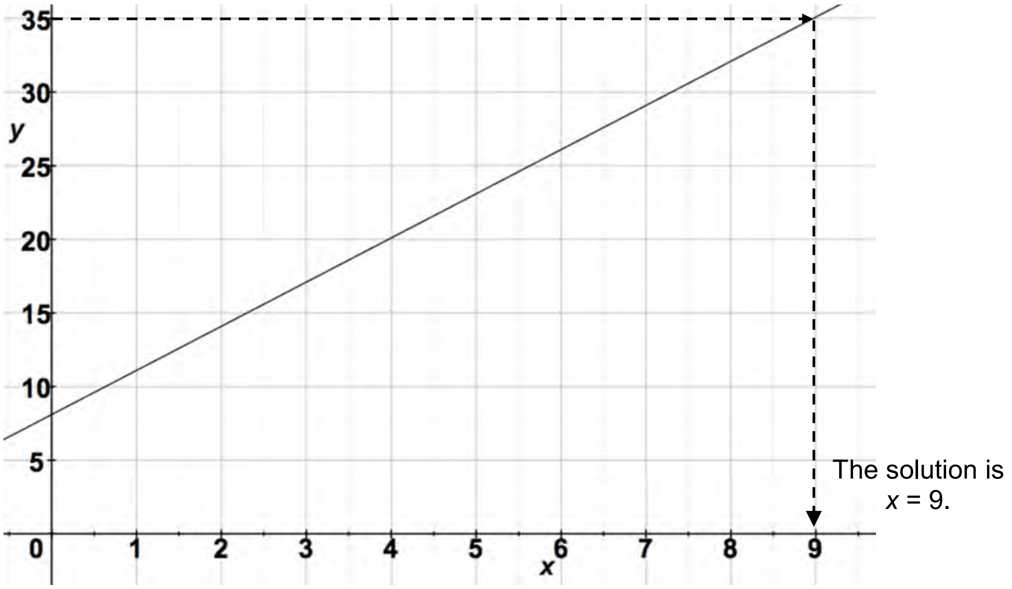
7.2.3 Solving a Problem Involving Two Relationships

Try This

A. Karma's mother is $6\frac{1}{2}$ times as old as he is. Karma's mother was 12 times as old as Karma two years ago. How old is Karma now?

- One way to solve an equation is to use a graph.

For example, suppose a problem can be modelled by the equation $3x + 8 = 35$. To solve the problem, you can graph $y = 3x + 8$ and then look for the x-coordinate of the point on the graph where the y-coordinate is 35.



- If the graphs of two related equations **intersect**, or pass through the same point, they have the same solution at the **intersection point**. If a problem situation can be modelled with two equations, you can draw two graphs and find the solution to the problem at the intersection point.

For example:

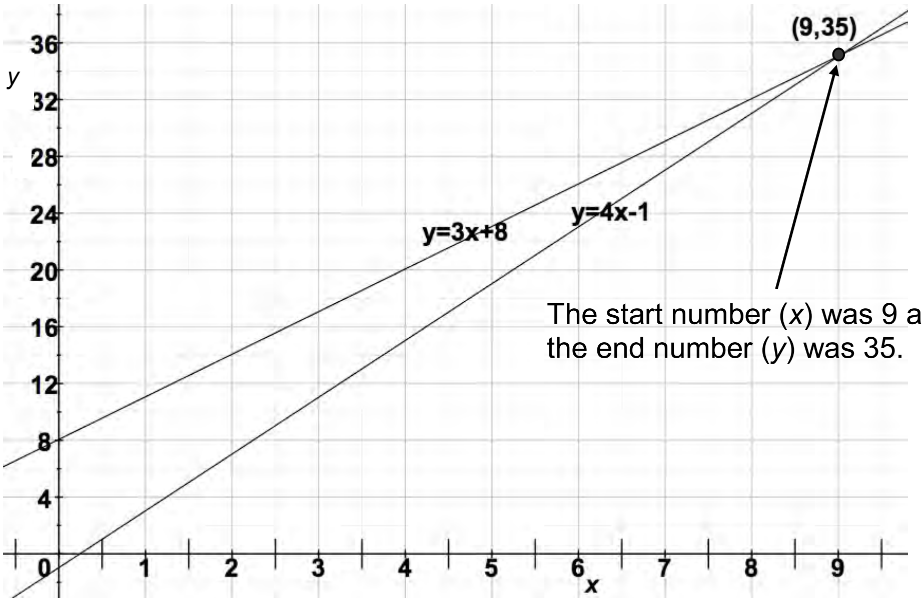
Dorji tripled a number and then added 8. He got the same answer when he multiplied the same number by 4 and then subtracted 1. What numbers did he start and end with?

You can let x represent the start number and y the end number.

Write the equation $3x + 8 = y$ to model tripling a number (x) and adding 8.

Write $4x – 1 = y$ to model multiplying a number (x) by 4 and subtracting 1.

If you graph $y = 3x + 8$ and $y = 4x – 1$ on the same axes, the x-coordinate of the intersection point tells you Dorji's start number and the y-coordinate of the intersection point tells you Dorji's end number.



The start number (x) was 9 and the end number (y) was 35.

- You can use other methods to find the common solution to two equations.

For example:

Samten bought a snack for Nu 50. Then he bought three chocolate bars.
Tenzin bought a snack for Nu 70. Then he bought two of the same chocolate bars.
The total cost for each boy was the same. How much did each chocolate bar cost?

To solve this problem, you could create one equation and then solve it using inverse operations:

If c is the price of one chocolate bar and t is the total cost, then the following two equations represent Samten's and Tenzin's purchases:

$$3c + 50 = t \qquad 2c + 70 = t$$

Since t has the same value in each equation, you can make a single equation from the two equations and then solve it:

$$3c + 50 = 2c + 70$$

Subtract 50 from both sides of the equation. $3c = 2c + 20$

Subtract $2c$ from both sides of the equation. $c = 20$

Each chocolate bar costs Nu 20.

- There are other ways to solve the chocolate bar problem.

For example, you can use some combination of these methods:

- guess and test to find a value that works
- a table of values (see the **example part a)**)
- a model (see the **example part b)**)

B. i) If you solve the problem in **part A** by creating one equation and then solving it, what equation would you use?

ii) How could you use a graph to solve the problem?

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Examples

Example Using a Table or Reasoning to Solve a Problem

Consider the chocolate bar problem on **page 211**.

Samten bought a snack for Nu 50. Then he bought three chocolate bars.
Tenzin bought a snack for Nu 70. Then he bought two of the same chocolate bars.
The total cost for each boy was the same. How much did each chocolate bar cost?

a) How could you solve the problem using a table of values?

b) How could you solve the problem using a model?

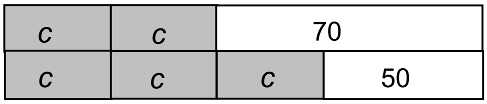
Solution

a)

Possible price of 1 bar	Total cost of 3 bars and Nu 50 more	Total cost of 2 bars and Nu 70 more
30	140	130
10	80	90
20	110	110 ✓

Each chocolate bar costs Nu 20.

b)



Since $c + 50 = 70$, then $c = 70 - 50 = 20$.

Each chocolate bar costs Nu 20.

Thinking

a) I made a double table of values.

- I figured out the total costs for both combinations for different chocolate bar prices.
- I started with a price that I thought was reasonable for a chocolate bar. I kept trying different prices until the total costs were the same.

b) I drew a picture to model the problem. It helped me visualize the problem.

- From my picture, I could see that Nu 70 had the same value as one chocolate bar plus Nu 50.

Practising and Applying

1. If you were to graph each pair of equations, what ordered pair would describe each intersection point?

a) $y = 2x + 9$ and $y = -4x - 3$

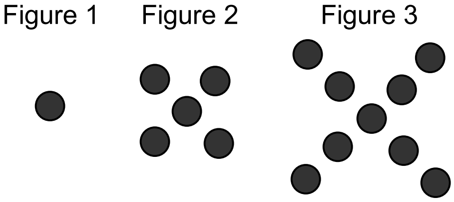
b) $y = 8 - x$ and $y = 5x - 40$

c) $y = 7x + 19$ and $y = 8x + 4$

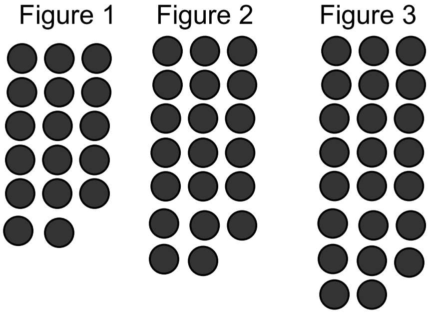
2. Explain why each intersection point in **question 1** is a common solution to the two equations.

3. The two patterns shown on the right have the same number of dots for one of the figures. What single equation could you write to determine which figure number it is? Solve the equation.

Pattern 1



Pattern 2



4. Yuden's and Lhaden's mothers make kiras. Here are the prices they charge for each kira:

- Yuden's mother charges Nu 30 for each hour of work plus Nu 120.
- Lhaden's mother charges Nu 20 for each hour of work plus Nu 200.

For how many hours of work would the total cost of a kira be the same for the two mothers? What equation did you use?

5. Sonam bought four tins of fish. Rinzin bought five tins of fish. Sonam spent Nu 40 more than Rinzin. How do you know that Sonam and Rinzin did not pay the same price for each tin of fish?

6. Write a word problem that could be solved by finding a common solution to two linear equations. Solve the problem.

GAME: Alge-Scrabble

Play with a partner.

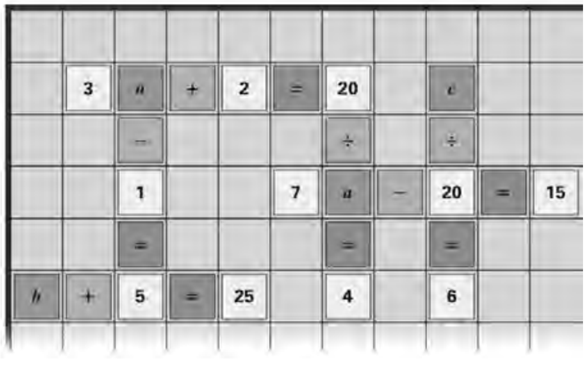
You need an Alge-Scrabble game board, which is a 12-by-12 grid, and Alge-Scrabble Game Tiles (slips of paper) in these four groups:

- 50 numbers, two of each number from 1 to 25
- 12 variables, three of each variable *a*, *b*, *c*, and *n*
- 12 operation signs, three of each sign +, – , ×, and ÷
- 12 equals signs =

This is how to play:

- Place each group of tiles in a pile face down. You should have four piles. Each player takes five operation tiles, five variable tiles, and five number tiles.
- Take turns placing tiles on the game board to form equations with one variable. You can use the equals sign tiles when you need them.
- Your score on each turn is equal to the solution to your equation.
- After each turn, take new tiles to replace the tiles you have used.
- Except for the first equation, each new equation must connect to an equation that is already on the board. If a variable that is already on the board is used in a new equation, its value stays the same.
- Play until each player has had 5 turns. The player with the higher score wins.

Here is an example:



3n + 2 = 20	n = 6, so Player A scores 6 points
n – 1 = 5	n = 6, so Player B scores 6 points
20 ÷ a = 4	a = 5, so Player A scores 5 points
7a – 20 = 15	a = 5, so Player B scores 5 points
b + 5 = 25	b = 20, so Player A scores 20 points
c ÷ 20 = 6	c = 120, so Player B scores 120 points

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Chapter 3 Linear Polynomials

7.3.1 Adding Polynomials

Try This

A. Which calculation is easier to do using mental math? Explain why.

$6 \times 423 + 4 \times 423$ or $6 \times 423 + 4 \times 517$

- A **linear polynomial** is an **algebraic expression** that includes a variable with an exponent of 1 and no other **powers**. It usually involves more than one term.

For example, each of these expressions is a linear polynomial:

one term	x	$2x$
two terms	$3x + 7$	$9 – 3n$
three terms	$2n – n + 2$	$5 – 2 – 2c$

- Just like you added numbers by representing them with materials (base ten blocks), you can use materials called algebra tiles to work with polynomials.

Algebra Tiles

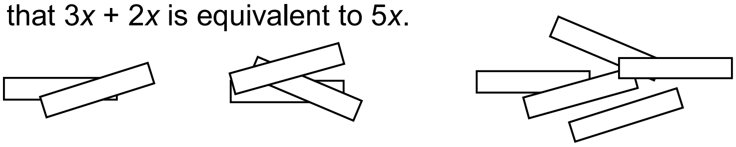
***x* tile**	**–*x* tile**	**+1 tile**	**–1 tile**
▭	▬	□	■

Dark tiles represent negative values. White tiles represent positive values.

- Here are the representations for the polynomials listed above:
(The *x*-tile represents the variable even if it is called *n* or *c* in the equation.)

x	▭
$2x$	▭ ▭
$3x + 7$	▭ ▭ ▭ □ □ □ □ □ □ □
$9 – 3n$	□ □ □ □ □ □ □ □ □ ▬ ▬ ▬
$2n – n + 2$	▭ ▭ ▬ □ □
$5 – 2 – 2c$	□ □ □ □ □ ■ ■ ▬ ▬

- If you use tiles to represent a polynomial such as $3x + 2x$, you use 3 *x*-tiles and 2 more *x*-tiles. You can combine the *x*-tiles because they are the same type of tile. You can see that $3x + 2x$ is equivalent to $5x$.



$2x \quad + \quad 3x \quad = \quad 5x$

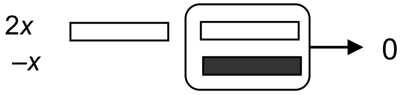
This is an example of adding **like terms**. Congruent tiles represent like terms.

- Note that $x + 3$ does not equal 4 or $4x$ because the x and the 3 are not like terms (they are not represented by the same type of tile). They cannot be combined.

$x + 3$

- You can use the **zero property** and the rules you know for adding integers to combine positive and negative terms if they are like terms.

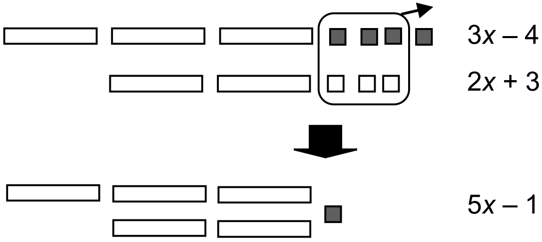
For example: $2x + (-x) = 2x - x$
$= 1x$
$= x$



The positive *x* term and negative *x* term add to 0, so the sum is *x*.

- When you combine like terms, it is called **simplifying** an expression because you end up with fewer terms and a simpler expression.

For example, to add $(3x - 4) + (2x + 3)$:



There are four terms to start ($3x$, –4, $2x$, and 3), but there are only two terms ($5x$ and –1) in the simplified expression.

$(3x - 4) + (2x + 3) = 5x - 1$

- Adding like terms is similar to adding numbers:

43 + 62 = (4 tens + 3 ones) + (6 tens + 2 ones) = 10 tens + 5 ones = 105

You add the tens together and the ones together since they are like terms.

B. How is the calculation you chose in **part A** related to adding like terms?

Examples

Example 1 Adding Polynomials Using Tiles

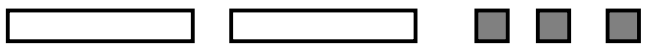
Simplify each by adding.

a) $(5 - 4x) + (2x - 3)$ **b)** $[8 + 3x + (-4x)] + (2x - 5)$

Solution

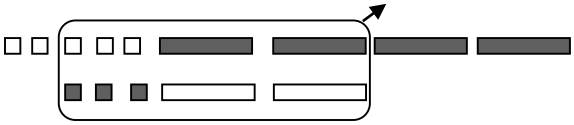
a) *Model the two polynomials*

$5 - 4x$



$2x - 3$

Use the zero property and combine like tiles (terms)



Write the simplified expression

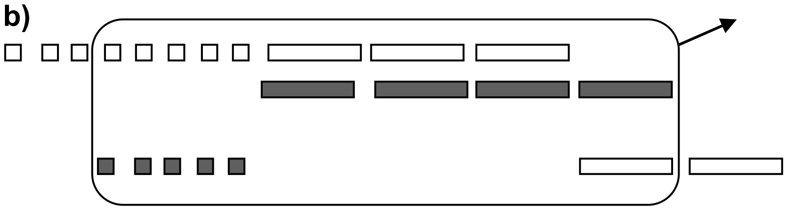
$2 - 2x$

$(5 - 4x) + (2x - 3) = 2 - 2x$

Thinking

I used algebra tiles to model each addition.

a) I put the *x*-tiles together and the 1-tiles together. Then I used the zero property to remove pairs of tiles that had a value of zero.

Example 1 Adding Polynomials Using Tiles [Continued]	
Solution	**Thinking**
b)  $(8 + 3x + (-4x)) + (2x - 5) = 3 + x$	**b)** I represented each polynomial with tiles. • Then I put together the same types of tiles and used the zero property to simplify.

Practising and Applying

1. Use algebra tiles to represent each polynomial.

a) $4x + 2$

b) $4x - 2$

c) $2 - 4x$

d) $-2 - 4x$

2. List the like terms in each expression.

a) $5x + 7 + 2x - 2$

b) $(-5x + 3) + 2x + (x - 2)$

c) $6 - 2x - 4x - 1$

3. Add the polynomials in **question 2**.

4. List five possible polynomials for each.

a) The polynomial can be represented using 6 black tiles.

b) The polynomial can be represented using 8 white tiles.

5. a) When you add $4x + 2$ and $-2x - 1$, you start with nine tiles but end with a sum that is represented with three tiles. Why does this happen?

b) Describe another polynomial addition that starts with nine tiles and ends with three tiles.

6. Simplify each.

a) $(4 + x) + (2 - 3x)$

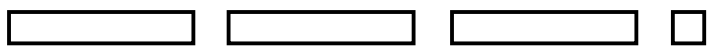
b) $(-7 + 5x) + (2x + 7)$

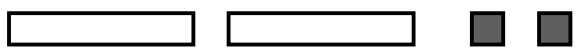
c) $(6x + 1) + (-4x - 3)$

d) $(-4x - 7) + (10 + 4x)$

7. Each model below represents the sum of two polynomials. What could the two polynomials be? Find three different answers for each.

a) 

b) 

c) 

8. Tashi added the measurement 3 m, 10 cm to the measurement 4 m, 3 cm. He said that adding the two measurements was just like adding like terms. What does Tashi mean?

9. Describe the role of the zero property in adding polynomials. Use an example to help you explain.

10. Why is there always more than one possible polynomial addition for any given sum? Use an example to help you explain.

7.3.2 Subtracting Polynomials

Try This

A. Predict the result of $(4x – 2) – (3x – 5)$. Explain your prediction.

- When you subtract integers, you can use any of the meanings of subtraction.

For example, $5 – 2$ can mean each of these things:

- Take away 2 from 5 and see what is left. $5 – 2 = 3$
- Compare how much greater 5 is than 2. $5 – 2 = 3$
- Find the missing addend to get from 2 to 5. $5 – 2 = ? \rightarrow 2 + ? = 5$
- Add the opposite. $5 – 2 = 5 + (–2) = 3$

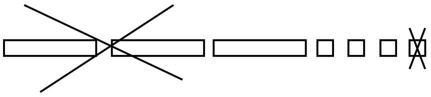
- This is also true for subtracting polynomials. You can use any of the meanings of subtraction, along with algebra tiles, to help you subtract.

For example:

This is how to use the take away meaning for $(3x + 4) – (2x + 1)$:

$(3x + 4) – (2x + 1)$ means taking away $2x + 1$ from $3x + 4$.

Model $3x + 4$ with three x-tiles and four 1-tiles. Then take away two x-tiles and one 1-tile.

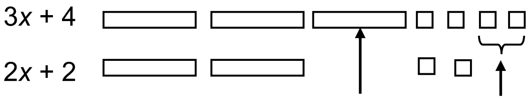


There are one x-tile and three 1-tiles left, so $(3x + 4) – (2x – 1) = x + 3$.

This is how to use the comparison meaning for $(3x + 4) – (2x + 2)$:

$(3x + 4) – (2x + 2)$ means "How much more is $3x + 4$ than $2x + 2$?"

Model both polynomials and then see how much longer one is than the other.

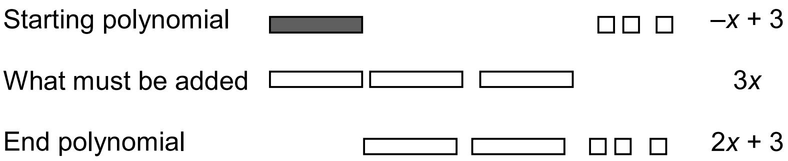


3x + 4 has one extra x-tile and two extra 1-tiles, so $(3x + 4) – (2x + 2) = x + 2$.

This is how to use the missing addend meaning for $(2x + 3) – (–x + 3)$:

$(2x + 3) – (–x + 3)$ means $(–x + 3) + ? = 2x + 3$.

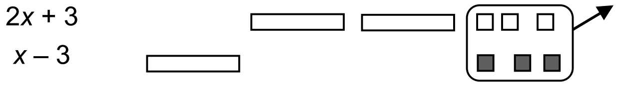
Model $–x + 3$ and then add tiles until you have $2x + 3$:



You have to add one $+x$-tile to get rid of the $–x$-tile and then add two more $+x$-tiles to get to $2x$. You do not have to add any ones, so $(2x + 3) – (–x + 3) = 3x + 0 = 3x$.

This is how to subtract by adding the opposite:

$(2x + 3) – (–x + 3) \rightarrow (2x + 3) + (x – 3)$ because $(x – 3)$ is the opposite of $(–x + 3)$.



$(2x + 3) + (x – 3) = 3x + 0 = 3x$

- Notice how each polynomial difference relates to the subtraction:

	Notice that …
$(3x + 4) – (2x + 1) =$ **$\underline{x + 3}$**	$3x – 2x =$ **$\underline{x}$** and $4 – 1 =$ **$\underline{3}$**
$(3x + 4) – (2x + 2) =$ **$\underline{x + 2}$**	$3x – 2x =$ **$\underline{x}$** and $4 – 2 =$ **$\underline{2}$**
$(2x + 3) – (–x + 3) =$ **$\underline{3x + 0}$** $= 3x$	$2x – (–x) =$ **$\underline{3x}$** and $3 – 3 =$ **$\underline{0}$**
$(2x + 3) – (–x + 3) =$ **$\underline{3x + 0}$** $= 3x$	$2x + x =$ **$\underline{3x}$** and $3 – 3 =$ **$\underline{0}$**

Each time, you can use what you know about integer subtraction to subtract the x terms and the constant terms separately.

B. Which meaning of subtraction would you use to calculate **part A**?

Examples

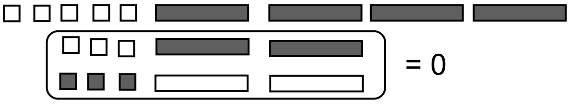
Example 1 Subtracting Polynomials Using the Zero Property

Simplify $(5 – 4x) – (2x – 3)$.

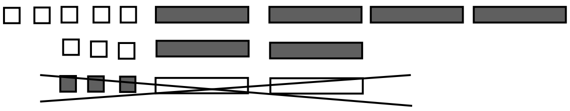
Solution

Model 5 – 4x *with tiles*

Add tiles using the zero property so you can take away 2x – 3



Take away 2x – 3



Count what is left

$(5 – 4x) – (2x – 3) = 8 – 6x$

Thinking

There were no positive x-tiles or negative 1-tiles to take away, so I added tiles that had a total value of 0:

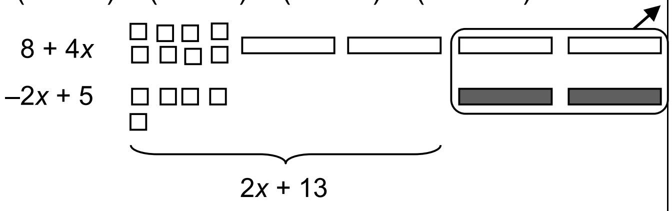
- Two $(–x)$-tiles and two $+x$-tiles: $2x + (–2x) = 0$
- Three +1-tiles and three (–1)-tiles: $3 + (–3) = 0$

Example 2 Subtracting Polynomials by Adding the Opposite

Simplify (8 + 4x) – (2x – 5).

Solution

(8 + 4x) – (2x – 5) = (8 + 4x) + (–2x + 5)



(8 + 4x) – (2x – 5) = 2x + 13

Thinking

- I added the opposite to subtract.
- I removed two +x-tiles and two (–x)-tiles because 2x + (–2x) = 0.

Practising and Applying

1. a) Explain how to use algebra tiles to subtract (5x + 2) – (2x + 1) each way.

i) take away

ii) comparison

iii) missing addend

iv) adding the opposite

b) Which method do you think worked best? Why do you think that?

c) Would you use the same method for 4x – (–2x + 3)? Why or why not?

2. Subtract each.

a) (8 + 4x) – (2x + 5)

b) (8 + 4x) – (2x – 5)

c) (8 + 4x) – (–2x – 5)

d) (8 + 4x) – (–2x + 5)

3. Complete each to make it true.

a) (10 + ☐ x) – (☐ x + 5) = (2x + 5)

b) (4 – ☐ x) – (☐ x – 5) = (2x + ☐)

c) (6 + ☐ x) – (3x + 5) = (–x + 1)

4. Simplify each by subtracting.

a) (1 + 2x) – (2 – 3x)

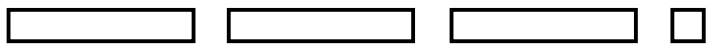
b) (–3 + 5x) – (4x + 9)

c) (x + 1) – (–4x – 5)

5. Tashi subtracted a polynomial that was modelled with only black tiles from a polynomial that was modelled with only white tiles. What colour were the tiles that modelled the difference? How do you know?

6. Each model below represents the result of a subtraction. For each difference, write three possible polynomial subtractions.

a) 

b) 

7. Suppose you subtracted one polynomial from another polynomial.

For example: (ax + b) – (cx + d) = A

Next, suppose you subtracted the opposites of those polynomials:

(–ax – b) – (–cx – d) = B

How do the two differences, A and B, compare?

8. When you subtract polynomials, A – B = C, Polynomial C could be represented by more than, less than, or the same number of tiles as Polynomial A. Show an example of each.

9. The sum of (3x + 3) and (–2x – 4) is modelled with two algebra tiles. The difference is modelled with 12 tiles.

Is it *always* true that you need more tiles to represent the difference between two polynomials than to represent their sum? Explain using an example.

10. Is it important to think about like terms when you subtract polynomials? Explain your thinking.

7.3.3 EXPLORE: Multiplying a Polynomial by an Integer

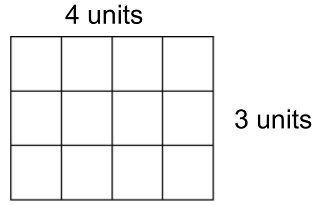
- You already know that multiplication can be represented in different ways.

For example:

Multiplication is repeated addition: $3 \times 4 = 4 + 4 + 4 = 12$

Multiplication is the area of a rectangle:

3 × 4 is the area of a rectangle that is 3 units by 4 units



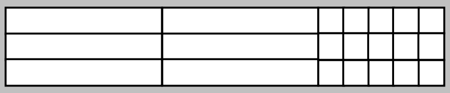
3 units by 4 units = 12 square units

- The exploration below shows how you can use these same ideas to multiply a linear polynomial by an integer.

A. Represent $3 \times (2x + 5)$ as repeated addition. What is the product?

B. **i)** What are the length and width of the *x*-tile? The 1-tile?

ii) Why can $3 \times (2x + 5)$ be represented using the rectangle below?



iii) How does the rectangle show the product of 3 and $(2x + 5)$?

C. How might you have predicted the product of 3 and $(2x + 5)$ before you completed **parts A and B**?

D. Predict each product and then a draw rectangle to test each prediction.

i) $4 \times (3x – 2)$ **ii)** $2 \times (6 – 2x)$

E. What is the product of $–2 \times (2x + 5)$? Explain your thinking.

F. How is multiplying a polynomial by an integer like multiplying two integers?

UNIT 7 Revision

1. A and B are whole numbers. You multiply A by 3. Then you add the product to the double of B. The final result is 60.

a) List three possible pairs of numbers for A and B that make this true.

b) How do you know that A cannot be an odd number?

c) How can you use a graph to find other possible pairs of whole numbers?

2. Represent the relationship between the term number and the term value for each pattern. Use an algebraic equation and a graph.

a) 6, 15, 24, 33, . . .

b) 37, 35, 33, . . .

c) 7, 18, 29, 40, . . .

d) 63, 59, 55, 51, . . .

3. Use an algebraic equation or a graph to predict or estimate the 20th term in each pattern.

a) 8, 15, 22, 29, . . .

b) 42, 36, 30, 24, . . .

c) 17, 22, 27, 32, . . .

d) 1.5, 2.3, 3.1, 3.9, . . .

4. Examine the pattern below.

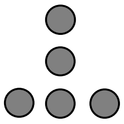


Figure 1

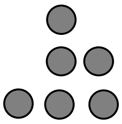


Figure 2

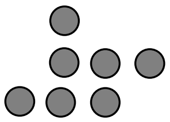


Figure 3

a) How many dots are in Figure 4?

b) Draw a graph to show the relationship between the figure number and the number of dots in that figure.

c) Use your graph to predict the number of dots in Figure 10.

d) Find a way to test your prediction.

5. Which relationship below is linear? How do you know?

A.

x	y
1	15
2	18
3	22
4	27

B.

x	y
1	15
2	18
3	21
4	24

6. Tell whether each relationship is linear or nonlinear. Tell how you know.

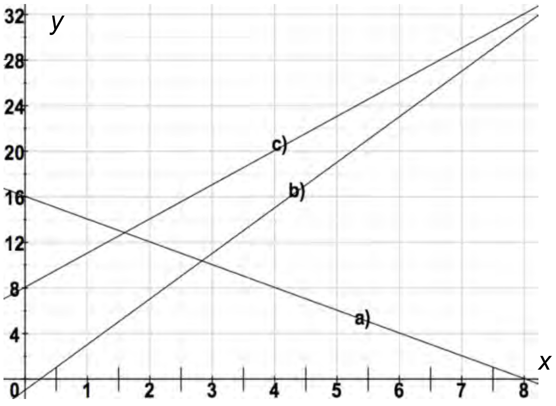
a) the side length of a rhombus compared to its perimeter

b) the side length of a square compared to the length of one of its diagonals

c) the side length of a square compared to its area

7. Is $y = 3x - 2x^2$ a linear equation? How do you know?

8. Calculate the slope of each graph.



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9. A line passes through each set of points. What is the slope of each line?

a) (2, 6) and (3, 8)

b) (1, –1), (2, –2), and (3, –3)

c) (4, 2), (6, 8), and (1, –7)

10. Draw a line with each slope.

a) –5 **b)** 3

c) $\frac{3}{4}$ **d)** –0.6

11. Complete each set of blanks to list three points on a line that has a slope of –2.

a) (5, 4), (3, ☐), and (☐, –8)

b) (3, 10), (8, ☐), and (☐, 28)

12. Solve each equation.

a) $8 - 3x = 23$

b) $5x - 16 = 34$

c) $8x = 5$

d) $2 - 5x = -37$

13. Create three possible equations for each solution below. Use a negative coefficient for the variable in at least one equation for each.

a) $m = 7$

b) $n = 0.2$

c) $x = \frac{3}{5}$

d) $k = -3$

14. a) Which operations would you use, in what order, to solve this equation? Tell why you would do each operation.

$$\frac{2x}{5} - 2 = 17$$

b) What is the solution to the equation?

15. Yangdon was 30 km from home when she started a trip that took her farther from home. She travelled at a speed of 24 km/h.

a) Write an equation you could use to determine how many hours she had travelled when she was 95 km from home.

b) Solve your equation.

16. At what point does each pair of lines intersect?

a) $y = 3x + 2$ and $y = -3x - 4$

b) $y = 6x + 13$ and $y = 8x - 7$

c) $y = 20 - 2x$ and $y = 5x - 30$

17. Represent each polynomial with algebra tiles. Sketch each model.

a) $3x + 1$ **b)** $2x - 5$

c) $3 - 2x$ **d)** $-3 - 3x$

18. List the like terms in each expression.

a) $6x + x + 2x - 8$

b) $(-3x + 13) + 5x + (9x - 2)$

c) $8 - 4x - 7x - 5$

19. Simplify each polynomial in **question 18**.

20. Bhagi subtracted two polynomials. The difference was modelled with three tiles. What could the two polynomials have been? How do you know?

21. a) $2x - 5$ is the difference between two polynomials. What could be the two polynomials? How do you know?

b) Use algebra tiles to model the subtraction in **part a)**. Sketch what you did. Explain each step.

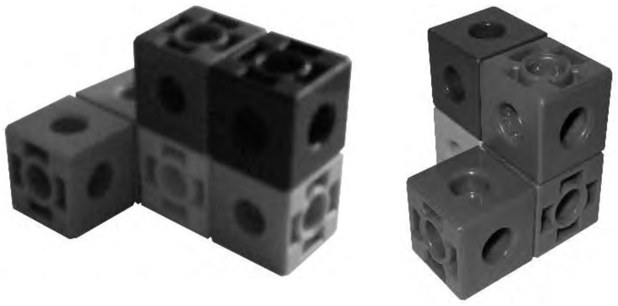
22. Multiply $4 \times (7 - 2x)$. Show your work.

UNIT 8 GEOMETRY

Getting Started

Use What You Know

A. Look at the two cube structures below. One structure is made with six cubes. The other structure is made with five cubes.



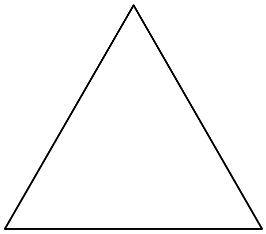
i) How many different structures can you make with two cubes?
ii) How many can you make with three cubes?
iii) How many can you make with four cubes?

B. Draw all the structures you found in **part A**. Use isometric drawings.

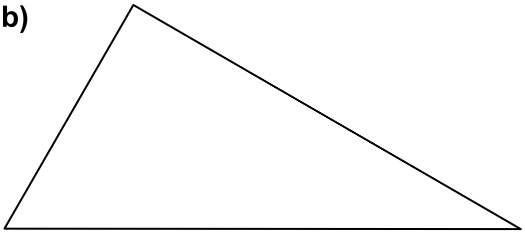
Skills You Will Need

1. Measure the angles (to the nearest degree) and sides (to the nearest millimetre) in each shape. Then classify each shape in as many ways as you can.

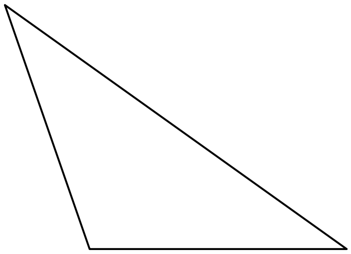
a)



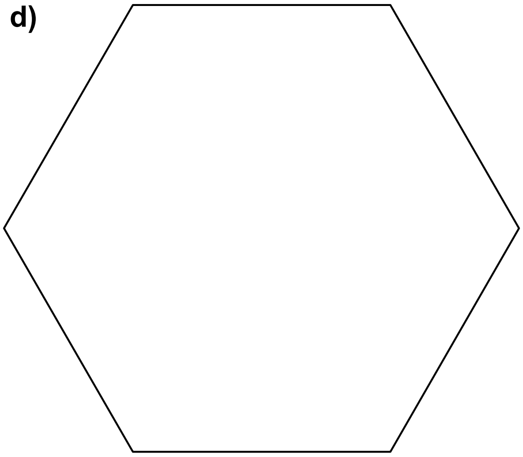
b)



c)



d)

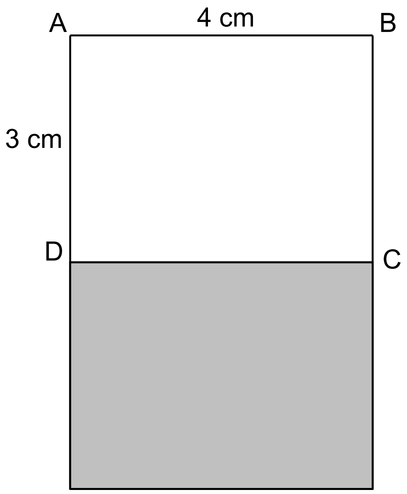


2. The bottom (grey) rectangle shown below is the image of rectangle ABCD after a transformation. More than one transformation can result in this same image.

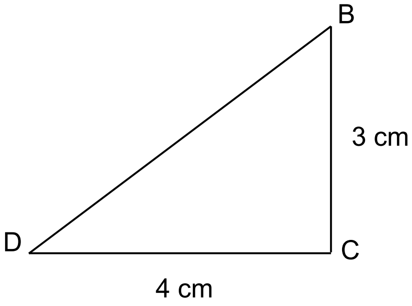
a) If ABCD was translated, describe the direction and distance of the translation.

b) If ABCD was rotated, describe the turn centre and angle of rotation.

c) If ABCD was reflected, describe the location of the reflection line.



3. Copy triangle BCD and then show each transformation described below.



a) Translate ∆BCD using the translation you described in **question 2 a)**.

b) Rotate ∆BCD using the rotation you described in **question 2 b)**.

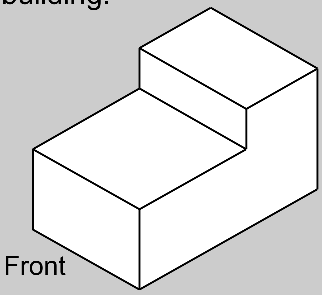
c) Reflect ∆BCD using the reflection line you described in **question 2 c)**.

Chapter 1 Representing Objects

8.1.1 Isometric Drawings

Try This

Some friends were discussing possible designs for a new building.
Here is a diagram of one design.



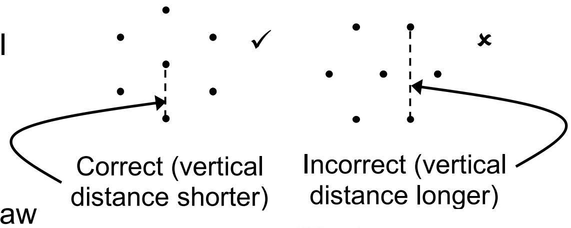
A. Use cubes to create a model of the design.

B. i) How did you decide how many cubes high to make the short and tall sections?

ii) How did you decide how many cubes long and wide to make the short and tall sections?

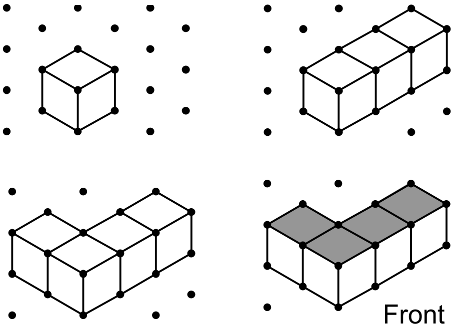
You can draw a **three-dimensional** (3-D) shape in different ways. One way is an **isometric drawing**, which shows three surfaces of the shape at once. An isometric drawing is a two-dimensional (2-D) drawing that appears 3-D.

- You can use isometric dot paper to help you make an isometric drawing:
  - Position the dot paper so the vertical distance between dots is shortest.



  - Position the structure you want to draw so that it lines up with the dots as much as possible.

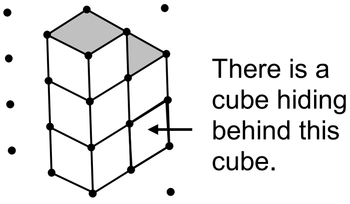
  - Choose a starting point. Draw what you see. As you draw, visualize the edges of each cube in the structure.
- You can colour the drawing to help people visualize the structure:
  - You could colour each cube a different colour or use alternating colours.
  - Or, you could colour the upper surface all one colour (as shown to the right).



Use whichever technique works for the structure you are drawing.

- It is not always possible to show every cube in a drawing if some cubes are hidden from view.

For example, the drawing to the right does not show the cube that is behind the bottom right cube in the structure.



C. Make an isometric drawing of your cube structure from **part A**.

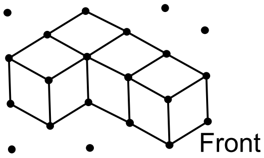
Examples

Example 1 Drawing a Structure on Isometric Dot Paper

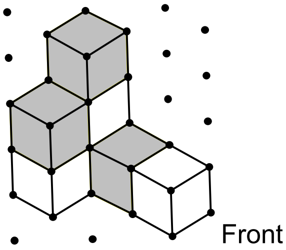
Use cubes to make this structure.
Then draw the structure on isometric dot paper.

Solution 1

Bottom layer



Whole structure

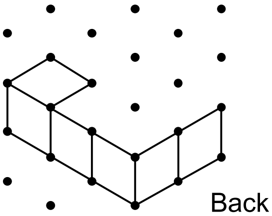


Thinking

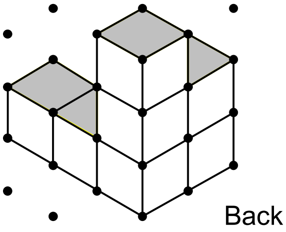
- After I created the structure, I positioned it in the same way as it was in the photo.
- I lined up the structure with the dots to help me visualize. I drew the bottom layer first.
- I added the label "Front" to match the structure.
- After I drew the upper layers, I erased parts of the drawing in the bottom layer.
- I coloured alternating cubes to make it easier to see the structure.

Solution 2

Bottom layer



Whole structure



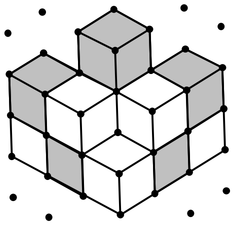
Thinking

- I made my structure face the opposite way from the photo, so I added the label "Back" to show this.
- I drew the bottom layer first. I only drew a face if I could see all of it.
- After I drew the upper layers, I coloured the top faces of the cubes to make it easier to see the structure.

Example 2 Building a Structure from an Isometric Drawing

a) What is the least number of cubes that could be in this structure? Explain your thinking.

b) What is the greatest number of cubes? Explain your thinking.



Front

Solution

Bottom layer

Bottom two layers

Whole structure

Front

a) The least number of cubes is 11: the 10 cubes you can see in the drawing and the hidden cube in the second layer that is holding up the cube in the top layer.

b) The greatest number of cubes is 18: You could add 7 more cubes behind the structure that would be hidden from view and not show up in the drawing.

These two views from the back show the structure without and with the 7 hidden cubes.

Back

Back

Thinking

- I built the structure.
- I started with the five cubes that I could see in the bottom layer.
- I added four cubes for the second layer.
- I added an extra cube to the second layer to hold up the cube in the top layer.

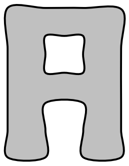
b) I realized that this drawing would not show some cubes that might be hidden.

- To see how many cubes might be hidden, I added cubes to the back. Then I held the structure exactly like it was in the drawing to see if I could see the cubes I had added, and I couldn't.

Practising and Applying

1. Use cubes to create this structure. Then draw the structure on isometric dot paper.

2. Model the letter A with cubes. Then draw the structure on isometric dot paper.



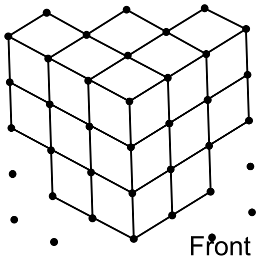
3. a) Use 10 cubes to build a structure.

b) Draw your structure on dot paper.

c) Exchange structures with a classmate. Draw your classmate's structure.

d) Compare your drawings. Explain why two drawings of the same structure could be different.

4. a) Build this structure with cubes.

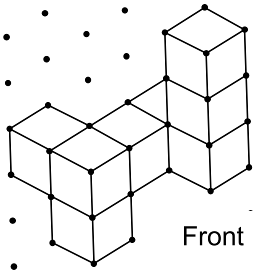


b) What is the least possible number of cubes in this structure? Explain your thinking.

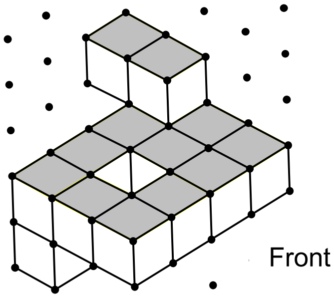
c) What is the greatest possible number of cubes? Explain your thinking.

5. Build a structure for each drawing. Then turn it to get a different view. Draw the new view.

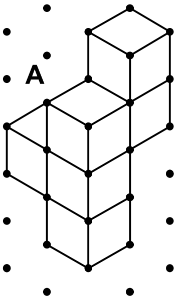
a)

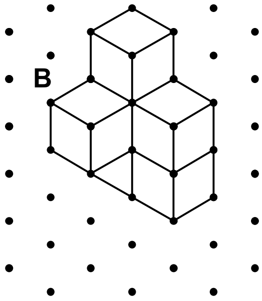


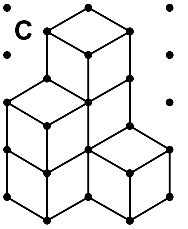
b)

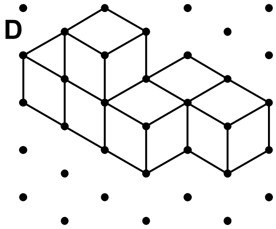


6. Each structure below is made with six cubes. Which structures are the same? How do you know?







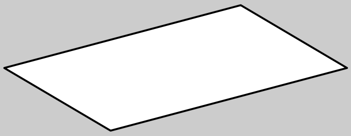


7. Why might someone use two or more isometric drawings to represent one structure?

8.1.2 Orthographic Drawings

Try This

This is a view of a rectangular desktop.



A. Describe where you could be standing in order to see the desktop in this way.

- Another way to represent a 3-D structure is with a set of **orthographic drawings**.

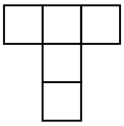
A set of orthographic drawings can include any of these orthographic **face views**:

- the top view
- the right view and left view
- the front view and back view

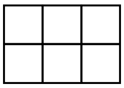
Each face view is a 2-D diagram of the structure drawn by looking directly at one face of the structure.

For example:

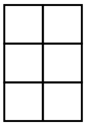
Here is a set of orthographic drawings for the structure shown on the right.



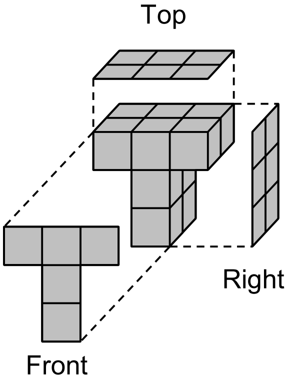
Front view



Top view



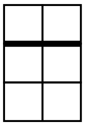
Right view



Note that the right and left face views are right and left for the person viewing the structure, *not* the right and left sides of the structure itself.

- To better represent a 3-D structure, you can use a thick line to represent a change in depth.

For example, a thick line in the right face view of the structure above shows that there is a change in depth.



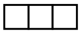
Right view

- Since structures are 3-D, a set of two face views is usually not enough to represent a structure. Two different structures could have some views that are the same.

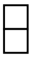
For example:

These two structures have the same top face view and left face view.
The third face view (from the front) shows the differences.

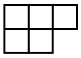
Front



Top view

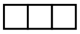


Left view

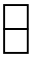


Front view

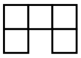
Front



Top view



Left view



Front view

- This is how to choose which face views to include to represent a structure:
  - Choose face views that give different information.
  - Try not to include two face views that are opposite each other.

For example, include either a left face view or a right face view, but not both.

- Suppose you want to create a structure to match a set of orthographic drawings. Visualize the structure as you try different combinations of cubes. When you think you have a structure that works, check that it matches each face view.

B. Use cubes to create a very simple model of a desk or table.

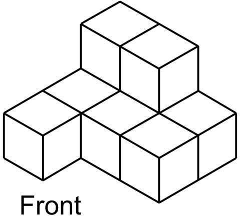
i) Draw three orthographic views of your model.

ii) Is the view shown in **part A** an orthographic view? Explain your thinking.

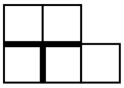
Examples

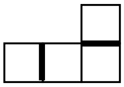
Example 1 Making a Set of Orthographic Drawings

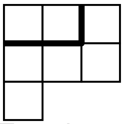
Create a set of orthographic drawings of this structure.
Include the top face view, the front face view, and
a side face view (a left view or a right view).



Solution 1

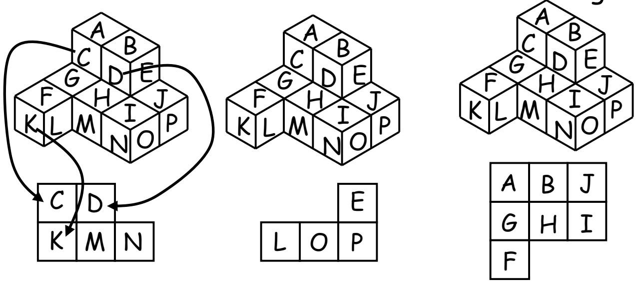

Front view


Right view


Top view

Thinking

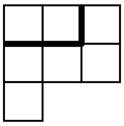
- I first built the structure with cubes so I could use it to draw each face view.
- I labelled the cubes in my drawings to show which cubes are the same cubes in the drawings.

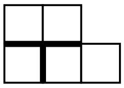


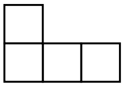
Front face view Right face view Top face view

- I drew thick lines to show changes in depth for each view.

Solution 2


Top view


Front view


Left view

Thinking

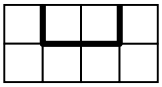
- I first built the structure.
- For each face view, I turned the structure so I was looking directly at it, seeing only one face at a time.
- I decided to use the top, front, and left views. The other views wouldn't add any more information.
- I used thick lines to show changes in depth for the top and front views.

Example 2 Building a Structure From Orthographic Drawings

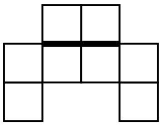
a) Build a structure that matches this set of orthographic drawings.

b) Is more than one structure possible?

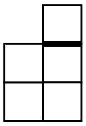
c) Make an isometric drawing of the structure.



Top view



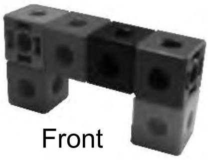
Front view

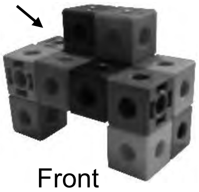


Right view

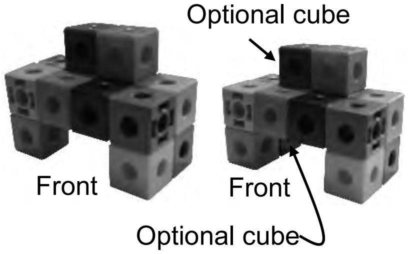
Solution

a)



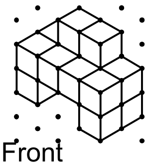


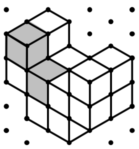
b) Yes, both these structures are possible:



The two optional cubes in the second structure would not have shown up in the top, front, or right view.

c)





The shaded cubes are optional.

Thinking

a) I started with the front view because I could visualize it best.

Step 1: I used six cubes to make the bottom two layers.

Step 2: I attached a cube to the back of the middle layer so I could attach one of the two top cubes to it.
Then I attached the second top cube to the other top cube.

Step 3: To match the right view, I added two cubes.

Step 4: To match the top view, I added one cube.

b) I could put another cube in the bottom layer. It would not show up in the top, front, or right view.

- I also could put another cube in the middle layer under the left top cube. It would not show up in the three views.
- A back view would have shown both of these optional cubes.

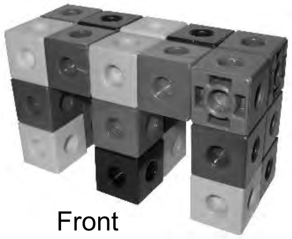
c) I made two isometric drawings:

- The first drawing could be of both possible structures.
- The second drawing shows only the structure with the two optional cubes. I drew it upside down and from the back to show the optional cubes.

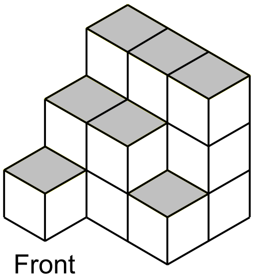
Reprint 2024

Practising and Applying

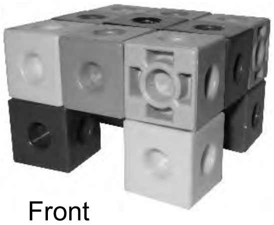
1. Build this structure. Then draw three orthographic face views.



2. Build this structure using 15 cubes. Then draw three orthographic face views.



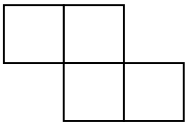
3. Lhamo made a model of this table using 13 cubes.



a) Draw three orthographic face views of Lhamo's table.

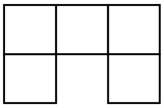
b) Build a model of a different table and then draw three orthographic face views.

4. a) Build two different cube structures that have one side face view like this:



b) For each structure, draw the top face view, the left face view, and the front face view.

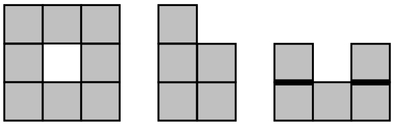
5. a) Build a structure that looks like this from the front and from one side.



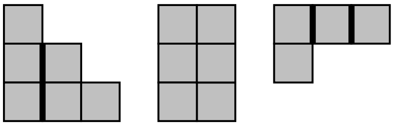
b) Draw the top face view.

6. Build a structure for each set of orthographic drawings.

a)



b)



7. Make an isometric drawing of each structure in **question 6**.

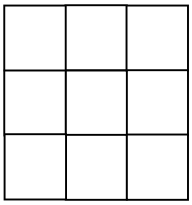
8. a) Use 12 cubes to build a structure.

b) Draw three orthographic face views.

c) Exchange drawings with a classmate. Use your classmate's drawings to build his or her structure.

d) Compare the structure you built in **part c)** with your partner's structure from **part a)**.

9. a) Describe a structure that has this view from the front, both sides, the top, and the bottom.



b) What is the greatest number of cubes the structure could have? What is the least number of cubes? How do you know?

10. Can you always build a unique structure if you have three orthographic face views? Explain your thinking.

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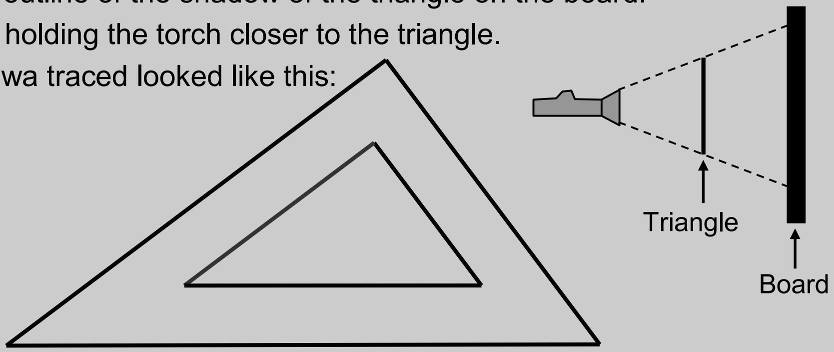
Chapter 2 Transformations

8.2.1 Dilatations

Try This

Lhamo held a triangle while Dorji shone a torch on it.
Dawa traced the outline of the shadow of the triangle on the board.
They did it again holding the torch closer to the triangle.
The shadows Dawa traced looked like this:

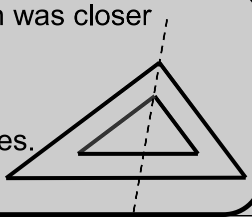


A. Which of the two shadow outlines was drawn when the torch was closer to the triangle? Explain how you know.

B. i) Trace the outlines of the two shadows.

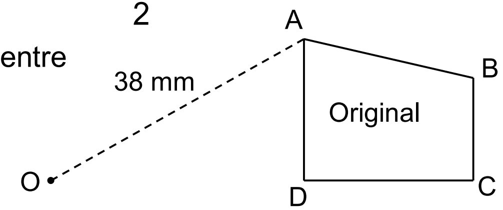
ii) Draw a line that passes through each pair of matching vertices.

iii) What do you notice about the three lines that you drew?



- A **dilatation** is a transformation that changes a shape to a larger size (an **enlargement**) or to a smaller size (a **reduction**) without changing anything else.
- A dilatation is described by its **dilatation centre** and **scale factor**. The scale factor describes how much the size has changed.
- To perform a dilatation on a **polygon**, you can locate the **dilatation image** of each **vertex** and then connect the images of the **vertices** to complete the image.
- If you know the scale factor and where the dilatation centre is, you can locate the images of the vertices in this way:

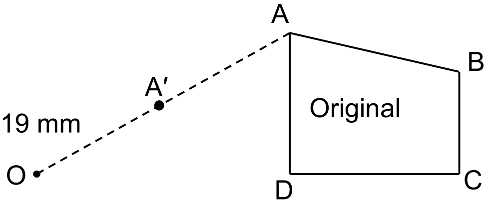
A dilatation of **trapezoid** ABCD with a scale factor of $\frac{1}{2}$ and dilatation centre at O:



- Draw a line that connects the dilatation centre to one of the vertices.
- Measure the distance from the dilatation centre to the vertex.
- Multiply the distance by the scale factor:

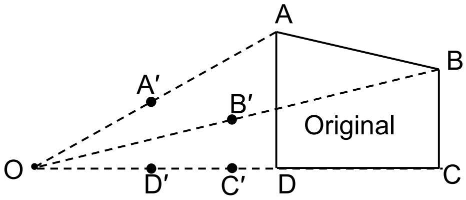
$$38 \text{ mm} \times \frac{1}{2} = 19 \text{ mm}$$

- The image vertex is on the line at a point 19 mm from the dilatation centre.

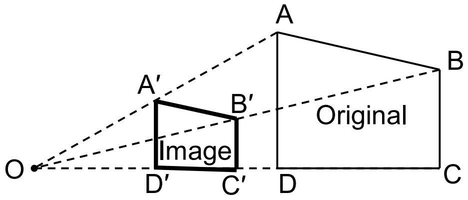


[Continued]

- Repeat this for the other vertices.



- Connect the image vertices to complete the dilatation image, trapezoid A′B′C′D′.



- These things are true for all dilatations:
  - You can draw a line through any point, its image, and the dilatation centre.
  - The image and the **original shape** are **similar**. That means **corresponding angles** are equal and the scale factor of the similar shapes (the ratio of the lengths of **corresponding sides**) is equal to the scale factor of the dilatation.
  - If the scale factor is greater than 1, the image is an enlargement.
  - If the scale factor is between 0 and 1, the image is a reduction.

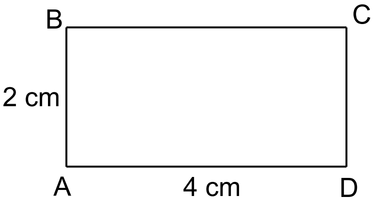
C. i) Suppose Dawa's larger triangle is a dilatation of the smaller triangle. What is the scale factor and where is the dilatation centre?

ii) Suppose Dawa's smaller triangle is a dilatation of the larger triangle. What is the scale factor and where is the dilatation centre?

Examples

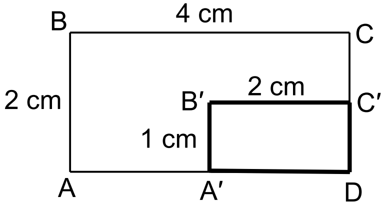
Example 1 Performing a Dilatation

Create a dilatation of rectangle ABCD so that the original vertex D is one of the image vertices. What is the scale factor and where is the dilatation centre? Show your work.



Solution 1

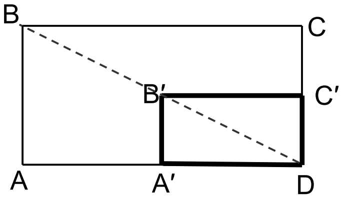
- For the dilatation image, I used a scale factor of $\frac{1}{2}$ to draw a similar shape inside ABCD with D as one of its vertices.



Thinking

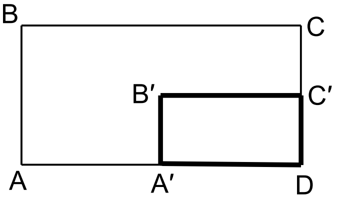
- I predicted A′B′C′D would work because it is similar to ABCD and it has a vertex at D.
- I used a scale factor of $\frac{1}{2}$ because it was easy to work with, but I could have used any scale factor.

- To locate the dilatation centre, I drew a line from B to B′ and doubled it because the scale factor was $\frac{1}{2}$. The dilatation centre was at the end of the line. That meant the dilatation centre was vertex D.



- I checked this by comparing the distances between DC′ and DC and between DA and DA′. DC′ was $\frac{1}{2}$ DC and DA′ was $\frac{1}{2}$ DA.

- A′B′C′D is the image of ABCD using dilatation centre D with scale factor $\frac{1}{2}$.

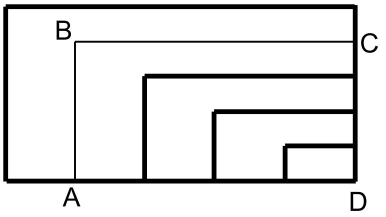


- Since I used a scale factor of $\frac{1}{2}$, I knew that each image vertex was halfway along the line that connects the dilatation centre to the corresponding vertex in the original shape.

- I also knew D was the centre because it is the only point that did not move.

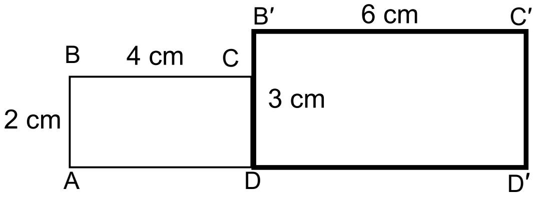
- Because vertex D is the same in the original shape and the image, I called it D in the image, but I could have renamed it D′.

- If I had used a different scale factor with the same dilatation centre, images like these would have been possible.



Solution 2

- For the dilatation image, I used a scale factor of $1\frac{1}{2}$ to draw a similar shape beside ABCD with D as one of its vertices.



[Continued]

Thinking

- I predicted D′B′C′D would work because it is similar to ABCD and it has a vertex at D.

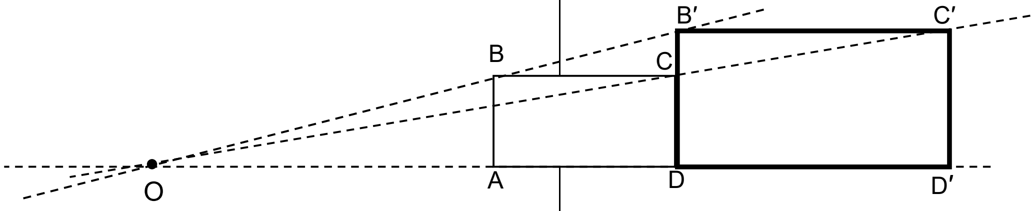
Example 1 Performing a Dilatation [Continued]

Solution 2

- To find the dilatation centre, I drew lines connecting corresponding vertices to see where they all met. The intersection point was the dilatation centre, O.

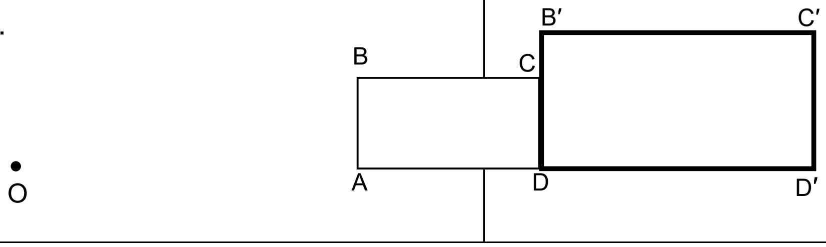
Thinking

- I knew that the dilatation centre was where all the lines connecting corresponding vertices intersected.



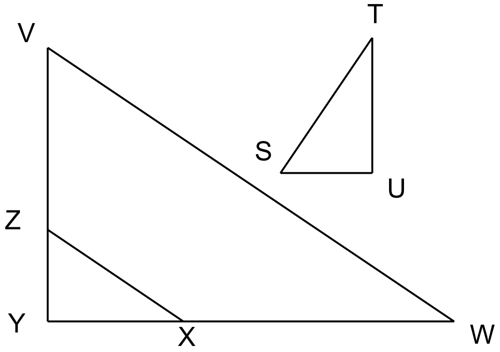
- DB′C′D′ is the image of ABCD using dilatation centre O and a scale factor of $1\frac{1}{2}$.

- I compared the lengths of AB and DB′ to find the scale factor.



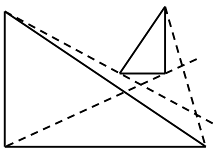
Example 2 Identifying Dilatations

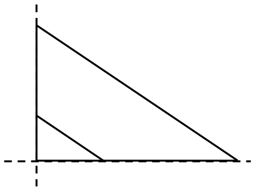
Are ∆UTS and ∆YZX dilatations of ∆YVW? If so, describe the dilatation. Show your work.



Solution

- I drew lines to connect pairs of vertices that might be corresponding vertices (based on the size of the angles at those vertices).





∆UTS is not a dilatation image because the lines do not intersect at one point.

∆YZX is a dilatation image because the lines ZV and XW intersect at vertex Y.

Thinking

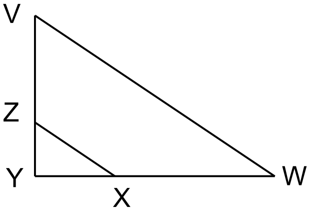
- I knew that the dilatation centre was the intersection point for all the lines that connect corresponding vertices.

- To find the dilatation scale factor, I calculated the ratio of the lengths of one pair of corresponding sides of the similar shapes.

YZ = 1.2 cm

YV = 3.6 cm

$1.2 \div 3.6 = \frac{1}{3}$

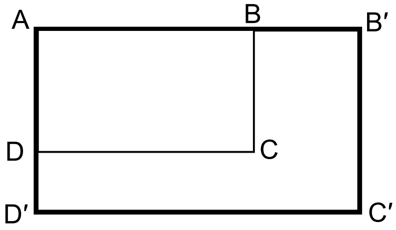


The scale factor is $\frac{1}{3}$.

- I knew that, if $\triangle YZX$ is a dilatation image of $\triangle YVW$, then $\triangle YZX$ is similar to $\triangle YVW$.
- I also knew that the scale factor of the similar shapes is the same as the scale factor of the dilatation.
- The ratio of corresponding sides of two similar shapes is the scale factor. If you know that two shapes are similar, you only need to find the ratio of one pair of corresponding sides. I used 1.2 ÷ 3.6 instead of 3.6 ÷ 1.2 because the image was smaller.

Practising and Applying

1. a) How do you know AB′C′D′ is a dilatation image of ABCD?

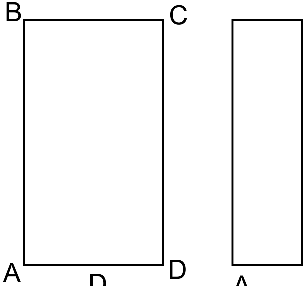


b) What are the dilatation centre and the scale factor?

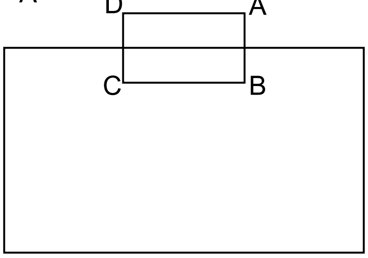
2. a) Which pair of shapes below shows
a dilatation? How do you know?

b) What are the dilatation centre and the scale factor?

A.

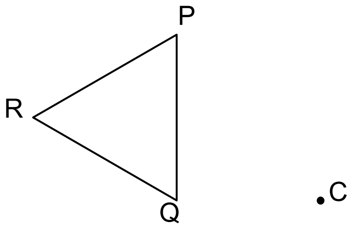


B.



3. Trace $\triangle PQR$ and point C.

a) Enlarge $\triangle PQR$ by a scale factor of 2 with dilatation centre C.



b) Reduce $\triangle PQR$ by a scale factor of $\frac{1}{2}$ with dilatation centre C.

4. a) Draw right triangle $\triangle LMN$ with $\angle L = 90°$, LM = 4 cm, and LN = 3 cm.

b) What is the area of $\triangle LMN$? Show your work.

c) Suppose you were to enlarge $\triangle LMN$ by a scale factor of 2 with dilatation centre L.

i) Predict the area of the image.

ii) Test your prediction by enlarging $\triangle LMN$ and calculating the area. How does the area of the image compare with the area of the original shape?

d) Repeat **part c)** for a scale factor of 3.

e) How could you have used the scale factor to predict the area of each image?

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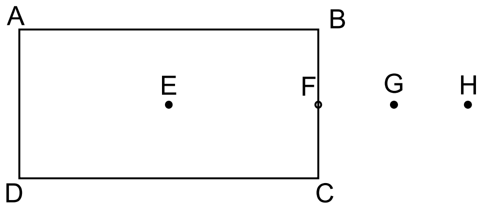
5. a) Trace ABCD and point E.

Reduce ABCD by a scale factor of $\frac{1}{3}$ using dilatation centre E.



b) Trace ABCD and point F.

Reduce ABCD by a scale factor of $\frac{1}{3}$ using dilatation centre F.

c) Trace ABCD and point G.

Reduce ABCD by a scale factor of $\frac{1}{3}$ using dilatation centre G.

d) Trace ABCD and point H.

Reduce ABCD by a scale factor of $\frac{1}{3}$ using dilatation centre H.

6. Suppose that you perform a dilatation on rectangle EFGH with scale factor $\frac{1}{2}$. Where might the dilatation centre be for each image described below? Sketch a diagram as part of each answer.

a) The image is inside rectangle EFGH.

b) The image is inside but touching EFGH.

c) The image is outside EFGH.

d) The image is outside but touching EFGH.

7. Suppose you perform a dilatation on a rectangle using a scale factor of 2. Is it possible for the image to be completely inside the original shape? Explain your thinking.

8.2.2 Combining Transformations

Try This

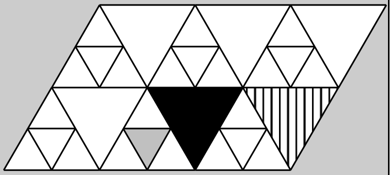
A. i) The black triangle is a dilatation image of the grey triangle. What is the scale factor and where is the dilatation centre?

ii) The black triangle is also a translation image of the striped triangle. What is the distance and direction of the translation?

iii) The black triangle is also a reflection image of the striped triangle. Describe the reflection line.

iv) The black triangle is also a rotation image of the striped triangle. Where is the turn centre and what is the angle of rotation?

Purna Bahadur is tiling a wall with triangular tiles as shown here.



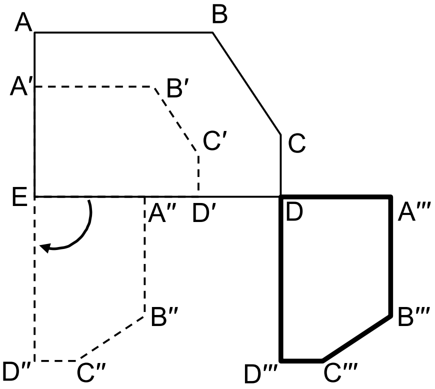
- When two shapes are similar or congruent, you can always use a combination of transformations to go from the one shape to the other shape.
- To perform or identify a combination of transformations, you will need to recall the information in this chart.

Transformation	How to describe	Properties of the image
Dilatation	• location of dilatation centre • scale factor	• similar • same **orientation**
Translation	• distance • direction	• congruent • same orientation
Reflection	• location of **reflection line**	• congruent • opposite orientation
Rotation	• location of **turn centre** • **angle of rotation** and **direction of rotation** (cw or ccw)	• congruent • same orientation

For example:

To transform **pentagon** ABCDE to A‴B‴C‴D‴D, you can use these transformations:

- Dilatate ABCDE to A′B′C′D′E using dilatation centre E and a scale factor of $\frac{2}{3}$.
- Rotate A′B′C′D′E to A″B″C″D″E 90° **clockwise** (cw) around turn centre E.
- Translate A″B″C″D″E to A‴B‴C‴D‴D to the right along ED.



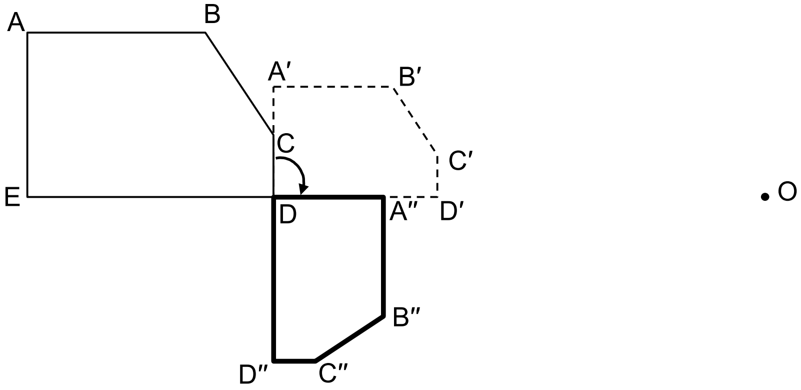
- When you perform a combination of transformations, it helps to draw the intermediate images.

- There is often more than one way to combine transformations for the same result.

For example:

To transform ABCDE to A″B″C″D″D, you can also use these transformations:

- Dilatate ABCDE to A′B′C′D′D using dilatation centre O and a scale factor of $\frac{2}{3}$.
- Rotate A′B′C′D′D 90° cw around turn centre D.

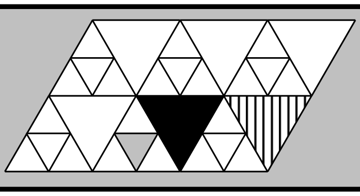


For some combinations of transformations, the order in which you do them makes a difference. For other combinations, the result is the same, no matter what the order of the transformations.

B. Describe a combination of transformations for each (from **part A**).

i) to transform the grey triangle to the striped triangle

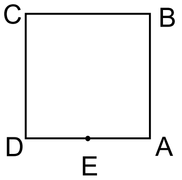
ii) to transform the striped triangle to the grey triangle



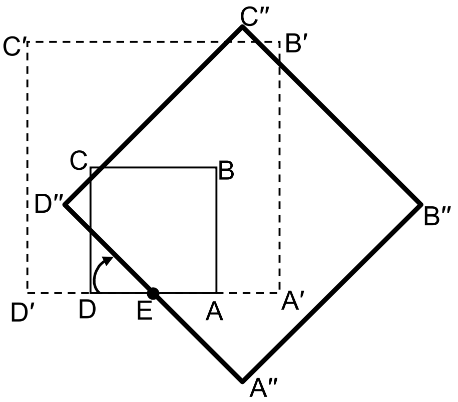
Examples

Example 1 Performing a Combination of Transformations

Enlarge square ABCD by a scale factor of 2 using dilatation centre E (the midpoint of AD). Then rotate the image 45° clockwise around turn centre E.



Solution



Thinking

- I enlarged ABCD to A′B′C′D′.
- Then I used tracing paper to rotate A′B′C′D′ to A″B″C″D″. I traced A′B′C′D′ and then held my pencil tip on E while I turned the tracing 45° cw.
- If I had done a rotation followed by a dilatation — the result would be the same (but I know that, with some combinations, the opposite order has a different result).

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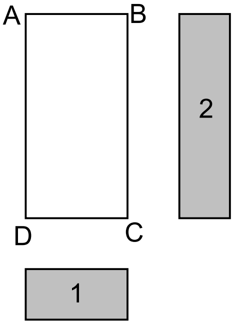
Example 2 Describing a Combination of Transformations

a) Which of the grey rectangles could be an image of ABCD? How do you know?

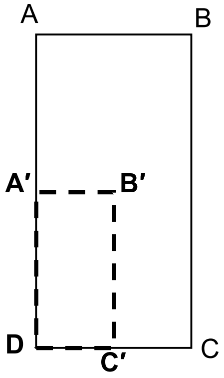
b) Describe a combination of transformations that would transform rectangle ABCD to the rectangle you chose in **part a)**. Show your work.

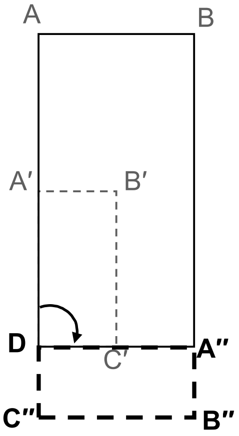


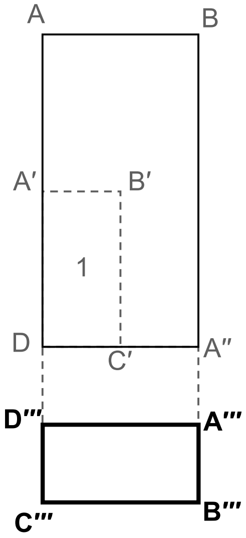
Solution

a) Rectangle 1 could be an image because it is similar. Its length and width are half of ABCD's length and width.

b)







To go from ABCD to A‴B‴C‴D‴, the transformations are:

- A dilatation with scale factor $\frac{1}{2}$ and centre D
- Then, a 90º cw rotation around D
- Finally, a translation down along DC″

Thinking

a) I knew that Rectangle 2 could not be an image because its width was half but its length was the same as ABCD.

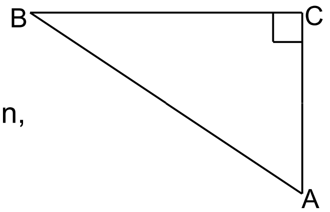
b) I knew there was a reduction because the image is smaller.

- I also knew there was a rotation because the image is turned.
- I knew the scale factor for the dilatation was $\frac{1}{2}$ because the image dimensions are half the dimensions of ABCD.
- I used vertex D as the dilatation centre so the image would end up in the bottom left corner of ABCD.
- I used vertex D as the turn centre and made a 90° cw turn.
- I realized that I also needed a translation to slide A′B′C′D down.
- I translated A′B′C′D down along the length of side DC″.

Example 3 Testing to See if the Order of Transformations Matters

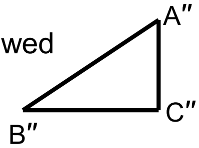
Pema was asked to dilatate ∆ABC with centre A and scale factor $\frac{1}{2}$ and then reflect in line BC.

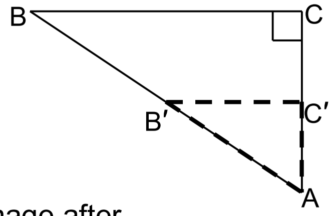
Would he get the same result if he first did the reflection, followed by the dilatation?



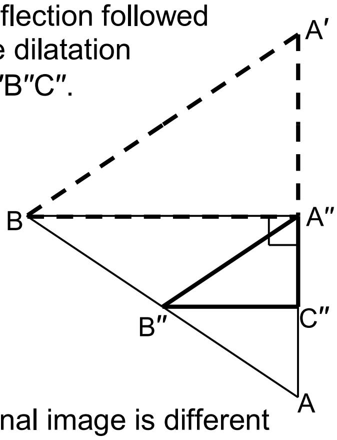
Solution

The image after the dilatation followed by the reflection is ∆A″B″C″.





The image after the reflection followed by the dilatation is ∆A″B″C″.



The final image is different if the transformations are done in the opposite order.

Thinking

- First I did the combination of transformations in the given order:

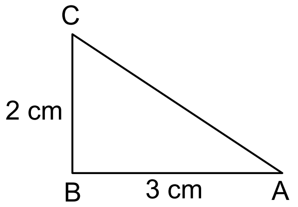
  - To dilatate with centre A, I knew point A would not move.
  - The other image points were B′ and C′, halfway from point A to points B and C.
  - I reflected AB′C′ in BC to get the final image triangle.

- Then I did it in the opposite order:

  - The reflection in line BC flipped the triangle across that line. I drew the image with a dashed line.
  - To dilatate the reflection, I knew all the points would move because no points were on the triangle.
  - Each image point was half the distance to A from its pre-image.
  - The final image was ∆A″B″C″. (A″ was the same point as point C.)

Practising and Applying

1. Copy this right triangle. Perform each combination of transformations.

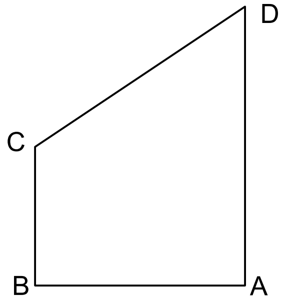


a) Dilatate ∆ABC by a scale factor of 2 using dilatation centre A. Then translate up 2 cm.

b) Translate ∆ABC up 2 cm. Then dilatate by a scale factor of 2 with centre A.

2. Explain why the final image for **question 1 part a)** is different from the final image for **part b)**.

3. Trace this trapezoid. Perform each combination of transformations.

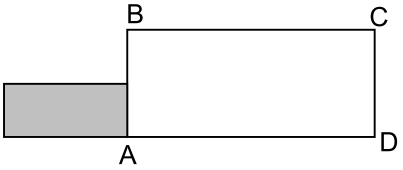


a) Dilatate trapezoid ABCD by a scale factor of $\frac{1}{2}$ using dilatation centre A. Then rotate 90º cw around vertex A.

b) Rotate trapezoid ABCD 90º cw around vertex A. Then dilatate by a scale factor of $\frac{1}{2}$ with centre A.

4. Explain why the final image for **question 3 part a)** is the same as the image from **part b)**.

5. The grey rectangle is the image of rectangle ABCD after a combination of transformations.



a) Describe a dilatation followed by a translation that would result in the grey image.

b) Describe a translation followed by a dilatation that would result in the grey image.

c) Is it possible to map ABCD onto the grey rectangle with one transformation? How do you know?

6. Use a triangle of your choice.

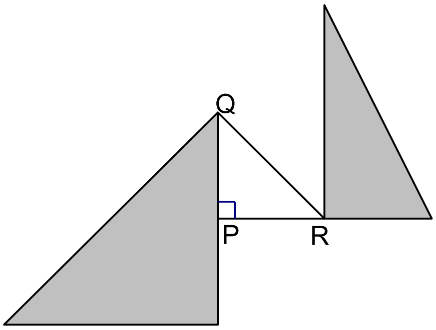
a) Find a combination of a dilatation and a reflection that you can do in either order to give the same final image.

b) Find a combination of a dilatation and a reflection that has a different final image when you do them in the opposite order.

7. Why do you think that the order of the transformations does not matter in **question 6 part a)**, but the order does matter in **part b)**?

8. a) Which grey triangle below could be an image of∆PQR? How do you know?

b) Describe a combination of transformations that would transform ∆PQR to the grey triangle you chose in **part a)**.



9. Suppose you are given a 2-D shape and its image after a combination of transformations. Explain how you would know each.

a) that one of the transformations was a dilatation

b) that one of the transformations was a rotation

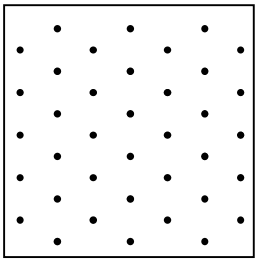
c) that one of the transformations was a reflection

GAME: Isometry

Play this game with a partner. You need isometric dot paper.

The goal of the game is to recognize transformations.

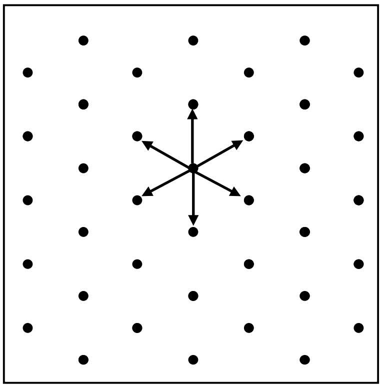
- Cut out or mark off a square area on isometric dot paper for a game board. You can only use the dots in this area.



Game board

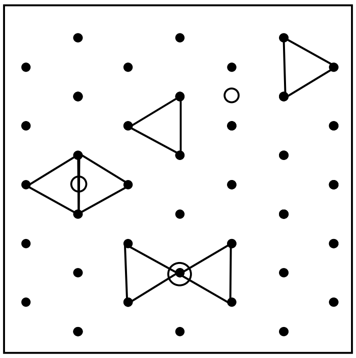
- Decide who will be Player A and Player B.
- Take turns. On your turn, draw a line segment to connect any dot to one of the six dots around it.
- If your line segment completes a triangle, write your player letter (A or B) in the triangle. Take another turn, up to a maximum of 3 turns in a row.
- You score points by naming the transformations between the triangle you just completed and the last triangle drawn:

- 1 point for a triangle that shows a single translation
- 2 points for a triangle that shows a single reflection
- 3 points for a triangle that shows a single rotation

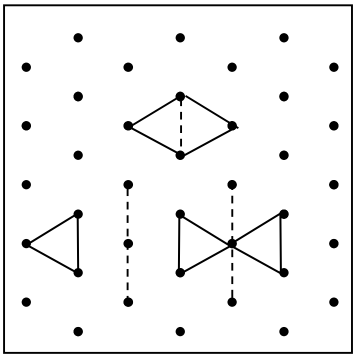


You can draw a line segment from a dot to any of the 6 dots around it.

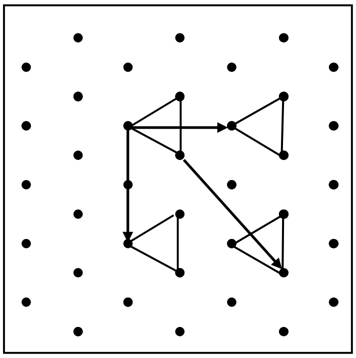
Here are three examples of each type of transformation:



A rotation around an open dot
3 points



A reflection across a dashed line
2 points



A translation along an arrow
1 point

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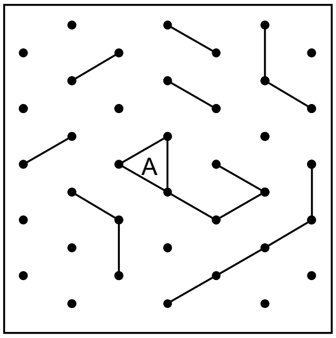
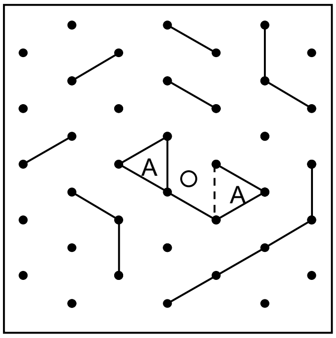
- Play ends when no more line segments can be drawn.
- The player with more points wins.

For example:

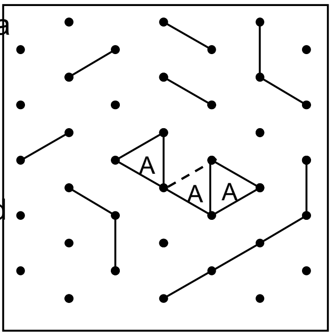
- Player A has just completed a triangle (labelled "A"), so she gets another turn. Since there are no other triangles on the board, there are no transformations to name. She scores no points.



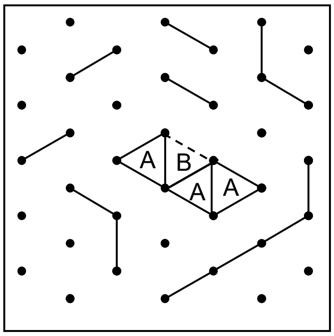
- If Player A draws her next line segment where there is a dashed line, she completes a second triangle and gets another turn. She gets 3 points if she names the rotation around the point (open dot) shown.



- If Player A draws her next line segment where there is a dashed line, she completes a third triangle, gets another turn, and scores these points:
  - 3 points if she names the rotation between the second and third triangles
  - 2 points if she names the reflection between the second and third triangles



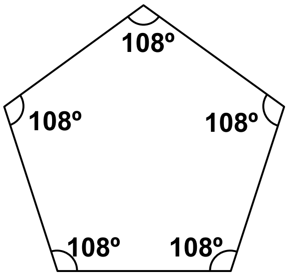
- It is now Player B's turn because Player A had three turns in a row. If Player B draws his line segment where there is a dashed line, he completes a triangle, gets another turn, and scores these points:
  - 3 points if he names the rotation between the fourth and third triangles
  - 2 points if he names the reflection between the fourth and third triangles



- Player B then draws another line segment. He cannot complete a triangle, so his turn ends.
- It is now Player A's turn.

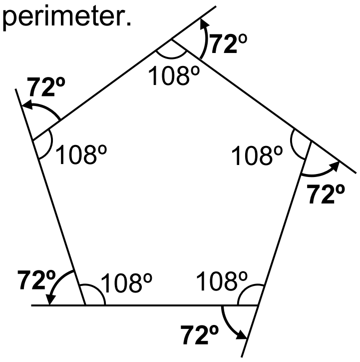
8.3.1 EXPLORE: Measuring Angles in Polygons

Kamala drew a pentagon. She measured the **interior angles**.



The sum of the interior angles was 540º.
108º + 108º + 108º + 108º + 108º = 540º

She measured the **exterior angles**. She visualized these angles as the amount she would turn at each vertex if she walked counterclockwise around the perimeter.



The sum of the exterior angles was 360º.
72º + 72º + 72º + 72º + 72º = 360º

A. i) Draw a different pentagon. Find the sum of the interior angles.
ii) Extend each side and then find the sum of the exterior angles.
iii) Compare your results with Kamala's results and your classmates'. What conclusions can you make about the interior and exterior angle sums of pentagons?

B. i) Draw other polygons and measure the angles to complete this table.

	Triangle	Quadrilateral	Pentagon	Hexagon
Number of vertices	3	4	5	6
Interior angle sum				
Exterior angle sum				

ii) Graph the relationship between the number of vertices and the interior angle sum. Is it a linear relationship? How do you know?
iii) What do you notice about the relationship between the number of vertices and the exterior angle sum? Is it a linear relationship? How do you know?

C. Examine your results from **part B**.
i) How can you use your graph to predict the interior angle sum of a polygon if you know the number of vertices?
ii) How can you use the table to make a prediction?
iii) What conclusion can you make about exterior angle sums and polygons?

D. How can you predict the size of each exterior angle and each interior angle of any <u>regular</u> polygon from the angle sums? Explain your thinking.

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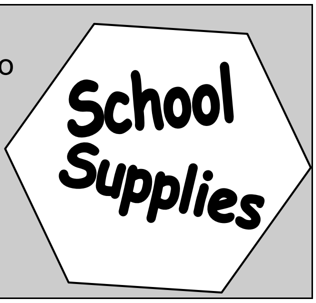
8.3.2 Angles in Polygons

Try This

Tandin wants to paint a regular hexagon sign so it will catch the attention of his customers. He wants to divide the sign into triangles and then paint each triangle a different colour.



A. i) What is the least number of triangles into which he can divide the sign? Show this in a diagram.

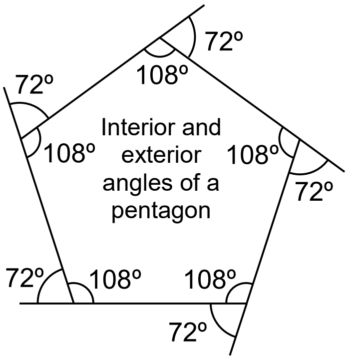
ii) What is the sum of all the interior angles from all the triangles?

- In the last lesson, you learned about these two angle relationships in any polygon:

- The sum of the **exterior angles** in any polygon is 360º.

- The sum of the **interior angles** in any polygon is related to the number of vertices:

  Interior angle sum = 180º × (number of vertices – 2)



- The reason why the sum of the interior angles is 180º × (number of vertices – 2) is explained below:

  - When you divide a polygon into the least number of triangles, there are 2 fewer triangles than the number of vertices in the polygon. That means a polygon with n vertices can be divided into $n - 2$ triangles.

  - The interior angles of each triangle total 180º, so the sum of the interior angles of the triangles is $180^\circ \times (n - 2)$.

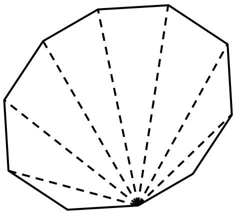
  - The sum of the interior angles of the triangles is equal to the sum of the interior angles of the polygon, so the sum of the interior angles of the polygon is $180^\circ \times (n - 2)$.

  For example:

  - A **decagon** has 10 vertices. It can be divided into 10 – 2 = 8 triangles.

  - The interior angles of each triangle total 180º, so the sum of the interior angles of the 8 triangles is 180º × (10 – 2) = 180º × 8 = 1440º.

  - Since the sum of the interior angles of the 8 triangles is equal to the sum of the interior angles of the decagon, the sum of the interior angles of the decagon is 1440º.



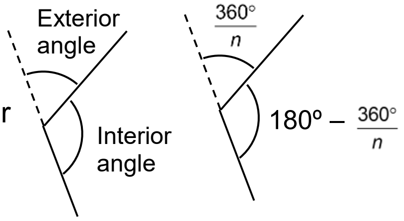
A decagon (10 sides) can be divided into 8 triangles.

- You can use the above formula to develop a formula for calculating the measure of an interior angle of any **regular polygon**:

Since a regular polygon with n sides has n congruent angles:

$$\text{The interior angle of a regular polygon} = \frac{180^\circ(n-2)}{n}$$

- You can develop the same formula using the exterior angles of a regular polygon:
  - The exterior angle in any regular polygon is 360º ÷ number of vertices, or $\frac{360°}{n}$.
  - At each vertex, the exterior and interior angles add to 180º because they form a straight line.
  - Since the exterior angle is $\frac{360°}{n}$, then the interior angle is $180° - \frac{360°}{n}$, or 180º – 360º ÷ *n*.



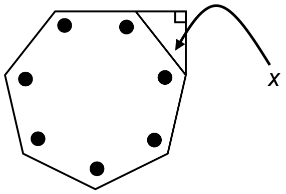
The interior angle in a regular polygon is $180° - \frac{360°}{n}$ or $\frac{180°(n-2)}{n}$.
Both formulas give the same result for any value of *n*.

B. i) Use a formula to find the sum of the interior angles in Tandin's hexagon.
ii) Use a formula to calculate the size of each exterior angle.
iii) Use a formula to calculate the size of each interior angle.

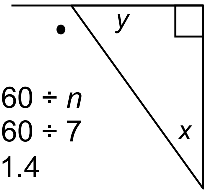
Examples

Example 1 Using Properties of a Polygon to Find an Angle Measurement

Determine the measurement of angle *x*.



Solution



$y = 360 \div n$
$= 360 \div 7$
≈ 51.4

$x + 51.4 + 90 = 180$
$x + 141.4 = 180$
$x = 180 - 141.4$
$x = 38.6°$

Angle *x* is about 39º.

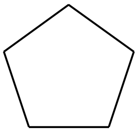
Thinking

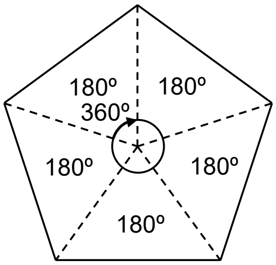
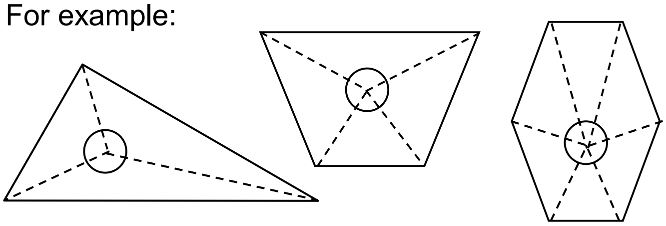
- I knew the heptagon had 7 sides. I also knew it was regular because all the angles have the same symbol.
- I labelled the other angle in the triangle *y*. I knew it was an exterior angle of the heptagon because the side of the triangle is an extension of a side of the heptagon.
- To find *y*, I used the formula for exterior angles in a regular polygon.
- Since the angles in a triangle total 180º, I was able to find *x* by subtracting.

Example 2 Developing a Formula for the Interior Angle Sum Differently

a) Divide a regular pentagon into five triangles. Find the sum of the interior angles. Show your work.

b) Use the five triangles to develop the formula for the sum of the interior angles in any polygon. Show your work.

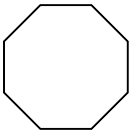


Solution	**Thinking**
a)  • Each of the 5 triangles has an interior sum of 180º. • The sum of the interior angles of the 5 triangles is 5 × 180º = 900º. • The sum of the 5 angles in the centre is 360º. • Since the angles in the centre are not part of the pentagon's interior angles, the sum of the interior angles of the pentagon is 900º – 360º = 540º.	 **a)** I drew a line from each vertex to the centre of the pentagon to divide the pentagon into five triangles.
b) You can divide any polygon into the same number of triangles as the number of vertices. For example:  You can find the interior angle sum of any polygon by multiplying the number of vertices by 180º and then subtracting 360º. The total angle sum = Number of vertices × 180° $= n \times 180°$ Interior angle sum $=$ Total angle sum – 360° $= n \times 180° - 2 \times 180°$ $= (n - 2) \times 180°$ Interior angle sum of any polygon $= (n - 2) \times 180°$	**b)** I knew that if I put a point somewhere inside the polygon and connected it to each vertex, there would be the same number of triangles as vertices. • I also knew that the sum of the angles around the point was 360°, or 2 × 180°. • I subtracted the sum of the angles around the centre point from the total angle sum. What was left was the sum of the interior angles of the polygon.

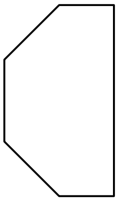
Practising and Applying

1. What is the sum of the interior angles of each polygon? Show your work.

a)



b)



c) a regular hexagon

d) a trapezoid

2. What is the measure of an exterior angle in each regular polygon? Show your work.

a) equilateral triangle

b) square

c) decagon (ten sides)

d) dodecagon (twelve sides)

3. Why can you *not* use this formula to find the measure of an exterior angle in an irregular polygon?

$$\text{Exterior angle} = \frac{360°}{n}$$

4. What is the measure of an interior angle in each regular polygon? Show your work.

a) equilateral triangle

b) nonagon (nine sides)

c) octagon (eight sides)

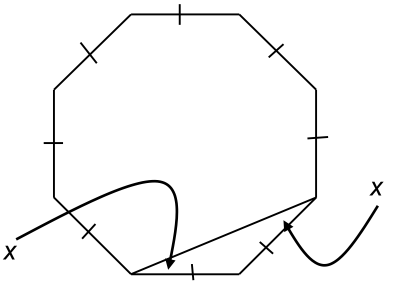
5. a) A friend tells you that he has drawn a regular polygon with an exterior angle of 40º. Can you tell what type of polygon it is? Explain your thinking.

b) What if he says the angle is 55º instead of 40º? Can you tell what type of polygon it is? Explain your thinking.

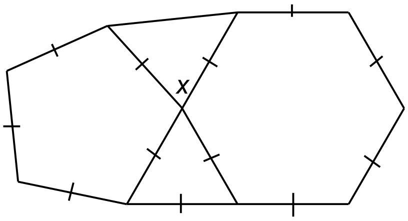
6. Tshering walks through fresh snow on a field. He walks six steps straight ahead, turns 45º cw, walks six more steps, turns 45º cw, and continues with this pattern until he returns to his starting point. What will be the shape of his path in the snow? Explain how you know.

7. Find the measure of angle x in each. Show your work.

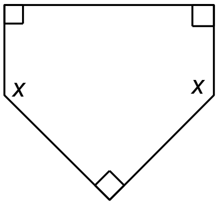
a)



b)



7. c)



8. You have two formulas for the interior angle of a regular polygon.

A. $\text{Interior angle} = \frac{180°(n-2)}{n}$

B. $\text{Interior angle} = 180° - \frac{360°}{n}$

a) Use Formula A to find an interior angle in a regular nonagon (nine sides).

b) Use Formula B to find an interior angle in a regular nonagon.

c) Use each formula to find the interior angle in a 120-sided regular polygon.

9. Which formula in **question 8** do you prefer to use? Why?

10. a) Which regular polygons have an exterior angle that is a whole number of degrees? Explain your thinking.

b) Which regular polygons have an interior angle that is a whole number of degrees? Explain your thinking.

11. a) Which polygon has the greatest interior angle sum?

- an equilateral triangle
- a regular dodecagon
- a regular polygon with 100 sides

How do you know?

b) Which polygon has the greatest exterior angle?

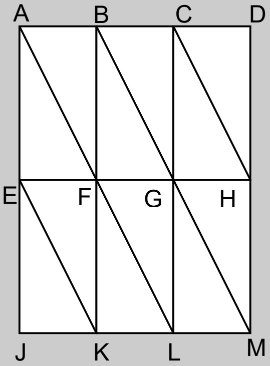
- an equilateral triangle
- a regular dodecagon
- a regular polygon with 100 sides

How do you know?

8.3.3 Angles With Parallel and Intersecting Lines

Try This

Kuenzang made this book cover design with congruent triangles.



A. i) Identify sets of parallel lines in the design.

ii) How do you know ∠ GFL + ∠ EFL = 180º?

B. What is the image of ΔFGL after each transformation?

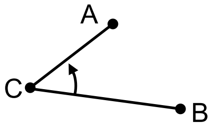
i) a translation along FA

ii) a rotation of 180º around point F

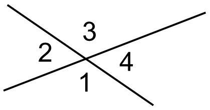
iii) a rotation of 180º around the midpoint of FL

- You can name an angle using its vertex letter or by using its vertex letter and the letters of two points on its arms (with the vertex letter in the middle).

∠ C or ∠ ACB or ∠ BCA



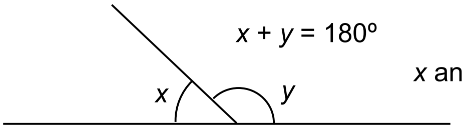
- Where lines intersect, there are some special relationships among the four angles formed.



\- **Opposite angles**, which are across from each other, are equal.

∠ 2 and ∠ 4 are opposite angles. So are ∠ 1 and ∠ 3.
That means ∠ 2 = ∠ 4 and ∠ 1 = ∠ 3.

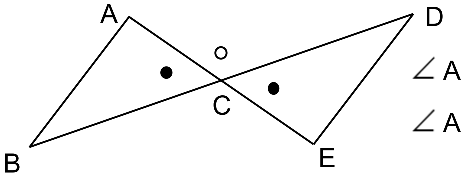
\- **Adjacent angles**, which are beside each other and share a vertex, add to 180º because together they form a straight angle, or a 180º angle.



x and y are adjacent angles.

In the diagram above, ∠ 1 and ∠ 2 are adjacent angles. So are ∠ 3 and ∠ 4, ∠ 1 and ∠ 4, and ∠ 2 and ∠ 3. That means ∠ 1 + ∠ 2 = 180º, ∠ 3 + ∠ 4 = 180º, ∠ 1 + ∠ 4 = 180º, and ∠ 2 + ∠ 3 = 180º.

For example:



∠ ACB = ∠ DCE because they are opposite.

∠ ACD + ∠ DCE = 180º because they are adjacent.

- Any two angles that have a sum of 180º are called **supplementary angles**.

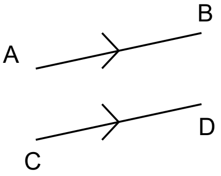
- **Parallel lines** are lines that will never intersect, no matter how far they are extended. These photographs show examples of parallel lines in the real world.

Rails for a train

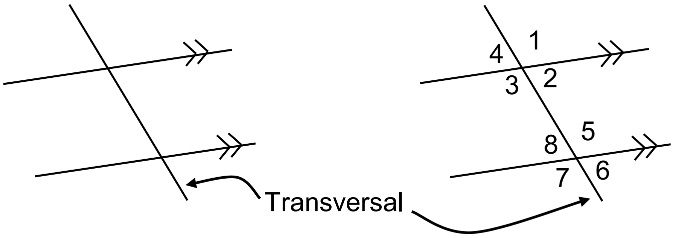
Electric power lines

- To show that lines are parallel, you can mark them with small arrows, as shown to the right.

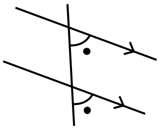
- To write that two lines are parallel, you can use the symbol ∥. AB∥CD means that AB is parallel to CD.



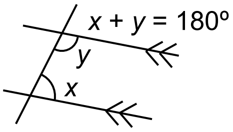
- A line that crosses other lines is called a **transversal**. When a transversal crosses two parallel lines, there are many relationships among the angles formed.



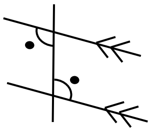
- **Corresponding angles** are in the same position along the transversal. Corresponding angles are equal.
∠1 = ∠5, ∠2 = ∠6, ∠4 = ∠8, and ∠3 = ∠7.



- **Interior angles** are between the parallel lines on the same side of the transversal. Interior angles are supplementary.
∠2 + ∠5 = 180º and ∠3 + ∠8 = 180º.

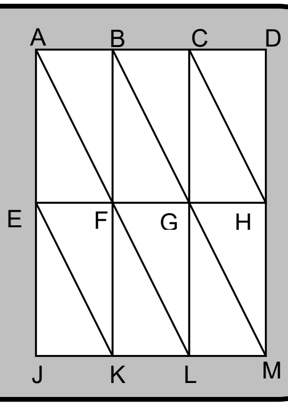


- **Alternate angles** are between the parallel lines on opposite sides of the transversal. Alternate angles are equal.
∠2 = ∠8 and ∠3 = ∠5.



C. Identify each in Kuenzang's pattern (from **part A**).

i) a pair of opposite angles
ii) a pair of corresponding angles
iii) a pair of interior angles
iv) a pair of alternate angles
v) a pair of adjacent angles that form a straight angle

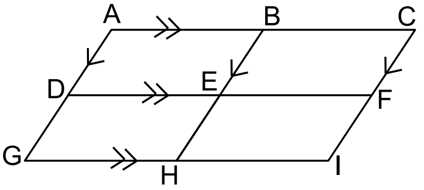


Examples

Example 1 Identifying Angle Relationships

a) Identify three angles equal to ∠ BED.
How do you know they are equal?

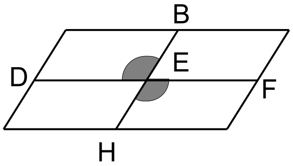
b) Identify two angles supplementary to ∠ BED.
How do you know they are supplementary?



Solution

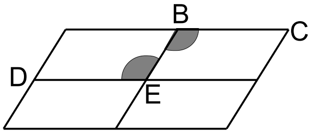
a) ∠ BED = ∠ FEH

They are opposite angles.



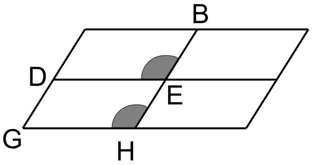
∠ BED = ∠ EBC

They are alternate angles.



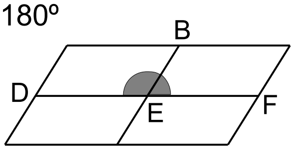
∠ BED = ∠ EHG

They are corresponding angles.



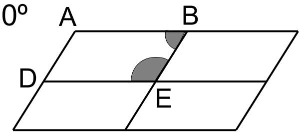
b) ∠ BED + ∠ BEF = 180º

They are adjacent angles that form a straight angle.



∠ BED + ∠ EBA = 180º

They are interior angles.



Thinking

a) I knew the angle opposite from ∠ BED would be equal to it.

- I used the fact that AC ∥ DF and BH was a transversal to find a pair of equal alternate angles.

- I used the fact that DF ∥ GI and BH was a transversal to find a pair of equal corresponding angles.

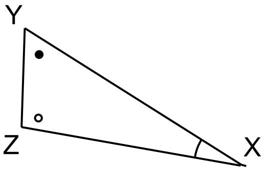
b) I knew that two adjacent angles were supplementary.

- I used the fact that DF ∥ AC and BH was a transversal to find a pair of supplementary interior angles.

Example 2 Proving That Vertically Opposite Angles are Equal

Rotate a triangle around one of its vertices to show that opposite angles are equal.

Solution

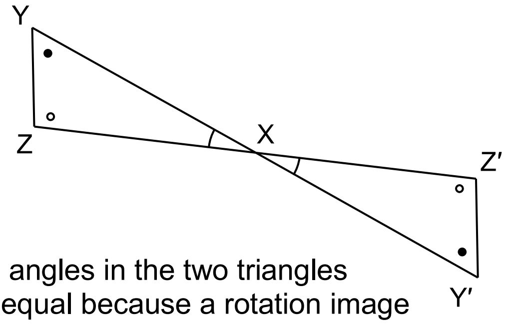


[Continued]

Thinking

- I used a scalene triangle so it would be easier to match corresponding vertices in the original shape and its image.

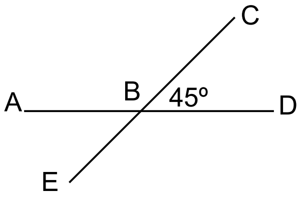
Example 2 Proving That Vertically Opposite Angles are Equal [Continued]

Solution	Thinking
Rotate ∆XYZ around X  The angles in the two triangles are equal because a rotation image is congruent to the original shape. ∠YXZ and ∠Y′XZ′ are opposite angles. ∠YXZ = ∠Y′XZ′ because ∠Y′XZ′ is a rotation image of ∠YXZ.	• I rotated ∆XYZ around X until ∠YXY′ made a straight line. • I marked congruent angles in the two triangles. • I noticed that ∠Y′XZ′ and ∠YXZ are opposite angles so they were equal (and these were the angles that I needed to show were equal).

Practising and Applying

1. Find the measure of each angle without measuring. Explain how you know you are right.

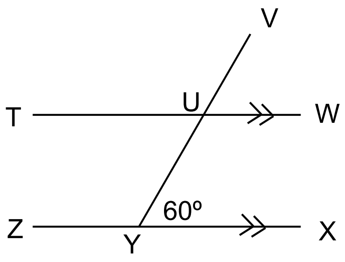
a) ∠ABC **b)** ∠ABE



2. Find the measure of each angle without measuring. Explain how you know you are right.

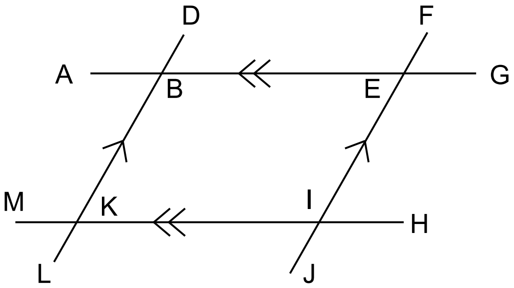
a) ∠WUY **b)** ∠TUY

c) ∠VUW **d)** ∠VUT



3. These pairs of parallel lines form a parallelogram. Each line is also a transversal. Which pair of parallel lines does each transversal cross?

a) LD **b)** HM **c)** AG **d)** FJ



4. Use the diagram from **question 3**.

a) Which angles are supplementary to ∠KBE?

b) Which angles are supplementary to ∠KIE?

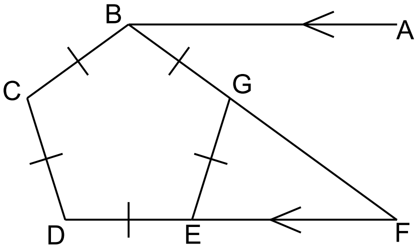
c) Use your answers to **parts a) and b)** to say something that is always true about angles in a parallelogram.

5. Use the diagram from **question 3**.

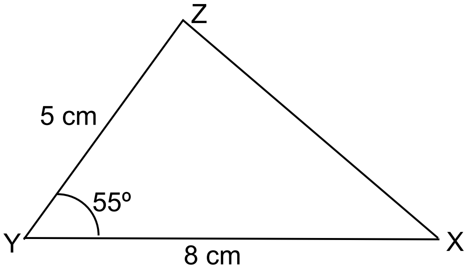
a) Which angles are corresponding angles to ∠KBE?

b) Which angles are alternate angles with ∠KBE?

6. Use what you know about regular polygons and parallel lines to find the measures of all the angles in this diagram. Show your work.



7. a) Draw ΔXYZ with XY = 8 cm, YZ = 5 cm, and ∠ XYZ = 55°.

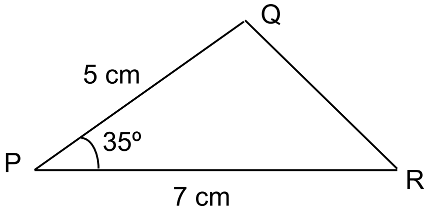


b) Translate ΔXYZ along side YZ so that Z is the image of Y, X′ is the image of X, and Z′ is the image of Z.

c) Identify a pair of parallel lines. How do you know they are parallel?

d) Identify a pair of corresponding angles formed by the transversal YZ′. Without measuring, how do you know they are equal?

8. a) Draw ΔPQR with PQ = 5 cm, PR = 7 cm, and ∠ P = 35°.



b) Find the midpoint of QR. Label it M.

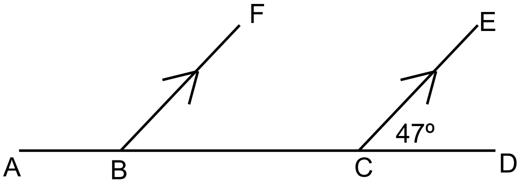
c) Rotate ΔPQR 180° around M. Label the image of P as P′.

d) Identify a pair of parallel lines. How do you know they are parallel?

8. e) Identify a pair of alternate angles formed by the transversal QR. Without measuring, how do you know they are equal?

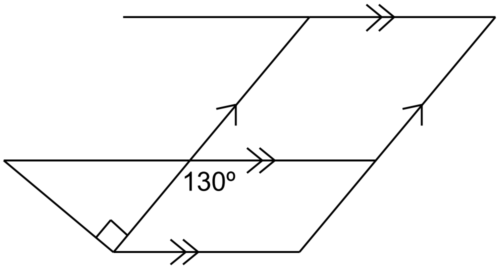
9. a) In the diagram below, what is the measure of ∠ ECB? How do you know?

b) What is the measure of ∠ FBC? How do you know?



c) ∠ FBC and ∠ ECB are interior angles. Explain why they are also supplementary angles.

10. Use what you know about polygons and parallel lines to find as many angle measures as you can in this diagram.



11. Sometimes a transversal crosses two lines that are not parallel.

a) Samten says that, when this happens, the corresponding angles and alternate angles are not congruent. Is this true? Explain your thinking.

b) However, he says that opposite angles are still congruent. Is this true? Explain your thinking.

12. a) Draw a diagram that gives only one or two angle measures. Make sure someone could determine five or more other angle measurements.

b) Show the angles that can be found in your diagram from **part b)**.

CONNECTIONS: Tools for Geometry

- Here are the standard tools used for studying geometry in mathematics classes.

- You use a compass (a pair of compasses) and a straight edge for constructions.
- You use a protractor to measure angles.
- You use a ruler to measure lengths.

- Other tools can also be useful.

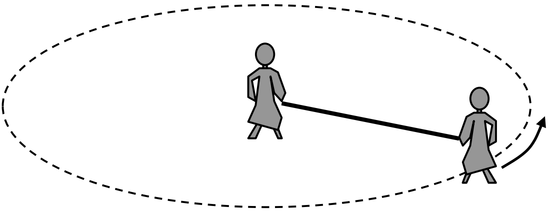
- You can use a measuring tape to measure a length that is longer than a ruler or a length that is not in a straight line, like the circumference of a tree.

- You can use a level to see whether objects are horizontal, vertical, and parallel.

- You can improvise your own tools.

For example, you can make a large compass:

- Stretch a rope tightly between two people to make a straight line.
- One person stands in one place, holding one end of the rope, while the other person walks around, holding the other end and keeping the rope tight.
- The path formed will be a circle. To show the path, you can drag your feet in the dirt. Or, you can use your free hand to draw on the ground with chalk.



1. Here are some examples of improvisation. Try each method and then describe what you did.

a) Fold a piece of paper to make a straight edge.

b) Fold a piece of paper to create a pair of parallel lines.

2. Describe how you could use improvised geometry tools to make each.

a) a set of parallel lines

b) a large square

c) a large regular hexagon

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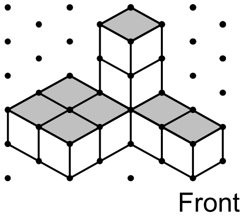
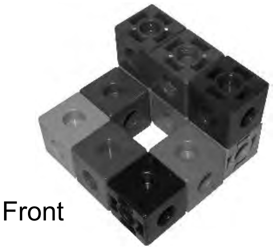
UNIT 8 Revision

1. Make an isometric drawing of this structure from a different view.

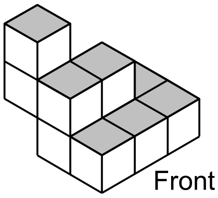


2. Make three different isometric drawings of this cube structure.

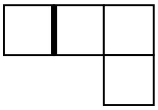


3. Make a set of orthographic drawings of the structure in **question 2** that has three or more face views.

4. Make a set of orthographic drawings of this structure, made of 10 cubes, that has three or more face views.



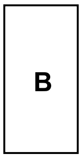
5. This is the top face view of a structure made from cubes.

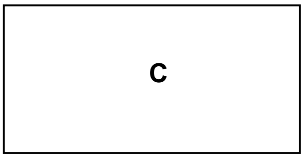


a) Build two possible structures that it could represent. Then, make an isometric drawing of each.

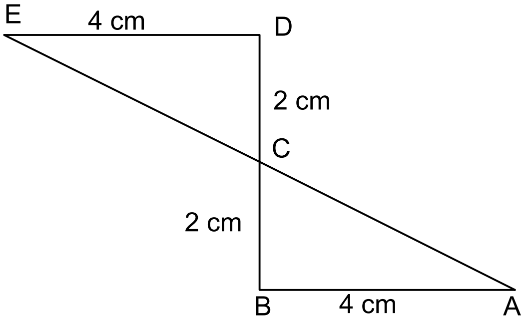
b) Draw a set of orthographic drawings for each structure in **part a)** that has two or more additional face views.

6. B is the original shape. Which of the other shapes could be a dilatation image? How do you know?





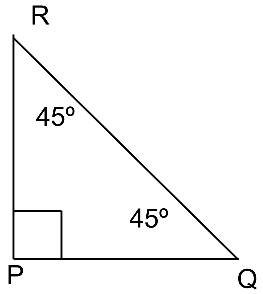
7. a) Dilatate ΔABC by a scale factor of 3 with centre D. Label the image ΔA′B′C′.



b) Dilatate ΔABC by a scale factor of $\frac{1}{2}$ with centre D.

c) Describe a dilatation of ΔABC for which the image has a vertex at A.

8. Transform ∆PQR with each combination of transformations.



a) Dilatate ∆PQR with centre P and scale factor 2. Then reflect the image in line PR.

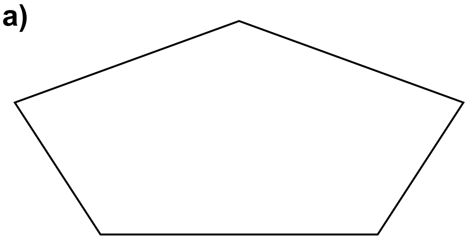
b) Perform the transformations in **part a)** in the opposite order. Do you get the same result?

c) Rotate ∆PQR 90º ccw with centre P. Then enlarge the image with a scale factor of 2 (choose a dilatation centre that results in a different final image, if the transformations are done in the opposite order).

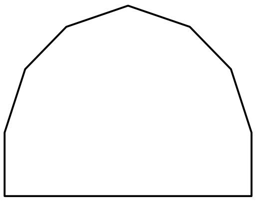
9. Use your work from **question 7 part a)**. Describe a combination of transformations that would transform ∆CDE to ∆C′B′A′.

10. What is the sum of the interior angles in each shape? Show your work.

a)



b)



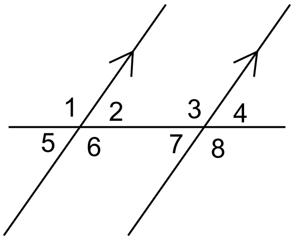
c) a regular dodecagon (twelve sides)

11. a) What is the measure of an interior angle in a regular octagon? Show your work.

b) What is the measure of an exterior angle in a regular nonagon (nine sides)? Show your work.

c) How many sides does a regular polygon have if one of its exterior angles is 10º? How do you know?

12. Find an angle in this diagram that fits each description below.

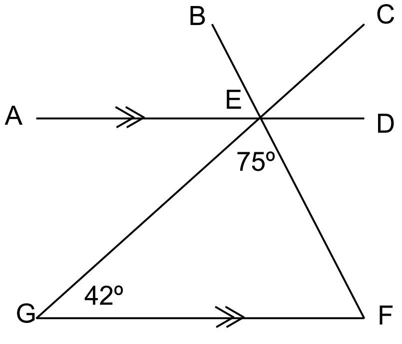


a) an angle that is a corresponding angle to ∠ 2

b) an angle that is an alternate angle to ∠ 2

c) an angle that is an interior angle to ∠ 2

13. What is the measure of each angle below? Explain how you know for each.



a) ∠ EFG
b) ∠ CEF
c) ∠ BEC
d) ∠ AEG
e) ∠ CED
f) ∠ DEG

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ANSWERS

UNIT 1 NUMBER pp. 1–20

Getting Started — Skills You Will Need p. 1

1. a) 2 × 2 × 2 × 2 × 3 × 5
b) 3 × 3 × 5
c) 2 × 2 × 2 × 2 × 3 × 3
d) 31 (prime number)

2. a) 4 hundred thousands + 1 ten thousand + 2 thousands + 1 hundred + 5 tens;
4 × 100,000 + 1 × 10,000 + 2 × 1000 + 1 × 100 + 5 × 10
b) 3 hundred thousands + 6 ten thousands + 5 thousands + 1 hundred + 2 tens + 4 ones;
3 × 100,000 + 6 × 10,000 + 5 × 1000 + 1 × 100 + 2 × 10 + 4 × 1
c) 1 million + 3 thousands + 1 ten;
3 × 1,000,000 + 3 × 1000 + 1 × 10
d) 1 billion + 9 hundred thousands + 1 thousand + 1 hundred + 4 tens + 2 ones;
1 × 1,000,000,000 + 9 × 100,000 + 1 × 1000 + 1 × 100 + 4 × 10 + 2 × 1

3. a) 8,052,000; 8 millions + 5 ten thousands + 2 thousands
b) 40,070,000,637; 4 ten billions + 7 ten millions + 6 hundreds + 3 tens + 7 ones

4. a) 1.07 **b)** 1.98 **c)** 0.21 **d)** 0.0096

5. a) 3740 **b)** 230 **c)** 0.03028
d) 0.6234 **e)** 4000 **f)** 821.13
g) 0.00312 **h)** 0.234

6. a) Ten thousandths **b)** Hundredths
c) Tens **d)** Hundreds

1.1.1 Negative Exponents p. 5

1. a) 0.1407; 1 tenth + 4 hundredths + 7 ten thousandths; 1 × 0.1 + 4 × 0.01 + 7 × 0.0001
b) 306.057008; 3 hundreds + 6 ones + 5 hundredths + 7 thousandths + 8 millionths; 3 × 100 + 6 × 1 + 5 × 0.01 + 7 × 0.001 + 8 × 0.000001
c) 0.00075; 7 ten thousandths + 5 hundred thousandths; 7 × 0.0001 + 5 × 0.00001
d) 5,060,030.047003; 5 millions + 6 ten thousands + 3 tens + 4 hundredths + 7 thousandths + 3 millionths; 5 × 1,000,000 + 6 × 10,000 + 3 × 10 + 4 × 0.01 + 7 × 0.001 + 3 × 0.000001

2. a) 0.011 **b)** 2.58 **c)** 0.000001

3. a) 3×10^{-2}
b) 2×10^{-3}
c) $10^{2} \times 10^{-2}$

4. a) 13 **b)** Yes
c) Greater ; $5 \times 10^{-4} + 6 \times 10^{-7} + 5 \times 10^{-9} + 9 \times 10^{-11} + 4 \times 10^{-13} > 5 \times 10^{-4}$

5. $\frac{1}{2}$

1.1.2 Scientific Notation p. 9

1. a) 2.3196×10^{2} **b)** 4.356×10^{6}
c) 2.1×10^{-4} **d)** 1.367×10^{-1}

2. a) 2×10^{-2} **b)** 2×10^{6} **c)** 1.99×10^{5}

3. 3.8×10^{3} m is a reasonable height

4. a) 4.7×10^{4} **b)** 7.982×10^{8}

5. Yes

6. Dorji; Nu 83,240 more

7. a) 1.496×10^{8}
b) 5.1×10^{8}
c) 7.4×10^{-4}

8. Powers of 10

1.2.1 Perfect Squares p. 11

1. 144

2. a) 0, 1, 4, 5, 6, and 9 **b)** No

3. a) 4, 9, 16, 25
b) The sums are perfect squares.
c) 36

4. a) 4 **b)** 9 **c)** 16 **d)** 36

5. a) No **b)** 14

6. a)		**b)**
1:	1	1 factor
2:	1, 2	2 factors
3:	1, 3	2 factors
4:	1, 2, 4	3 factors
5:	1, 5	2 factors
6:	1, 2, 3, 6	4 factors
7:	1, 7	2 factors
8:	1, 2, 4, 8	4 factors
9:	1, 3, 9	3 factors
10:	1, 2, 5, 10	4 factors
11:	1, 11	2 factors
12:	1, 2, 3, 4, 6, 12	6 factors
13:	1, 13	2 factors
14:	1, 2, 7, 14	4 factors
15:	1, 3, 5, 15	4 factors
16:	1, 2, 4, 8, 16	5 factors

c) The perfect squares each have an odd number of factors and the other numbers each have an even number of factors.

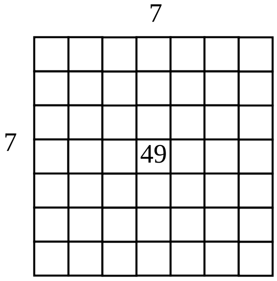
7. a) 20

8. Yes

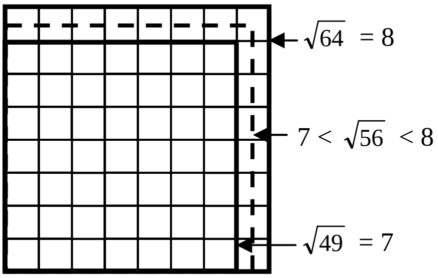
9. *Sample response*:
A perfect square
- has an odd number of factors
- can form a square with whole number length sides
- has factors that can be paired when factored into prime numbers

1.2.3 Interpreting Square Roots p. 15

1. a)



b)



2. a) 9 **b)** 10 **c)** 8 **d)** 11

3. a) 5 **b)** 6 **c)** 10 **d)** 12

4. No

5. *Sample response*: 625, 169, and 8100

6. a) 1 × 72, 2 × 36, 3 × 24, 4 × 18, 6 × 12, and 8 × 9
b) About 8.5
c) Yes

7. a) 1 × 95 and 5 × 19
b) About 12
c) No

8. *Sample response*:
About 5.4 cm; No

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1.2.4 Estimating and Calculating Square Roots p. 18

1. a) 6.2 **b)** 9.8

2. B

3. a) 82 **b)** 820

4. a) $70 \times 70 = 4900$ and $4900 \approx 4823$
b) Less than
c) 69.4 m

5. a) i) $\sqrt{64} = \sqrt{4} \times \sqrt{16}$ and $64 = 4 \times 16$
ii) $\sqrt{225} = \sqrt{9} \times \sqrt{25}$ and $225 = 9 \times 25$
iii) $\sqrt{324} = \sqrt{36} \times \sqrt{9}$ and $324 = 36 \times 9$

b) i) 25 **ii)** 81 **iii)** 576

6. Yes

7. a) 3 or 4 s **b)** 14 s **c)** 45 s

8. **a)** Yes
b) $9216 = 3 \times 3 \times 2 \times 2 \times 16 \times 16$
$= (3 \times 2 \times 16) \times (3 \times 2 \times 16)$, so
$\sqrt{9216} = 3 \times 2 \times 16 = 96$

9. a) Yes
b) No
c) The area of the square must have 7 or 8 digits.

CONNECTIONS: The Square Root Algorithm p. 19

1. a) 27 **b)** 51

UNIT 1 Revision p. 20

1. a) 90,040.057008;
9 ten thousands + 4 tens + 5 hundredths + 7 thousandths + 8 millionths;
$9 \times 10{,}000 + 4 \times 10 + 5 \times 0.01 + 7 \times 0.001 + 8 \times 0.000001$
b) 4.5007;
4 ones + 5 tenths + 7 ten thousandths;
$4 \times 1 + 5 \times 0.1 + 7 \times 0.0001$

2. a) 1.2×10^{-3} **b)** 6×10^{-2}

3. a) 3 **b)** 8

4. a) 2.0030905×10^{4}
b) 2.395×10^{3}
c) 2.0030905×10^{4} in **question 3** is greater

5. a) 5.198723×10^{4} **b)** 1.93567×10^{-1}
c) 7.4×10^{-3} **d)** 1.017×10^{1}

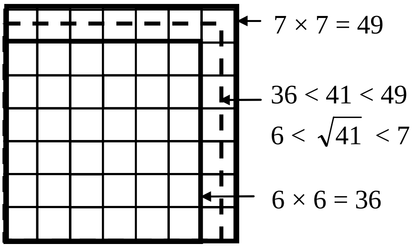
6. A. 1.39×10^{2} cm

7. a) 3.4×10^{7} **b)** 3.1×10^{-2}

8. B. 121

9. a) No
b) 11
c) $308 = 2 \times 2 \times 7 \times 11$

12.



13. a) 12 or 13 **b)** 7 **c)** 9

14. a) 12.4 **b)** 6.9 **c)** 9.1

15. a) $8.5 = \frac{7+10}{2}$
b) Estimate to one decimal place: 8.4
Both estimates are about the same.
c) $7.5 = \frac{5+10}{2}$;
Estimate to one decimal place: 7.1
The estimate of 7.1 suggests that the estimate of 7.5 is too high.

16. 270;
$72{,}900 = 729 \times 100 = 9 \times 81 \times 100$
$= 3 \times 3 \times 9 \times 9 \times 10 \times 10$
$\sqrt{72{,}900} = 3 \times 9 \times 10 = 270$

17. 3 or 4 digits

UNIT 2 PROPORTION AND PERCENT pp. 21–50

Getting Started — Skills You Will Need p. 21

1. 36 : 45 and 4 : 5

2. a) 32% **b)** 20%

3. 10 apples for Nu 100

4. a) 0.6 **b)** 2.5
c) 9.6 **d)** 25.5

5. a) 2000 **b)** 80
c) 50 **d)** 40

2.1.1 Solving Proportions pp. 24–25

1. a) 2 **b)** 9 **c)** 25
d) 30 **e)** 10 **f)** 6

2. 72 L

3. 39 people are sitting.

4. 32 kg

5. 6 times

6.

Concentrate (mL)	Water (mL)
1	3
350	**1050**
475	**1425**
175	525
400	1200

7. a) 20 g
b) *Sample response*:
I have a mass of 45 kg. If I were as strong as an ant, I could carry 2250 kg.

8. a) 150 seedlings
b) 20 min ($\frac{1}{3}$ h)
c) 12 min ($\frac{1}{5}$ h)

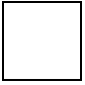
9. 343 students

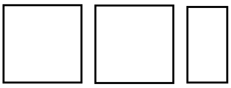
10. a) No **b)** 25 cm

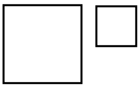
11. *Sample response*:
If 4 apples cost Nu 80, how much do 7 apples cost? (Answer: Nu 140)

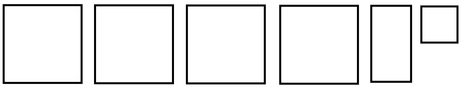
2.2.1 Percents Greater Than 100% p. 30

1. *Sample response*:


= 100%

a) 250% =


b) 125% =


c) 475% =


2. a) i) 8.4 **ii)** 84 **iii)** 840
b) 932.4 (840 + 84 + 8.4)

3. a) 286 **b)** 1375 **c)** 1329.8 **d)** 300

4 a) 500 **b)** 350 **c)** 150

5. 17,000 tourists

6. *Sample response*:
About 630,000 U.S. dollars

7. 1500 on Day 2 and 3375 on Day 3

8. a) 683,400
b) 697,068
c) 711,009

9. a) Nu 2500
b) Less than the original price.

10. No

2.2.2 Solving Percent Problems   pp. 33–34

1. a) 26.88 **b)** 307.2 **c)** 189

2. a) 30 **b)** 320 **c)** 160

3. a) 60% **b)** 115% **c)** 30%

4. a) 66 girls **b)** 54 boys **c)** 45%

5. a) 16 **b)** 24 **c)** 50%

6. a) 12,750 **b)** 30,000

7. 400 children

8. 20%

9. *Sample responses*:
a) About 167 cm (166.6...)
b) About 163 cm (163.3...)

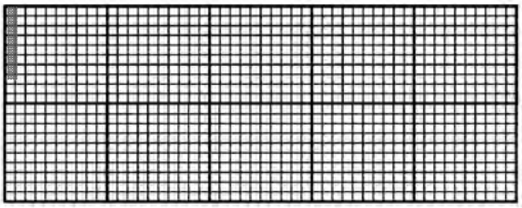
10. a) About 160 cm
b) *Sample response*: About 157 cm (156.8)

11. *Sample response*:
I was 153 cm tall when I was 11 years old. Because I am a girl, that is 90% of my adult height. I will be about 153 ÷ 9 × 10 = 170 cm tall.

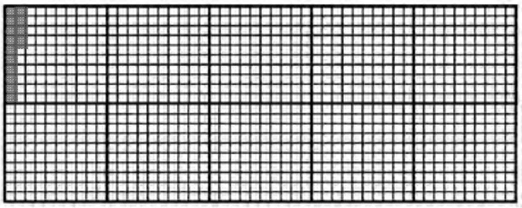
12. *Sample response*:
80% of the boys in my class play football.
There are 20 boys in my class.
How many of them play football? (16 boys)

2.2.3 Fractional Percents   p. 37

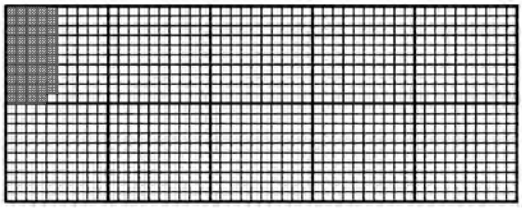
1. a)



b)

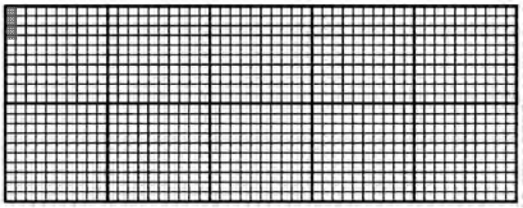


c)



2. 7.6%

4. b)



c) 1.08

5. 2.5 mL

6. a) 9.3 g **b)** 0.3 g

7. All the ways are correct.

8. b) 0.32%

9. Yes

11. No; it depends on the size of the number.

2.2.4 Solving Percent Problems Using Familiar Percents   pp. 39–40

1. a) 88 **b)** 648 **c)** 344 **d)** 1058

2. a) 400 m **b)** 80 g **c)** Nu 175
d) 30 kg **e)** 60 L

3. a) Nu 25 **b)** Nu 75 **c)** Nu 35
d) Nu 14 **e)** Nu 2700

4. a) About 637,500 **b)** About 1560 males

2.2.4 Solving Percent Problems Using Familiar Percents [Cont'd] pp. 39–40

5. 14 games

6. a) 168 g **b)** About 0.27 kg (270 g)

7. 25 questions

8. 35 students

9. 400 mL

10. a) 5,000,000
b) 30,000,000
c) 33,000,000

11. Archery, 180; Soccer, 120; Track & Field, 60; Other, 40.

2.3.1 Markup and Discount Consumer Problems p. 44

1. a) Discount amount: Nu 21; Sale price: Nu 119
b) Discount amount: Nu 18; Sale price: Nu 54
c) Discount amount: Nu 52; Sale price: Nu 598
d) Discount amount: Nu 5400; Sale price: Nu 12,600

2. a) Markup amount: Nu 6; Regular selling price: Nu 36
b) Markup amount: Nu 70; Regular selling price: Nu 350
c) Markup amount: Nu 75; Regular selling price: Nu 825
d) Markup amount: Nu 750; Regular selling price: Nu 3250

3. Nu 84 per kg

4. 10% **5.** 150%

6. a) Nu 240 **b)** Nu 192

7. Nu 1500

8. Shop 2

9. Yes

10. *Sample response*:
The cost price of an item is Nu 300. The markup is 25%. What is the selling price? (125% of Nu 300 = Nu 375)

2.3.2 Simple Interest and Commission pp. 47–48

1. a) 0.07 **b)** 0.045 **c)** 0.0625

2. a) Nu 1558 **b)** Nu 8100 **c)** Nu 2550

3. a) Nu 648 **b)** Nu 1820 **c)** Nu 2340

4. Nu 640

5. a) Interest: Nu 450; Total amount paid: Nu 6450
b) Interest: Nu 250; Total amount paid: Nu 12,750
c) Interest: Nu 2880; Total amount paid: Nu 10,880

6. Nu 3250

7. They both earn the same amount of interest.

8. a) Nu 150 **b)** Nu 300
c)

Number of years	Interest paid (Nu)
3	450
4	600
5	750
6	900
7	1050
8	1200
9	1350
10	1500

9. a) Nu 7200
b) Nu 36,000
c) Nu 1500

10. Dawa earned more.

CONNECTIONS: Currency Conversion p. 49

1. a) 8 baht **b)** 80 baht **c)** 800 baht

2. a) Nu 12.5 **b)** Nu 125 **c)** Nu 1250

3. Nu 1 = 0.025 Canadian dollars

4. *Sample response*:
1 US dollar = Nu 40.95
Nu 1 ≈ 0.024 US dollars
1 £ = Nu 82.55
Nu 1 = 0.121 £

UNIT 2 Revision p. 50

1. a) 5 **b)** 27 **c)** 28

2. 200 girls

3. 15 min

4. 350%

5. a) 1300 **b)** 1428 **c)** 1210

6. 50,400

7. a) 105 **b)** 180 **c)** 640
d) Nu 240 **e)** 20%

8. 40

9. 615 g

10. a) Nu 540 **b)** Nu 1260

11. 37.5%

12. No. It's less expensive at the end of June.

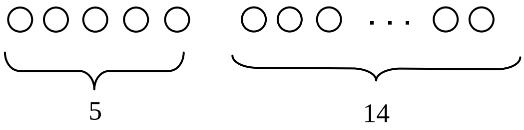
13. a) Nu 630 **b)** Nu 4130

14. Nu 4200 **15.** 8%

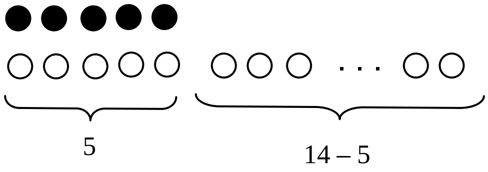
UNIT 3 INTEGERS pp. 51–72

Getting Started — Skills You Will Need p. 52

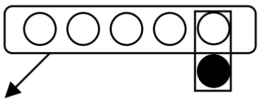
1. a) +19, *Sample response*:



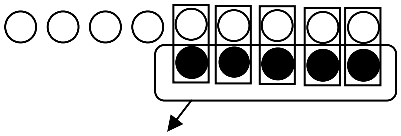
b) +9, *Sample response*:



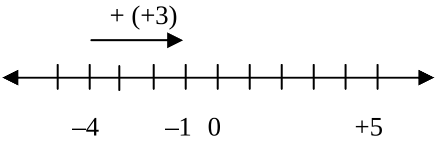
c) –1, *Sample response*:



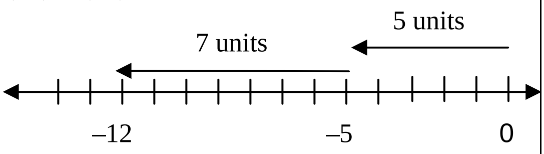
d) +9, *Sample response*:



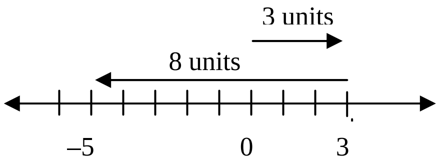
2. *Sample response*:
–4 + (+3) = –1



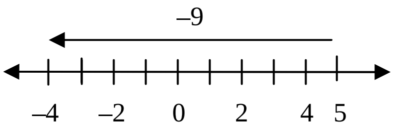
3. a) –12; *Sample response*:
(–5) + (–7) = –12



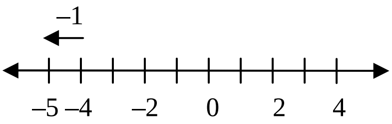
b) –9; *Sample response*:
(+3) + (–8) = –5



c) –9; *Sample response*:
(–4) – (+5) = –9



d) –1; *Sample response*:
(–5) – (–4) = –1



4. a) –14 **b)** 0 **c)** –35
d) +50 **e)** –70

3.1.1 Multiplying Integers Using Counters and Patterns p. 55

1. a) 6 × (–4)
b) 2 × (–4) or 4 × (–2)

2. a) –6; *Sample response*:
(–2) × 3 = 3 × (–2)
(● ●) + (● ●) + (● ●)

b) –25; *Sample response*:
5 × (–5)
(● ● ● ● ●) +
(● ● ● ● ●) +
(● ● ● ● ●) +
(● ● ● ● ●) +
(● ● ● ● ●)

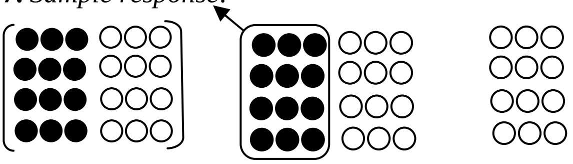
3. a) –4 **b)** –6 **c)** –6

4. No

5. a) 8 **b)** 0 **c)** 12 **d)** 20

6. a) 3 × (–5) **b)** –15°C

7. *Sample response*:

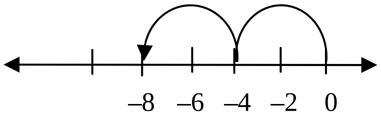


8. No

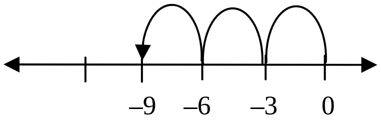
3.1.2 Multiplying Integers Using a Number Line p. 57

1. 5 × (–2) = –10

2. a) –8; *Sample response*:



b) –9; *Sample response*:



3. a) *Sample response*: 55 + (10 × 13)
b) Nu 185
c) –75, which means he will owe Nu 75.

4. 14 ways

5. a) Positive **b)** Negative
c) Positive **d)** Positive

6. –12°C

7. The product is the opposite of the original integer; *Sample response*: (–3) × (–1) = +3

10. Yes

11. a) Positive
b) Negative

3.1.4 Renaming Factors to Multiply Mentally p. 60

1. a) –450 **b)** 960 **c)** 140
d) –140 **e)** –480

2. a) 300 **b)** 240 **c)** 60
d) –80 **e)** 60

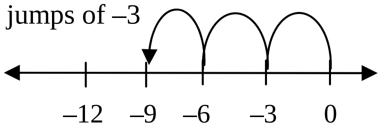
3. a) –115 **b)** 192 **c)** –372
d) 108 **e)** –572

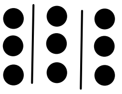
4. *Any two sample responses*:
a) –33 **b)** 80 **c)** 300
d) 300 **e)** –3100

3.2.1 Dividing Integers Using Models and Patterns p. 65

1. *Sample responses*:
a) (–12) ÷ (–3) = 4 **b)** (–6) ÷ (–2) = 3

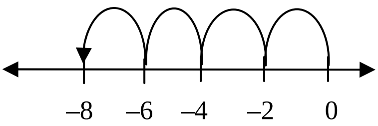
2. *Sample responses*:
a) 3 jumps of –3

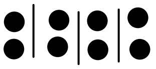




3 groups of
3 black counters

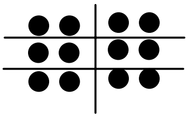
b) 4 jumps of –2





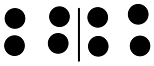
4 groups of
2 black counters

3. a) –2; *Sample response*:



6 groups of 2 black counters

b) 2; *Sample response*:



2 groups of 4 black counters

4. a) –6 **b)** 8 **c)** –2
d) 2 **e)** –4 **f)** –8

6. 9 h

7. *Sample response*:
(–21) ÷ (–7) = 3
(–14) ÷ (–7) = 2
(–7) ÷ (–7) = 1
0 ÷ (–7) = 0
7 ÷ (–7) = –1
14 ÷ (–7) = –2
21 ÷ (–7) = –3

8. When you divide a negative by a negative, or a positive by a positive, the quotient is positive. When the signs are different, the quotient is negative.

3.2.2 Relating Division of Integers to Multiplication pp. 67–68

1. a) –2 **b)** –17 **c)** –5

2. a) 4 × ■ = –64 **b)** (–7) × ■ = –84

3. a) –16 **b)** 12

4. 16 h

6. a) (–60) ÷ (–3) = 20 **b)** (–60) ÷ 4 = –15

7. *Sample responses*:
a) –2, –3 **b)** 2, 6 **c)** –2, –9 **d)** 2, 5

8. a) The first integer is 1 and the second integer is –1.

CONNECTIONS: Mean Temperatures p. 68

1. a) 11°C **b)** –3°C **2. a)** 12°C **b)** 24 weeks

3.2.3 Order of Operations with Integers pp. 70–71

1. a) 0 **b)** –65 **c)** 10
d) –2 **e)** 8 **f)** 12

2. First divide by –3, then subtract –9, and then multiply by 3; [0 ÷ (–3) – (–9)] × 3 = 27

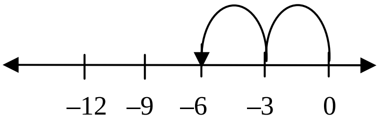
3. [(40 × 6) – 3] × (4 – 5) = –237

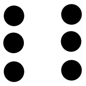
4. a) 36 **–** [4 **–** 1] **×** 2 = 30
b) (–12) **+** 4 **×** (–3) = –24

5. Different people may get different answers to the same question if the rules are not followed.

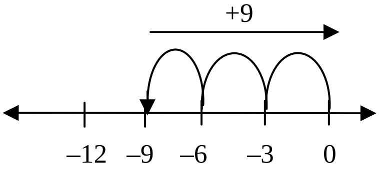
UNIT 3 Revision pp. 72

1. a) (–3) × 2 = –6





b) *Sample response*: (–3) × (–3) = +9



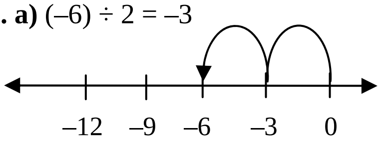
2. a) 30 **b)** –48 **c)** –28

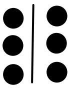
3. No

4. 14 ways

6. a) 2300 **b)** –2860 **c)** 670

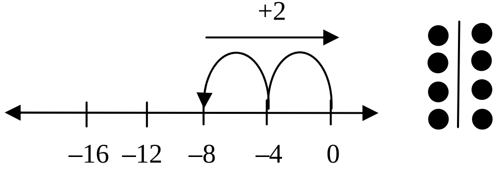
7. a) (–6) ÷ 2 = –3





UNIT 3 Revision [Continued] p. 72

b) (–8) ÷ (–4) = 2



8. a) (–92) ÷ 4 → ♦ × (4) = –92; ♦ = –23
b) (–91) ÷ (–7) → ♦ × (–7) = –91; ♦ = 13

9. 4 h **10.** 6°C

12. a) Positive **b)** Negative
c) Positive **d)** Negative

13. a) 112 **b)** –29 **c)** 153
d) 2 **e)** –1

14. 40 × (6 – 3) × 4 – 5 = 475

UNIT 4 FRACTIONS AND RATIONAL NUMBERS pp. 73–116

Getting Started — Skills You Will Need p. 74

1. *Sample responses*:
a) $\frac{4}{6}=\frac{6}{9}=\frac{8}{12}$ **b)** $\frac{3}{4}=\frac{6}{8}=\frac{9}{12}$
c) $\frac{10}{16}=\frac{15}{24}=\frac{20}{32}$ **d)** $\frac{3}{5}=\frac{6}{10}=\frac{15}{25}$

2. a) Yes **b)** No

3. a) $3\frac{3}{4}$ **b)** $2\frac{1}{2}$ **c)** $3\frac{1}{9}$

4. a) $\frac{1}{12}$ **b)** $\frac{13}{24}$ **c)** $4\frac{1}{8}$

d) $1\frac{1}{5}$ **e)** $2\frac{11}{12}$ **f)** $1\frac{7}{12}$

5. $2\frac{1}{4}$ cups

6. a) +10 **b)** +300 **c)** –79 **d)** –15

7. a) –268 **b)** +136 **c)** –110

8. a) 96 **b)** –126 **c)** –90

10. a) 180 **b)** –53 **c)** 3 **d)** –1

4.1.2 Adding and Subtracting Fractions Mentally p. 77

1. a) $\frac{7}{9}$ **b)** $\frac{7}{8}$ **c)** $8\frac{1}{2}$

2. a) $\frac{1}{6}$ **b)** $\frac{3}{8}$ **c)** $2\frac{4}{9}$

4. *Sample responses*:
a) and b)
i) 7; Higher **ii)** 6; Lower
iii) $8\frac{1}{2}$; Not sure **iv)** 5; Higher
v) $2\frac{1}{2}$; Higher **vi)** $\frac{1}{4}$; Lower

5. Yes **6.** No **7.** Yes

8. $\frac{1}{6}$ **9.** $18\frac{1}{2}$

10. *Sample responses*:
a) Wondering whether a 5-cup bowl will be big enough to hold $1\frac{1}{2}$ cups flour, $2\frac{1}{4}$ sugar, and $1\frac{2}{3}$ cups butter.
b) $1\frac{1}{4}+2\frac{3}{4}$

4.1.2 Adding and Subtracting Fractions Symbolically p. 81

1. a) $\frac{17}{24}$ **b)** $1\frac{5}{18}$ **c)** $10\frac{1}{20}$ **d)** $7\frac{3}{10}$

2. a) $\frac{5}{24}$ **b)** $\frac{11}{30}$ **c)** $1\frac{17}{24}$ **d)** $4\frac{5}{8}$

3. a) Day 1: $1\frac{1}{12}$ h, Day 2: $1\frac{7}{12}$ h **b)** $\frac{1}{2}$ h

4. $11\frac{3}{4}$ laps

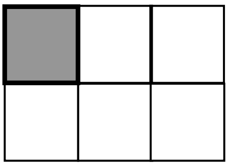
5. a) Yes
b) No

7.

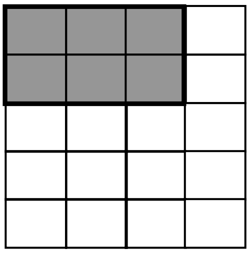
$1\frac{2}{3}$	$7\frac{1}{2}$	$3\frac{1}{3}$
$5\frac{5}{6}$	$4\frac{1}{6}$	$2\frac{1}{2}$
5	$\frac{5}{6}$	$6\frac{2}{3}$

4.2.2 Multiplying Fractions p. 86

1. a)



b)



2. a) $\frac{4}{15}$ **b)** $\frac{5}{16}$ **c)** $\frac{1}{2}$

d) $\frac{9}{35}$ **e)** $\frac{1}{5}$ **f)** $\frac{8}{15}$

3. a) About 12; $12\frac{4}{5}$ **b)** About 8; $8\frac{1}{4}$

c) About 20; $21\frac{7}{8}$ **d)** About 8; $8\frac{1}{3}$

4. $\frac{1}{5}$

5. *Sample response*:

$$\frac{2}{5}\times\frac{3}{8}=\frac{1}{40}\times\frac{6}{1}=\frac{3}{4}\times\frac{2}{10}=\frac{6}{40}$$

6. $\frac{3}{28}$

7. $\frac{3}{4}$ day

8. a) 0.12

b) $\frac{3}{10}\times\frac{4}{10}=\frac{12}{100}$

c) They are equal.

9. a) $\frac{1}{2}\times\frac{2}{3}=\frac{1}{3}$

$\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=\frac{1}{4}$

$\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}=\frac{1}{5}$

The numerator of the first fraction is the numerator of each product. The denominator of the last fraction is the denominator of each product.

b) $\frac{1}{100}$

10. a) i) 5000 g **ii)** 7500 g **iii)** 9170 g
b) $450 U.S.

11. The product is less than each fraction.

CONNECTIONS: The Sierpinski Triangle p. 87

1. a) $\frac{3}{4}$

b) i) $\frac{3}{4}$ **ii)** $\frac{9}{16}$

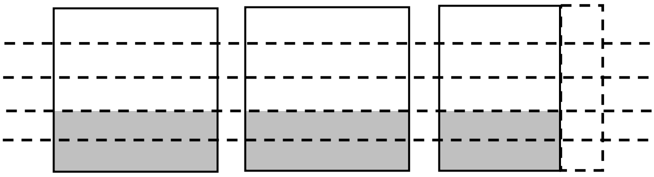
c) $\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$

2. a) $\frac{27}{64}$

b) I counted the total number of small white triangles (64) and the number that were not coloured (27). $\frac{27}{64}$ of the large triangle was not coloured.

4.2.3 Multiplying Mixed Numbers p. 91

1. *Sample response*:



2. a) $\frac{7}{12}$ **b)** 72 **c)** $2\frac{2}{7}$ **d)** 8

e) $1\frac{1}{3}$ **f)** 355 **g)** 16 **h)** $4\frac{7}{32}$

3. a) $\frac{7}{8}$ cups **b)** $4\frac{3}{8}$ cups

4. 33 eggs

5. 5 h

6. *Sample response*: About 8800 m

7. *Sample responses*:
a) About 333
b) About $20\frac{1}{2}$

8. a) $\frac{81}{100}$
b) $0.3 \times 2.7 = 0.81$
c) They are the same.

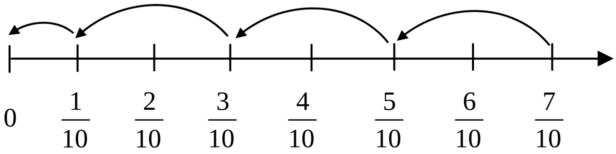
9. *Sample response*: $1\frac{1}{5}$

10. *Sample response*:
To find the area of a wall or a floor with dimensions $3\frac{1}{2}$ m × $2\frac{1}{3}$ m.

4.2.4 Dividing Fractions with a Common Denominator p. 94

1. a) $\frac{2}{4} \div \frac{1}{8} = 4$ **b)** $\frac{5}{6} \div \frac{1}{3} = 2\frac{1}{2}$

2. *Sample response*:



3. $1\frac{1}{5}$

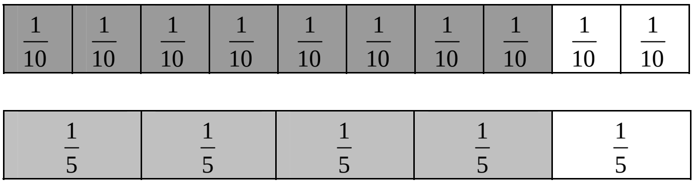
4. a) 3 **b)** 2
c) $1\frac{3}{4}$ **d)** 4
e) $2\frac{7}{10}$ **f)** $10\frac{1}{2}$

5. $1\frac{1}{2}$; $\frac{1}{2} \div \frac{1}{3} = 1\frac{1}{2}$

4.2.5 Dividing Fractions in Other Ways p. 98

1. a) $\frac{3}{4} \div 3 = \frac{3}{12}$ or $\frac{1}{4}$ **b)** $3 \div \frac{3}{8} = 8$

2. a) 4
b) *Sample response*:



3. a) 8 **b)** $\frac{1}{8}$ **c)** $\frac{3}{4}$
d) $\frac{4}{5}$ **e)** $\frac{9}{10}$ **f)** $2\frac{1}{10}$

4. 3 subjects

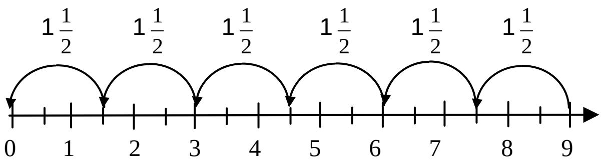
5. $\frac{7}{40}$ m

6. 4 parts

4.2.6 Dividing Mixed Numbers p. 101

1. $4 \div 1\frac{1}{3} = 3$

2.



3. a) 2 **b)** $1\frac{5}{6}$ **c)** $2\frac{1}{18}$ **d)** $1\frac{1}{3}$

4. a) A: $\frac{3}{2} = 1\frac{1}{2}$ and $\frac{2}{3}$ **B**: $\frac{5}{3} = 1\frac{2}{3}$ and $\frac{3}{5}$

b) The quotients are reciprocals.

c) *Sample response*:

$2\frac{1}{4} \div 4\frac{1}{2} = \frac{9}{4} \div \frac{9}{2} = \frac{9}{4} \times \frac{2}{9} = \frac{2}{4}$

$4\frac{1}{2} \div 2\frac{1}{4} = \frac{9}{2} \div \frac{9}{4} = \frac{9}{2} \times \frac{4}{9} = \frac{4}{2}$

5. a) 7 crops
b) No

6. 8 instruments

7. $1\frac{1}{4}$ h

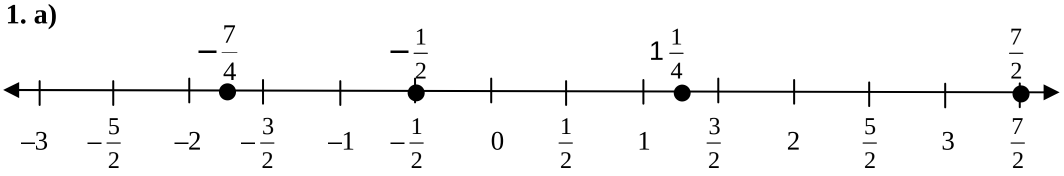
9. a) i) *Sample response*: about 9
ii) $9\frac{1}{9}$

b) i) *Sample response*: about $5\frac{1}{2}$

ii) $5\frac{2}{5}$

4.3.1 Introducing Rational Numbers p. 105

1. a)



b) $-\frac{7}{4}$

3. a) > **b)** < **d)** < **e)** < **f)** <

4. a) $-6\frac{1}{2}$, –3, $-\frac{9}{4}$, $-\frac{4}{5}$

b) $-\frac{5}{4}$, $-\frac{1}{2}$, 0, $\frac{7}{12}$, $\frac{11}{12}$

c) –5, –3, $\frac{15}{8}$, $\frac{11}{4}$, $\frac{15}{4}$

d) $-5\frac{2}{5}$, –5.2, 0, 4.7, $4\frac{3}{4}$

5. Paro **6.** Stock B

7. a) $-\frac{11}{2}$
b) “It is less than –3” is not needed.

8. *Sample response*: $-\frac{11}{4}$, $-\frac{5}{2}$, $-2\frac{1}{3}$, $-2\frac{1}{6}$

4.3.2 Operations with Rational Numbers p. 109

1. a) i) $-\frac{17}{24}$ **ii)** $-\frac{17}{24}$ **iii)** $-\frac{17}{24}$ **iv)** $-\frac{17}{24}$

b) Same: Each had the same answer. Different: Two involved adding two rational numbers but in different orders. The other two involved subtracting a rational number from a rational number but in different orders.

4.3.2 Operations with Rational Numbers [Continued] p. 109

2. a) 36.18 **b)** –134.55 **c)** –22.1 **d)** 0.95

3. a) $-\frac{3}{4}$ **b)** 83.13 **c)** $-\frac{7}{8}$ **d)** 9.5

4. a) i) Negative **ii)** Positive
iii) Negative **iv)** Positive

b) *Sample responses*:
i) About –900 **ii)** About 6
iii) About –9 **iv)** About 1

5. a) i) 4.61 **ii)** 36.80
iii) 198.75 **iv)** 6.50
b) +5.36
c) –10.70

4.3.3 Order of Operations p. 113

1. a) $\frac{5}{7}$ **b)** $\frac{33}{40}$ **c)** $\frac{13}{24}$
d) 5.9 **e)** –27.9 **f)** –11.5

2. a) **(**3.6 + 6**)** ÷ **(**3.5 – 1.1**)** + 3 = 7
b) $\frac{5}{8} \div \mathbf{(}\frac{1}{2} + \frac{1}{3}\mathbf{)} \times \frac{3}{5} = \frac{9}{20}$

3. a) Multiply by $\frac{3}{4}$, add $\frac{2}{3}$, then divide by $\frac{1}{2}$. ($2\frac{5}{6}$)
b) Divide by $\frac{1}{2}$, add $\frac{2}{3}$, then multiply by $\frac{3}{4}$. (2)

4. 11.2 – (–5.4) ÷ 2.7 + (–9) = 4.2

5. a) Positive
b) Negative

6. *Sample responses*:
a) About 54
b) About –30

7. a) $\frac{1}{10}$
b) $-\frac{1}{10}$

8. *Sample response*:
$-2.5 \times [4 + (-5.3)] \div \frac{3}{4}$;

UNIT 4 Revision pp. 115–116

1. a) i) $7\frac{5}{8}$ **ii)** $10\frac{7}{9}$ **iii)** $5\frac{1}{4}$ **iv)** $5\frac{1}{6}$

2. $9\frac{1}{2}$

3. a) $15\frac{7}{12}$ **b)** $2\frac{1}{12}$

4. a) $\frac{21}{32}$ **b)** $\frac{7}{18}$ **c)** $\frac{1}{3}$ **d)** $\frac{3}{5}$

5. $\frac{3}{28}$

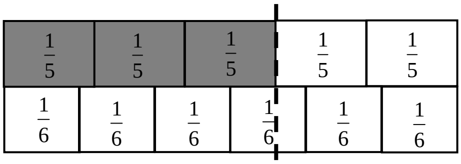
6. a) $5\frac{1}{7}$ **b)** $8\frac{2}{3}$

7. a) $\frac{5}{8}$ cups **b)** $4\frac{3}{8}$ cups

8. a) $\frac{2}{3}$ **b)** $\frac{3}{4}$ **c)** $\frac{4}{5}$ **d)** 16

9. 6 times

10. a) About $3\frac{1}{2}$ times
b) *Sample response*:



11. a) $1\frac{13}{14}$ **b)** $1\frac{5}{16}$

12. a) $1\frac{5}{9}$ **b)** $1\frac{4}{5}$ **c)** $\frac{5}{6}$

13. a) 5 **b)** Yes; $\frac{5}{8}$ m

14. a) Aluminum Corp.
b) Associated Int. Hotel
c) –0.50, –0.07, –0.03, +0.06, +1.18

15. a) $-11\frac{1}{6}$ **b)** 15.13

c) –15.95 **d)** $1\frac{1}{2}$

16. a) $-\frac{1}{2}$ **b)** –6

17. a) $(1\frac{3}{4} - \frac{5}{8}) \times \frac{2}{3} + \frac{4}{9} \div 1\frac{1}{3}$

b) $1\frac{3}{4} - \frac{5}{8} \times (\frac{2}{3} + \frac{4}{9} \div 1\frac{1}{3})$

UNIT 5 MEASUREMENT pp. 117–150

Getting Started — Skills You Will Need p. 121

1. a) 1200 **b)** 45

2. a) $A = l \times w$
b) *Sample response*: $A = b \times h \div 2$

3. a) 24 cm^2 **b)** $12\frac{1}{2}$ cm^2

4. a) *Sample response*:
The outside outline of the circle.
b) π : 1 or about $\frac{22}{7}$

c **c)** $C = 2 \times \pi \times r$ or $C = 2r\pi$

5. a) 22 cm **b)** 88 mm

6. a) 63 cm^3 **b)** 48 m^3

c

7. a) 1 mL **b) i)** 1000 mL **ii)** 1 L

8. 2 cm

9. No

5.1.1 The Pythagorean Theorem p. 121

1. a) AC **b)** EF **c)** GH

2. a) Yes **b)** No **c)** Yes

3. a) Yes
b) *Sample response*:
Multiply the values of Row 1 by 2 to get Row 2, multiply the values of Row 1 by 3 to get Row 3, multiply the values of Row 1 by 4 to get Row 4, and so on.
c) 18, 24, 30; 21, 28, 35
d) 2 triangles: 18, 24, 30 and 30, 40, 50

5.1.1 Applying the Pythagorean Theorem p. 124

1. a) 15 cm **b)** 26 m

2. a) 8.9 km **b)** 11.3 cm

3. 10.2 m

4. 9 km

5. About 2500 m (or 2482 m)

6. 1.3 m

5.1.1 Applying the Pythagorean Theorem [Continued] p. 124

7. a)

	a^2	b^2	c^2
i)	36	49	64
ii)	64	64	81
iii)	225	400	576

b) $a^2 + b^2 > c^2$

c)

	a^2	b^2	c^2
i)	100	144	400
ii)	36	36	100
iii)	225	400	676

d) $a^2 + b^2 < c^2$

8. No

5.2.1 Area and Perimeter Relationships pp. 127–128

1. a) P = 2.6 km or 2600 m
A = 0.3 km^2, or 30 ha, or 300,000 m^2
b) P = 18 cm, A = 12 cm^2
c) P = 16 m, A = 12.5 m^2
d) P = 80 cm, A = 300 cm^2

2. *Sample responses*:
a) Metres and squares metres or hectares
b) Millimetres and square millimetres (or centimetres and square centimetres)
c) Centimetres and square centimetres
d) Kilometres and square kilometres or hectares

3. a) 1 × 11

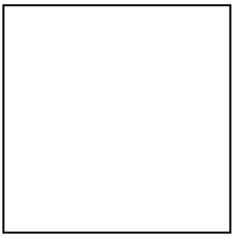
2 × 10

3 × 9

4 × 8

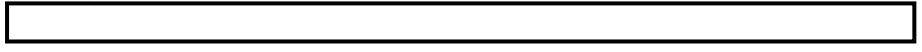
5 × 7

6 × 6

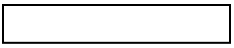


b) The smallest area (11 cm^2) is long and thin (1 cm by 11 cm) and the greatest area (36 cm^2) is a square (6 cm by 6 cm).

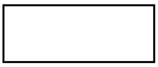
4. a) 1 × 24



2 × 12



3 × 8

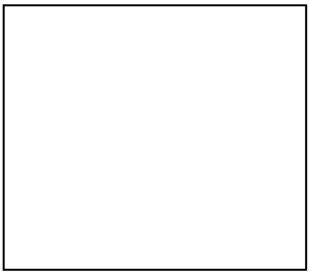


4 × 6

b) The greatest perimeter (50 cm) is long and thin (1 cm by 24 cm) and the least perimeter (20 cm) is closest to the shape of a square (4 cm by 6 cm).

5. *Sample responses*:
a) 0.5 cm by 2 cm **b)** 5 cm

6. 4 cm by 3.5 cm

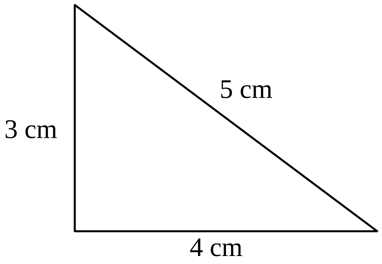


7. a) 45,000 km^2
b) It is close to the real area of 47,000 km^2
c) 900 km

8. a) The perimeter is twice the sum of length and the width, so it will be a multiple of 2.
b) The area will be an even number when one of the sides is an even number

9. a) 6.25 cm^2 **b)** 2 km

10. A 3-4-5 right triangle:

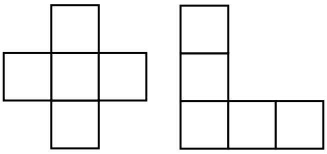


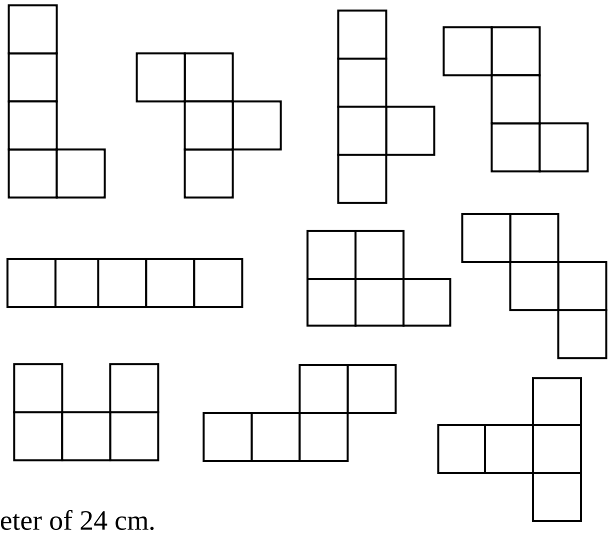
11. a) 6 cm by 8 cm
b) *Sample response*:
A 1-by-48 rectangle is long and thin and has a much greater perimeter than the rectangle in **part a)**.

12. No

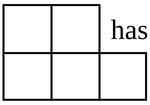
CONNECTIONS: Pentominos p. 128

1. There are 12 pentominos. The two from the student book, on the left, plus these 10:





2. 11 of the 12 pentominos have a perimeter of 24 cm.

Only  has a perimeter of 20 cm.

5.2.2 Scale Drawings p. 133

1. a) 1 cm represents 16 cm
b) 1 cm represents 12.5 cm
c) 1 cm represents 2 m
d) 1 cm represents 6 m
e) 1 cm represents 3 m
f) 1 cm represents 0.3 mm

2. a) The key is for a real object that is smaller than the scale drawing.
b) *Sample response*: To show details of very small things, like insects.

3. *Sample responses*:
a) Height: 5.5 m; width: 6.6 m

b)

1 cm represents 2 m

4. 4.5 cm by 6 cm

5. a) 160 km **b)** 2.5 cm

6. a) 1.8 m **b)** 3 m **c)** 2 cm

5.2.2 Scale Drawings [Continued] p. 133

7. a) About 4000 km
b) About 13 times

8. 22 cm represents 12 m or
1 cm represents about 0.6 m.

5.2.4 The Formula for the Area of a Circle pp. 137–138

1. a) 154 cm^2 **b)** 39 m^2
c) 50 km^2 **d)** 79 mm^2

2. a) 3 m^2 **b)** 11 m^2 **c)** 89 cm^2
d) 16 m^2 **e)** 19 cm^2

3. Centre: 314 cm^2; Inner band: 2512 cm^2; Outer band: 5024 cm^2.

4. Centre: 900 cm^2;
Each other section: 1200 cm^2.

5. a) 5 m **b)** 314 m^2

6. 2π cm

CONNECTIONS: The History of Pi p. 138

1. a) 0.0013
b) $\frac{22}{7}$ (3.142...) is closer to 3.1415... than 3.14 is.

2. a) 3.3397 (this may vary with different calculation methods)
b) No

5.2.5 Applying Area Formulas p. 141

1. 84 tins

2. a) 19.6 cm^2 **b)** 204 cm^2 **c)** 30.9 cm^2

3. B is greater

4. 8 cm by 32 cm

5. 35.7 m^2

6. A, the square; *Sample response*:
For both shapes, the area is the product of the height or width and the base or length. The square is bigger because the slanted sides of the rhombus make its height less than the square's height.

7. 2 cm **8.** 8 bags

9. about 30 tonnes

10. about 12.4 cm

11. No

12. *Sample response*:
A circle of diameter 4 cm is cut out of a triangle with a base of 10 cm and a height of 8 cm. What is the area of the resulting shape? (40 – 12.56 = 27.4 cm^2)

13. The area is multiplied by 9

CONNECTIONS: Tangrams p. 142

1. 25 cm^2 **2.** 12.5 cm^2 **3.** 6.25 cm^2 **4.** 12.5 cm^2 **5.** 12.5 cm^2

5.3.1 Volume of a Rectangular Prism p. 146

1. a) 12 cm^3; 12 mL; 12 g
b) 30 dm^3; 30 L; 30 kg
c) 25 cm^3; 25 mL; 25 g
d) 1000 dm^3 or 1 m^3; 1000 L; 1000 kg or 1 t

2.

V (cm^3)	**l (cm)**	**w (cm)**	**h (cm)**	**Capacity (mL)**
48	4	3	**4**	**48**
105	10	**3.5**	3	**105**
720	**12**	12	5	720

3. A has the greatest volume;
A. 729 cm^3 **B.** 720 cm^3 **C.** 704 cm^3

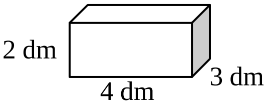
4. a) 512 cm^3 **b)** 2 cm on each edge

5. a) 4 cm **b)** 5 cm **c)** 100 cm

6. a) 1,000,000 cm^3 **b)** 1000 L
c) 1,000,000 g; 1,000 kg; 1

7. B; A is 1000 cm^3, B is 512 cm^3, C is 700 cm^3

8. *Sample response*:



9. 20 cm

10. a) 12 dice
b) 44 cm^3

11. No

12. *Sample response*:
A cube has a volume of 12 m^3. What are its dimensions to the nearest tenth of a metre? (2.3 m)

13. a) It is multiplied by 9.
b) It is multiplied by 27.

5.3.2 Surface Area of a Rectangular Prism p. 149

1. a) 32 cm^2 **b)** 26 cm^2
c) 59 cm^2 **d)** 600 dm^2

2.

***SA* (cm^2)**	***l* (cm)**	***w* (cm)**	***h* (cm)**
33	6	1.5	1
10	2	**1**	1
76	**5**	4	2
150	5	5	**5**

3. a) 216 cm^2 **b)** 216 cm^3
c) Numbers are the same; It's usually not true.

4. a) The area is multiplied by 4
b) The total surface area is multiplied by 9.

5. a) 96 cm^2 **b)** 96 cm^2
c) The cube has the greater volume;
A_{cube} = 64 cm^3 is greater than A_{prism} = 36 cm^3.

6. a) 1 by 1 by 12, 1 by 3 by 4, 1 by 2 by 6, 2 by 2 by 3
b) 1 by 1 by 12; It is flat, long, and wide.
c) 2 by 2 by 3; It is closest to cube-shaped.

7. 38,100 cm^2 or 3.81 m^2

8. a) The total volume does not change.

UNIT 5 Revision p. 150

1. a) Yes **b)** No

2. 11.4 cm **3.** 14.1 cm

4. 3 rectangles;
1 by 20: P = 42 cm
2 by 10: P = 24 cm
4 by 5: P = 18 cm

5. 5 rectangles;
1 by 9: A = 9 cm^2
2 by 8: A = 16 cm^2
3 by 7: A = 21 cm^2
4 by 6: A = 24 cm^2
5 by 5: A = 25 cm^2

6. 125 m^2

7. 1 : 13,000,000

8. 1500 km

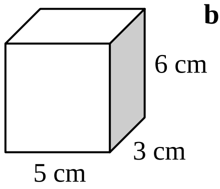
9. a) 38.5 cm^2 **b)** 38.5 km^2

10. 49 cm^2 **11.** 84 cm^2

12. 343 m^3 **13.** 7 cm

14. a) 19.3 L **b)** 17.9 L **c)** 17.9 kg

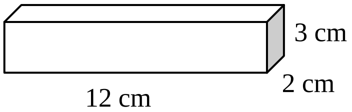
15. a)



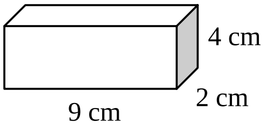
b) 126 cm^2

UNIT 5 Revision [Continued] p. 150

16. *Sample response*:



SA = 132 cm^2



SA = 124 cm^2

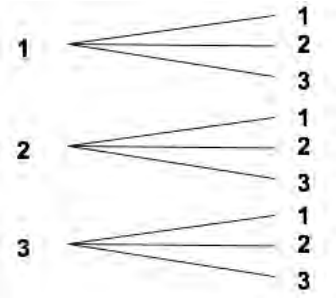
UNIT 6 PROBABILITY AND DATA pp. 151–182

Getting Started — Skills You Will Need p. 151

1. a) Tea; *Sample response*:
Fruit juice, but there might have been another less popular drink in “Other”.
b) Tea: 40%, Other: 25%, Water: 25%, Juice: 10%

2. a) 161.27
b) 161.7
c) 162.7
d) 47.9

3. a)



b) $\frac{4}{9}$

6.1.1 Complementary Events pp. 154–155

1. a) $\frac{1}{2}$ **b)** $\frac{1}{3}$ **c)** $\frac{1}{6}$
d) $\frac{1}{2}$ **e)** $\frac{2}{3}$ **f)** $\frac{5}{6}$

2. a) $\frac{1}{16}$ **b)** $\frac{15}{16}$ **c)** $\frac{1}{2}$

3. a) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

4. *Sample responses*:
a) spinning an odd number
b) spinning a number that is not a multiple of 4
c) spinning a number that is 10 or less
d) spinning a prime number or 1

5. a) 2, 4, 6, 8, 10, 12 **b)** 1, 3, 5, 7, 9, 11
c) 2, 3, 5, 7, 11 **d)** 1, 4, 6, 8, 9, 10, 12
e) 6, 12 **f)** 1, 2, 3

6. a) $\frac{1}{2}$ **b)** $\frac{1}{2}$ **c)** $\frac{5}{12}$

d) $\frac{1}{3}$ **e)** $\frac{2}{3}$ **f)** $\frac{5}{6}$

8. a) Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King

9. a) $\frac{12}{13}$ **b)** $\frac{10}{13}$ **c)** $\frac{4}{13}$

10. a) $\frac{1}{3}$ **b)** $\frac{5}{9}$
d) Card value ≥ 5

11. *Sample response*:

P(not 6 of clubs) = $\frac{12}{13}$

P(6 of clubs) = $\frac{1}{13}$

$1 - \frac{12}{13} = \frac{1}{13}$

CONNECTIONS: Simpson's Paradox p. 156

1. Situation 1

Bag 1: P(white) = $\frac{5}{11}$ or about 45%

Bag 2: P(white) = $\frac{3}{7}$ or about 43%

Situation 2

Bag 1: P(white) = $\frac{6}{9}$ or about 67%

Bag 2: P(white) = $\frac{9}{14}$ or about 64%

2. a) The combined Bag 1 has 11 white marbles and 9 black marbles.

P(white) = $\frac{11}{20}$ or 55%

b) The combined Bag 2 has 12 white marbles and 9 black marbles.

P(white) = $\frac{12}{21}$ or about 57%

c) I have a higher probability of winning with Bag 1 in each single situation. When I combine the bags, the probability of winning is higher with Bag 2.

6.1.2 Simulations p. 160

1. a) A, B (odd vs. even), C (odd vs. even), and D (odd vs. even)
b) C
c) B and D

2. *Sample responses*:
a) Flip a Nu 1 coin 3 times. Khorlo represents a girl being born. Repeat the experiment 20 times and count the percent of the time you get 3 Khorlos in 3 flips.
b) Cut out 3 pieces of paper and label one with RAIN. Put them in a bag. Draw one piece of paper from the bag, put it back, and then draw another. Repeat the drawing two more times. Repeat the entire experiment 25 times and count the percent of the time you choose the paper that says RAIN on each of the three draws.
c) Label 28 papers with BOY and 12 papers with GIRL. Put them into a bag. Draw two papers. Repeat the experiment 25 times and count the percent of the time you choose two papers that say GIRL.
d) Make a spinner with 10 equal sectors. Label 3 sectors as kareys. This is because 15 out of 50 is the same ratio as 3 out of 10. Spin the spinner two times. Repeat the experiment 20 times and count the percent of the time you spin karey both times.

3. *Sample response*:
The simulation could involve a spinner with four equal sectors. One sector is labelled CORRECT. This is because 1 out of 4 choices is correct. Spin the spinner five times and count how many times you get CORRECT. Repeat the experiment 20 times. Count the percent of the time that you got correct 3, 4, or 5 times.
The probability is likely to be close to 0.1 (actual answer ≈ 0.12).

4. *Sample response*:
The simulation could involve a spinner with 8 equal sectors. One sector is labelled HIT. This is because 25 out of 200 is the same as 1 out of 8. Spin the spinner five times and record whether all five spins are HITs. Repeat the experiment 20 times. Count the percent of the time that you got all HITs.
The probability is likely to be close to 0 (actual answer ≈ 0.00003).

5. *Sample response*:
5% is 1 out of 20. The simulation could involve 20 slips of paper, with one of the slips labelled BAD. Choose a slip of paper from a bag. Record whether it says BAD and return the slip to the bag. Repeat twice more. Record whether all of the slips chosen said BAD. Repeat the experiment 20 times. Count the percent of the time that you got all BADs.
The probability is likely to be close to (actual answer = 0.000125).

6.1.2 Simulations [Continued] p. 160

6. *Sample response*:
Print the numbers on cards and draw two numbers 20 times. Record the numbers chosen. Repeat the experiment 20 times. Count the percent of the time that each outcome occurs.
Actual probabilities should be close to:

a) $\frac{5}{6}$ **b)** $\frac{5}{6}$ **c)** $\frac{1}{2}$

7. *Sample response*:
Toss a Nu 1 coin until you get 4 outcomes the same, with Khorlo representing a win for Maaros. Do this 20 times. Use the fraction that describes each probability.
The probabilities might be:

a) $\frac{1}{10}$ **b)** $\frac{1}{5}$ **c)** $\frac{1}{2}$

6.2.2 Selecting a Random Sample p. 164

1. *Sample response*:
Assign each volunteer a number from 001 to 500. Use a random number table and select 100 three-digit numbers. You can start anywhere and keep moving three digits ahead. Ignore any numbers that are greater than 500 or that are repeats.

2. *Sample response*:
Decide on how big a sample you want, such as 10. Have the yaks pass in single file through a narrow opening or a gate. Select every tenth yak that passes through.

3. *Sample response*:
Put each renter's name on a piece of paper in a bag. Select 5 pieces of paper from the bag.

4. *Sample responses*:
a) Obtain a list of people old enough to buy a cell phone plan (e.g., no children) who live in the dzongkhag. Assign each person a number in sequence from 0000 to 9999. Choose 4-digit random numbers from a table.
b) Children should not be included because they are too young to own cell phones.

5. a) About 4

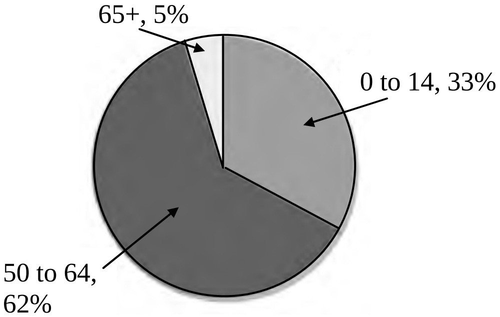
6.2.3 Circle Graphs pp. 166–167

1. a) and b)

Age group	*Persons*	*%*	*Angle*
0–14	210,000	33	119°
15–64	395,000	62	223°
65+	30,000	5	18°
Total	635,000	100	360°

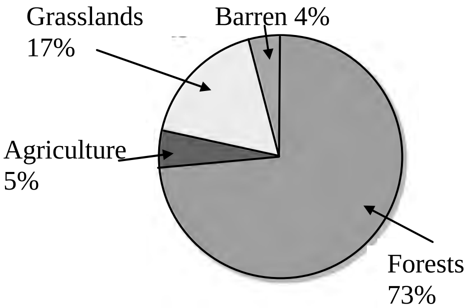
c)

Age Distribution

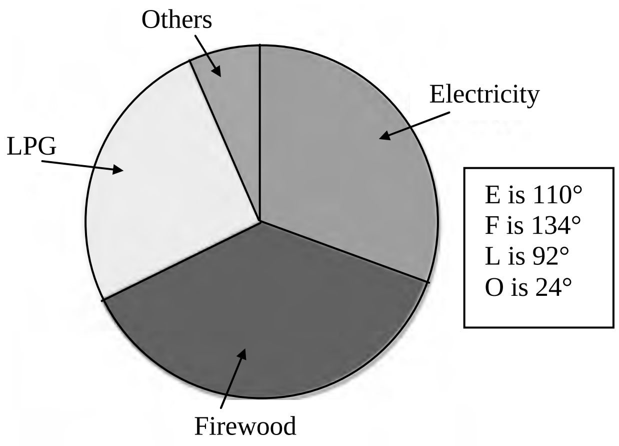


2.

Ecosystems in Bhutan

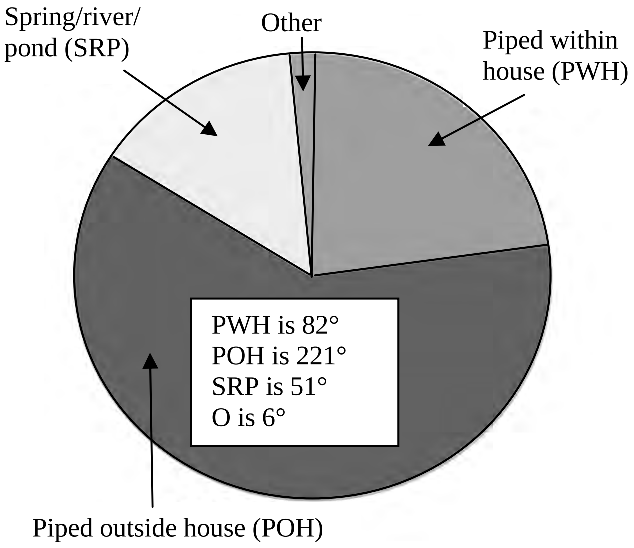


3. a)


Cooking Fuel

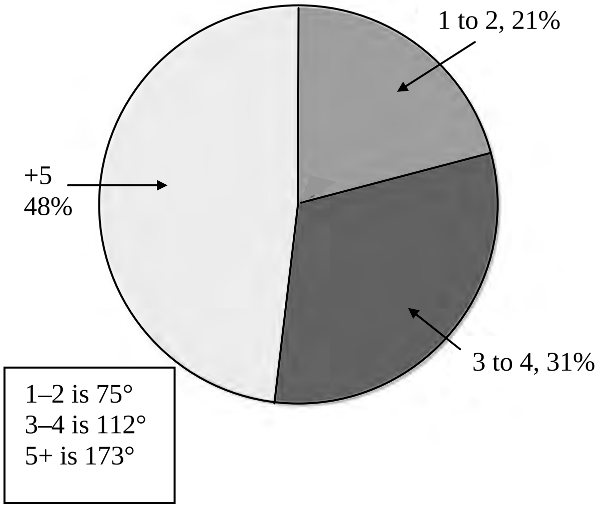
E is 110°
F is 134°
L is 92°
O is 24°

b)


Drinking Water Resources

PWH is 82°
POH is 221°
SRP is 51°
O is 6°

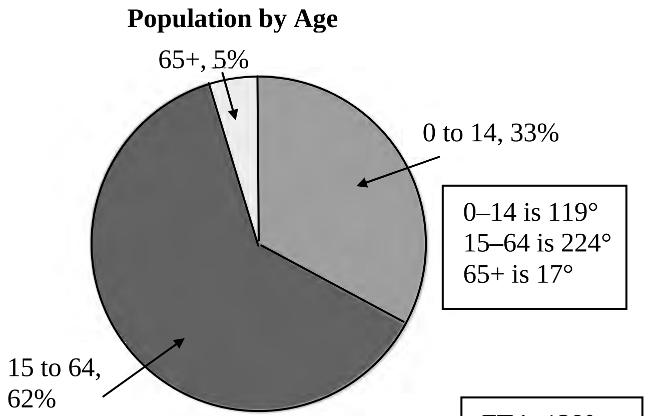
c)


People in Households

1–2 is 75°
3–4 is 112°
5+ is 173°

6.2.3 Circle Graphs [Continued] pp. 166–167

d)

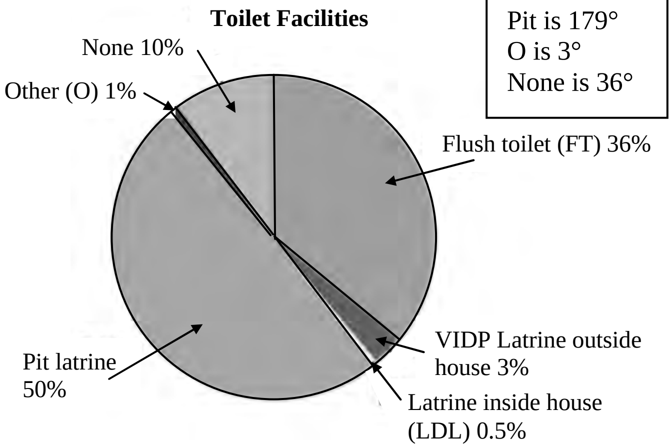


Population by Age

0–14 is 119°
15–64 is 224°
65+ is 17°

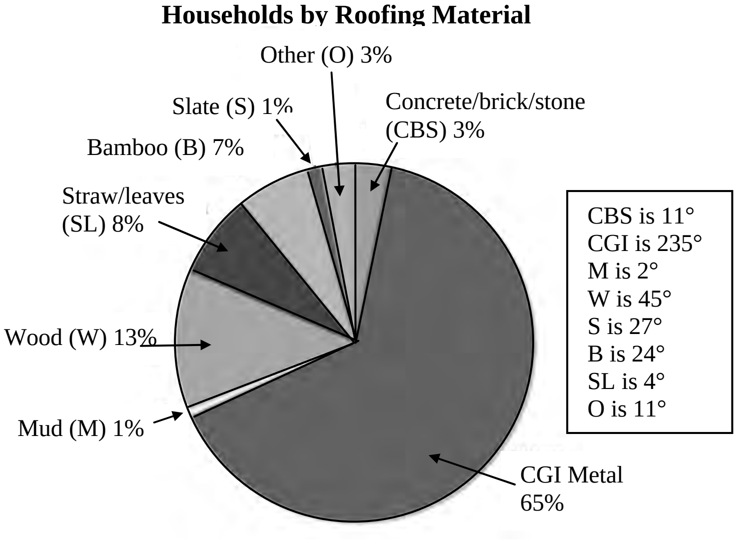
e)

FT is 129°
VIDP is 11°
LDL is 2°
Pit is 179°
O is 3°
None is 36°



Toilet Facilities

f)



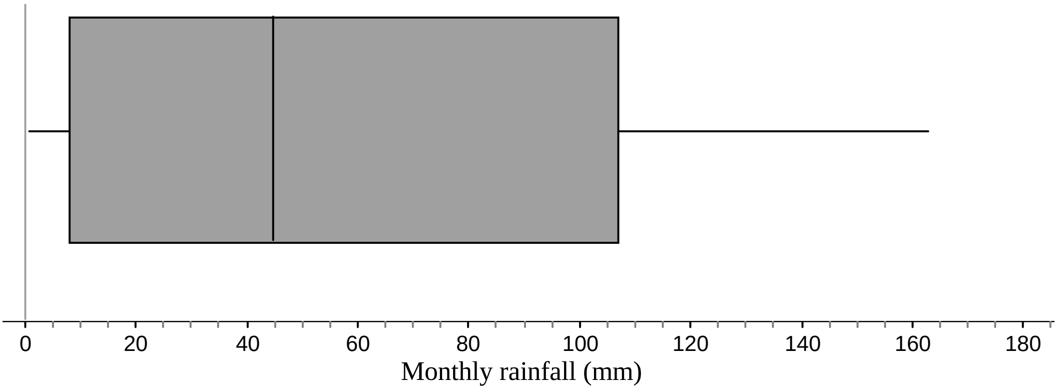
Households by Roofing Material

CBS is 11°
CGI is 235°
M is 2°
W is 45°
S is 27°
B is 24°
SL is 4°
O is 11°

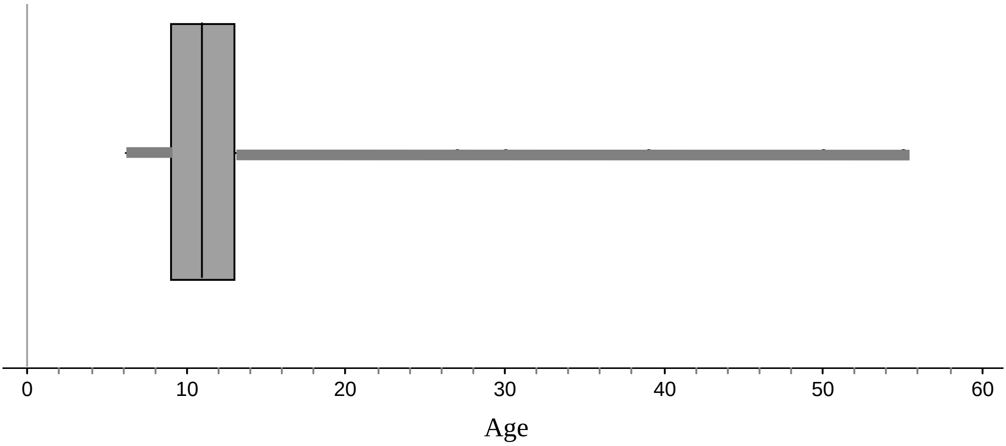
4. *Sample response*:
The percent of students in different classes of a school.

6.2.4 Box and Whisker Plots pp. 172–174

1.



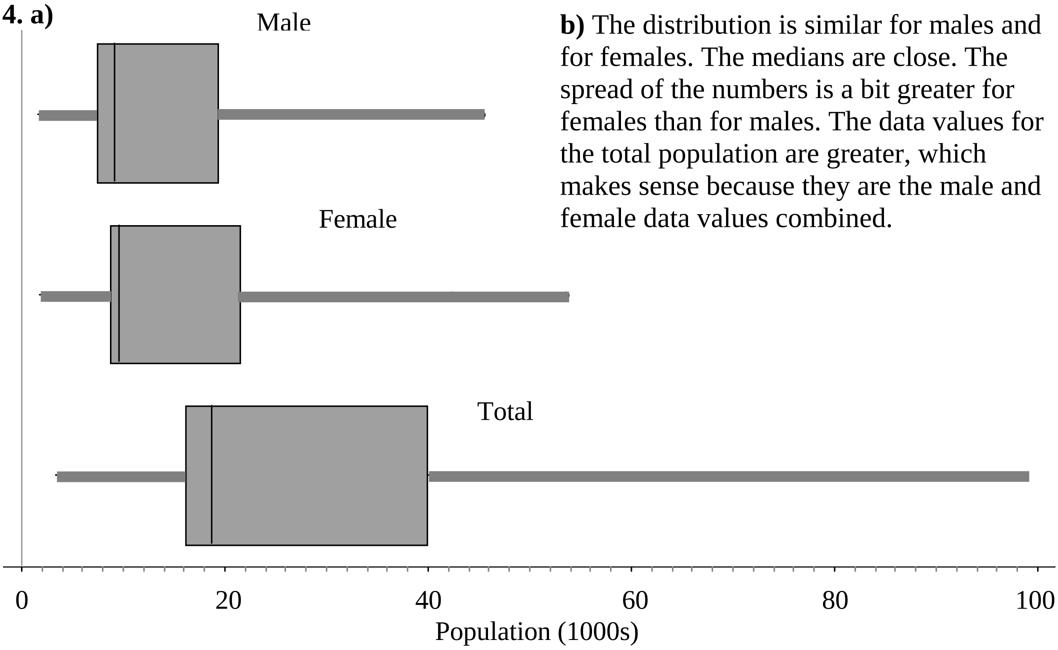
2. a)



b) *Sample response*:
Most of the data values are clustered close to the median but the very long right whisker shows that there are some very high values, which represent the teachers' ages. These values make the range of the data very large.

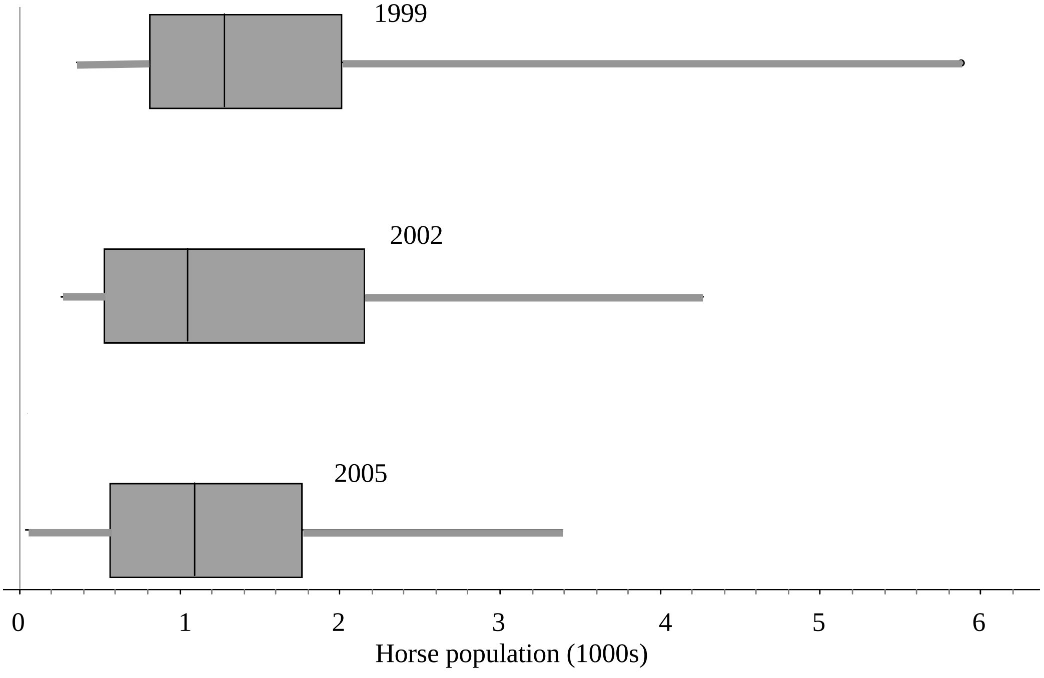
3. *Sample response*:
It looks like Econo Bulb has the best quality. They have fewer problem bulbs overall and the extremes are low values rather than high values, which means that fewer bulbs do not work.

4. a)



b) The distribution is similar for males and for females. The medians are close. The spread of the numbers is a bit greater for females than for males. The data values for the total population are greater, which makes sense because they are the male and female data values combined.

5. a)



b) *Sample response*:
The spread of the data was greatest in 1999. That year there was an unusually large number of horses in Trashigang that should likely be considered an outlier.
The distributions for 2002 and 2005 had almost identical medians, but there was a greater range of values in 2002.
Trashigang provided the extreme value each year, but it decreased from 1999 to 2005.

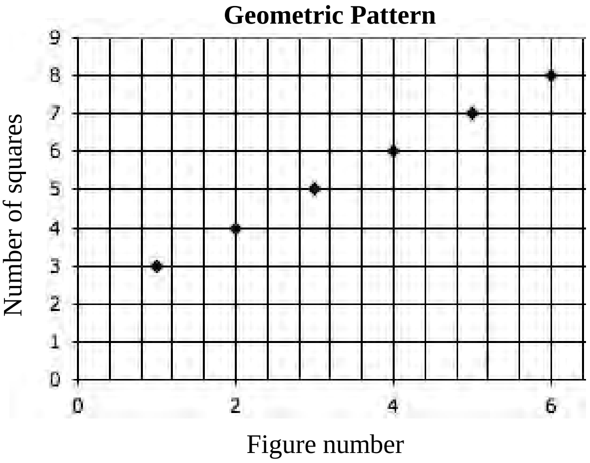
6. The right whisker will be a lot longer than the left whisker.
The box will be very narrow with no median line.

6.3.2 Using a Scatter Plot to Represent a Relationship pp. 179-180

1. a) i)

Figure number	Number of squares
1	3
2	4
3	5
4	6
5	7
6	8

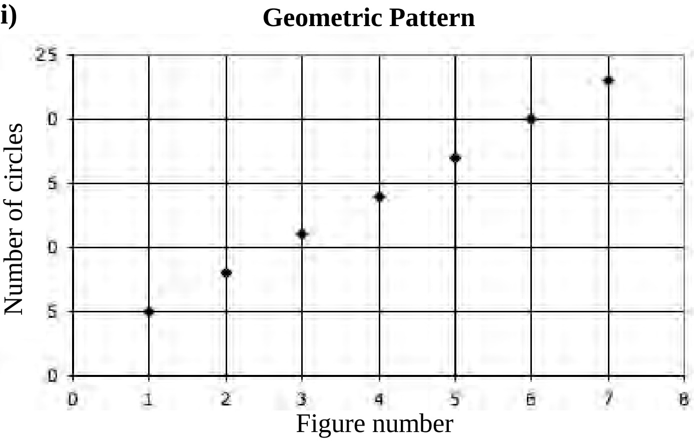
ii)



b) i)

Figure number	Number of circles
1	5
2	8
3	11
4	14
5	17
6	20
7	23
8	26

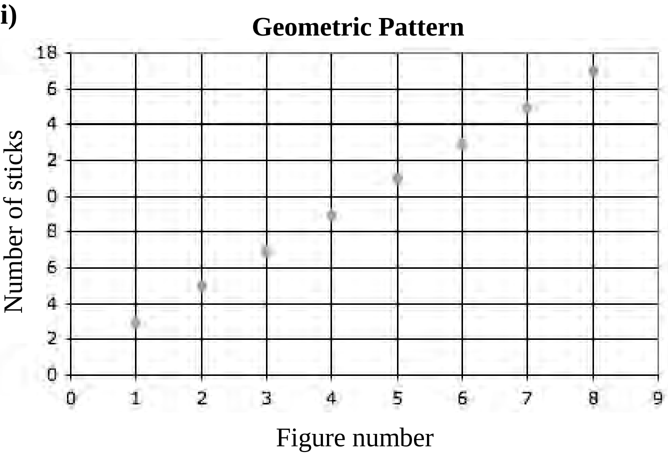
ii)



c) i)

Figure number	Number of sticks
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17

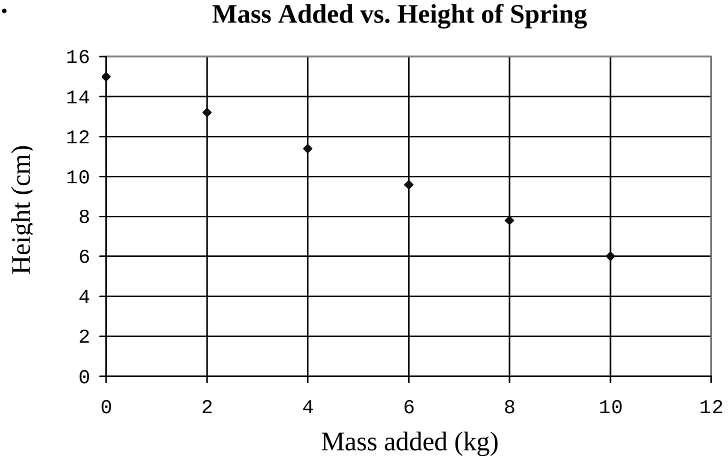
ii)



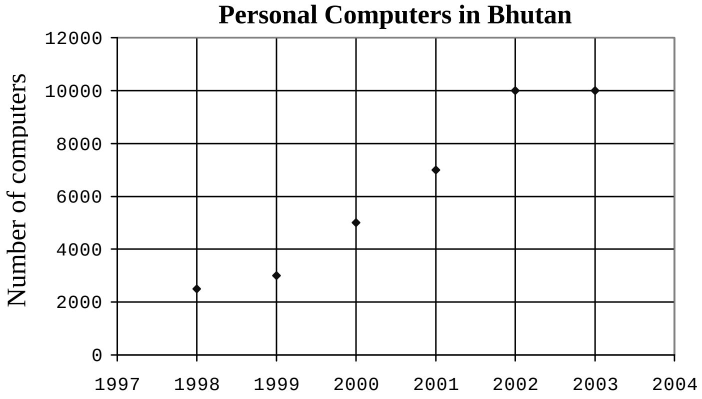
2. 5, 7, 11, 14

6.3.2 Using a Scatter Plot to Represent a Relationship [Cont'd] pp. 179-180

3.

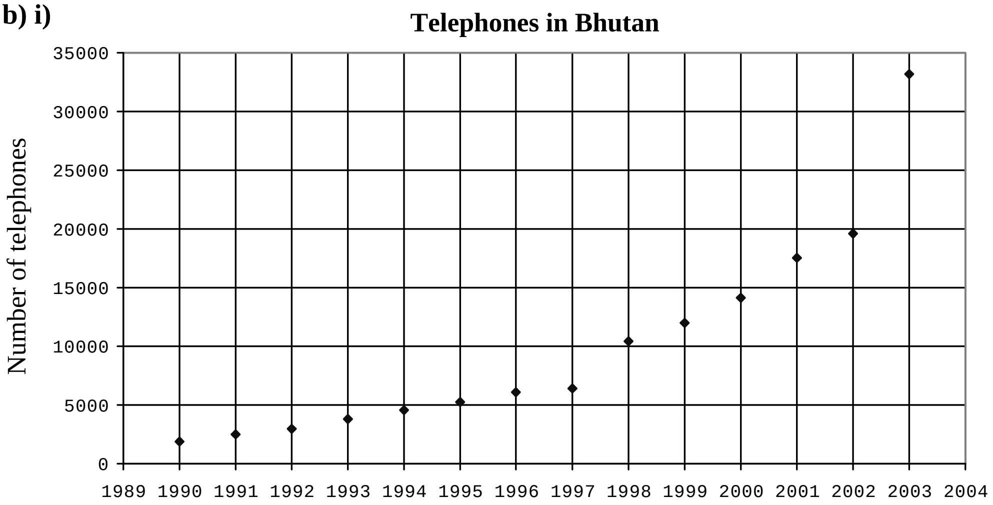


4. a) i)



ii) *Sample response*:
The number of computers grew quickly for a few years but then stopped growing.

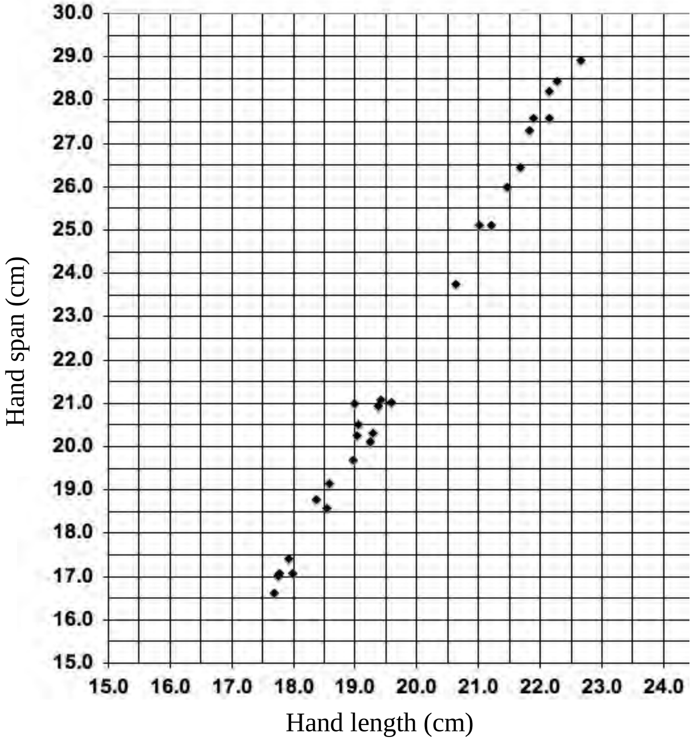
b) i)



ii) *Sample response*:
The number of telephones grew by an increasing amount each year. In the last few years, the change from one year to the next also increased.

5. A

6. a) *Sample response*:



Sample Class Data

b) If hand length increases, so does hand span. The plotted points seem to fall closely along a straight line.

7. *Sample responses*:

- The data values in one row go together. Selecting data values from two different rows will give a false piece of information, for example, a year with the incorrect population.
- Each column in a table o usually represents a specific type of data, for example, year (time) in one column and population in the other column. I must use the same coordinate for the same type data so that the data will be displayed accurately and the trend will be displayed correctly.

UNIT 6 Revision pp. 181–182

1. a) $\frac{4}{52} = \frac{1}{13}$

b) $1 - \frac{1}{13} = \frac{12}{13}$

c) $\frac{36}{52} = \frac{9}{13}$

2. a) Drawing the white ball or one of three grey balls.

b) i) $\frac{4}{10}$ **ii)** $\frac{9}{10}$

3. *Sample response*:

Use a coin, where one side represents “rain” and the other side represents “no rain”. Also use a 3-sector spinner with one sector marked “rain”.

Flip the coin and spin the spinner and record whether both come out as “no rain”. Repeat 20 times.

Record the fraction or percent of the 20 times that you got “no rain” for both the flip and the spin.

4. *Sample responses*:

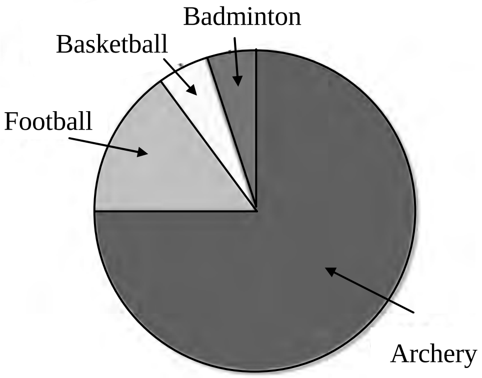
a) Write the 100 student names on identical slips of paper. Put them in a hat, mix them up, and draw out 10 names at random.

b) A sample that consists of only girls or boys.

5. a) and b)

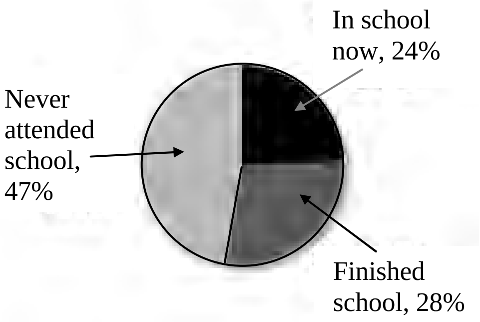
Favourite sport	Percent of students	Angle
Archery	75%	270°
Football	15%	54°
Basketball	5%	18°
Badminton	5%	18°

Favourite Sport



6.

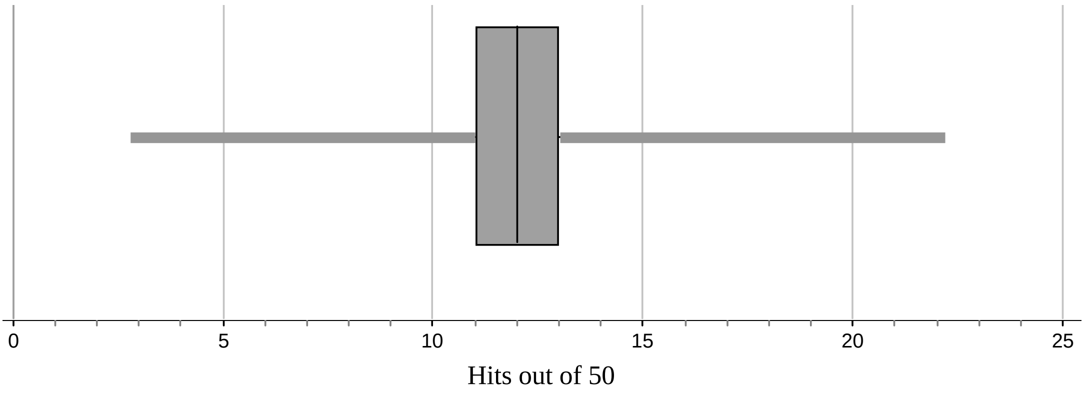
Education Categories



In school now is 88°
Finished school is 101°
Never attended school is 171°

7. a) Mean = 11.95, median = 12, range = 19

b)

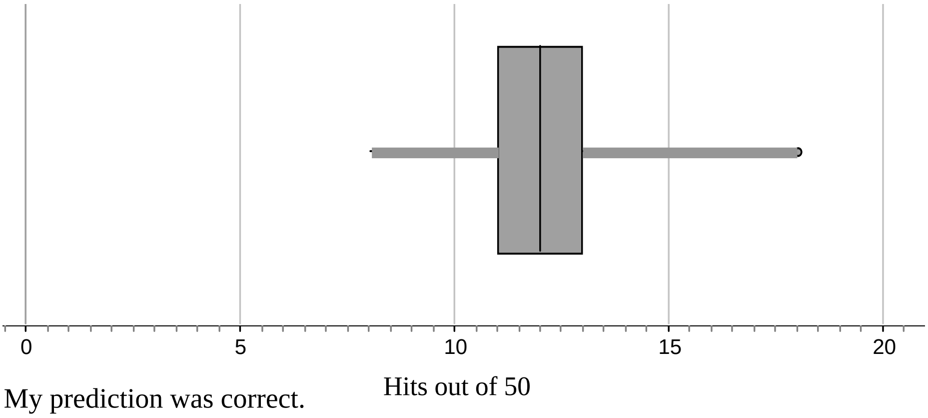


c) 3 and 22

d) The mean increases to about 12.4; the median does not change; the range decreases to 14.

e) The mean decreases to about 11.4; the median does not change; the range decreases to 15.

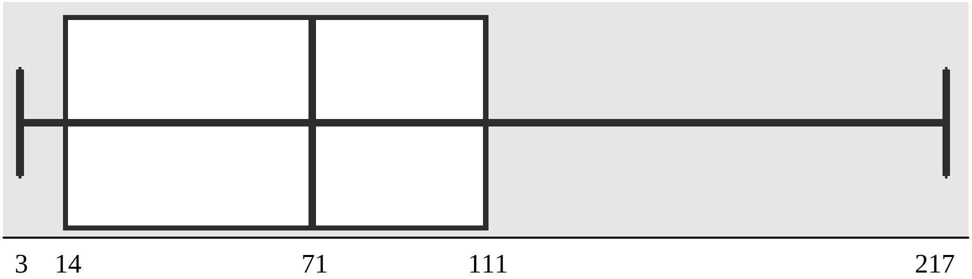
f) The whiskers would be shorter.



My prediction was correct.

8. Plot B

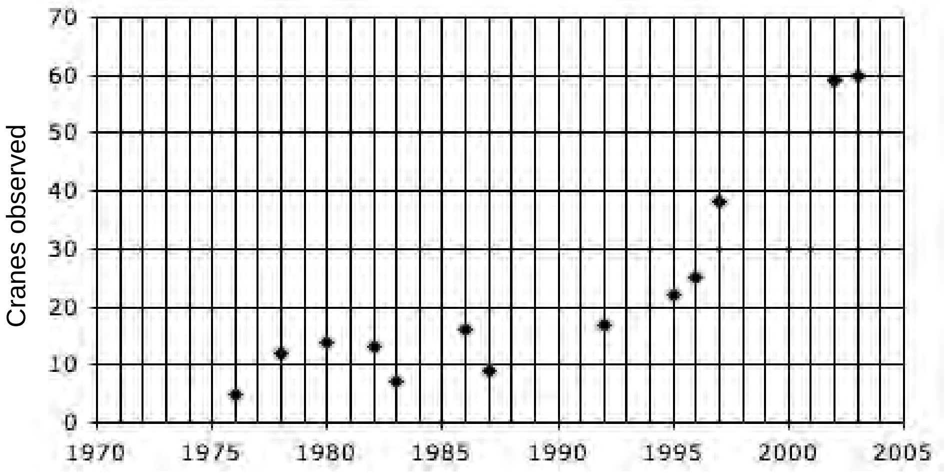
9. a)



b) *Sample response*:
The box plot shows that there is at least one month with very, very high rainfall compared to the median and that the high rainfall is much farther away from the median than the least rainfall amount is from the median.

10. a) The number was fairly constant until about 1987. Then it began to increase, with a large increase in 1996.
b) *Sample response*: 2

11. a) **Black-necked Crane Sightings**



b) The trends seem almost identical.

UNIT 7 ALGEBRA pp. 183–222

Getting Started — Skills You Will Need p. 184

1. a) i)

x	*y*
1	8
2	13
3	**18**
4	**23**
5	28

ii)

x	*y*
1	18
2	**16**
3	**14**
4	12
5	**10**

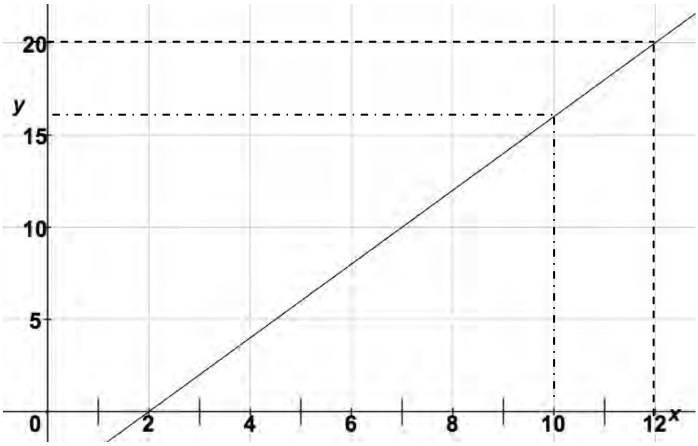
v

b) i) $y = 5x + 3$
ii) $y = 20 - 2x$

2. a)

x	*y*
1	– 2
2	0
3	2
4	4

2. b) i) $y = 2x - 4$, so $y = 2 \times 10 - 4 = 16$
ii) $20 = 2x - 4$, so $x = 12$
c)



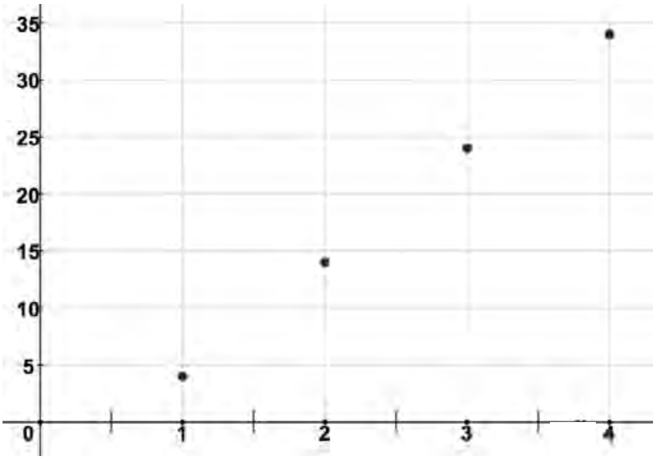
3. a) i) $7n - 3$
ii) $-n - 10$
b) i) $7 \times 8 - 3 = 56 - 3 = 53$
ii) $-8 - 10 = -18$

4. a) $90 = 4(f + 1) - 2$
b) $f = 22$

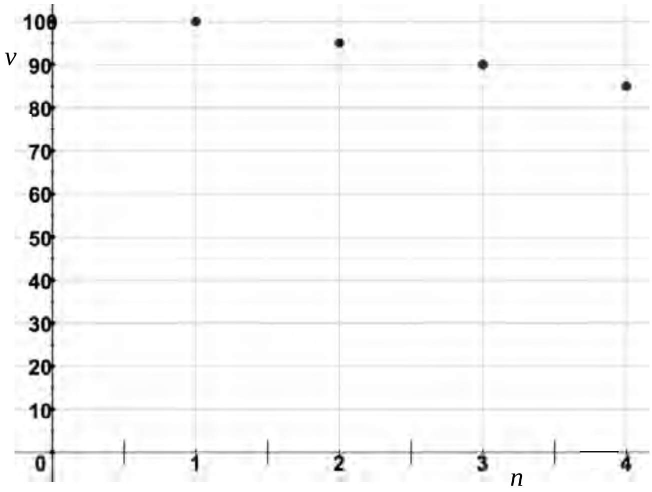
5. a) Coefficient = 2, constant = –3
b) Coefficient = 3, constant = 4
c) Coefficient = –3, constant = 6

7.1.2 Describing Relationships and Patterns p. 190

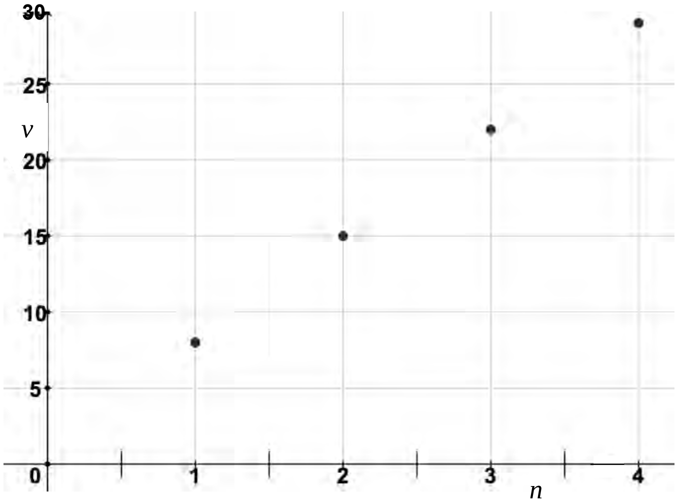
1. a) $v = 10n - 6$, where n is the term number and v is the term value.



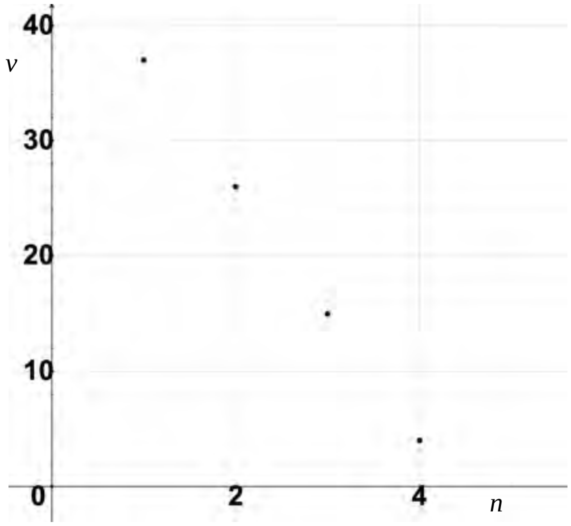
b) $v = 105 - 5n$, where n is the term number and v is the term value.



c) $v = 7n + 1$, where n is the term number and v is the term value.



d) $v = 48 - 11n$ or $v = -11n + 48$, where n is the term number and v is the term value.

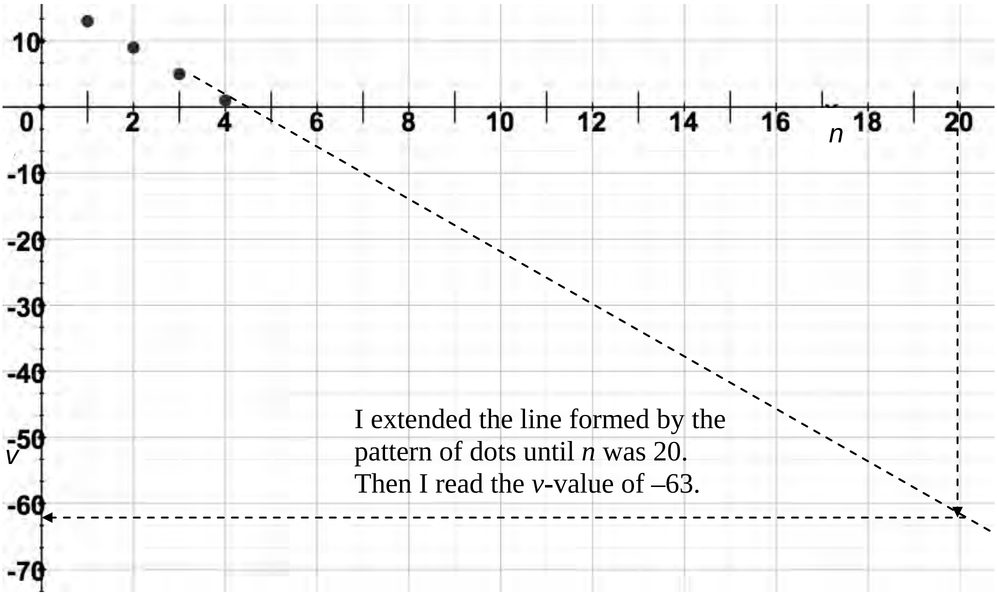


7.1.2 Describing Relationships and Patterns [Continued] p. 190

2. a) $v = 7n - 4$; The 20th term is 140 – 4 = 136.

b) $v = 22 - 7n$ (or $v = -7n + 22$); The 20th term is 22 – 140 = –118.

c)

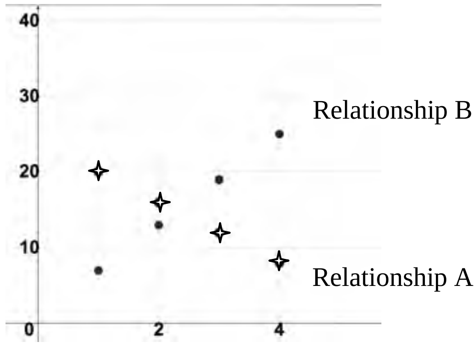


I extended the line formed by the pattern of dots until n was 20. Then I read the v-value of –63.

d) $v = \frac{-4n+13}{2}$; The 20th term is $\frac{-67}{2}$.

3. *Sample response*:

a) Relationship A: $v = -4n + 24$ or $v = 24 - 4n$
Relationship B: $v = 6n + 1$

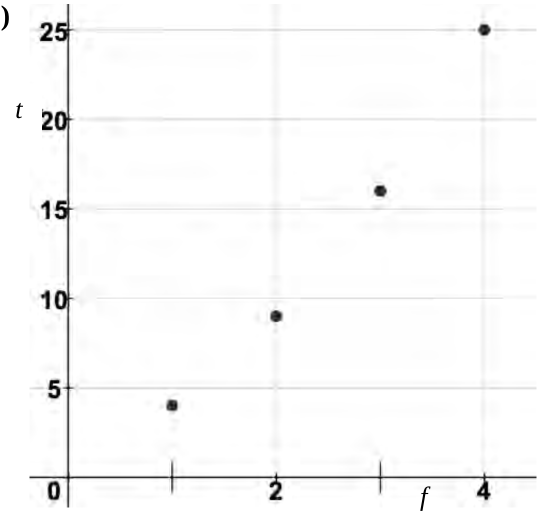


b) The equation for Relationship A involves multiplying by a negative number and then adding, while the equation for Relationship B involves multiplying by a positive number and then adding.

The graph for Relationship A slopes down, while Relationship B's graph slopes up.

4. a) No

b)



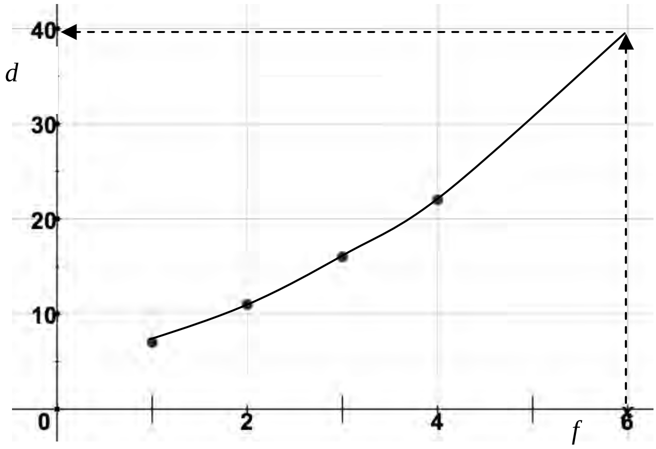
(*f* is the figure number and *t* is the number of triangles.)

c) The points form a curve that slopes upward.

5. Rinzin is right.

6. *Sample responses*:

a) and b)



(*f* is the figure number and *d* is the number of dots.)

I connected the points with a curve and then extended the curve to Figure 6. It looks like Figure 6 has 40 dots.

c) I noticed that the curve went up by 4, then by 5, then by 6. I started with the 7 dots in Figure 1 and added 4 + 5 + 6 + 7 + 8. I got 37, so I was close.

7. $d = 3f + 3$, where d is the number of dots and f is the figure number.

Figure number	Number of dots
1	6
2	9
3	12
4	15

8. *Sample responses*:

A. An algebraic equation

B. A graph

7.1.3 Recognizing Linear Relationships p. 195

1. a) Yes **b)** No **c)** Yes

2. a)

x	*y*
1	20
2	23
3	**26**
4	**29**

b)

x	*y*
1	20
2	**22.5**
3	**25**
4	27.5

4. a) Yes **b)** No **c)** Yes

5. a)

Cars	Nu
0	400
1	1400
2	2400
3	3400
4	4400

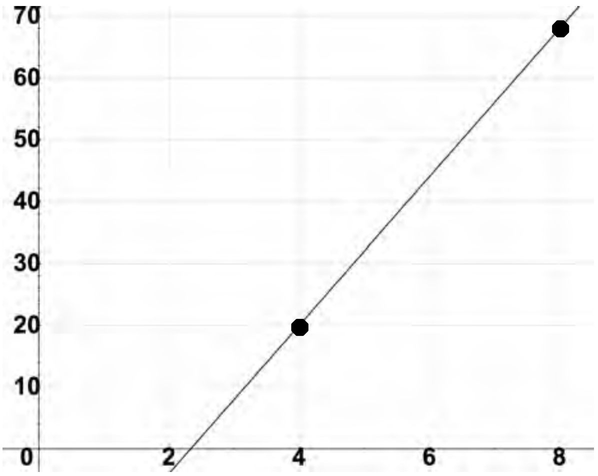
b) Yes

6. a) Graph 2 **b)** No

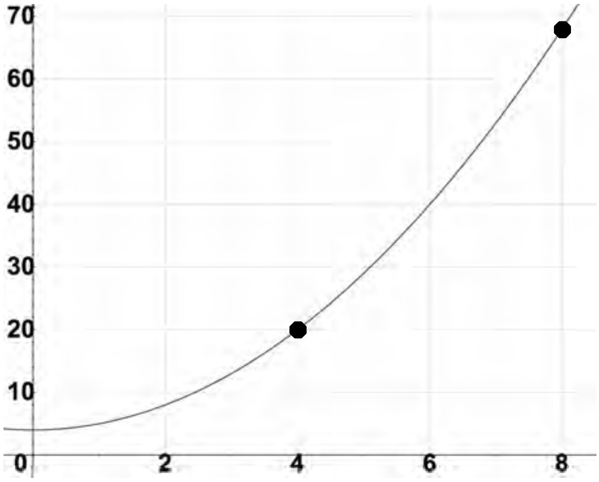
7. a) Yes **b)** No

8. *Sample responses*:

a) A straight line joining the two points shows a linear relationship.



b) A curve joining the two points does not show a linear relationship.



CONNECTIONS: Adding Values in a Linear Relationship p. 196

1. 50 × 101 = 5050

2. a) 420 **b)** 590 **c)** 14,340

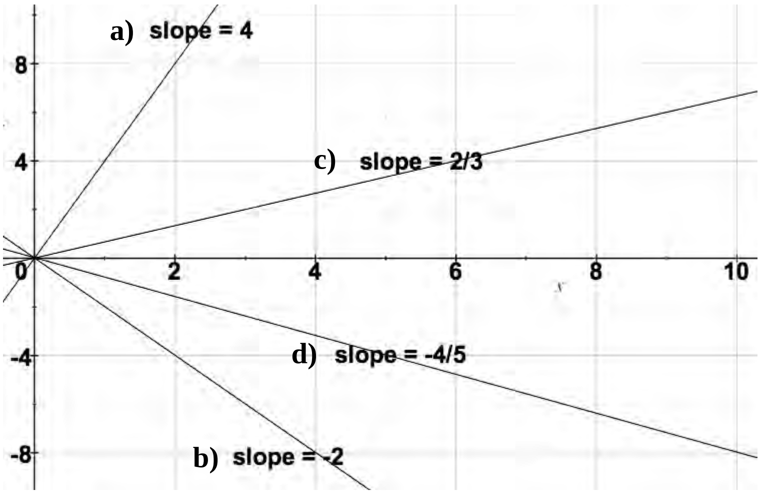
3. No

7.1.4 Slope p. 202

1. a) –1 **b)** 3 **c)** 4

2. a) 3 **b)** 3 **c)** 4

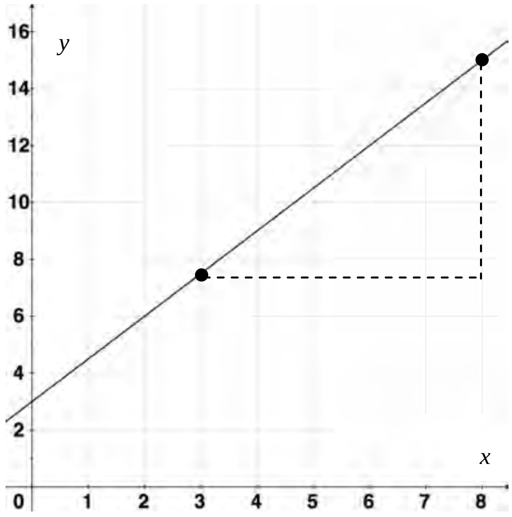
3. *Sample response*:



5. $y = 3x - 10$

6. 60

7. a) The change is $\frac{7.5}{5}$ cm per minute since it changes 7.5 cm in 5 minutes. That is 1.5 cm per minute.
b) The slope of the graph is 1.5.



8. a) (5, 4),
(3, **–8**),
(**7**, 16)
b) (3, 10),
(8, **12**),
(**18**, 16)

9. 15 km/h

10. $\frac{2}{3}$ m

7.2.1 Solving an Equation Using Inverse Operations p. 206

1. a) $3m - 4 = 12$
b) *Sample response*: $(x + 3) \times 2 = -6$
c) *Sample response*: $(n - 4) \times 3 = 1.5$

2. a) $x = -1$ **b)** $x = 7$ **c)** $x = \frac{5}{6}$ **d)** $x = 7$

3. *Sample responses*:
a) $3m = 33$; $2 - m = -9$; $m + 1 = 12$
b) $5n = 4$; $10n = 8$; $80 - 10n = 72$
c) $8x = 1$; $16x = 2$; $-16x = -2$
d) $2k = -16$; $-k = 8$; $30 - k = 38$

5. 1

7. *Sample responses*:
a) x is about 9
b) x is about 10
c) x is about 1

8. Pema

7.2.2 Using an Equation to Solve a Problem p. 209

1. *Sample responses*:
a) $24b = 744$
b) $k + k + 2 = 82$ or $2k + 2 = 82$
c) $4s + 4 = 28$
d) $4b + 7 = 103$

2. a) $b = 31$
b) $k = 40$
c) $s = 6$
d) $b = 24$

3. $100 = 2.2k$; $k \approx 45$

4. $400 - 112 = 32m$; $m = 9$ min

5. *Sample response*:
a) $l - 4 + l - 4 + l + l = l + 64$; $l = 24$ cm
b) $w = l - 4$; $w = 20$ cm

6. *Sample response*:
$150 = 80 + 28h$; $h = \frac{70}{28} = 2\frac{1}{2}$ h

7. *Sample responses*:
a) Four equal groups of students and another group of 97 students attended an event. Altogether, there were 489 students.
How many students were in each of the four equal groups?
b) Four pieces of string of equal length were cut from a string of length 100 cm, or 1 m. There were 44 cm of string left. How long was each of the equal pieces?
c) Dechen followed a recipe that called for two containers of flour plus another six cups. The total amount of flour was equal to four full containers of flour. How many cups of flour does a full container hold?

7.2.3 Solving a Problem Involving Two Relationships pp. 212–213

1. a) (–2, 5) **b)** (8, 0) **c)** (15, 124)

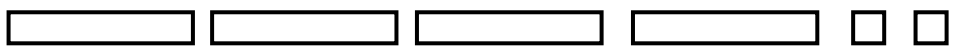
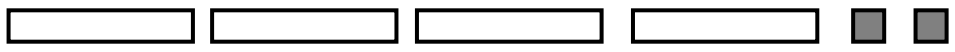
2. *Sample response*: Since the point is on both lines, the coordinates of that point make both equations true.

3. $4f - 3 = 3f + 14$; $f = 17$, so Figure 17 has the same number of dots for both patterns.

4. 8 h; $30h + 120 = 20h + 200$

6. *Sample response*:
An isosceles triangle has two equal long sides. Its third side is 5 cm shorter than each long side. The perimeter is the same as the perimeter of a square whose side length is the same as the short side of the triangle. What are the dimensions of each shape?
(The triangle has sides of 10 cm, 15 cm, and 15 cm. The square has four sides of 10 cm.)

7.3.1 Adding Polynomials p. 216

1. a)

b)

c)

d)


2. a) $5x$ and $2x$; 7 and – 2
b) $-5x$, $2x$, and x; 3 and – 2
c) $-2x$ and $-4x$; 6 and –1

3. a) $7x + 5$ **b)** $-2x + 1$ **c)** $-6x + 5$

4. *Sample responses*:
a) $-6x$, $-5x - 1$, $-4x - 2$, $-3x - 3$, $-2x - 4$
b) $8x$, $7x + 1$, $6x + 2$, $5x + 3$, $4x + 4$

5. b) *Sample response*:
$(-3x + 1) + (4x + 1) = x + 2$

6. a) $6 - 2x$ **b)** $7x$
c) $2x - 2$ **d)** 3

7. *Sample responses*:
a) $(4x + 2) + (-6x)$
$(2x + 1) + (-4x + 1)$
$(-2x + 1) + 1$
b) $(2x + 1) + x$
$4x + 2 + (-x + -1)$
$(6x + -2) + (-3x + 3)$
c) $x + (x - 2)$
$2x + (-2)$
$(7x - 9) + (-5x + 7)$

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7.3.2 Subtracting Polynomials p. 219

2. a) $2x + 3$ **b)** $2x + 13$
c) $6x + 13$ **d)** $6x + 3$

3. a) *Sample response*:
$(10 + \mathbf{10}x) - (\mathbf{8}x + 5) = (2x + 5)$
b) *Sample response*:
$(4 - \mathbf{2}x) - (-\mathbf{4}x - 5) = (2x + \mathbf{9})$
c) $(6 + \mathbf{2}x) - (3x + 5) = (-x + 1)$

4. a) $-1 + 5x$ **b)** $-12 + x$ **c)** $5x + 6$

5. All white tiles

6. *Sample responses*:
a) $4x + 2 - (6x)$, $x + 2 - (3x)$, $8x + 5 - (10x + 3)$
b) $9x + 3 - (6x + 2)$, $9x + 5 - (6x + 4)$, $10x + 3 - (7x + 2)$

7. The differences are opposites:
$5x - (3x + 2) = 2x - 2$
and
$-5x - (-3x - 2) = -2x + 2$

8. *Sample response*:
- More tiles in the difference:
For $x - (-2x) = 3x$, A uses 1 tile, but C uses 3 tiles.
- Fewer tiles in the difference:
For $3x - 2x = x$, A uses 3 tiles, but C uses 1 tile.
- The same number of tiles in the difference:
For $(2x + 4) - (x - 1) = x + 3$, A uses 6 tiles and C uses 6 tiles.

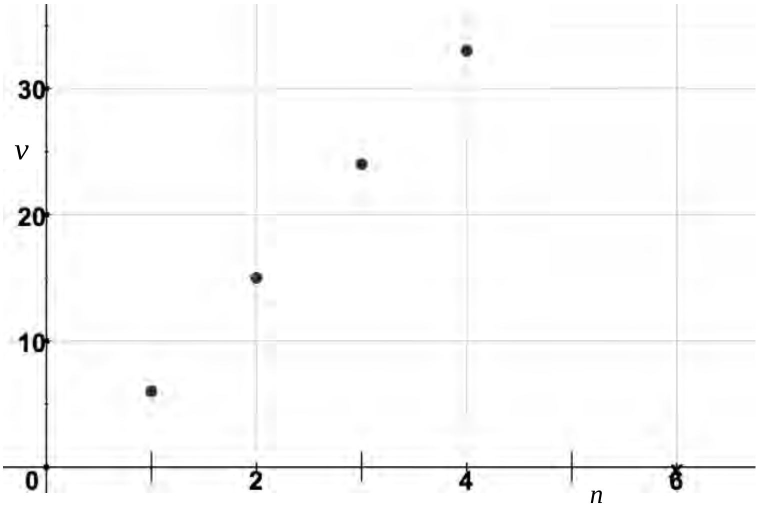
9. No

10. Yes

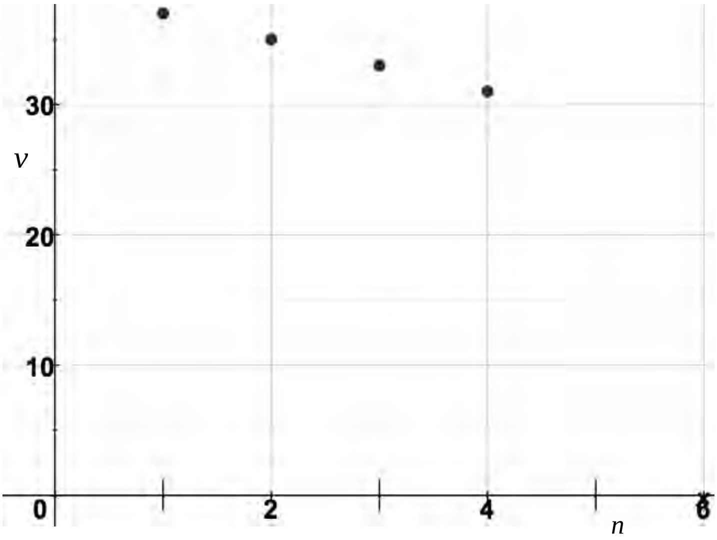
UNIT 7 Revision pp. 221–222

1. *Sample responses*:
a) 20 and 0; 2 and 27; 10 and 15

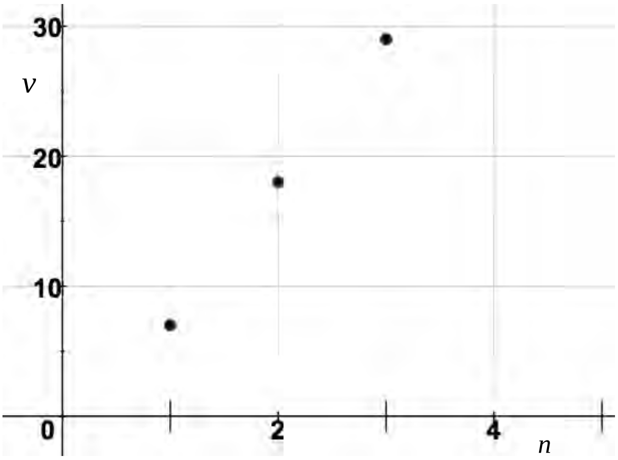
2. If n is the term number and v is the term value:
a) $v = 9n - 3$



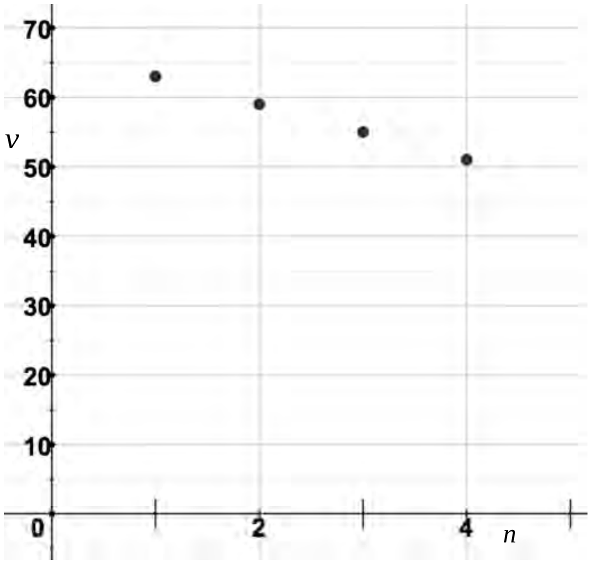
b) $v = 39 - 2n$



c) $v = 11n - 4$



d) $v = 67 - 4x$



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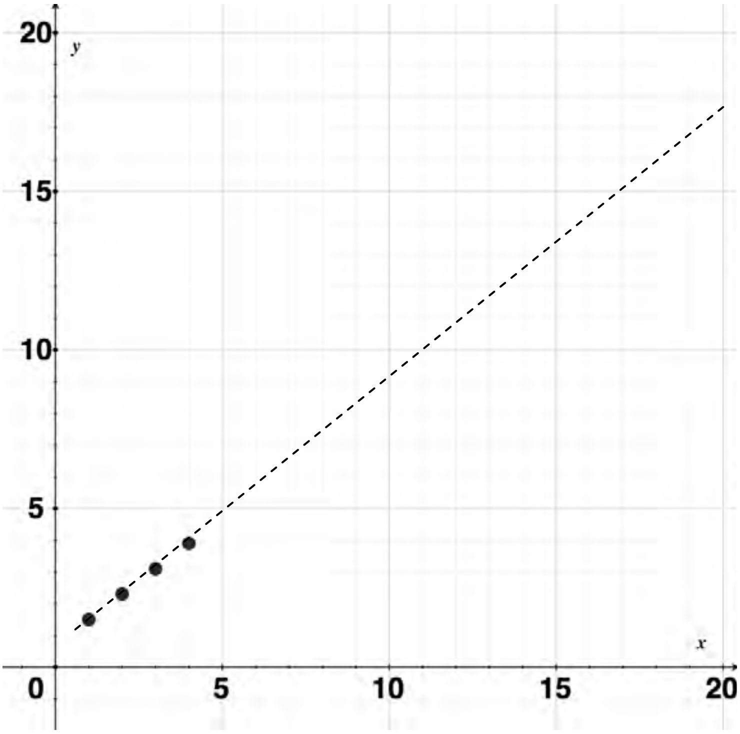
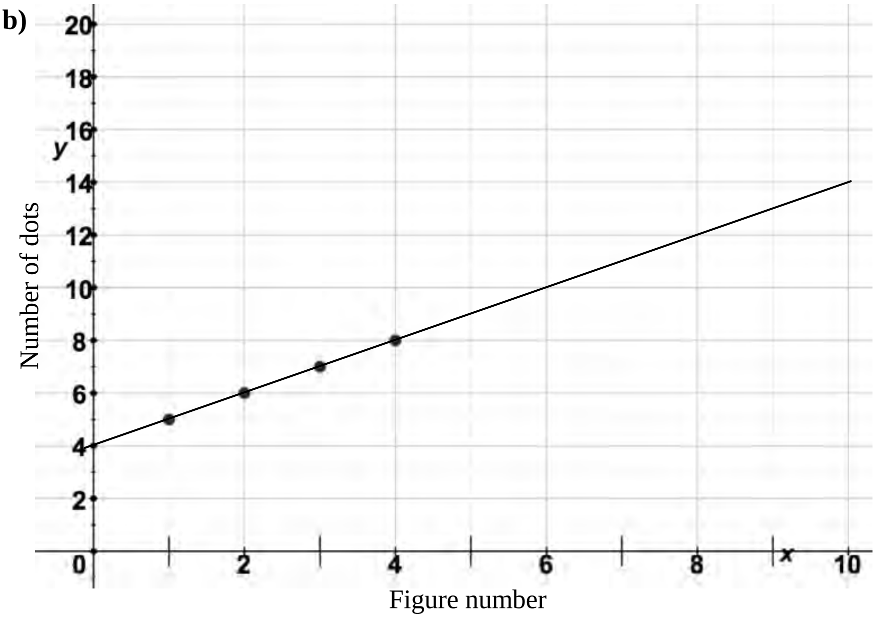
3. a) $v = 7n + 1$, so the 20th term is $7 \times 20 + 1 = 141$.
b) $v = 48 - 6n$, so the 20th term is $48 - 6 \times 20 = 48 - 120 = -72$.
c) $v = 5n + 12$, so the 20th term is $5 \times 20 + 12 = 112$.
d)



The y-value for $x = 20$ is about 16.5. (If you use an equation, it is actually 16.7.)

4. a) 8

b)



c) 14
d) *Sample response*:
I can test with the algebraic expression $y = x + 4$ (x is the figure number and y is the number of dots). $y = 10 + 4 = 14$.

5. B is linear

UNIT 7 Revision [Continued] pp. 221–222

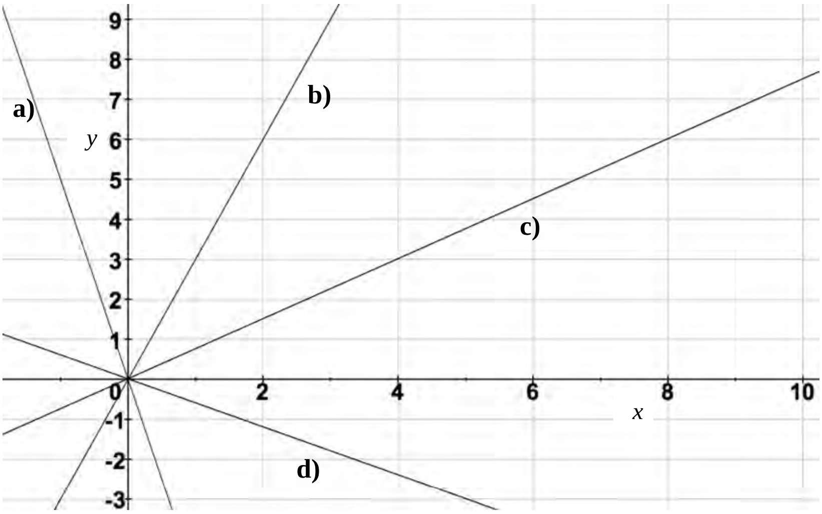
6. a) Linear **b)** Linear **c)** Not linear

7. No

8. *Sample responses*:
a) –2 **b)** 4 **c)** 3

9. a) 2 **b)** –1 **c)** 3

10.



11. a) (5, 4), (3, **8**), (**11**, –8)
b) (3, 10), (8**, 0**), (**–6**, 28)

12. a) $x = –5$
b) $x = 10$
c) $x = \frac{5}{8}$ or 0.375
d) $x = \frac{39}{5}$ or $7\frac{4}{5}$ or 7.8

13. *Sample responses*:

a) $2m = 14$	$8 – m = 1$	$3m + 7 = 28$
b) $5n = 1$	$10n = 2$	$11 – 5n = 10$
c) $5x = 3$	$10x = 6$	$–15x = –9$
d) $–k = 3$	$2k + 11 = 5$	$30 + 5k = 15$

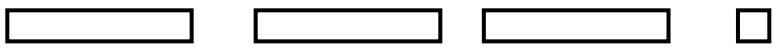
14. a)
- First, add 2 to both sides to get rid of –2 on the left side.
- Then multiply both sides by 5 to get rid of the denominator 5 on the left side.
- Then divide both sides by 2 to get rid of the coefficient of 2 on the left side.
- x is now alone on the left side so I know its value, which is on the right side.

b) $x = 47.5$

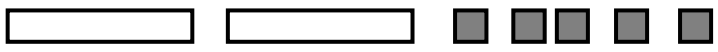
15. a) $30 + 24n = 95$; n is number of hours
b) $n = \frac{65}{24} = 2\frac{17}{24}$ h

16. a) (–1, –1)
b) (10, 73)
c) $\left(\frac{50}{7}, \frac{40}{7}\right)$ or $\left(7\frac{1}{7}, 5\frac{5}{7}\right)$

17. a)



b)



c)

d)

18. a) $6x$, x, and $2x$
b) $5x$, $9x$, and $–3x$; 13 and –2
c) $–4x$ and $–7x$; 8 and –5

19. a) $9x – 8$
b) $11x + 11$
c) $–11x + 3$

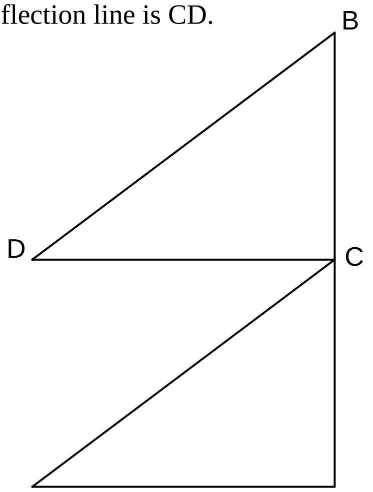
20. *Sample response*:
$(4x + 8)$ and $(3x + 6)$

Getting Started — Skills You Will Need p. 224

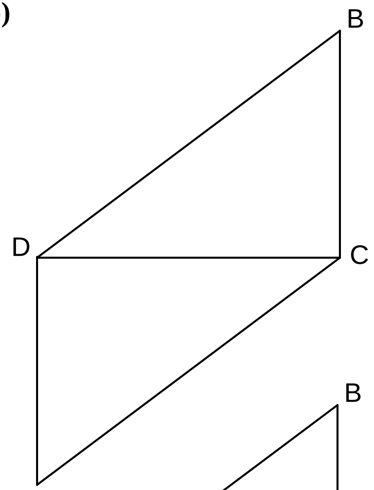
1. a) Angles are all 60º, side lengths are all 3.4 cm; Acute equilateral triangle.
b) Angles are 30º, 60º, and 90º, side lengths are 3.4 cm, 5.9 cm, and 6.9 cm; Scalene right triangle.
c) Angles are 35º, 35º, and 110º, side lengths are 3.4 cm, 3.4 cm, and 5.5 cm; Isosceles obtuse triangle.
d) Angles are all 120º, side lengths are all 3.4 cm; Regular hexagon.

2. a) 3 cm down along BC.
b) Turn centre is the midpoint of CD, angle of rotation is 180º.
c) Reflection line is CD.

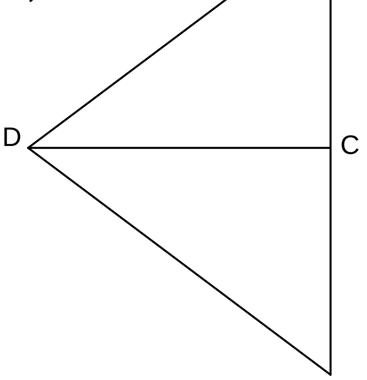
3. a)



b)

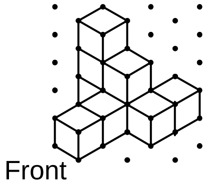


c)

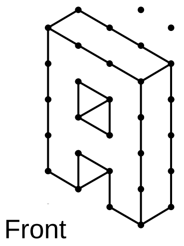


8.1.1 Isometric Drawings p. 228

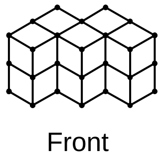
1. *Sample response*:



2. *Sample response*:



3. *Sample responses*:
a) and b)



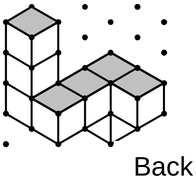
4. a)

b) 14
c) 25

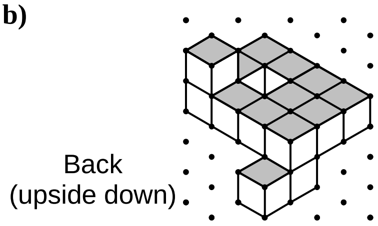
8.1.1 Isometric Drawings [Continued] p. 228

5. *Sample responses*:

a)



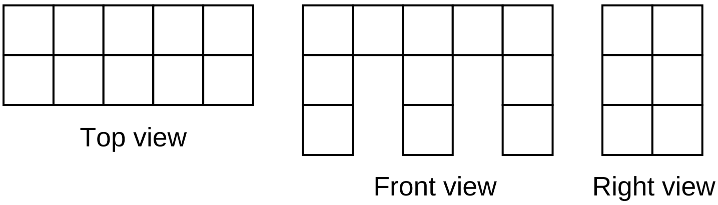
b)



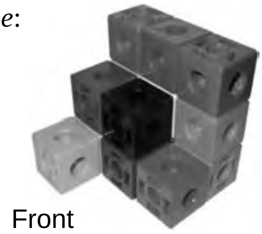
6. A and D

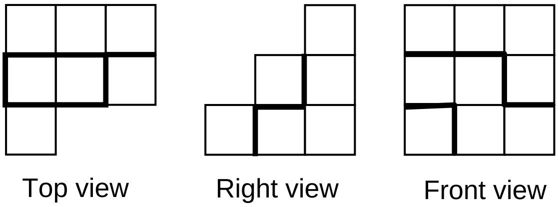
8.1.2 Orthographic Drawings p. 232

1. *Sample response*:

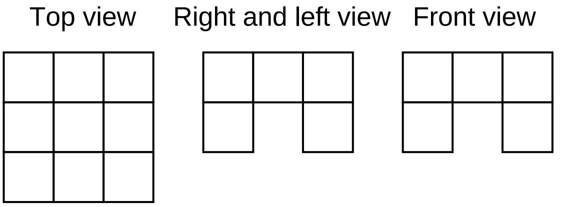


2. *Sample response*:





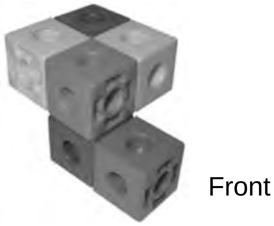
3. a)



b) *Sample response*: See **part B i)** above.

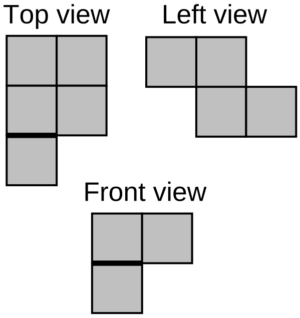
4. *Sample responses*:

a) Structure 1

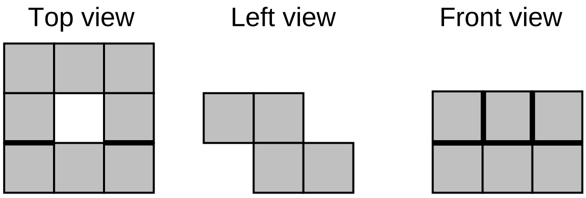


Structure 2

b) Structure 1



4. b) [Cont'd]
Structure 2

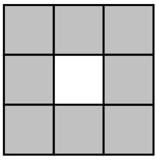


5. *Sample responses*:

a)

Front

b)



6. *Sample responses*:

a) Two different views of the structure:

Front

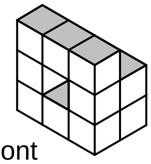
Back

b)

Front

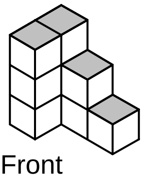
7. *Sample responses*:

a)



Front

b)



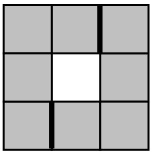
Front

8. *Sample responses*:

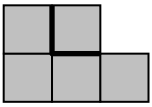
a)

Front

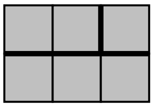
b)



Top view



Front view



Right view

9. a) A cube **b)** 27 cubes

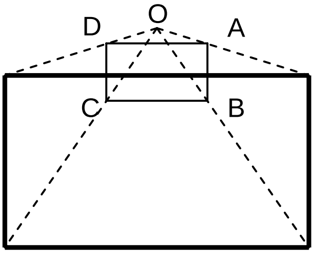
10. No

8.2.1 Dilatations pp. 237–238

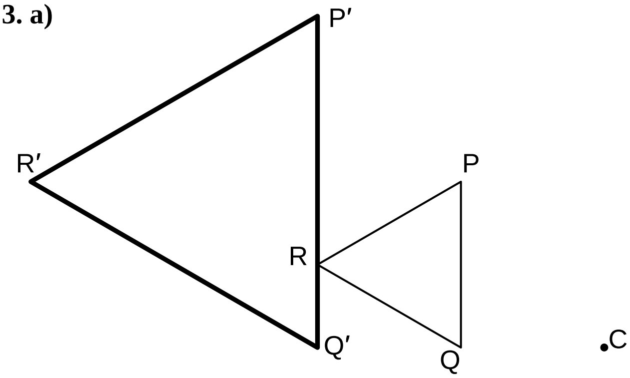
1. b) Dilatation centre A, scale factor $1\frac{1}{2}$

2. a) B

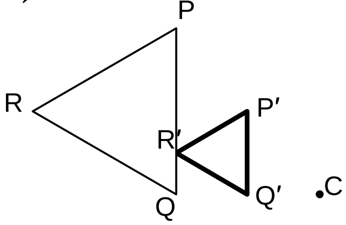
b) The dilatation centre is O; the scale factor is 3.



3. a)

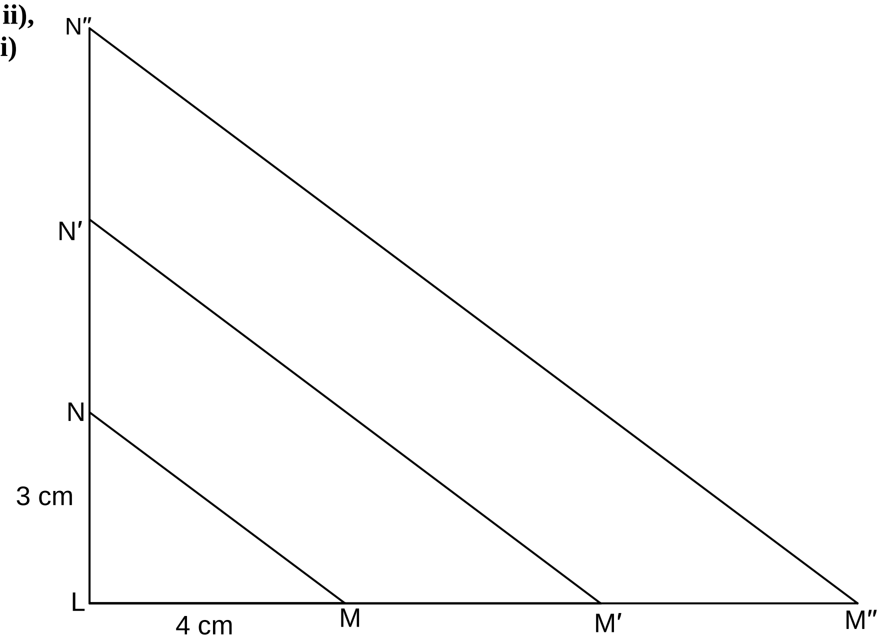


b)



8.2.1 Dilatations [Continued] **pp. 237–238**

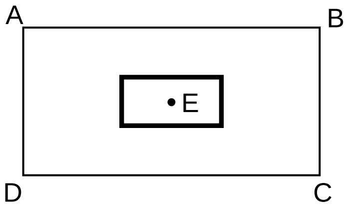
4. a), c) ii), and d) ii)



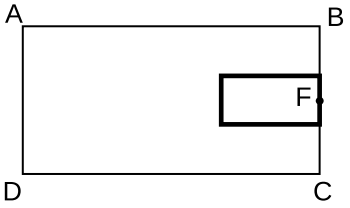
b) $A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6\text{ cm}^2$

c) i) *Sample response*: I predict the area will be 2 times the area, or 12 cm^2.
ii) The area is 24 cm^2, which is 4 times the area.
d) i) *Sample response*: I predict the area will be 3 times the area, or 18 cm^2.
ii) The area is 54 cm^2, which is 9 times the area.
e) Multiply the area by the square of the scale factor.

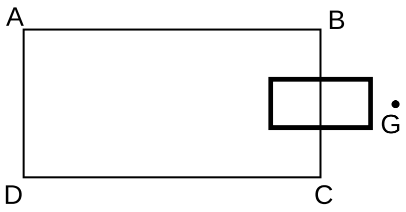
5. a)



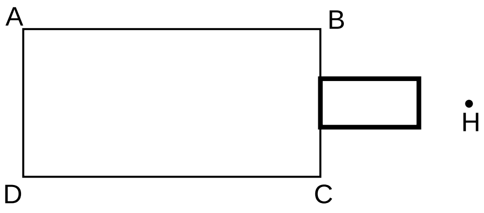
b)



c)

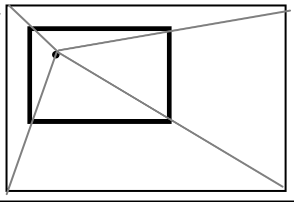


d)

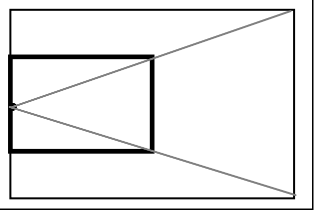


6. *Sample response*:

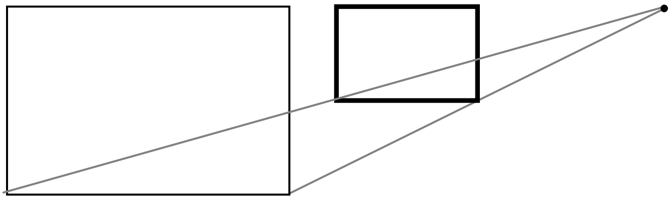
a) The dilatation centre is somewhere inside the rectangle.



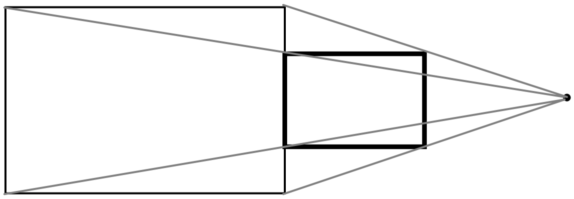
b) The dilatation centre is somewhere on the side of the rectangle.



c) The dilatation centre is somewhere outside both the rectangle and its image.



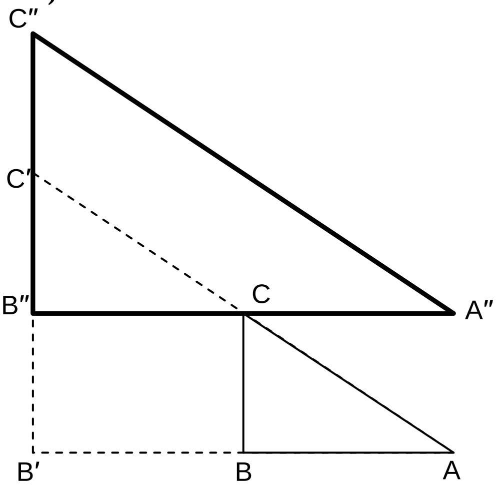
d) The dilatation centre is somewhere outside both the rectangle and its image.



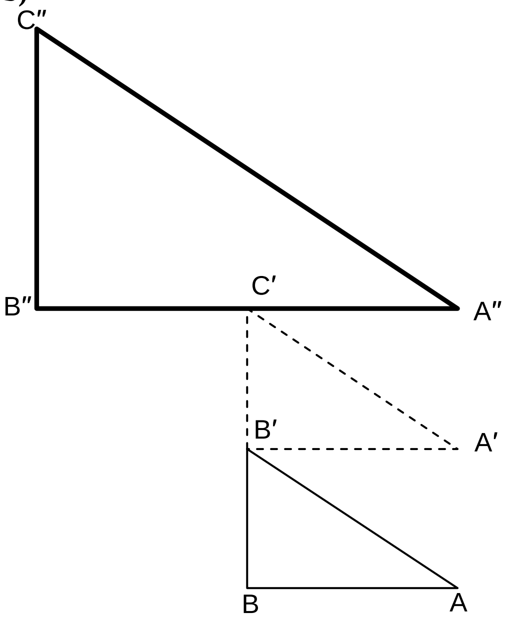
7. No

8.2.2 Combining Transformations pp. 242–243

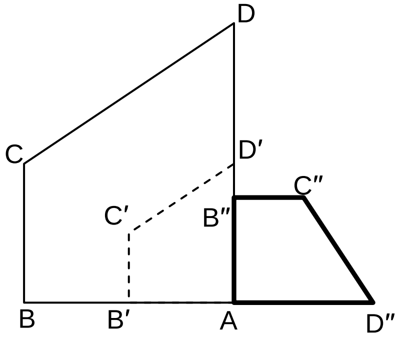
1. a)



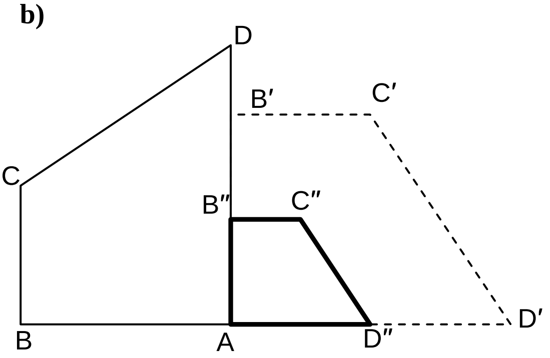
b)



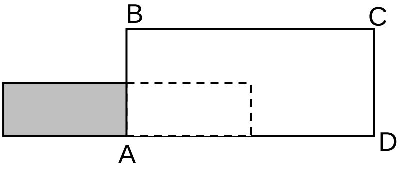
3. a)



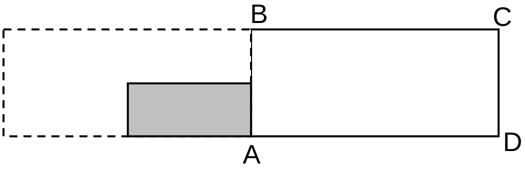
b)



5. a) A dilatation with scale factor $\frac{1}{2}$ and centre A results in the dashed rectangle. This is followed by a translation to the left half the length of DA.

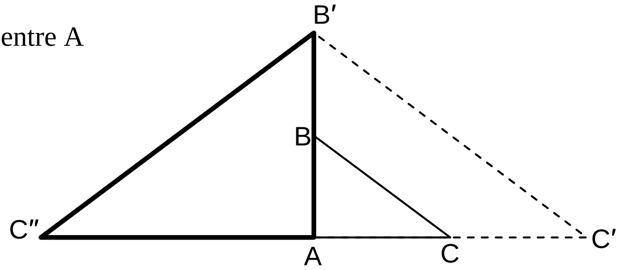


b) A translation to the left along DA results in the dashed rectangle. This is followed by a dilatation with scale factor $\frac{1}{2}$ and centre A.

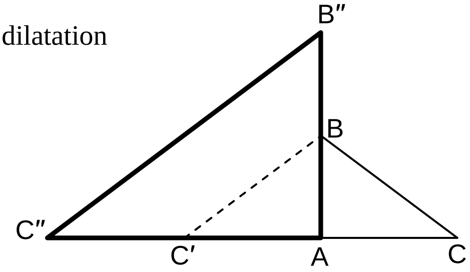


c) Yes

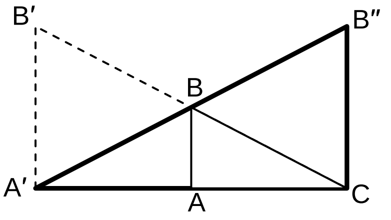
6. *Sample responses*:
a) A dilatation with scale factor 2 and centre A followed by a reflection in line AB.



A reflection in line AB followed by a dilatation with scale factor 2 and centre A.

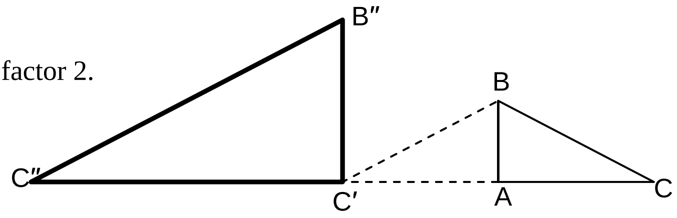


6. b) A dilatation with centre C and scale factor 2 followed by a reflection in line AB.

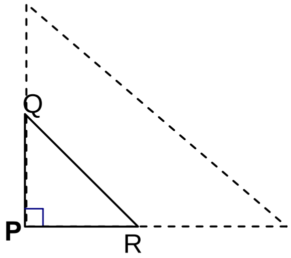


The final image of this combination in the opposite order is congruent, but in a different position.

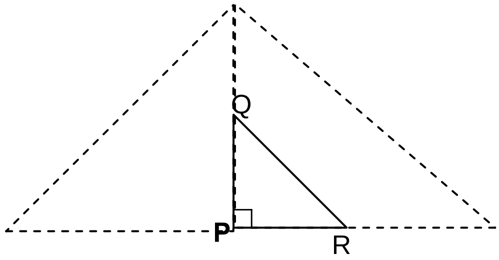
A reflection in line AB followed by a dilatation with centre C and scale factor 2.



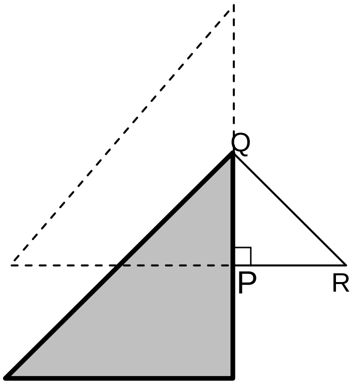
8. a) The triangle on the left
b) *Sample response*:
- Enlarge by a scale factor of 2 with centre P.



- Rotate 90° ccw around centre P.



8. b) [Continued]
- Translate down along QP.



8.3.2 Angles in Polygons pp. 249–250

1. a) 1080° **b)** 720° **c)** 720° **d)** 360°

2. a) 120° **b)** 90° **c)** 36° **d)** 30°

4. a) 60° **b)** 140° **c)** 135°

5. a) Yes **b)** No

6. An octagon

7. a) 22.5° **b)** 72° **c)** 135°

8. a) 140° **b)** 140° c

10. a) Regular polygons with these numbers of sides: 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360.
b) Regular polygons with these numbers of sides: 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360 (same as **part a)**).

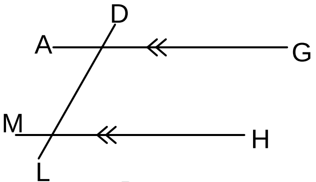
11. a) A regular polygon with 100 sides
b) An equilateral triangle

8.3.3 Angles With Parallel and Intersecting Lines pp. 254–255

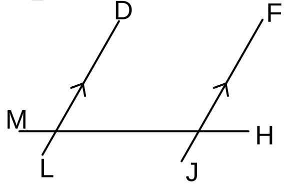
1. a) 135° **b)** 45°

2. a) 120° **b)** 60° **c)** 60° **d)** 120°

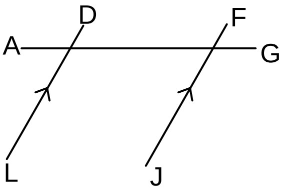
3. a) LD crosses parallel lines AG and MH.



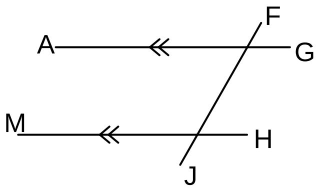
b) HM crosses parallel lines DL and FJ.



3. c) AG crosses parallel lines DL and FJ.



d) FJ crosses parallel lines AG and MH.



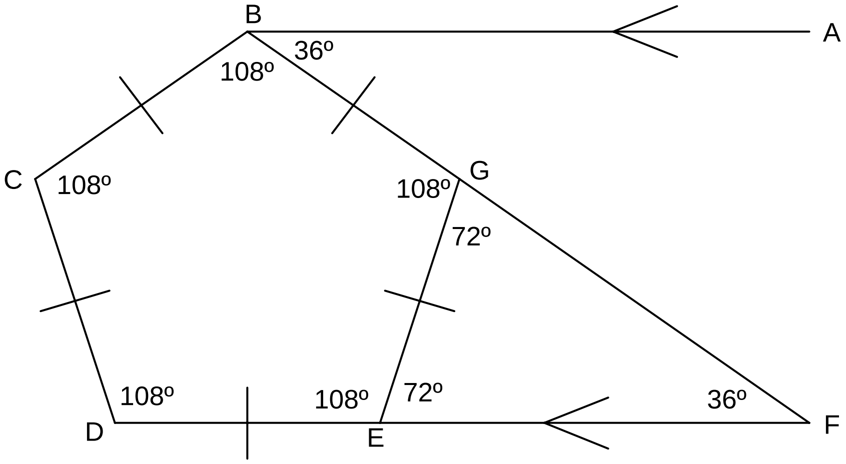
8.3.3 Angles With Parallel and Intersecting Lines [Cont'd] pp. 254–255

4. a) ∠ BKI, ∠ BEI, ∠ ABM, and ∠ LKM

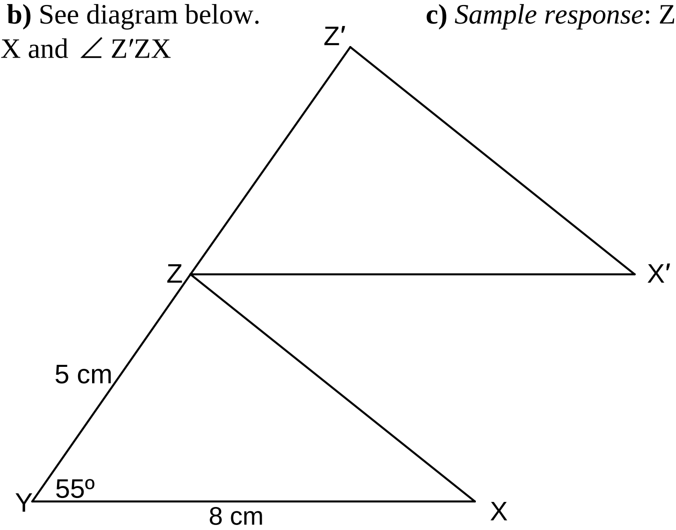
b) ∠ BKI, ∠ BEI, ∠ ABM, and ∠ LKM
c) *Sample response*:
The angles across from each other are equal.

5. a) ∠ LKI and ∠ GEI
b) ∠ MKB and ∠ BEF

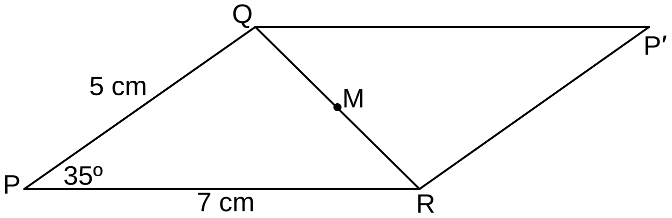
6.



7. a) and b) See diagram below.
c) *Sample response*: ZX′ ∥ YX
d) ∠ ZYX and ∠ Z′ZX

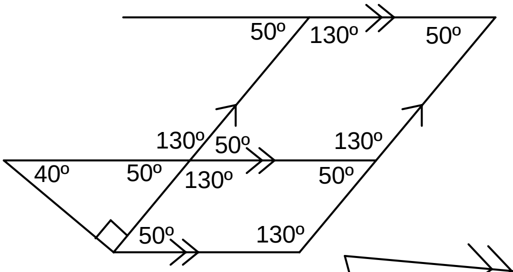


8. a), b), and c) See diagram below.
d) *Sample response*: PR ∥ QP′
e) ∠ QRP and ∠ RQP′



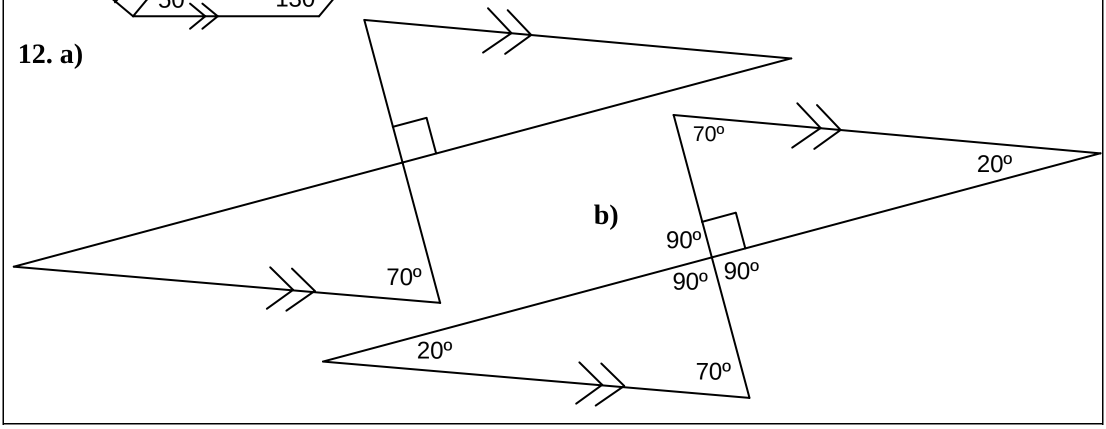
9. a) 133º **b)** 47º

10. *This diagram shows all possible angle measures*:



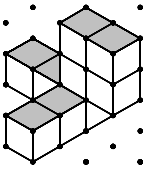
11. a) Yes **b)** Yes

12. a)



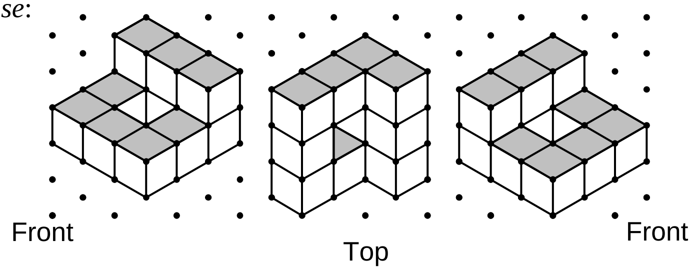
UNIT 8 Revision pp. 257–258

1. *Sample response*:



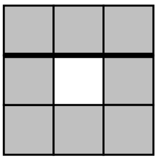
Back (but turned $\frac{1}{4}$ turn)

2. *Sample response*:

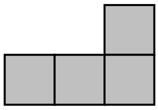


3. *Sample response*:

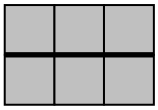
Top face view



Right face view

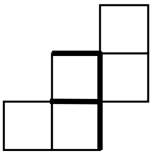


Back face view

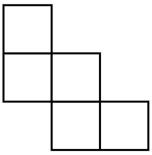


4. *Sample response*:

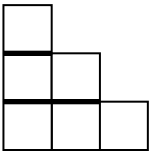
Right face view



Left face view



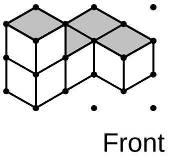
Front face view



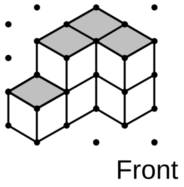
UNIT 8 Revision [Continued] **pp. 257–258**

5. *Sample responses*:

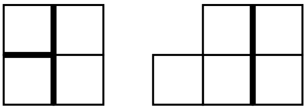
a) Structure A



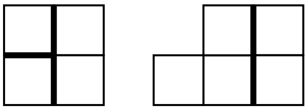
Structure B



b) Structure A

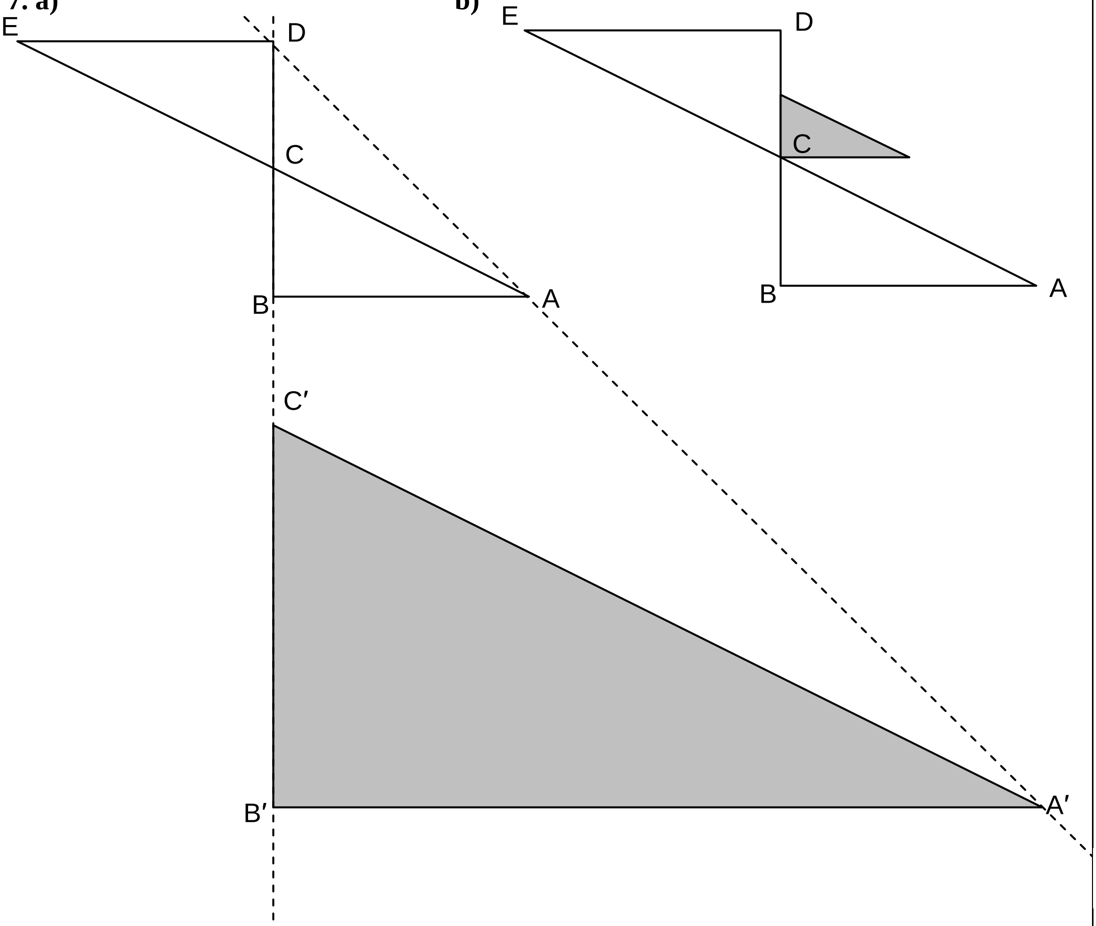

Left face view Front face view

Structure B


Left face view Front face view

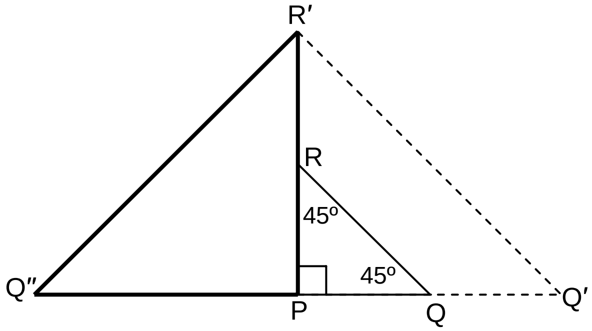
6. A

7. a) **b)**

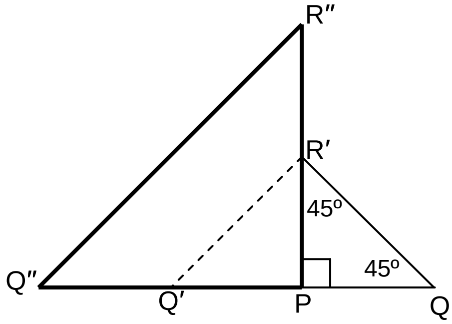


c) *Sample response*: Any dilatation of this triangle with centre A will have a vertex at A.

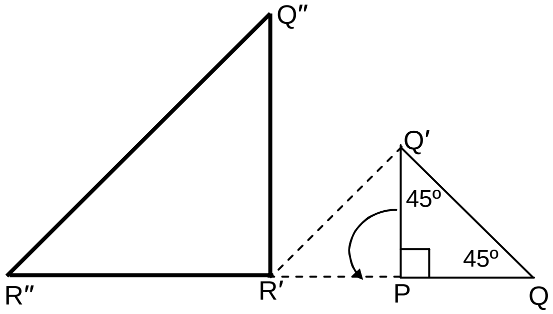
8. a)



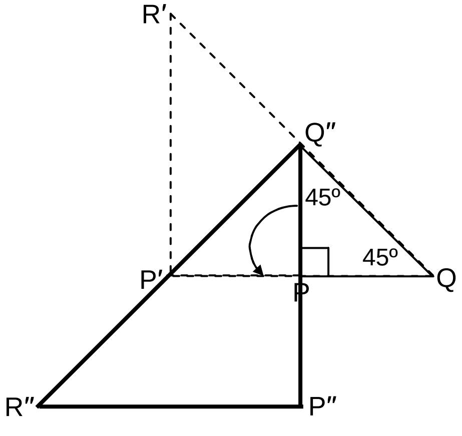
b) Yes.



c) If the centre of the dilatation is Q, then the order matters:
Rotation with centre P followed by dilatation with centre Q:



Dilatation with centre Q followed by rotation with centre P:



UNIT 8 Revision [Continued] **pp. 257–258**

9. *Sample response*:
A rotation of 180º (cw or ccw) around C followed by a dilatation with centre D′ and scale factor 3.

10. a) 540º
b) 1260º
c) 1800º

11. a) 135º
b) 40º
c) 36 sides

12. a) $\angle 4$
b) $\angle 7$
c) $\angle 3$

13. a) 63º
b) 105º
c) 75º
d) 42º
e) 42º
f) 138º

GLOSSARY

Instructional Terms

calculate: Figure out the number that answers a question; compute

compare: Look at two or more objects or numbers and identify how they are the same and how they are different; e.g., compare the numbers 6.5 and 5.6; compare the size of the students' feet; compare two shapes

conclude: Judge or make a decision after looking at all the data

construct: Draw using only a compass and straight edge

create: Make your own example or problem

describe: Tell, draw, or write about what something is or what something looks like; tell about a process in a step-by-step way

determine: Decide what the answer or result is for a calculation, a problem, or an experiment

draw: 1. Show something using a picture **2.** Take out an object without looking; e.g., draw a card from a deck

estimate: Use what you know to make a sensible decision about an amount; e.g., estimate how long it takes to walk from your home to school; estimate how many leaves are on a tree; estimate the sum of 3210 + 789

evaluate: 1. Determine whether something makes sense; judge **2.** Calculate the value as a number; e.g., evaluate the expression $m + 3$ for $m = 5$

explain (your thinking): Tell what you did and why you did it; write about what you were thinking; show how you know you are right

explore: Investigate a problem by questioning and trying new ideas

justify: Give convincing reasons for a prediction, an estimate, or a solution; tell why you think your answer is correct

measure: Use a tool to tell how much; e.g., use a ruler to measure a height or distance; use a protractor to measure an angle; use balance scales to measure mass; use a measuring cup to measure capacity; use a stopwatch to measure elapsed time

model: Show an idea using objects, pictures, words, and/or numbers; e.g., you can model a polynomial expression using algebra tiles:

Modelling $-2x + 2$ with algebra tiles

predict: Use what you know to figure out what is likely to happen; e.g., predict the number of times you will roll a sum of 5, when you roll two dice 30 times

relate: Describe how two or more objects, drawings, ideas, or numbers are similar

represent: Show information or an idea in a different way; e.g., draw a graph of an equation; make a model from a word description; create an expression to model a situation

show (your work): Record all the calculations, drawings, numbers, words, or symbols that you used to calculate an answer or to solve a problem

simplify: Write a number or expression in a simpler form; e.g., combine like terms of a polynomial, write an equivalent fraction with a lower numerator and denominator

sketch: Make a quick drawing to show your work; e.g., sketch a picture of a field with given dimensions

solution: The complete answer to a calculation or problem, showing all the work involved to get the answer

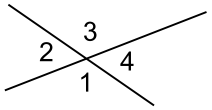
solve: Find an answer to a problem

visualize: Form a picture in your head of what something is like; e.g., visualize the number 6 as 2 rows of 3 dots like you would see on a die

Definitions of Mathematical Terms

A

adjacent angles: When two lines intersect, angles that are beside each other and share a vertex add to 180º and are called adjacent angles



Adjacent angles in this diagram are:

∠1 and ∠2 ∠3 and ∠4

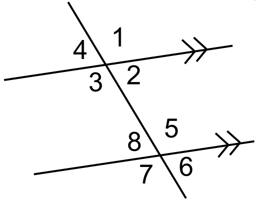
∠1 and ∠4 ∠2 and ∠3

algebraic equation: An equation that includes an algebraic expression and an equals sign; e.g., $3x + 5 = 8$

algebraic expression: A combination of one or more terms with at least one variable; it may include numbers and operation signs; e.g., $8x + 9$

algorithm: A specific set of instructions or procedure for finding a solution to a problem or an answer to a calculation

alternate angles: When a third line, a transversal, crosses two parallel lines, alternate angles are formed between the parallel lines and on opposite sides of the transversal; alternate angles are equal



Alternate angles in this diagram are:

∠2 = ∠8 ∠3 = ∠5

angle: A figure formed by two arms with a shared endpoint, or vertex; the measure of an angle is the amount of turn between the two arms; angles are often measured in degrees (º)

annual interest rate: See *Financial Terms* on **page 274**

anticlockwise: See *countercockwise*

area: The number of square units needed to cover a shape; often measured in square centimetres or square metres

C

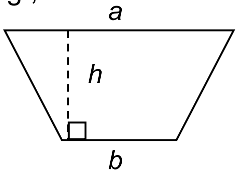
associative property (of multiplication): When you change the grouping of the factors in a multiplication, the product does not change; e.g., $(2 × 3) × 4 = 2 × (3 × 4)$

average: Average is a term we can use instead of the term mean See *mean*

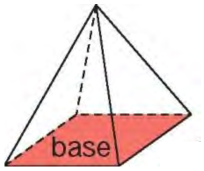
average rate: A rate expressed as a unit rate; an average rate assumes that the rate stays the same over the entire time period; e.g., if someone walks 15 km in 5 h, the average rate is 3 km in 1 h, or 3 km/h

B

base: 1. The number that is repeatedly multiplied in a power; e.g., in the power 5^3, 5 is the base **2.** In a 2-D shape, the line segment(s) that is perpendicular to the height **3.** In a 3-D shape, the face(s) that determines the name of a prism or pyramid; e.g.,

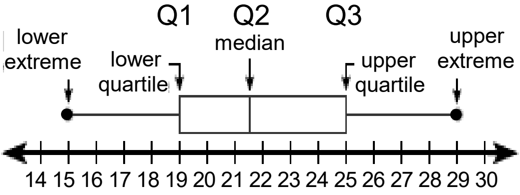


A trapezoid has two bases, *a* and *b*



A square-based pyramid

box and whisker plot: A graph that uses the median (Q2), the extremes, and the lower and upper quartiles (Q1 and Q3) to organize data into four groups or quartiles; each quartile has an equal number of data values



box plot: See *box and whisker plot*

common denominator: A common

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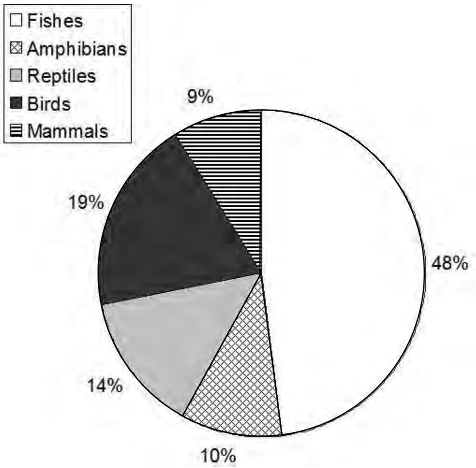
capacity: The amount that a container can hold, often measured in millilitres (mL) and litres (L)

census: A survey of an entire population; e.g., a government conducts a census of its people to collect information for making decisions about making laws and spending tax money

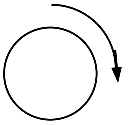
circle graph: A graph that shows how a complete set of data is broken into categories, each represented by a sector of a circle; e.g.,

Known Vertebrate Species



circumference: **1.** The boundary or perimeter of a circle **2.** the length of the boundary of a circle calculated using the formula $C = 2 \times \pi \times r$, where r is the radius, or $C = \pi \times d$, where d is the diameter See *circle*

clockwise (cw): The direction that the hands of a clock move; describes the direction of a rotation



coefficient: The number by which a variable is multiplied; e.g., in the term $3z$, the coefficient is 3

commission: See *Financial Terms* on **page 274**

multiple of the denominators of two or more fractions; e.g., you can use the common multiple of 6 for 2 and 3 to create fractions with a common denominator and then add them:

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

common factor: A whole number that divides into two or more other whole numbers with no remainder; e.g., 4 is a common factor of 8 and 12 because $8 \div 4 = 2$ and $12 \div 4 = 3$

common multiple: A whole number that is a multiple of two or more given whole numbers; e.g., 12, 24, and 36 are common multiples of 4 and 6

commutative property: When you change the order of numbers in a calculation, the result does not change; both addition and multiplication have the commutative property; e.g., $3 + 4 = 4 + 3$ and $3 \times 4 = 4 \times 3$

complement of an event: All possible outcomes that are not part of an event; sometimes called "Not E" as in P(E) + P(Not E) = 1; e.g., when you find the probability of rolling a prime number on a die, the event consists of the outcomes 2, 3, 5, and the complement of the event consists of the outcomes 1, 4, 6

congruent: Identical in size and shape; shapes, side lengths, and angles can be congruent

consecutive numbers: Whole numbers that follow each other; e.g., 3, 4, and 5 are consecutive numbers

constant: A numerical value that does not change **1.** Any number is a constant, whether a whole number or not, e.g., –2, 8, and π are all constants
2. In an algebraic expression, equation, or formula, a constant is a value that does not change when the variable changes; in algebraic expressions the term is usually used only for constants that are not coefficients, e.g., in $y = 3x + 7$, the constant is 7, not 3 (even though both 3 and 7 do not change)

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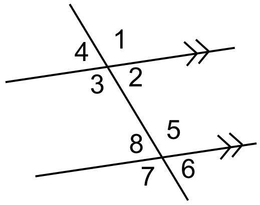
convert currency: See *Financial Terms* on **page 274**

corresponding angles: 1. When a third line (a transversal) crosses two parallel lines, corresponding angles are in the same position along the transversal; corresponding angles are equal



Corresponding angles in this diagram are:

$\angle 1 = \angle 5$ $\angle 2 = \angle 6$

$\angle 4 = \angle 8$ $\angle 3 = \angle 7$

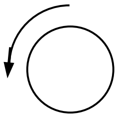
2. In two similar shapes, each angle in one of the shapes relates to, or matches an angle in the other shape; corresponding angles are equal

3. In a transformation, each angle in the original shape relates to, or matches an angle in the image; corresponding angles are equal

corresponding sides: 1. In two similar shapes, each side in one of the shapes relates to, or matches a side in the other shape; all corresponding sides have the same ratio **2.** In a transformation, each side in the original shape relates to, or matches a side in the image; corresponding sides are equal in translations, reflections, and rotations; in dilatations, corresponding sides have the same ratio

cost price: See *Financial Terms* on **page 274**

counterclockwise (ccw): The direction opposite to the direction the hands of a clock move; sometimes called anticlockwise; describes the direction of a rotation



credit: See *Financial Terms* on **page 274**

cuboid: Another name for a rectangular prism See *rectangular prism*

currency: See *Financial Terms* on **page 274**

dividend: A number that is being divided;

D

data: Information collected in a survey, in an experiment, or by observing; the word data is plural, not singular; e.g., a set of data can be a list of students' names or it can be a set of marks for a quiz

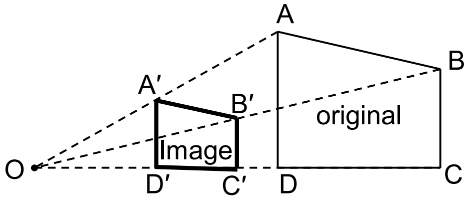
decagon: A 10-sided polygon

denominator: The number in a fraction that represents the total number of parts in a whole set or the number of parts the whole has been divided into; e.g., in $\frac{4}{5}$, the denominator is 5

diameter: 1. A line segment that joins two points on a circle and passes through the centre **2.** The length of the line segment described in **1** See *circle*

difference: The result of a subtraction; e.g., in 45 – 5 = 40, the difference is 40

dilatation: A transformation that enlarges or reduces a figure by a scale factor; lines that join corresponding points on the original and the dilatation image meet at the centre of dilatation (marked O below)



dilatation centre: See *dilatation*

dilatation image: See *dilatation*

direction of rotation: A rotation can be clockwise or counterclockwise See *rotation, clockwise*, and *counterclockwise*

dimension: The size or measure of an object, usually length; e.g., the width and length of a rectangle are its dimensions

discount: See *Financial Terms* on **page 274**

distribution of data: A description of a set of data that tells its range and how the data values are clustered

distributive property: When one operation is distributed over another operation, the answer does not change; e.g., multiplication over addition: 2(4 + 1) = 2 × 4 + 2 × 1; multiplication over subtraction: 2(4 – 1) = 2 × 4 – 2 × 1

expanded form: A way of writing

e.g., in 45 ÷ 5 = 9, the dividend is 45

divisor: The number by which another number is divided; e.g., in 45 ÷ 5 = 9, the divisor is 5

dodecagon: A 12-sided polygon

E

equally likely: If two events have the same probability, they are equally likely to happen; if one event has a probability of $\frac{1}{2}$, it is equally likely to happen as not to happen

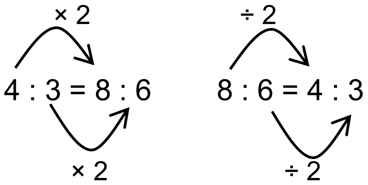
enlargement: See *dilatation*

equation: A mathematical statement in which the value on the left side of the equals sign is the same as the value on the right side of the equals sign; e.g., the equation $5n + 4 = 39$ means that 4 more than the product of 5 and a number equals 39

equivalent fractions: Fractions that represent the same part of a whole or set; e.g., $\frac{2}{4}$ is equivalent to $\frac{1}{2}$

equivalent rates: Rates that describe the same relationship; you can find an equivalent rate by multiplying or dividing each term by the same number; e.g., a rate of 26 km in 2 days is equivalent to a rate of 52 km in 4 days or 13 km in 1 day

equivalent ratios: Ratios that makes the same comparison; you can find an equivalent ratio by multiplying or dividing each term by the same number; e.g., 4 : 3 and 8 : 6 are equivalent ratios



event: A set of outcomes for a probability experiment; e.g., if you roll a die with the numbers 1 to 6, the event of rolling an even number has the outcomes 2, 4, and 6

factor: **1.** One of the numbers you multiply in a multiplication operation;

a number that shows the place value of each digit; e.g., 1209 in expanded form is 1 × 1000 + 2 × 100 + 9 × 1 or 1 thousand + 2 hundreds + 9 ones

expenses: See *Financial Terms* on **page 274**

experimental probability: The probability of an event based on the results of an experiment with many trials; it is calculated using this expression:

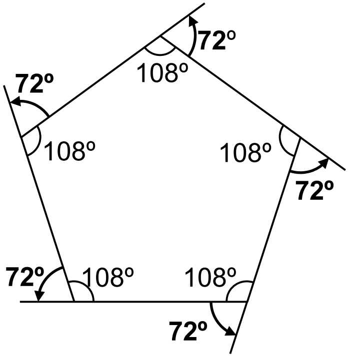
$$\frac{\text{Number of favourable results}}{\text{Number of trials}}$$

exponent: A superscript in mathematics that denotes repeated multiplication; e.g., 4^3 means $4 \times 4 \times 4$ since the exponent is 3; sometimes called a power or an index

exponential form: A way of writing a number that shows the value of each digit as a power of 10; e.g., in exponential form, 1209 is $1 \times 10^3 + 2 \times 10^2 + 9 \times 1$

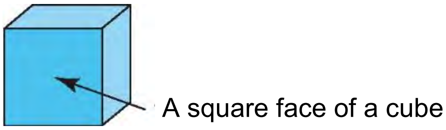
expression (numerical): A combination of numbers and operations; e.g., 3 + 5 × 6.3 See *algebraic expression*

exterior angle (of a polygon): An angle outside a polygon created by extending a side length; e.g., the regular pentagon below has five exterior angles of 72º



F

face: A 2-D shape that forms a flat surface of a 3-D object; e.g.,



face view: See orthographic drawings

G

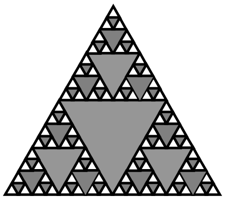
e.g., 3 and 4 are the factors in 3 × 4 = 12
2. A number that divides into another number with no remainder; e.g., the factors of 24 are 1, 2, 3, 4, 6, 8, and 12

favourable outcome: The desired outcome when you calculate a theoretical probability; e.g., when you find the theoretical probability of rolling a number less than 3 on a die, rolls of 1 and 2 are favourable outcomes

favourable result: The desired result when you calculate an experimental probability; e.g., when you find the experimental probability of rolling an even number on a die, rolls of 2, 4, and 6 are favourable results

formula: A general rule stated in mathematical language; e.g., the formula for the area of a rectangle is
Area = length × width, or $A = lw$

fractal: A geometric shape that is subdivided into parts, each of which is a reduced copy of the shape; e.g.,



frequency: The number of times a data value or range of data values occurs in a data set; e.g., in the frequency table below, ages between 0 and 11 happened 50 times so the frequency is 50

frequency table: A table that organizes a set of data into intervals and indicates the number of times data values occur in each interval

Age	Frequency
0 – 11	50
11 – 22	300
22 – 33	250
33 – 44	400
44 – 55	550

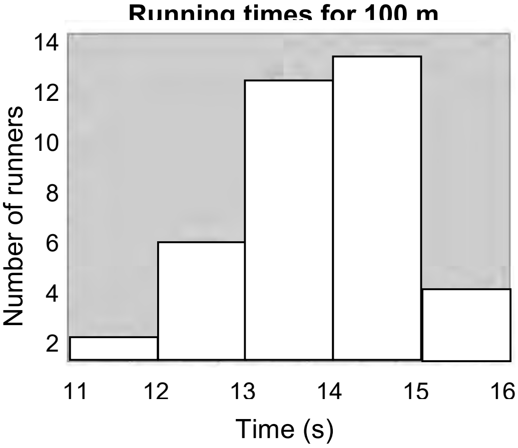
graph: A picture of a set of data or a mathematical relationship between two sets of data; e.g., when you create a box and whisker plot from a set of data, you create a graph of the data set; when you plot the ordered pairs in a table of values, you create a graph of the relationship between the two data sets in the table

greatest common factor (GCF): The greatest whole number that divides into two or more other whole numbers with no remainder; e.g., 4 is the greatest common factor of 8 and 12

H

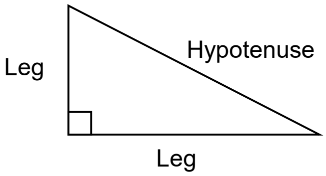
hexagon: A 6-sided polygon

histogram: A graph with vertical or horizontal bars that show frequencies of data organized into intervals; the bars line up side by side without gaps on the scale because there are no gaps between the intervals of data; e.g., the histogram below shows the number of runners that ran 100 m in different amounts of time



horizontal axis: See *x-axis*

hypotenuse: The side that is opposite to the right angle in a right triangle



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I

image: The new shape that results when you apply a transformation to a shape See *dilatation*, *reflection*, *rotation*, and *translation*

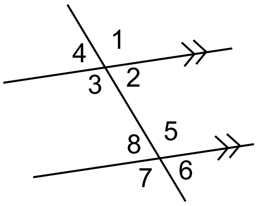
improper fraction: A fraction in which the numerator is greater than or equal to the denominator; e.g., $\frac{5}{4}$ and $\frac{6}{6}$

integers: The set of whole numbers and their opposites (zero is its own opposite): …, –2, –1, 0, 1, 2, …

interest: See *Financial Terms* on **page 274**

interior angle (of a polygon): An angle inside a polygon formed by two adjacent sides of the polygon

interior angles (of parallel lines and a transversal): When a third line (a transversal) crosses two parallel lines, interior angles are formed between the parallel lines on the same side of the transversal; interior angles add to 180º



Interior angles in this diagram are

$\angle 2 + \angle 5 = 180^\circ$ $\angle 3 + \angle 8 = 180^\circ$

intersect: When two lines, line segments, or graphs meet or cross, they intersect

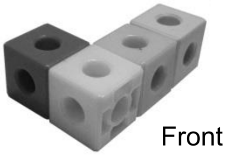
intersection point (of two graphs or equations): The point where two graphs cross or meet; when two equations are graphed, it is the point where the two graphs intersect

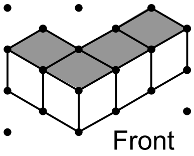
interval: A range of values, often used in creating a histogram; e.g., 0–10 is the interval from 0 to 10 See *frequency table* and *histogram*

inverse operation: An operation that "undoes" another operation, often used in solving an equation; e.g., addition is the inverse of subtraction

invest: See *Financial Terms* on **page 274**

isometric drawing: A 2-D drawing of a 3-D object where parallel edges of the object look parallel in the drawing and equal distances of the object are drawn as equal; e.g.,

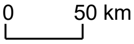




isosceles right triangle: A right triangle with two congruent sides

K

key (of map or scale drawing): A statement, diagram, or ratio that shows how a measurement on a map or drawing relates to the real measurement; e.g., a key of "1 cm represents 50 km" can be represented by the diagram shown below or as the scale ratio 1 : 5,000,000



L

legs: See *hypotenuse*

like terms: Terms of an expression that have the same variable raised to the same power but that may have different coefficients; e.g., in $2x + 6x + 5$, the like terms are $2x$ and $6x$

linear equation: An algebraic equation that represents a linear relationship; when graphed, a linear equation forms a straight line graph; each variable in a linear equation has an exponent of 1; e.g., $3x + 2 = 8$ is a linear equation (because $x^1 = x$)

linear relationship: A relationship between two variables that forms a straight line when graphed because a constant increase in one variable results in a constant increase in the other variable

linear polynomial: An algebraic expression that includes a variable with an exponent of 1 and no other powers; it usually involves more than one term; e.g., $2x$, $3x + 7$, and $2n - n + 2$ are linear polynomials (because $x^1 = x$ and $n^1 = n$)

linear unit: A unit for measuring a straight length or distance; perimeter uses a linear unit because it is a combination of several lengths or distances; e.g., metres and kilometres are linear units

line segment: A part of a line; it consists of two end points and all the points in between

lower quartile: See *box and whisker plot*

lowest common multiple: The least multiple that is common to two or more numbers; e.g., the lowest common multiple of 2 and 3 is 6:

Multiples of 2: 2, 4, **6**, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, ...

Multiples of 3: 3, **6**, 9, 12, 15, 18, 21, 24, 27, 30, ...

lowest terms: 1. When a fraction is in lowest terms, the only common factor of the numerator and the denominator is 1; e.g., $\frac{5}{10}$ is $\frac{1}{2}$ in lowest terms

2. When a ratio is in lowest terms, the only common factor of the terms is 1; e.g., 12 : 9 in lowest terms is 4 : 3

M

markdown: See *Financial Terms* on **page 274**

markup: See *Financial Terms* on **page 274**

mass: The measure of the amount of matter in an object; mass is measured in units such as grams and kilograms

mean: A single number that represents all the values in a data set; to calculate the mean, you add the values together and then divide the total by the number of values in the set; it is often called the average; e.g., the mean of the data set 3, 4, 5, and 6 is (3 + 4 + 5 + 6) ÷ 2 = 4.5

median: The middle value of a set of data arranged in order. If there is an even number of values in the set, the median is the mean of the two middle values; e.g., in the data set below, the median is 10:

1 7 9 11 11 13

The median is the mean of 9 and 11

In a box plot, the median is also called the middle, or second quartile, Q_2
See *box and whisker plot*

mixed number: A number made up of a whole number and a proper fraction; e.g., $5\frac{1}{7}$

multiple: The product of a whole number and any other whole number; e.g., when you multiply 10 by the whole numbers 0, 1, 2, 3, and 4, you get the multiples 0, 10, 20, 30, and 40

multiplier: See *scientific notation*

N

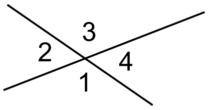
negative: 1. A negative integer is an integer less than 0: …, –4, –3, –2, –1
2. A negative rational number is a number less than 0 that can be represented by the quotient of two integers; e.g., -1.7, $-2\frac{1}{2}$, and -7 are all negative rational numbers

numerator: The number in a fraction that shows the number of parts of a given size the fraction represents; e.g., in $\frac{4}{5}$, the numerator is 4

O

octagon: An 8-sided polygon

opposite angles: When two lines intersect, angles that are across from each other and share a vertex are called opposite angles; opposite angles are equal



Opposite angles in this diagram are:

$\angle 1$ and $\angle 3$ $\angle 2$ and $\angle 4$

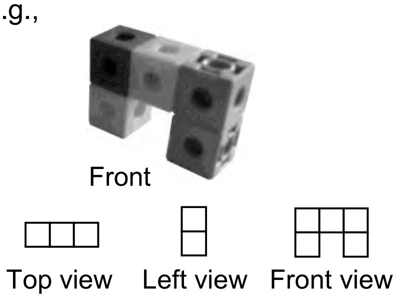
opposite integers: Two integers that are the same distance away from zero in opposite directions; e.g., 6 and –6 are opposite integers

order of operations (rules): Rules that describe the sequence to use to evaluate an expression:

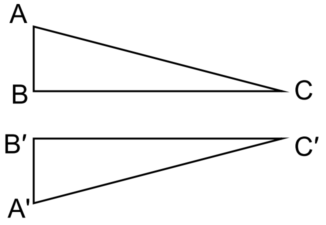
1 Evaluate within brackets
2 Divide and multiply from left to right
3 Add and subtract from left to right

ordered pair: A pair of numbers in a particular order that describe the location of a point in a coordinate grid; e.g., the ordered pairs (3, 5) and (5, 3) describe the locations of two different points on the grid shown on **page 273** for *x-axis*

orthographic drawings: A set of several different face views of a 3-D object; e.g.,



orientation: The direction around a shape when you name the vertices in order, clockwise or counterclockwise; e.g., the orientation of the vertices of ∆ABC is counterclockwise, but the orientation of the vertices of its reflection image, ∆A′B′C′, is clockwise



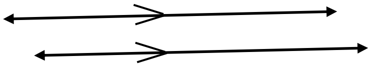
Original Shape: The shape you begin with in a transformation, sometimes called the pre-image

outliers: Data values that are much lower or much higher than the other data values in the set; e.g., the values 3, 23, and 24 appear to be outliers in this set of data:
3 11 11 13 13 13 15 15 23 24

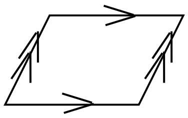
plot (a point): Locate a point on a graph

P

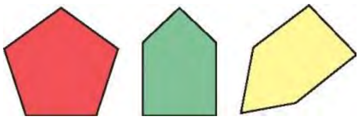
parallel (lines or line segments): When lines or line segments are always the same distance apart and never meet, they are parallel; matching symbols, such as << and << or > and > on two lines indicate that they are parallel; e.g.,



parallelogram: A quadrilateral with pairs of opposite sides that are parallel; e.g.,



pentagon: A polygon with five sides; a regular pentagon has five congruent sides and five congruent angles; e.g.,



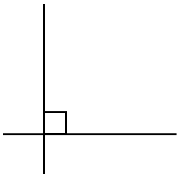
These are all pentagons. The first one is a regular pentagon.

percent: A special ratio that compares a number to 100 using the symbol %; e.g., if 3 out of 4 students are girls, then 75% are girls because $\frac{3}{4} = \frac{75}{100} = 75\%$

perfect square: The product of a whole number multiplied by itself; e.g., 25 is a perfect square because 5 × 5 = 25

perimeter: 1. The boundary or outline of a 2-D shape **2.** The length of the boundary

perpendicular: At a right angle; e.g., two line segments that are at a right angle to each other are perpendicular



π (pi): The result of dividing the circumference of any circle by its diameter; it has a value of 3.141592654 …, or about 3.14 or $\frac{22}{7}$

probability: A number from 0 (will never a coordinate grid using its coordinates

polygon: A closed 2-D shape with three or more sides; e.g., triangle, quadrilateral, pentagon, and so on

population: The entire group of subjects that you are interested in collecting data about; e.g., for collecting data about the favourite type of momo of students at a school, the population is all of the students in the school

positive: 1. A positive integer is an integer greater than 0: 1, 2, 3, 4, 5, …
2. A positive rational number is a number greater than 0 that can be represented by the quotient of two integers; e.g., 1.7, 7, and $2\frac{1}{2}$ are all positive rational numbers

possible outcome: A thing that could happen in a probability situation; e.g., when you roll a die, there are six possible outcomes: 1, 2, 3, 4, 5, and 6

power: A numerical expression that shows repeated multiplication; a power has a base and an exponent: the exponent tells how many equal factors there are in a power; sometimes the exponent is also called the power;
e.g., the power 5^3 is a shorter way of writing $5 \times 5 \times 5$:

3 is the exponent of the power

↓

$5^3 = 125$

↑

5 is the base of the power

power of 10: A number that can be represented by a power with a base of 10; e.g., 100 is a power of 10 because $100 = 10^2$

prime factors: The factors of a number that are prime numbers; usually written as a product; e.g., the prime factors of 24 are $2 \times 2 \times 2 \times 3$

principal: See *Financial Terms* on **page 274**

random numbers: A series of numbers that has no predictable pattern

happen) to 1 (certain to happen) that tells how likely it is that an event will happen; it can be a decimal or fraction; sometimes it is called chance

product: The result of multiplying numbers; e.g., in $5 \times 6 = 30$, the product is 30

profit: See *Financial Terms* on **page 274**

proper fraction: A fraction in which the denominator is greater than the numerator; e.g., $\frac{1}{7}, \frac{4}{5}, \frac{29}{40}$

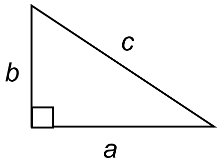
proportion: A statement that shows two equivalent ratios or equivalent rates is called a proportion; e.g.,

$3 : 4 = 18 : 24$ or $\frac{3}{4} = \frac{18}{24}$

protractor: A tool used to measure the size of an angle

Pythagorean theorem: The Pythagorean theorem states that this relationship exists among the three sides of a right triangle: the sum of the squares of the lengths of the two shorter sides is equal to the square of the length of the hypotenuse

$c^2 = a^2 + b^2$



Pythagorean triple: A set of three whole numbers that makes the Pythagorean theorem true; e.g., 3, 4, and 5 is a Pythagorean triple because $3^2 + 4^2 = 5^2$

Q

quartiles: See *box and whisker plot*

quotient: The result of dividing one number by another number; e.g., in $45 \div 5 = 9$, the quotient is 9

R

radius (plural is radii)**:** The name and length of the line segment that joins the centre of a circle to any point on its circumference See *circle*

reduction: See *dilatation*

reflection: A transformation that

random number table: A table of random numbers, sometimes use to create a random sample

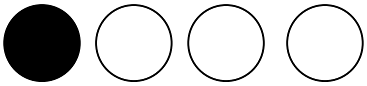
random sample: A sample chosen so that each member of the population has an equal chance of being selected to be part of the sample; e.g., to choose a random sample of five students from a class, put all the students' names into a bag, mix them up, and draw five names without looking

range: The difference between the extremes (minimum and maximum) of a set of data

rate: A comparison of two quantities measured in different units; unlike ratios, rates include units; e.g., 45 km/h

rate of exchange: See *Financial Terms* on **page 274**

ratio: A number or quantity compared with another, expressed in symbols as $a : b$ or $\frac{a}{b}$; it can be a part-to-part comparison or a part-to-whole comparison; e.g., all three ratios describe the set of counters below

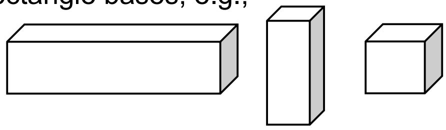


1 : 3, 1 : 4, and 3 : 4

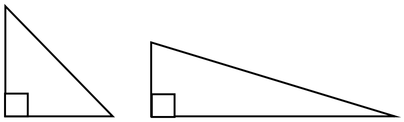
rational numbers: A rational number is a number that can be written as a quotient of two integers; the divisor cannot be zero; e.g., $\frac{3}{4}$ (0.75) and $\frac{-2}{3}$ (–0.666...) are rational numbers

reciprocal: The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$; e.g., multiplying by the reciprocal is a way to divide fractions: $3 \div \frac{2}{3} = 3 \times \frac{3}{2}$

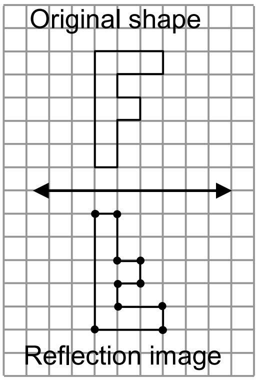
rectangular prism: A prism with rectangle bases; e.g.,



right triangle: A triangle with one right angle; e.g.,

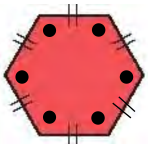


produces a mirror image of a shape across a reflection line; also called a flip; e.g., this is a reflection of the F-shape across a horizontal refection line:



reflection line: See *reflection*

regular polygon: A polygon with congruent sides and congruent angles; e.g.,



A regular hexagon

relation/relationship: A property that connects two sets of numbers or two variables; a relation can be represented mathematically as a table of values, a graph, or an equation; e.g., in the pattern below, the term value and the term number are related:

4, 7, 10, 13, ...

Term number	Term value
1	4
2	7
3	10
4	13

The term value = 3 × term number + 1

$$v = 3n + 1$$

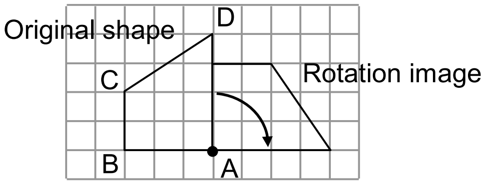
repeated addition: When the same number is added over and over again; multiplication is a way of doing repeated addition; e.g., 4 + 4 + 4 + 4 + 4 → 5 × 4

right angle: An angle that measures 90º; sometimes called a square corner See the right angles in *right triangle*

scale factor (of a proportion): The factor by which the corresponding terms in a proportion are related; e.g., the scale

rise: See *slope*

rotation: A transformation in which each point in a shape moves around a point (the turn centre) through the angle of rotation; e.g., this is a 90º cw rotation of trapezoid ABCD around vertex A:

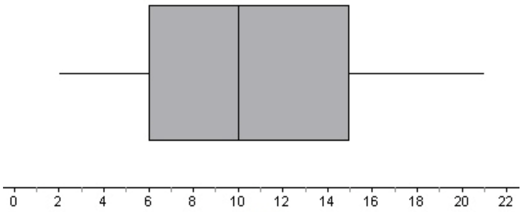


run: See *slope*

S

sample: If you cannot collect data from the entire population you are interested in, you can collect data from a carefully chosen random sample; e.g., to collect data about the favourite type of momo of all the students at a school, a good sample might be five students chosen randomly from each classroom

scale: 1. The numbers marked at regular integers along a number line or the axis of a graph; e.g., the scale of the box plot below goes from 0 to 22 **2.** The value of each interval on an axis; the scale tells how to interpret a graph; e.g., the scale of this box plot is 1 as it is marked in increments of 1:



scale drawing: A drawing that represents a real object or figure in two dimensions, usually at a reduced size, but sometimes at an enlarged size

sequence: A set of numbers, usually in a pattern, where each number can be identified by its position in the set

$$\frac{2}{3} = \frac{50}{\mathbf{75}}$$

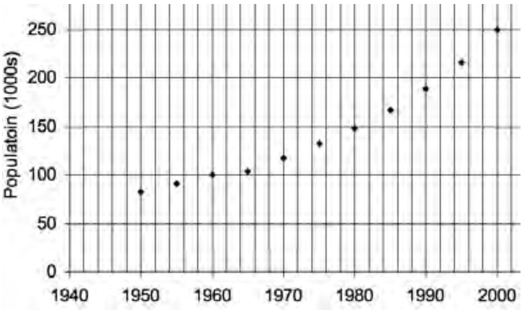
scale factor (of a dilatation): The number that describes how much the side lengths of a shape have changed; if the scale factor is between 0 and 1, it is a reduction; if it is greater than 1, it is an enlargement; if it is 1, the original shape and image are congruent

scale ratio (of a scale drawing or map): The relationship between a length on a scale drawing or map and the real object or distance, expressed as a ratio; e.g., a scale ratio of 1 : 20 means that 1 unit on a scale drawing of a figure represents 20 of the same units on the real figure

scalene triangle: A triangle with no congruent sides

scatter plot: A graph on a coordinate grid that can be used to see if there is a relationship between two variables; e.g., in the scatter plot below, there appears to be a relationship between time and population: as time progresses, the population grows

Population of Bhutan, Ages 10 to 14



scientific notation: A number written as the product of a multiplier, that is a whole number or decimal that is 1 or greater but less than 10, and a power of 10; e.g., $2300 = 2.3 \times 10^4$ and $0.009 = 9 \times 10^{-3}$

sector (of a circle): See *circle graph*

solve (an equation): Find the value of a variable in an equation; e.g., solve $3 + x = 7$ by finding the value of x that makes it true, which is 4

selling price: See *Financial Terms* on **page 274**

similar shapes: Shapes that are identical in shape and proportion, but not necessarily the same size; all congruent shapes are similar but not all similar shapes are congruent

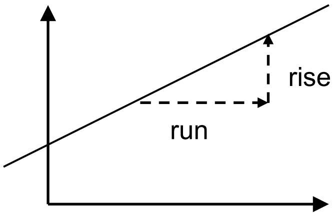
These rectangles are similar

simple interest: See *Financial Terms* on **page 274**

simplify: 1. To simplify a fraction means to write it in lowest terms or as a mixed fraction; e.g., you can simplify $\frac{18}{10}$ as $\frac{9}{5}$ and then as $1\frac{4}{5}$ **2.** To simplify an expression means to collect and combine like terms; e.g., to simplify $2x + 3x + 4 + 7$, combine the x terms and then combine the constants to get $5x + 11$

simulation (experiment): An experiment that models an event for which it is difficult or impossible to calculate the theoretical probability exactly; a simulation experiment can generate a reasonable experimental probability

slope: The measure of the amount of slant in a line graph; it is the ratio of $\frac{\text{rise}}{\text{run}}$ (or $\frac{\text{change in y}}{\text{change in x}}$) anywhere along the graph



solution: 1. The complete answer to a problem **2.** The value that makes an equation true; e.g., in $x + 4 = 39$, the solution is $x = 35$ because $35 + 4 = 39$

solve (a proportion): Find a missing term in a proportion; e.g., to solve the proportion $\frac{■}{7} = \frac{8}{28}$, means to find the value of ■: If $\frac{■}{7} = \frac{8}{28}$, then ■ = 2

speed: The rate at which a moving object changes position with time, often given as a unit or average rate; e.g., a sprinter who runs 100 m in 10 s has an average speed of 10 m/s

spread of data: How the values in a set of data are distributed; to describe the spread of a set of data, you need to consider things such as the mean, median, range, and extreme values

square of a number: The product of a number and itself; e.g., $4^2 = 4 \times 4 = 16$; to square a number means to multiply it by itself; e.g., when you square 5, you get 25

square root: A number that is multiplied by itself to get another number; e.g., the square root of 25 ($\sqrt{25}$) is 5 because $5 \times 5 = 25$; the square root of 15 ($\sqrt{15}$) is approximately 3.87 because $3.87 \times 3.87 \approx 15$

square units: Units used to measure area; some common square units are cm^2 and m^2

standard form (of a number): The usual way to write a number; e.g., 23,650 is in standard form See *exponential form* and *expanded form*

sum: The result of adding numbers; e.g., in $5 + 4 + 7 = 16$, the sum is 16

supplementary angles: Any two angles that share a vertex and have a sum of 180°

surface area: The total amount of area that covers all the surfaces of a 3-D object; surface area units are the same as area units

symmetry: A property of a shape; line or reflectional symmetry means that when a 2-D shape is folded or reflected across a line (the reflection line), the two sides of the shape match

T

table of values: An arrangement of numerical values, usually arranged in rows and columns, that represents a relationship between two variables

term: 1. Part of an algebraic expression that is separated from the rest of the expression by addition or subtraction signs; e.g., the expression 3x + 3 has two terms **2.** Each number or item in a sequence; e.g., in the sequence 1, 3, 5, 7, …, the third term is 5 **3.** The numbers in a ratio or rate; e.g., the ratio 2 : 3 has two terms

theorem: Something that is always true; e.g., the Pythagorean theorem is true for all right triangles

theoretical probability: A number from 0 to 1 that tells how likely it is that an event will occur; it is calculated using this expression

$$\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}};$$

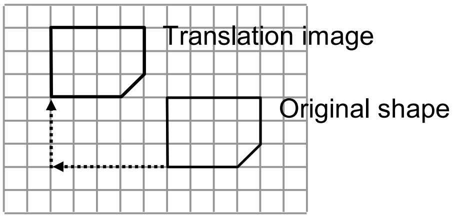
e.g., the theoretical probability of rolling a 4 on a six-sided die is $\frac{1}{6}$

three-dimensional (3-D): A shape with three dimensions: length, width (or breadth or depth), and height

total surface area: See *surface area*

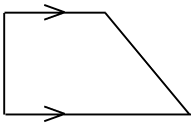
transformation: Changing a shape according to a rule; transformations include translations, rotations, and reflections See *dilatation*, *reflection*, *rotation, and translation*.

translation: A transformation in which each point of a shape moves the same distance and in the same direction; also called a slide; e.g., the pentagon has been translated 5 units left and 3 units up

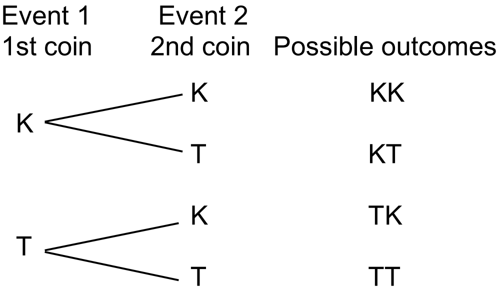


transversal: A line that intersects two or more lines at different points; if a transversal intersects parallel lines, there are special relationships between the angles formed See *alternate angles*, *complementary angles*, and *interior angles*

trapezoid: A quadrilateral in which one pair of opposite sides are parallel; e.g.,



tree diagram: A way to record and count all combinations of events in a probability experiment, usually in order to determine theoretical probability; e.g., the tree diagram below shows all the possible outcomes if you flip two coins



trial: Each repetition of an experiment in a probability experiment; e.g., if the experiment involves finding the probability of rolling two even numbers when two dice are rolled, each roll of a pair of dice is a trial

turn centre: The point around which all the points in a shape turn or rotate in a clockwise (cw) or counterclockwise (ccw) direction See *rotation*

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U

unit fraction: A fraction with a numerator of 1; $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, … are unit fractions

unit percent: The value that is equivalent to 1%; e.g., if 100% is 200 mL, the unit percent or 1% is 2 mL

unit rate: A rate with a second term of 1; e.g., 4 km/h is a unit rate because it means 4 km in 1 h

unit ratio: A ratio with a second term of 1; e.g., 4 : 1 is a unit ratio

upper quartile (Q_3): See *box and whisker plot*

V

variable: **1.** A letter or symbol, such as *a*, *b*, *x*, or *n*, that represents a number; e.g., in the formula for the area of a rectangle, A = *l* × *w*, the variables *A*, *l*, and *w* represent the area, length, and width of the rectangle **2.** The parts that interact in a relationship; e.g., when a car travels at 50 km/h, there is a relationship between the variables: time and distance

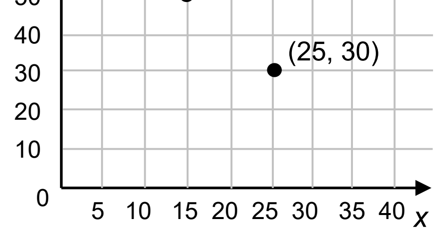
vertex (vertices): The point at the corner of an angle or shape where two or more sides or edges meet; e.g., a cube has eight vertices, a triangle has three vertices, and an angle has one vertex

vertical axis: See *y-axis*

volume: The amount of space occupied by an object; often measured in cubic centimetres or cubic metres

X

***x*-axis:** One of the two axes in a coordinate grid; sometimes called the horizontal axis; e.g., the *x*-axis below goes from 0 to 40



***x*-coordinate:** The first value in an ordered pair, representing the distance along the *x*-axis from (0, 0); e.g., in (15, 50), the *x*-coordinate is 15 See *x-axis*

Y

***y*-axis:** One of the two axes in a coordinate grid; sometimes called the vertical axis; e.g., the *y*-axis of the grid shown in *x-axis* goes from 0 to 60

***y*-coordinate:** The second value in an ordered pair representing the distance along the *y*-axis from (0, 0); e.g., in (15, 50), the *y*-coordinate is 50 See *y-axis*

Z

zero property: When you add two opposite integers or opposite terms in an algebraic expression, the sum is zero; the zero property is often used in operations with integers or for simplifying polynomial expressions; e.g.,

- to subtract –5 – (+2), you first add (–2) + (+2) because (–2) + (+2) = 0, so that you can subtract (+2):

  –5 – (+2)
= –5 **+ [(–2) + (+2)]** – (+2)
= –5 + (–2)

- to subtract (5 – 4*x*) – (2*x* – 3), you first add (2x – 3) + (–2x + 3), so that you can subtract (2x – 3):

  (5 – 4*x*) – (2*x* – 3)
= (5 – 4*x*) **+ [(2x – 3) + (–2x + 3)]** – (2*x* – 3)
= (5 – 4*x*) + (–2*x* + 3) = 8 – 6*x*

Financial Terms

annual interest rate: The percent interest that is either earned or charged on an investment or loan for one year; e.g., if you borrow Nu 1000 at a simple annual interest rate of 8% for two years, you must pay 8% of Nu 1000, or Nu 80 each year in interest for each of the two years

commission: The money a salesperson earns for a sale or for total sales; a commission can be an amount, or a percent or rate; e.g., if a salesperson earns a commission of 10% on total sales worth Nu 10,000, the commission rate is 10% and the commission amount is Nu 1000

convert currency: Exchange money from one country's currency into another country's currency; e.g., if you exchange Nu 1000 for Thai baht at a rate of exchange of Nu 1 = 0.8 Thai baht, you will get 800 Thai baht

cost price: The price that a shopkeeper pays for an item; the shopkeeper marks up the price before selling the item, in order to make a profit

credit: When you buy something now but pay for it later, you are buying the item on credit

currency: Another name for money that is used within a country; the currency of Bhutan is the ngultrum

discount: The percent or amount that the regular selling price of an item is marked down in order to make the item attractive to buyers; e.g., if a Nu 1000 item is discounted by 25%, the discount amount is Nu 250

expenses: The money a shopkeeper must pay to run his or her business; expenses can include rent, taxes, electricity, and water

interest: The extra money you pay or earn when you borrow money or invest money; interest can be an amount or a percent; e.g., if you borrow Nu 1000 at a simple interest rate of 5%, you must pay Nu 50 in interest

invest: To use money to try to earn money; e.g., if you deposit or invest Nu 5000 in a savings account at a bank, you invest Nu 5000 in order to earn interest; not all investments are certain, and some investments lose money

markdown: See *discount*

markup: See *cost price*

principal: The amount of money initially invested or borrowed

profit: See *cost price*

rate of commission: See *commission*

rate of exchange: See *convert currency*

regular price: The usual selling price of an item in a shop

selling price: Sometimes called regular selling price See *regular price*

simple interest: When the interest charged or interest earned is based only on the money that was originally borrowed or invested and does not change when more money is earned or paid back, the interest is simple interest; it can be an amount or a percent
See *annual interest rate*

MEASUREMENT REFERENCE

Measurement Abbreviations and Symbols

Time		**Capacity**		**Mass**	
second	s	millilitre	mL	milligram	mg
minute	min	centilitre	cL	gram	g
hour	h	decilitre	dL	kilogram	kg
		litre	L	tonne	t
Length		**Volume**		**Area**	
millimetre	mm	cubic millimetre	mm^3	square millimetre	mm^2
centimetre	cm	cubic centimetre	cm^3	square centimetre	cm^2
decimetre	dm	cubic decimetre	dm^3	square metre	m^2
metre	m	cubic metre	m^3	hectare (10,000 m^2)	ha
kilometre	km			square kilometre	km^2

Metric Prefixes

Prefix	kilo × 1000	hecto × 100	deka × 10	unit 1	deci × 0.1 or $\frac{1}{10}$	centi × 0.01 or $\frac{1}{100}$	milli × 0.001 or $\frac{1}{1000}$
Example	*kilo*metre km	*hecto*metre hm	*deka*metre dam	metre m	*deci*metre dm	*centi*metre cm	*milli*metre mm
	1000 m	100 m	10 m	1 m	0.1 m	0.01 m	0.001 m

Measurement Formulas and Relationships

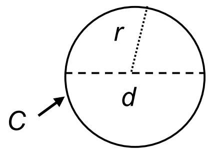
Perimeter and Circumference

rectangle $P = 2(l + w)$

square $P = 4\ s$

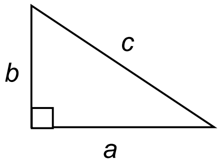
circle $C = \pi d$ or $C = 2\pi r$

($\pi \approx \frac{22}{7}$ or 3.14)



Pythagorean theorem

$c^2 = a^2 + b^2$



Area

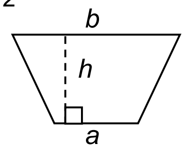
rectangle $A = lw$

square $A = s \times s$ or s^2

parallelogram $A = bh$

triangle $A = \frac{1}{2}bh$

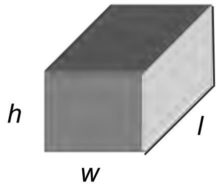
trapezoid $A = \frac{1}{2} \times h \times (a + b)$



circle $A = \pi r^2$ ($\pi \approx \frac{22}{7}$ or 3.14)

Volume of a rectangular prism $V = \text{Area of base} \times \text{height}$ or $V = l \times w \times h$

Surface area of a rectangular prism $SA = 2\ (h \times l + l \times w + w \times h)$



Volume, Capacity, and Mass of Water

1 cm^3 = 1 mL = 1 g

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